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## Editorial.

## Abstracts and References.

READERS will probably have noticed that in our last issue " Abstracts and References" formed a more important feature than in previous numbers. With the June number we initiated what we feel sure will prove a valuable extension of this section of $E . W$. \& $W . E$. It is a difficult matter for a worker in any branch of modern science to keep himself informed on the work being done and published by other workers in the same branch. The difficulty increases year by year, and can only be overcome bv a well organised system of Abstracts. The Department of Scientific and Industrial Research has for some time past prepared such an abstract of all the publications of scientific importance in the field of radio telegraphy and telephony for circulation among members of the Radio Research Board, and our readers will be pleased to learn that arrangements have been made whereby they will share in the benefits of this important service of references and abstracts. The abstracts will be published in E.W. \& $W . E$. simultaneously with the distribution to the members of the Radio Research Board and will thus be available at the earliest possible moment after the publication of the original articles. The Radio Research Board has adopted the system of classification employed consistently in this journal, which has always proved very
successful and is being continued. Although the space devoted to this feature has been increased, we feel convinced that every reader will agree that the value of $E . W . \mathcal{E} W . E$. has been greatly enhanced by this innovation. It will enable the reader, on receiving his copy, to scan in half an hour the wireless publications of the previous month in whatever language they may have been published and to take steps to consult the originals of any articles which appear to be of special interest to him. We venture to assert that no wireless publication in any language offers its readers such a complete and prompt service of abstracts and references.

## Esperanto.

A$S$ the result of our questionnaire on this subject we found that many readers regarded the Esperanto section as a valuable feature of $E . W . \mathcal{E} W . E$. and after considerable thought and discussion we decided that the most useful form that an Esperanto section could take would be a number of brief abstracts of the articles appearing in the previous issue. This will be of special service to those foreign readers who can read Esperanto but whose knowledge of English is not sufficient to enable them to gather the contents of an article by simply perusing the pages. Our readers will have noticed that the Esperanto section was resumed on these lines in the June issue.

# Use of Plate Current-Plate Voltage Characteristics in Studying the Action of Valve Circuits. 

By E. Green.

[R131

WHEN studying the action of valves in various circuits the characteristics usually used are those showing the variation of plate current with grid volts ( $I_{p}-E_{g}$ characteristics), the plate volts being kept constant for any individual curve.

Captain H. J. Round has pointed out elsewhere that in many cases it is preferable to use characteristics showing the variation of plate current with plate voltage ( $I_{p}-E_{p}$ characteristics) at constant grid volts. The method of obtaining $I_{p}-E_{p}$ characteristics from the $I_{p}-E_{g}$ characteristics usually provided by the valve manufacturers, is given in a note at the end of this instalment.

Fig. I shows such $I_{p}-E_{p}$ curves, (a) in an actual case, (b) in the ideal case of straight characteristics. Plate voltage and plate current will always be positive so that the whole series of curves are contained in one quadrant. The region to the left of the curve for $E_{\xi}=O$ will be a region where grid current occurs.


Fig. I.
We can also draw $I-E$ diagrams to represent the behaviour of. different electrical circuits when a simple harmonic voltage is applied. These are shown in Fig. 2.

Case (A). Pure Resistance. $E=R I$ or $E / I=$ R. $E$ and $I$ are in phase. Hence a straight
line $R^{\prime} O R$ (Fig. 2a) where $\cot \theta=E / I=R$ will show the relation between $E$ and $I$ at all points of the cycle. $\quad O M=$ maximum value of $E$, and $O N=$ maximum value of $I$.
$\therefore$ Power $=\frac{1}{2} I E=$ Area of triangle $O M R$.


Fig. 2.
Case (в). Pure Inductance. $E=L \omega I$ but $I$ lags $90^{\circ}$ behind $E$. The locus is composed of two simple harmonic motions at right angles and in quadrature. This locus is an ellipse $A B C D$ traversed in the counterclockwise direction, as this makes the maximum of $I \operatorname{lag} 90^{\circ}$ behind that of $E$.

Case (c). Similarly the locus for a pure capacity load $(I=E / C \omega)$ is an ellipse such as $A D C B$ itraversed in the clockwise direction.

Case (D). Resistance and Reactance in Serics. In Fig. $2 b$ let $O R$ be the line for the resistance alone and $A B C D$ the ellipse for the inductance alone.

At the instants represented by the current 'OF , the voltage across the resistance is $F J$ and that across the inductance either $F G$ or $F Z$ according as the current is increasing or decreasing. Hence the resultant voltage is obtained by adding these voltages and is either $F H$ or $F K$ where

$$
J H=F Z \text { and } J K=F G
$$

Hence the locus of $I$ and $E$ for resistance and inductance in series is the ellipse $A B^{\prime} C D^{\prime}$ obtained. by shearing the ellipse $A B C D$ so that $O B$ comes into the position OR.

Similarly if we have resistance and capacity. in series the locus is an ellipse such as $A D^{\prime} C B^{\prime}$ traversed in the clockwise direction.
$B^{\prime}$ and $D^{\prime}$ are the highest and:lowest points of the ellipse.


Fig. 2.
If $B^{\prime} O L=\theta, \cot \theta=R$ the resistance of the circuit whilst $O A=X . I$ where $X$ is the reactance, $O L=$ maximum volts on resistance, $L B^{\prime}=$ maximum current.

Power $=\frac{1}{2} O L . L B^{\prime}=$ Area triangle $O L B^{\prime}$.
Case ( E ). Resistance and Reactance in Parallel. This is shown in Fig. 2c. As before, the current through the resistance alone is represented by the line $O R$, whilst that through the reactance alone is represented by the ellipse $A B C D$. If we add these two currents together for any particular value of $E$ we shall get the resultant current. Thus for voltage $O F$, the current in the resistance is $F J$, whilst that in the reactance is $F G$ or $F Z$. Hence the resultant current is either $F H$ or $F K$ where,

$$
J H=F Z \text { and } J K=F G .
$$

Hence the locus of the relation between $I$. and $E$ for resistance and reactance in parallel is the ellipse $A^{\prime} B C^{\prime} D$. obtained by shearing the ellipse $A B C D$ so that $O A$ comes into the position $O R$. $A^{\prime}$ and $C^{\prime}$ will be the points of the ellipse most distant from OI.. Then if $A^{\prime} O E=\theta, \cot \theta=R$, the resistance of the circuit, whilst
$O B=$ maximum current ${ }^{\prime}$ in reactance.
$A A^{\prime}=$ maximum current in resistance.
$0.4=$ maximum voltage across resistance.
$\therefore \quad$ Power supplied to circuit $=\frac{1}{2} O A . A A^{\prime}$ $=$ Area of triangle $O A A^{\prime}$
The diagram for any combination of resistances and reactances in series and
parallel can be obtained by a series of such steps- as are outlined above.

If then we have a valve operating with a certain external load, we can combine the characteristics of the valve with that of the load to show the behaviour of the combination.

Take the circuit of Fig. $3 a$ - where the load in the plate circuit is a non-inductive resistance $R_{r}$.

In Fig. 3b, let
$E_{o}=$ voltage of $\mathrm{H}: \mathrm{T}$. supply,
and for any particular value of grid voltage $E_{g}$,

$$
I_{p}=\text { plate current }
$$

and $\quad E_{p}=$ voltage on plate.
Then if the grid volts are adjusted so that the current through the valve is zero, there will be no drop in voltage in $R_{e}$, and the voltage on the plate will be $E_{o}$ the full H.T. battery voltage. Hence in Fig. $3 b$ if $O A=E_{0}, A$ represents the working point for these conditions. Now if the grid volts are altered, so that a current $I_{p}$ flows through $R_{e}$ and the plate filament circuit, there will be a drop in volts across $R_{e}=R_{\varepsilon} I_{p}$, and the volts on the plate are reduced by this amount. The plate voltage is now therefore $E_{o}-R_{\epsilon} I_{p}$. The new working point $D$ is obtained as


Fig. 3.
follows: Make $A C=R_{e} I_{p}$ and $C D$ perpendicular to $A C$, and equal to $I_{p}$. Also, since $\quad \cot \boldsymbol{\theta}=\frac{C A}{C D}=\frac{R_{\epsilon} I_{p}}{I_{p}}=R_{e}$
$\theta$ is independent of $I_{p,}$, and therefore as $E_{g}$ is varied, the working point will move along the line drawn through $A$ and $D$. This is true whatever the law of variation of $E_{g}$.

We can express this more shortly thus, H.T. volts $=$ volts on plate + volts drop in external load,
or, $\quad E_{o}=E_{p}+R_{e} I_{p}$

$$
\begin{aligned}
& E_{o}-E_{p}=R_{\epsilon} I_{p} \\
& \frac{E_{o}-E_{p}}{R_{e}}=I_{p} \text { for any particular }
\end{aligned}
$$ grid voltage $E_{g}$.

Hence the relation between $E_{p}$ and $I_{p}$ for varying values of $\mathrm{E}_{g}$ is given by a straight line. In particular for

$$
\begin{aligned}
& I_{p}=O \text { we have } E_{p}=E_{o}=O A \\
& E_{p}=O \text { we have } I_{p}=E_{c} / R_{t}=O B
\end{aligned}
$$

The line $A B$ is the locus showing the relation of $E_{p}$ and $I_{p}$ when $E_{g}$ is varied in any manner. Its slope is given by

$$
\cot \theta=\frac{O A}{O B}=E_{o} / \frac{E_{o}}{R_{t}}=R_{e}
$$

If the grid has a steady voltage of $-x, D$ is the working point and $D C$ is perpendicular to $O X$,

$$
O C=E_{p} \text { plate volts }
$$

and $C A=E_{o}-E_{p}=$ voltage drop across $R_{e}$
Power lost in valve $=E_{p} I_{p}=O C . C D$
Power lost in load $R_{e}=R_{\ell} I_{p}{ }^{2}=C A . C D$
Now let a sinusoidal voltage be applied to the grid and let $e_{p}, i_{p}$ be the instantaneous values of the changes in plate voltage and current. We can see from the above that the working line will be along $A D B$.

Working it out in detail we have,

$$
E_{o}-\left(E_{p}+e_{p}\right)=R_{e}\left(I_{p}+i_{p}\right)
$$

whence by subtraction,

$$
e_{p}=R_{e} i_{p}
$$

This is the relation shown on the simple $I-E$ diagram for a non-inductive resistance, except for the minus sign which indicates that the positive direction for voltage across $R_{e}$ is opposite to that for the plate. The positive directions for applied voltage and current for $R_{\epsilon}$ are shown by the arrow lines through $D$, the mean position.

This line $A D B$ is entirely a characteristic of the external circuit and not of the valve itself. In particular it will be straight even if the valve characteristics are curved; though in this case the excursions on either side of $D$ for a sinusoidal voltage on the grid .will not be equal and there will be rectification and distortion.

As in the ordinary $I-E$ diagrams for a resistance,

$$
\cot \theta=\frac{-e_{p}}{i_{p}}=R_{e}
$$

If the voltage applied to the grid carries the valve over the working range $E D F$ the amplitude of the voltage variation on $R_{e}$ is $e_{p}=H F$ or $G E$ (Fig. 3) and of current variations in $R_{\epsilon}$ is $i_{p}=D H$ or $D G$.

Hence $A C$ power in $R_{c}=\frac{1}{2} e_{p} i_{p}=\frac{1}{2} D H . H F$ $=$ area of triangle $D H F$.
This power is in addition to the direct current power loss in $R_{t}$. And since the mean value of $I_{p}$ is unchanged, the total power from $/ \mathrm{H} . \mathrm{T}$. supply is unchanged. Hence the loss in the valve is diminished by the amount

$$
\frac{1}{2} e_{p} \cdot i_{p}=\text { triangle } D H F
$$


(a)

(b)

Fig. 4.
If the external load comprises resistance and inductance in series the equation showing the relation of the changes in $e_{p}$ and $i_{p}$ becomes,

$$
-e_{e}=R_{e} i_{p}+L \frac{d i_{p}}{d t}
$$

which, except for the minus sign, is the ordinary $A C$ relation between voltage and current. Hence the locus showing the behavioùr of the combination is an ellipse $E M F L$ (Fig. 4a) traversed in the clockwise direction. The shape and slope of this ellipse is entirely a characteristic of the external circuit, provided it lies in the region where the valve characteristics are straight. We can look at this from a rather different point of view. From the ordinary $I-E$ diagram, we know that such an ellipse represents the relation that must exist between an alternating (sinusoidal) current in the load circuit and the alternating voltage at its terminals. This ellipse, when
placed in position on the valve characteristics, shows what excursions of grid voltage are required to produce it. So long as the ellipse lies wholly in the region of the straight and evenly spaced characteristics, it will be seen that a sinusoidal grid voltage is required. But if the ellipse enters the region of curved characteristics it cannot be produced by a sinusoidal grid voltage. That is, a sinusoidal voltage on the grid will not produce a sinusoidal current in the load circuit. There is rectification and distortion. The position of the ellipse on the valve characteristics will be fixed by the grid and high tension voltages. If $E$ and $F$ are the highest and lowest points $\cot \theta=R e$ and $L D=D M$ $=L \omega i_{p} \max$.

If the load is capacitive the locus is a similar ellipse traversed in the counterclockwise direction.

In either case the $A C$ power in the load
$=\frac{1}{2} D T . T F=$ area of triangle $D T F$.
If $R_{e}=O$ the ellipse takes the position shown in Fig. $4 b$.

We shall however restrict the investigation to cases where the load is equivalent to a resistance. This is the condition usually aimed at in practice, and the modifications required if it is not fulfilled have been indicated.

We return therefore to the simple resistance circuit of Fig. 3 a.

## Maximum Output and Efficiency.

Still keeping $E_{o}$ and $R_{\epsilon}$ fixed, and with no limitations as to grid current, and restricting ourselves to sinu-


Fig. 5. soidal conditions, it is clear that the $A C$ power output will be a maximum when the working range extends from $A$ to $B$, Fig. $3 b$. To secure this with ideal characteristics, the grid bias should make $D$ the mean position, the mid point of $A B$. (See Fig. 5.)

$$
O C=C A=\frac{E_{n}}{2}
$$

Then from the figure,
Total power supplied $=E_{o} \quad I_{p}=$ area $O A . C D$.
Arith: mean volts on $R_{e}=C A=\frac{E_{o}}{2}$
D.C. power in $R_{e}=\frac{E_{o}}{2} I_{p}=$ area $C A . C D$.
A.C. power in $R_{\epsilon}=\frac{1}{2} C A$. C $D=\frac{1}{4} E_{o} I_{p}$

Overall efficiency $=\frac{1}{ \pm} \frac{E_{0}}{E_{0}} \frac{I_{p}}{I_{p}}=\underline{25 \%}$
Power loss in valve $=\frac{1}{2} E_{o} I_{p}-\frac{1}{4} E_{o} I_{p}$ $=\frac{1}{4} E_{o} I_{p}=$ A.C. output.


Fig. 6.
By avoiding the direct current loss in $R_{\epsilon}$ we can bring up the overall efficiency to $5^{\circ}$ per cent.

This can be done by circuits of the type shown in Fig. $6(a),(b),(c),(d)$ and (e).

All these with proper adjustments will behave as a resistance load. In (a) and (c) $L$ must be a choke with an impedance (at the frequency concerned) large compared with $R_{t}$.

In (b), (c) and (d) when the circuit is in resonance the equivalent resistance is

$$
\left.\begin{array}{rl}
R_{e} & =\frac{M^{2} \omega^{2}}{R} \\
\text { In }(e) \quad & R_{e}
\end{array}=R \cdot\left(\frac{n_{1}}{n_{2}}\right)^{2}\right) ~ \$
$$

In all these cases the mean plate voltage is $E_{o}$, the H.T. battery voltage, since there is no D.C. voltage drop in the load. In particular in (e) the resistance of the transformer primary will be small compared with $R_{c}$ for the valve. As the instantaneous
(a)

$$
\text { Fig. } 7 .
$$

(b)
plate voltage will not usually drop absolutely to zero the maximum possible variation in plate voltage without distortion is less than $E_{c}$, and the maximum instantaneous value of plate voltage is less than $2 E_{0}$.

Hence in Fig. 6 if $O A=2 E_{o}$ the working line cannot extend beyond $A M$. Also the slope of the working line must be given. by $\tan \theta=i_{p} / e_{p}=\mathrm{I} / R_{\varepsilon}$. (The detailed proof of this is left to the reader.) According to the grid bias it will be a line such as $A B$ or $A^{\prime} B^{\prime}$ or $A^{\prime \prime} B^{\prime \prime}$. The best line however will be $A B$ as involving the minimum power input for maximum output. We have for the line
$A B$, when working over the whole range $A$ to $B$.

Power input $=E_{o} I_{p}=C A . C D$
A.C. power in load $=\frac{1}{2} C A . C D=\frac{1}{2} E_{o} I_{p}$.

Loss in valve $=\frac{1}{2} E_{o} I_{p}$.
Efficiency $=50$ per cent.
The mean position of the working line must always lie on $C N$ (where $O C=E_{0}$ ) which cuts $A B$ at $D$ its midpoint. Note also that, so far, no restrictions have been placed on the value of $R_{e}$ and therefore the above efficiencies can be obtained for any value of $R_{e}$.

Variation in $R_{e}$. The slope of the working line is given by $\tan \theta=i_{p} / e_{p}=\mathbf{r} / R_{\ell}$. (See Fig. 7.)

When $R_{\ell}=O$ the line is vertical.
,, $\quad R_{\varepsilon}=\infty$ the line is horizontal.
", $R_{c}$ is negative the line slopes in the other direction.
Returning to circuit shown in Fig. $7 b$, we saw that so long as $E_{0}$ is fixed the working line will always pass through $A$ (where $O .4=E_{0}$ ) for any value of grid bias.

(a)

Fig. 8.

(b)

For the other type of circuit (Fig. 8b) altering $R_{e}$ does not change the conditions in the valve so long as there is no alternating. voltage on the grid. Hence in Fig. 8 when $R_{e}$ only is altered the working line will swing about $D$, and the grid bias would have to be altered to make the working line pass through $A$.

# Harmonics and their Effects on Wave Form. 

By J. F. Herd, A.M.I.E.E., M.I.R.E.

THE existence of harmonics accompanying a fundamental oscillation, electrical or mechanical, is very well known, but the exact effect which these harmonics have in influencing the final shape of the wave representing the oscillation is usually a matter of much less attention. It is only when oscillographic study renders the presence of harmonics obvious-frequently in circumstances where they are not desiredthat their effect on wave shape renders a knowledge of their properties desirable.

It is true that any curve containing fundamental and harmonics is capable of being represented by an appropriate Fourier series, but it is equally true that such a series is, to most minds, much inferior to a graphical representation in assisting the formation of a mental picture as to the effect of the harmonic terms. On the other hand, the
it is obviously impossible to deal with anything but the smallest fraction of what might well be encountered in practice. The cases illustrated may serve, however, to form a rough guide as to the effect of the chief harmonics. The last four cases especially show the ultimate approximation of some useful Fourier series.

The fundamental wave is throughout shown by light continuous lines, the harmonics by dotted lines, and the resultant by heavy continuous lines. Considerable uncertainty appears still to exist as to the naming of harmonics-e.g., one finds a harmonic of twice the fundamental frequency referred to variously as the first or as the second harmonic. The tendency is, how-ever-especially in wireless practice-more generally towards calling this the second harmonic, so that a harmonic of $n$ times the


Fig. 1.
$y=a \sin x$.


Fig. 2.
$y=a \sin x+\frac{a}{2} \sin 2 x$.


Fig. 3.
$y=a \sin x+\frac{a}{4} \sin 2 x$.
accurate drawing of a series of sine curves and harmonics is admittedly a tedious matter, and as a result is frequently omitted when it might be advantageous and illuminating.

In this article it is proposed to illustrate a few typical cases of the effect of harmonics by the graphical process of drawing one complete cycle of the fundamental, together with the harmonic or harmonics stated in each case. The resultant wave form is then obtained by their algebraic addition. Since the resultant depends upon both the amplitude and the phase of the harmonics present,
fundamental frequency would be called the $n$th harmonic. This practice will be adopted here.

In Fig. 1 is shown a simple sine curve. This has the equation

$$
y=a \sin x
$$

where $a$ is the amplitude attained at the maximum of the half cycle and $x=2 \pi f t$, where $f$ is the frequency.

Fig. 2 shows the addition of a harmonic of twice the frequency and half the amplitude of the fundamental, i.e., the function

$$
y=a \sin x+\frac{a}{2} \sin 2 x
$$

The distortion from sine wave purity is very pronounced. Figs. 2, 3 and 4 together show the effect which the amplitude of this same harmonic has on the resultant wave shape, in Fig. 3 the harmonic being one-quarter,
as compared with Fig. 2, and the effect of reversing the incidence of the " peaky" side of each half cycle is very noticeable. Fig. 6 shows the case of a harmonic shifted $90^{\circ}$ from that of either Figs. 2 or 5 (i.e., $45^{\circ}$ or


Fig. 4.

$$
y=a \sin x+\frac{3}{4} a \sin 2 x
$$



Fig. 5.
$y=a \sin x-\frac{a}{2} \sin 2 x$.


Fig. 6.
$y=a \sin x+\frac{a}{2} \sin 2\left(x-\frac{\pi}{4}\right)$.
and in Fig. 4 three-quarters the amplitude of the fundamental. From these the effect of intermediate amplitudes or even of an amplitude equal to that of the fundamental can readily be deduced. In passing it may be noted that such a harmonic amplitude is not beyond the bounds of possibility or of practical experience.
$\pi / 4$ radians with respect to the phase of the fundamental). Here it is seen that while each half wave is itself symmetrical, the two half waves are quite different from each other in appearance.

From these curves it is obvious that the effect of a harmonic of $2 f$, or, more generally, of an even multiple harmonic is to cause either


Fig. 7.

$$
y=a \sin x+\frac{a}{2} \sin 2\left(x-\frac{\pi}{8}\right)
$$



Fig. 8.

$$
y=a \sin x+\frac{a}{2} \sin 2 x+\frac{a}{4} \sin 4 x
$$



Fig. 9.

$$
y=a \sin x+\frac{a}{3} \sin 3 x
$$

In Figs. 2, 3 and 4 the phase of the harmonic has remained constant with respect to that of the fundamental. Figs. 2, 5, 6 and 7 together show the effect of varying this phase while the amplitude remains constant at a/2. In Fig. 5 the harmonic is anti-phased
a marked asymmetry of each half cycle or a marked dissimilarity between the half cycles. A more randomly chosen phase relation may indeed do both. This is shown in Fig. 7, where the harmonic has the phase of $22 \frac{1}{2}^{\circ}$, or $\pi / 8$ radians with respect to the fundamental.

The asymmetry of the half cycles is still clearer in Fig. 8, which shows the addition of the next even harmonic, that of frequency $4 f$, and amplitude shown as $a / 4$.

Fig. 9 illustrates the combination of the fundamental and a harmonic of frequency $3 f$
are still exactly similar to each other, in that the peaky portion occurs first in each (cf. the different incidence of Figs. 2 and 5 and the even multiples generally). Such similarity of the incident order of detail in the half cycles would be found still to exist even with


Fig. 10.
$y=a \sin x-\frac{a}{3} \sin 3 x$.


Fig. 11.
$y=a \sin x+\frac{a}{3} \sin 3\left(x-\frac{\pi}{6}\right)$.


Fig. 12.
$y=a \sin x+\frac{a}{3} \sin 3 x+\frac{a}{5} \sin 5 x$.
and amplitude $a / 3$, beginning in phase with the fundamental. It is now very noticeable that the half cycles are each symmetrical and mutually similar. There is also evident the beginning of a flat top, referred to later.


Fig. 13

$$
y=a \sin x+\frac{a}{3} \sin 6 x
$$

Fig. Io shows the effect of reversal of the phase of this harmonic, the half cycles still being symmetrical and mutually similar.

In Fig. II the harmonic is displaced $90^{\circ}$ from those of Figs. 9 and 10, i.e., $30^{\circ}$ or $\pi / 6$ radians with respect to the fundamental. It will now be observed that although the half cycles are no longer symmetrical they
any randomly chosen pbase relation, although, of course, the actual shape would differ from that actually illustrated, according to the phase. This is indeed true of all the odd multiple harmonics, $3 f$, $5 f$, etc., and the consequent similarity of the half cycles in such cases is of some importance in the work of making a Fourier analysis of a wave.
Fig. 12 combines a harmonic of frequency $5 f$ and amplitude $a / 5$ with the case of Fig. 9, and shows the still further development of the flat top, the half cycles still remaining symmetrical and similar.


