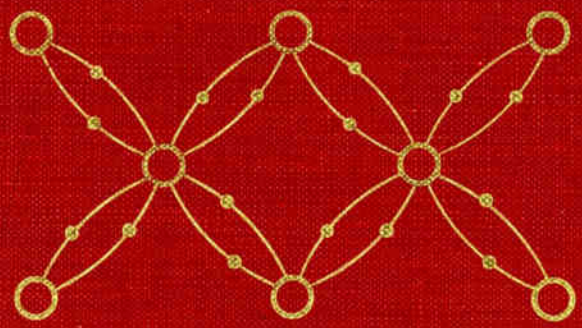
Essentials of Electricity—Electronics

THIRD EDITION



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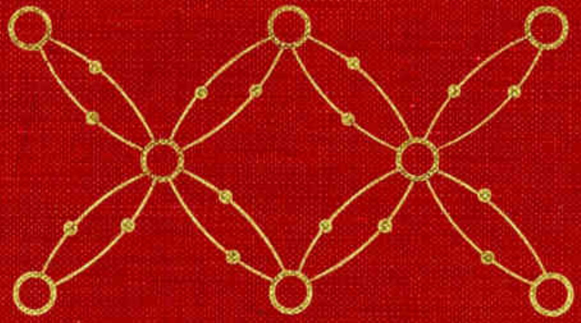

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Essentials of Electricity—Electronics

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ESSENTIALS OF RADIO—ELECTRONICS
SECOND EDITION

McGraw-Hill Book Company
New York
St. Louis
San Francisco
London
Toronto
Sydney
Mexico
Panama

Essentials of Electricity—Electronics

Formerly published under the titles of
Essentials of Electricity for Radio and Television

Electrical Essentials of Radio

Library of Congress Catalog Card Number: 64-8625
58260

567890(HD)721069

ESSENTIALS OF ELECTRICITY—ELECTRONICS

THIRD EDITION

Preface

Since the publication of the first edition of the authors' *Essentials of Electricity* in 1944, the many advances in the knowledge and uses of electricity have resulted in the present broad field of electronics. In order to provide a good basic knowledge for the worker in the electrical and electronic fields, this edition (the third) has been expanded to include chapters on electron tubes, transistors, motors, and generators. The coverage of circuit components, batteries, and magnetism and the solution of complex circuit problems have been brought up to date and/or expanded to meet the needs of modern technology.

The purpose of this book is to present at an intermediate level a comprehensive study of the essential principles of electricity and electronics to prepare a worker to enter and/or advance in the technological industrial and electronic industries of today. This text represents over 70 years of combined experience of the authors in industry and in teaching in the fields of electricity, electrical machinery, radio, radar, television, and industrial electronics.

The text covers: (1) an introduction to the vast scope of the field of electronics; (2) the basic atomic structure of materials in terms of the electron theory; (3) the field of electric circuits and circuit elements including resistors, inductors, and capacitors; (4) battery and generator power supplies; (5) a-c and d-c meters; (6) magnetism, electromagnetism, and magnetic circuits; (7) electrical devices, including relays, protective devices, transformers, a-c and d-c generators, motors, and controls; (8) the basic principles of electronic circuit elements, including vacuum tubes, gas tubes, phototubes, crystal diodes, transistors, photocells, and solid-state light-sensitive devices; (9) the basic vacuum-tube and transistor amplifier circuits; and (10) the basic electronic circuits.

This book is intended for: (1) students studying electricity or basic electronics in technical institutes, junior colleges, trade or vocational schools, and industrial training programs; or (2) persons not attending any regular school but who wish to study the subject at home on an intermediate level. This book is also intended to provide the essential preparation for further study in the fields of electricity and electronics. The range of mathematics required for the successful study of this text is within the comprehension of the average high school graduate.

The following features, not generally found in any one book, have been incorporated in this text.

1. A minimum knowledge of mathematics is required for so technical a subject. The presentation and application of mathematical principles are explained with their initial use.
2. Examples are used throughout the book to illustrate the applications of equations and principles discussed in the text. All major equations are followed by an illustrative example. The values used in the examples represent commercial values wherever practicable. Examples of complex as well as simple circuits are illustrated for d-c circuits, a-c circuits, vacuum-tube circuits, and transistor circuits.
3. The principle of operation of the various circuit elements and the analysis of electric, electron-tube, and transistor circuits are explained according to the electron theory.
4. A large number of line drawings and numerous photographs of commercial components are used to illustrate basic principles and applications as they are presented.
5. The chapter on batteries describes the newer types of cells as well as the older prototypes. Among the newer types of cells described are the mercury cell, silver oxide cell, alkaline cell, nickel-iron cell, nickel-cadmium cell, cadmium-silver oxide cell, solar cell, and fuel cell.
6. The solution of all types of direct-current circuit problems is first presented by use of the simplest type of mathematical operations. The solution of complex circuit problems by use of Kirchhoff's laws, simultaneous equations, quadratic equations, and Thévenin's theorem is presented later.
7. The chapter on magnetism and electromagnetism includes numerous examples illustrating the procedures used in calculating practical magnetic circuits, and includes the use of magnetization curves.
8. The chapter on capacitors describes the newer types of capacitors as well as the older prototypes. Among the newer types included are Mylar, Teflon, temperature compensating, dual dielectric, energy storage, tantalum, and niobium capacitors. Two charts give the overall characteristics of a large number of types of capacitors.
9. The solution of all types of alternating-current circuit problems is first presented by use of the simplest type of mathematical operations. The solution of complex circuit problems by use of the j-operator, polar vectors, and Thévenin's theorem is presented later.
10. The chapters on electron tubes and transistors present the basic principles of the essential electronic circuit elements that are used in many modern electronic circuit applications.
11. A complete chapter describes the principles of operation of a-c motors,

and d-c motors, generators, and controls. The principles of the selsyn or synchro unit are also presented in this chapter.

12. Eleven appendixes provide general reference data and serve as useful tools for working with electronic circuits and problems.
13. As an aid to the instructor and as a challenge to the more interested student, numerous questions and/or problems are provided at the end of each chapter. Answers to all the odd-numbered problems are given in the Appendix. The values used in the problems have been carefully selected and represent practical values.

Numerous industrial organizations have assisted in providing illustrations and technical information regarding their products, and this service is greatfully acknowledged. These organizations are: Aerovox Corporation; The Alliance Manufacturing Company; Automatic Electric Company; Cornell-Dubilier Electronics; Electric Storage Battery Company; Electro Dynamic Works; Erie Technological Products, Inc.; General Electric Company; Hammarlund Manufacturing Company, Inc.; P. R. Mallory and Company, Inc.; Ohmite Manufacturing Company; Radio Corporation of America; Simpson Electric Company; Sprague Electric Company; Standard Transformer Corporation; Thordarson-Meissner Corporation; Union Carbide Corporation; Westinghouse Electric Corporation; and Weston Instruments and Electronics.

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Appendix

Chapter 1

Introduction to Electricity

Numerous scientific developments have occurred in this century, many of which can be attributed to the phenomenal expansion of electrical principles and applications. In the early years of the twentieth century, electricity was just beginning to be used as a source of energy in the fields of (1) illumination, (2) power, and (3) communication. During the past forty years, the great advances made in the many new uses of electricity have affected the lives of men all over the world.

1-1 The Use of Electricity As a Source of Energy

Electrical energy is used in varying quantities and is produced by several methods in order to meet a wide range of power requirements. The large power plants near our population centers and industrial areas are familiar sights. Such power plants supply millions of kilowatthours of electrical energy for operating (1) lights, (2) home appliances, (3) industrial equipment, (4) communication equipment, and (5) railway transportation equipment. The large amounts of power required by these services may be produced by (1) converting heat energy (obtained from coal, oil, gas, or atomic reactors) to electrical energy and (2) converting water power to electrical energy.

Another widely used source of electrical energy is the chemical cell which converts stored chemical energy to electrical energy. Two familiar applications of this type of power source are (1) the primary cell, as used in transistor radio receivers and electronic test equipment, and (2) the secondary cell, as used in storage batteries for automobiles.

A relatively new source of electrical energy is the solar cell which converts light energy, obtained from the sun, to electrical energy. This type of power source is now being used in the control and monitoring equipment for satellite and space vehicles and is providing our scientists with much new information about outer space.

Another new source of electrical energy is the fuel cell. This type of cell, unlike the ordinary chemical cell, does not store energy but, rather, converts the energy from conventional fuel oxidation directly to electrical energy. The fuels used are of the inexpensive fossil types, such as coal and hydrocarbons or substances easily derived from them, as hydrogen, alcohol, carbon monoxide, etc.

1-2 Electronics

Classifications. The term *electronics*, which covers a broad and rapidly expanding field, may be divided into the following classifications: (1) communications, (2) industrial, (3) automation, (4) instrumentation, (5) computers, (6) data processing, (7) therapeutics, (8) bionics, (9) military, (10) space, and (11) cryogenics. Since these are very broad classifications, there are many subdivisions for each category.

Communications. Electricity performs a very important part in our modern communication systems. A wide variety of both visual and auditory means of communication has been developed, such as (1) the printed page of newspapers and books, (2) heliograph, (3) telegraph, (4) wireless telegraphy, (5) telephone, (6) radio, (7) public-address systems, (8) disk recorders, (9) tape recorders, (10) facsimile, (11) television, (12) motion pictures, (13) radar, (14) sonar, (15) loran, (16) shoran, and (17) telemetry. These are also broad classifications, and there are many subdivisions for each of these categories. The vastness and complexity of the field of communications are illustrated by the following example. The branch of radio may be divided into two divisions, namely, transmitters and receivers. However, there are different types of receivers and transmitters. For example, a radio receiver may (1) be amplitude- or frequency-modulated; (2) have commercial, high-fidelity, or stereophonic quality; (3) use vacuum tubes, transistors, or hybrid components; (4) operate from d-c power lines, a-c power lines, or batteries; (5) be designed for home, portable, personal, automobile, airplane, or ship use.

Although the applications of the principles of electronics to the field of communications are numerous, the basic circuits used are similar, but the purpose they are to serve may differ. Some of these circuits are (1) low- and high-voltage power supplies, (2) voltage and power amplifiers, (3) oscillator, (4) converter, (5) a-m and f-m detectors, (6) tuning, (7) pulse forming, (8) differentiator, (9) integrator, (10) sweep, (11) synchronizing, (12) filter, (13) coupling, (14) convergence, and (15) matrixing. These circuits are basic; therefore they are not limited to the field of communications but are common to all branches of electronics.

Industrial. The applications of electronic principles and circuits to industry are many and varied. A general description of a few examples will indicate the vastness of this field. The principles of resonance may be used to control the thickness, quality, weight, or moisture content of a material. Amplifier circuits may be used to increase the intensity of weak current impulses, produced by phototubes or photocells, and cause them to operate relays controlling circuits of door openers, lighting systems, power systems, safety devices, etc. Principles other than those common to the field of communications are also used, for example, stroboscopic lighting, which can cause fast-moving objects to appear motionless or make their movements appear similar to slow-motion moving pictures. This principle makes it possible to study the movements of various parts of a machine under their operating conditions. It is also used for high-speed photography applications in order to arrest the motion of fast-moving objects.

Automation. Automation is the process of having a machine perform operations previously performed by a person. These machines may be (1) simple and replace only a few manual operations or (2) complex and replace a number of human operations. This type of equipment is controlled electronically, using memory and associated circuits to perform a number of predetermined operations, by means of a punched card or tape. For example, one of a series of tapes could be inserted in the control circuit of equipment designed to manufacture machine screws to produce any quantity, size, or type of screw.

Instrumentation. The principles of electronics have made it possible to measure quantities that previously have not been measurable. The electronic voltmeter, cathode-ray oscilloscope, resonant-circuit checkers, and signal generators are a few of the many new types of instruments that have become synonymous with electronics. The electron microscope, electron telescope, etc., have become valuable aids to scientists in many fields other than electronics.

Computers. In today's complex society, many important economic, financial, scientific, or military decisions have to be made in a short time. These decisions may have to be based on the correct answer to a complex problem involving numerous calculations that, if solved manually, would take long periods of time.

A computer is a machine that performs mathematical operations previously performed by a person. It is possible for a computer to provide the correct answers for many intricate problems in several minutes or even several seconds. For example, when America's first orbiting astronaut made his historic flight into space, many decisions involving (1) safety of orbit, (2) the orbital path, (3) exact time of firing retrorockets, etc., had to be made within seconds. These decisions were based on millions of individual mathematical calculations performed by a computer.

In addition to using many of the basic circuits used in communications, electronic computers also use *memory* and *logic* circuits. The solution or answer to a problem may be indicated by (1) a typed record, (2) a system of binary lighting, (3) meter readings, (4) an oscilloscope pattern, or (5) a printed graph.

Data Processing. Electronic data processing involves the recording of information on cards or tape and the use of these recorded data for classification, accounting, or interpretive operations. Classifiers are used to determine which items or persons have certain characteristics. This type of equipment is employed by law-enforcement agencies to identify a person; by Federal, state, or local agencies for tax and license notifications; etc. Accounting equipment is used by large business organizations and government agencies for making out payroll checks, inventories, etc. The prediction of final election results from samplings of the returns is one of the many applications of interpretive equipment. The circuits used in electronic data-processing equipment are similar to those used in electronic computers.

Therapeutics. Medical doctors and scientists in the field of therapeutics are constantly finding new uses for the principles of electronics to develop instruments to aid in treating and preventing human ailments. These instruments include (1) X-ray machines, which are used for treatment of skin disorders and acute infections as well as for taking pictures of internal structures of the human body; (2) ultraviolet lamps for arresting harmful mold and bacteria; (3) short-wave diathermy units for healing sprains and fractures; (4) electrocardiographs for measuring and recording heartbeats; (5) electroencephalographs for measuring and recording impulses to and from the brain; (6) inductotherm units to generate artificial fever; (7) oscilloscopes for viewing muscular, respiratory, heart, and other body actions; (8) artificial heart; (9) artificial lung; (10) data-processing equipment to aid in the identification of a disease or illness; etc.

Bionics. The use of electronics as a tool for studying the biological sciences is called *bionics.* In bionics, many astounding electrochemical devices, as found in living entities, are simulated by an electronic circuit. These simulators aid the medical scientist better to understand the action of the original organism and provide the industrial scientist with information to build better control and computer equipment.

Our complex technology is demanding more operations to be performed by computer, control, and data-processing equipment. These machines do not possess the ability to think, speculate, reason, imagine, recognize, or screen information received as do humans and other living creatures. However, should it become possible to incorporate any of these abilities of a human being in a machine, it would also be possible to construct equipment that could surpass the perfections of many persons. This is possible because (1) two weak characteristics of humans, forgetting and fatigue, can be eliminated and (2) the exceptional abilities of other creatures can be used.

Military. The military applications of electronics are numerous and varied. Many applications are similar to those used in other fields, for example, radio and telephony. However, there are a number of applications that are peculiar to military requirements. Some of these are (1) radar, shoran, and loran for the detection and location of moving objects in space; (2) sonar for the detection and location of moving objects under water; (3) computers and control equipment for firing projectiles; (4) selsyn motors and their associated circuits for remote control movement of an antenna or the simultaneous movement of fire-control apparatus.

Space. Electronic circuits and components associated with the operation and control of space vehicles are designed to meet many exacting requirements. Some of these conditions are (1) split-second operation, (2) subminiature construction, and (3) stable operation with extreme variations of temperature, pressure, and humidity. However, some of the circuitry is basic and similar to that used in other fields of electronics. One of the circuits that is associated with space is *telemetry*. This circuit is used for controlling (1) some of the movements of a space vehicle and (2) the operation of various test, measurement, and recording apparatus from the ground control center of operations.

Cryogenics. Cryogenics is the science of producing and using extremely low temperatures. The temperatures used are those below the boiling point of liquid oxygen, −297°F. Because of the many useful properties that some materials possess at these low temperatures, scientists working in this new field envision (1) powerful superconducting electromagnets, (2) shoebox-sized computers able to do the work of today's giant-size equipment, (3) motors having practically no friction, and (4) power-transmission systems having almost a zero power loss. Although at present these are only predictions for the future, the fantastic rate at which electronic science has advanced in a relatively short time may make this future not too far away.

1-3 Need for a Knowledge of Electricity

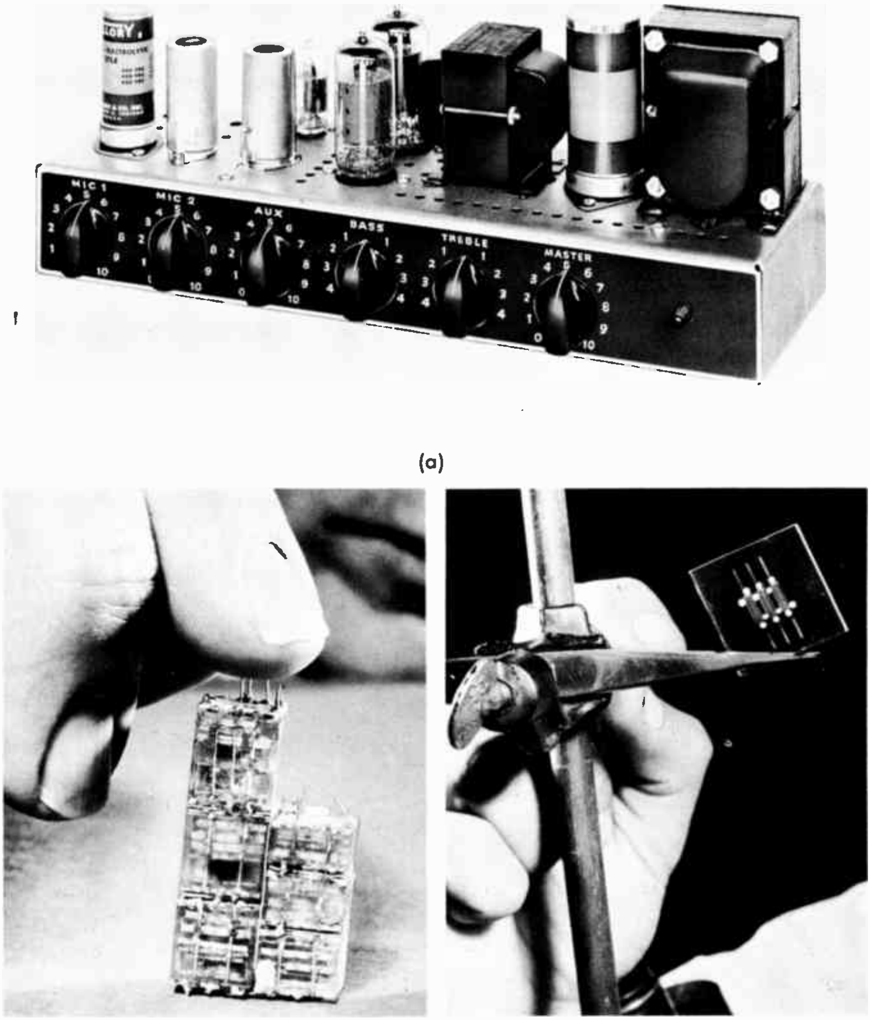
Electric Circuits. The description of the electronic industry presented in the preceding article indicates the vastness, diversity, and complexity of electronic applications and the circuits they employ. No matter how complex the circuit of an electronic unit, basically it will consist of two or more of the fundamental electronic circuits. Any two components can be connected only in either of the two basic electric circuits: (1) series and (2) parallel. Three or more components may be connected in either of these two circuits or in a complex arrangement of the two basic circuits. A complex circuit may be analyzed by (1) reducing it to an equivalent series or parallel circuit or (2) using an equivalent series or parallel arrangement of the components.

Direct-current Power Circuits. Components used in d-c power circuits generally possess only resistance, and therefore they may be considered as resistors.

Alternating-current Power Circuits. Alternating-current power circuits may use singly or in combination any arrangement of resistors, inductors, or capacitors. While most a-c power circuits have a frequency of 60 cycles per second (abbreviated cps), these components are usually operated in the relatively low frequency range of 25 to 400 cps.

Electronic Circuits. The components used in electronic circuits may be any combination of resistors, inductors, capacitors, electron tubes, metallic rectifiers, transistors, nuvistors, tunnel diodes, etc. It is possible for different portions of an electronic circuit or even the same portion to have currents of various frequencies ranging from zero (direct current) to hundreds of millions of cycles.

Circuit Elements. In addition to the continuous developments and improvements of electronic circuitry since its early application to radio,



(a)

(b) (c)

Fig. 1-1 Miniaturization of electronic units. (a) Vacuum-tube audio amplifier. (b) Micromodule with discrete components encapsulated in plastic. (c) Thin-film transistor with metal film evaporated to form an integrated circuit. (*RCA*) new developments and improvements were also constantly being made to the circuit components. Many of the circuit elements used in the early stages of electronics were quite large and inefficient and had limited operating characteristics. As the field of electronics expanded, the range of requirements for the circuit components also increased. New applications demanded circuit elements having (1) wider current, voltage, frequency, and temperature characteristics; (2) higher efficiency and stability of operation; and (3) smaller size.

The decrease in size of electronic circuit elements and the resulting complete unit has been phenomenal. The relatively simple home vacuum-tube radio receiver of the 1920s was quite large compared with the personal transistor radio receiver of the 1960s. The demand for smaller and more compact units has decreased both the size of the components and the amount of wiring used to connect them. Newer type elements, such as the transistor, nuvistor, compactron, and tunnel diode, have also been instrumental in the production of compact units. Miniaturization has been further accomplished by the use of printed circuitry and printed components that employ silk-screen, photoetching, and photolithographic techniques. Scientists envision that the future complex electronic component circuits of radio, television, space probes, computers, etc., will be contained in tiny electronic tiles smaller than an aspirin tablet.

Fundamental Principles. Although there has been a tremendous growth in the field of electronics, the principles of the basic components and circuits have remained the same. In order to understand the circuit operation of the many applications of these electronic principles, one must have a broad and thorough knowledge of electric circuit theory. It takes time to master these fundamental principles, and one should not be discouraged because he has not worked on or studied radio, television, industrial, or computer circuits at the beginning. Once the basic principles are mastered, it is possible to understand their applications to all kinds of electronic circuits.

A knowledge of some of the principles of physics, such as radio waves, sound, and light, is helpful toward a better understanding of the many applications of electronics.

1-4 Wave Propagation

Need for Concept of Wave Motion. In the study of electricity and electronics, frequent reference is made to the principles of wave motion as is indicated by such terms as (1) sine-wave alternating currents, (2) sound waves, (3) radio waves, (4) light waves, (5) carrier waves, (6) ultrasonic waves, etc. A simple analogy of wave motion in electricity and electronics is the wave motion produced by dropping a pebble in a small body of water, namely, that the energy moves away from the source in ripples or waves. An alternating current in a low-frequency electric power circuit

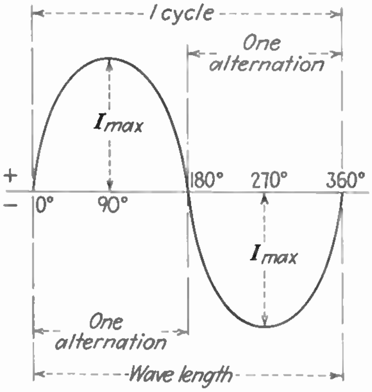


Fig. 1-2 An a-c wave.

and the high-frequency radio signals sent through space both follow the principles of wave motion.

Alternating Current in an Electric Circuit. An alternating current (abbreviated a-c) in an electric circuit reverses its direction at fixed intervals, hence the name *alternating current.* During each interval the current will rise from zero to its maximum value, then diminish to zero. Figure 1-2 shows that an a-c wave goes through two similar sets of changing values, one in a positive direction and one in a negative direction. The interval required for one set of values, in either direction, is called an *alternation* and corresponds to 180 electrical degrees. After two successive alternations, or 360 electrical degrees, the a-c wave has completed one *cycle.* This a-c wave is followed by a continuing succession of such waves. The number of cycles occurring in one second is called the *frequency* of the power source. In Fig. 1-2, I is the symbol denoting current and *max* is an abbreviation for maximum; I_{max} indicates the maximum amount of current which occurs at each 90-, 270-, 450-, etc., degree instant of an a-c sine wave.

Electric and Magnetic Waves in Space. Light, heat, X-ray, radio, and television waves are forms of radiant energy considered to be electromagnetic oscillatory disturbances in space. The frequency of oscillation determines whether a particular signal is evident as light, heat, X-ray, radio, or television waves. These forms of energy are referred to as *electromagnetic waves* because they emanate from a transmitter with a progressive action similar to waves in a body of water. The overall range of frequencies of these waves is referred to as the *electromagnetic spectrum.* The division of the electromagnetic spectrum is shown in Fig. 1-3. The division of the radio-frequency (abbreviated r-f) spectrum and the descriptive names applied to each portion of the spectrum are shown in Table 1-1.

Radio Waves. Radio and television transmitting stations convert sound waves and light waves to electrical impulses. The electrical impulses representing the original sound and light waves are sent out by the use

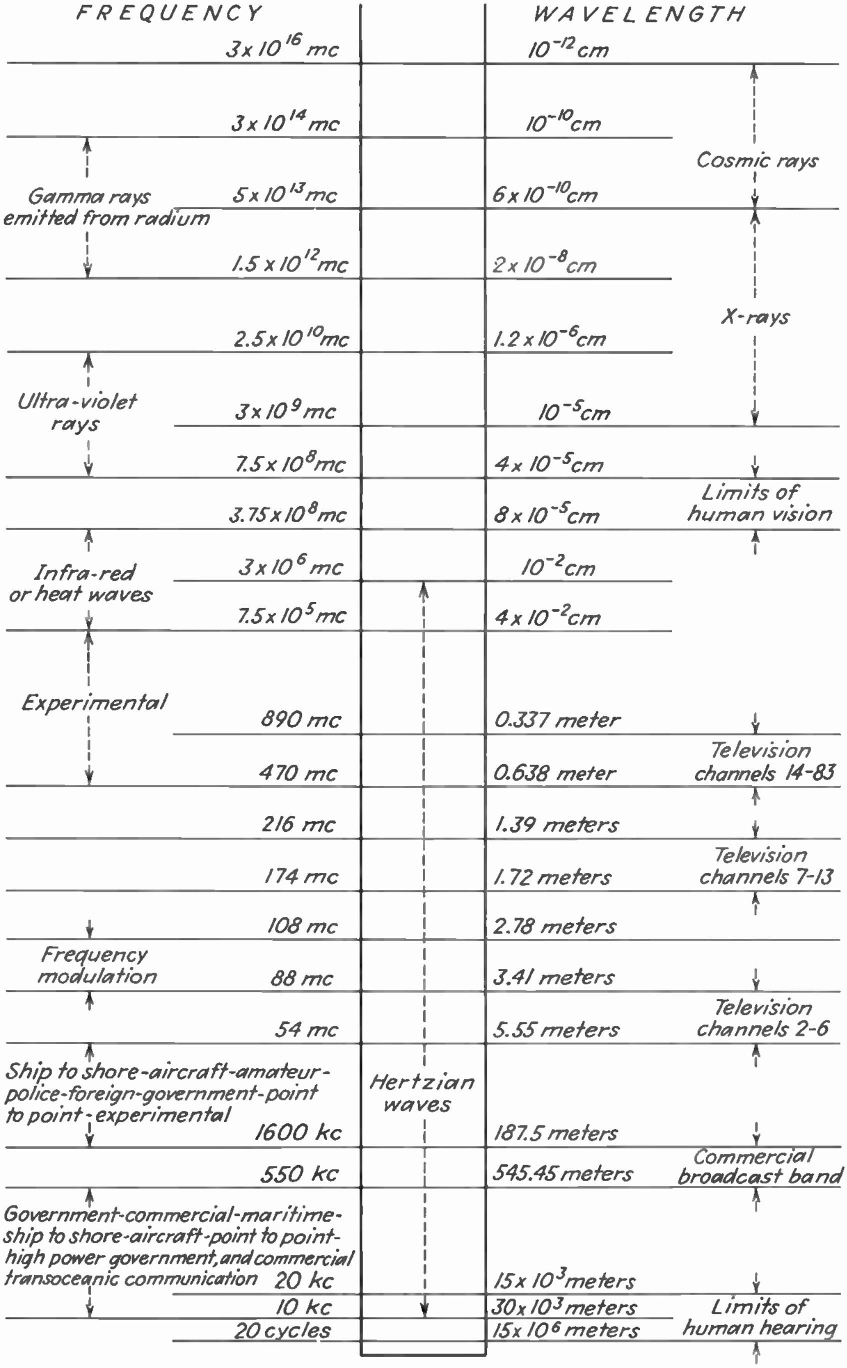


Fig. 1-3 The electromagnetic spectrum.

Table 1-1 Division of the Radio-frequency Spectrum

FREQUENCY, MC	WAVELENGTH, METERS	DESCRIPTION	ABBREVIATION
0.01–0.03	30,000–10,000	Very low frequency	vlf
0.03–0.3	10,000–1,000	Low frequency	l-f
0.3–3	1,000–100	Medium frequency	m-f
3–30	100–10	High frequency	h-f
30–300	10–1	Very high frequency	vhf
300–3,000	1–0.1	Ultrahigh frequency	uhf
3,000–30,000	0.1–0.01	Superhigh frequency	shf

of high-frequency alternating currents. These currents produce magnetic and electric fields that radiate in all directions over long distances. The magnetic and electric fields produced by high-frequency currents are called *radio waves.* The strength and frequency of a radio wave are dependent on the high-frequency alternating current producing it and will vary in the same manner as the alternating current.

In addition to its application for the transmission of sound and light signals, radio waves are also used by other types of transmitters employing frequencies in the r-f spectrum to send out information. For example, a radar transmitter sends out high-frequency pulses at regular intervals and the echo pulses, reflected from a target, are picked up by a receiving antenna.

Speed of Radio Waves. Radio waves travel at the same speed as light waves, or 186,000 miles per sec. In some calculations the metric system is used, and the speed of the radio waves is then expressed in meters per second.

EXAMPLE 1-1 Radio waves travel at the rate of 186,000 miles per sec. What is the rate in (*a*) feet per second, (*b*) meters per second? (One mile = 5,280 ft; also, 1 meter = 3.28 ft.)

GIVEN: Miles per sec = 186,000 Ft per mile = 5,280 Ft per m = 3.28

FIND: (*a*) Ft per sec (*b*) M per sec

SOLUTION:

(*a*) $$\text{Ft per sec} = 186{,}000 \times 5{,}280 \cong 982{,}000{,}000$$

(*b*) $$\text{M per sec} = \frac{982{,}000{,}000}{3.28} \cong 300{,}000{,}000$$

Note: ≅ means *is approximately equal to.*

Wavelength and Frequency Definitions

Wavelength. The distance that the radio wave travels in the time of one cycle is called its *wavelength;* it is expressed in meters and is represented by the symbol λ, a letter of the Greek alphabet pronounced "lambda."

Frequency. The number of cycles per second of a radio wave is called its *frequency* and is represented by the letter f. The frequency is often referred to as a number of *cycles*. Cycles per second is commonly abbreviated as cps. Recently, Hertz and Hz have been adopted as the name and abbreviation for frequency, or cycles per second.

Wavelength and Frequency Calculations

Wavelength. If the frequency of a wave is known, the distance it will travel in 1 cycle can be calculated by

$$\lambda = \frac{300{,}000{,}000}{f} \tag{1-1}$$

where λ = wavelength, m
f = frequency, cps

Kilocycles and Megacycles. The frequencies of the common radio waves are of high values, that is, in the hundreds of thousands or millions of cycles per second. For convenience these frequencies are generally expressed in kilocycles or megacycles and abbreviated as kc and mc respectively. *Kilo-* is a prefix meaning thousand; hence a kilocycle is equal to 1,000 cycles, which actually means 1,000 cycles per second. The prefix *mega-* means million; hence a megacycle means 1,000,000 cps.

When radio frequencies are expressed in kilocycles or megacycles, Eq. (1-1) becomes

$$\lambda = \frac{300{,}000}{f\,(\text{in kc})} \tag{1-1a}$$

$$\lambda = \frac{300}{f\,(\text{in mc})} \tag{1-1b}$$

EXAMPLE 1-2 What is the wavelength of a radio station which operates on an assigned frequency of 570 kc?

GIVEN: $f = 570$ kc

FIND: λ

SOLUTION:

$$\lambda = \frac{300{,}000}{f} = \frac{300{,}000}{570} = 526.3 \text{ m}$$

EXAMPLE 1-3 What is the wavelength of a television video carrier wave whose frequency is 77.25 mc?

GIVEN: $f = 77.25$ mc

FIND: λ

SOLUTION:

$$\lambda = \frac{300}{f} = \frac{300}{77.25} = 3.88 \text{ m}$$

Frequency. Equations (1-1), (1-1*a*), and (1-1*b*) can be transposed to solve for frequency instead of wavelength and become

$$f\,(\text{in cps}) = \frac{300{,}000{,}000}{\lambda} \tag{1-2}$$

$$f\,(\text{in kc}) = \frac{300{,}000}{\lambda} \tag{1-2a}$$

$$f\,(\text{in mc}) = \frac{300}{\lambda} \tag{1-2b}$$

EXAMPLE 1-4 If by definition a short radio wave is one whose wavelength does not exceed 200 m, what is the lowest frequency at which a short-wave radio receiver may operate?

GIVEN: $\lambda = 200$ m

FIND: f

SOLUTION:

$$f = \frac{300{,}000}{\lambda} = \frac{300{,}000}{200} = 1{,}500 \text{ kc}$$

If the wavelength in meters is changed to our more commonly used units of feet and inches, it should provide a better understanding of the length of the radio waves.

EXAMPLE 1-5 What is the length in feet of one radio wave of the broadcast station referred to in Example 1-2?

GIVEN: $\lambda = 526.3$ m

FIND: Length, ft

SOLUTION:

$$\text{Length} = \text{meters} \times 3.28 = 526.3 \times 3.28 = 1{,}726 \text{ ft}$$

EXAMPLE 1-6 A video carrier wave used in uhf transmission has a frequency of 867.25 mc. What is its length in inches? (One meter = 39.37 inches.)

GIVEN: $f = 867.25$ mc

FIND: Length, inches

SOLUTION:

$$\lambda = \frac{300}{f} = \frac{300}{867.25} = 0.346 \text{ m}$$

$$\text{Length} = 0.346 \times 39.37 = 13.62 \text{ inches}$$

The solutions of Examples 1-5 and 1-6 indicate that each wave transmitted by a 570-kc radio station is 1,726 ft long, or approximately one-third of a mile, and each wave transmitted by a 867.25-mc video carrier has a length of 13.62 inches, or approximately 1 ft.

Knowing that radio waves travel 186,000 miles per sec, the time required for a radio wave to get from one place to another can be readily calculated.

EXAMPLE 1-7 How long does it take a radio wave to travel from New York to San Francisco, a distance of approximately 2,600 miles?

GIVEN: Distance = 2,600 miles Rate = 186,000 miles per sec

FIND: Time, t

SOLUTION:

$$t = \frac{\text{miles}}{186{,}000} = \frac{2{,}600}{186{,}000} = 0.0139 \text{ sec}$$

Example 1-7 shows that it takes only about 1/70 sec for a radio program broadcast from New York to travel to San Francisco.

1-5 Sound

Characteristics of Sound. *Sound* is the sensation produced in the brain by sound waves. It makes use of one of our five fundamental senses, namely, that of hearing. The air in a room in which no sound is present is in a static condition; in other words, it is motionless. If a sound is made by a person, by a musical instrument, or by any other means, the air about it is set into vibration. These vibrations are transmitted to adjacent layers of air and so on until all the original energy is expended. Such air vibrations are called *sound waves*. When these vibrations strike the eardrum of a person, the eardrum too will vibrate in a similar manner. The auditory nerves will be stimulated and will communicate the sensation of sound to the brain.

Sound waves are produced by the mechanical vibration of any material

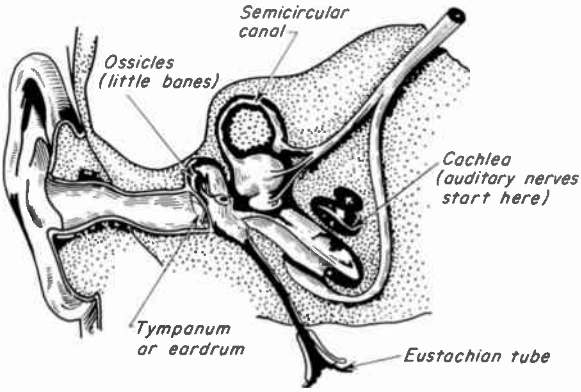


Fig. 1-4 Internal structure of the human ear.

in elastic media such as gases, liquids, and some solids, but they will not travel in a vacuum. Sound waves are longitudinal waves and travel outward in all directions from the source. A longitudinal wave may be defined as one in which the vibrating molecules or particles of the transmitting medium move back and forth in the same direction in which the sound wave is traveling.

Intensity. The *intensity*, or *loudness*, of a sound depends upon the energy of motion imparted to the vibrating molecules of the medium transmitting the sound. A greater amount of energy causes more violent movement of the molecules, which in turn exerts a greater pressure upon the eardrum, thus causing the auditory nerves to send the sensation of a louder sound to the brain. Loudness is affected by the distance between the listener and the source of the sound, and its intensity varies inversely with the square of this distance. For example, if the distance between the listener and the source of the sound is doubled, the intensity is reduced to one-quarter. Also, if the distance between the listener and the source of the sound is decreased to one-half the original amount, the intensity of the sound will be four times as great.

Speed of Sound. The speed at which sound waves travel varies with the kind of material through which it is traveling. For air, the most common medium used for transmitting sound waves, the speed is 1,130 ft per sec at the normal room temperature of 68° F.

Frequency, Pitch, and Wavelength. The vibration of the reeds in a harmonica, of the skin on a drum, of the strings on a violin, or of the cone of a radio loudspeaker will all send out various sound waves. These waves will produce different sounds, depending on the number of vibrations that the wave makes per second. The number of complete waves or vibrations

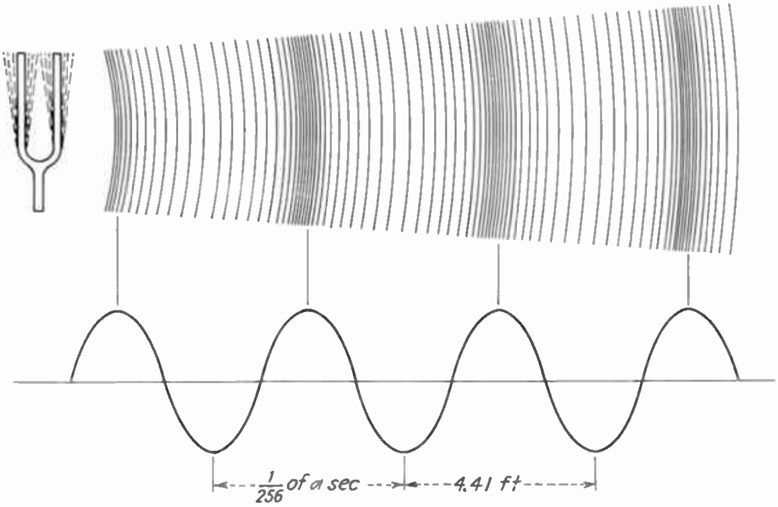


Fig. 1-5 Propagation of a sound wave of 256 cps in air.

created per second is known as the *frequency* of the sound and is generally expressed as the number of cycles per second. For example, a sound wave that is making 2,000 vibrations per second is the same as a sound whose frequency is 2,000 cps; this is also commonly referred to as a 2,000-cycle sound or a 2,000-cycle note.

If the sound is loud enough to be heard by the human ear, it is said to be *audible*. Its *pitch* will vary with the frequency. High frequencies produce sounds having a high pitch, and low frequencies produce sounds of low pitch.

Sound waves may also be referred to in terms of the length of a wave. Knowing that sound waves travel 1,130 ft per sec in air, the length of one wave can be calculated by dividing the number 1,130 by the frequency of the sound. Figure 1-5 illustrates a tuning fork producing sound waves whose frequency is 256 cps and whose wavelength is 4.41 ft.

EXAMPLE 1-8 The frequency range of a piano is from 25 to 8,000 cycles. (*a*) What is the range of wavelengths in feet? (*b*) In meters? (*c*) If the sound waves are converted to electrical waves by a microphone, what is the frequency range of the electric currents?

GIVEN: Sound waves = 25–8,000 cycles

FIND: (*a*) Wavelengths, ft (*b*) Wavelengths, m (*c*) Frequency range of electric currents, cycles

SOLUTION:

(*a*)
$$\text{Wavelength, 25 cycles} = \frac{\text{ft per sec}}{\text{cps}} = \frac{1{,}130}{25} = 45.2 \text{ ft}$$

$$\text{Wavelength, 8,000 cycles} = \frac{\text{ft per sec}}{\text{cps}} = \frac{1{,}130}{8{,}000} = 0.14125 \text{ ft}$$

(*b*)
$$\text{Wavelength, 25 cycles} = \frac{\text{wavelength, ft}}{3.28} = \frac{45.2}{3.28} = 13.7 \text{ m}$$

$$\text{Wavelength, 8,000 cycles} = \frac{\text{wavelength, ft}}{3.28} = \frac{0.14125}{3.28} = 0.043 \text{ m}$$

(*c*) 25 to 8,000 cycles (same frequencies as the sound waves)

Frequency Ranges of Sound Waves. The range of frequencies that the human ear is capable of hearing will vary with the individual, the lower limit being approximately 20 cycles and the upper limit 20,000 cycles. Some persons are able to hear the low-pitch sounds but cannot hear those of high pitch, while others can hear the high-pitch sounds but cannot hear those of low pitch. However, there are people able to hear sounds covering a wide range of frequencies.

A few of the common audible sounds and their approximate frequency ranges are listed in Table 1-2.

Table 1-2 Approximate Frequency Range of Common Sounds, cps

Human voice	80–10,000
Piano	30– 8,000
Violin	200– 8,000
Trombone	80– 7,000
Clarinet	150–10,000
Flute	250–10,000
Piccolo	500–10,000

Code signals may employ any audio frequency, but experience has shown that a signal with a frequency of 1,000 cycles will produce a pleasing sound for continual, easy listening; it will permit each dit, dah, or space to be quickly distinguished.

Musical Sounds and Noise. When sound waves are produced repeatedly at regular intervals, the result is a *musical sound* at some definite pitch which is more or less pleasant to the ear. The orderly repetition produces rhythm which is also a requirement to obtain a musical note. When sound waves of constant or varying frequencies are produced at haphazard irregular intervals, the result is an unpleasant sound called *noise.*

Quality, Fundamentals, and Overtones. The middle C of a piano has a frequency of 256 cps. A corresponding note of 256 cycles can also be produced on other musical instruments such as a violin, clarinet, or harmonica. The notes from the various instruments differ in *quality,* which depends upon the number and relative amplitude of the overtones blended with the fundamental.

The *fundamental* note is the lowest tone produced, which in the above example would be the 256-cycle note. The overtones, which are higher pitched notes, blend with the fundamental and give each instrument (and each human voice) its individuality. The *overtones* are vibrations whose frequencies are multiples of the fundamental. The frequencies of the overtones of a 256-cycle note would be 512, 768, 1,024, 1,280, etc., cycles. The overtones are also often referred to as *harmonics.*

Reflection of Sound. When sound waves strike a solid object such as a wall of a building, the side of a cliff, or the wall of a room, the sound will be reflected and may cause an echo. An *echo* is the effect produced when a reflected sound returns to the ear a fraction of a second after the original source of the sound has ceased. If the interval between the original sound and the reflected sound is 1/10 sec or greater, an echo is likely to result. Echoes do not appear in small rooms because the reflected sounds return to the ear too soon after the original sound to be distinguished. Large rooms or auditoriums, where the reflecting surfaces are more than 50 ft away from the source of the sound, often produce echoes. In such cases the walls and ceilings may be decorated or padded with tapestries or soft materials to eliminate or reduce the production of echoes.

Sympathetic Vibration. The sound waves set up by one sound-producing object can cause a nearby object to start vibrating and thereby also produce sound waves if both objects have the same natural frequency. The vibrations of the second object are said to be the result of *sympathetic vibration.* An example of this phenomenon may sometimes be observed when the music from a radio receiver causes a metal vase, picture frame, or other object to start vibrating and give off sound waves. Sympathetic vibration may also cause a loose part in sound-producing equipment to start vibrating.

Sympathetic vibration may be explained in the following manner: The original sound wave strikes all nearby objects and sets them in motion, even though the amount of movement is very slight. If the second wave strikes at the precise instant which causes it to add its motion in perfect unison to that of the preceding wave, the movement will be increased. If all the succeeding waves strike at the corresponding precise instants, the movement will be cumulative and the object will vibrate at the frequency of the source of the sound. If, however, successive waves do not strike the object at precisely the proper instant, some of the waves will neutralize the motion imparted to the object by some preceding waves and no sympathetic vibration will result.

Forced Vibrations. An object in the area near a source of sound may be set into vibration by the sound waves of frequencies other than the natural frequency of the object if the intensity of the sound is great enough. This phenomenon is called *forced vibration.* Examples illustrating forced vibrations are the sounding board of a piano, sound reflectors of musical instruments, and the loudspeaker of a radio or television receiver.

Resonance. When a sound wave sent out by an object is reflected in such a manner that it returns to the object at the proper instant, it will produce sympathetic vibration within that object and thereby increase the intensity of the sound. This phenomenon is called *resonance.* An example of resonance can be observed by causing air waves to flow through pipes as in a pipe organ. The resonance of a pipe (for a fixed value of frequency) depends upon the length of the pipe and whether it is open or closed. A pipe closed at one end will produce resonance when the length of the pipe is one-quarter of the wavelength of the sound. A pipe open at both ends will produce resonance when the length of the pipe is one-half of the wavelength of the sound.

Beats. In the preceding paragraphs on Sympathetic Vibration and Resonance it was shown that two sounds of the same frequency would reinforce each other. If two sounds of different frequency are considered, it can be shown that the two sounds when started at the same instant of time will at first reinforce each other and then, after a number of cycles have been completed, will be out of step and will neutralize each other. Over a relatively long span of time there will be periods of reinforcements producing a strong sound and periods of neutralizing effect during which no sound will be produced. The sounds produced in this manner are called *beats*. If the number of beats appears frequently and at regular intervals, a new sound or beat note will be produced. The frequency of the beat note will be equal to the difference between the frequencies of the two sounds; for example, sounds of 256 and 200 cycles will produce a beat note of 56 cycles.

1-6 Light

Characteristics of Light. *Light* is the sensation produced in the brain by light rays. It makes use of one of our five fundamental senses, namely, that of sight. In the study of physics light is considered as a form of energy that may be derived from mechanical, electrical, chemical, heat, or light energy. Objects are *visible* because light rays from them reach the eyes, which in turn stimulate the optic nerves and send the sensation of sight to the brain. When the visible object is the source of light energy, it is *luminous*. The electric light, the sun, and the picture tube of a television receiver are examples of luminous objects. Most objects are not luminous, and the light rays that reach the eyes from such objects are actually the reflected light from some luminous body. Objects visible because of the light they reflect are called *illuminated bodies*.

Light is transmitted by *transverse waves*, that is, waves in which the motion is transmitted in a direction at right angles to the vibrations. Light waves can be transmitted only by a transparent medium, including vacuum. In terms of the ability to transmit light, matter may be divided into three classifications: transparent, translucent, and opaque materials. Materials that transmit light so well that objects can be seen clearly through the material are called *transparent*. Materials that transmit light so poorly that objects cannot be seen clearly through the material are called *translucent*. Materials through which light cannot pass are called *opaque*.

Brightness and Intensity of Illumination. The first consideration in regard to the measure of light is usually its brightness. The *brightness* of a source of light is expressed in candlepower, and a light source of one candlepower produces the same amount of light as that emitted by a candle of standard dimensions. A more useful measure of light is the *intensity of*

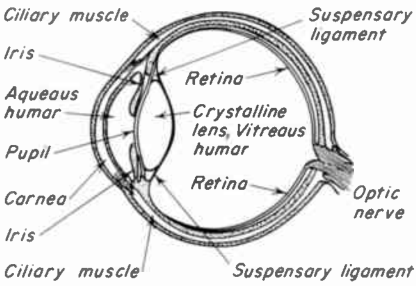


Fig. 1-6 Internal structure of the human eye.

illumination, which expresses the rate of flow of light energy upon a unit of surface. The unit of the intensity of illumination is the foot-candle (abbreviated ft-c), which is the amount of illumination produced on a surface that is one foot away from a standard candle of one candlepower.

The intensity of illumination varies inversely as the square of the distance between the light source and the surface to be illuminated. For example, a 100-cp lamp will provide an intensity of illumination of 100 ft-c at a distance of 1 ft, 25 ft-c at 2 ft, 6.25 ft-c at 4 ft, and 1 ft-c at 10 ft. In practice, the intensity of illumination is generally determined by use of a photometer.

Speed of Light. At one time scientists believed that light traveled instantaneously from its source to the observer. In 1675, Olaus Römer, a Danish astronomer, calculated that the speed of light through air was approximately 186,000 miles per sec. By more accurate means Albert A. Michelson (1852–1931), a noted American physicist, determined the speed of light to be 186,284 miles per sec. For general purposes, the speed of light through air is taken as 186,000 miles per sec.

The speed of light varies with the medium through which it travels. Its speed through various media depends upon the density of the medium to light rays, as is indicated by the fact that light will travel faster through vacuum than through air, water, glass, etc.

Frequency, Wavelength, and Color. If light is considered as wave motion, it can be expressed in terms of frequency and wavelength the same as sound waves. The color of light varies with its frequency just as the pitch of a sound varies with the frequency of the sound waves. Also as in the case of sound, when the velocity and frequency of the light waves are known, it is possible to calculate the wavelength of the light waves. The values listed in Table 1-3 indicate the average frequency and wavelengths of the various colors of light that make white light when combined.

From this table it can be observed that the frequency of light waves is very high compared with sound waves and that the wavelength of light waves is much shorter than sound waves. Furthermore, examination of the

Table 1-3 Average Frequency and Wavelength of Various Colors of Light

COLOR	FREQUENCY, CPS	WAVELENGTH		
		CENTIMETERS	INCHES	MICRO-INCHES
Red	423,000,000,000,000	0.000071	0.000028	28
Orange	483,000,000,000,000	0.000062	0.000024	24
Yellow	525,000,000,000,000	0.000057	0.000022	22
Green	576,000,000,000,000	0.000052	0.000020	20
Blue	639,000,000,000,000	0.000047	0.000018	18
Indigo	682,000,000,000,000	0.000044	0.000017	17
Violet	732,000,000,000,000	0.000041	0.000016	16

frequency spectrum chart of Fig. 1-3 reveals that the frequency of light waves is also much higher than radio waves. It may be observed that white is not included in Table 1-3; it is omitted because white light contains all seven colors listed in the table. The spectrum of visible light consists of a band of colors changing gradually from violet at one end to deep red at the other, just as in a rainbow.

The phenomenon of color is explained by the fact that different materials may transmit (or reflect) lights of different colors. For example, a red piece of glass appears red because it transmits only the red and absorbs all the other colors contained in white light. If a material transmits two or more colors, a new color results. When a material transmits all seven colors, it appears colorless, as illustrated by ordinary window glass.

Frequencies just below those of visible red light are classed as *infrared,* and frequencies just above those of violet are classed as *ultraviolet.* These classifications are being used in a rapidly increasing number of applications that include electronic equipment.

Propagation of Light. As in radio- and sound-wave motion, *propagation* of light refers to the transmitting or spreading out of wave motion. The outstanding factor concerning the propagation of light is that light rays travel in straight lines when the medium transmitting the light is homogeneous (which means that the transmitting medium must be uniform). That light travels in a straight line may be observed when rays of sunlight enter a darkened room through a small opening. The straight path of the light rays becomes visible owing to the illumination of the dust particles in the air.

Reflection of Light. When light energy from a source strikes the surface of an object, some of the light energy is reflected. The amount and the color of the light reflected will depend on the condition of the surface and the color of the reflecting body. Smooth bodies reflect light better than irregular ones; also light bodies reflect more light than dark ones. This further explains the theory of color as illustrated by the fact that an opaque red body appears red because it reflects only the red light and absorbs all others. In the case of black, an object appears black because it absorbs all colors and hence reflects none.

When the reflecting surface is smooth and flat, the reflected rays will be reproductions of the original and the reflection is *regular.* When the reflecting surface is irregular, the reflected rays will not have the same relation to one another as the original and the reflection is *diffused.*

Refraction of Light. Refraction is the name used to describe the effect that causes rays of light to bend when they pass from one medium to another of different optical density. For example, air and water have different optical densities, which is another way of saying that light travels through air and water at different rates of speed. (In air the speed of light is 186,000 miles per sec; in water it is approximately 140,000 miles per sec.)

Thus if an object such as a pencil is placed in a glass of water, the refraction of the light causes the pencil or object to appear bent or broken.

Lenses. A *lens* is a piece of transparent substance denser than the surrounding medium and with at least one of the two surfaces ground to conform to a definite curvature. Lenses are classified in terms of the curvature being convex or concave. *Convex lenses* are thicker at the center than at the edges, and *concave lenses* are thinner at the center than at the edges. Although lenses are usually made of glass, they can also be made of quartz crystals, water or other clear liquids, and a variety of other materials.

Fundamentally, the function of a lens is to change the direction of rays of light. Convex lenses cause the light rays to converge, that is, to come to a common point from different directions; the image may be enlarged or decreased in size depending upon the distances between the lens, the object, and the image. Concave lenses cause the light rays to diverge, that is, to extend from a common point in different directions; the size of the image is always decreased.

There are many applications of lenses in everyday life such as eyeglasses, magnifying glasses, cameras, motion-picture projectors, microscopes, and telescopes.

Persistence of Vision. An important property of the human eye that makes television and motion pictures possible is persistence of vision. The eye cannot observe or follow any sequence of changes that occur at a rate of 10 or more times per second. Anything in excess of this rate produces the effect of a continuous picture. This phenomenon is called *persistence of vision.*

In a television receiver the image appearing on the picture tube is not a steady picture but is actually a sequence of 60 individual pictures per second, each separated from the preceding one by a short interval of time during which the screen of the picture tube is dark. The viewing area of the picture tube is coated with a fluorescent substance, usually a phosphor, that will glow for only a very short period of time after the electron beam strikes the screen; hence the tube is said to have low persistency. Because the persistence of vision of the human eye is much greater than the persistence of the picture tube, the image at the television receiver appears as a continuous picture that also reproduces the movements of the objects being viewed.

QUESTIONS

1. (*a*) Name five applications of electrical energy as produced by large power plants. (*b*) Describe two methods of producing large amounts of electrical energy.
2. (*a*) What is meant by a chemical cell? Name an application of (*b*) the primary cell, (*c*) the secondary cell.
3. Describe (*a*) the basic principle of the solar cell, (*b*) an application of the solar cell.

4. (*a*) Describe the basic principle of the fuel cell. (*b*) Name some of the fuels used in this type of cell.
5. Name 11 classifications of the field of electronics.
6. (*a*) Name five applications of the visual method of communications. (*b*) Name ten applications of the auditory method of communications.
7. Describe how the vastness and complexity of the communication field are illustrated by an outline of its radio branch.
8. Name 15 circuits that are basic to the field of communications and also basic to other branches of electronics.
9. Name four applications to industrial electronics of (*a*) the principles of resonance, (*b*) amplifier circuits.
10. Describe (*a*) the principle of stroboscopic lighting, (*b*) two applications of this principle.
11. (*a*) What is meant by automation? (*b*) Describe briefly how automated equipment is controlled.
12. Name (*a*) four electronic instruments used in electronics, (*b*) two electronic instruments used in fields other than electronics.
13. (*a*) What is a computer? (*b*) What are its basic advantages?
14. (*a*) Name two circuits used in computers that are not common to the field of communication. (*b*) Name four methods of indicating the solution or answer to a problem.
15. (*a*) What is meant by data processing? (*b*) Name three of its basic operations.
16. Describe what is meant by and explain an application of (*a*) a classifier, (*b*) accounting equipment, (*c*) interpretive equipment.
17. Name and describe 10 electronic instruments used in the field of therapeutics.
18. What is meant by (*a*) bionics? (*b*) Simulators?
19. How do simulators aid (*a*) the medical scientist? (*b*) The industrial scientist?
20. Describe four applications of electronics that are peculiar to military purposes.
21. (*a*) Name three exacting requirements of electronic equipment used in space vehicles. (*b*) Name two uses of telemetry.
22. (*a*) What is meant by cryogenics? (*b*) What values of temperature are used in this field?
23. (*a*) Name two basic electric circuits. (*b*) How may a complex electric circuit be analyzed?
24. What circuit components may be used in (*a*) d-c power circuits? (*b*) A-c power circuits? (*c*) Electronic circuits?
25. Describe the factors that have contributed to the miniaturization of (*a*) circuit components, (*b*) circuit wiring.
26. Why is a knowledge of the theory of electric circuits necessary in order to study electronics?
27. Describe a simple analogy of electrical and electronic wave motion.
28. Define (*a*) alternating current, (*b*) electrical degree, (*c*) alternation, (*d*) cycle, (*e*) frequency, (*f*) I_{max}.
29. Name five forms of radiant energy that are electromagnetic oscillatory disturbances in space.
30. Using the chart of Fig. 1-3, determine the frequency band assigned for (*a*) commercial a-m broadcast, (*b*) f-m broadcast, (*c*) vhf television, (*d*) uhf television.

31. (*a*) How are radio waves produced? (*b*) What two factors make up a radio wave? (*c*) Name three applications of radio waves.
32. Define (*a*) wavelength, (*b*) kilocycle, (*c*) megacycle.
33. What is sound?
34. Explain what occurs when sound waves strike the human ear and produce the sensation of sound.
35. (*a*) How are sound waves produced? (*b*) What factors affect the intensity at which a sound is heard?
36. Define the following terms as used with sound: (*a*) frequency, (*b*) pitch, (*c*) wavelength, (*d*) audible.
37. What frequency is usually used for the audio wave of code signals? Why?
38. How does the speed of radio waves compare with the speed of (*a*) sound? (*b*) Light?
39. Define the following terms as used with sound: (*a*) quality, (*b*) fundamental, (*c*) overtone, (*d*) harmonic.
40. Explain what is meant by (*a*) reflection of sound waves, (*b*) echoes, (*c*) sympathetic vibrations, (*d*) forced vibrations.
41. What is meant by resonance?
42. How are beat notes produced?
43. What is light?
44. Explain how objects are made visible.
45. Define the following terms: (*a*) transparent, (*b*) translucent, (*c*) opaque.
46. (*a*) What is meant by the intensity of illumination? (*b*) What is its unit of measurement? (*c*) What is its relation to distance and the source of light?
47. (*a*) What is the relation between the color of light and its frequency? (*b*) Why is white omitted from the frequency-color-spectrum chart?
48. Explain the phenomenon of color.
49. What is meant by (*a*) propagation of light? (*b*) Reflection of light? (*c*) Refraction of light?
50. (*a*) What is a lens? (*b*) How are lenses usually classified? (*c*) Describe each of the classifications named in (*b*).
51. (*a*) What materials are used in making lenses? (*b*) Name six applications of lenses.
52. (*a*) What is meant by persistence of vision? (*b*) Explain two commercial applications of this phenomenon.

PROBLEMS

1. What is the wavelength of a carrier wave of a transmitter whose frequency is 1,200 kc?
2. What is the wavelength of a carrier wave of a transmitter whose frequency is 660 kc?
3. What is the frequency of the radio waves from a transmitter operating on a 10-m wavelength?
4. What is the frequency of the radio waves of a transmitter if its wavelength is 75 m?
5. The frequency of the video carrier wave for channel 42 is 639.25 mc. Find its wavelength in (*a*) meters, (*b*) feet, (*c*) inches.

6. The frequency of the audio carrier wave for channel 77 is 853.75 mc. Find its wavelength in (*a*) meters, (*b*) feet, (*c*) inches.
7. What is the frequency of the video carrier wave whose wavelength is 21.044 inches?
8. What is the frequency of the audio carrier wave whose wavelength is 15.464 inches?
9. A certain f-m radio station operates on an assigned frequency of 95.5 mc. Find its wavelength in (*a*) meters, (*b*) feet, (*c*) inches.
10. A certain f-m radio station operates on an assigned frequency of 101.1 mc. Find its wavelength in (*a*) meters, (*b*) feet, (*c*) inches.
11. An experimental radio wave has a frequency of 60,000 mc. Find its wavelength in inches.
12. An experimental radio wave has a frequency of 150,000 mc. Find its wavelength in inches.
13. A certain radio station located in New York operates on a carrier frequency of 30 mc. How long does it take for an audio signal being transmitted to reach (*a*) Honolulu, Hawaii (approximately 5,000 miles)? (*b*) Melbourne, Australia (approximately 10,000 miles)?
14. A certain radio station located in Chicago operates on a carrier frequency of 1,210 kc. How long does it take for an audio signal being transmitted to reach (*a*) New York (approximately 800 miles)? (*b*) San Francisco (approximately 2,200 miles)?
15. How long does it take the radio carrier waves from a television transmitter to travel a distance of 100 miles?
16. The time between the sending out of a radar pulse and the receiving of its echo pulse is 0.001 sec. How far away is the target?
17. If the frequency of middle C on a piano is 256 cycles, what is its wavelength in (*a*) meters? (*b*) Feet?
18. If the frequency of high C on a piano is 4,096 cycles, what is its wavelength in (*a*) meters? (*b*) Feet?
19. If an a-f wave of 256 cycles is superimposed on a carrier wave of 1,200 kc, how many cycles does the carrier wave make during the time it takes the a-f wave to complete 1 cycle?
20. If an a-f wave of 4,096 cycles is superimposed on a carrier wave of 600 kc, how many cycles does the carrier wave make during the time it takes the a-f wave to complete 1 cycle?
21. The creaking of a door makes a sound of approximately 15,000 cycles. What is its wavelength?
22. What is the wavelength of the sound waves being produced by an insect if the frequency of the sound is 12,000 cycles?
23. How many cycles will a carrier wave of 1,200 kc make during the time required for (*a*) 1 cycle of a 256-cycle a-f wave? (*b*) 1 cycle of a 15,000-cycle sound wave?
24. How many cycles will a carrier wave of 100 mc make during the time required for (*a*) 1 cycle of a 256-cycle a-f wave? (*b*) 1 cycle of a 12,000-cycle sound wave?
25. Radio programs are often presented to studio audiences as well as to the radio audience. (*a*) How long does it take the sound waves to reach a listener in the studio audience seated 100 ft away? (*b*) How long does it take the program to reach a listener at the loudspeaker of a radio receiver 200 miles away? (*c*) Which listener hears the program first?

26. A certain public event is being broadcast from a large arena. (*a*) How long does it take the sound waves to reach a listener who is seated in the arena 250 ft from the microphone? (*b*) How long does it take for the program to reach a listener at the loudspeaker of a radio receiver 2,500 miles away? (*c*) Which listener hears the program first?
27. How far would a radio wave travel in the time it takes for a sound wave to travel 10 ft?
28. How far would a sound wave travel in the time it takes for a radio wave to travel around the earth (approximately 25,000 miles)?
29. A pipe of an organ is measured and found to be 0.565 ft long. At what frequency is it resonant if the pipe is (*a*) closed at one end? (*b*) Open at both ends?
30. What length of organ pipe, open at both ends, is required to produce resonance for a frequency of (*a*) 5,000 cycles? (*b*) 500 cycles? (*c*) 50 cycles?
31. What is the frequency of the beat note produced when a sound having a frequency of 400 cycles is combined with one of (*a*) 300 cycles? (*b*) 350 cycles? (*c*) 450 cycles? (*d*) 500 cycles?
32. If it is desired to produce a beat note of 250 cycles, what frequency sound wave must be added if the original sound has a frequency of (*a*) 100 cycles? (*b*) 200 cycles? (*c*) 500 cycles?
33. What is the intensity of illumination from a 200-cp lamp at a distance of (*a*) 2 ft? (*b*) 5 ft? (*c*) 20 ft?
34. What size lamp must be used to produce an intensity of illumination of 10 ft-c: (*a*) 10 ft from the lamp? (*b*) 5 ft from the lamp? (*c*) 2 ft from the lamp?
35. A source of light having a color between red and orange has a frequency of 450,000,000 mc. What is its wavelength in (*a*) centimeters? (*b*) Inches? (*c*) Microinches?
36. A source of blue light has a wavelength of 17.5 microinches (abbreviated μin.). What is its frequency in (*a*) megacycles? (*b*) Cycles?

Chapter 2

Basic Theory of Electricity

Electricity cannot be perceived by any one of the five senses, as we cannot see, hear, taste, smell, or feel it. However, it is possible to observe the effects of electricity as (1) *seeing* its effects produced at an electric light or at a television receiver, (2) *feeling* its heating effects produced at a laundry iron or a soldering iron, (3) *hearing* its effects at the output of a radio receiver or a record player, (4) *feeling* its effects when a shock is received by coming in contact with certain electric wires.

The exact nature of electricity is not known. However, scientific investigations indicate that it consists of small negative charges called *electrons*. Electricity occurs in two basic forms: (1) *static electricity*, when electrons are at rest, and (2) *dynamic electricity*, when electrons are in motion. Dynamic electricity is the kind used in the home, school, and factory, in fact any place where electricity is used to provide energy for (1) light, heat, and ventilation; (2) many types of mechanical work; (3) communications systems; (4) operating the numerous types of electrical and electronic equipment.

2-1 Electrostatics

Electrostatics may be defined as the study of electricity at rest, or *static electricity*. Static electricity may be produced by friction, and there are many examples of this in normal daily occurrences. After one combs one's hair, the static electricity in the comb will cause it to attract bits of paper. After a person walks across a rug, very often a spark caused by static electricity will appear upon touching a radiator. Trucks carrying a load of gasoline accumulate a charge of static electricity, and the spark that may result can cause serious consequences. To prevent such a dangerous accumulation of static charge in a gasoline tank truck, a chain is dragged along the ground and passes the static electricity to the earth. Sometimes static electricity sparks are obtained from leather belts that are being used to operate machinery. The friction of clouds produces static electricity which is known as lightning.

Storing charges of electrons on the plates of a capacitor and later releasing the charge is one example of a useful application of static electricity.

Many of the advancements made in the field of dynamic electricity owe their development to knowledge that scientists obtained from electrostatics.

2-2 Structure of Matter

Matter. Matter is defined as anything which occupies space and has weight; it may occur in a solid, liquid, or gaseous state.

Molecule. Any bit of matter (hereafter called a *substance*) of such size that it can be recognized by means of one or more of the human senses can conceivably be subdivided into smaller pieces. The smallest particle into which a substance can be divided by mechanical, chemical, or other means and still retain the same chemical characteristics as the original substance is called a *molecule.* The molecules of some substances can be further divided into two or more parts having characteristics that differ from the original substance, while those of other substances cannot be broken down into smaller divisions by any simple means.

Elements. Any substance whose molecules cannot be subdivided by ordinary chemical means is called an *element.* At present, there are about 100 known elements. Examples of commonly used elements are hydrogen, oxygen, carbon, copper, iron, aluminum, silver, and gold.

Atom. The smallest particle into which an element can be subdivided and still retain all the properties of the original element is called an *atom.*

Compound. Any substance whose molecules can be further subdivided and thereby yield atoms of two or more elements in the process is called a *compound.* Conversely, the molecules of compounds consist of the union of two or more elements. The resulting combination can have physical and chemical characteristics differing from those of its component atoms. For example, when two atoms of hydrogen gas combine properly with one atom of oxygen gas, they form one molecule of water. Chemically this is generally expressed as H_2O, where H_2 indicates that there are two atoms of hydrogen and O indicates one atom of oxygen. The basic molecules of wood contain three elements, namely, carbon, hydrogen, and oxygen. The human body contains about 15 elements of which oxygen, carbon, hydrogen, nitrogen, calcium, and phosphorus make up about 99 per cent. The infinite number of possible combinations that can be made by using any two or more elements and also by varying the proportions of each element used explains why it is possible to have so many different substances.

Size of Molecules and Atoms. A molecule is so small that it cannot be seen with the naked eye or even with the use of lenses, because both the eye and the lenses are themselves made of a vast number of molecules. They would have to become molecular in size in order for the molecules to become visible. Molecules are so small that a drop of water contains millions of molecules of water. As small as molecules are, they are relatively large compared with an atom, as a molecule may be made up of a number of atoms.

2-3 The Electron Theory of Matter

According to the electron theory of matter every atom consists of one or more positively charged particles called *protons* and one or more negatively charged particles called *electrons* and that an atom in its normal or neutral state has an equal number of protons and electrons.

Structure of the Atom. Molecules are composed of one or more atoms which in turn are made up of still smaller particles. The atoms of all elements, except hydrogen, are composed of protons, electrons, and neutrons. The hydrogen atom contains only one proton and one electron.

The *electron* is a very small negatively charged particle of the atom; its diameter is in the order of 1/10,000,000,000,000 inch. The *proton* is a very small positively charged particle of the atom; its diameter is in the order of 1/100,000,000,000,000 inch. In most elements, a number of protons have a very strong bond with a similar number of electrons and form a like number of *neutrons*. Because the charges of the electron and proton are of equal strength and opposite polarity, the neutron has no charge, as its name indicates. There is, however, a great difference in the weights of the electron and proton; the proton is about 1,845 times heavier than the electron. A neutron has about the same weight as a proton.

A simplified explanation of the electron theory, which is sufficient for this text, compares the structure of the atom with the sun and its planets. In the atom the protons and neutrons form a stationary central mass about which a number of electrons rotate. This central mass is called the *nucleus* and is compared with the sun, and the electrons represent the planets. The nucleus always has a positive charge. The electrons move in orbits around the nucleus, the number of orbits depending upon the number of orbiting electrons. Each orbit can hold only a limited maximum number of electrons. The first four orbits can accommodate only 2, 8, 18, and 32 electrons, respectively. Figure 2-2 illustrates, in a simplified manner, the structures of the atoms of several elements.

Although the nucleus contains more than 99 per cent of the weight of an atom, it occupies only a very small fraction (one million billionth) of its

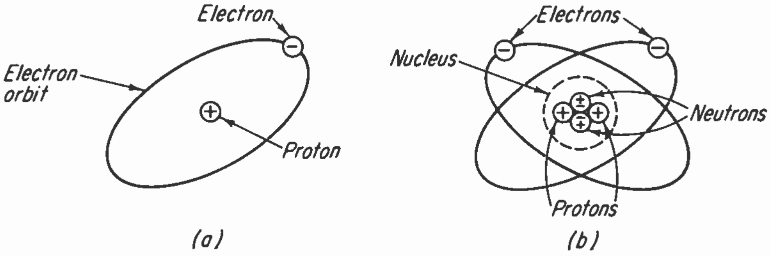


Fig. 2-1 Structure of the atom illustrating the electrons, protons, neutrons, nucleus, and electron orbits. (a) The hydrogen atom. (b) The helium atom.

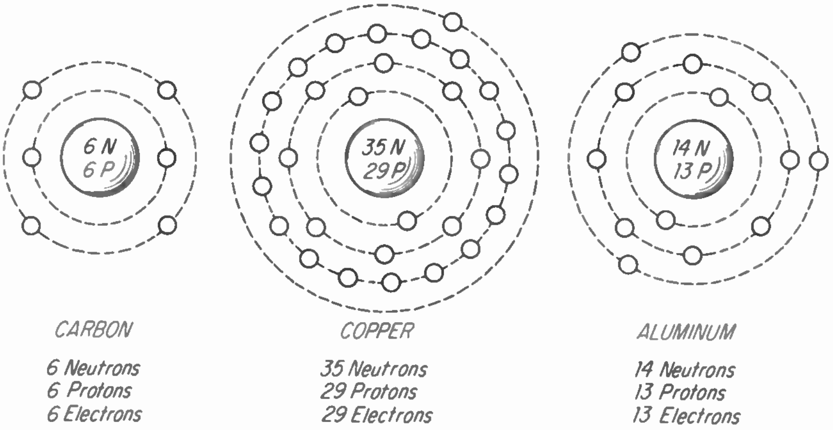


Fig. 2-2 Structure of carbon, copper, and aluminum atoms.

volume. The paths of the orbiting electrons, which move at very high speed, extend to relatively great distances from the nucleus. If an atom could be magnified by 10^{14} (one hundred thousand billion times), the electrons would be as large as basketballs and the spacing of the orbits would be about 12 miles.

Electrons and Current Flow. Electric current is described as electrons in motion or as a flow of electrons. Copper, carbon, and aluminum are conductors of electricity because in these materials electrons can be forced to move from atom to atom when an electrical pressure or voltage is applied.

In the atoms of the three elements shown in Fig. 2-2, the electrons in the inner orbits are tightly bound to the nucleus and cannot easily be forced out of their orbits; they are called *bound electrons.* The electrons in the outermost orbit, which contains fewer than its maximum possible number of electrons, are relatively loosely bound to the nucleus and are called *valence electrons.* Applying an electrical pressure to such materials will cause one or more valence electrons to move progressively from atom to

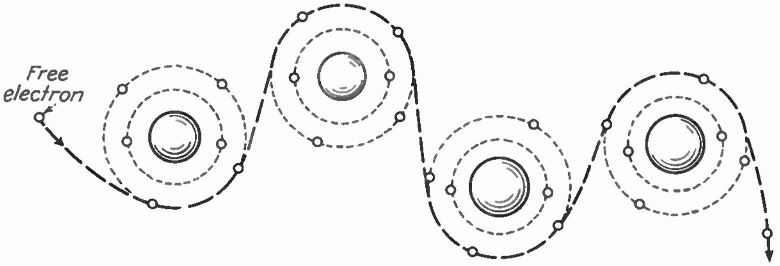


Fig. 2-3 Path of a free electron in a carbon conductor.

atom; these are called *free electrons.* Materials having only three, four, or five valence electrons are relatively good conductors of electricity.

When a current is flowing through a conductor, the individual free electrons drift from atom to atom along the conductor at a comparatively slow rate of speed, estimated to be in the order of a few inches per minute. However, free electrons in adjacent atoms pass along the impact, much like the cars of a freight train that is just starting, at about the speed of light or 186,000 miles per sec.

2-4 Positive and Negative Charges

Producing Charged Bodies. When a glass rod is rubbed briskly with a silk cloth, a number of valence electrons will be transferred from the glass rod to the silk cloth. Before the rubbing process, the number of electrons and protons in the atoms of each material were equal and the materials were each electrically in a *neutral state.* After the rubbing process, the atoms of the glass rod have a greater number of protons than electrons and the rod has a *positive charge.* The atoms of the silk cloth now have a greater number of electrons than protons and the cloth has a *negative charge.*

When a hard-rubber rod is rubbed with flannel, a number of valence electrons will be transferred from the flannel to the rod. The rod will become negatively charged, and the flannel positively charged.

Law of Charges. If a positively charged glass rod is suspended on a string (see Fig. 2-4) and a second positively charged glass rod is brought close to the suspended rod, the rods will repel each other. Two negatively charged hard-rubber rods will also produce a repelling action. But if a positively charged glass rod and a negatively charged hard-rubber rod are used, the two rods will attract each other as indicated in Fig. 2-5. These facts are the basis for the *law of charges;* namely, like charges repel each other, and unlike charges attract each other.

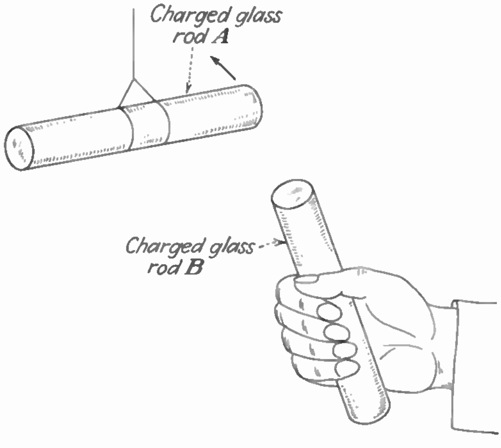


Fig. 2-4 Repulsion of two similarly charged rods.

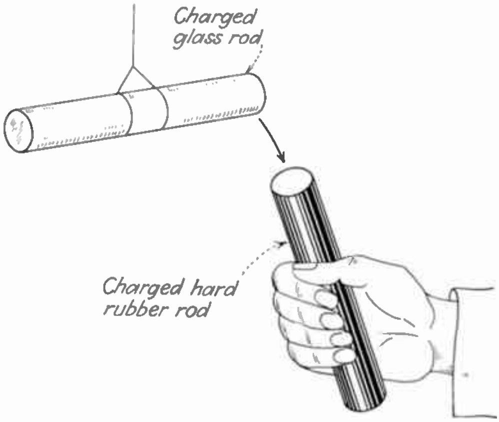


Fig. 2-5 Attraction of two dissimilarly charged rods.

2-5 Charging and Discharging

Charging by Contact. A charged body can transmit some of its charge to a neutral body in either of two ways, namely, by contact or by induction. If a negatively charged rod is placed in contact with a neutral body, some electrons will pass from the rod to the body and thereby charge that body. Figure 2-6*a* shows a rod with an equal number of protons and electrons. In Fig. 2-6*b*, a negatively charged rod is shown in contact with the neutral body, and electrons will pass from the charged rod to the neutral body. When the charged rod is removed from making contact with the body, a negative charge will have accumulated on the body (Fig. 2-6*c*). If a positively charged rod had been used, the neutral body would have lost some electrons to the positively charged rod and thus become positively charged. In either case the neutral body becomes charged by contact, and it takes on a charge of the same polarity as the charged rod.

Charging by Induction. If a negatively charged rod is brought near to but not in contact with a freely suspended neutral body, a repelling action is produced on the electrons of the neutral body. If the neutral body is connected with the ground (Fig. 2-7*b*), some electrons from the neutral body will move to the ground and the neutral body will become positively charged. If a positively charged rod had been used, the neutral body would have taken on some electrons (from the ground) and become negatively charged. In either case no electrons pass between the free body and the charged rod, and the free body is charged by induction. The polarity of the charge on the free body is opposite to that of the charged rod.

Discharging. The rate at which a charge passes from a body depends largely upon the shape of the body. If it is pointed, the charge passes off rapidly because the electrons are concentrated in a small area and build up considerable pressure. If a discharge takes place from a large area, such as a sphere or ball, the electrons are distributed over a large area and the pressure causing the discharge is relatively low.

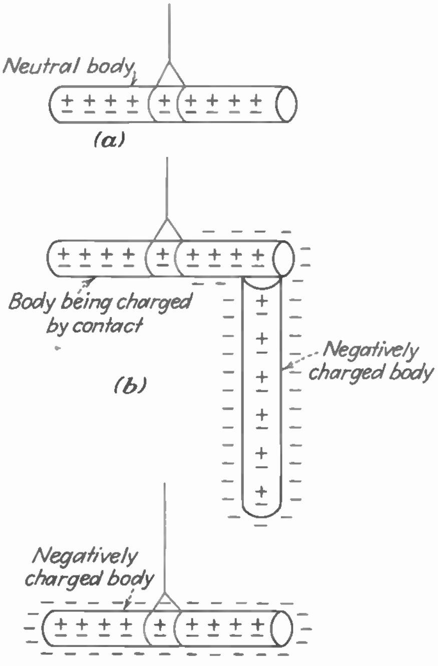


Fig. 2-6 Charging by contact.

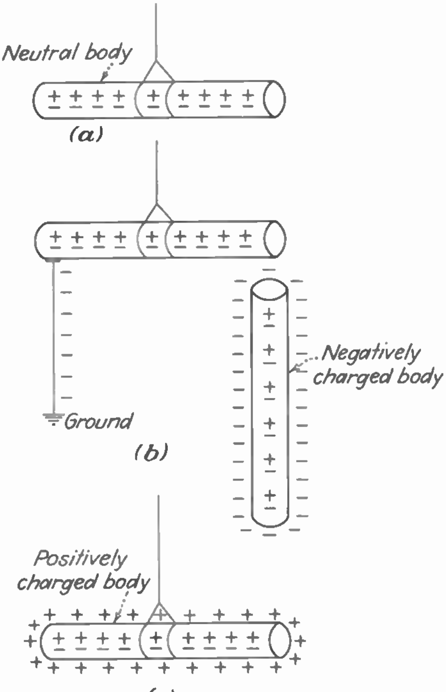


Fig. 2-7 Charging by induction.

Lightning. Lightning is the discharge that occurs between clouds of unlike charges or between a cloud and the earth. During the uprush of warm moist air from the earth, friction between the air and the tiny particles of water builds up charges. When drops of water are formed, the larger drops become positively charged and the smaller drops become negatively charged. When the drops of water accumulate, they form clouds, and hence clouds may possess either a positive or negative charge, depending upon the charge of the drops of water they contain. The charge on a cloud may become so great that it will discharge to another cloud or to the earth, and this discharge is called *lightning*. The thunder which accompanies lightning is caused by the sudden heating of the air, thereby causing it to expand. The surrounding air pushes the expanded air back and forth causing the wave motion of the air to produce the sound called ***thunder.***

Lightning Rods. The purpose of the lightning rod is to offer protection from lightning discharges by discharging small electrical charges as rapidly as they accumulate. Because pointed objects are relieved of charges more rapidly than any other shaped bodies, the lightning rod is made with a point.

In Fig. 2-8, the cloud has a positive charge, and it induces a negative charge on the lightning rods on the barn. The molecules surrounding the points of the rods become negatively charged (by induction) and repel one another as they accumulate. Also, the cloud, being of opposite charge, attracts some of these charges. Each charged molecule that reaches the cloud neutralizes some of the charge on the cloud. Usually this action completely discharges the cloud. However, if the cloud has a very strong charge, it will attract electrons with a terrific force. These electrons are drawn up from the earth through the lightning rod, which is intentionally made of a good conductor to provide a safe path. If the barn did not have the lightning rod, the electrons would rush through the material of which the barn is made. If the barn were made of wood (a poor conductor), the rush of electrons would cause much heat and would quite likely set the barn afire.

Lightning rods do not prevent lightning but rather prevent charges from accumulating on the buildings to which they are attached. The rod should be attached to a conductor embedded deeply enough in the ground so that it is always in moist earth. If it is not properly grounded, the lightning rod may be more of a menace than a protection because it will invite cloud discharges.

Single tall objects in open spaces such as a farmhouse or a single tree in a large field will accumulate charges rapidly and are therefore very likely to be struck by lightning. Farm buildings, church steeples, high brick chimneys, etc., are frequently protected by lightning rods. Steel buildings in a

Fig. 2-8 Application of the lightning rod.

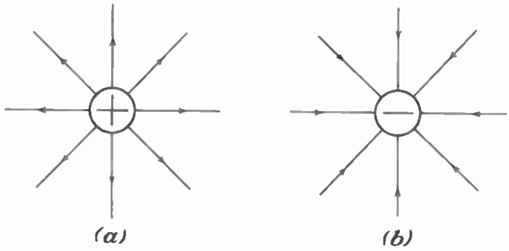


Fig. 2-9 Electrostatic fields. (a) Positive charge. (b) Negative charge.

city have numerous metal parts that extend into the ground and therefore have a good deal of protection.

2-6 Electrostatic Fields

Electrostatic Field. Figure 2-4 indicates that, when the positively charged glass rod *B* is brought close to the positively charged glass rod *A*, they will repel each other and cause the freely suspended rod *A* to move away from *B*. The fact that this action can take place without the rods making actual contact indicates that the air about the rods has been affected by the charges. The space about a charged body that is influenced by that charge is called its *electrostatic field*. As a force of repulsion (or attraction in the case of unlike charges) exists in this space it is also known as the *field of force*.

Electrostatic Lines of Force. The field about a charged body is generally represented by lines which are referred to as *electrostatic lines of force*. These are only imaginary lines and are used to represent the direc-

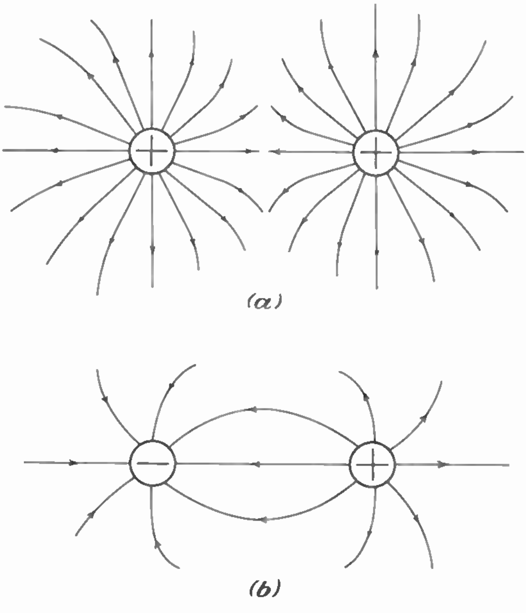


Fig. 2-10 Electrostatic fields. (a) Like charges. (b) Unlike charges.

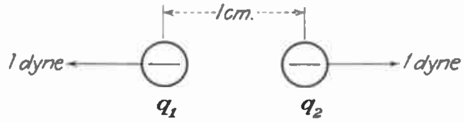


Fig. 2-11 Unit electrostatic charge.

tion and strength of the field. The lines about a positive charge are always shown leaving the charge, and for a negative charge they are shown as entering the charge. Figure 2-9 illustrates the use of lines to represent the field about a positive charge and a negative charge, while Fig. 2-10 illustrates the electrostatic field due to like charges and to unlike charges.

Force between Charges. Figure 2-11 shows two negative charges placed one centimeter apart. The charges repel each other with a force of one dyne. This illustrates the unit of electrostatic charge, which, by definition, is a charge that repels another equal and like charge one centimeter away with a force of one dyne. The electrostatic unit is generally abbreviated esu.

The force between two charges is directly proportional to the product of the charges and inversely proportional to the square of the distance between the charges. This is expressed mathematically as

$$F = \frac{q_1 q_2}{kd^2} \tag{2-1}$$

where F = force, dynes
q_1 = strength of charge 1, esu
q_2 = strength of charge 2, esu
d = distance between charges, cm
k = dielectric constant of the medium through which the force is exerted

Note: k for vacuum and air is 1; for other materials see Appendix VI.

EXAMPLE 2-1 What force is exerted between two negative charges of 10 and 20 esu, respectively, when placed 2 cm apart in air?

GIVEN: $q_1 = 10$ esu $\quad q_2 = 20$ esu $\quad k = 1 \quad d = 2$ cm

FIND: F

SOLUTION:

$$F = \frac{q_1 q_2}{kd^2} = \frac{10 \times 20}{1 \times 2 \times 2} = 50 \text{ dynes (repulsion)}$$

EXAMPLE 2-2 What force is exerted between two charges, one negative and one positive, each with a strength of 50 esu? They are placed in air 4 cm apart.

GIVEN: $q_1 = 50$ esu $\quad q_2 = 50$ esu $\quad k = 1 \quad d = 4$ cm

FIND: F

SOLUTION:

$$F = \frac{q_1 q_2}{kd^2} = \frac{50 \times 50}{1 \times 4 \times 4} = 156 \text{ dynes (attraction)}$$

2-7 Introduction to Dynamic Electricity

Potential and the Volt. In order to produce an electrostatic charge, either positive or negative, energy is required to move electrons from one position to another; the charge then possesses potential energy. In electrical terminology, *potential* is an abbreviation of the term potential energy. The practical unit for electrical potential is the *volt*, and the terms potential and voltage are sometimes used interchangeably.

Potential is measured relative to that of the earth, the potential of the earth being considered as zero. Notice that this is only a reference level of potential. Positively charged objects have a higher potential than that of the earth, and negatively charged objects have a lower potential than that of the earth.

Difference of Potential and Voltage. The effect of two unequal charges upon each other may be expressed in terms of their relative charges, and it is said that a difference of potential exists. This difference of potential is expressed in volts and is generally called *voltage.*

Article 2-5 describes how a charged object can transmit part of its charge to a neutral object. This can also take place between two charged objects if one has a greater potential than the other. A transfer of charge can occur only when there is a difference of potential between two points or objects.

In dynamic electricity the difference of potential is of great importance, for in order to maintain a flow of electrons there must be a continual difference of potential between the two points in which the electron flow is desired.

Electric Current. The continuous flow of electrons in a conductor is called an *electric current;* this is commonly shortened to just *current.* Such a movement of electrons occurs whenever a conductor (wire) is connected between two points of different potential. If one end of a wire is connected to a negative potential and the other end to a positive potential, electrons will flow from the negative to the positive potential. If the ends of the wire are connected to positive potentials but of different levels, electrons will flow from the lower positive potential to the higher one. If the ends of the wire are connected to negative potentials but of different levels, electrons will flow from the higher negative potential to the lower one.

Electron Motive Force or EMF. In order to maintain a flow of electrons in an electric circuit a pressure or force must be applied. This force is called *electromotive force* and is generally abbreviated as emf or EMF; its practical unit is the volt.

Voltage Drop. When a current is flowing through a number of elements in a circuit, a voltage, or difference of potential, will be present at the terminals of each circuit element; these voltages are often referred to as *voltage drops.*

EMF, Voltage Drop, and Difference of Potential. Because the practical unit of emf, voltage drop, and difference of potential in each case is the volt, their proper use may be confusing. Electromotive force is used to denote the voltage at a power source such as a battery or generator. Voltage drop is used to denote the voltage that exists between any two points of a circuit through which current is flowing. In any case, a difference of potential exists between two points.

2-8 Methods of Producing an Electric Current

In order to obtain an electron flow it is necessary to establish a difference of potential between the two points in which the electron flow is desired. There are various methods that may be used to set up a potential difference, such as friction, chemical, magnetic, thermal, light, and pressure.

Friction Method. Article 2-4 describes how rubbing together certain kinds of dissimilar materials produces static charges in the materials. Article 2-7 establishes the relationship between these static charges and difference of potential. Any discharge of static electricity is accompanied by an electric current, but for only a very short period of time.

Chemical Method. Batteries, fuel cells, etc., are examples of electrical energy produced by chemical action. If two dissimilar metals, such as copper and zinc, are immersed in a salt solution and they are connected by a wire or any other form of conductor, a chemical action will take place and cause an electric current to flow through the conductor. Such a unit is called a *cell*, and a combination of two or more cells properly connected is called a *battery*. Unless the battery is very large, the amount of electrical energy that can be obtained by this method is small. The chief uses of batteries are (1) where a small amount of current is needed, (2) where a portable source of electricity is needed, (3) for a reserve supply in case of an emergency.

Magnetic Method. An emf may be produced in a conductor when it is cut by or is itself cutting through magnetic lines of force. The act of forcing the electrons to move in a conductor by either of these means is called *electromagnetic induction*. The amount of emf obtained will depend upon the number of magnetic lines being cut, the number of conductors cutting them, and the speed with which it is done. This is the basic principle of the electric generator and the alternator, the former producing a direct current and the latter an alternating current. The flexibility as to the amount of emf obtained, the amount of current it can deliver, the ease of production, and the low cost make this the most practical method of obtaining large amounts of electrical energy.

Thermal Method. An electric current can be produced by heating two dissimilar metals at their junction point; this is called a *thermocouple*. The amount of electricity that can be obtained by this method will depend upon the amount of heat applied and the metals used, different metals giving different amounts of electricity. Practical combinations are made from antimony and bismuth, German silver and copper sulfide, copper and

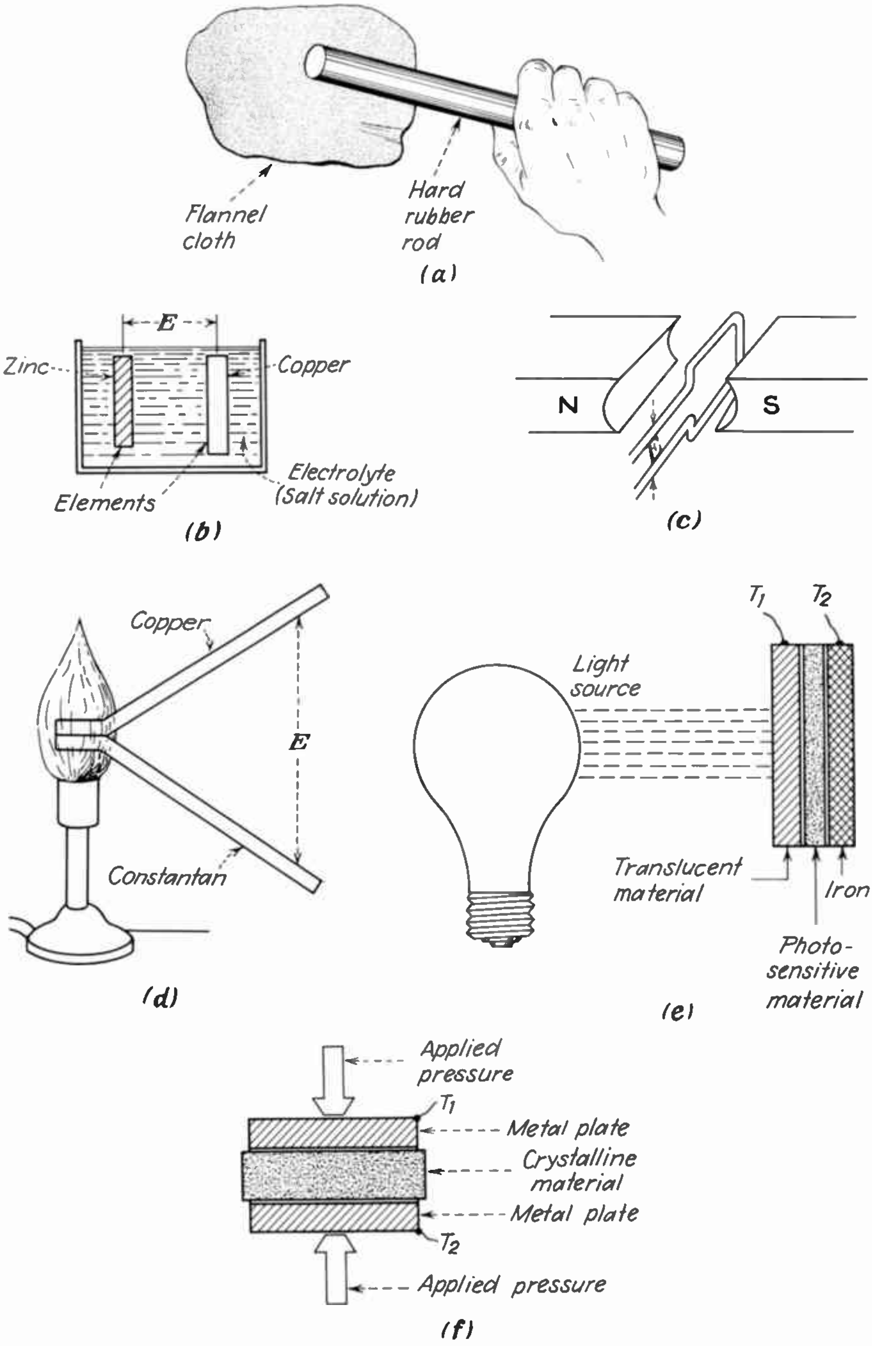


Fig. 2-12 Methods of producing an electric current. (a) Friction. (b) Chemical. (c) Magnetic. (d) Thermal. (e) Light. (f) Pressure.

constantan, iron and constantan. The amount of electricity produced at any one junction point is very small, and therefore the uses of thermocouples are limited. Three applications of thermocouples are (1) to measure the temperature where a thermometer may not be practical, (2) to measure high-frequency currents, (3) as a protective device.

Light Method. In this method, light energy is converted to electrical energy when light strikes the photosensitive material of a photocell. A basic photocell consists of three layers of material: (1) a translucent material, (2) a selenium alloy, and (3) iron. When light strikes the translucent material, it is focused onto the photosensitive selenium alloy and an electric charge is developed between the two outside layers which serve as the electrodes of the cell. The amount of electrical energy obtained from the photocell is small. The photographer's light meter is an example of one use of the photocell.

Pressure Method. When certain crystalline materials are placed under mechanical strain, an electrical difference of potential will be developed across opposite faces of the crystal. The material may be quartz, rochelle salts, or tourmaline, and the mechanical stress may be either compression or expansion. The basic pressure cell consists of a piece of crystalline material placed between two metal plates to which terminals are attached. Examples of the uses of pressure cells are the crystal microphone and the crystal cartridge of a record player.

2-9 Effects of Electric Current

When an electric current flows through a conductor, it produces a number of effects, the three most important being (1) thermal, (2) chemical,

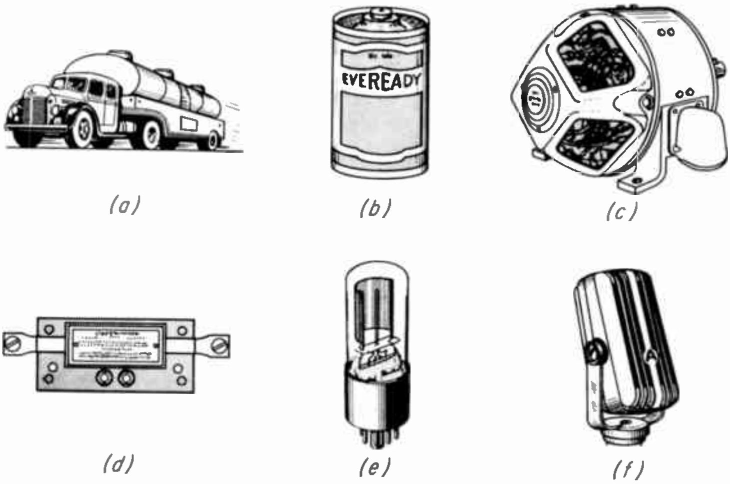


Fig. 2-13 Practical examples of various means of producing electric current. (a) Friction. (b) Chemical. (c) Magnetic. (d) Thermal. (e) Light. (f) Pressure.

(3) magnetic. The operation of all electrical appliances, devices, instruments, or machinery is based on one or more of these effects.

Thermal Effect. Any conductor through which an electric current flows will become heated owing to the fact that energy is used in forcing the current through the resistance offered by the conductor. The rise in temperature may be small or large, according to the current flowing and the resistance offered to the flow, but some heat is always produced. Because of this effect we are able to have the various electrical heating devices such as soldering irons, baking ovens, irons, toasters, and heating pads. The current flowing through an electric lamp heats the filament to incandescence causing the lamp to give off light. Vacuum tubes use this effect as their basic principle of operation, as any material will emit electrons when heated. This heat is obtained by having an electric current flow through a wire inside the tube.

Chemical Effect. Chemically, an electric current is capable of decomposing water, that is, breaking it up into its elements of hydrogen and oxygen. This effect, the decomposition of a chemical compound by an electric current, is called ***electrolysis*** and is used in applications of the chemical effect of electron flow. Battery cells, electroplating, electrotyping, and therapeutics are a few illustrations of the many applications of electrolysis.

Magnetic Effect. An electric current flowing in a conductor causes it to be surrounded by a magnetic field consisting of lines of force encircling the conductor at all points. This effect is the basis of operation of motors, generators, induction coils, and transformers; in fact, practically all electrical machinery and many electrical devices use this effect.

2-10 Kinds of Electric Current

All electrical currents are essentially the same in nature, but they may vary in their method of flow, direction, current strength, or a combination of these. Basically there are six different kinds of electric currents: (1) continuous, (2) pulsating, (3) direct, (4) alternating, (5) oscillatory, (6) interrupted.

Continuous Current. A continuous current is one in which the direction and amount of current flow do not vary with time. Referring to Fig. 2-14, it can be seen that the direction of flow does not change and that the intensity of the current is constant at all times as $I_1 = I_2 = I_3$, etc. Continuous currents are obtained from battery cells.

Pulsating Current. The direction of flow of a pulsating current is constant, but its strength rises and falls at fixed intervals. In Fig. 2-14, $t_1 = t_2 = t_3$, etc. Pulsating currents are obtained from any rectifier of alternating currents. The difference between the maximum and minimum values will depend on the rectifier and its filter circuit; the better the rectifier and its filter circuit, the smaller this difference becomes and the nearer the pulsating current approaches being a continuous current. There are numerous types of electric current of various wave forms used in electronics that are variations of pulsating current.

Direct Current. A direct current is a pulsating current whose flow varies so little that it is almost equivalent to a continuous current. Because the current flows in only one direction, it is a unidirectional current. This has been abbreviated by common usage to simply ***direct current.*** Direct-current generators actually generate an alternating current, but the commutator rectifies it to a direct current. Increasing the number of coils and commutator segments will reduce the difference between the value of the maximum and minimum currents.

