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V.h.f. aerial gain calculation using
tables of mutual resistance between
the radiating elements

by

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(Research Department, BBC Engineering Division)

BRITISH BROADCASTING CORPORATION

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FOREWORD

THIS is one of a series of Engineering Monographs published by the British Broadcasting Corporation. About six are produced every year, each dealing with a technical subject within the field of television and sound broadcasting. Each Monograph describes work that has been done by the Engineering Division of the BBC and includes, where appropriate, a survey of earlier work on the same subject. From time to time the series may include selected reprints of articles by BBC authors that have appeared in technical journals. Papers dealing with general engineering developments in broadcasting may also be included occasionally.

This series should be of interest and value to engineers engaged in the fields of broadcasting and of telecommunications generally.

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V.H.F. AERIAL GAIN CALCULATION USING TABLES OF MUTUAL RESISTANCE BETWEEN THE RADIATING ELEMENTS

SUMMARY

A method for calculating the gain of a v.h.f. aerial from the mutual resistances between the radiating elements is described. The computation of mutual resistances between dipoles and unipoles mounted on a support mast is discussed. A set of tables of mutual resistances is included, together with notes on their use.

1. Introduction

Directional transmitting aeriels are frequently used at v.h.f. to prevent co-channel interference and to concentrate the power radiated in the directions in which it is needed. One type of aerial commonly used consists of columns of dipole or unipole radiating elements mounted around a mast; in general the number of elements in each column is different, while the currents in the individual elements may also be unequal. The calculation of the power gain of such an arrangement is not straightforward.

Gain calculation by integration of the power radiation pattern would require the computation of patterns in a large number of planes and would be very laborious. The alternative method of gain calculation from the mutual resistances between radiating elements is therefore more attractive, but mutual resistance tables for dipoles in free space cannot be used because of the presence of the support mast. To overcome this difficulty, mutual resistances between dipoles and unipoles mounted on a conducting cylinder have been computed. The method of computation is described in this monograph, which also contains a set of tables and notes on their application to specific aerial problems. In calculating aerial gains the mutual resistances tabulated in this monograph are used in conjunction with tables of horizontal radiation patterns (h.r.p.s.) contained in Reference 1.

2. Gain Computation from Mutual Resistance

The radiating elements of a v.h.f. aerial may be of one of the three types shown in Fig. 1. Let us suppose that such an aerial consists of n identical radiating elements of the same type mounted at arbitrary positions on a cylindrical supporting structure, subject only to the restriction that the elements must all be at the same distance from the cylinder axis. Let the current in the p th element be I_p (a complex quantity) and the mutual impedance between the p th and q th elements be Z_{pq} , the self-impedance of the p th element being denoted by Z_{pp} . Then the power radiated by the p th element is

$$P_p = \text{Re} Z_p I_p I_p^* \quad (1)$$

where Re denotes 'the real part of', Z_p is the input impedance of the p th element and I_p^* is the complex conjugate of I_p .

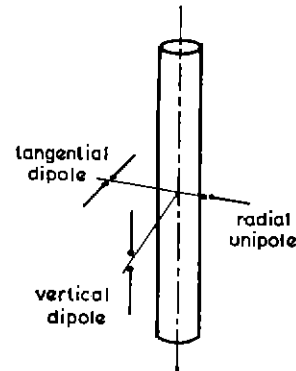


Fig. 1 — Types of radiating element

The input impedance of the p th element is the sum of its self-impedance and the mutual impedances of the other elements, modified in accordance with the current ratios. We therefore have

$$Z_p = \frac{1}{I_p} \sum_{q=1}^n I_q Z_{pq} \quad (2)$$

Substituting in Equation (1) we have

$$P_p = \text{Re} \sum_{q=1}^n I_q I_p^* Z_{pq} \quad (3)$$

The total power radiated is therefore

$$P = \text{Re} \sum_{p=1}^n \sum_{q=1}^n I_q I_p^* Z_{pq} \quad (4)$$

Now the imaginary part of Z_{pq} has no effect on this result, because the sum of the coefficients of Z_{pq} and Z_{qp} (which are equal) is $I_q I_p^* + I_p I_q^*$, a real quantity, while the coefficient of Z_{pp} is also real. The total power radiated is therefore

$$P = \sum_{p=1}^n \sum_{q=1}^n I_q I_p R_{pq} \quad (5)$$

where R_{pq} is the mutual resistance.

The power radiated by a single element carrying a current I (assumed real) when all the other elements carry zero current is

$$P_1 = I^2 R_{11} \quad (6)$$

where R_{11} is self-resistance of an element. From Equations (5) and (6) we may write

$$\frac{P}{P_1} = \frac{1}{I^2} \sum_{p=1}^n \sum_{q=1}^n I_p I_q^* r_{pq} \quad (7)$$

where r_{pq} denotes the ratio of the mutual resistance between the p th and q th elements to the self resistance of a single element.

We now apply this result to the calculation of the gain of the array.

Fig. 2 is a plan view of the cylinder and the p th element. The relative field strength in the horizontal plane due to the p th element alone may be expressed in the form

$$E_p = I_p f(\phi - \alpha_p) \quad (8)$$

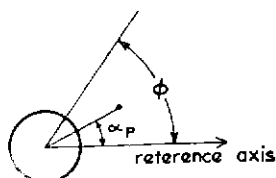


Fig. 2 — Cylinder and p th radiating element

where $f(\phi - \alpha_p)$ is the complex radiation pattern of a single element, tabulated elsewhere.¹ The relative field strength due to the complete array is therefore

$$E = \sum_{p=1}^n I_p f(\phi - \alpha_p) \quad (9)$$

A reference element situated at $\alpha = 0$ carrying a current I radiates, in a specified direction ϕ_s , a field strength given by

$$E_1 = I f(\phi_s) \quad (10)$$

The gain $G_R(\phi)$ of the array relative to that of a single element on the cylinder may now be derived from Equations (7), (9), and (10), as follows

$$\begin{aligned} G_R(\phi) &= \frac{P_1}{P} \left(\frac{|E|}{|E_1|} \right)^2 \\ &= \frac{\sum_{p=1}^n |I_p f(\phi - \alpha_p)|^2}{|f(\phi_s)|^2 \sum_{p=1}^n \sum_{q=1}^n I_p I_q^* r_{pq}} \quad (11) \end{aligned}$$

It should be noted that Equation (11) gives the gain of

the array in the direction ϕ relative to the gain in the direction ϕ_s . For vertical and tangential dipoles ϕ_s is specified as zero but for radial elements it is specified as 90° . The field from the reference element is not necessarily a maximum in the specified direction.

The gain $G(\phi)$ of the array relative to a $\lambda/2$ dipole in free space is the parameter of practical interest; this is given by the expression

$$\begin{aligned} G(\phi) &= G_1 G_R(\phi) \\ &= \frac{G_1 \sum_{p=1}^n |I_p f(\phi - \alpha_p)|^2}{|f(\phi_s)|^2 \sum_{p=1}^n \sum_{q=1}^n I_p I_q^* r_{pq}} \quad (12) \end{aligned}$$

where G_1 is the gain of an element at $\alpha = 0$, in the specified direction ϕ_s , relative to a $\lambda/2$ dipole in free space. The calculation of G_1 is discussed in Section 4.

It is important to note that the gain of a directional aerial is a function of the azimuth angle ϕ . The performance of a directional aerial is often expressed in terms of its effective radiated power (e.r.p.); this is equal to the power supplied to the aerial multiplied by $G(\phi)$ and is also a function of azimuth. Further quantities sometimes referred to are the mean or r.m.s. gain and the corresponding mean e.r.p.; the mean gain G_M is defined as the gain of an omnidirectional aerial whose h.r.p., plotted in polar co-ordinates, has the same area as the h.r.p. of the array. Stated formally

$$G_M = \frac{\int_0^{2\pi} |E(\phi)|^2 d\phi}{2\pi |E(\phi_0)|^2} G(\phi_0) \quad (13)$$

where ϕ_0 is a value of ϕ at which $G(\phi)$ and $E(\phi)$ are known.

3. The Computation of Relative Mutual Resistance

In order to facilitate the use of Equation (12), tables of r_{pq} for the three types of radiating elements have been computed and are contained in Section 9.2. The computation of these tables is discussed in this Section.

The method of computation involves integration of the radiation pattern of a pair of radiating elements in order to find the total power radiated. A comparison is then made with the power radiated by a single element; this enables the relative mutual resistance of the pair of elements to be deduced. Details of this procedure are as follows:

Fig. 3 shows a cylinder of radius a with two radiating elements A_1 and A_2 displaced vertically by a distance c and in azimuth by an angle α . Both elements are at the same distance b from the cylinder axis. The point O will be taken as the origin of a spherical polar co-ordinate system (r, θ, ϕ), the lower element being located at $(b, \frac{\pi}{2}, 0)$.

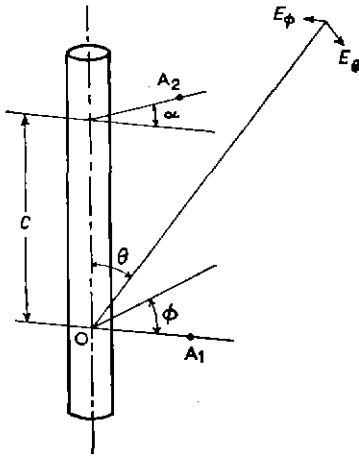


Fig. 3 — A cylinder with two radiating elements

We assume initially that the lower element carries unit current while the upper element carries zero current. In general the field at a great fixed distance due to the lower element will contain both E_θ and E_ϕ components; we may denote them by $E_\theta(\theta, \phi)$ and $E_\phi(\theta, \phi)$ respectively, since they are functions of both θ and ϕ . The power radiated by the single element is then given by

$$P_1 = K \int_S (|E_\theta(\theta, \phi)|^2 + |E_\phi(\theta, \phi)|^2) ds \quad (14)$$

where K is a constant and S denotes the surface of a sphere of large radius enclosing the aerial.

If the upper element carries unit current it produces a similar radiation pattern, but with ϕ replaced by $\phi - \alpha$ and the phase advanced by $\beta c \cos \theta$ radians, where $\beta = 2\pi/\lambda$. When both elements carry unit current the field components are therefore

$$E'_\theta(\theta, \phi) = E_\theta(\theta, \phi) + E_\theta(\theta, \phi - \alpha)e^{j\beta c \cos \theta} \quad (15)$$

$$E'_\phi(\theta, \phi) = E_\phi(\theta, \phi) + E_\phi(\theta, \phi - \alpha)e^{j\beta c \cos \theta} \quad (16)$$

The power P_2 radiated by the two elements is given by an expression similar to Equation (14) but with E'_θ and E'_ϕ in place of E_θ and E_ϕ .

Now P_1 is proportional to R_s and P_2 to $2(R_s + R_M)$, where R_s and R_M are the self and mutual resistances of the elements. It follows that the relative mutual resistance $r = R_M/R_s$ is given by

$$r = \frac{P_2}{2P_1} - 1 \quad (17)$$

E_θ and E_ϕ for doublets and dipoles near cylinders are complicated functions of θ and ϕ given by Carter's formulae.² The computation of P_1 and P_2 is not straightforward since it must be done by numerical integration. Attention is confined here to the following brief discussion of the methods of computation for vertical, tangential and radial radiating elements.

3.1 Vertical Radiating Elements

For vertical radiating elements Carter gives an expression for the radiation pattern of a short doublet; it contains only an E_θ component. The radiation pattern of a half-wave dipole is of more practical interest, however, and it can be obtained from Carter's formula by including the v.r.p. factor of a vertical dipole in free space. The result is

$$E_\theta = \frac{\cos(\frac{\pi}{2} \cos \theta)}{\sin \theta} [V_0 + 2 \sum_{n=1}^{\infty} j^n V_n \cos n\phi] \quad (18)$$

where

$$V_n = J_n(\beta b \sin \theta) - J_n(\beta a \sin \theta) \frac{H_n^{(2)}(\beta b \sin \theta)}{H_n^{(2)}(\beta a \sin \theta)} \quad (19)$$

In Equation (19) J_n and $H_n^{(2)}$ denote Bessel and Hankel functions of order n .

The computation of P_1 and P_2 from these formulae gives the relative mutual resistance between half-wave dipoles.

3.2 Tangential Radiating Elements

Carter's expressions for the distant field radiated by a tangential doublet contain two components given by

$$E_\phi = -j[W_0 + 2 \sum_{n=1}^{\infty} j^n W_n \cos n\phi] \quad (20)$$

$$E_\theta = \frac{-j2 \cos \theta}{\beta b \sin \theta} \sum_{n=1}^{\infty} j^n n V_n \sin n\phi \quad (21)$$

where

$$W_n = J'_n(\beta b \sin \theta) - J'_n(\beta a \sin \theta) \frac{H_n^{(2)'}(\beta b \sin \theta)}{H_n^{(2)'}(\beta a \sin \theta)} \quad (22)$$

in which J'_n and $H_n^{(2)'}$ denote the derivatives of the Bessel and Hankel functions with respect to the argument. V_n was defined in Equation (19).

These formulae cannot be extended to tangential dipoles since the currents flowing in the latter, although predominantly tangential, also contain radial components. Consequently relative mutual resistance tables can only be computed for tangential doublets. The use of these tables for dipoles may be justified by the fact that relative mutual resistances between dipoles in free space are very similar to those between doublets.

3.3 Radial Radiating Elements

Carter gives the following expressions for the distant field radiated by a radial doublet:

$$E_\phi = \frac{-j2}{\beta b \sin \theta} \sum_{n=1}^{\infty} j^n n B_n \sin n\phi \quad (23)$$

$$E_\theta = j \cos \theta [D_0 + 2 \sum_{n=1}^{\infty} j^n D_n \cos n\phi] \quad (24)$$

where

$$B_n = J_n(\beta b \sin \theta) - J_n'(\beta a \sin \theta) \frac{H_n^{(2)}(\beta b \sin \theta)}{H_n^{(2)'}(\beta a \sin \theta)} \quad (25)$$

$$D_n = J_n'(\beta b \sin \theta) - J_n(\beta a \sin \theta) \frac{H_n^{(2)'}(\beta b \sin \theta)}{H_n^{(2)}(\beta a \sin \theta)} \quad (26)$$

Radiating elements used in practice consist of base driven unipoles mounted on the surface of the cylinder; they are usually $\lambda/4$ long although greater lengths are sometimes used. Although Carter's formulae could be extended to unipoles by integrating along their length, this additional complication is hardly worth while because it is permissible to replace a short radiator by a doublet situated at the centroid* of its current distribution.³ Since the centroid of a $\lambda/4$ unipole is approximately 0.5 radians from its drive point, $\beta b - \beta a$ was set equal to 0.5 in all the computations.

It can be shown that the first term of Carter's formula for E_θ , Equation (24), tends to infinity at $\theta = 0$ and π , whilst the remaining terms tend to zero, as do all the terms of the series for E_ϕ . The E_θ component behaves as $1/(\theta \log \theta)$ in the neighbourhood of $\theta = 0$, and is therefore virtually dependent on θ only. This is equivalent to saying that the electric field near the cylinder is dominated by a radial component at distances from the origin large compared with the overall dimensions of the aerial. The associated magnetic field component is circumferential and the direction of travel of this dominant wave is along the axis of the cylinder.

When performing an outward-power-flow integration over an enclosing surface using Carter's formulae, it can be shown that the contributions due to the two portions of the surface which lie inside the cylinder are vanishingly small if these portions are sufficiently far from the origin.† In other words, the large E_θ component when θ is small does not contribute significantly to the total power flow. It appears, therefore, that the method used in this report for calculating mutual resistances remains valid for radial unipoles. It will be seen in Section 5 that measurements of mutual resistance between radial unipoles mounted on a conducting cylinder show very close agreement with the theory.

Because of the influence of the dominant wave mentioned above, the mutual resistance between a pair of well-spaced radial doublets decreases slowly (approximately as $1/\log \beta c$) as their separation along the cylinder axis increases and it is virtually independent of their angular displacement α .

* We have to imagine that the unipole has a mass distributed along its length, the mass per unit length at any point being proportional to the current at that point. Then the centre of gravity (or centroid) of this mass would correspond to the centroid of the current distribution.

† Over these portions of the surface the principal part of the power flow integral takes the form

$$\int_0^{a/R} \frac{2\pi\theta d\theta}{(\theta \log \theta)^2} = \frac{2\pi}{\log R - \log a}$$

where R is the distance from the origin of the surface of integration. As R tends to infinity this contribution to the integral tends to zero.

4. The Gain of a Single Element

The quantity of G_1 in Equation (12) was defined as the ratio of the gain of one element on the cylinder relative to that of a $\lambda/2$ dipole in free space. The calculation of this quantity is discussed in this section.

We consider first the gain of a single radiating element on a cylinder relative to an identical element in free space. Since the total power radiated by the element in free space is independent of its orientation, it will for convenience be assumed to be vertical. Its radiation pattern is then independent of ϕ and contains only an E_θ component, which may be written as $\sin \theta$ for a doublet or as $\cos(\frac{\pi}{2} \cos \theta)/\sin \theta$ for a $\lambda/2$ dipole. The power radiated by the element in free space is then given by

$$P_0 = 4\pi K \int_0^\pi E_\theta^2 \sin \theta d\theta \quad (27)$$

where K is the constant appearing in Equation (14).

The presence of the cylinder modifies the field radiated by the element, the field components now being given by the appropriate Carter formulae. The power radiated by the element is then equal to P_1 , given by Equation (14).

Carter's formulae are stated here in such a way that if the cylinder were removed, the element would radiate unit field in its direction of maximum radiation. It follows, therefore, that the quantities P_0 and P_1 may be directly compared. It also follows that the formulae give the field E_1 in the specified direction ϕ_s (defined in Section 2) correctly scaled to the maximum field radiated by an element in free space. The gain G_1 of a single element in the presence of the support cylinder, relative to an identical element in free space, is therefore given by

$$G_1 = |E_1|^2 \frac{P_0}{P_1} \quad (28)$$

The quantity G_1 is stated above each mutual resistance table.

In the tables for vertical elements this is the gain of a single element relative to a $\lambda/2$ dipole in free space, since Carter's formulae modified for $\lambda/2$ dipoles were used for the computation.

In the tables for tangential elements G_1 is the gain of a doublet on the cylinder relative to a doublet in free space. For cylinders of small radius G_1 is approximately equal to the gain of a $\lambda/2$ dipole on the cylinder relative to a $\lambda/2$ dipole in free space; thus the gain of an array of doublets relative to a doublet in free space may be assumed to be approximately equal to the gain of a similar array of dipoles relative to a dipole in free space.

In the tables for radial elements, G_1 is the gain of a single doublet on the cylinder relative to a $\lambda/2$ dipole in free space; the gain is referred to a $\lambda/2$ dipole rather than to a doublet since the doublet on the cylinder is a good approximation to a $\lambda/4$ unipole.

To summarize, substitution of the value of G_1 given above each table into Equation (12) gives the gain of an

array of dipoles or unipoles on the cylinder relative to a $\lambda/2$ dipole in free space for all three types of radiating elements; the gain is exact for vertical dipoles and a close approximation for tangential dipoles and radial unipoles.

