HERBERT J. REICH

FUNCTIONAL CIRCUITS AND OSCILLATORS

The Van Nostrand Series in Electronics and Communications

REICH

FUNCTIONAL CIRCUITS AND OSCILLATORS

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THE rapidly growing fields of instrumentation, control, and computation have placed great emphasis upon electronic circuits that generate or employ pulses and nonsinusoidal waves. This work deals primarily with the analysis, characteristics and applications of such circuits; it includes a comprehensive presentation of the theory and characteristics of negative-resistance circuits and devices. The last 22 sections treat sine-wave oscillators, since many of the basic principles of oscillators are closely related to those of circuits discussed earlier.

Professor Reich emphasizes the analysis and characteristics of basic circuits rather than circuits designed to meet specific requirements. The student or engineer who has mastered the basic principles should be able to synthesize circuits to meet special needs.

Tube and transistor circuits are discussed in the same sections except where differences in characteristics make this treatment impractical. In the examples of tube and transistor circuits, numerical values of circuit elements are provided wherever convenient.

Problems form an integral part of the presentation. In many cases, they outline details of analysis that are not completely worked out in the body of the text. The author refers to appropriate problems at points where their solutions will help the reader increase his understanding of the theory.

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FUNCTIONAL CIRCUITS AND OSCILLATORS

by

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THIS BOOK IS DEDICATED
TO ALL WHO HAVE PARTICIPATED IN ITS PREPARATION

PREFACE

A large portion of this book deals with the theory, characteristics, and applications of negative-resistance circuits and with the theory and characteristics of sine-wave oscillators. The book might have been restricted to these subjects and entitled, "Oscillators and Negative-Resistance Devices." Because many negative-resistance devices find their principal applications as counters, memory devices, and generators of pulses and nonsinusoidal waves, it seemed desirable to extend the coverage to other functional circuits, including summing circuits, differential amplifiers, differentiators, integrators, clippers, gates, and nonlinear-circuit simulators.

Much has been written on the subject of negative-resistance circuits and their applications. No previous attempt appears to have been made, however, to analyze generalized basic negative-resistance circuits and to derive principles and criteria that can be conveniently used in the synthesis of almost all presently-known practical negative-resistance circuits and devices and the bistable, astable, monostable, and sine-wave-oscillator circuits based upon them. When these principles and criteria are understood, it is a simple matter to synthesize negative-resistance circuits using any type of active element and to determine the ports at which the circuits have voltage-stable current-voltage characteristics, and those at which they have current-stable characteristics. The circuit elements that must be added in order to convert a particular negative-resistance circuit into a bistable, astable, monostable, or sine-wave-oscillator circuit can then be readily determined.

It is advantageous to consider all feedback oscillators from the point of view of a basic generalized circuit consisting of a broadband amplifier and a frequency-selective feedback network. In this manner a criterion for steady-state oscillation of all feedback oscillators may be derived in terms of the y and z parameters of two-port networks. The criterion may be readily applied to the analysis of almost all commonly-used feedback oscillators incorporating tubes, transistors, or other active elements. This approach affords an insight into the causes of frequency instability and suggests ways of increasing stability. It is also helpful in the synthesis of new or modified circuits. Another important aspect of oscillator theory treated in this book is the mechanism of amplitude stabilization by the use of circuit nonlinearities.

The desire to minimize the space devoted to routine mathematical derivations may tempt an author to rely unduly upon the statement, "It may be readily shown that . . . ," when, in truth, the proper procedure is obvious only after the analysis in question has been performed. In order to eliminate mathematical details without straining the ingenuity of the reader unnecessarily, many proofs and derivations throughout the book have been presented in the form of problems in which a general method of procedure is outlined.

It is hoped that this book may find application both as a text and as a reference and that it will prove useful in the synthesis of new circuits to serve specific functions. Although the author has attempted to make the book up-to-date at the time of publication and to include pertinent recent references, anyone working in any branch of the electronics field will be aware of the difficulty faced by a single author in achieving this goal at the present time of very rapid growth.

H. J. R.

New Haven, Conn.
April 1, 1961

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Introduction

For many years the field of electronics was concerned mainly with the use of electronic devices in the generation, amplification, and control of sinusoidal and modulated waves. More recently, the rapidly growing fields of instrumentation, control, and computing have placed great emphasis upon electronic circuits that generate or employ pulses and nonsinusoidal waves. The principal portion of this book deals with the analysis, characteristics, and applications of circuits of this type. The last 22 sections treat sine-wave oscillators, the basic principles of many of which are closely related to those of circuits discussed in earlier sections.

A knowledge of the basic principles and characteristics of the circuits treated throughout this book should enable the reader to synthesize circuits to meet specific requirements. Of even greater potential value as a discipline is the mathematical and phenomenological analysis of the circuits. For this reason, greater emphasis has been placed upon the analysis and characteristics of the basic circuits than upon specific circuits that meet individual requirements. Where convenient, numerical values of circuit elements have been provided in the examples of tube and transistor circuits.

Problems are numbered according to sections, and reference is made to the problems at points where their solution should be helpful to an understanding of the theory. Many of the problems consist of outlines of details of analysis that can be profitably performed by the reader, and that would take up considerable space if included in complete form in the body of the text. The reader is urged to work at least a substantial portion of the problems.

Chapter divisions have not been used in this book, because the material presented is in general such that breakdown into chapters would be difficult and often artificial.

1. Circuits for the Addition of Voltages

1.1. Summing Amplifiers. Among the desirable characteristics of adding circuits are negligible amplitude, frequency, and phase distortion; high input impedance; and small coupling between various sources of input voltage. These characteristics can be realized in circuits incorporating vacuum tubes and, with somewhat greater difficulty, in circuits incorporating transistors. As in other circuits in which vacuum tubes are used, amplitude distortion is minimized by the use of small input voltage, by the proper choice of load impedances, and by the use of inverse feedback. Small frequency and phase distortion necessitate that the bandwidth of the circuit be large in comparison with the reciprocal of the shortest rise or fall time of the input voltage. Frequency and phase distortion are caused by the presence of reactive circuit elements, usually shunt circuit capacitances. These types of distortion are minimized by the reduction of shunt capacitances and the resistances through which they charge and discharge, by the use of the inverse feedback, and by any other means of increasing the bandwidth of the circuit.

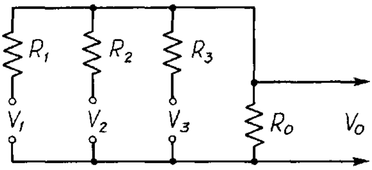


FIG. 1. Resistance type of summing circuit.

The simplest type of summing circuit is the resistor network shown in Fig. 1. Although the requirement of high input impedance can be satisfied in this circuit by making the resistances R_1, R_2, etc., large, cross-coupling between the sources of input voltage can be kept small only by making the resistance R_o (or the resistance of the external load shunting the output terminals) very small in comparison with R_1 and the other series resistances. Since this has the undesirable effect of reducing the output voltage, the output voltage must in general be amplified. Distortion and dependence of the output voltage upon amplifier supply voltage are minimized by the use of the inverse-feedback circuit of Fig. 2. The phase-inverting amplifier in this

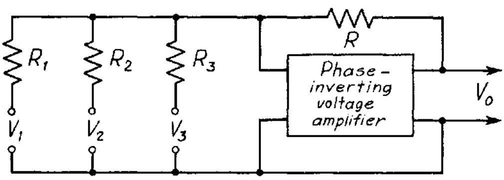


FIG. 2. Summing circuit incorporating an amplifier with inverse feedback.

circuit transforms the resistance R into a much lower effective resistance $R/(|A| + 1)$ across its input terminals and at the same time amplifies the voltage developed across this effective resistance.

If $R_1 = R/k_1$, $R_2 = R/k_2$, $\cdots$ $R_n = R/k_n$, and if $|A| \gg 1 + \sum_1^n k_h$, the output voltage V_o of the circuit of Fig. 2 is very nearly $-\sum_1^n k_h V_h$, the input resistance of lth input branch is very nearly the series resistance R/k_l of that branch, and the current through the mth input source produced by the lth input voltage V_l is very nearly $V_l k_l k_m / AR$ if each source resistance is negligible in comparison with the series resistance of its branch (Prob. 1.1-1). It is apparent that high input resistance and negligible cross-coupling may be obtained in this circuit by the use of high gain and a large value of R, and that the amplification of each input voltage V_h may be adjusted independently of that of the others by choice of the series resistance R_h of the input branch to which it is applied. The circuit has the additional desirable feature that the inverse feedback produced by R greatly reduces amplifier distortion and dependence of gain upon supply voltage and individual tube characteristics if the magnitude of the open-loop amplification A is high. A high-gain stabilized d-c amplifier suitable for use in this circuit will be discussed in Sec. 5.

1.2. Tubes or Transistors with Common Load. Loading of the sources of input voltage and cross-coupling between the sources can be made negligi-

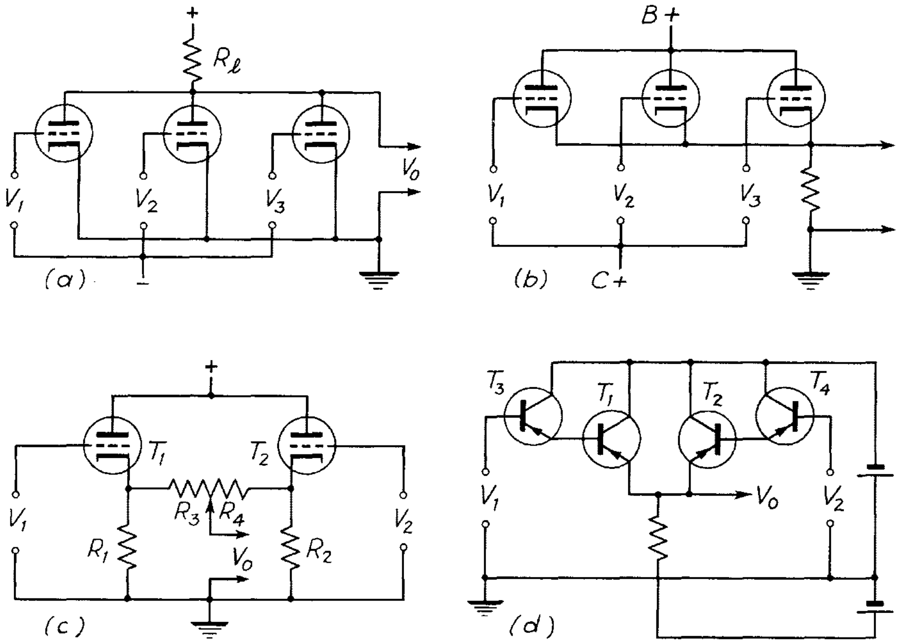


FIG. 3. Summing circuits consisting of several tubes with common load resistor: (a) plate output; (b) cathode output; (c) weighted cathode output; (d) emitter output with compound-connected input.

ble by impressing the input voltages upon the grids of individual vacuum tubes having a common load resistor, as shown in Fig. 3a. As long as the input voltages are so small that amplitude distortion is negligible, the component of output voltage resulting from any input voltage is independent of the other input voltages and the output voltage for n similar tubes is (Prob. 1.2–1):

$$-\frac{\mu R_l \sum_{h=1}^{n} V_h}{r_p + nR_l}$$

where V_h is the input signal voltage of the hth tube and r_p is the a-c plate resistance. Output impedance, distortion, and dependence of the output voltage upon the plate supply voltage can be reduced, at the expense of output amplitude, by using the tubes in the cathode-follower circuit of Fig. 3b. The output voltage of this circuit is approximately (Prob. 1.2–2):

$$\frac{1}{n}\sum_{h=1}^{n} V_h$$

Figure 3c shows a modified cathode-follower circuit that provides a weighted output approximately equal to $V_1R_4/(R_3 + R_4) + V_2R_3/(R_3 + R_4)$ when only two input voltages are required (Prob. 1.2–3).

Outputs proportional to the sum of two or more input voltages or currents can be obtained from transistor circuits analogous to the tube circuits of Fig. 3. The transistor circuit analogous to the tube circuit of Fig. 3a is one in which a collector resistor is shared by two or more transistors; the transistor circuit analogous to the cathode-follower circuit of Fig. 3b is one in which a single emitter resistor is shared by two or more transistors in the common-emitter connection. The input resistance of the transistor circuit is considerably lower than that of the tube circuits. The input resistance of a transistor circuit may be increased, however, by the use of the "compound connection," as shown in Fig. 3d.[1]

2. *Circuits for the Subtraction of Voltages*

2.1. The Differential Amplifier. The desired characteristics of subtracting circuits are the same as those for adding circuits. The circuit that is usually used to obtain an output voltage proportional to the difference between two input voltages is the differential amplifier, shown in basic form in Fig. 4a.[1] If the circuit of Fig. 4a is symmetrical in all respects, $R_k \ll 1/g_m$, and the internal impedance shunted across the terminals is assumed to be infinite, the plate-to-plate output voltage approximates $\mu R_l(V_1 - V_2)/(r_p + R_l)$ (Prob. 2.1–1). Zero output is then obtained for direct-voltage inputs of equal magnitudes and the same phase. Differences

[1] A. R. Pearlman, *Trans. I.R.E.*, Vol. ED-2, No. 1, p. 25 (January, 1955); S. Darlington, Pat. No. 2,663,806; B. Oliver, Pat. No. 2,663,830.

[1] Valley and Wallman, *Rad. Lab. Series*, Vol. 18, p. 441.

in the values of the amplification factors and plate resistances of the two triodes and in the values of the two load resistances cause the voltages developed across the two load resistors to be unequal for equal voltage inputs. Some output is then obtained for equal in-phase alternating input

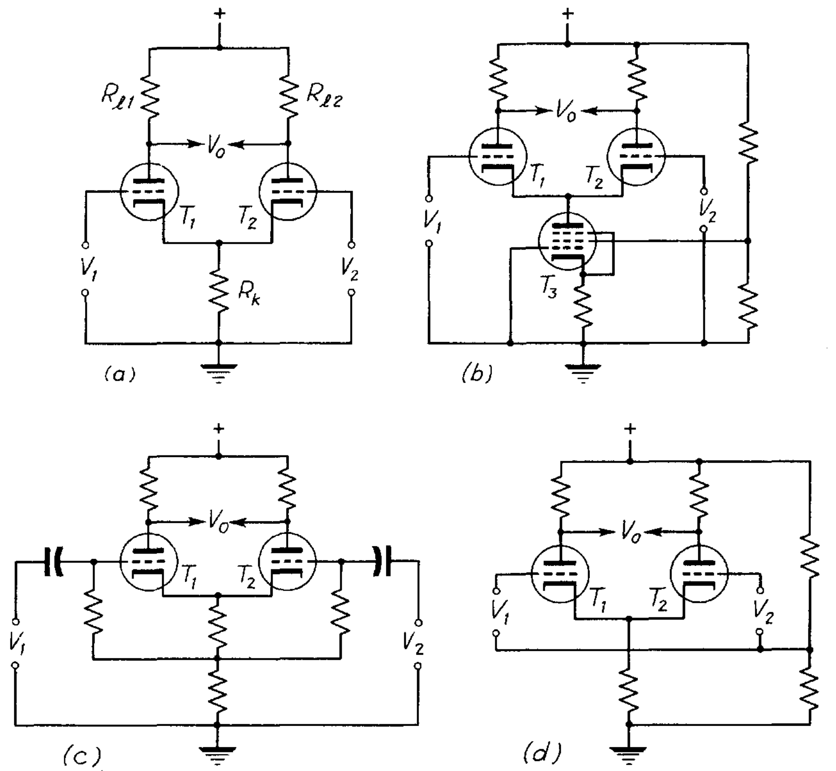


FIG. 4. Differential amplifier: (a) basic circuit; (b) circuit in which the cathode resistor is replaced by a pentode; (c) circuit in which a tapped cathode resistor is used to reduce the negative grid bias; (d) circuit using a tap on the voltage supply (if the lower input terminals are grounded, an additional negative source of voltage must be used in circuit (d).

voltages. The degree of balance of such a circuit is usually specified in terms of the *transmission factor* (*rejection ratio*), which is defined as the ratio of the *difference-mode output* to the *common-mode output* for equal input voltages; i.e., the ratio of the output voltage for equal input voltages of opposite phase to the output voltage for equal input voltages of the same phase.

The expression for the plate-to-plate output voltage of the circuit of Fig. 4a when the circuit is not symmetrical (Prob. 2.1–1) shows that the unbalance may be minimized by making μR_k large in comparison with $r_p + R_l$.[2] Excessive loss of direct voltage across the cathode resistor may be avoided by the use of a pentode in place of a resistor for R_k, as in Fig. 4b. Transmission factors as high as 100,000 are attainable in a carefully balanced circuit of this type.[3] The high value of cathode resistance essential to a high transmission factor results in a direct voltage drop across the cathode resistor that usually exceeds the desired grid bias, even when a pentode is used as the cathode circuit resistance. When the circuit is used only with varying input voltages, the correct bias may be established by means of resistance-capacitance coupling and a tap on the cathode resistor (or on a resistor shunting the pentode), as in Fig. 4c. When the output must indicate the difference between two steady direct input voltages, however, the lower input terminals must be connected to a positive point on the voltage supply, as in Fig. 4d.

2.2. Differential-Amplifier Balance. The differential amplifier may be balanced by changing the ratio of the two load resistances R_{l1} and R_{l2}, as shown in Fig. 5a. Decrease of either resistance is accompanied by decrease of r_p of the corresponding tube, caused by increase of operating plate voltage of that tube. The circuit may also be balanced by shunting the two tubes by two high resistances,[4] the ratio of which is adjustable, as shown in Fig. 5b.[5] A transmission factor of 50,000 is easily attainable in this manner. A third method of adjustment is by means of a voltage divider between the cathodes of the two tubes, as shown in Fig. 5c. This method has the advantage that the adjusting resistance is at low direct potential.

When voltage gain is not of prime importance improved stability of balance may be obtained by use of the cathode-follower variant shown in Fig. 5d. If the balancing resistance is small and the internal impedance shunted across the output terminals is large in comparison with $1/g_m$, the output voltage approximates $(V_1 - V_2)g_mR_k/(1 + R_kg_m)$. The output voltage approximates $V_1 - V_2$ if $R_k \gg 1/g_m$.

If the need for high amplification necessitates the use of tandem stages of the form of Fig. 4a, plate-to-ground output from the first stage for common-mode input of the first stage must also be minimized. The reason for this requirement is that plate-to-ground output from the first stage as the result of common-mode input to the first stage provides common-mode input to the second stage, even though the plate-to-plate output of the first stage

[2] F. F. Offner, *Proc. I.R.E.*, **35**, 306 (March, 1947).

[3] D. L. Johnson, *Wireless Engineer*, **24**, 231, 271, and 292 (August, September, and October, 1947); A. M. Andrew, *Wireless Engineer*, **32**, 73 (March, 1955).

[4] D. L. Johnson, *Wireless Engineer*, **24**, 231, 271, and 292 (August, September, and October, 1947); A. M. Andrew, *Wireless Engineer*, **32**, 73 (March, 1955).

[5] D. H. Parnum, *Wireless Engineer*, **27**, 125 (April, 1950).

is negligible. Unbalance of the second stage then results in some plate-to-plate output of the second stage, which is amplified by following stages. This effect may be made negligible by the use of a pentode as the cathode resistance of the first stage.[6] The constant-current characteristic of the

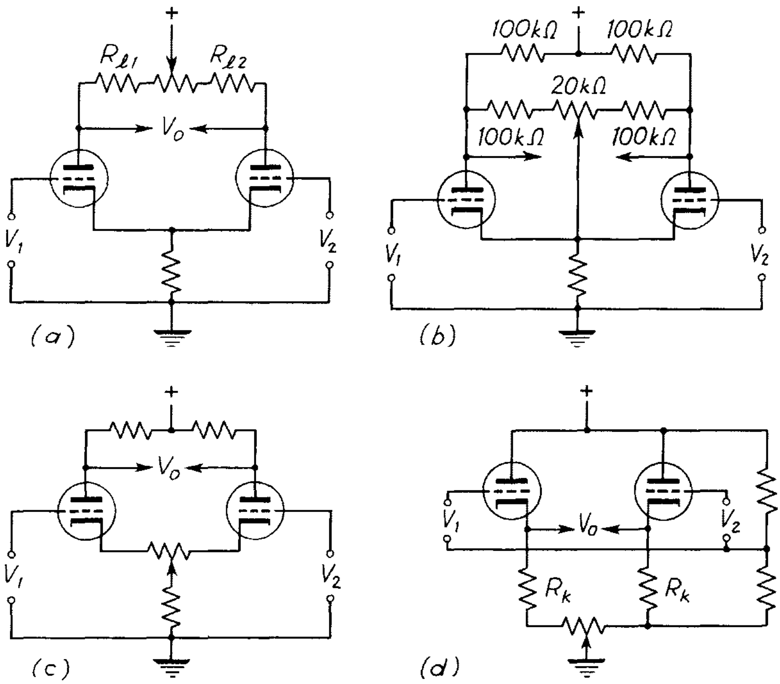


FIG. 5. Methods of balancing the differential amplifier.

pentode tends to prevent both plate currents from increasing or decreasing simultaneously, but does not inhibit the differential action.[7]

2.3. Current Output of Differential Amplifier. In some applications of differential amplifiers the desired output is not a voltage, but a current through a load shunted across the output terminals.[8] In the circuit of Fig. 4a the current through a load z connected between plates is

$$\frac{(V_1 - V_2)g_m}{2 + \left(\dfrac{z}{r_p} + \dfrac{z}{R_l + 2R_k} + \dfrac{zR_k}{R_l(R_l + 2R_k)}\right)}$$

[6] D. H. Parnum, *Wireless Engineer,* **27,** 125 (April, 1950).
[7] Johnson, *loc. cit.*
[8] M. Cooperstein, *Sylvania Technologist,* **4,** 88 (October, 1951).

In the circuit of Fig. 5d the current is

$$\frac{(V_1 - V_2)g_m}{2 + z(g_m + 1/R_k)}$$

In both circuits the current approaches $(V_1 - V_2)\,g_m/2$ as the impedance z approaches zero.

2.4. Differential Amplifier with Single-Sided Output. In many applications of the differential amplifier the output voltage must be that between one plate and ground in order that one output terminal will be at ground potential. The output between the plate of T_2 and ground in the circuit of Fig. 4a is given by the following expression (Prob. 2.1–1):

$$V_{o2} = \frac{\mu_1(\mu_2 + 1)R_{l2}R_kV_1 - \mu_2[r_{p1} + R_{l1} + (\mu_1 + 1)R_k]R_{l2}V_2}{(r_{p1} + R_{l1})[r_{p2} + R_{l2} + (\mu_2 + 1)R_k] + (\mu_1 + 1)R_k(r_{p2} + R_{l2})} \tag{1}$$

It can be seen from Eq. (1) that output is obtained for equal inputs of the same polarity or phase. It is also apparent that this common-mode output may be reduced by making R_{l1} zero. The improvement results not only from the removal of the term R_{l1} in the numerator of Eq. (1), but also from the accompanying reduction of r_{p1} caused by the increase in operating plate voltage of tube T_1. From the definition of transmission factor and Eq. (1), the following expression may be derived for the transmission factor under the assumption that the two triodes have equal amplification factors and that R_{l1} is zero (Prob. 2.1–1):

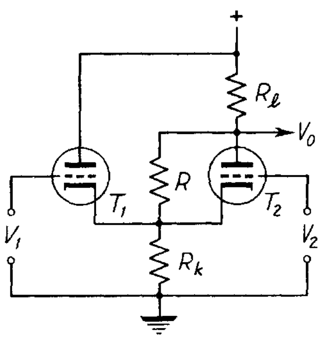


FIG. 6. Balanced differential amplifier with single-sided output.

$$\text{T.F.} = 1 + 2(\mu + 1)R_k/r_{p1} \approx 1 + 2g_{m1}R_k \tag{2}$$

Equation (2) shows that, when the output is taken from one plate of a differential amplifier, high transconductance and high cathode coupling resistance are essential unless the circuit is balanced in some manner.

One simple means of balancing the circuit for single-sided output is the addition of a resistance in shunt with the tube from which the output is taken, as shown in Fig. 6.[9] Solution of the equivalent circuit shows that

[9] R. McFee, *Rev. Sci. Instr.*, **21**, 770 (August, 1950).

the circuit of Fig. 6 is theoretically balanced when $R = R_k$ and that the output voltage is then

$$V_o = g_m(V_1 - V_2)/(2/R_l + 1/R_k + 1/r_p) \tag{3}$$

In practice, transmission factors of the order of several thousand may be obtained in this circuit. Another balanced circuit that has a high transmission factor for both single-sided and plate-to-plate output is the symmetrical circuit of Fig. 7.[10] The circuit is balanced when

$$R_2/R_1 = (R_l + R/2)/2R_k \tag{4}$$

The output voltage is then given by the relation:

$$V_{o2} = \frac{\mu R_1 R_l (V_1 - V_2)}{(r_p + R_l)(R_1 + R_2) + r_p R_l} \tag{5}$$

Practical values of transmission ratio for the circuit of Fig. 7 range from 2000 to 5000. The value of R_k' does not affect the balance or the magni-

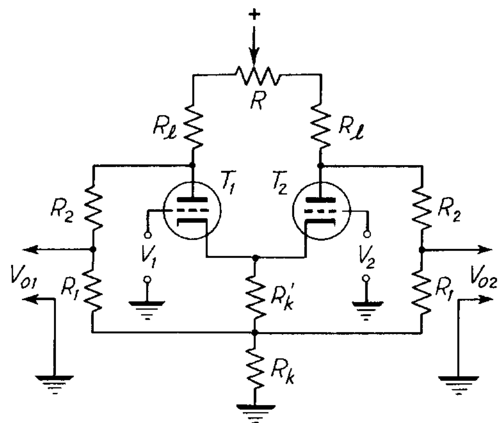


FIG. 7. Balanced differential amplifier.

tude of the output voltage, but affects the grid bias. The d-c level of the output is determined by the ratio R_1/R_2. When $R_1 = R_2$ and $R_k = R_l + R/2$ and the circuit is balanced, the output terminal is at the direct potential of the center of the plate supply, which can be grounded, and the output is not affected by equal changes in the characteristics of the two triodes or by changes of plate-supply voltage.

The basic differential amplifier may also be effectively balanced for single-

[10] McFee, *loc. cit.* See also "Waveforms," Chance, *et al.*, Rad. Lab. Series, Vol. 19, Fig. 18.16, p. 646 (McGraw-Hill Book Co., Inc., 1949).

sided output by the addition of a voltage divider in the input circuit of the tube from which the output is taken, as in Fig. 8.[11]

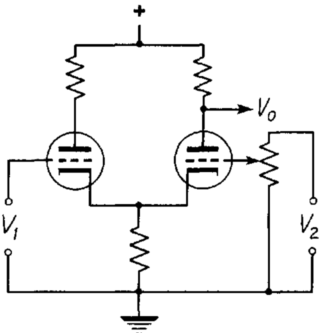


FIG. 8. Balanced differential amplifier with single-sided output.

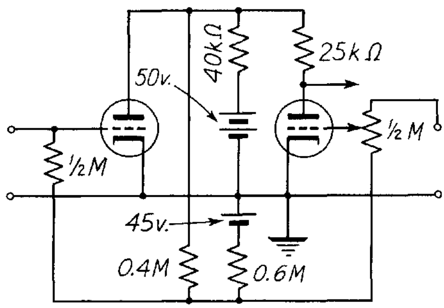


FIG. 9. Differential amplifier in which both the voltage supply and the inputs may be grounded.

Occasionally need arises for a differential amplifier circuit in which not only the voltage supply, but also the cathodes, can be grounded. One such circuit is shown in Fig. 9.[12]

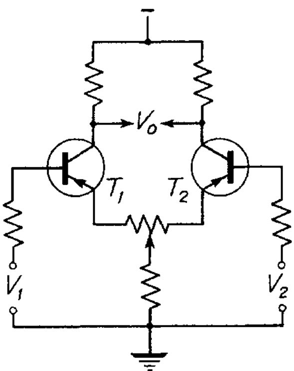


FIG. 10. Transistor differential amplifier.

It is important to bear in mind that interelectrode capacitances cause changes in gain and transmission factor as the frequency is increased. A compensated circuit that is flat and has a high transmission factor to approximately 5000 cps has been described by Barnette and Giacoletto.[13] The circuit uses one triode and one pentode and incorporates several compensating networks for the medium and high frequencies.

2.5. Transistor Differential Amplifiers. The vacuum-tube differential amplifiers that have been discussed can be converted into transistor circuits by replacing the tubes by transistors used in the common-emitter connection, as shown in Fig. 10.[14] By proper choice of transistor characteristics, temperature dependence of balance may be made small. The input imped-

[11] B. F. Davies, *Electronic Eng.*, **24**, 404 (September, 1952).

[12] E. E. Suckling, *Electronics*, February, 1948, p. 186. See also J. F. Toennies, *Rev. Sci. Instr.*, **9**, 95 (1938).

[13] W. E. Barnette and L. J. Giacoletto, *Electronics*, August, 1955, p. 148.

[14] D. W. Slaughter, *Trans. I.R.E.*, Vol. ED-3, No. 1, p. 51 (March, 1956). See also *Electronics*, May, 1955, p. 174.

ance of transistor circuits is lower than that of tube circuits, and the transistor circuits are susceptible to changes of balance resulting from temperature changes. Resistance may be used in series with the emitters for temperature stabilization. The theoretical expressions for the output voltage and transmission factor of transistor differential amplifiers are more complicated than those of tube circuits because the feedback and input parameters of transistors are not negligible.

3. Differentiating and Integrating Circuits

3.1. Resistance-Capacitance Differentiating Circuit. There are numerous applications of circuits that provide an output voltage proportional to the derivative of the input voltage. Because of circuit limitations, true differentiation can be only approximated at best, and the approximation fails if the input voltage has a discontinuity. The simplest form of differentiating circuit is the R-C circuit shown in Fig. 11. Analysis of the circuit shows that the output voltage is given by the following relation (Prob. 3.1–1):

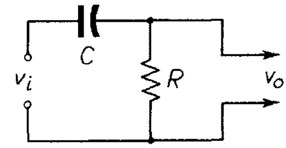


FIG. 11. Basic differentiating circuit.

$$v_o = \Sigma(-1)^{n+1}(RC)^n \frac{d^n v_i}{dt^n} \tag{6}$$

Unless one or more of the higher-order time derivatives of the input voltage are very large in comparison with the first derivative, the following approximation is valid:

$$v_o \approx RC(dv_i/dt) \tag{7}$$

if

$$RC|d^2v_i/dt^2| \ll |dv_i/dt| \tag{8}$$

and

$$RC\left|\frac{d^{n+1}v_i}{dt^{n+1}}\right| < \left|\frac{d^n v_i}{dt^n}\right| \tag{8A}$$

For a sine wave, Eq. (8) reduces to the relation

$$\omega RC \ll 1 \tag{9}$$

If the input voltage contains a discontinuity, the higher-order terms of Eq. (6) are not negligible in comparison with the first term and Eq. (7) consequently does not apply. Analysis of the circuit for specific discontinuous input voltages, such as a step voltage or a trapezoidal voltage, shows that the discontinuity initiates a transient, which is superimposed upon the steady-state solution. In certain applications of the R-C differentiator, notably in the conversion of a voltage step into a short pulse, the desired output voltage is the transient voltage, rather than the steady-state voltage.

Practically, a true step input voltage cannot be achieved, the leading or trailing edge of the input voltage often more nearly approximating a linearly rising or falling voltage of the forms shown in Figs. 12a and 12b. Direct

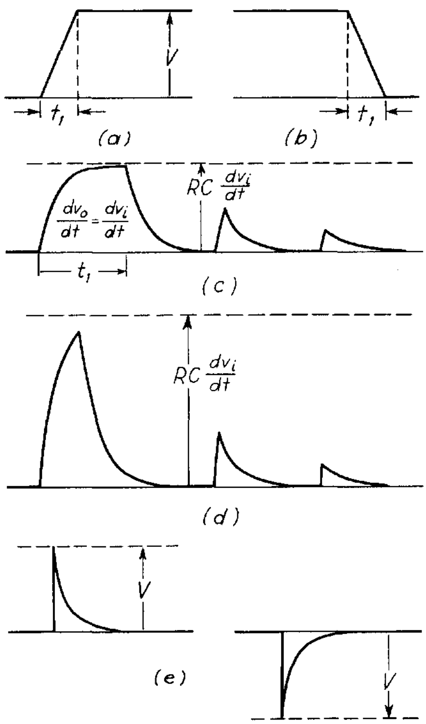


FIG. 12. (a) Approximation of a practical positive voltage step; (b) Approximation of a practical negative voltage step; (c) Response of the circuit of Fig. 11 to input voltage of the form of (a) for three values of V at constant dv_i/dt and RC; (d) Effect of increase of dv_i/dt upon outputs shown in (c); (e) Response of the circuit of Fig. 11 to ideal positive and negative input-voltage steps of magnitude V.

analysis of the circuit of Fig. 11, in which series input resistance and shunt capacitances are assumed to be negligible, shows that if the input voltage rises linearly from an initial value of zero, the output voltage is of the form:

$$v_o = RC(dv_i/dt)(1 - \epsilon^{-t/RC}) \tag{10}$$

in which dv_i/dt is the constant rate of rise of input voltage. The value of the output voltage at the termination of the leading or trailing edge of the input voltage is seen to depend upon t_1/RC, the ratio of the duration of the leading or trailing edge to the time constant of the circuit. After the input voltage reaches its constant value, the output voltage decays exponentially to zero at a rate determined by the time constant RC.

Figure 12c shows the manner in which the output pulse changes as the duration of the leading edge of the input voltage is decreased at constant slope and fixed time constant RC. It is apparent from Fig. 12c that the rise time of the output voltage pulse can be made short by making the rise time of the input voltage short, but that the amplitude of the output pulse decreases as the rise time of the input voltage is decreased. In order to insure adequate output-pulse amplitude, therefore, the steepness of the leading or trailing edge of the input voltage must be increased as its duration is decreased. If rapid decay of the output pulse is also desired, as it usually is, the time constant must be made small. It follows that an output spike of adequate amplitude and steep leading and trailing edges requires that the time constant be small (in comparison with the desired pulse length), that the rise or fall time of the input voltage be small in comparison with the time constant, and that the rate of rise or fall of the input voltage be large. Practically, it is difficult to meet these requirements simultaneously, particularly because source resistance and shunt capacitance, neglected in the foregoing analysis, become important as the time constant RC is reduced.

As the duration of the leading edge of the input voltage is decreased at constant voltage rise, the input voltage approaches a true step, and the leading edge of the output voltage approaches a vertical rise equal in magnitude to the change of input voltage, as shown in Fig. 12e. This is to be expected, inasmuch as the capacitor voltage cannot change instantaneously and an abrupt change in input voltage must therefore be accompanied by an equal change in voltage across the resistance. The capacitor at once starts to charge or discharge, and the initial output voltage change is followed by an exponential decay at a rate determined by the time constant RC. The smaller the time constant, the more rapidly the capacitor voltage changes following the abrupt change of input voltage, and the shorter is the output pulse.

3.2. Effect of Source Resistance and Load Capacitance. Usually the input to a resistance-capacitance differentiator is derived from a tube or transistor amplifier and the output of the differentiator is used to excite another tube or transistor circuit. The general form of the two most commonly used tube circuits is shown in Fig. 13. The transistor circuits are similar in form. The capacitance C_s is the effective input capacitance of the circuit that is driven by the differentiating circuit, and C_o is the output capacitance of the driving stage. If the driving stage is a cathode-follower or common-collector amplifier, as in Fig. 13b, the output resistance is usually so low that the time constant determined by the output resistance and C_o is very small in comparison with the principal time constant RC. This condition can also be approximated by the use of a low plate load resistance in the circuit of Fig. 13a, or of a low collector circuit resistance when a transistor is used. Under the assumption that the time constant of the driving circuit is small in comparison with RC, the effect of C_o may

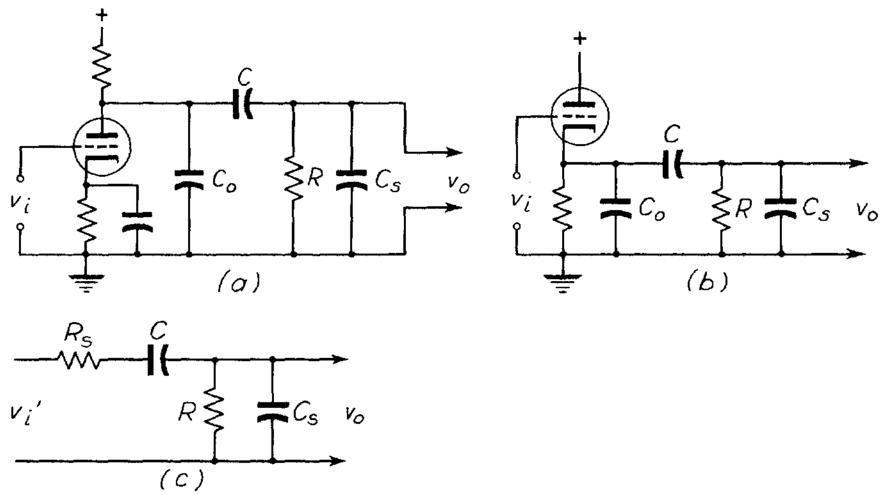


FIG. 13. (a) Differentiator with common-cathode driving stage; (b) Differentiator with cathode-follower driving stage; (c) Equivalent circuit for the circuits of (a) and (b).

be neglected and, by application of Thévenin's theorem, the equivalent circuit may be reduced to the form of Fig. 13c.

Under the additional assumption that R_s/R and $C_s/C \ll 1$, the output voltage of the circuit of Fig. 13c resulting from an input-voltage step of magnitude V may be shown to be

$$v_o = \frac{VR}{R + R_sC_s/C}\left[\epsilon^{-t/RC} - \epsilon^{-t/R_sC_s}\right] \approx V\left[\epsilon^{-t/RC} - \epsilon^{-t/R_sC_s}\right] \qquad (11)$$

The form of this output voltage, together with the form of the output voltage that would be obtained with zero series resistance and shunt capacitance, is shown in Fig. 14a. Figures 14b and 14c show the output voltage for a rectangular input voltage. It is seen that the effective series resistance of the input-voltage source and the shunt capacitance across the output terminals of the differentiator round the leading edge of the output pulse and decrease the amplitude of the pulse. As the series resistance and shunt capacitance are reduced, the rise time of the leading edge of the output pulse is decreased and the amplitude of the pulse is increased.

It must be concluded that the low output resistance of the cathode-follower or common-collector amplifier makes it preferable to the common-cathode or common-emitter circuit to drive the R-C differentiator. Furthermore, the series resistance of the source of input voltage and the shunt capacitance of the circuit connected across the output terminals must be taken into consideration in the selection of the parameters R and C of the differentiating circuit, particularly when a small time constant RC must be used in order to obtain a short output pulse or a good approximation to true differentiation of a continuous voltage wave.

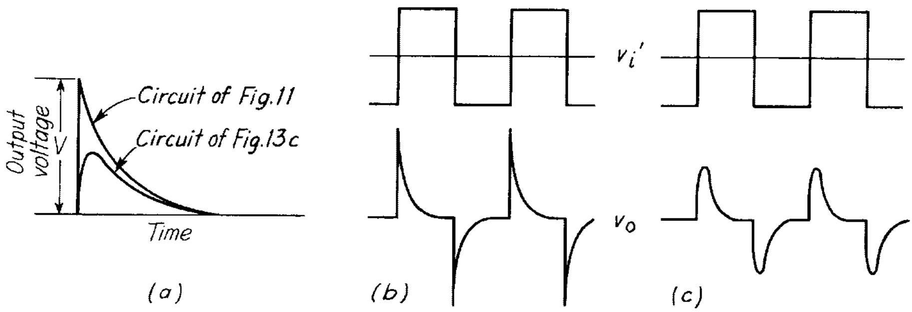


FIG. 14. (a) Comparison of the response of the circuit of Fig. 11 to a voltage step of magnitude V with that of Fig. 13c; (b) Response of the circuit of Fig. 11 to a rectangular voltage wave; (c) Response of the circuit of Fig. 13c to a rectangular voltage wave.

If C_s is negligible, but R_s is not, as may be true when the differentiator is driven by a common-cathode (plate output) amplifier, the output voltage produced by an input voltage step of magnitude V is $VR\epsilon^{-t/(R+R_s)}/(R + R_s)$. The source resistance therefore reduces the output voltage and increases the time constant.

When the capacitance C_o cannot be neglected, the following procedure may be used to determine the values of R_s, R, and C that will result in the sharpest output pulse from the circuit of Fig. 13a when C_s and C_o are fixed.[1] ("Sharpest pulse" is defined as the one having the largest value of d^2v_o/dt^2 at the peak of v_o. This definition is consistent with the highest amplitude of the crest in a short time interval about the crest.[1])

1. Choose C so that $C = \sqrt{C_oC_s/2}$.

2. Make $R_s = R$ and reduce both until the minimum useful output-pulse amplitude is obtained.

3.3 Use of Inverse-Feedback Amplifier. Equations (7) and (10) show that reducing the time constant RC in order more nearly to approximate

[1] H. Moss, *Wireless Engineer*, **28**, 345 (November, 1951).

distortionless differentiation, or to obtain a short output pulse from an input step, also reduces the magnitude of the output voltage. Amplification of the output voltage may therefore be necessary. Although the desired result can be obtained by using low resistance in the circuit of Fig. 11 and amplifying the output by means of a simple amplifier, distortion of the amplifier and dependence of amplifier gain upon supply voltages may produce adverse effects upon the circuit response. Better performance may be obtained by means of the circuit of Fig. 15. The amplifier used in this cir-

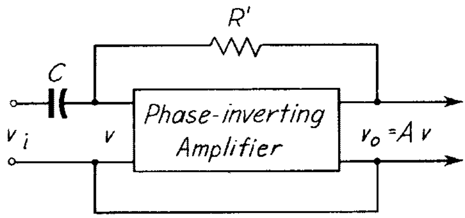


FIG. 15. Differentiating circuit incorporating an amplifier with inverse feedback.

cuit is phase-inverting; the voltage amplification A is therefore numerically negative.

Under the assumption that the input impedance of the amplifier is infinite and that the amplification of the amplifier is unaffected by loading produced by C and R', analysis of the circuit of Fig. 15 shows that the output voltage is given approximately by the following expression (Prob. 3.3–1):

$$v_o \approx \frac{A}{1 + |A|} R'C(dv_i/dt) \tag{12}$$

if

$$\frac{R'C}{1 + |A|} |d^2v_i/dt^2| \ll |dv_i/dt| \tag{13}$$

and

$$\frac{R'C}{1 + |A|} \left| \frac{d^{n+1}v_i}{dt^{n+1}} \right| < \left| \frac{d^n v_i}{dt^n} \right| \tag{13A}$$

If the voltage amplification is large in comparison with unity, the output voltage is nearly independent of amplifier characteristics and the magnitude of the output voltage is approximately the same as that obtained when R' and C are used in the basic circuit of Fig. 11. Equations (8) and (13) show, however, that the degree to which the circuit of Fig. 15 approximates an ideal differentiator is greater than that of the circuit of Fig. 11 by a factor $1 + |A|$. It is of interest to note that the amplifier of Fig. 15, in combination with the resistance R', produces an effective resistance $R'/(1 + |A|)$

between the amplifier input terminals and that the differentiation may be considered to be produced by the action of the capacitance and this low effective resistance (Prob. 3.3–2).

A practical form of the circuit of Fig. 15 is shown in Fig. 16a.[2] In order to make this circuit easier to understand, a simplified version is shown in Fig. 16b. Voltage amplification is provided by T_1, the cathode-coupled stage T_2 merely providing a means for connecting the feedback resistor R between the output and the input without loading the voltage amplifier and without applying a high positive voltage to the grid of T_1. In the circuit of Fig. 16a, T_1 is a pentode instead of a triode. Since the potentially large

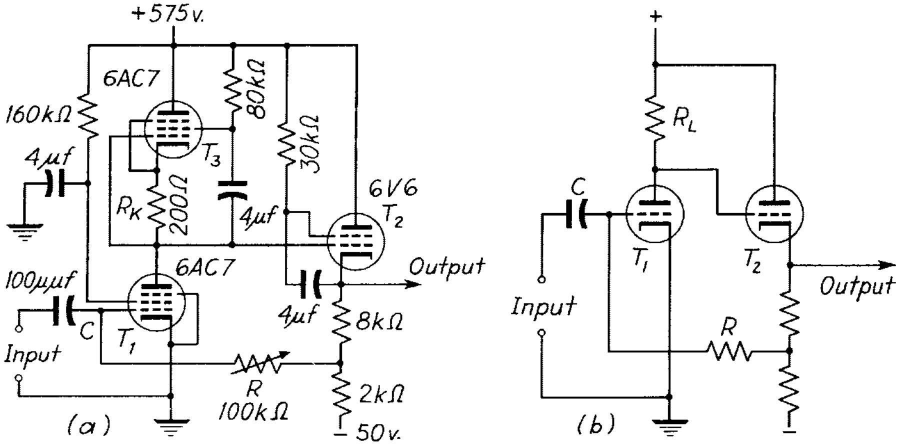


FIG. 16. (a) Practical form of the circuit of Fig. 15; (b) A simplified version of circuit (a).

voltage amplification of a pentode can be approximated only if the load resistance is comparable to the a-c plate resistance, the load resistor R_L is also replaced by a pentode, T_3, which provides a high a-c resistance without excessive direct-voltage drop. The high resistance provided by T_3 results not only from the high a-c plate resistance of this tube, but also from the fact that an increase of plate current is accompanied by an increase of voltage across the cathode resistor R_K of T_3 that makes the grid more negative and thus tends to prevent the increase of current. A stabilized high-gain d-c amplifier suitable for use in the circuit of Fig. 15 will be discussed in Sec. 4.

Figure 17a shows a differentiating circuit of the form of Fig. 15 in which amplification is accomplished by means of a transistor. This circuit was designed to convert a 1000-cps triangular wave into a rectangular wave.[2]

[2] J. B. Oakes, *Electronics*, September, 1954, p. 165.

The circuit is improved by the addition of a common-collector coupling stage, equivalent to the cathode-follower stage in the circuit of Fig. 16, as shown in Fig. 17b. The 10-kΩ resistor in series with the emitter in the circuits of Fig. 17 provides temperature stabilization.

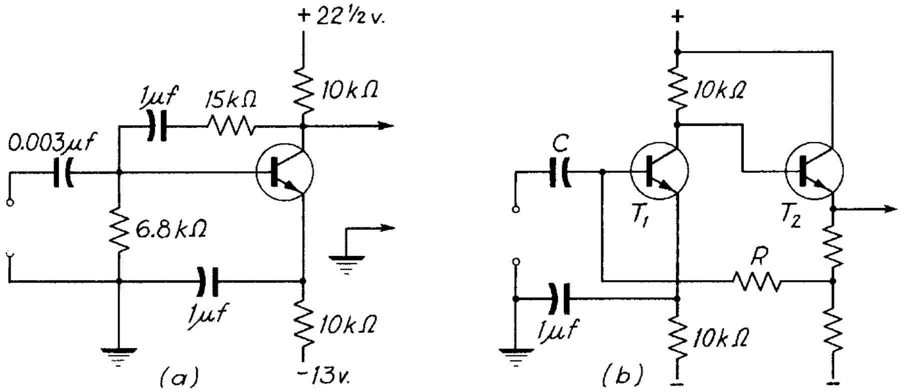


FIG. 17. (a) A transistor differentiating circuit; (b) An improved form of circuit (a) in which a common-collector coupling stage is used.

Differentiation can also be accomplished by impressing the input voltage upon a series combination of resistance and inductance that has a small time constant L/R, and taking the output from across the inductance. The resistance of the inductor causes distortion, however, and output capacitance may cause transient oscillation. One type of inductance-resistance differentiating circuit consists of a pulse transformer in the plate circuit of a vacuum tube. The input voltage is applied to the grid and the output voltage is taken from the secondary of the transformer. The plate resistance of the tube serves as the resistance of the L-R circuit.

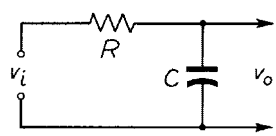


FIG. 18. Basic integrating circuit.

4. *Integrating Circuits*

4.1. Resistance-Capacitance Circuits. Integrating circuits find applications in many types of electronic instruments and controls, notably in analog computers and in circuits for the generation of linearly rising voltages. In the simplest type of circuit, shown in Fig. 18, the input voltage is impressed upon a series combination of resistance and capacitance, and the output is taken from across the capacitance. The output voltage $v_o(t)$ at any time t, in terms of the circuit parameters and the output voltage at the time t_o, is given by the relation:

$$v_o(t) = v_o(t_o) + \frac{1}{C}\int_{t_o}^{t} i\,dt = v_o(t_o) + \frac{1}{RC}\int_{t_o}^{t} (v_i - v_o)\,dt \tag{14}$$

If $v_o \ll v_i$, Eq. (14) reduces to the approximate form:

$$v_o(t) \approx v_o(t_o) + \frac{1}{RC}\int_{t_o}^{t} v_i\,dt \tag{15}$$

If the initial output voltage $v_o(t_o)$ is zero or very small, the requirement that $v_o \ll v_i$ can be satisfied by making the time constant RC very large in comparison with the time interval $t - t_o$ over which the integration is to be performed. The minimum ratio $RC/(t - t_o)$ at which the output voltage is a sufficiently close approximation to the true integral of the input voltage depends upon the form of the input voltage. Where a very close approximation is essential, as in analog computers, $RC/(t - t_o)$ must be of the order of 100 or more. Leakage, particularly through the capacitor, limits the extent to which the time constant of the basic circuit of Fig. 18 can be increased by increasing the values of the capacitance and resistance. Furthermore, increasing RC decreases the magnitude of the output voltage. For these reasons, the integrating amplifier of Fig. 19, which is similar in form to the differentiating circuit of Fig. 15, is usually used when accurate integration must be performed. Analysis of this circuit shows that it has a time constant $(1 + |A|)RC$ and that the output voltage of the circuit is given by the following approximate relation if $(1 + |A|)RC \gg t - t_o$:

$$v_o(t) \approx v_o(t_o) + \frac{|A|}{1 + |A|} \cdot \frac{1}{RC}\int_{t_o}^{t} v_i\,dt \tag{16}$$

It can be seen from Eq. (16) that amplifier distortion and dependence of the output voltage upon the voltage amplification of the amplifier can be made small by making the voltage amplification A high, because $v_o(t)$ is nearly independent of $|A|$ if $|A| \gg 1$.

Comparison of Eqs. (15) and (16) shows that the circuit of Fig. 19 provides the same output-voltage amplitude as that of Fig. 18 if equal values of RC are used in the two circuits. However, since Eq. (16) is valid if $(1 + |A|)RC \gg t - t_o$, whereas Eq. (15) is valid only if $RC \gg t - t_o$, it follows that the same degree of approximation to ideal integration is obtained in the two circuits if the RC product is smaller in the amplifier circuit of Fig. 19 by the factor $1/(1 + |A|)$. The possibility of using a smaller value of capacitance in the amplifier circuit than in the basic circuit reduces the leakage problem.

It is of interest to note that the high time constant of the circuit of Fig. 19 may be considered to be the effect of capacitance multiplication by the amplifier. If the input impedance of the amplifier is assumed to be infinite, the current produced by the input voltage v_i must all flow into the capacitor. The capacitor current is $C\frac{d(v_i - v_o)}{dt} = C(1 - A)\frac{dv_i}{dt}$. The amplifier being phase-inverting, A is negative and the current is $C(1 + |A|)\frac{dv_i}{dt}$. The amplifier and capacitance C are therefore equivalent to a capacitance $C(1 + |A|)$ between the input terminals of the amplifier. This capacitance multiplication is the same as the Miller effect, which may give rise to a large effective input capacitance of a vacuum tube as the result of relatively small grid-plate interelectrode capacitance.

Because relatively large values of R are normally used in the circuit of Fig. 19, loading of the amplifier by the R-C network is less likely to be objectionable than in the differentiating circuit of Fig. 15. If necessary, it

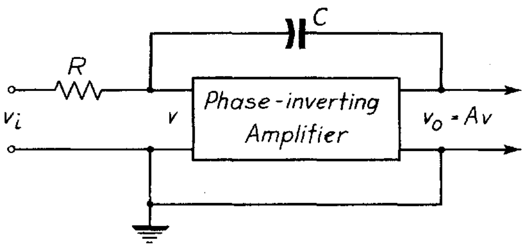


FIG. 19. Integrating circuit incorporating an amplifier with inverse feedback.

may be prevented by the use of a cathode-follower or grounded-collector stage in the output of the amplifier.

4.2. Miller and Bootstrap Circuits. Several practical forms of the circuit of Fig. 19 are shown in Fig. 20. The circuit of Fig. 20a is called the *Miller integrator* and that of Fig. 20b is known as the *bootstrap circuit.* (See also Sec. 59.3). The cathode resistor R_k, which provides bias, may be omitted if v_i has a suitable negative component. Although these two circuits may at first glance appear to be different, careful comparison shows that they actually differ only as to the points at which they are grounded. The ungrounded input of the bootstrap circuit may be a disadvantage in some applications. The circuit of Fig. 20c is a transistor version of the Miller integrator designed to convert 10-μsec rectangular pulses into a triangular wave.[1] Figure 20d shows a modified form of this circuit in which the input resistance of the amplifier is increased by the use of compound-connected[2] transistors T_1 and T_2 in the common-emitter voltage amplifier,

[1] J. B. Oakes, *Electronics,* September, 1954, p. 165.

[2] A. R. Pearlman, *Trans. I.R.E.,* Vol. ED-2, No. 1, p. 25 (January, 1955); S. Darlington, Pat. No. 2,663,806; B. Oliver, Pat. No. 2,663,830.

and loading of the amplifier by R and C is reduced by the addition of the common-collector stage T_3.

A high-gain stabilized d-c amplifier suitable for long-time-constant integrators will be discussed in the following section.

It should be noted that the circuits of Figs. 19 and 20 cease to be useful

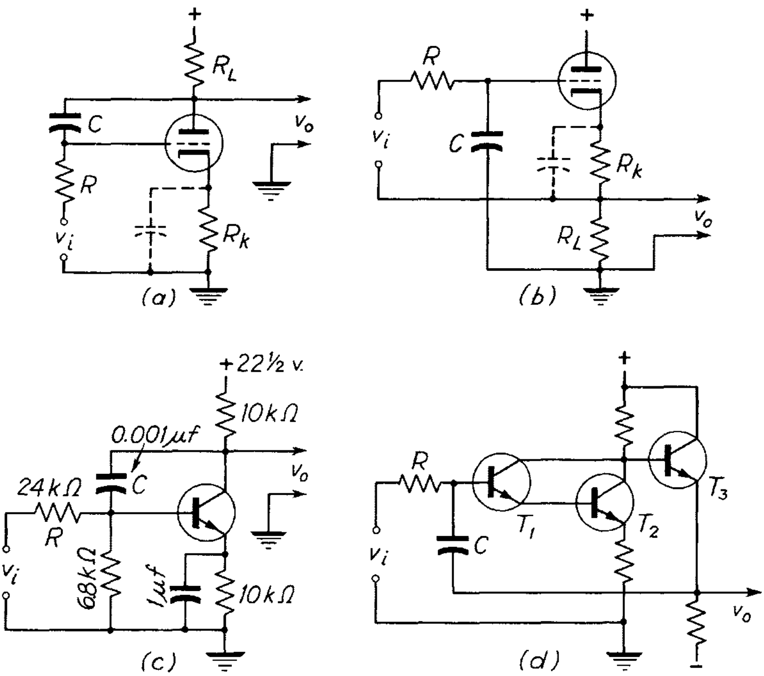


FIG. 20. (a) The Miller integrator; (b) the bootstrap circuit; (c) transistor version of the Miller integrator; (d) transistor Miller integrator incorporating a compound-connected transistor and a common-collector coupling stage.

when the amplifier is driven into cutoff or saturation or to a point where excessive amplifier input current flows. Frequency compensation of the amplifier may also be desirable or necessary.[3]

5. *Stabilized D-C Amplifier for Adding, Differentiating, and Integrating Circuits*

5.1. Need for High Stability. One of the principal applications of the adding circuit of Fig. 2 and the integrating circuit of Fig. 19 is in analog computers. Adding circuits used for this purpose must have high input impedance and negligible cross-coupling, and integrating circuits must closely

[3] J. W. Stanton, *Trans. I.R.E.*, Vol. CT-3, p. 65 (1956).

approximate ideal integrators. It follows from the discussions of Secs. 1 and 4 that the amplifiers incorporated in these circuits must have high gain. Furthermore, in order that the currents and voltages in the analog circuits may be recorded by means of a plotting table, the time constants of these circuits must often be of the order of minutes and the amplifiers must therefore have flat frequency response down to frequencies of the order of hundredths or thousandths of a cycle per second. High stability is also essential.

High and constant gain over a wide frequency band extending into fractional frequencies can be achieved only by means of multistage direct-coupled amplifiers. This type of amplifier is inherently unstable, however, because a small change in the tube or circuit parameters of the first stage may make a large change in the output voltage or current. Consequently, in order to maintain the output voltage zero in the absence of input, changes in tube characteristics and circuit elements as the amplifier warms up must be compensated by frequent adjustment unless the circuit is electronically stabilized.[1]

5.2. Basic Circuit. The block diagram of the stabilized d-c amplifier circuit that is usually used in analog computers is shown in Fig. 21a.[2] Stabilization is achieved in this circuit with the aid of an auxillary a-c amplifier, a mechanical chopper to convert direct voltage into alternating voltage that can be amplified by means of the a-c amplifier, and a synchronous rectifier to reconvert the output of the a-c amplifier into direct voltage.

If the zero-signal output voltage V_O is positive, a positive voltage is impressed upon the input of the a-c amplifier when the chopper contacts are open. Since the output voltage of the a-c amplifier is in phase with its input voltage, the capacitor C_1 is charged in the indicated polarity through the rectifier contacts, which are closed when the chopper contacts are open. When the rectifier contacts are open, the capacitor C_2 charges through R_1. The resulting relatively large negative stabilizing voltage V_S applied to the grid of T_2 is of the proper polarity to reduce V_O. The over-all loop gain of the two amplifiers, chopper, and rectifier being of the order of 10^8 at zero frequency, a high degree of stabilization results from the inverse feedback. The time constant R_1C_2 is large enough to prevent the application of signal-frequency or chopper-frequency voltages to the grid of T_2.

Analysis of the circuit of Fig. 21a (Prob. 5.2–1) shows that if $|A_1(A_2 - 1)| \gg (Z_f + Z_i)/Z_i$ and if A_2 is sufficiently high at the chopper frequency, at which $A_1 = v_O/v_E$ and $A_2 = v_S/v_E$, the zero-signal output voltage of the d-c amplifier is negligible and the signal output voltage is very nearly $v_I Z_f/Z_i$.

[1] A. J. Williams, Jr., R. E. Tarpley, and W. R. Clark, *Trans. A.I.E.E.*, **67**, 47 (1948).
[2] E. A. Goldberg, *RCA Rev.*, **11**, 296 (June, 1950).

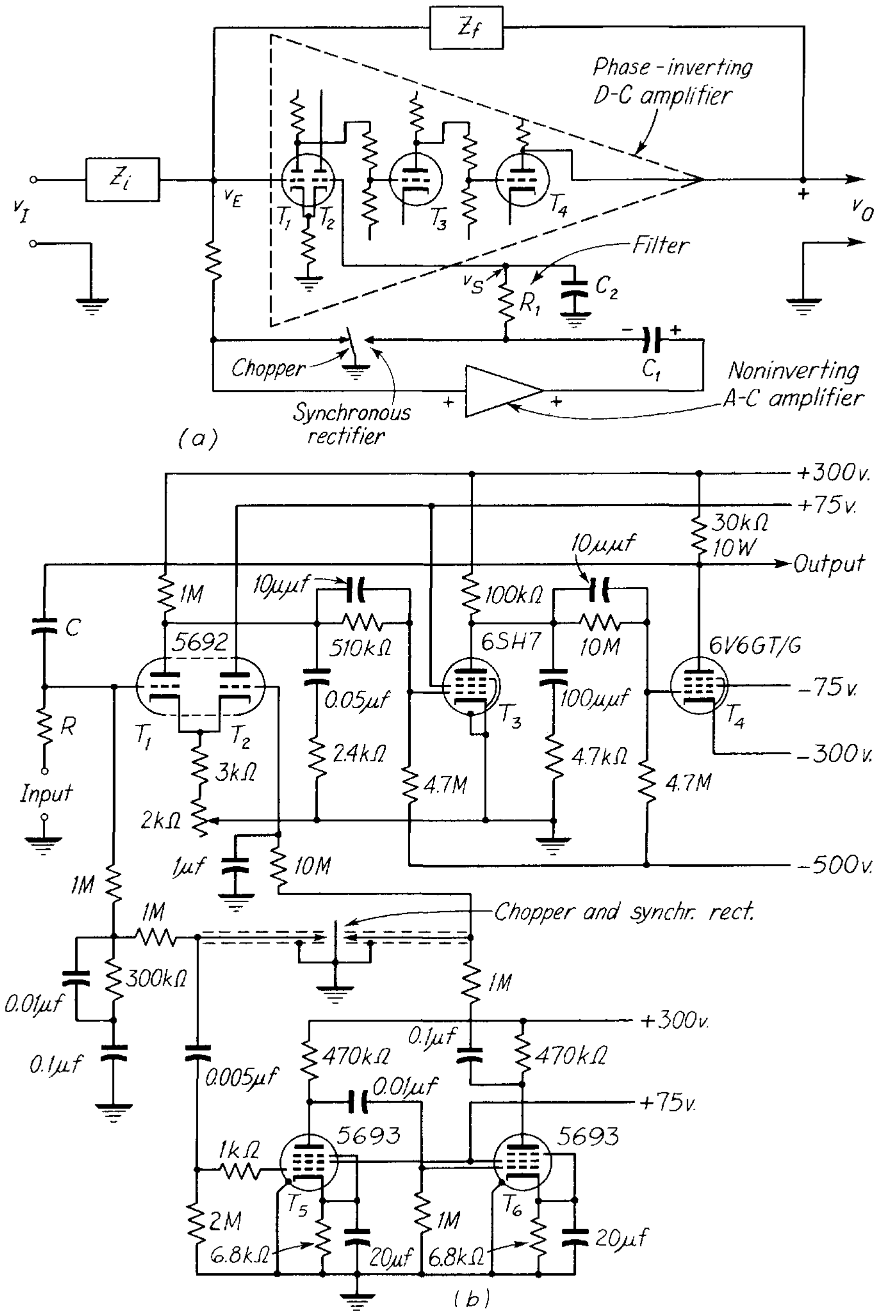


FIG. 21. Stabilized d-c amplifier: (a) block diagram of the basic circuit; (b) practical form of the circuit.

5.3. Practical Circuit. Figure 21b shows a practical form of the amplifier of Fig. 21a as it is used in the integrating circuit of Fig. 19.[3] This circuit may be converted into the summing amplifier of Fig. 2 by replacing the capacitance C by a suitable resistance and adding other input circuits. Similarly, interchanging C and R converts the circuit into the differentiator of Fig. 15. The amplifier is capable of producing a zero-frequency output swing from —200 to +200 volts and has a flat frequency response to at least 100 kc when Z_i and Z_f are equal resistances. Shielding of the chopper and rectifier leads is necessitated by the fact that any feedback from the output to the input of the a-c amplifier is regenerative and may therefore cause oscillation. In some applications of the circuit the mechanical chopper and rectifier may be replaced by transistor switches of the type discussed in Sec. 13.

Because complete stabilization cannot be obtained by the use of inverse feedback alone, and some unbalance may be caused by contact or thermal potentials in the chopper or by the flow of grid current of T_1 through Z_f, a variable resistor is provided in the cathode circuit of T_1 and T_2 in order that the circuit may be balanced initially. After initial balance the zero-signal output voltage, referred to the input terminals of the d-c amplifier, does not exceed 50 microvolts when the input and feedback impedances are of the order of megohms.

6. Characteristics of Diodes

6.1. Thermionic Diodes. The circuits to be discussed in Sec. 7 and a number of other types of circuits to be discussed in later sections of this book function by virtue of rectification or other useful properties of diodes. Either thermionic diodes (Figs. 24a and 24b) or semiconductor diodes (Figs. 24c, 24d, and 24e) can be used in most of these circuits, the choice between the two types depending upon which features are of primary importance in the individual circuits. A brief discussion of diodes at this point will aid the reader in choosing between thermionic and semiconductor diodes and will be helpful in some of the treatments presented in later sections. The technology of semiconductor devices is developing so rapidly, however, that great improvements may be made between the writing and the publication of any discussion of diodes. The reader is therefore urged to consult recent technical literature and the specification sheets of manufacturers before selecting a diode type.

The principal features in which thermionic diodes are at present superior to semiconductor diodes are their high reverse breakdown voltage, their rapid response to abrupt voltage changes, and the fact that their current-voltage characteristics are virtually independent of ambient tube tempera-

[3] Goldberg, *loc. cit.*

ture if the cathode heating current is maintained constant. It is also easier to obtain in a single thermionic diode the combination of low forward resistance, high reverse resistance, and small shunt capacitance.

In the forward direction, the plate current of an ideal thermionic diode with a unipotential cathode varies as the 3/2 power of the plate voltage, the perveance or ratio of the plate current to the 3/2 power of the plate voltage depending upon the cathode area and the cathode-to-plate spacing. Initial velocity of electron emission, contact potential difference, variations in electrode spacing, fringing electric fields at the ends of the electrodes, and differences of potential over the cathode surface in filamentary types cause the form of the characteristics of practical thermionic diodes to depart from that of the ideal diode. From the point of view of application of diodes to circuits, the most important departure is failure of the current-voltage characteristic to pass through the origin. Usually current cutoff occurs at a negative voltage of a quarter to three-quarters of a volt, the exact value varying with cathode temperature, from tube to tube, and during the life of a tube.

The forward resistance of thermionic diodes is of the order of 100 to 1000 ohms. The reverse resistance of a properly mounted thermionic diode exceeds 10^9 ohms. The plate-to-cathode capacitance is of the order of several micromicrofarads, and the heater-to-cathode capacitance of the order of 1 $\mu\mu$f. Although the heater-to-cathode capacitance is small, it may cause undesirable power-frequency pickup in high-impedance circuits in which the cathode must be operated above ground potential. Thermionic diodes may be designed so that they can withstand reverse voltages of thousands of volts without breakdown if they are properly mounted. The delay time in the response of the plate current to a positive or negative step of plate voltage, which is the result of electron transit time from the cathode to the plate, may be as small as a fraction of a millimicrosecond at plate voltages of the order of tens of volts.

6.2. Semiconductor Diodes. The desirable features of semiconductor diodes are lower forward resistance than that of thermionic diodes (1 to 500 ohms), smaller size and weight, lower cost, and the absence of heater-power requirements and of the attendant possibility of power-frequency pickup, and, in some types, lower shunt capacitance. On the other hand, presently available semiconductor diodes have lower values of reverse breakdown voltage (less than 200 volts in most types suitable for use in circuits discussed in later sections of this book), have current-voltage characteristics that depend upon operating temperature, and except for breakdown diodes are in general much slower than thermionic diodes to respond to abrupt changes of voltage.

The general form of the current-voltage characteristic of a semiconductor diode is shown in Fig. 22a. If the diode consisted of an ideal p-n junction, the reverse current would approach a small constant saturation value I_o and the characteristic would obey the law $i = I_o(\epsilon^{\gamma v} - 1)$. The factor γ may

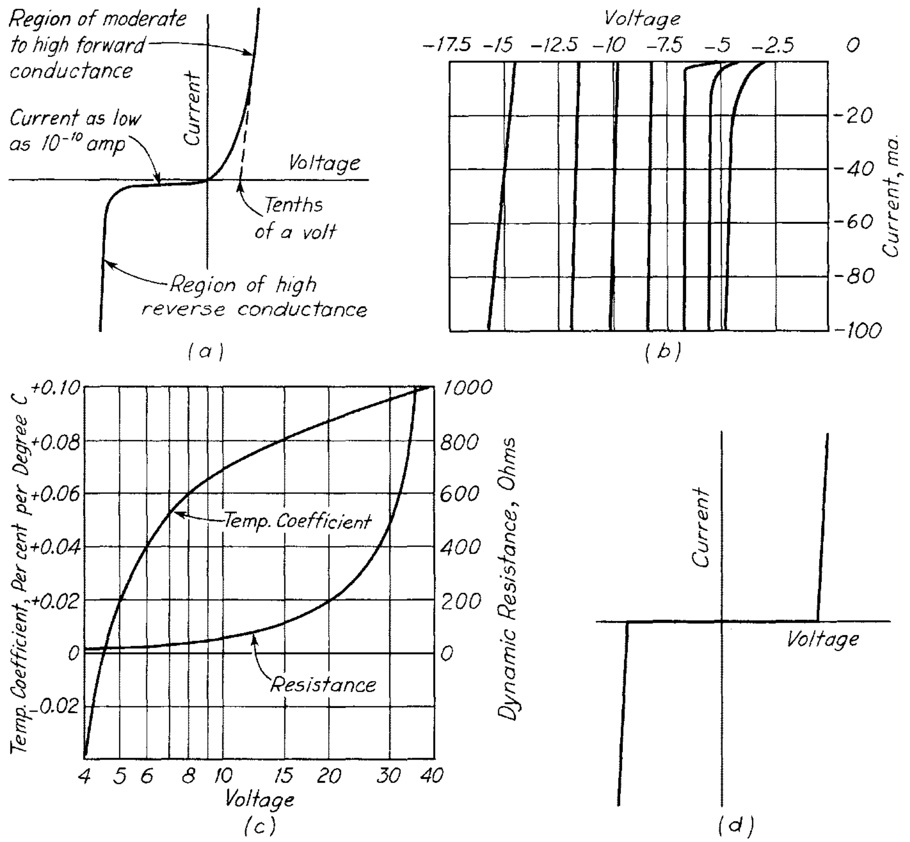


FIG. 22. (a) A typical semiconductor current-voltage characteristic; (b) a typical family of breakdown-diode characteristics; (c) typical curves of dynamic resistance and temperature coefficient of voltage vs. voltage; (d) current-voltage characteristic of two breakdown diodes connected in series, back-to-back.

have a value anywhere between q_e/kT and $q_e/2kT$, where q_e is the magnitude of the charge of an electron, k is Boltzmann's constant, and T is the absolute temperature.[1] The value of γ depends upon the junction structure and decreases from low values of voltage v to high values. Departures from this simple exponential law result from a number of causes, including the

[1] W. Shockley, *Bell Sys. Tech. J.*, **28,** 435 (1949); E. Spenke, *Z. Naturf.*, **11a,** 440 (1956); J. L. Moll, *Proc. I.R.E.*, **46,** 1076 (June, 1958); I. Ladany, *Proc. I.R.E.*, **47,** 589 and 1252 (April and July, 1949).

resistance of the p- and n-regions of the semiconductor and shunt leakage conductance across the junction. An important effect of the series resistance of the semiconductor is that the current-voltage characteristic of types in which the series semiconductor resistance is not negligible approaches a straight line at large positive values of current, as shown by the dotted line in Fig. 22a.[2] The voltage intercept of this line approximates a few tenths of a volt. Even for diodes that follow the exponential law closely, it is often convenient or necessary to assume, in a simplified approximate analysis of a diode circuit, that the current-voltage characteristic may be approximated by a straight line having a small positive voltage intercept.

The flow of reverse current in the saturation range is caused by the presence of some holes in the n-type semiconductor and some electrons in the p-type semiconductor. Because the presence of these carriers is mainly the result of thermal excitation of some of the intrinsic semiconductor atoms, the saturation current increases with diode temperature and eventually becomes so large as to render the diode useless. The saturation current of silicon diodes at room temperature may be as low as 10^{-10} amp;[3] that of germanium diodes is several orders of magnitude greater. The maximum temperature at which silicon diodes can be operated without excessive reverse current is considerably higher than that of germanium diodes (in the vicinity of 80°C for germanium diodes and 300°C for silicon diodes).

The reverse current of an ideal p-n junction would not exceed the saturation value even at large values of negative voltage. The reverse characteristics of actual semiconductor diodes depart from those of an ideal junction in that the characteristics may have appreciable slope throughout the negative-voltage range, and that a value of voltage is reached beyond which the slope of the characteristic increases and may become very large. The finite slope of the characteristic throughout the reverse-voltage range, which is caused by shunt leakage conductance, is small in a good diode. The rise in reverse-current magnitude above a critical value of reverse voltage is caused by avalanche multiplication of carriers as the result of high electric field within the diode junction. This phenomenon, which is called breakdown, is in some respects similar to the breakdown of a gas diode (see Sec. 22).

6.3. Breakdown Diodes. Reverse breakdown of a semiconductor diode is nondestructive if the current is limited to a value that does not cause the allowable dissipation to be exceeded. In silicon diodes, breakdown occurs

[2] R. G. Shulman and M. E. McMahon, *J. Appl. Phys.*, **24**, 1267 (October, 1953); H. L. Armstrong, E. D. Metz, and I. Weiman, *Trans. I.R.E.*, Vol. ED-3, p. 86 (April, 1956); F. S. Barnes, *Tech. Report No. 8*, Stanford Electronic Labs. (August 30, 1956); N. H. Fletcher, *J. Electronics*, **2**, 609 (May, 1957); N. H. Fletcher, *Proc. I.R.E.*, **45**, 862 (June, 1957); H. L. Armstrong, *Proc. I.R.E.*, **46**, 361 (January, 1958).

[3] G. L. Pearson and B. Sawyer, *Proc. I.R.E.*, **40**, 1348 (November, 1952).

much more abruptly with respect to voltage than in germanium diodes, and the slope of the current-voltage characteristic at negative voltages exceeding the breakdown voltage may be very steep. Diodes designed to serve useful functions by virtue of the sharp break in the current-voltage characteristic or the steepness of the characteristic in the reverse-current range beyond breakdown are called *breakdown (Zener) diodes.* The letter B within a circle, as in Fig. 25, is used to indicate that a diode is being used as a breakdown diode. Because the onset of breakdown is not in general abrupt, the breakdown voltage must usually be specified at an arbitrary value of current above the saturation value.

The breakdown voltage of a breakdown diode depends upon the conductivity of the silicon and can be controlled to a high degree in the process of manufacture.[4] A typical family of characteristics is shown in Fig. 22b.[5] The breakdown is gradual at voltages below 7 volts, but becomes sharper as the breakdown voltage is increased. The steepness of the characteristic decreases with increase of breakdown voltage. A typical curve of dynamic resistance (reciprocal of the slope of the current-voltage characteristic in the operating range) vs. voltage is shown in Fig. 22c.[6]

The voltage of a breakdown diode at constant current is in general dependent upon the temperature. A typical curve of temperature coefficient of voltage as a function of voltage at constant dissipation is shown in Fig. 22c.[7] Increase of current displaces the curve toward lower voltages. Zero temperature coefficient is obtainable only in units operating at low voltages and then only at a single current for each unit. At higher voltages complete temperature stabilization can be approximated by using a forward-biased diode in series with the breakdown diode, the two units being designed or selected so that the positive temperature coefficient of the breakdown diode is offset by the negative coefficient of the forward-biased diode. Packaged combinations of this type are available.

Because the ratio of the dynamic resistance to the breakdown voltage increases as the breakdown voltage is increased, and units having high breakdown voltage also have relatively high temperature coefficient of voltage, better stability is obtained by the use of two or more low-voltage units in series than by a single unit having the same voltage as the combination. The series combination has the additional advantage of higher dissipation capability, and therefore longer life at the same operating current. Available breakdown diodes have rated operating voltages extending from 3.6 volts to more than 150 volts and power ratings up to 10 watts.

[4] Pearson and Sawyer, *loc. cit.*
[5] G. Porter, *Internat. Rect. Corp. Engineering Handbook*, p. 101.
[6] D. H. Smith, *Communication and Electronics*, No. 16, 645 (January, 1955).
[7] Smith, *loc. cit.*

A breakdown diode can be used as a rectifier when it is biased either near zero voltage or near the breakdown voltage. The fact that the variation of voltage of a breakdown diode with current in the conducting state is small can be used to advantage in obtaining a nearly constant supply or reference voltage over a range of supply voltage or load current.[8] The supply voltage and series resistance are chosen so that the diode current remains within the operating range throughout the required range of load current, and the stabilized voltage is obtained across the diode. Breakdown diodes may also be used as direct-coupling elements, as in the circuit of Fig. 98. The breakdown voltage is chosen equal to the desired difference of potential between the coupled points. If the average current through the diode is well within the operating range, the alternating voltage across the diode is small and may be further reduced by means of shunt capacitance.

When two breakdown diodes are connected in series in opposite polarity (back-to-back), the resulting current-voltage characteristic is of the form of Fig. 22d. A number of applications of this symmetrical characteristic will be discussed in later sections.

Some breakdown diodes display instability in the form of random or periodic pulses at low values of operating current.[9]

6.4. Capacitor Diodes. The capacitance of a reverse-biased p-n junction is an inverse function of applied voltage,[10] as indicated by the relation $C = kv^{-n}$. The value of n depends upon the diode structure and may range from the order of ⅓ to 1. Maximum capacitance ranges from the order of 100 to thousands of micromicrofarads, and the capacitance may be varied over a ten-to-one range or greater by varying the voltage from approximately zero to the breakdown value. By proper design, the reverse leakage current can be made small and the breakdown voltage high.

Voltage-sensitive semiconductor capacitors find applications in oscillator frequency control and tuning, frequency modulation, and other processes. A problem encountered in their use is dependence of capacitance upon temperature at low magnitudes of reverse voltage, which may require the use of some compensating device, such as a forward-biased semiconductor diode.

6.5. Transient Response of Semiconductor Diodes. The performance of semiconductor diodes in pulse circuits is impaired by failure of the forward current to assume its final value instantaneously in response to a forward voltage step and by the flow of large reverse current when a

[8] Smith, *loc. cit.;* F. Finnegan, *Trans. I.R.E.*, Vol. ED-2, No. 1, p. 51 (January, 1955); C. N. Wulfsberg, *Electronics*, December, 1955, p. 192.

[9] K. Bewig and B. Salzberg. See J. E. Scobey, W. A. White, and B. Salzberg, *Proc. I.R.E.*, **44**, 1880 (December, 1956).

[10] A bibliography covering this subject is contained in an article by M. E. McMahon and G. F. Straube, *I.R.E. Wescon Convention Record,* Part 3, p. 72, 1958.

reverse voltage step is applied to a diode that is initially forward-biased. The forward transient is the result of a change of the semiconductor resistance caused by minority-carrier injection across the rectifying junction (electrons into p-type material and holes into n-type material). Thus the slope of the dashed characteristic of Fig. 23a is partially determined by the initial conductivity of the n-type and p-type semi-

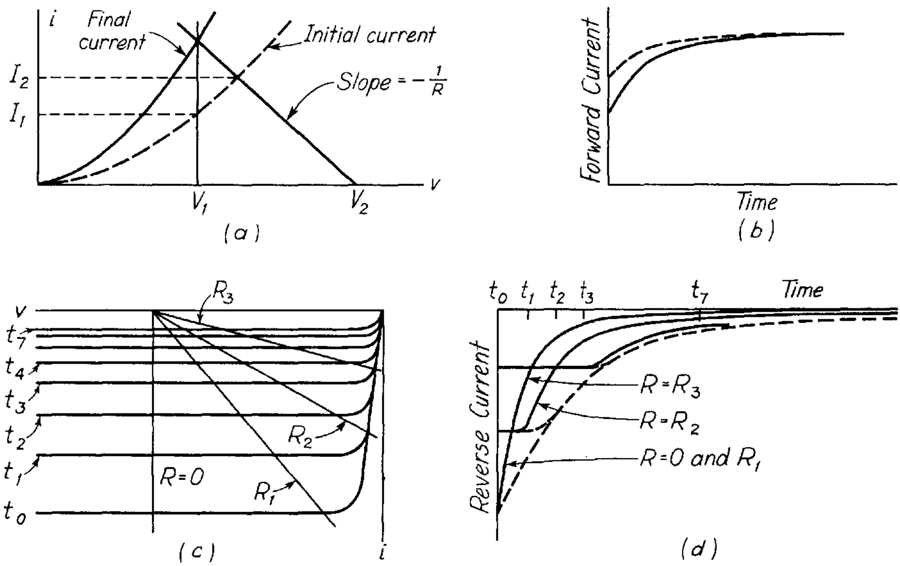


FIG. 23. (a) Effect of carrier injection upon the current-voltage characteristic of a semiconductor diode; (b) effect of circuit resistance upon diode forward transient; (c) reverse current-voltage characteristics of a semiconductor diode following interruption of forward current; (d) effect of V/R upon diode reverse transient.

conductors that make up the diode. As electrons diffuse into the p-type layer and holes into the n-type layer, the conductivity rises and the characteristic approaches the solid curve. This effect is more marked, both as to magnitude and the time required for the equilibrium characteristic to be assumed, in point-contact than in junction diodes, because of the relatively high "spreading resistance" in the vicinity of the contact, where the current density is high. The initial current rise in point-contact diodes ranges from as low as 50 to essentially 100 percent of the final value. The time constant of the slow rise may be of the order of 1 μsec.[11]

It can be seen from Fig. 23a that the ratio of the initial current rise to the final value should depend upon the circuit resistance and the applied

[11] M. C. Waltz, *Proc. I.R.E.*, **40,** 1483 (November, 1952); C. G. Dorn, *Trans. I.R.E.*, Vol. ED-3, No. 3, p. 153 (July, 1956).

voltage. Thus, if the circuit resistance is zero and the applied voltage is V_1, the initial current is I_1, and the transient is of the form of the solid curve of Fig. 23b. If the voltage is V_2, however, and there is a resistance R in series with the diode, the current first rises to the value I_2 and the transient is as shown by the dashed curve of Fig. 23b.

Reverse current, when a reverse voltage step is applied to a diode that is initially forward biased, is the result of minority-carrier storage.[12] Since some time is required for the disappearance of electrons that were injected into the p region and of holes that were injected into the n region during forward conduction, the instantaneous behavior of the junction following the interruption of forward current would be expected to resemble that of the base-collector junction of a transistor when there is minority-carrier injection from the emitter to the base. Immediately following the application of reverse voltage, therefore, the reverse current-voltage characteristic of the diode should be similar to a curve of collector-current vs. collector-base voltage of a transistor at constant emitter current.[13] The number of stored minority carriers decreases with time, however, and the current-voltage characteristic changes in a similar manner to the collector characteristic of the transistor when the emitter current is decreased. Figure 23c shows the general form of the reverse current-voltage characteristics of a diode at equally-spaced instants following interruption of forward current under the assumption that the loss of minority carriers results only from recombination.

The load lines in Fig. 23c correspond to a fixed value of applied reverse voltage and four values of series resistance. It is apparent that for any value of resistance in the range from zero to R_1 the initial current following application of reverse voltage is nearly independent of circuit resistance and that the current immediately starts to decay. If the resistance is increased to R_2, the initial reverse current is smaller and remains nearly constant until $t = t_1$. A further increase of resistance to the value R_3 results in a greater reduction of initial reverse current and an increase in the time during which the current remains nearly constant.

Figure 23c predicts that the rate at which current decays after the diode voltage leaves the saturation range should be nearly independent of V/R, as shown by the dashed curves of Fig. 23d. Curves obtained experimentally, however, are of the form of the solid curves. The discrepency is explained by the fact that the applied field causes the minority carriers to be swept away and the minority-carrier density consequently to fall more rapidly than if the carriers were lost only by recombination. Every characteristic curve of Fig. 23c except that for $t = 0$ should therefore be closer to the volt-

[12] Waltz, *loc. cit.;* R. H. Kingston, *Proc. I.R.E.*, **42**, 829 (May, 1954).

[13] L. P. Hunter, editor, *Handbook of Semiconductor Electronics,* McGraw-Hill Book Co., New York, 1956.

age axis by an amount that increases with increase of initial reverse current. The initial rate of decay of reverse current should consequently increase with initial current, as observed experimentally. Since the initial reverse current obtained following a given value of forward current is determined principally by V/R, the value of V/R also determines the form of the reverse transient.

Increase of the forward current prior to the application of reverse voltage increases the density of stored minority carriers and thus causes the characteristics of Fig. 23c to be displaced to proportionately higher values of current. Increase of the forward current that flows prior to reversal of the applied voltage should therefore increase the height of the reverse-current pulse if the pulse has no constant-current portion. It should also increase the duration of a constant-current portion of the pulse and the time required for the current to fall to a specified small value. The forms of measured transients are in agreement with this prediction.

The forward-current transient of most diodes is sufficiently small in comparison with the final current value, and the duration of the transient is small enough, so that the effect of the transient may usually be neglected. When point-contact diodes are used in circuits for the generation of pulses less than a microsecond in duration, forward transients should be taken into consideration if more than 5 percent variation of forward resistance is objectionable. The duration of diode reverse transients may range from as short as a millimicrosecond [14] to as long as tens of microseconds, and the reverse currents may be as high as several amperes.[15] The possibility of objectionable reverse transients must therefore always be kept in mind.

The minority-carrier storage effects that are responsible for forward and reverse transients in ordinary point-contact and junction diodes are not present in breakdown diodes used only in the breakdown mode. Oscillograms taken with an oscilloscope having a resolving time of 10^{-8} second show no observable delay in the response of breakdown diodes to a voltage step, and no observable reverse transient.[16] Theoretically, breakdown-diode recovery times shorter than 10^{-10} sec. should be possible.[17]

7. Clippers (Amplitude Comparators)

7.1. Diode Circuits. A very useful operation that may be performed by means of electronic circuits is the elimination or selection of portions of pulses or waves that exceed a reference value. Because such circuits can

[14] J. H. Forster and P. Zuk, *I.R.E. Wescon Convention Record,* Part 3, p. 122, 1958.
[15] Shulman and McMahon, *loc. cit.*
[16] B. Salzberg and E. W. Sard, *Proc. I.R.E.,* **45,** 1149 (1957).
[17] J. R. Madigan, *Electronic Ind.,* February, 1957, p. 78.

be used to determine with considerable accuracy the equality of an instantaneous value of input voltage and a reference voltage, they are also called *amplitude comparators.*

The simplest and most commonly used type of clipper consists of one or more biased diodes in series with a resistance. The circuits shown in basic form in Fig. 24 clip voltages of one polarity only. In the parallel diode circuit (a) the diode is reverse-biased when the input voltage is negative or when the input voltage is positive but smaller in magnitude than the biasing

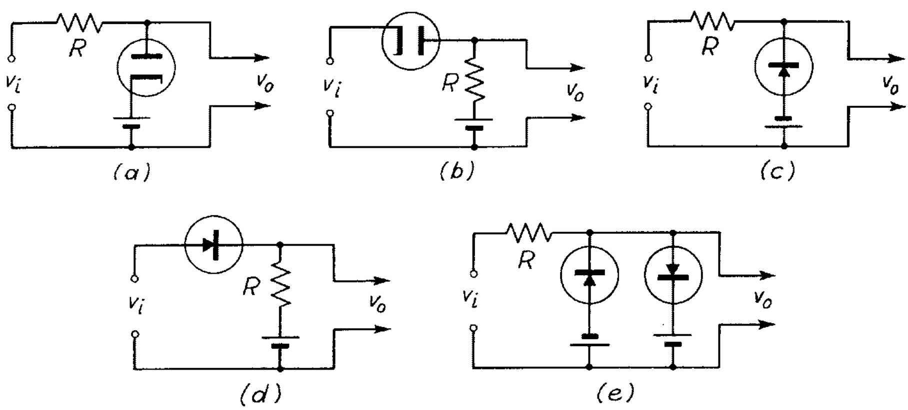


FIG. 24. Diode clipper circuits: (a) and (b) circuits that clip positive input voltages; (c) and (d) circuits that clip negative input voltages; (e) one form of circuit that clips both positive and negative input voltages.

voltage. If the reverse resistance of the diode and the load resistance shunted across the output terminals are very high in comparison with the resistance R and *if the shunt load capacitance is negligible,* nearly the full input voltage appears across the output during the time in which the diode is reverse-biased. If the forward resistance of the diode is small in comparison with R, the output voltage approximates the constant biasing voltage while the diode is forward-biased. The circuit therefore clips positive pulses of magnitude greater than the bias or the positive peaks of periodic waves having positive amplitudes greater than the bias. The series-diode circuit (b) produces the same effect as circuit (a), but output is obtained as the result of conduction, rather than nonconduction, of the diode. Circuits (c) and (d) clip negative input voltages. The series circuits (b) and (d) have the disadvantage that the interelectrode capacitance of the diode causes some output to appear during the time the diode is nonconducting and therefore prevents complete clipping. The shunt circuits are preferable to the series circuits when used with a source having high internal impedance, since negligible current is drawn from the source during the time in which output is obtained. The series circuits, on the other hand, are preferable when the input is obtained from a low-impedance source and the impedance level must be kept low during conduction.

A common application of diode clippers is in the elimination of voltages of one or the other polarity. This is accomplished by the circuits of Fig. 24 if the biasing voltage is zero. When a diode clipper is capacitance-coupled to the source of input voltage, charging of the coupling capacitor may cause undesired biasing of the diode and thus prevent proper functioning of the clipper. This phenomenon will become apparent in the discussion of clamping circuits in Sec. 8.

Circuits that clip both positive and negative voltages consist of combinations of circuits (a) and (c) or of circuits (b) and (d) of Fig. 24. Circuits (a) and (c) are combined by using two parallel oppositely connected diodes with a single series resistor, as shown in Fig. 24e. Circuits (b) and (d) of Fig. 24 are combined by connecting the output of one to the input of the other.

Although separate biasing supplies have been shown in the basic circuits of Fig. 24, the required values of bias can ordinarily be obtained from a tap on the main power supply or by equivalent means. Low resistance of the bias supply is essential except when the same resistances can be made to serve the function of the resistance R of the basic circuits and of the voltage divider of the biasing-voltage source. This can be accomplished readily with the series circuits (b) and (d) of Fig. 24.

The choice of the value of the resistance R in the circuits of Fig. 22 depends to some extent upon the purpose for which the clipper is to be used, upon the type of rectifying device used, and upon the shunt load capacitances, but usually ranges from the order of 10,000 ohms to several megohms. If R is made too large, diode interelectrode capacitance or load capacitance causes the circuit to act as an integrator during the time the diode is nonconducting. It is important to note that the circuits of Fig. 24 function properly only if they are essentially unloaded. When they are required to supply voltage to a circuit having appreciable shunt capacitance and conductance, a cathode-follower or common-collector isolating stage must be used between the clipper and the load.

7.2. The Use of Clippers in Rectangular-Wave Generation. One frequent application of double-acting clipper circuits is in the conversion of sinusoidal voltages into rectangular voltages. In order that the output shall be as nearly rectangular as possible, i.e., that the voltage rise and fall times shall be as short as possible, clipping must be done at a very low voltage level relative to the input amplitudes. If the required output voltage is large, and the available input voltage low, the clipper must be followed by an amplifier. Sometimes further improvement of waveform may be obtained by clipping and amplifying a second time, although a preferable procedure is to use a large input amplitude. It is very important to bear in mind, however, that the extent to which the output wave may be made to approximate rectangular form is limited by circuit capacitances. In the parallel-diode circuits of Figs. 24a and 24c, shunt load capacitance limits the rate at which the output voltage can rise and fall, whereas in the series diode circuits diode capacitance prevents the top and bottom of the output wave from being truly flat. These effects are reduced by lowering the resistance *R*, but *R* must always be large relative to the diode resistance in order that the output voltage will be negligible during the clipped portion of the input pulse or wave.

7.3. Variants of the Basic Diode Circuits. Since the breakdown-diode characteristic of Fig. 22d is of the same form as that of the two-diode

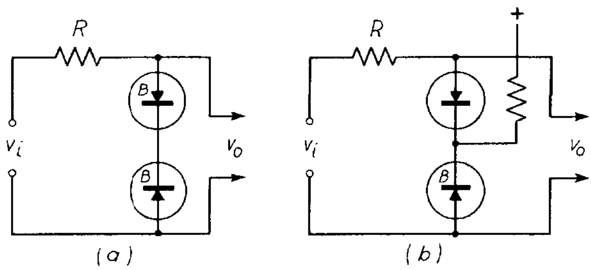


FIG. 25. (a) Breakdown-diode clipper circuit; (b) use of a breakdown diode to bias a conventional diode clipper.

circuit of Fig. 24d, two breakdown diodes connected in series in opposite polarity may be used to obtain full-wave clipping, as shown in Fig. 25a. In this manner the need for forward biasing voltages is avoided. An additional advantage in the use of breakdown diodes is the absence of objectionable transient effects.

If the diode of Fig. 24b is replaced by two breakdown diodes connected in series in opposite polarity and the biasing voltage is zero, the output consists of portions of the voltage input that exceed the diode breakdown voltages. Breakdown diodes may also be used to furnish bias in the circuits of Fig. 24, as shown in Fig. 25b. In this manner the need for a low-resistance bias supply is avoided.

Occasionally the output that is desired from a clipper is the portion of the input voltage in which the instantaneous input voltage exceeds the reference voltage, rather than the portion in which the input voltage is less than the reference voltage. The desired result may be obtained in the circuits of Fig. 24 by reversing the polarity of the reference voltage. This is equivalent to taking the output from across the resistor in the circuits of Figs. 24a and 24c and from across the diode in the circuits of Figs. 24b and 24d.

7.4. Clipping by Single Triodes or Pentodes. Clipping can also be accomplished by means of triodes and pentodes. Negative voltages may be readily clipped by making use of plate-current cutoff by negative grid voltage. If the output is taken from the plate of the tube, cutoff is not sharp. If a high-mu-triode cathode-follower is used, on the other hand, inverse feedback increases the sharpness of cutoff to the point where it is comparable with that of diodes. The cathode-follower has the advantage that it provides a high-impedance input and a low-impedance output. If a plate load resistor is also used, output at a high-impedance level may be obtained from the plate. The input voltage at which cutoff occurs can be controlled by means of additional biasing voltage introduced in series with the cathode resistor.

As in diode clipper circuits, the sharpness of clipping in triode and pentode circuits is limited by shunt output capacitance. When a cathode-follower is used to clip a negative input voltage, the output voltage falls more slowly than it rises. After the negative input voltage cuts the tube off, the shunt output capacitance discharges only through the cathode resistor (and any external path that shunts this resistor), which usually has a resistance of several thousand ohms. After the rising input voltage turns the tube on, however, the capacitance charges through the tube. Because the effective output resistance of a cathode-follower approximates $1/g_m$, or the order of only several hundred ohms, the charging time constant is only approximately one-tenth the discharging time constant.

Transistors in the common-emitter or common-collector connection can be used in a similar manner to grid-controlled tubes to produce clipping by collector-current cutoff. Presently available transistors have the disadvantage of lower input resistance than that of tubes, however.

Positive clipping with single triodes or multigrid tubes can be accomplished by means of the voltage drop produced by the flow of grid current through a high resistance in series with the grid. The action is similar to that in a diode circuit, but the output voltage is amplified. Because of dependence of the grid-current cutoff voltage upon plate voltage, grid-current clipping is inferior to diode clipping. A second type of positive clipping action is based upon the very sharp knee in the plate characteristics of voltage pentodes at zero and positive control-grid voltages. If a high value of plate load resistance is used, the plate voltage falls to a very low value at zero grid voltage and little further change occurs as the grid is driven positive. Clipping is not so sharp as with diodes, and is dependent upon the plate-supply voltage.

7.5. Transistor Clippers. The collector characteristics of junction transistors in the common-emitter connection being of the same general form as the plate characteristics of pentodes, *p-n-p* or *n-p-n* transistors may be used in a similar manner to pentodes to produce positive and negative clipping. Clipping by collector saturation has the disadvantage, however, that minority-carrier storage slows the response of the transistor when the magnitude of the base voltage falls below the value at which saturation occurs, and therefore reduces the sharpness of the trailing edge of the output voltage.

Transistor clipping circuits may also function by virtue of base-emitter rectification analogous to grid rectification in tube circuits.

7.6. The Cathode-Coupled Clipper. A symmetrical clipper that has a number of advantages relative to the diode clippers discussed in the preceding section is the *cathode-coupled clipper* shown in Fig. 26.[1] In this circuit T_1 serves as cathode-follower amplifier that drives T_2. Clipping of negative input voltage is the result of tube-1 cutoff. The input voltage at which tube 1 is cut off and negative clipping occurs is much smaller than in an isolated cathode-coupled stage, since the plate current of tube T_2 increases as that of tube T_1 decreases, and thus tends to maintain the voltage across R_K constant. (If tube T_2 were not present, the voltage across R_K would decrease by almost the same amount as the negative input voltage increased, and thus tend to prevent T_1 from cutting off.) Clipping of positive input voltage takes place when the voltage produced across R_K by the plate current of T_1 is sufficient to bias T_2 beyond cutoff.

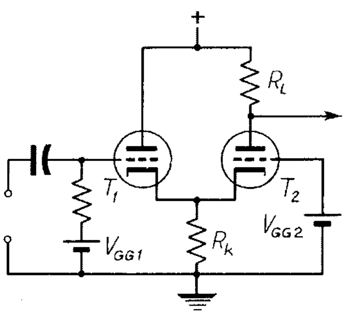


FIG. 26. The cathode-coupled clipper.

The advantages of the cathode-coupled clipper relative to diode circuits are that, with proper choice of circuit parameters and operating voltages, it has high input impedance over a wide range of input voltage and, because of amplification, provides much greater output. In comparison with the biased-diode circuits of Figs. 21 and 23 it has the further advantage that it does not require a low-impedance bias supply.

In order to obtain abrupt clipping, the criterion $(1 + \mu)R_K \gg r_p + R_L$ must be satisfied.[2] This is accomplished by the use of tubes having high

[1] For a detailed analysis of this circuit as a clipper, see L. A. Goldmuntz and H. L. Krauss, *Proc. I.R.E.*, **36,** 1172 (1948). For other applications of the cathode-coupled circuit, see G. C. Sziklai and A. C. Schroeder, *Proc. I.R.E.*, **33,** 701 (1945), and K. A. Pullen, *Proc. I.R.E.*, **34,** 402, (1946).

[2] Goldmuntz and Krauss, *loc. cit.*

transconductance and making R_K large in comparison with $r_p + R_L$. The resulting high negative grid bias produced by R_K may then necessitate the use of positive bias-supply voltage V_{GG2} in the grid circuit of tube 2 in order to obtain sufficient output amplitude. This biasing voltage, which may be obtained from a tap on the power supply, increases the positive clipping voltage by approximately V_{GG2} and decreases the negative clipping voltage by approximately V_{GG2}, as can be readily seen from the following discussion of Fig. 26. Insertion of V_{GG2} increases by an equal amount the voltage that must be produced across R_K by the plate current of tube 1 in order to cut tube 2 off. When tube 2 is cut off, however, tube 1 acts as a simple cathode follower, having a voltage amplification of nearly unity. The increased voltage required across R_K to cut tube 2 off therefore calls for an approximately equal increase of input voltage, or an increase of approximately V_{GG2}. When tube 1 is cut off, on the other hand, tube 2 acts as a cathode follower, introduction of the voltage V_{GG2} producing an approximately equal increase of voltage across R_K and hence reducing by this amount the negative input voltage required to maintain tube 1 cutoff. If symmetrical clipping is desired, therefore, a positive biasing voltage V_{GG1} approximately equal to V_{GG2} must also be used in the grid circuit of tube 1.

7.7. Input Ratio of Cathode-Coupled Clipper. In the use of clippers to convert sinusoidal input voltage into rectangular output voltage, short rise and fall times of the output voltage necessitate a high ratio of input voltage to the voltage at which clipping takes place. If the source of input voltage is coupled to the clipper directly or by means of a transformer, the flow of grid current does not affect the clipping action adversely. Then, so far as the clipping action is concerned, there is no limit to the input amplitude that can be used, provided sufficient series grid-circuit or source impedance is present to prevent excessive grid dissipation. When capacitive input coupling is used, however, as in the circuit of Fig. 16, grid rectification causes an undesirable change of grid bias. The analysis of Goldmuntz and Krauss (*loc. cit.*) shows that the *input ratio*, defined as the ratio of the positive input voltage at which grid current starts flowing to that at which positive clipping takes place, is given by the following approximate relation:

$$\text{Input ratio} = \frac{\mu\mu_o}{\mu - \mu_o}\left[\frac{R_K}{R_K + r_p} - \frac{V_{GG2}}{V_{PP}}\right] \tag{17}$$

in which μ and μ_o are the amplification factors at the operating point and at plate-current cutoff, respectively. Equation (17) shows that tubes used in the cathode-coupled clipper should have as small variation of μ as possible in the range of operation. It also indicates that high input ratio is favored by making R_K large in comparison with r_p and that the input ratio decreases as the positive biasing voltage V_{GG2} is increased to increase the output amplitude. Since increase of transconductance increases the output amplitude at constant V_{GG2}, high-transconductance tubes also favor a high input ratio. Typical values of the factor $\mu\mu_o/(\mu - \mu_o)$ are 70 for the 6J4, 50 for the 6J6, 45 for the 6SN7, and 200 for the 6SL7. Input ratios ranging from 30 to 50 may be readily attained.

The high cathode circuit resistance essential to abrupt clipping and to a high input ratio may be obtained by the use of a self-biased triode or pentode in place of the cathode resistor.[3] A lower direct voltage drop is thus ob-

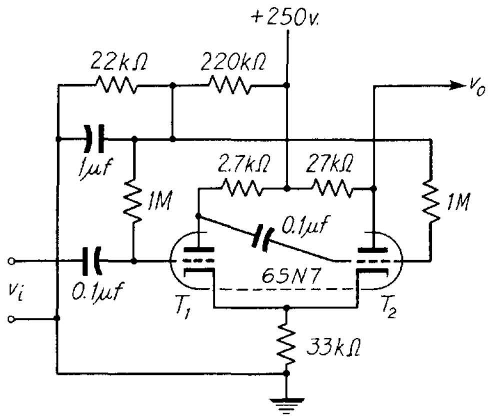


FIG. 27. Cathode-coupled clipper incorporating a feedback capacitor.

tained and lower supply voltage may be used. The disadvantages of this modification of the circuit are the greater complexity of the circuit and the increased capacitance shunting the cathode load resistance.

The input ratio can be increased by using feedback. This can readily be accomplished by using resistors in series with the plate of tube 1 and the grid of tube 2 and coupling the plate of tube 1 to the grid of tube 2 by means of a capacitor, as shown in Fig. 27. (Goldmuntz and Krauss, *loc. cit.*). The feedback also shortens the rise and fall times of the output voltage, provided that they are not determined by interelectrode and distributed capacitances. If sufficient feedback is used, the circuit oscillates, and the input voltage serves merely to control the frequency of oscillation. (The use of this circuit as an oscillator will be discussed in Sec. 51.)

7.8. Speed of Response of Cathode-Coupled Clipper. The speed of response of the cathode-coupled clipper to a rapidly rising or falling input voltage is determined mainly by the time constant of the output circuit, which is usually approximately equal to the product of R_L and the capacitance between the plate of tube 2 and ground.[4] Capacitance shunting R_K

[3] W. M. Grim and A. B. Van Rennes, *Rev. Sci. Instr.*, **23**, 563 (October, 1952).
[4] P. F. Ordung and H. L. Krauss, *Television Engineering*, April, May, and June, 1951.

has little effect upon the rise and fall times of the output voltage, but delays a rapidly falling output voltage relative to the input voltage and thus tends to lengthen a pulse of output voltage if the trailing edge is steep. Positive feedback from the plate of tube 1 to the grid of tube 2 does not increase the rate of rise or fall of output voltage if the input voltage rises and falls so rapidly that the rate of rise or fall of output voltage is determined by the output time constant. By the use of tubes with high transconductance and low interelectrode capacitance and a plate load resistance of the order of 1000 ohms, the cathode-coupled clipper may be made to function satisfactorily with sinusoidal input voltages at frequencies up to 5 Mc, the upper limit depending upon the required steepness of the leading and trailing edges of the output voltage.

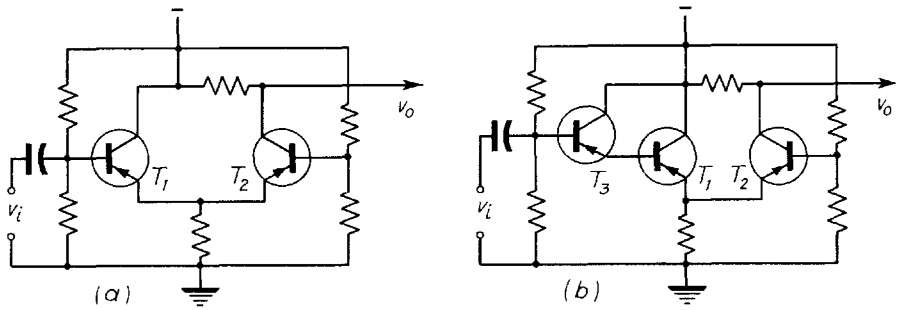


FIG. 28. (a) Emitter-coupled clipper; (b) emitter-coupled clipper incorporating a compound-connected transistor to increase the input impedance.

Figure 28a shows an emitter-coupled transistor clipper circuit that is the equivalent of the cathode-coupled circuit of Fig. 26. The transistor circuit has lower input impedance than the tube circuit. The input impedance may be increased by using compound-connected transistors in place of a single input transistor, as shown in Fig. 28b.

A combination of clipping and differentiating circuits by means of which a sinusoidal input voltage may be converted into positive and negative periodic pulses of short duration is shown in Fig. 29. The functions of the various portions of the circuit are indicated by the voltage waveforms.

8. Clamping Circuits

8.1. Operation of Clamping Circuits. It is sometimes necessary or desirable to ensure that the voltage variations at some point in a circuit take place relative to a constant direct reference potential. This can be accomplished by means of the diode *clamping* circuits of Figs. 30 and 31. If the resistance R in the circuit of Fig. 30a is infinite, the capacitor C

charges in the indicated polarity to a voltage equal to the crest positive value of the input voltage, after which the diode is not forward-biased at any time in the cycle and diode current consequently ceases to flow. The constant voltage across the capacitor lowers the output voltage relative to

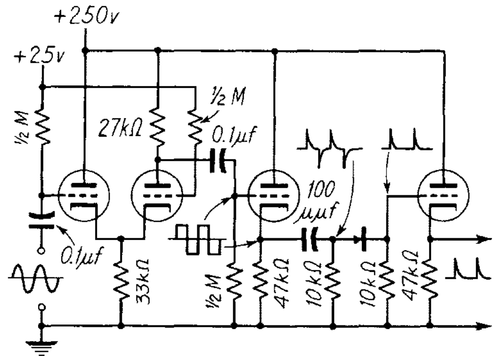


Fig. 29. The use of clipper and differentiating circuits to convert a sinusoidal input voltage into periodic positive pulses.

the input voltage by an amount equal to the positive crest input voltage, and the output voltage waveform is as shown in Fig. 30a.

If R is infinite, any subsequent reduction in amplitude of input voltage is not accompanied by a reduction of voltage across C, for there is no discharge path. As the amplitude of the input voltage is reduced, both the top and the bottom of the output wave approach a negative voltage equal to the fixed voltage across the capacitor. The resistor R allows the capacitor to discharge following a reduction of input voltage, and the positive crests of the output voltage wave remain clamped at approximately zero if the input voltage is not reduced too rapidly. The resistor also allows the

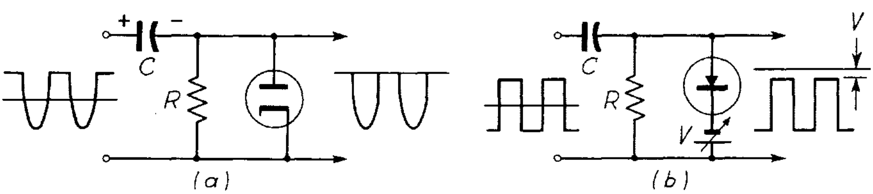


Fig. 30. (a) Unbiased clamping circuit; (b) biased circuit.

capacitor to discharge slightly during the portion of each cycle in which the diode does not conduct. The anode of the diode is therefore driven slightly positive near the positive crest of input voltage while the charge lost by the capacitor is restored. If the forward resistance of the diode is small in comparison with R, and the time constant RC is large in comparison with the period, the variation of capacitor voltage is small. There is then little distortion of the output voltage and the positive peak is clamped only slightly above zero voltage.

The addition of a forward biasing voltage V in series with the diode, as shown in Fig. 30b, allows the capacitor to charge to a voltage equal to the magnitude of the crest positive input voltage plus V and therefore lowers the output wave by the additional voltage V. The top of the wave is then clamped at the voltage $-V$, rather than zero. If the reference voltage V is reversed in polarity, the capacitor charges to a voltage that is less than the

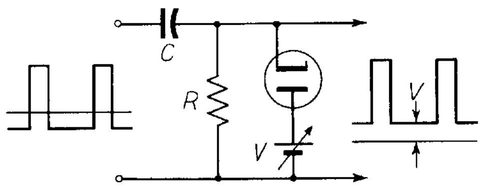


FIG. 31. Biased clamping circuit.

magnitude of the crest positive input voltage by the voltage V, and the top of the wave is clamped at the voltage $+V$. An alternative method of lowering or raising the clamped output wave is to connect the lower output terminal to a point having a positive or a negative direct voltage, rather than to ground.

When the polarity of the diode is reversed, as in Fig. 31, the capacitor charges to a voltage that approximates the crest negative input voltage plus or minus the reference voltage V, and the bottom of the output wave, rather than the top, is clamped at the reference voltage.

8.2. Distortion by Clamping Circuits. Perfect clamping of all types of periodic voltages without the introduction of distortion would require that the forward resistance of the diode be zero, the back resistance infinite, the resistance of the source of input voltage zero, and the time constant RC infinite. The objectionable effects of failure to satisfy these requirements are particularly severe when the input voltage is a periodic rectangular wave of low or high duty ratio. For this reason it is instructive to discuss the effects of reverse diode conduction, forward diode resistance, source resistance, and finite time constant upon the rectangular wave (a) of Fig. 32. Curve (b) shows the theoretical waveform of the output of the circuit of Fig. 31 under the assumption that the capacitance C is infinite and the diode biasing voltage zero. Under equilibrium conditions the average charge on the capacitor does not change from cycle to cycle. The following expressions for V_+ and V_- may therefore be derived by equating the charge gained by the capacitor to the charge lost in one period [1] (Prob. 8.2-1).

$$V_+ = V_i \frac{(1 - D)R_r'}{R_s + DR_f' + (1 - D)R_r'} \tag{18}$$

$$V_- = V_i \frac{DR_f'}{R_s + DR_f' + (1 - D)R_r'} \tag{19}$$

in which R_f' is the resultant resistance of R in parallel with the forward resistance R_f of the diode; R_r' is the resultant resistance of R in parallel with the reverse resistance R_r of the diode; R_s is the series resistance of the source of input voltage; and V_i, T, and D are the peak-to-peak amplitude, period, and duty ratio of the input wave.

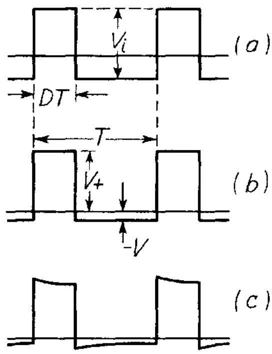


FIG. 32. Distortion of a rectangular wave by a clamping circuit: (a) input voltage; (b) output voltage of an ideal circuit; (c) output voltage of a practical circuit.

Failure to make the discharge time constant $R_r'C$ infinite causes the output voltage to vary exponentially during the portions of the cycle in which the input voltage is constant, and reduces the values of V_+ and V_- below those given by Eqs. (18) and (19). The general form of the output voltage for finite time constant is shown in Fig. 32c. The exponential fall of voltage during the positive portion of the wave is caused by the charging of C through the diode. The rise during the negative portion is caused by the discharging of C through R.

It follows from the definition of V_- in Fig. 32b that the negative peak of the output wave is clamped at zero voltage (or, in general, at the bias voltage) only if V_- is zero. Unless R_r' is infinite, therefore, the output wave will not only experience some exponential distortion, but will not be clamped at the desired level, and the clamping level will vary with duty ratio D. Accordingly, R should be large and the diode should have high reverse resistance. If the time constant RC is too large, on the other hand, appreciable time may be required for the output wave to assume its clamped position following a sudden reduction of input amplitude, the decrease in output amplitude taking place about the initial average value of the output wave.

8.3. Example of Clamper Application. An example of a typical application will serve to show the value of clamping circuits. Suppose that it is

[1] J. Van Bladel, *Electronic Eng.*, **25**, 524 (December, 1953).

necessary to amplify periodic negative pulses by means of an amplifier that incorporates a resistance-capacitance coupling network. Since the coupling network does not pass direct current, it will remove the direct component of the input wave and the output excursions will take place about the average value, as illustrated by the input wave of Fig. 30a, rather than from zero. The average value depends upon the amplitude of the pulses and upon their duty ratio. Consequently, variation of the pulse amplitude, length, or repetition frequency will cause a vertical displacement of the output wave that is extremely annoying if the wave is observed oscillographically and may be intolerable if the pulses are used to initiate action in a following circuit. Additional vertical "jitter" may result from random fluctuations of the amplifier supply voltage. If the periodic pulses delivered by the amplifier are passed through the clamping circuit of Fig. 30a, the direct component will be restored and the output will again consist of periodic negative pulses based at zero voltage. The output coupling capacitor of the amplifier may itself serve as the capacitor of the clamping circuit.

9. Pulse Sharpeners and Stretchers

9.1. Resonant-Circuit Pulse Sharpener. In Sec. 3 it was shown that exponential pulses of short duration can be derived from voltage steps or

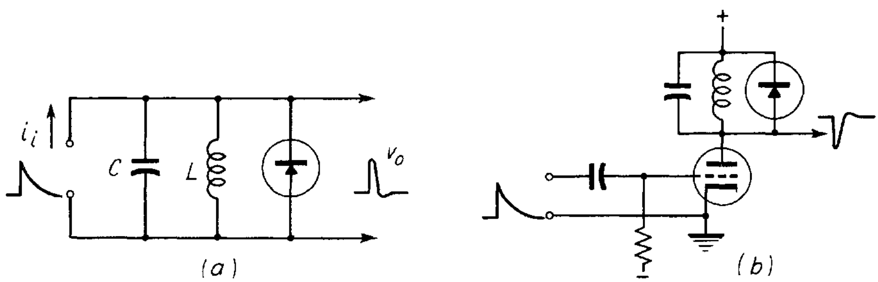


FIG. 33. (a) Current-driven diode pulse sharpener; (b) Voltage-driven triode pulse sharpener.

rectangular voltage waves. Although successive differentiation can also be used to sharpen exponential pulses, long exponential pulses can usually be sharpened with less loss in amplitude by using them to shock-excite a parallel resonant circuit shunted by a diode, as shown in Fig. 33.[1] This circuit is driven by a current pulse provided by a high-impedance circuit. Without the diode, the input pulse would initiate a high-frequency damped oscillation in the resonant circuit. After the first half-cycle of oscillation,

[1] L. Reiffel, *Rev. Sci. Instr.*, **22**, 214 (March, 1951); *Electronics*, July, 1951, p. 162 (abstract).

however, the diode provides a low-resistance path that prevents the appearance of negative output voltage. If the damping provided by the diode is great enough, the output consists of a single positive pulse of length equal to half the period of the resonant circuit.

9.2. Diode Pulse Stretcher. Circuits used to increase the length of pulses are called *pulse stretchers.* The simplest type of pulse stretcher is shown in Fig. 34a. The capacitor C is charged through the diode by the

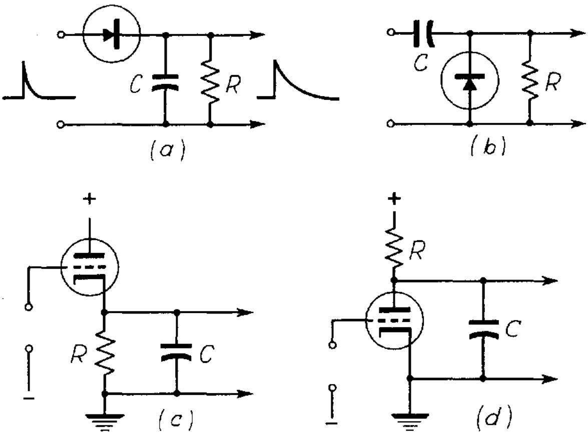


FIG. 34. Diode and triode pulse stretchers.

input voltage pulse, after which the diode isolates the capacitor from the low-impedance source of input voltage. The capacitor then discharges slowly through the high resistance R. The output of the circuit is therefore an exponential pulse, the duration of which exceeds that of the input pulse if the time-constant RC is large in comparison with the length of the input pulse. A variant of the circuit of Fig. 34a is shown in Fig. 34b. The triode circuits of Figs. 34c and 34d have the advantage of high input impedance.

9.3. Transmission-Line Pulse Stretcher. In some applications of pulse stretchers, an output pulse of essentially constant voltage, rather than of exponentially decreasing voltage, is required. It is then necessary to use a more sophisticated circuit, such as that of Fig. 35, which incorporates a transmission line.[2] The input pulse causes the cathode-follower to charge the line uniformly through the series diodes. Subsequent discharge of the capacitors produces a long pulse of essentially the same amplitude as the input pulse. The output voltage is nearly constant except for the rise and fall at the beginning and termination of the output pulse. Fall in output voltage during the pulse is mainly the result of leakage through the diodes and through the load admittance that shunts the output. In order to mini-

[2] J. F. Craib, *Electronics,* June, 1951, p. 129.

mize the effect of leakage, the line impedance should not exceed about one-tenth of the back resistance of the diodes. The time constant determined by the output resistance of the cathode-follower and the sum of the capacitances must be small enough to ensure charging of the line during the input

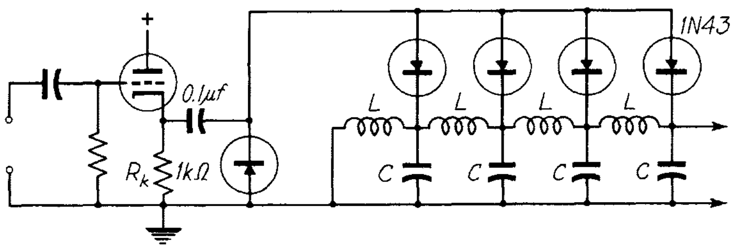


FIG. 35. Transmission-line pulse stretcher.

pulse, and the cathode-follower must be capable of supplying the peak current required to charge the capacitors. For pulse durations of 1 to 20 μsec, a 6J6 tube and a line impedance of 5000 to 20,000 ohms are satisfactory. A 2-volt input pulse is required, but the 6J6 can handle an input of up to 10 volts without grid-current flow.

Craib points out that the circuit of Fig. 35 tends to improve the apparent signal-to-noise ratio when the input pulses are accompanied by low-level background noise, but that the signal-to-noise level may be decreased by large noise pulses, which may reduce the crystal back resistance momentarily and thus allow the capacitors to discharge.

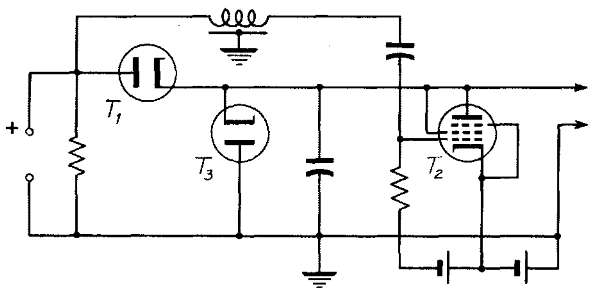


FIG. 36. Delay-line pulse stretcher.

Another pulse stretcher that delivers an essentially rectangular pulse is shown in Fig. 36.[3] The input pulse charges the capacitor C through the diode T_1. The input pulse is also applied to the delay line, which delivers a delayed positive pulse to the grid of T_2 and thus causes C to be discharged

[3] L. Reiffel and G. M. Burgwald, *Electronics*, February, 1952, p. 186.

through T_2. If the resistance shunting C is so large that little discharging takes place during the desired duration of the output pulse, the output pulse has essentially constant voltage. The length of the output pulse is equal to the delay time of the line. If desired, the delay line may be replaced by a monostable circuit (one-shot multivibrator) of the type discussed in Sec. 55. The function of the diode T_3 is to prevent reversal of the capacitor voltage as the result of the negative cathode voltage of T_2.

9.4. Monostable Circuits. Some monostable circuits are in themselves generators of approximately rectangular pulses of adjustable length and may therefore be used as pulse stretchers. (See Sec. 55.9.)

10. Electronic Switches (Gates)

10.1. Desirable Features of Electronic Switches. A very important type of electronic device is the electronic switch, or *gate,* which serves the same general functions as a mechanical switch but has the advantages that it can be operated in a very short time and can be controlled by means of a voltage with the expenditure of negligible or small control power. Gates can be designed so that they deliver output either in the presence of an activating voltage or in the absence of an inhibiting voltage. Variants of the basic gate circuits include *coincidence* or *and* circuits, from which an output is obtained only when two or more signals are applied simultaneously, *or* circuits, from which output is obtained when any of two or more voltages are impressed, and *anticoincidence* or *not* circuits, which deliver an output when one voltage is impressed, but not when a second voltage is simultaneously impressed. These variants will be discussed in Sec. 14.

The desirable properties of an electronic switch include (1) low leakage (output when the switch is open), (2) low attenuation when the switch is closed, (3) linear response, (4) attenuation that is independent of the magnitude of the control voltage when the switch is closed, (5) low required control voltage, (6) low control current, (7) low signal input current if the switch is used to provide an output voltage from a high-impedance source, and (8) negligible output arising from the control voltage.

The simplest types of electronic switches make use of the rectifying property of diodes. The diodes may be used either as shunt short-circuiting switches or as series connecting switches. The choice between thermionic and semiconductor diodes is based upon the characteristics discussed in Sec. 6.

10.2. Diode Gates. Figure 37 shows a number of gate circuits that are suitable for use with positive input signals. In circuit (a) the shunt diode conducts for a positive input voltage in the absence of a control pulse and, if the resistance R is large in comparison with the sum of the forward resistance of the diode and the resistance of the source of control voltage, the output voltage is negligible. A positive control pulse of amplitude equal to or greater than that of the input signal prevents conduction of the diode and allows essentially the entire input voltage to appear across the output terminals. If negative signals must be rejected, a second diode may be used in series with the output terminal, as shown in circuit (b). Alternatively, a second diode may be shunted across the output terminals in op-

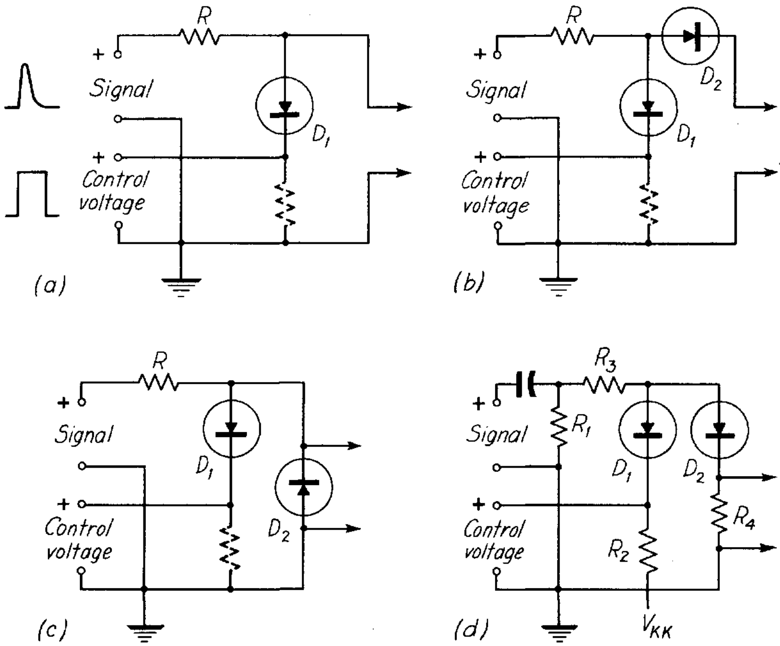


FIG. 37. Diode gate circuits for positive input signals.

posite polarity to the main diode, as shown in circuit (c). If the impedance of the source of control voltage is not small, the source should be shunted by a resistance, as shown by the dotted lines in Fig. 37a.

Because the extent to which the resistance R can be increased in the circuits of Figs. 37a to 37c without excessive attenuation is limited by the capacitance and reverse conductance of the diode or diodes and by the load that shunts the output, leakage may not be small enough for some purposes. This difficulty is avoided in the series diode circuit of Fig. 37d. In this circuit the parameters and the negative voltage V_{KK} are such that the current of D_1 flowing through R_2 and R_3 maintains the anode of D_2 below ground potential and therefore prevents the flow of current through R_4 in the absence of a positive control pulse. A positive control pulse of amplitude exceeding the sum of the biasing voltage V_{KK} and the potential of the anode of D_2 in the presence of signal input raises the potential of the cathode of D_1 sufficiently to prevent conduction of D_1. A positive input signal then causes D_2 to conduct and output to appear across R_4, the resistors R_3 and R_4 serving as a voltage divider. Negative input signals are rejected by this circuit.

Reversal of the polarities of the diodes and the control pulse, and of the biasing voltage in circuit (d), converts the circuits of Fig. 37 for use with negative signal input.

10.3. *Not* Circuits. An example of a circuit that delivers output in the absence of a control pulse, but not when a negative control pulse is applied

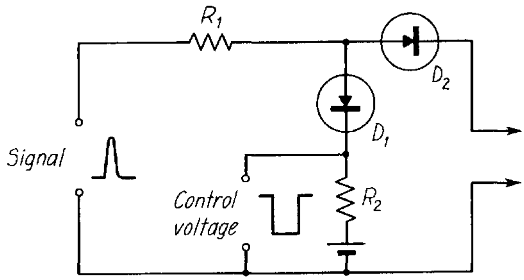


FIG. 38. "Not" circuit.

is that of Fig. 38. In the absence of a control voltage, D_1 is reverse-biased. Application of a negative control voltage causes D_1 to conduct and lowers the potential of the anode of D_1 below ground potential. The function of D_2 is to prevent the output voltage from falling below ground potential when the inhibiting pulse is applied.

10.4. Full-Wave Diode Circuits. A two-diode gate that can be used with input signals of either polarity, and therefore with alternating voltage, is shown in Fig. 39. In this circuit, which is based upon that of Fig. 37a, the output is normally short-circuited by the diodes. The control voltage cuts both diodes off and thus allows essentially the entire output voltage to appear across the output terminals. In some applications the control voltage may be impressed upon the circuit through a center-tapped transformer.

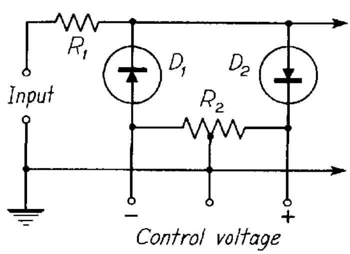


FIG. 39. Full-wave diode gate.

Another type of two-diode circuit that operates with signals of either polarity is shown in Fig. 40.[1] A control voltage of the polarity indicated causes both diodes to conduct and thus allows a part of the impressed signal voltage to be delivered to the output

[1] J. Millman and T. H. Puckett, *Proc. I.R.E.*, **43**, 29 (January, 1955).

terminals. A control voltage of opposite polarity, on the other hand, prevents both diodes from conducting and therefore isolates the output terminals from the input terminals except through the interelectrode capacitance of the diodes. Since the circuit is basically a bridge, the control voltage does not produce output voltage if the circuit is completely symmetrical and the two control voltages are equal. The adjustable tap on the resistance R is used to balance the circuit so that no output is obtained when the input is zero.

The choice of resistance values in the circuit of Fig. 40 is governed by the

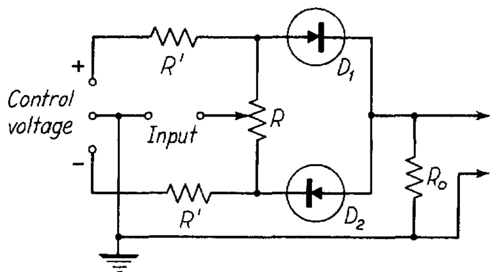


FIG. 40. Full-wave diode gate.

need to provide the desirable properties listed in the second paragraph of Sec. 10.1. Complete design formulas for this circuit have been developed, and the reader is referred to the literature for design details.[2] Because of the relatively low back resistance of germanium diodes, low leakage may not be obtained without loss of some of the other desirable properties, whereas the leakage can be made of the order of a tenth of a percent of the desired signal output voltage when thermionic or silicon diodes are used. For a circuit using thermionic diodes having a 250-ohm forward resistance and excited by a signal of 20-volt amplitude provided by a 1000-ohm source, Millman and Puckett suggest 100,000 ohms for R_o and R', and 133,000 ohms for R. These values result in a ratio of signal output voltage to signal input voltage of 0.5 and a ratio of output voltage to difference of the two control voltages of 0.16. The control voltage necessary to ensure adequate diode conduction with these values when the circuit is on is 10 volts, and that necessary to ensure that the diodes are biased beyond cutoff at the peak input voltage is 60 volts. The signal current is 0.3 ma during conduction and 9.24 ma during nonconduction of the diodes. The control current is 0.2 ma during conduction and 0.5 ma during nonconduction.

A reduction in the control voltage required to open the circuit at a fixed value of gain (output voltage divided by input voltage) can be achieved by addition of two more diodes, as shown in Fig. 41a. The additional diodes

[2] Millman and Puckett, *loc. cit.*

open the branch of the circuit containing R when the control-voltage polarity is such as to open the circuit, and thus reduce the control current. Figure 41b shows another four-diode circuit having the desirable characteristics that it is not affected by unbalance of the two control voltages and requires a relatively low value of control voltage to open the circuit. Although the circuit is sensitive to unbalance of the additional voltages $+V$ and $-V$, these voltages are fixed and may be obtained from regulated supplies. The circuit has the disadvantage of drawing large currents from the signal and control sources when the circuit is closed. The advantages of both of the four-diode circuits without their disadvantages are obtained by combining them. The resulting six-diode circuit is formed by adding a diode in series with each end of R, in the same manner as in the circuit of Fig. 41a.

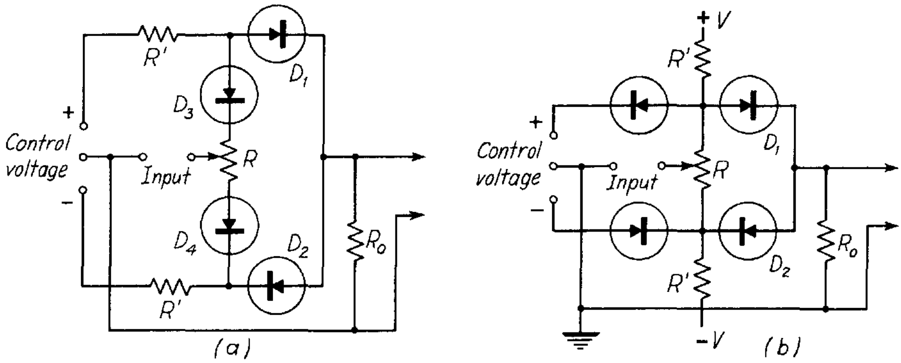


FIG. 41. Four-diode gates.

The effect of interelectrode capacitances in the circuits of Figs. 40 and 41 is to produce relatively high leakage at frequencies at which the reactance of the capacitances becomes comparable with the diode back resistance. The six-diode circuit is preferable to the others in this respect.

10.5. Effect of Load Capacitance and Conductance. Like diode clippers, diode gate circuits do not function properly when loaded by a high-admittance circuit. When a gate circuit must deliver voltage to a load of relatively high shunt capacitance or conductance, a cathode-follower or common-collector isolating stage must be used between the gate and the load.

11. *Vacuum Triode and Pentode Gates*

11.1. Full-Wave Triode Gate. A triode gate circuit, in which the tubes act as a full-wave series switch, is shown in Fig. 42.[1] Although this circuit

[1] M. W. Tobin, H. Grundfest, and R. L. Schoenfeld, *Rev. Sci. Instr.*, **22**, 189 (March, 1951). See also "Waveforms," Rad. Lab. Series, Vol. 19, pp. 375–378 (McGraw-Hill Book Company, 1949).

has the advantage that it draws very little current or power from the source of control voltage, it has a number of disadvantages relative to diode circuits. These include the high tube voltage drop, distortion of the output as the result of nonlinearity of the tube characteristics, and coupling of the control voltage to the output through interelectrode capacitances and grid-cathode conductance of tube 2.

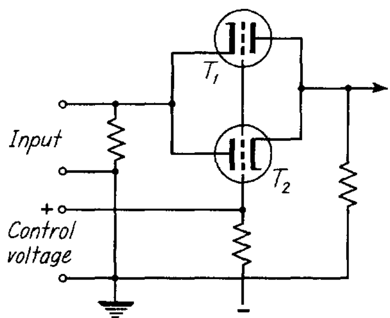


FIG. 42. Full-wave triode gate.

11.2. Cathode-Follower Gate. Figure 43 shows a cathode-follower circuit that has desirable features when only negative pulses must be gated.[2] The operation of the circuit resembles that of a cathode-coupled clipper. If the grid of T_2 is maintained at the potential V_{GG}, a very small negative increment of voltage of the grid of T_1 (relative to ground) cuts T_1 off, the voltage across R_K being maintained nearly constant by the plate current of T_2. If the voltage of the grid of T_2 is reduced considerably below V_{GG}, on the other hand, T_2 is cut off and T_1 acts as a conventional cathode-follower amplifier, delivering an output that is essentially a replica of the signal

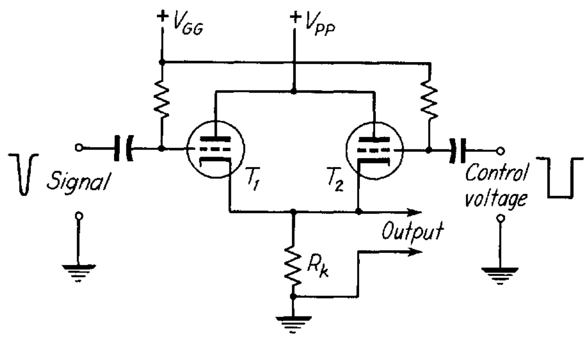


FIG. 43. Cathode-follower gate.

input. When a voltage step, rather than a pulse, is used for the selector voltage, it should be applied directly in series with the grid of T_2, instead of through the R-C coupling circuit shown in Fig. 43. A small output voltage is obtained in the absence of control voltage. Kurshan has shown that the magnitude of this output voltage does not exceed approximately $V_{k0}/2R_K g_m$, where V_{k0} is the quiescent voltage drop across R_K (see also Prob. 11.2–1.).

[2] J. Kurshan, *Rev. Sci. Instr.*, **18**, 647 (1947).

Typical values are of the order of 1 volt, as compared to useful output voltages of the order of 100 volts or more. To avoid distortion, R_K should be chosen so that $g_m R_K \gg 1$.

The circuit of Fig. 43 has the advantages of low distortion, high input impedance, low output impedance, and leakage output that is practically independent of input amplitude. Another characteristic that may be desirable in some applications is that the circuit acts as a clipper for input voltages of amplitude greater than that of the selector pulse. A disadvantage of the circuit is higher power consumption than that of diode circuits.

11.3. Pentode Gates. A circuit in some respects similar to that of Fig. 43 consists of two pentodes with a common high-resistance plate load resistor, operated at zero control-grid bias. A negative pulse applied to the control grid of one tube serves as the control voltage. The signal (negative) is applied to the control grid of the other tube.

It is also possible to use a single pentode as a gate. Best results are obtained when the signal is applied to the control grid and the control voltage to the suppressor grid, as shown in Fig. 44. In the absence of a control

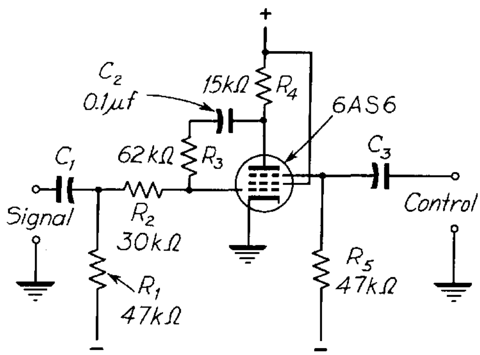


FIG. 44. Single-pentode gate.

pulse, the suppressor grid is biased sufficiently negative to cut off the plate current. The function of R_2, R_3, and C_2 is to reduce distortion by providing negative feedback. In the gating of alternating signal voltages, a disadvantage of this circuit relative to diode circuits is that the static operating grid voltage must be well above cutoff. Consequently, output is obtained from the switching voltage alone, and the signal output is superimposed upon a "pedestal." When the input consists of positive pulses, the pedestal can be prevented by biasing the control grid to cutoff. Appreciable distortion of the lower part of the pulses will then occur because of curvature of the transfer characteristic. In applications in which push-pull input and output signal voltages can be used, the pedestal problem can be solved by the use of two pentodes connected in push-pull, the control voltage being applied to the suppressor grids, in the same manner as in the circuit of Fig. 44.

12. Beam-Tube Gates

12.1. Gated-Beam Tube. Excellent gating action is obtained in the 6BN6 gated-beam tube, the structure of which is shown in Fig. 45.[1] Electrons that leave the cathode and are accelerated by the accelerator electrode are focused by the focusing electrode and the first slot in the accelerator into a thin sheet beam that is projected upon the first control grid. The beam is then refocused by the screen, the slot of the lens, and the first control grid and projected upon the second control grid. The focus, lens, and shield electrodes are internally connected to the cathode. Because the number of electrons leaving the cathode is independent of the potentials of the two control grids, the anode current saturates at very small positive voltage of the control grid. With saturation voltage applied to either control grid, the anode current can be controlled by the other control grid. Consequently the tube can be used as a gate by applying the signal voltage to one grid and the control voltage to the other. If the control voltage exceeds the saturation voltage, the output is independent of the control voltage.

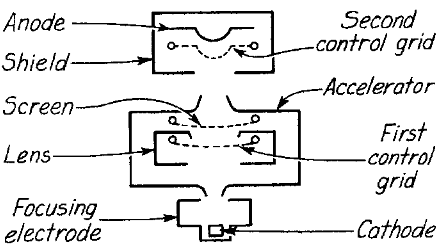


FIG. 45. Electrode structure of the gated-beam tube.

With 60 volts on the accelerator, the cathode current of the 6BN6 is about 5 ma, of which about 3 ma can reach the anode. Nearly full plate current is obtained with zero voltage on the first grid, which cuts off at a negative voltage slightly greater in magnitude than 2 volts. The knee and the cutoff voltage of the transfer characteristic of the second grid depend upon the anode voltage, since the second grid and the anode behave as a triode. Most of the electrons turned back by the first grid strike the inside of the accelerator, whereas those turned back by the second grid strike the outside of the accelerator. As both grid currents are relatively small and limited, being about 0.5 ma with 60 volts on the accelerator, there is no objection to driving the grids positive. The transconductance relating the grids and the anode is several thousand micromhos. Typical static transfer characteristics for the two grids are shown in Fig. 46. As in the pentode circuit of Fig. 44, the output of the gated-beam tube is superimposed upon

[1] R. Adler, *Electronics,* February, 1950, p. 82.

a pedestal unless the input consists of positive pulses and the grid to which the signal is applied is biased to cutoff.

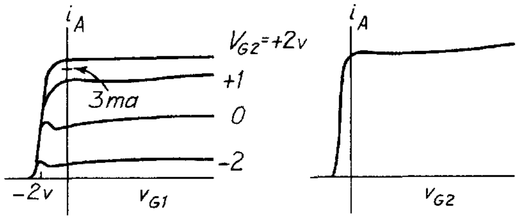


FIG. 46. Static transfer characteristics of the gated-beam tube.

12.2. Beam-Deflection Tube. Another type of beam tube that may be used as a gate is the beam-deflection tube, shown in basic form in Fig. 47. In this tube electrons emitted by the cathode are focused by grid 1 and accelerated by slotted grids 2, 3, and 4, grids 3 and 4 being connected internally in order to provide an equipotential shield around the deflection electrodes. When there is no difference of potential between the deflection electrodes, the electron beam passes through a slot in grid 4, through the suppressor grid, and strikes the anode. If a difference of potential is applied between the two deflection plates, the beam is deflected and is intercepted in whole or in part by grid 4 (also called the interceptor), a difference of potential of approximately 15 volts sufficing to reduce the anode current to zero when normal operating voltages are applied to the other electrodes. Signal voltage may be applied to either grid 1 or grid 2. Transfer characteristics relating the anode and grid 2 when the voltage of grid 1 is constant, and those relating the anode and one deflection electrode when the voltage of the other deflection electrode is constant, are shown in Fig. 48.

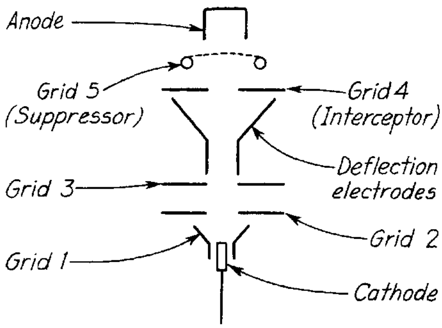


FIG. 47. Electrode structure of the beam-deflection tube.

Although the beam-deflection tube can be used in the same way as other gates by applying the signal voltage to grid 1 or grid 2, using an initial deflection of the beam to cut off the anode current, and centering the beam by means of a control voltage applied to either or both deflection electrodes, this tube is particularly useful in sampling the instantaneous value of the signal voltage by sweeping the beam rapidly across the slot in the interceptor.[2] Very short gating time is obtained by the use of a control pulse of small rise time and large amplitude. The amplitude of the output pulse is essentially independent of the amplitude of the control pulse, but depends upon the control-grid voltage. The rise time of the output pulse depends upon the rate of rise of the control pulse and is independent of the control-pulse amplitude, provided the amplitude exceeds that required to sweep the beam across the interceptor slot. Response to the signal voltage is linear over about 15 volts when the first grid is used as the control grid and 100 volts

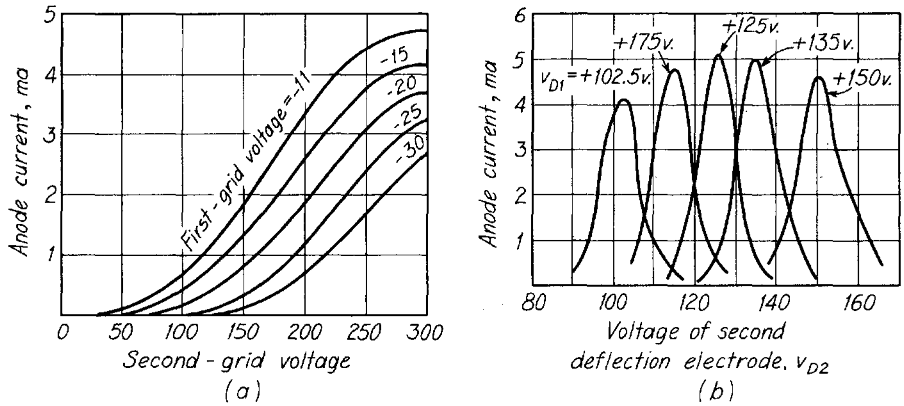


FIG. 48. Transfer characteristics of the beam-deflection tube: (a) second grid to anode with $v_{D1} = 0$, $v_{D2} = 120$ v, $v_{G3} = v_{G4} = 380$ v, and $v_A = 300$ v; (b) second deflection electrode to anode with $v_{G1} = -11$ v, $v_{G2} = v_A = 300$ v, and $v_{G3} = v_{G4} = 380$ v.

when the second grid is used. On the other hand, the first grid has higher transconductance and produces less defocusing than the second grid. The highest deflection sensitivity is obtained for an average deflection-electrode voltage of 125. Double gating may be avoided by the application of a delayed blanking pulse to the first grid in order to cut off the anode current during the return of the beam. Sperling and Tackett reported an output-pulse rise time of less than 7 μsec, independent of anode load resistance, and a maximum switching rate of 5 Mc. Feed-through of signal voltage when the gate was off was found to be approximately 0.12 percent.