Alternating Current. An alternating current reverses its direction of flow at fixed intervals. During each interval the current rises from zero to maximum, then diminishes from maximum to zero. Figure 2-14 shows that (1) the current flows in a positive direction during the intervals t_1, t_3, etc., and in a negative direction during the intervals t_2, t_4, etc.; (2) the intervals are all equal as $t_1 = t_2 = t_3$, etc.; (3) the maximum current in the positive direction is equal to the maximum current in the negative direction. Alternating currents are produced by a-c generators commonly called ***alternators.*** Because an alternating current can be readily transformed from low to high

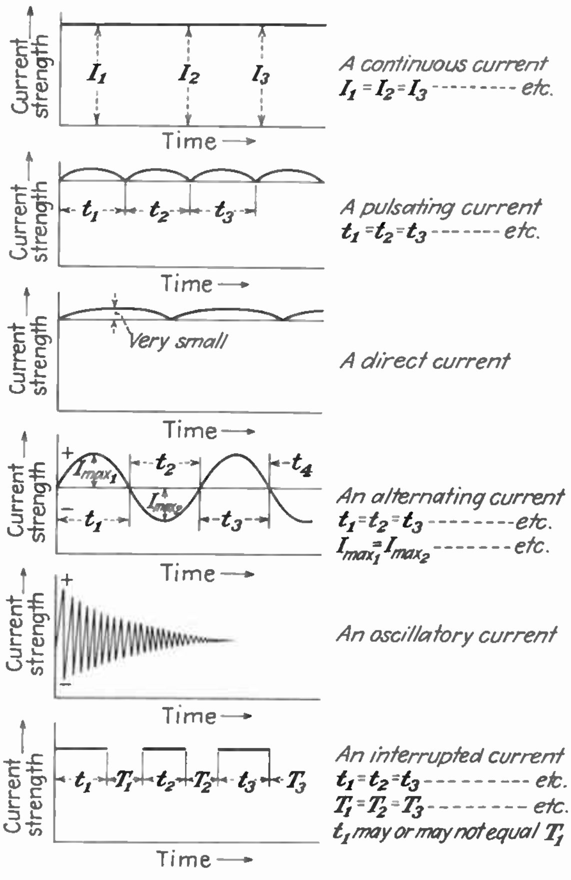


Fig. 2-14 Six kinds of electric current.

voltages, and vice versa, it is possible to send large amounts of power at a low current through conductors having a comparatively small diameter. It is, therefore, cheaper to transmit an alternating current over great distances than a direct current, and for this reason it is the type of current usually found in homes, offices, and other buildings.

Oscillatory Current. An oscillatory current is one which reverses its direction at fixed intervals and decreases in magnitude with each reversal until all current flow ceases. An example of an oscillatory current occurs when a capacitor is discharged through an inductor and a resistor. The energy stored in the capacitor will be transferred to the inductor, which will then in turn return its energy to the capacitor. This process repeats itself, but during each transfer some of the energy is dissipated (given off as heat) at the resistor. The time required for the capacitor to discharge completely and the maximum value of the current will depend on various factors in the circuit, such as the voltage applied and the size of the capacitor, inductor, and resistor.

Interrupted Current. An interrupted current is one in which the circuit is made and broken at fixed intervals. It may be an alternating, direct, pulsating, or continuous current. Interrupted currents can be obtained by either mechanical or electrical means. An example of the mechanical means of interrupting a current is the rotating drum which is used to break an electric circuit at definite intervals. The rotating-drum method may be used to control the lights on the various parts of an electric flashing sign. By this means it is possible to control the number of lights in various circuits and also the intervals at which these lights are turned on and off.

Interrupted currents can be obtained electrically by several methods. One method, used in some types of portable radio receivers and electronic devices, employs a vibrating coil in its power-supply circuit to increase the comparatively low battery voltage to a higher voltage required by the radio receiver or electronic device. Another method of interrupting a current by electronic means is to apply at the desired intervals of time an amount of voltage to the control grid of a vacuum tube sufficient to interrupt the flow of electrons in its plate circuit.

Complex Currents. Combinations of two or more of the various fundamental types of current described in the preceding paragraphs will result in a current of complex wave form. Currents having a complex wave form will be encountered in electronic circuits. Figure 2-15 illustrates a number of wave forms that are either variations of the fundamental waves or are complex waves.

2-11 Ohm's Law

The flow of electrons through a circuit resembles in many ways the flow of water through pipes. Therefore, the action of an electric current can be understood by comparing its flow with the flow of water.

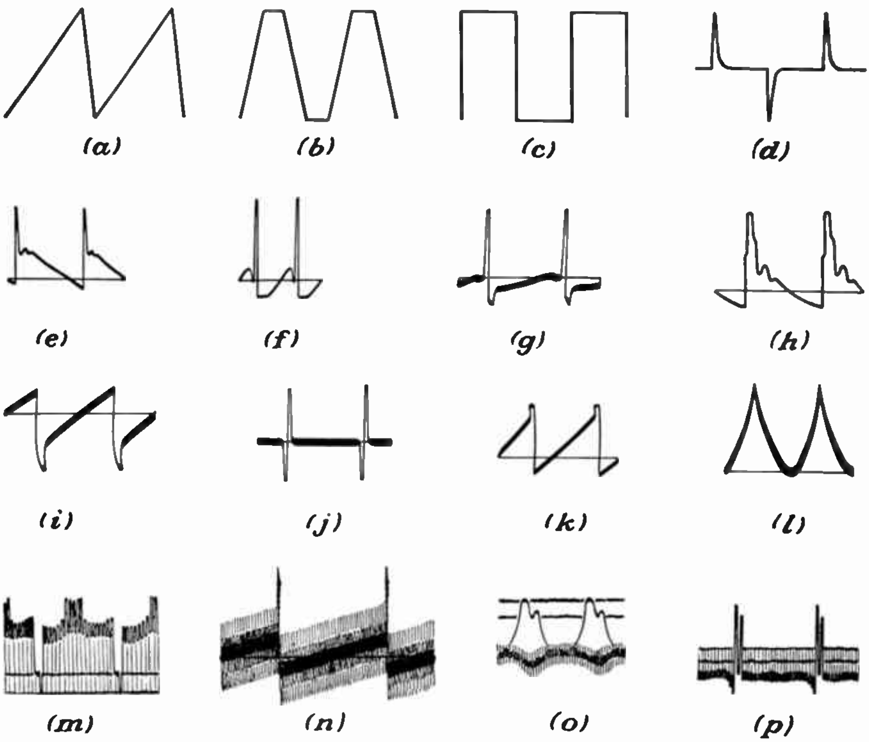


Fig. 2-15 Complex waves.

Hydraulic Analogy of Current Flow. In hydraulics, which is the scientific study of the flow of water in pipes, there are three fundamental considerations, namely, (1) the cause of the flow; (2) the amount of flow in a given time, or rate of flow; (3) the factors regulating the flow.

The rate of flow is controlled by the hydraulic pressure which may be obtained by means of pumps. If the pressure is increased, the amount of water flowing will be increased; if the pressure is decreased, the rate of water flow will decrease; and if there is no pressure, the water will stand still. The rate of water flow is, therefore, directly proportional to the pressure causing it to flow. If the resistance due to friction, bends, etc., is increased, the flow of water will be decreased, and if the resistance is decreased, the flow will increase. The rate of water flow is, therefore, inversely proportional to the resistance of the pipes through which it must flow.

The foregoing conclusions can be expressed by the simple mathematical equation

$$\text{Rate of water flow} = \frac{\text{hydraulic pressure}}{\text{resistance of the pipes}}$$

In the study of electricity there are three fundamental terms that compare with the three factors in hydraulics:

Electric pressure is called *voltage.*

Rate of electron flow is called *current.*

Electric resistance is called *resistance.*

Substituting the comparative electrical terms in the above equation,

$$\text{Current} = \frac{\text{voltage}}{\text{resistance}} \tag{2-2}$$

Ohm's Law. The mathematical relation between voltage, current, and resistance was discovered by Georg Simon Ohm and is called *Ohm's law.* This law is the foundation upon which the study of all branches of electricity is based. Ohm's law may be stated in three forms, the first as in Eq. (2-2) and the others as in Eqs. (2-3) and (2-4).

Equation (2-2) can be used to find the current that will flow in a circuit when the resistance and voltage are known. This equation can be transposed by algebra so that it will be possible to solve for the voltage when the current and resistance are known or to solve for the resistance when the voltage and current are known. Two new equations will be produced, namely,

$$\text{Voltage} = \text{current} \times \text{resistance} \tag{2-3}$$

and

$$\text{Resistance} = \frac{\text{voltage}}{\text{current}} \tag{2-4}$$

A study of these three equations will lead to the following conclusions:

1. From Eq. (2-2), the current flowing in a circuit having a fixed amount of resistance will increase with an increase in the voltage and decrease with a decrease in the voltage. Also, the current flowing in a circuit with a fixed amount of voltage will decrease with an increase in resistance and increase with a decrease in resistance. This may be stated more briefly. The current flowing in a circuit varies directly with the voltage and inversely with the resistance.
2. From Eq. (2-3), the voltage at any circuit or circuit element is directly proportional to the current and resistance of the circuit or circuit element.
3. From Eq. (2-4), the resistance requirements of a circuit or circuit element vary directly with the voltage and inversely with the current.

2-12 Fundamental Electrical Units

In every scientific field there are definite and precise units of measurement. In electrical work the basic units are the volt, ampere, and ohm. In practical work the numerical values of voltage and current of a circuit are obtained by use of a voltmeter and an ammeter. The resistance of a circuit may be obtained by the application of Ohm's law when the current and

Table 2-1 Comparison of Electrical and Hydraulic Terms and Units

HYDRAULIC ANALOGY			ELECTRICAL TERMS AND UNITS					
Term	Units of measure	How controlled	Term	Units of measure	How controlled	How measured	Symbols and abbreviations	Ohm's law relation
Pressure	Weight per unit area, lb/sq in., grams/sq cm	Pump	Electromotive force	Volt	D-C: batteries, generator A-C: alternator, transformer	Voltmeter	E = impressed voltage e = voltage drop	Current × resistance $I \times R$
Water	Quantity per unit time, gal/min, cu ft/sec	Valves	Current	Ampere	Rheostat, switch	Ammeter	i = branch current I = line current	$\frac{\text{Voltage}}{\text{Resistance}}$ $\frac{E}{R}$
Resistance	k, coefficient of friction	Turns, bends, friction	Resistance	Ohm	Length, cross-section area, material	Ohmmeter, bridge	r = branch resistance R = line resistance Ω or ω = ohm	$\frac{\text{Voltage}}{\text{Current}}$ $\frac{E}{I}$
Power	Horsepower-foot-pounds	Pressure or amount of water	Power	Horsepower, watt, kilowatt	Voltage or current	Wattmeter	P = power kw = kilowatt kva = kilovolt-ampere	Volts × amperes $E \times I$
Energy	Horsepower-hour	Power, time	Energy	Kilowatthour, watt-second	Power, time	Kilowatthour, meter	En = energy kwh = kilowatt-hour ws = watt-second	Power × time $P \times T$

voltage are known, or it may be measured directly with a bridge or with an ohmmeter. For voltmeters, ammeters, and ohmmeters to provide accurate indications of voltage, current, and ohms, they are calibrated to conform to standard meters of these quantities. Standard instruments are maintained by meter manufacturers and precision measuring laboratories, who in turn periodically have their instruments checked with United States standards at the National Bureau of Standards at Washington.

Standard Units. So that the volt, ampere, and ohm might represent the same value all over the world, standards were set up by an international commission in 1881 and made legal by the governments of all civilized countries. These standards have been corrected or revised at a number of subsequent international conferences.

The Ampere. When a conductor is connected to the terminals of a source of electrical energy, such as a battery or a generator, there will be a movement of free electrons from the negative terminal toward the positive terminal of the power source. The ampere is used to express the rate of flow of electrons, but as the electron represents a very small quantity, it is necessary to have 6,280,000,000,000,000,000 (6.28 quintillion) electrons flowing past a point to constitute one ampere. Because such a large number is difficult to use, it is replaced by the coulomb, which represents a charge of 6.28×10^{18} electrons (see Art. 4-16). Thus the *ampere*, named after André Marie Ampère, is the quantity of electricity equivalent to one coulomb passing a given point per second.

The Volt. The practical unit of electrical pressure (also of potential, potential difference, emf, and voltage drop) is the volt, named in honor of Alessandro Volta. The volt is equivalent to the electrical pressure required to force one ampere through a resistance whose value is one ohm. Through common usage, the word voltage is generally used in place of potential, potential difference, and emf.

The Ohm. Certain materials allow the free electrons to travel through them more easily than do other materials. These materials offer a comparatively small amount of resistance to the flow of electrons and are called *conductors*. The unit of electrical resistance is the ohm, named in honor of Georg Simon Ohm, and is equal to the amount of opposition offered by a conductor to the flow of one ampere of current when a pressure of one volt is applied across its terminals.

Practical Units. The practical units used in electricity and electronics have been adopted from the standards described above. These units are: (1) the ampere, for current flow; (2) the volt, for electrical pressure; (3) the ohm, for the resistance offered to the flow of current.

The mathematical relation between these terms is expressed by Ohm's law and has been stated as Eqs. (2-2), (2-3), and (2-4). It is common practice to express Ohm's law by using symbols for the words ampere, volt, resistance (see Art. 2-15). The symbol for ampere is I, for volt E, and for resistance R. Equations (2-2), (2-3), and (2-4) then become

$$I = \frac{E}{R} \qquad \text{or} \qquad \text{Amperes} = \frac{\text{volts}}{\text{ohms}} \tag{2-5}$$

$$E = IR \qquad \text{or} \qquad \text{Volts} = \text{amperes} \times \text{ohms} \tag{2-6}$$

$$R = \frac{E}{I} \qquad \text{or} \qquad \text{Ohms} = \frac{\text{volts}}{\text{amperes}} \tag{2-7}$$

EXAMPLE 2-3 How much current flows through an electric circuit that has a resistance of 20 ohms and is connected to a power supply whose pressure is 110 volts?

GIVEN: $R = 20$ ohms $\qquad E = 110$ volts

FIND: I

SOLUTION:

$$I = \frac{E}{R} = \frac{110}{20} = 5.5 \text{ amp}$$

EXAMPLE 2-4 What is the resistance of a circuit that has a current of 2.5 amp flowing through it when the electrical pressure is 125 volts?

GIVEN: $I = 2.5$ amp $\qquad E = 125$ volts

FIND: R

SOLUTION:

$$R = \frac{E}{I} = \frac{125}{2.5} = 50 \text{ ohms}$$

EXAMPLE 2-5 What pressure is required to force a current of 1.75 amp through a circuit whose resistance is 60 ohms?

GIVEN: $I = 1.75$ amp $\qquad R = 60$ ohms

FIND: E

SOLUTION:

$$E = IR = 1.75 \times 60 = 105 \text{ volts}$$

2-13 Work, Power, Energy

Work. Work is the accomplishment of motion against the action of a force tending to resist it. Work may be (1) useful, as drilling holes in a large sheet of steel; (2) destructive, as tearing one's clothes; (3) pleasant, as hitting a baseball for a home run; (4) accidental, as falling down a flight of stairs. In all cases, work is accomplished by the action of some force through a distance. The work done upon any body is equal to the product of the force exerted upon it and the distance through which it acts. Expressed mathematically,

$$\text{Work} = \text{force} \times \text{distance} \tag{2-8}$$

If the distance is expressed in feet and the force in pounds, then the work done will be expressed in foot-pounds.

Time is not a factor in the consideration of work done. For example, one person, being fairly strong, could move a 100-lb bag of sand across a room 10 ft long in one trip taking one minute, and another person, not being so strong, might have to do it in 10 trips, carrying 10 lb in each trip and taking one minute per trip. In both cases the work accomplished would be the same.

$$\text{Work done first case} = 100 \text{ lb} \times 10 \text{ ft} = 1{,}000 \text{ ft-lb}$$

$$\text{Work done second case} = 10 \text{ lb} \times 10 \text{ ft} \times 10 \text{ trips} = 1{,}000 \text{ ft-lb}$$

Power. Power is the rate of doing work per unit of time. Therefore, it may be said that the first person possessed more power than the second because he did the job at the rate of 1,000 ft-lb per min while the second one worked at the rate of 100 ft-lb per min to do the same job. Expressed mathematically,

$$\text{Power} = \frac{\text{work done}}{\text{time}} \tag{2-9}$$

It has been established that the average work horse could work at the rate of 33,000 ft-lb per min, or 550 ft-lb per sec. Thus the horsepower is a larger unit of power and is equal to

$$\text{Horsepower} = \frac{\text{foot-pounds of work per minute}}{33{,}000} \tag{2-10}$$

$$\text{Horsepower} = \frac{\text{foot-pounds of work per second}}{550} \tag{2-11}$$

The work done by an electric current may be the illumination of a room, the running of a motor for almost any kind of mechanical work, the operation of a radio receiver or transmitter, etc. The unit of electrical power is the watt, and it is equivalent to the work done in one second by a steady current of one ampere flowing under a pressure of one volt. Mathematically this is expressed as

$$1 \text{ watt} = 1 \text{ volt} \times 1 \text{ amp}$$

or $$P = E \times I \qquad \text{or} \qquad \text{watts} = \text{volts} \times \text{amperes} \tag{2-12}$$

If E/R, which is the equivalent of I as indicated in Eq. (2-5), is substituted for I in Eq. (2-12), it becomes $P = E \times E/R$, or

$$P = \frac{E^2}{R} \qquad \text{or} \qquad \text{watts} = \frac{(\text{volts})^2}{\text{ohms}} \tag{2-13}$$

Also, if IR, which is the equivalent of E as indicated in Eq. (2-6), is substituted for E in Eq. (2-12), it becomes $P = IR \times I$, or

$$P = I^2R \qquad \text{or} \qquad \text{watts} = (\text{amperes})^2 \times \text{ohms} \tag{2-14}$$

The above equations provide three means of calculating the power of a circuit. The equation to be used is best determined by first identifying the known quantities and then choosing the equation which best fits the conditions. A careful examination of the above equations will show that an equation has been provided for each combination of two known quantities of the Ohm's law group, namely, E, I, and R.

EXAMPLE 2-6 What power is consumed by a circuit that has a current of 2.5 amp when connected to a source whose pressure is 120 volts?

GIVEN: $I = 2.5$ amp $\quad E = 120$ volts

FIND: P

SOLUTION:

$$P = E \times I = 120 \times 2.5 = 300 \text{ watts}$$

EXAMPLE 2-7 What power is consumed by a circuit whose resistance is 80 ohms and which has a current of 1.5 amp?

GIVEN: $R = 80$ ohms $\quad I = 1.5$ amp

FIND: P

SOLUTION:

$$P = I^2R = 1.5 \times 1.5 \times 80 = 180 \text{ watts}$$

EXAMPLE 2-8 What power is consumed by a circuit whose resistance is 75 ohms and which is connected to a source whose pressure is 15 volts?

GIVEN: $R = 75$ ohms $\quad E = 15$ volts

FIND: P

SOLUTION:

$$P = \frac{E^2}{R} = \frac{15 \times 15}{75} = 3 \text{ watts}$$

Energy. Energy is the capacity to do work. The units for energy are, therefore, the same as those for work. The amount of energy in the universe is always constant, for, according to the law of conservation of energy, it can neither be created nor destroyed but is merely changed from one form to another. A battery changes chemical energy into electrical energy, and a generator changes mechanical energy into electrical energy. The energy of an electric current will be the energy of the moving electrons.

If a 100-watt lamp was placed in a room and no current flowed through its filament, the lamp would radiate no light. No work would be done, and therefore no energy would be used. If an electric current was caused to flow through its filament for 1 hr, the lamp would radiate light for 1 hr and would perform 100 watthours (whr) of work, thereby using 100 whr of electrical energy. Expressed mathematically,

$$\text{Energy (watthours)} = \text{power (watts)} \times \text{time (hours)} \qquad (2\text{-}15)$$

2-14 Other Units

The units of electricity such as the volt, ampere, ohm, and watt are basic quantities for the smaller and larger units that are used from time to time. The names of these units are formed by using the base name and a prefix to designate the quantity.

The prefix *milli-* means one-thousandth ($1/1{,}000$) and *micro-*, one-millionth ($1/1{,}000{,}000$). Both these prefixes are used to designate small amounts of voltage, current, and power in electrical instruments and various electronic circuits.

$$1 \text{ millivolt (mv)} = \frac{1}{1{,}000} \text{ of a volt}$$

$$1 \text{ milliampere (ma)} = \frac{1}{1{,}000} \text{ of an ampere}$$

$$1 \text{ milliwatt (mw)} = \frac{1}{1{,}000} \text{ of a watt}$$

$$1 \text{ microvolt } (\mu\text{v}) = \frac{1}{1{,}000{,}000} \text{ of a volt}$$

$$1 \text{ microampere } (\mu\text{a}) = \frac{1}{1{,}000{,}000} \text{ of an ampere}$$

$$1 \text{ microwatt } (\mu\text{w}) = \frac{1}{1{,}000{,}000} \text{ of a watt}$$

The prefix *kilo-* means one thousand (1,000) and is used to designate large amounts of voltage, power, and energy.

1 kilovolt (kv) = 1,000 volts
1 kilowatt (kw) = 1,000 watts
1 kilowatthour (kwhr) = 1,000 watthours

The prefix *meg-* means one million (1,000,000) and is used to designate high resistances.

1 megohm (mΩ) = 1,000,000 ohms

Electrical Equivalent of Horsepower. Although horsepower is basically a mechanical term, it is frequently desired to substitute an equivalent electrical amount of power. One horsepower is equivalent to 746 watts of electrical power, or

$$\text{Watts} = \text{horsepower} \times 746 \qquad (2\text{-}16)$$

2-15 Symbols and Abbreviations

Symbols. Symbols may be defined as abbreviations for any item or classification and may consist of letters, figures, emblems, or signs. The purpose of symbols is (1) to indicate definitely a particular item to the exclusion of all others and (2) to obtain brevity in the writing of equations, specifications, reports, drawings, etc. Symbols promote exactness, because each symbol is definitely associated with only one item. A description of an article or an electrical circuit may be vague, even though the words are used in the true sense of their dictionary definitions, for the reason that all persons do not ascribe the same meaning to words. It is easier to write $123.45 than "one hundred and twenty-three dollars and forty-five cents," and it would be quite a problem to add 20 or more different amounts of money if each amount had to be written out instead of using symbols.

Symbols used for the diagrammatic representation of electrical, radio, television, and other electronic apparatus have not all been standardized, and various modifications will be found. Those listed in Appendix I are the forms easily understood and easily made and are therefore the ones most commonly used. There is likewise no definite standard of size, and those listed are drawn to a scale used in general practice. At times, it may be necessary to enlarge or reduce the size of the symbols listed. When this is done, care must be taken so that all the symbols used are enlarged or reduced in the same proportion.

Abbreviations. The terms *voltage, current, resistance, power,* etc., are very seldom written out in the solution of problems or in an explanation of any electronic circuit. It is common practice to use the following abbreviations for the few terms that have been mentioned. A complete list will be found in Appendix II. In general, capital letters are used to denote the total current, resistance, power, etc., in a circuit and small letters to denote the resistance, current, power, etc., in any branch or part of a circuit.

E	The first letter of the abbreviation for electromotive force, used to designate the voltage applied to any circuit from a power supply. The subscript method of notation is used to denote the source of the applied emf as:
E_B	Voltage supplied from a battery
$E_{G \cdot A}$	Voltage supplied from generator *A*
E_L	Voltage supplied from a power line
e	The first letter of the abbreviation for electromotive force is used to designate voltage drops and all applied or output voltages other than those from power supplies. For example:
e_g	Input signal voltage, as applied to the grid circuit of a vacuum tube
e_o	Output voltage, as of an amplifier unit
e_r	Voltage drop, as the pressure lost in forcing a current through a resistor

I or i	Current (*intensity* of electron flow)
R or r	Resistance
Ω or ω	A symbol used in place of the word ohm
P or p	Power
En or en	Energy

2-16 How to Solve Problems

1. Read the problem very carefully.
2. Note the values that are given and the values to be found.
3. Draw a circuit diagram wherever possible.
4. Using the various equations that are known, find the one that involves the known and unknown values.
5. If none of the equations available will meet the needs, then find one that involves most of the knowns and that, when solved, will produce a value that can be used in another equation to solve for the desired unknown.
6. Write down the equation.
7. Substitute the known values.
8. Solve for the unknown, performing all necessary arithmetic on another sheet of paper.
9. Keep the work neat.
10. Underline the answer.

The following examples will illustrate the above rules:

EXAMPLE 2-9 If a 40-watt lamp operates from a 110-volt line, what current does it draw?

In this problem power and voltage are the known values and current the unknown value. This is recorded in the following manner, and a circuit diagram is drawn:

$$E = 110 \text{ volts} \qquad P = 40 \text{ watts} \qquad I = ?$$

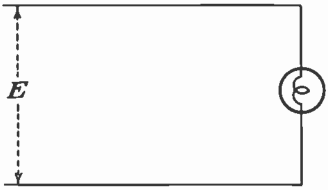


Fig. 2-16

As Eq. (2-12) contains just these three quantities, it may be used to solve this problem. The quantities are transposed in order to solve for the current, as

$$I = \frac{P}{E}$$

substituting the known values,

$$I = \frac{40}{110}$$

Dividing 40 by 110 on a separate piece of paper,

$$I = \underline{0.363 \text{ amp}}$$

The above problem should appear in the following form:

GIVEN: $E = 110$ volts $P = 40$ watts

FIND: I

SOLUTION:

$$I = \frac{P}{E} = \frac{40}{110} = \underline{0.363 \text{ amp}}$$

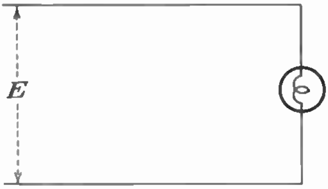


Fig. 2-16

The above example is very simple and requires the use of only one formula. When a problem involves the application of more than one formula, it is necessary to use step 5. This is illustrated by the following problem.

EXAMPLE 2-10 If a flatiron draws 5 amp from a 110-volt line, how much energy does it consume in 8 hr?

In this problem, current, voltage, and time are the known values and energy the unknown value.

The energy consumed can be found by Eq. (2-15), but this requires knowing the value of the power. It is therefore necessary to determine the power first, which can be found by use of Eq. (2-12). The solution of the problem will then appear in the following form.

GIVEN: $E = 110$ volts $I = 5$ amp $T = 8$ hr

FIND: En

SOLUTION:

$$P = E \times I = 110 \times 5 = 550 \text{ watts}$$

$$\text{En} = P \times T = 550 \times 8 = 4{,}400 \text{ whr} = \underline{4.4 \text{ kwhr}}$$

Fig. 2-17

If the cost of operation of any electrical appliance is desired, it is necessary to multiply the energy consumed by the rate at which electricity is sold. This is expressed mathematically as

$$\text{Cost} = \text{energy} \times \text{rate} \tag{2-17}$$

EXAMPLE 2-11 If electricity sells for 5 cents per kilowatthour, how much does it cost to operate the flatiron used in Example 2-10?

The solution of Example 2-10 shows that 4.4 kwhr is consumed, and the following step must be added:

$$\text{Cost} = \text{en} \times \text{rate} = 4.4 \times 0.05 = 0.22 = \underline{22 \text{ cents}}$$

These examples illustrate the general rules to be followed in solving problems. As there are any number of different kinds of problems, no one set of rules can hold for all of them; the above form may be varied somewhat, but the general thought always is kept intact.

Square Root. Solving problems involving Eqs. (2-13) and (2-14) may require extracting the square root of a number. For example, if in Eq. (2-13) the watts and ohms are known and it is required to determine the voltage, then

$$E^2 = P \times R \qquad \text{and} \qquad E = \sqrt{P \times R}$$

The process of deriving the square root can best be explained by working out an example.

EXAMPLE 2-12 If the value of E^2 was found to be 52,805.367, what is the value of E?

GIVEN: $E^2 = 52{,}805.367$

FIND: E

SOLUTION:

$$E = \sqrt{52{,}805.367}$$

The following procedure is recommended in working for the square root of a number.

1. Set down the number, and divide the digits on each side of the decimal point into groups of two; start at the decimal point, and work toward the left and then again from the decimal point work toward the right. It is possible for the extreme left-hand group to have only one digit. If the extreme right-hand group contains only one digit, a zero is added so that it will have two digits.

$$\sqrt{5\ 28\ 05.36\ 70}$$

2. Find the largest number that when multiplied by itself will go into the first group. In this example, the group is 5 and the largest number that can be used is 2. Now write the 2 above the 5. Next square the root number 2, and place it under the 5; its value is 4. Then subtract the 4 from 5, leaving a value of 1.

$$\begin{array}{l} \phantom{\sqrt{}}2 \\ \sqrt{5\ 28\ 05.36\ 70} \\ \phantom{\sqrt{}}\underline{4} \\ \phantom{\sqrt{}}1 \end{array}$$

3. Now bring down the next group of numbers alongside the remainder and draw a $\underline{\quad}\rfloor$ to the left of this complete number. Double that part of the square root already found, and place it in the $\underline{\quad}\rfloor$.

$$\begin{array}{rl} & \phantom{\sqrt{}}2 \\ & \sqrt{5\ 28\ 05.36\ 70} \\ & \phantom{\sqrt{}}4 \\ \underline{4} & \!\rfloor 1\ 28 \end{array}$$

4. The next step is to add a number alongside that part of the square root already found and also place this same number alongside the 4 in the ____|. This number *A* should be the largest that when multiplied by 4A| will go into the remainder 128. This step is done by trial-and-error procedure; therefore let us try the number 3.

```
      2  3
     √5 28 05.36 70
      4
   43|1 28
      1 29
```

At this point it is necessary to subtract again, but unfortunately 129 is just a bit too large. Therefore the number 2 must be used.

```
      2  2
     √5 28 05.36 70
      4
   42|1 28
        84
        44
```

5. Bring down the next group (05), and repeat the procedure. Double the root already found, and find the largest multiplier that can be used.

```
      2  2  9
     √5 28 05.36 70
      4
   42|1 28
        84
  449|  44 05
        40 41
         3 64
```

6. Continue until all places are filled above each group in the original number.

```
      2  2  9. 7  9
     √5 28 05.36 70
      4
   42|1 28
        84
  449|  44 05
        40 41
 4587|   3 64 36
         3 21 09
45949|     43 27 70
           41 35 41
            1 92 29
```

7. Notice that the decimal point in the root is placed directly above the decimal point in the original number. If desired, the solution can be carried to additional decimal places by adding groups of two zeros to the right of the existing digits, then proceeding as before.

$$\begin{array}{r} 2\ \ 2\ \ 9.\ 7\ \ 9\ \ 4 \\ \sqrt{5\ 28\ 05.36\ 70\ 00} \end{array}$$

Therefore the square root of 52,805.367 when worked to three decimal places is 229.794.

8. Should any of the divisors be too large to go into the remainder, a zero is placed in the root above that group and the next pair of numbers is brought down. This step is illustrated in the solution of the following example.

EXAMPLE 2-13 What is the square root of 1,620?

$$\begin{array}{r|r} & 4\ \ 1 \\ & \sqrt{16\ 20.} \\ & 16\quad\ \ \\ \hline \underline{81} & 0\ 20 \\ & 81 \end{array}$$

Note that 1 is too large; therefore

$$\begin{array}{r|r} & 4\ \ 0.\ 2 \\ & \sqrt{16\ 20.00} \\ & 16\qquad\ \ \\ \hline \underline{802} & 20\ 00 \\ & \underline{16\ 04} \\ & 3\ 96 \end{array}$$

The square root of 1,620 when worked to one decimal point is 40.2.

EXAMPLE 2-14 What is the voltage of an electrical circuit that takes 50 watts and whose resistance is 250 ohms?

GIVEN: $P = 50$ watts $\qquad R = 250$ ohms

FIND: E

SOLUTION: Using Eq. (2-13),

$$P = \frac{E^2}{R} \qquad \text{and} \qquad E^2 = PR$$

then $$E = \sqrt{PR} = \sqrt{50 \times 250} = \sqrt{12{,}500} = \underline{111.8 \text{ volts}}$$

EXAMPLE 2-15 What current is flowing through a 25-ohm resistance if the power being consumed is 500 watts?

GIVEN: $R = 25$ ohms $\qquad P = 500$ watts

FIND: I

SOLUTION: Using Eq. (2-14),

$$P = I^2R \qquad \text{and} \qquad I^2 = \frac{P}{R}$$

then

$$I = \sqrt{\frac{P}{R}} = \sqrt{\frac{500}{25}} = \sqrt{20} = \underline{4.47 \text{ amp}}$$

QUESTIONS

1. (*a*) Name two kinds of electricity. (*b*) In what main respect do they differ? (*c*) Which is used more widely?
2. How may static electricity be produced?
3. List some examples of static electricity.
4. Define the following terms: (*a*) matter, (*b*) molecule, (*c*) element, (*d*) atom, (*e*) compound.
5. Describe the composition of a compound.
6. State briefly the principle of the electron theory of matter in terms of protons and electrons.
7. What importance does the electron theory have in the study of electricity and electronics?
8. What are (*a*) electrons? (*b*) Protons?
9. Describe the structure of an atom in terms of electrons, protons, and nucleus.
10. Describe an atom of copper in terms of the electron theory.
11. What is the relationship between electric current and electrons?
12. Define (*a*) bound electrons, (*b*) valence electrons, (*c*) free electrons.
13. Describe the theory of electron flow along a conductor.
14. Compare the speed of movement of individual free electrons with the speed of the electric current.
15. Describe how electricity may be produced by friction.
16. When does a body have (*a*) a positive charge? (*b*) A negative charge?
17. State the law of charges.
18. (*a*) In what manner may charges be transferred from one body to another? (*b*) Describe each way.
19. Compare discharging from a pointed body with discharging from a sphere.
20. (*a*) What is lightning? (*b*) What is its cause?
21. Describe the use of the lightning rod.
22. What is meant by an electrostatic field?
23. What means are used to represent the electrostatic field?
24. Illustrate the field about (*a*) a positive charge, (*b*) a negative charge, (*c*) adjacent positive and negative charges.
25. Explain how the force between two charges is affected by (*a*) the strength of the charges, (*b*) the distance between the charges, (*c*) the medium through which the force is exerted, (*d*) the polarity of the charges.
26. What is meant by (*a*) potential? (*b*) Difference of potential?
27. Define electric current.
28. Describe the direction of current flow in relation to potential difference.

29. What is meant by (*a*) emf? (*b*) Voltage drop?
30. Name six ways in which a difference of potential may be established.
31. Explain how a difference of potential may be established by means of (*a*) friction, (*b*) chemical action, (*c*) magnetic fields.
32. Explain how a difference of potential may be established by means of (*a*) heat, (*b*) light, (*c*) pressure.
33. (*a*) Name and explain three important effects of electron flow. (*b*) Name some applications for each of these three effects.
34. Name and explain six kinds of electric current.
35. Explain where each of the above-mentioned currents may be found.
36. (*a*) What are complex currents? (*b*) Where are complex currents likely to be found?
37. In what respects is the flow of water similar to electric current?
38. What mathematical relationship does Ohm's law establish?
39. Why is a knowledge of Ohm's law essential to the study of electricity and electronics?
40. State the Ohm's law equations for (*a*) voltage in terms of current and resistance, (*b*) current in terms of voltage and resistance, (*c*) resistance in terms of voltage and current.
41. Is there any difference between the electrical units of measure in this country and any other country? Explain.
42. What is the relation between (*a*) an electron and an ampere? (*b*) A coulomb and an ampere?
43. Define the volt in terms of current and resistance.
44. What is the relationship of the term volt to potential difference, emf, voltage drop, and electrical pressure?
45. (*a*) What is meant by electrical resistance? (*b*) What is the ohm? (*c*) Define the ohm in terms of voltage and current.
46. State Ohm's law in its three forms in terms of the letter symbols E, I, and R.
47. Define the following terms: (*a*) work, (*b*) power, (*c*) energy.
48. Explain the difference between the terms work, power, and energy.
49. What is the basic unit of (*a*) electrical power? (*b*) Electrical energy?
50. Give the equations for finding the power in watts when (*a*) voltage and current are known, (*b*) voltage and resistance are known, (*c*) current and resistance are known.
51. In order to calculate the energy consumed by a circuit, what factor must also be known in addition to the power in watts?
52. What do the following prefixes mean: (*a*) milli-, (*b*) micro-, (*c*) kilo-, (*d*) meg-, (*e*) mega- ?
53. What is the electrical equivalent of a horsepower?
54. (*a*) What are symbols? (*b*) What is their purpose?
55. Have all electrical symbols been standardized? Explain.
56. What is the purpose of the subscript method of notation?
57. Name 10 steps suggested for the proper solution of problems.
58. When is it necessary to know how to solve for the square root of a number in electrical problems?

PROBLEMS

1. What is the magnitude and direction of the force between two positive charges of 200 and 300 esu, respectively, when placed in air 1.5 cm apart?
2. What is the magnitude and direction of the force between a positive and negative charge, each of 300 esu, when placed in air 2.5 cm apart?
3. What is the magnitude and direction of the force between two negative charges of 400 esu each when separated by a piece of fiber 2 cm thick?
4. If the fiber separator of Prob. 3 is replaced by one of common glass, what force would be exerted?
5. If the fiber separator of Prob. 3 is removed and not replaced by anything other than air, what force would be exerted?
6. What is the hot resistance of the filament of a vacuum tube if it draws 0.05 amp from a 1.4-volt power source?
7. How much current does the heater of a vacuum tube draw if it has a resistance of 42 ohms and is connected across a 6.3-volt source of power?
8. If a certain transistor circuit has a resistance of 1,500 ohms and requires a current of 1 ma, how much voltage is required of its power source?
9. What is the resistance of a circuit drawing 2.5 amp when 120 volts is impressed across its terminals?
10. The voltage drop across a rheostat is 3.6 volts when 1.8 ma is flowing through it. What is the resistance of the rheostat?
11. If a 2,250-ohm resistor causes a voltage drop of 1.5 volts, how much current is flowing through the resistor?
12. How much signal voltage is lost in a 2-megohm grid resistor when 0.1 μa flows through it?
13. What is the resistance of an ammeter whose full-scale reading is 10 amp and this maximum allowable current causes a voltage drop of 50 mv?
14. What is the rated maximum voltage of a voltmeter having a resistance of 150,000 ohms if the maximum rated current is 1 ma?
15. An 8,000-ohm resistance connected in the plate circuit of a certain vacuum tube causes a plate current of 22 ma. How much is the voltage drop across the 8,000-ohm resistance?
16. An electric lamp whose hot resistance is 240 ohms is to be used on a 120-volt circuit. How much current does it draw?
17. Which resistance is greater, one that requires 250 volts to force a current of 2.5 ma through it or one that requires 2.5 volts to force a current of 0.25 amp through it?
18. Through which resistor does the greater current flow, a 2-megohm resistor having a 250-mv drop or a 20-ohm resistor having a 25-volt drop?
19. Which resistance causes the greater voltage drop, a 250,000-ohm resistor having a current flow of 25 ma or a 250-ohm resistor having a current flow of 2.5 amp?
20. What is the rate of power loss in a 75-ohm resistor when 150 ma flows through it?
21. What is the power rating of an electric lamp operating from a 110-volt supply and drawing 0.909 amp?
22. If the total resistance effect of a radio set is 240 ohms and it operates from a 120-volt line, what is its rated power?
23. If the input of a d-c motor is 5 hp, how much current does it draw when operated from a 110-volt line?

24. If the input of a d-c motor is 5 hp, how much current does it draw when operated from a 220-volt line?
25. (*a*) If the output of a motor is 5 hp, what is its equivalent wattage rating? (*b*) If a motor draws 1,500 watts from a power line, what is the equivalent horsepower input to the motor?
26. How much does it cost to operate a flatiron that draws 10 amp from a 110-volt line if the iron is used 4 hr per day for 10 days and the cost of electricity is 4 cents per kilowatthour?
27. How much energy does a 7½-hp (input) motor consume in 1 month of 30 days if it is operated 2 hr each day?
28. What is the resistance of the output circuit of a power output tube if it delivers 15 watts and has a current flow of 40 ma?
29. If a radio set draws 0.85 amp from a 110-volt line, what is the cost of operation for 1 month of 30 days if it is used 3 hr each day and electricity costs 3 cents per kilowatthour?
30. A 600-watt toaster is used every day for a half hour. What does it cost to operate this toaster for one month of 30 days if electricity costs 3 cents per kilowatthour?
31. How much energy is used by an electric clock that runs every day for a month of 30 days if it operates from a 115-volt line and has a resistance of 4,000 ohms?
32. Which costs more to operate, a 5-hp (input) motor used 1 hr every other day for 1 month (30 days) or a 500-watt lamp used 4 hr each day for 1 month (30 days)?
33. Many power companies base their customers' bills on a rate schedule that varies with the amount of energy used over a 1-month period. The following is the rate schedule used by one power company for its residential customers:
    $1.05 for the first 14 kwhr (or less)
    3.9 cents for each of the next 26 kwhr
    2.77 cents for each of the next 60 kwhr
    2.56 cents for each of the next 100 kwhr
    2.05 cents for each kilowatthour in excess of 200 kwhr
    What is the electric bill if 368 kwhr of energy was consumed in 1 month?
34. Using the rate schedule given in Prob. 33, calculate the cost for 140 kwhr used in 1 month.
35. If kilowatthour meter readings of 11,552 and 12,018 were obtained at the start and finish, respectively, of a 1-month period, what is the electric bill based on the rate schedule given in Prob. 33?
36. The power output of a tube is 6.5 watts, and the current is 20 ma. What amount of voltage is developed across the circuit?
37. If the rated output power of a vacuum tube is 6.5 watts and the load resistance is 2,500 ohms, what amount of voltage is developed across the output circuit?
38. A 16,000-ohm resistor has a power rating of 10 watts. What is the voltage across its terminals when it is delivering its rated power?
39. How much current flows through the heater of a rectifier tube which has a resistance of 83.3 ohms and consumes 7.5 watts of power?
40. What is the current rating of a 10-watt 500-ohm resistor?
41. How much current must be flowing in a 7,500-ohm resistor if the power consumed is 25 watts?
42. What is the distance between two positive charges each of 250 esu if the repelling force is 750 dynes? The charges are separated only by the surrounding air.

43. What thickness of fiber separator would cause the force between the two charges of Prob. 42 to be 500 dynes?
44. (*a*) What is the resistance of the heater circuit of a television picture tube if its rated voltage is 6.3 volts and its rated current is 0.6 amp? (*b*) What is the power rating of the heater circuit of this tube?
45. (*a*) What is the resistance of the heater circuit of a rectifier tube if it is rated at 5 volts and 3 amp? (*b*) What is the power rating of the heater circuit of this tube?
46. A certain six-tube radio receiver has a power rating of 40 watts. What is the cost of operation for 30 days if the receiver is used on an average of 10 hr per day and the cost of electricity is 3 cents per kilowatthour?
47. A certain 12-tube a-m and f-m radio receiver has a power rating of 85 watts. What is the cost of operation for 30 days if the receiver is used 10 hr per day and the cost of electricity is 3 cents per kilowatthour?
48. A certain stereo high-fidelity music-reproducing system draws 185 watts from the power line. What is the cost of operation for 30 days at 6 hr per day if the cost of electricity is 3 cents per kilowatthour?
49. A certain portable television receiver with a 17-inch picture tube draws 180 watts from the power line. What is the cost of operation for 30 days at 6 hr per day if the cost of electricity is 3 cents per kilowatthour?
50. The d-c power supply of a certain television receiver delivers 250 ma at 325 volts and 60 μa at 16,000 volts. (*a*) What is the power rating of the low-voltage supply? (*b*) What is the power rating of the high-voltage supply? (*c*) What is the apparent resistance of the low-voltage load? (*d*) What is the apparent resistance of the high-voltage load?

Chapter 3

Batteries

The battery has played a very important part in the development of the various forms of electronic communication; it is also used to operate many types of electronic equipment. Military, medical, and space requirements have stimulated fantastic advances in the field of electronics, with an accompanying need for specialized battery power sources. Continuous research and development in these fields have produced new chemical-electrical systems. In addition to the improvements made to the chemical-electrical cell, two new methods of obtaining electrical energy have been developed: (1) the solar cell, which converts light energy from the sun to electrical energy, and (2) the fuel cell, which converts the energy from conventional fuel oxidation directly to electrical energy.

3-1 Battery Terms

Cell. A cell is a device that transforms chemical or light energy to electrical energy. Two examples of the conversion of chemical energy to electrical energy are the carbon-zinc (dry cell) and lead-lead peroxide (storage battery) cells. Two examples of the conversion of light energy to electrical energy are the selenium and solar cells.

Battery. A battery consists of two or more cells that are connected to each other and are usually placed in a common container. For example, the 12-volt batteries used in automobiles consist of six 2-volt cells connected in series, and a 4½-volt battery used in a transistor radio receiver may consist of six 1½-volt cells connected in a parallel-series arrangement of two groups of cells connected in parallel with each group having three cells connected in series.

Electrodes. The electrodes are the conductors through which current leaves or returns to the electrolyte.

Negative Electrode. The negative electrode usually consists of a single metallic element such as zinc, lead, iron, or cadmium. A common characteristic of these materials is the ease with which they release electrons to the external circuit, thereby becoming a source of positive-charged ions.

Positive Electrode. The positive electrode usually consists of a chemical compound such as manganese oxide, lead oxide, mercuric oxide, or silver chloride. A common characteristic of these materials, which also serve as depolarizers, is the ease with which they accept electrons, thereby becoming a source of negative-charged ions.

Electrolyte. The electrolyte is the solution in which the electrodes are placed and serves as an ion-transfer medium between the negative and positive electrodes. The electrolyte may be a salt, an acid, or an alkaline solution in either a liquid form as used in automobile storage batteries or a paste form as used in dry cells.

Primary Cell. A primary cell is one in which the chemical action decomposes one of the electrodes when the cell delivers current. When a relatively large portion of the electrode has been eaten away, the cell is no longer capable of delivering its rated voltage and current and has to be replaced.

Secondary Cell. A secondary cell is one in which the electrodes and electrolyte are altered by chemical action that takes place when the cell delivers current and may be restored to their original condition by sending a current through them in the opposite direction. The automobile storage battery and the nickel-cadmium rechargeable battery are examples of the secondary cell. Charging these batteries after they have become discharged represents restoring the cells to their original state.

3-2 Fundamental Principles of a Cell

The Voltaic Cell. The fact that chemical action could produce electric current was accidentally discovered in 1786 by Luigi Galvani, but it remained for Alessandra Volta to understand this chemical action, and in 1800 he constructed the first cell. This simplest form of cell, known as either the *galvanic* or *voltaic cell*, is shown in Fig. 3-1 and consists of a strip of copper and a strip of zinc placed in a jar of water to which a little sulfuric acid has been added.

Factors Affecting Cell Construction. If two electrodes of copper are

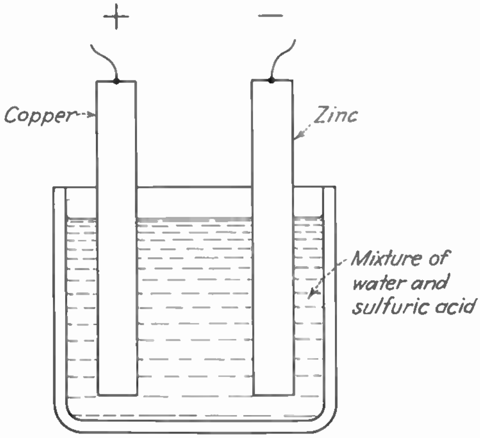


Fig. 3-1 Simple voltaic cell.

placed in a weak sulfuric acid solution, no electrical energy will result, but if one electrode of copper and one of zinc are placed in the solution, the chemical action will cause electrical energy to be available at the open ends of the electrodes. The basic characteristics of cells are:

1. The two electrodes must be of different materials.
2. The electrodes must be conductors of electricity.
3. The electrolyte must contain an acid, alkali, or salt that will conduct the current.
4. The voltage of the cell will vary with the materials used as electrodes and electrolyte but will not exceed approximately 2 volts.
5. The voltage of the cell is not affected by the size or spacing of its electrodes.
6. The current capacity of a cell may be raised by increasing the surface area of that part of its electrodes actually making contact with the electrolyte.

Chemical Action of a Simple Cell. If a cell as shown in Fig. 3-2 is made by placing a carbon rod and a strip of zinc into a jar containing water to which a small amount of ammonium chloride (sal ammoniac) has been added, certain chemical actions will result. When the ammonium chloride is placed in the water, many of the atoms become separated from their original molecules and ammonia and chlorine molecules are formed. Another action that takes place during this process is an unbalancing of the number of electrons and protons in each atom. When an atom is unbalanced, the number of electrons and protons no longer equal one another and it is then called an *ion.* If the protons are greater in number, the atom is called a *positive ion;* if the electrons are greater in number, the atom is called a

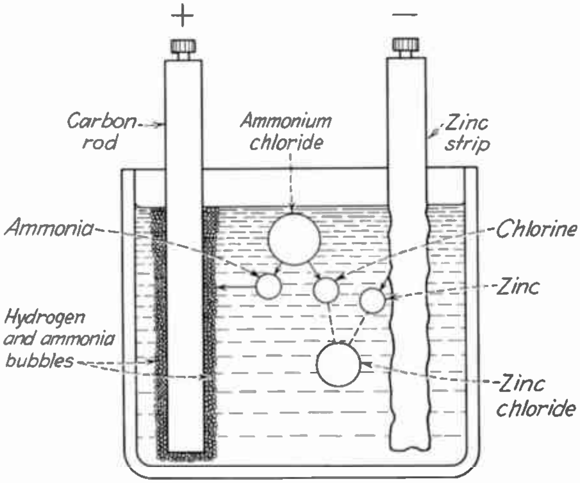


Fig. 3-2 Chemical action of a simple cell.

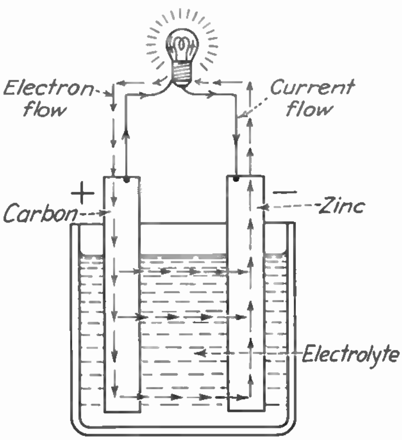


Fig. 3-3 Electron flow versus current flow.

negative ion. The process of unbalancing the atoms is called *ionization* of the electrolyte.

Current Flow. The ability of the cell to deliver electrical energy may be demonstrated by connecting a low-voltage flashlight bulb to its terminals (Fig. 3-3). The bulb will light, thus indicating that current is flowing. The excess electrons on the zinc strip repel one another and push one another around the external circuit where they are attracted by the positive carbon electrode. Chemical action in the cell maintains a constant flow of electrons as long as there is an external circuit.

Erosion of the Negative Electrode. While this chemical action is going on in the cell, the zinc ions unite with the chlorine ions to form zinc chloride, which remains in the solution. The fact that some of the zinc is continually being given off to the solution indicates that this electrode is being reduced or eaten away, and it is therefore a primary cell.

Electron Flow versus Current Flow. The action of a cell indicates that *externally* electrons flow from the zinc electrode through the external circuit to the carbon electrode and *internally* from the carbon electrode through the electrolyte to the zinc electrode to complete the circuit (Fig. 3-3). In Chap. 2, the flow of electrons has been described as the electric current. Hence, the electrons and therefore the current flow externally from the negative electrode (zinc) to the positive electrode (carbon). By definition, a *cathode* is the electrode from which electrons move within a cell, and an *anode* is the electrode toward which electrons move within a cell. Hence, internally the electrons flow from the cathode (positive electrode) to the anode (negative electrode).

Before the scientists developed and introduced the electron theory, the early experimenters arbitrarily chose the direction of current flow as being from positive to negative, and they called the positive electrode the anode and the negative electrode the cathode. This is opposite to the flow of electrons and also conflicts with the definitions for anode and cathode. This has caused and will continue for some time to cause confusion. As this text concerns the study of electricity for electronics, all study and references will be based on the electron flow.

3-3 Polarization and Local Action

Polarization. In the operation of the simple cell, positive hydrogen ions are liberated from the water in the electrolyte, and when they come in contact with the carbon electrode, these positive ions take on electrons from the carbon and form neutral hydrogen bubbles on the surface of the positive electrode. In a similar manner, the positive ammonia ions also take on electrons from the carbon, become neutral ammonia atoms, and also form bubbles on the surface of the positive electrode. The effect of the neutral ammonia and hydrogen bubbles at the carbon electrode is to reduce the active area of the electrode and hence lower the current capacity of the cell. Another effect of these bubbles is to set up a voltage which opposes the normal voltage of the cell and thereby reduces its effective voltage. This effect, which is called *polarization* and is common to all primary cells, is counteracted by adding a depolarizing agent to the cell. A *depolarizer* is a substance having a relatively large amount of oxygen which readily combines with hydrogen to form water, thus preventing the hydrogen from becoming attached to the positive electrode.

Local Action. If a cell is to be efficient, the chemical action that eats away the negative electrode should take place only when the cell is delivering electrical energy. As commercial metals contain some impurities such as carbon or iron, many small cells are formed on the electrode itself. The effect of these small cells is to eat away the metal whether the cell is delivering electrical energy or not; this effect is called *local action.* A method used in commercial carbon-zinc cells to reduce local action is to apply mercury to the surface of the zinc so that the impurities become covered with mercury. This process is called *amalgamation.* As the zinc is eaten away, the mercury works its way on to combine with the remaining zinc.

3-4 Primary Cells

Types of Cells. The earliest cell (Leclanché) consisted basically of carbon, zinc, and ammonium chloride and is commonly referred to as a *carbon-zinc cell.* Since the first Leclanché cell many different types of cells have been developed to meet specific needs. The following four types of cells are most generally used: (1) the carbon-zinc cell, (2) the mercury cell, (3) the silver oxide cell, and (4) the alkaline cell.

The Dry Cell. The dry cell is so called because its electrolyte is not in a liquid form. Actually the electrolyte is a moist paste, and if it should lose its moisture and really become dry, the cell would no longer be able to transform chemical energy to electrical energy. Though technically incorrect, most primary cells are commonly called *dry cells.*

Cell Life. The life of a dry cell is determined by the following five factors: (1) ***initial drain,*** the current that the cell is expected to deliver at full voltage; (2) ***operating schedule,*** the daily time interval or intervals during which the cell is required to deliver current; (3) ***cutoff voltage,*** the voltage below which the cell is no longer useful (this voltage varies with the application); (4) ***temperature,*** this factor is dependent on its operating characteristics at both low and high temperatures; (5) ***shelf life,*** a cell that is not put into use will gradually deteriorate because of unavoidable slow chemical reactions and changes in moisture content which take place in the cell.

Physical Characteristics. Primary cells are available in both cylindrical and rectangular shapes of various sizes. Regardless of the shape or size, the basic construction for each type of cell is always the same. The type of terminals used may be socket, snap, Fahnestock, flat contact, or plastic knurl. The size and shape of a cell are determined by its use. The larger its initial drain and the longer its operating schedule, the larger will be the size cell required. All cells are made in standard shapes and sizes that are the same for all manufacturers. Three commonly used sizes are: (1) the No. 6 or ***standard cell,*** (2) the *D* or ***unit-size cell,*** and (3) the *AA* or ***standard penlite cell.***

3-5 The Carbon-Zinc Cell

Construction of the Carbon-Zinc Cell. The construction of the carbon-zinc cell and other types of cell varies with the manufacturer and size of the cell. However, their basic structure is the same. The cross section of a typical carbon-zinc cell is shown in Fig. 3-4*a*. The cell is built in a cylindrical container which also serves as the anode and negative electrode. A cathode mix, which serves as both the positive electrode and depolarizer, occupies most of the cell. A gelatinous paste, which contains the electrolyte, separates the cathode mix from the zinc and functions as the ion-transfer medium between the positive and negative electrodes. The electrolyte is a solution of ammonium chloride, zinc chloride, and water. The cathode mix is a powder, consisting of manganese dioxide, zinc chloride, and graphite, and is not mechanically suitable for termination. This problem is overcome by placing a carbon rod in the center of the cathode mix to serve as a connection for the positive terminal. The carbon rod is used because it (1) is a good electrical conductor, (2) has a large surface area that provides a low-resistance conducting path, (3) is porous enough to permit the escape of gases accumulating in the cell, and (4) has sufficient density to prevent leakage of the electrolyte. As the zinc container is one of the electrodes, it is protected with an insulating covering of paper

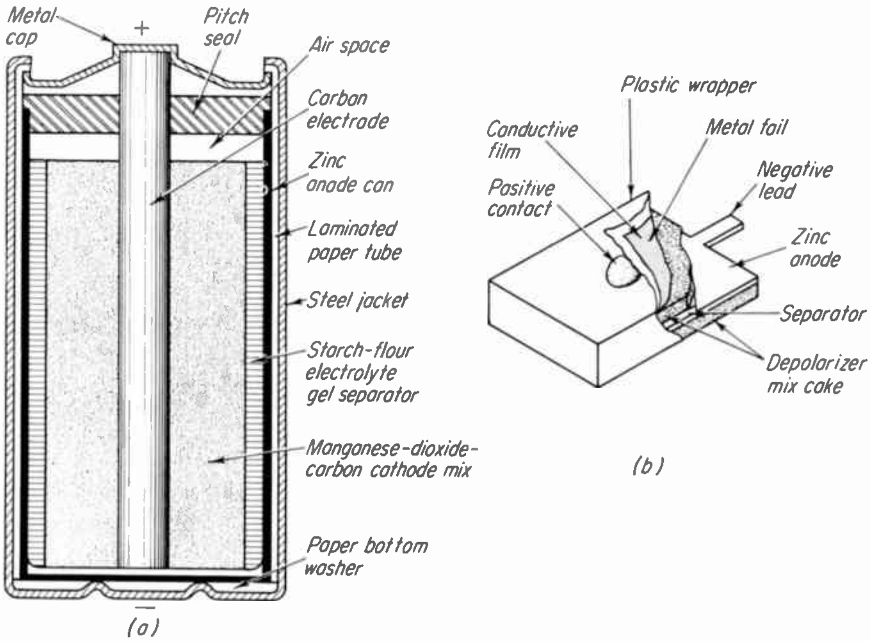


Fig. 3-4 (a) Cross-sectional view of a cylindrical-type carbon-zinc cell. (b) Cutaway view of a flat-type carbon-zinc cell. (RCA)

or cardboard and in some types of cell also with a steel jacket. To prevent the loss of the electrolyte due to spilling or evaporation, an insulating material is used to seal off the cell contents.

Action of the Carbon-Zinc Cell. The carbon-zinc cell is fundamentally the same as the simple cell described in Art. 3-2, and its action, therefore, will be the same. The action of the water and the ammonium chloride in the paste, together with the zinc and carbon electrodes, produces the electric current. The manganese dioxide is added to reduce the polarization due to hydrogen, and the zinc chloride is added to reduce the polarization due to the ammonia. The gelatinous paste contains materials, such as cornstarch and flour, to permit the electrolyte to filter through it slowly.

Characteristics of the Carbon-Zinc Cell. The carbon-zinc cell has a rated output of 1.5 volts and a low discharge rate, and its terminal voltage may decrease appreciably with the use of the cell. For similar ratings, it is both larger in size and heavier in weight than the newer types of primary cell. This cell is generally used where size, weight, and voltage fluctuation are not of great importance. Carbon-zinc cells are available in a wide variety of sizes and shapes, with a broad range of operating characteristics. Most primary cells used in radio and other electronic applications are of this type.

3-6 The Mercury Cell

Construction of the Mercury Cell. The fundamental components of the mercury cell (Fig. 3-5) are (1) a *depolarizing cathode* consisting of small pellets of mercuric oxide and a small percentage of graphite, (2) an *anode* of high-purity amalgamated zinc powder, (3) a *concentrated aqueous electrolyte* of potassium hydroxide and zinc oxide, and (4) a *sealed steel container.* It should be noted that the polarity of the electrodes of the mercury cell is reversed from that of the carbon-zinc cell.

In the cylindrical type of cell (Fig. 3-5*a*), the anode is pressed into a hollow cylindrical rod that forms the center portion of the cell. The cathode is pressed into a cylindrical sleeve that forms the outer portion of the cell. In the flat-type cell (Fig. 3-5*b*), the cathode is pressed into a flat shape and is consolidated with the bottom of the cell case. The anode is also pressed into a flat shape and forms the upper portion of the cell. In both types of cell the two electrodes are separated by an absorbent material that contains the electrolyte. A permeable barrier prevents the migration of any solid particles in the cell, thereby contributing effectively to long shelf and service life. The insulating and sealing gaskets are molded polyethylene or neoprene, depending on the cell application. The inner-cell tops are plated with a material that provides an inner surface to which the zinc forms a zinc amalgam bond. The cell cases and outer tops are made of nickel-plated steel in order to (1) resist corrosion and (2) provide greater inactivity to internal cell materials. The outer nickel-plated steel jacket is

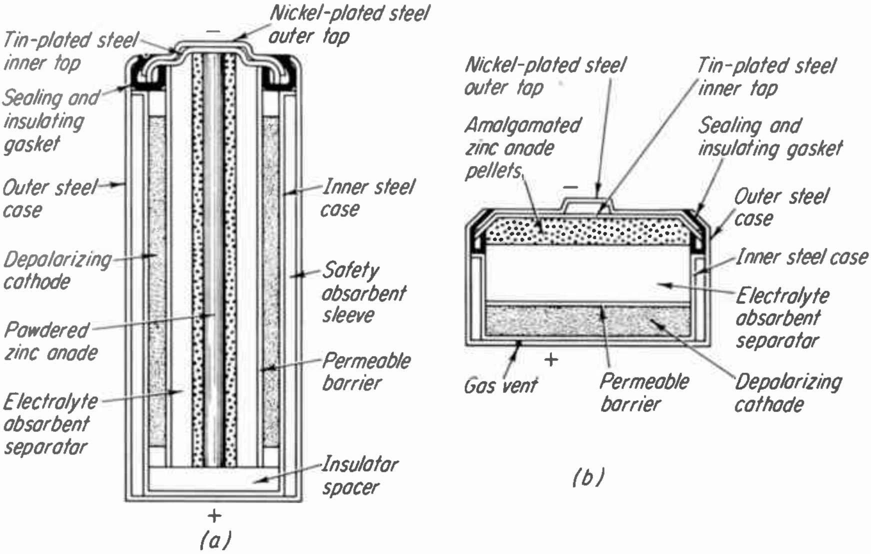


Fig. 3-5 Cross-sectional views of mercury cells. (a) Cylindrical type. (b) Flat type. (*Union Carbide Corporation, Consumer Products Division*)

a necessary component for self-venting construction. This vent provides a means for releasing the excessive gas in a cell that occurs during a reverse current or short circuit.

Characteristics of the Mercury Cell. The mercury cell may be obtained with a rating of either 1.35 or 1.40 volts. This voltage is relatively stable at rated current, having a voltage regulation of 0.5 per cent for long periods of time and 0.1 per cent for short intervals. A high-current drain or a momentary short circuit will not permanently damage the cell, and there is a complete recovery to its full open-circuit voltage within minutes.

In comparison with the carbon-zinc cell the mercury cell has (1) a much higher rate of discharge, (2) a more constant ampere-hour capacity that is approximately three times greater per unit volume, (3) a much better performance at high temperatures, (4) a lower and more substantially constant internal impedance, (5) a much longer shelf life, (6) less frequent replacement, and (7) a higher initial cost.

Applications of the Mercury Cell. Because of its superior characteristics, the mercury cell has replaced the carbon-zinc cell in some applications. Because of its relatively constant potential, the mercury cell may be used as a secondary standard of voltage. Some of its applications as a reference source are regulated power supplies, radiation detector meters, portable potentiometers, electronic computers, and voltage recorders. The flat-type cell is used in compact electronic equipment where physical size is an important factor.

3-7 The Silver Oxide Cell

Construction of the Silver Oxide Cell. A silver oxide cell basically consists of a depolarizing silver oxide cathode, a zinc anode, and a highly alkaline electrolyte (Fig. 3-6). It should be noted that the polarity of the electrodes of the silver oxide cell is reversed from that of the carbon-zinc cell. The depolarizing cathode is a mixture of silver oxide and manganese dioxide that is pressed into a flat shape and consolidated with the bottom of the cell case. The percentage of the manganese dioxide depolarizer that is used will depend upon whether the cell is to provide a flat-discharge current or increased service hours. Powdered zinc, which is pressed into a disk having a relatively large surface area, forms the upper portion of the

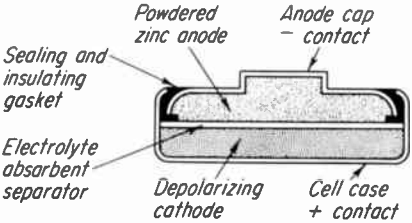


Fig. 3-6 Cross-section view of a silver oxide cell. *(Union Carbide Corporation, Consumer Products Division)*

cell. The two electrodes are separated by an absorbent material containing the electrolyte. This electrolyte may be either (1) potassium hydroxide for cells requiring maximum power density at rated currents, such as cells used in hearing aids, or (2) sodium hydroxide for cells requiring long-term reliability, such as cells used in electric timepieces.

Characteristics of the Silver Oxide Cell. The silver oxide cell is an excellent miniature power source. Its relatively high open-circuit and operating voltages, 1.6 and 1.5 volts, respectively, and its flat discharge characteristic enable the silver oxide cell to provide more service life or power than is available from other miniature cells of the same size. Its internal impedance is low and nearly constant. The silver oxide cell will withstand severe abuses with no hazard to personnel or equipment. It has an excellent shelf life, having approximately 90 per cent capacity after one year of storage at 70°F. Because of the relatively large surface area of the anode, this type of cell is capable of supplying a service capacity that is much greater than can be obtained from other types of cell of comparable size. The silver oxide cell is primarily used for hearing aids, electric watches, and reference voltage sources.

3-8 The Alkaline Manganese-Zinc Cell

Construction of the Alkaline Cell. The alkaline-manganese cell is commonly referred to as an *alkaline cell;* its basic construction is shown in Fig. 3-7. The alkaline cell is similar to the carbon-zinc cell in that they both have zinc negative electrodes and manganese dioxide positive electrodes. They differ only in the structure of the electrodes and the material used for the electrolyte. The cathode consists of a manganese dioxide cylindrical sleeve that forms the outer portion of the cell. A steel can, which makes direct contact to the outside surface of this electrode, serves as a cathode collector. The upper portion of this can makes contact to the metal cap, or positive terminal. The zinc anode is also a cylindrical sleeve and forms the inner portion of the cell. A cylindrical anode collector makes contact to the inner surface of this electrode and also to the outer metal bottom of the cell, or negative terminal. The alkaline cell uses potassium hydroxide to produce a highly alkaline electrolyte. The construction of this cell permits the electrolyte to make contact with a very large surface area of the zinc anode. To prevent leakage the cells are hermetically sealed and encased in a steel jacket.

Characteristics of the Alkaline Cell. The alkaline cell is designed to meet a need for a power source having a high-rate source of energy, with a high-service capacity. This cell is rated at 1.5 volts and has a relatively constant ampere-hour capacity over a wide range of current drains. The primary advantage of the alkaline cell is its ability to perform with a high degree of efficiency under continuous high-current service. Under certain conditions, the alkaline cell will provide more than ten times the service of

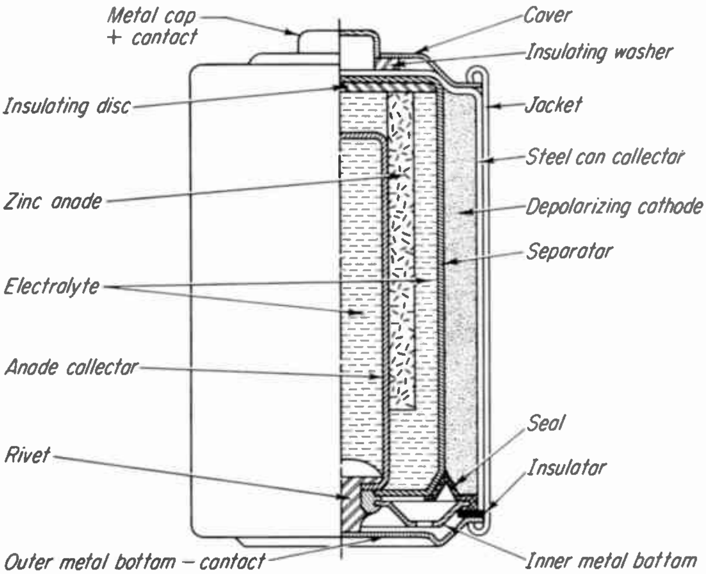


Fig. 3-7 Cutaway view of an alkaline manganese-zinc cell. (*Union Carbide Corporation, Consumer Products Division*)

a similar size carbon-zinc cell. The alkaline cell has an extremely low internal impedance that can improve radio reception by minimizing distortion. The low-temperature characteristic of this type of cell results in good service life even under outdoor winter conditions. The alkaline cell has a relatively long shelf life at normal temperatures and is better than the carbon-zinc cell for both high- and low-temperature applications.

Applications of the Alkaline Cell. Before the alkaline cell was developed, many devices were considered to be impractical to operate from a battery because a suitable power source was not available. These devices can now be operated from alkaline batteries which are ideal for high-drain heavy-discharge schedules such as (1) heater element applications; (2) bicycle lights and horns; (3) heavy-duty lighting; (4) motion-picture cranking; (5) glo-plug ignition; (6) toys; (7) model boats, airplanes, and automobiles; (8) electric shavers; and (9) electronic photoflash units.

3-9 Combination of Cells

Need for Extra Cells. A standard, or No. 6, dry cell has an emf of 1.5 volts and can deliver about ⅛ amp continuously. Such a cell can supply electrical energy to a circuit requiring 1.5 volts and not more than ⅛ amp. Many occasions arise where higher voltage or higher current or both are required. To meet these needs and maintain satisfactory service, a number of cells are joined into a series-connected group, a parallel-connected group,

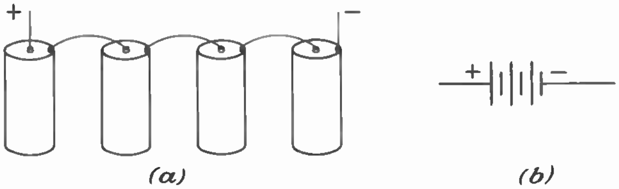


Fig. 3-8 Cells connected in series. (a) Pictorial representation. (b) Schematic diagram representation.

or a series-parallel-connected group, depending upon the voltage and current requirements.

Cells Connected in Series. Whenever the voltage required exceeds that of a single cell, it becomes necessary to use more than one cell and the cells must be connected in series as shown in Fig. 3-8*a*. Cells are connected in series by connecting the negative terminal of each successive cell to the positive terminal of the next following cell. Notice that the positive terminal of the first cell and the negative terminal of the last cell are free and that they become the terminals of the battery. Figure 3-8*b* shows the same group of series-connected cells in schematic-diagram form, a long line being used to represent the + terminal and a shorter line for the − terminal. If the battery consists of only a few cells, the number of pairs of long and short lines is made to equal the number of cells. In a series circuit, the battery voltage will equal the sum of the separate cell voltages.

$$E_B = E_{c1} + E_{c2} + E_{c3}, \text{ etc.} \tag{3-1}$$

If each cell has the same voltage, then

$$E_B = \text{number of cells} \times \text{volts per cell} \tag{3-2}$$

also

$$\text{Number of cells required} = \frac{\text{voltage of battery}}{\text{volts per cell}} \tag{3-3}$$

In the series arrangement of cells, the current rating of the battery will be the same as that of an individual cell; hence this arrangement can be used only where the continuous current requirement does not exceed the rated current of one cell.

Cells Connected in Parallel. Whenever a continuous current greater than the rated current of one cell is required, it becomes necessary to use more than one cell and the cells must be connected in parallel. The parallel arrangement of cells is shown in Fig. 3-9. Notice that now all the negative terminals are connected together and a line lead is brought off one end and that all the positive terminals are connected together and a line lead is brought off one of these ends. In the parallel circuit, the current rating of the battery is equal to the sum of the rated currents of the separate cells.

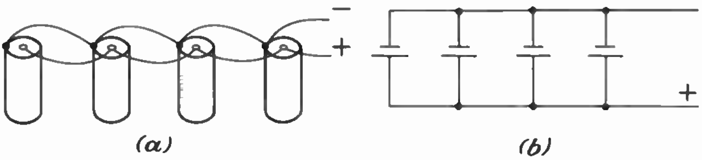


Fig. 3-9 Cells connected in parallel. (a) Pictorial representation. (b) Schematic diagram representation.

$$I_B = I_{c1} + I_{c2} + I_{c3}, \text{ etc.} \tag{3-4}$$

If each cell has the same current rating, then

$$I_B = \text{number of cells} \times \text{current per cell} \tag{3-5}$$

also
$$\text{Number of cells required} = \frac{\text{current of battery}}{\text{current per cell}} \tag{3-6}$$

In the parallel arrangement of cells, the voltage of the battery will be the same as that of a single cell; hence this arrangement can be used only where the voltage requirement is that of one cell.

Cells Connected in Series Parallel. When both the voltage and current required exceed the rated voltage and current of a single cell, it becomes necessary to use four or more cells connected in a series-parallel combination. To get the higher voltage, a number of cells must be connected in series, and to get the higher current rating, a number of series-connected groups must be connected in parallel.

$$\text{Number of cells in each series-connected group} = \frac{\text{voltage of battery}}{\text{volts per cell}} \tag{3-7}$$

$$\text{Number of parallel groups} = \frac{\text{current of battery}}{\text{current per cell}} \tag{3-8}$$

$$\text{Number of cells required} = \text{number of cells in a series group} \times \text{number of parallel groups} \tag{3-9}$$

For example, if a load current of ½ amp at 4½ volts is to be supplied by a battery composed of No. 6 dry cells, then the number of cells in each series-connected group will be 4½ divided by 1½, or 3, and the number of parallel groups will be ½ divided by ⅛, or 4. The total number of cells required will be 3 times 4, or 12 cells. The connections for this requirement will be four groups in parallel, each group consisting of three cells connected in series. This arrangement of the cells is shown in Fig. 3-10.

3-10 Batteries

At one time radio receivers required three separate batteries designated A, B, and C, each with a different voltage and current requirement. Each

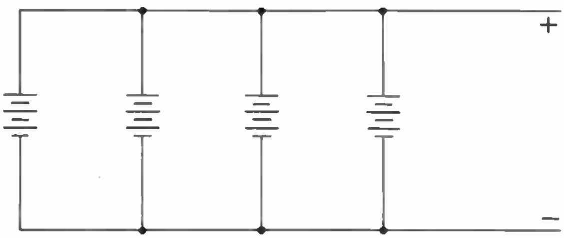


Fig. 3-10 Cells connected in series parallel.

of these types had a specific use, and while not used so widely today for radio receivers as in the past, there are many of these types still being used for various electronic applications.

A Batteries. The A battery has a low voltage rating and a relatively high current capacity. This type of battery is designed to supply current to the filaments or heaters of electron tubes.

B Batteries. The B battery has a high voltage rating and a very low current capacity. The amount of current required varies with the number and type of tubes used in the radio receiver and usually is under 50 ma. B batteries are made in a number of different voltage ratings, two commonly used ones being 22½ and 45 volts. The 22½-volt battery consists of 15 cells connected in series. The 45-volt battery has 30 cells connected in series and is sometimes provided with an extra terminal that is tapped off at 22½ volts.

The original type of 45-volt B battery consists of 30 (size D) unit cells assembled in a rectangular container (Fig. 3-11). A disadvantage of this type of unit is the great amount of inactive space, which results in a large battery. The demand for smaller batteries and the desire to utilize the unproductive space in the round-cell type of battery led to the development of a new type of cell construction. In this type of battery, the cells are made in thin squares (Fig. 3-12*a*), and are stacked one upon the other as shown in Fig. 3-12*b*. The 45-volt battery contains two stacks, 15 cells in each stack, connected in series.

C Batteries. The C battery is a medium-voltage unit and is required to supply only very small currents, usually so small that the battery has

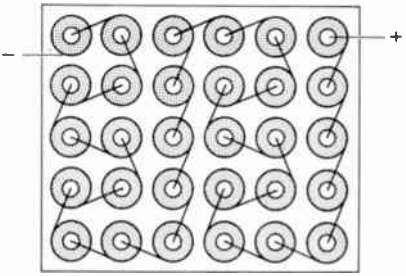


Fig. 3-11 Arrangement of cells in a B battery using cylindrical cells.

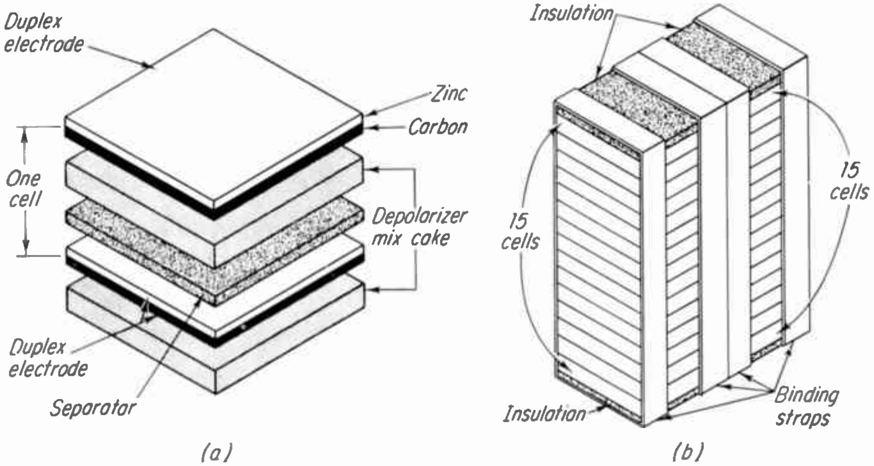


Fig. 3-12 A miniature B battery. (a) Arrangement of the parts of a cell. (b) 30 cells assembled in two stacks of 15 each. (Union Carbide Corporation, Consumer Products Division)

an exceptionally long life. C batteries can be used for grid voltage supplies or in electronic measuring instruments such as ohmmeters.

3-11 Secondary Cells and Storage Batteries

Secondary cells operate on the same principles as primary cells but differ in the method in which they may be renewed. Some of the materials of a primary cell are used up in the process of changing chemical energy to electrical energy, and they must be replaced to renew the cell. In the secondary cell, the materials are merely transferred from one electrode to the other, and they may be restored to their original state by sending an electric current from some other source through the cell in the opposite direction.

The storage battery consists of a number of secondary cells that are usually connected in series. Properly speaking, this battery does not store electrical energy, but it does store chemical energy which it can transform to electrical energy. Most secondary cells may be classified in one of the following groups: (1) lead-acid, (2) nickel-iron, (3) nickel-cadmium, and (4) cadmium–silver oxide.

3-12 Lead-Acid Cell

Construction of the Lead-Acid Cell. The construction of a commercial type of lead-acid cell is shown in Fig. 3-13. The negative electrodes are made of spongy lead, and the positive electrodes are made of lead peroxide. The electrolyte is a sulfuric acid solution.

The capacity of a lead-acid cell is expressed in ampere-hours, and this rating is proportional to the amount of active surface area of the electrodes, or *plates* as they are commonly called. The total active area of the plates is therefore an important factor in the rating of a cell. In a multiple-plate cell a positive plate is placed between two negative plates, and both sides of the positive plate become active and the cell can be made smaller. In order to make the lead-acid cell in convenient sizes, the total plate area is obtained by using a number of smaller plates set side by side and connected in parallel. In order to have both sides of each positive plate facing a negative plate, it is necessary to have one more negative plate than the number of positive plates. This accounts for the fact that lead-acid cells have an odd number of plates, such as 11, 13, 15, 17, etc. A further advantage of using both surfaces of the plates is that it prevents them from buckling, an undesirable action which takes place when only one side is active. By the use of a special design and method of manufacture, the active area of each plate is increased as much as six to ten times its apparent surface area.

To conserve space, the plates are placed close to each other, and insulators are placed between them to prevent a positive plate from making contact with either of its adjacent negative plates. These insulators, called *separators*, are generally made of hard rubber, wood, or plastics.

Each cell is placed in a container that is usually made of hard rubber. After the plates are placed in the container, a hard-rubber cover fitted with a filler tube and vent cap is set over the cell and sealed with a compound. The electrolyte is a dilute sulfuric acid solution mixed in such proportions

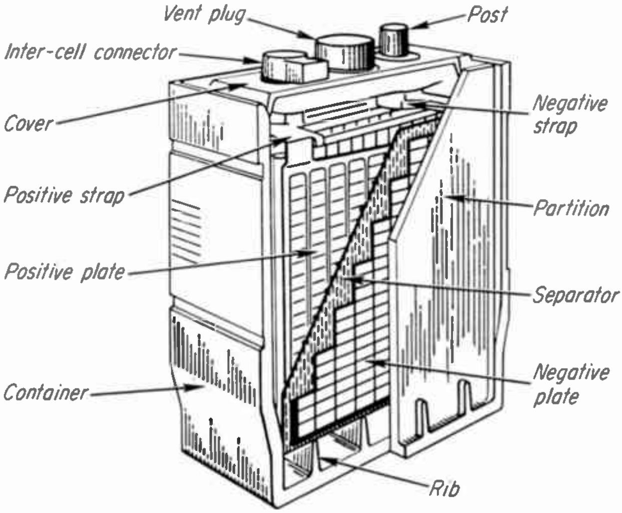


Fig. 3-13 A cutaway view of a lead-acid cell showing details of construction. (*Electric Storage Battery Co.*)

so that with a fully charged cell its specific gravity will be approximately 1,300.

Chemical Action of the Lead-Acid Cell during Discharge. Since the electrodes are of chemically dissimilar materials, the cell functions the same as a primary cell. As such, it can transform its chemical energy to electrical energy. During discharge the sulfuric acid is broken up into hydrogen ions and sulfate ions. The sulfate ions near the anode combine with the spongy lead to form lead sulfate at the negative electrode, and the sulfate ions near the cathode combine with the lead peroxide also to form lead sulfate at the positive electrode. All the hydrogen ions are attracted to the cathode and unite there with the oxygen to form water. When all the lead peroxide is changed to lead sulfate and all the spongy lead is changed to lead sulfate, the two electrodes are chemically similar materials and the cell is no longer capable of delivering electrical energy. When the cell is in this condition, it is said to be ***discharged***, and the sulfuric acid electrolyte will be changed to water. The condition of the electrolyte is therefore a means of determining the charge of a lead-acid cell.

Chemical Action of the Lead-Acid Cell during Charge. The statement that the chemical action of a secondary cell can be reversed by sending electrons through the cell in the opposite direction to discharge can be restated as follows: Electrical energy can be transformed to chemical energy. When electrons are sent through a lead-acid cell in the opposite direction to discharge, the current through the electrolyte divides the water into hydrogen ions and oxygen ions. The hydrogen ions near the anode combine with the sulfate ions to produce sulfuric acid, and the negative electrode is changed from lead sulfate to spongy lead. The hydrogen ions near the cathode combine with the sulfate ions and also produce sulfuric acid, and the positive electrode is changed from lead sulfate to lead. All the oxygen ions are attracted to the cathode and combine with the lead to make the positive electrode lead peroxide. During these actions the water has been changed to sulfuric acid. Both the electrodes and the electrolyte are restored to their original state (when charged), and the cell again has stored chemical energy that can be transformed to electrical energy.

Characteristics of the Lead-Acid Cell. The advantages of this type of cell are its (1) relatively high voltage per cell, (2) high current capacity, (3) relatively long life, and (4) relatively low initial cost. The disadvantages are (1) it is relatively large and heavy, (2) it cannot be hermetically sealed, (3) it has poor low-temperature characteristics, and (4) it cannot remain in the discharge state too long without damage.

Applications of the Lead-Acid Cell. The lead-acid cell is the most widely used secondary cell because of its capability of supplying large amounts of current (in the order of several hundred amperes) at relatively high voltages, approximately 2.1 volts per cell. The main application of the lead-acid cell is in the automotive field, where it is used to provide high currents for a few seconds or minutes for engine starting. Automotive batteries are rated at 6 volts (3 cells) and 12 volts (6 cells) and have a cycle service life of 250 to 400 cycles and a float charging life up to 4 years. The lead-acid cell is also used in motive batteries to supply energy for electric trucks, mine locomotives, and material-handling equipment. These batteries have a discharge rate of 3 to 10 hr with nearly complete discharge and have a cycle service life of 3 to 6 years. The stationary battery is another application of the lead-acid cell. These batteries are designed for (1) extremely long service life (14 to 25 years), (2) dependability, (3) minimum maintenance cost, and (4) excellent charge retention.

3-13 Nickel-Iron Cell

The nickel-iron cell is usually called the *Edison cell* after its inventor. This cell basically consists of an iron anode, a nickel oxide cathode, and a potassium hydroxide alkaline electrolyte. In a commercial cell, the positive plate consists of a number of long nickel tubes filled with nickel oxide. As nickel oxide is a poor conductor, very fine nickel flakes are mixed with it to produce the required conductivity. The negative plate consists of a number of flat perforated nickel-plated steel stampings containing finely divided metallic iron. The electrolyte is potassium hydroxide. As with the lead-acid cell, the capacity of this cell can be increased by connecting a number of negative plates and a number of positive plates in parallel.

The nickel-iron cell is relatively light in weight and extremely rugged and can withstand abuses such as overcharges and overdischarges. In addition, it can remain discharged for a long period of time or be subjected to freezing temperatures without chemical deterioration.

The Edison cell is rated on the basis of a 5-hr charging rate and has an average emf of 1.2 volts during discharge. The nickel-iron cell has a cycle service of 1,800 cycles, a float charging life of 7 to 12 years in heavy discharge service, and 14 to 25 years as a standby source of power. This type of cell is employed mainly in heavy-duty industrial and railway applications.

3-14 Nickel-Cadmium Cell

Action of the Nickel-Cadmium Cell. Basically, the nickel-cadmium cell is similar to the nickel-iron cell except for the use of cadmium as the negative electrode. In the charged condition, the cathode is nickelic hydroxide, the anode cadmium, and the electrolyte potassium hydroxide. In the discharge condition, the positive electrode becomes nickel oxide, the negative electrode cadmium hydroxide, and the electrolyte does not change and remains as potassium hydroxide.

Construction of the Nickel-Cadmium Cell. The nickel-cadmium cell is a sealed unit that is available in three basic types of construction: (1) button, (2) cylindrical, and (3) rectangular. The electrodes used in the button and cylindrical cells (Fig. 3-14) consist of molded screen encased active

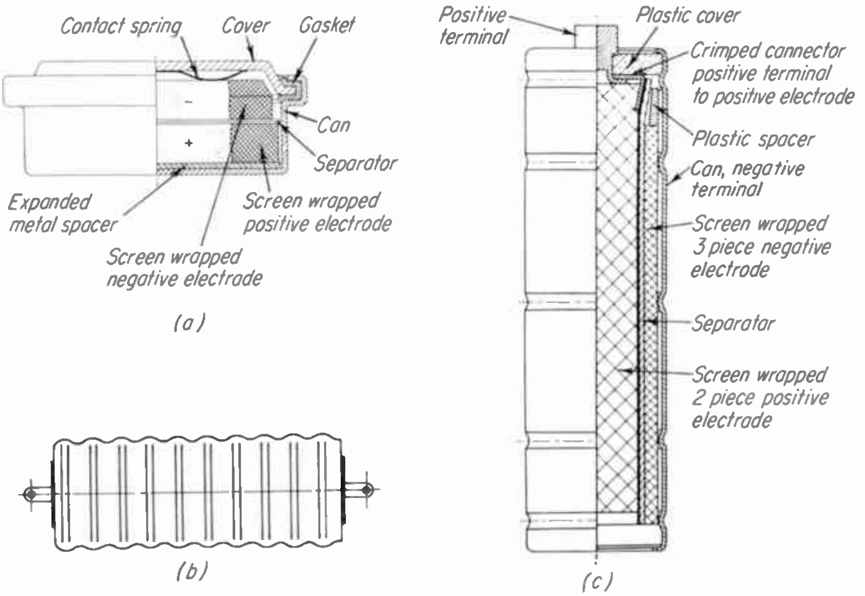


Fig. 3-14 Nickel-cadmium cells. (a) Cutaway view of a button cell. (b) Button stack assembly. (c) Cutaway view of a cylindrical cell. (*Union Carbide Corporation, Consumer Products Division*)

materials. In the rectangular cell the plates consist of pressed active materials held in perforated steel pockets that are locked into welded frames.

During the latter part of a recommended charge cycle and also during an overcharge, the nickel-cadmium cell generates gases. When the cell becomes fully charged, hydrogen is formed at the cadmium electrode, and after it is fully charged, oxygen is generated at the nickelic hydroxide electrode. In a conventional *vented-type cell* the hydrogen and oxygen gases, plus any entrained fumes from the electrolyte, are liberated through a valve. In a hermetically sealed cell these gases are used up inside the cell. This is accomplished in the following manner: (1) The cadmium electrode is constructed with an excess ampere-hour capacity. (2) The positive electrode will reach full charge first and will start to generate oxygen; however, the negative electrode will not reach full charge as yet, and therefore no hydrogen will be generated. (3) The cell is so designed that the oxygen reaches the surface of the cadmium electrode, where it reacts to form electrochemical equivalents of cadmium oxide. (4) During overcharge the cadmium is oxidized at a rate that offsets the input energy, thus keeping the cell in equilibrium at full charge. This process can continue for long periods.

Each of the three types of nickel-cadmium cell come in various sizes and may be used individually or connected in series to obtain a desired voltage. The button cell is made in series stacks (Fig. 3-14*b*), the cylindrical cell in assemblies of varying configurations, and the rectangular cell in a five-cell (6 volts) pack in a plastic case. The cylindrical cell incorporates a different electrode arrangement from the button cell (Fig. 3-14*c*). Its terminal connections are similar to those of small cylindrical dry cells, and some sizes may be used as a direct replacement for these primary cells. The rectangular cell is a heavier-duty cell than the other two types and covers a range of 1.5 to 23 amp-hr. The steel case is polarized positive and contains a safety vent that is activated only in case of severe misuse of the cell.

Characteristics of the Nickel-Cadmium Cell. Like the nickel-iron cell, the nickel-cadmium cell is mechanically rugged and has a long potential life. During discharge the average voltage is approximately 1.2 volts. Under conditions of very light or casual service the expected life of a cell is several years. With normal service and discharge of rated capacity, the cycle life is in excess of 100 for the button and cylindrical cells and in excess of 400 for the rectangular cell. The nickel-cadmium cell has a high effective capacitance; however, its impedance is very low. Although the sealed nickel-cadmium cell will lose some of its charge during storage, this type of cell has a lower self-discharge rate than any other cell. More important, the cell is not harmed chemically even if not used for long periods of time.

Applications of the Nickel-Cadmium Cell. Because the nickel-cadmium cell is (1) relatively small, (2) rechargeable, (3) hermetically sealed, (4) economical, and (5) trouble free, it is ideal for use in many types of battery-operated equipment. Some of these applications are (1) alarm systems, (2) amplifiers, (3) dictating machines, (4) electric shavers, (5) electronic

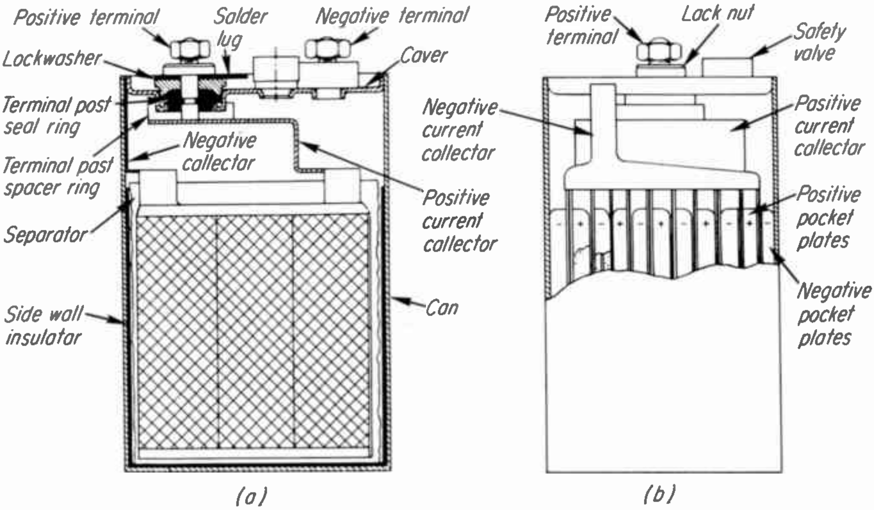


Fig. 3-15 Cutaway views of a rectangular nickel-cadmium cell. (a) Side view. (b) End view. (Union Carbide Corporation, Consumer Products Division)

photoflash, (6) emergency lighting, (7) instruments, (8) hearing aids, (9) motion-picture cameras, (10) radio receivers, (11) tape recorders, (12) telemetery, and (13) transmitters.

3-15 Cadmium–Silver Oxide Cell

The cadmium–silver oxide cell uses a cadmium anode for long operating life, a silver oxide cathode for high watthour capacity, and a potassium hydroxide electrolyte. This type of cell is capable of providing higher currents, a more level voltage, and up to three times greater watthour capacity per unit weight and volume than any other type of secondary cell. It also has a high efficiency on extended shelf life in either a charged or discharged condition, is mechanically rugged, and operates satisfactorily over a wide temperature range. Its use has been limited to applications where cost is not a factor and space and weight are the prime considerations. Cadmium–silver oxide cells can be used (1) to power a portable television receiver, (2) as storage batteries in satellite programs, and (3) in conjunction with solar-cell systems.

3-16 The Solar Cell

Basic Principle. The solar cell is based on the principle that, when a silicon crystal is exposed to light, light rays are absorbed by the crystal by liberating free-to-move negative charges, called *electrons*, and free-to-move positive charges, called *holes*. The direction of movement of the electrons and holes can be controlled to produce an electric current. The strength of this current is dependent upon the size of the crystal and the intensity of the source of light.

Basic Construction of a Solar Cell. The basic construction of a solar cell is shown in Fig. 3-16, and consists of two layers of impure silicon crystal. One layer contains a controlled amount of a chemical impurity that is rich in electrons; this is the *N-type* silicon layer, or negative electrode. The other layer contains a controlled amount of a chemical impurity that is deficient in electrons; this is the *P-type* silicon layer, or positive electrode. The two crystal layers are in direct contact with each other, and the junction formed between them exhibits the property of a strong built-in electric field. This field tends to contain the electrons in the N-silicon crystal and the holes in the P-silicon crystal, thus forming a barrier between them. The rays of light are absorbed through the thin P-type silicon crystal into this barrier region, where they liberate both electrons and holes. The built-in electric field forces the electrons to move to the N-type layer, making this crystal a negative electrode, and the holes to move to the P-type layer, making this crystal a positive electrode.

Characteristics of the Solar Cell. The efficiency of a solar cell in converting light energy to electrical energy is dependent upon the following factors: (1) the intensity of the light source, (2) the spectral content of the light

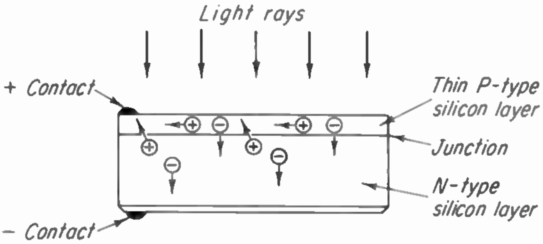


Fig. 3-16 Basic construction and operation of a silicon solar cell.

waves, (3) the angle at which the light waves strike the surface of the cell (angle of incidence), (4) the operating temperature of the cell, and (5) the characteristics of the external circuit. The solar cell has an output of approximately 0.4 volt, and cells may be connected in series to increase the voltage output and in parallel to increase the current output. The simplicity of a solar cell permits its construction to be relatively rugged. These cells have a high output level, an indefinite life, and a nonaging characteristic.

Applications of the Solar Cell. A highly publicized application of the solar cell is to provide the power requirements of satellites and space vehicles. Solar cells are used in these applications to charge storage batteries and to supply power during that portion of the orbit when the cells are in direct sunlight. The storage battery provides a continuity of power for instrumentation, transmitters, and telemetry systems during the periods of darkness when the cells are inoperative. Because of its advantages, the silicon solar cell is finding increased and varied industrial applications. This type of cell is used extensively in data-processing equipment to read punched tape, punched cards, or film. It is also used as power supplies for remote and unattended radio stations, lighthouses, beacons, and telephone repeaters in sunny areas (such as the polar regions). Other potential uses of solar cells are (1) light-operated transistor radio, (2) light-controlled toys, (3) automatic street lighting, (4) analog-to-digital encoders, (5) servomechanisms, (6) gyrocompasses, (7) light-operated burglar alarms, (8) modulated light-beam communication.

3-17 The Fuel Cell

Another source of energy is the fuel cell. This type of cell, unlike the chemical cell, does not store energy but rather converts energy from conventional fuel oxidation directly to electrical energy. The substances used are (1) of the inexpensive fossil types, such as coal and hydrocarbons, or (2) of more expensive substances, such as hydrogen, alcohol, and carbon monoxide, which may be derived from the fossil types.

The basic construction of a simple hydrocarbon fuel cell is shown in Fig. 3-17. Air and fuel enter opposite sides of the upper portion of the cell and are kept separated by the electrolyte. However, hydrogen ions, formed from the hydrocarbon fuel, flow through the electrolyte to unite with oxy-

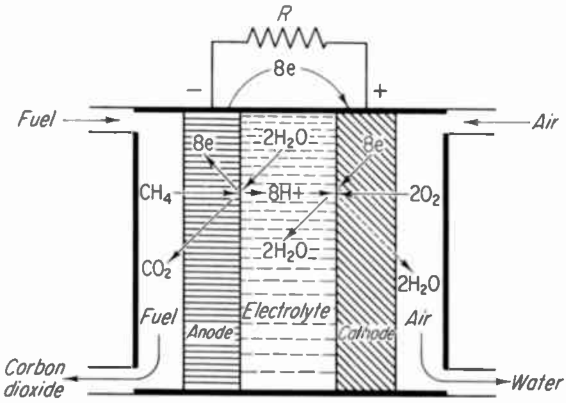


Fig. 3-17 Basic principle of operation of a hydrocarbon fuel cell. (General Electric Co.)

gen. At the same time, the electrons stripped from the hydrocarbon fuel travel through the external circuit. An electric current is thus produced *directly* without steam, combustion, noise, vibration, or moving parts. Carbon dioxide and water are the two harmless byproducts of the chemical reactions of this type of cell.

Hydrogen fuel cells are used as a power source in the two-man Gemini spacecrafts. This type of cell is expensive and is not practical for commercial applications. However, an experimental cell has been developed that uses a variety of hydrocarbons such as propane, natural gas, octane, gasoline, and diesel oil.

QUESTIONS

1. Why is it important to have a knowledge of batteries?
2. Name two examples of the conversion of (*a*) chemical energy to electrical energy, (*b*) light energy to electrical energy.
3. Define (*a*) cell, (*b*) primary cell, (*c*) secondary cell, (*d*) battery.
4. Define (*a*) electrode, (*b*) negative electrode, (*c*) positive electrode, (*d*) electrolyte.
5. Describe three factors regarding the electrodes and the electrolyte that are essential to the construction of a cell.
6. Describe the factors that affect (*a*) the voltage rating of a cell, (*b*) the current rating of a cell.
7. Define (*a*) ion, (*b*) positive ion, (*c*) negative ion, (*d*) ionization.
8. Explain why the zinc in a carbon-zinc cell is (*a*) the negative electrode, (*b*) the anode.
9. Explain why the carbon in a carbon-zinc cell is (*a*) the positive electrode, (*b*) the cathode.
10. (*a*) How is the difference in direction of electron flow and current flow explained? (*b*) Which is correct? (*c*) Which is used more and why?
11. (*a*) What is meant by polarization? (*b*) What is done to reduce it?

12. (*a*) What is meant by local action? (*b*) What is done to reduce it?
13. Describe five factors that determine the life of a cell.
14. Describe the construction of a commercial carbon-zinc cell.
15. Describe the construction of a mercury (*a*) flat-type cell, (*b*) cylindrical cell.
16. Explain why in a mercury cell (*a*) the zinc is the anode, (*b*) the mercuric oxide is the cathode.
17. Describe seven characteristics of comparison between the carbon-zinc and mercury cells.
18. Describe the construction of a commercial silver oxide cell.
19. Describe six favorable characteristics of the silver oxide cell.
20. Describe the construction of a commercial alkaline cell.
21. What are the basic differences between the carbon-zinc cell and the alkaline cell?
22. What are the primary advantages of the alkaline cell?
23. Draw a diagram showing (*a*) 6 cells connected in series, (*b*) 6 cells connected in parallel, (*c*) 12 cells connected in a parallel-series combination so that the voltage will be 6 volts.
24. Describe the construction of a commercial B battery using (*a*) cylindrical cells, (*b*) flat-type cells.
25. How do secondary cells differ from primary cells?
26. Describe the construction of a commercial lead-acid cell.
27. In a lead-acid battery, what is the function of (*a*) separators? (*b*) Containers? (*c*) Vent caps? (*d*) Connectors?
28. (*a*) How is a storage battery rated? (*b*) What are the factors that determine the rating of a lead-acid battery?
29. Describe the chemical action of a lead-acid cell during (*a*) discharge, (*b*) charge.
30. Explain how the condition of the electrolyte can be used to determine the charge of a lead-acid cell.
31. Does the battery store electrical or chemical energy? Explain your answer.
32. What is meant by the expression "the action of a secondary cell is reversible"?
33. Describe (*a*) four advantages of the lead-acid cell, (*b*) four disadvantages of the lead-acid cell.
34. What are the basic materials used in an Edison cell for (*a*) the cathode? (*b*) The anode? (*c*) The electrolyte?
35. Describe the construction in a commercial nickel-iron cell (*a*) of the positive plate, (*b*) of the negative plate.
36. Describe four advantages of the nickel-iron cell.
37. Describe the construction of a nickel-cadmium (*a*) button-type cell, (*b*) cylindrical-type cell, (*c*) rectangular-type cell.
38. Explain how the gases and fumes inside a hermetically sealed nickel-cadmium cell are consumed inside the cell.
39. Describe six advantages of the nickel-cadmium cell.
40. Describe the construction of the cadmium–silver oxide cell.
41. (*a*) What are the prime advantages of a cadmium–silver oxide cell? (*b*) What are the prime factors that determine its use?
42. Describe (*a*) the basic principle of a solar cell, (*b*) the basic construction of a simple silicon solar cell.
43. Describe how a silicon solar cell transforms light energy to electrical energy.
44. Describe five factors that determine the efficiency of a solar cell.