5. Verification of the Method of Computation

Because of the complexity of the computer programme required for this work it was considered prudent to check its accuracy by using the tables to compute the gains of

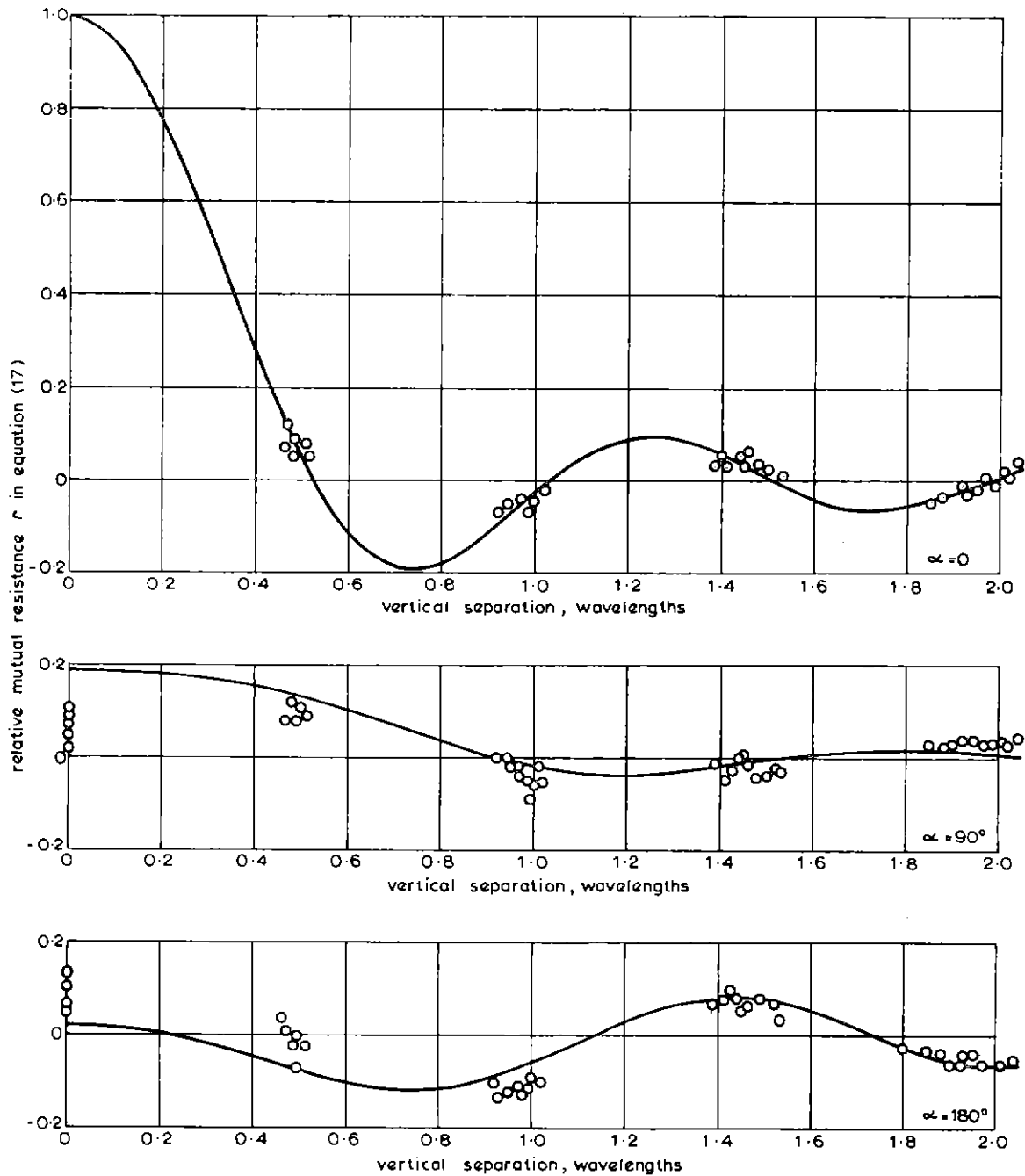


Fig. 4 — Comparison of computed and measured mutual resistances r between tangential elements

- Computed values for tangential doublets, $\beta a = 1.25$, $\beta b = 2.50$
- o Measured values for tangential dipoles, $\beta a = 1.18$, $\beta b = 2.50$

aerials whose gains could be calculated by other methods. This test was carried out for vertical dipoles since no approximations are involved in the computation of their mutual resistances. Gains calculated from the mutual resistance tables for rings of vertical dipoles were compared with gains calculated directly by radiation pattern integration and found to differ by less than 1 per cent; this agreement was considered satisfactory.

The assumption that mutual resistance tables for tangential doublets can be used for tangential $\lambda/2$ dipoles was tested by comparing the tabulated values with model measurements using dipoles. This comparison is shown in Fig. 4, in which the three curves show relative mutual resistance as a function of vertical separation for three different angular separations. The measurements, which were made during the development of the Norwich television aerial (Vision, 56.75 Mc/s; Sound, 53.25 Mc/s), occur in groups because the positions of the dipoles on the cylinder were fixed; the small ranges of separation within each group result from the measurements having been made over a band of frequencies. The comparison shows that the use of the doublet approximation results in an error when the vertical separation between the dipoles is small; elsewhere the approximation appears to be satisfactory.

It was also considered prudent to test the accuracy of the mutual resistance tables for radial doublets because it was uncertain whether the infinities in E_θ occurring at $\theta = 0$ and π (referred to in Section 3.3) rendered the method of computation invalid. Measurements, made during the development of the Peterborough FM aerial (90.1 Mc/s–94.5 Mc/s), of the impedance of a pair of unipoles mounted on a square-section cylinder* were used for this test. The unipoles were first mounted on separate cylinders 5λ apart so as to minimize their mutual impedance. They were then placed in the desired relative position on a common cylinder, the resulting change in impedance being assumed to be equal to their mutual impedance. These measurements, which were made between unipoles on the same and on adjacent faces, are compared in Fig. 5 with computed values for doublets near a circular cylinder having the same perimeter; the agreement shows that the theory is valid.

As a further check the radial doublet tables were used to calculate the gain of two doublets driven in antiphase and mounted diametrically opposite each other on a cylinder of diameter small compared with the wavelength. This arrangement simulates a half-wave dipole mounted on a boom, as in a Yagi aerial, and its gain should be approximately equal to that of a $\lambda/2$ dipole in free space. This was confirmed by calculating the gain of the pair of antiphase doublets in the direction $\phi = 90^\circ$ for $\beta a = 0.25$, $\beta b = 0.75$;

* Cylinders of other than circular cross-section may be regarded as having an 'equivalent radius' provided that the width W of each face is not greater than 0.5λ for a square section and 0.3λ for a triangular section. The equivalent radii are $0.59W$ and $0.42W$ for square and triangular sections respectively for vertical elements. With tangential and radial elements the equivalent circular cylinder may be assumed to be that which has the same perimeter as the polygonal cylinder.

the result obtained was a gain of 1.025 relative to a dipole in free space. In passing it should be noted that the gain G_1 (given above each table) of a single radial doublet on a cylinder is considerably less than unity because it generates a pair of axial cylindrical surface waves; with two antiphase doublets, however, the surface waves are suppressed.

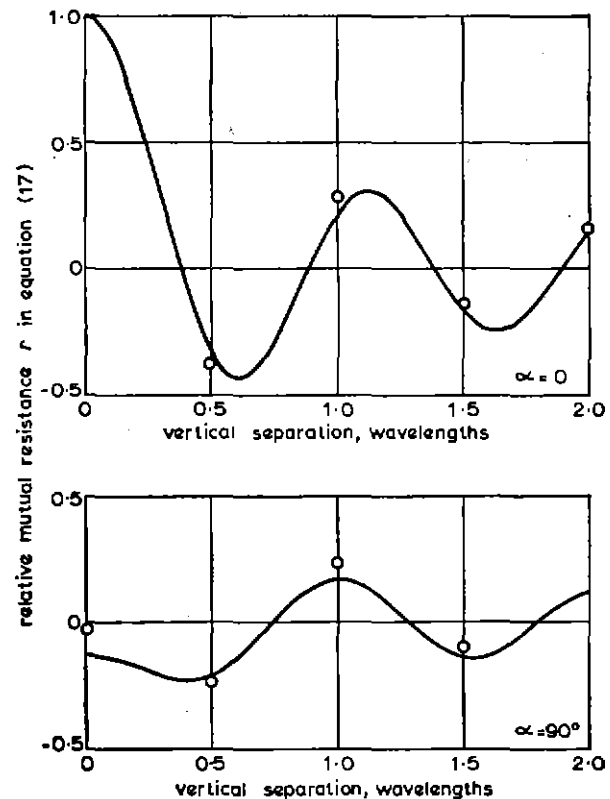


Fig. 5 — Comparison of computed and measured mutual resistances r between radial elements

— Computed values for radial doublets on a circular cylinder $\beta a = 1.5$, $\beta b = 2.0$
 o Measured values for radial unipoles on a square-section cylinder 0.37λ wide

6. Conclusions

Calculation of the gain of directional v.h.f. aerials is simplified by the use of the mutual resistance tables contained in this monograph because the need for radiation pattern integration is avoided. The tables cover the range of mast sizes and element spacings normally encountered at v.h.f. and can be used for any arrangement of radiating elements, provided all the elements have the same spacing from the axis of the mast. The tables may be applied to elements mounted on square and triangular section masts by assuming an equivalent circular cylinder. The accuracy of the tables has been verified both experimentally and by applying them to aerials whose gain can be calculated by other methods.

The mutual impedance between radial unipoles has been found to decrease very slowly as their separation increases because they are coupled by surface waves propa-

gating along the mast. The generation of surface waves can be avoided in an array by a suitable choice of inter-tier spacing, thereby preventing the reduction of gain which would otherwise result.

7. Acknowledgements

The measurements described in Section 5 with which the theoretical mutual resistances were compared were performed by Dr J. B. Izatt and Mr R. D. C. Thoday.

8. References

1. Tables of horizontal radiation patterns of dipoles mounted on cylinders, BBC Engineering Division Monograph, No. 35, February 1961.
2. Carter, P. S., 1941. Antenna arrays around cylinders, Proc. Inst. Radio Engrs., 31, No. 12, pp. 671-93, 1943.
3. Medhurst, R. G., 1947. Radiation from short aerials, Wireless Engr., 25, No. 299, pp. 260-6, 1948.

9. Appendices

9.1 Notes on the Use of the Tables in Section 9.2

The tables give the relative mutual resistance between two elements displaced vertically and circumferentially around a cylinder. The vertical separation in wavelengths (c/λ) is given in the extreme left-hand column and the angular separation in degrees in the headings of the other columns. Each table also gives the gain G_1 of a single element on the cylinder as discussed in Section 4. The examples below demonstrate the way in which the tables are used.

9.1.1 Aerials with Dipoles Carrying Equal Currents

The example chosen is a 90 Mc/s aerial consisting of eight tiers of horizontal dipoles mounted on a cylinder, as shown in Fig. 6. The cylinder radius is 1.0 m and the dipoles are 1.8 m distant from the axis of the cylinder; these dimensions correspond approximately to $\beta a = 2.0$ and $\beta b = 3.5$ (see Section 3). The inter-tier spacing is 0.5λ .

Referring to Fig. 6, it will be seen that (in the example chosen) the dipoles have been divided into eight groups, each denoted by a different letter, all the dipoles in a given group being identically situated and therefore having the same impedance when all dipoles carry equal currents. In order to determine the total power radiated by the aerial the relative resistance of the dipoles in each group must be found from the mutual resistance tables. This part of the calculation is set out below, it being assumed that values computed for doublets can be applied to dipoles with little error. The quantities $R_A, R_B,$ etc., are the ratios of resistance of a single doublet in each group to that of its self-resistance in the presence of the cylinder.

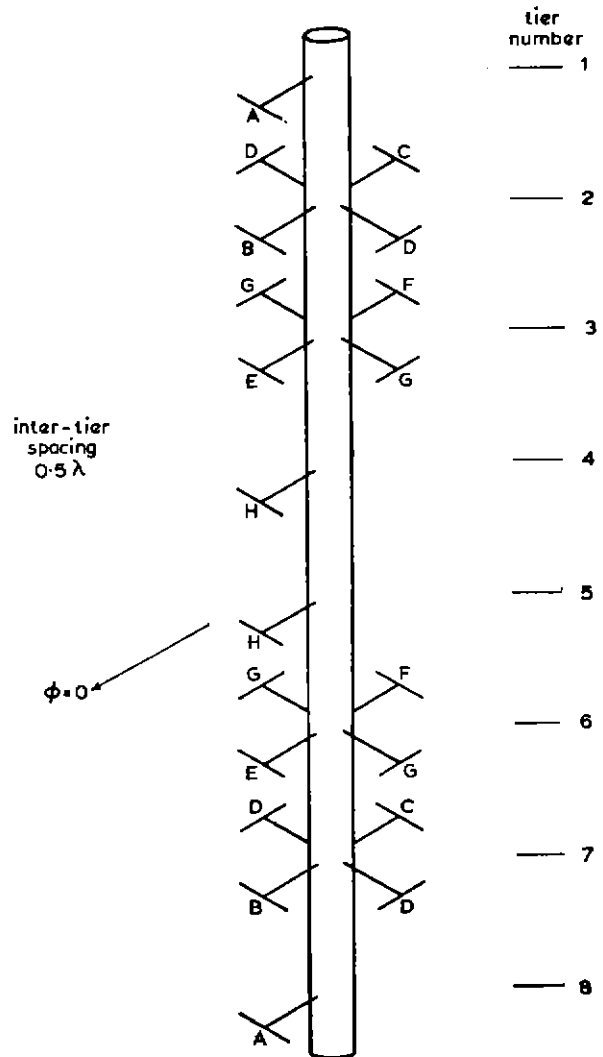


Fig. 6—V.h.f. aerial for which the gain is calculated in Section 9.1.1

Dipole A

TIER		=	
1	1.000	=	1.000
2	$0.078 + 2 \times 0.032 - 0.005$	=	0.137
3	$-0.056 - 2 \times 0.058 + 0.049$	=	-0.123
4	0.028	=	0.028
5	-0.017	=	-0.017
6	$0.011 - 2 \times 0.001 + 0.025$	=	0.034
7	$-0.005 + 2 \times 0.004 - 0.024$	=	-0.021
8	0.002	=	0.002
		<hr/>	
	R_A	=	1.040

Dipole B

TIER		=	
1	0.078	=	0.078
2	$1.000 + 2 \times 0.132 - 0.037$	=	1.227
3	$0.078 + 2 \times 0.032 - 0.005$	=	0.137
4	-0.056	=	-0.056
5	0.028	=	0.028
6	$-0.017 - 2 \times 0.006 - 0.011$	=	-0.040
7	$0.011 - 2 \times 0.001 + 0.025$	=	0.034
8	-0.005	=	-0.005
		<hr/>	
	R_B	=	1.403

Dipole F

TIER		=	
1	0.049	=	0.049
2	$0.078 + 2 \times 0.032 - 0.005$	=	0.137
3	$1.000 + 2 \times 0.132 - 0.037$	=	1.227
4	-0.005	=	-0.005
5	0.049	=	0.049
6	$0.028 + 2 \times 0.025 - 0.015$	=	0.063
7	$-0.017 - 2 \times 0.006 - 0.011$	=	-0.040
8	0.025	=	0.025
		<hr/>	
	R_F	=	1.505

Dipole C

TIER		=	
1	-0.005	=	-0.005
2	$1.000 + 2 \times 0.132 - 0.037$	=	1.227
3	$0.078 + 2 \times 0.032 - 0.005$	=	0.137
4	0.049	=	0.049
5	-0.015	=	-0.015
6	$-0.017 - 2 \times 0.006 - 0.011$	=	-0.040
7	$0.011 - 2 \times 0.001 + 0.025$	=	0.034
8	-0.024	=	-0.024
		<hr/>	
	R_C	=	1.363

Dipole G

TIER		=	
1	-0.058	=	-0.058
2	$0.078 + 2 \times 0.032 - 0.005$	=	0.137
3	$1.000 + 2 \times 0.132 - 0.037$	=	1.227
4	0.032	=	0.032
5	-0.058	=	-0.058
6	$0.028 + 2 \times 0.025 - 0.015$	=	0.063
7	$-0.017 - 2 \times 0.006 - 0.011$	=	-0.040
8	-0.001	=	-0.001
		<hr/>	
	R_G	=	1.302

Dipole D

TIER		=	
1	0.032	=	0.032
2	$1.000 + 2 \times 0.132 - 0.037$	=	1.227
3	$0.078 + 2 \times 0.032 - 0.005$	=	0.137
4	-0.058	=	-0.058
5	0.025	=	0.025
6	$-0.017 - 2 \times 0.006 - 0.011$	=	-0.040
7	$0.011 - 2 \times 0.001 + 0.025$	=	0.034
8	0.004	=	0.004
		<hr/>	
	R_D	=	1.361

Dipole H

TIER		=	
1	0.028	=	0.028
2	$-0.056 - 2 \times 0.058 + 0.049$	=	-0.123
3	$0.078 + 2 \times 0.032 - 0.005$	=	0.137
4	1.000	=	1.000
5	0.078	=	0.078
6	$-0.056 - 2 \times 0.058 + 0.049$	=	-0.123
7	$0.028 + 2 \times 0.025 - 0.015$	=	0.063
8	-0.017	=	-0.017
		<hr/>	
	R_H	=	1.043

Dipole E

TIER		=	
1	-0.056	=	-0.056
2	$0.078 + 2 \times 0.032 - 0.005$	=	0.137
3	$1.000 + 2 \times 0.132 - 0.037$	=	1.227
4	0.078	=	0.078
5	-0.056	=	-0.056
6	$0.028 + 2 \times 0.025 - 0.015$	=	0.063
7	$-0.017 - 2 \times 0.006 - 0.011$	=	-0.040
8	0.011	=	0.011
		<hr/>	
	R_E	=	1.364

For convenience, let us assume that a single dipole on the cylinder, carrying unit current, radiates one unit of power. Then the total power radiated by the aerial, when each dipole carries unit current, is

$$2[R_A + R_B + R_C + R_D + R_E + R_F + R_G + R_H] = 26.09$$

The h.r.p. of the aerial may be calculated from the h.r.p. table for a single dipole, given on p. 31 of Reference 1, and is shown in Fig. 7. The field is greatest at $\phi=0$. If we assume that a dipole in free space carrying unit current radiates unit field, then the tables of Reference 1 give the field radiated by a dipole, carrying unit current, on a cylinder. It

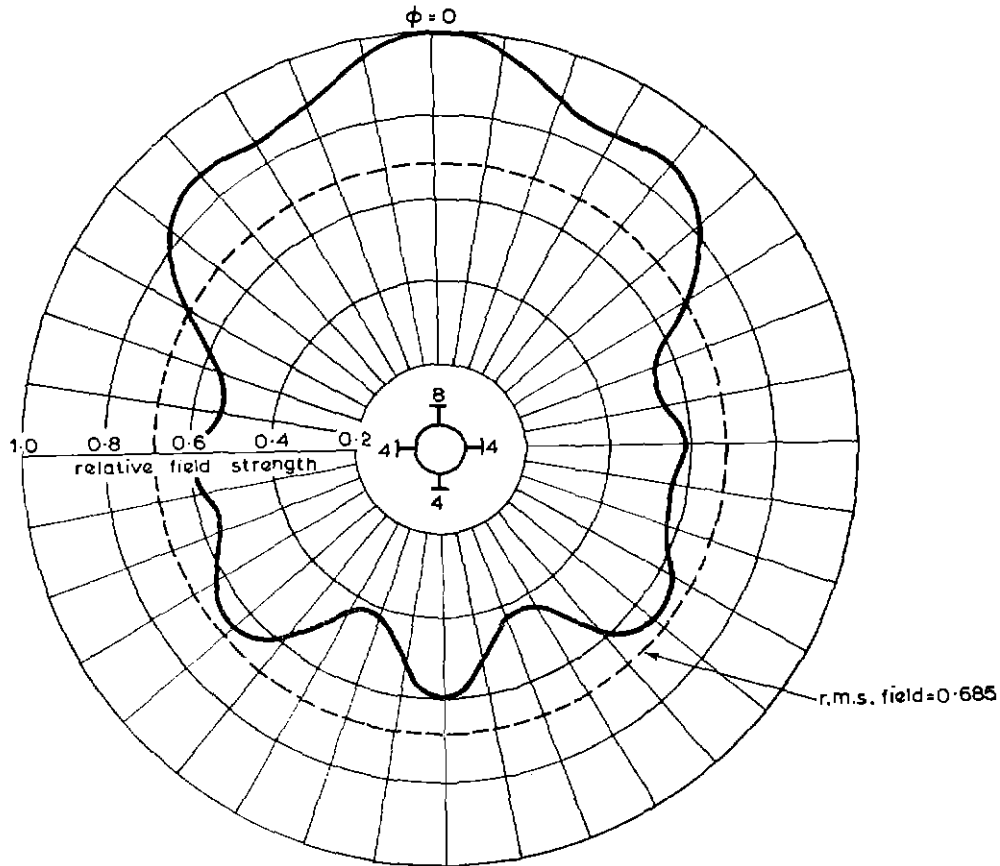


Fig. 7 — Theoretical h.r.p. of typical v.h.f. aerial
Numbers against dipoles indicate number of dipoles on that bearing

follows therefore that the field radiated by the aerial in the direction $\phi=0$ is equal to:

$$8(-1.444 - j0.713) + 8(-0.209 + j0.194) + 4(0.162 - j0.570) = -12.58 - j6.43$$

This field has an amplitude of 14.13.

The table in Reference 1 also shows that a single dipole on the cylinder, carrying unit current, produces a field of $-1.444 - j0.713$ (amplitude 1.610) in the direction $\phi=0$.

The gain of the array in the direction $\phi=0$ relative to a single $\lambda/2$ dipole on the cylinder is therefore:

$$\begin{aligned} \text{Gain} &= \frac{\text{Power radiated by dipole}}{\text{Power radiated by array}} \\ &\times \left[\frac{\text{Field radiated by array}}{\text{Field radiated by dipole}} \right]^2 \\ &= \frac{1.00}{26.09} \times \left(\frac{14.13}{1.610} \right)^2 = 2.952 \end{aligned}$$

The gain (in the direction $\phi=0$) of a doublet on the cylinder relative to a doublet in free space is the value denoted by G_1 (2.544) at the head of the table. This figure is substantially the same as the gain of a $\lambda/2$ dipole on the cylinder relative to a $\lambda/2$ dipole in free space. Thus the gain of the array (in the direction $\phi=0$) relative to a $\lambda/2$ dipole in free space is:

$$2.952 \times 2.544 = 7.51 \quad (8.8 \text{ dB})$$

The mean gain of the aerial may be calculated from its mean field, defined as the radius of the circle which has the same area as the h.r.p. In the example considered the ratio between the mean and maximum field is 0.685. The mean gain of the aerial is therefore:

$$7.51 \times 0.685^2 = 3.52 \quad (5.5 \text{ dB})$$

relative to a $\lambda/2$ dipole in free space.

