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## Editorials.

## Symbolical Algebra.

Ian editorial note in our issue of May 192\% we commented on a letter from Mr. W. A. Barclay, which appeared in the same issue dealing with the subject of $\sqrt{ }=\mathbf{I}$ and the Heaviside operator. In the present issue we publish an article on " Symbolical Algebra," by Mr. J. A. Ratcliffe, B.A., of Cambridge, in which he develops the subject step by step in the orthodox way and shows that, if properly employed, there is no need to distinguish between $j=\sqrt{-\mathrm{I}}$ and $j$, the so-called vector operator. No one can doubt the truth of his concluding remark that " provided sufficient care is taken in the interpretation of the results, this symbolical algebra is a very powerful weapon in the hands of the mathematician." There are difficulties, however, which do not seem to have occurred to Mr. Ratcliffe. Our editorial of a year ago was prompted by Mr. Barclay's suggestion to do away with $j$ as a vector operator and replace it by $D / \omega$, and we considered therefore three symbols, standing for three distinct ideas, viz., $D / \omega, j$ as an operator, and $\sqrt{-I}$. Nobody will deny that these symbols stand for three entirely different conceptions; the first, rate of change divided by cyclic frequency; the second, rotation of a vector through 90 degrees ; the third, an imaginary arithmetical process. If this be granted, then it follows that any such statement or assumption as
$D / \omega=j=\sqrt{-1}$ should be treated with suspicion. Mr. Barclay's letter dealt mainly with the first pair, as he wished to have nothing to do with $\sqrt{ }-\mathrm{I}$; Mr. Ratcliffe's article ignores $D / \omega$, but strongly supports $j=\sqrt{-\mathbf{I}} . \quad$ Mr. Barclay clained an enormous simplification by the use of the $D / \omega$ notation, but we showed not only the impossibility of treating the operator $D / \omega$ as an algebraic multiplier, as is obvious from the fact that $(D \sin \omega t)^{2}$ is not the same as $D^{2} \sin ^{2} \omega t$, but also the fallacy of assuming that because the effect of two similar operations carried out successively is represented by a certain symbol, viz., - I, it necessarily follows that a single operation is correctly represented by the square root of this symbol.

Mr. Ratcliffe does not appear to appreciate the difficulties connected with $j$. He defines it as $\sqrt{-I}$ and then proceeds to find a geometrical interpretation of it. But this is merely giving it another definition. Because multiplication twice in succession by $\sqrt{-\mathrm{I}}$ is equivalent to a change of sign, it is not natural but very artificial to assume that a single application-he wisely avoids saying "multiplication" at this pointto a magnitude will rotate it through 90 degrees. Later he is less precise and says that " multiplication of the rotating vector $e^{j p h}$ by $j$ will give another rotating vector

90 degrees ahead""; surely "orientation" would be a better word than " multiplication" to describe such a process.

Mr. Ratcliffe concludes with the hope that "it will be clear that $j$ means exactly the same when used in purely algebraical work and when used in symbolical algebra, and that there is no distinction between $j=\sqrt{-\mathrm{I}}$ and $j$, the so-called vector operator." In our opinion they mean very different things, and there is a very great distinction between them. The square root of a minus quantity is an arithmetical fiction which we call an imaginary quantity-
although this is hardly fair to the imagina-tion-whereas the operation of rotating a line through 90 degrees is a simple thing that a child can not only comprehend but actually do. One must surely be hypnotised by the unquestioned beauty and utility of symbolical algebra to suggest that there is no distinction between two such utterly different things. We leave our readers to judge of the correctness of the statement that the square root of a minus number, although called an imaginary number, " is not, in the everyday sense of the word, any less real than is the ordinary number."

## The Self-induction of Single-turn Circuits of Various Shapes.

$I^{\mathrm{N}}$N connection with Mr. Allen's article on formule for the self-induction of single-turn circuits of various shapes which we publish in this issue, we would draw our readers' attention to an article on the same subject by M. Bashenoff, the chief of the wireless section of the Soviet Research Institute at Moscow, published in the December issue of the Proceedings of the Institute of Radio Engineers. The inductances of various single-turn circuits are expressed in terms of their perimeters, that is, of the length of wire $l$, its radius $r$, permeability $\mu$, and a correction factor $\delta$ depending on the frequency $f$ which, of course, only modifies the flux within the material
of the wire itself; $\delta=0.25$ for D.C. and low frequencies and decreases as the frequency increases.

The results are all expressed in the convenient form

$$
L=2 l\left(\log _{e} \frac{2 l}{r}-a+\mu \delta\right)
$$

where $a$ has the following values:

| Circle . . 2.45 I | Square | 2.853 |
| :---: | :---: | :---: |
| Octagon. 2.561 | Equilateral triangle | 3.197 |
| Hexagon 2.636 | Isosceles right-angle |  |
| Pentagon 2.712 | triangle | $3 \cdot 3$ |

If $x=0.28 \mathrm{I} \quad r \sqrt{\mu f / \rho}$, where $\rho$ is the specific resistance of the wire in microhms per cm.-cube, then $\delta$ depends on $x$ in the following way:

G. W. O. H.

# Symbolical Algebra. 

By J. A. Ratcliffe, M.A.

THIS article draws attention to some mistakes which are often made in applying the processes of symbolical algebra to the treatment of problems containing sine and cosine functions. In the edition of this periodical for May, 1926, there appeared an editorial and a letter from Mr. W. A. Barclay, as a result of which it might be thought that the $j$ which appears in the operational equations is something quite different from the $j$ of ordinary algebra, and does not obey the same algebraical laws. In this article we shall show that, if the problem is looked at in a slightly different way, this apparent difference disappears, with the result that the method of symbolical algebra becomes much more useful than it is if we have continually to remember whether the $j$ which we use has its algebraical or its operational significance.

In symbolical algebra we try to attach some physical or geometrical meaning to an algebraical symbol (such, for example, as $j \cdot=\sqrt{-I})$. We then apply the ordinary rules of algebra to this symbol, and so deduce more complicated expressions, and these expressions may then be interpreted directly in terms of their geometrical or physical meaning. It is essential that, once a geometrical statement has been put into an algebraical form, we should be able to apply all the ordinary operations of algebra to the expression, weithout any thought as to the geometrical meaning of the symbols. The resulting equation can then be interpreted, according to the rules which we will lay down, and will give a result which is known to be true from a geometrical standpoint. From the treatment given in the Editorial previously referred to, it would appear that this is not the case, but that we must always bear in mind the geometrical meaning of the operator, and only perform on it such algebraical operations as do not violate the geometrical laws. If this were the case the method would lose all its use and beauty, but we shall show that the treatment previously referred to was incorrect and if the idea of the symbolical operator is correctly applied it gives a very powerful method for the algebraical solution of Physical and

Geometrical problems involving circular functions (sines and cosines).

Let us define $j$ by means of the algebraical equation $j=\sqrt{-I}$. As a result of this we see that $j^{2}=-\mathbf{I}$. Now it is obvious that there is no ordinary number whose square is equal to -I, and so we call $j$ an imaginary number. It must be clearly understood that the term " imaginary" is merely a name to distinguish this kind of number from the more usual hind. An imaginary number is not, in the every-day sense of the word, any less real than is the ordinary number. Having now defined $j$ in an algebraical manner, we may work with it just as we would with any other symbol and we may apply all the ordinary operations of algebra to it.

Suppose we have a number made up of the sum of an ordinary number and an imaginary number, such, for example, as $x+j y$; we call this a complex number and say that it consists of a real part $x$ and an imaginary part $j y$. If two complex numbers are equal to each other, then their real parts are separately equal and their imaginary parts are also equal. Thus if $x+j y=a+j b$ then $x=a$ and $y=b$. This is easily seen by transposing the equation thus $(x-a)=$ $j(b-y)$, i.e., a real number is equal to an imaginary, which cannot be true, unless both $x-a=0$ and $b-y=0$; which gives $x=a$ and $y=b$. Thus, if our algebraical work leads to an equation in which one complex number is equal to another, then we may equate the real part of the one side to the real part of the other side, and the imaginary part of one side to the imaginary part of the other and so obtain two independent equations from the one.

Having defined $j$ algebraically, let us now look for its geometrical symbolical meaning. From the equation $j^{2}=-\mathrm{I}$ we see that $j$ is a symbol which, when applied twice to a magnitude, converts it into a negative magnitude of the same size. In geometry, this is equivalent to rotating the line which represents the magnitude, through $180^{\circ}$. Hence applying the symbol to a magnitude twice in succession will rotate it through $180^{\circ}$, and so it is natural to suppose that
applying the symbol $j$ to a magnitude once will rotate it through $90^{\circ}$. It is important to notice that this is only a geometrical interpretation of the symbol $j$ and that it is still defined by the equation $j=\sqrt{-\mathbf{I}}$.


Fig. I
Now suppose that all positive real numbers are represented by lines of corresponding length drawn along $O X$. Then the number $a$ is represented by the line $O A$ (Fig. 1). The number $j a$ must be represented by a line of length " $a$ " in a direction perpendicular to $O A$, that is, it is represented by $O A^{\prime}$. Now since the quantities $O A$ and $O A^{\prime}$ have magnitude and direction they are of the kind called vectors. If we wish to add such quantities we must employ the parallelogram law for vector addition, which says that the sum of two vectors $O A$ and $O B$ (Fig. 2) is the vector represented by the diagonal $O B$ of the parallelogram having $O A$ and $A B$ as adjacent sides. Geometrically, this means that going a distance $O A$ along $O X$ and then a distance $A B$ perpendicular to $O X$ leads us to the same point as going a


Fig. 2
distance $O B$ along the direction of $O B$. Thus we may say $\overline{O A}+\overline{A B}=O B$ (a bar
over a symbol is used to denote that it represents a vector quantity). If we express this in terms of symbols it becomes $a+j b=Z$. The quantity $Z$ is a complex number and represents the vector $\overline{O B}$. Thus a real number corresponds to a vector along $O X$, an imaginary number to a vector along $O Y$, and a complex number to a vector in any intermediate position. It is easy to see that the complex vector $a+j b$ $(O B)$ has actual length given by $\sqrt{a^{2}+b^{2}}$ and makes an angle with $O X$ given by $\tan \theta=b / a$.

We are now in a position to work out a simple problem by means of symbolical algebra. Let us rotate the vector represented by $a+j b$ through $90^{\circ}$. By the processes


Fig. 3
of symbolical algebra we have merely to multiply it by $j$. This gives

$$
j(a+j b)=-b+j a
$$

If, now, we go back to the geometrical meaning of this, it indicates that we must go a distance $b$ along the negative direction of $O X^{\prime}$ (Fig. 3) and a distance " $a$ " perpendicular to $O X$. This leads to the point $R^{\prime}$ and it is easily seen that $O R$ and $O R^{\prime}$ are mutually perpendicular. Thus multiplication of a complex number by $j$ always corresponds to rotation of the corresponding vector through a right angle. If we wish to rotate the vector $O R^{\prime}$ through a further $90^{\circ}$ we must multiply $a+j b$ by a further $j$, and we obtain $j(-b+j a)=-a-j b$, which gives us at once the vector $O R^{\prime \prime}$ and we see that $O R$ has now been rotated through $180^{\circ}$.

Now let us consider what is meant by the expression a $\sin p t$. Algebraically it is a
real number which varies in a simple harmonic manner between the values $+a$ and $-a$, the time period being $2 \pi / p$. Since, algebraically, it is a real number, then we interpret it geometrically as a line drawn along $O X$, whose length varies periodically with period $2 \pi / p$; the end of this line moves. in fact, with simple harmonic motion. Let us now enquire what is the geometrical interpretation of the algebraical expression $j a \sin p t$. Since it is an imaginary number, it represents a line drawn along $O Y$. It is also periodic, and so represents a simple harmonic motion along $O Y$. The simple harmonic motion (S.H.M.) along $O Y$ (represented by a $\sin p t$ ) and the S.H.M. along $O Y$ $(j a \sin p t)$ are in phase with each other. The multiplication by $j$ has simply turned the S.H.M. through $90^{\circ}$ in space. There is no change of phase, and cos $p t$ does not appear in the equations at all. There is no reason, either algebraical or geometrical, for supposing that $j$ represents differentiation or for writing

$$
\frac{d}{d l} \sin p t=j p \sin p t
$$

as is often done, and in fact this statement is quite untrue.

We see from this that the use of the symbolical operator $j$ does not help us very much so far. By making a further extension, however, we arrive at a symbolical method which is very useful in A.C. circuit problems.

Let us consider the complex number $a(\cos p t+j \sin p t)$. From the exponential definitions of sine and cosine we see that this is equal to $e^{i p t}$ as is shown below:
$\left.\begin{array}{l}\cos p t=\frac{e^{j \mu t}+e^{-j p t}}{2} \\ j \sin p t=j \frac{e^{j \nu^{\prime} t}-e^{-j \nu^{t}}}{2 j}\end{array}\right\} \therefore \begin{aligned} & \cos p t \\ & +j \sin p t=e^{j p t}\end{aligned}$
It is important to notice that this equality holds because $j$ has been defined algebraically by the equation $j=\sqrt{-\mathrm{I}}$. The geometrical interpretation of the complex number $a(\cos p t+j \sin p t)$ (or ae.ipt) is as follows. It is a vector, whose absolute length is given by $a \sqrt{\cos ^{2} p t+\sin ^{2} p t}=a$, and which makes an angle $\theta$ with $O X$ such that

$$
\tan \theta=\frac{\sin p t}{\cos p t}=\tan p t
$$

and hence $\theta=p t$. Thus as the time $t$ increases the angle steadily increases, and
so the vector continually rotates, the angular velocity being $p$. We therefore see that ac $c^{j p i}$ represents a vector of constant magnitude $a$, which rotates with constant angular velocity $p$. Its end will trace out a circle at a constant speed. If now we take the projection of this rotating vector on the $x$ axis we shall, by the ordinary definition of S.H.M., arrive at the periodically changing vector represented by $a \cos p t$. We arrive at the same result, algebraically, by taking the real part of the complex number $a e^{j y t}[=a(\cos p t+j \sin p t)]$. We may write this $R\left[e^{j} t\right]=\cos p t$, which means that the real part of ejpt is $\cos p t$. The mistake is often made of writing $e^{j p t}=\cos p t$, which is quite untrue and leads at once to difficulties. We can see the nature of these difficulties by the following example. Let us enquire what is meant by $\left\{e^{i \mu t}\right\}^{2}$. We have the following algebraical equations:

$$
\begin{aligned}
& \left\{e^{i j t}\right\}^{2}=e^{2 j p t}=\cos 2 p t+j \sin 2 p t \\
& \text { and }\left\{e^{\operatorname{cin} t}\right\}^{2}=\{\cos p t+j \sin p t\}^{2} \\
& =\cos ^{2} p t-\sin ^{2} p t+2 j \cos p t \sin p t \\
& =\cos 2 p+j \sin 2 p t \text {. }
\end{aligned}
$$

Thus we see that we obtain the same result by squaring the expression eipt and then proceeding to its interpretation in terms of sines and cosines, or by interpreting it first in terms of sines and cosines and then squaring it. If, however, we restrict ourselves to the real parts of these expressions we obtain the equations

$$
\begin{aligned}
& e^{i p t}=\cos p t \\
& \left\{e^{i p t}\right\}^{2}=\cos ^{2} p t=e^{2 j p t}=\cos 2 p t
\end{aligned}
$$

which is obviously wrong. We should have written $R e^{j p t}=\cos p t$, and then we should have

$$
R\left[\left\{\rho^{j \nu t}\right\}^{2}\right]=R\left[e^{2 j p t}\right]=\cos 2 p t
$$

which we have seen to be true.
Let us now proceed further, and apply the process of differentiation to the complex number $e^{j p t}$. We have

$$
\frac{d}{d t} e^{\text {ipt }}=j p e^{i p t} .
$$

Thus differentiation of $e^{\text {ipt }}$ is equivalent to multiplication by ip. It would be quite wrong to say that differentiation of the real part of eipt was also equivalent to multiplying by $j p$, which is done if we write

$$
\begin{equation*}
\frac{d}{d t} \cos p t=j p \cos p t \tag{I}
\end{equation*}
$$

We have seen that differentiation of $e^{j p t}$ is equivalent to multiplication by $j p$. Let us enquire into the geometrical interpretation of this. Multiplication of the rotating vector $e^{i p t}$ by $j$ will give another rotating vector $90^{\circ}$ ahead of the original one, and multiplication by $p$ will increase its size $p$ times. Projection of this new vector on the $z$ axis will give the S.H.M. $p \cos (p t+\pi / 2)$, which is $90^{\circ}$ out of phase with the original S.H.M. and has amplitude $p$ times as large. The new S.H.M. can also be represented as $-p \sin p t$. We could have deduced this algebraically from the equation

$$
\begin{aligned}
& \frac{d}{d t} e^{j, p t}=j p e^{j \nu t} \text { or } \frac{d}{d t} \\
&(\cos p t+j \sin p t) \\
&=j p(\cos p t+j \sin p t)
\end{aligned}
$$

by taking the real parts of both sides, when we obtain

$$
\begin{equation*}
\frac{d}{d t} \cos p t=-p \sin p t \tag{2}
\end{equation*}
$$

It is important to notice the difference between the process employed here and the process employed in deducing the fallacious equation (I). In the first case we took the real parts of both sides and then differ-
entiated, while in the latter we have differentiated and then taken the real parts. We may express these two processes as follows

$$
\begin{align*}
& \frac{d}{d t} R\left[e^{j \mu t}\right]=j p R\left[e^{j \mu t}\right] \quad \text { INCORRECT } \quad \text { (Ia) } \\
& R\left[\frac{\bar{d}}{d t} e^{i \mu t}\right]=R\left[j p e^{j \mu t}\right] \quad \text { CORRECT } \tag{2a}
\end{align*}
$$

The results of this argument are as follows. We define $j$ algebraically, and as a result we can work with it just as we should with any other quantity, and our results are always correct algebraically. [Such a result as $j \sin p t=\cos p t$, which was given in the editorial referred to, is algebraically quite untrue.] We then find a geometrical interpretation of the symbol $j$, by means of which we can always pass from any true algebraical equation to a corresponding geometrical statement. It is hoped that it will be clear that $j$ means exactly the same when used in purely algebraical work and when used in symbolical algebra, and that (contrary to what is said at the end of the previous editorial) there is no distinction between $j=\sqrt{-1}$ and $j$, the so-called vector operator.

# The Reflecting Layer of the Upper Atmosphere. 

# An Estimation of the Height for Wireless Waves of 600 Metres Wavelength in New Zealand. 

By G. H. Munro, M.Sc.

DURING December, 1925, a series of special signals was transmitted from Wellington wireless station for the purpose of making direction-finding observations at a receiving station near Auckland, a distance of approximately 300 miles. The signals consisted of spark transmission on 600 metres for 5 -minute periods with intervals of 5 minutes, from 3.30 to 4.30 , sunrise at Auckland then being at about 4.25 a.m., and were observed on 21 days from the 7 th to the 3 Ist December.

The average intensity during each period was estimated aurally and recorded, allowance being made for rapid fading due to " night effect."

It was found that the rate of decrease in the strength of normal signals varied in much the same way each day. The observations for 21 days were therefore tabulated, a scale of numerical values assigned to the strengths as recorded, and an average strength for each period derived. This tabulation is given below.

The figures in the columns represent strengths ranging from 6 , the maximum night strength, to $I$, the normal daylight strength.

The time of sunrise varied, of course, during the month, being at $4.25 \mathrm{a} . \mathrm{m}$. on the 7 th, 4.26 on the 16th, 4.29 on the 24 th, and $4.33 \mathrm{a} . \mathrm{m}$. on the 30 th and 3ist ; but, considering the degree of accuracy obtainable, it was considered sufficient to take the average value of $4.27 \frac{1}{2}$ a.m. The difference between this and the middle of the period of observation was then taken as the time before sunrise, of the observation. These are the figures given at the top of the tabulation.

It is seen that until 45 minutes before sunrise the decrease in strength is small. During the next two Io-minute periods there is a considerable decrease, at a rate increasing with time. From 25 minutes before sunrise to 15 minutes before, the decrease is very great, the rate being then a maximum. After this the rate of decrease rapidly becomes less, as the strength is now reduced almost to the normal daylight strength.

It should be noted that the figures serve mainly to enable averages to be computed, and while they indicate the relative strengths, they are probably not indicative of the actual current-strength of the received signals.

EFFECT OF SUNLIGIIT ON SIGNAL STRENGTH.

| Date. | Average Time of Observation before Sunrise, in Minutes. |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| December. | 55. | 45. | 35. | 25. | 15. | 5. | $-5$. |
| 7 8 | 5 | 3 | 3 | 2 | 1 | 1 | 1 |
|  |  | 5 | 5 | 5 | 3 | 2 | 2 |
| 9 | 5 | 5 | 4 | 3 | 2 | 2 | 2 |
| 10 | 6 | 6 | 6 | 5 | 4 | 2 | 2 |
| 11 | 6 | 6 | 6 | 5 | 2 | 2 | I |
| 12 | 5 | 3 | 3 | 2 | 3 | 3 | 3 |
| 14 | 6 | 6 | 5 | 4 | 3 | 2 | I |
| 15 | 6 | 6 | 4 | 2 | 3 | 1 | 1 |
| 16 | 6 | 6 | 6 | 6 | 3 | 2 | 2 |
| 17 | 6 | 4 | 3 | 3 | 3 | 3 | 1 |
| 18 | 6 | 6 | 6 | 6 | 3 | 2 | 1 |
| 19 | 6 | 5 | 5 | 4 | 2 | 1 | 1 |
| 21 | 6 | 6 | 5 | 5 | 3 | 2 | 2 |
| 22 | 4 | 5 | 3 | 3 | 2 | 1 | 2 |
| 23 | 6 | 6 | 6 | 3 | 2 | 2 | 2 |
| 24 | 6 | 6 | 5 | 3 | 1 | 1 | I |
| 26 | 6 | 6 | 5 | 4 | 3 | 2 | 2 |
| 28 | 5 | 4 | 3 | 3 | 3 | I | I |
| 29 | 4 | 5 | 4 | 4 | 3 | 2 | 2 |
| 30 | 4 | 5 | 3 | 4 | 1 | I | 2 |
| 3 I | 2 | 5 | 4 | I | I | 2 | 2 |
| Total 21 | 112 | 109 | 94 | 77 | 51 | 37 | 34 |
| Average | 5.36 | 5.19 | 4.47 | 3.66 | 2.43 | 1.76 | I. 62 |

These results are interesting when considered with respect to the least height at which the sun is shining in the path of the received rays. Assuming a single reflection of the waves, it will occur approximately vertically over a point mid-way between the transmitting and receiving stations. The height of the sunlight above this point can be calculated from a knowledge of the time of sunrise at that point, its latitude, and the time of the year. As the degree of accuracy is not great, a mean time of sunrise for the month may be taken, and the calculations made as at midsummer in the southern hemisphere. The values found for the conditions of the observations are given below:

| Minutes before sunrise at receiving station | 55 | 45 | 35 | 5 |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
| Height of sunlight in miles at point mid-way between stations | 5 | 36 | 19 | 8 |  |

Since there is little effect in the first period, the corresponding height (i.e., 57 miles) should be that of the lower boundary for the Heaviside layer.

The heights at which the sun was shining


Fig. I
at the particular times were calculated by trigonometry. In case the method should be of interest, it is added as an appendix.

## APPENDIX.

Calculation of the height at which the sun is shining above a point on the earth's
surface at a given time before sunrise at that point, the latitude being known.

The calculation in this case is for midsummer in the southern hemisphere.

Consider the earth as a sphere WAEZBD (Fig. I), with axis of rotation $X Y$, and the sun's rays as a parallel beam making an angle $90^{\circ}-\theta$ with the earth's axis of rotation. Fig. I represents this as seen from a direction at right angles to $X Y$ and to the direction of the sun's rays.

In the figures, capital letters will represent


Fig. 2
points on the surface of the sphere; small letters, points on the section in the plane of the paper. $O$ is the centre of the sphere, $X Y$ the axis of rotation. $W Z$ will be the sunrise belt.

Let $\phi$ be the latitude of the point under consideration ( $P$ ), $R$ the radius of the sphere, and $A C B$ and $E S_{1} D$ plane sections through $P$ perpendicular to $X Y$ and $W Z$ respectively.

Then

$$
\begin{aligned}
& A c=R \cos \phi=r . \\
& O c=R \sin \phi \\
& c s=O c \tan \theta .
\end{aligned}
$$

Now considering the section $A C B$ (Fig. 2) :

$$
\left.a=\sin ^{-1} \frac{c s}{r} \text { (approx. since } a \text { is small }\right)
$$

and $\beta=360 \times \frac{t}{24}$ degrees. [Where $t=$ time
to go from $P$ to $S$, i.e., time before sunrise (in hours).]