12.3. Secondary-Emission Beam Tube. When the beam-deflection tube of Fig. 47 is used to gate alternating signals impressed upon one or both deflection electrodes, the beam is centered in the absence of signal voltage. Consequently, output is obtained when control voltage is applied, even in the absence of signal voltage, and the output is superimposed upon a pedestal. The output is also superimposed upon a pedestal when the signal is impressed upon one of the grids and the control voltage is impressed upon

[2] L. Sperling and R. W. Tackett, *Trans. I.R.E.*, Vol. ED-4, No. 1, p. 59 (January, 1957).

the deflection electrodes. This difficulty is avoided in the anode-dynode beam-deflection tube shown in Fig. 49, in which the output electrode consists of the combination of a normal anode and a dynode.[3]

Secondary electrons emitted from the dynode surface in the tube of Fig. 49 are collected by the collector, and the resulting output current contributed by the dynode is the difference between the primary beam current to the dynode and the secondary-emission current. The suppressor prevents secondary electrons from leaving the anode. When the beam is centered, half

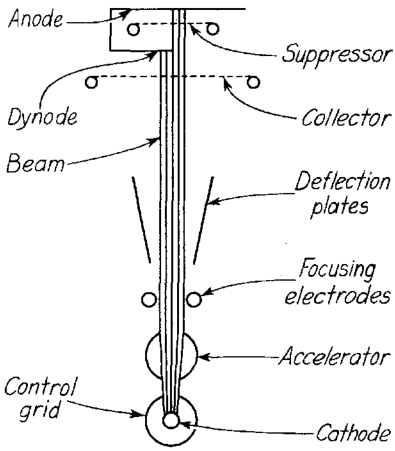


FIG. 49. Electrode structure of the anode-dynode beam-deflection tube.

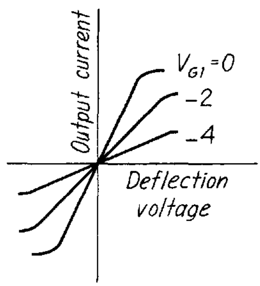


FIG. 50. Transfer characteristics of the anode-dynode beam-deflection tube.

of the beam electrons terminate upon the dynode surface, and the remainder upon the anode surface. If the ratio of secondary electrons emitted by the dynode to primary electrons striking it is 2, the output current is zero when the beam is centered. When the entire beam strikes the anode, the output current is equal to the beam current; when the entire beam strikes the dynode, the output current is equal to minus the beam current. The transfer characteristic passes through the origin and is symmetrical, as shown in Fig. 50. If the signal input voltage is impressed between the deflection electrodes and the beam is centered in the absence of signal voltage, no output is obtained in the absence of signal input voltage, regardless of the control voltage impressed upon the control grid. Therefore the output is not superimposed upon a pedestal.

The deflection transconductance obtainable in the anode-dynode beam-

[3] H. J. Walkstein and A. W. Kaiser, *Electronics*, August, 1955, p. 132.

deflection tube is twice as great as that obtainable when the dynode is replaced by an interceptor, and the transfer characteristic relating the output electrode with the deflection electrodes is less nonlinear and has the advantage of symmetry, as can be seen by comparison of Figs. 48 and 50. Because of the linearity and symmetry of the transfer characteristics, the tube makes an excellent differential amplifier, the two inputs being applied to the two deflection electrodes. It is evident from the form of the transfer characteristics that the tube can also be used as a symmetrical clipper, although the greater cost and complexity of the tube and circuit more than offset any possible advantage over diode or cathode-coupled clippers. The deflection-electrode transconductance of the tube of Fig. 49 is 1200 μmhos.

13. Transistor Switches[1]

13.1. Complete Collector Characteristic Curves. In order to understand fully the behavior of transistor gates, it is necessary to examine the collector characteristics of a junction transistor in both positive and negative ranges of collector and base voltages. Figure 51a shows typical characteristics of an *n-p-n* alloyed-junction transistor in the common-emitter connection. The assumed positive polarities of voltages and direction of collector current are indicated in Fig. 51b. The symbols on the various branches of the curves indicate whether the junctions 1 and 2 are biased in the forward direction (so as to cause the flow of electrons from the *n*-type material into the *p*-type material or holes from the *p*-type material into the *n*-type material) or in the reverse direction (so as to tend to prevent the flow of electrons from the *n*-type material or holes from the *p*-type material). Thus the symbol $1F$, $2R$ indicates that junction 1 is biased in the forward direction and junction 2 in the reverse direction. It can be seen from Fig. 51b that the *n-p-n* transistor is in its low-current, or off state only if both junctions are reverse-biased; i.e., if the base is negative relative to both the emitter and the collector.

The portions of the characteristics in the first quadrant, for which junction 1 (the emitter-base junction) is forward-biased and junction 2 (the base-collector junction) is reverse-biased, are those normally used in the application of the transistor as a common-emitter amplifier. In the third quadrant, however, junction 1 is reverse-biased and junction 2 is forward-biased. This means that the electrode which normally functions as the emitter now functions as the collector, and vice versa, as shown in Fig. 51c. The principal cause of the difference in the shapes of the characteristics in

[1] R. L. Bright, *Comm. and Electr.*, **17**, 111 (March, 1955); G. C. Sziklai, R. D. Lohman, and G. B. Herzog, *Proc. I.R.E.*, **41**, 708 (June, 1953); D. E. Deuitch, IRE-AIEE Conference on Transistor Circuits, 1954; T. A. Prugh, *Electronics*, January, 1955, p. 169; R. L. Bright and A. P. Kruper, *Electronics*, April, 1955, p. 135.

the first and third quadrants is that for the former the voltage of the base relative to the emitter does not vary as the collector-emitter voltage is varied, whereas for the latter the voltage of the base relative to the acting emitter

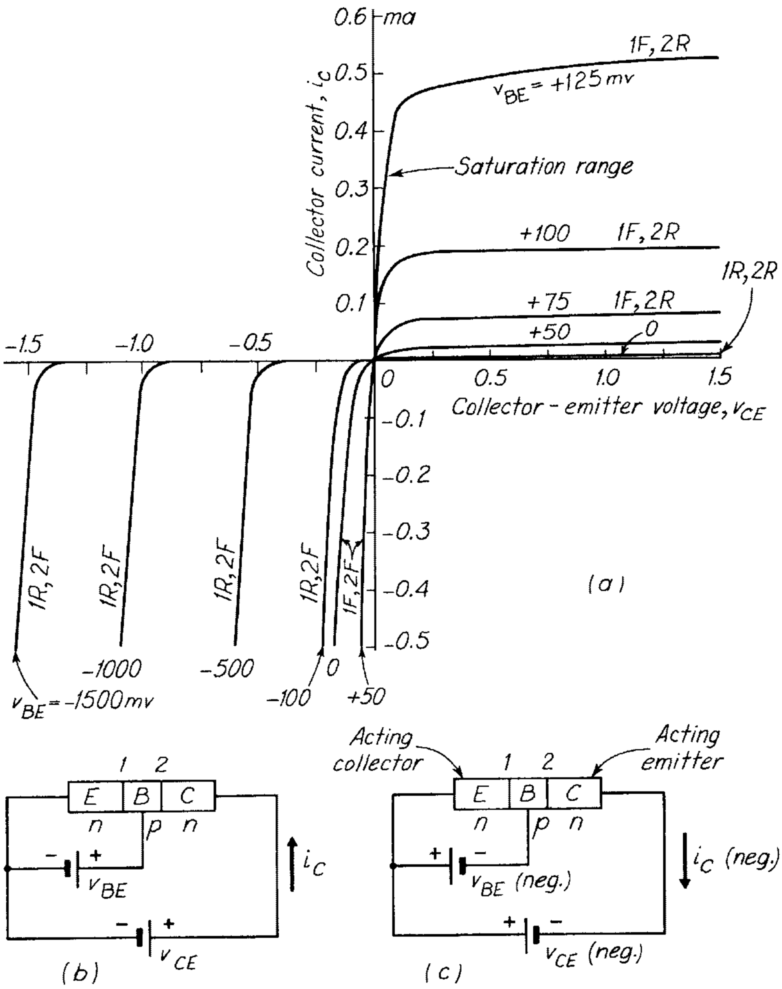


FIG. 51. (a) Collector characteristics of an *n-p-n* alloyed-junction transistor in the common-emitter connection; (b) voltage polarities in the first quadrant; (c) voltage polarities in the third quadrant.

(the collector) varies with collector-emitter voltage. This can be seen by comparison of Figs. 51b and 51c. As the magnitude of v_{CE} is increased in the third quadrant, the forward bias of the acting base-emitter junction also increases, causing a rapid increase of current across this junction. Although the common-emitter current amplification of the transistor in this inverted operation is usually considerably lower than in the normal connection, its magnitude is usually at least unity. Consequently, when junction 2 becomes forward-biased (i.e., when the magnitude of v_{CE} exceeds that of v_{BE}), the magnitude of the collector-emitter current rises rapidly with the magnitude of v_{CE}, and the third-quadrant characteristics are steep to the left of the break points, which correspond to equal magnitudes of negative v_{BE} and v_{CE}.

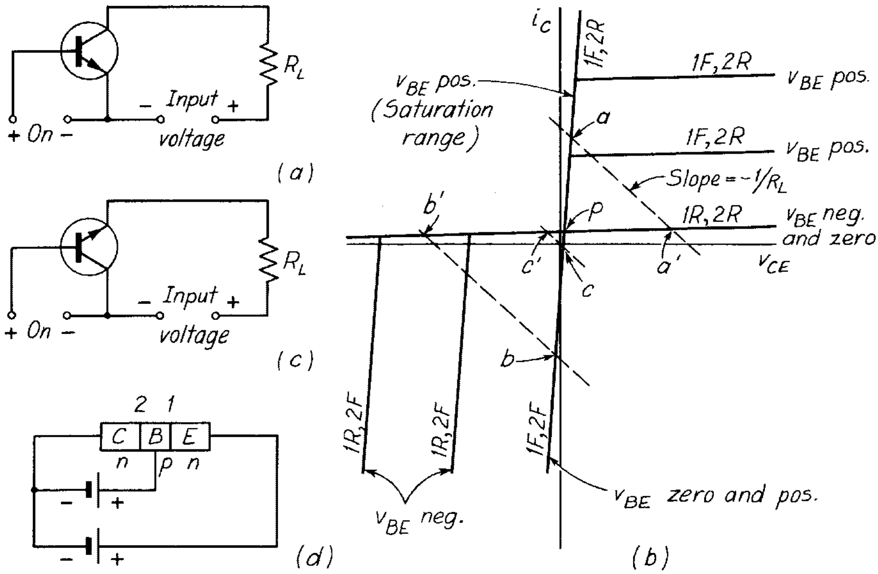


FIG. 52. (a) Common-emitter transistor switch; (b) idealized common-emitter characteristics; (c) common-collector transistor switch; (d) equivalent of the common-collector switch.

The characteristics of *p-n-p* transistors are similar to those of *n-p-n* transistors, but portions of the characteristics that appear in the first quadrant for an *n-p-n* transistor appear in the third quadrant for a *p-n-p* transistor, and vice versa. The collector current of a *p-n-p* transistor is cut off only if the base is positive relative to both the emitter and the collector.

13.2. Common-Emitter Switch. Figure 52a shows the manner in which the transistor may be used in the common-emitter connection as a switch to control current through a load resistance R_L. Load lines corresponding to the load resistance R_L and to positive, zero, and negative values of applied collector voltage are shown in Fig. 52b, in which the transistor characteristics have been idealized. When the applied collector voltage is positive, the application of a relatively small base voltage v_{BE}

(of the order of tenths of volts) carries the operating point to point a in the saturation range, where the load current is large, and the voltage drop through the transistor relatively small. The load current can be reduced to the low value at point a' by reducing v_{BE} to zero or to a small negative value. When the applied collector voltage is negative, the current and voltage correspond to point b if the base voltage v_{BE} is zero or positive. In order to reduce the current magnitude to the small value corresponding to point b', not only must the base voltage v_{BE} be negative, but its magnitude must exceed that of the applied collector voltage in order to bring the break in the characteristic to the left of the intersection of the characteristic with the load line. If the applied collector voltage is always positive, the load current can be turned on and off by means of a small change in base voltage. If the applied collector voltage is negative or alternating, however, the control voltage must be varied between a relatively small positive value and a negative value greater in magnitude than the applied collector voltage in order to turn the load current on and off.

It follows from the foregoing discussion that the transistor resembles a switch that can be opened and closed by means of a change of base voltage. The analogy would be complete if the saturation branch of the characteristic (the "on" characteristic) were coincident with the current axis and if the $1R$-$2R$ characteristic (the "off" characteristic) were coincident with the voltage axis. Failure of the characteristics to satisfy this desirable condition causes the transistor to be analogous to an ideal switch in series with a small voltage and a low resistance, and in shunt with a constant current source and a small conductance. The switch is also shunted by the collector capacitance. The equivalent series voltage source and the shunt current source are of importance only in low-level operation (small applied collector voltage).

13.3. Common-Collector Switch. Transistor switches are also used in the common-collector connection, shown in Fig. 52c. For zero load resistance the circuit of Fig. 52c is equivalent to that obtained by inverting the transistor in Fig. 51b, as in Fig. 52d. If the emitter current i_C is plotted against the emitter-collector voltage v_{EC} with the base-collector voltage v_{BC} as parameter, the characteristics for this inverted connection are of the same general form as those of Fig. 52b for the normal connection. However, differences in the size and other physical characteristics of the two junctions of most transistors cause the current amplification factors and other properties to differ in the normal and inverted connections. As the result of these differences, the intersection at point P is much closer to the origin, and the angle between the saturation characteristics and the current axis and that between the $1R$-$2R$ characteristic and the voltage axis are smaller in the inverted or common-collector connection. The characteristics in the third quadrant (first quadrant for *p-n-p* transistors) are steeper, and those in the first quadrant (third quadrant for *p-n-p* transistors) are more nearly horizontal and closer together for the same increments of base voltage than in the common-emitter connection. For germanium transistors the voltage corresponding to the intersection at P is of the order of 10 to 40 mv in the common-emitter connection, but only 1 to 4 mv in the common-collector connection. The voltage at P is larger in silicon transistors than in germanium transistors.

The closer proximity of the intersection P to the origin, the greater slope of the $1F$-$2R$ characteristic, and the smaller slope of the $1R$-$2R$ characteristic cause the behavior of the transistor to approximate much more closely that of an ideal switch in the common-collector connection than in the common-emitter connection. However, the closer spacing of the first-quadrant characteristics (third-quadrant for *p-n-p* transistors) in the common-collector connection causes this important advantage of the common-collector connection to be accompanied by an increase in the magnitude of the base voltage required to ensure that the load line intersects the characteristics in the saturation range.

13.4. Power Dissipation in Transistor Switches. Because the collector voltage is low when the transistor is biased fully on (into the saturation range) and the collector current is small when it is biased off, the collector dissipation is low in both the on and the off states of operation of a transistor switch. During switching, however, the operating point moves along the load line, and the instantaneous dissipation may be high. The power that can be controlled by a transistor in a switching circuit is large in comparison with that which may be controlled by the same transistor in an amplifier circuit, provided that the control voltage applied to the base is a close approximation to a step function. The problem of dissipation in switching transistors has been treated in detail in the literature.[2]

13.5. Two-Transistor Circuits. Greater symmetry in a-c operation is obtained if two transistors are connected in parallel, as shown in Fig. 53a.[3] In this circuit one transistor is used in the common-emitter connection, and the other in the common-collector connection. In order to avoid excessive collector dissipation, the base voltage used to turn the transistor on must be sufficient to drive both transistors into the saturation region, and the magnitude of the base voltage used to turn the transistors off must exceed the amplitude of the applied alternating collector voltage. The base circuit resistances must be used in order to ensure that the base current will not be excessive if the on and off amplitudes of applied control voltage are made equal because of convenience or necessity.

[2] R. L. Bright, *Comm. and Electr.*, **17**, 111 (March, 1955).
[3] Sziklai, Lohman, and Herzog, *loc. cit.*

The magnitude of the control voltage required to turn the load current off can be greatly reduced by using two transistors in series, back-to-back, as shown in Fig. 53b.[4] In this circuit the collector-emitter voltage of one transistor is negative when that of the other is positive. Consequently, one transistor is operating in the third quadrant of the characteristic diagram while the other is operating in the first quadrant. Although a large negative (for *n-p-n* transistors) base voltage would be required in order to cut off the transistor that is operating in the third quadrant, only a small negative voltage is sufficient to cut off the one that is operating in the first quadrant. Since the load current must pass through both transistors, the load current can be cut off by means of a small negative base voltage regardless of the

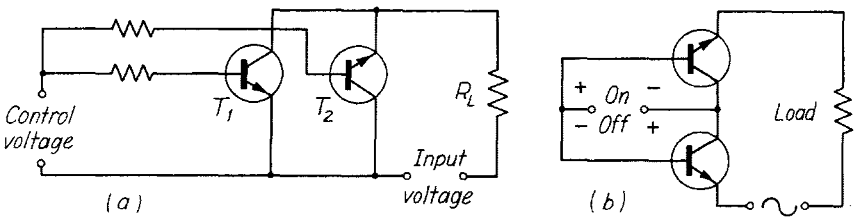


FIG. 53. (a) Parallel two-transistor switch; (b) series two-transistor switch.

polarity of the instantaneous voltage applied to the collector-emitter circuit. As in the single-transistor and parallel-transistor circuits, the positive base voltage required to turn on the load current in the circuit of Fig. 53b is only that required to drive either transistor into the saturation current range at the instant when the emitter voltage of that transistor has its maximum positive value. A disadvantage of the circuit of Fig. 53b is that the control voltage and the signal voltage sources cannot both be grounded unless one or the other is applied through a transformer.

13.6. Compensation for Unbalance. It can be seen from Fig. 52b that the load current is not in general zero when the impressed collector voltage is zero and that there is a change in the small collector current, and hence in the voltage across the load, when the control voltage is changed from its on value to its off value. This effect is much smaller in the common-collector circuit than in the common-emitter circuit. Because of differences in the common-emitter and common-collector characteristics, it is not avoided in the parallel two-transistor circuit of Fig. 53a. In the series circuit of Fig. 53b, the voltages across the two transistors are of opposite polarity when the applied load-circuit voltage is zero and balance out if the two transistors have like characteristics. Small differences between the two transistors can be compensated by means of a balancing voltage-divider,

[4] Bright, *loc. cit.;* Bright and Kruper, *loc. cit.*

as in the circuit of Fig. 54. Change of load voltage with control voltage is also avoided in the circuit of Fig. 55. When the input voltage is zero, the load resistance is shunted by the two transistors in parallel. Because the base voltage of one transistor is positive when that of the other is negative, and vice versa, the load current remains constant if the transistors have like characteristics. Transistor T_2 has negligible effect upon the load current when input voltage is applied, being cut off when T_1 conducts.

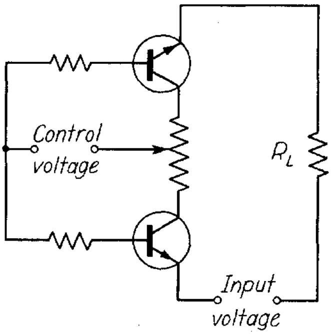


FIG. 54. Balanced series two-transistor switch.

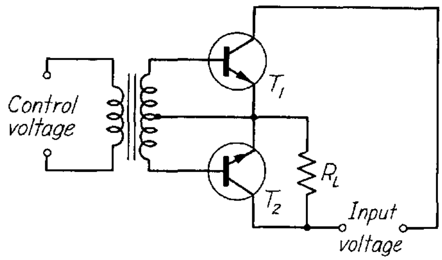


FIG. 55. Circuit in which the control voltage does not produce alternating voltage across the load in the absence of input voltage.

13.7. Modified Circuits. Although *p-n-p* transistors have been shown in the circuits of Figs. 51 to 55, *n-p-n* transistors can also be used, the polarities of all voltages being reversed.

If the output voltage is taken from across the transistors, the transistor circuits of Figs. 52 to 54 serve the same function as the diode voltage gates of Figs. 37a and 39.[5] When the control voltage and current are sufficient to drive the base into the saturation range of collector current, the resistance of a transistor is much lower than that of a diode. Consequently, a relatively low series resistance R may be used in the gate. Because of the low values of the transistor and series resistances, circuit capacitances produce little reduction of output voltage when the circuit is on, or leakage when the circuit is off. Balanced transistor circuits capable of operating on signal voltage of both polarities are in general simpler than the equivalent diode circuits.

13.8. Transient Effects in Transistor Switches. Two types of transient effects are observed in transistor switching circuits. One type of transient

[5] D. E. Deuitch, *IRE-AIEE* Conference on Transistor Circuits, Philadelphia, 1954; T. A. Prugh, *Electronics,* January, 1955, p. 168; D. E. Deuitch, *Electronics,* May, 1956, p. 160.

results from the storage of minority carriers when the collector current is in the saturation range.[6] If the collector current is initially in the saturation range and the voltage of the base relative to the emitter is abruptly reversed, the potential of the collector relative to the emitter remains unchanged at its initial low value for a short time, suddenly drops to approximately the base potential, and then rises exponentially to its final value, approximately the applied collector voltage. If the collector current is initially cut off and forward bias is applied abruptly to the base, the collector-emitter voltage decays smoothly to its final value. The time constants of the transients are of the order of 1 to 5 μsec. The second type of transient is the result of charging of the collector capacitance through the load resistance when the transistor is cut off. The effect of this type of transient is minimized in the circuit of Fig. 55, since the collector capacitance of each transistor charges through the low resistance of the other transistor.

13.9. Use of Transistor Switches as Choppers. Transistor switching circuits may be advantageously used in the "chopping" of direct voltages in order that they may be amplified by high-gain a-c amplifiers, rather than

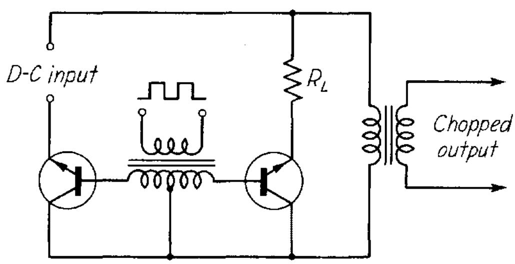


FIG. 56. Transistor chopper circuit.

by high-gain d-c amplifiers, which are much more difficult to stabilize against changes of supply voltage, temperature, and circuit parameters[7] (see also Sec. 5). An example of a transistor chopping circuit is shown in Fig. 56.

Figure 57 shows three circuits in which a transistor amplifier is switched on and off by means of a control voltage impressed upon the base of one or two switching transistors. In circuit (a), a positive voltage applied to the base of T_2 causes T_2 to conduct and thus short-circuit the input to T_1. In circuit (b) the output of transistor T_1 is short-circuited when a positive

[6] J. L. Moll, *Proc. I.R.E.*, **42,** 1773 (December, 1954).

[7] A. P. Kruper, *Comm. and Electr.*, **17,** 141 (March, 1955); R. L. Bright and A. P. Kruper, *Electronics,* April, 1955, p. 135; J. P. Reynolds, *Raytheon Tech. Inf. Bull.,* "Practical Considerations for the Application of Junction Transistors in Chopper Circuits," September, 1957.

voltage on the base of T_2 causes T_2 to conduct. An undesirable feature of these circuits is that the initiation of conduction of T_2 changes the direct collector voltage of T_1 and therefore causes a change in amplifier output voltage. This effect is avoided in the circuit of Fig. 57c, which combines the circuit of Fig. 53a with a single-stage amplifier.

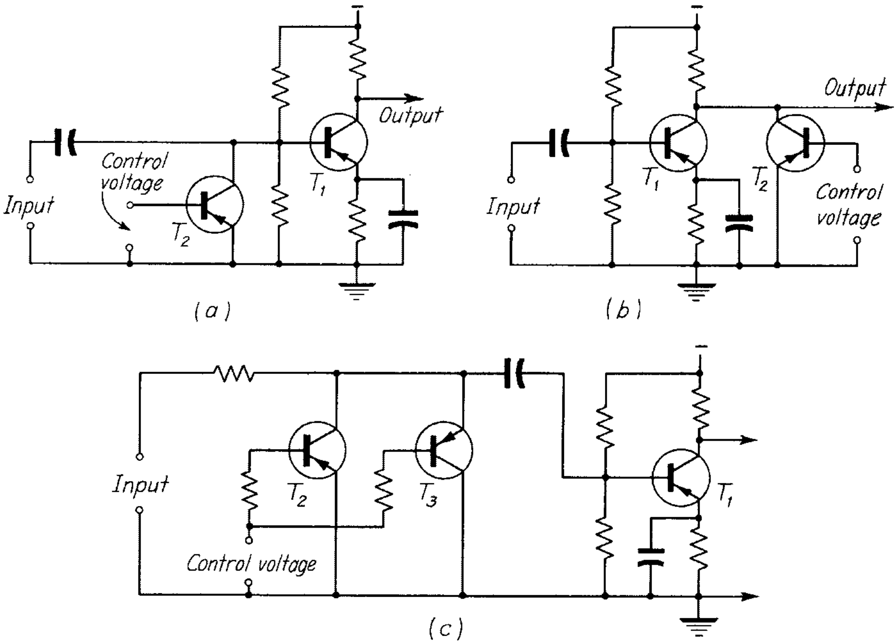


FIG. 57. Control of amplifiers by means of switching transistors.

14. Or Circuits, And (Coincidence) Circuits, and Not (Anticoincidence) Circuits

14.1. Or Circuits. Gate circuits can be readily modified so that output is obtained when any one of two or more control voltages is applied to the circuit, or when two or more of a number of control voltages are applied simultaneously. This is accomplished by connecting the control-voltage sources to the control terminals of the basic circuits through series isolating diodes that prevent interaction between the voltage sources and loading of each source by the admittances of the other sources. Two typical circuits, formed from the basic circuits of Figs. 37a and 53b, are shown in Fig. 58.

14.2. And Circuits. The basic gate circuits are in themselves *and* or *coincidence* circuits, since they deliver output voltage only when the signal voltage and the control voltage are impressed simultaneously. Some of the basic circuits may be readily modified so that signal output is obtained only in the presence of two or more control voltages, impressed simultaneously. This may be accomplished in the circuits of Fig. 37 by shunting the control diode T_1 and resistor R_2 with one or more additional diodes and resistors, as shown

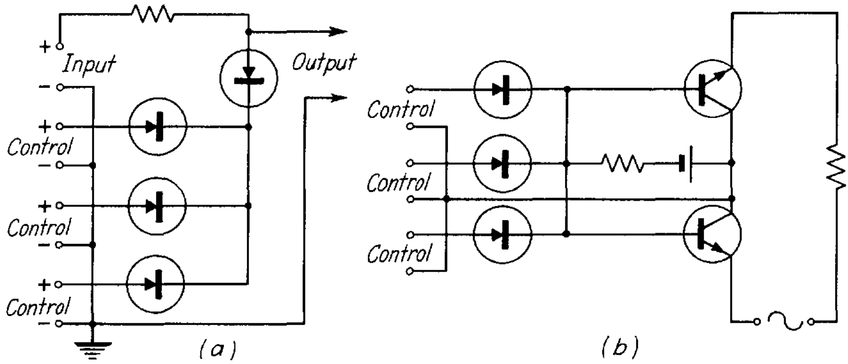


FIG. 58. (a) Diode "or" circuit; (b) diode-transistor "or" circuit.

in Fig. 59. The full-wave circuit of Fig. 39 and the transistor circuits of Fig. 52 to 55 may be modified in a similar manner. The cathode-follower circuit of Fig. 43 may be provided with additional shunt triodes, to which additional negative control pulses must be applied in order to cause the circuit to deliver signal output.

The pentode circuit of Fig. 44 can be used as a coincidence circuit by using zero operating screen voltage and applying a second positive control voltage

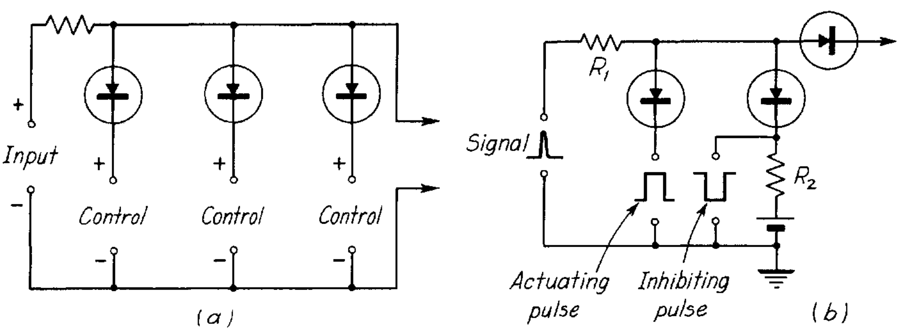


FIG. 59. Diode "and" (coincidence) circuit.

to the screen grid. However, since the control voltage applied to the screen causes a large change of plate current, this circuit has a relatively large output pedestal. In a similar manner, the beam-deflection tube of Fig. 47 may be controlled by voltages applied simultaneously to grids 1 and 2.

Figure 60 shows a coincidence circuit that consists essentially of a differential amplifier and two pulse stretchers.[1] A negative pulse applied to input terminals 1 charges capacitors C_1 and C_2 to equal voltages through diodes D_1 and D_2. If the time constants R_1C_1 and R_2C_2 are equal, the capacitors discharge at the same rate. The tap on R_1 may then be adjusted so that the effects of the voltages applied to the grids of the differential amplifier balance out, and no output is obtained from the plate of T_2. If negative pulses are applied simultaneously to input terminals 1 and 2, C_2 charges to a lower voltage than C_1, the magnitude of the negative grid

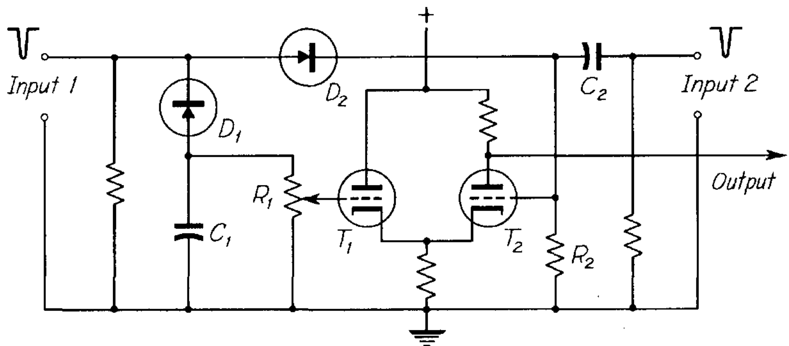


FIG. 60. Coincidence circuit.

voltage of T_1 exceeds that of T_2, and the voltage of the plate of T_2 is lowered throughout the discharge time of the capacitors. On the other hand, if a positive pulse is applied to input terminals 2 at the same time that a negative pulse is applied to input terminals 1, C_2 charges to a higher voltage than C_1, the grid voltage of T_2 is more negative than that of T_1, and the plate voltage of T_2 is raised throughout the discharge time of the capacitors. This circuit has high sensitivity and is capable of resolving pulses separated by an interval of only 3×10^{-10} second.

14.3. Not Circuits. The basic gate circuits may also be modified or combined in such a manner as to form anticoincidence circuits in which output is obtained when control voltage is applied to one control terminal, but not when another control voltage is applied simultaneously to another control terminal. This can be accomplished, for example, by combining the circuit of Fig. 37a with that of Fig. 38, as shown in Fig. 59b. In a similar manner, anticoincidence circuits may be formed from the full-wave diode circuit of Fig. 39, and the transistor circuits of Figs. 52 to 55.

In the cathode-follower circuit of Fig. 43, another, negatively biased, triode may be used in parallel with T_1 and T_2. If the negative bias in suf-

[1] Z. Bay, *Rev. Sci. Instr.*, **22**, 397 (June, 1951).

ficient to bias the additional tube beyond cutoff, this tube normally has no effect. When a positive inhibiting pulse is applied to its grid, however, the auxiliary tube conducts and thus prevents the appearance of output voltage. The pentode circuit of Fig. 44 acts as an anticoincidence circuit if the screen grid is positively biased through a resistor and a negative inhibiting voltage is applied to the screen grid. The beam-deflection tube of Fig. 47 may be used in a similar manner, control voltages of opposite polarity being applied to grids 1 and 2, which are suitably biased through resistors.

The principles discussed in this section may be extended to the design of combination coincidence-anticoincidence circuits that deliver output only in the presence or absence of various control voltages (see Probs. 14.3–1 to 14.3–3).

15. Negative-Resistance Circuits

15.1. Definition of Negative Resistance. Several types of circuits to be discussed in later sections, including circuits that have two or more stable states of equilibrium, relaxation oscillators, certain types of pulse generators, and certain types of sine-wave oscillators, are based upon tube and transistor circuits that provide a negative resistance over a portion of their operating range. It is desirable, therefore, to discuss at this point the general theory of negative-resistance production, and some of the tube and transistor negative-resistance circuits.

If the variational resistance of a circuit element is defined as the ratio of the fundamental component of the alternating voltage across the element to the fundamental component of the alternating current through it, the small-signal variational resistance of a circuit element at any operating point is to a first approximation equal to the reciprocal of the slope of the static current-voltage characteristic of the element at that operating point. Throughout any range in which the slope of the characteristic is negative, therefore, the variational resistance is negative. A negative value of variational resistance at any port of a circuit implies that an increment of current flows in opposition to the increment of voltage that produces it (or that a negative increment of voltage is produced across the port as the result of a positive increment of current into it), and that the circuit therefore contains a source of power. Typical current-voltage characteristics having a range in which the variational resistance is negative are shown in Fig. 61. The negative-slope ranges of such characteristics may have any position relative to the origin. Examples of such curves for a number of circuits and devices are shown in Figs. 66, 69, 72, 76, 77, 78, 79, and 80.

The majority of tube and transistor negative-resistance circuits are special forms of two basic feedback circuits. Rather than explain in detail the operation of the various individual circuits, it is advantageous to analyze the basic circuits. Before this is done, however, a particular form of each of these circuits will be discussed briefly in order to indicate the mechanism by means of which negative-resistance characteristics, such as those of Fig. 61, are produced.

15.2. Example of a Negative-Resistance Circuit. Figure 62 shows a circuit by means of which a current-voltage characteristic of the form of Fig. 61a may be produced. To simplify the explanation, the assumption will be made that the voltage amplifier of Fig. 62 is nonreciprocal, contains no reactive elements, and has negligible input admittance. Under these assumptions, the current i is the current through the resistance R and is

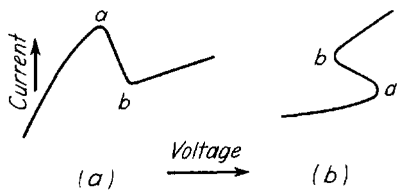


FIG. 61. (a) Voltage-stable current-voltage characteristic; (b) current-stable characteristic.

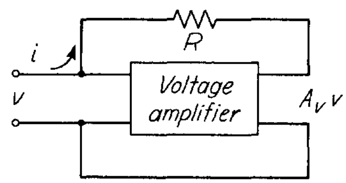


FIG. 62. One circuit that has a current-voltage characteristic of the form of Fig. 61a.

equal to $(v - A_v v)/R$, where A_v is the voltage amplification of the amplifier. The input resistance is therefore $R/(1 - A_v)$, which is negative if the amplifier is noninverting (i.e., if the output voltage of the amplifier is in phase with the input voltage), and if the magnitude of the voltage amplification A_v exceeds unity. In any voltage and frequency range in which these conditions are satisfied, a positive increment of voltage Δv is accompanied by a negative increment of current Δi. Inasmuch as the voltage range in which any practical amplifier can produce a voltage amplification greater than unity is limited at both ends by cutoff or saturation of the active element or elements, the range over which the current-voltage characteristic can have a negative slope is also limited. As the voltage v approaches a value at which one or more of the active elements is cut off or saturated, the magnitude of the amplification falls and eventually passes through unity. When $A_v = 1$, the slope of the current-voltage curve is zero. Beyond the value at which A_v is unity, an increment of v is accompanied by an increment of i of the same sign, and the slope of the current-voltage characteristic is positive. The entire current-voltage characteristic must therefore be of the general form of the curve of Fig. 61a.

It is apparent that the input resistance of the circuit of Fig. 62 is negative because an increment of input voltage is accompanied by a much larger increment of amplifier output voltage, which opposes the voltage v and thus causes current to flow through R in opposition to v. The power delivered to the source of the input voltage v must be derived from the amplifier power supply. Because a current-voltage characteristic of the form of Fig. 61a is single-valued with respect to voltage, any circuit port at which such a characteristic is observed is said to be *voltage-stable.* It will be shown in Sec. 23 that the current into a port having such a characteristic may in general have more than one value for a single value of source voltage if there is resistance in series with the source, but that there can be only one value of current if the external resistance across the port is zero. For this reason a port that has a characteristic of the form of Fig. 61a is also said to be *short-circuit stable.*

15.3. Another Example of a Negative-Resistance Circuit. Figure 63 shows an example of one circuit by means of which a current-voltage characteristic of the general form of Fig. 61b may be produced. Again, to simplify the analysis, the assumption will be made that the current amplifier of Fig. 63 is nonreciprocal and contains no reactive elements, and the input resistance will be assumed to be negligible. Under these assumptions the voltage v is the voltage across the resistance R and is equal to $(i + A_i i)R$, where A_i is the current amplification of the amplifier. The input resistance is $(1 + A_i)R$, which is negative if the current amplifier is phase-inverting (A_i is negative) and if the magnitude of A_i exceeds unity. In any current range in which these conditions are satisfied, a positive increment of input current i is accompanied by a negative increment of voltage v. As the current approaches a value at which one or more of the active elements of the amplifier is cut off or saturated, the amplification falls and the slope of the current-voltage characteristic rises, eventually passing through infinity when the magnitude of A_i is 1. Beyond the value of i at which $|A_i|$ is unity, an increment of current is accompanied by an increment of voltage of the same sign, and the slope of the characteristic is positive. The entire current-voltage characteristic must be of the general form of the curve of Fig. 61b. The negative input resistance in the circuit of Fig. 63 is the result of the fact that an increment of input current Δi is accompanied by a much larger increment of amplifier output current, which flows through R in the opposite direction to Δi and therefore produces a voltage *rise* across R with respect to Δi.

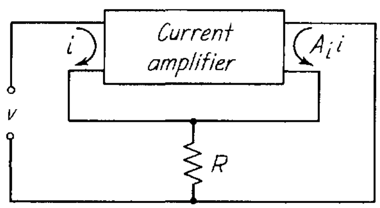


FIG. 63. One circuit that has a current-voltage characteristic of the form of Fig. 61b.

Because a current-voltage characteristic of the form of Fig. 61b is single-valued with respect to current, any circuit port at which such a characteristic is observed is said to be *current-stable.* It will be shown in Sec. 23 that, although the voltage across a port having such a characteristic may in general have more than one value for a single value of current into the port, there can be only one value of voltage if the external resistance shunting the port is infinite. For this reason a port that has a characteristic of the form of Fig. 61b is also said to be *open-circuit stable.*

The terms voltage-stable and current-stable, or short-circuit stable and open-circuit stable, will be applied to current-voltage characteristics of the forms shown in Fig. 61, as well as to the ports at which these characteristics are observed. However, since a circuit may in general have both voltage-stable and current-stable ports, these terms should not be applied to the circuit itself unless the circuit has only one port. (The tunnel diode and the glow tube are examples of one-port negative-resistance devices.)

The use of a circuit containing a voltage amplifier in the demonstration of the production of a voltage-stable characteristic, and of a circuit containing a current amplifier in the demonstration of the production of a current-stable characteristic, is a matter of convenience. It should not be interpreted as implying that voltage-stable characteristics are observed only in circuits that incorporate a voltage amplifier and current-stable characteristics only in circuits that incorporate a current amplifier. The circuits of Figs. 62 and 63, like most other negative-resistance circuits, have both voltage-stable and current-stable ports.

16. Some Characteristics of Voltage-Stable and Current-Stable Circuits

16.1. Effect of Gain Parameter Upon Current-Voltage Curves. The negative slope of the current-voltage characteristics of Fig. 61 in the range *a-b* is the result of amplification and positive feedback. The amplification may be produced by a single active element such as a tube or a transistor, or by two or more active elements and associated passive circuit elements. Whatever type of amplifier is used, the amplification, and therefore the slope of the characteristic in the range *a-b*, are dependent upon a gain parameter of the active element or elements, such as the transconductance of a tube or the current-amplification factor of a transistor. Figures 64a and 64b show the manner in which the negative-slope portion of the current-voltage characteristics of Fig. 61 changes as the gain parameter is changed in such a manner as to increase the amplification from zero. The following differences in the a-c behavior of voltage-stable and current-stable circuits operated at a point within the negative-resistance range of the current-voltage characteristics may be inferred from Fig. 64:

(1) As the gain parameter is gradually changed so as to increase the amplification from zero, the variational resistance of a voltage-stable element first increases with increases of amplification, passes through an in-

finite value, and then becomes negative. Within the negative-resistance range the magnitude of the variational resistance decreases with increase of amplification.

(2) As the gain parameter is gradually changed so as to increase the amplification from zero, the variational resistance of a current-stable element first decreases with increase of amplification, passes through zero, and then becomes negative. Within the negative-resistance range the magnitude of the variational resistance increases with increase of amplification.

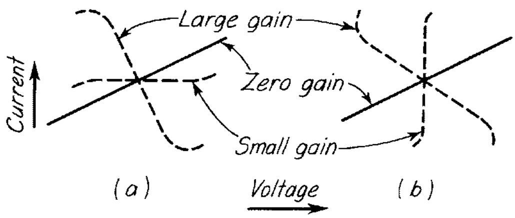


FIG. 64. Effect of change of gain upon the current-voltage characteristic of (a) a voltage-stable element and (b) a current-stable element.

16.2. Criteria for Voltage-Stable and Current-Stable Ports. The current-voltage characteristic for a voltage-stable port must have at least two points at which the slope is zero, corresponding to infinite input resistance of the port, but need have no points of infinite slope and zero resistance. It follows from this fact, or from (1), that (a) any port is voltage-stable if the algebraic expression for the input resistance of that port in terms of the parameters of the circuit and the amplifying device is of such form that there is a value of gain parameter at which the resistance given by the expression is infinite. Similar reasoning leads to the conclusion that (b) a port is current-stable if the expression for the input resistance of that port in terms of the circuit and amplifier parameters is of such form that there is a value of gain parameter at which the resistance given by the expression is zero.

Examination of Fig. 61a shows that if a constant-current source is connected to a port having a voltage-stable characteristic, i.e., if a voltage source is connected to the port through a high resistance, there may in general be three equilibrium values of voltage across the port for a single value of current into the port.* Since the various currents and voltages in the entire circuit are interdependent, there will in general then also be three corresponding equilibrium values of currents and voltages at other ports. If the source resistance shunting the first port is reduced to zero, however, there can be only one combination of port voltage and current for each value of supply voltage. Consequently, there can then be only

* See also Fig. 82. This behavior will be discussed in detail in Sec. 23.2.

one corresponding combination of current and voltage *at every other port.* (Otherwise, for each combination of current and voltage at some other port there would be a different combination of current and voltage at the first port.) Because the negative-slope portion of a current-voltage characteristic that makes possible multiple equilibrium combinations of current and voltage at the port at which it is observed is the result of amplification and positive feedback, it follows that short-circuiting a voltage-stable-port, or reducing the resistance shunting such a port, must reduce the amplification or the positive feedback in the circuit. A similar argument leads to the conclusion that open-circuiting a current-stable port, or increasing the resistance shunting such a port, must reduce the amplification or the positive feedback.

The following criteria follow from the foregoing analysis: (c) A port is voltage-stable (short-circuit stable) if short-circuiting that port, or reducing the resistance shunting it, reduces the amplification of the amplifier or the positive feedback from the output to the input of the amplifier. (d) A port is current-stable (open-circuit stable) if open-circuiting that port, or increasing the resistance shunting it, reduces the amplification of the amplifier or the positive feedback from the output to the input of the amplifier.

The foregoing criteria, which are verified by application to actual circuits, may be conveniently used to determine by inspection of an unfamiliar circuit, or of expressions for the input resistance at various ports, whether the ports are voltage-stable or current-stable (Prob. 16.2–1). In applying criteria (c) and (d), it is usually simpler to check for the feedback of voltage than for the feedback of current. Thus, short-circuiting the input port of the circuit of Fig. 62 prevents the feedback of any voltage from the output of the amplifier to the input, and criterion (c) indicates that the port is voltage-stable, in agreement with Fig. 61a. On the other hand, opening the input port of the circuit of Fig. 63 prevents the application of output voltage of the amplifier to the input, and criterion (d) indicates that the port is current-stable, in agreement with the characteristic of Fig. 61b.

Criteria (c) and (d) may give ambiguous results in circuits that have more than a single feedback path. Thus, a port may be established in the circuit of Fig. 65b between the upper input and output terminals of the amplifier. Increasing the resistance shunting this port reduces the direct feedback from the upper output terminal of the amplifier to the upper input terminal. Criterion (b) therefore leads to the conclusion that this port is open-circuit stable. However, even when the port is open, there is still feedback by way of the path in parallel with R_{12}, and this feedback may be great enough so that the circuit is unstable. One must conclude, therefore, that the port may be open-circuit unstable. The algebraic expression for the input resistance at this port is of such form that it may be either infinite or zero (Prob. 16.2–2). Criteria (a) and (b) thus indicate that the port may be either voltage-stable or current-stable, depending upon the amplifier and circuit parameters.

It should be noted that one or more ports that are of interest may lie within the amplifier. Thus, in the circuit of Fig. 68a a port may be established between the plate of T_1 and the grid of T_2. Since opening this port (necessary changes in the grid bias of T_2 being made) decreases the amplification of the amplifier, the port must be current-stable. Similarly, short-circuiting a port established between the grid of T_2 and ground reduces the amplification to zero. Criterion (c) indicates that this port is voltage-stable.

17. The Basic Negative-Resistance Circuits

17.1. Pi and Tee Circuits. Most vacuum-tube and transistor negative-resistance circuits are special forms of the generalized pi and tee feedback circuits of Fig. 65. Thus, the circuit of Fig. 62 is a form of the circuit of

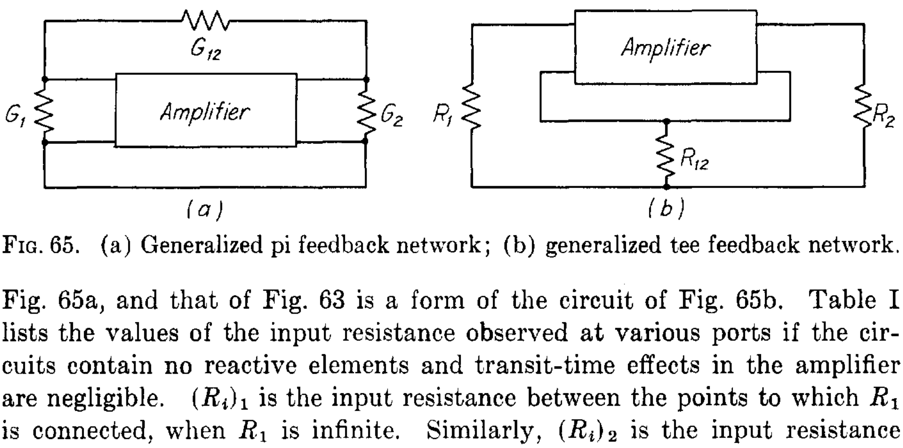


FIG. 65. (a) Generalized pi feedback network; (b) generalized tee feedback network.

Fig. 65a, and that of Fig. 63 is a form of the circuit of Fig. 65b. Table I lists the values of the input resistance observed at various ports if the circuits contain no reactive elements and transit-time effects in the amplifier are negligible. $(R_i)_1$ is the input resistance between the points to which R_1 is connected, when R_1 is infinite. Similarly, $(R_i)_2$ is the input resistance between the points to which R_2 is connected, when R_2 is infinite, and $(R_i)_{12}$ is the input resistance between the points to which R_{12} is connected, when R_{12} is infinite. Because conductance parameters are convenient to use for tubes, whereas hybrid parameters are convenient to use for transistors, the resistances are expressed in terms of both sets of parameters of the amplifier, which may consist of a single tube or transistor, or of two or more direct-coupled stages.

It is apparent from the expressions listed in Table I that there is a value of h_f and g_f that makes $(R_i)_{12}^T$, $(R_i)_1^\pi$, and $(R_i)_2^\pi$ infinite and a value that makes $(R_i)_1^T$, $(R_i)_2^T$, and $(R_i)_{12}^\pi$ zero. It follows from criteria (a) and (b) of Sec. 16 that $(R_i)_{12}^T$, $(R_i)_1^\pi$, and $(R_i)_2^\pi$ are voltage-stable, whereas $(R_i)_1^T$, $(R_i)_2^T$, and $(R_i)_{12}^\pi$ are current-stable.

TABLE I. LOW-FREQUENCY INPUT RESISTANCES OF PI AND TEE FEEDBACK CIRCUITS

Pi Circuit

$$(R_i)_1^\pi = \frac{h_i(G_{12} + G_2) + \Delta_h}{G_{12}(1 + h_iG_2 + h_f - h_r + \Delta_h) + h_o + G_2} \qquad \text{(v-s)}$$

$$(R_i)_2^\pi = \frac{h_i(G_{12} + G_1) + 1}{G_{12}(1 + h_iG_1 + h_f - h_r + \Delta_h) + G_1\Delta_h + h_o} \qquad \text{(v-s)}$$

$$(R_i)_{12}^\pi = \frac{1 + h_i(G_1 + G_2) + h_f - h_r + \Delta_h}{h_iG_1G_2 + h_o + G_1\Delta_h + G_2} \qquad \text{(c-s)}$$

$$(R_i)_1^\pi = \frac{y_o + G_{12} + G_2}{G_{12}(y_i + y_f + y_r + y_o + G_2) + y_1G_2 + \Delta_y} \qquad \text{(v-s)}$$

$$(R_i)_2^\pi = \frac{y_i + G_{12} + G_1}{G_{12}(y_i + y_f + y_r + y_o + G_1) + y_oG_1 + \Delta_y} \qquad \text{(v-s)}$$

$$(R_i)_{12}^\pi = \frac{y_i + y_f + y_r + y_o + G_1 + G_2}{y_iG_2 + y_oG_1 + G_1G_2 + \Delta_y} \qquad \text{(c-s)}$$

$$\Delta_h = h_ih_o - h_fh_r \qquad \Delta_y = y_iy_o - y_fy_r$$

Tee Circuit

$$(R_i)_1^T = \frac{R_{12}(1 + h_f - h_r + \Delta_h + h_oR_2) + h_i + R_2\Delta_h}{h_o(R_{12} + R_2) + 1} \qquad \text{(c-s)}$$

$$(R_i)_2^T = \frac{R_{12}(1 + h_f - h_r + \Delta_h + h_oR_1) + h_i + R_1}{h_o(R_1 + R_{12}) + \Delta_h} \qquad \text{(c-s)}$$

$$(R_i)_{12}^T = \frac{h_i + R_2\Delta_h + R_1(h_oR_2 + 1)}{1 + h_f - h_r + \Delta_h + h_o(R_1 + R_2)} \qquad \text{(v-s)}$$

$$(R_i)_1^T = \frac{R_{12}(y_i + y_f + y_r + y_o + R_2\Delta_y) + y_oR_2 + 1}{y_i + (R_{12} + R_2)\Delta_y} \qquad \text{(c-s)}$$

$$(R_i)_2^T = \frac{R_{12}(y_i + y_f + y_r + y_o + R_1\Delta_y) + y_iR_1 + 1}{y_o + (R_1 + R_{12})\Delta_y} \qquad \text{(c-s)}$$

$$(R_i)_{12}^T = \frac{y_iR_1 + y_oR_2 + R_1R_2\Delta_y + 1}{y_i + y_f + y_r + y_o + (R_1 + R_2)\Delta_y} \qquad \text{(v-s)}$$

Examination of the expressions listed in Table I shows that reduction of R_1 and R_2 and increase of R_{12} are favorable to the production of negative values of input resistance of the tee circuit, and that reduction of G_1 and G_2 and increase of G_{12} are favorable to the production of negative values of input resistance of the pi circuit. In the limit, when R_1 and R_2 are zero and R_{12} is infinite, or when G_1 and G_2 are zero and G_{12} is infinite, the expressions for R_i reduce to those listed in Table II. Since these expressions apply under the circuit conditions most favorable to negative values of R_i,

TABLE II. INPUT RESISTANCE. $G_1 = G_2 = 0; R_1 = R_2 = 0; G_{12} = R_{12} = \infty$

$$(R_i)_1^T = (R_i)_2^T = (R_i)_{12}^\pi = \frac{1 + h_f - h_r + \Delta_h}{h_o} \quad \text{(c-s)}$$

$$(R_i)_{12}^T = (R_i)_1^\pi = (R_i)_2^\pi = \frac{h_i}{1 + h_f - h_r + \Delta_h} \quad \text{(v-s)}$$

$$(R_i)_1^T = (R_i)_2^T = (R_i)_{12}^\pi = \frac{y_i + y_f + y_r + y_o}{\Delta_y} \quad \text{(c-s)}$$

$$(R_i)_{12}^T = (R_i)_1^\pi = (R_i)_2^\pi = \frac{1}{y_i + y_f + y_r + y_o} \quad \text{(v-s)}$$

they may serve as criteria for determining whether a particular circuit is capable of providing a negative resistance. If the amplifier parameters are such that the expressions of Table II yield positive or zero values of R_i, the circuit cannot produce a negative resistance with any values of R_1, R_2, and R_{12} or of G_1, G_2, and G_{12}. These expressions will be used in subsequent sections to determine the values of tube and transistor parameters essential to the production of negative resistance in practical forms of the circuits of Fig. 65.

17.2. Effect of Reactive Elements. Neglect of reactive elements and transit-time effects in the derivation of the expressions listed in Tables I and II limits their application to static conditions. They are useful in determining whether or not the static current-voltage characteristic has a negative-slope range and thus whether or not the circuit can have more than one state of equilibrium. Because they do not apply when the circuit currents and voltages change so rapidly that the effects of reactive elements and transit time cannot be neglected, they cannot be used to determine the behavior of the circuit when current or voltage increments take place in the vicinity of one of the equilibrium points, and therefore they cannot indicate whether the equilibrium states are stable or unstable. Analyses that include the effects of shunt capacitances and transit time show that under conditions that cause the input resistance or conductance to be negative, the circuits of Fig. 65 behave at a voltage-stable port as though they consist of a negative conductance in shunt with a capacitive susceptance (or negative inductive suspectance) and at a current-stable port as though they consist of a negative resistance in series with an inductive reactance (or negative capacitive reactance) (see Probs. 17.2-1 to 17.2-4). At a voltage-stable port, such a circuit may therefore be replaced by an equivalent circuit consisting of a conductance G_i in shunt with a capacitance C_i, and at a current-stable port by an equivalent circuit consisting of a resistance R_i in series with an inductance L_i.

TABLE III. LOW-FREQUENCY y PARAMETERS OF A VACUUM TUBE IN THE COMMON-CATHODE, COMMON-GRID, AND COMMON-PLATE (CATHODE-FOLLOWER) CONNECTIONS

	Common-cathode		Common-grid		Common-plate	
	Grid positive	Grid negative	Grid positive	Grid negative	Grid positive	Grid negative
y_i	$\frac{1}{r_g}$	0	$g_m + \frac{1}{r_g} + \frac{1}{r_p}$	$g_m + \frac{1}{r_p}$	$\frac{1}{r_g}$	0
y_r	~ 0	0	$-\frac{1}{r_p}$	$-\frac{1}{r_p}$	$-\frac{1}{r_g}$	0
y_f	g_m	g_m	$-\left(g_m + \frac{1}{r_p}\right)$	$-\left(g_m + \frac{1}{r_p}\right)$	$-\left(g_m + \frac{1}{r_g}\right)$	$-g_m$
y_o	$\frac{1}{r_p}$	$\frac{1}{r_p}$	$\frac{1}{r_p}$	$\frac{1}{r_p}$	$g_m + \frac{1}{r_g} + \frac{1}{r_p}$	$g_m + \frac{1}{r_p}$
Δ_y	$\frac{1}{r_g r_p}$	0	$\frac{1}{r_g r_p}$	0	$\frac{1}{r_g r_p}$	0

TABLE IV. LOW-FREQUENCY INPUT RESISTANCE OF VACUUM-TUBE TEE AND PI CIRCUITS. R_1, R_2, G_1, AND G_2 ZERO; R_{12} AND G_{12} INFINITE

Tube Connection	$(R_i)_1^T$, $(R_i)_2^T$, $(R_i)_{12}^\pi$		$(R_i)_{12}^T$, $(R_i)_1^\pi$, $(R_i)_2^\pi$	
	Grid positive	Grid negative	Grid positive	Grid negative
Common-cathode	$r_g(1+\mu) + r_p$	∞	$\frac{1}{g_m + 1/r_g + 1/r_p}$	$\frac{1}{g_m + 1/r_p}$
Common-grid	r_p	r_p	r_g	∞
Common-plate (cathode-follower)	r_g	∞	r_p	r_p

In tube circuits, in which the shunt capacitances usually shunt relatively high electrode and circuit resistances, the effects of shunt capacitances become important at frequencies much lower than those at which the effects of transit time are appreciable. At frequencies at which transit time is important in tube circuits, shunt capacitances usually cause the real component of the input impedance or admittance to be positive, rather than negative (see Probs. 17.2–1 to 17.2–4). Consequently the transit-time effects may usually be neglected in tube negative-resistance circuits. In transistor negative-resistance circuits, on the other hand, the transistor and circuit capacitances are usually shunted by relatively low circuit resistances, and the imaginary components of input impedance or admittance are the result of transit time (alpha cutoff).

18. Vacuum-Tube Negative-Resistance Circuits

18.1. Pentode Circuits. Table II may be used to determine how negative-resistance circuits may be formed by the use of vacuum tubes. Table III lists the conductance parameters for a vacuum tube in the common-cathode, common-grid, and common-plate (cathode-follower) circuits. If these values are used in the expressions listed in Table II, the values of input resistance listed in Table IV are obtained.* Because r_p and g_m are positive for triodes, negative resistance cannot be obtained with a single triode. Table IV shows, however, that voltage-stable and current-stable negative resistance could be produced in the common-cathode connection if the transconductance of the tube were negative. Although tubes do not have a negative transconductance between the control grid and the plate, a pentode may have a negative transconductance from the suppressor to the screen over a limited range of suppressor voltage. If the suppressor is negative, some electrons that would strike the plate in the absence of the suppressor are returned to the screen by the retarding field between the screen and the suppressor. Making the suppressor grid more negative causes more electrons to be turned back and thus increases the screen current.

Figures 66a and 66b show pentode pi and tee negative-resistance circuits. Voltage-stable current-voltage characteristics are obtained between either the suppressor or the screen and ground in the pi circuit of Fig. 66a and between the cathode and ground in the tee circuit of Fig. 66b. Current-stable characteristics are obtained between the suppressor and the screen of circuit (a) and between the screen and ground of circuit (b). Plate-circuit resistance reduces suppressor-screen gain. Criterion (d) of Sec. 16.2 therefore predicts a current-stable port between the plate and the cathode. Typical characteristics obtained between the screen and ground, between

* The values listed in Table IV may also be derived by inspection of the individual circuits. See Prob. 18.1–1.

the plate and ground, and between the suppressor and the screen in circuit (a) are shown in Figs. 66c, 66d, and 66e, respectively. Figure 66f shows a characteristic obtained between the cathode and ground in circuit (b). The

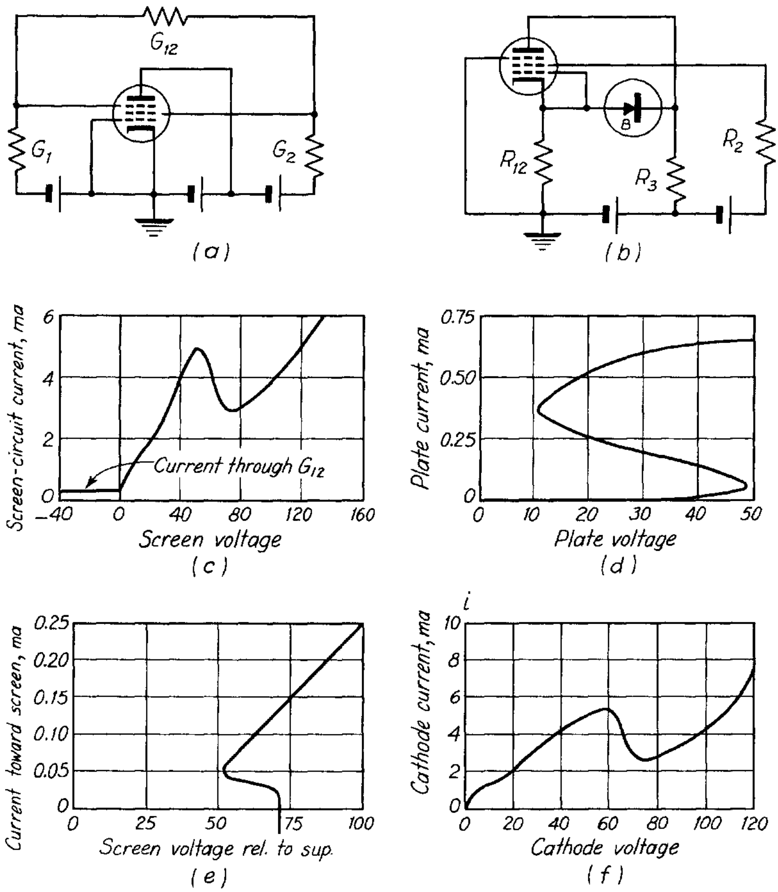


FIG. 66. (a) Pentode pi negative-resistance circuit; (b) pentode tee negative-resistance circuit; (c) current-voltage characteristic observed between the screen and ground in circuit (a); (d) characteristic observed between the plate and ground in circuit (a); (e) characteristic observed between the suppressor and the screen in circuit (a); (f) characteristic observed between cathode and ground in circuit (b).

shape of the characteristics of Fig. 66 and of other vacuum-tube negative-resistance circuits can be readily explained qualitatively. The general procedure is the same as that which will be used for transistor circuits in Sec. 19 (see Prob. 18.1–2).

Secondary emission from the plate causes the plate characteristics of some tetrodes to have a negative slope over a range of plate voltage. The slope of the plate characteristics of presently available tetrodes is so small in the negative-resistance range, however, that the tetrode is not a very useful negative-resistance device. Another disadvantage of the tetrode as a negative-resistance device results from the variation of secondary emission throughout the life of the tube, which causes the shape of the characteristic to change.

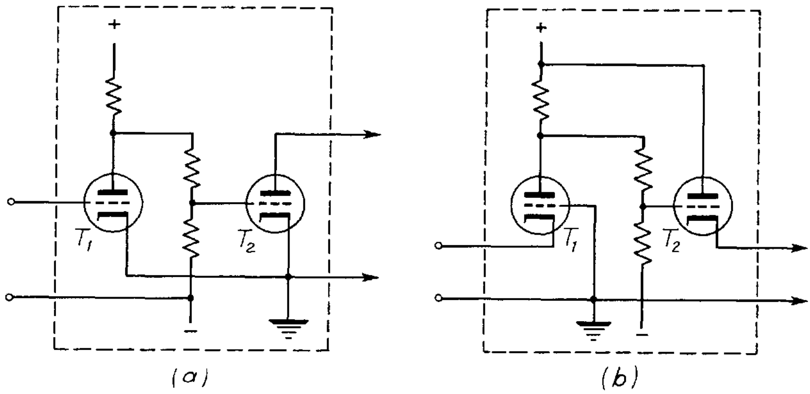
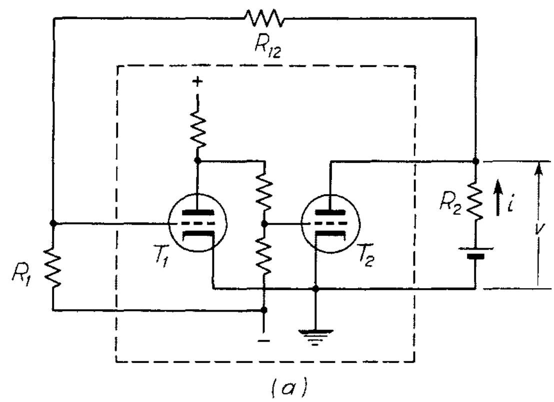


FIG. 67. Two amplifiers that can be used to produce negative resistance in the circuits of Fig. 65: (a) two direct-coupled common-cathode stages; (b) common-grid stage directly coupled to a common-plate (cathode-follower) stage.

18.2. Common-Cathode, Common-Cathode Two-Tube Circuit. Although a negative value of g_m cannot be attained in a triode, the negative value of y_f essential to the production of negative values of R_i in tube circuits may be obtained by the use of a two-stage direct-coupled amplifier, instead of a single tube. Two types of two-stage amplifiers that can be conveniently used for this purpose are shown in Fig. 67. The circuit of Fig. 67a consists of two direct-coupled common-cathode stages. Use of this amplifier in the pi feedback circuit of Fig. 65a leads to the Eccles-Jordan circuit of Fig. 68a, which may be redrawn in the form of Fig. 68b.[1] Typical current-voltage characteristics obtained in this circuit are shown in Figs. 69a to d. The characteristic of Fig. 69a is obtained between one plate and ground, as shown in Fig. 68a. The symmetrical voltage-stable characteristic of Fig. 69b is obtained between the two plates, as shown in Fig. 68b. The current-stable characteristic of Fig. 69c is obtained between the grid of one tube and the plate of the other, as shown in Fig. 68c, and that of Fig. 69d is obtained between one cathode and ground, as shown in Fig. 68d. The symmetrical

[1] W. H. Eccles and F. W. Jordan, *Radio Rev.*, **1**, 143 (1919).



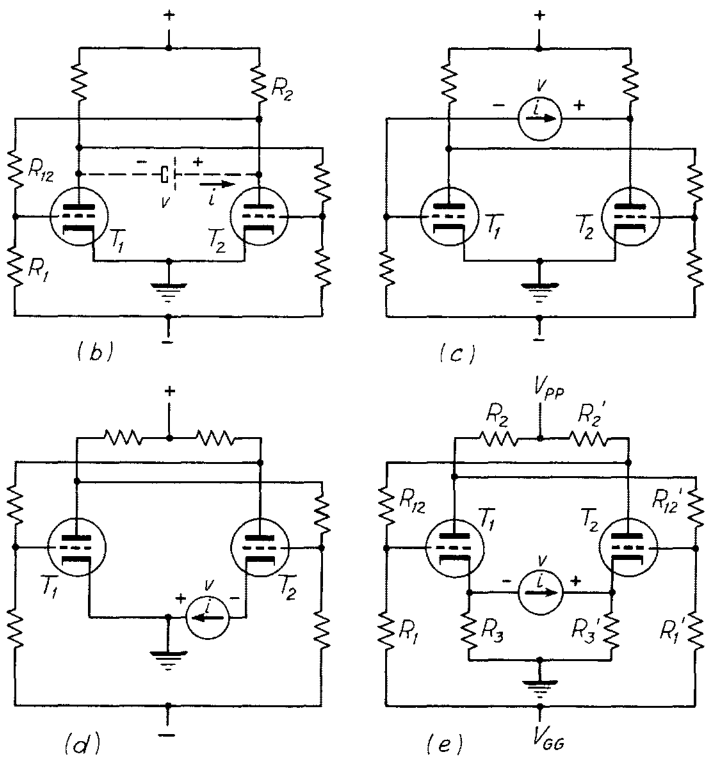


FIG. 68. (a) Circuit in which the amplifier of Fig. 67a is used in the pi circuit of Fig. 65a; (b) a symmetrical voltage-stable port in the circuit of Fig. 67a, which has been redrawn in symmetrical form; (c) a current-stable port; (d) a current-stable port; (e) a symmetrical current-stable port.

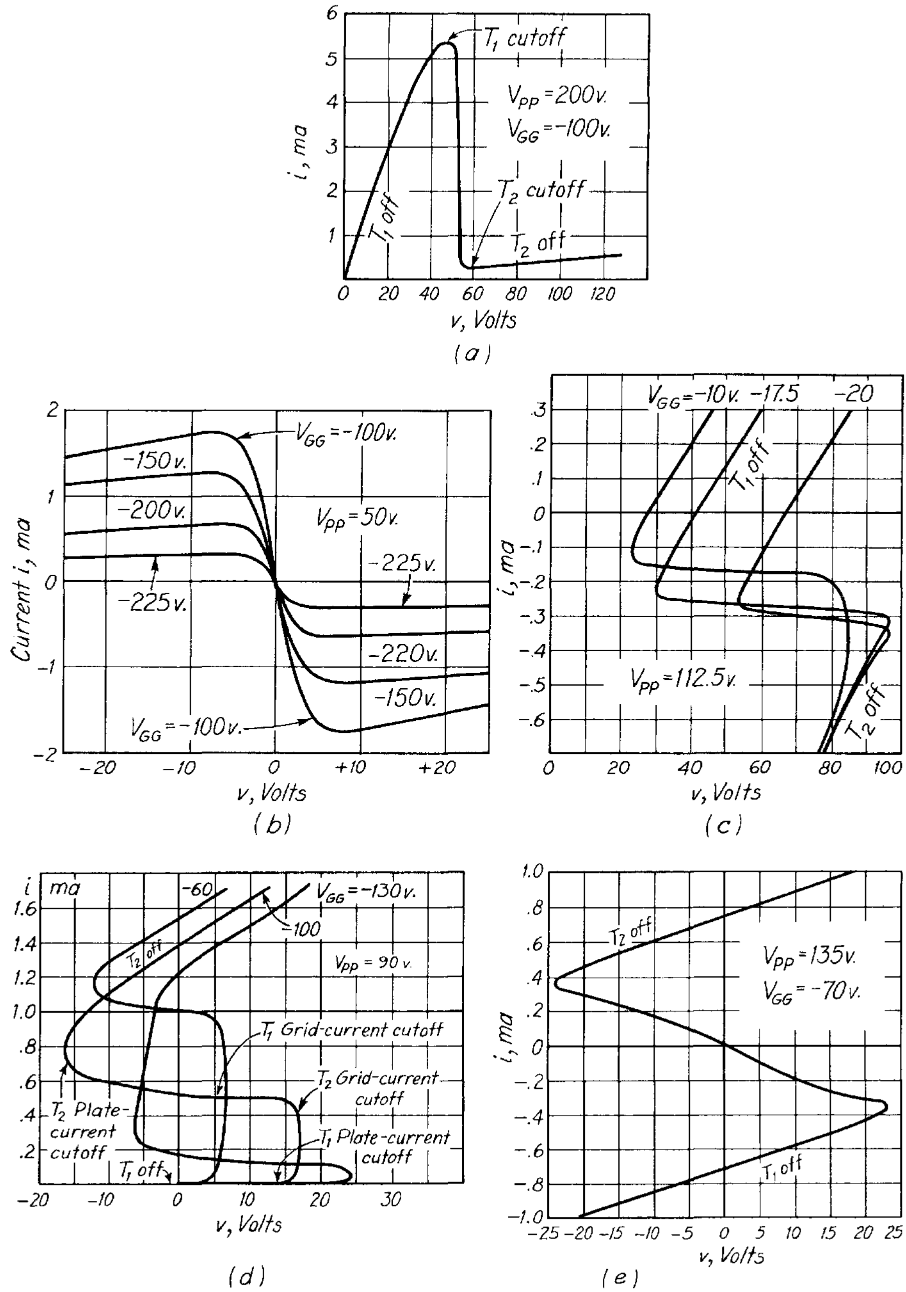


FIG. 69. (a) Voltage-stable characteristic measured as shown in Fig. 68a; (b) voltage-stable characteristic observed in the circuit of Fig. 68b; (c) current-stable characteristic observed in the circuit of Fig. 68c; (d) current-stable characteristic observed in the circuit of Fig. 68d; current-stable characteristic observed in the circuit of Fig. 68e. 6SN7 tube; $R_1 = R_1' = 470$ kΩ, $R_{12} = R_{12}' = 270$ kΩ, $R_2 = R_2' = 47$ kΩ, $R_3 = R_3' =$ zero or 47 kΩ.

current-stable characteristic of Fig. 69e is obtained between cathodes in the circuit of Fig. 68e.

Expressions for the input resistance R_i at various ports in the circuits of Fig. 68 can be derived by solving the equations of the network obtained by replacing the tubes by equivalent plate circuits (see Prob. 18.2–1). Under the assumption that the resistance $R_1 + R_{12}$ is much larger than R_2 and that the circuit is symmetrical, the input conductance $1/R_i$ measured between the two plates is $\frac{1}{2}[1/R_2 + 1/r_p - g_m R_1/(R_1 + R_{12})]$. The conductance is negative if $g_m > (1/R_2 + 1/r_p)(R_1 + R_{12})/R_1$.

Grid bias may be obtained in the circuit of Fig. 68b by the use of a common cathode resistor. If the circuit is symmetrical, increase of plate current of T_1 is accompanied by nearly equal decrease of plate current of T_2, and the voltage across the cathode resistor remains nearly constant. Consequently, the resistor has little effect upon the current-voltage characteristics at the various ports.

In some applications of the circuit of Fig. 68, particularly in bistable circuits, it is advantageous to use pentodes and to couple from the plate of each tube to the suppressor or screen of the other, instead of to the control grid. It is also possible to place the load resistors R_2 in the screen circuits and to couple from the screen of each tube to the control grid of the other. The plate circuits can then be used solely to deliver output.

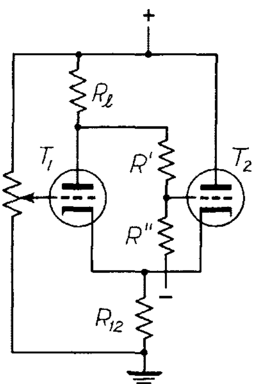


FIG. 70. Circuit in which the amplifier of Fig. 67b is used in the tee circuit of Fig. 65b with R_1 and R_2 zero.

18.3. Common-Grid, Common-Plate Two-Tube Circuit. The circuit of Fig. 67b consists of a common-grid stage directly coupled to a cathode-follower stage. If this amplifier is used in the tee feedback circuit of Fig. 65b, R_1 and R_2 being made zero, the resulting circuit is that of Fig. 70.[2] This circuit has voltage-stable characteristics between the plate of T_1 and ground ($B+$), between the grid of T_2 and ground ($C-$), and between the cathodes and ground, and it has a current-stable characteristic between the plate of T_1 and the grid of T_2, as well as between the plate of T_2 and ground. A current-stable characteristic is also observed at a break in either cathode lead. Unlike the circuits of Fig. 68, this circuit has no symmetrical current-voltage characteristic. If the two tubes are identical, the input resistance $(R_i)^{T}_{12}$ between the cathodes and ground is approximately $1/g_m(1 - A_{v1})$, where A_{v1} is the voltage amplification of the first stage between the cathode

[2] M. G. Crosby, *Electronics*, May, 1946, p. 136; P. Sulzer, *Proc. I.R.E.*, **36**, 1034 (August, 1948); P. Sulzer, *Proc. I.R.E.*, **38**, 540 (May, 1950).

of T_1 and the grid of T_2 and g_m is the transconductance of T_2 (see Prob. 18.3–1). The sign of the voltage amplification of a common-grid stage being positive, the input resistance is negative if the magnitude of the voltage amplification of the first stage exceeds unity. The magnitude of $1/g_m(A_{v1} - 1)$ may be of the order of 100 ohms or less if g_m is large, if the resistance of the combination of R_l in parallel with $R' + R''$ is large in comparison with r_p, and if $R''/R' \gg 1$.[3]

19. Transistor Negative-Resistance Circuits

19.1. Criteria for Negative Resistance in Transistor Circuits. Table II shows that when a transistor is used in the common-base connection in the tee or pi circuit of Fig. 65, negative resistance is obtained only if

$$\frac{1 + h_{fb} - h_{rb} + \Delta_{hb}}{h_{ob}} < 0 \tag{20}$$

or

$$\frac{h_{ib}}{1 + h_{fb} - h_{rb} + \Delta_{hb}} < 0 \tag{21}$$

If expressions analogous to Eqs. (20) and (21) are written for transistors used in the common-emitter connection in the circuits of Fig. 65, and the common-emitter h parameters are expressed in terms of the common-base h parameters, the following relations are obtained for the criterion for negative resistance (Prob. 19.1–1):

$$\frac{1 + \Delta_{hb}}{h_{ob}} < 0 \tag{20A}$$

or

$$\frac{h_{ib}}{1 + \Delta_{hb}} < 0 \tag{21A}$$

Similarly, the criteria for the production of negative resistance by the use of transistors in the common-collector connection in the circuits of Fig. 65 are (Prob. 19.1–1):

$$\frac{\Delta_{hb}}{h_{ob}} < 0 \tag{20B}$$

or

$$\frac{h_{ib}}{\Delta_{hb}} < 0 \tag{21B}$$

Numerical values of h_{ib} and h_{ob} of the usual types of junction transistors are positive, Δ_{hb} is positive, and h_{rb} is positive but small in comparison with unity. The value of h_{fb} in such transistors is negative, but smaller in magni-

[3] H. J. Reich, *Proc. I.R.E.*, **43**, 228 (February, 1955).

tude than unity. It follows that Eqs. (20) to (21B) cannot be satisfied with a single transistor of this type, and that a negative-resistance characteristic cannot therefore be obtained by the use of such a transistor. However, Eqs. (20) and (21) may be satisfied if the magnitude of h_{fb} can be made to exceed unity, h_{ib} and h_{ob} remaining positive, h_{rb} positive or small, and Δ_{hb} small.

A qualitative explanation of the fact that negative resistance cannot be obtained with a single transistor having a magnitude of h_{fb} less than unity is that such a transistor cannot produce in the circuits of Fig. 65 a voltage amplification that is at the same time positive and greater in magnitude than unity or a current amplification that is at the same time negative and greater in magnitude than unity. This can be accomplished, however, by the use of a two-stage amplifier. The function of one stage may be merely to reverse the polarity of an increment of amplifier output voltage or current, i.e., to reverse the sign of the over-all amplification. Alternatively, one may say that the use of two transistors each having a magnitude of h_{fb} less than unity results in an amplifier having a large negative value of h_f and small values of h_r and h_ih_o, so that Eqs. (20) and (21) are satisfied. A number of negative-resistance circuits based upon direct-coupled two-stage transistor amplifiers will be discussed later in this section.

19.2. Avalanche, Point-Contact, and *P-N-P-N* Transistors. Presently available transistors that have magnitudes of h_{fb} greater than unity include the avalanche junction transistor,[1] the *p-n-p-n* junction transistor,[2] and the point-contact transistor. In the avalanche transistor the collector current exceeds the emitter current because of charge multiplication in the base-collector junction. Carriers originating in the emitter are accelerated by high electric field in the base-collector junction and may thus gain sufficient kinetic energy to enable them to make other carriers available as the result of collision with semiconductor molecules in this junction. (This phenomenon is similar to ionization of the gas in a gas diode as the result of collision of gas molecules with electrons accelerated by the electric field between the anode and the cathode.)

The basic structure of a *p-n-p-n* transistor is shown in Fig. 71a. The high value of h_{fb} of this type of transistor may be explained by assuming that the behavior of such a transistor is similar to that of the transistor depicted in Fig. 71b, and that the latter may in turn be replaced by a *p-n-p* transistor directly coupled to an *n-p-n* transistor, as shown in Fig. 71c. The common-base current-amplification factor h_{fb} of the *p-n-p-n* transistor being defined as the ratio of a change of collector current to the corresponding change of

[1] S. L. Miller and J. J. Ebers, *Bell System Tech. J.*, **34,** 883 (September, 1955); H. Schenkel and H. Statz, *Proc. I.R.E.*, **44,** 360 (March, 1956).

[2] J. L. Moll, M. Tanenbaum, J. M. Goldey, and N. Holonyak, *Proc. I.R.E.*, **44,** 1174 (September, 1956).

emitter current when the collector voltage is held constant, some indication of the magnitude of h_{fb} to be expected may be obtained from the circuit of Fig. 71d, in which the two transistors of Fig. 71c are represented by conventional symbols. This circuit consists of a common-base stage directly connected to a common-collector stage. The current amplification I_{e2}/I_{b2} of the common-collector stage with zero load resistance is approximately $-1/(1 + h_{fb2})$, where h_{fb2} is the value of h_{fb} of the *n-p-n* transistor. The input resistance of this stage, which is the short-circuit input resistance h_{ic} of the transistor, is of the order of 2000 ohms. The current amplification I_{c1}/I_{e1} of the common-base stage with a load resistance of only 2000 ohms is very nearly equal to

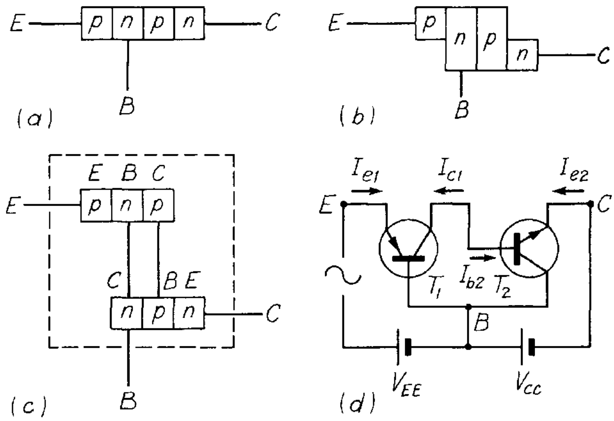


FIG. 71. (a) Basic structure of a *p-n-p-n* transistor; (b) and (c) transition from the *p-n-p-n* structure to a two-transistor equivalent; (d) conventional representation of circuit (c).

h_{fb1}, the value of h_{fb} of the *p-n-p* transistor. Since $I_{c1} = -I_{b2}$, the ratio I_{e2}/I_{e1} in the circuit of Fig. 71d approximates $h_{fb1}/(1 + h_{fb2})$, which is negative and may have a magnitude considerably larger than unity for typical values of h_{fb1} and h_{fb2}. (Note that numerical values of h_{fb1} and h_{fb2} are negative.) (See also Prob. 19.2-1.)

One explanation of the high values of $|h_{fb}|$ obtainable in point-contact transistors is that the process of "forming" the contacts results in a *p-n-p-n* structure similar to that of the *p-n-p-n* junction transistor. Because point-contact transistors have been displaced by junction transistors, they will not be discussed further.

19.3. *P-N-P-N*-Transistor Base Characteristics. The general forms of current-voltage characteristics obtained experimentally in the circuits of Fig. 65b with transistors having values of $|h_{fb}|$ greater than unity are shown in Fig. 72. The shapes of these characteristics can be explained by a qualitative examination of the circuits. Thus, Fig. 73a shows a *p-n-p-n* transistor circuit of the form of Fig. 65b, with the base circuit resistance replaced by an impressed base-to-ground voltage v_B. The indicated emitter and collector current directions are those in which these currents actually flow in the circuit. (Note that h_{fb} is negative.) If v_B is zero, a very small positive current i_B flows as the result of the collector supply voltage V_{CC}. Positive (relative to ground) voltage v_B biases the emitter-base junction in the reverse direction, causing a small reverse emitter current. It also increases the reverse bias of the base-collector junction and thus causes a small increase of collector current. Consequently, increase of positive v_b is accompanied by a very small increase of i_B, as shown in Fig. 72a. A negative

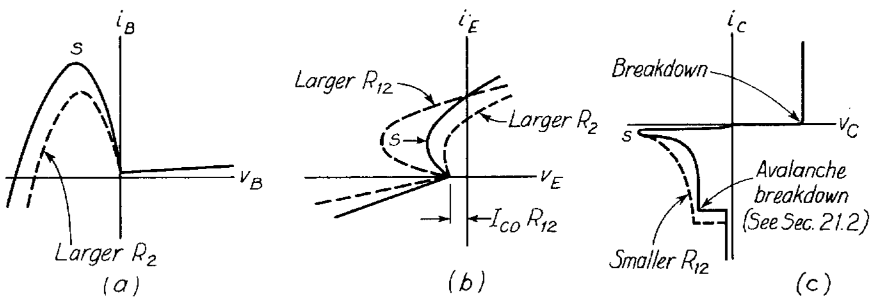


FIG. 72. (a) Characteristic observed in the circuit of Fig. 73a; (b) characteristic observed in the circuit of Fig. 73b; (c) characteristic observed between the collector and ground when R_2 is infinite in Fig. 73b and R_1 is connected between the emitter and ground.

value of v_B biases the emitter-base junction in the forward direction and thus causes current to flow into the emitter. The forward emitter current causes an increment of collector current, which flows out of the collector (since h_{fb} is negative). If $|h_{fb}|$ is greater than unity, the increment of collector current exceeds the increment of emitter current that causes it and the net increment of base current is into the base, or positive.

As v_B is made continuously more negative, i_B becomes increasingly positive until the collector current approaches saturation. The increment of collector current accompanying an increment of emitter current then becomes smaller and eventually becomes less than the increment of emitter current that produces it. For this reason, increase in the magnitude of negative base voltage v_B beyond the value at point s is accompanied by a decrease of i_B. The emitter current, however, continues to increase with magnitude of negative v_B even after i_C reaches its saturation value and will eventually exceed the saturation value of i_C. The base current i_B then becomes negative.

Increase of R_2 in the circuit of Fig. 73a causes the collector current to saturate at a lower magnitude of v_B and the saturation value of i_C to be lower. The current-voltage characteristic is altered as shown by the dotted curve of Fig. 72a. Increase of R_1 reduces the change of emitter current and, therefore, of collector current resulting from an increment of negative base voltage and consequently reduces the slope of the characteristic at negative values of v_B.

19.4. *P-N-P-N*-Transistor Emitter and Collector Characteristics. The emitter characteristic of Fig. 72b can be most readily explained by allowing the emitter current to be the independent variable. If i_E is zero in the

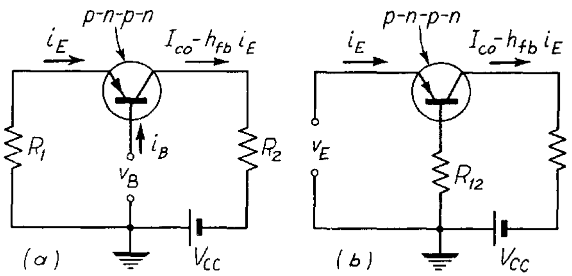


FIG. 73. Negative-resistance circuits in which a single *p-n-p-n* transistor is used as the amplifier in the tee circuit of Fig. 65b.

circuit of Fig. 73b, a small current I_{co} flows in the collector circuit and lowers the potential of the base relative to ground. The emitter-base voltage being very small in the normal range of emitter current, the emitter is negative relative to ground by approximately the voltage $I_{co}R_{12}$, as indicated in Fig. 72b. Negative emitter current is current in the reverse direction across the emitter-base junction and therefore requires a relatively high reverse voltage across this junction. For this reason, v_E rapidly becomes more negative as i_E is made negative.

If i_E is positive, on the other hand, a positive increment Δi_E of emitter current is accompanied by an increment of collector current flowing out of the collector and of magnitude $|h_{fb}|\,\Delta i_E$. The increment of emitter current flows through R_{12} from base to ground, tending to raise the potential of the base and of the emitter. The increment of collector current, however, flows from ground to base and thus tends to lower the potential of the base and of the emitter. If $|h_{fb}|$ is greater than unity, the net effect is a reduction of the potential of the emitter. As the emitter current is increased continuously, the emitter voltage continues to fall until the collector current approaches saturation (because of the voltage drop in R_2). The magnitude of h_{fb} then falls below unity, the increment of collector current becomes less than the increment of emitter current that causes it, and further increase of emitter current beyond the value at s is accompanied by reduction of voltage across R_{12} and consequently by a rise of voltage of the base and the emitter. If R_2 is increased, a smaller fraction of the applied collector voltage appears across R_{12}. Furthermore, collector-current saturation occurs at a lower value of emitter current. The effect of an increase of R_2 upon the characteristic is therefore to displace it to the right and to lower the current at which the voltage peak s occurs, as shown by the dotted curve of Fig. 72b. Increasing R_{12} increases the voltage across R_{12} at any value of emitter current and thus causes the curve to be displaced to the left at negative values of v_E and to the right at positive values, as shown by the dashed curve of Fig. 72b.

The collector current-voltage characteristic of Fig. 72c and the pi-circuit characteristics may be explained in a similar manner.

19.5. Two-Transistor and Three-Transistor Circuits. The two-transistor circuit of Fig. 71d, used in the explanation of the p-n-p-n transistor,

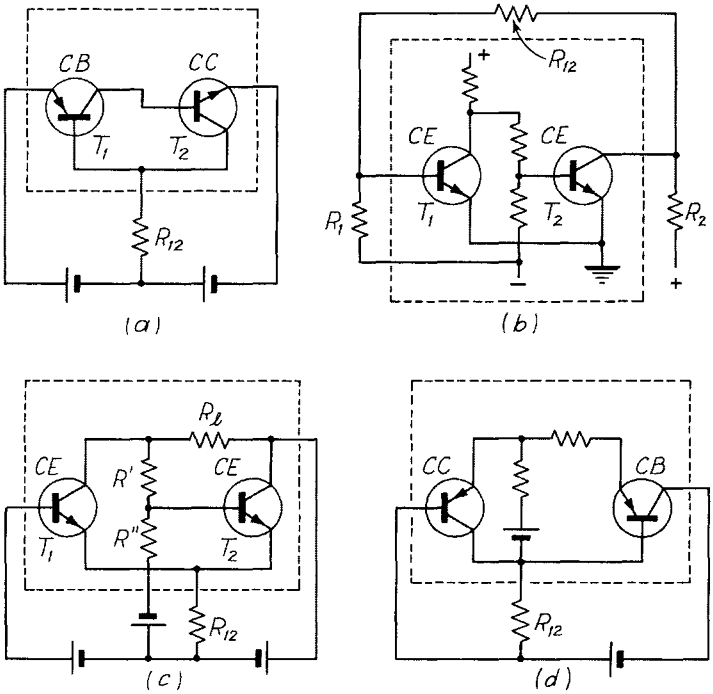


FIG. 74. Two-transistor negative-resistance circuits: (a) Common-base, common-collector tee circuit; (b) common-emitter, common-emitter pi circuit; (c) common-emitter, common-emitter tee circuit; (d) common-collector, common-base tee circuit.

may itself be used as the basis of a negative-resistance circuit. A simple form of the resulting circuit in which R_1 and R_2 are zero is shown in Fig. 74a. These resistances may be added if they are needed in the application of the circuit. Three circuits in which two p-n-p or two n-p-n transistors are used are shown in Figs. 74b to 74d. In circuits (b) and (c), which are the transistor equivalents of the tube circuits of Figs. 68a and 70, the portions of the circuits within the dotted lines consist of two direct-coupled common-emitter stages. Circuit (b) may be redrawn in the same form as Fig. 68b. In circuit (d) the portion of the circuit within the dotted lines consists of a common-collector stage coupled to a common-base stage. Either two-stage combination of transistors provides a value of h_f that is negative and greater in magnitude than unity.

The reader should determine the ports at which voltage-controlled and current-controlled negative resistance is observed in the circuits of Fig. 74. Criteria (c) and (d) of Sec. 16.2 may be conveniently used for this purpose (Prob. 19.5–1).

Transistors may be used in place of tubes in the circuits of Figs. 68d and 68e. Voltage-stable and current-stable current-voltage characteristics obtained at the various ports are similar to the tube-circuit characteristics of Fig. 69.

An interesting three-stage transistor circuit has been described in which the change from positive to negative resistance occurs because one stage changes from an inverting amplifier to a noninverting amplifier when the collector voltage approaches zero.[3] This change is the result of the large difference in collector resistance above and below the knee of the collector current-voltage characteristic. The stage is phase-inverting (voltage) at large values of collector resistance, but noninverting at small values.

20. The Unijunction Transistor

20.1. Structure of the Unijunction Transistor. Figure 75a shows the basic structure of the unijunction transistor,[1] which differs from the ordinary p-n-junction diode in that there are two nonrectifying contacts to the base, rather than a single contact. The base contacts are not opposite to the p-n junction, but are displaced from it laterally in opposite directions. Either base terminal may be considered to serve the function of the base terminal of an ordinary double-junction transistor, the other base terminal that of the collector terminal of the double-junction transistor, and the junction terminal that of the emitter terminal. The graphical symbol used for the unijunction transistor is shown in Figs. 95 and 172. Figure 76a shows characteristic curves relating emitter current i_E with emitter voltage v_E

[3] E. Keonjian and J. J. Suran, *Electronics*, July, 1955, p. 138.

[1] I. A. Lesk and V. P. Mathis, *I.R.E. Convention Record*, Part 6, p. 2 (1953); R. W. Aldrich and I. A. Lesk, *Trans. I.R.E.*, Vol. ED-1, February, 1954, p. 24; J. J. Suran, *Trans. I.R.E.*, Vol. ED-2, No 2, April, 1955, p. 40; J. J. Suran, *Electronics*, March, 1955, p. 198; J. J. Suran and E. Keonjian, *Proc. I.R.E.*, **43**, 814 (July, 1955).

at several values of collector voltage v_C relative to the base. The characteristics are current-stable.

The curves of Fig. 76a may be explained in the following manner: When i_E is zero, the voltage v_C impressed between the collector and the base produces a uniform potential gradient and, if the structure is symmetrical, the potential of the base region at a point adjacent to the emitter is half the impressed voltage. At any positive voltage v_E that is less than $v_C/2$, therefore, the junction is reverse-biased and only a small reverse current flows in the emitter circuit. When v_E approaches $v_C/2$, however, the junction becomes biased in the forward direction over a portion of the junction area nearest the base terminal and a positive current flows across the junction and through the base region to the base terminal. The presence of the additional carriers (holes) in the portion of the base between the emitter junction and the base terminal increases the conductivity of this part of the base. At small values of i_D, the reduction of the base resistance produces more effect upon the IR drop between the junction and the base terminal than does the increase in current. The resulting reduction of potential of the base region in the vicinity of the junction increases the forward bias of the junction and thus produces a further increase of i_E. If i_E is not limited by the external circuit, the current increases cumulatively. If i_E is limited by the external circuit, on the other hand, a small increase of i_E in the vicinity of zero current is accompanied by a reduction in potential of the base region in the vicinity of the junction. The junction resistance in the forward direction being small, the increment of current is accompanied by very little increase in voltage across the junction, and v_E falls by nearly the same amount as the voltage of the base region at the junction. At currents greater than that at the minimum-voltage point, the effect of the emitter current upon the base IR drop is greater than that of the decrease of resistance. Above this point the potential of the base region at the junction rises, and v_E must also rise.

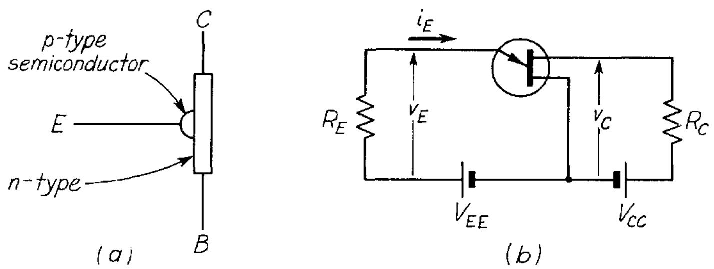


FIG. 75. (a) Basic structure of the unijunction transistor; (b) circuit used to obtain the characteristics of Fig. 76.

In some applications of the unijunction transistor, output voltage is obtained from a resistance in the collector circuit. Figure 76b, showing the manner in which the emitter current-voltage characteristic is altered by the presence of collector circuit resistance, is therefore of interest.

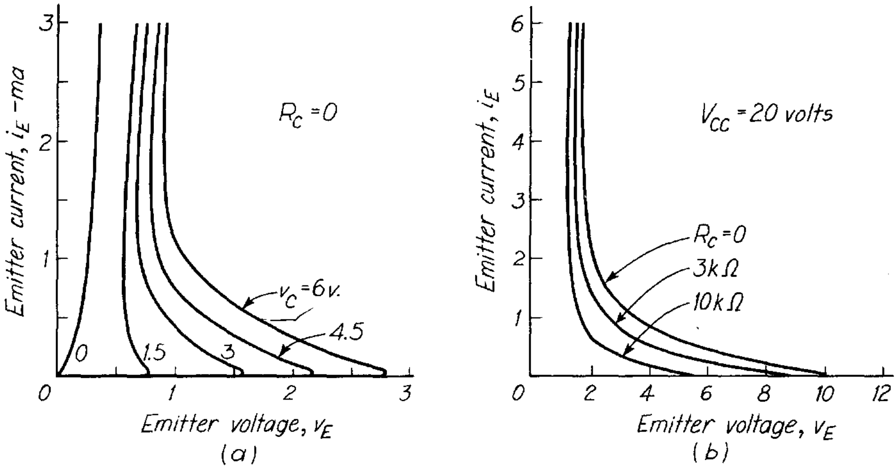


FIG. 76. Emitter characteristic curves of a unijunction transistor: (a) at zero R_c and several values of v_C; (b) at fixed V_{CC} and three values of R_c.

21. Diode Negative-Resistance Circuits

21.1. The Use of Transistors as Negative-Resistance Diodes. Table I shows that the expression for the resistance $(R_i)_{12}^{\pi}$ between the amplifier input and output terminals of the pi circuit of Fig. 65a reduces to the following form if G_1 and G_2 are zero:

$$(R_i)_{12}^{\pi} = \frac{h_i h_o + h_f - h_r(h_f + 1) + 1}{h_o} \tag{22}$$

If the amplifier consists of a transistor used in the common-base connection, this resistance is the resistance between the emitter and the collector when the base is open. Inasmuch as the numerical value of h_{fb} is negative, the resistance may be negative if the following relation is satisfied:

$$|h_{fb}| > \frac{h_{ib}h_{ob} - h_{rb} + 1}{1 - h_{rb}} \approx 1 \tag{23}$$

It follows that a transistor having a magnitude of h_{fb} greater than unity will exhibit negative resistance between the emitter and the collector if the base is left open. Avalanche, *p-n-p-n*, and point-contact transistors may be used as negative-resistance diodes in this manner. Special diffused-junction silicon *p-n-p-n* units without base connection are available, but the base connection is sometimes desirable when the unit is used as the basis of a bistable circuit.

21.2. Current-Voltage Characteristics of *P-N-P-N* Diodes. A typical current-voltage characteristic of a *p-n-p-n* silicon diode is shown in Fig. 77a, and a semilogarithmic plot of the forward characteristic is shown in Fig. 77b.[1] Since such a diode may be considered as a *p-n-p-n* transistor used in the common-base connection in the circuit of Fig. 65a with G_1 and G_2 zero, the diode resistance should be given approximately by Eq. (22) if the common-base parameters of a typical *p-n-p-n* transistor are used. For such a transistor the quantity $h_{ib}h_{ob} - h_{fb}h_{rb} - h_r$ is very small in comparison with unity, and the resistance in the open-base diode connection should approximate the value given by the following equation:

$$R_i = (h_{fb} + 1)/h_{ob} \qquad (24)$$

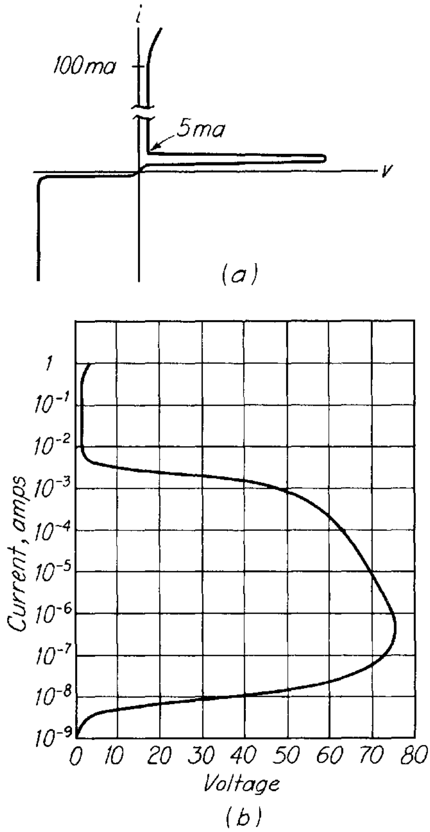


FIG. 77. Current-voltage characteristic of a *p-n-p-n* diode: (a) general form of the complete characteristic; (b) typical forward characteristic.

The shape of the characteristic of Fig. 77b will be explained with the aid of this equation and a knowledge of the manner in which h_{fb} depends upon the collector current and the collector-base voltage. The characteristic being current-stable, it can be analyzed most readily by assuming that current is the independent variable.

At a current of 10^{-7} ampere, h_{fb} at low collector voltages is of the order of -0.15 and varies relatively slowly with collector current. The value of h_{ob} is the order of 10^{-6} mho. Consequently, R_i is of the order of $+1$ megohm and the voltage across the diode rises rapidly with current in this range of current. The shape of the characteristic in this range is similar to

[1] J. L. Moll, M. Tanenbaum, J. M. Goldey, and N. Holonyak, *Proc. I.R.E.*, **44**, (September, 1956).

that of the common-base collector characteristic of a junction transistor for zero emitter current. As the collector voltage approaches the avalanche-breakdown voltage of the middle junction (which is reverse-biased and thus takes up most of the terminal voltage), the magnitude of h_{fb} increases rapidly. Hence R_i, and therefore the rate of change of terminal voltage with current, falls, R_i becoming equal to zero when $h_{fb} = -1$. At higher values of current R_i becomes negative and an increase of current is accompanied by a decrease of diode voltage. As the voltage continues to fall with increase of current, avalanche carrier multiplication has less and less effect upon h_{fb}, but the magnitude of the low-voltage value of h_{fb} increases with current, reaching a maximum of approximately 1.8 at a current between 10^{-3} and 10^{-2} ampere. As the voltage approaches values of the order of a few volts, the magnitude of h_{fb} begins to fall rapidly and eventually becomes less than unity. R_i then becomes positive and further increase of current must be accompanied by rise of voltage.

If the impressed voltage is reversed, the two outer junctions of the diode are reverse-biased, whereas the center junction is forward-biased. The reverse-biased *p-n-p-n* diode is therefore roughly the equivalent of two reverse-biased *p-n* diodes in series with a forward-biased *p-n* diode, and the current-voltage characteristic is essentially that of a reverse-biased *p-n* diode.

Figure 77b indicates that the current is of the order of 10^{-8} ampere in the low-current range and of the order of an ampere or higher in the high-current range. By proper design and fabrication, the diode may be made to have either a very low resistance in the high-current, low-voltage range, or a very rapid response to changes of impressed voltage. The time required to change the current from the low-current range to the high-current range is determined principally by the rate of diffusion of stored minority carriers. Rapid response is important in the use of the device in bistable circuits, relaxation oscillators, pulse generators, and computer circuits. It should be noted that the behavior of the *p-n-p-n* diode in the forward current range is similar to that of the gas diode, which will be discussed in Sec. 22. It can be used for all applications in which gas diodes can be used, and has the advantage of a much lower voltage drop than a gas diode in the high-current range and can therefore carry a much higher current without excessive dissipation.

21.3. Effect of Temperature and Illumination Upon *P-N-P-N* Diode Characteristics. The voltage peak of the current-voltage characteristic (breakdown voltage) of a *p-n-p-n* diode is moved to the left by increase of temperature or illumination of the diode. A displacement of the voltage peak is also observed when base current is allowed to flow in a *p-n-p-n* triode designed to have an open-base emitter-collector characteristic similar to the diode characteristic of Fig. 77. It will be shown in Sec. 26 that these changes in the shape of the characteristic may be used to advantage in controlling the current in bistable circuits. Other semiconductor devices having characteristics similar to those of the p-n-p-n diode and triode will be discussed in Sec. 26.

21.4. The Esaki Diode. A voltage-stable semiconductor diode that is unique among presently available negative-resistance devices in regard to the magnitude of negative conductance which it provides and the small voltage at which it operates is the Esaki diode (tunnel diode).[2] A typical current-voltage characteristic of this device is shown in Fig. 78. The voltage range in which the resistance is negative does not vary greatly in units made with the same semiconducting material and having various current capacities but units may be designed to have a current maximum anywhere in the range from milliamperes to amperes. It follows that the magnitude of the variational conductance, as determined from the slope of the current-voltage characteristic, may range from millimhos up to mhos and that the negative conductance can be obtained with a supply voltage of less than a volt. Negative conductances as large as 10 mhos have been reported by Sommers.[3] The shunt capacitance is large, being of the order of 100$\mu\mu$f in a unit having a junction 1.5 mils in diameter.

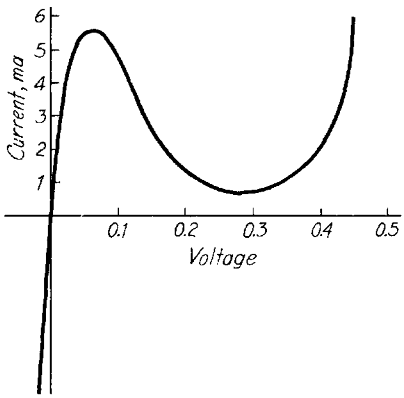


FIG. 78. Typical current-voltage characteristic of a tunnel diode.

The current peak of the Esaki diode characteristic and the high conductance at low values of voltages are caused by quantum-mechanics tunneling. In a p-n junction, the diffusion of electrons from the n-type semiconductor to the p-type semiconductor and of holes from the p-type to the n-type leaves a double layer of bound positive and negative charges on the two sides of the junction. The direction of the field between the positive and negative layers of bound charge is such as to prevent electrons from diffusing to the p-type semiconductor and holes to the n-type. A potential barrier therefore exists across the junction. In the ordinary p-n junction, only carriers that have sufficient kinetic energy to surmount the barrier can cross the junction against the field. In the Esaki diode, however, heavy doping of the semiconductors with p- and n-type impurities results in the

[2] E. Esaki, *Phys. Rev.*, **109**, 603 (1958); H. S. Sommers, Jr., *Proc. I.R.E.*, **47**, 1201 (July, 1959).

[3] Esaki, *loc. cit.*

existence of allowable energy levels on the far side of the junction that are equal to or lower than those on the near side, and there is consequently a finite probability that electrons will cross the junction even though they have insufficient energies to surmount the barrier. This phenomenon can occur only in junctions having thickness of the order of 200 Ångstrom units or less. Theory predicts that the number of electrons that can tunnel across the junction should pass through a maximum and become small at a few hundred millivolts. The second rise of current at higher voltages is the same as that observed in the usual type of *p-n* junction.

The current-voltage characteristics of Esaki diodes, unlike those of other negative-resistance diodes, are independent of frequency at frequencies extending into the kilomegacycle range. Theoretically, negative resistance should be obtained up to a frequency of the order of 10^{12} cps. The product of the magnitude of the negative resistance and the shunt capacitance of Esaki diodes is of the order of 5×10^{-12} second. Temperature has little effect upon the current-voltage characteristic at any temperatures likely to be encountered in normal applications.

The very high negative conductance provided by the Esaki diode makes possible its use in a variety of circuits, including microwave oscillators. The ease with which oscillation occurs in circuits using this type of negative-resistance device introduces practical problems in circuits in which oscillation is not desired.

22. *Glow and Arc Tubes*

22.1. Glow-Diode Current-Voltage Characteristics. Glow and arc tubes have long been used as negative-resistance elements. Glow tubes suitable for this application consist of two electrodes in an envelope containing one of the noble gases, usually helium, argon, or neon, at a pressure of a few millimeters Hg. A typical current-voltage characteristic for such a tube, shown in Fig. 79, is seen to be a current-stable characteristic similar to that of a *p-n-p-n* diode. The voltage in the high-current range is much greater than that of a *p-n-p-n* diode, however, ranging from about 40 volts to several hundred volts, depending upon the gas pressure, the type of gas, and the cathode-surface material. The negative-resistance range of the characteristic is explained by cumulative increase of current resulting from ionization of gas molecules by electrons accelerated in the electric field, and

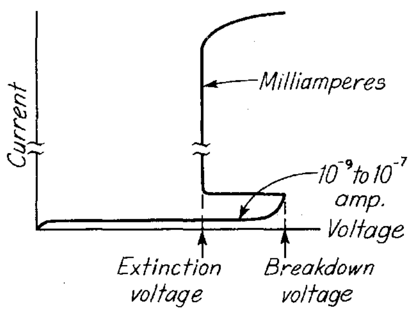


FIG. 79. General form of the current-voltage characteristic of a glow tube.

by emission of electrons at the cathode as the result of bombardment of the cathode by positive ions formed in the interelectrode space. The form of the characteristic in the low-current (dark-current) range is dependent upon the initial electron emission from the cathode. Increase of emission produced by illumination, radioactivity, and other external causes moves the voltage peak of the characteristic to the left. The voltage corresponding to the voltage peak of the glow-tube characteristic is called the *breakdown voltage* because the current jumps to a very high value if the voltage is raised to this value, unless the current is limited by circuit resistance. The voltage corresponding to the minimum-voltage point of the characteristic is called the *extinction voltage* because the current cannot be maintained in the high-current range at voltages below this value. The extinction voltage of a glow tube is several times the ionization potential of the gas. The shape of the current-voltage characteristic is unique only under static conditions. Because of time taken for ionization and deionization to take place, the form of a dynamic characteristic depends upon the rate of change of current and voltage and upon previous values of current.[1] The tube has been shown to behave as though it contained series inductance.[2]

22.2. Arc-Diode Current-Voltage Characteristics. The relatively large minimum voltage drop that is characteristic of a glow discharge is necessary in order to impart to positive ions sufficient energy so that they can produce the required secondary emission from the cathode. Consequently, if the cathode is heated sufficiently to provide the needed emission thermionically, the tube voltage drop falls to approximately the first ionization potential of the gas or vapor. Gas or vapor tubes in which the required emission at the cathode is provided by some copious source other than secondary emission are called *arc tubes*. The voltage drop through heated-cathode arc tubes containing mercury vapor is approximately 10 volts in the normal current range. In tubes containing argon, it is about 16 volts.

At voltages below the ionization potential, the anode current of an arc diode is approximately the same as that which would be obtained at the same voltage in a high-vacuum diode of similar structure. Above the first ionization potential, however, positive ions neutralize negative space charge present in the vicinity of the cathode, and the anode voltage remains very nearly constant until the anode current exceeds the thermionic emission current from the cathode, after which the anode voltage rises rapidly with further increase of current. In order to prevent damage to the cathode, the anode current must be limited by the external circuit to values less than the cathode emission current.

[1] H. J. Reich and W. A. Depp, *J. Appl. Phys.*, **9**, 421 (1938).
[2] R. S. Mackay and H. D. Morris, *Proc. I.R.E.*, **42**, 961 (June, 1954).

22.3. Current-Voltage Characteristics of Grid-Controlled Arc Tubes. The current-voltage characteristic of a heated-cathode arc diode in which the cathode is not shielded from the anode does not contain a range in which the slope is negative, the current increasing rapidly, but continuously, as the anode voltage is raised above the ionization potential of the gas or vapor. A negative-slope range is present, however, if there is a grid of suitable structure between the cathode and the anode. Grid-controlled heated-cathode arc tubes are called *thyratrons.* Typical characteristics relating anode current with anode voltage of a thyratron at several values of grid voltage are shown in Fig. 80. The on current in the low-voltage range is of the order of four to six magnitudes greater than the off current observed below the maximum-voltage point. If the anode voltage is gradually raised at fixed grid voltage, the current rises abruptly to a value determined by the circuit parameters when the anode voltage becomes equal to that corresponding to the maximum-voltage point of the characteristic. This voltage, which is called the *breakdown voltage,* increases as the grid is made more negative (or, in some tubes, less positive). If the anode voltage is maintained constant and the grid voltage is gradually made less negative (or more positive), the voltage peak of the characteristic is moved to the left. Breakdown occurs when the voltage of the peak becomes equal to the applied anode voltage. The grid voltage at which breakdown occurs is called the *critical grid voltage.* The critical grid voltage is obviously a function of anode voltage.

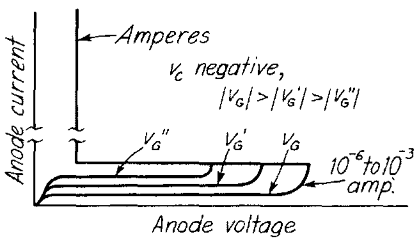


FIG. 80. General form of the current-voltage characteristics of a thyratron at several values of grid voltage.

The grid loses control of the anode current after the tube breaks down. In order to return the tube to its low-conduction state and allow the grid to regain control, the current must be reduced to a value in the off range for a sufficient time to allow the gas to become deionized. This may be accomplished by reducing the anode voltage to a value less than that in the high-current range of the characteristics or by opening the anode circuit completely. The minimum anode voltage at which the tube remains in its high-current (low-voltage) state is called the *extinction voltage.* The extinction voltage of a thyratron does not differ greatly from the first ionization potential of the gas or vapor.

23. Multistable Circuits

23.1. Definition of Multistable Circuit. A multistable circuit is one that, for fixed values of supply voltages and passive circuit parameters, has two or more stable states of equilibrium, characterized by different stable sets of circuit currents and voltages, flux densities, or frequencies of oscillation. Although most multistable circuits are bistable, circuits may be designed to have three or more stable states. One of the useful features of multistable circuits is that transition from one stable state of equilibrium to another may be caused to occur abruptly as the result of small impressed triggering voltages or currents or small changes in circuit parameters. Usually the desired output of a multistable circuit is a change of voltage or current caused by transition from one state of equilibrium to another, but changes of magnetic flux density, frequency, or other variables may be used. The following sections will treat only circuits in which the two or more states of equilibrium are characterized by differences in currents and voltages. Circuits in which the equilibrium states are characterized by differences in frequency of oscillation will be discussed in Sec. 75.3.

23.2. Graphical Explanation of Bistability. The existence of more than one state of equilibrium in circuits in which the equilibrium states are characterized by differences of currents and voltages may be explained by means of the simple series circuit of Fig. 81, in which N is a nonlinear circuit element having a current-voltage characteristic of one of the forms shown in Fig. 82. (The negative-slope ranges of the characteristics of Fig. 82 may have any position relative to the origin. Particular examples of such curves for a number of circuits and devices were shown in Secs. 18 to 22.) The line MN in Fig. 82 is the load line corresponding to the resistance R and the supply voltage V_S. By construction, the load line passes through the point on the

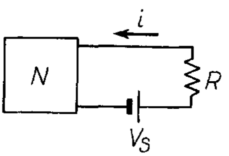


FIG. 81. Generalized bistable circuit.

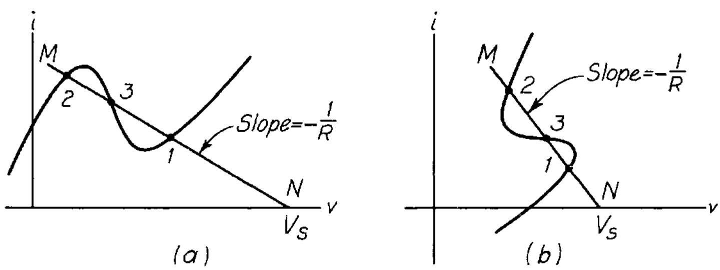


FIG. 82. Current-voltage diagrams for the circuit of Fig. 81: (a) for a voltage-stable element N; (b) for a current-stable element N.

voltage axis corresponding to the supply voltage, and the magnitude of its slope is $1/R$. The voltage corresponding to any point on this line is therefore $V_S - IR$, and at any point at which the load line intersects the characteristic of the nonlinear element the voltage v across the element satisfies the relation $v = V_S - iR$. Since Kirchhoff's law requires that this relation be satisfied in the circuit of Fig. 81, possible equilibrium values of current and voltage in the circuit are those corresponding to intersections of the load line and the current-voltage characteristic. If the current-voltage characteristic has a portion in which the slope is negative, as in Fig. 82, the load line may intersect the characteristic in three points, 1, 2, and 3, corresponding to three possible equilibrium sets of currents and voltages.

Additional intersections, and consequently additional states of equilibrium, may exist if the characteristic curve of the nonlinear element has additional points of inflection. In Sec. 24 it will be shown that the presence of reactive elements neglected in the simple circuit of Fig. 81 causes equilibrium states corresponding to point 3 or other intersections in the negative-slope ranges of the characteristic to be unstable, and that these states are therefore not observed in practical circuits. It will also be shown that the reactive elements are essential to transition from one stable state of equilibrium to another.

24. Stability of Equilibrium Points

24.1. Voltage-Stable Characteristic. Although the current-voltage characteristics of practical voltage-stable and current-stable circuits are curved in both the positive-resistance and negative-resistance ranges, it is convenient, in deriving stability criteria for the points of equlibrium, to make the assumption that the characteristics consist of three linear sections, as shown by the idealized voltage-stable characteristic of Fig. 83. Furthermore, although Eqs. (P-16) to (P-20) of Probs. 17.2–1 to 17.2–4 show that the magnitude of the real component of input impedance or admittance is a function of frequency and therefore of rate of change of current or voltage, the assumption will be made that the idealized static current-voltage characteristic gives the relation between the real component of input current and the input voltage, regardless of the rate of change of current and voltage. Under these assumptions, a voltage-stable circuit that is connected to a voltage source V_S through a series resistance R may be represented by the equivalent circuit of Fig. 84, in which the element N is a nonreactive, nonlinear element whose characteristic is that of Fig. 83, and C_i is the effective shunt input capacitance of the voltage-stable circuit.* Although C_i may be a function of frequency, as shown by Eq. (P-20) of Prob. 17.2–3, it will be assumed to be constant.

* Although some inductance is always present in series with R in any practical circuit, the ratio of R to this inductance is so great in bistable circuits that the inductance may be neglected in this analysis. The effect of large values of inductance will be considered in the analysis of relaxation oscillators, Sec. 45.

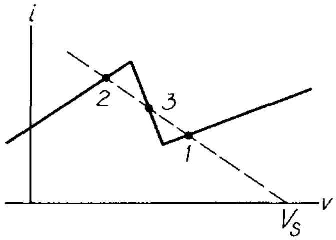


FIG. 83. Current-voltage diagram for an idealized voltage-stable element in the circuit of Fig. 81.

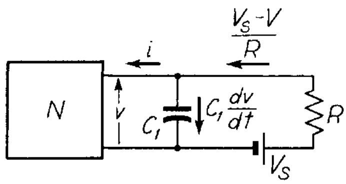


FIG. 84. Equivalent circuit for the circuit of Fig. 81 when N is a voltage-stable element.

The following differential equation may be written by inspection of Fig. 84:

$$\frac{V_S - v}{R} = i + C_i \frac{dv}{dt} \tag{25}$$

The current i in any of the three ranges of Fig. 83 is given by the relation:

$$i = I' + v/R_i \tag{26}$$

where I' is the current at the intersection of the current axis with the extension of the corresponding branch of the characteristic, and R_i is the reciprocal of the slope of that branch of the characteristic. Substitution of Eq. (26) into Eq. (25) and solution of the resulting differential equation yields the following expression for the voltage across the element at any time t:

$$v = V_o + A\epsilon^{-t(R+R_i)/RR_iC_i} \tag{27}$$

in which A is an arbitrary constant and

$$V_o = \frac{R_i}{R + R_i}(V_S - I'R) \tag{28}$$

By simple geometry, the voltage V_o may be shown to be the equilibrium value of voltage corresponding to an intersection of the load line of Fig. 83 with the applicable branch of the characteristic of the voltage-stable device. Whether an intersection corresponds to stable equilibrium or to unstable equilibrium may be determined by assuming that the voltage is $V_o + \Delta v$ when $t = 0$, where Δv is arbitrarily small. If these values of t and v are substituted into Eq. (27), the following expression is obtained for the voltage v at any subsequent time t:

$$v = V_o + \Delta v\epsilon^{-t(R+R_i)/RR_iC_i} \tag{29}$$

Equation (29) shows that, if R_i is positive or if R_i is negative but larger in magnitude than R, the exponent is negative and the voltage approaches the value V_o. Then V_o is a stable equilbrium value of voltage. On the other hand, if R_i is negative, but smaller in magnitude than R, the exponent is positive. A small positive or negative increment of voltage, however small, then initiates an exponential voltage change that carries the voltage away from that at the equilibrium point. Under this condition the equilibrium point is unstable. When the series resistance R is less than the magnitude of R_i in the negative-resistance range, the magnitude of the slope of the load line exceeds the slope of the negative-resistance portion of the charac-

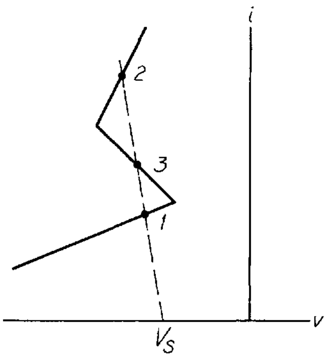


FIG. 85. Current-voltage diagram for an idealized current-stable element in the circuit of Fig. 81.

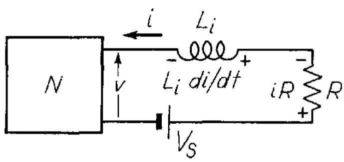


FIG. 86. Equivalent circuit for the circuit of Fig. 81 when N is a current-stable element.

teristic, and there is only one intersection. It follows from the foregoing criteria that this intersection is stable, regardless of its location.* When the magnitude of the series resistance R is greater than the magnitude of R_i in the negative-resistance range, the slope of the load line is less than that of the negative-resistance portion of the characteristic, and there may be either one or three intersections. The stability criteria show that if there is only one intersection, it is stable. If there are three intersections, the middle one, lying on the negative-resistance portion of the curve, is unstable, and the other two are stable. Thus, points 1 and 2 in Fig. 83 are stable points, whereas point 3 is unstable. If the circuit current and voltage are initially those corresponding to point 3, any small circuit disturbance, such as random noise, causes the current either to increase or to decrease continuously until intersection 1 or 2 is reached.

24.2. Current-Stable Characteristic. The current-stable circuit may be analyzed in a similar manner with the aid of the idealized characteristic

* A single intersection in the negative-resistance range is not necessarily stable if there is appreciable inductance in series with R_1. See Secs. 45 to 49.

of Fig. 85 and the equivalent circuit of Fig. 86.† If the current is $I_o + \Delta i$ when $t = 0$, where I_o is the current at an equilibrium point and Δi is arbitrarily small, the current at any time t is (Prob. 24.2-1):

$$i = I_o + \Delta i \epsilon^{-t(R+R_i)/L_i} \tag{30}$$

Equation (30) shows that i approaches I_o, and that the equilibrium point is therefore stable, if the value of R_i at that point is positive or if R_i is negative but smaller in magnitude than R. If R_i is negative and greater in magnitude than R, on the other hand, the second term of Eq. (30) grows and the equilibrium point is therefore unstable. Application of these criteria to Fig. 85 leads to the conclusion that, when the load line intersects the characteristic in only one point, that point is stable, regardless of its location.‡ When the load line intersects the characteristic in three points, however, the two outer intersections are stable, whereas the middle one, lying on the negative-resistance portion of the characteristic, is unstable. Thus, points 1 and 2 in Fig. 85 are stable, but point 3 is unstable.

24.3. Criteria for Bistability. It follows from the foregoing analyses that, if the circuit has a negative input resistance at a port, a bistable circuit may be formed by shunting the port by an external resistance. If the circuit is voltage-stable at that port, the external resistance must exceed the magnitude of the negative resistance; if the circuit is current-stable at that port, the external resistance must be smaller than the magnitude of the negative resistance. Application of these criteria to any port of the pi circuit of Fig. 65a shows that the circuit is bistable if the following relation is satisfied: §

$$h_f - h_r + h_i\left(G_1 + \frac{G_1G_2}{G_{12}} + G_2\right) + \frac{h_o}{G_{12}} + \left(1 + \frac{G_1}{G_{12}}\right)\Delta_h + \frac{G_2}{G_{12}} + 1 < 0 \tag{31}$$

in which $\Delta_h = h_ih_o - h_fh_r$. Equation (31) may also be written in the following form:

$$y_f + y_r + y_i\left(1 + \frac{G_2}{G_{12}}\right) + y_o\left(1 + \frac{G_1}{G_{12}}\right) + G_1 + \frac{G_1G_2}{G_{12}} + G_2 + \frac{\Delta_y}{G_{12}} < 0 \tag{32}$$

† Although there is always some shunt capacitance across R in any practical bistable circuit, the ratio of this capacitance to L_i is usually so small that the capacitance may be neglected in this analysis. The effect of large values of C will be considered in the analysis of relaxation oscillators, Secs. 45 to 49.

‡ A single intersection in the negative-resistance range is not necessarily stable if R is shunted by appreciable capacitance. See Secs. 45 to 49.

§ Since the circuit is either bistable or not bistable, the same relation must be obtained by application of these criteria at different ports.

in which $\Delta_y = y_i y_o - y_f y_r$. The tee circuit is bistable if

$$h_f - h_r + \frac{h_i}{R_{12}} + h_o\left(R_1 + \frac{R_1 R_2}{R_{12}} + R_2\right) + \left(1 + \frac{R_2}{R_{12}}\right)\Delta_h + \frac{R_1}{R_{12}} + 1 < 0 \tag{33}$$

$$y_f + y_r + y_i\left(1 + \frac{R_1}{R_{12}}\right) + y_o\left(1 + \frac{R_2}{R_{12}}\right) + \left(R_1 + \frac{R_1 R_2}{R_{12}} + R_2\right)\Delta_y + \frac{1}{R_{12}} < 0 \tag{34}$$

It is of interest that Eqs. (31) to (34) are also the criteria for loop current or voltage amplification greater than unity in the circuits of Fig. 65.

25. Transition Between States of Equilibrium

25.1. Change of Supply Voltage and Resistance. Figure 87 shows two ways in which an abrupt transition from one state of equilibrium to another may be made to occur in bistable circuits. The shapes of the current-voltage

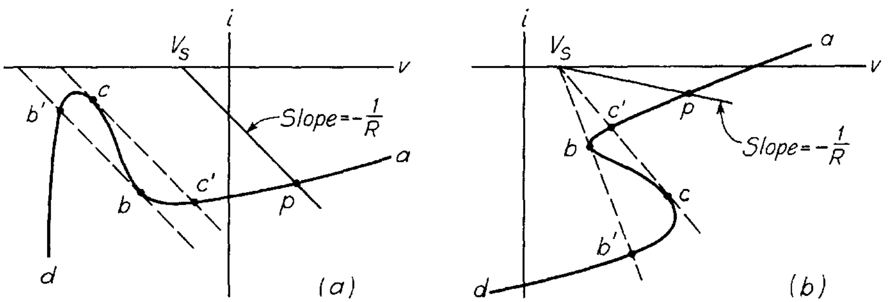


FIG. 87. (a) Transition initiated by change of supply voltage V_S; (b) transition initiated by change of resistance R.

characteristics have been purposely chosen so as to emphasize that the negative-resistance portion may have any position relative to the origin and that the characteristic need not pass through the origin. The initial impressed voltage and series resistance are those corresponding to the solid load line. In Fig. 87a the voltage V_S is assumed to be varied and the resistance to be maintained constant. As the impressed voltage V_S is made more negative, the intersection p moves along the branch ab of the characteristic toward b. When the load line becomes tangent at b, an infinitesimal positive increment of current through the negative-resistance circuit causes R_t to become less than R and the current to change abruptly to the value at b'. Further increase in the magnitude of V_S causes the intersection to move toward d. If V_S is now made progressively less negative, the intersection moves toward c, jumps from c to c', and then moves toward a. If V_S is maintained at such a value that there are three intersections, it is apparent that the current can be made to jump abruptly between the values corresponding to the upper and lower intersections by means of positive and negative voltage pulses of such polarities and magnitudes as to displace the load line alternately to the right of c and to the left of b.

Voltages higher than the impressed voltage V_S in the diagrams of Fig. 87 are made possible by a voltage source within the circuit whose current-voltage characteristic is shown.

Figure 87b shows that abrupt transitions of equilibrum values of current and voltage can also be made to occur by variation of the resistance R. The reader may readily show that similar behavior accompanies a vertical or horizontal displacement of the current-voltage characteristic or a change of slope of the negative-slope portion of the characteristic. In practical circuits, such changes in position or shape of the characteristic can be produced by changes of electrode voltages and currents of the active circuit elements.

25.2. Speed of Transition. It is seen from Eq. (29) that the rate of change of voltage during the transition from one state of equilibrium to another increases with the difference between R and the magnitude of negative R_i, and decreases with increase of C_i. Rapid transition is therefore favored by increasing R, and by decreasing C_i by reducing shunt electrode and circuit capacitances and raising the alpha-cutoff frequency of transistors (see Probs. 17.2–1 to 17.2–4). An insight into the physical reason why the speed of transition is favored by decreasing C_i and increasing R (decreasing the slope of the load line) can be gained from an examination of Fig. 84, which shows that at all times during a transition the current $C_i(dv/dt)$ through the capacitance C_i must equal the difference between the current $(V_S - v)/R$ through the resistance R and the current i through the voltage-stable circuit, as depicted in Fig. 88a. At first thought, it might appear that the instantaneous value of dv/dt at any value of v could be found from the difference between the ordinates of the load line and the characteristic and that a curve of current or voltage as a function of time could thus be plotted by a point-by-point method. However, it must be borne in mind that the analysis of this and the preceding section neglects the fact that the curve relating the inphase components of current and voltage is actually a function of frequency and that the slope of the current-voltage characteristic of Fig. 88a is consequently less for rapidly changing voltages than for incremental changes. The capacitance C_i is also in general a function of the rate of change of voltage, dv/dt.

Similarly, Eq. (30) shows that rapid transition in a current-stable circuit is favored by low L_i and by large difference between R and the magnitude of R_i. Consequently, rapid transition is favored by a decrease of R, by a decrease of shunt capacitances within the current-stable circuit, and by an increase in alpha-cutoff frequency of transistors (see Probs. 17.2–1 to 17.2–4). Examination of Fig. 86 discloses that, at all instants during transition, the voltage $L_i(di/dt)$ across L_i must be equal to the difference between $(V_S - iR)$ and v, as depicted in Fig. 88b.

Although low external resistance at a current-stable port is desirable to ensure rapid transition, biasing problems may necessitate the use of relatively large resistance. The equivalent effect of a low external resistance

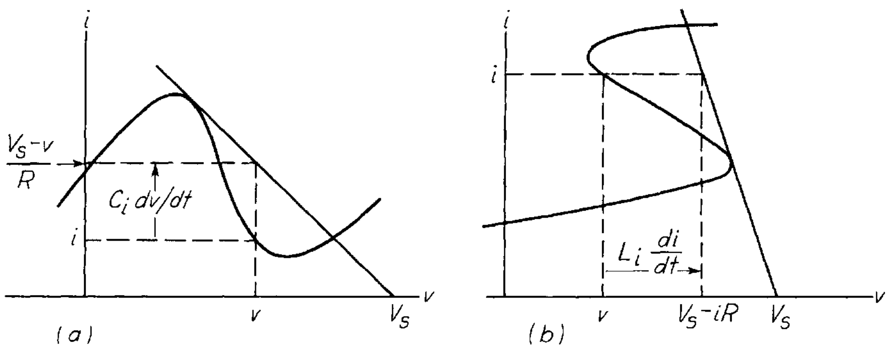


FIG. 88. (a) Current relations in the circuit of Fig. 84 during transition; (b) voltage relations in the circuit of Fig. 86 during transition.

may then be obtained by shunting the resistance by a small capacitance. Because the voltage across a capacitor cannot change instantaneously, the effect of the capacitance during transition is similar to that of a reduction of the external resistance. In general, the speed of transition is increased by the addition of shunt capacitance across any resistor in a bistable circuit if the capacitance increases the regenerative feedback during transition. In order to minimize the time required for the voltage across the capacitor to reach equilibrium subsequent to transition, and in order to reduce the likelihood of relaxation oscillation (Secs. 46 to 48), the capacitance should be no larger than necessary. Usually a capacitance of the order of 25 to 200 $\mu\mu$f is adequate.

25.3. Triggering by Rectangular Pulse. When a bistable circuit is triggered by means of an impressed voltage or current triggering pulse, transition continues to completion only if the duration of the pulse exceeds a critical value. This is shown by the curves of Fig. 89b, which were obtained for the circuit of Fig. 89a by means of an analog computer. The supply voltage V_S and resistance R were chosen so that the load line MN intersected the current-voltage characteristic in three points. With the initial values of current and voltage equal to those at point 1, a rectangular voltage pulse of magnitude Δv was impressed in series with V_S. The dashed curve shows the form of the static current-voltage characteristic. The solid curve shows the current i as a function of the voltage v for pulses of various durations. Curve a was obtained with a pulse of sufficient duration so that

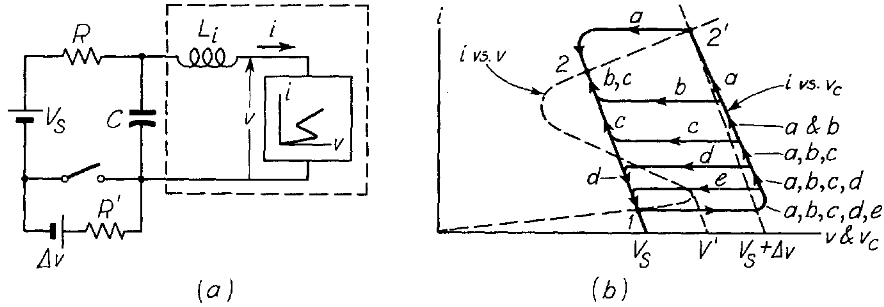


FIG. 89. (a) Circuit used to study transition in a bistable circuit based upon a current-stable element; (b) transition paths obtained by means of analog computer.

the current is equal to that at point 2′ when the pulse ends. Curves b, c, d, and e were obtained with successively shorter pulses. The latter curves depart from curve a at the instant the pulse is terminated. For each pulse amplitude, transition time is least if the pulse length is such that the current equals that at point 2 when the pulse ends. Similarly, for a fixed pulse length there is an optimum amplitude, as shown by Fig. 90a. Figure 90b shows that transition time decreases with increased pulse amplitude at optimum pulse length.

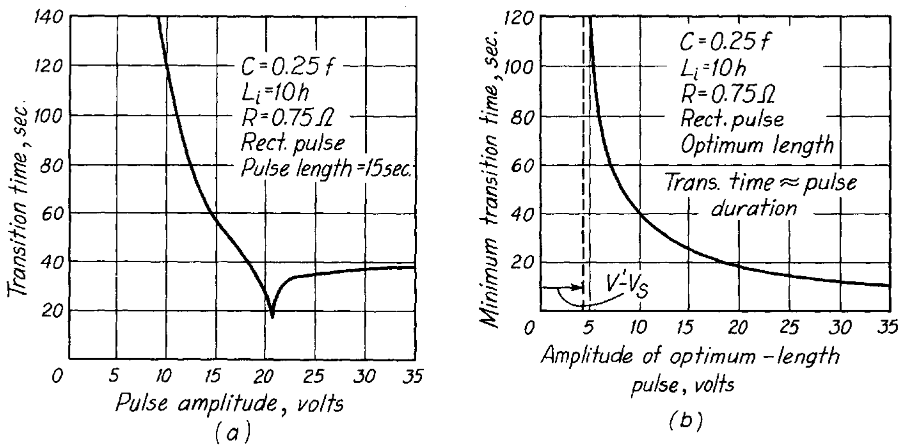


FIG. 90. (a) Experimental curve of transition time vs. amplitude of rectangular triggering pulse of fixed length for the circuit of Fig. 89a; (b) curve of minimum transition time vs. triggering-pulse amplitude at optimum pulse length.

Although the time of transition decreases as the pulse duration is increased, the fact that the pulse affects the manner in which the current varies with time sets an upper limit to desirable pulse length in some applications of bistable circuits. In particular, it may be undesirable to allow "overshooting" of the current to values above the final stable value at 2.

25.4. Triggering by Exponential Pulse. In most applications of bistable circuits the circuits are triggered by an exponential pulse generated

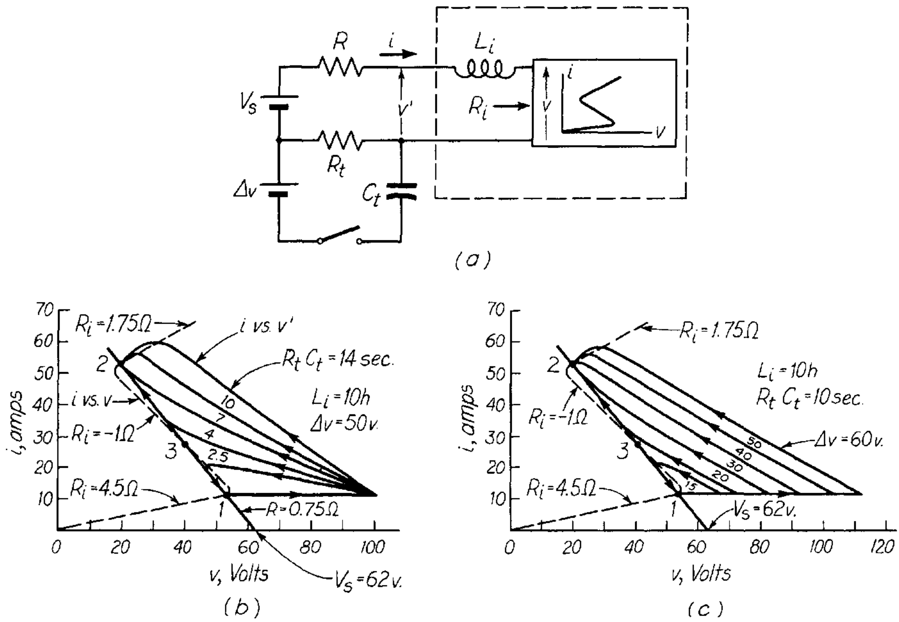


FIG. 91. (a) Circuit used to study transition in a bistable circuit based upon a current-stable element; (b) transition paths for exponential triggering pulses of fixed amplitude and several time constants; (c) transition paths for exponential pulses of fixed time constant and several amplitudes.

by differentiating a voltage step. A knowledge of the manner in which the pulse amplitude and time constant of the differentiating circuit affect the transition time is therefore of interest. Figures 91b and 91c show curves obtained for the circuit of Fig. 91a by means of an analog computer for various values of pulse amplitude Δv at fixed time constant R_tC_t and for various time constants at fixed pulse amplitude. The time required for the current to reach a value within an arbitrarily selected range encompassing the final stable value is plotted in Fig. 92 as a function of pulse amplitude at three values of time constant. Although the shapes of the curves of Fig. 92 are not necessarily typical of all bistable circuits, it seems likely that the following conclusions are generally applicable:

1. At any value of differentiating-circuit time constant R_tC_t there is an optimum pulse amplitude Δv that gives minimum transition time. The optimum pulse amplitude varies inversely with the time constant.

2. Since the transition time increase rapidly at pulse amplitudes below the optimum value, but increases relatively slowly at amplitudes that exceed a value only slightly higher than the optimum value, triggering-pulse amplitudes greater than the optimum value are desirable when short transi-

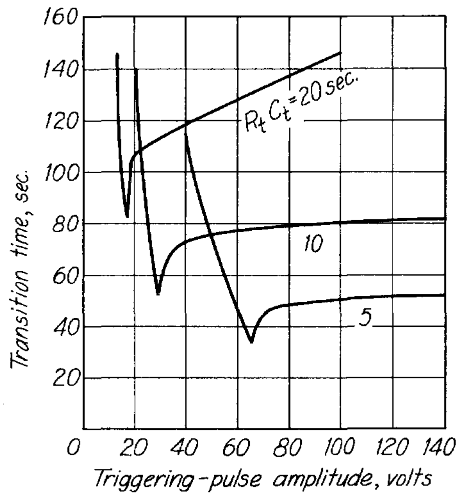


FIG. 92. Experimental curves of transition time vs. triggering-pulse amplitude for an exponential triggering pulse at three values of pulse time constant.

tion time is essential. Figure 91b shows, however, that pulse amplitudes greater than optimum value cause overshooting of the current to values above the final stable value. When overshooting is undesirable the pulse amplitude should be adjusted to a value only slightly greater than that of the knee of the curve of Fig. 92. The pulse amplitude corresponding to the knee varies inversely with the differentiator time constant.

3. The transition time at optimum pulse amplitude and at amplitudes above optimum decreases with decrease of time constant. When short transition time is essential, therefore, it is advisable to make the time constant of the differentiator small, even though greater triggering-pulse amplitude will be required.

If the voltage axis of Figs. 89 and 90 is relabeled current axis and vice versa, the current-voltage paths are typical of those obtained with a voltage-stable characteristic.

It can be seen from Fig. 88 that the decrease of transition time accompanying an increase of supply voltage, and therefore of triggering-pulse amplitude, is caused by the increase in the value of $L_i(di/dt)$ at all values of current in the current-stable circuit and of $C_i(dv/dt)$ at all values of voltage in the voltage-stable circuit.

26. Tube and Transistor Bistable Circuits

26.1. Synthesis of Bistable Circuits. Any of the negative-resistance circuits discussed in Secs. 18 to 22 may be used as the basis of bistable circuits. It is necessary only to choose the external resistance and supply voltage connected across a voltage-stable or current-stable port so that the load line intersects the current-voltage characteristic in three points. The circuit may be triggered by means of changes of circuit resistance or, more conveniently, by means of voltages applied in series with one or more electrodes of the active device or devices or currents sent between nodes of the circuit. Usually the smallest triggering voltage is required if the voltage is impressed upon the control grid of a tube or the base of a transistor. The voltage is conveniently applied through a small coupling capacitor, as shown in Fig. 101a. The speed of transition can usually be increased by the addition of small coupling capacitors in shunt with any coupling resistors connected across current-stable ports. Usually capacitors having capacitances of the order of 25 to 200 $\mu\mu$f are adequate. Since these capacitors must charge or discharge subsequent to transition, too large values of capacitance increase the time that must elapse before the circuit can respond properly to another triggering pulse. It will also be shown in Sec. 46 that relaxation oscillation may occur if these capacitances are too large.

Because of the relatively low electrode and circuit resistances of transistor circuits relative to tube circuits, greater current is required to trigger transistor circuits than tube circuits, and the triggering pulse should in general be obtained from a constant-current source, such as a transistor.

26.2. Criteria for Bistability. For many tube and transistor circuits it is possible to derive theoretical expressions for the negative resistance in terms of the tube and circuit parameters and therefore expressions for the minimum value of gain parameters of the tube or transistor below which the circuit cannot be bistable. Thus, the input resistance R_i between the screen and the cathode of the circuit of Fig. 93a may be quickly found from the formula for $(R_i)_2^\pi$ of Table I. In this tube connection y_i is the third-grid conductance, which may be assumed to be zero, y_f is the transconductance g_{23} from the third grid to the second grid, y_o is the output conductance g_{g2} of the second grid, and y_r is the reverse transfer conductance from the second grid to the third grid, which may also be assumed to be zero. Under the assumption that G_1 and G_2 are small in comparison with

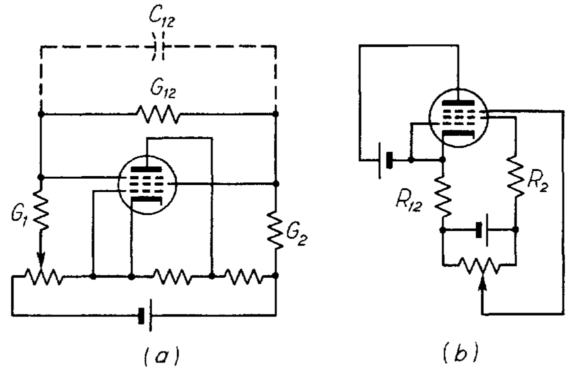


FIG. 93. Bistable circuits based upon the pentode negative-resistance circuits of Fig. 66.

g_{g2} and g_{23}, the input resistance (voltage-stable) between the screen and the cathode is given by the relation (Prob. 26.2–1)

$$\frac{1}{R_i} = g_{g2} + \frac{G_{12}}{G_{12} + G_1} g_{23} \tag{35}$$

In order for the circuit to be bistable, R_2 must exceed $|R_i|$. This is true if

$$|g_{23}| > \frac{G_{12} + G_1}{G_{12}} (g_{g2} + G_2) \tag{36}$$

Equation (36) can also be obtained by substituting the appropriate values of the y parameters into Eq. (32). The capacitor C_{12} speeds transition by increasing the coupling from the screen to the suppressor for rapidly changing voltages.