The gain figures stated above may be in error by about 0.2 dB because two assumptions were made in their calculation:

- (a) The mutual resistance values for doublets were assumed to be correct for dipoles.
- (b) The gain of a doublet on a cylinder relative to a doublet in free space was assumed to be equal to the gain of a $\lambda/2$ dipole on the cylinder relative to a $\lambda/2$ dipole in free space.

Had the gain of an array of vertical dipoles been calculated these assumptions would have been unnecessary and the gains would have been exact.

9.1.2 Aerials with Dipoles Carrying Unequal but Co-phased Currents

The radiation pattern of Fig. 7 can also be achieved with an aerial consisting of identical tiers, each tier containing four dipoles, but with the dipoles in the position $\phi = 0$ carrying twice the current of the other dipoles. Fig. 8 shows a single tier of this arrangement; we will proceed to calculate the gain of an aerial consisting of one tier. The gain of a twelve-tier aerial is calculated in Section 9.1.3.

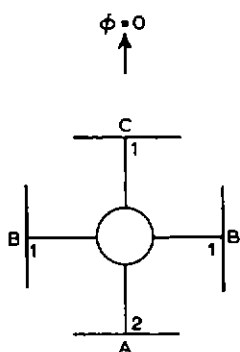


Fig. 8 — A tier of dipoles carrying unequal but co-phased currents
Numbers against dipoles indicate currents

Because of the unequal currents the dipoles must be divided into three groups, designated A, B, and C. Their relative resistances are:

$$R_A = 1 + r(0,90^\circ) + \frac{1}{2}r(0,180^\circ)$$

$$R_B = 1 + 3r(0,90^\circ) + r(0,180^\circ)$$

$$R_C = 1 + 2r(0,90^\circ) + 2r(0,180^\circ)$$

where $r(0,90^\circ)$ and $r(0,180^\circ)$ denote the relative mutual resistance between dipoles at the same level but with angular separations of 90° and 180° respectively.

If a single dipole on the cylinder, carrying unit current, radiates unit power, the total power radiated by the tier is:

$$4R_A + 2R_B + R_C = 7 + 12r(0,90^\circ) + 6r(0,180^\circ) \\ = 7 + 12 \times 0.132 + 6 \times 0.037 = 8.362$$

The field due to the four dipoles, in the direction $\phi = 0$, is (from Reference 1) equal to:

$$2(-1.444 - j0.713) + 2(-0.209 + j0.194) + 0.162 \\ - j0.570 = 3.144 - j1.608$$

This has an amplitude of 3.531

A single dipole on the cylinder carrying unit current radiates a field of 1.610 in the direction $\phi = 0$.

The gain of the four dipoles, in the direction $\phi = 0$, relative to a single dipole on the cylinder is therefore:

$$\frac{1.00}{8.362} \times (3.531)^2 = 0.575$$

The other gain figures are determined as in Section 9.1.1. Thus the gain in the direction $\phi = 0$ relative to a dipole in free space is:

$$0.575 \times 2.544 = 1.463 \quad (1.7 \text{ dB})$$

and the mean gain is:

$$1.463 \times 0.685^2 = 0.686 \quad (-1.6 \text{ dB})$$

9.1.3 Aerials with Large Numbers of Identical Tiers

Consider a 90 Mc/s aerial with twelve tiers of the type described in Section 9.1.2. Although its gain may be calculated by the method described in Section 9.1.1 (with due allowance for the unequal currents) this would be very laborious because the dipoles must be divided into eighteen different groups and calculation of the resistance of each group entails the summation of no less than thirty-six terms. However, as the tiers are identical, the calculation may be greatly simplified by first calculating the relative mutual resistances between complete tiers and then finding the gain of twelve tiers relative to one tier. The result obtained in Section 9.1.2 may then be used to derive the gain of the aerial relative to a $\lambda/2$ dipole in free space. The procedure is as follows:

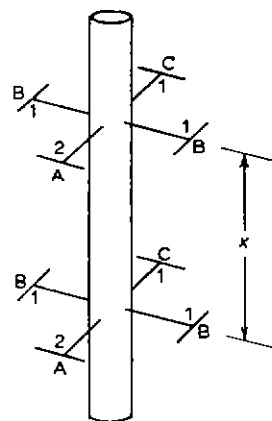


Fig. 9 — Two identical tiers

Consider two tiers separated by a distance x/λ as shown in Fig. 9. Because of the unequal currents the dipoles must be divided into three groups. The relative resistances of one dipole in each group are:

$$R_A = 1 + r(0,90^\circ) + \frac{1}{2}r(0,180^\circ) + r(x,0) + r(x,90^\circ) + \frac{1}{2}r(x,180^\circ)$$

$$R_B = 1 + 3r(0,90^\circ) + r(0,180^\circ) + r(x,0) + 3r(x,90^\circ) + r(x,180^\circ)$$

$$R_C = 1 + 2r(0,90^\circ) + 2r(0,180^\circ) + r(x,0) + 2r(x,90^\circ) + 2r(x,180^\circ)$$

where $r(x,a)$ denotes the relative mutual resistance between dipoles having an angular separation of a and a vertical separation of x/λ .

The total power radiated by one tier is

$$\begin{aligned} I^2(R_s + R_M) &= k[4R_A + 2R_B + R_C] \\ &= k[7 + 12r(0,90^\circ) + 6r(0,180^\circ) \\ &\quad + 7r(x,0) + 12r(x,90^\circ) + 6r(x,180^\circ)] \end{aligned}$$

where I is the feed current to one tier as a whole, R_s is its self-resistance, R_M is the mutual resistance between the two tiers, and k is a constant.

The power radiated by one tier on its own, carrying the same current, is:

$$I^2R_s = k[7 + 12r(0,90^\circ) + 6r(0,180^\circ)]$$

From these two equations it follows that:

$$I^2R_M = k[7r(x,0) + 12r(x,90^\circ) + 6r(x,180^\circ)]$$

The relative mutual resistance between tiers is therefore:

$$\frac{R_M}{R_s} = \frac{7r(x,0) + 12r(x,90^\circ) + 6r(x,180^\circ)}{7 + 12r(0,90^\circ) + 6r(0,180^\circ)} = R(x)$$

The aerial under consideration has twelve tiers spaced 0.7λ . The relative mutual resistance between all the tiers (i.e. values for separations which are multiples of 0.7λ) are calculated in the table below:

x/λ	$r(x,0)$	$r(x,90)$	$r(x,180)$	$f(x)$	$R(x)$
0	1.000	0.132	-0.037	8.362	1.000
0.7	-0.175	-0.027	0.022	-1.417	-0.169
1.4	0.068	0.011	0.004	0.632	0.076
2.1	0.016	-0.016	0.006	-0.044	-0.005
2.8	-0.028	0.013	-0.022	-0.172	-0.021
3.5	0.002	-0.004	0.022	0.098	0.012
4.2	0.017	-0.003	-0.007	0.041	0.005
4.9	-0.009	0.005	-0.012	-0.075	-0.009
5.6	-0.011	-0.001	0.015	0.001	0.000
6.3	0.011	-0.002	-0.002	0.041	0.005
7.0	0.003	0.002	-0.011	-0.021	-0.003
7.7	-0.011	0.000	0.010	-0.017	-0.002

$f(x) = 7r(x,0) + 12r(x,90^\circ) + 6r(x,180^\circ)$

From the tabulated values of $R(x)$, the total power radiated by the array is:

$$\begin{aligned} I^2R_s[12R(0) + 22R(0.7) + 20R(1.4) + 18R(2.1) \\ + 16R(2.8) + 14R(3.5) + 12R(4.2) + 10R(4.9) \\ + 8R(5.6) + 6R(6.3) + 4R(7.0) + 2R(7.7)] \end{aligned}$$

This is equal to 9.528 times the power radiated by a single tier.

The field radiated by the array is twelve times that radiated by a single tier.

Thus the gain of the array relative to a single tier is equal to:

$$\frac{12^2}{9.528} = 15.11$$

The results obtained in Section 9.1.2 may now be used to determine the gain of the array relative to a dipole in free space.

The gain in the direction $\phi = 0$ is therefore:

$$15.11 \times 1.463 = 22.1 \quad (13.4 \text{ dB})$$

The mean gain is

$$15.11 \times 0.686 = 10.4 \quad (10.2 \text{ dB})$$

9.1.4 Aerials with Dipoles Carrying Unequal Currents

If the dipole currents are not co-phased the calculation of the total power radiated is more complicated since the relative phases of the currents must be taken into account.

Fig. 10 shows an aerial consisting of two tiers spaced 0.8λ . Each tier contains two dipoles which carry currents of $1/0^\circ$ and $2/45^\circ$; the resulting h.r.p. is shown in Fig. 11. The maximum field occurs in the direction opposite the dipoles carrying the current of $2/45^\circ$ ($\phi = 0$). There are two groups of dipoles, A and B, as shown in Fig. 10.

We revert to the original assumption that a single dipole on the cylinder, carrying unit current, radiates unit power.

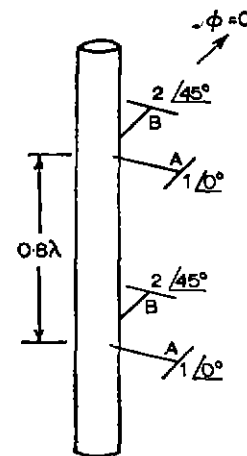


Fig. 10 — Aerial with unequal currents

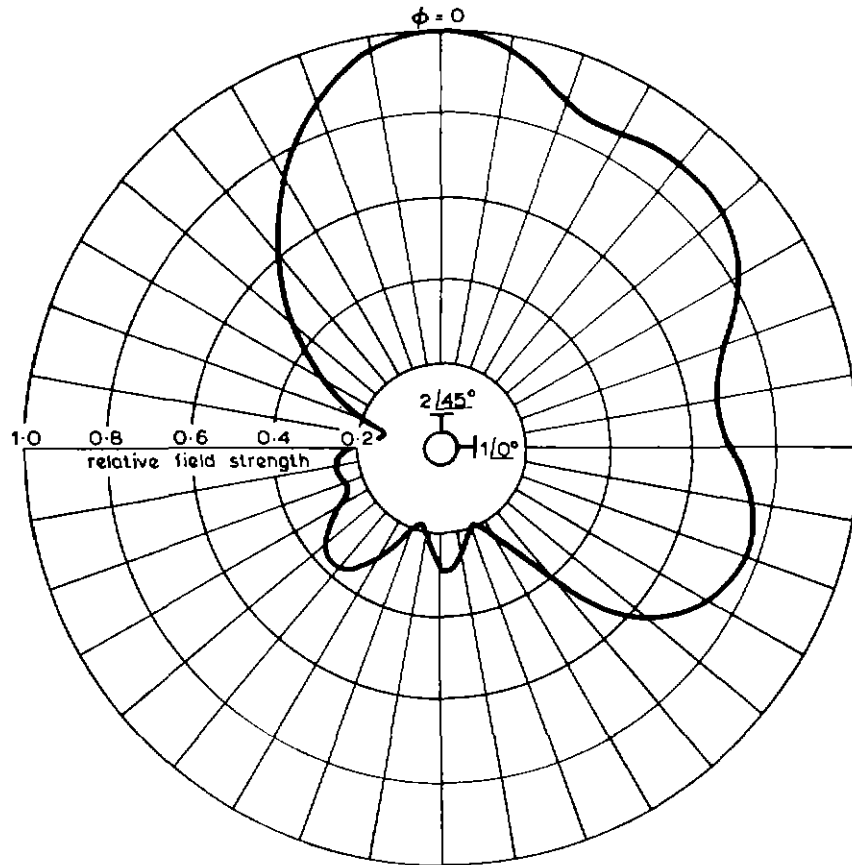


Fig. 11 — Theoretical h.r.p. of aerial with unequal currents

Applying Equation (5) and taking each dipole in turn, we find that the total power radiated is:

$$\begin{aligned}
 & 1/0^\circ [1 \times 1/0^\circ + r(0,90^\circ) \times 2/-45^\circ + r(0.8,0) \times 1/0^\circ + r(0.8,90^\circ) \times 2/-45^\circ] \quad (\text{dipole A, upper}) \\
 & + 2/45^\circ [1 \times 2/-45^\circ + r(0,90^\circ) \times 1/0^\circ + r(0.8,0) \times 2/-45^\circ + r(0.8,90^\circ) \times 1/0^\circ] \quad (\text{dipole B, upper}) \\
 & + 1/0^\circ [1 \times 1/0^\circ + r(0,90^\circ) \times 2/-45^\circ + r(0.8,0) \times 1/0^\circ + r(0.8,90^\circ) \times 2/-45^\circ] \quad (\text{dipole A, lower}) \\
 & + 2/45^\circ [1 \times 2/-45^\circ + r(0,90^\circ) \times 1/0^\circ + r(0.8,0) \times 2/-45^\circ + r(0.8,90^\circ) \times 1/0^\circ] \quad (\text{dipole B, lower}) \\
 & = 10 + r(0,90^\circ) \times 2(2/45^\circ + 2/-45^\circ) + r(0.8,0) \times 10 + r(0.8,90^\circ) \times 2(2/45^\circ + 2/-45^\circ) \\
 & = 10 + 0.132 \times 5.656 - 0.184 \times 10 - 0.047 \times 5.656 = 8.641
 \end{aligned}$$

The field radiated by the aerial in the direction $\phi = 0$ is (from Reference 1) equal to:

$$\begin{aligned}
 & 4(-1.444 - j0.713)(0.707 + j0.707) \\
 & + 2(-0.209 + j0.194) = -2.486 - j5.712
 \end{aligned}$$

This has an amplitude of 6.230.

A single $\lambda/2$ dipole on the cylinder carrying unit current radiates a field of 1.610 in the direction $\phi = 0$.

The gain of the aerial in the direction $\phi = 0$, relative

to a single $\lambda/2$ dipole on the cylinder, is therefore:

$$\frac{1.00}{8.461} \times \left(\frac{6.230}{1.610} \right)^2 = 1.77 \quad (2.5 \text{ dB})$$

The gain of a single dipole on the cylinder relative to a $\lambda/2$ dipole in free space is approximately 2.544 (see Section 9.1.1). Thus the gain of the aerial, in the direction $\phi = 0$, relative to a $\lambda/2$ dipole in free space, is:

$$1.77 \times 2.544 = 4.50 \quad (6.5 \text{ dB})$$

9.2 RELATIVE MUTUAL RESISTANCE TABLES

9.2.1 VERTICAL DIPOLES

$\beta a = 0.25, \quad \beta b = 1.75, \quad G_1 = 1.756$								$\beta a = 0.25, \quad \beta b = 2.25, \quad G_1 = 1.197$							
c/λ	0°	30°	60°	90°	120°	150°	180°	c/λ	0°	30°	60°	90°	120°	150°	180°
.0	1.000	.860	.538	.224	.026	-.062	-.085	.0	1.000	.781	.336	.000	-.123	-.130	-.120
.1	.968	.831	.519	.214	.022	-.063	-.084	.1	.966	.753	.320	-.005	-.122	-.127	-.117
.2	.877	.751	.463	.185	.012	-.063	-.082	.2	.869	.673	.275	-.018	-.119	-.118	-.108
.3	.740	.630	.380	.142	-.003	-.063	-.077	.3	.723	.552	.209	-.037	-.114	-.105	-.093
.4	.575	.484	.282	.092	-.019	-.062	-.071	.4	.549	.409	.133	-.056	-.105	-.088	-.074
.5	.403	.334	.182	.043	-.032	-.057	-.061	.5	.370	.264	.058	-.072	-.092	-.068	-.054
.6	.245	.197	.092	.002	-.040	-.050	-.050	.6	.208	.134	-.004	-.080	-.076	-.047	-.033
.7	.116	.085	.022	-.026	-.042	-.040	-.036	.7	.079	.032	-.047	-.078	-.057	-.026	-.014
.8	.023	.007	-.023	-.040	-.038	-.027	-.022	.8	-.010	-.034	-.068	-.067	-.037	-.009	.002
.9	-.031	-.036	-.044	-.041	-.028	-.015	-.010	.9	-.057	-.064	-.069	-.049	-.018	.005	.013
1.0	-.052	-.051	-.046	-.033	-.016	-.004	.001	1.0	-.068	-.067	-.054	-.028	-.002	.014	.018
1.1	-.048	-.044	-.034	-.019	-.004	.005	.008	1.1	-.055	-.049	-.032	-.008	.009	.017	.019
1.2	-.030	-.027	-.017	-.005	.005	.010	.011	1.2	-.030	-.024	-.009	.007	.015	.017	.016
1.3	-.009	-.007	.000	.006	.010	.011	.011	1.3	-.004	.000	.009	.016	.016	.012	.010
1.4	.007	.009	.011	.012	.011	.009	.008	1.4	.015	.017	.019	.018	.012	.006	.004
1.5	.017	.016	.015	.012	.009	.005	.004	1.5	.023	.023	.021	.015	.007	.001	-.002
1.6	.017	.016	.013	.009	.004	.001	.000	1.6	.022	.020	.015	.008	.001	-.004	-.006
1.7	.012	.011	.008	.004	.000	-.003	-.004	1.7	.014	.012	.007	.001	-.004	-.007	-.007
1.8	.004	.003	.001	-.002	-.004	-.005	-.005	1.8	.003	.002	-.002	-.005	-.007	-.007	-.007
1.9	-.003	-.004	-.005	-.005	-.005	-.005	-.005	1.9	-.006	-.007	-.008	-.008	-.007	-.005	-.004
2.0	-.008	-.008	-.007	-.006	-.005	-.004	-.003	2.0	-.011	-.011	-.010	-.008	-.005	-.002	-.001
2.2	-.006	-.006	-.004	-.002	.000	.001	.001	2.2	-.007	-.007	-.004	-.001	.001	.003	.003
2.4	.002	.002	.002	.003	.003	.003	.003	2.4	.003	.003	.004	.004	.004	.003	.003
2.6	.005	.005	.004	.003	.002	.001	.001	2.6	.006	.006	.005	.003	.001	.000	.000
2.8	.001	.001	.000	.000	-.001	-.001	-.002	2.8	.001	.001	.000	-.001	-.002	-.002	-.002
3.0	-.003	-.003	-.003	-.002	-.002	-.002	-.002	3.0	-.004	-.004	-.003	-.003	-.002	-.002	-.001
3.2	-.002	-.002	-.002	-.001	.000	.000	.000	3.2	-.003	-.003	-.002	-.001	.000	.001	.001
3.4	.001	.001	.001	.001	.001	.001	.001	3.4	.001	.001	.002	.002	.002	.002	.002
3.6	.002	.002	.002	.001	.001	.001	.001	3.6	.003	.003	.002	.002	.001	.000	.000
3.8	.000	.000	.000	.000	.000	-.001	-.001	3.8	.000	.000	.000	.000	-.001	-.001	-.001
4.0	-.001	-.001	-.001	-.001	-.001	-.001	-.001	4.0	-.002	-.002	-.002	-.001	-.001	-.001	-.001