Fig. 14.

$$
y=a \sin x+\frac{a}{2} \sin 2 x
$$

Ultimate approximation.
$+\frac{a}{3} \sin 3 x+\frac{a}{4} \sin 4 x+\frac{a}{5} \sin 5 x$

Since 6 , is a multiple of 2 and of 3 ; it is to be: expected that a harmonic of $6 f$ will partake of: the qualities of both $2 f$ and $3 f$. In Fig. 13 is shown the combination of a


Fig 15.

$$
\begin{gathered}
y=a \sin x-\frac{a}{2} \sin 2 x \\
+\frac{a}{3} \sin 3 x-\frac{a}{4} \sin 4 x+\frac{a}{5} \sin 5 x
\end{gathered}
$$

Ultimate approximation.
fundamental and harmonic of frequency $6 f$ and amplitude $a / 3$, when it is seen that although the second multiple effect is producing asymmetry, the influence of the third multiple is still traceable.

Examples of the effects of harmonics cannot be better concluded than by considering the effect of a few well-known Fourier series and the approximations toward which they tend if carried far enough.

Fig. I4 (a) shows the function

$$
\begin{aligned}
y=a \sin x & +a / 2 \sin 2 x+a / 3 \sin 3 x \\
& +a / 4 \sin 4 x+a / 5 \sin 5 x \ldots \text { etc. }
\end{aligned}
$$



Fig. 16.

$$
\begin{array}{r}
y=a \sin x+\frac{a}{3} \sin 3 x \quad \\
+\frac{a}{5} \sin .5 x+\frac{a}{7} \sin 7 x+\frac{a}{9} \sin 9 x .
\end{array}
$$

graphed up to the term $5 x$, while (b) shows the ultimate form to which the series
approximates, the continued addition of further terms tending to smooth the rippled structure which is still strongly evident in (a).

Fig. I5 illustrates the case of the same series with the even terms reversed in sign.

Fig. 16 is of very considerable wireless interest. It represents the Fourier series:-

$$
\begin{aligned}
y=a \sin x & +a / 3 \sin 3 x+a / 5 \sin 5 x \\
& +a / 7 \sin 7 x+a / 9 \sin 9^{x} \ldots \text { etc. }
\end{aligned}
$$

up to the term $9 x$. It is very evident that the waves is approaching still more closely to the flat-topped form already apparent in Figs. 9 and I2. The continued smoothing of the top by the addition of further terms of the series would ultimately give the approximation of Fig. 16 (b).

Fig. 17 is possibly of less wireless importance but is still of considerable mathematical and physical interest. - It shows the series

$$
\begin{aligned}
& y=a \sin x-a / 3^{2} \sin 3 x+a / 5^{2} \sin 5 x \\
&-a / 7^{2} \sin 7 x+a / 9^{2} \sin 9 x, \text { etc. }
\end{aligned}
$$

drawn to the term $9 x$. The reversal of the phase of alternate terms gives a very different form from that of Fig. I6, while the diminished amplitude of the harmonic terms is


Fig. 17.
noticeable in tending to the more rapid smoothing towards the final approximation of Fig. 17 (b).

While it is not thought that the comparatively few cases illustrated here can serve as a general guide to such an extensive subject as harmonics, it is hoped that these graphs may prove useful in familiarising the wireless experimenter with some of the more prominent cases likely to be: encountered. In particular, they may serve at some time to render unnecessary the Fourier analysis of a: suspect wave form-a process which, although mathematically simple, is perhaps: even more tedious to the busy man than the construction of the graphs.

## Germany's High-Power Broadcast Station. Königswusterhausen, Berlin.: <br> [R616.5

THE principal German broadeast station is sitmated at Königswusterhausen, 20 miles south-east of Berlin. It is a highpower station which can be received with tese anvwhere in Europe. In the afternoon it transmits a programme of scientific and educational lectures, and in the erening an entertainment prugramme.
and two of 7 kilowatt, these being the powers supplied to the valves in the absence of modulation. This is the methorl of stating the power of a station agreed upon at the intemational conference at Geneva. Where being modulated the average power is only about half this value. The whole equipment has been supplied by the Telefunken Company.


Fig. 1. The thansmitting station.

Köngswusterhausen is the wireless transmitting headquarters of the Cirman Post ) fifice, but the broadcast transmitters are housed in a special building (Fig. I). There are three separate transmitters, one of 14

[^0]The rectifiers, high frequency valves and circuits are placed hehind the switchboard. shown in Fig. 4. The thrce marble panels. on the right belong to one of the 7 -kilowatt sets, the next three to the other 7 -kilowatt set, whilst the panels of the ry-kilowatt set can be seen at the far end; the latter are shown in detail in Fig. 5.

The telephone currents on their arrival
by cable from Berlin are amplified on the apparatus shown in Fig. 2; there are three similar sets, one for each of the transmitters.
transformed to a high voltage and rectified. The filaments of the rectifiers are heated by 500-cycle alternating current supplied by a rokVA generator, but the filaments of the


Fig. 2. The audio-amplifiers for the $7 k W$ and the $14 k W$ transmitters.

The anode supply for the large transmitter, which is the one which we shall describe, is obtained from a 500 -cycle A.C. generator giving 42 kVA at 220 volts; the current is
power valves are supplied from a low voltage D.C. dynamo, and those of the modulating valves from accumulators. The various motor-generator sets are shown in Fig. 3.


Fig. 3. Machine room.

The transmitter consists of four parts :-
I. The independent oscillator.
2. The rectifiers and power valves.
are mounted on marble panels on a switchboard (Fig. 5) placed in front of the set.

The independent oscillator is shown in


Fig. 4. Panels of the three transmitters.
3. The intermediate circuit in the Fig. 6; its connections can be seen from anode circuit of the power valves.
4. The antenna tuning coil.

The necessary switches and instruments

Fig. 1o; its valve of Type RS47 has an output of I kilowatt at an anode voltage of Io,000 volts. Behind the same panel (Fig. 6)


Fig. 5. Panels of the I4kW transmitter.


Fig 6. Independent oscollator and modulating valves.
Fig. 7. Rectifiers and power valees.


Fig. 8. Main anode coil and aekial coupling.


Fig. 9. Aexial tuning coil.
are the two modulating valves (Type RV24) in parallel, having an output of about Io watts; these are normally controlled from the a nplifiers shown in Fig. 2, but the panel also has a microphone and telephone transformer, so that modulation can be carried out at the switchboard if necessary.

The modulation is carried out on the method patented by the Telefunken Company, and which they have found very successful. The usual high resistance gridleak is replaced by the variable resistance of the modulating valves, the resistance of which depends on their grid voltage, which, in turn, is controlled by the amplified microphone current.
valves; there are eight of the former (Type $\mathrm{RG}_{44}$ ) and eight of the latter (Type RS53). Each valve has an output of 2.5 kilowatts, with an anode voltage of Io,000 volts. Beneath the rectifiers and valves are the transformer, by means of which the 500cycle current is raised from 220 to $2 \times 10,000$ volts, the smoothing condenser and other auxiliary apparatus.

The intermediate circuit between the power valves and the aerial is shown in Fig. 8; the aerial coupling coil is seen at the top and the condenser at the bottom.

The aerial tuning coil (Fig. 9) can be adjusted in a number of steps, and also by means of the variometer coil seen below it.


Fig. Io. Simplified diagram of connections.

The oscillations produced by the independent oscillator are amplified by the main power valves which have a low resistance oscillatory circuit in the anode circuit. The aerial is coupled to this so-called intermediate circuit, which tends to reduce to a negligible amplitude the harmonics of the fundamental oscillation.

The modulation does not affect the independent oscillating circuit, which is the real oscillation generator, but the modulating valves replace the grid-leak on the power valves, which are really power amplifiers, and thus vary their amplification.

Fig. 7 shows the rectifiers and power

There are ammeters in both the intermediate and the aerial circuit.

Fig. Io is a simplified diagram of connections.
The aerial is of the T type, consisting of four wires about 80 metres long and Io metres wide, suspended between two masts 210 metres high. The length of the down lead is about 200 metres. The aerial has a capacity of $2,500 \mathrm{cms}$. $(0.0028 \mu \mathrm{~F})$ and a natural wavelength of about 1,500 metres. The station can work at any wave-length between 1,000 and 3,200 metres, but its normal wavelength is 1,300 metres.
G.W.O.H.

# Obtaining Anode Current from Mains. 

Points to be Observed.

ARTICLES have recently appeared in the wireless Press showing methods by which the high tension current required for the anode supply can be obtained from electric light mains. In carrying out many of these schemes no attention is paid to the Board of Trade regulations, which were made for the purpose of securing the safety of the public and an efficient supply of electrical energy. These regulations provide that where the pressure of a supply between the adjacent conductors of a three-wire system of mains exceeds 125 volts, the intermediate wire shall be connected to earth subject to the agreement of the Postmaster-General, and in accordance with the following conditions :-
(a) The connection with earth of the intermediate conductor shall be made at one point only on each distinct circuitnamely, at the generating station, substation or transformer, and the insulation of the circuit shall be efficiently maintained at all other parts.
(b) The current from the intermediate conductor to earth shall be continuously recorded, and if it at any time exceed one-- thousandth part of the maximum supply current, steps shall immediately be taken to improve the insulation of the system.
These regulations alone should be sufficient to show why the neutral main or any other main should not be earthed indiscriminately. As, however, there are people who disregard all rules and regulations, a word of warning as to the danger attendant upon the breaking of these regulations will not be inopportune.

It is usual to insert in the earthed connection an ammeter to comply with regulation (b) above. A little consideration will show that if an earth fault develops on one of the outer mains, the current will flow to earth at the fault and pass back to the generator through this ammeter. In order to protect this ammeter from injury by a very heavy fault current, an automatic switch is provided
to cut the ammeter out of circuit and introduce in its place a resistance whenever the current becomes too great for the ammeter to record. The reading on this ammeter is accepted as an indication of the condition of the mains. It is possible, however, for earth faults to develop simultaneously on both outers of the supply, and for the earth currents through these faults to balance, in which case no reading would be observed on the ammeter, but this condition is one which very seldom obtains.
It is clear that the current which flows to earth through the faults will pass back to the generator not only through the earth connection at the station, but also through any other earth connection. Should, therefore, a very heavy fault develop, the first thing which would happen on premises where the neutral conductor was earthed would be the blowing of the fuses on the local fuse board. If the fuses on the local fuse board are of fairly heavy copper wire, as is sometimes the case, it will be seen that there are many possibilities of accidents, such as the burning out of the earth connection and the firing of the premises, or the burning out of the wireless set.
Not only does this danger arise, but considerable trouble will be caused to the power station engineer whose duty it is to maintain the supply, as it can easily be seen that should a large number of experimenters earth the neutral main, the ammeter at the station will give false readings. In addition to this, certain methods of measuring the insulation resistance of the mains and the location of faults would be rendered ineffective.

In addition to the above, the earthing of the neutral main may cause trouble to all the receiving apparatus in the neighbourhood, particularly in the case of alternating current networks, and in the writer's opinion the phenomenon of fading might be experienced far more than it is at the present time.
It is to be hoped that these few notes will be sufficient to prevent the improper earthing of electric light mains.

# Effective Resistance of Inductance Coils at Radio Frequency.-Part III. 

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[R144

## 13. Multilayer Coils of Small Winding Depth.

Let the winding section of the coil be $b \times t$. Let $t / b$ be small and suppose at first that $b$ is small compared with the radius of the coil. Let there be $m$ layers in the depth $t$ and $n$ turns per layer.

The field at any point in the section will have two components, $H_{t}$ and $H_{b}$, parallel to $t$ and $b$ respectively, which will act independently in producing eddy losses.

As regards $H_{t}$, the field acting on a single wire is the same as that for a single layer coil for which $c=b / m n$. Thus the added resistance due to the action of $H_{t}$ is

$$
\frac{1}{3} \pi^{2} R G(m n d / b)^{2}
$$

the number of turns being assumed large.
As regards $H_{b}$, each layer behaves as a current sheet having current density $n I / b$. In the immediate neighbourhood of the sheet the component of the field parallel to the sheet is

$$
h=2 \pi n I / b
$$

and reverses its direction as we pass through the sheet.

If, as is assumed, $t / b$ is small, the value of $h$ due to any one layer will remain the same throughout the winding section, so that the field acting on the top layer due to the remaining layers is ( $m-\mathrm{I}$ ) $h$, on the next layer $(m-3) h$, on the third layer $(m-5) h$ and so on.
The mean square field for all the layers is therefore

$$
\begin{aligned}
h^{2}\left\{(m-1)^{2}\right. & \left.+(m-3)^{2}+(m-5)^{2}+\ldots\right\} / m \\
& =\left(m^{2}-1\right) h^{2} / 3=\frac{4}{3} \pi^{2}\left(m^{2}-\mathrm{I}\right)(n I / b)^{2} .
\end{aligned}
$$

Applying this result to the eddy loss formula it follows that the added resistance due to the action of $H_{b}$ is

$$
\frac{1}{3} \pi^{2}\left(m^{2}-\text { I }\right) R G(n d / b)^{2}
$$

The total resistance of a short shallow multilayer solenoid or disc coil is therefore got by
adding these two resistance terms to the skin resistance and we have

$$
\begin{equation*}
R_{c}=R\left\{\mathrm{I}+F+\frac{1}{3} \pi^{2}\left(2 m^{2}-\mathrm{I}\right) G\left(n d_{l} b\right)^{2}\right\} \tag{29}
\end{equation*}
$$

The same reasoning may be applied to the case where $b$ is not small compared with the radius of the coil and it is found that

$$
\begin{equation*}
R_{c}=R\left\{\mathrm{I}+F+\left(u+\frac{1}{3} \pi^{2 m^{2}-\mathrm{I}} m^{m^{2}}\right) G(m n d / b)^{2}\right\} \tag{30}
\end{equation*}
$$

In applying to solenoidal coils, the length $b$ is interpreted as the winding length of the coil and in applying to disc coils $b$ is the difference between the inner and outer radii. In either case, the width $t$ is supposed to be very small.

If the number of layers is large, and the total number of turns $(m \times n)$ is $N$, formula (30) becomes

$$
\begin{equation*}
R_{c}=R\left\{\mathrm{I}+F+\left(u+\frac{1}{3} \pi^{2}\right) G\left(N d^{\prime} b\right)^{2}\right\} \tag{3I}
\end{equation*}
$$

## 14. Multilayer Coils of Finite Winding Depth.

The theory of the losses in multilayer coils at very high frequencies has been given by Prof. Fortescue ${ }^{17}$ who, however, confinedhimself to coils for which the winding length lies between one and three times the overall radius, and, for solid wire coils, assumed the wire so thick that the square root law holds in regard to frequency. Otherwise his method is similar to that developed above. As regards the mean square field acting on the wires of the coil, Fortescue expresses this in the form

$$
\begin{equation*}
H^{2}{ }_{m}=K^{2} N^{2} I^{2} / D^{2} \tag{32}
\end{equation*}
$$

where
$D=$ overall diameter of coil.
$N=$ total turns,
and $K$ is a factor depending on the ratios

[^1]$b / D$ and $t / D$ in which $b$ is the winding length and $t$ the winding depth. Tables of values of $K$ are given covering the range $b \nmid D=0.5$ to $b / D=1.5$.

It turns out, however, that these longish coils are uneconomical both in regard to space, number of turns and length of wire, so that the writer considered it advisable to calculate the values for shorter coils and hoped that at the point of overlapping the values would link on to those of Prof. Fortescue. This, unfortunately, was not so, the discrepancy being such that in extreme cases the copper losses using Fortescue's values of $K$ are only 60 per cent. of those obtained with the writer's values. Some means of cheeking which of the values was the more reliable was therefore necessary.
The mode of arriving at the $K$ factors is long and tedious, and the writer was not prepared to spend the time recalculating the values over the range of Fortescue's coils. The check adopted was therefore to find whether the present or Fortescue's values trended satisfactorily towards the easily found values for infinitely long coils.
The factor $K$ becomes zero for such coils, but if we multiply $K$ by $b / D$ the limiting value for $b / D$ infinity is finite. The method of check adopted therefore was to plot curves for the various values of $t / D$ of the factor $K b / D$, using $D / b$ as abscissex, and to find which set of values was pointing to the proper value of $K b / D$ when $D / b=0$.
These curves, using the present values, are given in Fig. 6. It is seen that the portions of the curves linking the last calculated values of $K b / D$ for finite coils $(D / b=2)$ to the zero value are continuous with the remainder. This could not be done with Fortescue's values.

We are forced to conclude that the present values must be the correct ones. The curves of Fig. 6 may be used to get approximate values of $K$ for any length of coil.

The formula for $K b / D$ for infinitely long coils is

$$
\begin{equation*}
K b_{i} D=2 \pi\{(4 D-6 t)(3 D-3 t)\}^{\ddagger} \cdots \tag{33}
\end{equation*}
$$

For short coils the values of $K$ as found by the present writer are given in Table VII. In comparing with Fortescue's Table it should be noted that Fortescue uses the ampere as the unit of current, so that apart from the above mentioned discrepancy

Fortescue's $K$ values are ten times the present values.
For the case of solenoids, for which $t / D=0$, it is easily shown by the theory of the previous section that

$$
\begin{equation*}
K^{2}=\frac{4 D^{2}}{b^{2}}\left\{u+\frac{1}{3} \pi^{2}\left(\mathbf{I}-\frac{I}{m^{2}}\right)\right\} \quad \ldots \tag{34}
\end{equation*}
$$

and a similar formula holds for disc coils for which $b / D=0$. The mode of arriving at the values of $K$ in the general case is indicated in the Appendix.
We can now readily arrive at the A.C. resistance formula for multilayer coils by using the value of $H^{2}{ }_{m}$, given by (32) in the eddy loss formula (Io) and adding this loss to the ordinary skin loss. We then obtain

$$
\begin{equation*}
R_{c}=R\left\{\mathrm{I}+F+\frac{1}{4}\left(\frac{K N d}{D}\right)^{2} G\right\} \tag{35}
\end{equation*}
$$

## TABLE VII.

Values of the Factor $K$ in Formula (32).

| $b / D$ | 0.000 | 0.125 | 0.250 | 0.375 | 0.500 |
| :---: | :---: | :---: | :---: | :---: | :---: |
| $t / D$ |  |  |  |  |  |
| 0.0 | Inf. | 41.7 | 21.2 | 14.4 | 11.0 |
| 0.1 | 52.4 | 23.3 | 15.4 | 11.6 | 9.5 |
| 0.2 | 27.4 | 16.2 | 12.4 | 9.9 | 8.2 |
| 0.3 | 19.6 | 13.7 | 10.7 | 8.8 | 7.5 |
| 0.4 | 16.0 | 12.0 | 9.5 | 8.0 | 6.9 |
| 0.5 | 13.8 | 10.4 | 8.4 | 7.0 | 6.0 |

Note.-The column $b / D=0$ refers to many layered disc coils, and the row $t / D=0$ to many layered solenoids. When the layers are few the following values hold for $K$ :-

Solenoid with $m$ Layers.

| -0.000 | 0.125 | 0.250 | 0.375 | 0.500 |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
| $m$ | 0.0 |  |  |  |  |
| I | - | 30.1 | 15.6 | 10.7 | 8.3 |
| 2 | - | 39.2 | 20.0 | 13.6 | 10.7 |
| Inf. | - | 40.6 | 20.7 | 14.0 | 10.7 |
|  | - | 41.7 | 21.2 | 14.4 | 11.0 |

Disc Coil with $m$ Layers.

| $t / D$ | 0.1 | 0.2 | 0.3 | 0.4 | 0.5 |
| :---: | :---: | :---: | :---: | :---: | :---: |
| $m$ |  |  |  |  |  |
| 1 | 37.8 | 20.6 | 15.4 | 13.2 | 11.7 |
| 2 | 45.0 | 25.9 | 18.6 | 15.3 | 13.3 |
| 3 | 5 I .0 | 26.8 | 19.2 | 15.7 | 13.6 |
| Inf. | 52.4 | 27.4 | 19.6 | 16.0 | 13.8 |

This formula, together with Tables I. and VII. and the curves of Fig. 6, includes all the previous formulæ and tables and con-
with 350 turns of No. 24 D.s.c. wire, the dimensions of the coil being $D=7.5 \mathrm{~cm}$., $b=0.75 \mathrm{~cm} ., t=2.5 \mathrm{~cm}$.


Fig. 6.
stitutes the final solution of the A.C. resistance problem for solid wire coils with well spaced windings.

In application the self-capacity correction as given by (23) must also be made when the coil is worked at wave-lengths which are comparable with the natural wave-length of the coil.

## 15. Test of Formula by Comparison with Observed Resistances.

Our first example is a coil in which there is no space factor other than that due to the unavoidable insulation space. In this case the general field losses are the main term in the resistance, so that the comparison gives a very severe test of the general field loss formula.

A coil of 5,000 microhenries was wound

The D.C. resistance of the coil was 3.8 ohms and the following A.C. resistances were measured by the reactance-variation method in a resonant circuit :-

|  | Wave <br> length <br> (metres). | Measured <br> resistance <br> (ohms). | Calculated <br> copper <br> resistance <br> (ohms). |
| :---: | :---: | :---: | :---: |
|  |  | Total loss. |  |
| 2,000 | 370 | $\mathbf{3 2 2}$ | 0.87 |
| 2,500 | 283 | 242 | $0: 86$ |
| 3,000 | -222 | 193 | $0: 87$ |

The second example is for a series of coils of equal dimensions but of different inductances, each coil being measured at that wavelength which gives resonance with a condenser of $\mathrm{I}, 000 \mu \mu \mathrm{~F}$. The common dimensions are $D=8.3 \mathrm{~cm} ., b=1.5 \mathrm{~cm}$., $t=3.0 \mathrm{~cm}$.


The wire gauge and turns have been estimated from the measured D.C. resistance and inductance and the calculated A.C. resistance is based on these estimated values. There were also small variations in the measured dimensions, but for ease of calculation the average values of $D, b$ and $t$ throughout the series was used. Thus the loss ratio in the last column can only be taken as representing approximately the importance of the copper loss.

Other examples will be given later in illustrating points of design. The above examples are however sufficient to show that the basis of design must be that for minimum copper loss and not (as many people suppose) from the point of view of dielectric loss.

## 16. Design of Inductance Coils for Minimum Copper Losses-General Principles.

The general problem to be solved is to find what form of coil to use and what diameter of wire to employ to produce a coil having a given inductance and the smallest possible resistance at a given frequency. In regard to the form of the coil, factors other than the resistance and inductance also enter into the problem. Thus if we are considering a frame aerial the turn area enters into the question, as this determines the E.M.F. induced by a given external field. It is also sometimes considered that a flat type of coil is advantageous, as then we may obtain close coupling with another similar type of coil. It has even been suggested that the type of coil should be based upon the ease with which it may be adapted to existing types of plugs and plug holders. Relative costs of construction also play an important part in deciding between the relative merits of two coils so that, other things being equal, a coil which will give the requisite inductance with few turns is to be preferred to one requiring many turns, as the former coil will be quicker to wind. It is clearly therefore impossible to lay down a definite rule and say that a coil of any particular shape is the best coil for all purposes.

We may, however, prescribe certain conditions to fix our problem and find the " best" coil fulfilling these conditions.

For the purpose of comparing coils of
different shapes we will take three different assumptions as follows :-
(a) The volume of the copper is the same for all coils.
(b) The same length of wire is used in all coils.
(c) Each coil occupies the same space.

The first method of comparison is equivalent to the old D.C. problem in which a given amount of copper was available and it was required to utilise this copper to give the best D.C. time constant (ratio of inductance to resistance). The resulting shape of coil is sometimes referred to as the most economical shape. This would only be true if the cosf of the copper were the main cost of the coil or if the other costs remained constant. For the second method of comparison to be fair, the coil costs must be mainly proportional to the length of the wire. This seems fairly reasonable, especially in the case of stranded wire coils where the wire is expensive, and also takes into account the fact that the cost of winding depends largely on the length of wire. The third method is of value when compactness is an important consideration. A small difficulty arises in deciding how to define "space occupied." If we express this • by mere overall volume of coil our equations lead us to a single layer disc coil of very large diameter, but of negligible length, a form which is by no means compact. If we measure space occupied by the overall diameter (as was done by Prof. Fortescue) we are led to a very long solenoid. Prof. Fortescue got out of this difficulty by pointing out that after a certain length the gain in resistance was small and therefore recommended a coil of length equal to the diameter. If, however, we wish to give both length and diameter due weight the most suitable criterion appears to the writer to compare coils of equal surface area, and the Tables to be given refer to coils satisfying this condition. The coils compared on this basis therefore fall in diameter as the length increases, and the disc coil is kept of finite diameter. If sketches of a series of such coils be drawn they appear to the eye to be about equally compact, except perhaps for the long, small-diameter solenoids. However, it turns out that we have passed through the region of efficient coils before
reaching this stage. This mode of comparison may seem somewhat arbitrary, but it is necessary to make some such assumption to put the matter upon a numerical basis. In addition, it turns out that if the comparison be made on assumption (a), (b) or (c) the resulting best shape is nearly the same, and the difference between coils in the neighbourhood of the " best" shape are so slight in regard to increase of resistance that the designer is left with considerable latitude to fulfil conditions of good coupling or of accommodation into various shapes of spaces without thereby departing seriously from the best resistance conditions.