By substituting the common-cathode parameters of Table III into Eq. (34), the reader may show that the circuit of Fig. 93b is bistable if (Prob. 26.2–2)

$$|g_{23}| > \frac{1}{r_{g2}} + \frac{r_{g2} + R_2}{r_{g2}R_{12}} \tag{37}$$

in which r_{g2} is the variational resistance $1/g_{g2}$ of the screen grid. It is apparent from Eq. (37) that reduction of R_2 is favorable to bistability and that R_2 can therefore be made zero unless R_2 is needed in order to provide output voltage. A resistor may also be added in series with the plate for the same purpose in either of the circuits of Fig. 93. The circuits function best when the control-grid voltage is zero or slightly positive. The circuits may be adjusted so that one of the stable values of plate current is zero, as can be seen from Fig. 66d.

26.3. Switching Diodes and Triodes; Thyratron. An important feature of the characteristics of *p-n-p-n* diodes and triodes and of thyratrons is that the current may be of the order of only a microampere in the low range and of the order of an ampere or more in the high range, and may be quickly switched from the low or off value to the high or on value. Figure 94, in which *p-n-p-n*-triode current-voltage characteristics are plotted for various values of base current, shows that current switching from the off

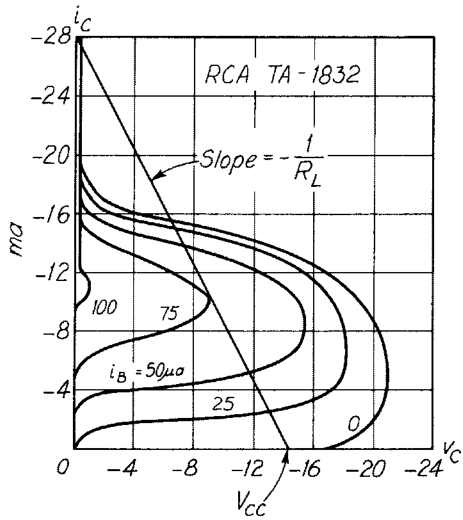


FIG. 94. Collector characteristics of a *p-n-p-n* transistor in the common-emitter connection at five values of base current.

state to the on state can be initiated by an increase of base current.[1] The supply voltage V_{CC} and load resistance R_L are chosen so that the load line intersects the zero-base-current characteristic in the low-current range. If the base current is increased sufficiently so that the current-voltage characteristic is tangent to the load line or intersects the load line only in the high-current range of the characteristic, the current increases rapidly to the on value. By proper adjustment of the circuit, the base current required to initiate switching may be made as small as a few microamperes. A diagram similar to Fig. 94 may be constructed for the thyratron characteristics of Fig. 80.

The *p-n-p-n* switching transistor has the important advantage over the thyratron that the voltage drop across the transistor in the high-current range is only of the order of a volt, as compared with 8 to 10 volts in a

[1] I. M. Mackintosh, *Trans. I.R.E.*, Vol. ED-5, 10 (January, 1958); I. A. Lesk, *Trans. I.R.E.*, Vol. ED-6, 28 (January, 1959).

thyratron. An additional advantage of the switching transistor is that it does not require cathode-heating current. Characteristics similar to those of the *p-n-p-n* transistor are also observed in some types of *p-n-p* transistors.[2]

Large values of load current cannot be interrupted by means of the base current of a *p-n-p-n* transistor or by means of the grid voltage of a thyratron. In order to turn the collector current of a switch transistor off, the base current must be made low enough so that the off portion of the characteristic intersects the load line, and the collector current must be reduced to a value within the off range for a sufficient time for the diffusion of stored carriers in the transistor. Similarly, the grid voltage of a thyratron must be made sufficiently negative so that the load line intersects the off portion of the characteristic and the current must be reduced to a value in the off range for sufficient time for the gas or vapor to deionize.

26.4. Fully Controllable Switching Devices. A semiconductor device similar to the *p-n-p-n* switching transistor, in which a metallic contact is used in place of one of the *n*-type semiconductor layers, can not only be turned on by means of base current, but can also be turned off.[3] The reverse base current required to stop the collector current in this device, called the *p-n-p-m* transistor, is less than the collector current, power gains of the order of 20 db being observed in turning the current off. The minimum voltage drop may be as low as 0.6 volt at a current density of 100 amperes per sq cm. In the high-current range the dynamic resistance is of the order of only a few hundredths of an ohm. A diode unit of this type only 2.5 mm in diameter has a breakdown voltage of 350 volts and can switch 10 amperes when suitable means are provided for heat dissipation. The same unit can carry 500-ampere, 5-μsec pulses. Larger units have switched 25 amperes, and higher-current units are feasible. The switch-on time is about a tenth of a microsecond and decreases with increase of triggering-pulse amplitude.

Another three-terminal semiconductor negative-resistance device, called the *thyristor*, has the very desirable feature that collector currents up to 100 ma or higher can be cut off by means of reverse base voltage with the expenditure of very little control power.[4] The desirable characteristics of the thyristor are obtained by the use of a special collector contact, which collects holes at low collector currents but emits electrons at high collector currents. The voltage drop in the high-current range is of the order of 0.5 volt and the resistance is about 3 ohms. The off current is approximately 2 μa. The device can be turned on by a 50-μsec pulse with an energy ex-

[2] C. G. Thornton and C. D. Simmons, *Trans. I.R.E.*, Vol. ED-5, p. 6 (January, 1958).
[3] J. Philips and H. C. Chang, *Trans. I.R.E.*, Vol. ED-5, p. 13 (January, 1958).
[4] C. W. Mueller and J. Hillibrand, *Trans. I.R.E.*, Vol. ED-5, p. 2 (January, 1958).

penditure of approximately 10^{-4} erg and can be turned off by a 100-μsec pulse with energy expenditure of 0.1 erg. The current-voltage characteristics are similar to the *p-n-p-n*-transistor characteristics of Fig. 94.

26.5. Limitation of Power Dissipation by Use of Diode. When a negative-resistance element of relatively low power-dissipation capability, such as a unijunction transistor, is used to switch the current through a

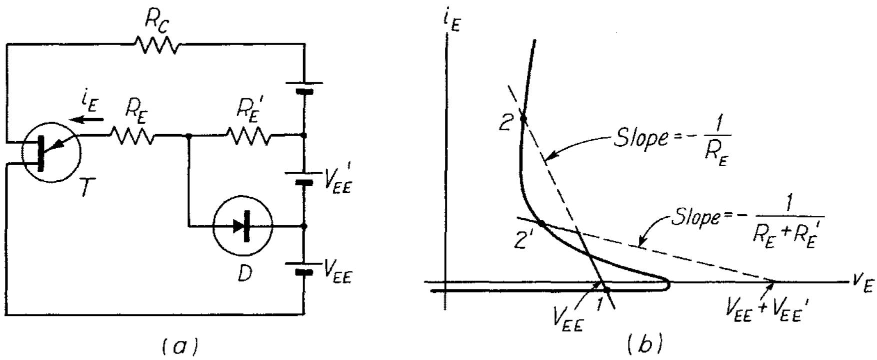


FIG. 95. (a) Unijunction-transistor bistable circuit in which the upper stable value of emitter current is limited by means of an added resistance R_E', shunted by a diode D; (b) current-voltage diagram for the circuit.

load of fixed resistance, the load resistance may be so low that the on current will cause the dissipation rating of the element to be exceeded. Figure 95a shows how this difficulty may be avoided in a unijunction-transistor circuit by the addition of a diode D and a resistor R_E' to the circuit.

Because the forward resistance of the diode is very small in comparison with the emitter-circuit resistances R_E and R_E', the emitter current is approximately the same as it would be if the diode and R_E' were omitted and R_E connected directly to the positive side of V_{EE}. For the same reason, the diode current is very nearly $V_{EE}'/R_E' - i_E$. If i_E exceeds V_{EE}'/R_E', the diode current must reverse and the diode resistance becomes its reverse resistance, which is very high in comparison with $R_{E'}$. As the emitter current rises above the value V_{EE}'/R_E', the emitter-circuit resistance therefore changes from approximately R_E to approximately $R_E + R_E'$. The addition of D and R_E' to the circuit consequently reduces the "on" emitter current from the value corresponding to point 2 of Fig. 45b to that corresponding to point $2'$. Although this same value could be obtained by the addition of R_E' and V_{EE}' without the diode, the current corresponding to point $2'$ would then be the only equilibrium value.

27. The Eccles-Jordan Circuit

27.1. Basic Circuit. The bistable circuit that is best known and probably has the greatest number of applications is the Eccles-Jordan circuit, which is the pi circuit of Fig. 68a. The following criterion for bistability of this circuit may be obtained by substituting the applicable y parameters into Eq. (32) (Prob. 27.1–1):

$$g_m > \frac{R_1 + R_{12}}{R_1}\left(\frac{1}{r_p} + \frac{1}{R_2} + \frac{1}{R_1 + R_{12}}\right) \tag{38}$$

Under the assumption that the conductance of the series-parallel combination of R_1, R_{12}, and R_2 is small in comparison with h_{oe}, a similar procedure leads to the following expression for the bistability criterion of the transistor version of the circuit of Fig. 68a (Prob. 27.1–2):

$$h_{fe} > \frac{h_{ie}(R_1 + R_{12} + R_2) + R_1(R_{12} + R_2)}{R_1R_2} \tag{39}$$

Equations (38) and (39) may also be obtained in a number of other ways. (See, for example, Probs. 27.1–3 and 27.1–4.)

Practical tube and transistor versions of the Eccles-Jordan circuit are shown in Fig. 96. It can be seen from an examination of Fig. 96a that two

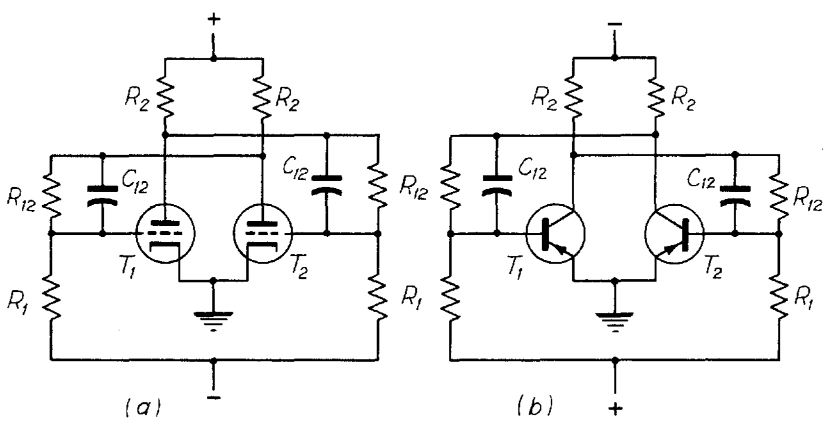


FIG. 96. (a) Vacuum-tube Eccles-Jordan circuit that requires separate grid and plate voltage supplies; (b) corresponding transistor circuit.

stable states must exist in this circuit if the voltage drop across the plate resistor R_2 produced by plate current in one tube depresses the grid voltage of the other tube sufficiently to cut the other tube off. Tube 1 then conducts in one stable state and tube 2 in the other. Transition from one stable state to the other can be initiated by impressing upon the circuit a voltage or current increment that raises the grid voltage of the off tube above cutoff, and in other ways, which will be discussed. Thus, if tube 1 is initially on and tube 2 is off, a pulse that raises the grid voltage of tube 2 above cutoff initiates plate current in tube 2 and consequently lowers the grid voltage of tube 1. The resulting reduction of plate current of tube 1 is accompanied by a reduction in the voltage drop across the plate resistor of tube 1, and therefore in further rise of grid voltage of tube 2. If the magnitude and duration of the triggering pulse are sufficient to raise the grid voltage of tube 2 far enough above cutoff so that the open-loop voltage amplification exceeds unity, the action becomes cumulative, terminating when tube 1 is cut off.

The value of R_2 may lie between the order of 1000 ohms and 100,000 ohms, depending upon the tubes used, the purpose for which the circuit is designed, the desired stability, and other practical considerations. Usually R_1 and R_{12} are approximately equal and considerably larger than R_2, but this is not necessarily so, provided the resistance of the parallel combination of R_2 and $R_1 + R_{12}$ is high enough to ensure bistability (Eqs. (38) and (39)).

The coupling capacitors C_{12} increase the coupling from the plate of one tube to the grid of the other for rapidly changing plate voltage and therefore reduce the transition time. (Because time is required for the capacitor to charge or discharge, a rapid change of plate voltage of one tube is accompanied by an almost equal change of grid voltage of the other tube.) They also aid in commutation when the circuit is triggered by successive pulses of one polarity. These functions will be discussed in detail. The value of C_{12} usually lies in the range from 25 $\mu\mu$f to 200 $\mu\mu$f.

27.2. Transistor Eccles-Jordan Circuit. The transistor circuit of Fig. 96b functions similarly to the tube circuit. However, the collector current of the off transistor is not zero, but small. The relatively high input conductance of transistors in comparison with tubes necessitates the use of lower circuit resistances than in the tube circuit of Fig. 96a. The shape of the collector characteristics of junction transistors is favorable to the use of transistors in Eccles-Jordan circuits. The collector voltage of the conducting transistor being of the order of a volt or less in the saturation range of collector current, the collector dissipation of the conducting transistor is small. Although the instantaneous collector dissipation may be high during transition, the transition time is so short that overheating is not likely to occur. Another consequence of the low collector voltage of the conducting transistor is that a large fraction of the supply voltage is available as output voltage. The small size, weight, and power requirements of transistors are obvious advantages in devices that require large numbers of bistable circuits, such as computers. On the other hand, the output voltage obtainable from transistor circuits is limited by the possibility of transistor breakdown. The values of R_1, R_{12}, and R_2 of transistor circuits are usually smaller than those of tube circuits.

Surface barrier and some alloyed-junction transistors cut off at a positive base voltage. When such transistors are used in the Eccles-Jordan circuit, the base of each transistor may be directly coupled to the collector of the other, without the use of a coupling network.[1]

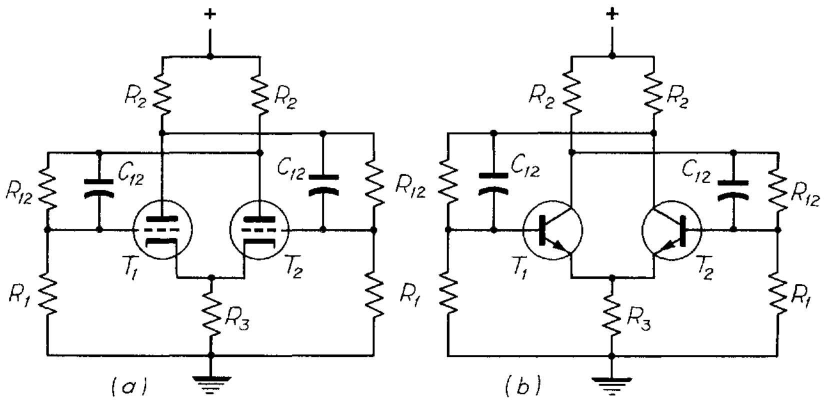


FIG. 97. Vacuum-tube and transistor Eccles-Jordan circuits that require only one voltage supply.

The need for a separate source of grid voltage is avoided in the tube circuit of Fig. 97a, which makes use of a cathode resistor R_3 common to both tubes. The rise of cathode current of one tube being accompanied by a fall of cathode current of the other, the variation of voltage across the cathode resistor is small, and a bypass capacitor across R_3 is unnecessary. It will be shown in Sec. 28 that a bypass capacitor may, in fact, sometimes be undesirable. A transistor counterpart of the tube circuit of Fig. 97a is shown in Fig. 97b.

27.3. Need for Coupling Capacitors. The voltage dividers consisting of the resistors R_1 and R_{12} in the circuits of Figs. 96 and 97 provide the proper grid or base bias. However, these resistors not only serve as dividers for constant voltages, but in conjunction with the interelectrode capacitances they act as dividers for changes of the plate voltages. Unless the grid or base supply voltage is made inconveniently large, R_{12} cannot be made small in comparison with R_1, and only a fraction of the change of plate or col-

[1] R. H. Beter, W. E. Bradley, R. B. Brown, and M. Rubinoff, *Electronics*, June, 1955, p. 132.

lector voltage of one tube or transistor is applied to the grid or base of the other. The reduction of coupling is partially prevented by the coupling capacitors C_{12}. Because these capacitors cannot charge or discharge instantaneously, rapidly changing plate voltages are accompanied by nearly equal changes of grid voltage. The maximum capacitance that can be used for C_{12} may be limited because increase of this capacitance increases the time required for the voltage across the capacitors to reach equilibrium subsequent to transition and thus raises the minimum interval that must elapse before the circuit will respond to another triggering pulse.

27.4. Use of Breakdown Diodes For Coupling. The proper grid or base voltage may be provided with negligible loss of coupling by the use of breakdown diodes in place of the coupling resistors R_{12}, as shown in Fig. 98.[2] Figure 22a shows that a breakdown diode acts like a constant-voltage source in series with a low resistance at reverse voltages above the breakdown value. The diodes in the circuit of Fig. 98 are selected to have a breakdown voltage equal to the required difference between the direct voltages of the collector and the base (or of the plate and the grid). Because of the relatively low variational resistance of the diodes, almost the entire change of plate or collector voltage is applied to the base (or the grid). The close coupling achieved by the use of breakdown diodes is favorable both to rapid transition and to static stability. Breakdown diodes lend themselves particularly to use in transistor bistable circuits, in which the voltages are usually relatively low. The function of the resistances R_3 and capacitances C_3 will be discussed in Sec. 29.3.

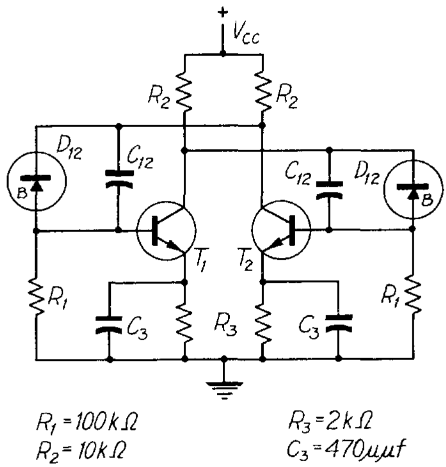


FIG. 98. Diode-coupled transistor Eccles-Jordan circuit.

27.5. Variants of The Eccles-Jordan Circuit. Figure 99a shows a modification of the circuit of Fig. 96a in which the tubes are tetrodes or pentodes and the plate of each tube is coupled to the screen grid of the other, instead of to the control grid.[3] The relatively high positive voltage at which the screen grids operate makes possible direct coupling to the

[2] J. G. Linvill, *Proc. I.R.E.*, **43**, 826 (July, 1955).

[3] J. M. Blair, *Rev. Sci. Instr.*, **14**, 64 (1943); V. H. Regener, *Rev. Sci. Instr.*, **17**, 180 (1946).

plates, instead of through coupling resistors. The control grids are used only for triggering the circuit. The advantage of increased simplicity of this circuit is offset by the low screen-plate transconductance, which makes the circuit more difficult to design and adjust, reduces the stability of the stable states of equilibrium, and increases the time required for transition, as will be shown.

In another modification of the Eccles-Jordan circuit using pentodes, the plate of each tube is coupled to the suppressor grid of the other, as shown

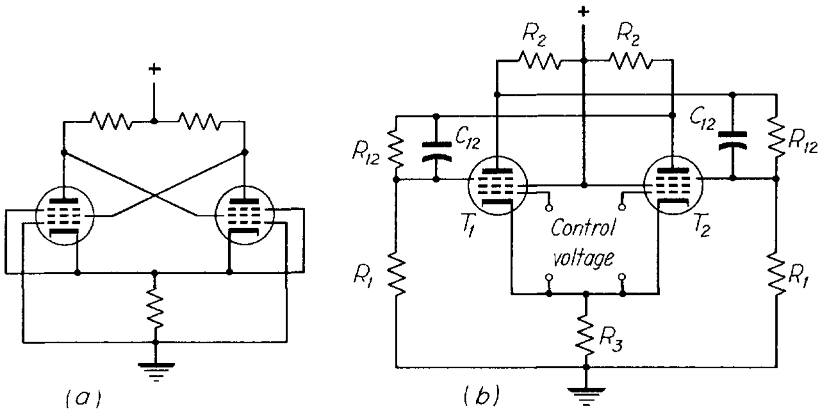


FIG. 99. (a) Screen-plate-coupled pentode Eccles-Jordan circuit; (b) suppressor-plate-coupled pentode Eccles-Jordan circuit.

in Fig. 99b.[4] Because the high negative voltage of the suppressor grid of the nonconducting tube relative to the cathode cuts off plate current regardless of the value of the control-grid voltage, the circuit is not triggered by the application of positive voltage to the control grid of the nonconducting tube. Application of negative voltage to the control grid of the conducting tube reduces its plate current and thus triggers the circuit. Use of the suppressor grids for coupling has the same disadvantages as use of the screen grids, but the selective response of the circuit to negative control voltage applied to the control grid of the conducting tube has advantages in certain applications.

In still another variant of the symmetrical Eccles-Jordan circuit the control grid of each pentode is coupled to the screen grid of the other, the suppressor grids being tied to the cathodes as in pentode voltage amplifiers, and output being taken from the plates. This form of the circuit has the advantage that the plate current is less dependent upon the control-grid voltage at positive values of control-grid voltage. This is a desirable char-

[4] H. J. Reich, *Rev. Sci. Instr.*, **9**, 222 (1938).

acteristic in the use of the circuit as the basis of a relaxation oscillator (see Sec. 51.4).

27.6. Cathode-Coupled Eccles-Jordan Circuit. A useful unsymmetrical form of the Eccles-Jordan circuit is shown in Fig. 100, which is based upon the cathode-coupled negative-resistance circuit of Fig. 70.[5] The coupling from the plate circuit of tube T_2 to the grid of tube T_1 takes place through the cathode resistor R_3. Since the grid voltage of tube T_1 can change only as the result of a change in current through R_3, the current carried by tube T_2 when tube T_1 is cut off must be greater than that carried by tube T_1 when tube T_2 is cut off. The proper degree of unbalance in plate currents is achieved by adjustment of the voltage applied to the grid resistor of tube T_1 and by the use of unequal plate load resistances R_2. The resistor in the plate circuit of tube T_2 is not essential to the switching action, but may be used to produce voltage output. Transistors in the common-emitter connection may also be used in the circuit of Fig. 100. Although the lack of symmetry of the cathode-coupled or emitter-coupled circuit may sometimes be objectionable, the circuit finds a number of important applications, which will be discussed in later sections.

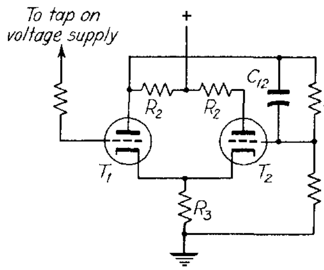


FIG. 100. Cathode-coupled Eccles-Jordan circuit.

28. Triggering of the Eccles-Jordan Circuits

28.1. Methods of Triggering. Eccles-Jordan circuits are usually triggered by means of voltages applied to one of the grids or plates through a small coupling capacitor, as shown in Fig. 101a. The capacitor C_t, in conjunction with the resistors of the bistable circuit, acts as a differentiating circuit. A step function or a rectangular or trapezoidal input pulse is thus converted into one or more short pulses of grid or plate voltage. The effect is equivalent to that of a pulse or pulses impressed in series with one of the electrodes. As explained in Sec. 25.4, if short transition time is essential, the pulse amplitude and the time constant of the differentiating circuit must be properly chosen, large pulse amplitude and small time constant usually being preferable to small pulse amplitude and large time constant. When the coupling capacitances C_{12} are large in comparison with the interelectrode capacitances and the time constant $R_{12}C_{12}$ is large in

[5] O. H. Schmitt, *J. Sci. Instr.*, **15**, 24 (1938).

comparison with the transition time, little change of voltage across the coupling capacitors takes place during transition, and the action is essentially the same whether the input capacitor C_t is connected to the grid of one tube or to the plate of the other.

28.2. Comparison of Negative and Positive Triggering. The circuit may be triggered by either a negative pulse applied to the grid of the conducting tube (or the plate of the nonconducting tube) or a positive pulse applied to the grid of the nonconducting tube (or the plate of the conducting tube), but the required magnitude of a negative pulse is usually considerably smaller than that of a positive pulse. Transition starts when the grid voltage of the nonconducting tube rises above cutoff. The amplitude of a positive pulse applied to the grid of the nonconducting tube must therefore exceed the difference between the equilibrium grid voltage of the nonconducting tube and the cutoff voltage. A negative pulse applied to the grid of the conducting tube, on the other hand, is amplified by the conducting tube. For a negative pulse applied to the grid of the conducting tube, the amplitude required to raise the grid of the nonconducting tube above cutoff is smaller than the required amplitude of a positive pulse applied to the grid of the nonconducting tube by a factor equal to the voltage amplification of the conducting tube. Although a trapezoidal voltage impressed upon the input of the circuit of Fig. 101a may cause the application of pulses of both polarities to the grid of T_1 if the time constant of the differentiator is short in comparison with the duration of the trapezoidal pulse, the difference in the response of the bistable circuit to positive and negative triggering pulses prevents retriggering at the termination of a negative trapezoidal voltage if the amplitude of the trapezoidal voltage is not too great.

When the circuit of Fig. 97 is triggered by means of a negative voltage applied to the grid of the conducting tube, less triggering voltage is required if the cathode resistor R_3 is not bypassed. The change in voltage across R_3 prior to conduction of the initially nonconducting tube is such as to lower the cathode voltage of that tube. The magnitude of the grid-cathode voltage of the initially nonconducting tube is thus reduced not only because of the rise in the voltage of the grid, but also because of the fall in voltage of the cathode.

29. Commutation of the Eccles-Jordan Circuit

29.1. Commutation by Coupling Capacitors. In many applications of the Eccles-Jordan circuit, the circuit must be triggered back and forth by means of periodic or random pulses of only one polarity, so that each tube conducts following alternate pulses. This may be accomplished by applying the pulses to the circuit in such a manner as to cause both tubes to be cut off throughout the duration of the pulses. The simplest method is to apply negative pulses simultaneously to both grids or positive pulses to both cathodes through small coupling capacitors (Figs. 101b and 101c). Unless some means is provided to ensure commutation, however, either tube may conduct at the termination of each pulse, and transition therefore

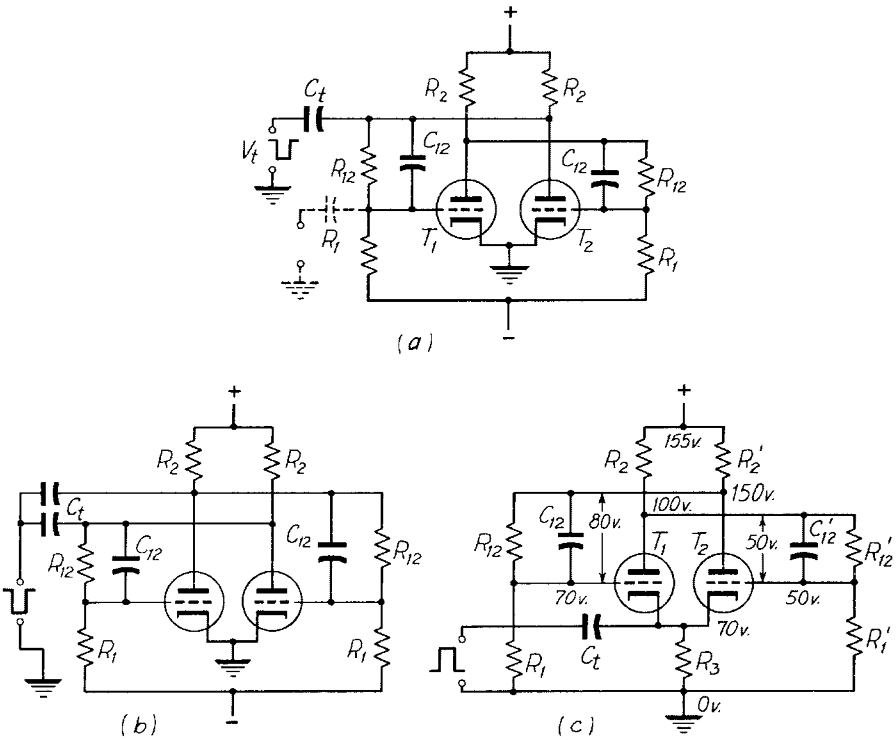


FIG. 101. Three methods of applying triggering pulses to an Eccles-Jordan circuit: (a) noncommutating circuit; (b) and (c) commutating circuits.

occurs in a random manner. If the coupling capacitances C_{12} are properly chosen, they ensure that the tubes conduct alternately.

The way in which the coupling capacitors ensure commutation can be readily explained with the aid of Fig. 101c. Tube T_1 is assumed to be conducting initially, and the tube and circuit parameters are assumed to have values such that the voltages of the electrodes relative to ground have the values shown. (The flow of grid current through R_{12} prevents the grid voltage of the conducting tube from rising greatly above the cathode voltage.) When a positive triggering pulse is applied to the cathodes, T_1 is cut off and its plate voltage rises to nearly 150 volts. (Charging current into C_{12}'

prevents the voltage from rising immediately to 150 volts.) The voltage across C_{12}' immediately starts rising toward 80 volts, but if the duration of the triggering pulse is not too great, there is still considerable difference between the voltages across C_{12} and C_{12}' at the termination of the pulse. Consequently the grid voltage of T_2 at the termination of the triggering pulse is higher than that of T_1 and more plate current flows in T_2 than in T_1. The greater voltage drop across R_2' than across R_2 increases the difference between the grid voltages, and cumulative action rapidly causes T_1 to be cut off. In order to ensure sufficient difference between the voltages across C_{12} and C_{12}' at the termination of the triggering pulse, the time constant $R_{12}C_{12}$ must be large in comparison with the triggering-pulse duration. C_{12} should also be large in comparison with the interelectrode capacitances. However, in order to ensure that the voltage across C_{12} can approach equilibrium in the time between the termination of transition and the application of the next triggering pulse, the time constant should be small in comparison with the minimum repetition period of the triggering pulses. Usually values of C_{12} in the range between 25 to 200 $\mu\mu$f are adequate.

In the circuit of Fig. 98 the coupling diodes maintain the voltage across the coupling capacitors practically constant. These capacitors therefore cannot aid in the computation process, but this function is served by the emitter-circuit capacitors C_3. For reliable commutation, the time constant R_3C_3 must be large in comparison with the duration of the triggering pulse.

29.2. Use of Diodes to Aid Commutation. Reliability of commutation can be greatly increased by impressing the triggering pulses upon the circuit through biased diodes, as shown in Fig. 102. In the circuit of Fig. 102a the biasing voltage applied to the high-resistance input resistor R_t is chosen so that when triode T_1 is conducting, diode T_3 conducts, but diode T_4 is cut off. A rapidly rising positive input pulse is coupled to the grid of T_2 through C_t and C_{12}'. If the triggering-pulse amplitude is great enough, conduction is initiated in T_2. It was shown in Sec. 18.1 that a negative input resistance exists between either plate and ground when both tubes of an Eccles-Jordan circuit are active, and that the plate current falls rapidly with increasing impressed plate voltage in this range (Fig. 69a). Although the rise in plate voltage of T_1 produced by the triggering voltage also causes some reduction of the current i flowing through R_2 from the voltage supply, the rate of fall of i is less than the rate of fall of plate current i_{p1}. At some value of the rising input voltage, the plate current i_{p1} becomes equal to $i - i'$ and the input current i_i is zero. Since T_3 does not conduct in the reverse direction, T_3 thereafter isolates the circuit from the source of triggering voltage and transition proceeds without interference from the triggering voltage or loading by the input circuit. When the plate voltage of T_2 falls below the plate voltage of

T_4, T_4 starts to conduct, connecting the input circuit to the plate of T_2. The point in the transition at which this occurs depends upon the form of the input pulse and upon the time constant R_tC_t. The difference in the volt-

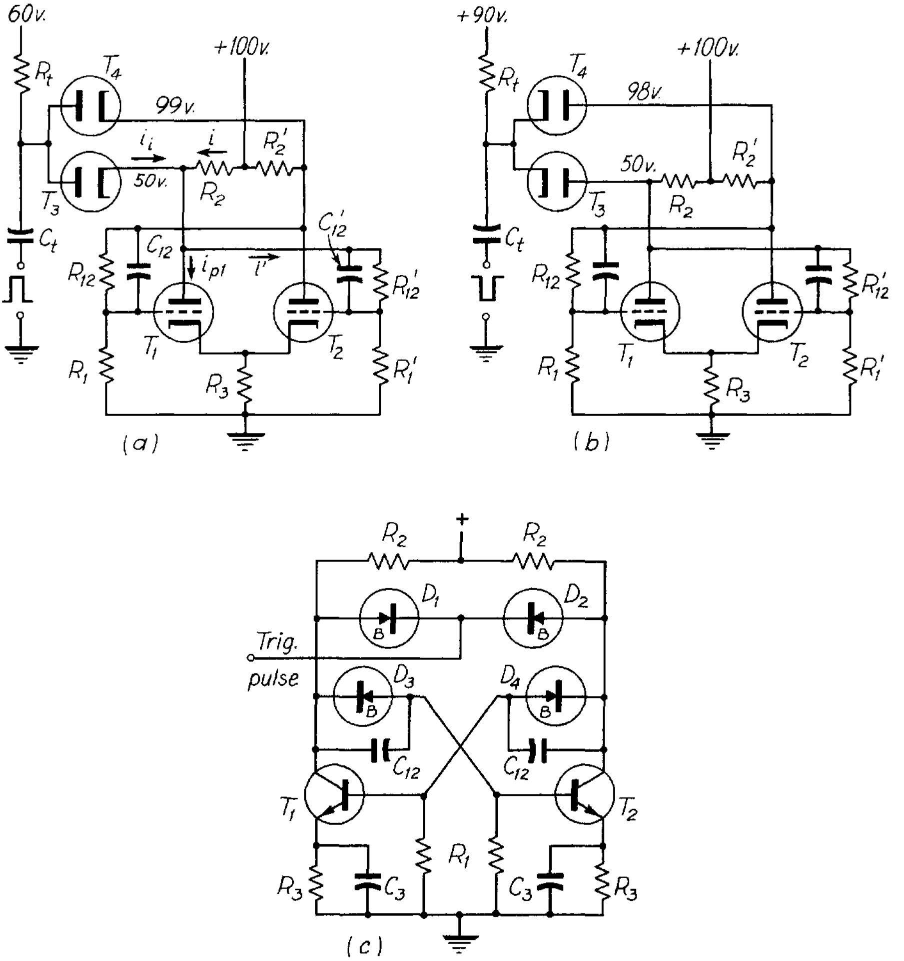


Fig. 102. Circuits in which diodes are used to increase the reliability of commutation: (a) tube circuit triggered by positive pulses; (b) tube circuit triggered by negative pulses; (c) transistor circuit. In (a) $R_1 = R_{12} = 100$ kΩ, $R_2 = 10$ kΩ, $R_3 = 2$ kΩ, $C_{12} = 4$ μf, and $C_3 = 470$ μμf.

ages across the coupling capacitors C_{12} and C_{12}' is such as to aid in the completion of the transition process, as already explained.

In the circuit of Fig. 102b, the diode polarities are opposite to those in the circuit of Fig. 102a and the biasing voltage applied to the input resistor R_t

is lower than the voltage of the nonconducting triode. When T_1 conducts, diode T_4 conducts and T_3 is cut off. A negative pulse applied to the input terminals depresses the grid voltage of the conducting triode T_1, and the resulting rise in the plate voltage of T_1 raises the grid voltage of T_2, initiating conduction in T_2. The remainder of the triggering process is similar to that in the circuit of Fig. 102a. In both circuits the diodes may be connected to the grids, instead of to the plates, the biasing voltage applied to R_t again being chosen so that only one diode conducts in each state of equilibrium. Both circuits can also be used with transistors, and the diodes may be either tubes or semiconductor diodes.

The resistance R_t in the circuits of Fig. 102a and 102b should be large in comparison with R_2, but must be small in comparison with the back resistance of the diodes. The time constant R_tC_t should be small enough in comparison with the repetition period of the triggering pulses to allow the input circuit to "settle" between pulses, but long enough to ensure that the plate voltage of the initially conducting triode rises sufficiently so that the initially conducting diode cuts off and disconnects the input circuit. The magnitude and rate of rise of the triggering voltage must also be sufficient to ensure that the initially conducting diode cuts off, but not so great as to cause both diodes to conduct simultaneously.

29.3. Use of Breakdown Diodes to Aid Commutation. Another commutating circuit that is particularly suitable for use with transistor Eccles-Jordan circuits is shown in Fig. 102c. In this circuit, breakdown diodes D_1 and D_2 are used in place of conventional diodes.[1] In each stable state of equilibrium one of these diodes conducts in the forward direction and the other is broken down and conducts in the reverse direction. Thus, if transistor T_1 conducts and the collector voltage of T_2 exceeds that of T_1 by more than the diode breakdown voltage, D_1 breaks down and conducts in the reverse direction, and D_2 conducts in the forward direction. Application of a positive pulse to the input terminals cuts D_2 off if the pulse amplitude exceeds the voltage that the collector of T_2 would have in the absence of the diodes. The diode D_1 continues to conduct, and serves the same function as the conducting diode in the circuits of Figs. 102a and 102b. The opening of D_1 disconnects the input circuit, and at a later time the breaking down of D_2 connects the input circuit to the collector of T_2. The difference between the voltages across the capacitors C_3 aids in the completion of the transition process, in the manner explained earlier in Sec. 29.1. The breakdown diodes D_1 and D_2 also limit the maximum and minimum values of collector voltage. The effect of this action upon the transition time of the circuit will be discussed in Sec. 32.2.

[1] J. G. Linvill, *Proc. I.R.E.*, **43**, 826 (July, 1955).

29.4. Commutation of Suppressor-Coupled Circuit. Reliable commutation is obtained in the circuit of Fig. 99b if the circuit is triggered by negative pulses impressed simultaneously upon the two control grids, which may be directly connected. The circuit is not affected by positive pulses impressed upon the control grids. This circuit has the advantage that the control circuit is independent of the feedback networks and may therefore have high input impedance. Its disadvantage is the relatively low suppressor-to-plate transconductance and the resulting relatively low transition speed.

30. Analysis of the Eccles-Jordan Circuit as an Amplifier

30.1. Use of Voltage Transfer Characteristics. Certain aspects of the operation of the Eccles-Jordan circuit can be readily explained by considering the circuit as a two-stage direct-coupled amplifier, the output of

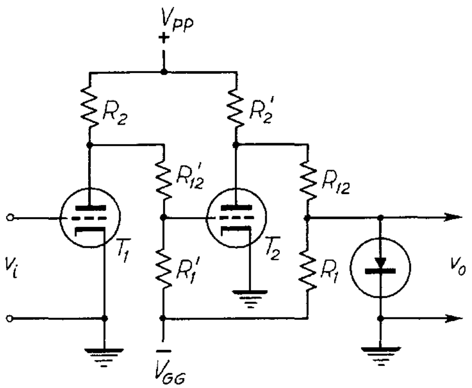


FIG. 103. Two-stage amplifier from which the circuit of Fig. 96a may be formed by connecting the output terminals to the input terminals and removing the diode.

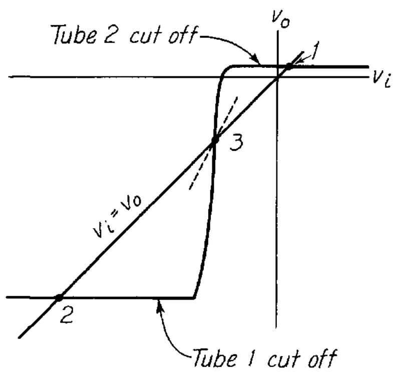


FIG. 104. Voltage transfer characteristic of the circuit of Fig. 103.

which is connected to the input. Thus, the circuit of Fig. 96a may be formed from that of Fig. 103 by connecting the output terminals to the input terminals and removing the diode, which is replaced by the grid-cathode path of T_1 in the closed circuit. The diode in the circuit of Fig. 103 and the grid-cathode path of T_1 in that of 96a prevent v_o from rising more than a fraction of a volt above zero. Figure 104 shows a curve of output voltage v_o of the amplifier as a function of input voltage v_i. Such a curve may be obtained experimentally, or it may be constructed graphically from the transfer characteristic of one tube and its load, which consists of the parallel combination of R_2 and $R_1 + R_{12}$.

Connecting the output terminals to the input terminals imposes the requirement that the input voltage v_i must at all times be equal to the output voltage v_o. Equilibrium values of output and input voltages are therefore those at which the voltage transfer curve of Fig. 104 intersects the straight line corresponding to the relation $v_i = v_o$. This line will be called the *feedback line* in the following discussion. Intersection 1 corresponds to the stable state of equilibrium in which tube 2 is cut off and tube 1 conducts. Intersection 2 corresponds to that in which tube 1 is cut off and tube 2 conducts. Intersection 3, which corresponds to the state of equilibrium in which the two tubes conduct equally, will now be shown to be unstable.

The voltage transfer characteristic of Fig. 104 is a static characteristic constructed by plotting graphically or experimentally determined steady values of direct voltage. If this curve also held for varying values of v_i and v_o, regardless of the rate of change or frequency of the voltages, point 3 would also correspond to stable equilibrium, since any departure of v_i from the value at point 3 would cause v_o to differ from v_i, in violation of the requirement that v_o must equal v_i. However, shunt capacitances in the circuit of Fig. 103 cause the ratio of a change of output voltage to a change of input voltage to fall off as the frequency or the rate of change of voltage is increased. Therefore, if the input and output voltages v_i and v_o initially have values corresponding to point 3 and the magnitude of v_i is changed rapidly from that value, the resulting curve of v_o vs. v_i will have a smaller slope, as indicated by the dotted curve of Fig. 104. If the time rate of change of v_i is increased, the slope of the dynamic voltage transfer characteristic is reduced further. If the dv_i/dt is great enough, the slope of the dynamic characteristic at point 3 may become unity, i.e., the variational (incremental) voltage amplification of the two-stage amplifier may become unity. An increment of input voltage may then take place without violating the requirement that v_o and v_i remain equal. Once this action starts, it must continue, inasmuch as any reduction of dv_i/dt below the instantaneous value at which dv_o/dv_i is unity would violate the requirement of equality of input and output voltages. The input and output voltages consequently continue to change, the rate of change of voltages at all instants being such as to allow the path of operation to follow the feedback line until point 1 or point 2 is reached. Points 1 and 2 are stable, because a change in v_i *from* the value at either of these points produces no change in v_o, whereas an equal change would be required in order to carry the point of operation away from these values along the feedback line.

It is apparent from Fig. 104 that the maximum slope of the voltage-transfer characteristic, i.e., the maximum zero-frequency voltage amplification of the two-stage amplifier of Fig. 103, must exceed unity in order for the circuit to be bistable. This requirement is consistent with Eq. (38) (see Prob. 27.1–1).

30.2. Triggering by Voltage Increment. If a positive voltage increment Δv is applied in series with the grid of tube 1 in the circuit of Fig. 96a, $v_i = v_o + \Delta v$. Thus the feedback line is displaced to the right, as shown in Fig. 105. A negative voltage increment, on the other hand, displaces the feedback line to the left. If tube 1 is initially cut off, i.e., if operation is at point 2 in Fig. 104, and a gradually increasing positive voltage Δv is impressed upon the grid of tube 1, points 2 and 3 approach each other, as shown in Fig. 105. At a value of voltage at which the feedback line is tangent to the transfer characteristic, points 2 and 3 merge into a single unstable point and the operating point moves rapidly along the feedback line to 1′, in the same manner as that for transfer from the unstable point 3 to either 1 or 2. Upon termination of the triggering voltage, the operating point moves to point 1, tube 2 being cut off. If a negative triggering voltage is now applied to the grid of tube 1, the the feedback line is displaced to the left. If the voltage is sufficient to make the line tangent to the characteristic, as shown by the dotted line, the operating point moves rapidly to point 2′, at which tube 1 is again cut off. Upon termination of the triggering voltage, the operating point returns to 2. Similar action takes place if the triggering voltage is greater than that necessary to make the feedback line tangent to the transfer characteristic.

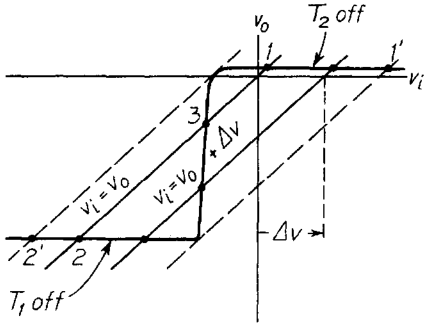


FIG. 105. Triggering by change of input voltage.

It can be seen from Fig. 105 that the minimum triggering voltage that can initiate transition is that which moves the operating point to the point at which the feedback line is tangent to the characteristic. At this point the slope dv_o/dv_i of the characteristic is unity, i.e., the variational (incremental) amplification of the amplifier is unity. Since the voltage amplification of the initially conducting tube may range from the order of 10 to 100 or higher, the amplification of the initially nonconducting tube need be only of the order 0.1 to 0.01, which is attained at a very low value of plate current. Consequently, to all intents and purposes, transition may be assumed to start at a value of triggering voltage that raises the grid voltage of the nonconducting tube to the cutoff value.

Figure 105 also shows that the positive triggering voltage required to cause a transition from point 2 to point 1′ is much greater than the negative voltage required to cause a transition from point 1 to point 2′. In other words, a smaller triggering voltage is required if triggering is accomplished by means of a negative voltage applied to the grid of the conducting tube than if it is accomplished by means of a positive voltage applied to the grid of the nonconducting tube. This fact, which is of importance in the design of devices making use of the Eccles-Jordan circuit, was shown in Sec. 28.2 to be a consequence of amplification of a negative triggering pulse applied to the grid of the conducting tube. The magnitudes of the required positive and negative triggering voltages can be made more nearly equal by reducing the negative grid-supply voltage.

30.3. Speed of Transition. It has already been pointed out that the rate of change of voltages during transition must at all instants be such as to reduce the variational amplification of the amplifier to unity. Because the reduction in amplification is the result of shunt interelectrode and other circuit capacitances, the required rate of change of input voltage necessary to reduce the variational amplification from its zero-frequency value to unity increases with decrease of shunt capacitance. It also increases with increase of zero-frequency amplification, and consequently with increase of transconductance. In applications in which short transition time is essential, therefore, a figure of merit of the tubes used in the Eccles-Jordan circuit is the ratio g_m/C, where C is an effective capacitance that increases with the interelectrode capacitances. This figure of merit will be derived more rigorously in Sec. 31.

It follows from the analysis of Sec. 30.1 that any circuit change that tends to prevent the amplification from falling at high rates of change of input voltage, and therefore at high frequency, increases the required rate of change of voltage and thereby reduces the transition time. One such change is the addition of coupling capacitors C_{12} in parallel with the coupling resistors R_{12}, as shown in Fig. 93. The function of these capacitors can also be readily seen if it is noted that the grid-cathode interelectrode capacitances tend to prevent the grid voltages from changing in response to changes of plate voltages, and that the grid-plate interelectrode capacitances couple the grid of each tube to its own plate and thus tend to make the grid voltages swing in the wrong direction. The coupling capacitors C_{12} increase the coupling of the plate of each tube to the grid of the other tube for voltage changes, and therefore increase the speed of transition. However, the change in voltage across the plate resistors R_2 during transition is followed by charging of one coupling capacitor and discharging of the other. The capacitor voltages must reach a certain percentage of their equilibrium values before the circuit will respond to another triggering pulse. The minimum time interval following transition before the circuit will respond to another triggering pulse is called the *settling time.* Settling time increases with the time constant of the coupling capacitances and associated circuit resistances. In the choice of coupling capacitance, therefore, a compromise may have to be made between the requirements of rapid transition and high repetition frequency. Coupling capacitances of the order of 25 to 200 $\mu\mu$f are usually adequate.

The conclusions drawn from the analysis presented in this section are also applicable to circuits that use transistors in place of tubes. Transistors being basically current-controlled devices, there is some advantage in using current transfer diagrams for transistor circuits, rather than voltage transfer diagrams, but the current transfer diagrams do not yield any additional important information. For this reason, they will not be discussed further.

31. Speed of Transition of Tube Eccles-Jordan Circuits [1]

31.1. Equivalent Circuit During Transition. The three most important considerations in the design of bistable circuits are the transition time, the settling time, and the standby stability. Although factors that affect the transition time have already been discussed qualitatively, it is desirable at this point to analyze the Eccles-Jordan circuit more rigorously. It has been pointed out previously that the settling time is determined by the charging time constants of the tube and circuit capacitances and associated resistances, and that settling time limits the value of coupling capacitance C_{12} that can be used. Standby stability will be treated in the following section.

The Eccles-Jordan circuit, like other circuits containing nonlinear elements, can be treated theoretically only by approximate analytical methods or by point-by-point graphical methods. The complexity of even an approximate analytical treatment in which the nonlinearity of the tubes or transistors is taken into account is so great as to overshadow the advantages of such a method. Useful information concerning the factors that govern the behavior of the circuit may be obtained, however, under the assumption that the tubes or transistors act as linear elements throughout the change from one state of equilibrium to the other, and that a constant-element equivalent circuit is therefore applicable. The analysis is still complicated by dependence of the behavior of the circuit during switching not only upon the amplitude of the pulse that initiates triggering, but also upon the shape of the pulse.

The analysis is greatly simplified by the assumption that the coupling capacitances C_{12} are so large in comparison with the interelectrode capacitances that little change in voltage across the coupling capacitors takes

[1] E. M. Williams, D. F. Aldrich, and J. B. Woodford, *Proc. I.R.E.*, **38**, 65 (January, 1950); R. Feinberg, *Wireless Engineer*, **26**, 153 and 325 (May and October, 1949).

place during transition. This is equivalent to the assumption that the grid of each tube is directly connected to the plate of the other. Under this assumption the plate-cathode capacitance of each tube is in parallel with the grid-cathode capacitance of the other, and the two grid-plate capacitances are in parallel. The equivalent circuit may then be reduced to that of Fig. 106, in which $C = C_{gk} + C_{pk}$ and R is the resistance of the parallel combination of R_1, R_2, r_p, which is assumed to be constant, and the grid resistance of the conducting tube when the grid is posi-

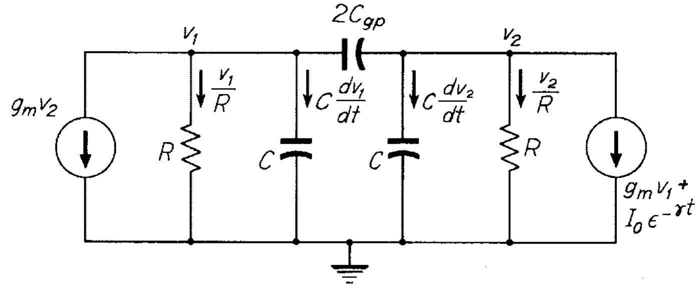


FIG. 106. Approximate equivalent circuit of the Eccles-Jordan circuit of Fig. 96a or 97a during transition.

tive. C should also include any capacitive load coupled to the plate, and may include C_t if the trigger input circuit is not disconnected from the bistable circuit by diodes during transition. The equivalent circuit of Fig. 106 also applies to a transistor Eccles-Jordan circuit if R is the effective resistance of the parallel combination of R_1, R_2, and the transistor input and output resistances (assumed to have constant values) and if g_m and the interelectrode capacitances are replaced by y_{fe} and the corresponding transistor capacitances, respectively. However, the transistor circuit is more conveniently analyzed in terms of the h parameters, and the speed of transition in the transistor circuit is usually determined by carrier transit time, rather than by interelectrode capacitances. For this reason, the transistor circuit will be analyzed separately in Sec. 32.

Although different investigators have made various assumptions as to the form of the triggering pulse, in most practical circuits the pulse is exponential, often being derived by differentiating the approximately rectangular output of another bistable circuit or a clipper. An exponential voltage pulse, applied to the grid or the plate of one of the tubes, causes an exponential pulse of current into the circuit. In Fig. 106 this is the current $I_0\epsilon^{-\gamma t}$ applied at the plate nodal point of tube 2.

31.2. Analysis for Rectangular Triggering Pulse. If the triggering pulse is impressed at the instant when $t = 0$, solution of the differential equations for the circuit of Fig. 106 yields the following expressions for the varying components of the plate-to-cathode voltages at any subsequent time t (see Prob. 31.2–1):

$$v_1 = I_o[A\epsilon^{at} + B\epsilon^{-bt} - (A + B)\epsilon^{-\gamma t}] \tag{40}$$

$$v_2 = I_o[-A\epsilon^{at} + B\epsilon^{-bt} + (A - B)\epsilon^{-\gamma t}] \tag{41}$$

where

$$A = \frac{1}{2(C + 4C_{gp})(\gamma + a)} \approx \frac{1}{2\gamma(C + 4C_{gp}) + 2g_m} \tag{42}$$

$$B = \frac{1}{2C(b - \gamma)} \approx \frac{1}{2g_m - 2\gamma C} \tag{43}$$

$$a = \frac{g_m R - 1}{R(C + 4C_{gp})} \approx \frac{g_m}{C + 4C_{gp}} = \frac{g_m}{C_{gk} + C_{pk} + 4C_{gp}} \tag{44}$$

$$b = \frac{g_m R + 1}{RC} \approx \frac{g_m}{C} = \frac{g_m}{C_{gk} + C_{pk}} \tag{45}$$

The approximate forms of Eqs. (42) to (45) hold when $g_m R \gg 1$, i.e., when the low-frequency amplification of the two-stage amplifier of Fig. 103 is much greater than unity. Because this condition normally holds, it follows that the rate of change of voltages and currents during transition is usually independent of the circuit resistances. Increase of R_2 does, however, increase the change in plate voltage during transition, and hence increases the *time* required for transition from one state of equilibrium to the other.

Equations (40) and (41) show that the plate voltages have a component associated with the triggering pulse, and that the time constant $1/\gamma$ of the triggering pulse should be short in comparison with the circuit time constants $1/a$ and $1/b$ in order that the input-pulse transient will fall to a low value as rapidly as possible. The second and third terms of Eqs. (40) and (41) represent decaying currents, and the first term rapidly predominates. The important time constant of the circuit is therefore $(C_{gk} + C_{gp} + 4C_{gp})/g_m$. Increase of g_m not only reduces the time constant, but also reduces the magnitude of the coefficient A. However, at values of t that exceed the time constant by only a small amount, the net effect of an increase of g_m is an increase of magnitude of v_1 and v_2. It follows that increase of g_m reduces the time taken for the voltages to change from one state of equilibrium to the other and that the ratio $g_m/(C_{gk} + C_{pk} + 4C_{gp})$ is a figure of merit for tubes used in the Eccles-Jordan circuit.

31.3. Analysis for Exponential Triggering Pulse. Finite rise time of the voltage applied to the differentiating circuit that generates the triggering pulse, together with shunt capacitance across the output of the differentiator and series resistance of the source of voltage (Sec. 3), may cause the form of the triggering pulse to resemble more nearly the rounded pulse shown in Fig. 14a, rather than the simple decaying exponential pulse assumed in the derivation of Eqs. (40) and (41). Such a pulse may be represented by the function $I_o(\epsilon^{-\gamma t} - \epsilon^{-\gamma' t})$, where γ' is much larger than γ. Under the assumption made in the derivation of Eqs. (40) and (41), that the tubes may be considered to be linear elements and g_m therefore constant, the principle of superposition is valid and the solution must contain terms in $\epsilon^{-\gamma' t}$, as well as additional terms in ϵ^{at} and ϵ^{-bt}. Because of the negative sign before the second term of the triggering-pulse function, the coefficients of the added terms will be opposite to those of Eqs. (40) and (41). Terms in ϵ^{at} will again rapidly predominate and the expression for v_1 will be:

$$v_1 \approx I_o \left[\frac{1}{2\gamma(C + 4C_{gp}) + 2g_m} - \frac{1}{2\gamma'(C + 4C_{gp}) + 2g_m} \right] \epsilon^{at} \tag{46}$$

Equation (46) shows that rounding of the leading edge of the triggering pulse reduces the transition speed. However, γ' is much larger than γ and the increase in transition time is not great if the leading edge of the triggering pulse is steep in comparison with the trailing edge.

31.4. Settling Speed. Inasmuch as the grid voltage of the nonconducting tube usually falls considerably beyond cutoff, there is a portion of the total transition time in which only one tube is active. Solution of the equations for the circuit of Fig. 106 under the assumption that g_m is zero for one tube shows that the varying plate voltages are of the form:

$$v = A'\epsilon^{-a't} + B'\epsilon^{-b't} + C\epsilon^{-\gamma t} \tag{47}$$

in which

$$a' = \frac{1}{2R[C + C_{gp}(2 + Rg_m)]} \tag{48}$$

and

$$b' = a' \left[\frac{4[C + C_{gp}(2 + Rg_m)]^2}{C^2 + 4CC_{gp}} - 1 \right] \tag{49}$$

When the circuit is triggered by means of a short negative pulse, the portion of the transition in which only one tube is active occurs toward the end of the transition period and the third term in Eq. (47) is negligible. It can be seen from Eq. (49) that $a' < b'$ and that the predominant time constant is therefore $2R[C_{gk} + C_{pk} + (Rg_m + 2)C_{gp}]$, which is usually much larger than the predominant time constant $(C_{gk} + C_{pk} + 4C_{gp})/g_m$ that applies when both tubes are active.* Consequently, short transition time is favored

* It is of interest to note that the term Rg_mC_{gp} represents the increase of effective capacitance caused by the Miller effect (see Sec. 4).

by reducing the amount by which the maximum negative grid voltage exceeds the cutoff voltage. This can be accomplished by reducing the resistances R_2, provided that the low-frequency voltage amplification Rg_m is not thereby reduced to too low a value. (The exact form of Eq. (44) shows that the time constant may become very large if the value of g_mR approaches unity.) An alternative method of shortening the transition time is the use of clipping diodes that prevent the plate voltage from falling below a minimum value. Thus, in the circuit of Fig. 107, if the plate voltage of the

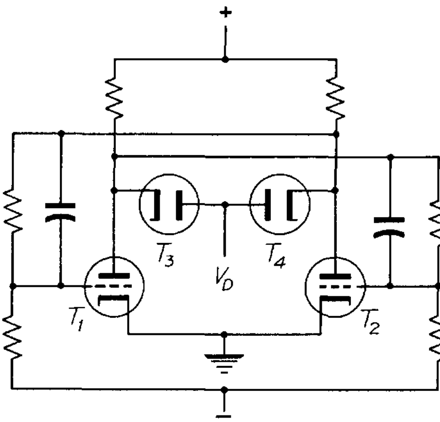


FIG. 107. The use of clipping diodes to reduce the transition time of an Eccles-Jordan circuit.

conducting triode falls below the voltage V_D applied to the diode plates, the corresponding diode conducts and any additional plate current flows almost entirely through the diode, rather than through the plate resistor R_2. The plate voltage consequently does not fall appreciably below the voltage V_D. The use of diodes to limit the minimum plate voltage decreases the transition time not only because it reduces the fraction of the transition time in which only one tube is active, but also because it reduces the plate-voltage change between the two states of equilibrium without reducing the rate of change of voltage. The voltage-limiting diodes of Fig. 107 may also serve as the commutating diodes of Fig. 102a.

If the amplitude and rate of rise of the triggering pulse are sufficient to bias the initially conducting tube beyond cutoff before the initially nonconducting tube starts to conduct, there is a period during which both tubes are inactive. The time constants during the period in which both tubes are off may be found from Eqs. (48) and (49) if g_m is made zero. The time constants are seen to be larger than when one or both tubes conduct. This situation also obtains if the trigger pulse is a large-amplitude step function.

The relatively large time constants in this type of triggering are not conducive to short transition time.

31.5. Effect of Nonlinearity. In the foregoing analyses the assumption was made that the transconductance g_m is constant. Actually, the transconductance decreases as the grid voltage is made more negative, and it approaches zero as the grid voltage approaches cutoff. The low value of g_m in the vicinity of cutoff of either tube and the reduction of R by the low grid resistance of the conducting tube at positive grid voltages cause the low-frequency amplification g_mR, and consequently the speed of transition, to be low near the beginning and end of the transition. As a result, the transition starts slowly and may not be completed if the triggering-pulse duration is too short. Figure 108 shows curves of plate voltages derived graphically under the assumption that the transfer characteristics are parabolic and the grid-plate capacitances negligible.[2] These curves show clearly that transition is completed only if the duration of the triggering pulse exceeds a critical value. This conclusion is consistent with the experimental curves of Figs. 89 and 91, which show that transition of state of equilibrium of a bistable circuit continues to completion only if the triggering-pulse duration exceeds a limiting value, which decreases with increase of pulse amplitude.

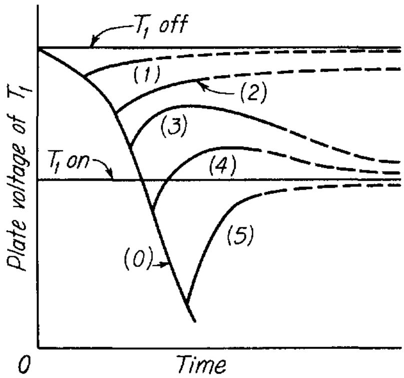


FIG. 108. Graphically derived curves of plate voltage of T_1 vs. time during transition in the circuit of Fig. 96a. Rectangular triggering pulse initiated when $t = 0$ and terminated at the instants when curves (1) to (5) depart from curve (0).

32. Speed of Transition of Transistor Eccles-Jordan Circuits

32.1. Analysis of Equivalent Circuit. If the coupling capacitors C_{12} in the transistor Eccles-Jordan circuit of Fig. 96b are large enough so that the base of each transistor may be considered to be coupled directly to the collector of the other during transition, the circuit may be represented by the simplified circuit of Fig. 109a. R_{ie} is the variational base resistance of the transistors in the common-emitter connection.* Under the assumption that the variational resistance of the source of collector voltage is

[2] J. R. Tillman, *Wireless Engineer*, **28**, 101 (April, 1951).

* To simplify the analysis, R_{ie} is assumed to be the same for the on transistor as for the off transistor.

negligible, the resistances R_1 and R_2 are effectively in parallel so far as current and voltage changes are concerned, and the circuit may be represented by the equivalent circuit of Fig. 109b, in which $R_2' = R_1R_2/(R_1 + R_2)$.

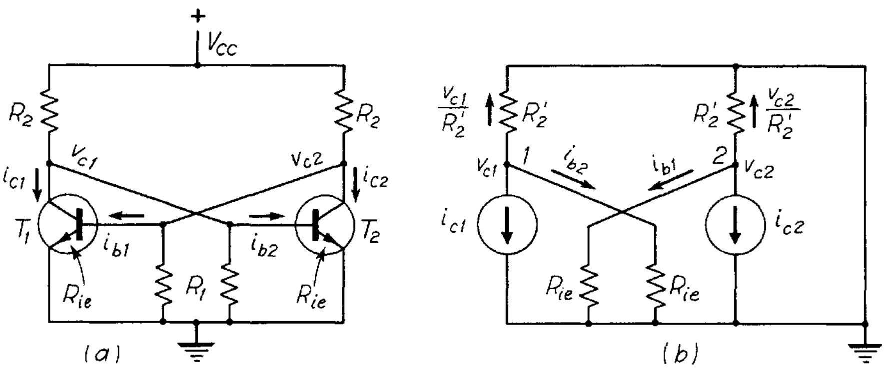


FIG. 109. (a) Simplified form of the circuit of Fig. 96b in which the base of each transistor is assumed to be directly coupled to the collector of the other; (b) equivalent circuit applicable during transition.

Summation of currents at node 1 of Fig. 109b yields the following equation:

$$v_{c1}/R_2' + i_{c1} + i_{b2} = 0 \tag{50}$$

But

$$v_{c1} = i_{b2}R_{ie} \tag{51}$$

Therefore,

$$i_{b2}(R_2' + R_{ie})/R_2' + i_{c1} = 0 \tag{52}$$

In eliminating the base current i_{b2} from Eq. (52), it is necessary to take into consideration the fact that carrier transit time affects the values of the transistor parameters at high frequencies or high rates of change of current. Although the transistors are used in the common-emitter connection in the circuit of Fig. 109, the high-frequency behavior of transistor parameters is usually indicated in terms of $f_{\alpha b}$, the frequency at which the magnitude of the common-base current-amplification factor h_{fb} is reduced to $1/\sqrt{2}$ times its low-frequency value h_{fbo}. The common-base parameter h_{fb}, rather than the common-emitter parameter h_{fe}, will therefore be used in relating i_b to i_c.

To a first approximation, h_{fb} and h_{fbo} are related as follows:[1]

$$h_{fb} = \frac{h_{fbo}}{1 + jf/f_{\alpha b}} \tag{53}$$

The current-amplification factor h_{fb} is the ratio of the collector current to the

[1] R. L. Pritchard, *Proc. I.R.E.*, **40**, 1476 (November, 1954).

emitter current when the collector-to-base load impedance is zero. The collector is not short-circuited to the base in the circuit of Fig. 109, but the circuit resistances are usually so small in comparison with the output impedance $1/h_{ob}$ of the transistor (of the order of a megohm) that $i_c \approx h_{fb}i_e$. We may therefore write

$$i_b = -(i_e + i_c) \approx -i_c(1/h_{fb} + 1) = -i_c \left[1 + \frac{1 + j(f/f_{\alpha b})}{h_{fbo}}\right] \qquad (54)$$

In differential form, Eq. (54) may be written as follows (Prob. 32.1–1):

$$i_b \approx -\left(1 + \frac{1}{h_{fbo}} + \frac{p}{h_{fbo}\omega_{\alpha b}}\right) i_c \qquad (55)$$

in which p is the differential operator d/dt and $\omega_{\alpha b} = 2\pi f_{\alpha b}$. Substitution of Eq. (52) into Eq. (55) gives the equation:

$$\frac{R_2' + R_{ie}}{R_2'}\left(1 + \frac{1}{h_{fbo}} + \frac{p}{h_{fbo}\omega_{\alpha b}}\right) i_{c2} - i_{c1} = 0 \qquad (56)$$

Similarly, summation of currents at node 2 and elimination of v_{c2} and i_{b1} leads to the equation

$$\frac{R_2' + R_{ie}}{R_2'}\left(1 + \frac{1}{h_{fbo}} + \frac{p}{h_{fbo}\omega_{\alpha b}}\right) i_{c1} - i_{c2} = 0 \qquad (57)$$

Solution of the simultaneous equations (56) and (57) yields the following equations:

$$i_{c2} = -i_{c1} \qquad (58)$$

$$\left[p + \omega_{\alpha b}\left(1 + h_{fbo} + \frac{h_{fbo}R_2'}{R_2' + R_{ie}}\right)\right] i_{c1} = 0 \qquad (59)$$

But

$$h_{fbo} \approx -1$$

Therefore, approximately,

$$\left[p - \omega_{\alpha b}\frac{R_2'}{R_2' + R_{ie}}\right] i_{c1} = 0 \qquad (60)$$

The currents in the circuit of Fig. 109 are therefore of the form:

$$i = A\epsilon^{at} + B \qquad (61)$$

where

$$a = \omega_{\alpha b}\frac{R_2'}{R_2' + R_{ie}} \qquad (62)$$

By an analysis in which the input resistance of the transistor was assumed to be negligible, Linvill showed that the principal time constant of the diode-coupled circuit of Fig. 98 is approximately $1/\omega_{\alpha b}$ if $\omega_{\alpha b}C_3 \gg R_3$ and $R_3 \ll R_2$.[2] If the input resistance is not assumed to be negligible, the time constant is found to have the approximate value $1/a$ determined from Eq. (62).

Equation (62) shows that the maximum speed of transition of a transistor Eccles-Jordan circuit is proportional to the cutoff frequency ω_{ab}. It also indicates that R_2 must be made considerably larger than R_{ie} in order that the maximum transition speed may be approached.

If R_2 is made too large, however, time constants involving the charging of capacitors may become larger than the time constant associated with transit time. A value of R_2 of the order of 10,000 ohms is satisfactory in most applications.

32.2. Use of Breakdown Diodes to Reduce Carrier-Storage Effects. The analysis of this section has neglected the effect of carrier storage, which may greatly increase the time taken to cut the collector current off. The effect of carrier storage upon transition time is greatest if the transistors are driven into the saturation (low-collector-voltage) range. Saturation may be prevented by the addition of two breakdown diodes, connected back-to-back between the collectors, as shown in Fig. 102c.[3] The manner in which the breakdown diodes limit the collector voltage excursions can be most readily explained by assuming that the circuit is in the process of transition from conduction of T_1 to conduction of T_2. At the instant of equality of collector currents, the collector voltages are equal and both diodes are cut off. As the transition proceeds, the collector voltage of T_1 rises and that of T_2 falls. When the difference in collector voltages becomes equal to the diode breakdown voltage, D_2 breaks down and conducts in the reverse direction, while D_1 conducts in the forward direction. The collector of T_1 is then connected to the base of T_1 through D_1, D_2, and D_4, all of which are conducting. The diode voltages being almost independent of current, the breaking down of D_2 applies essentially 100 percent inverse feedback to T_1 and thus stabilizes the collector current and voltage at approximately the values which obtained at the instant of breakdown. Similarly, 100 percent inverse feedback is applied to T_2 through D_1, D_2, and D_3.

The breakdown diodes D_1 and D_2 in the circuit of Fig. 102c reduce the transition time not only because they prevent the collector voltage from falling to the saturation range, but also because they limit the change in collector voltage to the diode breakdown voltage. It was shown in Sec. 29.2 that the diodes may also be used for the purpose of commutation.

[2] J. G. Linvill, *Proc. I.R.E.*, **43**, 826 (July, 1955).
[3] Linvill, *loc. cit.*

33. Static Stability of Vacuum-Tube and Transistor Eccles-Jordan Circuits

33.1. Method of Analysis of Tube Circuits. In some applications of the Eccles-Jordan circuit, a very important consideration is that the circuit be designed so that, throughout the useful life of the tubes or transistors, transition will not take place without the application of a triggering pulse, in spite of tube or transistor deterioration, differences in the characteristics of individual tubes or transistors, resistor tolerances, and variations of supply voltage. Methods of design that ensure static stability of vacuum-tube Eccles-Jordan circuits have been developed by a number of investigators.[1] Basically, the various methods are similar. Voltage and resistance tolerances are assumed, and simple d-c analysis of the circuit is used to determine ranges over which the circuit resistances can be varied without raising the grid voltage of the nonconducting tube above cutoff or lowering that of the conducting tube below zero, at which it is assumed to be clamped by the flow of grid current under equilibrium conditions.

In deriving the expressions for the maximum and minimum values of R_2 and R_{12} that can be used, it is necessary to make use of the d-c plate resistance R_p of the conducting tube, which is defined as the ratio of the plate voltage of the conducting tube to its plate current under equilibrium conditions. Under the assumption that the grid voltage of the conducting tube is clamped at approximately zero by the flow of grid current, the plate current and plate voltage of the conducting tube can be readily approximated from the plate diagram by constructing the load line corresponding to the assumed values of R_2 and supply voltage V_{PP} and determining its intersection with the zero-grid-voltage characteristic. Although the load line is dependent not only upon R_2, but also upon $R_1 + R_{12}$, R_2 is usually so small in comparison with the resistance of the coupling network that a sufficiently close approximation to R_p is obtained if the load line is drawn on the basis of R_2 alone.

33.2. Stability Criteria for Tube Circuits. If the plate and grid supply voltages are assumed to vary between the limits $V_{PP}(1 \pm \Delta_v)$ and $V_{GG}(1 \pm \Delta_v)$, respectively, and the resistances are assumed to vary between the limits $R(1 \pm \Delta_r)$, where R is the nominal value of any resistance, simple d-c circuit analysis yields the following expressions for the maximum and minimum allowable nominal values of R_{12} in terms of R_1, R_2, and other parameters:[2]

[1] R. Feinberg, *Wireless Engineer,* **26,** 325 (October, 1949); J. R. Tillman, *Wireless Engineer,* **28,** 101 (April, 1951); M. Rubinoff, *Communication and Electronics,* No. 1, July, 1952, p. 215 [also in *Electronic Eng.,* **71,** 905 (October, 1952)]; R. F. Johnston and A. G. Ratz, *Communication and Electronics,* No. 5, March, 1953, p. 54; R. Pressman, *Electronics,* April, 1953, p. 164; D. K. Ritchie, *Proc. I.R.E.,* **41,** 1614 (November, 1953).

[2] Johnston and Ratz, *loc. cit.*

$$R_{12} < STR_1 - R_2 \tag{63}$$

$$R_{12} > \frac{(K\mu_{co} + 2)R_1}{T(S\mu_{co} - 2)} - KR_2 \tag{64}$$

in which

$$K \equiv \frac{2R_p}{2R_p + R_2}$$

$$S \equiv \frac{V_{PP}}{V_{GG}}$$

$$T \equiv \frac{(1 - \Delta_v)(1 - \Delta_r)}{(1 + \Delta_v)(1 + \Delta_r)}$$

and μ_{co} is the ratio of the plate voltage to the magnitude of the negative grid voltage at plate-current cutoff when the plate voltage is equal to the plate-supply voltage, i.e., the ratio of the plate voltage to the grid voltage at the point where the load line intersects the plate-voltage axis. In the derivation of Eqs. (63) and (64), tube deterioration is taken into account by the assumption that the d-c plate resistance of the conducting tube will double during its life (the plate current will fall to half the value determined from the plate diagram) and that the cutoff grid voltage will double.

In some applications it is desirable that the plate voltage of the conducting tube fall at least as low as some selected value V_l and that the plate voltage of the nonconducting tube rise at least as high as some selected value V_u. These requirements will be satisfied if the following relations hold:

$$R_{12} > \frac{V_u}{V_{PP}} \cdot \frac{R_2}{T - V_u/V_{PP}} \tag{65}$$

$$R_1 + R_{12} < \frac{KT(S + V_l/V_{PP})}{K - TV_1/V_{PP}} - R_1 \tag{66}$$

33.3. Use of The Stability Criteria for Tube Circuits. In the design of an Eccles-Jordan circuit, the four lines corresponding to Eqs. (63) to (66), with equal signs substituted for the inequality signs, are plotted in a diagram of R_{12} vs. R_1. A combination of values of R_{12} and R_1 corresponding to any point within the area enclosed by the four lines may then be selected. Before the lines can be plotted, however, values of R_2, V_{PP}, V_{GG}, V_l, and V_u must be selected, and R_p must be found from the plate diagram. The choice of R_2 and V_{PP} is governed by such factors as desired output voltage (change of plate voltage during transition), allowable plate dissipation, and speed of transition (Rg_m should be large in comparison with unity, as shown in Sec. 31). The magnitudes of R_2 and V_{PP} necessary to provide a desired output can be approximated quickly from the plate diagram under the assumption that R_2 is small in comparison with $R_1 + R_{12}$. The analysis that leads to Eqs. (63) to (65) shows, however, that adequate stability against changes of supply voltages and circuit resistances necessitates that the following relations be satisfied:

$$R_2 > 4R_p \left[\frac{1 + \Delta_r}{T^2 \left(1 - \frac{2S}{\mu_{co}}\right) - \frac{2}{\mu_{co}}} \right] \tag{67}$$

$$S \equiv \frac{V_{PP}}{V_{GG}} > \frac{\mu_{co}}{2} \left[1 - \frac{K + 2\mu_{co}}{T^2} \right] \tag{68}$$

Usually a value of supply-voltage ratio S in the range between 0.5 and 2.0 is satisfactory.

Figure 110 shows a typical set of curves for a circuit using a 6J6 double triode, values of Δ_v and Δ_r of 0.05, $V_l = 65$ volts, and $V_u = 115$ volts. The value of R_2 was chosen as 33 kΩ, V_{PP} and V_{GG} as 200 volts, and R_p and μ_{co} were found to be 7500 ohms and 20, respectively. It can be seen from this diagram that suitable values of R_1 and R_{12} would be 150 kΩ and 87 kΩ, respectively. The freedom in the choice of R_1 and R_{12} would be greater if

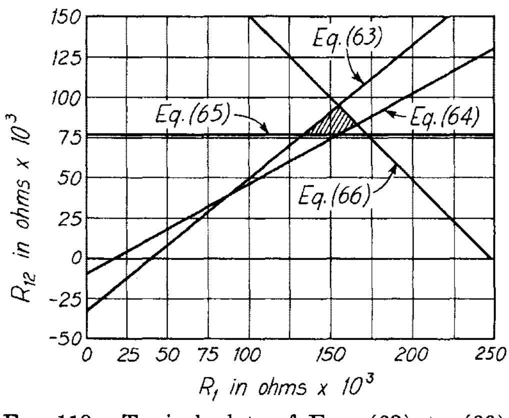


FIG. 110. Typical plots of Eqs. (63) to (66).

V_u and V_l were not specified, as two of the boundary lines would then be absent.

33.4. Static Stability of the Transistor Eccles-Jordan Circuit; Collector Diagram. Figure 111 shows the collector diagram for a transistor used in the circuit of Fig. 96b. The clamping of the collector voltage at point B is caused by base current in the other transistor and is similar to grid-current clamping in the vacuum-tube Eccles-Jordan circuit. Since the collector current does not rise above A or fall below B, these points correspond to values of base current at which h_{fe}, and consequently the current amplification, have fallen to zero. Although the loop amplification of the complete circuit is zero at either point and the circuit will have two stable equilibrium states if the circuit is adjusted so that either point A or B is reached by the collector of each transistor, it is desirable to operate in such a manner that the operating point of one transistor is at A when that of the other is at B, and vice versa. If this is accomplished for all values of transistor parameters that can be assumed by individual transistors throughout their usable life, and for all values of circuit resistances and supply voltages within the ranges of tolerances, complete static stability will be achieved.

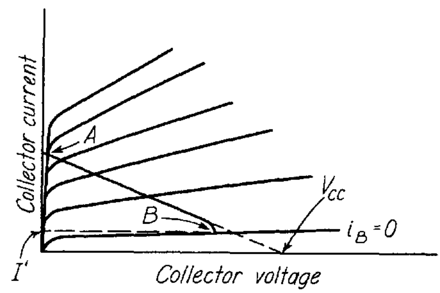


FIG. 111. Collector diagram for a transistor in the circuit of Fig. 96b.

In the following discussion, either transistor will be said to be nonconducting (off) when its collector current has the value I'; i.e., when the operating point is at B.

The collector diagram can be used to select the load resistance R_2 and the collector supply voltage on the basis of the desired change in collector voltage or current during transition and of allowable collector dissipation. It should be noted that the base supply voltage causes current to flow through the resistors R_1 and R_{12}. This current adds to the collector current in the collector load resistance. It therefore assures saturation of the conducting transistor at a lower value of base current than if it were zero. It also lowers the collector voltage below that of point A, causing the operating point to move down the characteristic to a lower value of collector current. In effect, some of the current flowing through R_2 is diverted from the collector to the coupling resistors, and the collector dissipation is reduced. The design is therefore conservative so far as static stability and collector dissipation are concerned if the load line in the collector diagram is drawn under the assumption that the effect of the coupling network may be neglected.

It was shown in Sec. 32 that R_2 should be large in comparison with the base input resistance R_{ie} if rapid transition is essential, and that R_2 should therefore be of the order of 10,000 ohms. The collector supply voltage can then be chosen so as to provide the desired change of collector voltage. It should be borne in mind, however, that the increase of collector-current and collector-voltage change accompanying an increase of supply voltage also increases the transition time.

33.5. Requirements for Static Stability of Transistor Circuits. One requirement for static stability of a transistor Eccles-Jordan circuit is that the base current of the conducting transistor be high enough to ensure saturation with the highest value of collector supply voltage and the lowest value of collector load resistance to be expected within the tolerance ranges. The base current required to drive the conducting transistor into saturation can be determined from the collector diagram constructed for the transistor having the lowest value of h_{fe} likely to be encountered in randomly selected transistors at the lowest operating temperature. If the assumption is made that R_1 is high in comparison with the transistor input resistance R_{ie} (of the order of 1000 ohms), the maximum value of R_{12} that will provide the required base current can be determined from the characteristics of base voltage vs. base current, the base-circuit load line being drawn through the point on the base-voltage axis corresponding to the collector supply voltage and through the point on the zero-collector-voltage characteristic corresponding to the required base current. Because the base current of the conducting transistor must flow through the collector load resistance of the nonconducting transistor, the slope of the load line determined in this manner corresponds to the sum of R_2 and R_{12}.

The second requirement for static stability is that the base current of the "nonconducting" transistor be reduced sufficiently to reduce the collector current to its minimum value, I'. With most types of junction transistors, the corresponding values of base voltage and current approximate zero and the second requirement is satisfied approximately if the base supply voltage and the resistance R_1 are such that the voltage at the junction of R_1 and R_{12} is zero when the voltage of the junction between R_{12} and R_2 is that of point A of the collector diagram. This should be true for the transistor having the lowest h_{fe} likely to be encountered in randomly selected transistors, and for the highest value of V_{CC}, lowest value of R_2, and lowest value of R_{12} within the assumed tolerances. If the value of R_1 selected is not large in comparison with the input resistance R_{ie}, some reduction in R_{12} below the value initially selected may be necessary.

If the circuit is to be used over a wide range of temperature, the second requirement must be satisfied at the highest temperature of operation, since both the minimum collector current I' and the corresponding value of base current vary appreciably with temperature. For this reason, collector characteristics obtained at the highest temperature of operation should be used in determining V_{BB} and R_1. At temperatures considerably above room temperature the base current and voltage may have to be reversed in order to reduce the collector current to its minimum value. The base current in R_1 must then be taken into consideration in determining V_{BB} and R_1.

33.6. Stability of Diode-Coupled Transistor Circuit. The circuit of Fig. 102c, which incorporates two sets of breakdown diodes, not only has high inherent static stability, but is relatively easy to design. For rapid transition, R_2 should be large in comparison with R_{ie} and with R_3. Suitable values of R_2 and R_3 are 10,000 ohms and 2000 ohms, respectively.[3] In order to ensure reliable commutation, C_3 must be chosen so that $R_3C_3\omega_{co} \gg 1$. Since the collector must be at a higher potential than the base, the breakdown voltage V_{12} of the coupling diodes D_3 and D_4 must exceed the breakdown voltage V_2 of diodes D_1 and D_2. Both diodes D_3 and D_4 being broken down at all times, whereas one of the diodes D_1 and D_2 is broken down and the other biased in the forward direction in either state of equilibrium, the voltage between the collector and the base of the conducting transistor is very nearly equal to $V_{12} - V_2$, and the voltage between the collector and the base of the nonconducting transistor is very nearly equal to $V_{12} + V_2$. The base-emitter voltage of transistors suitable for use in high-speed bistable circuits is of the order of only one- or two-tenths of a volt. Consequently, to a first approximation, the collector-to-emitter voltage of the conducting transistor may be assumed to be $V_{12} - V_2$ and that of the nonconducting transistor $V_{12} + V_2$, as shown in Fig. 112. In order to avoid slow transition caused by large carrier-storage effects in the saturation region, the breakdown diodes must be chosen so that the difference between the breakdown voltages exceeds the voltage corresponding to the intersection of the load line with the saturation portion of the collector characteristic curves. It is apparent from Fig. 112 that the choice of diode breakdown voltages and of supply voltage is not critical.

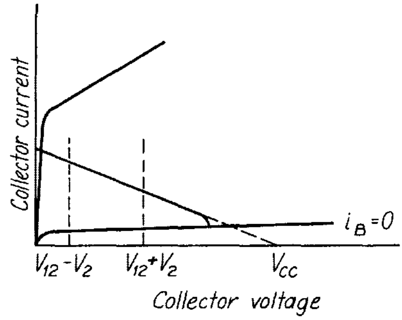


FIG. 112. Collector diagram for a transistor in the circuit of Fig. 102c.

34. Multistable Circuits

34.1. Tristable Eccles-Jordan Circuit. A modified form of the Eccles-Jordan circuit having three stable states of equilibrium is shown in Fig. 113.[1] The values of the cathode resistances R_3, R_4, R_3', and R_4' are such that the diodes D_1 and D_2 do not conduct when equal plate currents flow in the

[3] Linvill, *loc. cit.*

[1] R. Weissman, *Electronics,* October, 1952, p. 118.

two triodes and that the degenerative feedback produced by the resistance in the cathode circuits reduces the loop amplification below unity when the diodes do not conduct. Consequently, the circuit is in stable equilibrium when the triode plate currents are equal and the diodes are cut off. If the plate current of triode T_1 is decreased and that of T_2 correspondingly increased, the increase in current through R_3' and R_4' will raise the potential of the anode of D_2, and the decrease in current through R_3 and R_4 will lower the potential of the cathode of D_2. If the current difference is made large

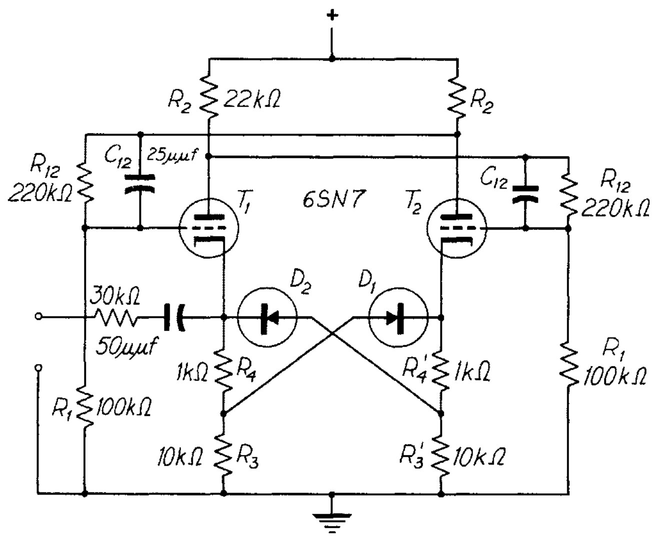


FIG. 113. Tristable Eccles-Jordan circuit.

enough, D_2 will conduct and some of the cathode current of T_2 will be diverted from R_3' to R_3 and R_4. The resulting increase of negative grid-to-cathode voltage of T_1 and decrease of that of T_2 causes further unbalance of the plate currents. If the resistor values have been properly chosen, the action becomes cumulative and causes T_1 to be cut off. Similarly, if the plate currents are unbalanced in the other direction sufficiently to allow D_1 to conduct, T_2 will be cut off. Therefore, in addition to the balanced-current state of stable equilibrium, there is a stable state in which triode T_1 and diode D_1 conduct, and one in which triode T_2 and diode D_2 conduct. It is apparent that the balanced-current state exists because cutting off of the diodes when the triode plate currents approach equality increases negative feedback and decreases positive feedback and thus makes the circuit stable.

34.2. Triggering of Tristable Circuit. The three states of stable equilibrium are assumed in sequence when negative triggering pulses are applied to the cathode of T_1 through the coupling capacitor. Assume that T_2 is conducting fully and T_1 is cut off. A short (1-μsec) negative pulse applied to the cathode of T_1 starts conduction in this tube and thus initiates transition in the same manner as in a bistable circuit. Before the triode plate currents become equal, however, diode D_1 is cut off and, if the input pulse has ended, the circuit is left in the balanced-current state. Application of a second triggering pulse increases the current in T_1 and reduces that in T_2 to the point where diode D_1 starts to conduct. Cumulative action then causes T_2 to be cut off, in the manner already explained. The flow of grid current in T_1 maintains the grid of T_1 at approximately zero voltage relative to its cathode in this state. The third pulse applied to the cathode of T_1 makes this cathode negative relative to the grid. The resulting grid current raises the voltage across C_{12} by an amount approximating the pulse amplitude. At the termination of the pulse, the additional voltage across C_{12} falls sufficiently slowly so that the transition is carried through the balanced-current state and into the initial state, in which T_1 is cut off.

The circuit of Fig. 113 can be triggered reliably at triggering-pulse frequencies up to 175,000 pulses per second. The triggering-pulse amplitude can range between 85 and 150 volts. Component tolerances are about the same as in bistable circuits. Symmetrically located components should be matched within 5 percent, except for the plate resistors, which need be matched within only 20 percent. If the circuit is symmetrical within these limits, the component tolerances may be 10 percent, except for those of the input-coupling components, which may have tolerances of 20 percent. The supply voltage may range between 250 and 325 volts.

34.3. Current-Voltage Characteristic of Tristable Circuit. The introduction of the additional cathode resistors and diodes into the Eccles-Jordan circuit in order to form the tristable circuit of Fig. 113 modifies the current-voltage characteristics of the circuit so as to introduce a short positive-slope portion into the part of the curves that originally has a negative slope. The manner in which the diagrams of Figs. 69a and 69e are modified is shown in Fig. 114. It is seen that the load line may now intersect the current-voltage characteristic in five points, three of which are stable, and two unstable. Other circuits that have current-voltage characteristics of the form shown in Fig. 61 between two terminals can be modified in a similar manner to the Eccles-Jordan circuit to provide three states of stable equilibrium. Furthermore, by the use of additional diodes and more complicated networks, the current-voltage characteristics of any of the negative-resistance circuits can be made to have a larger number of portions of alternate positive and negative slope, and the circuits therefore can be made to have more than three stable states of equilibrium.

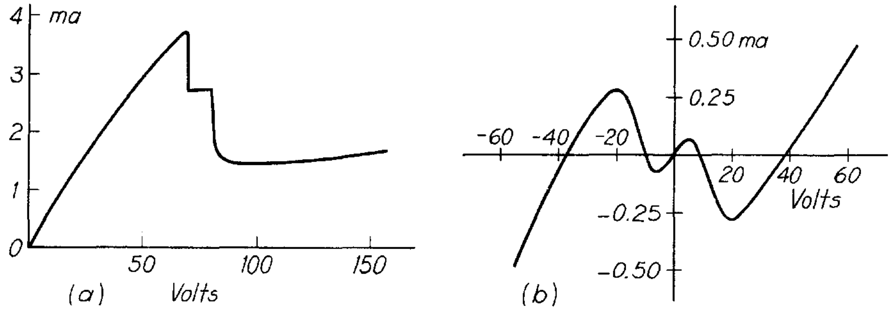


FIG. 114. Current-voltage characteristics of the circuit of Fig. 113: (a) between one plate and ground; (b) between plates.

34.4. Use of Nonlinear Load Resistance. Any circuit of the form of Fig. 81 in which the nonlinear element N has current-voltage curves of the form of Fig. 61a or 61b can also be made to have more than two stable states of equilibrium by replacing the resistance R by a network the load line of which consists of alternate sections having slope magnitude greater than and less than that of the negative-resistance portion of the current-voltage curve, as shown in Fig. 115. A diode network that accomplishes this result is shown in Fig. 116a and the current-voltage curve of this network is shown in Fig. 116b.[2] The circuit of Fig. 116a is most readily analyzed if the input current i is assumed to be the independent variable.

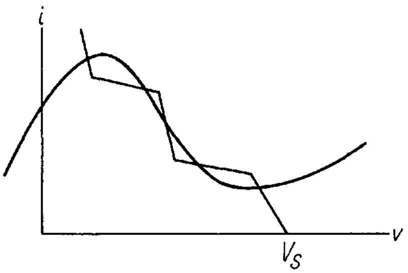


FIG. 115. Current-voltage diagram in which the load line has several sections of different slopes and intersects the current-voltage characteristic in three stable and two unstable points.

The positive supply voltages V_1, V_2, V_3, and V_4 maintain conduction in diodes D_1, D_3, and D_5 when no current enters the input terminal and they also bias D_2, D_4, and D_6 in the reverse direction. For zero input current the input voltage v is equal to V_1 less the voltage drop $V_1R_f/(R_f + R_1)$ in D_1, where R_f is the diode forward resistance. Because R_f is small in comparison with R_1, an input current that is small flows almost entirely through D_1, and thus the input current reduces the current in D_1. As the input current is increased, the current in D_1 falls and consequently the voltage drop across D_1 decreases slowly in a manner determined by the current-voltage characteristic of the diode. The input voltage must therefore rise as shown

[2] R. A. Henle, *Electrical Eng.*, **74**, 570 (July, 1955). See also Sec. 67.

in Fig. 116b, this portion of the characteristic being the diode characteristic rotated through 180 degrees. When v becomes equal to V_1, diode D_1 is cut off, and if the current is increased further the additional input current must flow through R_1. Under the assumption that the diode back resistances are high in comparison with R_1, the slope of this portion of the curve of Fig. 116b is $1/R_1$. When v becomes equal to V_2 minus the drop across D_3, D_2 starts to conduct and the current through D_3 to fall. The input voltage rises slowly in a manner determined by the current-voltage characteristic of D_2 and D_3 in series. When the input voltage equals V_2, D_3 is cut off and the current flows through R_1 in parallel with the series

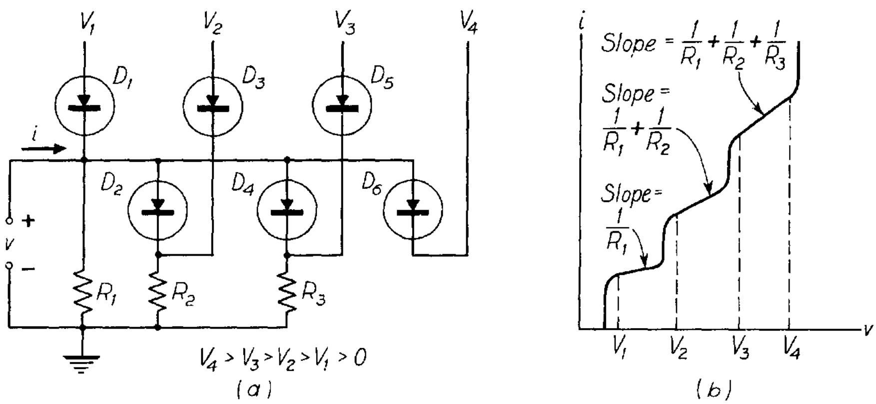


FIG. 116. (a) Nonlinear-load circuit; (b) current-voltage curve for the circuit.

combination of D_2 and R_2. In this region the slope of the curve is approximately $(R_1 + R_2)/R_1R_2$. The remainder of the curve of Fig. 116b may be explained in a similar manner.

If desired, additional biased diodes could be introduced in series with the lower ends of the resistors in the circuit of Fig. 116a in such a manner that the input current would flow through only one resistor at a time. The slopes of the various sections of the curve of Fig. 116b would then be determined by the individual resistances.

34.5. Use of Breakdown Diodes to Provide Nonlinear Load. A nonlinear load circuit that makes use of breakdown diodes is shown in Fig. 117a, and the current-voltage characteristic of the circuit in Fig. 117b.[3] The voltage V' must exceed the sum of the diode breakdown voltages. When the input current I is zero, the three diodes are broken down, V' causing currents to flow through the diodes and resistances in the indicated directions. The input voltage V is negative and equal to the sum of the diode voltages, which is somewhat greater than the sum of their breakdown voltages.

[3] Henle, *loc. cit.* See also Sec. 67.

If a small current enters the input terminal, it flows almost entirely through the low-resistance path of the diodes and therefore reduces the current through the diodes. The input resistance approximates the sum of the variational breakdown resistances of the three diodes. At point a of Fig. 117b the input current becomes equal to I_1, the current through D_1 becomes zero, and the voltage across D_1 is its breakdown voltage. As the current I is raised further, D_1 remains cut off and the input current flows through R_1, raising the voltage across R_1 and reducing the reverse voltage across D_1.

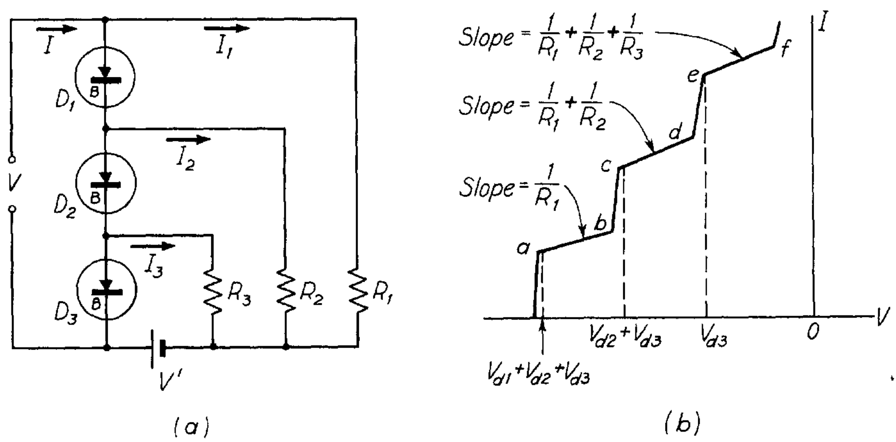


FIG. 117. (a) Breakdown-diode nonlinear-load circuit; (b) characteristic for the circuit. V_{b1} to V_{b3} are the breakdown voltages of D_1 to D_3, respectively.

The input resistance in the range between a and b is R_1. At point b the input voltage becomes equal to the sum of the voltages across D_2 and D_3 (which somewhat exceeds the sum of their breakdown voltages), the voltage across D_1 becomes zero, and further increase of input current causes D_1 to conduct in the forward direction. Any additional current then flows through the three diodes, and the input resistance is the sum of the forward resistance of D_1 and the breakdown resistances of D_1 and D_2. At point c the input current becomes equal to the sum of I_1 and I_2, D_2 cuts off, and the input current must flow through R_1 and through D_1 and R_2. In the range between c and d, the input resistance approximates $R_1R_2/(R_1 + R_2)$. Diode D_2 starts to conduct in the forward direction at d. Diode D_3 cuts off at point e and starts to conduct in the forward direction at f. The resistance in the range between e and f approximates that of R_1, R_2, and R_3 in parallel. Beyond f it approximates the sum of the forward resistances of the three diodes. The heights of the current steps in the characteristic of Fig. 117b increase with increase of V' and with reduction of the resistances R_1, R_2, and R_3. The relative heights of the current steps may be adjusted by changing the relative values of R_1, R_2, and R_3. The voltage steps are equal to the diode breakdown voltages. The entire curve may be displaced along the voltage axis by changing the direct potential of the lower input terminal.

Figure 118 shows multistable circuits formed by combining the transistor negative-resistance circuit of Fig. 74d with the load circuits of Figs. 116a and 117a.[4]

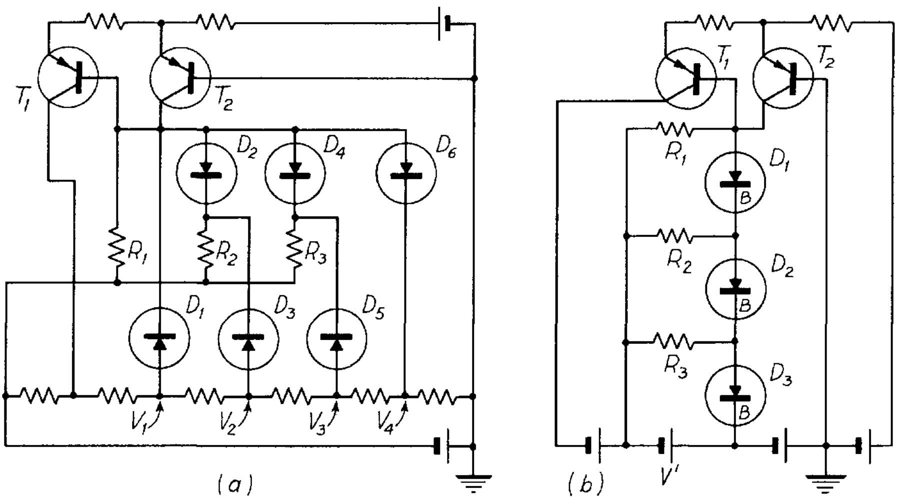


FIG. 118. Multistable circuits formed by combining the circuit of Fig. 74d with (a) the circuit of Fig. 116a and (b) the circuit of Fig. 117a.

34.6. Multiple-Tube Eccles-Jordan Circuit. A third type of multistable circuit is shown in Fig. 119a.[5] In this circuit the grid of each tube is coupled to the plate of the other two tubes, and the bias supply and circuit parameters are chosen so that the reduction of plate voltage resulting from the flow of plate current in any tube biases the other tubes beyond cutoff. The capacitors C_{12} between the plate of each tube and the grid of the following tube cause the tubes to conduct in sequence when periodic negative pulses are applied simultaneously to all grids. Thus, if tube 1 is conducting, the next negative pulse causes this tube to stop conducting and its plate voltage to rise, but has no effect upon the plate voltages of the other tubes. Because the commutating capacitors C_{12} cannot change voltage instantaneously, the rise in plate voltage of tube 1 causes an equal rise in grid voltage

[4] Henle, *loc. cit.*
[5] J. W. Christensen, *Very High-Frequency Techniques,* Staff of the Radio Research Laboratory, McGraw-Hill Book Co., Inc., 1947, p. 305.

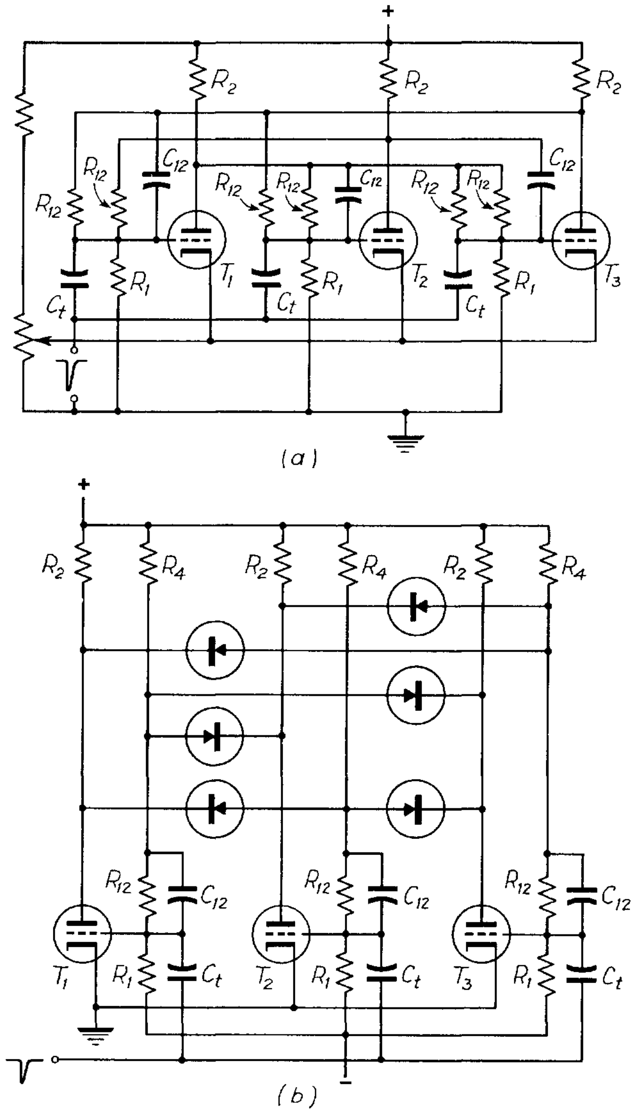


FIG. 119. (a) Multiple-tube Eccles-Jordan circuit; $R_1 = 500$ kΩ, $R_{12} = 1$ M, $R_2 = 150$ kΩ, $C_{12} = 250$ $\mu\mu$f, and $C_t = 10$ $\mu\mu$f.

of tube 2, but there is no change in grid voltage of tubes 1 and 3. If the duration of the input pulse is shorter than the time taken for the commutating capacitors to charge or discharge to the point where the grid voltage of tube 1 or 3 is equal to that of tube 2, tube 2 will conduct at the termination of the input pulse.

Although four tubes have been used successfully in this circuit, and it seems likely that more could be used if the resistance values were carefully chosen and the individual resistors were selected, the stability goes down as the number of tubes is increased. One cause of decreased stability is reduction of plate-voltage swing and therefore of grid-voltage swing resulting from reduction of effective plate load resistance by the coupling networks. A more important cause of reduced stability is associated with the fact that the grid of each nonconducting triode is connected not only to the plate of the conducting triode, but also to the plate of each other nonconducting triode, which is nearly at positive supply potential. The amount by which the grid voltage of the nonconducting triodes is depressed is therefore reduced by the addition of another triode and may become less than the cutoff grid voltage. This undesirable coupling can be prevented by the addition of coupling diodes, as shown in Fig. 119b.[6] All of the coupling diodes except those connected to the plate of the conducting triode are reverse-biased and therefore disconnect the grid of each nonconducting triode from the plate of the other nonconducting triode (triodes, if more than three triodes are used). The disadvantage of the circuit of Fig. 119b is the cost of the diodes, $n(n - 1)$ of which are required for a circuit having n triodes. Design equations for the circuit Fig. 119b have been derived by Beckwith.[6]

35. The Use of Multistable Circuits to Control Gated Amplifiers

Voltages derived from multistable circuits may be used to control gate circuits of the types discussed in Secs. 10 to 13. Figure 120 shows a circuit in which random or periodic pulses are used to control the application of two signal voltages alternately to the inputs of two amplifiers. The collector voltages of the Eccles-Jordan circuit, consisting of the transistors T_1 and associated resistors, control two gated-amplifier circuits of the form of Fig. 57c, the voltage divider P being adjusted so that the voltages applied to the bases of T_2 and T_3 alternate between the proper positive and negative values. Although separate output terminals are provided for the two signal voltages in the circuit of Fig. 120, a single collector-circuit resistor may be used for the two transistors T_4 and the two output voltages thus be delivered to a common pair of output terminals.

Figure 121 shows a similar circuit in which vacuum tubes are used. The

[6] H. Beckwith, *Electronics*, January, 1955, p. 149.

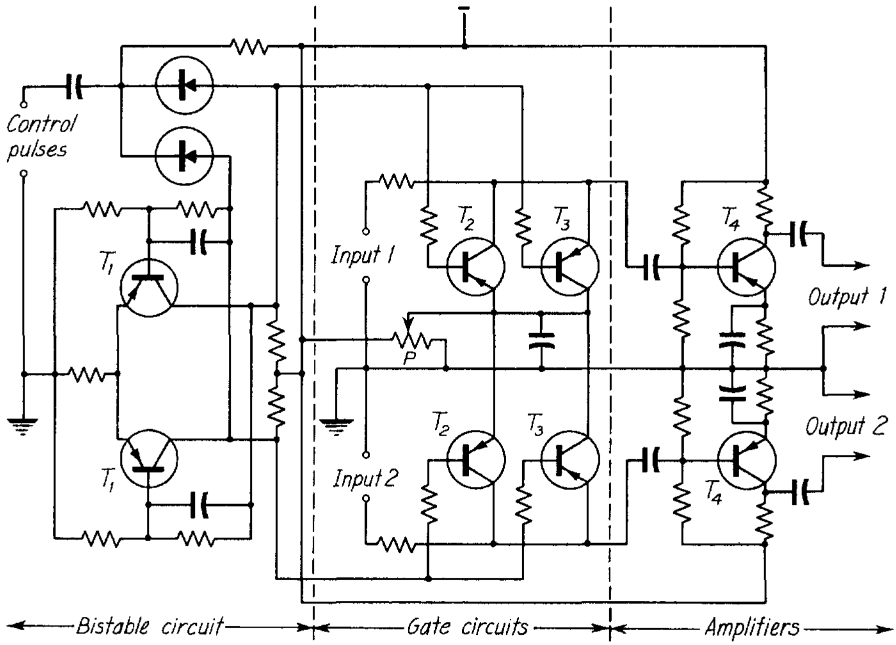


FIG. 120. Transistor circuit in which two amplifiers are activated alternately by means of an Eccles-Jordan circuit.

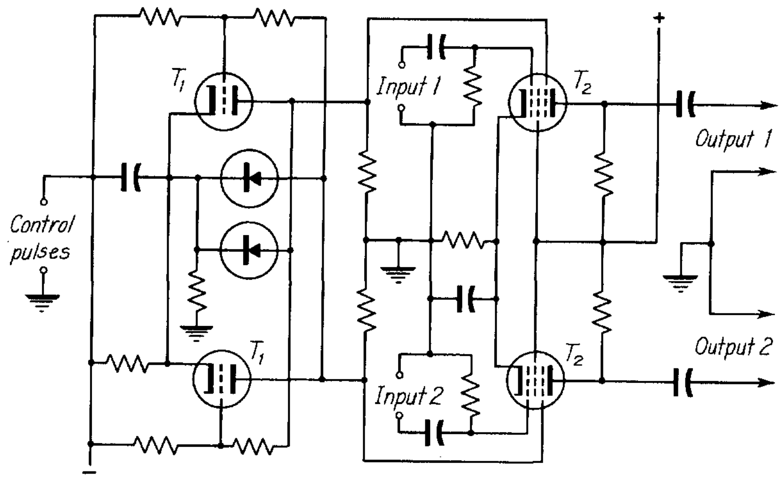


FIG. 121. Vacuum-tube circuit in which two amplifiers are activated alternately by means of an Eccles-Jordan circuit.

pentode amplifiers T_2 are gated by means of voltages derived from the plates of the triodes T_1 and applied to the suppressors of the amplifiers. The circuits of Figs. 120 and 121 may be readily adapted to use with three or more signal voltages by the addition of other amplifiers with separate or common output terminals and deriving the control voltages from a circuit of the form of Fig. 119.

36. Binary and Ternary Counting Circuits

36.1. Eccles-Jordan Binary Counter. An important application of bistable circuits is in the counting of pulses. The simplest type of circuit consists of a chain of Eccles-Jordan bistable circuits connected so that the

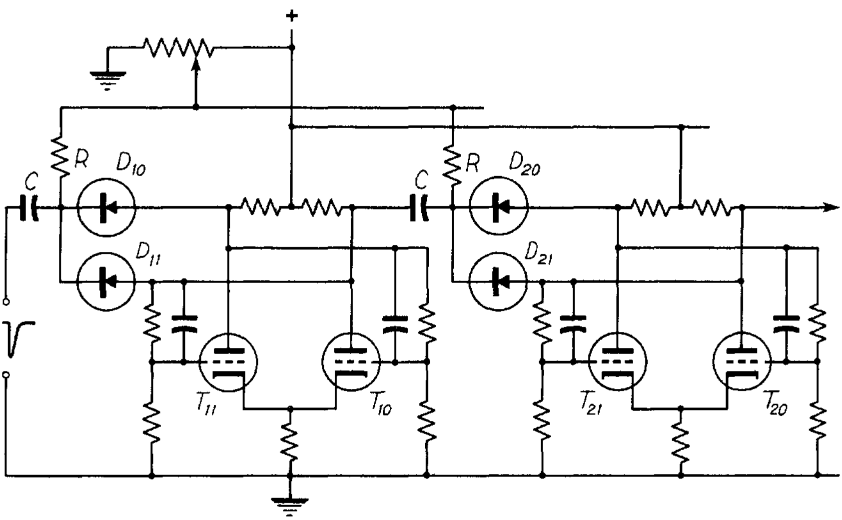


FIG. 122. Two-stage Eccles-Jordan binary counting circuit.

sudden drop in voltage caused by the firing of one tube of any stage causes triggering of the next stage. A two-stage circuit is shown in Fig. 122. Let us assume that tubes T_{10} and T_{20} are conducting initially and that a series of negative pulses is applied to the input terminals. The first pulse transfers conduction from T_{10} to T_{11}. Because the back-biased diode D_{21} prevents the application of the plate-voltage rise of T_{10} to the plate of T_{20} and the grid of T_{21}, the first input pulse does not trigger the second stage. The second pulse transfers conduction back to T_{10}. The time constant RC of the coupling network is low enough so that the resulting drop in plate voltage of T_{10} produces a negative pulse at the cathodes of the coupling diodes which transfers conduction from T_{20} to T_{21}, as explained in Sec. 29. The third pulse triggers only the first stage, and the fourth pulse returns the circuit to its initial state. Similarly, if n stages are used, the circuit goes through all equilibrium combinations and returns to its original state in 2^n pulses. The first five columns of Table V show the effect of a series of 16 pulses upon a four-stage circuit. The two tubes in each stage are represented by the symbols 0 and 1, and the table indicates which tubes conduct in each stage after each pulse. All the 0 tubes are assumed to be conducting prior to number-1 pulse. Each stage is assumed to be triggered by negative pulses only, and the triggering pulses for the second, third, and fourth stages are assumed to be derived from the plate voltage of the 0 tube of the preceding stages, so that the second, third, and fourth stages are triggered when the 0 tube of the preceding stage fires.

TABLE V CONDUCTION SEQUENCE IN THE CIRCUITS OF FIGS. 123 TO 125

Pulse Number	Stage 4	Stage 3	Stage 2	Stage 1	Pulse Number with Feedback		
					Circuit of Fig. 123	Circuit of Fig. 124	Circuit of Fig. 125
0	0	0	0	0	0	0	0
1	0	0	0	1	1	1	1
2	0	0	1	0	2	2	2
3	0	0	1	1	3	3	3
4	0	1	0	0	4	4	4
5	0	1	0	1	5	5	5
6	0	1	1	0	6	6	—
7	0	1	1	1	7	7	—
8	1	0	0	0	—	8	—
9	1	0	0	1	—	9	—
10	1	0	1	0	—	—	6
11	1	0	1	1	—	—	7
12	1	1	0	0	—	—	8
13	1	1	0	1	—	—	9
14	1	1	1	0	8	—	—
15	1	1	1	1	9	—	—
16	0	0	0	0	10	10	10

Table V shows that each successive pulse leaves the circuit with a unique combination of conducting tubes, and that this combination may therefore be used in determining the total number of pulses (up to 16) that have been impressed upon the input. Those familiar with the binary number system, in which each digit is either a 1 or a zero, will recognize that columns 2 to 5 in Table V express the pulse number in the binary system. (The numbers in the columns 6 to 8 will be explained below.) For this reason this type of circuit is called a binary counter. A three-stage circuit, which counts up to 8 pulses, is often called a "scale-of-eight" circuit; a four-stage circuit is called a "scale-of-sixteen" circuit; etc.

36.2. Addition and Subtraction of Counts. The order in which the various combinations of conducting tubes listed in Table V are assumed is reversed if the triggering voltages for the second, third, and fourth stages are obtained from the plate of the 1 tube of the preceding stages (Prob. 36.2–1). If the source of triggering voltage of the second, third, and fourth stages is switched from 0 tubes to 1 tubes of the preceding stages after n pulses have been impressed, and m additional pulses are then impressed, the combination of conducting tubes will be the same as though $n - m$ pulses had been impressed without making this change. It follows, therefore, that if provision is made to switch the triggering source of each stage but the first from 0 tubes to 1 tubes of preceding stages, and vice versa, the circuit can either add or subtract the input-pulse count. The required switching can be accomplished by mechanical switches, or by electronic switches of the type discussed in Secs. 10 to 13. At the termination of the count the circuit is reset by opening the plate circuits of all the 1 tubes, or by equivalent means.

Although the total count that can be obtained in an n-stage circuit is limited to 2^n, the total count can obviously be increased to any desired extent by the use of many stages. An alternative method of increasing the counting limit is by the use of an electromagnetic type of mechanical counter that is actuated, directly or indirectly, by the final stage of the binary counter. One mechanical count is then obtained for each 2^n input pulses, and the number of additional input pulses following the last mechanical count can be determined from the combination of conducting tubes in a manner that will be explained in Sec. 38. In this way a mechanical counter can be used at input frequencies far in excess of the maximum frequency at which the mechanical counter can function directly.

36.3. Ternary Counter. A ternary counter may be constructed by the use of two or more stages of the tristable circuit of Fig. 113.[1] The output from the plate of T_2 is differentiated and applied to the trigger input terminals of the next stage through a diode clipper that removes the positive pulse. In order to reduce the fall time of the plate voltage of T_2 during the transition that cuts T_1 off and turns T_2 on, a capacitor of approximately 450 $\mu\mu$f capacitance should be connected between the cathode of T_2 and

[1] R. Weissman, *Electronics*, October, 1952, p. 118.

ground. Relatively short fall time is necessary to ensure a sufficiently sharp output pulse to trigger the next stage.

37. Decade Counters

37.1. Conversion of Binary Circuit to Decade Circuit. Because 10 is not a multiple of 2^n, decade circuits, which complete one counting cycle in a multiple of ten input pulses, cannot be obtained directly by the use of unmodified n-stage circuits of the type discussed in Sec. 36. A decade circuit can be made, however, by modifying a four-stage circuit in such a manner that some of the combinations of conducting tubes listed in Table V are skipped.[1] This can be accomplished by applying to the inputs of the second and third stages an additional negative pulse derived from the plate of the 1 tube of the fourth stage, as shown in Fig. 123a.[2] The manner in which the operation of the circuit is affected by this feedback can be seen from an examination of Table V. Without the feedback pulse the eighth input pulse would cause transition in all stages and leave stages 2 and 3 in the 0 conduction state. Transition of the fourth stage, however, applies to the grids of the 0 tubes of the second and third stages a slightly delayed negative pulse that retriggers these stages and leaves them in the 1 conduction state. After the eighth input pulse, therefore, the circuit configuration is the same as that which would obtain after the fourteenth count without feedback, and the circuit returns to its original configuration after the tenth pulse. The counting sequence is shown in column 6 of Table V.

One practical form of the circuit of Fig. 123a is shown in detail in Fig. 123b. In this circuit and in those of Figs. 124 and 125, the first subscript in the letter symbol used to label individual tubes or transistors indicates the number of the stage and the second subscript indicates whether a particular tube is a 0 tube or transistor (initially conducting) or a 1 tube or transistor (initially cut off).

The maximum speed at which the circuit of Fig. 123 can operate is limited by the total delay time involved in the transition of all four stages and the retriggering of stages 2 and 3 following the eighth input pulse. The delay time can be reduced and the circuit consequently speeded up by applying the output pulse from the first stage directly to the fourth stage through a coincidence circuit (gate) controlled by stages 2 and 3[3] (Prob. 37.1-1). Since stages 2 and 3 are in the 1 configuration only prior to the eighth and tenth counts in the decade circuit, and the fourth stage must be triggered by the eighth and tenth input pulses, the desired operation is obtained if the coincidence circuit is connected so that it applies an input pulse to

[1] H. Lifschutz, *Phys. Rev.*, **57,** 243 (1940).
[2] I. E. Grosdoff, *RCA Rev.*, **7,** 438 (1946).
[3] A. S. Bagley, *Hewlett-Packard J.*, **2,** No. 2, October, 1950.

the fourth stage only when the second and third stages are in the 1 configuration. Feedback from the fourth stage retriggers the second and third stages after the eighth input pulse, as in the circuit of Fig. 123.

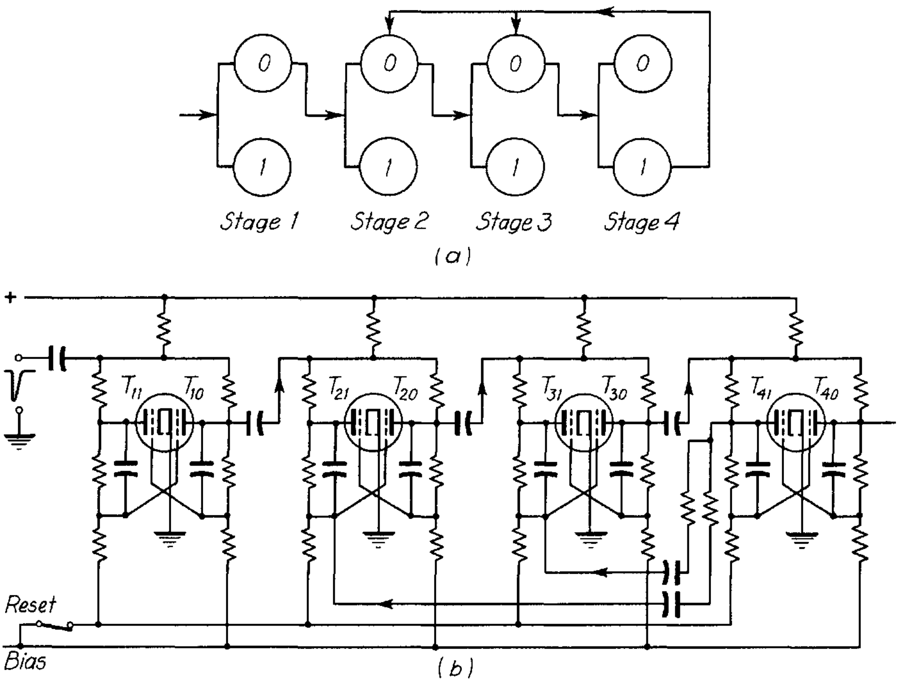


FIG. 123. Decade counter formed from a four-stage binary circuit by the addition of feedback paths: (a) basic circuit; (b) one form of practical circuit.

37.2. Alternate Form of Vacuum-Tube Circuit. Another way in which a four-stage binary circuit can be modified to provide a decade circuit is shown in Fig. 124.[4] Initially the 0 tubes T_{10}, T_{20}, T_{30}, and T_{40} are on. Forward coupling is from the plate circuit of the 0 tube of each stage to the grids of the following stage. In addition, feedback coupling is used from the plate circuit of T_{41} to the grid of T_{20} and additional forward coupling from the plate circuit of T_{10} to the grid of T_{41}.

The conduction sequence of the circuit of Fig. 124, which is shown in column 7 of Table V, is unaffected by the feedback path and the extra forward path during the first nine triggering pulses. When the tenth triggering pulse causes T_{10} to conduct, however, a negative pulse derived from a tap on the plate resistor of T_{10} is applied to the grid of T_{41} through the auxiliary forward path. The rise of plate voltage of T_{41} accompanying the

[4] J. T. Potter, *Electronics*, June, 1944, p. 110.

resulting transition of stage 4 causes the application of a relatively large positive pulse to the grid of T_{20} through the feedback path. This pulse overpowers the triggering pulse fed to stage 2 from stage 1 and thus prevents transition of stage 2. The tenth pulse therefore leaves T_{10}, T_{20}, T_{30}, and T_{40} in the conducting state, as at the beginning of the count. The position of the tap on the plate resistor of T_{10} must be such that the voltage fed forward will cause transition of stage 4 at the tenth count, but will be overpowered

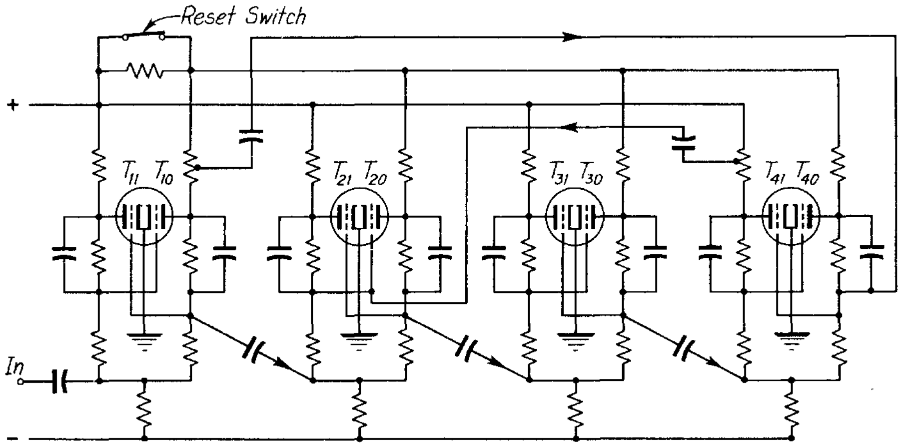


FIG. 124. Decade counter incorporating both a feedback path and an auxiliary forward path.

by the negative pulse from stage 3 at the eighth count. Temporary opening of the reset switch after any number of input pulses causes the plate voltage of all the 0 tubes to be reduced below that of the 1 tubes and thus ensures that all 0 tubes are conducting at the beginning of the next count.

37.3. Transistor Decade Circuit. A transistor decade counting circuit is shown in Fig. 125 and the conduction sequence in column 8 of Table V.[5] This circuit has an auxiliary forward coupling path from the collector of T_{10} to the base of T_{31} through the diode D_f and a reverse coupling path from the collector of T_{40} to the base of T_{21} through the diode D_r. The conduction sequence of the binary stages is unaltered by the auxiliary paths until after pulse 5, which leaves T_{30} off and thus raises the anode potential of D_f to nearly the supply voltage. Firing of T_{10} at the sixth pulse lowers the collector voltage of T_{10}. This drop in voltage not only triggers stage 2 in the normal manner, but also causes D_f to be biased in the forward direction, allowing D_f to transfer a sufficient portion of the voltage drop to the base of T_{31} to cut T_{31} off and make T_{30} conduct. The

[5] P. Krevitsky, *Electronics*, August, 1955, p. 112.

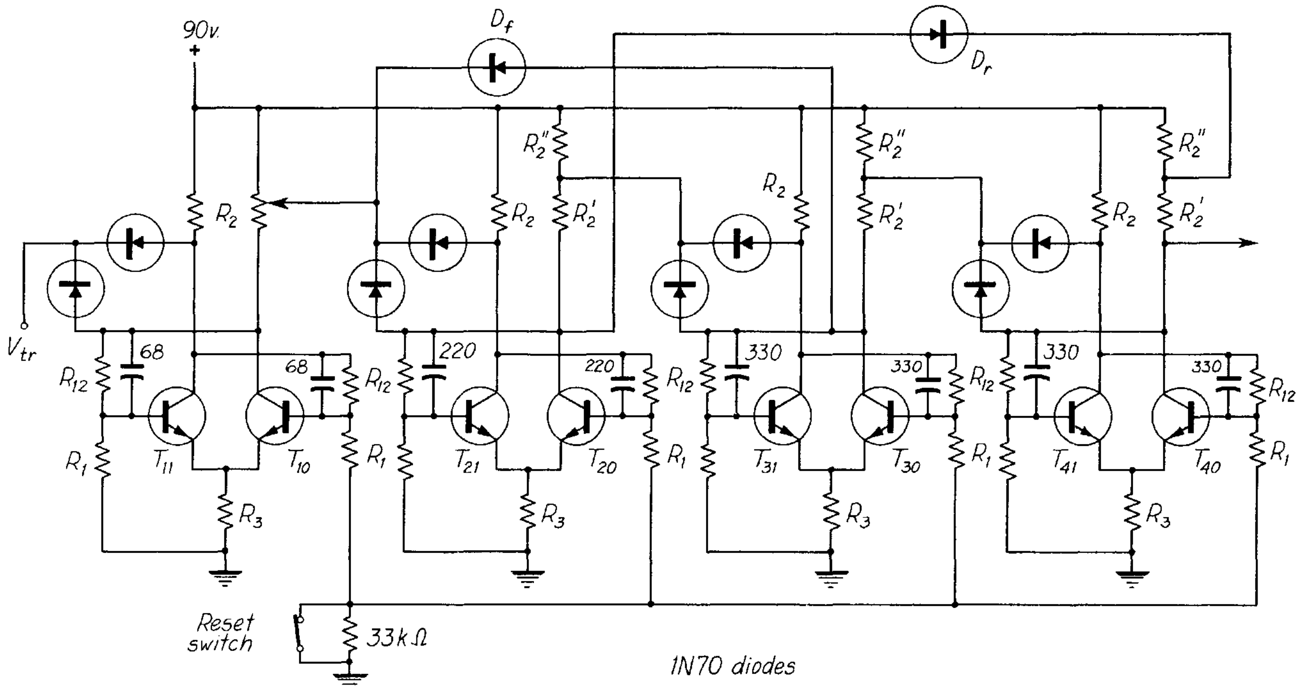


FIG. 125. Transistor decade counter incorporating both a feedback path and an auxiliary forward path. $R_1 = 4.7$ kΩ, $R_{12} = 22$ kΩ, $R_2 = 10$ kΩ, $R_2' = 5.3$ kΩ, $R_2'' = 4.7$ kΩ, $R_3 = 900\Omega$. Capacitances are in $\mu\mu$f.

resulting drop in the collector voltage of T_{30} triggers stage 4, causing T_{41} to conduct. The tenth pulse fires T_{10}, which in turn fires T_{21} and, through D_f, also T_{30}. The firing of T_{30} again initiates transition of stage 4, which is left with T_{40} on. The cutting off of T_{20} raises the anode potential of the feedback diode D_r, so that the firing of T_{40}, and consequent lowering of its collector voltage, allows D_r to conduct. A negative pulse is therefore applied through D_r to the base of T_{21} and conduction is transferred from T_{21} to T_{20}. All stages are now in their initial states. Opening of the reset switch after a series of input pulses raises the base voltage of all 0 transistors and thus ensures that they conduct at the beginning of the next series of pulses.

Typical parameters of the silicon transistors used by Krevitsky were $h_{ie} =$ 50 to 150 ohms, $h_{fe} = 16$, $h_{oe} = 0.3$ to 0.4×10^{-6} mho, and $h_{re} = 1$ to 4×10^{-4} for T_4 and T_6. For all other transistors h_{oe} was in the range from 0.5 to 1×10^{-6} mho. The allowable peak inverse voltage of the transistors was 90 volts. The maximum frequency of the input pulse at which the circuit operated reliably was 200 kc, and the minimum input pulse amplitude was 10 volts. The supply current was 40 ma, the collector dissipation 40 to 50 mw, the total dissipation 3.6 watts, and the operating temperature 20° to 70°C.

37.4. Decade Circuit Incorporating Electronic Switch. Another method of modifying a binary circuit in order to obtain a decade counter has been described by Kemp.[6] This counter is basically a three-stage binary circuit, but has an electronic switch between the first and second binary stages. The switch is controlled by an additional binary stage that is triggered by the output of the third stage. Initially the switch directs the output of the first stage to the second stage. Transition of the third stage at the eighth count, however, operates the switch-control binary stage, which changes the switch so that the ninth and tenth outputs of the first stage are impressed directly upon the switch-control binary, instead of upon the second stage. The output of the first stage produced by the ninth input pulse is of the wrong polarity to trigger the switch-control binary, but the tenth pulse causes transition of the switch-control binary and thus causes the switch to be reset. The circuit is then in its initial state. The output is taken from the switch-control binary on the tenth count. This circuit responds to successive pulses having a minimum separation of 2 μsec. The input pulse amplitude may range from 10 to 100 volts. For circuit details the reader is referred to Kemp's paper.

37.5. Circuits for Other Number Systems. The techniques described in this section may be extended and modified to provide circuits that complete one cycle of stable states in any desired number of counts (Probs. 37.5-1 and 37.5-2). Usually the desired result can be achieved in a num-

[6] E. L. Kemp, *Electronics*, February, 1953, p. 145.

ber of ways, but some methods are simpler than others.[7] In general, the indicated count of an n-stage binary circuit is increased or decreased by 1, 2, 4, or 8, depending upon whether an extra pulse is applied to the first, second, third, or fourth stage, respectively. The number of times the count is altered during one cycle by a particular auxiliary coupling path, and whether the indicated count is increased or decreased depends upon the point in the circuit from which the pulse is obtained, whether the pulse is applied to one or both grids of the stage that receives it, and whether or not the coupling path incorporates diodes. In the circuit of Fig. 123 an extra pulse is applied once in the counting cycle simultaneously to stages 2 and 3, causing the indicated count to be increased by $2 + 4$. In the circuit of Fig. 124 the extra pulse applied once to stage 4 increases the indicated count by 8 and the extra pulse applied once to stage 2, which inhibits transition of that stage, reduces the indicated count by 2. (Note that, if the feedback path from stage 4 to stage 2 were omitted, the combination of conducting tubes after ten input pulses would be the same as that assumed after twelve pulses in the decade circuit.) In the circuit of Fig. 125, two extra pulses applied to stage 3 through D_f increase the indicated count by $4 + 4$, and the extra pulse applied to stage 2 reduces the indicated count by 2.

The output of the 0 tube of the fourth stage of a decade counter can be used to drive another similar decade circuit, which receives an input pulse for each ten pulses of the first circuit. By the use of additional decade circuits the maximum count that can be recorded may obviously be extended to as many places as desired. The count may be displayed by means of neon tubes or other indicator lamps in a manner explained in the following section.

38. Count Indicators for Counting Circuits

38.1. Indicators for Binary Counters. The simplest type of count indicator for unmodified binary counters consists of glow tubes in series with high resistances connected between ground and the plates of the 0 tubes or the collectors of the 0 transistors. If each glow tube is assigned a numerical value $2^n/2$, where n is the number of the stage to which it is connected, the sum of the values of all glowing tubes at any time is equal to the number of input pulses that have elapsed since the circuit was reset, provided that this number is less than that required for one complete counting cycle. The difference between the breakdown and extinction voltages of the glow tubes must be less than the plate-voltage change of the counting-circuit tubes, and the glow tubes must be biased so that cutting off of the counter-circuit tube or transistor to which the glow tube is connected raises the voltage across the glow tube from below the extinction voltage to above the breakdown voltage. The resistances in series with the glow tubes must be large

[7] G. F. Montgomery, *J. Appl. Phys.*, **22**, 780 (June, 1951).

in comparison with the load resistances of the binary stages in order to prevent excessive loading.

38.2. Indicators for Decade Counters. Count indicators used with decade circuits of the types discussed in Sec. 40 depend upon the fact that after each input pulse there is a unique combination of tubes (or transistors), located in different stages, one nonconducting and the others conducting, and that this combination of nonconducting and conducting tubes

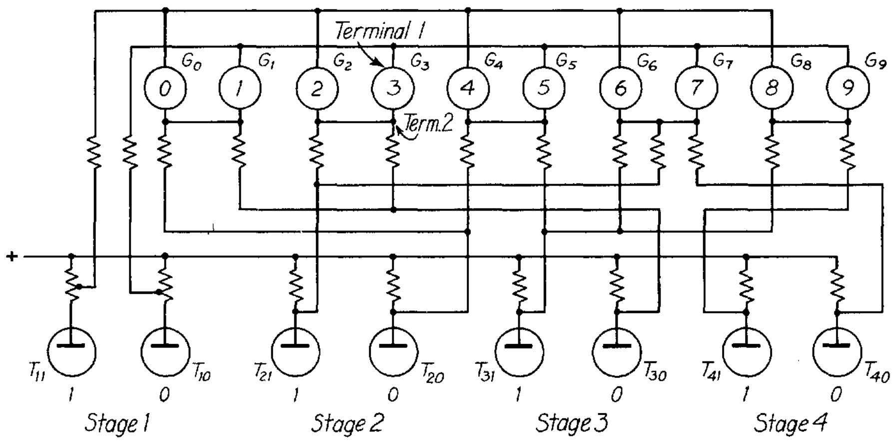


FIG. 126. The use of indicator glow tubes in the decade circuit of Fig. 123.

is not assumed after any other input pulse in a series of ten. The count is indicated by means of ten glow tubes, which are numbered from 0 to 9. Each glow tube is connected to the plates of the particular combination of counting-circuit tubes that is observed after the number of pulses equal to the glow-tube number, one side of the glow tube being connected through a resistor to a tap on the plate resistor of the nonconducting tube, and the other side through separate resistors to the plates of the conducting tubes. Thus the voltage applied to this glow tube at this point in the counting cycle is the difference between the plate voltage of the nonconducting triode (nearly supply voltage) and that of the conducting triodes. If this voltage exceeds the breakdown voltage of the glow tube, it glows. For any other combination of conduction and nonconduction that can be assumed by the counting-circuit tubes, the voltage across the glow tube is lower. If the glow tubes, supply voltage, and plate load resistances are properly selected, the lower voltages are insufficient to allow the glow tube to continue to glow.

Figure 126 shows how this indicating method may be applied to the decade circuit of Fig. 123. Table VI lists the tubes that are on after each input pulse, and also the tubes to which the two terminals of the glow tubes are connected. The operation can be most readily explained by considering one glow tube, say No. 3, which should light only after count 3. In order that the voltage applied to G_3 shall be a maximum at this time, one terminal of G_3 must connect to the plate of a triode that is off. Since T_{11} is on after pulse 3, this is accomplished by connecting terminal 1 to the plate of T_{10}. Terminal 2 of G_3 is connected to the plates of T_{21} and T_{30},

Table VI Conduction Sequence and Glow-Tube Connections for the Circuit of Fig. 126

Pulse Number	Conducting Tubes				Glow-Tube Connections	
	Stage 1	Stage 2	Stage 3	Stage 4	Terminal 1	Terminal 2
0	T_{10}	T_{20}	T_{30}	T_{40}	T_{11}	T_{20} and T_{30}
1	T_{11}	T_{20}	T_{30}	T_{40}	T_{10}	T_{20} and T_{30}
2	T_{10}	T_{21}	T_{30}	T_{40}	T_{11}	T_{21} and T_{30}
3	T_{11}	T_{21}	T_{30}	T_{40}	T_{10}	T_{21} and T_{30}
4	T_{10}	T_{20}	T_{31}	T_{40}	T_{11}	T_{20} and T_{31}
5	T_{11}	T_{20}	T_{31}	T_{40}	T_{10}	T_{20} and T_{31}
6	T_{10}	T_{21}	T_{31}	T_{40}	T_{11}	T_{21}, T_{31}, and T_{40}
7	T_{11}	T_{21}	T_{31}	T_{40}	T_{10}	T_{21}, T_{31}, and T_{40}
8	T_{10}	T_{21}	T_{31}	T_{41}	T_{11}	T_{31} and T_{41}
9	T_{11}	T_{21}	T_{31}	T_{41}	T_{10}	T_{31} and T_{41}
10	T_{10}	T_{20}	T_{30}	T_{40}	T_{11}	T_{20} and T_{30}

both of which conduct at this time and therefore have low plate voltage. If T_{10} is off and either T_{21} or T_{30} is on, as after pulse 1 and pulse 9, the voltage of terminal 2 is midway between the on and off values of plate voltage. If T_{10}, T_{21}, and T_{30} are all off, as after pulse 5, there is no voltage across the glow tube. If T_{10} is on and T_{21} and T_{30} are off, as after pulse 4, the voltage across G_3 is lower than after pulse 3 because terminal 1 is connected to a tap on the plate resistor of T_{10}, rather than directly to the plate. The reader may readily show that the voltage impressed upon G_3 initially and after pulses 2, 6, 7, 8 is also less than that impressed after pulse 3.

Terminal 2 of glow tubes G_6 and G_7 must be connected to the plates of three tubes of the counter, rather than to only two, because there are only eight combinations of one off and two on tubes not duplicated during a series of ten input pulses in the circuit of Fig. 123. On the other hand, if neither of the fourth-stage tubes (or transistors) is conducting after more than six in a series of ten input pulses, there is a set of ten combinations of three tubes none of which is duplicated in a series of ten counts. This is true of the transistor circuit of Fig. 125 and other decade circuits[1] (Prob. 38.2–1). In the circuit of Fig. 126, change of only one of three tubes T_{21}, T_{31}, or T_{40} from the conducting state to the nonconducting state raises the voltage of terminal 2 of G_6 and G_7 by only one-third the rise in plate voltage of that tube. Consequently the difference between the breakdown and extinction voltages of the glow tubes must be less than one-third the difference between the off and on values of plate voltage.

39. Decade Ring Counting Circuits

39.1. Basic Ring Circuit. A second type of decade circuit consists of a ring of ten tubes connected in such a manner that diagonally opposite tubes form Eccles-Jordan bistable pairs, as shown in Fig. 127, in which the screen-coupled Eccles-Jordan circuit of Fig. 99a is used. The resistances connecting the plates of adjacent tubes ensure that the five conducting tubes are adjacent.[1] Thus, if T_4, T_6, T_7, T_8, and T_0 were conducting in the circuit of Fig. 127, the plate voltage of T_9 would be depressed by the low plate voltages of T_8 and T_0. The accompanying reduction of screen voltage of T_4 would cause T_4 to be cut off and T_9 to conduct, and the circuit would be left with five adjacent conducting tubes.

Commutation in one direction is assured in the circuit of Fig. 127 by the capacitors that couple the plate of each tube to the suppressor of one adjacent tube. If tubes T_6, T_7, T_8, T_9, and T_0 are initially on, the capacitors connected to the plates of these tubes are charged to a lower voltage than those connected to the plates of the off tubes. The next negative pulse applied to the control grids cuts all tubes off during the duration of the pulse and thus causes the plate voltage of all tubes to rise to approximately the supply voltage. At the termination of the pulse, the higher voltage across the capacitors connected to the plates of T_1, T_2, T_3, T_4, and T_5 makes the suppressor voltage of T_2 to T_6 lower than that of the other five tubes and thus prevents tubes T_2 to T_6 from conducting. Consequently, T_7, T_8, T_9, T_0, and T_1 conduct and the series of five adjacent conducting tubes has been advanced by one tube in the clockwise direction. If each capacitor is connected from the plate of one tube to the suppressor of the next tube in the counterclockwise direction, rather than the clockwise direction used in Fig. 127, the direction of advance is reversed.

39.2. Control of Direction of Commutation. A modified form of the circuit of Fig. 127 uses triodes, instead of resistors between the plates of adjacent pentodes, triggering being accomplished by means of positive pulses

[1] See, for instance, Grosdoff, *loc. cit.*
[1] V. H. Regener, *Rev. Sci. Instr.*, **17**, 185 (May, 1946).

applied to the grids of the coupling tubes.[2] The direction of commutation is from pentodes adjacent to coupling-triode cathodes toward pentodes adjacent to coupling-triode plates. If two sets of coupling triodes of opposite polarity are used, the direction of advance may be readily reversed by switching the input pulses from the grids of one set of coupling triodes to the grids of the other set. The triode-coupled circuit has the additional

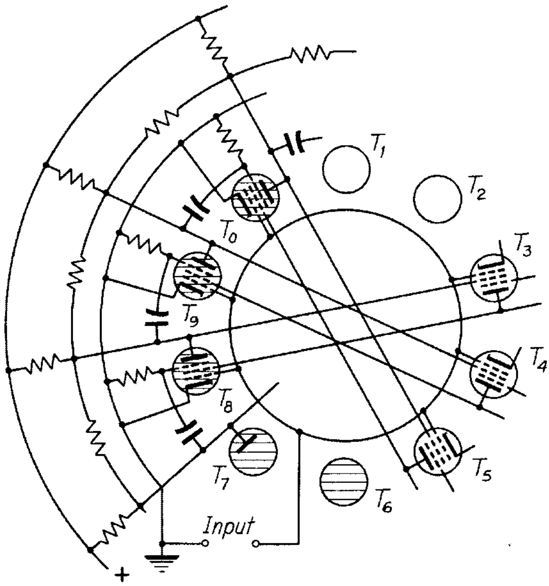


FIG. 127. Portion of a ring type of Eccles-Jordan decade counter based upon the screen-coupled circuit of Fig. 99a.

advantage of greater reliability of commutation, responding not only to sharp pulses, but also to sinusoidal triggering voltage. Counting rates up to 5×10^5 per second were reported by Regener and by Seren for this type of circuit.

Ring circuits similar to that of Fig. 127 have been developed in which triodes are used instead of pentodes,[3] and also circuits in which the screen grids and control grids of pentodes, instead of the plates and screen grids, are used to form the bistable pairs, the suppressors and plates being used for triggering and commutating purposes.[4]

A cathode-ray tube may be used to indicate the count in the circuit of

[2] V. H. Regener, *Rev. Sci. Instr.*, **17**, 375 (October, 1946); L. Seren, *Rev. Sci. Instr.*, **18**, 645 (1947); F. H. Martens, *Rev. Sci Instr.*, **20**, 424 (1949).
[3] G. T. Baker, *J. Sci. Instr.*, **25**, 127 (1948).
[4] B. L. Moore, *Rev. Sci. Instr.*, **21**, 337 (April, 1950).

Fig. 127 and its variants.[5] Milliammeters have also been used.[6] A somewhat more convenient method is the use of ten glow tubes G_0 to G_9, as indicated in the partial circuit of Fig. 128, in which the various grid circuits have been omitted for the purpose of simplification. The voltage impressed upon any glow tube is highest during conduction of the pentode the plate of which connects directly to that glow tube and simultaneous nonconduction

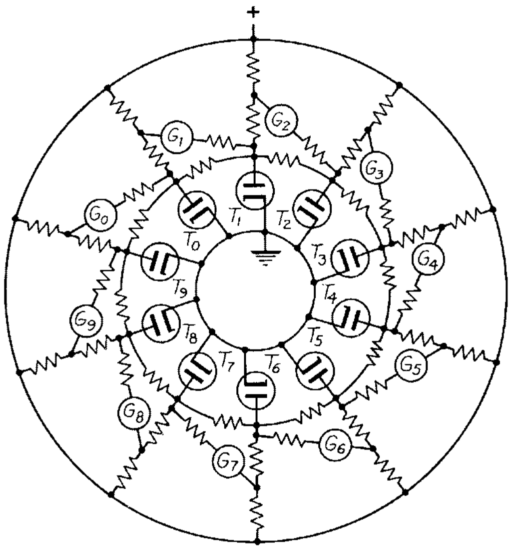


FIG. 128. Partial circuit showing the use of indicator glow tubes in a ring type of decade counter.

of the other pentode to which the glow tube connects (Prob. 39.2–1). If this maximum voltage exceeds the breakdown voltage of the glow tube, the glow tube lights. The voltages impressed upon all other glow tubes for this combination of conducting pentodes is lower. If the glow-tube characteristics, circuit resistances, and supply voltage are properly chosen, only one glow tube conducts for each combination of conducting pentodes. The resistances in series with the glow tubes should be high in comparison with the other circuit resistances.