Fig. 3
Also

$$
c p=r \sin (\alpha+\beta)
$$

and $\quad s t=c p-c s$;
from which we can find the distance $s p$.
Again, in Fig.,$p s_{\mathbf{1}}=s p \cos \theta$.
Also

$$
O s=O c / \cos \theta
$$

and

$$
s s_{1}=s p \sin \theta
$$

and

$$
s_{1} D=\sqrt{R^{2}-O_{s_{1}^{2}}}=r_{1},
$$

whence we get $r_{1}$, the radius of the section $E S_{1} D$.

Now consider the section $E S_{1} P D$, Fig. 3.
We have found $s_{1} p$, and $s_{1} S$ which $=$ $s_{1} D=r_{1}$.

Now $\quad \gamma=\sin ^{-1} \frac{s_{1} p}{r_{1}}$ (approx)
and

$$
\frac{r_{1}}{r_{1}+h}=\cos \gamma
$$

[Where $h=P K=$ height required.]
Therefore $r_{1}=\cos \gamma\left(r_{1}+h\right)$
and

$$
h=\frac{\gamma_{1}(I-\cos \gamma)}{\cos \gamma}
$$

Thus, by substituting the appropriate values for $\theta, \phi$ and $R, h$ can be calculated. By repeating the calculation for several values of $t$ a graph may be drawn showing the relation between $t$ and $h$.

# Retro-action in Amplifiers. 

By H. A. Thomas, M.Sc.

## Introductory.

THE study of the effects produced by retro-action or, as it is sometimes called, reaction, feed-back, or backcoupling, is one which is made exceedingly difficult by virtue of the complexity of the circuits involved and by the ambiguity which lies beneath the various assumptions usually made in such an analysis.

A true physical conception of all that is taking place in an amplifier is hard to obtain and one is forced to make certain assumptions in order to obtain a satisfactory working basis. These assumptions have not yet been shown to be generally truc, and only the simplest cases can as yet be applied. Certain experimental properties are well known and these properties can be partially explained by the use of certain theories.*

The object of this paper is to demonstrate some of the general properties of retro-action and to analyse what conditions have to be fulfilled so that retro-action may advantageously be used in an amplifier.

## Types of Retro-action.

Retro-action in an amplifier is its property of feeding back energy from the output end to the input end, and thus modifying the normal amplification which it would possess if no such feed-back of energy took place.

Suppose I/I,ooo of a volt is applied to the input of an amplifier, and the amplification is 1,000 , then the output will be 1 volt. Now suppose by some means I/ 4 ,oooth part of the output voltage leaks back to the input.
The input will now have $\left[\frac{I}{1000}+\frac{I}{4000}\right]$ of a volt impressed upon it, and this in turn is amplified $I, 000$ times, giving $I_{4}^{\frac{1}{4}}$ volts

[^0]output, which again gives $\frac{5}{16000}$ th part of a volt to the input, and so on. It is clear that the original $1 / 1$, oooth of a volt will give more than I volt at the output, and therefore the amplification has been increased above the normal amount.

In this case the effect of retro-action is to increase the overall amplification, and this happens when we deliberately insert special "reaction" coils whose function is to so feed back energy to the input. Due to the inherent capacities of the valves, we sometimes get a feed-back effect which may produce an increase in amplification, or if it is too great may produce instability and self-oscillation. This type of reaction is responsible for thei nstability of multi-stage high frequency amplifiers, and neutrodyne arrangements are intended to insert a negative reaction to counterbalance the inherent positive reaction due to the valves. Again, back reaction can take place from one coupling unit to the previous one, as in the case of open transformers. Screening is now adopted to minimise this type of coupling. In audio-frequency amplifiers, the reaction may be acoustic, in which case the output sound energy mechanically vibrates sympathetically the initial valves of the amplifier, producing a sustained ring or howl in the limiting case when self-oscillation is produced. This is partially overcome by anti-vibration valve holders.

Since this feed-back of energy may in many cases be a great advantage, enormously increasing the sensitivity of amplifier and yet in other cases may reduce the amplification or produce self-oscillation, we will examine the conditions necessary to produce any one of these three modifications to the normal behaviour of an amplifier.

## Examination of the Retro-action Effect.

For the purpose of this discussion, we are going to assume that the amplification is distortionless, and that we are applying sinusoidal waves of potential, i.e., we shall assume we are working on the straight
portions of the valve characteristics. We shall also assume that reaction from the output to the input takes place either by means of a special circuit for that purpose, or by means of the amplifier itself, but in the latter case we must assume that the amplifier is a unit. We cannot consider feed-back effects from stage to stage, as this would introduce too great a complexity.

Let $V_{0}$ be the peak value of the injected sine wave of E.M.F to the grid of the amplifier, whose instantaneous value is $v_{0}$ and let $E_{0}$ be the peak value of the sine wave of output E.M.F. produced by the amplifier. Then $\frac{E_{0}}{V_{0}}$ is the overall amplification of the system, neglecting any input circuit or reaction effects.

Let us call this normal amplification coefficient

$$
\begin{equation*}
k=\frac{E_{0}}{V_{0}} \cdots \tag{I}
\end{equation*}
$$

Now, if as a result of retro-action, the output is changed from $E_{0}$ to $E$, we shall call $\frac{E}{V_{0}}$, the total or final amplification coefficient, and thus we can call this coefficient

$$
\begin{equation*}
K=\frac{E}{V_{0}} \ldots \tag{2}
\end{equation*}
$$

Now $K$ may be greater or less than $k$ depending on whether the effect of back reaction is to increase or diminish the amplification.

If we put $K=C k, C$ will be a coefficient expressing the final amplification in terms of the normal amplification, and we can call it the reaction coefficient.

$$
\begin{equation*}
C=\frac{K}{h} \tag{3}
\end{equation*}
$$

If $C>\mathrm{I}$, the effect of the reaction is to increase the amplification, but if $C<I$ the final amplification is less than the normal.

Now consider the effect of a sinusoidal input applied to the grid of the amplifier of the form

$$
\begin{equation*}
v_{0}=V_{0} \sin (\omega t+\alpha) \tag{4}
\end{equation*}
$$

This will give rise to an output E.M.F. $k$ times this value, due to the normal amplifica-
tion coefficient, and therefore represented by

$$
\begin{equation*}
e_{0}=E_{0} \sin (\omega t-\beta) \tag{5}
\end{equation*}
$$

The angular difference between these two E.M.F. vectors is clearly $(a+\beta)$ and this must therefore represent the phase lag produced by the amplification process. This lag will, of course, depend upon the nature of the circuits and also upon the frequency. We shall assume that this frequency is constant throughout our investigation. We can, therefore, represent the total phase lag due to the amplifier by

$$
\begin{equation*}
\gamma=(\alpha+\beta) \tag{6}
\end{equation*}
$$

Now, this potential $e_{0}$ in the output circuit produces a small E.M.F. in the input due to the retro-action property which we are assuming to be condensed into one unit. We will call this fed-back E.M.F. $v_{1}$ and its peak value $V_{1}$, and we will denote the phase lag produced between the output and this new E.M.F. by $\theta_{0}$. Now this E.M.F. will be superimposed on the original $v_{0}$, but it must be added vectorially owing to its difference in phase.
$\frac{E_{0}}{V_{1}}$ represents the output E.M.F. divided * by this new fed-back E.M.F., and will be a large number in most practical cases. Call this ratio ' $x$.'

Then $\quad x=\frac{E_{0}}{V_{1}}=\frac{k V_{0}}{V_{1}}$.
If we denote the total phase lag from the first input to the new added fed-back input by $\theta$, we have

$$
\begin{equation*}
\theta=\theta_{\mathbf{0}}+\gamma=\theta_{0}+\alpha+\beta \tag{8}
\end{equation*}
$$

Now the type of coupling will vary this phase lag $\theta_{0}$ and therefore $\theta$, and the degree of coupling will vary $x$.

If $\frac{I}{x}$ is large, the coupling is tight, but if $\frac{I}{x}$ is nearly zero, the coupling is loose, and when $\frac{I}{x}=0$ there is no feed-back. $\frac{I}{x}$ may therefore be considered as the coupling coefficient.

The first fed-back component of E.M.F. can be represented by the equation

$$
\begin{equation*}
v_{1}=V_{1} \sin (\omega t+\alpha-\theta) \text { from (4) } \tag{9}
\end{equation*}
$$

since it lags by $\theta$ behind the original applied
E.M.F., and since $V_{1}=\frac{E_{0}}{x}$ we have

$$
\begin{equation*}
v_{1}=\frac{E_{0}}{x} \sin (\omega t+\alpha-\theta) \tag{IO}
\end{equation*}
$$

By the normal process of amplification, this E.M.F. $v_{1}$ is amplified $k$ times, giving an output

$$
\begin{align*}
c_{1} & =\frac{k}{x} E_{0} \sin (\omega t-\beta-\theta) \text { from }(5) \\
& =\frac{k^{2}}{x} V_{0} \sin (\omega t-\beta-\theta) \text { from }(7) \tag{II}
\end{align*}
$$

This output component of E.M.F. now establishes a new input component $v_{2}$ by virtue of the feed-back.

Therefore, as before, we have

$$
v_{2}=\frac{e_{1}}{x} \text { and lagging } \theta_{0} \text { behind } e_{1}
$$

So using (II) for $e_{1}$, we have

$$
\begin{align*}
v_{2} & =\frac{k^{2}}{x^{2}} V_{0} \sin \left(\omega t-\beta-\theta-\theta_{0}\right) \\
& =\frac{k^{2}}{x^{2}} V_{0} \sin (\omega t+\alpha-2 \theta) \tag{I2}
\end{align*}
$$

since $\quad \theta=\theta_{0}+\alpha+\beta$ from (8)
and this gives a second output E.M.F.

$$
\begin{equation*}
e_{2}=\frac{k^{2}}{x^{2}} E_{0} \sin (\omega t-\beta-2 \theta) \ldots \tag{I3}
\end{equation*}
$$

We can now write down the successive output components that make up the total output produced. They are

$$
\begin{align*}
& e_{0}=E_{0} \sin (\omega t-\beta) \\
& e_{1}=\frac{k}{x} E_{0} \sin (\omega t-\beta-\theta) \\
& e_{2}=\frac{k^{2}}{x^{2}} E_{0} \sin (\omega t-\beta-2 \theta)  \tag{I4}\\
& e_{3}=\frac{k^{3}}{x^{3}} E_{0} \sin (\omega t-\beta-3 \theta) \\
& e_{4}=\frac{k^{4}}{x^{4}} E_{0} \sin (\omega t-\beta-4 \theta)
\end{align*}
$$

and so on.
It is clear that the sum of these E.M.F. vectors gives the resultant output and the sum is that of a geometrical progression whose common ratio is $\frac{k}{x}$, whilst the phase angle lag between each term forms an
arithmetical progression with a common difference of $\theta$.

We can best examine the nature of this summation by graphical treatment.

In Fig. I, let $O V_{0}$ represent the applied E.M.F. to the amplifier delivered by the tuned circuit system. Then since $\gamma$ is the total phase lag due to the amplifier alone, we can represent the first output component by $O A=E_{0}=k V_{0}$.

The next output component will be $A B=\frac{k}{x} E_{0}$ in magnitude and displaced by an angle $\theta$ due to the lag caused by the feed-back and amplifier. $B C$ will be the next component and equals $\frac{k}{x} \times A B$ again displaced by $\theta$, and so we shall complete a polygon having an infinite number of sides and arrive at a resultant limiting point $R$. The vector $O R$ clearly represents the sum


Fig. I
of the progression, and in this case is greater than $O A$.

So in the diagram $\frac{O A}{O V_{0}}=k$ the normal amplification of the amplifier, $\frac{O R}{O V_{0}}=K$ the total amplification when feed-back is considered, and since $K=C k$ by definition (3), the reaction coefficient $C$ is represented by $\frac{O R}{O A}$, which is greater than unity in this case. It is clear that this condition is obtained when $\frac{k}{x}$ is less than unity and when the angle $\theta$ is acute.

In Fig. 2, the ratio $\frac{k}{x}$ is again less than unity, but $\theta$ is obtuse.

We see here that the resultant $O R$ is less than $O A$, and therefore $K$ is less than $k$ and the reaction coefficient $C$ is less than unity. The effect of the back coupling is


Fig. 2
thus to reduce the amplification to a value below the normal value $k$.

If the ratio $\frac{k}{x}$ is greater than unity, it is clear that whatever value $\theta$ may possess, the polygon increases without limit and the resultant $O R$ becomes infinitely large, however small $O A$ may be. In this case the system is unstable and self-oscillation commences.

We shall therefore only consider the case when $\frac{k}{x}$ is less than unity.

## The Maximum Value of Amplification.

We have seen that to obtain the value of $E$, the resultant output E.M.F. represented by $O R$ and the phase angle $\Phi$, it is necessary to obtain the vectorial sum of the series $\sum_{s=0}^{\infty} e_{s}$. To do this let us represent the vectors $E_{0}, E_{1}, E_{2}$, etc. by the straight lines $O E_{0}, O E_{1}, O E_{2}$, etc., respectively in Fig. 3, equal in magnitude to the vectors $E_{0}, E_{1}, E_{2}$, and such that the angles $A O E_{1}$, $A O E_{2}, A O E_{3}$, etc., are $\theta, 2 \theta, 3 \theta$, etc., respectively. Then we have to find the sum of these vectors.

Taking the general case of a vector $O E$, we can express this vector in terms of a real and an imaginary quantity, the real
value being the component of the vector along the axis $O A$ and the unreal value, the component perpendicular to $O A$.

Thus $E_{0} p^{s}(\cos s \theta+i \sin s \theta)=E_{0} p^{s i \theta \theta}\left(\mathrm{I}_{5}\right)$ where

$$
p=\frac{k}{x}
$$

The resultant sum $R$ can thus be expressed in the form

$$
\begin{gather*}
R=E_{0} \sum_{s=0}^{x} Z^{s}  \tag{I6}\\
Z=p e^{i \theta}
\end{gather*}
$$

where
The series is clearly a geometrical one, and the condition for convergence is as before that $p<\mathrm{I}$. Under these conditions, the sum

$$
\begin{equation*}
\sum_{t=0}^{x} Z^{x}=\frac{\mathrm{I}}{\mathrm{I}-Z} \tag{I8}
\end{equation*}
$$

and therefore the resultant $R=E_{0} \frac{\mathrm{I}}{\mathrm{I}-Z}$ (I9)
The phase lag between the resultant and the first E.M.F. component is given in terms of the ratio of the real and imaginary components of the complex factor $R$.

Using the trigonometrical form

$$
\begin{equation*}
\frac{\mathrm{I}}{\mathrm{I}-p(\cos \theta+i \sin \theta)}=\frac{\mathrm{I}}{\mathrm{I}-Z} \tag{20}
\end{equation*}
$$

where $p=\frac{k}{x}$, we can represent the complex

factor $R$ by the following equation

$$
\begin{equation*}
R=a+i b=\frac{\mathrm{I}}{(\mathrm{I}-p \cos \theta)-i p \sin \theta} \cdots \tag{2I}
\end{equation*}
$$

This gives the real component

$$
\begin{equation*}
a=\frac{I-p \cos \theta}{p^{2}-2 p \cos \theta+1} \tag{22}
\end{equation*}
$$

and the unreal component

$$
\begin{equation*}
b=\frac{p \sin \theta}{p^{2}-2 p \cos \theta+1} \tag{23}
\end{equation*}
$$

The magnitude of $R=a+i b$ is given by $\sqrt{a^{2}+b^{2}}$ and the angle $\Phi$ by $\sin \Phi=\frac{b}{\sqrt{ } a^{2}+b^{2}}$

Therefore

$$
\begin{align*}
R & =\sqrt{a^{2}+b^{2}} \\
& =\sqrt{\frac{\mathrm{I}+p^{2} \cos ^{2} \theta-2 p \cos \theta+p^{2} \sin ^{2} \theta}{\left(p^{2}-2 p \cos ^{2} \theta+\mathrm{I}\right)^{2}}} \\
& =\sqrt{\frac{p^{2}-2 p \cos \theta+\mathrm{I}}{\left(p^{2}-2 p \cos \theta+\mathrm{I}\right)^{2}}} \\
& =\frac{1}{\sqrt{\left(p^{2}-2 p \cos \theta+\mathrm{I}\right)}} \tag{2+}
\end{align*}
$$

and

$$
\begin{align*}
\sin \Phi & =\frac{p \sin \theta \sqrt{\left(p^{2}-2 p \cos \theta+\mathrm{I}\right)}}{\left(p^{2}-2 p \cos \theta+\mathrm{I}\right)} \\
& =\frac{p \sin \theta}{\sqrt{p^{2}-2 p \cos \theta+\mathrm{I}}}=R p \sin \theta \tag{25}
\end{align*}
$$

$$
\begin{equation*}
\text { So } E=\frac{E_{0}}{\sqrt{p^{2}-2 p \cos \theta+1}} \tag{26}
\end{equation*}
$$

and $K=\frac{k}{\sqrt{p^{2}-2 p \cos \theta+\mathrm{I}}}$
In the limiting case, on the point of oscillation $p=1$, and we get

$$
\begin{equation*}
E=\frac{E_{0}}{2 \sin \frac{1}{2} \theta} \text { and } K=\frac{k}{2 \sin \frac{1}{2} \theta} \ldots \tag{28}
\end{equation*}
$$

As $p$ tends towards unity-i.e, as the feedback is increased-and as $\theta$ tends towards zero, we see that the resultant output $E$ increases towards an infinite value and the amplification has greatly increased. Under these conditions the amplifier is very sensitive but very unstable. The slightest variation in $p$ or $\theta$ will produce the oscillatory condition. Now usually the angle $\theta$ is defined in terms of the circuit arrangements and an increase of coupling will vary $p=\frac{k}{x}$, since $\frac{1}{x}$ defines the degree of coupling. We will therefore assume $\theta$ to be constant and consider for any given value of $\theta$ the maximum amplification that is possible by adjusting the feed-back coupling.

To do this we equate $\frac{d E}{d p}$ to zero, and we find that the optimum condition is $p=\cos \theta$ and the maximum value of $E$ is given as
and

$$
\begin{align*}
E_{\max } & =\frac{E_{0}}{\sin \theta}  \tag{29}\\
K_{\max } & =\frac{k}{\sin \theta} \tag{30}
\end{align*}
$$

We see that further increase of coupling beyond this value leads to a reduction in amplification, and thus for a given circuit arrangement we cannot increase the amplification by merely adjusting the magnitude of the feed-back effect by any of the ordinary methods.


Summing up the analysis, we see that if we assume the reaction effect to be due to a specific coupling arrangement, the maximum amplification that can possibly be obtained is dependent upon $\frac{k}{x}$ and $\theta$. When $\frac{k}{x}$ is less than I , the amplifier is stable and the coupling may make $C$ greater or less than unity depending on whether $\theta$ is acute or obtuse respectively. When $\frac{k}{x}=\mathrm{I}$, the coupling is such as to compensate exactly for the losses in the circuit, and then the resistance of the input circuit has been reduced to zero, and finally, when $\frac{k}{x}$ is greater than unity, the coupling is such as to more than compensate for all the losses, and therefore oscillations are set up and maintained, and the amplifier becomes unstable. The resistance of the input circuit has been reduced to a negative value.

## Graphical Representation of the Retroactive effect.

We can represent the results we have obtained by a simple graphical construction as follows:-

In Fig. 4, consider a line $O M$ representing the vector $E_{0}$. Now from $O$ measure a length $O B$ equal in magnitude to the value $p$. Set out from $O$ the line $O N$, making an angle equal to $\theta$ with $O M$ and to the same scate measure $O A=\mathrm{I}$ along $O N$.

Thus

$$
\overline{O A}>\overline{O B}
$$

and in the triangle $O A B$

$$
\begin{gathered}
\overline{A B^{2}}=p^{2}+\mathrm{I}-2 p \cos \theta . \\
\text { Now } C=\frac{K}{k}=\frac{\mathrm{I}}{\sqrt{p^{2}-2 p \cos \theta+\mathrm{I}}} \text { from (27). }
\end{gathered}
$$

Hence

$$
C=\stackrel{\mathrm{I}}{A B}
$$

Since $\overline{O A}$ was made equal to unity, we require to know the ratio $\frac{O A}{A B}$ to determine $C$. Draw $O C_{1}$ at an angle $A O C_{1}$ equal to $A B O$ and produce $A B$ to $C_{1}$. From the similar triangles we see that $\frac{A C_{1}}{O A}=\frac{O A}{A B}$, and therefore $\overline{A C}_{1}=\mathrm{C}$. Now since the angle $O C A$ $=\theta$ and is constant when $p$ varies, the locus of point $C_{1}$ will be a circle in which $O A$ is a segment. When a becomes a right


Fig. 5
angle we know that the angle $A O C$ is also a right angle and thus $A C$ is the diameter of the circle. The middle point $O_{1}$ of this line gives the centre of the circle.

We see that the maximum value that $C$ may have is when $A C$ becomes the diameter of the circle. Then

$$
A C=\frac{O A}{\sin \theta}, \text { i.e. } C=\frac{\mathrm{I}}{\sin \theta}
$$

as given in equation (30). The phase angle $\Phi$ between the first output vector $E_{0}$ and the final resultant vector $O R$ in Fig. I is given by the angle $\beta$ in Fig. 4 .

It is important to note that the value


Fig. 6
of $C$ so obtained is only applicable to the case when $p<\mathrm{I}$, i.e., when $O B<O .1$. When $p>1$, the stable condition is no longer maintained.

Now suppose the angle $\theta$ is a right angle instead of being acute, as shown in Fig. 5. In this case $A O$ becomes the diameter of the circle and is made equal to unity. As " $p$ " rises from $O$ to unity $A C$ falls from $A O=\mathrm{I}$ to $A C$. So we see that, whatever coupling we may use, we cannot get a total amplification $K$ greater than the normal value $k$.

Lastly, when $\theta$ is obtuse we obtain the conditions shown in Fig. 6, from which we see that $A C$ is always less than $A O$, which is made unity as before. This means that, whatever value $p$ may have, the effect of the coupling is to reduce the overall amplification making $K$ less than the normal amplification coefficient $k$.

Referring to Fig. 4, we see that the maximum amplification is obtained when $A C$ is a maximum, that is when $O B_{1}=p$ is some definite optimum value less than $O A=\mathrm{I}$. The amplifier is stable, and we can increase $p$ until its value reaches unity. The maximum value that $O B$ may have is therefore $O B_{2}=O A=\mathrm{I}$, and then the value of $C$ is given by the length $A C_{2}$, which is less than $A C_{1}$, the maximum value. $C$ may thus lie on the arc of the circle $O C_{1} C C_{2}$ but cannot lie to the right of $C_{2}$. as this gives $p$ greater than I and therefore the oscillatory condition. It is interesting
to note that in this case the maximum amplification falls when near to the oscillation point, which means that if a coil or coupling condenser was being used for reaction, increase of this coupling past a certain point would reduce the overall amplification. Similarly in Figs. 5 and 6 for the case of $\theta$ equal to a right angle or an obtuse angle respectively, we see that the amplification falls from the value $O A$ to the value $A C_{2}$, where $O B_{2}=O A=\mathrm{I}$ in
both cases. Here the effect of reaction is to reduce the amplification until the fedback E.M.F. is equal to the component E.M.F. producing it, and when this condition is obtained the amplifier becomes unstable.

It will be noticed from Fig. + that if $\theta$ is very small, the point of maximum amplification occurs when the reaction is greatest, and this is usually so in most simple practical circuits.

## A German H.T. Mains Unit with Glow Discharge Rectifier.

Ainteresting mains unit made by Dr. Georg Seibt is described in Elektrotechnische Zeitschrift of 24th November, 1927. The connections are as shown in Fig. I, which also gives the value of the components employed. Fig. 2 shows two sectional views of the rectifying valve. It is called an "Anotron" and is a glow discharge valve with a cathode above and two anodes below. The cathode is coated with an


Fig. I.-Circuit connections of the mains unit.
alkali nitrogen compound which gives off nitrogen at about $300^{\circ} \mathrm{C}$. and frees the alkali; any traces of oxygen are thus


Fig. 2.-Sectional views of the "A notron" valve.
absorbed. It is claimed that, unlike other glow discharge valves, the "Anotron" maintains constant characteristics. The absence of any heated filament increases the life of the valve and also does away with the necessity of a special filament-heating secondary winding on the transformer. By means of a 10,000 ohm potentiometer, grid bias voltages from I to 15 , detector voltages from 30 to 70 , low-frequency amplifier voltages from So to 130 , and a power amplifier voltage of 180 can be tapped off.
G. W. O. H

# The Power in a Modulated Oscillation. 

By E. Howard Robinson.

MODULATED oscillations are becoming increasingly important both in radio engineering and line work where modulated carrier currents are employed. So far the present writer has not seen any published information which makes it possible to interpret voltmeter or ammeter readings in terms of power dissipated in the case of circuits carrying modulated high-frequency carrier currents.

In ordinary electrical engineering we deal with A.C. of simple sine-wave form and constant amplitude, the calculations in connection with which are well known. The relation between voltage $V$ and time $t$ is

$$
V=V_{0} \sin \omega t
$$

and we know that if this voltage is applied across a resistance $R$ the power dissipated in the resistance is $\frac{V_{0}{ }^{2}}{2 R}$, the value $\frac{V_{0}}{\sqrt{2}}$, termed the R.M.S. voltage, being usually employed in specifying alternating voltages.