45. Name six advantages of the solar cell.
46. Describe how solar cells are used in satellites and space vehicles.
47. Name six industrial applications of the solar cell.
48. (*a*) What is the basic principle of a fuel cell? (*b*) What substances are used for fuel in this type cell?
49. Describe the construction of a simple fuel cell.
50. Describe how a fuel cell converts fuel directly to electrical energy.

Chapter 4

Direct-current Circuits

An electric circuit is the path taken by an electric current from its source, through the conductors, and back to its starting point. From this definition, it can be seen that an electric circuit must be a closed path in order that the electrons leaving the starting point can return to that point upon completing the circuit.

The essential parts of any electric circuit are the source of power, the conductors used to transmit the electric current, and the appliance or appliances to be supplied with electrical energy.

In Chap. 2 it was shown that the controlled movement of the free electrons in a conductor forms an electric current. Certain materials emit free electrons more easily than do others and offer very little resistance to the flow of electrons. These materials are good conductors of electric currents. Other materials emit very few electrons and greatly oppose their flow. These materials are poor conductors of electric currents and are called *insulators.*

4-1 Resistance of Conductors

The resistance of any conductor will vary with its length, cross-sectional area, the material of which it is made, and its temperature.

Length. A conductor of any given material and cross-sectional area will offer a definite amount of resistance to the flow of electrons per unit length of the conductor. If the length of the conductor is increased, the distance the electrons must travel is increased and the resistance of the conductor is also increased. In a similar manner, the resistance of the conductor will decrease if the length of the conductor is decreased. The resistance of a conductor will therefore vary directly with its length.

Cross-sectional Area. A conductor of any given material, length, and cross-sectional area will offer a definite amount of resistance to the flow of electrons. If this area is increased and the same flow of electrons is maintained, the resistance offered to its flow decreases as the area of the path for the electron flow is increased. In a similar manner the resistance of a conductor would increase if the area is decreased. The resistance of a conductor will therefore vary inversely with its cross-sectional area.

Material. The resistance of any conductor depends upon the material of which it is made, as some materials offer greater or less resistance to the flow of an electric current than others. For example, a piece of steel wire offers more resistance to the flow of an electric current than a piece of copper wire of the same length and cross-sectional area. Therefore the resistance of a conductor will vary directly with a basic unit resistance depending on the material. This value is usually expressed by the letter K and is called the *specific resistivity* of the material.

Temperature. The resistance of most materials increases if their temperature is increased, and a few materials show a decrease in resistance with a temperature rise. There are also a few materials whose resistance is not affected by any temperature change. The resistance of a conductor will therefore vary directly with its temperature coefficient. In general, the change in resistance due to temperature variation is very slight and for most practical purposes may be disregarded. Where accurate results are desired, the following formula can be used. A list of temperature coefficients for the more common materials used will be found in Appendix IV.

$$R_F = R_i + [R_i \times T_c \times (t_f - t_i)] \tag{4-1}$$

where R_F = final resistance
R_i = initial resistance
T_c = temperature coefficient
t_i = initial temperature, °C
t_f = final temperature, °C

EXAMPLE 4-1 A resistor made of advance wire has a resistance of 10,000 ohms at 20°C. What is its resistance at 40°C?

GIVEN: Material = advance $t_i = 20°\text{C}$ $t_f = 40°\text{C}$ $R_i = 10{,}000$ ohms

FIND: R_F

SOLUTION:

$$T_c \text{ for advance wire} = 0.000018 \text{ (Appendix IV)}$$

$$\begin{aligned} R_F &= R_i + [R_i \times T_c \times (t_f - t_i)] \\ &= 10{,}000 + [10{,}000 \times 0.000018 \times (40 - 20)] = 10{,}000 + 3.6 = 10{,}003.6 \text{ ohms} \end{aligned}$$

EXAMPLE 4-2 A carbon resistor has a resistance of 250,000 ohms at 20°C. What is its resistance at 60°C?

GIVEN: Material = carbon $t_i = 20°\text{C}$ $t_f = 60°\text{C}$ $R_i = 250{,}000$ ohms

FIND: R_F

SOLUTION:

$$T_c \text{ for carbon} = -0.0003 \text{ (Appendix IV)}$$

$$\begin{aligned} R_F &= R_i + [R_i \times T_c \times (t_f - t_i)] = 250{,}000 + [250{,}000 \times (-0.0003) \times (60 - 20)] \\ &= 250{,}000 - 3{,}000 = 247{,}000 \text{ ohms} \end{aligned}$$

Relation of Material, Length, and Area to Resistance. From the foregoing explanations of the various factors affecting the resistance of conductors (disregarding temperature), the following mathematical expression has been derived:

$$\text{Resistance} = \frac{K \times \text{length}}{\text{area}} \qquad \text{or} \qquad R = \frac{KL}{A} \tag{4-2}$$

where R = resistance, ohms
K = specific resistance (Appendix IV)
L = length, ft
A = area, cir mils

Circular Mil Area. A mil is equal to one-thousandth of an inch. A square mil is equal to the area of a square whose sides are all one mil in length. A circular mil is an amount that is equal to the area of a circle whose diameter is one mil. To find the area in circular mils, square the diameter in mils. The circular mil is a smaller area than the square mil, and for convenience in arithmetic it is used to express the area of wire sizes.

EXAMPLE 4-3 Find the area in circular mils of a wire 0.25 inch in diameter.

GIVEN: d = 0.25 inch

FIND: Area, cir mils

SOLUTION:

$$0.25 \text{ in.} = 250 \text{ mils}$$
$$\text{Area} = 250 \times 250 = 62{,}500 \text{ cir mils}$$

4-2 Specific Resistance

Definition. It has been stated that the resistance of a conductor will depend on the specific resistance of the material of which it is made. The specific resistance of a wire is the resistance of a circular mil-foot of that wire, or the resistance of a wire whose diameter is one mil and whose length is one foot.

Shapes of Conducting Materials. A large percentage of the conductors used in the electrical and electronic industries is in the form of round wires. Conductors of various other shapes, particularly square and rectangular, are also used.

Because round wire is the most commonly used shape, the specific resistance in Appendix IV is given in ohms per circular-mil-foot. Equation (4-2) may be revised for use with round conductors, as

$$R = \frac{KL}{d^2} \tag{4-3}$$

where d is the diameter in mils.

EXAMPLE 4-4 What is the resistance of 1,000 ft of round copper wire that is 0.05 inch in diameter?

GIVEN: $K = 10.4$ (Appendix IV) $L = 1{,}000$ ft $d = 0.05$ inch or 50 mils

FIND: R

SOLUTION:

$$R = \frac{KL}{d^2} = \frac{10.4 \times 1{,}000}{50 \times 50} = 4.16 \text{ ohms}$$

Equation (4-2) may be revised in the following manner for calculating the resistance of square, rectangular, and other shape conductors.

$$R = \frac{0.785KL}{A'} \tag{4-4}$$

where A' is the area in square mils.

EXAMPLE 4-5 What is the resistance of 1,000 ft of 0.05-inch-square copper wire?

GIVEN: $K = 10.4$ (Appendix IV) $L = 1{,}000$ ft $A' = 50 \times 50$ sq mils

FIND: R

SOLUTION:

$$R = \frac{0.785KL}{A'} = \frac{0.785 \times 10.4 \times 1{,}000}{50 \times 50} = 3.26 \text{ ohms}$$

Wire Gauge. Round wire is designated according to definite gauge sizes or numbers. Each number represents a wire of a certain diameter. In the United States, the American Wire Gauge is used for designating the size of any kind of wire. It is based on a constant ratio of cross-sectional areas among wires of successive gauge numbers. An increase in the gauge number will correspond to a decrease in the cross-sectional area but will also result in an increase in the resistance. A wire table showing the resistance per 1,000 ft of copper wire (at 25°C) is given in Appendix V.

EXAMPLE 4-6 What is the resistance of a coil having an average diameter of 1.5 inches and consisting of 320 turns of No. 28 copper wire?

GIVEN: Diameter of coil = 1.5 inches Turns = 320 Wire = No. 28 copper

FIND: Resistance

SOLUTION:

$$\begin{aligned}\text{Length of wire} &= \text{diameter} \times \pi \times \text{number of turns}\\ &= 1.5 \times 3.14 \times 320 = 1{,}507 \text{ inches} = 125.58 \text{ ft}\end{aligned}$$

$$\text{Resistance of } 1{,}000 \text{ ft of No. 28 wire} = 66.17 \text{ ohms (Appendix V)}$$

$$\text{Resistance of } 125.58 \text{ ft} = \frac{125.58}{1{,}000} \times 66.17 = 8.31 \text{ ohms}$$

Conductance. Conductance is a term used to express the ease with which a material allows an electric current to flow through it. This is the opposite of resistance, and the unit of conductance, the mho, is obtained by spelling the unit of resistance, ohm, backward. By definition, conductance is the reciprocal of resistance. The reciprocal of any number is equal to one divided by that number. For example, the reciprocal of 5 is one-fifth; the reciprocal of 10 is one-tenth, etc.

$$\text{Conductance} = \frac{1}{\text{resistance}} \qquad \text{or} \qquad G = \frac{1}{R} \tag{4-5}$$

From this formula, it can be seen that a material having a high specific resistance would have a low specific conductance, and vice versa.

EXAMPLE 4-7 What is the specific conductance of copper wire?

GIVEN: Material = copper

FIND: Specific conductance

SOLUTION:

$$\text{Specific resistance} = 10.4 \text{ (Appendix IV)}$$

$$\text{Specific conductance} = \frac{1}{10.4} = 0.0961 \text{ mho}$$

4-3 Conductors

Conducting Material. A substance through which an electric current can flow easily is called a *conductor.* Every substance is a conductor of electricity, at least to some slight extent, but some substances are far better conductors than others. Theoretically, the best material to be used as a conductor would be the one having the highest specific conductance; this would be the one having the lowest specific resistance.

Factors Determining Choice of a Conducting Material. The use of conducting materials requires consideration of a number of other factors. Some of these factors are (1) cost, (2) specific resistance, (3) ability to be fused, (4) ability to withstand nature's elements, (5) flexibility, (6) melting point, (7) weight, (8) elasticity, (9) tensile strength. For example, if a conductor is to be used for instrument work where the losses due to resistance should be as low as possible, the cost would be disregarded and the material having the lowest specific resistance would be used.

If a material is used for general wiring purposes, other factors must be considered. The cost cannot be too high because of the large quantity to be used; the specific resistance must be low in order to minimize power (I^2R) losses; the material should be easily joined whether it be soldered, brazed, or welded; the elements of nature such as heat, cold, and dampness should have no effect on it. It should be flexible enough to be handled and shaped into the various necessary bends that are peculiar to all wiring. Its melting point must be high enough to withstand abnormal temperature rises. In general, copper is used for most wiring purposes because it meets the necessary requirements best.

When a material that is elastic is needed for such uses as spring contacts and circuit breakers, phosphor bronze is generally used.

Whenever a sliding or wiping contact is required, the material used must be soft enough to prevent excessive wear on the surface with which it makes contact. Carbon, graphite, and soft copper, used individually or in combination with one of the others, are the materials used for making brushes (sliding contacts).

If the material desired is to be used as a resistor to limit the flow of electric current as in the case of rheostats, heaters, and resistors, a material having a high specific resistance is used. The melting point must also be high, as this type of apparatus is constantly subjected to high temperatures.

The operation of a fuse depends on the ability of a material to melt easily and thus break the circuit when its rated current is exceeded. Materials used for fuses must have a low melting point.

4-4 Insulators

Insulating Material. A perfect insulator is a material through which no electric current can pass. All materials will pass a number of free electrons; therefore no such substance as a perfect insulator exists. However, very poor conductors are approximately such and are therefore used whenever an insulating material is required.

Breakdown Voltage of an Insulator. All materials offer some resistance to the flow of electric current. Insulators, or poor conductors, offer a high resistance to the flow of electricity; the higher the resistance, the better the insulator. If the voltage across an insulator is increased, the attractive force acting on the free electrons in the insulator will increase. This causes the free electrons to move at an increased speed. If the voltage is high enough to increase the velocity of the electrons to a point that causes them to collide against the atoms with sufficient force to detach other electrons from the neutral atoms, a steady stream of free electrons will rapidly form through the insulating material and the insulator is now a conductor. The voltage required to cause this electron flow, thus changing the insulator to a conductor, is called the *breakdown voltage* of the material.

Effect of Breakdown of the Insulation. This breakdown of the insulation between conductors is often accompanied by the passage of an electric spark. Sufficient heat is produced by this spark to burn a path through such insulating materials as paper, cloth, wood, or mica. Hard materials such as porcelain or glass will crack or allow a small path to be melted through them.

Dielectric Strength. Increasing the thickness of the insulator will increase

Fig. 4-1 Electronic applications of insulating materials. (*P. R. Mallory & Co., Inc.*)

the distance through which the voltage must force the electrons. This is the same as increasing the resistance of the insulator, for the resistance of any material will vary directly with its length. Therefore the breakdown voltage of a material will vary directly with its thickness. The dielectric strength of a material is expressed in volts per unit thickness of the material. In Appendix VI are listed some of the common materials used as insulators for electronic apparatus and their corresponding dielectric strength expressed in volts per thousandths of an inch thickness of the material.

Materials used for electrical insulation are generally manufactured compounds. The dielectric strength of a material will therefore vary with the manufacture and the proportion in which the required elements are combined. The physical structure of a material will vary with changes in weather, heat, cold, and moisture; this in turn will affect its dielectric strength.

Effect of Alternating Current on Insulating Properties. The insulating property of a material is different for direct current than for an alternating current. As the frequency of an alternating current is increased, the dielectric strength of a material will decrease. This decrease in dielectric strength not only varies inversely with the frequency but also decreases very rapidly with increases in frequency. The variation of the breakdown voltage of a material with a change in the frequency of the applied current must be taken into consideration in choosing a material for high-frequency applications.

Classification and Uses of Insulating Materials. Materials used as insula-

Table 4-1 Classification of Insulating Materials

Vitreous	Glass, enamel
Stone	Slate, marble, mica, porcelain, asbestos, ceramics
Resinous	Shellac, resins, gums
Bituminous	Asphalt, pitch
Wax	Beeswax, paraffin
Elastic	Rubber, ebonite
Oil	Oils of vegetable, animal, or mineral origin
Cellulose	Paper, wood, cotton, linen, cambric, celluloid, cellophane, plastics, lucite
Animal tissue	Silk, fiber, catgut

tion for electrical and electronic apparatus may be classified as shown in Table 4-1.

Materials used for electrical insulation usually have a dual purpose: (1) to provide mechanical protection and (2) to serve as electrical insulation. In selecting an insulating material its use is the important factor in determining its electrical, physical, and mechanical characteristics. For example, if the insulator is to be subjected to great heat, as in soldering irons or toasters, mica must be used; if it must be fireproof, as in the power cords of irons, etc., asbestos is used; if space, flexibility, and a fair dielectric strength are to be the deciding factors, as in the dielectric for small fixed capacitors, cellulose and animal tissue materials are used; if the material to be insulated is of intricate design, so that it can be insulated only with a liquid that will harden when dry, as on the coils in audio and small power transformers, resinous, bituminous, and waxy materials are used. Where a high dielectric strength is desired, as in the case of high-frequency or high-voltage equipment, glass and porcelain are used; where the insulation must remain liquid, as used in large switches and circuit breakers to quench the arc when the circuit is opened, the various oils are used.

4-5 Resistors

In d-c circuits, all appliances and devices are generally considered as having only resistance. In a-c circuits, two new factors, inductance and capacitance, must be taken into consideration. How these factors affect the electric circuit will be taken up in a later chapter on alternating currents. For the present, only d-c circuits and the corresponding resistance effect will be considered.

The number of resistors used in radio, television, and other electronic circuits is quite large. Improvements are constantly being made; new types are being developed and are continually replacing some of the older forms, thereby increasing the already vast number of types of resistors in use.

Classification of Resistors According to Material. There are two general types of construction employed in the manufacture of resistors, according to the material used, and resistors are referred to as *metallic* or *nonmetallic.*

The material used in metallic resistors is generally in the form of a wire or a ribbon, and these resistors are called *wire-wound resistors.* The wire or ribbon is wound around a supporting form made of insulating material. The wire is generally an alloy containing two or more elements such as copper, iron, nickel, chromium, zinc, and manganese.

The material used in the nonmetallic resistors is carbon or graphite, both of which have a very high specific resistance. Because of the high specific resistance, the resistors can be made smaller than wire-wound resistors. As both carbon and graphite exist in the form of fine powder, it is necessary to add a substance, usually called a *binder,* that will hold the fine particles of carbon together. It is then formed into rods which are next cut into short pieces to make up the resistor. Each piece is generally enclosed by some insulating material, and leads are attached to the ends. Nonmetallic resistors are used extensively in electronic circuits because of the ease with which high resistance can be obtained and their low cost of manufacture.

Classification of Resistors According to Control. Resistors may be further classified as (1) fixed, (2) variable, (3) adjustable, (4) tapped, (5) automatic resistance control.

A ***fixed resistor*** is one whose value cannot be changed by any mechanical means. Fixed resistors may be made of carbon or of a metallic wire. The nonmetallic resistor made of carbon or graphite powder held together by a suitable binding substance is a widely used type of fixed resistor. Low-power wire-wound fixed resistors are made by winding the wire on a plastic or fiber strip and attaching leads to each of the ends. High-power wire-wound resistors are made by winding the wire on large threaded porcelain tubes and attaching copper terminals at each end; the entire unit is then dipped in enamel or a porcelain solution and baked.

A ***variable resistor,*** commonly called a ***rheostat,*** is one whose value of resistance at its terminals may be varied. The rheostat has a sliding contact

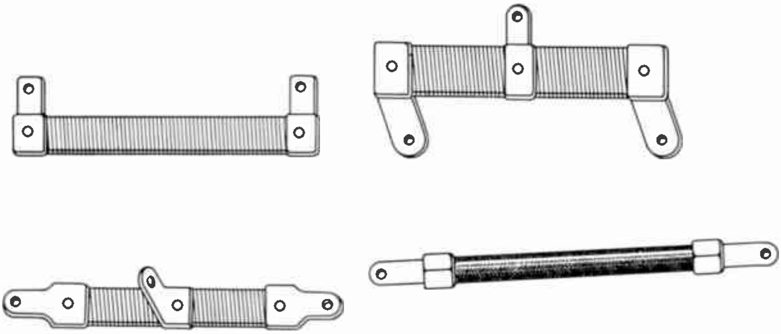


Fig. 4-2 Wire-wound fixed resistors.

arm that may be moved to any position along the resistor and has one of its terminals attached to the contact arm and the other to one end of the resistance. As the position of the contact arm is varied, the value of the resistance between the two terminals will vary.

An *adjustable resistor* is one that may be adjusted to a desired value and then set at that value. It differs from the variable resistor in that, once it is adjusted to the desired value, it is kept at that value. Adjustable resistors are always of the metallic type and are generally wound on porcelain forms. They are provided with one or more movable collars that may be clamped in a definite position after they have been adjusted to the desired value.

A *tapped resistor* is one that provides two or more definite values of resistance on a single unit. Tapped resistors are similar to adjustable resistors except that the collars are not made movable but are set at fixed positions along the resistor to give definite values of resistance. Tapped resistors are always of the metallic type; they may be of the low-power wire-wound variety that uses a fiber or plastic form or of the high-power wire-wound type that uses a porcelain form.

An *automatic resistance control resistor* is one whose resistance value changes automatically with a change in current or temperature. In the study of effects of temperature on the resistance of certain materials, it was

Fig. 4-3 Some of the various types of resistors used in electronics. (*Ohmite Manufacturing Company*)

found that, as the temperature of certain metals increased, the resistance increased. This principle is used in automatic line controls or *ballast resistors*, as they are usually called. A nickel or iron wire is placed inside a glass tube filled with an inert gas such as hydrogen or inside an air-cooled metal case. When the current flowing through this unit increases, it causes an increase in the temperature of the wire. The increase in temperature causes an increase in resistance which regulates the current and prevents it from rising excessively.

The resistance of most circuit elements increases with temperature increases. A *thermistor* is a device used to maintain the resistance of a circuit fairly constant by compensating for increases in resistance caused by temperature increases. Thermistors are made from a variety of semiconductors. A *semiconductor* is a solid having a value of resistivity midway between that of conductors and insulators. Their resistivity varies nonlinearly with temperature changes, and these solids may possess either a positive or negative temperature coefficient. Thermistors usually have a negative temperature coefficient; that is, as the temperature increases, the resistance decreases. Thermistors may also be obtained with a positive temperature coefficient and are then sometimes called *sensitors*. The resistance of a thermistor varies widely with changes in temperature. A typical thermistor may have a 5 per cent resistance change for each centigrade degree change in temperature.

A resistor whose resistance varies with changes in applied voltage is called a *varistor*. When the voltage across a varistor increases, its resistance is decreased. Varistors were originally used as lightning arrestors and have since found many applications in the fields of surge suppression and voltage stabilization. Varistors are manufactured by pressing a semiconductor material with a ceramic binder under high pressure and temperature.

Use of Resistors. Resistors and rheostats are used in a number of ways to adjust the current and voltage of electrical circuits. In radio, television, and other electronic circuits they are used as voltage dividers, loads for the output of vacuum tubes, resistors to provide the proper grid bias, current regulators in filament circuits, filter networks, etc. The type of resistor used is determined by its application and the tolerance permitted in its value of resistance.

Commercial resistors of the wire-wound type may be obtained with a tolerance as low as ±1 per cent, which means that their values of resistance are accurate within 1 per cent of their rated values. Such resistors are used as multipliers and shunts for converting low-range voltmeters and ammeters to higher range voltmeters and ammeters. They are sometimes used in high-quality radio and television receivers and in electronic equipment when exact resistance values are of importance.

In selecting a resistor, its power rating as well as its resistance value must be taken into consideration. For example, a resistor rated at 1 watt and

10,000 ohms should not be used to carry a current of more than 10 ma, as

$$I = \sqrt{\frac{P}{R}} = \sqrt{\frac{1}{10{,}000}} = 0.01 \text{ amp, or } 10 \text{ ma}$$

Standard Color Code for Resistors. A color code has been established to indicate the resistance value of fixed resistors. This color code is very useful for identifying the values of the carbon resistors used so extensively in electronic circuits especially since many resistors of different ohmic values have the same appearance. Appendix VII lists the standard color coding for resistors and illustrates the method used to determine the resistance value and tolerance from the color markings.

4-6 Protective Devices

Fuses. A *fuse* is a protective device designed to open the circuit in which it is connected when a dangerous overload condition occurs. The fuse will melt and open the circuit when the current exceeds its rated value for a definite period of time. Basically, a fuse is a small strip of metal such as zinc, lead, or lead-tin alloy that melts at a much lower temperature than a piece of copper of the same physical dimensions. Fuses may be classified as (1) plug and (2) cartridge. The cartridge type may be obtained as one-time fuses or as renewable fuses.

A *superlag fuse* is a renewable fuse having a fusible link that is designed to take a longer period of time to melt for a given current than does an ordinary link. The time-lag characteristic is desirable in motor circuits to prevent the fuse from opening the circuit during the short interval when the motor is being started. The current during this interval may be several times the rated motor current.

A cartridge-type fuse having a time lag much greater than the superlag fuse is the *fusetron.* It consists of a fusible link and a thermal cutout connected in series. The fusible link is designed to melt at a higher current than in an ordinary fuse of the same rating. The link is made of copper, which permits a smaller amount of metal to be used, and protects only against short circuits. On overloads the circuit is opened by the thermal cutout, which consists of a small copper block attached by a spring to a heating element. The fusible link is attached to the copper block by means of a low-melting-point solder. When an overload occurs, the heat generated by the heating element and the fusible link melts the solder connection and the spring pulls the copper block out of place, thereby opening the circuit.

Some electronic circuits use short lengths of small-size copper wire as fuses. It is not uncommon to find a 1-inch length of No. 22 wire (or smaller) connected in a circuit to serve as a fuse.

Circuit Breakers. The magnetic circuit breaker is another device used to protect a circuit against overload and short-circuit currents. A simple form of the magnetic circuit breaker consists of (1) a solenoid, (2) an iron plunger within the solenoid, (3) a set of contacts that are normally closed, (4) a toggle mechanism that holds the contacts in their normal position, and (5) a trigger to trip the mechanism. The solenoid is connected in series with the circuit to be protected. An excessive current in the solenoid causes the iron plunger to move upward and strike the trigger, which trips the holding mechanism and opens the breaker contacts.

The thermal circuit breaker is another protective device; it is basically a series circuit consisting of a bimetallic strip, a flexible connecting lead, and a contact bar. When the current exceeds the rating of the breaker, it causes the bimetallic strip to be heated sufficiently to bend, thus displacing the tripping end of the bimetallic strip and releasing the contact bar. A compression spring actuates the contact bar and plunger to open the breaker contacts.

Some breakers must be reset manually, some may be tripped or reset from a remote point, and others may be reset automatically.

4-7 Electric Circuits

Resistance of Electric Circuits. In terms of voltage and current, the total resistance of a circuit is equal to the total pressure applied to that circuit divided by the total current flowing in the circuit; expressed mathematically

$$R_T = \frac{E_T}{I_T} \tag{4-6}$$

where R_T = total line resistance, ohms
E_T = total line voltage, volts
I_T = total line current, amp

The resistance of any particular part of a circuit is equal to the voltage across the terminals of that part of the circuit divided by the current flowing through that part of the circuit; expressed mathematically

$$r = \frac{e}{i} \tag{4-7}$$

where r = branch resistance, ohms
e = branch voltage, volts
i = branch current, amp

Four Kinds of Circuits. There are four ways in which electrical appliances may be connected: (1) simple circuit, (2) series circuit, (3) parallel circuit, (4) combination circuit.

4-8 The Simple Circuit

A simple circuit is one in which a single resistance is connected across a power source as in Fig. 4-4.

In this circuit

$$R_T = \frac{E_T}{I_T}$$

$$P_T = E_T \times I_T$$

$$En_T = P_T \times T$$

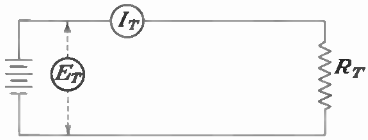


Fig. 4-4 A simple circuit.

4-9 The Series Circuit

A series circuit is one in which two or more resistances are connected in one continuous path so that the current passes in turn from one to another. In the circuit of Fig. 4-5, the current leaves the negative side of the generator, goes through each of the three resistors, and returns to the positive side of the generator, thus completing the electric circuit.

Currents in a Series Circuit. As there is only one path through which the current may flow and as all the current that leaves the generator must return to it, the same amount of current must flow in all parts of the circuit. Therefore

$$I_T = i_1 = i_2 = i_3 \tag{4-8}$$

Voltages of a Series Circuit. The *voltage drops* e_1, e_2, and e_3 indicate the pressures required to force the current through the resistors r_1, r_2, and r_3, respectively. As E_S represents the total voltage required at the power source to force the current through the complete circuit, the voltage supplied by the generator must be equal to the sum of the voltage drops of the circuit, or

$$E_S = e_1 + e_2 + e_3 \tag{4-9}$$

Resistance of a Series Circuit. The current in this circuit must pass through all the resistors before it can return to the starting point. The total resistance offered to the flow of current will therefore be the sum of all the individual resistances, or

$$R_T = r_1 + r_2 + r_3 \tag{4-10}$$

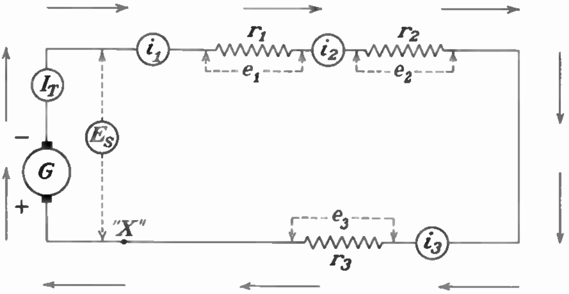


Fig. 4-5 A series circuit.

Power of the Series Circuit. Each of the resistors consumes power, and as all the power must come from the generator, the total power taken by the series circuit must be equal to the sum of the separate powers, or

$$P_T = p_1 + p_2 + p_3 \tag{4-11}$$

Energy of the Series Circuit. As electrical energy is transformed to heat energy at each resistor and as all the energy must be supplied by the generator, the total energy of the series circuit must be equal to the sum of the separate amounts of energy, or

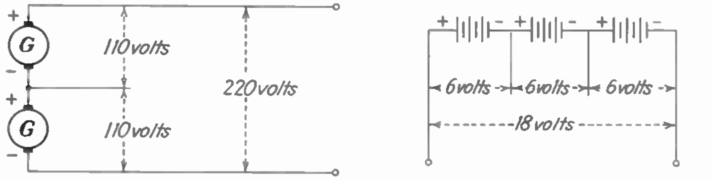
$$En_T = \text{en}_1 + \text{en}_2 + \text{en}_3 \tag{4-12}$$

Characteristics of the Series Circuit. The characteristics of the series circuit may be summarized as follows:

1. The current in all parts of the circuit is the same.
2. The voltage applied, or the line voltage, is equal to the sum of the separate voltage drops of the circuit.
3. The resistance of the complete circuit is equal to the sum of the separate resistances of the circuit.
4. The total power is equal to the sum of the powers of the separate resistors.
5. The total energy is equal to the sum of the energies of the separate resistors.

Uses of the Series Circuit. Series circuits are used in radio, television, and other electronic equipment and may be found in some of the following circuits: (1) the plate circuit of vacuum tubes, (2) the heater circuit of vacuum tubes, (3) a circuit to which a dropping resistor is connected to limit the voltage across a certain part of the circuit.

Power supplies are connected in series when a higher voltage is to be obtained from several lower voltage units. For example, two 110-volt generators may be connected in series in order to obtain a 220-volt power source, or three 6-volt storage batteries may be connected in series in order to obtain an 18-volt power source. When power supplies are connected in series, the terminals of opposite polarities must be connected together, as shown in Fig. 4-6.



(a)-Two generators connected in series

(b)-Three 6-volt storage batteries connected in series

Fig. 4-6 Power sources connected in series.

Disadvantages of the Series Circuit. If the wire should break at the point *X* (Fig. 4-5) or at any other point, the circuit would be broken and no current would pass through any part of the circuit. This is a serious disadvantage of the series circuit, for if a break occurs in any part of the circuit, the entire circuit becomes useless.

If a pressure of 110 volts is desired for each resistor of Fig. 4-5, the line voltage would have to be 330 volts. If four more resistors were added to the circuit and each required a pressure of 110 volts, the line voltage would then have to be 770 volts. It can readily be seen that a high line voltage would be required if a number of 110-volt lamps (in place of resistors) were to be connected in series; therefore the series circuit is not practical for lighting circuits.

Solution of Series-circuit Problems. Using the rules for series circuits as outlined and the fundamental principles of Ohm's law, it is possible to solve any problem involving series circuits. This can best be illustrated by the following example.

EXAMPLE 4-8 A 10-, a 15-, and a 30-ohm resistor are connected in series across a 110-volt line. (*a*) What is the resistance of the circuit? (*b*) If the circuit is used for 10 hr, how much energy is consumed by each resistor? (*c*) What is the total amount of energy consumed?

GIVEN: $E_T = 110$ volts $r_1 = 10$ ohms $r_2 = 15$ ohms $r_3 = 30$ ohms $T = 10$ hr

FIND: (*a*) R_T (*b*) en_1 en_2 en_3 (*c*) En_T

SOLUTION:

In solving circuit problems it is always best to draw a circuit diagram. The circuit diagram for this problem would be drawn as shown in Fig. 4-7.

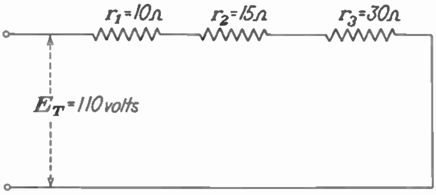


Fig. 4-7

From the rules for series circuits

(*a*) $$R_T = r_1 + r_2 + r_3 = 10 + 15 + 30 = 55 \text{ ohms}$$

(*b*) In order to find the energy, it is necessary to know the value of the power and the hours. The power of each resistor may be found by i^2r, and since only the value of r is known, it is now necessary to solve for the current,

$$I_T = \frac{E_T}{R_T} = \frac{110}{55} = 2 \text{ amp}$$

as $$I_T = i_1 = i_2 = i_3$$

then
$$p_1 = I_T^2 \times r_1 = 2 \times 2 \times 10 = 40 \text{ watts}$$
$$p_2 = I_T^2 \times r_2 = 2 \times 2 \times 15 = 60 \text{ watts}$$
$$p_3 = I_T^2 \times r_3 = 2 \times 2 \times 30 = 120 \text{ watts}$$
$$P_T = p_1 + p_2 + p_3 = 40 + 60 + 120 = 220 \text{ watts}$$

It is good practice continually to check the problem in order to locate any mathematical errors. In this problem, the total power can be checked by using the formula

$$P_T = E_T \times I_T = 110 \times 2 = 220 \text{ watts}$$

This checks with the value obtained before by using a different method, and therefore the solution is mathematically correct. Continuing the solution of the problem

$$\text{en}_1 = p_1 \times T = 40 \times 10 = 400 \text{ whr}$$
$$\text{en}_2 = p_2 \times T = 60 \times 10 = 600 \text{ whr}$$
$$\text{en}_3 = p_3 \times T = 120 \times 10 = 1{,}200 \text{ whr}$$
(c) $$En_T = \text{en}_1 + \text{en}_2 + \text{en}_3 = 400 + 600 + 1{,}200 = 2{,}200 \text{ whr}$$

Checking the total energy by using the formula

$$En_T = P_T \times T = 220 \times 10 = 2{,}200 \text{ whr, or } 2.2 \text{ kwhr}$$

This is the same answer as the above. Therefore the problem is mathematically correct.

4-10 The Parallel Circuit

When two or more resistances are connected so that the current will have two or more paths in which it may flow, it is a parallel circuit. In Fig. 4-8, the current leaves the generator at *A* and, when it reaches *B*, it divides into two paths, part of the current going toward *G* and the remainder toward *C*. The current going toward *C* upon reaching that point has two paths, and it will divide, part of this current going toward *F* and the remainder toward *D*. At *F*, the current from *E* joins with the current from *C* and flows toward *G*. At *G*, the current from *F* joins with the current from *B* and returns to the generator at *H*.

Currents in a Parallel Circuit. Figure 4-9 shows the distribution of the current in the circuit of Fig. 4-8. The values of current shown throughout the circuit indicate the amounts of current in those parts of the circuit. This diagram shows that the line current is equal to the sum of the branch currents, as 4 amp + 6 amp + 2 amp = 12 amp.

$$I_T = i_1 + i_2 + i_3 \tag{4-13}$$

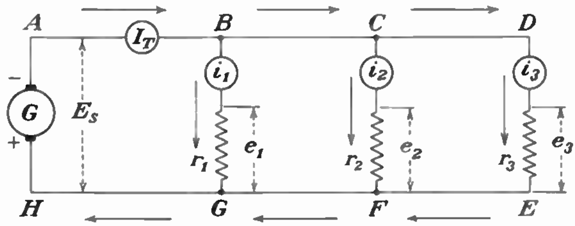


Fig. 4-8 A parallel circuit.

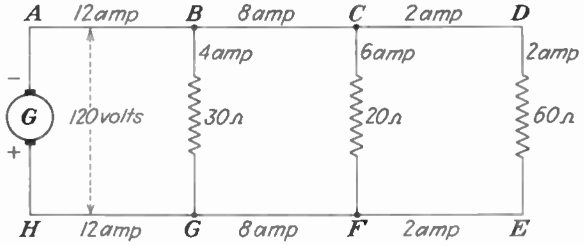


Fig. 4-9 Current distribution of the parallel circuit shown in Fig. 4-8.

Voltages of a Parallel Circuit. The current in flowing from the generator to r_1 does not pass through any appreciable amount of resistance; therefore the voltage drop is negligible and $e_1 = E_S$. Likewise as there is no appreciable resistance offered to the current going from r_1 to r_2 or from r_2 to r_3, the voltage across r_2 and r_3 is equal to that across r_1 and

$$E_S = e_1 = e_2 = e_3 \tag{4-14}$$

These conclusions are based on the assumption that the connecting wires have no resistance, which actually is not true; however, the resistance of these wires is ordinarily so low that the resulting voltage drop can be neglected. If more accurate values of the branch voltages are desired, the drop in the connecting wires can be calculated and subtracted from the line voltage, the remainder being the voltage at the resistor.

Resistance of a Parallel Circuit. The resistance of a parallel circuit is calculated by using the conductance method and may be obtained by using the formula

$$R_T = \frac{1}{\frac{1}{r_1} + \frac{1}{r_2} + \frac{1}{r_3}} \tag{4-15}$$

Substituting the values from Fig. 4-9 in the above formula,

$$R_T = \frac{1}{\frac{1}{30} + \frac{1}{20} + \frac{1}{60}} = \frac{1}{\frac{2+3+1}{60}} = \frac{1}{\frac{6}{60}} = \frac{60}{6} = 10 \text{ ohms}$$

The resistance of the line is found to be 10 ohms, and it should be noticed that this total resistance is lower than the lowest individual resistance in the circuit which is 20 ohms. This is characteristic of all parallel circuits.

When the parallel circuit contains only two circuit elements, Eq. (4-15) may be expressed as

$$R_T = \frac{r_1 r_2}{r_1 + r_2} \tag{4-15a}$$

$$r_1 = \frac{R_T r_2}{r_2 - R_T} \tag{4-15b}$$

$$r_2 = \frac{R_T r_1}{r_1 - R_T} \tag{4-15c}$$

Power of the Parallel Circuit. Each of the resistors consumes power, and as all the power must come from the generator, the total power taken by the parallel circuit must be equal to the sum of the separate powers (note that this is the same as for the series circuit), or

$$P_T = p_1 + p_2 + p_3 \tag{4-11}$$

Energy of the Parallel Circuit. As electrical energy is transformed to heat energy at each resistor and as all the energy must be supplied by the generator, the total energy of the parallel circuit must be equal to the sum of the separate amounts of energy (note that this is the same as for the series circuit), or

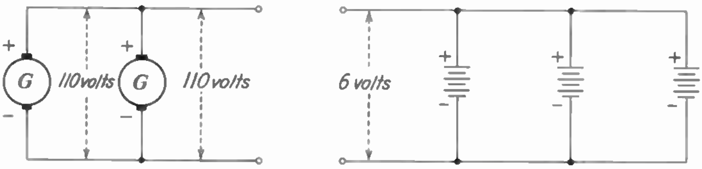
$$En_T = \mathrm{en}_1 + \mathrm{en}_2 + \mathrm{en}_3 \tag{4-12}$$

Characteristics of the Parallel Circuit. The characteristics of the parallel circuit may be summarized as follows:

1. The line current is equal to the sum of the currents in the separate branches.
2. The voltage across each branch is the same and is equal to the line voltage.
3. The resistance of a parallel circuit is equal to the reciprocal of the sum of the reciprocals of each branch resistance. It is always less than the lowest branch resistance.
4. The total power is equal to the sum of the powers of the separate branches.
5. The total energy is equal to the sum of the energies of the separate branches.

Uses of the Parallel Circuit. Parallel circuits are used wherever a constant-voltage power-supply system is needed. House wiring is a good example of the use of this type of circuit.

Power supplies are connected in parallel when a higher current is needed and the voltage is to remain constant. For example, two 110-volt generators may be connected in parallel in order to obtain an increased current output (and consequently power) at 110 volts. Only one of them need be used if the current required is equal to or is less than the current rating of one of the generators. Storage batteries may be connected in parallel if the current rating is to be increased without an increase in voltage. When power supplies are connected in parallel, all terminals of similar polarity must be connected together. The voltage across the combination is the same as the voltage of an individual unit, and the current rating becomes equal to the



(a)-Two generators connected in parallel

(b)-Three 6-volt storage batteries connected in parallel

Fig. 4-10 Power sources connected in porollel.

sum of the individual current ratings. Parallel connected power sources are illustrated in Fig. 4-10.

Disadvantage of the Parallel Circuit. A disadvantage of the parallel circuit is that, as additional appliances or loads are added to the circuit, the line current increases. If the increased amount raises the current above the safe carrying capacity of the wiring used, it becomes necessary to rewire the circuit with a larger size wire or to install additional feeder circuits.

Advantage of the Parallel Circuit. An advantage of the parallel circuit is that, if a break occurs in any one of the branch circuits (*BG*, *CF*, or *DE* in Fig. 4-8 or 4-9), it will have no effect on the other circuits. In house wiring, the use of parallel circuits makes it possible to switch any light or appliance on or off without affecting the other lights and appliances.

Solution of Parallel-circuit Problems. Using the rules for parallel circuits as outlined and the fundamental principles of Ohm's law, it is possible to solve any problem involving parallel circuits. This can best be illustrated by the following example.

EXAMPLE 4-9 A 10-, a 15-, and a 30-ohm resistor are connected in parallel across a 110-volt line. (*a*) What is the resistance of the circuit? (*b*) If the circuit is used for 10 hr, how much energy is consumed by each resistor? (*c*) What is the total amount of energy consumed?

GIVEN: $E_T = 110$ volts $r_1 = 10$ ohms $r_2 = 15$ ohms $r_3 = 30$ ohms $T = 10$ hr

FIND: (*a*) R_T (*b*) en_1 en_2 en_3 (*c*) En_T

The circuit diagram for this problem would be as shown in Fig. 4-11.

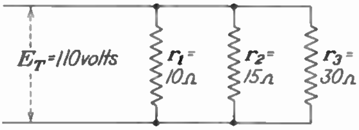


Fig. 4-11

SOLUTION: From the rules for parallel circuits,

$$(a) \qquad R_T = \frac{1}{\frac{1}{r_1}+\frac{1}{r_2}+\frac{1}{r_3}} = \frac{1}{\frac{1}{10}+\frac{1}{15}+\frac{1}{30}} = \frac{1}{\frac{6}{30}} = \frac{30}{6} = 5 \text{ ohms}$$

(b)
$$i_1 = E_T \div r_1 = 110 \div 10 = 11 \text{ amp}$$
$$i_2 = E_T \div r_2 = 110 \div 15 = 7.33 \text{ amp}$$
$$i_3 = E_T \div r_3 = 110 \div 30 = 3.67 \text{ amp}$$
$$I_T = i_1 + i_2 + i_3 = 11 + 7.33 + 3.67 = 22 \text{ amp}$$

Checking the answer:

$$I_T = \frac{E_T}{R_T} = \frac{110}{5} = 22 \text{ amp}$$

$$p_1 = E_T \times i_1 = 110 \times 11 = 1{,}210 \text{ watts}$$
$$p_2 = E_T \times i_2 = 110 \times 7.33 = 806.3 \text{ watts}$$
$$p_3 = E_T \times i_3 = 110 \times 3.67 = 403.7 \text{ watts}$$
$$P_T = p_1 + p_2 + p_3 = 1{,}210 + 806.3 + 403.7 = 2{,}420 \text{ watts}$$

Checking the total power:

$$P_T = E_T \times I_T = 110 \times 22 = 2{,}420 \text{ watts}$$

$$\text{en}_1 = p_1 \times T = 1{,}210 \times 10 = 12{,}100 \text{ whr, or } 12.1 \text{ kwhr}$$
$$\text{en}_2 = p_2 \times T = 806.3 \times 10 = 8{,}063 \text{ whr, or } 8.063 \text{ kwhr}$$
$$\text{en}_3 = p_3 \times T = 403.7 \times 10 = 4{,}037 \text{ whr, or } 4.037 \text{ kwhr}$$

(c)
$$En_T = \text{en}_1 + \text{en}_2 + \text{en}_3 = 12.1 + 8.063 + 4.037 = 24.2 \text{ kwhr}$$

Checking this answer:

$$En_T = P_T \times T = 2420 \times 10 = 24{,}200 \text{ whr, or } 24.2 \text{ kwhr}$$

4-11 Simple Combination Circuits

When a circuit contains both series and parallel circuits, it is a combination circuit. Combination circuits may be connected in series-parallel or parallel-series.

The Series-parallel Circuit. When groups of parallel circuits are connected in series, it is called a *series-parallel circuit.* In Fig. 4-12, the current leaves the generator at *A* and returns at *G*. The current divides at point *C* and joins at *D*, dividing and joining a second time at points *E* and *F* as indicated by the arrows.

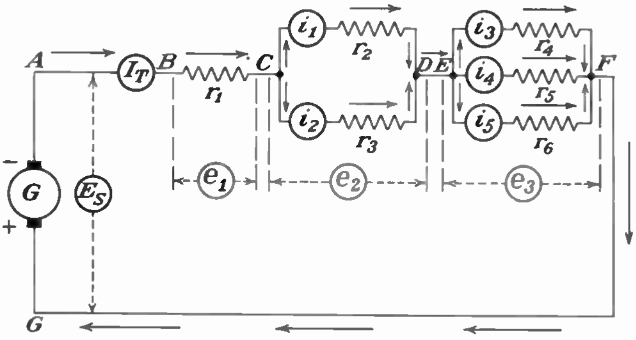


Fig. 4-12 A series-parallel circuit.

Expressed mathematically,

$$I_T = (i_1 + i_2) = (i_3 + i_4 + i_5) \tag{4-16}$$

Close observation of Fig. 4-12 will show that this circuit is fundamentally a series circuit; therefore the voltage drops should equal the line voltage, or

$$E_S = e_1 + e_2 + e_3 \tag{4-17}$$

Solution of Series-parallel Circuits. To solve series-parallel circuits, each group of parallel resistance values is first combined into an equivalent single resistance value by using the reciprocal method. The whole is then treated as a series circuit.

EXAMPLE 4-10 Find the voltage and current of each resistor in the circuit of Fig. 4-13.

GIVEN:

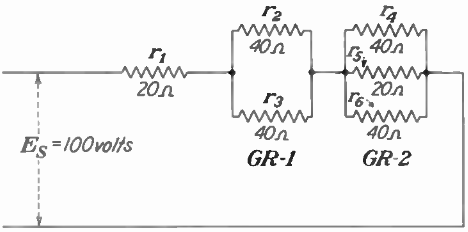


Fig. 4-13

SOLUTION:

$$R_{GR\cdot 1} = \frac{1}{\frac{1}{r_2} + \frac{1}{r_3}} = \frac{1}{\frac{1}{40} + \frac{1}{40}} = \frac{40}{2} = 20 \text{ ohms}$$

$$R_{GR\cdot 2} = \frac{1}{\frac{1}{r_4} + \frac{1}{r_5} + \frac{1}{r_6}} = \frac{1}{\frac{1}{40} + \frac{1}{20} + \frac{1}{40}} = \frac{40}{4} = 10 \text{ ohms}$$

Figure 4-13 then becomes a simple series circuit as illustrated in Fig. 4-14.

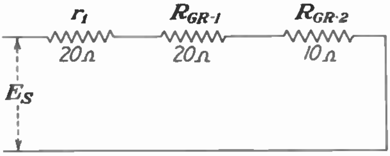


Fig. 4-14

$$R_T = r_1 + R_{GR\cdot 1} + R_{GR\cdot 2} = 20 + 20 + 10 = 50 \text{ ohms}$$
$$I_T = E_S \div R_T = 100 \div 50 = 2 \text{ amp}$$
$$e_1 = I_T \times r_1 = 2 \times 20 = 40 \text{ volts}$$
$$e_2 = I_T \times R_{GR\cdot 1} = 2 \times 20 = 40 \text{ volts}$$
$$e_3 = I_T \times R_{GR\cdot 2} = 2 \times 10 = 20 \text{ volts}$$

Checking:

$$
\begin{aligned}
E_S &= e_1 + e_2 + e_3 = 40 + 40 + 20 = 100 \text{ volts}\\
i_1 &= I_T = 2 \text{ amp}\\
i_2 &= E_{GR\cdot1} \div r_2 = 40 \div 40 = 1 \text{ amp}\\
i_3 &= E_{GR\cdot1} \div r_3 = 40 \div 40 = 1 \text{ amp}\\
I_{GR\cdot1} &= i_2 + i_3 = 1 + 1 = 2 \text{ amp}\\
i_4 &= E_{GR\cdot2} \div r_4 = 20 \div 40 = 0.5 \text{ amp}\\
i_5 &= E_{GR\cdot2} \div r_5 = 20 \div 20 = 1.0 \text{ amp}\\
i_6 &= E_{GR\cdot2} \div r_6 = 20 \div 40 = 0.5 \text{ amp}\\
I_{GR\cdot2} &= i_4 + i_5 + i_6 = 0.5 + 1.0 + 0.5 = 2 \text{ amp}
\end{aligned}
$$

The voltage across each resistance, each parallel group of resistances, and the entire circuit is now known. The value of the current flowing in each resistance, in each group of resistances, and in the entire circuit is also known. If the power used by the entire circuit or any part of it is desired, it can easily be obtained by using the power formulas.

The Parallel-series Circuit. When groups of series circuits are connected in parallel, it is called a parallel-series circuit. Such a circuit is shown in Fig. 4-15. The current leaves the generator at *A* and returns at *H*. It divides into three paths, *BG*, *CF*, and *DE*. The directions of the currents are shown by the arrows.

$$I_T = i_1 + i_2 + i_3 \tag{4-18}$$

As each group is a series circuit, the sum of the voltage drops in each group should equal the line voltage; also

$$E_S = E_{GR\cdot1} = E_{GR\cdot2} = E_{GR\cdot3} \tag{4-19}$$

Solution of Parallel-series Circuits. To solve parallel-series circuits, each group of series resistance values is first solved for its equivalent single resistance by adding all resistances in that group. The whole is then treated as a parallel circuit.

EXAMPLE 4-11 Find the total resistance of each branch, the total resistance of the circuit, the current in each resistor, and the voltage across each resistor of the circuit shown in Fig. 4-16.

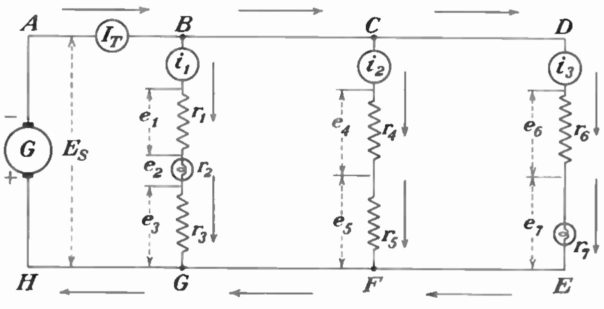


Fig. 4-15 A parallel-series circuit.

GIVEN:

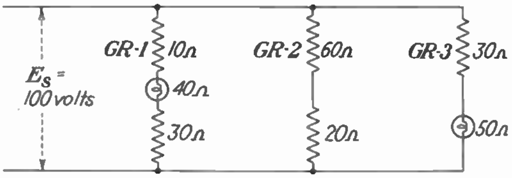


Fig. 4-16

SOLUTION:

$$R_{GR\cdot 1} = 10 + 40 + 30 = 80 \text{ ohms}$$
$$R_{GR\cdot 2} = 60 + 20 = 80 \text{ ohms}$$
$$R_{GR\cdot 3} = 30 + 50 = 80 \text{ ohms}$$

The circuit of Fig. 4-16 can now be simplified and becomes a simple parallel circuit as shown in Fig. 4-17.

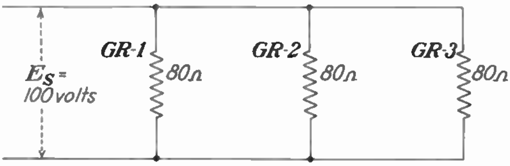


Fig. 4-17

$$R_T = \frac{1}{\frac{1}{R_{GR\cdot 1}} + \frac{1}{R_{GR\cdot 2}} + \frac{1}{R_{GR\cdot 3}}} = \frac{1}{\frac{1}{80} + \frac{1}{80} + \frac{1}{80}} = \frac{80}{3} = 26.66 \text{ ohms}$$

$$I_T = \frac{E_S}{R_T} = \frac{100}{26.66} = 3.75 \text{ amp}$$

$$i_{GR\cdot 1} = \frac{E_{GR\cdot 1}}{R_{GR\cdot 1}} = \frac{100}{80} = 1.25 \text{ amp}$$

$$i_{GR\cdot 2} = \frac{E_{GR\cdot 2}}{R_{GR\cdot 2}} = \frac{100}{80} = 1.25 \text{ amp}$$

$$i_{GR\cdot 3} = \frac{E_{GR\cdot 3}}{R_{GR\cdot 3}} = \frac{100}{80} = 1.25 \text{ amp}$$

Checking:

$$I_T = i_{GR\cdot 1} + i_{GR\cdot 2} + i_{GR\cdot 3} = 1.25 + 1.25 + 1.25 = 3.75 \text{ amp}$$

Solving for the individual voltage drops,

$$e_1 = i_{GR\cdot 1} \times r_1 = 1.25 \times 10 = 12.5 \text{ volts}$$
$$e_2 = i_{GR\cdot 1} \times r_2 = 1.25 \times 40 = 50 \text{ volts}$$
$$e_3 = i_{GR\cdot 1} \times r_3 = 1.25 \times 30 = 37.5 \text{ volts}$$
$$e_4 = i_{GR\cdot 2} \times r_4 = 1.25 \times 60 = 75 \text{ volts}$$
$$e_5 = i_{GR\cdot 2} \times r_5 = 1.25 \times 20 = 25 \text{ volts}$$
$$e_6 = i_{GR\cdot 3} \times r_6 = 1.25 \times 30 = 37.5 \text{ volts}$$
$$e_7 = i_{GR\cdot 3} \times r_7 = 1.25 \times 50 = 62.5 \text{ volts}$$

Checking:

$$E_{GR\cdot 1} = e_1 + e_2 + e_3 = 12.5 + 50 + 37.5 = 100 \text{ volts}$$
$$E_{GR\cdot 2} = e_4 + e_5 = 75 + 25 = 100 \text{ volts}$$
$$E_{GR\cdot 3} = e_6 + e_7 = 37.5 + 62.5 = 100 \text{ volts}$$

If the power used by any part of the circuit or by the entire circuit is desired, it can be obtained by using the power formulas.

Advantages of Combination Circuits. Combination circuits have the advantages of both series and parallel circuits and minimize their disadvantages. Generally, less copper is required and a smaller size wire can be used. Also, if one resistor should burn out and thereby cause an open circuit, it may affect only one or two branch circuits or it may affect the entire circuit depending upon the kind of circuit and the location of the faulty resistor.

Uses of Combination Circuits. Combination circuits are used whenever various types of circuits must be fed from the same power supply. In an automobile the starting, lighting, and ignition circuits are all individual circuits joined to make a combination circuit drawing its power from one battery.

Electronic equipment generally contains a number of separate circuits, such as voltage amplifiers, power amplifiers, tuning, oscillator, demodulator, synchronizing, sweep, integrator, differentiator, etc. Individually they may be simple series or parallel circuits or complex combination circuits. When an electronic unit is considered as a whole, the result is a complex combination circuit. In the solution of electronic circuit problems, each circuit is solved separately and then combined to obtain the final result.

Power supplies are connected in series to get a higher voltage and connected in parallel to obtain a higher current. Rows of series-connected battery cells may be so connected that the rows themselves are grouped in parallel to form a parallel-series arrangement as shown in Fig. 4-18.

Rows of parallel-connected battery cells may be so connected that the rows themselves are grouped in series to form a series-parallel arrangement as shown in Fig. 4-19.

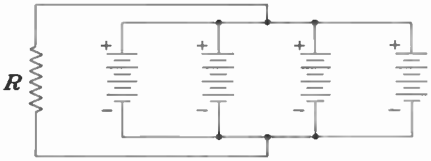


Fig. 4-18 Parallel-series grouping of battery cells.

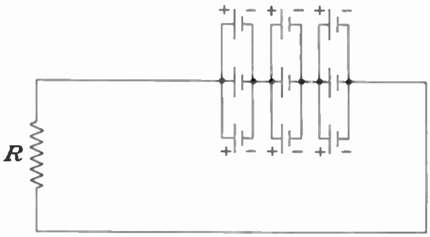


Fig. 4-19 Series-parallel grouping of battery cells.

4-12 More Advanced Combination Circuits

Many combination circuits are neither simple series-parallel nor parallel-series circuits. The resistances combine to form complicated combination circuits.

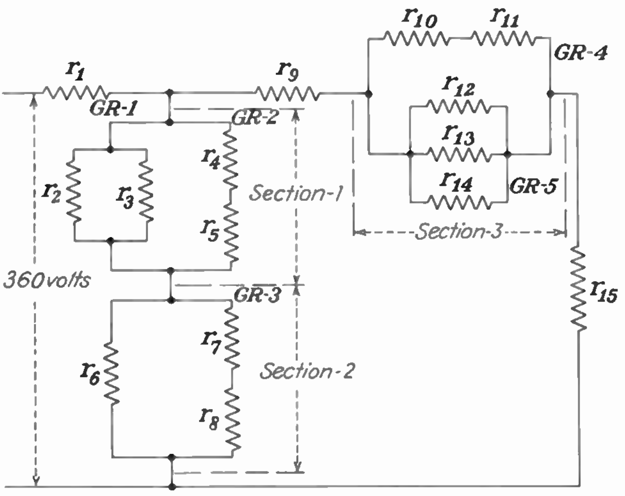
To solve such circuits, it is necessary to perform the following steps in the order listed:

1. Combine the resistance values in each group to obtain one single equivalent resistance value for each section.
2. Combine the resistance values of all sections to obtain one single equivalent resistance value for the line.
3. Solve for the line current.
4. Find the current flowing in each resistor.
5. Find the voltage across each resistor.

In the solution of some circuits, steps 4 and 5 may have to be interchanged.

EXAMPLE 4-12 Find the current distribution and the voltage drop across each resistor in the circuit shown in Fig. 4-20.

GIVEN:



$r_1 = 20\ \Omega$
$r_2 = 1{,}000\ \Omega$
$r_3 = 1{,}500\ \Omega$
$r_4 = 200\ \Omega$
$r_5 = 400\ \Omega$
$r_6 = 600\ \Omega$
$r_7 = 400\ \Omega$
$r_8 = 200\ \Omega$
$r_9 = 20\ \Omega$
$r_{10} = 200\ \Omega$
$r_{11} = 40\ \Omega$
$r_{12} = 100\ \Omega$
$r_{13} = 200\ \Omega$
$r_{14} = 600\ \Omega$
$r_{15} = 52\ \Omega$

Fig. 4-20

SOLUTION:

$$R_{GR\cdot 1} = \frac{1}{\dfrac{1}{r_2} + \dfrac{1}{r_3}} = \frac{1}{\dfrac{1}{1{,}000} + \dfrac{1}{1{,}500}} = \frac{3{,}000}{5} = 600 \text{ ohms}$$

$$R_{GR\cdot 2} = r_4 + r_5 = 200 + 400 = 600 \text{ ohms}$$

$$R_{sec\cdot 1} = \frac{1}{\dfrac{1}{R_{GR\cdot 1}} + \dfrac{1}{R_{GR\cdot 2}}} = \frac{1}{\dfrac{1}{600} + \dfrac{1}{600}} = \frac{600}{2} = 300 \text{ ohms}$$

$$R_{GR\cdot 3} = r_7 + r_8 = 400 + 200 = 600 \text{ ohms}$$

$$R_{sec\cdot 2} = \frac{1}{\frac{1}{r_6} + \frac{1}{R_{GR\cdot 3}}} = \frac{1}{\frac{1}{600} + \frac{1}{600}} = \frac{600}{2} = 300 \text{ ohms}$$

$$R_{GR\cdot 4} = r_{10} + r_{11} = 200 + 40 = 240 \text{ ohms}$$

$$R_{GR\cdot 5} = \frac{1}{\frac{1}{r_{12}} + \frac{1}{r_{13}} + \frac{1}{r_{14}}} = \frac{1}{\frac{1}{100} + \frac{1}{200} + \frac{1}{600}} = \frac{600}{10} = 60 \text{ ohms}$$

$$R_{sec\cdot 3} = \frac{1}{\frac{1}{R_{GR\cdot 4}} + \frac{1}{R_{GR\cdot 5}}} = \frac{1}{\frac{1}{240} + \frac{1}{60}} = \frac{240}{5} = 48 \text{ ohms}$$

By substitution of values found for sections 1, 2, and 3 the equivalent circuit may now be drawn as shown in Fig. 4-21.

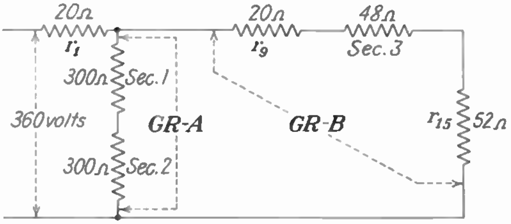


Fig. 4-21

Close observation of Fig. 4-21 will show that this circuit is now a simple parallel-series circuit having two groups of series resistors connected in parallel and a line-dropping resistor r_1.

$$R_{GR\cdot A} = R_{sec\cdot 1} + R_{sec\cdot 2} = 300 + 300 = 600 \text{ ohms}$$

$$R_{GR\cdot B} = r_9 + R_{sec\cdot 3} + r_{15} = 20 + 48 + 52 = 120 \text{ ohms}$$

$$R_t = \frac{1}{\frac{1}{R_{GR\cdot A}} + \frac{1}{R_{GR\cdot B}}} = \frac{1}{\frac{1}{600} + \frac{1}{120}} = \frac{1}{\frac{1+5}{600}} = \frac{600}{6} = 100 \text{ ohms}$$

$$R_T = r_1 + R_t = 20 + 100 = 120 \text{ ohms}$$

$$I_T = \frac{E_T}{R_T} = \frac{360}{120} = 3.0 \text{ amp}$$

$$e_{r\cdot 1} = I_T \times r_1 = 3.0 \times 20 = 60 \text{ volts}$$

$$E_{GR\cdot A} = E_{GR\cdot B} = E_T - e_{r\cdot 1} = 360 - 60 = 300 \text{ volts}$$

$$I_{GR\cdot A} = \frac{E_{GR\cdot A}}{R_{GR\cdot A}} = \frac{300}{600} = 0.50 \text{ amp}$$

$$I_{GR\cdot B} = \frac{E_{GR\cdot B}}{R_{GR\cdot B}} = \frac{300}{120} = 2.50 \text{ amp}$$

$$I_T = I_{GR\cdot A} + I_{GR\cdot B} = 0.50 + 2.50 = 3.00 \text{ amp}$$

$$e_{sec\cdot 1} = I_{GR\cdot A} \times R_{sec\cdot 1} = 0.50 \times 300 = 150 \text{ volts}$$

$$e_{sec\cdot 2} = I_{GR\cdot A} \times R_{sec\cdot 2} = 0.50 \times 300 = 150 \text{ volts}$$

$$E_{GR\cdot A} = e_{sec\cdot 1} + e_{sec\cdot 2} = 150 + 150 = 300 \text{ volts}$$

$$e_{r\cdot 9} = I_{GR\cdot B} \times r_9 = 2.50 \times 20 = 50 \text{ volts}$$

$$e_{sec\cdot 3} = I_{GR\cdot B} \times R_{sec\cdot 3} = 2.50 \times 48 = 120 \text{ volts}$$

$$e_{r\cdot 15} = I_{GR\cdot B} \times r_{15} = 2.50 \times 52 = 130 \text{ volts}$$

$$E_{GR\cdot B} = e_{r\cdot 9} + e_{sec\cdot 3} + e_{r\cdot 15} = 50 + 120 + 130 = 300 \text{ volts}$$

$$i_{GR\cdot 2} = \frac{e_{sec\cdot 1}}{R_{GR\cdot 2}} = \frac{150}{600} = 0.250 \text{ amp}$$

$$i_{r\cdot 2} = \frac{e_{sec\cdot 1}}{r_2} = \frac{150}{1{,}000} = 0.150 \text{ amp}$$

$$i_{r\cdot 3} = \frac{e_{sec\cdot 1}}{r_3} = \frac{150}{1{,}500} = 0.100 \text{ amp}$$

$$I_{sec\cdot 1} = i_{GR\cdot 2} + i_{r\cdot 2} + i_{r\cdot 3} = 0.250 + 0.150 + 0.100 = 0.500 \text{ amp}$$

$$e_{r\cdot 4} = i_{GR\cdot 2} \times r_4 = 0.250 \times 200 = 50 \text{ volts}$$

$$e_{r\cdot 5} = i_{GR\cdot 2} \times r_5 = 0.250 \times 400 = 100 \text{ volts}$$

$$e_{sec\cdot 1} = e_{r\cdot 4} + e_{r\cdot 5} = 50 + 100 = 150 \text{ volts}$$

$$i_{r\cdot 6} = \frac{e_{sec\cdot 2}}{r_6} = \frac{150}{600} = 0.250 \text{ amp}$$

$$i_{GR\cdot 3} = \frac{e_{sec\cdot 2}}{R_{GR\cdot 3}} = \frac{150}{600} = 0.250 \text{ amp}$$

$$I_{sec\cdot 2} = i_{r\cdot 6} + i_{GR\cdot 3} = 0.250 + 0.250 = 0.500 \text{ amp}$$

$$e_{r\cdot 7} = i_{GR\cdot 3} \times r_7 = 0.250 \times 400 = 100 \text{ volts}$$

$$e_{r\cdot 8} = i_{GR\cdot 3} \times r_8 = 0.250 \times 200 = 50 \text{ volts}$$

$$e_{sec\cdot 2} = e_{r\cdot 7} + e_{r\cdot 8} = 100 + 50 = 150 \text{ volts}$$

$$i_{GR\cdot 4} = \frac{e_{sec\cdot 3}}{R_{GR\cdot 4}} = \frac{120}{240} = 0.50 \text{ amp}$$

$$i_{GR\cdot 5} = \frac{e_{sec\cdot 3}}{R_{GR\cdot 5}} = \frac{120}{60} = 2.0 \text{ amp}$$

$$I_{sec\cdot 3} = i_{GR\cdot 4} + i_{GR\cdot 5} = 0.50 + 2.0 = 2.50 \text{ amp}$$

$$e_{r\cdot 10} = i_{GR\cdot 4} \times r_{10} = 0.50 \times 200 = 100 \text{ volts}$$

$$e_{r\cdot 11} = i_{GR\cdot 4} \times r_{11} = 0.50 \times 40 = 20 \text{ volts}$$

$$e_{sec\cdot 3} = e_{10} + e_{11} = 100 + 20 = 120 \text{ volts}$$

$$i_{r\cdot 12} = \frac{e_{sec\cdot 3}}{r_{12}} = \frac{120}{100} = 1.20 \text{ amp}$$

$$i_{r\cdot 13} = \frac{e_{sec\cdot 3}}{r_{13}} = \frac{120}{200} = 0.60 \text{ amp}$$

$$i_{r\cdot 14} = \frac{e_{sec\cdot 3}}{r_{14}} = \frac{120}{600} = 0.20 \text{ amp}$$

$$i_{GR\cdot 5} = i_{r\cdot 12} + i_{r\cdot 13} + i_{r\cdot 14} = 1.20 + 0.60 + 0.20 = 2.00 \text{ amp}$$

When each circuit is solved separately and combined whenever possible, the current through each resistor and the voltage drop across it may be obtained. Any reducible circuit, no matter how complex, can be solved in a similar manner.

4-13 Kirchhoff's Laws

Irreducible Circuits. The circuits discussed up to now have all been reducible. That is, they all could be reduced to a simple circuit of one equivalent resistance with a single power source. However, not all electric circuits are reducible; those that are not are called *irreducible circuits.*

There are two general types of irreducible circuits: (1) those containing a single power source and (2) those containing more than one separate power source. An example of the first type is illustrated by the Wheatstone-bridge circuit of Fig. 4-22*a* and of the second type by the resistance network of Fig. 4-22*b*. An analysis of the bridge circuit (Fig. 4-22*a*) will show that it is possible to reduce this circuit to a single resistance only when the bridge is balanced and the current in *G* is zero. In circuits similar to Fig. 4-22*b*, where there is more than one power source, it is possible to have two or more currents approach a single junction. The currents may either aid or oppose each other depending on the location of the power sources in the circuit. Because of this current relationship, it is not possible to replace two or more power sources with a single equivalent power supply, thus making this type of circuit irreducible.

Use of Kirchhoff's Laws. Kirchhoff's laws may be used in solving for unknown circuit values (voltage, current, resistance, etc.) of both reducible and irreducible circuits. These laws do not introduce any new theory but rather present another method of using the principles already known.

Kirchhoff's Voltage Law. The algebraic sum of all the voltages in any closed path of a circuit must be zero. This is another way of stating the

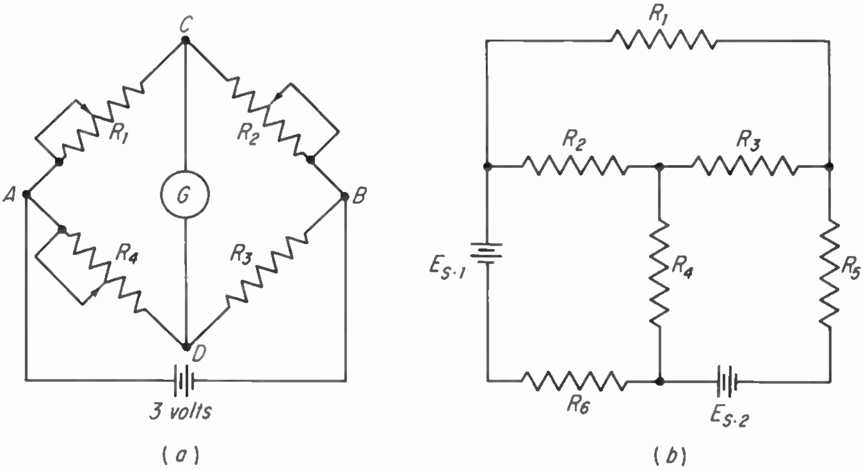


Fig. 4-22 Complex circuits. (a) Wheatstone bridge. (b) Resistance network.

voltage relationship of the series circuit originally expressed by Eq. (4-9), which therefore may also be expressed as

$$E_S - e_1 - e_2 - e_3 = 0 \tag{4-9a}$$

Kirchhoff's Current Law. The algebraic sum of all currents at any point in a circuit must be zero. This is another way of stating the current relationship of a parallel circuit originally expressed by Eq. (4-13), which therefore may also be expressed as

$$I_T - i_1 - i_2 - i_3 = 0 \tag{4-13a}$$

Method of Solving Problems. The use of Kirchhoff's laws to solve for unknown values of a circuit involves the application of simultaneous equations. Two or more separate equations are called *simultaneous equations* when they represent the relations among their unknown quantities that exist at the same instant. Simultaneous equations are solved by the elimination of one or more of the unknown quantities by using addition, subtraction, substitution, or comparison. The solution of circuit problems involving the application of simultaneous equations can best be obtained by using the following procedure:

1. Label all circuit elements as to name and value.
2. Label the current direction in each branch of the circuit by drawing an arrow alongside the branch indicating the direction of electron flow.
3. Mark all connecting points of elements with a reference letter.
4. List all current equations at each junction of three or more circuit elements. When current junction equations are being set up, the currents entering the junction are considered algebraically positive and the currents leaving the junction are considered algebraically negative.
5. List all voltage equations for each closed path in the circuit. Indicate unknown voltages in terms of current and resistance. Be sure to indicate the polarity of the voltages. When voltage loop equations are being set up, the following rules should be followed:

(*a*) The voltage of a power source is positive when the direction of the path being traced is from the negative to the positive terminal and negative when the path is from the positive to the negative terminal.

(*b*) The polarity of a voltage at a resistor will depend on the direction of electron flow through it. When the indicated direction of electron flow is opposite to the direction in which the voltage loop is being traced, the voltage at a resistor is negative. When the indicated direction of electron flow and the direction in which the voltage loop is being traced are the same, the voltage is positive.

6. From the current equations of step 4, note the number of unknown currents. Solve this number of independent current and voltage equations simultaneously.
7. Any unknown voltages may then be determined by using Ohm's law.

8. Check answers obtained by substituting their values in unused current or voltage equations so that all unknown current values are used at least once.

When proficiency in the use of Kirchhoff's laws has been achieved, steps 4 and 5 may be reduced to the listing of only those equations that are necessary for a solution.

The application of Kirchhoff's laws using this procedure to solve for unknown values of circuits may best be explained by examples.

Application to Reducible Circuits. The combination circuit of Fig. 4-20 used in Example 4-12 was reduced to that shown in Fig. 4-21. For simplicity, this equivalent circuit will be solved rather than the initial circuit.

EXAMPLE 4-13 Find the current distribution and the voltage drops across each resistor in the circuit shown in Fig. 4-23*a*.

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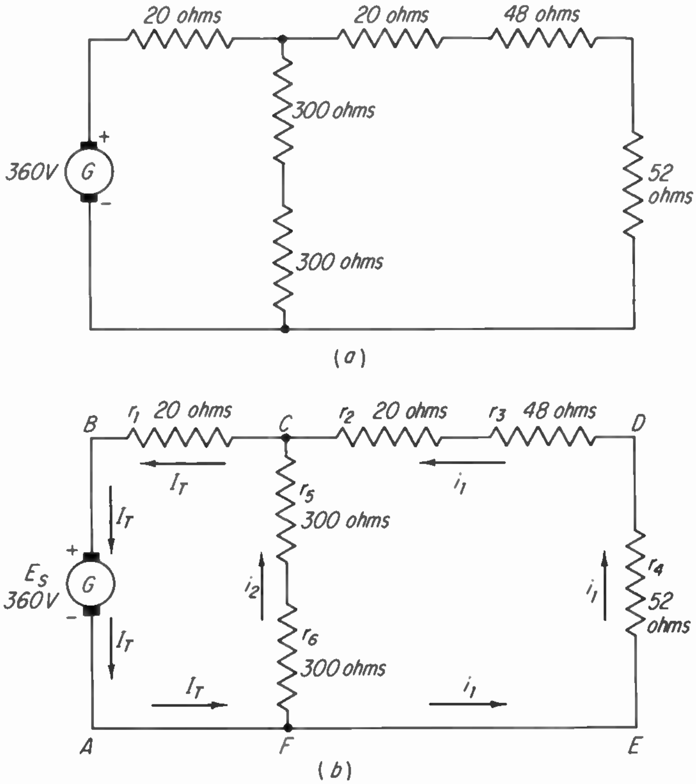

Fig. 4-23

SOLUTION: The circuit elements, currents, and junctions are labeled as shown in Fig. 4-23*b*.

At junction C

$$i_1 + i_2 - I_T = 0 \tag{I}$$

At junction F

$$I_T - i_1 - i_2 = 0 \tag{II}$$

In this case, except for the algebraic signs, the two equations are the same. The reason for this condition is that the number of *independent equations* that can be used is always one less than the total number of junctions in a circuit.

For loop $ABCDEFA$

$$E_S - I_T r_1 - i_1 r_2 - i_1 r_3 - i_1 r_4 = 0 \tag{III}$$

If the path of this loop is taken in the opposite direction as $AFEDCBA$, then

$$i_1 r_4 + i_1 r_3 + i_1 r_2 + i_T r_1 - E_S = 0$$

Substituting known values in Eq. (III),

$$360 - 20I_T - 20i_1 - 48i_1 - 52i_1 = 0$$

Combining like terms,

$$360 - 20I_T - 120i_1 = 0 \tag{IV}$$

In a similar manner, for loop $ABCFA$

$$E_S - I_T r_1 - i_2 r_5 - i_2 r_6 = 0$$

$$360 - 20I_T - 300i_2 - 300i_2 = 0$$

$$360 - 20I_T - 600i_2 = 0 \tag{V}$$

For loop $FCDEF$

$$i_2 r_6 + i_2 r_5 - i_1 r_2 - i_1 r_3 - i_1 r_4 = 0$$

$$300i_2 + 300i_2 - 20i_1 - 48i_1 - 52i_1 = 0$$

$$600i_2 - 120i_1 = 0 \tag{VI}$$

In this problem there are three basic unknowns, namely, the currents I_T, i_1, and i_2. Therefore, only three equations, which contain each unknown at least once, are necessary for the solution of the problem.

From one of the current equations, write one of the unknowns in terms of the other unknown(s).

$$i_1 + i_2 - I_T = 0 \tag{I}$$

or

$$I_T = i_1 + i_2 \tag{IA}$$

Substituting the quantity $i_1 + i_2$ for I_T in Eq. (IV),

$$360 - 20(i_1 + i_2) - 120i_1 = 0$$

or

$$360 - 20i_1 - 20i_2 - 120i_1 = 0$$

Combining like terms,

$$360 - 140i_1 - 20i_2 = 0 \tag{VII}$$

Using Eq. (VII), solve for one of the unknowns in terms of the other unknown. Solving for i_1

$$i_1 = \frac{360 - 20i_2}{140} = 2.57 - 0.143i_2 \tag{VIII}$$

Substituting the quantity $2.57 - 0.143i_2$ for i_1 in Eq. (VI) and solving for i_2,

$$600i_2 - 120(2.57 - 0.143i_2) = 0$$
$$600i_2 - 120(2.57) + 120(0.143i_2) = 0$$
$$600i_2 - 308.4 + 17.16i_2 = 0$$
$$617.16i_2 = 308.4$$
$$i_2 = \frac{308.4}{617.16} = 0.4997 \text{ amp}$$

Substituting 0.4997 for i_2 in Eq. (VIII) and solving for i_1,

$$i_1 = 2.57 - 0.143(0.4997) = 2.57 - 0.0714 = 2.4986 \text{ amp}$$

It should be noted that solving for i_1 in terms of i_2 and substituting this value in Eq. (VI) produce a cumbersome decimal process. The use of involved decimals may be avoided in some instances by first solving for a different unknown value, as is shown in the following steps. Solving for i_2 using Eq. (VII),

$$20i_2 = 360 - 140i_1$$
$$i_2 = 18 - 7i_1 \tag{IX}$$

Substituting the quantity $18 - 7i_1$ for i_2 in Eq. (VI) after first solving for i_1,

$$600i_2 - 120i_1 = 0$$
$$i_1 = \frac{600i_2}{120} = 5i_2$$
$$i_1 = 5(18 - 7i_1) = 90 - 35i_1$$
$$36i_1 = 90$$
$$i_1 = \frac{90}{36} = 2.5 \text{ amp}$$

Substituting 2.5 for i_1 in Eq. (IX),

$$i_2 = 18 - 7i_1 = 18 - 7(2.5) = 18 - 17.5 = 0.5 \text{ amp}$$

Substituting the known values for i_1 and i_2, 2.5 and 0.5, respectively, in Eq. (IA) and solving for I_T,

$$I_T = 2.5 + 0.5 = 3 \text{ amp}$$

The voltage drops may now be solved for by using Ohm's law.

$$e_{r\text{-}1} = I_T r_1 = 3 \times 20 = 60 \text{ volts}$$
$$e_{r\text{-}2} = i_1 r_2 = 2.5 \times 20 = 50 \text{ volts}$$
$$e_{r\text{-}3} = i_1 r_3 = 2.5 \times 48 = 120 \text{ volts}$$
$$e_{r\text{-}4} = i_1 r_4 = 2.5 \times 52 = 130 \text{ volts}$$
$$e_{r\text{-}5} = i_2 r_5 = 0.5 \times 300 = 150 \text{ volts}$$
$$e_{r\text{-}6} = i_2 r_6 = 0.5 \times 300 = 150 \text{ volts}$$

Check: The answers are correct, as the same values for i_1 and i_2 were obtained by using two different sets of equations.

Application to Irreducible Circuits. The circuit of Example 4-13 was solved by using Kirchhoff's laws. This same circuit had also been solved in

Example 4-12 by simple resistor combination and reduction. The irreducible circuit of Fig. 4-24*a* cannot be solved using this combination and reduction method.

EXAMPLE 4-14 Find the current distribution and the voltage drops across each resistor in the circuit shown in Fig. 4-24*a*.

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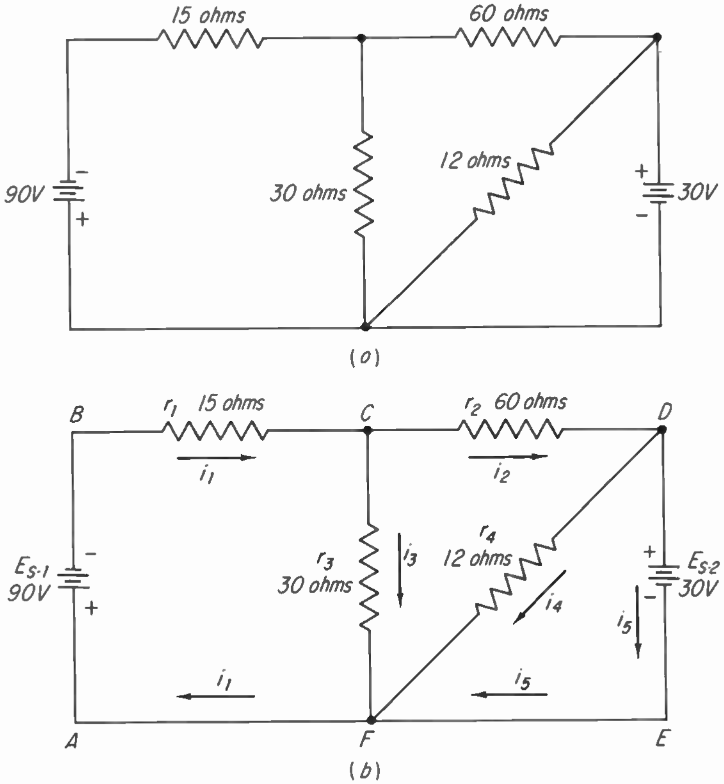


Fig. 4-24

SOLUTION: The circuit elements, currents, junctions, and power sources are labeled as shown in Fig. 4-24*b*. In this circuit, the directions of the currents are not known owing to the presence of two power sources. Current directions must therefore be assumed. If an incorrect direction is chosen, it will show up in the solution as a negative current. However, the magnitude of the current will be correct regardless of the direction chosen.

The assuming of current direction need not be a hit-or-miss proposition. Careful inspection of the magnitudes of both the power sources and resistances will generally indicate most of the correct current directions. In many circuits the current direction for only one circuit element need be assumed.

Listing all current and voltage equations, substituting known values, combining where necessary, and using the labeled circuit diagram of Fig. 4-24*b* at junction *C*

$$i_1 - i_2 - i_3 = 0 \tag{I}$$

At junction *D*

$$i_2 - i_4 - i_5 = 0 \tag{II}$$

At junction *F*

$$i_3 + i_4 + i_5 - i_1 = 0 \tag{III}$$

loop *ABCDEFA*

$$-E_{S\cdot 1} + i_1r_1 + i_2r_2 - E_{S\cdot 2} = 0$$
$$-90 + 15i_1 + 60i_2 - 30 = 0$$
$$-120 + 15i_1 + 60i_2 = 0 \tag{IV}$$

loop *ABCFA*

$$-E_{S\cdot 1} + i_1r_1 + i_3r_3 = 0$$
$$-90 + 15i_1 + 30i_3 = 0 \tag{V}$$

loop *EDCFE*

$$E_{S\cdot 2} - i_2r_2 + i_3r_3 = 0$$
$$30 - 60i_2 + 30i_3 = 0 \tag{VI}$$

loop *CDFC*

$$i_2r_2 + i_4r_4 - i_3r_3 = 0$$
$$60i_2 + 12i_4 - 30i_3 = 0 \tag{VII}$$

loop *EDFE*

$$E_{S\cdot 2} + i_4r_4 = 0$$
$$30 + 12i_4 = 0 \tag{VIII}$$

loop *ABCDFA*

$$-E_{S\cdot 1} + i_1r_1 + i_2r_2 + i_4r_4 = 0$$
$$-90 + 15i_1 + 60i_2 + 12i_4 = 0 \tag{IX}$$

Rearranging Eq. (I)

$$i_1 = i_2 + i_3 \tag{IA}$$

Rearranging Eq. (II)

$$i_2 = i_4 + i_5 \tag{IIA}$$

Rearranging Eq. (V)

$$90 = 15i_1 + 30i_3 \tag{VA}$$

Substituting Eq. (IA) in Eq. (VA)

$$90 = 15(i_2 + i_3) + 30i_3$$

or

$$90 = 15i_2 + 15i_3 + 30i_3$$

Combining like terms

$$90 = 15i_2 + 45i_3$$

Solving for i_2

$$i_2 = \frac{90 - 45i_3}{15} = 6 - 3i_3 \tag{X}$$

Solving for i_3 in terms of i_2 using another equation, namely Eq. (VI),

$$i_3 = \frac{60i_2 - 30}{30} = 2i_2 - 1 \tag{XI}$$

Substituting $2i_2 - 1$ for i_3 in Eq. (X)

$$i_2 = 6 - 3(2i_2 - 1) = 6 - 6i_2 + 3$$

$$7i_2 = 9$$

$$i_2 = \frac{9}{7} = 1.286 \text{ amp}$$

Substituting 1.286 for i_2 in Eq. (XI)

$$i_3 = 2(1.286) - 1 = 2.572 - 1 = 1.572 \text{ amp}$$

Substituting the known values for i_2 and i_3, 1.286 and 1.572, respectively, in Eq. (IA) and solving for i_1

$$i_1 = 1.286 + 1.572 = 2.858 \text{ amp}$$

Rearranging Eq. (VIII)

$$i_4 = \frac{-30}{12} = -2.5 \text{ amp}$$

The negative current indicates that the direction chosen for i_4 was incorrect. However, i_4 is kept as a negative quantity when used in any remaining calculations.

Substituting the known values for i_2 and i_4, 1.286 and -2.5, respectively, in Eq. (IIA) and solving for i_5

$$1.286 = -2.5 + i_5$$

$$i_5 = 1.286 + 2.5 = 3.786 \text{ amp}$$

The voltage drops may now be determined by using Ohm's law.

$$e_{r\text{-}1} = i_1r_1 = 2.858 \times 15 = 42.87 \text{ volts}$$

$$e_{r\text{-}2} = i_2r_2 = 1.286 \times 60 = 77.16 \text{ volts}$$

$$e_{r\text{-}3} = i_3r_3 = 1.572 \times 30 = 47.16 \text{ volts}$$

$$e_{r\text{-}4} = i_4r_4 = 2.500 \times 12 = 30.00 \text{ volts}$$

Check: Current junction Eqs. (I) and (II) were used; therefore substituting known values in Eq. (III)

$$i_3 + i_4 + i_5 - i_1 = 1.572 - 2.5 + 3.786 - 2.858 = 0$$

Voltage loop Eqs. (V), (VI), and (VII) were used; therefore substituting known values in Eq. (IX)

$$-90 + 15i_1 + 60i_2 + 12i_4 = 0$$

$$-90 + 15(2.858) + 60(1.286) + 12(-2.5) = 0$$

$$-90 + 42.87 + 77.16 - 30 \cong 0$$

4-14 Rheostats and Potentiometers

Rheostats. A rheostat is a variable resistor constructed so that the value of its resistance may be varied by means of a sliding contact arm. Rheostats are generally used to control the amount of current flowing in the load to which they are connected. Two circuits illustrating the use of rheostats are shown in Fig. 4-25.

Figure 4-25*a* shows a rheostat connected in series with the load. When the sliding contact arm *B* is moved toward *A*, the resistance of section *AB* is decreased, thus decreasing the resistance of the circuit, which in turn increases the load current and the voltage across the load. When the sliding contact arm is moved toward *C*, the resistance of section *AB* is increased, which decreases both the load current and the voltage available at the load.

Figure 4-25*b* shows a circuit with a rheostat connected in parallel with the load and the power obtained between the terminals of an antenna and ground. When current from the antenna reaches *C*, it has two paths through which it may flow. The proportion of the current that will flow in either path depends upon the relative amounts of resistance offered by each path. When the sliding contact arm is moved toward *A*, the resistance in the path *BC* is increased and it will take a smaller portion of the antenna current, thereby increasing the load current. When the contact arm is moved toward *C*, the resistance in section *BC* is decreased and it will take a greater portion of the antenna current, thereby decreasing the load current.

It should be observed that in the series circuit the load current is increased by decreasing the amount of rheostat resistance being used while in the parallel circuit the load current is increased by increasing the amount of

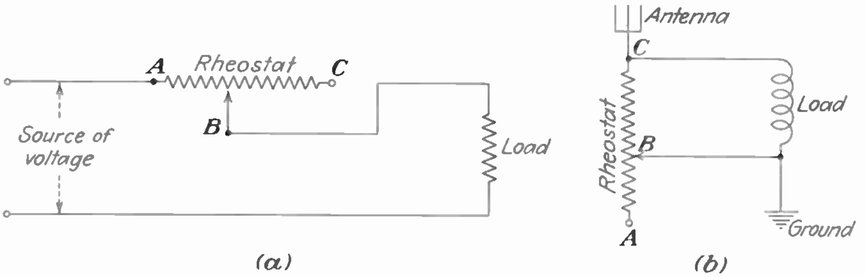


Fig. 4-25 Circuits illustrating uses of rheostats. (a) Rheostat in series with a load. (b) Rheostat in parallel with a load.

rheostat resistance being used. It should also be noticed that only two of the three terminals (marked *ABC* on the diagram) are used and that current passes through only that part of the resistance actually between the sliding contact arm and that end of the resistor being used as a terminal. Use of terminals *A* and *B* in the series circuit of Fig. 4-25*a* and terminals *B* and *C* in the parallel circuit of Fig. 4-25*b* results in obtaining an increase in the load current by rotating the sliding contact arm of the rheostat in a clockwise direction (see Fig. 4-29). In a similar manner, the use of terminals *B* and *C* in the series circuit and *B* and *A* in the parallel circuit would cause the load current to decrease when the sliding contact arm is moved in a clockwise direction.

An open circuit could occur if the contact arm of a rheostat can be moved past the conductor material and onto the insulator form on which it is wound. To prevent this condition, the three terminals of the rheostat are used by connecting the terminal of the sliding contact arm to either of the other two terminals as shown in Fig. 4-26. The section of the rheostat that is not being used is thereby short-circuited, and a permanent connection is maintained between the sliding contact arm and the resistor.

Potentiometers. A potentiometer is a variable resistor connected so that it may be used for subdividing a voltage. Figure 4-27 shows how a potentiometer is connected to the line and the load.

The points *A* and *C* of the potentiometer are connected to the line, and the load terminals are connected to the sliding contact arm and to terminal *C*. By varying the position of the sliding contact arm, it is possible to obtain any voltage from zero to full line voltage at the load. The voltage across the load will be equal to the voltage across section *BC* because they are connected in parallel with each other. The amount of voltage across *BC*, and hence across the load, will depend upon the resistance between *BC* and the current flowing through *BC*. It may be expressed as

$$E_{\text{load}} = E_{BC} = I_{BC} \times R_{BC} \tag{4-20}$$

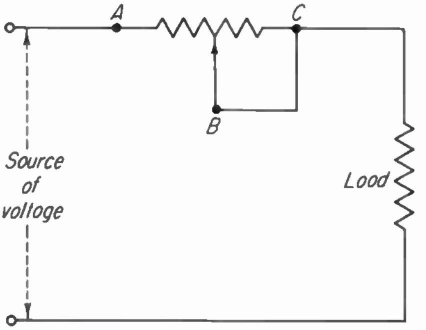


Fig. 4-26 Three-terminal rheostat connection.

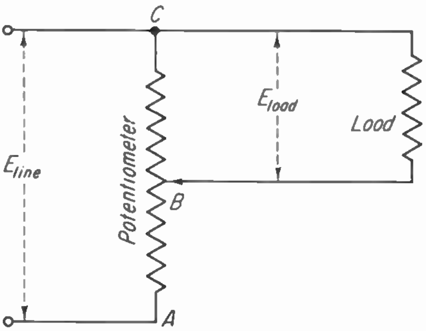


Fig. 4-27 Circuit illustrating the use of a potentiometer.

When the sliding contact arm is moved toward *A*, the amount of resistance between *B* and *C* is increased and the voltage supplied to the load is increased. When the arm is moved toward *C*, the resistance of section *BC* is decreased and the voltage supplied to the load is decreased.

The current flowing through section *AB* will be equal to the sum of the currents flowing through section *BC* and the load. This may be expressed mathematically as

$$I_{AB} = I_{BC} + I_{\text{load}} \tag{4-21}$$

A potentiometer should be large enough to carry the current drawn by the load plus the current drawn by the potentiometer itself. As the power consumed by a potentiometer is all lost, its current should be kept at a minimum. This can be accomplished by increasing the resistance between *A* and *C* to a very high value in order to reduce the current in *BC* to a minimum.

The solution for unknown values in a potentiometer circuit may result in an equation having factors in the first and second powers of the unknown. The equation is often a variation of the regular quadratic expression

$$ax^2 + bx + c = 0 \tag{4-22}$$

In such a case, the value of the unknown can be found as

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \tag{4-23}$$

The use of the quadratic equation can best be explained by an example.

EXAMPLE 4-15 A 100,000-ohm potentiometer having a uniform resistance is used to obtain 50 volts across a load from a power source rated at 100 volts. What is the current in each part of the potentiometer of Fig. 4-27 if the load resistance is 5,000 ohms?

GIVEN: $R_{CA} = 100{,}000$ ohms $\quad R_L = 5{,}000$ ohms $\quad E_L = 50$ volts $\quad E_S = 100$ volts

FIND: $i_{CB} \quad i_{BA}$

SOLUTION:

$$i_L = \frac{E_L}{R_L} = \frac{50}{5{,}000} = 0.01 \text{ amp} \tag{I}$$

$$i_{CB} = i_{BA} - i_L = i_{BA} - 0.01 \tag{II}$$

$$r_{CB} = \frac{E_{CB}}{i_{CB}} \tag{III}$$

Substituting known value for E_{CB} and the equivalent value for i_{CB} as expressed in Eq. (II)

$$r_{CB} = \frac{50}{i_{BA} - 0.01} \tag{IV}$$

$$r_{BA} = \frac{50}{i_{BA}} \tag{V}$$

$$r_{CB} + r_{BA} = 100{,}000 \text{ ohms} \tag{VI}$$

Substituting the equivalent values for r_{CB} and r_{BA} [Eqs. (IV) and (V)] in Eq. (VI)

$$\frac{50}{i_{BA} - 0.01} + \frac{50}{i_{BA}} = 100{,}000 \tag{VII}$$

Cross-multiply numerators and denominators in the left-hand side of Eq. (VII) to obtain a common denominator.

$$\frac{50i_{BA} + 50(i_{BA} - 0.01)}{i_{BA}(i_{BA} - 0.01)} = 100{,}000$$

Multiply values to remove parentheses.

$$\frac{50i_{BA} + 50i_{BA} - 0.5}{i_{BA}^2 - 0.01i_{BA}} = 100{,}000$$

Multiply both sides of the equation by $i_{BA}^2 - 0.01i_{BA}$.

$$50i_{BA} + 50i_{BA} - 0.5 = 100{,}000(i_{BA}^2 - 0.01i_{BA})$$

Multiply values to remove parentheses.

$$50i_{BA} + 50i_{BA} - 0.5 = 100{,}000i_{BA}^2 - 1{,}000i_{BA}$$

Combining and rearranging like terms,

$$100{,}000i_{BA}^2 - 1{,}100i_{BA} + 0.5 = 0 \tag{VIII}$$

Using the quadratic equation [Eq. (4-23)] and the values in Eq. (VIII),

$$i_{BA} = \frac{1{,}100 \pm \sqrt{1{,}100^2 - 4 \times 100{,}000 \times 0.5}}{2 \times 100{,}000}$$

$$= \frac{1{,}100 \pm \sqrt{1{,}210{,}000 - 200{,}000}}{200{,}000} = \frac{1{,}100 \pm \sqrt{1{,}010{,}000}}{200{,}000}$$

$$= \frac{1{,}100 \pm 1{,}005}{200{,}000}$$

$$i_{BA} = \frac{1{,}100 + 1{,}005}{200{,}000} = \frac{2{,}105}{200{,}000} = 0.010525 \text{ amp} = 10.525 \text{ ma} \tag{IX}$$

$$i_{BA} = \frac{1{,}100 - 1{,}005}{200{,}000} = \frac{95}{200{,}000} = 0.000475 \text{ amp} = 0.475 \text{ ma} \tag{X}$$

Inasmuch as i_{BA} must be greater than 0.01 amp [Eq. (II)], the answer obtained using Eq. (IX) is correct and the answer obtained using Eq. (X) is incorrect.

$$i_{CB} = i_{BA} - 0.01 = 0.010525 - 0.01 = 0.000525 \text{ amp} = 0.525 \text{ ma}$$

Check:

$$r_{CB} = \frac{E_{CB}}{i_{CB}} = \frac{50}{525 \times 10^{-6}} \cong 95{,}250 \text{ ohms}$$

$$r_{BA} = \frac{E_{BA}}{i_{BA}} = \frac{50}{10.525 \times 10^{-3}} \cong 4{,}750 \text{ ohms}$$

$$R_{CA} = r_{CB} + r_{BA} = 95{,}250 + 4{,}750 = 100{,}000 \text{ ohms}$$

Uses of Rheostats and Potentiometers. Rheostats and potentiometers are used to control various types of circuits used in electronics such as antenna, volume, tone, brilliance, plate voltage, synchronizing, balancing, etc. As the amount of current in these circuits is very small, carbon resistors can be used (see Fig. 4-28).

When higher currents are required, metallic or wire-wound resistors are used (see Fig. 4-29). An objection to metallic resistors is that noisy operation of sound equipment may result when the contact arm moves from one turn of wire to another. This occurs when there is an appreciable amount of voltage drop between adjacent turns of wire. Carbon controls do not present such conditions, as the resistance change progresses smoothly.

Taper. Rheostats and potentiometers used for control circuits may vary in direct ratio, or they may taper. In a direct ratio potentiometer, the resistance value varies directly with the degree of rotation. That is, at quarter rotation the resistance value is one-quarter of the total resistance, and similarly at half rotation it is one-half of the total resistance. When a potentiometer is tapered, the resistance does not vary directly with the rotation. The potentiometer shown in Fig. 4-28 has a total resistance of 500,000 ohms. At half rotation the resistance is only 50,000 ohms, and at quarter rotation it will be less than 25,000 ohms, as the resistance is tapered and not uniform between the OFF position and the mid-point. In a similar manner, the resistance at three-quarters rotation would not be equal to one-half of 450,000 plus 50,000 (or 275,000), because the resistance between the mid-point and the ON position is not uniform but is tapered.

It is necessary to taper the resistance of an audio-volume control in order to obtain an apparent uniform control of the signal. When the con-

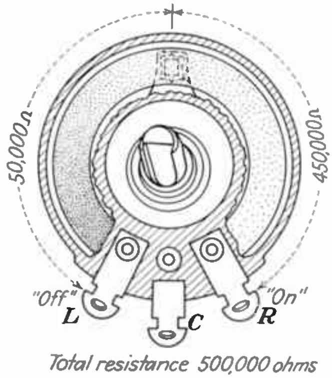


Fig. 4-28 A carbon-type potentiometer with a left-hand taper.

Fig. 4-29 A wire-wound potentiometer with a left-hand taper.

trol is turned to the halfway position, it is generally expected that the signal volume will be one-half that obtained at the full position of the control. In order to double a given volume of sound, an increase of approximately ten times the original intensity is required. At one-half full volume, only one-tenth of the full volume voltage is required, and therefore one-tenth of the total resistance is all that is needed.

Potentiometers have either left-hand or right-hand tapers, depending on which side is tapered out. In Figs. 4-28 and 4-29, the left hand of the control is tapered out; therefore each is a left-hand taper.

Comparison of Wire-wound and Carbon Controls. Wire-wound and carbon controls have a number of advantages and disadvantages. The choice of a control will depend on its use in a circuit. Some of the advantages of carbon controls are (1) ease of obtaining a taper, (2) ease of obtaining high resistance values, and (3) silent operation. Two of their disadvantages are that (1) their resistance varies with heat, humidity, and wear and (2) they have a low current-carrying ability. Some of the advantages of wire-wound controls are (1) ease of obtaining accurate values of resistance, (2) ease of obtaining low resistance values, and (3) their high current-carrying ability. Two of their disadvantages are that (1) it is more difficult to obtain a taper and (2) their noisy operation.

4-15 The Voltage Divider

If a tapped resistor is connected across a power supply, a number of loads requiring different amounts of voltage can be connected to the taps along the body of the resistor. The voltage between any two points will be equal to the product of the current flowing through that part of the resistor and the value of the resistance between the two points (see Fig. 4-30). A resistor used in this manner is called a *voltage divider.*

Voltage dividers are used to divide the voltage of the power supply into such values of potential as are required by the various parts of the circuit. Voltage dividers also act as a safety load to protect the capacitors from having too high a voltage placed across their terminals. In order to obtain a uniform voltage output, the power supply should be operated as near as possible to the rated output of its transformer. This can be controlled by the amount of current drawn by the voltage divider, which is known as the *bleeder current.*

The calculation of the resistance values and the power rating of the voltage divider may be accomplished by the use of Ohm's law. The following procedure should be followed.