39.3. Another Type of Ring Circuit. Another type of ring circuit, in which n binary stages ($2n$ triodes) are used in order to obtain an n-count cycle, is shown in basic form in Fig. 129.[7] In this circuit only one of the right-hand triodes, T_1', T_2', T_3', etc., conducts at any time. This property

[5] Regener, *loc. cit.*
[6] Seren, *loc. cit.*
[7] T. K. Sharpless, *Electronics*, March, 1948, p. 122.

is achieved by the use of a common cathode resistor for T_1, T_2, and T_3 and another for T_1', T_2', and T_3' and making the resistance of the former $1/(n-1)$ as great as that of the latter. The voltages across the two resistors are equal if the right tube of one bistable pair conducts and the left tubes of the other $n-1$ pairs conduct. The circuit parameters are chosen so that if two right tubes were to conduct simultaneously, the voltage across the 15-kΩ cathode resistor would cause the grid bias of these tubes to be

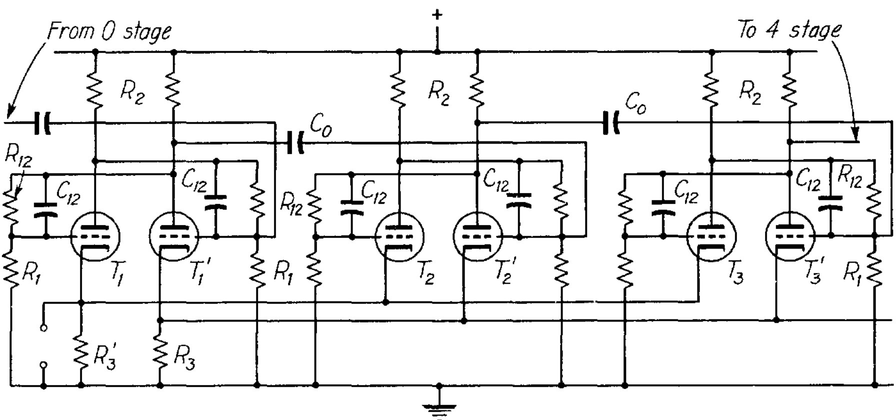


FIG. 129. Another ring type of decade counter. $R_1 = R_{12} = R_2 = 50$ kΩ, $R_3 = 15$ kΩ, $R_3' = 15/(\mathrm{n}-1)$ kΩ, $C_{12} = 25$ $\mu\mu$f, and $C_o = 50$ $\mu\mu$f.

so great that their plate currents would not produce sufficient voltage drop across their plate resistors to bias their companion left tubes beyond cutoff. The circuit is triggered by means of negative pulses applied to the cathodes of all of the left tubes. Thus, if T_1' is initially on and all other right tubes are off, a negative triggering pulse causes T_1 to conduct. The rise in the plate voltage of T_1' accompanying the cutting off of T_1' applies a positive pulse to the grid of T_2'. The effect of this pulse overrides that of the negative pulse applied to the cathode of T_2 and therefore conduction is transferred from T_2 to T_2'. If the cathode resistors are bypassed by capacitors, the circuit may also be triggered by pulses applied simultaneously to all the grids.[8] The circuit of Fig. 129 operates reliably at a maximum counting rate of 180,000 pulses per second, but the triggering pulses must be properly shaped and the pulse amplitude controlled.

An n-stage ring circuit is formed from the basic circuit of Fig. 129 by connecting the plate of T_n to the grid of T_1' through a 50-$\mu\mu$f coupling capacitor. The count may be readily indicated by means of n glow tubes connected, through high resistances, from the plates of T_1', T_2', etc., to a properly se-

[8] R. Weissman, *Electronics,* May, 1949, p. 84.

lected source of biasing voltage. Each glow tube is assigned the number corresponding to that of the stage to which it is connected.

The number of tubes required in an n-stage circuit of the form of Fig. 129, in which n is even, can be reduced from $2n$ to $n + 2$ by combining a ring having $n/2$ stages with a single binary stage. In particular, a decade counter is readily formed by preceding or following a five-stage ring by a binary stage.[9] If the binary stage follows the ring, the binary stage is triggered by the output of the ring at the fifth and tenth counts. The count is indicated by means of ten glow tubes. Glow tubes G_0 to G_4 are connected, through high resistances, between the plates of T_1' to T_5', respectively, and a tap on the plate load resistor of one of the binary tubes; glow tubes G_5 to G_9 are connected between the plates of T_1' to T_5', respectively, and a tap on the plate load resistor of the other binary tube.

40. Glow-Diode Counting Circuits

40.1. Glow-Tube Binary Stage. When high counting rates are not essential, glow-diode circuits may be conveniently used. The maximum counting rate of glow-tube circuits is limited by time required for the tubes to deionize. Advantages of glow tubes over vacuum tubes and transistors in this application are low current drain, low cost, and the fact that no additional tubes are needed to indicate the count. Although a single glow tube in series with a high resistance is in itself bistable (Sec. 22), this simple circuit cannot be used directly as the basis of a counting circuit, since it is not symmetrical with respect to current magnitudes in the two states of equilibrium.

A symmetrical four-tube bistable circuit is shown in Fig. 130a.[1] The operation of the circuit may be explained with the aid of Fig. 130b, in which T_1 and T_2 are two glow tubes of like characteristics and R_1 and R_2 have equal values of resistance. If the voltages V have values intermediate between the breakdown and extinction voltages of the glow diodes, T_1 and T_2 are initially off. Application of a positive pulse to the input terminals causes T_1 to fire. The resulting voltage drop across R_1 raises the voltage across T_2 and causes it to fire. If a negative pulse is applied to the input terminals, T_2 fires first, and the drop across R_1 causes T_1 to fire immediately thereafter.

The circuit of Fig. 130a is formed by combining two of the circuits of Fig. 130b and adding a commutating capacitor C. The resistors R_4 and R_5 serve only as a voltage divider for the supply voltage. Because of small differences between individual tubes, application of a positive triggering pulse to the input of the circuit causes either T_1 and T_2 to fire before T_3

[9] Weissman, *loc. cit.*
[1] H. A. Vuylsteke, *Electronics*, April, 1953, p. 248.

and T_4, or vice versa. Assume that T_1 and T_2 fire first. The sudden drop in voltage at junction P is transferred to P' through C and thus lowers the voltage impressed across T_3 and T_4. If the time constant R_2C is large enough in comparison with the length of the triggering pulse, T_3 and T_4 do not fire. Application of a second triggering pulse causes T_3 and T_4 to fire. The sudden drop in voltage of P' is transmitted to P through C and causes T_1 and T_2 to be extinguished. An n-stage binary circuit is formed by connecting point P' of each stage to the input of the following stage.

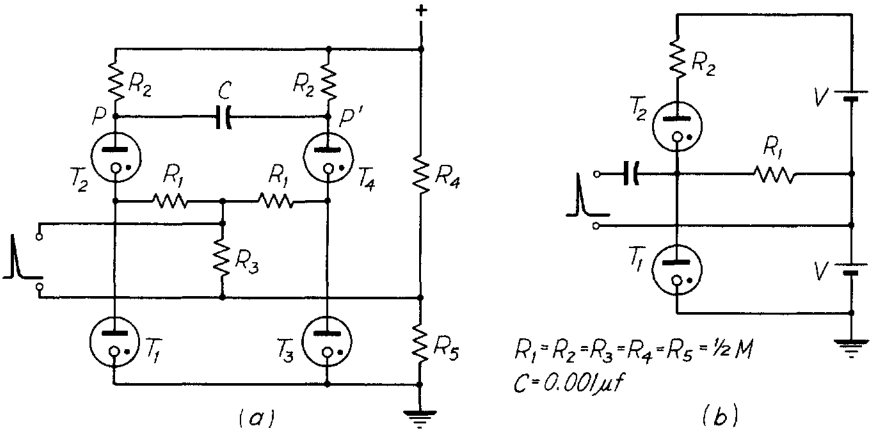


FIG. 130. (a) Symmetrical bistable circuit using four glow tubes; (b) Two-tube circuit from which circuit (a) is formed; R_1 to R_5 = ½ M, C = 0.001 μf.

In order to ensure that each pair of tubes will not refire after it is extinguished by the firing of the other pair, the time constant R_2C must be made large in comparison with the deionization time of the tubes. However, increase in this time constant lengthens the settling time that must elapse following the application of each pulse before the voltage across C has changed sufficiently to allow the circuit to respond to the next pulse. It follows that the maximum frequency at which the circuit will respond reliably to each pulse is an inverse function of the ionization time.

40.2. Glow-Tube Decade Counter. An interesting ring type of glow-tube counting circuit for use at frequencies up to 500 cps is shown in Fig. 131.[2] The supply voltage exceeds the breakdown voltage of the neon tubes, but the supply voltage and the resistance R_o are so chosen that, when one glow tube conducts, the voltage drop through R_o is great enough to reduce the voltage between P and ground below the breakdown voltage of the tubes. Consequently only one tube conducts at a time. Assume that T_1 is initially conducting. The voltage drop across R_1 caused by the current through T_1

[2] J. C. Manley and E. F. Buckley, *Electronics*, January, 1950, p. 85.

charges the capacitor C_1 with the polarity shown. Application of a negative pulse to the trigger input terminals reduces the potential of point P below the extinction potential of the glow tubes, causing T_1 to be extinguished. The voltage across C_1 then causes the potential of the lower terminal of T_2 to be lower than that of the lower terminals of the other tubes. Consequently, as the input voltage rises toward the end of the triggering pulse,

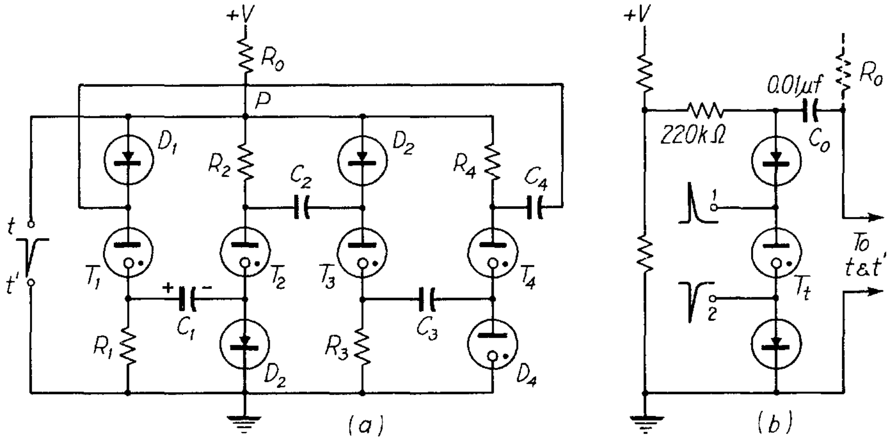


FIG. 131. (a) Two-stage glow-tube counting circuit; $R_0 = 82$ kΩ, R_1 to $R_4 = 68$ kΩ, C_1 to $C_4 = 0.01\ \mu$f, T_1 to T_4 = type NE 96; (b) Low-impedance source of triggering pulses for circuit (a).

the voltage across T_2 reaches the breakdown value before that across the other tubes, and T_2 therefore fires. The voltage across R_2 caused by the current through T_2 charges C_2 with the right-hand terminal positive, so that the voltage across T_3 near the termination of the second triggering pulse will exceed the voltages across the other tubes and T_3 will be left in the conducting state. Similarly, C_3 and C_4 ensure that T_4 and T_1 will conduct following the third and fourth input pulses, respectively. The function of the semiconductor diodes D_1 to D_3 is to allow the commutating capacitors to charge quickly, but to discharge slowly during the triggering pulses.

The circuit of Fig. 131a will function with any even number of stages and may therefore be used as a decade counter if ten tubes are used. The order in which the tubes fire can be reversed by changing the commutating-capacitor connections so that C_1 connects from the lower terminal of T_1 to the lower terminal of T_4, C_2 connects from the upper terminal of T_2 to the upper terminal of T_1, C_3 from the lower terminal of T_3 to the lower terminal of T_2, and C_4 from the upper terminal of T_4 to the upper terminal of T_3. Thus, the circuit can be changed from addition to subtraction (Prob. 40.2–1) by means of a multiple-pole double-throw switch.

The triggering pulses for the circuit of Fig. 131a may be obtained by differentiating a step voltage, as explained in Sec. 3, or by any other convenient means, but the impedance of the pulse source should be low in comparison with the combined resistance of R_0 in parallel with one of the other resistances. Figure 131b shows a circuit by means of which an additional glow tube may be used in providing a relatively low-impedance source of pulse voltage.[3] The auxiliary glow tube T_t is normally off, but may be fired by the application of a positive voltage pulse to terminal 1 or a negative pulse to terminal 2. The sudden drop in voltage across T_t is followed by a flow of current through R_0, the 0.01-μf capacitor, and T_t. The additional voltage drop across R_0 produced by this current provides the reduction in voltage at P necessary to trigger the circuit. Reduction of the voltage of P below the glow-tube extinction voltage when the next ring-circuit glow tube fires near the termination of the triggering pulse causes T_t to stop conducting. Because of the relatively high back resistance of the semiconductor diodes in series with T_t, very little current is required to fire T_t. The required triggering current can be reduced even further by substituting a small thyratron in place of the glow tube T_t and applying the triggering pulse to the thyratron grid.

40.3. Semiconductor-Diode Circuits. Much higher counting rates can be obtained by the use of semiconductor diodes of the proper type instead of glow tubes in the circuits of Figs. 130 and 131. Any device having a current-voltage characteristic similar to that of the glow tube may be used. Comparison of Figs. 75 and 79 shows that *p-n-p-n* diodes should be suitable for this purpose.

41. Counter Circuits Using Starting-Anode Glow Tubes, Thyratrons, and Switching Transistors

41.1. Circuits Using Starting-Anode Glow Tubes. Binary and ring counting circuits may also make use of starting-anode glow tubes, such as the OA4-G and the 5823. Breakdown of tubes of this type is initiated by a positive voltage applied to the starting anode. If the voltage between the main anode and the cathode of the 5823 does not exceed approximately 290 volts, the tube does not break down unless a minimum voltage of +75 volts is also applied to the starting anode, the required starting-anode voltage decreasing with the magnitude of the main-anode voltage. The main-anode extinction voltage is approximately 85 volts.

Figure 132 shows a binary stage making use of two starting-anode glow tubes.[1] By means of the voltage divider R_3-R_4, the starting anodes of the tubes are positively biased at a voltage insufficient to cause the tubes to fire.

[3] Manley and Buckley, *loc. cit.*

[1] H. L. Foote, *Communication and Electronics,* No. 18, 161 (May, 1955).

Because of slight differences between the two tubes, one tube will fire before the other when a triggering pulse is applied to the trigger input terminals. If T_1 fires first, the sudden flow of current through R_1 raises the potential of the cathode of T_1. The abrupt rise in voltage of the cathode of T_1 immediately initiates current through C_1 and R_2 and is accompanied by an equal rise in voltage of the cathode of T_2, which prevents T_2 from firing. As C_1 charges in the indicated polarity, the current through R_2 decreases and the voltage of the cathode of T_2 falls. If the time constant C_1R_2 is large relative to the length of the triggering pulse, however, the potential of the starting anode of T_2, which is essentially equal to the potential of the

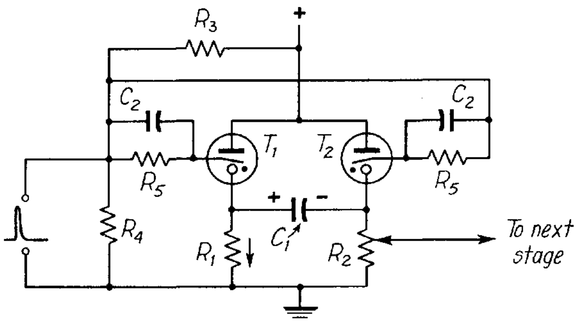


FIG. 132. Symmetrical bistable circuit using starting-anode glow tubes.

trigger input terminal, falls more rapidly than the potential of the cathode, and T_2 does not fire.

After the lapse of a minimum settling time, the cathode voltage of T_2 becomes low enough so that application of a second triggering pulse causes T_2 to fire. When T_2 fires, its anode-cathode voltage falls to a value only a little larger than the extinction voltage (see Fig. 79). Since the anodes of the two tubes are at the same potential, the voltage across C_1 reduces the anode-cathode voltage of T_1 below the extinction value, and may make it negative. If the deionization time of T_1 is small in comparison with the time constant C_1R_1, the reignition voltage of T_1 rises more rapidly than the applied anode-cathode voltage and T_1 remains off. In a similar manner, a third triggering pulse again fires T_1 and extinguishes T_2.

As in vacuum-tube binary counting circuits, the output voltage of one stage is applied to the input terminals of the following stage. The resistors R_5 limit the starting-anode current to a safe value, the capacitors C_2 being used to prevent excessive loss of triggering voltage across R_5 as the result of starter-capacitance charging current. The resistor R_5 and capacitor C_2 may be omitted if R_3 and the resistance of the triggering-voltage source are great enough to prevent excessive starting-anode current.

The maximum frequency at which the circuit of Fig. 132, and counting circuits based upon it, can be operated is determined by the deionization time of the tube. In order to ensure the extinction of one tube when the other fires, the time constant of the C_1 and R_1 or R_2 must be large in comparison with the deionization time. However, increase of this time constant increases the minimum settling time that must elapse between triggering pulses, and therefore reduces the maximum frequency at which the circuit will respond reliably to each input pulse.

41.2. Ring Circuit Using Starting-Anode Tubes. A ring circuit that makes use of starting-anode glow tubes is shown in Fig. 133.[2] Although

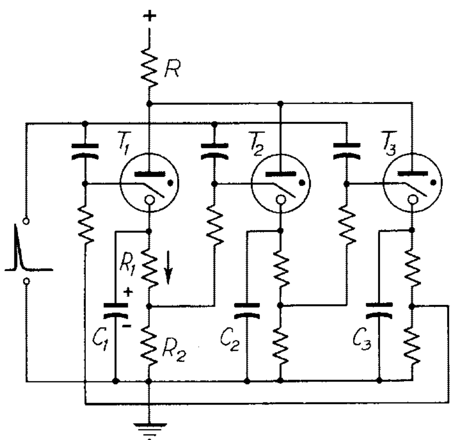


FIG. 133. Three-stage ring type of counting circuit using starting-anode glow tubes.

three stages are shown in Fig. 133, any number of stages greater than one may be used. Deionization time of the tubes limits the maximum counting frequency to a few hundred pulses per second.

The applied voltage in the circuit of Fig. 133 must be insufficient to allow the tubes to fire in the absence of positive starting-anode voltage. When the first triggering pulse (positive) is applied, small differences in the individual tubes cause one tube to fire before the others, and the resulting drop across the common anode resistor R prevents the other tubes from firing. Assume that T_1 is conducting and that the current through the cathode resistors R_1 and R_2 has charged the capacitor C_1 in the polarity shown. The voltage drop across R_2 provides a positive bias on the starting anode of T_2 of insufficient magnitude to fire T_2 in the absence of a triggering pulse.

If the trigger-pulse amplitude is sufficient to fire T_2 with the aid of the

[2] G. H. Hough and D. S. Ridler, *Electrical Communication,* **27,** 214 (September, 1950); Foote, *loc. cit.*

initial starting-anode bias, but insufficient to fire T_3 (and other tubes if the circuit has more than three stages) in the absence of starting-anode bias, the next input pulse causes T_2 to conduct. Because C_2 is uncharged at the instant T_2 fires, whereas C_1 is charged, the cathode potential of T_2 at the instant of firing is lower than that of T_1. The anode potentials are equal. Consequently, when the firing of T_2 reduces the tube drop of T_2 to a value only a little greater than the extinction voltage (see Fig. 79), the voltage across T_1 is reduced below the extinction value and T_1 is cut off. If the

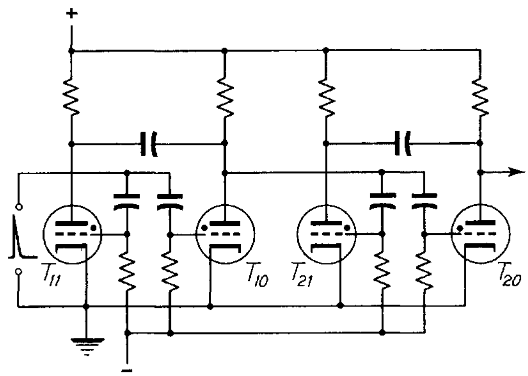


FIG. 134. Two stages of a thyratron binary counting circuit.

time constant of C_1 and associated resistances and capacitances is large in comparison with the triggering-pulse length, the starting-anode voltage of T_1 falls more rapidly than the cathode voltage of T_1. If deionization is also sufficiently rapid so that the reignition voltage rises more rapidly than the applied anode-cathode voltage, T_1 remains off. In a similar manner, the next pulse fires T_3 and extinguishes T_2.

41.3. Thyratron and Switching-Transistor Circuits. The circuits of Figs. 132 and 133 may also be used with thyratrons.[3] The operation of the thyratron circuit of Fig. 134 is basically the same as that of Fig. 132, and the thyratron circuit of Fig. 135 is essentially the same as that of Fig. 133. Much higher counting rates can be attained by the use of switching transistors, such as those discussed in Secs. 19 and 26.4, in place of thyratrons in the circuits of Figs. 134 and 135 and similar circuits.

42. *Multiple-Electrode Glow Counting Tubes*

42.1. Tubes with Two Sets of Transfer Electrodes. A number of special glow tubes have been developed that in effect consist of ten controlled glow tubes in a single envelope, arranged in the form of a ring.

[3] C. E. Wynn-Williams, *Proc. Roy. Soc. (London)*, **136**, 312 (1932).

Available tubes have a single main anode, ten main cathodes, and either one or two sets of *transfer,* or *stepping* cathodes, the function of which is to cause the glow discharge to progress from one main cathode to the next in response to triggering pulses. The chief problem involved in this type of tube is that of ensuring that the discharge progresses consistently in the same direction.

In the type of tube in which two sets of transfer cathodes are used, the structure of the stepping cathodes is identical with that of the main cathodes, successive main cathodes being separated by two transfer cathodes.[1] The

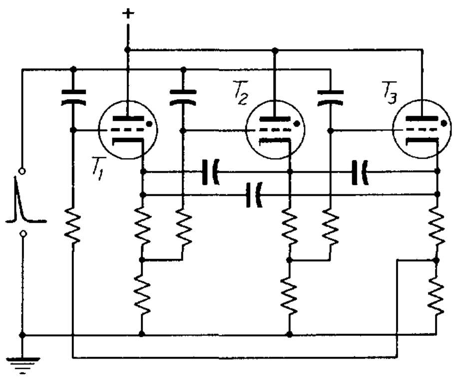


FIG. 135. Three-stage ring type of counting circuit using thyratrons.

anode is usually in the form of a cylinder or circular disk, as shown in the line drawing of Fig. 136.

The basic circuit used with a tube having two sets of transfer cathodes is shown in Fig. 137. For simplicity only three stages are shown and these are presented in an open linear array. The explanation of the operation of the circuit applies to any number of stages, and in tubes designed for counting purposes the array closes upon itself, as in other types of ring counters. The two sets of transfer electrodes are necessary when all cathodes are identical, in order to ensure that the glow will advance consistently in the same direction. This is accomplished by applying a triggering pulse first to one set of transfer electrodes and shortly thereafter a similar pulse to the second set. Thus, if the main cathode K_1 is initially conducting, the application of a negative pulse to the transfer terminal 1 lowers the potential of the first set of transfer cathodes, k_1, k_2, and k_3, below that of K_1. If the ionization were the same in the vicinity of these three electrodes, the

[1] J. J. Lamb and J. A. Brustman, *Electronics,* November, 1949, p. 92; G. H. Hough and D. S. Ridler, *Electrical Communication,* **27,** 214 (September, 1950); J. E. Adams and D. H. Simon, *Sylvania Microwave Newsletter,* **1,** No. 4, 1957.

discharge could transfer to any one of them, but ionization produced by the current to K_1 ensures that the current will transfer to k_2. If a negative pulse is now applied to terminal 2, before the termination of the first pulse, ionization in the vicinity of k_2 will allow firing of k_2'. At the termination of the pulse applied to k_2 current to k_2 stops. At the termination of the

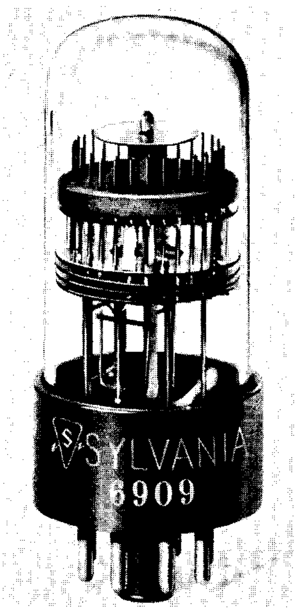


FIG. 136. Multiple-cathode counting glow tube. (Courtesy of General Telephone and Electronic Labs., Inc.)

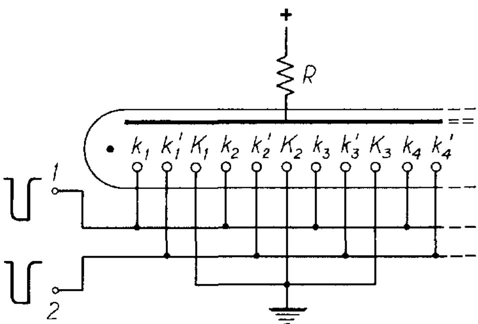


FIG. 137. Circuit for counting glow tube having two sets of transfer cathodes.

pulse applied to k_2', current to k_2' stops, and the resulting reduction of voltage drop in the anode resistor R raises the voltage of the anode relative to the main cathodes. Ionization in the vicinity of k_2' ensures that K_2 then conducts, rather than one of the other cathodes. Although cathode resistors are not essential to the operation of this type of tube, they may be used in series with one or more main cathodes in order to make output voltage available.

Some tubes with two sets of transfer cathodes are capable of operation at frequencies as high as 100,000 pulses per second. Typical supply voltages used with these tubes are of the order of 450 volts, anode currents are of the order of a milliampere, and output voltages across cathode resistors range from 12 to 30 volts. Tubes are available both in small sizes for use in circuits in which only voltage output is desired, and in larger sizes where visual indication of the count is also desired.

42.2. Generation of Triggering Pulses for Glow-Tube Circuits. There are a number of ways in which the required overlapping triggering pulses can be initiated by single input triggering pulses. One of the simpler circuits is shown in Fig. 138, in which the two sets of transfer electrodes are

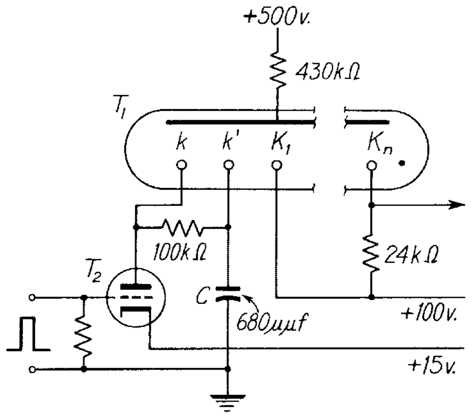


FIG. 138. Circuit for the generation of overlapping triggering pulses for the two sets of transfer cathodes.

for simplicity shown as only two individual electrodes. Initially the triode T_2 is biased beyond cutoff. Application of a positive triggering pulse to the grid of T_2 causes it to conduct and lower the potential of the transfer electrodes k from a potential approximately the same as that of the conducting cathode to a value intermediate between 15 and 100 volts. The reduction of plate voltage of T_2 is followed by a discharge of C and an exponential drop in the voltage of the second set of transfer cathodes k'. Upon the termination of the triggering pulse applied to the grid of T_2, T_2 stops conducting, and the 100-kΩ resistor prevents the flow of appreciable capacitor current to k, which consequently stops conducting. A path for k' current is provided by the capacitor C, however, which charges up. The potential of k' consequently rises exponentially and at some value the next main cathode fires. The capacitance of C must be large enough to ensure deionization of k before k' reaches 100 volts. Otherwise transfer in the reverse direction may occur. An input pulse of 30 μsec or longer is required.

Another method of obtaining delayed pulses is to apply the input pulse directly to k and to generate a delayed negative pulse by inverting the input pulse and differentiating the trailing edge.[2] Other, more complicated R-C or L-C networks may also be used. Figure 139 shows a circuit capable of operating at frequencies up to 100 kc.

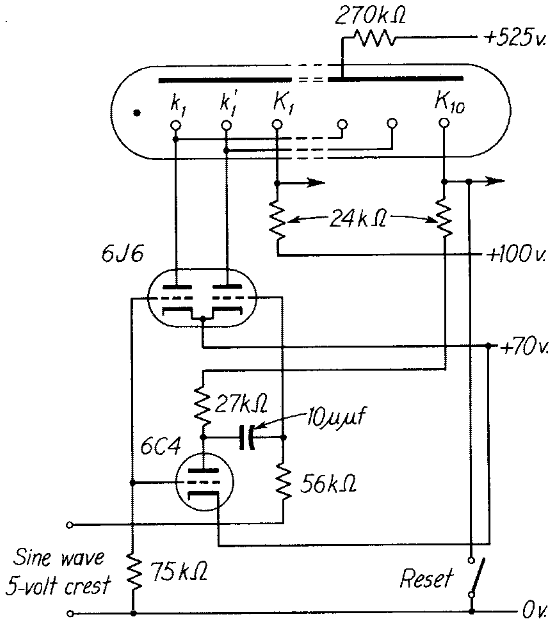


FIG. 139. Circuit for the generation of overlapping triggering pulses for the two sets of transfer cathodes.

42.3. The Use of Directional Cathodes to Ensure Commutation. It is apparent that the requirement of two sets of transfer pulses greatly complicates the circuit for tubes in which two sets of transfer electrodes are required. However, two sets of transfer electrodes are needed to ensure transfer in the proper direction unless some other mechanism is used to ensure proper progression. It can be seen from Fig. 137 and the explanation based upon it that, if the second set of transfer electrodes k' is omitted, the discharge is just as likely to return to K_1 as to K_2 at the termination of a triggering pulse unless the tube itself embodies some mechanism for favoring transfer in the forward direction. This can be achieved by designing the cathodes so that the glow is restricted to the portion of the cathode adjacent to the next electrode in the desired direction of advance.[3] This is accomplished by the use of a cathode that has a partially enclosed portion

[2] Adams and Simon, *loc. cit.*
[3] M. A. Townsend, *Electrical Eng.*, **69**, 810 (September, 1950); D. S. Peck, *Electrical Eng.*, **71**, 1136 (December, 1952).

and an open portion. The mechanism of glow maintenance is such that the glow is concentrated in the enclosed portion.

The structure of a tube containing directional cathodes is shown in Fig. 140. The glow concentrates in the cylindrical portion of the wire cathodes. For this reason the cathode adjacent to the projecting wire of a conducting cathode is a relatively long distance away from the source of ions, whereas the cathode adjacent to the helical portion of the conducting cathode is close

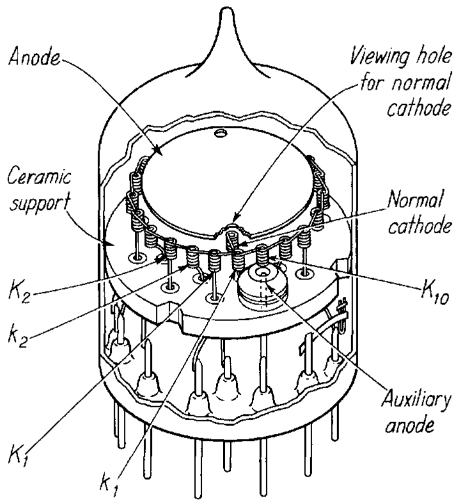


FIG. 140. Counting glow tube with directional cathodes. (Courtesy of the Bell Telephone Laboratories, Inc.)

to the source of ions. Consequently, conduction transfers to the cathode adjacent to the glow when the adjacent cathode is only about 10 volts negative relative to the glowing cathode, whereas the cathode on the other side of the glowing cathode must be made 60 to 80 volts negative relative to the glowing cathode in order that conduction will transfer to it. If the two cathodes adjacent to a conducting cathode are both made negative relative to the conducting cathode by a voltage exceeding 10 volts, transfer in the forward direction is ensured.

The circuit used with a counting tube of this type is shown in Fig. 141. The individual cathode resistors are not essential to the performance of the circuit, but are used to provide output voltage. If K_1 is initially conducting, transfer to k_2 requires that k_2 be made more than 10 volts negative relative to k_1. Since the voltage of K_1 falls toward zero as the current to K_1 falls, the voltage pulse applied to k_2 must actually be negative in order to ensure transfer. Transfer to K_2 then requires that K_2 be negative relative to k_2

by at least 10 volts. Because the voltage of K_2 rises as current starts to flow to K_2, the positive voltage applied to k_2 in order to ensure transfer must be at least 10 volts plus the voltage drop that will exist across the cathode resistor in series with K_2 when K_2 is fully conducting.

In ring counting tubes of this type an alternative way of obtaining an output voltage at the termination of one series of input pulses (at the tenth pulse in a decade tube) is by means of an auxiliary anode adajcent to number-ten cathode. This anode conducts only when number-ten cathode conducts. A circuit by means of which an exponential output pulse is obtained

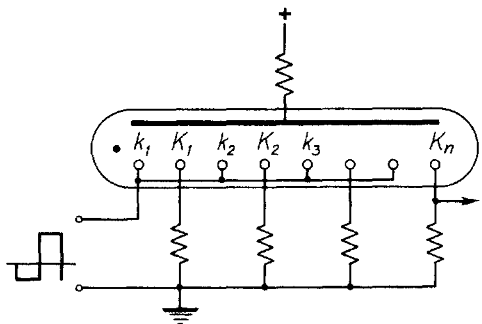


FIG. 141. Circuit for a counting glow tube with directional cathodes.

from the auxiliary anode when the tenth cathode fires is shown in Fig. 142. The auxiliary anode may also be used to reset the tube to zero following any number of input pulses. This is accomplished by applying a high negative voltage to the auxiliary anode, which then acts as cathode, the discharge transferring to the tenth cathode upon removal of the voltage. The structure of the tube may be considerably simplified if output is obtained from only one cathode or from an auxiliary anode, since most or all of the cathodes may then be connected to a single cathode lead.

42.4. Performance of Directional-Cathode Tubes. The circuit simplification achieved by the use of directional cathodes is accompanied by a reduction of the maximum frequency of operation. The frequency is limited by the time required for deionization to take place in the vicinity of a cathode after interruption of current. If K_1 is initially conducting, it may refire following the application of a triggering pulse if the residual ion density remaining in the vicinity of K_1 from its initial conduction exceeds the ion density in the vicinity of K_2 produced by conduction of k_2. The tube illustrated in Fig. 140 has a maximum frequency limit of 2000 to 2500 pulses per second. This frequency can be extended to about 12,000 pulses per second by shunting the individual cathode resistors of the circuit of

Fig. 141 with capacitors. The maximum frequency of operation is reduced by use of the auxiliary anode, since additional time is required for breakdown to take place to this electrode after breakdown to the tenth cathode.

The anode-to-cathode breakdown voltage of the tube of Fig. 140 is in the range of 180 to 300 volts, and the extinction voltage (minimum maintaining voltage) is about 110 volts. The anode current is of the order of 1 to 3 ma. The output voltage obtainable from a cathode resistor is limited to about 45 volts, larger voltage causing direct transfer from a conducting

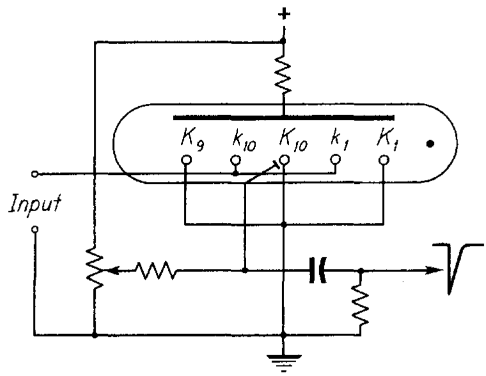


FIG. 142. Use of an auxiliary anode to obtain an output pulse when the tenth cathode conducts.

cathode to one of the adjacent main cathodes without conduction of the intervening transfer cathode. Pulse durations of the order of a millisecond are required to ensure transition.

43. Multistable Magnetrons

43.1. Negative-Resistance Magnetrons. It has long been known that negative resistance is observed in split-anode magnetrons, which consist of a cylindrical cathode, surrounded by two hemicylindrical anodes, operated in a magnetic field parallel to the axis of the structure, as shown in Fig. 143. If both portions of the anode are at the same positive potential relative to the cathode, electrons that leave the cathode move in cycloidal paths under the action of the combined electric and magnetic fields. At low ratios of magnetic flux density to anode voltage, most of the electrons strike the anodes and anode current flows in the external circuit. If the ratio of flux density to anode voltage exceeds a critical value, however, the electrons do not strike the anodes, but return to the cathode. The critical ratio at which the electrons graze the anode surfaces and the anode currents are exactly at cutoff is called the *cutoff ratio.*

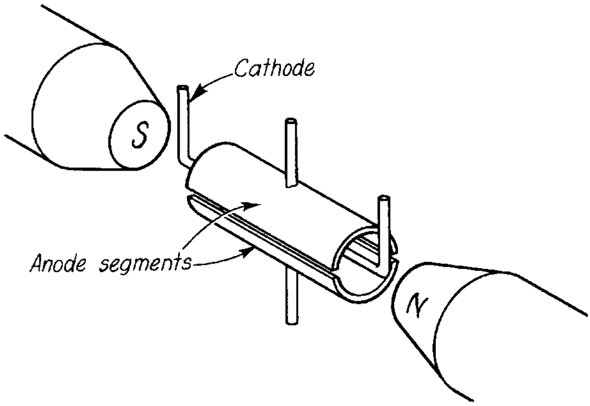


FIG. 143. Basic structure of a split-anode magnetron.

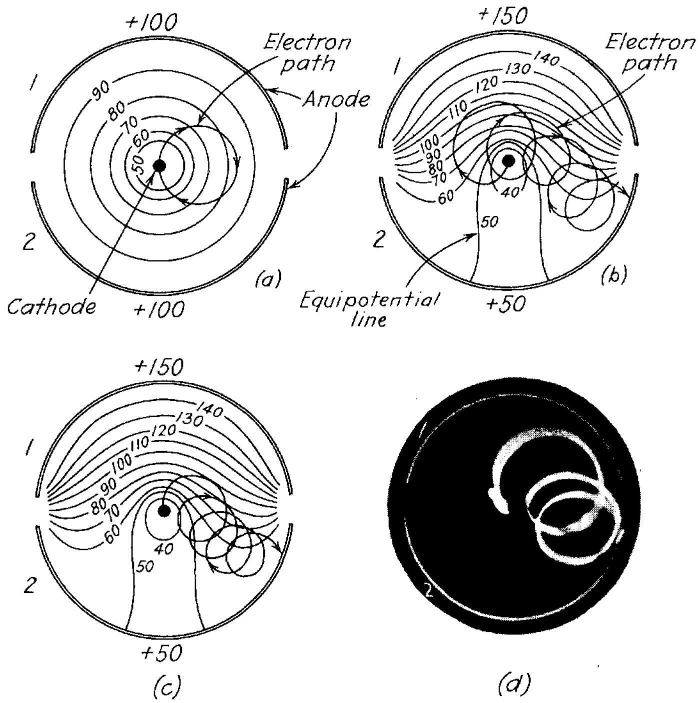


FIG. 144. Electron paths in a split-anode magnetron (a) when the two anode segments are at the same potential; (b) and (c) when the potential of one anode segment is considerably lower than that of the other; (d) photograph of electron paths made visible by ionization of gas.

Figure 144a shows the path of an electron when both anode segments are at the same voltage, the value of which is below the cutoff value. Figures 144b and 144c show the paths of electrons leaving opposite sides of the anode when the potential of one anode segment is reduced considerably below that of the other. It is apparent from Fig. 144 that the anode segment which is at the *lower* potential conducts. The general form of a typical curve of current to one anode segment vs. voltage of that segment when the voltage of the other segment is maintained constant at a value below the cutoff value is shown in Fig. 145. It is apparent from the form of this curve that a bistable circuit should result from the introduction of a sufficiently high resistance in series with either anode segment. If the supply voltage is high enough, the lower stable value of current is zero.

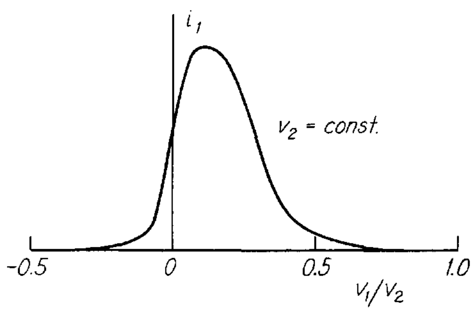


FIG. 145. General form of the current-voltage characteristic of one anode segment.

If equal resistances are used in series with both anode segments, the circuit has three stable conditions of equilibrium, corresponding to zero current to both anode segments and zero current to either segment and high current to the other. A fourth equilibrium state, in which current flows to both anode segments, does not exist, since the ratio of anode voltage to flux density is such that the tube is cut off unless the voltage of one segment is lower than that of the other. It was shown that current then flows only to the segment that has the lower voltage. Transition from one equilibrium state to another can be initiated by suitable triggering voltages applied to one or both anode segments.

Because of the complication, weight, size, and cost resulting from the need of a permanent magnet for each tube, two-anode magnetrons have not been developed for use in counting circuits. However, the principle of operation of split-anode-magnetron bistable circuits can be readily extended to tubes having more segments. The relatively small size and simple structure of a decade ring counting tube of this type more than compensates for the disadvantage of a peramanent magnet.

43.2. Magnetron Decade Switching Tube. The structure of one type of magnetron decade switching tube is shown in Fig. 146a.[1] In this tube the simple, segmented, cylindrical anode structure has been replaced by ten "spades" surrounded by a cylindrical anode. The spades serve the same

[1] S. Kuchinsky, *I.R.E. Convention Record,* Part 6, p. 43, 1953.

function as the two segments of a split-anode tube, but the combination of the spade shape of these electrodes and the presence of the cylindrical outer anode produces a desirable shaping of the electric field. Figure 146b shows the equipotential lines and the electron beam when one spade is at zero potential and all other spades and the anode are at the same higher potential. Because the equipotential lines are nearly parallel to the sides of the spades, the beam moves parallel to the sides of the spades and a large fraction of the electrons strike the anode. If the anode contains a slot at the point where the electrons reach the anode surface, many electrons pass through the slot and may be collected by a target electrode beyond the anode. The advantage of this arrangement is that the current to the low-potential spade is nearly independent of the target potential. Hence, the switching action from one spade to another is not affected by changes of load in the target circuits, from which output is obtained. Furthermore, the target current is almost independent of target voltage over a wide voltage range.

If suitable resistances are used in series with each spade and a common anode supply, there are eleven stable states of equilibrium. In the symmetrical state, all spades are at supply potential and no current flows to any spade. In the other ten states, current flows to one spade, the voltage of which is maintained low by *IR* drop. When the supply voltage is first impressed upon the spade circuit, the symmetrical state of equilibrium is assumed. Any spade may be made to conduct by depressing its voltage momentarily. Subsequent application of a negative voltage pulse to all spades or the anode or a positive pulse to the cathode causes the beam to advance in the direction in which the magnetic field causes the electrons to rotate about the cathode. The number of spades by which the beam advances depends upon the length of the pulse. If the pulse is too short, the beam returns to its initial position. If the pulse is too long, it may cause the beam to move through two or more positions. Consequently, in counting applications the triggering pulse must be carefully shaped. The speed at which switching can be accomplished is limited by the time constant of the spade capacitance and the series spade-circuit resistance. The mechanism of switching will be explained in Sec. 43.4.

In most applications of the magnetron decade switching tube, nine of the spade circuit resistors are connected only to the spade voltage supply and may therefore be mounted within the tube, a single lead sufficing for connection to the supply. The other spade resistor may also be mounted internally, but a separate lead to this resistor is necessary in order that the beam may be set to the zero position initially. If an output pulse is required only for each ten input pulses, only one target is required. As few as six tube leads may then be used. If an output indication must be ob-

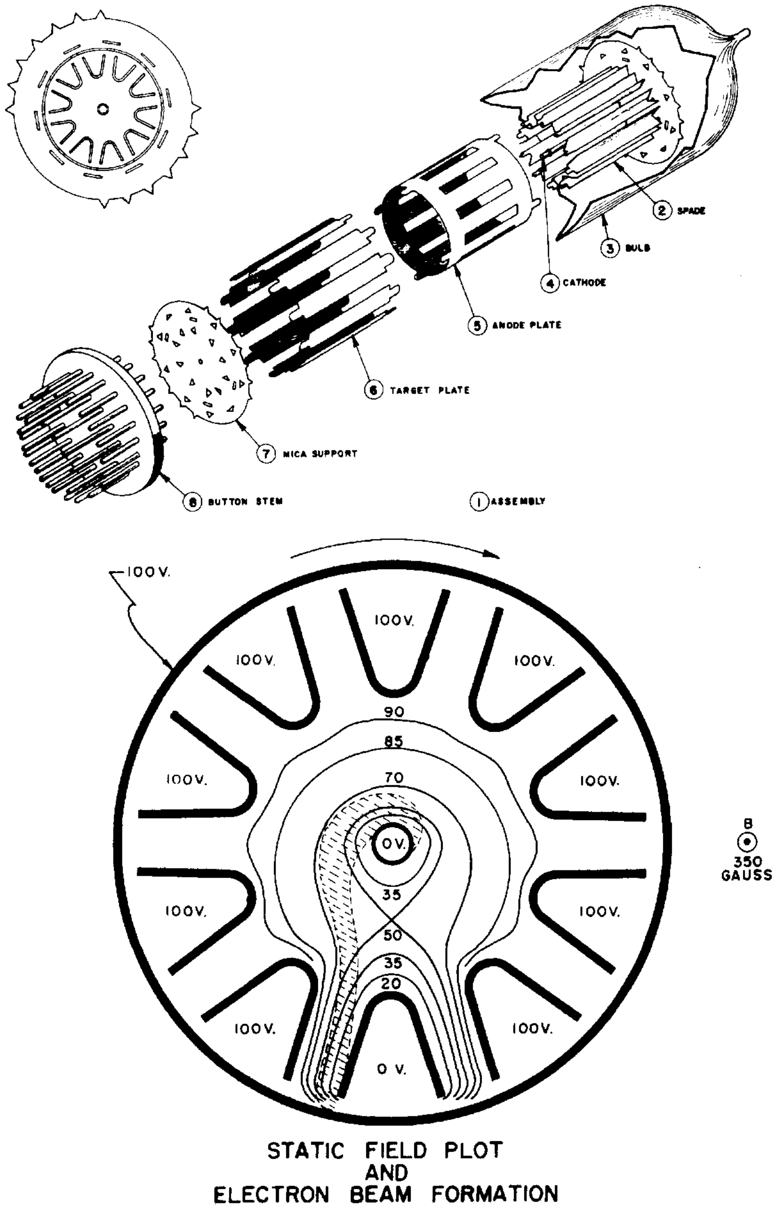


FIG. 146. (a) Structure of a magnetron decade switching tube; (b) static field plot and electron-beam configuration. (Courtesy of Burroughs Corp.)

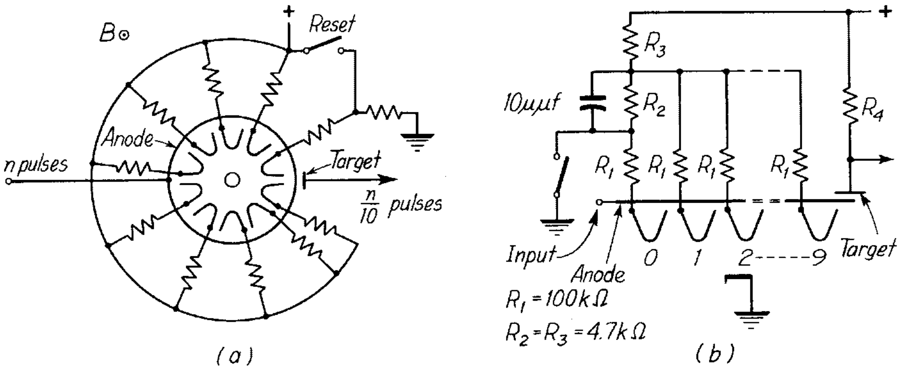


FIG. 147. (a) Basic circuit of a six-lead magnetron decade counting tube; (b) circuit incorporating means for resetting to zero.

tained after each pulse, ten targets are required and fifteen leads are necessary. In some applications it may also be necessary to provide a lead for each spade. Internal mounting of the resistors not only reduces the number of leads required, but also increases the reliability of the resistors and reduces the spade capacitances and thus increases the switching speed. The circuit of a six-lead counting tube is shown in Fig. 147a. The triggering pulses used for this circuit should have an amplitude approximately equal to the positive supply voltage and a duration of 0.3 μsec. The circuit can operate at frequencies up to the order of a million pulses per second.

The beam cannot be reliably reset to the zero position merely by the application of a negative reset voltage to the zero spade. The reason for this is that the beam is unaffected by the voltage of a spade lying on the opposite side of the cathode. Reliable resetting requires that the spade supply voltage be momentarily lowered sufficiently so that all spade currents are cut off and that the zero spade be maintained at a low voltage during or immediately following reapplication of the spade supply voltage. One simple circuit by means of which this is accomplished is shown in Fig. 147b. Closing of the switch, which is normally open, reduces the voltage of the zero spade to zero and reduces the voltage of all other spades to half the supply voltage. The capacitor charges to a voltage equal to half the supply voltage. When the switch is opened, the voltage of all but the zero spade rises to the supply voltage, but the voltage across the capacitor prevents the voltage of the zero spade from rising initially above half the supply voltage. The beam therefore forms to the zero spade when the switch is opened.

43.3. Modifications of Magnetron Switching Tube. Another useful modification of the magnetron decade switching tube is shown in Fig. 148. This tube has four cylindrical targets, which are assigned the code numbers 1, 2, 4, and 8, respectively. At each beam position the holes in the anode are such that electrons strike a different combination of targets. Thus, in the zero beam position there are no holes in the anode, and current does not flow to any target. In the 1 position, current flows to target 1; in

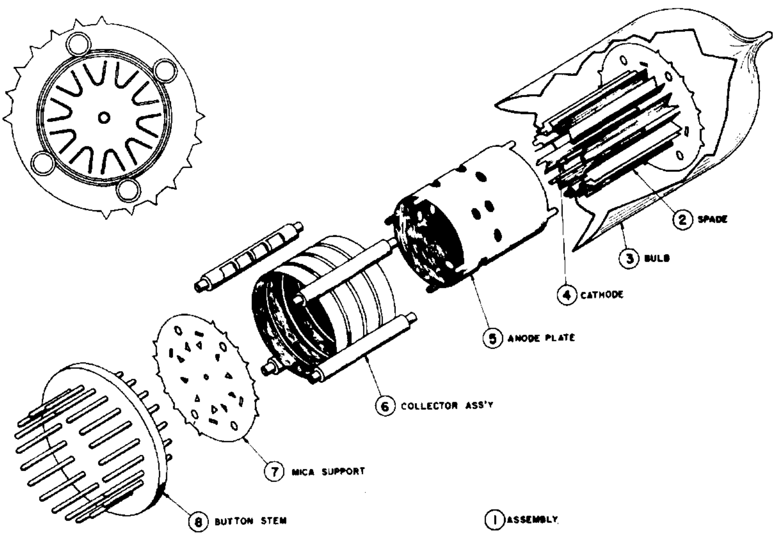


FIG. 148. Magnetron decade switching tube that provides binary-coded output.

the 5 position, current flows to targets 1 and 4; in the 7 position, current flows to targets 1, 2, and 4; etc. In this manner, binary-coded output is obtained from the tube.

Magnetron decade tubes may also be used for signal switching. Up to 25-percent modulation of the beam current may be produced by modulating voltages applied to the cathode, to the individual spades, or to two or more spades. A tube in which all spade leads and all target leads are individual may therefore be used to switch two or more input signals to a single output terminal or a single input to two or more output terminals.

43.4. Mechanism of Switching. Some insight into the switching mechanism can be gained from the curves of Fig. 149.[2] The lower curve of Fig. 149 is a typical curve of current to any spade as a function of the voltage of that spade when all other spades are at supply potential. It therefore applies to a spade that is in the conducting state of equilibrium. The upper

[2] J. Bethke, *Electronics,* April, 1956, p. 122.

curve is the plot of the current to any spade as a function of the voltage of that spade when the adjacent lagging spade (lying in the direction opposite to that of beam advance) is at zero potential relative to the cathode and all other spades are at supply potential. It applies to the spade to which it is desired to switch the beam from the adjacent conducting spade. Thus, if the beam terminates on the k spade, the lower curve applies to the k spade and the upper curve to the $k + 1$ spade. For spade load resistance R and supply voltage V_s corresponding to the solid load line, points 1 and 2 correspond to stable equilibrium values of current to the k spade. Since the k spade is assumed to be conducting, the current has the value corresponding to point 2. (Point 1 corresponds to the state in which no spades conduct.) The stable point for the $k + 1$ spade is 1.

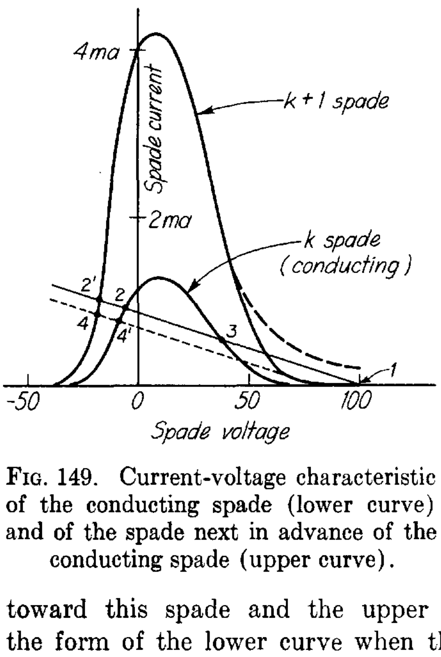


FIG. 149. Current-voltage characteristic of the conducting spade (lower curve) and of the spade next in advance of the conducting spade (upper curve).

If the voltage applied to the $k + 1$ spade circuit is reduced sufficiently to displace its load line to the left beyond the point of tangency with the upper curve, as indicated by the dotted line of Fig. 149, current to the $k + 1$ spade will increase very rapidly toward the value at 4, in the manner explained in Sec. 25.* As the current to the $k + 1$ spade increases, the beam advances toward this spade and the upper curve decreases in height, assuming the form of the lower curve when the beam reaches the $k + 1$ spade. If the supply voltage of the $k + 1$ spade is still depressed, the equilibrium current to the $k + 1$ electrode assumes the value at 4′ and subsequently rises to the value at 2 when the triggering voltage is terminated. If the triggering voltage is in the form of a short pulse, on the other hand, the current may assume the value at 2 without passing through that at 4′.

Like circuits making use of other types of negative-resistance devices, the magnetron circuit has more than one stable state of equilibrium only if the load line intersects the lower curve in more than one point. If the spade load resistance is too low, the load line intersects the lower curve only at point 1, and the only stable state of equilibrium is that in which all spades are cut off. Physically, this means simply that the IR drop in the spade resistors is insufficient to depress the spade voltage to the point where electrons strike the spade in sufficient numbers to maintain the cur-

* Anode-voltage change displaces the curves, instead of the load line.

rent. Increase of spade circuit resistance decreases the amount by which the load line of the $k + 1$ spade must be displaced in order to initiate transition of current to that electrode. At very large values of resistance the required triggering voltage may be so small that random noise initiates transition. The beam will then advance spontaneously from spade to spade at a rate determined by the spade-circuit time constant.

43.5. Triggering Grids. The necessity of limiting the length of the triggering pulse in order to prevent the beam from advancing by more than

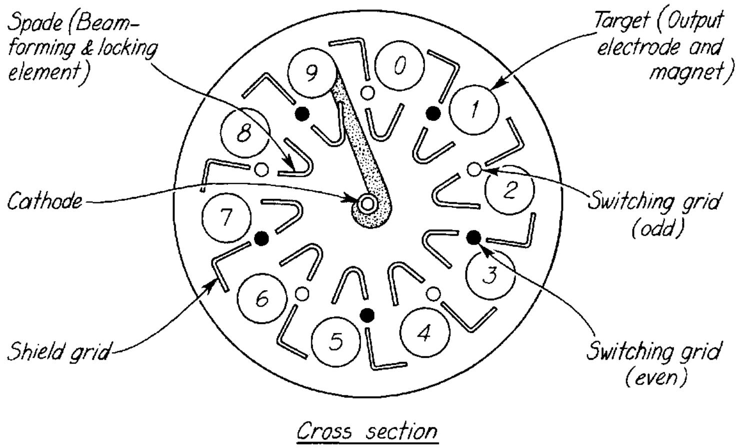


FIG. 150. Structure of a magnetron decade switching tube incorporating triggering grids. (Courtesy of Burroughs Corp.)

one spade may be prevented in tubes that are provided with triggering grids. These grids may consist of straight wires adjacent to the spades, as shown in Fig. 150.[3] An additional difference between the structures of Figs. 146 and 150 is the omission of the cylindrical anode in the latter, made possible by proper shaping of the target electrodes. As in the tube of Fig. 146, the target and spade currents are nearly independent of target voltage over a range of target voltage from ½ volt to the maximum rated target voltage.

Under standby conditions, the grids of the tube of Fig. 150 are operated at a positive biasing voltage intermediate between the cathode and spade voltages. If the voltage of the grid adjacent to the conducting target is reduced, or if the voltage is made negative, the upper curve of Fig. 149 is altered as shown by the dashed curve. A negative increment of grid voltage sufficient to raise the tail of the characteristic above the point of tangency with

[3] Bethke, *loc. cit.*

the load line causes the current to the leading spade adjacent to the conducting spade to rise abruptly to the value at point 2′ and thus allows the beam to advance. As the beam advances from the initially conducting spade to the leading adjacent spade, the upper curve falls, assuming the form of the lower curve when the beam has advanced to the leading spade. During this transition the current to the leading spade falls to the equilibrium value at point 2. If the negative increment of grid voltage is applied simultaneously to all grids and is maintained for a longer time than that required for the beam to advance by one spade, a similar process will cause the beam to advance to the next spade. In this manner the beam will continue to advance from spade to spade, at a rate determined by the time constant of the spade circuits, until the voltage of the grids is again raised to the standby value.

The beam may be prevented from advancing by more than one spade at a time without limitation of the triggering pulse length by applying the triggering voltage alternately to all grids adjacent to the even-numbered spades and to all grids adjacent to the odd-numbered spades. This may be accomplished by connecting all even-numbered grids to one plate or collector of an Eccles-Jordan bistable circuit and all the odd-numbered grids to the other plate or collector and using the input pulses to trigger the Eccles-Jordan circuit.

Visual indication of the elapsed count may be obtained by means of small neon tubes, in series with suitable current-limiting resistors, shunted across either the spade load resistors or the target load resistors. Connecting the glow tubes to the targets is preferable, since the increase of spade capacitance increases the spade-circuit time constant and thus reduces the speed of transition when the glow tubes are connected to the spades.

44. Multistable Electrostatic-Deflection Tube

44.1. Current-Voltage Characteristic. A current-voltage characteristic having many regions of negative slope can be obtained by means of a beam-deflection tube shown in basic form in Fig. 151. A rectangular electron beam produced by a suitable electron gun passes between a pair of electrostatic deflection plates and strikes a masking electrode containing a series of equally spaced apertures. The electrons that pass through the apertures strike the anode and thus cause current to flow in the anode circuit. It is apparent that if the potential of one or both deflection electrodes is varied, the beam will be swept across the apertures and the anode current will vary in the manner indicated by the curve of Fig. 152a. The addition of a tapered horizontal slot below the rectangular apertures adds a component of anode current that increases continuously with deflection voltage and thus changes the curve of anode current vs. deflection voltage to the form shown in Fig. 152b.

The anode current is practically independent of anode voltage over a wide range of anode voltage, but depends almost entirely upon the percentage of the beam that passes through the mask electrode. Consequently, if one of the deflection electrodes is tied to the anode, as shown in Fig. 153, the

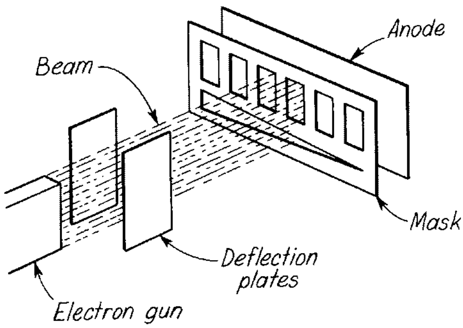


FIG. 151. Basic electrode structure of a beam-deflection tube.

curve of anode current vs. anode voltage is identical in form with the curve of anode current vs. deflection voltage. It follows from principles discussed in Secs. 23 and 24 that the circuit can be made multistable by the addition of sufficient anode circuit resistance so that the corresponding load line intersects the characteristic curve in more than one point, as shown in Fig. 154.

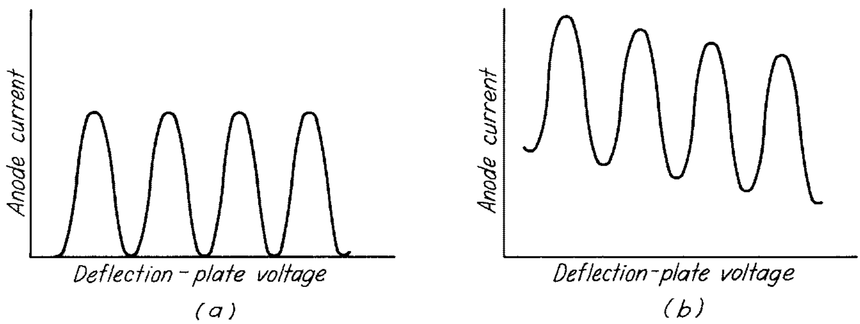


FIG. 152. Anode current of the tube of Fig. 151 vs. deflection-plate voltage (a) when the mask has only rectangular apertures; (b) when the mask also has a tapered slot.

Because of the tilt of the characteristic, the voltages of the various stable points can be made to differ by approximately equal increments.

Each stable equilibrium point of the current-voltage diagram corresponds to a stable equilibrium position of the electron beam in which a portion of the beam passes through one of the apertures and strikes the anode. Any departure of the beam from this position changes the anode current and therefore the IR drop in the anode load resistance. The resulting change of deflection voltage is such as to tend to return the beam to its equilibrium position.

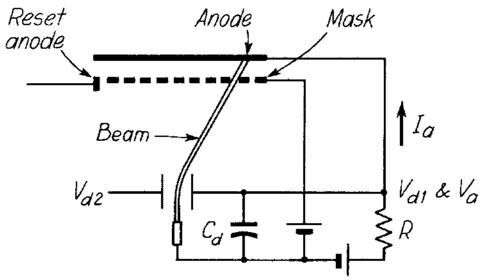


FIG. 153. Multistable circuit based upon the beam-deflection tube of Fig. 151.

44.2. Mechanism of Commutation. If the beam is in an equilibrium position and the voltage V_{d2} of the left deflection electrode is slowly increased, the beam is displaced to the right, more electrons are intercepted at the edge of the aperture, and the anode current is decreased. The resulting reduction in IR drop in the anode load resistance raises the voltage V_{d1} of the right deflection electrode and thus limits the displacement of the beam to a very small amount. A slow change of V_{d2} therefore does not displace the beam to a different aperture.

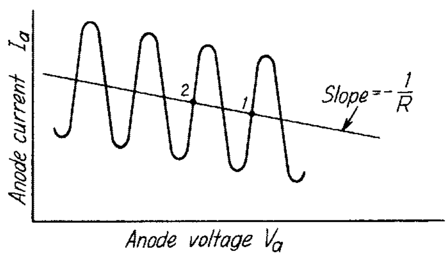


FIG. 154. Current-voltage diagram for the circuit of Fig. 153, showing several points of stable equilibrium.

If the voltage V_{d2} is increased abruptly, on the other hand, the capacitance C_d of the right deflection electrode and the anode tends to maintain the voltage of those electrodes constant. When the time constant RC_d is large in comparison with the rise time of the voltage increment ΔV_{d2}, the accompanying change of V_{d1} is much smaller than ΔV_{d2} and the beam displacement is nearly as great as it would be if V_{d1} remained constant. The beam may therefore be displaced to the vicinity of another equilibrium position by a sudden increment of V_{d2}. In order that the beam will remain in the new equilibrium position, the triggering voltage must be terminated so slowly that V_{d1} can change nearly as rapidly as V_{d2}. Otherwise the beam may be returned to its original position (or to an intermediate position if it was displaced by more than one step) during the fall of V_{d2}.

The effects of slow and rapid changes of V_{d2} can also be explained by means of the current-voltage diagram of Fig. 154. Since the position of the beam is equally dependent upon the voltages of the two deflection electrodes, increasing the positive voltage of the left deflection electrode in the circuit of Fig. 153 displaces the characteristic of Fig. 154 to the right by an amount equal to the voltage increment. If the operating point is initially at 1 and the voltage V_{d2} is slowly increased, the operating point remains

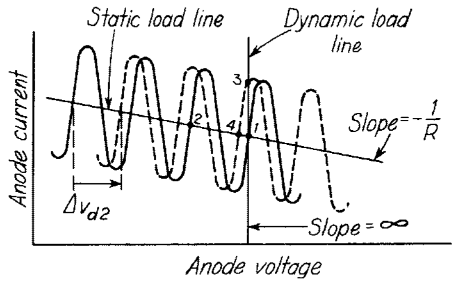


FIG. 155. Current-voltage diagram used to explain the mechanism of commutation in the circuit of Fig. 153.

on the same section of the characteristic as the characteristic moves to the right. The anode voltage rises and the anode current falls as the point moves to the right along the static load line.

The fact that the anode voltage remains essentially constant when the anode current changes rapidly and the time constant RC_d is large means that for rapid changes of current, the dynamic load line approximates a vertical line through point 1 during a rapid change in V_{d2}. In Fig. 155 the initial position of the characteristic is shown solid and the initial equilibrium point is assumed to have been at 1. The voltage V_{d2} has been increased abruptly by an amount slightly less than the difference in voltage between two equilibrium points at the initial value of V_{d2}. Immediately following the change in V_{d2} the new operating point is 3. Since this point does not lie on the static load line, change in V_{d2} is followed by a gradual change of the operating point from 3 to 4 as the capacitance C_d discharges and the anode voltage falls to the static equilibrium value. If V_{d2} is now decreased *slowly*, the dashed characteristic gradually moves to the left along the static load line and the equilibrium point moves to 2.

It is seen that the magnitude of the triggering voltage required to ensure transition of the beam by one step is any value that causes the next positive-slope section of the characteristic to intersect the vertical dynamic load line. If the initial intersection is near the center of a positive-slope section, the pulse amplitude may lie in the range equal to the differences between two adjacent equilibrium voltages plus or minus approximately one-fourth this

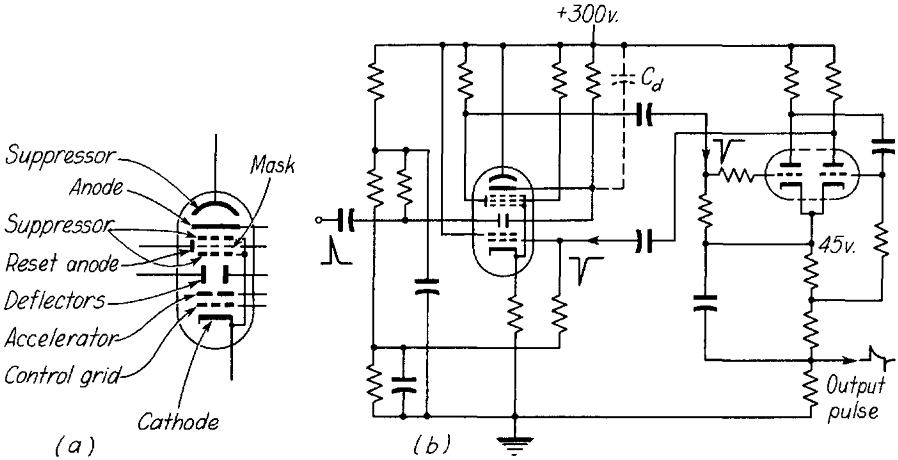


FIG. 156. (a) Structure of the multistable beam-deflection tube; (b) Complete decade counting circuit based upon this tube.

difference. If the pulse is too small, the beam will return to its original position; if it is too large, the beam may be advanced by more than one step.

44.3. Practical Tube. A commercially available tube of this type, the Amperex 6370, has ten stable equilibrium positions of the beam so that it may be used in decade counting circuits.[1] The anode is provided with holes in order that a portion of the beam may pass through the anode and impinge upon a fluorescent screen on the inner surface of the tube envelope. Numbers from 0 to 9 on the screen give a direct indication of the count.

Some means must be provided to return the beam to the zero position, corresponding to the highest-voltage intersection of the static load line with the characteristic, when the tenth triggering pulse is impressed. This may be accomplished by means of a reset anode beyond the 9 aperture. The tenth input pulse causes the beam to strike this electrode, and the resulting voltage drop in reset-anode load resistance is used to trigger a mono-

[1] Irwin Rudich, *I.R.E. Convention Record,* Part 3, p. 74, 1954.

stable Eccles-Jordan pulse-forming circuit of the type to be discussed in Sec. 55. The complete circuit is shown in Fig. 156b. The pulse circuit provides an output pulse for the next stage and also impresses a negative voltage upon a grid that forms a portion of the electron gun. Cutting off of the electron beam by this grid reduces the anode current to zero and raises the anode voltage to the supply value. The capacitance C_d, which may be simply the anode and deflection-electrode capacitance, prevents the anode and deflection voltage V_{d1} from falling instantaneously, and hence the beam forms in the zero position at the termination of the grid pulse.

The general arrangement of the various electrodes of the Amperex 6370, including those essential to beam formation, shielding, and the prevention of the effects of secondary emission, is shown in Fig. 156a.

45. Analysis of Negative-Resistance Circuits Containing both L and C

45.1. Generalized Equivalent Circuits. In Secs. 23 to 25 it was shown that any single-port device that has a negative static resistance in a portion of its operating range may be used as the basis of a bistable or multistable

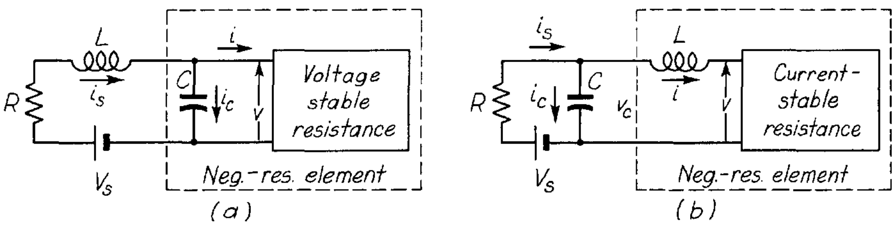


FIG. 157. Simplified equivalent circuit that can be used in the analysis of circuits based upon (a) a voltage-stable negative-resistance element; (b) a current-stable element.

circuit. It will now be shown that such a device may also be used to form astable circuits (relaxation oscillators), monostable circuits (single-pulse generators), and sine-wave oscillators. Later sections will treat these circuits in detail and will include explanations of the physical mechanism of operation of particular tube and transistor circuits.

Figure 157 shows two simplified equivalent circuits to which it is possible to reduce all practical bistable, astable, monostable, and sine-wave negative-resistance circuits. Although the two circuits of Fig. 157 are in most respects equivalent mathematically, the circuit of Fig. 157a is not adequate when the negative-resistance element is current-stable, because the circuit does not take into account the internal series inductance of the element that is essential to the transition process of bistable circuits (Sec. 24). Similarly, the circuit of Fig. 157b is not adequate when the negative-resistance element is voltage-stable, since it neglects the internal shunt capacitance of the negative-resistance element that is essential to the transition process. Comparison of the circuits of Fig. 157 with those of Figs. 84 and 86 shows that the former may be formed from the latter by the addition of the inductance L to the circuit of Fig. 84 and of the capacitance C to the circuit of Fig. 86. The bistable circuits of Figs. 84 and 86 are, therefore, merely simplified forms of the more generalized circuits of Fig. 157. The capacitance C of Fig. 157a and the inductance L of Fig. 157b may, of course, also be assumed to include additional shunt circuit capacitance and series circuit inductance not present in the negative-resistance elements. In most circuits, however, the additional capacitance or inductance is negligible and will consequently be assumed to be absent.

45.2. Analysis of Equivalent Circuit Having Voltage-Stable Element. If the static current-voltage characteristic of the voltage-stable resistance of Fig. 157a is assumed to be of the idealized form shown in Fig. 83, application of the network laws and elimination of the voltage v by means of Eq. (26) leads to the following differential equation:

$$\left[p^2 + \left(\frac{1}{R_{iv}C} + \frac{R}{L}\right)p + \frac{R + R_{iv}}{R_{iv}LC}\right] i = \frac{V_s}{R_{iv}LC} + \frac{I'}{LC} \tag{69}$$

in which R_{iv} is the reciprocal of the slope of that section of the static voltage-stable current-voltage characteristic in which operation is occurring, I' is the current at the intersection of the current axis with the extension of that section of the characteristic, and p represents the time derivative.

The solution of Eq. (69) is oscillatory if the following relation is satisfied (see Prob. 45.2–1):

$$\sqrt{\frac{L}{C}}\frac{1}{R_{iv}} + 2 > \sqrt{\frac{C}{L}}R > \sqrt{\frac{L}{C}}\frac{1}{R_{iv}} - 2 \tag{70}$$

The solution is then written most conveniently in the following form:

$$i = \epsilon^{\alpha t}(A \sin \omega t + B \cos \omega t) + I_o = A'\epsilon^{\alpha t} \sin (\omega t + \theta) + I_o \tag{71}$$

in which A and B or A' and θ are arbitrary constants and

$$\alpha = -\frac{1}{2}\left(\frac{1}{R_{iv}C} + \frac{R}{L}\right) = -\frac{1}{2\sqrt{LC}}\left(\sqrt{\frac{L}{C}}\frac{1}{R_{iv}} + \sqrt{\frac{C}{L}}R\right) \tag{72}$$

$$\omega = +\sqrt{\frac{R + R_{iv}}{R_{iv}LC} - \alpha^2} \tag{73}$$

$$I_o = \frac{V_s + I'R_{iv}}{R + R_{iv}} \tag{74}$$

By geometry, I_o may be shown to be the value of the current i at the intersection of the static load line corresponding to the resistance R with the section of the current-voltage characteristic over which operation is taking place. Consequently, the complete solution consists of an oscillation superimposed upon the static equilibrium current I_o. Equations (71) and (72) show that, if R_{iv} is positive, the amplitude of oscillation decays and the current approaches the stable value I_o. If R_{iv} is negative, however, as it is in the middle section of the current-voltage characteristic, the amplitude of oscillation may decrease, remain constant, or increase, depending upon whether the magnitude of $\sqrt{L/C}\,(1/R_{iv})$ is less than, equal to, or greater than $\sqrt{C/L}\,R$, respectively.

The solution of Eq. (69) is exponential if either of the following relations is satisfied:

$$\sqrt{\frac{C}{L}}R < \sqrt{\frac{L}{C}}\frac{1}{R_{iv}} + 2 \tag{75}$$

or

$$\sqrt{\frac{C}{L}}R < \sqrt{\frac{L}{C}}\frac{1}{R_{iv}} - 2 \tag{75A}$$

The solution is then written most conveniently in the following form:

$$I = A'\epsilon^{(\alpha+\omega')t} + B'\epsilon^{(\alpha-\omega')t} + I_o \tag{76}$$

in which

$$\omega' = +\sqrt{\alpha^2 - \frac{R + R_{iv}}{R_{iv}LC}} \tag{77}$$

If the first two terms of Eq. (76) both decay, the intersection of the load line corresponding to the resistance R with the section of the static current-voltage characteristic over which operation is taking place is stable, and the current returns to the value at this point following any disturbance of the circuit. If either the first or the second term grows, however, the intersection of the load line with the characteristic is an unstable point of equilibrium. If the current has this value initially, any disturbance of the circuit causes the current to depart from this value exponentially.

It is possible for the circuit parameters and initial conditions to be such that the two exponential terms are of like sign and that one is initially large but decays rapidly, whereas the other is initially small but grows rapidly.

Under these conditions the current will first change in one direction with a decreasing rate of change, pass through a maximum or minimum, and then change in the other direction at an increasing rate. In any case, however, if either exponential term grows, the current will eventually increase in magnitude and will continue to do so unless the increasing current magnitude causes a change in one or more of the circuit parameters.

Whether the exponential terms of Eq. (76) decay, remain constant, or grow depends upon the sign of α, and upon whether ω' is larger or smaller than $|\alpha|$. The sign of α depends in turn upon whether the magnitude of $\sqrt{L/C}\ (1/R_{iv})$ is larger or smaller than $\sqrt{C/L}\,R$, i.e., upon whether $|R_{iv}RC/L|$ is less than or greater than unity. The magnitude of ω' relative to α depends upon the magnitude of R_{iv} relative to R, i.e., upon whether R_{iv}/R is smaller than or greater than unity. Table VII shows the ranges of the parameters over

TABLE VII RANGES OF PARAMETERS IN WHICH THE EXPONENTIAL TERMS OF EQ. (76) GROW AND DECAY

R	R_{iv}	$\lvert R_{iv}/R\rvert$	$\left\lvert\frac{R_{iv}RC}{L}\right\rvert$	α	ω'	$\alpha+\omega'$	$\alpha-\omega'$	1st term	2nd term
>0	>0			<0	$<\lvert\alpha\rvert$	<0	<0	Decays	Decays
>0	<0	<1	<1	>0	$>\lvert\alpha\rvert$	>0	<0	Grows	Decays
>0	<0	<1	>1	<0	$>\lvert\alpha\rvert$	>0	<0	Grows	Decays
>0	<0	>1	<1	>0	$<\lvert\alpha\rvert$	>0	>0	Grows	Grows
>0	<0	>1	>1	<0	$<\lvert\alpha\rvert$	<0	<0	Decays	Decays
>0	<0	1	1	0	0	0	0	Constant	Constant
<0	<0			>0	$<\lvert\alpha\rvert$	>0	>0	Grows	Grows

which the exponential terms of Eq. (76) grow and decay. The ranges of the parameters throughout which the varying component of current in a voltage-stable circuit has the form of a growing oscillation (G.O.), a damped oscillation (D.O), a growing exponential (G.E.), and a decaying exponential (D.E.) are shown in Fig. 158 (Prob. 45.2-2).

Constant-Q contours may be added to Fig. 158 by use of the relation

$$Q^2 = \left(\frac{\omega}{2\alpha}\right)^2 = \frac{4-(x-y)^2}{4(x+y)^2} \tag{78}$$

in which x and y are the abscissas and ordinates, respectively, of Fig. 158. For a constant value of Q, Eq. (78) is that of an ellipse with axes along the lines $x = y$ and $x = -y$.

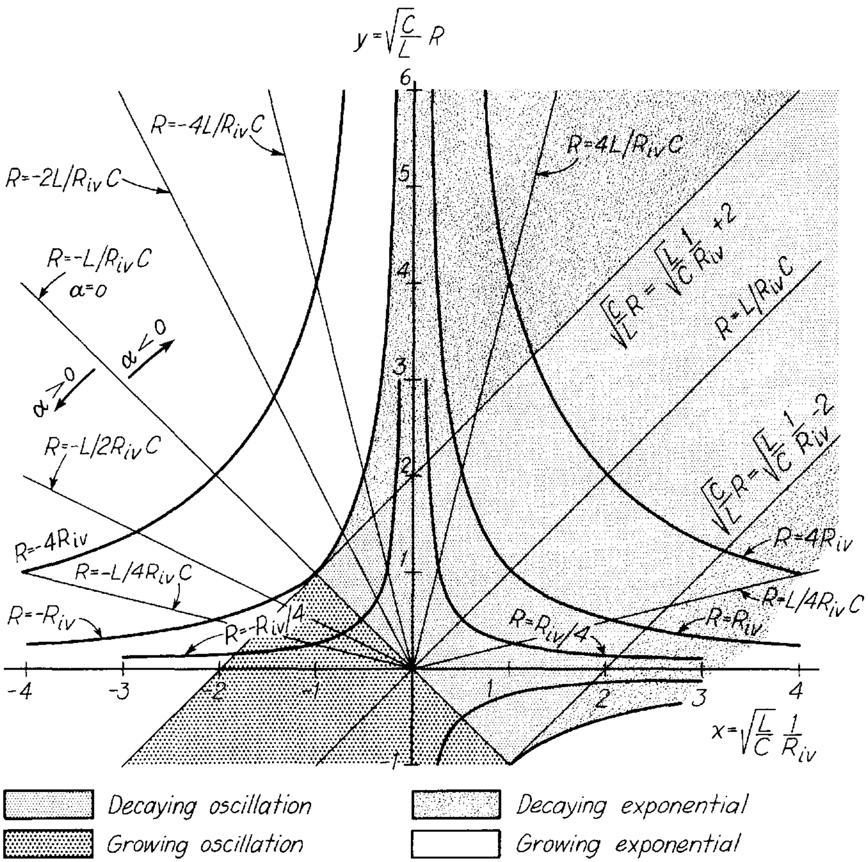


FIG. 158. Diagram showing the parameter regions in which the currents and voltages in the circuit of Fig. 157a are oscillatory with growing or decaying amplitude, and the regions in which they are exponential, under the assumption that all parameters are constant.

45.3. Analysis of Equivalent Circuit Having Current-Stable Element. A similar analysis of the circuit of Fig. 157b under the assumption that the static current-voltage characteristic of the current-stable negative-resistance element is of the form of Fig. 85 leads to the following differential equation:

$$\left[p^2 + \left(\frac{1}{RC} + \frac{R_{ic}}{L}\right)p + \frac{R + R_{ic}}{RLC}\right] i = \frac{V_s}{RLC} + \frac{I'R_{ic}}{RLC} \tag{79}$$

in which R_{ic} is the reciprocal of the slope of that section of the current-stable static characteristic in which operation is taking place, and I' is again the current at the intersection of the current axis with the extension of that section of the characteristic.

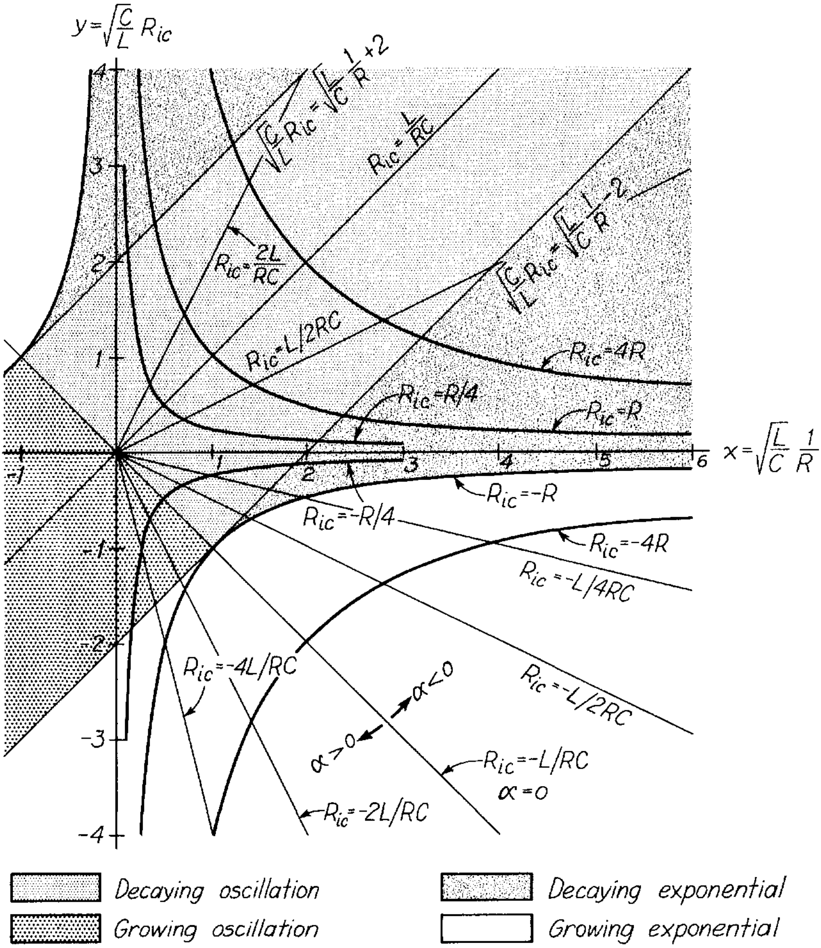


FIG. 159. Diagram showing the parameter regions in which the currents and voltages in the circuit of Fig. 157b are oscillatory with growing or decaying amplitude, and the regions in which they are exponential, under the assumption that all parameters are constant.

The solution of Eq. (79) is also given by Eqs. (71) and (76), but α, ω, ω', and I_o are now given by the following relations:

$$\alpha = -\frac{1}{2}\left(\frac{1}{RC} + \frac{R_{ic}}{L}\right) = -\frac{1}{2\sqrt{LC}}\left(\sqrt{\frac{L}{C}}\frac{1}{R} + \sqrt{\frac{C}{L}}R_{ic}\right) \tag{80}$$

$$\omega = +\sqrt{\frac{R_{ic} + R}{RLC} - \alpha^2} \tag{81}$$

$$\omega' = +\sqrt{\alpha^2 - \frac{R_{ic} + R}{RLC}} \tag{82}$$

$$I_o = \frac{V_s + I'R_{ic}}{R + R_{ic}} \tag{83}$$

A table similar to Table V may be constructed for the current-stable circuit (Prob. 45.3-1). The types of behavior of the circuit in the various ranges of the circuit parameters are shown by Fig. 159.* Reference will be made to Figs. 158 and 159 in sections that follow.

45.4. Oscillatory, Multistable, Astable, and Monostable Operation. The foregoing analysis is based upon the assumption that R_i is constant. It therefore gives no direct indication of the effect of change of magnitude or sign of R_i. In any actual circuit, R_i is a function of voltage and current and consequently it changes in magnitude and may change in sign over the range of operation. These changes have very important effects upon the behavior of the circuit, as might be surmised.

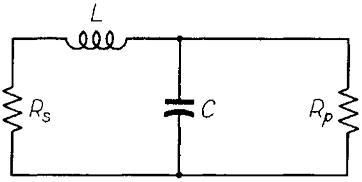


FIG. 160. Single equivalent circuit equivalent to the two circuits of Fig. 157.

When the circuit values are such that the linear analysis predicts an oscillation of growing amplitude, one effect of nonlinearity is the limitation of amplitude of oscillation. This phenomenon will be discussed in detail in Secs. 74 and 83. It has already been shown in Secs. 23 to 25 that the existence of adjacent regions of the current-voltage characteristic in which R_i is alternately positive and negative makes possible multistable circuits. Two other useful types of operation that depend upon nonlinearity of the current-voltage characteristic are astable operation and monostable operation.

In astable operation (relaxation oscillation) the circuit values are chosen so that the load line intersects the current-voltage characteristic only in the

* It is of interest to note that Eqs. (80) to (81) differ from Eqs. (72), (73), and (77) only in that R_{iv} in the latter is everywhere replaced by R in the former, and R in the former is replaced by R_{ic} in the latter. For this reason the variational behavior of both the voltage-stable circuit and the current-stable circuit can be determined from the single equivalent circuit of Fig. 160, and the two diagrams of Figs. 158 and 159 can be combined into a single diagram plotted in coordinates of $\sqrt{C/L}\,R_s$ *vs.* $\sqrt{L/C}\,(1/R_p)$. For the voltage-stable circuit R_s represents the resistance R, and R_p represents the resistance R_{iv} of the negative-resistance element; for the current-stable circuit R_s represents R_{ic} and R_p represents R. The constant component of current I_o can be found from the static current-voltage characteristic and the load line.

negative-resistance range. (An exception will be discussed in Secs. 46.2 and 57.2.) If the values are also chosen so that the linear analysis predicts that operation corresponding to this point lies in a growing-exponential region of the diagram of Fig. 158 or 159, this point is unstable and the circuit currents and voltages cannot be constant. In Sec. 46 the currents and voltages in astable operation will be shown to change in such a manner that the current through the negative resistance varies periodically over a range extending from one positive-slope branch of the current-voltage characteristic into the other branch. The change in sign and magnitude of R_i causes the currents and voltages to alternate between growing-exponential and decaying-exponential form.

The periodic voltages and currents obtained in astable circuits have a wide variety of forms, including rectangular, triangular, and quasisinusoidal. The useful properties of relaxation oscillators include their ability to generate nonsinusoidal waves of desired form, the high harmonic content of their voltages and currents, their susceptibility to tuning and modulation by means of a direct voltage, and the ease with which the frequency of oscillation can by synchronized to a control voltage of frequency approximating a multiple or submultiple of the natural frequency of oscillation of the circuit. Astable circuits will be discussed in detail in Secs. 47 to 54 and in Secs. 57 to 60.

The operation of monostable circuits is similar to that of astable circuits. As in astable circuits, the circuit values are such that the load line can intersect the current-voltage characteristic in only one point, but the intersection must lie in one of the positive-slope regions. This intersection corresponds to a stable state of equilibrium. Application of a suitable triggering pulse to the circuit carries the operating point into the negative-slope range. The currents and voltages then vary in a manner similar to that in astable circuits, but only a single pulse of voltages and currents is obtained, the operating point returning to the stable intersection. Monostable circuits will be discussed in Secs. 55 to 58.

46. Graphical Analysis of Astable and Monostable Circuits

46.1. Path of Operation. Astable and monostable circuits, like other circuits that contain nonlinear elements, can be analyzed only by approximation methods or by a point-by-point graphical procedure. The second method has the advantage that it indicates directly the path followed in the current-voltage diagram. An equation that may be used for this purpose is obtained by eliminating the time variable from the circuit equations. In the voltage-stable circuit of Fig. 157a the capacitance C usually represents only the shunt capacitance of the voltage-stable element, and the current flowing into the complete element is therefore i_s. The most useful equation is one relating the current i_s into the element with the voltage v across the element, which has the following form (Prob. 46.1–1):

$$\frac{di_s}{dv} = \frac{(v + i_sR - V_s)R_{iv}C}{(v + I'R_{iv} - i_sR_{iv})L} \tag{84}$$

The corresponding equation for the current-stable circuit of Fig. 157b is

$$\frac{di}{dv_c} = \frac{(v_c + I'R_{ic} - iR_{ic})RC}{(V_s - v_c - iR)L} \tag{85}$$

It is apparent from Eqs. (84) and (85) that if the circuit parameters L, C, and R, the slope $1/R_{iv}$ or $1/R_{ic}$ and current intercept I' of the static current-voltage characteristic, and the supply voltage V_s are known, the slope of the dynamic path of operation can be found for any point on the path, i.e., for the values of i_s and v or of i and v_c corresponding to that point. Normally L, C, and R are independent of current and voltage, but the values of I' and R_{iv} or R_{ic} are different in the three sections of the static characteristic and in general vary over each section unless the characteristic is assumed to be an ideal one consisting of three straight lines.

Even in the relatively simple case in which the characteristic is assumed to be ideal, the work involved in plotting a single path is great, and in certain critical regions of the diagram a relatively small error in plotting may result in a very large error in the complete curve. If many curves are desired, the work of plotting becomes almost prohibitive. It is then much simpler, if in some respects somewhat less satisfying, to resort to an analog computer. In this manner a series of curves may be obtained in a relatively short time, and the effect of changes of parameters, supply voltage, and triggering voltage or current may be quickly and accurately determined. An alternative, though less accurate, method is to study an actual circuit with the aid of an oscilloscope. Once a series of dynamic paths has been plotted, considerable information concerning the behavior of the circuit becomes obvious and may be readily explained. The explanations contained in the following sections will be based upon curves obtained by means of an analog computer.

46.2. Choice of R and V_s for Astable Operation. The static current-voltage characteristics of most negative-resistance elements are of such form that the varying currents and voltages in the positive-resistance ranges are rapidly decaying exponentials or highly damped oscillations. Under these conditions, relaxation oscillation (astable operation) is observed only if the supply voltage and the external resistance are such that the load line intersects only the negative-slope section of the static characteristic. Most vacuum-tube and transistor negative-resistance elements fall into this category. When glow tubes, thyratrons, and thyratron-like *p-n-p-n* transistors having current-stable characteristics of the form of those of Figs. 79, 80, and 94 are used as negative-resistance elements, on the other hand, the varying currents and voltages in the high-current positive-resistance range may be slowly decaying oscillations. The static characteristics of these devices are characterized by a very high magnitude of negative resistance (of the order of megohms or greater), a small range of current in which the resistance is negative, and a very low positive resistance in the high-current range (of the order of a few ohms). Because of the small current range in which the characteristic has negative slope, the supply voltage and series resistance are difficult to adjust so that the load line intersects the characteristic only in the negative-resistance section. However, it will be shown in Sec. 57 that relaxation oscillation can also occur in such a circuit if the load line intersects only the high-current section of the characteristic. This type of relaxation oscillator generates sawtooth waves.

Because of the smaller size, weight, and cost and the greater ease of adjustment of capacitors in comparison with inductors, most relaxation oscillators are of the type shown in basic form in Fig. 157b. For this reason, astable circuits based upon current-stable negative-resistance elements will be discussed in somewhat greater detail than those based upon voltage-stable elements.

47. Typical Paths of Operation for Astable Circuits Based upon Current-Stable Elements

47.1. Analysis of Path of Operation. Figure 161a shows a typical current-stable static characteristic and a typical path of operation obtained when the circuit parameters and the supply voltage of the circuit of Fig. 157b are chosen so that the circuit is astable and the currents and voltages are rapidly changing exponentials in all three current ranges. The parameters were chosen so that the x and y coordinates in the diagram of Fig. 159 in the low-current, negative-resistance, and high-current ranges of operation, respectively, are 0.25 and 11.75, 0.25 and -2.35, and 0.25 and 4.0. These values correspond to rapidly decaying exponentials in the positive-resistance ranges, and a rapidly growing exponential in the negative-resistance range. The general aspects of the path of operation are not greatly altered if the parameters are such that the currents are rapidly decaying or growing oscillations, rather than rapidly decaying or growing exponentials. Furthermore, any change in the values of the parameters that does not alter the values of x and y will not change the path of operation, but will change the time constants and hence the time required to complete the path of operation (one cycle of operation). Since L is assumed to be merely the series inductance of the negative-resistance element, its value was chosen to be small. In agreement with most practical astable circuits, the other parameters were chosen so that $R'C \gg L/R_{ic}$, where R' is the resistance of R and R_{ic} in parallel.

In the plotting of Fig. 161a by means of the analog computer, the supply voltage was initially zero and was abruptly increased to the value V_s by closing a switch. A rigorous analysis shows that if $R'C \gg L/R_{ic}$ and R_{ic} is positive, the varying voltages and currents approximate the sum of a

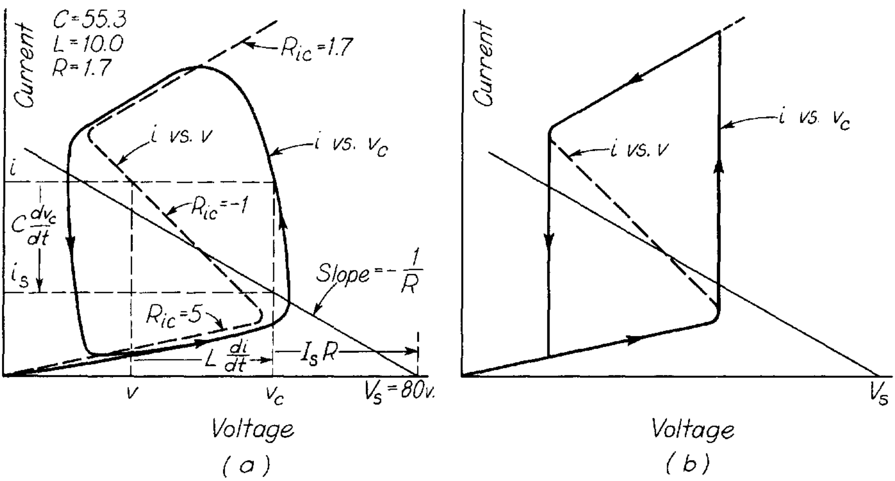


FIG. 161. (a) Typical path of operation (limit cycle) for the circuit of Fig. 157b when the circuit parameters and the supply voltage are chosen so that astable operation obtains; (b) path of operation for a very large C/L ratio.

negligibly small component having a time constant L/R_{ic} and a large component having a time constant $R'C$.* To a first approximation, therefore, v, v_c, and i may be assumed to have the time constant $R'C$ in the range corresponding to the low-current section of the characteristic. The analysis also shows that a very small fraction of v_c appears across L. This is in agreement with the fact that the observed path of operation, i vs. v_c, nearly coincides with the low-current section of the characteristic, i vs. v.

When the current i enters the range in which R_{ic} is negative, the curves of currents and voltages vs. time change from decaying exponentials to

* See Prob. 47.1-1. This fact may also be shown less rigorously as follows: Since v_c is a decaying exponential of the form $K(1 - \epsilon^{-t/\tau})$, for the values of x and y selected for the low-current section of the characteristic, the fraction of v_s that appears across L decreases with decrease of $\tau L/R_{ic}$. Furthermore, the time constant τ of the exponential voltage v_c may be seen by inspection of Fig. 157b to decrease with decrease of L and increase of R_{ic}. Consequently, if $R'C \gg L/R_{ic}$, v_c should consist almost entirely of the voltage across R_{ic} and its form should be nearly independent of C.

growing exponentials and i rises at an exponentially increasing rate. In this current range the voltage across the negative resistance decreases, but the large capacitance C tends to maintain the voltage v_c constant. The decrease of v must therefore be accounted for almost entirely by increasing voltage induced in the inductance and the time constant of the increasing current through L and R_{ic} must approximate L/R_{ic} (see also Prob. 47.1–1).

Equation (85) shows that the slope of the path of operation is infinite at the point at which $v_c = V_s - iR = V_s - i_sR$.† Since this is the equation of the load line, one must conclude that v_c reaches its maximum value at the point at which the path of operation crosses the load line. (At higher values of i, i exceeds i_s and current must flow out of the capacitance. Therefore, $C dv_c/dt$ must be negative.)

When the rising current enters the upper range, in which the slope of the characteristic is again positive, the curves of v_c and v vs. time revert to decaying exponential form. The rate of rise of current and rate of fall of voltage therefore decrease. The changing voltage across R_{ic} is still accounted for almost entirely by voltage induced in L, and the time constant of the decreasing current through L and R_{ic} remains approximately L/R_{ic}. Equation (85) indicates that the slope of the path of operation is zero at the point at which $v_c = iR_{ic} - I'R_{ic}$. This is the point at which the path of operation crosses the static characteristic.

From the point of maximum current to the point at which the current again enters the negative-resistance range of the characteristic, the behavior is similar to that during the initial rise of current, the time constant approximating $R'C$. Within the negative-resistance range and until the path of operation crosses the low-current section of the characteristic, the time constant again approximates L/R_{ic}. After passing through this minimum point the path differs only slightly from the initial path. Unless some change is made in the circuit parameters or in the supply voltage, the circuit rapidly approaches steady-state conditions in which the path closes and repeats. Waveforms of the currents i and i_s and of the voltage v_c are shown in Fig. 162a. Because the values of R_{ic} in the high-current and low-current sections of the characteristic are not in general equal, the time constants and durations of the positive and negative parts of the wave are not necessarily equal.

47.2. Effect of Change of C/L Ratio. As the ratio of C to L is increased, the left and right portions of the path of operation approach vertical lines tangent to the static characteristic, and the upper and lower portions approach the positive-resistance sections of the characteristic, as shown in Fig. 161b. The time constants of the left and right portions also become

† Note that v_c must be constant if the slope of the path of operation is infinite. Consequently $i_c = 0$ and $i = i_s$ at this point.

negligible in comparison with the other portions, and the current and voltage waves approach the forms shown in Fig. 162b. Since most of the period is taken up by the slowly rising and falling portions of the current wave and the time constant of these portions approximates $R'C$, the frequency of oscillation decreases with increase of R and C. If the form of the static current-voltage characteristic is known, the change in voltage may be determined and the period of oscillation computed (Prob. 47.2–1).

As the ratio of C to L is decreased, the time constants depart from the approximate values $R'C$ and L/R_{ic} that apply when $R'C \gg L/R_{ic}$, and

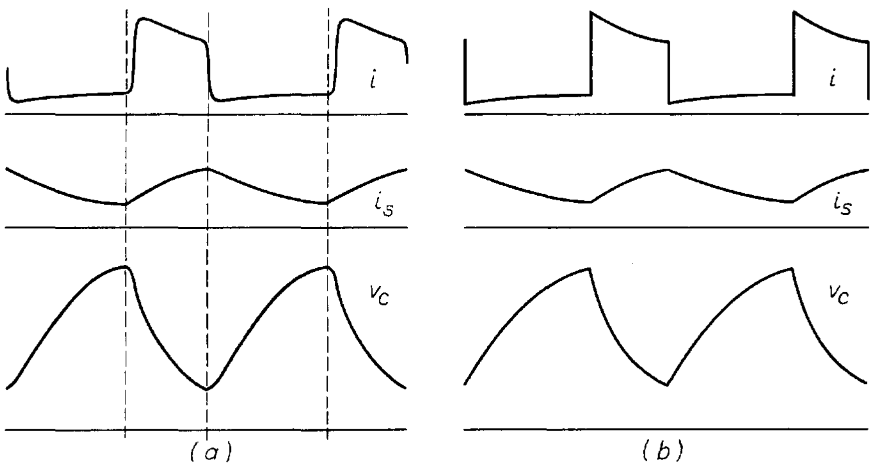


FIG. 162. (a) Waveforms of the currents i and i_s and of the voltage v_c corresponding to the diagram of Fig. 161a; (b) waveforms corresponding to the diagram of Fig. 161b.

the two time constants become more nearly equal. A typical path of operation and the corresponding waveforms are shown in Figs. 163 and 164. Depending upon the values of x and y of Fig. 159 for the parameters chosen, the various sections of the voltage and current waves may consist of decaying or growing exponentials or of portions of decaying or growing oscillations. The wave may, in fact, closely approximate sinusoidal form. Sinusoidal oscillation will be considered in detail in Sec. 73 and following sections. If the capacitance is made negligibly small, the circuit approaches that of Fig. 86 and becomes bistable. Since the frequency of oscillation depends mainly upon the time constant $R'C$ and there is a minimum value of R at which the load line intersects the negative-slope section of the characteristic only in the negative-slope range, and a minimum value of C at which relaxation oscillation is possible, it follows that there is a maximum frequency of relaxation oscillation for a device having a particular current-voltage characteristic.

47.3. Useful Properties of Astable Circuits. Usually, relaxation oscillators are used because waves of the form of Fig. 162 have desirable properties for some applications, and because the circuits are readily locked into step with a synchronizing voltage when the waves have the forms of those of Fig. 162b. For these reasons, the parameters of a relaxation oscillator based upon a current-stable negative-resistance element are usually chosen so that $R'C \gg L/R_{ic}$ and the current and voltage waveforms approximate as nearly as possible those of Fig. 162b. It is important to note, however, that the parameters need not be so chosen and that the path of operation

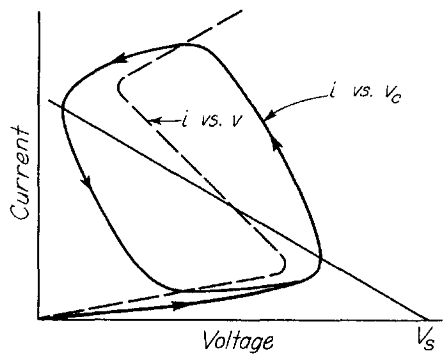


FIG. 163. Path of operation for the circuit of Fig. 157b for a very small C/L ratio.

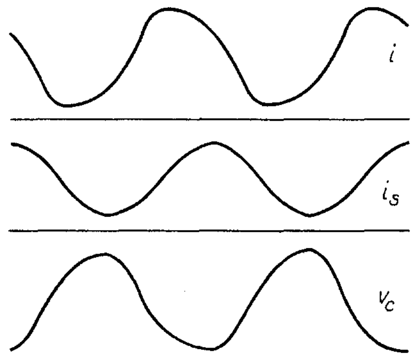


FIG. 164. Current and voltage waveforms corresponding to the diagram of Fig. 163.

and the waveforms may be more nearly of the forms shown in Figs. 163 and 164. This fact is sometimes overlooked.

48. Paths of Operation for Astable Circuits Based upon Voltage-Stable Elements

48.1. Duality of Current-Stable and Voltage-Stable Circuits. Comparison of the voltage-stable circuit of Fig. 157a with the current-stable circuit of Fig. 157b discloses that the two circuits are duals. Furthermore, comparison of Eqs. (72), (73), and (77) with Eqs. (80), (81), and (82), respectively, shows that the solutions of the equations for the two circuits have identical forms if the numerical values of x and y in the diagram of Fig. 157 are the same, respectively, as the numerical values of y and x in that of Fig. 158. Under these conditions, the path of operation, i_s vs. v, for the voltage-stable circuit of Fig. 157a must be identical with the path of operation, i vs. v_c, of the current-stable circuit of Fig. 157b if the shape of the voltage-stable characteristic, i vs. v, is identical in form with the current-stable characteristic, plotted as v_s vs. i. The waveforms of i_s and v in the voltage-stable circuit are then also identical with those of v_c and i, respectively, in the current-stable circuit.

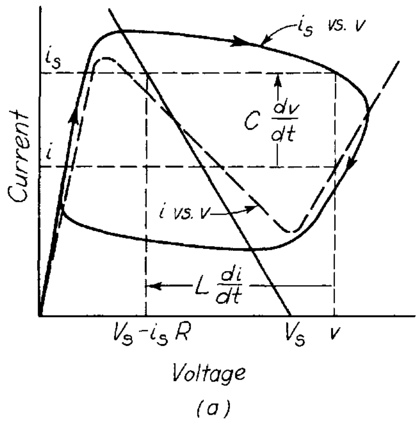


FIG. 165. (a) Typical path of operation (limit cycle) for the circuit of Fig. 157a when the circuit parameters and the supply voltage are chosen so that astable operation obtains; (b) path of operation for a very large L/C ratio.

The static characteristic, path of operation, and current and voltage waveforms for circuit parameters equivalent to those used in plotting Figs. 161a and 162a are shown in Figs. 165a and 166a. For these circuit parameters, $L/(R + R_{iv}) \gg R_{iv}C$. The value of C, which represents the shunt capacitance of the negative-resistance element, is small.

Figures 165a and 166a can be analyzed in a manner analogous to that used in Sec. 47 for Figs. 161a and 162a. In order to avoid needless repeti-

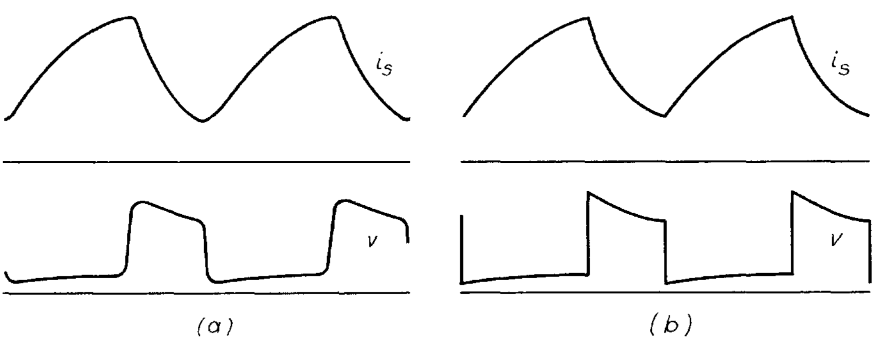


FIG. 166. (a) Waveforms of i_s and v corresponding to Fig. 165a; (b) waveforms corresponding to Fig. 165b.

tion of almost identical wording, this explanation is left to the reader as an exercise (see Probs. 48.1–1 and 48.1–2). This analysis shows that the currents and voltages vary slowly with the approximate time constant $L/(R+R_{iv})$ in the ranges in which R_{iv} is positive, and rapidly in accordance with the approximate time constant $R_{iv}C$ in the range in which R_{iv} is negative.

48.2. Effect of Change of L/C Ratio. As the ratio of L to C is increased, the upper and lower portions of the path of operation approach horizontal lines tangent to the static characteristic, and the left and right portions approach the positive-resistance sections of the characteristic, as shown in Fig. 165b. The time constants of the upper and lower portions of the path also become negligible in comparison with those of the other portions, and the waveforms of the waves of v and i_s approach the forms shown in Fig. 166b. Since most of the period is taken up by the slowly rising and falling portions of the voltage wave and the time constant of these portions approximates $L/(R + R_{iv})$, the frequency of oscillation increases with increase of R and with decrease of L. As in the current-stable circuit, the magnitude of the voltage excursion and the frequency of oscillation can be predicted approximately if the static characteristic is known.

Decrease of the ratio of L to C causes the time constants to depart from the values $L/(R + R_{iv})$ and $R_{iv}C$ that apply when $L/(R + R_{iv}) \gg R_{iv}C$, and to become more nearly equal. The path of operation changes in an analogous manner to that of the current-stable circuit. Typical waveforms are similar to those in Fig. 164. If the value of L approaches zero, the circuit approaches that of Fig. 84 and becomes bistable. The fact that there is a lower limit to the value of L and an upper limit to the value of R at which relaxation oscillation is possible for a given current-voltage characteristic means that there is an upper limit to the frequency of relaxation oscillation.

The parameters of relaxation oscillators based upon voltage-stable circuits, like those of relaxation oscillators based upon current-stable circuits, are usually, but not necessarily, chosen so that the wave has quasidiscontinuities.

49. Some General Observations Concerning Astable Circuits

49.1. Synthesis of Astable Circuits. It follows from the analysis presented in Secs. 47 and 48 that a relaxation oscillator may be formed by connecting an inductance in series with a resistance across a voltage-stable negative-resistance element, or a capacitance in parallel with a resistance across a current-stable element and adjusting the resistance and the supply voltage so that the load line corresponding to the resistance intersects the static current-voltage characteristic only in the negative-resistance section. If the ratio of the series inductance to the internal shunt capacitance of the voltage-stable element or the ratio of the shunt capacitance to the internal series inductance of the current-stable element is made large enough, the path of operation and the current or voltage waves have abrupt changes that approximate discontinuities. These abrupt changes are similar to those which occur in a bistable circuit during transition from one stable state of equilibrium to the other. Relaxation oscillators may, therefore, be thought of as circuits that have two quasistable states of equilibrium, in each of which the circuit currents and voltages change very slowly until critical values are reached at which transition takes place very rapidly to the other state, as in bistable circuits. This point of view is an aid to understanding the mechanism of operation of particular relaxation oscillators and will for this reason be used in this and following sections.

49.2. Conversion of Bistable Circuits into Astable Circuits. It was pointed out that the generalized bistable circuits of Figs. 84 and 86 differ from the generalized astable circuits of Figs. 157a and 157b only as to the presence of an additional capacitance in shunt with R or of an inductance in series with R in the astable circuit and in the way in which the load line intersects the static characteristic. Any bistable circuit can therefore be converted into a relaxation oscillator by the addition of a capacitor in parallel with any resistor that shunts a current-stable port of the negative-resistance element or by the addition of an inductor in series with any resistor that shunts a voltage-stable port.

49.3. Choice of V_s and R for Astable Operation. The choice of the particular current-voltage characteristics used in the plotting of the diagrams of Figs. 161, 163, and 165 was governed in part by convenience in setting up the analog computer. The forms of these characteristics and their positions relative to the current and voltage axes is such that a positive supply voltage V_s is required in order to make the load line intersect the negative-slope section of the characteristic. In general, however, the supply voltage in series with R may be positive, negative, or zero. For example, in Fig. 72b a load line may be drawn so that it intersects only the negative-slope section of the dashed characteristic and intersects the voltage axis at zero or either a positive or a negative point.

It is important to note that there is no upper limit to the magnitude of R at which the load line can intersect only the negative-slope section of a current-stable characteristic. Furthermore, increase of R in an astable circuit based upon a current-stable element is favorable to the inequality $R'C \gg L/R_{ic}$ that must be satisfied in order that the transition time between quasistable states shall be short and the current waves have quasidiscontinuities. Consequently, relaxation oscillation of desirable waveform may be obtained if R is infinite. Since infinite R means that the load line coincides with the voltage axis, however, infinite resistance can be used only if the negative-slope section of the static characteristic intersects the voltage axis. An example of such a characteristic is the symmetrical current-voltage characteristic of Fig. 69e. Usually the use of a variable resistor in parallel with C is desirable, since it affords a simple and convenient means of changing the frequency of oscillation. Circuits in which no resistance is used in parallel with the capacitor will be shown in the next section.

Similarly, there is no lower limit to the magnitude of R at which the load line can intersect only the negative-slope section of a voltage-stable characteristic, and reduction of R is favorable to the inequality $L/(R + R_{iv}) \gg R_{iv}C$. Consequently, resistance need not be used in series with the inductance in a relaxation oscillator based upon a voltage-stable element if the negative-slope section of the characteristic intersects the current axis, as in Fig. 69b, and if the resistance is not required for frequency adjustment or the production of biasing voltage.

49.4. Advantages of Current-Stable Circuits. Although practical relaxation oscillators are based upon both current-stable and voltage-stable elements, the smaller size, weight, cost, and ease of adjustment of capacitors in comparison with inductors usually results in the choice of circuits that make use of current-stable current-voltage characteristics.

50. Practical Astable Circuits

50.1. Pentode Relaxation Oscillator. Very simple relaxation oscillators can be formed from the pentode negative-resistance circuits of Fig. 66 (or

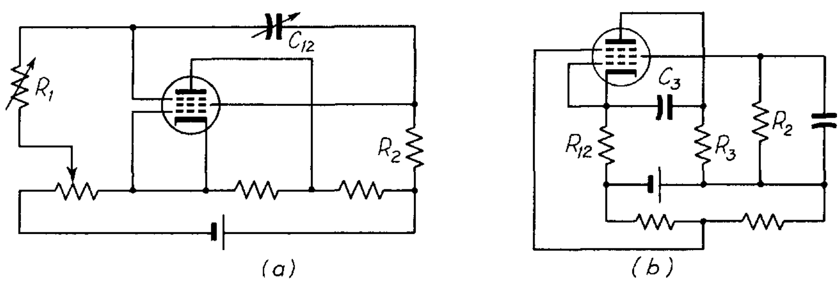


FIG. 167. Pentode relaxation oscillators based upon the negative-resistance circuits of Fig. 66.

the bistable circuits of Fig. 93). Figure 167a shows a circuit that makes use of the current-stable characteristic of Fig. 66e.[1] The suppressor-screen capacitor in this circuit need not be shunted by an additional resistor because, as far as changing currents and voltages are concerned, such a re-

[1] B. van der Pol, *Phil. Mag.*, **2**, 978 (1926); R. M. Page and W. F. Curtis, *Proc. I.R.E.*, **18**, 1921 (1930); E. W. Herold, *Proc. I.R.E.*, **23**, 1201 (1935).

sistor would merely parallel the series combination of R_1 and R_2, which serves as the necessary shunt resistance. The circuit of Fig. 167b makes use of the current-stable characteristic between the suppressor and ground in the circuit of Fig. 66b, which is similar to the characteristic of Fig. 66c. The resistor R_3 and capacitor C_3 serve to provide the proper plate biasing voltage and should be chosen so that the time constant R_3C_3 is large in comparison with the period of oscillation. A breakdown diode may be used in place of C_3, as in Fig. 66b.

The current-stable characteristic of Fig. 66d indicates that a relaxation oscillator can also be formed by inserting parallel resistance and capacitance in series with the plate in the circuit of Fig. 66a (or 93a). Although a current-stable port is formed by opening the suppressor lead in the circuit of Fig. 66b (or 93b), an oscillator cannot be formed by inserting parallel resistance and capacitance in series with the suppressor, because the suppressor current is negligible (i.e., the current-stable negative resistance between the suppressor and ground approaches an infinite value).

Less convenient relaxation oscillators may be formed by inserting an inductor in series with the suppressor or screen resistor in the circuit of Fig. 66a (or 93a) or a resistor in series with an inductor in the cathode lead in the circuit of Fig. 66b (or 93b).

50.2. Phenomenological Explanation of The Operation of The Pentode Circuit. The general analysis of Sec. 47 is applicable to the circuit of Fig. 167, but the following explanation of the operation of circuit (a) is of interest and typifies similar explanations that can be made of the operation of all relaxation oscillators.

Transition from the higher to the lower quasistable value of screen current causes an abrupt rise of screen voltage. Because the voltage across the capacitor cannot change instantaneously, there is an initial rise of suppressor voltage equal to the change of screen voltage. The decreased magnitude of negative suppressor voltage maintains the lower screen current (by allowing more electrons to go to the plate). The sudden increase of screen voltage also initiates a charging current into the capacitor through R_2 and R_1. As the voltage across the capacitor rises, the suppressor voltage becomes more negative and finally reaches a critical value at which transition occurs back to the higher quasistable value of screen current. The resulting abrupt fall of screen and suppressor voltages is immediately followed by discharging of the capacitor and consequently by exponential rise of suppressor voltage until transition again occurs. Figure 168 shows a typical wave of capacitor current or of suppressor or screen voltage, and the corresponding wave of capacitor voltage. The wave is asymmetrical because the screen current is lower during charging of C_{12} than during discharging and because the suppressor conducts if the suppressor becomes positive. The suppressor current, together with lower screen current and the resulting increase in the portion of the current through R_2 that flows into C_{12}, causes the capacitor voltage to change more rapidily than during the discharge of the capacitor, in which interval the suppressor does not conduct and the screen current is higher. (Alternatively, the more rapid change of capacitor voltage may be explained by the lower value of R_{ic} in the low-current range of the static current-voltage characteristic, Fig. 66e, and the consequent lower value of $R'C$.)

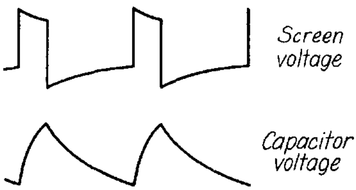


FIG. 168. Waveforms of screen voltage and voltage across the capacitor in the circuit of Fig. 167a.

Normally R_1 is large in comparison with the parallel combination of R_2 and the screen resistance, and it has relatively small effect upon the difference between the two suppressor-screen voltages at which transition occurs (Fig. 66e). The frequency of oscillation therefore increases with decrease of R_1 and C_{12}. The frequency is affected by the change of R_2 not only because increase of R_2 produces some change in the charging and discharging time constants, but also because increase of R_2 changes the difference between the suppressor-screen voltages at which transition begins. As R_2 is increased from a low value, the frequency first decreases, passes through a minimum, and then increases.[1]

50.3. Cathode-Coupled Relaxation Oscillator. Figure 169 shows two relaxation oscillators based upon the cathode-coupled circuit of Fig. 70. Circuit (a) makes use of the current-stable negative-resistance characteristic between the plate of T_1 and the grid of T_2.[2] Circuit (b) makes use of the current-stable negative resistance observed at a break in the cathode lead of T_2. The coupling capacitance C' between the plate of T_1 and the grid of T_2 in circuit (b) may be added in order to reduce the transition time between quasistable states. Alternatively, a large capacitance may be used in place of R', the time constant $R''C'$ of the coupling circuit being made so large that it does not affect the oscillation frequency.

Relaxation oscillators that make use of voltage-stable current-voltage characteristics may be developed from the circuit of Fig. 70 by the use of inductance in series with R_l, R_{12}, or R''.

In the circuits of Fig. 169 the capacitor current does not flow through the plate resistor of T_2. Furthermore, if T_2 is a pentode and R_l' is so large that

[1] Page and Curtis, *loc. cit.*

[2] J. L. Potter, *Proc. I.R.E.*, **26,** 713 (1938); M. G. Crosby, *Electronics,* May, 1946, p. 136; P. Sulzer, *Proc. I.R.E.*, **36,** 1034 (August, 1948); P. Sulzer, *Proc. I.R.E.*, **38,** 540 (May, 1950).

the load line intersects the plate characteristics of T_2 in the range in which the plate current is essentially independent of control-grid voltage, the plate current of T_2 is essentially constant during the time in which T_2 conducts. The output voltage then closely approximates a rectangular wave.

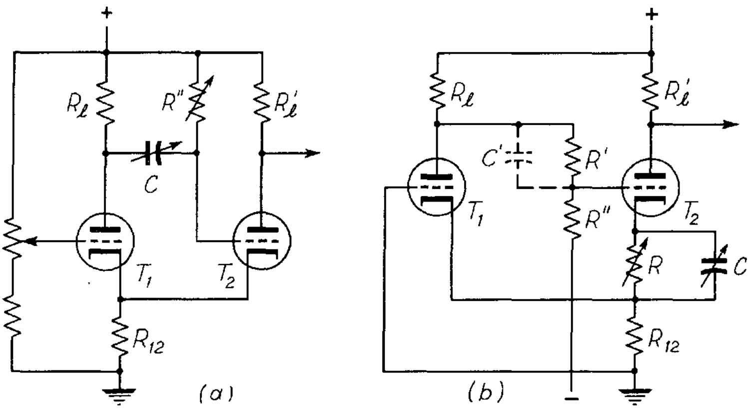


FIG. 169. Relaxation oscillators based upon the cathode-coupled negative-resistance circuit of Fig. 70.

50.4. *P-N-P-N*-Transistor Relaxation Oscillator. The oscillators of Fig. 170 are based upon the *p-n-p-n*-transistor negative-resistance circuits of Fig. 73. Those of Fig. 171 are derived from the two-transistor circuits of Fig. 74. The circuit of Fig. 171d, which is equivalent to the cathode-coupled tube circuit of Fig. 169a, covers the frequency range from 6 to 6000 cps with the circuit parameters shown.[3] Omission of the resistor in parallel with C is made possible by the fact that the negative-slope section of the current-stable characteristic observed between the collector of T_1 and the base of T_2 intersects the voltage axis. Rectification of the transistors allows C to charge to the voltage corresponding to the point at which the characteristic crosses the voltage axis. A resistance in shunt with C helps to stabilize the circuit against changes of transistor characteristics caused by temperature changes and also provides a convenient means of frequency adjustment. The base of T_1 is also operated at zero biasing current.

The circuit of Fig. 171e, which is equivalent to the tube circuit of Fig. 169a, gives a nearly rectangular output voltage of 11-volt swing over the frequency range of 50 cps to 20 kc.[4] The relatively complicated resistance network used for frequency adjustment maintains the duty factor essentially constant throughout the frequency range.

[3] F. C. Alexander, *Electronics*, December, 1954, p. 188.
[4] C. Martin, *Electronics Res. Lab. Rpt.* No. 78, Stanford Univ. (August, 1954).

50.5. Unijunction-Transistor Relaxation Oscillator. Figure 172a shows the extreme simplicity that is possible in a unijunction-transistor (double-base-diode) relaxation oscillator. Frequencies up to the megacycle range can be obtained with this circuit at supply voltages as low as 1.5.[5] The emitter current in the low-current range of the unijunction transistor being essentially independent of junction voltage, the capacitor charges nearly linearly. Because R_{ic} is much lower in the high-current range than in the low-current range and the time constant during most of the charging

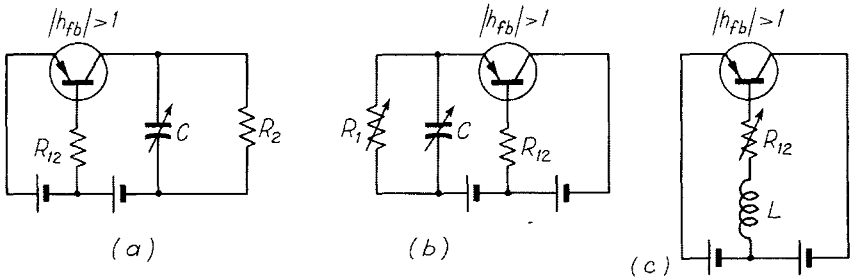


FIG. 170. Three relaxation oscillators based upon the *p-n-p-n*-transistor negative-resistance circuits of Fig. 73.

and discharging periods is $R_{ic}C$, the discharging time constant is much smaller than the charging time constant and the waveform approximates sawtooth form. Greater freedom from the effects of characteristic changes caused by temperature changes is achieved by the use of the slightly more complicated circuit of Fig. 172b.

An interesting modification of the circuit of Fig. 172b that delivers an approximately rectangular output voltage is shown in Fig. 172c.[6] The diagram of Fig. 173, which shows the paths of operation with the diode D short-circuited and with D in the circuit, is helpful in explaining the operation of the circuit. With D short-circuited, the path of operation, shown dashed, is of the same general form as that shown in Fig. 161a, the maximum capacitor voltage occurring at the point where the path of operation crosses the load line corresponding to the effective supply voltage $V_S R'/(R + R')$ and the resistance $RR'/(R + R')$. (At any value of current i_E, the difference between the voltage v_E and the voltage on the current-stable characteristic is accounted for by voltage induced across the effective internal inductance of the transistor.) When the diode D is in the circuit, C charges through R and the diode D, the diode current being equal to the sum of the current into the capacitor and that through R'. If R' is large in comparison with the forward resistance of D, the charging time constant is approxi-

[5] J. J. Suran, *Electronics*, March, 1955, p. 199.
[6] J. J. Suran and E. Keonjian, *Proc. I.R.E.*, **43**, 814 (July, 1955).

mately the same as that with D short-circuited, namely $RR'C/(R + R')$. Beyond the maximum-voltage point m, the capacitor current reverses and thereafter reduces the current through D. At n the current out of the capacitor equals the current through R', and the current through D is zero. Point n must lie on the load line corresponding to the resistance R and the

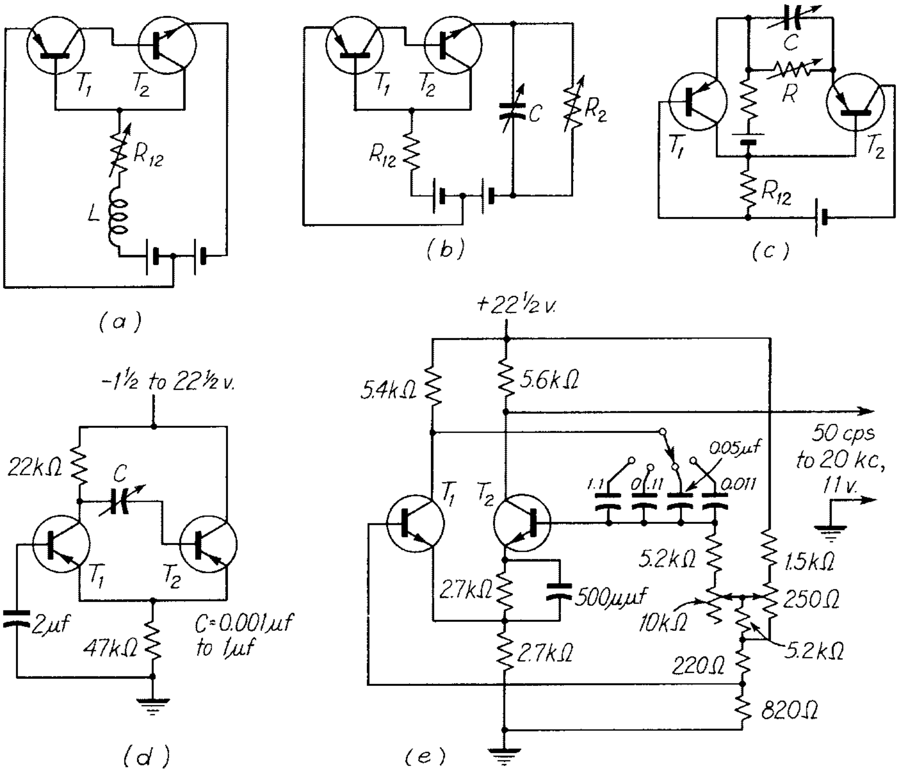


FIG. 171. Relaxation oscillators based upon the two-transistor negative-resistance circuits of Fig. 74.

supply voltage V_S, for the current at this instant all passes through R and the voltage v_E is $V_S - i_E R$. Further increase of i_E causes the current through D to reverse. Because the reverse resistance of the diode D is very high, C and R' are in effect disconnected from the rest of the circuit.

After D opens the circuit, the two parts of the circuit function independently, the capacitor discharging through R' with the time constant $R'C$, and the transistor emitter voltage and current changing to the values at point P along a path and with a time constant determined by R and the effective emitter inductance. When the capacitor voltage becomes equal to the voltage at P, the diode D begins to conduct again, and C and R' are again connected to the emitter. The emitter current and voltage then follow along a path similar to the left portion of the path obtained with D short-circuited, but starting from point p, rather than the low-voltage point of the characteristic. The capacitor, emitter, and output voltages of the

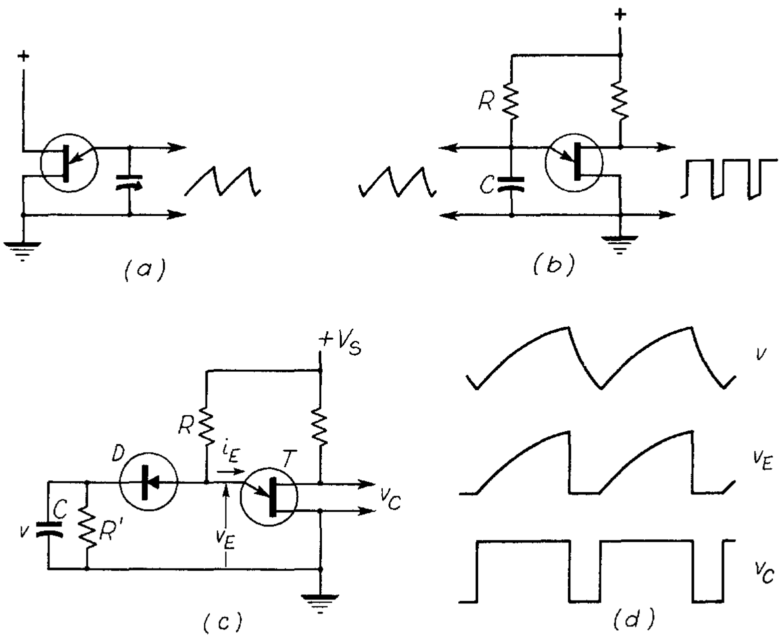


FIG. 172. (a), (b), and (c) Unijunction-transistor relaxation oscillators; (d) voltage waveforms for circuit (c).

circuit of Fig. 172c are shown in Fig. 172d. The output (collector) voltage is rectangular because the collector current is essentially independent of emitter current in the low-current range of the emitter characteristic followed during the charging of the capacitor, and the emitter is disconnected from the capacitor during the discharging of the capacitor. The rise time of the output voltage is aproximately 2 μsec and the fall time approximately 1 μsec. The waveform is good up to approximately 50 kc. Complete design details for this circuit have been given by Suran and Keonjian.[7]

51. The Multivibrator

51.1. Basic Circuit. It was pointed out in Sec. 18 that a current-stable current-voltage characteristic is observed between the plate of one tube of

[7] Suran and Keonjian, *loc. cit.*

an Eccles-Jordan circuit and the grid of the other (Fig. 69c), or between the collector of one transistor and the base of the other. Astable operation is therefore to be expected if $R_{12}C_{12}$ is made sufficiently large in the circuits of Fig. 96. The two astable states then become quasistable, switching of

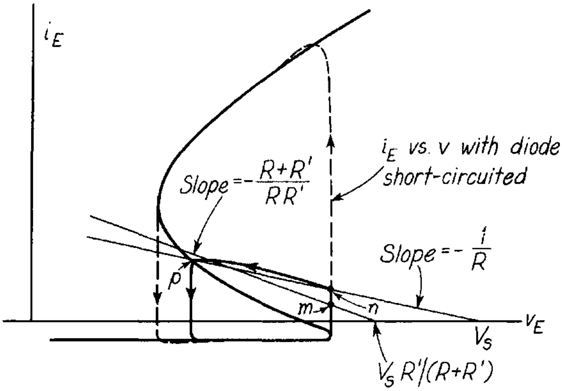


FIG. 173. Current-voltage diagram for the circuit of Fig. 172c.

the current from one tube or transistor to the other being followed by exponential charging of one capacitor and discharging of the other. Because the frequency of oscillation can be readily adjusted by means of the resistors R_1, the coupling resistors R_{12} may be omitted. The resulting circuit, called

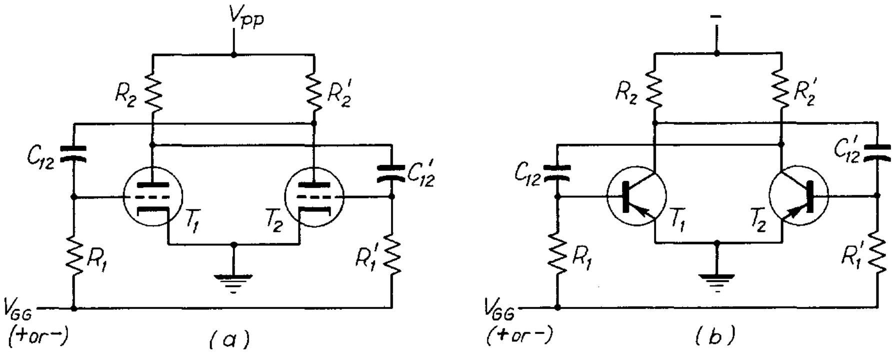


FIG. 174. Vacuum-tube multivibrator; (b) transistor multivibrator.

the *multivibrator,* is shown in basic form in Fig. 174. The reason for the use of forward grid or base bias in the multivibrator will be explained.

The analyses presented in this and following sections are based upon tube circuits. These analyses may, with minor modifications, be extended to transistor circuits. The conclusions that are drawn from the analyses are therefore in general also applicable to transistor circuits.

51.2. Analysis of Multivibrator Operation. Transistion between the equilibrium states of a multivibrator is similar to that in bistable circuits. It will first be assumed to take place in a time that is negligible in comparison with the period of oscillation and to start when the grid voltage of the initially nonconducting tube reaches the cutoff value.

Figure 175 shows the current directions and typical voltages in a vacuum-

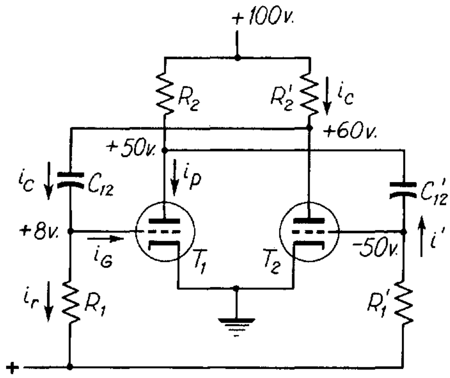


FIG. 175. Current directions and typical voltages in a vacuum-tube multivibrator immediately following switching of current from tube T_2 to tube T_1.

tube multivibrator immediately following switching of current from tube T_2 to tube T_1. Immediately prior to this transfer no current flowed through R_2 and the voltage of the plate of T_1 was therefore 100; that of the plate of T_2 was 50. The flow of grid current through R_1' prior to switching maintained the grid of T_2 at approximately zero potential. Since the voltage across C_{12}' cannot change instantaneously, the 50-volt drop in plate voltage of T_1 was accompanied by an equal drop in the grid voltage of T_2 to a value of approximately -50 volts.

Voltage drop in R_2' caused by i_c prevents the plate voltage of T_2 from rising immediately to 100 volts when T_2 is cut off. The initial rise depends upon the ratio of R_2' to the resistance of R_1 in parallel with the grid resistance of T_1. Because this grid resistance is relatively low, the plate voltage of T_2 rises rapidly toward 100 volts. The initial value of plate voltage of T_2 following transition is assumed to be 60 volts. The abrupt rise in plate voltage of T_2 from 50 volts to 60 volts is accompanied by a 10-volt rise in the grid voltage of T_1 from the cutoff value, at which transition was initiated and which is assumed to be -2 volts. The switching is followed by an exponential discharging of C_{12}' through R_1' and the parallel combination of R_2 and the plate-cathode path of T_1 and by an exponential charging of C_{12} through R_2' and the parallel combination of R_1 and the grid-cathode path of T_1. Usually R_1' is large in comparison with R_2 and the a-c plate resistance of T_1, and the rate of rise of grid voltage of T_2 is determined largely by the time constant $R_1'C_{12}'$. Because the grid resistance of T_1 is small in comparison with R_2', on the other hand, the rate of fall of grid voltage of T_1 is determined largely by the relatively small time constant $R_2'C_{12}$. Plate current switches from T_1 to T_2 when the grid voltage of T_2 reaches the cutoff value. (See discussion of Fig. 105, Sec. 30.2.) Switching of current from T_1 to T_2 is again followed by exponential changes of voltage and currents, the cycle being completed when the grid voltage of T_1 has risen to the cutoff value.

51.3. Waveforms of Multivibrator Voltages and Currents. Figure 176 shows typical waveforms of currents and voltage in the circuit of Fig. 175 when the circuit is symmetrical, the grid supply voltage positive, and switching time negligible in comparison with the period of oscillation. Two representative waveforms of plate-to-plate and grid-to-grid voltage for an asymmetrical circuit are shown in Figs. 177a and 177b. The negative spikes of grid current in Fig. 176 are caused by the charging of the grid-cathode tube capacitance during switching, which was neglected in the foregoing analysis. As the period of oscillation is decreased, the transition time becomes appreciable in comparison with the period. Figures 177c and 177d show two representative waveforms of plate-to-plate and grid-to-grid voltage of a symmetrical circuit in which the transition time is not negligible.

The waveforms of multivibrator currents and voltages, particularly the plate-to-plate voltage, depend in such a complicated manner upon the circuit constants and the tube characteristics that they are very difficult to predict. A discussion of some of the factors that affect the plate-to-plate voltage is of interest. (See also Prob. 51.3–1.)

The plate-to-plate voltage is the difference between the voltage drops across the two plate resistors. Its waveform therefore depends upon the waveforms of the two plate voltages. From Fig. 175 and the discussion of that figure it can be seen that variation of the plate voltage of the conducting tube T_1 occurs for two reasons. Unless the plate current is independent of the grid voltage at positive grid voltages, positive grid swing of the conducting tube causes the plate voltage of the conducting tube to drop to a value below that corresponding to zero grid voltage and to rise rapidly subsequently as the grid voltage decays toward zero. Secondly, the discharge current of the capacitor C_{12}' connected to the plate of the conducting tube causes the plate voltage of that tube to be higher than it would be otherwise, and to fall slowly as the capacitor discharges. The plate voltage of the nonconducting tube varies because charging current of the capacitor

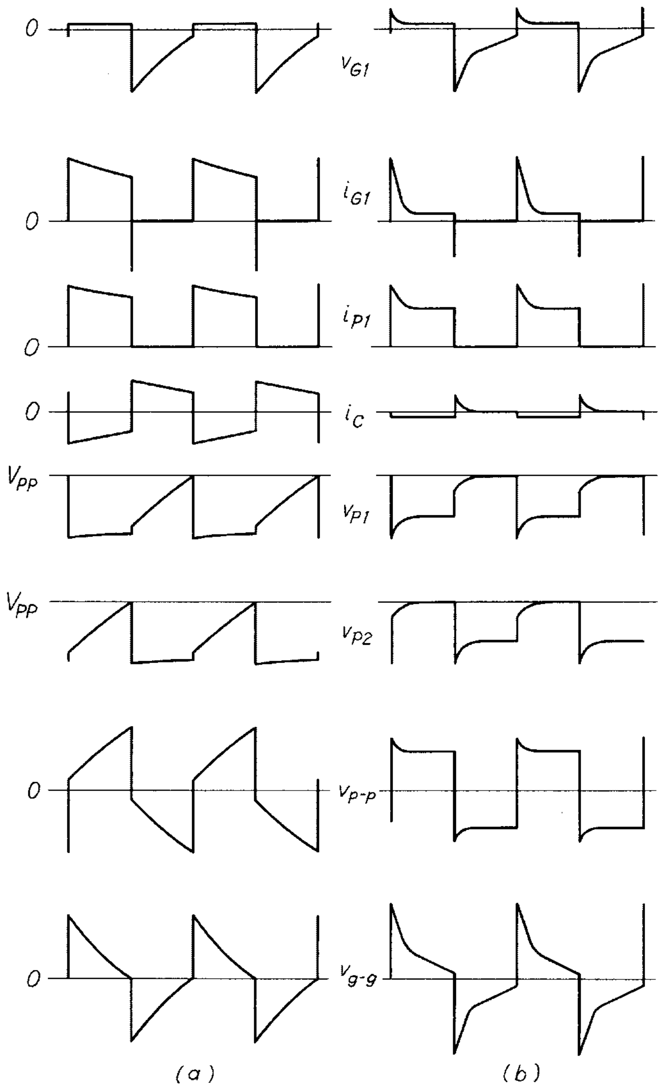


FIG. 176. Typical current and voltage waveforms in the circuit of Fig. 174 when the switching time is negligible in comparison with the period of oscillation: (a) $R_1 = R_1' = 47$ kΩ, $R_2 = R_2' = 47$ kΩ, $C_{12} = C_{12}' = 0.2$ μf, $V_{PP} = 100$ v., $V_{GG} = +50$ v., (b) $R_1 = R_1' = 470$ kΩ, $R_2 = R_2' = 2.7$ kΩ, $C_{12} = C_{12}' = 0.1$ μf, $V_{PP} = 100$ v., $V_{GG} = +50$ v.; 6SN7 tube.

C_{12} connected to the plate of that tube prevents the plate voltage from rising instantaneously to the supply voltage during transition. The voltage first rises abruptly to a value less than the supply voltage and then rises exponentially toward the supply voltage.

51.4. Screen-Coupled Pentode Multivibrator. Useful variants of the basic multivibrator circuit are based upon the various modifications of the

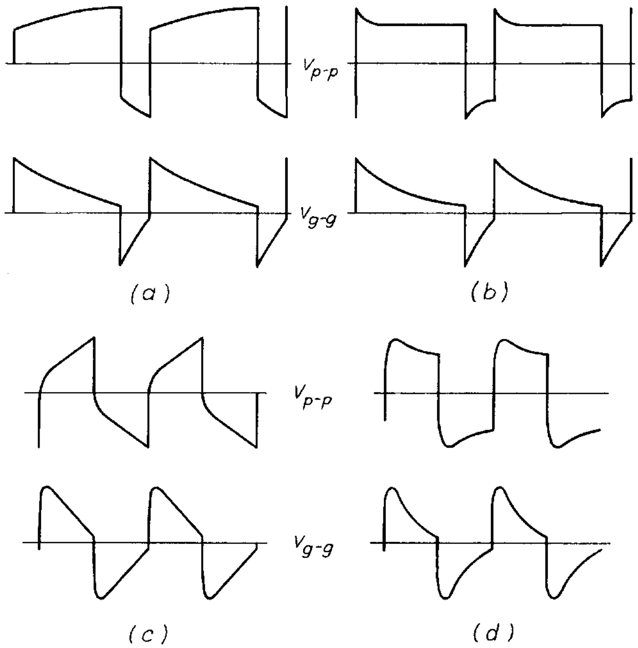


FIG. 177. (a) and (b) Representative plate-to-plate and grid-to-grid voltages in an asymmetrical vacuum-tube multivibrator; (c) and (d) representative plate-to-plate and grid-to-grid voltages in a symmetrical vacuum-tube multivibrator in which transition time is not negligible relative to the period of oscillation.

Eccles-Jordan circuit in which tetrodes and pentodes are used in place of triodes. One circuit that merits special mention is that in which the capacitors are connected between the screen grid of one tube and the control grid of the other and the output is taken from the plates.[1] Because the plate current is largely independent of positive control-grid voltage and because the capacitor currents do not flow through the plate load resistors, the plate-voltage waveform is more nearly rectangular than in the usual triode circuit.

[1] N. W. Mather, *Electronics*, October, 1946, p. 136.