As regards the equally compact coils (condition (c)) the vertical columns of the Tables are absolutely valid and tell us the best depth of winding to employ for any given ratio of length to overall diameter.

## 1\%. Inductance of Coil.

As we are going to compare coils of equal inductance, we require an equation connecting the inductance with the dimensions and turns of the coil. The equation for the inductance will be written

$$
\begin{equation*}
L=\frac{L_{\mathrm{n}} N^{2} D}{\mathrm{I}, \mathrm{OOO}} \quad \ldots \quad \ldots \tag{36}
\end{equation*}
$$

in which $L_{0}$ is a shape factor and is given by the following Table, and $L$ is the required inductance in $\mu \mathrm{H}$.

TABLE VIII.
Values of $L_{0}$.

| $b / D$ | 0.000 | 0.125 | 0.250 | 0.375 | 0.500 |
| :---: | ---: | ---: | ---: | ---: | ---: |
| $t / D$ |  |  |  |  |  |
| 0.0 | Inf. | 18.68 | 14.43 | 12.02 | 10.37 |
| 0.1 | 17.46 | 12.92 | 10.52 | 8.93 | 7.78 |
| 0.2 | 11.51 | 9.10 | 7.58 | 6.49 | 5.68 |
| 0.3 | 7.82 | 6.33 | 5.31 | 4.57 | 4.00 |
| 0.4 | 5.26 | 4.27 | 3.59 | 3.08 | 2.69 |
| 0.5 | 3.48 | 2.82 | 2.37 | 2.03 | 1.78 |
|  |  |  |  |  |  |

## 18. Design for Given Volume of Copper.

We will first compare coils which are all wound with the same length and diameter of wire. If the length of the wire be $l$ we have

$$
\begin{equation*}
l=\pi N(D-t)=\pi N D(\mathrm{r}-\mathrm{t} / D) \ldots \tag{37}
\end{equation*}
$$

Now for each possible shape of coil equation (36) will determine $N^{2} D$ in terms of the
inductance, and equation (37) will determine $N D$ in terms of the length of wire, so that the two equations may be used to find $N$ and $D$ separately. As we pass from one shape of coil to another the values of $N$ and $D$ will vary, as (36) and (37) involve shape factors. Now in the resistance equation (35) the D.C. resistance will be the same for all the coils and also the functions $F$ and $G$ will be the same. In fact the only quantity that varies for different shapes of coils is the factor $K N / D$. The best shape of coil is clearly therefore the one which yields the minimum value for $K N / D$. If we use (36) and (37) to express $N$ and $D$ in terms of shape factors we find that $K N / D$ is proportional to $K(\mathrm{I}-t / D)^{3} / L_{0}{ }^{2}=\phi$ say.
The values of $\phi$ are given in Table IX.
The Table shows that the best coil is a single-layer coil having a winding length equal to one-third its diameter. The best single-layer disc coil should have a winding depth equal to one-quarter the external diameter. If a multilayer coil is desirable the Table shows that there is a wide range

## TABLE IX.

Values of $\phi=K(\mathrm{I}-t D)^{3}{ }^{\prime} L_{0}{ }^{2}$.

$$
\text { S.L. }=\text { Single-layer. } \quad \text { M.L. }=\text { Multi-layer. }
$$

| $b / D$ | 0.000 |  | 0.125 | 0.250 | 0.375 | 0.500 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $t / D$ |  |  |  |  |  |  |
| O | S.L. M.L. |  | $\begin{aligned} & 0.087 \\ & 0.120 \end{aligned}$ | $\begin{aligned} & 0.075 \\ & 0.102 \end{aligned}$ | $\begin{aligned} & 0.075 \\ & 0.100 \end{aligned}$ | $\begin{aligned} & 0.077 \\ & 0.102 \end{aligned}$ |
|  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |
| O. 1 | 0.090 | 0.125 | 0.102 | 0.101 | 0.106 | 0.114 |
| 0.2 | 0.080 | 0.106 | O.101 | 0.110 | 0.120 | 0.130 |
| 0.3 | 0.079 | 0.110 | 0.118 | 0.130 | O. 144 | 0.161 |
| 0.4 | O.IOI | 0.124 | 0.142 | 0.158 | 0.182 | 0.206 |
| 0.5 | 0.120 | 0.142 | 0.163 | 0.187 | 0.212 | 0.236 |
|  |  |  |  | 1 |  |  |

of choice, and if we make $5 t+3 b=D$ we
shall never depart greatly from the condition of maximum efficiency.

Next, keeping the shape constant, let us suppose a series of coils wound with different diameters of wire, the length of wire being chosen so that the volume of copper remains constant ; that is, $l$ must be proportional to $I / d^{2}$ Again the number of turns and the overall diameter will vary from coil to coil. The inductance equation (36) shows that $N^{2} D$ remains constant and (37) shows
that $N D$ is proportional to $l$. Hence $N$ is proportional to $I_{i l}^{\prime l}$ and $D$ to $l^{2}$ so that $N / D$ is proportional to $I l^{3}$, that is to $d^{6}$. In the resistance equation (35) the D.C. resistance $R$ varies as $l / d^{2}$ or as $\mathrm{I} / d^{2}$. The functions $F$ and $G$ also vary. For low frequencies $F$ is negligible and $G$ varies as $d^{4}$.

Then from these variations and (35), the skin resistance $R_{s}\{=R(\mathrm{I}+F)\}$ is of the form $A / d^{4}$ and the general fiek resistance $R_{h}$ is of the form $B d^{1 t}$ where $A$ and $B$ are independent of diameter.
The whole resistance may therefore be written

$$
\begin{equation*}
R_{c}=R_{s}+R_{h}=\frac{A}{d^{4}}+B d^{4} \tag{38}
\end{equation*}
$$

The best diameter of wire is found by differentiating (38) with respect to $d$ and equating to zero. Doing this we find

$$
\begin{equation*}
2 R_{s}=7 R_{h} \tag{39}
\end{equation*}
$$

For high frequencies $F$ and $G$ are large and both proportional to $d$. In this case $R_{s}$ is of the form $A^{\prime} / d^{3}$ and $R_{h}$ of the form $B^{\prime} d^{11}$. The minimum occurs when

$$
3 R_{s}=\operatorname{II} R_{h} \quad \ldots \quad(40)
$$

The difference between conditions (39) and (40) is so slight that we may say that for any frequency the best diameter is such that $R_{s}=3.6 R_{h}$.

The practical method of deducing the proper diameter is deferred to a later section.

## 19. Design for Fixed Length of Wire.

If coils of equal length of wire are compared, the best shape is found exactly as in the last section. The best diameter is however different, as we now suppose the different coils wound with the same length of wire in all cases.
$R$ now varies as I/ $d^{2}$ and for a given shape $N / D$ remains constant. Hence at low frequencies $R_{s}$ is proportional to $I / d^{2}$ and $R_{h}$ to $d^{+}$so that by the method of the previous section the condition for the best diameter becomes

$$
\begin{equation*}
R_{s}=2 R_{h} \tag{4I}
\end{equation*}
$$

At high frequencies $R_{s}$ is proportional to $I / d$ and $R_{h}$ to $d$ and the best diameter condition is

$$
\begin{equation*}
R_{s}=R_{h} \tag{42}
\end{equation*}
$$

## 20. Design for Fixed Overall Surface.

The overall surface $S$ of the coil is given by the formula

$$
S=\frac{1}{2} \pi D^{2}(\mathrm{r}+2 b / D) \quad \ldots
$$

It is clear that if we take a coil of given shape and inductance the condition of constant surface fixes the diameter of the coil and the inductance then fixes the number of turns. Hence the length of wire is fixed and the best diameter of wire is therefore the same as in the last section.
The best shape of coil depends to some extent upon the frequency.

For low frequencies the condition $R_{s}=2 R_{h}$ makes $d^{3}$ proportional to $D / K N$, and when the condition is satisfied

$$
\begin{equation*}
R_{c}=6 \rho N(D-t) / d^{2} \tag{44}
\end{equation*}
$$

or on elimination of $d, R c$ is proportional to

$$
(\mathrm{I}-t / D)\left(K^{2} N^{3} D\right)^{4}
$$

The inductance equation (36) and the surface equation (43) enable us also to express $N$ and $D$ in terms of shape factors, and we obtain finally

$$
R_{c}=A K^{3}(\mathrm{I}-t D)(\mathrm{I}+2 b / D)^{4} L_{0}{ }^{5}{ }^{6}(45)
$$ in which $A$ is proportional to $f^{3} L^{5 / 6} / S^{\ddagger}$.

By a similar process of reasoning in the case of high frequencies we obtain

$$
\begin{equation*}
R_{c}=B K\left(\mathrm{I}-t_{l}^{\prime} D\right)\left(\mathrm{I}+2 b_{i}^{\prime} D\right)^{b^{\prime}} L_{0} \tag{46}
\end{equation*}
$$

in which $B$ is proportional to $f=L / S^{3}$
To compare various shapes of coils we must therefore calculate the shape factors (45) and (46). The results are given in Table $\mathbf{X}$.

On comparing these Tables with Table IX., it is seen that the best coils for a given space are somewhat longer than the best coils for a given amount of copper, but, owing to the slow variation in the neighbourhood of minimum resistance, the same general rules may be taken to apply in both cases. The advantage of single layer coils is again apparent.

A study of the numbers in the vertical columns is instructive, as the numbers in any one column refer to coils of equal outside surfaces and we therefore see how far it is profitable or otherwise to increase the winding depth. Thus if we have decided that it is convenient to use a winding length equal to one-quarter the overall diameter, the column $b / D=0.25$ tells us the best resistance we can get for various winding depths.

The first four numbers show that as we pass from single to multilayer coils, keeping .the winding depth very small, there is a progressive increase in resistance. The
remaining; five numbers give the: effect of diminishing the internal diameter keeping: the external diameter constant. There is: a diminution, in resistance until $t / D=0.15$ and then the: resistance increases again, but the gain obtained by increasing the winding: depth is not enough in this case to counterbalance the loss in passing from a: single layer to a multilayer coil.

It must be emphasised that these comparisons refer to coils of equal inductance; so that for each winding depth the turns: are always adjusted to satisfy this condition; also the diameter of wire in each case is such as to give the minimum resistance for each shape. In this respect the comparison differs from that usually made in comparing various shapes of coils, where itis nearly always assumed that the diameter of the wire is constant.

## 21. Large and Small: Coilgcompared.

The forms of the $A$ and $B$ factors in (45) and (46) enable us to answer: the question as: to what: advantage may be expected by increasing the diameter of the coil.

Since $S$ is proportional to $D^{2}$ for a given shape, the form of $A$ shows that for lowe frequencies the "best" resistance varies inversely as the square root of the diameterand for high frequencies: the "best." resistance varies inversely as the diameter as is seen from the form of $B$.

The reason for the slow increase in efficiency with increase of coil diameter at low frequencies is seen when we consider the variation of " best" space factor with coil diameter.

The area of winding section available per turn is proportional to $D^{2} / N$, and, if the inductance is held constant, $N^{2}$ is

TABLE X.
Relative Resistances of Coils of Equal Inductange and Equal Surface.
(A) Low Frequencies.

(b) High Freguencies.

| $b / D$ | 0.000 |  | 0.125 | 0.250 | 0.375 | 0.500 | 0.625 | 0.750 | 0.875 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $t / D$ |  |  |  |  |  |  |  |  |  |
|  | ( one | yer | 1.81 | 1.32 | r. 18 | 1.12 | 1.12 | 1.12 | 1.14 |
|  | <two | yers | 2.36 | - 1:68 | T. 49 | 1.40 | 1.39 | 1.38 | r. 39 |
| 0.0 | $\zeta$ three | ayers | 2.44 | r. 74 | 1:44 | I.43 | 1:42 | 1.42 | I:43 |
|  | Cmult | yer | 2.51 | r. 79 | I. 49 | r. 49 | 1.45 | 1.45 | 1.46 |
|  | S.L. | M.L. |  |  |  |  |  |  |  |
| O.I | 1.95: | 2.70 | 1.82 | r:60 | I'54 | 1.55 | - | - | - |
| 0.2 | 1.43 | 1.90 | I. 59 | 1. 59 | 1.61 | 1.64 | - | - | - |
| 0.3 | 1.26. | 1.76 | 1:70 | 1.72 | r. 78 | I. 86 | - | - | - |
| 0.4 | r. 50 | 1.82 | 1.88 | 1.93 | 2.06 | 2:17 | - | - | - |
| 0.5 | 1. 68 | 1. 98 | 2:06 | 2.16 | 2:27 ${ }^{\circ}$ | $2.37{ }^{\text {, }}$ | - | - | —. |

proportional to $I / D$. Hence as the coil diameter increases the area commanded by a turn varies as $D^{5 / 2}$. On the other hand the best diameter of wire is proportional to $(D / N)^{\frac{1}{d}}$ that is to $D^{\frac{1}{2}}$ when the inductance is held constant, so that the copper area only increases as the diameter of the coil.

Thus although there is a rapid increase in available space for the turns we cannot take advantage of this as we would increase the A.C. resistance of the coil by filling the space with copper. The only way to make further use of the available space is to use stranded wire instead of solid wire.

Note.-Since the publication of the previous instalment of this article, Mr. A. L. M. Sowerby has published an article in which inductance coils of various forms are compared experimentally (E.W. \& W.E., June, 1926). He arrives at the conclusion that there is a " best" spacing but not a " best" ratio of length to diameter. His results are not, however, in contradiction with theory as
the coils prepared by him all had equal wire diameters. In Table I. of Mr. Sowerby's paper it is shown that the H.F. resistance remained constant for a large variation of length-diameter ratio. Fortunately, Mr. Sowerby also adds a column giving the D.C. resistance of the various coils, which is shown to pass through a definite minimum. Since the diameter of the wire is constant, this means that the length of wire employed passes through a minimum, and therefore there is a "best" shape, viz., that which will produce the coil with the minimum amount of wire. Similarly, if we calculated the spaces occupied by each coil as defined by condition (c) above, we should find a "most compact" coil in Mr. Sowerby's series. These remarks seem to emphasise how careful one must be in deciding what is the best shape of coil, and shows why so many different recommendations have been made from time to time.

## S. Butterworth.

$$
\text { 12th June, } 1926 .
$$

## Among the Experimental Transmitters.

THE indiscriminate output of QSL cards by listeners is already becoming a nuisance to those amateur transmitters who are engaged in serious research. A great number of these cards undoubtedly fulfil the desired object of furnishing transmitters with useful reports concerning their tests, but we fear that an ever-increasing number is sent out merely with a view of collecting " wallpaper," and resembles the quest by small boys for cigarette cards. The senders of these worthless cards appear aggrieved if the recipients fail to reply. One amateur informs us that if he were to send a reply to every QSL card he receives, it would be necessary for him to engage a secretary for that purpose only, and the cost in postage would be very considerable.
Perhaps the best solution to this difficulty would be for transmitters to devise a code which would signify the nature of the reports desired. An experimenter working on comparatively high power would probably only be interested in reports from distant stations, while one conducting tests on low-power might welcome cards from listeners at all ranges; another might only be interested in fading effects or meteorological observations, and cards merely stating that his signals had been heard would be of no material use. We suggest that the T. \& R. Section of the R.S.G.B. should consider this matter and arrange some such code. Listeners who wish to help experimenters will then know what special records are desired, while
mere collectors of wallpaper will tacitly be warned that no contributions to their collections may be expected from that particular source.

## Norwegian and Danish Amateurs.

We understand from correspondents in Norway and Denmark that amateurs in those countries are now able to obtain transmitting licences.
In Norway licences are being issued for transmission on 3-6, 29-35, 43-47, 69-75 and 100-120 metres. The aerial power must not exceed 20 watts. The licencee must possess certain technical qualities and be able to send and receive at least 12 words per minute. The annual fee is fixed at 30 kroner. QSL cards should be sent via the Norsk Amatoer Sender Union, Oslo. The nationality prefix adopted by Norwegian amateurs is LA.

Our Danish correspondent states that the maximum input allowed to amateurs is 100 watts, and the wavelengths allotted are: Below 15 metres, $43-47,70-75$ and $95-115$ metres. Transmission is not permitted between 7.30 and 10.30 p.m. (18.30-21.30 G.M.T.). Through the courtesy of the Telegraph Department, the callsigns allotted will begin with the figure 7 , and the letters used will, as far as possible, be the applicant's initials; this arrangement allows previously unlicensed experimenters to retain the call-signs they have hitherto used. QSL cards may be sent via Mr. James Steffensen ( $\mathrm{D}_{7} \mathrm{JS}$ ), Ehlersvej 8. Hellerup, Denmark.

# The Multi-Range Ammeter of Constant Resistance. 

By L. G. A. Sims, M.Sc.(Eng.), and M. Heyzood Hunt, M.Sc.(Eng.).

THE practice of extending the range of an ammeter, by placing shunts of suitable value across the instrument, is one which appeals particularly to the radio experimenter, since a single reliable meter can readily be arranged to perform the work of several, with greatly reduced outlay in the first instance.

The presence of such a shunt naturally alters the resistance of the complete instrument, as far as the external circuit is concerned ; a fact which, perhaps, is not always fully appreciated, since the cases in which it assumes an importance are comparatively infrequent.

For really accurate work, however, and in certain types of circuit, this alteration of resistance of ammeter and shunt, when used on different ranges, should be taken into account, and it is the object of the present article to indicate certain modifications of the ordinary shunted instrument which, while in no way limiting the utility of the meter as a multi-range instrument, yet ensure that the total resistance of the circuit will not be altered appreciably when changing from one range to another.

Before proceeding to describe the proposed modifications of the ordinary shunted instrument it would, perhaps, assist the reader in following the subsequent argument if a brief consideration of the case of the simple shunted instrument is recalled.

Suppose that the instrument gives a certain deflection when a current $i_{1}$ flows through it, and suppose that we wish to shunt it in such a manner that we get the same reading when a current $I=n . i_{1}$ is flowing through the combination.

Let the resistance of the instrument alone $=r_{\mathrm{g}}$. and the resistance of the shunt to be added $=r_{s}$.

Then the combined resistance of ammeter and shunt is given by

$$
\begin{equation*}
R_{c}=\left[\frac{r_{g} \cdot r_{s}}{r_{g}+r_{s}}\right] \tag{I}
\end{equation*}
$$

The shunt must have such a value that the current flowing through the instrument is still $i_{1}$ while the total current is $n . i_{1}$.

Hence, for direct current, the current through the shunt is ( $n-\mathrm{I}$ ). $i_{1}$.

Since $\quad(n-1) . i_{1}, r_{s}=i_{1}, r_{g}$
we have $\quad r_{s}=\left[\frac{r_{g}}{(n-\mathrm{I})}\right] \quad$.
Substituting in ( I ) for $r_{s}$ we get :-
whence

$$
R_{c}=\left[\frac{r_{g} \cdot \frac{r_{g}}{(n-\mathrm{I})}}{r_{g}+\left\{\frac{r_{g}}{(n-\mathrm{I})}\right\}}\right]
$$

We want to arrange matters so that the combined resistance of the ammeter and shunt shall be the same as that of the


Fig. 1.


Fig. 2.
instrument when used by itself, and in order to secure this condition we shall have to insert an extra resistance in series with the combination. Consider now the circuit shown in Fig. I. Let $R_{t}$ be the total resistance between the line terminals under the new conditions.

We have

$$
R_{l}=r_{2}+\left[\frac{r_{g} \cdot r_{1}}{r_{g}+r_{1}}\right]
$$

But as on its lowest range we shall require the instrument to read directly, without a shunt, and as on all ranges the total resistance between line terminals is to be constant, then $R_{l}=r_{g}$.

From (3) we have :-

$$
\begin{align*}
& R_{t}=r_{2}+\frac{\mathrm{T}}{n} \cdot r_{g} \quad \ldots \quad . \quad(4) \\
& \text { Whence } \quad r_{2}=\left[\frac{n-1}{n} \cdot r_{g}\right]  \tag{4~A}\\
& \text { also } \\
& {\left[\frac{r_{\mathrm{g}} \cdot r_{1}}{r_{\mathrm{g}}+r_{1}}\right]={ }_{n}^{\mathrm{I}} . r_{g}} \\
& \text { or } \\
& {\left[\begin{array}{c}
r_{g}+r_{1}+r_{1} \\
\hdashline r_{2}
\end{array}\right]=\frac{I}{n} .} \tag{5}
\end{align*}
$$

These expressions are quite general, and we may, as a check, take the case when the instrument is reading directly. Then $n=\mathrm{I}$.

From (5) $\left[\frac{r_{1}}{r_{g}+r_{1}}\right]=\frac{I}{n}=I$ for this case. This is only satisfied when $r_{1}$ is infinitely large compared with $r_{g}$, that is to say when the instrument is not shunted.

Also from (4) $r_{2}+\frac{\mathrm{x}}{n} \cdot r_{g}=r_{g}$, giving $r_{2}=0$, which is also correct for this case.

Equations (4A) and (2) provide the necessary information concerning the values of the shunt and series resistances required, in terms of the ammeter internal resistance, and the particular multiplying factor desired. Accordingly, a set of such resistances could be constructed and mounted in the case containing the instrument, together with suitable switches for changing the series and shunt resistances, according to the required ratio.

The arrangement would then be as shown in Fig. 2 in which a rotating wiper moves over two sets of contact studs. It is needless to add that, for accurate work, this switch should be a sound mechanical job, since small contact resistances will easily upset the values of the shunt resistances, especially on the higher ranges, and will falsify the readings. An alternative method would be to employ plugs, or even terminals, though the latter would be undesirable for rapid changing from one range to another.

There is, however, a method which is superior to that previously described, so far
as the question of changing the shunts and series resistances is concerned. It will be seen on reference to Fig. I that the two resistances $r_{1}$ and $r_{2}$ can really be regarded as one single resistance, with the ammeter placed across one end and a tapping point.

Now it is possible, by taking suitable tapping points, so to arrange matters that, when a portion of the resistance in shunt with the ammeter is cut out, it is added, at the same time, to the total resistance in series. The amount which is removed from the shunt can be arranged to be almost exactly the required addition to the series branch. It cannot be arranged to be quite the correct amount, as will be shown below, but the error involved is very small ; smaller in most cases than the error involved in reading the instrument, and the simplification obtained is considerable. The arrangement in this case will take the form shown in Fig. 3.


When using the instrument direct, the link between terminals $b$ and $c$ is opened, and the flexible lead from one side of the ammeter is connected to terminal $b$ (No. I). For other ranges, it is connected to terminals 2, 3, 4, etc., and the link between $b$ and $c$ is replaced.

The resistance between the line terminals will be practically constant for all ranges; and equal to the resistance of the ammeter itself within very close limits.

We will now consider the design of suitable resistances to convert an ordinary ammeter into a constant resistance, multi-range instrument.

Taking first the case of a fairly high resistance instrument, such as would be employed for the measurement of small currents, we have :-

Resistance of instrument, $\boldsymbol{r}_{\mathrm{g}}=30$ ohms.

Required : $r_{1}$ and $r_{2}$ for the following ranges :-

$$
n=I O, \quad I O O, I, O O O
$$

For the case where $n=1$ the instrument is used by itself.
For $n=10$ :

$$
\begin{aligned}
& r_{1}=30 / 9=3.333 \text { ohms. } \\
& r_{2}=\left[\frac{n-\mathrm{I}}{n}\right] r_{g}=27.0 \text { ohms. }
\end{aligned}
$$

For $n=100$ :
$r_{1}=0.303 \mathrm{ohm}$.
$r_{2}=29.70$ ohms.
For $n=1,000$ :
$r_{1}=0.03003$ ohm.
$r_{2}=29.95$ ohms.
The above values are absolutely correct for constant resistance between terminals. For the approximate method described, we shall take the value of the shunt resistance as correct. This will mean that the amount of series resistance on each range will not be exactly correct, but will have the values calculated below.

