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Fundamentals of
COMPUTER MATH

by ALLAN LYTEL • edited by A. A. WICKS

An introduction to the mathematics used in modern digital computer technology.

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FUNDAMENTALS OF COMPUTER MATH

by ALLAN LYTEL

CHAPTER 1. Numbers and the Computer—Introduces the concept of the radix, or base, of a numbering system. Discusses numbering systems having various bases and the sectional functions of a digital computer, as they relate to handling numerical data.

CHAPTER 2. Numbering Systems—Presents the conventional notation of the numbering systems important to digital computer technology. The binary, octal, and decimal systems are emphasized, and the methods of conversion between these systems are discussed.

CHAPTER 3. Numerical Operations—Explains coded systems, including the binary-coded decimal, and discusses the arithmetic operations pertaining to these systems.

CHAPTER 4. Polynomials and Linear Forms—Presents techniques for the evaluation of the roots of a polynomial, a short-cut method of evaluation of Taylor's expansions, and the methods of solution of simultaneous linear equations, both the method of elimination of variables and the method of solution by determinants.

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CHAPTER 5. Interpolation—Introduces trigonometric functions as a prerequisite to the representation of a continuous function. The method of differences is given as a useful instrument for interpolation, and Newton's formulas for backward and forward interpolation are developed by using the method of differences.

CHAPTER 6. Other Techniques—Gives a short survey of various techniques in numerical analysis; the method of iteration for approximate solutions of simultaneous linear equations, the use of vectors and matrices, linear programming, and the essentials of relaxation methods.

CHAPTER 7. Other Mathematical Forms—Includes a brief discussion of symbolic logic and its application to switching circuits. Also presents various acronyms used in computer programming.

APPENDICES

Appendix I—A glossary of terms in common use in computer technology.

Appendix II — Fraçtional Decimal-to-Octal Conversion Tables.

Appendix III—Octal-to-Decimal Conversion Tables.

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Fundamentals of COMPUTER MATH

by ALLAN LYTEL

The advantage of an electronic digital computer is its capability to perform mathematical operations at great speeds with maximum reliability. Properly guiding the computer in its work is the key problem in utilizing its capabilities.

The mathematical techniques associated with computer technology are covered in this text. The binary (two-valued) numbering system, whose values correspond to the on-off states of an electronic gate, is first developed so that number conversions between it and other systems can be made. Then it is possible to utilize the modern abstract mathematical techniques with a digital computer. Some of the methods covered in this text are iteration, linear programming, relaxations, numerical integration and differentiation, and interpolation. They are presented so as to allow for maximum latitude in the choice of techniques for problem-solving on the digital computer.

Written as a self-teaching guide for students and technicians, *Fundamentals of Computer Math* can also be used as a textbook in courses on modern digital computer technology, as well as a convenient reference for anyone involved with computer mathematics.

ABOUT THE AUTHOR

Allan Lytel, a graduate of both Temple and Syracuse Universities, holds a Master of Science degree in Journalism. He has taught mathematics and electronics at the Technical Institute of Temple University and radar maintenance in the Army Signal Corps. Author of more than a dozen books and innumerable articles, he has also prepared and conducted a course on digital computers. Mr. Lytel is currently manager of U.S. Electronics Publications, Inc. His other SAMS books include: *ABC's of Computers; ABC's of Computer Programming; ABC's of Boolean Algebra; Handbook of Electronic Charts & Nomographs; Handbook of Transistor Circuits; ABC's of Model Radio Control; Industrial X-Ray Handbook; Transistor Circuit Manual;* and *Automotive Electronics Test Equipment.*

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Fundamentals of COMPUTER MATH

Preface

A computer system can receive input information in many forms, perform arithmetical computations, and make logical decisions. Since the ultimate advantage of an electronic digital computer is its capability to perform simple mathematical operations at terrific speeds and with high reliability, the key problem in utilizing this power is in guiding the computer in its work. This is not, as might be expected, in knowing the correct controls to operate, but in analyzing a problem and preparing a list of instructions (programs) for the computer. The great amount of information that has been developed for computer programming has led to many programs and subprograms which have been thoroughly worked out and tested.

This book is concerned with the mathematical methods of providing instructions for the computer, and the mathematically based computer operations guided by these instructions. The first area discussed is that of how the computer handles the data. This leads naturally to a coverage of numbering systems and numbering representations, including the basic binary (two-valued) and octal (eight-valued) numbering systems. Then the basic mathematical operations within these systems are discussed and are subsequently incorporated into the mathematical techniques of problem-solving, such as the approximation of continuous functions, solutions of linear equations, and numerical integration and differentiation.

The subjects are categorized as follows: Numbers and the Computer; Numbering Systems; Numerical Operations; Polynomials and Linear Forms; Interpolation; Other Techniques, including simultaneous linear equations, use of vectors and matrices, and linear programming; and Other Mathematical Forms, including symbolic logic and its applications to switching circuits.

It is felt that this book will make it easier to solve the ever-growing problems of utilizing digital computers in data processing work. This content is designed to be used as either a self-teaching text for the technician and student or as a textbook for courses covering modern digital computer technology.

ALLAN LYTEL

January, 1964

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CHAPTER 1

Numbers and the Computer

The fundamental operations of a digital computer are exceedingly simple. As a system the computer can receive input information in many forms, perform arithmetical computations, and make logical decisions. The basic arithmetic operation of the computer is addition; subtraction is performed as an inverse of addition, and multiplication is performed as a series of additions. Division, by the same means, is performed as a series of subtractions.

INTRODUCTION

Since the ultimate advantage of an electronic digital computer is its capability to perform simple mathematical operations at terrific speeds and with high reliability, the key problem in utilizing this power is in guiding the computer in its work. This is not, as might be expected, in knowing the correct controls to operate (however, this is certainly extremely important), but in analyzing a problem and preparing a list of instructions for the computer. This function, programming, details a long series usually of fundamental and individual steps. (In contrast to this, the analog computer is more sophisticated, at least in terms of the mathematics that can be performed, since it is possible to integrate or differentiate quite easily in an analog device.)

Before considering the types of mathematics that can be accomplished on the digital computer, it is necessary to consider the special features as well as the limitations of the computer. In many cases, the digital computer is used where the input is a series of discrete individual counts or pulses and the problem is one of manipulation of these input data to perform the necessary computations. In other cases, where the input information is essentially analog, or a continuous function, the input data are necessarily broken up into a series of discrete pulses which are then used for the computation inside the machine. It is possible, of course, to graph the function by plotting the individual values and then connecting these individual values with a curve. This curve (Fig. 1-1) will closely approximate the function under discussion if a large number of points are plotted with great care. This, of course, is the method of aproximation, and there is extensive literature on the various techniques and methods of approximating functions for use in the digital computer.

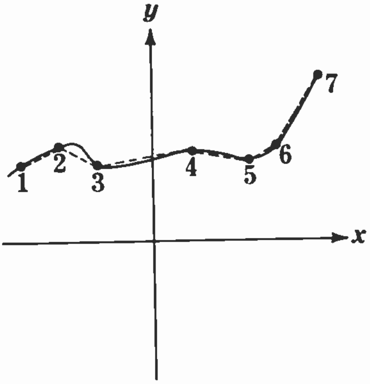


Fig. 1-1. Approximating a function.

COMMON NUMBERING SYSTEMS

The base of a number system is determined by the number of different digits used. Thus, in our common (Arabic) system of numbering, this is based on 10; hence, the decimal (deci = tenth) system.

We are most familiar with this system, but there are other, equally valid numbering systems using other bases. These include *biquinary*, using 10 by pairs, and the Roman system of numerals. Our decimal system is used for most desk calculators; the Roman numeral system is still used for building-cornerstone dates, and the biquinary system is the basis of the abacus and the IBM 650 computer.

The decimal system bears closer examination. Take the two digits 23, for example; in the decimal system we take this to mean three 1's and two 10's. To the base 5, however, this would mean decimal 13, because 23 is then three 1's and two 5's. Using the base 8, 23 would be decimal 19, for this is two 8's and three 1's.

This is shown in Table 1-1, where in the first column we count in tens, the second in fives, and the third in eights. Counting in decimal is 1, 2, 3, 4, 5, 6, 7, 8, 9, 10. To the base 5, the count is 1, 2, 3, 4, 10; and to the base 8, it is 1, 2, 3, 4, 5, 6, 7, 10.

The radix of a numbering system therefore being its base, the radix of the decimal system becomes 10. Because we use numbers so very often, we tend to forget that decimal numbers are not used in all applications. When we see 53 or VIII, what do we see? The number 53 means 3 more than 5 tens; VIII is *not* 53, but the eighth in the series of Roman numerals. It is one 5 and three 1's.

Let us examine this aspect further by using a trinary numbering system in which we count by A, B, C. As shown in Table 1-2, A is decimal 1, B is 2, while C is 3. Thus we count 1, 2, 3, or A, B, C. The next "number" is AA, which in decimal is one 3 plus one 1, or decimal 4.

In the same way:

$$\begin{array}{cccl}
(3^2) & (3^1) & (3^0) & \\
A & B & C & = 9+6+3=18 \\
B & A & C & = 18+3+3=24 \\
C & A & B & = 27+3+2=32 \\
C & C & B & = 27+9+2=38
\end{array}$$

This numbering system, as others, is an ordered set of digits used for counting. When all numbers, in sequence, have been used in the first position, the second position is occupied by the next symbol.

Thus our normal numbering system has 10 as its base. Any decimal number may be written as the sum of a power series

Table 1-1. Counting in Three Numbering Systems

	BASE	
10	5	8
1	1	1
2	2	2
3	3	3
4	4	4
5	10	5
6	11	6
7	12	7
8	13	10
9	14	11
10	20	12

of numbers, each of which is a power of 10. This may be seen by making columns of powers of tens and writing the digits in the proper place. The decimal numbers 352, 4,167, and 50,213 are written this way in Table 1-3. Each column, reading from right to left, has an increase of one in the power of 10, so that 4,167 is, for example, 4(1,000)+1(100)+6(10)+7(1). For mechanization, the decimal system of numbering (to the base ten) requires ten different and recognizable states for each digit. Any numbering system requires *n* states, where *n* is the numbering base.

Table 1-2. Trinary Numbering System

Decimal	Trinary	Decimal	Trinary
1	A	21	ACC
2	B	22	BAA
3	C	23	BAB
4	AA	24	BAC
5	AB	25	BBA
6	AC	26	BBB
7	BA	27	BBC
8	BB	28	BCA
9	BC	29	BCB
10	CA	30	BCC
11	CB	31	CAA
12	CC	32	CAB
13	AAA	33	CAC
14	AAB	34	CBA
15	AAC	35	CBB
16	ABA	36	CBC
17	ABB	37	CCA
18	ABC	38	CCB
19	ACA	39	CCC
20	ACB		

A computer built to accommodate the decimal system is certainly technically possible, but it would be unwieldy and expensive from a practical point of view. This is because the storing of figures in the decimal system would require ten different states, each needing to be easily identifiable and recoverable.

Any number can be expressed as a power series. However, any base can be used to express a given number; a base of 10 is normally used for ordinary computation. But, suppose the base is 5, as an example: a count from decimal 0 to 15 would be, to the base 5, as shown in Table 1-4. Representing the decimal number 2,469 to the base 5 would be easier as far as computer construction is concerned, requiring fewer states than the

Table 1-3. Decimal Notation

10^4	10^3	10^2	10^1	10^0
0	0	3	5	2
0	4	1	6	7
5	0	2	1	3

decimal system. If we write the base of a numbering system as the subscript of any number in that system, the preceding example becomes:

$$2{,}469_{10} = 34{,}334_5$$
$$34{,}334_5 = 3(5^4) + 4(5^3) + 3(5^2) + 3(5^1) + 4(5^0)$$
$$2{,}469_{10} = 1{,}875 + 500 + 75 + 15 + 4$$

Binary Numbering System

Electrically, there can be produced two basic states that may indicate "on" or "off," "go" or "stop," "white" or "black," or—and this is what permits a mathematical usage—"one" or "zero." Because of the practical considerations, then, digital computers use and represent numbers in the base 2, from which we derive the binary system.

The binary number system, to the base (or radix) 2, therefore, uses two characters: 0 and 1. The binary 1 and the binary 0 characters are referred to as *bits*, or *binary digits*. A binary 1 indicates the presence of a bit of information. A binary 0 indicates the absence of a bit of information. This does not mean

Table 1-4. Counting to the Base 5

Decimal	5^1	5^0
1	0	1
2	0	2
3	0	3
4	0	4
5	1	0
6	1	1
7	1	2
8	1	3
9	1	4
10	2	0
11	2	1
12	2	2
13	2	3
14	2	4
15	3	0

that the input to the computer must be in the binary system; it may be decimal or, for that matter, alphabetical or word-oriented. This is a matter of electronic conversion prior to processing by the computer. Other inputs, such as coded binary and coded decimal, may be used; these will be explained later.

Just as the first four powers of 10 are 1, 10, 100 and 1,000, the first four powers of 2 are 1, 2, 4, and 8, as in Table 1-5. Any quantity or magnitude which is expressed in the decimal system can be expressed in the binary system:

$$\begin{aligned} \text{Decimal:} \quad 19_{10} &= 1 \times 10^1 + 9 \times 10^0 \\ &= 10 + 9 \\ \text{Binary:} \quad 19_{10} &= 1 \times 2^4 + 0 \times 2^3 + 0 \times 2^2 + 1 \times 2^1 + 1 \times 2^0 \\ &= 16 + 0 + 0 + 2 + 1 \\ &= 1011_2 \end{aligned}$$

Note that for any integer n, 10^n is greater than 2^n. Thus, it is obvious that in the binary system many more positions are required to represent a given magnitude or quantity than are required in the decimal system.

Table 1-5. Powers of 10 and 2

System	Powers (Exponents)				
Decimal	1	10	100	1,000	10,000
Binary	1	2	4	8	16

COMPUTER PROGRAMMING

Computer programming is a technique and a field of its own, but a computer-user should know something of basic principles. Programming is the technique of setting up a specific series of steps by which a digital computer solves problems. To program a computer you must know the meaning of each individual step the computer can do. You must understand how to feed information into the machine, arrange the sequence of events, and let the machine solve your specific problem.

The individual and distinct functions of a digital computer are input, memory, arithmetic, control, and output.

Input devices read information into the computer from punched cards, punched tapes, or magnetic tapes. This information must first be translated into numbers, usually binary, to be read into the computer.

Information is read into some form of the computer *memory*. The input is either a set of instructions which tell the computer what to do, or information representing data for computation. Instructions, in total, form the computer program. Input data represent the information upon which the computer operates to perform its calculations.

The memory must be large enough to contain all of the necessary information and large enough that the information can be retrieved as fast as required. Magnetic cores are used in the large machines for the main memory, whereas magnetic drums, which are much slower, are used in some of the smaller machines. Magnetic tapes are also a form of computer memory.

The *arithmetic* section of the computer actually does the computations; it can add, subtract, multiply, divide, and make certain decisions. A register, which is often used as part of the arithmetic unit, is simply a place for the temporary storage of information. Information is placed in a register while it is being operated upon. Many computers have several registers where numbers are placed before, after, and during the various arithmetic operations.

Instructions are interpreted in the *control* section of the computer. In an "add" instruction, for example, the control section takes the instruction and internally arranges the circuits so that the computer will add two numbers.

The *output* of the computer takes information and stores it until it is needed. It also translates the information from computations to the necessary form, which may be printed sheets of paper, magnetic tapes, punched cards, or punched tapes. Computers can do several things simultaneously—read information from one input, provide information to several outputs, and at the same time compute another section of the problem.

Programming of a computer reduces to a coding problem after the problem has been defined in detail. Computers are of great value in solving problems that involve either a great many calculations or a large amount of data. Coded instructions are required for actual computer operation of a problem, and although the first electronic data-processing systems could be used only by experts, it now is possible to write machine-language programs that enable a computer to recognize instructions written in other languages; this led to the development of programming languages that were simpler to use than the machine language.

These "languages" are problem-oriented and permit the programmer to write convenient equivalents of machine instructions, using mnemonic symbols to represent them. The computer, acting under the control of previously written machine-language programs, then "translates" these instructions into equivalent machine instructions that are used in solving problems.

One such language is FORTRAN (*Mathematical FORmula TRANslating System*) developed by IBM. This language is written in algebraic and English notations; after the steps are written in this manner, the program is processed, by a computer, into a computer program.

CHAPTER 2

Numbering Systems

Numbering systems may be considered as being of two types: additive and positional. The Roman, Hebrew, Greek and Egyptian systems are additive. The decimal and binary systems are positional.

ADDITIVE NOTATION

An additive system is one in which symbols have different weights. In the Roman system, XVII means 17, whereas the Roman number XVII stands for 10 + 5 + 1 + 1. In this manner each symbol has a special, weighted value. Some symbols and their weights in this system are shown in Table 2-1.

Additive systems such as the Roman can be used for addition or subtraction without great difficulty, but they are extremely awkward for division and multiplication. To add 17 and 11, we have:

$$\begin{array}{l} \text{XVII} \\ \text{XI} \\ \hline \text{XXVIII} \end{array}$$

But, to multiply two numbers, such as MCXVII and MMCCI, is most difficult, although it can be done.

Table 2-1. Roman Numerals and Their Decimal Weights

Roman Symbol	Decimal Weight
I	1
V	5
X	10
L	50
C	100
D	500
M	1,000

POSITIONAL NOTATION

With the introduction of the concept of zero, the positional notation was developed. Note the absence of zero in the Roman system. In the positional notation, there is the concept of base, such as the base 10 in the decimal system. Other bases have been used in the past; consider angular measure with 60 seconds in a minute, 60 minutes in a degree, and 6 × 60, or 360, degrees in a circle.

Thus, in decimal, for the units' place we count from 0 to 9; for the next count, a 1 is used in the tens' place, while the units' place reverts to zero; thus 10 means $1 \times 10 + 0 \times 1$.

Any decimal number C can be expressed as:

$$C = A_n 10^n + A_{n-1} 10^{n-1} + \cdots + A_1 10^1 + A_0 10^0$$

For example: $156 = 1 \times 10^2 + 5 \times 10^1 + 6 \times 10^0$

$138 = 1 \times 10^2 + 3 \times 10^1 + 8 \times 10^0$

In general, for additive systems, any number N may be expressed as:

$$N = A_n X^n + A_{n-1} X^{n-1} + A_{n-2} X^{n-2} + \cdots + A_1 X^1 + A_0 X^0$$

where,

N is the number,
X is the base,
A is any digit that can be used.

In every case, the weight of a digit is X times the weight of the digit to its right. Thus, for base 10 each digit has 10 times the weight of the preceding (right-hand) digit. Table 2-2 shows the notations for bases 3, 4, 5, and 10. Thus for decimal 5, that number expressed in a system to the base is 012 $(1 \times 3 + 2 \times 1)$, to the base 4 is 011 $(1 \times 4 + 1 \times 1)$, and to the base 5 is 10 $(1 \times 5 + 0 \times 1)$. For decimal 12, that number expressed in a system to the base 3 is 110 $(1 \times 9 + 1 \times 3 + 0 \times 1)$, to the base 4 is 030 $(3 \times 4 + 0 \times 1)$, and to the base 5 is 22 $(2 \times 5 + 2 \times 1)$.

Computation (Table 2-3) follows from the structure of each positional notation for each base. Here are some examples for addition:

Decimal *Base 3*

$1 + 2 = 3 = 10_3 \; (1 \times 3^1 + 0 \times 3^0)$

$2 + 2 = 4 = 11_3 \; (1 \times 3^2 + 1 \times 3^0)$

Decimal *Base 4*

$$2 + 2 = 4 = 10_4 \ (1 \times 4^1 + 0 \times 4^0)$$

$$3 + 2 = 5 = 11_4 \ (1 \times 4^1 + 1 \times 4^0)$$

Decimal *Base 5*

$$3 + 3 = 6 = 11_5 \ (1 \times 5^1 + 1 \times 5^0)$$

$$4 + 3 = 7 = 12_5 \ (1 \times 5^1 + 2 \times 5^0)$$

Table 2-2. Positional Notation for Bases 3, 4, 5, and 10

Base 3	Base 4	Base 5	Base 10
000	000	00	00
001	001	01	01
002	002	02	02
010	003	03	03
011	010	04	04
012	011	10	05
020	012	11	06
021	013	12	07
022	020	13	08
100	021	14	09
101	022	20	10
102	023	21	11
110	030	22	12
111	031	23	13
112	032	24	14
120	033	30	15
121	100	31	16

For multiplication:

Decimal *Base 3*

$$2 \times 1 = 2 = 2_3 \ (2 \times 3^0)$$

$$2 \times 2 = 4 = 11_3 \ (1 \times 3^1 + 1 \times 3^0)$$

Decimal *Base 4*

$$2 \times 3 = 6 = 12_4 \ (1 \times 4^1 + 2 \times 4^0)$$

$$3 \times 3 = 9 = 21_4 \ (2 \times 4^1 + 1 \times 4^0)$$

Decimal *Base 5*

$$3 \times 3 = 9 = 14_5 \ (1 \times 5^1 + 4 \times 5^0)$$

$$4 \times 4 = 16 = 31_5 \ (3 \times 5^1 + 1 \times 5^0)$$

Table 2-3. Addition and Multiplication Tables From Positional Notation

ADDITION							MULTIPLICATION					
Augend / Addend	0	1	2			BASE	Multiplicand / Multiplier	0	1	2		
0	0	1	2			3	0	0	0	0		
1	1	2	10				1	0	1	2		
2	2	10	11				2	0	2	11		
Augend / Addend	0	1	2	3		BASE	Multiplicand / Multiplier	0	1	2	3	
0	0	1	2	3		4	0	0	0	0	0	
1	1	2	3	10			1	0	1	2	3	
2	2	3	10	11			2	0	2	10	12	
3	3	10	11	12			3	0	3	12	21	
Augend / Addend	0	1	2	3	4	BASE	Multiplicand / Multiplier	0	1	2	3	4
0	0	1	2	3	4	5	0	0	0	0	0	0
1	1	2	3	4	10		1	0	1	2	3	4
2	2	3	4	10	11		2	0	2	4	11	13
3	3	4	10	11	12		3	0	3	11	14	22
4	4	10	11	12	13		4	0	4	13	22	31

From these tables one could construct a complete set of tabular values for computation, if required.

Binary Notation

The value 2 is useful as a notational base. Switches for industrial control systems operate on a two-valued system, such as *on-off* or *open-closed* states. Vacuum tubes or transistors in modern digital computers also operate as simple switches which, as an example, either permit or prevent current flow. As stated in Chapter 1, these switching devices have only two

Table 2-4. Increasing Powers of 2

Powers of 2	←	2^4	2^3	2^2	2^1	2^0
Decimal Value	←	16	8	4	2	1

states and cannot easily be used for representing the decimal system.

The numbering system that is most easily developed for use in industrial control systems or in digital computers is therefore based on successive powers of 2. In this binary notation only two digits are used, 1 and 0. The first six powers of 2 are 1, 2, 4, 8, 16, and 32. Table 2-4 shows the increasing powers of 2 as 1, 2, 4, 8, and 16.

It is necessary that the base that is being used is known; this is in order to know what a quantity of 0's and 1's represents, since 10 represents 2 for a binary base and 10 for a decimal base; the number 3 expressed in the binary system is represented as 11, and it is *not* the decimal 11. Thus the decimals 1, 2, 3, 4, 5, and 6 correspond to binary 1, 10, 11, 100, 101, and 110. The binary number 101 is $4_{10} + 0_{10} + 1_{10}$, or decimal 5, written 5_{10}. In the same way 10111 is 23_{10}, or $16 + 0 + 4 + 2 + 1$. A table for binary numbers is developed in exactly the same way the powers of 10 were shown for the decimal system. From 1 to 10 decimal is (in binary) 01, 10, 11, 100, 101, 110, 111, 1000, 1001, 1010, as in Table 2-5.

Table 2-5. Decimal 1 Through 10 in Binary

Decimal	Binary			
	2^3	2^2	2^1	2^0
1	0	0	0	1
2	0	0	1	0
3	0	0	1	1
4	0	1	0	0
5	0	1	0	1
6	0	1	1	0
7	0	1	1	1
8	1	0	0	0
9	1	0	0	1
10	1	0	1	0

For the binary system the general expression for any number N is:

$$N_2 = A_n 2^n + A_{n-1} 2^{n-1} + \cdots + A_1 2^1 + A_0 2^0$$

Referring to Table 2-5, we have:

$$\begin{aligned} 010111_2 &= 1 \times 2^4 + 0 \times 2^3 + 1 \times 2^2 \\ &\quad + 1 \times 2^1 + 1 \times 2^0 \\ &= 16 + 0 + 4 + 2 + 1 \\ &= 23_{10} \end{aligned}$$

The basic technique of expression is the same for binary as for decimal; 48_{10} means 4 tens plus 8 ones. In binary this is 110000, or one decimal 32 plus one decimal 16. Table 2-6 shows the binary representation of $2{,}469_{10}$; this is the sum of 2,048, 256, 128, 32, 4, and 1.

Binary numbers are powers of 2. Table 2-7 shows the values of positive and negative exponents of 2 from 0 to 500. Just as 10^{-1} is 0.1, so 2^{-1} is 0.5.

A negative exponent has this significance:

$$10^{-1} = \frac{1}{10^1} = \frac{1}{10} = 0.1$$

or:

$$10^{-2} = \frac{1}{10^2} = \frac{1}{100} = 0.01$$

For the binary system, this is:

$$2^{-1} = \frac{1}{2^1} = \frac{1}{2} = 0.5$$

or:

$$2^{-2} = \frac{1}{2^2} = \frac{1}{4} = 0.25$$

or:

$$2^{-3} = \frac{1}{2^3} = \frac{1}{8} = 0.125$$

To express a digit in the binary system or a bit only, a single indicator is needed in either the *on* (or 1 state) or *off* (which is the 0 state). Binary numbers are used for computation in a great many digital machines. The input data are converted to binary, usually in a coded system, the computations are made in the binary system, and the output data are converted from binary to the decimal system, for use external to the machine.

Binary Arithmetic

Binary numbers require very few rules and no tables for arithmetic operations. One advantage of the binary notation is the simplicity of operations.

Table 2-6. Binary Representation of $2{,}469_{10}$

2^{11} 2048	2^{10} 1024	2^9 512	2^8 256	2^7 128	2^6 64	2^5 32	2^4 16	2^3 8	2^2 4	2^1 2	2^0 1
1	0	0	1	1	0	1	0	0	1	0	1

Table 2-7. Powers of 2

n	2^n	2^{-n}
1	2	.50000 00000 x 10^{0}
2	4	.25000 00000 x 10^{0}
3	8	.12500 00000 x 10^{0}
4	16	.62500 00000 x 10^{-1}
5	32	.31250 00000 x 10^{-1}
6	64	.15625 00000 x 10^{-1}
7	128	.78125 00000 x 10^{-2}
8	256	.39062 50000 x 10^{-2}
9	512	.19531 25000 x 10^{-2}
10	1,024	.97656 25000 x 10^{-3}
11	2,048	.48828 12500 x 10^{-3}
12	4,096	.24414 06250 x 10^{-3}
13	8,192	.12207 03125 x 10^{-3}
14	16,384	.61035 15625 x 10^{-4}
15	32,768	.30517 57812 x 10^{-4}
16	65,536	.15258 78906 x 10^{-4}
17	131,072	.76293 94531 x 10^{-5}
18	262,144	.38146 97266 x 10^{-5}
19	524,288	.19073 48633 x 10^{-5}
20	1,048,576	.95367 43164 x 10^{-6}
21	2,097,152	.47683 71582 x 10^{-6}
22	4,194,304	.23841 85791 x 10^{-6}
23	8,388,608	.11920 92896 x 10^{-6}
24	16,777,216	.59604 64478 x 10^{-7}
25	33,554,432	.29802 32239 x 10^{-7}
26	67,108,864	.14901 16119 x 10^{-7}
27	134,217,728	.74505 80597 x 10^{-8}
28	268,435,456	.37252 90298 x 10^{-8}
29	536,870,912	.18626 45149 x 10^{-8}
30	1,073,741,824	.93132 25746 x 10^{-9}
35	3,435,973,837 x 10^{1}	.29103 83046 x 10^{-10}
40	1,099,511,628 x 10^{3}	.90949 47018 x 10^{-12}
45	3,518,437,209 x 10^{4}	.28421 70943 x 10^{-13}
50	1,125,899,907 x 10^{6}	.88817 84197 x 10^{-15}
60	1,152,921,505 x 10^{9}	.86736 17380 x 10^{-18}
70	1,180,591,621 x 10^{12}	.84703 29473 x 10^{-21}
80	1,208,925,820 x 10^{15}	.82718 06126 x 10^{-24}
90	1,237,940,039 x 10^{18}	.80779 35669 x 10^{-27}
100	1,267,650,600 x 10^{21}	.78886 09052 x 10^{-30}
125	4,253,529,586 x 10^{28}	.23509 88702 x 10^{-37}
150	1,427,247,693 x 10^{36}	.70064 92322 x 10^{-45}
175	4,789,048,565 x 10^{43}	.20880 97430 x 10^{-52}
200	1,606,938,044 x 10^{51}	.62230 15278 x 10^{-60}
300	2,037,035,976 x 10^{81}	.49090 93465 x 10^{-90}
400	2,582,249,878 x 10^{111}	.38725 91915 x 10^{-120}
500	3,273,390,608 x 10^{141}	.30549 36363 x 10^{-150}

Addition has three rules. These are illustrated below:

$$\begin{array}{r}0\\ \underline{0}\\ 0\end{array}\quad\begin{array}{r}0\\ \underline{1}\\ 1\end{array}\quad\text{or}\quad\begin{array}{r}1\\ \underline{0}\\ 0\end{array}\quad\begin{array}{r}1\\ \underline{1}\\ 10\end{array}$$

When these rules are followed, any two binary numbers may be added directly. For example, to add decimal 12 and decimal 5:

$$\begin{array}{r}1100\\ \underline{101}\\ 10001\end{array}$$

Also, for example, to add 01011 and 00110:

$$\begin{array}{r}01011\\ \underline{00110}\\ 10001\end{array}$$

Fractions (negative exponents of 2) are treated in the same way:

$$\begin{array}{rcl}10.11 &=& 2.75_{10}\\ \underline{1.01} &=& \underline{1.25_{10}}\\ 100.00 &=& 4.00_{10}\end{array}$$

For *multiplication* there are four rules which reduce to two: 1 times 1 is 1, and all other combinations are zero:

$$\begin{array}{r}1\\ \underline{\times 1}\\ 1\end{array}\qquad\begin{array}{r}0\\ \underline{\times 0}\\ 0\end{array}\qquad\begin{array}{r}1\\ \underline{\times 0}\\ 0\end{array}\qquad\begin{array}{r}0\\ \underline{\times 0}\\ 0\end{array}$$

Table 2-8. Binary Multiplication

		Multiplicand Digit	
	×	0	1
Multiplier Digit	0	0	0
	1	0	1

The binary multiplication table is shown in Table 2-8.

Because 0 and 1 are the only alternatives for the multiplier digit, either 0 or the multiplicand itself is used in forming the partial product. The partial products are added to provide the final product.

An example of binary multiplication would be 26 multiplied by 19:

$$\begin{array}{lr}
\qquad\qquad \textit{Decimal} & \textit{Binary} \\
26 = 16 + 8 + 0 + 2 + 0 = & 11010 \\
19 = 16 + 0 + 0 + 2 + 1 = & 10011 \\
\cline{2-2}
 & 11010 \\
 & 11010 \\
 & 00000 \\
 & 00000 \\
 & 11010 \\
\cline{2-2}
 & 111101110
\end{array}$$

Interpreting the binary result of the multiplication by using the ones, twos, fours system:

$$256 + 128 + 64 + 32 + 0 + 8 + 4 + 2 + 0 = 494$$

The product will be obtained by a series of adding the multiplicand and shifting whenever a 1 is found in the multiplier. For example, multiply 1011 by 0101:

$$\begin{array}{rl}
1011 & = 11 \\
0101 & = 5 \\
\cline{1-1}
1011 & \\
1011 & \\
\cline{1-1}
110111 & = 55
\end{array}$$

To multiply decimal 12 and 5:

$$\begin{array}{rl}
1100 & = 12 \\
101 & = 5 \\
\cline{1-1}
1100 & \\
1100 & \\
\cline{1-1}
111100 & = 60
\end{array}$$

With binary multiplication there are no carries (except in adding partial products), and every product is equal to 0 except for 1 × 1, which is 1. Multiplication is a series of additions, and multiplying 14 by 8 in decimal is the same as adding 14 eight times. In binary, 10101 can be multiplied by 101, as shown below:

$$\begin{array}{r}
10101 \\
101 \\
\hline
10101 \\
00000 \\
10101 \\
\hline
1101001
\end{array}$$

This multiplication is a series of shifts and additions. The 101 is a multiplier; as in the preceding example, a 1 is an addition and shift left; the 0 is a shift but no addition.

Shifting right or left is just a form of multiplication. Starting with 101100, a shift left is 1011000, or twice 101100. Dividing by 2 is a shift right, or 10110:

$$\begin{aligned} 1011000 &= 88 \\ 101100 &= 44 \\ 10110 &= 22 \end{aligned}$$

Table 2-9 is a tabular summary of addition and multiplication.

Division is the inverse of multiplication. In decimal division, we subtract the divisor from appropriate orders of the dividend, and to build the quotients, count the number of subtractions that do not produce a negative remainder. In binary division only one subtraction is necessary for each quotient digit. If it produces a positive remainder, then the quotient digit is 1. If it produces a negative remainder, the quotient digit is 0. The counting process is not required.

Consider the following example using the binary system to divide decimal 45 by decimal 9. Then we have 101101 ÷ 1001 = 101, or:

$$\begin{array}{r} 101 \\ 1001 \,\overline{)\,101101} \end{array}$$

			Quotient Digit
	101101		
Subtract	$\underline{100100}$		
	001001		1st = 1
Shift left	010010		
Subtract	$\underline{100100}$		
	101100	Negative	2nd = 0
Restore Add	$\underline{100100}$		
	010010		
Shift left	100100		
Subtract	$\underline{100100}$		
	000000		3rd = 1

Table 2-9. Binary Operations

Addition					Multiplication			
	+	0	1		×	0	1	
	0	0	1		0	0	0	
	1	1	10		1	0	1	

Two further examples are:

```
          110
  101 | 11110
        101
        ---
         101
         101
         ---
          00
```

and:

```
          101.11
  101 | 11101.00
        101
        -------
         1001
          101
          ----
          1000
           101
           ----
            110
            101
            ---
              1 Remainder
```

Table 2-10 shows a comparison of base 2 and bases 3, 4, 5, and 10 for addition and multiplication.

Table 2-10. Comparison of Bases for Addition and Multiplication

Decimal	Base 2	Base 3	Base 4	Base 5
Conversion				
54 23	110110 10111	2000 212	312 113	204 43
Addition				
54 (+)23 77	110110 10111 (carries) 1 1 1001101	2000 212 2212	312 113 1 431	204 43 11 302
Multiplication				
54 (+)23 162 108 1 (carry) 1242	1 1 0 1 1 0 1 0 1 1 1 1 1 0 1 1 0 1 1 0 1 1 0 1 1 0 1 1 0 0 0 0 0 0 0 1 1 0 1 1 0 1 1 1 10 10 1 1 1 (carries) 1 0 0 1 1 0 1 1 0 1 0	2000 212 1000 2000 11000 1 (carry) 1201000	312 113 2202 312 312 11 (carries) 103122	204 43 1122 1331 14432

Octal Notation

The octal number system is useful as a shorthand notation of the binary number system because eight is an integral power of two ($8 = 2^3$). One octal digit is always equal to three binary digits (and vice versa). Since a long string of ones and zeros cannot be effectively transmitted from one individual to another, some shorthand method is desirable.