If, however, the main frequency voltage is not of constant amplitude but is varied harmonically in accordance with another frequency, our usual A.C. calculation does not hold. It might appear at first sight, since the A.C. peak voltages are varied equal amounts above and below the normal value, that the power would average out to the same value as that obtained in the absence of modulation. The following simple calculations show that this is by no means the case.

Whatever the particular circuit under consideration-it may be simply a condenser or inductance or a more complex system of capacities, inductances and resistances-it may be reduced to one equivalent reactive component in series with an equivalent non-reactive component which we shall designate as a resistance $R$. It is required to arrive at the watts dissipated in $R$ :

Let $V=$ the voltage across $R$ at any instant.
$t=$ the instantancous time value.
$\omega_{1}=2 \pi$ times the modulation frequency.
$\omega=2 \pi$ times the carrier frequency.
$V_{0}=$ the peak voltage.
$m=$ the modulation ratio, i.e., roo $m=$ percentage modulation.

For simplicity only the case of one modulation frequency at a steady amplitude is considered. We have three important cases to consider, namely: (a) ordinary modulation, where the carrier frequency and two side-bands are present; (b) two side-bands without carrier, and (c) one side-band alone.

## (a) Ordinary Modulation.

We have a carrier-frequency oscillation modulated by a simple harmonic variation of its normal amplitude at a frequency of $\frac{\omega_{1}}{2 \pi}$,

$$
\begin{align*}
& \text { i.e., }=\left(V_{0}+m V_{0} \sin \omega_{1} t\right) \sin \omega t \\
& =V_{0} \sin \omega t+\frac{m V_{0}}{2} \cos \left(\omega-\omega_{1}\right) t \\
&
\end{aligned} \quad \begin{aligned}
{ }^{2} & \frac{m V_{0}}{} \cos \left(\omega+\omega_{1}\right) t .
\end{align*}
$$

The first of these terms represents the steady carrier component, while the other two represent the two side-bands.

Now it is well known that if we have a number of simple sine (or cosine) waves of R.M.S. values $V_{1}, V_{2}, V_{3}$, etc., the resultant R.M.S. value $V$ is given by the square-root of the sum of the squares of the component R.M.S. values, i.e.,

$$
V=\sqrt{V_{1}^{2}+V_{2}^{2}+V_{3}{ }^{2}+\ldots \ldots}
$$

Remembering, therefore, that with a simple sine term the R.M.S. is ${ }_{\sqrt{2}}$ i of the amplitude, we have for the resultant R.M.S. voltage of the three terms in (1):

$$
\begin{aligned}
V_{\text {R.M. }} & =\sqrt{\frac{V_{0}{ }^{2}}{2}+m^{2} V_{0}{ }^{2}+\frac{m^{2} V_{0}{ }^{2}}{8}} \\
& =V_{0} \sqrt{\frac{m^{2}+2}{4}}
\end{aligned}
$$

The average power expended in a resist-
ance $R$ is the mean square of the voltage divided by $R$, i.e. :

$$
\begin{equation*}
\text { Power }=\frac{V_{0}^{2}}{R} \times \frac{m^{2}+2}{4} \tag{2}
\end{equation*}
$$

## (b) Two Side-bands without Carrier.

An ordinary modulated carrier voltage is, as previously shown, represented by

$$
\begin{aligned}
& V=V_{0} \sin \omega t+\frac{m V_{0}}{2} \cos \left(\omega-\omega_{1}\right) t \\
&-\frac{m V_{0}}{2} \cos \left(\omega+\omega_{1}\right) t
\end{aligned}
$$

If the steady carrier component $V_{0} \sin \omega t$ is suppressed we have for the residual voltages

$$
\begin{align*}
& V=\frac{m V_{0}}{2} \cos \left(\omega-\omega_{1}\right) t \\
& \quad-\frac{m V_{0}}{2} \cos \left(\omega+\omega_{1}\right) t \tag{3}
\end{align*}
$$

The two terms in this expression represent the two side-bands without the carrier component, and, using the same theorem as before, the resultant R.M.S. voltage is given by

$$
\begin{aligned}
V_{\mathrm{R}, \mathrm{~S}, \mathrm{~S} .} & =\sqrt{\frac{m^{2} V_{0}^{2}}{8} \times 2} \\
& =\frac{m V_{0}}{2}
\end{aligned}
$$

and

$$
\begin{equation*}
\text { Power }=\frac{V_{0}^{2}}{R} \times \frac{m^{2}}{4} \tag{4}
\end{equation*}
$$

(c) One Side-band Alone.

The expression for two side-bands is given in (3), i.e. :

$$
V=\frac{m V_{0}}{2} \cos \left(\omega-\omega_{1}\right) t-\frac{m V_{0}}{2} \cos \left(\omega+\omega_{1}\right) t
$$

One side-band is therefore given by

$$
\begin{equation*}
V=\frac{m V_{0}}{2} \cos \left(\omega-\omega_{1}\right) t \tag{5}
\end{equation*}
$$

This represents an ordinary cosine wave and therefore

$$
\begin{align*}
\text { Power } & =\frac{1}{2}\left(\frac{m V_{0}}{2}\right)^{2} \times \frac{\mathrm{I}}{R} \\
& =\frac{V_{0}^{2}}{R} \times \frac{m_{2}}{8} \ldots \tag{6}
\end{align*}
$$

Conclusions.
The problem is really that of determining
root-mean-square voltages in terms of the modulation ratio $m$ and the peak voltage $V_{0}$ of the original unmodulated carrier component. We see that the ratio $\frac{\sqrt{2}}{I}$ for $\frac{\text { peak voltage }}{\text { R.M.S. voltage }}$ only holds for an unmodulated sine wave, or single side-band, which amounts to the same thing for a steady value of $m$. The results may be usefully summarised as in the following table. The same relations, of course, hold for corresponding current values as well as for voltages, i.e., I's may be substituted for $V$ 's everywhere.

We see that even with 100 per cent. modulation ( $m=\mathrm{I}$ ) in an ordinary radio telephone transmitter only one-third of the total high-frequency power is transmitted in the form of useful modulation component. The other two-thirds carries no intelligence at all. When the modulation is 50 per cent. ( $m=\frac{1}{2}$ )-quite a reasonably high value for good telephony-the useful power is only one-ninth of the total power sent out.

The term " useful power" as applied to the side-band components may perhaps need some explanation. The steady carrier component need not, in theory, be radiated or even ever generated at the transmitter as its function can perfectly well be fulfilled by a small local oscillator at the receiver. The intensity of the received telephony in such circumstances depends entirely upon the power radiated from the transmitter in the form of side-band components. The presence of the carrier component in the transmission will not in any way add to the strength of reception where a correctly adjusted local homodyne is used. We have an analogy in the case of ordinary line telephony. The direct current from the microphone battery is not sent right along the line to produce the steady flux in the receiver which is necessary for intelligible reception. The A.C component is separated from the local D.C. by means of a transformer and is transmitted alone, while the necessary steady flux is supplied at the receiver by a permanent magnet.

It can be shown that with a two-side-band transmission half the transmitting power which would be necessary with a one-sideband transmission is required to produce the same effect at the receiver, provided that
the homodyne at the receiver is of adequate strength. It is for this reason that a two-side-band system is taken as the basis of usefulness for the present discussion. Of course the practical difficulties in working

An approximate method of measuring the modulation ratio $m$ suggests itself in the case of an ordinary telephony transmitter where a rapidly-responding thermo-ammeter is available. Let $I_{1}$ be the thermo-ammeter

| Type of Waveform. | R.M.S. Voltage (Peak Value of Carrier $=V_{0}$ ). | Power in a Load $R$. | Maximum Peak Volts <br> R.M.S. Volts. |
| :---: | :---: | :---: | :---: |
| Carrier and both sidebands. | $V_{0} \times \sqrt{\frac{m^{2}+2}{4}}$ | $\frac{V_{0}{ }^{2}}{R} \times \frac{m^{2}+2}{4}$ | $\frac{2(m+1)}{\sqrt{m^{2}+2}}$ |
| Both side-bands without carrier. | $V_{0} \times \frac{m}{2}$ | $\frac{V_{0}{ }^{2}}{R} \times \frac{m^{2}}{4}$ | 2 |
| One side-band alone. | $V_{0} \times \frac{m}{\sqrt{8}}$ | $\frac{V_{0}^{2}}{R} \times \frac{m^{2}}{8}$ | $\sqrt{2}$ |
| Carrier alone (no modulation). | $V_{0} \times \frac{1}{\sqrt{2}}$ | $\frac{V_{0}{ }^{2}}{R} \times \frac{1}{2}$ | $\sqrt{2}$ |

without the carrier component in the transmission medium in the case of radio telephony constitute an important secondary consideration which, however, does not enter the present discussion.

An interesting point arises in connection with the interpretation of meter readings in circuits carrying modulated currents. A thermal instrument, such as the usual hot-wire or thermo-junction aerial ammeter, depends upon the rate at which heat is produced in a resistance and it gives therefore an accurate measure of the mean square of the current whatever the waveform. The dial is generally calibrated in amperes, the actual reading being the R.M.S. current, and this is correct both for modulated and unmodulated currents.

When the microphone of an ordinary radio-telephone transmitter is spoken into the reading of a thermal ammeter in the aerial circuit should increase slightly. Systems of modulation where the reading either stays dead constant or decreases when the microphone is spoken into must operate in some freak manner and give rise to distortion.
reading in the absence of modulation and $I_{2}$ the reading when some fairly steady modulation is applied. If $I_{0}$ is the peak value of the unmodulated carrier current ( $I_{0} \sin \omega t$ ) we have

$$
\begin{aligned}
& I_{1}=I_{0} \times \frac{1}{\sqrt{2}} \\
& I_{2}=I_{0} \times \sqrt{\frac{m^{2}+2}{4}} \quad \text { (see table). }
\end{aligned}
$$

Eliminating $I_{0}$ between these two equations,

$$
m=\sqrt{\frac{2\left(I_{2}{ }^{2}-I_{1}{ }^{2}\right)}{I_{1}{ }^{2}}} .
$$

In the particular case of 100 per cent. modulation ( $m=\mathrm{I}$ )

$$
\frac{I_{2}}{I_{1}}=\sqrt{\frac{3}{2}}=\mathrm{I} .22
$$

The simple relations given in this article do not, of course, hold for completely interrupted carrier waves of the "I.C.W." type.

# Dielectric Losses in Single Layer Coils at Radio Frequencies. 

By W. Jackson, M.Sc.

THE results of resistance measurements on single layer coils at radio frequencies have always shown a serious discrepancy between the measured values of resistance and the values computed from the high-frequency resistance formulæ which S. Butterworth $\dagger$ has derived from purely theoretical considerations.

The importance of the dielectric necessarily associated with inductance coils in introducing a source of loss into the coil circuit and therefore in increasing the effective resistance of the coil at radio frequencies has been fully realised. The experiments to be described have been an attempt to measure the magnitude of this added resistance and to enquire whether it was of sufficient magnitude to account for part of this discrepancy.

The dielectric loss may be associated with either the insulated covering on the wire or the mechanical support or former on which the coil is wound. The wire covering may be of silk, cotton or enamel, while the materials used frequently as formers are tubes of ebonite, micarta, paxolin, wood, glass and cardboard.

Any dielectric loss associated with the coil is exhibited as an increase in the effective resistance of the coil, and the magnitude of this increase has been measared at radio frequencies for coils, of exactly similar spacing, inductance and direct current resistance, wound on formers of the abovementioned materials, using both bare and covered wire.

For this purpose it was necessary to construct a coil having a minimum of solid dielectric, which could be regarded as sensibly devoid of dielectric loss and with which the exactly similar former wound coils could be compared. The photograph of Fig. I shows the former constructed for the purpose of winding this standard coil. A brass tube of four inches external diameter

[^1]and four inches in length was provided at one end with a brass flange and was then grooved approximately if turns to the inch. After grooving, the tube and its associated flange was cut axially into 6 sectors, a circumferential space of one-quarter inch being cut between sectors. The sectors were mounted on a brass base plate and the true position of each sector located by two fastening screws passing through radial slots in the flanges. By loosening the screws each sector could be slid radially inwards along the face of the base plate. The sectors were supported at their extreme ends by a removable brass ring placed inside the end of the tube and to which each sector was fastened by screw. A spindle passing through the base plate enabled the former to be supported and at the same time rotated, thus rendering the coil winding a simple process.


Fig. I-Former for coil winding.
The coil used throughout as the standard of comparison consisted of 46 turns of No. 20 S.W.G. bare copper wire, the diameter of which was .0357 inch. The pitch of the turns being . 068 f inch, provided sufficient spacing between wires to allow the construction of a similar coil of double cotton and enamel-covered wire.

In order to retain the spacing of the wires
when the coil was removed from the former a thin stream of molten paraffin wax was poured axially along the coil at each circumferential space between sectors. This was found to give sufficient rigidity to enable measurements to be made on the coil. The coil could be taken from the former by removing the end supporting ring and


Fig. 2.-Coils used in tests.
collapsing the sectors of the former a small distance along the base plate.

Two coils wound on the former and used in the tests for comparison with Butterworth's theoretical formule are shown in Fig. 2; the smaller was wound with bare No. 20 S.W.G. wire, and the larger with silk covered No. 20 wire.

Coils of the same number of turns as the standard were wound on the various formers given below. Each former had an external diameter of 4 inches, and was grooved in the same manner as the brass former previously described, ensuring exact similarity in overall diameter and in spacing between turns for all the coils tested. The brass former was also used in winding similar coils using silk, cotton, and enamel-covered wire.
(a) Ebonite former. Tube, $\frac{1}{4}$ in. thick.
(b) Micarta former. Tube, $\frac{1}{4} \mathrm{in}$. thick.
(c) Teak wood former. Tube, $\frac{1}{4}$ in. thick.
(d) Leatheroid former (consisting of layers of leatheroid sheet held together by shellac varnish and baked). Tube, $\frac{1}{4} \mathrm{in}$. thick.
(e) Paxolin former. Tube, $\frac{1}{16}$ in. thick.
( $f$ ) Cardboard former. Tube, $\frac{1}{8}$ in. thick.
The common direct-current resistance was .380 ohm and the true inductance 173.5 microhenries.

## High-frequency Resistance Measurement.

The method of measurement adopted was the "Resistance Variation Method,"
as this method had been found reliable and accurate. Measurements covered a wavelength range of 300 to 900 metres. The standard resistances used consisted of short lengths of Eureka wire, which could be inserted between two mercury cups connected in the measuring circuit. Normally, the cups were short circuited by a piece of thick copper wire of the same shape as the standard resistances. The latter had values of $888 ; 1.785 ; 2.329$ ohms, and results using the three resistances in turn were found to agree within I per cent.

Measurements on air coils wound with No. 20 s.w.G. single silk-covered, double silk-covered, double cotton-covered, and double cotton- and enamel-covered wire when compared with those on the bare wire standard coil indicated an inappreciable loss due to the wire covering over the entire wavelength range, 300 to 900 metres. This is substantially in agreement with Wilmotte's results on a square single layer coil.*


Fig. 3.
A coating of wet shellac to the double cottoncovered wire coil was found to increase the effective resistance by . 18 ohm at 300 metres, a percentage increase of 6.0 per cent.

[^2]The effect of the different formers was, however, very pronounced, as will be seen from the curves of Fig. 3. These show the variation of effective resistance of the coils (b), (c), (d), and (f), and of the standard air coil with wavelength.

It is seen that the cardboard former is capable of introducing a resistance of 2.30 ohms into the circuit at 300 metres, which represents a 76 per cent. increase in effective resistance. The extent to which moisture may affect the dielectric loss in such a former will be seen from the following results at a wavelength of 500 metres.

|  | Effective <br> Resistance (ohms) |
| :---: | :---: |
| Standard bare wire, air coil | т. 96 |
| D.C.C. wire. Cardboard former |  |
| (dry) ............................. | 3.35 |
| (damp) | 7.03 |

The ebonite and paxolin formers were found to increase the effective resistance of the coil only slightly, the increase at 300 metres in each case being of the order of 3.0 per cent.

## Variation of Added Resistance Due to Dielectric Loss with Frequency.

For purposes of theoretical analysis, a coil can be represented by the equivalent circuit of Fig. 4, where $L$ represents the pure inductance, and $R$ the true resistance of the coil ; $C$ is a small condenser representing the self-capacity and $r$ a resistance accounting for the dielectric loss.

If this is replaced by an equivalent series circuit consisting of an effective resistance $R^{1}$ and an effective inductance $L^{1}$, it can readily be shown that \|

$$
R^{1}=\frac{R+r C^{2} \omega^{2}\left(L^{2} \omega^{2}+R^{2}+r R\right)}{\left(\mathrm{I}-L C \omega^{2}\right)^{2}+(R+r)^{2} C^{2} \omega^{2}}
$$

where $\omega$ is $2 \pi$ times the frequency.
Provided that the natural frequency of the coil is not approached; that is, provided $L C \omega^{2}$ is small compared with unity, and since $C$ is only of the order of a few micro-

[^3]micro-farads, the above expression may be simplified to
\[

$$
\begin{equation*}
R^{1}\left(\mathrm{I}-2 L C \omega^{2}\right)=R+r L^{2} C^{2} \omega^{4} \tag{I}
\end{equation*}
$$

\]

The dielectric circuit consisting of $C$ and $r$


Fig. 4
has a power factor $P=r C \omega$, which allows equation (I) to be written

$$
\begin{equation*}
R^{1}\left(\mathrm{I}-2 L C \omega^{2}\right)=R+P L^{2} C \omega^{3} \tag{2}
\end{equation*}
$$

The term $P L^{2} C \omega^{3}$ represents the added series resistance due to dielectric loss, and, since $P$ is fairly constant with change in frequency, is seen, theoretically, to vary as the cube of the frequency.

The results given in Fig. 3 on the various coils enable the variation of added resistance due to dielectric loss with frequency to be checked. Since all the coils are similar, the true high-frequency resistance $R$, at a given frequency, will be the same in all cases. Measurements of self-capacity indicated that the type of former had no effect on the value of $C$, the latter being $4.5 \mu \mu f$ for all the coils.


Fig. 5.
It follows then that if the effective resistances of the standard air coil and of a former wound coil at a frequency $\omega / 2 \pi$ are respectively $R_{1}{ }^{1}$ and $R_{2}{ }^{1}$, the added series resist-
ance due to dielectric loss in the former wound coil is given by

$$
\left(R_{2}{ }^{1}-R_{1}{ }^{1}\right)\left(\mathrm{I}-2 L C \omega^{2}\right)
$$

provided that the dielectric loss in the standard coil is negligibly small in comparison, which is the case.

This added resistance has been derived from the results of Fig. 3 for the coils (b), (c) and (d). It is seen from Fig. 5, where the added series resistance is plotted against the cube of the frequency, that this resistance sensibly follows the cube law previously derived theoretically.

This conclusion does not hold in the case of the cardboard former. As the former was not baked before the test, this is probably accounted for by the presence of moisture in the cardboard.
The results serve to show that dielectric loss may be an important factor in determining the effective resistance of a coil at radio frequencies, and to emphasise the deleterious effect of moisture on dielectric loss.

Since the added series resistance due to dielectric loss is given by the expression $P L^{2} C \omega^{3}$, the average power factor $P$ of the dielectric circuit of the various coil formers can be calculated from the curves of Fig. 5 . These give values for $P$ of .032 for the leatheroid former, .02I for the micarta former and .oI 6 for the teak wood former.

## Comparison of Results on Bare Wire Coil with Theoretical Values.

Since the standard bare wire coil can be regarded as devoid of dielectric loss, the measured values of effective resistance, when corrected for self-capacity, can be compared with the calculated values of high-frequency resistance given by Butterworth's equation for short single layer coils.

$$
R_{n}=R_{0}\left\{\mathrm{I}+F+u G \frac{d^{2}}{D^{2}}\right\}
$$

where $R_{n}$ is the resistance at frequency $n$ and $R_{0}$ is the direct current resistance. In this formula $\mathrm{I}+F=\frac{\sqrt{2} z+\mathrm{I}}{4} ; G=\frac{\sqrt{2 z-\mathrm{I}}}{8}$ and $z=\pi d \sqrt{\frac{2 n}{\rho}}, d$ being the wire diameter in cms., and $D$ the pitch of the turns. The factor $u=3.29+b / a$, where $b$ is the length of the coil and $a$ its radius.

The measured values of high-frequency resistance can be corrected for self capacity by the formula

$$
R=R^{1}\left(\mathrm{I}-2 L C \omega^{2}\right) .
$$

The curves of Fig. 6 show the experimental results along with the calculated values. It may be seen that the experimental values are everywhere too high. This may be accounted for by the unknown resistance of the condenser included in the measuring circuit. At a wavelength of 600 metres a condenser power factor of $20.5 \times 10^{-5}$ is sulficient to account for the discrepancy, although at 300 metres the condenser power factor would require to be $38.2 \times 10^{-5}$.

Measurements on other air coils of different turns to the above indicated that power factors of this order would account in all


Fig. 6.
cases for the difference between measured and calculated results.
It has been stated§ that the power factor of a good air condenser is of the order of $30 \times 10^{-5}$, which justifies the conclusion that if the condenser loss be allowed for Butterworth's formula gives close agreement with experimental values.

The author wishes to express his thanks to the Principal of the Technical College, Bradtord, for facilities to carry out the experimental work recorded in the paper.

[^4]
# The Establishment of Formulx for the Selfinductance of Single-turn Circuits of Various Shapes. 

By R. G. Allen, B.Sc., A.R.C.Sc.I., M.I.E.E.

THE more elementary fundamental principles required in this treatment are given in text-books on electrical principles and will be assumed as known. They include :-
(a) Laplace's formula which gives the strength of the magnetic field at a point outside a conductor carrying


Fig. I. a current of electricity due to a small element of the circuit.
(b) That the equivalent number of magnetic lines due to the internal system of lines of magnetic force, or induction, in the material of a conductor carrying current is $\frac{\mu}{2}$ per to amperes per linear centimetre of the conductor, $\mu$ being the magnetic permeability of the material of the conductor.
(c) The strength of the magnetic field at a point $P$, Fig. I, outside a limited length $A B$ of a straight conductor carrying current is

$$
\begin{equation*}
\frac{I}{P M}\left(\sin \theta_{\mathbf{1}}+\sin \theta_{\mathbf{2}}\right) \tag{I}
\end{equation*}
$$

$I$ being the current in c.g.s. units and $P M$ the distance in cms . of $P$ from the axis of the conductor. The latter formula may be readily proved by using Laplace's formula.

## The Self-inductance of a Circular Coil having One Turn.

The radius of the circle is $a$ cms. and the radius of the wire $r$ cms. In Fig. 2, $A B$ is a part of the turn of wire and is taken so small that it may be regarded as straight. Its length is taken as $s$.

The first step is to find the number of lines due to $A B$, which threads the element of area between the lines $O M$ and $O N$.

By Laplace's formula the strength of the magnetic field due to $A B$ at $P$ is

$$
\frac{A B \cos \theta}{O P^{2}}=\frac{s \cos \theta}{x^{2}}
$$

the current being io amperes.
It follows then that the number of lines of magnetic force threading the shaded element at $P$ will be

$$
\frac{s \cos \theta}{x^{2}} \times x d \theta d x=s \cos \theta d \theta \frac{d x}{x}
$$

Therefore the number of external lines due to $A B$ threading the area $O M N$ will be,

$$
s \cos \theta d \theta \int_{o n}^{o M} \frac{d x}{x}=s d \theta \cos \theta \log \frac{O M}{O H}
$$

Now $O M=2 a \cos \theta$ and $O H=\frac{r}{\cos \dot{\theta}}$.