1. Determine the voltage required at each tap and the current to be drawn from it.

2. Determine the amount of bleeder current desired. This is the difference between the total current required by the load and the current necessary to operate the power supply at approximately 90 per cent of its rated value.

3. Determine the current in each section of the divider.
4. Calculate the resistance of one section at a time.
5. Determine the power rating of the voltage divider.

4-16 Use of Exponents in Calculations

When calculations involve the use of very large or very small number the method of expressing these numbers and performing arithmetic operations can be simplified by the use of exponents. This is really a shorthand method of mathematics.

Calculations involving current in milliamperes or microamperes and resistance in kilohms or megohms may be performed more easily by this method.

The following examples illustrate the use of the exponent method of expressing common numbers:

1. 5 ma = 0.005 amp = 5×10^{-3} amp
2. 25 μa = 0.000025 amp = 25×10^{-6} amp
3. 3.9 mc = 3,900,000 cycles = 3.9×10^{6} cycles
4. 8,500,000 = 8.5×10^{6}
5. 0.0035 = 3.5×10^{-3}
6. 6,280,000,000,000,000,000 = 6.28×10^{18}

Note: This is the number of electrons corresponding to 1 amp (Art. 2-12).

Numbers that have similar exponent characteristics may be added or subtracted as indicated by the following illustrations:

7. $(4.5 \times 10^6) + (8.25 \times 10^6) + (0.25 \times 10^6) = 13 \times 10^6$
8. $(8.5 \times 10^3) - (3.5 \times 10^3) = 5 \times 10^3$

When numbers are multiplied, the exponents are added. The exponents do not have to be the same.

9. $650{,}000 \times 3{,}000 = (6.5 \times 10^5) \times (3 \times 10^3) = 19.5 \times 10^8$

Table 4-2 Exponent Method of Notation

NUMBER	EXPONENT METHOD	NUMBER	EXPONENT METHOD
100,000,000	10^8	1	10^0
10,000,000	10^7	0.1 = 1/10	10^{-1}
1,000,000	10^6	0.01 = 1/100	10^{-2}
100,000	10^5	0.001 = 1/1,000	10^{-3}
10,000	10^4	0.0001	10^{-4}
1,000	10^3	0.00001	10^{-5}
100	10^2	0.000001	10^{-6}
10	10^1	0.0000001	10^{-7}
1	10^0	0.00000001	10^{-8}

10. $2{,}500{,}000 \times 0.005 = (2.5 \times 10^6) \times (5 \times 10^{-3}) = 12.5 \times 10^3$
11. $0.015 \times 0.0006 = (1.5 \times 10^{-2}) \times (6 \times 10^{-4}) = 9 \times 10^{-6}$

When numbers are divided, the exponents are subtracted. The exponents do not have to be the same.

12. $750{,}000 \div 150 = (7.5 \times 10^5) \div (1.5 \times 10^2) = 5 \times 10^3$
13. $2{,}500 \div 50{,}000 = (25 \times 10^2) \div (5 \times 10^4) = 5 \times 10^{-2}$
14. $5{,}000 \div 0.025 = (5 \times 10^3) \div (2.5 \times 10^{-2}) = 2 \times 10^5$

4-17 Calculation of a Typical Voltage Divider

The application of the procedure outlined in Art. 4-15 and the use of exponents as described in Art. 4-16 are illustrated by the following example.

EXAMPLE 4-16 Determine the resistance values of a voltage divider for a small superheterodyne receiver that employs a 6BE6 oscillator-mixer tube, a 6BA6 i-f amplifier tube, a 6AV6 detector-amplifier tube, and a 6AQ5-A power-output tube. The operating voltages and currents are to be obtained from a tube manual for plate voltages of 250 volts. A power transformer rated at 100 ma is to be operated at 90 per cent of its rated value.

GIVEN: Tubes: 6BE6, 6BA6, 6AV6, 6AQ5-A $I_T = 100$ ma $I_S = 0.9\, I_T$

FIND: R of each section

SOLUTION: See Table 4-3.

Using the values of voltage and current listed in Table 4-3, the voltage and current at each tap of the voltage divider will be as shown in Fig. 4-30. The total voltage required from the power-supply unit will be the sum of the highest plate voltage and the highest grid-bias voltage.

$$E_o = 250 + 12.5 = 262.5 \text{ volts}$$
$$I_{250} = 2.9 + 11 + 1.2 + 45 + 4.5 = 64.6 \text{ ma}$$
$$I_{100} = 6.8 + 4.2 = 11 \text{ ma}$$
$$I_o = 64.6 + 11 = 75.6 \text{ ma}$$
$$I_{\text{bleeder}} = 0.9\, I_T - I_o = (0.9 \times 100) - 75.6 = 14.4 \text{ ma}$$
$$R_{\text{sec-}1} = \frac{e_1}{i_1} = \frac{250 - 100}{25.4 \times 10^{-3}} = 5{,}905 \text{ ohms}$$

Table 4-3 Values from a Tube Manual

TUBE	6BE6	6BA6	6AV6	6AQ5-A
E_b, volts	250	250	250	250
$E_{c\text{-}2}$, volts	100	100	---	250
$E_{c\text{-}1}$, volts	−1.5	− 12.5	−2	−12.5
I_b, ma	2.9	11	1.2	45
$I_{c\text{-}2}$, ma	6.8	4.2	---	4.5

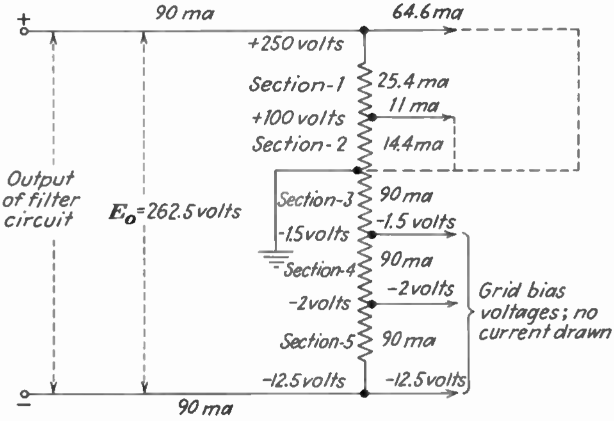


Fig. 4-30 A voltage-divider circuit used for providing plate and grid-bias voltages.

$$R_{\text{sec}\cdot 2} = \frac{e_2}{i_2} = \frac{100 - 0}{14.4 \times 10^{-3}} = 6{,}944 \text{ ohms}$$

$$R_{\text{sec}\cdot 3} = \frac{e_3}{i_3} = \frac{1.5 - 0}{90 \times 10^{-3}} = 16.7 \text{ ohms}$$

$$R_{\text{sec}\cdot 4} = \frac{e_4}{i_4} = \frac{2 - 1.5}{90 \times 10^{-3}} = 5.5 \text{ ohms}$$

$$R_{\text{sec}\cdot 5} = \frac{e_5}{i_5} = \frac{12.5 - 2}{90 \times 10^{-3}} = 117 \text{ ohms}$$

If a single resistor of uniform wire size is to be used, the wire size must be based on the maximum current in any section of the voltage divider. However, the voltage divider could also be made with separate resistors for each section, in which case the wire size for the separate resistors is determined by the currents in the individual sections.

As the voltage divider is usually mounted under the chassis of the radio receiver and therefore does not have much ventilation, it is recommended that its power rating be approximately double that of the load it is to carry.

EXAMPLE 4-17 Determine the power rating of the voltage divider used in Example 4-16 for (*a*) a single resistor having four taps, (*b*) five separate resistors.

GIVEN: Resistance and current values determined in Example 4-16

FIND: (*a*) P_R, single resistor (*b*) $P_{R\cdot\text{sec}}$, separate resistors

SOLUTION:

(*a*)

$$\begin{aligned} R_T &= R_{\text{sec}\cdot 1} + R_{\text{sec}\cdot 2} + R_{\text{sec}\cdot 3} + R_{\text{sec}\cdot 4} + R_{\text{sec}\cdot 5} \\ &= 5{,}905 + 6{,}944 + 16.7 + 5.5 + 117 = 12{,}988 \text{ ohms} \\ P_R &= 2I_{\max}{}^2 R_T = 2(90 \times 10^{-3})^2 \times 12{,}988 = 210 \text{ watts} \end{aligned}$$

(*b*)

$$
\begin{aligned}
P_{R\cdot \text{sec}\cdot 1} &= 2i_1{}^2R_1 = 2(25.4 \times 10^{-3})^2 \times 5{,}905 = 7.62 \text{ watts}\\
P_{R\cdot \text{sec}\cdot 2} &= 2i_2{}^2R_2 = 2(14.4 \times 10^{-3})^2 \times 6{,}944 = 2.87 \text{ watts}\\
P_{R\cdot \text{sec}\cdot 3} &= 2i_3{}^2R_3 = 2(90 \times 10^{-3})^2 \times 16.7 = 0.27 \text{ watt}\\
P_{R\cdot \text{sec}\cdot 4} &= 2i_4{}^2R_4 = 2(90 \times 10^{-3})^2 \times 5.5 = 0.088 \text{ watt}\\
P_{R\cdot \text{sec}\cdot 5} &= 2i_5{}^2R_5 = 2(90 \times 10^{-3})^2 \times 117 = 1.88 \text{ watts}
\end{aligned}
$$

Thus, if the voltage divider is made of uniform wire size designed to withstand the maximum current in any part of the unit, the voltage divider is capable of carrying 210 watts even though it is required to carry only 12.7 watts. A voltage divider designed to meet only the needs of the individual sections is less expensive and more practical.

QUESTIONS

1. What are the essential parts of any electric circuit?
2. Explain by use of the electron theory why the electric circuit has to be a closed circuit.
3. What are the factors that affect the resistance of an electrical conductor?
4. For practical purposes, why can the change in resistance due to variations in temperature be disregarded?
5. What is meant by a negative, positive, and zero temperature coefficient?
6. Name several materials that have (*a*) a relatively high temperature coefficient, (*b*) a relatively low temperature coefficient.
7. What is the mathematical relation among the factors, other than temperature, affecting the resistance of a conductor?
8. What is the relation among a mil, square mil, circular mil, inch, square inch?
9. (*a*) How is the diameter of a wire usually expressed? (*b*) How is the cross section of a wire usually expressed?
10. What is meant by specific resistance?
11. How is the size of a wire designated?
12. (*a*) What is meant by conductance? (*b*) What is its unit of measurement?
13. What is the difference between a conductor and an insulator?
14. What factors determine the material to be used (*a*) as a conductor? (*b*) As an insulator?
15. What is the difference between the dielectric strength of a material and its breakdown voltage?
16. Name five conductors used in radio, television, and other electronic apparatus, and explain where and why they are used.
17. Name five insulators used in radio, television, and other electronic apparatus, and explain where and why they are used.
18. Name five metals used in making alloys to be used as resistors.
19. What material is generally used to obtain high resistances?
20. (*a*) What are the advantages and disadvantages of a fixed metallic resistor? (*b*) Of a fixed carbon resistor?
21. Explain what is meant by the following terms used in conjunction with a resistor: fixed, variable, adjustable, tapped, and rheostat.
22. Explain what is meant by the following terms: (*a*) automatic-resistance control, (*b*) thermistor, (*c*) varistor.

23. What is meant by (*a*) a low-power resistor? (*b*) A high-power resistor?
24. In selecting a resistor, what factor other than resistance should be taken into consideration? Explain your answer.
25. Explain the color code system for identifying resistance values of fixed carbon resistors.
26. Explain the color code system for identifying tolerance ratings of fixed carbon resistors.
27. Describe the following terms: (*a*) fuse, (*b*) superlag fuse, (*c*) fusetron.
28. Describe the basic principle of operation of (*a*) a magnetic circuit breaker, (*b*) a thermal circuit breaker.
29. What are the four ways of connecting electrical circuit components?
30. What is meant by a series circuit?
31. What are the advantages and disadvantages of series circuits?
32. Name and explain three uses of the series circuit.
33. (*a*) Why are power supplies connected in series? (*b*) Name an application of the series connection of power supplies.
34. Why is it good practice to check the answers continually in solving circuit problems?
35. What is meant by a parallel circuit?
36. What are the advantages and disadvantages of parallel circuits?
37. Name and explain three uses of the parallel circuit.
38. (*a*) Why are power supplies connected in parallel? (*b*) Name an application of the parallel connection of power supplies.
39. What is meant by (*a*) a combination circuit? (*b*) A series-parallel circuit? (*c*) A parallel-series circuit?
40. (*a*)Where are combination circuits used? (*b*) What are their advantages?
41. What is the general procedure to be used in solving (*a*) series-parallel circuits? (*b*) Parallel-series circuits? (*c*) Complex combination circuits?
42. Describe the difference between a reducible circuit and an irreducible circuit.
43. Describe two types of irreducible circuits.
44. Explain the comparison between (*a*) Kirchhoff's voltage law and the relationship among the voltages of a series circuit and (*b*) Kirchhoff's current law and the relationship among the currents in a parallel circuit.
45. (*a*) What is meant by simultaneous equations? (*b*) Why must simultaneous equations be used in solving some types of electric circuit problems?
46. What is the general procedure to be followed in solving problems involving the application of simultaneous equations?
47. What is meant by (*a*) a rheostat? (*b*) A potentiometer?
48. Explain an advantage in using a three-terminal rheostat connection rather than a two-terminal connection.
49. Why is the quadratic equation used in solving for unknown values in some types of potentiometer circuit problems?
50. Name and explain some of the uses of rheostats and potentiometers.
51. What is meant by (*a*) taper? (*b*) Left-hand taper? (*c*) Right-hand taper?
52. Why is it necessary to use resistances that are tapered?
53. What are the advantages and disadvantages of carbon controls?
54. What are the advantages and disadvantages of wire-wound controls?
55. What is the main purpose of the voltage divider?

56. What other purposes does the voltage divider perform?
57. Where are voltage dividers generally used?
58. (*a*) What is meant by the bleeder current? (*b*) Explain two functions of the bleeder current.
59. What general procedure should be followed in solving for resistance values and power ratings of the various sections of a voltage divider?
60. (*a*) What is meant by the exponent method of notation? (*b*) What is the advantage in using this method of notation in solving electric circuit problems?
61. How is the power rating of a voltage divider determined?
62. Why are voltage dividers generally wire-wound?

PROBLEMS

1. What is the operating resistance of a ¼-megohm carbon resistor if its temperature increases from 20°C when the equipment is not being used to 45°C when it is being operated? (T_c for carbon $= -0.0003$.)
2. A 2,500-ohm resistor made of nichrome wire is used in a certain electronic unit. What is its operating resistance if the temperature within the unit increases from 20°C when it is not being used to 60°C when it is in operation? (T_c for nichrome $= 0.0004$.)
3. What is the voltage drop across the resistor in Prob. 1 when 20 μa is flowing through it (*a*) at 20°C? (*b*) At 45°C?
4. What is the voltage drop across the resistor in Prob. 2 when 40 ma is flowing through it (*a*) at 20°C? (*b*) At 60°C?
5. A resistance of 17.27 ohms is made of a rectangular hard steel wire having a cross section of 0.1 by 0.05 inch. What length of wire is needed to make this resistor?
6. A 50-ft length of square aluminum wire has a resistance of 0.267 ohm. What is the length of each side of its cross section?
7. How many turns of No. 28 copper wire are necessary to wind a coil having an average diameter of 2 inches and a resistance of 88 ohms?
8. What is the average diameter of a coil wound with No. 24 copper wire having 150 turns and a resistance of 2.56 ohms?
9. A 300-, a 500-, and a 400-ohm resistor are connected to form a series circuit across a 240-volt line. Find (*a*) the total resistance of the circuit, (*b*) the current flowing through the circuit, (*c*) the voltage drop across each resistor, (*d*) the power taken by each resistor, (*e*) the power taken by the circuit.
10. A 1,500-, a 2,500-, a 1,000-, and a 5,000-ohm resistor are connected in series across a 250-volt power supply. Find (*a*) the total resistance of the circuit, (*b*) the current flowing through the circuit, (*c*) the voltage drop across each resistor, (*d*) the power taken by each resistor, (*e*) the power taken by the circuit.
11. The current flow in a series circuit consisting of three resistances is 375 ma. The voltage drop across resistance *A* is 40 volts and across *B* 25 volts. If the voltage across the line is 100 volts, find (*a*) the voltage drop across resistance *C*; (*b*) the resistance of *A*, *B*, *C*; (*c*) the resistance of the entire circuit; (*d*) the power taken by each resistor; (*e*) the power taken by the circuit.
12. A resistor is connected between the cathode of a radio tube and the negative terminal of the B power supply to produce a negative voltage (grid bias) on the grid of the tube. When the plate current, which also passes through the bias resistor,

is 24 ma, what must be the value of the resistor in order to produce a grid bias of 12 volts?

13. A four-tube radio set has its heaters connected in series. The rated voltages are 25, 6.3, 6.3, and 12.6 volts, and they all draw 0.3 amp. (*a*) What value of resistance must be connected in series with these heaters in order to operate them directly from a 110-volt line? (*b*) How much power is consumed by the dropping resistor?
14. A 30,000-ohm resistor is connected in series with a 250-volt B power supply and the plate circuit of a radio tube. If the plate current is 2.5 ma, what is the voltage across (*a*) the resistor? (*b*) The plate circuit of the tube?
15. What value of resistance must be connected in series with the heater of an electron tube rated at 2.0 volts and 0.12 amp if it is to be operated from two 1.5-volt cells connected in series with each other?
16. Two electron tubes have their filaments connected in series with a 6.3-volt battery and a filament resistor. If the filaments are rated at 2.5 volts and 2.5 amp, what is the value of the filament resistor?
17. The heaters of a five-tube radio receiver are connected in series and are rated at 12.6, 12.6, 12.6, 35, and 35 volts. A 2.2-volt panel lamp is connected in series with the heaters and the 110-volt line. What is the resistance of each of the heaters and the filament of the panel lamp in order that they draw their rated current of 0.15 amp?
18. A voltmeter reading its maximum rated voltage of 150 volts draws 10 ma. What value of resistance is connected in series with the moving coil of the voltmeter if the resistance of the coil is 20 ohms (see Fig. 4-31)?
19. The voltmeter in the previous problem is to have a 30- and 75-volt terminal in addition to the 150-volt terminal (see Fig. 4-31). If the current drawn is to be 10 ma, at what value of resistance must the resistor *R* be tapped?
20. If 10 ma is drawn by the voltmeter in Prob. 18 and an external resistance connected in series with it, what is the value of this resistance when the voltmeter reads (*a*) 300 volts? (*b*) 450 volts?
21. A 300-, a 500-, and a 400-ohm resistor are connected in parallel across a 240-volt power line. Find (*a*) the total resistance of the circuit, (*b*) the current flowing in each resistor, (*c*) the line current, (*d*) the power taken by each resitor, (*e*) the power taken by the circuit.

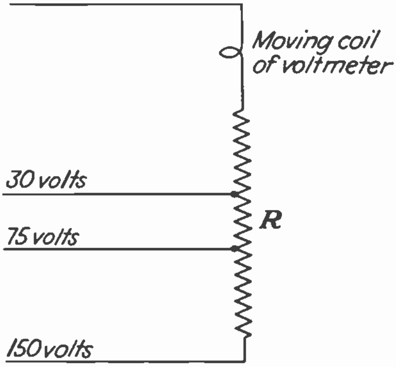


Fig. 4-31

22. Four resistors are connected in parallel across a 180-volt source of power. The current flowing in each circuit is 20, 250, 180, and 300 ma. Find the value of each resistor and the total resistance of the circuit.
23. An electric circuit in a home has a 100-watt lamp, a 550-watt toaster, and a 660-watt broiler connected in parallel with a 110-volt line. Find (*a*) the current drawn by each appliance and the total current flowing in the circuit, (*b*) the resistance of each appliance and the resistance of the circuit.
24. The resistance of the moving coil of an ammeter is 5 ohms. What value resistor must be connected in parallel with this coil in order that the voltage drop across the meter will be 50 mv when 1 amp is flowing through the line?
25. The resistor connected in parallel with an ammeter coil is called a ***shunt***. How much current is flowing through the coil and through the shunt of Prob. 24 when the line current is 1 amp?
26. If the ammeter in Prob. 24 is to have the same voltage drop across it, what value of shunt resistor must be connected in parallel with it in order that it read (*a*) 5 amp? (*b*) 10 amp?
27. Three resistors are connected in parallel across a 250-volt source of power. The total current flowing in the line is 760 ma. Two of the resistors have a value of 1,250 and 25,000 ohms. (*a*) What is the current flowing in each resistor? (*b*) What is the value of the third resistor?
28. A radio receiver has four tubes whose heaters each draw 0.3 amp. The heaters are connected in parallel to a 2.5-volt tap of the power transformer. (*a*) What is the resistance of each heater? (*b*) What is the combined resistance of all the heaters? (*c*) What is the total current taken from the power transformer?
29. The circuit shown in Fig. 4-32 has the following resistance values: $r_1 = 10$ ohms; $r_2 = 30$ ohms; $r_3 = 40$ ohms; $r_4 = r_8 = 15$ ohms; $r_5 = r_7 = 45$ ohms; $r_6 = 60$ ohms. Find the following quantities: (*a*) resistance of each group, (*b*) resistance of the entire circuit, (*c*) current taken by the entire circuit, (*d*) current in each resistance, (*e*) voltage drop across each resistance.
30. The circuit shown in Fig. 4-32 has the following resistance values: $r_1 = r_5 = 40$ ohms; $r_2 = 60$ ohms; $r_3 = r_6 = r_7 = r_8 = 20$ ohms; $r_4 = 80$ ohms. Find the following quantities: (*a*) resistance of each group, (*b*) resistance of the entire circuit, (*c*) current taken by the entire circuit, (*d*) current in each resistance, (*e*) voltage drop across each resistance.
31. The circuit shown in Fig. 4-33 has the following resistance values: $r_1 = 10$ ohms; $r_2 = 30$ ohms; $r_3 = 40$ ohms; $r_4 = r_8 = 15$ ohms; $r_5 = r_7 = 45$ ohms; $r_6 = 60$ ohms. Find the following quantities: (*a*) resistance of each group, (*b*) resistance of the entire circuit, (*c*) current taken by the entire circuit, (*d*) voltage drop across each group, (*e*) current in each resistance.

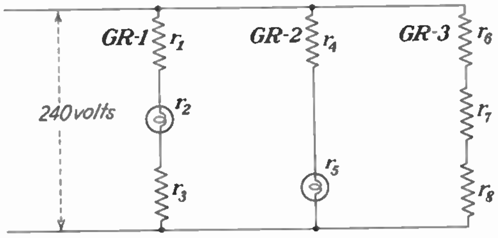


Fig. 4-32

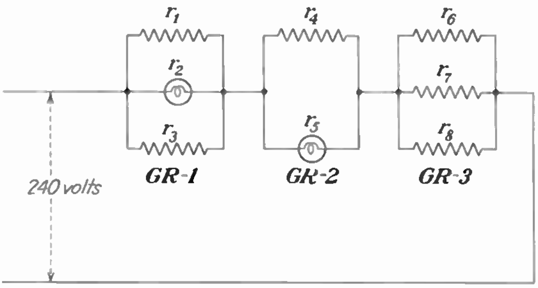


Fig. 4-33

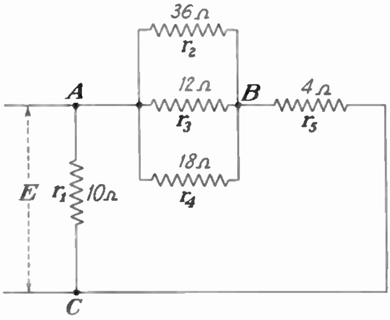


Fig. 4-34

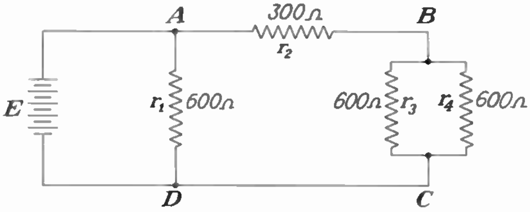


Fig. 4-35

32. The circuit shown in Fig. 4-33 has the following resistance values: $r_1 = r_6 = r_8 =$ 30 ohms; $r_2 = r_4 = 60$ ohms; $r_3 = 20$ ohms; $r_5 = 40$ ohms; $r_7 = 10$ ohms. Find the following quantities: (*a*) resistance of each group, (*b*) resistance of the entire circuit, (*c*) current taken by the entire circuit, (*d*) voltage drop across each group, (*e*) current in each resistance.
33. Find the total resistance of the circuit shown in Fig. 4-34.
34. Find the total resistance of the circuit shown in Fig. 4-35.
35. In the circuit used in Prob. 33, $E = 4$ volts. Find (*a*) line current; (*b*) current in branch *AC* through r_1; (*c*) current through branch *AC* through r_5; (*d*) voltage across *AC*, *AB*, and *BC*; (*e*) current in r_2, r_3, r_4.
36. In the circuit used in Prob. 34, $E = 9$ volts. Find (*a*) line current; (*b*) current in branch *AD*; (*c*) current in branch *AC*; (*d*) voltage across *AD*, *AB*, and *BC*; (*e*) current in r_3 and r_4.
37. In the circuit shown in Fig. 4-36 find (*a*) resistance of the entire circuit; (*b*) I_1, I_2, and I_3; (*c*) voltage across *AB*, *BC*, and *BD*.

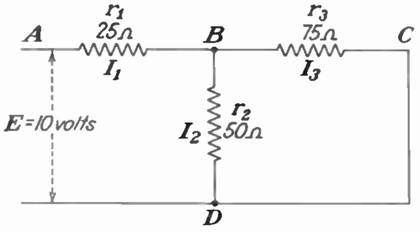


Fig. 4-36

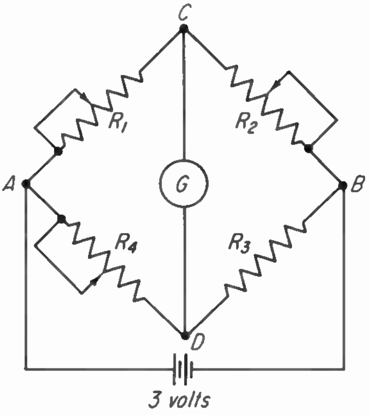


Fig. 4-37

38. A Wheatstone bridge, shown in Fig. 4-37, is used to find the resistance of R_3. The two ratio arms R_1 and R_2 have a resistance of 200 and 800 ohms. The variable resistance R_4 reads 292 ohms when no current flows between C and D. Find (*a*) I_1, I_2, I_3, I_4; (*b*) R_3.
39. Find the resistance between *AB*, *BC*, *CD*, and *AD* of the circuit shown in Fig. 4-38.
40. Find the resistance between *AB*, *CD*, and *DE* and the total resistance of the circuit shown in Fig. 4-39.
41. The circuit in Prob. 39 is connected to a 300-volt power supply. Find the current flowing through each resistor.
42. In Prob. 40, $E = 150$ volts; what is the current flowing through each resistor?
43. In Prob. 41 what is the voltage between *AB*, *BC*, and *CD*?
44. What is the voltage between the points *AF*, *FG*, *GB*, *CD*, and *DE* of the circuit used in Prob. 42?
45. What is the power used by sections *AB*, *BC*, and *CD* and the total power for the circuit used in Prob. 41?
46. What is the power used by sections *AF*, *FG*, *GB*, *CD*, and *DE* and the total power for the circuit used in Prob. 42?

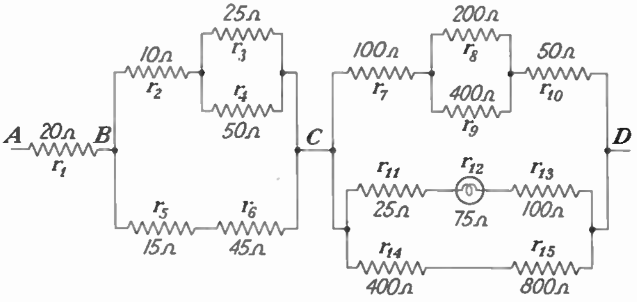


Fig. 4-38

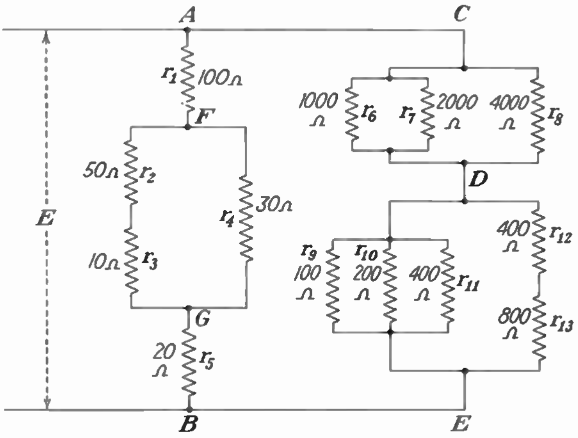


Fig. 4-39

Note: The following eight problems should be solved by using Kirchhoff's laws and simultaneous equations.

47. In the circuit shown in Fig. 4-40, $E = 3$ volts and $r_1 = r_2 = r_3 = r_4 = 12$ ohms. (*a*) What is the voltage across each resistor? (*b*) What are the voltage and polarity, with respect to ground, at junctions *A* and *B*?
48. In the circuit shown in Fig. 4-41, $E = 9$ volts and $r_1 = r_2 = r_3 = r_4 = 30$ ohms. (*a*) What is the voltage across each resistor? (*b*) What are the voltage and polarity, with respect to ground, at junctions *A* and *B*?
49. The circuit shown in Fig. 4-42 has the following values: $r_1 = r_2 = 60$ ohms; $r_3 = r_4 = 40$ ohms; $r_5 = 120$ ohms; $E = 3$ volts. What is the voltage drop across each resistor?
50. The circuit shown in Fig. 4-43 has the following values: $r_1 = 60$ ohms; $r_2 = r_4 = 40$ ohms; $r_3 = 100$ ohms; $r_5 = 120$ ohms; $E = 12$ volts. What is the voltage drop across each resistor?
51. The circuit shown in Fig. 4-44 has the following values: $r_1 = r_4 = r_8 = 2{,}000$ ohms; $r_2 = r_7 = r_9 = 1{,}000$ ohms; $r_3 = r_{10} = 500$ ohms; $r_5 = 3{,}000$ ohms; $r_6 = 4{,}000$ ohms; $E = 12$ volts. Find (*a*) the current in each resistor, (*b*) the voltage across each resistor.

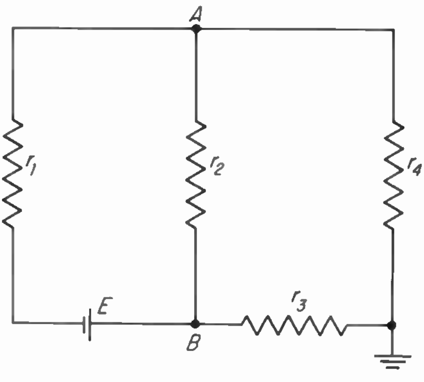


Fig. 4-40

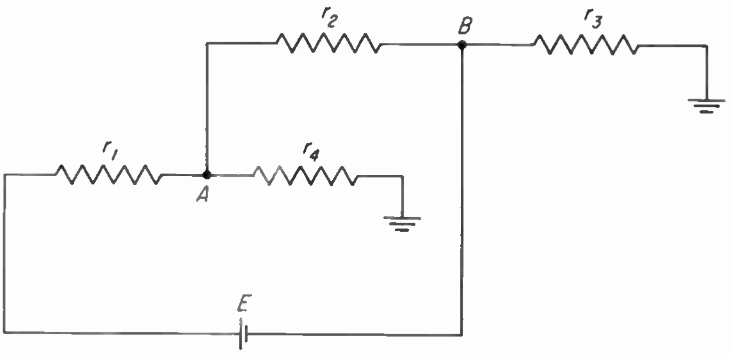


Fig. 4-41

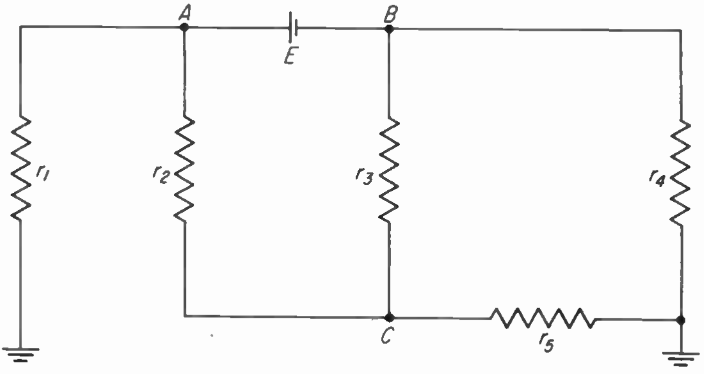


Fig. 4-42

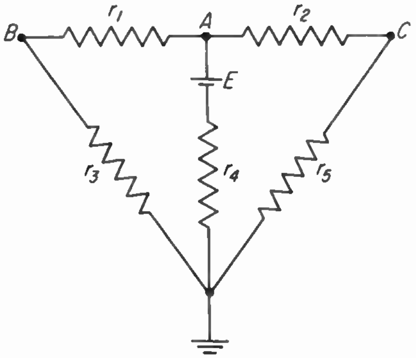


Fig. 4-43

52. The circuit shown in Fig. 4-44 has the following values: $r_1 = r_{10} = 600$ ohms; $r_2 = r_7 = r_8 = 400$ ohms; $r_3 = r_4 = 200$ ohms; $r_5 = 1{,}200$ ohms; $r_6 = 2{,}400$ ohms; $r_9 = 800$ ohms; $E = 24$ volts. Find (*a*) the current in each resistor, (*b*) the voltage across each resistor.

53. The circuit shown in Fig. 4-45 has the following values: $r_1 = r_2 = r_3 = r_4 = r_5 =$ 10 ohms; $E_1 = E_2 = E_3 = 100$ volts. Find (*a*) the current in each resistor, (*b*) the voltage across each resistor.
54. The circuit shown in Fig. 4-46 has the following values: $r_1 = r_2 = r_3 = r_4 = r_5 =$ 10 ohms; $E_1 = E_3 = 100$ volts, $E_2 = 50$ volts. Find (*a*) the current in each resistor, (*b*) the voltage across each resistor.
55. A 50,000-ohm potentiometer connected to a 45-volt battery supplies a 4,000-ohm load with 20 volts. The resistance of section *AB* (Fig. 4-27) is 4,595 ohms. Find the current in each part of the potentiometer.
56. If a 21,850-ohm potentiometer with R_{AB} set at 4,081 ohms is used in Prob. 55, what is the current in each part of the potentiometer?
57. The load in Prob. 55 is changed to 6,000 ohms and R_{AB} is set at 6,650 ohms. What is the current in each part of the potentiometer?
58. A 20,000-ohm potentiometer connected to a 45-volt battery supplies a 2,000-ohm load with 20 volts. The resistance of section *AB* (Fig. 4-27) is 2,247 ohms. Find the current in each part of the potentiometer.
59. A 500,000-ohm potentiometer having a uniform resistance is used to obtain 50 volts across a load from a power source rated at 200 volts. (*a*) What is

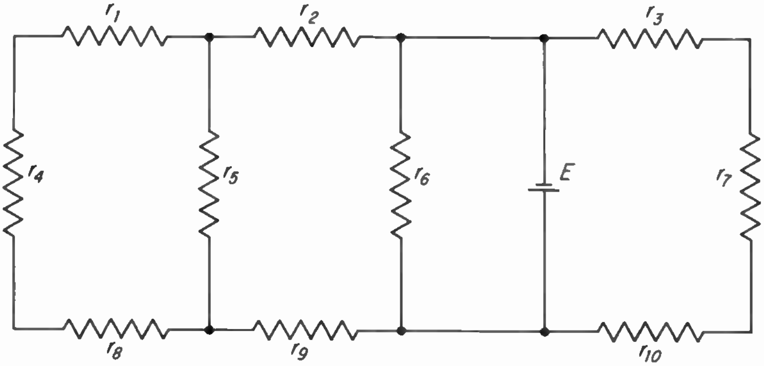


Fig. 4-44

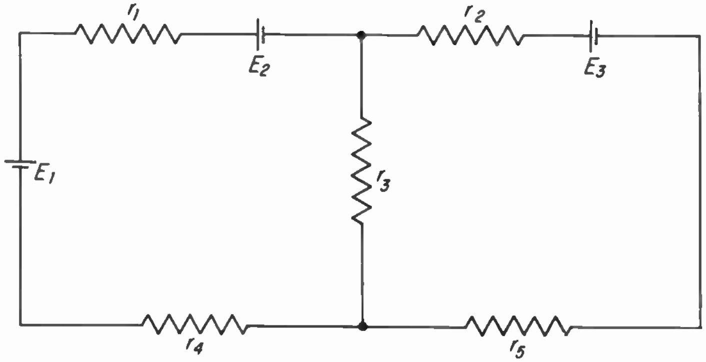


Fig. 4-45

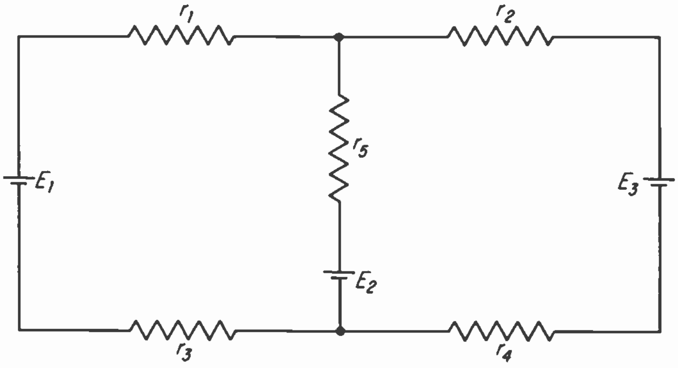


Fig. 4-46

the current in each part of the potentiometer of Fig. 4-27 if the load resistance is 50,000 ohms? (*b*) What is the resistance of each section of the potentiometer?

60. A 500,000-ohm potentiometer having a uniform resistance is used to obtain 100 volts across a load from a power source rated at 250 volts. (*a*) What is the current in each part of the potentiometer of Fig. 4-27 if the load resistance is 25,000 ohms? (*b*) What is the resistance of each section of the potentiometer?

61. A radio receiver has five tubes, each having a plate voltage of 250 volts. Their plates draw the following currents: 7, 3.5, 7, 1.1, and 34 ma. The screen grids of two tubes operate at 100 volts and 1.7 ma, one tube at 250 volts and 6.5 ma, and one tube at 50 volts and 1.6 ma. One of the tubes has no screen grid. The control grids of three tubes operate at -3 volts and one at -16.5 volts. One tube used as a diode detector has no control-grid bias. A transformer rated at 75 ma is used in the power supply. Find (*a*) the resistance of each section of the voltage divider if the transformer is to be operated at 90 per cent of its rated value, (*b*) the power rating of the voltage divider using the largest current flowing in any part of the resistor, (*c*) the power rating of the voltage divider if it is made of individual resistors each rated according to the current actually flowing in the section.

●

62. A radio receiver has six tubes, each having a plate voltage of 250 volts. Their plates draw the following currents: 7, 7, 3.5, 1.1, 32, and 32 ma. The screen grids of four tubes operate at 100 volts, and their currents are 1.7, 1.7, 2.2, and 0.35 ma. The screen grids of the remaining two tubes operate at 250 volts, and their currents are 5.5 ma each. A transformer rated at 120 ma is used in the power supply. Find (*a*) the resistance of each section of the voltage divider if the transformer is to be operated at 90 per cent of its rated value, (*b*) the power rating of the voltage divider using the largest current flowing in any part of the resistor, (*c*) the power rating of the voltage divider if it is made of individual resistors each rated according to the current actually flowing in the section.

Chapter 5

Magnetism and Electromagnetism

Magnetism has been known to man for many centuries, and the Chinese are said to have been aware of some of its effects as early as 2600 B.C. Its first practical use, the magnetic compass, is credited to the Chinese and was introduced in Europe about A.D. 1200. Dr. William Gilbert (1540–1603) made further discoveries about magnetism and is also credited with being the first to publish records of his work. After Gilbert's discoveries many scientists have made numerous contributions to the study of magnetism. The principles they discovered have made possible the many applications of magnetism as used in electrical and electronic equipment.

5-1 Relation of Magnetism to Electricity

Magnetism is so closely related and so important to electricity that the two are often called twins. The study of electrical principles in Chaps. 2 and 4 established the Ohm's law relations and presented methods for solving series and parallel circuits. The study of magnetism and the solutions of series and parallel magnetic circuits are quite similar to their electrical counterparts. Electricity is so dependent upon magnetism that without it very few of our modern devices would be possible. Without the aid of magnetism, it would be impossible to generate and transmit power in large enough quantities to meet the needs of our industrial and home use.

5-2 Magnetism, Magnets, Magnetic Materials

Magnetism is generally defined as the property or power of a material to attract and hold pieces of iron or steel. While this is true, it would be better to consider magnetism as the study of all the properties and actions of magnets and magnetic materials.

A magnet is defined as a body that has the property of polarity and the power of attracting iron and steel.

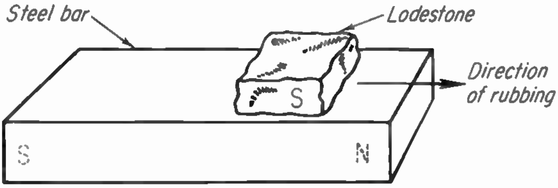
Magnetic materials are those which will be attracted to a magnet; they may or may not possess the property of polarity and may or may not have the power of attracting other magnetic materials.

These two definitions lead to the conclusion that all magnets are magnetic materials but not all magnetic materials are magnets.

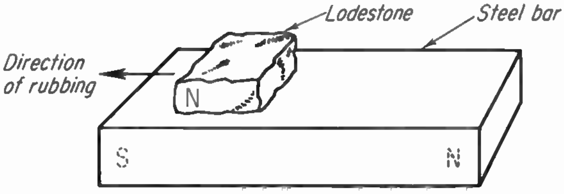
5-3 Natural and Artificial Magnets

Natural Magnets. Centuries ago it was discovered that certain stones taken from the earth had two peculiar properties. One was that they possessed the power to attract and hold to them other bits of similar stones or iron. The other was that, when an elongated piece of this stone was suspended from a cord, it would always come to rest with one end pointing north. The Chinese were the first to discover and use this stone to aid in determining directions. However, its later common use in navigation resulted in the name of *lodestone,* meaning leading stone. This substance taken from the earth is now called *magnetite.* The name *magnet* was given to the lodestone because large deposits of the stone were found near the city of Magnesia in Asia Minor. These stones are called *natural magnets* because they possess magnetic power when taken from the earth. Natural magnets no longer have any practical value, as it is now possible to manufacture powerful artificial magnets in a variety of shapes to meet definite requirements.

Artificial Magnets. The lodestone possessed the property of being able to pick up bits of steel, and though each bit of steel could be attracted to the lodestone, it was found that there was no attraction between the various bits of steel before being touched by the lodestone. If a bar of steel is rubbed or stroked with a piece of lodestone (Fig. 5-1), it will be found that the steel bar has the same properties as the lodestone and is



FIRST STROKING



SECOND STROKING

Fig. 5-1 Making a bar magnet with a lodestone.

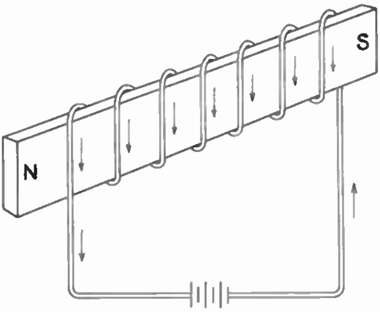


Fig. 5-2 Magnetizing a steel bar by means of an electric current.

able to attract some bits of steel to it. A material magnetized in this manner is called an *artificial magnet*. The magnet produced by rubbing with a lodestone would be weak in terms of modern magnets; such magnets are now made by inserting the steel bar in a coil of wire that has an electric current flowing through it (Fig. 5-2).

5-4 Permanent and Temporary Magnets

Permanent Magnets. If a piece of steel is hardened by heat treatment and is then made an artificial magnet by being placed in a coil of wire carrying an electric current, it will be found that the hardened steel will remain a magnet for a long time after the magnetizing force has been removed. This type of artificial magnet is called a *permanent magnet*.

Temporary Magnets. If a piece of iron, soft steel, or nickel is made an artificial magnet by means of a coil carrying an electric current, it will be found that these materials will lose practically all their magnetism almost immediately after being taken away from the magnetizing force. This type of artificial magnet is called a *temporary magnet*.

Uses of Magnets. Temporary magnets are generally used where the magnet has a coil of wire wound around it and an electric current is flowing through the coil. Examples are generators, motors, transformers, electric bells, buzzers, telegraph sounders, relays, microphones, magnetic phonograph pickups, and deflection and focusing coils used with the picture tubes of television receivers.

Permanent magnets are used in compasses, earphones, loudspeakers, electric meters, electric tachometers, miniature-sized motors, etc.

5-5 Polarity of a Magnet

North and South Poles. If an elongated lodestone or bar magnet is suspended so that it can turn freely, it will come to rest in such a position that one end will point approximately to the earth's geographical north pole. If the magnet is turned and comes to rest of its own accord, it will settle in the same position as at first. The end of the magnet that points

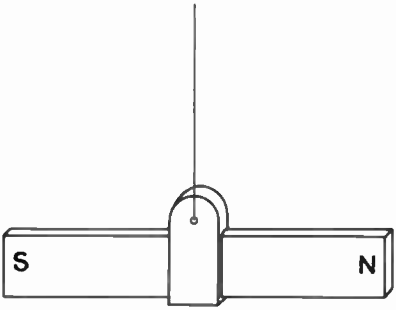


Fig. 5-3 Suspended bar magnet used as a compass.

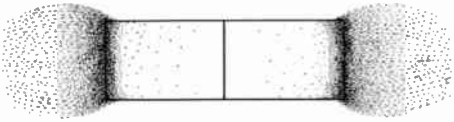


Fig. 5-4 Illustrating the poles of a magnet by the use of iron filings.

toward the earth's north geographical pole is called the *north-seeking pole,* or in short the *north pole.* The other end of the magnet, which points toward the earth's south geographical pole, is called the *south-seeking pole,* or simply the *south pole.*

If a bar magnet is placed upon a flat surface and a large quantity of iron filings is sprinkled over it, most of the filings will accumulate at two areas. If the magnet is lifted and rotated gently, it will be seen that many of the iron filings in the middle portion will drop off while those at the ends will cling to the magnet (Fig. 5-4). This is but another way of describing the poles of a magnet; that is, the two points where the magnetic strength is greatest are called the *poles* of the magnet.

The Compass. The compass operates on the principle that the earth's magnetic field will always cause a freely suspended bar magnet, or lodestone, to take a position parallel with the earth's magnetic lines, that is, in a north-south position. The simple compass is shown in Fig. 5-5 and consists of a small bar magnet, usually called the *needle,* mounted on a pivot so that it is free to rotate above a printed scale of the compass points N, E, S, and W. The compass was already in general use in Columbus's

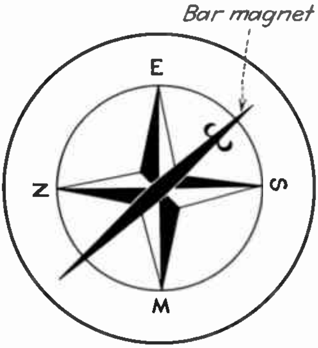


Fig. 5-5 The simple compass.

time and was used by him at the time of his historic voyage to America. The mariner's compass has its printed scale attached to the needle, and its indication is noted against a mark on the rim of the compass. These compasses must have special mountings to keep them level at all times. Aviation and ship compasses are set in liquid to keep them level.

The compass is also a useful tool in the electrical industry because of its ability readily to identify the polarity of a magnet. For example, adjacent poles of a d-c motor or generator must be of opposite polarity; therefore, for a four-pole motor the polarity of the poles must be N, S, N, and S. If an error in the series connection of the field coils produced polarities of N, N, N, and S, the location of the incorrectly connected coil could be easily determined with the aid of a compass.

5-6 Theory of Magnetism

Magnetic Theories. There have been various theories developed from time to time in the scientist's search for the explanation of magnetism. Three of the theories currently being used are (1) ampere, (2) domain, and (3) molecular. The first two theories are based on principles used in the study of advanced physics and therefore are beyond the scope of this text. The *molecular theory,* also called *Weber's theory* after its discoverer, is the most popular explanation.

Molecular Theory. This theory is based on the assumption that the molecules of a magnetic substance are all individual minute magnets. If a magnetic substance lacks the property of polarity and the power of attraction, it is believed that the many tiny magnets are arranged in a disorganized manner as shown in Fig. 5-6*a*. However, when a magnetic substance possesses polarity and power of attraction, it is believed that the molecular magnets are arranged in orderly rows, each with its north pole in the same direction as shown in Fig. 5-6*b*. Also, according to this theory, the molecules in a magnetic substance such as steel, iron, cobalt, or nickel can readily rearrange themselves from a disorganized manner to orderly rows.

Explanation of Magnetic Actions. A lodestone when taken from the earth has a large majority of its molecular magnets lined up in rows and

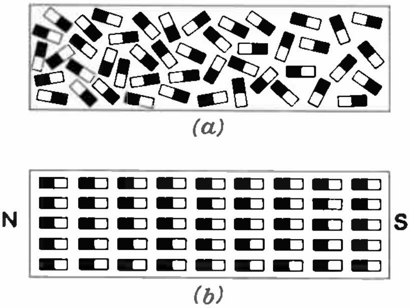


Fig. 5-6 The molecular theory of magnetism. (a) Unmagnetized bar. (b) Magnetized bar.

possesses the powers of magnetism. As the many tiny magnets are lined up in orderly rows, they all aid in building up the strength of the magnet and in forming its poles.

In a piece of iron, however, the molecular magnets ordinarily are not lined up in an orderly fashion but instead settle in a haphazard manner. Because of this haphazard arrangement, the tiny magnets neutralize one another and the iron will not possess the powers of magnetism.

If the lodestone is brought near the iron, it will attract the iron to it, and if the piece of iron is not too large and heavy, the lodestone will be able to lift it. The explanation for this action is that the magnetism of the lodestone influences the tiny magnets of the iron and causes them to rearrange themselves in orderly rows. The iron then becomes a magnet.

If the lodestone is taken away from the iron, the molecular magnets of the iron will shift about and again fall into a haphazard position. The iron is no longer a magnet but is merely a magnetic material.

The basic magnetic actions are explained by this theory. The natural magnet has its molecules arranged in an orderly way when taken from the earth, while the artificial magnet has to have its molecules lined up by some artificial means. In the permanent magnet, once the molecules are set into orderly rows, they will retain their positions, while in the temporary magnet, the molecules stay in line only as long as there is some external magnetic influence to keep them in line.

5-7 Magnetic Attraction and Repulsion

Law of Attraction and Repulsion. If two bar magnets with their poles marked *N* and *S* are used, the laws of magnetic attraction and repulsion may be derived from them. If one of the magnets is suspended by means of a string so that it can move freely and the second magnet is brought near to it so that similarly marked poles (such as two norths or two souths) are brought close together, the suspended magnet will be repelled (Fig. 5-7). When poles of unlike markings such as an *N* and an *S* are brought close together, the suspended magnet will be attracted by the other one. This

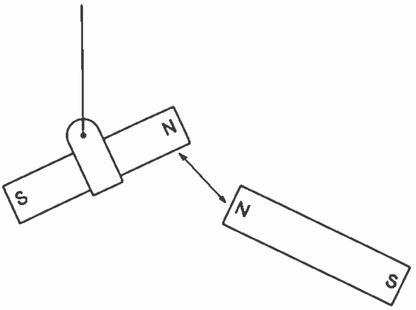


Fig. 5-7 Repulsion between magnetic poles of like polarity.

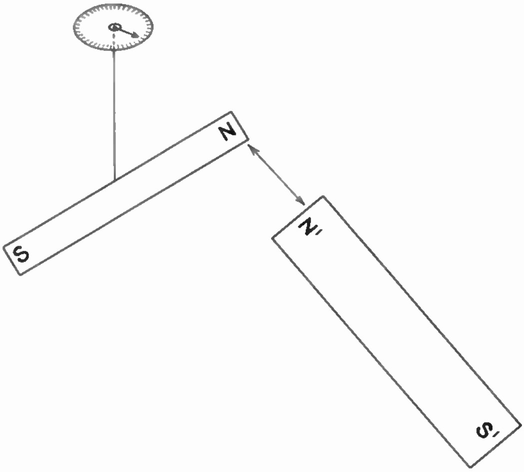


Fig. 5-8 Measuring the force of repulsion by the torsion balance.

action is commonly stated as the magnetic law: *Like poles repel each other and unlike poles attract each other.*

Pole Strength. The force with which two poles will attract or repel each other depends upon the strength of the poles and the distance between them. The pole strength is measured in unit poles which is described as follows: *A unit magnetic pole is one which, if placed in air one centimeter from a similar pole of the same strength, will repel it with a force of one dyne.* (981 dynes = 1 gram; 454 grams = 1 pound.)

Force of Attraction and Repulsion. The force of attraction or repulsion between two poles varies inversely as the square of the distance between them. For example, if two poles 4 cm apart exert a force of 2 dynes, cutting the distance in half so that they are only 2 cm apart, the force will become four times (2 × 2) as great, or 8 dynes. If, however, the distance is increased to 8 cm, or double the original, the force will become one-quarter (½ × ½) of the original amount, or ½ dyne. This relation among force, pole strength, and distance is expressed by Coulomb's law, which states that the force between two magnetic poles is directly proportional to the strengths of the poles and inversely proportional to the square of the distance between the poles. This is shown by the equation

$$f = \frac{m_1 m_2}{d^2} \tag{5-1}$$

where f = force between two poles in air, dynes
m_1 = strength of first pole, unit poles
m_2 = strength of second pole, unit poles
d = distance between the poles, cm

EXAMPLE 5-1 A north pole with a strength of 20 unit poles is placed 5 cm from a south pole whose strength is 30 unit poles. What is the force acting between these poles?

GIVEN: $m_1 = 20$ unit poles $m_2 = 30$ unit poles $d = 5$ cm

FIND: f

SOLUTION:

$$f = \frac{m_1 m_2}{d^2} = \frac{20 \times 30}{5 \times 5} = 24 \text{ dynes (attraction)}$$

5-8 Magnetic Field Characteristics

Magnetic Fields. The properties of a magnet are not restricted to the magnet itself but also influence an area surrounding it. This may readily be shown by placing a piece of glass or paper over a bar magnet and then sprinkling iron filings over the glass or paper. The iron filings will take positions in a definite pattern similar to Fig. 5-9. The filings become tiny magnets under the influence of the bar magnet, and the pattern therefore represents the bar's magnetism. The space surrounding the magnet in

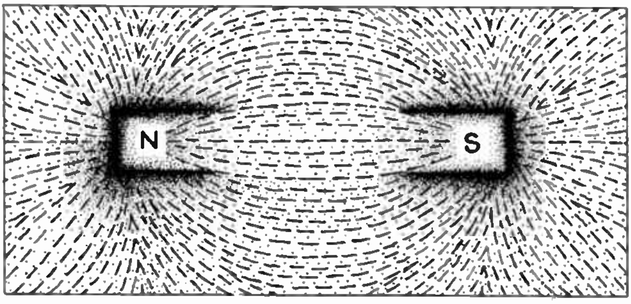


Fig. 5-9 Magnetic field about a bar magnet illustrated by iron filings.

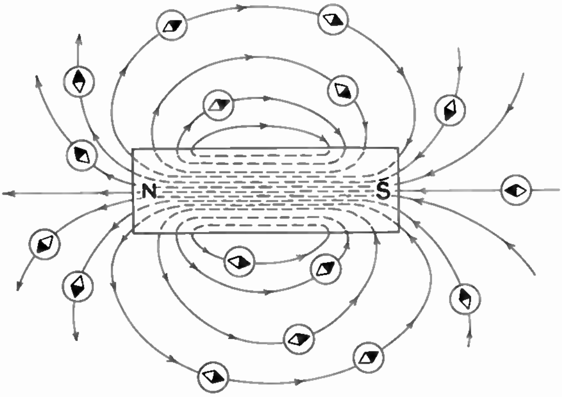
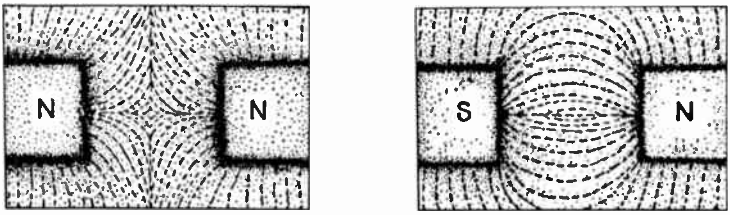


Fig. 5-10 Magnetic field about a bar magnet illustrated by small compasses.



(a) (b)

Fig. 5-11 Magnetic fields about the poles of two bar magnets as illustrated by iron filings. (a) Two like poles. (b) Two unlike poles.

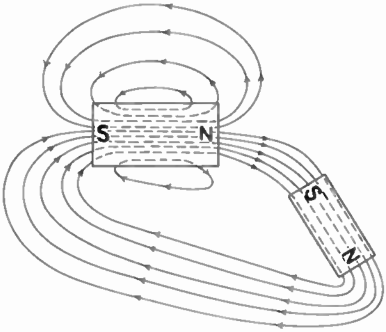


Fig. 5-12 Path taken by magnetic lines when a magnetic substance is near the magnet.

which this influence exists is called its *magnetic field.* The magnetic field can also be shown by a number of small compasses placed about the magnet as shown in Fig. 5-10. The magnetic fields between like poles and unlike poles of two magnets are shown in Fig. 5-11.

Magnetic Lines. A careful examination of the magnetic fields shown in Figs. 5-9 to 5-11 leads to the conclusion that the magnetic field may be represented as lines arranged in an orderly fashion. These lines are commonly referred to as *lines of magnetism* or *lines of induction.* A magnetic line, called a *maxwell,* represents the path along which an isolated unit north pole will tend to move when it is placed in a magnetic field. The total number of lines leaving or entering a pole is called its *magnetic flux* and is usually represented by the Greek letter ϕ, pronounced phi. While these lines are invisible and said to be imaginary, their effect or existence may readily be shown by the iron filings and the compass. The magnetic lines follow definite rules as listed below.

1. Magnetic lines always form a closed loop. The lines leave the magnet at the north pole, travel along definite paths outside the magnet, enter the magnet at the south pole, and travel through the magnet to the starting point at the north pole (Fig. 5-10).
2. Magnetic lines never cross one another (Figs. 5-9 to 5-12).
3. Magnetic lines can pass through any material, but they will take the

path that offers the least resistance (Fig. 5-12), tending to concentrate in magnetic materials and not concentrating in nonmagnetic materials. Hence, magnetic materials offer a lower resistance to the passage of magnetic lines than do nonmagnetic materials.

4. Magnetic lines act like rubber bands. They will stretch outward if a force is exerted upon them and will contract when that force is removed. Since magnetic lines do not cross one another, they push adjacent lines away from them.

Lines of Force. If a pole of a second magnet is brought into the magnetic field, a force will be exerted upon it by that field. The force will be proportional to the lines per square centimeter acting at right angles to the field. These lines are called *lines of force,* and they extend only from one pole, along the external path, and end at the other pole. They differ in this respect from lines of induction, which are always closed loops. The lines of force and the lines of induction are the same when their path is through air, but they will differ when the path is a magnetic substance.

Field Intensity. The strength of a magnetic field, which is also called the *field intensity,* is expressed in terms of the force it will exert upon a magnetic pole of unit strength. The unit of field intensity is the oersted, and a magnetic field is said to be of unit intensity when it is capable of exerting a force of exactly 1 dyne upon a unit pole. This may be expressed by the equation

$$f = m \times H \tag{5-2}$$

where f = force acting upon a magnetic pole placed in a magnetic field, dynes
m = strength of the pole, unit poles
H = field intensity, dynes per unit pole, or oersteds

EXAMPLE 5-2 What is the intensity of a magnetic field that exerts a force of 500 dynes upon a magnet of 40-unit pole strength placed in this field?

GIVEN: $f = 500$ dynes $m = 40$ unit poles

FIND: H

SOLUTION:

$$f = m \times H$$

Therefore,

$$H = \frac{f}{m} = \frac{500}{40} = 12.5 \text{ oersteds}$$

In free space the field intensity will have the same numerical value as the number of lines per square centimeter in a plane at right angles to these lines. Within magnetic materials these two quantities will not have equal values, hence the two terms cannot be substituted for each other indiscriminately. The field at *ABCD* of Fig. 5-13 has a unit field intensity

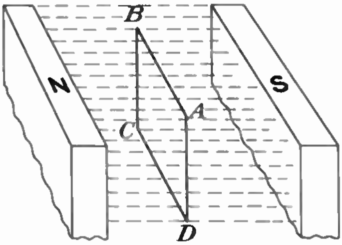


Fig. 5-13 Field intensity of a magnet.

of 1 oersted when one line per square centimeter passes in free space through this section perpendicular to it.

EXAMPLE 5-3 The two parallel pole sides shown in Fig. 5-13 are each 4 by 6 cm, and the magnetic field consists of 72,000 lines uniformly distributed and passing from the north to the south pole. (*a*) What is the field intensity? (*b*) What force would be exerted upon a pole of 25-unit pole strength placed in this field in dynes and in ounces?

Note: 1 oz = 28.4 grams 981 dynes = 1 gram

GIVEN: $\phi = 72{,}000$ lines $A = 4 \times 6$ sq cm $m = 25$ unit poles

FIND: (*a*) H (*b*) f

SOLUTION:

(*a*) Field intensity H equals oersteds and lines per square centimeter when in free space.

$$H = \frac{\phi}{\text{area}} = \frac{72{,}000}{4 \times 6} = 3{,}000 \text{ oersteds}$$

(*b*)
$$f = m \times H = 25 \times 3{,}000 = 75{,}000 \text{ dynes}$$

$$= \frac{75{,}000}{981 \times 28.4} = 2.69 \text{ oz}$$

Flux Density. The number of magnetic lines per square centimeter in a plane perpendicular to the direction of the magnetic field is commonly called the *flux density* and is designated by the symbol B. When the magnetic field is uniform, that is, each square centimeter contains the same number of lines, then the flux density may be expressed as

$$B = \frac{\phi}{A} \tag{5-3}$$

where B = flux density, gausses
ϕ = total flux
A = area, sq cm

A *gauss* represents the flux density of one line per square centimeter.

EXAMPLE 5-4 A magnetic pole has a flux of 150,000 maxwells. If the field is uniformly distributed and the pole is 5 cm wide and 10 cm long, what is the flux density?

GIVEN: $\phi = 150{,}000$ maxwells $A = 5 \times 10$ sq cm

FIND: B

SOLUTION:

$$B = \frac{\phi}{A} = \frac{150{,}000}{50} = 3{,}000 \text{ gausses}$$

Flux density is often expressed in lines per square inch to correspond to the English units in place of the metric units. In such cases, the flux density is still found by dividing the total flux by the area, but as the area is in square inches, the flux density will be expressed in lines per square inch and not in gausses.

EXAMPLE 5-5 What is the flux density of a magnetic pole 1 by 3 inches that has an evenly distributed flux of 4,500 maxwells?

GIVEN: $\phi = 4{,}500$ maxwells $A = 1 \times 3$ sq in.

FIND: B

SOLUTION:

$$B = \frac{\phi}{A} = \frac{4{,}500}{1 \times 3} = 1{,}500 \text{ lines per square inch}$$

The magnetic lines of the bar magnet of Fig. 5-9 represent its magnetic field in only one plane. If the bar were set on its edge and iron filings were again used, they would show a similar magnetic field extending out from the magnet. It can be shown by this means that the magnetic field extends in all directions from the pole of a magnet. If a unit magnetic pole is placed in the center of a sphere of 1 cm radius as shown in Fig. 5-14, it will have a field of 4π or 12.57 ($4 \times 3.1416 = 12.5664$) lines. This is true because according to the definition of the unit pole an equal and like pole placed anywhere on the sphere which would be 1 cm away would be repelled with a force of 1 dyne. Also the field must be of unity intensity,

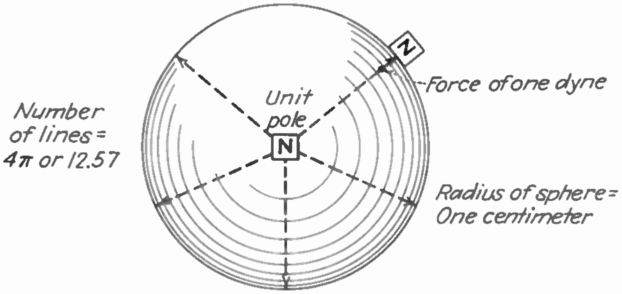


Fig. 5-14 Magnetic field of a unit pole (in air) in a sphere 2 cm in diameter.

one line per square centimeter in free space, in order to exert a force of 1 dyne upon the unit pole. As the area of a sphere is $4\pi r^2$, there will be $4\pi(1)^2$ or 12.57 sq cm and therefore 12.57 lines. The number of lines from a pole whose strength is m becomes

$$\phi = 4\pi m \tag{5-4}$$

EXAMPLE 5-6 How many magnetic lines are emitted by a magnetic pole whose strength is 20 unit poles?

GIVEN: $m = 20$ unit poles

FIND: ϕ

SOLUTION:

$$\phi = 4\pi m = 12.57 \times 20 = 251.4 \text{ maxwells}$$

5-9 Magnetic Induction

Basic Principles. As a magnet is brought close to a piece of iron or steel, that piece becomes magnetized by induction. Figure 5-15*a* shows an iron screw *A* brought close to a magnet. Some of the magnetic lines leaving the north pole of the magnet find that their path of least resistance is through the screw *A* and hence take such a path. The screw becomes magnetized by induction, and the lines entering at the head of the screw make it a south pole and upon leaving at the point make it a north pole. If a second screw is now placed in contact with the first screw (Fig. 5-15*b*), it will cling to it. The second screw has become magnetized by magnetic induction through its contact with the first screw. The head will be a south pole because the magnetic lines enter there, and the point will be a north pole because the lines leave the screw at the point. If the space between the magnet and the first screw is increased, the two screws will no longer hold

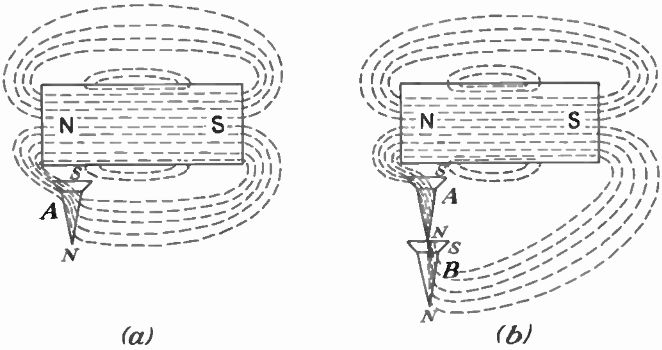


Fig. 5-15 Magnetic induction. (a) Screw *A* magnetized by induction. (b) Screw *B* magnetized by contact with screw *A*.

together and the second one will fall off, thus showing that they were magnetized only while in the influence of the bar magnet.

Poles Produced by Magnetic Induction. Magnetic induction always causes a south pole to be produced in that part of a magnetic substance nearest to the north pole of the magnet and a north pole at that end nearest to the south pole of the magnet. As the induced pole nearest to the magnet is always of a polarity opposite that of the inducing pole, there will be a force of attraction between the two poles. Furthermore, as the magnetic lines always tend to shorten themselves, they will try to pull the induced pole against the inducing pole. This explains why a magnet draws a piece of iron to it.

If a comparatively weak north pole of one magnet is brought near a strong north pole of another magnet, it is possible that there will be an attraction between the two poles instead of the repulsion that would be expected. This is explained by the fact that, when the weak north pole is brought near the strong north pole, the strong north induces a south in the first magnet that exceeds its own weak north and attraction results. The polarity of the weaker magnet becomes reversed under this condition. For this reason, a compass needle or other weak magnet should not be brought too close to a strong pole of another magnet of similar polarity.

Magnetism may be induced in several ways. In Fig. 5-15*a*, the magnetism is induced in the screw *A* by its mere presence in the magnetic field of the bar magnet. In Fig. 5-15*b*, the magnetism is induced in the screw *B* by its contact with the magnet, screw *A*. In Fig. 5-1, magnetism is induced in the bar by stroking it with a lodestone.

5-10 Magnetic Properties and Classification of Materials

The properties of magnetic materials most commonly referred to are reluctance, reluctivity, permeance, permeability, and retentivity.

Reluctance. Reluctance is the opposition offered by a material to the passage of magnetic lines. This corresponds to the term resistance in the electric circuit, which is the opposition to the flow of electric current. Reluctance, however, has the additional characteristic that its value for magnetic materials is not constant but varies with the flux density. The reluctance of nonmagnetic materials is constant. The symbol for reluctance is the script letter $\mathcal{R}$; no name has been assigned for its unit of measurement.

Reluctivity. Reluctivity is the specific reluctance, or the reluctance per centimeter cube. For nonmagnetic materials, its value is 1, and for magnetic materials, its value varies with changes in flux density. Reluctivity corresponds to resistivity in the electric circuit. The symbol for reluctivity is the Greek letter ν, pronounced nu; no name has been assigned for its unit of measurement.

Permeance. Permeance is the ability of a material to carry magnetic lines. Its value is equal to the reciprocal of the reluctance. Permeance corresponds to conductance in the electric circuit. The symbol for permeance is the script letter $\mathcal{P}$; no name has been assigned for its unit of measurement.

Permeability. Permeability is a measure of the ease with which magnetic lines can pass through a material. Numerically it is equal to the reciprocal of the reluctivity. The permeability of nonmagnetic materials is 1, while for magnetic materials it is a variable quantity depending upon the flux density. It may also be considered as the ratio of the lines of force passing through a material to the lines of force passing through air for the same conditions. Good magnetic materials will have a high value of permeability. The symbol for permeability is the Greek letter μ, pronounced mu; no name has been assigned for its unit of measurement.

Retentivity. Retentivity is the ability of a material to retain magnetism after the magnetizing force is removed. Permanent magnets have a high degree of retentivity, and temporary magnets are low in retentivity. The magnetism that remains in a material after the magnetizing force is removed is called the *residual magnetism*. This property is very useful in the operation of electric generators.

Classification. It has been common practice in the past to classify materials as being either magnetic or nonmagnetic. Iron, steel, nickel, and such other materials that may easily be magnetized were classed as magnetic; air, copper, brass, and such materials which seemed impossible to magnetize were classed as nonmagnetic. The present practice is to classify materials into one of three groups, namely, ferromagnetic, paramagnetic, and diamagnetic.

Ferromagnetic materials are those which become strongly magnetized in the direction of the magnetizing field. They have high values of permeability. Included in this classification are iron, steel, nickel, cobalt, magnetite, and alloys such as Heusler's alloys, Permalloy, and alnico.

Paramagnetic materials are those which become very weakly magnetized in the direction of the magnetizing field. The permeability of these materials is greater than 1 but the values are low. Included in this classification are aluminum, platinum, oxygen, air, manganese, and chromium.

Diamagnetic materials are those which will become very weakly magnetized but in a direction opposite to that of the magnetizing field. The permeability of these materials is less than 1. Included in this classification are bismuth, antimony, copper, zinc, mercury, gold, and silver.

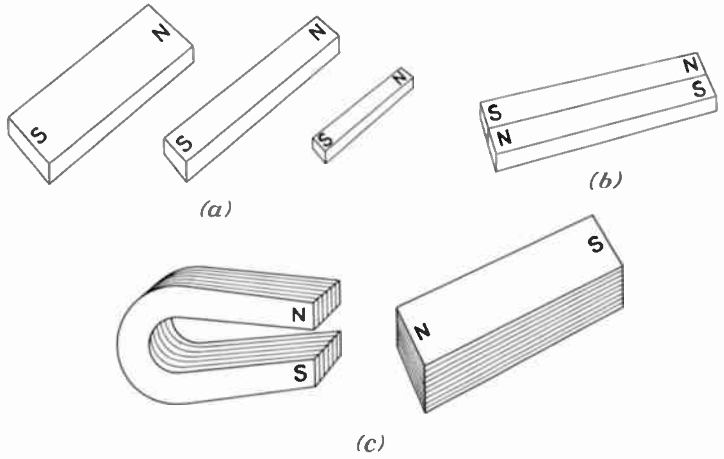
In general only iron, steel, Permalloy, alnico, and a few other alloys are considered as magnetic materials, and practically all other materials are considered nonmagnetic. Pure annealed iron has a permeability of 200 to 5,000, and silicon iron has a range of 600 to 10,000. Permalloy, which is an alloy containing about 78 per cent nickel and 22 per cent iron, is highly magnetic and has permeability values as high as 100,000.

Alnico magnets, which are very powerful, are made of an alloy containing aluminum, nickel, iron, and cobalt. Sintered alnico magnets, made by pressing finely powdered component alloy metals together and heating them almost to melting point, are more powerful than ordinary alnico magnets. Their magnetic qualities are so great that they can lift 500 times their own weight. When a magnet is assembled in a special manner, it has been found capable of lifting as much as 4,000 times its own weight.

5-11 Magnetic Shapes

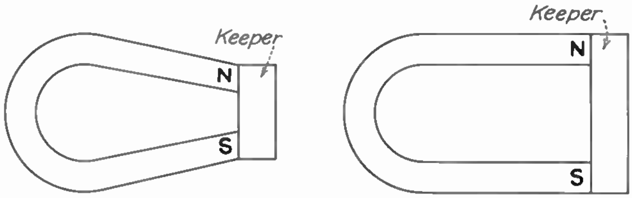
Bar Magnets. Magnets are used for many purposes and are therefore found in a large variety of shapes. The bar magnets shown in Fig. 5-16*a* are commonly used in school laboratories, as they provide a very satisfactory means of demonstrating the laws of magnetism and the path of magnetic lines. When bar magnets are not being used, it is desirable to place them so that adjacent ends will be of opposite polarity (Fig. 5-16*b*) in order that they may retain their magnetic strength.

Horseshoe Magnets. Magnets for commercial use are generally in the form of a horseshoe or some variation of this form because they provide a much stronger magnet than a bar magnet of an equal weight and size. This



(a) (b) (c)

Fig. 5-16 Shapes of magnets. (a) Bar magnets. (b) Proper method of storing bar magnets. (c) Laminated magnets.



(a) (b)

Fig. 5-17 Forms of horseshoe magnets. (a) Close-pole type. (b) U-shape.

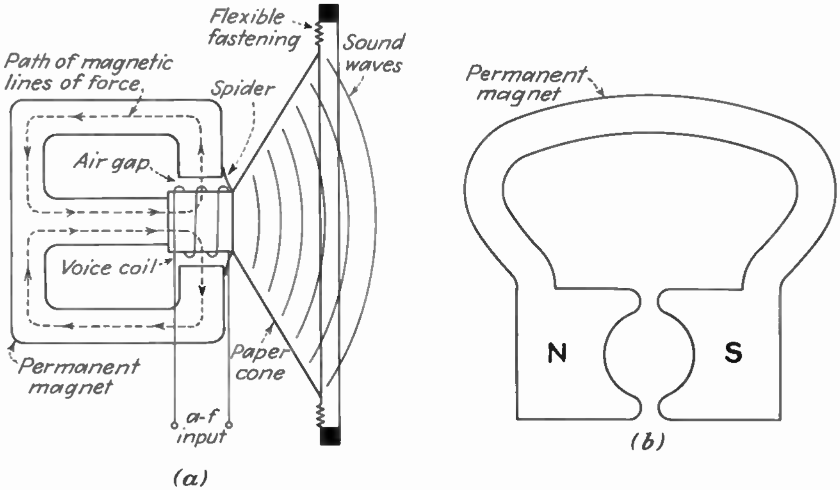


Fig. 5-18 Applications of magnets. (a) As used in a permanent-magnet dynamic loudspeaker. (b) As used in an electrical measuring instrument.

is true because the two poles are closer to each other and also because both poles can be utilized. When the horseshoe magnet is not in use, a soft-iron keeper should be placed across its poles to keep the magnet from losing its strength. Two basic forms of the horseshoe magnet are shown in Fig. 5-17. Two variations of the basic horseshoe magnet as used in a permanent-magnet loudspeaker and an electrical measuring instrument are shown in Fig. 5-18. The compound horseshoe magnet of a magneto used for some types of ignition systems is illustrated in Fig. 5-19.

Laminated Magnets. The strength of magnets may be further increased by constructing a magnet of a number of thin sheets whose total thickness

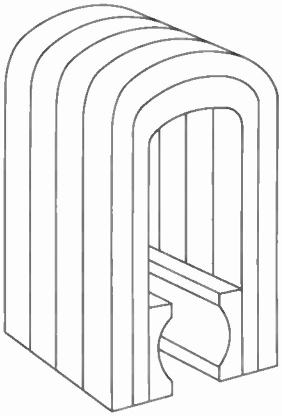


Fig. 5-19 Compound horseshoe magnet.

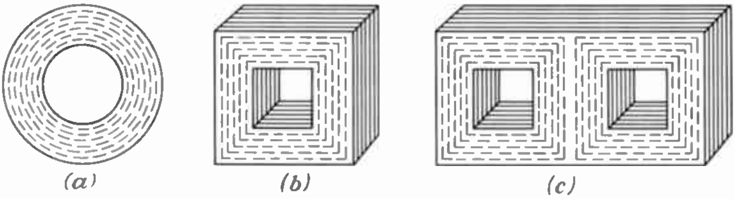


Fig. 5-20 Magnetic cores. (a) Ring. (b) Square. (c) Shell.

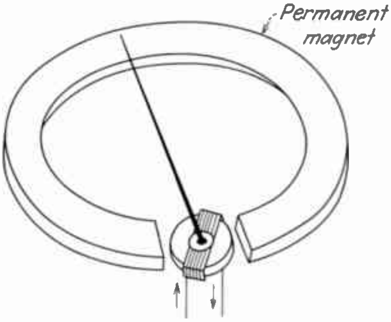


Fig. 5-21 Ring magnet as used in a meter.

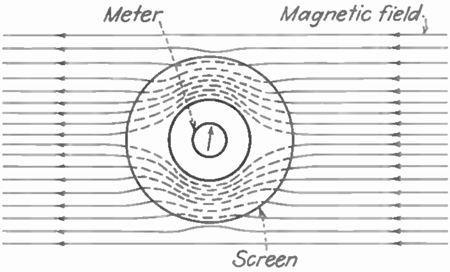


Fig. 5-22 Magnetic screen.

is equivalent to the thickness it would be if made in one piece. This is referred to as a *laminated magnet* and is illustrated in Fig. 5-16*c*.

Ring Magnets. Magnets are sometimes made in a circular form as shown in Fig. 5-20*a* and are called *ring magnets*. This magnet has no poles, and its magnetic lines form closed loops around the ring. This form of magnet may be used to demonstrate the operating principle of the transformer. The magnetic cores of transformers shown in Fig. 5-20*b* and *c* are variations of the ring magnet.

If a small piece is cut out of a ring magnet as in Fig. 5-21, it will then have two strong magnetic poles. This type of magnet is sometimes used in electrical instruments.

Magnetic Screens. Another use of a circular (usually a temporary) magnet is the magnetic screen. Its purpose is to shield any object from magnetic lines as illustrated by the meter and its screen in Fig. 5-22. As there is no known insulator of magnetic lines, shielding is the practical substitute and is accomplished by placing the object in the center of a ring or other closed circuit. As the screen is usually made of soft iron, it provides the easiest path for the magnetic lines and thereby keeps the object in the center free from these magnetic lines.

5-12 Magnetic Field about a Wire Carrying a Current

In 1819, Oersted made the discovery that a magnetic field always exists about a wire that is carrying a current and that certain relations existed between the magnetic and electric conditions. The principle of Oersted's discovery may be demonstrated by placing a wire vertically through a piece of cardboard and determining its magnetic characteristics when a current is flowing through the wire. This is shown in Fig. 5-23, where the wire *AB* is placed through the board *D* and connected to a d-c power source *S*, with an adjustable resistor *R* connected in series to control the amount of current. If a current of approximately 50 amp is permitted to flow from *A* to *B* and iron filings are sprinkled over the cardboard, the filings will take an orderly circular form about the wire as shown in Fig. 5-23. It will also be observed that, if compasses are set at the four *C* positions, the compass needles will come to rest at right angles to the wire and all north poles will point in the same direction of rotation. If the current is removed by opening the switch, the compass needles will no longer stay perpendicular to the wire but will take positions in accordance with the earth's or any other nearby magnetic field. If the board is gently tapped, the iron filings will take a haphazard form, thus proving that the magnetic field exists about the wire only when a current is flowing through it.

5-13 Relation of Magnetic Field and Electron Flow

Effect of Amount of Electron Flow. If the current flowing through the wire of Fig. 5-23 is raised from 50 to 80 amp, it will be found that, while the iron filings still take the same form, a greater quantity of filings will be affected. If the compasses are moved, first closer to the wire and then farther from the wire, it will be found that the magnetic effect will extend farther from the wire when the current is increased. This shows that the strength of the magnetic field about a current-carrying wire increases as the current increases and decreases as the current decreases.

Effect of Direction of Electron Flow. If the leads connected to the wire *AB* are interchanged and the circuit is adjusted so that 50 amp flow from *B* to *A*, the sprinkling of iron filings on the board will produce the

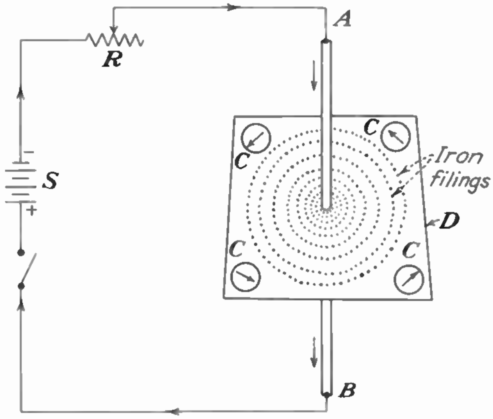


Fig. 5-23 Magnetic field about a wire carrying a current.

same results as before. Close observation of the compass needles will, however, indicate that a change has taken place, for they will now have their north poles pointing in the direction opposite to that when the current was flowing from *A* to *B*. This indicates that the direction of the magnetic field about a current-carrying wire depends upon the direction in which current flows through the wire.

Left-hand Rule for a Wire Carrying a Current. The relation between the direction of electron flow and the magnetic field is easily remembered by the left-hand rule for a wire, which states: *Grasp the wire with the left hand so that the thumb points in the direction of the electron flow, and the fingers will point in the direction of the magnetic field.* Thus, if either the direction of electron flow or the direction of the magnetic field is known, the other may be obtained. Figure 5-24 illustrates this rule. It should be observed that ⊕ indicates electrons entering the wire and ⊙ indicates electrons leaving the wire.

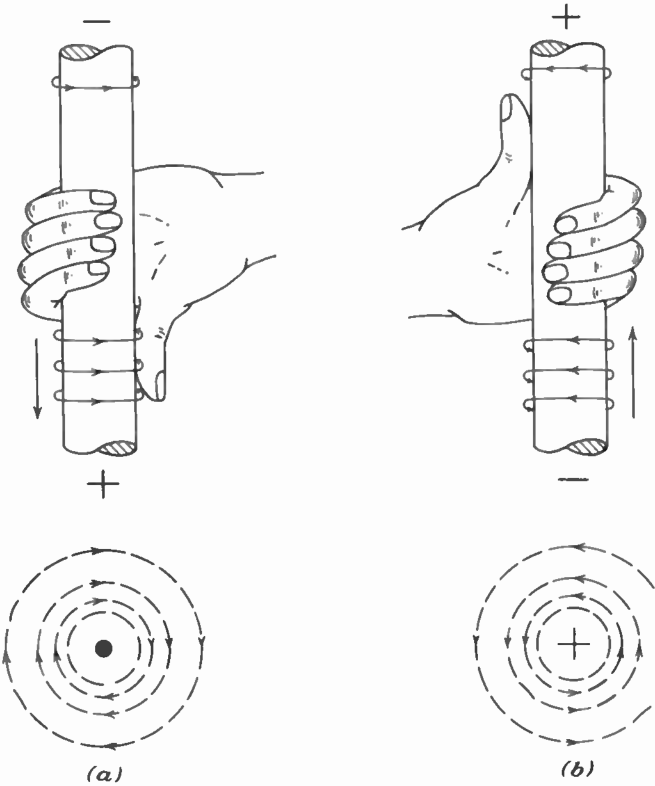


Fig. 5-24 Left-hand rule for a wire carrying a current. (a) Electrons flowing downward or out of page. (b) Electrons flowing upward or into the page.

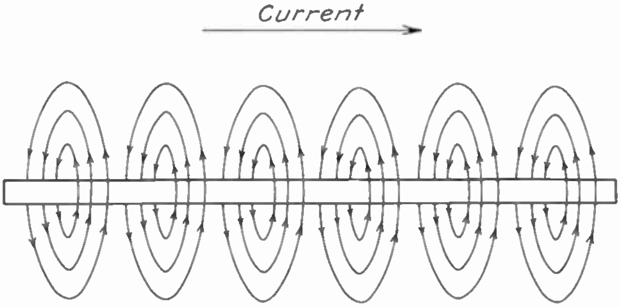


Fig. 5-25 Magnetic field about a straight wire carrying a current.

5-14 Magnetic Characteristics of a Coil

Magnetic Field about a Coil. The magnetic field around a straight wire carrying a current exists at all points along the length and consists of concentric circles in a plane perpendicular to the wire (Fig. 5-25). If such a long piece of wire is wound on a core as shown in Fig. 5-26, it is called a *coil*, a *solenoid*, or a *helix*. When a current flows through this coil, additional magnetic characteristics result.

Considering first the two turns of wire, *A* and *B* of Fig. 5-27, it will be seen that the current is leaving at these points. If the turns are not

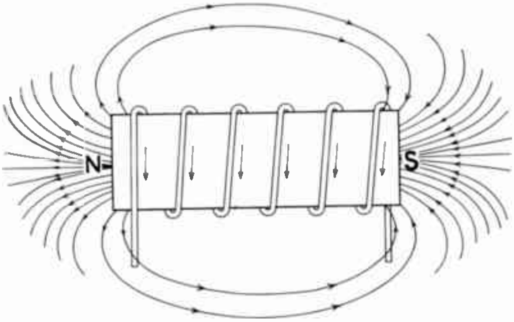


Fig. 5-26 Magnetic field about a coil carrying a current.

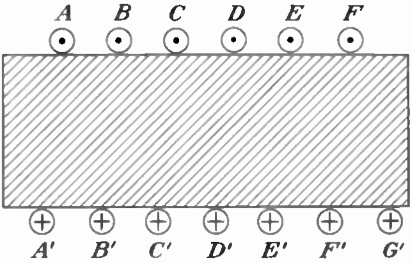


Fig. 5-27 View showing the direction of the current in the conductors of the coil of Fig. 5-26.

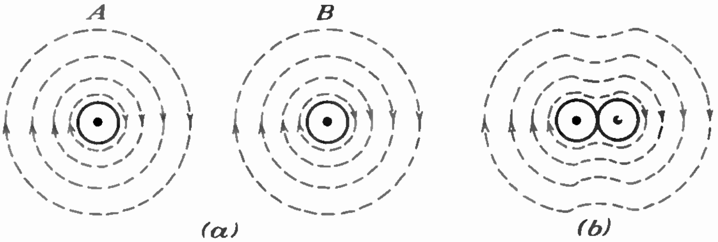


Fig. 5-28 Magnetic fields about adjacent conductors of a coil carrying a current. (a) Conductors separated. (b) Conductors adjoining.

too close to each other, their magnetic fields will take the form as illustrated in Fig. 5-28*a*. Notice that the magnetic lines at the right of *A* are going downward while the adjacent lines at the left of *B* are going upward. When the two turns *A* and *B* are adjacent to each other, the effect of the one's downward and the other's upward lines will neutralize each other at this point, but the field about the two wires will be strengthened as indicated in Fig. 5-28*b*.

When the entire coil, *A* to *F* and *A′* to *G′*, is considered, the resulting magnetic field is shown in Fig. 5-26. The strength of the magnetic field will increase with an increase in the number of turns and also with an increase in current. Note that the field is the strongest at the ends of the coil and that this corresponds to the definition for a pole; hence the coil has two magnetic poles. The direction of the magnetic lines indicated in Fig. 5-26 shows them leaving the coil at the left side and entering again at the right. According to the magnetic laws, this indicates that the left pole is a north and the right pole is a south.

Left-hand Rule for a Coil. As the direction of the magnetic lines was determined by the direction of electron flow, it is apparent that reversing the direction of electron flow will also reverse the polarity of the coil. The relation between the direction of the current flow and the poles of the coil is known as the *left-hand rule for a coil* and is: *Grasp the coil with the left hand so that the fingers point in the direction of the electron flow, and the thumb will point in the direction of the coil's north pole.* This is illustrated in Fig. 5-29*a* and *b*.

Magnetic Core Materials. The magnetic strength of a coil is affected by the type of core upon which the coil is wound. If a ferromagnetic material is used for the core, the coil will produce much stronger magnetic properties than if the coil had a paramagnetic core. In general, various forms of soft iron having a high permeability, such as silicon iron, are used for making magnetic cores. These materials are made in thin sheets, from 0.001 to 0.05 inch thick, and the sheets are stacked to form a laminated core of required thickness and design. The sheets are insulated electrically from one another with a thin coating of varnish.

At very high frequencies laminated cores cause an excessive power loss, and cores made of powdered material have been developed to reduce these losses. Powdered-material cores are produced either from a very fine metallic powder or from a ceramic powder. Metallic powders have a particle size as small as 2 microns and are mixed with a binder which insulates the particles electrically from one another. Ceramic powdered cores are called *ferrites* and have a very high electrical resistivity and high permeability. Ferrite cores are made from a suitable mixture of oxides of iron and other metals by pressing these powders together to a required shape and then firing at an intense heat to produce a porcelainlike material. The oldest known ferrite is magnetite, or lodestone. The most widely used ferrites are nickel ferrite, manganese ferrite, and zinc-manganese ferrite.

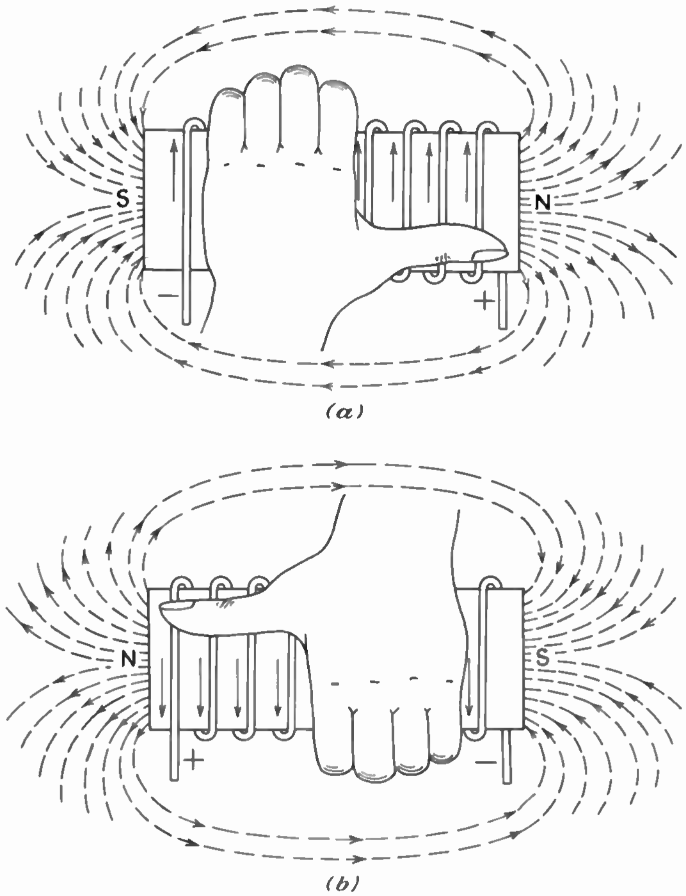


Fig. 5-29 Left-hand rule for a coil. (a) Electrons entering the coil at the left terminal. (b) Electrons entering the coil at the right terminal.

5-15 Magnetic Circuit Calculations

Magnetic Circuits. Many electrical devices depend upon magnetism for their operation, and to have these devices function efficiently, various intricate designs have been made for different magnetic conditions. The magnets must supply the required strength and must be provided with paths or circuits of suitable shapes and materials. The magnetic circuit is defined as the path or paths taken by the magnetic lines of induction leaving a north pole, passing through the entire circuit, and returning to the starting point. A magnetic circuit may be a simple, series, or parallel circuit or may be a complex combination circuit depending upon how intricate the device may be.

Basic Equations. Many of the magnetic-circuit calculations are similar to the electric-circuit calculations as is apparent in the following table.

Table 5-1

UNIT	ELECTRIC CIRCUIT	MAGNETIC CIRCUIT
Pressure	Volt (E)	Gilbert (F)
Quantity	Ampere (I)	Maxwell (ϕ)
Resistance	Ohm (R)	$\mathfrak{R}$
Mathematical relation	$E = IR$	$F = \phi\mathfrak{R}$
	$I = \frac{E}{R}$	$\phi = \frac{F}{\mathfrak{R}}$
	$R = \frac{E}{I}$	$\mathfrak{R} = \frac{F}{\phi}$
Specific resistance, or reluctance	Resistivity (K)	Reluctivity (ν)
	$R = \frac{Kl}{A}$	$\mathfrak{R} = \frac{\nu l}{A}$

Examination of Table 5-1 will show that the magnetic-circuit equations and the electric-circuit equations are very similar. There is, however, one point in which the two systems differ, and this occurs in the calculation of the reluctance of the magnetic circuit. In the resistance equation of the electric circuit, K represents the specific resistance of the material used to conduct the electric current. The value of K is obtained from Appendix IV; for example, K for copper is 10.4, and this value is the same whether the conductor carries 1 or 5 amp. In the reluctance equation of the magnetic circuit, ν represents the specific reluctance of the material used to conduct the magnetic lines. This value of ν is not a constant even for a given material but varies with the number of magnetic lines per unit of area and must therefore be found separately for each condition considered. This may be done by use of the permeability curve of the kind of material being used. Curves for various materials may be found in engineering handbooks or may be obtained from the manufacturer of the material. A sample permea-

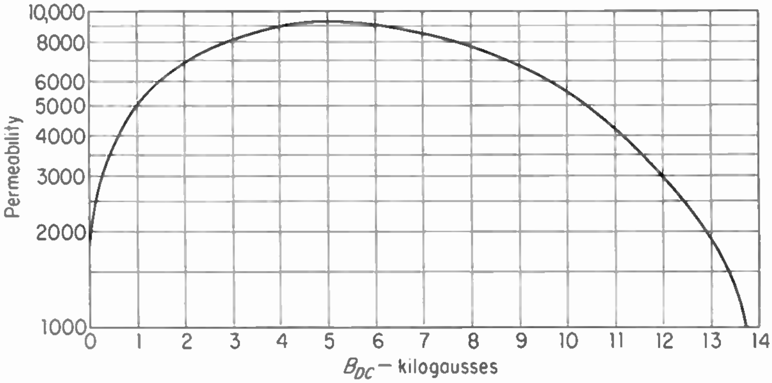


Fig. 5-30 Permeability curve for transformer-grade steel.

bility curve is given in Fig. 5-30. The specific reluctance or reluctivity is the reciprocal of the permeability, or

$$\nu = \frac{1}{\mu} \tag{5-5}$$

where ν = specific reluctance (or reluctivity)
μ = permeability

EXAMPLE 5-7 What is the reluctivity of a piece of transformer-grade steel 2 sq cm in cross section and carrying 24,000 maxwells?

GIVEN: ϕ = 24,000 maxwells A = 2 sq cm

FIND: ν

SOLUTION: To find ν, we must have μ, and to find μ from Fig. 5-30, we must have B [see Eq. (5-3)].

$$B = \frac{\phi}{A} = \frac{24{,}000}{2} = 12{,}000 \text{ gausses}$$

$$\mu = 3{,}000 \text{ (from curve, Fig. 5-30)}$$

$$\nu = \frac{1}{\mu} = \frac{1}{3{,}000} = 0.000333$$

The reluctance of a magnetic circuit is expressed as

$$\mathfrak{R} = \frac{\nu l}{A} \tag{5-6}$$

where $\mathfrak{R}$ = reluctance of the circuit
ν = reluctivity of the magnetic path
l = length of the magnetic path, cm
A = area of the magnetic path, sq cm

EXAMPLE 5-8 If the length of the magnetic path of the piece of transformer-grade steel of Example 5-7 is 16 cm, what is its reluctance?

GIVEN: $\nu = 0.000333$ (from Example 5-7) $l = 16$ cm $A = 2$ sq cm

FIND: $\mathcal{R}$

SOLUTION:

$$\mathcal{R} = \frac{\nu l}{A} = \frac{0.000333 \times 16}{2} = 0.002664$$

When the reluctance and flux are known, it is possible to find the magnetomotive force required to push the flux through the magnetic circuit by use of the equation

$$F = \phi \mathcal{R} \tag{5-7}$$

where F = magnetomotive force, gilberts
ϕ = flux, maxwells
$\mathcal{R}$ = reluctance

EXAMPLE 5-9 What magnetomotive force is required to push the flux through the magnetic circuit of Example 5-8?

GIVEN: $\phi = 24{,}000$ maxwells (from Example 5-7) $\mathcal{R} = 0.002664$ (from Example 5-8)

FIND: F

SOLUTION:

$$F = \phi \mathcal{R} = 24{,}000 \times 0.002664 = 63.9 \text{ gilberts}$$

The magnetizing force in an electromagnet is supplied by passing an electric current through a coil wound around a portion of the magnetic circuit. The strength of this magnetizing force will depend upon the number of turns and the amount of current and is expressed by the equation

$$F = 1.26NI \tag{5-8}$$

where N = number of turns on the coil
I = current through the coil, amp

EXAMPLE 5-10 How many ampere-turns are required for a coil to supply the magnetic circuit of Example 5-9?

GIVEN: $F = 63.9$ gilberts (from Example 5-9)

FIND: NI

SOLUTION:

$$F = 1.26NI$$

Therefore, $$NI = \frac{F}{1.26} = \frac{63.9}{1.26} = 50.7 \text{ amp-turns}$$

EXAMPLE 5-11 If the coil of Example 5-10 has 100 turns, how much current would be required?

GIVEN: $NI = 50.7$ amp-turns (from Example 5-10) $N = 100$ turns

FIND: I

SOLUTION:

$$NI = 50.7$$

Therefore $$I = \frac{50.7}{N} = \frac{50.7}{100} = 0.507 \text{ amp}$$

B-H Curves. The materials used in magnetic circuits are tested to determine the number of gausses set up by a magnetizing force of a few oersteds to values far beyond those generally used. The results obtained are plotted to produce curves showing the relation between the magnetizing force H, oersteds, and the corresponding flux density B, gausses. These curves are called *d-c magnetization, saturation,* or *B-H curves.* Figure 5-31 shows d-c magnetization curves for three different grades of silicon steel.

The permeability of a material may be expressed as the flux density set up by one unit of magnetizing force, or

$$\mu = \frac{B}{H} \tag{5-9}$$

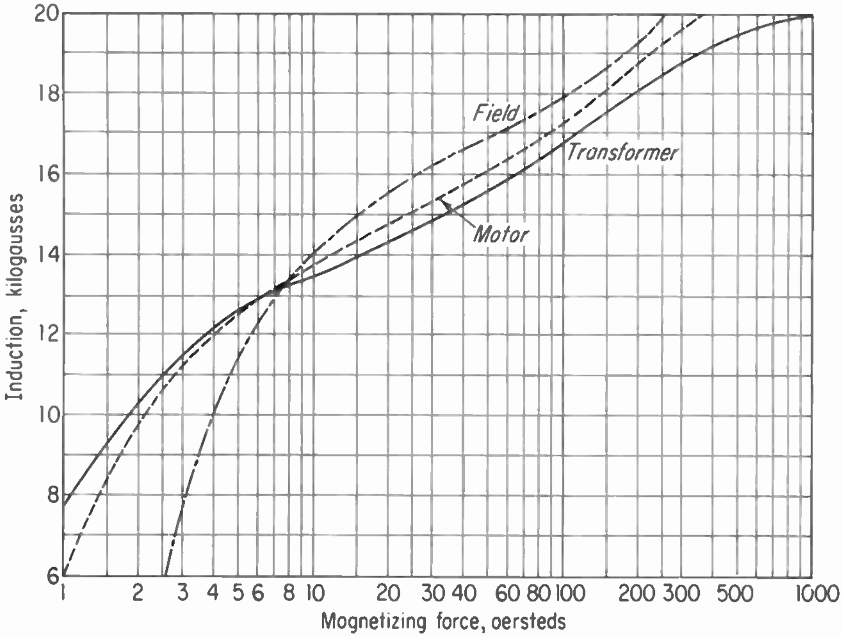


Fig. 5-31 Direct-current magnetization curves for transformer-, motor-, and field-grade steel. *Note:* These curves could have been plotted with an abscissa of ampere-turns per centimeter. To convert oersteds to NI per centimeter multiply oersteds by 0.796.