The octal number system fills this need. Because of its simple relationship to binary, numbers can be converted from one system to another by inspection. The base or radix of the octal system is 8. This means there are eight symbols: 0, 1, 2, 3, 4, 5, 6, and 7. Notice that there are no 8's or 9's in this number system. The important relationship to remember is that three binary positions are equivalent to one octal position. Table 2-11 shows the equivalent of decimal numbers from 0 to 25 in binary and octal notation. This is a table that may be used when working in octal notation on a computer.

Table 2-11. Octal and Binary Equivalents

Decimal	Octal	Binary	Decimal	Octal	Binary
0	0	000	13	15	001101
1	1	001	14	16	001110
2	2	010	15	17	001111
3	3	011	16	20	010000
4	4	100	17	21	010001
5	5	101	18	22	010010
6	6	110	19	23	010011
7	7	111	20	24	010100
8	10	001000	21	25	010101
9	11	001001	22	26	010110
10	12	001010	23	27	010111
11	13	001011	24	30	011000
12	14	001100	25	31	011001

Beginning at the binary point, mark off the binary number into groups of three digits. Then replace each group with its octal equivalent. For example, convert 10110011 to decimal.

010	110	011	*Binary*
2	6	3	*Octal*
(2×8^2)	(6×8^1)	(3×8^0)	*Decimal*

Hence this is decimal 179, or octal 263.

A carry is also required in octal, for 001000_2 is 10_8 (since there is no 8 or 9 in octal). Thus:

$$\begin{aligned} 001001_2 &= 11_8 \\ 001010_2 &= 12_8 \\ 001011_2 &= 13_8 \\ 001100_2 &= 14_8 \end{aligned}$$

The octal numbering system to the base 8 is closely related to the binary system. Table 2-12 indicates the possibilities using octal addition. Thus, to add 7_8 and 1_8, we have:

$$\begin{array}{rcr} 7_8 & = & 111 \\ 1_8 & = & 001 \\ \hline 10_8 & = & 1000 \end{array}$$

Thus, in the octal system:

$$\begin{aligned} 1 + 1 &= 2 \\ 2 + 2 &= 4 \\ 3 + 3 &= 6 \\ 4 + 4 &= 10 \\ 5 + 5 &= 12 \\ 6 + 6 &= 14 \\ 7 + 7 &= 16 \end{aligned}$$

Binary, octal, and decimal notations may be compared as in the following:

$$\begin{array}{rrr} \textit{Binary} & \textit{Decimal} & \textit{Octal} \\ 010 & 2 & 2 \\ \underline{111} & \underline{7} & \underline{7} \\ 1001 & 9 & 11 \\ & & \\ 101 & 5 & 5 \\ \underline{110} & \underline{6} & \underline{6} \\ 1011 & 11 & 13 \end{array}$$

For example, to add 321_8 and 405_8 we have:

$$\begin{array}{ccc} 3 & 2 & 1 \\ 4 & 0 & 5 \\ \hline 7 & 2 & 6 \end{array}$$

Table 2-12. Octal Addition

+	0	1	2	3	4	5	6	7
0	0	1	2	3	4	5	6	7
1	1	2	3	4	5	6	7	10
2	2	3	4	5	6	7	10	11
3	3	4	5	6	7	10	11	12
4	4	5	6	7	10	11	12	13
5	5	6	7	10	11	12	13	14
6	6	7	10	11	12	13	14	15
7	7	10	11	12	13	14	15	16

Or, to add 767 and 654 in the octal system, we have:

$$\begin{array}{rrr} 7 & 6 & 7 \\ 6 & 5 & 4 \\ \hline 16 & 4 & 3 \end{array}$$

This octal system can be used on special desk calculators. Adding 321$_8$ and 405$_8$ in binary would be as follows:

$$\begin{array}{ccccccc} 011 & 010 & 001 & = & 3 & 2 & 1 \\ 100 & 000 & 101 & = & 4 & 0 & 5 \\ \hline 111 & 010 & 110 & = & 7 & 2 & 6 \end{array}$$

Notice that the translation from binary to octal is quite simple, as 111 is 7$_8$, 010 is 2$_8$, and 110 is 6$_8$.

Table 2-13 shows octal multiplication. Perhaps an example of this can best be seen by steps: consider 457 multiplied by 263:

$$\begin{array}{rcrcr} 457 & \quad & 457 & \quad & 457 \\ 3 & & 6 & & 2 \\ \hline 25 & & 52 & & 16 \\ 17 & & 36 & & 12 \\ 14 & & 30 & & 10 \\ \hline 1615 & & 3432 & & 1136 \end{array}$$

Table 2-13. Octal Multiplication

×	0	1	2	3	4	5	6	7
0	0	0	0	0	0	0	0	0
1	0	1	2	3	4	5	6	7
2	0	2	4	6	10	12	14	16
3	0	3	6	11	14	17	22	25
4	0	4	10	14	20	24	30	34
5	0	5	12	17	24	31	36	43
6	0	6	14	22	30	36	44	52
7	0	7	16	25	34	43	52	61

By addition of these partial products we have:

```
  1615
 3432
1136
------
151735
```

Thus, while the digital computer itself uses the binary system of numbering, the octal system can be used by a human operator (with or without a desk calculator) to check on the operation of the computer during the testing of a new program. Conversion from the binary (computer language) to octal system, which can be used by the human operator, is direct and simple. Conversion from decimal to binary is the last remaining step needed for going in any direction between decimal, binary, and octal.

NUMBER CONVERSIONS

For simplification, numbering conversions are frequently made, including binary, octal and decimal. There are five different number conversions in use:

1. Decimal to Octal
2. Octal to Decimal
3. Octal to Binary (Binary to Octal)
4. Binary to Decimal
5. Decimal to Binary

Decimal-to-Octal Conversion

Decimal integers can be converted to octal by successively dividing the integer by 8, and noting the remainders. The remainders are ignored in the following divisions, and the divisions are continued until the decimal integer is reduced to zero. The remainders are then combined to form the octal equivalent. Here is an example:

```
              R
8 | 159       7     ↑
   8 | 19     3     |
      8 | 2   2     |
          0         |
```

Read the answer *up* as 237_8.

The rules are as follows:

1. Divide the original decimal number by 8.

2. This remainder (7) becomes the low order (least significant) digit.
3. The quotient of the first division (19) is divided again by 8.
4. This remainder (2) is the next order digit.
5. When the quotient (2) is less than the divisor, this quotient becomes the highest order digit.

Thus in this case the answer is 237_8, which is equal to 159_{10}.

Decimal fractions can be converted to octal fractions by means of successive multiplications by 8. The integers produced by the multiplication become the octal digits. When the multiplications are continued, the integers are ignored. The multiplications are performed until the fraction becomes zero. Here is an example in converting 0.149_{10} to octal:

$$\begin{array}{c|r}
I & \\
 & .149 \\
 & \underline{8} \\
1 & .192 \\
 & \underline{8} \\
1 & .536 \\
 & \underline{8} \\
4 & .288 \\
 & \underline{8} \\
2 & .304
\end{array}$$

Thus the octal number is .1142+, which is binary .001001100010.

The steps are:

1. Multiply .149 by 8 to get 1.192; consider 1 as the first significant digit.
2. Multiply the .192 by 8 to get 1.536; consider the 1 as the next significant digit.
3. Multiply .536 by 8 to get 4.288; consider 4 as the next digit.
4. In the same way, the last digit is 2.

Table 2-14 shows decimal values from 10^0 to 10^{10} and their octal conversions. Here are some examples:

$$10^0 = 001_8 = 001_2$$
$$10^2 = 144_8 = 001100100_2$$
$$= 100_{10}$$

Table 2-15 shows negative decimal to octal values. Table 2-16 gives some constants that often appear in calculations.

If a decimal fraction cannot be represented exactly as an octal fraction, the multiplication process will never produce a result of 0. In this case, multiplication is performed until a satisfactory approximation is reached. This approximation limit is determined by the computer programmer at the time the program is developed. When the computer reaches this point it will do one of two things: it will stop, or it will continue with the next step.

Octal-to-Decimal Conversion

Octal integers can be converted to decimal integers by multiplying each octal digit by the correct power of 8. The lowest order digit is multiplied by 8^0, the next higher order digit by 8^1, the next by 8^2, and so on. When the products are added together, the equivalent decimal integer is obtained. For example, consider 143_8:

Table 2-14. Decimal Values and Octal Conversions

DECIMAL	OCTAL
10^0	+ 000000 000001
10^1	+ 000000 000012
10^2	+ 000000 000144
10^3	+ 000000 001750
10^4	+ 000000 023420
10^5	+ 000000 303240
10^6	+ 000003 641100
10^7	+ 000046 113200
10^8	+ 000575 360400
10^9	+ 007346 545000
10^{10}	+ 112402 762000

$$\begin{aligned} 143_8 &= 1(8^2) + 4(8^1) + 3(8^0) \\ &= 1(64) + 4(8) + 3(1) \\ &= 64 + 32 + 3 \\ &= 99_{10} \end{aligned}$$

There is a short cut, which is the following series of steps:

1. Multiply the highest order digit by 8.
2. Add the next highest order digit.
3. Multiply this sum by 8.
4. The process is completed when the lowest order digit has been added.

In the preceding example of 143_8:

$$\begin{array}{rl} 1 & \text{the highest order digit (1)} \\ \underline{8} & \\ 8 & \text{multiplied by 8.} \\ \underline{4} & \\ 12 & \text{the next highest order digit (4) is added.} \\ \underline{8} & \\ 96 & \text{the sum (12) is multiplied by 8.} \\ \underline{3} & \\ 99 & \text{the final, or lowest order digit (3) is added.} \end{array}$$

$$143_8 = 99_{10}$$

For octal numbers less than 1_8, the highest order digit (i.e., the digit nearest the point) is multiplied by 8^{-1}, the next highest order by 8^{-2}, the next by 8^{-3}, and so on. When the products are added together, the equivalent decimal fraction is obtained. As an example, consider $.1142_8$:

$$\begin{aligned} .1142_8 &= 1(8^{-1}) + 1(8^{-2}) + 4(8^{-3}) + 2(8^{-4}) \\ &= \frac{1}{8} + \frac{1}{64} + \frac{4}{512} + \frac{2}{4096} \\ &= \frac{610}{4096} \\ &= .1489_{10} \end{aligned}$$

Conversions are given in Table 2-17 for $\frac{1}{n}$ in octal and decimal for the range of n from 2 to 30. From Table 2-17, we can see that:

$$\begin{aligned} 0.5_{10} &= 0.4_8 \\ 0.25_{10} &= 0.2_8 \\ 0.125_{10} &= 0.1_8 \end{aligned}$$

For assistance in conversions several appendices are included. Appendix III overs integers for octal-to-decimal conversions from 00000 octal (which is 00000 decimal). Appendix II covers fractional decimal-to-octal conversions for decimal values from 0.000 to 0.999; it provides octal values with factors of 1, 10^{-3}, and 10^{-6}.

Binary–Octal Conversion

This process is performed almost by inspection and is quite simple. The binary number (for binary to octal) is divided into groups of three bits, starting from the right, and then

Table 2-15. Negative Powers of 10 (Octal)

10^{-1}	.06314	63146	31463	14631	46314	63146
10^{-2}	.00507	53412	17270	24365	60507	53412
10^{-3}	.00040	61115	64570	65176	76355	44264
10^{-4}	.00003	21556	13530	70414	54512	75170
10^{-5}	.00000	24761	32610	70664	36041	06077
10^{-6}	.00000	02061	57364	05536	66151	55323
10^{-7}	.00000	00153	27745	15274	53644	12742
10^{-8}	.00000	00012	57143	56106	04303	47375
10^{-9}	.00000	00001	04560	27640	46655	12263
10^{-10}	.00000	00000	06676	33766	35367	55653
10^{-11}	.00000	00000	00537	65777	02745	44453
10^{-12}	.00000	00000	00043	13631	40226	75204
10^{-13}	.00000	00000	00003	41134	11502	22732
10^{-14}	.00000	00000	00000	26411	15606	50226
10^{-15}	.00000	00000	00000	02200	72763	67165
10^{-16}	.00000	00000	00000	00163	22545	13731
10^{-17}	.00000	00000	00000	00013	41675	24311
10^{-18}	.00000	00000	00000	00001	11622	73507
10^{-19}	.00000	00000	00000	00000	07301	71124
10^{-20}	.00000	00000	00000	00000	00571	62410
10^{-21}	.00000	00000	00000	00000	00045	61664
10^{-22}	.00000	00000	00000	00000	00003	61622
10^{-23}	.00000	00000	00000	00000	00000	30133
10^{-24}	.00000	00000	00000	00000	00000	02326
10^{-25}	.00000	00000	00000	00000	00000	00174
10^{-26}	.00000	00000	00000	00000	00000	00014
10^{-27}	.00000	00000	00000	00000	00000	00001

each three-bit number is converted into its octal equivalent. For example, the binary number 101010 is converted to its octal equivalent as follows:

$$\begin{array}{cc} 1\ 0\ 1 & 0\ 1\ 0 \\ (2^2 + 1) & (2^1) \\ = 5 & = 2 \end{array}$$

$$= 52 \text{ (octal)}$$

Table 2-16. Octal Representation for Common Constants

Miscellaneous Constants	Decimal	Octal
$\sqrt{2}$	1. 414 213 562 4	1. 324 047 463 201
$\sqrt{3}$	1. 732 050 807 6	1. 566 636 564 132
$\sqrt{5}$	2. 236 067 977 5	2. 170 673 633 460
$\sqrt{6}$	2. 449 489 742 8	2. 346 107 024 023
$\sqrt{7}$	2. 645 751 311 1	2. 512 477 651 650
$\sqrt{8}$	2. 828 427 124 8	2. 650 117 146 402
$\sqrt{10}$	3. 162 277 660 2	3. 123 054 072 667
π	3. 141 592 653 6	3. 110 375 524 211
2π	6. 283 185 307 1	6. 220 773 250 413
$1/\ \pi$	. 318 309 886 2	. 242 763 015 564
$1/2\pi$	. 159 154 943 1	. 121 371 406 672
1° = 1/360 of a circle	. 002 777 777 8	. 001 330 133 015
e	2. 7 8 281 828 5	2. 557 605 213 053
1/e	. 367 879 441 2	. 274 265 306 615
$\log_{10}e$	. 434 294 481 9	. 336 267 542 512
$\log_e 10$	2. 302 585 093 0	2. 232 730 673 533
$\log_e 2$	. 693 147 180 6	. 542 710 277 600
$\log_{10}\pi$	. 497 149 872 7	. 376 424 666 307
$\log_e\pi$	1. 144 729 885 8	1. 112 064 044 344

Octal-to-binary conversions are made by simply reversing the process. Each digit of the octal number is replaced by its three-bit binary equivalent. Thus, the octal number 725 is converted to its binary equivalent:

$$\begin{array}{ccc} 7 & 2 & 5 \\ (111) & (010) & (101) \end{array}$$

$$= 111010101 \text{ (binary)}$$

Fractions are treated in the same way:

$$.001 \quad 001 \quad 100 \quad 010 \quad = \quad .1142_8$$

Binary-to-Decimal Conversion

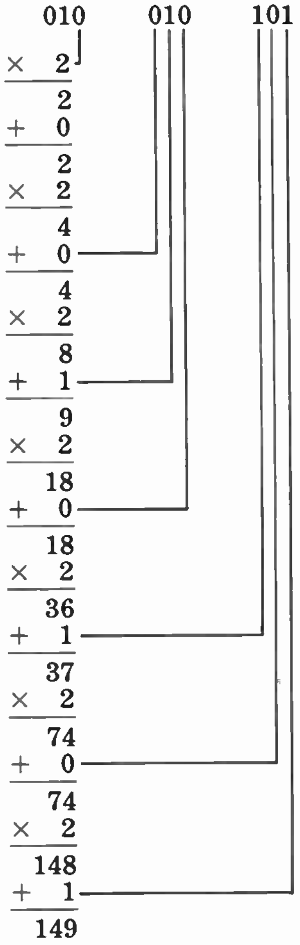
There are two methods for this conversion. In one, binary integers can be converted to decimal numbers by means of successive multiplications and additions. First, the highest order bit is multiplied by 2. Then the product is added to the next lower order bit and this sum multiplied by 2. The new product is then added to the next lower order bit and the re-

Table 2-17. Octal-to-Decimal Conversion

n	1/n (octal)	1/n (decimal)
2	.400000000000	.5000000000
3	.252525252525	.3333333333
4	.200000000000	.2500000000
5	.146314631463	.2000000000
6	.125252525252	.1666666667
7	.111111111111	.1428571428
8	.100000000000	.1250000000
9	.070707070707	.1111111111
10	.063146314631	.1000000000
11	.056427213505	.0909090909
12	.052525252525	.0833333333
13	.047304730473	.0769230769
14	.044444444444	.0714285714
15	.042104210421	.0666666667
16	.040000000000	.0625000000
17	.036074170360	.0588235294
18	.034343434343	.0555555555
19	.032745032745	.0526315789
20	.031463146314	.0500000000
21	.030303030303	.0476190476
22	.027213505642	.0454545454
23	.026205441310	.0434782609
24	.025252525252	.0416666667
25	.024365605075	.0400000000
26	.023542354235	.0384615384
27	.022755022755	.0370370370
28	.022222222222	.0357142857
29	.021517345410	.0344827586
30	.021042104210	.0333333333

sultant sum multiplied by 2. This multiplication and addition is continued bit by bit to the right until the lowest order bit is added. No multiplication follows the adding of the lowest order bit because this is in the units column. The resulting sum is the decimal equivalent of the binary integer. An example is given below:

Convert 010010101 to its decimal equivalent:



OR, 10010101

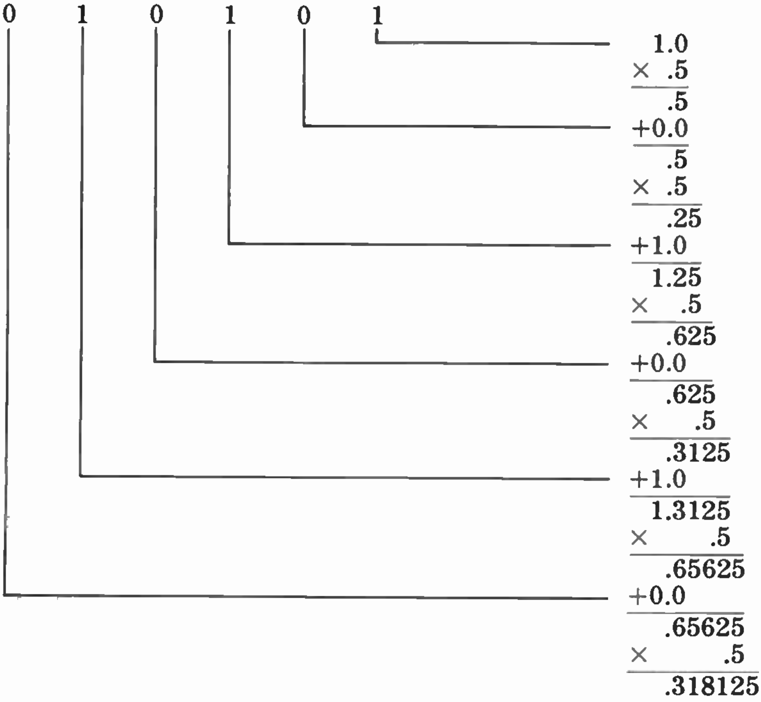
$$= 1(2^7) + 0(2^6) + 0(2^5) + 1(2^4) + 0(2^3) + 1(2^2) + 0(2^1) + 1(2^0)$$

$$= 128 + 16 + 4 + 1$$

$$= 149_{10}$$

Binary fractions can be converted to decimal fractions by means of successive multiplications by 0.5 and by the performance of bit additions.

An example for the conversion of .010101 is shown below:



```
0     1     0     1     0     1
                                      1.0
                                    ×  .5
                                       .5
                                     +0.0
                                       .5
                                    ×  .5
                                       .25
                                     +1.0
                                      1.25
                                    ×   .5
                                       .625
                                     +0.0
                                       .625
                                    ×    .5
                                       .3125
                                     +1.0
                                      1.3125
                                    ×      .5
                                       .65625
                                     +0.0
                                       .65625
                                    ×       .5
                                       .318125
```

During the conversions, however, each bit is reduced to its correct power of 2 because of the successive multiplications by 0.5. The conversion is begun at the extreme right-hand bit. This bit is multiplied by 0.5, and the resultant product is added to the next higher-order bit. This sum is then multiplied by 0.5, and the new product is added to the next higher-order bit. This multiplication and the addition is continued bit by bit to the left until the highest-order fraction bit is added. This digit is then multiplied by 0.5. The resultant sum is the decimal equivalent of the binary fraction.

Fractions may also be converted as in the following example for .001001100010:

$$
\begin{aligned}
.001001100010 &= 1(2^{-3}) + 1(2^{-6}) + 1(2^{-7}) + 1(2^{-11}) \\
&= \frac{1}{8} + \frac{1}{64} + \frac{1}{128} + \frac{1}{2048} \\
&= \frac{305}{2048} \\
&= .1489_{10}
\end{aligned}
$$

Double-Dabble—This is a system for binary-to-decimal conversion. You start with a digit; if the next is a 0, you double what you have; if the next is a 1, you dabble (double plus 1). For example, consider 10101:

1. Start with the MSD (most significant digit), which is equal to 1.
2. Next is 0; double 1, which is 2.
3. Next is 1; dabble 2, which is 5.
4. Next is 0; double 5, which is 10.
5. Next is 1; dabble 10 which is 21.

Thus $10101_2 = 21_{10}$.

Decimal-to-Binary Conversion

Decimal integers can be converted to binary numbers by successively dividing the integer by 2. After each division, the 0 or 1 remainder is recorded. In succeeding divisions the remainders are ignored and the divisions are continued until zero is obtained. The remainders are the resultant binary number when they are combined in an order such that the first remainder is the lowest-order binary digit, the second remainder the next higher-order digit, etc.

For example, convert 270_{10} to its binary equivalent:

```
2|270          Remainder 0   ↑  Read
 2|135         Remainder 1   |  Up
  2|67         Remainder 1   |
   2|33        Remainder 1   |
    2|16       Remainder 0   |
     2|8       Remainder 0   |
      2|4      Remainder 0   |
       2|2     Remainder 0   |
        2|1    Remainder 1   |
           0
```

Thus, $270_{10} = 100001110_2$.

Decimal fractions can be converted to binary fractions by successively multiplying the fraction by 2. A 0 is recorded each time the product remains smaller than 1; a 1 is recorded each time the product exceeds 1. The integer portion of the result is dropped when the multiplication is continued. This multiplication process is continued until the answer becomes exactly 1. The bits are then combined to form the binary fraction. For example, consider $.149_{10}$:

$$\begin{array}{c c|r}
\text{Read} & & .149 \\
\text{Down} & & \times\ 2 \\ \cline{3-3}
\Bigg\downarrow & 0 & .298 \\
& & \times\ 2 \\ \cline{3-3}
& 0 & .596 \\
& & \times\ 2 \\ \cline{3-3}
& 1 & .192 \\
& & \times\ 2 \\ \cline{3-3}
& 0 & .384 \\
& & \times\ 2 \\ \cline{3-3}
& 0 & .768 \\
& & \times\ 2 \\ \cline{3-3}
& 1 & .536 \\
& & \times\ 2 \\ \cline{3-3}
& 1 & .072 \\
& & \times\ 2 \\ \cline{3-3}
& 0 & .144 \\
& & \times\ 2 \\ \cline{3-3}
& 0 & .288 \\
& & \times\ 2 \\ \cline{3-3}
& 0 & .576 \\
& & \times\ 2 \\ \cline{3-3}
& 1 & .152 \\
& & \times\ 2 \\ \cline{3-3}
\downarrow & 0 & .304
\end{array}$$

Thus, $.149_{10} = .001\ 001\ 100\ 010\ +$

CHAPTER 3

Numerical Operations

As we have seen, digital computers use binary digits during manipulation in the computer to produce the basic operations of addition, subtraction, multiplication, and division. The representation we have discussed thus far has been "pure binary." That is, the ones and zeros making up a number have a place-value that represents some power of two. However, in many instances the pure binary system is not practical for computers to implement.

CODED SYSTEMS

Obviously, a pure binary system would require the manipulation of large digital quantities, both in and out of the computer. For instance, the decimal number 147 expressed in pure binary is 10010011. Various systems have therefore been devised to reduce this manipulation of large notations. These systems are essentially coded systems, and we have within these binary-coded decimal codes the excess-three code (shortened to XS-3), the 8421 and 7421 codes, and the Gray code. There are also special error-detecting codes and others which have been devised.

Binary-Coded Decimal (BCD) Systems

Let us consider the coding of the preceding example, 147. If we represent each individual decimal digit in binary but keep their decimal values regarding position, we have:

0001	0100	0111
1	4	7

With computers operating in coded decimal, the information can be placed in or removed from the computer directly. But if

Table 3-1. Decimal, Binary, and BCD Notations

Decimal Notation	Binary Notation	Binary-Coded Decimal Notation	
00	00000	0000	0000
01	00001	0000	0001
02	00010	0000	0010
03	00011	0000	0011
04	00100	0000	0100
05	00101	0000	0101
06	00110	0000	0110
07	00111	0000	0111
08	01000	0000	1000
09	01001	0000	1001
10	01010	0001	0000
11	01011	0001	0001
12	01100	0001	0010
13	01101	0001	0011
14	01110	0001	0100
15	01111	0001	0101
16	10000	0001	0110
17	10001	0001	0111
18	10010	0001	1000
19	10011	0001	1001
20	10100	0010	0000

we were using pure binary, this would have to be converted to decimal form. Although there are a number of inefficiencies in using them, coded forms are very easily converted by the computer operator or programmer.

A *weighted code* is one in which each binary digit contributes a specific decimal value to the number. A *non-weighted code* is a code in which a constant decimal value cannot be assigned to each binary digit position. In the first case, the 8421 code is a weighted code because each digit has a decimal value of 8, 4, 2 or 1. An example of a non-weighted code is the XS-3 code, which will be described.

In Table 3-1, each decimal digit is given a binary-coded decimal representation (the binary notation is also shown). You will note that zero is represented as all zeros. This is a disadvantage in either pure binary or BCD, since there is no clear-cut definition of the difference between true zero and a failure.

XS-3 Code—In the XS-3 code each number is represented by a binary number that is three more than its actual value.

Table 3-2. Binary to XS-3 Code

BINARY	XS-3	
00	0011	0011
01	0011	0100
02	0011	0101
03	0011	0110
04	0011	0111
05	0011	1000
06	0011	1001
07	0011	1010
08	0011	1011
09	0011	1100
10	0100	0011
11	0100	0100
12	0100	0101
13	0100	0110
14	0100	0111
15	0100	1000
16	0100	1001
17	0100	1010
18	0100	1011
19	0100	1100
20	0101	0011

This permits direct subtraction using complements, which will be described later. In the XS-3 code, binary 0011 (3_{10}) is added to each number to produce the code. Table 3-2 shows this code. To subtract 3, which is 0110, from 1100, or 9, the 0110 is changed to 1001, which is 6. In the same manner, 2 or 0101 from 9 is 7 or 1010. This is a simplification of computer arithmetic.

The 7421 Code—In addition to the XS-3 code already mentioned, there is the 7421 code in which the fourth digit represents 7 rather than 8. This simple change, it turns out, never

Table 3-3. Relationship of 7421 and 8421 Codes

Decimal	7421 Code	8421 Code
0	0000	0000
1	0001	0001
2	0010	0010
3	0011	0011
4	0100	0100
5	0101	0101
6	0110	0110
7	1000	0111
8	1001	1000
9	1010	1001

requires more than two ones for any number up to 9. Thus it requires less power consumption than pure binary. Table 3-3 shows this code.

Biquinary Code—This code has been used in a number of computers, notably the IBM 650. It has an advantage in providing easy error detection. Each valid digit has only two binary ones, with one bit in the "bi" portion and one in the "quinary" part. Therefore all single- and triple-bit errors in each part are detected, as well as many double-bit errors. In the IBM 650, the biquinary code is converted to a five-level code for storage. Table 3-4 shows this code.

Table 3-4. Decimal to Biquinary Code

Decimal	Binary	Biquinary
0	0000	01 00001
1	0001	01 00010
2	0010	01 00100
3	0011	01 01000
4	0100	01 10000
5	0101	10 00001
6	0110	10 00010
7	0111	10 00100
8	1000	10 01000
9	1001	10 10000

Gray Code—This code, also known as the *cyclic code*, or *reflective code*, is used where a conversion of a shaft rotation (analog data) is being converted to digital data. The chief advantage of this code is that only a single binary digit is changed in passing from the representation of one decimal number to the next greater or lesser number.

In the design of analog-to-digital converters it is convenient to change one digit at a time as the shaft rotates, and for this reason the Gray code is widely used for equipment such as shaft encoders or decoders.

In pure binary there are times when only one digit changes, as from 2 to 3 in decimal, which is from 0010 to 0011 in binary; or from 8 to 9 in decimal, which is from 1000 to 1001 in binary. However, there are many other cases where more than one digit changes in binary, as from 9 to 10 (decimal), which is 1001 to 1010; or 1011 to 1100, from 10 to 11. Because of this the Gray code is used, since in this code there is always a change of just one digit in going from one number to the next, higher or lower. Table 3-5 shows the decimal, pure binary, and Gray codes.

By inspection, conversion from binary to the Gray code may be deduced to follow these rules:

1. The binary number is added to itself (shifted or indexed one place to the right).
2. The right-most digit is dropped.
3. All carries are ignored.

For example, 9_{10} is 1001_2. For the Gray code, we have:

$$\begin{array}{r l} 1001 & \\ \underline{1001} & \text{(indexed left)} \\ 1001x & \end{array}$$

where x is the dropped digit.

Thus for decimal 9 the Gray code representation is 1101. In the same way for 11_{10}, we have:

$$\begin{array}{r} 1011 \\ \underline{1011} \\ 1110x \end{array}$$

And so 1110 in the Gray code represents 11 in decimal.

Table 3-5. Decimal to Gray Code

Decimal	Binary	Gray Code
0	0000	0000
1	0001	0001
2	0010	0011
3	0011	0010
4	0100	0110
5	0101	0111
6	0110	0101
7	0111	0100
8	1000	1100
9	1001	1101
10	1010	1111
11	1011	1110
12	1100	1010

Error-Detecting Codes—In general, computers manufactured today provide automatic means to prevent equipment-caused errors from continuing, by stopping the computation, an alarm, or both. However, errors can occur in the control portion of the computer or in the input, and a means must be provided to detect these errors.

One method of error-checking has already been noted: use of the biquinary code. Another, which is very commonly used, is the *parity* (or *odd-even*) *code*. In this method an extra binary digit, called the check digit, is used for each character code. This bit is assigned so that the total number of 1's is always odd. For example, in the case of 0001 nothing is added, and it appears as 0001. For 0011 (3_{10}) a 1 is added, so it appears as 10011 with the first bit being the parity bit.

Thus, by checking each number for an odd number of bits (1, 3, 5, 7, and so on), a check for errors is obtained to prevent dropping a 1.

Table 3-6. Two-out-of-Five Error-Detection Code

Decimal	Two-out-of-Five Code
0	00011
1	00110
2	01100
3	11000
4	10001
5	10010
6	10100
7	01010
8	00101
9	01001

For the same technique with even-parity, 0001 becomes 10001, 0011 is unchanged, 0100 becomes 10100, and so on.

It is also possible to use a two-out-of-five code in which only two of the five digits are 1's, which is a further error check. One type of two-out-of-five code is shown in Table 3-6.

ARITHMETIC OPERATIONS

The ordinary arithmetic operations (addition, subtraction, division, and multiplication) can be straightforward with decimal and with binary numbers. But there are some special cases used by computers. Some of these are: *subtraction by complements; binary complements; fixed radix-point numbers; floating radix-point numbers;* and *shifting.*

Shorthand Decimal Operations

We can examine briefly some of the basic operations with decimal numbers to see how these operations may be approached in different ways.

Division offers several possibilities if it is considered as repeated subtraction. Consider 157 divided by 12. By long division this is:

$$\begin{array}{r} 13 \\ 12\overline{)157} \\ \underline{12} \\ 37 \\ \underline{36} \\ 1 \end{array}$$

But, as repeated subtraction, we have:

$$\begin{array}{rcl} 157 & & \\ \underline{120} & 1 & (12 \times 10) \\ 37 & & \\ \underline{12} & & \\ 25 & 1 & (12 \times 1) \\ \underline{12} & & \\ 13 & 1 & (12 \times 1) \\ \underline{12} & & \\ 1 & = & \text{Remainder} \end{array}$$

As the first step the divisor (12) is multiplied by 10 (or 100 or 1,000 as the case may be) and subtracted from the dividend. Note that 1,200 would be too large to give a positive difference. The first digit of the quotient is 1, and, since 12 was written as 120, the 1 appears as 10.

Next, the 12 is subtracted from 37 as many times as is necessary to obtain a remainder less than 12. In this case, 12 is subtracted 3 times, with a remainder of 1. Thus, the quotient is 10 + 3, or 13, with a remainder of 1.

Consider 10,763 divided by 53; this yields 203 + 4R.

$$\begin{array}{rcl} 10763 & & \\ \underline{5300} & 1 & (53 \times 100) \\ 5463 & & \\ \underline{5300} & 1 & (53 \times 100) \\ 163 & & \\ \underline{53} & 1 & (53 \times 1)^{*} \\ 110 & & \\ \underline{53} & 1 & (53 \times 1) \\ 57 & & \\ \underline{53} & 1 & (53 \times 1) \\ 4 & R & \end{array}$$

* Note that 53 × 10, or 530, is too large; hence, it is not used.

In summation, we obtain:

$$\begin{aligned} 2(53 \times 100) + 3(53 \times 1) + 4R &= 200 + 3 + 4R \\ &= 203 + 4R \end{aligned}$$

Multiplication can also be shortened by using the right-hand digits and left-hand digits with no carries.

Multiplying 97 by 83 in the usual manner gives 8,051. Using the left-hand digits only (such as 2 for 21, 3 for 37, or 4 for 46, for example) one obtains:

$$\begin{array}{rl} 97 & \text{(LHD)} \\ \underline{83} & \\ 22 & \\ \underline{75} & \\ 772 & \end{array}$$

For right-hand digits:

$$\begin{array}{rl} 97 & \text{(RHD)} \\ \underline{83} & \\ 71 & \\ \underline{26} & \\ 331 & \end{array}$$

The final sum is:

$$\begin{array}{r} 331 \\ \underline{772} \\ 8051 \end{array}$$

This product can be written so that all carries are deferred until the final step:

$$\begin{array}{rl} 97 & \\ \underline{83} & \\ 220 & \\ 7500 & \text{LHD} \\ 71 & \\ \underline{260} & \text{RHD} \\ 8051 & \end{array}$$

Consider a further example. In the normal manner of multiplication, we have:

$$\begin{array}{r} 437 \\ \underline{752} \\ 874 \\ 2185 \\ \underline{3059} \\ 328{,}624 \end{array}$$

By left- and right-hand digits, we have:

```
      RHD           LHD

      437           437
      752           752
      ---           ---
      864           001
     055           213
    819           224
    -----         -----
    83314         24531
```

Adding the partial products, we obtain:

```
      83314
     24531
     -------
     328,624
```

COMPLEMENTS

The arithmetic operations of a computer are forms of addition and subtraction, and this function is enhanced by making it possible for the computer to process these operations by using complement arithmetic. Complement arithmetic was mentioned in the discussion on XS-3 codes and is that form of arithmetic that makes it possible to perform subtraction by means of addition, or addition by means of subtraction.