For cases $n=10$ and $n=100$ respectively, $r_{1}=3.333 \mathrm{ohms}$, and 0.303 ohm .

The amount of resistance eliminated, in changing from range $n=10$ to range $n=100$, is therefore equal to 3.030 ohms , which value becomes the added series resistance. For $n=10, r_{2}=27.0 \mathrm{ohms}$.

The actual series resistance for $n=100$, is therefore equal to 30.03 ohms.

The value required from the theoretical equation is 29.7 ohms. It will be evident that we have added slightly more resistance than is actually required.

Similarly, the amount of shunt resistance eliminated in changing from range $n=100$ to range $n=1,000$, is equal to 0.273 ohm , very nearly.

The added resistance now becomes 30.303 ohms, the theoretical value being 29.95 ohms.

Taking the expression :-

$$
\left[r_{2}+\frac{r_{g} \cdot r_{1}}{r_{k}+r_{1}}\right]=R_{t}
$$

and inserting the values found for $r_{1}$ and $r_{2}$ for the various ranges, we can find the actual resistance of the combination on any range.

Thus, for $n=$ io, $R_{t}$ (total resistance)
$=30$ ohms.

$$
\begin{array}{ll}
\text { for } n=100, R_{t} & =30.324 \mathrm{ohms} \\
\text { for } n=1,000, R_{t} & =30.3325 \mathrm{ohms}
\end{array}
$$

The percentage errors for the respective ranges are therefore:-

$$
0.0 \%, 1.07 \%, \text { I.II } \%, \text { very nearly. }
$$

We can, however, obtain a useful general expression for the error in the total resistance when this approximate method is employed.

It is very important to notice that, because the shunt resistance is always accurate, the ratio " $n$ " is accurate in all cases, and the approximation has no effect upon the reading of the instrument on any range.

In considering the error in overall resistance, it is advisable to change slightly the symbols previously employed.

Let $K=$ the first ratio of the series.
$n=$ any ratio of the series, as previously.
$R=$ shunt resistance, calculated from the equations we have derived, and applying at ratio $K$.
$R_{s}=$ series resistance calculated from the equations at ratio $K$.
$R_{i}=$ total resistance between terminals on any ratio.
$\boldsymbol{r}_{\mathrm{g}}=$-instrument resistance alone, as previously.
As the ratio is changed from the first value $K$ let the shunt resistance $R$ assume successive values $r_{1}, r_{2}, r_{3}$, etc. Then, referring to Fig. 4 we see that the following applies.
On Ist Range :

$$
\begin{aligned}
& R=\infty \quad \text { Instrument alone. } \\
& R_{s}=0
\end{aligned}
$$

On 2nd Range :

$$
\begin{aligned}
& R=\left(\frac{r_{g}}{n-\mathbf{I}}\right) \\
& R_{s}=\left(\frac{n-\mathbf{I}}{n} \cdot \gamma_{g}\right)
\end{aligned}
$$

from equations (2) and (4A).
Up to this point everything is correct, but on succeeding ranges the value of $R_{\mathrm{s}}$ is not calculated, but is simply increased by the amount of shunt resistance eliminated on changing to those ranges.

Hence, added series resistance as $R$ changes
to $r_{N}$ (where $N$ is the number of ranges counted after the second)
$=R-r_{1}+\left(r_{1}-r_{2}\right)+\left(r_{2}-r_{3}\right)+\ldots+\left(r_{N-1}-r_{N}\right)$

$$
=\left(R-r_{N}\right) \text {. }
$$

Thus total series resistance is given by

$$
R_{s}+\left(R-r_{v}\right)
$$

Hence on any range where the approximation is used we have

$$
R_{t}=R_{s}+\left(R-r_{N}\right)+\left[\begin{array}{l}
r_{g} \cdot r_{N} \\
r_{g}+r_{N}
\end{array}\right]
$$

Thus we have

$$
\frac{R_{t}}{r_{g}}=\left(\frac{K-\mathrm{I}}{K}\right)+\frac{\mathrm{I}}{K-\mathrm{I}}-\frac{\mathrm{I}}{n-\mathrm{I}}+\frac{\mathrm{I}}{n}
$$

This simplifies down to

$$
\frac{R_{t}}{r_{g}}=\left[\frac{K^{2}-K+\mathrm{I}}{K(K-\mathrm{I})}-\frac{\mathrm{I}}{n(n-\mathrm{I})}\right]^{*}
$$

As $\boldsymbol{r}_{g}$ is the constant value of resistance which we have approximated, the above expression gives the error at once. It will be noticed that the error depends only on the ratios used, and that it becomes slightly greater as the value of the ratio increases, in a given range of ratios. For ratios of the orders 1o, 100, etc., the error is about I per cent., rising to about 5 per cent. for ratios of the orders 5, 25, etc. A short consideration will show how vastly

$$
\begin{aligned}
& \text { *[This formula may be written } \\
& \qquad \frac{R}{r_{g}}-\mathrm{I}=\frac{\mathrm{I}}{K(K-I)}-\frac{\mathrm{I}}{n(n-1)} .
\end{aligned}
$$

Since the ideal is to make $R_{t} / r_{\mathrm{g}}=\mathrm{I}$ this expression gives the error as a fraction, or, if multiplied by 100 , as the percentage error. Since $n$ is usually at least 5 or to times $K$, the second term is negligible and the percentage error is equal to $100 / K(K-I)$. If $K=5$ this gives an error of 5 per cent., whereas if $K=$ io the error becomes I.II per cent. A comparison of these figures with those given by the authors in the last column of the table shows the accuracy of the approximation.-G. W. O. H.]
superior to the case of the simple shunted instrument this is, and it should be remembered that the error can be entirely eliminated if the approximation is not employed. The following table has been compiled from the foregoing equations for two high resistance instruments, such as the radio experimenter would be likely to employ, and for a low resistance instrument. Equations (2) and (4A) are those from which the required shunt and series resistances are calculated.

| Ratio. | Instrument resistance. | Shunt <br> resistance. | Series resistance. | Per cent. error from constant resistance. |
| :---: | :---: | :---: | :---: | :---: |
| I to I instr. alone .. | 1 <br> I 16 ohms | $\infty$ | 0.0000 | 0.00 |
| Io to I . . | " | 12.8870 | 104.4000 | 0.00 |
| IOO to I .. | " | I.1717 | II6.1I70 | I. 110 |
| 1,000 to 1.. | " | 0.1161 | 117.1830 | I. I I I |
| I to I instr. alone .. | 30 ohms | $\infty$ | 0.0000 | 0.00 |
| Io to I .. | " | 3.3333 | 27.0000 | 0.00 |
| 100 to I .. | " | 1.3030 | 30.0300 | 1.110 |
| 1,000 to I.. | " | 0.0300 | 30.3030 | I.III |
| I to I instr. alone | 1. 0 ohm | $\infty$ | 0.0000 | 0.000 |
| 5 to I | " | 0.2500 | 0.8100 | 0.000 |
| 25 to 1 . | " | 0.4170 | 1.0083 | 4.975 |
| 125 to I. | " | 0.0081 | 1.0419 | 4.999 |

## A New Theory.

The Effect of the Moon on Radio Reception.

By Derek Shannon (5PX).

[R113.8

ONE of the outstanding problems still to be solved in connection with the transmission and reception of wireless or ether waves, is the effect experienced on long-distance reception of variation of signal strength from a given input power to the transmitter and stable and unaltered conditions at both receiving and transmitting ends.
For instance, a station will be received quite satisfactorily for a considerable period of time, perhaps for some weeks, and then for some unknown reason the signals from that station will become quite or nearly inaudible on the same receiver; this condition may last for some days, and then for no apparent reason the signal strength will again become normal, without anything having been altered in either transmitter or receiver.

This effect has nothing to do with the wellknown phenomenon of increased signal strength at night, which is believed to be due to an ionised layer of gas in the upper atmosphere, known as the Heaviside layer, so called after the originator of this theory, the late Oliver Heaviside.
My purpose here is to suggest an entirely new theory to account for the cause of this variation in signal strength at different periods of time, of which nothing very much appears to be known.
I have carried out a great deal of experimental work to prove my theory, and I have now arrived at some definite results, which other workers in the field can follow up for the benefit of radio science.
I have noticed from time to time when listening to radio transmissions from distant stations that on very moonlight nights results always appear to be much better than at other times, especially on transmissions from American or other far-away stations. This led me to wonder whether the moonlight had something to do with this effect, so I commenced to make notes of
good nights and bad nights for reception, and after a certain period of time I found that good reception always occurred during the period of full moon, even when locally the moon was obscured by clouds or bad weather.
So I have come to the conclusion that it is not the moonlight which has the effect of increased signal strength, but the rotation of the moon round the earth.

From observations I have taken on the reception of KDKA, the East Pittsburg station, U.S.A., at a distance of 3,500 miles, over a definite period of time using a very constant receiver and a special device for measuring the received signal strength, I have noticed that as the moon commences to decline from full so does the received signal strength also decline (this, of course, is in direct opposition to our present knowledge, that as the rays of the sun grow stronger over the path of the received signal, the strength of the signal declines). As the moon declines still further the signal strength diminishes and at the time when the moon is invisible, i.e., when the moon is between the earth and the sun, the signal strength is at zero. As the moon begins to move towards the first quarter the signals show an increase in strength, at first slight, but rapidly increasing until about three days before the moon is full, when maximum strength is registered which remains until about three days past the full. Signals then begin to fade and the process is repeated through the next lunar month.
I will now proceed with the description of the apparatus used for these tests. There were many factors to be considered in the choice of a constant and stable receiver, and the problem was especially difficult on the short wavelength on which KDKA transmits, namely 63 metres. I finally decided to make these tests on this station, as it is a fairly constant transmission, and at a sufficient distance to make fading and changes
in signal strength apparent. After much consideration and many tests, I finally decided to use a simple one-valve circuit with two stages of low-frequency amplification. The greatest trouble I considered would be the use of reaction, which of course had to be used to enable reception to be obtained from KDKA.

However, I overcame this difficulty by using the Reinartz circuit and method of reaction control, and fitting it with a very fine screw vernier adjustment, and a fine scale with a pointer for accurate setting.

Once this was set at a critical point it was never touched again during the whole test period, and the set remained quite constant in its action. Also, the set was
the test bench. No loose leads were allowed to hang about, and the whole of the wiring was done with 4 ft . of square tinned copper wire. V24 valves were used both for detector and low frequency. The low-frequency amplifier was used for listening to the transmission on phones. For the actual measurement of the signal strength an extremely sensitive mirror galvanometer was used.

The deflection was not very great, but was magnified by focusing the light spot on a special screen.
I so arranged matters that a full scale deflection of the light spot travelled over a scale of ro inches. The screen was of ground glass with the scale marked on it, and


Fig. I. Circuit arrangement employed for the observations.
fitted with a filament voltmeter and ammeter and a plate voltmeter and milliammeter (Weston) to ensure always having the exact conditions in the receiver every time it was used.

The aerial consisted of a single wire $L$ type $7 / 22$, 50 ft . long with 20 ft . down lead, and 40 ft . high at both ends. The aerial was stretched as tightly as possible between the masts and kept strained by means of a weight on one halyard, so that no capacity change should take place due to the aerial swinging or otherwise moving. The aerial was aperiodically coupled to the receiver.
The earth connection was only to ft . long, single 14 -gauge copper, taken straight to a main waterpipe right below the set. Every part of the apparatus was rigidly fixed to
divided into 10 divisions, each division being again split into ro, the object of this being to enable a graph to be prepared on a squared paper chart.
The galvanometer was inserted in series with the plate of the detector valve, and shunted by a variable resistance to enable adjustments to be made. The light spot was then adjusted to zero on the scale with no signals coming in, but with the set switched on and tuned to 63 metres. When the carrier wave of the station came in, the galvanometer dipped, and the scale was so arranged that the dip showed as an increased reading, that is to say, the scale was arranged actually the reverse way round, so that the reading should commence from zero and finish at 10 .

Fig. I shows the circuit arrangements.
When all was prepared a preliminary trial was made every night for one week to test if the apparatus would remain stable without adjustments, and this was found to be so, provided all values of voltage and current

23rd February, the night of the new moon, and continued every night until 24th March. the night of the next new moon. Owing to KDKA not transmitting on the short wavelength on Sundays, no reading is shown for this day.


Fig. 2.
on filaments and anodes were kept adjusted exactly as when the apparatus was first set. A start was made on .Ioth January, I925, at II. 30 p.m., this date being the night of full moon, and observations were taken until 12 o'clock. Slight variations took place during the half hour, but a reading was taken every five minutes, and an average was struck, this average being entered on the

Fig. 3 shows the graph of these observations.

It will be noticed that in each case a rise in signal strength is indicated towards full moon, which falls off again as the moon declines. Of course the graph does not show a straight line up and down, but we must take into consideration the fact that slight alterations may take place in the actual


Fig. 3.
graph. This test was made every night until 8th February, the night of the next full moon.

Fig. 2 shows the graph of these observations.

A second test was made commencing on
transmitter, which would account for this, but this does not alter the fact that the strength rises as the moon increases and falls as the moon declines.

At the present time I am occupied in trying to devise a set with a number of
stages of H.F. amplification, and a carborundum rectifier in which no reaction will be used. This will be used on a higher wavelength, and I would suggest that anyone who wishes to carry out experiments in this line of research should proceed on these lines, as the elimination of reaction would remove the greatest variable factor. I would also lay great stress on the necessity of making everything as rigid as possible, and the whole apparatus must be untouched during the period of the test. I must also add that the readings indicate the carrier wave strength, and not the amount of modulation; this is very important, as the modulation of a telephone transmitter, especially at a broadcasting station, is constantly being altered, and would provide no indication of strength of reception.

On many occasions during these tests, the actual speech transmission was nearly inaudible, but the carrier wave was extremely strong. I mention this fact so that if anyone makes such tests they will take no notice of the actual strength of the speech or music received, but will only deal with the strength as actually shown on the galvanometer, which is the carrier wave strength.

Referring to the circuit shown in Fig. I, the values of components used are as follows :-

Li-5 turns 16 D.c.c. Close wound on 3 in . former.
L2-1o turns 16 d.c.c. Close wound on 3 in. former. Reaction tap at third turn.
$\mathrm{L}_{3}-200$ Burndept plug-in coil.
CI-.00025 Sterling vernier.
$\mathrm{C}_{2}-.00025$ Sterling vernier.
$\mathrm{C}_{3}-.0003$ Dubilier.
$\mathrm{C}_{4}$-. 002 Dubilier.
$\mathrm{C}_{5}-.002$ Dubilier.
$\mathrm{C} 6-2 \mu \mathrm{~F}$ Mansbridge.
$\mathrm{C}_{7}$-. 002 Dubilier.
L.F. transformers (Ashley Wireless Telephone Co.)
Grid-leak. (. 5 megohm.)
Valves. (V24 Marconi-Osram.)
Meters. (Weston.)
Filament voltmeter (o-6).
Filament ammeter (0-10).
H.T. voltmeter (o-Ioo).

Plate milliammeter ( $0-15$ ).
The mirror galvanometer must be of a
very sensitive type, and the variable resistance placed in shunt must be of sufficiently low resistance to pass the plate current. Great care must be taken, when adjusting this resistance to obtain the zero reading, that the galvanometer is not burnt out; it is advisable to set the resistance at its lowest value so that the galvo is practically short circuited when first switching on the set.

The station required is then tuned in, and all meter readings should be carefully noted; then wait until the station has closed down, and adjust the galvanometer to zero by means of the shunt resistance.

When the next transmission takes place the galvanometer will read the strength of the carrier wave, and you can then make your tests night by night during the lunar month.

One more word of warning. When taking the readings of the galvanometer, it is advisable to stand as far away from the apparatus as possible in order that the set may not be affected by body capacity, as this would give a false reading.

I have spent many months making these observations, and have carefully checked the results of the nightly tests, and there seems to be no possible doubt that this effect is produced by the moon. I suggest that the variation of signal strength is caused by the gravitational effect of the moon and the sun, acting on the ether waves and bending or deflecting them from their course, in the same manner that light is bent or deflected by gravitation. This, of course, is for mathematicians to decide, and I only advance it as a probable theory which seems to be backed up by my actual observations.

A fact which, to my mind, helps to bear out my theory is that daylight reception is also affected and shows this increase and decrease, as the moon moves through her various phases. There is, however, one small factor which must not be overlooked, which is that usually during very bright moonlight nights the atmosphere is much more free from moisture than at other times; and this may have some bearing on the alteration of signal strength, by reason of the fact that the insulating materials used in the construction of the transmitter and receiver would be more efficient when free from moisture. But I do not think this is the reason, as even when the moon is full, we get bad weather
at times, yet the signal strength remains good when the moon is full even on damp nights. So I am led to the conclusion that the increase and decrease is due to the combined effect of the gravitation of the moon and sun, which as I have stated is backed by my actual observations and experiments.

I am at the present time engaged on a fresh series of observations, using more delicate and accurate instruments* for measuring and recording the variations of signal strength, and I am taking much greater precautions to exclude any factors which might give false results. I hope later to place on record the results of these further tests. However, in the meantime I hope

[^2]these few notes will open a new field of radio research and will enable other investigators to follow up what may prove to be a help to the advancement of radio science. In conclusion, I would add that I have questioned a very large number of people as to whether they have noticed any definite period for this increase and decrease of signal strength, and in all cases I find they can call to mind that they have noticed a certain time period, but they have never attached much importance to it. One experimenter whom I questioned (who lives in North Wales on the sea-coast) did say he had always noticed a distinct rise in signal strength as the tide came to high water and diminished as the tide receded. This, of course, would point to some lunar influence, and I am perfectly sure that this influence plays a much greater part in radio reception than has hitherto been suspected.

# Wireless Aerials and the Landlord. 

By a Barrister.

[R540.07

TTHERE is, unfortunately, always a certain opposition set up against a new invention. There was a time when motorists were opposed by property owners and by magistrates who did not hesitate to express their public disapproval of "that noisome and loathsome disturber of the peace-the motor-car." And wireless experimenters will doubtless have their troubles of a similar kind until the voice of universal popularity drowns the cry of the opposition.

An instance of the type of difficulty that may be met with by the innocent listener-in is furnished by the disputes which are arising in certain quarters between landlords and their tenants. The landlords feel apprehensive that when a tenant puts up a wireless outfit some danger will result and damage will be done to the property. The attaching of wires to chimney stacks, fences, and trees is enough to cause alarm in the minds of some landlords, and among others there is the not uncommon fear that wireless apparatus attracts lightning and may become thereby a source of danger.

The landlords accordingly attempt to prohibit wireless fixtures of any kind, and seek to enforce the prohibition by threatsnof ejectment or increased rent. It is useful to inquire, therefore, what rights the landlord has in respect of a tenant's radio outfit, and how far the tenant can go without risk of legal action. .

It should first of all be remembered that where a tenant has no written lease for a definite period of years, or is unprotected by the Rent Restrictions Acts, the landlord can, on giving proper notice (a month or a week, as the case may be), turn the tenant out without giving any reason at all.

And, of course, if the tenant has promised the landlord not to put up any wireless apparatus, or has accepted a lease containing a clause prohibiting wireless fittings, the tenant will be bound by such undertakings on his part, and can be sued for damages if he puts up any apparatus.

But in general a tenant to-day is either protected by the Rent Act and cannot be turned out, or he has a lease for a number
of years and, therefore, cannot be ejected until his lease ends or is terminated by some breach on his part.

Now let us see what breaches of the terms of a lease would be committed by a tenant who sets up a wireless aerial. It depends, of course, on the terms of the lease, but most leases have somewhat similar terms, namely, that the tenant shall keed the premises in "tenantable repair and condition," that he shall do no damage, that he shall insure against dangers of various sorts, and other terms of that nature.

In general, all these terms embody a promise by the tenant not to injure the property. And if these terms were not expressly agreed upon, they would always be implied. But it must be noticed that such terms do not protect the landlord against future damage. They only give him a right of compensation for damage which has actually been done.

So when a tenant puts up an aerial support and damages the property in doing so, the landlord can sue him for the damage, and can obtain from the court an injunction preventing the aerial from being put up again. But this action by the landlord can only be brought when damage has been done.

There may be cases where a landlord might bring an action to prohibit the continuance of a structure or of something which is bound to do damage ultimately. But it is not at all certain yet that the mere erection of a wireless aerial is bound to be a source of danger to property. Consequently, so long as a tenant does no actual injury, his wireless apparatus should be considered in the same class with domestic fitting's which he has every right to put up so long as they are put up in a reasonable and proper manner.

With regard to insurance, this is a matter for the tenant in any case, whether he is under an obligation to insure or not. For if the premises are damaged, or struck by lightning, the tenant would in general be under an obligation to renew the damaged premises. So it is better to be insured
against all risks. Some insurance companies make no objection to wireless apparatus, some make a small extra charge to cover whatever risk there may be. But it is better for the tenant to cover himself in every case.

Sometimes there is included in the tenant's lease a term forbidding the erection of any " building or thing" upon the premises. Such a term would, of course, cover a wireless. mast and perhaps aerial wires. But a clause in the lease prohibiting building would not prohibit masts or aerials. . Every tenant should read his lease carefully and see exactly what he has promised his landlord before defying him, or even before fitting an aerial.

This is especially necessary in the case of flats and tenement houses, where the terms are rather strict. It should be remembered that the tenant of a flat has only his own rooms and the outer walls of those rooms. The roof generally remains the property of the landlard. Consequently the tenant would have no right to go upon the roof, much less attach his wires to it. The same might also apply to semi-detached houses with a common chimney-stack, and the tenant of one house would need to get the consent of his neighbour before fitting the aerial supports to the chimney.

The case of a tenant of furnished rooms is somewhat similar to that of a tenant of a flat, but a distinction must be drawn between the tenant of a flat let furnished and a. lodger who has his own room. A lodger in such a case has no rights over the premises and ought not to so much as stick a nail in the wall without the landlord's permission.

Wireless is, however, growing more and more popular and is beginning to beregarded as an object of domestic convenience if not a necessity to modern comfort, and most of the difficulties foreshadowed above will disappear as wireless gets more universal and as landlords and tenants in their tenancy agreements arrange terms which include the fitting of wireless apparatus.

# Low-Frequency Intervalve Transformers. 

Paper read by Mr. P. W. WILLANS, M.A., before the Wireless Section, I.E.E., on 2nd June, 1926.

## [R132.1


#### Abstract

.

T(HE paper gives a detailed consideration of the low-frequency intervalve transformer from the combined aspects of theoretical considerations, a method and result of experimental measurement, and the application of both to the actual design of a commercial pattern of transformer developed by the author. In the latter part of the paper notes are also given on various methods of connecting the transformer, especially with a view to an increased range of constancy of amplification.