Fig. 2.
The number of lines for $O M N$ will be then equal to

$$
s d \theta \cos \theta \log \frac{2 a \cos ^{2} \theta}{\gamma}
$$

that is, to

$$
s d \theta \cos \theta\left(\log \frac{2 a}{r}+2 \log \cos \theta\right)
$$

The total number of lines threading the whole circular enclosed area due to $A B$ will be the integration of the number of lines due to $A B$ threading all the elemental areas such as $O M N$. The number of external lines due to $A B$ threading the circle will therefore be
$2 s \log \frac{2 a}{r} \int_{0}^{\frac{\pi}{2}} \cos \theta d \theta+4 s \int_{0}^{\pi} \cos \theta \log \cos \theta d \theta$ which equals
$2 s \log \frac{2 a}{r}[\sin \theta]_{0}^{\frac{\pi}{2}}+4 s \int_{0}^{\frac{\pi}{2}} \cos \theta \log \cos \theta d \theta$, that is :

$$
2 s \log \frac{2 a}{r}+4 s \int_{0}^{\frac{\pi}{2}} \cos \theta \log \cos \theta d \theta .
$$

The integration of the second term may be found by integrating by parts, and using

$$
\sin ^{2} \theta=I-\cos ^{2} \theta
$$

and

$$
\int \frac{d \theta}{\cos \theta}=\log \frac{\mathrm{I}+\sin \theta}{\cos \theta}
$$

Its value is

$$
\sin \theta \log \cos \theta+\int \frac{d \theta}{\cos \theta}-\int \cos \theta d \theta
$$

Therefore this integration is
$[\sin \theta \log \cos \theta-\sin \theta+\log (\mathrm{r}+\sin \theta)$
$-\log \cos \theta]_{0}^{\frac{\pi}{2}}$
$=-\mathrm{I}+\log 2=-\mathrm{I}+2.3 \times 0.3=-0.3 \mathrm{I}$.
Thus the number of external lines threading the circle due to $A B$ is

$$
2 s \log \frac{2 a}{r}-4 s \times 0.3 \mathrm{I}
$$

Now as each part of the circuit equal to $A B$ produces the same number of lines of magnetic force in the coil, $s$ will be replaced by $2 \pi a$ in the value for $L$, the self-inductance of the circuit. Thus

$$
\begin{aligned}
L & =4 \pi a\left(\log \frac{2 a}{r}-0.62\right)+\pi a \mu \\
& =4 \pi a\left(\log \frac{8 a}{\gamma}-2.0\right)+\pi a \mu
\end{aligned}
$$

The second term is the internal magnetic linkages for the circuit. This result multiplied by $10^{-9}$ will give the self-inductance of the coil in henries, or by $10^{-3}$ in microhenries. The logarithm is to the base $e$.

## The Self-inductance of a Square Coil having One Turn.

The mean length of a side is $a \mathrm{cms}$. and $r$ is the radius of the wire. The strength of


Fig. 3.
the magnetic field at $B$ due to $A C$ (Fig. 3). is by formula (I) :

$$
\frac{\mathrm{I}}{b}(\sin C B D+\sin D B A) d x d b
$$

that is

$$
\left(\frac{a-x}{\sqrt{b^{2}+(a-x)^{2}}} d x+\frac{x}{\sqrt{b^{2}+x^{2}}} d x\right) \frac{d b}{b} .
$$

Therefore the number of lines of magnetic force threading the shaded area due to $A C$ is

$$
\frac{d b}{b} \int_{0}^{a}\left(\frac{a-x}{\sqrt{b^{2}+(a-x)^{2}}} d x+\frac{x}{\sqrt{b^{2}+x^{2}}} d x\right)
$$

which equals

$$
\frac{d b}{b}\left[-\sqrt{b^{2}+(a-x)^{2}}+\sqrt{b^{2}+x^{2}}\right]_{0}^{a}
$$

that is

$$
\frac{2 d b}{b}\left(\sqrt{b^{2}+a^{2}}-b\right)
$$

The total external lines threading the square due to $A C$ will then be

$$
\begin{equation*}
2 \int\left(\frac{\sqrt{a^{2}+b^{2}}}{b}-\mathbf{I}\right) d b \tag{2}
\end{equation*}
$$

Now the integration of $\frac{\sqrt{a^{2}+b^{2}}}{b} d b$ is

$$
a\left\{\frac{\sqrt{a^{2}+b^{2}}}{a}+\log \frac{b}{\sqrt{a^{2}+b^{2}}+a}\right\}
$$

so that (2) becomes
$2\left[\sqrt{a^{2}+b^{2}}+a \log \frac{b}{\sqrt{a^{2}+b^{2}}+a}-b\right]_{r}^{a} \cdots$
which equals

$$
\begin{aligned}
& 2 a\left(\sqrt{2}-2+\log \frac{2 a}{r(\sqrt{2}+1)}\right) \\
& =2 a\left\{\log \frac{2 a}{r}-[2-\sqrt{2}+\log (\sqrt{2}+\mathrm{I})]\right\} \\
& =2 a\left(\log \frac{2 a}{r}-1 .+7\right)=2 a\left(\log \frac{4 a}{r}-2.16\right)
\end{aligned}
$$

As each side produces the same number of lines of magnetic force in the coil,

$$
L=8 a\left(\log \frac{4 a}{\gamma}-2.16\right)+2 a \mu
$$

This multiplied by $10^{-3}$ will give the selfinductance of the square coil in microhenries.

## The Self-inductance of a Rectangular Coil having One Turn.

The mean lengths of the adjacent sides are taken as $A$ and $B$ cms., and the radius of the wire $r \mathrm{cms}$. This value of $L$ is readily found from the expression (3), using limits $B$ and $r$ and multiplying by 2 for the two like sides.

Thus

$$
\begin{array}{r}
4\left[\sqrt{A^{2}+B^{2}}+A \log \frac{B}{\sqrt{A^{2}+B^{2}}+A}-B\right. \\
\left.-A-A \log \frac{r}{2 A}\right]
\end{array}
$$

and for the other two sides

$$
\begin{array}{r}
4\left[\sqrt{A^{2}+B^{2}}+B \log \frac{A}{\sqrt{A^{2}+B^{2}+B}}-A\right. \\
\left.-B-B \log \frac{r}{2 B}\right] .
\end{array}
$$

This gives for $L$ the value

$$
\begin{aligned}
& 8 \sqrt{A^{2}+B^{2}}-8(A+B) \\
& \quad-4 A \log \left(\sqrt{A^{2}+B^{2}}+A\right) \\
& -4 B \log \left(\sqrt{A^{2}}+B^{2}+B\right) \\
& \quad+4(A+B) \log \frac{2 A B}{r}+(A+B) \mu .
\end{aligned}
$$

Multiply by Io $^{-3}$ to get $L$ in microhenries.
The Self-inductance of a Coil having the Form of an Equilateral Triangle with One Turn of Wire.
The mean length of a side is $a \mathrm{cms}$. and
the radius of the wire is $r \mathrm{cms}$. The strength of the magnetic field at $P$ due to side $A B$ (Fig. 4) is $\frac{\mathrm{I}}{b}\left(\sin \theta_{1}+\sin \theta_{2}\right)$ for a current of Io amperes.


Fig. 4
The number of lines of magnetic force threading the shaded element due to $A B$ will be therefore

$C D$ being equal to $\left(a-\frac{2 b}{\sqrt{ } 3}\right)$.
This equals

$$
\begin{aligned}
& \frac{d b}{b}\left[\sqrt{b^{2}+\left(z+\frac{b}{\sqrt{3}}\right)^{2}}\right. \\
&\left.-\sqrt{b^{2}+\left\{a-\left(z+\frac{b}{\sqrt{3}}\right)\right\}^{2}}\right]_{0}^{C D}
\end{aligned}
$$

that is

$$
\frac{2 d b}{b}\left(\sqrt{b^{2}+\left(a-\frac{b}{\sqrt{3}}\right)^{2}}-\frac{2}{\sqrt{ } 3} b\right)
$$

or,

$$
\begin{equation*}
\frac{4}{\sqrt{3}} \frac{d b}{b}\left(\sqrt{\left(b-\frac{\sqrt{3}}{4} a\right)^{2}+\left(\frac{3}{4} a\right)^{2}}-b\right) \ldots \tag{4}
\end{equation*}
$$

The integration of

$$
\sqrt{(b-\sqrt{ } 3 a)^{2}+\left(\frac{3}{4} a\right)^{2}} \frac{d b}{b}
$$

may be found as follows:
Let $y=b-\frac{\sqrt{3}}{4} a, A=\frac{3}{4} a$, and $k=\frac{\sqrt{3}}{4} a$.
This gives the form

$$
\begin{equation*}
\int \frac{\sqrt{y^{2}+A^{2}}}{y+k} d y \ldots \tag{5}
\end{equation*}
$$

Assuming that $\sqrt{y^{2}+A^{2}}=x-y$,

$$
\begin{equation*}
y^{2}+A^{2}=x^{2}-2 x y+y^{2} \tag{6}
\end{equation*}
$$

so that $\quad x^{2}-2 x y=A^{2} \quad$.
By differentiation,

$$
2 x d x-2 y d x-2 x d y=0
$$

Thus,

$$
d y=\frac{(x-y)}{x} d x
$$

Also from (6) $\quad y=\frac{x^{2}-A^{2}}{2 x}$
Therefore (5) becomes

$$
\begin{aligned}
& \text { I }
\end{aligned} \int_{x^{4}-x^{2} A^{2} x^{2}+A^{4}}^{\frac{x^{4}}{} x^{3}} d x
$$

which equals, by division,

$$
\frac{\mathrm{I}}{2} \int\left(\mathrm{I}+\frac{3 x^{2} A^{2}+A^{4}-2 k x^{3}}{x^{2}\left(x^{2}+2 k x-A^{2}\right)}\right) d x
$$

Resolving the second term by the method of partial fractions gives the result as $\frac{I}{2} \int\left\{I-\left(\frac{2 k x+A^{2}}{x^{2}}\right)+\frac{4\left(A^{2}+k^{2}\right)}{x^{2}+2 k x-A^{2}}\right\} d x$.

That is,

$$
\begin{gathered}
\frac{\mathrm{I}}{2} \int\left[\mathrm{I}-\frac{2 k}{x}-\frac{A^{2}}{x^{2}}+2 \sqrt{A^{2}+k^{2}} \times\right. \\
\left\{\begin{array}{r}
(x+k)-\sqrt{A^{2}+k^{2}}
\end{array}\right. \\
\left.\left.-\frac{\mathrm{I}}{(x+k)+\sqrt{A^{2}+k^{2}}}\right\}\right] d x
\end{gathered}
$$

which equals

$$
\begin{aligned}
\frac{I}{2} \int\left(1-\frac{\sqrt{3} a}{2 x}\right. & -\frac{9}{16} \frac{a^{2}}{x^{2}} \\
& \left.+\frac{\sqrt{3} a}{x-\frac{\sqrt{3}}{4} a}-\frac{\sqrt{3} a}{x+\frac{3 \sqrt{3}}{4} a}\right) d x
\end{aligned}
$$

Solving equation (6) gives

$$
\begin{aligned}
x & =y \pm \sqrt{A^{2}}+y^{2} \\
& =\left(b-\frac{\sqrt{3}}{4} a\right) \pm \sqrt{\frac{9}{\mathrm{I} 6} a^{2}+\left(b-\frac{\sqrt{3}}{4} a\right)^{2}}
\end{aligned}
$$

Now the limits of $b$ are $\frac{a \sqrt{3}}{2}$ and $r$, so that the corresponding limits of $x$ taking the root with the positive sign are $3 \sqrt{3} a$ and

$$
r-\frac{\sqrt{3}}{4} a+\sqrt{\frac{9}{16} a^{2}+\left(r-\frac{\sqrt{3}}{4} a\right)^{2}}
$$

or $r-\frac{\sqrt{3}}{4} a+\frac{\sqrt{3} a}{2}\left\{\mathrm{I}+\frac{4 r^{2}}{3 a^{2}}-\left.\frac{2}{\sqrt{3}} \frac{r}{a}\right|^{\frac{1}{2}}\right.$
As $r$ is taken very small compared with $a$, this second limit becomes $r+\frac{\sqrt{3} a}{4}-\frac{r}{2}$ very nearly.

For more accurate results, the whole value of the lower limit may be used.

It will be noted that the root with the positive sign has been taken for $x$. Either root will give the same result, but the former is the more convenient.

Thus the upper limit of $x$ is $\frac{3 \sqrt{3}}{4} a$ and the lower limit is $\frac{r}{2}+\frac{\sqrt{3}}{4} a$. Integrating the former result gives

$$
\begin{aligned}
& \frac{I}{2}\left[x-\frac{\sqrt{3}}{2} a \log x+\frac{9}{16} a^{2} x\right. \\
& \left.\quad+\sqrt{3} a \log \frac{x-\frac{\sqrt{3}}{4} a}{x+\frac{3 \sqrt{3}}{4} a}\right]_{\frac{\gamma}{2}+\frac{3^{-}}{4} a}^{4} a
\end{aligned}
$$

which is equal to

$$
\frac{\sqrt{3}}{2} a \log \frac{2 a}{3 r}
$$

Finally, the number of lines of magnetic force threading the triangle due to $A B$ will be from (4) equal to

$$
\frac{4}{\sqrt{3}}\left(\frac{\sqrt{3}}{2} a \log \frac{2 a}{3^{r}}-\frac{\sqrt{3}}{2} a\right)
$$

and the total number of external lines thread-
ing the triangle will be three times this, namely
or

$$
\begin{gathered}
6 a \log \frac{2 a}{3^{r}}-6 a \\
6 a\left(\log \frac{3^{a}}{r}-2.5\right)
\end{gathered}
$$

Therefore

$$
L=6 a\left(\log \frac{3 a}{r}-2.5\right)+1.5 a \mu
$$

This will be multiplied by $10^{-3}$ to give microhenries.

## The Self-inductance of a Plane Coil having One Turn and the Form of a Hexagon.

The mean side length of the hexagon is taken as $a$ cms. and the radius of the wire $r$ cms. On account of the limitations of space and the length of the solution, the steps of the latter are merely outlined

The number of lines of magnetic force in the elemental area $P Q$ (Fig. 5) due to the current in $A B$ was first obtained, and then integrated between the limits $\frac{a \sqrt{3}}{2}$


Fig. 5
and $r$. Those in the elemental area $R S$ due to $A B$ were then obtained and integrated between the limits $a \sqrt{3}$ and $\frac{a \sqrt{3}}{2}$.

The result was obtained finally by adding these two integrations, multiplied by 6 for the six sides of the hexagon, and the equivalent internal system of lines of magnetic force, namely, $3 a \mu$.

That is, the solution was of the form
$\frac{24}{\sqrt{3}}\left[\int_{r}^{a \sqrt{3} / 2} A d b+\int_{a \sqrt{3} / 2}^{a \sqrt{ } 3}(B-C) d b_{1}\right]+3 a \mu$, in which the integrations involving $A, B$, and $C$ are of the same form as expression (5) and solved in the same way. By this method the following formula was obtained.

$$
L=12 a\left(\log \frac{6 a}{\gamma}-\mathrm{I} .95\right)+3 a \mu
$$

This result, divided by $\mathrm{IO}^{-3}$ gives $L$ in microhenries.

A summary of the preceding results is as follows:

Circle mean radius a cms.

$$
L=4 \pi a\left(\log \frac{8 a}{r}-2.0\right)+\pi a \mu
$$

Square mean side length $a$ cms.

$$
L=8 a\left(\log \frac{4 a}{r}-2.16\right)+2 a \mu .
$$

Rectangle mean side lengths $a$ and $b \mathrm{cms}$.

$$
\begin{aligned}
L=8 & (P-Q)-4\{a \log (P+a) \\
& +b \log (P+b)\}+4 Q\left(\log \frac{2 a b}{r}+\begin{array}{l}
\mu \\
4
\end{array}\right)
\end{aligned}
$$

in which $P=\sqrt{a^{2}+b^{2}}$ and $Q=a+b$.
Equilateral triangle mean side length $a$ cms.

$$
L=6 a\left(\log \frac{3 a}{r}-2.5\right)+1.5 a \mu
$$

Hexagon mean side length $a$ cms.

$$
L=\mathrm{I} 2 a\left(\log \frac{6 a}{r}-\mathrm{I} .95\right)+3 a \mu
$$

In these formulæ $r$, the radius (cms.) of the wire, is regarded as very small compared with the perimeter of the circuit; $\mu$ is the magnetic permeability of the material of the wire and is assumed constant ; the logarithms are to base $e$; and the value of $L$ given has to be multiplied by $\mathrm{IO}^{-3}$ to give the result in microhenries.

# The Harmonic Comparison of Radio-Frequencies by the Cathode-Ray Oscillograph. 

By T. S. Rangachari, M.A. (Indian Institute of Science, Bangalore, India).

ONE of the many uses of the Cathode-Ray Oscillograph in a high-frequency laboratory is the harmonic comparison of radio-frequencies. For this purpose there are a number of known methods.

## Lissajous' Figure Method.

For instance, at the Bureau of Standards a radio-frequency wavemeter was calibrated in terms of a standard tuning fork by producing Lissajous' figures, from which the frequency ratios were determined (Hazon and Kenyon, Scientific Papers of the Bureau of Standards, No. 489). The chief limitation of this method is that ratios greater than about 15: I introduce such complicated patterns that to recognise the ratio between the two frequencies which are compared is difficult, if not impossible.

## Method of the Western Electric Co.

The Western Electric Company developed a slightly different method (Kipping "Electrical Communications," July, 1924). The principle of this method is briefly as follows: The motion of a particle described by $x=a \cos n t, y=b \cos (n t+\epsilon)$ is an ellipse. Now if $a=b$ and, further, if $\epsilon=\frac{\pi}{2}$, the ellipse becomes a circle. Now suppose $x=a \cos n t+b \cos p t$ and $y=a \sin n t$ and that $p / n$ is integral, then it is readily seen that the path is a circle with a serrated edge. If the cathode-ray beam is subjected to forces as defined above, a pattern with serrated edges is produced. By counting the number of edges the frequency ratio is determined.

## Rotating Ray Methods.

A valuable contribution was made by D. W. Dye, of the National Physical Laboratory, by developing his rotating ray method (Dye, Proc. Phy. Soc., 1925, 37, 158). This method possesses great advantages and
is particularly suitable for the purpose of setting a low radio-frequency to an exact value for purposes of calibration. Briefly, the rotating ray method consists in producing first a circular or elliptical time trace by means of two-phase voltage of low frequency. The radio-frequency is then caused to operate on the ray in one of three different ways. These are as follows :-
(I) A small voltage (about 20-30) at the radio-frequency is introduced into the steady anode voltage supply of the cathode-ray tube. The ray thus receives a radial vibratory displacement due to the radio-frequency at whatever position it may be in its circular or elliptical path at the lower frequency. This is due to the variation in velocity of the electrons forming the ray consequent upon the variation of the anode voltage.
(2) A small circular movement is given to the ray at the radio-frequency, whilst the larger circular or elliptical movement is provided by the lower frequency.
(3) A vibratory movement in a fixed direction is given by the radio-frequency, whilst the large circular or elliptical movement takes place at the audiofrequency.
Of these three methods the first one is not convenient in practice. One of the disadvantages of this method of superposition is that a rather high voltage at the radio-frequency is necessary to produce the requisite radial deflection, and a loss of definition occurs when the voltage reaches its minimum value. The radial movement permissible is also very limited.

The second method is the most convenient of the three in practice. The advantage of this method is the much smaller highfrequency voltage necessary as compared with the first method, and, further, the formation of a looped pattern renders counting easier. However, some disadvantages of this method will be referred to below.

The third method is suited more for the examination of waveform than for the comparison of frequencies.

Since the present article is concerned with


Fig. I. Cathode-ray tube arrangement.
the second method, the arrangement by which it is carried out is shown in Fig. I. Here the low-frequency circular or elliptical motion of the cathode ray is produced by applying the low-frequency P.D's in quadrature at $a b$ and $b c$. The high-frequency rotation of the cathode ray is produced by means of field coils $F_{1}, F_{2}$. These produce a magnetic deflection in a plane which can be rotated by turning the coils round the axis of the tube. The quadrature deflection at the high-frequency is produced by introducing the high-frequency voltage at $e$ in the lead marked $c$. When the two frequencies are in simple ratio a looped pattern is produced, the loops being either inside or outside the low-frequency circular motion according as the two circular motions of the cathode ray are in the same or in opposite directions.

Two disadvantages are encountered when this method is put into practice. One is that it requires a special mounting of the oscillograph with the coils $F_{1}, F_{2}$ arranged so that they can be rotated around the axis of the cathode-ray tube for phase adjustments. The other is that, unlike in pure electrostatic
deflection, considerable power is needed to produce sufficient magnetic deflection.

It appears, therefore, that a method of superimposing both the circular motions on the same two pairs of plates of the oscillograph without at the same time sacrificing convenience of independent adjustment of the amplitudes of the two circular motions is desirable. A number of arrangements were tried with this object in view. The circuit represented in Fig. 2 was found to be satisfactory. The oscillograms shown in Figs. 3 and 4 were produced with the help of this circuit, the ratio of the two frequencies compared being $I: 6$. The absolute values of the two frequencies were of the order of 3,000 cycles per second and 18,000 cycles per second. This circuit was found convenient in practice and allowed independent adjustment of amplitudes such that the high-frequency loops could be reduced to bright spots as was done by Dye (loc. cit.).

The action of the circuit may be explained as follows: Circuit $M N O Q$, containing a condenser $C_{1}$ and resistance $R_{1}$ in series, forms a phase-splitting device for the low frequency. Similarly, circuit $O P S Q$, containing the condenser $C_{2}$ and resistance $R_{2}$ in series, forms a phase-splitting device for the high frequency. The point $N$ is connected to the common plates of the oscillograph.


Fig 2. Cathode-ray oscillograph plutes.
Since there is no coupling between the two circuits $M N O Q$ and $O P S Q$ because of the short-circuit $O Q$, the low-frequency current may be considered to circulate in the former circuit and the high-frequency current in the latter circuit.
where $\quad \theta=-\frac{\omega D}{c}=-\frac{2 \pi D}{\lambda}$,
$\lambda$ being the wavelength, while $D=a^{\prime}-a$ and $t^{\prime}=t-a / c$.

If this E.M.F. is amplified linearly and rectified by a detector having a "square law " characteristic, the mean signal current

$$
\begin{equation*}
\bar{\imath}=A\left(H_{0}^{2}+{ }_{4} H_{1}^{2}+{ }_{4} H_{0} H_{1} \cos \theta\right) \ldots \tag{4}
\end{equation*}
$$

The fact of night variations in such a loop indicates that $H_{1}$ or $\cos \theta$ or both must be changing, assuming $H_{0}$ to be approximately constant. The ratio of day and night signals may thus be written

$$
\begin{equation*}
\frac{\bar{i}_{N}}{\bar{i}_{D}}=\mathrm{I}+4\left(\frac{H_{1}}{H_{0}}\right)^{2}+{ }^{4} H_{1} \cos \theta \ldots \tag{5}
\end{equation*}
$$

The effect of changing the wavelength of the transmitter is then considered. It is shown that for slow diminution of the

wavelength the signal current $i_{N}$ passes through a maximum and a minimum value. The number, $n$, of signal maxima (or minima) encountered in changing the wavelength from $\lambda_{1}$ to $\lambda_{2}$ is

$$
n=\frac{\theta_{1}-\theta_{2}}{2 \pi}
$$

where $\theta_{1}=2 \pi D / \lambda_{1}$ and $\theta_{2}=2 \pi D / \lambda_{2}$,
so that $\quad n=\frac{D}{\lambda_{1}}-\frac{D}{\lambda_{2}}$
A signal maximum $M$, artificially produced by change of wavelength, indicates that $\theta$ is $0,2 \pi, 4 \pi$, etc. ; a minimum $m$ indicates that it is $\pi, 3 \pi, 5 \pi$, etc.

Thus from (5)

$$
\begin{equation*}
\frac{2 H_{1}}{H_{0}}=\frac{\sqrt{ }(M / m)-\mathrm{I}}{\sqrt{ }(M / m)+\mathrm{I}} \tag{7}
\end{equation*}
$$

Almost similar considerations apply to the determination of $\phi_{1}$, the angle of incidence. Study of signal variations on an aerial set and on a loop set simultaneously permit the determination of $\bar{i}_{N} / \bar{i}_{D}$ for both receivers.

If signal maxima and minima due to wavelength change, are recorded either simultancously or in rapid succession on both loop and aerial sets, by using only measurements at maximum and minimum values, we get (7) for the loop, and for the aerial

$$
\begin{equation*}
\frac{2 E_{1} \sin \phi_{i}}{E_{0}}=\left[\frac{\sqrt{ }(M / m-I)}{\sqrt{ }(M / m+I)}\right]_{\text {Aeriai }} \tag{9}
\end{equation*}
$$

Hence, since $E_{1} / E_{0}$ is equal to $H_{1} / H_{0}$, we get

$$
\begin{equation*}
\sin \phi=\frac{\left[\frac{\sqrt{ }(M / m-\mathrm{I}}{\sqrt{ }(M / m)+\mathrm{I}}\right]_{\text {Aerial }}}{\left[\frac{\sqrt{ }(M / m)-\mathrm{I}}{\sqrt{ }(M / m)+\mathrm{I})}\right]_{\text {Loop }}} \tag{го}
\end{equation*}
$$

In this way $\phi_{1}$ may be found.
By using the method of wavelength change, attention may be confined to maximum and minimum values, permitting solutions not obtainable by simple intensity measurements. The method does not require a knowledge of the strength of the ground ray, and is a sensitive test of the presence of downcoming waves, especially when the latter are weak.

Complete records of variations in the downcoming waves could be obtained by record of the interference maxima and minima on various loop and aerial systems, but the apparatus would become somewhat complicated. It is thus desirable to study the temporal variations of the intensities of the two components of the downcoming wave.

To receive signals due to the normally polarised component $E_{1}$, we may use a loop and aerial combination possessing a polar reception diagram of cardioid form. In this case, if the assembly is adjusted to cut out the ground ray during the daytime, only the downcoming wave is received at night and its intensity variations may be studied directly. To receive signals due to the abnormally polarised component $H^{\prime}{ }_{3}$, a vertical loop at right angles to the plane of propagation is used.