The *complement* of a number is that quantity which, when added to the number, produces a power of the base of the number, or a power of the base minus one. The former is called the *ten's complement* in binary arithmetic. The latter is called the *nine's complement* in decimal arithmetic and the *one's complement* in binary arithmetic. This is more fully explained later.

Counting

Counting is a basic arithmetic operation and is also another way of viewing addition. A counter with two digits starts from 00 and progresses to 01 as the first count, through 99 for a full count, after which it returns to 00. There are 100 possible numbers, hence it has a modulus 100, or 10^2. Thus a counter with modulus 10^5 would count to 99,999, or 10^5-1.

Consider a cyclic counter as in Fig. 3-1 where positive (clockwise) counting, starting from 0, goes in unit steps until 100 is reached, which is the point 0 again. A count of 25 is one-quarter around the circle, 50 is half-way, and so on. Negative counting is done counterclockwise. Thus, on this counter to subtract 35 from 50 (add a negative 35 and positive 50) one would go clockwise 50 units, then counterclockwise from this point 35 units to stop at plus 15. From this, one can see that the outer numbers (positive) and the inner numbers (negative) are complements. For, in every case, we have:

$$| n_1 | + | n_2 | = M$$

where,

n_1 is any inner number,
n_2 is the corresponding outer number,
M is the modulus.

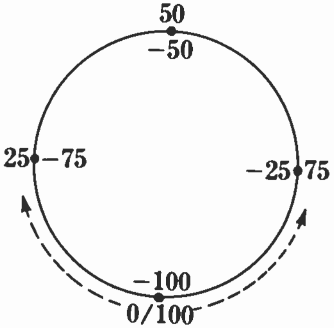


Fig. 3-1. Operation of a cyclic counter.

Counting is related to addition, because addition is an extension of counting where the last possible number is infinity. There is always a finite last number in any usable system, and it depends on the number of columns in the counting device. Consider a counter which counts the 100,000 numbers from 00,000 to 99,999; after 99,999 the counter goes back to 00,000. This counter has a modulus of 100,000 or 10^5, where 5 is the number of columns in the counter.

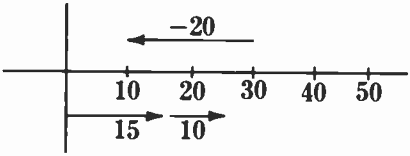


Fig. 3-2. Counting represented linearly.

Counting can be performed in two directions as in Fig. 3-2. If counting to the right is counting forward, counting to the left is counting backward. A count to the right means an addition, a count to the left is subtraction.

A practical counter with a fixed modulus is represented by a circle in which counting counterclockwise may be called forward counting and counting clockwise may be called backward counting. Positive numbers, or forward counting, and the negative numbers, or backward counting, can be placed side by side around the circle. This was shown in Fig. 3-1 where the corresponding numbers are "complements" of each other.

The value of the complement concept is very significant in the performance of subtraction by a mechanized system where subtraction may be performed by reversing the direction of movement of a wheel. By converting negative numbers to their complements it is possible to convert all problems into addition problems for which the wheel always turns in the same direction.

When the modulus is given in the decimal system, the complement is called the ten's complement, and it is defined as that number obtained when each digit is subtracted from 9, and a 1 added to the result. Here is an example showing the ten's complement of 456:

$$\begin{array}{rr} & 999 \\ - & 456 \\ \hline & 543 \\ + & 1 \\ \hline & 544 \end{array}$$

In the binary system there is a two's complement, which is defined as that number obtained when each bit is subtracted from 1 and a 1 added to the result. Here is an example deriving the two's complement of 1010:

$$\begin{array}{rr} & 1111 \\ - & 1010 \\ \hline & 0101 \\ + & 1 \\ \hline & 0110 \end{array}$$

The number obtained by subtracting each digit from 9 is called the nine's complement, and the number obtained by subtracting each bit from 1 is called the one's complement (there is no addition of 1, as is the case for the two's complement). The nine's complement of 456 is therefore 543; and the one's complement of 1010 is 0101. Note that in the binary system, obtaining the one's complement is very easy; an interchange of 0's and 1's is all that is required.

Subtraction By Complements

A complement is a useful quantity for operations such as subtraction. The *true complement* of any quantity in positional notation is that quantity which, when added to the first quantity, gives the least quantity containing one more place. The *base-minus-one's complement* of any quantity is the quantity which, when added to the first quantity, gives the largest quantity containing the same number of places.

For the decimal system the true complement of a one-digit number m is $(10 - m)$. Thus, subtracting 427 from 564:

$$\begin{array}{r} 564 \\ -\ 427 \\ \hline 137 \end{array}$$

The ten's or true complement of 427 is 573, since we have:

$$\begin{array}{r} 573 \\ 427 \\ \hline 1{,}000 \end{array}$$

From the previous definition, 573 when added to 427 gives the smallest number (1) in the fourth place.

Thus, subtracting 427 from 564, we have:

$$\begin{array}{r} 564 \\ -427 \\ \hline 137 \end{array}$$

Complementing 427 and adding to 564 gives:

$$\begin{array}{r} 564 \\ 573 \\ \hline 1137 \end{array}$$

The MSD, or extra 1, is dropped (since the answer is just 1,000 too large).

Consider another example:

$$\begin{array}{r} 392 \\ -141 \\ \hline 251 \end{array} \quad \text{or,} \quad \begin{array}{r} 392 \\ +859 \\ \hline 1{,}251 \end{array} \text{ (which is 251)}$$

The nine's complement is the difference between each digit and 9. Thus, for 451 the nine's complement is 548. Subtracting 451 from 863 yields:

$$\begin{array}{r} 863 \\ -451 \\ \hline 412 \end{array}$$

But, by using nine's complement, we obtain:

$$\begin{array}{r} 863 \\ +548 \\ \hline 1{,}411 \\ \hookrightarrow 1 \\ \hline 412 \end{array}$$

The "extra" 1 is an end-around carry to complete the sum. Table 3-7 shows the decimal complements. Consider now an example using the ten's complement: Subtracting 427 from 564 yields:

$$\begin{array}{r} 564 \\ -427 \\ \hline 137 \end{array}$$

Table 3-7. Decimal Complements

	Decimal	
Digit	True Complement	Nine's Complement
0	10	9
1	9	8
2	8	7
3	7	6
4	6	5
5	5	4
6	4	3
7	3	2
8	2	1
9	1	0

The ten's complement of 427 is 573; and adding this to 564:

$$\begin{array}{r} 564 \\ +573 \\ \hline 1{,}137 \end{array}$$

Thus, the answer is 137, by dropping the extra first digit. The nine's complement of 427 is 572, or just 1 less than the true ten's complement (573). Adding again:

$$\begin{array}{r} 564 \\ +572 \\ \hline 1{,}136 \end{array}$$

But here the extra 1 is added to the 136, which is the sum without the first 1 digit:

$$\begin{array}{r} 136 \\ +1 \\ \hline 137 \end{array}$$

Thus, subtraction is somewhat more complicated than addition; it involves "borrowing" from the next higher column whenever a subtrahend digit is greater than the corresponding minuend digit. But it is possible to obtain the same result by adding the minuend to the complement of the subtrahend and ignoring the final carry.

Note that 4 can be subtracted from 7 in either of the following manners.

By normal subtraction:

$$\begin{array}{r} 0007 \\ -0004 \\ \hline 0003 \end{array}$$

Adding the true complement gives:

$$\begin{array}{r} 0007 \\ +9996 \\ \hline 10{,}003 \end{array}$$

Note that all places, including nonsignificant digits, of the subtrahend must be complemented, and that the resulting sum always exceeds the capacity of the register. When the true complement of the subtrahend is used, the carry from the highest order of the register is ignored. Because the base-minus-one's complement is more easily obtained, subtraction is often accomplished by adding the minuend to the nine's complement of the subtrahend, and performing an end-around carry from the highest order column to the lowest order column. For example, to subtract 4 from 7, add the nine's complement and 1:

$$\begin{array}{r} 0007 \\ +9995 \\ \hline 0002 \\ \llcorner\!\!\longrightarrow 1 \\ \hline 0003 \end{array}$$

Binary Complements—In binary, the nine's complement of 1011 is 0100; this is obtained by interchanging the 1's and 0's. To subtract 1011 from 101101 directly:

$$\begin{array}{r} 101101 \\ -001011 \\ \hline 100010 \end{array}$$

And since the complement of 001011 is 110100, subtraction is also possible by addition:

$$\begin{array}{rl} 101101 & \\ +110100 & \\ \hline 1100001 & \\ +\qquad 1 & \text{(end-around carry)} \\ \hline 100010 & \end{array}$$

Another example is subtracting 0101 from 1011:

$$\begin{array}{r} 1011 \\ -0101 \\ \hline 0110 \end{array}$$

or,

$$\begin{array}{r} 1011 \\ +1010 \\ \hline 10101 \\ +1 \\ \hline 0110 \end{array}$$

Table 3-8 shows the complements of the binary-coded decimal.

Table 3-8. Complements of BCD Numbers

Decimal	Binary-Coded Decimal		
	Digit	True Complement	Nine's Complement
0	0000	1010	1001
1	0001	1001	1000
2	0010	1000	0111
3	0011	0111	0110
4	0100	0110	0101
5	0101	0101	0100
6	0110	0100	0011
7	0111	0011	0010
8	1000	0010	0001
9	1001	0001	0000

Some codes have the advantage of being self-complementing. Each binary digit of any decimal representation is the complement of the corresponding binary digit in the representation of the decimal complement. One of these self-complementing codes is the previously mentioned XS-3 code, in which each decimal digit is represented by the binary equivalent of the digit plus three.

The XS-3 code with its complements is shown in Table 3-9. In XS-3 coding, the binary representations of each decimal place are added in binary fashion. Carries from one decimal place to the next are determined by carries from the highest-order binary place. Because the decimal digits are each expressed in XS-3 code, the sum of two digits will be expressed in XS-6 code, which can be converted back to XS-3 code by subtracting three. Table 3-10 shows the complement pairs of the XS-3 code.

Table 3-9. XS-3 Code and Nine's Complement

Decimal Digit	Excess-Three	Nine's Complement
0	0011	1100
1	0100	1011
2	0101	1010
3	0110	1001
4	0111	1000
5	1000	0111
6	1001	0110
7	1010	0101
8	1011	0100
9	1100	0011

RADIX-POINT OPERATIONS

The significance of the radix must not be lost in numerical operations. In manual arithmetic, the decimal point locates the units' order of a number, and separates the negative and positive powers of the base. For example, in the decimal number 724.32, the quantity to the left of the decimal point equals $(7 \times 10^2) + (2 \times 10^1) + (4 \times 10^0)$; and the right-hand portion: $(3 \times 10^{-1}) + (2 \times 10^{-2})$. This sharp definition is not required for computer use, however, because the power coefficients are placed in a predetermined position, and all operations are maintained on this basis. This is known as *fixed-point representation* and is described under the next heading. It is customary in this type of operation to place the decimal point to the left of the most significant digit, so that *all* numbers in the computer manipulation are considered to have values of less than 1.

In all probability such an arrangement would limit the range of numbers to be used in a computer. Therefore, in program-

Table 3-10. Complement Pairs of XS-3 Code

Decimal	Binary	XS-3
0	0000	0011 ←
1	0001	0100 ←
2	0010	0101 ←
3	0011	0110 ←
4	0100	0111 ←
5	0101	1000 ←
6	0110	1001 ←
7	0111	1010 ←
8	1000	1011 ←
9	1001	1100 ←

ming a problem for the computer, the programmer must (after initially determining the location he will use for the decimal point) position all numbers in correct relationship to the position of the decimal point selected. As long as each step or operation is complete within itself, the decimal-point position can be changed as the program progresses.

The step whereby the decimal point of a number is located so as to be within the capability of a computer is known as *scaling*, which is essentially varying the units or scale that defines the number. Given a decimal number of, say, 500.0, we may also express it as 5×10^2. If we move the position of the decimal point, we also move the position of the power of the base. Therefore, we could also state that the 5 in 50.0 is equal to 5×10^1. In scaling, the digits of the number are moved in direct relationship to some fixed but assumed position of the decimal point. This is the same as multiplying by a power of the base, which will be equal to the same number of positions that the digits have been shifted. If the shift of the decimal point is made to the right of its normal position, this parallels a multiplication of a positive power of the base; if it is to the left, it is the same as a multiplication by a negative power of the base.

Fixed-Point Numbers

In fixed-point operation the point (decimal point or binary point) is established for a particular run or program on a computer as a convenience. Suppose that you were operating with these numbers:

$$
\begin{aligned}
A &= 13{,}740.49\\
B &= 5{,}067.78\\
C &= 1{,}548.90\\
D &= 330.54
\end{aligned}
$$

It is possible to work with these, as has been mentioned, by dividing each by 100 and instead use:

$$
\begin{aligned}
A &= 137.4049\\
B &= 50.6778\\
C &= 15.4890\\
D &= 3.3054
\end{aligned}
$$

You must, however, recognize the problems this creates. Take A = 100.0 and B = 200.0 (two three-digit numbers). We add them and get a three-digit number:

$$\begin{array}{r} 100.0 \\ +200.0 \\ \hline 300.0 \end{array}$$

Or we subtract them and also get a three-digit number:

$$\begin{array}{r} 100.0 \\ -200.0 \\ \hline -100.0 \end{array}$$

But, notice that when we multiply we no longer obtain a three-digit number:

$$\begin{array}{r} 100.0 \\ \times 200.0 \\ \hline 20{,}000.0 \end{array}$$

The product is at least a five-digit number.

If we had operated with A as 1.0 and B as 2.0, by dividing 100 and 200 each by 100, then the multiplication would have been:

$$\begin{array}{r} 1.0 \\ \times 2.0 \\ \hline 2.0 \end{array}$$

This product (2.0) cannot be multiplied by 100, it must be multiplied by 100 × 100, or 10,000, to get the proper answer. Thus:

$$\begin{array}{lrcr} 1.0 \times & 100 & = & 100 \\ 2.0 \times & 100 & = & 200 \\ \hline 2.0 \times & 10{,}000 & = & 20{,}000 \end{array}$$

This, of course, comes about from the following rule:

In multiplication, we add exponents. (Conversely, in division, we subtract exponents.) Therefore:

$$\begin{array}{l} 1.0 \times 10^2 \\ 2.0 \times 10^2 \\ \hline 2.0 \times 10^4 = 20{,}000 \end{array}$$

Floating-Point Numbers

Computers utilizing floating-point arithmetic were devised in order to avoid the problems involved in scaling. In floating-point computers, numbers are stored in a fixed format, including the number itself and the power of the base. There are some disadvantages in this type of computer, since additional equipment is required in order to perform floating-point arithmetic.

The use of floating-point numbers is, in general, quite similar to using a slide rule, where numbers are converted to powers of ten, since there is no radix point on the slide rule.

Thus, to multiply 451 and 372, first express the numbers as:

$$451 = 4.51 \times 10^2$$

$$372 = 3.72 \times 10^2$$

Read the product as 16.7, which is multiplied by 10^4, or:

$$16.7 \times 10^4$$

Thus, a number X is converted into $X = N \times 10^m$ as shown in Table 3-11.

Table 3-11. Conversion to Floating-Point Numbers

X	N	10^m
88.0	.88	10^2
4.6	.46	10^1
.14	.14	10^0
705.0	.705	10^3

Storage space for numbers within the computer can be utilized to the best advantage by moving the point to some finite number of positions to the left, let us say, 50 places. In this case, a number such as 828 would be represented as $.00\cdots0828 \times 10^{50}$. It follows that negative powers of ten will be indicated as exponents less than 50. In computers in use, which utilize floating-point arithmetic, the first two digits or the last two digits are reserved for the exponent. Also, within the computer the power of ten of a number is automatically noted, with the exponent being operated upon separately from the number. Correct shifting is made by the computer if, in adding and subtracting, the exponents are not the same. During multiplication the exponents are added; and, in division, they are subtracted.

SHIFT OPERATIONS

With a number such as 53746.942, a shift right to 5374.6942 is a division by 10; a shift left to 537469.42 is a multiplication by 10. Binary shifts correspond to division or multiplication by 2, as shown in Table 3-12.

Table 3-12. Binary Shifting

Shift	Binary Number	Decimal
	00001011010.0000	90
Left Once	00010110100.0000	180
Left Twice	00101101000.0000	360
Reset	00001011010.0000	90
Right Once	00000101101.0000	45
Right Twice	00000010110.1000	22.5

A computer shift register is used to multiply or divide, since shifting can be used for both operations. For example, consider 21_{10}. In the binary system, it is:

$$010101 = (0 \times 32 + 1 \times 16 + 0 \times 8 + 1 \times 4 + 0 \times 2 + 1 \times 1) = 21_{10}$$

A shift left is:

$$101010 = (1 \times 32 + 0 \times 16 + 1 \times 8 + 0 \times 4 + 1 \times 2 + 0 \times 1) = 42_{10}$$

A decimal shift left is a multiply-by-ten operation, whereas a binary shift left, as above, is a multiply-by-two operation. In the same manner, the binary shift right is a divide-by-two operation:

$$010101.0 = 21_{10}$$

$$001010.1 = 1 \times 8 + 0 \times 4 + 1 \times 2 + 0 \times 1 + 1 \times .5 = 10.5_{10}$$

CHAPTER 4

Polynomials and Linear Forms

From the techniques of the various numbering systems in the first three chapters it is now possible to delve into some of the classical mathematical techniques and to see how these are handled on a modern digital computer. This chapter first discusses various polynomial forms and simple equations, simultaneous linear equations, and then the techniques for handling them follow. The binomial expansion is explained as a basic, well known technique, which has a number of applications to evaluation of simple expressions in a digital machine.

POLYNOMIAL FORMS

A polynomial in the variable x and of degree n is the function:

$$f(x) = a_0x^n + a_1x^{n-1} + a_2x^{n-2} + \cdots + a_{n-1}x + a_n$$

where

$a_0, a_1, \cdots, a_n$ are constants,
a_0 is not 0,
n is a positive integer.

Some examples of polynomials are:

$$3x^6 - 4x^5 + 17x^4 + 12x^3 - 7x^2 + 2x - 3$$
$$12x^4 + 6x^3 - x^2 - x + 15$$

The first is of degree 6 $(n = 6)$, whereas the second is of degree 4 $(n = 4)$. If we set $f(x) = 0$ we have an *integral rational equation* of degree n. Any root of $f(x) = 0$ is by definition zero of $f(x)$.

The roots of an equation (Fig. 4-1) are often necessary for the solution of a specific problem. For example, consider a polynomial, $f(x)$. If $f(x)$ is divided by $x - r$, for any $r \neq x$, the remainder R is equal to the value of the polynomial when x is replaced by r. If $Q(x)$ is the quotient, then:

$$\frac{f(x)}{x-r} = Q(x) + \frac{R}{x-r}$$

For $x = r$, this equation is:

$$\begin{aligned} f(x) &= (x-r)Q(x) + R \\ f(r) &= (r-r)Q(r) + R \\ f(r) &= R \end{aligned}$$

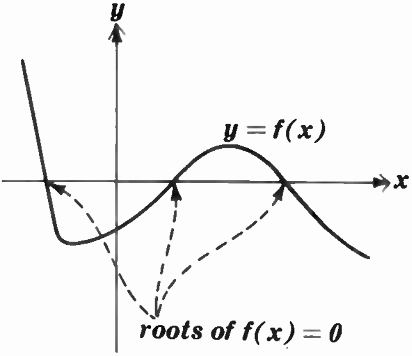


Fig. 4-1. Roots of the equation $y = f(x)$.

Consider $f(x) = x^3 + 2x^2 + x + 3$, and take $r = 2$. Then, dividing $f(x)$ by $x - 2$, we have:

$$\begin{array}{r|l}
 & x^2 + 4x + 9 \\ \hline
x - 2 & x^3 + 2x^2 + x + 3 \\
 & x^3 - 2x^2 \\ \cline{2-2}
 & \quad 4x^2 + x \\
 & \quad 4x^2 - 8x \\ \cline{2-2}
 & \qquad 9x + 3 \\
 & \qquad 9x - 18 \\ \cline{2-2}
 & \qquad\quad 21
\end{array}$$

Thus:

$$\begin{aligned} r &= 2 \\ R &= 21 \\ Q &= x^2 + 4x + 9 \end{aligned}$$

Substituting $x = r = 2$ in $f(x)$ gives:

$$\begin{gathered} 2^3 + 2(2)^2 + 2 + 3 = 21 \\ f(r) = 21 \\ R = 21 \end{gathered}$$

Factors—A factor is one of several values whose product equals the number in question.

Thus, the factors of 12 are:

$$\begin{aligned} 12 \times 1 &= 12 \\ 6 \times 2 &= 12 \\ 3 \times 4 &= 12 \\ 3 \times 2 \times 2 &= 12 \end{aligned}$$

Note that if r is a root of $f(x) = 0$, then $f(r) = 0$, and therefore $R = 0$. Thus:

$$f(x) = (x - r)\, Q(x)$$

That is, $x - r$ is a factor of $f(x)$.

Conversely, if $x - r$ is a factor of $f(x)$, then:

$$f(x) = (x - r)\, P(x)$$

where $P(x)$ is the other factor(s) of $f(x)$.

Then:

$$\frac{f(x)}{x - r} = P(x)$$

And thus $R = 0$. But $f(r) = R = 0$. Hence r is a root of $f(x) = 0$. We have just proved that r is a root of $f(x) = 0$ if, and only if, $x - r$ is a factor of $f(x)$.

Often division by $x - r$ is required in evaluating the roots of $f(x)$; hence a short-cut technique of division is useful. For example, consider the division of $2x^3 - 3x^2 - 13x + 5$ for $r = 3$. In the long-division manner, this is:

$$\begin{array}{r|rrrr}
 & 2x^2 & +\ 3x & -\ 4 & \\ \cline{2-5}
x - 3 & 2x^3 & -\ 3x^2 & -\ 13x & +\ 5 \\
 & 2x^3 & -\ 6x^2 & & \\ \cline{2-3}
 & & 3x^2 & -\ 13x & \\
 & & 3x^2 & -\ 9x & \\ \cline{3-4}
 & & & -\ 4x & +\ 5 \\
 & & & -\ 4x & +\ 12 \\ \cline{4-5}
 & & & & -\ 7
\end{array}$$

However, a shorter method, known as *synthetic division*, may be used. In general, if $f(x) = a_0 x^n + \cdots + a_{n-1} x + a_n$ is a polynomial of degree n, then $f(x)$ divided by $x - r$, for any real number r, is given by:

$$\frac{f(x)}{x-r} = A_0 x^{n-1} + A_1 x^{n-2} + \cdots + A_{n-1} + \frac{R}{x-r}$$

where, for $i = 1, \cdots, n$,

$$A_0 = a_0,$$
$$A_i = A_{i-1}\, r + a_i$$
$$A_n = R \text{ (remainder).}$$

This is commonly written as:

$$\begin{array}{r|llll} 1 - r & a_0 & a_1 & a_2 \cdots & a_n \\ & & a_0 r & A_1 r \cdots & A_{n-1} r \\ \hline & a_0 & A_1 & A_2 \cdots & R \end{array}$$

In the preceding example for $r = 3$, $f(x) = 2x^3 - 3x^2 - 13x + 5$, this is:

$$\begin{array}{r|rrrr} 1 - 3 & 2 & -3 & -13 & +5 \\ & & -6 & -9 & +12 \\ \hline & 2 & +3 & -4 & -7 \end{array}$$

The expression $2 + 3 - 4 - 7$ is understood to mean $2x^2 + 3x - 4 = Q(x)$, while $R = -7$. Any missing term is written as with a zero coefficient, for each power of x is in descending order. For example, $6x^4 + 7x^3 + 3x + 17$ is written as $6x^4 + 7x^3 + 0x^2 + 3x + 17$.

Here are some rules for determining the roots of an equation.

1. Every integral rational equation of degree n has n roots (but the same root may occur more than once).
2. If an integral rational equation $f(x) = 0$, with real coefficients, has one root of the form $a + bi$ ($i^2 = -1$), it also has the conjugate $(a - bi)$ as a root.
3. An integral rational equation $f(x) = 0$, with real coefficients, has the same number of positive roots as it has variations of signs (or less by an even number), where some roots may be multiple.

For example, consider:

$$3x^4 + 7x^3 - 8x^2 + 9x + 7 = 0 \qquad (1)$$

$$7x^5 - 3x^4 + 2x^3 + x^2 - 12x + 14 = 0 \qquad (2)$$

By rule 1, equation 1 has four roots, and by rule 3 it has either two positive roots or no positive roots. Equation 2 has five roots, of which four, two, or none are positive roots.

The equation in Fig. 4-2 has two real roots. In Fig. 4-3, $y = f(x)$ has three real roots; y' has two real roots, and y'' has one real root.

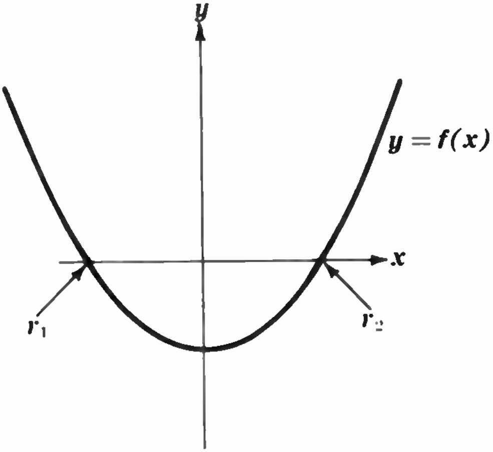


Fig. 4-2. The real roots (r_1, r_2) of a parabola $y = f(x)$.

Evaluation of Polynomials

Since synthetic division may be performed with fewer steps than most methods, it is a type of factoring often used to evaluate a polynomial. These steps generally include successive multiplications and additions.

Any analytic function $f(x)$ may be written as a Taylor's series. For example, consider:

$$\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \cdots$$

Thus, if sin 0.5 is required, the series is:

$$\sin 0.5 = 0.5 - \frac{0.5^3}{3!} + \frac{0.5^5}{5!} - \frac{0.5^7}{7!} + \cdots$$

$$= 0.5 - 0.0208333 + 0.00026042 - 0.00000155 + \cdots$$

$$\sin 0.5 = 0.4794$$

This is accurate enough for a specific case, and if for example we wanted sin x to within 0.0001, the series is used until a term less than 0.0001 is reached.

However, synthetic division provides a simplification. The series for e^x is as follows:

$$e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \frac{x^4}{4!} + \frac{x^5}{5!} + \frac{x^6}{6!} + \frac{x^7}{7!} + \frac{x^8}{8!} + \cdots$$

1. Multiply the coefficient of x^N, the highest power of x, by x. This gives:

$$\frac{1}{8!}x$$

2. Next, the coefficient of the next highest power of x is added to produce:

$$\frac{1}{8!}x + \frac{1}{7!}$$

And this is again multiplied by x to obtain:

$$\left(\frac{1}{8!}x + \frac{1}{7!} \right)x = \frac{x}{7!} + \frac{x^2}{8!}$$

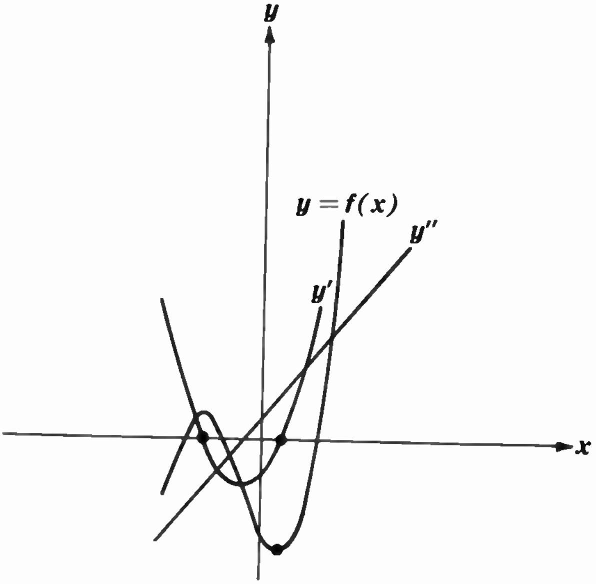


Fig. 4-3. Graph of $y = x^3 + 3x^2 - 10x - 24$.

3. The next lowest coefficient is added, and the expression is again multiplied by x:

$$\left(\frac{1}{6!} + \frac{x}{7!} + \frac{x^2}{8!} \right)x = \frac{x}{6!} + \frac{x^2}{7!} + \frac{x^3}{8!}$$

4. The succeeding step results in:

$$\left(\frac{1}{5!} + \frac{x}{6!} + \frac{x^2}{7!} + \frac{x^3}{8!} \right)x = \frac{x}{5!} + \frac{x^2}{6!} + \frac{x^3}{7!} + \frac{x^4}{8!}$$

5. When the highest desired power of x has been reached (which is x^8 in this case), the polynomial is of the form:

$$x+\frac{x^2}{2!}+\frac{x^3}{3!}+\frac{x^4}{4!}+\frac{x^5}{5!}+\frac{x^6}{6!}+\frac{x^7}{7!}+\frac{x^8}{8!}$$

And when 1 is added to this, the value for e has been obtained.

This is faster and requires fewer steps than the more common way of evaluating the approximation of e^x that is given below:

1. To 1 add x.
2. Multiply x by x and divide by 2!
3. Add (1) and (2) above to obtain the sum of the first three terms.
4. Take (2) above, multiply it by x and divide by 3.
5. Add (4) to (3). This result is the sum of the first four terms.
6. This continues for all required steps.

The same general approach can be used for the cos x series:

$$\cos x=1-\frac{x^2}{2!}+\frac{x^4}{4!}-\frac{x^6}{6!}+\frac{x^8}{8!}+\cdots$$

$$\left(\frac{1}{8!}\right)x^2=\frac{x^2}{8!}$$

$$\left(\frac{x^2}{8!}-\frac{1}{6!}\right)x^2=\frac{x^4}{8!}-\frac{x^2}{6!}$$

$$\left(\frac{x^4}{8!}-\frac{x^2}{6!}+\frac{1}{4!}\right)x^2=\frac{x^6}{8!}-\frac{x^4}{6!}+\frac{x^2}{4!}$$

$$\left(\frac{x^6}{8!}-\frac{x^4}{6!}+\frac{x^2}{4!}-\frac{1}{2!}\right)x^2=\frac{x^8}{8!}-\frac{x^6}{6!}+\frac{x^4}{4!}-\frac{x^2}{2!}$$

Adding 1 and rearranging the terms gives the desired form:

$$\cos x=1-\frac{x^2}{2!}+\frac{x^4}{4!}-\frac{x^6}{6!}+\frac{x^8}{8!}$$

SUBROUTINES FOR EVALUATION

The evaluations, as before, can be used as programming loops or subroutines in digital computers. Rather than storing long tables in a limited computer memory, this technique is often used. Two examples are given: one for square root, and one for $\log_e x$.

Mathematical Description of Square-Root Subroutine—We wish to find $\sqrt{N}$ for $0 < N < 1$. First, normalize N so that $N' = 2^n N$ *and* $\frac{1}{2} \leqq N' < 1$, for some integer n. Now, if n is even, we have:

$$\sqrt{N} = 2^{-n/2}\sqrt{N'}$$

If n is odd, then we have:

$$\sqrt{N} = 2^{-\frac{(n-1)}{2}}\sqrt{\frac{N'}{2}}$$

In the following discussion, let N^* denote N' if n is even, or $\frac{N'}{2}$ if n is odd. The first estimate of $\sqrt{N^*}$ is E_0, which is given by:

$$E_0 = \frac{1}{2} + \frac{N^*}{4}$$

The next estimates are:

$$E_1 = \frac{N^*}{2E_0} + \frac{E_0}{2}$$

$$E_2 = \frac{N^*}{2E_1} + \frac{E_1}{2}$$

$$\cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot$$

$$E_k = \frac{N^*}{2E_{k-1}} + \frac{E_{k-1}}{2}$$

It follows that:

$$\sqrt{N^*} \approx E_k \text{ if } | E_k - E_{k-1} | \leqq 2^{-35}$$

Natural Logarithm—Evaluation of the logarithm function may be made by using a simple expansion. Hence, for $\log_e x$, we have:

$$\log x = 2\left[\frac{x-1}{x+1} + \frac{1}{3}\left(\frac{x-1}{x+1}\right)^3 + \frac{1}{5}\left(\frac{x-1}{x+1}\right)^5 + \cdots\right] \qquad x > 0$$

$$\log x = \frac{x-1}{x} + \frac{1}{2}\left(\frac{x-1}{x}\right)^2 + \frac{1}{3}\left(\frac{x-1}{x}\right)^3 + \cdots \qquad x > \frac{1}{2}$$

$$\log x = (x-1) - \frac{(x-1)^2}{2} + \frac{(x-1)^3}{3} - \cdots \qquad 0 < x \leqq \frac{1}{2}$$

HIGHER-DEGREE EQUATIONS

The second-degree equation (or quadratic), which is a parabola, has a pair of roots. Consider:

$$x = \frac{-b \pm \sqrt{b^2 - 4\,ac}}{2a}$$

This is the solution of the general equation $ax^2 + bx + c = 0$. In this solution, $b^2 - 4ac$ is termed the *discriminant (D)* of the equation. The roots of the quadratic equation may be characterized by the sign of D: If D is greater than 0, the roots are real and unequal; if D is equal to 0, the roots are real and equal; if D is less than 0, the roots are conjugate imaginaries.

Consider the equation $4x^2 - 3x + 2 = 0$. In this case:

$$D = 9 - 4(8)$$
$$D = -23$$

By the preceding rules the roots are of the form $a \pm bi$. Therefore, the roots are:

$$x = \frac{3 \pm (-23)^{1/2}}{8}$$

or,

$$x = \frac{3 + i(23)^{1/2}}{8}, \frac{3 - i(23)^{1/2}}{8}$$

The Cubic Equation

The general form of a cubic equation is $y^3 + py^2 + qy + r = 0$. This equation may be reduced to:

$$x^3 + ax + b = 0$$

This is done by substituting for y the relation:

$$y = x - \frac{p}{3}$$

Therefore:

$$(x - \frac{p}{3})^3 + p(x - \frac{p}{3})^2 + q(x - \frac{p}{3}) + r = 0$$

From this, we derive:

$$a = \frac{3q - p^2}{3}$$

$$b = \frac{2p^3 - 9pq + 27r}{27}$$

We define A and B as:

$$A \equiv \sqrt[3]{-\frac{b}{2}+\sqrt{\frac{b^2}{4}+\frac{a^3}{27}}}$$

$$B \equiv \sqrt[3]{-\frac{b}{2}-\sqrt{\frac{b^2}{4}+\frac{a^3}{27}}}$$

From this there are three values for x:

$$x_1 = A + B$$

$$x_2 = -\frac{A+B}{2}+\frac{A-B}{2}\sqrt{-3}$$

$$x_3 = -\frac{A+B}{2}-\frac{A-B}{2}\sqrt{-3}$$

We may define the *discriminant (D)* of a cubic equation by:

$$D = \frac{b^2}{4}+\frac{a^3}{27}$$

If D is greater than 0, one root is real and two are complex conjugates. If D is equal to 0, all three roots are real, and at least two are equal. If D is less than 0, all three roots are real and unequal.

For another approach, consider the following:

$$f(x) = x^3 - 3x^2 - 12x + 1 = 0$$

Table 4-1 gives values for $f(x)$ determined by various values of x.

Table 4-1. Values of $f(x)$

x	−4	−2	−1	0	1	2	3	8
$f(x)$	−63	5	9	1	−15	−27	−35	225

There are three real roots. This may be seen from Table 4-1. Note that $f(x)$ must equal zero for some value of x between $x = -4$ and $x = -2$; for some value of x between $x = 0$ and $x = 1$; and for some value of x between $x = 3$ and $x = 8$. Each of these roots may be approximated. Consider the root between $x = 0$ and $x = 1$; let us call the first approximation d (Table 4-2).