After a brief note of previous work on the subject, the author reviews theoretical considerations. From discussion of Fig. I * 

Fig. I. $v g=$ input sinusoidal voltage. $v_{g}^{\prime}=$ output sinusoidal voltage. $R_{1}=$ primary resistance. $L_{1}=$ primary inductance. $L_{2}=$ secondary close-coupled inductance. $l_{2}{ }^{2}=$ secondary leakage inductance. $r_{0}=$ secondary effective resistance (shunt). $C_{2}=$ secondary effective capacity. $m=$ valve magnification factor. $\rho=$ valve internal resistance. $\sigma=$ step-up ratio.


and its equivalent circuit, the following approximate expression is derived for the effective voltage step up:-

$$
\begin{aligned}
& -v_{g}^{\prime} / v_{g} \\
& =\frac{m \sigma}{\mathrm{I}+\frac{\rho \sigma^{2}}{r_{2}}-\omega^{2} l_{2} C_{2}+j \rho \sigma^{2}\left[\omega\left(C_{2}+\frac{l_{2}}{r_{2} \rho \sigma^{2}}\right)-\frac{\mathrm{I}}{\omega L_{2}}\right]}
\end{aligned}
$$

Special cases are then discussed in detail under the general headings: r. Loss-free close-coupled transformer; 2. Effect of

[^3]damping on close-coupled transformers; 3 . Transformer with magnetic leakage but no losses; 4. Transformer with both losses and leakage. The conclusions reached on the general theory and special cases are :-
r. The performance of a single-stage transformer amplifier may be expressed as a vector voltage amplification ratio, consisting of a fraction whose numerator is independent of frequency and whose denominator consists of a real part $X$ and an imaginary part $Y$, both, in general, varying with frequency.
2. The imaginary term $Y$ vanishes for a certain frequency (the resonant frequency) determined mainly by $L_{2}$ and $C_{2}$. The extent of variation on either side of this frequency is determined by the ratio $C_{2} / L_{2}$, the step-up ratio $\sigma$ and the valve resistance $\rho$, the constancy being greater the smaller these quantities. The values of $Y$ are identical for any two frequencies bearing inverse ratios to the resonant frequency. It is important that $C_{2}$ and $\mathrm{r} / L_{2}$ should separately be made as small as possible. The value of $\sigma$ can then be adjusted in conformity with $\rho$ so that the required degree of constancy of $Y$ is attained.
3. The real term $X$ vanishes for a certain frequency which is dependent upon the secondary leakage inductance $l_{2}$, capacity $C_{2}$ and resistance $r_{2}$. The variation is steep above this critical frequency but tends to constancy at low frequencies. The lower the values of $l_{2}, C_{2}$ and $r_{2}$, the higher is the frequency for which $X$ vanishes, but a low value of $r_{2}$ impairs the efficiency.
4. Since $X$ is not governed by $\sigma$ we cannot ensure constant amplification by a reduction in ratio. The best that can be achieved is the adjustment of $\sigma$ until $\left(X^{2}+Y^{2}\right)$ is constant over a band of frequencies, but the width of this band is determined by the degree of constancy achieved for $X$.
5. Setting aside the alternative of applying a damping resistance to the transformer
windings, the problem of design resolves itself into increasing $L_{2}$ to the utmost, and at the same time minimising $C_{2}$ and $l_{2}$. This necessitates a determination of the winding space occupied by the primary winding, but not of the number of turns with which this space is filled.


Fig. 3.
The paper next deals with experimental measurements and results. The method of measuring the performance of a transformer is shown in Fig. 3, where it will be seen that the method is a bridge one, employing a rule observation. By using the dotted inductance $L$, the $C L$ circuit can be used with the bridge for frequency determination. By setting $S_{1}$ to its upper position and $S_{2}$ to its right-hand position the connections of Fig. 5 are utilised for balancing the two valves $B$ and $B^{\prime}$, the filament of one or the other being varied until extinction is


Fig. 5.
obtained. Earth balance is effected on the Wagner principle by the variable resistance and $C_{0}$ and measurement finally adjusted. With $S_{1}$ and $S_{2}$ in their lower and right-hand positions respectively, the connections of Fig. 6 are given for final measurement of
the ratio $v_{g}^{\prime} / v_{g}$. This ratio is equal and opposite to the ratio of the voltage across the arm 2-4 and that across I-4, and is given hy

$$
-\frac{v_{g}^{\prime}}{v_{g}}=\frac{\cdot R^{\prime}+R}{R^{\prime}-R-j \omega R^{2} C}
$$

and

$$
-\frac{v_{g}^{\prime}}{v_{g}}=\frac{R^{\prime}+R}{R^{\prime}-R+\omega^{2} R^{3} C^{2}+j \omega R^{2} C}
$$

according to whether the condenser switch is at (a) or (b). Calculation of the scalar amplification and of phase angle are alsu illustrated in the paper.

During the course of the work, a large number of transformers were tested. Some were already on the market, others were tentative designs, while others were actual samples drawn from production of the final design adopted by the author's firm.


Fig. 6.
A specimen tabulation of results and the method of calculating the scalar amplification factor and other functions of the transformer constants are shown in a table. A set of results are shown graphed in Fig. 7.

The author then describes a low-leakage transformer developed by himself in conjunction with Mr. M. Ward, B.Sc. Both $l_{2}$ and $C_{2}$ had to be minimised, the former being a point that had not previously received sufficient consideration. In order to reduce leakage inductance the customary method of divided and interleaved windings was adopted. The first attempt, without special precautions, gave too high mutual capacity. The sections were accordingly spaced apart until no appreciable gain in this respect was obtained by further separation. The result was a transformer having the characteristics shown in Fig. Io.

A sectional view of the transformer is shown in Fig. II. The sections $C$ each consisted of about 5,000 turns of 44 s.w.G.
enamelled wire, while the gauge and number of turns of the section $B$ varied according to transformer ratio. Separation of sections was effected by means of the star-shaped

The defect of the transformer characteristic is its inefficiency at lower frequencies, which has been made good in a later design. Arrangements are also described and


Fig. 7.


Fig $\mathbf{r o}$.
millboard separators shown. Four different ratios were specified, i.e., $2.7: 1$ for a 40,000 ohm valve, 4 : I for a 16,000 to 18,000 ohm valve, $6:$ I for a 7,000 ohm valve, and $8:$ I for a 4,000 or 5,000 ohm valve.
illustrated for the use of the bridge arrangement for the determination of the voltage factor $m$ and internal impedance $\rho$ of the valve. In a section on the choice of a valve, it is concluded that considerable gain is to
be expected from the employment of a valve of high amplification factor and a transformer of conservative step-up ratio. As an offset, however, it is a matter of great difficulty to produce such a valve with a sufficiently straight characteristic to avoid asymmetric distortion at the high and low frequencies respectively. Such a valve is therefore of greater advantage in the earlier stages of an amplifier where the voltage is small.

The effect of inter-electrode capacity is then considered, especially from the standpoint of the impedance introduced into the


Fig. 1 r.
grid circuit by the reactive effect of this capacity. It is shown that this impedance consists of two parallel branches: (a) A capacity equal to $C_{0}$ (the anode-grid capacity) which can be counted as part of the transformer capacity, and (b) an impedance that can be represented by a condenser of value $m C_{0}$, in series with a resistance the value of which varies with frequency and may be positive or negative. This resistance has zero value at the resonant frequency of the transformer, is negative below this frequency, and positive above it, remaining positive however high the frequency is
raised. In the absence of magnetic leakage the only effect of any consequence is at high frequencies. With magnetic leakage (as $l_{2}$ of Fig. 1), if $l_{2} C_{2} \omega^{2}>1$ the resistance is again negative, and may give regeneration at higher frequencies. This appears to be the cause of whistling and distortion on the upper frequencies that is sometimes encountered.


Fig. 15.
Finally, the author, devotes a section to various intervalve transformer connections. The primary series condenser circuit of Fig. I5 is described, the anode here being fed through a choke coil. The equivalent circuit is discussed, and an expression derived for the amplification. From this it is concluded that the disadvantage of the arrangement would appear to be an increased tendency towards asymmetric distortion at low frequencies, although test results quoted in a table shows considerable uniformity of scalar voltage amplification over the range 200 to 1,000 cycles.

The use of low frequency reaction is illustrated in Fig. 17. The transformer windings are oppositely connected so that $\sigma$ is negative, and $L_{0}$ should be arranged so that its impedance at practical low audiofrequencies is large compared with $R$. In practice $L_{0}$ may be fixed and $R$ a variable


Fig. 17.
resistance of the graphite leak type. The arrangement gives a good range of constant amplification, experimental results on this point being shown in a table. The circuit is quite workable in practice. If $R$ be increr sed, a setting is reached where oscillations cccur
on a very low frequency. On setting $R$ to a value close to but definitely below this point, a stable arrangement is obtained, with a great increase in the lower frequency amplification.

The last arrangement discussed is an auto-transformer circuit shown in Fig. 18, due to Messrs. Graham '\& Ricketts. The connection is such that $\sigma$ is positive; and


Fig. 18.
the effect of the arrangement is roughly to increase $\sigma$ to $(\sigma+1)$. Results of measurements, shown in a tabulation, reveal a fairly flat-topped curve of amplification.

## DISCUSSION.

Mr. P. K. Turner thanked the author for the excellent detailed work on transformers, and congratulated him on the fact that by his work he had achieved a transformer giving results formerly considered impossible. He criticised some of the symbols used in the mathematical work as being non-standard, and sought information on the values of magnetisation encountered in such transformers. He considered that a fundamental objection to the bridge method of measurement was that it had no A.C. load in the anode circuit of the valve joined to the transformer.

Mr. H. A. Thomas said that Mr. Turner's reference to the bridge method had covered some of his proposed remarks, notably as regards the absence of load in the output valve. He had tried a similar
bridge method and quoted changes which he had observed according to the conditions of experiment,. and sought information from the author on thissubject.

Mr. H. I. Kirke dealt with the series condenser system, especially from the point of view of amplifiers working to land lines for broadcast relay. The arrangement had proved very free from distortion. Iron distortion was also non-apparent if certain values of magnetisation were used.

Mr. C. G. Garton spoke of transformer design with large leakage and large self-capacity. With these it had been found possible to get a flat curve up to 5 kC , while the arrangement was less expensive. He strongly maintained the soundness of the bridge method with no reactive load. This gave a measure of the effect of the transformer, which was the chief point of measurement.

Mr. C. F. Philips considered the bridge method elaborate and involving much calculation for the arrival at results. He outlined a double slide-back system for giving stage magnification, which he said gave good results and was easy and convenient in practice.

Dr. R. L. Smith-Rose congratulated the author on the rational and scientific manner in which he had tackled the problem of transformer design. He asked for information as to the lowest frequency used far test, pointing out the need for improved features of the curve below 250 cycles. Considered on a pitch basis this was very important. The existence of load in the second anode gave a measure of complete amplifier performance, and the chief necessity was not merely for a transformer but for a total amplifier giving linear response.

Mr. Willans briefly replied to some of the points raised. He particularly defended the bridge method as showing the transformer performance and permitting the sorting out of different effects. In practice, it had shown great constancy of results and close agreement between theory and experimental verification.

On the motion of the Chairman (Major B. Binyon) the author was accorded a very cordial vote of thanks.

# Constructing a Self-Contained Ballistic or Deadbeat Spot Galvanometer. 

By F. A. Boyce.
[R384.6

SPOT galvanometers on the second-hand market are eagerly snapped up, while new ones of the ordinary deadbeat type range from $£_{15}$ to $£_{25}$; it appeared, therefore, worth while to construct one. By so doing one could at any rate incorporate one's own ideas.


Fig. 1.
Apart from being inexpensive the galvanometer to be described has several outstanding advantages:-

1. Self-contained.-That is to say, no external focused beam or scale is required. The spot-light is permanently focused upon a ground glass screen forming part of the containing cabinet, 10 inches from the mirror.
2. Ballistic or Deadbeat.-Spare bobbin units are unnecessary. The instrument is converted from ballistic to deadbeat and vice versa by a switch on the front terminal panel.
3. Simplicity.-Skill is of less importance than patience in its construction.


Fig. 2.
The suspended coil type of galvanometer with a taut wire at the top and a loose connecting wire below is extremely susceptible to mechanical jars; moreover, levelling screws must be provided to obtain freedom for the coil. The galvanometer under consideration has, however, taut wires both top and bottom and once set up takes very rough treatment to disturb its action.

The general assembly is clearly shown in Figs. I and 2; the parts in Fig. 2 have been numbered and are described in detail subsequently.

## Item 1.-The Magnets.

There are two magnets, each made up of two laminations three-eighths of an inch thick. The whole was bought from a "surplus" dealer for 2 s . 6 d ., which, we should imagine, is the lowest figure one can expect to give.
up to $2 \frac{3}{4}$ by $2 \frac{1}{4}$ inches to fit exactly between the poles of the magnets, and then left in a dying fire to anneal. Two $\frac{1}{8}$-inch brass plates were cut and squared to the same dimensions and fixed to the steel by the method shown. In this connection it is important to fix the centres of the screws so as to avoid the side screws which hold the steel to the magnets.

The marking out to exact centre, drilling or boring of the central I -in. hole through $2 \frac{3}{4}$ inches should be done accurately; in fact,


Fig. 3.

It would appear that sizes are more or less standard, the dimensions of succeeding items therefore have been given in order that the original may be followed if desired without modification.

Apparently the degree to which the original magnetism is retained is not important, since before the galvanometer is finally assembled the magnets can be re-magnetised for two shillings or thereabouts. On the other hand, it is essential to select magnets having convenient fixing holes; this should not be difficult.

## Item 2.-Pole Shoes.

A piece of $\mathrm{I}-\mathrm{in}$. mild steel was next obtained from a local engineer; this was squared
it is probable that the time expended by oneself on this job is out of all proportion to the labour charge of an engineer who has a precision lathe amongst his equipment.

After the side fixing holes have been drilled and tapped $\frac{1}{4}$-in. Whitworth, the plates can be removed and the sharp edge filed away from the shoes to the $\frac{3}{4}$ inch given in the sketch.

## Item 3.-Central Core.

This is also mild steel, turned from rod, and placed, like the shoes, in a dying fire to soften. A single fixing hole was used.

The core is seated upon a distance piece shaped from brass $\frac{3}{32}-\mathrm{in}$. thick to $\frac{1}{8}$-in. at the middle line. When later assembling, a
screw of such a length must be chosen that it will pass through the arm (Item 5) and one of the brass plates of the pole shoes (Item 2), as well as the distance piece mentioned.

## Item 4.-The Bobbin.

Bobbins of insulating material and of metal were tried, as well as the alternativea former wound coil-but it was decided that a carefully constructed former of relatively substantial aluminium was the best in order to obtain uniformity and balance throughout. Hard sheet aluminium $\frac{1}{64}-\mathrm{in}$. thick was


Fig. 4.
obtained and carefully marked out as shown in Fig. 4 (a). A sharp knife was used on the narrow parts and shears with extreme discretion on the end pieces. Two small rivet holes were made to coincide and tiny lead rivets used for fixing.

Two pieces of high grade ebonite were next squared up to fit between the projections on the aluminium former, and when assembled should leave a winding space of at least $\frac{1}{16} \mathrm{in}$.

The pivot pins as well as the cross rivets are of $\mathbf{1} 6$ s.w.g. brass wire. If a No. $\mathbf{5 2}$ drill be used it will be found that the wire pins can be driven home by gentle taps of a small hammer. Accuracy is essential, of course, in fitting the pivot pins, but the side holes, for holding pins only, can be drilled with the ebonite in position.

The bobbin should be examined before
winding, in order to make sure that it is rigid and well balanced. The balance can be more or less tested by pressing the bobbin by the pivots lightly between the thumb and second finger and causing it to spin by a tap on one edge. Thus far, the side pins may be drawn out and the aluminium given a heavy covering of shellac, left to dry and finally baked.

The winding consists of 250 to 300 turns of 44 s.w.g. enamelled wire. The long sides must be watched closely to see that no turns slip over the edge. An ebonite former was made to fit snugly inside the bobbin, and through a central clearance hole a length of 4 B.A. studding was inserted, while a nut on either side held the former squarely in position. A small hand-drill was mounted in the vice to take the other end of the studding. After testing for continuity the end pieces were reassembled, the wire ends soldered to the pivot pins and the bobbin covered with shellac and baked again.

## Item 5.-Suspension Arm.

A straight io inches of square section brass rod ( $\frac{3}{8}$ by $\frac{1}{4}$ inch) was drilled with five clearance holes, the dimensions and details of which are clearly shown in Fig. 4 (b). It will be seen that the arm is fixed to one of the shoe-plates; marking-out, drilling and counter-sinking of the arm, and the tapping of two holes in the plate again obviously calls for accuracy.

## Items 6 and 7.-Suspension Blocks.

These are of brass to the dimensions given in Fig. $5(a)$, $\frac{1}{2}$-in. square section, recessed at one end to fit closely over the suspension arm, while at the other a hole is made the centre of which is located exactly over the centre of the space between the pole shoes.

The assembly of the top block is also shown in Fig. 5(a). The ebonite knob and terminal top can, of course, be any other shape than that shown so long as the latter runs easily, but without side play over the 4 B.A. studding which passes through a clearance hole in the block. The side set screw is usetul for securing the adjustment.

Particular note should be made of the lower end of the studding which is filed flat to its centre and tinned in readiness for the torsion wire or strip.

The suspension block (Item 7) at the lower
end of the arm differs somewhat in detail from the top member. The clearance hole has been widened to take the studding, which is inserted into a short length of Sistoflex or other good insulating sleeving having fairly thick walls. At the same time two locking nuts, having between them a light soldering tag, butt against two ruby mica washers, thus insulating the studding and fittings from the block. The studding at this point is also filed to its centre to receive the wire.

When the instrument was finished it was found that considerably more thought could have been given to the design of the top


Fig. 5.
block to facilitate adjustment of the bobbin to give a spot at central zero. However, setting is not troublesome as it stands.

## Item 8.-Suspension Wires.

For the purpose of the initial assembly and try-out, 44 s.w.g. bare wire can be used to suspend the coil between the pole shoes only. If the work has been accurately carried out and the winding nicely packed the bobbin will swing freely. On the other hand, there may be one or two mistakes to rectify with a half-round file applied lightly to the inner sides of the shoes. In such a case one must not forget the filings, which perhaps will not be noticed until the magnets are added and may be found to block completely the free action of the coil. Later, wire or strip for the suspension can be fitted as required according to the sensitivity sought.

Certain grades of tinsel are obtainable which are composed of plated copper strip, and the micrometer gives the thickness as 0.02 mm . with a width up to 0.5 mm .

The tinsel was tried for suspensions both top and bottom, and it was gratifying to find that the galvanometer was extremely sensitive though requiring very careful adjustment to prevent the coil fouling the pole shoes. It was then realised that if a break occurred while the instrument was in use a great deal of time would be wasted in readjustment, and since the aim was to obtain nearly one centimetre per microampere it appeared that some metal could be spared from the shoes. The central hole, therefore, was increased by one-eighth of an inch, this being accomplished in a lathe by packing up the side plates to give clearance to the tool, otherwise, of course, the plates would have been cut away.

Such an increase appears to be a drastic measure, but it followed on observations taken. The scale was marked in 0.5 centimetre divisions with the intention of adding millimetres later.
Reassembling, using tinsel drawn taut: both top and bottom, a deflection of $2 \mu \mathrm{~A}$ per cm. was obtained deadbeat, and as a ballistic instrument the coil had ample room ; moreover, the instrument required to be levelled approximately only. Further, deflections taken to the right and left of zero tallied beautifully.

It is recommended, of course, that the metal parts be lacquered or protected by any other means. The brass can be treated with hot or cold lacquer, while the mild steel can be blued or blue-lacquered.

## Item 9.-The Mirror.

This was purchased from Messrs. Negretti: and Zambra for 4 s .9 d ., and it is understood that they made two or three for stock when supplying the original. The mirror is half an inch in diameter, having a focal length of ro inches. The top of the bobbin was covered on the front lightly with shellac and the mirror pressed upon it, in a very short time the shellac was tacky enough to allow the mirror to be adjusted correctly.*

## Cabinet and Fittings.

The drawing of the cabinet was entrusted to Mr. Carrington, of the Carrington Manufacturing Company, Normans' Buildings, Central Street, E.C. The finished article-

[^4]was found to be exactly to dimensions given in the accompanying sketch (Fig. 6), together with a wooden block measuring 4 by 4 by $5 \frac{1}{4}$ inches.

The cabinet is of $\frac{3}{8}$-inch polished teak with the exception of a teak-finished 5-ply back board. The top is fastened with wood screws through brass bushes, and is therefore removable as required. At first sight such a construction appears to be weak, but it must be remembered that at no time will the instrument be roughly handled,
while the sketch Fig. 5(b) gives outside dimensions. When cutting away the box in front to take the panel ample holding space for the wood screws must be left.

Though the bobbin in itself is a shortcircuited turn of generous cross sectional area the currents set up therein do not react sufficiently on the field to render the action deadbeat, it is therefore necessary in this particular instrument to provide an external load in the form of a noninductive resistance. Six yards of 42


Fig. 6.
and except for the fitting of the parts, the top will always be on.

There is an aperture of I inch across the top of the front, the exposed edges being grooved to take a strip of finely ground glass. The latter was marked off in 0.5 and 1 cm . divisions, and no difficulty was experienced in printing the figures backwards. It did not appear necessary to use a fixing solution since the ground side of the glass is inside and the Indian ink can come to no harm.

Also, on the front is a small ebonite panel which carries a two-way switch, a telephone jack, and, at the back, a batten holder for a small screw-type lamp. Fig. 7 shows the general location of the panel,
s.w.g. covered Concordin was used, having a resistance of 8 I 3 ohms, wound back on itself in the usual way, arranged as a shunt to be thrown in or out of use by the switch mentioned above. The ballistic stud of the switch is blank, of course. In this case the shunt reduces the deflection for a given current by 25 per cent., but it kills any oscillation of the bobbin instantly; moreover, the actual resistance is not critical. If one seeks, therefore, to make the galvanometer deadbeat to a reasonable degree only, an adjustment of the resistance when first assembling provides means to set the deflection to multiples of a centimetre to coincide with multiples of a microampere.

The lamp is the ubiquitous $4 \frac{1}{2}$-volt flash
type fed by an Ever Ready battery of generous capacity held inside and against the side of the cabinet by a brass strip and two wood screws.

A plug-and-jack comnection to the galvanometer does duty in three ways. First, the double contact is utilised to give direct connection from the galvanometer to external instruments; secondly, the act of inserting the plug switches on the filanent current to the lamp by the usual method of a double


Fig. 7.
circuit jack, and finally, by no means the least important, the method itself provides protection to the instrument since an accidental tug on the flex simply pulls out the plug. The capacity of the flex itself does not matter, for when taking observations of resonance curves and the like it is usual to shunt the galvanometer leads with a condenser.

The insulating properties of the distance pieces used in the construction of the jack must be carefully watched. It is essential to get a jack of good quality. The first one tried in this case leaked, upsetting readings to the extent of one and a half microamperes.

## The Focus.

For the spot light upon the ground glass screen we have not attempted to project a moon bisected by a sharply focussed hair following, we believe, the usual practice. A
watchmaker's glass of $2 \frac{1}{2}-\mathrm{in}$. focus has been strapped and mounted on a slotted brass arm. The lens itself has been covered with tinfoil across which was carefully drawn a knife, which procedure gives a hair line of brilliant light upon the scale; in fact, the line of light is no thicker than the division lines.

Finally, a brass cover was added to protect the top adjustment. The item seen in the photograph is perhaps of larger diameter than necessary, but the tube and other brass were to hand, and it must be added that after fixing it will be found that in removing the cover from its push-fit base there is always a final jerk, so that if the tube were smaller the top suspension would probably suffer.

## General Remarks.

In experimental work a spot galvanometer is essential. Its obvious d.c. job is in the measurement of high resistances or leakage paths by Ohm's law, or as a sensitive balancing galvanometer used in conjunction with a Wheatstone bridge.

It is admirably suited to work in the measurement of inductance and capacity at radio frequencies. The personal element present with telephones is eliminated by placing the deadbeat instrument in series with a carborundum detector or valve rectifier (no H.T.), the outside terminals then being connected across the circuit to be measured. Care must be taken to make the galvanometer circuit of as high a resistance as possible consistent with a readable deflection. A calibrated valve oscillator set up and coupled loosely to the circuit under test completes such an arrangement. Knowing the wavelength, capacity and inductance values can be calculated; this test at once suggests others involving the same terms. In the plotting of resonance curves for the calculation of high frequency resistance the galvanometer is invaluable.

On the ballistic side, the logarithmic decrement can be calculated from observation and the capacity of a condenser determined absolutely, or commercial results obtained by comparison with a standard.*

[^5]
# Mathematics for Wireless Amateurs. 

By F. M. Colebrook, B.Sc., A.C.G.I., D.I.C.

(Continued from page 378 of June issue.)
(D) The Fundamental Laws of Algebra. Multiplication.

BEFORE plunging into this subject the writer would like to anticipate a criticism which is likely to be brought against this and perhaps some other parts of the series, namely, that a deal of breath is being wasted in expounding the obvious and the familiar. For instance, the conclusion

$$
3 \times 4=4 \times 3
$$

may be regarded as a poor return for the investment of a large number of words. There are probably many readers who will say or think, "I know all about simple multiplication. Let's get on to the calculus." The implicit assumption that one needs to know all about multiplication before going on to the calculus is right enough, but such readers are invited to reconsider the first part of the statement. Anyone who knows all about multiplication should be able to give clear answers to the following questions and to many like them :-
I. Since one can multiply volts by amperes and call the product " watts," can one multiply oranges by apples? If not, why not ?
2. Since one can multiply inches by inches and call the product "square inches," can one multiply apples by apples and call the product "square apples"? If not, why not?
3. Can one multiply a number by a negative number? (The writer would be prepared to offer long odds against this one being answered correctly.)

The following discussion of the subject is recommended to the attention of all those who feel at all uncertain about the answers to the above questions.