$\beta a = 0.25, \quad \beta b = 2.75, \quad G_1 = 0.676$							$\beta a = 0.5, \quad \beta b = 2.0, \quad G_1 = 1.906$								
c/λ	0°	30°	60°	90°	120°	150°	180°	c/λ	0°	30°	60°	90°	120°	150°	180°
.0	1.000	.658	.060	-.229	-.204	-.106	-.065	.0	1.000	.825	.448	.129	-.027	-.067	-.070
.1	.963	.630	.050	-.228	-.199	-.102	-.061	.1	.969	.798	.432	.121	-.028	-.066	-.068
.2	.859	.552	.022	-.222	-.185	-.090	-.051	.2	.879	.721	.384	.101	-.032	-.062	-.063
.3	.704	.437	-.017	-.212	-.163	-.071	-.035	.3	.743	.606	.313	.072	-.036	-.057	-.056
.4	.520	.302	-.059	-.195	-.134	-.049	-.016	.4	.580	.467	.229	.039	-.040	-.050	-.046
.5	.333	.168	-.095	-.172	-.102	-.025	.002	.5	.410	.323	.144	.007	-.042	-.041	-.035
.6	.167	.052	-.118	-.143	-.068	-.003	.019	.6	.252	.191	.068	-.019	-.041	-.031	-.023
.7	.039	-.032	-.123	-.108	-.036	.015	.030	.7	.122	.084	.009	-.035	-.036	-.020	-.013
.8	-.043	-.079	-.111	-.071	-.009	.027	.037	.8	.029	.008	-.028	-.041	-.028	-.010	-.003
.9	-.080	-.091	-.085	-.037	.011	.033	.037	.9	-.027	-.035	-.044	-.037	-.019	-.002	.004
1.0	-.081	-.077	-.051	-.007	.024	.032	.032	1.0	-.050	-.050	-.044	-.028	-.009	.004	.008
1.1	-.057	-.048	-.018	.015	.028	.026	.023	1.1	-.048	-.044	-.032	-.015	.000	.008	.010
1.2	-.024	-.014	.009	.026	.026	.017	.012	1.2	-.032	-.027	-.015	-.003	.006	.009	.009
1.3	.006	.013	.026	.029	.019	.007	.002	1.3	-.011	-.008	.000	.006	.008	.007	.006
1.4	.026	.029	.031	.024	.010	-.002	-.006	1.4	.006	.007	.010	.011	.008	.005	.003
1.5	.032	.032	.026	.014	.001	-.008	-.011	1.5	.016	.015	.014	.011	.006	.001	.000
1.6	.027	.024	.016	.003	-.006	-.011	-.012	1.6	.017	.016	.013	.008	.002	-.001	-.003
1.7	.014	.011	.003	-.006	-.010	-.010	-.010	1.7	.013	.011	.007	.003	-.001	-.003	-.004
1.8	.000	-.002	-.007	-.011	-.010	-.007	-.006	1.8	.005	.004	.001	-.002	-.003	-.004	-.004
1.9	-.011	-.012	-.013	-.012	-.007	-.003	-.001	1.9	-.002	-.003	-.004	-.004	-.004	-.003	-.003
2.0	-.016	-.015	-.013	-.009	-.003	.001	.003	2.0	-.007	-.007	-.007	-.005	-.003	-.002	-.001
2.2	-.009	-.007	-.003	.001	.004	.005	.006	2.2	-.006	-.006	-.004	-.002	.000	.001	.002
2.4	.005	.006	.007	.006	.005	.003	.002	2.4	.001	.002	.002	.002	.002	.002	.002
2.6	.008	.008	.006	.003	.000	-.002	-.002	2.6	.005	.004	.004	.002	.001	.000	.000
2.8	.001	.000	-.001	-.003	-.003	-.003	-.003	2.8	.001	.001	.001	.000	-.001	-.001	-.001
3.0	-.005	-.005	-.005	-.004	-.002	-.001	-.001	3.0	-.002	-.002	-.002	-.002	-.001	-.001	-.001
3.2	-.004	-.003	-.002	.000	.001	.002	.002	3.2	-.002	-.002	-.002	-.001	.000	.000	.000
3.4	.002	.002	.003	.003	.002	.002	.002	3.4	.001	.001	.001	.001	.001	.001	.001
3.6	.004	.003	.003	.002	.001	.000	.000	3.6	.002	.002	.002	.001	.001	.000	.000
3.8	.000	.000	.000	-.001	-.001	-.002	-.002	3.8	.001	.000	.000	.000	.000	-.001	-.001
4.0	-.003	-.003	-.002	-.002	-.001	-.001	-.001	4.0	-.001	-.001	-.001	-.001	-.001	-.001	-.001

9.2.1 Vertical Dipoles (cont.)

$\beta a = 0.5, \quad \beta b = 2.5, \quad G_1 = 1.281$									$\beta a = 0.5, \quad \beta b = 3.0, \quad G_1 = 0.680$								
c/λ	0°	30°	60°	90°	120°	150°	180°		c/λ	0°	30°	60°	90°	120°	150°	180°	
.0	1.000	.740	.246	-.062	-.118	-.077	-.053		.0	1.000	.605	-.027	-.248	-.153	-.034	.007	
.1	.966	.713	.233	-.064	-.117	-.075	-.050		.1	.963	.579	-.034	-.245	-.149	-.032	.008	
.2	.870	.636	.196	-.071	-.111	-.068	-.044		.2	.859	.505	-.054	-.235	-.136	-.025	.013	
.3	.725	.521	.142	-.079	-.102	-.057	-.034		.3	.704	.396	-.081	-.219	-.117	-.014	.019	
.4	.552	.385	.080	-.086	-.090	-.044	-.023		.4	.520	.268	-.109	-.195	-.092	-.002	.026	
.5	.374	.247	.020	-.090	-.075	-.030	-.010		.5	.333	.142	-.130	-.166	-.065	.010	.032	
.6	.212	.123	-.028	-.087	-.058	-.016	.001		.6	.167	.034	-.139	-.131	-.038	.020	.035	
.7	.082	.026	-.060	-.077	-.040	-.003	.010		.7	.038	-.044	-.133	-.094	-.014	.027	.036	
.8	-.007	-.036	-.073	-.061	-.022	.006	.016		.8	-.045	-.086	-.112	-.057	.005	.029	.033	
.9	-.055	-.065	-.068	-.042	-.007	.012	.018		.9	-.082	-.095	-.082	-.024	.019	.028	.026	
1.0	-.068	-.067	-.052	-.022	.004	.015	.017		1.0	-.083	-.078	-.046	.002	.025	.023	.018	
1.1	-.056	-.050	-.029	-.004	.011	.014	.013		1.1	-.059	-.048	-.013	.020	.026	.015	.009	
1.2	-.032	-.025	-.007	.009	.014	.011	.008		1.2	-.026	-.014	.013	.028	.022	.007	.001	
1.3	-.006	-.001	.010	.016	.013	.006	.003		1.3	.005	.013	.028	.028	.014	.000	-.006	
1.4	.013	.016	.019	.017	.009	.007	-.002		1.4	.026	.029	.032	.022	.006	-.006	-.009	
1.5	.023	.023	.020	.013	.004	-.003	-.005		1.5	.033	.032	.026	.012	-.002	-.009	-.010	
1.6	.022	.020	.015	.007	-.001	-.005	-.006		1.6	.028	.025	.015	.002	-.007	-.009	-.009	
1.7	.015	.012	.007	.000	-.004	-.005	-.005		1.7	.015	.012	.002	-.006	-.009	-.007	-.006	
1.8	.004	.002	-.002	-.005	-.006	-.004	-.004		1.8	.001	-.002	-.008	-.011	-.008	-.004	-.002	
1.9	-.005	-.006	-.007	-.007	-.005	-.002	-.001		1.9	-.010	-.011	-.013	-.011	-.006	.000	.002	
2.0	-.010	-.010	-.009	-.007	-.003	.000	.001		2.0	-.016	-.015	-.013	-.008	-.002	.003	.004	
2.2	-.008	-.007	-.004	-.001	.001	.003	.003		2.2	-.009	-.007	-.003	.001	.004	.004	.004	
2.4	.003	.003	.004	.004	.003	.002	.002		2.4	.005	.006	.006	.006	.004	.001	.000	
2.6	.006	.006	.005	.003	.001	-.001	-.001		2.6	.009	.008	.006	.003	.000	-.002	-.002	
2.8	.002	.001	.000	-.001	-.002	-.002	-.002		2.8	.001	.001	-.001	-.002	-.003	-.002	-.002	
3.0	-.004	-.003	-.003	-.002	-.002	-.001	.000		3.0	-.005	-.005	-.005	-.003	-.002	.000	.000	
3.2	-.003	-.003	-.002	-.001	.000	.001	.001		3.2	-.004	-.003	-.002	.000	.001	.002	.002	
3.4	.001	.001	.001	.001	.001	.001	.001		3.4	.002	.002	.002	.002	.002	.001	.001	
3.6	.003	.003	.002	.001	.001	.000	.000		3.6	.004	.003	.003	.002	.000	.000	-.001	
3.8	.001	.000	.000	.000	-.001	-.001	-.001		3.8	.001	.000	.000	-.001	-.001	-.001	-.001	
4.0	-.002	-.002	-.002	-.001	-.001	-.001	.000		4.0	-.003	-.002	-.002	-.002	-.001	.000	.000	

$\beta a = 0.75, \quad \beta b = 2.25, \quad G_1 = 2.039$									$\beta a = 0.75, \quad \beta b = 2.75, \quad G_1 = 1.370$								
c/λ	0°	30°	60°	90°	120°	150°	180°		c/λ	0°	30°	60°	90°	120°	150°	180°	
.0	1.000	.787	.357	.047	-.054	-.048	-.035		.0	1.000	.695	.159	-.109	-.105	-.035	-.006	
.1	.969	.761	.343	.043	-.054	-.047	-.034		.1	.967	.669	.148	-.109	-.103	-.034	-.005	
.2	.880	.688	.303	.030	-.054	-.043	-.030		.2	.871	.597	.119	-.110	-.096	-.029	-.002	
.3	.746	.577	.244	.013	-.052	-.038	-.024		.3	.727	.488	.077	-.110	-.086	-.022	.003	
.4	.584	.445	.174	-.007	-.049	-.030	-.017		.4	.556	.359	.029	-.107	-.072	-.014	.008	
.5	.415	.308	.103	-.025	-.045	-.022	-.010		.5	.378	.228	-.016	-.100	-.057	-.005	.013	
.6	.258	.182	.041	-.037	-.038	-.014	-.003		.6	.216	.111	-.051	-.088	-.040	.003	.016	
.7	.128	.079	-.006	-.043	-.030	-.006	.003		.7	.086	.020	-.071	-.072	-.024	.009	.018	
.8	.033	.007	-.034	-.041	-.020	.000	.007		.8	-.004	-.039	-.075	-.053	-.010	.012	.017	
.9	-.024	-.034	-.045	-.033	-.011	.004	.009		.9	-.053	-.065	-.066	-.032	.001	.014	.015	
1.0	-.048	-.048	-.041	-.022	-.003	.007	.009		1.0	-.067	-.066	-.047	-.013	.009	.012	.011	
1.1	-.047	-.043	-.029	-.010	.003	.007	.008		1.1	-.057	-.049	-.024	.002	.013	.010	.006	
1.2	-.032	-.027	-.013	.001	.007	.006	.005		1.2	-.033	-.024	-.003	.012	.013	.006	.002	
1.3	-.012	-.008	.001	.008	.008	.004	.002		1.3	-.007	-.001	.012	.017	.010	.002	-.002	
1.4	.005	.007	.010	.010	.006	.002	.000		1.4	.012	.015	.019	.016	.006	-.002	-.004	
1.5	.015	.015	.014	.010	.004	-.001	-.002		1.5	.022	.022	.019	.011	.002	-.004	-.005	
1.6	.017	.016	.012	.006	.001	-.002	-.003		1.6	.022	.020	.014	.005	-.002	-.005	-.005	
1.7	.013	.011	.007	.002	-.002	-.003	-.003		1.7	.015	.012	.006	-.001	-.004	-.004	-.003	
1.8	.005	.004	.001	-.002	-.003	-.003	-.002		1.8	.005	.003	-.002	-.005	-.005	-.003	-.001	
1.9	-.002	-.003	-.004	-.004	-.003	-.002	-.001		1.9	-.004	-.006	-.007	-.007	-.004	-.001	.001	
2.0	-.007	-.007	-.006	-.005	-.002	.000	.000		2.0	-.010	-.010	-.009	-.006	-.002	.001	.002	
2.2	-.006	-.006	-.004	-.001	.000	.001	.002		2.2	-.008	-.007	-.004	-.001	.002	.002	.002	
2.4	.001	.001	.002	.002	.002	.001	.001		2.4	.002	.003	.003	.004	.002	.001	.000	
2.6	.005	.004	.003	.002	.001	.000	-.001		2.6	.006	.006	.004	.002	.000	-.001	-.001	
2.8	.002	.001	.001	.000	.001	-.001	-.001		2.8	.002	.001	.000	-.001	-.001	-.001	-.001	
3.0	-.002	-.002	-.002	-.002	-.001	-.001	.000		3.0	-.003	-.003	-.003	-.002	-.001	.000	.000	
3.2	-.002	-.002	-.001	-.001	.000	.000	.001		3.2	-.003	-.003	-.002	-.001	.000	.001	.001	
3.4	.000	.001	.001	.001	.001	.001	.001		3.4	.001	.001	.001	.001	.001	.001	.001	
3.6	.002	.002	.001	.001	.000	.000	.000		3.6	.003	.002	.002	.001	.000	.000	.000	
3.8	.001	.001	.000	.000	.000	.000	-.001		3.8	.001	.001	.000	.000	-.001	-.001	-.001	
4.0	-.001	-.001	-.001	-.001	-.001	.000	.000		4.0	-.002	-.002	-.001	-.001	-.001	.000	.000	

9.2.1 Vertical Dipoles (cont.)

$\beta a = 0.75,$		$\beta b = 3.25,$		$G_1 = 0.700$			$\beta a = 1.0,$		$\beta b = 2.5,$		$G_1 = 2.157$				
c/λ	0°	30°	60°	90°	120°	150°	180°	c/λ	0°	30°	60°	90°	120°	150°	180°
.0	1.000	.550	-.104	-.248	-.103	.006	.035	.0	1.000	.746	.271	-.015	-.063	-.026	-.006
.1	.964	.526	-.108	-.243	-.100	.007	.035	.1	.969	.722	.259	-.017	-.062	-.025	-.006
.2	.860	.456	-.120	-.230	-.089	.011	.036	.2	.881	.652	.227	-.023	-.059	-.022	-.004
.3	.705	.354	-.136	-.209	-.073	.015	.036	.3	.748	.547	.178	-.032	-.054	-.018	-.001
.4	.521	.234	-.150	-.181	-.053	.021	.036	.4	.587	.421	.121	-.040	-.048	-.013	.003
.5	.334	.117	-.157	-.147	-.032	.025	.035	.5	.419	.290	.065	-.046	-.040	-.007	.006
.6	.168	.017	-.153	-.110	-.012	.027	.032	.6	.262	.171	.016	-.048	-.031	-.002	.008
.7	.038	-.054	-.137	-.073	.005	.027	.027	.7	.132	.073	-.020	-.046	-.021	.002	.009
.8	-.045	-.091	-.109	-.038	.018	.025	.020	.8	.036	.004	-.040	-.038	-.012	.005	.010
.9	-.083	-.097	-.075	-.008	.025	.020	.013	.9	-.022	-.035	-.045	-.028	-.004	.007	.009
1.0	-.084	-.078	-.038	.013	.027	.013	.005	1.0	-.047	-.048	-.038	-.016	.002	.007	.006
1.1	-.061	-.047	-.006	.026	.024	.006	-.001	1.1	-.047	-.042	-.025	-.005	.006	.006	.004
1.2	-.028	-.013	.017	.031	.017	.000	-.006	1.2	-.033	-.026	-.010	.004	.007	.004	.001
1.3	.004	.014	.030	.027	.010	-.005	-.009	1.3	-.013	-.008	.003	.009	.007	.002	-.001
1.4	.025	.029	.031	.019	.002	-.007	-.010	1.4	.004	.006	.011	.010	.005	.000	-.002
1.5	.033	.032	.025	.009	-.004	-.008	-.008	1.5	.014	.014	.013	.008	.002	-.002	-.003
1.6	.028	.025	.013	-.001	-.008	-.007	-.005	1.6	.016	.015	.011	.005	.000	-.002	-.003
1.7	.016	.012	.001	-.008	-.008	-.004	-.002	1.7	.013	.011	.006	.001	-.002	-.002	-.002
1.8	.002	-.002	-.008	-.011	-.007	-.001	.001	1.8	.006	.004	.000	-.003	-.003	-.002	-.001
1.9	-.010	-.011	-.013	-.010	-.004	.001	.003	1.9	-.001	-.002	-.004	-.004	-.003	-.001	.000
2.0	-.015	-.015	-.013	-.007	-.001	.003	.004	2.0	-.006	-.006	-.006	-.004	-.002	.000	.001
2.2	-.009	-.007	-.003	.002	.004	.003	.003	2.2	-.006	-.005	-.003	-.001	.001	.001	.001
2.4	.005	.005	.006	.006	.003	.000	-.001	2.4	.001	.001	.002	.002	.002	.001	.000
2.6	.009	.008	.006	.002	-.001	-.002	-.002	2.6	.004	.004	.003	.002	.000	.000	-.001
2.8	.001	.001	-.001	-.002	-.002	-.002	-.001	2.8	.002	.001	.000	.000	-.001	-.001	-.001
3.0	-.005	-.005	-.005	-.003	-.001	.000	.001	3.0	-.002	-.002	-.002	-.002	-.001	.000	.000
3.2	-.004	-.003	-.002	.000	.001	.001	.001	3.2	-.002	-.002	-.001	-.001	.000	.000	.001
3.4	.002	.002	.002	.002	.001	.001	.000	3.4	.000	.000	.001	.001	.001	.000	.000
3.6	.004	.003	.003	.001	.000	-.001	-.001	3.6	.002	.002	.001	.001	.000	.000	.000
3.8	.001	.000	.000	-.001	-.001	-.001	-.001	3.8	.001	.001	.000	.000	.000	.000	.000
4.0	-.002	-.002	-.002	-.002	-.001	.000	.000	4.0	-.001	-.001	-.001	-.001	.000	.000	.000

$\beta a = 1.0,$		$\beta b = 3.0,$		$G_1 = 1.445$			$\beta a = 1.0,$		$\beta b = 3.5,$		$G_1 = 0.714$				
c/λ	0°	30°	60°	90°	120°	150°	180°	c/λ	0°	30°	60°	90°	120°	150°	180°
.0	1.000	.648	.080	-.135	-.082	-.007	.016	.0	1.000	.494	-.167	-.226	-.055	.024	-.033
.1	.967	.624	.072	-.134	-.079	-.007	.016	.1	.964	.471	-.169	-.221	-.052	.024	.033
.2	.872	.555	.050	-.130	-.073	-.004	.017	.2	.860	.406	-.174	-.206	-.044	.025	.032
.3	.729	.452	.019	-.124	-.063	.000	.018	.3	.705	.310	-.180	-.183	-.032	.026	.030
.4	.558	.331	-.016	-.113	-.050	.004	.018	.4	.521	.199	-.181	-.153	-.017	.027	.027
.5	.381	.207	-.047	-.099	-.036	.008	.018	.5	.335	.090	-.176	-.119	-.003	.026	.023
.6	.220	.097	-.069	-.082	-.022	.011	.017	.6	.168	-.001	-.161	-.083	.011	.025	.018
.7	.089	.012	-.079	-.062	-.009	.013	.015	.7	.038	-.065	-.136	-.048	.021	.021	.012
.8	-.002	-.042	-.076	-.041	.001	.013	.012	.8	-.046	-.096	-.103	-.017	.027	.017	.006
.9	-.052	-.066	-.062	-.021	.009	.011	.008	.9	-.084	-.098	-.066	.007	.029	.011	.000
1.0	-.067	-.065	-.042	-.004	.013	.008	.004	1.0	-.086	-.078	-.030	.024	.026	.005	-.004
1.1	-.057	-.048	-.019	.008	.013	.005	.000	1.1	-.063	-.046	.000	.031	.020	.000	-.008
1.2	-.034	-.024	.000	.015	.012	.002	-.003	1.2	-.029	-.012	.021	.031	.013	-.004	-.009
1.3	-.009	-.001	.013	.017	.008	-.001	-.005	1.3	.003	.014	.031	.025	.005	-.007	-.009
1.4	.011	.015	.019	.014	.004	-.003	-.005	1.4	.024	.029	.031	.015	-.002	-.007	-.007
1.5	.021	.021	.018	.009	.000	-.004	-.004	1.5	.032	.032	.023	.005	-.006	-.006	-.004
1.6	.022	.020	.012	.003	-.003	-.004	-.003	1.6	.028	.024	.011	-.004	-.008	-.004	-.001
1.7	.015	.012	.004	-.003	-.004	-.003	-.001	1.7	.017	.012	-.000	-.009	-.008	-.002	.001
1.8	.005	.003	-.003	-.006	-.004	-.001	.000	1.8	.003	-.002	-.009	-.011	-.006	.001	.003
1.9	-.004	-.005	-.007	-.007	-.003	.000	.002	1.9	-.009	-.011	-.013	-.010	-.003	.002	.004
2.0	-.009	-.010	-.009	-.005	-.001	.001	.002	2.0	-.015	-.015	-.012	-.006	.001	.003	.004
2.2	-.008	-.007	-.003	.000	.002	.002	.001	2.2	-.010	-.007	-.002	.003	.004	.002	.001
2.4	.002	.002	.004	.003	.002	.000	.000	2.4	.004	.005	.006	.005	.002	-.001	-.002
2.6	.006	.006	.004	.002	.000	-.001	-.001	2.6	.008	.008	.005	.002	-.001	-.002	-.002
2.8	.002	.001	.000	-.001	-.001	-.001	-.001	2.8	.002	.001	-.001	-.003	-.002	-.001	.000
3.0	-.003	-.003	-.003	-.002	-.001	.000	.000	3.0	-.005	-.005	-.004	-.003	-.001	.001	.001
3.2	-.003	-.003	-.002	.000	.000	.001	.001	3.2	-.004	-.003	-.002	.000	.001	.001	.001
3.4	.001	.001	.001	.001	.001	.000	.000	3.4	.002	.002	.002	.002	.001	.000	.000
3.6	.003	.002	.002	.001	.000	.000	.000	3.6	.004	.003	.002	.001	.000	-.001	-.001
3.8	.001	.001	.000	.000	-.001	-.001	.000	3.8	.001	.000	.000	-.001	-.001	-.001	-.001
4.0	-.001	-.001	-.001	-.001	-.001	.000	.000	4.0	-.002	-.002	-.002	-.001	-.001	.000	.000