By the straight-line formula of analytic geometry, we have:

$$d = \frac{x_1 - x_2}{y_2 - y_1} = \frac{0-1}{-15-1} = \frac{1}{16}$$

$$= 0.06$$

Table 4-2. Approximation of x

x	1	0+d	1
f(x)	1	0	−15

By synthetic division:

$$\begin{array}{r|rrrr} .06 & 1 & -3.00 & -12.00 & +1.00 \\ & & +0.06 & +\ 1.76 & -1.76 \\ \hline & 1 & +2.49 & -10.24 & -0.76 \end{array}$$

and,

$$\begin{array}{r|rrrr} .07 & 1 & -3.00 & -12.00 & +1.00 \\ & & +0.07 & -\ 2.05 & -0.98 \\ \hline & 1 & -2.93 & -14.05 & +0.02 \end{array}$$

Thus this desired root is between 0.06 and 0.07. Table 4-3 lists values of $f(x)$ for making the next approximation.

$$\frac{d^{(1)}}{0.07-0.06}=\frac{0-(-0.76)}{0.02-(-0.76)}$$

$$\frac{d^{(1)}}{0.01}+\frac{0.76}{0.78}$$

$$d^{(1)}=0.0097$$

Since the earlier approximation was 0.06, the next approximation is 0.06 + 0.0097, or 0.0697.

Table 4-3. The Second Approximation of x for which f(x) = 0

x	−0.76	0	0.02
f(x)	0.06	0.06 + $d^{(1)}$	0.07

SIMULTANEOUS LINEAR EQUATIONS

Many examples can be given of a fixed series of steps that will solve a class of problems. An example is simultaneous linear equations. Consider the simple case of two equations:

$$2x + 4y = -10 \qquad (1)$$

$$3x - 9y = \quad 45 \qquad (2)$$

These could be solved by equating each for x:

$$x = -\ 5 - 2y \qquad (3)$$

$$x = \quad 15 + 3y \qquad (4)$$

And equating these to solve for y:

$$-5 - 2y = 15 + 3y$$
$$-5y = 20$$
$$y = -4$$

Substituting $y = -4$ in (3) gives the solution:

$$x = -5 + 8$$

The complete solution is:

$$x = 3$$
$$y = -4$$

Fig. 4-4 shows the graphical solution of this intersection.

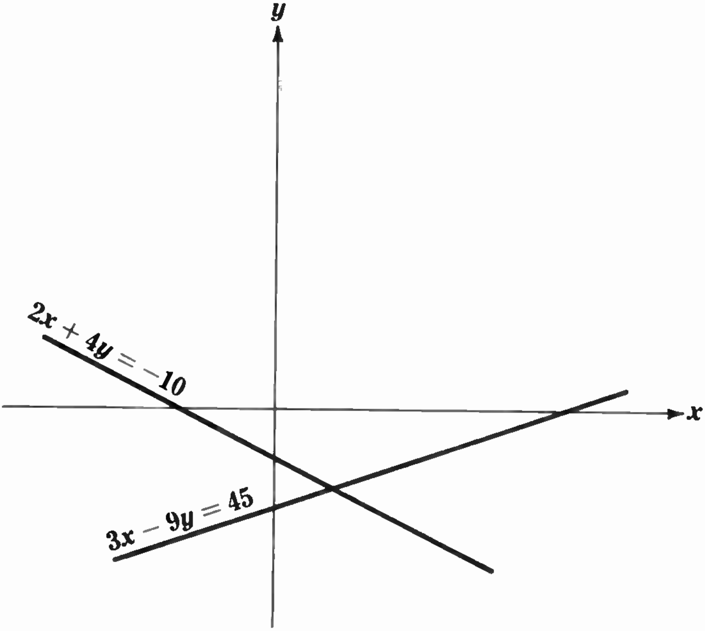


Fig. 4-4. Graph solution of text problem.

A second method involves removing the x term by multiplying (1) by 3 and (2) by 2:

$$6x + 12y = -30 \qquad (5)$$
$$6x - 18y = 90 \qquad (6)$$

Subtracting (6) from (5) yields:

$$30y = -120$$
$$y = -4$$

By substitution of $y = -4$ in (5), we obtain:

$$6x + 12(-4) = -30$$
$$6x = \quad 18$$

Hence:

$$x = \quad 3$$
$$y = \quad -4$$

Neither method can be used for a series of n linear equations in n unknowns (where n is greater than 2) in any direct and easily mechanized system that can be applied in the general case. But the second method can be expanded into a usable technique. Consider the following system of equations:

$$\begin{aligned} a_{11}x_1 + a_{12}x_2 + \cdots + a_{1n}x_n &= b_1 \\ a_{21}x_1 + a_{22}x_2 + \cdots + a_{2n}x_n &= b_2 \\ \cdots\cdots\cdots\cdots&\cdots\cdots \\ a_{n1}x_1 + a_{n2}x_2 + \cdots + a_{nn}x_n &= b_n \end{aligned}$$

These can be reduced by one unknown at a time to obtain a complete solution to the system. For example, consider:

$$2w - 2x + 6y - 2z = \quad 8 \qquad (1)$$
$$3w + 3x + 4y - 6z = -3 \qquad (2)$$
$$7w - \quad x + 4y + 2z = \ 25 \qquad (3)$$
$$8w + 4x + 2y - 3z = \ 10 \qquad (4)$$

Divide both sides of (1) by 2 so that:

$$w - \quad x + 3y - \quad z = \quad 4 \qquad (5)$$

We wish to eliminate the w-term, so we multiply (5) by 3 and subtract this from (2):

$$\begin{array}{r} 3w + 3x + 4y - 6z = - \ 3 \\ \underline{3w - 3x + 9y - 3z = \quad 12} \\ 6x - 5y - 3z = -15 \end{array} \qquad (6)$$

In the same manner we multiply (5) by 7 and subtract this from (3):

$$\begin{array}{r} 7w - \quad x + \quad 4y + 2z = \ 25 \\ \underline{7w - 7x + 21y - 7z = \ 28} \\ 6x - 17y + 9z = -3 \end{array} \qquad (7)$$

Also in the same manner we multiply (5) by 8 and subtract this from (4):

$$\begin{array}{r} 8w + 4x + 2y - 3z = 10 \\ 8w - 8x + 24y - 8z = 32 \\ \hline 12x - 22y + 5z = -22 \end{array} \qquad (8)$$

This results in three equations in three unknowns:

$$6x - 5y - 3z = -15 \qquad (6)$$

$$6x - 17y + 9z = -3 \qquad (7)$$

$$12x - 22y + 5z = -22 \qquad (8)$$

Now, subtracting (6) from (7), we have:

$$-12y + 12z = 12 \qquad (9)$$

And multiplying (6) by 2 and then subtracting this result from (8):

$$\begin{array}{r} 12x - 22y + 5z = -22 \\ 12x - 10y - 6z = -30 \\ \hline -12y + 11z = 8 \end{array} \qquad (10)$$

Now there are two equations in two unknowns:

$$-12y + 12z = 12 \qquad (9)$$

$$-12y + 11z = 8 \qquad (10)$$

Subtracting (10) from (9) gives:

$$z = 4 \qquad (11)$$

To evaluate these equations for w, x, and y, substitute (11) in (10):

$$-12y = -36$$

Or.

$$y = 3$$

From (8), substitution for y and z yields:

$$12x - 66 + 20 = -22$$

or,

$$x = 2$$

In the same manner, by using (5) we have:

$$w - 2 + 9 - 4 = 4$$

or,

$$w = 1$$

The set of equations which are actually solved is:

$$w - x + 3y - z = 4 \tag{5}$$
$$6x - 5y - 3z = -15 \tag{6}$$
$$-12y + 12z = 12 \tag{9}$$
$$z = 4 \tag{11}$$

The complete solution is:

$$x = 2 \qquad w = 1$$
$$y = 3 \qquad z = 4$$

A general solution of this type can easily be automated or programmed for use by a computer.

DETERMINANTS

The example previously given for the solution of n simultaneous equations in n unknowns implicitly uses determinants.

Consider two equations in two unknowns:

$$a_{11}x_1 + a_{12}x_2 = b_1 \tag{1}$$
$$a_{21}x_1 + a_{22}x_2 = b_2 \tag{2}$$

Multiplying (1) by a_{22}, and (2) by a_{12}:

$$a_{22}a_{11}x_1 + a_{22}a_{12}x_2 = a_{12}b_1 \tag{3}$$
$$a_{21}a_{12}x_1 + a_{22}a_{12}x_2 = a_{12}b_2 \tag{4}$$

Thus:

$$x_1 = \frac{a_{22}b_1 - a_{12}b_2}{a_{22}a_{11} - a_{21}a_{12}}$$
$$x_2 = \frac{a_{11}b_2 - a_{21}b_1}{a_{22}a_{11} - a_{21}a_{12}}$$

The terms in the denominators (which are the same) may be expressed in an array as:

$$\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}$$

This is defined as being equal to $a_{11}a_{22} - a_{12}a_{21}$.

Thus, following this rule, we can write:

$$\begin{vmatrix} 3 & 4 \\ -7 & 2 \end{vmatrix} = 6 - (-28) = 34$$

And, in the same manner, we have:

$$\begin{vmatrix} 2 & 5 \\ 4 & 6 \end{vmatrix} = 12 - 20 = -8$$

The diagonal from upper left to lower right is called the *principal diagonal*; the determinant's value is the product of the principal diagonal minus the product of the other diagonal. The use of determinants permits a rapid solution of simultaneous equations.

Consider the general equations (1) and (2). The solutions are:

$$x_1 = \frac{\begin{vmatrix} b_1 & a_{12} \\ b_2 & a_{22} \end{vmatrix}}{\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}}, \qquad \text{and} \qquad x_2 = \frac{\begin{vmatrix} a_{11} & b_1 \\ b_{21} & b_2 \end{vmatrix}}{\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}}$$

For example, consider the following equations:

$$\begin{aligned} 2x - 3y &= 16 \\ 5x + 2y &= \ \ 2 \end{aligned}$$

Then:

$$x = \frac{\begin{vmatrix} 16 & -3 \\ 2 & 2 \end{vmatrix}}{\begin{vmatrix} 2 & -3 \\ 5 & 2 \end{vmatrix}} = \frac{32 - (-6)}{4 - (-15)} = \frac{38}{19}$$

$$x = 2$$

$$y = \frac{\begin{vmatrix} 2 & 16 \\ 5 & 2 \end{vmatrix}}{\begin{vmatrix} 2 & -3 \\ 5 & 2 \end{vmatrix}} = \frac{4 - 80}{4 - (-15)} = \frac{-76}{19}$$

$$y = -4$$

One can operate on the determinant with specific rules to simplify their solution.

(1) The determinant's value is unchanged if corresponding rows and columns are interchanged.

From the previous example, by interchanging the first row and first column of the determinant in the numerator and interchanging the first row and first column in the denominator, we have:

$$x = \frac{\begin{vmatrix} 16 & 2 \\ -3 & 2 \end{vmatrix}}{\begin{vmatrix} 2 & 5 \\ -3 & 2 \end{vmatrix}} = \frac{32 - (-6)}{4 - (-15)} = \frac{38}{19}$$

$$x = 2$$

(2) If two columns or rows are interchanged, the sign of the determinant is changed.

Interchanging the first and second columns of the determinant in the numerator of x gives:

$$x = \frac{\begin{vmatrix} -3 & 16 \\ 2 & 2 \end{vmatrix}}{\begin{vmatrix} 2 & -3 \\ 5 & 2 \end{vmatrix}} = \frac{-6 - 32}{4 - (-15)}$$

$$x = -2$$

(3) If all the numbers in a row or column are multiplied by a constant k, the value of the determinant is multiplied by this constant.

Multiplying by 3 the second column of the determinant in the numerator of x:

$$3x = \frac{\begin{vmatrix} 16 & -9 \\ 2 & 6 \end{vmatrix}}{\begin{vmatrix} 2 & -3 \\ 5 & 2 \end{vmatrix}} = \frac{96 - (-18)}{4 - (-15)}$$

$$3x = \frac{114}{19} = 6$$

(4) If the numbers in a row or column are added to another row or column, the determinant is unchanged in value:

$$x = \frac{\begin{vmatrix} 16 & -3 \\ 2 & 2 \end{vmatrix}}{\begin{vmatrix} 2 & -3 \\ 5 & 2 \end{vmatrix}}$$

By Rule 4:

$$x = \frac{\begin{vmatrix} 16 & 13 \\ 2 & 4 \end{vmatrix}}{\begin{vmatrix} 2 & -1 \\ 5 & 7 \end{vmatrix}}$$

Hence:

$$x = \frac{64 - 26}{14 - (-5)} = \frac{38}{19}$$

$$x = 2$$

Using these and other rules, one can reduce a determinant of order greater than 2 in order to allow more simple evaluation.

Determinants of Higher Order

As before, the second-order determinant is expanded as:

$$\begin{vmatrix} a_1 & b_1 \\ a_2 & b_2 \end{vmatrix} = a_1b_2 - a_2b_1$$

For the third-order determinant, expansion is possible either directly or by means of minors. By direct expansion, we have:

$$\begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix} = a_1b_2c_3 + a_2b_3c_1 + a_3b_1c_2 - a_3b_2c_1 - a_2b_1c_3 - a_1b_3c_2$$

Minors can also be used for expansion; consider only the terms having a_1 as a coefficient. These are $a_1b_2c_3 - a_1b_3c_2$. This difference is defined as equal to:

$$a_1 \begin{vmatrix} b_2 & c_2 \\ b_3 & c_3 \end{vmatrix}$$

The expansion by minors is:

$$\begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix} = a_1 \begin{vmatrix} b_2 & c_2 \\ b_3 & c_3 \end{vmatrix} - a_2 \begin{vmatrix} b_1 & c_1 \\ b_3 & c_3 \end{vmatrix} + a_3 \begin{vmatrix} b_1 & c_1 \\ b_2 & c_2 \end{vmatrix}$$

This is also equal to:

$$-b_1 \begin{vmatrix} a_2 & c_2 \\ a_3 & c_3 \end{vmatrix} + b_2 \begin{vmatrix} a_1 & c_1 \\ a_3 & c_3 \end{vmatrix} - b_3 \begin{vmatrix} a_1 & c_1 \\ a_2 & c_2 \end{vmatrix}$$

This is, in turn, also equal to:

$$+c_1 \begin{vmatrix} a_2 & b_2 \\ a_3 & b_3 \end{vmatrix} - c_2 \begin{vmatrix} a_3 & b_3 \\ a_1 & b_1 \end{vmatrix} + c_3 \begin{vmatrix} a_1 & b_1 \\ a_2 & b_2 \end{vmatrix}$$

THE BINOMIAL EXPANSION

In general, it is possible to expand $(a+b)^n$ as:

$$(a+b)^n = a^n + \frac{n}{1} a^{n-1} b + \frac{n(n-1)}{2!} a^{n-2} b^2$$
$$+ \frac{n(n-1)(n-2)}{3!} a^{n-3} b^3 + \cdots + b^n$$

Thus, for $(a+3)^4$ one can write:

$$(a+3)^4 = a^4 + 4a^3 (3) + \frac{4(3)}{2} a^2 (3)^2 + \frac{4(3)(2)}{3\cdot 2} a (3)^3 + 3^4$$
$$= a^4 + 12a^3 + 54a^2 + 108a + 81$$

The binomial expansion can be used for calculation of certain values, such as the following:

$$\begin{aligned} (102)^4 &= (100+2)^4 \\ &= 100^4 + 4(100)^3 (2) + \frac{4(3)}{2} 100^2 (2)^2 \\ &\quad + \frac{4(3)(2)}{2\cdot 3} 100 (2)^3 + 2^4 \\ &= \begin{array}{r} 100{,}000{,}000 \\ 8{,}000{,}000 \\ 240{,}000 \\ 3{,}200 \\ 16 \\ \hline \end{array} \\ (102)^4 &= 108{,}243{,}216 \end{aligned}$$

In the same manner, one can evaluate, say, $(506)^6 = (500+6)^6$, or $(1.4)^4 = (1.0 + 0.4)^4$.

Consider the binomial expansion term with b^r; this is the $r+1$st term in the expansion. In general, this term can be expressed as:

$$\frac{n(n-1)(n-2)\cdots(n-r+1)}{r!} a^{n-r} b^r$$

As an example, find the third term of $(2x - 3y)^{12}$. The third term will have $r = 2$:

$$\frac{12(12-2+1)}{2!}(2x)^{12-2}(-3y)^2 = 607{,}256x^{10}y^2$$

Coefficients for the binomial expansion can be expressed in *Pascal's Triangle* (Table 4-4) where the first row represents the coefficients of $(a+b)^0$; the second row represents the coefficients of $(a+b)^1$; the third, $(a+b)^2$, and so on. Any number can be determined by adding the two adjacent numbers above it, as $1 + 6 = 7$, $15 + 20 = 35$, $21 + 35 = 56$, and so on. If this is applied for $a = 1$ and $b = x$, the binomial series becomes:

$$(1+x)^n = 1 + nx + \frac{n(n-1)}{2!}x^2 + \frac{n(n-1)(n-2)}{3!}x^3 + \cdots + x^n$$

For example, consider $(1+x)^3$:

$$(1+x)^3 = 1 + 3x + \frac{3\ (2)}{2}x^2 + \frac{3(2)\ (1)}{6}x^3$$
$$= 1 + 3x + 3x^2 + x^3$$

If $n = -2$, for example, we have the infinite series:

$$(1+x)^{-2} = 1 - 2x + \frac{-2(-3)}{2}x^2 + \frac{-2(-3)(-4)}{3 \cdot 2}x^3 + \cdots$$
$$= 1 - 2x + 3x^2 - 4x^3 + \cdots$$

Consider $\sqrt{21} = (25-4)^{1/2}$. To place this in the $(1+x)^n$ form, we express it as:

$$\sqrt{21} = [\ 25\ (1 - \frac{4}{25})\ ]^{1/2}$$
$$= 5\ (1 - \frac{4}{25})^{1/2}$$
$$= 5\ [\ 1 + 1/2(-.16) + \frac{1/2(-1/2)}{2}(.16)^2\ ]$$
$$= 5\ [\ 1 - .08 - .0032\ ]$$
$$= 5\ [\ 1 - .0832\ ]$$
$$= 5\ [\ .9168\ ] = 4.5840$$

Table 4-4. Pascal's Triangle

								1								
							1		1							
						1		2		1						
					1		3		3		1					
				1		4		6		4		1				
			1		5		10		10		5		1			
		1		6		15		20		15		6		1		
	1		7		21		35		35		21		7		1	
1		8		28		56		70		56		28		8		1

In the same manner, one can find the root of 1.2:

$$\begin{aligned}\sqrt{1.2} &= (1 + 0.2)^{1/2}\\ &= 1 + 1/2\ (0.2) + \frac{1/2\ (-1/2)}{2}\ (0.2)^2 + \cdots\\ &= 1 + 0.1 - 0.005 + \cdots\\ &= 1.095\end{aligned}$$

Note that with one term, the root is 1; to two terms, it is 1.1; to three terms, it is 1.095. Thus, the value approaches the correct value as more terms are used.

CHAPTER 5

Interpolation

In a general sense, interpolation involves finding a certain value of a function by using the values that this function assumes for specific values of the independent variable. The mathematical relation between the independent and dependent variables is itself not known. This, then, is actually a method of curve-fitting, and for most curves a good fit can be found.

INTRODUCTION

In 1885, the German mathematician Karl Theodor Weierstrass proved the following two theorems:

1. Any function $f(x)$ that is continuous over the interval (a,b) can always be represented (approximated) over this interval by some polynomial. That is, there exists a polynomial $P(x)$ such that $|f(x) - P(x)|$ is less than ϵ for every value of x in the interval (a,b) and where ϵ can be *any* preassigned positive number (Fig. 5-1).
2. Any periodic continuous function $f(x)$ with period 2π can be represented by a trigonometric series whose form is:

$$\begin{aligned} g(x) &= a_0 + a_1 \sin x + a_2 \sin 2x + a_3 \sin 3x + \cdots \\ &\quad + b_1 \cos x + b_2 \cos 2x + b_3 \cos 3x + \cdots \\ &= a_0 + \sum_{n=0}^{\infty} (a_n \sin nx + b_n \cos nx) \end{aligned}$$

where,

$|f(x) - g(x)| < \epsilon$,

ϵ is any preassigned positive number.

The first theorem means that one can always find or determine a function to fit data which are continuous; the second means that for approximations a trigonometric series is often very valuable. For this reason some basic trigonometric functions will be reviewed.

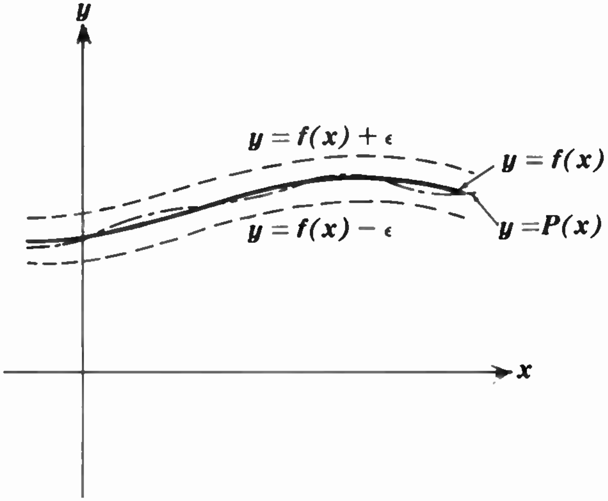


Fig. 5-1. Illustration of Weierstrass' Theorem.

TRIGONOMETRIC FUNCTIONS

Basic trigonometric functions (sine, cosine, and tangent) can be stored in the computer memory, although this is wasteful of valuable memory space. External (magnetic tape) memories may be used to store trigonometric tables, but there are other techniques.

Consider the relations between the trigonometric functions of an angle:

$$\sin^2\theta + \cos^2\theta = 1 \qquad (1)$$

$$1 + \tan^2\theta = \sec^2\theta \qquad (2)$$

$$1 + \cot^2\theta = \csc^2\theta \qquad (3)$$

These relations are well known and can be used to find one function if the other is given as sine or cosine in (1), tangent or secant (2), or cotangent or cosecant (3).

As in the preceding chapter, the series expression for sin x can be evaluated as:

$$\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \cdots$$

One can also develop all necessary functions, given a single function. For example, given sin θ, we can write:

$$\cos\theta = \sqrt{1-\sin^2\theta} = \frac{1}{\sec\theta}$$

$$\tan\theta = \frac{\sin\theta}{\sqrt{1-\sin^2\theta}} = \frac{1}{\cot\theta}$$

$$\csc\theta = \frac{1}{\sin\theta}$$

Solutions to trigonometric equations require finding an angle to satisfy the given equation. Two simple equations will serve as examples:

$$\sin^2\theta - \frac{3}{2}\sin\theta + \frac{1}{2} = 0$$

$$(\sin\theta - \frac{1}{2})\ (\sin\theta - 1) = 0$$

$$\sin\theta = \frac{1}{2}, \qquad \text{and} \qquad \sin\theta = 1$$

$$\sin^{-1}\frac{1}{2} = 30°, \qquad \text{and} \qquad \sin^{-1}1 = 90°$$

Consider another example:

$$\cos^2\theta + 2\cos\theta - 3 = 0$$

$$(\cos\theta + 3)\ (\cos\theta - 1) = 0$$

$$\cos\ \theta = 1, \qquad \text{and} \qquad \cos\theta = -3$$

But $\cos\theta = -3$ is an impossible value; hence, the only solution is $\cos\theta = 1$.

Reduction of Angles—Instead of the full set of values of trigonometric functions, a restricted range of values suffices for many applications. By reducing the given angle to one less than 90°, many angles can be used, given only a small tabular list of functions. This reduction is made as follows:

1. Express the given angle β as 90° ± θ, where θ is less than 90°, and n is some positive integer (Fig. 5-2).
2. If n is even, write the given function of β as a function of θ. If n is odd, write the *cofunction* of the given function as a function of θ.
3. The sign of the angle is determined by (a) the function of the angle and (b) the quadrant in which the angle is located.

4. If the original angle β is negative, then the expression for any trigonometric function may be derived from $\sin(-\theta) = -\sin\theta$, and $\cos(-\theta) = \cos\theta$.

Consider the following examples:

$$\begin{aligned}\cos 300^\circ &= \cos(3 \times 90 + 30)^\circ \\ &= \sin 30^\circ \\ \sin 370^\circ &= \sin(4 \times 90 + 10)^\circ \\ &= \sin 10^\circ\end{aligned}$$

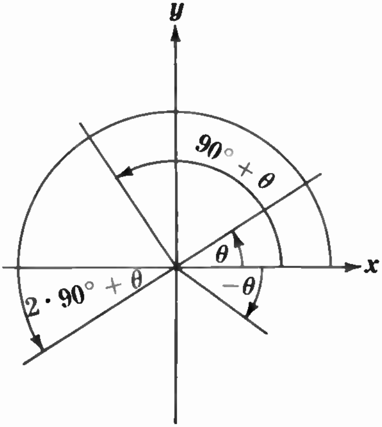


Fig. 5-2. Representation of $n90^\circ \pm \theta$.

Another way of expressing the reduction of angles is $\sin x = \cos(90 - x) = \sin(180 - x)$. For example, we have:

$$\sin 30^\circ = \cos 60^\circ = \sin 150^\circ$$
$$\sin 20^\circ = \cos 70^\circ = \sin 160^\circ$$

And:

$$\cos x = \sin(90 - x) = -\cos(180 - x)$$
$$\tan x = \cot(90 - x) = -\tan(180 - x)$$

Often it is useful to express the function of an angle as a sum of two angles. For example:

$$\sin(x + y) = \sin x \cos y + \cos x \sin y$$
$$\cos(x + y) = \cos x \cos y - \sin x \sin y$$
$$\tan(x + y) = \frac{\tan x + \tan y}{1 - \tan x \tan y}$$

For the differences between two angles, rather than their sum, the expressions on the right have all signs (shown) changed.

Further, functions of multiple angles can be expressed in terms of a single angle if that particular function of a single angle is known.

$$\begin{aligned}\sin 2x &= 2\sin x\cos x\\ \cos 2x &= 2\cos^2 x - 1\\ \sin 3x &= 3\sin x - 4\sin^3 x\\ \cos 3x &= 4\cos^3 x - 3\cos x\end{aligned}$$

DIFFERENCES

Interpolation of intermediate values in an equation is a powerful tool. The fundamentals of interpolation require consideration of the table of differences of the function.

Consider a function $f(x)$ whose values for equidistant points $x, x + h, x + 2h, \cdots, x + mh$ of the independent variable x are known. If we subtract $f(x)$ from $f(x+h)$, we establish the *delta* of f of x, of $\Delta f(x)$, where Δ is the symbol for the difference. In this way:

$$\begin{aligned}\Delta f(x) &= f(x+h) - f(x)\\ \Delta f(x+h) &= f(x+2h) - f(x+h)\\ \Delta f(x+2h) &= f(x+3h) - f(x+2h)\\ \Delta f(x+3h) &= f(x+4h) - f(x+3h)\\ \Delta f(x+mh) &= f[x+(m+1)h] - f(x+mh)\end{aligned}$$

These are the *first differences* of the function f of x. Consider $y = 2x^2 - x + 1$. Let $x = 0$, and $h = 1$. Then:

$$\begin{aligned}y_0 &= f(x) &&= 1\\ y_1 &= f(x+h) &&= 2\\ y_2 &= f(x+2h) &&= 7\\ y_3 &= f(x+3h) &&= 16\\ y_4 &= f(x+4h) &&= 29\\ y_5 &= f(x+5h) &&= 46\end{aligned}$$

The corresponding values of x and y are given in Table 5-1.

$$\begin{aligned}f(x+h) - f(x) &= 1 = \Delta f(x)\\ f(x+2h) - f(x+h) &= 5 = \Delta f(x+h)\\ f(x+3h) - f(x+2h) &= 9 = \Delta f(x+2h)\\ f(x+4h) - f(x+3h) &= 13 = \Delta f(x+3h)\\ f(x+5h) - f(x+4h) &= 17 = \Delta f(x+4h)\end{aligned}$$

Table 5-1. Values of x and y

x	y
0	1
1	2
2	7
3	16
4	29
5	46

The corresponding values of x, y, and Δf are given in Table 5-2. Note the values in the last column (Δf) lie between the values for x and y in the table 5-2.

Table 5-2. Values of Δf

x	y	Δf
0	1	
		1
1	2	
		5
2	7	
		9
3	16	
		13
4	29	
		17
5	46	

The *second differences* (or differences of order two) of $f(x)$ are:

$$\begin{aligned}\Delta^2 f(x) &= \Delta[\Delta f(x)] = \Delta[f(x+h) - f(x)]\\ &= f(x+2h) - f(x+h) - [f(x+h) - f(x)]\\ &= f(x+2h) - 2f(x+h) + f(x)\end{aligned}$$

In the same manner, we can write:

$$\Delta^2 f(x+h) = f(x+3h) - 2f(x+2h) + f(x+h)$$
$$\Delta^2 f(x+2h) = f(x+4h) - 2f(x+3h) + f(x+2h)$$

The *third differences* are the differences of the second differences, and this sequence then continues for the higher differences.

The complete list of differences for this example is given in Table 5-3.

Table 5-3. First and Second Differences of *f(x)*

x	y	Δf	$\Delta^2 f$
0	1		
		1	
1	2		4
		5	
2	7		4
		9	
3	16		4
		13	
4	29		4
		17	
5	46		

We need not obtain differences higher than n, where n is the highest exponent in the original function.

One can use differences to find the terms in a given series if it is known that differences of a specific order are constant.

For positive integral values for m, any function of $f(x)$ is given by:

$$f(x + mh) = f(x) + m\Delta f(x) + \frac{m(m-1)}{2!}\Delta^2 f(x)$$

$$+\frac{m(m-1)(m-2)}{3!}\Delta^3 f(x) + \cdots + \Delta^m f(x)$$

If $f(x)$ is a polynomial of degree n, this expression terminates at $\Delta^m f(x)$ and the nth order differences are constants.

Consider the following series:

$$5, 3, 1, 5, 21, 55, \ldots$$

Suppose $x = 0$ and $h = 1$. Then, since $x = 0$ and $h = 1$, $x + mh = m$, and we can construct the differences as in Table 5-4.

Table 5-4. Differences of *f(m)*

m	f(m)	Δf	$\Delta^2 f$	$\Delta^3 f$
0	5			
		− 2		
1	3		0	
		− 2		6
2	1		6	
		4		6
3	5		12	
		16		6
4	21		18	
		34		
5	55			

The values of $f(m)$ are given as 5, 3, 1, 5, 21, 55. Thus:

$$\begin{aligned}
\Delta f(x) &= 3 - 5 = -2\\
\Delta f(x+h) &= 1 - 3 = -2\\
\Delta f(x+2h) &= 5 - 1 = 4\\
\Delta f(x+3h) &= 21 - 5 = 16\\
\Delta f(x+4h) &= 55 - 21 = 34
\end{aligned}$$

For $x = 0$ and $m = 0$, $f(x + mh) = f(x) = f(m) = 5$.
Hence

$$\begin{aligned}
f(0+5) &= 5 + m(-2) + \frac{m(m-1)}{2!}(0)\\
&\quad + \frac{m(m-1)(m-2)}{3!}(6)\\
&= 5 - 2m + 0 + (m^2 - m)(m-2)\\
&= 5 - 2m + m^3 - m^2 - 2m^2 + 2m\\
&= 5 + m^3 - 3m^2\\
f(m) &= m^3 - 3m^2 + 5
\end{aligned}$$

Thus,

$(m = 0)$	$f(0) = 5$
$(m = 1)$	$f(1) = 1 - 3 + 5 = 3$
$(m = 2)$	$f(2) = 8 - 12 + 5 = 1$
$(m = 3)$	$f(3) = 27 - 27 + 5 = 5$
$(m = 4)$	$f(4) = 64 - 48 + 5 = 21$
$(m = 5)$	$f(5) = 125 - 75 + 5 = 55$

Consider, as an example, the determination of the common logarithm of 7.075. Now, we have:

$$\log 7.0 = 0.8451$$
$$\log 7.1 = 0.8513$$

Let $x = 7.0$ and $h = 0.1$. (Table 5-5 gives the first, second, and third differences using these values for $m = 0, \ldots, 5$.)
To obtain log 7.075 we write:

$$\begin{aligned}
x + mh &= 7.075\\
7.0 + m(0.1) &= 7.075\\
0.1m &= 0.075\\
m &= 0.75
\end{aligned}$$

Table 5-5. Calculation of log $(x + mh)$

m	x + mh	log (x + mh)	Δ	Δ^2	Δ^3
0	7.0	0.8451			
			.0062		
1	7.1	0.8513		−.0002	
			.0060		.0002
2	7.2	0.8573		.0000	
			.0060		−.0001
3	7.3	0.8633		−.0001	
			.0059		.0001
4	7.4	0.8692		.0000	
			.0059		
5	7.5	0.8751			

Hence:

$$\log 7.075 = 0.8451 + 0.75(0.0062) + \frac{0.75(-0.25)}{2}(-0.0002)$$

$$= 0.8451 + 0.004650$$

$$= 0.84975$$

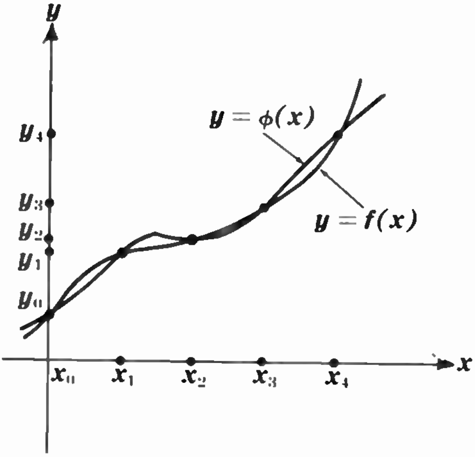


Fig. 5-3. Polynomial $\phi(x)$ approximating $f(x)$.

NEWTON'S FORMULAS

Differences can be used in many ways for interpolation. One use is in Newton's formulas for interpolation.

Consider $y = f(x)$, which, for equidistant values of x_0, x_1, x_2, x_3, . . . ,x_n, takes the values y_0, y_1, y_2, y_3, . . . , y_n (Fig. 5-3).

For a polynomial $\phi(x)$ of degree n, $\phi(x)$ can be written as:

$$\phi(x) = a_0 + a_1(x - x_0) + a_2(x - x_0)(x - x_1)$$
$$+ a_3(x-x_0)(x-x_1)(x-x_2) + a_4(x-x_0)(x-x_1)(x-x_2)(x-x_3) +$$
$$\cdots + a_n(x - x_0)(x - x_1)(x - x_2)(x - x_3) \cdots (x - x_{n-1})$$

When $\Delta x = h$ one can write Newton's formula for *forward interpolation.* First, the coefficients $a_0, a_1, a_2, \ldots, a_n$ are determined so that:

$$\phi(x_0) = y_0, \phi(x_1) = y_1, \phi(x_2) = y_2, \ldots, \phi(x_n) = y_n$$

where $x_1 - x_0 = h$, $x_2 - x_0 = 2h$, and so on.
One form of Newton's formula appears as:

$$\phi(x) = y_0 + \frac{\Delta y_0}{h}(x - x_0) + \frac{\Delta^2 y_0}{2!h^2}(x - x_0)(x - x_1)$$
$$+ \frac{\Delta^3 y_0}{3!h^3}(x - x_0)(x - x_1)(x - x_2)$$
$$+ \frac{\Delta^4 y_0}{4!h^4}(x - x_0)(x - x_1)(x - x_2)(x - x_3)$$
$$+ \cdots + \frac{\Delta^n y_0}{n!h^n}(x - x_0)(x - x_1)(x - x_2) \cdots (x - x_{n-1})$$

Also, if we write $\dfrac{x - x_0}{h} = u$, then the preceding formula becomes:

$$\phi(x) = y_0 + u\Delta y_0 + \frac{u(u-1)}{2!}\Delta^2 y_0 + \frac{u(u-1)(u-2)}{3!}\Delta^3 y_0$$
$$+ \frac{u(u-1)(u-2)(u-3)}{4!}\Delta^4 y_0 + \cdots$$
$$+ \frac{u(u-1)(u-2)(u-3) \cdots (u-n+1)}{n!}\Delta^n y_0$$

These equations are known as the *forward interpolation* formulas of Newton, since they contain values of y_0 forward (to the right).