Multiplication in algebra is a generalisation
of the corresponding process in arithmetic. Let us therefore go back for a while to the multiplication tables: $3 \times 5=15$. Three multiplied by five is fifteen. We learned it by heart a, long while ago and have probably never given it another thought. Giving it another thought, we realise that it means

$$
3 \times 5=3+3+3+3+3=15
$$

i.e., five groups of three ones combined together make up the group that we have agreed to call fifteen. In other words, multiplication is addition, a rather special case of addition in which all the numbers added together are the same as one another. We owe a great deal to the ingenious person (probably a Greek or an Arab) who realised that this special case of addition could be written down in a very short way.

Expressing the process in algebra terms, i.e., by means of letters which for the present will be taken to mean any positive whole numbers,

$$
a \times b=a+a+a+a
$$

written down $b$ times.
Notice that although $a$ and $b$ are both numbers in the case of pure arithmetical multiplication, there is a difference between them when the process refers to groups of things, which things must of course be things all of the same kind, whether volts, amperes, cabbages or kings, " $a$ " is the number of things in each group, and " $b$ " is the number of groups, so that $a \times b$ means " $b$ " groups of " $a$ " things. On the other hand $b \times a$ means " $a$ " groups of " $b$ " things. We can see that the total number of things is the same in the two cases. Starting with " $b$ " groups of " $a$ " things, take one out of each group. This will give one group of " $b$ " things, and since this process can be repeated until all the " $a$ " things in each group are used up, i.e., " $a$ " times, the final result of
the rearrangement will be " $a$ " groups of " $b$ " things. In symbols this is

$$
(a \times b)=(b \times a) .
$$

It is one of the most important properties of the process of multiplication and is called the Law of Commutation. In $(a \times b)$, " $a$ " is called the multiplicand and " $b$ " the multiplier. $(a \times b)$ is called the product of " $a$ " by " $b$." In practice the explicit multiplication sign or St. Andrew's Cross is often omitted or replaced by a dot, thus-

$$
a \times b=a . b=a b .
$$

The product $(a \times b)$ can be considered as a single group and as such can be multiplied again, e.g., $(a \times b) \times c$. Further, it can be shown that

$$
(a \times b) \times c=a \times(b \times c) .
$$

The proof is not immediately obvious, but it will not be given in full as it would take up rather a lot of space. The easiest steps are

$$
(a \times b) \times c=(b \times a) \times c .
$$

By writing out the right-hand side fully and by regrouping the symbols, it can be shown that

$$
\begin{aligned}
(b \times a) \times c & =(b \times c) \times a \\
& =a \times(b \times c)
\end{aligned}
$$

so that

$$
(a \times b) \times c=a \times(b \times c)
$$

or

$$
(a b) c=a(b c)
$$

This is known as the Associative Law for multiplication. By combining this with the Commutative Law it can be shown that-

$$
a b c=b c a=c a b=c b a=\mathrm{etc} . \text {, etc. }
$$

and since the whole argument can then be repeated with $(a \times b \times c) \times d$ and so on indefinitely, we may say at once that the result of multiplying together any number of positive whole numbers is independent of the order in which the operations are performed.

Returning to $a \times b$, suppose $a$ is the sum of two numbers $c$ and $d$, i.e.,

$$
a=c+d
$$

Then

$$
\begin{aligned}
a \times b & =(c+d) \times b \\
& =(c+d)+(c+d)+(c+d)
\end{aligned}
$$

written down $b$ times.

Therefore by the Associative Law for addition-

$$
(c+d) \times b=(c+c+c+c+c+\ldots
$$

written down $b$ times)
$+(d+d+d+d+d+$
written down $b$ times)

$$
=(c \times b)+d \times b)
$$

and since

$$
(c+d) \times b=b \times(c+d)
$$

we have

$$
b \times(c+d)=(b \times c)+(b \times d)
$$

or

$$
b(c+d)=b c+b d
$$

This is known as the Distributive Law, for a fairly obvious reason. The process can clearly be continued. Thus if, in the above,

$$
\begin{aligned}
b & =e+f \\
b(c+d) & =(e+f)(c+d) \\
& =(e+f) c+(e+f) d \text { as already shown. } \\
& =e c+f c+e d+f d .
\end{aligned}
$$

The general character of the process will not be expressed in words. It would take too long. The algebraic symbols tell the whole truth much more concisely.

## (DI) Multiplication and Negative Numbers.

In terms of the original definition and of the interpretation which has already been found for - $a$ (see para. 3B) the meaning of $-a \times b$, where $a$ and $b$ are positive whole numbers, does not present any difficulty, for

$$
\begin{gathered}
(-a) \times b=(-a)+(-a)+(-a)+(-a \ldots \\
\text { written down } b \text { times }) \\
=-(a+a+a+a \ldots \text { etc., } \\
\quad \text { written down } b \text { times. }) \\
=-(a \times b)
\end{gathered}
$$

But what is $a \times(-b)$ ? According to the original definition it would be

$$
\begin{aligned}
a \times(-b)=a & +a+a+a, \text { etc., } \\
& \text { written down }-b \text { times. }
\end{aligned}
$$

But this does not mean anything. It is literally nonsense. Nor can we get over the difficulty by writing-

$$
a \times(-b)=(-b) \times a=-(b \times a)
$$

for this law was only proved for positive numbers. Actually there is no way out of the difficulty, or rather, there is no difficulty. There is simply the plain statement that one
cannot multiply a number by a negative number.* (See question 3, para. D.) But then, who wants to ?

It is true that later on there will often be occasion to pretend to multiply by a negative number. For instance, a number " $a$ " can be multiplied by " $b$ " giving $(a \times b)$. Its sign can then be reversed, giving $-(a \times b)$. To save time this can be described as multiplying by - $b$, but actually it consists of two quite separate operations. To take another example, if

$$
x=y
$$

multiplying each of these equal numbers by a gives

$$
x \times a=y \times a
$$

or

$$
a x=a y
$$

If these positive numbers are equal the corresponding negative numbers will also be equal, i.e.

$$
-a x=-a y
$$

Now the same true conclusion can be reached by assuming that the Law of Commutation applies to multiplication by a negative number, in which case a meaning can be found for multiplication by a negative number as shown above. Thus, since

$$
\begin{aligned}
x & =y \\
x \times(-a) & =y \times(-a)
\end{aligned}
$$

i.e.,

$$
(-a) \times x=(-a) \times y
$$

or

$$
-a x=-a y
$$

Actually the writer has never yet come across any case in which the application of the laws of multiplication to positive or negative numbers indifferently has lead to a false

[^6]conclusion, but that is almost certainly due to the fact that in every case the apparent process of multiplication by a negative number admits of some alternative explanation of the kind illustrated above. In all that follows, therefore, it will be considered possible to multiply by a negative number on this understanding.

## (D2) The Multiplication of Number Groups. Zero in Multiplication.

Consider the product $a \times(b-c)$. In view of the discussion in para. (DI), it will be considered permissible, irrespective of the sign of $(b-c)$, i.e., whether $b>c$ or $b<c$, to put

$$
a \times(b-c)=(b-c) \times a
$$

in which form the right-hand side can be written out in full exactly as in para. (D) of this section. In this way it can be shown that

$$
(b-c) \times a=(b \times a)-(c \times a)
$$

i.e.,

$$
a(b-c)=(b-c) a=b a-c a=a b-a c
$$

If $b>c$, there is another way of arriving at the same result which will give a further conclusion. The group $a \times(b-1)$ will obviously contain one $a$ less than that represented by the group $a \times b$, so that-

$$
\{a \times(b-\mathrm{I})\}+a=(a \times b)
$$

Similarly

$$
\{a \times(b-2)\}+(a \times 2)=(a \times b)
$$

and so on up to

$$
\{a \times(b-c)\}+(a \times c)=(a \times b)
$$

Now adding to each equal number the negative number - $(a \times c)$,
$\{a \times(b-c)\}+(a \times c)-(a \times c)=(a \times b)-(a \times c)$ and since

$$
\begin{gathered}
+(a \times c)-(a \times c)=0 \\
a \times(b-c)=(a \times b)-(a \times c)
\end{gathered}
$$

Now continue the process further, by increasing $c$ until it is equal to $b$. Then-

$$
a \times(b-b)=(a \times b)-(a \times b)
$$

i.e.,

$$
a \times 0=0 .
$$

Also from the original definition,

$$
\begin{aligned}
\mathrm{o} \times a= & 0+\mathrm{o}+\mathrm{o}+\mathrm{o} \text { etc., . . . . } \\
& \text { written down } a \text { times. } \\
& =\mathrm{o}
\end{aligned}
$$

This shows that the symbol o obeys the Commutative Law, and that the product of any number with zero is zero.

Returning to the multiplication of number groups, in the product

$$
(b-c) \times a=(b \times a)-(c \times a),
$$

suppose that $a$ is itself the sum of $d$ and -e, i.e.,

$$
a=(d-e)
$$

Then

$$
\begin{aligned}
(b-c) \times(d-e) & =\{b \times(d-e)\}-\{c \times(d-e)\} \\
& =\{(d-c) \times b\}-\{(d-e) \times c\} \\
& =\{(d \times b)-(e \times b)\}-\{(d \times c)- \\
& (e \times c)\} \\
= & (d \times b)-(e \times b)-(d \times c)+(e \times c)
\end{aligned}
$$

i.e.,

$$
(b-\dot{c})(d-e)=d b-c d-b e+e c
$$

in which it is seen that each number in the first group is multiplied by every number in the second group, signs being combined according to the rules already established in the section dealing with the combination of positive and negative numbers.

As in the case of addition, the meaning of the letter symbols can now quite legitimately be extended so as to include both positive and negative numbers.

An interesting special case should be noted at this point.

$$
\begin{aligned}
(x-a)(x-b) & =x x-b x-a x+a b \\
& =x x-(a+b) x+a b
\end{aligned}
$$

and if $b=a$,

$$
\begin{aligned}
(x-a)(x-a) & =x x-(a+a) x+a a \\
& =x x-2 a x+a a .
\end{aligned}
$$

In this last expression, we have for the first time algebraic symbols and ordinary numbers in association. There is obviously no need whatever for any given expression to be made up either of letters or explicit numbers exclusively. Nothing more need be said about this for it introduces no new ideas, though as a matter of fact it does introduce a new word. A number associated with a letter symbol in the way that - 2 is associated with $x$ in $x x-2 x+4$ is often called the "coefficient" of the given letter. This name is also applied more generally to one member of a product which remains constant while the other is allowed to vary in magnitude under given conditions. Thus if $x$ is allowed to vary, the $-2 a$ in $x x-2 a x+a a$ could
be called the coefficient of $x$. It is not an entirely fortunate choice of word, for "efficient" is always used adjectively in common speech, whereas it appears substantively in "coefficient," the literal meaning of which is " co-worker."

The form of the two products just considered should be very carefully observed, for they play a large part in the practical applications of algebra. It will be seen later that there is a shorter way of writing them, which does not of course affect their general character.

## (D3) Resolution into Factors.

The reverse (not the inverse, which is something quite different) of the process described in the last paragraph is at least as important as the direct process. It is called " factorisation" or " resolution into factors " and consists of the expression of a more or less complex group of numbers in the form of a product of two or more terms, called "factors." Thus the factors of $x x-(a+b) x+a b$ are $(x-a)$ and $(x-b)$. Take for instance $x x-7 x+12$. This can be written $x x-(4+3) x+4 \times 3$, and comparing this with the general form it is clear that the factors of this expression are $(x-4)$ and $(x-3)$, i.e.,

$$
x x-7 x+12=(x-4)(x-3)
$$

At present the only method available for factorising such expressions is that illustrated in this example, namely, " inspired guesswork," or what the disrespectful or possibly envious poet described as "cunning low, meet for things problematical." Later on some more certain though, in consequence, rather less exciting methods will be discovered.

## (D4) The Physical Application of Multiplication.

The main interest of those for whom these articles are being written will be the physical application of the processes of mathematics. This is what distinguishes practical or applied mathematics from pure mathematics considered as a science in itself.

Two questions involving the physical meaning of the process of multiplication have already been raised in para. (D), and these questions we are now in a position to answer.

The physical aspect of addition presented no great difficulty. It was clear that things
could only be added together in the arithmetical sense if they were things of the same kind. What is the corresponding rule for multiplication? The fact that one cannot multiply three oranges by four apples and equally cannot multiply three apples by three apples suggests that one can multiply neither things of the same kind nor things of different kinds. And yet we talk of multiplying volts by amperes and inches by inches. Does this mean that in some cases one can and in others cannot multiply together things of the same or things of different kinds? That if the result means anything one can do either of these things, and that if it does not one cannot! Some will perhaps be willing to accept this as the true explanation. Actually it is not an explanation at all. It is a mere statement, and not even a true statement.

The true answer has already been indicated in para. (D). Multiplication is the addition of a number of groups and, physically, a :group means a group of things which must .all be of the same kind. Thus one of the factors of a product must be interpretable as a number of groups, and the other as a group of similar things. But how is this to be related to the apparent process of multiplying volts by amperes and calling the product " watts"?

To trace the connection we will take an instance which is sufficiently novel and ridiculous to be free from any pre-conceptions. Returning to our three apples, let it be supposed that one of these, following a well-known precedent, falls on the head of a philosopher and makes him think. It can even be supposed that the harder it hits him the more it will make him think, so that if it falls from a height of one foot it produces .one theory, if from two feet, two theories, and so on. Suppose now that each of these three apples falls on the head of the philosopher from a height of five feet. Each apple now produces five theories. Each apple, so to speak, represents a group of five theories, and to find the total number of
theories produced by this profitable though perhaps painful episode, we combine together as many groups of five theories as there are apples; that is, three groups of five theories. Actually, however, life is too short to permit our being as explicit as this on every occasion, and we would quickly form the habit of saying that we multiply the height by the apples and call the product theories, just as we have formed the habit of saying that we multiply the volts by the amperes and call the product watts-a very good habit too, provided we do not make the mistake of thinking that we really mean what we say.

In the electrical example it is a matter of definition and of experience that each ampere in " falling through" a potential difference of five volts would give rise to. a group of five watts, and three amperes would therefore give three groups of five watts.

Thus the physical meaning of the process of multiplication is derivable from the ideal or purely mathematical meaning, and the application of multiplication in physics is seen to be of the same essential character as ordinary arithmetical or algebraic multiplication.

## Examples.-I.

I. Distribute, i.e., " multiply out "-$(a-b)(b-c)(c-a)$.
2. Find the magnitude of the number$(x x x-6 x x+11 x-6)$ when $x=1,2,3$, and 4.
3. Under what conditions will $(x-a)(x-b)$ be zero?
4. Find the factors of 一
(a) $x x-7 x+10$.
(b) $x x-3 x-10$.
(c) $x x+3 x-10$.
(d) $a a$-100ab $+99 b b$.
(e) $a a-b b$.

Note.-On p. 373 of June issue:-
For $a-b$, last line, col. 1, read $a \sim b$.
,, $(a-b)$ line 15, col. 2, read $(a \sim b)$.
$\because=+(a-b)$, line 26 , col. 2 , read $+(a \sim b)$.
$\because, \quad-(d-c)$, line 29 , col. 2 , read $-(c \sim d)$.

# Abstracts and References. 

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R000.-WIRELESS IN GENERAL.
Rogo.2.-Radio Communication and Imperial Development.-Dr. Eecles. (Nature, 8th May, 1926, Pp. 659-662.)

## R100.-GENERAL PRLNCIPLES AND THEORY.

RiI2.-Transmission and Reception without Aerial or Earth. (Wireless World, 2nd June, 1926, pp. 725-727.)
An account of a demonstration recently given by Mr. Derek Shannon to the Wireless World, when he justified the claims made for his apparatus.
Riiz.-Receiving Aerials. A Simple Theory of Induced Aerial Currents.-F. M. Colebrook. (Wireless World, 5 th May, 1926, pp. 665-668.)
Ríi 3 .-Versuche t'ber die Ausbreitung der Elektromagnetischen Wellen (Experiments on the propagation of electromagnetic waves).-M. Bäumber and J. Zenneck. (Electrische Nachrichten-Technick April, 1926, pp. 139-141. Zeitschr. f. Hochfrequenz., 27, 4, pp. 1 17-119.)
A description of experiments made in August, 1924, chiefly to study the behaviour of radio waves as they pass over from sea to land. The transmitter was erected on the island Heligoland, and receiving stations were set up at several places on and near the coast between Cuxhaven and Bremerhaven. A plan of these is shown in the diagram.

The site was particularly favourable for the following reasons :-
I. On Heligoland a valve transmitter for several wavelengths could be set up at the Post Office without much difficulty.
2. The distance between transmitter and coast was so great, on the one hand, that the waves radiated could spread out over a large surface of water before they reached the unhomogeneous tract in front of and at the coast, and on the other hand, sufficiently small to permit a convenient intensity measurement.
3. Assuming rectilinear propagation, the waves met the coast at all possible angles-approximately at right angles at receiver I, and approximately at grazing incidence at receiver 9 .
4. The relatively large difference in level of the North Sea at low and high tide made it possible to use the same tract under two different conditions: at high tide the sands and flats stretching away from the shore lay one to two metres below the water's surface, while at low tide they were as much above it.
5. In between transmitter and two of the receivers (3 and 4) there lay two islands (Neuwerk and Scharhörn) that could exert an influence.
The receiving stations are numbered in the figure and were selected so that every two, viewed
from the transmitter, lie in the same direction, one of them always being on the coast and the other inland.
For the observations two similar receiving arrangements were employed, based on that of Hollingworth (Journ. Inst. Elect. Engineers, 61, 501, 1923)-the chief differences being Auders' addition of a current transformer to the circuit for measuring the weak frame voltages (E.N.T., 2, $401,1925)$ and the use of a unifilar electrometer as indicating instrument.


The results are tabulated on the next page.
Here $K$ stands for coast station, and $L$ for inland station. The measurements show no effect of the level of the water on the intensity or direction of the radio waves, nor can any change of intensity. be detected with certainty as the waves pass from. water to land. In any case the change cannot be great. It thus appears to make no perceptible difference to the propagation of waves whether

they travel over sands and flats or smooth and sandy solid ground-or whether the sands and flats lie $1-2$ metres above the sea level or the same distance below it. The effect of the weather on 20th August is however remarkable. The great changes of directions and field strength observed during the storm are exceptionally striking even with the small distance between transmitter and receiver that there was here ( 56 km .). The readings show that direction-finding in such weather is out of the question.

With regard to direction-finding, all bearings gave too small an angle compared with the geographical direction-which goes to show that even right on the coast refraction takes place, and therewith distortion of the wave-front, as has been observed at stations further inland. The observations agree as to size with the German navy's directional error for the North Sea coast-these likewise giving too small an angle compared with the geographical direction. It is not certain whether the specially large error observed at Station 3 is to be attributed to the islands Neuwerk and Scharhörn owing to the low degree of accuracy of the measurements.

Rif3.-The Propagation of Radio Waves.J. Hollingworth. (Journ. I.E.E., May, 1926, pp. 579-595.)
The full text of the paper is given here of which there was an abstract in E.W. $\mathcal{E} W . E$. for March. The discussion following the reading of the paper is also given.

Rir3--La Propagation des Ondes Electromagnétiques dans la Theorie de la Relativite Generale.-Guido Beck. (Archives des Sciences Physiques et Naturelles, Mars-Avril, 1926, pp. 75-77.)
A mathematical note.
Rif3.-Application of Radio Transmission Phenomena to the Problems of Atmospheric Electricity.-J. H. Dellinger. (Journ. Washington Acad. Sciences, 19th March, 1926, pp. 162-167.)
A brief progress report on the inter-relations of the problems of radio transmission phenomena and those of atmospheric electricity. These interrelations are tabulated as follows:-

| Atmospheric Electricity. | Radio Phenomena. |
| :---: | :---: |
| Lightning and thunderclouds, | Atmospheric disturbances. |
| Aurora and magnetic storms. | Radio and electric line telegraph <br> disturbances. |
| Atmospheric potential gradient, <br> and conductivity diurnal <br> and annual variations. | Similar variations in atmos- <br> pheric disturbances and field <br> intensities. |
| Earth's magnetic field and <br> upper air conductivity. | Differing radio wave propaga- <br> tion at various frequencies, <br> distances and directions. |

The writer states that he might equally have entitled his remarks " Application of atmosphericelectric phenomena to the problems of radio transmission," but that the reasons for putting it in the way he did are two :-

1. Radio gives us a direct means of conducting controlled experiments on phenomena affected by the electrical conditions of the atmosphere. a means which is wholly impossible in the field of direct atmospheric-electric measurements; and
2. Atmospheric electricity, rather than radio, is the science which must take the responsibility and the labour of deciphering the inner relations between these various phenomena and the deduction of the underlying causes thereof.
Rir3.-Preliminary Note on Proposed Changes in the Constants of the Austin-Cohen Transmission Formula.-L. W. Austin. (Journ. Washington Acad. Sciences. 9th April, 1926, pp, 228-23r.)
The original formula of i910-1914 for daylight signals over salt water was written

$$
E=\mathrm{I} 20 \pi \frac{h I}{\lambda d} \sqrt{\frac{\theta}{\sin \theta}} \varepsilon^{-u} \text { (volts km. amp.) }
$$

where

$$
u=\frac{0.0015 d}{\lambda^{0.5}}
$$

The constants in $u$ were determined empirically from shunted telephone observations for distances up to $2,000 \mathrm{~km}$. and frequencies between $1,000 \mathrm{kC}$. $(\lambda=300 \mathrm{~m}$.$) and 80 \mathrm{kC} .(\lambda=3,750 \mathrm{~m}$.$) .$

The author has naturally been desirous of bringing the formula into better agreement with the observations. The idea of altering the Hertzian portion of the formula has been given up, since this is the part that rests on a theoretical basis, and attention given only to possible changes in the values of the constants of the exponential term.

The very considerable amount of experimental data on signal field strength collected during recent years has been used to obtain at least tentative constants for a new formula. Up to the present a value of

$$
u=\frac{0.0014 d}{\lambda^{0.6}}
$$

seems to give fairly satisfactory results. This may be slightly varied as more and better observational data are obtained.
Ril3.-The Magnetic and Electric Survey of the Earth: Its Physical and Cosmical Bearings and Development.-J. A. Fleming. (Journ.Washington Acad.Sciences, $4^{\text {th }}$ March, 1926, pp.109-I 32 .)
Rii3.i.-The Observation of Fading Effects : Measurement of Signal Strength with Simple Apparatus.-Prof. E. V. Appleton. (Wireless World, 2 Ist April, 1926, pp. 581-582.)
Rif3.2.-On the Diurnal Variation of Ultrashort Wave Wireless Transmission.Prof. Appleton. (Proc. Cam. Phil. Soc., April, 1926, pp. 155-161.)
An interpretation of ultra-short wave wireless phenomena is given which indicates that the maximum number of electrons per cc. in the atmospheric ionised layer is of the order $10^{5}$ to $10^{6}$. (Cf. A Hoyt Taylor and E. O. Hulbert in February issue of the Physical Review, where similar conclusions are arrived at regarding the maximum electronic content of the ionised layer.)

Rir3.2.-Fading.-Prof. Marchant. (E.W. \& W.E. May, 1926, Pp: 288-299.)
A lecture given to the Radio Society of Great Britain on 24th March, together with the discussion that followed.

Ril3.4.-High Frequency Rays of Cosmic Origin. (I.) Sounding Balloon Observations at Extreme Altitudes.-R. Millikan and I. Bowen. (Physical Review, April, 1926, pp: 353-361.)
This first of three articles on the cosmic radiation, recently investigated by Millikan, considers the observations recorded by two balloons which reached altitudes of $I I .2$ and 15.5 km . respectively. These give an average rate of production of ions that is only 25 per cent. of the value to be expected from the observations of Hess and Kolhörster and show the absorption coefficient of the extremely penetrating radiation to be as low as . 18 per metre of water instead of .57 as estimated previously.

Rii3.4.-The Kennelly-Heaviside Layer and Radio-Wave Propagation.-Dr. E. O. Hulburt. (Journ. Franklin Inst., May, 1926, pp. 597-634.)
Description of a theory of wave transmission, in many respects quantitative, adequately explaining several numerical facts of the propagation, distortion and polarisation of radio waves. The earlier part of the article is based on a paper by Taylor and Hulbuat (Physical Review, February, 1926)-the latter part, dealing with the polarisation of the received wave, is claimed to be new. This new part chiefly discusses reception with the Hertz antenna and those errors in radio-direction finding that arise from refraction in the upper atmosphere.