## Experimental Details.

The essentials of the receiving apparatus are shown in Fig. 4, the apparatus consisting of a high frequency amplifier followed by a simple rectifier and galvanometer. For wavelength-change experiments, resistance capacity amplifiers with low capacity valves
have been used. For observations on constant wavelength, neutrodyne amplifiers have been found satisfactory. For the rectifier a crystal has been used, and, more recently, the valve voltmeter arrangement shown in Fig. 4, which is a Wheatstone bridge assembly, balanced when no signal is being received.

For recording natural signal variations due to a constant-wavelength transmission, a


Fig. 4.
Pye galvanometer with a 5 -second period has been used, together with a Cambridge drum camera. For recording signals received during a wavelength change, which is usually made to take place in about 5 seconds, a Cambridge Einthoven galvanometer has been employed.

## Some Experimental Results and Conclusions.

Study of downcoming waves by the methods described has led to the following general conclusions.
(I) Fading is due to the effects of variable rays deviated by the upper atmosphere. The fact of interference maxima and minima being obtained when the wavelength is changed indicates the presence of two waves, one of which must be the ground wave. The fact that interference maxima are greater when receiving on a loop aerial indicates that one set of waves reaches the ground at an angle of incidence less than $\frac{1}{2} \pi$, i.e., comes down from above.
(2) The equivalent height of the ionised layer may be determined by a simple calculation from the value of $D$, the path difference between ground and atmospheric waves, or from $\phi_{1}$, the angle of incidence. Measurements made during the dark hours show that for most nights, after sunset, the height graclually increases, reaching its maximum value about an hour before sunrise, when a somewhat rapid change takes place to the lower daytime value. During a normal night the height may vary from 90 to 130
km., but, on occasions, in winter, heights of an entirely different magnitude, such as 250 to 350 km ., have been measured during the three hours before dawn.
(3) Variations may occur due to changes in (a) angle of incidence, (b) intensity, (c) phase, (d) polarisation of the downcoming waves. By simultaneous study of the signals on a vertical aerial and on the suppressed ground ray system, it is possible to show that changes in intensity of the downcoming waves ("intensity fading ") are more frequent than changes in the phase difference between ground and downcoming rays (" phase fading"). It is also concluded that rotation of the plane of polarisation is not responsible in any marked degree for signal fading on these wavelengths, and that on the whole intensity variations and, to a lesser degree, phase variations are the chief causes of signal fading.
(4) From experiments on the polarisation of the downcoming waves, it is concluded that it may be described as approximately circularly polarised with a left-hand rotation. This is possibly due to the influence of the earth's magnetic field, but a critical test would be to repeat the experiments in the Southern Hemisphere, when right-hand rotation would be looked for.

Observations on the Occasion of the Solar Eclipse, 29th Junle, 1927.
The observations made on the occasion of last year's eclipse are quoted as typical of the kind of records obtained. These were made on special transmissions arranged by the B.B.C. :
(i) Transmissions of an unmodulated carrier wave, the wavelength of which could be changed continuously through a small range with but small variation of amplitude. Such transmissions took place from the Newcastle and Birmingham B.B.C. stations, and were used for the special measurements of the type described above.
(ii) Transmissions of a carrier wave of constant amplitude, unmodulated except for the announcement of time signals. The signal variations at various distances were studied by observers using galvanometric methods. Such transmissions took place from the London and Manchester B.B.C. stations.

As the region of totality in the ionised layer differed from that on the ground, observations in the region of ground totality were made at Liverpool on the Newcastle transmissions, and in the region of layer totality at Peterborough on the Birmingham transmission.

Observations on the transmissions of type (ii) were made at St. Albans, Nottingham, Peterborough, Newcastle-on-Tyne, Giggleswick and Aberdeen.

## Experimental Results.

(a) Observations at Peterborough on the Birmingham Transmissions.
Using the wavelength-change method the following quantities were determined:
(I) $H_{1} / H_{0}$, deduced from the amplitude of the "fringes" recorded when the loop acrial was used.
(2) Path difference $D$.
(3) Angle of incidence $\phi_{1}$ of downcoming waves, deduced from the relative amplitudes of the "fringes" recorded using loop and vertical aerial in rapid succession.

Check observations were made on the days before and after the eclipse, and as the eclipse was so soon after sunrise, observations began each day at 2 a.m. G.M.T. so as to cover the period of normal sunrise effects.

Fig. 6, for the morning of the eclipse, shows he variation of intensity of the downcoming


Fig. 6.
ray as a fraction of the ground ray intensity. Normal daytime conditions had been reached before the effects of the eclipse were noticeable. At the time of the large increase due to the eclipse, the observations show that the equivalent path difference between ground and atmospheric waves had increased, while the angle of incidence of the downcoming waves at the ground had been reduced. The first of these effects is illustrated in Fig. 7, in which $\delta n / \delta \lambda$, the number of fringes per metre wavelength change, a
quantity which is proportional to the path difference ( $D$ ) between ground and atmospheric rays, is plotted as a function of the time. The increase in the value of $\delta n / \delta \lambda$ caused by the eclipse is equal to an increase in the equivalent height of the layer from


Fig. 7.
75 to $9+\mathrm{km}$. The decrease in the angle of incidence at which the downcoming waves reach the ground also indicates an increase in the height of the stratum responsible for deviating the waves. We therefore conclude that the increased intensity of the downcoming ray was due to the removal of ionisation in the lower layers of the atmosphere, together with an increase in the height at which the waves were turned back. The large " fringes" recorded during the period of the eclipse effect were, however, smooth and therefore similar to those recorded about 3.45 a.m. rather than to those representative of night-time conditions, when "secondaries" are almost always present. We must therefore regard the effect of the eclipse as only a partial return to proper night-time conditions.

Examples of the signal maxima and minima obtained before, during, and after the eclipse effect are shown in Fig. 8. The records show the marked increase in the intensity of the downcoming ray at totality.

## (b) Observations at Liverpool on the Newcastle

 Transmissions.During the middle of the eclipse period these observations were rendered quite impossible by interference from a neighbouring amateur transmitting station (6NI), so that no observations could be made during the critical period of $4.50 \mathrm{a} . \mathrm{m}$. to $5.25 \mathrm{a} . \mathrm{m}$. on 29th June. The readings that were obtained at the beginning and end of the eclipse period entirely confirm the Peter-
borough results, but the interference prevented the comparison of the eclipse effects in the regions of layer totality and ground totality respectively.
(c) Observations on the constant wavelength transmissions from London and Manchester. As already stated, these observations were made at St. Albans, Nottingham, Peterborough, Newcastle, Giggleswick and Aberdeen. Readings of rectified signal current were taken every io seconds throughout each alternate io minutes during the period of the transmissions, except during the eclipse when readings were taken every 5 seconds continuously. The results ob-


Fig. 8.-Signal variations received alternately on loop and aevial, beginning on loop.
tained confirm the variations in downcoming ray intensity noted at Peterborough.

Two particular cases most simply illustrate the main effect.

At Nottingham a strong ground ray was received and the downward ray was evidenced by small variations about a constant mean value. Fig. 9 shows the mean departure of signal from daytime value, which gives an inferior limit for the downcoming ray intensity relative to that of the ground ray, for the eclipse morning and for the same period on ist July.

It is to be noted that, as in the case of the Peterborough measurements, the greatest amplitude of downcoming ray was detected before totality.


Fig. 9.
In the case of observations at Giggleswick on 2 LO , the reception was almost solely on indirect ray or rays, so that signal-current readings may be taken as indicating the strength of downcoming waves. These are shown in Fig. Io. A similar increase of signal intensity was reported at the more distant station of Aberdeen, where the normal daytime signal is negligible.

At Giggleswick the maximum of downcoming ray was just about the time of layer totality, while at Aberdeen it was at 5.29, that is 7 or 8 minutes afterwards. Another observer at a distant station in Pembroke-


Fig. ${ }^{10}$.
shire found maximum signal intensity at 5.27 a.m.

Discussion of Eclipse Results.
The observations show that the eclipse produced a very definite effect on the properties of the layer responsible for
deflecting waves of $300-400 \mathrm{~m}$. back to the earth. The most striking feature was the large increase in intensity of the downcoming ray at both near and at distant stations. This is ascribed partly to increase in height of the stratum responsible and partly to the rapid removal of ionisation in the lower layer of the atmosphere consequent on the removal of the solar ionising agents. The results suggest that the more southerly stations experienced the maximum influence a little earlier than the northern stations.

A striking feature was the short time that the eclipse effect lasted, the period varying from 20 to 50 minutes at different stations, while the total time taken for the moon's shadow to pass across the sun was nearly 2 hours. This means that a large fraction of the sun's radiation may be cut off before the effect can be detected by wireless methods.

## DISCUSSION.

In opening the discussion which followed the paper, Mr. T.L. Eckersley congratulated the author on the very interesting work which represented the maximum return for the energy expended. He would be interested to have a comparison of layer height, as determined by fringes and as determined by the angle of incidence. The methods suggested one downcoming ray, but observations showed that the downcoming ray was very composite, and he thought that this point should be cleared. Fading was intrinsic to the downward ray as was confirmed by short-wave work. Comparison of fading of short waves on horizontal and on vertical acrials were suggestive of rotation of the electric vector. On short waves there was also more trace of right-hand rotation in the polarisation than of the left-hand rotation suggested by the author. Observations had actually been made in which this varied in certain cases with the season. Could the author from his measurements give a figure for the height of the layer in daytime ?

Prof. G. W. O. Howe expressed great appreciation of the author's work and of his energy and enthusiasm. The paper pointed to difficulties still remaining. There was nothing to indicate the causes of the phenomena. What was happening to cause changes in the intensity of the downcoming wave? The very high rise at the eclipse time did not seem as if the night effect was only partially restored.

Dr. E. H. Rayner said the paper marked the beginning of radio measurements applied to upper air measurements, and emphasised the importance of the eclipse observations in these problems. He snowed a model of the eclipse effect, illustrating the distribution of totality, and outlined the work of
the Committee which had been dealing with the arrangements for the radio measurements. Such observations should form part of future eclipse observations.
Capt. P. P. Eckersley said that fading limited the service area of broadcasting stations, and the B.B.C. gave 100 miles as a figure beyond which fading might be expected. Hilly country appeared to have some effect on this, as at Carnarvon, for example, 5 GB fader badly, but in Norfolk at 145 miles from ${ }_{5} \mathrm{~GB}$ he had found no trace of fading. Was it possible to get an aerial to receive the direct ray only eliminating the indirect ray ? He was interested in the study of telephony over long distances where only the indirect ray was available. Did the author consider that there was differential fading as between side bands and carrier ? Such fading had been noted in work on two broadcast transmitters working on the same wavelength.

Mr. J. Hollingworth referred to the simplicity of the apparatus used. As the effects were different on different wavelengths, the author should specify that this paper was limited to certain wavelengths. With reference to the direction of the rotation of polarisation, he used both left hand and right hand, and found that these sometimes changed during the day. The disadvantage of the wavelength change method was the large ether band which it occupied, especially on longer waves, since the percentage change varied with wavelength. He hoped shortly, by co-operation with the Post Office, to observe on a io per cent. change on 8,000 metres.

Mr. R. H. Barfield said he had been working on parallel lines, and had got confirmation of the author's results by independent methods. Three methods of measuring the angle of incidence had been used: (a) the forward tilt, measured by a rotating Hertzian rod, (b) the tilt of the magnetic field measured by a rotating coil which could also be rotated about a horizontal axis, (c) a method of comparing the magnetic and electric fields. All angles gave heights approximately in agreement with the author.

Col. H. P. T. Lefroy suggested the distance of two layers, one higher and going round with the earth, and the lower and more dense layer, being a function of sunlight, following like the tail of a comet.

Mr. G. H. Munro referred to direction-finding measurements (at the time of the eclipse) at Giggleswick, Ditton Park and Bristol. The results confirmed the author's experiments, directions becoming very unstable with the eclipse. Intensity measurements also gave results similar to those of the author.

Prof. Appleton replied to several of the points raised in the discussion, especially with reference to the remarks of Mr. T. L. Eckersley.

On the motion of the Chairman (Lt.-Col. A. G. Lee, O.B.E., M.C.) the author was cordially thanked for his paper.

# A Short Survey of Some Methods of Radio Signal Measurement. 

By K. Sreenivasan, B.Sc.<br>(Concluded from page 210 of April issue.)


#### Abstract

4. Baümler's Method.*_At the Telegraphentechnische Reichsamt in Berlin, Baümler developed apparatus for measuring the field strength of Transatlantic stations like Marion, Tuckerton, etc. The observations recorded are those of a complete year with hour to hour readings throughout the twenty-four hours of the day. These observations form perhaps the most continuous and extended investigation on the variation of field strength of long-distance radio stations. Baümler's conclusions acquire additional weight since he employs an objective method free from personal equation as with the aural method.


heterodyne circuit, and an auxiliary transmitter with a measuring circuit. The amplifiers are all of the transformer coupled type. In the primary winding of the transformer in the last stage of the low-frequency amplifier, a telephone is inserted for detecting the station to be measured. In the secondary winding of the same transformer is a single string electrometer which is used for measurement. The thread of the electrometer follows accurately the code signals of the station under observation.

Method for Obtaining Small H.F. Voltages. --The auxiliary transmitter consists of a


Fig. 5.-Receiving arrangement employed in Germany by Baümler, 1923-1924, for reception of Tuckerton

Method of Measurement (Fig. 5).-For measurement, an open antenna and a loop aerial are used. The receiving apparatus, in addition to the receiving aerials, consists of a small coupling coil, a secondary circuit, a two-stage high-frequency amplifier, a two to four-stage low-frequency amplifier, a

[^5]triode oscillator and two aperiodic or untuned coils, $P$ and $Q$, coupled to the measuring circuit over a calibrated mutual inductance $M$. By this means small known H.F. voltages are introduced into the receiving system for purposes of comparison with the incoming signal. The calibrated mutual inductance can be replaced by a potentiometer arrangement across the secondary of a small radio frequency current transformer. Arrangements with either of these two methods permit the measurement of antenna currents up to $10-7$ ampere.

Undesirable incluctive couplings are avoided by winding all H.F. coils as differential coils ; stray capacitative couplings are kept down by suitably arranging the circuits. The residual inductive and capacitative couplings are
practically all the high power long-wave stations in the world.

For measurement, both an open antenna $A$ and a coil aerial $B$ are used (Fig. 6). $C$ is a filter circuit for minimising troublesome


Fig. 6.-Circuit of Dr. L. W. Austin, used at Washington for long-wave reception.
avoided by placing the individual elements of the apparatus in separate boxes lined with copper plate. The shielding being as thorough as possible, measurements can be carried on at any wavelength without any lurking doubt about stray electromagnetic disturbances.

The recciving arrangements are adjusted to give deflections on the electrometer directly proportional to the aerial current due to the incoming signal. Then coil $Q$ is inserted in the auxiliary transmitter and the couplings in it are adjusted till the electrometer is brought to the same deflection as before, the receiving arrangement remaining unchanged throughout. The arrangement, besides being objective, is remarkably independent of atmospheric disturbances. The signal to be measured is easily distinguished from these disturbances.
5. Austin's Mcthod.*-The method used by Dr. I. W. Austin some time ago and the results obtained by him are of great interest.

The apparatus used by Dr. Austin has some notable departures from the more usual practice. With this set, he has measured

[^6]interferences. $D$ is the detector, and $E$ a three-stage radio frequency amplifier, adjusted to prevent self-oscillation. $F$ is a heterodyne oscillator to produce any desired beat note in the telephones.

Calibration of the receiving set is effected by the triode generator $G$. The current in the oscillating circuit, measured in the usual way by a thermojunction and galvanometer, passes through an attenuation box $I$ and a one-ohm resistance $S$ across the loop aerial. Thus, an accurately known, easily adjustable small radio frequency E.M.F. is injected into the receiving aerial-exactly the same arrangement used by Messrs. Englund, Bown, and Friis.

The peculiarity of the method consists (a) in making all measurements at the same pitch of sound in the telephones, and (b) in the arrangement to measure the telephone current. The telephone current measuring device (Fig. 7) consists of a tuning fork generator of 1,000 cycles, which is measured by a thermo-galvanometer and then passed through a potential divider arrangement. By this means any known 1,000 cycle E.M.F. can be put on the telephones.

Whatever be the frequency of the incoming signal, the heterodyne $F$ is always adjusted to give a beat frequency equal to that of the telephone comparator. Intensity of the telephone note due to the comparator is made equal to that due to the signal by
adjustment of the potential divider and by switching the telephones rapidly from the receiver to the telephone comparator.

With this arrangement, Dr. Austin has covered very long distances.* From San Diego in California, where Austin conducted his experiments, Cavite in the Philippine Islands is $11,000 \mathrm{~km}$., with a time difference of 8 hours, while the high power station Malabar in Java is $14,700 \mathrm{~km}$., with a time difference of 9 hours. Malabar has at San Diego a field strength of $4.02 \mu \mathrm{v} . / \mathrm{m}$., and Cavite a field strength of $2.04 \mu \mathrm{v} . / \mathrm{m}$. The corresponding calculated values on the basis of the Austin-Cohen attenuation factor are $1.83 \mu \mathrm{v} . / \mathrm{m}$. and $0.69 \mu \mathrm{v} . / \mathrm{m}$.


Fig. 7.-Telephone current measuring device employed by Dr. L. W. Austin.

These values were about the lowest measured up to that time.

The Marconi Company Method. $\dagger$--During

[^7]1922-23, Messrs. Round, T. L. Eckersley Tremellen and Lunnon, of the Marconi research staff, successfully carried out a most ambitiously planned expedition to study the complicated phenomena attending electro-magnetic wave propagation over long distances and to secure data necessary to establish communication between any two given places on the earth. The expedition, starting from England, went to North America, througl the Panama Canal to New Zealand, thence along the South and the West coasts of Australia to Ceylon; and thence through the Suez Canal and the Mediterranean round Spain back to England. The conclusions arrived at from the huge mass of data collected during this big expedition throw a flood of light on the varied and complex factors affecting radio wave propagation.

Apparatus and Method of Measurement.* The receiving aerial is tuned to the incoming signal frequency; coupled to the aerial circuit is a tuned circuit followed by an amplifier and a separate heterodyne. The signals from the latter are introduced in the last stage of the amplifier. To prevent stray inductive and capacitative E.M.F.s, and also to avoid direct pick up of the signal by the amplifier, the whole apparatus is well screened by being enclosed in a metal box.

To determine the field strength, i.e., primarily, to measure the E.M.F. induced in the aerial by the signal, an auxiliary transmitter and a dummy aerial are employed. The electrical constants of the dummy or non-radiating aerial are made as nearly the same as those of the actual receiving aerial.

The auxiliary transmitter is in three mutually screened compartments of a well shielded box. In the first compartment is a triode oscillator coupled to an intermediate circuit, the current in which is measured by the potentiometer slide-back method due to Captain Round. $\dagger$ The intermediate circuit is in the second compartment.

In the third compartment are two sets of mutual inductances which are calibrated over a reasonably large range of frequencies.

[^8]The mutual capacity coupling between the various coils was found to be negligible.

It is through this mutual inductance that the intermediate circuit of the auxiliary transmitter is coupled to the common part of the receiving and dummy aerial circuits. Knowing the value of $M$ used, the frequency of the signal and the current in the intermediate circuit, the voltage induced in the dummy aerial is known. By proper adjustment, the artificial signal is made equal to the incoming signal with a pair of telephones ; then from the known constants of the nonradiating aerial, the field strength is calculated

There are a few points of interest in this apparatus :-
I. The method being subjective on account of the use of telephones to ascertain equality of the local and incoming signals, errors due to the observer and to the inherent defects of the telephone are inevitable.
2. As the authors have pointed out, the artificial signals in the dummy aerial are not in the same condition as the incoming signals on the receiving aerial as regards jamming and atmospheric disturbances.
3. Under the best conclitions, it is difficult to secure identity, or at least even approximate equality, between the constants of the two aerials. For the above two reasons, the authors have whenever possible dispensed with the dummy aerial.


Fig. 8.-General scheme of comnections in the N.P.L method, 1925.
4. Doubts have not infrequently been expressed with regard to precision in the measurement of radio frequency voltages by the slide-back method.* Itlappears as

[^9]though considerable experience and practice are necessary to secure a fair degree of accuracy by this method.
5. To avoid direct pick up by the amplifier and to prevent stray inductive and capacitative E.M.F.s from getting into the receiving apparatus, efficient and elaborate screening arrangements are necessary.


Fig. 9.-Aerial and tuming circuits of the N.P.I. method.

The N.P.I. Method.*-About the year 1922, Mr. J. Hollingworth set up at the National Physical Laboratory measuring apparatus primarily to determine at Teddington the field strength of the U.R.S.I. signals. The arrangement has certain features distinguishing it from the others.

The aerial consists of a loop of 8o turns on a 5 ft . square frame with a three-way switch, by which 30,50 or 80 turns can be brought into the circuit. Although the E.M.F. induced in a coil aerial of this kind is smaller than in an open aerial, the constants of the coil can be determined more accurately and they remain far steadier. There is the question of portability also, in cases where the apparatus has to be shifted from place to place. Fig. 9 shows the switching arrangements and the tuning condensers and series resistance, while Fig. 8 gives the general scheme of connections.

The incoming signal from the station under question is tuned in by the variable air condenser across the coil ; the voltage across the condenser is then applied across the grid and filament of the first triode of a resistance capacity coupled amplifier

[^10](see Fig. 10). There are no low-frequency stages, and for purposes of listening in a separate heterodyne is used. For wavelengths greater than 2.5 km . this type of amplifier has several advantages:-
(a) Larger effective wavelength range as compared with the transformer type.
(b) The absence of any necessity to screen the amplifier since there are no coils to be affected inductively by externa! fields.
(c) Stability of working over a large range of frequencies.
It is essential that the amplifier should be adjusted to work well away from regeneration, though this may involve a slight lowering of the amplification factor.

In the anode circuit of the last stage of the amplifier there is a sensitive galvanometer; a micro-ammeter in the case of special transmission or a string galvanometer with routine transmission. The normal anode current is balanced by an accumulator and potentiometer, so that ordinarily the galvanometer reads zero. When the signals come in, the grid of the last tube gets polarised, causing a diminution


Fig. Io.--Circuit of the high-frequency amplifict used in N.P.L. measurements, 1925.
in the anode current. This is indicated as a deflection in the galvanometer. If routine transmission is received on a string galvanometer, the deflections will be in accordance with the dots and dashes composing the signal.

To determine primarily the E.M.F. induced in the coil, and from that deduce the field intensity, the coil is switched off from the amplifier and an adjustable known E.M.F. from the secondary of a calibrated mutual inductance belonging to a local oscillator is put on. By impressing varying voltages, the respective galvanometer deflections give the calibration curve of the amplifier. From this curve, the voltage corresponding to the signal is read off.

The local oscillator has several interesting features. Ordinarily a few turns of the main oscillating inductance form the primary, well screened from the rest of the oscillating inductance. In this apparatus, the whole of the oscillating inductance forms the primary of the mutual inductance and screening is avoided. Assumptions of any sort and suspicions regarding stray capacity coupling are entirely avoided by determining the value of the mutual inductance under normal working conditions. The mutual inductance has an untuned or aperiodic secondary, thus requiring the measurement of the high-frequency resistance of the receiving circuit for every measurement This is effected as follows :-

A multi-way switch fitted in the coil circuit introduces into the oscillating circuit a series of known resistances. Deflections on the galvanometer due to a local or incoming signal are noted for all the positions of the switch. The corresponding voltages are then measured by the local calibrating circuit and the effective resistance calculated in the usual way. The measurement of high-frequency resistance, although a com plication, acts as an effective check on the observations and avoids assumptions of uncertain accuracy. From this point it view, it is really an advantage.
L.et $D=$ field strength of the incoming signal.
$n=$ number of turns in the coil.
$a=$ area of a single turn.
$L=$ inductance of receiving aerial.
$R=$ effective resistance of receiving circuit.
E.M.F. induced in the coil due to the in-

$$
\text { coming signal }=\frac{2 \pi a n}{\lambda} D .
$$

Current in the coil $=\frac{2 \pi a n D}{R \lambda}$.

Voltage across the tuning condenser at resonance $=\frac{2 \pi a n D}{R \lambda} \sqrt{R^{2}+\omega^{2} L^{2}}$

$$
\bumpeq \frac{2 \pi a n D \omega L}{R \lambda} .
$$

If the calibrated oscillator is adjusted to give the same deflection as the incoming signal, then resonance volts
$=$ volts applied to amplifier by the secondary of the calibrated mutual inductance,
i.e., $\frac{2 \pi a n D \omega L}{R \lambda}=\omega M I_{0}$, where $I_{0}$ is the current in the primary of the calibrated mutual inductance $M$.

$$
\therefore \quad D=\frac{M \lambda}{2 \pi a n L} R I_{0}
$$

The great advantage of this method is that there is no need to screen anything. There is at any time only one oscillating
circuit in action: either the local oscillator is on the amplifier or else the incoming signals. Screening involves inconvenient complications and means bulky apparatus, and no small cost. Perfect screening, specially in the neighbourhood of a powerful amplifier, is at best a very difficult matter. The best efforts may some time leave an uneasy doubt regarding its effectiveness. A further advantage is that the method is objective and does not depend upon the person carrying out the experiment.