9.2.1 Vertical Dipoles (cont.)

c/λ	$\beta a = 1.25,$		$\beta b = 2.75,$		$G_1 = 2.249$		
	0°	30°	60°	90°	120°	150°	180°
.0	1.000	.704	.192	-.055	-.056	-.009	.008
.1	.969	.681	.183	-.056	-.055	-.008	.008
.2	.882	.614	.157	-.057	-.051	-.007	.009
.3	.749	.515	.118	-.059	-.045	-.004	.010
.4	.589	.395	.074	-.059	-.038	-.001	.010
.5	.421	.271	.030	-.057	-.029	.002	.010
.6	.265	.158	-.006	-.052	-.020	.004	.010
.7	.135	.066	-.032	-.043	-.011	.006	.009
.8	.039	.001	-.044	-.032	-.004	.007	.007
.9	-.020	-.035	-.044	-.020	.002	.006	.005
1.0	-.046	-.047	-.035	-.009	.006	.005	.002
1.1	-.047	-.041	-.021	.000	.007	.003	.000
1.2	-.033	-.025	-.007	.007	.007	.001	-.002
1.3	-.014	-.008	.004	.010	.006	.000	-.003
1.4	.003	.006	.011	.010	.003	-.002	-.003
1.5	.013	.014	.013	.007	.001	-.002	-.003
1.6	.016	.015	.010	.003	-.001	-.002	-.002
1.7	.013	.011	.005	.000	-.002	-.002	-.001
1.8	-.006	.004	.000	-.003	-.003	-.001	.000
1.9	-.001	-.002	-.004	-.004	-.002	.000	.001
2.0	-.006	-.006	-.006	-.004	-.001	.001	.001
2.2	-.006	-.005	-.003	.000	.001	.001	.001
2.4	.001	.001	.002	.002	.001	.000	.000
2.6	-.004	.004	.003	.002	-.000	-.001	-.001
2.8	-.002	.001	.000	-.001	-.001	-.001	.000
3.0	-.002	-.002	-.002	-.001	-.001	.000	.000
3.2	-.002	-.002	-.001	.000	.000	.000	.000
3.4	.000	.000	.001	.001	.001	.000	.000
3.6	-.002	-.002	.001	.001	.000	.000	.000
3.8	.001	.001	.000	.000	.000	.000	.000
4.0	-.001	-.001	-.001	-.001	.000	.000	.000

c/λ	$\beta a = 1.25,$		$\beta b = 3.25,$		$G_1 = 1.500$		
	0°	30°	60°	90°	120°	150°	180°
.0	1.000	.599	.011	-.141	-.052	.007	.018
.1	.967	.576	.005	-.139	-.051	.008	.018
.2	.872	.512	-.010	-.132	-.045	.009	.018
.3	.730	.415	-.032	-.121	-.037	.010	.017
.4	.560	.301	-.054	-.107	-.027	.012	.015
.5	.383	.185	-.073	-.089	-.016	.012	.013
.6	.222	.082	-.084	-.069	-.005	.013	.011
.7	.091	.003	-.085	-.047	.004	.012	.008
.8	.000	-.046	-.075	-.027	.010	.010	.004
.9	-.051	-.067	-.057	-.009	.014	.007	.001
1.0	-.067	-.065	-.036	.005	.015	.004	-.002
1.1	-.058	-.047	-.014	.013	.013	.001	-.004
1.2	-.035	-.023	.004	.017	.009	-.001	-.005
1.3	-.039	.000	.015	.016	.005	-.003	-.005
1.4	.010	.015	.019	.012	.001	-.004	-.004
1.5	.021	.021	.017	.006	-.002	-.003	-.003
1.6	.022	.019	.011	.000	-.004	-.002	-.001
1.7	.015	.012	.003	-.004	-.005	-.001	.001
1.8	.006	.003	-.003	-.006	-.004	.000	.002
1.9	-.003	-.005	-.007	-.006	-.002	.001	.002
2.0	-.009	-.009	-.008	-.004	.000	.002	.002
2.2	-.008	-.006	-.003	.001	.002	.001	.001
2.4	.002	.002	.004	.003	.002	.000	-.001
2.6	.006	.005	.004	.002	.000	-.001	-.001
2.8	.002	.001	.000	-.001	-.001	-.001	.000
3.0	-.003	-.003	-.003	-.002	-.001	.000	.001
3.2	-.003	-.002	-.001	.000	.000	.001	.001
3.4	.001	.001	.001	.001	.001	.000	.000
3.6	.002	.002	.002	.001	.000	.000	.000
3.8	.001	.001	.000	.000	-.001	.000	.000
4.0	-.001	-.001	-.001	-.001	.000	.000	.000

c/λ	$\beta a = 1.25,$		$\beta b = 3.75,$		$G_1 = 0.725$		
	0°	30°	60°	90°	120°	150°	180°
.0	1.000	.436	-.216	-.188	-.012	-.026	.019
.1	.964	.414	-.216	-.183	-.010	.026	.018
.2	.860	.354	-.216	-.169	-.005	.026	.017
.3	.705	.265	-.212	-.146	.003	.025	.014
.4	.522	.163	-.203	-.117	.012	.023	-.011
.5	.335	.063	-.187	-.085	.020	.021	.008
.6	.168	-.020	-.163	-.052	.027	.017	.004
.7	.038	-.076	-.131	-.022	.031	.013	-.001
.8	-.046	-.102	-.094	.003	.031	.008	-.004
.9	-.085	-.100	-.055	.021	.028	.004	-.007
1.0	-.087	-.078	-.020	.031	.022	-.001	-.009
1.1	-.064	-.045	.007	.033	.015	-.004	-.009
1.2	-.030	-.011	.025	.029	.007	-.006	-.008
1.3	.002	.015	.032	.021	.000	-.007	-.006
1.4	.024	.030	.030	.011	-.005	-.006	-.004
1.5	.032	.032	.021	.001	-.008	-.004	-.001
1.6	.029	.024	.009	-.007	-.008	-.002	.001
1.7	.017	.011	-.002	-.011	-.007	.000	.003
1.8	.003	-.002	-.010	-.011	-.004	.002	.004
1.9	-.009	-.011	-.013	-.008	-.001	.003	.003
2.0	-.015	-.015	-.012	-.004	.002	.003	.002
2.2	-.010	-.007	-.001	.004	.004	.001	.000
2.4	.004	.005	.006	.005	.002	-.001	-.002
2.6	.008	.008	.005	.000	-.001	-.002	-.001
2.8	.002	.001	-.001	-.003	-.002	.000	.000
3.0	-.005	-.005	-.004	-.002	.000	.001	.001
3.2	-.004	-.003	-.001	.000	.001	.001	.001
3.4	.001	.002	.002	.002	.001	.000	.000
3.6	.004	.003	.002	.001	.000	-.001	-.001
3.8	.001	.001	.000	-.001	-.001	.000	.000
4.0	-.002	-.002	-.002	-.001	.000	.000	.000

c/λ	$\beta a = 1.5,$		$\beta b = 3.0,$		$G_1 = 2.328$		
	0°	30°	60°	90°	120°	150°	180°
.0	1.000	.660	.121	-.076	-.040	.002	.010
.1	.970	.638	.114	-.076	-.039	.002	.010
.2	.882	.575	.094	-.074	-.035	.003	.010
.3	.750	.481	.065	-.071	-.030	.004	.010
.4	.591	.368	.032	-.065	-.023	.005	.009
.5	.424	.251	.000	-.058	-.016	.006	.008
.6	.267	.144	-.026	-.048	-.008	.007	.006
.7	.137	.057	-.041	-.037	-.002	.007	.004
.8	.041	.003	-.047	-.024	.003	.006	.002
.9	-.019	-.037	-.042	-.013	.007	.004	.001
1.0	-.045	-.047	-.032	-.003	.008	.003	-.001
1.1	-.047	-.040	-.018	.005	.008	.001	-.002
1.2	-.033	-.025	-.004	.009	.006	.000	-.003
1.3	-.015	-.007	.006	.010	.004	-.001	-.003
1.4	.002	.006	.011	.008	.001	-.002	-.002
1.5	.013	.014	.012	.005	-.001	-.002	-.001
1.6	.016	.014	.009	.002	-.002	-.001	-.001
1.7	.013	.010	.004	-.002	-.003	-.001	.000
1.8	.006	.004	-.001	-.004	-.002	.000	.001
1.9	-.001	-.002	-.004	-.004	-.002	.001	.001
2.0	-.006	-.006	-.005	-.003	-.001	.001	.001
2.2	-.006	-.005	-.003	.000	.001	.001	.000
2.4	.000	.001	.002	.002	.001	.000	-.001
2.6	.004	.004	.003	.001	.000	-.001	-.001
2.8	.002	.001	.000	-.001	-.001	.000	.000
3.0	-.002	-.002	-.002	-.001	.000	.000	.000
3.2	-.002	-.002	-.001	.000	.000	.000	.000
3.4	.000	.000	.001	.001	.001	.000	.000
3.6	.002	.002	.001	.001	.000	.000	.000
3.8	.001	.001	.000	.000	.000	.000	.000
4.0	-.001	-.001	-.001	-.001	.000	.000	.000

9.2.1 Vertical Dipoles (cont.)

c/λ	$\beta a = 1.5,$		$\beta b = 3.5,$		$G_1 = 1.552$		
	0°	30°	60°	90°	120°	150°	180°
.0	1.000	.549	-.047	-.131	-.023	.013	.011
.1	.967	.528	-.051	-.128	-.022	.013	.011
.2	.873	.467	-.060	-.120	-.018	.013	.010
.3	.731	.376	-.073	-.108	-.012	.013	.009
.4	.561	.269	-.085	-.091	-.005	.013	.007
.5	.385	.161	-.094	-.072	.002	.012	.005
.6	.224	.066	-.095	-.052	.009	.010	.003
.7	.093	-.006	-.087	-.031	.013	.008	.000
.8	-.001	-.051	-.072	-.013	.016	.006	-.002
.9	-.051	-.069	-.052	.001	.016	.003	-.003
1.0	-.067	-.064	-.029	.011	.014	.001	-.004
1.1	-.058	-.046	-.009	.017	.010	-.001	-.005
1.2	-.035	-.022	.007	.017	.006	-.003	-.004
1.3	-.010	.000	.016	.014	.002	-.003	-.003
1.4	.010	.015	.019	.009	-.001	-.003	-.002
1.5	.021	.021	.016	.003	-.004	-.002	-.001
1.6	.022	.019	.009	-.002	-.005	-.001	.001
1.7	.015	.011	.002	-.005	-.004	.000	.002
1.8	.006	.002	-.004	-.006	-.003	.001	.002
1.9	-.003	-.005	-.007	-.006	-.001	.001	.002
2.0	-.009	-.009	-.008	-.003	.000	.001	.001
2.2	-.008	-.006	-.002	.001	.002	.001	.000
2.4	.001	.002	.004	.003	.001	.000	-.001
2.6	.006	.005	.004	.001	-.001	-.001	-.001
2.8	.002	.001	.000	-.001	-.001	.000	.000
3.0	-.003	-.003	-.003	-.002	.000	.000	.001
3.2	-.003	-.002	-.001	.000	.001	.000	.000
3.4	.001	.001	.001	.001	.001	.000	.000
3.6	.002	.002	.002	.001	.000	.000	.000
3.8	.001	.001	.000	.000	-.001	.000	.000
4.0	-.001	-.001	-.001	-.001	.000	.000	.000

c/λ	$\beta a = 1.5,$		$\beta b = 4.0,$		$G_1 = 0.740$		
	0°	30°	60°	90°	120°	150°	180°
.0	1.000	.377	-.252	-.142	.019	.021	.003
.1	.964	.357	-.250	-.138	.020	.021	.003
.2	.860	.301	-.244	-.124	.023	.020	.002
.3	.706	.220	-.234	-.104	.027	.018	.000
.4	.522	.126	-.216	-.078	.031	.016	-.002
.5	.335	.036	-.191	-.050	.034	.013	-.004
.6	.168	-.038	-.160	-.023	.035	.009	-.006
.7	.038	-.087	-.122	.001	.033	.005	-.008
.8	-.047	-.107	-.082	.019	.030	.002	-.009
.9	-.086	-.101	-.044	.031	.024	-.002	-.009
1.0	-.088	-.077	-.011	.035	.016	-.004	-.008
1.1	-.065	-.043	.014	.033	.008	-.006	-.007
1.2	-.031	-.009	.029	.026	.001	-.006	-.005
1.3	.001	.016	.033	.016	-.004	-.006	-.002
1.4	.023	.030	.028	.006	-.008	-.004	.000
1.5	.032	.032	.018	-.003	-.009	-.002	.002
1.6	.029	.024	.007	-.009	-.008	.000	.003
1.7	.018	.011	-.004	-.011	-.005	.001	.003
1.8	.003	-.002	-.011	-.010	-.002	.002	.003
1.9	-.008	-.011	-.013	-.007	.001	.003	.002
2.0	-.015	-.015	-.011	-.003	.003	.002	.001
2.2	-.010	-.007	-.001	.004	.003	.000	-.001
2.4	.004	.005	.006	.005	.001	-.001	-.002
2.6	.008	.007	.004	.000	-.002	-.001	-.001
2.8	.002	.001	-.002	-.003	-.002	.000	.001
3.0	-.005	-.005	-.004	-.002	.000	.001	.001
3.2	-.004	-.003	-.001	.001	.001	.001	.000
3.4	.001	.002	.002	.002	.001	.000	-.001
3.6	.004	.003	.002	.001	.000	-.001	-.001
3.8	.001	.001	.000	-.001	-.001	.000	.000
4.0	-.002	-.002	-.002	-.001	.000	.000	.000

c/λ	$\beta a = 1.75,$		$\beta b = 3.25,$		$G_1 = 2.397$		
	0°	30°	60°	90°	120°	150°	180°
.0	1.000	.615	.060	-.082	-.021	.007	.006
.1	.970	.594	.054	-.080	-.021	.007	.006
.2	.883	.535	.040	-.077	-.018	.007	.005
.3	.751	.446	.019	-.071	-.014	.007	.005
.4	.592	.339	-.004	-.062	-.009	.007	.004
.5	.426	.229	-.026	-.052	-.003	.007	.003
.6	.270	.129	-.041	-.040	.002	.006	.001
.7	.139	.048	-.049	-.028	.006	.005	.000
.8	.043	-.008	-.048	-.016	.008	.004	-.001
.9	-.017	-.038	-.040	-.005	.009	.002	-.002
1.0	-.044	-.047	-.028	.003	.008	.001	-.003
1.1	-.047	-.039	-.014	.008	.007	.000	-.003
1.2	-.034	-.024	-.001	.010	.004	-.001	-.002
1.3	-.015	-.007	.008	.009	.002	-.002	-.002
1.4	.002	.006	.012	.007	.000	-.002	-.001
1.5	.012	.013	.011	.003	-.002	-.001	.000
1.6	.016	.014	.008	.000	-.003	-.001	.001
1.7	.013	.010	.003	-.002	-.003	.000	.001
1.8	.006	.004	-.001	-.004	-.002	.000	.001
1.9	.000	-.002	-.004	-.004	-.001	.001	.001
2.0	-.005	-.006	-.005	-.003	.000	.001	.001
2.2	-.006	-.005	-.002	.001	.001	.000	.000
2.4	.000	.001	.002	.002	.001	.000	-.001
2.6	.004	.004	.003	.001	.000	.000	.000
2.8	.002	.001	.000	-.001	-.001	.000	.000
3.0	-.002	-.002	-.002	-.001	.000	.000	.000
3.2	-.002	-.002	-.001	.000	.000	.000	.000
3.4	.000	.000	.001	.001	.000	.000	.000
3.6	.002	.002	.001	.001	.000	.000	.000
3.8	.001	.001	.000	.000	.000	.000	.000
4.0	-.001	-.001	-.001	-.001	.000	.000	.000

c/λ	$\beta a = 1.75,$		$\beta b = 3.75,$		$G_1 = 1.599$		
	0°	30°	60°	90°	120°	150°	180°
.0	1.000	.498	-.094	-.110	.000	.012	.002
.1	.967	.478	-.096	-.107	.001	.012	.002
.2	.873	.421	-.100	-.099	.003	.011	.001
.3	.732	.337	-.106	-.086	.007	.011	.000
.4	.562	.237	-.109	-.070	.011	.010	-.001
.5	.386	.137	-.108	-.052	.014	.008	-.002
.6	.225	.050	-.100	-.033	.017	.006	-.003
.7	.094	-.016	-.087	-.016	.018	.004	-.004
.8	.002	-.056	-.067	-.001	.017	.002	-.004
.9	-.050	-.070	-.045	.010	.015	.000	-.005
1.0	-.067	-.063	-.023	.016	.011	-.002	-.004
1.1	-.058	-.044	-.003	.018	.007	-.003	-.004
1.2	-.036	-.020	.010	.016	.002	-.003	-.003
1.3	-.011	.001	.018	.012	-.001	-.003	-.001
1.4	.009	.015	.019	.006	-.004	-.002	.000
1.5	.020	.021	.014	.001	-.005	-.001	.001
1.6	.022	.018	.008	-.003	-.005	.000	.002
1.7	.016	.011	.001	-.006	-.004	.001	.002
1.8	.006	.002	-.005	-.006	-.002	.001	.002
1.9	-.003	-.005	-.008	-.005	.000	.001	.001
2.0	-.009	-.009	-.007	-.002	.001	.001	.001
2.2	-.008	-.006	-.002	.002	.002	.000	-.001
2.4	.001	.002	.004	.003	.001	-.001	-.001
2.6	.006	.005	.003	.001	-.001	-.001	.000
2.8	.002	.001	-.001	-.002	-.001	.000	.000
3.0	-.003	-.003	-.003	-.001	.000	.000	.001
3.2	-.003	-.002	-.001	.000	.001	.000	.000
3.4	.000	.001	.001	.001	.001	.000	.000
3.6	.002	.002	.001	.001	.000	.000	.000
3.8	.001	.001	.000	-.001	.000	.000	.000
4.0	-.001	-.001	-.001	-.001	.000	.000	.000