Knowing the value of either B or H, the corresponding value can be found from the B-H curve. Furthermore, since the reluctivity of magnetic materials varies with the flux density, the value of reluctivity for any flux density can be obtained by use of the B-H curve for that material.

EXAMPLE 5-12 What is the reluctivity of a sheet of transformer-grade silicon steel, whose characteristics are shown in Fig. 5-31, if the magnetizing force is 15 oersteds?

GIVEN: $H = 15$ oersteds

FIND: ν

SOLUTION:

$$B = 14{,}000 \text{ gausses (from Fig. 5-31)}$$

$$\mu = \frac{B}{H} = \frac{14{,}000}{15} = 933$$

$$\nu = \frac{1}{\mu} = \frac{1}{933} = 0.00107$$

EXAMPLE 5-13 A magnetic core made of motor-grade silicon steel sheets, whose characteristics are shown in Fig. 5-31, is 8 cm in length, has a cross section of 1.5 by 1.5 cm, and has a flux density of 13,000 gausses. What is its reluctance?

GIVEN: $B = 13{,}000$ gausses $l = 8$ cm $A = 1.5 \times 1.5$ sq cm

FIND: $\mathfrak{R}$

SOLUTION:

$$H = 6 \text{ oersteds (from Fig. 5-31)}$$

$$\mu = \frac{B}{H} = \frac{13{,}000}{6} = 2{,}166$$

$$\nu = \frac{1}{\mu} = \frac{1}{2{,}166} = 0.00046$$

$$\mathfrak{R} = \frac{\nu l}{A} = \frac{0.00046 \times 8}{1.5 \times 1.5} = 0.0016$$

Hysteresis. When a ferromagnetic material is subjected to varying magnetic fields, such as changes in direction and intensity, the magnetic induction will lag behind the changes in the magnetizing force. This lag in magnetization is called *hysteresis* and is the result of an internal friction within the substance. The heat produced by this friction represents a power loss. When a ferromagnetic material is used with a-c equipment, such as transformers, or with rotating equipment, such as motors or generators, the material is subjected to a cyclical change in both direction and intensity. The energy loss occurring during each cycle is represented by a hysteresis loop.

A hysteresis loop is obtained by plotting the magnetic intensity H against the resulting magnetic induction B for a complete magnetization cycle (Fig. 5-32). When a magnetizing force is first applied to a magnetic material and the magnetic intensity *H* is gradually increased in a positive direction to a value that will produce the maximum value occurring at point *A*, the magnetic induction *B* will follow the line *OA*. If the value of *H* is then gradually decreased from this value to zero, the value of *B* will decrease along the line *ABCD* and not along the line *OA*. When *H* is decreased to zero, the magnetic induction *B* decreases to only the value *OB*. The value *OB* is called the *remanence*, the *residual induction*, or the *residual magnetism*. If the value of *H* is then varied an equal amount in the negative direction, the magnetic induction *B* will reach zero when *H* has a value of *OC*. The value *OC* is called the *coercive force*. When *H* reaches its maximum negative value, point *D* will have negative values of *B* and *H* equal to the positive values of *B* and *H* at point *A*. Any repeating cycles of *H* from $+H_{\text{max}}$ $-H_{\text{max}}$ will produce the loop *ABCDEFA*, and the enclosed area is called the *hysteresis loop* and represents the energy loss during each cycle.

Eddy Currents. Eddy currents are circulating currents that are induced in a material by a varying magnetic field. These currents are undesirable, as they produce heat and thus represent a loss in energy. The iron cores of transformers and the poles, armature, rotors, and stators of rotating elec-

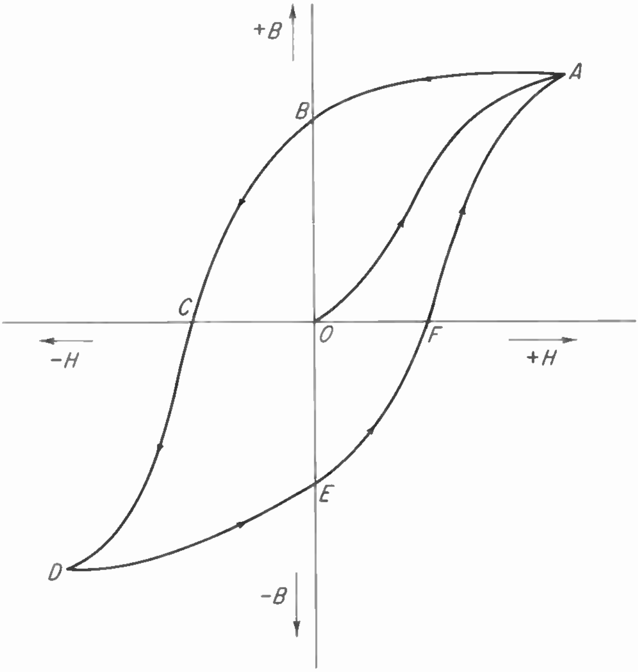


Fig. 5-32 Hysteresis loop.

trical equipment are constructed of laminated sheets that are electrically insulated from each other, thus restricting the eddy currents to individual sheets. The thinner the laminations, the greater will be the reduction of the eddy currents. Eddy currents are reduced in powdered cores by use of a binder that insulates each metallic particle electrically from one another.

5-16 Magnetic Circuits

Simple Magnetic Circuit. The magnetic flux follows a closed path in a magnetic circuit just as an electric current follows a closed path in an electric circuit. A simple magnetic circuit is the iron-ring core of Fig. 5-20*a* or the square core of Fig. 5-20*b*. The magnetic flux is set up by the flow of current in a winding of insulated wire surrounding all or part of the core. Except for a very slight leakage, all the loops of flux lie entirely within the core. Since no flux leaves or enters a ring or rectangular core, they have no polarity.

Series Magnetic Circuit. When a section is removed from a ring magnet for an armature (Fig. 5-21), an air gap is produced between the armature and each end of the magnet. To make a complete circuit, the lines of flux must pass through the magnet to one end, through an air gap, the armature, a second air gap, to the other end of the magnet. This is a *series magnetic circuit*, in that the flux has only one path and it must pass in turn through each reluctance in order to complete the circuit. This is the same as a series electrical circuit in which the current must pass in turn through each resistance to complete the circuit. The total reluctance of a series magnetic circuit is

$$\mathfrak{R}_T = \mathfrak{R}_1 + \mathfrak{R}_2 + \mathfrak{R}_3, \text{ etc.} \tag{5-10}$$

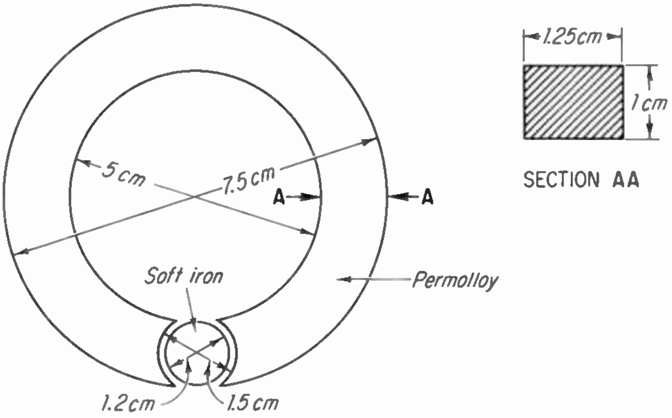


Fig. 5-33

EXAMPLE 5-14 The magnetic circuit of a ring magnet as used in an electrical measuring instrument has the configuration and dimensions shown in Fig. 5-33. The permeability of the Permalloy ring magnet is 10,000 and the soft-iron core 1,000. Determine the reluctance of the magnetic circuit.

GIVEN: Fig. 5-33 μ of Permalloy = 10,000 μ of soft iron = 1,000

FIND: $\mathfrak{R}_T$

SOLUTION:

$$\text{Reluctivity of Permalloy} = \frac{1}{\mu} = \frac{1}{10{,}000} = 0.0001$$

$$\text{Reluctivity of soft iron} = \frac{1}{\mu} = \frac{1}{1{,}000} = 0.001$$

$$\text{Reluctivity of air} = \frac{1}{\mu} = \frac{1}{1} = 1$$

$$\text{Average diameter of Permalloy magnet} = \text{ID} + \frac{\text{OD} - \text{ID}}{2}$$

$$= 5 + \frac{7.5 - 5}{2} = 6.25 \text{ cm}$$

Approximate average length of magnetic path in Permalloy magnet

$$= (\pi \times \text{average diameter}) - 1.5 = (3.14 \times 6.25) - 1.5 = 18.12 \text{ cm}$$

$$\text{Cross-sectional area } AA = 1 \times 1.25 = 1.25 \text{ sq cm}$$

$$\mathfrak{R} \text{ of Permalloy path} = \frac{\nu l}{A} = \frac{0.0001 \times 18.12}{1.25} \cong 0.00145$$

Approximate average length of armature path = 1.2 × 0.7854 = 0.943 cm

Note: The length of the magnetic path is 1.2 cm only for the average diameter of 6.25 cm, and the length decreases with both the larger and smaller diameters. The value of 0.7854 is a factor that provides for the average length of the path through the armature.

$$\mathfrak{R} \text{ of soft iron path} = \frac{\nu l}{A} \cong \frac{0.001 \times 0.943}{1.25} \cong 0.000754$$

Length of two air gaps = 1.5 − 1.2 = 0.3 cm

$$\mathfrak{R} \text{ of air gaps} = \frac{\nu l}{A} \cong \frac{1 \times 0.3}{1.25} \cong 0.24$$

$$\mathfrak{R}_T = \mathfrak{R}_{pp} + \mathfrak{R}_{sip} + \mathfrak{R}_{ag} \cong 0.00145 + 0.000754 + 0.24 \cong 0.242$$

Parallel Magnetic Circuit. Some of the magnetic circuits used in electrical and electronic equipment are simple parallel circuits having only two paths, as the shell-type transformer core of Fig. 5-20*c*. However, there are many other types of parallel circuits that are more complex, as the permanent magnet of the loudspeaker of Fig. 5-18*a* or the two-pole generator of Fig. 7-23. The total reluctance of a parallel magnetic circuit is

$$\mathfrak{R}_T = \frac{1}{\dfrac{1}{\mathfrak{R}_1} + \dfrac{1}{\mathfrak{R}_2} + \dfrac{1}{\mathfrak{R}_3}, \text{ etc.}} \tag{5-11}$$

EXAMPLE 5-15 What is the reluctance of a parallel magnetic circuit made of reluctances of 0.0025, 0.0005, and 0.0025?

GIVEN: $\mathfrak{R}_1 = 0.0025$ $\quad\mathfrak{R}_2 = 0.0005$ $\quad\mathfrak{R}_3 = 0.0025$

FIND: $\mathfrak{R}_T$

SOLUTION:

$$\mathfrak{R}_T = \frac{1}{\dfrac{1}{\mathfrak{R}_1} + \dfrac{1}{\mathfrak{R}_2} + \dfrac{1}{\mathfrak{R}_3}} = \frac{1}{\dfrac{1}{0.0025} + \dfrac{1}{0.0005} + \dfrac{1}{0.0025}} = \frac{0.0025}{7} = 0.000357$$

The magnetic circuit of the two-pole generator (Fig. 7-23) can be considered as two series circuits in parallel with each other. The total reluctance of this type of circuit is obtained by first solving for the reluctance of each series circuit and then substituting these values in Eq. (5-11).

EXAMPLE 5-16 A shell-type transformer core having the dimensions shown in Fig. 5-34 uses silicon-steel laminations having a permeability of 400. What is the total reluctance of the magnetic circuit?

GIVEN: Fig. 5-34 $\quad\mu = 400$

FIND: $\mathfrak{R}_T$

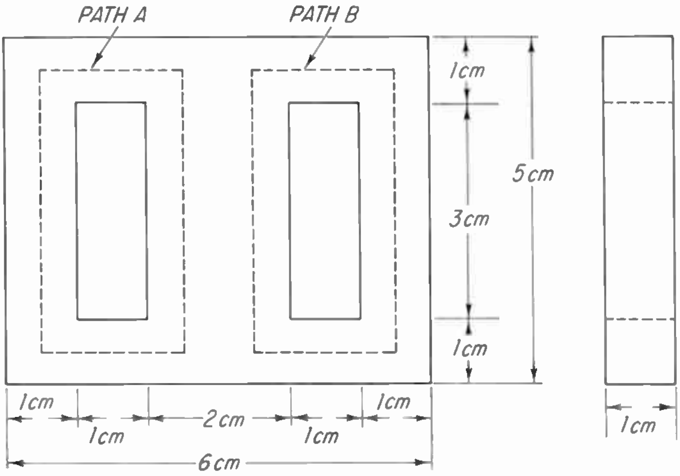


Fig. 5-34

SOLUTION:

$$\nu = \frac{1}{\mu} = \frac{1}{400} = 0.0025$$

Average length of each path = 4 + 4 + 2 + 2 = 12 cm

$$\mathcal{R} \text{ of each path} = \frac{\nu l}{A} = \frac{0.0025 \times 12}{1 \times 1} = 0.03$$

$$\mathcal{R}_T = \frac{\mathcal{R}_A \mathcal{R}_B}{\mathcal{R}_A + \mathcal{R}_B} = \frac{0.03 \times 0.03}{0.03 + 0.03} = 0.015$$

EXAMPLE 5-17 How many ampere-turns would be required of a coil on a ring magnet similar to Fig. 5-20*a* that is to have a flux of 28,800 lines? The average length of the magnetic path is 20 cm, its cross section is 2 sq cm, and it is made of transformer-grade steel, whose characteristics are shown in Fig. 5-31.

GIVEN: $\phi = 28{,}800$ lines $l = 20$ cm $A = 2$ sq cm
Material = transformer-grade steel

FIND: NI

SOLUTION:

$$B = \frac{\phi}{A} = \frac{28{,}800}{2} = 14{,}400 \text{ gausses}$$

$$H = 20 \text{ oersteds (from Fig. 5-31)}$$

$$NI = NI/\text{cm} \times l = 0.796 \times 20 \times 20 = 318 \text{ amp-turns}$$

Note: 0.796 × oersteds = NI per cm.

EXAMPLE 5-18 A coil having 500 turns and carrying 2 ma is wound on a core having the configuration and dimensions shown in Fig. 5-34. The unit is to be used in a circuit in which the flux density will be very low, that is, less than 1,000 lines per square centimeter. Under these conditions, calculations are based on the principle that the initial permeability of the core material, in this case 475, will be constant in this low-flux density range. Determine the flux in each path; consider the core as having three legs and two equal parallel paths.

GIVEN: Fig. 5-34 $N = 500$ $\mu = 475$ $I = 2$ ma

FIND: ϕ center leg ϕ each outside leg

SOLUTION:

Average length of each path = 4 + 4 + 2 + 2 = 12 cm

Cross-sectional area of each path = 1 × 1 = 1 sq cm

$$\mathcal{R} \text{ of each path} = \frac{\nu l}{A} = \frac{0.0021 \times 12}{1} = 0.0252$$

$$\text{where } \nu = \frac{1}{\mu} = \frac{1}{475} = 0.0021$$

$$\mathcal{R}_T = \frac{1}{\frac{1}{\mathcal{R}_A} + \frac{1}{\mathcal{R}_B}} = \frac{1}{\frac{1}{0.0252} + \frac{1}{0.0252}} = \frac{0.0252}{2} = 0.0126$$

$$F = 1.26NI = 1.26 \times 500 \times 2 \times 10^{-3} = 1.26 \text{ gilberts}$$

$$\phi \text{ center leg} = \frac{F}{\mathcal{R}_T} = \frac{1.26}{0.0126} = 100 \text{ maxwells}$$

$$\phi \text{ each outside leg} = \frac{F}{\mathcal{R}_p} = \frac{1.26}{0.0252} = 50 \text{ maxwells}$$

5-17 Relays

The Basic Relay. A relay is basically an electromagnetic switch that is operated by a variation in the conditions in one electric circuit and that changes the operation of other devices in the same or in other electric circuits. The fundamental components of a common type of relay are shown in Fig. 5-35. Whenever current flows through the coil, the core becomes magnetized. When the current is of sufficient intensity to produce a magnetic force that is strong enough to overcome the counterforce of the spring, the armature is pulled down, thus closing the two contacts. The length of the air gap between the armature and the core can be varied by adjusting the back rest on the armature. Decreasing this distance produces a greater magnetic pull for the same ampere-turns; however, it also reduces the distance the armature must travel from *rest position* to *operate position*. A closed path for the magnetic flux is provided by the core, the air gap, the armature, and the frame. In order to secure maximum concentration of flux at the air gap, the three magnetic components are made of materials having a high permeability, such as soft iron.

Vibrating Bell. The familiar vibrating bell (Fig. 5-36) is the forerunner of the basic relay. In the bell, the contacts are included in the circuit

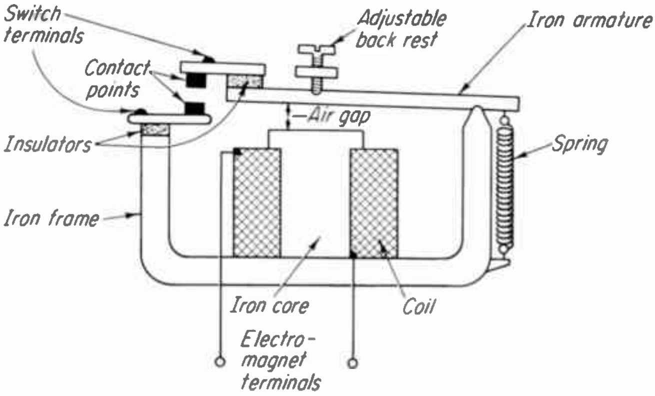


Fig. 5-35 Basic relay.

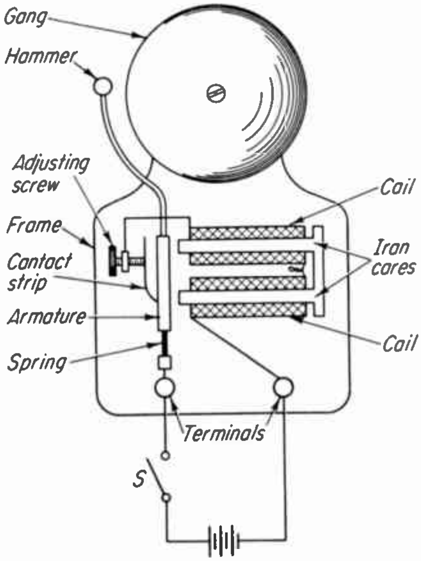


Fig. 5-36 Vibrating bell.

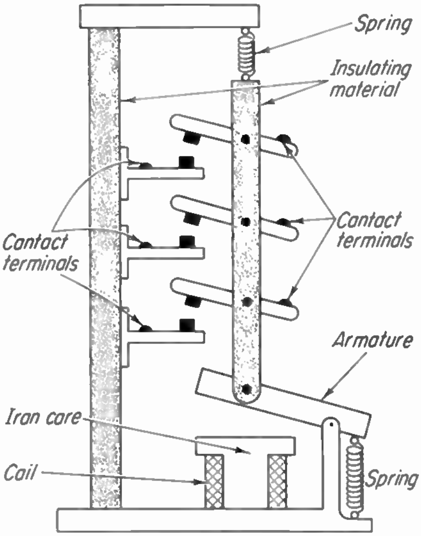


Fig. 5-37 Multiple contacts of a relay.

of the electromagnet. When switch S is closed, the two iron cores are energized by the electron flow through the coils and attract the armature to them. This movement of the armature causes the hammer to strike the gong and also causes the contact between the adjustable screw and the contact strip to open, thus breaking the circuit. Since current no longer flows in the coils, the two cores are no longer energized. They therefore release the armature, and the spring pulls it back to its rest position. The two contacts are now closed and in their original rest position. This cycle is repeated for as long as switch S is closed.

Multiple Contacts. The basic relay and the vibrating bell operated only one set of contacts. Relays are available that can close or open as many as 100 sets of contacts. These types of relay use multiple sets of contacts as illustrated in Fig. 5-37. Each movable contact is called a *pole*, and the relay of Fig. 5-35 is called a *single-pole* relay, abbreviated SP; the relay of Fig. 5-37 is a three-pole relay, abbreviated 3P.

Contact Arrangements. The contacts in a relay may be arranged to open or close in a number of different ways. A combination of a stationary contact and a movable contact that are disengaged when the coil is unenergized is referred to as being *normally open*, abbreviated NO. A combination of a stationary contact and a movable contact that are engaged when the coil is unenergized is referred to as being *normally closed*, abbreviated NC. Normally open and normally closed relays are called *single-throw contacts*, abbreviated ST. Relay contact notation is usually given in the following order: (1) poles, (2) throw, and (3) normal position. The relay of Fig. 5-35 is a single-pole single-throw relay with the contacts normally open, abbreviated SPSTNO. The vibrating bell of Fig. 5-36 is a single-pole single-throw relay with the contacts normally closed, abbreviated SPSTNC.

A combination of two stationary contacts and a movable contact, which engages one of them when the coil is unenergized and the other when the coil is energized, is called a *transfer* or *double-throw contact*, abbreviated DT. In the contact arrangement of Fig. 5-38*a*, electric contact is transferred from the NO contacts to the NC contacts when the relay is energized. Since the NO contacts must close before the NC contacts open, this arrangement is called a *make-before-break* or simply *make-break contacts*. In the contact arrangement of Fig. 5-38*b*, electric contact is transferred from the NC contacts to the NO contacts when the relay is energized. Since the NC contacts must open before the NO contacts close, this arrangement is called *break-make contacts*.

A contact arrangement in which a movable contact simultaneously makes and breaks connections between two stationary contacts is called *double-break contacts*, abbreviated DB (items *U* to Z, Fig. 5-39). For normally open contacts, a similar arrangement would be called *double-make contacts*, abbreviated DM.

NARM Nomenclature. Relay contact descriptions become quite lengthy and cumbersome when complex arrangements and multiple sets of poles are used. To simplify these descriptions the National Association of Relay Manufacturers (NARM) has adopted a standard nomenclature for 18 basic contact forms (Fig. 5-39) with each arrangement having a letter of the alphabet designation. A complete description of multiple-pole contact arrangements is obtained by prefixing the letter designating the basic arrangement used with a number indicating the number of contact groups. Thus, the relay shown in Fig. 5-37, 3PSTNO, is referred to as 3A.

Resistance-Capacitance Filters. When contacts are separated, an arc is formed between them. As the contacts move farther apart, the arc will stretch out and finally break. The spacing at which the arc is extinguished

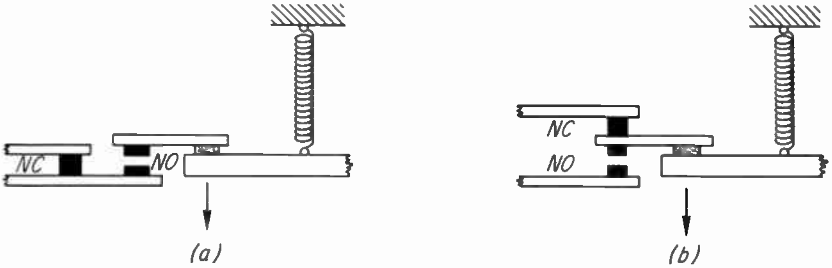


Fig. 5-38 Double-throw transfer contacts. (a) Make-break arrangement. (b) Break-make arrangement.

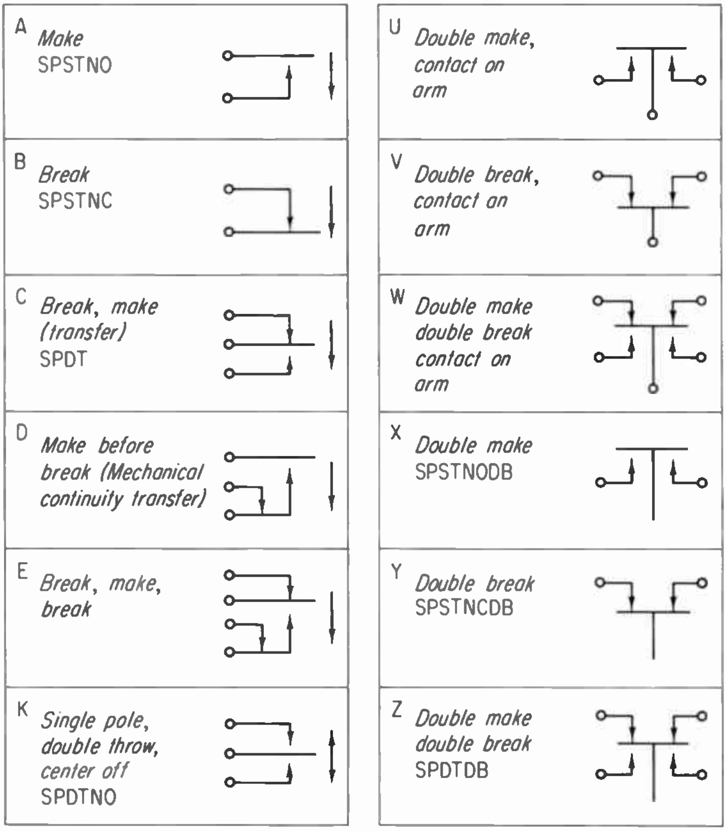


Fig. 5-39 NARM nomenclature for basic relay circuits. (*Automatic Electric Company*)

is determined by the load current and the source voltage. The final normal open distance between contacts should be large enough to extinguish the arc. Since this arc produces heat, which may damage the contacts and may also interfere with their operation, various means have been devised to minimize the amount of arcing between contacts.

One of the methods used to suppress the formation of arcing between contacts is to use a resistance-capacitance filter circuit as shown in Fig. 5-40. When the relay is not energized, its contacts will be open and the load will be inoperative; also, the capacitor will become charged to the voltage of the controlled circuit. When the relay is energized, its contacts will close and the load will become operative; also, any charge on the capacitor will be dissipated through the resistor and the relay contacts. If the relay controlling circuit is opened, the relay will become unenergized and the relay contacts will open. The instant the contacts open, they are short-circuited by the uncharged capacitor and the resistor, and arc formation is prevented.

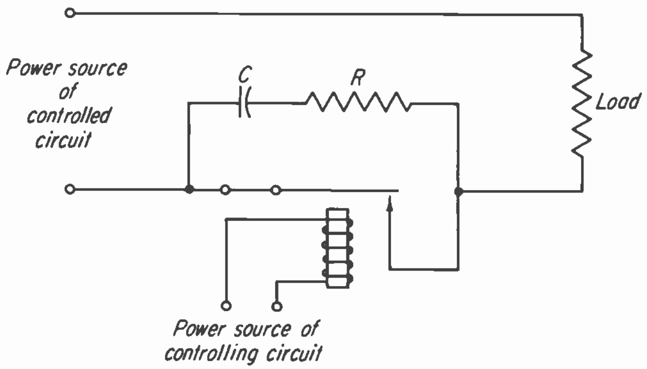


Fig. 5-40 Relay with a resistance-capacitance filter circuit.

As the charge on the capacitor increases, the voltage across the contacts also increases. However, by the time the voltage across the contacts reaches an amount sufficient to cause an arc, the contacts will have opened far enough to prevent the formation of an arc.

QUESTIONS

1. What is the relative importance of magnetism and electricity to the study of electronics?
2. Define (*a*) magnet, (*b*) magnetic materials.
3. What is meant by (*a*) a natural magnet? (*b*) An artificial magnet?
4. Distinguish between permanent and temporary magnets.
5. (*a*) Where are temporary magnets generally used? (*b*) Where are permanent magnets generally used?
6. (*a*) Describe the poles of a magnet. (*b*) By what name is each pole known?
7. Describe two applications of the compass.
8. Explain the molecular theory of magnetism.
9. State the law of magnetic polarity.
10. How does the distance between two magnets affect the force of attraction between them?
11. (*a*) What is meant by the magnetic field? (*b*) How may the field of a magnet be illustrated?
12. What are magnetic lines?
13. State four rules to which the actions of magnetic lines conform.
14. What are lines of force?
15. Distinguish between lines of induction and lines of force.
16. (*a*) Define field intensity. (*b*) What is its unit of measurement?
17. (*a*) Define flux density. (*b*) What is its unit of measurement?
18. Explain why a magnet draws a piece of magnetic material to it?
19. Describe several methods of inducing magnetism in a steel bar.
20. Define the following properties of magnets: (*a*) reluctance, (*b*) permeability, (*c*) retentivity.

21. (*a*) Define reluctivity. (*b*) With what electrical term does it compare?
22. (*a*) Define permeance. (*b*) What is its relation to reluctance?
23. Give the meaning of ferromagnetic, paramagnetic, and diamagnetic as they apply to classification of materials.
24. (*a*) What is Permalloy? (*b*) What are its characteristics?
25. (*a*) What is alnico? (*b*) What are its characteristics?
26. Name three shapes of magnets, and give some uses of each.
27. State some precautions to be observed in the use of magnets.
28. (*a*) What was Oersted's discovery? (*b*) To what important uses is it applied?
29. What effect does a change in the amount of current flowing in a conductor have upon the magnetic field?
30. What effect does a change in the direction of current flow in a conductor have upon the magnetic field?
31. State the left-hand rule for a wire carrying a current.
32. What is a solenoid?
33. What factors determine the magnetic strength of a solenoid?
34. State the left-hand rule for a coil.
35. Describe the following magnetic core materials: (*a*) laminated, (*b*) metallic powder, (*c*) ceramic powder.
36. Compare the equations for a magnetic circuit with those for an electric circuit.
37. In what manner does the reluctance of a magnetic circuit differ from the resistance of an electric circuit?
38. (*a*) What is meant by a *B-H* curve? (*b*) For what purposes are d-c magnetization curves used?
39. Define the following terms: (*a*) hysteresis, (*b*) hysteresis loop, (*c*) residual induction, (*d*) coercive force.
40. (*a*) Describe eddy currents and their effects. (*b*) Describe two methods used to minimize the effects of eddy currents.
41. Describe a series magnetic circuit.
42. Describe a parallel magnetic circuit.
43. How is the reluctance of a magnetic circuit affected (*a*) by the length of the circuit? (*b*) By the cross-sectional area of the magnetic path?
44. How do the number of turns and the current affect the magnetomotive force of an electromagnet?
45. How are the reluctances of a series magnetic circuit added?
46. How are the reluctances of a parallel magnetic circuit added?
47. Describe the construction and operation of a basic relay.
48. Describe the construction and operation of a vibrating bell.
49. Define the following: (*a*) multiple-contact relay, (*b*) pole, (*c*) SP relay, (*d*) 6P relay.
50. Define the following: (*a*) NO, (*b*) NC, (*c*) ST, (*d*) DT.
51. Define the following: (*a*) transfer contacts, (*b*) make-break contacts, (*c*) break-make contacts.
52. What is meant by (*a*) double-break contacts? (*b*) Double-make contacts?
53. (*a*) What is the NARM nomenclature? (*b*) What is its purpose?
54. What is the purpose of a resistance-capacitance filter as used with relays?
55. Describe the circuit arrangement and operation of a resistance-capacitance filter.

PROBLEMS

1. Draw a diagram showing how a bar magnet can be used to magnetize a steel rod by stroking the rod with the bar magnet. Indicate the area of the steel rod that must be stroked by one pole of the magnet and the area that must be stroked by the other pole. Indicate the polarity of the magnet and also the polarity that the steel rod will have after the stroking operation is completed.
2. If two steel bars were found on a workbench and their magnetic characteristics were not known, describe a simple method of testing whether (*a*) neither bar was a magnet, (*b*) both bars were magnets, (*c*) only one bar was a magnet.
3. If it was found that only one of the two bars of Prob. 2 was a magnet, describe a method to determine (*a*) which bar is the magnet, (*b*) which of its poles is the north pole, (*c*) which of its poles is the south pole.
4. Draw a diagram of two bar magnets in the relative positions shown in Fig. 5-41, and sketch the magnetic field that will result.
5. Draw a diagram of the two bar magnets in the relative positions shown in Fig. 5-42, and sketch the magnetic field that will result.
6. Sketch the magnetic field of the permanent magnet shown in Fig. 5-18*b*.
7. Sketch the magnetic field of the magnet and the soft steel bar shown in Fig. 5-43. What is the polarity of the bar at *A* and at *B*?
8. A magnetic north pole of 250-unit pole strength and a south pole of 100-unit pole strength are placed 10 cm apart in air. (*a*) What force in dynes is acting on the poles? (*b*) Is the force attraction or repulsion?
9. Two magnetic north poles of 500- and 250-unit poles, respectively, are placed 15 cm apart in air. (*a*) What force in dynes is acting on the poles? (*b*) Is the force attraction or repulsion?
10. What is the distance between two magnetic poles of like polarity having strengths of 100- and 200-unit poles, respectively, if the force acting on the poles is 2,000 dynes?
11. How far apart must two magnetic poles of unlike polarity of 250-unit pole strength each be spaced in air if the force of attraction is to be 3,000 dynes?

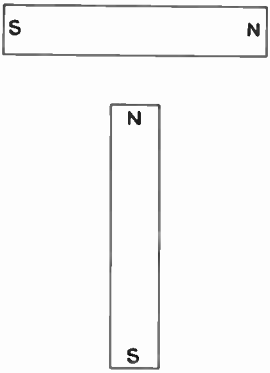


Fig. 5-41

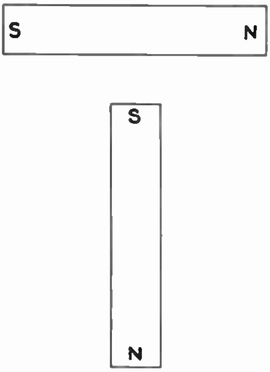


Fig. 5-42

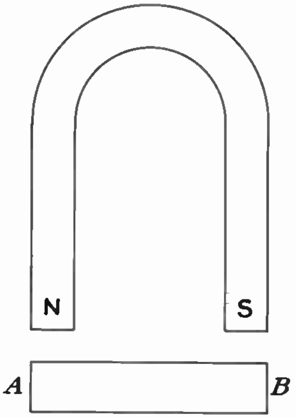


Fig. 5-43

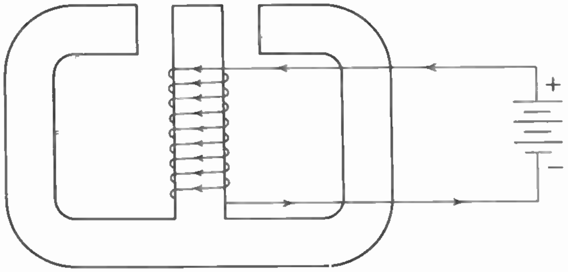


Fig. 5-44

12. If two poles of similar strength are to have a force of attraction of 1,500 dynes through a distance of 6 cm, what is the strength of each pole?
13. What strength is required of a magnet if it is to exert a repulsion force of 6,250 dynes upon a magnet of 250-unit poles that is 2 cm away?
14. If each pole of the horseshoe magnet of Fig. 5-43 has a strength of 250-unit poles, what is the total force acting on the bar *AB* when the distance between the bar and the magnet is (*a*) 2 cm? (*b*) 1 cm? (*c*) ½ cm?
15. What is the force in dynes on a magnet of 75-unit pole strength when placed in a field whose intensity is 100 lines per square centimeter?
16. What is the force in ounces on a magnet of 125-unit poles when placed in a field whose intensity is 3,000 oersteds?
17. If a magnet of 40-unit pole strength is to be acted upon with a force of 3 oz when placed in a magnetic field, what must be the intensity of the field?
18. What is the strength of a magnet that is acted upon with a force of ¼ lb when placed in a field whose intensity is 6,450 lines per square inch?
19. What is the flux density in gausses of a magnet that has a cross-sectional area of 4 sq cm and a flux of 5,000 maxwells?
20. What is the flux density in lines per square inch of a magnet that has a cross-sectional area of 1½ by 1½ sq in. and that has a flux of 50,000 maxwells?
21. What cross-sectional area must a magnet have if it must carry a flux of 250,000 maxwells and the flux density is to be 50,000 lines per square inch?
22. If a magnet is to be made of an iron rod (round) and is to carry a flux of 980 maxwells, what diameter rod must be used if the flux density is to be 5,000 lines per square inch?
23. How many lines of force extend outward from a pole of 50-unit pole strength?
24. (*a*) How many lines of force extend outward from a pole of 75-unit pole strength? (*b*) What is the flux density 1 cm away? 2 cm away? 4 cm away? (*c*) What force will it exert on a unit pole 1 cm away? 2 cm away? 4 cm away?
25. Sketch the magnetic field of the loudspeaker magnet shown in Fig. 5-18*a*. Describe the action (*a*) when a current is flowing inward at the left-hand lead and (*b*) when a current is flowing outward at the left-hand lead.
26. Sketch the magnetic field of the electromagnet shown in Fig. 5-44, and indicate the north and south poles.
27. A horseshoe magnet made of transformer-grade steel has a cross-sectional area of 2 by 4 sq cm and carries a flux of 32,000 maxwells. (*a*) What is its flux density? (*b*) What is its permeability (see curve, Fig. 5-30)? (*c*) What is its reluctivity?

28. A horseshoe magnet made of transformer-grade steel has a cross-sectional area of 1.5 by 2 sq cm and carries a flux of 3,000 maxwells. (*a*) What is its flux density? (*b*) What is its permeability (see curve, Fig. 5-30)? (*c*) What is its reluctivity?
29. The magnet of Prob. 27 has an average length of 24 cm. (*a*) What is the reluctance of the magnet? (*b*) What is its permeance?
30. The magnet of Prob. 28 has an average length of 12 cm. (*a*) What is the reluctance of the magnet? (*b*) What is its permeance?
31. (*a*) What magnetomotive force is required to push the flux through the magnetic circuit of Prob. 29? (*b*) How many ampere-turns are required for a coil to supply this magnetic circuit?
32. (*a*) What magnetomotive force is required to push the flux through the magnetic circuit of Prob 30? (*b*) How many ampere-turns are required for a coil to supply this magnetic circuit?
33. (*a*) If the coil of Prob. 31 has 40 turns, how much current would be required? (*b*) If this coil is connected in a circuit in which 56 ma is flowing, how many turns would the coil need?
34. (*a*) If the coil of Prob. 32 has 100 turns, how much current would be required? (*b*) If this coil is connected in a circuit in which 50 ma is flowing, how many turns would the coil need?
35. A piece of transformer-grade steel 1.5 cm in diameter and 10 cm in length carries 17,700 maxwells. (*a*) What is the reluctance? (*b*) How many ampere-turns are required for a coil to supply this magnetic circuit?
36. A bar of transformer-grade steel 0.5 inch square and 2 inches long carries 3,000 maxwells. (*a*) What is its reluctance? (*b*) How many ampere-turns are required to supply this magnetic circuit?
37. A magnetic core made of field-grade silicon-steel sheets, whose characteristics are shown in Fig. 5-31, is 0.8 cm in diameter and 4 cm in length and carries 5,000 maxwells. (*a*) What is its reluctance? (*b*) If the core is wound with a coil drawing 63.5 ma, how many turns does the coil have?
38. A magnetic core made of motor-grade silicon-steel sheets, whose characteristics are shown in Fig. 5-31, is 6 cm square and 8 cm in length and requires a magnetizing force of 70 oersteds. (*a*) What is the reluctance of the circuit? (*b*) If the core is wound with a coil having 1,000 turns, how much current flows through the coil?
39. The electromagnet shown in Fig. 5-45 is made of transformer-grade silicon steel,

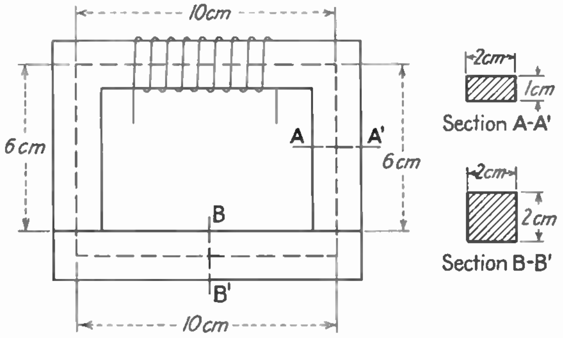


Fig. 5-45

whose characteristics are shown in Fig. 5-30, and carries a flux of 24,000 maxwells. What is the reluctance of (*a*) the U-shaped magnet? (*b*) The straight bar? (*c*) The complete circuit?

40. The electromagnet shown in Fig. 5-45 is made of motor-grade steel, whose characteristics are shown in Fig. 5-31, and carries a flux of 36,000 maxwells. What is the reluctance of (*a*) the U-shaped magnet? (*b*) The straight bar? (*c*) The complete circuit?
41. The coil of the electromagnet of Prob. 39 is connected in a circuit whose current is 250 ma. How many turns are required on the coil?
42. The coil of the electromagnet of Prob. 40 has 2,000 turns. How much current is required to supply this magnetic circuit?
43. The coil of the electromagnet of Prob. 41 is wound with No. 28 copper wire. (*a*) What is the resistance of the winding if the average length of turn is 2.5 inches? (*b*) What voltage is needed to produce the required current?
44. The coil of the electromagnet of Prob. 42 is wound with No. 22 copper wire. (*a*) What is the resistance of the winding if the average length of turn is 2.5 inches? (*b*) What voltage is needed to produce the required current?
45. When the straight bar is separated from the U-shaped magnet of Prob. 39, additional reluctances in the form of air gaps are introduced at each pole. If the air gap at each pole is ⅛ cm, find (*a*) the reluctance of each air gap (permeability of air = 1), (*b*) the reluctance of the complete circuit, (*c*) the magnetomotive force required by the magnetic circuit.
46. When the straight bar is separated from the U-shaped magnet of Prob. 40, additional reluctances in the form of air gaps are introduced at each pole. If the air gap at each pole is 3/16 cm, find (*a*) the reluctance of each air gap (permeability of air = 1), (*b*) the reluctance of the complete circuit, (*c*) the magnetomotive force required by the magnetic circuit.
47. The coil of the electromagnet of Prob. 45 is connected in a circuit carrying 1.5 amp. (*a*) How many turns are required of the coil? (*b*) If 1 amp per 400 cm is permitted, what size copper wire should be used? (*c*) What is the resistance of the coil if the average length per turn is 2.5 inches? (*d*) What voltage is needed to produce the required current?
48. The coil of the electromagnet of Prob. 46 is connected in a circuit carrying 3.0 amp. (*a*) How many turns are required of the coil? (*b*) If 1 amp per 400 cm is permitted, what size copper wire should be used? (*c*) What is the resistance of the coil if the average length per turn is 2.5 inches? (*d*) What voltage is needed to produce the required current?
49. A series magnetic path consisting of three reluctances of 0.0065, 0.0032, and 0.0018 is to carry a flux of 5,000 maxwells. (*a*) What is the reluctance of the circuit? (*b*) How many ampere-turns are required to produce the flux?
50. A series magnetic path consisting of five reluctances of 0.00025, 0.00125, 0.0025, 0.0075, and 0.0050 is to carry a flux of 8,000 maxwells. (*a*) What is the reluctance of the circuit? (*b*) How many ampere-turns are required to produce the flux?
51. The magnetic circuit of a ring magnet as used in an electrical measuring instrument has the configuration shown in Fig. 5-33 and the following dimensions: OD = 8 cm, ID = 5 cm, thickness = 1 cm, diameter of the soft-iron core = 1.5 cm, each air gap = 0.15 cm. The permeability of the ring magnet, which is made of alnico, is 30,000, and the permeability of the soft-iron core is 3,000. Determine the reluctance of the magnetic circuit.

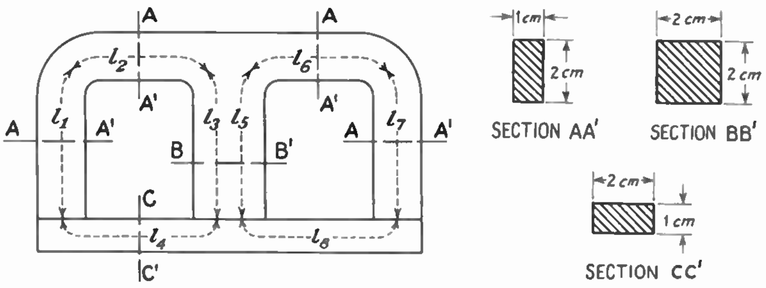


Fig. 5-46

52. The magnetic circuit of a ring magnet as used in an electrical measuring instrument has the configuration shown in Fig. 5-33 and the following dimensions: OD = 6 cm, ID = 5 cm, thickness = 1 cm, diameter of the soft-iron core = 1 cm, each air gap = 0.1 cm. The permeability of the ring magnet, which is made of Permalloy, is 25,000, and the permeability of the soft-iron core is 5,000. Determine the reluctance of the magnetic circuit.
53. A parallel magnetic circuit contains reluctances of 0.005, 0.0025, and 0.0075. (*a*) What is the reluctance of the circuit? (*b*) How many ampere-turns are required if the total flux is to be 15,000 maxwells?
54. A parallel magnetic circuit contains reluctances of 0.0005, 0.0015, 0.00025, 0.0003, and 0.0005. What is the reluctance of the circuit?
55. The electromagnet shown in Fig. 5-46 is made of motor-grade steel and carries a flux of 26,000 maxwells in sections AA' and CC' and 52,000 maxwells in section BB'. The length of paths indicated as l_1 to l_8 is 5 cm each. Assume that the bar fits tightly against the poles of the magnet. (*a*) What is the reluctance of each path? (*b*) How many ampere-turns are required for a coil to be wound around the center pole?
56. The electromagnet shown in Fig. 5-46 is made of field-grade steel and carries a flux of 25,000 maxwells in sections AA' and CC' and 50,000 maxwells in section BB'. The length of paths indicated as l_1 to l_8 is 7.5 cm each. Assume that the bar fits tightly against the poles of the magnet. The area of AA' is 1 by 1.5 cm, BB' is 2 by 1.5 cm, CC' is 1.5 by 1.5 cm. (*a*) What is the reluctance of each path? (*b*) How many ampere-turns are required for a coil to be wound around the center pole?
57. The coil in Prob. 55 is to carry 0.50 amp. (*a*) How many turns are required on this winding? (*b*) What size wire is to be used with 600 cm per amp? (*c*) What voltage is needed to produce the required current if the average length per turn is 3.5 inches?
58. The coil in Prob. 56 is to carry 0.25 amp. (*a*) How many turns are required on this winding? (*b*) What size wire is to be used with 400 cm per amp? (*c*) What voltage is needed to produce the required current if the average length per turn is 3.0 inches?
59. Repeat Prob. 55 for a condition when the bar is ⅛ cm away from the poles of the electromagnet.
60. Repeat Prob. 56 for a condition when the bar is 3/16 cm away from the poles of the electromagnet.

Chapter 6

Meters

Electrical instruments are essential in order that the amount of voltage, current, resistance, power, etc., in a circuit can be measured, adjusted, or controlled. There are so many types of electric meters that a complete description of each would not be practical in this text. Therefore, only those types of meters that are necessary for general electrical- and electronic-circuit measurements will be considered.

6-1 Electrical Instruments

Voltage and current are the two quantities most generally measured, as it is possible to calculate other quantities such as resistance and power by using these two values. The basic mechanism in practically all meters is the *galvanometer*, which may be defined as a sensitive instrument used for detecting and/or measuring very small amounts of electric current. Meters may be classified according to their principle of operation as (1) electrostatic, (2) electrothermal, and (3) electromagnetic.

6-2 Electrostatic Meters

Electrostatic meters are used for measuring high voltages and are based on the theory that charged bodies of like polarity repel one another and those of unlike polarity attract one another. The basic principle of such a meter is illustrated by Fig. 6-1. Two hollow cylinders are connected by a pivoted bar that also supports the pointer, and when a difference of potential is applied at AB, the two plates P_1 and P_2 become charged and a force of attraction is set up causing the bar to move. The amount of motion of the pointer is indicated on a scale calibrated in volts. The electrostatic mechanism is the only one used for electrical indications that functions because of the presence of a voltage. All other mechanisms are current-sensitive, producing pointer movements due to current flowing through the instrument circuit. The electrostatic voltmeter is useful on a-c circuits, where the current taken by other mechanisms would result in distorted values because of the IR drop of the instrument.

6-3 Electrothermal Meters

Hot-wire Type. Hot-wire meters depend upon the expansion and contraction of a wire carrying an electric current. A typical meter of this type

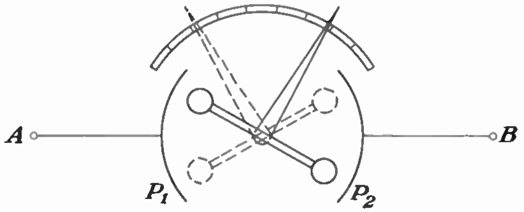


Fig. 6-1 Basic principle of the electrostatic voltmeter. (*Weston Instruments & Electronics, Division of Daystrom, Inc.*)

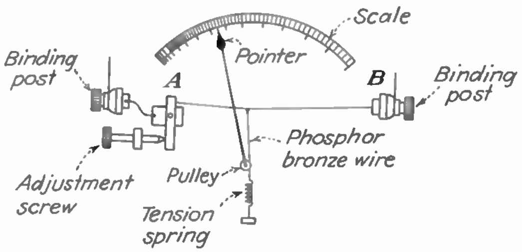


Fig. 6-2 Principle of the hot-wire meter. (*Weston Instruments & Electronics, Division of Daystrom, Inc.*)

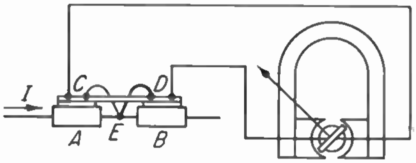


Fig. 6-3 Principle of the thermocouple meter. (*Weston Instruments & Electronics, Division of Daystrom, Inc.*)

is shown in Fig. 6-2. The current to be measured is passed through a platinum-alloy wire *AB*. A phosphor-bronze wire is arranged with one end attached to the wire *AB*, one turn of the wire wound around the pulley supporting the needle, and the other end attached to a tension spring. The wire sags as it becomes hot and causes the pointer to move across the scale. The purpose of the spring is to maintain a steady pull so that the movement of the pointer is proportional to the lengthening of the wire. The expansion of the wire is proportional to the heat, and as the heat is proportional to the current, the scale can be calibrated to indicate amperes. As the voltage is directly proportional to the current, the scale can also be calibrated to indicate volts.

Thermocouple Type. A practical application of the thermal effect of electron flow is obtained by using the principle of the thermocouple. A typical meter of this type is shown in Fig. 6-3. The current to be measured is forced to flow through *AB* and in so doing flows through the junction of the two dissimilar metals *EC* and *ED*. The heat produced by the current in *AB* causes a direct voltage to be produced across *CD*. This voltage forces a current through a very sensitive moving-coil instrument. The movement of the pointer is calibrated to indicate the amount of current flowing in *AB*.

Uses of Thermal-type Meters. Thermo instruments measure the effective values of current and therefore can be used with both direct and alternating currents. They are used where the wave form and/or high values of frequency cause errors in other types of meters. Instruments giving accurate readings at frequencies up to 100 mc can be obtained (Fig. 6-4).

The amount of heat that is developed in a circuit varies as the square of the current. Consequently, on the meter scale, the divisions for equal changes of current will be farther apart at the high readings than at the low readings.

6-4 Permanent-magnet Moving-coil Meters

Most commercial meters are based on the electromagnetic principle. These meters may be further classified as (1) permanent magnet, (2) iron vane, (3) dynamometer. As the name implies, the operation of the permanent-magnet movable-coil meter depends upon the presence of a magnetic field which acts upon a movable element. In this type of instrument (Fig. 6-5), a coil of wire *C*, wound on a cylindrical frame of aluminum, carries the current to be measured. The frame is so pivoted that it will revolve freely between the soft-iron core *D* and the pole pieces *N* and *S* of the permanent magnet *P*. A pointer, which is attached to the aluminum frame, moves across the scale, thus indicating the deflection made. This type of meter is applicable only to d-c measurements, as no cumulative resultant force is set up when alternating current flows in coil *C*. Instruments of this type may be calibrated to indicate current, or if they are connected in series with a proper resistance, voltages may be measured. Most galvanometers use the permanent-magnet moving-coil type of mechanism (Fig. 6-6).

Fig. 6-4 Commercial multirange thermocouple meter. (*Weston Instruments & Electronics, Division of Daystrom, Inc.*)

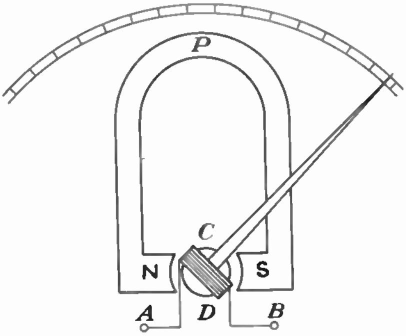


Fig. 6-5 Essential parts of the permanent-magnet moving-coil meter.

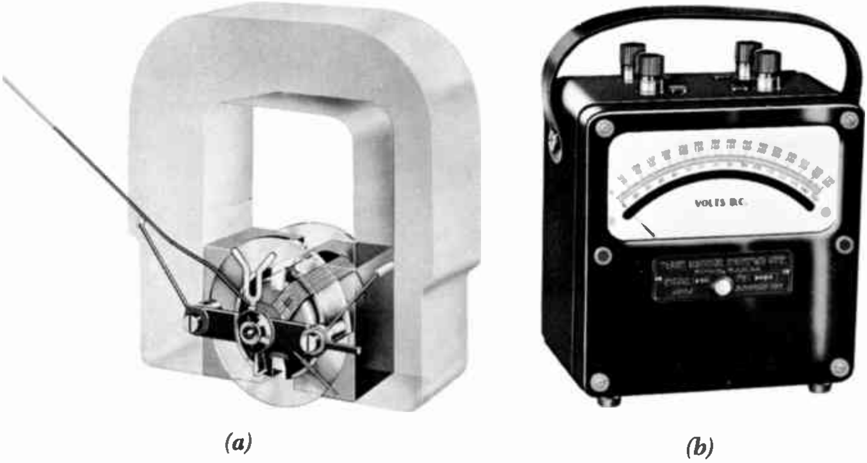


Fig. 6-6 Permanent-magnet moving-coil meter. (a) Mechanism of a commercial meter. (b) Commercial meter using mechanism shown in (a). (*Weston Instruments & Electronics, Division of Daystrom, Inc.*)

6-5 Iron-vane Meters

Thomson Inclined-coil Instrument. The Thomson inclined-coil instrument illustrated in Fig. 6-7 consists of an energizing coil *A*, which is located about 45° from the horizontal, and a rotating rectangular piece of thin soft iron mounted on a shaft. Whenever the coil *A* is energized, the soft-iron vane *V* seeks to place itself parallel with the magnetic flux of the coil; the shaft is then forced to rotate, causing the pointer *P* to indicate the deflection on a calibrated scale. This type of instrument may be used as an ammeter or voltmeter for either direct or alternating currents. However, it is generally used for measuring alternating currents. The deflection of the pointer will vary as the square of the power, thus making the scale irregular.

Radial- and Concentric-type Movable Iron Vanes. If two similar adjacent iron bars are similarly magnetized, a repelling force is developed between them which tends to move them apart. In the moving iron-vane mechanism, this principle is used by fixing one bar in space and pivoting the second so that it will tend to rotate when current flows through the

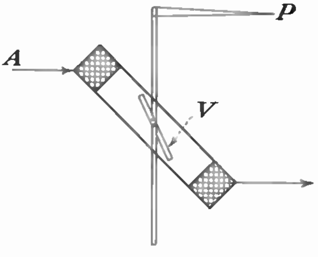
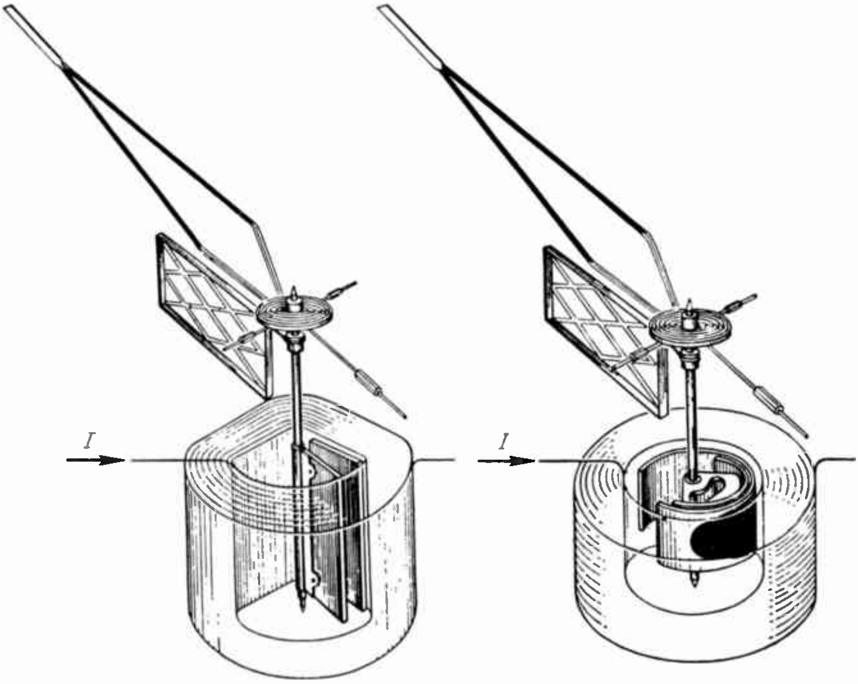


Fig. 6-7 Essential parts of the Thomson inclined-coil instrument.

coil. A spring attached to the moving vane opposes its motion and permits the scale to be calibrated in terms of the current flowing. The movable iron vane may be of the radial type (Fig. 6-8*a*) or of the concentric type (Fig. 6-8*b*). Comparing these two types, the radial vane is the more sensitive and has almost a linear scale. The concentric vane causes the pointer to move by the square law, but its vanes can be shaped to secure special scale characteristics; its shorter magnetic vanes result in smaller errors. Both types can be used as ammeters or voltmeters to measure either direct or alternating currents. However, they are generally used for measuring alternating currents.

6-6 Electrodynamometer Meters

The electrodynamometer-type meter is composed of two stationary coils and a movable coil which also supports the pointer. A diagram illustrating the basic principle of this type of instrument is shown in Fig. 6-9. When current flows in all the meter windings, the movable coil C tends to place itself parallel to the two stationary coils S_1 and S_2 but is opposed by the action of a spiral spring. This type of meter can be used for a-c as well as d-c measurements. When it is used as an ammeter or voltmeter, all coils



(a) (b)

Fig. 6-8 Moving iron-vane mechanisms. (a) Radial type. (b) Concentric type. (Weston Instruments & Electronics, Division of Daystrom, Inc.)

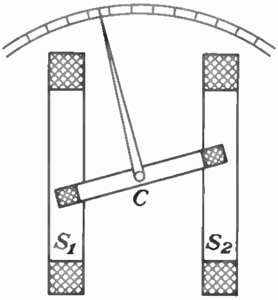


Fig. 6-9 Essential parts of the dynamometer-type instrument.

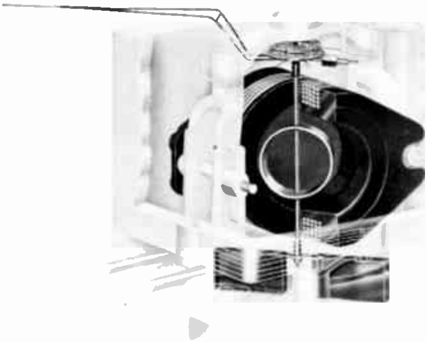


Fig. 6-10 Cutaway view illustrating the construction of a commercial type of dynamometer mechanism. (*Weston Instruments & Electronics, Division of Daystrom, Inc.*)

are connected in series; when it is used as a wattmeter, the stationary coils form the current element and the movable coil the voltage element.

Distortion of alternating currents does not occur because of the complete absence of magnetic materials such as iron, and the indications are true effective values. As the mechanism is current sensitive, the pointer moves because of current flowing through turns of wire. It is the most versatile of all the basic mechanisms, since it can be used to indicate current, voltage, or power for either alternating or direct current. Using a crossed-coil movement, the power factor, phase angle, frequency, or capacitance measurements can be obtained.

6-7 Rectifier-type Meters

Rectifier-type meters are used for measuring alternating currents and voltages of small magnitude, such as milliamperes, millivolts, microamperes, and microvolts. They are used in telephone, telegraph, and signal systems; for monitoring in broadcast studios; and for measuring loss or gain on transmission lines of communications systems.

A rectifier is a device that changes an alternating current to a direct current. It operates on the principle that the resistance to current flow is high in one direction and low in the opposite direction. Metallic rectifiers used for instrument work are usually of the copper oxide type. A disk of copper is oxidized on one side by a special heat treatment. Resistance to current flow from oxide to copper is very low, while the resistance to current flow in the opposite direction is very high. If only one disk is used, an interrupted pulsating current will flow, as conditions permitting the flow of current will exist during only one-half of each cycle of the power source. In order to obtain full-wave rectification, that is, to utilize both halves of the a-c cycle, a bridge circuit is used. Such a circuit using four disks is shown in Fig. 6-11*a* and *b*. The rectified current is then used to activate a permanent-magnet moving-coil instrument. This type of instrument can be used as a voltmeter by connecting a resistance in series with the line to limit the current to the rating of the rectifier unit and the galvanometer. The rectifier-type instrument can be used to measure alternating currents of frequencies up to 20,000 cps and has an error of less than 1 per cent per 1,000 cycles when calibrated at a definite frequency.

6-8 Ammeters

How to Connect an Ammeter in a Circuit. An ammeter is an instrument used for measuring electric currents. It is always connected in series with that part of a circuit whose current is to be measured. In Fig. 6-12, ammeter 1 is connected in series with the line to indicate the line current, and ammeters 2 and 3 are connected in series with the lamp and fixed resistance, respectively, to obtain their individual currents. Being connected in series, an ammeter must carry the current passing through that part of the circuit in which it is connected.

EXAMPLE 6-1 Assume the resistance of ammeter 1 (Fig. 6-12) to be 1 ohm, the line current 10 amp, and the line voltage 120 volts. (*a*) What is the voltage drop across the ammeter? (*b*) What is the voltage applied to the lamp?

GIVEN: $V_1 = 120$ volts $A_1 = 10$ amp $R_M = 1$ ohm

FIND: (*a*) e_M (*b*) V_2

SOLUTION:

(*a*) $$e_M = A_1 R_M = 10 \times 1 = 10 \text{ volts}$$

(*b*) $$V_2 = V_1 - e_M = 120 - 10 = 110 \text{ volts}$$

Example 6-1 indicates that, because of the resistance inherent in an ammeter, the voltage at the load will be lower than the voltage at the power source by the amount of the *IR* drop at the ammeter. Consequently,

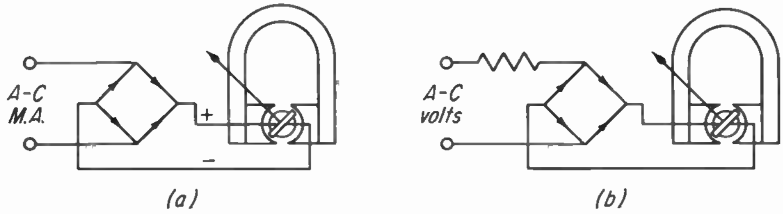


Fig. 6-11 Metallic-rectifier bridge circuits. (a) As used for a milliammeter. (b) As used for a millivoltmeter.

the resistance of ammeters is kept very low, generally so that the voltage drop at the meter does not exceed from 50 to 250 mv with rated current flowing.

Precautions in the Use of Ammeters. If an ammeter is erroneously connected across a power source instead of in series with the load whose current is to be measured, the amount of current will be so high that the meter can be badly damaged in less than 1 sec.

EXAMPLE 6-2 A certain 10-amp ammeter is designed with a 50-mv drop at rated current. (*a*) What is the resistance of the meter? (*b*) What possible current flow is indicated by Ohm's law if the meter is connected to a 110-volt power source?

GIVEN: $E = 110$ volts $I = 10$ amp $e_M = 50$ mv

FIND: (*a*) R_M (*b*) I_P

SOLUTION:

(*a*)
$$R_M = \frac{e_M}{I} = \frac{0.050}{10} = 0.005 \text{ ohm}$$

(*b*)
$$I_P = \frac{E}{R_M} = \frac{110}{0.005} = 22{,}000 \text{ amp}$$

The amount of current obtained for part (*b*) of Example 6-2 would not be reached because the winding of the meter would have burned out before this current was reached. Therefore, ***never connect an ammeter across the line.***

The current due to an unforeseen short circuit, an overload, or even the starting of a motor may be high enough to damage an ammeter if left connected in the circuit. To prevent damage due to factors that cannot be foreseen, ***the ammeter should always be protected by connecting a short-circuiting switch across it,*** as illustrated by S_1, S_2, and S_3 in Fig. 6-12. The switch is kept closed except when a reading is to be taken. If, upon opening the switch, the needle swings backward or completely across the meter scale, it should be closed instantly to prevent damaging the meter. The

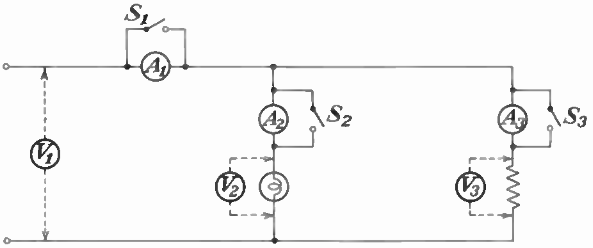


Fig. 6-12 Correct method of connecting ammeters and voltmeters.

necessary changes to the circuit should then be made before the switch is again opened. If the meter needle moves backward, it indicates a reversal of polarity, and the lead-in wires should be interchanged to make it move in the proper direction.

6-9 Voltmeters

How to Connect a Voltmeter in a Circuit. A voltmeter is an instrument used for measuring voltage. It is always connected across that part of the circuit whose voltage is to be measured. Figure 6-12 shows the correct way to connect voltmeters in a parallel circuit. Voltmeter 1 indicates the line voltage, voltmeter 2 indicates the voltage across the lamp, and voltmeter 3 indicates the voltage across the fixed resistance.

The construction of a voltmeter does not differ materially from that of an ammeter in so far as the movements and magnets are concerned. As the voltmeter is connected directly across the line, it is desirable that it take as little current as is practicable. The current in the moving coil of a voltmeter is generally in the order of 50 μa to 1 ma. Because of its comparatively low resistance, the moving element cannot be connected directly across the line. It is therefore necessary to connect a high resistance in series with it. The value of this resistance depends upon the resistance of the coil or moving element, the current rating of the instrument, and the full-scale voltage desired.

EXAMPLE 6-3 If the resistance of the moving coil of a voltmeter is 50 ohms and the current rating of the instrument is 1 ma, what amount of resistance must be added to produce a full-scale reading of 150 volts?

GIVEN: $E = 150$ volts $\quad R_c = 50$ ohms $\quad I_M = 1$ ma

FIND: R_S

SOLUTION:

$$R_{\text{total}} = \frac{E}{I_M} = \frac{150}{0.001} = 150{,}000 \text{ ohms}$$

As the instrument has a resistance of 50 ohms, this means that

$$R_S = R_{\text{total}} - R_c = 150{,}000 - 50 = 149{,}950 \text{ ohms}$$

When the value of the series resistance is 100 or more times greater than the resistance of the moving coil, the resistance of the moving coil can usually be ignored; and in Example 6-3, a 150,000-ohm series resistor would be used.

Methods of Obtaining Additional Voltage Ranges. Voltmeters are generally made with more than one voltage range. To accomplish this, the high resistance is tapped to give the voltage range desired (Fig. 6-13).

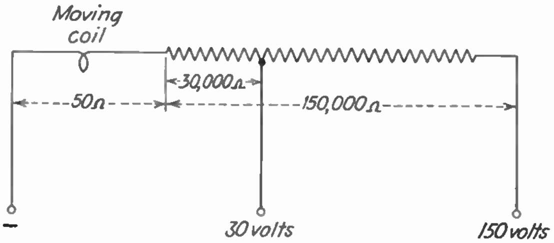


Fig. 6-13 Internal circuit of a voltmeter having a 30- and 150-volt range.

EXAMPLE 6-4 If the voltmeter of Example 6-3 is to have a 30-volt scale in addition to the 150-volt scale, where must the 150,000-ohm resistance be tapped?

GIVEN: $E = 30$ volts $R_S = 150{,}000$ ohms $I_M = 1$ ma

FIND: R_{tap}

SOLUTION:

$$R_{\text{tap}} = \frac{E}{I_M} = \frac{30}{0.001} = 30{,}000 \text{ ohms}$$

Multirange voltmeters, such as those used in electronic test equipment, use the above principle. The resistance must be large enough to withstand the highest voltage and must have a tap for each additional voltage range desired.

Sensitivity of Voltmeters. The sensitivity of a voltmeter is indicated by its *ohms per volt rating* which may be found by (1) dividing the resistance of the meter by its full-scale voltage rating or (2) obtaining the reciprocal of the current required for full-scale deflection. Mathematically

$$\text{Meter sensitivity} = \frac{R_M}{E_{f-s}} \qquad (6\text{-}1)$$

or

$$\text{Meter sensitivity} = \frac{1}{I_{f-s}} \qquad (6\text{-}2)$$

where R_M = resistance of the voltmeter, ohms
E_{f-s} = full-scale rating, volts
I_{f-s} = current required for full-scale deflection, amp

EXAMPLE 6-5 What is the sensitivity of a voltmeter having a resistance of 300,000 ohms for a 300-volt-range?

GIVEN: $E_{f-s} = 300$ volts $R_M = 300{,}000$ ohms

FIND: Meter sensitivity

SOLUTION:

$$\text{Meter sensitivity} = \frac{R_M}{E_{f-s}} = \frac{300{,}000}{300} = 1{,}000 \text{ ohms per volt}$$

EXAMPLE 6-6 What is the sensitivity of a voltmeter that produces full-scale deflection when the meter current is 50 μa?

GIVEN: $I_{f-s} = 50\ \mu\text{a}$

FIND: Meter sensitivity

SOLUTION:

$$\text{Meter sensitivity} = \frac{1}{I_{f-s}} = \frac{1}{50 \times 10^{-6}} = 20{,}000 \text{ ohms per volt}$$

When voltages in power circuits are being measured, accurate values can be obtained with a voltmeter having a low sensitivity, such as 1,000 ohms per volt, as the resistance of these circuits is usually less than 1,000 ohms and often less than 100 ohms. The resistance of many electronic circuits is very high, being of the order of more than 100,000 ohms. When voltages in these circuits are being measured, accurate values can be obtained only with a voltmeter having a sensitivity of 20,000 ohms per volt or higher. The sensitivity required of a voltmeter depends largely upon the resistance and the type of circuit whose voltage is to be measured.

When a voltmeter is used to measure the voltage across a circuit element, a parallel circuit is formed because the voltmeter acts as a resistor connected in parallel with the circuit element whose voltage is being measured. The effect produced and the error caused by using a low-sensitivity voltmeter are illustrated in the following examples.

EXAMPLE 6-7 A voltmeter having a 150-volt range with a sensitivity of 1,000 ohms per volt is used to measure the voltage across the individual elements of a series circuit consisting of two 500,000-ohm resistors connected to a 120-volt power source. (*a*) What is the actual voltage across each resistor, that is, without the voltmeter connected to either resistor? (*b*) When the voltmeter is connected across one resistor, what voltage will the meter indicate? (*c*) What is the per cent error caused by connecting the voltmeter across the resistor?

GIVEN: $E_M = 150$ volts $\quad E_L = 120$ volts $\quad R_1 = R_2 = 500{,}000$ ohms
Sensitivity = 1,000 ohms per volt

FIND: (*a*) e_1, e_2 (*b*) e_1 (*c*) Per cent error

SOLUTION:

(*a*)
$$e_1 = e_2 = \frac{E_L R_1}{R_1 + R_2} = \frac{120 \times 500{,}000}{500{,}000 + 500{,}000} = 60 \text{ volts}$$

(*b*) Connecting the voltmeter across R_1 produces a parallel circuit whose resistance is

$$R_{eq\cdot 1} = \frac{R_1 R_M}{R_1 + R_M} = \frac{500{,}000 \times 150{,}000}{500{,}000 + 150{,}000} \cong 115{,}000 \text{ ohms}$$

$$e_1 = \frac{E_L R_{eq\cdot 1}}{R_{eq\cdot 1} + R_2} = \frac{120 \times 115{,}000}{115{,}000 + 500{,}000} \cong 22.4 \text{ volts}$$

(c) $$\text{Error} = \frac{e_{\text{meas}} - e_{\text{actual}}}{e_{\text{actual}}} \times 100 = \frac{22.4 - 60}{60} \times 100 \cong -62.7\%$$

EXAMPLE 6-8 What results will be obtained if a voltmeter having a 20,000-ohms-per-volt sensitivity is substituted for the meter used in Example 6-7?

GIVEN: $E_M = 150$ volts $E_L = 120$ volts $R_1 = R_2 = 500{,}000$ ohms
Sensitivity = 20,000 ohms per volt

FIND: (a) e_1, e_2 (b) e_1 (c) Per cent error

SOLUTION:

(a) $$e_1 = e_2 = \frac{E_L R_1}{R_1 + R_2} = \frac{120 \times 500{,}000}{500{,}000 + 500{,}000} = 60 \text{ volts}$$

(b) $$R_{eq\cdot 1} = \frac{R_1 R_M}{R_1 + R_M} = \frac{500{,}000 \times 150 \times 20{,}000}{500{,}000 + (150 \times 20{,}000)} \cong 429{,}000 \text{ ohms}$$

$$e_1 = \frac{E_L R_{eq\cdot 1}}{R_{eq\cdot 1} + R_2} = \frac{120 \times 429{,}000}{429{,}000 + 500{,}000} \cong 55.4 \text{ volts}$$

(c) $$\text{Error} = \frac{e_{\text{meas}} - e_{\text{actual}}}{e_{\text{actual}}} \times 100 = \frac{55.4 - 60}{60} \times 100 = -7.66\%$$

Examples 6-7 and 6-8 show that, unless the resistance of a voltmeter is many times greater than the resistance of the circuit element whose voltage is being measured, the error introduced will be so great that it will make the readings obtained worthless.

Precautions in the Use of Voltmeters. Voltmeters do not form a definite part of a circuit as do ammeters. It is therefore not necessary to connect voltmeters permanently in the circuit. Because a voltmeter usually is con-

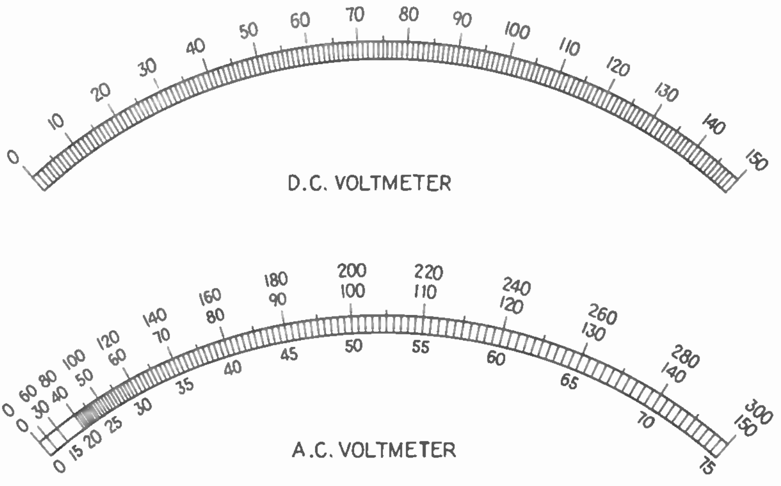


Fig. 6-14 Meter scales. Top, uniform scale. Bottom, irregular scale.

Fig. 6-15 A combination a-c/d-c voltmeter, d-c ammeter, and ohmmeter illustrating an application of the complex scale. (*Simpson Electric Co.*)

nected only when a reading is to be taken, fewer meters are needed, as one meter may be used to take readings of several circuits and circuit elements. A voltmeter may be damaged by excessive voltage, but since it is connected only when a reading is to be taken, it may be disconnected instantly if any overvoltage condition is apparent.

6-10 Meter Scales

Uniform and Irregular Scales. In the selection of meters, it is well to choose a meter of such a range that the readings will be in the middle two-thirds of the scale. The readings taken from this part of the scale are more accurate than those at the extreme ends, and there is also less danger of damaging the needle. Meter-scale divisions may be uniform or irregular. A uniform scale is one that has the entire scale divided into equal divisions. If full-scale deflection is 150 volts, the middle of the scale will be 75 volts, one-sixth of the scale will be 25 volts, and two-thirds of the scale will be 100 volts.

On an irregular scale, the divisions are not equal. On the irregular scale shown in Fig. 6-14 the scale divisions are very close at the low readings, and the space between divisions increases as the voltage readings increase. Examination of this scale shows that the middle two-thirds of the 150-volt scale extends from approximately 63 to 135 volts.

Multiple and Complex Scales. A common practice is to have one scale indicate more than one value. On such a scale, each division will represent two or more values, each one being a multiple or submultiple of the others.

Scales of this type are referred to as *multiple scales*. Referring to Fig. 6-14, when the needle is over the 100 mark on the center set of numbers, it will indicate 100 volts if the 150-volt range is being used, 50 volts for the 75-volt range, and 200 volts for the 300-volt range.

Test instruments used in radio, television, and electronics usually consist of one meter that is capable of indicating a number of different units, such as direct current, direct voltage, alternating voltage, and resistance. Instruments of this type have two or more multiple scales and are therefore known as *complex-scale meters* (Fig. 6-15). Care must be taken in using a complex-scale meter to read the correct scale, that is, the one corresponding to the setting for which the meter is adjusted.

6-11 Parallax

To read a meter correctly, the eyes must be parallel to the scale of the instrument. If the eyes are not parallel with the meter scale, a false reading will be obtained because of the space between the scale and the needle. Referring to Fig. 6-16 a correct reading can be taken only when the eyes are parallel to the scale; readings taken at any other position will be incorrect. This error in reading meters is due to *parallax*.

To avoid error in reading meters due to parallax, a mirror is placed below the scale to be read, as shown in Fig. 6-16. When the eyes are in a direct line with the needle and the scale, the reflection of the needle cannot be seen in the mirror and the reading taken at this point is correct.

6-12 Shunts

Calculation of the Resistance of the Shunt. The current which may safely be led into an ammeter mechanism is limited by the current-carrying

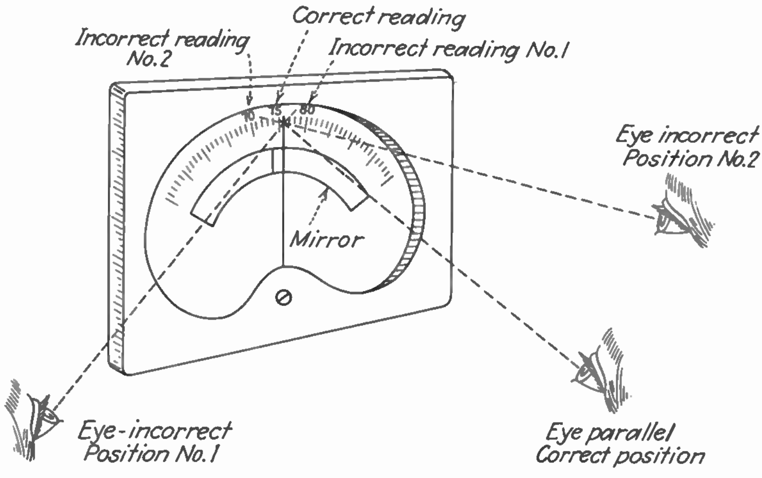


Fig. 6-16 Errors possible in reading meters because of parallax.

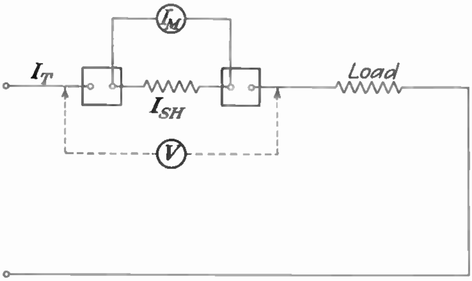


Fig. 6-17 Proper connections for use of a shunt with a millivoltmeter.

capacity of the coil in the moving element. To increase the range of such instruments, shunts and current transformers are used, the former with direct currents, the latter with alternating currents.

The amount of current permitted to flow in the coil of the mechanism of an ammeter is very small, generally less than 10 ma. This is achieved by connecting a low resistance in parallel with the meter. The ammeter is in reality a voltmeter indicating the voltage drop across a resistance (Fig. 6-17). The resistance, called a *shunt,* forms a definite part of all ammeters.

$$R_{SH} = \frac{I_M \times R_M}{I_{\text{line}} - I_M} \qquad (6\text{-}3)$$

where R_{SH} = resistance of shunt, ohms
R_M = resistance of meter coil, ohms
I_M = current in meter coil, amp
I_{line} = current to be measured, amp

EXAMPLE 6-9 If an instrument that is to be used as an ammeter has a resistance of 50 ohms and the meter current required to obtain full-scale deflection is 1 ma, what resistance shunt must be used to give full-scale deflection with 10 amp flowing?

GIVEN: $R_M = 50$ ohms $I_M = 1$ ma $I_{\text{line}} = 10$ amp

FIND: R_{SH}

SOLUTION:

$$R_{SH} = \frac{R_M \times I_M}{I_{\text{line}} - I_M} = \frac{50 \times 0.001}{10 - 0.001} = 0.005 \text{ ohm}$$

Types of Shunts. When a shunt is connected permanently inside an ammeter, it is called an *internal shunt.* When it is desired to increase the range of an ammeter, an external shunt is used. External shunts consist of a low resistance, usually manganin brazed to two comparatively large copper blocks. Two sets of binding posts are fastened to these blocks, one for carrying the line current through the shunt and the other for the ammeter leads. These ammeter leads are calibrated to be used with a particular instrument, and therefore no other leads should be used.

For greater convenience, a multiple-range rotary-switch type of shunt can be used. It is so constructed that any one of a number of ranges may be selected by merely turning the switch handle until the index points to the desired range. Multirange ammeters use a similar rotary-type adjustable shunt that is permanently connected to the galvanometer.

6-13 Multipliers

Calculation of the Resistance of a Multiplier. The range of a voltmeter having its resistance incorporated within the instrument may be increased by the use of an external resistance connected in series with the instrument (Fig. 6-18). External resistances used in this way are called *multipliers* and are usually placed within a perforated box, the terminals being brought outside. The multiplying power of each terminal is indicated by a number. The equation showing the relation of the resistance of the multiplier R_{EX}, the resistance of the meter R_M, and the multiplying power M may be expressed as

$$M = \frac{R_{EX} + R_M}{R_M} \tag{6-4}$$

EXAMPLE 6-10 If a voltmeter has a resistance of 150,000 ohms and an external resistance of 450,000 ohms is connected in series with it, what is the multiplying power of the external resistance?

GIVEN: $R_M = 150{,}000$ ohms $R_{EX} = 450{,}000$ ohms

FIND: M

SOLUTION:

$$M = \frac{R_{EX} + R_M}{R_M} = \frac{450{,}000 + 150{,}000}{150{,}000} = \frac{600{,}000}{150{,}000} = 4$$

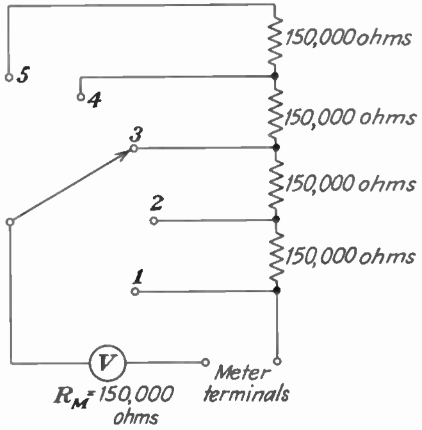
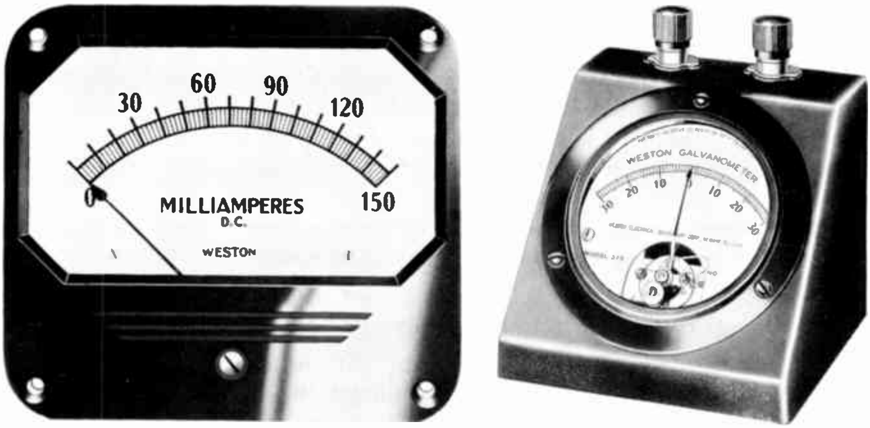


Fig. 6-18 Connections for using a multiplier with a voltmeter. This multiplier used with a 150-volt-range meter would extend the maximum range to 300, 450, 600, or 750 volts.



(a) (b)

Fig. 6-19 Low-range meters. (a) Milliammeter. (b) Galvanometer. (*Weston Instruments & Electronics, Division of Daystrom, Inc.*)

The amount of resistance required to extend the range of a voltmeter can be calculated by

$$R_{EX} = \frac{R_M(E_N - E_O)}{E_O} \qquad (6\text{-}5)$$

where E_N = new maximum range of voltmeter, volts
E_O = original maximum range of voltmeter, volts

EXAMPLE 6-11 How much resistance must be provided by an external resistor to extend the range of a voltmeter from 300 to 450 volts if the resistance of the voltmeter for the 300-volt range is 3 megohms?

GIVEN: $E_O = 300$ volts $E_N = 450$ volts $R_M = 3$ megohms

FIND: R_{EX}

SOLUTION:

$$R_{EX} = \frac{R_M(E_N - E_O)}{E_O} = \frac{3 \times 10^6(450 - 300)}{300} = 1.5 \text{ megohms}$$

6-14 Low-range Meters

It is sometimes necessary to measure a current of only a few thousandths or a few millionths of an ampere or a voltage of only a few thousandths or a few millionths of a volt. Measurements of these kinds are sometimes required in testing radio, television, and other electronic circuits.

Instruments calibrated to indicate currents in millionths of an ampere are called *microammeters.*

cate the resistance of the external circuit in ohms. The highest reading that can be made with any degree of accuracy is approximately ten times the center reading, and the lowest reading is approximately one-tenth of the center reading. The ohmmeter being discussed will therefore have an approximate range of 450 to 45,000 ohms.

Variable Series-resistor Method of Compensating for Variations in Cell Voltage. As the voltage of unit cells will diminish with their age, the resistance R_A is made adjustable to compensate for the variations in voltage of the power supply. If the voltage E_B decreases to 4.2 volts, then $R_1 + R_A$ must be equal to 4,200 ohms in order to obtain 1 ma of current. If the milliammeter indicates that 0.5 ma is flowing in the circuit, then

$$R_1 + R_A + R_X = \frac{4.2}{0.0005} = 8{,}400 \text{ ohms}$$

as

$$R_1 + R_A = 4{,}200 \text{ ohms}$$

then

$$R_X = 8{,}400 - 4{,}200 = 4{,}200 \text{ ohms}$$

As the ohmmeter has been calibrated to indicate 4,500 ohms at this point, the reading will be in error, as it now should read only 4,200 ohms. The readings at all other points on the scale will be decreased accordingly. Resistance values obtained with instruments using this method of adjusting for decreases in battery voltage will not be accurate when the battery voltage drops because of the aging of the cell. When the cells become too weak, it will no longer be possible to adjust the resistance R_A to obtain a flow of 1 ma, and new cells must be substituted in place of the old ones. In the ohmmeter being considered, this will be necessary when the total voltage of the cells drops below 4 volts.

Variable Parallel-resistor Method of Compensating for Variations in Cell Voltage. In order to overcome the disadvantages of the series-resistor method of compensation, an adjustable low resistance R_P is connected in parallel with the milliammeter as indicated by the dotted lines in Fig. 6-22. This resistor is used in place of the adjustable high resistance R_A that was connected in series with the meter and the fixed resistor R_1. By making $R_1 = 4{,}500$ ohms, the meter can be adjusted to zero by varying the position of the movable contact on R_P.

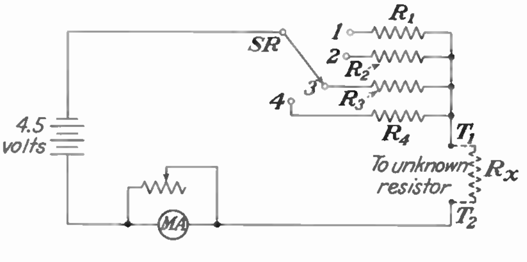


Fig. 6-23 Diagram showing the use of resistors and switch to select the range of a multi-range ohmmeter.

Multirange Ohmmeters. If the resistance of $R_1 + R_A$ is increased to 9,000 ohms and the battery voltage E_B is increased to 9 volts, then the range of the ohmmeter will increase to approximately 900 to 90,000 ohms. By using different values of resistance and battery voltage, an ohmmeter can be made to indicate any value of resistance. In Fig. 6-23, R_1, R_2, R_3, and R_4 are multiples of one another; therefore it is only necessary to have a single scale to indicate the values of the lowest range and a multiplying factor for each of the other ranges. The switch *SR* is used to connect or to disconnect these resistors into the circuit. The points 1, 2, 3, and 4 are marked to indicate the multiplying factor to be used for the resistor that is connected into the circuit.

6-16 Combination Meters

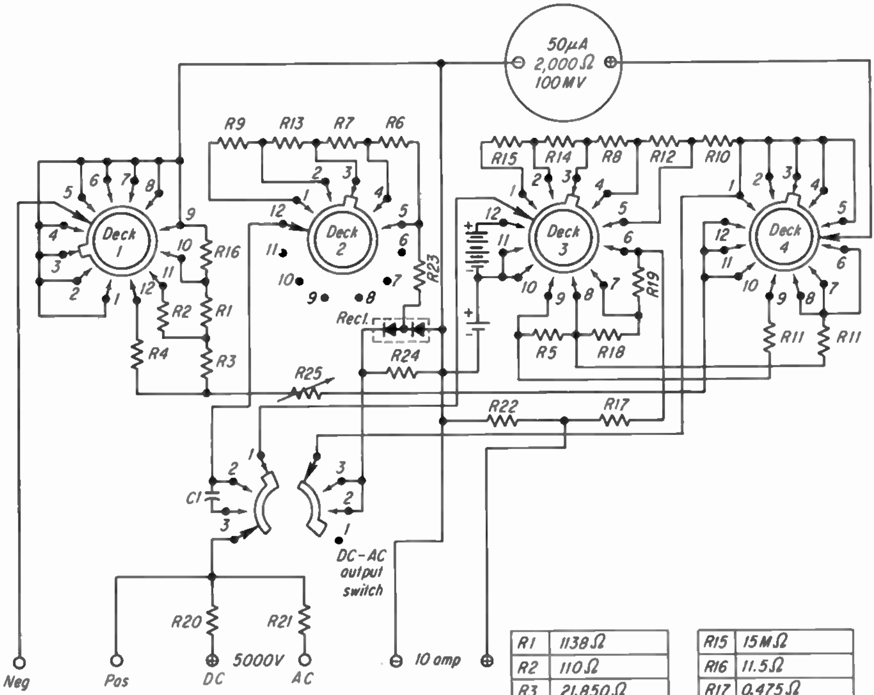
The construction of a voltmeter, ammeter, or ohmmeter does not differ materially as far as the movements and magnets are concerned. With the proper switching and circuit arrangements it is therefore possible to use a single d-c milliammeter as a multirange d-c voltmeter, d-c ammeter, or ohmmeter. A meter used in this manner is called a *combination meter*. The values of the multipliers, shunts, and ohmmeter resistors can be calculated by using the methods explained for shunts, multipliers, and ohmmeters. By the use of a metallic rectifier, the d-c milliammeter can also be used to measure alternating currents and voltages. In radio and other electronic test and service work, combination meters are very useful, since one instrument is made to take the place of several meters.

Figure 6-24 shows the circuit diagram of a combination meter. This instrument has six direct- and alternating-voltage ranges, 2.5, 10, 50, 250, 1,000, and 5,000 volts; five direct-current ranges, 100 μa, 10 ma, 100 ma, 500 ma, and 10 amp; and three ranges of resistance, 2,000 ohms, 200,000 ohms, and 20 megohms. The direct-voltage readings have a sensitivity of 20,000 ohms per volt, and the alternating-voltage readings 1,000 ohms per volt. Direct-current readings are taken with a 250-mv drop across the instrument for full-scale deflection. This instrument may also be used to measure volume levels in volts and in decibels and is calibrated for a reference level of 500 ohms and 6 mw.

When combination meters are used, care should be taken to see that all switches and dial settings are in their correct positions before the instrument is connected into the circuit. If this is not done, the instrument may be damaged, as it might be set for use on direct current and be used on alternating current or set to be used as an ammeter and connected for use as a voltmeter or set in some other incorrect manner.

6-17 Wattmeters

D-C Power. The unit of measurement of electric power is the watt. The power for any d-c circuit can be computed by multiplying the amperes in



Selector switch positions			
1	1000V	7	100 MA
2	250V	8	10 MA
3	50V	9	100μA
4	10V	10	R x 1
5	2.5V	11	R x 100
6	500MA	12	R x 10,000

DC-AC output switch positions
1.-DC 2.-AC 3.-output

Switches shown in 50VDC position

R1	1138Ω
R2	110Ω
R3	21,850Ω
R4	117,700Ω
R5	4975 Ω
R6	7500 Ω
R7	40KΩ
R8	800KΩ
R9	750KΩ
R10	48KΩ
R11	(Two) 3000Ω
R12	150KΩ
R13	200KΩ
R14	4MΩ

R15	15MΩ
R16	11.5Ω
R17	0.475Ω
R18	22.5Ω
R19	2Ω
R20	80MΩ
R21	4MΩ
R22	0.025Ω
R23	Rect. series calibrated
R24	Rect. shunt calibrated
R25	10KΩ pot.
C1	.1MFD 400V

Fig. 6-24 Schematic diagram of the combination meter shown in Fig. 6-15. (*Simpson Electric Co.*)

a circuit by the voltage across the circuit. These two quantities may be measured by using separate instruments, namely, an ammeter and a voltmeter, and the value obtained from one is multiplied by the value obtained from the other. However, where the amount of power used varies continuously and its value must be known almost instantly, a wattmeter is used and it performs three functions at once: (1) takes into account the value of the current in *amperes,* (2) takes into account the value of the electromotive force in *volts,* and (3) indicates the product of these two quantities in *watts* directly on the scale of the meter.

The mechanism used in most commercial wattmeters is similar to the electrodynamometer type shown in Fig. 6-10. This type of instrument is used as a wattmeter by connecting its two fixed coils in the same manner as the current coil of an ammeter and its movable coil in the same manner as the potential coil of a voltmeter (Fig. 6-25). A high-ohmage resistor is connected in series with the potential coil to serve the same purpose as the high-resistance unit used with the ordinary voltmeter; this resistance unit is mounted within the meter case. This type of wattmeter is basically a single meter unit containing a voltage-sensitive element and a current-sensitive element arranged so that their combined magnetic effects produce a torque on the pointer of the meter causing it to travel over a scale calibrated in watts.

A-C Power. In an a-c circuit, the power is equal to the product of the volts and amperes for only certain limited conditions (Art. 7-7). Because of this, the wattmeter is universally used to determine the power consumed by an a-c circuit. The connections for the wattmeter and the principles of operation are the same as explained in the preceding paragraph.

Precautions in the Use of Wattmeters. The current side of the wattmeter should be protected by a short-circuiting switch for the same reason and in the same manner as explained under the use of ammeters. Figure 6-25 shows a SPST switch connected so that it protects both the ammeter and the current side of the wattmeter simultaneously.

When a wattmeter is used with d-c circuits, it is possible that either the current or the voltage element of the meter may be overloaded without causing the meter to reach full-scale indication. When a wattmeter is used with a-c circuits, it is possible that the current element, the voltage element, or both may be overloaded without causing the meter to reach full-scale indication. In order to know whether the current and voltage elements of a wattmeter are being used within their rated safe ranges, it is recommended that an ammeter and a voltmeter be included in the circuit.

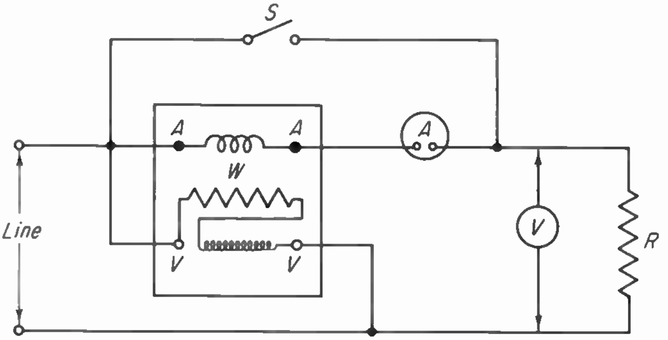


Fig. 6-25 Ammeter, voltmeter, and wattmeter connections for measuring the current, voltage, and power for the load resistor *R*.

6-18 Wheatstone Bridge

The ohmmeter is a simple and convenient means for measuring resistance, but the values obtained are not extremely accurate, although for most practical purposes the error involved can be ignored. When precise measurement of resistance is required, the instrument most generally used is the *Wheatstone bridge.* Essentially, this instrument consists of four resistors connected to form a diamond. A source of voltage, usually a battery (of one or two cells), is connected in series with a switch and across two terminals of opposite junctions as points A and B in Fig. 6-26. A sensitive galvanometer is connected in series with another switch and across the two terminals of opposite junctions shown as points C and D in Fig. 6-26. The unknown resistor R_X is used as one of the four resistors forming the diamond; the other three resistors are of the adjustable type whose value is indicated as it is varied. The settings of the three adjustable resistors are varied until, with switches S_1 and S_2 closed, no current flow is indicated on the galvanometer. Under this condition the bridge is balanced. The value of the unknown resistance is then determined from the values to which the adjustable resistors were set in order to produce a balance.

The principle of the Wheatstone bridge can be explained in the following manner: When the switch S_1 is closed, the current from the battery will divide into two paths, namely, I_1 through the path ACB and I_2 through path ADB. The voltage drops across the individual resistors are

$$E_{AC} = I_1R_1 \tag{6-7}$$

$$E_{CB} = I_1R_2 \tag{6-8}$$

$$E_{AD} = I_2R_S \tag{6-9}$$

$$E_{DB} = I_2R_X \tag{6-10}$$

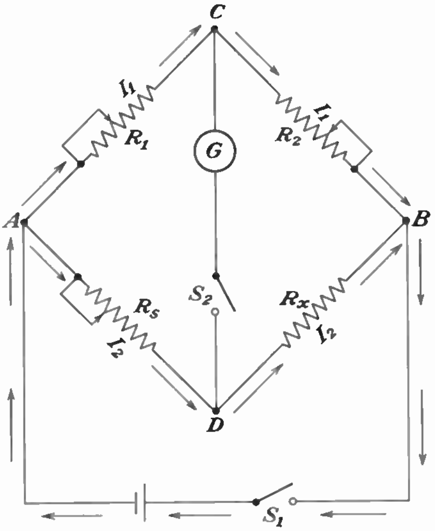


Fig. 6-26 Wheatstone-bridge method for measuring resistance.

When the bridge is balanced, the junction points C and D are at the same potential. This is evident from the fact that the galvanometer reading is zero, since current will flow between two points only when there is a difference of potential between the two points. Under this condition,

$$I_1R_1 = I_2R_S \tag{6-11}$$

and

$$I_1R_2 = I_2R_X \tag{6-12}$$

Dividing Eq. (6-11) by Eq. (6-12),

$$\frac{I_1R_1}{I_1R_2} = \frac{I_2R_S}{I_2R_X} \tag{6-13}$$

or

$$\frac{R_1}{R_2} = \frac{R_S}{R_X} \tag{6-14}$$

Solving for R_X as the unknown resistance,

$$R_X = R_S \times \frac{R_2}{R_1} \tag{6-15}$$

This is the fundamental equation for the Wheatstone bridge. The ratio R_2/R_1 is generally made to equal 1 or a multiple or a fraction of 10. For example, R_2/R_1 may be made equal to 1, 1/10, 1/100, 10, 100, etc. The value of the unknown resistance is then found by multiplying the value of R_S required to produce a balance by the ratio R_2/R_1.

EXAMPLE 6-12 A balance is obtained in a Wheatstone-bridge circuit similar to Fig. 6-26 when $R_1 = 10$ ohms, $R_2 = 1{,}000$ ohms, and $R_S = 75$ ohms. What is the value of the unknown resistance R_X?