51.5. Cathode-Loaded Multivibrator. Another variant of the basic multivibrator, which makes use of the symmetrical current-stable characteristic, Fig. 69e, of the circuit of Fig. 68e, is shown in Fig. 178.[2] Advantages of this circuit include the single tuning capacitor and the fact that the plate-voltage waveform is more nearly rectangular than in the circuit of Fig. 174. A physical analysis of the operation of this circuit is of interest. Assume that conduction has just changed from T_1 to T_2. Voltage across C caused by the flow of current through R when T_1 was conducting raises the potential of the cathode of T_1 above that of T_2 and thus helps to maintain T_1 cut off. The voltage across R' produced by the plate current of T_2 discharges C and charges it in reverse polarity. As the voltage across C changes, the negative grid bias of T_1 decreases. When it becomes equal to the cutoff voltage, T_1 starts to conduct and the current switches rapidly from T_2 to T_1.

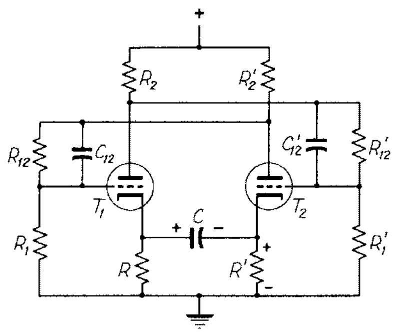


FIG. 178. Symmetrical single-capacitor multivibrator formed from the negative-resistance circuit of Fig. 68e.

Although the capacitor current in the circuit of Fig. 178 passes through R and the combination of R' in parallel with T_2 and its plate resistor while T_2 conducts, $r_p + R_2'$ is so small in comparison with R' that the time constant approximates RC. Furthermore, the small fraction of the capacitor current that flows through the cathode resistor of the conducting tube has very little effect upon the grid voltage of the conducting tube. For this reason the plate current of the conducting tube is nearly constant and the plate-to-ground voltage is very nearly rectangular. The cathode-to-ground voltage also approximates rectangular form. The degree to which the plate-to-ground and cathode-to-ground voltages approximate rectangular form can be increased by the use of pentodes with cathode-resistor bias in place of the resistors R and R' in order to make the charging and discharging current of the capacitor nearly constant. Under this condition the voltage across the capacitor rises and falls linearly and the capacitor-voltage wave is triangular. The coupling capacitors C_{12} and C_{12}' serve the same function as in bistable circuits, namely, to reduce the transition time. Their capacitance is therefore of the order of 25 to 200 $\mu\mu$f. Alternatively, capaci-

[2] Rad. Lab. Series, Vol. 19, Sec. 5.6, McGraw-Hill Book Co., Inc., New York, 1949; G. W. Gray, *Electronics*, February, 1952, p. 101.

tors may be used in place of R_{12} and R_{12}', the time constants R_1C_{12} and $R_1'C_{12}'$ being made large in comparison with the oscillation frequency.[3] The circuit of Fig. 178 may also be used with transistors.[4]

52. Stabilization of Multivibrator Frequency

52.1. Importance of Frequency Stability. In some applications of multivibrators, good frequency stability is important. In other applications, the frequency must be controlled by means of a voltage. It is therefore important to determine the manner in which the frequency of oscillation depends upon supply voltages and the circuit parameters.

The manner in which the circuit operates is somewhat different for negative grid (or reverse base) supply voltages than for positive grid (or forward base) supply voltages. In either type of operation, however, if the period of oscillation is long in comparison with the transition time between the two quasistable states, the period of the positive or negative portion of the cycle approximates the time taken for the grid voltage of the nonconducting tube to rise from its maximum negative value to the cutoff value. Operation with negative grid supply voltage has no advantage over operation with positive grid supply voltage, but complicates the power-supply problem and makes stabilization of frequency against changes of supply voltage more difficult. For this reason, only positive-voltage operation will be analyzed in detail. Some aspects of negative-grid-supply operation will be discussed in Sec. 52.4, further details being presented in Probs. 52.2–1 and 52.4–1.

52.2. Period of Oscillation. Figure 179 shows the form of the grid-voltage wave of T_1 or T_2 in Fig. 174a when V_{GG} is positive and transition time is negligible in comparison with both halves of the cycle. Under the assumption that the transition time is negligible and that R_1 and R_1' are large in comparison with R_2 and R_2', the change of grid voltage kV_{PP} during transition is approximately equal to the change of plate voltage. Its approximate value can be determined from the family of plate characteristics and the load line corresponding to the resistance R_2 or R_2'.* The factor m is merely the ratio of the grid supply voltage V_{GG} to the plate voltage V_{PP} and is used in order that all voltages may be expressed in terms of V_{PP}. In the following analysis, m will be assumed to be positive throughout. The magnitude of the cutoff voltage is V_{PP}/μ, μ being measured at cutoff.

When the grid supply voltage is positive, the grid voltage of the conducting

[3] Gray, *loc. cit.*

[4] E. W. Sard, *I.R.E. Convention Record,* Part II, p. 119 (1954).

* When R_1 and R_1' are not large in comparison with R_2' and R_2, k is smaller because the effective load resistance during transition is then $R_2R_1'/(R_2 + R_1')$ instead of R_2.

tube following transition falls rapidly from its initial positive value to a value in the vicinity of zero, where it is maintained by the flow of grid current through the grid resistor until the circuit again switches. Since the grid voltage of the conducting tube remains essentially constant throughout most of the time in which the tube conducts, and the change of plate voltage resulting from exponentially varying capacitor current produces a relatively small change of plate current, an approximate expression for the instantaneous grid voltage of the nonconducting tube can be readily deter-

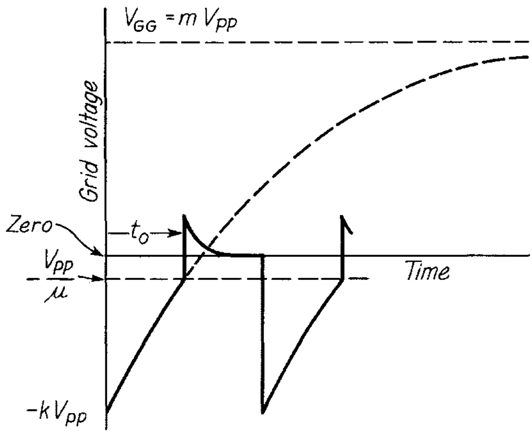


FIG. 179. Grid-voltage waveform for the circuit of Fig. 174a with positive grid supply voltage.

mined by assuming that the plate voltage of the conducting tube is constant throughout the conduction period.* Under the additional assumption that R_1 is large in comparison with the parallel combination of R_2' and the plate resistance of T_2, the grid voltage of T_1 rises exponentially with the approximate time constant R_1C_{12} following the switching of current from T_1 to T_2. The following expression for the time t_o in which the grid voltage of the nonconducting tube rises from its maximum negative value $-kV_{PP}$ to the cutoff grid voltage $-V_{PP}/\mu$ may be readily found from the expression for the voltage across the resistance R_1 when the voltage $(k + m)V_{PP}$ is applied suddenly to R_1 in series with C_{12}:

$$V_{PP}(k - 1/\mu) \approx V_{PP}(k + m)(1 - \epsilon^{-t_o/R_1C_{12}}) \qquad (86)$$

* Note that, if the plate load resistance in pentode circuits is such that the load line intersects the zero-grid-voltage plate characteristic well below the knee, driving the grid positive has very little effect upon the minimum plate voltage. A similar limiting of minimum collector voltage occurs when the base voltage of a transistor is driven into the saturation range.

or

$$\epsilon^{-t_o/R_1C_{12}} \approx \frac{m + 1/\mu}{m + k} \tag{87}$$

52.3. Frequency Instability of Multivibrators. Usually the most important cause of frequency instability of multivibrators is the dependence of frequency upon supply voltages. Since t_o, and therefore f, is constant if the right side of Eq. (87) is constant, the variation of frequency with supply voltages can be minimized by minimizing the dependence of the right side of Eq. (87) upon supply voltages. If the grid and plate supply voltages are obtained from separate sources, they may vary independently and any change in either one produces a proportional change in m. Equation (87) shows that considerable variation in frequency may then be expected. On the other hand, if the grid supply voltage is obtained from a tap on a voltage divider across the plate supply voltage, m is constant. The amplification factor of tubes normally used in multivibrators is also essentially constant. The only factor in the right side of Eq. (87) that is dependent upon V_{PP} is then k, and we may write:

$$\frac{d}{dV_{PP}} \frac{m + 1/\mu}{k + m} = -\frac{m + 1/\mu}{(k + m)^2} \frac{dk}{dV_{PP}} \tag{88}$$

Equation (88) shows that the frequency stability against changes of supply voltage is increased by decreasing the dependence of k upon V_{PP} and by increasing μ or k. Since $1/\mu$ is usually small relative to k, stability is also favored by making m either large or small relative to k. However, small m is unfavorable to stability against noise (Sec. 52.6). Consequently, a large value of m is preferable. It will now be shown that increase of m or k produced by increasing V_{GG} or R_2 does not increase dk/dV_{PP}.

Under the assumption that R_1 and R_1' are so large that grid current reduces the grid voltage of the conducting tube to nearly zero in a time that is short in comparison with the period of conduction, kV_{PP} approximates the difference between V_{PP} and the plate voltage at the point where the load line in the plate diagram of the conducting tube intersects the zero-grid-voltage plate characteristic, as shown in Fig. 180. The following facts are apparent from an inspection of Fig. 180: (1) Because the grid voltage of the conducting tube following transition remains at a value close to zero during most of the conduction period if V_{GG} is positive, and this value changes very slowly with V_{GG} when R_1 and R_1' are large, k is affected very little by an increase of V_{GG}. For the same reason, increase of m by means of V_{GG} has a negligible effect upon dk/dV_{PP}. (2) The value of k would be independent of V_{PP}, and dk/dV_{PP} would therefore be zero, if the zero-grid-voltage characteristic were a straight line through the origin. The zero-grid-voltage characteristic of pentodes approximates a straight line through the origin at plate voltages below the knee of the characteristic. With triodes, the variation of k with V_{PP} is reduced by an increase of R_2, which brings the intersection of the load line with the zero-grid-voltage characteristic close to the origin, but is increased by an increase of V_{PP} unless this increase is accompanied by an increase of R_2.

Because increasing V_{GG} increases m without increasing dk/dV_{PP} or decreasing k, the frequency stability of the multivibrator of Fig. 174a against changes of supply voltage is increased by increasing m. For this reason, R_1 and R_1' should be connected to the positive side of the plate-voltage

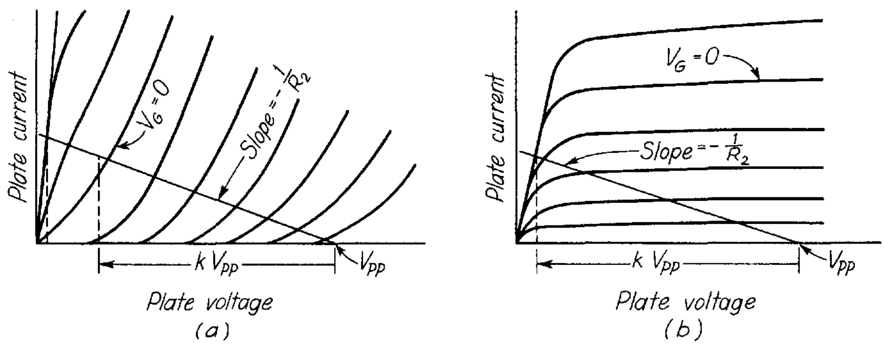


FIG. 180. Plate diagram for the circuit of Fig. 174a: (a) triode circuit; (b) pentode circuit.

supply, or R_2 and R_2' should be connected to a tap on a voltage divider across a positive grid-voltage supply. With triodes, the supply voltage and the values of R_2 and R_2' should be chosen so that the load line intersects the zero-grid-voltage characteristic close to the origin. With pentodes, the intersection should be well below the knee. In general, the dependence of k upon V_{PP} can be made smaller with pentodes than with triodes, and the frequency stability therefore greater.

52.4. Frequency Stability for Negative Grid Supply Voltage. When V_{GG} is negative, the grid voltage of the conducting tube is not clamped at or near zero by grid current, but falls exponentially toward V_{GG}. As the grid voltage of the conducting tube swings negative, the plate voltage rises and helps to raise the grid voltage of the nonconducting tube. Because of amplification, the grid voltage of the conducting tube need fall by only a few volts in order to raise the grid voltage of the nonconducting tube above cutoff. Consequently, the time during which the nonconducting tube remains off is determined almost entirely by the rate at which the grid voltage of the conducting tube falls, rather than by the rate at which the grid voltage of the nonconducting tubes rises. The period of each portion of the cycle is determined by the time constant of the grid circuit of the conducting tube, rather than by the time constant of the grid circuit of the nonconducting tube. It follows that Fig. 179 and Eqs. (86) to (88) are not applicable when m is negative.

An analysis similar to that used in the derivation of Eqs. (86) to (88) shows that, when the grid supply voltage is negative, the frequency stability is also improved by obtaining the grid supply voltage from the same source as the plate supply voltage and that the stability increases with the magnitude of m (Prob. 52.4–1). However, there is a maximum magnitude of m, which cannot exceed k, above which oscillation ceases. There is no advantage in the use of a negative grid supply voltage, and the use of a single tapped voltage source in order to maintain negative m constant may involve grounding problems.

52.5. Frequency Stability of Transistor Multivibrator. A similar analysis for the transistor circuit of Fig. 174b shows that, for good frequency stability against changes of supply voltage, R_1 and R_1' should be connected directly to the collector voltage supply or that the collector supply voltage should be obtained from a tap on a voltage divider across the base voltage supply.

52.6. Stabilization of Frequency Against Noise. Another important consideration in the stabilization of multivibrator frequency is the effect of noise. Noise raises the grid or base voltage of the nonconducting tube or transistor above cutoff before the instant at which this would normally occur, and therefore advances the onset of transition by an amount that is dependent upon the amplitude of the noise voltage. The dependence of the time of advance upon noise amplitude causes a random variation in the period of oscillation when the noise is random. This phenomenon is called *frequency jitter*, because the pattern observed on the screen of an oscilloscope used to display the output wave moves from side to side in a random manner.

Superposition of a periodic or random increment of grid voltage of constant magnitude upon the grid-voltage curve of Fig. 179 shows that the maximum change of the instant of switching that can be produced by voltage increments of fixed magnitude decreases with increase of steepness of the grid-voltage wave in the vicinity of the cutoff grid voltage. Increase of V_{GG} therefore helps to stabilize the frequency against noise, as well as against variation of supply voltage.

52.7. Stabilization Against Variation of Load. The effect of the resistance and capacitance of external load coupled to a multivibrator upon the frequency and waveform can be best prevented by the use of a cathode-follower or common-collector stage between the multivibrator and the load.

53. Tuning of the Multivibrator

53.1. Frequency of Oscillation. When the parameters and supply voltages of a multivibrator are such that the transition times between the two quasistable states of equilibrium are small in comparison with the times in which the grid voltages of the nonconducting tubes rise from their most negative values to the cutoff value, the period of oscillation approximates the sum of the rise times t_o and t_o' in the two halves of the cycle. If the oscillator is asymmetrical because of differences in circuit parameters or supply voltages, t_o and t_o' are in general different. The following expression for the oscillation frequency when the grid supply voltage is positive and R_1 and R_1' are much larger than R_2 and R_2' may be found (Prob. 53.1–1) by solving for t_o and t_o' in Eq. (87):

$$\frac{1}{f} \approx t_o + t_o' = R_1C_{12}\log_\epsilon \frac{m+k}{m+1/\mu} + R_1'C_{12}'\log_\epsilon \frac{m'+k'}{m'+1/\mu} \tag{89}$$

The primed symbols in Eq. (89) refer to one-half of the circuit, and the unprimed symbols to the other half. If the circuit is completely symmetrical, Eq. (89) becomes

$$\frac{1}{f} \approx 2R_1C_{12}\log_\epsilon \frac{m+k}{m+1/\mu} \tag{90}$$

When R_1 and R_1' are not large in comparison with R_2 and R_2', Eq. (90) has the more general form

$$\frac{1}{f} = 2\left(R_1 + \frac{r_pR_2}{r_p+R_2}\right)C_{12}\log_\epsilon \frac{m+k}{m+1/\mu} \tag{90A}$$

Equations (89) to (90A) show that the frequency of oscillation of a multivibrator with positive grid supply voltage increases with decrease of R_1, R_1', C_1, C_1' and k, and with increase of m and therefore of V_{GG}. Since negative grid supply voltage is not ordinarily used, a detailed analysis will not be made for this method of operation. The following simple qualitative analysis is, however, instructive. It was pointed out in Sec. 52.4 that the grid voltage of the conducting tube is not clamped at or near zero when the grid supply voltage is negative, but that it continues to fall toward V_{GG}, and that the resulting rise in plate voltage of the conducting tube is then the main cause of the rise in grid voltage of the nonconducting tube. Increasing the magnitude of negative V_{GG} increases the rate of fall of grid voltage of the conducting tube and therefore the rate of rise of grid voltage of the nonconducting tube and thus reduces the time taken by the grid voltage of the nonconducting tube to reach the cutoff value. Consequently, increase of magnitude of V_{GG} increases the frequency of oscillation not only when V_{GG} is positive, but also when it is negative. Figure 179 indicates that the frequency of oscillation should be increased by decrease of k regardless of whether V_{GG} is positive or negative.

An experimental curve of frequency of a symmetrical multivibrator as a function of grid supply voltage is shown in Fig. 181a. This curve is in agree-

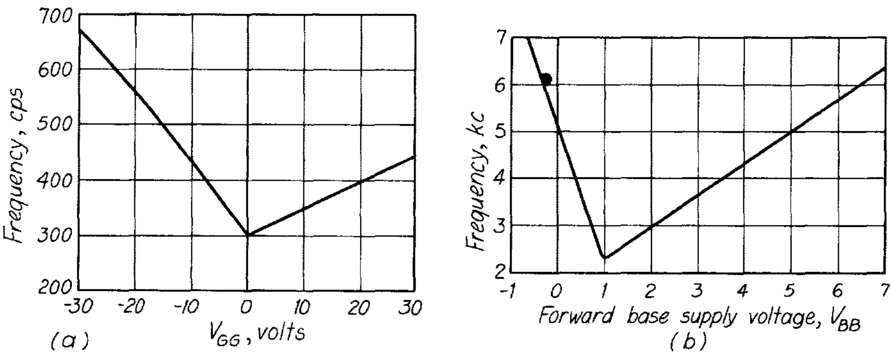


FIG. 181. (a) Experimental curve of frequency vs. grid supply voltage for a symmetrical vacuum-tube multivibrator; $R_1 = 75$ kΩ, $R_2 = 20$ kΩ, $C_{12} = 0.01$ μf, $V_{PP} = 250$ v., 6SN7 tube. (b) Experimental curve of frequency vs. base supply voltage for a symmetrical transistor multivibrator; $R_1 = 4.7$ kΩ, $R_2 = 1$ kΩ, $C_{12} = 0.02$ μf, $V_{CC} = 9.5$ v., *n-p-n* switching transistors.

ment with the theoretical analysis. An oscilloscopic examination of the plate-voltage wave of one tube of a multivibrator also confirms the theoretical prediction that the conducting time of either tube should depend mainly upon the time constant of the grid circuit of the nonconducting tube when V_{GG} is positive and mainly upon that of the grid circuit of the conducting tube when V_{GG} is negative.

53.2. Limits of Frequency of Oscillation. The extent to which the oscillation frequency of a multivibrator can be increased by reduction of C_{12} and C_{12}' is limited by interelectrode capacitances and by the fact that operation in the negative-resistance portion of the current-voltage characteristic is not in a growing-exponential region of Fig. 159 if C_{12} and C_{12}' are too small. The extent to which R_1 and R_1' can be reduced, on the other hand, is limited by grid dissipation when positive grid supply voltage is used, and by reduction of the factor k. As R_1 and R_1' are reduced, k is reduced because of reduction of the effective plate load resistance of the tubes during transition. The lower limit of frequency is usually determined by capacitor leakage.

53.3. Voltage Tuning of the Multivibrator. The frequency of oscillation of the tube multivibrator of Fig. 174a may be varied by changing either supply voltage or the voltage V_d applied to diodes connected to the grids or to the plates, as in the bistable circuit of Fig. 107. Varying either supply voltage changes the value of m, the ratio of the grid supply voltage to the plate supply voltage. Varying the diode voltage V_d in the astable form of the circuit of Fig. 107 causes a proportional change of k, since the plate voltages cannot fall appreciably below V_d. Frequency variation by means of V_d is of necessity accompanied by amplitude variation.

Equations (89) to (90A) show that the frequency does not in general vary linearly with m or k and therefore with control voltages applied in series with the supply voltages, or in series with the diode reference voltage when clipping diodes are used. The variation of frequency with m may, however, be made approximately linear by keeping t_o small in comparison with the time constant R_1C_{12}. Figure 179 shows that this can be accomplished by making V_{GG} equal to the quiescent value of V_{PP} and by making k small by the use of small values of R_2 and R_2' or by the use of diodes to limit the plate or grid swings. If $t_o \ll R_1C_{12}$, the left side of Eq. (87) may be replaced by the first two terms of the series expansion for the exponential. This procedure leads to the following expression for the oscillation frequency of a symmetrical circuit:

$$f \approx \frac{1}{2t_o} \approx \frac{k + m}{2R_1C_{12}(k - 1/\mu)} \tag{91}$$

The value of k was shown in Sec. 52.3 to be nearly independent of V_{GG} when V_{GG} is positive, and to vary only slowly with V_{PP}. Equation (92) therefore shows that the frequency varies approximately linearly with m when m approximates unity and k is small in comparison with unity, and that nearly linear tuning is then obtained if the control voltage is introduced in series with V_{GG} or V_{PP}.

A high degree of frequency stability of the multivibrator can be achieved by applying the output voltage to a frequency discriminator tuned to the desired fundamental frequency and applying the direct-voltage output of the discriminator to the multivibrator control grids in such a manner that the discriminator voltage resulting from a change in frequency produces a frequency change of opposite sign.

53.4. Linearization of Tuning. The linearity of the relation between frequency and positive grid supply voltage of a multivibrator can be increased by making the charging current of the coupling capacitors as nearly constant as possible. One way in which this can be done is illustrated by the asymmetrical circuit of Fig. 182a in which the parameters are chosen so that the time of conduction of T_1 is very large in comparison with that of T_2 and the period of oscillation is therefore determined almost entirely by the rate at which C_{12}' charges.[1] When T_1 fires, the voltage of the grid of T_3 is driven negative to the value $-kV_{PP}$. Since T_3 is a cathode-follower amplifier, its cathode voltage falls by nearly the same amount as its grid voltage, and the cathode of diode T_4 is made negative. If the d-c resistance of T_3 and T_4 is small, C_3 charges rapidly to a potential difference equal to

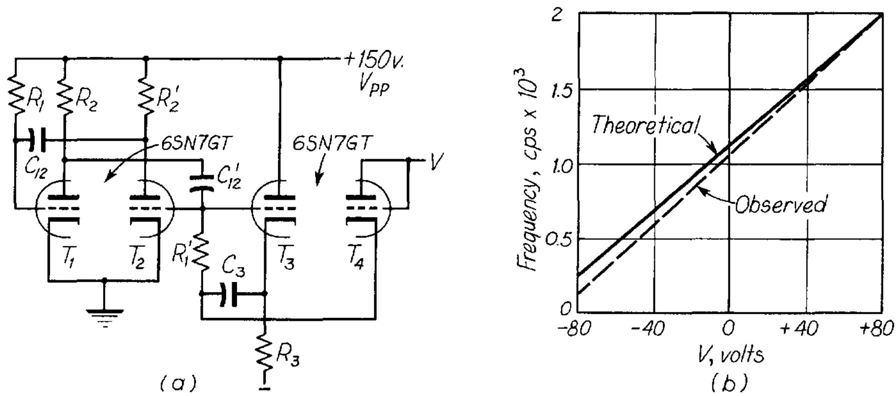


FIG. 182. (a) Circuit having a linear voltage-tuning curve; $R_1 = 330$ kΩ, $R_2 = R_2' = 22$ kΩ, $R_1' = 1$ M, $C_{12} = 250$ μμf, $C_{12}' = 1000$ μμf, $C_3 = 0.1$ μf. (b) Tuning curve for this circuit.

nearly $mV_{PP} - kV_{PP}$, where m is determined by the voltage V. The capacitor C_{12}' immediately starts to discharge through R_1'. The resulting rise in grid voltage of T_3 is accompanied by a practically equal rise in cathode voltage and, if $C_3 \gg C_{12}'$, by a nearly equal rise in the voltage of the cathode of T_4. The voltage across R_1', and consequently the charging current of C_{12}', therefore remains essentially constant, and the grid voltage of T_2 continues to rise approximately linearly at the rate $V_{PP}(m + k)/R_1'C_{12}'$, which is the same as the initial rate in the absence of the cathode-follower stage. Figure 182b shows an experimentally determined tuning curve for the circuit of Fig. 181a.

When a symmetrical circuit is desired, cathode-follower feedback stages may be added to both halves of the circuit and controlled by the same voltage. If the control voltages applied to the diodes are changed in opposite directions by equal amounts, the change in frequency is proportional to the square of the change of control voltages[2] (see Prob. 53.3–1).

[1] J. M. Sturtevant, *Electronics*, December, 1949, p. 144.
[2] Sturtevant, *loc. cit.*

53.5. Transistor Circuit. Figure 182A shows a transistor multivibrator in which positive base supply voltage is obtained by shunting C_{12} and C_{12}' by resistors R_{12} and R_{12}'.[3] The values of R_1 and R_1' used in this circuit are not negligible in comparison with R_2 and R_2'. The frequency of oscillation of a symmetrical circuit of this type is given by the relation:

$$\frac{1}{f} \approx \frac{2R_{12}C_{12}(R_1 + R_2)}{R_1 + R_{12} + R_2} \log_\epsilon \frac{k}{1-k} \cdot \frac{R_{12}}{R_1 + R_2} \tag{92}$$

Complete design procedure was developed by Suran and Reibert.

A curve of frequency vs. base supply voltage V_{BB} of a transistor multivibrator using *n-p-n* switching transistors is shown in Fig. 181b. With some other types of transistors the curve is of the same form as that of Fig. 181a, the minimum occurring at or near zero base supply voltage. Oscillation may not take place at reverse values of V_{BB}.

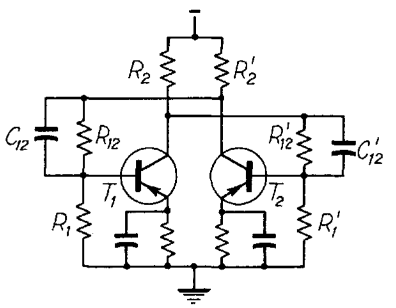


FIG. 182A. Modified transistor multivibrator.

The curve of Fig. 181b is in agreement with an extension of the theoretical analysis of Secs. 52 and 53.1 in which account is taken of the fact that the loop amplification of the transistor circuit may rise above unity at a forward value of base voltage, rather than at a reverse value. For values of V_{BB} to the right of the minimum of the tuning curve the frequency is determined mainly by the time constant of the base circuit of the off transistor; for values to the left of the minimum it is determined mainly by the time constant of the base circuit of the on transistor. Because considerable base current may flow at the value of base voltage at which the loop amplification is unity if this is a forward value, the displacement of the minimum of the tuning curve from the frequency axis may increase appreciably with R_1 and R_1'.

54. Synchronization of Relaxation Oscillators

54.1. Use of Relaxation Oscillators in Frequency Transformation. Relaxation oscillators are of great value in frequency transformation. Introduction into the oscillator circuit of a small voltage of frequency slightly higher than a multiple or submultiple of the oscillation frequency causes the relaxation oscillator to "lock in." This locking action is possible because in

[3] J. J. Suran and F. A. Reibert, *Trans. I.R.E.*, Vol. CT-3, p. 26 (March, 1956).

all relaxation oscillators transition commences at certain critical values of some voltage in the circuit. If a peak of the control voltage comes just before the transition normally begins, the added voltage is enough to trigger the circuit. The control frequency may be a multiple or submultiple of the oscillator frequency because the only peak of locking voltage that affects the oscillator is that which comes just as transition is about to occur. If the frequency of the control voltage is lower than that of the oscillator, every nth cycle of the oscillator will be controlled, where n is the ratio of the oscillator frequency to the control frequency. With reasonable care in

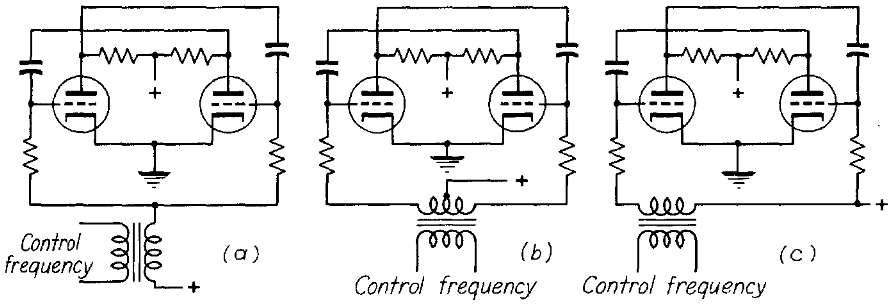


FIG. 183. Three methods of introducing synchronizing voltage into a multivibrator.

circuit adjustment, relaxation oscillators may be controlled when the frequency ratio is as great as 50. The fundamental output of one controlled oscillator may be used to control a second relaxation oscillator, which may in turn control a third, etc. By this means it is possible to obtain an audio-frequency voltage of great stability from the final relaxation oscillator when the first one is stabilized by a crystal-controlled radio-frequency oscillator. The a-f output may be used to drive a synchronous clock.[1] By observation of the clock over a long period of time, a very accurate determination may be made of the frequency of the crystal-controlled oscillator or of any lower frequency controlled by it. By separating and amplifying the various harmonics of the controlled relaxation oscillators, a large number of frequencies of high constancy may be obtained from one crystal oscillator.

The multivibrator has proved to be the most satisfactory type of relaxation oscillator for use in frequency conversion. Its use in this manner has been discussed by a number of investigators.[2] Figure 183 shows three ways in which the control voltage may be introduced into a vacuum-tube multivibrator. Transistor circuits are similar. In circuit a the control frequency

[1] J. W. Horton and W. A. Marrison, *Proc. I.R.E.*, **16**, 137 (1928).

[2] H. Abraham and E. Bloch, *Ann. Physik*, **12**, 237 (1919); M. Mercier, *Compt. rend.*, **174**, 448 (1922); J. K. Clapp, *J. Opt. Soc. Am. and Rev. Sci. Instruments*, **15**, 25 (1927); W. A. Marrison, *Proc. I.R.E.*, **17**, 1103 (1929).

must be an even integral multiple of the multivibrator frequency, in circuit *b* it must be an odd integral multiple, and in circuit *c* it may be any integral multiple. Resistance-capacitance coupling may be used in place of transformer coupling in impressing the control frequency upon the multivibrator circuit. An excellent analysis of the control of multivibrator frequency has been given by Hull and Clapp, and further details by Andrew.[3]

54.2. Mechanism of Synchronization. As pointed out in Secs. 30.2 and 51.2, the onset of transition of a multivibrator is determined by the voltage of the grid (or base) that is reverse biased. For this reason, the action of the stabilized circuits can be analyzed by reference to the negative portion of the wave of grid (or base) voltage. Under equilibrium conditions, each cycle of oscillation of the stabilized circuit must be identical with the previous cycle, and the grid voltage at the beginning of each cycle must be the same as at the beginning of the previous cycle. Furthermore, in a symmetrical circuit, the number of cycles of control voltage per half cycle must be the same in both halves of the cycle. The instantaneous grid voltage must therefore be the same for both tubes at the beginning of their respective half cycles. In the circuit of Fig. 183a, in which the control voltage is applied in like phase to the grids of the two tubes, the requirement is satisfied if the instantaneous value of the control voltage of one tube at the end of the negative half cycle of grid voltage of that tube is the same as at the beginning of that cycle, i.e., if there is an integral number of cycles of control voltage per half cycle of oscillation. (This assumes that the transition time between quasistable states is negligible.)

Figure 184 shows how the frequency of oscillation of the circuit of Fig.

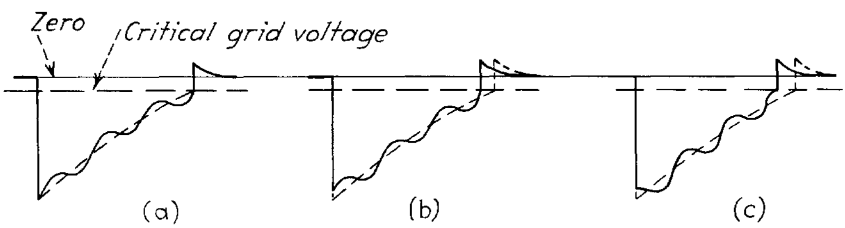


FIG. 184. Control of period of oscillation of the circuit of Fig. 183a. The dotted curves show the grid-voltage wave of one tube in the absence of control voltage. The solid curves show the wave for three values of control frequency.

183a is changed by an increase of control frequency at a frequency ratio of 6. The dotted curve shows the manner in which the negative grid voltage of one tube varies without control voltage. The solid curve shows the varia-

[3] L. M. Hull and J. K. Clapp, *Proc. I.R.E.*, **17**, 252 (1929); V. J. Andrew, *Proc. I.R.E.*, **19**, 1911 (1931).

tion of grid voltage when control voltage is impressed, and the dashed line indicates the critical grid voltage at which triggering occurs. It can be seen that, as the frequency of the control voltage is increased, the beginning of the negative half-cycle shifts in phase relative to the control voltage in such a manner that there are always three cycles of control voltage per half cycle of oscillation. If the frequency is increased beyond that corresponding to curve c in Fig. 184, the grid-voltage curve will not intersect the line of triggering voltage in three cycles, but at some time in the fourth cycle, and the frequency ratio will jump from 6 to 8. As the control frequency is increased, therefore, the oscillation frequency will also increase over a certain range, above which the frequency ratio will jump to a higher value at which control will again be attained throughout a similar frequency range. Further analysis of Fig. 184 shows that the range of control frequency throughout which the oscillator "locks in" increases with amplitude of control voltage. It can be seen from Fig. 184 that there must always be a whole number of cycles of control voltage per half cycle of oscillation, and so the frequency ratio for the circuit of Fig. 183a must be an even integer.

In the circuit of Fig. 183b, the control voltage is applied to the two grids (or bases) in phase opposition, and so the control voltage at the end of the negative half cycle must be of equal magnitude but opposite polarity to that at the beginning of the cycle in order to make the grid voltage the same in both tubes at the beginning of their respective negative half cycles. There must be $(n - \frac{1}{2})$ cycles of control voltage per half cycle of oscillation, and hence the frequency ratio will be $2n - 1$, where n is any positive integer. The frequency ratio is therefore odd. In the circuit of Fig. 183c the control voltage is applied to only one tube. The voltage at the beginning of the negative half cycle of grid voltage of that tube must be the same in each succeeding cycle, which is true for any integral frequency ratio.

In a multivibrator based upon the circuit of Fig. 99b, the control frequency may be applied to one or both control grids. In the van der Pol oscillator of Fig. 167a, the control frequency is best applied to the suppressor grid.[4] In the circuit of Fig. 167b, the control frequency may be applied to any electrode, preferably the control grid or the suppressor grid. In the transistor circuits of Figs. 170 and 171, the control voltage may also be introduced in series with any of the electrodes.

54.3. Synchronization by Periodic Pulses. It can be seen from Fig. 184 that variations in the amplitude of the sinusoidal control voltage may not only cause changes in the ratio of the oscillation frequency to the control frequency, but may also cause random variatons in the period of oscillation. It was pointed out in Sec. 52.6 that the same effect is produced by random noise. The resulting random variation of frequency is called

[4] Page and Curtis, *loc. cit.*

frequency jitter. Changes in the ratio of oscillation frequency to control frequency may be avoided and frequency jitter minimized by the use of sharp periodic pulses of constant amplitude, instead of sinusoidal voltage, as the control voltage. If the primary source of control voltage is a sine-wave generator, the desired periodic pulses may be obtained by clipping and differentiating the sinusoidal voltage in the manner explained in Sec. 7 (Fig. 29, for example) and, if necessary, removing positive or negative pulses by means of a diode clipper. Then, if the control frequency exceeds the free-running frequency of the relaxation oscillator, the oscillator is triggered at the beginning of each cycle or half cycle of the control voltage, regardless of the amplitude and frequency of the sinusoidal control voltage.

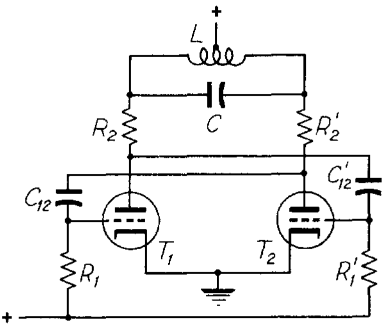


FIG. 185. Multivibrator in which the frequency is controlled by a resonant circuit.

54.4. Control by Resonant Circuit. The control frequency of a multivibrator need not be supplied by an external source, but may be generated by a high-Q resonant circuit connected in series with the plate resistor of one or both tubes, as shown in Fig. 185, which is based upon the circuit of Fig. 174. The resonant circuit, which is shock-excited by the plate-current pulses, introduces a sinusoidal component into the grid voltages. If the resonance frequency of the resonant circuit exceeds the free-running frequency of the multivibrator, the oscillation frequency is that of the resonator, or a submultiple thereof. Further stabilization may be obtained by applying the oscillator voltage to a frequency discriminator, the direct-voltage output of which is impressed in series with the grid supply voltage in the proper polarity to produce parial compensation of frequency variations, as explained in Sec. 53.3.

Frequency control and stabilization of a multivibrator may also be achieved by the use of the pulse-sharpening circuit of Fig. 33a in place of the plate-circuit resistors R_2 and R_2'.[5]

55. Monostable Circuits

55.1. Theory of Operation. Although the term "monostable" should logically apply also to circuits that have only one state of equilibrium, it is usually applied only to circuits that have one stable state and one quasistable state. Application of a triggering voltage or current to a monostable circuit initiates transition from the normal stable state to the quasistable

[5] A. E. Johnson, *Bell Lab. Record,* **28,** 208 (May, 1950).

state. After a time interval that depends upon the circuit parameters, the circuit returns to its stable state, where it remains until it is again triggered.

From the point of view of negative resistance, a monostable circuit differs from an astable circuit only in that the load line intersects the current-voltage characteristic in one of the positive-slope sections, rather than in the negative-slope section. A typical current-voltage diagram and path of operation for a current-stable circuit triggered by an exponential voltage pulse of the form $Ve^{-\alpha t}$ is shown in Fig. 186. The circuit is normally in the

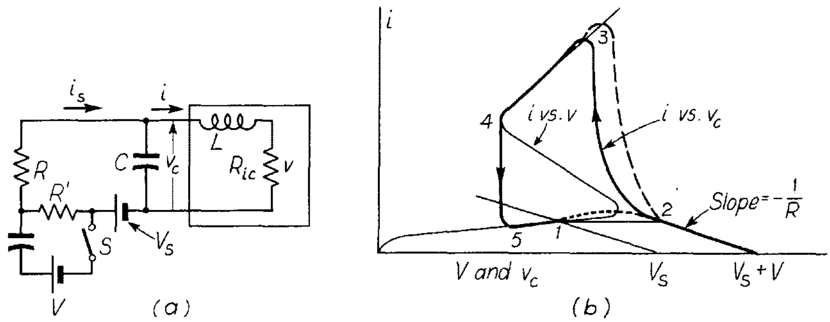


FIG. 186. (a) Equivalent circuit for a monostable circuit using a current-stable negative-resistance element. (b) Current-voltage diagram for this circuit obtained by means of an analog computer.

stable state of equilibrium corresponding to point 1. Closing of the switch S initiates the triggering pulse across the small resistance R' and thus moves the operating point abruptly to point 2, from which rapid transition takes place to the high-current branch of the i-v curve. Transition is followed by exponential decrease of voltage and current, with the approximate time constant RC, to point 4, from which rapid transition occurs to point 5. The voltage then rises exponentially, with the approximate time constant RC, to the value at the stable point 1.

The dashed curve in Fig. 186, obtained with a triggering pulse of considerably longer time constant, shows that increase of triggering-pulse duration moves point 3 to higher values of current and voltage and thus tends to lengthen the time in which the circuit remains in the quasistable state. The circuit is affected similarly, but to a lesser degree, by increase of pulse amplitude. If the pulse amplitude or duration (time constant) is too small, transition is not completed and the operating point returns to point 1, as shown by the dotted line in Fig. 186. There is, however, a range of pulse amplitude and duration over which transition is assured and for which the time in which the circuit remains in the quasistable state is essentially independent of the triggering-pulse characteristics.

The path of operation in the current-voltage diagram is similar when the stable point is in the high-current section of the current-voltage characteristic.

Figure 187 shows a typical diagram for a voltage-stable circuit triggered by a short exponential current pulse.

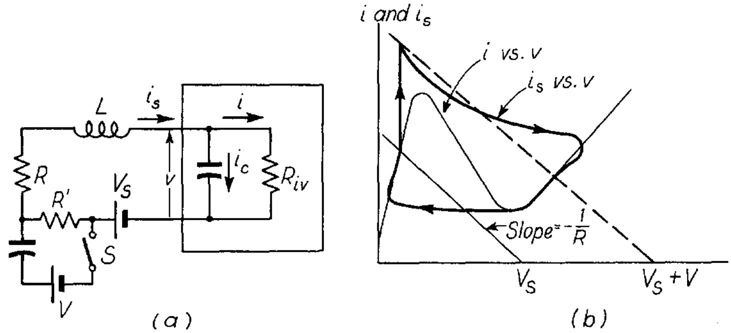


FIG. 187. Current-voltage diagram for a monostable circuit using a voltage-stable negative-resistance element.

55.2. Conversion of Astable Circuits Into Monostable Circuits. It follows from the foregoing discussion that any astable circuit (relaxation oscillator) can be converted into a monostable circuit merely by changing one or more supply voltages in order to bring the intersection of the load line with the current-voltage characteristic into one of the positive-slope ranges. However, the most commonly used monostable circuits are based upon the negative-resistance circuits of Figs. 68b or 70 or upon the equivalent transistor circuits of Figs. 74b or 74c. The basic vacuum-tube monostable circuits of these types are shown in Fig. 188. The basic transistor circuits are similar in form. Circuit (a), which is basically an asymmetrical multivibrator, is often called a "one-shot multivibrator." This name is also frequently applied to circuit (b) which is a modification of the astable circuit of Figs. 169 and 171. It is sometimes applied to monostable circuits formed by modifying single-tube or single-transistor astable circuits.

55.3. Analysis of One-Shot Multivibrator. The operation of the circuits of Fig. 188 is in some respects similar to that of the multivibrator of Fig. 174. The circuit values are chosen so that the circuit has more than one state of equilibrium, T_1 conducting in one state and T_2 in another. Under standby conditions, the positive grid supply voltage of T_1 maintains T_1 in the conducting state and T_2 is cut off. Application of a positive triggering pulse to the grid of T_2 or of a negative pulse to the plate of T_2 or the grid of T_1 initiates transition. The sudden drop in voltage of the plate of T_2 during transition is accompanied by a practically equal drop in voltage of the grid of T_1 that biases T_1 beyond cutoff. The capacitor C_{12} immediately starts to discharge and the voltage of the grid of T_1 rises exponentially until it reaches the cutoff voltage and reverse transition takes place. The abrupt rise in plate voltage of T_2 causes C_{12} to recharge rapidly through R_2' and the grid of T_1. Normally the resistance R_1 is large in comparison with the d-c grid resistance of T_1, and the standby value of grid voltage of T_1 relative to the cathode of T_1 is close to zero. The plate voltage of T_1 is therefore normally low and rises to approximately the supply voltage when T_1 is cut off (if $R_1' + R_{12}' \gg R_2$). The output voltage is a positive pulse of duration equal to the time during which T_1 is off and T_2 on.

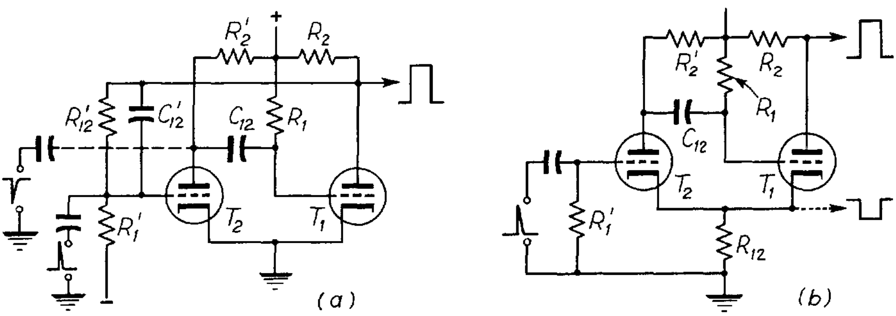


FIG. 188. Monostable circuits based upon the negative-resistance circuits of (a) Fig. 68b and (b) Fig. 70.

The diagram of Fig. 179 applies to the rise of grid voltage of T_1. Under the assumption that R_1 is large in comparison with the resistance of R_2' in parallel with the plate resistance of T_2, the following approximate expression for the time during which T_1 remains cut off may be found by solving Eq. (87) for t_o:

$$t_o = R_1 C_{12} \log_\epsilon \frac{m + k}{m + 1/\mu} \tag{93}$$

in which m is the ratio of the grid supply voltage of T_1 to the plate supply voltage (unity when R_1 is connected to the positive terminal of the plate supply voltage, as in Fig. 188), and k is the ratio of the negative swing of the grid voltage of T_1 to the plate supply voltage. An approximate value for k may be found from the plate diagram for T_2, the approximate load resistance R_2' being used to determine the slope of the load line when $R_1 \gg R_2'$.

Equation (93) shows that the length of the output pulse may be changed by means of R_1, C_{12}, or the ratio of the grid supply voltage of T_1 to the plate supply voltage. In most applications of the circuits of Fig. 188 large changes of pulse length are made by changing C_{12} and continuous adjustment is made by varying R_1. The extent to which C_{12} can be reduced in the production of very short pulses is limited by the input capacitance of T_1, which tends to prevent transition. Furthermore, if C_{12} is made too small, operation in the negative-resistance portion of the current-voltage characteristic may change from the growing-exponential region of the diagram of Fig. 159 to a decaying-exponential region. The circuit then becomes completely stable. The extent to which R_1 can be reduced is limited by the allowable value of standby grid current of T_1 and by the reduction of k when R_1 approaches R_2' (see Sec. 52.3). As R_1 is reduced, a value of k is reached at which the negative grid-voltage swing of T_1 is insufficient to cut T_1 off. As this value is approached, the response of the circuit to a triggering pulse becomes erratic. Another factor that may determine the minimum allowable value of R_1 is the change from monostable to bistable operation if the slope of the load line becomes greater than the slope of the current-voltage characteristic in the negative-resistance range.

The maximum attainable pulse length is limited largely by capacitor leakage, which reduces the effective value of R_1. Except at low values of plate supply voltage, the extent to which R_1 may be increased may also be limited by the maximum grid-circuit resistance of T_1 that can be used without danger of cumulative increase of grid current. This value is specified by the tube manufacturer. The minimum time interval between pulses, and therefore the maximum pulse-repetition frequency, is limited by the time required for C_{12} to recharge following termination of the pulse. Short settling time is favored by small C_{12} and R_2'.

An analysis similar to that of Sec. 53.3 shows that if the circuit values are properly chosen, the length of the pulse generated by the circuits of Fig. 188 may be made to vary very nearly linearly with grid supply voltage of T_2.[1] The pulse length may be conveniently varied by means of a voltage divider shunted across the plate power supply.

The analysis of Sec. 52, which also applies to the circuits of Fig. 188, shows that stability of pulse length against changes of supply voltage is favored by making m large. For this reason R_1 is usually connected to the positive terminal of the plate supply voltage unless the pulse length is varied by means of the grid supply voltage.

The time required for conduction to switch from T_1 to T_2 can be reduced by applying the triggering pulse to the circuit through a diode. Cutting off of the diode disconnects the source of triggering voltage from the circuit near the beginning of transition, and thus allows transition to proceed freely (see Sec. 29.2). As in the Eccles-Jordan bistable circuit, less trig-

[1] K. Glegg, *Proc. I.R.E.*, **38**, 655 (1950). Note that $R_2/R_1 + R_2$ should have been written $R_2/(R_1 + R_2)$ in this article.

gering voltage is required if a negative triggering pulse is used than if a positive pulse is used (see Sec. 28.2).

55.4. Causes of Nonrectangular Pulse Form. As the conduction time of T_2 is reduced, the ratio of the transition times to the conduction time of T_2 becomes larger. When the conduction time is of the same order of magnitude as the transition times, the rise and fall times of the output pulse are an appreciable fraction of the pulse duration and the pulse is no longer approximately rectangular, but is of the general form of the positive pulses of Fig. 14c. As in bistable circuits (Secs. 30 and 31), the transition times can be minimized by the use of tubes having high transconductance and low interelectrode capacitances, by adding diodes to limit plate-voltage swings, and by shunting R_{12}' by a coupling capacitor C_{12}' having a capacitance of the order of 25 to 200$\mu\mu$f. Too large a value of C_{12}' may increase the rounding of the leading edge of the pulse and decrease the maximum repetition frequency, however. Transition in response to the triggering pulse can also be speeded up by applying the triggering pulse through a diode that becomes reverse biased and disconnects the triggering circuit as soon as transition starts. Such a circuit is shown in Fig. 189. The diode may serve the additional function of eliminating any positive input pulse that might cause termination of the output pulse before it is terminated by the timing circuit.

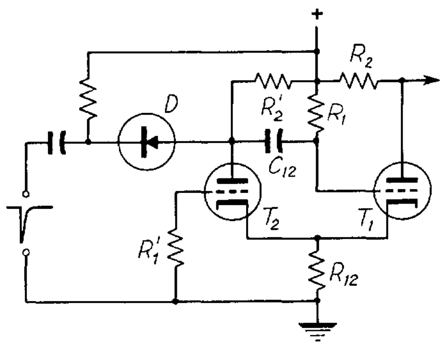


FIG. 189. One-shot multivibrator in which a diode disconnects the source of triggering voltage after initiation of transition.

Reduction in the rise time may be achieved in the circuits of Fig. 188 by the use of a highly damped resonant circuit in place of R_1.[2] If the grid supply voltage of T_1 is zero, the first half cycle of the transient oscillation set up in the tuned circuit by transition initiated by the triggering pulse drives the grid of T_1 beyond cutoff, but the grid-cathode path of T_1 short-circuits the resonant circuit during the second half cycle and thus dissipates the energy stored in the tuned circuit, eliminating the remainder of the transient. The length of the output pulse is equal to half the period of oscillation of the tuned circuit and may be varied by means of the capacitance of the tuned circuit. A disadvantage of this circuit is that the output

[2] F. A. Benson, *Wireless Engineer*, **29**, 12 (June, 1952).

waveform is critically dependent upon the values of the inductance and C_{12}. A 250-μh choke, shunted by a 650-ohm resistance was found to be suitable, and provided a rise time of a half microsecond.

Nonrectangular pulse shape is also caused by variation of plate voltage following transition. In the circuit of Fig. 188b, and in that of Fig. 188a when C_{12}' is small, the principal cause of variation of plate voltage of T_1 is the positive swing of the grid voltage of T_1 following the termination of conduction of T_2 and the subsequent decay of grid voltage to approximately zero. The principal cause of variation of plate voltage of T_2, on the other

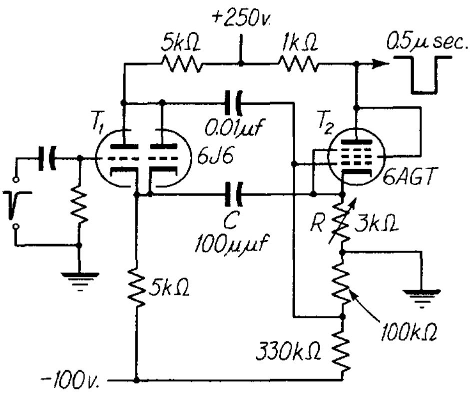


FIG. 190. Monostable circuit capable of generating pulses as short as 0.1 μsec.

hand, is the flow of charging current of C_{12} through R_2' following the conduction of T_2. Because the effect of positive swing of grid voltage of T_1 can be made small by the use of a large value of R_2, particularly with pentodes, whereas the effect of charging current upon the plate voltage of T_2 is difficult to avoid, better waveform can usually be obtained if the output is taken from the plate of T_1. An additional advantage of taking the output from T_1, rather than from T_2, is that variation of resistance or capacitance of load shunted across the output terminals has negligible effect upon the plate-voltage swing of T_2 and hence upon the grid swing of T_1 and the pulse length. When close approach to rectangular form is essential, therefore, a negative-going pulse is best obtained by the use of a phase-inverting tube to invert the pulse derived from the plate of T_1, rather than by using the plate of T_2 as the output terminal. Clipping may also be used to improve the output waveform. In the circuit of Fig. 188b, a low-amplitude negative-going pulse of good form can be obtained from the cathodes of T_1 and T_2, as shown by the dotted line.

Another cause of variation of output voltage during or following the pulse is capacitance of the external load shunted across the output terminals.

The effect of load capacitance can be best minimized by the use of a cathode-follower stage between the load and the output of the monostable circuit. This is particularly desirable when pulse lengths of the order of a microsecond or shorter are desired. The cathode-follower stage serves the additional function of preventing the load capacitance or resistance from affecting the pulse length.

When the output pulses are not required to be approximately rectangular, pulses as short as a fraction of a microsecond are attainable by the use of monostable circuits of the general form of those of Fig. 188. Figure 190

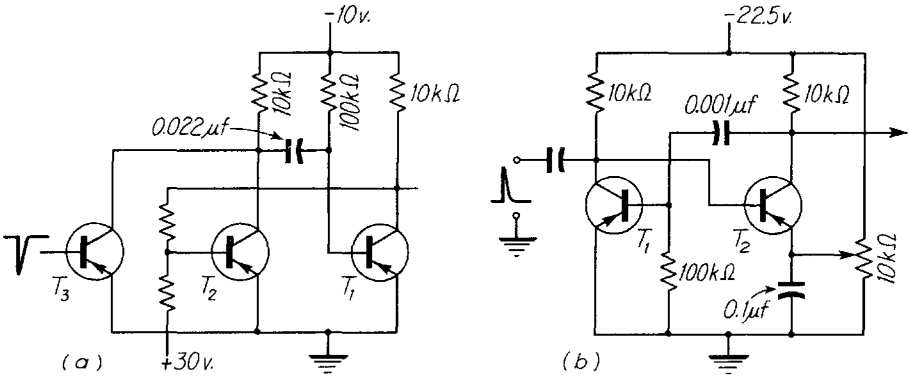


FIG. 191. Transistor one-shot multivibrators.

shows a circuit by means of which pulse lengths as small as 0.1 μsec and practically independent of load resistance can be attained.[3] This circuit is based upon the negative-resistance circuit of Fig. 68e (or astable circuit of Fig. 178).

55.5. Transistor One-Shot Multivibrators. Figure 191 shows two transistor circuits analogous to the vacuum-tube circuit of Fig. 188a. The main difference between circuit (a) [4] and circuit (b) [5] lies in the method of providing the bias for the base of T_2. The direct coupling from the collector of T_1 to the base of T_2 in circuit (b) is favorable to rapid transition. The use of an additional transistor in the application of the triggering pulse is also shown in circuit (a). Circuit (b) delivers a 250-μsec pulse with a 2-μsec rise time at a maximum repetition frequency of 1000 pulses per sec.

When surface-barrier and some alloy-junction transistors are used, considerable circuit simplification is made possible by the fact that these transistors cut off when the base is still forward-biased relative to the emitter.

[3] Rad. Lab. Series, Vol. 19, Fig. 5.27 (McGraw-Hill Book Co., Inc., New York, 1949).
[4] T. A. Prugh, *Electronics*, January, 1955, p. 168.
[5] P. G. Sulzer, *Electronics*, August, 1953, p. 170.

The base of transistor T_2 may then be connected directly to the collector of T_1 without the use of an emitter resistor to raise the emitter potential of T_2 above ground, as shown in Fig. 192.[6] For some values of R_1 this circuit is astable, rather than monostable.

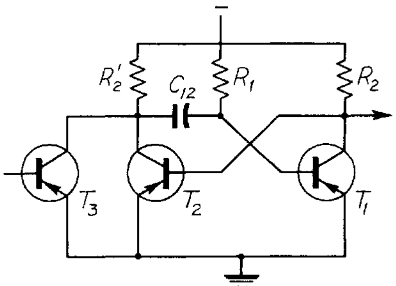


FIG. 192. Direct-coupled transistor one-shot multivibrator in which both emitters are at ground potential.

55.6. Nonsaturating Transistor Circuit. Figure 193 shows a more complicated transistor version of the circuit of Fig. 188a in which diodes are used to prevent the transistors from being driven into the saturation range of collector current (below the knee of the collector characteristic).[7] The resulting reduction of minority-carrier storage reduces transition time and thus makes possible the generation of pulses of shorter rise and fall times and shorter duration. The diode D_1 prevents the collector voltage of T_1 from falling to zero, whereas diode D_2 prevents a sufficient rise of the collector voltage of T_1 to drive T_2 into the saturation range. (Diodes D_1 and D_2 may be breakdown diodes if they are reversed in polarity, the

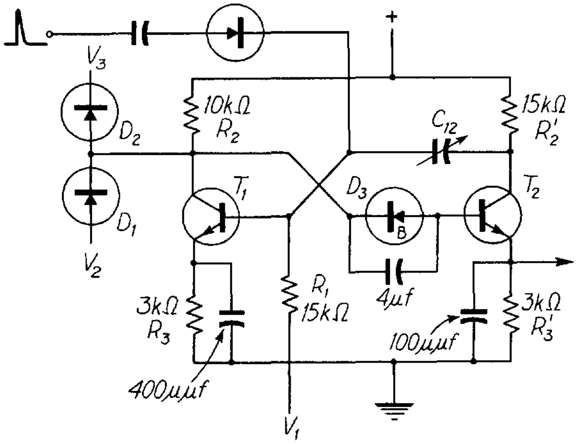


FIG. 193. Transistor one-shot multivibrator in which diodes are used to prevent transistor saturation.

cathode of D_1 being connected to the positive terminal of the voltage supply and the anode of D_2 to ground.) The breakdown diode D_3 is used to

[6] R. H. Beter, W. E. Bradley, and M. Rubinoff, *Electronics*, June, 1955, p. 132.
[7] J. G. Linvill, *Proc. I.R.E.*, **43**, 826 (July, 1955).

couple the base of T_2 directly to the collector of T_1. In combination with D_1 and D_2 it also ensures that the maximum and minimum voltages of the output wave are independent of the transistor characteristics.

When the voltage V_1 in the circuit of Fig. 193 is zero, or close to zero, T_2 normally conducts, and T_1 is cut off. Diode D_2 also conducts. Since the emitter-to-base voltage of T_2 is very small, the output voltage V_o approximates $V_3 - V_{d3}$, where V_{d3} is the breakdown voltage of diode D_3. A positive triggering pulse initiates transition, which leaves T_1 conducting and T_2 cut off. The reduction of collector voltage of T_1 cuts D_2 off and causes D_1 to conduct. The output voltage then approximates $V_2 - V_{d3}$. Transition is followed by charging of C_{12} through R_2' and through R_1 and the base of T_1. When the base voltage of T_1 has fallen to the value at which D_1 becomes reverse-biased, D_1 ceases to conduct, the collector voltage of T_1 starts to rise, and the circuit switches back to its initial state of equilibrium.

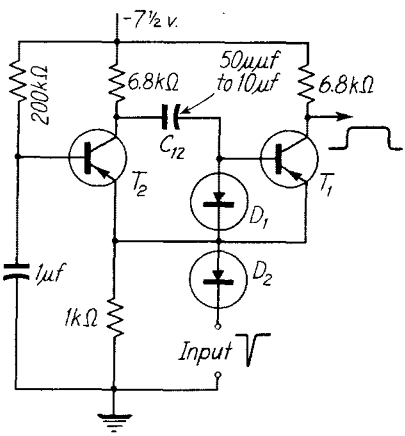


FIG. 194. Transistor circuit analogous to the tube circuit of Fig. 188b.

The circuit of Fig. 193 can provide a minimum pulse length of approximately 1 μsec with a ¼-μsec rise time. The maximum repetition frequency is about 10^6 pulses per sec. When the voltage V_1 is raised sufficiently to open D_2 the circuit becomes astable, delivering an output voltage of swing equal to $V_3 - V_2$. Both the monostable and the astable circuits deliver an essentially rectangular output voltage.

A transistor circuit analogous to the tube circuit of Fig. 188b is shown in Fig. 194.[8] Use of the diode D_1 in place of a resistor R_1 ensures rapid charging of C_{12} following termination of the output pulse. The rise time of the output pulse is 2 μsec and the pulse length is adjustable from 5 μsec to several seconds by variation of C_{12}.

55.7. High-Current Transistor Circuit. The analysis of Sec. 32 shows that rapid transition of transistor monostable circuits necessitates the use of transistors having high values of h_{fb} and f_{ab}. At the present time, however, such transistors do not have high current ratings and therefore are incapable of providing high-current pulses. Figure 195 shows a circuit that avoids this difficulty by the use of one low-power *n-p-n* transistor having

[8] F. C. Alexander, *Electronics*, December, 1954, p. 188.

high values of h_{fb} and f_{ab} and two medium-power *p-n-p* transistors having a relatively low value of f_{ab}.[9] The circuit is based upon the negative-resistance circuit of Fig. 71. Transistor T_1, which is normally cut off, is rapidly driven into saturation by a positive triggering pulse. The resulting drop in collector voltage drives T_2 into saturation, conduction in T_1 being then maintained by the rise in potential of the collector of T_2. This transition is followed by charging of C_2 and therefore by falling of base potential of T_3, which is driven into saturation. Since the collector-emitter resistance

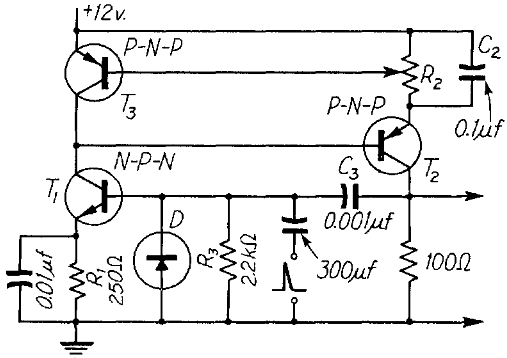


FIG. 195. Monostable transistor circuit capable of providing high-current output pulses.

of T_3 is only about 50 ohms in the saturation range, the base of T_2 is connected to the emitter through a low resistance. Consequently, T_2 is rapidly cut off and causes T_1 to be cut off. The coupling capacitor C_3 then discharges through the diode.

The pulse length is adjusted by means of the arm of R_2. The purpose of R_3 is to improve the temperature stability of the circuit. This circuit provides a ½-volt output pulse of 2.5-μsec minimum length at repetition frequencies up to 100 kc. The rise time is 0.7 μsec at the upper limit of repetition frequency and falls to 0.5 μsec as the frequency is reduced. The fall time is 0.6 μsec for a 2.5-μsec pulse and 1 μsec for a 5-μsec pulse. A 3-ma triggering pulse is required.

55.8. Monostable Unijunction-Transistor Circuit. Very simple monostable circuits are made possible by the use of double-base diodes. Figure 196 shows a circuit that provides pulses ranging in length from 50 μsec to 2000 μsec at repetition frequencies up to 5 kc.[10] The pulse length varies

[9] F. Rozner, *Electronic and Rad. Eng.*, **34**, 8 (January, 1957).
[10] J. J. Suran and E. Keonjian, *Proc. I.R.E.*, **43**, 814 (July, 1955).

linearly with the capacitance C. Other monostable circuits may be formed from the basic negative-resistance circuit of Fig. 75.[11]

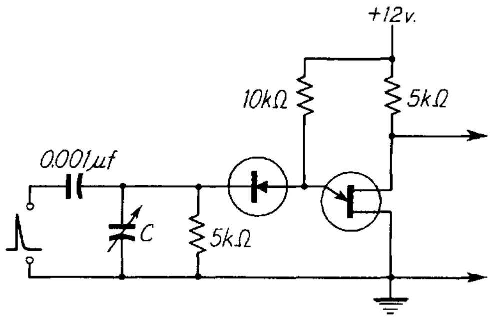


FIG. 196. Monostable unijunction-transistor circuit.

55.9. Use of Monostable Circuits as Delay Circuits. A very useful application of monostable circuits is in the production of delayed pulses. If the output of a monostable circuit is applied to an R-C differentiating circuit of the form discussed in Sec. 3, the output of the differentiator consists of a short pulse initiated by the leading edge of the pulse generated by the monostable circuit and a second short pulse of opposite polarity initiated by the falling edge. The delay between the second pulse and the triggering pulse applied to the monostable circuit can be adjusted by changing the length of the pulse generated by the monostable circuit. The first pulse, which is usually only slightly delayed relative to the triggering pulse, may be eliminated from the output of the differentiator by a diode clipper. The delayed pulse can in turn be used to trigger a second monostable circuit, which then generates a pulse of adjustable length delayed relative to the initial triggering pulse by an adjustable interval (Prob. 55.9–1).

56. Precision Pulse Generators

56.1. Use of Miller Integrator for Timing. The Miller integrator circuit of Fig. 20a, because it provides a timing voltage that varies essentially linearly with time, makes possible more accurate timing of pulse length over a wide range than does a simple R-C circuit. The requirement of high voltage amplification necessitates that the capacitor in a pentode Miller circuit be connected between the plate and the control grid, rather than between the plate and the screen or the suppressor. For this reason the control grid cannot be used for feedback purposes in the negative-resistance circuit from which the pulse generator is derived. Negative-resistance cir-

[11] J. J. Suran, *Electronics,* March, 1955, p. 198.

cuits in which the coupling is from screen to suppressor, as in Fig. 66a, from plates to screens, as in Fig. 99a, or from plates to suppressors, as in Fig. 99b, are suitable for this purpose, however. Only the phantastron pulse generator, based upon the single-pentode trigger circuit of Fig. 66a, will be discussed.[1]

56.2. The Phantastron. Comparison of the basic *phantastron* circuit of Fig. 197 with the bistable circuit of Fig. 93a shows that the former may be formed from the latter by coupling the grid to the plate through the capacitor C and adding a coupling capacitor in parallel with R_{12} to increase

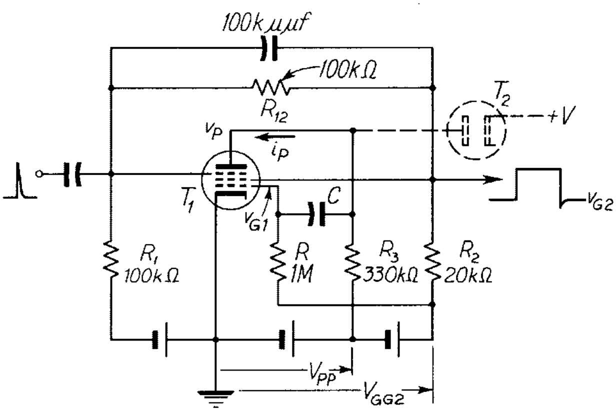


FIG. 197. Phantastron circuit based upon the bistable circuit of Fig. 93a.

speed of transition. The flow of grid current through the resistor R, the resistance of which is of the order of a megohm, prevents the control-grid voltage from rising appreciably above zero. At zero control-grid voltage the screen current has its higher equilibrium value and the plate current has its lower value, zero. Application of a positive triggering pulse to the suppressor or the screen or of a negative pulse to the control grid causes the circuit to switch to its other equilibrium state, in which plate current flows and the screen current has its lower value. The manner in which the plate current and the control-grid, plate, and screen voltages vary following transition is illustrated in Fig. 198.

The magnitude of the change of plate and control-grid voltage during transition is reduced by the degenerative coupling of the plate to the grid, the abrupt decrease of plate voltage being accompanied by an equal reduction of control-grid voltage that tends to prevent the increase of plate current. The grid voltage v_{G1} therefore drops to a negative value of magni-

[1] For a treatment of other pulse generators incorporating the Miller integrator, see Rad. Lab. Series, Vol. 19, Secs. 5.16 to 5.18 (McGraw-Hill Book Co., Inc., 1949).

tude V' that is of the order of only 5 or 6 volts. The capacitor C immediately starts to charge and the grid voltage to rise from an initial value $-V'$ toward a positive value V_{GG2}. Inasmuch as the tube in conjunction with R, R_3, and C constitutes a Miller integrator circuit, the effective time constant is $RC(|A| + 1)$, where $|A|$ is the magnitude of the voltage amplification of the circuit between the control grid and the plate, and the grid voltage rises nearly linearly at its initial rate $(V_{GG2} + V')/RC(|A| + 1)$.

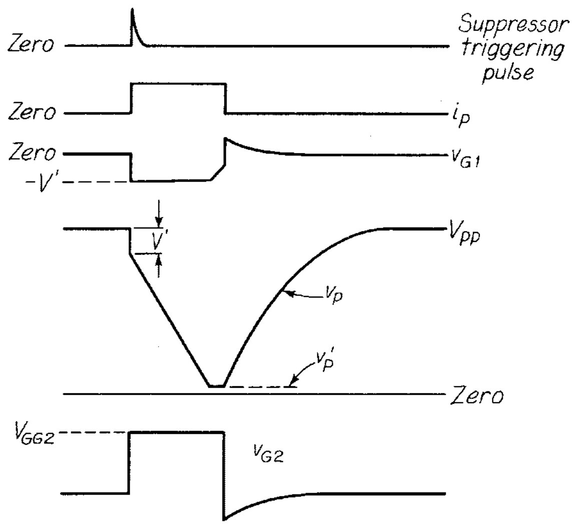


FIG. 198. Current and voltage waveforms for the circuit of Fig. 197.

As the grid voltage rises, the plate voltage falls approximately linearly at the much higher rate $|A|(V_{GG2} + V')/RC(|A| + 1)$ until it reaches the limiting value v_P' equal to the supply voltage V_{PP} minus the product of R_3 and the upper equilibrium value of plate current. The Miller effect then stops and the time constant of the control-grid circuit changes from $RC(|A| + 1)$ to RC. Thereafter the control-grid voltage rises rapidly to the value at which the circuit switches back to its initial state of equilibrium. The rise in plate voltage resulting from the interruption of plate current swings the control-grid voltage to a positive value. As the capacitor C then discharges through R_3 and the grid-cathode path, the grid voltage falls exponentially and the plate voltage rises exponentially toward its standby value V_{PP}. The grid resistance being low, the charging time constant approximates R_3C.

The positive grid-voltage swing and subsequent exponential decay causes the screen voltage to fall below its standby value and then to rise exponentially. The negative exponential portion of the screen-voltage wave may be removed by means of a diode having its cathode connected to the screen and its anode to a point on the voltage supply corresponding to the standby voltage of the screen. The length of the rectangular pulse of screen voltage is nearly equal to the time taken by the plate voltage to fall from its initial value $V_{PP} - V'$ to its minimum value v_P'. The approximate pulse length is therefore $(V_{PP} - V' - v_p')RC/(V_{GG2} + V')$. Since v_P' is nearly independent of V_{PP} and approximates only a volt or two if R_3 is of the order of a quarter of a megohm or greater, the pulse length varies nearly linearly with V_{PP}.

The initial value of plate voltage, and therefore the length of the pulse, can be conveniently controlled by means of a reverse-biased diode T_2 connected to the plate of T_1, as shown by the dotted lines of Fig. 197. Diode current limits the maximum pentode plate voltage to the biasing voltage V. The pulse length is therefore $(V - v_P')RC/(V_{GG2} + V')$, and linear pulse-length modulation can be produced by varying V. Limitation of the plate-current swing by the diode serves the additional function of reducing the transition time. The transition time can also be reduced by applying the triggering pulse through a diode, as in the bistable circuit of Fig. 102.

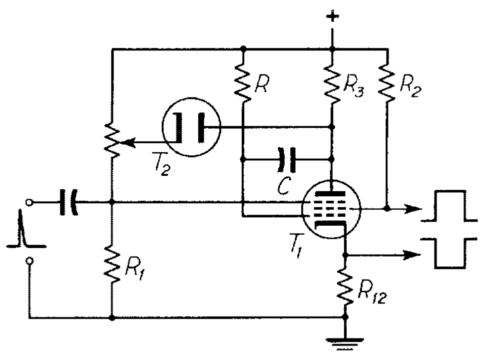


FIG. 199. Phantastron circuit based upon the bistable circuit of Fig. 93b.

56.3. Modified Phantastron. Figure 199 shows a modification of the phantastron based upon the pentode bistable circuit of Fig. 93b, in which the screen is coupled to the suppressor by means of a cathode resistor, instead of a voltage-dividing network. The advantages of this circuit are that the screen (output) circuit is not loaded by the screen-suppressor coupling network, that both positive and negative output voltages may be obtained from the circuit, and that it does not require a negative supply voltage. Because of the lower amplification of the tube as the result of de-generation, however, the approximation to linear variation of grid and plate voltages is not so close and the pulse-length timing accuracy is therefore not so good as in the circuit of Fig. 197.

57. Sawtooth-Voltage Generators

57.1 Types of Sawtooth-Voltage Generators. Sawtooth-voltage generators find their principal applications as timing devices and as sweep generators for cathode-ray oscilloscopes, radar displays, and television receivers. They are of two basic types: astable or monostable circuits, and circuits in which a capacitor is charged relatively slowly and approximately linearly by means of an approximately constant current and discharged relatively rapidly by means of a short-circuiting tube. The astable and monostable circuits are basically of the form of the circuit of Fig. 157b and make use of negative-resistance elements having a current-stable characteristic of the general form of those of Figs. 75, 79, and 80. The important features of these characteristics in this application are very high resistance in the negative-resistance range, and very low resistance in the high-current range. For convenience of reference, Fig. 157b is repeated as Fig. 200a.

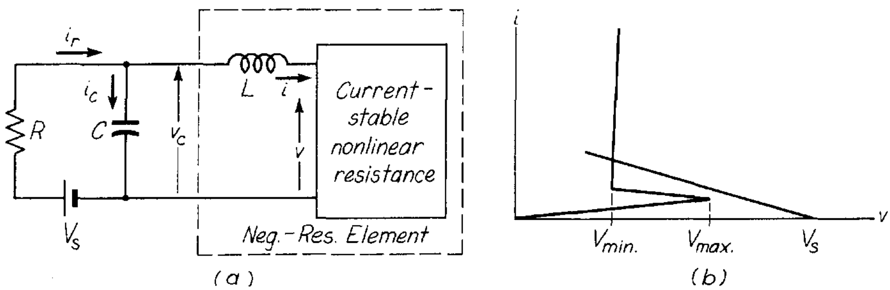


FIG. 200. (a) Equivalent circuit for astable or monostable circuits based upon a current-stable negative-resistance element; (b) current-voltage characteristic suitable for sawtooth-voltage generation.

57.2. Analysis of Astable Sawtooth-Voltage Generators. Figure 200b shows an idealized current-stable current-voltage characteristic suitable for sawtooth-wave generation, together with a load line that intersects the characteristic in the high-current range. In the low-current range the resistance R_{ic} of such a characteristic is usually so high that operation of the circuit of Fig. 200a is in the upper decaying-exponential range of the diagram of Fig. 159. When voltage is first applied to the circuit, the voltage across the capacitor therefore rises exponentially with a time constant approximating $R'C$, where R' is the resistance of R in parallel with R_{iL}, the value of R_{ic} in the low-current range (see Sec. 47.1). Usually $R \gg R_{iL}$ and the time constant is very nearly RC.

When the voltage across the capacitor reaches the value $V_{\max}$, the current i enters the negative-resistance range of the characteristic and the currents and voltages become growing exponentials. The rapid drop in the voltage v across the current-stable element with increase of current i is balanced by voltage Ldi/dt induced in the inductance. As shown in Sec. 47.1, the time constant of the growing exponentials in this range approximates L/R_{iN}, where R_{iN} is the value of R_{ic} in the negative-resistance range. The ratio of this time constant to the approximate time constant RC of the low-current range is $L/R_{ic}RC$. Typical values of L, R_{iN}, R, and C are 10^{-4} henry, -10^5 ohms, 10^5 ohms, and 10^{-8} farad, respectively. With these values the magnitude of the ratio is 10^{-6}. Because of the low value of this ratio and the small current range covered by the negative-resistance portion of the characteristic, the current passes through this range in a time very short in comparison with that in which it passed through the low-current range. The effect of the transition from the low-current range to the high-current range is therefore equivalent to a sudden reduction of the voltage v across the current-stable resistive element of Fig. 200a from $V_{\max}$ to $V_{\min}$, and an abrupt reduction of the resistance R_{ic} from the high value in the low-current range to the low value in the high-current range. The subsequent behavior of the circuit is similar to that of the equivalent circuit of Fig. 201 if the switch S in the latter circuit is moved from the left position to the right position at the instant the capacitor voltage reaches the value $V_{\max}$. The resistance R_{iH} in Fig. 201 is the resistance of the current-stable element in the high-current range.

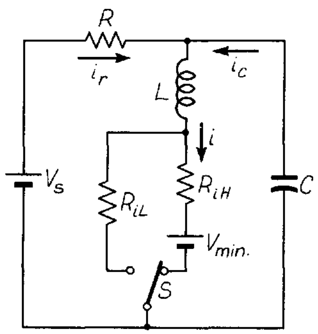


FIG. 201. Approximate equivalent circuit applicable following transition to the high-current range.

The resistance R_{iH} is so low and the resistance R so high that the currents and voltages in the circuit of Fig. 201 following switching are oscillatory with small damping. Because of the large value of R, the current i_r through the resistance R is small in comparison with the capacitor current i_c and the capacitor current and voltage waveforms approximate those observed when a charged capacitor is discharged through an inductance in series with a low resistance. Typical waveforms during the first two periods are shown by curves (a) and (b) of Fig. 202. The variation of capacitor voltage is accompanied by an equal variation of voltage across R and therefore of current i_r, as shown by curve (c) of Fig. 202. The current i through the current-stable element, which is the sum of the currents i_c and i_r, is shown in curve (d). In the actual circuit of Fig. 200a the current i does not swing to negative values. When this current reaches the value corresponding to the upper end of the negative-resistance range of the characteristic, the currents and voltages again become growing exponentials and the current falls through this range in a very short interval, at the termination of which the voltage v is equal to V_{max}, the voltage across the capacitor is approximately $2V_{min} - V_{max}$, and the resistance R_{ic} is again the high value R_{iL} of the low-current range. The subsequent behavior of the circuit is similar to that of the equivalent circuit, Fig. 201, if the switch S in the latter is moved to the left position when the current i re-enters the negative-resistance range. The current i and the capacitor voltage v_c at this instant are those corresponding to points p and p' on curves (d) and (a), respectively, of Fig. 202. The capacitor then recharges exponentially with the approximate time constant RC. The path of operation rapidly approaches a closed loop.

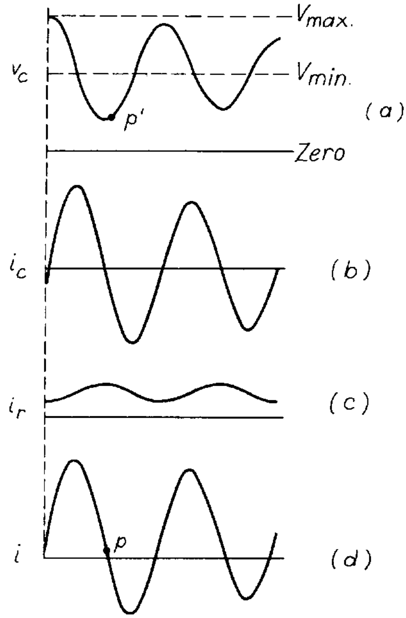


FIG. 202. Typical waveforms of currents in the circuit of Fig. 201 following switching of S to the right-hand position.

The current i and voltage v_c in the high-current range approximate portions of sine waves that are nearly 90 degrees out of phase. The curve of i vs. v_c in the high-current range should therefore resemble a portion of an ellipse, as shown in the typical curve of Fig. 203a. If the circuit Q in the high-current range is high, so that the damping is small, as assumed in the foregoing analysis, the voltage across the capacitor may reverse if V_{min} is smaller than $V_{max}/2$, as it is in the characteristics of Figs. 75 and 80.

57.3. Frequency of Oscillation of Astable Circuit. The approximate frequency of oscillation in the high-current range is $1/2\pi\sqrt{LC}$. The ratio of the period of the oscillation to the time constant in the low-current range is approximately $(2\pi/R)\sqrt{L/C}$. For the circuit parameters used earlier in this section, this ratio is 6.28×10^{-3}. Since it was shown that the ratio of the time required for the current to pass through the negative-resistance range to the time constant in the low-current range is 10^{-6} with these parameters, it follows that most of the complete period of oscillation is taken up in charging the capacitor from its minimum voltage (which may be negative if V_{min} is less than $V_{max}/2$) to the voltage V_{max}. The waveform of the capacitor voltage is therefore similar to that shown in Fig. 204a.

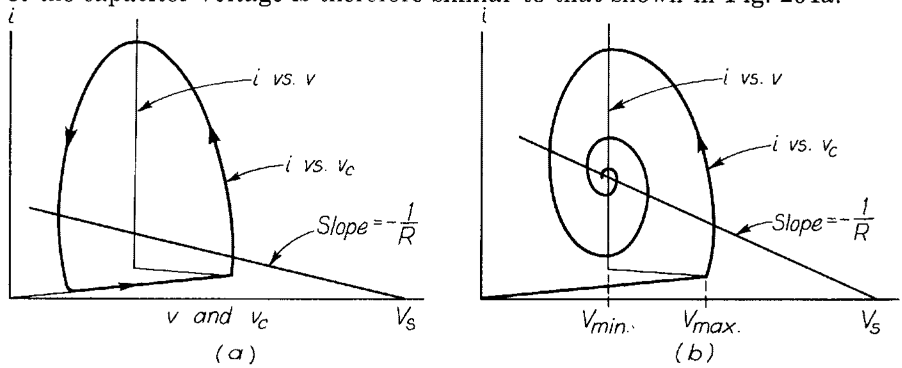


FIG. 203. Current-voltage diagram for the circuit of Fig. 201: (a) for a high-Q circuit; (b) for a low-Q circuit.

It can be seen from curve (a) of Fig. 202 that the total change of capacitor voltage approximates $2(V_{max} - V_{min})$ if the damping of the oscillatory circuit in the high-current range is small. The frequency of oscillation is then given approximately by the relation (Prob. 57.3–1):

$$\frac{1}{f} = RC \log_\epsilon \frac{V_s + V_{max} - 2V_{min}}{V_s - V_{max}} \tag{94}$$

As the damping in the high-current range is increased, the capacitor-voltage swing becomes smaller, approaching $V_{max} - V_{min}$ as a limit, and the frequency approaches the value given by the relation:

$$\frac{1}{f} = RC \log_\epsilon \frac{V_s - V_{min}}{V_s - V_{max}} \tag{94a}$$

If the current i_r is maintained constant, the capacitor charges linearly and the waveform is that of Fig. 204b. With small damping in the high-current range, the frequency then has the approximate value (Prob. 57.3–1):

$$f = \frac{I}{2C(V_{max} - V_{min})} \tag{95}$$

in which I is the constant current. With increase of damping the frequency goes up, approaching twice the value given by Eq. (95). The current i_r can be made approximately constant during charging of the capacitor by making R and V_s so large that the change of capacitor voltage is small in comparison with V_s. Alternatively, a constant-current device, such as a pentode, may be used in place of R. This method will be discussed in Sec. 58.2.

57.4. Effect of Changes of R and C. Decreasing the resistance R in the circuit of Fig. 200a increases the current i_r and also increases the damping of the oscillatory currents in the high-current range. The amplitude of curve b of Fig. 202 is consequently decreased, and the minimum of curve d is raised. A value of R is reached at which the current i does not re-enter the low-current range. The curve of i vs. v_C is then of the form shown in Fig. 203b. Application of the voltage V_s to the circuit is followed by a single transient, and the current and voltage terminate at the values corresponding to the intersection of the load line with the characteristic.

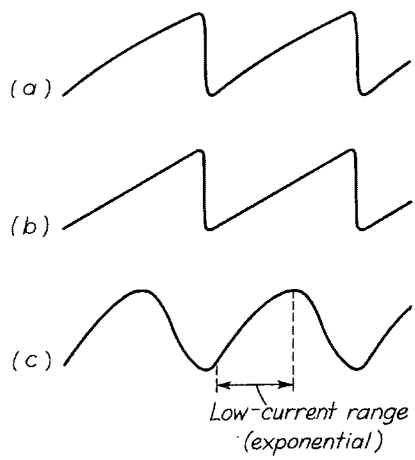


FIG. 204. (a) Capacitor voltage of the astable circuit of Fig. 200a for a current-voltage characteristic of the form of Fig. 200b; (b) voltage waveform if i_r is maintained constant; (c) voltage waveform if C/R is very large.

Decreasing the value of C decreases the amplitude of i_C and at the same time decreases the Q of the circuit in the high-current range and thus increases the damping of i_C. The extent to which the current i swings to negative values is consequently also decreased by a decrease of C, and there is a minimum value of C at which relaxation oscillation can occur at a fixed value of R. This value may, however, be smaller than the inherent shunt capacitance of the negative-resistance element.

Increasing C increases the period of the damped oscillation in the high-current range. Decreasing R reduces the capacitor charging time in the low-current range. Consequently, if C is increased and R decreased, the portion of the cycle of relaxation oscillation in which the capacitor charges becomes a smaller fraction of the entire period of oscillation. If the capacitor charging time becomes of the same order of magnitude as the time required for the current to pass through the negative-resistance and high-current ranges, the waveform of the capacitor voltage is of the general form shown in Fig. 204c, which, on casual inspection, appears to resemble a sine wave.

Theoretical predictions of the manner in which the circuit should respond to changes of R and C are in agreement with observations of the behavior of actual circuits. As R is decreased, the frequency increases until the circuit stops oscillating. Increase of C then allows the circuit to oscillate again through an additional range of R. With large values of C and values of R near those at which the circuit stops oscillating, the waveform of the capacitor voltage is of the form of Fig. 204c.

57.5. Choice of V_s and R. If the resistance R_{iH} in the high-current range is low, the behavior of the circuit of Fig. 200a is essentially the same whether the load line intersects the characteristic in the high-current range, as assumed in the analysis of Sec. 57.2, or in the negative-resistance range. Because of the difficulty of adjusting the values of V_s and R so that the intersection is in the negative-resistance range if this range is small, this type of operation is usually observed only if the slope of the current-voltage characteristic in the negative-resistance range is relatively high. Transition from the low-current range to the high-current range then requires a larger portion of the period of oscillation, and the waveform of the capacitor voltage is intermediate between those of Figs. 204a and 204c.

57.6. Monostable Operation. When the current-voltage characteristic and the parameters of the circuit of Fig. 200a are such that sawtooth relaxation oscillation is obtained, the circuit may be made monostable by reducing the supply voltage so that the load line intersects the current-voltage characteristic in three points, as shown in Fig. 205. The intersection of the load line with the low-current section of the characteristic then corresponds to a stable state of equilibrium. Application of a short voltage pulse in series with V_s, of sufficient magnitude to cause the load line to be displaced to the right of the voltage peak of the characteristic, as shown by the dashed line of Fig. 205, is followed by a single discharge of the capacitor and subsequent slow exponential recharging to the voltage corresponding to the stable point. The general form of the curve of I vs. v_C is shown in Fig. 205. If the duration of the triggering pulse is short in comparison with the time in which the current passes through the negative-resistance and high-current ranges, the recharging of the capacitor is not affected by the triggering pulse, and the waveform of the capacitor voltage is that of one period of the wave of Fig. 204a or 204b.

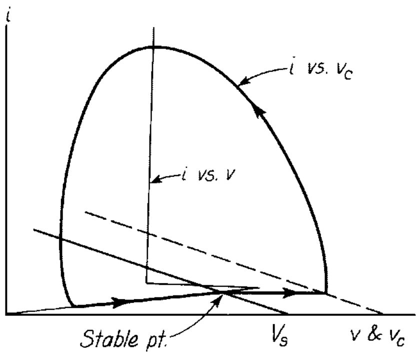


FIG. 205. Current-voltage diagram for the circuit of Fig. 200 in monostable operation.

57.7. Synchronization. Like other relaxation oscillators, sawtooth-wave oscillators may be synchronized to a control frequency slightly higher than the uncontrolled frequency of the oscillator or a multiple or submultiple thereof (see Sec. 54). The control voltage may be applied to the circuit in series with the supply voltage V_s, or it may be applied to a control electrode of the negative-resistance element in such a manner as to cause the breakdown voltage V_{max} to vary at the control frequency.

58. Practical Forms of Sawtooth-Voltage Generators

58.1. Glow-Tube Circuit. The first circuit to be used as a sawtooth-voltage generator made use of a glow tube as the negative-resistance element. The circuit is shown in Fig. 206. (The inductance L, which is essential to the operation of a circuit of the form of Fig. 200a, is not shown, since it is internal to the tube.) The glow-tube current-voltage characteristic of Fig. 79 is a static characteristic and does not apply when the tube is used in a relaxation oscillator. The form of the dynamic characteristic depends upon the ionization of the gas within the tube and therefore upon the time elapsed between the extinction of an earlier discharge and reapplication of voltage to the electrodes, magnitude of current in earlier discharges, rate of change of voltage applied to the tube, tube temperature, and other factors.[1] The dynamic breakdown voltage is always lower than the static value, and decreases with increase of frequency when the tube is used in a relaxation-oscillator circuit. For this reason the amplitude of oscillation decreases with increase of frequency, and Eqs. (94) and (95) cannot be used to predict the frequency of oscillation of a glow-tube circuit.

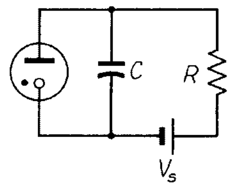


FIG. 206. Glow-tube sawtooth-voltage generator.

As a generator of sawtooth voltages, the glow-tube relaxation oscillator has been displaced by other circuits that are more stable as to frequency and amplitude and can be used over a wider frequency range. The glow-tube oscillator has found recent application as a flashing warning light. The very small amount of power required is an obvious desirable feature of the circuit in this application.

58.2. Thyratron Circuit. Considerable improvement in operating characteristics results from replacing the glow tube of Fig. 206 by a low-current thyratron. Advantages of a thyratron over a glow tube in this application include breakdown voltage that is independent of frequency but controllable by means of grid voltage, and much lower extinction voltage and therefore higher amplitude of varying capacitor voltage and shorter transition time from the low-current range to the high-current range. Because the breakdown voltage of a thyratron may be hundreds of volts, whereas the extinction voltage is of the order of only 10 to 20 volts, Fig. 202 shows that the

[1] H. J. Reich and W. A. Depp, *J. Appl. Phys.*, **9**, 421 (1938).

capacitor voltage swings to negative values in a thyratron sawtooth-voltage circuit. If the breakdown voltage V_{max} is large in comparison with the extinction voltage V_{min} and damping is small in the high-current range, the negative plate-voltage swing approximates the breakdown voltage.

The dependence of thyratron breakdown voltage upon grid voltage makes possible the variation of frequency and amplitude of oscillation by means of the grid voltage. The frequency of oscillation may be readily synchronized to the frequency of an external source by introducing a small amount of the synchronizing voltage in series with the grid. The synchronizing grid voltage causes the breakdown voltage to vary at the control frequency.

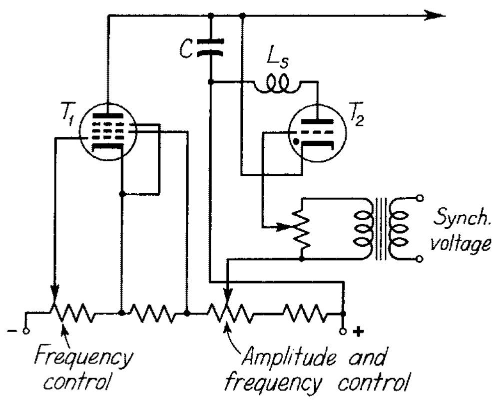


FIG. 207. Thyratron linear sawtooth-voltage generator.

Figure 207 shows a thyratron sawtooth-voltage generator in which approximately linear charging of the capacitor is obtained by the use of a pentode in place of the charging resistor R. In this circuit charging of the capacitor raises the cathode potential of the thyratron relative to the grid. The breakdown voltage therefore falls as the applied anode-to-cathode voltage rises. The resulting rapid rise of anode-cathode voltage relative to the breakdown voltage is desirable from the point of view of frequency stability. The small inductance L_s in series with the thyratron plate serves to limit the peak thyratron plate current and also increases the Q of the circuit in the high-current range of the current-voltage characteristic and thus assures that the capacitor-voltage swing approaches twice the difference between the breakdown and extinction voltages. Continuous variation of frequency is accomplished in the circuit of Fig. 207 by changing the pentode control-grid voltage; large changes are made by changing the capacitance C. The amplitude of the output voltage may be changed by means of the grid bias of the thyratron.

58.3. *P-N-P-N*-Transistor Circuit. Examination of the *p-n-p-n* transistor characteristics of Fig. 94 indicates that a *p-n-p-n* transistor can also serve as the active element of a sawtooth-voltage generator. The transistor has the advantage over thyratrons of not requiring a source of heater voltage. The transistor circuit therefore has the simplicity of the glow-tube circuit, together with the desirable feature of controllability of breakdown voltage by means of base current. Unlike thyratrons, however, *p-n-p-n* transistors break down at relatively low reverse collector voltage. Proper operation of the circuit requires that the maximum reverse collector-voltage be less than the reverse breakdown voltage. Since the extinction or "break-off" voltage V_{min} of a *p-n-p-n* transistor is independent of the breakdown voltage V_{max}, the reverse collector-voltage swing decreases with decrease of forward breakdown voltage and may be adjusted to a suitable value by means of the base current. Synchronizing voltage may be applied to the base circuit.

58.4. Eccles-Jordan Circuit. The grid bias of the Eccles-Jordan circuit can be adjusted so that the current-voltage characteristic observed in the

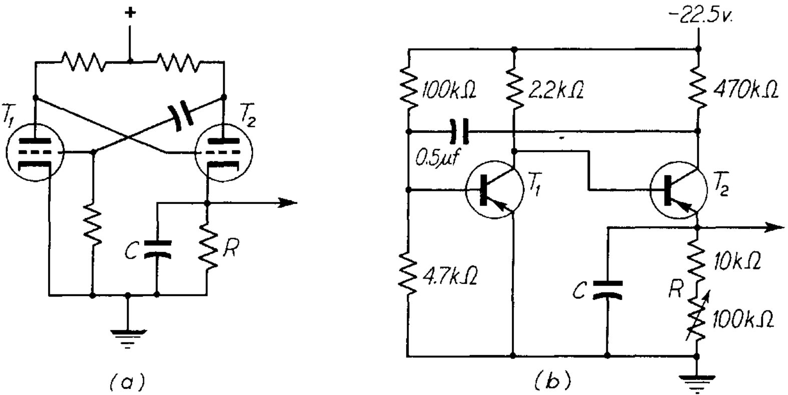


FIG. 208. Tube and transistor sawtooth-voltage generators that make use of the current-stable current-voltage characteristic observed in the circuit of Fig. 68d.

cathode lead of either tube has the general form of the characteristic of Fig. 200b (see Figs. 68d and 69d). Figure 208 shows tube [2] and transistor [3] circuits that make use of the current-voltage characteristics of Fig. 69d.

[2] O. S. Puckle, *J. Sci. Instr.*, **13,** 78 (1936).
[3] P. G. Sulzer, *Electronics,* August, 1953, p. 170.

59. Switch-Type Sawtooth-Voltage Generators

59.1. Basic Circuit. Certain types of cathode-ray oscilloscopes, radars, and similar equipment make use of circuits that generate a linearly rising or falling voltage initiated by a triggering pulse that may not be periodic. The basic circuit ordinarily used for this purpose is shown in Fig. 209. Under

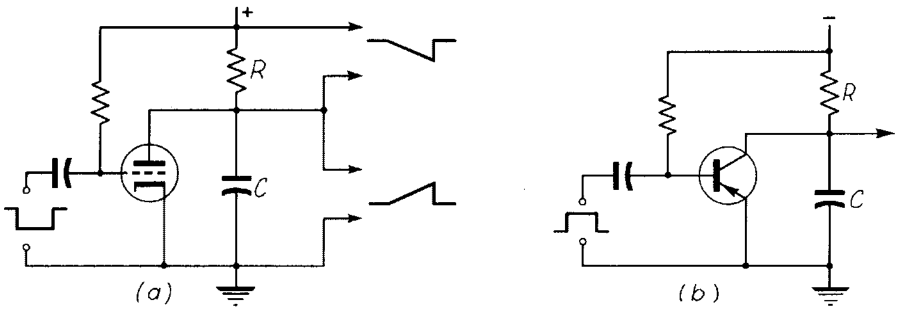


FIG. 209. Tube and transistor versions of the switch-type sawtooth-voltage generator.

standby conditions the capacitor is short-circuited by a triode, the grid of which is biased positively, or by a transistor, the base of which is forward-biased. Application of a rectangular negative pulse to the grid of the triode or of a reverse-voltage pulse to the base of the transistor biases the tube or transistor beyond cutoff and allows the capacitor to charge throughout the duration of the pulse. If the time constant RC is large in comparison with the pulse duration, the voltage rises approximately linearly. The nonlinearity may be reduced by the addition of inductance in series with the resistance.

In the tube circuit of Fig. 209a an approximately linear positive-going voltage is obtained across the capacitor and a negative-going voltage across the resistor. The availability of both *n-p-n* and *p-n-p* transistors suitable for use in the circuit of Fig. 209b makes possible the generation of either positive-going or negative-going voltage across either the capacitor or the transistor.

As the ratio of the time constant RC to the charging period of the capacitor is increased in order to linearize the output voltage, the amplitude of the output voltage decreases unless the supply voltage is simultaneously increased. In order to avoid the necessity of using inconveniently large supply voltage, the resistor R may be replaced by a constant-current device, such as a self-biased tube or transistor, as shown by the tube circuit of Fig. 210a. The regulating action is the result of increase of negative grid voltage of T_2 with plate current. A high degree of current stabilization may call for such a large value of resistance R_k that the grid bias is excessive. This problem is even more acute when a transistor is used as the constant-current element, since the transistor requires a forward-biasing voltage, whereas the voltage drop across the resistor in series with the emitter produces a reverse bias.

Figure 210b shows a modification of the circuit of Fig. 210a in which the biasing problem is solved by the use of a charged capacitor as a means of providing biasing voltage.[1] The standby grid voltage is maintained at the

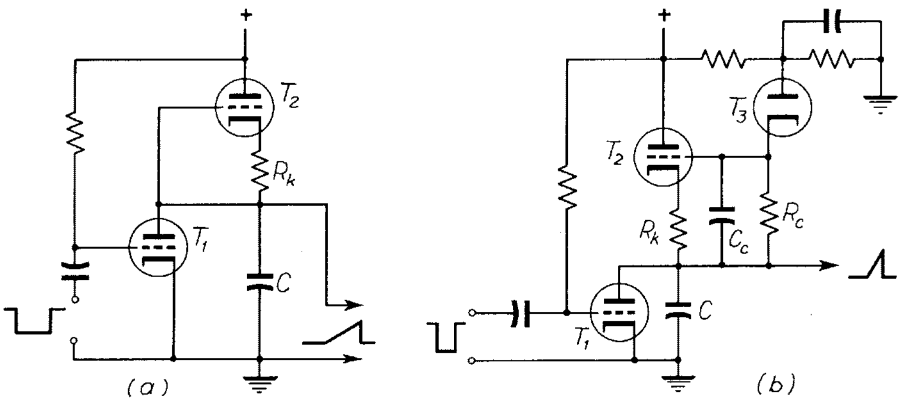


FIG. 210. (a) A modification of the circuit of Fig. 209a in which a current-stabilizing tube is used to linearize the output-voltage wave; (b) a variant of circuit (a) in which T_2 is biased by means of a charged capacitor C_c.

desired value by means of the diode T_3. As the capacitor C charges following application of the input pulse to T_1, the diode cathode becomes positive relative to the anode and T_3 becomes nonconducting. The capacitor then discharges through R_c, but if the time constant R_cC_c is large in comparison with the length of the control pulse, little change in voltage across C_c takes place during the charging of C. The rate of rise of output voltage approximates the initial voltage across C_c divided by R_kC. At the termination of the pulse, C discharges rapidly through T_1. When a transistor in the common-emitter connection is used for the constant-current device, the flow of base current necessitates the use of a larger value of C_c than that required when a triode is used. The required value of C_c can be reduced by the use of compound-connected transistors for T_2.

59.2. Bootstrap Circuit. Another type of linear-voltage generator, based upon the bootstrap circuit of Fig. 20b, is shown in Fig. 211a.[2] The circuit

[1] Rad. Lab. Series, Vol. 19, Fig. 7.16 (McGraw-Hill Book Co., Inc., New York, 1949).
[2] Rad. Lab. Series, Vol. 19, 7.13 (McGraw-Hill Book Co., Inc., New York, 1949).

of Fig. 211a is formed from that of Fig. 20b by replacing the input voltage v_i by a constant direct voltage V and shunting the capacitor C by a short-circuiting tube. As the capacitor C charges following application of the control pulse, the grid voltage of tube T_2 rises at the same rate. Because the tube and the load resistor R_L form a cathode-follower amplifier, the cathode voltage rises at very nearly the same rate as the grid voltage, and the voltage across R remains essentially constant. Consequently, the current through R is nearly constant and the voltage across C rises approximately linearly. The cathode-follower amplifier increases the effective

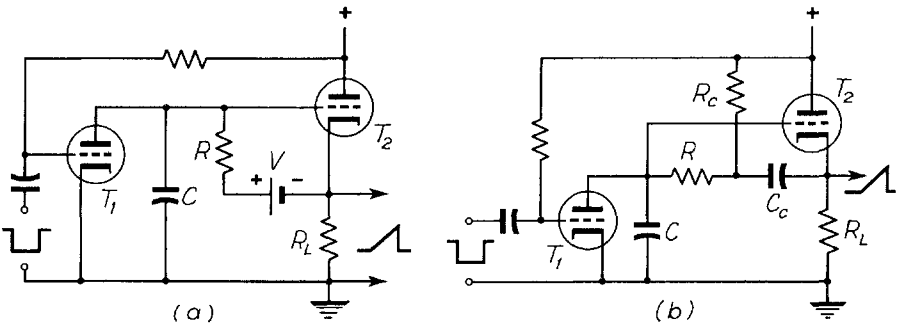


FIG. 211. (a) Bootstrap sawtooth-voltage generator; (b) variant of circuit (a) in which the voltage source V is replaced by a charged capacitor C_c.

time constant governing the charging of C from the value RC to the value $RC[1 + \mu R_L/(r_p + R_L)]$. Low-impedance output may be taken from across the resistor R_L.

Figure 211b shows a practical form of the circuit of Fig. 211a in which the need for a separate voltage source to charge the capacitor C is eliminated by the use of a charged capacitor C_c in place of the voltage supply V. If C_c is much larger than C and the time constant $RR_cC_c/(R + R_c)$ is large in comparison with the duration of the control pulse, the voltage across C_c does not change greatly during the charging of C, and the linearity of the output voltage is comparable with that obtained from the circuit of Fig. 211a. As a result of the time required to recharge C_c between input pulses, however, the initial voltage across C_c at the beginning of each pulse is a function of the time between pulses, and therefore the rate at which the output voltage rises is also dependent upon the time between pulses. If the control pulses are periodic, the rate of rise of output voltage depends upon the repetition frequency and the duty factor. This effect is small if the period between pulses is large in comparison with the approximate time constant

$C_cRR_c/(R + R_c)$ and it can be reduced by substituting a diode for the resistor R_c, as in the transistor circuit of Fig. 212.[3]

For the duration of the control pulse the diode of Fig. 212 is cut off by the rise in emitter voltage of T_2, and C_c discharges only through R and the base of T_3. The base current is made small by the use of the compound-connected transistors T_2 and T_3, instead of a single transistor. The use of the diode in place of R_c therefore increases the discharging time constant from approximately $C_cRR_c/(R + R_c)$ to approximately C_cR and thus reduces the change in voltage across C_c during the charging of C. At the

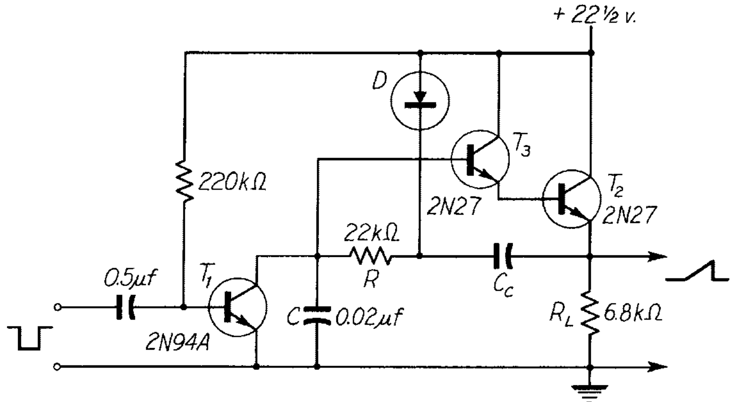


FIG. 212. Transistor version of the circuit of Fig. 211b in which R_c is replaced by a diode D.

termination of the input pulse the sudden discharge of C lowers the potential of the diode cathode and C_c recharges rapidly through the diode. The low resistance of the diode in comparison with the resistance R_c of the circuit of Fig. 211b results in much more rapid recharging of C_c, and hence in less dependence of the rate of rise of output voltage upon the interpulse interval.[4]

59.3. Miller Circuit. A linear voltage generator based upon the Miller circuit of Fig. 20a is shown in Fig. 213a. A pentode is used in place of the triode shown in Fig. 20a, and the positive supply voltage is used for the voltage v_i of Fig. 20. Under standby conditions, current through the high resistance R reduces the plate voltage of T_2 to a low value and the capacitor C charges in the indicated polarity. When the control pulse is applied to the grid of T_2, T_2 is cut off and C discharges through R and the parallel paths of R_l and the plate of T_1. As shown in Sec. 4, the amplifier T_1, to-

[3] J. S. Sherwin, Univ. of California, Inst. of Eng. Res., Rpt. Series No. 60, Issue No. 181, May 15, 1957.

[4] For a detailed analysis of the frequency dependence of this circuit, see Rad. Lab. Series, Vol. 19, Sec. 7.8 (McGraw-Hill Book Co., Inc., New York, 1949).

gether with the feedback capacitance C, acts like a much higher capacitance between the grid of T_1 and ground. Consequently the grid voltage of T_1 rises slowly and approximately linearly and the plate voltage falls approximately linearly at a much higher rate.* At the termination of the input

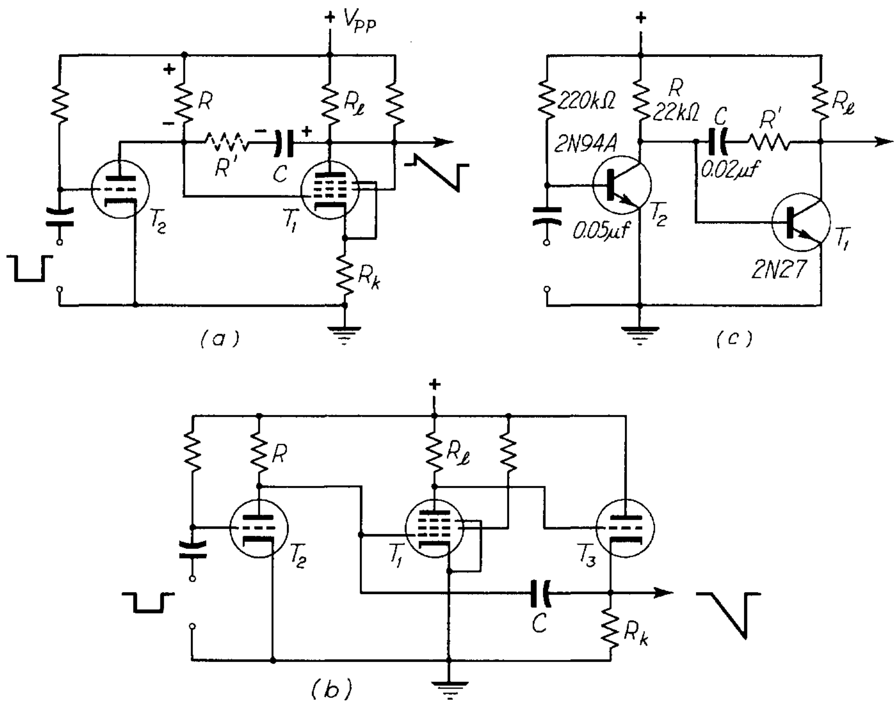


FIG. 213. (a) Miller sawtooth-voltage generator; (b) Modified form of the Miller circuit in which the step at the beginning of the output pulse is reduced by a cathode-follower stage; (c) transistor version of circuit (a).

pulse T_2 again conducts and C recharges through T_2 and through R_l in parallel with the plate path of T_1. Because the plate resistance of T_2 is relatively low, the charging time constant approximates the product of C and the resistance of R_l in parallel with the plate resistance of T_1.

Before T_2 is cut off by the control pulse, the current through R approximates V_{PP}/R. When T_2 is cut off, the reduction in current through R tends to raise the grid voltage of T_1 and therefore the plate current of T_1. A

* A simple qualitative explanation of the operation of the Miller circuit is that any increment of grid voltage produces a much larger increment of plate voltage of opposite sign, which is coupled back to the grid through the capacitor and thus tends to prevent the increment of grid voltage. The grid voltage of T_1 consequently rises much more slowly than it would if T_1 did not amplify.

portion of the increment of plate current appears as discharging current of C and maintains a voltage drop through R. However, because the output resistance of the amplifier stage is much higher than the resistance of the short-circuiting tube T_2 in the absence of an input pulse, the initial discharging current is less than the standby current through R, and there is an abrupt rise in the control-grid voltage of T_1 at the instant of application of the control pulse. Since the capacitor cannot change voltage instantaneously, there must be an equal initial rise in output voltage. The initial output voltage step may be eliminated by the addition, in series with C, of a resistance R' equal to the reciprocal of the transconductance of T_1. An abrupt decrease of control-grid voltage of T_1 is then accompanied by an equal drop in voltage across the series resistance, and the output voltage does not change (Prob. 59.3-1).

The output voltage step at the beginning of the control pulse can also be reduced by connecting the capacitor C to the output of T_1 through a cathode-follower stage, which has a low output resistance. The circuit is shown in Fig. 213b.[5] This circuit has the additional advantage that the lower output resistance of the cathode-follower stage allows C to recharge more rapidly following the termination of the control pulse and therefore results in a more rapid drop in voltage at the termination of the control pulse.

A transistor version of the circuit of Fig. 213a is shown in Fig. 213c.[6]

59.4. Suppressor-Controlled Miller Circuit. Figure 214a shows a single-tube Miller linear-voltage generator in which the control pulse is applied to the suppressor of the pentode. The suppressor bias is sufficient to make the standby plate current zero. Initially the control-grid voltage approximates zero, and the plate voltage is equal to the positive supply voltage. The voltage across the capacitor is therefore the supply voltage. Application of a rectangular positive pulse to the suppressor first causes the plate current to increase abruptly to a value determined by the zero voltage of the control grid, and the plate voltage to drop. The capacitor then discharges and the plate voltage falls nearly linearly at a rate equal to the initial voltage across the capacitor divided by RC. At the termination of the control pulse the plate current is cut off abruptly and the capacitor recharges through R_L and the control grid. The plate voltage therefore rises exponentially with the approximate time constant R_LC.

The initial drop in plate current at the instant of application of the control pulse can be reduced by making the standby grid voltage negative, rather than zero. A circuit by means of which this can be done is shown in Fig. 214b.[7] Under standby conditions the drop in R caused by the plate current

[5] Rad. Lab. Series, Vol. 19, p. 199 (McGraw-Hill Book Co., Inc., New York, 1949).

[6] J. S. Sherwin, Univ. of California Electronics Research Laboratory Series No. 60, Issue No. 182, Div. of Electrical Eng., Berkeley, Calif., May 15, 1957.

[7] H. A. Dell, *Electronic Eng.*, **25**, 94 (March, 1953).

of T_2 lowers the control-grid voltage of T_1. Conduction of the diode D, however, prevents the control-grid voltage of T_2 from falling below the negative voltage applied to the anode of D. Application of a negative control voltage to the grid of T_2 cuts T_2 off. The resulting rise in plate voltage of T_2 cuts the diode off and allows C to start charging linearly, as in the circuit of Fig. 214a. At the termination of the control pulse, C recharges through R_L and the plate of T_2.

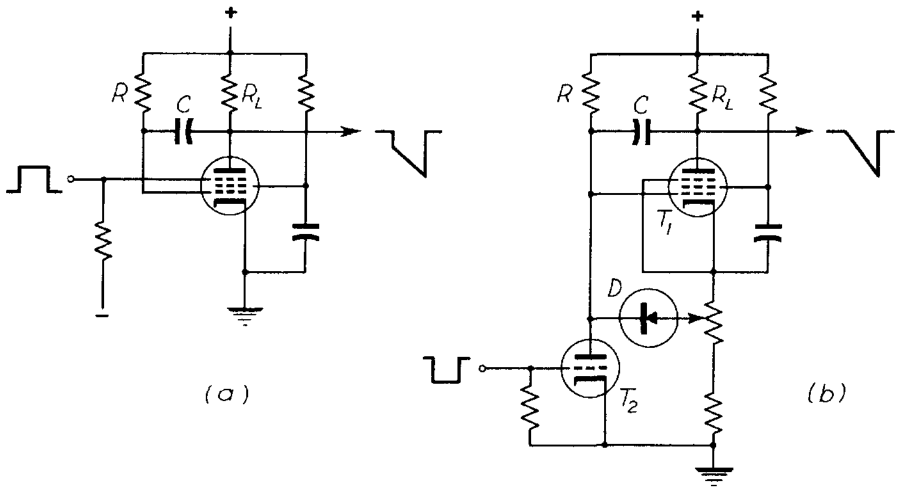


FIG. 214. Suppressor-controlled Miller circuit: (a) circuit controlled by a positive-going input pulse; (b) circuit in which the initial output-voltage step is reduced.

The operation of the circuits of Fig. 214 is in some respects similar to that of the phantastron circuit of Fig. 198, and the output voltage is of the same form as the plate voltage of the phantastron, shown in Fig. 198. The phantastron circuit has the advantage that it delivers both a linearly falling voltage and a rectangular positive voltage pulse.

59.5. Triangular-Wave Circuit. When the circuits of Figs. 209 to 214 are used with periodic control pulses, the output wave consists of linearly rising or falling sections separated by constant-voltage sections. In some applications it is desirable to make the interval between the linear sections as small as possible. This can be accomplished by making the interval between rectangular control pulses very small or, if it is not essential that the linear sections be initiated by the leading edge of the control pulses, by biasing the short-circuiting tube or transistor beyond cutoff and applying short, positive pulses to the grid of the tube or forward pulses to the base of the transistor. A modification of the circuit of Fig. 210b that operates in this manner is shown in Fig. 215.[8] The positive biasing voltage necessary to compensate for the large voltage drop across R_k is furnished by the capacitor C_c, which is charged through the diode D, to a voltage approximating the negative supply voltage, during the time in which T_1 is made conducting by the control pulse. The rate of rise of output voltage approximates the magnitude of the negative supply voltage divided by R_kC. If

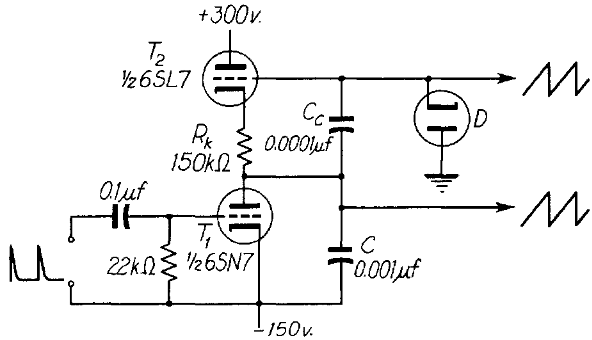


FIG. 215. Sawtooth-wave generator based upon the circuit of Fig. 210b.

the output is taken from the cathode of the diode, the output voltage rises from approximately zero; if it is taken from the plate of T_1, it rises from approximately the negative supply voltage.

59.6. Pentode-Triode Circuit. Figure 216 shows a self-excited sawtooth-voltage generator in which the capacitor is charged through a pentode and discharged through a triode in series with a resistance.[9] The constant-current feature of the pentode is used to obtain nearly linear charging of the capacitor. During charging of the capacitor C the voltage drop through R_1 produced by the pentode screen current biases the triode T_2 beyond cutoff. As the capacitor charges, the potential of the cathode of T_2 falls and eventually reaches a value at which T_2 starts to conduct. The flow of capacitor discharge current through R_2 then drives the control grid of T_1 in the negative direction. The resulting reduction of screen current of T_2 raises the grid voltage of T_2. Cumulative action terminates with T_1 cut off and T_2 highly conducting. Discharge of C' causes the grid voltage of T_1 to rise to a point where T_1 starts to conduct, and cumulative action returns the circuit rapidly to the initial state in which T_1 conducts and T_2 is cut off.

[8] D. Sayre, *Electronics*, July, 1950, p. 172.
[9] *Loc. cit.*

Fine control of frequency in the circuit of Fig. 216 can be obtained by means of a variable 1000-ohm resistor in series with the cathode of T_1. For short discharge time, the time constant R_3C' should be small. Shorter discharge time can be obtained by using a high-transconductance tetrode or pentode for T_2 and connecting R_2 and C' to the screen. The discharge

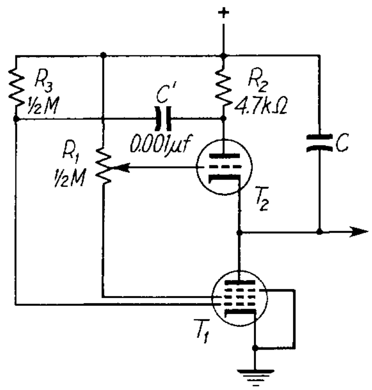


FIG. 216. Pentode-triode sawtooth-voltage generator.

time may then be adjusted by means of a variable 2000-ohm resistor in series with the plate of T_2. The capacitor discharge may be made linear by restricting the operation of the pentode T_2 to the constant-current range.

Synchronizing voltage may be conveniently applied to the circuit of Fig. 216 by the use of a series grid resistor between R_1 and the grid of T_2, and a small coupling capacitor to the grid.

60. Sawtooth-Current Generators

A sawtooth wave of current through a nonreactive load can be readily obtained by using the load in the output of an amplifier excited by one of the sawtooth-voltage generators discussed in Secs. 58 and 59. With an inductive load, however, such as the coils used to produce electromagnetic deflection of the beam of a cathode-ray tube, the amplifier exciting voltage required to produce a sawtooth wave of load current is not sinusoidal. The sawtooth-voltage generator used to drive the amplifier must therefore be modified in order that the load current will have the desired sawtooth form.

Figure 217a shows the equivalent circuit of a tube with an inductive load. The resistance R includes the inherent resistance of the inductor. If the current is of the form Kt, where $K = di/dt =$ constant, it produces a constant voltage KL across the inductance and a linearly rising voltage $K(R + r_p)t$ across the resistances. The required exciting voltage therefore consists of the sum of a constant component of magnitude KL and a linearly rising component having a rate of change $K(R + r_p)$, as shown in Fig. 217b. An exciting voltage of this form can be generated by sending a constant current I_k through a series combination of resistance R' and capacitance C. The current produces a constant voltage I_kR' across the resistance and a

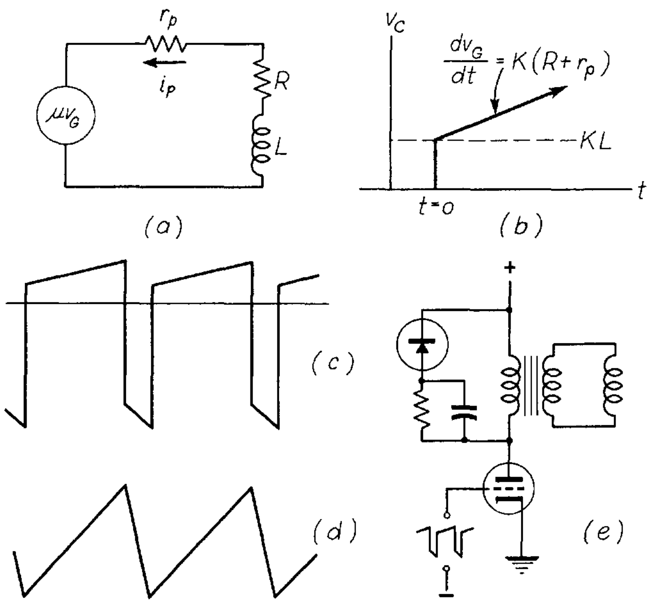


FIG. 217. (a) Equivalent circuit of a vacuum triode with inductive load; (b) voltage v_G required in circuit (a) in order to produce a linearly rising current; (c) waveform of voltage across a series combination of capacitance and resistance when the capacitance is periodically charged by a constant current and discharged by a much larger constant current; (d) current in circuit (a) when v_G is of the form shown in (c); (e) the use of an R-C-diode combination to damp out transients.

linearly rising voltage I_kt/C across the capacitance. The load current will have the desired form Kt if $R' = L/(R + r_p)C$ and $I_k = KRC/\mu$. A similar analysis may be readily made for a transistor circuit.

If the capacitor C of the exciting-voltage generator is periodically charged slowly by means of a relatively small current I_k and discharged rapidly by means of a much larger current of opposite sign, the voltage produced across R' and C is of the form of Fig. 217c and the load current is of the form of Fig. 217d. It follows that a sawtooth wave of current through an inductive load can be approximated closely by driving the load by means of an amplifier excited by one of the sawtooth-voltage generators discussed in Secs. 58

and 59 if a resistance of magnitude $L/(R + r_p)C$ is inserted in series with the capacitance C of the sawtooth-voltage generator.

When an impedance-matching transformer is used between the amplifier and the inductive load, the primary of the transformer should be shunted by a diode in series with a parallel combination of capacitance and resistance, as shown in Fig. 217e, in order to damp out transient oscillations initiated in the transformer windings during the rapidly changing portion of the current wave.

61. Blocking-Oscillator Pulse Generator

61.1. Principle of Operation. An approximately rectangular pulse of large amplitude and of duration ranging from a fraction of a microsecond to several hundred microseconds and a rise time as short as 10^{-10} sec may be generated by the feedback circuit shown in Fig. 218. Because the circuit is basically a feedback oscillator in which the parameters are chosen so that relaxation oscillation or monostable operation is obtained, it is called the *blocking oscillator.*

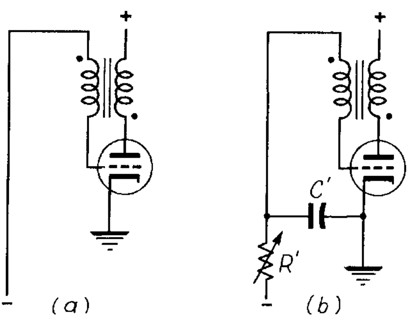


FIG. 218. Common-cathode blocking oscillator: (a) basic circuit; (b) circuit using resistance and capacitance to control the pulse length.

If the grid-bias in the vacuum-tube circuit of Fig. 218a is gradually raised from a large negative value, plate current starts flowing when the grid voltage reaches the cutoff value. A further increment of grid voltage causes an increase of plate current that induces a voltage in the transformer secondary which is added to the impressed grid-voltage increment. The transconductance, and therefore the voltage amplification, increase as the grid voltage is raised. At some value of grid voltage the open-loop voltage amplification of the tube and transformer exceeds unity and the rise in grid voltage and plate current become cumulative. The plate current then rises rapidly to a value determined by the characteristics of the tube and by the internal resistance of the B supply. One of the most important factors that limit the plate current is the diversion of current from the plate to the control grid at large positive values of grid voltage.

Because of the rapidity with which the plate current rises to its maximum value, the subsequent behavior of the circuit is similar to that which would obtain if the plate-cathode path of the tube were replaced by a switch in series with a resistance. The sudden rise in plate current, which flows through the primary winding, is accompanied by the sudden flow of grid current through the secondary. Since there is no source of direct voltage in the secondary circuit, the secondary current decays toward zero and the primary current rises toward the value determined by the voltage supply and the primary circuit resistance. If the grid and plate resistances of the tube and the transformer inductances were constant, the secondary current would decay exponentially. Actually, however, the grid resistance is an inverse function of the grid current, and core saturation reduces the transformer inductance as the secondary current dies out. For these reasons the secondary current in the circuit of Fig. 218a decreases more rapidly than if the decrease followed an exponential law. Oscillographic studies indicate that in some circuits the decay follows more nearly a linear law than an exponential.

As the grid current and voltage fall, the transconductance of the tube rises and eventually becomes so large that the open-loop amplification of the tube and transformer again exceeds unity. The reduction of grid voltage and plate current caused by a small decrease of secondary current then produces an additional decrease of secondary current equal to the initial decrease and the action becomes cumulative. The plate current is therefore rapidly reduced to zero, i.e., the equivalent switch is opened.

61.2. Use of Resistor and Capacitor to Control Pulse Length. In the circuit of Fig. 218a, the time taken for the secondary voltage to decay to the value at which the circuit switches back to its initial state and the plate-current pulse terminates may be of the order of several hundred microseconds unless very small transformers are used. Much shorter pulses are produced by the modified circuit of Fig. 218b. If the time constant $R'C'$ is small, the capacitor charges rapidly following the first transition; hence the grid voltage falls quickly to the value at which transition is again initiated and the plate current falls to zero. The circuit may be triggered by voltage pulses applied to any electrode, in a manner similar to that for Eccles-Jordan circuits.

61.3. Modes of Operation. If the grid biasing voltage of a blocking oscillator is sufficient to keep the tube cut off, plate current pulses are obtained only when a positive triggering pulse is applied to the grid. The characteristics of the circuit then resemble those of the monostable multivibrator discussed in Sec. 55, but the circuit is capable of producing pulses having much shorter rise time and duration than those generated by the monostable multivibrator. Because low transconductance in the vicinity of cutoff causes the current to rise slowly near cutoff, short rise time is favored by making the amplitude of the triggering pulses high.

If the biasing voltage is less than the cutoff value, the blocking oscillator acts as a relaxation oscillator. One application of the astable blocking oscillator is as a frequency divider. Its advantage over the multivibrator is the much higher upper limit of oscillation frequency.

The grid of a blocking oscillator may also be biased positively, so that the tube conducts in the standby state. The positively biased blocking oscillator is called the *multiar*. Usually the flow of grid current through the grid resistor prevents the grid voltage from rising greatly above zero. Application of a negative triggering pulse to the grid circuit initiates a reduction of plate current that in turn induces in the grid winding of the transformer a voltage of correct polarity to depress the grid voltage further. The action

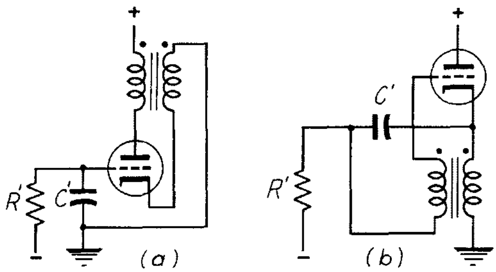


FIG. 219. (a) Common-grid blocking oscillator; (b) common-plate blocking oscillator.

becomes cumulative and the plate current falls rapidly. The negative grid voltage during the fall of plate current prevents the flow of grid current. Hence the secondary of the transformer is essentially unloaded and there is no long transformer transient to maintain the circuit in a quasistable state, as in the negatively biased circuit. As the plate current approaches zero, the reduction of transconductance causes the rate of change of plate current to be reduced and the induced secondary voltage to become smaller. Unless the amplitude and duration of the triggering pulse are sufficient to hold the grid voltage below cutoff, therefore, the plate current stops falling before it reaches zero. The subsequent increase of plate current induces a positive voltage in the secondary of the transformer. The plate current then rises at an increasing rate, and the circuit returns to the standby state.

Common-grid and common-plate variants of the circuits of Fig. 218 are shown in Fig. 219.[1] Transistor blocking oscillators will be discussed in Secs. 63 to 66.

61.4. Blocking-Oscillator Transformers. The size, structure, and parameters of transformers used in blocking oscillators vary greatly with the rise time and pulse length required. For extremely short pulses and

[1] B. Chance, V. Hughes, E. F. MacNichol, and D. Sayre, *Waveforms*, Rad. Lab. Series, Vol. 19, Chap. 6.

rapid rise times, the transformers may be very small, consisting, for example, of five primary turns and five secondary turns wound upon a toroidal ferromagnetic dust core approximating $\frac{5}{16}$ in. in outside diameter and $\frac{1}{4}$ in. in inside diameter. Such a transformer may have a shunt capacitance of the order of 10^{-16} farad, a maximum inductance of 10^{-6} henry, and a coupling coefficient of 0.8. Because the lead inductance of such a transformer is obviously appreciable in comparison with the winding inductance, the transformer must be mounted as close to the tube or transistor as possible, preferably directly on the electrode leads. Interelectrode and interwinding capacitances are also an important consideration in tube circuits, the latter because it may cause degenerative feedback coupling unless great care is taken in the arrangement of windings.

62. Analysis of the Vacuum-Tube Blocking Oscillator

62.1. Analysis During Switching Periods. Although the vacuum-tube blocking oscillator is basically simple in form, rigorous analysis of the circuit is greatly complicated by the fact that the effect of shunt and inter-

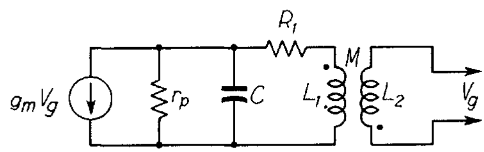


FIG. 220. Equivalent circuit for the common-cathode blocking oscillator during the time in which grid current is negligible.

winding capacitances cannot be neglected at the high rates of change of voltage that are obtainable in practical circuits. Furthermore, for the very short rise times observed in tube circuits using miniature transformers, such as that described in Sec. 61.4, the electron transit time is of the same order of magnitude as the rise time and the transadmittance of the tube cannot be considered to be a pure transconductance.

Some indication of the manner in which the plate-current rise and fall times are affected by the circuit parameters of a blocking oscillator may be determined by analyzing a greatly simplified equivalent plate circuit in which interwinding coupling capacitances are neglected and the transadmittance is assumed to be nonsusceptive. During the time in which the grid current is negligible, the equivalent circuit is of the form of Fig. 220, in which C represents the distributed capacitance of the windings and the grid-cathode capacitance of the tube. Solution of the network equations for this equivalent circuit shows that the primary current is of the form (see also Prob. 62.1-1)

$$i = A\epsilon^{pt} \tag{96}$$

in which

$$p = -\frac{r_pR_1C + L_1 - \mu M}{2r_pL_1C}\left[1 \pm \sqrt{1 - \frac{4r_pL_1C(R_1 + r_p)}{(r_pR_1C + L_1 - \mu M)^2}}\right] \tag{97}$$

If $\mu M \gg r_pR_1C + L_1$ and $\mu M > 4r_p(R_1 + r_p)LC$ (which is true when the tube has high transconductance, the transformer is tightly coupled, and the winding capacitance is low), the second term under the radical in Eq. (97) is small in comparison with unity and by application of the binomial expansion Eq. (97) may be written in the form,

$$p_1 \approx \frac{\mu M - L_1 - r_pR_1C}{r_pL_1C} - \frac{R_1 + r_p}{\mu M - r_pR_1C - L_1}$$

$$\approx Mg_m/L_1C - 1/Mg_m \tag{98}$$

$$p_2 \approx \frac{R_1 + r_p}{\mu M - r_pR_1C - L_1} \approx 1/Mg_m \tag{99}$$

The primary current is of the form

$$i = A_1[\epsilon^{(Mg_m/L_1 - 1/Mg_m)t} - \epsilon^{t/Mg_m}] \tag{100}$$

If $Mg_m/L_1C \gg 1/Mg_m$, the time constant of the first term of Eq. (100) is positive and very small, whereas the time constant of the second term is much greater. Consequently the current increases rapidly from its initial value.

The plate current may be shown to be of the form

$$i_p = A_2\epsilon^{(Mg_m/L_1C - 1/Mg_m)t} - B\epsilon^{t/Mg_m} \tag{101}$$

in which A_2 is much larger than B. The magnitudes of A_1, A_2, and B depend upon the disturbance that initiates the current change.

In the derivation of Eqs. (100) and (101) the dependence of circuit and tube parameters upon current magnitudes was neglected. For values of plate current just above cutoff, the plate resistance is high and therefore g_m is low. Although the assumptions made in the derivation of Eqs. (100) and (101) are not satisfied, Eqs. (96) and (97) indicate that current caused by a small initial disturbance then dies out exponentially. If the magnitude of the negative grid-biasing voltage is reduced, however, g_m reaches a value at which an initial disturbance causes an exponentially increasing current. The rise of current is accompanied by rapid increase of g_m to the high value assumed in the derivation of Eqs. (100) and (101). As the current continues to rise, the high positive grid voltage accompanying the rise of primary current results in a diversion of current from the plate to the grid and therefore in a reduction of μ and g_m. At the same time, loading of the transformer secondary by the relatively low grid resistance causes a reduction of the effective primary inductance L_1 of the transformer. Although the reduction of μ may result in a violation of the assumptions made in the simplification of Eq. (97), these equations still show in a qualitative way that reduction of g_m and L_1 with increase of current causes the time constant of the first term of Eqs. (100) and (101) to increase and that of the second term to decrease. Eventually, therefore, the magnitude of the second term of these equations increases at least as rapidly as that of the first term, and the currents stop increasing and may start to decrease.

62.2. Analysis During Settling Periods. Because practical values of circuit and tube parameters are such that rise times as short as 10^{-9} sec are obtainable, the subsequent behavior of the circuit may be analyzed under the assumption that the tube may be replaced by a switch in series with the plate supply voltage V_{PP}, a constant voltage V_P', and the resistance r_p. The voltage V_P' and the resistance r_p are the direct plate voltage and the a-c plate resistance, respectively, at the termination of the abrupt rise of plate current. Inasmuch as the currents are observed to vary slowly subsequent to the initial rise, winding capacitance may be neglected. The equivalent circuit is then that of Fig. 221, in which r_g represents the a-c grid resistance, which is a function of the grid voltage. The winding resistances R_1 and R_2 have been neglected in comparison with the much larger resistances r_p and r_g.

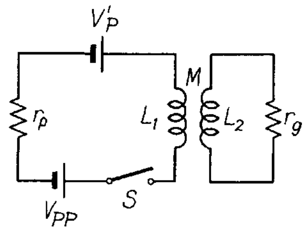


FIG. 221. Equivalent circuit for the common-cathode blocking oscillator during the time of high plate conduction.

Solution of the differential equations for the circuit of Fig. 221 yields the following expressions for the plate and grid currents under the assumption that the transformer coupling is perfect ($M = L_1/N$ and $L_2 = L_1/N^2$):

$$i_p = \frac{V_{PP} - V_P'}{r_p}\left[1 - \frac{r_g}{N^2 r_p + r_g}\,\epsilon^{-\frac{t}{L_1(1/r_p + N^2/r_g)}}\right] \tag{102}$$

$$i_g = \frac{N(V_{PP} - V_P')}{N^2 r_p + r_g}\,\epsilon^{-\frac{t}{L_1(1/r_p + N^2/r_g)}} \tag{103}$$

Equation (102) shows that the plate current continues to rise slowly following the initial abrupt rise, and that the grid voltage falls. When the tube is in its high-conduction state, the plate current varies relatively slowly with plate voltage, but the grid current rises rapidly with grid voltage. Consequently, r_p is high and r_g is low, and the time constant with which the plate and grid currents change approximates N^2L_1/r_g. As the grid current and voltage fall, r_g increases rapidly. Furthermore, although the primary and secondary ampere-turns are initially equal in magnitude and opposite in sign, the secondary ampere-turns decrease in magnitude as the transient dies out and hence the core becomes saturated by the flux produced by the primary current. Therefore L_1 decreases as the secondary current falls. The reduction of L_1 and increase of r_g cause the time constant to decrease. Consequently the secondary current falls, and the primary current rises, more rapidly than if they changed exponentially. In some practical circuits the secondary current is observed to fall nearly linearly.

When the grid voltage has fallen to a value at which the transconductance is again large enough so that the bracketed term in Eq. (100) increases with time, the current falls abruptly to zero.

63. Transistor Blocking Oscillators

63.1. Transistor Blocking-Oscillator Circuits. Transistor versions of the blocking oscillator are shown in Fig. 222. The common-emitter, common-base, and common-collector circuits of Figs. 222a, 222b, and 222c are equivalent to the common-cathode, common-grid, and common-plate circuits, respectively, of Figs. 218b, 219a, and 219b.[1] Figure 219d shows a variant of the common-base circuit in which a portion of the voltage produced across the 10,000-ohm collector load resistance is applied to the base by means of a tapped inductor.

To eliminate transient oscillations initiated in the transformer windings by the rapid fall of collector current during transition out of the high-current state, the secondary of the transformer may be shunted by a diode or a diode and a series resistor, as shown by the dotted lines in Figs. 222a and 222b. (The relatively low input resistance of the transistor following transition to the high-current state prevents transformer transient oscillation at that time.)

Although the transistor blocking oscillator functions in a manner that is basically the same as that of the analogous tube circuits, the speed of transition from the off to the on state in the transistor circuits is limited principally by delays caused by diffusion effects in the transistor, rather than by capacitance of the transformer windings or the electrodes. Another difference between transistor and tube circuits is the result of the fact that the base of the transistor is forward-biased during the entire transition period and the input conductance is therefore not negligible.

[1] P. Sulzer, *Electronics*, August, 1953, p. 170; T. A. Prugh, *Electronics*, January, 1955, p. 169; J. E. Flood, *Wireless Engineer*, **32**, 122 (May, 1955); W. S. Ekess, J. E. Deavenport, and K. J. Sherman, *Electronics*, November, 1955, p. 132.

63.2. Simplified Analysis. Before a detailed analysis of transistor blocking oscillators is presented, it is desirable to make a rough determination of the upper limit of the speed of transition by means of a greatly simplified analysis in which the circuits are considered as current amplifiers, the output of which is applied to the input. Thus, if the resistance R in the

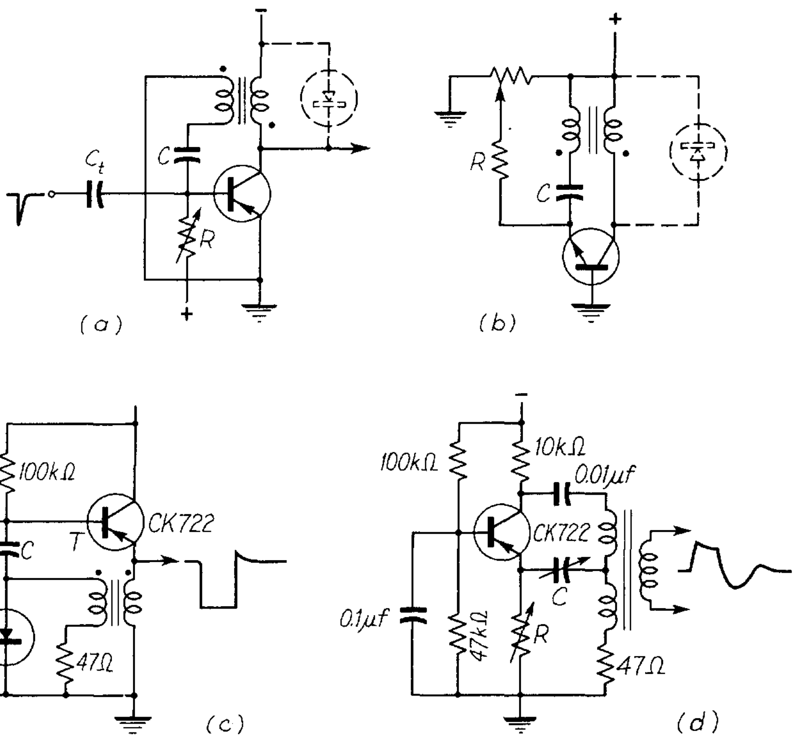


FIG. 222. Transistor blocking oscillators: (a) common-emitter; (b) common-base; (c) common-collector; (d) modified common-base.

circuit of Fig. 222a is large in comparison with h_{ib} and the time constant $h_{ib}C$ is large in comparison with the transition period, the circuit may be represented by the simplified circuit of Fig. 223 during transition. The magnetizing inductance of transformers used in transistor blocking oscillators is sufficiently high so that negligible magnetizing current builds up during the short transition period, and the coefficient of coupling k is nearly unity. Therefore $i_O \approx Ni_C$ as the collector current builds up from its initial value, which approximates zero. Furthermore, for values of N normally used, $R_1 + N^2(h_{ie} + R_2)$ is so small in comparison with $1/h_{oe}$ that the collector current is very nearly equal to the short-circuit collector current and

$i_O = Nh_{fe}i_B$. In the actual circuit of Fig. 222a the restriction is imposed that i_B must equal i_O at all instants during transition. In the simplified circuit of Fig. 223 this can be true only because h_{fe} becomes smaller as the rate of change of collector current increases. (It will be shown in Sec. 64 that the speed of transition may be reduced by transistor capacitance, which is neglected in this simplified circuit.) The dependence of h_{fe} upon rate of change of collector current may be taken into account by the use of Eq. (55). We may therefore write

$$\left(1 + \frac{1}{h_{fbo}} + \frac{p}{h_{fob}\omega_{\alpha b}}\right) i_C = i_B = i_O = Ni_C \tag{104}$$

where $p = d/dt$.

Solution of Eq. (104) leads to the following expression for the collector current at any instant following the onset of transition:

$$i_C = A\epsilon^{-\omega_{\alpha b}[1+(N+1)h_{fbo}]t} \tag{105}$$

in which A is a constant. Since h_{fbo} approximates -1 in the usual types of junction transistors, Eq. (105) may be reduced to the approximate form:

$$i_C \approx A\epsilon^{N\omega_{\alpha b}t} \tag{106}$$

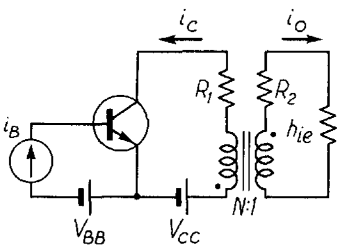


FIG. 223. Simplified equivalent circuit for the common-emitter blocking oscillator during transition.

Equations (105) and (106) indicate that, as long as the transition speed is not greatly dependent upon transistor or circuit capacitance, the speed increases with the alpha-cutoff frequency ω_{ab} of the transistor and with the transformer turns ratio N. It can be seen from Eq. (105) that the current increases with time (i.e., the exponent is positive) only if

$$N > \frac{1 + h_{fb}}{h_{fb}} = \frac{1}{h_{fe}}$$

The collector winding of the transformer must therefore have a greater number of turns than the base winding.

Similar analyses (Prob. 63.2–1) lead to the following approximations for the collector current during transition in common-base and common-collector blocking oscillators:

$$i_C \approx A\epsilon^{(N-1)\omega_{\alpha b}t} \qquad \text{(common-base)} \tag{107}$$

$$i_C \approx A\epsilon^{[N/(1-N)]\omega_{\alpha b}t} \qquad \text{(common-collector)} \tag{108}$$

These equations show that the speed of transition of the common-base circuit and of the common-collector circuit increases with the alpha-cutoff frequency. In the common-base circuit, N must exceed unity; the collector winding of the transformer must have a greater number of turns than the emitter winding. The speed of transition increases with N. In the common-collector circuit, N must be less than unity; i.e., the base winding must have a greater number of turns than the emitter winding. The speed of transition increases as N approaches unity. Because of the difficulty of maintaining high coefficient of coupling as the turns ratio is increased, it would appear that somewhat higher transition speed should be possible in the common-collector circuit than in either the common-emitter or the common-base circuit.

The constant A in Eqs. (106) to (108) is the value of the collector current when $t = 0$. If the circuit is initially biased beyond cutoff, therefore, the current cannot build up unless a triggering pulse causes some initial current to flow.