Newton's formula for *backward interpolation,* used for values to the left of a given y (backward from the y), can also be very useful. Consider the polynomial $\phi(x)$ in the form:

$$\phi(x) = a_0 + a_1(x - x_n) + a_2(x - x_n)(x - x_{n-1})$$
$$+ a_3(x - x_n)(x - x_{n-1})(x - x_{n-2})$$
$$+ a_4(x - x_n)(x - x_{n-1})(x - x_{n-2})(x - x_{n-3})$$
$$+ \cdots + a_n(x - x_n)(x - x_{n-1})(x - x_{n-2}) \cdots (x - x_1)$$

By proper substitution, the formula for backward interpolation usually appears as:

$$\phi(x) = y_n + u\Delta^1 y_n + \frac{u(u+1)}{2!}\Delta^2 y_n + \frac{u(u+1)(u+2)}{3!}\Delta^3 y_n$$

$$+ \frac{u(u+1)(u+2)(u+3)}{4!}\Delta^4 y_n + \cdots$$

$$+ \frac{u(u+1)(u+2)(u+3)\cdots(u+n-1)}{n!}\Delta^n y_n$$

LAGRANGE'S FORMULA

Another method of interpolation involves the use of Lagrange's formula, which can be expressed as:

$$y(x) = \frac{(x-x_1)(x-x_2)\cdots(x-x_n)}{(x_0-x_1)(x_0-x_2)\cdots(x_0-x_n)}y(x_0)$$

$$+ \frac{(x-x_0)(x-x_2)\cdots(x-x_n)}{(x_1-x_0)(x_1-x_2)\cdots(x_1-x_n)}y(x_1)$$

$$+ \frac{(x-x_0)(x-x_1)\cdots(x-x_n)}{(x_2-x_0)(x_2-x_1)\cdots(x_2-x_n)}y(x_2) + \cdots$$

$$+ \frac{(x-x_0)(x-x_1)\cdots(x-x_{n-1})}{(x_n-x_0)(x_n-x_1)\cdots(x_n-x_{n-1})}y(x_n)$$

This formula is useful in finding any desired value of a given function for which there are not equidistant values of the independent variable, or when one needs to locate values of the independent variable for a given value of the function.

Consider the following example. Suppose we wish to find the logarithm of 565.6, and we have the following values:

x	560.0	564.3	571.0	573.6
log x	2.74819	2.75151	2.75664	2.75861

Thus:

$$x = 565.6$$

$$x_0 = 560.0$$

$$x_1 = 564.3$$

$$x_2 = 571.0$$

$$x_3 = 573.6$$

And the differences of these values are as follows:

$$\begin{array}{lll}
x - x_1 = 1.3 & x - x_0 = 5.6 & x_2 - x_3 = -2.6 \\
x - x_2 = -5.4 & x_1 - x_0 = 4.3 & x_3 - x_0 = 13.6 \\
x - x_3 = -8.0 & x_1 - x_2 = -6.7 & x_3 - x_1 = 9.3 \\
x_0 - x_1 = -4.3 & x_1 - x_3 = -9.3 & x_3 - x_2 = 2.6 \\
x_0 - x_2 = -11.0 & x_2 - x_0 = 11.0 & \\
x_0 - x_3 = -13.6 & x_2 - x_1 = 6.7 &
\end{array}$$

$$y = \log x = \frac{(1.3)(-5.4)(-8.0)}{(-4.3)(-11.0)(-13.6)}\,2.74819$$

$$+\frac{(5.6)(-5.4)(-8.0)}{(4.3)(-6.7)(-9.3)}\,2.75151$$

$$+\frac{(5.6)(1.3)(-8.0)}{(11.0)(6.7)(-2.6)}\,2.75664$$

$$+\frac{(5.6)(1.3)(-5.4)}{(13.6)(9.3)(2.6)}\,2.75861$$

$$\log 565.6 = 2.75251$$

There are other interpolation formulas (Stirling, Bessel, and Gauss) which have different degrees of accuracy for approximating a function. For periodic functions, trigonometric formulas are often used. *Hermite's formula* for periodic functions is:

$$y(x) = \frac{\sin(x-x_1)\sin(x-x_2)\cdots\sin(x-x_n)}{\sin(x_0-x_1)\sin(x_0-x_2)\quad\sin(x_0-x_n)}\,y(x_0)$$

$$+\frac{\sin(x-x_0)\sin(x-x_2)\cdots\sin(x-x_n)}{\sin(x_1-x_0)\sin(x_1-x_2)\cdots\sin(x_1-x_n)}\,y(x_1)+\cdots$$

$$+\frac{\sin(x-x_0)\sin(x-x_1)\cdots\sin(x-x_{n-1})}{\sin(x_n-x_0)\sin(x_n-x_1)\cdots\sin(x_n-x_{n-1})}\,y(x_n)$$

NUMERICAL DIFFERENTIATION AND INTEGRATION

One can calculate the derivative of a function, given a set of values for that function. By representing the function as an interpolation formula and differentiating, one can find any desired derivative.

For example, from Stirling's interpolation formula, one can derive:

$$\left(\frac{dy}{dx}\right)_{x_0}=\frac{1}{h}\left(\frac{\Delta y_{-1}+\Delta y_0}{2}-\frac{1}{3!}\frac{\Delta^3 y_{-2}+\Delta^3 y_{-1}}{2}\right.$$

$$\left.+\frac{1\cdot 2^2}{5!}\frac{\Delta^5 y_{-3}+\Delta^5 y_{-2}}{2}+\cdots\right)$$

$$\left(\frac{d^2y}{dx^2}\right)_{x_0}=\frac{1}{h^2}\left(\frac{2\Delta^2 y_{-1}}{2!}-\frac{2}{4!}\Delta^4 y_{-2}\right.$$

$$\left.+\frac{2\cdot 2^2}{6!}\Delta^6 y_{-3}-\frac{2\cdot 2^2\cdot 3^2}{8!}\Delta^8 y_{-1}-\cdots\right)$$

$$\left(\frac{d^3y}{dx^3}\right)_{x_0}=\frac{1}{h^3}\left(\frac{\Delta^3 y_{-2}+\Delta^3 y_{-1}}{2!}-\frac{30}{5!}\frac{\Delta^5 y_{-3}+\Delta^5 y_{-2}}{2!}+\cdots\right)$$

In much the same general manner one can find the value of a definite integral $\int_a^o y(x)dx$ by replacing $y(x)$ with the interpolation formula and integrating. Although the general formulas are complex, there are certain simplifications such as *Simpson's Rule*:

$$\int_{x_0}^{x_0+nh} y(x)dx=\frac{h}{3}\sum_{k=0}^{n} c_k y(x_k)$$

where,

c_k is 1, 4, 2, ..., 2, 4, 1.

Another formula for numerical integration is Weddle's Rule:

$$\int_{x_0}^{x_0+nh} y(x)dx=\frac{3h}{10}\sum_{i=0}^{n} k_i y(x_i)$$

where for $i=0, 1, 2, \ldots, k_i$ is 1, 5, 1, 6, 1, 5, 2, 5, 1, 6, 1, 5, 2, ...

NEWTON-RAPHSON METHOD OF APPROXIMATE SOLUTION OF EQUATIONS

Consider a function $f(x)$, where the first approximation of the desired root x is x_0. The correction required (h) is such that $x = x_0 + h$. By Taylor's Series the expansion of $f(x_0 + h)$ is:

$$f(x_0 + h) = f(x_0) + hf'(x_0) + \frac{h_2}{2!}f''(x_0) + \cdots$$

Putting $f(x_0 + h) = 0$, i.e., $x_0 + h$ is a root of $f(x)$, and neglecting terms in h of the second order and higher, we have:

$$f(x_0) + hf(x_0) = 0$$

The first approximation (h_1) of h is then:

$$h_1 = \frac{f(x_0)}{f'(x_0)}$$

The next step is similar to the first:

$$x_1 = x_0 + h_1 = x_0 - \frac{f(x_0)}{f'(x_0)}$$

This process may be continued to any desired approximation of x. For example, consider $f(x) = ax^2 + bx + c = 0$. Then:

$$f'(x) = 2ax + b$$

$$x_1 = \frac{ax_0^2 - c}{2ax_0 + b}$$

If the coefficients of the preceding equation are $a = 1$, $b = 0$, and $c = -u$, then the algebraic solution is $x = \sqrt{u}$. And the first numerical approximation is:

$$x_1 = \frac{1}{2}\left(x_0 + \frac{u}{x_0} \right)$$

Consider another example:

$$f(x) = 2x^3 - 7x^2 - 10x + 20$$

By graphical means, the roots are located at approximate values of -2, 1, and 4. Now:

$$f'(x) = 6x^2 - 14x - 10$$

Take $x_0 = -2$. Then:

$$\begin{aligned} h_1 &= -\frac{f(x_0)}{f'(x_0)} \\ &= -\frac{2(-8) - 7(4) + 20 + 20}{6(4) - 14(-2) - 10} \\ &= -\frac{-16 - 28 + 40}{24 + 28 - 10} = -\frac{-4}{42} \\ &= .095 \end{aligned}$$

Hence:

$$\begin{aligned} x_1 &= -2 + .095 \\ &= -1.905 \end{aligned}$$

The process can be continued as many times as required.

CHAPTER 6

Other Techniques

This chapter covers some additional techniques and methods. While it is not feasible, because of the limitations of space, to include all methods possible, iteration, matrices, and linear programming are covered in this chapter. All three are techniques used for problem solving in computers.

ITERATION

One of the most powerful tools for problem solving is iteration. For an equation such as $3x - 4 \log x = 17$, for example, one can write $x = 4 \log x + 17$ and then substitute a "good guess" for x. The equation is then evaluated, and the resulting value is used in a further substitution that results in a more nearly correct value. This method is self-correcting for errors in computation, and it is not difficult to program for a computer.

Consider a system of three equations as:

$$30x + 6y - z = 40 \qquad (1)$$

$$4x + 20y + 2z = 63 \qquad (2)$$

$$x + 2y + 40z = 50 \qquad (3)$$

Solving for each variable, we have:

$$x = \frac{1}{30}(40 - 6y + z) \qquad (4)$$

$$y = \frac{1}{20}(63 - 4x - 2z) \qquad (5)$$

$$z = \frac{1}{40}(50 - 2y - x) \qquad (6)$$

Let $y = 0$ and $z = 0$ in Equation (4). Thus:

$$x^{(1)} = \frac{40}{30} = 1.33$$

This is the *first iteration* for x, or $x^{(1)}$.

In (5), let $z = 0$ and $x = 1.33$. Then we have:

$$y^{(1)} = \frac{1}{20}(63 - 5.32) = 2.88$$

This is the first iteration for y, or $y^{(1)}$.

Let $x = 1.33$ and $y = 2.88$ in (6):

$$z^{(1)} = \frac{1}{40}(43.09) = 1.08$$

This is the first iteration for z, or $z^{(1)}$. Note that each new value is used in the following equations as soon as that value has been determined:

$$x^{(2)} = \frac{1}{30}(41.08 - 17.28) = 0.793$$

$$y^{(2)} = \frac{1}{20}(63 - 5.332) = 2.883$$

$$z^{(2)} = \frac{1}{40}(50 - 6.559) = 1.086$$

And the third iterations are:

$$x^{(3)} = \frac{1}{30}(41.086 - 17.298) = 0.793$$

$$y^{(3)} = \frac{1}{20}(63 - 5.344) = 2.883$$

$$z^{(3)} = \frac{1}{40}(50 - 6.559) = 1.086$$

The successive iterations are listed in Table 6-1.

Note that the true values are given as I_2 or I_3, and, in general, the iteration is continued until the unknown values do not change.

Table 6-1. Iterated Solutions of Equations 1-3

I	x	y	z
I_1	1.33	2.88	1.08
I_2	0.793	2.883	1.086
I_3	0.793	2.883	1.086

Iteration can also be used for certain equations. Consider an equation, such as $f(x) = 0$, that can be expressed as $x = l(x)$. A value for x_0 is determined by an approximation, which is substituted so that:

$$x^{(1)} = l(x_0)$$

Succeeding approximations are:

$$x^{(2)} = l(x^{(1)})$$
$$x^{(3)} = l(x^{(2)})$$
$$x^{(4)} = l(x^{(3)})$$
$$\cdot \quad \cdot \quad \cdot$$
$$x^{(n)} = l(x^{(n-1)})$$

Consider an example of the form $2x - \log x = 6$. This can be rewritten as:

$$y_1 = \log x, \qquad y_2 = 2x - 6$$

It is possible to plot the intersection of this pair to obtain an approximate root, or other means can be used. A plot of these equations would provide a value of about 3.25, but suppose that a first guess is incorrect, such as 4.8. This then can be used to obtain the proper value.

Starting with the approximation $x = 4.8$, we obtain:

$$x^{(1)} = \frac{1}{2}(\log 4.8 \quad + 6) = \frac{1}{2}(6.6812) = 3.3406$$

$$x^{(2)} = \frac{1}{2}(\log 3.34 + 6) = \frac{1}{2}(6.5237) = 3.2618$$

$$x^{(3)} = \frac{1}{2}(\log 3.26 + 6) = \frac{1}{2}(6.5132) = 3.2566$$

$$x^{(4)} = \frac{1}{2}(\log 3.25 + 6) = \frac{1}{2}(6.5119) = 3.2559$$

$$x^{(5)} = \frac{1}{2}(\log 3.25 + 6) = \frac{1}{2}(6.5119) = 3.2559$$

Thus, after five iterations we have $x^{(5)} = 3.2559$. But, using five-place logarithms, we obtain:

$$x^{(5)} = \frac{1}{2}(\log 3.256 \quad + 6) = \frac{1}{2}(6.51268) = 3.25634$$

$$x^{(6)} = \frac{1}{2}(\log 3.2563 + 6) = \frac{1}{2}(6.51272) = 3.25636$$

Therefore, the result is 3.2559 ($x^{(4)}$ and $x^{(5)}$), while succeeding iterations show that the value is between 3.25634 and 3.25636. Fig. 6-1 shows to four figures the approach of x to the final value.

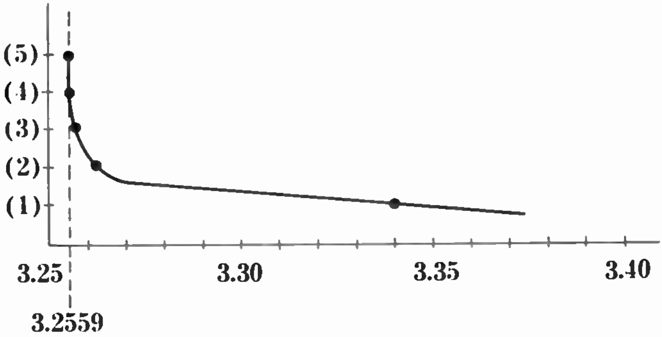


Fig. 6-1. Iteration of $x = f(x)$.

MATRICES

A matrix X is a rectangular array of numbers such as:

$$\mathbf{X} = \begin{pmatrix} x_{11} & x_{12} & \cdots & x_{1n} \\ x_{21} & x_{22} & \cdots & x_{2n} \\ \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot \\ x_{m1} & x_{m2} & \cdots & x_{mn} \end{pmatrix}$$

X represents an ordered array of real numbers, while m and n are integers. This is an $m \times n$ *matrix* of m rows and n columns. If $m = n$, the matrix is called a *square* matrix.

An example of a 1 × 3 matrix, which is also a row-vector, is:

$$(7, \ 0, \ 6)$$

And a 3 × 1 matrix, or a column-vector, is:

$$\begin{pmatrix} 7 \\ 0 \\ 6 \end{pmatrix}$$

Consider the example of a book publisher who has three types of books, A, B, and C. Suppose in a specific case there are 5 of kind A, 3 of kind B, 10 of kind C. Each kind takes certain hours of typesetting, illustrations, printing, and editing, as in Table 6-2.

Table 6-2. Factors in Book Publishing

Books	Type	Illustrations	Print	Edit
A	140	100	50	200
B	200	50	15	100
C	350	200	500	30

X is the row-vector (5, 3, 10), and R is a 3 × 4 matrix. We now wish to find XR.

$$XR = (5,3,10)\begin{pmatrix} 140 & 100 & 50 & 200 \\ 200 & 50 & 15 & 100 \\ 350 & 200 & 500 & 30 \end{pmatrix}$$

$$= (5(140) + 3(200) + 10(350), 5(100) + 3(50) + 10(200), 5(50) + 3(15) + 10(500), 5(200) + 3(100) + 10(30)\ )$$

$$= (4{,}800, 2{,}650, 5{,}295, 1{,}600)$$

Thus there are 4,800 hours of typesetting, 2,650 hours of illustrations, 5,295 hours of printing, and 1,600 hours of editing required.

Now, for example, it is possible to establish a cost-per-hour of $8 for typesetting, $6 for illustrating, $5 for printing, and $7 for editing. This is a matrix Z such that:

$$Z = \begin{pmatrix} 8 \\ 6 \\ 5 \\ 7 \end{pmatrix}$$

We want to find the product RZ:

$$RZ = \begin{pmatrix} 140 & 100 & 50 & 200 \\ 200 & 50 & 15 & 100 \\ 350 & 200 & 500 & 30 \end{pmatrix}\begin{pmatrix} 8 \\ 6 \\ 5 \\ 7 \end{pmatrix}$$

$$= \begin{pmatrix} 8(140) + 6(100) + 5(50) + 7(200) \\ 8(200) + 6(50) + 5(15) + 7(100) \\ 8(350) + 6(200) + 5(500) + 7(30) \end{pmatrix}$$

$$RZ = \begin{pmatrix} 3{,}370 \\ 2{,}675 \\ 6{,}710 \end{pmatrix}$$

So that the cost of the "A" books is $3,370; the "B" books cost $2,675, and the "C" books cost $6,710.

VECTORS

A vector is a special form of numerical notation for an ordered collection of numbers. A column-vector, for example, is a series of numbers written as:

$$\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} \qquad \begin{pmatrix} 1 \\ 3 \end{pmatrix} \qquad \begin{pmatrix} 9 \\ -6 \\ 2 \end{pmatrix} \qquad \begin{pmatrix} 1 \\ 6 \\ 0 \end{pmatrix}$$

The components of the vector are the individual numbers, such as x_1, x_2, x_3. Thus, the first vector above is a three-component column-vector, the last two are three-component column-vectors, and the second is a two-component column-vector. In the same manner, (1,7), (6,0), (4,–3, 2), and (0,0,6) are row-vectors.

Addition is defined on a component-addition basis. If $\mathbf{x}$ and $\mathbf{y}$ are:

$$\mathbf{x} = \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} \qquad \text{and} \qquad \mathbf{y} = \begin{pmatrix} y_1 \\ y_2 \\ y_3 \end{pmatrix}$$

then:

$$\mathbf{x} + \mathbf{y} = \begin{pmatrix} x_1 + y_1 \\ x_2 + y_2 \\ x_3 + y_3 \end{pmatrix}$$

The addition of row-vectors is defined similarly.

Multiplication is also on a component basis. If:

$$\mathbf{x} = \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix}$$

then:

$$a\mathbf{x} = \begin{pmatrix} ax_1 \\ ax_2 \\ ax_3 \end{pmatrix}$$

Note that since a can be negative, subtraction is defined as negative addition.

The *zero-vector* is defined as:

$$\mathbf{0} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}, \qquad \text{or} \qquad \mathbf{0} = (0,0,0)$$

And thus $\mathbf{x} + \mathbf{0} = \mathbf{x}$.

Vectors allow group of numbers to be treated as a single unit, which is very useful in a range of problems. Here are some examples:

$$\mathbf{x} = \begin{pmatrix} 3 \\ 7 \\ 1 \end{pmatrix}, \quad \mathbf{y} = \begin{pmatrix} 2 \\ 0 \\ 1 \end{pmatrix}, \quad \mathbf{z} = \begin{pmatrix} 1 \\ 2 \\ 4 \end{pmatrix}$$

To evaluate $2\mathbf{x} + 3\mathbf{y} - \mathbf{z}$ we form the expression:

$$\begin{aligned} 2\mathbf{x} + 3\mathbf{y} - \mathbf{z} = 2 & \begin{pmatrix} 3 \\ 7 \\ 1 \end{pmatrix} + 3 \begin{pmatrix} 2 \\ 2 \\ 1 \end{pmatrix} - \begin{pmatrix} 1 \\ 2 \\ 4 \end{pmatrix} \\ = & \begin{pmatrix} 6 \\ 14 \\ 2 \end{pmatrix} + \begin{pmatrix} 6 \\ 6 \\ 3 \end{pmatrix} - \begin{pmatrix} 1 \\ 2 \\ 4 \end{pmatrix} \\ = & \begin{pmatrix} 2 + 3 - 4 \\ 14 + 6 - 2 \\ 6 + 6 - 1 \end{pmatrix} = \begin{pmatrix} 1 \\ 18 \\ 11 \end{pmatrix} \end{aligned}$$

Multiplication of row-vectors and column-vectors is defined as follows. If:

$$\mathbf{x} = (x_1, x_2, x_3) \qquad \text{and} \qquad \mathbf{y} = \begin{pmatrix} y_1 \\ y_2 \\ y_3 \end{pmatrix}$$

then:

$$\mathbf{xy} = x_1y_1 + x_2y_2 + x_3y_3$$

Thus the product of two vectors is not itself a vector, but is instead a number. As an example, consider:

$$\mathbf{x} = (3, -2, 1), \qquad \text{and} \qquad \mathbf{y} = \begin{pmatrix} 1 \\ 4 \\ 2 \end{pmatrix}$$

Then:

$$\mathbf{xy} = 3 - 8 + 2 = -3$$

Another means of using vectors can be seen from the simple drawing in Fig. 6-2. There is a pair of axes, x_1 and x_2, as shown.

And two vectors **x** and **y**, where $\mathbf{x} = (a, b)$ and $\mathbf{y} = (c, d)$. From this point $\mathbf{x} + \mathbf{y}$ is:

$$\mathbf{x} + \mathbf{y} = (a, b) + (c, d)$$
$$\mathbf{x} + \mathbf{y} = (a + c, b + d)$$

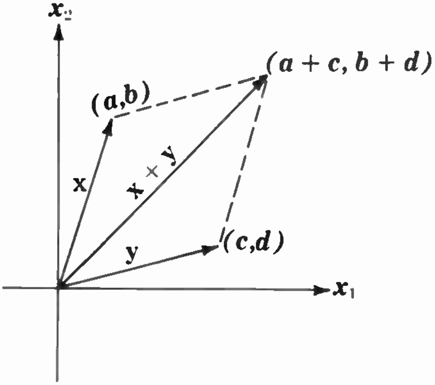


Fig. 6-2. Equation-solving by vectors.

LINEAR PROGRAMMING

Consider the linear equation $3x + 2y = 12$ as in Fig. 6-3. This equation may be plotted as a straight line, in a very simple and direct manner, since for $x = 0$, $y = 6$ and for $y = 0$, $x = 4$.

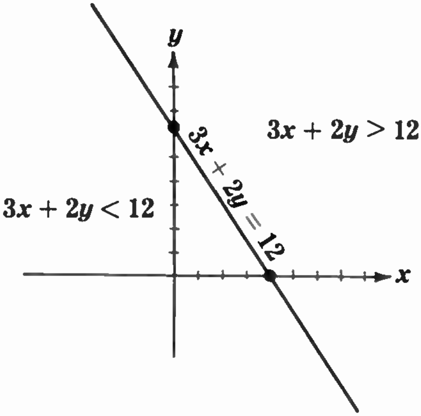


Fig. 6-3. Solving a linear equation.

Thus this straight line is the locus of all points that satisfy this equation, such as (6,0), (4,0), and (2,3). Any number of points can be found by assuming a value for x or y, and substituting it in the equation.

Now, suppose that $3x + 2y = 12$ is now written as an inequality of the form $3x + 2y < 12$. We wish to find the locus of all points that satisfy this inequality. Take a given point such as (2,3) on this line. For $x = 2$ any y less than 3 will satisfy this inequality. Any value of y greater than 3, for $x = 2$, will be above the line and will *not* satisfy the inequality. If we take $y = 3$, any $x < 3$ will again satisfy the inequality, while any $x > 3$ will not. Thus any (x, y) point below the line will satisfy the inequality $3x + 2y < 12$, and this area is known as an open half-plane whose area contains all these points.

One can show that all points above the line $3x + 2y = 12$ are in the other open half-plane corresponding to the inequality $3x + 2y > 12$. An inequality of the form $3x + 2y \leqq 12$ is made up of the open half-plane of all points below the line and the line itself; this combination is known as a closed half-plane.

Thus, for the general equation $ax + by = c$, we have:

(1) The locus of solutions is always a straight line.
(2) The inequalities $ax + by > c$ and $ax + by < c$ each have an open half-plane as its locus.
(3) The inequalities $ax + by \geqq c$ or $ax + by \leqq c$ each have a closed half-plane as its locus.

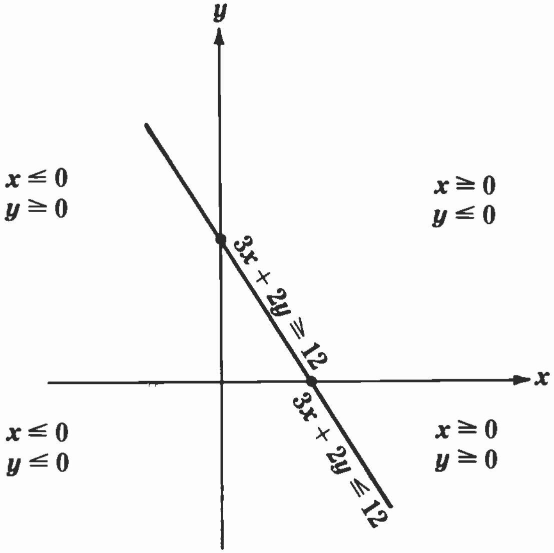


Fig. 6-4. Area defined by a system of inequalities.

Consider now a system of inequalities (Fig. 6-4):

$$x \geqq 0$$
$$y \geqq 0$$
$$3x + 2y \leqq 12$$

The locus of $x \geqq 0$ is the first and fourth quadrants, which make up the closed half-plane; the locus of $y \geqq 0$ is the first and second quadrants. Hence, only the first quadrant corresponds to both $x \geqq 0$ and $y \geqq 0$. The locus of $3x + 2y \leqq 12$ is the closed half-plane below the straight line (but including it).

From this the area that includes all points of the loci of all three inequalities is the triangle (0,6), (0,0), (4,0). This area, which is the intersection of closed half-planes, is known as a *polygonal convex set.*

Note that at any corner of the triangle, there are two and only two inequalities which become equalities at that corner. At any point on a boundary, only one of the inequalities becomes an equality; and at any interior point of the triangle there is no inequality which becomes an equality.

Consider another example. The inequalities are:

$$x > -1$$
$$y > 1$$
$$x < 2$$
$$2x + y \leqq 6$$

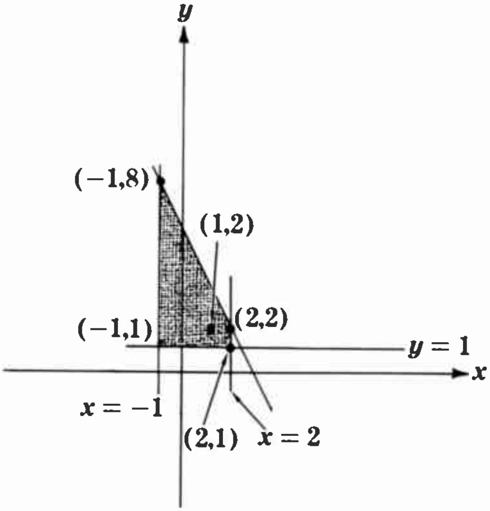


Fig. 6-5. Boundaries of *x* and *y*.

By considering $x = -1$ and $x = 2$ we establish the vertical lines as shown in Fig. 6-5. Considering $y = 1$ and $2x + y = 6$ the other boundaries are established. The inequalities for points at the vertices of the figure are shown in Table 6-3.

Table 6-3. Values at the Vertices of the Trapezoid

Point	Inequality			
	$x \geqq -1$	$y \geqq 1$	$x \leqq 2$	$2x + y \leqq 6$
(1, 8)	$x = 1$	$y = 8$	$x = 1$	$10 < 6$
(−1, 1)	$x = -1$	$y = 1$	$x = -1$	$-1 < 6$
(2, 1)	$x = 2$	$y = 1$	$x = 2$	$5 < 6$
(2, 2)	$x = 2$	$y = 2$	$x = 2$	$6 = 6$

RELAXATION METHODS

A powerful technique of problem solving, which can be used if other methods fail, is the method of *relaxation*. Consider a two-dimensional surface or membrane as in Fig. 6-6 as shown on the grid structure. Some points on this curve, as shown, are (x_1, y_2), (x_2, y_1), (x_2, y_4), (x_3, y_3), and so on. Just as a single line $(x + 3y)$ or a curve $(x^2 - 3y + xy)$ represents a set of points which can form an open loop, so the set of points above can define (enclose) an area.

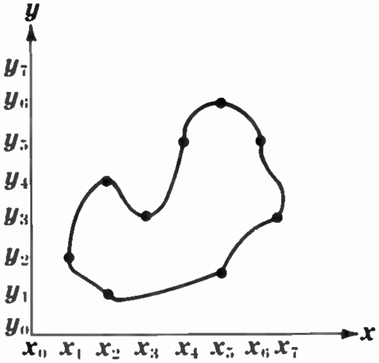


Fig. 6-6. Use of relaxation method.

Consider, as a simplification, that the curve in the figure represents a boundary of a surface with x, y, and z coordinates, but we are concerned with only the x, y relations over the surface (the z coordinates are ignored). Boundary conditions are:

$$\begin{array}{lll}
x_0y_0 = 3.0 & x_0y_1 = 17.0 & x_2y_3 = 9.0 \\
x_1y_0 = 0.0 & x_0y_2 = 10.0 & x_3y_1 = 7.5 \\
x_2y_0 = 0.0 & x_0y_3 = 5.0 & x_3y_2 = 5.0 \\
x_3y_0 = 6.0 & x_1y_3 = 7.5 & x_3y_3 = 11.0
\end{array}$$

In this example (Table 6-4) the unknown points, shown as open boxes, are to be determined. Any "guess" could be used for these values, say 5. This value, which is not correct since it is at best an approximation of the proper value, is "relaxed" toward its surrounding values. In the upper left box we use 5 for the first guess.

Table 6-4. Solving Problem by Relaxation Method

y_3	5.0	7.5	9.0	11.0
y_2	10.0	□	□	5.0
y_1	17.0	□	□	7.5
y_0	3.0	0.0	0.0	6.0
	x_0	x_1	x_2	x_3

(A)

y_3	5.0	7.5	9.0	11.0
y_2	10.0	7.3	6.5	5.0
y_1	17.0	6.7	4.8	7.5
y_0	3.0	0.0	0.0	6.0
	x_0	x_1	x_2	x_3

(B)

y_3	5.0	7.5	9.0	11.0
y_2	10.0	7.7	6.6	5.0
y_1	17.0	7.3	5.3	7.5
y_0	3.0	0.0	0.0	6.0
	x_0	x_1	x_2	x_3

(C)

y_3	5.0	7.5	9.0	11.0
y_2	10.0	7.9	6.7	5.0
y_1	17.0	7.5	5.4	7.5
y_0	3.0	0.0	0.0	6.0
	x_0	x_1	x_2	x_3

(D)

To obtain a closer value we average the surrounding numbers as 1/4 (7.5 + 10.0 + 5.0 + 5.0) = 7.3. This "better" value replaces 5, and by the same technique, the other "first-guesses" are replaced by nearer approximations. The result is the set of values shown in B. Here the central square of:

5	5
5	5

has become:

7.3	6.5
6.7	4.8

After another relaxation, the table in C results in:

7.7	6.6
7.3	5.3

And, after another relaxation (D) this is:

7.9	6.7
7.5	5.4

In this way one can arrive at a final set of values for the initial conditions.

CHAPTER 7

Other Mathematical Forms

The material in the preceding chapters has dealt with the classical mathematics of computation or numerical analysis. There are, however, other methods and techniques—indeed other forms of mathematics—that can be programmed and used on the digital computer to solve many classes of problems.

One of these is the use of symbolic logic or Boolean algebra, which is a mathematical expression of a logical system that has become exceedingly useful in circuit design, such as for control systems using complex switching arrangements. Symbolic logic is used in the programming of certain specialized types of computer language, such as COBOL (*Common Business Oriented Language*), FORTRAN (*FORmula TRANslation*), and ALGOL (a special form of algebraic language).

SYMBOLIC LOGIC

Boolean algebra derives its name from George Boole, who, in 1847, first introduced it in a paper on the mathematical analysis of logic. Until 1938, however, no practical use was found for this new algebra. Then Claude E. Shannon pointed out its application to telephone and computer switching circuits.

This extremely simple algebra provides a convenient means of representing a switching circuit without a drawing of the circuit. It also provides a means for quickly finding many circuits that will do the same thing. With a little practice a designer using Boolean algebra may quickly design "good" switching circuits. Although Boolean algebra will always suggest a good circuit, it will not always give the best circuit. (The best circuit is the circuit with the least number of components.)

Symbolic logic (Boolean algebra) is a language that can be easily programmed for a certain class of problems. The symbolic logic equation often can be manipulated into a simpler form, saving switches or gates in the implementation. The symbolic logic equation expresses the operation of a system or a circuit in terms of the basic symbolic logic operations AND, OR, and NOT. To allow a single logical function to be used for a complete system, two contemporary techniques involve NOR and NAND logic (NOT-OR and NOT-AND). The equation is developed by use of logic charts, truth tables, or simple statements of fact and can be implemented by switches and gates.

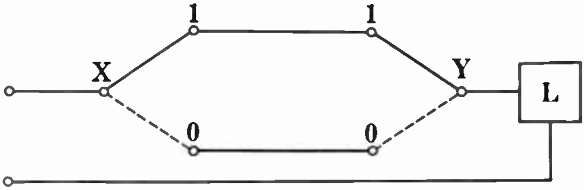


Fig. 7-1. Light controlled by 2 two-position switches in series.

If X and Y are independent variables, and A is the dependent variable, some typical equations are:

$A = X \cdot Y$ (A equals X and Y, as where X and Y are switches or relays in series.)

$A = X \vee Y$ (A equals X OR Y, as where X and Y are switches or relays in parallel.)

Consider a "1" equal to a closed switch, or a true statement, while a "0" represents an open switch, or a false statement. If A = 1, then $\overline{A}$ (NOT A) = 0.

Table 7-1. Truth Table for Light Controlled by Two Switches

X	Y	Light	Condition for Light On
0	0	1	$\overline{X} \cdot \overline{Y}$
0	1	0	
1	1	1	X•Y
1	0	0	

For example, Fig. 7-1 shows a light switch that is controlled by 2 two-position switches. Since each switch can have only two conditions, 0 and 1, there are only four possibilities, which are shown in the truth table of Table 7-1.