GIVEN: $R_1 = 10$ ohms $\quad R_2 = 1{,}000$ ohms $\quad R_S = 75$ ohms

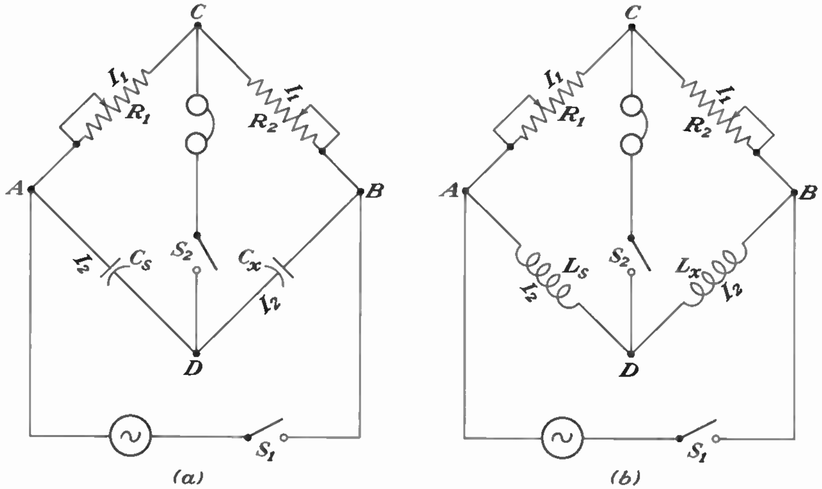
FIND: R_X

SOLUTION:

$$R_X = R_S \times \frac{R_2}{R_1} = 75 \times \frac{1{,}000}{10} = 7{,}500 \text{ ohms}$$

6-19 A-C Bridge

The unknown value of the inductance of an inductor or the capacitance of a capacitor can also be measured by means of the bridge circuit. The basic circuits for measuring inductance or capacitance are shown in Fig. 6-27. The voltage source is a steady a-c voltage, usually 1,000 cycles. The indicating instrument may be a set of earphones or a sensitive a-c milliammeter. Resistors R_1 and R_2 are also used in this circuit as ratio arms as in the Wheatstone bridge. A known standard value of inductance L_S or of capacitance C_S is used in place of the standard resistor R_S of Fig. 6-26. The unknown inductor or capacitor is connected in a manner similar to R_X of Fig. 6-26. A balance is obtained by adjusting the ratio resistors R_1 and



(a) (b)

Fig. 6-27 Basic a-c bridge circuits. (a) For capacitance measurements. (b) For inductance measurements.

R_2 until the sound in the earphones or the indication on the a-c milliammeter is at its minimum. The unknown value of inductance or capacitance can then be found by using the following equations:

$$L_X = L_S \times \frac{R_2}{R_1} \tag{6-16}$$

$$C_X = C_S \times \frac{R_2}{R_1} \tag{6-17}$$

If a standard value of resistance R_S and an unknown value of resistance R_X are substituted for L_S and L_X in the circuit shown in Fig. 6-27, the circuit can also be used for measuring the unknown resistance value of a resistor.

QUESTIONS

1. Why are electrical instruments essential to the study of electricity and electronics?
2. (*a*) Define galvanometer. (*b*) What are its uses?
3. How may electrical instruments be classified in terms of their principle of operation?
4. (*a*) Explain the principle of operation of an electrostatic meter. (*b*) What type of instruments operates on this principle?
5. (*a*) Explain the principle of operation of an electrothermal meter. (*b*) Name the different types of current it can measure, and explain why this is possible.
6. Explain the principle of operation of the permanent-magnet moving-coil meter.
7. Explain the principle of operation of an iron-vane meter.
8. Explain the operation of (*a*) a radial moving iron-vane meter, (*b*) a concentric moving iron-vane meter.

9. Explain the principle of operation of an electrodynamometer meter.
10. What type of electromagnetic meter is (*a*) most versatile? (*b*) Limited to d-c operation? (*c*) Most widely used?
11. (*a*) What is the principle of operation of the copper oxide rectifier? (*b*) How is this principle used in electrical measuring instruments?
12. (*a*) How is an ammeter connected in a circuit? (*b*) How is it protected when readings are not being taken?
13. Is the resistance of an ammeter high or low? Explain.
14. (*a*) How is a voltmeter connected in a circuit? (*b*) How is it protected when readings are not being taken?
15. What is the essential difference between the construction of a voltmeter and that of an ammeter?
16. Is the resistance of a voltmeter high or low? Explain.
17. (*a*) What is meant by the sensitivity of a voltmeter? (*b*) How is the sensitivity rating of a voltmeter usually indicated?
18. Why is it desirable to use a voltmeter having a high sensitivity rating when voltages are being measured in electronic circuits?
19. What is meant by a multirange voltmeter?
20. What is meant by (*a*) a uniform scale? (*b*) An irregular scale?
21. On what part of the meter scale is it desirable to take readings? Explain.
22. Define (*a*) multiple scale, (*b*) complex scale.
23. What errors can easily be made when reading multiple or complex scales?
24. (*a*) What is meant by parallax? (*b*) How are errors due to parallax counteracted?
25. Can an ammeter be used as a millivoltmeter? Explain.
26. (*a*) What is the purpose of a shunt? (*b*) What is the difference between an internal and an external shunt?
27. (*a*) What is meant by a multiplier? (*b*) How does it serve its purpose?
28. What instruments are used to measure (*a*) low values of current? (*b*) Low values of voltage?
29. (*a*) Describe the ammeter-voltmeter method of measuring resistance. What variation in connections is recommended for measuring (*b*) high values of resistance? (*c*) Low values of resistance?
30. How can high resistances be measured with a voltmeter and a power source?
31. (*a*) What is an ohmmeter? (*b*) Explain its basic principle of operation.
32. How does a decrease in the battery voltage affect the readings of an ohmmeter?
33. Describe the variable series-resistor method of compensating for the effects caused by variations in battery voltage.
34. Describe the variable parallel-resistor method of compensating for the effects caused by variations in battery voltage.
35. What is meant by a combination meter?
36. Why are combination meters useful in test and service work on radio, television, and other electronic equipment?
37. What precautions are necessary in using combination meters?
38. What three functions are simultaneously performed by a wattmeter?
39. (*a*) What type of mechanism is generally used in commercial wattmeters? (*b*) Describe the connections for this mechanism when it is to be used as a wattmeter.
40. What precautions should be taken in the connection of a wattmeter to measure power for (*a*) d-c circuits? (*b*) A-c circuits?

41. For what purpose is the Wheatstone bridge used?
42. Explain the principle of operation of the Wheatstone bridge.
43. Name three types of circuit elements whose value may be determined by means of a bridge circuit.
44. Describe the basic differences in the circuit arrangement for a Wheatstone bridge and an a-c bridge.

PROBLEMS

1. In the circuit shown in Fig. 6-12, ammeter 2 indicates 2 amp; ammeter 3, 12 amp; and voltmeter 1, 120 volts. If the resistance of ammeter 1 is assumed to be 0.5 ohm, determine (*a*) the voltage drop across ammeter 1, (*b*) the voltage applied to the lamp.
2. A certain 5-amp ammeter is designed with a 100-mv drop at rated current. (*a*) What is the resistance of the meter? (*b*) What possible current flow is indicated by Ohm's law if the meter was accidentally connected to a 120-volt power source?
3. The resistance of the moving coil of a certain voltmeter is 200 ohms, and the current rating of the instrument is 1 ma. Determine the amount of resistance that must be added to produce a full-scale reading of 100 volts.
4. If the voltmeter of Prob. 3 is to have a 25-volt scale in addition to the 100-volt scale, where must the external resistance be tapped?
5. A 150/15-scale voltmeter has a total resistance of 150,000 ohms. (*a*) What is the resistance between its 15-volt terminals? (*b*) What resistance multiplier is necessary to give this instrument a range of 600 volts?
6. If the resistance of an instrument is 1,200 ohms and its current rating is 0.1 ma, what amount of resistance must be added to give a full-scale reading of (*a*) 50 volts? (*b*) 150 volts? (*c*) 250 volts?
7. A voltmeter having a sensitivity of 10,000 ohms per volt has ranges of (*a*) 10, (*b*) 50, (*c*) 100, (*d*) 250, and (*e*) 1,000 volts. What is the resistance of the voltmeter for each of these ranges?
8. A voltmeter having a sensitivity of 5,000 ohms per volt has ranges of (*a*) 1, (*b*) 2.5, (*c*) 25, (*d*) 50, (*e*) 100, and (*f*) 250 volts. What is the resistance of the voltmeter for each of these ranges?
9. What is the sensitivity of a voltmeter having a resistance of 750,000 ohms for a 150-volt range?
10. Determine the sensitivity of a voltmeter having a resistance of 100,000 ohms for a 10-volt range.
11. What is the sensitivity of a voltmeter that produces full-scale deflection when the meter current is 200 μa?
12. Determine the sensitivity of a voltmeter that produces full-scale deflection when the meter current is 50 μa.
13. A voltmeter having a sensitivity of 1,000 ohms per volt is used to measure a voltage drop of approximately 40 volts that exists across a 50,000-ohm resistor. What is the per cent decrease in the resistance of this circuit when the voltmeter is connected across the resistor and the full-scale range of the meter is (*a*) 50 volts? (*b*) 100 volts?
14. What is the per cent decrease of the resistance of a circuit when a voltmeter having a sensitivity of 20,000 ohms per volt is used to measure the voltage drop

across a 200,000-ohm resistor (*a*) using its 100-volt range? (*b*) Using its 250-volt range?

15. Two 50,000-ohm resistors are connected in series across a 100-volt source of power. A voltmeter having a sensitivity of 1,000 ohms per volt is used to measure the voltage drop across each resistor. (*a*) What voltage will the voltmeter indicate when using its 100-volt range? (*b*) What is the per cent error?
16. Two 200,000-ohm resistors are connected in series across a 120-volt source of power. A voltmeter having a sensitivity of 1,000 ohms per volt is used to measure the voltage drop across each resistor. (*a*) What voltage will the voltmeter indicate when using its 100-volt range? (*b*) What is the per cent error?
17. A voltmeter having a sensitivity of 20,000 ohms per volt is substituted for the meter used in Prob. 15. (*a*) What voltage will the voltmeter indicate when using its 100-volt range? (*b*) What is the per cent error?
18. A voltmeter having a sensitivity of 10,000 ohms per volt is substituted for the meter used in Prob. 16. (*a*) What voltage will the voltmeter indicate when using its 100-volt range? (*b*) What is the per cent error?
19. A galvanometer has a resistance of 125 ohms. What resistance shunt must be used with this galvanometer if it is desired that (*a*) one-tenth of the line current should pass through the galvanometer? (*b*) One-twenty-fifth of the line current should pass through the galvanometer?
20. An instrument has a rating of 50 mv, and its moving coil has a resistance of 20 ohms. If it is to measure a current of 1 amp, how much current flows through (*a*) the meter? (*b*) The shunt?
21. An instrument has a resistance of 25 ohms, and full-scale deflection is obtained when 25 ma flows through the instrument. Determine the value of shunt resistance required to produce full-scale deflection when the line current is (*a*) 250 ma, (*b*) 500 ma, (*c*) 1 amp, (*d*) 10 amp.
22. It is desired to extend the range of a microammeter having a full-scale deflection of 50 μa and whose resistance is 1,200 ohms. Determine the value of shunt resistance required to obtain full-scale deflection when the line current is (*a*) 100 μa, (*b*) 500 μa, (*c*) 1 ma, (*d*) 100 ma, (*e*) 1 amp.
23. External resistances are connected in series with a voltmeter having a sensitivity of 20,000 ohms per volt and a full-scale reading of 2.5 volts. Determine the multiplying power of the external resistance when its value is (*a*) 450,000 ohms, (*b*) 950,000 ohms, (*c*) 1,950,000 ohms.
24. A voltmeter having a full-scale reading of 10 volts has a sensitivity of 5,000 ohms per volt. What value resistance must be connected in series with the meter to extend its range to (*a*) 50 volts? (*b*) 100 volts? (*c*) 250 volts?
25. The voltmeter-ammeter method is used to determine the resistance of a 2-ohm resistor. The voltmeter has a sensitivity of 1,000 ohms per volt and a full-scale deflection of 10 volts. The ammeter has a 50-mv drop for its full-scale deflection of 5 amp. The voltage of the power supply is 6 volts. Two sets of readings are taken under the following conditions: (1) The voltmeter is connected so that the ammeter reads the current taken by the voltmeter and the resistor as in Fig. 6-20*a*; (2) the voltmeter is connected so that it reads the voltage drop across the resistor and the ammeter as in Fig. 6-20*b*. Find the voltmeter and ammeter readings, the resistance calculated from these readings, and the approximate per cent of error for the condition when the meters are connected (*a*) as in (1), (*b*) as in (2).

26. Repeat Prob. 25 for the following conditions: A 100-volt power source, a 1,000-ohm-per-volt voltmeter with a 150-volt full-scale deflection, and a 50-ma ammeter having a 50-mv drop used to determine the resistance of a 10,000-ohm resistor.
27. Repeat Prob. 25 for the following conditions: A 1.5-volt power source, a 50-ma ammeter whose resistance is 2 ohms, and a 1,000-ohm-per-volt voltmeter with a 3-volt full-scale deflection used to determine the resistance of a 3-ohm resistor.
28. Repeat Prob. 25 for the following conditions: A 60-volt power source, a 1-ma ammeter whose resistance is 60 ohms, and a 1,000-ohm-per-volt voltmeter with a 150-volt full-scale deflection used to determine the resistance of a 150,000-ohm resistor.
29. A voltmeter having a sensitivity of 10,000 ohms per volt and a full-scale reading of 150 volts is connected as shown in Fig. 6-21. Determine the ohmic value of R_X if the voltmeter indicates 120 volts with the key closed and 100 volts with the key open.
30. A voltmeter having a sensitivity of 20,000 ohms per volt and a full-scale reading of 100 volts is connected as shown in Fig. 6-21. Determine the ohmic value of R_X if the voltmeter indicates 75 volts with the key closed and 60 volts with the key open.
31. A voltmeter having a sensitivity of 10,000 ohms per volt and a full-scale reading of 150 volts is connected as shown in Fig. 6-21 to measure the resistance values of three unknown resistors. The voltage of the power source is 100 volts, and the meter reads (*a*) 10 volts, (*b*) 50 volts, (*c*) 60 volts, respectively, when each of the resistors is connected in the circuit. What is the value of each resistance?
32. If the voltmeter of Prob. 31 had a sensitivity of 1,000 ohms per volt (*a*) could it be used to measure these resistances? (*b*) If so, what voltages would be indicated for each of the resistors measured?
33. In the circuit shown in Fig. 6-22, what should the resistance of $R_1 + R_A$ be in order that the milliammeter indicate 9,000 ohms in the center of the scale when the applied voltage is 9 volts?
34. In the circuit shown in Fig. 6-22, $E_B = 15$ volts, $R_1 + R_A = 15{,}000$ ohms, $I = 0.25$ ma. What is the ohmic value of R_X?
35. A voltmeter has a full-scale reading of 15 volts and a sensitivity of 1,000 ohms per volt. This meter is connected in series with a 30-volt battery and unknown resistances. What are the values of these unknown resistances if the meter indicates (*a*) 5 volts? (*b*) 10 volts? (*c*) 15 volts?
36. A voltmeter has a full-scale reading of 15 volts and a sensitivity of 20,000 ohms per volt. This meter is connected in series with a 15-volt battery and unknown resistances. What are the values of these unknown resistances if the meter indicates (*a*) 3 volts? (*b*) 5 volts? (*c*) 10 volts?
37. A d-c milliammeter having a resistance of 30 ohms and a full-scale deflection of 1 ma is to be connected to various shunts and multiplier resistors so that it will indicate the following direct voltages and direct currents when a rotating switch connects the proper unit into the circuit: 10, 50, 100, 250, and 500 volts; 1, 10, 50, 100, and 1,000 ma. (*a*) Draw a circuit diagram of the meter, shunts, multipliers, and rotary switch. (*b*) Indicate the resistance value required for each shunt and multiplier.
38. A d-c microammeter having a resistance of 2,000 ohms and a full-scale deflection of 50 μa is to be connected to various shunts and multiplier resistors so that it will

indicate the following direct voltages and direct currents when a rotating switch connects the proper unit into the circuit: 2.5, 25, 100, 250, and 500 volts; 50 μa, 1, 10, 100, and 1,000 ma. (*a*) Draw a circuit diagram of the meter, shunts, multipliers, and rotary switch. (*b*) Indicate the resistance value required for each shunt and multiplier.

39. A balance is obtained in a Wheatstone bridge whose circuit is similar to Fig. 6-26 when $R_1 = 100$ ohms, $R_2 = 1$ ohm, and $R_S = 30$ ohms. What is the value of the unknown resistor R_X?
40. A balance is obtained in a Wheatstone bridge whose circuit is similar to Fig. 6-26 when $R_1 = 10{,}000$ ohms, $R_2 = 10$ ohms, and $R_S = 78.5$ ohms. What is the value of the unknown resistor R_X?
41. A bridge circuit similar to Fig. 6-27*a* is used to measure the unknown value of a capacitor. When a balance is obtained, it is found that $R_1 = 250$ ohms, $R_2 = 100$ ohms, and $C_S = 10$ μf. What is the value of the unknown capacitor?
42. A bridge circuit similar to Fig. 6-27*a* is used to measure the unknown value of a capacitor. When a balance is obtained, it is found that $R_1 = 3{,}750$ ohms, $R_2 = 100$ ohms, and $C_S = 10$ μf. What is the value of the unknown capacitor?
43. A bridge circuit similar to Fig. 6-27*b* is used to measure the unknown value of an inductor. When a balance is obtained, it is found that $R_1 = 80$ ohms, $R_2 = 1{,}200$ ohms, and $L_S = 100$ mh. What is the value of the unknown inductor?
44. A bridge circuit similar to Fig. 6-27*b* is used to measure the unknown value of an inductor. When a balance is obtained, it is found that $R_1 = 4$ ohms, $R_2 = 1{,}680$ ohms, and $L_S = 100$ mh. What is the value of the unknown inductor?

Chapter 7

Electrical Power Systems

The two chief sources of electrical energy for operating electronic equipment are (1) commercial power-supply systems and (2) batteries. The theory and operation of various types of batteries were presented in Chap. 3. The purpose of this chapter is to present the basic principles of operation of the major types of electric power systems that use rotating electromagnetic machines as generators of electricity.

7-1 Types of Power Supply and Equipment

Of the six kinds of electric currents described in Art. 2-10, only the continuous, direct, and alternating are used as sources of power supply.

Continuous Current. The continuous current, which is obtained from battery cells, is used largely for operating portable equipment. The method of producing this type of current has been discussed in Chap. 3, and the circuit characteristics studied in Chap. 4 will apply to this type of current.

Direct Current. The direct current, usually referred to as *d-c*, is that type obtained from a rotating machine called the *d-c generator*. The theory of operation of this machine is based upon the principle of electromagnetic induction and is described in the following articles. Direct-current power systems are found mainly in rural and isolated sections or in some instances where an individual building may have its own power plant. Such power systems are generally 110 volts. Another common d-c system is the farm-lighting system used where no other source of electrical power is available. To have power available 24 hr per day without running a gasoline-engine-driven generator continuously, it becomes necessary to use storage batteries. The power is supplied by the batteries, and the generator is operated periodically to keep the batteries charged. To keep the number of cells at a reasonable amount, it has been common practice to make these systems 32 volts. Most electronic equipment that uses batteries as the prime power source is designed for 28-volt operation. The external-circuit characteristics of the d-c generator systems are similar to those of battery systems and have been studied in Chap. 4.

Alternating Current. The alternating current, usually referred to as *a-c*, is that type obtained from a rotating machine called an *alternator* or *a-c generator*. The theory of this machine is also based on the principle of electromagnetic induction and is described in the following articles. Over 90 per cent of the electrical power is generated as alternating current, and hence most power systems are of this type. The circuit characteristics of a-c systems differ in some respects from the d-c systems, and these new characteristics will be explained in the following chapters.

Voltage of A-C Systems; the Transformer. The generated voltage of the alternator in the large modern power plant is much higher than the voltage supplied to the homes; it is not unusual for this to be as high as 16,600 volts. This higher voltage increases the efficiency of operation for the power companies but makes it necessary to reduce the voltage before connecting it to the home. The extensive use of a-c systems is due largely to the ability to raise or lower the voltage easily by means of a transformer. The operation of this device is also based on the principle of electromagnetic induction and is explained in this chapter.

7-2 Electromagnetic Induction

Faraday's Discovery. The operation of electrical power apparatus such as the d-c generator, the alternator, and the transformer is based on the principle of electromagnetic induction. The discovery of this principle is credited to Michael Faraday, who, in 1831, found that, if a conductor which was part of a closed circuit was moved through a magnetic field, a current would flow in the conductor.

Faraday's discovery may be demonstrated by moving a conductor through the field of a strong electromagnet as shown in Fig. 7-1. If the conductor

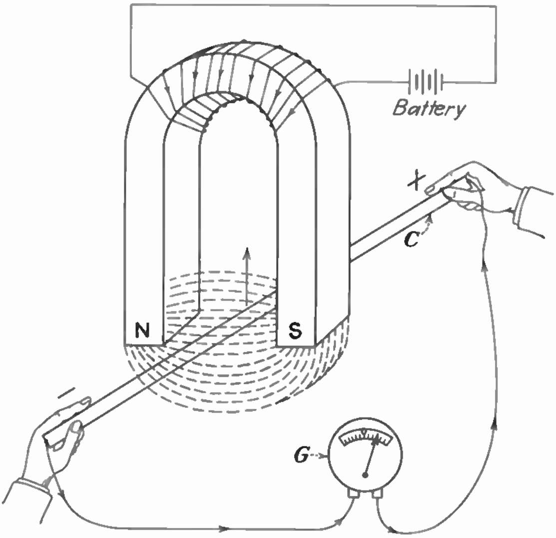


Fig. 7-1 Demonstration of Faraday's discovery.

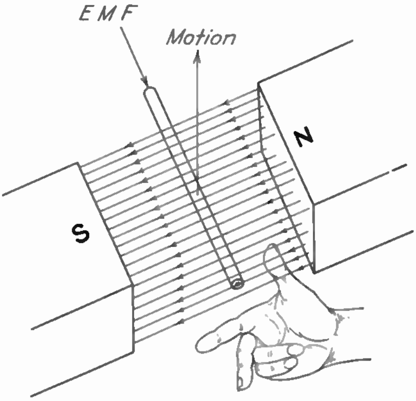


Fig. 7-2 Fleming's right-hand rule for determining the direction of the induced emf.

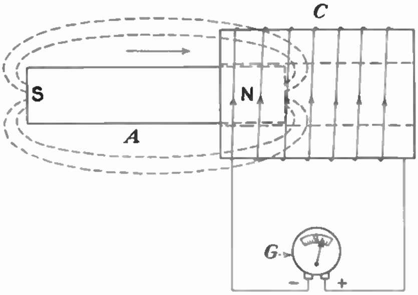


Fig. 7-3 Electromotive force induced by magnetic lines cutting a conductor.

C is moved upward through the magnetic field, it will cause a voltage to be induced in the conductor, and the galvanometer *G* will indicate the presence of this induced voltage. This induced voltage is called the *induced emf.* If the conductor is held stationary in the magnetic field, the galvanometer will show zero, indicating that no voltage is being induced. Moving the conductor downward through the magnetic field will cause the galvanometer to indicate a voltage but in the opposite direction to that when the conductor was moved upward. If the conductor is moved in the field in a sidewise direction, that is, parallel to the field from *N* to *S*, the galvanometer will show zero, indicating that no voltage is being induced. Reversing the polarity of the magnet, that is, interchanging the location of the *N* and *S* poles, would also cause a reversal of the galvanometer indication.

Fleming's Right-hand Rule. The relation among the direction of motion of the conductor, the direction of the magnetic field, and the direction of the induced emf is expressed by Fleming's right-hand rule. This relation is shown in Fig. 7-2 and may be stated as follows:

1. *Hold the thumb, forefinger, and middle finger of the right hand at right angles to one another.*
2. *Point the thumb in the direction of motion of the conductor.*
3. *Point the forefinger in the direction of the magnetic field.*
4. *The middle finger will then point in the direction of the induced voltage, namely, toward the positive terminal.*

Methods of Producing an Induced EMF. It has been shown that a voltage is induced when a conductor is moved through a magnetic field. An induced emf can also be set up in a stationary conductor by having a magnetic field move so that its lines cut the conductor. If the magnet *A* (Fig. 7-3) is moved into the center of the coil *C*, the field of the magnet will cut the conductors of the coil, and if the coil circuit is closed through a galvanometer *G*, a current flow will be indicated by the galvanometer. When the left hand is used in place of the right, Fleming's rule may be applied to finding the direction of the induced emf when the moving magnetic field cuts the conductor.

The principle of electromagnetic induction may now be restated as follows: Whenever there is motion between a conductor and a magnetic field, an emf will be induced in the conductor; the motion may be produced by moving the conductor through a stationary magnetic field or by having the magnetic field cut a stationary conductor.

7-3 The Simple Generator

The Fundamental Generator. The principle of electromagnetic induction as it applies to generators is illustrated in Fig. 7-4. This fundamental generator has two poles, *N* and *S*, which set up the magnetic field. The coil *C* is mounted on a shaft *S* so that it can be rotated in the magnetic field; this part is called the ***armature.*** When the armature is rotated, its conductors, which are really the coil sides *aa′* and *bb′*, cut through the lines of the magnetic field and a voltage is induced in the conductors.

Induced EMF. The amount of the induced emf in any conductor is proportional to the rate of cutting lines. A fundamental electrical law states that, whenever a conductor cuts lines (or is cut by lines) at the rate of 100 million lines per second, an emf of 1 volt is induced in that conductor. Since this is a rate of cutting lines, the speed of rotation as well as the number of lines cut will affect the induced emf.

In Fig. 7-4, the armature coil *C* consists of only one turn of wire, but it has two conductors which are connected in series. In many generators, the

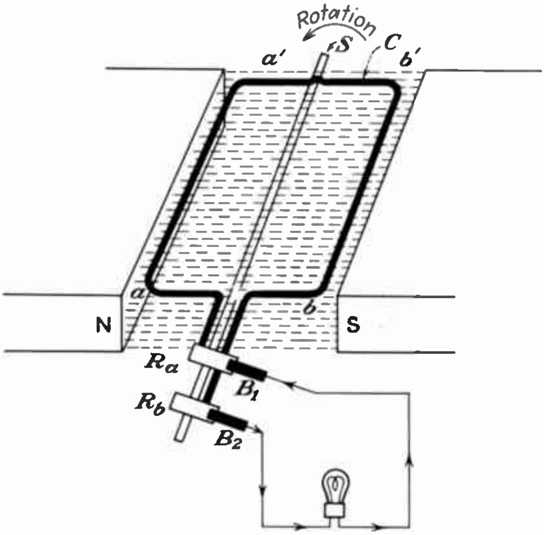


Fig. 7-4 A simple generator.

coils have more than one turn and hence have a greater number of conductors. This results in a higher induced emf, because when these conductors are connected in series, their voltages will add. Each conductor will cut the flux twice in one revolution. These facts may be expressed mathematically as

$$E = \frac{2\phi CS}{60 \times 10^8} \tag{7-1}$$

where E = induced emf of the generator
ϕ = total flux going from the N to S pole
C = number of conductors connected in series
S = speed, rpm

EXAMPLE 7-1 A generator operating at 1,800 rpm has 500 conductors on its armature. If the flux from the N pole to the S pole is 500,000 lines, what will the induced emf of the generator be?

GIVEN: $\phi = 500{,}000$ lines $C = 500$ conductors $S = 1{,}800$ rpm

FIND: E

SOLUTION:

$$E = \frac{2\phi CS}{60 \times 10^8} = \frac{2 \times 500{,}000 \times 500 \times 1{,}800}{60 \times 10^8} = 150 \text{ volts}$$

EXAMPLE 7-2 If the armature of the generator of Example 7-1 had only one single-turn coil, what would the induced emf of the generator be?

GIVEN: $\phi = 500{,}000$ lines $C = 2$ conductors $S = 1{,}800$ rpm

FIND: E

SOLUTION:

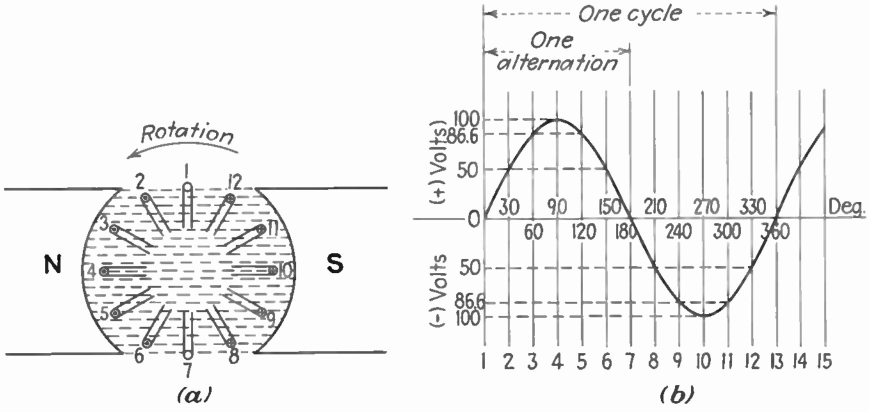
$$E = \frac{2\phi CS}{60 \times 10^8} = \frac{2 \times 500{,}000 \times 2 \times 1{,}800}{60 \times 10^8} = 0.6 \text{ volt}$$

The above examples illustrate that the practical generator requires a large number of series-connected conductors in order to obtain a standard commercial voltage.

Collector Rings and Brushes. If the coil C of Fig. 7-4 is to be rotated and its induced emf applied to an external circuit, it will be necessary to connect the conductors aa' and bb' to the collector rings R_a and R_b, respectively, and these rings must be insulated from the shaft. The external circuit is connected to the brushes B_1 and B_2 which make sliding contact with the collector rings.

7-4 The A-C Generator

The simple generator of Fig. 7-4 is really an a-c generator as will be shown in the following discussion. The coil is assumed to be rotating at a uniform speed in a counterclockwise direction, and the magnetic field is considered to be uniform; that is, each square inch of pole surface emits the same number of magnetic lines. Figure 7-5*a* shows one conductor in 12 positions 30° apart, and Fig. 7-5*b* shows the voltage corresponding to each of these positions. At position 1, the voltage is zero, owing to the fact that, when the conductor moves a very small amount, as from zero to 1°, or 1/360 of a revolution, its motion is practically parallel with the magnetic lines. Under this condition, it cuts none of the lines and the induced emf is zero. At position 4, the conductor has moved 90°, and the voltage as indicated on the curve is 100 volts. In this zone, any small amount of motion, as from 89 to 91°, will be in practically a vertical direction, and the motion of the conductor will be perpendicular to the magnetic lines. The conductor will now cut the lines at the greatest rate possible, and the voltage will be at its maximum value, assumed for convenience as 100 volts. Applying Fleming's right-hand rule shows that the direction of the induced emf is outward as indicated on Fig. 7-5*a* by ⊙. For any position between 1 and 4, the direction of the conductor's motion will be neither parallel nor perpendicular to the magnetic lines but will be at an angle to the lines. Therefore the induced emf will have values greater than zero but less than 100 volts. Figure 7-5*b* shows that at 30° the induced emf is 50 volts and at 60° it is 86.6 volts. When the conductor has passed position 4, the induced emf decreases until it reaches position 7, when it is again zero. After passing position 7, the conductor cuts lines again, but its motion is now upward. Applying Fleming's rule shows that the direction of the induced emf has been reversed. The values of the emf for positions between 7 and 13 (same as 1) are all negative as indicated on Fig. 7-5*b*. Any continued motion will merely result in additional similar voltage cycles. As the flow of electric current is dependent upon the voltage impressed, its flow will vary in a manner similar to the voltage.



(*a*) (*b*)

Fig. 7-5 Induced emf of a conductor in the simple a-c generator.

From the foregoing, it is now possible to set up a definition for an alternating current. An alternating current is one that is continually changing in magnitude and alternates in direction or polarity at regular intervals.

7-5 A-C Characteristics

Electrical Degrees. The a-c voltage of Fig. 7-5*b* is shown starting at zero volts, increasing in a positive direction to its maximum value at 90°, then decreasing to zero at 180° when it reverses in polarity, or alternates; next it increases to its maximum negative value at 270° and again decreases to zero at 360°. From this point on indefinitely, the voltage repeats this procedure. The degrees mentioned here are called *electrical degrees.* In this case, they also correspond to the mechanical degrees of coil motion, a condition that occurs only with two-pole alternators. One revolution of a four-pole alternator will produce 720 electrical degrees, or two complete cycles.

Alternation, Cycle, Frequency. The term *alternation* is used to define a period of 180 electrical degrees. The span of one complete set of values, that is, all the positive and all the negative values, is called a *cycle.* The cycle corresponds to 360 electrical degrees and also corresponds to two alternations. The number of times these complete sets of values occur in a second is called the *frequency* and is expressed in cycles per second.

The frequency of an alternator depends on its speed of rotation and the number of poles. Expressed mathematically, this is

$$f = \frac{P \times S}{120} \tag{7-2}$$

where f = frequency, cps
P = number of poles of the alternator
S = speed of the alternator, rpm

EXAMPLE 7-3 What is the frequency of a four-pole alternator which is being driven at 1,800 rpm?

GIVEN: $P = 4$ poles $S = 1{,}800$ rpm

FIND: f

SOLUTION:

$$f = \frac{P \times S}{120} = \frac{4 \times 1{,}800}{120} = 60 \text{ cps}$$

Period. The time required for a voltage (or current) to complete one cycle is called the *period* and is expressed mathematically as

$$t = \frac{1}{f} \tag{7-3}$$

where t = time required to complete one cycle, sec
f = number of cycles per second.

For a 60-cycle circuit *t* equals 1/60 sec, while for a 25-cycle circuit *t* would be 1/25 sec.

Frequencies Used in Power-supply Systems. The frequency of power systems is low, the most common being 60 cycles, although 25, 30, 40, and 50 are also used. The 60-cycle is the most popular because it gives good results when used for operating both lights and machinery. Alternators can be built to produce frequencies of 500 cycles and in a few special cases have been built for a frequency of 20,000 cycles.

Frequencies Used in Communications Systems. In radio, television, telemetry, etc., higher frequencies such as hundreds of thousands of cycles, millions of cycles, and up into the hundreds of millions of cycles per second are used. These high frequencies are obtained by means of generating systems (oscillators) that use vacuum tubes or transistors.

7-6 A-C Voltage and Current Characteristics

Instantaneous Values. When an emf is produced by rotating a coil at constant speed in a uniform magnetic field, the value of the voltage at any instant of time may be found by the equation

$$e_\theta = E_{\max} \times \sin\theta \tag{7-4}$$

where e_θ = instantaneous value of emf when coil has gone through θ electrical degrees

$E_{\max}$ = maximum value of emf

$\sin\theta$ = value obtained from table in Appendix XI.

EXAMPLE 7-4 An alternator produces a sine-wave voltage whose maximum value is 500 volts. What is the instantaneous value at (*a*) 7°, (*b*) 73°, (*c*) 162.5°, (*d*) 195°, (*e*) 322.5°?

GIVEN: $E_{\max} = 500$ volts $\theta =$ (*a*) 7° (*b*) 73° (*c*) 162.5° (*d*) 195° (*e*) 322.5°

FIND: e

SOLUTION:

$$e_\theta = E_{\max} \times \sin\theta$$

(*a*) $e_{7^\circ} = 500 \sin 7^\circ = 500 \times 0.122 = 61$ volts

(*b*) $e_{73^\circ} = 500 \sin 73^\circ = 500 \times 0.956 = 478$ volts

(*c*) $e_{162.5^\circ} = 500 \sin 162.5^\circ = 500 \times 0.301 = 150.5$ volts

(*d*) $e_{195^\circ} = 500 \sin 195^\circ = 500 \times -0.259 = -129.5$ volts

(*e*) $e_{322.5^\circ} = 500 \sin 322.5^\circ = 500 \times -0.609 = -304.5$ volts

Note: For values of sine, see Appendixes X and XI.

The sine-wave voltage of Fig. 7-5*b* has a maximum value of 100 volts and instantaneous values of 50 volts at 30°, 86.6 volts at 60°, etc. These values may be verified by use of Eq. (7-4).

The Sine Wave. If a large number of instantaneous values obtained by use of Eq. (7-4) are plotted and a curve is drawn, the curve is called a *sine wave.* A voltage corresponding to this shape is called a *sine-wave voltage.*

A simple method of drawing a sine-wave voltage is by means of the wheel diagram illustrated in Fig. 7-6. This may be done by first drawing a circle whose radius is made equal to the value of $E_{\max}$. Next, set off a number of equally spaced spokes. The number used must correspond to the number of points desired to draw the sine wave, and the greater the number of points, the more accurate the sine-wave drawing. Figure 7-6*b* is drawn with as many equally spaced vertical lines as there are spokes. When horizontal lines are projected from the ends of the spokes in Fig. 7-6*a* to the corresponding vertical lines on Fig. 7-6*b*, points of the sine-wave curve are obtained and may then be connected by a smooth line.

Most alternators produce a voltage that is a sine-wave voltage or so close to the sine wave that the entire study of alternating current from this point on will be based on sine-wave voltages and currents.

The Maximum Value. The highest value reached in a cycle is called the *maximum value* (see Fig. 7-7). This value is important in some parts of the study of alternating voltages and current, but it is not used as the rated value. It is designated as $E_{\max}$ and $I_{\max}$.

The Average Value. The sine wave of Fig. 7-7 is not a straight line from zero to maximum value but a smooth curved line. The average value of a complete cycle is zero, because the positive and negative areas under the curve are equal to each other. The average value generally referred to is, however, for only one-half of a cycle. This value may be determined by finding a large number of equally spaced instantaneous values from 0 to 180° (or from 0 to 90°), getting their sum, and then dividing it by

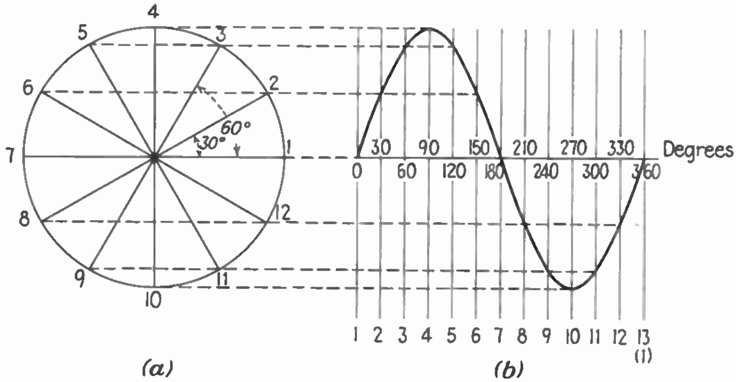


Fig. 7-6 Wheel-diagram method of drawing a sine-wave voltage.

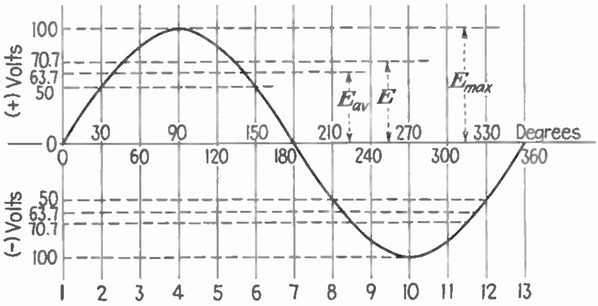


Fig. 7-7 Relative values of an a-c sine-wave voltage.

the number of cases used. Expressed mathematically,

$$E_{\text{ave}} = \frac{e_1 + e_2 + e_3 + e_n}{n} \tag{7-5}$$

Figure 7-8 illustrates this method of finding the average value. The accuracy of the result increases as the number of instantaneous values used is increased. A more accurate result may be obtained by higher mathematics (calculus), which shows that the average value is equal to $2/\pi$ times the maximum. This is the commonly accepted value and is expressed as

$$E_{\text{ave}} = 0.637\ E_{\text{max}} \qquad \left(\textit{Note: } \frac{2}{\pi} = 0.637\right) \tag{7-6}$$

The average value of the sine-wave voltage of Fig. 7-7 is 63.7 volts and is indicated by the line drawn through this point. While the average value is used in some engineering calculations, it is not the value used to represent the sine wave.

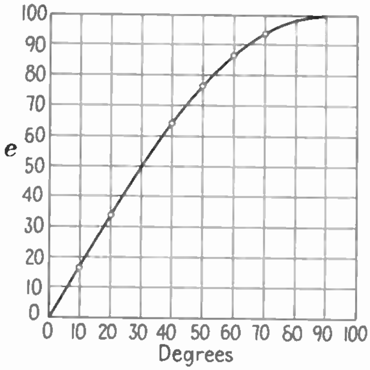
The A-C Ampere. The current flowing in a circuit is proportional to the voltage, and therefore the maximum, instantaneous, and average values as described for sine-wave voltages will also apply to sine-wave currents. Thus Eqs. (7-4) and (7-6) may be stated as

$$i_\theta = I_{\text{max}} \times \sin\theta \tag{7-7}$$

$$I_{\text{ave}} = 0.637 I_{\text{max}} \tag{7-8}$$

The maximum, instantaneous, and average values of current and voltage are used in some engineering calculations, but they are not used in practical work because they do not provide means of comparison with direct current.

The a-c ampere, which is the practical unit, is based upon the heating effect of the current. As the heating effect is the same regardless of the direction of the current flow, it provides a suitable means of comparing a-c and d-c effects. The a-c ampere may then be defined as that amount of



(*a*)

θ°	SIN θ	*e*
0	0.000	0.0
10	0.173	17.3
20	0.342	34.2
30	0.500	50.0
40	0.643	64.3
50	0.766	76.6
60	0.866	86.6
70	0.940	94.0
80	0.985	98.5
90	1.000	100.0

Total = 621.5

$$\text{Average} = \frac{621.5}{10} = 62.15$$

(*b*)

θ°	SIN θ	*e*
0	0.000	0.0
5	0.087	8.7
10	0.173	17.3
15	0.259	25.9
20	0.342	34.2
25	0.422	42.2
30	0.500	50.0
35	0.574	57.4
40	0.643	64.3
45	0.707	70.7
50	0.766	76.6
55	0.819	81.9
60	0.866	86.6
65	0.906	90.6
70	0.940	94.0
75	0.966	96.6
80	0.985	98.5
85	0.996	99.6
90	1.000	100.0

Total = 1195.1

$$\text{Average} = \frac{1195.1}{19} = 62.9$$

(*c*)

Fig. 7-8 Method of determining the average value of a sine-wave voltage. (a) Portion of a sine-wave voltage curve. (b) Average value obtained by taking 10° intervals. (c) Average value obtained by taking 5° intervals.

alternating current which will produce the same effect as one ampere of continuous or direct current.

The Effective Value. The instantaneous value of the sine wave chosen to represent the a-c ampere is called the *effective value,* since it must be such an amount that it will produce the same *heating effect* as 1 amp of direct current.

The heating effect of any current, whether alternating or direct current, is equal to the current squared, times the resistance ($P = I^2R$; $p = i^2R$). In alternating current, the heating effect will vary continually because the current is varying continually. The comparison of a-c and d-c heating effect is therefore based on the average rate at which the heat is produced. This may be expressed mathematically as

$$I_{dc}^2R = \frac{i_1^2R + i_2^2R + i_3^2R + i_4^2R + i_n^2R}{n} \tag{7-9}$$

In order to make a comparison, R must have the same value for both sides of the above equation and mathematically the equation may be simplified as

$$I_{dc}^2 = \frac{i_1^2 + i_2^2 + i_3^2 + i_4^2 + i_n^2}{n} \tag{7-10}$$

Examining the right-hand member of this equation, it will be seen that by adding the instantaneous i^2 values and dividing this sum by the number of cases, n, the average of the instantaneous i^2 values is obtained. Thus the equation may be restated as

$$I_{dc}^2 = \text{average of the instantaneous squares} \tag{7-11}$$

or

$$I_{dc}^2 = (\text{ave } i^2) \tag{7-11a}$$

As the a-c and d-c amperes are to have the same heating effect, the equation may also be stated as

$$I_{dc}^2 = I_{ac}^2 = (\text{ave } i^2) \tag{7-12}$$

Taking the square root of each member of this equation, it becomes

$$I_{dc} = I_{ac} = \sqrt{(\text{ave } i^2)} \tag{7-13}$$

The effective value of a sine wave is therefore equal to the square root of the average of the instantaneous squares. Figure 7-9 illustrates a simple method of finding the effective value. The effective value may also be obtained by higher mathematics (calculus), which shows that the effective value of a sine wave is always equal to the maximum value divided by $\sqrt{2}$. This is the commonly accepted value and is expressed as

$$I = \frac{I_{max}}{\sqrt{2}} \tag{7-14}$$

This may be simplified as

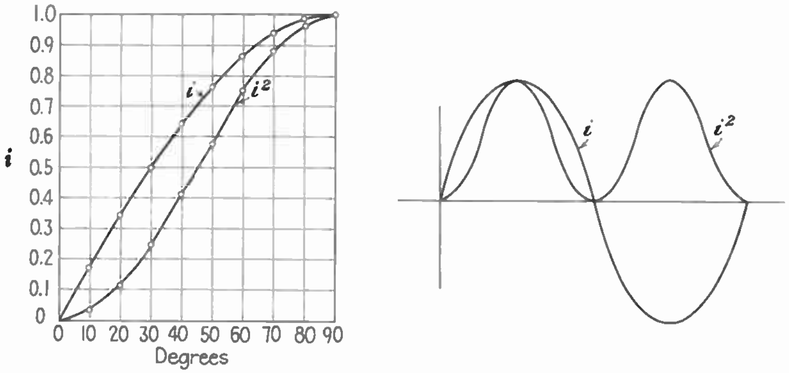
$$I = \frac{1}{\sqrt{2}} \times I_{max} = \frac{1}{1.414} \times I_{max} = 0.707\ I_{max}$$

$$I = 0.707\ I_{max} \tag{7-15}$$

Since alternating voltages and currents both follow sine-wave forms, the same relation applies to voltages; thus

$$E = 0.707\ E_{max} \tag{7-16}$$

$$E = \frac{E_{max}}{\sqrt{2}} = \frac{E_{max}}{1.414} \tag{7-17}$$



(a) (b)

θ°	SIN θ	i	i²
0	0.000	0.000	0.000
10	0.173	0.173	0.030
20	0.342	0.342	0.117
30	0.500	0.500	0.250
40	0.643	0.643	0.413
50	0.766	0.766	0.587
60	0.866	0.866	0.750
70	0.940	0.940	0.884
80	0.985	0.985	0.970
90	1.000	1.000	1.000

Total = 5.001

$$\text{ave } i^2 = \frac{5.001}{10} = 0.5001$$

$$\sqrt{\text{ave } i^2} = \sqrt{0.5001} = 0.707$$

(c)

Fig. 7-9 Method of determining the effective value of a sine-wave current. (a) Portion of a sine-wave current showing the variation of i and i^2. (b) Curves of i and i^2 for one complete cycle. (c) Effective value obtained by taking 10° intervals.

The effective value is sometimes called the *rms* (root-mean-square) *value* because it is found by taking the square root of the mean (or average) of the squares of a large number of instantaneous values. The effective value of the voltage wave of Fig. 7-7 is 70.7 volts and is indicated by the line drawn through this point. The effective value is represented by the capital letters E and I; note that no subscript is used as was the case in E_{ave} and I_{max}.

Alternating-current measuring instruments such as voltmeters or ammeters are calibrated to indicate the effective values; thus if the voltage measurement on an a-c circuit is 110 volts, it will produce the same effects on an incandescent lamp as a 110-volt d-c circuit even though the a-c circuit at some instants reaches the maximum of 110 × 1.414, or 155 volts.

EXAMPLE 7-5 (*a*) What is the effective value of the voltage of the alternator in Example 7-4? (*b*) What is its average value?

GIVEN: $E_{max} = 500$ volts

FIND: (*a*) E (*b*) E_{ave}

SOLUTION:

(*a*) $E = 0.707\ E_{max} = 0.707 \times 500 = 353.5$ volts

(*b*) $E_{ave} = 0.637\ E_{max} = 0.637 \times 500 = 318.5$ volts

EXAMPLE 7-6 The rms value of a current in an a-c circuit is 10 amp. What is the (*a*) maximum value? (*b*) Average value?

GIVEN: $I = 10$ amp

FIND: (*a*) I_{max} (*b*) I_{ave}

SOLUTION:

(*a*) $I_{max} = 1.414 \times I = 1.414 \times 10 = 14.14$ amp

(*b*) $I_{ave} = 0.637\ I_{max} = 0.637 \times 14.14 = 9.00$ amp

7-7 Volt-amperes, Power Factor, Power

Phase Relation of Voltage and Current. Because alternating voltages and currents are continually varying in a sine-wave manner, it is possible that the voltage and current waves of a circuit may or may not be in step, or in phase. Figure 7-10 shows several possible phase relations of voltage and current. Whether the voltage and current of a circuit are in phase or

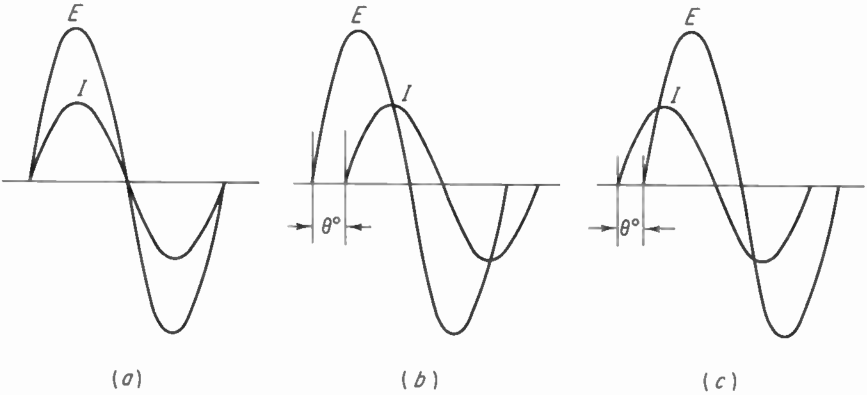


Fig. 7-10 Typical voltage-current phase relations. (a) Voltage and current in phase. (b) Current lagging the voltage by θ°. (c) Current leading the voltage by θ°.

out of phase depends upon the type of circuit elements forming the load of that circuit. The study of inductance and capacitance (Chaps. 8 and 9) will show why various phase relations may exist.

Volt-amperes. In d-c circuits, where the voltage and current are constant in value, the product of volts and amperes is equal to the power in watts [Eq. (2-12)]. In a-c circuits the product of volts and amperes will be equal to the power in watts *only* when the volts and amperes are in phase. When the voltage and current are out of phase, the product of the volts and amperes will be greater than the watts consumed by the circuit. For this reason, the product of volts and amperes is called the *volt-amperes* of the circuit. This product is also frequently called the *apparent power* of a circuit.

$$\text{Volt-amperes} = \text{volts} \times \text{amperes} \qquad \text{or} \qquad VA = E \times I \tag{7-18}$$

Power Factor. The ratio of the actual watts consumed by a circuit to the volt-amperes of that circuit is called the *power factor,* thus

$$\text{Power factor} = \frac{\text{actual power}}{\text{apparent power}} \qquad \text{or} \qquad \text{PF} = \frac{P}{E \times I} \tag{7-19}$$

The numerical value of the power factor varies and may range from zero to 1. Power companies strive to keep the power factor of their load as high as possible in order to attain the best operating efficiency for their system.

Power. Rearranging the terms in Eq. (7-19) shows that the power consumed by a circuit may be calculated by

$$P = E \times I \times \text{PF} \tag{7-20}$$

When the resistance and the voltage or current of a circuit are known, the power consumed by an a-c circuit may also be calculated by use of Eq. (2-13) or (2-14).

7-8 A-C Generators

Single-phase System. The simple a-c generator described in Art. 7-4 has only one coil. Such an a-c generator, hereafter called an *alternator,* would have a very low efficiency, as only a small portion of the iron core of the rotating member would be doing useful work. Placing additional coils around the periphery of the rotating member would raise the efficiency of the alternator. If all the coils are connected in series in a single circuit, a single output voltage would be obtained and the alternator would be classed as *single phase.*

The electrical power supplied for general residential use is single phase and may be either a two-wire system for 110 volts or a three-wire system for both 110 and 220 volts. Equations (7-18) to (7-20) apply to single-phase systems.

The study of the effects of inductance and capacitance in a-c systems and the study of a-c circuit characteristics are presented in this text in terms of a single-phase system. The principles and characteristics established for single-phase systems may readily be applied to polyphase systems.

Polyphase Systems. The efficiency of a multicoil alternator may be further increased by dividing the coils into two or more groups and connecting them to form two or more separate circuits. Each circuit has independent voltage characteristics and is called a *phase.* The phases may be interconnected to produce certain desirable system characteristics. Although systems having two, three, four, six, etc., phases have been developed, the three-phase system is used most extensively.

7-9 Two-phase System

A simple two-phase alternator having two coils placed 90° apart is shown in Fig. 7-11. The alternator will have four collector rings which are omitted to avoid complicating the drawing. Each coil, or phase, will produce an alternating voltage as described in Art. 7-4. When the voltage of phase 1 is zero, the voltage of phase 2 will be at its maximum value, and when the voltage of phase 1 is at its maximum value the voltage of phase 2 will be zero. Because the coils are placed 90° apart, the voltages induced in the two coils will be 90 electrical degrees apart (see Fig. 7-11).

The output terminals of the alternator may be arranged in several ways, as is shown in Figs. 7-12 to 7-14. The conventional two-phase four-wire system (Fig. 7-12) provides two equal independent voltages 90° apart. This system can supply power to either or both single-phase or two-phase loads.

A two-phase three-wire system is produced by joining one terminal of each phase and providing only three line wires, as shown in Fig. 7-13. The voltage between line wires 1 and 3 will be the sum of the voltages of

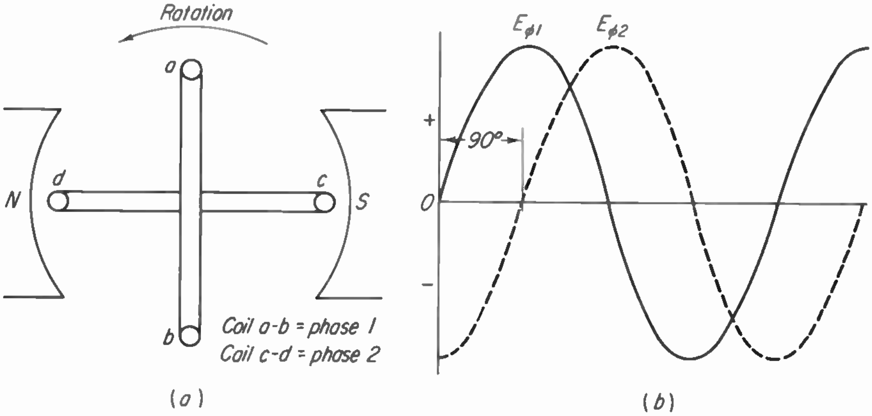


Fig. 7-11 Two-phase system. (a) The simple alternator. (b) The output voltages of the two phases.

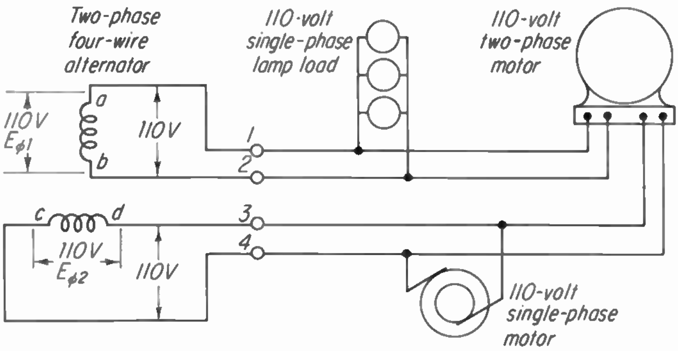


Fig. 7-12 Two-phase four-wire system supplying power to a two-phase motor, a single-phase lamp load, and a single-phase motor.

phase 1 and phase 2, but because these two voltages are 90° out of phase, their resultant sum will be only 1.41 times the phase voltage. This system also can supply power to both single-phase and two-phase loads. An advantage of this system is in the reduction of the amount of copper required by a three-wire distribution system over a four-wire system.

Figure 7-14 shows a two-phase five-wire system in which the fifth wire is obtained by joining the center points of the two phases. This system can be used to supply 110-volt single-phase lighting circuits and 220-volt two-phase motors. The 110-volt single-phase loads can be supplied from the common wire (line 3) and any one of the other four wires. The 220-volt two-phase load must be connected to line wires 1 and 4 for phase 1 and line wires 2 and 5 for phase 2. The 155-volt single-phase sources obtained at line wires 1-2, 2-4, 4-5, and 5-1 are not ordinarily used because it is an uncommon voltage.

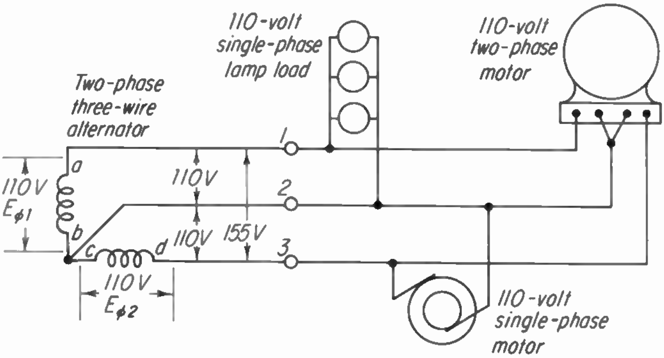


Fig. 7-13 Two-phase three-wire system supplying power to a two-phase motor, a single-phase lamp load, and a single-phase motor.

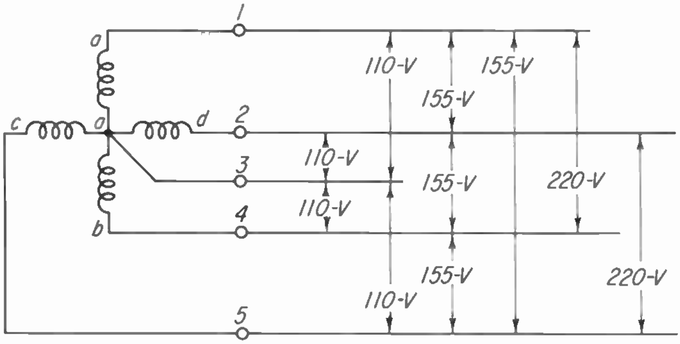


Fig. 7-14 Two-phase five-wire system that can also serve as a four-phase system.

7-10 Four-phase System

The alternator of Fig. 7-14 also provides four phases of output from the common wire (line 3) and each of the other line wires. A four-phase output can also be obtained from the four combinations of line wires, namely, 1-2, 2-4, 4-5, and 5-1. By proper design, a standard value of output voltage could be obtained.

7-11 Three-phase Systems

Simple Alternator. A simple three-phase alternator having three coils placed 120° apart is shown in Fig. 7-15. This basic alternator would have six collector rings, which have been omitted to avoid complicating the drawing. Each coil, or phase, will produce an alternating voltage as described in Art. 7-4. Because the coils are placed 120° apart, the voltages induced

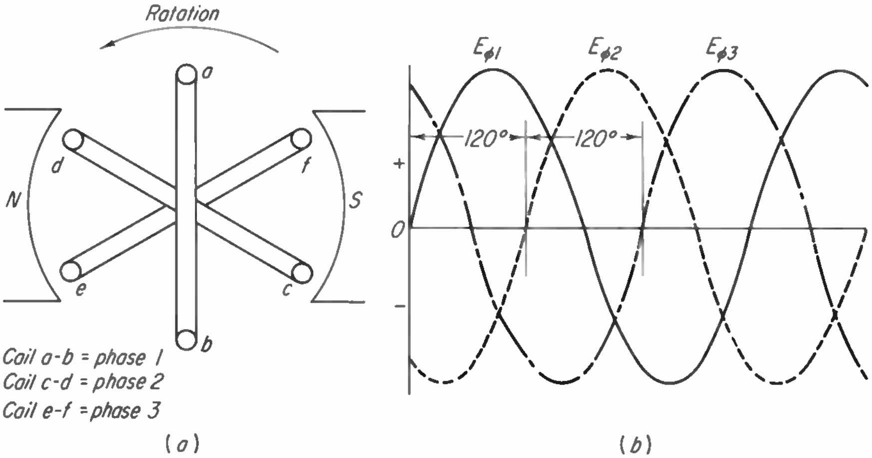


Fig. 7-15 Three-phase system. (a) The simple alternator. (b) The output voltages of the three phases.

in the three coils will be 120 electrical degrees apart (see Fig. 7-15). When the voltage of phase 1 is zero, the voltage of phase 2 will be at 86.6 per cent of its maximum value with a negative polarity and the voltage of phase 3 will be at 86.6 per cent of its maximum value with a positive polarity.

The three-phase alternator can supply power to both single- and three-phase loads. Although three-phase systems generally have only three line wires, the fundamental concepts were presented here in terms of six wires for ease of explanation. In general practice the six wires are interconnected to produce (1) a three-wire delta-connected system, (2) a three-wire wye-connected system, or (3) a four-wire wye-connected system. When single-phase loads are connected to three-phase alternators, it is recommended that these loads be evenly divided among the three phases. All the equations presented in this article are based on the assumption that the loads are balanced.

Delta System. The six terminals of the basic three-phase alternator may be interconnected as shown in Fig. 7-16. Because the schematic diagram produced resembles the letter delta (Δ) of the Greek alphabet, this system is referred to as being *delta connected.* Three-phase loads are connected to the three line wires 1, 2, 3, and single-phase loads may be connected to any pair of these line wires.

The voltages across the line wires 1-2, 2-3, and 3-1 are equal to the individual phase voltages $E_{\phi 1}$, $E_{\phi 2}$, $E_{\phi 3}$, or

$$E_{\text{line}} = E_{\text{phase}} \tag{7-21}$$

The current in the line wires is equal to 1.73 times the current in an individual phase, or

$$I_{\text{line}} = 1.73 I_{\text{phase}} \tag{7-22}$$

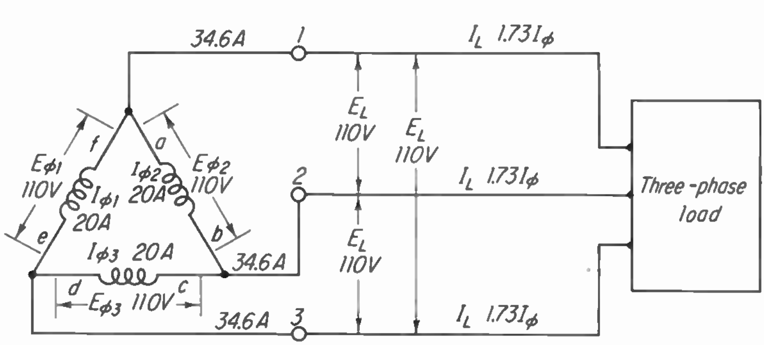


Fig. 7-16 Three-phase delta-connected system showing the relations between phase and line values of the voltages and currents.

For example, the current flowing in line wire 1 is composed of the current flowing from b to a and the current flowing from e to f, or

$$\underset{\cdot}{I}_{\text{line}} = \underset{\cdot}{I}_{ba} + \underset{\cdot}{I}_{ef} \tag{7-23}$$

Note: The dot under the letter I indicates that this is not necessarily the arithmetic sum. Because the individual phase currents do not correspond in phase (meaning time), their actual sum will be less than their arithmetic sum.

The volt-amperes delivered by this system may be found by

$$VA = 1.73E_LI_L \tag{7-24}$$

also,

$$VA = 3E_\phi I_\phi \tag{7-25}$$

Note: ϕ is often used to represent the word *phase.*

The power delivered by this system may be found by

$$P = 1.73E_LI_L \text{ PF} \tag{7-26}$$

also,

$$P = 3E_\phi I_\phi \text{ PF} \tag{7-27}$$

The power factor for this system may be found by

$$\text{PF} = \frac{P}{1.73E_LI_L} \tag{7-28}$$

also,

$$\text{PF} = \frac{P}{3E_\phi I_\phi} \tag{7-29}$$

EXAMPLE 7-7 A three-phase delta-connected alternator is supplying a three-phase 110-volt load with a balanced line current of 34.6 amp. (*a*) What is the three-phase volt-ampere rating of the load? (*b*) What is the three-phase power rating if the load power factor is 80 per cent? (*c*) What is the amount of current in each individual phase of the alternator?

GIVEN: $E_L = 110$ volts $I_L = 34.6$ amp PF = 80%

FIND: (*a*) VA_L (*b*) P_L (*c*) I_ϕ

SOLUTION:

(*a*) $VA_L = 1.73E_LI_L = 1.73 \times 110 \times 34.6 = 6{,}584$

(*b*) $P_L = 1.73E_LI_L\text{PF} = 1.73 \times 110 \times 34.6 \times 0.8 = 5{,}267$ watts

(*c*) $I_\phi = \dfrac{I_L}{1.73} = \dfrac{34.6}{1.73} = 20$ amp

Three-wire Wye-connected System. The six terminals of the basic three-phase alternator may be interconnected as shown in Fig. 7-17. Because the schematic diagram produced resembles the letter Y, this system is referred

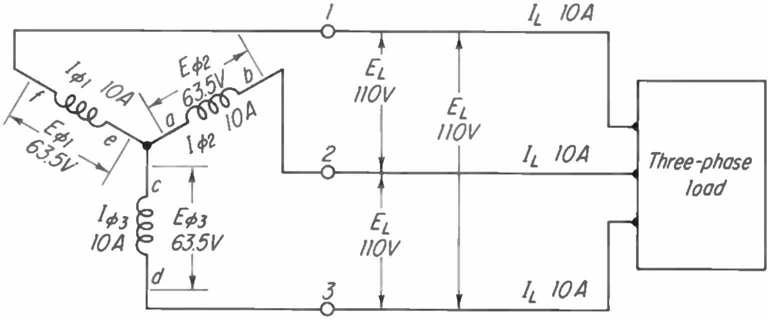


Fig. 7-17 Three-phase wye-connected system showing the relations between phase and line values of the voltages and currents.

to as being *wye-connected.* Three-phase loads are connected to the three line wires 1, 2, 3, and single-phase loads may be connected to any pair of line wires.

The voltages across the line wires 1-2, 2-3, and 3-1 are equal to the sum of two phase voltages; however, this is not their arithmetic sum because the voltages are not in phase with each other.

$$E_{\text{line}} = 1.73E_{\text{phase}} \tag{7-30}$$

The current in the line wires is the same as the phase current, or

$$I_{\text{line}} = I_{\text{phase}} \tag{7-31}$$

The equations for volt-amperes, power, and power factor are the same as those for the delta-connected system and are given as Eqs. (7-24) to (7-29).

EXAMPLE 7-8 Meter readings taken for a certain three-phase wye-connected alternator supplying power to a motor load show that the line voltages are 220 volts, the line currents are 20 amp, and the line power is 5,500 watts. (*a*) What is the load in volt-amperes? (*b*) What is the power factor of the load? (*c*) What is the phase voltage of the alternator? (*d*) What is the phase current of the alternator?

GIVEN: $E_L = 220$ volts $I_L = 20$ amp $P = 5{,}500$ watts

FIND: (*a*) VA (*b*) PF (*c*) E_ϕ (*d*) I_ϕ

SOLUTION:

(*a*) $$VA = 1.73E_LI_L = 1.73 \times 220 \times 20 = 7{,}612$$

(*b*) $$\text{PF} = \frac{P}{1.73E_LI_L} = \frac{5{,}500}{1.73 \times 220 \times 20} = 0.72 \text{ or } 72\%$$

(*c*) $$E_\phi = \frac{E_L}{1.73} = \frac{220}{1.73} = 127 \text{ volts}$$

(*d*) $$I_\phi = I_L = 20 \text{ amp}$$

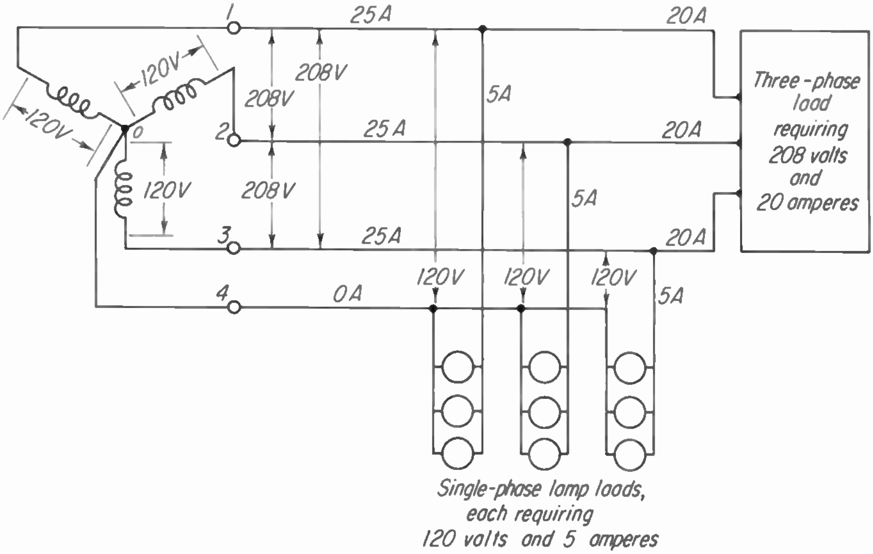


Fig. 7-18 Three-phase four-wire wye-connected power system with typical voltage and current distributions indicated. (These values will be valid only if all loads have the same power factor value.)

Four-wire Wye-connected System. The three-wire wye-connected system of Fig. 7-17 can be converted to a four-wire system by adding an additional line wire and connecting it to the mid-point of the wye. Figure 7-18 shows a four-wire wye-connected alternator supplying a typical load. The neutral wire (connected to point o) has no current in it because the loads are balanced. Under this condition the neutral wire could be omitted; however, it is used in order to keep the voltage distribution of the three single-phase loads balanced when the currents in these three loads are unbalanced.

A power system with the voltage values shown in Fig. 7-18 is frequently used to supply 120-volt single-phase lighting loads and 208-volt three-phase motor loads.

7-12 The D-C Generator

The Simple Generator. The current produced by a d-c generator is unidirectional; that is, one terminal of the generator is always positive and the other always negative. On the a-c generator described in Art. 7-4, one brush, B_1, for example, is positive during the first half cycle and negative during the second half cycle, while the other brush B_2 is negative during the first half cycle and positive during the second. The simple a-c generator can be made a d-c generator by installing a rotary reversing switch in place of the two slip rings. Such a device, called a *commutator*, is used to reverse the connections automatically between the coil leads and the brushes.

Figure 7-19 shows a simple d-c generator with a single armature coil, a commutator, and two brushes. The commutator consists of two segments K_1 and K_2 that are assembled onto the shaft but insulated from it and also insulated from each other. The coil side *aa'* is connected to the commutator bar K_1, and coil side *bb'* is connected to bar K_2. The carbon brushes B_1 and B_2 are mounted in a stationary position and make a sliding or wiping contact with the commutator.

Applying Fleming's right-hand rule shows that the conductor moving downward under the *N* pole will be positive because its induced emf is outward in direction. This is true whether it is the conductor *aa'* or the conductor *bb'*. As the brush B_1 is always making contact with that commutator bar which is connected to the conductor moving downward under the *N* pole, B_1 will always be the positive terminal of the generator. By similar reasoning, it can be shown that B_2 will be negative.

The voltage of the d-c generator *coil* will be exactly the same as for the a-c generator if the coil, poles, magnetic field, and speed are considered to be the same for each. However, since the commutator has been added in place of the slip rings, the voltage output of the generator will be unidirectional as shown in Fig. 7-20. This figure is similar to Fig. 7-5*b* with its negative loop reversed.

Voltage of Single-coil and Multicoil Generators. The voltage of this single-coil generator is a pulsating voltage, and it does not approach the continuous current as is expected of a d-c generator. If the generator is built with two coils as shown in Fig. 7-21*a*, it will result in an improved output voltage. The voltages of the two coils individually are shown as C_1

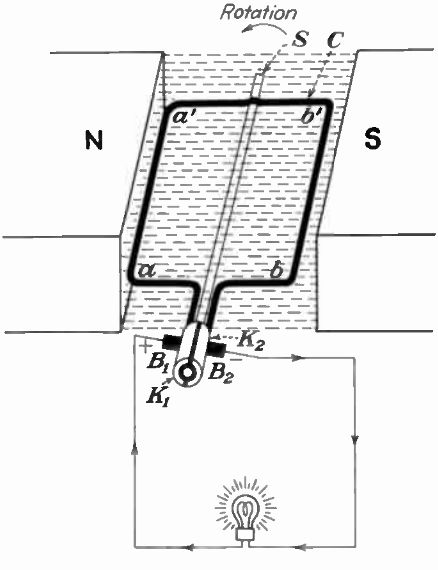


Fig. 7-19 A simple d-c generator.

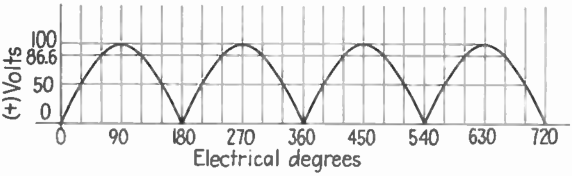


Fig. 7-20 Voltage of a single-coil d-c generator.

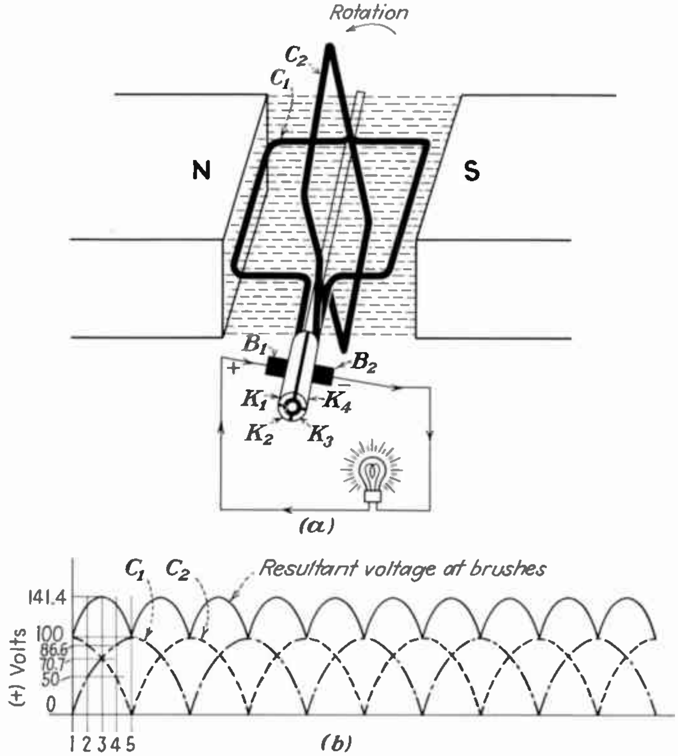


Fig. 7-21 A two-coil d-c generator. (a) The generator. (b) Voltage of the generator.

and C_2 in Fig. 7-21*b*. The voltage at the brushes is obtained by adding these two. For example, at point 1 the resultant voltage is $0 + 100 = 100$ volts; at point 2 the resultant is $50 + 86.6 = 136.6$ volts; at point 3 the resultant is $70.7 + 70.7 = 141.4$ volts; at point 4 the resultant is $86.6 + 50 = 136.6$ volts; at point 5 the resultant is $100 + 0 = 100$ volts.

Figure 7-22 shows the voltages of four separate coils and also their resultant voltage. These illustrations show that, as more coils are added, the resultant voltage approaches a straight line and becomes similar to the continuous current.

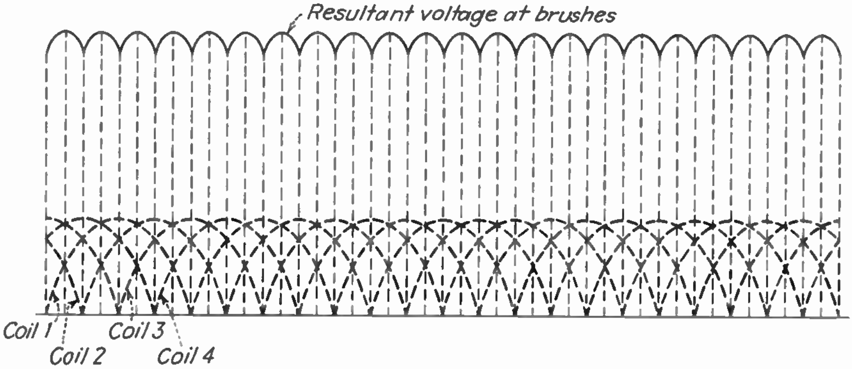


Fig. 7-22 Voltage of a four-coil d-c generator.

Use of Direct Current. When this current is used for operating motors, lights, heaters, and other power devices, it is usually considered equivalent to the continuous current, but when used to operate sound devices such as radio equipment, public-address systems, etc, it will not produce satisfactory results. In these cases, the wave shape of the output voltage, referred to as *commutator ripple,* causes disturbing noises in sound equipment, and special filter circuits must be used.

7-13 Commercial Generators

The simple generators discussed in the preceding articles are the fundamental machines; that is, the poles are only shown as bar magnets, the armatures have only single coils, and in general they contain only the bare necessities. This is done intentionally to make it easier to understand the theory of the generators.

In commercial machines, the simple poles are replaced by a well-designed magnetic circuit as shown in Fig. 7-23. It consists of two or more laminated pole pieces bolted to a frame. The frame in addition to supporting the bearings and armature also is a part of the magnetic circuit. In order to get a strong magnetic field, the bar magnets are replaced by electromagnets. The coils of the electromagnets are called the *field coils* and usually consist of a large number of turns of fairly small wire. The armature is constructed of a number of coils placed in the slots of a laminated iron core. The iron core provides a low-reluctance path for the magnetic lines. The air gap between the armature and the pole pieces is kept as small as possible to reduce the reluctance of the magnetic circuit. An air gap of approximately 0.015 inch is often found in small machines, while air gaps up to 0.125 inch may be found in the medium sizes. The method of constructing and mounting the slip rings or the commutator and the brush holders is also varied to provide adequate current-collecting methods (see also Chap. 13).

Alternating-current generators, with the exception of very small sizes, are usually made with the field poles on the rotating unit and the armature winding, now called the *stator*, as the stationary part. This requires fewer slip rings, provides a better magnetic circuit for the armature, permits higher speeds, and makes higher voltages possible. A disassembled view of an a-c generator of the rotating-field type showing its stator, rotating field unit, bearing brackets, and brush holders is given in Fig. 7-24. An a-c generator, whether of the rotating-field or rotating-armature type, requires

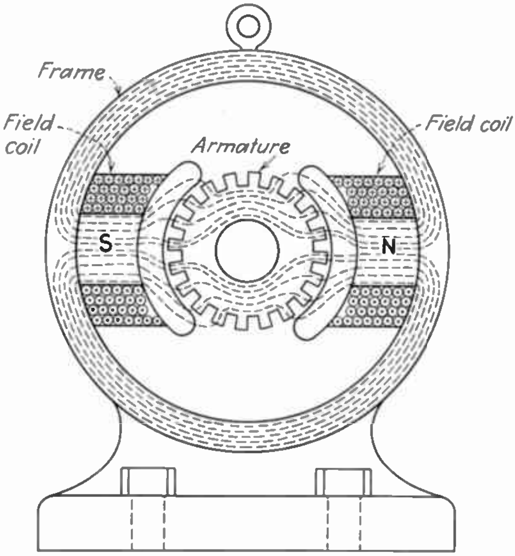


Fig. 7-23 The magnetic circuit of a commercial two-pole generator.

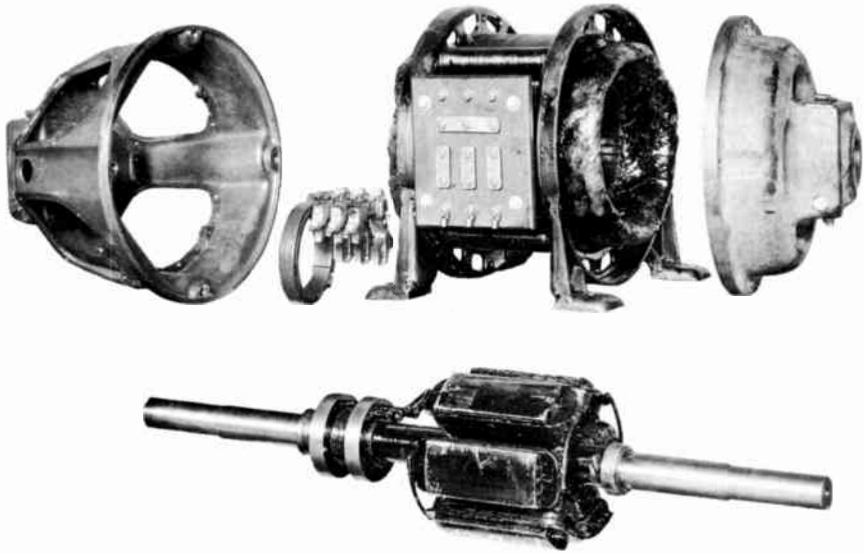


Fig. 7-24 Disassembled view of an a-c generator. (*General Electric Company*)

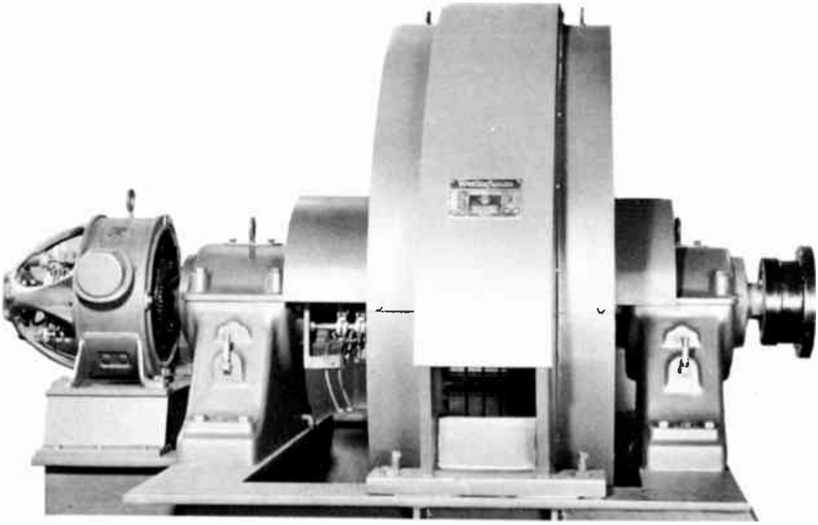


Fig. 7-25 A 1,250-kva 720-rpm 2,400-volt 60-cycle a-c generator with a direct-connected exciter. (*Westinghouse Electric Corporation*)

direct current for its field circuit. In many installations, no d-c power line is available, and under these conditions it is common practice to mount a small d-c generator on the end of the alternator so that it may be operated from the same driving force. This d-c generator need only supply current to the field coils of the alternator and is called the *exciter*. Its rating is usually less than 5 per cent of the alternator rating. Figure 7-25 shows an alternator with its own direct-connected exciter.

Direct-current generators are made only of the stationary-field type. Figure 7-26 shows a typical d-c generator.

7-14 Transformers

Use of Transformers in Power Systems. An important reason for the greater use of alternating current over direct current is the ease with which the voltage can be raised or lowered by use of transformers. This makes it possible to generate power in large quantities at the source of energy such as a hydroelectric station. The voltage may then be raised to transmission-line values as high as 500,000 volts and thereby efficiently transmit power to cities several hundred miles from the generating station. At the outskirts of each city, a transformer substation is installed to reduce the voltage to reasonable amounts for distribution throughout the city, and it is then further stepped down by additional transformers for supplying consumers with power.

Principle of Operation. Electromagnetic induction is also the basis of operation of the transformer. Figure 7-27 illustrates the fundamental transformer consisting of a core and two windings called the *primary* and *secondary* windings. The core provides a path for the magnetic field and is generally built up of a large number of thin high-grade sheet-steel laminations. The primary is the winding that receives the energy from the line, and the secondary is the winding that delivers the energy to the load.

The theory of operation of the transformer is as follows:

1. When the primary is connected to a source of alternating emf, an alternating current is caused to flow in that winding.

2. Whenever a current flows in a conductor, a magnetic field is set up about the conductor. When the current is continually changing in magnitude

Fig. 7-26 A d-c generator. (*Electro Dynamic Works, Division of General Dynamics*)

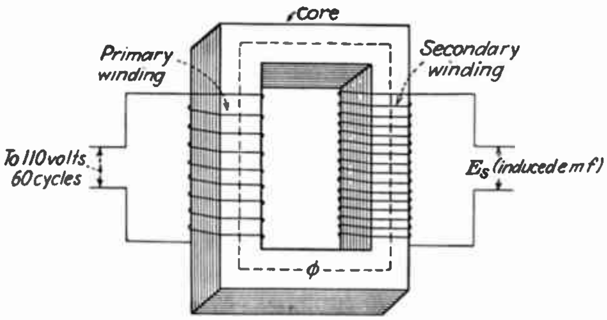


Fig. 7-27 A fundamental transformer.

and alternating in polarity, the magnetic field that it sets up in the iron core will do likewise.

3. The alternating magnetic field is therefore continually expanding and contracting. As the magnetic circuit is a closed path, the variation of the magnetic field is the same anywhere on the core.

4. The expanding and contracting magnetic lines will therefore cut conductors placed anywhere on the core, and according to Faraday's experiment, an induced emf will be set up in the conductors.

5. As the same flux cuts each conductor on the core, the induced emf per turn will be the same. Therefore the voltage of each winding will be proportional to the number of turns; expressed mathematically, this is

$$\frac{E_P}{E_S} = \frac{N_P}{N_S} \tag{7-32}$$

6. From this equation, it may be seen that the secondary voltage can be raised or lowered by using the proper ratio of turns.

EXAMPLE 7-9 It is desired to have a radio power transformer step up the voltage from 110 to 750 volts. How many turns will be required on the secondary winding if the primary has 120 turns?

GIVEN: $E_P = 110$ volts $E_S = 750$ volts $N_P = 120$ turns

FIND: N_S

SOLUTION:

$$\frac{E_P}{E_S} = \frac{N_P}{N_S}$$

Therefore

$$N_S = N_P \frac{E_S}{E_P} = 120 \times \frac{750}{110} = 818 \text{ turns}$$

Operation of Transformer When Loaded. If the secondary is connected to a load, a current will flow through the load and also through the secondary winding. The power consumed by the load must come from the line; hence the primary load must vary in the same manner as the secondary. Examination of Fig. 7-27 shows that there is no electrical connection between the primary and secondary windings. The power consumed by the load is transferred from the primary winding to the secondary winding by means of the magnetic flux. The efficiency of the transformer is very high, often above 95 per cent; hence the watts on the secondary side are nearly equal to the watts on the primary side. Under this condition, the currents vary inversely with the voltages. Mathematically, this is

$$\frac{E_P}{E_S} = \frac{I_S}{I_P} \qquad \text{or} \qquad E_P I_P = E_S I_S \tag{7-33}$$

This equation shows that as the voltage is stepped up by a transformer the current is stepped down. This is a decided advantage in power-transmission systems.

EXAMPLE 7-10 A certain city requires 5,000 kva for its lighting load. If the transmission line bringing the power to the city is operated at 132,000 volts, how much current will flow in the transmission lines?

GIVEN: kva = 5,000 $E_P = 132{,}000$ volts

FIND: I_P

SOLUTION:

$$I_P = \frac{\text{kva} \times 1{,}000}{E_P} = \frac{5{,}000 \times 1{,}000}{132{,}000} = 37.8 \text{ amp}$$

EXAMPLE 7-11 A transformer substation is used to reduce the voltage to 4,400 volts for the power system of Example 7-10. What is the secondary current?

GIVEN: $E_P = 132{,}000$ volts $E_S = 4{,}400$ volts $I_P = 37.8$ amp

FIND: I_S

SOLUTION:

$$\frac{E_P}{E_S} = \frac{I_S}{I_P}$$

Therefore $$I_S = I_P\frac{E_P}{E_S} = 37.8 \times \frac{132{,}000}{4{,}400} = 1{,}134 \text{ amp}$$

EXAMPLE 7-12 The power of Example 7-11 is passed through additional transformers to reduce the voltage from 4,400 to 110 volts. What is the current at 110 volts?

GIVEN: $E_P = 4{,}400$ volts $E_S = 110$ volts $I_P = 1{,}134$ amp

FIND: I_S

SOLUTION:

$$I_S = I_P\frac{E_P}{E_S} = 1{,}134 \times \frac{4{,}400}{110} = 45{,}360 \text{ amp}$$

The transformer operates because of the changing magnetic field, and it is therefore an a-c device and will not operate on direct current. Also, the frequency of the secondary circuit will be the same as that of the primary.

Commercial transformers will differ in construction from that shown in Fig. 7-27, which is merely a convenient form of illustration. Cutaway views of two commercial transformers are shown in Fig. 7-28.

7-15 Efficiency

Every generator, motor, transformer or other device that transforms energy from one form to another loses some of the energy in the process.

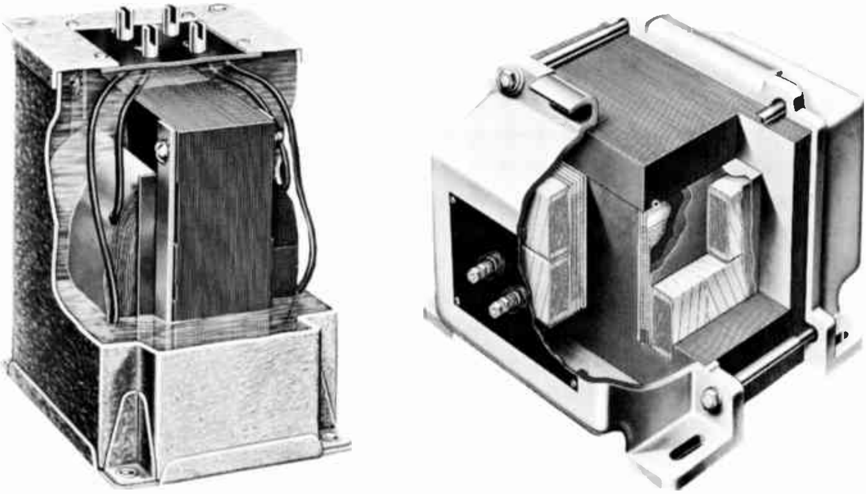


Fig. 7-28 Cutaway views of two types of radio power transformers. (*Standard Transformer Corporation*)

If a gasoline engine is used to rotate a generator, the engine supplies mechanical energy to the shaft of the generator, and this energy is converted to the electrical energy supplied to the load. Only part of the mechanical energy will be transformed to electrical energy because of the friction, iron, and copper losses of the generator. Efficiency is the expression used to indicate what portion of the energy received by a device can be given out by it. Efficiency may be defined as the ratio of the output to input of any device; mathematically it is expressed as

$$\text{Efficiency} = \frac{\text{output}}{\text{input}} \tag{7-34}$$

It is more commonly expressed in per cent, as

$$\text{Per cent efficiency} = \frac{\text{output}}{\text{input}} \times 100 \tag{7-34a}$$

EXAMPLE 7-13 A generator that is delivering 10 kw to a load requires the gasoline engine driving it to supply 15 hp to the generator shaft. What is the per cent efficiency of the generator?

GIVEN: Output = 10 kw Input = 15 hp

FIND: Per cent eff

SOLUTION:

$$\text{Per cent eff} = \frac{\text{output}}{\text{input}} \times 100 = \frac{10 \times 1{,}000}{15 \times 746} \times 100 = 89.3$$

Note: To use the efficiency equation, both the output and input must be in the same kind of units. To satisfy this requirement, both the kilowatts and horsepower were converted to watts.

The efficiency of a transformer is the ratio of the power output (secondary watts) to the power input (primary watts). As the transformer has no moving parts, there will be no friction loss, but there will be losses in the iron core and in the copper wire.

EXAMPLE 7-14 A transformer is used to supply the heater current to ten 6.3-volt tubes. The tubes are connected in parallel, and each requires 0.3 amp. The voltage at the secondary of the transformer is 6.3 volts. What is the efficiency of the transformer if its primary draws 22 watts from the line?

GIVEN: $E_S = 6.3$ volts $\quad I_S = 0.3 \times 10$ amp $\quad P_P = 22$ watts

FIND: Per cent eff

SOLUTION:

$$\text{Per cent eff} = \frac{\text{watts output}}{\text{watts input}} \times 100 = \frac{6.3 \times 0.3 \times 10}{22} \times 100 = \frac{18.9}{22} \times 100 = 85.9$$

Wiring systems carrying current are actually being used to transport electrical energy from a source to a load. In doing this, some energy is lost at the wire. This loss appears in the form of heat which the wire gives off to the surrounding air, and it is equal to I^2R.

EXAMPLE 7-15 The wires from the secondary of the transformer to the first tube of Example 7-14 have a resistance of 0.02 ohm. (*a*) How much power is lost in the wires? (*b*) How much power is available at the tubes? (*c*) How efficiently is the wiring doing its job? (*d*) What is the voltage at the heaters of the tubes?

GIVEN: $I = 0.3 \times 10$ amp $\quad R_{\text{wire}} = 0.02$ ohm $\quad P_S = 18.9$ watts
$E_S = 6.3$ volts

FIND: (*a*) Power loss in the wire (*b*) Power at the tubes (*c*) Per cent eff
(*d*) Voltage at tubes

SOLUTION:

(*a*) $$P_{\text{wire}} = I^2R = (0.3 \times 10)^2 \times 0.02 = 0.18 \text{ watt}$$

(*b*) $$P_{\text{tubes}} = P_S - P_{\text{wire}} = 18.9 - 0.18 = 18.72 \text{ watts}$$

(*c*) $$\text{Per cent eff} = \frac{P_{\text{tubes}}}{P_S} \times 100 = \frac{18.72}{18.9} \times 100 = 99.05$$

(*d*) $$E_{\text{tubes}} = E_S - IR = 6.3 - (0.3 \times 10 \times 0.02) = 6.3 - 0.06 = 6.24 \text{ volts}$$

The efficiency of most electrical apparatus is high, and generally the larger the device, the higher its efficiency. Large generators and motors have efficiencies around 90 per cent, and large transformers as used by power companies may have efficiencies in the order of 98 per cent.

QUESTIONS

1. (*a*) What kinds of current may be used to operate radio receivers, television receivers, and other electronic equipment? (*b*) What are the chief sources of these currents?
2. Name four applications where batteries are used to operate electronic equipment.
3. Where is the d-c power system most likely to be found?
4. (*a*) Why are a-c systems used so extensively? (*b*) What percentage of power is generated as alternating current?
5. (*a*) With what fundamental principle of electricity is Faraday's name associated? (*b*) Describe a simple experiment that illustrates this principle.
6. State Fleming's right-hand rule.
7. What conditions are necessary in order to have a voltage induced in a conductor?
8. Describe the fundamental generator.
9. Under what conditions will the induced emf have a value of 1 volt?
10. To what factors is the induced emf proportional?
11. Describe the voltage induced in a conductor that is being rotated at a constant rate of speed through a uniform magnetic field. (Consider this as a simple two-pole a-c generator.)
12. Give a definition of an alternating current.
13. Define (*a*) alternation, (*b*) cycle, (*c*) frequency, (*d*) period.
14. Compare the values of frequencies used in power systems with those used in radio and television.
15. What is meant by a sine-wave alternating voltage?
16. Define (*a*) maximum value, (*b*) instantaneous value, (*c*) average value, (*d*) effective value.
17. Describe a simple method of drawing a sine wave.
18. Describe the a-c ampere.
19. (*a*) Must an alternating current always be in step, or in phase, with its voltage? (*b*) Why?
20. (*a*) What is meant by the volt-amperes of a circuit? (*b*) What is the numerical relationship between volt-amperes and apparent power?
21. What are the relative numerical values of the volt-amperes and the watts of a circuit when the current and voltage are (*a*) in phase? (*b*) The current lags the voltage by 60°? (*c*) The current leads the voltage by 60°?
22. (*a*) What is meant by the power factor of a circuit? (*b*) Why are power companies interested in the power factor of their customers' loads?
23. (*a*) What is meant by a single-phase power source? (*b*) What is its chief use?
24. (*a*) What is meant by a polyphase power source? (*b*) How many phases may it have? (*c*) What number of phases is most frequently used?
25. Describe the principle of operation of a two-phase power source.
26. Describe three arrangements of line wires for a two-phase power system.
27. Describe the principle of operation of a three-phase power source.
28. (*a*) How are the phases of a three-phase power source interconnected to produce a three-wire delta-connected power source? In a delta-connected system, what is the numerical relation between (*b*) line voltage and phase voltage? (*c*) Line current and phase current?

29. (*a*) How are the phases of a three-phase power source interconnected to produce a three-wire wye-connected power source? In a wye-connected system, what is the numerical relation between (*b*) line voltage and phase voltage? (*c*) Line current and phase current?
30. (*a*) How are the phases of a three-phase power source interconnected to produce a four-wire wye-connected system? (*b*) What is the advantage of this system when used for both lighting and motor loads?
31. How does the construction of a simple d-c generator differ from that of a simple a-c generator?
32. (*a*) What is a commutator? (*b*) How is it constructed? (*c*) What is its purpose?
33. (*a*) What is the disadvantage of a single-coil generator? (*b*) How is this disadvantage overcome?
34. (*a*) What is meant by commutator ripple? (*b*) What effect does it have on the operation of sound-producing equipment? (*c*) What must be done to correct it?
35. Describe the field and frame construction of a commercial generator.
36. Describe the armature construction of a commercial generator.
37. (*a*) What is the location of the armature and field poles of the commercial a-c generator? (*b*) What are the advantages of this type of construction?
38. Why is an exciter required with an alternator?
39. How does the rating of the exciter compare with that of the alternator?
40. What is the importance of transformers to a-c systems?
41. What are the fundamental parts of the transformer?
42. (*a*) What is the purpose of the core of a transformer? (*b*) How is it constructed?
43. Define (*a*) primary, (*b*) secondary.
44. Explain the principle of operation of the transformer.
45. (*a*) Can a transformer be operated on direct current? (*b*) Why?
46. What is the relation of the primary voltage and turns to the secondary voltage and turns?
47. (*a*) How does the voltage per turn on the secondary side of a transformer compare with that on the primary side? (*b*) Why?
48. How do the primary and secondary currents vary with the voltages of a transformer?
49. (*a*) What is meant by efficiency? (*b*) How is it usually expressed?
50. (*a*) Is the efficiency of most electrical apparatus high or low? (*b*) Why?

PROBLEMS

1. A generator having 20 coils, each consisting of 24 turns, operates at a speed of 1,200 rpm, and the flux per pole is 650,000 lines. What is the value of the induced emf? (Each turn has two conductors; therefore $C = 2 \times 24 \times 20$, or 960.)
2. A generator having 15 coils of eight turns each is operated at a speed of 3,600 rpm. What is the induced emf if the flux per pole is 765,000 maxwells?
3. How many turns per coil are required on each of the coils of a generator that is to have an emf of 125 volts if it has 15 coils? The flux per pole is 1,150,000 maxwells, and the speed is 1,800 rpm.
4. What would the voltage of the generator of Prob. 3 be if the speed is increased to 3,600 rpm?

5. What is the flux per pole of a generator that has 33 coils of eight turns each and that produces an emf of 115 volts when rotated at a speed of 1,500 rpm?
6. What is the frequency of a four-pole alternator operating at a speed of (*a*) 1,800 rpm? (*b*) 1,500 rpm? (*c*) 750 rpm?
7. What is the frequency of a six-pole alternator operating at a speed of (*a*) 1,200 rpm? (*b*) 800 rpm? (*c*) 500 rpm?
8. At what speed must a two-pole alternator be driven in order to have a frequency of (*a*) 25 cycles? (*b*) 60 cycles? (*c*) 120 cycles?
9. How many poles must an alternator have if it is to produce 500 cycles when driven at (*a*) 1,500 rpm? (*b*) 3,000 rpm?
10. What is the period of an alternating voltage whose frequency is (*a*) 30 cycles? (*b*) 50 cycles? (*c*) 120 cycles?
11. What is the frequency of an alternating current whose period is (*a*) 0.04 sec? (*b*) 0.0167 sec? (*c*) 0.002 sec?
12. An a-c sine-wave voltage has a maximum value of 250 volts. What is the instantaneous value at (*a*) 10°? (*b*) 41.5°? (*c*) 107.5°? (*d*) 311°? (*e*) 342.5°?
13. An a-c sine-wave current has a maximum value of 15 amp. What is the instantaneous value of current at (*a*) 63°? (*b*) 218.5°? (*c*) 270°? (*d*) 290.5°? (*e*) 345°?
14. What is the maximum value of a sine-wave voltage whose value is 75 volts at 14.5°?
15. A sine-wave voltage has an instantaneous value of 106 volts at 32°. What is its value at 10.5°?
16. Draw one cycle of a sine-wave voltage whose maximum value is 300 volts. Use at least 36 points to draw the curve.
17. What is the average value of a voltage whose maximum is 250 volts?
18. Using the method illustrated in Fig. 7-8, find the average value of a sine-wave voltage whose maximum is 300 volts. Use values 2° apart. What is the ratio of the average to the maximum value?
19. What is the average value of a voltage whose instantaneous value is 97 volts at 29°?
20. What is the maximum value of a current whose average value is 7.5 amp?
21. Using the method illustrated in Fig. 7-9, find the effective value of the sine wave of Prob. 18. What is the ratio of effective to maximum value?
22. What is the effective value of a current whose maximum value is 3.54 amp?
23. What is the effective value of a voltage that has an instantaneous value of 152 volts at 49.5°?
24. What is the effective value of a current that has an instantaneous value of −3.00 amp at 270°?
25. What is the effective value of a voltage that has an instantaneous value of −150 volts at 345°?
26. What is the effective value of a voltage whose average value is 200 volts?
27. A sine-wave voltage has an instantaneous value of 145 volts at 133.5°. Find (*a*) the maximum value, (*b*) the average value, (*c*) the effective value.
28. A 120-volt single-phase lamp load draws 6.25 amp and consumes 750 watts. (*a*) How many volt-amperes are taken by the circuit? (*b*) What is the power factor of this load?
29. A 220-volt single-phase motor is drawing 4.5 amp and 825 watts from the power source. (*a*) How many volt-amperes are taken by the circuit? (*b*) What is the power factor of this motor?