A set exactly following the description given by Hollingworth was installed in the Radio Laboratory of the Indian Institute of Science, and worked very satisfactorily during observations being made every day on the strength of Madras Radio.
The foregoing paper is the substance of a lecture given by the auther to the Electrical Enginecring Society of the Indian Inst. of Science in Dec. 1925, and submitted to the Editor in the following year.

# The Demonstration of a New Precision Wavemeter Condenser. 

By W. H. F. Griffitk; A.M.I.E.E., Mem.I.R.E.

AT the Physical Society's recent exhibition of apparatus, the new SullivanGriffiths variable air condenser was demonstrated as a component of a substandard wavemeter. The instrument exhibited was the first model built on the lines indicated in the author's article in the February issue of this journal, and interesting results were obtained.

The principle, it will be remembered, is that of series complementary gaps in which the inverse law connecting capacity with gap distance is neutralised by the inverse law connecting reactance with capacity. It will also be remembered that a necessary feature of design is the screening of adjacent plate sections in order to eliminate all capacities to moving plates which, by remaining constant, tend to prevent the complementary changes in the capacities of adjacent gaps which form the arms of a number of Wheatstone bridges connected in parallel.

The condenser had, for the purpose of the demonstration, a fine screw thread bearing by which the entire moving system could be lowered relatively to the fixed system, thus providing a sim-


Fig. I ple means by which the wearing of the bearing or other mechanical changes with age could be imitated.
An actual displacement amounting to 5 per cent. of the total dielectric gap distance was in this manner made while the heterodyne wavemeter, of which it formed part, was oscillating. The change of frequency produced by this displacement was indicated by the change of frequency of
beating with the 20 kilocycle harmonic of a standard multivibrator wavemeter, the beats being recorded on a syphon recorder tape on which, for time marking, seconds were also being recorded by impulses from a pendulum. The initial beat frequency recorded was 5 per second, and this increased to 6 per second upon effecting the displacement-a frequency change of I part in 20,000 . The demonstration was made with the condenser at its mid-scale ( 90 degrees) setting.

The change that occurred was chiefly


PERCENTAGE AXIAL DISPlACEMENT
Fig. 2
due to the presence of constant capacities $C_{11}, C_{12}$ of the paralleled Wheatstone bridge connected condenser plates of Fig. I, owing
to the fact that a metal central shaft had been employed. It can be shown that shaft capacities $C_{11}, C_{12}$ of the order 1.o $\mu \mu \mathrm{F}$ per plate will produce an inconstancy of this order by virtue of the fact that, being constant, they tend to prevent the complementary changes of reactance of the series gap capacities $C_{5}, C_{6}$ and $C_{7}, C_{8}$ upon which the whole principle of constancy depends.

The degree of inconstancy represented by this change of frequency is, of course, extremely small, especially when compared with that which would be obtained with a similar displacement on an ordinary condenser. Such a comparison is effected in Fig. 2 in which $P G_{0} P G_{2}$ and $P G_{5}$ are calculated curves for an ordinary parallel gap condenser with correctly set plate systems, 2 per cent. initially displaced systems, and 5 per cent. initially displaced systems respectively. $S G_{1}$ is a calculated curve for a series complementary gap condenser without inter-section screening and $S G_{2}$ the curve plotted from actual results of the demonstration. Curve $S G_{1}$ will, of course, vary greatly with scale reading, the frequency change becoming much greater for lower scale settings.

The advantage of inter-section screening is, however, sufficiently marked by the comparison of the curves $S G_{1}$ and $S G_{2}$, and it is interesting to note that this screening may be effected in a very simple manner by displacing, relatively, through 180 degrees, alternate moving plates. The latter may then be completely screened from one another by suitably shaped fixed plates without the introduction of any special screening plates, such as those shown at $A$ and $A_{1}$ in Figs. 15 and 16 of the February: article.

# A Bridge for the Measurement of Inductance and Capacity. 

By Dr. G. Zickner.

$\mathrm{T}^{0}$the wireless amateur engaged on experimental work, the knowledge of the inductance and capacity of a circuit is of great importance, for the frequency and wavelength of an oscillatory circuit are primarily fixed by these two magnitudes. Although sufficiently exact measurements of capacity can be performed without difficulty with the aid of the convenient capacity bridge which is generally used, there has not hitherto been available a corresponding arrangement for the measurcment of inductance. The measuring device described below is capable of measuring in a very short time any inductance whose value lies between ten and a hundred thousand microhenries ( $1 \mathrm{o}^{4}$ to $10^{8} \mathrm{~cm}$.) In addition, the apparatus has the advantage that it can be converted, by the turn of a switch, into a capacity bridge with which capacities from about $50 \mu \mu \mathrm{~F}$. to about I $\mu \mathrm{F}$. can be measured in the usual simple manner. The accuracy of measurement, whether for inductance or capacity, amounts to about one to a few units per cent. The commercial production of the apparatus has been undertaken by the firm of Dr. Georg Scibt, BerlinSchöneberg; it can, however, be made at home without much difficulty by any reasonably skilled constructor.*

The apparatus is an application of the well-known Maxwell bridge-circuit. Two opposite arms of the bridge contain only resistances, the third contains a variable condenser in parallel with a resistance, while the fourth consists of the inductor with a resistance in series. (Fig. 1.) The variable condenser covers a capacity range from about 50 to about $1,100 \mu \mu \mathrm{~F}$. The resistance in arm I has the value of 10,000 ohms; in arm + there are four resistances, of 10,000, I,000, 100, and 1o ohms, which

[^11]can be interchanged by means of a switch. The apparatus thus possesses four separate ranges making up the total range of $10^{-5}$ to $10^{-1}$ henry.

Regarded as an alternating current bridge, the fulfilment of two conditions is required. One is satisfied by adjustment of the variable condenser, the other by adjustment of the high resistance connected in parallel with it. Since a reliable and continuously variable resistance of high ohmic value is not available, a resistance variable in steps is em-

ployed, and coarse adjustment is made with this. Fine adjustment is then obtained by the small resistance in the inductance arm, which is furnished in addition with a special vernier adjustment. It is therefore necessary to adjust the variable condenser and the resistance in turn until complete silence is obtained in the telephones.

It has been found, both by calculation and from practical experience, that it is necessary to attend to the following points :-

The resistance $r$ in the capacity arm must
be almost completely free from capacity, or it will falsify the reading of the condenser. The resistance in the inductance arm must be as free as possible from inductance, for


Fig. 2
otherwise its inductance $l$ will be added to that of the coil and measured with it. It follows that a correction must sometimes be introduced in the measurement of small inductances. The time-constants $\theta_{1}$ and $\theta_{4}$ of the resistances in the remaining arms I and + of the bridge enter into the bridgerelations.*

The fundamental equation of the bridge reads:

$$
L+l=r_{1} r_{4}\left(C+\underset{r}{\theta_{1}+\theta_{4}}\right) .
$$

It is therefore necessary, by employing a special mode of winding, to keep the timeconstants of these resistances sufficiently small to ensure that the correction $\frac{\theta_{1}+\theta_{4}}{\gamma}$ shall be negligible in comparison with $C$; further assistance in this direction can also be had by choosing the largest possible value for $r$. By so doing a small value for the correction $l$ is simultaneously achieved.

The apparatus is worked by a buzzer, which is connected through a transformer. The lay-out is so chosen that any possible stray field that may be present due to the transformer affects but little the leads to the bridge. Fig. 2 gives an illustration of the apparatus.

[^12]Since the capacity curve of the condenser is practically a straight line, the calibration curves of the bridge are also straight lines.

By the movement of a reversing switch the two arms 3 and $\&$ can be interchanged, thereby converting the apparatus into an ordinary capacity bridge of the type shown in Fig. 3. The terminals to which, in the previous application of the bridge, the inductor under examination was connected, are now joined to the condenser which is to be measured.

These considerations lead to the following procedure for the measurement of inductance :-

After connecting the inductor to be measured and the telephones, and earthing the terminal provided for that purpose, the main switch is set for inductance measurement and the buzzer is set going. Then, first with the range-switch and condenser, and possibly also with the switch controlling the high resistance, the range within which the silent point lies is determined. The point of complete silence in the telephones is then sought by alternate


Fig. 3
adjustment of the condenser and of the coarse or fine adjustment of the resistance in the arm containing the inductance, and at the same time the stepped resistance is brought, in the interests of accuracy, to as high a value as possible. The value of the inductance is very simply determined
from the setting of the condenser and the calibration curves provided. For coils of low inductance the correction $l$, mentioned above, must be introduced; its value is read off from an extra correction-curve.

Mutual inductance can also be determined by the apparatus. For this purpose primary and secondary coils are connected in series in the same sense, and the selfinductance of the two together is measured. Let this value be $L_{a}$. The connections to one of the coils are then reversed, and the measurement is repeated, obtaining the value $L_{l,}$. Then the mutual inductance $M$
between the two coils is given by $M=\begin{gathered}L_{\alpha}-L_{b} \\ 4\end{gathered}$
For measurements of capacity the main switch is set for this purpose. Coarse and fine-adjusting resistances in the testing-arm are then short-circuited. The stepped resistance is set to infinity. Then the measurement is performed in the usual way by adjustment of the variable condenser alone. It is only in the measurement of very large condensers, with high losses, that, to a limited extent, the resistances in arms 2 and 4 need be brought into use to improve the sound-minimum.

## Correspondence.

Letters of interest to experimenters are always welcome. In publishing such communications the Editors do not necessarily endorse any technical or general statements which they may contain

## The Performance of Valves in Parallel.

To the Editor, E.W. E W.E.
Sir,-Col. Edgeworth's courteous explanation of the reasons which prompted him to challenge niy treatment of " The Performance of Valves in Parallel ' makes it desirable for me to add a final word in apology for a title which was perhaps rather too general for the text which followed.
R. P. G. Denman.

Science Museum, S.W.7

## Good Quality in H.F. Amplifiers.

To the Editor, E.W. \& W.E.
Sir,-Mr. C. C. Inglis' article in your March issue "Good Quality in H.F. Amplifiers" appears to be entirely based on the tacit assumption that with a tuned anode coupling the voltage passed on to the grid of the next valve is proportional simply to the impedance of the tuned circuit to an externally applied E.M.F.

But surely this is not correct except in the extreme case where the impedance of the part of the circuit external to the tuned circuit is itself very large compared with that of the tuned circuit?

In the case of a tuned anode coupling the A.C. anode resistance of the valve constitutes this external circuit and is to be regarded as the seat of the alternating E.M.F. to which the potential difference developed across the tuned circuit is due. This tuned circuit, consisting of an inductance with resistance, in parallel with a capacity, is in series with the resistance of the valve, and the proportion of the total E.M.F. due to the valve which is impressed on the grid of the next valve is given in all circumstances by the vector expression

$$
\begin{equation*}
\frac{Z}{\rho+Z} \tag{I}
\end{equation*}
$$

where $Z$ denotes the impedance of the tuned circuit and $\rho$ the A.C. anode resistance of the valve.

When the circuit is tuned for maximum potential difference across the tuned circuit, this P.D. is in phase with the E.M.F. and the impedance $Z$ is equal to $\frac{L}{R C}$, where $R$ is the actual resistance of the tuned circuit. so that the above expression reduces to the scalar one

$$
\begin{gather*}
L \\
R C  \tag{2}\\
\rho+\frac{L}{R C}
\end{gather*}
$$

In determining the fraction of the total E.M.F. which is passed on to the next valve at a nonresonant frequency, (I) should strictly be evahuated as a vector expression, but so long as the impedance of the tuned circuit still has a value comparable with that at the resonant frequency, no great error is introduced by treating (I) as a scalar function. In any case the result of calculating the P.D. at the non-resonant frequency on this basis will be to give a value lower than would be obtained by the more accurate calculation.

Making the calculation on this approximate basis for the example given by Mr. Inglis of a tuned circuit of $183 \mu \mathrm{H}$ inductance, $0.0002 \mu \mathrm{~F}$ capacity and 5 ohms resistance, with a valve resistance of 20,000 ohms, $\frac{L}{R C}=183,000$, so that the value of
(I) at the resonant frequency is

$$
\frac{183,000}{20,000+183,000}=0.902
$$

while at the non-resonant frequency which reduces the impedance of the tuned circuit by 5 per cent.
( 1 ) becomes (or is strictly slightly greater than)

$$
\frac{173,850}{20,000+173,850}=0.897
$$

i.e., a reduction of only 0.005 , or rather more than one-half of 1 per cent. of the first value.

In calculating the band of frequencies corresponding to a 5 per cent. reduction of the impedance of the tuned circuit, the actual resistance of the tuned circuit must be used as in the first of Mr. Inglis' examples, so that the band of frequencies covered is 1,500 cycles-with a reduction, however, of the P.D. of about half of r per cent.

A reduction of 5 per cent. of this P.D. would require a reduction of the impedance of the tuned circuit by 34.6 per cent., i.e., $u=\frac{\mathrm{I}}{0.654}=\mathrm{I} .53$, giving a band of frequencies of 5,030 cycles. Mr. Inglis' value for this case is $\mathbf{1} 5$,0oo cycles.

In the case of his third example, using a value of 100,000 ohms, a reduction of $12{ }_{4}^{3}$ per cent. of the tuned circuit impedance gives a 5 per cent. reduction of the P.D.
In this case $n=\frac{1}{0.8725}=1.145$, and the band of frequencies covered is only $2,4,30$. Thus in both examples the band of frequencies covered is very much less than obtained by Mr. Inglis' method.

This, of course, merely emphasises his point of view, if one agrees with his opinion that the P.D. must not be allowed to fall by more than 5 per cent. over the band of frequencies it is required to use, but most people will probably agree with the Editor's remark that this limitation is much too stringent.

Where only one stage of H.F. amplification is used one would think that a reduction of the P.D. by at least 20 per cent should be permissible without the ear being able to detect any appreciable difference in quality.

This value leads to values of $n$ in the two examples of 1.706 and 3.53 , giving frequency bands of $I_{4}, 700$ and 6,000 cycles respectively. For a band of 12,000 cycles the maximum permissible valve resistance is about 26,000 ohms, or only half the value which Mr. Inglis finds for a 5 per cent. reduction of the tuned circuit imperlance.
E. A. Biedermann.

Brighton.

## To the Editor, E.W. \& W.E.

Sir,-I read with interest the article on " Good Quality in H.F. Amplifiers" by Mr. C. C. Inglis in the March issue of E.W. \& W.E. In my opinion the author is not quite correct in calculating the impedance of the plate circuit for clifferent frequencies, and considering the valve as a parallel resistance, for really the valve acts as a series resistance with respect to the E.M.F. Considered from the point of view of selectivity he should have determined the voltage at the terminals of the impedance in the plate circuit.

This voltage is :

$$
\begin{equation*}
v_{1}=\mu v_{g} \stackrel{Z}{R_{i}+Z} \tag{I}
\end{equation*}
$$

where: $\mu=$ amplification constant
$v_{g}=$ grid voltage.
$R_{i}=$ valve resistance.
Now considering the resulting impedance, with $R_{i}$ parallel to $Z$, we find:

$$
\begin{equation*}
Z_{t}=\frac{Z R_{i}}{R_{i}+Z}=R_{i} \frac{Z}{R_{i}+Z} \tag{2}
\end{equation*}
$$

Both formulæ ( 1 ) and (2) give the same curve but on different scales if $v_{1}$ or $Z_{t}$ is plotted against frequency for constant $R_{i}$, and for this reason the expression

$$
\omega_{A}-\omega_{B}=\frac{R}{L} \sqrt{ } n^{2}-\overline{\mathrm{I}}
$$

derived by the author is correct.*
If, however, the influence of a variable $R_{i}$ had been studied, the result would have been quite wrong: Formula (I) shows that for small values of $Z, R_{;}$ has a large influence, whereas in form (2) exactly the reverse is the case. The exact method is therefore to consider the voltage variations, and not the impedance variations.

Moreover, I derived the expression of $\omega_{\boldsymbol{A}}$ - $\omega_{r}$ in the following way and arrived at somewhat deviating results.

The voltage drop in $Z$ is proportional to $\frac{Z}{Z+R}$
$Z$ is composed of an inductance $Z$ and resistance $r$, shunted by a capacity $C$. Thus:

$$
Z=\frac{(j \omega L+r) \frac{\mathrm{I}}{j \omega C}}{j \omega L+\frac{I}{j \omega C}+r}
$$

Now $r$ can always be neglected with respect to $\omega L_{\text {, }}$ and in that case :

$$
\begin{align*}
& Z=\frac{L / C}{r+j\left(\omega L-\begin{array}{c}
\mathrm{I} \\
\omega C
\end{array}\right)} \\
& \text { and } \begin{array}{c}
Z \\
R_{i}+Z
\end{array}=\frac{\frac{L / C}{r+j\left(\omega L-\frac{\mathbf{I}}{\omega C}\right)}}{\left.R_{i}+\frac{L / C}{r+j\left(\omega L-{ }^{\mathrm{I}} \mathrm{LC}\right.}\right)} \\
& =\frac{L / C_{r}}{\frac{L}{C_{r}}+R_{i}+\jmath \frac{R_{i}}{r}\left(\omega L-\frac{\mathrm{I}}{\omega C}\right)} \\
& \frac{L / C_{r}}{\sqrt{\left(\frac{L}{C_{r}}+R_{i}\right)^{2}+\frac{R_{i}^{2}}{r^{2}}\left(\omega L-{ }_{\omega} \bar{C}\right)^{2}}} \tag{3}
\end{align*}
$$

In the resonance case :

$$
\begin{equation*}
\frac{Z_{r}}{R_{i}+Z_{r}}=\frac{L / C_{r}}{\frac{L}{C_{r}}+R_{i}} \tag{4}
\end{equation*}
$$

Putting $\frac{(4)}{(3)}=n$, we find easily :

$$
\begin{aligned}
\left(\omega L-\frac{\mathrm{I}}{\omega C}\right)^{2} & =\left(r+\frac{L}{C R_{i}}\right)^{2}\left(n^{2}-\mathrm{I}\right) \\
\omega L-\frac{\mathrm{r}}{\omega C} & = \pm\left(v+\frac{L}{C R_{i}}\right) \sqrt{n^{2}-\mathrm{r}}= \pm A .
\end{aligned}
$$

* For this statement I am indebted to Mr . Posthumus.

From this we find

$$
\begin{aligned}
& \text { I. } \omega=\frac{A}{2 L} \pm \sqrt{A_{4}^{2}+\frac{\mathrm{I}}{L C}} . \\
& \text { II. } \omega=-\frac{A}{2 L} \pm \sqrt{A_{4}^{2}+\frac{\mathrm{I}}{2}} .
\end{aligned}
$$

The only roots which give positive values of $\omega$ are

$$
\begin{aligned}
& \omega_{A}=\frac{A}{2 L}+\sqrt{\frac{A^{2}}{4 L^{2}}+\frac{\mathrm{I}}{L C^{\prime}}} \\
& \omega_{B}=-\frac{A}{2 L}+\sqrt{\frac{A^{4}}{4 l^{2}}+\frac{\mathrm{I}}{L C}}
\end{aligned}
$$

Hence :

$$
\omega_{A}-\omega_{B}=\frac{A}{L}=\frac{\gamma+\frac{L}{C R} ;}{L} \sqrt{n^{2}-\mathrm{I}}
$$

which is the same expression as found by Mr. Inglis. For $\frac{L}{C R_{i}}$ is nothing but the (parallel) valve resistance reduced into a series resistance.
We see, however, that it is not necessary to assume the resonance curve to be symmetrical, and that the expression applies for any value of $n$ and not only for small ones. The only approsimation made is that $r$ is small compared with $\omega L$ The formula appears to be much more general than the author assumes

Of course a 5 per cent. decrease is, as you remarked in an Editorial footnote, far too stringent. A better method is to calculate $n$ for a given value of the valve resistance, of, say, 100,000 ohms and see what is the result. Assuming the same values as the author for $l$ and $C$, but a coil resistance of 20 ohms, which in practice is nore probable than 5 ohms, we find $\omega_{A}-\omega_{B}=$ $2 \pi \times 20,000$, that is a band width of 10,000 cycles on each side of the resonant frequency, $n=1.27$, which, of course, is still quite permissible. So there need not be any fear of distortion by the introduction of a valve of the screened grid type in the H.F. amplifier
A. Van Sluiters.

Eindhoven Holland.

## The Radiation Resistance, etc., of Half-wave Aerials.

To the Editor, E.W \& W.E

Sir,-The February issue contains an article by Mr. E. Green on the radiation resistance of half-wave aerials.

It is there stated that this quantity comes ont to be 80 ohms, which the author proves by calculating the average current along the antenna and so on

Now we have in the formula of Mr. van der Pol, jun. (Jahrbuch der D.T. xiii, 1918, p. 229) a more accurate tool for calculation. The formula referred to is rather complicated, but in the special case under considcration $\left(h=\frac{\lambda}{2}\right)$ it can be shown that

$$
R_{s}=46.77+45 \cdot \log _{e} \pi+15 . C i_{4 \pi-60 . C i} 2 \pi \text { ohms, }
$$

which gives a value in the close proximity of 100 ohms. The function $C i$ is the integral-cosine

$$
\left(C i a=-\int_{a}^{\infty} \frac{\cos x}{x} \cdot d x\right)
$$

The formula of van der Pol is only correct when we can assume a perfectly conducting earth, and cannot consequently be used on very short wavelengths.

Nevertheless, by taking into account the real emission in the Hertzian case, a radiation resistance of 100 ohms may be expected, differing from the value of So ohms found by Mr. Green, who only considers the geometrical distribution of current in the well-known Lipol formula, but not the true phase-values of the contributions from different parts of the antenna.
E. T. Glas

Kungsbacka, Sweden

## Resistance-capacity Amplification with Screened Valves.

To the Editor, E.W. \& W.E.

Sir,-In the February number of your journal $E . W$. © W.E., there is an article entitled, " A New Method of Using Resistance-Capacity Amplification with Screened Grid Valves," by John J. Dowling. I take the liberty of drawing your attention to the fact that the matters dealt with in this article are given in my book, " Der Bau von Widerstandsverstärkern" (The Construction of Resistance Amplifiers). In the first edition of my book, published in the winter of 1925, I dealt shortly with the subject on pp. 47 and +8 . In the second edition, which appeared in March, 1926, the same question is touched upon on pp. 70 and 7 I . It is there stated: " It is adrisable not to raise the screengrid voltage appreciably above some 30 volts, for otherwise it is possible for the actual voltage on the anode to be considerably lower than the voltage on the screening grid, in the middle of the working range, on account of the voltage-drop in the external resistance. For one thing this influences unfavourably the current-distribution in the valve, and it also encourages the production of secondary electrons, which certainly may increase the amplification, but are a source of distortion through rendering the amplification unsymmetrical."

Since at that time there were not available ans screened-grid valves really suitable for the purpose. I did not go into the matter more thoroughly: In common with Mr. Dowling I regarded the amplifier, on account of the danger of distortion by this method, as particularly suitable for telegraphy. More recently I have nevertheless pursued this problem further, and it appears that very stable operation can be obtained with the high anode resistances that I have emploved. With the Telefunken screened-grid valve I have for example obtained by this method a voltage-amplification of 400 times. Highly promising results have been obtained by applying this method to an aperiodic high-frequency amplifier, although there remain under the conditions existing at present certain difficulties in connection with self-oscillation.

Berlin.
Manfred von Ardenne.

## Abstracts and References.

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 of Scientific and Industrial Research.
## PROPAGATION OF WAVES.

On Round-the-World Signals.-E. O. Hulburt. (Proc. Inst. Radio Engineers, 16, pp. 287-289, March, 1928.)
A note raising the question whether, for the round-the-world ray, a path following the curvature of the carth, as tacitly assumed by Quäck and Howe, is not the real path taken by the radio signal. The writer shows that the ray might quite well go round the earth in a sort of polygon when, incidentally, small movements of the KennellyHeaviside tayer would be expected to cause violent fading of the signal-as is observed.

The fact that the actual path taken by the ray depends upon the distribution of electrons in the upper atmosphere makes the question of particular interest.