9.2.2 TANGENTIAL DOUBLETS

$\beta a = 0.25,$		$\beta b = 1.75,$		$G_1 = 1.060$				$\beta a = 0.25,$		$\beta b = 2.25,$		$G_1 = 1.133$			
c/λ	0°	30°	60°	90°	120°	150°	180°	c/λ	0°	30°	60°	90°	120°	150°	180°
.0	1.000	.796	.410	.182	.171	.238	.272	.0	1.000	.761	.371	.224	.261	.307	.320
.1	.923	.734	.378	.177	.178	.249	.283	.1	.924	.701	.343	.216	.259	.304	.316
.2	.710	.561	.291	.161	.197	.275	.309	.2	.714	.537	.266	.195	.251	.292	.301
.3	.414	.321	.170	.138	.216	.301	.333	.3	.422	.309	.160	.164	.234	.266	.269
.4	.105	.071	.045	.110	.223	.308	.336	.4	.116	.072	.050	.126	.202	.221	.217
.5	-.149	-.131	-.055	.079	.208	.280	.301	.5	-.135	-.120	-.038	.086	.154	.153	.142
.6	-.298	-.248	-.111	.051	.166	.214	.223	.6	-.285	-.229	-.087	.049	.093	.069	.050
.7	-.326	-.264	-.117	.026	.101	.116	.113	.7	-.315	-.244	-.091	.019	.025	-.021	-.045
.8	-.248	-.195	-.080	.008	.027	.005	-.009	.8	-.242	-.179	-.059	-.003	-.038	-.100	-.127
.9	-.107	-.076	-.021	-.004	-.041	-.092	-.115	.9	-.106	-.067	-.007	-.015	-.084	-.152	-.177
1.0	.043	.047	.039	-.010	-.088	-.154	-.180	1.0	.039	.049	.043	-.019	-.105	-.165	-.184
1.1	.153	.135	.079	-.011	-.104	-.167	-.189	1.1	.146	.131	.074	-.017	-.097	-.138	-.148
1.2	.194	.165	.088	-.009	-.088	-.132	-.145	1.2	.187	.157	.078	-.011	-.066	-.079	-.078
1.3	.162	.134	.066	-.005	-.048	-.062	-.064	1.3	.157	.126	.055	-.005	-.020	-.005	.005
1.4	.079	.061	.024	-.001	.001	.018	.027	1.4	.077	.056	.015	.001	.026	.063	.079
1.5	-.021	-.024	-.021	.002	.044	.084	.100	1.5	-.020	-.026	-.026	.005	.059	.106	.124
1.6	-.101	-.090	-.055	.003	.068	.116	.134	1.6	-.098	-.089	-.054	.006	.070	.113	.128
1.7	-.137	-.118	-.066	.003	.067	.108	.122	1.7	-.133	-.114	-.062	.005	.058	.086	.093
1.8	-.120	-.101	-.053	.002	.044	.065	.071	1.8	-.116	-.096	-.047	.003	.030	.034	.033
1.9	-.062	-.050	-.022	.001	.009	.005	.002	1.9	-.060	-.046	-.017	.000	-.006	-.024	-.033
2.0	.013	.014	.013	.000	-.026	-.052	-.062	2.0	.013	.016	.016	-.002	-.036	-.069	-.082
2.2	.106	.091	.052	-.002	-.053	-.088	-.100	2.2	.103	.089	.050	-.002	-.049	-.077	-.085
2.4	.050	.041	.020	-.001	-.011	-.013	-.012	2.4	.049	.038	.016	-.001	-.002	.006	.011
2.6	-.061	-.054	-.033	.001	.038	.067	.078	2.6	-.059	-.053	-.033	.001	.040	.070	.081
2.8	-.078	-.067	-.036	.001	.033	.053	.060	2.8	-.076	-.064	-.034	.001	.027	.040	.044
3.0	.007	.008	.007	.000	-.013	-.025	-.031	3.0	.007	.009	.008	.000	-.018	-.035	-.042
3.2	.073	.063	.036	-.001	-.037	-.062	-.072	3.2	.071	.061	.035	-.001	-.035	-.058	-.066
3.4	.036	.030	.015	.000	-.011	-.016	-.017	3.4	.035	.029	.013	.000	-.006	-.006	-.005
3.6	-.044	-.038	-.023	.000	.026	.046	.054	3.6	-.043	-.038	-.023	.000	.027	.048	.056
3.8	-.058	-.049	-.028	.000	.026	.042	.048	3.8	-.056	-.048	-.026	.000	.022	.035	.040
4.0	.005	.005	.004	.000	-.008	-.016	-.019	4.0	.005	.006	.005	.000	-.011	-.021	-.026
4.2	.055	.048	.028	.000	-.028	-.048	-.055	4.2	.054	.047	.027	.000	-.027	-.046	-.052
4.4	.028	.023	.012	.000	-.010	-.015	-.016	4.4	.027	.022	.011	.000	-.007	-.009	-.009
4.6	-.034	-.030	-.018	.000	.020	.035	.040	4.6	-.033	-.030	-.018	.000	.020	.036	.042
4.8	-.046	-.039	-.022	.000	.021	.035	.046	4.8	-.045	-.038	-.021	.000	.019	.030	.034
5.0	.004	.004	.003	.000	-.005	-.011	-.013	5.0	.004	.005	.004	.000	-.007	-.015	-.018
5.2	.045	.039	.022	.000	-.023	-.039	-.045	5.2	.044	.038	.022	.000	-.022	-.037	-.043
5.4	.023	.019	.010	.000	-.008	-.013	-.015	5.4	.022	.018	.009	.000	-.006	-.009	-.010
5.6	-.028	-.025	-.015	.000	.016	.028	.032	5.6	-.028	-.024	-.015	.000	.016	.029	.034
5.8	-.038	-.033	-.018	.000	.017	.029	.034	5.8	-.037	-.032	-.018	.000	.016	.026	.030
6.0	.004	.004	.003	.000	-.004	-.008	-.010	6.0	.004	.004	.003	.000	-.006	-.011	-.013
6.2	.038	.033	.019	.000	-.019	-.033	-.038	6.2	.037	.032	.018	.000	-.018	-.031	-.036
6.4	.019	.016	.009	.000	-.007	-.012	-.013	6.4	.018	.015	.008	.000	-.006	-.009	-.010
6.6	-.024	-.021	-.012	.000	.013	.023	.027	6.6	-.024	-.021	-.012	.000	.014	.024	.028
6.8	-.032	-.028	-.016	.000	.015	.025	.029	6.8	-.031	-.027	-.015	.000	.014	.023	.026
7.0	.004	.003	.002	.000	-.003	-.007	-.008	7.0	.003	.003	.003	.000	-.004	-.009	-.011
7.2	.033	.028	.016	.000	-.016	-.028	-.032	7.2	.032	.027	.016	.000	-.016	-.027	-.031
7.4	.016	.014	.007	.000	-.007	-.011	-.012	7.4	.016	.013	.007	.000	-.005	-.008	-.009
7.6	-.021	-.018	-.011	.000	.011	.020	.023	7.6	-.021	-.018	-.011	.000	.012	.021	.024
7.8	-.028	-.024	-.014	.000	.013	.022	.026	7.8	-.027	-.023	-.013	.000	.012	.020	.023
8.0	.003	.003	.002	.000	-.003	-.006	-.007	8.0	.003	.003	.002	.000	-.004	-.007	-.008

9.2.2 Tangential Doublets (cont.)

$\beta a = 0.25, \quad \beta b = 2.75, \quad G_1 = 1.117$									$\beta a = 0.5, \quad \beta b = 2.0, \quad G_1 = 1.478$								
c/λ	0°	30°	60°	90°	120°	150°	180°		c/λ	0°	30°	60°	90°	120°	150°	180°	
.0	1.000	.718	.322	.214	.207	.164	.137		.0	1.000	.769	.367	.184	.221	.300	.330	
.1	.924	.661	.298	.206	.200	.157	.129		.1	.926	.711	.340	.180	.223	.301	.330	
.2	.716	.505	.231	.183	.181	.133	.103		.2	.723	.551	.266	.166	.227	.300	.325	
.3	.425	.289	.139	.148	.148	.093	.062		.3	.439	.328	.164	.146	.225	.287	.307	
.4	.121	.065	.046	.107	.104	.041	.008		.4	.142	.096	.059	.120	.211	.258	.267	
.5	-.131	-.116	-.029	.065	.051	-.020	-.054		.5	-.105	-.093	-.026	.092	.178	.201	.200	
.6	-.281	-.218	-.069	.027	-.005	-.081	-.114		.6	-.253	-.203	-.073	.063	.126	.121	.109	
.7	-.313	-.230	-.070	-.002	-.056	-.132	-.161		.7	-.287	-.222	-.078	.037	.060	.027	.005	
.8	-.242	-.166	-.041	-.020	-.093	-.161	-.183		.8	-.220	-.163	-.048	.016	-.007	-.064	-.091	
.9	-.109	-.059	.003	-.028	-.110	-.161	-.175		.9	-.094	-.058	-.001	.000	-.062	-.132	-.160	
1.0	.035	.052	.045	-.028	-.102	-.131	-.134		1.0	.042	.050	.045	-.010	-.095	-.162	-.185	
1.1	.142	.128	.068	-.021	-.074	-.076	-.070		1.1	.141	.126	.073	-.014	-.098	-.149	-.164	
1.2	.183	.151	.068	-.011	-.031	-.009	.006		1.2	.178	.149	.075	-.013	-.074	-.098	-.102	
1.3	.155	.119	.044	-.002	.014	.054	.073		1.3	.147	.118	.051	-.010	-.032	-.026	-.019	
1.4	.076	.050	.007	.005	.050	.099	.117		1.4	.069	.050	.012	-.005	.014	.046	.061	
1.5	-.019	-.028	-.030	.008	.069	.113	.127		1.5	-.023	-.028	-.027	.000	.050	.097	.115	
1.6	-.096	-.088	-.053	.009	.066	.095	.101		1.6	-.098	-.088	-.054	.003	.067	.113	.129	
1.7	-.130	-.111	-.057	.006	.044	.051	.050		1.7	-.129	-.111	-.060	.005	.061	.093	.103	
1.8	-.114	-.092	-.040	.003	.011	-.003	-.012		1.8	-.111	-.092	-.045	.005	.035	.045	.046	
1.9	-.059	-.042	-.011	-.001	-.022	-.053	-.066		1.9	-.056	-.043	-.015	.003	.001	-.013	-.020	
2.0	.012	.018	.020	-.003	-.045	-.082	-.095		2.0	.015	.018	.017	.001	-.030	-.061	-.074	
2.2	.101	.087	.047	-.003	-.042	-.059	-.062		2.2	.101	.087	.049	-.002	-.049	-.080	-.090	
2.4	.048	.036	.012	.000	.009	.027	.036		2.4	.046	.036	.015	-.002	-.006	-.001	.003	
2.6	-.058	-.053	-.033	.002	.042	.070	.080		2.6	-.059	-.053	-.033	.000	.038	.067	.078	
2.8	-.075	-.062	-.031	.001	.020	.024	.024		2.8	-.074	-.062	-.033	.002	.029	.044	.049	
3.0	.007	.009	.010	-.001	-.023	-.045	-.053		3.0	.008	.009	.009	.001	-.015	-.031	-.037	
3.2	.070	.060	.034	-.001	-.033	-.051	-.057		3.2	.070	.060	.034	-.001	-.035	-.058	-.067	
3.4	.034	.027	.011	.000	-.001	.006	.010		3.4	.034	.027	.013	-.001	-.008	-.009	-.009	
3.6	-.042	-.038	-.024	.001	.029	.050	.058		3.6	-.042	-.037	-.023	.000	.026	.046	.054	
3.8	-.055	-.047	-.025	.001	.019	.027	.029		3.8	-.055	-.047	-.025	.001	.023	.037	.042	
4.0	.005	.006	.007	.000	-.014	-.028	-.033		4.0	.006	.006	.006	.001	-.009	-.019	-.023	
4.2	.053	.046	.026	-.001	-.026	-.042	-.048		4.2	.053	.046	.026	.000	-.027	-.045	-.052	
4.4	.027	.022	.010	.000	-.003	-.001	.000		4.4	.026	.022	.011	-.001	-.008	-.011	-.011	
4.6	-.033	-.029	-.018	.000	.021	.038	.044		4.6	-.033	-.029	-.018	.000	.019	.035	.041	
4.8	-.044	-.037	-.020	.000	.016	.025	.028		4.8	-.044	-.037	-.021	.000	.019	.031	.035	
5.0	.004	.005	.005	.000	-.010	-.019	-.023		5.0	.005	.005	.004	.000	-.006	-.013	-.016	
5.2	.043	.037	.021	.000	-.021	-.035	-.040		5.2	.043	.037	.021	.000	-.021	-.037	-.042	
5.4	.022	.018	.008	.000	-.004	-.004	-.004		5.4	.021	.018	.009	.000	-.007	-.011	-.011	
5.6	-.027	-.024	-.015	.000	.017	.030	.035		5.6	-.027	-.024	-.014	.000	.016	.028	.032	
5.8	-.036	-.031	-.017	.000	.014	.023	.025		5.8	-.036	-.031	-.017	.000	.016	.027	.030	
6.0	.004	.004	.004	.000	-.007	-.014	-.017		6.0	.004	.004	.003	.000	-.005	-.010	-.012	
6.2	.036	.031	.018	.000	-.018	-.030	-.034		6.2	.036	.031	.018	.000	-.018	-.031	-.036	
6.4	.018	.015	.007	.000	-.004	-.005	-.005		6.4	.018	.015	.008	.000	-.006	-.010	-.011	
6.6	-.023	-.021	-.012	.000	.014	.025	.029		6.6	-.023	-.020	-.012	.000	.013	.023	.027	
6.8	-.031	-.026	-.014	.000	.013	.020	.023		6.8	-.031	-.026	-.015	.000	.014	.023	.026	
7.0	.003	.004	.003	.000	-.006	-.011	-.013		7.0	.004	.003	.003	.000	-.004	-.008	-.009	
7.2	.031	.027	.016	.000	-.015	-.026	-.030		7.2	.031	.027	.016	.000	-.015	-.027	-.031	
7.4	.015	.013	.006	.000	-.004	-.005	-.006		7.4	.015	.013	.007	.000	-.006	-.009	-.010	
7.6	-.020	-.018	-.011	.000	.012	.021	.025		7.6	-.020	-.018	-.011	.000	.011	.020	.023	
7.8	-.027	-.023	-.013	.000	.011	.018	.021		7.8	-.027	-.023	-.013	.000	.012	.021	.024	
8.0	.003	.003	.003	.000	-.004	-.009	-.011		8.0	.003	.003	.002	.000	-.003	-.006	-.007	

9.2.2 Tangential Doublets (cont.)

$\beta a = 0.75,$		$\beta b = 2.25,$		$G_1 = 2.015$				$\beta a = 0.75,$		$\beta b = 2.75,$		$G_1 = 1.735$			
c/λ	0°	30°	60°	90°	120°	150°	180°	c/λ	0°	30°	60°	90°	120°	150°	180°
.0	1.000	.752	.345	.181	.198	.222	.224	.0	1.000	.731	.343	.207	.157	.082	.043
.1	.933	.700	.323	.178	.197	.218	.218	.1	.933	.682	.323	.201	.153	.076	.037
.2	.747	.558	.264	.167	.192	.204	.199	.2	.749	.546	.267	.184	.139	.059	.018
.3	.486	.360	.182	.151	.180	.177	.166	.3	.490	.354	.189	.158	.115	.030	-.012
.4	.208	.151	.096	.129	.157	.135	.116	.4	.213	.152	.107	.125	.082	-.007	-.049
.5	-.027	-.024	.025	.103	.121	.078	.051	.5	-.024	-.020	.037	.089	.042	-.048	-.089
.6	-.177	-.132	-.018	.075	.073	.012	-.020	.6	-.180	-.129	-.009	.054	.000	-.087	-.124
.7	-.227	-.163	-.030	.048	.019	-.053	-.088	.7	-.236	-.164	-.027	.023	-.039	-.116	-.146
.8	-.186	-.126	-.016	.023	-.031	-.106	-.138	.8	-.203	-.133	-.020	-.001	-.068	-.128	-.148
.9	-.090	-.049	.011	.003	-.068	-.135	-.160	.9	-.111	-.063	.001	-.018	-.081	-.119	-.127
1.0	.021	.035	.038	-.011	-.084	-.133	-.148	1.0	-.003	.017	.023	-.026	-.076	-.088	-.085
1.1	.104	.095	.053	-.018	-.077	-.101	-.104	1.1	.083	.076	.036	-.027	-.055	-.041	-.029
1.2	.137	.113	.049	-.019	-.057	-.047	-.039	1.2	.123	.098	.034	-.022	-.022	.011	.029
1.3	.116	.088	.028	-.016	-.014	.015	.030	1.3	.110	.080	.018	-.013	.012	.057	.077
1.4	.054	.034	-.002	-.009	.023	.066	.085	1.4	.057	.034	-.005	-.004	.039	.085	.102
1.5	-.021	-.028	-.030	-.002	.049	.095	.112	1.5	-.011	-.021	-.026	.004	.053	.089	.100
1.6	-.081	-.074	-.046	.004	.058	.093	.105	1.6	-.067	-.062	-.038	.009	.050	.069	.072
1.7	-.105	-.090	-.046	.007	.047	.064	.068	1.7	-.092	-.078	-.036	.010	.033	.031	.026
1.8	-.090	-.072	-.030	.008	.023	.018	.014	1.8	-.081	-.063	-.022	.008	.007	-.013	-.023
1.9	-.043	-.030	-.005	.006	-.007	-.029	-.040	1.9	-.041	-.026	-.001	.005	-.018	-.049	-.062
2.0	.016	.019	.020	.003	-.031	-.064	-.077	2.0	.012	.017	.020	.001	-.036	-.068	-.080
2.2	.085	.073	.040	-.003	-.040	-.062	-.068	2.2	.076	.065	.034	-.005	-.032	-.042	-.043
2.4	.036	.027	.008	-.004	.000	.012	.018	2.4	.033	.023	.004	-.004	.008	.027	.036
2.6	-.052	-.047	-.030	.000	.034	.061	.071	2.6	-.047	-.043	-.028	.001	.033	.056	.064
2.8	-.062	-.052	-.026	.003	.023	.031	.032	2.8	-.057	-.046	-.022	.003	.015	.016	.014
3.0	.009	.010	.010	.002	-.016	-.033	-.040	3.0	.008	.010	.011	.001	-.019	-.039	-.046
3.2	.060	.052	.030	-.001	-.030	-.049	-.055	3.2	.055	.048	.027	-.001	-.026	-.040	-.044
3.4	.028	.022	.009	-.002	-.005	-.002	.000	3.4	.025	.019	.006	-.002	.000	.008	.012
3.6	-.037	-.034	-.021	-.001	.023	.042	.049	3.6	-.034	-.031	-.020	.000	.023	.041	.047
3.8	-.047	-.040	-.021	.001	.019	.029	.031	3.8	-.043	-.036	-.018	.001	.015	.020	.021
4.0	.006	.007	.006	.001	-.009	-.020	-.025	4.0	.006	.007	.007	.001	-.011	-.024	-.029
4.2	.046	.040	.023	.000	-.023	-.039	-.044	4.2	.043	.037	.021	-.001	-.021	-.034	-.038
4.4	.022	.018	.008	-.001	-.006	-.006	-.005	4.4	.020	.016	.006	-.001	-.002	.000	.002
4.6	-.029	-.026	-.016	.000	.018	.032	.037	4.6	-.027	-.024	-.015	.000	.017	.031	.036
4.8	-.038	-.032	-.017	.001	.016	.025	.028	4.8	-.035	-.029	-.015	.001	.013	.019	.021
5.0	.005	.005	.005	.001	-.006	-.014	-.017	5.0	.004	.005	.005	.001	-.008	-.016	-.020
5.2	.037	.032	.019	.000	-.019	-.032	-.036	5.2	.035	.030	.017	.000	-.017	-.028	-.032
5.4	.018	.015	.007	-.001	-.005	-.007	-.007	5.4	.017	.013	.006	-.001	-.003	-.002	-.001
5.6	-.024	-.021	-.013	.000	.014	.025	.030	5.6	-.022	-.020	-.012	.000	.014	.025	.029
5.8	-.031	-.027	-.014	.000	.013	.022	.024	5.8	-.029	-.024	-.013	.001	.011	.018	.019
6.0	.004	.004	.003	.001	-.005	-.010	-.013	6.0	.004	.004	.004	.000	-.006	-.012	-.015
6.2	.031	.027	.016	.000	-.016	-.027	-.031	6.2	.029	.025	.014	.000	-.014	-.024	-.027
6.4	.015	.013	.006	-.001	-.005	-.007	-.007	6.4	.014	.011	.005	-.001	-.003	-.003	-.003
6.6	-.020	-.018	-.011	.000	.012	.021	.024	6.6	-.019	-.017	-.010	.000	.011	.020	.024
6.8	-.027	-.023	-.012	.000	.012	.019	.022	6.8	-.025	-.021	-.011	.000	.010	.016	.018
7.0	.003	.003	.003	.000	-.004	-.008	-.010	7.0	.003	.003	.003	.000	-.005	-.010	-.012
7.2	.027	.024	.014	.000	-.013	-.023	-.026	7.2	.025	.022	.013	.000	-.012	-.021	-.024
7.4	.013	.011	.005	.000	-.004	-.006	-.007	7.4	.012	.010	.005	.000	-.003	-.004	-.004
7.6	-.018	-.016	-.010	.000	.010	.018	.021	7.6	-.017	-.015	-.009	.000	.010	.017	.020
7.8	-.023	-.020	-.011	.000	.010	.017	.019	7.8	-.021	-.018	-.010	.000	.009	.014	.016
8.0	.003	.003	.002	.000	-.003	-.006	-.008	8.0	.003	.003	.002	.000	-.004	-.008	-.009