30. A 220-volt single-phase motor is taking 1,100 watts from the line. How much current is being drawn if the power factor of the motor is (*a*) 85 per cent? (*b*) 65 per cent? (*c*) 100 per cent?
31. A 110-volt single-phase motor is taking 1,100 watts from the line. If the power factor of the motor is 85 per cent, (*a*) how much current is it drawing from the line? (*b*) What is the apparent power of the motor?
32. A 220-volt three-phase delta-connected alternator is supplying power to a three-phase motor whose line current is 7.1 amp, and the motor is taking 2,000 watts from the power source. (*a*) What are the volt-amperes of the circuit? (*b*) What is the power factor of the motor? (*c*) How much current is flowing in the phase windings of the alternator?
33. Meter readings taken for a certain three-phase delta-connected alternator that is supplying power to a motor load show that the line voltages are 220 volts, the line currents are 10 amp, and the line power is 3,000 watts. (*a*) What is the load in volt-amperes? (*b*) What is the power factor of the load? (*c*) What is the phase voltage of the alternator? (*d*) What is the phase current of the alternator?
34. A 440-volt three-phase wye-connected alternator is supplying power to a three-phase motor that has a power factor of 75 per cent and is drawing 11,400 watts from the power source. (*a*) How much current flows in the line wires? (*b*) What is the phase voltage of the alternator? (*c*) What is the phase current of the alternator? (*d*) What is the apparent power of the motor?
35. For the values given on Fig. 7-17, find (*a*) the volt-amperes at each phase of the alternator, (*b*) the volt-amperes of the three-phase system, (*c*) the watts delivered by the alternator if the load power factor is 65 per cent, (*d*) the power factor of the load if it consumes 1,650 watts.
36. For the values given on Fig. 7-18, find (*a*) the volt-amperes of the three-phase load, (*b*) the volt-amperes of each single-phase load, (*c*) the volt-amperes of the three single-phase loads, (*d*) the volt-amperes being supplied by the alternator.
37. (*a*) What is the rated line current of a 1,250-kva 2,400-volt three-phase delta-connected alternator? (*b*) What is its phase voltage? (*c*) What is its phase current? (*d*)What is its kilowatt rating for an 80 per cent power-factor load?
38. (*a*) What is the rated line current of a 1,250-kva 2,400-volt three-phase wye-connected alternator? (*b*) What is its phase voltage? (*c*) What is its phase current? (*d*) What is its kilowatt rating for an 80 per cent power-factor load?
39. Draw a diagram showing the voltage of a two-coil d-c generator. Each coil delivers a sine-wave voltage whose maximum value is 85 volts, and the individual coil voltages are 90 electrical degrees apart. What is the maximum and minimum value of the resultant voltage?
40. Repeat Prob. 39 for a three-coil d-c generator. Each coil voltage has a maximum value of 60 volts and is 60° from its adjacent voltage.
41. Repeat Prob. 39 for a four-coil d-c generator. Each coil voltage has a maximum value of 100 volts and is 45° from its adjacent voltage.
42. A transformer is required to step up the voltage from 110 to 480 volts. How many turns are required on the secondary winding if the primary has 150 turns?
43. A transformer is required to step up the voltage from 120 to 1,500 volts. How many turns are required on the secondary winding if the primary has 140 turns?
44. How many turns are required on the primary winding of a 120/1,500 volt step-up transformer if the secondary has 1,200 turns?

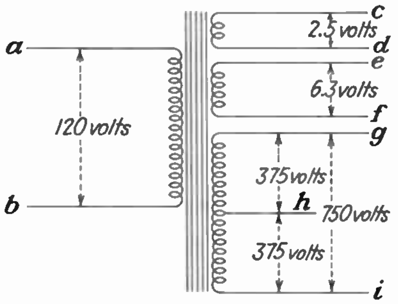


Fig. 7-29

45. The transformer shown in Fig. 7-29 has 192 turns on its primary winding. (*a*) How many turns are there on winding *cd*? (*b*) How many turns are there on winding *ef*? (*c*) How many turns are there on winding *gi*? (*d*) How many turns are there at *gh* and *hi*?
46. A 110/700 volt step-up transformer has a current of 150 ma on the secondary side. What is the primary current, assuming the losses to be negligible?
47. A 120/1,500 volt step-up transformer has a current of 350 ma on the secondary side. What is the primary current, assuming the losses to be negligible?
48. A 4,400/110 volt step-down transformer is used to supply a lighting load. (*a*) If the primary current is 5 amp, what current will flow in the secondary assuming the losses to be negligible? (*b*) What is the kilovolt-ampere load on the transformer? (*c*) How many 100-watt lamps can it supply with electrical energy?
49. What is the current in the primary winding of the transformer in Prob. 45 if the current in section *cd* is 6 amp, in *ef* is 3.5 amp, and in *gi* is 180 ma?
50. What is the efficiency of a transformer that draws 28 watts from the line when it delivers 9 amp at 2.5 volts?
51. What is the efficiency of a transformer similar to that of Prob. 45 if it takes 90 watts from the line when it supplies 5 amp at 2.5 volts, 3.3 amp at 6.3 volts, and 120 ma at 375 volts (center-tapped winding)?
52. A power transformer of a radio set has an efficiency of 78 per cent when it delivers a load of 40 watts. How much power does it draw from the line?
53. What is the power input of a transformer that has an efficiency of 83 per cent when its load is 65 watts?
54. How many watts will a transformer deliver at its secondary if it takes 75 watts from the line and its efficiency is 87.5 per cent?
55. How many horsepower of mechanical energy are required to drive the alternator of Prob. 37 if its efficiency is 90 per cent and the power factor of the load is 80 per cent?
56. A 400-watt generator requires 3/4 hp from its gasoline-driven engine. What is the efficiency of the generator?

Chapter 8

Inductance

The discoveries of Oersted and Faraday are among the most important in the entire fields of electricity and electronics. Understanding the principle of the magnetic field about a current-carrying conductor and that of electromagnetic induction led to the development of electrical machines. The development of electronic circuits also was made possible by the same principles. In electric and electronic circuits these principles are employed in the functioning of transformers and inductance coils, or choke coils as they are generally called.

8-1 Inductance, Lenz's Law

Inductance. In Chap. 5 the effects of the magnetic field about a conductor and about a group of conductors in the form of a coil were studied. At that point, the study was based entirely on the effects produced by a current that was constant in amount. When the current is changing in amount, an effect called *inductance* must be considered. *Inductance is the property of a circuit that opposes any change in the amount of current.*

When an alternating voltage is applied to the coil shown in Fig. 8-1, it will cause an alternating and therefore continually changing current to flow in the coil. If the conditions concerning conductor *A* of Fig. 8-2 are considered, electrons will be flowing outward at this conductor during the positive half-cycle of the alternating voltage. During this half-cycle, the current starts at zero, gradually increases to its maximum value, and then gradually decreases to zero.

Circuit Reactions with Increasing Field. Considering first the increasing values of current occurring from 0 to 90° (Fig. 8-3), it follows that the magnetic field too will be increasing in strength. As the current is increasing, the magnetic lines about conductor *A* will be expanding, and in doing so they will cut conductor *B*, which is adjacent to it. Whenever there is motion between a conductor and magnetic lines, an emf will be induced in that conductor; conductor *B* will therefore have an emf induced in it. Applying the left-hand rule (Fig. 8-3*b*) shows that the polarity of the induced emf at *B* is positive. The effect of this induced emf may be compared with the effect produced by cutting the conductor at *B* and inserting a voltage source. The overall effect would be that of having a

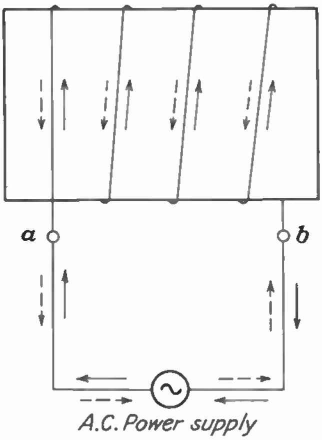


Fig. 8-1 Alternating current flowing in a coil. Solid-line arrows indicate electron flow when *b* is positive, and broken-line arrows indicate electron flow when *a* is positive.

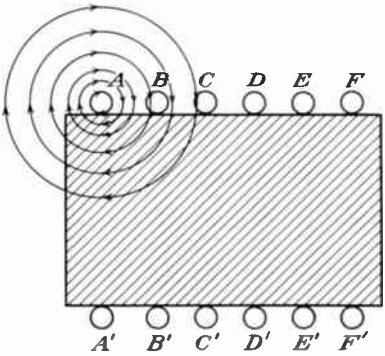


Fig. 8-2 Magnetic field set up when electrons are flowing outward at conductor A. This corresponds to terminal a of Fig. 8-1 being positive.

coil and two voltage sources, namely, the impressed emf and the induced emf, connected in series (Fig. 8-3*c*) with the positive terminals of the two voltage sources connected to each other. Accordingly, the induced emf is in the opposite direction of the impressed emf and thus reduces the effect of the impressed emf to push current through the coil. The more rapid the change in the amount of current, the greater this induced emf will be and the greater is the opposition to the change in current. In general, then, the induced emf opposes any increase in the amount of current. The value of the induced emf is equal to the product of the number of turns of the coil and the magnetic flux divided by the time in seconds necessary for the flux to change from its maximum value to zero. Expressed mathematically,

$$e = \frac{N\phi}{t \times 10^8} \tag{8-1}$$

where e = induced emf, volts
N = number of turns
ϕ = number of lines linking the coil
t = time, sec

EXAMPLE 8-1 A flux of 1,800,000 lines links a coil having 300 turns. The flux in the coil decreases from its maximum value to zero in 0.18 sec. What is the value of the induced voltage?

GIVEN: $N = 300$ turns $\qquad \phi = 1{,}800{,}000$ lines $\qquad t = 0.18$ sec

FIND: e

SOLUTION:

$$e = \frac{N\phi}{t \times 10^8} = \frac{300 \times 1{,}800{,}000}{0.18 \times 10^8} = \frac{3 \times 1.8}{0.18} = 30 \text{ volts}$$

The above example shows that the voltage induced in a coil is proportional to the number of turns in that coil and the rate of change of the flux.

Circuit Reactions with a Decreasing Field. During the period from 90 to 180°, the impressed voltage is decreasing and the current in conductor *A* will then also decrease (Fig. 8-4). The magnetic lines produced by this current will now be collapsing, and in doing so they will again cut conductor *B*, but the direction of motion of the magnetic lines is reversed. Applying the left-hand rule (Fig. 8-4*b*) indicates that the polarity of the induced emf will be negative at conductor *B*. This induced emf is now in the same direction as the impressed voltage (Fig. 8-4*c*), and it will aid the line voltage in pushing the current through the coil, thereby opposing the decrease in current. The more rapid the decrease in current, the greater the induced emf and the greater will be its effort to oppose the

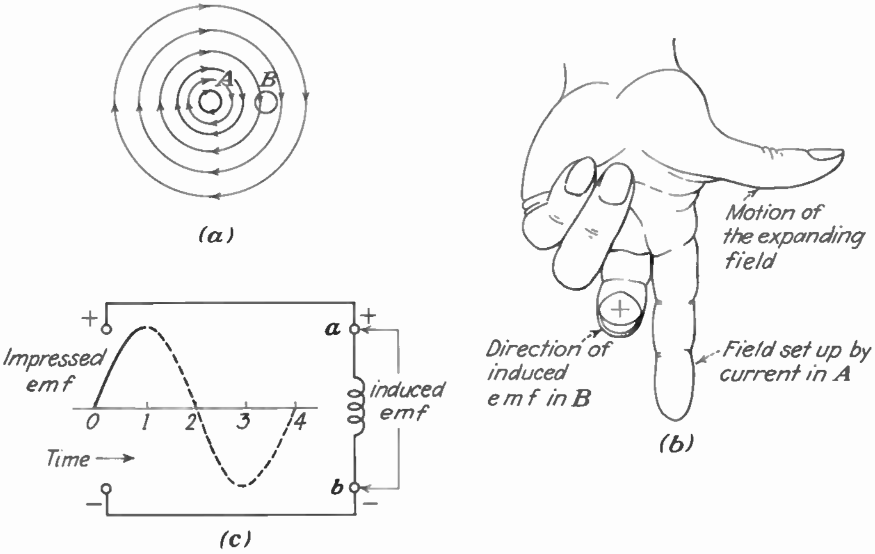


Fig. 8-3 Voltage induced in conductor *B* when the current in conductor *A* is increasing in strength. (a) Magnetic field set up when electrons are flowing outward at conductor *A*. (b) Left-hand rule showing that the polarity of the induced emf at conductor *B* is positive. (c) Relation of the polarities of the impressed emf and the induced emf while the impressed emf is increasing as during the time interval 0 to 1.

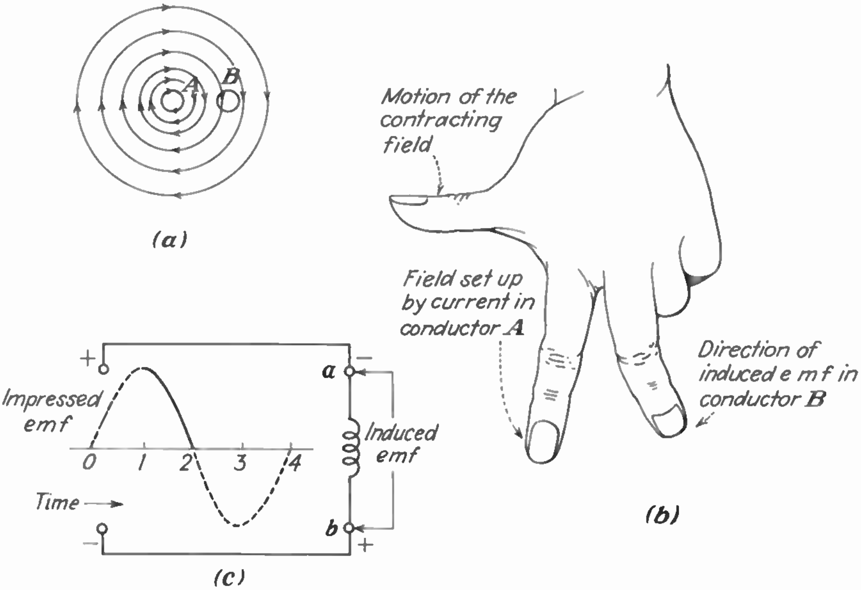


Fig. 8-4 Voltage induced in conductor B when the current in conductor A is decreasing in strength. (a) Magnetic field set up when electrons are flowing outward at conductor A. (b) Left-hand rule showing that the polarity of the induced emf at conductor B is negative. (c) Relation of the polarities of the impressed emf and the induced emf while the impressed emf is decreasing as during the time interval 1 to 2.

change in current. In general, then, the induced emf opposes any decrease in the amount of current.

These two explanations show that, when the current increases, the induced emf is in such a direction that it opposes the increase and that, when the current decreases, the induced emf is in such a direction that it opposes the decrease. This effect is called *inductance* and conforms to the definition that inductance is the property of a circuit which opposes any change in the amount of current. It should be noticed that *change* is a very important word in this definition.

Lenz's Law. The induced emf in a circuit always tends to oppose any change in the amount of current in that circuit. This is commonly referred to as *Lenz's law*, which may be stated as follows: *When the current in a circuit is increasing, the induced emf opposes the applied voltage and tends to keep the current from increasing, and when the current is decreasing, the induced emf aids the line voltage and tends to keep the current from decreasing.* Another way of stating Lenz's law is: *When a current flowing through a circuit is varying in magnitude, it produces a varying magnetic field which sets up an induced emf that opposes the current change producing it.*

Presence of Inductance. The above discussion was based upon the actions taking place at conductor A during the positive half-cycle. A similar action takes place at conductor A for the negative half-cycle. Applying this reasoning to any other conductor will show that the same actions occur at all conductors.

It is now evident that inductance is effective only when there is a changing current present in a circuit. With d-c circuits, this condition usually exists only at the instant of time when a circuit is being closed or opened by means of a switch; therefore, the inductance in such circuits is generally disregarded. In a-c circuits, the current is continually changing and the effect of inductance is always present; therefore, inductance must be considered in all a-c circuits.

8-2 Self-inductance

Self-inductance is defined as the property of a single circuit that opposes any change in the amount of current in that circuit. The preceding article actually presented an explanation of self-inductance. The discussion in the following paragraphs will present the unit of inductance and the factors affecting the value of inductance.

Unit of Inductance. The unit of inductance is the *henry;* it was named in honor of an early American scientist, Joseph Henry. A circuit has a self-inductance of one henry when a current changing at the rate of one ampere per second induces an average of one volt. The symbol used to represent inductance is the capital letter L.

Factors Affecting the Inductance of a Coil. While practically all circuits are likely to have some inductance, it is common to think of inductance most frequently in terms of a coil. When a coil is used expressly for its property of inductance, it is called an *inductor.* The self-inductance of a coil depends upon its physical characteristics, that is, its dimensions, number of turns, and the magnetic qualities of its core. Because it is sometimes difficult accurately to predict the magnetic conditions in a circuit, several equations are used to express the relations of the factors affecting the inductance of a coil.

When the length of a coil is several times its diameter, it is called a *solenoid.* For a solenoid whose length is at least 10 times its diameter, the inductance may be calculated by the equation

$$L = \frac{1.26N^2\mu A}{10^8 l} \tag{8-2}$$

where L = inductance of the coil, henrys
N = number of turns
μ = permeability of the core
A = area of the core, sq cm
l = length of the core, cm

EXAMPLE 8-2 What is the inductance of a tuning coil that has 300 turns wound on a cardboard tubing 4 cm in diameter and 40 cm long?

NOTE: As cardboard is nonmagnetic, $\mu = 1$.

GIVEN: $N = 300$ turns $\mu = 1$ $d = 4$ cm $l = 40$ cm

FIND: L

SOLUTION:

$$A = \frac{\pi d^2}{4} = \frac{3.14 \times 4 \times 4}{4} = 12.56 \text{ sq cm}$$

$$L = \frac{1.26 N^2 \mu A}{10^8 l} = \frac{1.26 \times 300 \times 300 \times 1 \times 12.56}{10^8 \times 40} = 0.000356 \text{ henry} = 356\ \mu\text{h}$$

EXAMPLE 8-3 What inductance would the coil of Example 8-2 have if it was wound on an iron core whose permeability was 4,000?

GIVEN: $N = 300$ turns $\mu = 4{,}000$ $A = 12.56$ sq cm $l = 40$ cm

FIND: L

SOLUTION:

$$L = \frac{1.26 N^2 \mu A}{10^8 l} = \frac{1.26 \times 300 \times 300 \times 4{,}000 \times 12.56}{10^8 \times 40} = 1.424 \text{ henrys}$$

EXAMPLE 8-4 What inductance would the coil of Example 8-3 have if there were 900 turns of wire on the coil?

GIVEN: $N = 900$ turns $\mu = 4{,}000$ $A = 12.56$ sq cm $l = 40$ cm

FIND: L

SOLUTION:

$$L = \frac{1.26 N^2 \mu A}{10^8 l} = \frac{1.26 \times 900 \times 900 \times 4{,}000 \times 12.56}{10^8 \times 40} = 12.81 \text{ henrys}$$

The above examples indicate both that the inductance can be increased considerably by winding the coil on a ferromagnetic core and that it also increases as the square of the number of turns. Changing the turns from 300 to 900 or making their number three times as great increases the inductance by 3 × 3, or nine times.

Multilayer, Pancake, Solenoid Coils. Equation (8-2) was used because it illustrates by the simplest mathematics how the various factors affect the inductance of a coil. The dimensions of coils used in radio circuits are such that they seldom conform to the conditions of this equation. In most cases it becomes necessary to resort to one of the three following equations. Figure 8-5 shows three classifications of coil shapes for which the three equations are used. Figure 8-5*a* is called a *multilayer coil*, and its inductance may be found by use of the equation

$$L = \frac{0.8a^2N^2}{6a + 9b + 10c} \tag{8-3}$$

Figure 8-5*b* shows a flat or pancake coil whose inductance may be calculated by the equation

$$L = \frac{a^2N^2}{8a + 11c} \tag{8-4}$$

Figure 8-5*c* shows a form of solenoid in which the length does not exceed its diameter by any great amount. The inductance of such a coil may be found by the equation

$$L = \frac{a^2N^2}{9a + 10b} \tag{8-5}$$

where L = inductance of coil, μh
N = number of turns
a, b, c = dimensions, inches (Fig. 8-5)

EXAMPLE 8-5 What is the inductance of a multilayer coil that has 1,200 turns and whose dimensions are $a = 1\frac{1}{2}$, $b = \frac{3}{4}$, $c = 1\frac{1}{2}$ inches?

GIVEN: $N = 1{,}200$ turns $a = 1.5$ inches $b = 0.75$ inch $c = 1.5$ inches

FIND: L

SOLUTION:

$$L = \frac{0.8a^2N^2}{6a + 9b + 10c} = \frac{0.8 \times 1.5 \times 1.5 \times 1{,}200 \times 1{,}200}{6 \times 1.5 + 9 \times 0.75 + 10 \times 1.5} = \frac{2{,}592{,}000}{30.75} = 84{,}292\ \mu\text{h}$$

The above equations will give reasonably accurate results for coils without iron cores such as may be found in r-f circuits. When iron cores are used, more accurate results can be obtained by securing readings of volts, amperes, and either watts or resistance, as is explained later. Numerous tables and charts have been prepared to facilitate calculations of inductance or number of turns by short-cut methods.

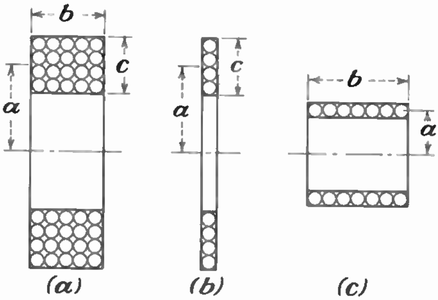


Fig. 8-5 Types of inductance coils. (a) Multilayer. (b) Flat or pancake. (c) Solenoid.

8-3 Inductive Reactance, Resistance, Impedance

The effects of inductance in an a-c circuit are twofold: namely, it sets up an opposition to the flow of current, and it causes a delay or lag in the current.

Inductive Reactance. The study of inductance shows that a changing magnetic field induces a voltage in such a direction that it opposes any change in the amount of current. This results in the current being lower than if inductance were not present, and inductances must therefore introduce an opposition to the flow of current. This opposition is called *inductive reactance* and is expressed in ohms; its symbol is X_L. The value of the inductive reactance is affected by two factors, one being the inductance of the circuit and the other the rate or speed at which the current is changing. The inductance of a circuit depends on the physical characteristics of the circuit as explained in Art. 8-2. The rate of speed at which the current is changing is directly proportional to the frequency of the power supply to which it is connected. The effect of these two factors results in the equation

$$X_L = 2\pi f L \tag{8-6}$$

where X_L = inductive reactance, ohms
f = frequency, cps
L = inductance, henrys

If a circuit is assumed to consist of inductance only, the amount of current flowing in such a circuit would be equal to its voltage divided by the inductive reactance, or

$$I_L = \frac{E_L}{X_L} \tag{8-7}$$

EXAMPLE 8-6 The choke coil of a filter circuit has an inductance of 30 henrys. (*a*) What is its inductive reactance on a 60-cycle circuit? (*b*) What current will flow when the voltage across the coil is 250 volts?

GIVEN: $L = 30$ henrys $f = 60$ cps $E = 250$ volts

FIND: (*a*) X_L (*b*) I_L

SOLUTION:

(*a*) $$X_L = 2\pi f L = 2 \times 3.14 \times 60 \times 30 = 11{,}304 \text{ ohms}$$

(*b*) $$I_L = \frac{E_L}{X_L} = \frac{250}{11{,}304} = 22.1 \text{ ma}$$

EXAMPLE 8-7 The primary of an r-f transformer has an inductance of 350 μh. (*a*) What is its inductive reactance at 1,200 kc? (*b*) What current will flow when the voltage across the primary is 10 volts?

GIVEN: $L = 350 \times 10^{-6}$ henry $f = 1{,}200 \times 10^3$ cps $E = 10$ volts

FIND: (*a*) X_L (*b*) I_L

SOLUTION:

(*a*) $X_L = 2\pi fL = 2 \times 3.14 \times 1{,}200 \times 10^3 \times 350 \times 10^{-6} = 2637.6$ ohms

(*b*) $I_L = \dfrac{E_L}{X_L} = \dfrac{10}{2637.6} = 3.79$ ma

Effect of the Resistance of an Inductive Circuit. In actual practice it is impossible to have a circuit containing only inductance because the wire of which the inductor is wound has some resistance. The resistance is usually so small compared with the inductive reactance that it is ignored and the circuit is assumed to contain inductance only.

Impedance. If the resistance is to be considered, its ohmic effect must be combined with the ohmic effect of the inductive reactance. The combined ohmic effect is called the *impedance* and is represented by the symbol Z. Mathematically it is equal to

$$Z = \sqrt{R^2 + X_L^2} \tag{8-8}$$

where Z = impedance of the circuit, ohms
R = resistance of the circuit, ohms
X_L = inductive reactance of the circuit, ohms

When both the resistance and the inductive reactance of a circuit are taken into consideration, the current flowing in the circuit will be equal to the voltage of the circuit divided by its impedance, or

$$I = \frac{E}{Z} \tag{8-9}$$

where I = current flowing in the circuit, amp
E = voltage of the circuit, volts
Z = impedance of the circuit, ohms

EXAMPLE 8-8 If the coil of Example 8-6 has a resistance of 400 ohms, (*a*) what is the impedance of the coil? (*b*) What current will flow when the voltage across the coil is 250 volts?

GIVEN: $X_L = 11{,}304$ ohms $R = 400$ ohms $E = 250$ volts

FIND: (*a*) Z (*b*) I

SOLUTION:

(*a*) $Z = \sqrt{R^2 + X_L^2} = \sqrt{400^2 + 11{,}304^2} = \sqrt{160{,}000 + 127{,}780{,}416}$
$= \sqrt{127{,}940{,}416} = 11{,}311$ ohms

(*b*) $I = \dfrac{E}{Z} = \dfrac{250}{11{,}311} = 22.1$ ma

Comparison of the results of Examples 8-6 and 8-8 shows that the impedance and inductive reactance are practically equal and that the current, too, is practically the same whether the resistance is considered or neglected. This is always the case when the inductive reactance is ten or more times greater than the resistance.

8-4 Time Constant and Angle of Lag

Time Constant. That inductance causes a delay or lag in the current is shown in the following manner. If a length of wire is arranged so that it has no inductance (for example, kept straight so that at no point will two sections of wire be near each other), its only effect will be that of resistance. If the wire has a resistance of 12 ohms and it is connected through a switch and an ammeter to a 6-volt d-c power source (Fig. 8-6*a*), a current of $6/12$, or 0.5, amp will flow when the switch is closed. Furthermore, because the circuit contains resistance only, the current will reach this value practically instantly as indicated by the graph of Fig. 8-6*b*.

If the piece of wire or an identical piece is wound around a cylindrical form, it will have inductance as well as its resistance of 12 ohms. If the coil is connected to a circuit as in Fig. 8-7*a*, the current upon closing the switch will again become $6/12$, or 0.5, amp. The current, however, will not attain this value instantly because of the inductance now present in the circuit. The amount of time required for the current to reach its final value depends upon the relative amount of inductance and resistance in the circuit. If the inductance of this circuit is 0.36 henry, the time

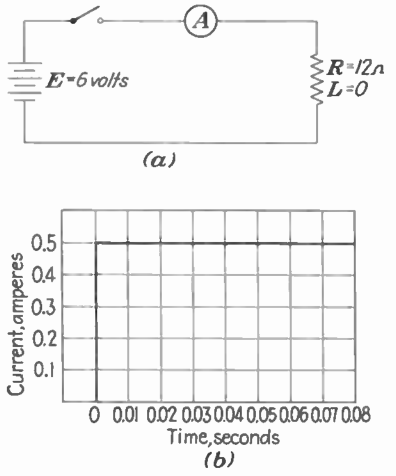


Fig. 8-6 Time required for the current to build up in a circuit containing only resistance. (a) The circuit diagram. (b) A graph showing the relation between current and time.

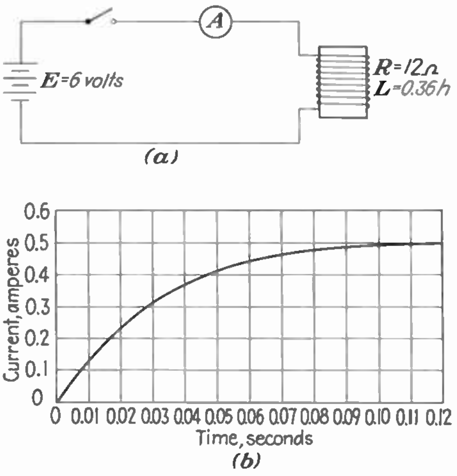


Fig. 8-7 Time required for the current to build up in a circuit containing resistance and inductance. (a) The circuit diagram. (b) A graph showing the relation between current and time.

required for the current to build up is that which is shown in Fig. 8-7*b*. The ratio of inductance to resistance is called the *time constant* and represents the time in seconds required for the current to build up to 63.2 per cent of its final value. Mathematically it is expressed

$$t = \frac{L}{R} \tag{8-10}$$

where t = time for current to reach 63.2 per cent of its final value, sec
L = inductance of the circuit, henrys
R = resistance of the circuit, ohms

EXAMPLE 8-9 What time is required for the current in the circuit of Fig. 8-7*a* to build up to 63.2 per cent of its final value?

GIVEN: $L = 0.36$ henry $R = 12$ ohms

FIND: t

SOLUTION:

$$t = \frac{L}{R} = \frac{0.36}{12} = 0.03 \text{ sec}$$

The application of the principle of time constants is presented in greater detail in Art. 12-12.

Angle of Lag. If the battery in Fig. 8-7*a* is replaced with a source of alternating current, the effect of the inductance in the circuit will cause the current to lag continually behind the voltage. The amount of lag is dependent upon the relative amounts of inductance and resistance in the circuit and is generally expressed in electrical degrees instead of time in seconds. It is determined mathematically by

$$\cos\theta = \frac{R}{Z} = \frac{R}{\sqrt{R^2 + X_L{}^2}} \tag{8-11}$$

EXAMPLE 8-10 If the circuit of Fig. 8-7*a* is connected to a 60-cycle 6-volt a-c power supply, find (*a*) the inductive reactance, (*b*) the impedance, (*c*) the angle of current lag, (*d*) the current.

GIVEN: $R = 12$ ohms $L = 0.36$ henry $E = 6$ volts $f = 60$ cps

FIND: (*a*) X_L (*b*) Z (*c*) θ (*d*) I

SOLUTION:

(*a*) $X_L = 2\pi fL = 2 \times 3.14 \times 60 \times 0.36 = 135.6$ ohms

(*b*) $Z = \sqrt{R^2 + X_L{}^2} = \sqrt{12^2 + 135.6^2} = \sqrt{144 + 18{,}387} = 136.1$ ohms

(*c*) $\cos\theta = \dfrac{R}{Z} = \dfrac{12}{136.1} = 0.0881$

$\theta = 85°$ (from Appendix XI)

(*d*) $I = \dfrac{E}{Z} = \dfrac{6}{136.1} = 0.0440$ amp

Equation (8-11) shows that, if it were possible to build an inductance without any resistance, the value of cos θ would be zero and the angle by which the current would lag the voltage would be 90°. Also, if no inductance is present, the value of cos θ would be 1 and the angle would be 0°. In actual practice, inductance coils cannot be built without resistance; therefore, a 90° angle of lag cannot be obtained. Practical inductances often achieve angles up to 80 or 85°.

The angle between the current and voltage may be illustrated by sine waves as in Fig. 10-2 or by vector diagrams as in Fig. 10-11*a*. The vector method of representation is the one most commonly used.

8-5 Mutual Inductance

When two windings are placed so that a change of current in one will cause its changing magnetic field to cut the turns of the other, an induced emf will be set up in the second coil. The two circuits are then said to possess *mutual inductance.*

Principle of Mutual Inductance. This may be demonstrated by the circuits shown in Fig. 8-8. A coil *AA′* is formed by winding a number of turns on a core of cardboard tubing. A second piece of cardboard tubing, one whose inside diameter is slightly larger than the outside diameter of the completed coil, is then placed over the coil to act as an insulator and as a form for the second winding *BB′*. The coil *BB′* is then wound over this tubing. If the winding *AA′* is connected to a source of alternating current and if a voltmeter is connected to the winding *BB′*, the voltmeter will indicate that a voltage is being induced in the winding *BB′* by the

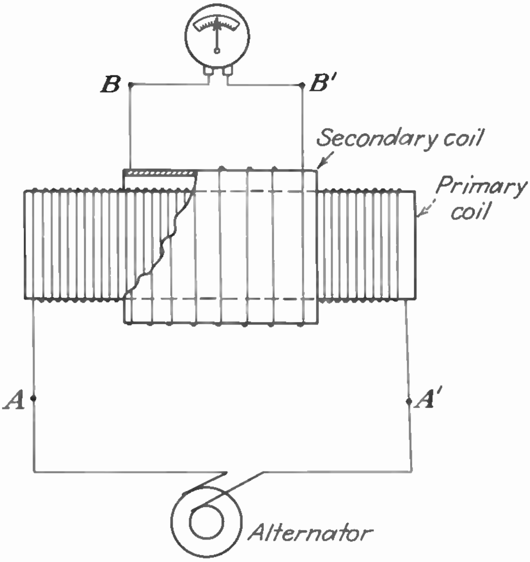


Fig. 8-8 Principle of mutual inductance.

expanding and contracting magnetic field caused by the current flowing in the winding AA'.

Calculation of Mutual Inductance. If the circuit BB' is connected to a load, it will cause a current to flow in the winding BB' and set up a magnetic field of its own. Each circuit can, and under such condition will, have an inductance of its own. The inductance of each may be calculated separately by a suitable equation from the group (8-2) to (8-5). The winding that receives the energy from the power line is called the *primary winding*, and its self-inductance is usually designated as L_1. The other winding is called the *secondary*, and its self-inductance is designated as L_2.

Unit of Mutual Inductance. When the current changes in one circuit as AA', at the rate of 1 amp per sec and it induces an average emf of 1 volt in the second circuit, as BB', the two circuits have a mutual inductance of 1 henry. If the self-inductances of the two coils are known and it is assumed that all the magnetic lines set up by the first coil cut all the turns of the second coil, the mutual inductance may be expressed by

$$M = \sqrt{L_1 \times L_2} \qquad (8\text{-}12)$$

where M = mutual inductance of the coils, henrys
L_1 = self-inductance of first coil, henrys
L_2 = self-inductance of second coil, henrys

EXAMPLE 8-11 What is the mutual inductance of two coils wound adjacent to one another? (Assume that all the magnetic lines set up in the first coil cut all the turns of the second coil.) The primary coil consists of 2,000 turns wound on a cardboard core 2 inches in diameter and 4 inches long. The secondary coil consists of 4,000 turns wound on a cardboard core 2¼ inches in diameter and 3½ inches long.

GIVEN: $N_1 = 2{,}000$ turns $a_1 = 1$ inch $b_1 = 4$ inches $N_2 = 4{,}000$ turns
$a_2 = 1⅛$ inches $b_2 = 3½$ inches

FIND: M

SOLUTION:

Note: The coils meet conditions of Fig. 8-5*c*; therefore use Eq. (8-5).

$$L_1 = \frac{a_1^2N_1^2}{9a_1 + 10b_1} = \frac{1 \times 1 \times 2{,}000 \times 2{,}000}{9 \times 1 + 10 \times 4}$$

$$= \frac{4{,}000{,}000}{49} = 81{,}632\ \mu\text{h} = 81.6\ \text{mh}$$

$$L_2 = \frac{a_2^2N_2^2}{9a_2 + 10b_2} = \frac{1.125 \times 1.125 \times 4{,}000 \times 4{,}000}{9 \times 1.125 + 10 \times 3.5}$$

$$= \frac{20{,}250{,}000}{45.125} = 448{,}753\ \mu\text{h} = 448.75\ \text{mh}$$

$$M = \sqrt{L_1 \times L_2} = \sqrt{81.6 \times 448.75} = \sqrt{36{,}618} = 191.3\ \text{mh}$$

8-6 Coefficient of Coupling

Calculation of Coefficient of Coupling. When two circuits are arranged so that energy from one circuit may be transferred to the other, the circuits are said to be *coupled*. Mutual inductance is an example of coupled circuits.

In the case of mutual inductance, if all the magnetic lines set up by the current in the first circuit cut all the turns of the second circuit, the circuits are coupled perfectly. If only half the lines set up in the first circuit cut the turns of the second circuit, the coupling is only 50 per cent. The percentage of coupling is referred to as the *coefficient of coupling* and is designated by the letter K. It is expressed mathematically by

$$K = \frac{M}{\sqrt{L_1 \times L_2}} \tag{8-13}$$

where K = coefficient of coupling (expressed as a decimal)
M = mutual inductance of two circuits
L_1 = self-inductance of first coil
L_2 = self-inductance of second coil

EXAMPLE 8-12 What is the coefficient of coupling of two coils whose mutual inductance is 1.0 henry and whose self-inductances are 1.2 and 2.0 henrys?

GIVEN: $M = 1.0$ henry $L_1 = 1.2$ henrys $L_2 = 2.0$ henrys

FIND: K

SOLUTION:

$$K = \frac{M}{\sqrt{L_1L_2}} = \frac{1.0}{\sqrt{1.2 \times 2.0}} = \frac{1.0}{\sqrt{2.4}} = \frac{1.0}{1.55} = 0.645$$

The coefficient of coupling depends upon the construction of the coils and also largely upon whether the coils are wound on an iron core or on an air core. The highest possible value is 1, and the lowest is zero. The power transformers described in Art. 7-14 often achieve the high value of 0.98, which is considered excellent. The coefficient of coupling for air-core transformers used in radio circuits is very low and will vary considerably depending upon the design of the coils and the frequency of the circuits in which they are used. In radio circuits, a low value of coefficient of coupling is often desired, as it aids sharpness of tuning.

Calculation of Voltage Induced in the Secondary. The voltage induced in the second circuit by a change of current in the first may be expressed by the equation

$$e_2 = \frac{N_2K\phi}{t \times 10^8} \tag{8-14}$$

where e_2 = voltage induced in second circuit
N_2 = turns on coil in second circuit
K = coefficient of coupling
ϕ = flux set up by current in first circuit
t = time for current in first circuit to change from maximum value to zero, or vice versa, sec

EXAMPLE 8-13 Two coils, the first having 200 turns and the second 350 turns, are placed so that only 40 per cent of the lines set up by coil 1 link coil 2. If 600,000 lines are set up when 3 amp flow through coil 1, what voltage will be induced in coil 2 if the current decreases from 3 amp to zero in 0.10 sec?

GIVEN: $N_2 = 350$ turns $K = 0.40$ $\phi = 600{,}000$ lines $t = 0.10$ sec

FIND: e_2

SOLUTION:

$$e_2 = \frac{N_2 K \phi}{t \times 10^8} = \frac{350 \times 0.40 \times 600{,}000}{0.10 \times 10^8} = 8.40 \text{ volts}$$

Many methods have been devised for varying the inductance in a circuit. The inductance of the unit shown in Fig. 8-9 is varied by changing the effective number of turns by means of an adjustable clip or by a sliding contactor. The inductance of the *variocoupler* or *variometer* (Fig. 8-10*c*) is varied by changing the position of one coil with respect to another, which, in effect, really varies the coefficient of coupling.

8-7 Series and Parallel Inductances

Basic Series and Parallel Connections. Circuits often contain several inductances connected in series or in parallel. When the separate inductors are located far enough apart so that there is no coupling between them, the inductances may be added in the same manner in which resistances are added. The inductance of a circuit containing several inductors con-

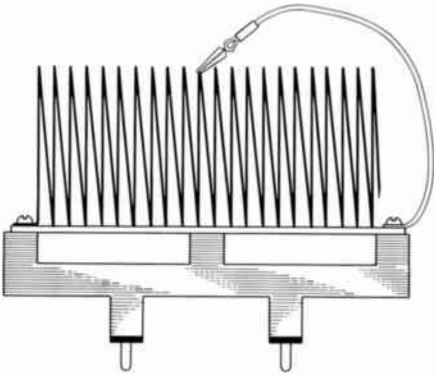


Fig. 8-9 A variable inductor.

nected in series and far enough apart so that no coupling exists between them will be

$$L_T = L_1 + L_2 + L_3, \text{ etc.} \tag{8-15}$$

The inductance of a circuit containing two or more inductors connected in parallel but far enough apart so that no coupling exists between them will be

$$L_T = \frac{1}{\dfrac{1}{L_1} + \dfrac{1}{L_2} + \dfrac{1}{L_3}, \text{ etc.}} \tag{8-16}$$

When a circuit has only two inductors connected in parallel, Eq. (8-16) can be simplified to

$$L_T = \frac{L_1 L_2}{L_1 + L_2} \tag{8-16a}$$

Reactance of Series- and Parallel-connected Inductors. The inductive reactance of two or more inductors connected either in series or parallel can be calculated by

$$X_{L\cdot T} = 2\pi f L_T \tag{8-17}$$

where $X_{L\cdot T}$ = inductive reactance at the line terminals of the group, ohms
L_T = inductance of the group, henrys

When the individual inductive reactances of a number of inductors connected in series are known,

$$X_{L\cdot T} = X_{L\cdot 1} + X_{L\cdot 2} + X_{L\cdot 3}, \text{ etc.} \tag{8-18}$$

When the individual inductive reactances of a number of inductors connected in parallel are known,

$$X_{L\cdot T} = \frac{1}{\dfrac{1}{X_{L\cdot 1}} + \dfrac{1}{X_{L\cdot 2}} + \dfrac{1}{X_{L\cdot 3}}, \text{ etc.}} \tag{8-19}$$

When a circuit has only two inductors connected in parallel, Eq. (8-19) can be simplified to

$$X_{L\cdot T} = \frac{X_{L\cdot 1} X_{L\cdot 2}}{X_{L\cdot 1} + X_{L\cdot 2}} \tag{8-19a}$$

EXAMPLE 8-14 A circuit having a 40-mh inductor and a 60-mh inductor connected in series is to be used on a 5,000-cycle power source. Find (*a*) the inductance of the circuit, (*b*) the reactance of each inductor, (*c*) the reactance of the circuit.

GIVEN: $L_1 = 40$ mh $L_2 = 60$ mh $f = 5{,}000$ cps

FIND: (*a*) L_T (*b*) $X_{L\cdot 1}$, $X_{L\cdot 2}$ (*c*) $X_{L\cdot T}$

SOLUTION:

(*a*) $L_T = L_1 + L_2 = 40 + 60 = 100 \text{ mh}$

(*b*) $X_{L\cdot 1} = 2\pi f L_1 = 6.28 \times 5{,}000 \times 40 \times 10^{-3} = 1{,}256 \text{ ohms}$

$X_{L\cdot 2} = 2\pi f L_2 = 6.28 \times 5{,}000 \times 60 \times 10^{-3} = 1{,}884 \text{ ohms}$

(*c*) $X_{L\cdot T} = 2\pi f L_T = 6.28 \times 5{,}000 \times 100 \times 10^{-3} = 3{,}140 \text{ ohms}$

EXAMPLE 8-15 A circuit having a 40-mh inductor and a 60-mh inductor connected in parallel is to be used on a 5,000-cycle power source. Find (*a*) the inductance of the circuit, (*b*) the reactance of each inductor, (*c*) the reactance of the circuit.

GIVEN: $L_1 = 40$ mh $L_2 = 60$ mh $f = 5{,}000$ cps

FIND: (*a*) L_T (*b*) $X_{L\cdot 1}$, $X_{L\cdot 2}$ (*c*) $X_{L\cdot T}$

SOLUTION:

(*a*) $$L_T = \frac{L_1 L_2}{L_1 + L_2} = \frac{40 \times 60}{40 + 60} = 24 \text{ mh}$$

(*b*) $X_{L\cdot 1} = 2\pi f L_1 = 6.28 \times 5{,}000 \times 40 \times 10^{-3} = 1{,}256 \text{ ohms}$

$X_{L\cdot 2} = 2\pi f L_2 = 6.28 \times 5{,}000 \times 60 \times 10^{-3} = 1{,}884 \text{ ohms}$

(*c*) $X_{L\cdot T} = 2\pi f L_T = 6.28 \times 5{,}000 \times 24 \times 10^{-3} = 753.6 \text{ ohms}$

Effect of Coupling on Series-connected Inductors. When two coils are located close enough to each other so that there will be coupling between them, the total inductance of the two coils when connected in series must be found in a different manner. The total inductance will depend on how the coils are connected, that is, whether they are aiding or opposing, and also upon their mutual inductance and coupling. Figure 8-10 shows three conditions that may exist. In Fig. 8-10*a*, two coils are connected in series so that their magnetic fields aid each other; the inductance of the circuit will then be

$$L = L_1 + L_2 + 2K\sqrt{L_1 L_2} \tag{8-20}$$

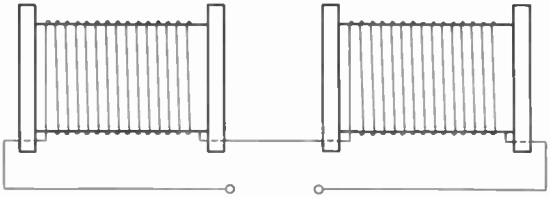
where L = inductance of circuit
L_1 = inductance of first coil
L_2 = inductance of second coil
K = coefficient of coupling

In Fig. 8-10*b*, the two coils are connected in series so that their magnetic fields oppose each other; the inductance of the circuit will then be

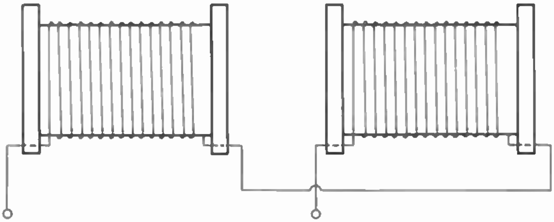
$$L = L_1 + L_2 - 2K\sqrt{L_1 L_2} \tag{8-21}$$

In Fig. 8-10*c*, the second coil is mounted on a shaft so that it may be rotated. If it can be rotated through 180°, it may be changed from aiding to opposing or it may also be stopped at any point in between. The inductance of the circuit will then be

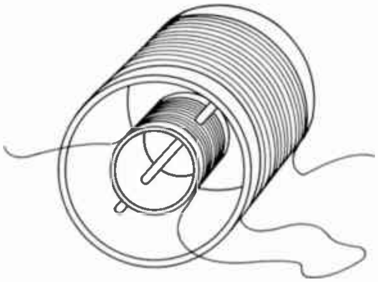
$$L = L_1 + L_2 \pm 2K\sqrt{L_1 L_2} \tag{8-22}$$



(a)-Coils aiding



(b)-Coils opposing



(c)-Variable coupling

Fig. 8-10 Two coupled inductors connected in series.

If two similar coils are used for Fig. 8-10*c* and perfect coupling could be achieved, the inductance could be varied between zero and $4L_1$. This is true because when the coefficient of coupling is unity, the value of $2K\sqrt{L_1L_2}$ is equal to $2L_1$ (or L_2 since L_1 and L_2 are equal). When the two coils are aiding,

$$L = L_1 + L_2 + 2K\sqrt{L_1L_2} = L_1 + L_1 + 2L_1 = 4L_1 \tag{8-23}$$

and when the two coils are opposing,

$$L = L_1 + L_2 - 2K\sqrt{L_1L_2} = L_1 + L_1 - 2L_1 = 0 \tag{8-24}$$

When the coils are at a 90° angle, the value of $2K\sqrt{L_1L_2}$ is zero and $L = L_1 + L_2 \pm 0 = 2L_1$. This is the basis of the variometer and variocoupler shown in Fig. 8-10*c*.

EXAMPLE 8-16 Two coils, each with an inductance of 4 henrys, are arranged so that they may be connected in series in the various ways shown in Fig. 8-10. What is the inductance of the circuit when the two coils are connected in series so that they are (*a*) aiding and the coupling is 100 per cent? (*b*) Opposing and the coupling is 100 per cent? (*c*) In a position that produces zero coupling? (*d*) Aiding and the coupling is 50 per cent?

GIVEN: $L_1 = 4$ henrys $\quad L_2 = 4$ henrys

FIND: (*a*) L (*b*) L (*c*) L (*d*) L

SOLUTION:

(*a*) $L = L_1 + L_2 + 2K\sqrt{L_1L_2} = 4 + 4 + (2 \times 1 \times \sqrt{4 \times 4})$
$= 4 + 4 + 8 = 16$ henrys

(*b*) $L = L_1 + L_2 - 2K\sqrt{L_1L_2} = 4 + 4 - 8 = 0$

(*c*) $L = L_1 + L_2 \pm 2K\sqrt{L_1L_2} = 4 + 4 \pm 2 \times 0 \times 4 = 8$ henrys

(*d*) $L = L_1 + L_2 + 2K\sqrt{L_1L_2} = 4 + 4 + 2 \times 0.5 \times 4 = 12$ henrys

8-8 Low-frequency Inductance Coils

Power-supply Applications. Inductance coils used in filter circuits of power supplies and in a-f circuits are generally classed as *low-frequency coils*. These coils must have a high value of inductance in order to get the desired value of impedance at the frequencies for which they are used. Examples 8-6 and 8-8 show that the impedance is practically equal to the inductive reactance, and thus it can be seen from Eq. (8-6) that at low frequencies the inductance must be high in order to obtain the desired high values of impedance. In order to get a high enough value of inductance, low-frequency inductors generally use some type of iron core and have a relatively large number of turns of wire on their coils.

Inductors used in power-supply filter circuits have a pulsating current flowing through them, and the rate of pulsations is usually in the order of

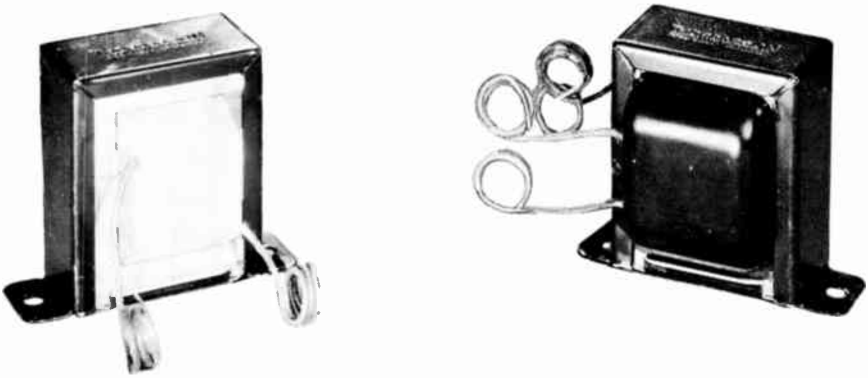


Fig. 8-11 Low-frequency filter chokes. (*Thordarson-Meissner*)

60 or 120 pulsations per second. These inductors are frequently referred to as *chokes* (Fig. 8-11).

A-F Circuit Applications. In a-f circuits, inductance coils may be used as a coupling device in order to transfer energy from one circuit to another or as part of a filter circuit to separate the signals of one frequency from those of other frequencies. Application of coils in such circuits includes (1) transformers used to transfer energy from one stage to another, (2) parallel feed supply of the *B* voltage to the plate of a vacuum tube, (3) inductors for impedance coupling, (4) filter circuits, (5) output transformers to provide impedance matching of the plate circuit of a vacuum tube with a loudspeaker.

Transformers may be used in the a-f circuits of various types of sound-reproducing equipment. The frequency handling requirements extend from a low range of 100 to 5,000 cycles for low-cost radio receiver applications to a range of 20 to 20,000 cycles for high-fidelity applications; some high-fidelity equipment may use transformers with a frequency range as high as 7 to 50,000 cycles. These transformers have a primary and secondary winding that are wound on a laminated iron or steel core. The ratio of the number of turns on the primary and secondary windings may range from a low of 1 to 1 to a high of 100 to 1 or more. The turns ratio and whether the primary turns are greater in number than the secondary turns or vice versa are determined by the requirements of each specific application.

The cores of transformers operating at frequencies up to 50,000 cps are made of a higher grade of steel than is used for the inductors in the lower-frequency power-supply-circuit applications. The higher grades of steel are more suitable for the higher-frequency applications because they have lower losses.

Other Applications. There are many other inductor and transformer applications in the fields of (1) communications, (2) television, (3) high-fidelity sound systems, (4) power-supply and rectifier equipment, (5) test equipment, (6) magnetic amplifiers, (7) automation controls, (8) computers, (9) telemetering, (10) military equipment, (11) aerospace equipment.

Q of an Inductor. A perfect inductor, that is, one having only inductance, cannot be achieved because a practical inductor will always have

Fig. 8-12 Audio-frequency inductors and transformers. (*Thordarson-Meissner*)

some resistance in its coil and some losses in its magnetic core. The ratio of the reactance of a coil to its resistance is called the Q of the coil, and

$$Q = \frac{X_L}{R} = \frac{2\pi fL}{R} \tag{8-25}$$

where Q = figure of merit
X_L = inductive reactance of coil or inductor, ohms
R = effective resistance of inductor, ohms

The term Q has several slightly different meanings depending on whether it is being associated with only a coil or with a complete circuit. Consequently, the term Q is known by various names such as figure of merit, quality factor, magnification factor, and energy factor.

The Q of an inductor is dependent upon factors other than the ohmic resistance of its coil. These factors include skin effect, eddy-current losses in the conductor, distributed capacitance of the coil, leakage resistance, type of core material, and shape of the coil. The numerical value of Q for a coil extends over a wide range with values under 20 considered as low Q and values of several hundred or more classed as very high Q.

8-9 High-frequency Inductance Coils

R-F Choke Coils. Choke coils used in r-f circuits and transformers used in r-f and i-f circuits operate at frequencies above 100 kc and are classed as *high-frequency coils.* Because the magnetic effects of high-frequency currents are difficult to predict and to control, various designs of coils have been developed.

Choke coils are used in r-f circuits to provide a high impedance to the r-f currents and a low impedance to direct current. To accomplish this, r-f chokes have a high inductive reactance and a low resistance. Because they operate on high frequencies, a high inductive reactance can be obtained with a relatively low inductance as compared with low-frequency coils. For example, an 80-mh choke operated at 500 kc has an inductive reactance of approximately 250,000 ohms. Radio-frequency chokes are available at ratings from a fraction of 1 μh to hundreds of millihenrys and have low values of resistance compared with their impedance. Many of the r-f coils used in f-m, television, and high-frequency electronic circuits operate at frequencies of 5 to 500 mc or more. Special single-layer coils wound on cores made of steatite, ceramics, or phenolic materials are used for these high-frequency applications. Coils wound on nonmagnetic cores are often referred to as *air-core coils.*

Magnetic-core Coils. Some r-f coils are wound on a specially prepared magnetic core consisting of finely powdered iron particles held together with a magnetic insulating binding substance. This type of iron core reduces the losses in the iron, thereby making it suitable for high-frequency appli-

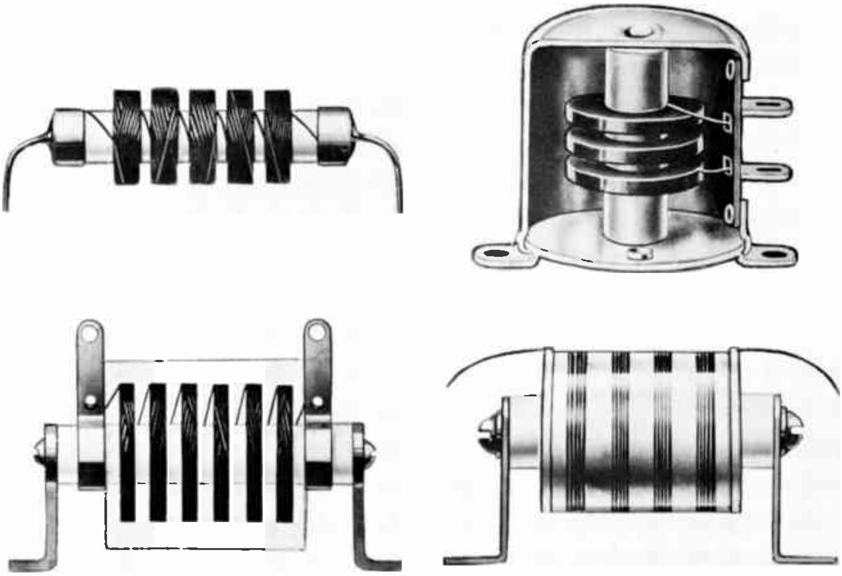


Fig. 8-13 Commercial r-f chokes. (*Hammarlund Manufacturing Company, Inc.*)

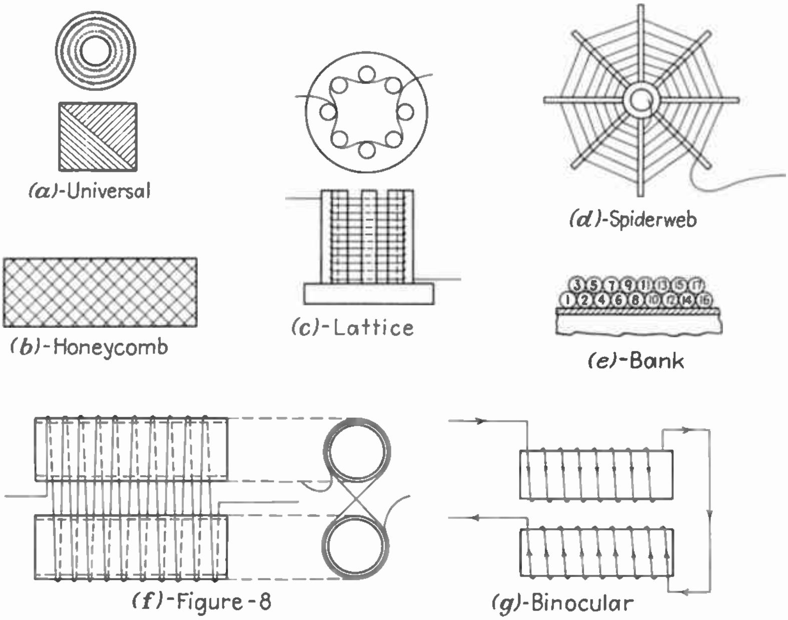
cations. Because of the iron core, a high inductance can be obtained with a smaller number of turns, resulting in lower resistance, smaller coil size, and a higher value of Q.

Coils with Adjustable Magnetic Cores. When it is required that the inductance of a unit be adjustable, the iron core of the inductor is arranged so that its position can be varied (Fig. 11-11*b*). The inductance of the coil is then controlled, within fixed limits, by varying the amount of iron inserted in the coil. When the core is all the way in the coil, the inductance will be at its maximum value, and when the core is withdrawn from the coil, the inductance will be at its minimum value. Some universal oscillator coils use this principle to permit adjustment of the oscillator circuit to its proper frequency. It is also used in coils requiring a high-Q factor, such as the i-f transformers used in f-m receivers and the peaking coils in the video circuits of a television receiver.

Methods of Winding. Another feature of the r-f coils is the special way in which they are wound. Instead of just winding the turns alongside each other and the layers on top of one another, special winding methods have been developed to reduce the distributed capacitance (see Chap. 9) of the coil. Among the methods used are the universal, honeycomb, spider-web, bank, figure-of-eight, and binocular types of windings (Fig. 8-14). The universal and honeycomb windings are the types most commonly used.

R-F Transformers. Radio-frequency transformers are used to couple one stage of a tuning circuit to the next stage. The transformer windings are usually of the universal type, and the core may be either an air core or the powdered-iron type. Often they are wound with a special type of wire, called *litz wire*, and the windings are treated with a special material to help them withstand varying temperature and humidity conditions. These transformers have a low number of turns, and the primary is of a lower number than the secondary. The r-f transformers operate through the frequency range of the receiver in which they are connected; thus, for an a-m broadcast-band receiver, the minimum operating range should be 530 to 1,650 kc. For an f-m receiver the minimum frequency range should be 88 to 108 mc, and for a television receiver the range to be covered is 54 to 216 mc for channels 2 to 13 and 470 to 890 mc for channels 14 to 83. Receivers used by amateur radio operators generally cover two or more of the numerous bands allocated to the amateurs. When a receiver, such as a television or multiband radio receiver, is to operate on several frequency ranges, separate r-f coils may be used for each channel or band. In some instances, a group of coils may be mounted in a single container or several separate coils of the plug-in type may be used.

I-F Transformers. Intermediate-frequency transformers are a form of r-f transformers that are designed for operation at a definite frequency. Standard values of intermediate frequency have been set up for each of the various types of service for which i-f transformers are used. For a-m radio



(a)-Universal (b)-Honeycomb (c)-Lattice (d)-Spiderweb (e)-Bank (f)-Figure-8 (g)-Binocular

Fig. 8-14 Various methods of winding high-frequency coils.

(a) (b) (c)

Fig. 8-15 Commercial r-f and i-f coils. (*Thordarson-Meissner*)

receivers, i-f values of 175, 262, 370, 456, and 465 kc have been used; of these, the 262- and 456-kc transformers are most frequently used. For f-m receivers, 10.7 mc is used. In many television receivers two or more i-f stages of slightly different frequency values are used in order to obtain the necessary wide-band characteristics by the method of *stagger tuning*. The i-f values used in television receivers cover a band in the order of 41 to 46 mc. The i-f transformers used in television receivers are of the adjustable type in order to obtain the necessary wide-band amplifier characteristics.

Radio-frequency and i-f transformers (Fig. 8-15) are usually mounted in shielding containers, and their leads are generally each of a different color to conform with a standard color code (Appendix IX). Transformers must be selected according to their frequency range, and in addition, their primary and secondary impedances should be chosen to match the parts of the circuits into which they are to be connected.

Bifilar Windings. The principle of bifilar windings, which means winding with two wires, is applied to some types of wire-wound resistors and transformers. One type of noninductive winding described in Art. 8-13 and illustrated in Fig. 8-21 is actually a bifilar coil. Winding the primary and secondary coils of a transformer simultaneously and directly adjacent to each other produces a bifilar-wound transformer (Fig. 8-16). Two advantages of this type of transformer are its high coefficient of coupling and a reduction in the leakage inductance with respect to the primary inductance.

Toroidal Inductors and Transformers. One type of inductor used in electronic equipment has its coil of wire wound around the entire length of a doughnut-shaped iron core (Fig. 8-17). These inductors are generally called *toroids* or *toroidal inductors*. Advantages of toroids are (1) high values of Q can be obtained; (2) except for minor stray losses, their entire magnetic field is contained within the coil; (3) inductance values can be kept to within 1 per cent tolerance; and (4) the temperature stability is excellent with inductance variations of less than 1 per cent over a temperature range of -55 to $+85°$C being attainable.

Toroids with a single winding are used for choke coil applications, and toroids having two or more windings are used as transformers. They are adaptable to many electronic applications, such as (1) instrument transformers, (2) variable-voltage autotransformers, (3) special power-supply equipment, (4) oscillators, (5) pulse circuits with high repetition rates, (6) computers, and (7) data-processing equipment.

8-10 Shielding

High-frequency circuits, such as the i-f and r-f circuits using the transformers just described, often have undesired coupling between adjacent circuits due to their magnetic fields. To prevent this undesired magnetic coupling, a metal shield, usually of aluminum or brass, is placed around the coil. When a magnetic field passes through such a shield, an emf is induced in it, and as the shield acts as a closed circuit, a current will flow in the shield. This current will set up its own magnetic field which, according to Lenz's law, will oppose the original magnetic field and tend to keep it from spreading beyond the shield.

Shields should be carefully designed as to size because the currents set up in them act as a loss that must be subtracted from the power in the coil circuit. This result has a tendency to increase the resistance and also to reduce the inductance. Both these effects result in a lower Q and hence reduce the effectiveness of the coil. The shield should be of a heavy material that is a good conductor, such as copper, brass, or aluminum, and should be large enough so that it is not too close to the coil. Aluminum is used in preference to copper because of its greater resistance to corrosion, greater mechanical strength, lower cost, and good conductivity.


Fig. 8-16 Bifilar-wound transformer.

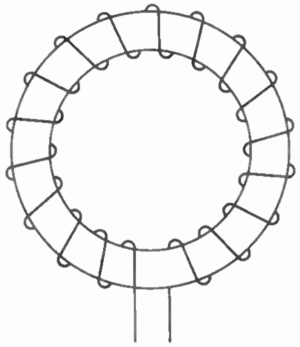


Fig. 8-17 A toroidal inductor.

8-11 Resistance of Coils

D-C Resistance. The resistance of coils that operate on direct-current, low-frequency power-supply circuits and audio frequencies is generally taken to be that amount which may be calculated by Ohm's law from voltage and current readings obtained when direct current is flowing through the coil. This resistance is often referred to as the *d-c resistance of the coil* or as the *ohmic resistance.*

When coils such as r-f chokes and transformers are used in high-frequency circuits, the resistance to the high-frequency currents is much greater than the d-c resistance of the coils. There are several reasons for this increased resistance to the higher frequency currents.

Skin Effect. Skin effect, which is one cause for this increased resistance, is explained in the following manner: Any flow of current through a conductor is considered as a flow of electrons. When the nature of the electron flow is a direct current, the electrons will be evenly distributed throughout the entire cross-sectional area of the conductor, as shown in Fig. 8-18*a*. However, when the nature of the electron flow is a high-frequency current, the electrons will be concentrated near the surface of the wire and the center of the cross-sectional area will carry practically no electrons, as shown in Fig. 8-18*b*. This reduces the effective area of the conductor, thereby increasing the resistance. The electrons of high-frequency currents move near the surface of the conductor because at high frequencies the magnetic lines expand and contract so rapidly that they cause sufficient induced voltage in the conductor itself to repel and push electrons from the center of the conductor to the outer area.

Eddy Currents. The varying concentric magnetic fields shown in Fig. 5-25 also induce voltages at many points along the conductor itself, and these voltages, even though they are small, set up additional currents in the con-

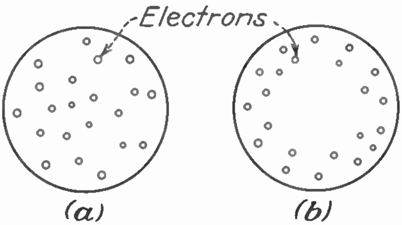


Fig. 8-18 Distribution of electrons over the area of a conductor carrying a current. (a) With direct current or low-frequency alternating current. (b) With high-frequency currents.

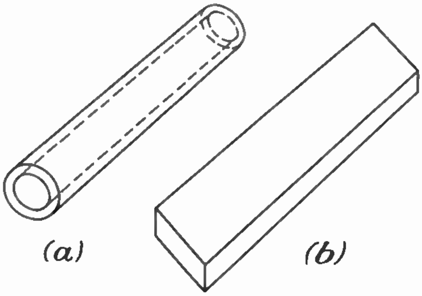


Fig. 8-19 Conductors used for high-frequency currents. (a) Hollow conductor. (b) Flat-strip conductor.

ductor. These currents, known as *eddy currents*, flow back and forth in small areas of the conductor and therefore do no useful work but instead are actually a loss in the conductor. The eddy currents cause additional heating of the conductor, and, as they are a loss, they have the same effect as an increased resistance.

Combined High-frequency Resistance. At high frequency, it is common practice to combine all the resistance effects, that is, the d-c or ohmic resistance, the skin effect, and the eddy-current effect, into a single value called the *a-c resistance of the coil.* This a-c resistance of a coil increases as the frequency at which the coil is operated is increased. The skin effect and hence the a-c resistance increase at a greater rate for large conductors than for small ones. To reduce the value of the a-c resistance, a special type of conductor called *litz wire* is made by weaving a large number of small insulated wires to form the conductor. As the individual wires are insulated, they act as separate conductors and will result in a fairly uniform distribution of current at broadcast and even at short-wave frequencies, but they are not highly effective for ultrahigh frequencies. Another method used to reduce the value of the a-c resistance is to use hollow conductors or thin, flat strip conductors as shown in Fig. 8-19.

8-12 Measuring Inductance

Methods of Measuring Inductance. The method of calculating the inductance from the physical characteristics of a coil was described in Art. 8-2. It is often desired to determine the inductance of a coil, but some factor required for calculating the inductance may not be readily obtained. In such cases it becomes necessary to measure the inductance of the coil. There are two methods available for measuring the inductance of a coil, the comparison and the impedance methods.

Comparison Method. The comparison method involves the use of a standard inductor and some form of a bridge circuit. There are a number of commercial bridges available for measuring inductance. Use of such a bridge is not difficult, and quite accurate results may be obtained (see Art. 6-19).

Impedance Method. The impedance method involves the taking of voltmeter, ammeter, frequency, and wattmeter readings of the coil when connected to an a-c power source. The inductance may then be calculated by use of Eqs. (8-9), (2-14), (8-8), (8-6), as illustrated in the following example.

EXAMPLE 8-17 It is desired to determine the inductance of a coil by the impedance method. The coil is connected to the circuit as shown in Fig. 8-20, and the following meter readings are obtained: voltmeter, 250 volts; ammeter, 0.100 amp; wattmeter, 0.50 watt; frequency, 60 cycles. (*a*) What is the inductance of the coil? (*b*) What is the Q of the coil?

GIVEN: $E = 250$ volts $I = 0.100$ amp $W = 0.500$ watt $f = 60$ cycles

FIND: (*a*) L (*b*) Q

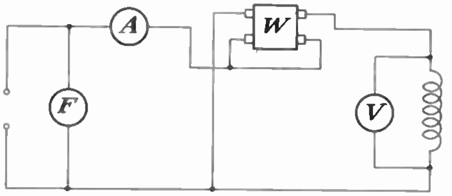


Fig. 8-20

SOLUTION:

(*a*)

$$Z = \frac{E}{I} = \frac{250}{0.100} = 2{,}500 \text{ ohms}$$

$$R = \frac{P}{I^2} = \frac{0.500}{0.100 \times 0.100} = 50 \text{ ohms}$$

$$X_L = \sqrt{Z^2 - R^2} = \sqrt{2{,}500^2 - 50^2} = \sqrt{6{,}250{,}000 - 2{,}500}$$

$$= \sqrt{6{,}247{,}500} = 2{,}499 \text{ ohms}$$

$$L = \frac{X_L}{2\pi f} = \frac{2{,}499}{2 \times 3.14 \times 60} = 6.63 \text{ henrys}$$

(*b*)

$$Q = \frac{X_L}{R} = \frac{2{,}499}{50} = 50 \text{ (approx)}$$

If a wattmeter is not available, the resistance can be determined by connecting the coil to a d-c power supply and taking voltmeter and ammeter readings; the resistance can then be calculated by Ohm's law. If the coil is then connected to an a-c power supply of known frequency and voltmeter and ammeter readings are taken, the inductance can be found in the manner indicated in Example 8-17. The results of Example 8-17 show that when R is small compared with Z, then X_L is practically equal to Z. Under this condition, R may be neglected, and yet it will not produce any appreciable error in the value of X_L and L. Further examination of the results of Example 8-17 shows that the Q of the coil is approximately 50. As the value of Q for coils ranges from 30 to several hundred and since with such values of Q the resistance is low compared with the inductive reactance, it is permissible to neglect the resistance entirely in the equation $X_L = \sqrt{Z^2 - R^2}$ and X_L is then considered equal to Z. On this basis, the inductance can be found by taking only voltmeter and ammeter readings with the coil connected to an a-c power supply of known frequency.

EXAMPLE 8-18 What is the inductance of a high-Q coil that draws 5 ma when connected to a 25-volt 1,000-cycle power supply?

GIVEN: $E = 25$ volts $I = 0.005$ amp $f = 1{,}000$ cycles

FIND: L

SOLUTION:

$$Z = \frac{E}{I} = \frac{25}{0.005} = 5{,}000 \text{ ohms}$$

$$X_L = Z = 5{,}000 \text{ ohms (approx)}$$

$$L = \frac{X_L}{2\pi f} = \frac{5{,}000}{2 \times 3.14 \times 1{,}000} = 0.796 \text{ henry}$$

The three steps used in the solution of Example 8-18 can be combined into a single equation

$$L = \frac{E}{2\pi f I} \tag{8-26}$$

EXAMPLE 8-19 What is the inductance of a high-Q coil that draws 5 ma when connected to a 25-volt 1,000-cycle power supply?

GIVEN: $E = 25$ volts $I = 0.005$ amp $f = 1{,}000$ cycles

FIND: L

SOLUTION:

$$L = \frac{E}{2\pi f I} = \frac{25}{2 \times 3.14 \times 1{,}000 \times 0.005} = 0.796 \text{ henry}$$

In the measurement of inductance of low-frequency coils wound on iron cores, it is recommended that the amount of current used for this measurement be about the same as the rated current of the coil. If the coil has a direct current flowing in addition to the a-f currents under its normal operating conditions, then a similar current should be applied when taking measurements to be used for determining its inductance. Measuring the inductance without its proper d-c component may result in errors as great as several hundred per cent.

The inductance of high-frequency coils, between 1 and 100 mh, may be measured at ordinary power frequencies, but a special low-current-drain voltmeter such as a vacuum-tube voltmeter should be used. Inductances below 1 mh should be measured with high-frequency currents applied, and care should be exercised that suitable high-frequency thermocouple-type meters are used.

8-13 Noninductive Windings

The effect of inductance is useful in radio, television, and other electronic circuits, as is indicated by the preceding description of the various choke coils and transformers; this will be further explained in the chapter on Resonance. There are cases, however, in which as little inductance effect as possible is desired.

When it is desired that a coil of wire have the lowest possible amount of inductance, the coil is wound so that each turn of wire with a current flowing in one direction has its inductive effect neutralized by an adjacent turn with its current flowing in the opposite direction. The coil is then said to be *noninductively wound.*

If a coil requires only a few turns, it may be noninductively wound by first looping the wire as shown in Fig. 8-21*a* and then winding the loop around the core. The completed coil will be as shown in Fig. 8-21*b*. When the coil requires a large number of turns, it may be difficult to loop the wire first. In this case, the coil is usually wound in two sections, each having two leads. When the coil is completed, the two winding sections are connected in series in such a manner that the currents in each section set up magnetic fields of opposite directions which will neutralize each other.

The wires used to connect various parts of a piece of equipment sometimes cause unwanted inductance. This is usually true of the heater circuits of vacuum tubes in a-c-operated equipment, especially if the wires are spaced a small distance apart and are parallel to each other. The inductive effect may be reduced to a negligible amount by twisting the wires, as shown in Fig. 8-22*a*, or even by running the wires parallel to each other but right alongside each other, as shown in Fig. 8-22*b*.

8-14 Use of Inductors in Radio, Television, and Electronic Circuits

Radio Circuits. Inductors in the form of transformers and chokes are important components in radio systems such as (1) a simple a-m receiver, (2) a high-fidelity f-m receiver, (3) a combination a-m and f-m receiver, (4) an f-m stereo receiver, (5) a broadcast transmitter. Trans-

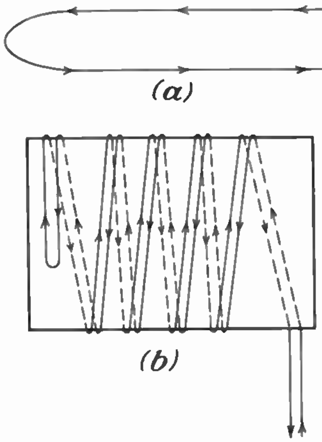


Fig. 8-21 Noninductive winding. (a) A loop of wire. (b) A noninductively wound coil.

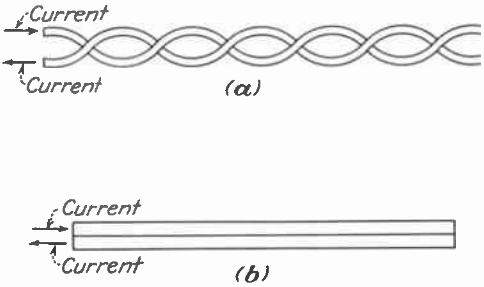


Fig. 8-22 Noninductive wiring. (a) Wires twisted to reduce the inductance. (b) Wires placed parallel to each other to reduce the inductance.

formers and chokes may be found in such circuit applications as (1) antenna, (2) r-f coupling, (3) i-f coupling, (4) a-f coupling, (5) oscillator, (6) a-f power output, (7) loudspeaker coupling, (8) discriminator, (9) heater, (10) power supply, (11) modulator, (12) r-f power output, (13) microphone, (14) transistor. The type and number of inductors and transformers used will depend upon the requirements of the equipment being considered.

Television Circuits. A television receiver is required to reproduce both video and audio signals. Therefore, most of the various types of inductors used in radio circuits will also be found in the audio circuits of a television receiver.

The video circuits have an additional number of inductor applications, among which are (1) picture-tube deflection coils, (2) horizontal oscillator coil, (3) vertical oscillator coil, (4) focus-control coil, (5) coils for vertical and horizontal width control, (6) coils for vertical and horizontal linearity control, (7) vertical- and horizontal-output transformers, (8) video series- and shunt-peaking coils, (9) filter and trap coils for various frequency currents, (10) heater-circuit filter chokes, (11) time-delay-circuit inductors, (12) convergence coils, (13) phasing coils, (14) high-voltage-supply transformer and chokes.

Electronic Circuits. In industrial electronic equipment a number of different types of inductors are used. Many of these inductors are similar to those used in radio and television except for their current and voltage ratings and the frequency at which they are designed to operate. In addition to the inductors that are common to radio and television circuits, various types of control and timing circuits use special inductors and relays that are designed for a particular purpose.

QUESTIONS

1. Define inductance.
2. Why is a knowledge of inductance important in the study of electronics?
3. (*a*) What is meant by induced emf? (*b*) How is it produced?
4. (*a*) What determines the magnitude of the induced emf? (*b*) What determines its polarity?
5. Explain the relation between the induced emf and the impressed voltage during the time that the magnetic field is expanding.
6. Explain the relation between the induced emf and the impressed voltage during the time that the magnetic field is collapsing.
7. State Lenz's law.
8. Why is inductance always present in a-c circuits?
9. When may the effects of inductance be disregarded?
10. Define the basic unit of inductance.
11. Define (*a*) millihenry, (*b*) microhenry.
12. Define the following terms: (*a*) self-inductance, (*b*) inductor, (*c*) solenoid.
13. What are the factors that determine the inductance of a coil?
14. Describe (*a*) multilayer coil, (*b*) pancake coil.

15. (*a*) What is meant by inductive reactance? (*b*) In what unit is it expressed? (*c*) What is its symbol?
16. What two factors affect the amount of inductive reactance of a circuit?
17. (*a*) What is meant by the impedance of a coil? (*b*) What is the unit of impedance? (*c*) What is its symbol?
18. (*a*) Under what condition may the resistance of an inductor be ignored? (*b*) Why?
19. Explain how inductance causes a delay or lag in current flow.
20. (*a*) What is meant by the time constant of a circuit? (*b*) Upon what factors is the numerical value of the time constant dependent? (*c*) In what unit of time is the time constant generally expressed?
21. How is the amount of delay or lag usually expressed?
22. What are the factors affecting the amount of lag in current flow?
23. Define the following terms: (*a*) mutual inductance, (*b*) primary winding, (*c*) secondary winding.
24. Give a definition for the unit of mutual inductance.
25. What is meant by the coefficient of coupling?
26. What factors affect the magnitude of the voltage induced in the secondary circuit?
27. (*a*) How is the inductance of two or more coils connected in series determined? (*b*) How is the inductive reactance of two or more coils connected in series determined?
28. (*a*) How is the inductance of two or more coils connected in parallel determined? (*b*) How is the inductive reactance of two or more coils connected in parallel determined?
29. Explain what is meant by coils connected in series: (*a*) aiding, (*b*) opposing.
30. Describe the principle of the variometer and the variocoupler.
31. Give three classifications into which low-frequency inductance coils may be grouped in terms of the nature of the current flowing in the coils.
32. (*a*) Name some applications of inductors in the a-f classification. (*b*) Give three frequency ranges likely to be encountered in a-f applications.
33. (*a*) Is the inductance of a low-frequency coil high or low? (*b*) Why? (*c*) What construction features are used to obtain the values of inductance required?
34. (*a*) Describe the construction of a-f transformers. (*b*) Why must the impedance of the primary be high in value? (*c*) What is meant by the turns ratio? (*d*) What determines the turns ratio to be used?
35. Name some applications of low-frequency inductors other than the a-f circuits of a radio receiver.
36. (*a*) What does the *Q* of an inductor represent? (*b*) How may inductors be classified in terms of their numerical values of *Q*? (*c*) By what other names is the term *Q* known?
37. What ranges of frequencies are included in the term high-frequency inductance coils?
38. (*a*) Is the inductance of a high-frequency inductance coil high or low? (*b*) Why? (*c*) What construction features are used to obtain the values of inductance required?
39. Compare the features of r-f chokes with those of a-f chokes.
40. (*a*) Describe the magnetic core material used in some high-frequency inductors. (*b*) What are the advantages of using powdered-iron-core r-f coils in place of air-core r-f coils?

41. (*a*) What method is used to make the inductance of a coil adjustable? (*b*) For what application is the adjustable inductor suitable?
42. What are the various types of construction used to reduce the effects of distributed capacitance in r-f coils?
43. (*a*) What is the purpose of r-f transformers? (*b*) What are their constructional features? (*c*) Through what frequency ranges do they operate in radio and television applications?
44. (*a*) What is meant by an i-f transformer? (*b*) At what frequencies do they operate in a-m, f-m, and television receiver applications?
45. (*a*) What system is used to provide an easy means of identifying the leads of r-f and i-f transformers? (*b*) Give an example.
46. How are the values of frequency and impedance used to determine the particular r-f transformer to be used?
47. (*a*) What is meant by bifilar windings? (*b*) What are two advantages of a bifilar-wound transformer?
48. (*a*) What is a toroidal inductor? (*b*) What is a toroidal transformer? (*c*) What are some of the advantages of toroids? (*d*) Name some applications of toroids.
49. Why must r-f and i-f coils be shielded?
50. Explain the principle of shielding.
51. What is meant by the d-c resistance of a coil?
52. What is meant by skin effect?
53. What is meant by eddy-current effect?
54. What methods of construction are used to reduce the skin and eddy-current effects?
55. What is meant by the a-c resistance of a coil?
56. Name two methods that may be used to determine the inductance of a coil.
57. (*a*) Describe two impedance methods of determining the inductance of a coil. (*b*) What are the advantages of each?
58. What precautions should be observed in measuring the inductance of low-frequency coils?
59. What precautions should be observed in measuring the inductance of high-frequency coils?
60. How are coils wound when it is desired to reduce their inductance to practically zero?
61. How are undesirable inductive effects reduced in the general wiring of electronic equipment?
62. Name some applications of transformers and chokes in radio receiving and transmitting equipment.
63. Name some applications of transformers and chokes in television receivers.
64. How do the transformers and chokes used in industrial electronic equipment compare with those used in radio and television applications?

PROBLEMS

1. A flux of 2,000,000 lines links a coil having 250 turns. The flux in the coil decreases from maximum value to zero in 0.125 sec. What is the value of the induced voltage?

2. How many turns must a coil have if its induced voltage is to be 125 volts when a flux of 500,000 lines changes from maximum value to zero in 0.004 sec?
3. A flux of 10 lines links a coil having 50 turns. The flux in the coil varies in accordance with an alternating current having a frequency of 465 kc. What is the value of the induced voltage? (The flux varies from zero to maximum and vice versa in one-quarter of a cycle.)
4. What is the maximum value of the flux in a coil having 75 turns if it induces an emf of 12 volts? The flux varies in accordance with an alternating current having a frequency of 175 kc.
5. What is the inductance of a tuning coil that has 250 turns wound on cardboard tubing 4 cm in diameter and 20 cm long? [Use Eq. (8-2).]
6. What would the inductance of the coil in Prob. 5 be if the number of turns were increased to 750?
7. How many turns are necessary to obtain an inductance of 250 μh if a coil is to be wound on fiber tubing 3 cm in diameter and its length is to be 24 cm?
8. What would the inductance of the coil in Prob. 7 be if it were wound on an iron core whose permeability was 3,600?
9. A transformer has 150 turns on its primary winding and 875 turns on its secondary. The length of the core is 30 cm, its cross-sectional area is 8 sq cm, and the permeability is 3,500. (*a*) What is the inductance of the primary winding? (*b*) What is the inductance of the secondary winding? [Use Eq. (8-2).]
10. What would the inductance of the coil in Prob. 8 be if the number of turns were doubled?
11. What is the inductance of a multilayer coil (Fig. 8-5*a*) having 750 turns if its dimensions are $a = 1\frac{1}{2}$ inches, $b = \frac{3}{4}$ inch, $c = 1$ inch?
12. What is the inductance of a multilayer coil (Fig. 8-5*a*) having 1,500 turns if its dimensions are $a = 1\frac{1}{2}$ inches, $b = 1\frac{1}{2}$ inches, $c = 1$ inch?
13. What is the inductance of a flat coil (Fig. 8-5*b*) having 200 turns if its dimensions are $a = 2$ inches, $c = 1\frac{1}{2}$ inches?
14. What is the inductance of a flat coil (Fig. 8-5*b*) having 100 turns if its dimensions are $a = 1\frac{3}{4}$ inches, $c = 1$ inch?
15. What is the inductance of a solenoid (Fig. 8-5*c*) having 600 turns if its dimensions are $a = 1\frac{1}{2}$ inches, $b = 5$ inches?
16. What is the inductance of a solenoid (Fig. 8-5*c*) having 1,200 turns if its dimensions are $a = 2$ inches, $b = 8$ inches?
17. What is the inductive reactance of a 30-henry choke coil at (*a*) 60 cycles? (*b*) 120 cycles?
18. What is the inductive reactance of a 5-henry choke coil at (*a*) 50 cycles? (*b*) 500 cycles? (*c*) 1,000 cycles? (*d*) 5,000 cycles?
19. What is the inductive reactance of a 250-μh choke coil at (*a*) 550 kc? (*b*) 1,000 kc? (*c*) 1,500 kc? (*d*) 7.5 mc?
20. What is the inductance of a coil that has an inductive reactance of 4,500 ohms at 60 cycles?
21. If the transformer of Prob. 9 is operated at 60 cycles, what is the inductive reactance of (*a*) the primary winding? (*b*) The secondary winding?
22. The secondary of an r-f transformer has an inductance of 185 μh. What is its inductive reactance at (*a*) 500 kc? (*b*) 1,000 kc? (*c*) 1,500 kc?
23. A 10-henry choke coil that has a resistance of 475 ohms is connected to a

110-volt 60-cycle power source. (*a*) What is its inductive reactance? (*b*) What is its impedance? (*c*) How much current does it take from the line?

24. A 6-henry choke coil that has a resistance of 200 ohms is connected to a 110-volt 60-cycle power source. (*a*) What is its inductive reactance? (*b*) What is its impedance? (*c*) How much current does it take from the line?
25. How much current will a 4-henry choke coil, whose resistance is 160 ohms, take when connected to a 120-volt 60-cycle power source?
26. A coil having an inductance of 300 μh and a resistance of 6 ohms is connected to a 3-volt battery. How much time is required for the current to build up to 63.2 per cent of its final value?
27. A coil having an inductance of 30 henrys and a resistance of 250 ohms is connected to a d-c power source. How much time is required for the current to build up to 63.2 per cent of its final value?
28. If the coil of Prob. 26 is connected to a 1,000-kc 3-volt a-c power source, find (*a*) the inductive reactance, (*b*) the impedance, (*c*) the angle of lag, (*d*) the current.
29. If the coil of Prob. 27 is connected to a 60-cycle 300-volt a-c power source, find (*a*) the inductive reactance, (*b*) the impedance, (*c*) the angle of lag, (*d*) the current.
30. Two coils are wound adjacent to each other in such a manner that all the magnetic lines set up in the first coil cut all the turns of the second coil. The primary coil consists of 800 turns wound on a cardboard core 1 inch in diameter and 4 inches long. The secondary coil consists of 1,600 turns wound on a cardboard core 1¼ inches in diameter and 4 inches long. (*a*) By use of Eq. (8-5) find the inductance of the primary winding. (*b*) By use of Eq. (8-5) find the inductance of the secondary winding. (*c*) What is the mutual inductance of the two coils?
31. The transformer of Prob. 9 has a primary inductance of 0.265 henry and a secondary inductance of 9 henrys. What is the mutual inductance if the coefficient of coupling is 0.95?
32. What is the coefficient of coupling of two coils whose self-inductances are 0.04 and 0.06 mh and whose mutual inductance is 0.01 mh?
33. What is the coefficient of coupling of two coils whose self-inductances are 50 and 75 μh and whose mutual inductance is 5 μh?
34. What is the coefficient of coupling of two coils whose self-inductances are 5 and 8 mh and whose mutual inductance is 75 μh?
35. Two coils, the first having 300 turns and the second 450 turns, are placed so that only 5 per cent of the lines set up by coil 1 link coil 2. If 1,200 lines are set up when 3 ma flow through coil 1, how much voltage will be induced across coil 2 if the current decreases from its maximum value to zero in 0.00003 sec?
36. Two coils, the first having 20 turns and the second 40 turns, are placed so that only 10 per cent of the lines set up by coil 1 link coil 2. If 200 lines are set up by the current flowing in the first coil, how much voltage will be induced across coil 2 if the current changes from zero to its maximum value in 10 μsec?
37. Two chokes, whose inductances are 6 and 10 henrys, are to be used in series on a 60-cycle power source. Find (*a*) the total inductance, (*b*) the total inductive reactance.
38. Three chokes, whose inductances are 6, 10, and 14 mh, are to be used in series

on a 5-kc power source. Find (*a*) the total inductance, (*b*) the total inductive reactance.

39. Two chokes, whose inductances are 300 and 600 μh, are to be operated in parallel from a 3-mc power source. Find (*a*) the total inductance, (*b*) the total inductive reactance.
40. Three chokes, whose inductances are 20, 30, and 60 mh, are to be operated in parallel from a 5-kc power source. Find (*a*) the total inductance, (*b*) the total inductive reactance.
41. Two coils, whose inductances are 100 and 400 μh, are arranged so that they may be connected in series in the various ways shown in Fig. 8-10. What is the inductance of the circuit when the two coils are connected in series so that they are (*a*) aiding and the coupling is 100 per cent? (*b*) Opposing and the coupling is 100 per cent? (*c*) In a position that produces zero coupling? (*d*) Aiding and the coupling is 50 per cent?
42. Two coils, whose inductances are 200 and 400 mh, are arranged so that they may be connected in series in the various ways shown in Fig. 8-10. What is the inductance of the circuit when the two coils are connected in series so that they are (*a*) aiding with 80 per cent coupling? (*b*) Aiding with 20 per cent coupling? (*c*) Opposing with 60 per cent coupling? (*d*) Opposing with 10 per cent coupling?
43. Two identical coils are connected in series aiding with a coupling coefficient of 0.20. What must the inductance of each coil be in order to produce a circuit inductance of 185 μh?
44. What is the inductance of each coil in Prob. 43 if the coefficient of coupling is reduced to 0.02?
45. What is the value of Q for the choke coil of Prob. 23?
46. What is the value of Q for the choke coil of Prob. 25?
47. It is desired to determine the inductance of a coil by the impedance method. The coil is connected to a circuit as shown in Fig. 8-20, and the following meter readings are obtained: voltmeter 115 volts, ammeter 1.0 amp, wattmeter 5 watts, frequency meter 60 cycles. (*a*) What is the inductance of the coil? (*b*) What is the Q of the coil?
48. The coil of Prob. 47 is connected to a signal source of 12 volts and 500 cycles. (*a*) What is the inductive reactance? (*b*) What is the resistance of the coil if the coil Q is assumed to be the same as in Prob. 47? (*c*) What is the impedance of the coil? (*d*) How much current is taken by the coil?
49. What is the inductance of a high-Q coil that draws 3 ma when connected to a 10-volt 5,000-cycle power source?
50. A coil having a Q of 150 and an inductive reactance of 1,200 ohms at 60 cycles is connected to a 15-volt 1,000-cycle source of power. Find (*a*) the inductance of the coil, (*b*) the resistance of the coil, (*c*) the current taken by the coil.
51. A coil is connected as shown in Fig. 8-20 in order to determine its inductance by the impedance method. The following meter readings were obtained: voltmeter 110 volts, ammeter 10 ma, wattmeter 0.012 watt, frequency meter 60 cycles. (*a*) What is the inductance of the coil? (*b*) What is the Q of the coil?
52. If the inductance of the coil of Prob. 51 was determined by assuming that the inductive reactance is approximately equal to the impedance, what is the percentage of error in the answer obtained by this method?

Chapter 9

Capacitance

Communication, industrial, and computer types of electronic equipment require resistance, inductance, and capacitance in one or more of their circuits to make them operate satisfactorily. The effects of resistance and inductance have been presented in preceding chapters. Capacitance and its effects in a circuit will now be presented.

9-1 Capacitance, Capacitor Action

Principle of Capacitance. Capacitance is the property of a circuit that opposes any change in the amount of voltage. Notice how this definition compares with the one for inductance, which states that inductance is the property of a circuit that opposes any change in the amount of current. Whereas inductance is caused by the magnetic field and it opposes any change in the amount of current, capacitance is caused by the electrostatic field and it opposes any change in the amount of voltage.

Capacitance is present in practically all a-c circuits; in some instances it is desired while in others it is not. When capacitance is desired, it is provided by a device called a *capacitor*, also formerly called a *condenser*. A capacitor is formed whenever two conductors are placed close to each other but are separated by an insulator which is called the *dielectric*. The conductors are usually made of thin aluminum-foil plates, and the dielectric is a very thin piece or film of insulating material.

Electron Theory of Capacitor Action. According to the electron theory, all matter is made up of electrons and the flow or movement of electrons constitutes an electric current. Materials in which electrons are free to move and which offer little resistance to the flow of electrons are classed as conductors. Materials in which the electrons have a very strong attachment to their nucleus and hence are not free to move far away from the nuclei are classed as insulators. Insulators offer considerable resistance to the flow of electrons.

Figure 9-1***a*** shows a capacitor, consisting of conductors *A* and *B* separated by the dielectric *C*, connected to a switch, a meter, and a battery. With the switch open, the battery emf can have no effect on the capacitor and the meter indicates zero current. The capacitor has no charge, and in terms of the electron theory the materials of the plate and the dielectric are in a neutral state. The nucleus and some orbital electrons of two atoms of the dielectric are shown. As the dielectric has too large a number of atoms to attempt to show all of them, only two are shown. Furthermore, since the actual material of the dielectric has not been named, no specific electron structure of the atom can be shown and the atom is represented by a nucleus and six orbital electrons.

If the switch is closed at *M* (Fig. 9-1*b*), the following actions take place:

1. The emf of the battery causes electrons from plate *A* to flow through the circuit to plate *B* as indicated by the arrows; this electron flow will be indicated by a movement of the needle of the meter.

2. The transfer of electrons from plate *A* to plate *B* causes plate *A* to have less than its normal amount of electrons, and it will have a *positive charge.* As plate *B* has taken on some electrons, it has an excess of electrons and it will have a *negative charge.* Also, an electrostatic field will exist between plates *A* and *B* of the capacitor.

3. The dielectric *C*, being in the electrostatic field, will be affected by it. Because of the nature of the dielectric material, its electrons do not become separated from their nuclei, but the electrostatic field does cause their orbital motion to become distorted as indicated in Fig. 9-1*b*.

4. This distortion causes an additional electrostatic field within the dielectric that is opposite in direction to the field between the plates, which in turn causes more electrons to flow from plate *A* to plate *B* and restores the electrostatic field between the plates to its original strength. The dielectric has thereby increased the charge on the capacitor.

The action just described is referred to as *charging a capacitor*, because through this action plate *A* will become positively charged and plate *B* negatively charged. This entire action takes place in a very short time, usually a small fraction of a second. The flow of electrons from plate *A* to plate *B* is completed in this short interval, and hence the current exists only for that time. The movement of the meter needle will be only a

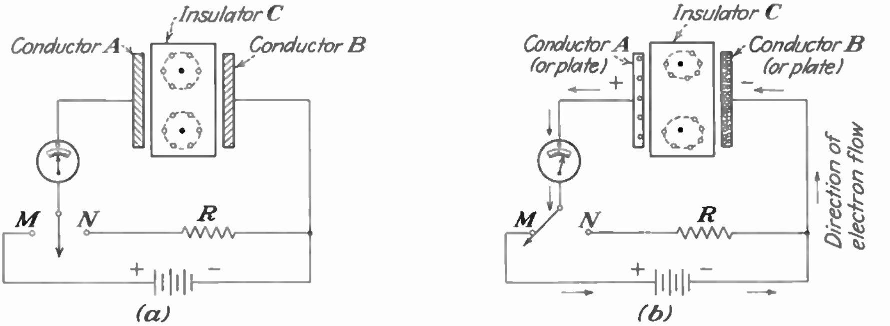


Fig. 9-1 Principles of capacitor action. (a) Capacitor in neutral electron state. (b) A charged capacitor.

momentary movement; that is, the needle will indicate a current flow for just a short time and then return to zero indication.

The discussion so far concerning the action of the capacitor should make clear two important characteristics: (1) A capacitor is a device that actually stores electrical energy. (2) Current does not flow through the capacitor, but the current flow indicated by the meter represents the transfer of electrons from one plate to another.

Effect of Capacitor Losses. If the switch is now opened so that it does not make contact with either *M* or *N* (Fig. 9-1*a*), the capacitor will be disconnected from the line. At the instant of time when the switch is opened, the capacitor is charged; that is, one plate has an excess of electrons, and the other has a deficiency of electrons. Also, electrostatic fields exist between the plates and in the dielectric. If the capacitor is assumed to be a perfect one, that is, having no losses, it will remain in a charged condition until connected into a closed circuit and then it will discharge through that circuit. Unfortunately, all capacitors have some losses, as is described later. However, the losses are small, and some types of high-grade capacitors may hold their charges for a long period of time.

Electron Flow in a Capacitor Circuit. If a capacitor is connected in a circuit similar to Fig. 9-1*b* and the switch is closed through point *M*, the capacitor will become charged. If the switch is opened and kept in the open position, the capacitor will remain charged. If the switch is now closed through point *N*, it will form a closed circuit from plate *B* through the connecting wires, the resistor *R*, the switch, and the ammeter to plate *A*. As plate *B* of the charged capacitor has an excess of electrons, some electrons will now flow through the circuit back to plate *A*. This electron flow constitutes a current and will cause an indication on the ammeter. The current of a capacitor discharging through a resistor will decrease quite rapidly. The time required to discharge the capacitor through a resistor will be dependent on the product of *R* and *C* (Art. 9-4). In most cases, however, the current will decrease so rapidly that the ammeter will merely show an indication for an instant and then return to zero.

If the battery terminals in Fig. 9-1 are interchanged and the switch is closed through *M*, the capacitor will become charged, but now plate *A* will have a negative charge and plate *B* will be positive.

If the capacitor is connected in a circuit similar to Fig. 9-2*a* and the switch is closed toward *F*, a momentary current flow will be indicated on the meter and plate *A* will take on a positive charge while plate *B* will become negatively charged. If the switch is changed so that it is closed toward *G*, a momentary current flow will again be indicated on the meter, but this time the needle will move in the opposite direction to that when the switch was closed toward *F*. Plate *A* of the capacitor will now have a negative charge, and plate *B* a positive charge. Each time the switch position is changed, a momentary current flow will be indicated by the meter.

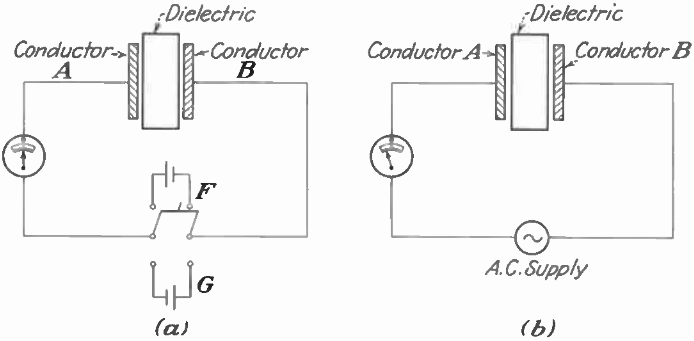


Fig. 9-2 Effect of reversing the polarity at the terminals of a capacitor. (a) Reversals produced by means of a switch. (b) Reversals produced by connecting the capacitor to an a-c power supply.