63.3. Transition Time. Usually the transition time, rather than the transition speed, is the more important consideration. The transition time decreases with increase of transition rate and with decrease of collector current in the high-conduction state. The factors that influence the current in the high-conduction state can be determined from the collector diagram of Fig. 224. Under the assumption that the transformer leakage inductance is negligible, the collector load in the common-emitter circuit during transition to the high-current state consists of the effective input resistance of the transformer, $R_1 + N^2(R_2 + h_{ie})$, where R_1 is the resistance of the collector winding and R_2 is that of the base winding. The increase in the slope of the load line in Fig. 224 with collector current is caused by decrease of the transistor input resistance h_{ie} as the base current i_B increases. Collector-current transition terminates when the collector voltage has fallen to saturation. The collector current in the common-emitter circuit at the end of transition is seen from Fig. 224 to be

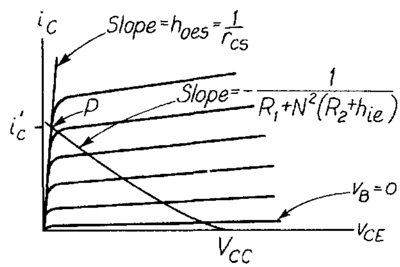


FIG. 224. Collector diagram for the common-emitter blocking oscillator.

$$i_C' = V_{CC}/[r_{cs} + R_1 + N^2(R_2 + h_{ie}')] \qquad (109)$$

where r_{cs} is the value of r_c in the saturation range and h_{ie}' is a value of h_{ie} near the midpoint of the load line. For the common-base and common-collector circuits, h_{ie}' in Eq. (109) must be replaced by h_{ib}' and h_{ic}', respectively. Equation (109) indicates that the value of the collector current at the termination of transition, and therefore the transition time, can be decreased by increasing R_1 or R_2 or, more practically, by adding resistance R_c, R_b, or R_e in series with the collector, the base, or the emitter. The addition of circuit resistance decreases the transition time only up to the point where the decrease in amplification causes a decrease in rate of change of current that offsets the reduction of current change. The transition time can also be decreased by decreasing V_{CC}. Since the current at every instant increases with the coefficient A in Eqs. (107) and (108), the transition time is also decreased by increasing the amplitude and duration of the triggering pulse.

It should be noted that lag in the response of the collector current to an increase of base current does not change the path of operation, but causes the collector current at any instantaneous value of base current to be smaller than if the change took place so slowly that diffusion delay were negligible. The effect upon the collector diagram can be approximated by compressing the characteristic curves downward at every instant during transition by a sufficient amount so that the current amplification Nh_{fe} at that instant is reduced to unity.

64. Linear Analysis of Transition of Transistor Blocking Oscillators

64.1. Transition to the High-Current State. Figure 225 shows approximate equivalent circuits of common-emitter, common-base, and com-

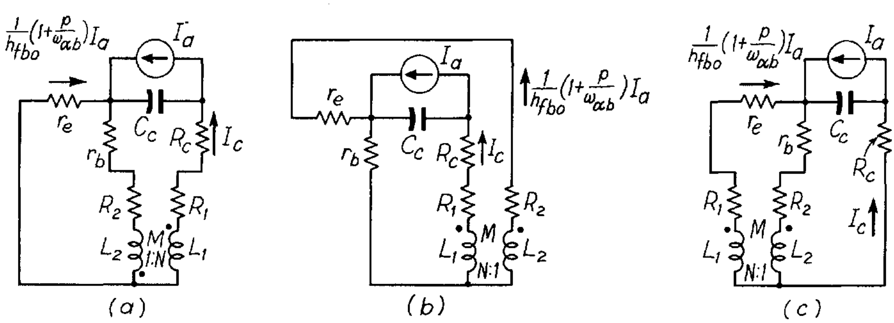


FIG. 225. Approximate equivalent circuits for transistor blocking oscillators applicable below the alpha-cutoff frequency: (a) common-emitter; (b) common-base; (c) common-collector.

mon-collector blocking oscillators that are valid at frequencies below and approaching the alpha-cutoff frequency.[1] Although Eqs. (106) to (108) indicate that transition time constants smaller than the reciprocal of the

[1] See, for example, R. F. Shea, editor, *Transistor Circuit Engineering,* Chap. 2, John Wiley and Sons, New York, 1957.

TABLE VIII. COEFFICIENTS FOR EQUATION (110)

COMMON EMITTER

a_4	$1 - k^2$
a_3	$(1 - k^2)\omega_{ab} + \dfrac{1}{L_1}[(N^2 + 2Nk + 1)r_e + N^2 r_b + R_c + R_1]$
a_2	$\dfrac{N^2}{L_1^2}[r_e(r_b + R_c + R_1) + r_b(R_c + R_1)$
	$+ \dfrac{\omega_{ab}}{L_1}[(N^2 + 2Nk + 1)r_e + N^2 r_b + R_c + R_1] + \dfrac{1}{L_1 C_c}$
a_1	$\dfrac{N^2}{L_1^2}\left\{\omega_{ab} r_e(r_b + R_c + R_1) + \dfrac{r_e}{C_c} + r_b\left[\omega_{ab}(R_c + R_1) + \dfrac{1}{C_c}\right]\right\} \dfrac{\omega_{ab}}{L_1 C_c}[1 + (Nk + 1)h_{fbo}]$
a_0	$\dfrac{N^2\omega_{ab}}{L_1^2 C_c}[r_e + r_b(1 + h_{fbo})]$

COMMON BASE

a_4	$1 - k^2$
a_3	$(1 - k^2)\omega_{ab} + \dfrac{1}{L_1}[N^2(r_e + R_2) + (N^2 - 2Nk + 1)r_b + R_c + R_1]$
a_2	$\dfrac{N^2}{L_1^2}(r_e + R_2)(r_b + R_c + R_1) + r_b(R_c + R_1)$
	$+ \dfrac{\omega_{ab}}{L_1}[N^2 r_e + (N^2 - 2Nk + 1)r_b + R_c + R_1] + \dfrac{1}{L_1 C_c}$
a_1	$\dfrac{N^2}{L_1^2}\left\{\omega_{ab} r_e(r_b + R_c + R_1) + \dfrac{r_e}{C_c} + r_b\left[\omega_{ab}(R_c + R_1) + \dfrac{1}{C_c}\right]\right\} + \dfrac{\omega_{ab}}{L_1 C_c}(1 + Nkh_{fbo})$
a_0	$\dfrac{N^2\omega_{ab}}{L_1^2 C_c}[r_e + r_b(1 + h_{fbo})]$

COMMON COLLECTOR

a_4	$1 - k^2$
a_3	$(k^2 - 1)\omega_{ab} + \dfrac{1}{L_1}[r_e + R_1 + N^2 r_b + (N - 1)^2 R_c]$
a_2	$\dfrac{N^2}{L_1^2}[r_e + R_1)(r_b + R_c) + r_b R_c]$
	$+ \dfrac{\omega_{ab}}{L_1}[r_e + R_1 + N^2 r_b + (N - 1)^2 R_c] + \dfrac{(N - 1)^2}{L_1 C_c}$
a_1	$\dfrac{N^2}{L_1^2}\left[\omega_{ab} r_e(r_b + R_c) + \dfrac{r_e + R_1}{C_c} + r_b\left(\omega_{ab} R_c + \dfrac{1}{C_c}\right)\right]$
	$+ \dfrac{\omega_{ab}}{L_1 C_c}[(N - 1)^2 + (1 - Nk)h_{fbo}]$
a_0	$\dfrac{N^2\omega_{ab}}{L_1^2 C_c}[r_e + R_1 + (1 + h_{fbo})r_b]$

alpha-cutoff frequency are to be expected, the more complete analysis based upon the circuits of Fig. 225 yield equations which indicate that the collector capacitance C_c prevents reduction of the transition time constant below $1/\omega_{ab}$. For this reason, and because of the fact that any linear equivalent circuit is at best only an approximation, the relatively simple circuits of Fig. 225 are adequate.

During most of the transition period to the high-current state, the emitter-base junction of the transistor is forward-biased to such an extent that the equivalent emitter resistance is of the order of only a few ohms. The emitter diffusion capacitance C_e is approximately equal to $1/1.2r_e\omega_{ab}$.[2] It will be shown that the results obtained when this capacitance is neglected indicate that other resistances with which r_e is effectively in series make its effect upon the circuit negligible. It is therefore valid to assume that C_e is negligible in order to simplify the analysis.

Analysis of the circuits of Fig. 223 with C_e neglected lead to the following characteristic equation:

$$a_4p^4 + a_3p^3 + a_2p^2 + a_1p + a_0 = 0 \tag{110}$$

The coefficients of Eq. (110) are listed in Table VIII. These coefficients were obtained with the aid of the relations $M^2 = k^2L_1L_2$, $L_2 = L_1/N^2$, $M = kL_1/N$. The winding resistances R_1 and R_2, which are of the order of a few ohms, have been neglected where they are in series with r_b, which is of the order of 100 ohms, but not where they are in series with r_e, which is of the order of an ohm or two, or with R_c, which may be zero.

Typical values of transformer parameters are $k^2 = 0.9$ to 0.98, $N = 1.5$ to 10 for common-emitter and common-base circuits and approximately unity for common-collector circuits, $L_1 = 10^{-4}$ to 10^{-3} henry, and R_1 and $R_2 = 1$ to 5 ohms. Typical transistor parameters are $h_{fbo} = -0.98$, $r_e = 1$ to 2 ohms, $r_b = 100$ to 200 ohms, $C_c = 10^{-11}$ farad, and $\omega_{\alpha bo} = 10^7$ rps. Substitution of these values into the coefficients listed in Table VIII shows that a_0 and terms involving $L_1{}^2$ and r_e may be neglected in an approximation, and that R_1 and R_2 may be assumed to be zero. The characteristic equation then reduces to the following forms for the common-emitter, common-base, and common-collector circuits, respectively:

$$\begin{aligned}(1 - k^2)p^3 &+ \left[(1 - k^2)\omega_{\alpha b} + \frac{1}{L_1}(N^2r_b + R_c)\right]p^2 \\ &+ \left[\frac{\omega_{\alpha b}}{L_1}(N^2r_b + R_c) + \frac{1}{L_1C_c}\right]p - \frac{\omega_{\alpha b}}{L_1C_c}[1 + (Nk + 1)h_{fbo}] = 0\end{aligned} \tag{111}$$

[2] Shea, *op. cit.*, Chap. 2.

$$(1 - k^2)p^3 + \left[(1 - k^2)\omega_{\alpha b} + \frac{1}{L_1}(N - 1)^2 r_b + \frac{R_c}{L_1}\right] p^2$$
$$+ \left\{\frac{\omega_{\alpha b}}{L_1}[(N - 1)^2 r_b + R_c] + \frac{1}{L_1 C_c}\right\} p + \frac{\omega_{\alpha b}}{L_1 C_c}(1 + Nkh_{fbo}) = 0 \quad (112)$$

$$(1 - k^2)p^3 + \left[(k^2 - 1)\omega_{\alpha b} + \frac{N^2 r_b}{L_1} + \frac{(N - 1)^2 R_c}{L_1}\right] p^2$$
$$+ \left\{\frac{\omega_{\alpha b}}{L_1}[N^2 r_b + (N - 1)^2 R_c] + \frac{(N - 1)^2}{L_1 C_c}\right\} p$$
$$+ \frac{\omega_{\alpha b}}{L_1 C_c}[(N - 1)^2 + (1 - Nk)h_{fbo}] = 0 \quad (113)$$

For abrupt transition from the off state to the on state, the collector current should be of the form of an exponential with positive exponent. One root of Eqs. (111) to (113) must therefore be positive. This can be true only if the final term of the characteristic equation is negative. For the common-emitter circuit, this requirement is satisfied if $-Nkh_{fbo}/(1 + h_{fbo}) > 1$, i.e., if $NKh_{fe} > 1$. The criterion for a growing-exponential solution in the common-base circuit is that $-Nkh_{fbo} > 1$. In the common-collector circuit, Nk must be less than unity and $|h_{fbo}| > (N - 1)^2/(1 - Nk)$. If k approximates unity, the latter criterion reduces to $N/(1 + h_{fbo}) > 1$, which is equivalent to $-Nh_{fco} > 1$. It follows that the turns ratio must exceed approximately 0.02 in the common-emitter circuit, and unity in the common-base circuit, and that it must lie in the range from approximately 0.02 to unity in the common-collector circuit. It was pointed out in Sec. 63.2 that the speed of transition increases with N at a fixed value of k up to the point where the speed is limited by capacitance.

64.2. Solution of Approximate Characteristic Equations. Examination of Eqs. (111) to (113) shows that the magnitude of the positive root, and therefore the rate of increase of current, increases with the coupling coefficient k. This is to be expected, since the open-loop amplification of the circuit increases with the primary-to-secondary coupling and the rate of change of current at all instants during transition must therefore be greater in order to satisfy the requirement that the loop amplification be unity in the closed circuit. Although practical values of k are not ordinarily large enough to justify neglecting the first term of the characteristic equation, it is instructive to examine the ideal case in which the coupling is perfect. If the two terms involving $1 - k^2$ are neglected and h_{fbo} is assumed to be -1, Eqs. (111) to (113) reduce to:

(common-emitter circuit)

$$p^2 + \left(\omega_{\alpha b} + \frac{1}{(N^2 r_b + R_c)C_c}\right)p - \frac{N\omega_{\alpha b}}{(N^2 r_b + R_c)C_c} \approx 0 \qquad (114)$$

(common-base circuit)

$$p^2 + \left(\omega_{\alpha b} + \frac{1}{[(N-1)^2 r_b + R_c]C_c}\right)p + \frac{(1-N)\omega_{\alpha b}}{[(N-1)^2 r_b + R_c]C_c} \approx 0 \qquad (115)$$

(common-collector circuit)

$$p^2 + \left(\omega_{\alpha b} + \frac{(N-1)^2}{[N^2 r_b + (N-1)^2 R_c]C_c}\right)p + \frac{N(N-1)\omega_{\alpha b}}{[N^2 r_b + (N-1)^2 R_c]C_c} \approx 0 \qquad (116)$$

Equations (114) to (116) are all of the following general form:

$$p^2 + \left(\omega_{\alpha b} + \frac{\eta_1}{\tau}\right)p - \frac{\omega_{\alpha b}\eta_2}{\tau} \approx 0 \qquad (117)$$

Equation (117) may be solved by application of the quadratic formula, the radical being expanded by means of the binomial expansion. Since $\dfrac{4\omega_{\alpha b}\eta_2\tau}{(\tau\omega_{\alpha b} + \eta_1)^2} \ll 1$, all but the first two terms of the expansion may be neglected, and the following approximate expressions for the roots obtained:

$$p_+ \approx \frac{\eta_2}{\eta_1/\omega_{\alpha b} + \tau} \qquad (118)$$

$$p_- \approx -\left(\omega_{\alpha b} + \frac{\eta_1}{\tau}\right) \qquad (119)$$

The growing transient corresponding to the positive root is therefore

$$i_C = A\epsilon^{\eta_2 t/(\eta_1\omega_{\alpha b} + \tau)} \qquad (120)$$

Substitution of the applicable expressions for η_1, η_2, and τ into Eq. (120) shows that, for the ideal common-emitter, common-base, and common-collector circuits with perfect coupling, the speed of transition is increased by an increase of N in the allowable range of N. In any practical circuit, however, the coupling coefficient is not only less than unity, but tends to decrease as N departs from unity. As suggested in Sec. 63.2, this fact would appear to indicate that more rapid transition should be possible in the common-collector circuit than in either the common-emitter or common-base circuits.

Substitution of typical values of transformer and transistor parameters into Eq. (120) shows that the transition time constant is determined principally by the alpha-cutoff frequency ω_{ab}, but that it is increased, and the speed of transition therefore reduced, by the collector capacitance C_c.

64.3. Graphical Solution of Characteristic Equations. Although the third-order Eqs. (111) to (113) cannot be solved directly, graphical methods of solution are possible for known values of transistor parameters. One convenient method consists of replacing p in Eqs. (111) to (113) by the normalized variable p/ω_{ab} by dividing through by ω_{ab}, and then solving the equations for L_1. For known values of R_c and the transistor parameters and selected values of p_+/ω_{ab}, L_1 may then be plotted as a function of N.[3] Typical curves obtained in this manner for zero R_c are shown in Fig. 226.[3] It is apparent from Fig. 226 that the speed of transition of the common-base circuit does not increase continuously with N when k is less than unity, but passes through a maximum, the value of which increases with L_1.

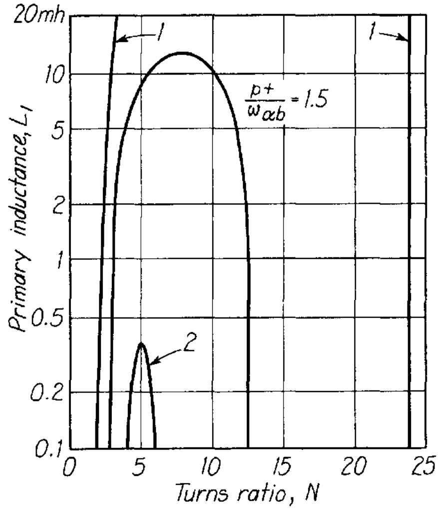


FIG. 226. Typical curves of L_1 vs. N at three values of p_+/ω_{ab}.

64.4. Effect of Loading. Solution of the equations of the equivalent circuits of Fig. 225 with a resistance shunting the transformer primary or secondary leads to characteristic equations that indicate that loading reduces the positive root of the equations.[4] Loading therefore reduces the speed of transition. This is to be expected, inasmuch as conductance shunting the transformer diverts current from the transistor input and thus reduces the current amplification of the circuit. Consequently, the amplification falls to unity at a lower rate of change of collector current, and the rate of change of current is less at all instants during transition.

64.5. Method of Triggering. It was shown in Sec. 63.3 that the transition time is decreased by an increase in the magnitude or duration of the triggering pulse. The transition time is not dependent upon the point in the circuit at which the triggering pulse is applied, provided that the pulse produces the same initial increment of collector current, and that the impedance of the source of the triggering pulse is such that it does not affect the time

[3] J. G. Linvill and R. H. Mattson, *Proc. I.R.E.*, **43**, 1632 (November, 1955).
[4] Linvill and Mattson, *loc. cit.*

constant appreciably. Thus, a low-impedance voltage source may be connected in series with one of the transistor electrodes, or a high-impedance current source between two of the electrodes. For the same transition time, the amount of triggering energy required is about the same, regardless of the type of triggering used.[5]

65. Linear Analysis of Transistor Blocking Oscillators During Conduction

65.1. Analysis of Equivalent Circuit. Prior to the onset of transition to the high-current state, the transistor emitter-base junction is reverse-biased and transistor currents are sufficiently small in comparison with those which flow after transition that they may be assumed to be zero. If C and R are infinite in Fig. 222, transition takes place so rapidly that the subsequent behavior of the circuit is essentially the same as though the supply voltages were suddenly applied to the equivalent circuits of Fig. 227,

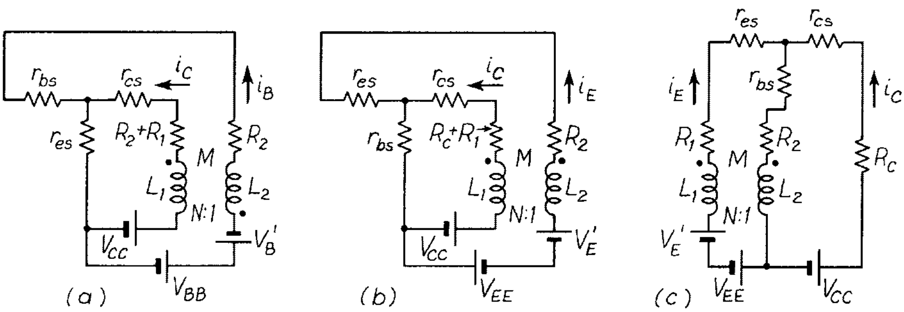


FIG. 227. Approximate equivalent circuits applicable following transition to the high-current state: (a) common-emitter circuit; (b) common-base circuit; (c) common-collector circuit.

in which the resistances r_{bs}, r_{cs}, and r_{es} are the equivalent tee-circuit resistances of the transistor in the saturation range. The emitter resistance r_{es}, being of the order of only an ohm or two, may be neglected in comparison with r_{cs} and r_{bs}, which are of the order of 5 to 25 ohms and 50 to 75 ohms. The equivalent reverse voltage V_B' is required because the input characteristics of the transistor are curved, as shown in Fig. 228 (see also Sec. 6.2). The magnitude of V_B' is of the order of 0.1 to 0.3 volt. Since change of base current does not affect the collector current in the saturation range, the equivalent circuit does not contain an equivalent current generator; i.e., h_{fe} is zero.

The equivalent circuit for the common-emitter circuit following transition to the on state may be simplified to the form of Fig. 229, in which $r_{es} = 0$,

[5] Linvill and Mattson, *loc. cit.*

$R_1' = R_1 + r_{cs} + R_c$, $R_2' = R_2 + r_{bs}$, and $V_{BB}' = V_{BB} - V_B'$. Solution of the equations for the circuit of Fig. 229 subject to the conditions that the primary and secondary ampere turns are initially equal in magnitude and

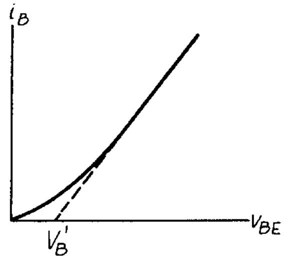


FIG. 228. Input characteristic of a transistor in the common-emitter connection.

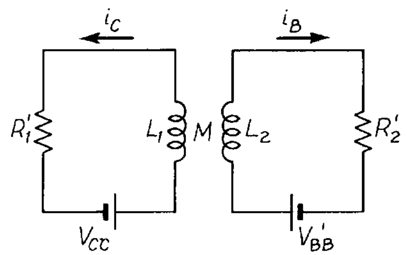


FIG. 229. Simplified equivalent circuit of the common-emitter blocking oscillator following transition to the high-current state.

of opposite sign (the initial flux is zero) and that steady-state primary and secondary currents are V_{CC}/R_1' and V_{BB}'/R_2', and under the assumption that the coupling is perfect leads to the following expression for the collector and base currents:

$$i_B = \frac{V_{BB}'}{R_2'}\left[1 + \frac{R_1'/N}{R_1'/N - NR_2'}\,\epsilon^{-t/\tau}\right] + \frac{V_{CC}}{NR_2' + R_1'/N}\,\epsilon^{-t/\tau} \tag{121}$$

$$i_C = \frac{V_{BB}'}{NR_2' + R_1'/N}\,\epsilon^{-t/\tau} + \frac{V_{CC}}{R_1'}\left[1 - \frac{NR_2'}{NR_2' + R_1'/N}\,\epsilon^{-t/\tau}\right] \tag{122}$$

where

$$\tau = L_2\left(\frac{N^2}{R_1'} + \frac{1}{R_2'}\right) \tag{123}$$

Equation (121) shows that the collector current continues to increase after its initial rise during transition, but that the base current decreases after its initial rise. (Note that V_{BB}' is negative in a monostable circuit and that the magnitude of V_{BB}' is much less than that of V_{CC}.) The path of operation in the collector diagram of Fig. 224 after transition to point P first follows the saturation characteristic. As the collector current rises and the base current falls, however, values are eventually reached that correspond to a point to the right of the saturation curve. The collector current then again becomes dependent upon the base current. The falling base current reduces the rate of rise of collector current and finally causes the collector current to start to fall. When the operating point has moved far enough to the right of the saturation curve so Nkh_{feo} again exceeds unity, the action becomes cumulative and the current falls abruptly to zero in a manner similar to that during the transition to the high-current state. The value of k being approximately unity, the period during which the circuit remains in the high-current state approximates the time required for the collector and base currents to reach values such that $i_C = Nh_{feo}i_B$.[1] (This approximation is based upon the assumption that h_{feo} does not vary during transition.) The length of the pulse generated may therefore be determined by combining this relation with Eqs. (121) to (123).

Solution of the equations for the circuits of Figs. 227b and 227c leads to equations that are similar to those of Eqs. (121) to (123). The time constants during the high-conduction period are given by the approximate relations:

$$\tau \approx L_1 \frac{(N-1)^2 r_{bs} + R_1' + N^2 R_2'}{N^2[r_{bs}(R_1' + R_2') + R_1'R_2']} \qquad C\text{-}B \qquad (124)$$

in which

$$R_1' = R_1 + r_{cs} + R_c \quad \text{and} \quad R_2' = R_2 + r_{es}$$

$$\tau \approx L_1 \frac{(N-1)^2 (r_{cs} + R_c) + R_1' + N^2 R_2'}{N^2[(r_{cs} + R_c)(R_1' + R_2') + R_1'R_2'} \qquad C\text{-}C \qquad (125)$$

in which

$$R_1' = R_1 + r_{es} \quad \text{and} \quad R_2' = R_2 + r_{bs}$$

Increase of the transistor input resistance as the secondary current decays, as well as saturation of the transformer core as the ratio of primary to secondary ampere turns increases, causes the waveforms of the primary and secondary currents to depart from the exponential forms predicted by Eqs. (121) and (122) and similar equations for the common-base and common-collector circuits. The current variation may, in fact, be more nearly linear than exponential.

65.2. Use of R-C Timing Circuit. The analysis of Sec. 65.1 applies to the circuits of Fig. 225 only when the time constant RC is large in comparison with the time constants given by Eqs. (123) to (125). If RC is considerably smaller than the value given by one of these equations, on the other hand, the pulse length is determined primarily by RC. The addition of R and C to the circuit therefore reduces the minimum pulse length below that attainable when the resistor and capacitor are omitted (R and C made

[1] D. O. Pederson, Univ. of California, Inst. of Research, Series No. 60, Issue No. 161, September 10, 1956. Since $i_O \approx Ni_C$ when $k \approx 1$, the relation $i_C = Nh_{feo}i_B$ states that the current amplification of the transistor and transformer in the circuit of Fig. 223 is unity.

infinite), and allows the pulse length to be varried by change of resistance or capacitance. The pulse length may also be varied by changing V_{BB}, because the steady-state value toward which i_B falls, and therefore the time taken for i_B to become equal to i_C/kNh_{feo}, depend upon V_{BB}.[2]

65.3. Transition to Low-Current State. During the transition from the high-current state, a high reverse bias is developed across the emitter-base junction. Eqs. (123) to (125) are applicable to the period following transistion from the high-current state if r_{bs} is replaced by the reverse resistance of the emitter-base junction. Because this resistance is of the order of thousands of ohms unless the reverse bias is sufficient to cause breakdown of the junction, the time constant during the period following transition from the high-current state is much smaller than that following transition to the high-current state in common-emitter circuits in which the timing resistor R and capacitor C are not used.

If reverse base-biasing supply voltage is used, the transistor remains cut off following transition from the high-current state until transition is again initiated by a triggering pulse. If forward base-biasing supply voltage is used, on the other hand, the base voltage rises above cutoff at some time following transition from the high-current state, and the circuit again switches to the high-current state without the application of a triggering pulse. The circuit is therefore astable and the collector current consists of periodic pulses.

66. Nonsaturating Transistor Blocking Oscillators

66.1. Use of Diodes to Reduce Saturation. The rate at which the collector current falls during the transition from the high-current state is reduced by the presence of stored minority carriers when the collector voltage is allowed to fall into the saturation range. Some decrease in pulse fall time may therefore be achieved by the use of a diode to limit the minimum collector voltage, as shown in the circuit of Fig. 230.[1] The manner in which this circuit functions can be explained with the aid of the collector diagram of Fig. 231, in which the characteristic curves are drawn for constant values of emitter voltage. At low values of collector current the diode is reverse-biased and has negligible effect upon the operation. The path of operation is therefore the load line corresponding to the effective input resistance R_l of the primary, where $R_l = R_1 + N^2(R_2 + h_{ib})$. When the col-

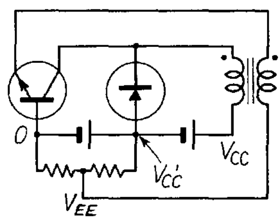


FIG. 230. Nonsaturating common-base blocking oscillator.

[2] S. H. Dinsmore and D. O. Pederson, Univ. of California, Inst. of Eng. Research, Series No. 60, Issue No. 190, July 25, 1957.

[1] Linvill and Mattson, *loc. cit.*

lector voltage reaches V_{CC}', however, the diode starts to conduct and the forward resistance of the diode is shunted across the primary.

In the ideal case in which the diode is assumed to have zero forward resistance, the diode short-circuits the transformer primary at currents greater than that corresponding to the intersection P of the load line with the constant-voltage line $v_C = V_{CC}'$ and thus prevents further increase of emitter voltage. Under the assumption that there is negligible buildup of the transformer magnetizing current during the short transition time, the emitter voltage when the operating point reaches P approximates $(V_{CC} - V_{CC}')/N$. Because of the delay in the response of the transistor to increasing base voltage, the collector current at P is much lower than it would be with the same emitter voltage under static conditions, and the static characteristic for the emitter voltage $(V_{CC} - V_{CC}')/N$ lies far above P. The collector current therefore continues to increase to the value at P' of Fig. 231a, where

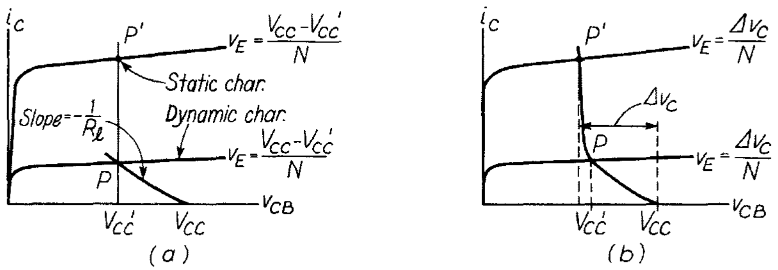


FIG. 231. Collector diagram for the circuit of Fig. 230: (a) for an ideal diode having zero forward resistance; (b) for an actual diode. $R_l = R_1 + N^2(R_2 + h_{ib})$.

the static characteristic intersects the constant-voltage line $v_C = V_{CC}'$, the additional current flowing through the diode. The rate at which the current builds up in this range is determined by diffusion phenomena in the transistor, and is lower than the rate below P.

In the actual circuit the resistance of the diode allows the collector voltage to continue to decrease somewhat as the current rises above the value at P. If the diode resistance is small in comparison with $R_1 + N^2(R_2 + h_{ib})$, the path of operation in this range resembles the mirror image of the diode characteristic, as shown in Fig. 231b. The collector current at the termination of transition corresponds to the intersection of the path of operation with the static characteristic for $v_E = \Delta v_C/N$. The increasing voltage across the diode as the current rises causes the emitter voltage to continue to rise and thus increases the rate at which the collector current rises. The rise time would not be expected to be shorter than with zero diode resistance, for the emitter voltage at P', and hence the collector current at P', increase rapidly with diode resistance.

It can be seen from Fig. 231b that the diode resistance must be considerably lower than $R_1 + N^2(R_2 + h_{ib})$ in order to be effective in preventing saturation, and that V_{CC}' should not be too much smaller than V_{CC}. Lower diode resistance can be achieved by the use of a breakdown diode, rather than the usual type of diode. The breakdown diode has the additional advantage of much smaller minority-carrier-storage effects.

66.2. Behavior Following Transition to the High-Current State. Following transition to P' the primary current increases and the secondary current decreases, in the same manner as in the saturating circuits of Fig. 222. Reduction of emitter (secondary) current causes the collector current

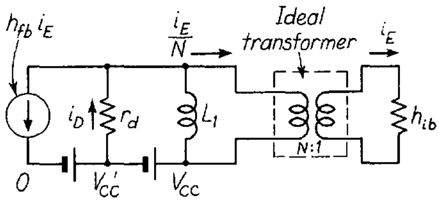


FIG. 232. Approximate equivalent circuit applicable following transition of the circuit of Fig. 230 to the high-current state.

to fall. The diode current, which is the difference between the collector current and the primary current, falls and would eventually become $-(V_{CC} - V_{CC}')/(r_d + R_1)$ at zero collector current if the diode acted like a constant nonrectifying resistance r_d. As the diode current falls, however, its resistance increases and at some instant before the diode current passes through zero the resistance becomes high enough so that the loop amplification of the circuit exceeds unity. Rapid transition to cutoff then takes place.

An approximate equivalent circuit that applies during the period following the transition to the high-current state is shown in Fig. 232. (The parameter h_{rb} is assumed to be zero and h_{ob} to be negligible in comparison with $1/r_d$ and $1/N^2h_{ib}$. The coupling is also assumed to be perfect.) Solution of the equations for this circuit leads to the following expression for the root of the characteristic equation (see also Prob. 66.2–1):

$$p_1 = -\frac{Nr_d(h_{ib} + R_2 + R_1/N^2)}{L_1[r_d(h_{fbo} + 1/N) + N(h_{ib} + R_2 + R_1/N^2)]}$$

$$\approx -\frac{Nr_dh_{ib}}{L_1[r_d(h_{fbo} + 1/N) + Nh_{ib}]} \tag{126}$$

Since h_{fbo} is negative and N exceeds unity, p_1 is negative only if

$$r_d < N^2h_{ib}/|1 + Nh_{fbo}|,$$

infinite if r_d equals this critical value, and positive if r_d exceeds this value.

If the perveance of the diode is so low that r_d exceeds the critical value at the termination of transition to the high-current state, but high enough to prevent the collector voltage from falling to saturation, the currents following this transition are growing exponentials. The growing rate of fall of collector current indicates that transition to the high-current state is at once followed by transition to cutoff. The collector-current pulse then does not have a flat portion, and its duration is the sum of the two transition times.

If the perveance of the diode is high enough so that r_d is less than the critical value at the termination of transition to the high-current state, on the other hand, p_1 is negative and the currents following this transition are decaying exponentials. As the diode current falls, r_d increases and the magnitude of p_1 increases. If h_{fb} remained constant at the value h_{fbo} and the collector and circuit capacitances had no effect, the rates of change of the currents would become infinite at a value of diode current such that r_d equaled the critical value. Transition to cutoff may be assumed to start at this instant. Typical values of h_{fbo}, h_{ib}, and N are -0.98, 35 ohms, and 2, respectively. For these values the critical value of r_d at which transition to cutoff begins is 146 ohms, which is obtained at a relatively low value of current in a diode suitable for this application. Thus transition to cutoff may be assumed to begin when the diode current becomes zero.

Actually, as the diode current approaches the critical value, the rapid change of collector current reduces $|h_{fb}|$ below the static value. This reduction and the effect of collector capacitance limit the rate of change of currents during transition, as explained in Sec. 64.

66.3. Conduction Period. The diode current at the termination of transition to the high-current state is the difference between the collector current at point P' of Fig. 231a and that at point P. If $h_{ib} \gg R_1$ and R_2, the diode current at this instant is (Prob. 66.3–1) (note that h_{fbo} is negative and $N > 1$):

$$i_{Do} \approx \frac{V_{CC}' - V_{CC}}{h_{ib}'N}(h_{fbo} + 1/N) \tag{127}$$

where h_{ib}' is the value of the short-circuit input resistance h_{ib} at a point near the midpoint of the section of the path of operation below P. If the diode behaved like a nonrectifying constant resistance, the diode current would decay toward the value $-(V_{CC} - V_{CC}')/(r_{do} + R_1)$, where r_{do} is the initial value of r_d, and the decay curve would be of the form shown by the solid curve of Fig. 233. The following approximate expression for the diode current subsequent to the termination of transition to the high-current state may be obtained by inspection of Fig. 233:

$$i_D = V_{CC} - V_{CC}' \left[\frac{1}{r_d + R_1} + \frac{h_{fbo} + 1/N}{h_{ib}'N} \right] \epsilon^{-p_1 t} - \frac{V_{CC} - V_{CC}'}{r_d + R_1} \tag{128}$$

For typical values of h_{fbo}, h_{ib}, r_d, R_1, and N, the initial diode current is so small relative to the magnitude of the negative current toward which the current decays that current may be assumed to fall nearly linearly for an appreciable portion of the period of conduction. As pointed out in the preceding section, however, increase of r_d causes the rate of decay to increase toward a value limited by decrease in h_{fb}, and the decay curve is of the general form of the dotted curve of Fig. 233. Under the assumption that the

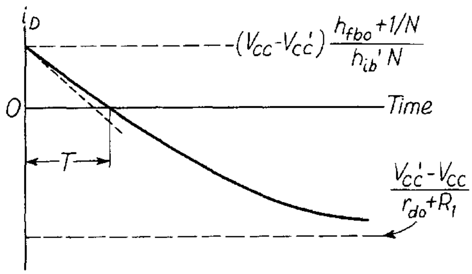


FIG. 233. Curve of diode current vs. time following transition of the circuit of Fig. 230 to the high-current state.

current decays linearly to zero, the period during which the circuit remains in the high-conduction state is (Prob. 66.2–1c):

$$T \approx -\frac{(h_{fbo}N + 1)L_1}{N^2 h_{ib}} \tag{129}$$

As in other blocking oscillators, the conduction period may be shortened by the addition of R-C timing circuits.

67. *Nonlinear-Circuit Simulators*

67.1. Advantages of Diode Circuits. Need frequently arises in analog computers, electronic instruments and controls, and other electronic circuits for circuit elements that have special nonlinear current-voltage relations. A number of special circuits have been devised in which the amplifying properties of vacuum tubes or transistors or the physical properties of vacuum or semiconductor devices have been used to provide continuous parabolic, exponential, or other special current-voltage characteristics. These circuits generally have the disadvantage of relatively high complexity or dependence of their characteristics upon the characteristics of individual active circuit elements. It is obviously desirable to make use of circuits in which the vacuum tubes or semiconductor devices serve only as switches and in which the current-voltage characteristic is dependent almost solely upon passive circuit elements. For this reason combinations of resistances and biased diodes are used in order to approximate a desired current-voltage curve by means of a number of straight-line segments. For some purposes one or two diodes and resistors suffice to provide a sufficiently close approximation to the desired curve. The maximum departure of the actual characteristic from the desired characteristic can be reduced to any desired degree by the addition of more circuit elements.

67.2. Characteristics of Multiple-Branch Circuits. In general, biased-diode circuits may contain combinations of parallel branches and series branches. Since the voltage across parallel branches is common and the total current is the sum of the individual branch currents, the resultant current-voltage characteristic of parallel circuits of known characteristics can be conveniently derived graphically by the simple process of adding the currents of the individual characteristic curves at equal values of voltage. Similarly, since the current through elements connected in series is common and the total voltage across the combination is equal to the sum of the individual voltages, the resultant current-voltage characteristic of a series combination of circuits of known current-voltage characteristics can be derived by adding the voltages of the individual characteristics at equal values of current over the range in which all the component circuits conduct.

The current-voltage characteristic of any diode-resistance network can be displaced parallel to the voltage axis by means of a constant voltage in series with the network. Similarly, the characteristic can be displaced parallel to the current axis by a constant-current source in parallel with the network. Although a constant-current source can be approximated by means of a self-biased pentode or transistor, usually a high resistance in series with a voltage source is adequate. If necessary, the effect of shunting the network by the additional finite resistance can be compensated by small changes in the network resistances.

67.3. Basic Diode Circuits. The basic voltage-biased resistance-diode circuits and their current-voltage characteristics are shown in Figs. 234a and 235a. Under the assumption that the diode is a perfect rectifier and that the diode perveance and the circuit resistance are sufficiently high so that the voltage across the diode is negligible in comparison with that across the resistance when the diode is forward-biased, the current-voltage characteristics of the circuit are of the form shown in Figs. 234b and 235b. As the resistance R is reduced, the forward characteristic of the circuit becomes steeper, and it gradually departs from approximate linear form, approaching

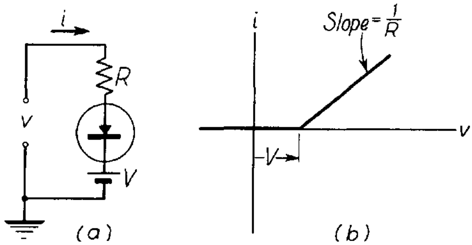


FIG. 234. (a) Voltage-biased resistance-diode circuit; (b) current-voltage characteristic of the circuit.

the form of the diode characteristic at very low values of resistance. Reversal of the biasing voltage displaces the break point to the left of the origin.

Current-biased circuits are formed by paralleling the circuit of Fig. 234 or that of Fig. 235 by a resistance in series with a voltage source, as shown

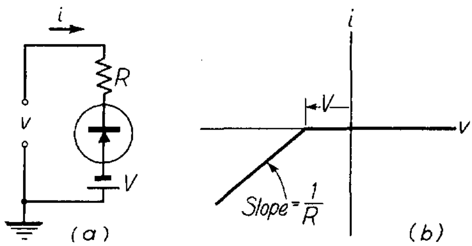


FIG. 235. (a) Voltage-biased resistance-diode circuit; (b) current-voltage characteristic of the circuit.

in Fig. 236a. The current-voltage characteristic of the circuit of Fig. 236a, shown by curve (3) of Fig. 236b, may be constructed by inspection or by adding the current values of curve (1) for branch 1 to the current values of curve (2) for branch 2 at equal values of voltage (Prob. 67.3–1). The characteristic may be displaced horizontally by changing the magnitude or

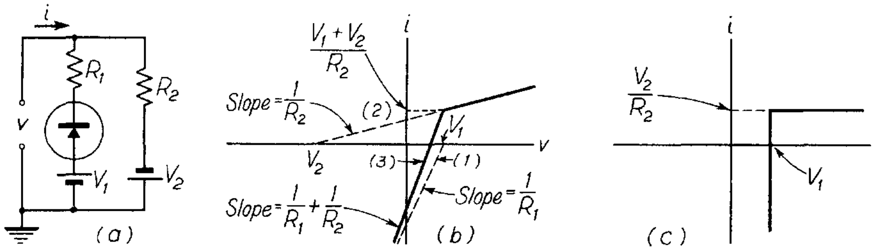


FIG. 236. (a) Current-biased resistance-diode circuit; (b) current-voltage characteristic of the circuit.

polarity of V_1 or by an additional voltage source in series with the circuit; it may be displaced vertically by changing the magnitude or polarity of V_2. If V_2 and R_2 are made very large, branch 2 of the circuit approaches a constant-current source I and the slope of the upper branch of the curve approaches zero. If R_1 is zero, the lower branch of the curve becomes the diode characteristic, displaced from the origin, and approximates a vertical line if the voltage scale is large and the current scale small. The characteristic obtained with zero R_1 and infinite R_2 is shown in Fig. 236c. If the polarities of the diode and of both voltages are reversed, the characteristics of Figs. 236b and 236c are rotated 180 degrees about the origin.

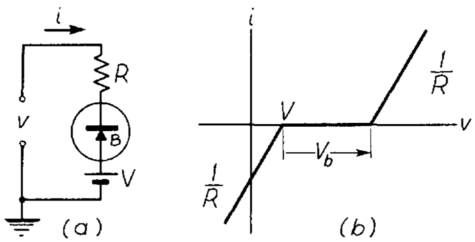


FIG. 237. (a) Breakdown-diode circuit; (b) current-voltage characteristic of the circuit. (V_b is the diode breakdown voltage.)

Figure 237 shows the current-voltage characteristic of a breakdown diode in series with a resistance. The characteristic may be made symmetrical about the origin by making $V = -V_b/2$, where V_b is the breakdown voltage of the diode, or by using two breakdown diodes of opposite polarity in series and making V zero. Breakdown diodes are useful in obtaining characteristics that are symmetrical with respect to the voltage axis or with respect to both axes (Prob. 67.3-2). Although the same characteristics can be obtained by the use of ordinary diodes, only half as many breakdown diodes are required, and fewer sources of biasing voltage may be required.

67.4. Parallel and Series Circuits. Figures 238 and 239 show examples of parallel and series circuits by means of which exponentials or curves of the form $i = v^n$ may be approximated. Characteristics similar to those of Figs. 238 and 239, but lying in the third quadrant, are obtained if the polarities of all diodes and biasing voltages are reversed. The various biasing voltages may be obtained from a single source by means of a low-resistance voltage divider.

Figure 240 shows the characteristic obtained by the series combination of the voltage-biased circuit of Fig. 234 and the current-biased circuit of Fig. 236. The voltage V_1 produces the same effect as a simultaneous increase in V_2 and decrease in V_3, and may therefore be omitted.

Curves having some sections that are concave upward and others that are concave downward can be approximated by circuits consisting of parallel combinations of the circuits of Figs. 238 and 239. Thus, if V_0 and I_0 are zero in the circuit of Fig. 241a, the current voltage characteristic is that of

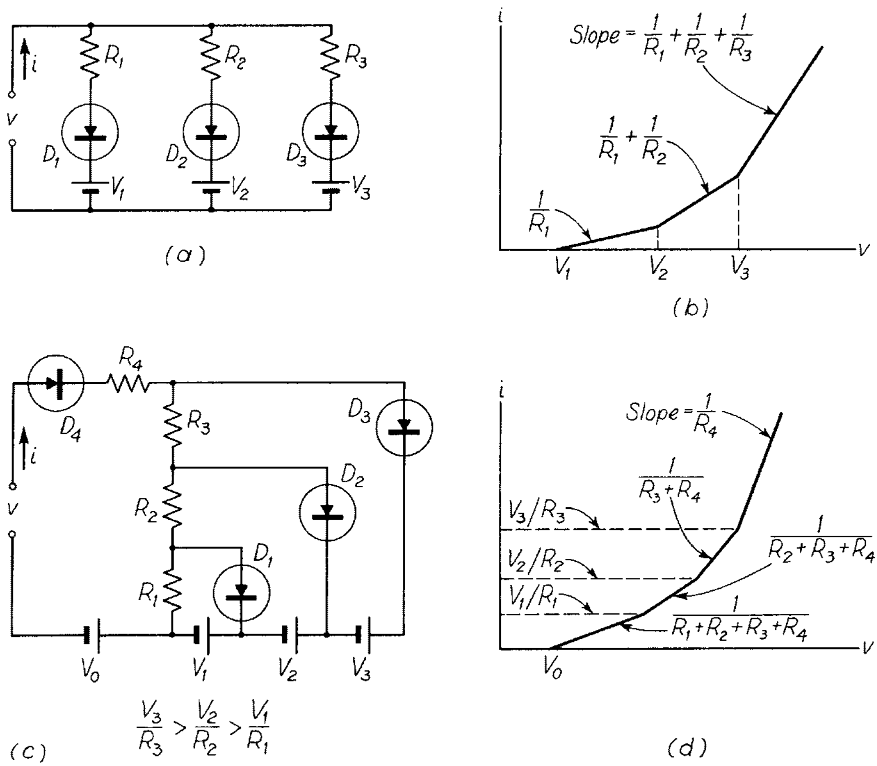


FIG. 238. (a) Reverse-biased parallel circuit; (b) current-voltage characteristic of the circuit; (c) reverse-biased series circuit; (d) current-voltage characteristic of the circuit.

Fig. 241b. (Note that the lowest break point in Fig. 239d is at the origin if $V_0 = -(V_1 + V_2 + V_3)$, i.e., if the lower input terminal voltage in Fig. 239c is $V_1 + V_2 + V_3$.) The biasing voltage V_0 and current I_0 displace the characteristic, as shown in Fig. 241c. The input diode prevents the flow of current when the input voltage is negative. Similar S and reverse-S characteristics may be obtained by paralleling two circuits of the form of Fig. 238a, 238c, or 239a. Circuits having "stair" characteristics were discussed in Secs. 34.4 and 34.5.

A typical characteristic obtained by the use of two breakdown-diode circuits in parallel is shown in Fig. 242. If the two resistances are equal, the characteristic has the interesting property that sections of slope $2/R$ alter-

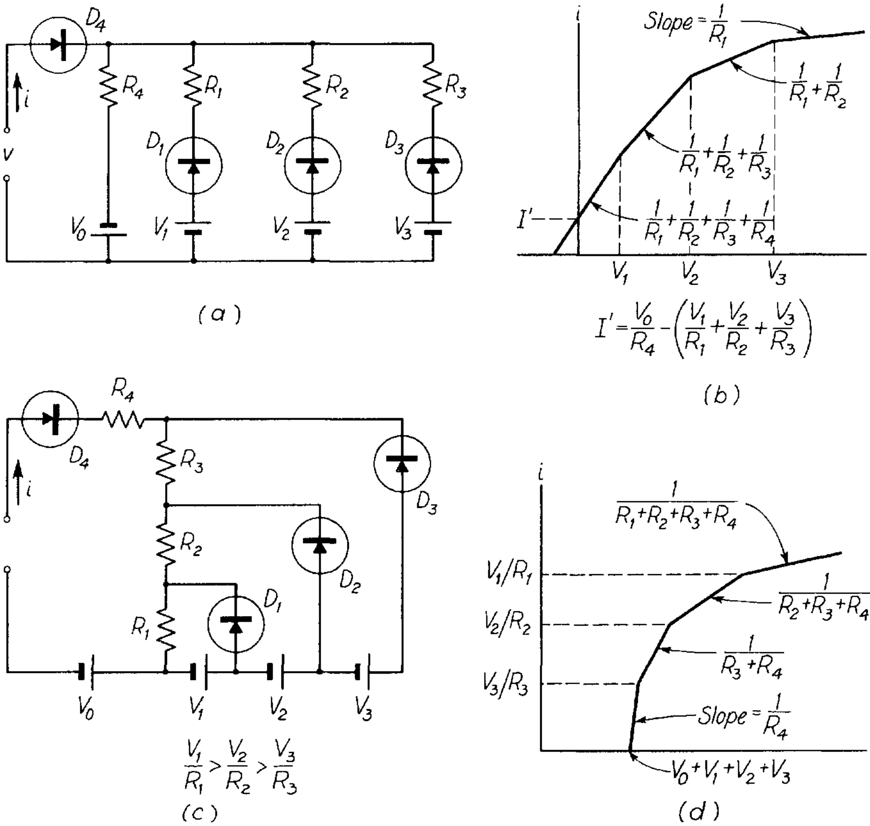


FIG. 239. (a) Forward-biased parallel circuit; (b) current-voltage characteristic of the circuit; (c) forward-biased series circuit; (d) current-voltage characteristic of the circuit.

nate with sections of slope $1/R$. Similar properties are displayed by curves obtained by the use of three or more breakdown-diode circuits in parallel.

67.5. Synthesis of Circuits Having Desired Characteristics. It is apparent that, if the desired slopes and break-point-positions are known in the broken curve of Figs. 241, the resistances and biasing voltages necessary to obtain this characteristic can be determined without great difficulty (Probs. 67.5-1 and 67.5-2). Furthermore, the characteristic can be made symmetrical by paralleling the network by a similar network in which polarities of all diodes and biasing voltages are reversed, and the characteristic can be displaced horizontally or vertically by connecting a voltage source in series with it or a current source (high voltage in series with a high resistance) in parallel with it. It follows that a wide variety of curves having only positive slopes can be simulated by the use of diode circuits. The simplest procedure is to draw the curve that is to be simulated and then to approximate the curve with the minimum number of linear sections that will keep the maximum error within the allowable value. This can usually be done with sufficient accuracy by eye (Probs. 67.5–3 to 67.5–5). The general form of the required circuit can then be determined and the values of the resistances and biasing voltages determined from the slopes of the segments and the positions of the break points. With practice, considerable facility can be developed. In designing the complete circuit, care must be taken to ensure that the maximum inverse-voltage and power-dissipation ratings of the diodes will not be exceeded.

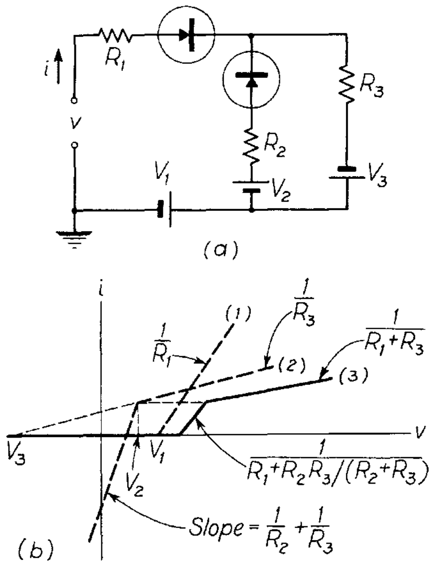


FIG. 240. (a) Voltage-biased circuit in series with a current-biased circuit; (b) current-voltage characteristic of the circuit.

Even relatively simple characteristics consisting of three or four segments can usually be obtained in a number of ways. The characteristic of Fig. 243a, for example, may be obtained by means of the three-diode circuit of Fig. 243b. The characteristic may, however, be synthesized by adding the ordinates of the two characteristics of Fig. 243c. Thus the simpler two-diode circuit of Fig. 243d may be used to obtain the desired characteristic. The desired characteristic can also be obtained by means of the even simpler breakdown-diode circuit of Fig. 243e.

The reader will find it instructive to determine the characteristics of various modifications of the circuits of Figs. 234 to 239 and combinations of these circuits, and also to synthesize circuits that will provide characteristics of desired forms.

The application of diode-resistance circuits to multistable circuits was discussed in Sec. 35 (Prob. 67.5–6).

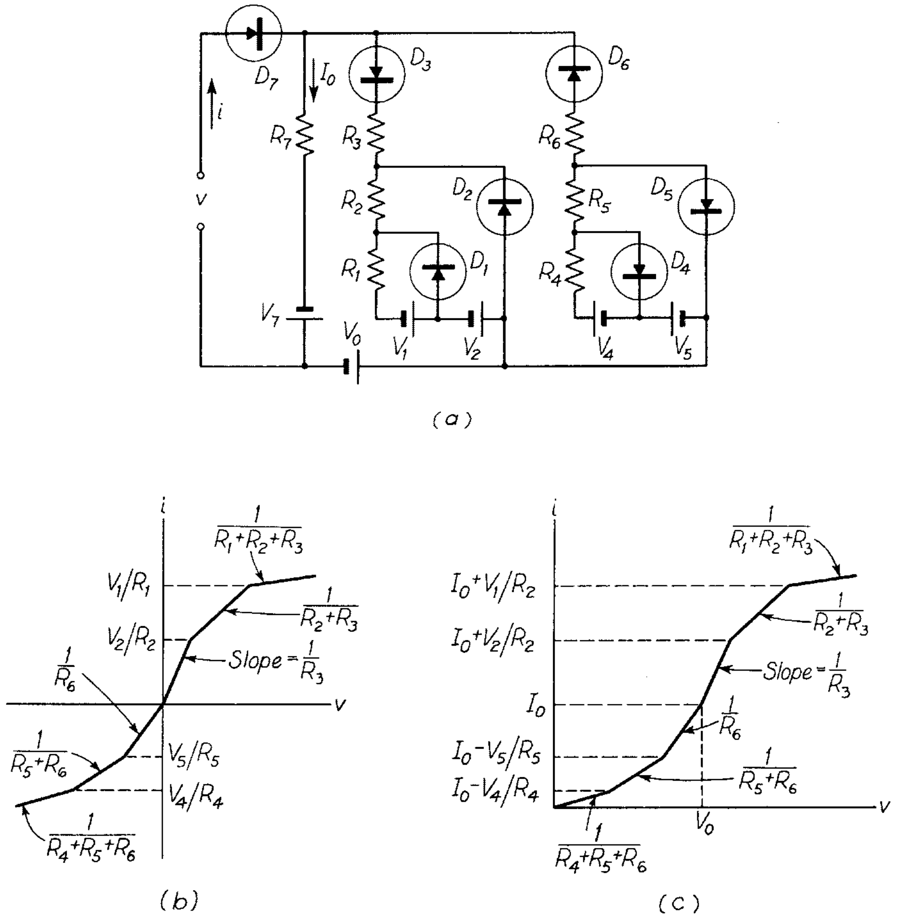


FIG. 241. (a) Circuit consisting of two series branches in parallel; (b) current-voltage characteristic of the circuit when V_0 and I_0 are zero; (c) approximate current-voltage characteristic when V_0 and I_0 are not zero and $R_7 \gg R_1$ to R_6.

67.6. The Photoformer. An interesting device that is capable of providing a voltage transfer fuction of arbitrary form $v_2 = f(v_1)$, where v_2 is single-valued with respect to v_1, is the *photoformer,* which consists basically of a cathode-ray tube, a phototube, a d-c amplifier, and a mask having the contour of the desired transfer function.[1] Light produced by the cathode-ray-tube spot falls upon the phototube, the voltage output of which is amplified by the d-c amplifier and applied to the vertical-deflection plates

[1] D. J. Mynall, *Electronic Eng.,* June, 1947, p. 178; D. E. Sunstein, *Electronics,* February, 1949, p. 100.

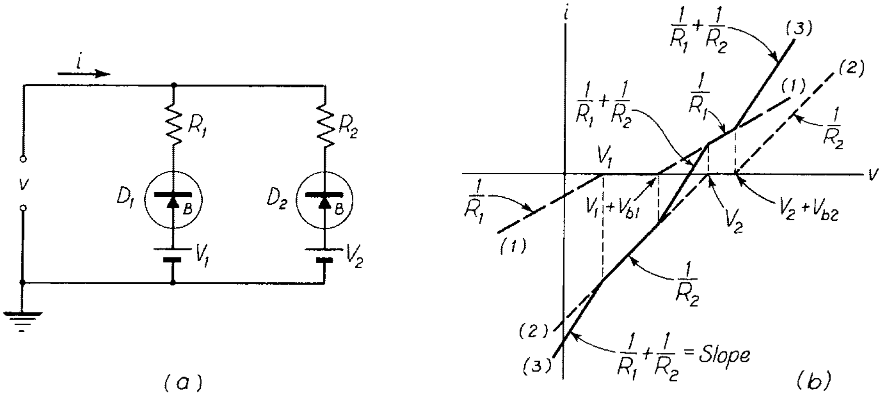


FIG. 242. (a) Circuit in which two breakdown-diode circuits are used in parallel; (b) current-voltage characteristic of the circuit.

in the proper polarity to displace the spot toward the edge of the mask, as shown in Fig. 244. The spot assumes a vertical equilibrium position such that it is partially masked. If a varying voltage is applied to the horizontal-deflection plates, the spot moves along the upper edge of the mask. Vertical deflection being proportional to v_2, the functional relation between the output voltage v_2 of the amplifier and the voltage applied to the horizontal-deflection plates is the relation $y = f(x)$ of the upper edge of the mask. The

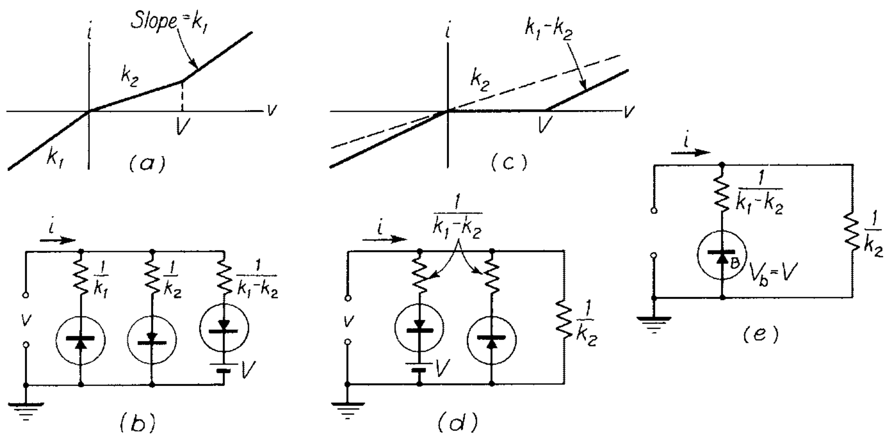


FIG. 243. (a) Desired current-voltage characteristic; (b) a three-diode circuit that has this characteristic; (c) two current-voltage characteristics with which the desired characteristic can be synthesized; (d) two-diode circuit that has the desired characteristic; (e) single-diode circuit that has the desired characteristic.

minimum rise and fall times of the output voltage produced by this device is limited by the high-frequency amplitude and phase characteristics of the

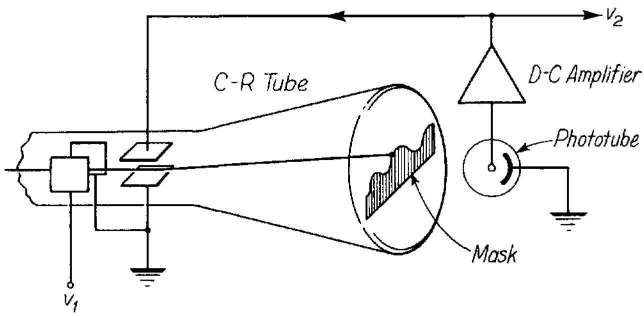


FIG. 244. Diagram showing the principle of operation of the photoformer.

amplifier. In the circuit described by Sunstein, the minimum rise time was approximately 10 μsec.[2]

68. Pulse Amplifiers

68.1. Requirements of Pulse Amplifiers; Step Response. An ideal pulse amplifier would be one capable of amplifying pulses having short rise and fall times without changing the pulse shape. The amplification and delay time of such an amplifier would have to be independent of frequency. Unfortunately, the shunt tube and circuit capacitances of vacuum-tube amplifiers and the diffusion delay of transistors make such an ideal amplifier impossible of attainment. One well known and convenient method of examining amplifiers for frequency and delay distortion is the determination of the response of the amplifier to a step input voltage or to a rectangular input pulse or wave. The response to a voltage step is particularly significant for pulse amplifiers because the pulses to be amplified often approximate rectangular form.

The equivalent circuit of the resistance-capacitance-coupled tube amplifier stage of Fig. 245a is shown in Fig. 245b. The undesired shunt capacitance C across the output may consist of the input capacitance of a following stage or of distributed circuit capacitance. The equivalent circuit may be redrawn in the form of Fig. 245c, which is identical in form with the differentiating circuit of Fig. 13c. The output of the amplifier for a step input voltage should, therefore, be of the form shown in Fig. 14a. The time constant $R'C'$ is normally much larger than the time constant $r_pRC/(r_p + R)$. The rising part of the output pulse is then determined principally by $r_pRC/(r_p + R)$ and the falling portion by $R'C'$, as indicated by Eq. (11). Usually no

[2] Mynall, *loc. cit.;* Sunstein, *loc. cit.*

difficulty is encountered in making $R'C'$ so large in comparison with the period in which the output voltage is of interest that the output voltage may be assumed to remain constant after its initial rise and the equivalent cir-

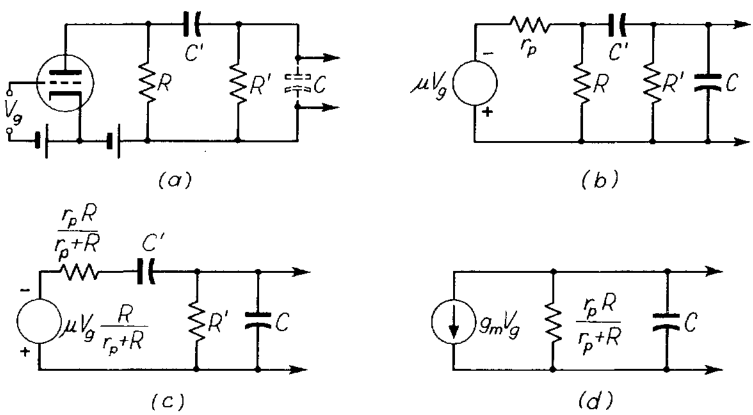


FIG. 245. (a) One stage of a resistance-capacitance-coupled vacuum-tube amplifier; (b) equivalent circuit for the stage; (c) alternate form of the equivalent circuit; (d) simplified approximate equivalent circuit.

cuit may be reduced to that of Fig. 245d. The output voltage for an input voltage step of magnitude V is then

$$\frac{Vg_mr_pR}{r_p + R}[1 - \epsilon^{-(r_p+R)t/r_pRC}]$$

The rise time of the output voltage is reduced by reducing the time constant $r_pRC/(r_p + R)$, which can be accomplished, at the expense of reduced output amplitude, by reducing R. Usually the rise time of a pulse is defined as the time during which the voltage or other variable rises from one-tenth to nine-tenths of its maximum value. In accordance with this definition, the product of the rise time and the upper cutoff frequency, at which the amplification is 0.707 times its midband value, is 0.35 for the amplifier of Fig. 245a (see Prob. 68.1–1).

68.2. Compensation of Shunt Capacitance by Inductance. The response of a resistance-capacitance-coupled amplifier to a step input voltage can be improved considerably by the addition of a small amount of inductance in series with the resistance R, as shown in Fig. 246a.[1] An equivalent

[1] H. M. Lane, *Proc. I.R.E.*, **26**, 722 (1932); G. D. Robinson, *Proc. I.R.E.*, **21**, 833 (1933); J. Beardsall, *Television and Short-wave World* (London), **5**, 95 (1936); L. E. Q. Walker, *Television and Short-wave World*, **9**, 305 (1936); S. W. Seeley, and C. N. Kimball, *RCA Rev.*, **2**, 171 (1937); P. Nagy, *Television and Short-wave World*, **16**, 160, 220 (1937); R. L. Freeman and J. D. Schantz, *Electronics*, August, 1937, p. 22; F. A. Everest, *Electronics*, January, 1938, p. 16; May, 1938, p. 24.

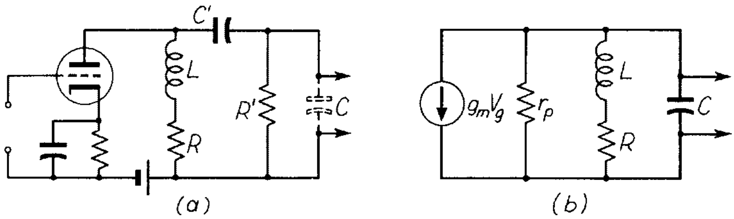


FIG. 246. (a) Inductance-compensated stage; (b) equivalent circuit of the compensated stage applicable at high frequencies.

circuit applicable at high frequencies is shown in Fig. 246b. Solution of the differential equation for this circuit leads to Eqs. (P-58) to (P-60) of Prob. 68.2-1 for the output voltage resulting from the application of a voltage step V to the input. These equations have been plotted in terms of normalized variables $t/2\pi RC$ and v_o/g_mRV in Fig. 247. It is apparent from Eqs. (P-58) to (P-60) and Fig. 247 that the rise time can be reduced appreciably by the use of series inductance, but that overshoot occurs if the inductance is so large that Q exceeds 0.5. The overshoot that can be tolerated depends, of course, upon the application of the amplifier.

Since the change in $t/2\pi RC$ corresponding to change in v_o/g_mRV from 10 to 90 percent of its final value depends only upon Q, it is also evident from Fig. 247 that, at fixed Q, the rise time is decreased by a decrease of C or R.

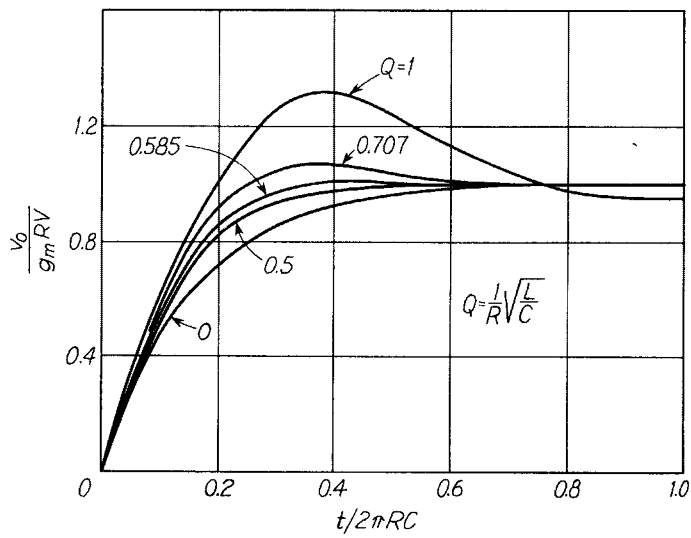


FIG. 247. Normalized curves of the response of the circuit of Fig. 246 to a step input voltage.

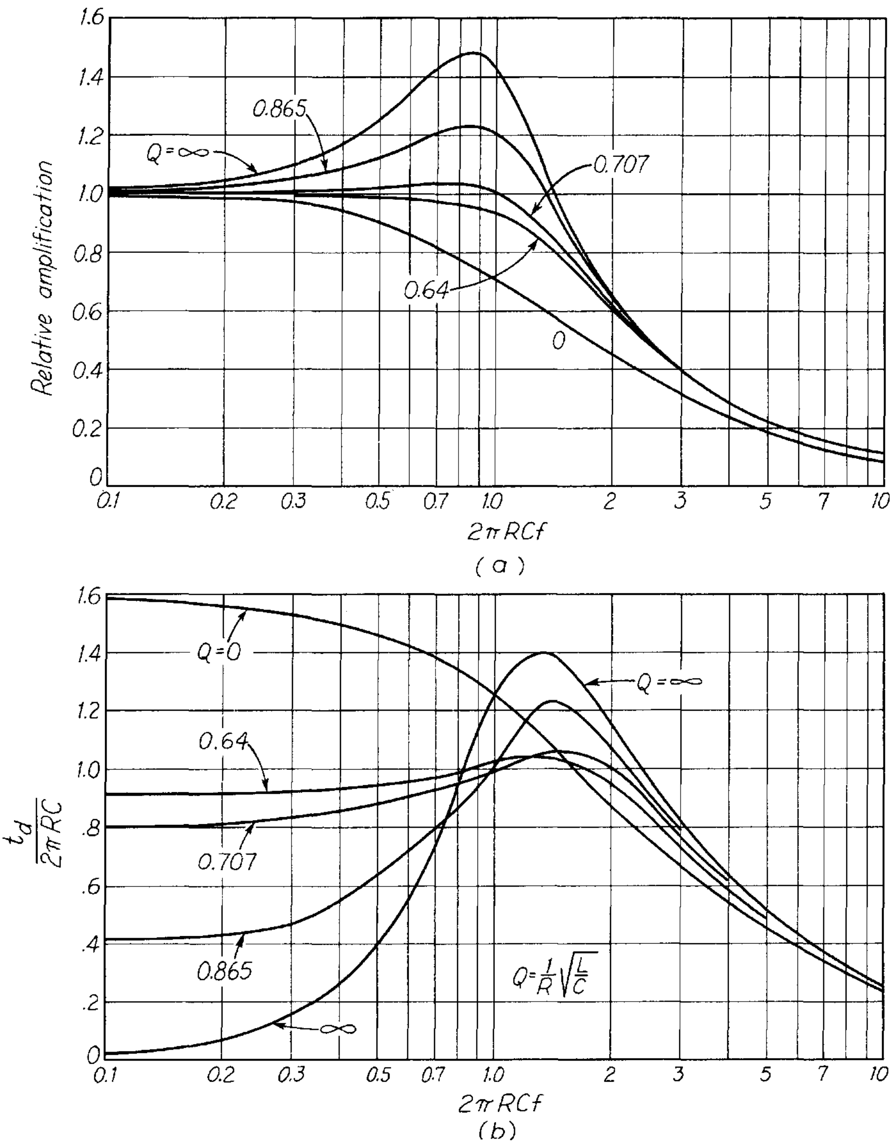


FIG. 248. Normalized curves of (a) voltage amplification and (b) phase shift of the circuit of Fig. 246 vs. frequency.

Reduction of R is, however, accompanied by loss of output-voltage amplitude Vg_mR. The extent to which R can be reduced in order to decrease the rise time is therefore dependent upon how much reduction in gain can be tolerated.

Solution of the network equations for the equivalent circuit of Fig. 246b with sinusoidal input voltage leads to Eqs. (P-62) and (P-64) of Prob. 68.2–2 for the relative amplification (ratio of the voltage amplification K_v at high frequencies to the midband amplification K_{vm}) and the delay time t_d as functions of the normalized frequency variable $2\pi RCf$. These equations have been plotted in Fig. 248 for several values of Q. From general circuit theory it is to be expected that improvement of the response of the amplifier to a voltage step by the use of series inductance L is accompanied by an increase in the bandwidth over which the gain and delay time are constant. This is confirmed by examination of Figs. 247 and 248. In fact, these curves show that the bandwidth-risetime product remains essentially 0.35 over the range of from zero to 0.7 (see Prob. 68.1–1d). It is of interest to note that minimum rise time without overshoot (critical damping) is obtained with $Q = 0.25$, the least dependence of delay time upon frequency is obtained with $Q = 0.585$, and the least dependence of amplification upon frequency is obtained with $Q = 0.645$.

68.3. Use of Cathode-Follower Coupling Stages Between Pentodes. An amplifier combination that is useful in the amplification of short pulses consists of a pentode common-cathode stage preceded and followed by cathode-follower stages. Because of the low effective input capacitance of the first cathode-follower stage, capacitance loading of the preceding circuit is minimized. The low output resistance of the first cathode-follower, together with the moderate input capacitance of the pentode stage make for good high-frequency response of the first cathode-follower; the low input capacitance of the second cathode-follower ensures good response of the pentode stage, particularly if the pentode load resistance is made as low as allowed by the required amplification.

68.4. Response of Stages in Tandem. When two or more stages of amplification are used in tandem, the rise time is increased over that of a single stage, the beginning of the output pulse is delayed relative to that of the input pulse, and if the individual amplifiers are designed so that overshoot is present, the total overshoot is increased above that for a single stage. This is illustrated by Fig. 248A, which shows the output of a 16-stage amplifier, a 32-stage amplifier, and a 64-stage amplifier at three values of Q when the input is a unit voltage step.[2]

68.5. Determination of Step-Voltage Response. A convenient method of determining the step-voltage response of an amplifier is to impress a

[2] A. V. Bedford and G. L. Fredendall, *Proc. I.R.E.*, **27**, 277 (April, 1939).

rectangular or triangular periodic wave upon the input and to observe the output waveform by means of an oscilloscope.[3] Each step in the input voltage then produces an output of the general form shown by Figs. 247 and 248A, which shows as a stationary pattern on the oscilloscope. The fundamental frequency of a rectangular input voltage used for this purpose should in general be close to the highest value that allows the output voltage to approach its steady value during a half cycle.

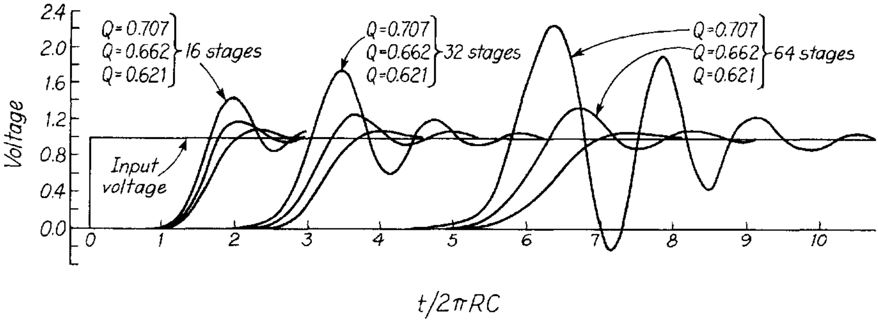


FIG. 248A. Response of a 16-stage, a 32-stage, and a 64-stage inductance-compensated amplifier to a step input voltage.

68.6. Transistor Pulse Amplifiers. Distortion of short pulses by transistor amplifiers is usually the result of alpha cutoff (diffusion delay effects), rather than capacitance. Distortion is minimized by the use of transistors having high alpha-cutoff frequency, by low collector-circuit resistance, and by use of the common-base connection, rather than the common-emitter connection ($f_{ab} = h_{fe}f_{ae}$). Limitations upon transistor amplifier performance are being rapidly removed by the development of transistors having high cutoff frequency. When such transistors are used, inductance compensation of the type discussed in the preceding sections may be profitably employed in order to extend the upper cutoff frequency of transistor amplifiers.

69. Distributed Amplifiers

69.1. Principle of Operation. Considerable increase in gain-bandwidth product over that attainable with conventional amplifiers, and therefore a corresponding improvement in the step-voltage response, can be obtained with distributed amplifiers.[1] A distributed amplifier consists of two lumped-

[3] H. J. Reich, *Proc. I.R.E.*, **19**, 401 (1931); A. C. Stocker, *Proc. I.R.E.*, 25, 1012 (1937); G. Swift, *Communications*, February, 1939, p. 22; A. V. Bedford and G. L. Fredendall, *Proc. I.R.E.*, **27**, 277 (1939); L. B. Arguimbau, *Gen. Rad. Expt.*, **14**, December, 1939, p. 1; D. L. Waidelich, *Proc. I.R.E.*, **32**, 339 (1944).

[1] W. S. Percival, Brit. Patent Spec. No. 460,562, applied for, July 24, 1936; E. L. Ginzton, W. R. Hewlett, J. H. Jasberg, and J. D. Noe, *Proc. I.R.E.*, **36**, 1956 (August, 1948).

element transmission lines having equal propagation velocities and incorporating the tube capacitances as shunt elements, as shown in Fig. 249 for one type of line. In order to prevent reflections from the ends of the lines, the lines are terminated in matched loads. The velocities of the waves in the two lines being equal, each grid is excited in the proper phase so that the resulting varying component of plate current adds to the current wave in the plate line. Because the tube capacitances serve as essential line elements, they do not in themselves directly limit the attainable bandwidth.

The plate load of each tube is $Z_o/2$, where Z_o is the characteristic impedance of the plate line. In the absence of line losses, therefore, the output voltage of the line would be $\frac{1}{2}ng_mZ_ov$, where n is the number of line sec-

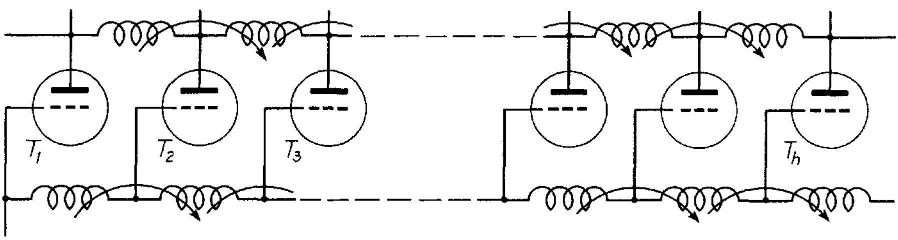


FIG. 249. A distributed amplifier.

tions, g_m is the transconductance of the tube, and v is the input voltage. For the constant-K-section lines used in the circuit of Fig. 249, $Z_o = 1/\pi C\sqrt{f_c^2 - f^2}$, where C is the capacitance of each line section and f_c is the cutoff frequency of the line. The voltage amplification of a stage consisting of n sections is therefore:

$$A_v = ng_m/2\pi C_{pk}\sqrt{f_c^2 - f^2} \tag{130}$$

The amplification A_v reduces to $ng_m/2\pi C_{pk}f_c$ at frequencies that are small in comparison with f_c, and the gain-bandwidth product is $ng_m/2\pi C_{pk}$. It is apparent that tubes used in distributed amplifiers, like those used in conventional amplifiers, should have high transconductance and low interelectrode capacitance in order that a high gain-bandwidth product may be achieved.* The advantage of the distributed amplifier over conventional amplifiers lies in the fact that the bandwidth may be made large at the expense of gain per section, the required over-all gain being achieved by the use of a large number of sections per stage.

* C_{gk} should be small in order to avoid loading of the source of input voltage and in order to avoid loss of gain in matching transformers between stages.

69.2. Number of Tubes Required. It has been shown[2] that the fewest tubes are required in order to achieve a desired gain-bandwidth product if the distributed amplifier is designed so that the amplification per stage is the Naperian logarithmic base ϵ, and the number of sections per stage is $2\pi f_c\sqrt{C_{gk}C_{pk}/g_m}$. The total amplification of m stages is then ϵ^m.

Equation (130) indicates that the voltage amplification rises as the frequency approaches the cutoff frequency. It can be made constant to the cutoff frequency by connecting the grids or the plates of all even-numbered tubes to the grids or plates, respectively, of the preceding odd-numbered tubes and replacing the shunt capacitances thus removed from alternate line sections by "dummy" capacitances. The frequency and phase characteristics of a distributed amplifier can also be improved by the use of more complicated filter sections.[3]

69.3. Attainable Bandwidth. The bandwidth attainable in practical distributed amplifiers is limited by input conductance of the tubes, which introduces high line attenuation at frequencies at which the conductance is comparable with the characteristic impedance of the grid line. Input conductance is caused by transit-time effects in the grid-cathode region of the tubes and by the cathode-lead inductance in conjunction with the grid-cathode capacitance of the line inductors and, to some extent, by inductor losses. By proper design the loss in amplification in the vicinity of cutoff, resulting from grid conductance and other causes, can be made to offset the rise in amplification of the ideal constant-K line, so that the gain remains essentially constant to the vicinity of cutoff. In this manner amplifiers that have a nearly flat amplitude response curve to about 180 Mc can be constructed with standard tetrodes or pentodes.[4]

The frequency and phase characteristics of distributed amplifiers are affected adversely by mismatched terminations. Proper matching is essential in amplifiers used in the amplification of pulses, because transient oscillation (ringing) may occur when the lines are not correctly terminated. The difficulty of matching may be complicated by the relatively low values of characteristic impedance of the lines, which necessitate the use of very large coupling capacitors between stages in order to obtain good low-frequency response.

Commercially built traveling-wave amplifiers are available.

70. Gain Controls for Pulse Amplifiers

When resistance voltage dividers are used to control the gain of pulse amplifiers, the input capacitance of the following circuit is shunted between

[2] Ginzton, Hewlett, Jasberg, and Noe, *loc. cit.*
[3] Ginzton, Hewlett, Jasberg, and Noe, *loc. cit.*
[4] W. H. Horton, J. H. Jasberg, and J. D. Noe, *Proc. I.R.E.*, **38**, 748 (July, 1950).

the divider tap and ground, as shown in Fig. 250a. If the circuit is replaced by the equivalent circuit of Fig. 250b, it is seen to be an integrating circuit. Unless very low values of resistance can be used, therefore, the leading and trailing edges of pulses with short rise and fall times may be rounded to an undesirable extent. Although this effect may be prevented by shunting R_1 by a capacitance of value $C' = CR_2/R_1$, this is not a convenient

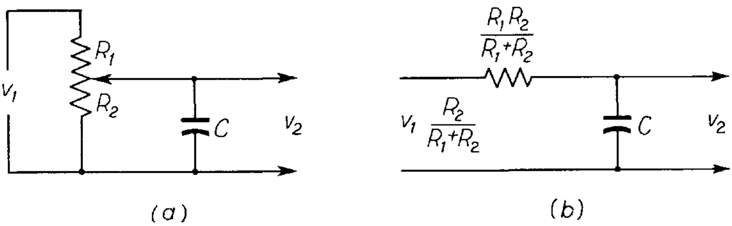


FIG. 250. (a) Voltage-divider gain control, including input capacitance of the following circuit; (b) equivalent circuit.

solution when the output voltage must be varied over a wide range. A more satisfactory solution is to use the voltage divider in the output of a cathode-follower or common-collector stage. The voltage-divider resistance can then be made low without excessive loss of amplification.

71. One-Port and Two-Port Sine-Wave Oscillators

Although there are many types of electronic sine-wave oscillators, for the purposes of analysis and design they may, with very few exceptions, be included in two broad classes: one-port oscillators, and two-port oscillators.

A *one-port oscillator* consists of a single-port circuit element that exhibits a negative real component of input admittance or impedance in a portion of its operating range, shunted by a parallel or series resonator (resonant circuit). Examples of suitable negative-conductance or negative-resistance elements were given in Secs. 18 to 22.

A *two-port oscillator* consists of an amplifying device, such as one or more tubes or transistors, and some form of frequency-sensitive two-port passive network that couples a portion of the amplifier output back to the input in the proper phase and magnitude to ensure oscillation at a frequency determined by the network.

One-port and two-port oscillators are usually called *negative-resistance* and *feedback oscillators,* respectively. Although the latter terms will be used in the remainder of this book, they are likely to be misleading in that they suggest that one-port oscillators do not involve feedback and that two-port oscillators do not involve negative resistance. As pointed out in Sec. 17, negative-resistance circuits involve regenerative feedback of some sort. It may also be shown that feedback oscillators exhibit negative variational resistance at one or more points in the circuits. Some oscillators may be analyzed equally well as two-terminal oscillators and as four-terminal oscillators.

72. Sine-Wave Oscillator Characteristics

72.1. Desirable Features of Sine-Wave Oscillators. Oscillators for laboratory or commercial use must not only be capable of providing the required power or voltage output at the desired frequency, but must generate only a small percentage of harmonic voltages, must have good frequency and amplitude stabilities, and must have accurate and constant calibration. These requirements will be discussed in this and following sections.

The output voltage or power, frequency of oscillation, and harmonic content of an oscillator are not in general independent. The amplitude of oscillation of a directly loaded oscillator varies with load and therefore with power output. Direct loading of an oscillator in general also affects the frequency of oscillation, both because one or more parameters of the resonant circuit are dependent upon the load, and because dynamic tube or transistor parameters depend upon the amplitude of oscillation, which varies with load. Harmonic content in general increases with amplitude of oscillation. Both harmonic content and frequency stability are dependent upon the Q of the resonant circuit and therefore upon coupled load.

72.2. Power Output of Oscillators. The power output that can be obtained from a Class A oscillator greatly exceeds the output from a Class A power amplifier with the same tube or transistor. This is so because the oscillator tube or transistor remains continuously excited and can normally be kept under constant load, whereas an amplifier usually has excitation of variable amplitude and may deliver zero output for long periods of time. The maximum power that the amplifier tube or transistor is required to dissipate is therefore equal to the plate-circuit or collector-circuit input power. Since the oscillator load is constant, on the other hand, the maximum power that the oscillator tube or transistor is required to dissipate is equal to the d-c power input minus the power output. At 22-percent efficiency, which is readily attainable without excessive harmonic generation, the dissipation is 78 percent of the input. The Class A oscillator can therefore use about 28 percent higher input power and thus deliver 28 percent more output with the same maximum plate or collector dissipation. If the circuit drops out of oscillation, however, the tube or transistor is likely to be damaged, both because of the increase of dissipation resulting from the elimination of power output and because of the rise in plate or collector current resulting from a change of bias when the bias is produced by rectification of the alternating voltage in a manner that will be discussed in Sec. 74.

72.3. Harmonic Content. Low harmonic content of the output of an oscillator is favored by maintaining the amplitude of oscillation as small as possible. When both large output and low harmonic content are required, it is best to use small-amplitude operation and to obtain the needed output by means of a low-distortion power amplifier. Advantage can be taken of the filtering action of the resonator in minimizing distortion. Thus, the impedance of a parallel resonant circuit is high at the fundamental frequency and low at harmonic frequencies. Consequently lower harmonic content is obtained if voltage output is taken from across the resonator or from a coil coupled to the resonator than if it is taken from some other point in the circuit, such as the plate of a vacuum tube or the collector of a transistor. The purity of the voltage waveform across the resonator increases with the resonator Q. The output from an oscillator that uses a series resonant circuit should be taken from across the resonator capacitor, which has a lower reactance at harmonic frequencies than at the fundamental frequency, or from a small resistance in series with the resonant circuit.

72.4. Frequency Stability. Undesired changes of frequency of oscillation of an oscillator result from three major causes:

(1) Changes in the mechanical arrangement of the elements of the oscillating circuit, which may be produced by vibration, by mechanical, electrostatic, or electromagnetic forces, or by temperature changes. Frequency changes resulting from this cause can be minimized by careful mechanical and electrical design and by stabilizing the oscillator temperature.

(2) Variations in the values of circuit parameters as the result of changes of temperature and humidity of inductors and capacitors and as the result of changes of load. Changes of inductance and capacitance can be minimized by temperature control, by use of thermally compensated inductors and temperature-controlled compensating capacitors, and by the careful selection of component parts and their judicious location. The effect of stray capacitances can be reduced by the use of a high C/L ratio in the resonator. The most obvious method of preventing the reaction of load upon an oscillator is to use the oscillator only to excite a "buffer amplifier" that requires negligible driving power and is capable of delivering the required power to the load. Other methods will be discussed in conjunction with specific oscillator circuits.

(3) In feedback oscillators, changes of phase angle of the forward transfer parameter of the amplifying device. Since the over-all phase shift around the closed circuit must be zero or 360 degrees, changes in amplifier phase shift must be accompanied by equal changes of phase shift of the frequency-

sensitive feedback circuit, and hence by changes in frequency. This type of frequency instability and methods of reducing it will be discussed in detail in Secs. 82, 84, and 85.

(4) Changes of amplifier parameters with temperature. The dependence of amplifier parameters upon temperature is particularly severe when transistors are used. Methods of stabilizing the frequency of transistor oscillators against temperature change will be discussed.

72.5. Amplitude Stability. Changes of amplitude of oscillation are usually the result either of changes of load coupled to the resonator or of changes of transconductance of a tube or of a current amplification factor h_f of a transistor. Short-time changes in the transconductance of a tube are usually the result of changes of supply voltage or of grid bias, although changes of cathode temperature may also have appreciable effect. In transistors, change in current amplification factor may be caused by changes of collector supply voltage or base biasing current, or by changes of temperature. Long-time changes of parameters occur as the result of normal aging or of excessive dissipation. Methods of stabilizing amplitude of oscillation will be discussed in Sec. 74.

72.6. Constancy and Accuracy of Calibration. Constancy and accuracy of calibration of an oscillator depend not only upon frequency stability, but also upon details of mechanical design, which will not be discussed.

72.7. Use of a Power Amplifier. In order to ensure good amplitude and frequency stability in an oscillator used with variable load it is usually best to combine a low-level, high-stability oscillator with a low-distortion power amplifier, intervening voltage-amplifier and driver stages being used if necessary. In this manner the oscillator stage can be designed for good waveform and stability, and the power stage for high power output and efficiency. When appreciable power must be obtained directly from a feedback oscillator, it should be borne in mind that such an oscillator is actually a self-excited amplifier (Sec. 78) and that the laws which govern the design of power amplifiers also apply to feedback power oscillators. Special methods of loading feedback oscillators in such a manner as to minimize the effects of load upon amplitude, frequency, and harmonic content will be discussed in Sec. 78.

73. Negative-Resistance (One-Port) Oscillators

73.1. Analysis of Two-Terminal Oscillators. Sine-wave oscillators may be formed by using any of the voltage-stable negative-resistance elements of Secs. 18 to 22 in the circuit of Fig. 157a or any of the current-stable elements in the circuit of Fig. 157b. It is necessary only to choose the circuit parameters to correspond with the growing-oscillation region of the diagram of Fig. 158 or 159 and to adjust the supply voltages so that the load line intersects the current-voltage characteristic in only the negative-resistance range.

For the sake of convenience of reference, the basic single-port oscillator circuits of Fig. 157 are shown again in Fig. 251, and some of the equations derived in Sec. 45 will be repeated. Usually the capacitance inherent in the voltage-stable element is too small to make sustained oscillation possible at the desired frequency. The capacitance C of Fig. 251a is therefore

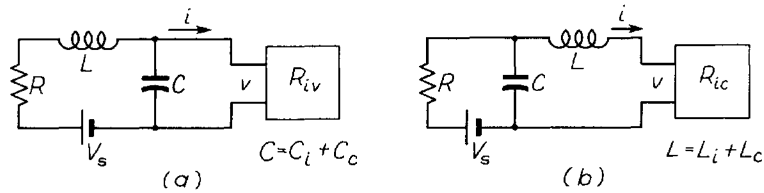


FIG. 251. Single-port sine-wave oscillators using (a) a voltage-stable negative-resistance element and (b) a current-stable element. (See also p. 450.)

assumed to be the sum of the inherent capacitance C_i and whatever additional circuit capacitance C_c is required in order to achieve sinusoidal oscillation at the desired frequency. Similarly, the inductance L in Fig. 251b is the sum of the inherent inductance L_i of the negative-resistance element and whatever additional circuit inductance L_c is required. The series winding resistance of the external inductor may be taken into account most readily by adding it directly to R_{ic}. The winding resistance is ordinarily so small in comparison with the magnitude of R_{ic} that it may be neglected.

If the parameters of the circuits of Fig. 251 are chosen so that operation is in one of the oscillation regions of the diagram of Fig. 158 or 159, the current into the negative-resistance element is of the form:

$$i = A'\epsilon^{\alpha t} \sin (\omega t + \theta) + I_o \tag{131}$$

in which A' and θ are the amplitude and the phase of the oscillation when t is zero, and I_o is the current corresponding to the intersection of the load line with the current-voltage characteristic (see Sec. 45). For the circuit of Fig. 251a, α and ω are:

$$\alpha = -\frac{1}{2}\left(\frac{1}{R_{iv}C} + \frac{R}{L}\right) = -\frac{1}{2C}\left(\frac{1}{R_{iv}} + \frac{RC}{L}\right) \tag{132}$$

$$\omega = 2\pi f = \sqrt{\frac{R + R_{iv}}{R_{iv}LC} - \alpha^2} \tag{133}$$

For the circuit of Fig. 251b, α and ω are:

$$\alpha = -\frac{1}{2}\left(\frac{1}{RC} + \frac{R_{ic}}{L}\right) = -\frac{1}{2L}\left(\frac{L}{RC} + R_{ic}\right) \tag{134}$$

$$\omega = 2\pi f = \sqrt{\frac{R + R_{ic}}{RLC} - \alpha^2} \tag{135}$$

Equation (131) also applies to the other currents and to the voltages in the circuits of Fig. 251, but the constants for the various currents and voltages differ.

Equations (131) and (132) show that if R_{iv} is positive in the circuit of Fig. 251a, or if it is negative but greater in magnitude than L/RC, α is negative and any oscillation initiated by noise, closing of a switch, or other disturbance dies out. If R_{iv} is negative and equal in magnitude to L/RC, α is zero and oscillation initiated in any manner remains constant in amplitude. If R_{iv} is negative and smaller in magnitude than L/RC, on the other hand, α is positive and oscillation initiated in any manner, and however small, grows in amplitude. Similarly, in the circuit of Fig. 251b any initial oscillation decays if R_{ic} is positive or if it is negative but smaller in magnitude than L/RC, remains constant if $R_{ic} = -L/RC$, and grows if R_{ic} is negative and larger in magnitude than L/RC.

If the values of the circuit elements are such that the amplitude of oscillation is constant, α is zero in Eqs. (133) and (135). It is apparent, therefore, that the frequency of oscillation is always lower during transient periods in which the amplitude is growing or decaying than it is under equilibrium conditions. Equations (133) and (135) also show that the steady-state frequency of oscillation is dependent upon the ratio R/R_{iv} or R_{ic}/R. Usually these ratios are small and the frequency of oscillation approximates $1/2\pi\sqrt{LC}$. Nevertheless, the dependence of the oscillation frequency upon R/R_{iv} or R_{ic}/R, however small, cannot be neglected when frequency stability is important, since R_{iv} and R_{ic} in general vary with supply voltage, and the effective value of R is increased by load coupled to the resonator. Frequency stability will be discussed further in Sec. 74.5.

The seeming absurdity that, if the values of the circuit elements are chosen so that the amplitude of oscillation grows, it will continue to grow without limit is the result of the assumption that the input resistance R_{iv} or R_{ic} of the element is constant. This is equivalent to the assumption that the current-voltage characteristic of the voltage-stable or current-stable element has constant slope. In any actual circuit, R_{iv} or R_{ic} is a function of amplitude, both because of curvature of the negative-slope branch of the characteristic and because operation is not confined to the negative-slope branch if the amplitude becomes sufficiently high.

73.2. Determination of R_i. A more elegant analysis[1] than that presented in Sec. 45, in which the nonlinearity of the negative-resistance device is taken into consideration, shows that R_{iv} and R_{ic} are dynamic or effective values and should be defined as in Sec. 15.1: as the ratio of the fundamental component of voltage across the port of the negative-resistance element to the fundamental component of current flowing into the port. The manner in which R_i varies depends upon the form of the current-voltage characteristic and the position of the intersection of the load line with the current-voltage characteristic; i.e., upon the location of the static operating point. The input resistance R_i of a particular negative-resistance element may be determined from measurements of fundamental components of voltage and current, or may be found graphically from the current-voltage characteristic.

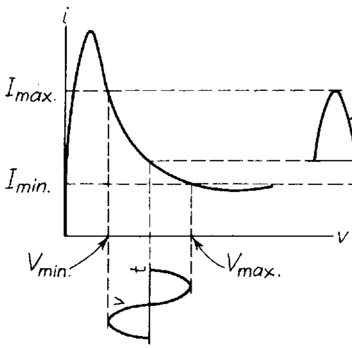


FIG. 252. Determination of current waveform by means of the current-voltage characteristic curve.

Figure 252 shows how the current waveform corresponding to a sinusoidal applied voltage may be determined from a voltage-stable current-voltage characteristic. The fundamental component of current may be determined to any desired degree of accuracy by any of the well known selected-point methods of graphical harmonic analysis. To a first approximation, which is very rough if the percentage of third or higher-order harmonics is high, the crest value of the fundamental component of current is equal to $\frac{1}{2}(I_{max} - I_{min})$. The effective value of R_i is therefore $(E_{max} - E_{min})/(I_{max} - I_{min})$, which is the reciprocal of the slope of the chord joining the limits of the path of operation on the characteristic. If the nonlinearity of the characteristic over the range of operation is great, a greater number of points must be used in determining the fundamental component of current (see Prob. 73.2–1). The procedure for the graphical determination of R_i from a current-stable characteristic is similar, but current is used as the independent variable and the fundamental component of voltage is found.

Although the chord-slope approximation is not in general useful in the accurate determination of R_i as a function of amplitude, it does serve to indicate that R_i varies with voltage or current amplitude. At small amplitudes the magnitude of R_i may either decrease or increase with amplitude, de-

[1] See, for example, J. Cunningham, *Introduction to Nonlinear Analysis,* McGraw-Hill Book Co., Inc., New York, 1958.

pending upon the shape of the current-voltage characteristic and the choice of the static operating point. When the amplitude becomes so large that the path of operation about a static operating point in the negative-slope region of a voltage-stable characteristic extends into the positive-slope regions, however, the slope of the chord joining the limits of the path of operation decreases with increase of amplitude and the magnitude of R_{iv} consequently increases. It follows that if the parameters of the circuit of Fig. 251a are chosen so that the magnitude of R_{iv} is less than L/RC and α therefore positive, the amplitude will eventually reach a value such that the magnitude of R_{iv} has increased to L/RC and α is zero. The amplitude then remains constant. Similar reasoning shows that decrease of the magnitude of R_{ic} at large amplitudes limits the amplitude of oscillation of the circuit of Fig. 251b.

73.3. Type of Resonator Required. Since oscillation of constant or growing amplitude is possible only if R_i is negative, the static operating point must lie on the negative-slope section of the current-voltage characteristic. The value of R in the circuit of Fig. 251a must therefore be less than the reciprocal of the slope of the negative-slope section of the voltage-stable characteristic at its steepest point. This is consistent with the requirement that R must be smaller than $|R_{iv}|$ in order for sustained sinusoidal oscillation to occur (Prob. 73.3–1). Similarly, the requirement that R must be large in the circuit of Fig. 251b in order to ensure that the load line intersects only the negative-slope section of the current-stable characteristic is consistent with the requirement that R be greater than $|R_{ic}|$ in order for sustained sinusoidal oscillation to occur.

Sinusoidal oscillation cannot be obtained by the use of a series resonant circuit across a voltage-stable element or a parallel resonant circuit across a current-stable element. If the capacitance of a series resonant circuit is shunted by a resistance small enough to ensure that the load line intersects the voltage-stable current-voltage characteristic only in the negative-resistance range, or if the sufficient resistance is used in series with the inductance of a parallel resonant circuit to ensure that the load line intersects the current-stable characteristic only in the negative-resistance range, the combination of circuit parameters will correspond to a point in Fig. 158 or Fig. 159 that is outside of the growing-oscillation range (see Prob. 73.3–2).

73.4. Dynamic-Negative-Resistance Circuits. The negative-resistance devices discussed in Secs. 18 to 22 all have static current-voltage characteristics that have a negative-slope range. Such devices exhibit negative variational resistance over a frequency range that extends to zero, a negative resistance being observed even when incremental measurements are made under static conditions. There is another class of one-port devices that exhibit negative resistance only under dynamic conditions, and then only at very high frequencies. Some of these devices, of which the reflex klystron is an outstanding example, have negative resistance by virtue of electron-transit-time effect and are therefore useful only as generators of microwave power. For a discussion of these devices the reader is referred to the literature in the microwave field.[2]

73.5. Shunt and Series Equivalent Circuits. As far as the varying components of current and voltage are concerned, the circuit of Fig. 251a may be approximated by an equivalent circuit of the form of Fig. 253a. The

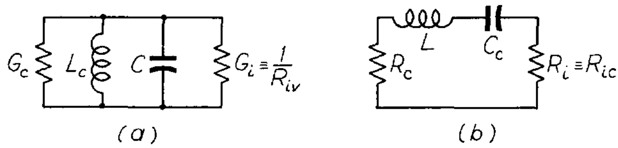


FIG. 253. (a) Equivalent circuit for the oscillator of Fig. 251a; (b) equivalent circuit for the oscillator of Fig. 251b.

damping factor α and the angular frequency of oscillation ω for the two circuits with the same value of R_{iv} are identical if $G_c = RC/L$ and $L_c = LR_{iv}/(R_{iv} + R)$ (see Prob. 73.5–1). Ordinarily R is the winding resistance of the inductor and is therefore several orders of magnitude smaller than R_{iv}; L_c may then be assumed to be equal to L. Similarly, the circuit of Fig. 251b may be represented by the circuit of Fig. 253b if $R_c = L/RC$ and $C_c = CR/(R_{ic} + R) \approx C$. The resistance R_c may also include the winding resistance of the inductor, which was taken into account in the circuit of Fig. 251b as an equivalent change in the magnitude of R_{ic}, usually negligible. Both G_c and R_c may include a component caused by loading of the resonator. The advantages of the circuits of Fig. 253 over those of Fig. 251 lie in their slightly simpler form and the fact that they may be much more conveniently used to represent distributed-element circuits such as those employed at microwave frequencies. Equivalent values of G, L, and C for a cavity resonator, for example, may be determined from measured values of resonance frequency, Q, and shunt conductance.

The circuits of Fig. 253, rather than those of Fig. 251 will be used in the remainder of the discussion of single-port oscillators. In Secs. 74 and 75 only the shunt circuit of Fig. 253a will be treated in detail. The discussion of the circuit of Fig. 253b is similar, but resistances replace conductances, and reactances replace susceptances.

[2] See, for example, H. J. Reich, J. G. Skalnik, P. F. Ordung, and H. L. Krauss, *Microwave Principles*, D. Van Nostrand Co., Inc., Princeton, N. J., 1957.

74. Amplitude and Frequency Stabilization of Parallel-Resonator Negative-Resistance Oscillators

74.1. Amplitude Diagrams. The criteria for decreasing, constant, and increasing amplitude of oscillation in the circuit of Fig. 253a are obtained from Eqs. (131) and (132) if RC/L is replaced by G_c and $1/R_{ip}$ is replaced by G_i, or they may be obtained directly by solution of the nodal equations for the circuit. They are:

$$\begin{aligned} &\text{If } |G_i| > G_c, \text{ the amplitude increases} \\ &\text{If } |G_i| = G_c, \text{ the amplitude remains constant} \\ &\text{If } |G_i| < G_c, \text{ the amplitude decreases} \end{aligned} \tag{136}$$

In practically all electron-tube and transistor oscillators, the relation between G_i and amplitude (or between R_i and amplitude) is not sufficiently simple so that the equilibrium amplitude can be predicted analytically. A simple graphical method may be used, however, if the curve of $|G_i|$ vs. amplitude can be constructed from measured values of fundamental components of current and voltage or by the application of graphical methods to the current-voltage characteristic of the negative-resistance element. Although such a procedure is not likely to be warranted in the design of an oscillator, much useful information concerning the general behavior of single-port oscillators can be gained from an analysis based upon generalized curves of $|G_i|$ vs. amplitude.

Application of the chord-slope method of determining the approximate effective value of G_i shows that, if the operating point lies at or near a point on the negative-slope portion of the current-voltage characteristic where the slope has a maximum value, as in Fig. 254a, the magnitude of G_i decreases continuously with increase of amplitude. The curve of $|G_i|$ vs. amplitude is then of the general form of that shown in Fig. 255a. If the operating point is located as in Fig. 254b, on the other hand, the magnitude of G_i increases with amplitude at small amplitude, reaches a maximum value, and then decreases when the amplitude becomes so great that the range of operation extends to the positive-slope sections of the characteristic. (In many negative-resistance circuits the transition from negative to positive slope is caused by the cutting off of one or more tubes or transistors. Consequently, when the path of operation extends into the positive-slope sections of the characteristic the tubes or transistors are cut off during portions of the cycle. In other words, Class AB operation is being used.) The curve of $|G_i|$ vs. amplitude is of the form of that of Fig. 255b.

The equilibrium amplitude of oscillation can be determined by adding curves of G_c vs. amplitude to the diagrams of Fig. 255. Since L, R, and C are ordinarily independent of amplitude, G_c is normally constant and the

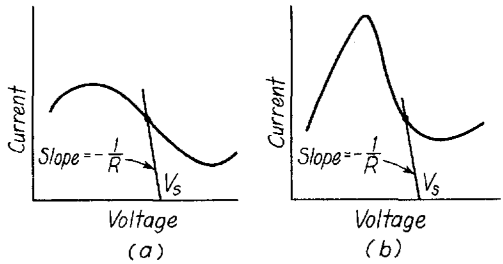


FIG. 254. (a) Operating point for which $|G_i|$ decreases continuously with increase of amplitude; (b) operating point for which $|G_i|$ first increases with increase of amplitude.

curve of G_c vs. amplitude is a horizontal line. The value of G_c may, however, be made to vary with amplitude by introducing an additional nonlinear element, such as a biased diode or a current-sensitive resistor into the circuit.

In order for the amplitude to build up following noise or some other disturbance in the circuit, $|G_i|_0$, the magnitude of G_i at zero amplitude, must exceed G_c. Since the amplitude continues to grow as long as $|G_i| > G_c$, the amplitude will increase to a value corresponding to the intersection P of the $|G_i|$ and G_c curves. Further increase of amplitude would cause $|G_i|$ to become smaller than G_c and the amplitude to decrease to the value corresponding to the intersection. It can be seen from Fig. 255a that the amplitude of oscillation can be controlled by increasing or decreasing G_c by changing R or the load coupled to the resonator. It can also be increased or decreased by raising or lowering the $|G_i|$ curve. This is ordinarily accomplished by changing one or more operating parameters of the electronic device that produces the negative conductance, such as the grid bias of an electron tube or the base current of a transistor. Figure 255a also shows that when the magnitude of G_i decreases continuously with increase of amplitude, the equilibrium amplitude may be varied continuously from

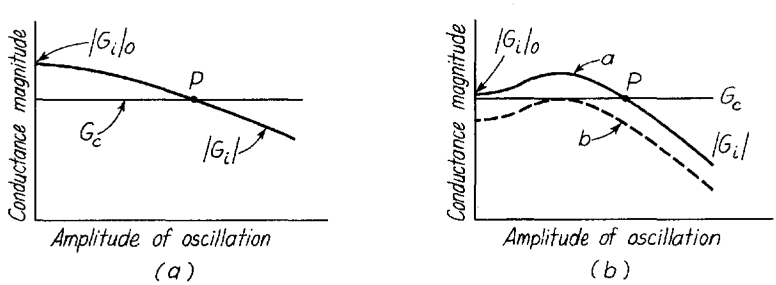


FIG. 255. (a) Conductance-amplitude diagram for the operating point of Fig. 254a; (b) diagram for the operating point of Fig. 254b.

zero to the maximum value of which the oscillator is capable and back to zero by changing G_c or some operating parameter of the source of negative resistance.

74.2. Amplitude Hysteresis. When the curve of $|G_i|$ vs. amplitude is of the form of that in Fig. 255b, equilibrium amplitude of oscillation cannot be varied continuously from zero to its maximum value. For curve (a) of Fig. 255b, $|G_i|_0$ barely exceeds G_c, and α in Eq. (131) is small, but positive. The amplitude again builds up to an equilibrium value corresponding to the intersection of the curves. If the $|G_i|$ curve is then raised by changing some parameter of the source of electronic conductance, the amplitude increases continuously with change of parameter. When the parameter is changed in the opposite direction, so as to lower the $|G_i|$ curve, the amplitude decreases continuously with change of parameter until the curve becomes

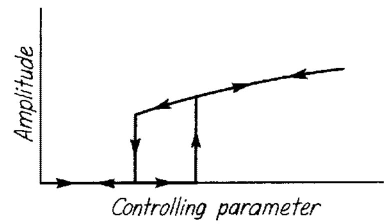


FIG. 256. Variation of amplitude of oscillation of the oscillator of Fig. 253a with a parameter of the negative-resistance element when the curve of $|G_i|$ vs. amplitude is of the form of Fig. 255b.

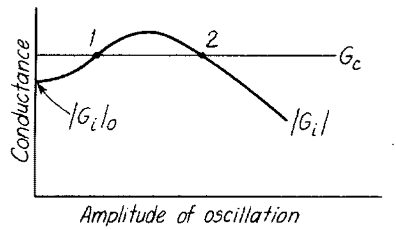


FIG. 257. Diagram for an oscillator in which oscillation builds up only if the circuit is initially excited.

tangent to the G_c line, as shown by curve (b) of Fig. 255b. Any further lowering of the $|G_i|$ curve causes $|G_i|$ to be smaller than G_c, α to be negative, and the oscillation consequently to die out. It will then start spontaneously only if the curve is raised until $|G_i|_0$ exceeds G_c, and the amplitude again jumps to that corresponding to point P. The minimum attainable amplitude of sustained oscillation is therefore that corresponding to the maximum of the $|G_i|$ curve. Variation of the controlling parameter will cause the amplitude to vary in the general manner shown in Fig. 256. Similar abrupt changes of amplitude occur when G_c is changed continuously. They are also observed when G_i is constant over a range of amplitude, i.e., when the current-voltage characteristic is linear over a region about the operating point. The variation of amplitude in the manner shown in Fig. 256 is called *amplitude hysteresis*. Amplitude hysteresis may be observed in most oscillators under some conditions of operation.

If the curve of $|G_i|$ vs. amplitude has two or more maxima, there may be several discontinuities in amplitude as the controlling parameter is gradually varied. When the variation of conductance G_i with amplitude is accompanied by variation of susceptance B_i, as is usually true in microwave oscillators, the abrupt changes in amplitude are accompanied by abrupt changes in frequency.

When the $|G_i|$ curve has a maximum and $|G_i|_0$ is adjusted to a value that is somewhat smaller than G_c, as in Fig. 257, oscillation is not self-starting. However, if the circuit is driven by external means to an amplitude equal to or greater than that corresponding to intersection 1 of the conductance curves, the amplitude will increase abruptly to the value corresponding to intersection 2 and oscillation will continue when the driving source is removed.

74.3. Minimization of Harmonic Content. As in amplifiers, the ratios of the amplitudes of harmonics generated in an oscillator to that of the fundamental increase with amplitude of oscillation and may become excessively large at high amplitudes. For this reason it is usually desirable to design and operate the negative-resistance element in such a manner that the curve of $|G_i|$ vs. amplitude is of the form shown in Fig. 255a rather than Fig. 255b. This can be accomplished with most of the negative-resistance circuits of Figs. 18 to 22 merely by proper choice of operating point, and low harmonic content can therefore be assured by adjusting the operating parameters and voltages so that the amplitude of oscillation is small. Symmetrical current-voltage characteristics, such as the one in Fig. 69b, are particularly desirable, since the curve of $|G_i|$ vs. amplitude obtained with a device having such a characteristic is of the desired form if the operating point is placed on the voltage axis. This is usually achieved simply by making the circuit completely symmetrical, as in Fig. 68b, and using zero external supply voltage in series with the port.

74.4. Amplitude Stabilization. Unfortunately, the magnitude of the negative resistance of practical negative-resistance elements, such as those discussed in Secs. 18 to 22, is dependent upon supply voltages. In general, the principal effect of a small change of supply voltage upon the $|G_i|$ curves of Fig. 255 is a vertical displacement. A change of supply voltage is therefore in general accompanied by change of equilibrium amplitude of oscillation. The stability of amplitude against changes of supply voltage may be poor if the rate of change of $|G_i|$ with amplitude is small; i.e., if the negative slope of the curve of $|G_i|$ vs. amplitude is small. The instability is particularly objectionable if $|G_i|_0$ is made only slightly larger than G_c in order to ensure low amplitude of oscillation, since a small change in the vertical position of the $|G_i|$ curve will then result in a large percentage change in amplitude, and may even result in cessation of oscillation.

It is apparent from an examination of Fig. 255a that the amplitude stability can be increased by increasing the angle at which the $|G_i|$ and G_c curves intersect, either by modifying the circuit in such a manner as to cause G_c to increase with amplitude, or by modifying the negative-conductance element in such a manner as to cause $|G_i|$ to fall more rapidly with increase of amplitude. The circuit conductance G_c can be made to rise with amplitude of oscillation by shunting the resonator with a resistance that has a large negative temperature coefficient of resistance.* The rise of temperature with oscillation amplitude, resulting from increase of current through the resistor, then causes the shunt resistance to decrease and G_c consequently to increase.

There are two ways in which the magnitude of G_i can be made to fall more rapidly with increase of amplitude. The first consists in rectifying the resonator voltage, or a voltage proportional to the resonator voltage, and using the resulting direct voltage to bias the negative-resistance tube or transistor in such a manner as to reduce g_m or h_f and thus the magnitude of G_i (see Table I). Although the rectification may often be accomplished by making use of the rectifying property of one of the grids of a tube used in the negative-resistance circuit, the use of a separate diode may be advantageous. Because the emitter-base junction of transistors must be forward-biased, a separate diode is required in transistor circuits (see Sec. 76.3). The manner in which the steepness of the negative-conductance curve is increased can be seen from Fig. 258a. The curve marked 0 is assumed to be that obtained in the unstabilized circuit. The curves marked 1, 2, 3, and 4 are those obtained with an additional constant biasing voltage equal to that obtained by rectifying the resonator voltage at amplitudes of 1, 2, 3, and 4 units, respectively. The curve of $|G_i|$ vs. amplitude for the stabilized circuit is the locus of the intersection of the curve for each value of bias with the abscissa corresponding to the amplitude necessary to produce that bias, as shown by the dashed curve of Fig. 258a. Figure 258b shows that if sufficient stabilizing control can be provided, undesirable characteristics of the form of Fig. 255b can be modified so as to prevent amplitude

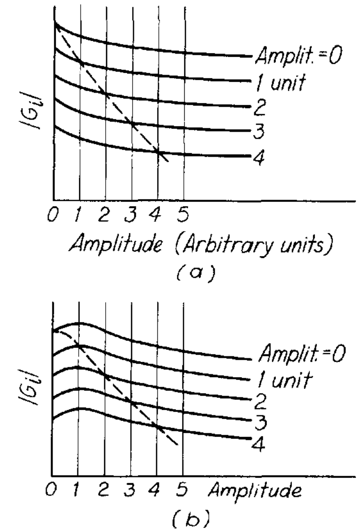


FIG. 258. Increase of steepness of the curve of $|G_i|$ vs. amplitude by the use of a biasing voltage that increases with amplitude of oscillation.

* Negative-coefficient resistors, made of semiconducting material and called *thermistors,* are available in a variety of resistance and power ratings.

hysteresis and make relatively small amplitude of oscillation possible. Problem 74.4–1 illustrates the application of this method of amplitude stabilization to an oscillator based upon the symmetrical current-voltage characteristic of Fig. 69b.

The second method of increasing the slope of the curve of $|G_i|$ vs. amplitude consists in incorporating a resistor having a high positive * or negative temperature coefficient of resistance into the circuit of the negative-resistance device in such a manner that the change of resistance accompanying the increase of temperature with amplitude reduces the magnitude of G_i. This method lends itself readily to use with negative-resistance devices that make use of voltage-divider feedback, such as the pentode circuit of Fig. 66, in which the screen grid is coupled to the suppressor grid by means of a resistance voltage divider. If one of the resistances is a temperature-sensitive resistor of proper temperature coefficient of resistance, increase of amplitude reduces the magnitude of the coupling and thus the magnitude of the negative conductance. Figure 258 also applies to this type of stabilization if each solid curve is assumed to be that obtained with a fixed coupling resistance of magnitude equal to the resistance of the thermal resistor at the amplitude corresponding to the number with which the curve is marked.

Examples of oscillators making use of these methods of amplitude stabilization will be given in Sec. 76.

74.5. Frequency Stabilization. Because α is zero when the amplitude of oscillation has reached its equilibrium value, equilibrium frequency of oscillation for the circuit of Fig. 251a, as given by Eq. (133), is

$$\omega = \sqrt{\frac{R + R_{iv}}{R_{iv}}} \frac{1}{\sqrt{LC}} \tag{137}$$

The factor $\sqrt{(R + R_{iv})/R_{iv}}$, which was assumed to approximate unity in the equivalent circuit of Fig. 253a, may depend both upon load and upon supply voltage. The dependence upon load arises because the effective value of R may include a component associated with load coupled to the resonator. The dependence upon supply voltage is caused by change of R_{iv} with transistor or tube electrode voltages.

Dependence of oscillation frequency upon load can be prevented by use of a buffer amplifier stage. The fractional change of frequency approximates $R/2R_{iv}$ times the fractional change in R_{iv} (Prob. 74.5–1). If a buffer amplifier is used, $R/2R_{iv}$ may be readily made of the order of 10^{-4} except when high-inductance inductors must be used in order to obtain frequencies in the low audio range or below. For a typical negative-resistance element,

* Tungsten-filament lamps have a positive temperature coefficient of resistance and may be used for this purpose.

the fractional change in resistance is of the same order of magnitude as the fractional change of operating voltage of an electrode. It is therefore not difficult to keep the change of frequency less than 10^{-4} percent for a 1-percent change of supply voltage, particularly since compensating effects usually reduce the change of R_{iv} if all electrode voltages are obtained from a single source.

74.6. Amplitude and Frequency Stabilization of Series-Resonator Negative-Resistance Oscillators. As pointed out in Sec. 73.5, treatments similar to those presented in Sec. 74 are possible for oscillators in which current-stable negative-resistance elements and series resonant circuits are used. The conductances G_i and G_c must be replaced by resistances R_i and R_c in Eqs. (136) and in the amplitude diagrams of Figs. 255 and 257. The discussion is analogous to that of Sec. 74 and will therefore be omitted.

75. Admittance Diagrams for Negative-Resistance Oscillators

75.1. Theory of Admittance Diagrams for Parallel-Resonator Oscillators. It has already been shown that G_t must be negative in order that sustained sinusoidal oscillation may take place, and that the magnitude of G_i is equal to G_c under equilibrium conditions. The following relation must therefore hold under equilibrium:

$$G_i = -G_c \tag{138}$$

If L_c is assumed to be equal to L in the circuit of Fig. 253a; i.e., if $\sqrt{(R+R_{iv})/R_{iv}}$ is assumed to be equal to unity, the angular frequency of oscillation is

$$\omega = \sqrt{\frac{1}{L_cC} - \alpha^2} \tag{139}$$

Under equilibrium conditions, α is zero and Eq. (139) reduces to

$$\omega = 1/\sqrt{L_cC} \tag{140}$$

In Sec. 73.1 C was defined as the sum of internal capacitance C_i of the negative-resistance element and additional circuit capacitance C_c. If $C_c + C_i$ is substituted for C in Eq. (140), the equation may be rewritten in the following form:

$$\omega C_i = \frac{1}{\omega L_c} - \omega C_c \tag{141}$$

In some devices in which negative resistance is the result of electron transit time, notably in the reflex-klystron microwave oscillator, the susceptive component of the input admittance of the negative-resistance element may be either capacitive or inductive, depending upon the magnitudes of the electrode voltages, and may pass through zero as the frequency is varied. Equation (141) should therefore be generalized by the addition of another inductive-susceptance term. (This is equivalent to the addition of a second shunt inductance, L_i, to the circuit of Fig. 253a.) The following general equation therefore applies under equilibrium amplitude of oscillation:

$$\omega C_i - \frac{1}{\omega L_i} = -\left(\omega C_c - \frac{1}{\omega L_c}\right) \tag{142}$$

or

$$B_i = -B_c \tag{143}$$

Equations (138) and (143) may be combined into the following single criterion for equilibrium oscillation:

$$Y_i = -Y_c \tag{144}$$

Although Y_i may vary considerably over a wide frequency range (Probs. 17.2–1 to 17.2–4), the variation is negligible over the frequency range in which the admittance of the resonator is high. Both G_i and B_i are, however, functions of amplitude of oscillation. Unless the resonator incorporates some type of nonlinear element, on the other hand, G_c and B_c are independent of amplitude, but vary with frequency. If the amplitude dependence of Y_i and the frequency dependence of Y_c are sufficiently simple, Eqs. (138) and (143) can be written in terms of amplitude and frequency and solved simultaneously to find the equilibrium values of amplitude and frequency. This procedure is possible with some oscillators, notably the reflex klystron, but Y_i is in general not known or is such a complicated function of amplitude that G_i and B_i cannot be readily expressed in terms of amplitude. Much useful general information concerning the behavior of oscillators may be obtained from a graphical analysis, nevertheless, even when the exact forms of the expressions for Y_i and Y_c are not known. The graphical method consists in plotting the conductive and susceptive components of Y_i at various amplitudes and of $-Y_c$ at various frequencies in rectangular coordinates of admittance and susceptance; i.e., in plotting B_i vs. G_i and $-B_c$ vs. $-G_c$, in the same diagram. Because the two curves intersect at points at which Eqs. (138) and (143) are satisfied, the points of intersection indicate possible equilibrium values of amplitude and frequency.

The presence of the term α^2 in Eq. (139) indicates that during the build-up and decay of oscillation the frequency of oscillation is less than the value at which B_i and B_c are equal in magnitude. Since α^2 is usually small in comparison with $1/L_cC$, however, the following treatment will be simplified by the assumption that B_i and B_c are equal in magnitude not only under steady-state oscillation, but also during the transient periods. Under this assumption, the ordinate of the admittance point on the $-Y_c$ curve in the admittance diagram must be equal to the ordinate of the admittance point on the Y_i curve at all times.

75.2. Analysis of an Admittance Diagram. Figure 259 shows an admittance diagram in which the curves of circuit admittance Y_c and *electronic admittance* Y_i are of arbitrary form. Equilibrium oscillation obtains for values of frequency and amplitude corresponding to point 3, since Eqs. (138) and (143) are satisfied at this point. The zero-amplitude electronic-admittance point is at 0. Under the assumption that $B_i + B_c = 0$ during the build-up of oscillation, the circuit admittance and frequency at zero amplitude correspond to point a on the curve of $-Y_c$. Because the magnitude of the electronic conductance at point 0 exceeds that of the circuit conductance at point a, α is positive and any disturbance, such as circuit or tube noise, initiates an oscillation of increasing amplitude. As the amplitude increases, the electronic-admittance point moves toward point 1 and the circuit-admittance point moves toward point b. Inasmuch as the frequency increases along the circuit-admittance curve from a to b, increase of amplitude from 0 to 1 is accompanied by increase of frequency of oscillation. With increase of amplitude beyond that corresponding to point 1, the electronic-admittance point moves back toward the conductance axis, the displacement being accompanied by an equal displacement of the circuit-admittance point and consequently by a decrease of frequency. The amplitude continues to increase and the frequency to decrease until the amplitude reaches the value corresponding to the intersection at 3. This point is stable, inasmuch as further increase of amplitude would cause the magnitude of G_i to become less than G_c and the amplitude therefore to decrease.

The rate at which the amplitude increases during the build-up of oscillation to the stable value corresponding to point 3 is at every instant proportional to the difference in magnitudes of the electronic and circuit conductances (Prob. 75.2–1). In the foregoing example, the greatest difference between the magnitudes of G_i and G_c occurs for an amplitude corresponding to a point on the Y_i curve not far from that at which G_i has its greatest magnitude. Figure 259b shows the general manner in which the amplitude and frequency vary with time during the growth of oscillation. The frequency f_0 is that at which the circuit susceptance is zero, and is therefore the resonance frequency of the resonator alone.

If the electronic conductance at zero amplitude is smaller in magnitude than the circuit conductance, as in Fig. 259c, α is negative at zero amplitude and oscillation cannot start spontaneously. Should the circuit be driven to

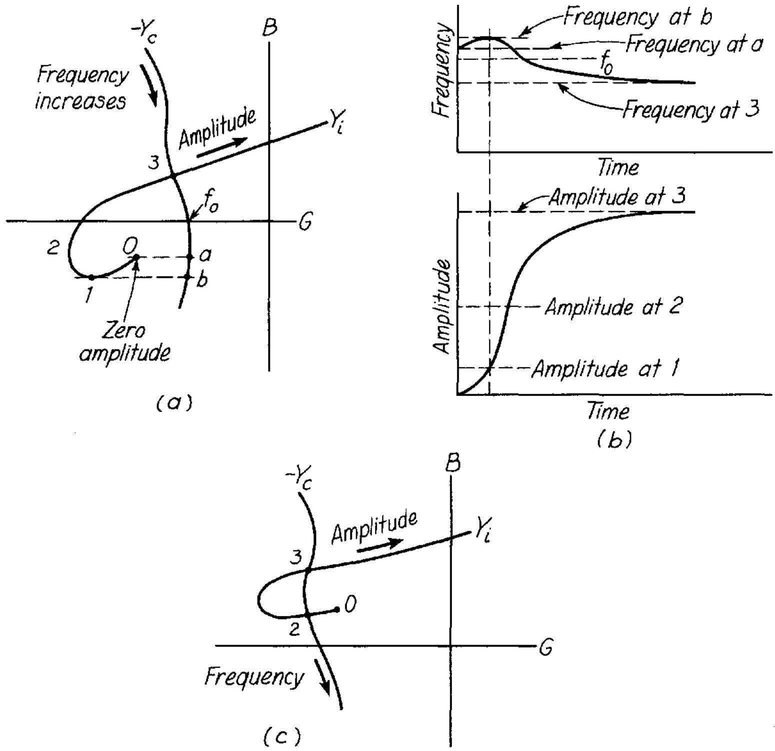


FIG. 259. (a) Admittance diagram for an oscillator that is self-starting; (b) variation of frequency and amplitude of oscillation during the growth of oscillation; (c) admittance diagram for an oscillator that is not self-starting.

an amplitude equal to or greater than that at point 2, however, the amplitude would increase to the equilibrium value at point 3.

75.3. Oscillation at More than one Frequency. When the Y_c curve is of such a form that there are two or more values of G_c for each value of B_c over a range of B_c, oscillation may start simultaneously at two or more frequencies. The manner in which oscillation builds up is then complicated by the fact, shown both theoretically and experimentally, that when two or more voltages of different frequencies are impressed simultaneously across a nonlinear circuit element such as the elements used to produce a negative conductance, the electronic admittance for any of the voltages is changed by the presence of the other voltages. An example of an admittance diagram that predicts simultaneous starting at two frequencies is shown in Fig. 260a, in which the circuit admittance is that of a section of parallel-wire or coaxial transmission line.

In Fig. 260a, the zero-amplitude electronic conductance is greater in magnitude than the circuit conductance of both point a and point a'. Oscillation therefore starts at the frequencies corresponding to these points. Because of the lower magnitude of G_c at point a than at point a', however, the oscillation corresponding to point a has a larger value of α and hence builds up much more rapidly than that corresponding to point a' and eventually reaches equilibrium at point 3. The electronic conductance for the a' frequency is reduced not only as a result of increase of amplitude of the a' oscillation, but also as the result of the presence of the higher-amplitude

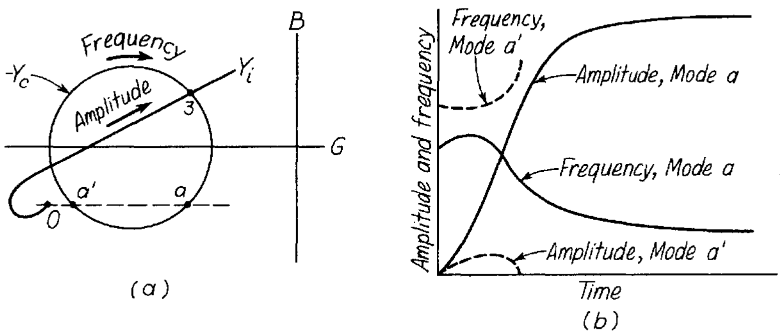


FIG. 260. (a) Admittance diagram for an oscillator that may start at two frequencies; (b) variation of frequency and amplitude in the two modes of oscillation during growth of the dominant mode.

a oscillation. At some time before the amplitude of the a oscillation reaches that corresponding to point 3, $|G_i|$ for the a' oscillation falls below the corresponding magnitude of G_c, and the a' oscillation therefore dies out. Figure 260b shows the general manner in which the amplitude and frequency of the two modes of oscillation vary with time.

Under certain conditions oscillations may exist simultaneously at two frequencies under equilibrium conditions. Oscillations may also take place alternately in two modes, one dying down as the other builds up.

The particular curves used in Figs. 259 and 260 were selected only because they are convenient in explaining important points in the analysis. The graphical analysis based upon admittance diagrams may be applied to any type of oscillator, regardless of the type of energy-converting device, type of resonator, or type of loading. The form of the electronic-admittance curve depends upon the type of electronic device and the operating voltages and currents; that of the circuit-admittance curve depends upon the characteristics of the resonator and the extent and manner in which it is loaded.

75.4. Use of Admittance Diagrams. In general, oscillator admittance diagrams are of principal value in explaining qualitatively the observed phenomena of oscillator behavior, rather than in making numerical predictions of power output and efficiency. However, when the electronic and circuit admittance curves can be drawn accurately and the amplitude corresponding to each point on the electronic-admittance curve is known or

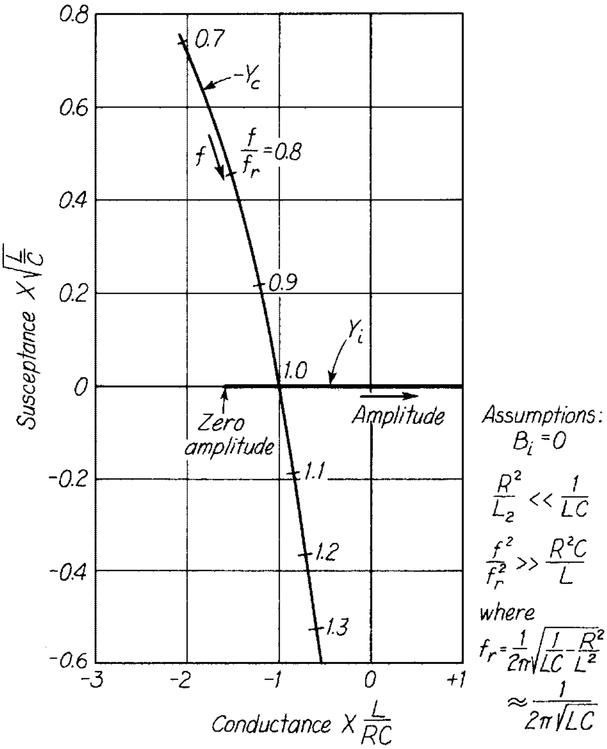


FIG. 261. Admittance diagram for the oscillator of Fig. 251a under the assumption that B_i is zero.

can be computed, the power delivered to the circuit conductance G_c can be readily found from the diagram, since the equilibrium point determines both the voltage amplitude and the circuit conductance. Furthermore, if the frequency corresponding to each point on the circuit-admittance curve is known or can be computed, the frequency of oscillation can be found from the equilibrium point. This graphical method of predetermining oscillator performance is illustrated by Prob. 75.4–2.

Figure 261 shows the admittance diagram for the oscillator of Fig. 251a under the assumption that the electronic susceptance B_i is zero. In order to generalize the diagram, G_i and G_c have been normalized relative to RC/L, and B_i and B_c have been normalized relative to $\sqrt{C/L}$ (see Prob. 75.4–1).

Equilibrium oscillation is obtained at the point $-1,0$ corresponding to a circuit conductance of RC/L and a frequency

$$\frac{1}{2\pi}\sqrt{1/LC - R^2/L^2} = \frac{1}{2\pi\sqrt{LC}}\sqrt{1 + \frac{R}{R_{iv}}}$$

This value of frequency agrees with that given by Eq. (133). It is apparent from Fig. 261 that increasing RC/L causes an equal increase in $|G_i|$ and therefore a decrease of amplitude of oscillation.* If the shunt conductance RC/L is changed by changing R/L, the resonance frequency f_r, and therefore the frequency of oscillation, is changed. This is also in agreement with Eq. (133) and is an important consideration in frequency stability of oscillators of this type, practical forms of which will be discussed in Sec. 76.

At frequencies such that transit-time effects in the negative-resistance element are not negligible, the admittance Y_i of the negative-resistance element has a susceptive component. The $|Y_i|$ curve is then no longer horizontal. At a fixed value of $|Y_i|$ the susceptive component alters the frequency of oscillation and reduces the amplitude (Prob. 75.4–2). If the phase angle of Y_i differs sufficiently from 180 degrees, the Y_i and Y_c curves no longer intersect. The admittance diagram therefore predicts that transit-time effects impose an upper limit upon the frequency of oscillation at fixed values of L_c, C_c, and G_c. In general, the upper frequency limit is the result not only of the change in the phase angle of Y_i, but also of a reduction in the magnitude of Y_i by transit-time effects (Probs. 17.2–1 to 17.2–4).

75.5. Admittance Diagrams for Oscillators Based Upon Current-Stable Negative-Resistance Elements. An analysis similar to that presented in Secs. 75.1 to 75.4 for the parallel-resonator oscillator of Fig. 253a justifies the use of diagrams analogous to those of Figs. 259 and 260 for the series-resonator oscillator of Fig. 253b (Prob. 75.5–1). Curves of X_i vs. R_i and $-X_c$ vs. $-R_c$ replace those of B_i vs. G_i and $-B_c$ vs. $-G_c$.

76. Practical Negative-Resistance Oscillators

76.1. Choice of Circuits. Once it is understood that a sine-wave oscillator can be formed by shunting a voltage-stable port with a parallel resonant circuit or a current-stable port with a series resonant circuit, it becomes apparent that a large number of oscillator circuits can be formed from the various negative-resistance devices and circuits discussed in Secs. 18 to 22. The chief merit of many of these circuits lies in the fact that they differ in some respects from other known circuits and are therefore patentable. The desirable features of others may include extreme simplicity, high oscillation frequency, small size, low cost, good waveform, high stability,

* Note that $G_iL/RC = -1$ at the equilibrium point. Therefore $|G_i| = RC/L$. The amplitude must decrease in order to increase $|G_i|$.

and the ability to operate on very low supply voltage. Only a few of the many possible circuits will be discussed, the derivation of the others being left to the reader as an exercise (Probs. 76.1–1 and 76.1–2).

The principal problems encountered in the devising of negative-resistance-oscillator circuits are those of providing the proper voltage or current biases to ensure that the operating point lies at the best point in the negative-resistance portion of the current-voltage characteristic and of selecting the circuit parameters so that sinusoidal, rather than relaxation, oscillation is obtained. In order that the amplitude of oscillation, and therefore the harmonic output, may be readily kept small, the operating point should be chosen so that the curve of $|G_i|$ vs. amplitude is of the form of that of Fig. 255a. This is most readily accomplished by the use of a negative-resistance element that has a symmetrical characteristic, such as the characteristics of Figs. 69b and 69e, and using the point of symmetry as the operating point.

76.2. Pentode Negative-Resistance Oscillators. Two single-tube oscillators based upon the pentode negative-resistance circuit of Fig. 66a are

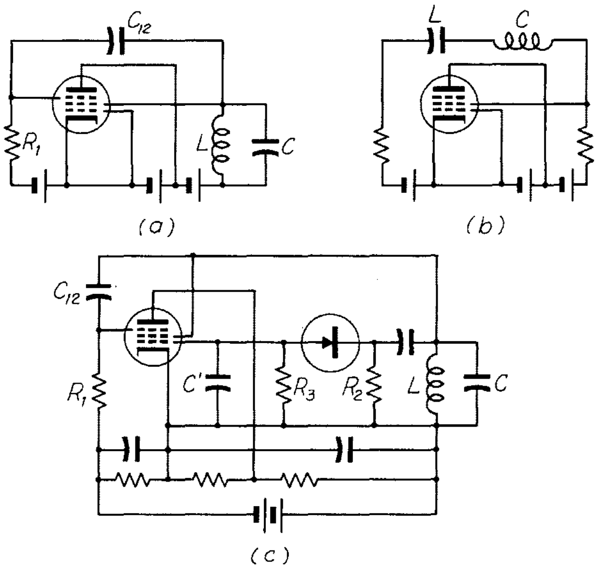


FIG. 262. (a) Pentode oscillator that makes use of the voltage-stable current-voltage characteristic of Fig. 66c; (b) pentode oscillator that makes use of the current-stable characteristic of Fig. 66e; (c) amplitude-stabilized version of circuit (a).

shown in Fig. 262. Circuit (a), which is known as the *transitron* oscillator, makes use of a parallel resonator across a voltage-stable port. Circuit (b) makes use of a series resonator across a current-stable port. The time constant R_1C_{12} of the coupling path of circuit (a) should be large enough in comparison with the period of oscillation so that relatively little alternating voltage appears across C_{12}. If R_1C_{12} is made too small in comparison with the period of resonance of the resonator, relaxation oscillation will take place at a frequency determined by R_1 and C_{12}.

The operating voltages and circuit parameters of the circuits of Fig. 262 may be chosen so that the amplitude of oscillation is small and the harmonic content therefore low, even when all operating voltages are constant, as in circuit (a). The dependence of amplitude upon supply voltage may be greatly reduced by use of control-grid bias that increases with amplitude of oscillation. One way in which this can be accomplished is by means of a diode excited by voltage from the resonator, as shown in Fig. 262c. In order to prevent excessive inverse feedback resulting from the application of oscillation-frequency voltage to the control grid, the time constant R_3C' should be large in comparison with the period of oscillation. R_2 and R_3 should be large enough to prevent excessive loading of the resonator.

76.3. Balanced Negative-Resistance Oscillators. The circuits of Fig. 263 make use of the symmetrical current-voltage characteristics of Figs. 69b and 69e obtained in the circuits of Figs. 68b and 68e, respectively, or their transistor counterparts. Because the magnitude of the negative conductance G_{iv} decreases continuously with increase of amplitude in the circuits of Figs. 263a and 263b (see Fig. 69b and Sec. 74.1) and the magnitude of R_{ic} decreases continuously with increase of amplitude in the circuits of Figs. 263c and 263d (see Fig. 69e), these circuits may be very easily adjusted for low amplitude of oscillation and resulting low harmonic content.

In order to avoid relaxation oscillation at a frequency determined by R_1 and C_{12}, instead of sinusoidal oscillation at a frequency determined by L and C, the time constant R_1C_{12} must be large in comparison with the period $2\pi LC$ in the circuits of Fig. 263. Alternatively, C_{12} and C_{12}' may be replaced by resistors. However, the resulting reduction of $|G_{iv}|$ or $|R_{ic}|$ increases the value of resonator Q required in order to satisfy the criterion for oscillation, and therefore reduces the frequency range in which sustained oscillation is possible.

As pointed out in Sec. 74.4, the amplitude stability against changes of supply voltage can be greatly increased by modifying the circuit so that the grid or base bias varies with amplitude of oscillation. This is most readily accomplished in the tube circuit of Fig. 263a by reducing the fixed bias to zero by making R_k zero. The flow of grid current during the halves of the cycle in which each grid is positive increases the charge on the capacitor connected to that grid and therefore the voltage across the capacitor. Under the assumption that grid current starts flowing at zero grid voltage, the increase of capacitor voltage is equal to the crest value of the alternating voltage of the plate of the other tube. If the time constant R_1C_{12} is large in comparison with the oscillation period, little discharge of the coupling capacitors takes place during the portions of the cycle in which each grid is reverse-biased. Consequently the grid bias is approximately equal to the

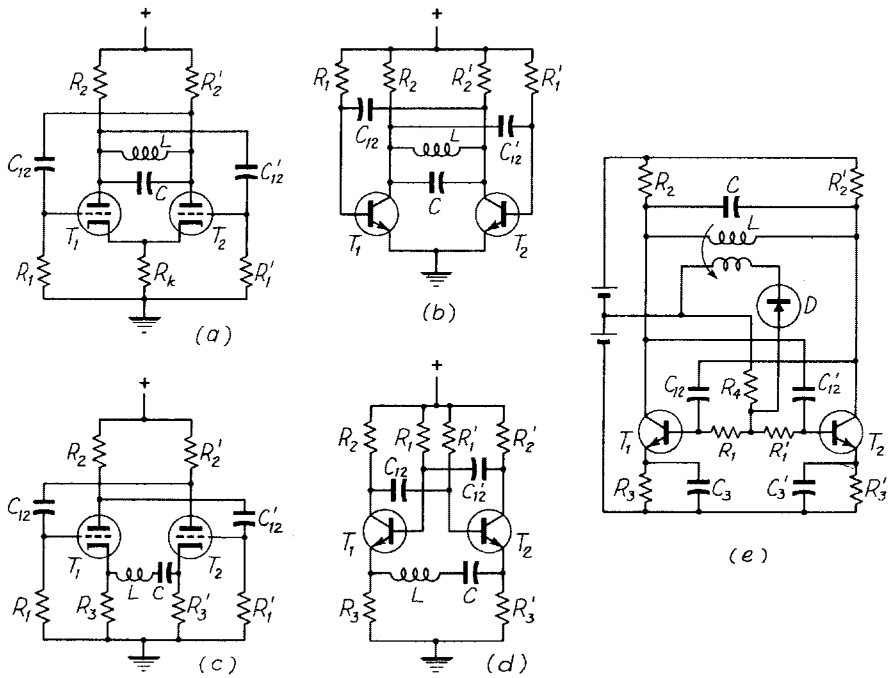


FIG. 263. (a) Symmetrical oscillator that makes use of the voltage-stable characteristic of Fig. 69b; (b) equivalent transistor circuit; (c) oscillator that makes use of the current-stable characteristic of Fig. 69e; (d) equivalent transistor circuit.

amplitude of the voltage across the resonator. At zero amplitude the bias is zero, g_m is large, and the expression for $1/R_{iv}$ given in Sec. 18.2 shows that the magnitude of G_{iv} is large. As the amplitude increases, the negative grid bias increases, g_m becomes smaller, and the magnitude of G_{iv} decreases. The curve of G_{iv} vs. amplitude is relatively steep, as required for good amplitude stability (Prob. 74.4–1).

Similar amplitude stabilization is produced by grid rectification in the circuit of Fig. 263c. In the circuit as shown, grid rectification and stabilization occur only at amplitudes of oscillation greater than the grid bias produced by the cathode resistors. The stabilization can be extended to small amplitudes by connecting the grid resistors to a point in the voltage supply that is positive relative to ground by an amount equal to the average voltage of the cathodes, instead of to ground.

Although rectification by the emitter-base junction can be used to stabilize the amplitude of the transistor circuits of Figs. 263b and 263d, the behavior of the circuit is complicated by the fact that forward bias is required in the normal operation of the transistors and that collector-current cutoff occurs in the vicinity of zero base-emitter voltage. Consequently stabilization as the result of base-emitter rectification is likely to be associated with Class C operation of the transistors and therefore with relatively high harmonic generation. This difficulty may be avoided by using adequate forward base bias and reducing this bias with increasing amplitude by means of rectified voltage provided by an auxiliary diode, as shown in Fig. 263e.

76.4. Cathode-Coupled and Emitter-Coupled Oscillators. Figure 264 shows transistor and tube oscillators based upon the negative-resistance circuit of Fig. 70 and its transistor counterpart. The merit of the circuits of

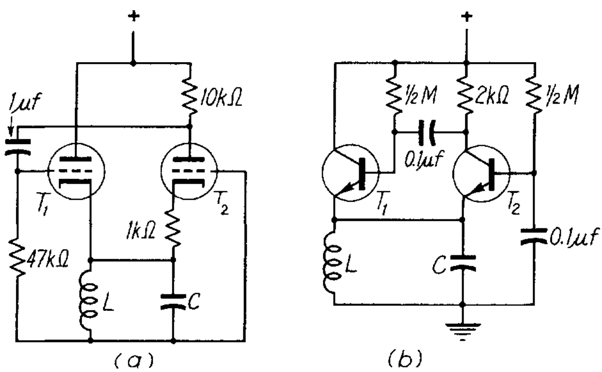


FIG. 264. (a) Oscillator based upon the circuit of Fig. 70; (b) equivalent transistor circuit.