Measurements of the Effective Heights of the Conducting Layer and the Disturbances of 19th August, 1927.-O. Dahl and L. Gebhardt. (Proc. Inst. Radio Engineers, 16, pp. 290-296, March, 1928.)
An account is given of further improvements in the echo method of observing effective heights of the reflecting layer. A table of values is given showing effective heights at various times of day from 15 th to 25 th August, 1927, covering a period of general disturbance in transmission phenomena. The table shows an increase of height after the disturbance as compared with the days preceding it. The data obtained are compared with those furnished by Mount Wilson on disturbances in sun spots as well as the general condition of radio reception. An unusually active spot was observed at Nount Wilson several days before 19th August. If it was responsible for the disturbance its effect must have been cumulative. More systematic data are necessary to ascertain whether the rise in heights observed is characteristic for radio disturbances covering large areas.
Une Recrudescence Importante des Taches Solaires dans la Deuxitime Quinzaine de Fevrier, 1928 (Significant renewed activity of sun spots in the last fortnight of February, 1928).-H. Memery. (Comptes Nendus, 186, pp. 629-631, 5 th March, 1928.)
Correlation of Long Wave Transatlantic Radio Transmission with other Factors affected by Solar Activity.-C. N. Anderson. (Proc. Inst. Radio Engineers, 16, pp. 297-347, March, 1928.$)$
A detailed presentation of the results of a study of long wave transatlantic radio transmission in its relation to a number of phenomena commonly thought to be manifestations of solar activity, namely: the occurrence of sun spots, the variations of the sun's radiation, disturbances of the earth's magnetic field, atmospheric electricity, the aurora, and earth currents. The results of the
correlation are set out at length and are best understood by referring to the numerous graphs that are reproduced and the tables given. A bibliograply of the subject is appended.
Report of the Chairman of the Commission on Radio Vave Propagation, International Union of Scientific Radio Telegraphy.-L. W. Austin. (Proc. Inst. Radio Engineers, 16, pp. 348-358, March, 1928.)

The work of the Commission on Radio Wave Propagation covers such a wide field of investigation that it was thought best to confine this report to a discussion of the subjects on which the various workers are not yet entirely in accord, or those in which the conclusions are not yet definitely established. The report is followed by a list of some of the questions suggested by the experimental material.

The Attenuation of Wireless Waves over Land.-R. H. Barfield. (Journ. Inst. Elect. Eng., 66, pp. 204-218, February, 1928.)
A paper read before the Wireless Section of the Institution, 7 th December, 1927 , together with the discussion that followed. An abstract of the paper appeared in E.W. E W.E. of January, pp. 25-30.
Discussion on Long Distance Radio Receiving Measurements at the Buread of Standards in 1925 (L. W: Austin).-B. H. Kynaston. (Proc. Inst. Radio Engineers, 16, pp. 359-360, March, 1928.)

Sur la Vitesse de Propagation des Ondes Radiotelegraphigues (On the velocity of propagation of radio waves).-A. Lambert. (Comptes Rendus, 186, pp. 686-688, I2th March, 1928.)
It is pointed out that the values for the speed of propagation of radio waves decluced from other data all lead to figures systematically and perceptibly less than $300,000 \mathrm{~km} . / \mathrm{sec}$., the general mean coming out to $247,000 \mathrm{~km} . / \mathrm{sec}$. (上9, $9,000 \mathrm{~km}$.).
Étude de la Couche d'Ozone de la Haute Atmosphere pendant la Nuit (Investigation of the layer of ozone in the upper atmosphere during the night).-D. Chalonge. (Comptes Rendus, 186, Pp. 446-448, I $3^{\text {th }}$ February, 1928.)
It is known that the terrestrial atmosphere contains ozone which is practically localised in the upper regions at an altitude of 40 to 50 kilometres. Brought to atmospheric pressure, it would constitute a thin film of varying thickness, in the neighbourhood of 3 millimetres, all round the earth. The permanence of this layer of gas at high altitude, as well as certain of its fluctuations in thickness, can be explained by the opposite effects on the molecules of oxygen and ozone of two
kinds of ultraviolet solar radiation : that creative and that destructive of ozone. The purpose of the present investigation is to find out whether the proportion of ozone changes when solar radiation ceases to act, that is whether it differs by night and by day. The method employed is outlined and the preliminary results are given. These indicate that while the thickness of the ozone layer remains constant during a same night, it is perceptibly greater by night than by day. It is stated that these first results require confirmation by a longer series of experiments.

## TRANSMISSION.

Strahlungsdichte und Empfangsfläche (Radiation density and receiving Surface)M. Dieckmann. (Zeitschr. f. Hochfrequenz., 31, pp. 8-15, January, 1928.)
A contribution to the rating of radio stations. It is shown that in the usual formula for calculating receiving efficiency in relation to the constants of transmitting and receiving antennx, as well as the current in the transmitting antenna, the factors entering in can be gathered into two groups, one of which gives the energy density at the place of reception and the other the magnitude of the Rüdenberg absorption or receiving surface. At the same time a means of rating transmitters one that has been suggested previously-is derived from the energy density reduced to a distance of one kilometre from the transmitter.

The dependence of the size of the receiving surface, as well as the current received, upon the antenna constants, is discussed with the help of curves and photographs of models.

In order to consider the degree of efficiency of receiving stations, the notion of the receiving value of a station is developed as the quotient of outgoing energy and energy density at the place of reception.

The deductions at first drawn only for open antennæ are extended to frame antennx.

The representation of the phenomena by means of radiation density and receiving surface is applied to secondary radiators and reflector arrangements
Compensation des Courants lnduits entre Antennes Emettrices Voisines (Compensating the currents induced between neighbouring transmitting antennæ).- H . Chireix and R. Villem. (Revue Generale de l'Electricité, 23, pp. 523-536, 24th March, 1928.)

The phenomena connected with mutual induction between neighbouring antennæ are analysed qualitatively together with their reciprocal electrostatic action. Two methods of diplex transmission are then indicated, one by electromagnetic coupling between the antennæ by means of either a mutual inductance or a self-inductance common to the two antenna circuits, the other by using a selfinductance forming an anti-resonant circuit with the capacity between the antenne. The authors give the precise signification of this capacity, and the method for calculating it, as well as the expressions for the self-inductances and mutual inductances of the antenn $x$, and conclude with some practical results.
I.E.E. Wirelless Section: Chairman's Adpress. -A. G. Lee. (Journ. Inst. Elect. Eng., 66, pp. 12-24.)
Address delivered before the Wireless Section, 2nd November, 1927, considering chiefly the problem of atmospherics and transatlantic telephony.

## RECEPTION.

Untersuchungen an einen Empfangsgerät fur KURZE WELLEN (Investigation of a receiving instrument for short waves).-O. Cords. (Zeitschr. f. Hochfrequenz., 31, pp. I-7 and 39-43, January and February, 1928.)
Abridgment of a Jena dissertation dealing in detail with the generation, reception and measurement of waves 3-6 metres in length.
Working on 8 Metres.-C. D. Abbott. (Wireless World, 22, pp. 135-138, 8th February, 1928.)
Description of a receiver and transmitter for ultra-short waves.
On the Distortionless Reception of a Modulated Wave and its Relation to Selec-tivity.-F. Vreeland. (Proc. Inst. Radio Engineers, 16, pp. 255-280, March, 1928.)
The paper is summarised as follows:
The importance of overtones in the faithful reproduction of speech and music. Overtones are transmitted by the extreme side bands of a modulated wave. Distortionless reception of the full side bands necessary for faithful reproduction. Crowding of the air channels brings adjacent waves into such close juxtaposition that selectivity demands a sharp cut-off at the limits of the band Selectivity by the usual resonance methods trims the side bands. Attempts at compromise by employing damped resonant circuits do not achieve distortionless reception and sacrifice selectivity. Description of an amplifier giving uniform amplification over the entire band width with a sharp cut-off. Description of a band selector having an approximately rectangular frequency characteristic. Frequency control and volume control. Various applications of the band amplifier and the band selector to broadcast reception.

On the Theory of Power Amplification.-
M. von Ardenne. (Proc. Inst. Radio Engin-
eers, $16, \mathrm{pp}$. $193-207$, February, 1928.)
An English rendering of the paper " Zur Theorie der Endverstarkung" that appeared in the Zeitschrift für Hochfrequenztechnik of last October, pp. II6-123.
Direct Coupled Detector and Amplifters with Automatic Grid Bias.-E. H. Loftus and S. Y. White. (Proc. Inst. Radio Engineers, 16, pp. 281-286, March, 1928.)
Description of a system for the direct coupling of valves to give composite detection and amplification. The system is designed to avoid frequency discrimination in amplifying audio frequency, and to be free from electrical and acoustical feed-back effects. High $\mu$ valves are used to control the output of the power amplifier through the aid of the very high filament-to-plate impedance of such
valves with extremely small expenditure of energy therein, and in such a way as to avoid effect of so-called " detector overloading." Microphonic effects are avoided. A unique method of automatically regulating the system to be responsive to carrier currents of different intensities is described, and includes an extension of the automatic effect to volume control. The entire system is designed to be inexpensive and extremely simple in the matter of construction.

Calculations for Resistance Amplifiers.A. L. M. Sowerby. (E.W. \& W.E., 5, pp. 201-203, April, 1928.)

High-Note Loss in Resistance Amplifiers.A. L. Sowerby. (Wiveless World, 22, pp. 213-25I and 285, February and March, 1928.)

A paper in three parts dealing respectively with the important effect of incidental capacities, the measurement of working impedances and capacities, and practical applications to amplifier design.

Anode and Grid Rectification.-A. L. Sowerby. (Wiveless World, 22, pp. 309-312, 21 st March, 1928.)

A survey based on practical measurement, showing that each type of detector has its particular sphere of usefulness.

The Indirectly Heated Cathode ReceiverA. P. Castellain. (Wiveless World, 22, p. 238 and 7 th and 21st March, 1928.)

Smooth Reaction Control.-A. P. Castellain. (Wiveless World, 22, pp. 361-364, 4th April, 1928.)

Discussion of the problem of the design of receivers without high-frequency amplification.

Volume Control-(Wiveless World, 22, p. 360 $4^{\text {th }}$ April, 1928.)
Brief description of methods for reducing signal intensity.

Progress in Radio Receiving during 1927.A. N. Goldsmith. (General Electric Revieu', 31, pp. $6_{4}$-69, January, 1928.)

Über den Schwingkristall (On the oscillating crystal).-K. Sixtus. (Zeitschr. f. Techn. Physik, 9, pp. 70-74.)
The static characteristic of a zincite-steel combination is investigated under various conditions. Since the deductions from a formula that takes account only of thermic effects are in agreement with the experimental results, it is concluded that the cause of the falling characteristic in the oscillating crystal is to be found in heat processes at the point of contact. The oscillograms obtained from the oscillations show that the periodic paths of current and tension are the same for both oscillating crystal and arc.

## VALVES AND THERMIONICS.

The Approximate Theory of the Screengrid Valve.-B. C. Brain. (E. $W$. © W.E., 5, pp. I79-183, April, 1928.)
A paper showing how the general shape of the characteristic curves and the static constants, viz., amplification factor, differential anode resistance, mutual conductance, and interelectrode capacity can be predicted from consideration of the structural constants in the special type of fourelectrode valve called the screen-grid valve.

Les Lampes ì Trois Grilees et leurs Montages ( V alves with three grids and their circuits).M. Chauvierre. (Radio-Revue, 7, pp. 561572, March, 1928.)
Valves with three gricls have been known for some time, at least abroad, one type having been constructed by the firm of Siemens in Germany and another put on the market by the firm of Vatea in Czecho-Slovakia. It is stated that in the classic functions of detector and amplifier, the three-grid valve may be regarded as the perfection of the two-grid value. It is not with the valve in these rôles, however, that this paper deals, but as a frequency changer. The author tried out several frequency-changing circuit-arrangements : the one he found the most interesting is reproduced below:


Experiments with this circuit are described, and its many advantages, which include great purity, suppleness and sensitivity.
Frequency Variations of the Triode Oscil-lator.-K. E. Edgeworth. (Phil. Mag., 5. pp. $783-784$, April, 1928.)

A note on Mr. D. F. Martin's paper in Phil. Mag. for November, 1927
Characteristic Curves of the Four-electrode Valve - N. R. Hall. (E.W. \& W.E., 5, pp. 198-200, April, 1928.)

The Frenotron Valve-A Vienna Novelty.(E.W. ÉW.E., 5, p. 214, April, 1928.)

Are there too many Types of Valies? - E. V. Appleton. (Wiyeless World, 22, pp. 243244, 7th March, 1928.)

Valve Classification.-A. L. M.-S. (Wiveless World, 22, pp. 321-322, 2rst March, 1928.)

A suggested new scheme based on impedance and magnification factor.

Theory of the Internal Action of Thermionic Systems at Moderately High Fre-Quencies.-Part I.-W. E. Benham. (Phil. Mag., 5, pp. 641-662, March, 1928.)
Equations are given applying to a parallel plane thermionic system subject to potentials varying in time. These equations are similar to Child's equations for steady potentials, but contain extra terms. A solution of the alternating case is obtained, taking into account space-charge but neglecting emission velocities, in the special case of a small oscillatory potential superimposed on a large steady potential. The time of transit of the electrons appears in the solution. The theory is used to explain certain experimental results on the frequency variation of rectified current.

The qualitative agreement between theory and experiment is satisfactory, but the observed effect is much the greater. The discrepancies between theory and experiment are discussed at some length. It is pointed out that electrons emitted with certain velocities are liable to execute a to-andfro motion about the surface of minimum potential. This effect would increase with the frequency, with a corresponding modification of rectified current.

## On Electrical Fields near Metallic Surfaces. -]. Hecker and D. Mueller. (Physical Reviere, 3I, pp. 431-440, March, 1928.)

An abstract of this paper was given in the Review for February (these abstracts, April, 1928, p. 225).

## DIRECTIONAL WIRELESS.

Sur un Procrde de Visée Radioélectrigue applicable à la Geodésie (On a method of radio direction-finding applicable to geodesy).-P. Schwartz. (Comptes Rendus, 186, p. 73, 9th January, 1928.)
This is stated to be a first application of the radiogoniometer to precision measurement. Employing a receiver that was very sensitive, though without a divided circle, the author "sighted radioelectrically " stations both near and distant, and by progressively diminishing the heating of the valves, was able to determine cxactly the azimuth of a station. It is estimated that the precision obtained compares with that of optical sighting. Two methods of observation are recommended: "sighting" the transmitting station directly by bringing the sound heard to an audible minimum and determining the "sights" of the audible limits of the station to the right and to the left, listening to a sound of medium intensity. A radiogoniometer, specially constructed for this purpose, would permit the application of the method of series to " sightings " and the obtaining of results comparable to those reached in the actual establishment of geodesic chains.

Sur un Nouveau Dispositif d'Alignement par Emissions Hertziennes (On a new method of course-setting by wireless).J. Aicardi. (Comptes Rendus, 186, pp. 305-307, 3oth January, 1928.)
Description of means to enable a ship or aero plane to follow a given course in foggy weather The device consists of two antenne a small distance apart transmitting waves of the same length but with a well-defined phase difference. When the two waves are continuous and emitted with the same energy, we know that nodal lines are formed along which the field and consequently reception are zero. In the present system one antenna emits continuous waves, while the other waves modulated at a musical frequency. Owing to this difference between the two emissions, it is no longer necessary for the fields produced by the two antemare to be equal for the nodal lines of the interference field to be clearly defined, places of silence occurring whenever the difference in phase of the two emissions reaches a certain value. It is explained why the precision of the arrangement is greater than that of the ordinary interference device, being capable of giving a result correct to one degree.

Rotating-Loop Radio Transmitters, and their Aptlication to Direction-Finding and Navigation.-T. H. Gill and N. F. Hecht. (Journ. Inst. Elect. Eng., 66, pp. 24I-255, March, 1928.)
A paper read before the Wireless Section of the Institution, $4^{\text {th }}$ January, 1928, an abstract of which was published in E.W. \& W.E. for February, pp. $85-88$.

Some Experiments on the Application of the Rotating-Beacon Transmitter to Marine Navigation.--R. L. Smith-Rose and S. R. Chapman. (Joum. Inst. Elect. Eng., 66, pp. 256-269, March, 1928.)
A paper read before the Wireless Section of the Institution, $4^{\text {th }}$ January, 1928, an abstract of which was published in E.W. © W.E. for February, pp. 88-90.

## A Theoretical Discussion of Various Possible

 Aerial Arrangements for RotatingBeacon Transmitters.-R. L. Smith-Rose. (Journ. Inst. Elect. Eng., 66, pp. 270-279, March, 1928.)A paper read before the Wireless Section of the Institution, $4^{\text {th }}$ January, 1928, followed by the discussion on it and the two preceding papers. An abstract of the paper appeared in E.W. \& W.E. for February, 1928, pp. 90-92.

Radio Communication Develofment in America. -(Electrical Review, 102, p. 468, I6th March, 1928.)

The enginecrs of the Radio Corporation of America have developed a reliable method of highfrequency directive transmission, known as the R.C.A. Projector Systemi, which is now in operation on several circuits. This system is stated to com-
pare most favourably with the performance of the Marconi beam system in efficiency, and to offer some advantages in simplicity of construction and adjustment. The development of the system i; the patent of the R.C.A., and other companies using it will have to do so on some arrangement with regard to royalties.

## MEASUREMENTS AND STANDARDS.

Stabilisateurs de Fréquence Pifzo-electriques pour Emetteurs d'Ondes Courtes (Piezo-electric frequency-stabilisers for shortwave transmitters).- I. Jammet. (L'Onde Electrique, 7, pp. 5 and 63, January and February, 1928.)
After an elementary account of piezo-electric phenomena, and a rapid survey of some of their applications, the author gives a detailed account of the means he has employed to achieve stabilisa-

tion of frequency in short-wave transmitters. The circuit-arrangement developed, comprising pilot oscillator, frequency multiplier, and power valve, is shown below.
Piezoeieistrische Kristalle als Frequenznormale (Piezo-electric crystals as frequency standards).-E. Giebe and $A$. Scheibe. (Elekt. Nachr. Techuik, 5, pp. 65-82, February, 1928.)
In all piezo-electric resonators and oscillators, up to now, longitudinal oscillations have been exclusively employed. The present paper shows how transverse (flexural) and torsional oscillations can also be excited piezo-electrically, and throws light on the piezo-electric phenomena linked up with the clastic oscillations of quartz plates and on the laws of the natural high-frequency oscillations. Transverse and torsional oscillations should also be of value for practical measurements for the reason, among others, that their employment would considerably extend the range of frequency in which piezo-electric resonators can find application as frequency standards.
Der piezoelektrische Resonator in Hochfrequenzschwingungskreisen (The piezoelectric resonator in high-frequency oscillatory circuits).-Y. Watanabe. (Elekt. Nachr. Technik, 5, pp. 4.5-64. February, 1928.)

In a previous paper (J.I.E.E. of Japan, May,
1927) the author deait witl some new methods of measuring motional admittance and investigated the characteristics of the resonator for various special cases. In the present paper he considers the oscillation conductivity of the piezo-electric resonator, the properties of the crystal coupling, the piezo-electric oscillation generator and the crystal frequency stabiliser.
Some Practical Applications of Quartz Reso-NATORS.-(E.W. © W.E., 5, pp. 215-219, April, 1928.)
Abstract of a paper read by Messrs. G. IV. Cobbold and A. E. Underdown, before the Wireless Section, I.E.E., 7 th March, Ig28.

A Short Survey of some Methods of Radio Sigival Measurement.-K. Sreenivasan. (E.W. © W.E., 5, pp. 205-210, April, 1928.)

Apparatus Standards of Telephonic Transmission, and the Technigue of Testing Microphones and Receivers.- B. S. Cohen. (Journ. Inst. Elect. Eng., 66, pp. 165-203, February, 1928.)
A paper read before the Institution, 17 th November, 1927 , together with the discussion that followed.
Mutual Inductance in Radio Circuits.-L Hartshorn (E.W. E W.E., 5, Pp. 18+-188, April, 1928.)

## SUBSIDIARY APPARATUS.

Microphone
Eiectrocapillaire
(Electrocapillary microphone).-M Latour. (Comptes Rendus, 186, pp. 223-224, 23rd January, 1928.)

Description of a simple microphone, the principle of which is shown below.

The capillary tube $t$, whose cross section increases towards its upper opening, is filled with an electrolyte (salt or acidulated water) and dips into mercury. When talking at some distance from the apparatus, the speech is electrically reproduced in a receiver, inserted between the conductors $c$ and $c^{\prime}$. It is remarked that operation continues just the same when the circuit is broken by inserting
 a capacity, showing that it certainly is a question of an alternating electromotive force of capillary origin and not of the variation of a continuous current produced by the variation of a resistance.

Further, conversely, if an alternating tension of musical frequency is applied to the conductors $c$ and $c^{\prime}$, the instrument works as a telephone receiver.

Untersuchungen Über Monotelephone (Research on mono-telephones).-R. Bauder and A. Ebinger. (Zeitschr.f. Techn. Physik, 9, pp. 65-69, February, 1928.)
Un Nouveau Haut-Parleur (A new loud-speaker). -(Radio-Revue, 7, p. 572, March, 1928.)
Brief description of a novelty loud-speaker called the "Phragmophone," which has the advantage of occupying very little space. Its principle in outline is that the air is set in motion by a stretched piece of cloth, caused to vibrate by two parallel wires carrying an iron disc, which is influenced by an excited electro-magnet.
Smoothing Condensers in Eliminators.-(Wireless World, 22, pp. 323-32.5, 2 Ist March, 1928.)

Discussion of the effect of voltage rises due to inductive surges.
Gleichstrom-Hochspannungsmaschinen als Anodengeneratoren (High-voltage, directcurrent machines for anode-potential supply in transmitters).-E. Rappel. (E.T.Z., 48, pp. 1285-1290.)
Description of modern requirements in generators for transmitting stations, and up-to-date designs. The winding, connection, and commutation, and the cause and suppression of higher harmonics are discussed in detail. Data are given of a 3 kw . 3,000 volt, 1,500 r.p.m. motor-generator, built by a Swedish firm, which is in daily use at the Falun radio station. It is stated that no commutator ripple can be detected by headphones in the current from this generator.
Sur quelques Proprietés du Demultiplicateur (On some properties of the frequency sub-multiplier).-E. Rouelle. (Comptes Revdus, 186, pp. 224-226, 23 rl January, 1928.)

Continuation of an investigation, the first results of which were given in Comptes Rendus, 185, p. 1450.

## STATIONS : DESIGN AND OPERATION.

la Station Radiotélf́fhonigue de Prague (The Prague broadcasting station).-E. M. Deloraine. (L'Onde Electrique, 7, pp. 21 and 45, January and February, 1928.)
A paper in two parts, the first of which is clevoted to general considerations of the problem of broadcast transmission, the second to a description of the Prague installation. After describing successively the microphones and amplifiers, the broadcast transmitter, the alternators, multiple antenna and earth, the methods of measuring the antenna output are explained, and the degree of modulation for the different frequencies transmitted.
Der Deutschlandsender bei Königswusterhausen (The "Deutschland" transmitter near Königswusterhausen).-W. Meyer. (Tele-funken-Zeitung, 9, pp. 69-89, January, 1928.)
A detailed account of this new high power broadcasting station constructed by the Telefunken Company for the German State to replace the
station at Königswusterhausen of the same designation. Several illustrations are shown of the new station, which will have about six times the efficiency of the old one.
London's New Air Port.-W. G. W. Mitchell. (Wiveless World, 22, pp. 130 and 170, 8th and 15 th February, 1928.)
llustrated description of the new directionfinding equipment installed at the Croydon Aerodrome for locating aircraft, and of the new transmitting aerials and plant for communicating with machines in flight erected at Mitcham Common, two miles away, in order to avoid obstruction to aircraft in landing and lessen interference with wireless reception.
220-Ky. Carrier-Telephony.-R. B. Ashbrook and R. E. Henry. (Electrical World, 9r, pp. 495-497, Ioth March, 1928.)
Description of the Southern California Edison Company installation embodying single-frequency duplex operation with selective ringing.
Short-Wave Transmissions.-(Wiveless World, 22, pp. 150-152, 8th February, 1928.)
A list of stations throughout the world working below ioo metres, arranged alphabetically according to their call-signs.
The Design and Distribution of Wireless Broadcasting Stations for a National. Service.-(E.W. Eo W. E., 5, pp. 189-197, April, 1928.)
Abstract of a paper read by Capt. I' P. Eckersley, before the Wireless Section, I.E.F., ist February, 1928.

## GENERAL PHYSICAL ARTICLES.

The Wider Aspects of Cosmogony.-J. H. Jeans (Supp. to Nature, 121, pp. 463-470, 24 th March, 1928.)
The Trueman Wood Lecture delivered before the Royal Society of Arts, 7 th March.

Referring to the cosmic radiation, the author states that, in a sense, this radiation is the most fundamental physical phenomenon of the whole universe, most regions of space containing more of it than of visible light or heat, and further, that there is no reason to doubt that it originates in the great nebule, and its amount is about what it ought to be, if it is evidence of the whole universe melting away into radiation (cf. Millikan, these Abstracts for March, p. 167).

Some Further Problems in Potential Differ-ence.-G. W. O. Howe. (E.W. E. W.E., 5 , pp. 175-178, April, 1928.)
Sur les Oscillations d'Ordre Supérieur d'un Circuit Oscillant (On the oscillations of an oscillating circuit of higher order).-M. Chenot. (Comptes Rendus, I 86, pp. 743-745, 19th March, 1928.)
An examination of the conclusions drawn by Lord Rayleigh (Theory of Sound, I, p. 200) as to the possible oscillations of a given mechanical system. by means of tests on an analogous use of electrical
oscillations, relating to the question whether the different wavelengths possible correspond to a harmonic series.