Rif3.6.-The Electro-Magnetic Equations of Light Propagation in Molecular Media of Varying Density.-Louis V. King. (Physical Review, April, 1926, p. 518.)
Abstract of paper presented to the American Physical Society.

Ril4.-A Lightining Strike.-N. E. Dorsey. (Journ. Washington Acad. Sciences, 16, pp. 87-93.)
Years ago a lightning flash was supposed to take place along the path of minimum electric resistance. Afterwards Lodge gave reasons for supposing that it followed the path of minimum electric inductance, and this is the orthodox theory at the present time. In this paper the author describes what happens during a lightning flash as follows: There is at first a rush of electrons. These blaze a conducting path along which flows a more leisurely current of the ordinary type. Possibly this conducting current conveys a far larger quantity of electricity than is carried by the dant of electrons. The direction in which the dart flies is the direction in which the blow is delivered. The effects produced where the stroke stants differ characteristically from those produced where it ends. These conclusions were suggested by the close examination of a tree that had recently been struck by lightning.

Rir4.-On Lightning.-Dr. Simpson. (Proc. Roy. Soc., May, 1926, pp. 56-67.)
A discussion of the discharge of electricity through air at atmospheric pressure leading to the following conclusions :-
(a) The conducting channel of a lightning flash originates in the region of maximum electric field and develops only in the direction of the seat of negative electricity.
(b) A negatively charged cloud can only be discharged by a discharge which originates in a positively charged cloud or in the induced positive charge on the earth's surface.
(c). A positively charged cloud may be discharged by discharges starting in the cloud and terminating. either in the surrounding atmosphere or on the earth's surface.
(d) If a lightning flash is branched, the branches are always directed towards the seat of negative electricity.
(e) The application of these conclusions to 442 photographs of lightning discharges reveals the fact that the preponderance of the lower clouds from which lightning discharges proceed are positively charged.

## RiI4-Ri25.-The Directional Recording of Atmospherics.-R. A.Watson Watt. (Journ.

 I.E.E., May, 1926, pp. 596-610.)The full text of the paper is given here of which there was an abstract in E.W. \& W.E. for April. The discussion following the reading of the paper is also given (p. 617).

RiI4-Ri25.-An Instantaneous Direct-Reading Radiogoniometer.-R. A. Watson Watt and J. F. Herd. (Journ. I.E.E., May, 1926, pp. 61I-622.)
A paper read before the Wireless Section, 3rd March, together with the discussion that followed. An abstract was given in E.W. © W.E. for April.

Ri27.-Berechnung der Induktivität einer einwickeligen Rahmenantenne beliebiGER FORM (Calculation of the inductance of a single-layer frame antenna of any shape).-V. Bashenoff. (Zeitschr. f. Hochfrequenz., 27, 4, PP. 110-i 17 .)
From his long experience in constructing receiving stations the writer has found that a large frame with few windings has many advantages over a small frame with several windings, and the advantage of the triangular form, which, however, has not yet been considered, in the literature of the subject. Accordingly, in this article, he deduces a formula for calculating the coefficient of self-inductance of a right-angled triangle on the basis of the BioSavart law. This formula is employed to calculate the coefficient of self-inductance of all plane figuxes which can be converted into right-angled triangles of the same area and perimeter; the conditions for the admissibility of such a transformation being also found. Further, an approximate method is given for calculating the coefficient of self-inductance of figures which do not satisfy the conditions of transformation. The experimental tests of the formulæ deduced for triangles and other figures
show a small percentage deviation from the calculated values, which for the most part falls within the limits of the degree of accuracy of the measurements.

Rr30.-Characteristics of Shielded-Grid Plio-trons.-A. Hull and N. Williams. (Physical Review, April, 1926, pp. 432-438.)
Description of valves specially designed with the plate and grid electrostatically shielded from each other to prevent internal " feed-back." The capacity between plate and control grid was found to be only $.006 \mu \mu \mathrm{~F}$ in the beșt models. These valves were found to behave as strictly unidirectional devices at all frequencies. Plate-resistance characteristics are given, and it is shown that the internal plate conductance is negligible compared with the conductance of practical circuits. The plate current thus depends on grid voltage only, i.e., in place of the seven parameters of the threeelement tube there is but one : mutual conductance.

Rizo.--Supplement to " On the Input Admittance" with Reference to the Voltage Amplification Ratio. - Y. Watanabe. (Journ. I.E.E., Japan,, April, 1926, pp. 397-405.)
In the original paper (previous number) the input admittance circle diagram was treated mathematically and experiments were described in confirmation.

In this supplementary paper the author considers the voltage amplification ratio circle diagram for various cases of plate loading, and experiments are made at 100,000 cycles with the C.R. type A.C. potentiometer.

Ri31.-Untersuchungen an RaumladegitterRÖHREN (Investigation of valves with spacecharge grid.-E. Alberti. (Electrische Nachrichten-Technik, April, 1926, pp. I49154.


Fig. 1.
To steepen the characteristic in amplifying valves by reducing the space-charge, Langmuir has introduced a positively-charged space-charge
grid between cathode and control grid. The theory of these valves is much more complicated than that of three-electrode valves and up to the present has only been treated with certain assumptions, such as that the wire of the control grid is very thin and also that no space-charge effects occur between the two grids and the anode-which condition is only fulfilled when the control and anode tensions have large positive values.

In this article some phenomena with these fourelectrode valves are described and explained that perhaps are not generally known. They. refer to measurements with the grid voltage negative, for which, of course, the assumptions of the theory are not fulfilled. The relation between anode current and heating is represented in Fig. I.


Fig. 2.
The explanation of the curve first rising and then falling is to be looked for in the presence of spacecharge between space-charge and control grids, which increases with the heating.

The way in which the space-charge grid current depends upon the space-charge grid voltage was also investigated and is shown in Fig. 2.

The characteristic obtained here likewise first rises and then falls, which can also be accounted for by the space-charge with negative grid voltage.

Riji.-Measurements of High Frequency Amplification with Shielded Grid Pliotrons —Albert W. Hull. (Physical Review, April, 1926, pp. 439-454.)
Description of tests of cascade high frequency amplification using specially constructed valves in which the control grid is electrostatically shielded from the plate. These valves are free trom internal feed-back so that any number may be operated in series. The voltage amplification per stage depends only on the circuit impedance, and was found to be 200 at $50 \mathrm{kC}, 40$ at $1,000 \mathrm{kC}$, and 7 at $10,000 \mathrm{kC}$. Total amplifications as high as $2,000,000$ in voltage were obtained and measured at $1,000 \mathrm{kC}$. It is shown that this is the largest amplification that can be usefully employed at any frequency. At $10,000 \mathrm{kC}$ a total amplification of 10,000 was obtained. Examples are given of other applications where shielding is advantageous, such as in the amplification of small photo-electric currents.

## Ri33-1.-SUR Les Variations D'Intensité du Courant Thermionique lorsqu'on Change la Distance entre le Filament et l'Anode.-T. Jonescu. (Comptes Rendus, 26th April, 1926, pp. 1016-1018.)

The filament employed in this investigation was of platinum covered with calcium oxide, 6 mm . long and . 1 mm in diameter. The circular anode had a diameter of 5 mm . The distance between filament and plate was varied by turning a micrometer screw and ranged for these experiments between 1.2 and . 01 mm .

The current obtained as a function of the distance is shown graphically for 60,120 and 180 volts difference of potential.


Fig. 1.
$\therefore$ As the distance diminishes, the current intensity is seen to pass through a maximum and then decrease again as the distance becomes very small. This decrease was found to be caused partly by the cooling of the filament due to the close approach of the anode.

The variations in current intensity produced by changing the distance between filament and anode have been applied to the investigation of small movements. For instance, a branch of a tuning fork was used as anode and the distance between it and the filament was adjusted so that the point of operation lay on the steepest part of the curve (Fig. r). By inserting a telephone in the anode circuit and causing the tuning fork to vibrate, the fundamental note and harmonics could be heard and the positions of the nodes of vibrations determined. The method appears a good one for finding the nodal lines of a plate.

Also a microphone was made by employing as anode a thin metal sheet stuck to a sheet of mica in front of which one speaks. The sound heard in a telephone inserted in the anode circuit is loud and very clear. The variation of intensity of the current obtained is of the order of $10^{-4}$ amperes.

Ri34.2.-La Détection par Caracteristique de Grille Peut-on l'Améliorer? (Can detection by grid characteristic be improved ?). -S. Luroff. (Radio Revue, 9th May, 1926, pp. 33-36.)
The mathematical theory of detection by grid characteristic has been developed by Laüt and Beauvais. This article deals with the same question, but with certain modifications at the outset.

Beauvais and Laüt start from a difference of potential between grid and variable filament. Here the author supposes that the difference of potential filament-grid is determined by a given incident E.M.F. superimposed on the constant E.M.F. of the heating battery.

A graph is thus obtained which is slightly different from that to which the theory of Laüt and Beauvais leads.

From the investigation some considerations as to the possible improvement of detection by grid characteristic are deduced.
Ri34.4.-High Frequency Resistance: Damping Effects in Receiving Circuits and how They may be reduced by Reaction. E. Mallett. (Wireless World, 28th April, 1926, pp. 618-620.)
Ri35.-Sur la Modulation Fixe des Postes Emetteurs i Triodes Alimentés par Courant Continu (On the fixed modulation of triode transmitters supplied with direct current).-A. Blondel. (Comptes Rendus, 26th April, 1926, pp. 997-999.)
Modulating devices for telephony transmitters respond to complex difficulties and the depth of the modulation has to be sacrificed to the purity. On the other hand, for the transmission of a steady note by a signalling station, such as a radio beacon, attempt should be made to obtain the maximum depth of modulation by simple means. From this point of view the author has investigated different methods of modulating the grid current of triode transmitters in the case where the plate is supplied with direct current. The usual methods employ low frequency devices, such as the tikker or vibrator or low frequency heterodyne, all of which necessitate the grid working in the neighbourhood of the point where oscillations just cannot take place, requiring fairly delicate adjustment, and reduce the useful power that can be obtained from triodes.

A better method is described here consisting of coupling a high frequency heterodyne to the grid circuit of an autodyne transmitter, the plate circuit being separately adjusted to a neighbouring wavelength. This employment of the high frequency heterodyne is said to improve the output of modulated waves and to permit the musical note obtained to be varied easily and within wide limits by a simple adjustment of the wavelength of the heterodyne and also to produce sharper sounds than with an alternator.
Ri38.-A Study of the Concurrent Variations in the Thermionic and Photo-Electric Emission from Platinum and Tungsten with the state of the Surfaces of these Metals.-T. Harrison. (Proc. Phys. Soc., Lond., 15 th April, 1926, pp. 214-233.)
Attempt is made to measure the thermionic
and photo-electric work functions for the same specimens of tungsten and platinum. It is found that in all cases the results depend greatly on the previous heat treatment of the material. In the case of platinum the curves showing the dependence of photo-electric sensibility on the wavelength of the irradiation are of four different types, while as regards thermionic properties the specimens can take up either a " large emission" or a "small emission" state, according to their treatment. The photo-electric work function of platinum came out greater than the thermionic, but no definite results were obtained for tungsten.

Ri38.-Reconciliation of Experiments on Probability of Ionisation by Electron Impact.-K. Compton and C. Van Voorhis. (Physical Review, April, 1926, p. 516.)
Abstract of paper read at the Montreal meeting of the American Physical Society last February.

It is shown that the recent work in this field by Hughes and Klein and that of the authors are brought into fair agreement by correcting for three sources of error:-
I. Effect of the electric field on the effective area of holes in the grid through which primary electrons pass.
2. Presence of slow secondary electrons and lack of homogeneity in velocities of primary electrons.
3. Warming of gas by filament.

Ri38.-Analysis of Positive Ions Emitted by A New Source.-G. Harnwell and $H$. Barton. (Physical Review, April, 1926, p.514.)
Abstract of paper read at the Montreal meeting of the American Physical Society last February.

Certain iron oxide crystals containing about i per cent. of some alkali metal or alkaline earth have been found to be constant and abundant thermionic emitters of positive ions. The alkali metal ions were emitted at lower temperatures than the alkaline earth ions. A preliminary treatment consisting of reduction at red heat in an atmosphere of hydrogen increased the emission.

The emission was analysed by means of a mass spectrograph described by Smyth.

Ri42. - Gekoppelte Schwingungsgebilde (Coupled oscillatory systems).-H. Hecht. (Elektrische Nachrichten-Technik, April, 1926, pp. 12I-I 38 .)
In this article the theory of coupled oscillatory systems is given with a view to the practical application of the theoretical results, but without reference to any particular technical problems. Since forced oscillations are the more important in practice, these are considered in detail, and free oscillations only mentioned occasionally. The ideas and expressions actually employed are those for a mechanical oscillatory system, but these apply equally well to an electrical oscillatory system for investigating theoretically oscillation phenomena occasioned exclusively through coupling.

Resonance is found to occur as follows (all
coupling is supposed close and of the same value) : A simple oscillatory system has only one position of resonance. Two coupled systems have two resonance positions symmetrical to the resonance position of the one-wave system, and a minimum coming at the position of resonance of the one-wave system. With three oscillatory systems, the number of resonance positions increases to three, the lowest one coming below the lower of the twowave system, the medium one at the dip, and the highest above the higher of the two-wave system, while the two minima fall at the two positions of resonance of this latter system. In the same way one passes from three-wave to four-wave systems, and the general law can be formulated, that the resonance or minima positions correspond to the minima or resonance positions respectively, of an arrangement that is greater or less by one system.

Ri44--Effective Resistance of Inductance Coils at Radio Frequency-Part II.S. Butterworth. (E.W. E W.E., May, 1926, pp. 309-316.)
Ri44.-Losses in Inductance Coils: The Need for a Standard of Efficiency.S. Butterworth. (E.W. E W.E., May, 1926, pp. 267-268.)
Ri48.i.-Comment Éviter les Déformations de la Voix en Télephonie sans Fil (How to avoid speech distortion in wireless telephony). (Radio Electricité, Ioth May, 1926, pp. 163-166.)
A first part of an examination of the different causes of distortion proceeding from the receiving end, leaving out of account those due to the transmitter and propagation.

This distortion can arise from :-
I. Effects due to the antenna and the high frequency circuits associated with it.
2. Effects due to the detection.
3. The low frequency amplifier.
4. The loud-speaker.

In addition, account must be taken of the characteristics of the human ear, in the appreciation of this deformation.

This article discusses the effects due to the antenna and high frequency circuits.

The unfavourable action is shown of too sharp resonance, of circuits with very little resistance, and of reaction that is pushed too far, leading consequently to a consideration of the employment of lightly damped circuits and the amplification of harmonics.

Causes of distortion 2, 3 and 4 are being discussed in a later number.

## Ri48.i.-Distortion in Telephones: Effect of Series and Parallel Connections on Quantity.-W. Griffiths. (Wireless World, 5th May, 1926, pp. 650-652.)

Ri49.-A New Contact Detector: The John-sen-Rahbek Contact as a Rectifier. (Wireless World, 2 Ist April, 1926, p. 588.)

R200.-MEASUREMENTS AND STANDARDS.
Ra4o.-On the Method or Measuring Phase Angle with a Triode.-T. Kumazawa. (Journ. I.E.E., Japan, April,' 1926, Pp. 362-3.96.)
Four methods are investigated both experimentally and theoretically.

Rz8i.--Theory of Absorption in Solid Dielec-TRICS.-V. Karapetoff. (Journ. Amer. I.E.E., March, 1926, pp. 236-243.)
Most solid dielectrics are imperfect in the sense that when a constant D.C. voltage is suddenly applied, a displacement of electricity first takes place almost instantly to a certain value, and then continues to increase asymptotically towards an ultimate magnitude. Accordingly, an initial electric charge and a greater final charge may be distinguished, with the corresponding values of initial and final permittivities. The purpose of the present investigation is to establish certain general properties of the function which expresses the increase in the initial electric displacement with the time. The initial and the final leakage conductivities are also taken into consideration.

## R300.-APPARATUS AND EQUIPMENT.

R320.-LUFTLEITERMESSUNGEN AM RuNDfUNKsender Witzleben (Aerial measurements at the Witzleben Transmitting Station).W. Schaffer and G. Lubsgynski. (E.N.T., 3. 4, 1926, pp. 155-160.)

Detailed numerical data of the Witzleben aerial are given and the conclusions drawn from them, preceded by a short general description of the station.

R342.4-A Reflexed Receiver with Resistance Audio Coupling.-L. W. Hatry. (Q.S.T., May, 1926, pp. 23-26.).
The principle is explained and constructional hints are given. Unlike reflexing a single valve, the multi-tube reflex is a troublesome device to build, but it represents a considerable saving in valve cost and battery consumption.

R344.-Piezo Crystal Control. (Electrician, 21st May, 1926, p. 524.)
Details are given of the method used in the United States for overcoming the problem of broadcast interference.

R344.-Adjusting the Crystal-controlled Transmitter.-S. McMinn. (Q.S.T., May, 1926, pp. 43-45.)

R355.51.-Taming the Synchronous Rectifier. -R. Kruse. (Q.S.T., May, 1926, pp. 9-16.)
Methods are discussed for the prevention of sparking in the employment of the synchronous rectifier for plate supply.
R355.54.-A Dry Electronytic Rectifier.S. Kruse. (Q.S.T., May, 1926, pp. 30-32.)

Description of the Ruben rectifier (or EIkorn charger) which uses a pair of discs, one of magne-
sium and the other a composition, between which a film is formed that effects the rectification, no moisture being present other than that in the air.

R355.55.-A New Valve Rectifier: Inexpensive Accumulator Charging from A.C. Mains.-Dr. Sayce. (E.W. © W.E., May, 1926, pp. 269-270.)

R384.1.-Neon Wavemeter: A Simple, Inexpensive Instrument for the Transmitter. -A. P. Castellain. (Wireless World, 2 Ist April, 1926, pp. 576-579.)
R387.-High Voltage Insulators.-W. Ure. (Electrician, 21st May, 1926, pp. 522523.)

Unfortunately, good mechanical and high electrical qualities are nearly always antagonistic. This article indicates some of the more common factors that enter into insulation failure.

R388.-Phase and Magnitude of Deflections of Braun Tube Beam with Internal and External Electrodes.-R. Jack and L. Jones. (Physical Review, May, 1926, p. 642.)
Abstract of paper presented at a recent meeting of the American Physical Society.

At the lower frequencies the deflection is found to be a function of the frequency, while at the higher range the deflection becomes constant. At high frequencies the deflections with external and internal electrodes are apparently equal. By the use of Lissajous' figures, with the internal and external sets of electrodes at right angles, the phase difference is found to disappear at high frequencies.

## R400.-SYSTEMS OF WORKING.

R420.-On the Wireless Beam of Short Electric Waves-II.-S. Uda. (Journ. I.E.E., Japan, April, r926, pp. 335-351.)
The investigation is continued (see last number), using metal rods as reflectors.

R440.-The Problem of Remote Control : Relays and Circuits for Switching the L.T. Current:-A. P. Castellain. (Wireless World, 2nd June, 1926, pp. 729-730.)
R440.-Automatic Valve Replacement : Auxiliary Valve Circuits for Remote Con-trol.-Dr. McLachlan. (Wireless World, 28th April, 1926, pp. 630-632.)

## R500.-APPLICATIONS AND USES.

Rsia.-Marine Wireless Eguipment: Moderk Practice Exemplified dy Siemens' Transmitting and Receiving Apparatus. (Wireless World, 21st April, 1926, pp. 585588.)

R520.-Wireless on the Polar Airship: Transmitting, Receiving and Direction-finding Eguipment of the " Norge." (Wixeless: World, 5 th May, 1925, pp. 669-671.)

R 582 .-Transatlantic Photo-Telegraphy. (Electrical Review, 4th June, 1926, pp. 820822.)

Apparatus has been installed in London, New Fork and San Francisco for the establishment of a regular, reliable service for the transmission and reception of news' photographs, facsimile printed and written documents, designs, etc., via the 220 -mile land line from London to the Carnarvon station of Marconi's Wireless Telegraph Co., thence across the Atlantic Ocean to the Riverhead station of the Radio Corporation of America on Long Island, and further on by land lines. Some account is given here of the system of transmitting and receiving photo-radiograms developed by Capt. Ranger and the Radio Corporation of America, of which there was a full description in Proc.I.R.E. for April (three abstracts, p.389, June, 1926).

Abridged accounts of picture transmission by the Ranger system are also to be found in Wiveless World, 26th May, 1926, pp. 686-688, and Electrician, 21st May, 1926, p. 526.

R582.-Wireless Photo-Telegraphy. (E.W \& W.E., May, 1926, p. 3OI.)

A short account of the Thorne-Baker system.
R586.-Television Apparatus: A Description of the Jenkins System.-A. Dinsdale. (Wiveless World, 5 th May, 1926, pp. 642645.)

## R600.-STATIONS : DESIGN, OPERATION AND MANAGEMENT.

R6if.-.The Rugby Radio Station of the British Post Office.-.E. Shaughnessy. (E.W.\&W.E., May, 1926, pp. 27 I-28I.)

Abstract of paper read before the Wireless Section, I.E.E., on 14th April.

R6if.-The Rugby Radio Station.-E. Shaughnessy. (Electrician, 23rd April, 1926, pp. 468-469; Electrical Review, 3oth * April and 7 th May, 1926, pp. 716 and 753 respectively.)
Abstract of a paper read before the Wireless Section of the Institution of Electrical Engineers.

R6i2.-Australia: Ballan "Beam" Station, (Electrical Review, 4 th June, 1926, p. 836.)
The site at Ballan is about 1,800 feet above sea level, and is free from "shielding" hills. The station will have two separate transmitters, one for communication direct with England, and the -other for working with Canada. Each aerial system will be supported by three masts, $25^{\circ} \mathrm{ft}$. in height, and the angle of the beam will be about 45 deg. The power used by the transmitters will be about 25 kilowatts. Messages will be sent from the offices of Amalgamated Wireless (Australasia), Ltd., Melbourne, by whom the station is owned, by telegraph line to Ballan, at a speed of roo words a minute. A very short wavelength, probably less than 40 metres, will be employed, and it is expected that the station will be ready for tests to be carried cout next month.

R6i6.57.-The New Vienna High-Power Station: A Description of Austria's "Daventry."-P. Gordon Fischal. (Wireless World, 28th April, 1926, pp. 625-629.)

R62o.-The Central Radio Office.-A. Irwin. (Post Office Electrical Engineers' Journ., April, 1926, pp. 67-73.)
A description of the C.R.O. situated on the second floor of G.P.O. West.

R620.-The Post Office Wireless Services.E. H. S. (Post Office Electrical Engineers' Journ., April, 1926, pp. 58-66.)
An account of the wireless services conducted by the Post Office by means of its point-to-point and coast stations, and the organisation by which these services are operated.

R62T.385.-The Telephone, I876-1926.-J. E. Kingsbury (Electrical Reyiew, 5th March, 1926.)

A review of the progress of the telephone since the first patent granted on 7 th March, 1876 , to Alexander Graham Bell.

## R800.-NON-WIRELESS SUBJECTS.

621.355.-Experimentelle Untersuchungen uber Magnetische Frequenzwandler (Experimental investigation of magnetic frequency changers).-H. Plendl, F. Sammer and J. Zenneck. (Zeitschr. f. Hochfrequenz., 27, 4, pp. гол-тіо.)
The article is summarised as follows :-
I. In frequency multiplication it is the saturation of the iron core that is important, while the tuning of the primary circuit is immaterial. For practical reasons, however, working generally takes place in the neighbourhood of resonance.
2. The large iron-free self-inductance in the primary circuit that is usually employed to obtain favourable conditions can often be replaced with advantage by a blocking circuit tuned to the third harmonic of the generator frequency.
3. Study is made of circuiting an iron-free selfinductance in parallel with the generator. This circuit arrangement, which has been described by Punp and Schmidt, gives favourable working conditions with correct tuning.
4. Considerable improvement in the frequency amplification can be obtained by placing a capacity parallel to the secondary winding of the iron core.
5. The higher the frequency multiplication, the closer is the most favourable coupling between primary and secondary circuits.
6. Multiplying frequency in two stages was found to be better than in one stage both as concerns the relation of the secondary current to the primary and the transient course of the secondary current.
7. Thorough investigation with the Braun tube was made of the physical phenomena and transference of energy in frequency multiplication with special reference to the circuit arrangement given in (4) and the question of the effect of tuning the secondary circuit.

## Esperanto Section.

Abstracts of the Technical Articles in our last Issue. Esperanto-Sekcio.