Fig. 264 lies in their ability to oscillate at very low values of supply voltage. This is the result of the low magnitude of negative resistance attainable between cathodes and ground in the negative-resistance circuit of Fig. 70 (see Sec. 18.3). Supply voltage as low as 10 may be used in the circuit of Fig. 264a [1] and 1.5 in that of Fig. 264b.

76.5. Negative-Resistance-Diode Oscillators. Figure 265 shows the relative simplicity that can be achieved by the use of a *p-n-p-n* diode or its two-transistor equivalent in a negative-resistance oscillator. Even greater

[1] H. J. Reich, *Proc. I.R.E.*, **43**, 228 (February, 1955).

simplicity is achieved in the unijunction-transistor circuit of Fig. 266a. The ultimate in oscillator simplicity is probably afforded by the use of a tunnel diode as the negative-resistance element, as in the circuit of Fig. 266b. Tunnel-diode circuits may be used over a frequency range extending into the microwave frequencies and require a supply voltage of the order of only a quarter of a volt. (See also Addendum, page 450.)

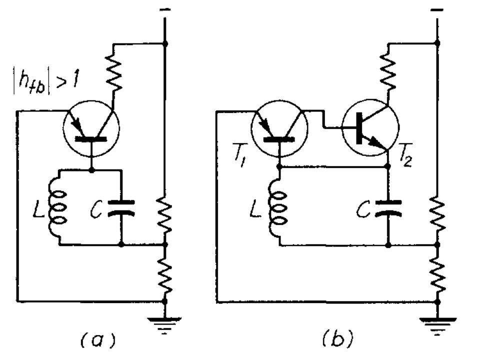


FIG. 265. (a) Oscillator using a *p-n-p-n* transistor; (b) equivalent two-transistor circuit.

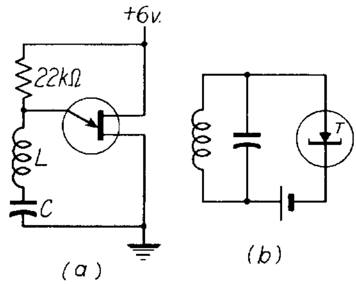


FIG. 266. (a) Unijunction-transistor oscillator; (b) tunnel-diode oscillator.

76.6. Stabilization of Transistor Oscillators Against Change of Temperature. It is important to bear in mind that the presence of R_{iv} or R_{ic} in Eqs. (132) to (135) is an indication that the amplitude of oscillation and, to a lesser extent, the frequency of oscillation of negative-resistance oscillators are dependent upon the magnitude of the negative resistance and therefore upon the parameters of the tubes or transistors used in the negative-resistance element. The fact that transistor parameters depend upon temperature necessitates that stabilization against temperature change be used when frequency and amplitude stability are important. The simplest method of stabilization, which is not always sufficiently effective, is the use of bypassed resistance (1000 to 2000 ohms), in series with the emitters of transistors. Other methods of stabilization against change of temperature are discussed in Sec. 84.7.

77. Electronic Tuning and Modulation of Negative-Resistance Oscillators

77.1. Tuning by Direct Variation of Susceptance. Negative-resistance oscillators may be tuned or frequency-modulated electronically in a number of ways. At frequencies at which carrier transit times are of the order of magnitude of the period of oscillation the input admittance of the negative-conductance element will in general have a susceptive component that is comparable in magnitude with the negative conductance, and the magnitude of the admittance will be a function of the transit times and consequently of the electrode voltages. Variation of one or more electrode voltages changes the susceptance shunting the parallel resonant circuit and thus the frequency of oscillation. Simultaneous change of the magnitude of the negative conductance causes the frequency change to be accompanied by an amplitude change unless the resonator is shunted by a voltage-controlled conductance element that varies with the control voltage in such a manner as to compensate for the variation of negative conductance. An outstanding example of a negative-resistance oscillator in which the frequency may be varied by means of an electrode voltage is the reflex-klystron microwave oscillator.[1] (See Prob. 75.4–3.)

A second method of varying the frequency of negative-resistance oscillators is by means of voltage-controlled variable capacitors, such as the reverse-biased semiconductor diodes (Sec. 6.4). The low leakage conductance attainable in capacitor diodes makes possible frequency control of modulation with relatively little change of amplitude.

77.2. Tuning by Variable-Admittance Tube and Transistor Circuits. A third commonly used means of controlling or modulating oscillator fre-

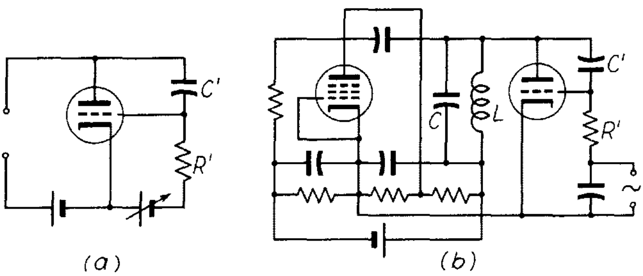


FIG. 267. (a) Variable-admittance circuit; (b) use of this circuit to vary the frequency of the oscillator of Fig. 262a.

quency is by means of a vacuum-tube or transistor variable-admittance circuit, of which the tube circuit of Fig. 267a is an example. Analysis of this circuit (Prob. 77.2–1) shows that the input admittance has a capacitive component and that the magnitude of the admittance depends upon the transconductance of the tube and hence upon the control-grid bias. The effective capacitance has a maximum value when $R' = 1/\omega C'$. For this value of R' the effective capacitance approximates $g_m/2\omega$ and the input con-

[1] See, for example, H. J. Reich, J. G. Skalnik, P. F. Ordung, and H. L. Krauss, *Microwave Principles*, D. Van Nostrand Co., Inc., Princeton, N. J., 1957.

ductance is $g_m/2$. Figure 267b shows how this circuit may be used to vary the frequency of the pentode negative-resistance oscillator of Fig. 262. The dependence of input conductance of the variable-admittance circuit upon g_m causes frequency modulation to be accompanied by amplitude modulation unless the variation of input conductance is compensated by means of another voltage-controlled conductance. A transistor may be used in place of the tube in the circuit of Fig. 267a and other similar variable-admittance circuits.

A somewhat simpler electronic variable-admittance circuit consists of a capacitor in series with the plate of a tube or the collector of a transistor.[2] Variation of the grid or base bias changes the effective susceptance of the series combination. The conductance variation produced by this circuit is relatively high, however. Frequency modulation is therefore accompanied by considerable amplitude modulation.

78. Feedback (Two-Port) Oscillators

78.1. Dependence of Type of Amplifier Upon Feedback-Network Phase Shift. In feedback oscillators the output of an electron tube or transistor, or of a more complicated amplifier incorporating two or more tubes or transistors, is coupled back to the input through a two-port frequency-determining network. Various types of feedback oscillators differ as to the type of feedback network that they incorporate and as to the details of the amplifier. Some feedback networks have a 180-degree difference in phase between the input and output voltages at or near resonance and therefore require an amplifier that produces a 180-degree voltage phase shift in order that the feedback voltage shall be in phase with the amplifier input voltage. Single-stage amplifiers are almost always used in such oscillators. Other feedback networks produce zero phase shift at or near resonance and consequently must be used with amplifiers that produce zero voltage phase shift. Either a two-stage amplifier or a single-stage of a tube having a negative value of g_m or of a transistor having a negative value of h_f exceeding unity in magnitude may be used in these oscillators.

78.2. Analysis of Generalized Feedback Oscillator. Any feedback oscillator may be represented by the block diagram of Fig. 268. In Fig. 268a the feedback loop has been opened and the amplifier excited by means of an external voltage V_i. The input admittance y_{in} of the amplifier, which normally loads the output of the feedback network, has been replaced by an equivalent admittance. If the amplifier and feedback-network parameters are adjusted so that $V_o = V_i$ and $I_o = I_i$ in both phase and magnitude, the amplifier cannot discriminate between the external voltage V_i and the

[2] For a transistor circuit of this type, see L. L. Koros and R. F. Schwartz, *Electronics,* July, 1951, p. 130.

output voltage V_o if the latter is used in place of the external voltage. Hence, if the double-pole, double-throw switch is instantaneously thrown to the left and the external driving voltage is simultaneously removed, all voltages and currents in the system will be unchanged. In other words, the circuit will oscillate and deliver a constant output voltage V_o. The requirement that the voltage at the output port of the feedback network be equal to the input voltage of the amplifier prior to closing of the feedback loop is equivalent to the requirement that the feedback in the closed system must be regenerative and that the closed-loop amplification under steady-state conditions must be unity.

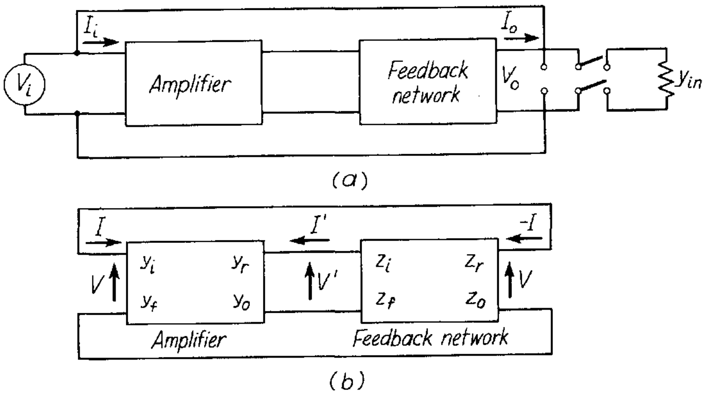


FIG. 268. Generalized representation of feedback oscillators.

A commonly used method of determining the frequency of oscillation and the criterion for oscillation of constant amplitude in a feedback oscillator is to replace the amplifier by an equivalent circuit and to solve the loop or nodal network equations simultaneously. An alternative procedure is to relate the currents and voltages in the circuit of Fig. 268 by means of generalized two-port network equations. Although any set of parameters may be used for the amplifier and any set for the feedback network, it is convenient to use y parameters for the amplifier and z parameters for the feedback network. The advantage of y parameters for the amplifier is that the y parameters g_m and $1/r_p$ have long been used for tubes and that there is no difficulty in converting y parameters to h parameters in the equations obtained in the analysis of transistor circuits. For many feedback networks the z parameters can be more readily found than can the y parameters.

The following four equations completely specify the conditions that must be satisfied in order that the output current and voltage of the feedback network shall be equal in phase and magnitude to the input current and voltage of the amplifier, as required in the circuit of Fig. 268b:

$$I = Vy_i + V'y_r \tag{145}$$

$$I' = Vy_f + V'y_o \tag{146}$$

$$V' = -I'z_i - Iz_r \tag{147}$$

$$V = -I'z_f - Iz_o \tag{148}$$

Solution of the simultaneous equations (145) to (148) leads to the following relation between the two sets of parameters:

$$y_f z_f + y_i z_o + y_o z_i + y_r z_r + \Delta_y \Delta_z + 1 = 0 \tag{149}$$

in which $\Delta_y = y_i y_o - y_f y_r$ and $\Delta_z = z_i z_o - z_f z_r$. Since $z_r = z_f$ for a passive network and $y_f \gg y_r$ for amplifiers suitable for use in the circuit of Fig. 268, the term $y_r z_r$ may ordinarily be neglected in Eq. (149). Therefore,

$$y_f z_f + y_i z_o + y_o z_i + \Delta_y \Delta_z + 1 \approx 0 \tag{149A}$$

For particular circuits the impedance parameters of the feedback network may be found and substituted into Eq. (149). Because the sum of the real terms and the sum of the imaginary terms of Eq. (149) must each be equal to zero, Eq. (149) leads to two equations that may be solved simultaneously for the frequency of oscillation and the criterion for oscillation of constant amplitude. Frequently the dropping of negligible terms leads to considerable simplification, particularly in the analysis of tube circuits. Examples of the use of Eq. (149) or (149A) in the analysis of feedback oscillators will be given in sections that follow.

It is important to note that the use of Eqs. (145) to (148) implies the assumption that the amplifier is a linear circuit. The application of Eq. (149) or (149A) can therefore yield no information concerning harmonic generation or the limitation of amplitude as the result of dependence of circuit parameters upon amplitude. The assumption that the amplifier is linear is not valid at large amplitudes. At large amplitudes the y parameters must therefore be defined as the ratios of fundamental components of current to fundamental components of voltage.

Interelectrode capacitances and coupling impedances used in the input and output of the amplifier can usually most conveniently be assumed to form parts of the feedback network. The analysis of some circuits, however, is simplified somewhat by considering the input and output coupling resistances to be included in the amplifier.

78.3. Analysis of Circuits in Which y_i and y_r Are Small. If z_r is replaced by its equivalent z_f in the term $y_r z_r$, Eq. (149) may be rewritten in the form:

$$y_f\left(1 - \frac{y_r}{z_f}\Delta_z\right) + \frac{1}{Z_f} + y_r = 0 \tag{150}$$

where

$$\frac{1}{Z_f} = \frac{1}{z_f}(y_i z_o + y_o z_i + y_i y_o \Delta_z + 1) \tag{151}$$

It may be shown that Z_f is the open-circuit forward transfer impedance of the network of Fig. 269a, in which the feedback network of Fig. 268 has been augmented by shunting the input terminals by y_o and the output terminals by y_i (Prob. 78.3–1). In common-cathode vacuum-tube circuits in which the grids are biased sufficiently negative to prevent the flow of grid current, y_i and y_r are zero and the augmented feedback network of Fig. 269a simplifies to that of Fig. 269b. If y_i and y_r are made zero in Eqs. (150) and (151) the following relation is obtained:

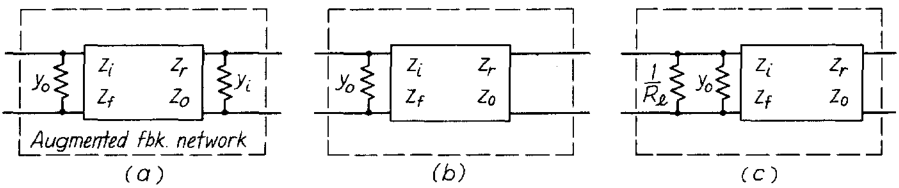


FIG. 269. (a) Augmented feedback network; (b) augmented feedback network applicable when $y_i \ll 1/z_o$; (c) augmented network that includes R_l.

$$y_f = -1/Z_f = -\frac{1}{z_f}(y_o z_i + 1) \tag{152}$$

Although the second term within the parentheses of Eq. (150) is not ordinarily negligible when transistors are used, useful qualitative information can sometimes be obtained under the assumption that Eq. (152) is also applicable to common-emitter transistor feedback oscillators.

The value of Eq. (152) rests partly in the fact that Z_f may sometimes be determined by inspection of the augmented feedback network. The approximate frequency of oscillation and the approximate criterion for oscillation of constant amplitude may thus be obtained with a minimum of effort. Equation (152) is also useful in the construction of admittance diagrams similar to those discussed in Sec. 75 for negative-resistance oscillators. Admittance diagrams for feedback oscillators will be discussed in Sec. 79.

In many feedback oscillators, a coupling resistance R_l in the output circuit of a tube or transistor shunts the input to the feedback network. This resistance can either be considered to be a part of the feedback network, as in Fig. 269c, or it may be considered to be a part of the amplifier. When it is considered to be a part of the amplifier, y_o in Eqs. (151) and (152) must be replaced by $y_o + 1/R_l$. Equation (152) then becomes

$$y_f = -\frac{1}{z_f} + \frac{z_i}{z_f}(y_o + 1/R_l) \tag{152A}$$

In the analysis of some circuits it is simpler to find the z parameters of the feedback network alone and to use these in Eq. (152A) than it is to find Z_f for the augmented network, including R_l.

78.4. Analysis by Differential Equations. An alternative method of analysis of feedback oscillators that is analogous to the method used in Sec. 73 for negative-resistance oscillators consists in writing the differential equations for a circuit in which the amplifier is replaced by an equivalent circuit containing one or more equivalent current or voltage generators. The advantage of this method lies in the fact that the solution of the equations for the equivalent circuit yields not only the frequency and the criterion for oscillation of constant amplitude, but also the criteria for increasing and decreasing amplitude of oscillation. These criteria are essential to an understanding of amplitude limitation and amplitude hysteresis. An example of this method of analysis is given in Sec. 81.2. The analysis of Sec. 81.2 and similar analyses for other types of feedback oscillators show that the amplitude of oscillation increases, remains constant, or decreases if $|y_f|$ is greater than, equal to, or less than, respectively, a critical value. In oscillators in which y_i and y_r may be considered to be zero, as in vacuum-tube circuits in which the grids are negatively biased, the critical value is equal to $|1/Z_f|$, the magnitude of the reciprocal of the forward open-circuit transfer impedance of the augmented feedback network of Fig. 269b or c. In oscillators in which y_i and y_r cannot be assumed to be zero, as in transistor circuits, the critical value approximates $|1/Z_f|$ sufficiently closely so that it may be assumed to be equal to $|1/Z_f|$ in rough qualitative analyses. When y_i is not negligible, the augmented feedback network must include y_i, as shown in Fig. 269a. The analyses also show that when the circuit values are such that the amplitude increases or decreases, the frequency of oscillation is slightly lower than when it is constant.

79. Admittance Diagrams for Feedback Oscillators

79.1. Justification for Admittance Diagrams. Considerable useful qualitative information concerning the behavior of feedback oscillators, particularly microwave oscillators, can be obtained by the use of admittance diagrams based upon Eq. (152). Over any single frequency range in which the open-circuit transfer impedance of the feedback network is high (i.e., in the vicinity of any one resonance frequency of the feedback network), the dependence of y_f upon frequency is ordinarily negligible. The magnitude of y_f, however, and in some amplifiers the phase of y_f, depends upon amplitude. Although y_f may increase in magnitude with increase of amplitude at low amplitudes, amplifier nonlinearities associated with saturation and cutoff invariably cause the magnitude to decrease with increase of amplitude beyond some critical value of amplitude. Unless the feedback network incorporates a nonlinear element, such as a diode, varistor, or thermistor, on the other hand, Z_f is independent of amplitude and is a rapidly varying function of frequency in the vicinity of a resonance frequency. It follows from Eq. (152) that, if the susceptive component of y_f is plotted against the conductive component over a suitable range of amplitude and if the susceptive component of $-1/Z_f$ is plotted against the conductive component in the same diagram over a suitable frequency range in the vicinity of a resonance frequency, any intersections of the two curves correspond to equilibrium values of frequency and amplitude.

79.2. Example of an Admittance Diagram for a Feedback Oscillator. Figure 270a is an admittance diagram in which the curve of $-1/Z_f$ is typical

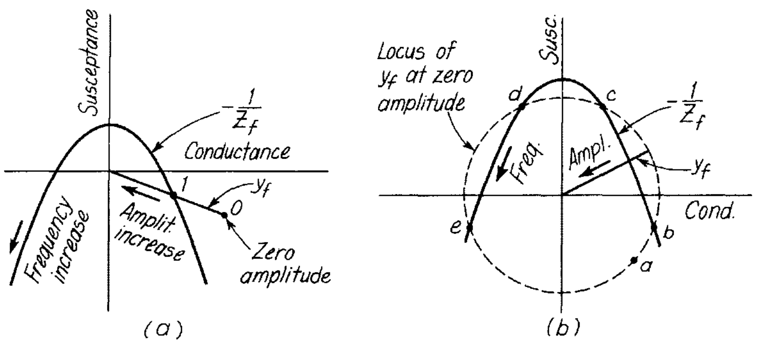


FIG. 270. (a) Admittance diagram for one type of feedback oscillator; (b) diagram showing the locus of y_f at zero amplitude.

of a feedback network consisting of a transformer tuned by means of capacitance across both the primary and the secondary,[1] and in which the y_f curve is typical of a vacuum tube operated at a sufficiently high frequency so that the transadmittance has a susceptive component. Point 0 on the y_f curve corresponds to zero amplitude. Since $|y_f|$ at zero amplitude exceeds $|1/Z_f|$, any small oscillation initiated by noise or some other circuit disturbance increases in amplitude.* The operating point on the y_f curve therefore moves to the left until it reaches point 1, at which $|y_f| = |1/Z_f|$

[1] See H. J. Reich, P. F. Ordung, H. L. Krauss, and J. G. Skalnik, *Microwave Theory and Techniques*, Sec. 10-17, D. Van Nostrand Co., Inc., Princeton, N. J., 1953.

* See Secs. 78.4 and 81.2.

and the amplitude is constant. If the amplitude scale for the y_f curve and the frequency scale for the $-1/Z_f$ curve are known, the equilibrium values of amplitude and frequency may be read from the curves. Usually, however, admittance diagrams such as Fig. 270a are of principal value in explaining or predicting general operating characteristics, rather than in predicting numerical values of frequency or amplitude (Prob. 79.2-1). It can be seen from Fig. 270a, for example, that the effect of the susceptive component of y_f caused by electron transit time is to reduce the frequency of oscillation below the value that would obtain if y_f were nonsusceptive.

79.3. Klystron Admittance Diagram. Admittance diagrams are particularly instructive in the analysis of two-resonator klystron oscillators [2] in which the phase of y_f can be varied through all angles by means of the anode voltage. The dotted circle in the diagram of Fig. 270b shows the locus of the zero-amplitude magnitude of y_f as the phase is varied. The curve of $-1/Z_f$ approximates that for two coupled cavity resonators. It can be seen that if the phase of y_f is gradually advanced from the value corresponding to point a, the oscillation builds up when the phase advances beyond the value at b. The equilibrium amplitude increases, passes through a maximum value at an angle approximately halfway between b and c, and becomes zero at c. Oscillation again starts at d, the amplitude rises to a second maximum value, and becomes zero at e. If the magnitude of y_f is increased to a value that exceeds $|1/Z_f|$ at the peak of the curve of $-1/Z_f$, oscillation takes place at all phases of y_f in the range $bcde$. This predicted behavior is in agreement with the observed behavior of two-resonator klystrons.

80. Parallel-Resonator Feedback Oscillators

80.1. Tuned-Output, Tuned-Input, Hartley, and Colpitts Circuits. Feedback oscillators, like negative-resistance oscillators, are characterized by great variety of usable circuits, many of which have no particular advantage over others. The desirable features of individual oscillator circuits may include good frequency and amplitude stability, low harmonic output, simplicity, low cost, and small size and weight.

A class of feedback oscillators that has found more applications than any other over the period of years since the invention of the vacuum triode is that in which the amplifier consists of a single tube or transistor and the feedback network consists essentially of a tuned parallel resonant circuit, the proper phase of z_f being obtained by the use of an untuned coil coupled to the resonator, a tapped inductor, or a capacitor voltage divider. The four most commonly used circuits are shown in basic form in Fig. 271. Circuit

[2] Reich, Ordung, Krauss, and Skalnik, *op. cit.*, Chap. 12.

(a), in which the transformer winding that is in the output circuit of the amplifier is tuned, will be called the *tuned-output* oscillator. Circuit (b), in which the transformer winding that is in the input circuit is tuned, will be called the *tuned-input* oscillator. In the *Hartley* circuit (c) either a two-winding transformer or an autotransformer may be used. Coupling from the output of the amplifier to the input in this circuit is both inductive and capacitive. The *Colpitts* oscillator (d) uses a single inductor.

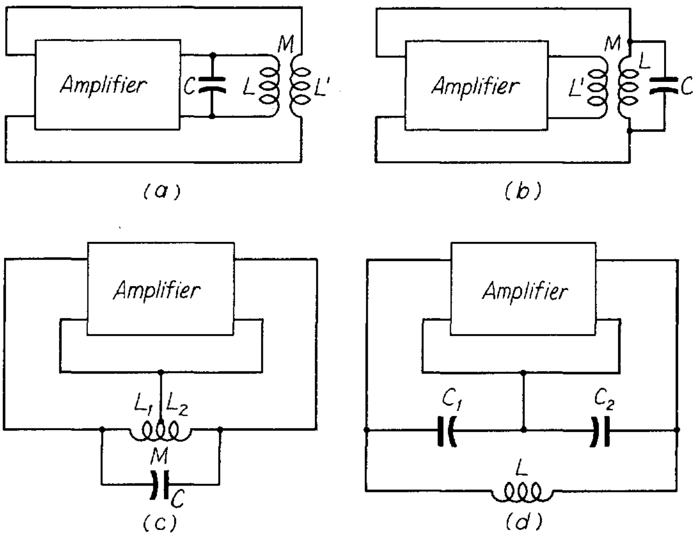


FIG. 271. (a) Tuned-output oscillator; (b) tuned-input oscillator; (c) Hartley oscillator; (d) Colpitts oscillator.

In order that the feedback shall be regenerative in the Hartley and Colpitts circuits, the algebraic sign of the forward transfer admittance or forward current transfer ratio of the amplifier must be positive. Hence, when a single tube or a single transistor is employed in either of these circuits, it must be used in the common-cathode or common-emitter connection. In the tuned-output or tuned-input circuits, on the other hand, the algebraic sign of the forward transfer admittance or forward current transfer ratio may be either positive or negative, the sign of the coupling being chosen so that the feedback is regenerative. A single tube may therefore be used in these circuits in either the common-cathode, the common-grid, or the common-plate connection, and a single transistor may be used in these circuits in either the common-emitter, the common-base, or the common-collector connection.

Tuned-output oscillators in which a single tube is used in the common-cathode connection or a single transistor in the common-emitter connection are called *tuned-plate* and *tuned-collector* oscillators, respectively. Similarly, common-cathode and common-emitter tuned-input oscillators are called *tuned-grid* and *tuned-base* oscillators, respectively. Examples of tuned-plate, tuned-base, Hartley, and Colpitts oscillators are shown in Fig. 272.

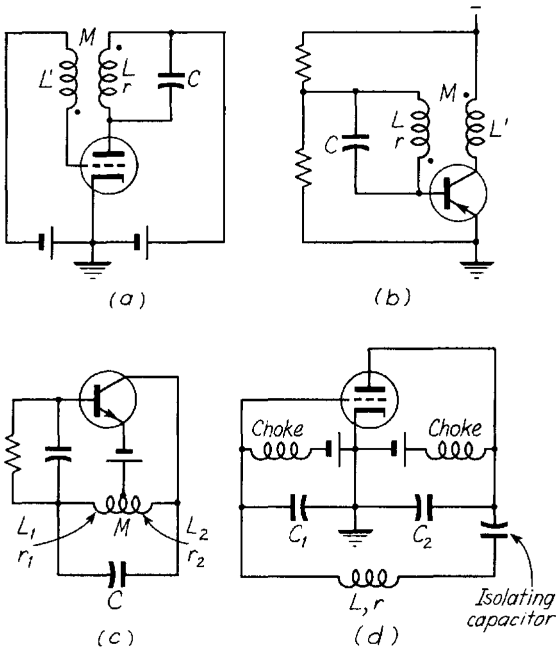


FIG. 272. (a) Tuned-plate oscillator; (b) tuned-base oscillator; (c) transistor Hartley oscillator; (d) tube Colpitts oscillator.

80.2. Choice Between Types. There is little difference in performance between tuned-input and tuned-output oscillators or in the performance of either type between the common-cathode and common-grid connections or between the common-emitter and common-base connections. Usually the choice between circuits is governed by practical considerations of grounding and provision of proper operating voltages and currents. In tube circuits, for example, it is usually convenient to maintain the cathode at ground potential. For this reason the common-cathode connection is usually used except in microwave oscillators, in which physical considerations of the resonator structure necessitate the use of the common-grid connection. The frequency stability of the common-plate or common-collector tuned-output and tuned-input oscillators against changes of amplifier parameters is somewhat lower than that of the other tuned-input and tuned-output circuits (Prob. 80.2–1).

Because both the input and the output inductances are included in the resonator of the Hartley circuit, this circuit requires less total inductance at a given frequency than do the tuned-output and tuned-input circuits. For this reason the Hartley circuit found frequent application at low audio frequencies before the advent of the resistance-capacitance-tuned oscillators to be discussed in Secs. 85, 86, and 88. The Colpitts oscillator is particularly useful in tube circuits at frequencies approaching the upper limit at which transit-time effects reduce the dynamic transconductance to the minimum value capable of satisfying the criterion for sustained oscillation. The difficulties involved in mutual coupling between small coils is avoided in this circuit. The inductance L may be that of a single loop of wire, and the capacitances C_1 and C_2 may be the grid-cathode and plate-cathode capacitances of the tube. As a capacitance-tuned, variable-frequency oscillator, the Colpitts oscillator is less convenient to use than other types because of the necessity of varying both C_1 and C_2 in order to maintain the correct coupling for sustained oscillation. It has been found suitable for use in marine transmitters because the capacitors may be sealed against moisture and the circuit tuned by means of a variometer type of inductance.

80.3. Doubly Tuned and Push-Pull Feedback Oscillators. In another type of parallel-resonator oscillator, parallel resonators are used in both the input and the output of the amplifier, the only coupling between the output and the input being the input-to-output capacitance of the amplifier. The use of two resonators, rather than a single resonator, favors frequency stability, but the necessity of tuning two circuits is a disadvantage in tunable oscillators. Furthermore, the presence of two coupled resonant circuits, which have two frequencies of resonance if the coupling between them exceeds the critical value, may result in undesirable frequency jumps when the circuit is tuned.

The circuits of Fig. 272 and similar circuits may be modified to use two tubes or transistors in push-pull operation. As in amplifiers, the use of push-pull circuits increases the power output and decreases the harmonic content of the output. The frequency stability of push-pull circuits is also generally higher than that of single-sided circuits. In general, however, when large power output, low harmonic content, and good frequency stability are required simultaneously, it is simpler to use a low-power oscillator of high stability to excite a low-distortion power amplifier.

80.4. Methods of Obtaining Bias. In order to emphasize the basic form of the feedback network, rather than the details of supplying the required electrode biasing voltages, fixed grid and base biasing voltages have been indicated in Fig. 272. It will be shown that limitation of amplitude to a low value in order to minimize harmonic content of the oscillator output requires the use of grid or base biasing voltage that varies with amplitude of oscillation. For this reason, fixed grid or base bias is seldom used. Practical circuits in which variable bias is used to control amplitude will be discussed in Sec. 83.

81. Analysis of Parallel-Resonator Feedback Oscillators

81.1. Methods of Analysis. Equation (149) applies only under steady-state conditions and its use in the analysis of any oscillator circuit implies the assumption that the circuit and amplifier parameters are such that sinusoidal oscillation of constant amplitude is obtained. As in the analysis of negative-resistance oscillators, the conditions that must be satisfied in feedback oscillators in order to ensure sustained sinusoidal oscillation can be determined only by solution of the differential equations for the circuits. It is desirable to introduce the general analysis of feedback oscillators by a simple analysis of this type, because such an analysis demonstrates clearly that various modes of operation other than sustained sinusoidal oscillation may occur in feedback-oscillator circuits. The differential-equation method also indicates the conditions under which the amplitude of oscillation increases and decreases.

In general, the frequency of oscillation and the criterion for oscillation with constant amplitude are more readily obtained by application of Eq. (149) or by an equivalent method of analysis than by solution of the differential equations. Furthermore, the general behavior of various circuits with respect to the various modes of operation is similar. For these reasons the following analysis will be made as simple as possible by the choice of a circuit that reduces the complexity of the differential equations to a minimum. This is the tuned-plate oscillator with sufficiently negative grid bias to ensure that negligible grid current flows. The assumption will also be made that the frequency of oscillation is so low that electron transit time and the interelectrode capacitances have a negligible effect upon the behavior of the circuit. The method of analysis is applicable to other feedback-oscillator circuits.

If appreciable grid current flows, it affects the behavior of the circuit both by introducing additional power losses and by increasing the harmonic content of the output as the result of the large nonlinearity of the curve of grid current vs. grid voltage in the vicinity of grid-current cutoff. These same effects result from the flow of transistor base current. The effects of nonlinear current-voltage characteristics can be determined only by approximation methods of analysis applicable to nonlinear circuits and will not be considered further.[1] The loading effect of grid or base current can be taken

[1] See, for example, J. Cunningham, *Introduction to Nonlinear Analysis*, McGraw-Hill Book Co., Inc., New York (1958).

into account by adding the grid or base admittance to the feedback network, as explained in Sec. 78.

81.2. Analysis of Tuned-Plate Oscillator by Differential Equations. An equivalent circuit for the tuned-plate oscillator under the assumption that grid current is negligible is shown in Fig. 273. The circuit equations are the following:

$$v_g = M\frac{di}{dt} \tag{153}$$

$$r_p i_p + ri + L\frac{di}{dt} = \mu v_g = \mu M\frac{di}{dt} \tag{154}$$

$$ri + L\frac{di}{dt} + \frac{1}{C}\int (i - i_p)\,dt = 0 \tag{155}$$

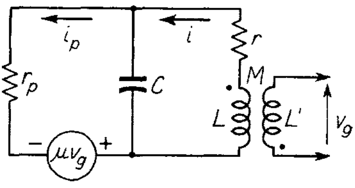


FIG. 273. Equivalent plate circuit of the tuned-plate oscillator under the assumption that grid current is negligible.

If Eq. (155) is differentiated and the result substituted into Eq. (154), the following second-order differential equation is obtained:

$$\frac{d^2i}{dt^2} + \left(\frac{r}{L} + \frac{1}{r_pC} - \frac{g_mM}{LC}\right)i + \frac{r_p + r}{r_pLC} = 0 \tag{156}$$

The solution of Eq. (156) is

$$i = A\epsilon^{\alpha t}\sin(\omega t + \theta) \tag{157}$$

in which A and θ are constants determined by the starting conditions and

$$\alpha = \frac{1}{2}\left(\frac{g_mM}{LC} - \frac{r}{L} - \frac{1}{r_pC}\right) \tag{158}$$

$$\omega = \sqrt{\frac{r_p + r}{r_pLC} - \alpha^2} \tag{159}$$

The solution given by Eq. (159) is oscillatory only if ω is real. The amplitude of oscillation increases if $\alpha > 0$, is constant if $\alpha = 0$, and decreases if $\alpha < 0$. Equation (155) therefore leads to the following amplitude criteria:

$$\text{Amplitude increases if } g_m > \frac{rr_pC + L}{Mr_p} \tag{160}$$

$$\text{Amplitude is constant if } g_m = \frac{rr_pC + L}{Mr_p} \tag{161}$$

$$\text{Amplitude decreases if } g_m < \frac{rr_pC + L}{Mr_p} \tag{162}$$

Under equilibrium conditions of oscillation (constant amplitude), for which α is zero, the frequency of oscillation is

$$f = \frac{1}{2\pi\sqrt{LC}}\sqrt{\frac{r + r_p}{r_p}} \tag{163}$$

Unless the effective value of r is increased by loading of the resonator or by the flow of appreciable grid current, r is normally very small in comparison with r_p, and the frequency of oscillation closely approximates $1/2\pi\sqrt{LC}$. However, because the value of r_p depends upon the operating point of the tube and upon amplitude, Eq. (163) shows that the frequency of oscillation is to some extent dependent upon supply voltage and load.

If the circuit parameters are such that Eq. (159) gives an imaginary value of ω, the solution is exponential in form. The currents are then either decaying exponentials or growing exponentials. As in negative-resistance oscillators, if the solution has the form of a growing exponential, variation of g_m with current leads to relaxation oscillation. A diagram similar to Figs. 158 and 159 may be constructed to show the ranges of parameters in which the various types of behavior occur (Prob. 81.2–1). The value of such a diagram lies in its emphasis of the fact that, if the circuit parameters of feedback oscillators are not properly chosen, relaxation oscillation, rather than sinusoidal oscillation, will occur. Analysis of Eqs. (158) and (159) shows that relaxation oscillation will not occur if the following relation is satisfied (Prob. 81.2–1):

$$rC + \frac{L}{r_p} - 2\sqrt{\frac{LC(r_p + r)}{r_p}} < g_m M < rC + \frac{L}{r_p} + 2\sqrt{\frac{LC(r_p + r)}{r_p}} \tag{164}$$

81.3. Analysis of Tuned-Plate Oscillator by Means of Eq. (152). Although Eq. (149) will be used in Sec. 82 to derive generalized expressions for the frequency of oscillation and the criterion for oscillation that are applicable to all tuned-output feedback oscillators, it is of interest at this point to demonstrate the ease with which the unloaded tuned-plate oscillator with negative grid bias can be analyzed with the aid of Eq. (152). The method is applicable to other vacuum-tube feedback-oscillator circuits.

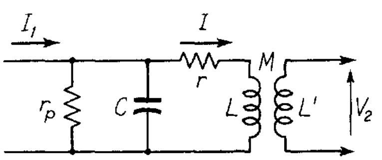


FIG. 274. Augmented feedback network of the tuned-plate oscillator under the assumption that grid current is negligible.

Figure 274 shows the augmented feedback network of the tuned-plate oscillator under the assumption that grid current does not flow and that the resonator is otherwise unloaded. The ratio of the current I through L to the input current I_1 is equal to the ratio of the admittance of the L-r branch of the circuit to the sum of this admittance, ωC, and $1/r_p$. The output voltage V_2 is $-j\omega MI$. The forward open-circuit transfer admittance $1/Z_f$ is readily found from these two relations.

$$1/Z_f = -\frac{rr_pC + L}{Mr_p} + j\frac{r_p + r - r_p\omega^2LC}{\omega Mr_p} \qquad (165)$$

At frequencies at which the susceptive component of the short-circuit transfer admittance y_f of the tube is negligible, Eq. (152) becomes for this oscillator:

$$g_m = \frac{rr_pC + L}{Mr_p} - j\frac{r_p + r - r_p\omega^2LC}{\omega Mr_p} \qquad (166)$$

The real part of Eq. (166) leads immediately to Eq. (161), and the imaginary part to Eq. (163).

It follows from the assumptions made in the derivation of Eq. (152) that the use of this equation to obtain the frequency of oscillation and the criterion for oscillation is equivalent to determining the conditions under which the feedback is regenerative and the loop amplification equal to unity. An equivalent method of analysis is to write the steady-state equations for the equivalent circuit of Fig. 273.

81.4. Analysis of Tuned-Plate Oscillator; Transit-Time Effects Considered. At frequencies at which the interelectrode transit times of electrons in the amplifier tube are comparable with the period of oscillation, the criterion for sustained oscillation and the frequency of oscillation are affected by transit time. Transit time causes the plate current to lag the grid voltage and may reduce the magnitude of the plate current. The short-circuit forward transfer admittance therefore departs from the low-frequency value both in phase and in magnitude.[2] The factor that causes the magnitude of the forward transfer admittance of the tube eventually to fall to a very low value as the frequency is increased is not interelectrode transit time in itself, but differences of the transit times of individual electrons. The expressions for forward transfer admittance as a function of frequency are too involved for a rigorous theoretical oscillator analysis. An indication of the general manner in which the criterion for oscillation and the frequency of oscillation are affected by transit time can be readily obtained, however, under the assumption that the short-circuit forward transfer admittance at any frequency is given by the following formula:

[2] F. B. Llewellyn and L. C. Peterson, *Proc. I.R.E.*, **32**, 144 (1944). For a résumé of this article, see H. J. Reich, J. G. Skalnik, P. F. Ordung, and H. L. Krauss, *Microwave Principles*, D. Van Nostrand Co., Inc., Princeton, N. J., 1957.

$$y_f = \frac{g_{mo}}{1 + jf/f_c} \tag{167}$$

in which f_c is the frequency at which the magnitude of the transfer admittance has fallen to $1/\sqrt{2}$ of its low-frequency value g_{mo}. The analysis will be further simplified by the assumptions that the oscillator is not loaded either as the result of the flow of grid current or by a resistance shunted across either winding of the transformer, that r is very small in comparison with r_p, and that C is much greater than the interelectrode capacitances. Under these assumptions Eq. (152) becomes:

$$\frac{g_{mo}}{1 + j\omega/\omega_c} = \frac{rr_pC + L}{Mr_p} + j\frac{\omega^2 LC - 1}{\omega M} \tag{168}$$

The real and the imaginary parts of Eq. (168) yield the following expressions for the criterion for sustained oscillation and the frequency of oscillation:

$$g_{mo} = \frac{1}{M}\left(rC + \frac{L}{r_p} + \frac{r/L + 1/r_pC}{\omega_c(\omega_c + r/L + 1/r_pC)}\right) \tag{169}$$

$$f = \frac{1}{2\pi\sqrt{LC + (rC + L/r_p)/\omega_c}} \tag{170}$$

Since ω_c is an inverse function of transit time, it is apparent from Eq. (169) that transit time increases the value of low-frequency transconductance g_{mo} required for sustained oscillation and that there must be an upper frequency limit beyond which the maximum attainable value of g_{mo} is insufficient to make oscillation possible. Equation (170) shows that transit time reduces the frequency below the value that would obtain if the transit time were negligible in comparison with the period of oscillation.

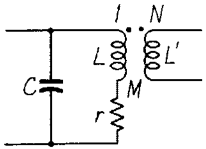
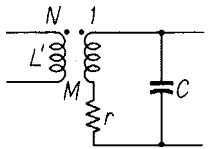
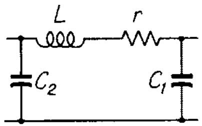
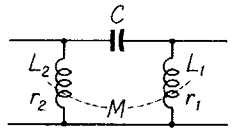
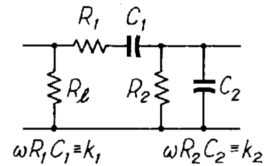
82. Generalized Analysis of Tuned-Output Circuits

82.1. Derivation of Expressions for Frequency and Criterion for Oscillation. Substitution of the impedance parameters listed in column 1 of Table IX into Eq. (149A), and the solution of the simultaneous equations obtained by separating real and imaginary terms of this equation leads to the following expressions for the frequency of oscillation of tuned-output feedback oscillators and the criterion for oscillation with constant amplitude:

$$\omega^2 = \frac{1 + y_o r}{LC + \Delta_y(LL' - M^2) + y_i L' r C} \tag{171}$$

$$y_f = -\frac{rC + y_i(M^2/L + y_o L' r) + y_o L}{M - y_r L' r} \tag{172}$$

TABLE IX. IMPEDANCE PARAMETERS OF FIVE COMMONLY USED OSCILLATOR FEEDBACK NETWORKS

	Tuned-Output, including Tuned-Plate and Tuned-Collector	Tuned-Input, including Tuned-Grid and Tuned-Base	Colpitts	Hartley	Wien-Bridge
					
z_i	$\frac{1}{\omega^2C^2z} - j\frac{1}{\omega C}$	$\frac{\omega^2M^2}{z} + j\omega L'$	$-\frac{1}{z}\left[\frac{1}{\omega^2C_1C_2} + j\left(\frac{r}{\omega C_2}\right) - \frac{L}{C_2}\right]$	$\frac{1}{z}\left[r_1r_2 + \frac{L_2}{C} - \omega^2L_1L_2 + \omega^2M^2\right] + j\frac{1}{z}\left[\omega(L_1r_2 + L_2r_1) - \frac{r_2}{\omega C}\right]$	$\frac{R_lR_1(k_1 + k_1k_2^2) + R_lR_2k_1}{z} - j\frac{R_lR_1(k_2^2 + 1) + R_lR_2k_1k_2}{z}$
z_r z_f	$\frac{M}{Cz}$	$\frac{M}{Cz}$	$-\frac{1}{\omega^2C_1C_2z}$	$\frac{1}{z}\left[r_1r_2 - \frac{M}{C} - \omega^2L_1L_2 + \omega^2M^2\right] + j\frac{\omega}{z}(L_1r_2 + L_2r_1)$	$\frac{R_lR_2(k_1 - jk_1k_2)}{z}$
z_o	$\frac{\omega^2M^2}{z} + j\omega L'$	$\frac{1}{\omega^2C^2z} - j\frac{1}{\omega C}$	$-\frac{1}{z}\left[\frac{1}{\omega^2C_1C_2} + j\left(\frac{r}{\omega C_1}\right) - \frac{L}{C_1}\right]$	$\frac{1}{z}\left[r_1r_2 + \frac{L_1}{C} - \omega^2L_1L_2 + \omega^2M^2\right] + j\frac{1}{z}\left[\omega(L_1r_2 + L_2r_1) - \frac{r_1}{\omega C}\right]$	$\frac{(R_l + R_1)R_2k_1 - R_1R_2k_2}{z} - j\frac{R_1R_2 + (R_l + R_1)R_2k_1k_2}{z}$
Δ_z	$\frac{L'}{C} + j\frac{1}{Cz} \times \left(\frac{L'}{\omega C} - \omega M^2\right)$	$\frac{L'}{C} + j\frac{1}{Cz} \times \left(\frac{L'}{\omega C} - \omega M^2\right)$	$-\frac{1}{z}\left[\frac{r}{\omega^2C_1C_2} + j\frac{L}{\omega C_1C_2}\right]$	$\frac{1}{Cz}(L_1r_2 + L_2r_1) + \frac{j}{\omega Cz}(\omega^2L_1L_2 - \omega^2M^2 - r_1r_2)$	$\frac{R_lR_1R_2}{z}[k_1 - k_2 - j(k_1k_2 + 1)]$
z	$r + j\left(\omega L - \frac{1}{\omega C}\right)$	$r + j\left(\omega L - \frac{1}{\omega C}\right)$	$R + j\left(\omega L - \frac{1}{\omega C_1} - \frac{1}{\omega C_2}\right)$	$r_1 + r_2 + j\left(\omega L_1 + \omega L_2 + 2\omega M - \frac{1}{\omega C}\right)$	$k_1(k_2^2 + 1)(R_l + R_1) + k_1R_2 - j[(k_2^2 + 1)R_1 + k_1k_2R_2]$

When the coils are wound on high-permeability cores or when the two coils are wound upon forms of equal diameters, $L' = N^2L$, where N is the ratio of the number of turns of L' to the number of turns of L. Furthermore, $M = kLN$, where the coefficient of coupling k is equal to $M/\sqrt{LL'}$. By use of these expressions for L' and M, Eqs. (171) and (172) may then be written in the following alternate form:

$$\omega^2 = \frac{1 + y_o r}{LC + N^2(1 - k^2)\Delta_y L^2 - N^2 y_i LrC} \tag{171A}$$

$$y_f = -\frac{rC/L + N^2 y_i(k^2 + y_o r) + y_o}{Nk - N^2 y_r r} \tag{172A}$$

For vacuum tubes in the common-cathode connection $y_i = 1/r_g$, $y_r = 0$, $y_f = 1/g_m$, and $y_o = 1/r_p$. Substitution of these values into Eqs. (171) and (172) yields the following equations for the frequency of oscillation and the criterion for oscillation of the tuned-plate oscillator: *

$$f = \frac{1}{2\pi\sqrt{LC}}\sqrt{\frac{r_p + r}{r_p}\cdot\frac{r_g}{r_g + \dfrac{L'}{r_pC} + \dfrac{L'r}{L} - \dfrac{M^2}{r_pLC}}} \tag{173}$$

$$g_m \approx \frac{M}{r_gL}\cdot\frac{r_p + r}{r_p} + \frac{L}{M(r + r_p)} + \frac{Crr_p}{M(r + r_p)} \approx \frac{M}{r_gL} + \frac{L}{Mr_p} + \frac{Cr}{M} \tag{174}$$

The principal value of Eqs. (173) and (174), which reduce to Eqs. (163) and (161), respectively, when r_g is infinite, is that they show that the frequency of oscillation is to some extent dependent upon r_g, as well as upon r_p, and that grid current increases the value of g_m required to maintain oscillation. Since the amplitude of oscillation increases if g_m exceeds the right side of Eq. (174) [compare with Eq. (160)], grid current also helps to limit the amplitude of oscillation by increasing the term M/r_gL with amplitude. This action will be discussed further in Sec. 83.

It is apparent from Fig. 269a that load shunting the grid or plate coil may be taken into account by replacing r_g or r_p by the resultant resistance of the parallel combination of the load resistance and r_g or r_p. A load coupled to the resonator may be taken into account by an equivalent increase in the resistance r of the resonator coil. Loading of the oscillator therefore changes the frequency of oscillation and increases the value of g_m necessary to maintain oscillation.

* In the derivation of Eq. (174) the assumption is made that $\omega^2 = 1/LC$.

82.2. Tuned-Collector Oscillator. Because the values of transistor admittance parameters are not usually available, Eqs. (171) and (172) are more useful for transistor circuits if they are written in terms of h parameters by substitution of the following identities: [1]

$$\begin{aligned} y_i &= 1/h_i & & & y_r &= -h_r/h_i \\ y_f &= h_f/h_i & \Delta_y &= h_o/h_i & y_o &= \Delta_h/h_i \end{aligned} \tag{175}$$

With these substitutions, Eqs, (173) and (174) become:

$$\omega^2 \approx \frac{h_i + \Delta_h r}{h_i LC + h_o(LL' - M^2) + L'rC} \tag{176}$$

$$h_f \approx \frac{h_i rC}{M} + \frac{\Delta_h L}{M} + \frac{M}{L} \tag{177}$$

For the tuned-collector oscillator, Eqs. (176) and (177) must be written in terms of the common-emitter parameters. In the common-emitter connection, terms in Eqs. (176) and (177) involving r are usually so small that they may be neglected in comparison with other terms. The frequency of oscillation of the tuned-collector oscillator and the criterion for oscillation are then given by the following approximations:

$$f \approx \frac{1}{2\pi\sqrt{LC}} \sqrt{\frac{1}{1 + (LL' - M^2)h_{oe}/LCh_{ie}}} \tag{178}$$

$$h_{fe} \approx \frac{L\Delta_{he}}{M} + \frac{M}{L} \tag{179}$$

It is apparent from Eqs. (176) and (178) that the frequency of oscillation is dependent upon the transistor parameters and therefore upon the supply voltages and the amplitude of oscillation. Equation (179) demonstrates that h_{fe}, like g_m in the tuned-plate circuit, is an important figure of merit in the tuned-collector oscillator. It will be seen that these parameters are also important figures of merit in other types of feedback oscillators. Another conclusion that can be drawn from Eqs. (177) and (179) is that there is in general not only a minimum value of coupling at which oscillation can occur at fixed values of transistor parameters, but also a maximum value. Although increasing the mutual inductance M increases the positive feedback, it also increases the loading effect of the transistor input resistance

[1] See, for example, W. W. Gärtner, *Transistors,* Chap. 7, D. Van Nostrand Co., Inc., Princeton, N. J., 1960.

upon the resonator and may therefore increase the required value of h_{fe} at large values of coupling.

Load conductance shunted across either the collector coil or the base coil or coupled to the resonator by means of an additional coil may be taken into account by an equivalent increase of the resistance r of the collector coil. Loading of the oscillator therefore changes the frequency of oscillation and increases the value of h_{fe} required to maintain oscillation.

82.3. Effect of Carrier Transit Time. The effects of carrier transit time and electrode capacitance upon the frequency of oscillation and the criterion for oscillation can be determined either by solving the network equations for an equivalent circuit in which the transistor is replaced by a high-frequency approximate equivalent tee circuit,[2] or by using in Eq. (149) or its h-parameter equivalent the complex expressions for the transistor parameters found from the equivalent tee circuit. Either procedure leads to complicated equations that are difficult to solve. However, the fact that the magnitudes of h_{fe} and h_{ie} are reduced and those of h_{re} and h_{oe} are increased as the frequency approaches the alpha cutoff frequency, h_{fe} approaching zero at very high frequencies, indicates that transit-time effects set an upper limit to the attainable frequency and that this limit is increased by an increase in the low-frequency value of h_{fe}. It is also to be expected that the imaginary component of h_{fe}, together with increased loading of the resonator as the result of the reduction of input resistance of the transistor, will reduce the frequency of oscillation below the value predicted by Eq. (176) or Eq. (178) (cf. Eq. (170) for the tuned-plate circuit).

82.4. Other Tuned-Output Oscillators. Expressions for the frequency of oscillation and the criterion for oscillation of common-grid and common-plate tuned-output oscillators can be derived by substituting the appropriate values of the y parameters listed in Table III into Eqs. (171) and (172) or (171A) and (172A) (Prob. 82.4–1). Similarly the corresponding expressions applicable to the common-base and common-collector tuned-output oscillators may be obtained by using the common-base or common-collector h parameters in Eqs. (176) and (177). If preferred, the expressions may be written in terms of the common-emitter h parameters by use of the identities listed in Appendix, Table A-1 (Prob. 82.4–2).

82.5. Tuned-Input, Colpitts, and Hartley Analyses. Tuned-input, Colpitts, and Hartley oscillators may be analyzed in a similar manner to tuned-plate and tuned-collector oscillators by using in Eq. (149) the appropriate z parameters listed in Table IX. For convenience, the expressions for the frequency of oscillation and the criterion for oscillation of the tuned-

[2] For a discussion of high-frequency equivalent circuits, see, for example, R. F. Shea, editor, *Transistor Circuit Engineering,* John Wiley and Sons, Inc., New York, 1957.

grid, tuned-base, Colpitts, and Hartley circuits, together with those for the tuned-plate and tuned-collector circuits, are listed in Table X (Probs. 82.5–1 to 82.5–3).

83. Amplitude Limitation in Parallel-Resonator Feedback Oscillators

83.1. Amplitude Diagrams for Vacuum-Tube Feedback Oscillators. Equations (160) to (162) and similar equations for other *LC* feedback oscillators, such as those listed in Table X, show that if the circuit and amplifier parameters are initially adjusted so that oscillation can start, amplitude limitation can result only from the dependence of the dynamic values of the circuit or tube parameters upon amplitude.

The most commonly used method of amplitude limitation in vacuum-tube parallel-resonator oscillators is based upon the dependence of the dynamic transconductance upon amplitude of the alternating grid voltage. Considerable useful qualitative information concerning the amplitude behavior of feedback oscillators can be deduced from a diagram based upon a generalized criterion for oscillation derived directly from Eq. (152). Equation (152) states that, at frequencies at which the tube transadmittance is nonsusceptive and equal to g_m, the magnitude of g_m under equilibrium conditions must be equal to the magnitude of the real component of $1/Z_f$, the reciprocal of the open-circuit forward transfer impedance of the augmented feedback network of Fig. 269b. For convenience, the magnitude of the real part of $1/Z_f$ may be represented by the symbol G_f. If g_m exceeds G_f, the amplitude of oscillation increases with time; if $g_m = G_f$, the amplitude remains constant; and if g_m is less than G_f, the amplitude decreases. For the oscillators listed in Table X, G_f is the right side of the criterion for oscillation. It can be seen from Table X that G_f involves r_p and that G_f therefore depends to some extent upon amplitude of oscillation. (The exact criterion for the Hartley oscillator also contains r_p.) However, unless the feedback network contains a nonlinear resistor, such as a thermistor or thyristor, the dependence of G_f upon amplitude is so small in comparison with the dependence of g_m that G_f may be assumed to be constant. At amplitudes exceeding the magnitude of the grid bias, grid current may cause G_f to increase rapidly with amplitude of grid voltage.

As pointed out in Sec. 78.2, the y parameters of the amplifier must in general be defined in terms of fundamental components of current and voltage. The dynamic value of transconductance that appears in the criteria for sustained oscillation is the ratio of the fundamental component of plate current to the alternating grid voltage at constant plate voltage. (At small amplitudes the dynamic transconductance approaches the slope of

TABLE X. FREQUENCY OF OSCILLATION AND CRITERION FOR OSCILLATION OF FOUR TYPES OF FEEDBACK OSCILLATORS. FOR CIRCUITS, SEE FIG. 272.

Type of Oscillator	ω^2	Criterion for Oscillation
Tuned-Plate, zero grid current	$\frac{1}{LC}\cdot\frac{r+r_p}{r_p}$	$g_m = \frac{rC}{M}+\frac{L}{Mr_p}$
Tuned-Collector	$\frac{h_{ie}+r\Delta_{he}}{LCh_{ie}+(LL'-M^2)h_{oe}+L'rC} \approx \frac{1}{LC}\cdot\frac{1}{1+(LL'-M^2)h_{oe}/LCh_{ie}}$	$h_{fe}\approx \frac{rCh_{ie}}{M}+\frac{L\Delta_{he}}{M}+\frac{M}{L}\approx\frac{L\Delta_{he}}{M}+\frac{M}{L}$
Tuned-Grid, zero grid current	$\frac{1}{LC}\cdot\frac{Lr_p}{Lr_p+L'r}$	$g_m=\frac{rC}{M}+\frac{M}{r_pL}$
Tuned-Base	$\frac{h_{ie}+r}{LCh_{ie}+(LL'-M^2)h_{oe}+L'rC\Delta_{he}}\approx\frac{1}{LC}\cdot\frac{1}{1+(LL'-M^2)h_{oe}/LCh_{ie}}$	$h_{fe}\approx rCh_{ie}+\frac{L}{M}+\frac{M\Delta_{he}}{L}\approx\frac{L}{M}+\frac{M\Delta_{he}}{L}$
Tube Colpitts, zero grid current	$\frac{1}{LC}\cdot\frac{rC+r_pC_2}{r_pC_2}$ where $C=C_1C_2/(C_1+C_2)$	$g_m=\frac{r(C_1+C_2)}{L}+\frac{C_1}{r_pC_2}$
Transistor Colpitts	$\frac{1}{LC}+\frac{r}{L}\left(\frac{1}{C_1h_{ie}}+\frac{\Delta_{he}}{C_1h_{ie}}\right)+\frac{h_{oe}}{C_1C_2h_{ie}}\approx\frac{1}{LC}\left(1+\frac{LCh_{oe}}{C_1C_2h_{ie}}\right)$ where $C=C_1C_2/(C_1+C_2)$	$h_{fe}\approx\frac{r(C_1+C_2)h_{ie}}{L}+\frac{C_2}{C_1}+\frac{C_1\Delta_{he}}{C_2}\approx\frac{C_2}{C_1}$
Tube Hartley, zero grid current	Approximately, $\frac{1}{LC}\cdot\frac{(r_p+r_2)/r_p}{1+(L_1r_2+L_2r_1)\left(\frac{rC}{(L_1+M)(L_2+M)}+\frac{1}{Lr_p}\right)}\approx\frac{1}{LC}\cdot\frac{r_p+r_2}{r_p}$ where $L=L_1+L_2+2M$	$g_m\approx\frac{rCL}{(L_1+M)(L_2+M)-r_1r_2LC}$ $\approx\frac{rCL}{(L_1+M)(L_2+M)}$
Transistor Hartley	Approximately, $\frac{h_{ie}}{C[Lh_{ie}+(L_1r_2+L_2r_1)h_{fe}]+(L_1L_2-M^2)h_{oe}}$ $\approx\frac{1}{LC+(L_1L_2-M^2)h_{oe}/h_{ie}}$ where $L=L_1+L_2+2M$	$h_{fe}\approx\frac{rLCh_{ie}+(M+L_1)^2+(L_1r_2+L_2r_1)\Delta_{he}/h_{ie}}{(L_1+M)(L_2+M)}$ $\approx\frac{L_1+M}{L_2+M}\approx\frac{1+kN}{1/N^2+kN}$ where $k=M/\sqrt{L_1L_2}$ and $N=\sqrt{L_2/L_1}$

the grid-plate transfer characteristic at the operating point.) The dynamic transconductance may be found either by measurement or, at frequencies at which transit-time effects are negligible, from the characteristic curves by graphical methods.

Figure 275 shows a curve of dynamic transconductance g_m vs. amplitude of alternating grid voltage that is typical of fixed-grid-bias operation, together with a typical curve of G_f vs. amplitude. In order for oscillation to start, g_{m0}, the transconductance at zero amplitude (the static transconductance, determined from the slope of the transfer characteristic at the operating point) must exceed G_{f0}. Since g_m then exceeds G_f at amplitudes less than that corresponding to the intersection of the curves, the amplitude will build up to the value corresponding to the intersection, the rate of increase depending upon the amount by which g_m exceeds G_f at each instantaneous amplitude, as shown by Eqs. (157) and (158) or similar equations for other feedback oscillators. Any increase of amplitude beyond the value corresponding to the intersection would cause g_m to become less than G_f and the amplitude consequently to decrease to the value at the intersection. Once oscillation has built up, the amplitude can be reduced by an increase of grid bias, which causes the curve of g_m to be lowered. However, if the bias is increased beyond the value at which the curves are tangent, as indicated by the dotted curve, oscillation will cease. The minimum steady-state amplitude therefore corresponds approximately to the value at the peak of the g_m curve, but at this value of bias the oscillation is not self-starting. Since the minimum grid bias at which oscillation is self-starting allows the amplitude to build up to the large value corresponding to the intersection of the curves, it is apparent that the small amplitude of oscillation essential to low harmonic content cannot readily be attained when fixed bias is used.* It is also apparent that amplitude hysteresis is observed as the grid bias is varied.

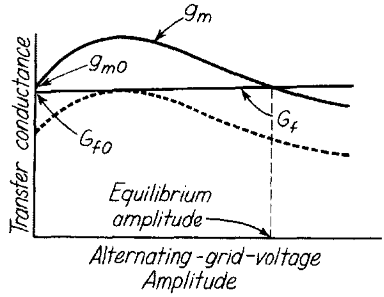


FIG. 275. Transconductance-amplitude diagram typical of a vacuum-tube feedback oscillator with fixed bias.

83.2. Capacitor-Gridleak Bias. Small amplitude of oscillation, consistent with self-starting, can be obtained by the use of a grid capacitor and resistor, as shown in Fig. 276a, in which the voltage v is the alternating

* Note that the distortion at high amplitude results not only from curvature of the transfer characteristic, but also from the flow of grid current during the positive peaks of alternating grid voltage.

grid voltage derived from the resonator, directly or by coupling. Before oscillation starts, the grid bias is zero and the corresponding large value of transconductance assures starting and rapid growth of amplitude. Flow of grid current during the positive half cycles of grid voltage causes the capacitor C_b to be charged in the indicated polarity and therefore introduces

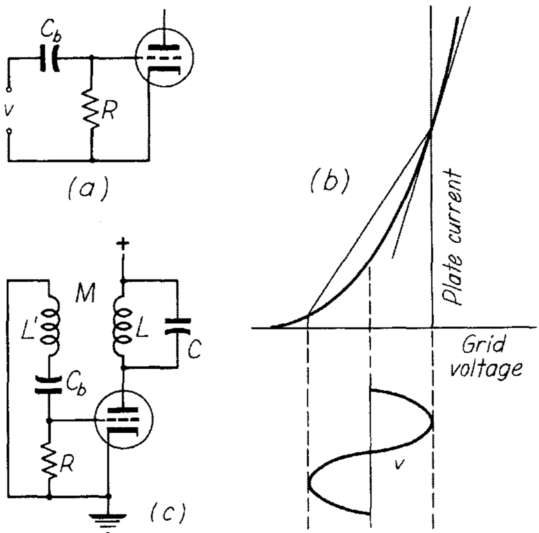


FIG. 276. (a) Capacitor-gridleak biasing circuit; (b) diagram showing the increase of dynamic transconductance with amplitude in a circuit in which capacitor-gridleak bias is used; (c) tuned-plate oscillator with capacitor-gridleak bias.

negative grid bias. If the resistor R were omitted, the capacitor voltage would build up to a value equal to the crest value of the impressed voltage v, but there would be no way for the capacitor to discharge if the amplitude of oscillation were reduced or the oscillator turned off. The addition of the high resistance R provides a discharge path for C_b and causes the biasing voltage to be reduced only slightly below the crest value of v.† Since R is large, the flow of grid current during positive peaks of alternating grid voltage is small and the distortion produced by the flow of grid current is negligible.

Under the assumption that the second and higher plate-current harmonics may be neglected, the dynamic transconductance approximates the slope of the chord joining the end points of the path of operation on the transfer

† Reference to Sec. 8 shows that the gridleak-capacitor biasing circuit is actually a clamping circuit that clamps the positive peak of grid voltage at or near zero voltage.

characteristic relating plate current and grid voltage at constant plate voltage (see Prob. 83.2–1). Figure 276b shows that, if the grid bias is maintained approximately equal to the crest value of the impressed grid voltage v, the slope of the chord decreases continuously with increase of v. When capacitor-gridleak bias is used, therefore, the transconductance decreases continuously with increase of amplitude of oscillation, as shown by the g_m curve of Fig. 277.‡ It can be seen from Fig. 277 that the amplitude of oscillation can be reduced to a low value by choosing the operating plate voltage and the circuit parameters so that g_{m0} is only slightly greater than G_{f0}, as indicated by the dotted curve, and that the small amplitude is attained without preventing the spontaneous starting of oscillation.

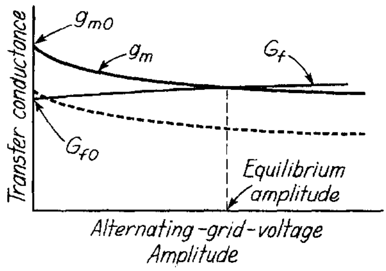


FIG. 277. Transconductance-amplitude diagram typical of a vacuum-tube feedback oscillator in which capacitor-gridleak bias is used.

83.3. Choice of C_b and R; Squegging. Choice of the values of biasing capacitance C_b and resistance R is governed by a number of considerations. In order to prevent excessive discharging of the capacitor during the negative half cycle of the impressed grid voltage, the time constant RC_b must be large in comparison with the period of oscillation. The value of R should be large in order to prevent loading of the oscillator. If C_b is too large, the charging time constant, which is determined principally by the capacitance C_b and the grid resistance of the tube when R is large, may be so large that the amplitude of oscillation builds up much more rapidly than the bias. The amplitude of the voltage across the grid coil may then reach a value that greatly exceeds the cutoff bias of the tube while the biasing voltage produced across the capacitor is still small. The bias continues to increase, and eventually becomes so large that g_m is no longer large enough to satisfy the criterion for sustained oscillation. Because of the initially large amplitude of oscillation, however, if the resonator Q is high, the amplitude of the voltage across the grid coil may still exceed the cutoff value of grid bias even after the requirement for self-sustained oscillation is no longer satisfied. As the resonator oscillation gradually dies out, the capacitor continues to charge up and the bias may reach a value exceeding cutoff. Since RC_b is large in comparison with the period, the capacitor discharges relatively slowly, and the transient oscillation may reach a very low value

‡ Figure 277 can be derived from Fig. 274 by a procedure analagous to that illustrated for a two-terminal oscillator in Fig. 258.

and may even die out completely before the magnitude of the bias becomes sufficiently low so that the criterion for self-sustained oscillation is again satisfied. Oscillation then again builds up rapidly, and the cycle repeats. The periodic interruption of oscillation in this manner is called *motorboating* or *squegging.* The likelihood of motorboating can be prevented by reducing C_b, reducing RC, and reducing resonator Q.

83.4. Amplitude Limitation by Temperature-Controlled Resistance. Another method of limiting and stabilizing the amplitude of oscillation is by the use of a thermistor or other resistor, the resistance of which varies rapidly with temperature and therefore with current through it.[1] The temperature-sensitive element is incorporated in a voltage divider or a more complicated bridge in such a manner that the change of resistance resulting from an increase of amplitude of oscillation reduces the feedback voltage impressed upon the grid. The effect of the variable feedback is to cause G_f to increase rapidly with amplitude. If the slope of the G_f curve is high enough, low amplitude of oscillation and freedom from amplitude hysteresis are obtained without the use of capacitor-gridleak bias, as can be seen from Fig. 278a.

Figure 278b shows a stabilized Hartley circuit in which increase of voltage across the resonator raises the current through the nonlinear resistor N, increasing its resistance and thus reducing the alternating voltage impressed upon the grid. In some circuits, of which Fig. 292 is an example, the nonlinear resistor may form a portion of a frequency-determining bridge. Although the resistance change that produces the stabilization is usually caused by alternating current, a higher degree of stabilization may be attained by the use of direct current obtained by rectifying the oscillator output voltage and using the resulting direct voltage as grid bias to vary the plate current, which flows through the control element, as in the circuit of Fig. 279.[2]

83.5. Amplitude Stability of Vacuum-Tube Parallel-Resonator Feedback Oscillators. It is apparent that a vertical displacement of either the G_f curve or the g_m curve in Fig. 277 results in a change of amplitude of oscillation unless both curves are displaced in the same direction by equal amounts. Vertical displacement of the g_m curve is caused by a change of the tube operating point and hence by a change of supply voltage. A vertical displacement of the G_f curve occurs when the oscillator is tuned by the variation of a single circuit element, such as a variable capacitance, because G_f depends upon L and C. Although it is possible to vary one or more operating voltages as the circuit is tuned and thus to maintain the amplitude of oscillation nearly constant, this method of amplitude stabilization is not in general feasible over wide frequency ranges. The use of capacitor-gridleak

[1] P. R. Aigrain, *Proc. I.R.E.*, **36**, 16 (1948). Contains a bibliography on stabilization by the use of nonlinear resistances.

[2] Aigrain, *loc. cit.*

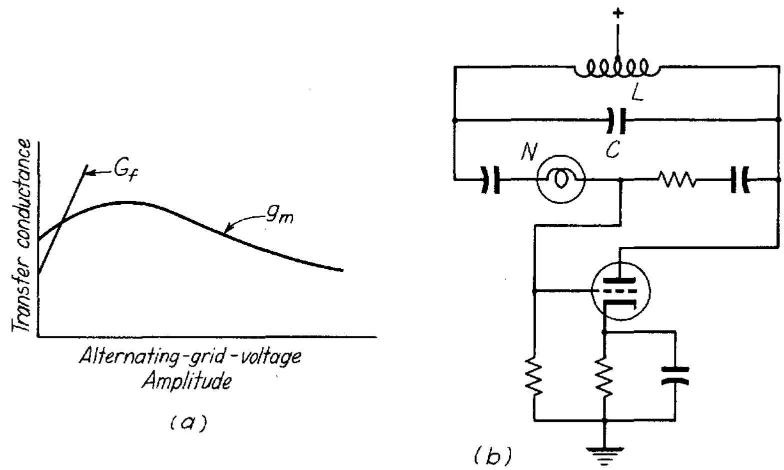


FIG. 278. (a) Transconductance-amplitude diagram typical of an oscillator in which a temperature-sensitive resistor is incorporated in the feedback network; (b) oscillator incorporating a temperature-sensitive resistor in a voltage divider.

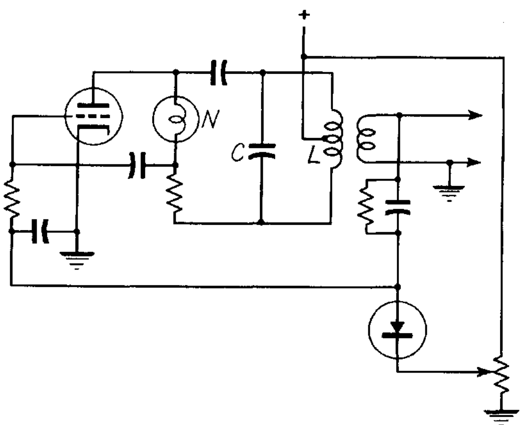


FIG. 279. Thermistor-stabilized oscillator in which the stabilization is increased by the use of a diode rectifier and a d-c amplifier.

bias results in considerable improvement in amplitude stability against changes in either supply voltage or tuning capacitance or inductance, because it increases the slope of the curve of g_m vs. amplitude.

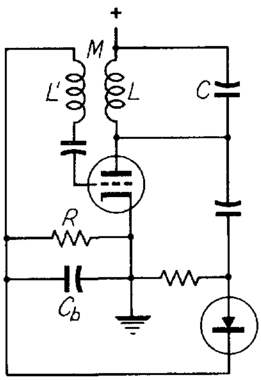


FIG. 280. Oscillator in which rectified output voltage is used to reduce the dynamic transconductance.

If the amplitude of the output voltage is greater than that of the alternating grid voltage, the grid bias can be made to vary more rapidly with amplitude by using rectified output voltage, instead of rectified grid voltage, to provide bias that increases with amplitude. The slope of the curve of g_m vs. amplitude is thus increased and the amplitude stability of the oscillator improved. Figure 280 shows an example of the use of this method of stabilization. Considerable improvement in amplitude stability may also be obtained by the simultaneous use of capacitor-gridleak bias and amplitude-dependent feedback networks, such as those of Figs. 278 and 279.

83.6. Amplitude Limitation in Transistor Feedback Oscillators. The form of the transfer characteristic relating collector current with base current for junction transistors is such that the magnitude of h_{fe} decreases continuously with increase of amplitude. The amplitude of oscillation of transistor feedback oscillators can therefore be more readily limited to small values than that of vacuum-tube oscillators, and amplitude hysteresis is not ordinarily observed. This advantage is offset by the fact that the amplitude stability of transistor oscillators against changes of circuit elements and supply voltages cannot be increased by the use of a simple resistance-capacitance biasing circuit equivalent to the capacitor-gridleak bias used with vacuum tubes. The reason for this is that the emitter-base junction is forward-biased in Class A operation and therefore does not act as a complete rectifier. Although some rectification is obtained as the result of curvature of the base current-voltage characteristic, it is insufficient to produce stabilization comparable with that obtained in vacuum-tube circuits. However, stabilization can be achieved by the use of amplitude-sensitive feedback networks equivalent to those of Figs. 278 and 279, as in the circuit of Fig. 281. If the values of R and N in Fig. 281 are such that R can provide the proper base bias, C' and R' may be omitted and the base connected to the junction between R and N.

Transistor feedback oscillators can also be stabilized against changes of amplitude by the use of a separate diode rectifier, as in the circuit of Fig. 282.[3] In this circuit, which employs a modified form of Colpitts feedback

[3] E. Keonjian, *Electrical Eng.*, **74**, 672 (August, 1955).

network to be discussed in Sec. 84.4, increase of amplitude of oscillation causes a decrease of base current, and therefore of h_{fe}, as the result of

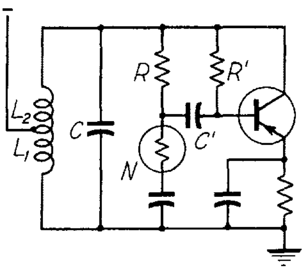


FIG. 281. Transistor oscillator in which amplitude is stabilized by means of a temperature-sensitive resistor.

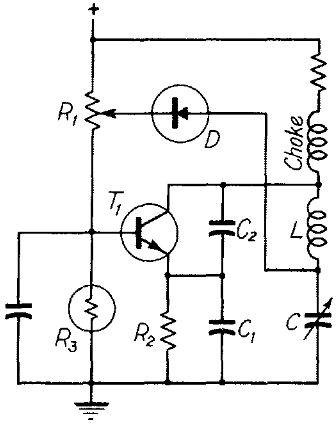


FIG. 282. Transistor oscillator in which a diode rectifier decreases the base current with increase of amplitude.

rectification of the diode D. The function of the thermistor R_3 will be discussed in Sec. 84.7.

The low variational resistance of the emitter-base junction of a transistor when the junction is forward-biased makes possible excellent stabilization of amplitude of oscillation of a transistor oscillator at a value determined by a constant reference voltage, such as the breakdown voltage of a breakdown diode.[4] The stabilization of a tuned-base oscillator by this method is illustrated in Fig. 283. The breakdown diode starts to conduct when the amplitude of the voltage V'' induced in L'' exceeds the breakdown voltage V_d of the diode D_1 minus the base-emitter biasing voltage V_{BE}. The variational base-emitter resistance of the forward-biased base being very low in comparison with R_1 and R_2, the average diode current I flows almost entirely out of the base of the transistor, and therefore reduces the average base current. The resulting reduction of average collector cur-

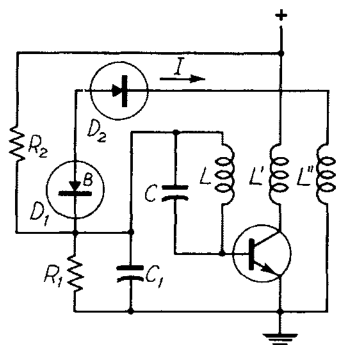


FIG. 283. Transistor oscillator in which the amplitude is stabilized at a value determined by the voltage of a breakdown diode.

[4] E. R. Kretzmer, *Proc. I.R.E.*, **42**, 291 (February, 1954).

rent reduces h_{fe} and therefore tends to prevent further increase of amplitude. The amplitude is stabilized at a value such that $V'' = V_d - V_{BE}$, where V_{BE} is of the order of 0.1 to 0.2 volt. The amplitude of oscillation may be changed by voltage applied to the lower terminal of L''. The capacitance of C_1 must be great enough so that essentially all the varying component of diode current flows through C_1, rather than through the base. The function of D_2 is to prevent current from flowing through D_1 when the voltage induced in L'' is positive.

84. Frequency Stabilization of Parallel-Resonator Feedback Oscillators

84.1. Stabilization Against Change of Load. The expressions for the frequency of oscillation of the feedback oscillators listed in Table X involve r, the resistance of the resonator coil, as well as h_{ie} and h_{oe} or r_g and r_p. Because loading of the resonator may be taken into account by an equivalent change in one of these parameters, it follows that the frequency of oscillation of parallel-resonator feedback oscillators will in general vary with load. Dependence of frequency of oscillation upon load can be best prevented or made negligible by not loading the oscillator directly, but using the oscillator only to excite a power amplifier that supplies the load power.

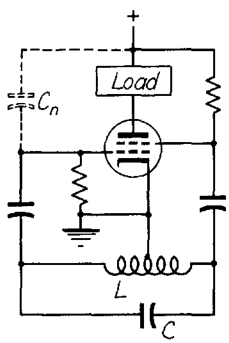


FIG. 284. Hartley oscillator with electron-coupled load.

Another method of making the frequency of oscillation of vacuum-tube oscillators independent of load is the use of electron coupling in loading the oscillator.[1] A Hartley oscillator with electron-coupled load is shown in Fig. 284. The control grid and the screen of a tetrode replace the grid and the plate of a triode in the oscillating circuit. The plate circuit serves only to deliver the output. The flow of electrons through the screen grid varies with screen current, which in turn varies at the oscillation frequency. The electrons that pass through the screen are attracted to the plate by the plate field, and hence the plate current has an alternating component whose frequency is equal to the oscillation frequency. Since the field of the plate terminates almost entirely on the screen, the plate load has very little effect upon the oscillating circuit.

Because of capacitive coupling of the load to the oscillating circuit through the screen-plate capacitance, the simple electron-coupled circuit, of which Fig. 284 is an example, is not entirely free from the effects of reaction of load upon the oscillating circuit. This difficulty may be eliminated by

[1] J. B. Dow, *Proc. I.R.E.*, **19**, 2095 (1931).

the addition of a neutralizing capacitor C_n, as shown by the dotted lines in Fig. 284. The capacitor C_n and the screen-plate capacitance form a voltage divider, the ends of which are connected to the control grid and the screen. The control-grid and screen alternating voltages being opposite in phase, a setting of C_n may be found at which no alternating voltage is applied to the plate when the plate supply voltage is zero. The same result may be accomplished by using a pentode in place of a tetrode, the third grid, which is made positive, acting as a screen between the second grid and the plate.

If the plate and oscillator-anode voltages are properly chosen, the change in frequency resulting from a change in plate voltage is equal and opposite to that resulting from a proportional change of oscillator-anode voltage.[2] Therefore, if the supply voltage for the oscillator anode (the screen in Fig. 284) is obtained from a tap on a voltage divider across the plate supply voltage, a setting of the tap can be found such that the frequency is independent of supply voltage. Thus, the electron-coupled oscillator not only prevents reaction of load upon the oscillating circuit but may also be designed to minimize the effect of supply-voltage variation upon frequency.

The advantages of the electron-coupled oscillator are gained at the expense of good wave form. The high harmonic content of electron-coupled oscillators results from two causes. Inasmuch as the oscillator-anode current is in itself not sinusoidal, the space current beyond it is not likely to be. Secondly, the plate current is influenced by secondary emission effects and may even reverse. Oscillographic studies show that it is difficult to obtain sinusoidal output from electron-coupled oscillators without the use of a resonant circuit or other form of filter in the output circuit.

84.2. Dependence of Oscillation Frequency upon Supply Voltage. Table X shows that the frequency of oscillation of parallel-resonator feedback oscillators is to some extent dependent upon the plate resistance in vacuum-tube circuits and upon all the transistor parameters in transistor circuits. The frequency is also to some extent dependent upon the interelectrode capacitances and, at extremely high frequencies, upon electron or hole transit time. Since tube or transistor parameters, interelectrode capacitances, and electron or hole transit time are dependent upon operating voltages and currents, the frequency of oscillation is in general not independent of supply voltages. (Interelectrode capacitances are altered by changes of electrode separation with electrode temperature and by changes of space-charge density with electrode voltages or currents.)

Dependence of frequency of oscillation upon transit time is predicted by Eq. (149). Carrier transit time affects the phase angle of the y parameters of the amplifier. In order that Eq. (149) shall hold when the phase

[2] Dow, *loc. cit.*

angle of one or more y parameters is changed, there must be a compensating change in one or more z parameters of the feedback network. Unless the required changes of z parameters are made by altering one or more of the circuit elements, they must result from a change of frequency. This can be seen most readily from Eq. (152), which applies to vacuum-tube oscillators with negative grid bias and may also be used as a first approximation for transistor oscillators. Any change in the phase angle of y_f must be accompanied by an equal change of phase angle of Z_f and therefore by a change of frequency.

A second cause of dependence of oscillation frequency upon supply voltages is the variation of the magnitudes of the amplifier parameters with operating voltages and currents. Because the input port of the feedback network is shunted by the output admittance of the amplifier and the output port of the feedback network is shunted by the input admittance of the amplifier, changes of the amplifier parameters in general affect the phase angles of the z parameters of the feedback network and therefore the frequency of oscillation.

The reason for the dependence of oscillation frequency upon amplifier parameters is most readily understood for the special case of a vacuum-tube oscillator with negative grid bias. Equation (152) then applies and the oscillator may be represented by the equivalent circuit of Fig. 285. If y_f

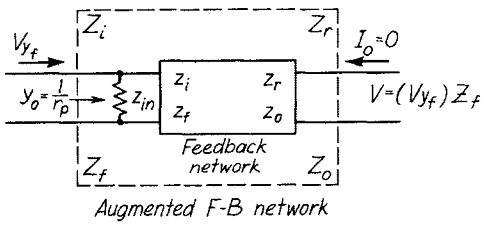


FIG. 285. Equivalent circuit of a vacuum-tube oscillator with negative grid bias.

is nonsusceptive, the frequency of oscillation is that at which $1/Z_f$ is nonsusceptive; i.e., the frequency at which the output voltage of the augmented feedback network is in phase with, or in phase opposition to, the driving current Vy_f. Unless the input impedance z_{in} of the feedback network is nonreactive at the frequency of oscillation, shunting the feedback network by r_p causes the phase angle of Z_f to differ from that of z_f and to vary with r_p even at frequencies at which the plate impedance is nonreactive. In general, z_f and z_{in} are not nonreactive at the same frequency, and the phase angle of Z_f is determined in part by r_p. A change of r_p must then be accompanied by a change of oscillation frequency in order that $1/Z_f$ remain nonsusceptive.

The frequency at which z_f is nonreactive is not necessarily the series resonance frequency of the feedback network (the frequency at which the expression for z listed in Table IX does not contain a reactive term). For this reason, the oscillation frequency may differ from the resonance frequency and be dependent upon r_p even when z_{in} is nonreactive at the resonance frequency.

A third cause of variation of oscillation frequency with supply voltages that is very important in transistor oscillators is the dependence of electrode capacitances upon operating voltages and currents.

Change of magnitude of y_f (or h_f) may have a second-order effect upon the frequency of oscillation because the resulting change of amplitude of oscillation results in a change of the dynamic values of the other amplifier parameters or of the values of any nonlinear resistances that may be contained in the feedback network.

84.3. Stabilization of Frequency Against Changes of Voltage. The dependence of the phase angle of Z_f upon the input and output admittances of the amplifier and therefore upon the supply voltages can be reduced by (1) reducing the input and output impedances of the feedback network, (2) decreasing the output and input admittances y_o and y_i (decreasing h_o and increasing h_i), or (3) adding resistances in series with the output and the input of the amplifier. Decreasing the input and output impedances of the feedback network also decreases the effect of the amplifier input and output capacitances upon Z_f and therefore increases the frequency stability against changes of interelectrode capacitance. Another method of stabilizing the frequency against changes of amplifier parameters consists in the addition of one or more properly selected reactances to the feedback network. One way in which this can be done is by the addition of reactances of proper sign and magnitude to cause the z parameters of the feedback network to be nonreactive at the series resonance frequency of the network. In some oscillators with inductive coupling, the frequency stability is improved by increasing the coefficient of coupling. These various methods of frequency stabilization will be discussed further in following sections.

84.4. Clapp-Gouriet Oscillator. An example of a circuit in which good frequency stability against changes of interelectrode capacitances and plate resistance is achieved by the use of low feedback-network input impedance is the modified Colpitts oscillator shown in vacuum-tube form in Fig. 286.[3] In this circuit the capacitances C_1 and C_2 are made large and the frequency is kept within the desired range by the introduction of the relatively small series capacitance C_3, which serves as the tuning element. Although the same frequency stability can be obtained in the Colpitts circuit by decreas-

[3] J. K. Clapp, *Proc. I.R.E.*, **36**, 356 and 1261 (March, 1948); W. A. Roberts, *Proc. I.R.E.*, **36**, 1261 (October, 1948); G. G. Gouriet, *Wireless Engineer*, **27**, 105 (April, 1950); J. K. Clapp, *Proc. I.R.E.*, **42**, 1259 (August, 1954).

ing the inductance to compensate for the increase of C_1 and C_2 at a desired frequency, the single small tuning capacitor in the Clapp-Gouriet circuit affords a cheaper and more convenient means of tuning than do the two large capacitors in the Colpitts circuit.

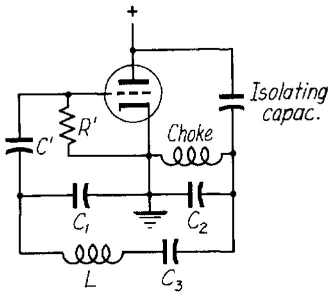


FIG. 286. Clapp-Gouriet oscillator.

84.5. Resistance Stabilization. Frequency stabilization by use of a large resistance in series with the input or output of the amplifier is called *resistance stabilization.* Two typical resistance-stabilized vacuum-tube circuits are shown in Fig. 287. The equivalent circuit of Fig. 285 represents the resistance-stabilized circuits if y_o is $1/(r_p + r_s)$. The increase of stability results not only from the fact that a decrease in the magnitude of the conductance shunting the input of the feedback network reduces the rate of change in phase of Z_f with this conductance, but also from the fact that a certain percentage change in $1/r_p$ produces a much smaller percentage change in $1/(r_p + r_s)$, since r_s is constant.

When the stabilizing resistance is present, the transconductance g_m must

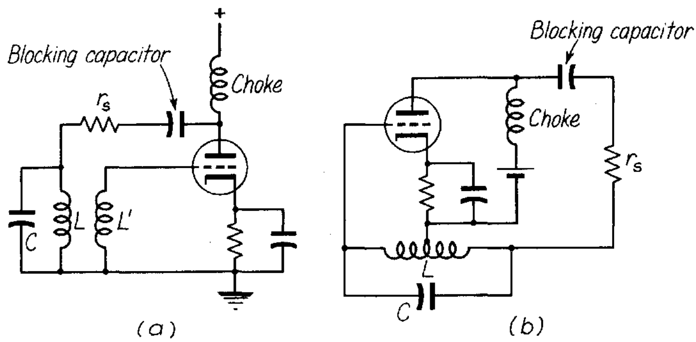


FIG. 287. Resistance-stabilized vacuum-tube oscillators: (a) tuned-plate; (b) Hartley.

be replaced by $\mu/(r_p + r_s)$ and r_p by $r_p + r_s$ in the criteria for oscillation listed in Table X. This indicates that the variation of amplitude with undesired changes of operating voltages is reduced by the stabilizing resistance, but that the criterion for self-sustained oscillation is more difficult to satisfy. Furthermore, because the variation of $\mu/(r_p + r_s)$ with grid bias when r_s is large in comparison with r_p is much smaller than the variation of g_m, the stabilizing resistance prevents much of the benefit of capacitor-gridleak bias. However, if the stabilizing resistance is made so large that oscillation barely starts, the additional losses resulting from the flow of grid current when the grid starts to swing positive limit the amplitude to a value that approximates the grid bias. Satisfactory waveform can therefore be obtained by proper choice of fixed bias.

Design details for resistance-stabilized vacuum-tube oscillators have been published. It is desirable to use tubes having an amplification factor ranging from 5 to 8, unity turn ratio of grid and plate coils, close coupling between grid and plate coils, resonator shunt impedance of the order to 5000 to 20,000 ohms, and stabilizing resistance of the order of two to five times the plate resistance.[4]

84.6. Reactance Stabilization. Examination of Table IX shows that the four z parameters of the commonly used feedback networks are not all nonreactive at the same frequency. The phase angle of the open-circuit forward transfer impedance Z_f of the augmented feedback network of Fig. 269a is therefore dependent upon the magnitudes of y_i and y_o, even though y_i and y_o are nonreactive. Consequently, a change in the y parameters of the tube or transistor accompanying a change of supply voltage results in a change of oscillation frequency. The dependence of oscillation frequency upon the tube or transistor parameters can be minimized by adding to the feedback network one or more reactive elements in such a manner that the z parameters of the modified network are all nonreactive at some frequency. If the y parameters of the tube or transistor are also nonreactive, oscillation will occur at this frequency.

The stabilization of feedback oscillators by the addition of reactive elements to the feedback network was analyzed by Llewellyn.[5] His method of analysis consists of setting up equivalent circuits of vacuum-tube feedback oscillators and determining from a general solution of the circuit equations how the frequency of oscillation may be made independent of the tube factors by the addition of one or more inductances or capacitances to the circuits. The effect of grid current is taken into account in the analysis, but distortion resulting from nonlinearity of tube characteristics is neglected. It is assumed that losses in the oscillating circuit are negligibly small and that the load is applied to the oscillator through a buffer stage that draws no power from the oscillator. Llewellyn's method has been extended to transistor circuits.[6]

Llewellyn also showed that frequency stability could be increased in inductively coupled circuits by the use of close coupling. This is apparent

[4] F. E. Terman, *Electronics*, July, 1933, p. 150.

[5] F. B. Llewellyn, *Proc. I.R.E.*, **19**, 2063 (1931); see also G. H. Stevenson, *Bell Sys. Tech. J.*, **17**, 458 (1938); and H. Jefferson, *Wireless Engineer*, **22**, 384 (August, 1945).

[6] J. B. Oakes, *Proc. I.R.E.*, **42**, 1235 (August, 1954).

also from the expressions for the frequency of oscillation of tuned-collector, tuned-base, and transistor Hartley circuits listed in Table X. The term involving $(LL' - M^2)$ or $(L_1L_2 - M^2)$ approaches zero as the coefficient of coupling approaches unity.

The improvement in frequency stability predicted by Llewellyn's analysis is verified by measurements. Typical of the performance of a stabilized vacuum-tube oscillator is a frequency change of less than 10 cps at 1 Mc when the plate voltage is reduced by 50 percent. Since the frequency of oscillation is independent of resistance shunting the input or output of the feedback network when stabilization is complete, an external load may be applied across either the input or the output without a reduction of stabilization. A practical problem in reactance-stabilized oscillators is the necessity of varying at least one stabilizing capacitance or inductance in variable-frequency oscillators. Fortunately, in some circuits the required stabilizing capacitance or inductance is proportional to the tuning capacitance or inductance.

84.7. Stabilization of Frequency of Transistor Oscillators Against Change of Temperature. The need for frequency stabilization is particularly great in transistor circuits, since the transistor operating currents and parameters depend upon temperature. Some stabilization against temperature change can be obtained by the use of bypassed resistance in series with the amplifier emitter. The stability against changes of temperature can be greatly increased by incorporating a thermistor in the amplifier. In the Clapp-Gouriet circuit of Fig. 282 [7] the frequency tends to decrease with increase of temperature. However, increase of temperature also decreases R_3 and thus causes a decrease of base and emitter currents of the transistor T_1. The decrease of emitter current reduces the base-emitter capacitance shunting C_2 and thus tends to prevent the reduction of frequency. In this manner the frequency stability is improved by a factor of from 20 to 35. Increase of C_1 tends to improve the stability, but the maximum value of C_1 is limited by feedback considerations.

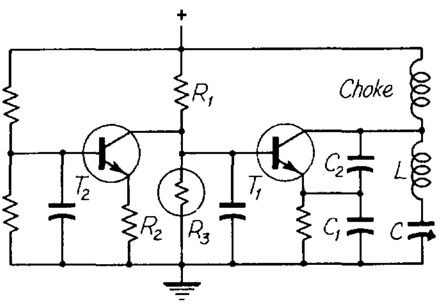


FIG. 288. Thermistor-stabilized transistor Clapp-Gouriet oscillator.

Another thermistor-stabilized transistor Clapp-Gouriet oscillator is shown in Fig. 288.[7] In this circuit stabilization against changes of supply voltage

[7] E. Keonjian, *Electrical Eng.*, **74**, 672 (August, 1955); also *Trans. I.R.E.*, Vol. CT-3, No. 1, 38 (March, 1956).

is improved by the addition of a second transistor T_2. Increase of supply voltage increases the base voltage of T_2 and therefore its collector current. The increased voltage drop in R_1 tends to compensate for the increase of supply voltage and thus maintain the base voltage of T_1 constant. With proper choice of circuit values, the frequency can be made nearly independent of supply voltage, an improvement in stability by a factor of the order of 25 being possible. The thermistor R_3 serves the same function as in the circuit of Fig. 282. Reverse-biased junction diodes may also be used as the temperature-sensitive element in the stabilization of transistor circuits.[8]

84.8. Use of Common-Collector and Common-Base Coupling Stages. Some improvement in the frequency stability of transistor parallel-resonator feedback oscillators can be achieved by the use of a common-collector or compound-connected stage ahead of the common emitter amplifier in order to increase h_i, and a common-base stage following the common-emitter amplifier in order to decrease h_o.

84.9. Electronic Tuning and Modulation of Feedback Oscillators. The frequency of parallel-resonator feedback oscillators may be controlled or modulated by the same methods as negative-resistance oscillators (Sec. 77). Some microwave feedback oscillators, of which the two-resonator klystron is an example, may be tuned by varying one or more electrode operating voltages. The effect of the voltage variation is to change the transit time of electrons and therefore the phase angle of the transfer admittance y_f. Since the phase of $1/Z_f$ must change by the same amount, the frequency changes. Voltage-controlled capacitors and variable-admittance circuits such as those of Fig. 267 may be used to control or modulate the frequency of parallel-resonator oscillators electronically.

85. Null-Network Oscillators

85.1. Desirable Features of Null-Network Oscillators. Excellent frequency of stability of feedback oscillators can be obtained by the use of null networks to provide the feedback path. Because the phase of the transfer impedance of null networks relative to the input voltage varies rapidly with frequency, only a small change in frequency is then necessary in order to compensate for an undesired change of phase of y_f or Z_f. Null networks include the Wheatstone bridge circuits of Figs. 289a and 289b, the bridged-tee network of Fig. 289c,[1] and the parallel-tee (twin-tee) network of Fig. 289d.[2] The resistance-capacitance bridged-tee network of Fig. 289e, although it

[8] J. W. Stanton, *Trans. I.R.E.*, Vol. CT-3, 65 (1956).

[1] H. H. Scott, *Proc. I.R.E.*, **26**, 226 (February, 1938); Augustadt, U. S. Patent 2,106,785; W. N. Tuttle, *Proc. I.R.E.*, **28**, 23 (January, 1940); P. M. Honnell, *Proc. I.R.E.*, **28**, 88 (February, 1940).

[2] Tuttle, *loc. cit.*, W. G. Shepherd and R. O. Wise, *Proc. I.R.E.*, **31**, 256 (June, 1943); A. E. Hastings, *Proc. I.R.E.*, **34**, 126P (March, 1946); L. Stanton, *Proc. I.R.E.*, **34**, 447 (July, 1946).

is not a true null network, has similar characteristics.[3] Oscillator null networks cannot be adjusted for a true null, but must be unbalanced sufficiently so that the magnitude of the transfer admittance $1/Z_f$ does not exceed the amplifier transconductance y_f at zero amplitude. The rate of change of phase of Z_f with frequency in the vicinity of the resonance frequency increases as the balanced condition is approached, being infinite when the network is adjusted for a true null. For this reason the frequency stability

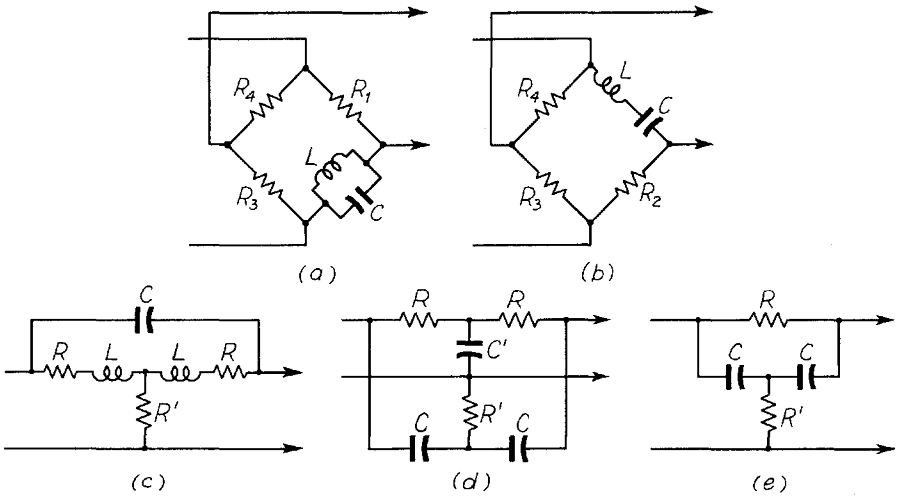


FIG. 289. Null networks suitable for use in feedback oscillators: (a) and (b) Wheatstone-bridge; (c) bridged-tee; (d) parallel-tee (twin-tee); (e) resistance-capacitance bridged-tee.

of a null-network oscillator increases with the transfer admittance of the amplifier.

The output voltage of a null network at resonance may be either in phase with or in phase opposition to the input voltage, depending upon the manner in which it is unbalanced. Consequently, null-network oscillators may use either single-stage (voltage-inverting) or two-stage (noninverting) amplifiers. Because of the greater amplifier transfer admittance attainable, two-stage oscillators have greater frequency stability, but two-stage, wideband amplifiers are more difficult to design than single-stage amplifiers.

A second desirable feature of null-network oscillators is that amplitude stabilization can be readily accomplished by the use of a temperature-dependent resistor as one of the network resistors. The sign of the temperature

[3] Scott, *loc. cit.*; Augustadt, *loc. cit.*; Tuttle, *loc. cit.*; Honnell, *loc. cit.*; H. S. McGaughan and C. B. Leslie, *Proc. I.R.E.*, **35**, 974 (September, 1947); P. G. Sulzer, *Proc. I.R.E.*, **39**, 819 (July, 1951).

coefficient of resistance must be such that increase of temperature resulting from increase of oscillation amplitude reduces the unbalance of the network and thus increases the attenuation. Amplitude stabilization may also be accomplished by using a temperature-dependent resistor to provide a negative-feedback path. The sign of the temperature coefficient of resistance must then be such that the negative feedback increases with oscillation amplitude.

A third desirable characteristic of null-network oscillators is that the feedback may be degenerative for all harmonics. Harmonics lying within the frequency-response band of the amplifier are then attenuated.

85.2. Parallel-Resonator Bridge Oscillator. An insight into the reason for the rapid rate of change of the phase of null-network transfer impedance with frequency can be gained by an examination of the oscillator circuit of Fig. 290, which makes use of the bridge of Fig. 289a.[4] If arms 1, 3, and 4 of

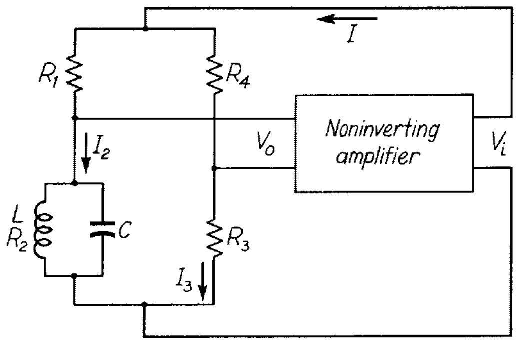


FIG. 290. Meacham oscillator.

the bridge are nonsusceptive, the impedance parameters of the bridge are nonsusceptive at the resonance frequency of arm 2. Consequently, if the forward transfer admittance y_f of the amplifier is nonsusceptive, oscillation occurs at the resonance frequency of arm 2. If y_f has a susceptive component, as it will have at frequencies near the limits of the amplifier pass band, the output voltage V_o of the bridge must be out of phase with the input voltage V_i, as shown in Fig. 291. In the construction of Fig. 291 it is assumed that the transfer admittance of the amplifier is large and that the required magnitude of the ratio of the bridge output voltage to the input voltage is therefore small. The phase angle between V_o and V_i is greatly exaggerated in Fig. 291, and the input admittance of the amplifier is assumed to be negligible. Figure 291 shows clearly that only a very small difference in phase between I_2Z_2 and I_3R_3 produces a large difference in

[4] L. A. Meacham, *Proc. I.R.E.*, **26**, 1278 (October, 1938).

phase between V_o and V_i and that the ratio of the phase angles increases with the magnitude of V_i/V_o. Consequently the frequency departure from the resonance value required to compensate for a susceptive component of y_f is small, and decreases with increase of y_f. (Note that $z_f = z_{in}V_o/V_i$.)

Since the ratio of the phase angle between V_o and V_i to that between I_2Z_2 and I_3R_3 increases with the magnitude of V_i/V_o, change in y_f caused by changes of operating voltage or of amplitude of oscillation must result in a

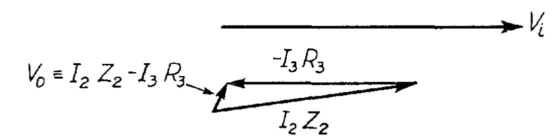


FIG. 291. Phasor diagram for the bridge used in the circuit of Fig. 290.

frequency change. This effect is small, however, unless the amplifier phase shift departs greatly from 180 degrees. For high stabilization, the amplifier should be designed for high, nonsusceptive forward transfer admittance y_f, and the resonator Q should be as high as possible.

It can be seen from Fig. 291 that in order for V_o to have a component in phase with V_i, as required for oscillation, the magnitude of I_2Z_2 must exceed that of I_3R_3. Since Z_2 is very low at harmonic frequencies, I_2Z_2 is much smaller than I_3R_3 at these frequencies. Harmonics are therefore passed through the bridge with negligible attenuation and are fed back to the amplifier degeneratively. Consequently, if the bandwidth of the amplifier is great enough to include the lower harmonics, considerable reduction of harmonic output may be obtained.

A practical form of the oscillator of Fig. 290 is shown in Fig. 292. The amplitude of oscillation is limited in the circuit of Fig. 292 by the use of a

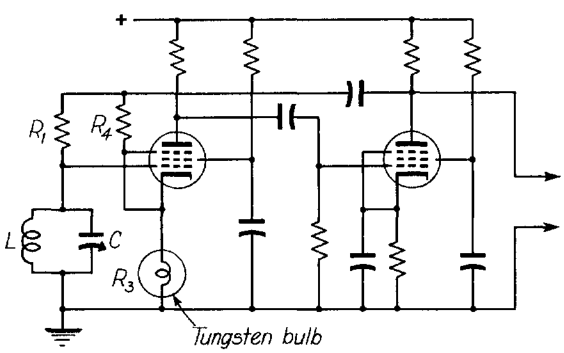


FIG. 292. Circuit diagram of a Meacham oscillator with amplitude stabilization.

temperature-controlled resistor for R_3. The zero-amplitude adjustment of the bridge is such that the attenuation is smaller than the amplification of the amplifier ($1/Z_f < y_f$ at zero amplitude). As the amplitude of oscillation builds up, increase of resistance of R_3 resulting from the increase in temperature reduces the bridge unbalance to the point where the bridge attenuation is equal to the amplifier amplification ($1/Z_f = y_f$). A tungsten-filament bulb has the positive temperature coefficient required in this circuit. A thermistor, which has a negative temperature coefficient, can be used if it is placed in arm 4 instead of arm 3 of the bridge.

85.3. Bridged-Tee Feedback Oscillators. A vacuum-tube feedback oscillator based upon the bridged-tee null circuit of Fig. 289c, is shown in Fig. 293a.[5] The resistance R' is made sufficiently smaller than the null value

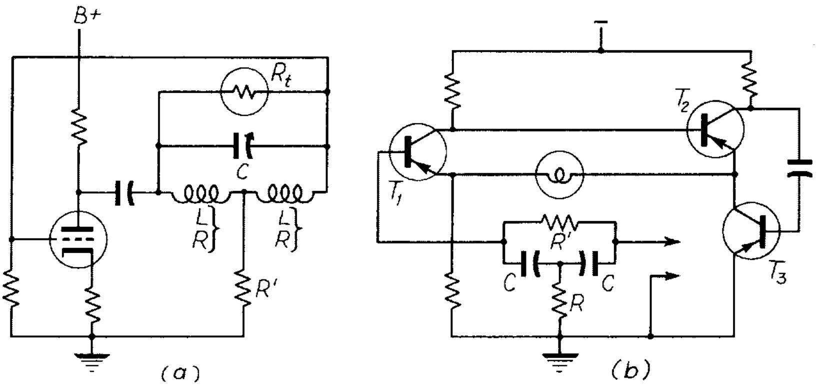


FIG. 293. Vacuum-tube and transistor bridged-tee oscillators.

to provide the required positive feedback voltage. Amplitude-dependent negative feedback is provided by the thermistor R_t. Increase of amplitude raises the temperature of R_t and the resulting decrease in R_t increases the inverse feedback, and thus stabilizes the amplitude. The frequency of oscillation is approximately $1/2\pi\sqrt{2LC}$. The rate of change of phase of Z_f with frequency, and therefore the stability of this type of oscillator increases with the inductor Q, $2\pi fL/R$.

A practical oscillator based upon the circuit of Fig. 293a showed a frequency instability of less than one part per million per volt, high amplitude stability, and very low harmonic output.

A transistor bridge oscillator analogous to the tube circuit of Fig. 293a is shown in Fig. 293b.[6] Transistors T_2 and T_3 serve as a push-pull, common-collector, power-output stage.

[5] W. G. Shepherd and R. O. Wise, *Proc. I.R.E.*, **31**, 256 (June, 1943).
[6] P. G. Sulzer, *Electronics*, September, 1953, p. 171.

A noninverting (voltage) amplifier may be used in the circuit of Fig. 293 if the resistance R' is made larger than the null value. Shepherd and Wise reported a stability improvement of two or three times and a corresponding decrease of harmonic content as the result of the use of a two-stage amplifier. Ordinarily this improvement is not sufficient to compensate for the increased complexity of the amplifier.

Although the resistance-capacitance bridged-tee network of Fig. 289e is not a true null network, at small ratios of R'/R its characteristics are similar to those of an unbalanced null network.[7] The ratio of input voltage to output voltage passes through a minimum and the rate of change of phase of z_f with frequency passes through a maximum at the frequency $f = 1/2\pi\sqrt{n}\,RC$, where $n = R'/R$. The phase angle of the voltage transfer ratio V_i/V_o is zero at this frequency. The maximum attenuation and the rate of change of phase of z_f with frequency increase with decrease of n, but the rate of change of phase at a given maximum attenuation is less than in the null networks of Fig. 289a, b, c, and d. Consequently, the frequency stability and waveform of positive-feedback oscillators based upon this circuit are inferior to those of null-network oscillators. Since the output voltage of the network is in phase with the input voltage at the resonance frequency, a feedback oscillator based upon this network must use a noninverting amplifier. To avoid change of amplitude and the ganging of dissimilar tuning elements, the circuit should be tuned by means of ganged capacitors rather than ganged resistors.

85.4. Parallel-Tee, Resistance-Capacitance-Tuned Oscillator. Figure 294 shows the circuit of a resistance-capacitance-tuned oscillator in which the feedback network is the parallel-tee null network of Fig. 289d.[8] Analysis of the parallel-tee circuit shows that an output-voltage null is obtained when the following relations are satisfied: [9]

$$f = 1/2\pi\sqrt{2n}\,RC \tag{180}$$

$$R'/R = C'/4C = n \tag{181}$$

in which n is any positive number. The open-circuit input and output impedances of the network are then:

$$z_i = z_o = \frac{2nR}{n+1}\left(1 - j\sqrt{1/2n}\right) \tag{182}$$

[7] H. S. McGaughan and C. B. Leslie, *Proc. I.R.E.*, **35**, 974 (September, 1947); P. G. Sulzer, *Proc. I.R.E.*, **39**, 819 (July, 1951).

[8] W. G. Shepherd and R. O. Wise, *Proc. I.R.E.*, **31**, 256 (June, 1943). See also L. G. Cowles, *Proc. I.R.E.*, **40**, 1712 (December, 1952); A. O. Behnke, *Proc. I.R.E.*, **41**, 935 (July, 1953); A. P. Bolle, *Brit. Inst. of Rad. Eng.*, **13**, 571 (December, 1953).

[9] A. E. Hastings, *Proc. I.R.E.*, **34**, 126P (March, 1946).

In order for oscillation to take place, $C'/4C$ must exceed R'/R by a sufficient amount so that the attenuation of the network at the null frequency is smaller than the gain of the amplifier ($g_m > 1/Z_f$) at zero amplitude. Amplitude is stabilized in the circuit of Fig. 294 by means of the thermistor R_t, which provides negative feedback. Increase of amplitude of oscillation raises the temperature of R_t, and the resulting decrease of resistance increases the negative feedback. Alternatively, the amplitude may be stabilized by using for R' a thermally controlled resistor having a positive temperature coefficient of resistance. This type of stabilization is less satisfactory than negative-coefficient stabilization because the relatively low

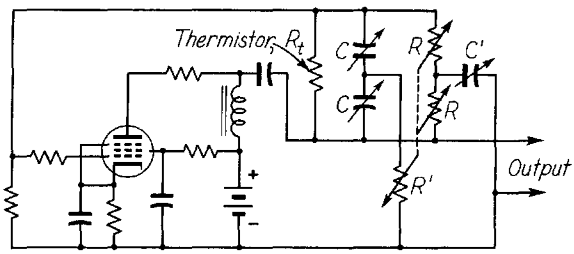


FIG. 294. Parallel-tee resistance-capacitance-tuned feedback oscillator.

resistance of positive-coefficient resistors results in a low value of z_f and thus necessitates greater unbalancing of the bridge in order that $g_m > 1/Z_f$. The frequency stability is therefore lower than when the amplitude is stabilized by means of a thermistor. Positive-coefficient control cannot be conveniently used when tuning is accomplished by ganged variable resistors.

Ganged-capacitor tuning of the oscillator of Fig. 294 is preferable to ganged-resistor tuning because it does not vary the magnitude of z_f and therefore results in less amplitude variation, and because of lower likelihood of tuning noise. Although the greatest rate of change of phase of z_f with frequency is obtained when n is $\frac{1}{2}$, tuning is simplified if n is chosen as $\frac{1}{4}$ in order to make all capacitors alike. The function of the series grid and plate resistors in the circuit of Fig. 294 is to suppress spurious high-frequency oscillations in the tube circuit. The phase angle of the input impedance of the parallel-tee network, like that of the Wien bridge circuit to be discussed in Sec. 86, is large. Consequently much of the discussion of Sec. 86 is also applicable to the parallel-tee oscillator. For additional details of design of this type of oscillator, the reader should refer to the literature.[10] An obvious disadvantage of the parallel-tee oscillator relative to the Wien-bridge oscillator is the necessity of ganging three tuning capacitors, instead of only two.

[10] Shepherd and Wise, *loc. cit.*

86. Wien-Bridge Oscillator

86.1. Analysis of General Form of Circuit. A resistance-capacitance-tuned feedback oscillator that has a number of desirable features is shown in basic form in Fig. 295.[1] Because the feedback network has the same form as two arms of the Wien bridge, this oscillator is usually called the Wien-bridge oscillator.

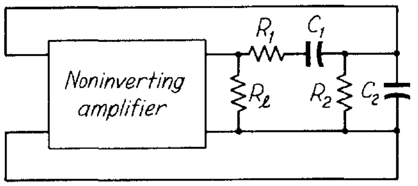


FIG. 295. Basic circuit of the "Wien-bridge" oscillator.

The frequency of oscillation and the criterion for oscillation with constant amplitude may be determined by substituting the appropriate z parameters listed in Table IX into Eq. (149) and separating the real and imaginary terms. In general form, the equations obtained in this manner are complicated and difficult to interpret. For practical reasons, however, R_2 is usually made equal to R_1 and C_2 to C_1. The analysis can be made somewhat more general without making the equations too complicated by the assumption that $R_2C_2 = R_1C_1$. Equation (149) then leads to the following simultaneous equations:

$$R_l[k^4 + R_1(1 + y_oR_l)] + k^2[R_l - KR_1^2(y_i + R_l\Delta_y) - [R_1(1 + y_oR_l) + KR_1^2(y_i + R_l\Delta_y)] = 0 \qquad (183)$$

$$y_fR_lKk^2 + k^2 + 1 + Kk^2 + y_i[KR_1 + (R_l + R_1)Kk^2] + y_oR_l[k^2 + 1 + Kk^2] + \Delta_yR_lR_1K(k^2 + 1) = 0 \qquad (184)$$

in which

$$K = \frac{R_2}{R_1} = \frac{C_1}{C_2} \qquad (185)$$

and

$$k = \omega R_1C_1 = \omega R_2C_2 \qquad (186)$$

86.2. Vacuum-Tube Wien-Bridge Oscillator. For tube circuits with negative grid bias, $y_i = y_r = \Delta_y = 0$. Equation (184) then reduces to

$$k^4 + \frac{k^2}{m+1} - \frac{m}{m+1} = 0 \qquad (187)$$

[1] F. E. Terman, R. R. Buss, W. R. Hewlett, and F. C. Cahill, *Proc. I.R.E.*, **27**, 649 (1939). See also J. A. B. Davidson, *Electrical Eng.* (London), **16**, 316 and 361 (1944); C. M. Edwards, *Proc. I.R.E.*, **39**, 277 (March, 1951); D. A. Bell, *Electronic Eng.*, **23**, 274 (July, 1951); J. A. B. Davidson, *Proc. I.R.E.*, **40**, 1124 (September, 1952); K. K. Clarke, *Proc. I.R.E.*, **41**, 246 (February, 1953); W. J. Wray, *Proc. I.R.E.*, **41**, 801 (June, 1953); H. L. Armstrong, *Electronics*, July, 1953, p. 43; J. M. Diamond, *Proc. I.R.E.*, **42**, 1449 (September, 1954); J. M. Diamond, *Proc. I.R.E.*, **42**, 1807 (December, 1954); N. Kovalevski and B. M. Oliver, *Hewlett-Packard J.*, **8**, No. 5, January, 1957.

in which

$$m = R_1(y_o + 1/R_l) \tag{188}$$

The solution of Eq. (187) is

$$k^2 = \frac{m}{m+1} \tag{189}$$

Therefore

$$f = \sqrt{\frac{m}{m+1}}\,\frac{1}{2\pi R_1 C_1} = \sqrt{\frac{R_1(y_o + 1/R_l)}{R_1(y_o + 1/R_l) + 1}}\,\frac{1}{2\pi R_1 C_1} \tag{190}$$

Examination of Eq. (190) shows that the best way to minimize the dependence of oscillation frequency upon y_o, and therefore upon supply voltage, is to make $R_l \ll 1/y_o$. When decade tuning is used, it is also desirable that the frequency of oscillation be independent of the ratio of R_l to R_1 so that frequency ranges can be conveniently changed by proportionate changes of R_1. The frequency can be made essentially independent of R_l by choosing R_l so that it is much smaller than R_1 in the highest frequency range (smallest value of R_1). The value of k^2 then approximates unity and Eq. (190) reduces to

$$f \approx 1/2\pi R_1 C_1 \tag{191}$$

If y_i, y_r, and Δ_y are zero and k^2 is unity, Eq. (184) becomes

$$y_f = -\frac{K+2}{K}(y_o + 1/R_l) - \frac{1}{KR_1} \tag{192}$$

Since the amplitude builds up to such a value that Eq. (192) is satisfied, making R_l small in comparison with R_1, not only tends to stabilize the frequency, but also the amplitude of oscillation. If $R_l \ll R_1$, the last term of Eq. (192) may be neglected and the equation written in the form:

$$-\frac{y_f R_l}{1 + y_o R_l} \approx \frac{K+2}{K} \tag{193}$$

If $R_1 = R_2$ and $C_1 = C_2$, K is unity and the criterion for oscillation reduces to

$$-\frac{y_f R_l}{1 + y_o R_l} \approx 3 \tag{193A}$$

Under the previous assumption that $R_1 \gg R_l$, the left sides of Eqs. (193) and (193A) is the voltage amplification of the amplifier. Equation (193A) therefore states that under equilibrium conditions of oscillation the voltage amplification of the amplifier must be approximately 3. In order for oscillation to build up initially, the zero-signal voltage amplification must exceed 3.

The minus sign in Eqs. (192) and (193) indicates that the forward short-circuit transfer admittance of the amplifier must be negative. Since the voltage amplification must also exceed 3, a conventional single-stage amplifier cannot be used in this circuit. It is possible to use a single pentode with the input voltage applied to the suppressor and the output taken from the screen, but much better performance is obtained with a two-stage, resistance-capacitance-coupled, common-cathode amplifier, as in the circuit of Fig. 296. In such a circuit y_o is the plate resistance of the second stage and y_f is the product of the voltage amplification of the first stage and the transconductance of the second tube. By the use of pentodes, ample voltage amplification is obtained even when $R_l \ll r_p$.

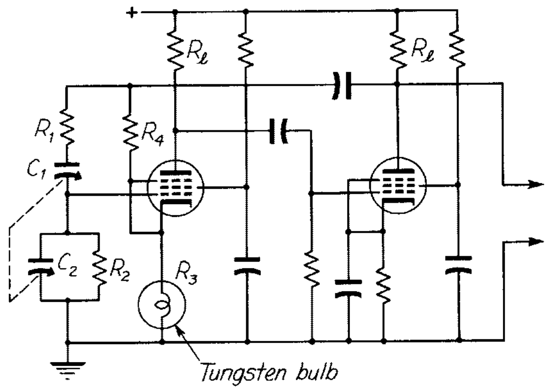


FIG. 296. Detailed circuit diagram of a vacuum-tube Wien-bridge oscillator.

The function of the tungsten-filament bulb R_3 in the circuit of Fig. 296 is to limit the amplitude of oscillation by reducing the input to the amplifier as the amplitude of oscillation increases. Alternatively, a thermistor may be used for R_4. In order to minimize tuning noise and to achieve accuracy of calibration, tuning is accomplished by simultaneous variation of C_1 and C_2, rather than R_1 and R_2. The tuning range is changed by simultaneous change of R_1 and R_2.

The tuning range of a Wien-bridge oscillator is limited at the low-frequency end by practical limitations on the size of the variable capacitors and by input conductance of the amplifier, which, although small, becomes important when R_1 and R_2 are of the magnitude of megohms or larger. It is limited at the high-frequency end by interelectrode capacitance and by the desirability of keeping R_1 and R_2 large relative to R_l. If the magnitude of R_1 and R_2 becomes comparable with that of R_l, the change in frequency between ranges is not proportional to $1/R_1$. The resulting complication in the choice of the values of R_1 and R_2 necessary to achieve decade tuning may be avoided by introducing a resistance equal to or slightly larger than $r_pR_l/2(r_p + R_l)$ in series with the parallel combination of R_2 and C_2.[2]

The high-frequency limit of oscillation can be extended considerably by the use of a cathode-follower stage between the main amplifier and the input to the feedback network.[3] A very low value of R_l may then be used without excessive loss of amplification, and R_1 and R_2 may be correspondingly reduced without departure of the tuning curve from that predicted

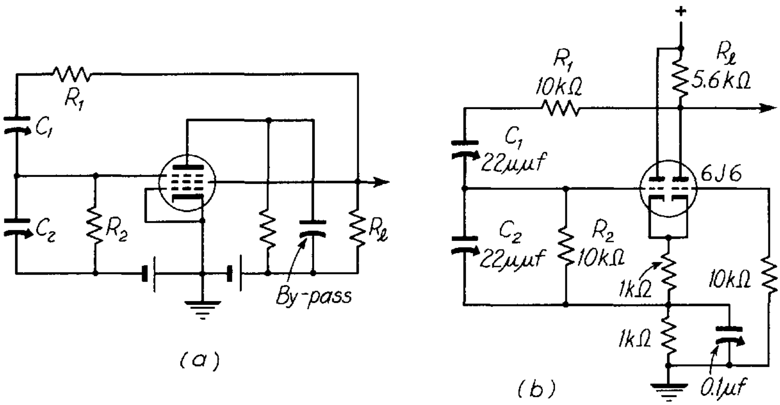


FIG. 297. (a) Single-pentode Wien-bridge oscillator; (b) Wien-bridge oscillator using a cathode-coupled amplifier.

by Eq. (191). The use of the cathode-follower stage has the additional advantage that it reduces the dependence of amplification upon R_1 and thus reduces change of amplitude from one frequency range to another (Eq. 192). Changes of amplitude between ranges in the circuit of Fig. 296 may be compensated by suitable changes in the value of R_4.

Figure 297a shows a single-pentode Wien-bridge oscillator and Fig. 297b a circuit in which a cathode-coupled (common-plate, common-grid) amplifier is used.[4] The principal merit of these circuits is their simplicity.

86.3. Transistor Wien-Bridge Oscillator. The transistor version of the Wien-bridge oscillator operates basically in the same way as the vacuum-tube version. The amplifier must therefore have a forward short-circuit transfer admittance that is negative and must be capable of voltage amplification in excess of 3. With ordinary junction transistors having a magnitude of h_{fb} smaller than unity, this requirement can be met only by the use

[2] J. A. B. Davidson, *Proc. I.R.E.*, **40**, 1124 (September, 1952).
[3] J. M. Diamond, *Proc. I.R.E.*, **42**, 1449 (September, 1954).
[4] P. S. De Laup, *Electronics*, January, 1941, p. 43; H. J. Reich, *Theory and Applications of Electron Tubes*, p. 398, McGraw-Hill Book Co., Inc., New York, 1939; H. L. Armstrong, *Electronics*, January, 1941, p. 43.

of more than one stage. Three possible combinations are a common-base stage followed by a common-collector stage, a common-collector stage followed by a common-base stage, and two common-emitter stages. It will be shown that good frequency stability necessitates that both the input impedance and the output impedance of the amplifier be high relative to the feedback-network resistances. For this reason the common-base, common-collector amplifier is unsuitable. The common-collector, common-base amplifier has somewhat higher input and output resistances than the common-emitter, common-emitter amplifier, but this advantage is more than offset by a much lower value of voltage amplification, together with a much higher value of h_r. In the analysis that follows, therefore, the amplifier will be assumed to have two common-emitter stages.

If the collector load resistance of the first common-emitter stage is of the order of 2000 ohms or less, the value of h_r of the two-stage amplifier is less than half the value of h_{re}^2, or less than 2×10^{-6}, and h_f is approximately equal to h_{fe}^2, or 2500. The value of h_o for the amplifier is the output admittance of the second-stage transistor for an input-source resistance approximately equal to the interstage coupling resistance. For a coupling resistance of 2000 ohms, h_o approximates 5×10^{-5} mho. The parameter h_i is the input resistance of the first stage, which is loaded by the coupling resistance in parallel with the input resistance of the second stage. For a coupling resistance of 2000 ohms, h_i approximates 2000 ohms. For these values of h_i, h_r, h_f, and h_o, $\Delta_h = h_ih_o$. Under this assumption, Eq. (183) may be written in the form

$$k^4(1 + m) + k^2\left(1 - \frac{KR_1}{h_i}m\right) - \left(1 + \frac{KR_1}{h_i}\right)m = 0 \qquad (194)$$

in which

$$m = R_1(h_o + 1/R_l) \qquad (195)$$

In order to minimize the dependence of oscillation frequency upon h_i, KR_1/h_i must be much smaller than unity. Equation (194) then reduces to Eq. (187), h_o in Eq. (195) being nearly equivalent to y_o in Eq. (188).* As in the vacuum-tube circuit, dependence of oscillation frequency upon h_o can be minimized by making R much smaller than $1/h_o$, and it is desirable to choose R_l so that it is much smaller than R_1 in the highest frequency range. Then $k^2 = 1$ and the frequency of oscillation is given by Eq. (191).

The criterion for oscillation of constant amplitude is derived by expressing Eq. (184) in terms of h parameters and eliminating k^2 by means of Eqs. (188) and (189):

* This is true only when $\Delta_h = h_oh_i$.

$$-\frac{h_f R_l}{h_i} \approx \frac{K+2}{K}(1 + h_o R_l) + \frac{R_l}{KR_1} + \frac{2}{h_i}[R_1(1 + h_o R_l) + R_l] \quad (196)$$

Under the previous restrictions that KR_1/h_i, $R_l h_o$, and R_l/R_1 are small in comparison with unity and that K is equal to unity, Eq. (196) reduces to

$$-\frac{h_f R_l}{h_i} \approx 3 \quad (196A)$$

The left side of Eq. (196A), like that of Eq. (193A), approximates the voltage amplification of the amplifier, and the criterion for oscillation again indicates that the zero-amplitude voltage amplification of the amplifier must exceed 3 in order for oscillation to build up.

The requirement that R_1 must be much smaller than h_i, which approximates 2000 ohms when a simple common-emitter input stage is used, greatly complicates the design of a frequency-stable transistor Wien-bridge oscillator and practically necessitates the use of one or more common-collector stages or of a compound-connected transistor before the two-stage, common-emitter amplifier. In this manner, h_i may be increased to a value between a hundred thousand ohms and a megohm. A common-collector stage between the common-emitter amplifier and the feedback network also allows the use of a value of R_l less than 100 ohms, and therefore correspondingly low values of R_1 and R_2. Increased stabilization against changes of h_i can be achieved by the use of series resistance between the output of the feedback network and the input of the amplifier. The improvement is the result of both increased effective input resistance of the amplifier and a reduction of the percentage change of effective input resistance with h_i.

Figure 298 shows two transistor Wien-bridge oscillators. Circuit (a) is the equivalent of the vacuum-tube circuit of Fig. 296; circuit (b) is the equivalent of the tube circuit of Fig. 297b.

86.4. Desirable Features of Wien-Bridge Oscillators. An obvious advantage of the Wien-bridge oscillator and of other oscillators having resistance-capacitance frequency-determining networks is their small size, weight, and cost as compared to those of oscillators with inductance-capacitance and inductance-resistance networks. This advantage is particularly important in wide-range, low-frequency oscillators, in which the size and weight of suitable inductors may be prohibitive. The absence of tuning inductors is also desirable because inductors may intercept stray magnetic fields from power transformers and other sources. The fact that the frequency of oscillation of resistance-capacitance-tuned oscillators is proportional to $1/C$, rather than to $1/\sqrt{C}$, makes possible a 10-to-1 tuning range with a single control.

At frequencies that are high in comparison with the resonance frequency of the feedback network, the output voltage of the network has a component that is in phase opposition to the input voltage. Harmonics whose frequencies lie within the response band of the amplifier can consequently be reduced appreciably by degenerative feedback. For this reason, and because the amplitude of oscillation can be readily stabilized at a low value, the waveform of the Wien-bridge oscillator is excellent.

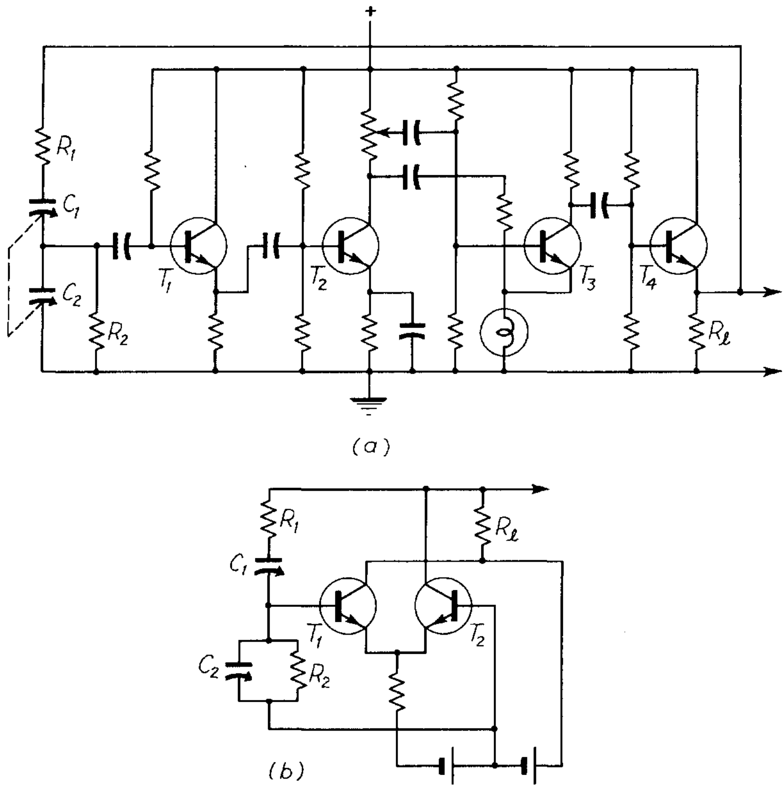


FIG. 298. Two transistor Wien-bridge oscillators.

The Wien-bridge oscillator circuit of Fig. 296 is well suited to application as a precision decade oscillator covering a wide frequency range.[5,6] A typical vacuum-tube decade oscillator of this type has the following performance characteristics: alignment maintainable to within 0.1 percent; frequency change less than 0.1 percent for a 10-percent change of line voltage; harmonic output less than 1 percent throughout most of the frequency range and less than 2 percent throughout the remainder; output level constant to

[5] Edwards, *loc. cit.*

[6] Davidson, *loc. cit.*

within 1 percent over most of the frequency range and within 5 percent over the remainder.[5] Another decade oscillator has been designed to cover a frequency range from 1 to 100,000 cps.[6]

The Wien-bridge oscillator may be frequency-modulated if R_2 is replaced by the cathode-to-ground path of a cathode-follower amplifier.[7]

87. Double-Feedback Null-Network Oscillators

If a null network is used to provide negative feedback in a high-gain amplifier, the feedback is zero at the null frequency of the network, but increases rapidly as the frequency departs from the resonance value. Consequently the amplifier has a sharp peak in gain at the resonance frequency of the network. If additional positive feedback is provided through a wideband path having an attenuation less than the maximum gain of the amplifier, oscillation takes place at the resonance frequency of the null network.[1] In order to ensure constant amplitude of oscillation throughout the tuning range, the amplifier response curve must be flat throughout this range. A Wien bridge is suitable for the negative-feedback path.[1]

A frequency-modulated double-feedback oscillator has been described in which the negative-feedback path is through a resistance-capacitance bridged-tee network of the form of Fig. 289e.[2] The frequency is varied by using as R' a balanced, tube-controlled, variable resistance incorporating nonlinear resistance. McGaughan and Leslie reported "reasonably linear frequency variation over a range from ½ to 1½ times the carrier frequency with less than 5-percent amplitude modulation and negligible harmonic distortion."

88. Phase-Shift (Ladder-Network) Oscillators

88.1. Vacuum-Tube Phase-Shift Oscillators. *Phase-shift* oscillators, two forms of which are shown in Fig. 299, consist of a single-stage amplifier and a ladder-type feedback network having three or more sections.[1] Al-

[7] C. K. Chang, *Proc. I.R.E.*, **31,** 22 (January, 1943). See also M. Artzt, *Proc. I.R.E.*, **32,** 409 (July, 1944).

[1] H. H. Scott, *Proc. I.R.E.*, **26,** 226 (1938); *Gen. Radio Expt.*, **13,** April, 1939, p. 1; **14,** January, 1940, p. 6.

[2] H. S. McGaughan and C. B. Leslie, *Proc. I.R.E.*, **35,** 974 (September, 1947).

[1] E. L. Ginzton and L. M. Hollingsworth, *Proc. I.R.E.*, **29,** 43 (February, 1941); W. W. Kunde, *Electronics*, November, 1943, p. 132; *Rad. Lab. Series*, Vol. 19, Sec. 4–4 (McGraw-Hill Book Co., Inc., New York, 1949); A. Blanchard, *Proc. I.R.E.*, **32,** 641 (October, 1944); P. G. M. Dawe and A. S. Gladwin, *Wireless Engineer*, **24,** 125 (April, 1947); W. C. Vaughan, *Wireless Engineer*, **26,** 391 (December, 1949); W. R. Hinton, *Electronic Eng.*, **22,** 13 (January, 1950); W. R. Hinton, *Wireless Engineer*, **27,** 65 (February, 1950); W. P. N. Court, *Wireless Engineer*, **27,** 65 (February, 1950); J. D. Tucker, *J. Brit. I.R.E.*, **11,** 22 (January, 1951); W. W. Holbrook, *Electronic Eng.*, **25,** 509 (December, 1953); W. Bacon, *Wireless Engineer*, **31,** 100 (April, 1954); S. Sherr, *Proc. I.R.E.*, **42,** 1169 and 1568 (July and October, 1954); D. Barbiere, *Proc. I.R.E.*, **43,** 679 (June, 1955).

though the rate of change of phase shift with frequency in this type of network is lower than in null networks, the frequency stability of phase-shift oscillators is adequate for many applications, and the distortion may be made low. The output voltage of the ladder network lags or leads the input

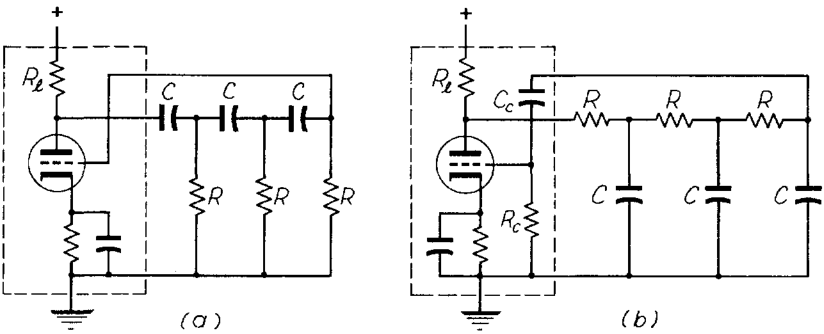


FIG. 299. Two forms of the *phase-shift* (ladder network) oscillator.

voltage by 180 degrees at some frequency. Consequently the amplifier must be a voltage-inverting amplifier, and either a single common-cathode or common-emitter stage can be used.

The vacuum-tube circuits of Fig. 299 may be analyzed by application of Eq. (152). The z parameters of the generalized ladder network of Fig. 300 are:

$$z_i = \frac{k^3 + 5k^2 + 6k + 1}{k^2 + 4k + 3} Z_2 \tag{197}$$

$$z_r = z_f = \frac{Z_2}{k^2 + 4k + 3} \tag{198}$$

$$z_o = \frac{k^3 + 5k^2 + 6k + 1}{k^2 + 3k + 1} \tag{199}$$

in which

$$k = Z_1/Z_2 \tag{200}$$

The plate load resistance R_l will be considered to be part of the amplifier, rather than of the feedback network. Then, under the assumptions that the impedance of the biasing network is negligible in comparison with R_l in the circuit of Fig. 299a, that R_c is very large in comparison with R in the circuit of Fig. 299b, and that the susceptances of the interelectrode capacitances are negligible throughout the desired frequency range, Eq. (152) for these circuits is:

$$g_m = -(k^3 + 5k^2 + 6k + 1)(1/r_p + 1/R_l) - (k^2 + 4k + 3)/Z_2 \tag{201}$$

The frequency of oscillation and the criterion for equilibrium oscillation are obtained by substituting the appropriate values for k and Z_2 into Eq. (201) and separating the real and imaginary terms. This procedure leads to the following expressions for the circuit of Fig. 299a (Prob. 88.1-1):

$$f = \frac{1}{2\pi RC\sqrt{6 + 4/R(1/r_p + 1/R_l)}} = \frac{0.065}{RC}\,\frac{1}{\sqrt{1 + 2/3R(1/r_p + 1/R_l)}} \tag{202}$$

$$g_m = 29\left(\frac{1}{r_p} + \frac{1}{R_l}\right) + \left[23 + \frac{4}{R(1/r_p + 1/R_l)}\right]\frac{1}{R} \tag{203}$$

The corresponding relations for the oscillator of Fig. 299b are (Prob. 88.1-2):

$$f = \frac{0.39}{RC}\sqrt{\frac{\dfrac{1}{r_p} + \dfrac{1}{R_l} + \dfrac{1}{2R}}{\dfrac{1}{r_p} + \dfrac{1}{R_l} + \dfrac{1}{R}}} \tag{204}$$

$$g_m = 29\left[\frac{1}{r_p} + \frac{1}{R_l} + \frac{24}{29R}\right]\frac{\dfrac{1}{r_p} + \dfrac{1}{R_l} + \dfrac{1}{2R}}{\dfrac{1}{r_p} + \dfrac{1}{R_l} + \dfrac{1}{R}} \approx 29\left(\frac{1}{r_p} + \frac{1}{R_l} + \frac{1}{2R}\right) \tag{205}$$

88.2. Stability of Vacuum-Tube Phase-Shift Oscillator. It is apparent from Eqs. (202) to (205) that both the frequency and the amplitude of oscillation of the circuits of Fig. 299 are to some extent dependent upon r_p and therefore upon the operating voltages of the tube. It can also be seen that the frequency and amplitude stabilities may be improved to some extent by making R_l smaller than r_p. However, the highest values of amplification factor of available tubes are such that Eqs. (203) and (205) are not satisfied if R_l is less than $r_p/2$ to $r_p/3$. Although the use of a common-grid stage following the common-cathode amplifier in the circuits of Fig. 299 would increase both the output resistance and the voltage amplification while preserving the proper phase relation between the amplifier input and output voltages, biasing difficulties make this method of obtaining a high ratio of r_p to R_l unfeasible.

An alternative method of increasing the frequency stability against changes of r_p is to make $R \gg r_pR_l/(r_p + R_l)$. Equations (202) and (204) then reduce to

$$f \approx 1/2\pi\sqrt{6}\,RC = 0.065/RC \qquad \text{(Fig. 299a)} \tag{206}$$

$$f = \sqrt{6}/2\pi RC = 0.39/RC \qquad \text{(Fig. 299b)} \tag{207}$$

and the criterion for sustained oscillation becomes, for both circuits,

$$\frac{g_m r_p R_l}{r_p + R_l} \approx 29 \tag{208}$$

Equation (208) states that, if R is large enough so that the loading effect of the feedback network upon the amplifier is negligible, the amplitude of oscillation will build up to such a value that the voltage amplification of the amplifier approximates 29. The zero-amplitude voltage amplification must evidently exceed this value, which is approximately ten times that required in the Wien-bridge oscillator.

The extent to which the frequency may be raised by reducing C is limited by the effects of tube and other stray shunt capacitances. For this reason, very high frequencies can be obtained only by reducing R also. Consequently, if frequency stability is important, R_l must be correspondingly reduced. The extent to which R_l can be reduced is limited by the necessity of satisfying Eq. (208). It is apparent from this equation that a high ratio of R to $r_pR_l/(r_p + R_l)$ can be obtained more readily with a tube of specified g_m if r_p is small, and that triodes are therefore preferable to pentodes in phase-shift oscillators if frequency stability is important.

It is of interest to note that the frequency dependence upon r_p is caused by the susceptive component of the input admittance of the phase-shifting network, which produces a susceptive component of transfer admittance at the frequency given by Eq. (202) or (204). Alternatively, one may say that the loading of the amplifier tube by the bridge input susceptance causes the amplifier phase shift to depart from 180 degrees. Since the frequencies given by Eqs. (206) and (207) are those at which the ladder-network phase shift is 180 degrees, the frequencies must differ sufficiently from these values to make the network phase shift equal to the amplifier phase shift.

Loading of the amplifier by the feedback network can be greatly reduced by the use of a cathode-follower stage between the amplifier plate and the input to the network. The effect of the circuit change upon Eqs. (202) to (205) is to replace $1/r_p$ by $(\mu + 1)/r_p$, and g_m by g_mA, where μ, r_p, and g_m now refer to the cathode-follower tube, and A is the voltage amplification of the amplifier stage. The requirement for good frequency stability then becomes that $R \gg r_pR_l/[r_p + (\mu + 1)R_l]$. This requirement is much easier to satisfy than that $R \gg r_pR_l/(r_p + R_l)$.

88.3. Use of Modified Ladder Networks. In the ladder network of Fig. 300, in which the three sections are alike, the impedance that shunts Z_2 in the first and second sections is of the same order of magnitude as Z_2. If the impedance of the second section is made higher than that of the first, and the impedance of the third section higher than that of the second, the loading of one section by the following sections is reduced. This has the desirable effect of increasing Z_i and Z_f. The increase of Z_i reduces the loading of the amplifier by the network and therefore increases the frequency stability of the oscillator; the increase of Z_f reduces the required amplification below the minimum of 29 required when all sections are alike.[2] As the impedance ratio of succeeding sections is made increasingly large, the required value of amplification approaches the limiting value of 8. (The factor 29 is replaced by 8 in Eq. (208).) The reduction in the required value of amplification allows the use of a lower value of R_l. The upper limit of the tuning range may consequently be raised by the use of smaller R without excessive frequency instability. The lower value of R_l also raises the frequency at which shunt tube capacitance lowers the amplification and thus the amplitude of oscillation.

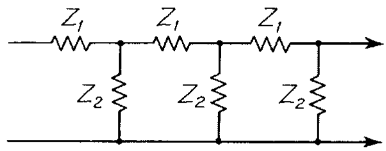


FIG. 300. Generalized ladder network.

The maximum value of the impedance ratio of succeeding sections of the ladder network is limited in practice because the maximum impedance of the final section is limited by the allowable grid-circuit resistance of the tube, and the input impedance of the first section must be large in comparison with $r_pR_l/(r_p + R_l)$. By the use of an impedance ratio of 5, a frequency range from 1 cps to 1 Mc with good frequency and amplitude stability can be readily attained.[3]

The principal objection to the use of an impedance ratio greater than unity between sections is the greater difficulty of ganging dissimilar variable resistors or capacitors. Although the required amplification can also be reduced by the use of four network sections, instead of three, this alternative also results in increased tuning complication. A third method of reducing the required amplification is to use three amplifier stages, one between the output of each network section and the input of the next, instead of a single amplifier. The frequency and the criterion for oscillation of an oscillator of this type in which the ladder network consists of three equal series capacitances C and shunt resistances R are (see Prob. 88.3–1, part (a)):

$$f = \frac{0.092}{RC} \frac{1/r_p + 1/R_l}{1/r_p + 1/R_l + 1/R} \tag{209}$$

$$g_m = 2(1/r_p + 1/R_l + 1/R) \tag{210}$$

[2] R. W. Johnson, *Proc. I.R.E.*, **33**, 597 (1945); P. G. Sulzer, *Proc. I.R.E.*, **36**, 1302 (October, 1948); P. W. Ward, *Electronic Eng.*, **26**, 318 (July, 1954).

[3] Johnson, *loc. cit.*

Considerably better frequency stability is obtained if the required amplification is obtained in a single common-cathode amplifier and a cathode-follower stage is used to drive each section of the ladder network, as shown in Fig. 301.[4] The frequency of oscillation and criterion for sustained oscillation are then (Prob. 88.3–2, part (a)):

$$f = \frac{0.092}{RC}\,\frac{g_m + 1/r_p + 1/R_k}{g_m + 1/r_p + 1/R_k + 1/R} \tag{211}$$

$$A = 8\left[\frac{g_m + 1/r_p + 1/R_k + 1/R}{g_m}\right]^3 \tag{212}$$

in which A is the amplification of the common-cathode amplifier stage and g_m, r_p, and R_k refer to the cathode-follower stages. Equations (209) to (212) hold for analogous shunt-capacitor phase-shift oscillators if the factor 0.092 is replaced by 0.276 (Probs. 88.3–1 and 88.3–2, parts (b)).

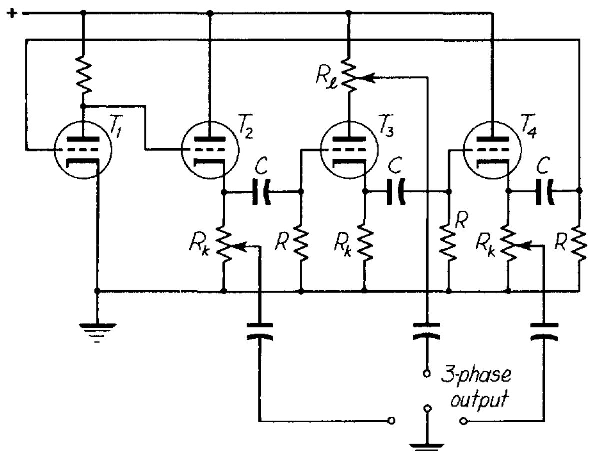


Fig. 301. Phase-shift oscillator with cathode-follower-coupled ladder network.