9.2.2 Tangential Doublets (cont.)

$\beta a = 0.75,$		$\beta b = 3.25,$		$G_1 = 1.185$				$\beta a = 1.0,$		$\beta b = 2.5,$		$G_1 = 2.160$			
c/λ	0°	30°	60°	90°	120°	150°	180°	c/λ	0°	30°	60°	90°	120°	150°	180°
.0	1.000	.689	.300	.174	.062	-.063	-.115	.0	1.000	.740	.342	.191	.160	.108	.075
.1	.930	.639	.281	.168	.057	-.067	-.116	.1	.937	.693	.324	.188	.158	.103	.069
.2	.738	.502	.229	.148	.042	-.078	-.125	.2	.763	.564	.274	.178	.150	.088	.052
.3	.466	.311	.156	.119	.020	-.094	-.136	.3	.517	.383	.204	.162	.135	.063	.023
.4	.177	.109	.079	.083	-.008	-.111	-.147	.4	.252	.188	.129	.141	.111	.029	-.014
.5	-.070	-.061	.013	.046	-.038	-.125	-.153	.5	.021	.021	.063	.114	.078	-.011	-.055
.6	-.230	-.166	-.028	.012	-.064	-.131	-.149	.6	-.135	-.090	.018	.084	.038	-.052	-.093
.7	-.284	-.195	-.043	-.015	-.082	-.125	-.132	.7	-.200	-.134	-.004	.053	-.003	-.066	-.121
.8	-.243	-.158	-.034	-.033	-.088	-.104	-.100	.8	-.181	-.116	-.004	.024	-.038	-.106	-.132
.9	-.140	-.079	-.011	-.040	-.079	-.069	-.055	.9	-.106	-.060	.007	.000	-.060	-.107	-.121
1.0	-.018	.008	.012	-.039	-.057	-.024	-.003	1.0	-.013	.008	.021	-.017	-.066	-.086	-.088
1.1	.081	.074	.027	-.031	-.025	.023	.047	1.1	.064	.060	.028	-.026	-.055	-.048	-.039
1.2	.130	.101	.028	-.019	.008	.062	.086	1.2	.102	.080	.024	-.027	-.031	-.002	.015
1.3	.123	.087	.016	-.006	.037	.086	.105	1.3	.093	.066	.009	-.022	-.001	.042	.062
1.4	.071	.041	-.004	.004	.053	.090	.100	1.4	.049	.028	-.010	-.013	.026	.071	.090
1.5	.001	-.014	-.022	.011	.054	.072	.073	1.5	-.008	-.018	-.026	-.003	.043	.080	.093
1.6	-.060	-.057	-.032	.014	.041	.037	.030	1.6	-.057	-.053	-.034	.006	.045	.066	.070
1.7	-.091	-.075	-.031	.012	.017	-.005	-.018	1.7	-.079	-.066	-.030	.011	.033	.034	.030
1.8	-.083	-.063	-.017	.008	-.009	-.042	-.058	1.8	-.069	-.052	-.016	.012	.012	-.006	-.016
1.9	-.045	-.028	.002	.003	-.031	-.066	-.079	1.9	-.033	-.020	.003	.009	-.012	-.041	-.054
2.0	.007	.015	.020	-.002	-.041	-.069	-.077	2.0	.012	.017	.021	.004	-.029	-.061	-.073
2.2	.074	.062	.030	-.007	-.024	-.021	-.016	2.2	.066	.057	.030	-.005	-.031	-.041	-.043
2.4	.035	.023	.001	-.003	.017	.043	.054	2.4	.027	.018	.001	-.006	.004	.022	.031
2.6	-.045	-.042	-.027	.002	.033	.051	.056	2.6	-.043	-.040	-.026	-.001	.030	.052	.060
2.8	-.056	-.045	-.019	.004	.008	.000	-.005	2.8	-.050	-.040	-.018	.004	.016	.017	.016
3.0	.007	.011	.013	.001	-.023	-.045	-.053	3.0	.009	.011	.012	.003	-.016	-.034	-.042
3.2	.055	.047	.025	-.002	-.023	-.032	-.034	3.2	.050	.043	.024	-.001	-.024	-.038	-.042
3.4	.025	.018	.005	-.002	.005	.018	.024	3.4	.022	.016	.005	-.003	-.002	.005	.009
3.6	-.034	-.032	-.020	.000	.024	.041	.047	3.6	-.032	-.029	-.019	-.001	.021	.038	.044
3.8	-.043	-.036	-.017	.002	.011	.012	.010	3.8	-.039	-.032	-.016	.002	.015	.020	.021
4.0	.006	.007	.008	.001	-.015	-.029	-.035	4.0	.006	.007	.007	.002	-.010	-.021	-.026
4.2	.043	.037	.021	-.001	-.020	-.030	-.033	4.2	.039	.034	.019	-.001	-.019	-.032	-.036
4.4	.020	.015	.005	-.001	.001	.007	.010	4.4	.018	.014	.005	-.002	-.003	-.001	.001
4.6	-.027	-.025	-.016	.000	.018	.032	.037	4.6	-.025	-.023	-.015	-.001	.016	.028	.033
4.8	-.035	-.029	-.015	.001	.011	.014	.015	4.8	-.031	-.026	-.014	.001	.013	.019	.020
5.0	.004	.005	.006	.001	-.010	-.021	-.025	5.0	-.005	.005	.005	.001	-.007	-.014	-.018
5.2	.035	.030	.017	.000	-.016	-.026	-.030	5.2	.032	.028	.016	.000	-.016	-.026	-.030
5.4	.017	.013	.005	-.001	-.001	.002	.004	5.4	.015	.012	.005	-.001	-.004	-.003	-.002
5.6	-.023	-.020	-.013	.000	.015	.026	.030	5.6	-.021	-.019	-.012	.000	.012	.023	.026
5.8	-.029	-.025	-.013	.001	.010	.014	.015	5.8	-.026	-.022	-.012	.001	.011	.017	.019
6.0	.004	.004	.004	.000	-.007	-.015	-.018	6.0	.004	.004	.004	.001	-.005	-.011	-.013
6.2	.030	.026	.015	.000	-.014	-.023	-.026	6.2	.027	.023	.013	.000	-.013	-.022	-.026
6.4	.014	.011	.005	-.001	-.002	.000	.001	6.4	.013	.010	.005	-.001	-.004	-.004	-.004
6.6	-.019	-.017	-.011	.000	.012	.022	.025	6.6	-.018	-.016	-.010	.000	.010	.019	.022
6.8	-.025	-.021	-.011	.000	.009	.014	.015	6.8	-.023	-.019	-.010	.001	.010	.015	.017
7.0	.003	.004	.003	.000	-.006	-.012	-.014	7.0	.003	.003	.003	.001	-.004	-.008	-.010
7.2	.026	.022	.013	.000	-.012	-.020	-.023	7.2	.023	.020	.012	.000	-.011	-.019	-.022
7.4	.012	.010	.004	.000	-.002	.001	-.001	7.4	.011	.009	.004	-.001	-.003	-.004	-.004
7.6	-.017	-.015	-.009	.000	.010	.018	.021	7.6	-.015	-.014	-.008	.000	.009	.016	.019
7.8	-.022	-.018	-.010	.000	.008	.013	.014	7.8	-.020	-.017	-.009	.000	.009	.014	.015
8.0	.003	.003	.003	.001	-.005	-.010	-.012	8.0	.003	.003	.002	.000	-.003	-.007	-.008

9.2.2 Tangential Doublets (cont.)

$\beta a = 1.25, \quad \beta b = 3.75, \quad G_1 = 0.931$									$\beta a = 1.5, \quad \beta b = 3.0, \quad G_1 = 2.186$								
c/λ	0°	30°	60°	90°	120°	150°	180°		c/λ	0°	30°	60°	90°	120°	150°	180°	
.0	1.000	.638	.244	.110	-.019	-.092	-.103		.0	1.000	.708	.329	.207	.105	-.028	-.086	
.1	.930	.591	.227	.104	-.022	-.091	-.101		.1	.940	.666	.314	.203	.102	-.030	-.087	
.2	.736	.460	.182	.086	-.030	-.090	-.096		.2	.774	.550	.273	.190	.092	-.036	-.090	
.3	.461	.277	.118	.059	-.043	-.087	-.087		.3	.537	.384	.213	.168	.075	-.045	-.093	
.4	.167	.083	.049	.027	-.057	-.081	-.073		.4	.279	.203	.146	.139	.052	-.055	-.095	
.5	-.087	-.082	-.010	-.007	-.069	-.070	-.053		.5	.048	.043	.083	.104	.025	-.063	-.093	
.6	-.256	-.186	-.050	-.036	-.077	-.054	-.028		.6	-.116	-.071	.034	.066	-.004	-.067	-.084	
.7	-.317	-.217	-.065	-.058	-.078	-.032	.000		.7	-.195	-.124	.001	.028	-.029	-.064	-.068	
.8	-.280	-.182	-.060	-.069	-.069	-.006	.030		.8	-.194	-.122	-.013	-.006	-.047	-.053	-.044	
.9	-.175	-.104	-.039	-.069	-.052	.020	.057		.9	-.133	-.080	-.015	-.031	-.055	-.034	-.014	
1.0	-.046	-.013	-.015	-.059	-.028	.044	.077		1.0	-.046	-.022	-.010	-.046	-.051	-.009	.017	
1.1	.067	.062	.006	-.042	-.002	.061	.085		1.1	.034	.030	-.005	-.049	-.037	.016	.044	
1.2	.133	.100	.017	-.021	.021	.067	.080		1.2	.083	.059	-.002	-.042	-.016	.038	.063	
1.3	.142	.099	.017	-.001	.037	.059	.061		1.3	.092	.061	-.003	-.028	.005	.050	.067	
1.4	.102	.064	.010	.015	.043	.040	.032		1.4	.067	.039	-.007	-.011	.023	.050	.058	
1.5	.036	.015	-.001	.025	.037	.013	-.003		1.5	.022	.006	-.010	.006	.031	.038	.036	
1.6	-.030	-.031	-.009	.027	.022	-.015	-.035		1.6	-.023	-.023	-.010	.018	.030	.016	.006	
1.7	-.073	-.057	-.011	.022	.003	-.039	-.058		1.7	-.051	-.039	-.007	.023	.019	-.008	-.023	
1.8	-.081	-.057	-.007	.013	-.016	-.052	-.066		1.8	-.055	-.038	.000	.021	.004	-.029	-.045	
1.9	-.059	-.036	.001	.002	-.028	-.051	-.057		1.9	-.037	-.021	.008	.014	-.012	-.041	-.053	
2.0	-.017	-.004	.010	-.007	-.031	-.036	-.034		2.0	-.007	.002	.014	.004	-.022	-.041	-.046	
2.2	.052	.042	.013	-.014	-.010	.014	.026		2.2	.040	.033	.012	-.011	-.018	-.008	-.001	
2.4	.037	.022	-.003	-.006	.019	.045	.054		2.4	.024	.013	-.006	-.011	.007	.030	.040	
2.6	-.025	-.025	-.016	.006	.023	.025	.023		2.6	-.023	-.023	-.016	.001	.020	.030	.033	
2.8	-.043	-.032	-.007	.008	.000	-.020	-.030		2.8	-.032	-.024	-.006	.008	.007	-.004	-.011	
3.0	.001	.006	.012	.001	-.021	-.038	-.044		3.0	.004	.008	.012	.005	-.013	-.031	-.037	
3.2	.039	.033	.015	-.005	-.014	-.011	-.007		3.2	.032	.027	.014	-.003	-.015	-.018	-.018	
3.4	.020	.013	-.002	-.005	.009	.027	.034		3.4	.014	.009	-.002	-.006	.002	.015	.021	
3.6	-.025	-.024	-.016	.001	.019	.030	.032		3.6	-.022	-.020	-.014	-.001	.015	.027	.031	
3.8	-.033	-.026	-.009	.004	.005	-.003	-.008		3.8	-.026	-.020	-.008	.004	.008	.005	.003	
4.0	.004	.007	.009	.002	-.015	-.029	-.035		4.0	.005	.007	.008	.003	-.009	-.022	-.027	
4.2	.033	.028	.015	-.002	-.014	-.018	-.018		4.2	.027	.023	.013	-.001	-.013	-.019	-.021	
4.4	.015	.010	.001	-.003	.004	.015	.020		4.4	.011	.008	.000	-.003	.000	.037	.011	
4.6	-.022	-.021	-.014	.000	.016	.026	.030		4.6	-.019	-.017	-.012	-.001	.012	.022	.026	
4.8	-.027	-.022	-.009	.002	.006	.004	.002		4.8	-.022	-.018	-.008	.002	.008	.008	.008	
5.0	.005	.006	.007	.001	-.010	-.022	-.026		5.0	.005	.006	.006	.002	-.007	-.016	-.019	
5.2	.028	.024	.013	-.001	-.013	-.018	-.020		5.2	.023	.020	.011	-.001	-.011	-.018	-.020	
5.4	.013	.009	.002	-.002	.002	.008	.012		5.4	.010	.007	.001	-.002	-.001	.003	.005	
5.6	-.019	-.018	-.012	.000	.013	.022	.026		5.6	-.016	-.015	-.010	-.001	.010	.019	.022	
5.8	-.023	-.019	-.009	.002	.007	.007	.006		5.8	-.019	-.016	-.007	.001	.007	.009	.009	
6.0	.004	.005	.005	.001	-.008	-.017	-.020		6.0	.004	.005	.005	.002	-.005	-.012	-.014	
6.2	.025	.021	.012	-.001	-.011	-.017	-.019		6.2	.020	.018	.010	.000	-.010	-.016	-.018	
6.4	.011	.008	.002	-.001	.000	.005	.007		6.4	.009	.006	.002	-.002	-.001	.001	.002	
6.6	-.017	-.015	-.010	.000	.011	.019	.022		6.6	-.014	-.013	-.008	-.001	.008	.016	.018	
6.8	-.021	-.017	-.008	.001	.007	.008	.008		6.8	-.017	-.014	-.007	.001	.007	.009	.009	
7.0	.004	.004	.004	.001	-.006	-.013	-.016		7.0	.003	.004	.004	.001	-.004	-.009	-.011	
7.2	.022	.019	.010	.000	-.010	-.016	-.018		7.2	.018	.015	.009	.000	-.009	-.014	-.016	
7.4	.010	.007	.002	-.001	.000	.002	.004		7.4	.008	.006	.002	-.001	-.002	.000	.001	
7.6	-.015	-.013	-.009	.000	.009	.016	.019		7.6	-.012	-.011	-.007	.000	.007	.013	.015	
7.8	-.018	-.015	-.007	.001	.006	.008	.008		7.8	-.015	-.012	-.006	.001	.006	.008	.009	
8.0	.003	.004	.004	.001	-.005	-.011	-.013		8.0	.003	.003	.003	.001	-.003	-.007	-.009	

9.2.2 Tangential Doublets (cont.)

$\beta a = 1.5,$		$\beta b = 3.5,$		$G_1 = 1.661$				$\beta a = 1.5,$		$\beta b = 4.0,$		$G_1 = 1.007$			
c/λ	0°	30°	60°	90°	120°	150°	180°	c/λ	0°	30°	60°	90°	120°	150°	180°
.0	1.000	.674	.290	.156	.030	-.065	-.090	.0	1.000	.607	.198	.050	-.057	-.061	-.037
.1	.937	.631	.275	.150	.026	-.065	-.089	.1	.930	.561	.183	.045	-.059	-.060	-.035
.2	.761	.511	.232	.133	.017	-.066	-.086	.2	.738	.436	.143	.029	-.064	-.057	-.029
.3	.512	.340	.170	.106	.002	-.067	-.080	.3	.465	.260	.086	.006	-.072	-.051	-.019
.4	.241	.156	.101	.073	-.016	-.066	-.070	.4	.172	.074	.025	-.021	-.079	-.042	-.005
.5	.001	-.005	.039	.037	-.035	-.061	-.056	.5	-.082	-.084	-.027	-.047	-.084	-.030	.012
.6	-.166	-.115	-.008	.001	-.052	-.052	-.037	.6	-.251	-.183	-.061	-.069	-.084	-.014	.030
.7	-.242	-.161	-.035	-.029	-.062	-.038	-.013	.7	-.315	-.212	-.073	-.082	-.077	.004	.049
.8	-.230	-.148	-.042	-.050	-.064	-.019	.012	.8	-.260	-.177	-.065	-.065	-.062	.023	.064
.9	-.154	-.094	-.035	-.061	-.057	.003	.036	.9	-.176	-.100	-.043	-.077	-.042	.039	.074
1.0	-.052	-.024	-.020	-.060	-.041	.024	.056	1.0	-.046	-.010	-.016	-.059	-.017	.051	.076
1.1	.041	.038	-.006	-.050	-.020	.041	.067	1.1	.068	.064	.007	-.036	.007	.055	.068
1.2	-.099	.073	.004	-.032	.003	.050	.067	1.2	.136	.103	.020	-.011	.027	.050	.050
1.3	.112	.076	.007	-.012	.021	.049	.055	1.3	.147	.102	.023	.011	.039	.036	.025
1.4	.084	.052	.005	.006	.032	.037	.033	1.4	.108	.069	.017	.027	.041	.015	-.004
1.5	.032	.015	.000	.020	.034	.018	.005	1.5	.041	.019	.007	.035	.034	-.008	-.031
1.6	-.020	-.021	-.004	.026	.025	-.005	-.023	1.6	-.027	-.028	-.003	.033	.018	-.028	-.050
1.7	-.056	-.043	-.005	.025	.011	-.026	-.044	1.7	-.074	-.057	-.008	.025	.000	-.042	-.058
1.8	-.065	-.045	-.002	.018	-.005	-.039	-.053	1.8	-.087	-.061	-.008	.012	-.016	-.044	-.053
1.9	-.048	-.029	.004	.008	-.018	-.041	-.048	1.9	-.067	-.042	-.003	-.002	-.026	-.036	-.035
2.0	-.016	-.005	.008	-.003	-.024	-.032	-.032	2.0	-.026	-.011	.003	-.012	-.027	-.019	-.010
2.2	.040	.032	.008	-.015	-.013	.007	.018	2.2	.048	.038	.007	-.018	-.008	.023	.037
2.4	.030	.018	-.005	-.009	.011	.035	.044	2.4	.042	.026	-.002	-.005	.017	.038	.044
2.6	-.018	-.019	-.013	.004	.019	.022	.021	2.6	-.017	-.018	-.010	.009	.019	.012	.006
2.8	-.034	-.025	-.004	.009	.003	-.014	-.022	2.8	-.041	-.030	-.004	.010	-.002	-.024	-.035
3.0	.000	.005	.010	.003	-.015	-.031	-.036	3.0	-.006	.001	.009	.001	-.018	-.031	-.034
3.2	.030	.025	.012	-.005	-.013	-.011	-.008	3.2	.032	.027	.010	-.007	-.011	-.002	.004
3.4	.016	.010	-.003	-.006	.005	.020	.027	3.4	.021	.013	-.003	-.006	.009	.027	.034
3.6	-.019	-.019	-.013	.000	.015	.024	.027	3.6	-.018	-.018	-.012	.002	.016	.022	.022
3.8	-.025	-.020	-.006	.005	.005	-.001	-.005	3.8	-.029	-.022	-.006	.006	.003	-.009	-.015
4.0	.004	.006	.008	.003	-.011	-.024	-.028	4.0	.001	.005	.009	.002	-.013	-.026	-.031
4.2	.026	.022	.012	-.002	-.012	-.015	-.015	4.2	.027	.023	.011	-.003	-.011	-.011	-.009
4.4	.012	.008	.000	-.003	.002	.011	.016	4.4	.014	.009	-.001	-.004	.005	.017	.022
4.6	-.018	-.017	-.011	.000	.013	.022	.025	4.6	-.018	-.017	-.012	.000	.014	.022	.024
4.8	-.021	-.017	-.007	.003	.006	.004	.002	4.8	-.023	-.018	-.007	.003	.004	-.001	-.004
5.0	.004	.005	.006	.002	-.008	-.018	-.021	5.0	.003	.005	.007	.002	-.010	-.021	-.025
5.2	.023	.020	.011	-.001	-.011	-.015	-.017	5.2	.024	.021	.011	-.002	-.011	-.013	-.014
5.4	.010	.007	.001	-.002	.001	.006	.009	5.4	.011	.007	.000	-.003	.003	.011	.014
5.6	-.016	-.015	-.010	-.001	.010	.019	.021	5.6	-.017	-.015	-.010	.000	.012	.020	.022
5.8	-.019	-.015	-.007	.002	.006	.006	.006	5.8	-.020	-.016	-.007	.002	.005	.003	.002
6.0	.004	.005	.005	.001	-.006	-.013	-.016	6.0	.004	.005	.006	.001	-.008	-.016	-.020
6.2	.020	.017	.010	-.001	-.009	-.014	-.016	6.2	.022	.019	.010	-.001	-.010	-.014	-.015
6.4	.009	.006	.001	-.002	.000	.003	.005	6.4	.009	.007	.001	-.002	.001	.007	.009
6.6	-.014	-.013	-.008	.000	.009	.016	.018	6.6	-.015	-.014	-.009	.000	.010	.017	.020
6.8	-.016	-.014	-.006	.001	.006	.007	.007	6.8	-.018	-.015	-.006	.001	.005	.005	.004
7.0	.003	.004	.004	.001	-.005	-.011	-.013	7.0	.003	.004	.005	.001	-.006	-.013	-.016
7.2	.018	.015	.009	.000	-.008	-.013	-.015	7.2	.019	.017	.009	-.001	-.009	-.013	-.014
7.4	.008	.006	.002	-.001	-.001	.002	.003	7.4	.008	.006	.001	-.001	.000	.004	.006
7.6	-.012	-.011	-.007	.000	.008	.014	.016	7.6	-.013	-.012	-.008	.000	.008	.015	.017
7.8	-.015	-.012	-.006	.001	.005	.007	.007	7.8	-.016	-.013	-.006	.001	.005	.006	.006
8.0	.003	.003	.003	.001	-.004	-.009	-.011	8.0	.003	.004	.004	.001	-.005	-.011	-.013