The symbolic-logic equation for having the light on is:

$$L = (\bar{X}\cdot\bar{Y}) \vee (X\cdot Y)$$

This is the result of two possible "light on" conditions which are $\bar{X}\cdot\bar{Y}$ as one and $X\cdot Y$ as the other.

It is desirable, of course, to reduce the symbolic logic equation to its simplest form, in order to implement it most economically. One valuable method is *De Morgan's Theorem*, which states that one can invert a statement by doing the following: (1) Change each OR into an AND; (2) change each AND into an OR; and (3) invert each term, (A, B, etc). That is, the inversion of A v B is:

$$\overline{A \vee B} = \bar{A}\cdot\bar{B}$$

This means NOT (A OR B) equals NOT A and NOT B.

By using De Morgan's Theorem, all AND-circuit gates can be changed into OR-circuit gates (or vice versa). Table 7-2 shows the truth table for the conversion of $\overline{A \text{ OR } B}$ into $\bar{A}$ AND $\bar{B}$. De Morgan's Theorem states that the expression (A v B v C) can be inverted as follows:

Table 7-2. Truth Table for $\overline{AvB} = \bar{A}\cdot\bar{B}$

A	B	A or B	$\overline{A \text{ or } B}$	$\bar{A}$	$\bar{B}$	$\bar{A}$ and $\bar{B}$
1	1	1	0	0	0	0
1	0	1	0	0	1	0
0	1	1	0	1	0	0
0	0	0	1	1	1	1

$$\overline{A \vee B \vee C} = \bar{A}\cdot\bar{B}\cdot\bar{C}$$

As the inverse of any logic expression is usually readily obtainable, De Morgan's Theorem becomes a useful method for changing the form of the expression. In the implementation of the two logic expressions, one is equivalent to the other, by De Morgan's Theorem. The proof of this equivalence is shown in the logic chart of Fig. 7-2.

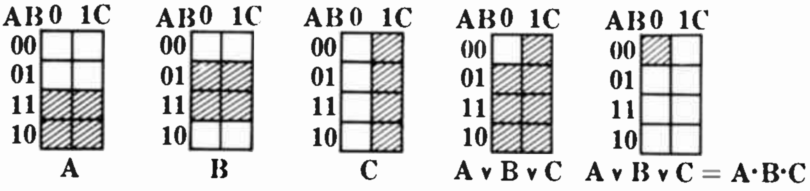


Fig. 7-2. Logic chart for $\overline{A \vee B \vee C} = \bar{A}\cdot\bar{B}\cdot\bar{C}$.

Some logic expressions can be shown to be redundant. Consider:

$$X \cdot X = X \quad (1 \text{ AND } 1 = 1;\ 0 \text{ AND } 0 = 0)$$
$$X \vee X = X \quad (1 \text{ OR } 1 = 1;\ 0 \text{ OR } 0 = 0)$$

Another redundant expression is:

$$X \cdot (X \vee Y) = X \vee (X \cdot Y) = X$$

The logic chart (Fig. 7-3) shows that this expression is the same as X. The switch analogy for this is shown in Fig. 7-4.

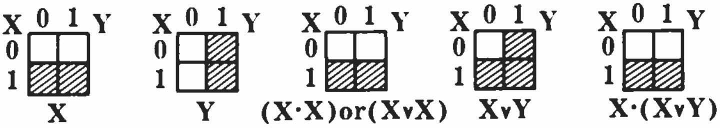


Fig. 7-3. Redundancy in logic.

Note that both $X \cdot (X \vee Y)$ and $X \vee (X \cdot Y)$ are equivalent to the simple expression X. In other words, if X is required, then other OR expressions involving X are redundant. From this it follows that:

$$X \cdot (X \vee Y \vee W) = X$$
$$X \cdot (X \vee W \vee T \vee Z) = X$$

Hence, each of these equations is redundant.

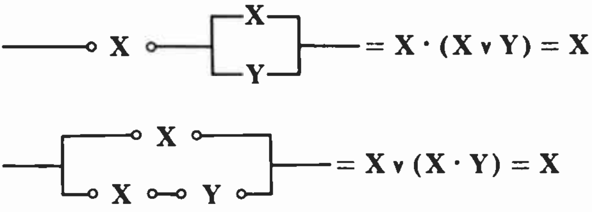


Fig. 7-4. Logically equivalent circuits.

SWITCHING CIRCUIT SIMPLIFICATION

Consider the expression:

$$X \cdot Y \cdot (X \vee Z) \cdot (Y \vee (X \cdot Z) \vee W) \cdot T$$

By a simple rearrangement of terms, this can be written:

$$Y \cdot (X \cdot (X \vee Z)) \cdot (Y \vee (X \cdot Z) \vee W) \cdot T$$

The first two terms $(X \cdot Y) \cdot (X \vee Z) = Y \cdot (X \cdot (X \vee Z))$ are equivalent to X, and the next term $(Y \vee (X \cdot Z) \vee W)$ is equivalent to Y, as shown in Fig. 7-5. Hence, this expression reduces to $X \cdot Y \cdot T$. All that this means is that if X is open (equal to 0), the entire circuit is open; this is regardless of the status of Y, Z, W, or T. The same thing applies to Y or T; each alone can open the circuit. This is something that cannot be said for W or for Z. Note that if Z is open, there can still be a complete circuit, and the same is true for W. Thus the original circuit reduces to its simplest form, $X \cdot Y \cdot T$.

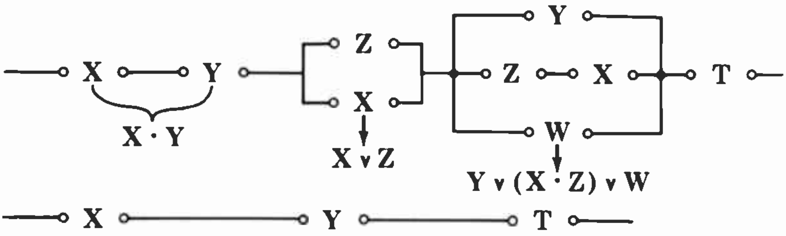


Fig. 7-5. Redundant switching circuit.

Consider the bridge circuit shown in Fig. 7-6A. The complete circuit paths possible are:

$X \cdot Y$	(1, 2, 3)
$X \cdot W \cdot S$	(1, 2, 4, 3)
$V \cdot W \cdot Y$	(1, 4, 2, 3)
$V \cdot S$	(1, 4, 3)

These are shown in Fig. 7-6B, which represents the same circuit in terms of function as that in Fig. 7-6A. One can show that this bridge circuit is the equivalent of the sum (AND's) of all complete "break-lines" characterized as *a*, *b*, *c*, or *d*. These may be represented as:

a is $Y \vee V \vee W$
b is $X \vee W \vee S$
c is $X \vee V$
d is $Y \vee S$

Their sum is:

$$(Y \vee S) \cdot (X \vee V) \cdot (X \vee W \vee S) \cdot (V \vee W \vee Y)$$

The circuit for this equation is shown in Fig. 7-5C. It is:

$$X \vee (V \cdot (W \vee S)) \cdot [Y \vee \{S \cdot (W \vee V)\}]$$

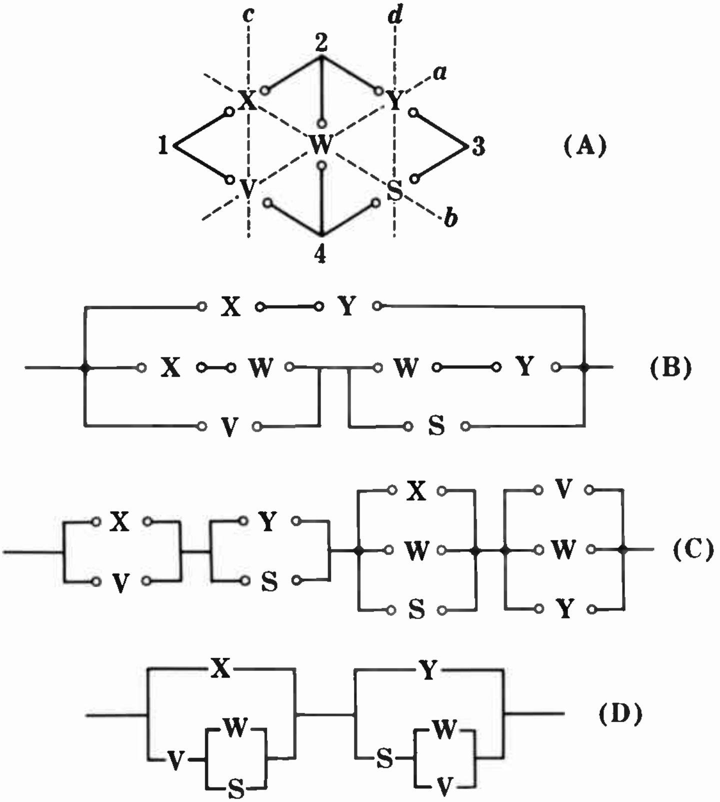


Fig. 7-6. Bridge circuit with associated logic.

This more simple version is shown in Fig. 7-6D. It can be shown that the circuits in Fig. 7-6A, B, C, and D are all equal.

LOGIC IN COMPUTER PROGRAMS

Various form of symbolic logic are used in programs for computers where "reasoning" or decisions are required.

COBOL

The COBOL (*COmmon Business Oriented Languages*) is an English-like programming language which can be used with many different types of data-processing systems. Computer programs can be written directly in machine language or indirectly in a problem-oriented language, which is a shorthand type of notation which requires translation into computer language.

A compiler or compiling system has an input language made up of abstract terms, a stored machine program (a translator) for changing or translating this input language into an output language. This output language may be in machine language that can be used directly on a specific computer, or it can be a language requiring translation. Compilers should also be considered different from interpreters which operate directly on an artificial or abstract language form to produce the computation which is the end purpose of the program.

COBOL is a shorthand for the computer's instructions. Any language is made up of words with certain meanings, and a computer language has specific words, each with a certain meaning; they can only be used with these meanings and no others. The COBOL language is derived from English; it looks like English. The programmer can work with it easily, without having to learn a long list of special symbols and codes, and the rules for using them.

For a computer to be able to interpret a COBOL sentence, the sentence must first be "translated" into the machine's language. As the COBOL system is designed, the COBOL-language program need be translated only once, and the resulting machine-language program can be used and re-used indefinitely without further translation.

A verb form in a COBOL machine language, for example, causes a certain action, and most verbs will cause some action to be taken at the time the program is executed. As an example, the verb *display* will cause specified information to be printed or otherwise displayed, and the verb *stop* will cause the computer to halt.

But there are other words and symbols besides verbs that cause action. The expression A + B − C directs that C is to be subtracted from the sum of A and B. Thus, the symbols + and − are *operators*. Operators usually specify actions or relationships without actually expressing them in verb form. There are three basic kinds of operators: arithmetic operators, relational operators, and logical operators.

The following is a list of arithmetic operators:

+	addition
−	subtraction
*	multiplication
/	division
**	exponentiation

The way in which an arithmetic expression is to be evaluated can best be specified by parentheses. Thus, the expression A*B + C might be considered ambiguous. Does the programmer mean (A*B) + C, or does he mean A*(B + C) ?If parentheses are not written to specify the order of computation, COBOL will evaluate an arithmetic expression using the following rules:

1. All exponentiation is performed first.
2. Then, multiplication and division are performed.
3. Finally, addition and subtraction are performed.
4. In each of the three preceding steps, computation starts at the left of the expression and proceeds to the right. Thus, A*B/C is computed as (A*B)/C, and A/B*C is computed as (A/B)*C.
5. When parentheses are present, computation begins with the innermost set and proceeds to the outermost. Items grouped in parentheses will be evaluated in accordance with the previous rules, and the result will then be treated as if the parentheses were removed.

Relational operators are used to express tests that are to be performed. The statement *if salary = zero* is built around the relationship implied by the equal sign, which is a relational operator. Another statement could be *if age is greater than 21 add A to B*. The words *is greater than* form a relational operator.

The relational operators in the COBOL language are: IS GREATER THAN, IS EQUAL TO, and IS LESS THAN. The words which are not underlined may be omitted, if the programmer desires, with no resulting effect on the meaning of the operator. Relational operators are combined with data to create conditional expressions and statements.

The three logical operators are AND, OR, and NOT; AND and/or OR are used when two or more tests are specified in the same expression; NOT is used to specify the negative of a condition.

Logic is also used for compound expressions. If a compound conditional expression consists of several consecutive, simple relational conditions, and if these conditions have common subjects and/or common relational operators, the common factors may be implied, instead of explicitly repeated in each condition.

For example, the compound conditional expression *married or divorced* means that either or both of these two conditions must be true for the expression as a whole to be true. The expression A AND (B OR C) OR D means one of the following must be true for the expression to be true:

1. D must be true, or
2. A must be true and either B or C (or both) must be true, or
3. Both the above must be true.

COBOL allows the four operations of addition, subtraction, multiplication, and division. For example, consider a case of writing: ADD RECEIPTS TO STOCK-ON-HAND. SUBTRACT SHIPMENTS FROM STOCK ON HAND. COMPUTE STOCK-ON-HAND VALUE. This is clear in meaning to both the computer processor and the casual reader. It is also possible to write this as: COMPUTE STOCK-VALUE = UNIT-PRICE * (STOCK-ON-HAND + RECEIPTS − SHIPMENTS. This will result in the following steps:

1. Take the stock-on-hand (number) and add the receipts.
2. From this number subtract the shipments.
3. Multiply (*) this number of units by the unit price to obtain the stock value.

This is an example of a part of a COBOL program.

ALGOL

ALGOL is a language used for programming algebraic computations much as COBOL is used for problems with business orientation. ALGOL stands for *ALGOrithmic Language*. It has been accepted as a common international language for use with algebraic problems. As with other specialized languages it is not possible to explain its use unless certain general aspects have been established.

A typical use of ALGOL would be to evaluate an expression such as $15x^3 - 7x^2 + 2x - 5$. The range of x may be a series of values such as 3, 10, 15, 41, and 62. Each value is substituted for x, and the expression is worked out. The results are printed out or punched out on cards or punched tape. ALGOL permits a direct means of writing out the required program.

There are various types of ALGOL programs. Those that are intended just for the human reader and not for use on a computer are called the *publication form* of the language. There are various machine (computer) programs by which ALGOL is turned into a program that can solve the specific problem. For simple equations or relationships these steps seem trivial. Actually, for complex programs ALGOL not only saves time, but it also allows the program to be determined without regard to the actual computer's construction and features.

The beginning is the basic letter, number, or symbol that is the character. The characters are of different types; the letters are upper-case A through Z and lower-case a through z. The digits are 0 through 9. The logical values are T for true, and F for false; these are the same as letters T and F in the series of letters, but they are told apart by their use. The typical arithmetic operators are as below:

+	plus (addition)
–	minus (subtraction)
×	multiplication
/	division
	exponentiation (B3 means B^3)

These operators can be used with letters to produce meaningful relations such as a + b, or d – f, or others.

Certain other ALGOL words are treated like symbols and have specific meanings. Words are made up of characters and, in turn, words make up expressions.

An automatic programming statement is like an English sentence or an instruction in computer language. Statements are used in the ALGOL program, which is often a succession of related statements. For example, "read (*a*, *b*);" means read the two numbers available as the input and assign these values to variables *a*, *b*. Thus "read (*a*, *b*);" is a statement, and all statements are separated by semicolons (;), as shown. The symbol : = is a single symbol.

The use of arithmetic operators such as +, –, × and others in expressions requires care, for more than one interpretation is possible in some cases. Here are some examples and their ALGOL meaning:

$a + b - c$	*means*	$(a + b) - c$
$a - b + c$	*means*	$(a - b) + c$
$a + b \quad c + d$	*means*	$a + (b \quad c) + d$
$-a \quad b$	*means*	$-(a \quad b)$
$-a - b - c$	*means*	$((-a) - b) - c$

FORTRAN

FORTRAN (*FORmula TRANslation*) is a mathematical language for computer programming. Using symbols, a program, which is coded in FORTRAN, might be, for example, D = A + X – Y. The number of instructions required for the complete solution of a problem may be a few hundred or many thousands, depending on the problem. The computer refers to them one after another, or it can be instructed to repeat, modify or skip over certain instructions, depending on immediate results or circumstances. However, such circumstances must be anticipated, and appropriate instructions included in the program.

Each FORTRAN program is composed of a number of *statements*. Each statement deals with one aspect of the problem; it may cause data to be fed into the computer, calculations to be performed, decisions to be made, results to be printed, etc.

Some statements written by the programmer do not cause specific computer action, but rather provide information to the processor. Some examples of FORTRAN statements and their effects are:

READ 1,A — This causes the computer to read an IBM card and handle the data on it in such a way that if the card read has the number 106.7 on it, then A will have the value 106.7.

C = 3.*A — The asterisk (*) indicates multiplication; so this statement means multiply A by 3.0, and set C equal to the result.

To find the roots of a quadratic equation, the computer must be told how to find the roots:

$$root = \frac{-B \pm \sqrt{B^2 - 4AC}}{2A}$$

Since there are two roots for the quadratic equation, the computer must be instructed how to find each root separately. Thus, by using the formula, the computer may be instructed as follows:

$$root\ 1 = (-B+\mathrm{SQRTF}\ (B{*}{*}2-4.\ {*}A{*}C))/(2.{*}A)$$

$$root\ 2 = (-B-\mathrm{SQRTF}\ (B{*}{*}2-4.\ {*}A{*}C))/(2.{*}A)$$

Parentheses may be used to specify the order of operation in an expression. Where parentheses are omitted, the order is taken to be from left to right as follows:

**	exponentiation
* and /	multiplication and division
+ and −	addition and subtraction

APPENDIX I

Glossary

accumulator—A portion of the digital computer which can store a number and, upon the proper signal, add a second number to its contents, so that the sum is formed. The accumulator can also be used for storage as well as for addition.

accuracy—This is a measure of the freedom from error. This is not the same as precision, since precision is the degree of fineness, which is a different concept. A five-place logarithm table, for example, can be just as accurate as a six-place table, but the six-place table is more precise than the five-place.

addend—This is a quantity or number to be added to a second number, the augend, to produce a result known as the *sum*.

address—A name or label designates a register used for storage of a number.

analog computer—A type of computer, often electronic, where numerical quantities are represented by electrical variables, and problem solutions are obtained by manipulating these variables.

AND—A logical connection, as in the statement A AND B, which means that the statement is true if, and only if, A is true and B is true simultaneously.

arithmetic operation—Various manipulations of numerical quantities, which include the fundamental operations of addition, subtraction, multiplication, and division.

arithmetic shift—The multiplication or division of a quantity by the power of the base notation. For example, since 1011_2 represents 11 in binary notation, the result of two shifts to the left is 101100_2, which represents 44 in binary notation.

base—The radix or the numbering base. Also a number used to define a positional notation of numbering representation.

binary—A numbering system based on two values or a numbering system to the base of 2.

binary digit—A digit in the binary scale of notation. This digit may be a 0 (zero) or a 1 (one). Abbreviated as *Bit.*

binary notation—The writing of numbers in the scale of two. The first dozen numbers, zero to eleven, are written, 0, 1, 10, 11, 100, 101, 110, 111, 1000, 1001, 1010, 1011. The positions of the digits represent powers of 2; thus, 1010 means 1 times 2^3, or 8, plus 0 times 2^2, or 0, plus 1 times 2^1, or 2, and 0 times 2^0, or 0; thus, 1010 is equal to 8 +2, or 10.

binary number—A digit or group of digits representing a number to the base 2 and having one of two possible values that are 0 and 1.

binary point—In a binary number, the point marks the place between the integral powers of two and the fractional powers of two, analogous to the decimal point in the decimal system. Thus 10.101_2 means 2 + 1/2 + 1/8.

binary to decimal conversion—The mathematical process of converting a number written in binary notation to the equivalent number written in decimal notation.

bit—A binary digit.

carry—A process of bringing forward. The carry digit or the digit which is to be added to the next higher column, or a special condition which occurs when the sum of two digits in a single column is equal to or greater than the numbering base.

code—As used in this book, a system of symbols, usually numerical, used to represent various values in a system of weighted values.

code, binary decimal—A coding system in which each individual decimal digit (rather than the entire number) is represented by a binary or two-valued code.

code, excess 3 (XS-3)—One of the series of numerical codes where each of the decimal digits is represented as the binary number plus 3 in a coded decimal notation. For example, 0 is binary 3, 1 is binary 4, and 7 is binary 10.

code, gray—A special type of coded or reflected binary system where there is a change in only one digit in going from one decimal number to the next higher decimal number.

complement—A quantity derived from a given number in such a way so that the sum of the two individual numbers is equal to the next higher place to the particular radix. To the base 10, the complement of 7 is 3; and to the base 10, the complement of 8 is 12.

conversion—A method of changing the representation of information from one system to another, as in converting a number from the binary system to the decimal system.

data—Information, particularly as taken in, operated on, or put out by a computer or other machine for handling information.

decimal code, binary—A type of decimal notation where the individual decimal digits are represented by means of a binary code rather than the binary representation of the entire decimal number.

decimal digit—One of the symbols 0, 1, 2, 3, 4, 5, 6, 7, 8, 9 when used in numbering in the scale of ten.

digit—Considered as one of n symbols, having an integral value and ranging from 0 to n-1, such as the decimal digits 0, 1, 2, 3, and so forth.

digit, binary—A 1 or 0 in the binary, or two-valued, numbering system.

dividend—A number, which when divided by another number known as the divisor, produces a result known as the quotient.

fixed-point—A system of arithmetic or notation in which all of the numerical quantities are represented by a number of digits that are predetermined with the radix point at some fixed location.

gate—An electronic circuit designed to produce an output under specific conditions, such as the AND gate, which requires both inputs to be present; and the OR gate, which requires either of two inputs to be present to produce an output.

instruction—Considered as a group of characters which are used to divide an operation, such as addition, and the address or addresses where the information specified is to be obtained.

msd—Most significant digit.

multiplicand—A quantity or number that is to be multiplied by a second number known as the multiplier to produce the result known as the product.

octal—Relating to a numbering system of radix 8.

operand—Any one of the quantities entering into or arising from an operation. An operand may be a result, a parameter, or a storage location.

operator—Usually a mathematical symbol representing a particular mathematical process to be performed.

OR circuit—A circuit which has a number of input lines and one output line, so that whenever a signal is present on one or more of the input lines, a signal is present on the output line.

parity—The condition of a number being either even or odd.

point—In a scale of numeric notation, the position designated by a dot that marks the separation between the integral and fractional parts of the number.

precision—As contrasted to accuracy, the degree of exactness to which it is possible to measure a quantity; the precision is often related to the number of significant digits representing a given value.

program—A precise sequence of coded instructions directing a computer to solve a given type of problem.

programmer—A person who prepares sequences of instructions for computer.

quantity—A positive or negative real number in the mathematical sense.

radix—One of the schemes for representing numbers, characterized by the arrangement in sequence of digits with the understanding that successive digits are multiplied by coefficients of successive powers of an integer called the base of the number system. The base of the decimal system is 10, and the base of the binary system is 2.

scale (scaling)—A technique used to alter or change the measure of units so that all variables are expressed within a certain range of magnitude.

self-checking code—A code that utilizes certain expressions so that an error produces a "forbidden" combination, thus stopping the computer or producing an alarm.

significant digits—If the digits of a number are ranked according to their ascending higher-powers of the base, then the significant digits are those ranging from the highest-power digit (different from zero) and ending with the lowest-power digit.

word—A group of characters as used in the computer so that the group is treated as and acts as a single unit.

APPENDIX II

Fractional Decimal-Octal Conversions

$(n)_{10}$	$(n)_8$	$(n \times 10^{-3})_8$	$(n \times 10^{-6})_8$
0.000	0.00000 00000 0	0.00000 00000 0	0.00000 00000 0
.001	.00040 61115 7	.00000 02061 6	.00000 00001 1
.002	.00101 42233 5	.00000 04143 4	.00000 00002 1
.003	.00142 23351 4	.00000 06225 2	.00000 00003 2
.004	.00203 04467 2	.00000 10307 0	.00000 00004 2
0.005	0.00243 65605 1	0.00000 12370 6	0.00000 00005 3
.006	.00304 46723 0	.00000 14452 4	.00000 00006 4
.007	.00345 30040 6	.00000 16534 2	.00000 00007 4
.008	.00406 11156 5	.00000 20615 7	.00000 00010 5
.009	.00446 72274 3	.00000 22677 5	.00000 00011 5
0.010	0.00507 53412 2	0.00000 24761 3	0.00000 00012 6
.011	.00550 34530 1	.00000 27043 1	.00000 00013 6
.012	.00611 15645 7	.00000 31124 7	.00000 00014 7
.013	.00651 76763 6	.00000 33206 5	.00000 00016 0
.014	.00712 60101 4	.00000 35270 3	.00000 00017 0
0.015	0.00753 41217 3	0.00000 37352 1	0.00000 00020 1
.016	.01014 22335 1	.00000 41433 7	.00000 00021 1
.017	.01055 03453 0	.00000 43515 5	.00000 00022 2
.018	.01115 64570 7	.00000 45577 3	.00000 00023 3
.019	.01156 45706 5	.00000 47661 1	.00000 00024 3
0.020	0.01217 27024 4	0.00000 51742 7	0.00000 00025 4
.021	.01260 10142 2	.00000 54024 5	.00000 00026 4
.022	.01320 71260 1	.00000 56106 3	.00000 00027 5
.023	.01361 52376 0	.00000 60170 0	.00000 00030 6
.024	.01422 33513 6	.00000 62251 6	.00000 00031 6
0.025	0.01463 14631 5	0.00000 64333 4	0.00000 00032 7
.026	.01523 75747 3	.00000 66415 2	.00000 00033 7
.027	.01564 57065 2	.00000 70477 0	.00000 00035 0
.028	.01625 40203 1	.00000 72560 6	.00000 00036 1
.029	.01666 21320 7	.00000 74642 4	.00000 00037 1
0.030	0.01727 02436 6	0.00000 76724 2	0.00000 00040 2
.031	.01767 63554 4	.00001 01006 0	.00000 00041 2
.032	.02030 44672 3	.00001 03067 6	.00000 00042 3
.033	.02071 26010 2	.00001 05151 4	.00000 00043 3
.034	.02132 07126 0	.00001 07233 2	.00000 00044 4
0.035	0.02172 70243 7	0.00001 11315 0	0.00000 00045 5
.036	.02233 51361 5	.00001 13376 6	.00000 00046 5
.037	.02274 32477 4	.00001 15460 4	.00000 00047 6
.038	.02335 13615 2	.00001 17542 2	.00000 00050 6
.039	.02375 74733 1	.00001 21623 7	.00000 00051 7
0.040	0.02436 56051 0	0.00001 23705 5	0.00000 00053 0
.041	.02477 37166 6	.00001 25767 3	.00000 00054 0
.042	.02540 20304 5	.00001 30051 1	.00000 00055 1
.043	.02601 01422 3	.00001 32132 7	.00000 00056 1
.044	.02641 62540 2	.00001 34214 5	.00000 00057 2

(Courtesy Computer Control Co.)

$(n)_{10}$	$(n)_8$	$(n \times 10^{-3})_8$	$(n \times 10^{-6})_8$
0.045	0.02702 43656 1	0.00001 36276 3	0.00000 00060 3
.046	.02743 24773 7	.00001 40360 1	.00000 00061 3
.047	.03004 06111 6	.00001 42441 7	.00000 00062 4
.048	.03044 67227 4	.00001 44523 5	.00000 00063 4
.049	.03105 50345 3	.00001 46605 3	.00000 00064 5
0.050	0.03146 31463 2	0.00001 50667 1	0.00000 00065 5
.051	.03207 12601 0	.00001 52750 7	.00000 00066 6
.052	.03247 73716 7	.00001 55032 5	.00000 00067 7
.053	.03310 55034 5	.00001 57114 3	.00000 00070 7
.054	.03351 36152 4	.00001 61176 0	.00000 00072 0
0.055	0.03412 17270 3	0.00001 63257 6	0.00000 00073 0
.056	.03453 00406 1	.00001 65341 4	.00000 00074 1
.057	.03513 61524 0	.00001 67423 2	.00000 00075 2
.058	.03554 42641 6	.00001 71505 0	.00000 00076 2
.059	.03615 23757 5	.00001 73566 6	.00000 00077 3
0.060	0.03656 05075 4	0.00001 75650 4	0.00000 00100 3
.061	.03716 66213 2	.00001 77732 2	.00000 00101 4
.062	.03757 47331 1	.00002 02014 0	.00000 00102 5
.063	.04020 30446 7	.00002 04075 6	.00000 00103 5
.064	.04061 11564 6	.00002 06157 4	.00000 00104 6
0.065	0.04121 72702 4	0.00002 10241 2	0.00000 00105 6
.066	.04162 54020 3	.00002 12323 0	.00000 00106 7
.067	.04223 35136 2	.00002 14404 6	.00000 00110 0
.068	.04264 16254 0	.00002 16466 4	.00000 00111 0
.069	.04324 77371 7	.00002 20550 1	.00000 00112 1
0.070	0.04365 60507 5	0.00002 22631 7	0.00000 00113 1
.071	.04426 41625 4	.00002 24713 5	.00000 00114 2
.072	.04467 22743 3	.00002 26775 3	.00000 00115 2
.073	.04530 04061 1	.00002 31057 1	.00000 00116 3
.074	.04570 65177 0	.00002 33140 7	.00000 00117 4
0.075	0.04631 46314 6	0.00002 35222 5	0.00000 00120 4
.076	.04672 27432 5	.00002 37304 3	.00000 00121 5
.077	.04733 10550 4	.00002 41366 1	.00000 00122 5
.078	.04773 71666 2	.00002 43447 7	.00000 00123 6
.079	.05034 53004 1	.00002 45531 5	.00000 00124 7
0.080	0.05075 34121 7	0.00002 47613 3	0.00000 00125 7
.081	.05136 15237 5	.00002 51675 1	.00000 00127 0
.082	.05176 76355 5	.00002 53756 7	.00000 00130 0
.083	.05237 57473 3	.00002 56040 5	.00000 00131 1
.084	.05300 40611 2	.00002 60122 3	.00000 00132 2
0.085	0.05341 21727 0	0.00002 62204 0	0.00000 00133 2
.086	.05402 03044 7	.00002 64265 6	.00000 00134 3
.087	.05442 64162 6	.00002 66347 4	.00000 00135 3
.088	.05503 45300 4	.00002 70431 2	.00000 00136 4
.089	.05544 26416 3	.00002 72513 0	.00000 00137 4
0.090	0.05605 07534 1	0.00002 74574 6	0.00000 00140 5
.091	.05645 70652 0	.00002 76656 4	.00000 00141 6
.092	.05706 51767 6	.00003 00740 2	.00000 00142 6
.093	.05747 33105 5	.00003 03022 0	.00000 00143 7
.094	.06010 14223 4	.00003 05103 6	.00000 00144 7

$(n)_{10}$	$(n)_8$	$(n \times 10^{-3})_8$	$(n \times 10^{-6})_8$
0.095	0.06050 75341 2	0.00003 07165 4	0.00000 00146 0
.096	.06111 56457 1	.00003 11247 2	.00000 00147 1
.097	.06152 37574 7	.00003 13331 0	.00000 00150 1
.098	.06213 20712 6	.00003 15412 6	.00000 00151 2
.099	.06254 02030 5	.00003 17474 4	.00000 00152 2
0.100	0.06314 63146 3	0.00003 21556 1	0.00000 00153 3
.101	.06355 44264 2	.00003 23637 7	.00000 00154 4
.102	.06416 25402 0	.00003 25721 5	.00000 00155 4
.103	.06457 06517 7	.00003 30003 3	.00000 00156 5
.104	.06517 67635 6	.00003 32065 1	.00000 00157 5
0.105	0.06560 50753 4	0.00003 34146 7	0.00000 00160 6
.106	.06621 32071 3	.00003 36230 5	.00000 00161 7
.107	.06662 13207 1	.00003 40312 3	.00000 00162 7
.108	.06722 74325 0	.00003 42374 1	.00000 00164 0
.109	.06763 55442 7	.00003 44455 7	.00000 00165 0
0.110	0.07024 36560 5	0.00003 46537 5	0.00000 00166 1
.111	.07065 17676 4	.00003 50621 3	.00000 00167 1
.112	.07126 01014 2	.00003 52703 1	.00000 00170 2
.113	.07166 62132 1	.00003 54764 7	.00000 00171 3
.114	.07227 43247 7	.00003 57046 5	.00000 00172 3
0.115	0.07270 24365 6	0.00003 61130 2	0.00000 00173 4
.116	.07331 05503 5	.00003 63212 0	.00000 00174 4
.117	.07371 66621 3	.00003 65273 6	.00000 00175 5
.118	.07432 47737 2	.00003 67355 4	.00000 00176 6
.119	.07473 31055 0	.00003 71437 2	.00000 00177 6
0.120	0.07534 12172 7	0.00003 73521 0	0.00000 00200 7
.121	.07574 73310 6	.00003 75602 6	.00000 00201 7
.122	.07635 54426 4	.00003 77664 4	.00000 00203 0
.123	.07676 35544 3	.00004 01746 2	.00000 00204 1
.124	.07737 16662 1	.00004 04030 0	.00000 00205 1
0.125	0.10000 00000 0	0.00004 06111 6	0.00000 00206 2
.126	.10040 61115 7	.00004 10173 4	.00000 00207 2
.127	.10101 42233 5	.00004 12255 2	.00000 00210 3
.128	.10142 23351 4	.00004 14337 0	.00000 00211 3
.129	.10203 04467 2	.00004 16420 6	.00000 00212 4
0.130	0.10243 65605 1	0.00004 20502 3	0.00000 00213 5
.131	.10304 46723 0	.00004 22564 1	.00000 00214 5
.132	.10345 30040 6	.00004 24645 7	.00000 00215 6
.133	.10406 11156 5	.00004 26727 5	.00000 00216 6
.134	.10446 72274 3	.00004 31011 3	.00000 00217 7
0.135	0.10507 53412 2	0.00004 33073 1	0.00000 00221 0
.136	.10550 34530 1	.00004 35154 7	.00000 00222 0
.137	.10611 15645 7	.00004 37236 5	.00000 00223 1
.138	.10651 76763 6	.00004 41320 3	.00000 00224 1
.139	.10712 60101 4	.00004 43402 1	.00000 00225 2
0.140	0.10753 41217 3	0.00004 45463 7	0.00000 00226 3
.141	.11014 22335 1	.00004 47545 5	.00000 00227 3
.142	.11055 03453 0	.00004 51627 3	.00000 00230 4
.143	.11115 64570 7	.00004 53711 1	.00000 00231 4
.144	.11156 45706 5	.00004 55772 7	.00000 00232 5