High-Frequency Currents.-E. W. Marchant (Journ. Inst. Elect. Eng., 65, pp. 977-988.)
The eighteenth Kelvin Lecture, delivered before the Institution, April, 1927.

Úber Koppelschwingungen kontinuierlicher Teilsysteme (Coupled oscillations of continual partial systems).-E. Waetzmann and K. Schuster. (Annalen der Physik, 84, pp. 507-524.)

Sur les Equations du Champ ElectromagNETique (On the equations for the electromagnetic field).-E. Brylinski. (Revue Genéral de l'Electricité, 23, pp. 167-168.)

Zur Theorie der Koppelschwingungenzwischen Telephonmembranen und Lufträumen (Theory of coupled oscillations between telephone membranes and air spaces).-K. Schuster. (Annalen der Physik, 84, pp. 52 5-552.)

## MISCELLANEOUS.

La Telévision Electrique (Electric television). -A. Dauvillier. (Revue Générale de l'Electricité, 23, pp. 5, 61 and $11_{7}$, January, 1928.)

In the first part of the paper the author outlines the history of television from 1875 to the present day, surveying critically the various systems that have been put forward. In the second part he describes his personal research resulting in the development of the "téléphote" and the "radioplote," where the principles of television are extended to the domain of X -rays.

Die neuesten fortschritte des Bildtelegraphie - Systems - Telefunken - Karo-lus-Siemens (The latest developments in the Siemens-Karolus Telefunken system of picture telegraphy).-F. Schröter. (Telefun-ken-Zeilung, 9, pp. 5-10, January, 1928.)
Phototelegraphy.-(Wireless World, 22, pp. 356359, $4^{\text {th }}$ April, 1928.)
Details are given of an important new process for telegraphing newspaper illustrations.

Quelques Procédís d'Amplification des Courants Photo-Electriques et Applications a l'Emission des Belinogrammes (Some methods of amplifying photo-electric currents with application to the transmission of "Belinogrammes").-M. P. Toulon. (L'Onde Electrique, 7, pp. 72-89, February, 1928.)

Paper read at a meeting of the S.A.T.S.F., December, 1927.
1). E. H.

## Esperanto Section.

## Abstracts of the Technical Articles in Our Last Issue. Esperanto-Sekcio.

Resumoj de la Teknikaj Artikoloj en Nia Lasta Numero.

PROPRECOJ DE CIRKVITOJ.
Komuna Indukteco en Radio-cirkvitoj.-L. Hartshorn.
Unue konsiderita estas la ĝenerala ekzemplo de du bobenoj kun komuna indukteco kun la diversaj mem-kaj aliaj kapacitoj, kiuj ekzistas en praktiko. Per la ekvivalenta cirkvito montrita en ĉi-tiu ekzemplo, la aŭtoro traktas lañvice pri kommnaj induktecaj ekvacioj, aplikado al variometroj, efektivaj rezistecaj ekvacioj, mem-kapacitaj ekvacioj, la determino de la konsistantaj kapacitoj de transformatoroj, mem-induktecaj ekvacioj, k.t.p.
Kelkaj Praktikaj Aplikadoj de Kvarcaj Resonatoroj.
Resumo de prelego legita de S-roj. G. N. Gobbold, M.A., kaj A. E. Underdown ce la Senfadena Sekcio de la Institucio de Elektraj Ingenieroj, Londono, je 7 a Marto, 1928a.

Post enkonduka revuo pri fruaj aplikadoj de la kvarca kristalo, la aŭtoroj priskribas eksperimentan laboron faritan de ili ĉ́e la Eksperimenta Fako de Signaloj (Woolwich) de la Brita Armeo. La priskribo anipleksas la cirkvitojn uzitajn, la
efekton de aera interspaco, cirkvitajn konstantojn, temperaturon, k.c.

Ili poste priskribas kelkajn instrumentojn desegnitajn por utiligi kristalajn oscilatorojn, lan̆jene: (1) Kvarcaj Kristaloj kiel Aludaj Normoj kune kun Sub-norma Ondometro, (2) Kvarca kristalo en Ondometro donanta serion de Ondolongoj, kiuj estas ĉiuj Obloj de ioo metroj, (3) Kvarca Kístalo en instrumento donanta serion de frekvencoj, kiuj estas ĉiuj obloj de i,ooo Kilocikloj. La cirkvitoj estas cíuj ilustritaj, kaj raporto de la diskuto, kiu sekvis la legadon de la prelego, estas donita.

Kelkaj Pluaj Problemoj pri Potenciala Diferenco.
Redakcia artikolo, super la cefliteroj de Prof. Howe, pritraktanta kelkajn ekzemplojn de potenciala diferenco, kiuj eble povas esti miskomprenitaj. La unua ekzemplo estas tiu de magneta poluso enmetita en ringon metalan, kiam la ringo estas nefermita kaj fermita respektive. La rezonado estas poste aplikita al la kondiĉoj de ringo de izola materialo.

Kalkulado de Rezistecay AmplifikatorojA. L. M. Sowerby

La ekzemplo pritraktita estas tiu de rezisteca kapacita stupo funkcianta en grandpotencan valvon ( $\mathrm{LS}_{5} \mathrm{~A}$ ).

La traktado sin koncernas pri la limoj de anodkurenta svingado, kradtensia svingado, krada potencialo, ŝtupa amplifado, k.t.p. Definitiva ekzemplo estas ellaborita por du malsamaj valvoj de konataj karakterizoj, montrante la metodon de elekto de la valvo por plenumi la bezonitajn kondiĉojn.

## SENDADO.

Ifa Desegnado kaj Distribuado de Senfadenay Brodkastaj Stacioj por Nacia Servado.
Resumo de prelego legita de Kap. P. P. Eckersley, M.I.E.E., êe la Senfadena Sekcio de la Institucio de Elektraj Ingenieroj, Londono, je la. Feburaro, 1928a.

Post enkonduka historia sekcio, la aŭtoro traktas pri la serva amplekso de brodkasta stacio kaj la diversaj detaloj pri desegno necesigitaj. Li poste dişkutas aliajn generalajn problemojn de brodkasta distribuado, t.e., limigo de onclolongo, provizo de alternativaj programoj, kunigado de stacioj, unuobla ondolonga funkciado, kaj la proponita "regiona plano" por Britujo.
La dua parto de l'prelego estas dediĉita al la desegno de brodkastaj sendiloj. Post postulo pri la generalaj bezonoj por la sendiloj, la aŭtoro diskutas "altpotencajn" kaj " malaltpotencajn" modulajn sendilojn respektive, clonante, sub la ci-lasta tipo, la kompletan cirkviton de la Daventry'a ${ }_{5} \mathrm{~GB}$ Stacio, kiu utiligas chi tiun sistemon.

Komparo inter la du sistemoj estas resumita, kaj la aŭtoro finas per kelkaj ciferoj pri la kosto de potenca provizado por brodkastaj stacioj.

Raporto pri la diskutado, kiu sekvis la legadon de la prelego, estas ankaŭ donita.

## VALVOJ KAJ TERMIONIKO.

la Proksimuma Teorio de la Valvo kun Skrenita Kadro.-B. C. Brain.
La prelego traktas pri la generala formo de la karakterizaj kurvoj kaj statikaj konstantojamplifa faktoro, diferenciala anoda rezisteco, interelektroda kapacito, k.t.p.-de la valvo kun skrenita krado, kaj la antaŭdiro pri ĉi tiuj per strukturaj konstantoj.

Post generala enkonduka sekcio, interelektroda kapacito kaj voltkvanta faktoro estas pritraktitaj, la aŭtoro konkludante, ke la kapacito estas preskaŭ inverse proporcia je la distanco inter la skrenita krado kaj la kontrola krado. Sekundaria emisio estas poste konsiderita sub la rubrikoj, Ekzemplo 1, tuta sekundaria emisio, kaj Ekzemplo II, la sekundaria emisio estas malpli ol saturigo. La generalaj konkludoj laŭ la antaŭa rezonado estas poste fine montritaj.

## Karakterizaj Kurvoj de la Kivar-Elektroda Valvo.-N. R. Hall.

La prelego pritraktas la familion de karakterizaj kurvoj de la kvar-elektroda valvo sammaniere generale kiel pri la triodo, dividante la kurvojn en du grupojn, en unu el kiuj la interna, kaj en la alia la ekstera krado estas tenita je konstanta potencialo.

Kompleta serio de kurvoj estas donita por
kolekto de valoroj de ĉiu el ĉi tiuj tensioj, kaj la konstantoj de la valvoj estas derivitaj.

Oni ankaŭ donas kurvojn de l'anoda tensio montrita kontraŭ internkrada tensio kaj eksterkrada tensio respektive, dum la alia krado estas konservita konstanta.
La " Freneton" Valvo.-Viena Novajo,
Mallonga artikolo super la cefliteroj de Prof Howe priskribanta valvon elpensitan de D-ro Pollak-Rudin de Vieno. La aranĝo konsistas el la apliko trans valvenmeta cirkvito de stabiligil? de ne-linia karakterizo. La elpensajo de D-ro. Pollak-Rudin enkorpigas ekstran anodon en la valvon, servitan per plilongigo de la filamento, tiel ke la valvo konsistas el triodo kaj diodo en la unu koverto, la diodo funkciante kiel la stabiliga elpensaĵo aludita.

## MEZUROJ KAJ NORMOJ.

Malifonga Priskribo de Kelkaj Metodoj de Radio-signala Mezurado.-K. Sreeniasan.
En Sekcio A de l'artikolo, la aŭtoro unue donas generalan diskutadon pri la problemo, kun la nomoj de tiuj asociitaj kun kaj la teoria kaj la eksperimenta flankoj. Li poste diskutas plifruajn kaj malplifruajn eksperimentojn, kondukante gis la plimodernaj anstata ŭigaj metodoj.

En Sekcio B, li pasas al revuo pri la diversaj metodoj. En la nuna numero la metodoj pritraktitaj estas (I) la metodo Vallauris, i919a, farante observadojn en Italujo koncerne la sendadoj el Annapolis, (2) la aranĝoj uzitaj de Len̆t. Guierre de la Franca Militmaristaro sur la "Aldebaran" en 1919-20a, (3) la metodoj kaj cirkvitoj de linglund, Bown kaj Friis de la Western Electric Company en 1922-3a por la mezurado de signala forteco $\hat{\hat{c}} \mathrm{e}$ Nauen, Marion, kaj New Brunswick.

La artikolo estas daŭrigota en la venonta numero.

## DIVERSAĴOJ.

## Resumoj Kaj Aludoj.

Kompilita de la Radio Research Board (RadioEsplorada Komitato), kaj publikigita laŭ aranĝo kun la Brita Registara Fako de Scienca kaj Industria Esplorado.
Dimensieco je Senfadenaj Ekvacioj.-W. A. Barclay.
La artikolo pritraktas la problemon havi ambaŭ flankojn de ekvacio samdimensie, kaj traktas pri kelkaj ekzemploj en senfadena cirkvita praktikado.

Oni montras, ke la principo de dimensieco donas utilan rimedon kontroli ekvaciojn per nura inspekto, kvankam ĝi ne estas per si sufîca provo de korekteco.

## LIBRO-RECENZO.

Recenzoj estas donitaj de la jenaj verkoj:
Thermionic Vacuum Tube Circuits (Termionaj Vakuaj Tubaj Cirkvitoj), de L. J. Peters.

Practical Radio Construction and Repairing (Praktika Radio-Konstruado kaj Riparado), de Mover kaj Wostrel.

Elements of Radio Communication (Elementoj de Radio-Komunikado), de W. E. W. Stone.

Electric Rectifiers and Valves (Elektraj Rektifikatoroj kaj Valvoj), de N. A. de Bruyne, tradukita el la germana de Gunterschulze.

The Four Electrode Valve (La Kvar-Elektroda Valvo), de Fred Goddard.

## Some Recent Patents.

The following abstracts are prepared, with the permission of the Controller of H.M. Stationery Office, from Specifications obtainable at the Patent Office, 25, Southampton Buildings, London, W.C.2, price I/-each.

## A LOUD-SPEAKER SUBSTITUTE.

(Convention date (Germany), 22nd January, 1926. No. 2648 II )
A defect inherent in all known forms of diaphragm is that owing to the elasticity and distributed mass of the vibrating body the system as a whole is bound to possess one or more fundamental resonances, which are emphasised in action and so introduce distortion. It is accordingly proposed to convert electrical current variations into corresponding sound-waves without the intermediary of any mechanical vibration, apart from that of the air itself.

This is accomplished by utilising the brush or corona discharge from a pointed electrode. A distinction is drawn between this kind of discharge and the ordinary disruptive or spark discharge. In the former case, the current of electrified ions streaming away from the point is found to set the adjacent air into steady motion. At a certain critical pressure of the charging voltage the discharge stream becomes highly responsive to fluctuations such as can be derived from a current of acoustic frequency. An equivalent wave motion is thus communicated clirectly to the adjacent air, and thereby reproduces the original sound.

Patent issued to International General Electric Co.

## PIEZO-CRYSTAL RECEIVERS.

(Convention date (Germany), $7^{\text {th }}$ November, 1925. No. $26 \mathrm{r}, 04 \mathrm{r}$.)
At certain of the fundamental longitudinal vibration-frequencies of a piezo-crystal a luminous glow is created, and can be used to give visual indication of received signals. There is a similar acoustic effect in which the crystal emits a sonorous note, which can be accentuated by means of a suitable low-frequency amplifier to operate a loudspeaker, thereby giving an audible record of incoming signals.


Both phenomena occur only when there is exact identity between the received signals and the fundamental crystal frequency, so that the method is ultra-selective and also largely free from the effect of atmospheric disturbances. Further, no heterodyne oscillation is necessary when the audible effect is utilised. As shown, incoming signals are
amplified at $A$ and applied across the crystal $Q$, which is preferably enclosed in an evacuated or gas-filled bulb. $A_{1}$ is a low-frequency amplifier feeding the sonorous crystal vibrations to a loudspeaker $L S$.

Patent issued to the Radiofrequenz Co.

## SCREENED GRID VALVES.

(Convention date (Holland), 19th November, 1926. No. 280,851.)
A relatively small screening-grid $S G$ is interposed between a cylindrical control grid $C G$ and a similarly-shaped plate $P$. In order to ensure that all the electrostatic lines of force existing between the plate and control grid shall pass through the

screening-grid, metal shields $M M$ are provided and surround the electrodes as shown. The shields $M$ are insulated from the screening grid, but areconnected to the filament or some other point of constant potential. They are supported by glass rings formed on the cylindrical plate.

Additional metal rings formed on the shields $M$ project inwardly between the control and shielding grids, in order to increase the neutralising effect. The input lead $T$ for the control grid is located at the top of the bulb.

Patent issued to N. V. Philips Glowlamp Co.

## SWITCH-CONTROLLED RECEIVER.

(Application date, 23 rd March, 1927. No. 284,887.)
Relates to a multivalve receiver of the kind in which the tuming is entirely controlled by switch mechanism so that a selected station can be brought in merely by setting the control knob to a definite position, as distinct from the ordinary fexible
adjustment of one or more rotary condensers. In the present invention this is accomplished by mounting a number of definitely tuned circuit elements in the receiver and then selecting the desired combination.
quency is not limited by the inductance-capacity, values of the external circuits. Such "internal " oscillations are created by interaction between the electron stream, a positively-charged grid, and a negatively-charged anode. In the case of gas-


Each circuit element comprises a coil $L$, on the top of which is mounted a fixed condenser, Fig. A. A certain degree of elasticity may be provided by forming the condenser with a series of partial armatures, shown shaded in Fig. B.

In the complete receiver the various tuned elements are assembled as shown at $T_{1}-\quad T_{8}$, Fig. C. They are brought successively into circuit by means of a series of spring contacts carried on a pair of sliding arms $L_{-}, L_{1}$ controlled by the master switch $S$. (The latter is also shown in elevation). The elements $T_{t}$ and $T_{s}$ utilise graded condensers of the type shown in Fig. B, the remaining elements being invariable.

Patent issued to J. A. Baertsoen.

## ULTRA HIGH-FREQUENCY GENERATORS.

(Convention date (Germany), $29^{\text {th }}$ October.
It has already been proposed to generate very high frequencies by causing the electrons forming the discharge strean inside a vacuum tube to vibrate individually, so that the resultant fre-
filled tubes, secondary effects between the electrons and adjacent gas-molecules may also be brought into play.

According to the present invention the available range of oscillations is stabilised by placing the generator tube in the field of a powerful electromagnet $N_{9}$, which may be energised either by

direct or alternating current. In this way it is also found possible to vary the internal oscillation energy within wide limits, and so to maintain the amplitude of the output oscillations fed to the wave circuit $d, d$ at a manageable value.

Patent issued to Dr. A. Esau.

## RECTIFYING INSTALLATIONS.

(Convention date $\begin{array}{r}\text { (Belgium), 29th January, } 1927 . \\ \text { No. 284,306.) }\end{array}$
The rectifier circuit is so arranged that full-wave rectification can be utilised for a low-charging voltage, with the alternative of using half-wave rectification if a higher charging-voltage is required.


When the battery $B$ is connected across the terminals $P$ and $N$, rectified current corresponding to one half-wave of the supply flows from $P$ through the battery, variable resistance $R$ and secondary coil $S$ to the anode $A$ of the rectifier, and back through the filament $F$ to complete the circuit.
case only alternate half-waves of the supply are rectified, the circuit being from terminals $P$ to $N_{1}$, then to point $M$ and through coil $S_{1}$, resistances $R_{1}, R$ and coil $S$, to the anode $A$, and so back through the filament $F$ to the positive terminal $P$.

Patent issued to the N. V. Philips Glowlamp Co.

## A MAINS-FED RECEIVER.

(Convention date (U.S.A.), $17^{\text {th }}$ August, 1925. No. 256,994.)
The Figure shows a five-valve set, comprising two stages of high-frequency amplification $V_{1}, V_{2}$, and a power amplifier $V_{\mathfrak{s}}$. A three-electrode tube $R$, with plate and grid connected, serves as a rectifier to supply the plate circuits through a filter circuit $F$, comprising series resistances and condensers. Unrectified current for the filaments of the first four valves is drawn directly from a section of $S_{2}$, the secondary winding, a highervoltage supply of A.C. current for the filament of the power stage $V_{5}$ being derived from the outer terminals $S_{1}, S_{3}$ of the full secondary winding.

Disturbance due to the use of A.C. filament supply is largely minimised by using filaments having a high heat-inertia, i.e., filaments which carry a relatively heavy current in proportion to the applied voltage. So far as the first two stages of HF amplification are concerned the discrepancy between the frequency of supply and the signal frequencies is such that little disturbance will arise here. Similarly the effect on the power amplifier is comparatively insignificant.


The next half-wave passes through the resistance $R_{1}$ and coil $S_{1}$ to the anode $A_{1}$, the circuit being completed as before.

For a higher charging-voltage the battery is connected across the terminals $P$ and $N_{1}$. In this

In the intermediate stages, hum due to the $\mathrm{A} C$. supply is dealt with as follows: The plate circuit of the H.F. amplifier $V_{2}$ comprises a low-frequency path through a lead $L$, to the upper part of the L.F transformer $T$ in the output of the valve $V_{4}$. The
two halves of the primary of this transformer are oppositely wound, so that the disturbances from the valves $V_{2}$ and $V_{4}$ tend to balance out. The output from valves $V_{1}$ and $V_{5}$ can be similarly opposed.

Patent issued to B. F. Miessner.

## SELECTIVE RECEPTION.

(Application date, I ith October, 1926. No. 285, 108.)
In order to ensure selectivity, advantage is taken of the fact that the a mount of energy absorbed by a resonant device, or circuit, coupled to a highfrequency generator depends upon the approximation of the frequency of the absorber to that of the oscillator. Consequently, any change in the frequency of one, if sufficiently rapid, can be indicated by the action of the other, say upon a rectifying valve feeding a telephone or other recording device.

As shown in the Figure, incoming signals are fed through an amplifier $V$ to a circuit $C_{1}, L_{1}$ tuned approximately to the frequency of an oscillating valve $V_{1}$. The valve $V$ may be regarded as a variable resistance across the circuit $C_{1}, L_{1}$, so that the amount of energy absorbed by the latter front the oscillator $V_{1}$ will depend upon the plate current of $V, i . e$., upon the strength of the incoming signals.

at $D$. The finished coil is suitably mounted in a holder. The self-capacity of the winding is admittedly high.

Patent issued to F. Rogers and E. H. Griffths.

## FOUR-ELECTRODE VALVES.

(Application date, 23 rd November, 1926. No. 285,975)
A four-clectrode valve is employed in order to introduce reaction in a resistance-coupled circuit where the value of the plate resistance is normally too high to permit of this being done with the standard type of valve. The main anode $A$ is connected to the high-tension through a resistance $R$ of 5 megohms, the coupling circuit being completed through a condenser $C$ as usual.


A piezo-electric crystal $Q$ is coupled to the grid circuit of a detector valve $D$, which is also coupled to the plate coil of the oscillator $V_{1}$. If the ircquency of valve $V_{1}$ is normally adjusted to that of the crystal $Q$, when no signals are being received, then there will be maximum absorption by the crystal. The effect of incoming signals, by altering the frequency of $V_{1}$, reduces absorption by the crystal $Q$ and so operates the telephones $T$ in the plate circuit of the detector $D$.

Patent issued to J. Robinson.

## COIL-WINDINGS.

(Application date, I -th January, 1927. No. 286,047.) A method of winding inductance coils, in which the external field of the completed coil is stated to remain approximately the same as that of the original straight wire, is illustrated in the Figure. The formation $A$ is obtained from a single circular loop by twisting it locally at each end of a diameter. By bending inwards each of the small loops the stage $B$ is reached. This is then twisted sidewavs to give $C$. The two remaining loops are then bent one over the other to complete the coil as shown

The reaction coil $L$ is inserted in series with the second anode $A_{1}$, and is back-coupled to the aerial. The valve is so constructed that the portion of ithe

cylindrical grid $G$, which lies between the flament and the inner and outer anodes $A, A_{1}$, is of coarser mesh than the remainder of the grid.

Patent issued to The Edison Swan Electric Co.


[^0]:    *C. Gutton, " On the amplification and maintenance of oscillations in an amplifier," L'Onde Elect. I, 261,347, 1922.
    V. G. Vallauri, " On the operation of 3 -electrode tubes in radio teleg.," L'Elettrotecnica 3, 1917.
    G. Filippini, "Contribution to the general theory of amplifiers with reaction," L'Elettrotecnica ti, 914-922, 1924.
    L. Brillouin, "Resistance Amplifiers," L'Onde Elect. I, 7, iOI, 1922.

[^1]:    $\dagger$ E.W E W.E. Vol. 3, April, May and July, 1926.

[^2]:    * E.W. \& W.E., Vol. 2, 1925, p. 48 r .

[^3]:    $\| E . W . \& W . E .$, Vol. 2, 1925, p. 483.

[^4]:    § '"Radio Frequency Measurements," Moullin, p. ${ }^{175}$.

[^5]:    * I. M. Baümier, " Recent investigations on the propagation of Electromagnetic Waves," Proc. Inst. Rad. Engs., Vol. 13, No. 1, 1025 , p. 5.

    2. G. Anders, "Quantitative Emphangsmessungen in der Funkentelegraphie," Elektrische Nachrichten Technik, Vol. 2, No. 12, 1925 , p. 416.
[^6]:    * L. W. Austin and E. B. Judson, "Method of Measuring Radio Signals used at the Radio Physical Laboratory, Bureau of Standards, Washington," Proc. Inst. Radio Engs, Vol. 12, No. 5, Oct. 1924, pp. 521-533.

[^7]:    * 1. L. W. Austin, Proc. Inst. Radio Engs., Vol I2, 1924, p. 52 I.

    2. L. W. Austin, Proc. Inst. Radio Engs., Vol. 13, No. 2, 1925, p. 151.
    3. L. W. Austin, Proc. Inst. Radio Engs., Vol 13. No. 3, June 1925, p. 283.
    $\dagger$ Report on Measurements made on Signal Strength at Great Distances during 1922 and 1923 by an Expedition sent to Australia: by Round, Eckersley, Tremellen, and Lunnon. - Jour. Inst. Eilec. Engs.
[^8]:    * 1. Ibid

    2. " Discussion on Long distance Transmission," Jour. Inst. Elec. Engs., 192 1, Vol. 59, p. 677. $\dagger$ Round, "Valve in Wireless Measurements. Rad. Rev., Vol. II, June 1921, pp. 303-7.
[^9]:    * E. B. Moullin, Jour. Inst. Elec. Engs., 1923. Vol. 6I, p. 295.

[^10]:    * J.I.E.E., Vol. 6i, No. 517, April 1923, p. 501.

[^11]:    * On this point see G. Zickner, " Eine technische Induktivitäts messbrücke," Zeitschrift für Fernmeldetechnik, Vol. S, p. 59, 1927.

[^12]:    * If $l$ is the inductance and $c$ the self-capacity of a resistance $r$, then the expression $\theta=\frac{l}{\gamma}-c r$ is called the time-constant of the resistance.