## Resumoj de la Teknikaj Artikoloj en nia lasta Numero.

## ROOO.-RADIO GENERALE.

Roio.-La Kunrilatigo de Kelkaj Lastatempaj Progresojen Radio-D-ro. B. Van der Pol.
Ci tiu artikolo estas traduko el la holanda lingvo, aperinta en Polytechnisch Weekblad, igan de Nov. 1925a. $\hat{G i}$ donas revuon pri la fundamentaj principoj de kelkaj malsamaj metodoj por generi oscilojn, kaj kunrilatigas diversajn arangojn, kiuj ŝajnas havi, unuavide, nenian rilaton unu al alia. La metodoj estas generale dividitaj en tiuj okazoj kie (a) la rezisteco kaj, (b) la induktanco estas funkcioj de l'kurento. En la unua klaso troviĝas la arko, triodo kun reakcio, dinatrono, Multibibratoro, Neona lampo, k.t.p. En la dua klaso troviĝas diversaj aranĝoj por la sango de frekvenco, k.t.p., kun aparta aludo al diversaj specialaj aranĝoj.

Roso.-Resumoj kaj Aludoj.
La komenciĝo de nova serio de Resumoj kaj Aludoj, kompilita de la Radio Research Board (Radio-Esplorada Konsilantaro), kaj publikigita de la Brita Registara Fako de Scienca kaj Industria Esplorado.
Ro82.-Alliniigaj Grafikajoj por Selektivaj Amplifikatoroj.-W. A. Barclay.
La grafika joj presitaj estas por ilustri kaj solvi esprimojn uzitajn en artikolo pri "Selektivaj Amplifikatoroj" de S-ro. P. K. Turner en E.W. © W.E., de Oktobro, 1925 a. La grafika $\hat{0} \mathrm{oj}$ reproduktitaj estas por la rezonanca ekvacio por malagorditaj signaloj, por agorditaj signaloj, kaj por la komparo de malagorditaj signaloj kun la agordita portanta ondo.

## R100.-GEENERALAJ PRLNOIPOJ KAJ TEORIO.

Ri32.-Kelkaj Notoj pri Intervalvaj Kuploj. --Lekcio de S-ro. H. L. Kirke, de la Brita Brodkasta Kompanio, al la Radio-Societo de Granda Britujo, je 28a Aprilo, 1926 a.
La lekcio amplekse esploris la temon de intervalva kuplo je radio-kaj aüd-frekvencoj. La temo de amplifado estas unue konsiderita, laŭ la simpla okazo de rezisteca kuplo, kaj la rezonado etendita al soka kuplo, kaj al agordita anoda kuplo. La efekto de reakcio estas poste diskutita; interalie, la aranĝo por uzado de ekstra valvo sole por reakciado. Pritraktita estas la efekto de interelektroda kapacito, sekvita de tre ampleksa revuo pri neŭtrodino. Oni priskribas kaj ilustras diversajn metodojn por obteni la neŭtrodinan alĝustigon, kun tre klaraj teoriaj diagramoj por klarigi ilian funkciadon. Poste konsideritaj estas malaltfrekuenca amplifado, rezisteco, kaj ŝoka kuplo (ferkerna).

Laste estas diskutita malaltfrekvenca transformatora kuplo, la argumento estante bone ilus-, trita per konsiderado de "ekvivalentaj cirkvitoj," La havigo de plata responda kurvo estas diskutita kaj montrita grafike, kaj mallonga fina aludo farita al la efikoj ce la fera kerno.
La diskutado kiu sekvis la lekcion estas presita.
Rifo.-La Kaŭzo Kaj Elimino de Noktaj Eraroj en Radia Direkto-Trovado.Resumo de Referato de D-ro. R. L. SmithRose kaj S-ro. R. H. Barfield, antaŭ la Senfadena Sekcio, Institucio de Elektraj Inĝenieroj, je 5 a Majo, $1926 a$.
La laboro priskribita estas esploro pri deflankiĝo de la alvenantaj ondoj. Ricevantena aranĝo por la kompenso de la horizonta membro de l'elektra kampo estas priskribita. Ci tio estas pro S-ro. F. Adcock, kaj konsistas el kvar Hertzaj oscilatoroj, aranĝitaj tiel, ke ili kompensas la efektojn ce siaj horizontaj sekcioj, la neta voltkvanto estanta pro la faza diferenco de la elektromovaj fortoj ce la vertikalaj membroj. Kurvoj de rezultoj montras tre malgrandajn variojn de la direktoj ricevitaj pere de ci-tiu sistemo, dum grandegaj varioj estas ricevitaj per unubobena aparato. Oni konkludas, ke variantaj noktaj eraroj okazas, ne pro deflankiĝo, sed pro la alveno de malsuprenirantaj ondoj polarizitaj, kun la elektra forto horizonta.

## R200.-MEZUROJ KAJ NORMOJ.

R220.-La Daŭreco de Normigo de Variebla Kondensatoro.-E. Simeon.
La artikolo diskutas la variecon kapacitan pro vario de la distanco inter la plataro, kaŭzita de suprena kaj malsuprena movado de la centra spindelo. Diversaj mekanikaj detaloj por malgrandigi tiun-ĉi specon de movlibereco estas diskutitaj, kaj ilustrita estas desegnajo kie, kiom ajn da movo, tio ne kaŭzos kapacitan sanĝon.
R240.-Inductancaj Bobenoj Kvante Kom-paritaj.-A. L. M. Sowerby.
En Parto Ia (en la Aprila numero), la aŭtoro citas rezultojn de mezurado de diversaj bobenoj, kompare kun normo de konata malgranda perdo, je la frekvenco de la Londona Brodkasta Stacio. La rezultoj generale montras, ke ju pli maldike la fadeno, des pli alta la rezisteco, kaj ke, se estas optimuma valoro de la diametra/longeca proporcio, $\hat{g} i \quad$ estas, $\hat{c} e$ tiuj-ĉi mezuradoj, kasita de aliaj faktoroj.

En Parto IIa (Junio), pluaj mezuroj estas montritaj, ilustrantaj la efikon de (a) Dielektriko, (b) formo de bobeno, (c) interspacado, kaj (d) dikeco
de fadeno. La komparo inter bobeno de malgranda perdo kaj bobeno vindita per sonorila fadeno vakse kovrita montras, ke la diferenco inter ili estas nur $2.5 \%$, kio sugestas, ke dielektrika perdo ne estas grava ce unutavolaj bobenoj je brodkastaj ondolongoj.

Rilate al la formo de la bobeno, la mezuroj malkaŝas nenian indikon pri optimuma valoro de D/I (proporcio diametro/longeco). Rilate al interspacado, kurvo por No. 22 kaj por No. 28 Standard

- Wire Gauge (brita Norma Fadená Dikeco) malkaŝas optimuman kondiĉon ĉe interspacado de ĉirkaŭ 1.2-I. 4 diametroj de la fadeno. Por optimuma interspacado la dikeco de fadeno faras malpli da diferenco ol estas generale kredita, la kurvo citita montranta, ke fadeno No. 22 estas nur $6 \%$ plibona ol la malpli dika No. 28 ce optimuma interspacado. Fina sekcio pritraktas praktikan desegnadon.


## Rz90.-Mezuroj Aŭd-Frekvencaj.

Tiu-ĉi artikolo konsistas el pri-radiaj resumoj el referato " Frekvencaj Karakterizoj de Telefonaj Sistemoj kaj Aǔd-Frekvenca Aparataro, kaj ilia Mezurado." legita antaŭ la Institucio de Elektraj Inĝenieroj, je Aprilo 29a, de S-roj. Cohen, Aldridge, kaj West, de la Brita Cef-Poŝtoficeja Esplorada Fako.

Priskribita estas metodo por obteni konstantan elektran elmeton, kontinue varieblan de o ĝis 5,000 cikloj. Du radio-frekvencaj oscilatoroj estas aranĝitaj por interbati, la rezulto estante rektifita, filtrita kaj amplifita por doni la bezonitan elmeton. Eksperimentajn rezultojn oni priskribas kaj ilustras,
de la respondo/frekvencaj karakterizoj de kelkaj laŭtparoliloj kaj intervalvaj transformatoroj.

## R300.-APARATO KAJ EKIPAĴO. <br> R383.-Fadenaj Anod-Rezistancoj.-F. M. Colebrook.

Priskribo pri konstruo de neindukta rezistanco de malgranda mem-kapacito kaj de valoro de cirkaŭ 50,000 omoj. Fendita fundamento estas farita per interfoliigo de ebonitaj diskoj de $1 / 16$-cola dikeco, kaj I $\frac{1}{4}-\mathrm{kaj}$ 2 $\frac{3}{4}$-cola diametro, kaj la fendoj vinditaj per No. 47 a "Eureka" fadeno, duoble kovrita per silko, la vindajoj estante inversitaj je ĉiu 50a turno. La mem-kapacito, oni trovis, estas inter 6 kaj $7 \mu \mu \mathrm{~F}$ por rezistancoj de ĉirkaŭ 50,000 omoj.

## R500.-APLIKOJ KAJ UZOJ.

R545.009.2.-Long-Distanca Laborado.-H. N. Ryan.
La perioda rakonto pri longdistanca interkomunikada laboro inter amatoroj.

## R800.-NE-RADIAJ TEMOJ.

5io.-Matematikoj por Senfadenaj Amatoroj. -F. M. Colebrook.
Daǔrigo de la serio komencita en la antaǔa numero. La temo pri Elementa Algebro daŭras, kaj la jenaj sekcioj pritraktitaj: (I) Krampoj kaj la Negativa Signo, (2) Ripetita Kombino, (3) Etendita signifo de la literaj simboloj, (4) Fizika ekzemplo, de la kombino de pozitivaj kaj negativaj numeroj, kaj (5) Komprenado, aŭ sensperta kalkulado.

## Correspondence.

## Letters of interest to experimenters are always welcome. In publishing such communications the Editors do not necessarily endorse any technical or general statements which they may contain.

## Inductance Coils Quantitatively Compared.

Sir,-With reference to the articles by Mr . Sowerby in E.W. © W.E. for April and June, I wish to draw attention to the small amount of information gained from the large amount of work, due, presumably, to an insufficient knowledge of the subject.

With the possible exception of his test to compare dielectric losses, there is hardly a result of serious value to other workers, largely on account of the simultaneous change of several variables in each of his series of tests.

After all, the following " postulates" are generally accepted by now among those who have studied the subject; they are based on theory, and have repeatedly been confirmed by test:-

1. For any coil of given dimensions and inductance, there is one optimum size of wire for any given frequency.
2. For geometrically similar coils, the H.F. resistance decreases, to a rough approximation, proportionally as the linear dimensions increase.
3. For single-layer solenoids there is an optimum $D / l$ ratio, of the order of 3 .
4. In any coil well designed for the broadcast band of frequencies, the copper losses are of the order of $80-90$ per cent. of the total ; the dielectric losses are not important.

No. 4 above confirms the test given on page 303 of the June issue, but this test is none the less quite unreliable, for the two coils compared differ not only as to their dielectric losses, but are also unlike in the cross-section of former, in the mean diameter, and in the shape ratio $D / l$.

Again, consider Tables I. and Ia. Each separate $D / l$ ratio should have its own optimum spacing, ratio in order to find which gives the " best best" coil. But all the coils are "close-wound," so the results tell us nothing; this apart from the fact that the diameters as well as the lengths of the coils vary.

In Tables II, and IIa. there is the same trouble : the relative efficiencies of various spacing ratios cannot be isolated, for the coils are all of different shapes.

Apart from these particular objections, it appears to me that the accuracy of his method of measurement is seriously open to question. In every test 'the resistance of the "short indoor aerial" is in the circuit. This may easily have been of the order of 20 to 40 ohms , and if so would mask any small differences of resistance between various coils. Again, all the coils have apparently been tested at the same frequency on the same condenser setting. There may be large differences of selfcapacity between the various coils, and if so there will have been corresponding differences of inductance. If this latter is the case, one would not
expect equal resistances nor equal deflections; in any case, the resistances should be corrected for the apparent increase due to self-capacity, and I see no statement as to this.

Lastly, with the source distant io ft., there is every possibility that equally good coils may give different deflections due to variations of their absolute size, which will affect the amount of energy directly picked up by them from the source.

I would suggest to Mir. Sowerby that he would produce much more useful results on the following lines:-
(a) Several $D / l$ ratios should be selected, say, between 0.2 and 5 .
(b) A set of coils should be built to each ratio. Each such set of coils should be identical in size, shape and number of turns, and should differ from the others in its set only in the gauge of wire.
(c) The gauges chosen should be such that the ratio

$$
\frac{\text { distance between centres }}{\text { diameter. of wire. }}
$$

varies betwee close winding and, say, 2.
(d) The inductance and self-capacity should be measured first.
(e) The H.F. resistance should be actually measured.
( $f$ ) This should be corrected for self-capacity, and expressed as power factor or " magnification," which is the reciprocal of power factor.

A set of accurate results on these lines, even if not corrected for the other resistances in the circuit (leads, condenser, etc.), would be of immense value, and if Mr. Sowerby or any other reader has the time and energy to undertake it I shall be only too glad to pass on a few hints based on a considerable experience of such work.

Blackheath. P. K. Turner.

## j, the Heaviside Operator, and $\sqrt{-1}$.

> The Editor, E.W. \& W.E.

Sir,-The fundamental importance of the topic discussed so ably in your Editorial of May issue must plead my excuse for craving yet further the courtesy of your columns.

It is, I think, greatly to be deplored that to any symbol so widely used as $j$ can be attached two distinct meanings. Unfortunately the confusion in this case is rendered greater since the two meanings are not widely dissimilar; indeed, it is their very resemblance that constitutes for the tyro his main difficulty.

If, for example, in the rectangular form of vector representation

$$
E=a+j b
$$

in which $j$ is the vector operator, we substitute the values

$$
a=E_{1} \cos \phi, \quad b=E_{1} \sin \phi
$$

we obtain the polar form

$$
E=E_{1}(\cos \phi+j \sin \phi)=E_{1} \varepsilon^{j \phi}
$$

often a convenient mode of expression. In this equation, however, $j$ can no longer be regarded as the same operator. For if it were, its behaviour,
as you correctly state in equation (2) of your Editorial, would have the effect

$$
j \sin \phi=\cos \phi
$$

which is clearly inadmissible.
The source of confusion which lies around this point is a very real one for the student, and in the interests of clear thinking, it behoves us to give earnest consideration to any means of avoiding the difficulty.

The Heaviside notation retains the use of $j$ for the algebraic imaginary $\sqrt{-1}$, and proposes for the vectorial operator the symbol $D / \omega$. As you. point out, this is a change of label without any change of principle; and this precisely meets the necessities of the case.

Besides this enormous advantage, the substitution of $D / \omega$ for $j$ in vector operations will in most cases simplify the working. For example, the operators $L D$ and $I / C D$ will supersede $j \omega L$ and $\mathrm{r} / j \omega C$. Moreover, the symbol $D$ is a constant reminder of the operational character of our work, which cannot always be said of $j$.

In conclusion, may I be allowed to point out an obvious slip in your Editorial regarding the derivation of $D^{2}=-\omega^{2}$ ? This, of comrse, is obtained by a repetition of the operator $D$ to the fundamental equations, and not by sqwaring and adding them.
W. A. Barclay.

## Murtle, Aberdeenshire.

[With reference to the concluding paragraph of Mr. Barclay's letter, we have pointed out elsewhere that the relation $D^{2}=-\omega^{2}$ can be obtained by simply multiplying the two equations
$D \sin \omega t=\omega \cos \omega t$
$D \cos \omega t=-\omega \sin \omega t$
together, without any repetition of the operator $D$. G. W.O.H.]

## ERRATA.

June issue:-
Page 350, Figs. 1 and 1 a. "Some Notes on Intervalve Couplings." Caption should read:

$$
I_{a}=\mu V_{g} /\left(R+R_{a}\right)
$$

magnification is

$$
R I_{a} / V_{g}=\mu /\left(i+\frac{R_{a}}{R}\right)
$$

Page 351, Fig. 3.

$$
I_{a}=\mu V_{g} /\left(R_{a}+j x\right)
$$

magnification is

$$
\mu /\left(\mathrm{I}+\frac{R_{a}}{j x}\right)=\mu j x /\left(R_{a}+j x\right)
$$

Page 351. In Fig. 4 the horizontal scale is marked off in units of henries in the anode circuit per thousand ohms of valve resistance.
Page 351, col. r; line 1. For ratio read percentage.
Page 352, Fig. 6. Mägnification is

$$
\mu /\left(\mathrm{I}+R_{u} R C / L\right)
$$

Page 370. "Audio-frequency Measurements." Col. 2, line II, should reat," "A variable condenser $C_{2}$ of $0.0005_{\mu} \mathrm{F}$ maximum is in parallel with one of the condensers $C_{i}$ to cover the beat range of 5,000 cycles" (instead of 50,000 cycles as printed).

## Some Recent Patents.

## THE REISZ MICROPEONE.

(Application date, 25th June, 1925. No. 250,430.)
The construction of the Eugen Reisz microphone is described in the above British Patent. The microphone is of the carbon granule type, and the novelty of the invention lies in the particular arrangement of the granules, diaphragm and marble supporting block. A good idea of the device can be obtained by reference to the accompanying illustration. The microphone consists of a large heavy block of marble $M$, provided with two tunnels $T$, which contain electrodes $E$ which communicate with terminals placed at the back of the marble block. A channel $C$ extends between the two tunnels, and the tunnels and the channel are filled with coal dust. The granules are retained in position by means of a stretched rubber diaphragm $D$, which is further covered by gauze $G$, attached to a solid heavy ring $R$. The ring is so placed that it protects the ends of the electrodes $E$ and the granules surrounding them from direct

action of sound waves impinging upon them. The specification states that the distance between the stretched rubber diaphragm and the adjacent parallel surface of the channel is of the order of 3 to 4 mm . The size of the grain is given as larger than would pass a sieve of two hundred meshes per square inch, and the pressure of the rubber diaphragm is that of a rubber skin 0.15 mm . thick, and slightly stretched. It is stated that the sound waves of the lower frequencies are absorbed in the deepest layers, while those of the highest frequencies disappear in the uppermost layers. This is supposed to be due to the fact that air passages are formed between the granules, which are permeable by the lower tones but not by the higher tones. We understand that this type of microphone is employed by the British Broadcasting Company.

## A MULTIPLE CONE DIAPHRAGM.

(Convention date (France), 3rd May, 1924.

$$
\text { No. } 233,356 .)
$$

The Société Française Radio Electrique describe in this British Patent Specification the construction of a diaphragm comprising a number of paper cones. It will be seen from the accompanying illustration that a number of small cones $C$, which are folded from sheets of stiff paper, are attached
at their bases to a ring $R$ of substantial dimensions, which is supported in any convenient manner. The apices of the cones are rigidly attached to a circular member $K$ which is attached by a link $E$ to a vibrating system $V$. The specification states

that although the diaphragm is formed of a number of. sectors which vibrate more or less as a whole, owing to the fact that the sectors are combined rigidly near their point of actuation the maximum flexibility occurs at their extremities.

PLATE AERIAL.
(Application date, 2oth February, 1925.
No. 250,714.)
The construction of an aerial consisting of a number of insulated plates is described in the above British Patent Specification. by W. E.


Durrant and K. I. Rayment. Essentially the aerial consists of a number of plates of appreciable area, supported co-axially one upon the other. Thus, in the accompanying illustration, a stand $S$ is provided with an upright member $U$ which
carries the plates. The plates $P$ may be of circular, rectangular, or other shape, and may be spaced apart by means of distances pieces $D$ of insulating material, a flange $F$ of some description preferably being attached to the centre of the plates. The plates are connected at their peripheries by means of short leads $L$, while in another modification a portion $X$ may be stamped and bent from the actual plate itself to take the place of the connecting leads $L$.

## AN AMPLIFICATION SCHEME.

(Application date, $4^{\text {th }}$ February, 1925. No. 25 1,041.)
A system of amplification is described in the above British Patent by the Western Electric Company, Limited. The object of the invention is to overcome oscillation or singing effects which

are liable to occur, particularly in high frequency amplification, more especially in power amplifiers if used with high power transmitters. The accompanying illustration indicates the scheme, and also includes a device for coupling the alternating current supply to be used for heating the filament of the power amplifier. Thus, in the accompanying illustration, a valve $V_{1}$ is arranged as an oscillator. and is coupled by an inductance $L_{1}$ and a capacity $C_{1}$ with a centre tap. This is coupled to a resonant circuit $L_{2} C_{2} R$, the resistance $R$ being connected between the grid and filament of the amplifying valve $V_{2}$, the anode circuit being supplied with a source of power $P$. The anode circuit of the valve $V_{2}$ contains the inductance $L_{3}$ coupled to the aerial inductance $L_{4}$. The filament of the valve $V_{2}$ is heated by alternating current derived through the medium of a transformer $T$ from a source $A$. The grid of the valve $V_{2}$ is negatively biased by a source of negative potential $B$, the amount of which is controlled by a potentiometer $R_{1}$. Another transformer $T_{1}$ has its secondary winding included in the grid circuit of the valve $V_{2}$, the primary winding of the transformer being energised from the source of supply $A$. The voltage applied to the primary of this transformer is controlled by a potentiometer $R_{2}$. It is well known that the
emission from the filament of the valve $V_{2}$ will vary according to the potential existing across the filament. a time lag occurring due to the thermal inertia, maximum emission occurring just after maximum current, and vice versa. In order to compensate for the variation in emission and prevent variation in output, which would give rise to a modulation effect and variation of received current, the grid potential is purposely modulated in such a way as to obtain substantially constant anode current. Thus it will be seen that the primary of the transformer $T_{1}$ which is energised from the source of filament current $A$ is connected through a resistance $R_{2}$, capacity $C_{3}$ and a variable inductance $L_{5}$. The inductance and capacity act as a phasing device, while the resistance, or potentiometer $R_{2}$, controls the amplitude of the potential applied to the grid. By suitably adjusting the value of $R_{2}, C_{3}$ and $L_{5}$ a substantially constant output can be obtained.

## A CONICAL DIAPHRAGM.

(Convention date (U.S.A.), 2 rst October, 1924. No. 241,869 .)
The construction of a loud-speaker embodying a conical diaphragm is described by the British Thomson-Houston Company Limited and W. B. Potter in the above British Patent. A good idea

of the general arrangement of the loud-speaker can be gathered by reference to the accompanying illustration, in which it will be seen that the diaphragm $D$ is in the form of a cone, and is made of stiff paper, or similar light material, is flanged at the periphery, and is reinforced with a metal or similar ring at $R$. A number of holes $H$ are arranged round the periphery and contain lengths of cords $C$ by means of which it is further supported from a circular framework $F$, attached to the base $B$. The apex of the cone is fixed to a conical clamp $K$, which is connected by a link $L$ to a movement $M$. In one modification of the invention the movement $M$ comprises an ordinary electromagnetic receiver provided with a diaphragm to which the link is attached. The specification includes constructional details of various minor points, upon which the success of the loud-speaker no doubt depends.


[^0]:    * From material supplied by the Telefunken Company. We are also indebted to the Telefunken Company for the photographs which illustrate the article.

[^1]:    ${ }^{17}$ Loc. cit. ${ }^{11}$.

[^2]:    * A suitable instrument for use as the plate galvanometer is the suspended moving coil; micro-volt-ammeter (ex-Government), which can be obtained from Messrs. Leslie Dixon \& Co., 9. Colonial Avenue, Minories, London, E.i.

[^3]:    * The author's original figure numbers are adhered to throughout this abstract.

[^4]:    * [This method of fixing is likely to lead to distortion of the mirror, giving a very bad spot. A better way is to make a small aluminium foil clamp.-ED]-

[^5]:    * Two ballistic uses, the above and the measurement of mutual inductance, are given in The Calculation on Measurement of Inductance and Capacity (Nottage), Chaps. 4 and 3 respectively.

[^6]:    * This statement is calculated to raise clouds of dust from the pages of agitated and outraged text-books. It is possibly a position that could be blown to pieces by the "big guns" of pure mathematics, but this series is being written with a strong practical bias and the writer hopes to be able to avoid the introduction of incomprehensible ideas into the fundamental definitions. The standard text-books usually take a perfectly plausible but not very useful way out of this apparent difficulty. Having shown that $a \times b=b \times a$ for positive numbers, they say it shall be so for negative numbers also, saying it three times if necessary to secure conviction. In other words the symbol $x$ is so defined. But since we can then no longer understand the process we are not much forrader.