Three-phase output can be obtained from the circuit of Fig. 301, as shown. In a similar manner, polyphase output may be obtained from circuits using a larger number of phase-shift sections. Polyphase output may also be obtained from a phase-shift oscillator having N identical R-C phase shifters

[4] H. J. Reich, *Proc. I.R.E.*, **43**, 229 (February, 1955); L. Fleming, *Proc. I.R.E.*, **43**, 754 (June, 1955); J. L. Stewart and K. S. Watkins, *Proc. I.R.E.*, **43**, 1527 (October, 1955).

coupled by single-stage, common-cathode amplifiers.[5] To minimize dependence of frequency upon plate-supply voltage, it is desirable to follow each phase-inverting amplifier by a cathode-follower amplifier.

A modified form of the Barrett circuit covering a frequency range from 0.01 to 1000 cps has been developed by Smiley.[6] In this circuit use is made of the Miller effect in order to reduce the size of the capacitors required to achieve these low frequencies. Need for very large coupling capacitors is avoided by the use of glow-tube coupling and cathode-follower coupling. Amplitude is limited by means of diodes.

88.4. Tuning of Phase-Shift Oscillators. The frequency of oscillation of phase-shift oscillators may be varied continuously by means of either ganged resistors or ganged capacitors, but capacitor tuning is preferable in oscillators designed to cover a wide frequency range. Equations (202) to (212) show that resistance tuning is accompanied by an increased dependence of frequency upon plate resistance and an increase in the required value of g_m as the frequency is increased. Since g_m must be large enough to satisfy the criterion for sustained oscillation at the highest frequency in the tuning range, the amplitude of oscillation increases as the frequency is decreased by increasing R. This problem is not serious, however, when the phase-shift network is driven by a cathode-follower stage or when continuous tuning is confined to a range of the order of 10 to 1. An additional disadvantage of resistor tuning, however, is the greater likelihood of noise as the result of poor contacts. Resistor tuning is preferable at very low frequencies because of the relatively high cost of large variable capacitors. With either type of tuning, frequency ranges may be selected by changing the magnitudes of the fixed elements of the feedback network.

Because the frequency of the shunt-capacitor circuit is approximately six times that of the shunt-resistor circuit for the same value of RC, the shunt-resistor circuit is preferable at low frequencies and the shunt-capacitor circuit at high frequencies. An advantage of the shunt-capacitor circuit at high frequencies is that the input capacitance of the tube is in shunt with the capacitance of the output section of the ladder network and therefore does not produce an adverse effect upon the phase shift. Another advantage that is important when frequency is varied by means of ganged capacitors is that the three capacitors have a common circuit point, which is at ground potential. The filtering action of the networks causes lower harmonic distortion to be obtained in the shunt-resistor circuit when the output is taken from the plate of the tube, and in the shunt-capacitor circuit when it is taken from the grid end of the ladder network. Although the second-

[5] R. M. Barrett, *Proc. I.R.E.*, **33**, 541 (August, 1945).

[6] G. Smiley, *Proc. I.R.E.*, **42**, 677 (April, 1954). See also M. D. Armitage, *Wireless Engineer*, **32**, 173 (June, 1955).

harmonic distortion in the shunt-capacitor circuit is somewhat lower than in the shunt-resistor circuit, the over-all distortion is greater.[7]

88.5. Transistor Phase-Shift Oscillators. In the analysis of transistor phase-shift oscillators, the input and reverse short-circuit transfer admittances of the amplifier cannot be neglected and Eq. (152) is not applicable. The analysis and the resulting equations for the frequency of oscillation and the criterion for sustained oscillation are considerably more complicated than those for the vacuum-tube phase-shift oscillator and for the transistor Wien-bridge oscillator.[8] The analysis shows, however, that the frequency and amplitude of oscillation are dependent upon h_i and h_o, and that good frequency stability requires that $R_l/(h_oR_l + 1) \ll R \gg h_i$. This requirement cannot be met without the use of a common-collector stage to drive the feedback network and preferably also another common-collector stage or a compound-connected transistor between the output of the network and the input of the common-emitter stage. When the common-collector stages are used, the frequency of oscillation is approximated by Eqs. (206) and (207) for the series-capacitance and shunt-capacitance circuits, respectively, and the criterion for sustained oscillation is again that the voltage amplification of the entire amplifier must exceed 29.

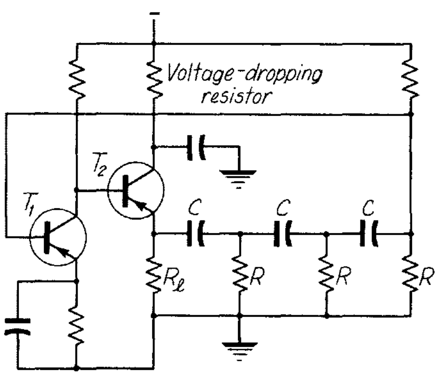


FIG. 302. Transistor phase-shift oscillator in which a common-collector stage is used to drive the feedback network.

A transistor phase-shift oscillator incorporating a common-collector stage to drive the feedback network is shown in Fig. 302.[9] A transistor circuit equivalent to the tube circuit of Fig. 301 is shown in Fig. 303. The frequency of oscillation of this circuit approximates $0.092/RC$, where $R = R'R''/(R' + R'')$. The voltage amplification of the common-emitter amplifier must be somewhat greater than 8.

Transistor phase-shift oscillators should be stabilized against temperature changes by the use of series emitter resistance or by more involved methods equivalent to those used in the circuits of Fig. 282 and 288. In some circuits, of which Fig. 303 is an example, the tuning resistors may also serve as stabilizing resistors.

[7] Chance, *et al.*, Rad. Lab. Series, Vol. 19, Sec. 4-4 (McGraw-Hill Book Co., New York, 1949).

[8] See, for example, L. J. Giacoletto, *Trans. I.R.E.*, Vol. AU-5, 60 (May–June, 1957).

[9] Giacoletto, *loc. cit.*

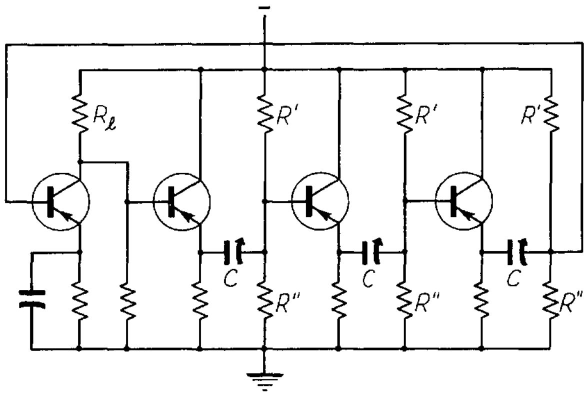


FIG. 303. Transistor phase-shift oscillator with emitter-follower-coupled ladder network

88.6. Two-Section Phase-Shift Oscillator. An interesting modification of the phase-shift oscillator is obtained by using a two-section network consisting of one R-C section and one C-R section. The phase shift produced in the second section is then equal and opposite to that produced in the first section, and the net phase shift is consequently zero. Oscillation takes place if such a network is used to couple the output of a noninverting pentode or cathode-coupled (common-cathode, common-grid) amplifier to the input, as shown in Fig. 304a. If $R_1 = R_2 = R$, $C_1 = C_2 = C$, and $R \gg r_pR_l/(r_p + R_l)$, the frequency of oscillation is very nearly $1/2\pi RC$, and the zero-amplitude voltage amplification of the amplifier must exceed 3 (Prob. 88.6–1). An equivalent variant of the circuit is obtained by interchanging the R-C and C-R sections of the network. Loading of the first section of the network by the second may be avoided by inserting one section into each grid

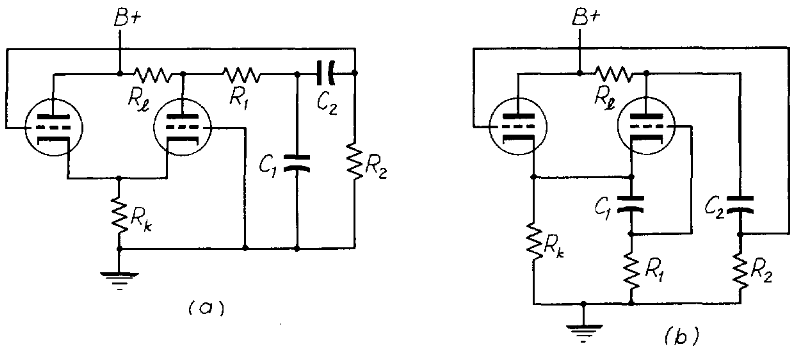


FIG. 304. Phase-shift oscillators using a two-section feedback network.

circuit, as in Fig. 304b. The required amplification for such a circuit in which equal resistances and equal capacitances are used is reduced in this manner from 3 to 2. A slightly more complicated circuit similar to that of Fig. 304b, but with the R-C and C-R sections interchanged, was described by J. H. Owens and analyzed from a different point of view.[10]

A modified form of ladder-network phase-shift oscillator using a balanced two-tube (push-pull) circuit has been described by D. A. Bell.[11] The advantage of such a circuit is very low even-harmonic distortion.

88.7. Other Types of Phase-Shift Oscillators. Another type of phase-shift oscillator is based upon the phase-shifting network shown in Fig. 305. If $R_2 = R_3$ and the impedance of the capacitance branch of the network is high in comparison with $R_1 + R_2$, the magnitude of the output voltage remains essentially constant and equal to the voltage across R_2 or R_3 as either C or R is increased from zero to infinity, but the phase of the output voltage changes from 180 degrees lead to zero. The oscillator, shown in Fig. 306a, uses two similar phase-shifting stages, each of which gives a phase shift of 90 degrees when $f = 1/2\pi RC$ [12] (see Prob. 88.7–1). In each phase-shifting stage, R_l and R_k correspond to R_2 and R_3 of Fig. 305, the a-c plate resistance of the tube corresponds to R_1, and μV_g corresponds to V_i. The resistors R_l and R_k should preferably be matched. The frequency is changed over a 10-to-1 range by either ganged resistors R or ganged capacitors C.

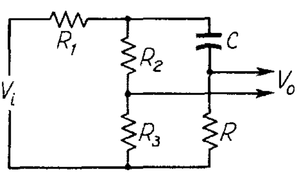


FIG. 305. All-pass phase-shift network.

Figure 306b shows a circuit that incorporates a more complicated phase-shifting network which produces a phase shift of 180 degrees and has a voltage ratio of approximately unity.[13] At the oscillation frequency, $1/2\pi\sqrt{C_1R_1C_2R_2}$, a 90-degree phase shift occurs in the R_1-C_1 bridge, and an additional 90-degree shift in the R_2-C_2 voltage divider. If R_2 and C_2 are interchanged, the R_2-C_2 phase shift is opposite to the R_1-C_1 shift, and the output voltage of the network is in phase with the grid voltage of tube 1. Oscillation is then obtained if the triode T_2 is replaced by a noninverting (voltage) amplifier, such as a cathode-coupled (common-cathode, common-grid) or a pentode noninverting amplifier.

Willoner and Tihelke have described a phase-shift oscillator in which the

[10] J. H. Owens, *Electronics*, March, 1954, p. 176.

[11] D. A. Bell, *Electronic Eng.*, **23**, 274 (July, 1951).

[12] O. G. Villard, Jr., *Electronics*, July, 1949, p. 77.

[13] P. Kundo, *J. Brit. I.R.E.*, **11**, 233 (June, 1951); **12**, 393 (July, 1952); H. Stibbe, *J. Brit. I.R.E.*, **12**, 392 (July, 1952).

phase-shifting network is a resistance-shunted ladder type of all-pass filter. The various sections of the filter are chosen so that each section functions over a portion of the entire frequency range. This oscillator has the advantage that it is tuned by means of a single tapped potentiometer, but it is basically more complicated than other oscillators discussed in this and preceding sections. For further details the reader is referred to the paper by Willoner and Tihelke.[14]

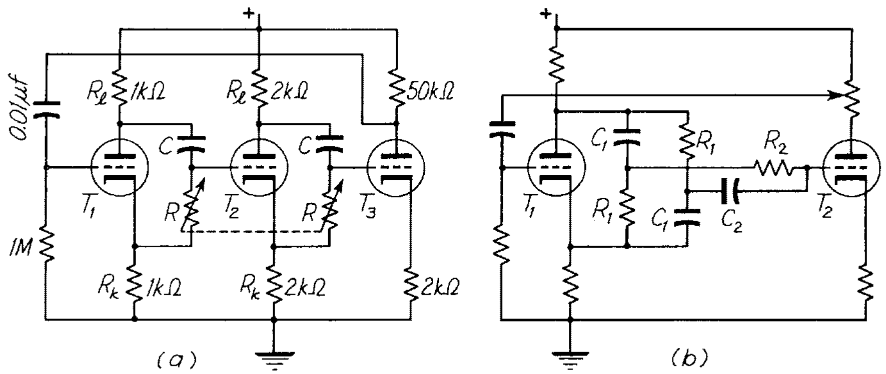


FIG. 306. (a) Oscillator based upon the phase-shift network of Fig. 305; (b) another resistance-capacitance-tuned oscillator.

88.8. Frequency-Modulation of Phase-Shift Oscillators. A number of methods have been developed for frequency modulation of phase-shift oscillators. A frequency-modulated oscillator capable of a frequency deviation of 40 percent of the carrier frequency with negligible amplitude modulation in the audio-frequency range is shown in Fig. 307.[15] In this circuit, tube T_1 provides the required amplification, T_2 is a cathode-follower used to prevent loading of the amplifier by the network, and T_3 and T_4 modulate the frequency by changing the effective shunt resistance of one section of the filter. The resistance R_2 is made sufficiently larger than the other shunt resistances of the phase-shifting network so that the network attenuation is approximately the same with R_2 open-circuited and grounded. When the control-signal voltage is zero, the plate current of T_3 and T_4 is high, and their plate resistance low. So far as alternating voltage is concerned, therefore, R_2 is connected to ground through a low resistance, and the oscillator frequency is high. When the control voltage is made negative, the plate current of T_3 and T_4 is reduced and the plate resistance of the tubes is in-

[14] G. Willoner and F. Tihelke, *Proc. I.R.E.*, **36**, 1096 (September, 1948).
[15] M. Artzt, *Proc. I.R.E.*, **32**, 409 (July, 1944).

creased. The oscillation frequency consequently falls, reaching its lowest value when T_3 and T_4 are cut off and their a-c resistance is infinite. Examination of the circuit discloses that T_4 serves as a cathode follower with T_3 as its load resistance. Since the grid of T_4 is connected to a fixed point on the voltage divider between $B+$ and ground, cathode-follower action tends to maintain the cathode of this tube at constant potential relative to

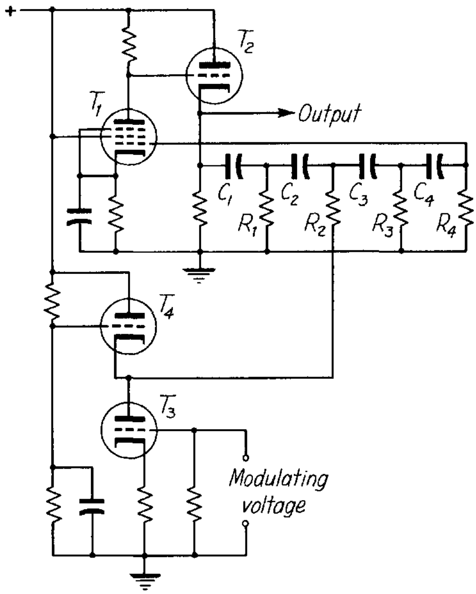


FIG. 307. Frequency-modulated phase-shift oscillator.

ground. Hence very little modulation-frequency voltage is introduced into the oscillator through R_2 and amplitude modulation of the oscillator is negligible if the operating bias of T_3 is properly chosen and the grid swing of T_3 is limited to the range in which the network attenuation remains essentially constant.

Figure 308 shows typical curves of frequency vs. grid voltage of T_3. Curve A was obtained by varying the effective value of R_1, whereas curve B was obtained by varying the value of R_2. When both R_1 and R_2 are connected to the cathode of T_4, the curves are similar, but extend over a wider frequency range. These curves show that linear frequency modulation over a wide range is possible. Corresponding curves of network attenuation vs. grid voltage of T_3 have a broad minimum occurring near the center of the linear range. Amplitude modulation is therefore small. The time taken for the frequency to respond to a change in grid voltage was found by Artzt to be negligible in comparison with the oscillation period. Useful design curves and procedures are given in the original paper.

Another form of phase-shift oscillator that can be tuned electronically over a 100-to-1 frequency range and that is capable of linear frequency modulation over a wide range is shown in basic form in Fig. 309.[16] In this oscillator the phase-shift network consists of four cathode-followers with capacitive load. Analysis of the equivalent plate circuit for one stage of the phase-shift network yields the following expression for the ratio of the alternating output (cathode-to-ground) voltage V_o to the alternating input (grid-to-ground) voltage V_i of one stage under the assumptions that the effects of interelectrode capacitances are negligible and that $g_m \gg 1/R$ (Prob. 88.8–1):

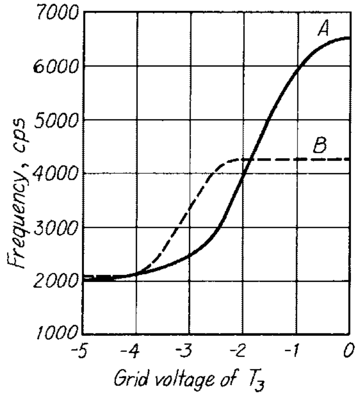


FIG. 308. Frequency-modulation characteristics for the circuit of Fig. 307. For curve A, T_3 was a 6SJ7, T_4 a 6J5, and the cathode of T_4 was connected to R_1. For curve B, T_3 and T_4 were 6C8G tubes and the circuit was connected as in Fig. 307.

$$\frac{V_o}{V_i} = \frac{\dfrac{1+\mu}{\mu} - \dfrac{j\omega C}{g_m}}{\left(\dfrac{1+\mu}{\mu}\right)^2 + \dfrac{\omega^2 C^2}{g_m{}^2}} \tag{213}$$

Because the network phase shift must be 180 degrees throughout the frequency range in which the amplification of the voltage amplifier is constant, the phase shift per stage is 45 degrees if g_m and μ are the same in all stages. Separating the real and imaginary components of Eq. (213) gives the following expression for the frequency of oscillation:

$$f = \frac{g_m}{2\pi C} \frac{\mu + 1}{\mu} \tag{214}$$

Under the assumption that the amplification factor μ is constant, therefore, the frequency of oscillation is directly proportional to g_m. Under the same assumptions, Eq. (213) indicates that the magnitude of the voltage ratio

[16] M. E. Ames, *Electronics*, Vol. 22, p. 96, May, 1949.

V_o/V_i has the constant value given by the following relation:

$$\frac{V_o}{V_i} = \frac{\mu}{(\mu + 1)\sqrt{2}} \tag{215}$$

Consequently, the amplitude of oscillation does not vary as the frequency is varied. The zero-amplitude amplification of the amplifier stage must exceed $(V_i/V_o)^2$, or somewhat more than 4.

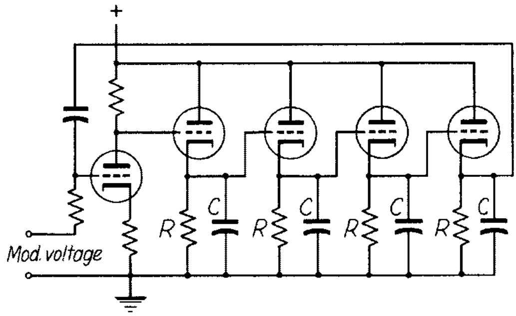


FIG. 309. Voltage-tuned phase-shift oscillator.

In the circuit of Fig. 309 the plate resistance of the cathode-followers is varied by changing grid-bias voltage. The modulation frequency is ordinarily so much smaller than the oscillation frequency that the quantity $\omega C/g_m$ is negligible in comparison with $(1 + \mu)/\mu$ at the modulation frequency and V_o/V_i approximates unity if μ is large. The variation of grid bias of the four cathode-follower stages is therefore nearly equal. However, the fact that the amplification is not exactly unity and that the variation of grid bias of each successive cathode-follower is less than that of the preceding one, means that the phase shifts of the four stages can be equal at only one frequency. As a consequence, the quantity $\omega C r_p(1 + \mu)$ varies in Eq. 213 as the oscillator frequency is changed, and frequency modulation is accompanied by some amplitude modulation. This difficulty is eliminated by replacing the cathode resistors in the circuit of Fig. 309 by triodes and applying the frequency-control voltage to the grids of these tubes, as shown in Fig. 310. The change in grid voltage of the control tubes results in a change in plate current of the cathode-follower tubes and thus in a change of their plate resistance. The increase of control-tube plate resistance with cathode-follower plate resistance makes it possible to satisfy the requirement that the transconductance of the cathode-follower be large in comparison with the cathode-follower load conductance at much lower frequency than in the circuit of Fig. 309.

The small curvature of the frequency-modulation characteristic of this type of oscillator is shown by the experimental curve of Fig. 311. Equation (215) shows that the required amplification of the voltage-amplifier stage

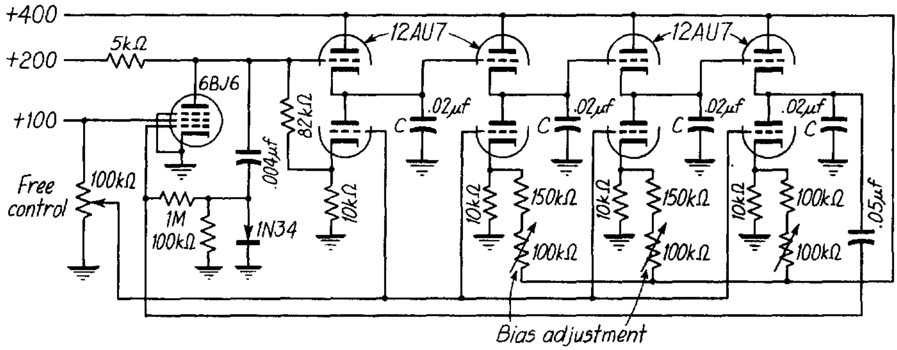


FIG. 310. Modified form of the circuit of Fig. 309 in which the cathode resistors are replaced by triodes in order to reduce amplitude modulation.

of the circuits of Figs. 309 and 310 is only slightly greater than 4 when μ is large in comparison with unity. This low value of amplification is readily attainable over a wide range of frequency. As the frequency is reduced by lowering g_m, however, a point is reached beyond which $1/R$ is no longer negligible in comparison with g_m, as assumed in the derivation of Eq. (213). The magnitude of the voltage ratio is then appreciably smaller than the value given by Eq. (215), and the required amplification of the amplifier stage greater. The amplitude of oscillation therefore falls with decrease of g_m, until oscillation ceases. The constant-amplitude range can be increased by making the low-frequency amplification of the amplifier stage considerably greater than 4 and reducing the amplification as the frequency is raised. This may be accomplished by using the output impedance of a cathode-follower in shunt with the plate load resistor of the amplifier stage and varying the grid voltage of the cathode-follower concomitantly with that of the control tubes.[17] At high frequencies, interelectrode capacitances, neglected in the foregoing analysis, cause the oscillation frequency to depart from the value given by Eq. (214).

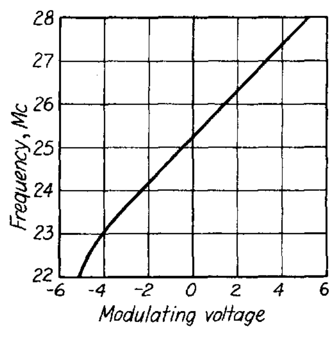


FIG. 311. Frequency-modulation characteristic for the circuit of Fig. 309.

[17] A. Cormack, *Wireless Engineer*, **28**, 266 (September, 1951).

Another method of frequency-modulating, phase-shift oscillators is shown by the circuit of Fig. 312, in which the input to the phase-shifting network is shunted by a reactance tube.[18] The required 90-degree phase relation between the grid and plate voltages of the reactance tube is obtained by taking the alternating grid voltage from the midpoint of the four-section phase-shifting network. Dennis and Felch reported linear modulation over

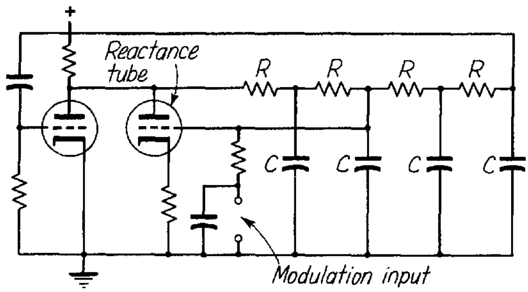


FIG. 312. Phase-shift oscillator tuned by means of a reactance tube (c.f. Fig. 267a).

a ±9-Mc band at 80 Mc and over a ±3-Mc band at 25 Mc. Although the tubes are shown as triodes in Fig. 312 for the sake of simplification, pentodes are more suitable because of their high transconductance.

89. Sum- and Difference-Circuit Oscillators

89.1. "Seven-League" Oscillator. A number of useful oscillators incorporate a sum or difference circuit as one element of the phase-shifting circuit. One such oscillator is the "seven-league" oscillator, which is based upon the circuit of Fig. 313.[1] If the R-C network in the grid circuit of tube T_1 contains a number of parallel R-C sections and the network parameters are properly chosen, the reactive component of the voltage V_a of point A relative to ground is nearly constant over a wide frequency range, but the resistive component increases with freqency. The voltage V_b of point B relative to ground is purely resistive and at any frequency may be made equal to the resistive component of V_a. The difference between V_a and V_b is then the constant reactive component of V_a. Conversely, for any setting of the voltage divider, there is one frequency at which $V_a - V_b$ leads V_i by 90 degrees, and the magnitude of $V_a - V_b$ at this frequency is nearly independent of the voltage-divider setting. Since tubes T_1 and T_2 and their associated plate and cathode resistances form a differential amplifier (Sec. 2), the plate voltage of T_1 at the balance frequency leads V_i by 90 degrees and that of T_2 lags V_i by 90 degrees.

[18] F. R. Dennis and E. P. Felch, *Bell Sys. Tech. J.*, **28**, 601 (October, 1949).
[1] F. B. Anderson, *Proc. I.R.E.*, **39**, 881 (August, 1951).

The seven-league oscillator consists of two stages of the form of Fig. 313 with ganged voltage dividers. The output of *tube 1* of the first stage is capacitance-coupled to the input of the second stage, and the output of *tube 2* of the second stage is capacitance-coupled to the input of the first stage. The output of the two stages is taken from opposite tubes in order that the 90-degree lead of the output voltage of the first stage relative to its input voltage is offset by the 90-degree lag of the output voltage of the second stage relative to its input voltage at the balance frequency. At this frequency, therefore, the total phase shift around the loop is zero, and oscillation takes place if the amplification of the differential amplifiers equals or exceeds the attenuation of the R-C networks. The frequency is determined by the setting of the ganged voltage dividers.

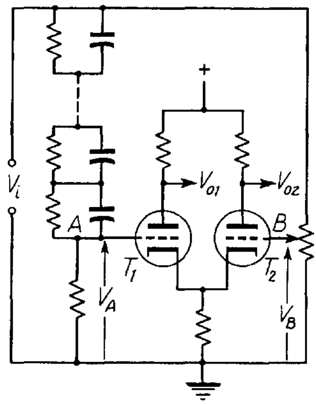


FIG. 313. Phase-shifting circuit used in the "seven-league" oscillator.

An experimental oscillator described by Anderson covered a frequency range from 20 cps to 3 Mc. The frequency variation with tube gain was 2 percent per decibel and the setting accuracy was 1 percent in a thermistor-stabilized circuit. The frequency varied logarithmically with the balancing voltage V_b. The frequency range increases with the number of parallel R-C sections used in the phase-shift network. Tentative frequency limits obtainable in practical circuits were given as 0.01 cps and 10 Mc. For details of the network analysis and design and other design details, the reader is referred to Anderson's article.

89.2. De Lange Circuit. Figure 314 shows the block diagram of an oscillator that incorporates an adding circuit. The parameters of the two phase-shifting circuits are such as to produce opposite phase shifts θ_1 and θ_2, one of which increases with frequency and the other decreases. When k_1 equals k_2, there is a frequency at which $\theta_1 = \theta_2$, and the sum of the voltages $k_1V\epsilon^{j\theta_1}$ and $k_2V\epsilon^{j\theta_2}$ is in phase with V, as shown in the phasor diagram (a) of Fig. 315. If the amplification of the amplifier is equal to the attenuation of phase shifters, voltage dividers, and the adding circuit, oscillation will take place at this frequency.

If k_2 exceeds k_1, there will be some other frequency, corresponding to a larger value of θ_1 and a smaller value of θ_2, at which the sum of V_1 and V_2 is again in phase with V, as depicted in diagram (b) of Fig. 315. Oscillation will take place at this new frequency if the circuit attenuation does not exceed the amplifier amplification. Similarly, increase of k_1 and decrease of k_2 results in an opposite change of frequency. For a properly chosen ratio of k_2 to k_1, the magnitude of the voltage sum V' will be the same as for equal k_2 and k_1 and therefore the amplitude of oscillation will be the same. The

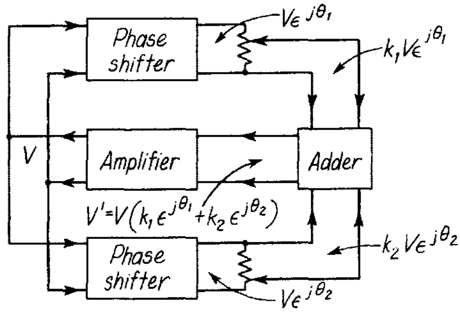


FIG. 314. Schematic diagram of a feedback oscillator in which the required phase shift is obtained with the aid of an adding circuit.

frequency of oscillation can then be varied over a wide range by simultaneous adjustment of the potentiometers P_1 and P_2. Alternatively, if the adding circuit consists of two triodes or pentodes with a common load resistor, the frequency can be varied by raising the bias of one tube and lowering that of the other. Wideband frequency modulation may be accomplished with the aid of a phase-inverter stage to convert a single-sided modulation voltage into a push-pull voltage.

The proper phase-shift characteristics can be obtained by the use of a series R-L voltage divider for one phase shifter and a series R-C voltage

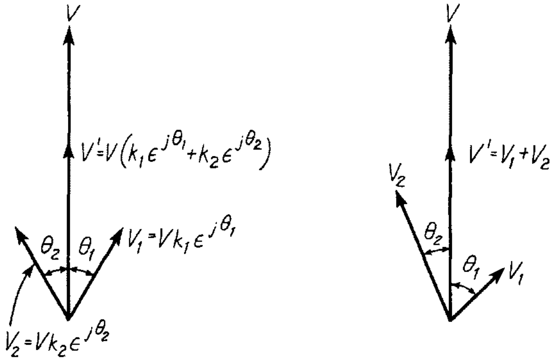


FIG. 315. Phasor diagrams for the circuit of Fig. 314.

divider for the other phase shifter.[2] The load impedance presented to the amplifier by the two phase shifters is nonreactive and has the value R at all frequencies if the resistances both have the value R, equal to $\sqrt{L/C}$, where L is the inductance of the R-L circuit, and C is the capacitance of the R-C circuit. The frequency of oscillation is then $1/2\pi\sqrt{LC}$, when $k_1 = k_2$. For a 65-Mc frequency-modulation circuit, De Lange reported a frequency deviation of ± 2 Mc, small amplitude deviation, second-harmonic amplitude 32 db below the fundamental, and third-harmonic amplitude 37 db below the fundamental.

89.3. Villard-Holman Oscillators. The necessary phase relation between the adding-circuit input voltages in Fig. 314 can also be obtained by

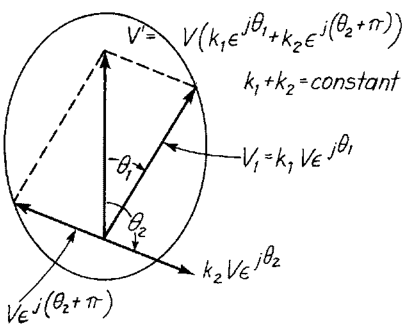


FIG. 316. Phasor diagram that suggests another method of obtaining the required phase of the feedback voltage.

the use of similar phase-shift circuits of different time constants and a phase inverter to provide an additional 180-degree shift of one voltage. The phasor diagram of Fig. 316 shows that the desired phase angle between V and V_2 can be obtained by means of a phase shift θ_2 of the same sign as θ_1, but of greater magnitude, and an additional 180-degree shift.[3] The resultant phasor V' is of constant magnitude if $k_1 + k_2$ is constant. The locus of the ends of the phasors V_1 and V_2 is then an ellipse, as shown in Fig. 316. The circuit of an oscillator based upon this principle is shown in Fig. 317.[2] The phase-shift networks used in this oscillator are of the same form as one stage of the phase-shift network of Fig. 306a, which is based upon the all-pass R-C network of Fig. 305. The phase shift θ_2 is made larger than θ_1 by making C_2 larger than C_1, and the additional 180-degree phase shift is obtained

[2] O. E. De Lange, *Proc. I.R.E.*, **37**, 1328 (November, 1949).

[3] O. G. Villard and F. S. Holman, *Proc. I.R.E.*, **21**, 368 (March, 1953); J. L. Stewart, *Proc. I.R.E.*, **43**, 589 (May, 1955).

by taking the input of T_4 from the plate of T_1, rather than directly from the junction of R_1 and C_1. The 180-degree phase shift in the amplifier T_5 is offset by the 180-degree shift in T_1 and T_2. If the circuit of Fig. 317 is to be used as a tunable oscillator, rather than as an FM circuit, the mixer tubes T_3 and T_4 may be replaced by triodes and the frequency controlled by means

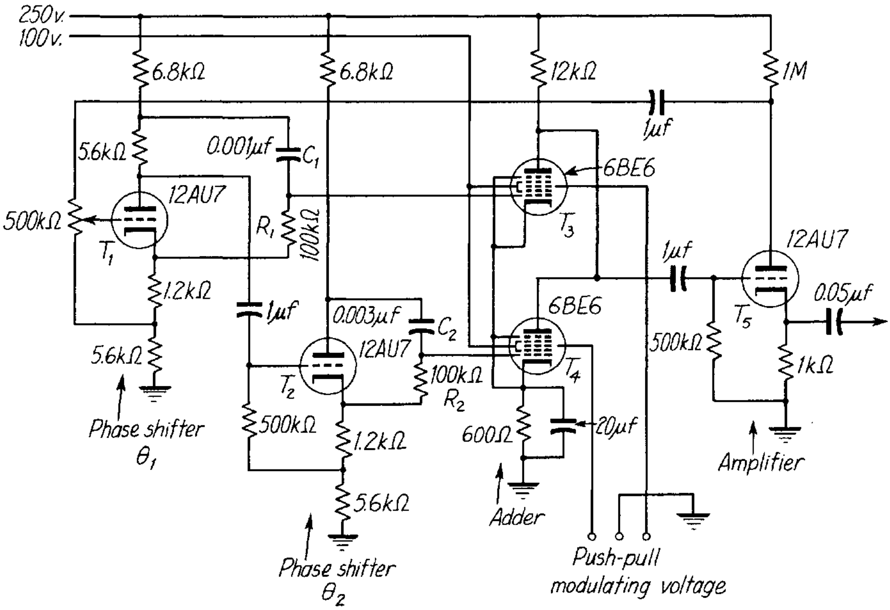


FIG. 317. Oscillator based upon the phasor diagram of Fig. 316.

of ganged potentiometers, as in the circuit of Fig. 314. Although an ideal oscillator of this type is theoretically tunable from zero to infinite frequency, Villard and Holman reported that bandwidth limitations on the amplifier and other circuit elements limit the practical frequency ratio to about four. Specific performance figures were not given.

Another interesting type of oscillator that incorporates either an adding circuit or a subtracting circuit is shown in Fig. 318.[4] Phasor diagrams for this oscillator are shown in Fig. 319. It can be seen from diagram (a) that if the voltage V is shifted in phase through an angle θ smaller than 180 degrees and a voltage $kV\epsilon^{j\theta/2}$ is subtracted from the shifted voltage $V\epsilon^{j\theta}$, the resultant is equal to $-V$ if k is properly chosen. Similarly, if θ exceeds 180

[4] Villard and Holman, *loc. cit.*

degrees, the resultant voltage is $-V$ if a voltage $kV\epsilon^{j\theta/2}$ of proper magnitude is added to the voltage $V\epsilon^{j\theta}$. Since θ is a function of frequency, the value of k required to make the resultant equal to $-V$ is also a function of frequency. Conversely, for each value of k there is one freouency at which $V\epsilon^{j\theta} \pm kV\epsilon^{j\theta/2} = -V$. Inasmuch as the amplifier provides an additional

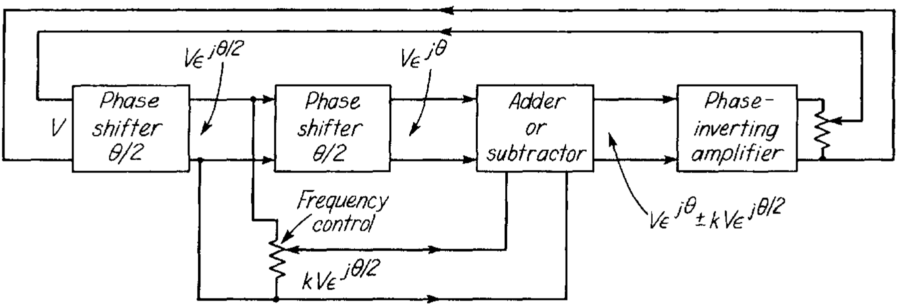


FIG. 318. Schematic circuit of a feedback oscillator that incorporates either an adding circuit or a differential amplifier.

180-degree phase shift, oscillation takes place at this frequency. A practical form of the circuit of Fig. 318 is shown in Fig. 320. Like that of the circuit of Fig. 317, the tuning ratio of this circuit approximates 4. The circuit may be adapted to frequency modulation by inserting an amplitude modulator ahead of T_4 in order to vary the value of k at modulation frequency.

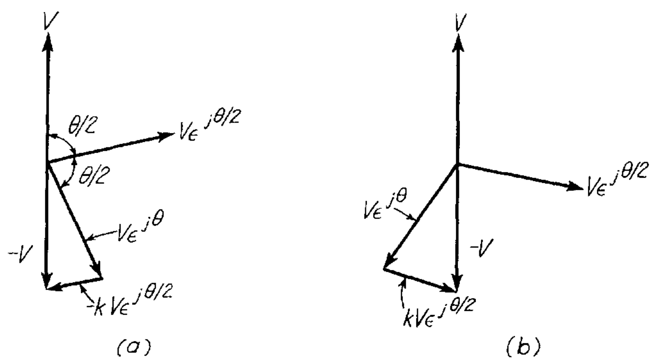


FIG. 319. Phasor diagrams for the circuit of Fig. 318.

90. *Crystal-Controlled Oscillators*

90.1. Principle of Operation.[1] Very high frequency stability can be obtained in relatively simple circuits by the use of mechanical resonance to determine the frequency. Radio-frequency oscillators in which the frequency-determining element is a quartz crystal have long been used in radio

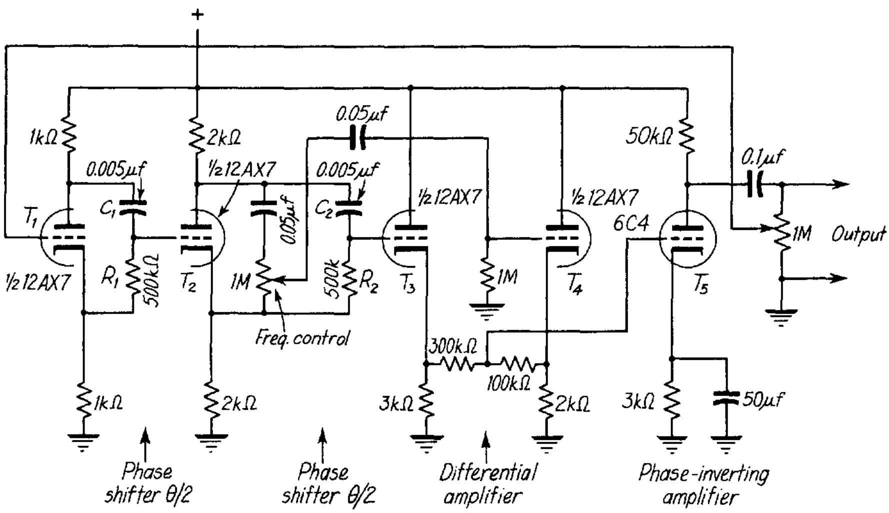


FIG. 320. Circuit diagram of an oscillator of the form of Fig. 318.

broadcasting. The value of a crystal in controlling frequency lies in the sharpness of its resonance curve. The equivalent Q of a crystal resonator is of the order of a hundred times that which can be readily obtained in lumped-element electrical circuits. When the temperature of the crystal is maintained constant by means of a temperature-control chamber, the frequency drift may be made less than 2 parts in 10 million.

The control of oscillator frequency by means of crystals is based upon the piezoelectric effect. When some crystals, notably quartz, are compressed

[1] W. G. Cady, *Proc. I.R.E.*, **10**, 83 (1922); G. W. Pierce, *Proc. Am. Acad. Arts Sci.*, **59**, 81 (1923); A. W. Hull, *Phys. Rev.*, **27**, 439 (1926); D. W. Dye, *Proc. Phys. Soc. (London)*, **38**, 399, 457 (1926); A. Hund, *Proc. I.R.E.*, **14**, 447 (1926), and **16**, 1072 (1928); L. P. Wheeler and W. E. Bower, *Proc. I.R.E.*, **16**, 1035 (1928); K. S. Van Dyke, *Proc. I.R.E.*, **16**, 742 (1928); J. R. Harrison, *Proc. I.R.E.*, **16**, 1455 (1928); E. M. Terry, *Proc. I.R.E.*, **16**, 1468 (1928); W. G. Cady, *Proc. I.R.E.*, **16**, 521 (1928) (with bibliography); J. W. Wright, *Proc. I.R.E.*, **17**, 127 (1929); H. R. Meahl, *Proc. I.R.E.*, **22**, 732 (1934); I. E. Fair, *Bell Sys. Tech. J.*, **24**, 161 (April, 1945); D. Makow, *Proc. I.R.E.*, **44**, 1031 (August, 1956). For other references, see A. Hund, *Phenomena in High-frequency Systems*, p. 118, McGraw-Hill Book Co., Inc., New York, 1936 and 1952.

or stretched in certain directions, electric charges appear on the surfaces of the crystal that are perpendicular to the axis of strain. Conversely, when such crystals are placed between two plane, conducting surfaces between which a difference of potential is applied, the crystals expand or contract. If the applied potential is alternating, the crystal is set into vibration. Because of compressional waves set up in the crystal, mechanical resonance is observed in the crystal, the resonance frequency being inversely proportional to thickness of the crystal. The amplitude of vibration of the crystal is greatest when the frequency of the impressed alternating voltage is equal to a resonance frequency of the crystal. So far as their effect upon the circuit is concerned, the crystal and its holder may be represented by the equivalent circuit of Fig. 321, in which L_x, C_x, and r_x are equivalent circuit elements for the crystal, C_m is the capacitance of the mounting plates, and C_g is an equivalent capacitance that represents the effect of the air gap between the crystal and the mounting plates.

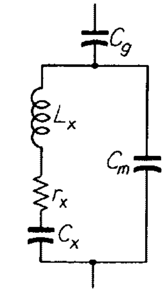


FIG. 321. Equivalent circuit for a piezoelectric crystal and holder.

90.2. Use of Crystals in Negative-Resistance Circuits. Since a crystal normally behaves like a series resonant circuit, it may be used as the resonator across any current-stable port of any negative-resistance circuit, provided that the magnitude of the negative resistance exceeds the effective series resistance of the crystal at its resonance

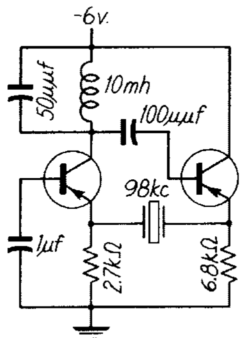


FIG. 322. Two transistor negative-resistance crystal oscillators.

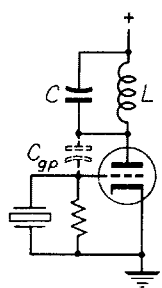


FIG. 323. Vacuum-tube negative-resistance crystal oscillator.

frequency. Figure 322 shows a crystal oscillator based upon the transistor version of the symmetrical current-stable circuit of Fig. 69e.[2] The reader

[2] F. C. Alexander, *Electronics,* December, 1954, p. 188.

can readily devise other crystal oscillators based upon the negative-resistance circuits discussed in Secs. 18 to 22.

A commonly used vacuum-tube circuit, shown in Fig. 323, makes use of the negative input conductance of a triode resulting from the grid-plate capacitance when the plate load is inductive.

90.3. Crystal-Controlled Feedback Oscillators. In the original Pierce circuit, shown in Fig. 324a, the crystal is connected between the grid and

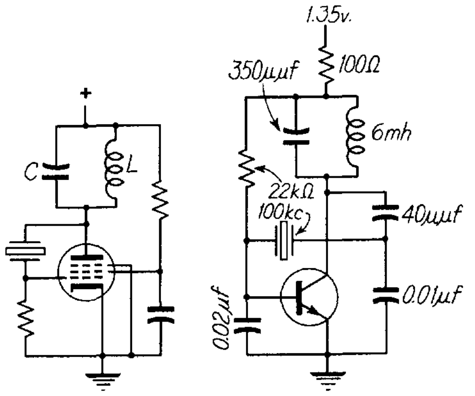


FIG. 324. Pierce crystal-controlled feedback oscillators: (a) vacuum-tube circuit; (b) transistor circuit.

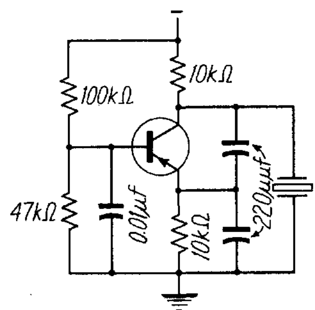


FIG. 325. A variant of the circuit of Fig. 286 in which a quartz crystal is used in place of the series resonant circuit.

the plate of a vacuum tube. This circuit oscillates only when the plate load is capacitive, and therefore the plate resonator must be tuned to a slightly lower frequency than the resonance frequency of the crystal. A transistor Pierce circuit is shown in Fig. 324b.[3] The frequency instability of this circuit is only 1.5×10^{-9} cps per degree C and 10^{-7} cps per volt at a frequency of 100 kc, and the drift is within 10^{-8} cps per week. It seems likely that even higher stability against change of temperature could be attained by the insertion of resistance in series with the emitter.

A crystal may also be used in place of the series resonator in the Clapp-Gouriet oscillator of Fig. 286, as shown in the transistor circuit of Fig. 325.[4]

90.4. Crystal-Controlled Null-Network Oscillator. A circuit that combines the high Q of the quartz crystal with the phase-shift-multiplication property of null networks is obtained by using a crystal in place of the

[3] P. G. Sulzer, *Electronics*, May, 1953, p. 206.
[4] P. G. Sulzer, *Electronics*, August, 1953, p. 172.

resonant circuit in the bridge oscillator of Fig. 290.[5] For a 100-kc bridge-stabilized crystal oscillator in which the crystal had a Q of approximately 10^5, Meacham reported a measured frequency change of one part in 10^8 for a plate-supply-voltage change from 120 to 140 volts. In Meacham's circuit, the tetrode amplifier, which was coupled to the bridge input and output terminals through singly tuned transformers, had an amplification of approximately 425.

91. Magnetostriction Oscillators

The phenomenon of magnetostriction is the expansion or contraction of magnetic materials as the result of magnetization, and the converse change of magnetization as the result of strain. The phenomenon may be used as the basis of an oscillator having high frequency stability.[1] The circuit diagram of a typical magnetostriction oscillator is shown in Fig. 326. The operation is as follows: The steady component of plate current produces a steady strain in the rod. Any small change in plate current results in a change of magnetization of the end of the rod that is within the plate coil. This results in an elongation or contraction of this portion of the rod and causes a compressional wave to move toward the other end of the rod. When the wave reaches the portion of the rod that lies inside of the grid coil, the resulting change in magnetic flux induces a voltage in the grid coil, which causes a change of plate current. This change in plate current in turn starts another compressional wave in the bar. The compressional waves are reflected from the grid end of the rod and return to the plate end, where they are again reflected. If the polarity of the grid and plate coils is correct, the induced and reflected waves at the plate end are in phase and will reinforce one another. The tendency for the amplitude to build up is increased to the point of oscillation by tuning the electrical circuit so that it has the same natural frequency as the rod. The action is not dependent upon inductive coupling of the coils. In contrast with that in the feedback oscillators of Fig. 271, the orientation of the coils must be such that the inductive coupling is degenerative. The inductive coupling should preferably be made small by the use of shielding. Considerable improvement results from the use of a two-stage amplifier in place of the single tube shown in Fig. 326.[2]

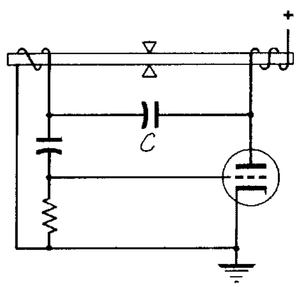


FIG. 326. Magnetostriction oscillator.

[5] L. A. Meacham, *Proc. I.R.E.*, **26**, 1278 (October, 1938).

[1] G. W. Pierce, *Proc. Am. Acad. Arts Sci.*, **63**, 1 (1928); *Proc. I.R.E.*, **17**, 42 (1929); W. W. Salisbury and C. W. Porter, *Rev. Sci. Instruments*, **10**, 142 (1939).

[2] G. W. Pierce and A. Noyes, Jr., *JASA*, **9**, 185 (1938).

Because the change in length of an initially unmagnetized rod is of the same sign for both polarities of magnetization, the rod will vibrate at twice the frequency of the electrical circuit if it is not polarized. To ensure that the rod is polarized throughout its length, it is usually magnetized permanently and placed in the coils so that the field of the plate coil increases the magnetization.

Nickel, monel metal, nichrome, invar, stoic metal, and other metal alloys may be used to make magnetostrictive rods. Difficulties of design of suitable rods having high natural frequency of oscillation restrict the use of the magnetostriction oscillator to frequencies within the audible and supersonic ranges. Very low audio frequencies are produced by the use of rods that are loaded at the ends or that consist of an outer shell of magnetostrictive material filled with lead or other material having a low velocity of propagation of compressional waves. If the temperature of the rod is kept constant, a frequency stability of 1 part in 30,000 may be obtained.[3] Dependence of frequency upon temperature can be reduced by employing special alloys of low temperature coefficient [4] or rods made up of a core and a shell of two magnetostrictive metals of opposite temperature coefficients of expansion.

92. Beat-Frequency (Heterodyne) Oscillators

In the beat-frequency oscillator, shown schematically in Fig. 327, the outputs of two radio-frequency oscillators of slightly different frequencies are applied simultaneously to a detector. The output of the detector contains, in addition to the impressed radio frequencies, their sum and difference. By means of a filter, the fundamental radio frequencies and their sum are removed, leaving only the difference frequency in the output, which may be suitably amplified by audio-frequency amplifiers. The principal advantage of the beat-frequency oscillator is that a very wide frequency range can be covered with a single dial. Other desirable characteristics that can be obtained with careful design include good waveform, constant output level, lightness, and compactness. By proper variable-capacitor design, a logarithmic frequency scale can be obtained, a considerable advantage when

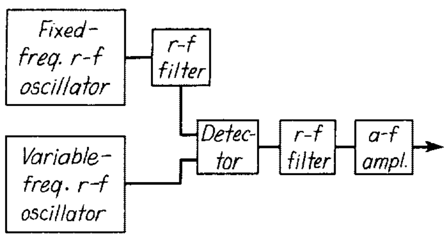


FIG. 327. Schematic diagram of the beat-frequency oscillator.

[3] Pierce, *loc. cit.*
[4] J. M. Ide, *Proc. I.R.E.*, **22**, 177 (1934).

the oscillator is to be used in obtaining amplifier-response curves. Unless extreme care is taken in the design and construction, however, this type of oscillator is likely to have relatively poor frequency stability, which necessitates frequent setting against a standard frequency during the period of use, particularly during the time required to establish temperature equilibrium.

The design of a heterodyne oscillator involves a number of special problems, among which are the prevention of interaction between the oscillators, elimination of harmonics and other undesired frequencies, improvement of frequency stability, and the prevention of variation of output level. Interaction of the two r-f oscillators may cause them to pull into synchronism when their frequency difference is small and thereby prevent the production of low audio-frequency. It also tends to distort the output wave into a sawtooth wave at output frequencies somewhat greater than that at which the oscillators pull into step. Interaction may be prevented by adequate shielding, proper location of component parts, use of chokes or decoupling resistors and bypass capacitors in voltage-supply leads, and by correct methods of coupling the output of the r-f oscillators to the detector tube. Coupling methods include the use of buffer amplifiers between the oscillators and the detector; electron-coupled oscillators; a balanced modulator circuit; a mixing bridge; or a multigrid mixer tube, the oscillator outputs being applied to two grids that are shielded from each other.

If the harmonics are removed from the output of one of the r-f oscillators of a heterodyne oscillator and detector distortion is negligible, the only undesired output frequencies lie above the beat-frequency range and are removed by the r-f filter following the detector.

Frequency instability in heterodyne oscillators results from the same causes as in other types of oscillators, but it is likely to be greater because a small percentage variation in the frequency of either r-f oscillator produces a much greater percentage variation of output frequency. The stability of the r-f oscillators should be as high as possible, and the two oscillators should be as nearly alike as possible in order that they will respond similarly to changes of supply voltage and of temperature.

Variation of amplitude of the oscillator output results from change of amplitude of the variable oscillator over the tuning range, from the action of improperly designed r-f filters in the plate circuit of the detector, and from frequency distortion of the a-f amplifier. The first effect can be made small by using a linear detector and making the amplitude of the variable-frequency input to the detector much greater than that of the fixed-frequency input. Audio-frequency attenuation by the r-f filter in the detector plate circuit can be avoided most readily by the use of a radio frequency that is several times the highest output frequency. The choice of the best value of frequency of the r-f oscillators of a heterodyne oscillator is governed largely by frequency stability, ease of filtering of r-f voltages from the detector output, and desired frequency range.

If a beat-frequency oscillator incorporates two or more fixed-frequency oscillators, instead of the single fixed-frequency oscillator shown in Fig. 327, it can generate two or more variable output frequencies having fixed differences. Such an instrument is useful in testing amplifiers for nonlinearity distortion.

93. Microwave Oscillators

Microwave oscillators are those operating at frequencies such that the circuits must be of the distributed-element type, rather than lumped-element. Such oscillators differ from those operating at lower frequencies in two important respects. The first is that microwave oscillators can usually be made to function efficiently only if the tube elements serve as portions of the resonators. The second important difference lies in the fact that, whereas electron transit time is ordinarily small in comparison with the period at the lower frequencies, the transit time between the electrodes of microwave tubes is of the same order of magnitude as the period and may be considerably greater. Although large transit time has objectionable effects in oscillators designed for lower frequencies, it is essential to the functioning of most types of microwave oscillators.

The mechanism of operation of various types of microwave oscillators differs in various details, but in all types using electron streams it involves the transfer of power from a source of direct voltage to a source of alternating voltage by means of a density-modulated (bunched) stream of electrons. (The word "source" as used here is not restricted to a primary source of power, but includes passive networks, such as resonators.) This is accomplished by accelerating electrons in a direct electric field and retarding most of them in an alternating electric field. The density modulation of the beam allows more electrons to be retarded than accelerated by the alternating field. In addition to triodes and tetrodes, microwave oscillators include klystrons, magnetrons, and traveling-wave tubes.

Whereas oscillators operating at frequencies below the microwave range can usually be analyzed most readily by considering the amplifying devices as circuit elements, as has been done throughout this book, microwave oscillators are usually analyzed by studying the mechanism by means of which an initially uniform-density electron beam is formed into bunches of electrons, and the mechanism by means of which energy is delivered to individual electrons by the direct field and extracted from individual electrons by the alternating field. For descriptions of microwave oscillators and explanations of their operation, the reader is referred to books covering the microwave field.[1]

[1] See, for example, H. J. Reich, J. G. Skalnik, P. F. Ordung, and H. L. Krauss, *Microwave Principles,* D. Van Nostrand Co., Inc., Princeton, N. J., 1957.

APPENDIX—TWO-PORT PARAMETERS

The relations between the input and output currents and voltages of a two-port (four-terminal) network, Fig. A-1, can be expressed in terms of a number of sets of parameters, of which the h parameters and y parameters are examples. The defining equations of these two sets of parameters are the following:

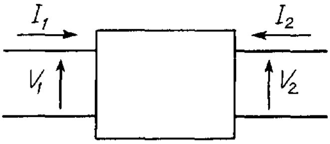


FIG. A-1.

$$V_1 = I_1h_i + V_2h_r \tag{A-1}$$

$$I_2 = I_1h_f + V_2h_o \tag{A-2}$$

$$I_1 = V_1y_i + V_2y_r \tag{A-3}$$

$$I_2 = V_1y_f + V_2y_o \qquad \text{A(-4)}$$

Any of the parameters can be expressed as the ratio of one dependent variable to one independent variable. Thus h_i, which is called the *short-circuit input* resistance, is the ratio of V_1 to I_1 when V_2 is zero; i.e., when the output terminals are short-circuited to alternating current. The *open-circuit inverse voltage transfer ratio* or *open-circuit inverse voltage amplification factor* h_r is the ratio of V_1 to V_2 with I_1 zero; i.e., with the input terminals open-circuited to alternating current. The *short-circuit forward current-amplification factor* h_f is the ratio of I_2 to I_1 with the output terminals short-circuited to alternating current. The *open-circuit output admittance* h_o is the ratio of I_2 to V_2 with the input terminals open to alternating current. The *short-circuit input admittance* y_i, the *short-circuit reverse transfer admittance* y_r, the *short-circuit forward transfer admittance* y_f, and the *short-circuit output admittance* y_o are defined in a similar manner.

There is a set of h parameters and a set of y parameters for each of the three possible connections of a transistor or tube. These are specified by means of additional subscripts that indicate which electrode is common to the input and the output: $_e$, $_b$, and $_c$ for transistors, and $_k$, $_g$, and $_p$ for tubes. Any one set of parameters suffices to determine the performance of a transistor or tube, the parameters for any circuit connection being readily expressible in terms of the parameters for the other two connections. For convenience, the relations among the three sets of h parameters and among the three sets of y parameters are given in Table A-1, and the relations between the h parameters and the y parameters in Table A-2.

It should be borne in mind that the circuit parameters are in general complex quantities and that the complete performance of an active circuit element over a frequency range can therefore be determined only if the real and imaginary components of each of the four parameters of any set is known as a function of frequency. The parameters are also a function of the operating voltages and currents and, in transistors, of temperature. The difficulty of measuring and publishing

TABLE A-I. COMMON-BASE AND COMMON-COLLECTOR PARAMETERS IN TERMS OF COMMON-EMITTER PARAMETERS

$y_{ib} = y_{ie} + y_{re} + y_{fe} + y_{oe}$	$y_{ic} = y_{ie}$
$y_{rb} = -(y_{re} + y_{oe})$	$y_{rc} = -(y_{ie} + y_{re})$
$y_{fb} = -(y_{fe} + y_{oe})$	$y_{fc} = -(y_{ie} + y_{fe})$
$y_{ob} = y_{oe}$	$y_{oc} = y_{ie} + y_{re} + y_{fe} + y_{oe}$
$\Delta_{yb} = \Delta_{ye} \equiv y_{ie}y_{oe} - y_{fe}y_{re}$	$\Delta_{yc} = \Delta_{ye} \equiv y_{ie}y_{oe} - y_{fe}y_{re}$
$h_{ib} = \dfrac{h_{ie}}{\Delta_{hc}} \approx \dfrac{h_{ie}}{h_{fe} + 1}$	$h_{ic} = h_{ie}$
$h_{rb} = \dfrac{\Delta_{he} - h_{re}}{\Delta_{hc}} \approx \dfrac{\Delta_{he} - h_{re}}{h_{fe} + 1}$	$h_{rc} = 1 - h_{re} \approx 1$
$h_{fb} = -\dfrac{\Delta_{he} + h_{fe}}{\Delta_{hc}} \approx -\dfrac{h_{fe}}{h_{fe} + 1}$	$h_{fc} = -(h_{fe} + 1)$
$h_{ob} = \dfrac{h_{oe}}{\Delta_{hc}} \approx \dfrac{h_{oe}}{h_{fe} + 1}$	$h_{oc} = h_{oe}$
$\Delta_{hb} = \dfrac{\Delta_{he}}{\Delta_{hc}} \approx \dfrac{\Delta_{he}}{h_{fe} + 1}$	$\Delta_{hc} = h_{fe} - h_{re} + \Delta_{he} + 1 \approx h_{fe} + 1$

TABLE A-II. RELATIONS BETWEEN h AND y PARAMETERS

$h_i = \dfrac{1}{y_i}$	$y_i = \dfrac{1}{h_i}$
$h_r = -\dfrac{y_r}{y_i}$	$y_r = -\dfrac{h_r}{h_i}$
$h_f = \dfrac{y_f}{y_i}$	$y_f = \dfrac{h_f}{h_i}$
$h_o = \dfrac{\Delta_y}{y_i}$	$y_o = \dfrac{\Delta_h}{h_i}$
$\Delta_h \equiv h_ih_o - h_fh_r = \dfrac{y_o}{y_i}$	$\Delta_y \equiv y_iy_o - y_fy_r = \dfrac{h_o}{h_i}$

curves of the four real components and the four imaginary components for various operating voltages and currents makes advantageous the use of approximate relations between the high-frequency and low-frequency values of the parameters, such as Eq. (55) and the analogous relations given in Prob. 63.2-1, or of approximate equivalent circuits.[1]

[1] See, for example, R. F. Shea, editor, *Transistor Circuit Engineering,* Chap. **2,** John Wiley and Sons, Inc., New York, 1957.

PROBLEMS

1.1–1. (a) Show that, if the input resistance of the amplifier of Fig. 2 is infinite when R is infinite, the effective input resistance with finite R is $R/(1 - A)$ and that the input resistance is therefore positive and has the magnitude $R/(1 + |A|)$ when A is negative, i. e., when the amplifier is phase-inverting.

(b) Show that the output voltage of a circuit with n inputs is

$$V_o = \frac{A \sum_1^n k_h V_h}{|A| + 1 + \sum_1^n k_h} \approx -\sum_1^n k_h V_h \tag{P-1}$$

if $|A| \gg 1 + \sum_1^n k_h$, in which V_h is the input voltage to the hth input branch and k_h is the ratio of the resistance R to the series resistance R_h of that input branch.

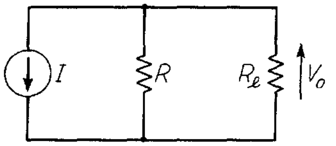
(c) Show that the effective input resistance of the lth input branch is

$$V_{il} = R\left[\frac{1}{k_l} + \frac{1}{|A| + 1 - k_l + \sum_1^n k_h}\right] \approx \frac{1}{k_l} \tag{P-2}$$

(d) Show that the current I_m in the mth input branch caused by a voltage V_l applied to the lth input branch has the following value if the input source resistances are negligible in comparison with the series input-branch resistances:

$$I_m = \frac{-V_l k_l k_m}{R(|A| + 1 + \sum_1^n k_h)} \approx \frac{V_l k_l k_m}{AR} \tag{P-3}$$

1.2–1. (a) Justify the use of the equivalent circuit of Fig. P-1 for n tubes with a common plate load resistance R_l. In Fig. P-1, g_{mh} and r_{ph} represent the transconductance and plate resistance of the hth tube and V_h the signal voltage impressed upon the grid of that tube.



$$I = \sum_{h=1}^{n} g_{mh} V_h$$

$$1/R = \sum_{h=1}^{n} 1/r_{ph}$$

FIG. P-1.

(b) Show that $V_o = -\dfrac{\sum_{h=1}^{n} V_h g_{mh}}{\sum_{h=1}^{n} 1/r_{ph} + 1/R_l}$ (P-4)

(c) Show that, if the tubes are identical,

$$V_o = -\frac{\mu R_l \sum_{h=1}^{n} V_h}{r_p + nR_l} \tag{P-5}$$

1.2–2. (a) Justify the use of the equivalent circuit of Fig. P-2 for n similar triodes connected as cathode followers with a common cathode load resistance R_k.

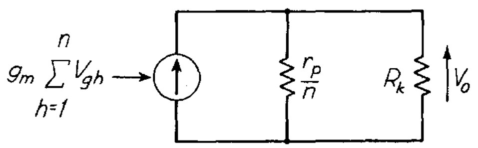


FIG. P-2.

(b) Evaluate the signal grid voltage V_{gh} of the hth tube in terms of the signal voltage V_h impressed upon the grid circuit of the hth tube and the output voltage V_o, and show that

$$V_o = \frac{\mu R_k}{r_p + n(\mu + 1)R_k} \sum_{h=1}^{n} V_h \tag{P-6}$$

(c) Under what conditions does Eq. (P-6) reduce to the following approximation?

$$V_o \approx \sum_{h=1}^{n} V_h \tag{P-7}$$

1.2–3. Show that, if $R_3 + R_4$ is much larger than the output resistance $1/(g_m + 1/r_p)$ of the cathode-followers T_1 and T_2 in the circuit of Fig. 3c,

$$V_o \approx \frac{V_1 R_4 + V_2 R_3}{R_3 + R_4} \tag{P-8}$$

2.1–1. (a) Find the output voltage V_{o1} developed across R_{l1} and the voltage V_{o2} developed across R_{l2} by the voltage V_1 in the circuit of Fig. 4a.

(b) Taking advantage of circuit symmetry, make use of the expressions derived in (a) to obtain Eq. (1).

(c) Show that the ratio of V_{o2} produced by V_2 to that produced by V_1 is $[r_p + R_l + R_k(\mu + 1)]/R_k(\mu + 1)$, and that this ratio approximates unity when $R_k \gg (r_p + R_l)/(\mu + 1)$.

(d) Derive Eq. (2) from Eq. (1).

(e) Show that, if the two halves of the circuit of Fig. 4a are identical and $R_k \ll 1/g_m$, the plate-to-plate output voltage V_o approximates $\mu R_l(V_1 - V_2)/(r_p + R_l)$ and that the signal voltage developed across the cathode resistor approximates $(V_1 + V_2)/2$.

(f) Derive Eqs. (4) and (5). (Suggestion: Find V_{o1} and V_{o2} produced by V_1 and invoke the superposition theorem, taking advantage of circuit symmetry.)

3.1–1. (a) Derive Eq. (6). (Suggestion: Expand the voltage transfer function in the form of a series.)

(b) Derive Eq. (10).

(c) Show that the characteristic equation for the circuit of Fig. 13c is:

$$p^2 + \frac{1}{R_sC_s}\left(\frac{R_s}{R} + 1 + \frac{C_s}{C}\right)p + \frac{1}{RCR_sC_s} = 0 \qquad \text{(P-9)}$$

(d) Show that, if R_s/R and $C_s/C \ll 1$, the roots of Eq. (P-9) are:

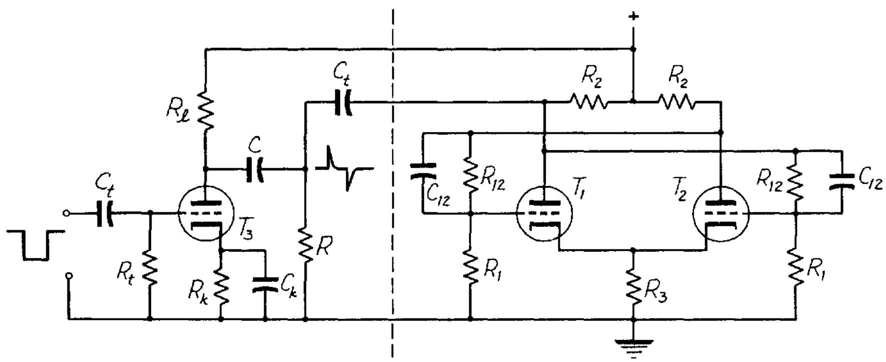
$$p = \frac{1}{2R_sC_s}\left[1 \pm \sqrt{1 - \frac{4R_sC_s}{RC}}\right] \qquad \text{(P-10)}$$

(e) By application of the binomial expansion, show that

$$p \approx -\frac{1}{RC} \quad \text{and} \quad -\frac{1}{R_sC_s} \qquad \text{(P-11)}$$

(f) Derive Eq. (11).

3.1–2. (a) Determine the amplitude and waveform obtained by clipping a 100-cps, 100-volt sine wave at 10 volts and differentiating the resulting approximately trapezoidal wave by means of an RC circuit having a 1-μsec time constant.



$R_1 = R_{12} = \frac{1}{2}$ megohm
$R_2 = R_3 = 10\ \text{k}\Omega$
$R_k = 1\ \text{k}\Omega$
$R_l = 20\ \text{k}\Omega$
$R_t = \frac{1}{2}$ megohm

$R = 5\ \text{k}\Omega$
$C = 100\ \mu\mu\text{f}$
$C_{12} = 25\ \mu\mu\text{f}$
$C_t = 0.01\ \mu\text{f}$

FIG. P-3.

(b) Repeat (a) if the trapezoidal wave is amplified by a factor of 10 and again clipped at 10 volts before being differentiated.

(c) Repeat (a) if another similar amplifier and 10-volt clipper are added ahead of the differentiator.

3.2–1. The characteristics of the circuit to the right of the dotted line of Fig. P-3 are such that either tube T_1 or tube T_2 conducts, and that conduction is changed from T_1 to T_2 by a short exponential positive pulse applied to the plate of T_1 and from T_2 to T_1 by a short negative pulse. The resistance R and capacitance C are used to differentiate the rectangular input pulse to produce the short pulses necessary to cause T_2 to start conducting at the beginning of the rectangular input pulse and to stop at the end of the pulse.

(a) Which of the circuit and tube parameters affect the differentiator time constant during differentiation of (1) the leading edge of the input pulse and (2) the trailing edge?

(b) Determine the approximate time constants.

3.3–1. Derive Eq. (12).

3.3–2. Under the assumption that the input resistance of the amplifier of Fig. P-4 is infinite, find the input resistance of the circuit.

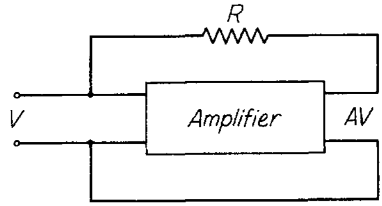


FIG. P-4.

5.2–1. (a) Zero-signal output voltage from the circuit of Fig. 21 can be eliminated by the application of a compensating voltage to the grid of T_1. Zero-signal output voltage may therefore be assumed to be the result of an unbalancing voltage v_U impressed in series with the grid of T_1. Since the first stage of the d-c amplifier, consisting of T_1 and T_2 and associated resistors, is a differential amplifier of the type discussed in Sec. 2.4, the output voltage v_O in the presence of an input voltage v_I approximates the difference between the voltage $v_E + v_U$ at the grid of T_1 and the stabilizing voltage v_S at the grid of T_2, multiplied by the amplification A_1 of the entire d-c amplifier (a function of frequency).

Under the assumption that v_O is exactly equal to $(v_E + v_U - v_S)A_1$, show that

$$v_O = \frac{v_I Z_f(1 - A_2) + v_U(Z_f + Z_i)}{(Z_f + Z_i)/A_1 + Z_i(A_2 - 1)} \tag{P-12}$$

in which A_2, the over-all amplification of the chopper, a-c amplifier, and rectifier, is equal to v_S/v_E.

(b) Show that, if $A_1(A_2 - 1) \gg (Z_f + Z_i)/Z_i$, Eq. (P-12) reduces to

$$v_O = v_I \frac{Z_f}{Z_i} + v_U \frac{Z_f + Z_i}{(A_2 - 1)Z_i} \tag{P-13}$$

(c) What requirement must the a-c amplifier, chopper, and filter meet in order that the second term of Eq. (P-13) shall be negligible and the zero-signal output voltage therefore be maintained negligible?

8.2–1. Derive Eq. (18).

11.2–1. With no control voltage applied to the grid circuit of T_2, the voltage across R_K in the circuit of Fig. 43 is minimum when T_1 is cut off by the negative signal; T_2 then acts as a cathode-follower. Under the assumption that the tube is approximately linear, the voltage across R_K approximates the voltage amplification of T_2, acting alone, multiplied by the change in grid-to-ground voltage between the cutoff value V_{PP}/μ and V_{GG}. The voltage across R_K is maximum when the signal voltage and the control voltage are both zero. The two tubes then pass equal currents and the current through R_K is the same as it would be with a single tube having the same amplification factor but twice the transconductance of each of the two tubes.

Show that if the negative signal voltage is increased in magnitude from zero to a value that cuts T_1 off, the control voltage remaining zero, the change in output voltage is

$$\Delta V_K \approx -(V_{GG} + V_{PP}/\mu)\left[\frac{2g_m}{2g_m + 2/r_p + 1/R_K} - \frac{g_m}{g_m + 1/r_p + 1/R_K}\right]$$

$$\approx -\frac{V_{ko}}{2R_K(g_m + 1/r_p + 1/R_K)} \approx -\frac{V_{ko}}{2g_mR_K}\cdot\frac{\mu}{\mu+1} \qquad \text{(P-14)}$$

in which V_{ko} is the voltage across R_K when both the signal voltage and the control voltage are zero:

$$V_{ko} = (V_{GG} + V_{PP}/\mu)\frac{2g_m}{2g_m + 2/r_p + 1/R_K} = \frac{\mu V_{GG} + V_{PP}}{\mu + 1 + r_p/2R_K} \qquad \text{(P-15)}$$

13.2–1. By the use of the collector curves of Fig. 51a, determine the collector current waveform when a 1-volt sinusoidal voltage is applied to the input terminals of the circuit of Fig. 52a and the base voltage is (a) +125 mv, (b) zero, (c) −500 mv, (d) −1.5 volts. R_l = 2500 ohms.

14.3–1. (a) Design the circuit of Fig. P-5 so that output is obtained when a positive control voltage is applied to terminal B' or C', or both, but not when a negative voltage is also applied to terminal D'. There should not be coupling between the various sources of voltage.

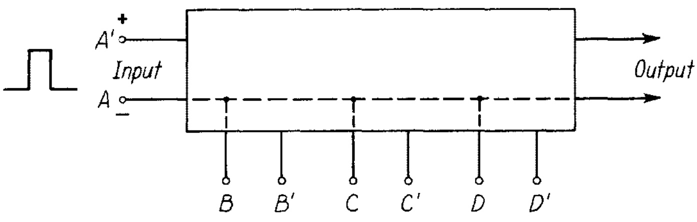


FIG. P-5.

(b) Show how the circuits of Fig. 37 may be modified so that output is obtained only in the absence of an inhibiting control pulse.

14.3–2. Indicate which of the following statements are correct:

(a) In the circuit of Fig. P-6, pulse output is obtained when (1) both input pulses

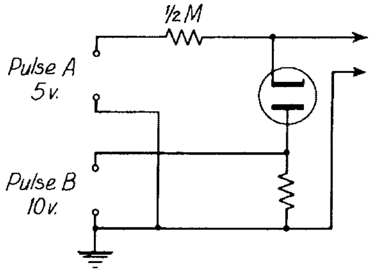


FIG. P-6.

are positive, (2) both input pulses are negative, (3) pulse A is positive and pulse B is negative, (4) pulse B is positive and pulse A is negative, (5) pulse B is zero and pulse A is negative.

(b) In the circuit of Fig. P-7, pulse output is obtained when (1) both input pulses are negative, (2) both input pulses are positive, (3) pulse A is positive and pulse B is negative, (4) pulse B is positive and pulse A is negative, (5) pulse B is zero and pulse A is negative.

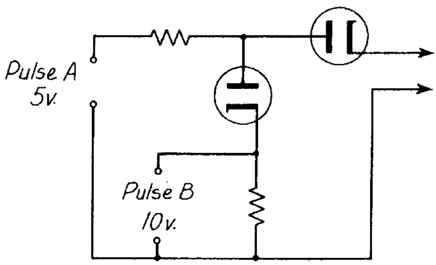


FIG. P-7.

14.3-3. Which of the following statements are correct?

(a) In the circuit of Fig. P-8, pulse output is obtained when (1) both input pulses

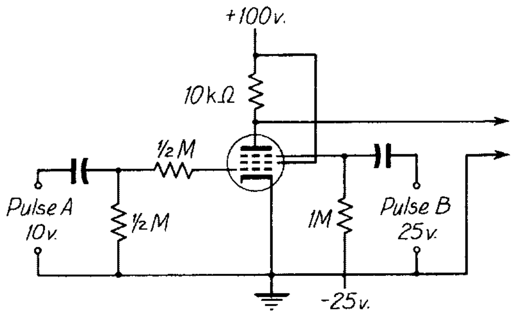


FIG. P-8.

are positive, (2) both input pulses are negative, (3) pulse A is positive and pulse B is negative, (4) pulse A is negative and pulse B is positive, (5) pulse B is zero and pulse A is negative. (Plate current is cut off when the suppressor voltage is -25.)

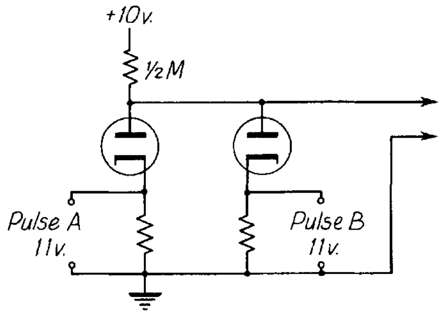


FIG. P-9.

(b) In the circuit of Fig. P-9, positive output is obtained when both input pulses are positive, (2) positive output is obtained when both input pulses are negative, (3) negative output is obtained when both input pulses are negative, (4) negative output is obtained when one input pulse is negative and one is positive, (5) positive output is obtained when one input pulse is negative and one positive.

16.2–1. By means of the criteria given in Sec. 16.2, determine at which points the circuits of Fig. 74 are voltage-stable, and at which points they are current-stable. Examine ports both within and outside of the amplifiers.

16.2–2. (a) The amplifier in the circuit of Fig. 65b consists of a single tube having negligible input conductance, a plate resistance r_p, and a negative transconductance of magnitude greater than unity.

(b) Determine the algebraic expression for the input conductance of the port whose terminals are the upper input and output terminals of the amplifier.

(c) By application of criteria (a) and (b) of Sec. 16.2, show that by proper choice of g_m and the circuit resistances, the port may be made either voltage-stable or current stable.

(d) Is this consistent with criteria (c) and (d) of Sec. 16.2?

17.2–1. (a) Show that, if the resistors R_{12} of the circuit of Fig. 68b are shunted by capacitors of sufficiently high capacitance so that the plate of each tube may be considered to be directly coupled to the grid of the other for rapidly changing voltages, the circuit may be represented by the equivalent circuit of Fig. P-10.

(b) Show that the input admittance is given by the following relation:

$$Y_i = I/V = \tfrac{1}{2}[1/R_1 + 1/R_2 + 1/r_p - g_m + j\omega(C_{gk} + C_{pk} + 4C_{gp})] \qquad \text{(P-16)}$$

(c) By means of the criteria stated in Sec. 16.2, determine whether the port is voltage-stable or current-stable.

(d) Discuss the validity of the use of the circuit of Fig. 84 as an equivalent for a bistable circuit formed from the vacuum-tube circuit of Fig. 68b.

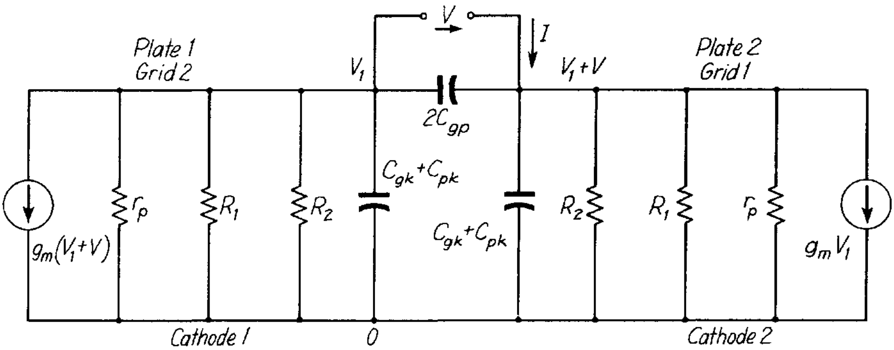


FIG. P-10.

17.2-2. (a) Show that, if the resistors R_{12} of the circuit of Fig. 68e are shunted by capacitors of sufficiently high capacitance so that the plate of each tube may be considered to be directly coupled to the grid of the other for rapidly changing voltages, and if $R_1 \gg R_2$, the circuit may be represented by the equivalent circuit of Fig. P-11.

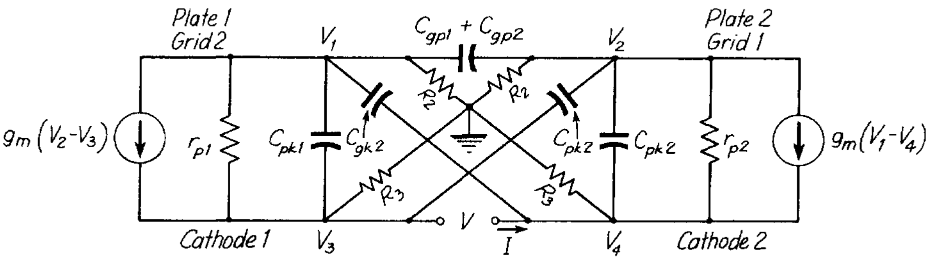


FIG. P-11.

(b) If the circuit is completely symmetrical with respect to circuit and tube parameters, $V_2 = -V_1$, $V_3 = -V/2$, and $V_4 = +V/2$. By summing currents at nodes 1 and 3, confirm the following equations relating V_1, V, and I:

$$V_1\left[\frac{1}{r_p} - g_m + j\omega(C_{pk} - C_{gk})\right] + \frac{V}{2}\left[\frac{1}{r_p} + \frac{1}{R_3} + g_m + j\omega(C_{gk} + C_{pk})\right] = I$$

$$V_1\left[\frac{1}{r_p} + \frac{1}{R_2} - g_m + j\omega(C_{gk} + C_{pk} + 4C_{gp})\right] + \frac{V}{2}\left[g_m + \frac{1}{r_p} + j\omega(C_{pk} - C_{gk})\right] = 0$$

(c) Derive the expression for the input impedance V/I, and show that it reduces to the following form if $R_2 = R_3 \equiv R$ and $C_{gk} = C_{pk} = C_{gp} \equiv C$:

$$Z_i = \frac{g_p + g - g_m + j6\omega C}{g(g_p + g/2) + 6\omega^2C^2 + j\omega C(4g_p + 4g + 2g_m)} \qquad \text{(P-17)}$$

in which $g_p = 1/r_p$ and $g = 1/R$.

(d) By means of criteria (a) and (b) of Sec. 16.2, determine whether the port is voltage-stable, or current-stable.

(e) For the following values of circuit and tube parameters, show that $Z_i = -543$ kΩ at zero frequency, $-416 + j232$ kΩ at 10^5 rps, $-15.4 + kj\,94$ kΩ at 10^6 rps, and $1.25 + j9.65$ kΩ at 10^7 rps. Plot X_i vs. G_i with frequency as a parameter.

$$g_m = 2.5 \times 10^{-3} \text{ mho} \qquad R = 5 \times 10^4 \text{ ohms}$$

$$r_p = 5 \times 10^3 \text{ ohms} \qquad C = 4 \times 10^{-12} \text{ farad}$$

(f) How can the inductive component of input impedance be explained in a circuit that contains no inductance?

17.2–3. (a) Under the assumption that the coupling capacitors C_{12} of the circuit of Fig. 96b are sufficiently large so that the collector of each transistor is in effect directly coupled to the base of the other for rapidly changing voltages, show that the circuit may be represented by the equivalent circuit of Fig. P-12, in which R_{ie} is the

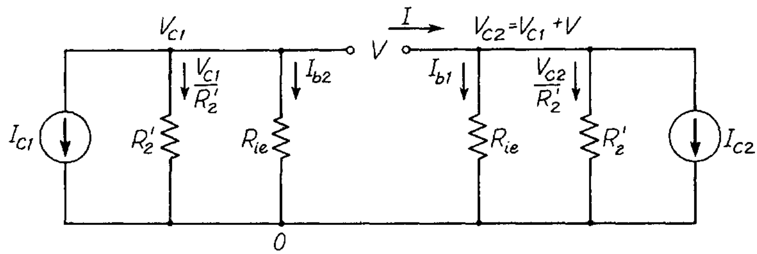


FIG. P-12.

base-to-emitter input resistance of the transistors and R_2' is the resistance of the parallel combination of R_1, R_2, and $\dfrac{1}{h_{oe}}$.

(b) Under the assumption that Eqs. (53) and (54) are valid, show that the input admittance I/V is

$$Y_i = \frac{1}{2R_2'} + \frac{1}{2R_{ie}} + \frac{1 + 1/h_{fbo} - jf/f_{\alpha b}h_{fbo}}{2R_{ie}\left[\left(1 + \dfrac{1}{h_{fbo}}\right)^2 + \dfrac{f^2}{f_{\alpha b}{}^2 h_{fbo}{}^2}\right]} \tag{P-18}$$

(c) Show that h_{fbo} must be negative and that its magnitude must exceed unity in order that the conductive component of Y_i shall be negative.

(d) If the value of h_{fbo} is such as to make the input conductance negative, is the input susceptance capacitive or inductive?

(e) By means of the criteria stated in Sec. 16.2, determine whether the port is voltage-stable or current-stable.

17.2–4. (a) Table II shows that when a transistor in the common-base connection is used in the T circuit of Fig. 65b the input resistances at two of the ports are

$$(R_i)_1^T = (R_i)_2^T = \frac{1 + h_{fb} - h_{rb} + \Delta_{hf}}{h_{ob}} \tag{P-19}$$

Under the assumptions that $1 + h_{fb} \gg \Delta_{hb} - h_{rb}$, that the frequency of the impressed voltage is such that Eq. (53) is valid but that the effects of transistor capacitances are small, and that h_{ob} remains nonsusceptive,[1] show that the input impedance at these ports is

$$(Z_i)_1^T = (Z_i)_2^T = \frac{1}{h_{ob}} + \frac{h_{fbo}(1 - jf/f_{ab})}{h_{ob}(1 + f^2/f_{ab}^2)} \tag{P-20}$$

(b) What are the restrictions on h_{fbo} if the resistive component of the input impedance is to be negative?

(c) If the value of h_{fbo} is such as to make the input resistance negative, is the input reactance inductive or capacitive?

(d) How do the magnitudes of the input resistance and susceptance depend upon frequency?

18.1–1. By inspection of pi and tee circuits using a single triode in the common-cathode, common-grid, and common-plate connections, verify the values of R_i listed in Table IV.

18.1–2. By means of qualitative analyses of the circuits of Fig. 66, explain the general aspects of the current-voltage characteristics of Figs. 66c, 66d, and 66e. Note that current-stable characteristics must be explained by choosing the current as the independent variable and determining how the voltage varies as the current is changed.

18.2–1. Construct the a-c equivalent circuits for the Eccles-Jordan circuits of Fig. 68 and from them determine the expressions for the input resistance (the ratio of the alternating voltage V impressed upon the port to the resulting alternating current I).

18.3–1. (a) By inspection, show that the input resistance R_i between the cathodes and ground (when R_{12} is infinite) in the circuit of Fig. 70 approximates $1/g_{m2}(1 - A_{v1})$, where g_{m2} is the transconductance of T_2 and A_{v1} is the voltage amplification of T_1 between the cathode of T_1 and the grid of T_2.

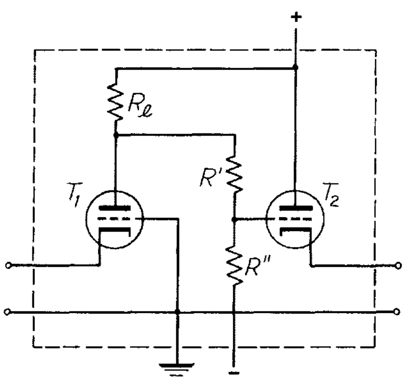


FIG. P-13.

(b) Show that the voltage amplification A_{v1} between the cathode of T_1 and the grid of T_2 in the circuit of Fig. P-13 is

$$A_{v1} \approx \frac{(\mu + 1)R_l}{r_p + R_l} \times \frac{R''}{R' + R''} \tag{P-21}$$

[1] Actually, the collector capacitance should not be neglected, and the output admittance h_{ob} has both a real and an imaginary part. It is assumed to be nonsusceptive in order to simplify the proof that the frequency dependence of h_{fb} is such as to make possible a reactive component of input impedance of the expected sign in this transistor circuit.

(c) Show that, if $R' + R'' \gg R_l$, the y parameters of the circuit of Fig. P-13 are:

$$y_f = -A_{v1}g_{m2} \qquad y_r = 0$$
$$y_o = g_{m2} + 1/r_{p2} \qquad y_i = (g_{m1} + 1/r_{p1})\frac{r_{p1}}{r_{p1} + R_l} \tag{P-22}$$

(d) By the use of the expression for $(R_i)_{12}^T$ listed in Table II, show that when the circuit of Fig. P-13 is used in a tee feedback circuit and $R_1 = R_2 = 0$, the value of $(R_i)_{12}^T$ is given by the following relation if $g_{m1} \gg 1/r_{p1}$ and $g_{m2} \gg 1/r_{p2}$:

$$(R_i)_{12}^T \approx \frac{1}{g_{m1}r_{p1}/(r_{p1} + R_l) + g_{m2}(1 - A_v)} \tag{P-23}$$

(e) Show that, if the tubes are identical and $R_l \gg r_p$, Eq. (P-23) reduces to:

$$(R_i)_{12}^T \approx \frac{1}{g_m[1 - \mu R''/(R' + R'')]} \tag{P-24}$$

(f) Assume reasonable values of the circuit parameters and determine the numerical value of R_i for several high-transconductance tubes.

19.1–1. Using the expressions listed in Table II and the transformations listed in Table A-1, derive Eqs. (20A), (21A), (20B), and (21B).

19.2–1. (a) Express the h parameters of the circuit of Fig. 71d in terms of the h parameters of the two transistors.

(b) If $h_{ib} = 40$ ohms, $h_{rb} = 5 \times 10^{-4}$, $h_{fb} = -0.98$, and $h_{ob} = 4 \times 10^{-7}$ mho for both transistors, show that Eqs. (20) and (21) are satisfied if the two-stage amplifier of Fig. 71d is used in the pi and tee feedback circuits of Fig. 65.

19.5–1. (a) By inspection of the circuit of Fig. 74d, determine whether the circuit is voltage-stable or current-stable at the port across which R_{12} appears.

(b) By inspection, determine the general form of the current-voltage characteristic to be expected at this port over a voltage range extending into both positive and negative values.

24.2–1. Derive Eq. (30).

26.1–1. Figure P-14 shows a circuit that differs from a commonly used two-stage voltage stabilizer only in the absence of a glow tube in the cathode lead of T_1 to furnish a reference biasing voltage.

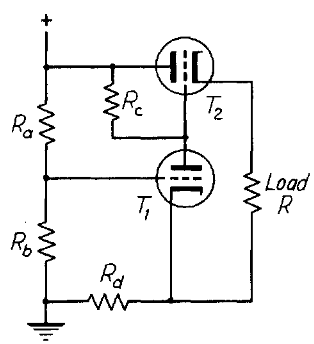


FIG. P-14.

(a) Show that the circuit of Fig. P-14 may be bistable.

(b) What types of amplifier circuits are used in the two stages (common-cathode, common-grid, or common-plate)?

(c) At which ports is the circuit voltage-stable, and at which ports is it current-stable?

26.2-1. (a) With the aid of superposition, derive Eq. (35) by inspection of Fig. 93.

(b) Derive Eq. (36) by substituting the appropriate y parameters into Eq. (32).

26.2-2. By substituting the common-cathode y parameters of Table III into Eq. (34), derive Eq. (37).

27.1-1. (a) Show that the low-frequency voltage amplification of the circuit of Fig. P-15 between the grid of T_1 and the grid of T_2 is

$$A_v = -g_m R_l R_1/(R_1 + R_{12}) \tag{P-25}$$

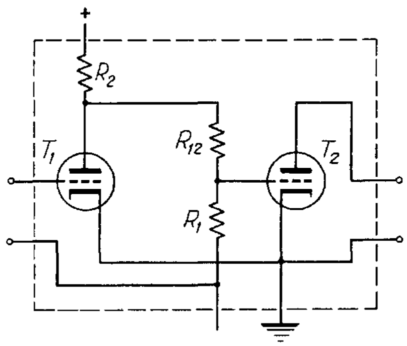


FIG. P-15.

in which

$$1/R_l = 1/r_p + 1/R_2 + 1/(R_1 + R_{12}) \tag{P-26}$$

(b) Show that the y parameters of the circuit of Fig. P-15 are

$$\begin{aligned} y_i &= 0 & y_o &= 1/r_p \\ y_f &= -g_m{}^2 R_l R_1/(R_1 + R_{12}) & y_r &= 0 \\ & & \Delta y &= 0 \end{aligned} \tag{P-27}$$

(c) The amplifier of Fig. P-15 is used in the pi circuit of Fig. 65b to form the symmetrical Eccles-Jordan circuit of Fig. 96b. By the use of Eq. (32), show that the circuit is bistable if

$$-g_m{}^2 R_l R_1/(R_1 + R_{12}) + \frac{1}{r_p}\left(1 + \frac{R_{12}}{R_1}\right) + \frac{1}{R_1} + \frac{R_{12}}{R_1 R_2} + \frac{1}{R_2} < 0 \tag{P-28}$$

(d) Transform Eq. (P-28) into Eq. (38).

(e) With the aid of Eq. (P-25), show that the criterion of Eq. (P-28) is equivalent to the requirement that the low-frequency open-circuit loop voltage amplification of the circuit of Fig. 96b exceed unity. Is this conclusion verified by Fig. 104?

27.1–2. (a) Show that, if $R_l h_{oe} \ll 1$, the low-frequency current amplification A_i between the base of T_1 and the base of T_2 in the circuit of Fig. P-16 is

$$A_i = \frac{Kh_{fe}}{1 + R_l h_{oe}} \approx Kh_{fe} \tag{P-29}$$

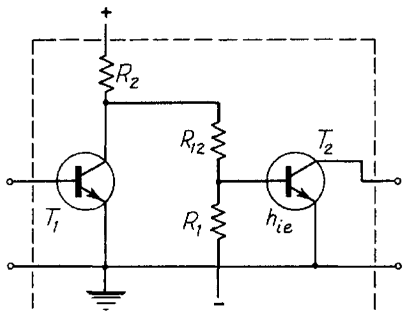


FIG. P-16.

in which R_l is the resistance of the series-parallel combination of R_1, R_{12}, R_2, and the input resistance of T_2, which may be assumed to approximate h_{ie}.

$$R_l = \frac{R_2[R_1R_{12} + h_{ie}(R_1 + R_{12})]}{h_{ie}(R_1 + R_{12} + R_2) + R_1(R_2 + R_{12})} \tag{P-30}$$

and

$$K \approx \frac{R_2}{R_2 + R_{12} + R_1h_{ie}/(R_1 + h_{ie})} \times \frac{R_1}{R_1 + h_{ie}} = \frac{R_1R_2}{h_{ie}(R_1 + R_{12} + R_2) + R_1(R_2 + R_{12})} \tag{P-31}$$

(b) Show that the h parameters of the amplifier circuit of Fig. P-16 have the following values:

$$h_i \approx h_{ie} \qquad h_r \approx 0$$
$$h_f \approx -h_{fe}^2K \qquad h_0 = h_{oe} \tag{P-32}$$
$$\Delta h \approx h_{ie}h_{oe}$$

(c) The amplifier of Fig. P-16 is used in the pi circuit of Fig. 65a to form the n-p-n version of the symmetrical Eccles-Jordan circuit of Fig. 96b. By the use of Eq. (31), show that the circuit is bistable if

$$-K^2h_{fe}^2 + 1 + Kh_{oe}\left[R_{12} + \frac{(R_1 + R_{12})h_{ie}}{R_1}\right] < 0 \tag{P-33}$$

(d) Show that $K\left[R_{12} + \frac{(R_1 + R_{12})h_{ie}}{R_1}\right] = R_l$ and that Eq. (P-33) therefore reduces to

$$h_{fe} > \frac{h_{ie}(R_1 + R_{12} + R_2) + R_1(R_{12} + R_2)}{R_1R_2} \tag{P-34}$$

(e) With the aid of Eq. (P-29) show that the criterion of Eq. (P-34) is equivalent to the requirement that the low-frequency current amplification of the circuit of Fig. P-16 exceed unity when the output load resistance is h_{ie}. (This amplification might be termed the "open-loop current amplification" of the circuit of Fig. 96b.)

27.1–3. (a) By inspection of the circuit of Fig. P-17, show that if the resistances R_2 in the Eccles-Jordan circuit of Fig. 93b are made infinite and the circuit is symmetrical, the necessary collector biasing voltages being supplied by batteries in series with the

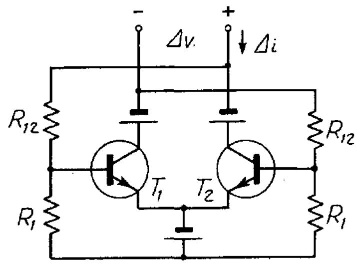


FIG. P-17.

collectors, the current Δi is given by the following expression (note that the base-emitter input resistance approximates h_{ie}):

$$\Delta i \approx \frac{h_{oe}\Delta v}{2} + \frac{\Delta v}{2R_{12} + 2\dfrac{R_1 h_{ie}}{R_1 + h_{ie}}}\left(1 - \frac{R_1}{R_1 + h_{ie}} h_{fe}\right) \tag{P-35}$$

and that the input resistance R_i is therefore given by the expression:

$$\frac{1}{R_i} \approx \frac{1}{2}\left[h_{oe} + \frac{R_1(1 - h_{fe}) + h_{ie}}{R_{12}(R_1 + h_{ie}) + R_1 h_{ie}}\right] \tag{P-36}$$

(b) A resistance $2R_2$ is connected between the collectors to form the circuit of Fig. 96b. Making use of the fact that the circuit is bistable if $2R_2 > |R_i|$, show that the criterion for bistability is

$$h_{fe} > 1 + \frac{h_{ie}}{R_1} + \frac{(1 + R_2 h_{oe})[R_{12}(R_1 + h_{ie}) + R_1 h_{ie}]}{R_1 R_2} \tag{P-37}$$

(c) Show that Eq. (P-37) reduces to Eq. (39) if $R_2 h_{oe} \ll 1$.

27.1–4. Using the parameters of Eqs. (P-32) in the appropriate relations listed in Table I, find the values of R_i corresponding to the negative-slope portions of transistor curves equivalent to the tube-circuit characteristics of Figs. 69a and 69c.

31.2–1. Derive Eqs. (40) to (45).

32.1–1. Are the following relations consistent?

$$I_e = \frac{1}{h_{fbo}}(1 + jf/f_{ab})I_c \tag{P-38}$$

and

$$i_e = \frac{1}{h_{fbo}}\left(i_c + \frac{1}{f_{ab}}\frac{di_c}{dt}\right) \tag{P-39}$$

36.2–1. By listing the states of equilibrium of the various stages of the circuits of Figs. 123 to 125 following each input pulse, show that the order in which the various combinations are assumed is reversed if the triggering voltages for the second, third, and fourth stages are obtained from the plate of the 1 tube of the preceding stages, instead of the plate of the 0 tube.

37.1–1. (a) Show a block diagram of the Bagley circuit discussed in Sec. 37.1 and explain its operation.

(b) Devise a practical decade circuit of this type.

37.5–1. Show a block diagram (similar to Fig. 123a) of a counting circuit that completes one cycle of stable states in twelve counts.

37.5–2. Show a block diagram of a circuit that completes one cycle of stable states in six counts.

38.2–1. Show how count-indicator glow tubes may be connected in the circuit of Fig. 125, each glow tube being connected to the collector circuit of only three transistors. (Note that the voltages available in a transistor circuit would probably be insufficient for reliable operation of the glow indicators.)

39.2–1. Assume that pentodes T_0 to T_4 of the circuit of Fig. 128 are conducting. Assume a supply voltage and determine reasonable possible values of voltage at the plates of the pentodes and at the junctions between the resistors connected in series from each plate to the positive supply terminal. Show that the voltage impressed upon the glow-tube branches is highest for the branch containing G_0.

40.2–1. Show that the order in which the tubes of the circuit of Fig. 131a conduct can be reversed by changing the commutating-capacitor connections as explained in Sec. 40.2.

45.2–1. (a) By reference to Eq. (69), show that, if x and y are defined as in Fig. 158, the characteristic equation for the circuit of Fig. 157a may be written in the form

$$p^2 + \frac{x+y}{\sqrt{LC}}\,p + \frac{xy+1}{LC} = 0.$$

(b) Show that the roots of the characteristic equation are complex, and the solution of Eq. (69) therefore oscillatory, if $(x - y + 2)(x - y - 2) < 0$, and that this inequality can be satisfied only if Eq. (70) holds.

45.2–2. With the aid of Eqs. (70) to (75) and Table VII, verify the general aspects of Fig. 158.

45.3–1. By analyzing Eqs. (71) and (76) in conjunction with Eqs. (80) and (81), construct a table analogous to Table VII for the current-stable circuit of Fig. 157b.

46.1–1. Derive Eq. (84) by eliminating the time variable from the differential equations for the circuit of Fig. 157a.

47.1–1. (a) Show that, if $L/(R + R_{iv})R_{iv}C \gg 1$, Eq. (72) reduces to:

$$\alpha \approx -\frac{1}{2R_{iv}C} \tag{P-40}$$

(b) By expanding $\left(1 - \dfrac{R + R_{iv}}{\alpha^2 R_{iv}LC}\right)^{1/2}$ in series form, show that if $L/(R + R_{iv})R_{iv}C \gg 4$, Eq. (77) reduces to:

$$\omega' \approx \alpha - \frac{R + R_{iv}}{2\alpha R_{iv}LC} \approx \alpha + \frac{R + R_{iv}}{L} \tag{P-41}$$

$$\alpha - \omega' \approx -\frac{R + R_{iv}}{L} \tag{P-42}$$

$$\alpha + \omega' \approx -\frac{1}{R_{iv}C} \tag{P-43}$$

(c) In a similar manner, show that if $R'R_{ic}C/L \gg 4$,

$$\alpha - \omega' \approx -\frac{1}{R'C} \tag{P-44}$$

$$\alpha + \omega' \approx -\frac{R_{ic}}{L} \tag{P-45}$$

where $R' = RR_{ic}/(R + R_{ic})$.

(d) Show that, if $R'R_{ic}C/L \gg 4$ and R_{ic} is positive, A' in Eq. (76) is very small in comparison with B' and Eq. (76) therefore reduces to

$$I = B'\epsilon^{-t/R'C} + I_o \tag{P-46}$$

(e) Show that, if R_{ic} is negative, Eq. (76) reduces to

$$I = A'\epsilon^{-R_{ic}t/L} \tag{P-47}$$

47.1–2. Under the assumption that the time constants in the various ranges are known, show how the general forms of the curves of Fig. 162a may be derived from Fig. 161a.

47.2–1. A current-voltage characteristic of the general form of Fig. 69e may be assumed to be made up of a section through the origin with a slope corresponding to a resistance of $-70{,}000$ ohms, and two sections having slopes corresponding to a resistance of 70,000 ohms and current-axis intercepts of 0.7 ma. The port at which the characteristic is observed is shunted by a capacitance of 0.01 μf and a resistance of 100 kΩ. Under the assumption that the internal inductance at the port is 10^{-3} henry, determine the approximate frequency of relaxation oscillation.

48.1–1. Using the information presented in the first paragraph of Sec. 48.1, analyze Figs. 165a and 165b in a manner analogous to that used in Sec. 47 to explain Figs. 161a and 161b.

48.1–2. Under the assumption that the time constants in the various ranges are known, show how the general forms of the curves of Fig. 166 may be derived from Fig. 165.

51.3–1. (a) Explain the change of slope in the negative half of the wave of grid voltage v_{G1} of Fig. 176b. (Note that the rate of change of v_{G1} is dependent upon both the rate of change of plate voltage of T_2 and the rate at which C_{12} discharges.)

(b) Why is this effect not apparent in Fig. 176a?

52.2–1. Assume that the negative grid supply voltage V_{GG} is used in the multivibrator circuit of Fig. 175 and that transition from T_2 to T_1 has just been initiated. During transition from T_2 to T_1, the grid voltage of T_1 rises abruptly from cutoff to a value which is usually positive, and then falls relatively rapidly to the value at which grid current ceases. It then falls toward the negative value V_{GG} with a time constant $(R_1 + R_l')C_{12}$, where $R_l' = r_{p2}R_2'/(r_{p2} + R_2')$. Make the following assumptions:

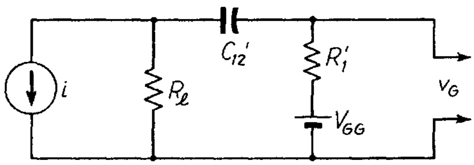
1. The load resistances R_2 and R_2' are so large relative to the grid resistance of the conducting tube that the positive grid swing of the conducting tube is small. Under this assumption the grid voltage of T_1 at the termination of transition is zero, and the drop in plate voltage kV_{PP} is the difference between V_{PP} and the intersection of the load line with the zero-grid-voltage plate characteristic.

2. The grid resistance of the conducting tube while the grid is positive is so small in comparison with R_1 that the time taken for the grid voltage of T_1 to fall to zero is a negligible portion of the cycle.

3. Grid-current cutoff occurs at zero grid-cathode voltage. (Actually, the cutoff voltage is usually approximately $-\frac{1}{2}$ to $-\frac{3}{4}$ volt.)

Under these assumptions the grid voltage of T_1 rises from the cutoff value $-V_{PP}/\mu$ to zero during transition from T_2 to T_1 and then falls exponentially toward V_{GG}. The varying component of this voltage following the termination of transition is $V_{GG}(1 - \epsilon^{-t/(R_1+R_l')C_{12}})$. Immediately preceding transition, the grid voltage of T_2 was falling toward V_{GG} and had a value somewhere between zero and the plate-current cutoff value $-V_{PP}/\mu$, probably close to the latter. If it is assumed to have had the value $-V_{PP}/\mu$ at the initiation of transition, the value immediately following transition is $-V_{PP}/\mu - kV_{PP}$, where kV_{PP} is the magnitude of the change in plate voltage of T_1 accompanying the rise of grid voltage of T_1 from cutoff to zero.

If the circuit is assumed to be linear, the grid voltage v_{G2} of T_2 following transition



$$i = g_m V_{GG}(1 - \epsilon^{-t/(R_1+R_l)C_{12}'})$$

$$R_l = r_{p1}R_2/(r_{p1} + R_2)$$

$$v_{G2} = -V_{PP}(k + 1/\mu) \quad \text{when} \quad t = 0$$

FIG. P-18.

from T_2 to T_1 may be determined from the equivalent circuit of Fig. P-18, in which $R_l = r_{p1}R_2/(r_{p1} + R_2)$.

(a) For a completely symmetrical circuit, show that

$$v_{G2}/V_{PP} = m - \left(k + 1/\mu + m + \frac{A_v m}{(R_1 + R_l)C_{12}}t\right)\epsilon^{-t/(R_1-R_l)C_{12}} \qquad \text{(P-48)}$$

in which

$$m = |V_{GG}|/V_{PP}$$

$$A_v = g_m R_l \qquad \text{(voltage amplification of } T_1\text{)}$$

(b) Show that the rate of change of v_{G2} when $t = 0$ is given by the relation:

$$\frac{d}{dt}\left(\frac{v_{G2}}{V_{PP}}\right)\bigg|_0 = [k + 1/\mu + m(1 - A_v)]/(R_1 + R_l)C_{12} \qquad \text{(P-49)}$$

(c) Show that the maximum value of v_{G2} occurs when

$$t = t_m \equiv (R_1 + R_l)C_{12}\frac{m(A_v - 1) - k - 1/\mu}{A_v m} \qquad \text{(P-50)}$$

and that the maximum value is

$$\text{Max } v_{G2}/V_{PP} = m - (k + m + 1/\mu + A_v m)/\epsilon \qquad \text{(P-51)}$$

(d) Choose a suitable tube and circuit parameters and determine k and A_v. Plot the curve of v_{G2}/V_{PP} for several negative values of m.

52.4–1. (Sequel to Prob. 52.1–1.) Except at very small values of $|m|$, the time t_o required for the grid voltage of the nonconducting tube to rise from its approximate maximum negative value $-V_{PP}(k + 1/\mu)$ to the cutoff value $1/\mu$ is small in comparison with $(R_1 + R_l)C_{12}$.

(a) Under the assumption that the variation of grid voltage during this interval is sufficiently linear so that rate of change of voltage remains approximately constant at its initial value, show that

$$t_o \approx \frac{k(R_1 + R_l)C_{12}}{k + 1/\mu + m(1 - A_v)} \tag{P-52}$$

and that the frequency of oscillation is given by the relation

$$f \approx \frac{k + 1/\mu + m(1 - A_v)}{2k(R_1 + R_l)C_{12}} \approx \frac{k + 1/\mu + m(1 - A_v)}{2kR_1C_{12}} \tag{P-53}$$

(b) Justify the statement that the frequency stability of a multivibrator with negative grid-supply voltage against changes of line voltage is favored by deriving the grid-supply voltage from the same source as the plate-supply voltage and making the magnitude of the grid-supply voltage large.

(c) Account for the difference between Eqs. (91) and (P-53) for $m = 0$. (Is the value of v_{G2} assumed at the termination of transition from T_2 to T_1 valid when $V_{GG} = 0$?)

(d) Is the difference between Eqs. (91) and (P-53) in agreement with the experimental curve of Fig. 181?

(e) Give a physical explanation of the difference between the values of $\partial f/\partial V_{GG}$ predicted by Eqs. (91) and (P-53).

53.1–1. Derive Eq. (89) by solving for t_o and t_o' in Eq. (87).

53.3–1. (a) Show that the frequency of the asymmetrical multivibrator of Fig. 182a is given by the approximate formula:

$$f \approx \frac{m + k}{(k - \mu)R_1'C_{12}'} \tag{P-54}$$

(b) The circuit of Fig. 182a is made symmetrical by making the circuit elements of the two halves equal and adding another cathode follower, T_5, and another diode, T_6, in the grid circuit of T_1. Show that the period of oscillation is

$$\frac{1}{f} \approx (k - \mu)R_1C_{12}\left(\frac{1}{m + k} + \frac{1}{m' + k}\right) \tag{P-55}$$

where $m = V/V_{PP}$ and m' is a similar ratio for the control voltage applied to the diode T_6.

(c) Show that, if $m = m_o + \Delta m$ and $m' = m_o - \Delta m$,

$$f \approx \frac{(m_o + k)^2 - \overline{\Delta m}^2}{2(m_o + k)(k - \mu)R_1C_{12}} \tag{P-56}$$

55.9–1. (a) Show, in block form, a circuit that will provide two periodic rectangular output voltages of the approximate form shown in Fig. P-19. The circuit is to be

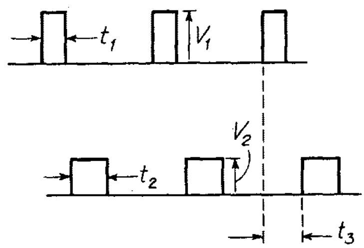


FIG. P-19.

driven by a sinusoidal voltage of 10-volt crest value and adjustable frequency ranging from 1000 cps to 5000 cps. The values of t_1, t_2, t_3, V_1, and V_2 are to be independently adjustable and should not be affected by the driving frequency. (Change of any one should not affect any other.) The values of t_1 and t_2 should be adjustable from 5 to 10μsec, and t_3 should be adjustable from 50 to 100 μsec.

(b) Design the complete circuit, which is to be operated from a single high-voltage power supply. Determine the proper values of the circuit resistances, capacitances, and operating voltages. (If monostable circuits are used, determine the required time constants with the aid of Fig. 179.)

(c) Inspect the complete circuit to make sure that all circuit elements have been taken into account in determining differentiator time constants, that diodes are not continuously forward-biased, that coupling capacitors and diodes do not cause undesirable clamping, and that electrode dissipations are not excessive.

(d) If possible, build the circuit and test it.

57.3-1. (a) Derive Eq. (94) by determining the time required to charge the capacitor C of Fig. 201 from its approximate minimum voltage $2V_{\min} - V_{\max}$ to its maximum voltage $V_{\max}$ through R.

(b) Derive Eq. (95) by finding the time required to charge a capacitor of capacitance C from the voltage $2V_{\min} - V_{\max}$ to the voltage $V_{\max}$ if the charging current I is constant.

59.3-2. (a) Justify the use of the equivalent circuit of Fig. P-20 in determining the

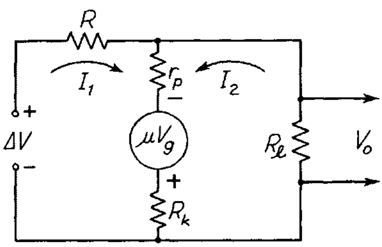


FIG. P-20.

initial response of tube T_1 of the circuit of Fig. 213a to a voltage step ΔV applied to the control grid.

(b) Show that the initial output is zero if $R = \dfrac{1}{g_m} + \dfrac{1+\mu}{\mu} R_k$.

62.1–1. (a) Derive the characteristic equation for the circuit of Fig. 220 for the portion of the transition period in which the grid conducts. Assume the grid resistance r_g to be constant.

(b) By application of the quadratic formula and of the binomial expansion, derive approximate expressions for the roots of the characteristic equation.

(c) Use the relations $M^2 = kL_1L_2$ and $L_2 = N^2L_1$ in order to express the roots in terms of L_1, k, and N, rather than L_1, L_2, and M.

63.2–1. (a) Construct approximate equivalent circuits analogous to that of Fig. 223 for the common-base and common-collector blocking oscillators.

(b) Making use of the following relations, derive Eqs. (107) and (108):

$$i_E = \frac{1}{h_{fbo}}\left[1 + \frac{p}{\omega_{ab}}\right] i_C$$

$$i_B = \left[1 + \frac{h_{fbo}\omega_{ab}}{p + \omega_{ab}}\right] i_E$$

66.2–1. (a) An approximate equivalent circuit for the transistor circuit of Fig. 230 in the high-current state, if perfect coupling is assumed and $R_1/N^2 + R_2 \ll h_{ib}$, is shown in Fig. P-21. Using this circuit, derive an expression for the root of the characteristic equation.

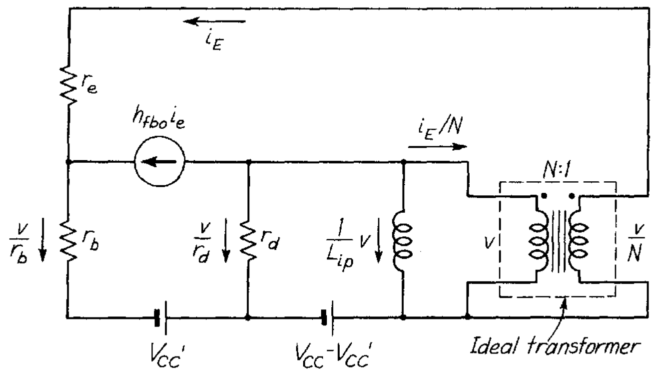


FIG. P-21.

(b) Making use of the fact that the short-circuit input resistance h_{ib} of the transistor is $r_e + (h_{fb} + 1)r_b$, show that the expression derived in (a) is equivalent to Eq. (126).

(c) Under the assumption that the diode current decays linearly from the initial value given by Eq. (127) and that transition to the low-current state begins when the diode current is zero, derive Eq. (129).

66.2–2. (a) Show the common-emitter form of the circuit of Fig. 230.

(b) Construct an equivalent circuit analogous to that of Fig. 232 or Fig. P-21.

(c) Derive the characteristic equation.

(d) What is the critical value of diode resistance at which transition to the cutoff state begins?

66.2–3. Repeat Prob. 66.2–1 for the common-collector form of the circuit of Fig. 230.

66.3–1. The emitter current i_E of a transistor in the common-base connection and with zero collector-circuit resistance is equal to v_E/h_{ib}', where v_E is the collector voltage and h_{ib}' is the value of h_{ib} at some value of emitter current between zero and i_E. To a first approximation, the collector current i_C is equal to $h_{fb}i_E$.

(a) Find the approximate value of the collector current at point P' of Fig. 231a.

(b) Find the approximate value of the collector current at point P in terms of V_{CC}, V_{CC}', and h_{ib}' under the assumption the h_{ib} is so much larger than R_1 and R_2 that $R_1 \approx N^2h_{ib}$. (Assume that the path of operation below P has a constant slope of magnitude $1/h_{ib}'$.)

(c) Show that the difference between the collector currents at P' and at P is given by Eq. (127). Derive Eq. (129).

67.3–1. (a) Plot the current-voltage characteristic of the circuit of Fig. 236a for the following circuit values: $R_1 = 1000$ ohms, $R_2 = 25{,}000$ ohms, $V_1 = 5$ volts, and $V_2 = 25$ volts.

(b) Determine the effect upon the characteristic of reversing the polarity of V_1 and V_2, individually and simultaneously.

67.3–2. (a) Plot the current-voltage characteristic of two parallel circuits of the form of Fig. 237a. The breakdown voltage of one diode is 10 volts and the series resistance is 10,000 ohms. The breakdown voltage of the other diode is 20 volts and the series resistance is 20,000 ohms. The diodes are biased so that the characteristic of each branch is symmetrical with respect to the origin.

67.5–1. Design a resistance-diode network that will provide the current-voltage characteristic shown in Fig. P-22.

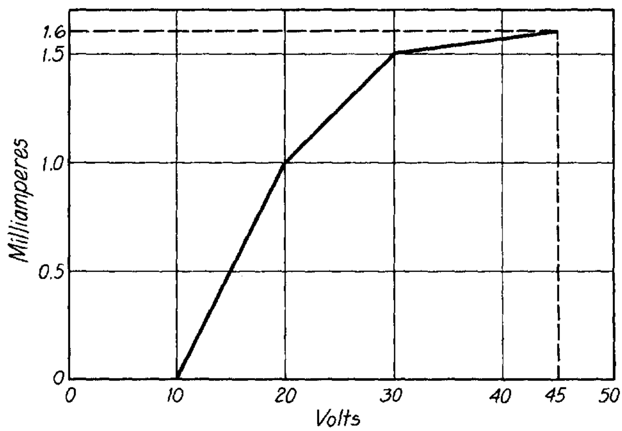


FIG. P-22.

67.5–2. Design a resistance-diode network that will provide the current-voltage characteristic shown in Fig. P-23.

67.5–3. A parabolic current-voltage characteristic passes through the origin and through the point 10 volts, 1 ma.

(a) Approximate this characteristic by means of two linear segments, adjusting the lengths and positions of the segments visually to minimize error at voltages ranging from zero to 20.

(b) Design a resistance-diode network that will provide the linear approximation.

(c) Determine the maximum current error.

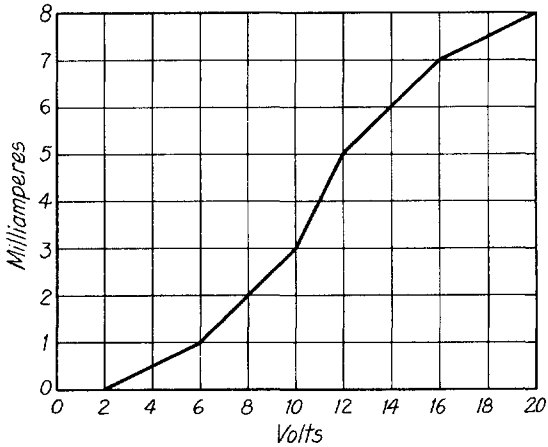


FIG. P-23.

67.5–4. (a) Repeat Prob. 67.5–3, using a three-segment approximation.

(b) Repeat Prob. 67.5–3, using a four-segment approximation.

67.5–5. Design a resistance-diode network that will provide a three-segment approximation of the plate characteristic curve of a 6F6 pentode at zero grid voltage.

67.5–6. Show how two breakdown-diode circuits in parallel may be used with a symmetrical Eccles-Jordan circuit to obtain a circuit with three stable states.

68.1–1. (a) Determine the time required for the output voltage of the circuit of Fig. 245 to increase from 0.1 to 0.9 of its final value when the input voltage is a step.

(b) Determine the frequency f_c at which the amplitude of the output voltage has fallen to 0.707 of its midband value if the input voltage is sinusoidal.

(c) Show that the product of the rise time t_r found in (a) and the upper cutoff frequency f_c found in (b) is 0.35.

(d) Using values of t_r and f_c found from Figs. 247 and 248, determine the values of t_rf_c for $Q = 0.585, 0.662, 0.71$, and 1.

68.2–1. (a) Show that the roots of the characteristic equation for the circuit of Fig. 246b are

$$p = -\frac{\pi f'}{Q^2}[1 \pm \sqrt{1 - 4Q^2}] \tag{P-57}$$

where $f' = 1/2\pi RC$ and $Q = \dfrac{1}{R}\sqrt{\dfrac{L}{C}}$.

(b) Show that, when $Q > 0.5$,

$$-\frac{v_o}{g_mRV} = 1 + \epsilon^{-\pi f't/Q^2}\left[\frac{1 - 2Q^2}{\sqrt{4Q^2 - 1}}\sin\frac{\pi\sqrt{4Q^2 - 1}}{Q^2}f't + \cos\frac{\pi\sqrt{4Q^2 - 1}}{Q^2}f't\right] \tag{P-58}$$

in which V is the amplitude of a step voltage impressed upon the circuit, and v_o is the instantaneous output voltage.

(c) Show that, when $Q = 0.5$,

$$-\frac{v_o}{g_m RV} = 1 - \epsilon^{4\pi f' t} - 2\pi f' t \epsilon^{-4\pi f' t} \tag{P-59}$$

(d) Show that, when $Q < 0.5$,

$$-\frac{v_o}{g_m RV} = 1 - \frac{2Q^2 + \sqrt{1 - 4Q^2} - 1}{2\sqrt{1 - 4Q^2}} \epsilon^{p_1 t} + \frac{2Q^2 - \sqrt{1 - 4Q^2} - 1}{2\sqrt{1 - 4Q^2}} \epsilon^{p_2 t} \tag{P-60}$$

where $p_1 = -(1 + \sqrt{1 - Q^2})\pi f'/Q^2$ and $p_2 = -(1 - \sqrt{1 - Q^2})\pi f'/Q^2$.

68.2–2. (a) Using the high-frequency equivalent circuit shown in Fig. 246b, show that the high-frequency voltage amplification of the resistance-coupled amplifier stage of Fig. 246a is

$$A_v = g_m R \frac{1 + jQ^2F}{1 - Q^2F^2 + jF} \tag{P-61}$$

where $Q = \dfrac{1}{R}\sqrt{\dfrac{L}{C}}$ and $F = 2\pi RCf$.

(b) Show that the ratio of the magnitude of the voltage amplification at high frequencies to the magnitude of the voltage amplification at zero frequency (actually the midband amplification when the coupling capacitor C' is used) is

$$\left|\frac{A_v}{A_{vo}}\right| = \sqrt{\frac{1 + Q^4F^4}{Q^4F^4 + (1 - 2Q^2)F^2 + 1}} \tag{P-62}$$

(c) Show that the phase shift of the output voltage relative to the input voltage is

$$\theta = \tan^{-1}[(Q^2 - 1)F - Q^4F^3] \tag{P-63}$$

and that the delay time is given by the relation

$$f't_d = \frac{1}{2\pi F}\tan^{-1}[(Q^2 - 1)F - Q^4F^3] \tag{P-64}$$

where $f' = 1/2\pi RC$.

73.2–1. A three-point analysis indicates that, if the voltage of a nonlinear two-terminal resistance is varied sinusoidally about an operating value, the magnitude of the fundamental component of current through the resistance is to a first approximation equal to $\frac{1}{2}(I_{max} - I_{min})$, where I_{max} and I_{min} are the instantaneous currents corresponding to the positive and negative crests of voltage. A five-point analysis shows that a closer approximation to the magnitude of the fundamental component of current is $\frac{1}{3}(I_{max} + I_{\frac{1}{2}} - I_{-\frac{1}{2}} - I_{min})$, where $I_{\frac{1}{2}}$ is the current at the instant the alternating voltage is half its positive crest value and $I_{-\frac{1}{2}}$ is the current at the instant the alternating voltage is half its negative crest value.

(a) Using the three-point approximation, construct a curve of $|G_i|$ vs. voltage amplitude at an operating voltage of 12 volts if the nonlinear resistance has a current-voltage characteristic corresponding to the values listed below.

(b) Repeat, using the five-point approximation.

Values for Prob. 73.2–1

Volts	*Milliamperes*	*Volts*	*Milliamperes*
0	0	12	5.2
2	2.7	14	3.25
4	6.5	16	2.0
6	11.2	18	1.4
6.6	12.0	20	1.1
7	12.2	21	1.0
7.4	11.9	22	1.2
8	11.3	24	2.0
10	8	26	4.0

73.3–1. (a) By reference to Fig. 158, show that the circuit of Fig. 157a is nonoscillatory if the value of R exceeds the magnitude of the negative resistance.

(b) By reference to Fig. 159, show that the circuit of Fig. 157b is nonoscillatory if the value of R is less than the magnitude of the negative resistance.

73.3–2. (a) Show that, if a series resonant circuit is connected across a voltage-stable negative-resistance element having a very small value of internal shunt capacitance C_i, the circuit approximates that of Fig. 251b with R_{ic} replaced by R_{iv}.

(b) By reference to Fig. 159, show that sustained sinusoidal oscillation is possible in this circuit only if $R > |R_i|$.

(c) Is this requirement consistent with the requirement that the load line intersect the voltage-stable current-voltage characteristic only in the negative-resistance range?

(d) With the aid of Figs. 251a and 158, repeat (a), (b), and (c) for a parallel resonant circuit connected across a current-stable element.

73.5–1. Show that the damping factor α and the angular frequency of oscillation ω for the circuits of Figs. 251a and 253a are identical if $G_c = RC/L$ and $L_c = LR_{iv}/(R_{iv} + R)$.

74.4–1. Figure P-24b shows a family of current-voltage characteristics for the circuit of Fig. P-24a with several values of grid bias. The curves were derived dynamically at a frequency high in comparison with $1/R_1C_{12}$.

(a) Using the chord-slope approximation for the negative conductance G_i, plot curves of $|G_i|$ vs. alternating-voltage amplitude at the various values of grid bias.

(b) If $L/rC = 5000$ ohms, determine the amplitude of oscillation of a parallel resonant circuit connected between the plates, when the grid bias is zero.

(c) Determine the curve of $|G_i|$ vs. v if the voltage across the resonator is rectified and the rectified voltage is added to the bias in such a manner that the negative grid bias is equal to the amplitude of the voltage across the resonator at all amplitudes of oscillation. From this curve, find the amplitude of oscillation if $L/rC = 5000$ ohms.

(d) Compare the rate of change of amplitude with r with and without the automatic bias.

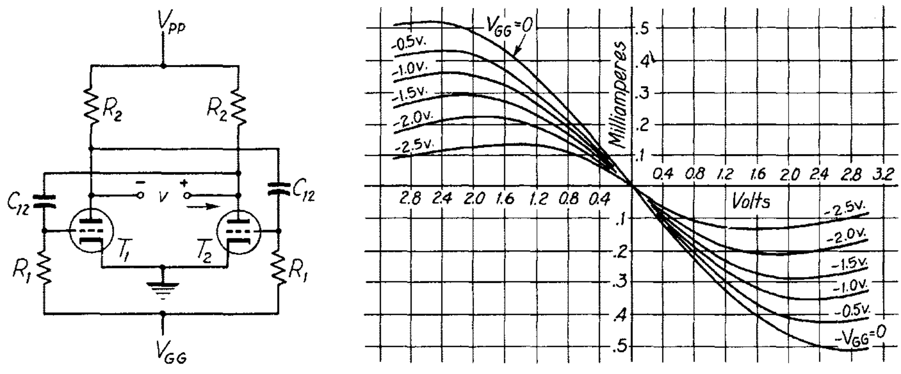
74.5–1. Equation (137) may be conveniently written in the form

$$f = f_o\left(\frac{R + R_{iv}}{R_{iv}}\right)^{1/2} \tag{P-65}$$

By differentiating Eq. (P-65) with respect to R_{iv} and eliminating f_o from the result by means of Eq. (P-65), show that

$$\frac{df}{f} = -\frac{R}{2(R + R_{iv})}\frac{dR_{iv}}{R_{iv}} \approx -\frac{R}{2R_{iv}}\frac{dR_{iv}}{R_{iv}} \tag{P-66}$$

75.2–1. By differentiating Eq. (131) with respect to time and making use of Eq. (132), show that the instantaneous amplitude of oscillation is proportional to $RC/L - 1/|R_{iv}| \equiv G_c - |G_i|$.



(a) (b)

FIG. P-24.

75.4–1. (a) Show that the effective shunt conductance of the L, R, C portion of the circuit of Fig. 251a is

$$G_c = \frac{R}{R^2 + \omega^2 L^2} \tag{P-67}$$

(b) Show that the susceptance is

$$B_c = \omega C - \frac{\omega L}{R^2 + \omega^2 L^2} \tag{P-68}$$

(c) Show that the resonance frequency, at which B_c is zero, is

$$\omega_r = \sqrt{1/LC - R^2/L^2} \tag{P-69}$$

and that the shunt conductance at resonance is

$$G_{cr} = RC/L \tag{P-70}$$

(d) Show that, if $1/LC \gg R^2/L^2$, i.e., if $Q \gg 1$,

$$G_c \approx \frac{G_{cr}}{RG_{cr} + \omega^2/\omega_r^2} \tag{P-71}$$

$$B_c \approx \frac{\omega}{\omega_r}\sqrt{\frac{C}{L}} - \frac{(\omega/\omega_r)\sqrt{L/C}}{R^2 + (\omega^2/\omega_r^2)L/C} \tag{P-72}$$

(e) Show that, if also $\omega^2/\omega_r^2 \gg RG_{cr}$, i.e., if $\omega^2/\omega_r^2 \gg R^2C/L$, Eqs. (P-71) and (P-72) reduce to:

$$G_c \frac{L}{RC} \approx \left(\frac{f_r}{f}\right)^2 \tag{P-73}$$

$$B_c\sqrt{\frac{L}{C}} \approx \frac{f}{f_r} - \frac{f_r}{f} \tag{P-74}$$

75.4–2. In the circuit of Fig. 251a, $L = 8$ mh, $C = 1\ \mu$f, and $R = 20$ ohms. The values of G_i and B_i at several amplitudes of voltage across the negative-resistance element are given in the following table:

Amplitude in volts	G_i *in millimhos*	B_i *in millimhos*
0	−3.0	1.0
1	−2.8	0.78
2.5	−2.6	0.60
5	−2.4	0.45
10	−2.2	0.35
15	−2.0	0.25

(a) With the aid of Eqs. (P-69), (P-70), (P-73), and (P-74), construct the admittance diagram, indicating the scale of frequency on the $-Y_c$ curve and the scale of amplitude on the Y_i curve.

(b) Repeat (a) if B_i is zero at all amplitudes.

(c) Repeat (a) for $R = 40$ ohms.

75.4–3. The admittance diagram for a reflex-klystron oscillator, with the load coupled to the resonator through an unmatched transmission line, is shown in Fig. P-25.[1] Decrease of direct operating voltage of one of the electrodes causes the $-Y_i$

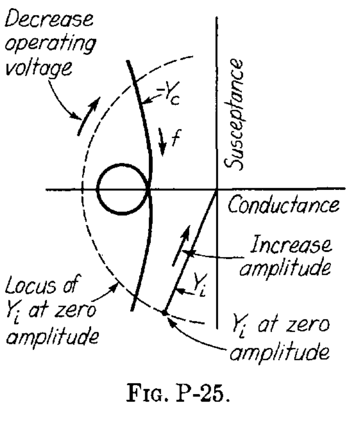


FIG. P-25.

curve to be rotated clockwise, as shown by the dotted curve. Discuss the way in which the amplitude and frequency of oscillation would be expected to change with the electrode operating voltage as the electrode voltage is first gradually raised and then gradually lowered over the indicated range.

75.5–1. By a procedure analogous to that used in the derivation of Eqs. (138), (143), and (144), show that equilibrium amplitude and frequency of oscillation are obtained in the series circuit of Fig. 253b when $R_i = -R_c$ and $X_i = -X_c$.

[1] H. J. Reich, J. G. Skalnik, P. F. Ordung, and H. L. Krauss, *Microwave Principles*, Sec. 10-32, D. Van Nostrand Co., Inc., Princeton, N. J., 1957.

76.1–1. Devise parallel-resonator oscillators based upon the negative-resistance circuits of Figs. 68, 70, 73, 74, and 75.

76.1–2. Devise series-resonator oscillators based upon the negative-resistance circuits of Figs. 68, 70, 73, 74, and 75.

77.2–1. (a) Show that the input admittance of the circuit of Fig. 267a is given by the following expression when the grid current is negligible:

$$Y_i = \frac{R'^2\omega^2C'^2(1+\mu) + r_pR'\omega^2C'^2 + 1 + j\omega C'(\mu R' + r_p)}{r_p(R'^2\omega^2C'^2 + 1)} \tag{P-75}$$

(b) Show that, if $g_mR' \gg 1$, the effective input capacitance is maximum when $\omega R'C' = 1$, that the maximum capacitance is equal to $g_m/2\omega$, and that the corresponding input conductance is $g_m/2$.

78.3–1. Show that the impedance Z_f defined by Eq. (151) is the open-circuit forward transfer admittance of the network of Fig. 269a.

80.2–1. The following circuit and transistor values apply to a tuned-output oscillator:

$r = 0.1\ \Omega$	$h_{ie} = 2000\ \Omega$
$L = L' = 10^{-3}$ h	$h_{re} = 16 \times 10^{-4}$
$C = 10^{-9}$ f	$h_{fe} = 50$
$M^2 = 10^{-7}\ h^2$	$h_{oe} = 5 \times 10^{-5}$ mho

(a) With the aid of Tables A-I and A-II, determine the values of the transistor y parameters in the common-emitter, common-base, and common-cathode connections.

(b) Find the frequency of oscillation of the oscillator in the common-emitter, common-base, and common-collector connections.

(c) Compare the departure of the oscillation frequency from the resonance frequency of the feedback network, and the dependence of this departure upon the transistor parameters, in the three connections.

81.2–1. By the general method used in Sec. 45 in the derivation of Figs. 158 and 159, construct a similar diagram for the tuned-plate oscillator, starting with Eqs. (157) to (159).

82.4–1. By substituting the common-grid y parameters (Table III) into Eqs. (171A) and (172A) and solving the resulting equations for f and g_m, determine the frequency of oscillation and the criterion for oscillation of the common-grid tuned-output oscillator.

82.4–2. (a) By substituting the common-collector h parameters into Eqs. (176) and (177), derive the expressions for the frequency of oscillation and the criterion for oscillation of the common-collector tuned-output oscillator.

(b) By means of the identities listed in Table A-I, write the expressions of part (a) in terms of the common-emitter h parameters.

82.5–1. (a) By using in Eq. (149) the appropriate z parameters listed in Table IX, derive the expressions for the frequency of oscillation and the criterion for oscillation of the tuned-input oscillator.

(b) By using the appropriate tube y parameters listed in Table III, convert the expressions of (a) into the proper forms for a tuned-grid oscillator.

(c) Use Tables A-I and A-II to convert the expressions of (a) into the proper forms for the tuned-base oscillator, in terms of the common-emitter h parameters.

82.5–2. Repeat Prob. 82.5–1 for the tube and transistor Colpitts oscillators.

82.5–3. Repeat Prob. 82.5–1 for the tube and transistor Hartley oscillators.

83.2-1. Show that the dynamic (large-signal) value of g_m may be approximated by the slope of the chord joining the limits of the path of operation on the characteristic relating plate current with grid voltage.

88.1-1. Derive Eqs. (202) and (203) by substituting $1/j\omega C$ and R for z_1 and z_2, respectively, in Eqs. (200) and (201) and solving the real and imaginary parts of the resulting equation.

88.1-2. Derive Eqs. (204) and (205) by substituting R and $1/j\omega C$ for z_1 and z_2, respectively, in Eqs. (200) and (201) and solving the real and imaginary parts of the resulting equation.

88.2-1. A phase-shift oscillator consists of a closed chain of three stages of the form of Fig. P-26. Under steady-state conditions of oscillation the output voltage of each

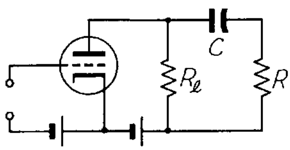


FIG. P-26.

stage must lead the input voltage of that stage by 120 degrees and the magnitude of the output voltage must be equal to the magnitude of the input voltage. (a) By making use of these relations, show that the frequency of oscillation and the criterion for oscillation are given by Eqs. (209) and (210).

(b) Show that the factor 0.092 in Eq. (209) must be replaced by 0.276 if the positions of C and R are interchanged in Fig. P-26.

88.2-2. (a) Derive Eqs. (211) and (212).

(b) Show that the factor 0.092 in Eq. (211) must be replaced by 0.276 if the positions of the capacitances C and the resistances R are interchanged in the circuit of Fig. 301.

88.7-1. Derive the voltage transfer function of the circuit of Fig. 305.

88.8-1. Derive Eqs. (213) to (215).

ADDENDUM

Maximum Oscillation Frequency of Negative-Resistance Oscillators

The equivalent circuit of Fig. 251a, in which C is assumed to be the sum of the external tuning capacitance C_c and the internal capacitance C_i of the voltage-stable negative-resistance element, neglects the series inductance L_s in the leads of the element. With most negative-resistance devices or circuits this is permissible because, as the frequency of oscillation is increased, oscillation ceases (the operating point in the diagram of Fig. 158 moves out of the region of growing sinusoidal oscillation) before the inductance and capacitance of the external resonator have been reduced to values such that the lead inductance has an appreciable effect upon the frequency of oscillation. This is true either if C_i/L_s is large or if the magnitude of the dynamic value of R_{iv} decreases with increase of frequency at high frequency as the result, for example, of reduction of amplification or positive feedback within the negative-resistance element (Probs. 17.2–1 to 17.2–4). The circuit of Fig. 251a also neglects series lead resistance R_s, which is small in comparison with R and $|R_{iv}|$ in most negative-resistance oscillators.

With some voltage-stable negative-resistance devices, notably the tunnel diode, the ratio of C_i to L_s is relatively small, and the dynamic negative resistance is independent of frequency to a very high frequency. With such a device oscillation may take place even if the external circuit is reduced to a short circuit across the terminals. The oscillation frequency is then the self-resonance frequency of the device, which may be of the order of 1 to 2 kMc in tunnel diodes that are on the market in 1961. The equivalent circuit of a short-circuited tunnel diode is shown in Fig. A-2a. If the frequency of oscillation is reduced somewhat below the self-resonance frequency by the use of a high-frequency external resonator in place of the short circuit, the complete equivalent circuit is that of Fig. A-2b. As the frequency is lowered by increasing C_c and L_c (and R_c), the effect of L_s and R_s upon the frequency of oscillation decreases and the circuit approaches that of Fig. 251a.

The frequency of oscillation can be raised considerably above the self-resonance frequency by offsetting a portion of the inductive reactance of L_s by a capacitive reactance across the terminals, in place of the short circuit.[1]

[1] F. Sterzer and D. E. Nelson, *Proc. I.R.E.*, **49**, 744 (April, 1961). In this paper Sterzer and Nelson have also taken into consideration the voltage-dependence of C_i in tunnel diodes.

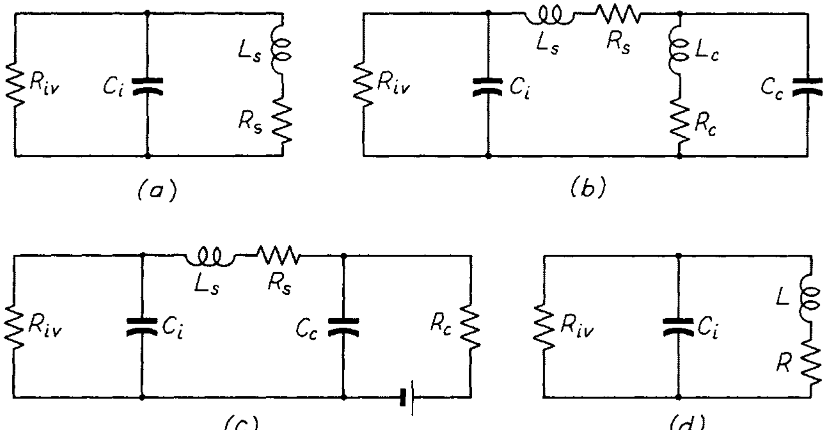


FIG. A-2.

To insure that the load line in the current-voltage diagram intersects the voltage-stable characteristic only in the negative-resistance range, the external capacitor must be shunted by a resistance smaller than the magnitude of R_{iv}. The resulting circuit is that of Fig. A-2c. This circuit may at first glance appear to incorporate a series resonator and thus to violate the rule, established in Sec. 73.3, that a parallel resonator must be used in a negative-resistance oscillator based upon a voltage-stable negative-resistance element. However, since R_c is small and the function of C_c is to reduce the effective inductive reactance of L_s, the circuit of Fig. A-2c is equivalent, at the oscillation frequency, to the circuit of Fig. A-2d, in which $L < L_s$ and $R > R_s$. The form of this circuit is identical with that of the parallel-resonator circuit of Fig. 251a.

The upper frequency of oscillation of the circuit of Fig. A-2c is determined by the extent to which the inductive reactance of L_s can be offset by the capacitive reactance of the parallel combination of R_c and C_c without making the effective shunt resistance of the combination of C_i, L_s, R_s, C_c, and R_s less than $|R_{iv}|$.

LETTER SYMBOLS

Standard letter symbols for transistor parameters, currents, and voltages are used throughout this book.[1] For the sake of consistency, standard tube letter symbols have been modified to agree with transistor symbols. Thus the symbols v and V are used in place of e and E for tube electrode voltages, and the same system is used for tubes as for transistors to distinguish between the various components of currents or voltages by means of lower-case, capital, and subscript symbols.[1] The tube subscripts $_g$, $_G$, $_p$, $_P$, $_k$, and $_K$ correspond to the transistor subscripts $_b$, $_B$, $_c$, $_C$, $_e$, and $_E$, respectively.

The symbol R_i is used to represent the input resistance of a circuit or device capable of producing negative resistance. Where necessary, parentheses with subscripts and superscripts are used to indicate whether the negative resistance is that of a pi feedback circuit or of a tee feedback circuit, and to indicate the port at which the resistance is observed (page 75). In some derivations it is convenient to employ subscripts $_c$ and $_v$ to show whether the port at which the negative resistance R_i is observed is current-stable or voltage-stable (pages 198 and 201).

The subscript $_1$ or $_2$ following the symbol for a resistance, conductance, or capacitance indicates that the circuit element in question is used in the input circuit or output circuit, respectively, of an amplifier (Fig. 65). The subscript $_{12}$ indicates that the circuit element serves to provide feedback from the output circuit to the input circuit (Fig. 65) or that it couples the output of one amplifier stage to the input of another (Figs. 68 and 96).

The following index, which lists the first page on which special symbols are defined or used, does not include many symbols that appear on only one or two pages.

[1] IRE Standards on Letter Symbols for Semiconductor Devices (56 IRE 28. S1), *Proc. I.R.E.*, **14,** 934 (July, 1956). Reprints of this Standard may be purchased while available from the Institute of Radio Engineers, 1 East 79 Street, New York 21, N. Y.

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