$(n)_{10}$	$(n)_8$	$(n \times 10^{-3})_8$	$(n \times 10^{-6})_8$
0.145	0.11217 27024 4	0.00004 60054 5	0.00000 00233 6
.146	.11260 10142 2	.00004 62136 2	.00000 00234 6
.147	.11320 71260 1	.00004 64220 0	.00000 00235 7
.148	.11361 52376 0	.00004 66301 6	.00000 00236 7
.149	.11422 33513 6	.00004 70363 4	.00000 00240 0
0.150	0.11463 14631 5	0.00004 72445 2	0.00000 00241 0
.151	.11523 75747 3	.00004 74527 0	.00000 00242 1
.152	.11564 57065 2	.00004 76610 6	.00000 00243 2
.153	.11625 40203 1	.00005 00672 4	.00000 00244 2
.154	.11666 21320 7	.00005 02754 2	.00000 00245 3
0.155	0.11727 02436 6	0.00005 05036 0	0.00000 00246 3
.156	.11767 63554 4	.00005 07117 6	.00000 00247 4
.157	.12030 44672 3	.00005 11201 4	.00000 00250 5
.158	.12071 26010 2	.00005 13263 2	.00000 00251 5
.159	.12132 07126 0	.00005 15345 0	.00000 00252 6
0.160	0.12172 70243 7	0.00005 17426 6	0.00000 00253 6
.161	.12233 51361 5	.00005 21510 3	.00000 00254 7
.162	.12274 32477 4	.00005 23572 1	.00000 00256 0
.163	.12335 13615 2	.00005 25653 7	.00000 00257 0
.164	.12375 74733 1	.00005 27735 5	.00000 00260 1
0.165	0.12436 56051 0	0.00005 32017 3	0.00000 00261 1
.166	.12477 37166 6	.00005 34101 1	.00000 00262 2
.167	.12540 20304 5	.00005 36162 7	.00000 00263 3
.168	.12601 01422 3	.00005 40244 5	.00000 00264 3
.169	.12641 62540 2	.00005 42326 3	.00000 00265 4
0.170	0.12702 43656 1	0.00005 44410 1	0.00000 00266 4
.171	.12743 24773 7	.00005 46471 7	.00000 00267 5
.172	.13004 06111 6	.00005 50553 5	.00000 00270 5
.173	.13044 67227 4	.00005 52635 3	.00000 00271 6
.174	.13105 50345 3	.00005 54717 1	.00000 00272 7
0.175	0.13146 31463 2	0.00005 57000 7	0.00000 00273 7
.176	.13207 12601 0	.00005 61062 4	.00000 00275 0
.177	.13247 73716 7	.00005 63144 2	.00000 00276 0
.178	.13310 55034 5	.00005 65226 0	.00000 00277 1
.179	.13351 36152 4	.00005 67307 6	.00000 00300 2
0.180	0.13412 17270 3	0.00005 71371 4	0.00000 00301 2
.181	.13453 00406 1	.00005 73453 2	.00000 00302 3
.182	.13513 61524 0	.00005 75535 0	.00000 00303 3
.183	.13554 42641 6	.00005 77616 6	.00000 00304 4
.184	.13615 23757 5	.00006 01700 4	.00000 00305 5
0.185	0.13656 05075 4	0.00006 03762 2	0.00000 00306 5
.186	.13716 66213 2	.00006 06044 0	.00000 00307 6
.187	.13757 47331 1	.00006 10125 6	.00000 00310 6
.188	.14020 30446 7	.00006 12207 4	.00000 00311 7
.189	.14061 11564 6	.00006 14271 2	.00000 00312 7
0.190	0.14121 72702 4	0.00006 16353 0	0.00000 00314 0
.191	.14162 54020 3	.00006 20434 6	.00000 00315 1
.192	.14223 35136 2	.00006 22516 3	.00000 00316 1
.193	.14264 16254 0	.00006 24600 1	.00000 00317 2
.194	.14324 77371 7	.00006 26661 7	.00000 00320 2

$(n)_{10}$	$(n)_8$	$(n \times 10^{-3})_8$	$(n \times 10^{-6})_8$
0.195	0.14365 60507 5	0.00006 30743 5	0.00000 00321 3
.196	.14426 41625 4	.00006 33025 3	.00000 00322 4
.197	.14467 22743 3	.00006 35107 1	.00000 00323 4
.198	.14530 04061 1	.00006 37170 7	.00000 00324 5
.199	.14570 65177 0	.00006 41252 5	.00000 00325 5
0.200	0.14631 46314 6	0.00006 43334 3	0.00000 00326 6
.201	.14672 27432 5	.00006 45416 1	.00000 00327 7
.202	.14733 10550 4	.00006 47477 7	.00000 00330 7
.203	.14773 71666 2	.00006 51561 5	.00000 00332 0
.204	.15034 53004 1	.00006 53643 3	.00000 00333 0
0.205	0.15075 34121 7	0.00006 55725 1	0.00000 00334 1
.206	.15136 15237 6	.00006 60006 7	.00000 00335 2
.207	.15176 76355 5	.00006 62070 4	.00000 00336 2
.208	.15237 57473 3	.00006 64152 2	.00000 00337 3
.209	.15300 40611 2	.00006 66234 0	.00000 00340 3
0.210	0.15341 21727 0	0.00006 70315 6	0.00000 00341 4
.211	.15402 03044 7	.00006 72377 4	.00000 00342 4
.212	.15442 64162 6	.00006 74461 2	.00000 00343 5
.213	.15503 45300 4	.00006 76543 0	.00000 00344 6
.214	.15544 26416 3	.00007 00624 6	.00000 00345 6
0.215	0.15605 07534 1	0.00007 02706 4	0.00000 00346 7
.216	.15645 70652 0	.00007 04770 2	.00000 00347 7
.217	.15706 51767 6	.00007 07052 0	.00000 00351 0
.218	.15747 33105 5	.00007 11133 6	.00000 00352 1
.219	.16010 14223 4	.00007 13215 4	.00000 00353 1
0.220	0.16050 75341 2	0.00007 15277 2	0.00000 00354 2
.221	.16111 56457 1	.00007 17361 0	.00000 00355 2
.222	.16152 37574 7	.00007 21442 5	.00000 00356 3
.223	.16213 20712 6	.00007 23524 3	.00000 00357 4
.224	.16254 02030 5	.00007 25606 1	.00000 00360 4
0.225	0.16314 63146 3	0.00007 27667 7	0.00000 00361 5
.226	.16355 44264 2	.00007 31751 5	.00000 00362 5
.227	.16416 25402 0	.00007 34033 3	.00000 00363 6
.228	.16457 06517 7	.00007 36115 1	.00000 00364 6
.229	.16517 67635 6	.00007 40176 7	.00000 00365 7
0.230	0.16560 50753 4	0.00007 42260 5	0.00000 00367 0
.231	.16621 32071 3	.00007 44342 3	.00000 00370 0
.232	.16662 13207 1	.00007 46424 1	.00000 00371 1
.233	.16722 74325 0	.00007 50505 7	.00000 00372 1
.234	.16763 55442 7	.00007 52567 5	.00000 00373 2
0.235	0.17024 36560 5	0.00007 54651 3	0.00000 00374 3
.236	.17065 17676 4	.00007 56733 1	.00000 00375 3
.237	.17126 01014 2	.00007 61014 6	.00000 00376 4
.238	.17166 62132 1	.00007 63076 4	.00000 00377 4
.239	.17227 43247 7	.00007 65160 2	.00000 00400 5
0.240	0.17270 24365 6	0.00007 67242 0	0.00000 00401 6
.241	.17331 05503 5	.00007 71323 6	.00000 00402 6
.242	.17371 66621 3	.00007 73405 4	.00000 00403 7
.243	.17432 47737 2	.00007 75467 2	.00000 00404 7
.244	.17473 31055 0	.00007 77551 0	.00000 00406 0

$(n)_{10}$	$(n)_8$	$(n \times 10^{-3})_8$	$(n \times 10^{-6})_8$
0.245	0.17534 12172 7	0.00010 01632 6	0.00000 00407 1
.246	.17574 73310 6	.00010 03714 4	.00000 00410 1
.247	.17635 54426 4	.00010 05776 2	.00000 00411 2
.248	.17676 35544 3	.00010 10060 0	.00000 00412 2
.249	.17737 16662 1	.00010 12141 6	.00000 00413 3
0.250	0.20000 00000 0	0.00010 14223 4	0.00000 00414 3
.251	.20040 61115 7	.00010 16305 2	.00000 00415 4
.252	.20101 42233 5	.00010 20367 0	.00000 00416 5
.253	.20142 23351 4	.00010 22450 5	.00000 00417 5
.254	.20203 04467 2	.00010 24532 3	.00000 00420 6
0.255	0.20243 65605 1	0.00010 26614 1	0.00000 00421 6
.256	.20304 46723 0	.00010 30675 7	.00000 00422 7
.257	.20345 30040 6	.00010 32757 5	.00000 00424 0
.258	.20406 11156 5	.00010 35041 3	.00000 00425 0
.259	.20446 72274 3	.00010 37123 1	.00000 00426 1
0.260	0.20507 53412 2	0.00010 41204 7	0.00000 00427 1
.261	.20550 34530 1	.00010 43266 5	.00000 00430 2
.262	.20611 15645 7	.00010 45350 3	.00000 00431 3
.263	.20651 76763 6	.00010 47432 1	.00000 00432 3
.264	.20712 60101 4	.00010 51513 7	.00000 00433 4
0.265	0.20753 41217 3	0.00010 53575 5	0.00000 00434 4
.266	.21014 22335 1	.00010 55657 3	.00000 00435 5
.267	.21055 03453 0	.00010 57741 1	.00000 00436 5
.268	.21115 64570 7	.00010 62022 6	.00000 00437 6
.269	.21156 45706 5	.00010 64104 4	.00000 00440 7
0.270	0.21217 27024 4	0.00010 66166 2	0.00000 00441 7
.271	.21260 10142 2	.00010 70250 0	.00000 00443 0
.272	.21320 71260 1	.00010 72331 6	.00000 00444 0
.273	.21361 52376 0	.00010 74413 4	.00000 00445 1
.274	.21422 33513 6	.00010 76475 2	.00000 00446 2
0.275	0.21463 14631 5	0.00011 00557 0	0.00000 00447 2
.276	.21523 75747 3	.00011 02640 6	.00000 00450 3
.277	.21564 57065 2	.00011 04722 4	.00000 00451 3
.278	.21625 40203 1	.00011 07004 2	.00000 00452 4
.279	.21666 21320 7	.00011 11066 0	.00000 00453 5
0.280	0.21727 02436 6	0.00011 13147 6	0.00000 00454 5
.281	.21767 63554 4	.00011 15231 4	.00000 00455 6
.282	.22030 44672 3	.00011 17313 2	.00000 00456 6
.283	.22071 26010 2	.00011 21374 7	.00000 00457 7
.284	.22132 07126 0	.00011 23456 5	.00000 00461 0
0.285	0.22172 70243 7	0.00011 25540 3	0.00000 00462 0
.286	.22233 51361 5	.00011 27622 1	.00000 00463 1
.287	.22274 32477 4	.00011 31703 7	.00000 00464 1
.288	.22335 13615 2	.00011 33765 5	.00000 00465 2
.289	.22375 74733 1	.00011 36047 3	.00000 00466 2
0.290	0.22436 56051 0	0.00011 40131 1	0.00000 00467 3
.291	.22477 37166 6	.00011 42212 7	.00000 00470 4
.292	.22540 20304 5	.00011 44274 5	.00000 00471 4
.293	.22601 01422 3	.00011 46356 3	.00000 00472 5
.294	.22641 62540 2	.00011 50440 1	.00000 00473 5

$(n)_{10}$	$(n)_8$	$(n \times 10^{-3})_8$	$(n \times 10^{-6})_8$
0.295	0.22702 43656 1	0.00011 52521 7	0.00000 00474 6
.296	.22743 24773 7	.00011 54603 5	.00000 00475 7
.297	.23004 06111 6	.00011 56665 3	.00000 00476 7
.298	.23044 67227 4	.00011 60747 1	.00000 00500 0
.299	.23105 50345 .3	.00011 63030 6	.00000 00501 0
0.300	0.23146 31463 2	0.00011 65112 4	0.00000 00502 1
.301	.23207 12601 0	.00011 67174 2	.00000 00503 2
.302	.23247 73716 7	.00011 71256 0	.00000 00504 2
.303	.23310 55034 5	.00011 73337 6	.00000 00505 3
.304	.23351 36152 4	.00011 75421 4	.00000 00506 3
0.305	0.23412 17270 2	0.00011 77503 2	0.00000 00507 4
.306	.23453 00406 1	.00012 01565 0	.00000 00510 5
.307	.23513 61524 0	.00012 03646 6	.00000 00511 5
.308	.23554 42641 6	.00012 05730 4	.00000 00512 6
.309	.23615 23757 5	.00012 10012 2	.00000 00513 6
0.310	0.23656 05075 4	0.00012 12074 0	0.00000 00514 7
.311	.23716 66213 2	.00012 14155 6	.00000 00515 7
.312	.23757 47331 1	.00012 16237 4	.00000 00517 0
.313	.24020 30446 7	.00012 20321 2	.00000 00520 1
.314	.24061 11564 6	.00012 22402 7	.00000 00521 1
0.315	0.24121 72702 4	0.00012 24464 5	0.00000 00522 2
.316	.24162 54020 3	.00012 26546 3	.00000 00523 2
.317	.24223 35136 2	.00012 30630 1	.00000 00524 3
.318	.24264 16254 0	.00012 32711 7	.00000 00525 4
.319	.24324 77371 7	.00012 34773 5	.00000 00526 4
0.320	0.24365 60507 5	0.00012 37055 3	0.00000 00527 5
.321	.24426 41625 4	.00012 41137 1	.00000 00530 5
.322	.24467 22743 3	.00012 43220 7	.00000 00531 6
.323	.24530 04061 1	.00012 45302 5	.00000 00532 7
.324	.24570 65177 0	.00012 47364 3	.00000 00533 7
0.325	0.24631 46314 6	0.00012 51446 1	0.00000 00535 0
.326	.24672 27432 5	.00012 53527 7	.00000 00536 0
.327	.24733 10550 4	.00012 55611 5	.00000 00537 1
.328	.24773 71666 2	.00012 57673 3	.00000 00540 1
.329	.25034 53004 1	.00012 61755 0	.00000 00541 2
0.330	0.25075 34121 7	0.00012 64036 6	0.00000 00542 3
.331	.25136 15237 6	.00012 66120 4	.00000 00543 3
.332	.25176 76355 5	.00012 70202 2	.00000 00544 4
.333	.25237 57473 3	.00012 72264 0	.00000 00545 4
.334	.25300 40611 2	.00012 74345 6	.00000 00546 5
0.335	0.25341 21727 0	0.00012 76427 4	0.00000 00547 6
.336	.25402 03044 7	.00013 00511 2	.00000 00550 6
.337	.25442 64162 6	.00013 02573 0	.00000 00551 7
.338	.25503 45300 4	.00013 04654 6	.00000 00552 7
.339	.25544 26416 3	.00013 06736 4	.00000 00554 0
0.340	0.25605 07534 1	0.00013 11020 2	0.00000 00555 1
.341	.25645 70652 0	.00013 13102 0	.00000 00556 1
.342	.25706 51767 6	.00013 15163 6	.00000 00557 2
.343	.25747 33105 5	.00013 17245 4	.00000 00560 2
.344	.26010 14223 4	.00013 21327 1	.00000 00561 3

$(n)_{10}$	$(n)_8$	$(n \times 10^{-3})_8$	$(n \times 10^{-6})_8$
0.345	0.26050 75341 2	0.00013 23410 7	0.00000 00562 4
.346	.26111 54657 1	.00013 25472 5	.00000 00563 4
.347	.26152 37574 7	.00013 27554 3	.00000 00564 5
.348	.26213 20712 6	.00013. 31636 1	.00000 00565 5
.349	.26254 02030 5	.00013 33717 7	.00000 00566 6
0.350	0.26314 63146 3	0.00013 36001 5	0.00000 00567 6
.351	.26355 44264 2	.00013 40063 3	.00000 00570 7
.352	.26416 25402 0	.00013 42145 1	.00000 00572 0
.353	.26457 06517 7	.00013 44226 7	.00000 00573 0
.354	.26517 67635 6	.00013 46310 5	.00000 00574 1
0.355	0.26560 50753 4	0.00013 50372 3	0.00000 00575 1
.356	.26621 32071 3	.00013 52454 1	.00000 00576 2
.357	.26662 13207 1	.00013 54535 7	.00000 00577 3
.358	.26722 74325 0	.00013 56617 5	.00000 00600 3
.359	.26763 55442 7	.00013 60701 3	.00000 00601 4
0.360	0.27024 36560 5	0.00013 62763 0	0.00000 00602 4
.361	.27065 17676 4	.00013 65044 6	.00000 00603 5
.362	.27126 01014 2	.00013 67126 4	.00000 00604 6
.363	.27166 62132 1	.00013 71210 2	.00000 00605 6
.364	.27227 43247 7	.00013 73272 0	.00000 00606 7
0.365	0.27270 24365 6	0.00013 75353 6	0.00000 00607 7
.366	.27331 05503 5	.00013 77435 4	.00000 00611 0
.367	.27371 66621 3	.00014 01517 2	.00000 00612 0
.368	.27432 47737 2	.00014 03601 0	.00000 00613 1
.369	.27473 31055 0	.00014 05662 6	.00000 00614 2
0.370	0.27534 12172 7	0.00014 07744 4	0.00000 00615 2
.371	.27574 73310 6	.00014 12026 2	.00000 00616 3
.372	.27635 54426 4	.00014 14110 0	.00000 00617 3
.373	.27676 35544 3	.00014 16171 6	.00000 00620 4
.374	.27737 16662 1	.00014 20253 4	.00000 00621 5
0.375	0.3000 00000 0	0.00014 22335 1	0.00000 00622 5
.376	.30040 61115 7	.00014 24416 7	.00000 00623 6
.377	.30101 42233 5	.00014 26500 5	.00000 00624 6
.378	.30142 23351 4	.00014 30562 3	.00000 00625 7
.379	.30203 04467 2	.00014 32644 1	.00000 00627 0
0.380	0.30243 65605 1	0.00014 34725 7	0.00000 00630 0
.381	.30304 46723 0	.00014 37007 5	.00000 00631 1
.382	.30345 30040 6	.00014 41071 3	.00000 00632 1
.383	.30406 11156 5	.00014 43153 1	.00000 00633 2
.384	.30446 72274 3	.00014 45234 7	.00000 00634 3
0.385	0.30507 53412 2	0.00014 47316 5	0.00000 00635 3
.386	.30550 34530 1	.00014 51400 3	.00000 00636 4
.387	.30611 15645 7	.00014 53462 1	.00000 00637 4
.388	.30651 76763 6	.00014 55543 7	.00000 00640 5
.389	.30712 60101 4	.00014 57625 5	.00000 00641 5
0.390	0.30753 41217 3	0.00014 61707 2	0.00000 00642 6
.391	.31014 22335 1	.00014 63771 0	.00000 00643 7
.392	.31055 03453 0	.00014 66052 6	.00000 00644 7
.393	.31115 64570 7	.00014 70134 4	.00000 00646 0
.394	.31156 45706 5	.00014 72216 2	.00000 00647 0

$(n)_{10}$	$(n)_8$	$(n \times 10^{-3})_8$	$(n \times 10^{-6})_8$
0.395	0.31217 27024 4	0.00014 74300 0	0.00000 00650 1
.396	.31260 10142 2	.00014 76361 6	.00000 00651 2
.397	.31320 71260 1	.00015 00443 4	.00000 00652 2
.398	.31361 52376 0	.00015 02525 2	.00000 00653 3
.399	.31422 33513 6	.00015 04607 0	.00000 00654 3
0.400	0.31463 14631 5	0.00015 06670 6	0.00000 00655 4
.401	.31523 75747 3	.00015 10752 4	.00000 00656 5
.402	.31564 57065 2	.00015 13034 2	.00000 00657 5
.403	.31625 40203 1	.00015 15116 0	.00000 00660 6
.404	.31666 21320 7	.00015 17177 6	.00000 00661 6
0.405	0.31727 02436 6	0.00015 21261 4	0.00000 00662 7
.406	.31767 63554 4	.00015 23343 1	.00000 00663 7
.407	.32030 44672 3	.00015 25424 7	.00000 00665 0
.408	.32071 26010 2	.00015 27506 5	.00000 00666 1
.409	.32132 07126 0	.00015 31570 3	.00000 00667 1
0.410	0.32172 70243 7	0.00015 33652 1	0.00000 00670 2
.411	.32233 51361 5	.00015 35733 7	.00000 00671 2
.412	.32274 32477 4	.00015 40015 5	.00000 00672 3
.413	.32335 13615 2	.00015 42077 3	.00000 00673 4
.414	.32375 74733 1	.00015 44161 1	.00000 00674 4
0.415	0.32436 56051 0	0.00015 46242 7	0.00000 00675 5
.416	.32477 37166 6	.00015 50324 5	.00000 00676 5
.417	.32540 20304 5	.00015 52406 3	.00000 00677 6
.418	.32601 01422 3	.00015 54470 1	.00000 00700 7
.419	.32641 62540 2	.00015 56551 7	.00000 00701 7
0.420	0.32702 43656 1	0.00015 60633 5	0.00000 00703 0
.421	.32743 24773 7	.00015 62715 2	.00000 00704 0
.422	.33004 06111 6	.00015 64777 0	.00000 00705 1
.423	.33044 67227 4	.00015 67060 6	.00000 00706 2
.424	.33105 50345 3	.00015 71142 4	.00000 00707 2
0.425	0.33146 31463 2	0.00015 73224 2	0.00000 00710 3
.426	.33207 12601 0	.00015 75306 0	.00000 00711 3
.427	.33247 73716 7	.00015 77367 6	.00000 00712 4
.428	.33310 55034 5	.00016 01451 4	.00000 00713 4
.429	.33351 36152 4	.00016 03533 2	.00000 00714 5
0.430	0.33412 17270 3	0.00016 05615 0	0.00000 00715 6
.431	.33453 00406 1	.00016 07676 6	.00000 00716 6
.432	.33513 61524 0	.00016 11760 4	.00000 00717 7
.433	.33554 42641 6	.00016 14042 2	.00000 00720 7
.434	.33615 23757 5	.00016 16124 0	.00000 00722 0
0.435	0.33656 05075 4	0.00016 20205 6	0.00000 00723 1
.436	.33716 66213 2	.00016 22267 3	.00000 00724 1
.437	.33757 47331 1	.00016 24351 1	.00000 00725 2
.438	.34020 30446 7	.00016 26432 7	.00000 00726 2
.439	.34061 11564 6	.00016 30514 5	.00000 00727 3
0.440	0.34121 72702 4	0.00016 32576 3	0.00000 00730 4
.441	.34162 54020 3	.00016 34660 1	.00000 00731 4
.442	.34223 35136 2	.00016 36741 7	.00000 00732 5
.443	.34264 16254 0	.00016 41023 5	.00000 00733 5
.444	.34324 77371 7	.00016 43105 3	.00000 00734 6

$(n)_{10}$	$(n)_8$			$(n \times 10^{-3})_8$			$(n \times 10^{-6})_8$		
0.445	0.34365	60507	5	0.00016	45167	1	0.00000	00735	7
.446	.34426	41625	4	.00016	47250	7	.00000	00736	7
.447	.34467	22743	3	.00016	51332	5	.00000	00740	0
.448	.34530	04061	1	.00016	53414	3	.00000	00741	0
.449	.34570	65177	0	.00016	55476	1	.00000	00742	1
0.450	0.34631	46314	6	0.00016	57557	7	0.00000	00743	1
.451	.34672	27432	5	.00016	61641	5	.00000	00744	2
.452	.34733	10550	4	.00016	63723	2	.00000	00745	3
.453	.34773	71666	2	.00016	66005	0	.00000	00746	3
.454	.35034	53004	1	.00016	70066	6	.00000	00747	4
0.455	0.35075	34121	7	0.00016	72150	4	0.00000	00750	4
.456	.35136	15237	6	.00016	74232	2	.00000	00751	5
.457	.35176	76355	5	.00016	76314	0	.00000	00752	6
.458	.35237	57473	3	.00017	00375	6	.00000	00753	6
.459	.35300	40611	2	.00017	02457	4	.00000	00754	7
0.460	0.35341	21727	0	0.00017	04541	2	0.00000	00755	7
.461	.35402	03044	7	.00017	06623	0	.00000	00757	0
.462	.35442	64162	6	.00017	10704	6	.00000	00760	1
.463	.35503	45300	4	.00017	12766	4	.00000	00761	1
.464	.35544	26416	3	.00017	15050	2	.00000	00762	2
0.465	0.35605	07534	1	0.00017	17132	0	0.00000	00763	2
.466	.35645	70652	0	.00017	21213	6	.00000	00764	3
.467	.35706	51767	6	.00017	23275	3	.00000	00765	3
.468	.35747	33105	5	.00017	25357	1	.00000	00766	4
.469	.36010	14223	4	.00017	27440	7	.00000	00767	5
0.470	0.36050	75341	2	0.00017	31522	5	0.00000	00770	5
.471	.36111	56457	1	.00017	33604	3	.00000	00771	6
.472	.36152	37574	7	.00017	35666	1	.00000	00772	6
.473	.36213	20712	6	.00017	37747	7	.00000	00773	7
.474	.36254	02030	5	.00017	42031	5	.00000	00775	0
0.475	0.36314	63146	3	0.00017	44113	3	0.00000	00776	0
.476	.36355	44264	2	.00017	46175	1	.00000	00777	1
.477	.36416	25402	0	.00017	50256	7	.00000	01000	1
.478	.36457	06517	7	.00017	52340	5	.00000	01001	2
.479	.36517	67635	6	.00017	54422	3	.00000	01002	3
0.480	0.36560	50753	4	0.00017	56504	1	0.00000	01003	3
.481	.36621	32071	3	.00017	60565	7	.00000	01004	4
.482	.36662	13207	1	.00017	62647	4	.00000	01005	4
.483	.36722	74325	0	.00017	64731	2	.00000	01006	5
.484	.36763	55442	7	.00017	67013	0	.00000	01007	6
0.485	0.37024	36560	5	0.00017	71074	6	0.00000	01010	6
.486	.37065	17676	4	.00017	73156	4	.00000	01011	7
.487	.37126	01014	2	.00017	75240	2	.00000	01012	7
.488	.37166	62132	1	.00017	77322	0	.00000	01014	0
.489	.37227	43247	7	.00020	01403	6	.00000	01015	0
0.490	0.37270	24365	6	0.00020	03465	4	0.00000	01016	1
.491	.37331	05503	5	.00020	05547	2	.00000	01017	2
.492	.37371	66621	3	.00020	07631	0	.00000	01020	2
.493	.37432	47737	2	.00020	11712	6	.00000	01021	3
.494	.37473	31055	0	.00020	13774	4	.00000	01022	3

$(n)_{10}$	$(n)_8$	$(n \times 10^{-3})_8$	$(n \times 10^{-6})_8$
0.495	0.37534 12172 7	0.00020 16056 2	0.00000 01023 4
.496	.37574 73310 6	.00020 20140 0	.00000 01024 5
.497	.37635 54426 4	.00020 22221 5	.00000 01025 5
.498	.37676 35544 3	.00020 24303 3	.00000 01026 6
.499	.37737 16662 1	.00020 26365 1	.00000 01027 6
0.500	0.40000 00000 0	0.00020 30446 7	0.00000 01030 7
.501	.40040 61115 7	.00020 32530 5	.00000 01032 0
.502	.40101 42233 5	.00020 34612 3	.00000 01033 0
.503	.40142 23351 4	.00020 36674 1	.00000 01034 1
.504	.40203 04467 2	.00020 40755 7	.00000 01035 1
0.505	0.40243 65605 1	0.00020 43037 5	0.00000 01036 2
.506	.40304 46723 0	.00020 45121 3	.00000 01037 2
.507	.40345 30040 6	.00020 47203 1	.00000 01040 3
.508	.40406 11156 5	.00020 51264 7	.00000 01041 4
.509	.40446 72274 3	.00020 53346 5	.00000 01042 4
0.510	0.40507 53412 2	0.00020 55430 3	0.00000 01043 5
.511	.40550 34530 1	.00020 57512 1	.00000 01044 5
.512	.40611 15645 7	.00020 61573 7	.00000 01045 6
.513	.40651 76763 6	.00020 63655 4	.00000 01046 7
.514	.40712 60101 4	.00020 65737 2	.00000 01047 7
0.515	0.40753 41217 3	0.00020 70021 0	0.00000 01051 0
.516	.41014 22335 1	.00020 72102 6	.00000 01052 0
.517	.41055 03453 0	.00020 74164 4	.00000 01053 1
.518	.41115 64570 7	.00020 76246 2	.00000 01054 2
.519	.41156 45706 5	.00021 00330 0	.00000 01055 2
0.520	0.41217 27024 4	0.00021 02411 6	0.00000 01056 3
.521	.41260 10142 2	.00021 04473 4	.00000 01057 3
.522	.41320 71260 1	.00021 06555 2	.00000 01060 4
.523	.41361 52376 0	.00021 10637 0	.00000 01061 5
.524	.41422 33513 6	.00021 12720 6	.00000 01062 5
0.525	0.41463 14631 5	0.00021 15002 4	0.00000 01063 6
.526	.41523 75747 3	.00021 17064 2	.00000 01064 6
.527	.41564 57065 2	.00021 21146 0	.00000 01065 7
.528	.41625 40203 1	.00021 23227 5	.00000 01066 7
.529	.41666 21320 7	.00021 25311 3	.00000 01070 0
0.530	0.41727 02436 6	0.00021 27373 1	0.00000 01071 1
.531	.41767 63554 4	.00021 31454 7	.00000 01072 1
.532	.42030 44672 3	.00021 33536 5	.00000 01073 2
.533	.42071 26010 2	.00021 35620 3	.00000 01074 2
.534	.42132 07126 0	.00021 37702 1	.00000 01075 3
0.535	0.42172 70243 7	0.00021 41763 7	0.00000 01076 4
.536	.42233 51361 5	.00021 44045 5	.00000 01077 4
.537	.42274 32477 4	.00021 46127 3	.00000 01100 5
.538	.42335 13615 2	.00021 50211 1	.00000 01101 5
.539	.42375 74733 1	.00021 52272 7	.00000 01102 6
0.540	0.42436 56051 0	0.00021 54354 5	0.00000 01103 7
.541	.42477 37166 6	.00021 56436 3	.00000 01104 7
.542	.42540 20304 5	.00021 60520 1	.00000 01106 0
.543	.42601 01422 3	.00021 62601 6	.00000 01107 0
.544	.42641 62540 2	.00021 64663 4	.00000 01110 1

$(n)_{10}$	$(n)_8$			$(n \times 10^{-3})_8$			$(n \times 10^{-6})_8$		
0.545	0.42702	43656	1	0.00021	66745	2	0.00000	01111	1
.546	.42743	24773	7	.00021	71027	0	.00000	01112	2
.547	.43004	06111	6	.00021	73110	6	.00000	01113	3
.548	.43044	67227	4	.00021	75172	4	.00000	01114	3
.549	.43105	50345	3	.00021	77254	2	.00000	01115	4
0.550	0.43146	31463	2	0.00022	01336	0	0.00000	01116	4
.551	.43207	12601	0	.00022	03417	6	.00000	01117	5
.552	.43247	73716	7	.00022	05501	4	.00000	01120	6
.553	.43310	55034	5	.00022	07563	2	.00000	01121	6
.554	.43351	36152	4	.00022	11645	0	.00000	01122	7
0.555	0.43412	17270	3	0.00022	13726	6	0.00000	01123	7
.556	.43453	00406	1	.00022	16010	4	.00000	01125	0
.557	.43513	61524	0	.00022	20072	2	.00000	01126	1
.558	.43554	42641	6	.00022	22154	0	.00000	01127	1
.559	.43615	23757	5	.00022	24235	5	.00000	01130	2
0.560	0.43656	05075	4	0.00022	26317	3	0.00000	01131	2
.561	.43716	66213	2	.00022	30401	1	.00000	01132	3
.562	.43757	47331	1	.00022	32462	7	.00000	01133	4
.563	.44020	30446	7	.00022	34544	5	.00000	01134	4
.564	.44061	11564	6	.00022	36626	3	.00000	01135	5
0.565	0.44121	72702	4	0.00022	40710	1	0.00000	01136	5
.566	.44162	54020	3	.00022	42771	7	.00000	01137	6
.567	.44223	35136	2	.00022	45053	5	.00000	01140	6
.568	.44264	16254	0	.00022	47135	3	.00000	01141	7
.569	.44324	77371	7	.00022	51217	1	.00000	01143	0
0.570	0.44365	60507	5	0.00022	53300	7	0.00000	01144	0
.571	.44426	41625	4	.00022	55362	5	.00000	01145	1
.572	.44467	22743	3	.00022	57444	3	.00000	01146	1
.573	.44530	04061	1	.00022	61526	1	.00000	01147	2
.574	.44570	65177	0	.00022	63607	6	.00000	01150	3
0.575	0.44631	46314	6	0.00022	65671	4	0.00000	01151	3
.576	.44672	27432	5	.00022	67753	2	.00000	01152	4
.577	.44733	10550	4	.00022	72035	0	.00000	01153	4
.578	.44773	71666	2	.00022	74116	6	.00000	01154	5
.579	.45034	53004	1	.00022	76200	4	.00000	01155	6
0.580	0.45075	34121	7	0.00023	00262	2	0.00000	01156	6
.581	.45135	15237	6	.00023	02344	0	.00000	01157	7
.582	.45176	76355	5	.00023	04425	6	.00000	01160	7
.583	.45237	57473	3	.00023	06507	4	.00000	01162	0
.584	.45300	40611	2	.00023	10571	2	.00000	01163	1
0.585	0.45341	21727	0	0.00023	12653	0	0.00000	01164	1
.586	.45402	03044	7	.00023	14734	6	.00000	01165	2
.587	.45442	64162	6	.00023	17016	4	.00000	01166	2
.588	.45503	45300	4	.00023	21100	2	.00000	01167	3
.589	.45544	26416	3	.00023	23161	7	.00000	01170	3
0.590	0.45605	07534	1	0.00023	25243	5	0.00000	01171	4
.591	.45645	70642	0	.00023	27325	3	.00000	01172	5
.592	.45706	51767	6	.00023	31407	1	.00000	01173	5
.593	.45747	33105	5	.00023	33470	7	.00000	01174	6
.594	.46010	14223	4	.00023	35552	5	.00000	01175	6

$(n)_{10}$	$(n)_8$	$(n \times 10^{-3})_8$	$(n \times 10^{-6})_8$
0.595	0.46050 75341 2	0.00023 37634 3	0.00000 01176 7
.596	.46111 56457 1	.00023 41716 1	.00000 01200 0
.597	.46152 37574 7	.00023 43777 7	.00000 01201 0
.598	.46213 20712 6	.00023 46061 5	.00000 01202 1
.599	.46254 02030 5	.00023 50143 3	.00000 01203 1
0.600	0.46314 63146 3	0.00023 52225 1	0.00000 01204 2
.601	.46355 44264 2	.00023 54306 7	.00000 01205 3
.602	.46416 25402 0	.00023 56370 5	.00000 01206 3
.603	.46457 06517 7	.00023 60452 3	.00000 01207 4
.604	.46517 67635 6	.00023 62534 0	.00000 01210 4
0.605	0.46560 50753 4	0.00023 64615 6	0.00000 01211 5
.606	.46621 32071 3	.00023 66677 4	.00000 01212 5
.607	.46662 13207 1	.00023 70761 2	.00000 01213 6
.608	.46722 74325 0	.00023 73043 0	.00000 01214 7
.609	.46763 55442 7	.00023 75124 6	.00000 01215 7
0.610	0.47024 36560 5	0.00023 77206 4	0.00000 01217 0
.611	.47065 17676 4	.00024 01270 2	.00000 01220 0
.612	.47126 01014 2	.00024 03352 0	.00000 01221 1
.613	.47166 62132 1	.00024 05433 6	.00000 01222 2
.614	.47227 43247 7	.00024 07515 4	.00000 01223 2
0.615	0.47270 24365 6	0.00024 11577 2	0.00000 01224 3
.616	.47331 05503 5	.00024 13661 0	.00000 01225 3
.617	.47371 66621 3	.00024 15742 6	.00000 01226 4
.618	.47432 47737 2	.00024 20024 4	.00000 01227 5
.619	.47473 31055 0	.00024 22106 2	.00000 01230 5
0.620	0.47534 12172 7	0.00024 24167 7	0.00000 01231 6
.621	.47574 73310 6	.00024 26251 5	.00000 01232 6
.622	.47635 54426 4	.00024 30333 3	.00000 01233 7
.623	.47676 35544 3	.00024 32415 1	.00000 01235 0
.624	.47737 16662 1	.00024 34476 7	.00000 01236 0
0.625	0.50000 00000 0	0.00024 36560 5	0.00000 01237 1
.626	.50040 61115 7	.00024 40642 3	.00000 01240 1
.627	.50101 42233 5	.00024 42724 1	.00000 01241 2
.628	.50142 23351 4	.00024 45005 7	.00000 01242 2
.629	.50203 04467 2	.00024 47067 5	.00000 01243 3
0.630	0.50243 65605 1	0.00024 51151 3	0.00000 01244 4
.631	.50304 46723 0	.00024 53233 1	.00000 01245 4
.632	.50345 30040 6	.00024 55314 7	.00000 01246 5
.633	.50406 11156 5	.00024 57376 5	.00000 01247 5
.634	.50446 72274 3	.00024 61460 3	.00000 01250 6
0.635	0.50507 53412 2	0.00024 63542 0	0.00000 01251 7
.636	.50550 34530 1	.00024 65623 6	.00000 01252 7
.637	.50611 15645 7	.00024 67705 4	.00000 01254 0
.638	.50651 76763 6	.00024 71767 2	.00000 01255 0
.639	.50712 60101 4	.00024 74051 0	.00000 01256 1
0.640	0.50753 41217 3	0.00024 76132 6	0.00000 01257 2
.641	.51014 22335 1	.00025 00214 4	.00000 01260 2
.642	.51055 03453 0	.00025 02276 2	.00000 01261 3
.643	.51115 64570 7	.00025 04360 0	.00000 01262 3
.644	.51156 45706 5	.00025 06441 6	.00000 01263 4