9.2.3 RADIAL UNIPOLES

$\beta a = 0.25,$		$\beta b = 0.75,$		$G_1 = 0.486$			$\beta a = 0.5,$		$\beta b = 1.0,$		$G_1 = 0.598$				
c/λ	0°	30°	60°	90°	120°	150°	180°	c/λ	0°	30°	60°	90°	120°	150°	180°
.0	1.000	.919	.712	.460	.241	.100	.052	.0	1.000	.891	.620	.310	.062	-.086	-.133
.1	.876	.801	.609	.377	.175	.046	.003	.1	.889	.787	.536	.251	.023	-.111	-.153
.2	.545	.486	.336	.156	.004	-.092	-.124	.2	.590	.510	.313	.093	-.076	-.172	-.202
.3	.112	.075	-.016	-.121	-.206	-.256	-.271	.3	.194	.143	.022	-.105	-.195	-.239	-.251
.4	-.286	-.299	-.328	-.356	-.370	-.373	-.373	.4	-.181	-.201	-.242	-.273	-.280	-.272	-.267
.5	-.529	-.522	-.500	-.464	-.422	-.388	-.375	.5	-.430	-.423	-.397	-.350	-.291	-.243	-.224
.6	-.555	-.536	-.483	-.409	-.335	-.281	-.261	.6	-.493	-.469	-.402	-.308	-.215	-.146	-.122
.7	-.377	-.354	-.293	-.213	-.136	-.081	-.062	.7	-.374	-.344	-.266	-.165	-.070	-.005	.018
.8	-.076	-.058	-.010	.052	.108	.146	.159	.8	-.134	-.110	-.048	.029	.096	.139	.154
.9	.230	.239	.261	.287	.309	.321	.325	.9	.128	.141	.171	.203	.227	.238	.242
1.0	.429	.427	.421	.410	.396	.383	.378	1.0	.318	.317	.311	.297	.277	.258	.251
1.1	.454	.444	.417	.378	.338	.308	.297	1.1	.373	.361	.327	.279	.228	.190	.176
1.2	.305	.292	.256	.207	.159	.124	.112	1.2	.282	.266	.221	.160	.100	.057	.042
1.3	.047	.035	.004	-.036	-.075	-.102	-.112	1.3	.090	.076	.036	-.014	-.062	-.094	-.105
1.4	-.219	-.225	-.240	-.260	-.277	-.289	-.293	1.4	-.125	-.133	-.153	-.178	-.198	-.210	-.214
1.5	-.391	-.390	-.387	-.381	-.374	-.368	-.365	1.5	-.281	-.280	-.277	-.270	-.260	-.250	-.247
1.6	-.408	-.401	-.383	-.357	-.331	-.311	-.303	1.6	-.322	-.314	-.292	-.259	-.226	-.200	-.191
1.7	-.269	-.259	-.234	-.200	-.165	-.140	-.131	1.7	-.238	-.227	-.195	-.152	-.109	-.078	-.067
1.8	-.031	-.023	.000	.030	.059	.080	.087	1.8	-.067	-.056	-.028	.010	.046	.071	.080
1.9	.213	.217	.229	.244	.259	.268	.272	1.9	.124	.130	.145	.164	.181	.192	.196
2.0	.369	.369	.367	.363	.359	.355	.353	2.0	.261	.260	.258	.254	.248	.242	.239
2.2	.247	.240	.221	.194	.167	.148	.141	2.2	.213	.204	.179	.146	.113	.088	.080
2.4	-.208	-.212	-.221	-.234	-.246	-.254	-.257	2.4	-.123	-.128	-.140	-.156	-.170	-.179	-.183
2.6	-.363	-.359	-.348	-.333	-.317	-.305	-.301	2.6	-.275	-.270	-.256	-.237	-.217	-.202	-.196
2.8	-.016	-.010	.004	.024	.043	.057	.062	2.8	-.043	-.037	-.018	.006	.030	.047	.053
3.0	.344	.343	.342	.340	.338	.336	.335	3.0	.239	.238	.237	.234	.231	.228	.227
3.2	.223	.218	.205	.186	.168	.154	.149	3.2	.184	.178	.161	.138	.115	.098	.092
3.4	-.201	-.204	-.210	-.219	-.228	-.234	-.236	3.4	-.122	-.125	-.133	-.145	-.155	-.162	-.165
3.6	-.340	-.337	-.329	-.318	-.307	-.299	-.296	3.6	-.251	-.248	-.238	-.224	-.210	-.199	-.195
3.8	-.010	-.006	.005	.020	.034	.045	.048	3.8	-.032	-.027	-.014	.004	.022	.035	.040
4.0	.328	.328	.327	.326	.324	.323	.322	4.0	.225	.225	.224	.222	.220	.218	.217
4.2	.210	.206	.196	.182	.168	.157	.154	4.2	.168	.163	.150	.133	.115	.102	.098
4.4	-.195	-.197	-.202	-.209	-.216	-.221	-.223	4.4	-.119	-.122	-.128	-.137	-.145	-.151	-.153
4.6	-.326	-.324	-.317	-.309	-.300	-.293	-.291	4.6	-.237	-.234	-.226	-.215	-.204	-.196	-.193
4.8	-.007	-.004	.005	.016	.028	.036	.039	4.8	-.026	-.023	-.012	.003	.017	.027	.031
5.0	.317	.317	.316	.315	.314	.313	.312	5.0	.216	.216	.215	.214	.212	.210	.210
5.2	.202	.199	.190	.179	.168	.159	.156	5.2	.158	.154	.144	.129	.115	.105	.101
5.4	-.190	-.191	-.195	-.201	-.207	-.211	-.212	5.4	-.117	-.119	-.124	-.131	-.138	-.143	-.145
5.6	-.316	-.314	-.309	-.301	-.294	-.289	-.287	5.6	-.227	-.224	-.218	-.209	-.200	-.193	-.190
5.8	-.007	-.004	.003	.013	.022	.029	.032	5.8	-.023	-.020	-.011	.001	.013	.021	.024
6.0	.308	.308	.308	.307	.305	.305	.304	6.0	.209	.209	.208	.207	.206	.204	.204
6.2	.196	.194	.187	.177	.168	.161	.158	6.2	.151	.148	.139	.127	.115	.106	.103
6.4	-.185	-.186	-.189	-.194	-.199	-.202	-.203	6.4	-.114	-.116	-.120	-.126	-.132	-.136	-.138
6.6	-.308	-.307	-.302	-.296	-.290	-.285	-.284	6.6	-.219	-.217	-.212	-.204	-.196	-.190	-.188
6.8	-.007	-.005	.001	.009	.017	.023	.025	6.8	-.021	-.019	-.011	-.001	.009	.016	.019
7.0	.301	.301	.300	.300	.299	.298	.297	7.0	.204	.203	.203	.202	.200	.199	.199
7.2	.193	.191	.185	.176	.168	.162	.160	7.2	.146	.143	.136	.126	.115	.108	.105
7.4	-.180	-.181	-.184	-.188	-.192	-.195	-.196	7.4	-.112	-.113	-.117	-.122	-.127	-.131	-.132
7.6	-.302	-.301	-.297	-.292	-.286	-.282	-.281	7.6	-.213	-.212	-.207	-.200	-.193	-.188	-.186
7.8	-.008	-.006	-.001	.006	.013	.018	.020	7.8	-.020	-.018	-.011	-.003	.006	.012	.015
8.0	.295	.295	.294	.293	.293	.292	.292	8.0	.199	.199	.198	.197	.196	.195	.194

9.2.3 Radial Unipoles (cont.)

$\beta a = 1.25,$		$\beta b = 1.75,$		$G_1 = 0.514$				$\beta a = 1.5,$		$\beta b = 2.0,$		$G_1 = 0.520$			
c/λ	0°	30°	60°	90°	120°	150°	180°	c/λ	0°	30°	60°	90°	120°	150°	180°
.0	1.000	.768	.290	-.061	-.154	-.106	-.070	.0	1.000	.722	.190	-.126	-.133	-.020	.037
.1	.901	.683	.239	-.080	-.154	-.098	-.060	.1	.902	.640	.145	-.141	-.133	-.016	.041
.2	.632	.455	.102	-.129	-.151	-.074	-.032	.2	.634	.420	.025	-.177	-.130	-.007	.050
.3	.269	.149	-.074	-.185	-.138	-.038	.009	.3	.274	.126	-.128	-.214	-.119	.009	.063
.4	-.089	-.147	-.233	-.222	-.112	.006	.054	.4	-.080	-.156	-.260	-.231	-.095	.030	.077
.5	-.348	-.352	-.324	-.218	-.069	.050	.094	.5	-.336	-.349	-.325	-.208	-.055	.052	.088
.6	-.451	-.419	-.318	-.165	-.014	.086	.120	.6	-.437	-.405	-.299	-.141	-.003	.071	.091
.7	-.391	-.344	-.221	-.073	.043	.106	.124	.7	-.377	-.325	-.191	-.044	.049	.079	.082
.8	-.209	-.166	-.067	.031	.087	.102	.103	.8	-.197	-.148	-.036	.059	.089	.073	.059
.9	.020	.044	.092	.118	.106	.076	.061	.9	.027	.058	.116	.138	.104	.049	.024
1.0	.213	.214	.204	.161	.093	.031	.006	1.0	.214	.220	.217	.170	.088	.013	-.017
1.1	.309	.292	.237	.148	.052	-.021	-.048	1.1	.303	.288	.236	.147	.046	-.029	-.056
1.2	.286	.259	.185	.087	-.005	-.066	-.087	1.2	.274	.248	.174	.077	-.010	-.064	-.081
1.3	.162	.137	.073	-.001	-.060	-.092	-.102	1.3	.148	.122	.057	-.014	-.062	-.082	-.086
1.4	-.008	-.023	-.055	-.083	-.094	-.091	-.087	1.4	-.021	-.038	-.072	-.095	-.093	-.077	-.068
1.5	-.161	-.161	-.154	-.132	-.096	-.062	-.048	1.5	-.169	-.171	-.166	-.138	-.092	-.048	-.030
1.6	-.242	-.229	-.191	-.131	-.065	-.014	.004	1.6	-.244	-.232	-.194	-.131	-.059	-.004	.017
1.7	-.228	-.209	-.155	-.083	-.012	.037	.054	1.7	-.224	-.204	-.150	-.077	-.006	.042	.059
1.8	-.132	-.113	-.066	-.007	.044	.076	.086	1.8	-.122	-.103	-.055	.002	.048	.075	.083
1.9	.007	.017	.043	.068	.084	.089	.090	1.9	.019	.030	.055	.077	.086	.083	.081
2.0	.133	.133	.129	.116	.094	.074	.065	2.0	.143	.144	.139	.121	.092	.065	.053
2.2	.192	.177	.135	.078	.022	-.017	-.032	2.2	.191	.175	.132	.073	.016	-.025	-.039
2.4	-.008	-.017	-.038	-.060	-.077	-.085	-.087	2.4	-.018	-.027	-.048	-.068	-.080	-.083	-.083
2.6	-.177	-.169	-.146	-.111	-.073	-.044	-.033	2.6	-.183	-.175	-.150	-.112	-.069	-.037	-.024
2.8	-.094	-.082	-.051	-.011	.026	.051	.060	2.8	-.090	-.077	-.045	-.004	.033	.057	.065
3.0	.107	.107	.105	.099	.089	.079	.075	3.0	.116	.116	.113	.104	.090	.077	.071
3.2	.149	.138	.110	.071	.033	.005	-.006	3.2	.150	.139	.109	.068	.027	-.003	-.013
3.4	-.013	-.019	-.034	-.051	-.066	-.074	-.077	3.4	-.020	-.026	-.042	-.059	-.072	-.078	-.080
3.6	-.146	-.140	-.124	-.100	-.074	-.055	-.047	3.6	-.152	-.146	-.128	-.102	-.073	-.051	-.042
3.8	-.073	-.065	-.042	-.012	.017	.036	.043	3.8	-.070	-.061	-.036	-.005	.024	.043	.050
4.0	.095	.095	.093	.090	.084	.078	.075	4.0	.102	.102	.101	.096	.087	.079	.076
4.2	.125	.117	.096	.066	.037	.016	.008	4.2	.126	.118	.095	.064	.032	.010	.002
4.4	-.017	-.021	-.032	-.046	-.058	-.066	-.068	4.4	-.023	-.028	-.040	-.054	-.065	-.072	-.074
4.6	-.129	-.124	-.111	-.093	-.073	-.059	-.053	4.6	-.134	-.129	-.116	-.095	-.074	-.057	-.051
4.8	-.060	-.054	-.036	-.012	.011	.027	.032	4.8	-.057	-.050	-.031	-.006	.018	.034	.040
5.0	.087	.087	.086	.083	.079	.075	.073	5.0	.095	.095	.093	.090	.084	.079	.076
5.2	.110	.103	.086	.063	.039	.022	.016	5.2	.111	.104	.086	.061	.036	.018	.011
5.4	-.019	-.022	-.032	-.043	-.053	-.060	-.062	5.4	-.025	-.029	-.039	-.051	-.060	-.066	-.068
5.6	-.117	-.113	-.103	-.088	-.072	-.060	-.056	5.6	-.122	-.118	-.107	-.091	-.074	-.060	-.055
5.8	-.052	-.046	-.031	-.012	.007	.020	.025	5.8	-.049	-.043	-.027	-.006	.013	.027	.032
6.0	.082	.082	.081	.079	.075	.072	.071	6.0	.090	.090	.088	.086	.081	.077	.075
6.2	.099	.094	.080	.060	.040	.026	.021	6.2	.101	.095	.080	.059	.038	.023	.017
6.4	-.020	-.023	-.031	-.040	-.049	-.055	-.057	6.4	-.027	-.030	-.038	-.048	-.057	-.062	-.064
6.6	-.109	-.106	-.097	-.084	-.071	-.061	-.057	6.6	-.114	-.111	-.101	-.088	-.073	-.062	-.058
6.8	-.046	-.041	-.028	-.012	.004	.016	.020	6.8	-.042	-.037	-.024	-.007	.010	.022	.026
7.0	.079	.078	.077	.075	.072	.070	.069	7.0	.086	.086	.085	.082	.079	.076	.074
7.2	.092	.087	.075	.058	.041	.029	.024	7.2	.093	.088	.075	.057	.039	.026	.021
7.4	-.021	-.024	-.030	-.038	-.046	-.051	-.053	7.4	-.028	-.031	-.037	-.046	-.054	-.058	-.060
7.6	-.103	-.100	-.092	-.081	-.069	-.061	-.057	7.6	-.108	-.105	-.097	-.085	-.072	-.063	-.059
7.8	-.041	-.037	-.026	-.012	.002	.012	.016	7.8	-.038	-.034	-.022	-.007	.008	.018	.022
8.0	.076	.075	.074	.072	.070	.068	.067	8.0	.083	.083	.082	.080	.077	.074	.073

9.2.3 Radial Unipoles (cont.)

$\beta a = 1.75, \quad \beta b = 2.25, \quad G_1 = 0.588$								$\beta a = 2.0, \quad \beta b = 2.5, \quad G_1 = 0.688$							
c/λ	0°	30°	60°	90°	120°	150°	180°	c/λ	0°	30°	60°	90°	120°	150°	180°
.0	1.000	.670	.089	-.178	-.114	.018	.071	.0	1.000	.615	.002	-.189	-.069	.034	.057
.1	.904	.593	.053	-.185	-.111	.021	.072	.1	.905	.543	-.027	-.191	-.065	.035	.057
.2	.642	.386	-.044	-.201	-.101	.027	.074	.2	.646	.347	-.104	-.192	-.053	.038	.054
.3	.287	.109	-.165	-.212	-.083	.036	.076	.3	.295	.086	-.197	-.185	-.033	.042	.049
.4	-.063	-.156	-.265	-.203	-.053	.047	.075	.4	-.053	-.165	-.267	-.159	-.005	.044	.040
.5	-.320	-.337	-.304	-.163	-.015	.055	.069	.5	-.311	-.334	-.284	-.109	.027	.044	.027
.6	-.426	-.390	-.265	-.091	.028	.059	.056	.6	-.421	-.382	-.234	-.038	.059	.039	.011
.7	-.375	-.313	-.157	-.001	.066	.054	.036	.7	-.376	-.305	-.126	.041	.081	.027	-.009
.8	-.205	-.144	-.012	.086	.090	.038	.009	.8	-.213	-.142	.009	.109	.089	.010	-.030
.9	.010	.051	.125	.146	.091	.014	-.021	.9	-.002	.046	.131	.148	.078	-.010	-.047
1.0	.194	.206	.210	.161	.069	-.015	-.048	1.0	.180	.196	.202	.146	.049	-.030	-.058
1.1	.285	.272	.221	.128	.028	-.043	-.067	1.1	.275	.261	.203	.104	.009	-.044	-.059
1.2	.263	.235	.157	.058	-.020	-.060	-.071	1.2	.260	.227	.137	.033	-.032	-.049	-.048
1.3	.147	.117	.044	-.026	-.061	-.063	-.059	1.3	.152	.115	.030	-.043	-.063	-.042	-.027
1.4	-.013	-.034	-.075	-.096	-.081	-.048	-.032	1.4	-.002	-.029	-.080	-.103	-.073	-.023	.001
1.5	-.155	-.160	-.160	-.131	-.074	-.019	.004	1.5	-.141	-.150	-.156	-.126	-.061	.003	.029
1.6	-.230	-.219	-.183	-.118	-.042	.017	.039	1.6	-.217	-.208	-.173	-.107	-.030	.029	.050
1.7	-.214	-.194	-.140	-.066	.003	.048	.063	1.7	-.206	-.185	-.129	-.054	.010	.047	.058
1.8	-.119	-.099	-.049	.007	.046	.065	.069	1.8	-.119	-.096	-.041	.015	.046	.052	.050
1.9	.014	.026	.054	.074	.073	.061	.054	1.9	.007	.023	.056	.075	.065	.041	.029
2.0	.134	.135	.132	.111	.074	.038	.024	2.0	.123	.127	.128	.106	.061	.017	-.001
2.2	.184	.168	.125	.064	.006	-.034	-.048	2.2	.176	.161	.117	.057	-.001	-.038	-.051
2.4	-.013	-.023	-.044	-.062	-.069	-.066	-.063	2.4	-.009	-.020	-.044	-.060	-.060	-.049	-.043
2.6	-.174	-.166	-.140	-.099	-.052	-.016	-.002	2.6	-.166	-.158	-.133	-.091	-.040	.000	.015
2.8	-.090	-.077	-.044	-.002	.033	.054	.061	2.8	-.088	-.075	-.041	-.001	.032	.049	.055
3.0	.107	.107	.104	.092	.073	.055	.048	3.0	.101	.101	.098	.085	.061	.038	.028
3.2	.146	.135	.103	.059	.016	-.014	-.025	3.2	.141	.130	.098	.053	.009	-.022	-.033
3.4	-.014	-.020	-.036	-.052	-.062	-.065	-.066	3.4	-.010	-.017	-.033	-.048	-.054	-.054	-.052
3.6	-.144	-.138	-.118	-.089	-.057	-.031	-.022	3.6	-.138	-.131	-.112	-.080	-.045	-.017	-.006
3.8	-.072	-.062	-.037	-.005	.024	.043	.050	3.8	-.072	-.062	-.036	-.004	.024	.042	.048
4.0	.093	.093	.090	.083	.072	.061	.056	4.0	.086	.086	.084	.075	.060	.046	.040
4.2	.123	.114	.089	.056	.022	-.002	-.010	4.2	.120	.110	.085	.050	.015	-.010	-.019
4.4	-.015	-.020	-.033	-.047	-.057	-.062	-.063	4.4	-.011	-.016	-.029	-.042	-.050	-.053	-.053
4.6	-.126	-.120	-.105	-.083	-.059	-.040	-.033	4.6	-.120	-.114	-.098	-.074	-.047	-.026	-.018
4.8	-.060	-.052	-.032	-.005	.019	.035	.041	4.8	-.061	-.053	-.032	-.005	.019	.035	.040
5.0	.084	.084	.083	.078	.070	.063	.060	5.0	.078	.077	.075	.069	.059	.049	.045
5.2	.107	.100	.080	.053	.026	.007	.000	5.2	.104	.097	.076	.047	.019	-.002	-.009
5.4	-.017	-.021	-.032	-.043	-.053	-.058	-.059	5.4	-.012	-.016	-.027	-.038	-.046	-.050	-.051
5.6	-.113	-.109	-.097	-.079	-.059	-.044	-.039	5.6	-.107	-.103	-.090	-.070	-.048	-.032	-.025
5.8	-.051	-.045	-.028	-.006	.015	.028	.033	5.8	-.053	-.046	-.028	-.005	.015	.029	.034
6.0	.079	.079	.077	.074	.068	.063	.060	6.0	.072	.071	.070	.065	.058	.051	.048
6.2	.097	.090	.074	.051	.029	.012	.006	6.2	.094	.087	.070	.045	.021	.004	-.002
6.4	-.019	-.022	-.031	-.041	-.049	-.054	-.056	6.4	-.013	-.017	-.026	-.036	-.044	-.048	-.049
6.6	-.105	-.101	-.091	-.076	-.059	-.047	-.043	6.6	-.098	-.094	-.083	-.067	-.049	-.035	-.030
6.8	-.045	-.039	-.025	-.006	.011	.024	.028	6.8	-.046	-.041	-.025	-.006	.012	.024	.029
7.0	.075	.075	.074	.071	.066	.062	.060	7.0	.068	.067	.066	.062	.056	.051	.049
7.2	.089	.083	.069	.050	.030	.016	.011	7.2	.086	.080	.065	.044	.023	.009	.003
7.4	-.020	-.023	-.030	-.039	-.046	-.051	-.052	7.4	-.014	-.017	-.025	-.034	-.041	-.045	-.046
7.6	-.098	-.095	-.086	-.073	-.059	-.049	-.045	7.6	-.092	-.088	-.079	-.064	-.049	-.038	-.033
7.8	-.040	-.035	-.023	-.007	.009	.020	.023	7.8	-.041	-.036	-.023	-.006	.010	.021	.025
8.0	.072	.072	.071	.068	.064	.061	.060	8.0	.065	.064	.063	.060	.055	.051	.049

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