$(n)_{10}$	$(n)_8$	$(n \times 10^{-3})_8$	$(n \times 10^{-6})_8$
0.645	0.51217 27024 4	0.00025 10523 4	0.00000 01264 4
.646	.51260 10142 2	.00025 12605 2	.00000 01265 5
.647	.51320 71260 1	.00025 14667 0	.00000 01266 6
.648	.51361 52376 0	.00025 16750 6	.00000 01267 6
.649	.51422 33513 6	.00025 21032 4	.00000 01270 7
0.650	0.51463 14631 5	0.00025 23114 1	0.00000 01271 7
.651	.51523 75747 3	.00025 25175 7	.00000 01273 0
.652	.51564 57065 2	.00025 27257 5	.00000 01274 1
.653	.51625 40203 1	.00025 31341 3	.00000 01275 1
.654	.51666 21320 7	.00025 33423 1	.00000 01276 2
0.655	0.51727 02436 6	0.00025 35504 7	0.00000 01277 2
.656	.51767 63554 4	.00025 37566 5	.00000 01300 3
.657	.52030 44672 3	.00025 41650 3	.00000 01301 4
658	.52071 26010 2	.00025 43732 1	.00000 01302 4
.659	.52132 07126 0	.00025 46013 7	.00000 01303 5
0.660	0.52172 70243 7	0.00025 50075 5	0.00000 01304 5
.661	.52233 51361 5	.00025 52157 3	.00000 01305 6
.662	.52274 32477 4	.00025 54241 1	.00000 01305 7
.663	.52335 13615 2	.00025 56322 7	.00000 01307 7
.664	.52375 74733 1	.00025 60404 5	.00000 01311 0
0.665	0.52436 56051 0	0.00025 62466 3	0.00000 01312 0
.666	.52477 37166 6	.00025 64550 0	.00000 01313 1
.667	52540 20304 5	.00025 66631 6	.00000 01314 1
.668	.52601 01422 3	.00025 70713 4	.00000 01315 2
.669	.52641 62540 2	.00025 72775 2	.00000 01316 3
0.670	0.52702 43656 1	0.00025 75057 0	0.00000 01317 3
.671	.52743 24773 7	.00025 77140 6	.00000 01320 4
.672	.53004 06111 6	.00026 01222 4	.00000 01321 4
.673	.53044 67227 4	.00026 03304 2	.00000 01322 5
.674	.53105 50345 3	.00026 05366 0	.00000 01323 6
0.675	0.53146 31463 2	0.00026 07447 6	0.00000 01324 6
.676	.53207 12601 0	.00026 11531 4	.00000 01325 7
.677	.53247 73716 7	.00026 13613 2	.00000 01326 7
.678	.53310 55034 5	.00026 15675 0	.00000 01330 0
.679	.53351 36152 4	.00026 17756 6	.00000 01331 1
0.680	0.53412 17270 3	0.00026 22040 4	0.00000 01332 1
.681	.53453 00406 1	.00026 24122 1	.00000 01333 2
.682	.53513 61524 0	.00026 26203 7	.00000 01334 2
.683	.53554 42641 6	.00026 30265 5	.00000 01335 3
.684	.53615 23757 5	.00026 32347 3	.00000 01336 3
0.685	0.53656 05075 4	0.00026 34431 1	0.00000 01337 4
.686	.53716 66213 2	.00026 36512 7	.00000 01340 5
.687	.53757 47331 1	.00026 40574 5	.00000 01341 5
.688	.54020 30446 7	.00026 42656 3	.00000 01342 6
.689	.54061 11564 6	.00026 44740 1	.00000 01343 6
0.690	0.54121 72702 4	0.00026 47021 7	0.00000 01344 7
.691	.54162 54020 3	.00026 51103 5	.00000 01346 0
.692	.54223 35136 2	.00026 53165 3	.00000 01347 0
.693	.54264 16254 0	.00026 55247 1	.00000 01350 1
.694	.54324 77371 7	.00026 57330 7	.00000 01351 1

$(n)_{10}$	$(n)_8$	$(n \times 10^{-3})_8$	$(n \times 10^{-6})_8$
0.695	0.54365 60507 5	0.00026 61412 5	0.00000 01352 2
.696	.54426 41625 4	.00026 63474 2	.00000 01353 3
.697	.54467 22743 3	.00026 65556 0	.00000 01354 3
.698	.54530 04061 1	.00026 67637 6	.00000 01355 4
.699	.54570 65177 0	.00026 71721 4	.00000 01356 4
0.700	0.54631 46314 6	0.00026 74003 2	0.00000 01357 5
.701	.54672 27432 5	.00026 76065 0	.00000 01360 6
.702	.54733 10550 4	.00027 00146 6	.00000 01361 6
.703	.54773 71666 2	.00027 02230 4	.00000 01362 7
.704	.55034 53004 1	.00027 04312 2	.00000 01363 7
0.705	0.55075 34121 7	0.00027 06374 0	0.00000 01365 0
.706	.55136 15237 6	.00027 10455 6	.00000 01366 0
.707	.55176 76355 5	.00027 12537 4	.00000 01367 1
.708	.55237 57473 3	.00027 14621 2	.00000 01370 2
.709	.55300 40611 2	.00027 16703 0	.00000 01371 2
0.710	0.55341 21727 0	0.00027 20764 6	0.00000 01372 3
.711	.55402 03044 7	.00027 23046 3	.00000 01373 3
.712	.55442 64162 6	.00027 25130 1	.00000 01374 4
.713	.55503 45300 4	.00027 27211 7	.00000 01375 5
.714	.55544 26416 3	.00027 31273 5	.00000 01376 5
0.715	0.55605 07534 1	0.00027 33355 3	0.00000 01377 6
.716	.55645 70652 0	.00027 35437 1	.00000 01400 6
.717	.55706 51767 6	.00027 37520 7	.00000 01401 7
.718	.55747 33105 5	.00027 41602 5	.00000 01403 0
.719	.56010 14223 4	.00027 43664 3	.00000 01404 0
0.720	0.56050 75341 2	0.00027 45746 1	0.00000 01405 1
.721	.56111 56457 1	.00027 50027 7	.00000 01406 1
.722	.56152 37574 7	.00027 52111 5	.00000 01407 2
.723	.56213 20712 6	.00027 54173 3	.00000 01410 3
.724	.56254 02030 5	.00027 56255 1	.00000 01411 3
0.725	0.56314 63146 3	0.00027 60336 7	0.00000 01412 4
.726	.56355 44264 2	.00027 62420 5	.00000 01413 4
.727	.56416 25402 0	.00027 64502 2	.00000 01414 5
.728	.56457 06517 7	.00027 66564 0	.00000 01415 5
.729	.56517 67635 6	.00027 70645 6	.00000 01416 6
0.730	0.56560 50753 4	0.00027 72727 4	0.00000 01417 7
.731	.56621 32071 3	.00027 75011 2	.00000 01420 7
.732	.56662 13207 1	.00027 77073 0	.00000 01422 0
.733	.56722 74325 0	.00030 01154 6	.00000 01423 0
.734	.56763 55442 7	.00030 03236 4	.00000 01424 1
0.735	0.57024 36560 5	0.00030 05320 2	0.00000 01425 2
.736	.57065 17676 4	.00030 07402 0	.00000 01426 2
.737	.57126 01014 2	.00030 11463 6	.00000 01427 3
.738	.57166 62132 1	.00030 13545 4	.00000 01430 3
.739	.57227 43247 7	.00030 15627 2	.00000 01431 4
0.740	0.57270 24365 6	0.00030 17711 0	0.00000 01432 5
.741	.57331 05503 5	.00030 21772 6	.00000 01433 5
.742	.57371 66621 3	.00030 24054 3	.00000 01434 6
.743	.57432 47737 2	.00030 26136 1	.00000 01435 6
.744	.57473 31055 0	.00030 30217 7	.00000 01436 7

$(n)_{10}$	$(n)_8$	$(n \times 10^{-3})_8$	$(n \times 10^{-6})_8$
0.745	0.57534 12172 7	0.00030 32301 5	0.00000 01437 7
.746	.57574 73310 6	.00030 34363 3	.00000 01441 0
.747	.57635 54426 4	.00030 36445 1	.00000 01442 1
.748	.57676 35544 3	.00030 40526 7	.00000 01443 1
.749	.57737 16662 1	.00030 42610 5	.00000 01444 2
0.750	0.60000 00000 0	0.00030 44672 3	0.00000 01445 2
.751	.60040 61115 7	.00030 46754 1	.00000 01446 3
.752	.60101 42233 5	.00030 51035 7	.00000 01447 4
.753	.60142 23351 4	.00030 53117 5	.00000 01450 4
.754	.60203 04467 2	.00030 55201 3	.00000 01451 5
0.755	0.60243 65605 1	0.00030 57263 1	0.00000 01452 5
.756	.60304 46723 0	.00030 61344 7	.00000 01453 6
.757	.60345 30040 6	.00030 63426 4	.00000 01454 7
.758	.60406 11156 5	.00030 65510 2	.00000 01455 7
.759	.60446 72274 3	.00030 67572 0	.00000 01457 0
0.760	0.60507 53412 2	0.00030 71653 6	0.00000 01460 0
.761	.60550 34530 1	.00030 73735 4	.00000 01461 1
.762	.60611 15645 7	.00030 76017 2	.00000 01462 2
.763	.60651 76763 6	.00031 00101 0	.00000 01463 2
.764	.60712 60101 4	.00031 02162 6	.00000 01464 3
0.765	0.60753 41217 3	0.00031 04244 4	0.00000 01465 3
.766	.61014 22335 1	.00031 06326 2	.00000 01466 4
.767	.61055 03453 0	.00031 10410 0	.00000 01467 4
.768	.61115 64570 7	.00031 12471 6	.00000 01470 5
.769	.61156 45706 5	.00031 14553 4	.00000 01471 6
0.770	0.61217 27024 4	0.00031 16635 2	0.00000 01472 6
.771	.61260 10142 2	.00031 20717 0	.00000 01473 7
.772	.61320 71260 1	.00031 23000 6	.00000 01474 7
.773	.61361 52376 0	.00031 25062 3	.00000 01476 0
.774	.61422 33513 6	.00031 27144 1	.00000 01477 1
0.775	0.61463 14631 5	0.00031 31225 7	0.00000 01500 1
.776	.61523 75747 3	.00031 33307 5	.00000 01501 2
.777	.61564 57065 2	.00031 35371 3	.00000 01502 2
.778	.61625 40203 1	.00031 37453 1	.00000 01503 3
.779	.61666 21320 7	.00031 41534 7	.00000 01504 4
0.780	0.61727 02436 6	0.00031 43616 5	0.00000 01505 4
.781	.61767 63554 4	.00031 45700 3	.00000 01506 5
.782	.62030 44672 3	.00031 47762 1	.00000 01507 5
.783	.62071 26010 2	.00031 52043 7	.00000 01510 6
.784	.62132 07126 0	.00031 54125 5	.00000 01511 6
0.785	0.62172 70243 7	0.00031 56207 3	0.00000 01512 7
.786	.62233 51361 5	.00031 60271 1	.00000 01514 0
.787	.62274 32477 4	.00031 62352 7	.00000 01515 0
.788	.62335 13615 2	.00031 64434 4	.00000 01516 1
.789	.62375 74733 1	.00031 66516 2	.00000 01517 1
0.790	0.62436 56051 0	0.00031 70600 0	0.00000 01520 2
.791	.62477 37166 6	.00031 72661 6	.00000 01521 3
.792	.62540 20304 5	.00031 74743 4	.00000 01522 3
.793	.62601 01422 3	.00031 77025 2	.00000 01523 4
.794	.62641 62540 2	.00032 01107 0	.00000 01524 4

$(n)_{10}$	$(n)_8$	$(n \times 10^{-3})_8$	$(n \times 10^{-6})_8$
0.795	0.62702 43656 1	0.00032 03170 6	0.00000 01525 5
.796	.62743 24773 7	.00032 05252 4	.00000 01526 6
.797	.63004 06111 6	.00032 07334 2	.00000 01527 6
.798	.63044 67227 4	.00032 11416 0	.00000 01530 7
.799	.63105 50345 3	.00032 13477 6	.00000 01531 7
0.800	0.63146 31463 2	0.00032 15561 4	0.00000 01533 0
.801	.63207 12601 0	.00032 17643 2	.00000 01534 1
.802	.63247 73716 7	.00032 21725 0	.00000 01535 1
.803	.63310 55034 5	.00032 24006 5	.00000 01536 2
.804	.63351 36152 4	.00032 26070 3	.00000 01537 2
0.805	0.63412 17270 3	0.00032 30152 1	0.00000 01540 3
.806	.63453 00406 1	.00032 32233 7	.00000 01541 3
.807	.63513 61524 0	.00032 34315 5	.00000 01542 4
.808	.63554 42641 6	.00032 36377 3	.00000 01543 5
.809	.63615 23757 5	.00032 40461 1	.00000 01544 5
0.810	0.63656 05075 4	0.00032 42542 7	0.00000 01545 6
.811	.63716 66213 2	.00032 44624 5	.00000 01546 6
.812	.63757 47331 1	.00032 46706 3	.00000 01547 7
.813	.64020 30446 7	.00032 50770 1	.00000 01551 0
.814	.64061 11564 6	.00032 53051 7	.00000 01552 0
0.815	0.64121 72702 4	0.00032 55133 5	0.00000 01553 1
.816	.64162 54020 3	.00032 57215 3	.00000 01554 1
.817	.64223 35136 2	.00032 61277 1	.00000 01555 2
.818	.64264 16254 0	.00032 63360 6	.00000 01556 3
.819	.64324 77371 7	.00032 65442 4	.00000 01557 3
0.820	0.64365 60507 5	0.00032 67524 2	0.00000 01560 4
.821	.64426 41625 4	.00032 71606 0	.00000 01561 4
.822	.64467 22743 3	.00032 73667 6	.00000 01562 5
.823	.64530 04061 1	.00032 75751 4	.00000 01563 5
.824	.64570 65177 0	.00033 00033 2	.00000 01564 6
0.825	0.64631 46314 6	0.00033 02115 0	0.00000 01566 7
.826	.64672 27432 5	.00033 04176 6	.00000 01566 7
.827	.64733 10550 4	.00033 06260 4	.00000 01570 0
.828	.64773 71666 2	.00033 10342 2	.00000 01571 0
.829	.65034 53004 1	.00033 12424 0	.00000 01572 1
0.830	0.65075 34121 7	0.00033 14505 6	0.00000 01573 2
.831	.65136 15237 6	.00033 16567 4	.00000 01574 2
.832	.65176 76355 5	.00033 20651 2	.00000 01575 3
.833	.65237 57473 3	.00033 22733 0	.00000 01576 3
.834	.65300 40611 2	.00033 25014 5	.00000 01577 4
0.835	0.65341 21727 0	0.00033 27076 3	0.00000 01600 5
.836	.65402 03044 7	.00033 31160 1	.00000 01601 5
.837	.65442 64162 6	.00033 33241 7	.00000 01602 6
.838	.65503 45300 4	.00033 35323 5	.00000 01603 6
.839	.65544 26416 3	.00033 37405 3	.00000 01604 7
0.840	0.65605 07534 1	0.00033 41467 1	0.00000 01606 0
.841	.65645 70652 0	.00033 43550 7	.00000 01607 0
.842	.65706 51767 6	.00033 45632 5	.00000 01610 1
.843	.65747 33105 5	.00033 47714 3	.00000 01611 1
.844	.66010 14223 4	.00033 51776 1	.00000 01612 2

$(n)_{10}$	$(n)_8$	$(n \times 10^{-3})_8$	$(n \times 10^{-6})_8$
0.845	0.66050 75341 2	0.00033 54057 7	0.00000 01613 2
.846	.66111 56457 1	.00033 56141 5	.00000 01614 3
.847	.66152 37574 7	.00033 60223 3	.00000 01615 4
.848	.66213 20712 6	.00033 62305 1	.00000 01616 4
.849	.66254 02030 5	.00033 64366 6	.00000 01617 5
0.850	0.66314 63146 3	0.00033 66540 4	0.00000 01620 5
.851	.66355 44264 2	.00033 70532 2	.00000 01621 6
.852	.66416 25402 0	.00033 72614 0	.00000 01622 7
.853	.66457 06517 7	.00033 74675 6	.00000 01623 7
.854	.66517 67635 6	.00033 76757 4	.00000 01625 0
0.855	0.66560 50753 4	0.00034 01041 2	0.00000 01626 0
.856	.66621 32071 3	.00034 03123 0	.00000 01627 1
.857	.66662 13207 1	.00034 05204 6	.00000 01630 2
.858	.66722 74325 0	.00034 07266 4	.00000 01631 2
.859	.66763 55442 7	.00034 11350 2	.00000 01632 3
0.860	0.67024 36560 5	0.00034 13432 0	0.00000 01633 3
.861	.67065 17676 4	.00034 15513 6	.00000 01634 4
.862	.67126 01014 2	.00034 17575 4	.00000 01635 5
.863	.67166 62132 1	.00034 21657 2	.00000 01636 5
.864	.67227 43247 7	.00034 23740 7	.00000 01637 6
0.865	0.67270 24365 6	0.00034 26022 5	0.00000 01640 6
.866	.67331 05503 5	.00034 30104 3	.00000 01641 7
.867	.67371 66621 3	.00034 32166 1	.00000 01642 7
.868	.67432 47737 2	.00034 34247 7	.00000 01644 0
.869	.67473 31055 0	.00034 36331 5	.00000 01645 1
0.870	0.67534 12172 7	0.00034 40413 3	0.00000 01646 1
.871	.67574 73310 6	.00034 42475 1	.00000 01647 2
.872	.67635 54426 4	.00034 44556 7	.00000 01650 2
.873	.67676 35544 3	.00034 46640 5	.00000 01651 3
.874	.67737 16662 1	.00034 50722 3	.00000 01652 4
0.875	0.70000 00000 0	0.00034 53004 1	0.00000 01653 4
.876	.70040 61115 7	.00034 55065 7	.00000 01654 5
.877	.70101 42233 5	.00034 57147 5	.00000 01655 5
.878	.70142 23351 4	.00034 61231 3	.00000 01656 6
.879	.70203 04467 2	.00034 63313 1	.00000 01657 7
0.880	0.70243 65605 1	0.00034 65374 6	0.00000 01660 7
.881	.70304 46723 0	.00034 67456 4	.00000 01662 0
.882	.70345 30040 6	.00034 71540 2	.00000 01663 0
.883	.70406 11156 5	.00034 73622 0	.00000 01664 1
.884	.70446 72274 3	.00034 75703 6	.00000 01665 1
0.885	0.70507 53412 2	0.00034 77765 4	0.00000 01666 2
.886	.70550 34530 1	.00035 02047 2	.00000 01667 3
.887	.70611 15645 7	.00035 04131 0	.00000 01670 3
.888	.70651 76763 6	.00035 06212 6	.00000 01671 4
.889	.70712 60101 4	.00035 10274 4	.00000 01672 4
0.890	0.70753 41217 3	0.00035 12356 2	0.00000 01673 5
.891	.71014 22335 1	.00035 14440 0	.00000 01674 6
.892	.71055 03453 0	.00035 16521 6	.00000 01675 6
.893	.71115 64570 7	.00035 20603 4	.00000 01676 7
.894	.71156 45706 5	.00035 22665 2	.00000 01677 7

$(n)_{10}$	$(n)_8$	$(n \times 10^{-3})_8$	$(n \times 10^{-6})_8$
0.895	0.71217 27024 4	0.00035 24746 7	0.00000 01701 0
.896	.71260 10142 2	.00035 27030 5	.00000 01702 1
.897	.71320 71260 1	.00035 31112 3	.00000 01703 1
.898	.71361 52376 0	.00035 33174 1	.00000 01704 2
.899	.71422 33513 6	.00035 35255 7	.00000 01705 2
0.900	0.71463 14631 5	0.00035 37337 5	0.00000 01706 3
.901	.71523 75747 3	.00035 41421 3	.00000 01707 4
.902	.71564 57065 2	.00035 43503 1	.00000 01710 4
.903	.71625 40203 1	.00035 45564 7	.00000 01711 5
.904	.71666 21320 7	.00035 47646 5	.00000 01712 5
0.905	0.71727 02436 6	0.00035 51730 3	0.00000 01713 6
.906	.71767 63554 4	.00035 54012 1	.00000 01714 6
.907	.72030 44672 3	.00035 56073 7	.00000 01715 7
.908	.72071 26010 2	.00035 60155 5	.00000 01717 0
.909	.72132 07126 0	.00035 62237 3	.00000 01720 0
0.910	0.72172 70243 7	0.00035 64321 0	0.00000 01721 1
.911	.72233 51361 5	.00035 66402 6	.00000 01722 1
.912	.72274 32477 4	.00035 70464 4	.00000 01723 2
.913	.72335 13615 2	.00035 72546 2	.00000 01724 3
.914	.72375 74733 1	.00035 74630 0	.00000 01725 3
0.915	0.72436 56051 0	0.00035 76711 6	0.00000 01726 4
.916	.72477 37166 6	.00036 00773 4	.00000 01727 4
.917	.72540 20304 5	.00036 03055 2	.00000 01730 5
.918	.72601 01422 3	.00036 05137 0	.00000 01731 6
.919	.72641 62540 2	.00036 07220 6	.00000 01732 6
0.920	0.72702 43656 1	0.00036 11302 4	0.00000 01733 7
.921	.72743 24773 7	.00036 13364 2	.00000 01734 7
.922	.73004 06111 6	.00036 15446 0	.00000 01736 0
.923	.73044 67227 4	.00036 17527 6	.00000 01737 0
.924	.73105 50345 3	.00036 21611 4	.00000 01740 1
0.925	0.73146 31463 2	0.00036 23673 1	0.00000 01741 2
.926	.73207 12601 0	.00036 25754 7	.00000 01742 2
.927	.73247 73716 7	.00036 30036 5	.00000 01743 3
.928	.73310 55034 5	.00036 32120 3	.00000 01744 3
.929	.73351 36152 4	.00036 34202 1	.00000 01745 4
0.930	0.73412 17270 3	0.00036 36263 7	0.00000 01746 5
.931	.73453 00406 1	.00036 40345 5	.00000 01747 5
.932	.73513 61524 0	.00036 42427 3	.00000 01750 6
.933	.73554 42641 6	.00036 44511 1	.00000 01751 6
.934	.73615 23757 5	.00036 46572 7	.00000 01752 7
0.935	0.73656 05075 4	0.00036 50654 5	0.00000 01754 0
.936	.73716 66213 2	.00036 52736 3	.00000 01755 0
.937	.73757 47331 1	.00036 55020 1	.00000 01756 1
.938	.74020 30446 7	.00036 57101 7	.00000 01757 1
.939	.74061 11564 6	.00036 61163 5	.00000 01760 2
0.940	0.74121 72702 4	0.00036 63245 3	0.00000 01761 3
.941	.74162 54020 3	.00036 65327 0	.00000 01762 3
.942	.74223 35136 2	.00036 67410 6	.00000 01763 4
.943	.74264 16254 0	.00036 71472 4	.00000 01764 4
.944	.74324 77371 7	.00036 73554 2	.00000 01765 5

$(n)_{10}$	$(n)_8$			$(n \times 10^{-3})_8$			$(n \times 10^{-6})_8$		
0.945	0.74365	60507	5	0.00036	75636	0	0.00000	01766	5
.946	.74426	41625	4	.00036	77717	6	.00000	01767	6
.947	.74467	22743	3	.00037	02001	4	.00000	01770	7
.948	.74530	04061	1	.00037	04063	2	.00000	01771	7
.949	.74570	65177	0	.00037	06145	0	.00000	01773	0
0.950	0.74631	46314	6	0.00037	10226	6	0.00000	01774	0
.951	.74672	27432	5	.00037	12310	4	.00000	01775	1
.952	.74733	10550	4	.00037	14372	2	.00000	01776	2
.953	.74773	71666	2	.00037	16454	0	.00000	01777	2
.954	.75034	53004	1	.00037	20535	6	.00000	02000	3
0.955	0.75075	34121	7	0.00037	22617	4	0.00000	02001	3
.956	.75136	15237	6	.00037	24701	1	.00000	02002	4
.957	.75176	76355	5	.00037	26762	7	.00000	02003	5
.958	.75237	57473	3	.00037	31044	5	.00000	02004	5
.959	.75300	40611	2	.00037	33126	3	.00000	02005	6
0.960	0.75341	21727	0	0.00037	35210	1	0.00000	02006	6
.961	.75402	03044	7	.00037	37271	7	.00000	02007	7
.962	.75442	64162	6	.00037	41353	5	.00000	02010	7
.963	.75503	45300	4	.00037	43435	3	.00000	02012	0
.964	.75544	26416	3	.00037	45517	1	.00000	02013	1
0.965	0.75605	07534	1	0.00037	47600	7	0.00000	02014	1
.966	.75645	70652	0	.00037	51662	5	.00000	02015	2
.967	.75706	51767	6	.00037	53744	3	.00000	02016	2
.968	.75747	33105	5	.00037	56026	1	.00000	02017	3
.969	.76010	14223	4	.00037	60107	7	.00000	02020	4
0.970	0.76050	75341	2	0.00037	62171	5	0.00000	02021	4
.971	.76111	56457	1	.00037	64253	2	.00000	02022	5
.972	.76152	37574	7	.00037	66335	0	.00000	02023	5
.973	.76213	20712	6	.00037	70416	6	.00000	02024	6
.974	.76254	02030	5	.00037	72500	4	.00000	02025	7
0.975	0.76314	63146	3	0.00037	74562	2	0.00000	02026	7
.976	.76355	44264	2	.00037	76644	0	.00000	02030	0
.977	.76416	25402	0	.00040	00725	6	.00000	02031	0
.978	.76457	06517	7	.00040	03007	4	.00000	02032	1
.979	.76517	67635	6	.00040	05071	2	.00000	02033	2
0.980	0.76560	50753	4	0.00040	07153	0	0.00000	02034	2
.981	.76621	32071	3	.00040	11234	6	.00000	02035	3
.982	.76662	13207	1	.00040	13316	4	.00000	02036	3
.983	.76722	74325	0	.00040	15400	2	.00000	02037	4
.984	.76763	55442	7	.00040	17462	0	.00000	02040	4
0.985	0.77024	36560	5	0.00040	21543	6	0.00000	02041	5
.986	.77065	17676	4	.00040	23625	4	.00000	02042	6
.987	.77126	01014	2	.00040	25707	1	.00000	02043	6
.988	.77166	62132	1	.00040	27770	7	.00000	02044	7
.989	.77227	43247	7	.00040	32052	5	.00000	02045	7
0.990	0.77270	24365	6	0.00040	34134	3	0.00000	02047	0
.991	.77331	05503	5	.00040	36216	1	.00000	02050	1
.992	.77371	66621	3	.00040	40277	7	.00000	02051	1
.993	.77432	47737	2	.00040	42361	5	.00000	02052	2
.994	.77473	31055	0	.00040	44443	3	.00000	02053	2
0.995	0.77534	12172	7	0.00040	46525	1	0.00000	02054	3
.996	.77574	73310	6	.00040	50606	7	.00000	02055	4
.997	.77635	54426	4	.00040	52670	5	.00000	02056	4
.998	.77676	35544	3	.00040	54752	3	.00000	02057	5
.999	.77737	16662	1	.00040	57034	1	.00000	02060	5

APPENDIX III

Octal-Decimal Conversions

OCTAL	0	1	2	3	4	5	6	7
00000	00000	00001	00002	00003	00004	00005	00006	00007
00010	00008	00009	00010	00011	00012	00013	00014	00015
00020	00016	00017	00018	00019	00020	00021	00022	00023
00030	00024	00025	00026	00027	00028	00029	00030	00031
00040	00032	00033	00034	00035	00036	00037	00038	00039
00050	00040	00041	00042	00043	00044	00045	00046	00047
00060	00048	00049	00050	00051	00052	00053	00054	00055
00070	00056	00057	00058	00059	00060	00061	00062	00063
00100	00064	00065	00066	00067	00068	00069	00070	00071
00110	00072	00073	00074	00075	00076	00077	00078	00079
00120	00080	00081	00082	00083	00084	00085	00086	00087
00130	00088	00089	00090	00091	00092	00093	00094	00095
00140	00096	00097	00098	00099	00100	00101	00102	00103
00150	00104	00105	00106	00107	00108	00109	00110	00111
00160	00112	00113	00114	00115	00116	00117	00118	00119
00170	00120	00121	00122	00123	00124	00125	00126	00127
00200	00128	00129	00130	00131	00132	00133	00134	00135
00210	00136	00137	00138	00139	00140	00141	00142	00143
00220	00144	00145	00146	00147	00148	00149	00150	00151
00230	00152	00153	00154	00155	00156	00157	00158	00159
00240	00160	00161	00162	00163	00164	00165	00166	00167
00250	00168	00169	00170	00171	00172	00173	00174	00175
00260	00176	00177	00178	00179	00180	00181	00182	00183
00270	00184	00185	00186	00187	00188	00189	00190	00191
00300	00192	00193	00194	00195	00196	00197	00198	00199
00310	00200	00201	00202	00203	00204	00205	00206	00207
00320	00208	00209	00210	00211	00212	00213	00214	00215
00330	00216	00217	00218	00219	00220	00221	00222	00223
00340	00224	00225	00226	00227	00228	00229	00230	00231
00350	00232	00233	00234	00235	00236	00237	00238	00239
00360	00240	00241	00242	00243	00244	00245	00246	00247
00370	00248	00249	00250	00251	00252	00253	00254	00255
00400	00256	00257	00258	00259	00260	00261	00262	00263
00410	00264	00265	00266	00267	00268	00269	00270	00271
00420	00272	00273	00274	00275	00276	00277	00278	00279
00430	00280	00281	00282	00283	00284	00285	00286	00287
00440	00288	00289	00290	00291	00292	00293	00294	00295
00450	00296	00297	00298	00299	00300	00301	00302	00303
00460	00304	00305	00306	00307	00308	00309	00310	00311
00470	00312	00313	00314	00315	00316	00317	00318	00319
00500	00320	00321	00322	00323	00324	00325	00326	00327
00510	00328	00329	00330	00331	00332	00333	00334	00335
00520	00336	00337	00338	00339	00340	00341	00342	00343
00530	00344	00345	00346	00347	00348	00349	00350	00351
00540	00352	00353	00354	00355	00356	00357	00358	00359
00550	00360	00361	00362	00363	00364	00365	00366	00367
00560	00368	00369	00370	00371	00372	00373	00374	00375
00570	00376	00377	00378	00379	00380	00381	00382	00383
00600	00384	00385	00386	00387	00388	00389	00390	00391
00610	00392	00393	00394	00395	00396	00397	00398	00399
00620	00400	00401	00402	00403	00404	00405	00406	00407
00630	00408	00409	00410	00411	00412	00413	00414	00415
00640	00416	00417	00418	00419	00420	00421	00422	00423
00650	00424	00425	00426	00427	00428	00429	00430	00431
00660	00432	00433	00434	00435	00436	00437	00438	00439
00670	00440	00441	00442	00443	00444	00445	00446	00447
00700	00448	00449	00450	00451	00452	00453	00454	00455
00710	00456	00457	00458	00459	00460	00461	00462	00463
00720	00464	00465	00466	00467	00468	00469	00470	00471
00730	00472	00473	00474	00475	00476	00477	00478	00479
00740	00480	00481	00482	00483	00484	00485	00486	00487
00750	00488	00489	00490	00491	00492	00493	00494	00495
00760	00496	00497	00498	00499	00500	00501	00502	00503
00770	00504	00505	00506	00507	00508	00509	00510	00511

(Courtesy Computer Control Co.)

OCTAL	0	1	2	3	4	5	6	7
01000	00512	00513	00514	00515	00516	00517	00518	00519
01010	00520	00521	00522	00523	00524	00525	00526	00527
01020	00528	00529	00530	00531	00532	00533	00534	00535
01030	00536	00537	00538	00539	00540	00541	00542	00543
01040	00544	00545	00546	00547	00548	00549	00550	00551
01050	00552	00553	00554	00555	00556	00557	00558	00559
01060	00560	00561	00562	00563	00564	00565	00566	00567
01070	00568	00569	00570	00571	00572	00573	00574	00575
01100	00576	00577	00578	00579	00580	00581	00582	00583
01110	00584	00585	00586	00587	00588	00589	00590	00591
01120	00592	00593	00594	00595	00596	00597	00598	00599
01130	00600	00601	00602	00603	00604	00605	00606	00607
01140	00608	00609	00610	00611	00612	00613	00614	00615
01150	00616	00617	00618	00619	00620	00621	00622	00623
01160	00624	00625	00626	00627	00628	00629	00630	00631
01170	00632	00633	00634	00635	00636	00637	00638	00639
01200	00640	00641	00642	00643	00644	00645	00646	00647
01210	00648	00649	00650	00651	00652	00653	00654	00655
01220	00656	00657	00658	00659	00660	00661	00662	00663
01230	00664	00665	00666	00667	00668	00669	00670	00671
01240	00672	00673	00674	00675	00676	00677	00678	00679
01250	00680	00681	00682	00683	00684	00685	00686	00687
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16600	07552	07553	07554	07555	07556	07557	07558	07559
16610	07560	07561	07562	07563	07564	07565	07566	07567
16620	07568	07569	07570	07571	07572	07573	07574	07575
16630	07576	07577	07578	07579	07580	07581	07582	07583
16640	07584	07585	07586	07587	07588	07589	07590	07591
16650	07592	07593	07594	07595	07596	07597	07598	07599
16660	07600	07601	07602	07603	07604	07605	07606	07607
16670	07608	07609	07610	07611	07612	07613	07614	07615
16700	07616	07617	07618	07619	07620	07621	07622	07623
16710	07624	07625	07626	07627	07628	07629	07630	07631
16720	07632	07633	07634	07635	07636	07637	07638	07639
16730	07640	07641	07642	07643	07644	07645	07646	07647
16740	07648	07649	07650	07651	07652	07653	07654	07655
16750	07656	07657	07658	07659	07660	07661	07662	07663
16760	07664	07665	07666	07667	07668	07669	07670	07671
16770	07672	07673	07674	07675	07676	07677	07678	07679