If a capacitor is connected to an a-c power supply as in Fig. 9-2*b*, the polarity as applied to the plates is continually being reversed. The number of reversals will be double the number of cycles; hence a capacitor connected to a 60-cycle power source will have 120 reversals per second. With alternating current applied to a capacitor, electrons flow continually from one plate through the external circuit to the other plate and then back through the external circuit to the first plate, etc.

Charge on a Capacitor. The amount of charge put on a capacitor is expressed in coulombs, or ampere-seconds, and is equal to the product of the average charging current and the time, or

$$Q = It \tag{9-1}$$

where Q = quantity of electricity, coulombs, or amp-sec
I = average charging current, amp
t = charging time, sec

The charge on a capacitor may also be expressed in terms of the capacitance and the voltage charge, as

$$Q = CE \tag{9-1a}$$

where C = capacitance, farads
E = volts

EXAMPLE 9-1 (*a*) What is the charge in coulombs on a 20-μf capacitor when it is charged to 300 volts? (*b*) What is the average charging current if the capacitor is charged in 0.01 sec?

GIVEN: $C = 20\ \mu\text{f}$ $E = 300$ volts $t = 0.01$ sec

FIND: (*a*) Q (*b*) I

SOLUTION:

(*a*)
$$Q = CE = 20 \times 10^{-6} \times 300 = 0.006 \text{ coulomb}$$

(*b*)
$$I = \frac{Q}{t} = \frac{0.006}{0.01} = 0.6 \text{ amp}$$

9-2 Factors Affecting the Capacitance

Unit of Capacitance. The unit of capacitance is the *farad,* named in honor of an early scientist, Michael Faraday. A circuit has a capacitance of 1 farad when a voltage changing at the rate of 1 volt per second causes an average current of one ampere. The symbol used to represent capacitance is the letter *C*. The farad is too large a unit for practical purposes, and the microfarad (one-millionth of a farad) is commonly used. Several forms of abbreviations such as μf, mf, and mfd are used to represent the word *microfarad;* the symbol μf will be used in this text.

The microfarad is often still too large a unit, and the micromicrofarad (one-millionth of a millionth of a farad), abbreviated as μμf, is then used. Additional abbreviations for lower values of capacitance are *nano,* or *n,* for 10^{-9} and *pico,* or *p,* for 10^{-12}; thus, 470 μμf may also appear as 470 pf or as 0.47 nf, or even simply as 470 p or 0.47 n. The symbols pf and μμf will be used in this text.

Factors Affecting the Capacitance. Since a capacitor or a circuit has a capacitance of 1 farad when a change of 1 volt per second causes a current of 1 amp to flow, the capacitance depends upon the number of electrons set in motion by the specified rate of voltage change. From the preceding discussion, it should be evident that the number of electrons flowing from one conductor to the other will be directly proportional to the active area of the conductor. This is true whether the conductor has only one plate or several plates connected in parallel. A second factor that affects the capacitance is the thickness of the dielectric, because if its thickness is reduced, fewer distorted electron orbits will oppose the electrostatic field. This results in a greater number of electrons being stored on the conductor, which increases the capacitance. As a decrease in the thickness causes an increase in capacitance, it may be stated that the capacitance varies inversely with the thickness of the dielectric. The third factor affecting the capacitance is the kind of material used as the dielectric. Because the atoms of different materials have different orbital electron arrangements, the effect upon the charge on the plates will vary. This effect is accounted for by a value called the *dielectric constant K* of the material. The capacitance is directly proportional to the value of *K*. A list of *K* values for materials used in the construction of capacitors is given in Appendix VI. These three factors affecting the capacitance are represented in each of the following equations for capacitance.

The capacitance of a simple capacitor, that is, one having two flat plates separated by a dielectric, may be found by

$$C = \frac{22.45KA}{10^8 t} \tag{9-2}$$

where C = capacitance, μf
K = dielectric constant (Appendix VI)
A = area of one plate, sq in.
t = thickness of dielectric, inches

In order to obtain a large surface area for the plates with reasonable overall dimensions for the capacitor, multiple-plate and rolled construction methods are used (Figs. 9-6 and 9-8). In a multiple-plate capacitor, both sides of the plates, except each outside plate, are active and the total active area is equal to the area of one plate multiplied by the total number of plates minus 1. The capacitance of a multiplate capacitor having N plates is

$$C = \frac{22.45KA(N - 1)}{10^8 t} \tag{9-2a}$$

EXAMPLE 9-2 What is the capacitance of a capacitor made up of 103 plates of aluminum foil, each 2 inches square and separated by layers of mica 0.01 inch thick?

GIVEN: $K = 5.5$ $A = 2 \times 2$ or 4 sq in. $N = 103$ plates $t = 0.01$ inch

FIND: C

SOLUTION:

$$C = \frac{22.45KA(N - 1)}{10^8 t} = \frac{22.45 \times 5.5 \times 4 \times 102}{10^8 \times 0.01} = 0.05\ \mu\text{f}$$

When a capacitor having two plates is rolled into a cylindrical or flat form (Fig. 9-8), one of the plates will have two active surfaces and the total active area is two times the area of one plate.

EXAMPLE 9-3 What is the capacitance of a rolled capacitor consisting of two plates, each 4 inches wide and 26 ft long, separated by a 0.005-inch-thick paraffined paper dielectric whose value of K is 2.2?

GIVEN: $l = 26$ ft $w = 4$ inches $K = 2.2$ $t = 0.005$ inch

FIND: C

SOLUTION:

$$A = l \times w \times 2 = 26 \times 12 \times 4 \times 2 = 2{,}496 \text{ sq in.}$$

$$C = \frac{22.45KA}{10^8 t} = \frac{22.45 \times 2.2 \times 2{,}496}{10^8 \times 0.005} = 0.247\ \mu\text{f}$$

9-3 Capacitive Reactance, Resistance, and Impedance

Capacitive Reactance. When a capacitor is connected to a d-c power source, current will flow momentarily and the capacitor will become charged. If the voltage is increased, the capacitor will tend to oppose the change in the magnitude of the voltage. However, if the increased voltage is maintained, a momentary current flow will again occur and increase the voltage charge on the capacitor. This shows that capacitance offers an opposition to the flow of current and that capacitance delays a change in voltage just as inductance delays a change in current. When a capacitor is connected to an a-c power supply, the opposition to the flow of current is present continuously.

The opposition to the flow of alternating current offered by a capacitance is called the *capacitive reactance.* It is expressed in ohms, and its symbol is X_C. The value of the capacitive reactance is affected by two factors: (1) the capacitance of the circuit, which depends on the physical characteristics of the capacitor (or circuit), and (2) the rate of speed at which the voltage is changing, which is proportional to the frequency of the power source. The effect of these two factors is indicated in the equation

$$X_C = \frac{1}{2\pi fC} \tag{9-3}$$

where X_C = capacitive reactance, ohms
f = frequency, cps
C = capacitance, farads

When the capacitance is expressed in microfarads,

$$X_C = \frac{10^6}{2\pi fC} = \frac{159{,}000}{fC} \tag{9-4}$$

EXAMPLE 9-4 What is the capacitive reactance of a 10-μf capacitor when connected to a 60-cycle power source?

GIVEN: $C = 10\ \mu\text{f}$ $\quad f = 60$ cycles

FIND: X_C

SOLUTION:

$$X_C = \frac{10^6}{2\pi fC} = \frac{10^6}{6.28 \times 60 \times 10} = 265 \text{ ohms}$$

If a capacitor could be built without any losses, the only opposition to the flow of current would be the capacitive reactance, and

$$I_C = \frac{E_C}{X_C} \tag{9-5}$$

Substituting $10^6/2\pi fC$ for X_C in Eq. (9-5), then

$$I_C = 2\pi fCE_C\, 10^{-6} \qquad \text{when } C \text{ is in } \mu\text{f} \tag{9-6}$$

EXAMPLE 9-5 How much current will the capacitor of Example 9-4 take when connected to a 110-volt 60-cycle power source?

GIVEN: $X_C = 265$ ohms (from Example 9-4) $E = 110$ volts

FIND: I

SOLUTION:

$$I = \frac{E}{X_C} = \frac{110}{265} = 0.415 \text{ amp}$$

EXAMPLE 9-6 How much current will a 0.05-μf capacitor take when connected to an a-f circuit of 25 volts and 5,000 cycles?

GIVEN: $C = 0.05\ \mu$f $E = 25$ volts $f = 5{,}000$ cycles

FIND: I

SOLUTION:

$$I = 2\pi fCE\, 10^{-6} = 6.28 \times 5{,}000 \times 0.05 \times 25 \times 10^{-6} = 0.0392 \text{ amp} = 39.2 \text{ ma}$$

Resistance. In actual practice, capacitors cannot be built without having some amount of resistance. Because this resistance is very small compared with the capacitive reactance, the resistance is sometimes ignored and the capacitor is then considered as having capacitive reactance only.

Impedance. If the resistance of a capacitor is to be considered, its ohmic effect must be combined with the ohmic effect of the capacitive reactance. The combined ohmic effect is called the *impedance* and is represented by the symbol Z. Mathematically it is equal to

$$Z = \sqrt{R^2 + X_C^2} \tag{9-7}$$

where Z = impedance of the circuit, ohms
R = resistance of the circuit, ohms
X_C = capacitive reactance of the circuit, ohms

When both the resistance and the capacitive reactance of a circuit are taken into consideration, the current flowing in the circuit will be

$$I = \frac{E}{Z} \tag{9-8}$$

where I = current flowing in the circuit, amp
E = voltage of the circuit, volts
Z = impedance of the circuit, ohms

EXAMPLE 9-7 A 10-μf capacitor, which also has a resistance of 10 ohms, is connected to a 110-volt 60-cycle power source. Find (*a*) the capacitive reactance, (*b*) the impedance, (*c*) the current.

GIVEN: $C = 10\ \mu f$ $R = 10$ ohms $E = 110$ volts $f = 60$ cycles

FIND: (a) X_C (b) Z (c) I

SOLUTION:

(a) $$X_C = \frac{159{,}000}{fC} = \frac{159{,}000}{60 \times 10} = 265 \text{ ohms}$$

(b) $$Z = \sqrt{R^2 + X_C{}^2} = \sqrt{10^2 + 265^2} = 265.2 \text{ ohms}$$

(c) $$I = \frac{E}{Z} = \frac{110}{265.2} = 0.414 \text{ amp}$$

The results of this example show that the values of the capacitive reactance and the impedance are practically the same, and hence the current of the circuit, too, will be practically the same whether the resistance is considered or ignored. This is always the case when the capacitive reactance is ten or more times greater than the resistance.

9-4 Power Factor, Angle of Lead, Time Constant

Power Factor. The power factor of a circuit (Art. 7-7) is the ratio of the power in watts to the product of the volts and amperes [Eq. (7-19)]. By mathematical operations, it can be shown that the power factor may also be expressed by the ratio of the resistance of a circuit to the impedance of the circuit; thus

$$\text{PF} = \frac{R}{Z} \tag{9-9}$$

Angle of Lead. If it were possible to build a capacitor without losses, the flow of electrons (caused by a change in the applied voltage) would lead the change in the voltage charge at the capacitor by 90 electrical degrees. When both the resistance and the capacitive reactance of a circuit are taken into consideration, the angle by which the current leads the voltage will depend upon the relative amounts of resistance and capacitive reactance in the circuit. The angle between the current and the voltage is expressed in electrical degrees and is referred to as the *phase angle.* Its value may be determined by

$$\cos\theta = \frac{R}{Z} = \frac{R}{\sqrt{R^2 + X_C{}^2}} \tag{9-10}$$

It should be noted that the equations for power factor and $\cos\theta$ are the same. Consequently, the power factor is equal to the cosine of the phase angle θ; also, θ may be referred to as the *power factor angle.*

EXAMPLE 9-8 What is the power factor and the phase angle for the capacitor in Example 9-7?

GIVEN: $R = 10$ ohms $\quad Z = 265.2$ ohms

FIND: PF $\quad \theta$

SOLUTION:

$$\mathrm{PF} = \cos\theta = \frac{R}{Z} = \frac{10}{265.2} = 0.0377$$

then, from Appendix XI

$$\theta = 88^\circ$$

Time Constant. If a perfect capacitor were connected to a d-c power source (Fig. 9-3*a*), an instantaneous surge of current would immediately charge the capacitor to the full amount of the impressed voltage as indicated in Fig. 9-3*b*. When a circuit contains both resistance and capacitance (Fig. 9-4*a*), the capacitor will not attain its full voltage charge instantly but will attain the full charge after a short period of time (Fig. 9-4*b*). The amount of time required to attain full charge depends upon the relative amounts of resistance and capacitance in the circuit. The product of the resistance in ohms and the capacitance in farads is called the *time*

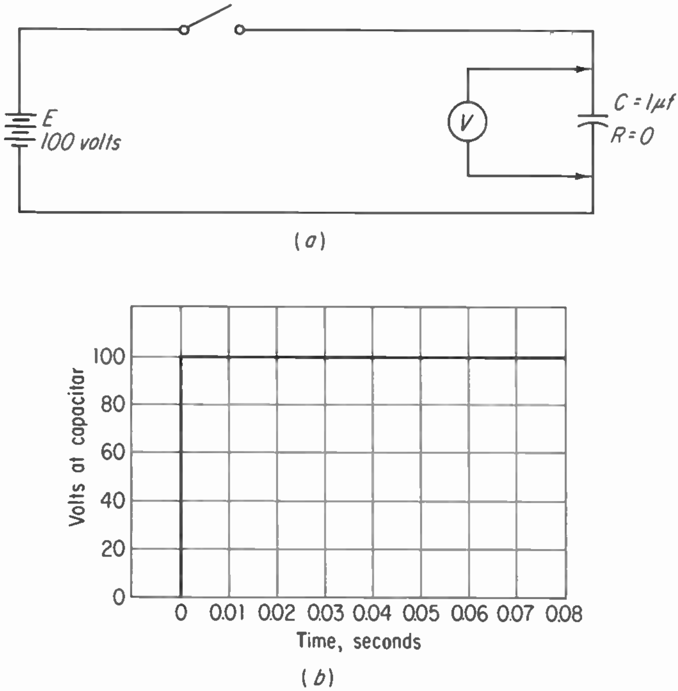


Fig. 9-3 Time required to charge a capacitor in a circuit containing only capacitance. (a) Circuit diagram. (b) Relation between the capacitor voltage and time.

constant and represents the length of time required for the capacitor voltage to reach 63.2 per cent of the applied voltage. The time constant is expressed mathematically as

$$t = CR \tag{9-11}$$

where t = time for capacitor voltage to reach 63.2 per cent of its final value, sec
C = capacitance, farads
R = resistance, ohms

For practical purposes, a capacitor may be considered to be fully charged (actually 99.3 per cent) in a period of time equal to five times the amount of one time constant.

EXAMPLE 9-9 (*a*) How much time is required for the voltage at the capacitor in the circuit of Fig. 9-4 to become charged to 63.2 per cent of its final value? (*b*) How much time is required for the capacitor to become practically fully charged?

GIVEN: $C = 1\ \mu f$ $\quad R = 30{,}000$ ohms

FIND: (*a*) t $\quad$ (*b*) t

SOLUTION:

(*a*) $t = CR = 1 \times 10^{-6} \times 30{,}000 = 0.03$ sec

(*b*) $t = 5CR = 5 \times 0.03 = 0.15$ sec

When a charged capacitor is connected to a resistance, as in Fig. 9-1*b* with the switch in position *N*, the time taken for the capacitor to discharge depends on the values of capacitance and resistance. The product of the capacitance and the resistance represents the time during which 63.2 per cent of the total possible voltage change takes place. Thus, at the end of the first time-constant interval, the voltage at the capacitor will decrease to 100 − 63.2, or 36.8, per cent of the original value. In the second time-constant interval, the charge on the capacitor will decrease by 63.2 per cent of the remaining charge and hence will be 36.8 − (36.8 × 0.632), or 13.5, per cent of the original voltage charge (see also Table 12-2). The current during both the charge and discharge of an *R-C* circuit follows the descending curve of Fig. 9-4*b* and Table 12-2.

The time constant of an *R-C* circuit may thus be defined as the time taken for a change of 63.2 per cent of the possible amount of change in voltage, current, or charge at a capacitor. The application of the principles of time constants is presented in Art. 12-12.

9-5 Fixed Capacitors

Classification of Capacitors. Capacitors are manufactured in various forms and may be divided into two fundamental classes, namely, fixed

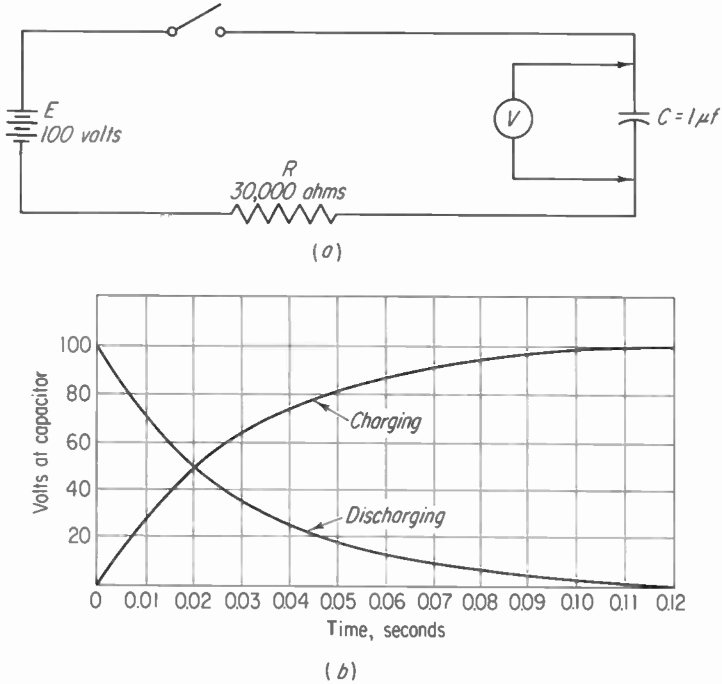


Fig. 9-4 Time required to charge a capacitor in a circuit containing resistance and capacitance. (a) Circuit diagram. (b) Relation between the capacitor voltage and time.

capacitors and variable capacitors. A capacitor constructed so that it can have only one value of capacitance is called a *fixed capacitor.* Fixed capacitors may be further classified as to the general type of material used for the dielectric, as (1) solid dielectric capacitors and (2) electrolytic capacitors. In order to meet the wide variety of requirements for solid-dielectric fixed-capacitor applications, a number of kinds of dielectric materials are used, such as (1) mica, (2) paper, (3) synthetic films, (4) ceramics, (5) oil, (6) glass. Electrolytic capacitors are generally classified as (1) aluminum foil, (2) tantalum foil, (3) wet tantalum, (4) solid or dry tantalum, (5) niobium.

Although it would be more proper to refer to capacitors as being mica-dielectric capacitors, paper-dielectric capacitors, etc., through common usage the word dielectric has been omitted and capacitors are most frequently referred to as being mica capacitors, paper capacitors, etc.

Characteristics. Fixed capacitors are used in widely divergent applications such as radio, television, industrial electronics, computers, aerospace vehicle control, and communications systems. Accordingly, different degrees of relative importance are placed upon such individual characteristics of capacitors as (1) reliability, (2) stability of capacitance value, (3) operating temperature, (4) ruggedness, (5) physical size, (6) cost. Table 9-1 provides a convenient comparison of the characteristics for various types of fixed capacitors.

9-6 Voltage Ratings of Capacitors

Effect of Voltage. Because capacitance varies inversely with the thickness of the dielectric, it is necessary to use very thin dielectrics in order to obtain the desired capacitance within a reasonable physical size for the capacitor. On the other hand, the dielectric must be of sufficient thickness to withstand the electrical pressure applied to the capacitor.

Breakdown Voltage. The voltage applied to the plates of a capacitor causes a strain on the electron orbits of the dielectric separating the plates. At some value of voltage the strain will be sufficient to cause a breakdown of the dielectric and thereby provide a path for electrons to flow from one plate to the other. This causes a discharge through the dielectric, usually in the form of a spark, which burns a hole through the dielectric. In some types of capacitors this causes permanent damage to the unit, while other types of capacitors have self-healing abilities. The voltage at which the dielectric becomes punctured is called the ***breakdown voltage.*** The breakdown voltage of a material is expressed for a specific thickness of the material, usually one-thousandth of an inch, and is called the ***dielectric strength.*** Appendix VI gives the dielectric strength for a number of insulating materials.

Voltage Ratings of Solid-dielectric Capacitors. Capacitors are rated for the maximum voltage at which they may be safely operated as well as for their capacitance. Solid-dielectric capacitors are rated for their (1) d-c working voltage or (2) both d-c and a-c working voltages. The d-c working voltage rating will be higher than the a-c because the a-c rating is for effective values, not the maximum value of the a-c wave form. Because the maximum voltage of an a-c sine-wave source is 1.414 times as great as its effective value, a capacitor rated at 450 volts d-c would have an a-c rating of only 450/1.414, or 318 volts. The a-c working voltage rating may be further reduced to account for (1) non-sine-wave voltages, which may cause the maximum voltage to be more than 1.414 times the effective value; (2) heating of the capacitor, which reduces the dielectric strength; (3) operation at higher frequencies, which increases the losses and heating of a capacitor.

Voltage Ratings of Electrolytic Capacitors. The voltage rating of an electrolytic capacitor is determined by the character of the oxide film, the forming voltage, and the electrolyte used. This type of capacitor is rated at its continuous d-c working voltage. Other important voltage ratings are (1) maximum overall peak voltage, (2) maximum superimposed a-c component or ripple voltage, and (3) surge voltage.

D-C Working Voltage. This is the maximum d-c voltage that the capacitor will withstand satisfactorily under continuous operating conditions within its normal temperature range.

A-C Working Voltage. For nonpolarized capacitors this is the effective value of an alternating sine-wave voltage whose maximum value is equal to the rated d-c working voltage. This value may also be further reduced as previously described under Voltage Ratings of Solid-dielectric Capacitors.

Peak Ripple Voltage or A-C Component. This is the maximum instantaneous value of alternating voltage across the capacitor due to the a-c component in the applied voltage of the capacitor.

Peak Voltage. This represents the sum of the direct voltage and the peak alternating ripple voltage and refers to continuous operation.

Surge Voltage. This is the maximum voltage which may be applied to the capacitor for short periods of time without causing damage to the capacitor. This includes peak ripple and transients at the highest voltage applied. The surge voltage permitted is in the order of 10 to 30 per cent higher than the rated d-c working voltage.

Voltage Derating. Some types of capacitors require a decrease in the rated operating voltage at the higher operating temperatures. This is called *voltage derating* and is expressed as a percentage of the rated voltage at 25°C. For example, a certain polyester-film capacitor may be rated for full voltage operation at temperatures from −55 to 85°C, with 10 per cent voltage derating at 100°C and 50 per cent derating at 125°C.

9-7 Losses in a Capacitor

Capacitors cannot be built without some amount of losses. The losses of a capacitor may be divided into two groups: (1) dielectric losses, (2) resistance losses.

Dielectric Losses. If a perfect capacitor was connected to a d-c power source, became charged, and then was disconnected from the power source, the capacitor would remain charged indefinitely. Actually, this condition is not achieved because all capacitors have some losses. Two types of losses occur at the dielectric; they are called *leakage loss* and *absorption loss.*

Dielectric Leakage Loss. At the instant that a capacitor is disconnected from a d-c power source, it will be fully charged to the potential of the power source. Because there is no perfect dielectric available, a relatively small number of electrons will flow from the negative plate through the dielectric and onto the positive plate. This flow of electrons produces heat, though only a small amount, and is considered as a loss, which is commonly called the *leakage loss.* The amount of this loss varies with the kind of material used as the dielectric. Mica, synthetic films, and ceramics produce very little leakage loss; paper dielectrics have a higher loss; and electrolytic capacitors have the greatest leakage loss.

Dielectric Absorption Loss. When a capacitor is connected to a d-c power source, it should become fully charged in a short period of time and then cease taking current from the line (except for a very small amount due to the dielectric leakage loss). The amount of time theoretically required for the capacitor to become fully charged can be calculated when certain parameters of the capacitor are known. Actually, current will flow into the capacitor for a slightly longer amount of time than the time calculated for the capacitor to become fully charged. The extra amount of energy taken from the power source is described as being absorbed by the dielectric and is called the *dielectric absorption loss* or simply *dielectric absorption.*

The effect of dielectric absorption can also be explained in the reverse order. If the terminals of a charged capacitor are connected to each other momentarily, the capacitor will be discharged, as may be seen by connecting a voltmeter to its terminals. If the capacitor is left disconnected and a short time later again tested with the voltmeter, it will be found that the capacitor has some amount of charge. Although the capacitor apparently had been fully discharged, some charge was retained owing to the dielectric absorption. (Repeatedly short-circuiting the capacitor at its terminals will ultimately result in its complete discharge.) The effect of dielectric absorption may be expressed as a percentage by

$$\mathrm{DA} = \frac{E_r}{E_c} \times 100 \tag{9-12}$$

where DA = dielectric absorption, per cent
E_r = recovery voltage
E_c = charging voltage

One method of determining the percentage of dielectric absorption specifies that (1) the capacitor is to be charged at rated voltage and at 25°C for 5 minutes, (2) then discharge the capacitor through a 5-ohm resistor for 10 seconds, and (3) after 1 minute, measure the recovery voltage through a 10^{11}-ohm resistor. The time for charge, discharge, and recovery may be varied to meet the requirements of a particular specification. For example, another specification requires (1) 60-minute charge at rated voltage at 25°C, (2) 10-second discharge through a 5-ohm resistor, and (3) after 15-minute recovery, read the voltage through a resistor of 10^{11} ohms. The values of dielectric absorption vary with the type of capacitor and range from approximately 0.02 per cent for synthetic-film capacitors to nearly 1 per cent for paper capacitors.

EXAMPLE 9-10 A mica capacitor being tested for dielectric absorption was charged at 300 volts and showed a recovery voltage of 0.75 volt. What is the per cent dielectric absorption?

GIVEN: $E_c = 300$ volts $E_r = 0.75$ volt

FIND: DA

SOLUTION:

$$\text{DA} = \frac{E_r}{E_c} \times 100 = \frac{0.75}{300} \times 100 = 0.25\%$$

When a capacitor is connected to an a-c power source, the residual charge, due to dielectric absorption, left in the capacitor when the applied voltage is going through its zero value constitutes a loss of energy. This energy loss increases with increases in the frequency of the power source. The residual charge in the capacitor may readily be compared with residual magnetism. In each case, when a-c power is considered, a loss occurs and results in heat at the capacitor and the magnet. Because of the similarity of effects, the dielectric absorption is sometimes referred to as *dielectric hysteresis.*

Resistance Losses. The plates of the capacitor, the connections to the plates, and the capacitor leads all carry the capacitor current, and all have some amount of resistance. Electrolytic capacitors have, in addition, the resistance of the electrolyte. All these resistances cause losses and result in additional heat in the capacitor.

Combined Losses. The effects of the individual capacitor losses are usually combined and called the *capacitor losses.* Figure 9-5 shows how the various losses may be represented by resistances. The combined effect of these resistances is called the *equivalent series resistance* of the capacitor. In the equivalent circuit of a capacitor (Fig. 9-5), R_1 represents the resistance of the plates, the connections to the plates, and the lead wires; R_2 represents the resistance effect of the dielectric leakage loss; and R_3 represents the resistance effect of the dielectric absorption loss and the resistance of the electrolyte with electrolytic capacitors.

With solid-dielectric capacitors, the dielectric leakage loss is so small that R_2 is extremely high and may therefore be disregarded. The equivalent series resistance of the capacitor will then be

$$R_{eq} = R_1 + R_3 \tag{9-13}$$

With electrolytic capacitors, the resistance effect of R_2 may be taken as

$$R_2 = \frac{E_{\text{d-c}\cdot w}}{I_{\text{DCL}}} \tag{9-14}$$

where $E_{\text{d-c}\cdot w}$ = rated d-c working voltage, volts
I_{DCL} = direct leakage current, amp

Figure 9-5 shows R_2 to be a parallel-connected resistance; R_2 may be changed to an equivalent series resistance by

$$R_{2\cdot eq\cdot s} = \frac{X_c{}^2}{R_2} \tag{9-15}$$

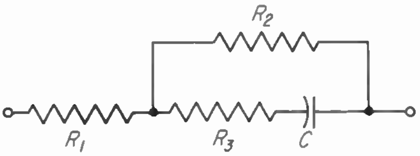


Fig. 9-5 Equivalent circuit of a capacitor. R_1 represents the resistance of the plates, the connections to the plates, and the lead wires; R_2 represents the dielectric leakage loss; R_3 represents the dielectric absorption loss and resistance of the electrolyte; C represents the true capacitance.

The equivalent series resistance of the capacitor may then be expressed as

$$R_{eq} = R_1 + R_{2\cdot eq\cdot s} + R_3 \tag{9-16}$$

EXAMPLE 9-11 A certain 40-μf 150-volt electrolytic capacitor has a normal direct leakage current of 1 ma, its plate and lead resistance is 0.5 ohm, and the resistance effect of dielectric absorption and the electrolyte is 6.8 ohms. (*a*) What is the parallel resistance effect of the leakage loss? (*b*) What is the series equivalent of the parallel resistance effect, using 120 cycles as the frequency value? (*c*) What is the equivalent series resistance of the capacitor?

GIVEN: $C = 40\ \mu f$ $E = 150$ volts $I_{DCL} = 1$ ma $R_1 = 0.5$ ohm
$R_3 = 6.8$ ohms $f = 120$ cycles

FIND: (*a*) R_2 (*b*) $R_{2\cdot eq\cdot s}$ (*c*) R_{eq}

SOLUTION:

(*a*)
$$R_2 = \frac{E_{d\text{-}c\cdot w}}{I_{DCL}} = \frac{150}{1 \times 10^{-3}} = 150{,}000 \text{ ohms}$$

(*b*)
$$R_{2\cdot eq\cdot s} = \frac{X_C^2}{R_2} = \frac{33.1^2}{150{,}000} = 0.0073 \text{ ohm}$$

where
$$X_C = \frac{159{,}000}{fC} = \frac{159{,}000}{120 \times 40} = 33.1 \text{ ohms}$$

(*c*)
$$R_{eq} = R_1 + R_{2\cdot eq\cdot s} + R_3 = 0.5 + 0.0073 + 6.8 = 7.3073 \text{ ohms}$$

The results of this example indicate that for normal conditions the effect of R_2 on the value of the equivalent series resistance is so low that it may be ignored.

The equivalent series resistance can also be calculated from the dissipation factor (Art. 9-8), the capacitance, and the frequency. When these three values are known, then

$$\text{ESR} = \frac{\text{DF} \times 10^6}{2\pi fC} = \frac{159{,}000\ \text{DF}}{fC} \tag{9-17}$$

where ESR = equivalent series resistance, ohms
DF = dissipation factor, decimal value
f = frequency, cps
C = capacitance, μf

EXAMPLE 9-12 What is the equivalent series resistance of an 80-μf 250-volt capacitor having a dissipation factor of 0.11? (The dissipation factor is based on a frequency of 120 cycles.)

GIVEN: $C = 80\ \mu\text{f}$ $\quad$ DF $= 0.11$ $\quad$ $f = 120$ cycles

FIND: ESR

SOLUTION:

$$\text{ESR} = \frac{159{,}000\ \text{DF}}{fC} = \frac{159{,}000 \times 0.11}{120 \times 80} = 1.82\ \text{ohms}$$

The term equivalent series resistance is used in studying some of the operating characteristics of capacitors. Because it may be difficult to ascertain accurately the values of all the losses separately, the resistance is sometimes calculated from Eq. (2-14). When the power and current taken by a capacitor are known, the resistance value found when the power is divided by the current squared is assumed to represent the equivalent series resistance.

9-8 Ratings and Electrical Characteristics of Capacitors

Capacitance and D-C Working Voltage. These are the two basic ratings of a capacitor. The circuit diagram and the parts list of electronic equipment always specify these two parameters.

Tolerance. Tolerance is the deviation from the rated capacitance that will be permitted and is expressed in (1) per cent of rated value, (2) a specified deviation in micromicrofarads or picofarads, or (3) a combination of (1) and (2). The tolerance ratings of capacitors cover a wide range (see Table 9-1), extending from ± 1 per cent for mica and synthetic-film solid-dielectric capacitors to $+150$ per cent for aluminum electrolytic capacitors. The most common standard tolerances for general-purpose applications are ± 20 and ± 10 per cent; when closer tolerances are required, they are accompanied by increased cost.

Operating-temperature Range. This is the range in temperature through which the capacitor may be safely operated. Table 9-1 shows a range of -55 to $250°$C; some tantalum capacitors have a lower temperature limit of $-80°$C. Many of the general-purpose capacitors have a temperature range of -55 to $85°$C.

Capacitance-temperature Stability. Capacitance-temperature stability expresses the maximum amount of change in the capacitance over the permissible operating temperature range. This may be expressed as a percentage or as a relative amount such as small, medium, or large with respect to the capacitance at $25°$C. Approximate percentages are (1) mica 1 per cent, (2) paper ± 10 per cent, (3) synthetic films 2 to 5 per cent, (4) metallized ± 5 per cent, (5) ceramic ± 10 to ± 20 per cent, (6) aluminum 50 per cent, (7) foil and wet-slug tantalum -50 to $+30$ per cent, (8) solid electrolyte tantalum ± 10 per cent.

Temperature Coefficient. The temperature coefficient represents the relative change in capacitance for a temperature change of one degree centigrade. It is usually expressed in parts per million per degree centigrade, which is abbreviated as ppm per °C, or it may occasionally be expressed in per cent. Capacitors featuring a very closely controlled temperature coefficient are available with limits as close as ± 5 ppm per °C of a desired normal value and in either + or − direction. The use of capacitors with controlled-temperature characteristics is presented in Art. 9-15.

Power Factor. The power factor is sometimes used as a capacitor rating to express the merit or quality of a capacitor. Power factor may be thought of as a relative measure of the losses in a capacitor. Mathematically, it is equal to the ratio of the equivalent series resistance to the impedance of the capacitor and may be expressed as a decimal or as a percentage. Because the impedance varies with the frequency, the power factor is stated for a specific frequency value such as 60, 120, 1,000, or 1,000,000 cps.

$$\mathrm{PF} = \frac{\mathrm{ESR}}{Z} = \frac{\mathrm{ESR}}{\sqrt{\mathrm{ESR}^2 + X_C^2}} = \frac{\mathrm{ESR}}{\sqrt{\mathrm{ESR}^2 + \left(\frac{159{,}000}{fC}\right)^2}} \tag{9-18}$$

where PF = power factor, decimal value
ESR = equivalent series resistance, ohms
Z = impedance, ohms
X_C = capacitive reactance, ohms
C = capacitance, μf
f = frequency, cps (generally 1,000 cps for capacitors of 1 μf or less and 60 cps for capacitors over 1 μf)

Power-factor rating is used to some extent with reference to solid-dielectric capacitors, but it is rarely used with reference to electrolytic capacitors. (Dissipation factor is more widely used.) For solid-dielectric capacitors the power factor ranges from 0.02 to 1 or 2 per cent, depending on the type of dielectric and the reference frequency.

Dissipation Factor. The dissipation factor, like power factor, expresses the merit or quality of a capacitor; it, too, may be expressed as either a decimal or a percentage. It differs from the power factor mainly in that it is calculated by dividing the equivalent series resistance of the capacitor by its capacitive reactance instead of dividing by the impedance as with power factor; thus

$$\mathrm{DF} = \frac{\mathrm{ESR}}{X_C} = \frac{\mathrm{ESR}\ 2\pi fC}{10^6} \tag{9-19}$$

where DF is the dissipation factor, decimal value, and ESR, X, C, and f are the same as for Eq. (9-18).

NOTE: The frequency value generally used is 120 cps.

The dissipation-factor rating is used frequently with reference to solid-dielectric capacitors and is used almost exclusively with electrolytic capacitors as compared with the use of power factor as a rating. For solid-dielectric capacitors the dissipation factor ranges from 0.02 to 1 or 2 per cent, depending on the type of dielectric and the reference frequency. For electrolytic capacitors up to approximately 2,000 μf, the dissipation factor will range from about 5 to 35 per cent depending upon the (1) dielectric, (2) electrolyte, (3) capacitance and voltage rating of the unit, (4) value of the applied voltage, (5) temperature. For low-voltage high-capacitance units, the dissipation factor may exceed a value of 5; for example, a 150,000-μf 2.5-volt capacitor may have a dissipation factor in the order of 5 or 6.

EXAMPLE 9-13 A certain capacitor rated at 150,000 μf and 2.5 volts has a dissipation factor of 6 when measured at 120 cycles. Find (*a*) the equivalent series resistance, (*b*) the capacitive reactance at 120 cycles, (*c*) the impedance at 120 cycles, (*d*) the power factor at 120 cycles.

GIVEN: $C = 150{,}000\ \mu\text{f}$ $\quad$ $\text{DF} = 6$ $\quad$ $f = 120$ cycles

FIND: (*a*) ESR $\quad$ (*b*) X_C $\quad$ (*c*) Z $\quad$ (*d*) PF

SOLUTION:

(*a*) $$\text{ESR} = \frac{10^6\ \text{DF}}{2\pi fC} = \frac{10^6 \times 6}{6.28 \times 120 \times 150{,}000} = 0.053 \text{ ohm}$$

(*b*) $$X_C = \frac{10^6}{2\pi fC} = \frac{10^6}{6.28 \times 120 \times 150{,}000} = 0.00884 \text{ ohm}$$

(*c*) $$Z = \sqrt{\text{ESR}^2 + X_C^2} = \sqrt{0.053^2 + 0.00884^2} = 0.0537 \text{ ohm}$$

(*d*) $$\text{PF} = \frac{\text{ESR}}{Z} = \frac{0.053}{0.0537} = 0.98$$

***Q* of a Capacitor.** The term *Q* is sometimes used to express the *figure of merit* or *storage factor* of a capacitor. Mathematically, it is equal to the reactance of a capacitor divided by its equivalent series resistance; thus

$$Q = \frac{X_C}{\text{ESR}} = \frac{10^6}{2\pi fC\ \text{ESR}} \tag{9-20}$$

Also, *Q* is equal to the reciprocal of the dissipation factor; thus

$$Q = \frac{1}{\text{DF}} \tag{9-21}$$

EXAMPLE 9-14 What is the *Q* of a paper-dielectric capacitor whose dissipation factor is 0.5 per cent?

GIVEN: DF = 0.5 per cent, or 0.005

FIND: *Q*

SOLUTION:

$$Q = \frac{1}{\text{DF}} = \frac{1}{0.005} = 200$$

Insulation Resistance. This is the resistance of the capacitor measured from (1) terminal to terminal or (2) terminal to container. It is normally a very high resistance and is generally expressed in (1) megohms, (2) megohms per microfarad, or (3) megohms times microfarads (see Table 9-1). The insulation resistance of a capacitor is specified at a standard temperature of 25°C and decreases rapidly with an increase in temperature. The amount of change in insulation resistance with temperature varies with the type of capacitor. For example, at 85°C the insulation resistance of a mica capacitor will decrease to one-fifth of its value at 25°C, while for a paper capacitor the resistance will decrease to one one-hundredth of its value at 25°C. Insulation resistance decreases with the aging of a capacitor, and with some types of capacitors it is also affected by the voltage.

Direct Leakage Current. When an electrolytic capacitor is connected to a d-c power source, with proper polarities observed, a small amount of current will flow continuously; this is called the *direct leakage current*, abbreviated as DCL. The leakage current, which may be considered as an indication of the quality of the oxide film, is an important characteristic of electrolytic capacitors and represents the amount of direct current flowing through a capacitor other than its momentary charging current.

The leakage current is measured by applying rated voltage to the capacitor for a specified length of time and then measuring the current. One test specification requires that (1) a series-connected current-limiting resistor should be used, (2) the resistance should be of such value that rated voltage will be applied to the capacitor within 1 min, and (3) the leakage current should be read after the rated voltage has been applied for 5 min.

Normal Leakage Current. This represents the direct leakage current when the capacitor is in actual service. The amount of leakage current varies with (1) the type and grade of the capacitor, (2) its rated capacitance, (3) its rated voltage, (4) its operating temperature, (5) the frequency, and (6) the age of the capacitor.

A number of methods are used to calculate the leakage current of a capacitor. One method used with aluminum-foil electrolytic capacitors is

$$I_{\text{DCL}} = K\sqrt{CE} \tag{9-22}$$

where I_{DCL} = direct leakage current, ma
C = rated capacitance, μf
E = rated d-c working voltage, volts
K = variable factor, depending on type, grade, size, and operating temperature of capacitor. For 25°C, the factor ranges in the order of 0.003 to 0.006; at 85°C, the factor increases by about four times.

For tantalum and niobium electrolytic capacitors, the leakage current may be calculated by

$$I_{DCL} = KCE \tag{9-23}$$

where I_{DCL} = direct leakage current, μa
C = rated capacitance, μf
E = rated d-c working voltage, volts
K = variable factor, depending on type, size, and operating temperature of capacitor. For 25°C, approximate values for K are 0.0005 for wet-anode type, 0.004 for foil type, and 0.02 for solid type. At maximum rated temperatures, these factors increase to 0.006, 0.025, and 0.2, respectively.

EXAMPLE 9-15 (*a*) What is the leakage current of a 100-μf 300-volt aluminum-foil electrolytic capacitor at 25°C if the value of K is 0.006? (*b*) What is the approximate leakage current at 85°C?

GIVEN: $C = 100$ μf $E = 300$ volts $K = 0.006$ at 25°C

FIND: (*a*) I_{DCL} at 25°C (*b*) I_{DCL} at 85°C

SOLUTION:

(*a*) At 25°C, $I_{DCL} = K\sqrt{CE} = 0.006\sqrt{100 \times 300} = 1.038$ ma

(*b*) At 85°C, $I_{DCL} \cong 4 \times I_{DCL}$ at 25°C $\cong 4 \times 1.038 \cong 4.14$ ma

EXAMPLE 9-16 (*a*) What is the leakage current of a 250-μf 30-volt foil-type tantalum capacitor at 25°C? (*b*) What is the leakage current at its maximum operating temperature of 85°C?

GIVEN: $C = 250$ μf $E = 30$ volts $K = 0.004$ at 25°C $K = 0.025$ at 85°C

FIND: (*a*) I_{DCL} at 25°C (*b*) I_{DCL} at 85°C

SOLUTION:

(*a*) At 25°C, $I_{DCL} = KCE = 0.004 \times 250 \times 30 = 30$ μa

(*b*) At 85°C, $I_{DCL} = KCE = 0.025 \times 250 \times 30 = 187.5$ μa

Initial Leakage Current. This represents the amount of direct leakage current taken by a capacitor when it is first applied to the voltage source, particularly after a long inactive period. The initial current is high compared with the normal leakage current, decreases rapidly for a short period of time, and then decreases more slowly until it reaches the normal leakage value.

9-9 Mica-dielectric Capacitors

Mica capacitors are made by stacking aluminum-foil plates and thin sheets of mica in alternate layers. Figure 9-6 shows that alternate plates 1, 3, 5, 7, 9 are connected as one terminal and plates 2, 4, 6, 8 form the

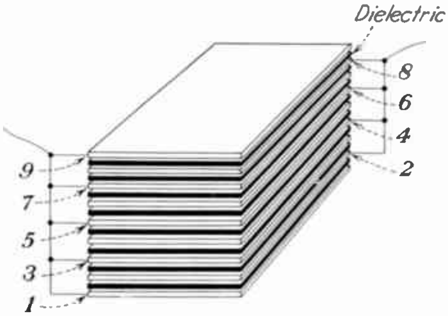


Fig. 9-6 A parallel-plate capacitor.

second terminal. The complete capacitor, consisting of plates, dielectric, and connections to the terminals, is enclosed in a molded Bakelite or other container to keep out moisture. Advantages of mica capacitors are (1) low dielectric losses, (2) low power factor, (3) high voltage ratings, (4) low self-inductance. They are adaptable to high-frequency circuit applications. Mica capacitors can be obtained in values ranging from 1 pf to 1 μf and at voltage ratings of 50 to 50,000 volts. The physical size of these capacitors covers a wide range (Fig. 9-7), as the size is dependent upon both the rated capacitance and voltage.

The standard mica capacitor is enclosed in a Bakelite case and is colored

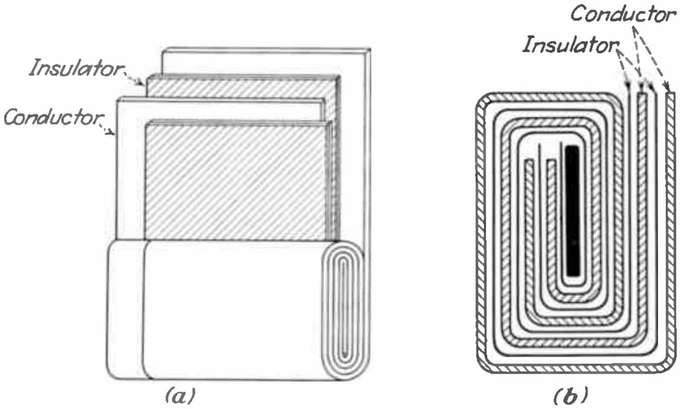
Fig. 9-7 Commercial types of mica dielectric capacitors. The large capacitors are for industrial control equipment and communications transmitters. The small capacitors are for general purposes and for communications receiving applications. (*Aerovox Corporation*)

brown to indicate that it is a standard stacked-plate mica capacitor. For applications which require higher insulation resistance, higher Q, or more stable characteristics, the capacitor is enclosed in a special low-loss case of Bakelite or equivalent material. Such capacitors can be identified by their yellow-colored case.

Silvered-mica Capacitors. In the silvered-mica capacitor, the stacks of metal foil and mica sheets are replaced by a process of applying a thin coating of silver to the mica and bonding the two at high temperature. These capacitors can be distinguished by their red, low-loss, molded-Bakelite cases. Advantages of the silvered-mica-type capacitors are (1) standard tolerance of ±5 per cent (also available down to ±1 per cent) compared with a tolerance of ±20 per cent for the stacked-plate mica capacitors, (2) very low temperature coefficient, (3) negligible capacity drift with time, (4) higher permissible operating temperatures, (5) smaller physical size, (6) more stable and dependable operation in high-humidity environments.

9-10 Paper-dielectric Capacitors

Capacitors made with aluminum-foil conductors and thin sheets of kraft paper (usually impregnated with wax, oil, resin, or a synthetic compound) for the dielectric are called *paper capacitors*. The metal foil and the paper are made into long strips and then are rolled together to form a compact unit (Fig. 9-8). The completed capacitor is placed in a metal, plastic, cardboard, or other type of container to protect the unit and to keep out moisture. Placing the strips of metal foil as shown in Fig. 9-8 permits attaching a lead at each end in such a manner that each turn of a conductor is connected to the terminal for that conductor. As this reduces the inductive effect of the various turns, such a capacitor is often called a *noninductive type.*



(a) (b)

Fig. 9-8 Basic construction of rolled paper-dielectric capacitors. (a) Front view. (b) End view.

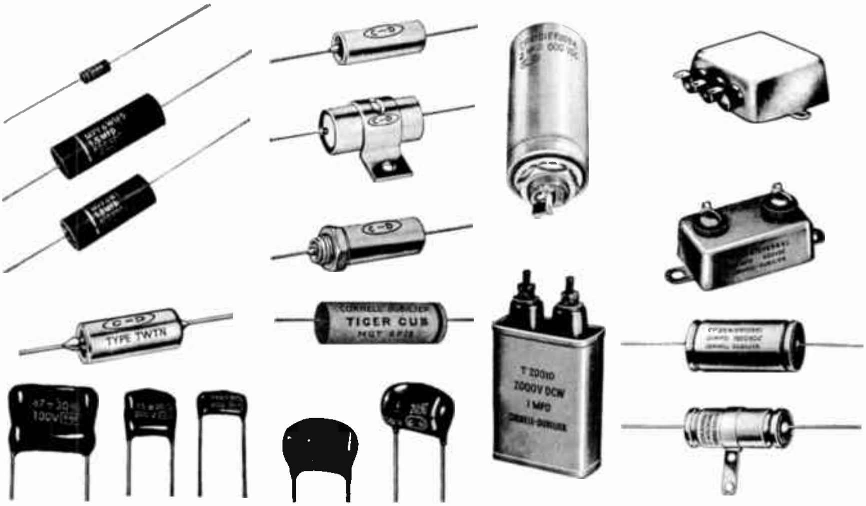


Fig. 9-9 Commercial capacitors representative of paper, film, and metallized types. (*Cornell-Dubilier Electronics, Division Federal Pacific Electric Company*)

Paper capacitors are used extensively in noncritical applications where size is not an important factor but where low cost is important. Paper capacitors are smaller in size and lower in cost than mica capacitors of similar rating and are used in many low- and medium-frequency applications. For general-purpose applications, they are available in sizes ranging from 0.0001 to 1 μf, with voltage ratings from 50 to 1,000 volts. They may also be obtained for special applications in ratings up to 200 μf and 200,000 volts.

9-11 Synthetic-film-dielectric Capacitors

In this type of capacitor, the paper dielectric is replaced by a very thin film of a synthetic material such as a polystyrene, polyester, or polycarbonate; two commonly used polyester dielectrics are the Du Pont Mylar and Teflon. The ability to produce dielectrics with a thickness in the order of 0.00015 inch has made it possible to make smaller size capacitors for use in miniaturized equipment. These capacitors have also extended the operating temperature range from a maximum of 85°C for the paper-dielectric type to a maximum of 150°C for Mylar and 250°C for Teflon capacitors. Other advantages are (1) low dissipation factor, (2) low dielectric absorption, (3) high insulation resistance, (4) good capacitance stability with temperature change and high operating temperatures, (5) small size with some types of film but at a higher cost. Capacitors with film-type dielectrics are adaptable for application to transistorized and printed circuits, computers, and high-grade military and aerospace equipment.

9-12 Dual-dielectric Capacitors

In order to combine the best features of both the paper and synthetic-film types of dielectrics, some capacitors are made with the dielectric consisting of both a paper and a polyester film. The manufacturers' trade names assigned to these capacitors generally indicate that two dielectrics are used. These capacitors are usually noninductively wound and utilize a manufacturing process that produces a solid impregnant for the dielectric and results in a rugged capacitor construction. Considerable reductions in size and weight, compared with the paper-oil type of dielectric, are obtained in the dual paper and film capacitors. Other advantages of the dual-dielectric-type capacitors are (1) low dissipation factor, (2) high insulation resistance, (3) large capacitance and/or high voltage ratings, (4) high relia-

Table 9-1 Characteristics of Solid-dielectric Capacitors†

Values and ranges shown are generally typical or average. Actual limits may vary considerably.

CHARACTERISTIC OF CAPACITOR	DIELECTRIC				
	PAPER	MYLAR*	PAPER AND MYLAR* (DUAL DIELECTRIC)	POLYSTYRENE	TEFLON*
Capacitance:					
Range, μf	0.001–200	0.01–20	0.01–30	0.01–10	0.01–4
Range, pf					
Volts, d-c operating	50–200,000	50–1,000	100–15,000	100–2,000	50–1,000
Tolerance:					
Per cent, standard	±20	±20	±20	±10	±10
Per cent, minimum	±2	±1	±2	±1	±2
Temperature, operating range, °C	−55 +125	−55 +150	−55 +125	−55 +85	−55 +250
Dissipation factor:					
Per cent at 60 cps	0.2–0.5	0.3	0.3	Less than 0.1	Less than 0.1
Per cent at 1 kc	0.2–0.5	0.5	0.5	0.02–0.05	0.02–0.05
Per cent at 1 mc, low-capacitance values	Higher; varies with type	Relatively high	Relatively high	0.05–0.1	0.04–0.07
Insulation resistance, megohms per μf: At 25°C	3,000–20,000	50,000	20,000	Over 100,000	Over 100,000
At 85°C compared with 25°C	$\frac{1}{100}$	$\frac{1}{25}$	$\frac{1}{40}$	$\frac{1}{15}$	$\frac{1}{10}$
Dielectric absorption, per cent at 25°C	0.6–3‡	0.5	0.9	0.02–0.05	0.02–0.05
Stability, capacitance change with temperature and aging	Medium	Medium	Medium	Small	Medium
Size, for equivalent CV rating	Medium small	Small	Medium small	Medium large	Large
Cost, relative cost for equivalent CV rating	Low	Moderately high	Moderately high	Moderately high	Very high

* Du Pont trademark.

† For similar relative characteristics of electrolytic capacitors, see Table 9-2.

bility, (5) long life, (6) little variation in capacitance over a relatively wide operating temperature range, (7) broad range of applications.

9-13 Metallized-dielectric Capacitors

Instead of using separate strips of metal foil and dielectric materials, capacitors can also be made by applying a very thin metallic coating directly on a paper or film-type dielectric. When rolled into a compact form, the metallic coatings on alternate strips of dielectric form the conductors. Capacitors using this type of construction are called *metallized capacitors*, and they are much smaller than foil-type capacitors of similar ratings.

An outstanding feature of the metallized capacitor is its *self-healing*

DIELECTRIC						
METALLIZED PAPER	METALLIZED MYLAR*	CERAMIC, LOW-VOLTAGE	CERAMIC, GENERAL-PURPOSE	CERAMIC, TEMPERATURE COMPENSATING	MICA RECEIVING	MICA TRANSMITTING
0.01–20	0.01–20	0.005–2.2				
			1–20,000	1–2,500	1–50,000	10–1,000,000
50–600	50–600	3–50	500–5,000	500–5,000	50–2,500	200–50,000
±20	±20	±20 to +100, −0	±5 to +100, −0	±5 to ±20	±10	±5
±5	±2	±20	±5	+0.25 pf	±1	±1
−55 +125	−55 +125	−55 +85	−55 +125	−55 +125	−55 +150	−55 +70
0.4–0.6	0.2–0.3	- - -	- - -	- - -	Seldom used	Seldom used
0.6–0.8	0.4–0.5	2.5–10	2–2.5	- - -	Less than 0.1	0.04–0.07
Relatively high	Relatively high	- - -	- - -	0.05–0.2	Less than 0.1	0.03–0.06
600–1,200	5,000–50,000	Variable with voltage	Over 30,000 megohms/unit	Over 50,000 megohms/unit	20,000–50,000 megohms/unit	15,000 megohms/unit
$\frac{1}{60}$	$\frac{1}{40}$	$\frac{1}{20}$	$\frac{1}{60}$	$\frac{1}{50}$	$\frac{1}{5}$	$\frac{1}{7}$
- - -	- - -	- - -	- - -	- - -	0.3 maximum	0.3 maximum
Medium	Medium	Medium	Small to medium	Small	Very small; excellent	Very small; excellent
Small	Small	Very small	Small	Small	Large	Large
Moderately high	Moderately high	Low	Low	Low	High	High

‡ Depending on impregnant,

Source: Cornell-Dubilier Electronics, Division of Federal Pacific Electric Company.

ability. When a breakdown of the dielectric due to high voltage occurs in a foil-type capacitor, the paper or film dielectric is punctured and allows the two metal conductors to make contact. This causes a short circuit in the capacitor, and it is permanently damaged. When a breakdown of the dielectric occurs in a metallized capacitor, the metal in the immediate vicinity of the dielectric puncture burns away with the paper or film. Under this condition, no short circuit occurs and the capacitor retains a satisfactory operating condition. Although the burning off of some metal will theoretically cause a reduction in the capacitance value of the capacitor, the amount of reduction is imperceptible.

Advantages of the metallized capacitor are (1) self-healing, (2) small size, (3) low dissipation factor, (4) reliability, (5) relatively small capacitance change over a wide operating temperature range, (6) relatively low cost. These capacitors are available in a wide range of capacitance and voltage and are more adaptable than the foil-type capacitors for sizes above 1 μf. They are used for filter, bypass, decoupling, and smoothing-action circuits but are not recommended for coupling, logic, and other types of applications where the effect of occasional sparking and subsequent self-healing cannot be tolerated.

Metallized capacitors are also available with a dual dielectric, such as a combination of metallized paper and a polyester film. These capacitors provide special operating characteristics such as (1) unusual self-healing abilities, (2) satisfactory operation at very low voltages where ordinary metallized capacitors fail to operate properly.

9-14 Ceramic-dielectric Capacitors

This type of capacitor is generally made by applying a silver coating on a piece of baked ceramic. A number of types of ceramic material have been developed with dielectric constants ranging from 10 to 10,000. Such high values of K make it possible to produce very small-size capacitors. In the manufacture of these capacitors, the ceramic material is molded into a thin circular, cylindrical, or rectangular form and baked to produce a very hard substance. A thin layer of silver is deposited on each side to form the plates of the capacitor. After the leads are attached, the unit is enclosed in a suitable insulated case. The advantages of ceramic capacitors include (1) low cost, (2) small size, (3) low losses, (4) very low self-inductance, (5) low temperature coefficient, (6) good capacitance stability, (7) long life. They are available in standard ratings from 1 to 20,000 pf at voltages of 500 to 5,000 volts and from 0.005 to 2 μf at voltages of 3 to 50 volts.

Voltage-variable Ceramic Capacitors. Certain ceramic capacitors, depending upon the composition of the ceramic, will have a varying value of capacitance when a variable direct voltage is applied. Such a characteristic can be useful in controlling the actions of a circuit or in the design of automatic control features for electronic circuits.

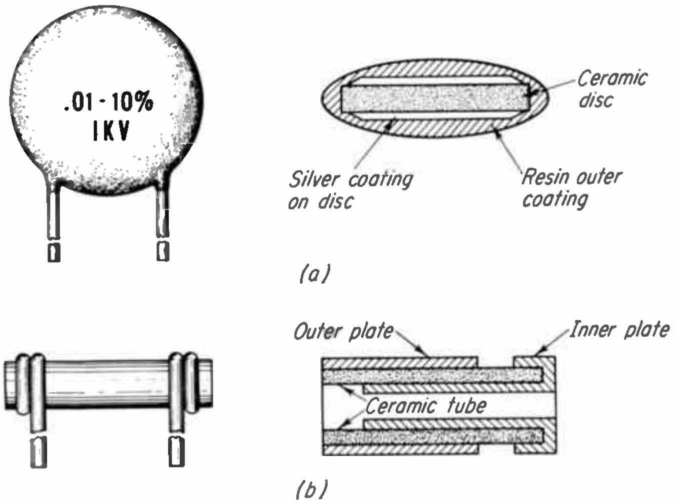


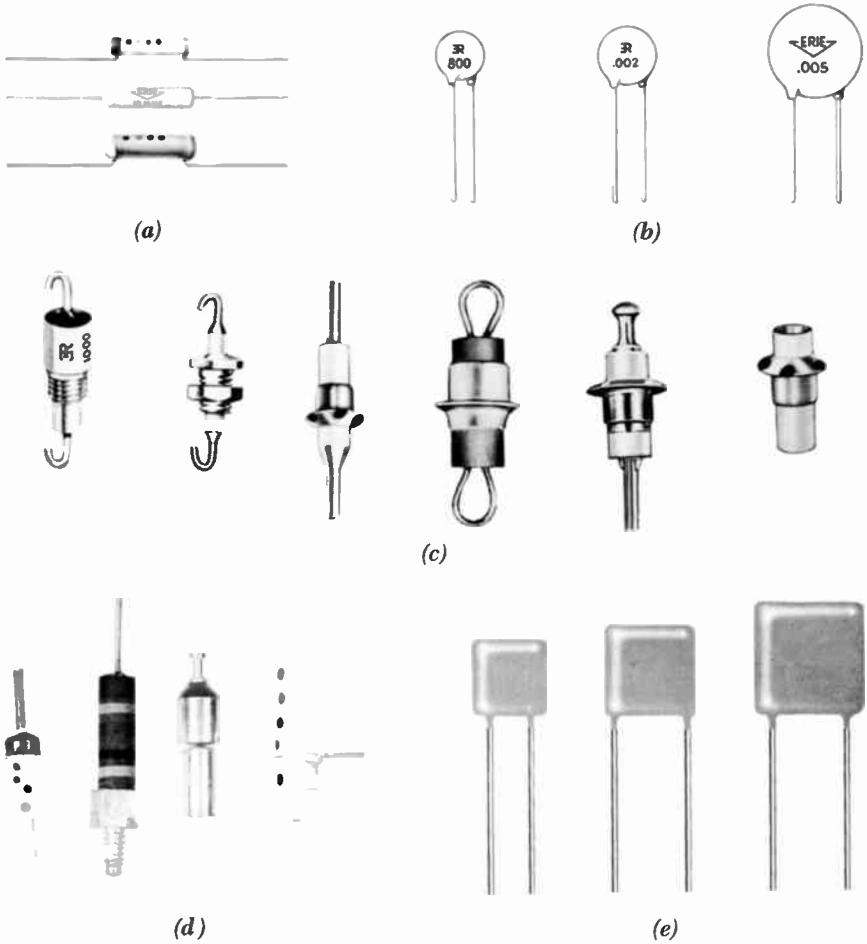
Fig. 9-10 Basic construction of ceramic capacitors. (a) Disc type. (b) Tubular type.

9-15 Temperature-compensating Capacitors

Most electronic circuit components, such as resistors, inductors, and capacitors, normally have a positive temperature coefficient, meaning that their values in ohms, henrys, and microfarads increase with an increase in temperature. Such changes are often likely to alter appreciably the operating characteristics of the equipment in which they are used. One method of correcting such a condition is to use one or more components that have a negative temperature coefficient. Both ceramic and mica capacitors can be manufactured to provide different types and degrees of temperature coefficients; such capacitors are called *compensating capacitors*. The ceramic-type compensating capacitors are used more extensively than the mica type.

The temperature coefficient of a ceramic capacitor can be controlled by the composition of the ceramic material, and it is possible to produce capacitors with zero, positive, or negative temperature coefficients. These capacitors are available with various temperature coefficients ranging from P100 to N5600. The letters P and N indicate positive and negative, respectively, and the number following the letter indicates the temperature coefficient in parts per million per degree centigrade rise. Thus, N750 indicates a decrease in capacitance of 750 units per million units for each 1° increase in temperature, based on an ambient (surrounding) temperature of 25°C. The rating NP0 indicates zero or no change in either positive or negative direction.

By using a parallel combination of NP0 and N750 capacitors, it is pos-



(a) (b) (c) (d) (e)

Fig. 9-11 Commercial types of ceramic capacitors such as (a) tubular, (b) disc, (c) feed-through, (d) stand-off, and (e) plate. (*Erie Technological Products, Inc.*)

sible to obtain practically any temperature coefficient between zero and −750 ppm. Type NPO capacitors are available in ratings from 1 to 300 pf, and type N750 in ratings from 2.2 to 750 pf. The following equations and example illustrate the manner of determining the values of the individual capacitors.

$$C_2 = \frac{C_1 \times T_{c\cdot 1}}{T_{c\cdot 2}} \tag{9-24}$$

$$C_0 = C_1 - C_2 \tag{9-25}$$

where C_1 = desired capacitance with temperature coefficient of $T_{c\cdot 1}$
$T_{c\cdot 1}$ = temperature coefficient desired for circuit
C_0 = capacitance of unit having a temperature coefficient of $T_{c\cdot 0}$
$T_{c\cdot 0}$ = temperature coefficient of zero, thus type NP0
C_2 = capacitance of unit having a temperature coefficient of $T_{c\cdot 2}$
$T_{c\cdot 2}$ = temperature coefficient of compensating capacitor; among the types available are N750, N330, N80, P100

EXAMPLE 9-17 What ratings of N750 and NP0 capacitors are required to produce 240 pf with a temperature coefficient of N250? Note: It will be necessary to have a parts catalog in order to observe the standard rated values for which capacitors are available.

GIVEN: $C_1 = 240$ pf $T_{c\cdot 1}$ = N250 ppm per °C $T_{c\cdot 2}$ = N750 ppm per °C

FIND: C_{N750} C_{NP0}

SOLUTION:

$$C_2 = \frac{C_1 \times T_{c\cdot 1}}{T_{c\cdot 2}} = \frac{240 \times 250}{750} = 80 \text{ pf}$$

From the manufacturer's catalog, the nearest standard capacitor available is 82 pf.

$$C_0 = C_1 - C_2 = 240 - 82 = 158 \text{ pf}$$

The nearest standard value in the catalog is 160 pf.

Checking results:

$$C_1 = C_0 + C_2 = 160 + 82 = 242 \text{ pf}$$

9-16 Oil-dielectric Capacitors

When capacitors are required to withstand very high voltages, as in radio and television transmitters and some industrial applications, an oil-impregnated paper is generally used as the dielectric. In addition, the higher voltage capacitors are enclosed in hermetically sealed oil-filled containers. Available standard ratings range from 0.1 to 10 μf at voltages from 600 to 50,000 volts. They may also be obtained in special ratings up to 200,000 volts.

9-17 Glass-dielectric Capacitors

The earliest capacitor, 1745, was the Leyden jar. A modern development has returned to the use of glass through the introduction of a low-loss glass ribbon approximately 0.001 inch thick. Layers of aluminum foil and glass are stacked to provide the required area. After the leads are attached, the unit is enclosed in glass, which is then fused with pressure and heat to form a practically indestructible unit. Standard ratings range from 1 to 10,000 pf at 300 to 500 volts. The high cost of this type of capacitor has limited its use to military, space, and other critical applications.

9-18 Color Codes for Capacitors

When the physical size of a capacitor permits, information about its rated values of capacitance and voltage is usually printed on the container. This is the case with most paper capacitors. When the available space on the capacitor does not permit stamping the information thereon, the important data are provided by a series of colored bands or dots. A complete description of the methods of color coding is given in Appendix VIII.

9-19 Electrolytic Capacitors

Basic Principle of Electrolytic Capacitors. An electrolytic capacitor consists of two metallic plates separated by an electrolyte. The electrolyte is not the dielectric material but is actually the negative electrode. The dielectric is a thin oxide film which is formed on the surface of the positive capacitor plate. The second plate, sometimes erroneously called the negative electrode, provides the means of making contact with the electrolyte (the actual negative electrode) and serves as the negative terminal.

The capacitance of the electrolytic capacitor depends on (1) the area of the plates, (2) the thickness of the dielectric, and (3) the dielectric constant of the oxide film.

Action of the Fundamental Electrolytic Capacitor. If two aluminum plates are immersed in a suitable electrolyte (Fig. 9-12) and connected to a d-c power source, current will pass from one plate to the other. At the instant of closing the circuit, the current will be very high for a short time and will then taper off to nearly zero. This action is due to the process called *forming*, which means that an insulating film is being formed on the surface of one of the plates. If the plates are of aluminum, this film will form on the plate connected to the positive terminal. The formation of this film on the plate retards the flow of current. If the polarity of the voltage applied to the terminals is reversed, the film will form on the opposite

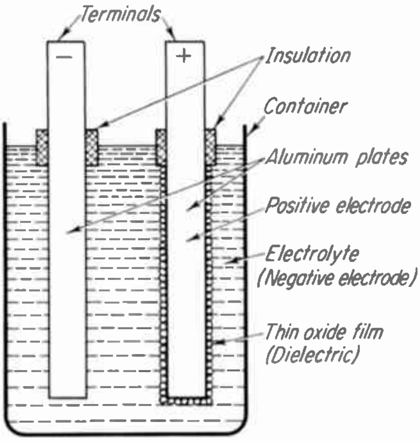


Fig. 9-12 The fundamental electrolytic capacitor.

plate and current will flow in the circuit until this forming process is sufficient to build up enough resistance to reduce the current flow to nearly zero. The film acts as an insulator only as long as the same polarity used in forming is maintained at its terminals.

Dielectric. The dielectric used in electrolytic capacitors is the film formed on the surface of the positive electrode. The thickness of this film will depend on the material used for the plates and the amount of voltage used in the forming process. The higher the applied voltage, the thicker the film becomes and the lower will be the resultant capacitance. The maximum working voltage rating of an electrolytic capacitor must be less than the voltage used in its forming process. If this voltage is exceeded, then, in addition to the possibility of puncturing the insulation, the thickness of the oxide film will be increased and the capacitance of the capacitor will be decreased. As long as the applied voltage is lower than that used in forming, the thickness of the oxide film and the capacitance will remain constant.

Forming Voltage. The maximum voltage that may be used in forming will depend on the electrolyte used; the maximum voltage commercially attainable is in the order of 500 volts. Solutions of borax are used extensively because of the relatively high voltages that they can withstand and because of their noncorrosive properties. Inasmuch as the thickness of the oxide film is determined by the forming voltage, the capacitance of a capacitor also depends on this voltage.

Self-healing Characteristic. The thickness of the oxide film formed on the surface of the plate is in the order of 0.00001 inch. Since this is only a fraction of the thickness of most other dielectrics of similar voltage rating, a relatively high capacitance can be obtained in a small space. When the applied voltage exceeds the maximum critical voltage that the dielectric can withstand, the thin oxide film will be punctured. This causes a short circuit between its terminals. When the applied voltage is removed or reduced to a value less than its maximum critical voltage, the break will be mended by the formation of a new insulating film. This self-healing characteristic is an important advantage of electrolytic capacitors.

9-20 Wet Electrolytic Capacitors

The electrolyte for an electrolytic capacitor may be in a liquid, paste or semiliquid, or solid form. The aluminum wet-electrolytic capacitor, which has a liquid electrolyte, was used to some extent at one time but has been replaced largely by the dry-type electrolytic capacitors. Some tantalum and niobium electrolytic capacitors use a liquid electrolyte.

9-21 Aluminum Electrolytic Capacitors

Construction. By the use of a jellylike electrolyte, electrolytic capacitors can be constructed in a substantially dry form. They are considered dry in the same manner that dry-cell batteries are considered dry; that is, the electrolyte cannot be spilled from its container. Dry electrolytic capacitors produce high values of capacitance in relatively small dimensions and are the most economical type for many applications.

In general, a dry electrolytic capacitor consists of a positive foil, a negative foil, and a separator containing the electrolyte, which are all wound into a roll and provided with a means for electrical connection, housing, and mounting (see Fig. 9-13). The positive foil is of high-purity aluminum and is subjected to a special electrochemical forming process which completely covers it with an extremely thin oxide film. The nature and thickness of this film will govern its voltage and the capacitance per unit area. The separator is made of some absorbent material, usually gauze, paper, nonfibrous cellulose, or a combination of these; it serves to hold the electrolyte in position and to keep the positive and negative foils from making physical contact. The electrolyte consists of a chemical solution in a moist paste form and serves as the negative electrode. The negative foil, generally of aluminum, acts merely as a means of making contact with the electrolyte and becomes the negative terminal of the capacitor.

The positive plate of the capacitor is made with either a plain or a specially prepared aluminum foil. An *etched aluminum plate* is formed by an etching process which roughens the surface of the foil and increases its effective area, thereby providing increased capacitance. Another method of increasing the effective surface area is the special *fabricated plate* which uses a metallized cotton gauze. Capacitors using a fabricated plate have a normal capacitance ratio, as compared with plain aluminum foil, in the

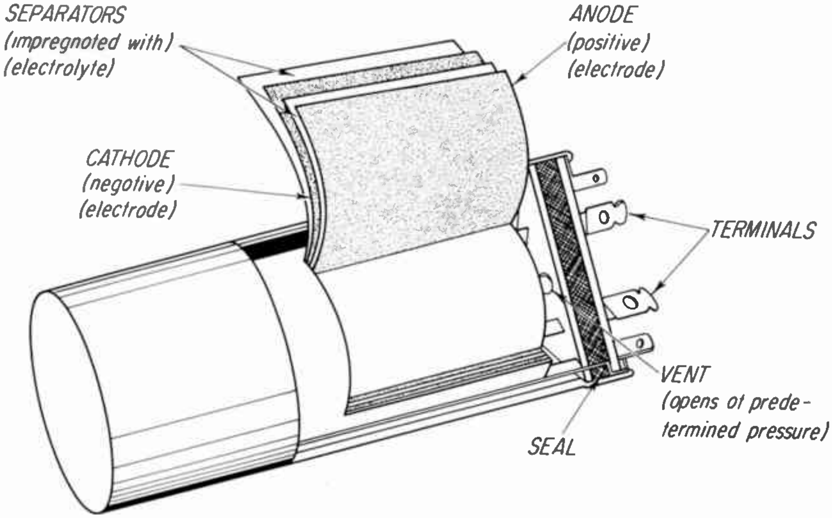


Fig. 9-13 Construction of aluminum dry-electrolytic capacitor. (P. R. Mallory & Co., Inc.)

Fig. 9-14 Comparative size of plain-, etched-, and fabricated-type capacitors. (*P. R. Mallory & Co., Inc.*)

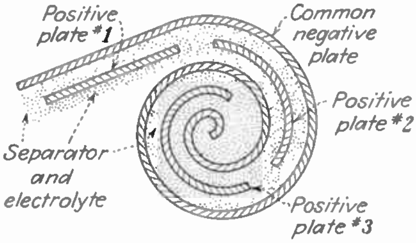


Fig. 9-15 A capacitor block with a common negative and three positive plates.

order of 10 to 1. Figure 9-14 illustrates the comparative size of capacitors using the three different types of plate construction.

Capacitor Blocks. Many capacitor applications require or permit the negative terminals of various sections to be connected to one common point in a circuit. Since in electrolytic capacitors the electrolyte itself is the negative electrode, several positive electrodes can be associated with one common electrolyte. In commercial capacitors, this type of unit has one long negative foil and the required number of positive foils laid end to end and arranged parallel to it. Together with the proper separators, the negative and positive foils are rolled into one complete unit (Fig. 9-15); each piece of foil is provided with its own terminal.

9-22 Tantalum Capacitors

Electrodes. Tantalum, which is a transition element, may be used in place of aluminum for the electrodes of electrolytic capacitors. Tantalum has a much higher rating than aluminum in microfarads per volt per cubic inch, thereby permitting extensive advances in the field of miniaturization. Furthermore, tantalum foil plates can be made with a thickness in the order of 0.0005 inch as compared with thicknesses of 0.002 to 0.005 inch for aluminum foil. Use of sintered tantalum provides further gains in miniaturization. Sintering is a process whereby finely powdered particles of a substance are mixed with an organic binder and pressed into a desired unit of any shape. This unit is then heated in a vacuum oven, which causes the organic binder to decompose and evaporate, thereby producing a strong, yet highly porous material.

Types of Capacitors. Tantalum capacitors are made in three basic types: (1) foil, (2) wet anode, (3) solid electrolyte.

Advantages. Among the advantages of tantalum capacitors are (1) very small size, (2) closer capacitance tolerances than with aluminum electrolytic capacitors, (3) high capacitance stability, (4) low leakage current, (5) low power factor, (6) long shelf life, (7) long service life, (8) ruggedness, (9) high reliability, (10) operating temperature range from −80 to 125°C, (11) freedom from possibility of electrolyte leakage and corrosion in the solid-electrolyte type. For some applications, one or more of these advantages can outweigh the disadvantage of the higher cost of tantalum capacitors.

Ratings. Tantalum capacitors are available in ratings ranging from 0.002 to 2000 μf and at voltages ranging from 3 to 600 volts. Generally, the higher capacitance ratings are available only in the lower voltage ranges. The solid-electrolyte types are available chiefly in voltage ratings of 6 to 100 volts.

9-23 Tantalum Foil-type Capacitors

The principle of the foil-type tantalum capacitor is the same as for the aluminum electrolytic capacitor. This type of capacitor consists of two tantalum foils, paper separators, and an electrolyte in either a liquid or paste form. The general construction features are similar to those for the aluminum capacitor shown in Fig. 9-13. The foil serving as the anode may be of either the plain- or etched-foil types. The dielectric is a thin tantalum oxide film which has been formed on the surface of the anode by an electrochemical process before assembly of the capacitor components. The electrolyte is the actual cathode of the capacitor and, because it is a liquid or paste, makes contact with the entire surface area of the dielectric. The second foil provides the means of connecting the external negative terminal of the capacitor with a low-resistance current-distribution path to the electrolyte along the entire active area of the anode. The separators serve two purposes: (1) They prevent damage to the oxide film during assembly and also in service, and (2) being a porous-type material, they serve as a carrier for the electrolyte. The complete assembly is enclosed in a suitable container.

The electrodes may be of either the plain-foil or the etched-foil type. Because of its smooth surface, the plain foil permits the formation of a more uniform layer of tantalum oxide on the anode. Consequently, electrical characteristics such as low power factor, close capacitance tolerances, and capacitance stability with temperature changes and age will be more stable with the plain-foil type than with the etched-foil type.

An etched-type electrode is produced by chemically etching the foil prior to formation of the tantalum oxide dielectric. As the etching process approximately triples the active area of an electrode, a capacitor with etched-foil electrodes will provide three times the amount of capacitance provided by a similar size capacitor having plain-foil electrodes.

Foil-type tantalum capacitors are available in ratings ranging from 0.25 to 580 μf at voltages ranging from 3 to 300 volts.

Table 9-2 Characteristics of Electrolytic Capacitors*
Values and ranges shown are generally typical or average. Actual limits may vary considerably.

CHARACTERISTIC OF CAPACITOR	ALUMINUM FOIL	NIOBIUM FOIL	TANTALUM FOIL	TANTALUM WET ANODE	TANTALUM DRY ANODE
Capacitance, range, μf	1–150,000	0.5–580	0.25–580	1.7–560[a]	0.05–330
Volts, d-c operating	2.5–500	6–50	3–150	4–125	6–50
Tolerance:					
Per cent, standard	[b]	[c]	[d]	[e]	±20
Per cent, minimum	±25	±15	±15	±10	±5
Temperature, operating range, °C	−40 +85	−55 +85	−55 +125	−55 +125	−55 +85[f]
Dissipation factor (maximum), per cent at 120 cps and 25°C	6–35[g]	Etched, 20 plain, 15	10–20[h]	10–20	6
Leakage current (maximum) Average at 25°C	Milliamperes $0.006\sqrt{CV}$	Microamperes 0.02/μf/volt	Microamperes 0.02/μf/volt	Microamperes 0.003/μf/volt	Microamperes 0.02/μf/volt
Increase at maximum rated temperature	4 times	4 times	4 times	5 times	10 times
Stability, capacitance change with temperature and aging	Relatively large	Large ±25%	Large ±25%	Large ±25[i]	Medium ±10%
Size, for equivalent CV rating	Very small	Very small	Very small	Very small	Very small
Cost, relative cost for equivalent CV rating	Very low	Moderate	Moderate	Moderate	Moderate

[a] To 5,000 μf in multiple-section units.
[b] Depends on voltage: 3 to 50 volts, +150 −10; 51 to 350 volts, +100 −10; over 350 volts, +50 −10.
[c] For plain foil, ±20; for etched foil, +75 −15.
[d] For plain foil, ±20; for etched foil: 15 to 49 volts, +75 −15; 50 to 99 volts, +50 −15; over 100 volts, +30 −15.
[e] Mostly ±20; for some types may increase up to +50 −15.
[f] Up to +125°C, with some voltage derating.
[g] Depends on voltage, type, and capacitance.
[h] Varies with capacitance and voltage; also, whether plain or etched foil.
[i] May increase up to −60 +25, depending on type, voltage, and capacitance.
* For similar relative characteristics of solid-electric capacitors, see Table 9-1.

Source: Cornell-Dubilier Electronics, Division of Federal Pacific Electric Company.

9-24 Wet-anode Tantalum Capacitors

The wet-anode tantalum capacitor uses a sintered tantalum slug in place of the tantalum foil as its anode. A tantalum oxide film is developed on the surface of the slug, and because of the porous nature of the sintered slug, it is possible to get a relatively large active area on the anode. The anode is immersed in a suitable electrolyte which is the cathode of the capacitor. The anode and electrolyte are enclosed in a hermetically sealed container. In some cases, when the container is of metal, it also serves as the negative terminal of the capacitor. Because of the relatively large active area achieved with the sintered slug, this type of capacitor is smaller than a foil-type capacitor of similar rating.

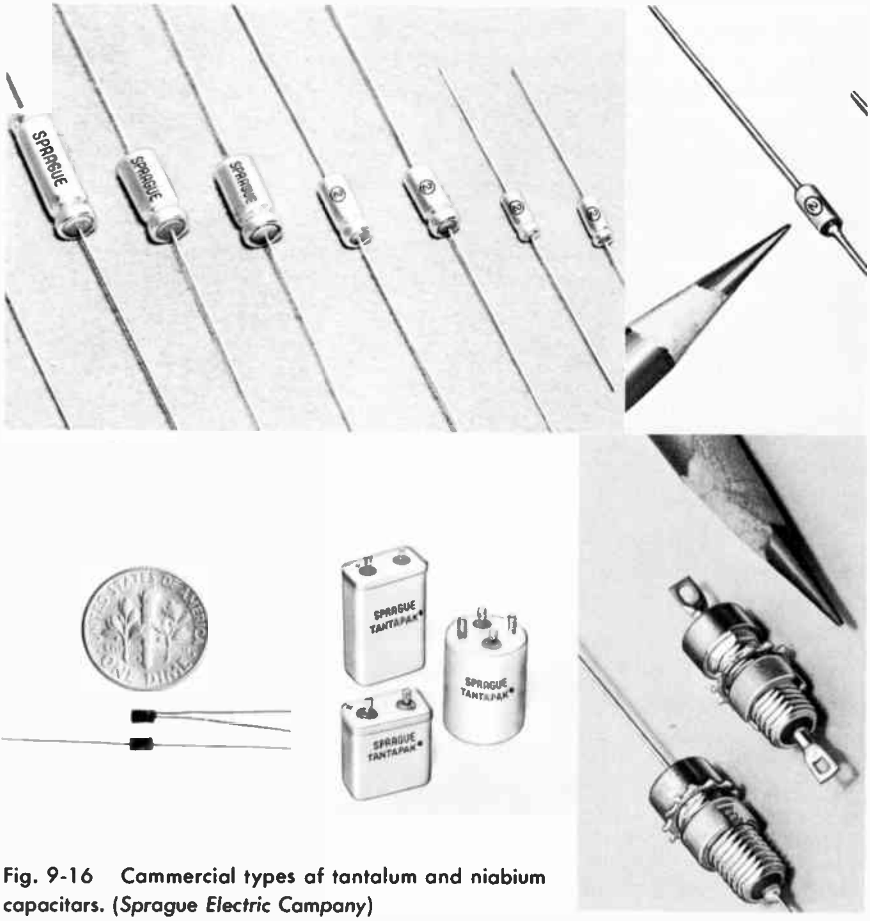


Fig. 9-16 Commercial types of tantalum and niobium capacitors. (*Sprague Electric Company*)

Wet-anode tantalum capacitors are available in ratings ranging from 1 to 2,200 μf at voltages ranging from 3 to 600 volts. The higher capacitance ratings are available only at the lower voltage ratings.

9-25 Solid-electrolyte Tantalum Capacitors

This type of capacitor uses for its anode a sintered tantalum slug on which a tantalum oxide film has previously been formed. The oxide film acts as the dielectric of the capacitor. A solid semiconductive material, deposited on the anode, serves as the electrolyte and becomes the cathode of the capacitor. The cathode is formed by immersing the anode into an aqueous manganese salt solution and allowing it to penetrate into the pores of the anode. Then, by means of a heat-treatment process, a thin layer of manganese dioxide is formed on the surface of the anode, hence the name *solid-electrolyte capacitor.* The manganese dioxide layer is then covered with a carbon compound to provide a means of obtaining a uniform current-distribution path between the cathode and the negative terminal of the capacitor. The completed unit is enclosed in an hermetically sealed metal container which may also serve as the negative terminal of the capacitor. As the capacitor consists of only solid materials, it eliminates the possibility of corrosion or leakage of the electrolyte even in the event of damage to the container.

The solid-electrolyte capacitor is very small, has a long life expectancy and excellent electrical characteristics. It is available in ratings from 0.005 to 330 μf at voltages ranging from 6 to 100 volts. The higher capacitance ratings are available only at the lower voltage ratings.

9-26 Niobium Capacitors

Niobium, which also is a transition element, has characteristics very similar to those of tantalum. The oxide film produced on a niobium anode has a dielectric constant of approximately double the value of that of the tantalum oxide film. Niobium is used in the manufacture of foil-type capacitors, both plain and etched, in the same manner as used for tantalum capacitors. Niobium capacitors are up to 25 per cent lighter in weight than equivalent tantalum foil capacitors and are also lower in cost. They are available in ratings ranging from 0.5 to 600 μf at voltages ranging from 6 to 50 volts.

9-27 Polarized and Nonpolarized Electrolytic Capacitors

Resistance Characteristic of the Dielectric. The dielectric of an electrolytic capacitor, that is, the aluminum, tantalum, or niobium oxide film formed on the anode, has a unidirectional current characteristic. These dielectrics offer a low resistance to the flow of current in one direction and a high resistance to the flow of current in the opposite direction.

Polarized Capacitors. Because of the resistance characteristics of the dielectric materials, the capacitors described in Arts. 9-19 to 9-26 are limited in use to applications where the polarity of the voltage applied to the capacitor terminals is never reversed. These capacitors are classed as being *polarized*, or *polar*, and have their terminals marked in some manner to indicate the positive and negative terminals. Applying a voltage of reversed polarity to this type of electrolytic capacitor will permit a high current to flow and damage the capacitor.

Polar-type tantalum capacitors generally have a relatively low-voltage oxide film on their cathode and can tolerate occasional small random voltage reversals of short duration.

Nonpolarized Capacitors. If the construction of an electrolytic capacitor is modified, it can be safely used in circuit applications where the voltage reverses polarity periodically or where the polarity of a power supply may become reversed and remain so indefinitely. Such capacitors are classed as being *nonpolarized*, or *nonpolar*. The change in construction is simply a matter of forming an equal oxide film on both the anode and cathode plates and then polarizing their dielectrics in opposite directions.

Although nonpolar capacitors are designed mainly for d-c applications, they may also be made for such intermittent a-c applications as starting capacitors for certain types of single-phase motors. Nonpolar capacitors are larger in size and higher in cost than polar capacitors of equal capacitance and voltage ratings.

9-28 Energy-storage Capacitors

Purpose. One of the characteristics of a capacitor is its ability to store a charge of electrical energy. For most radio, television, and other electronic applications this characteristic is not of prime importance, although it does indirectly aid in fulfilling the primary objective. There are, however, some applications, such as flash photography, that depend primarily on the energy-storage ability of a capacitor.

Principle. The basic principle of energy-storage-capacitor applications is that a charged capacitor can, under controlled conditions, deliver a large amount of energy to a desired load for a short period of time. Generally, the time involved is measured in microseconds or millimicroseconds and the current may be in the hundreds, thousands, or even in the millions of amperes. The amount of stored energy is expressed by

$$\mathrm{En} = \frac{CE^2}{2} \tag{9-26}$$

$$\mathrm{En} = \frac{QE}{2} \tag{9-27}$$

where En = energy, watt-sec or joules (1 watt-sec = 1 joule)
C = capacitance, farads
E = potential to which capacitor is charged, volts
Q = charge, coulombs

Capacitor Construction. Energy-storage capacitors are designed for short-time application of voltage and very high current discharge, and therefore special considerations are given to their construction. The quality of the dielectric is very important in determining the amount of charge the capacitor will take and the length of time that the capacitor will retain the full charge. The self-inductance of the capacitor must be kept very low, as it is one of the factors that limits the speed of the discharge. A stronger container is needed to withstand the greater forces produced during discharge.

Ratings. Because the usual mode of discharge of an energy-storage capacitor produces an oscillatory current, the peak value of the inverse voltage on the first reversal is an important consideration in the rating and the design of the capacitor. Energy-storage capacitors are often rated for 90 per cent voltage reversal. The life of the capacitor is affected by the number of times it is discharged, which therefore often becomes one of the ratings of the capacitor; this rating may be for 1,000, 5,000, or several million times. High values of capacitance and voltage are required, and often banks of parallel-connected units are used to obtain the desired capacitance rating. The rating of the energy-storage-type capacitor may include (1) voltage, (2) capacitance, (3) watt-seconds, (4) self-inductance, (5) reverse peak voltage, (6) number of discharges in the life of the capacitor.

Both solid-dielectric- and electrolytic-type capacitors may be used, the choice depending upon the requirements of the particular application. Electrolytic capacitors can provide high capacitance values, often in the tens of thousands of microfarads, and their relatively small size is an important advantage in small, lightweight portable equipment.

Typical Applications. Among the varied applications of energy-storage capacitors are (1) flash photography, (2) welding, (3) power supplies, (4) flasher signaling, (5) generating intense magnetic fields, (6) pulsing circuits, (7) computers, (8) plasma generators, (9) ballistic accelerators, (10) high-temperature research, (11) metal forming, (12) thermonuclear experimentation. One type of photoflash unit uses a capacitor rated at 525 μf, 450 w.v.d-c, and 53 watt-sec. A certain plasma research project uses a capacitor rated at 480 μf and 10,000 joules; the capacitor is composed of a bank of forty 12-μf units connected in parallel. A research project used to generate shock waves in an air tunnel uses a bank of 2,000 capacitors to obtain 390,000 μf at 20,000 volts. The capacitor bank provides a 5,000,000-amp arc that is used to produce the shock wave.

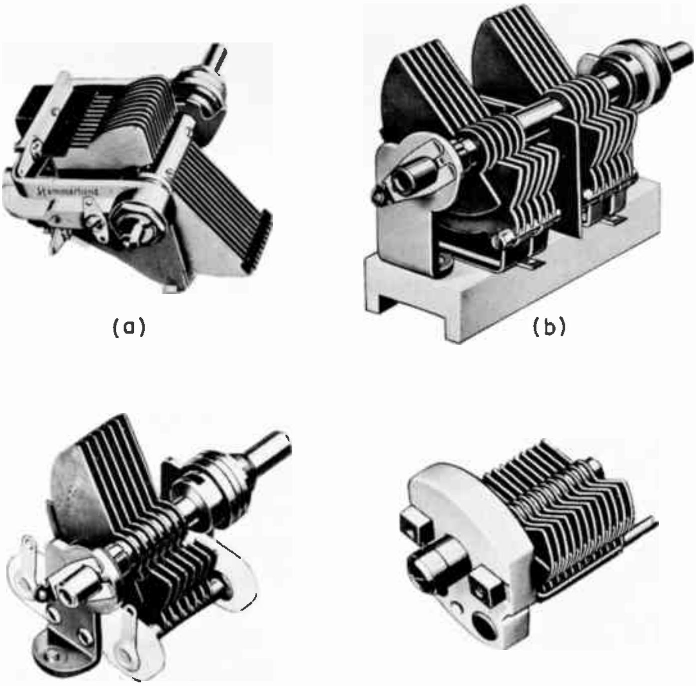
9-29 Variable and Adjustable Capacitors

Construction of Variable Capacitors. In some circuits, it is necessary to vary or adjust the capacitance in order to meet the operating requirements of that circuit; variable or adjustable capacitors fulfill these requirements. A variable capacitor has two sets of plates, a rotating set called the *rotor* and a stationary set called the *stator* (Fig. 9-17); the stator generally has one more plate than the rotor. The capacitor is so constructed that the rotor plates will move freely between the stator plates, thus causing the capacitance to be varied.

The plates of a variable capacitor should be (1) a good conductor, (2) noncorrosive, (3) strong and rigid in order to maintain a uniform capacitance and to prevent possible short circuits between plates. Capacitors are generally made with aluminum plates or with silver-, nickel-, or cadmium-plated brass plates. The thickness of the capacitor plates has no appreciable effect on the capacitance; however, in order to reduce the skin effect and eddy-current losses the plates are made as thin as is practical.

As the capacitance will vary inversely with the distance between the rotor plates and their adjacent stator plates, it is desirable to make this distance as small as practicable, usually in the order of 0.025 inch.

Ranges of Variable Capacitors. For a-m radio broadcast receivers, the range of the type of tuning capacitor most commonly used is in the order



(a) (b)

(c) (d)

Fig. 9-17 Commercial types of variable capacitors. (a) Standard-size capacitor. (b) Midget split-stator capacitor. (c) Midget single-unit capacitor. (d) Micro capacitor. (*Hammarlund Manufacturing Company, Inc.*)

of 10 to 365 pf. For short-wave reception, the maximum capacitance is generally less than 150 pf. For high frequencies and ultrahigh frequencies, correspondingly lower values of capacitance are required. Variable capacitors are available in three basic sizes called *standard, midget,* and *micro,* differing chiefly in the size and surface area of their plates. The general range of variable capacitors is from 5 to 500 pf.

Transmitting Capacitors. In radio and television transmitting equipment, the voltage between the plates of a capacitor is very high and double spacing must be used. Thus, the spacing between adjacent rotor and stator plates will be in the order of 0.05 inch. In other respects, the construction is similar to that of any other variable capacitor.

Split-stator Capacitors. In certain capacitor applications, it is required that the capacitance of several circuits be kept perfectly balanced. In order to fulfill this requirement, a split-stator capacitor is used. For this type of capacitor, the stator is separated into two equal parts, each half being electrically insulated from the other.

When the rotor of a variable capacitor has a large number of semicircular-shaped plates, the full weight of the rotor will be located on one side. This concentration of weight will tend to move the rotor out of the position to which it had been adjusted. To overcome this difficulty, the rotor is balanced mechanically by having half of the stator and rotor plates mounted 180° mechanically from each other.

Multiple or Gang Capacitors. Radio receivers generally contain more than one stage of tuning, and a separate variable capacitor is used for each stage. In order to tune all the circuits simultaneously, all the sets of rotating plates are mounted on a common shaft. These capacitors may have two or more units and are called *multiple* or *gang capacitors* (Fig. 9-18).

Calculation of Capacitance. The capacitance of a variable capacitor can be calculated by using Eq. (9-2*a*). The area is that of the rotor plates when fully in mesh with the stator. When the rotor plates are completely in mesh with the stator, maximum capacitance is obtained. In variable capacitors, the dielectric is air and K is equal to 1.

EXAMPLE 9-18 What is the maximum capacitance of a 15-plate variable capacitor if each rotor plate has an area of 2.75 sq in. and each air gap is 0.025 inch?

GIVEN: $A = 2.75$ sq in. $N = 15$ plates $K = 1$ $t = 0.025$ inch

FIND: C

SOLUTION:

$$C = \frac{22.45 \times K \times A \times (N-1)}{10^8 t} = \frac{22.45 \times 1 \times 2.75 \times 14}{10^8 \times 25 \times 10^{-3}} = 0.000346\ \mu\text{f} = 346\ \text{pf}$$

Adjustable Capacitors. Adjustable capacitors, sometimes referred to as *trimmers* or *padders,* are used to adjust the capacitance in a circuit, balance

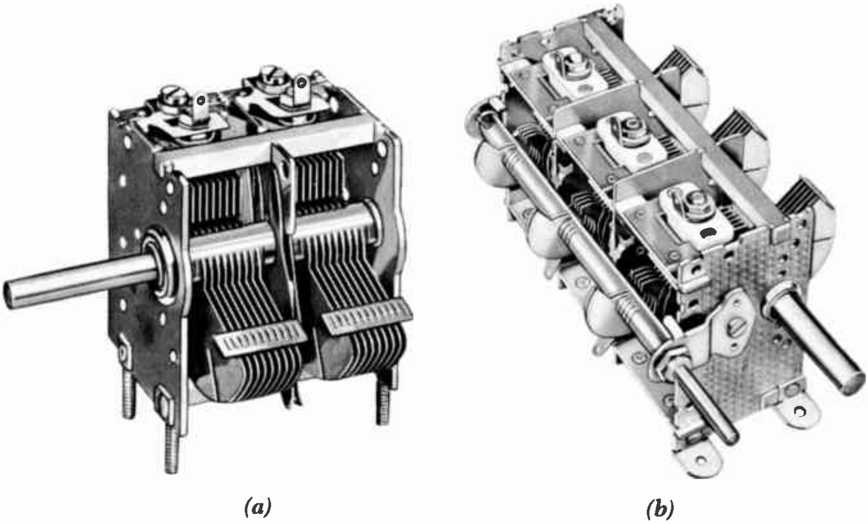

(a) (b)

Fig. 9-18 Commercial types of multiple, or gang, capacitors. (a) Two-gang capacitor. (b) Three-gang capacitor. (Meissner)

the capacitance between two or more tuning circuits, or neutralize a circuit. This type of capacitor consists of two plates insulated from each other by means of a mica sheet and constructed so that the distance separating the two plates can be varied by adjusting a small setscrew (Fig. 9-19). Trimmer and padder capacitors are rated as to their minimum and maximum capacitances. Adjustable capacitors can be obtained with a minimum capacitance as low as 0.5 pf and with maximum capacitances up to 500 pf.

All variable and adjustable capacitors have a definite amount of minimum capacitance. This is ordinarily very small in relation to the maximum value and often may be ignored. Capacitors used for trimmers and padders generally have a very low maximum capacitance value; therefore, in order to obtain adjustments over a wide range, it is essential that the minimum capacitance value be as low as possible.

9-30 Capacitors in Series

Current and Voltage Distribution. As in any series circuit, the magnitude of the current in the line and the current at each circuit element is the same. The voltage at each element will vary in proportion to its impedance when connected to an a-c power source and in proportion to its resistance when connected to a d-c power source. When two 0.5-μf. 150-volt capacitors are connected in series across a 300-volt 60-cycle power source, each capacitor will have approximately 150 volts at its terminals, as the voltage distribution is dependent mainly upon their individual capacitive reactances. When two capacitors of equal capacitance are connected to a d-c power source, the voltage distribution may vary considerably because the d-c resistance of capacitors varies considerably.

EXAMPLE 9-19 A 0.5-μf 150-volt paper capacitor having an insulation resistance of 1,500 megohms is connected in series with a 0.5-μf 150-volt metallized capacitor having an insulation resistance of 500 megohms. What is the distribution of voltage if these capacitors are connected to a 300-volt d-c power source?

GIVEN: $R_1 = 1{,}500$ megohms $R_2 = 500$ megohms $E = 300$ volts

FIND: E_1 E_2

SOLUTION:

$$E_1 = \frac{ER_1}{R_1 + R_2} = \frac{300 \times 1{,}500}{1{,}500 + 500} = 225 \text{ volts}$$

$$E_2 = \frac{ER_2}{R_1 + R_2} = \frac{300 \times 500}{1{,}500 + 500} = 75 \text{ volts}$$

When two or more capacitors are connected in series, the voltage across any one capacitor should not be permitted to exceed its rated voltage. The results of Example 9-19 indicate that these two capacitors do not make a satisfactory combination in terms of the voltage distribution. When a 1-megohm resistor is connected across each capacitor, satisfactory voltage distribution can be obtained.

Fig. 9-19 Commercial types of adjustable capacitors. (Hammarlund Manufacturing Company, Inc.)

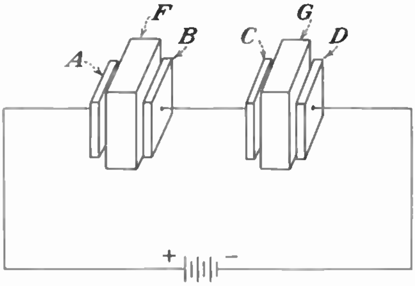


Fig. 9-20 Effect of two capacitors connected in series.

Capacitance of a Series-connected Group. When two capacitors are connected in series, the combined capacitance will be lower than either individual capacitance. This is true because connecting them in series has the same effect as increasing the thickness of the dielectric. If two capacitors are connected in series as in Fig. 9-20, the electrons from plate A are transferred through the external circuit to plate D and the orbital electrons of both dielectrics F and G are under strain. The effect is therefore the same as one dielectric of a thickness equal to the sum of F and G.

The combined capacitance of capacitors connected in series may be found by

$$C_T = \frac{1}{\frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}, \text{ etc.}} \tag{9-28}$$

When only two capacitors are connected in series, Eq. (9-28) may be simplified, as

$$C_T = \frac{C_1 C_2}{C_1 + C_2} \tag{9-28a}$$

and

$$C_1 = \frac{C_T C_2}{C_2 - C_T} \tag{9-28b}$$

Capacitive Reactance of a Series-connected Group. The combined capacitive reactance of capacitors connected in series may be found by

$$X_{C\cdot T} = \frac{10^6}{2\pi f C_T} \tag{9-29}$$

or

$$X_{C\cdot T} = X_{C\cdot 1} + X_{C\cdot 2} + X_{C\cdot 3}, \text{ etc.} \tag{9-30}$$

EXAMPLE 9-20 Two 10-μf and two 5-μf capacitors are connected in series across a 110-volt 60-cycle power source. Find (*a*) the capacitance of the group, (*b*) the capacitive reactance of each capacitor, (*c*) the capacitive reactance of the group, (*d*) the current at each capacitor, (*e*) the voltage at each capacitor.

GIVEN: $C_1, C_2 = 10\ \mu f$ $C_3, C_4 = 5\ \mu f$ $E = 110$ volts $f = 60$ cycles

FIND: (*a*) C_T (*b*) X_C at each capacitor (*c*) $X_{C\cdot T}$ (*d*) I at each capacitor
(*e*) E at each capacitor

SOLUTION:

(*a*)
$$C_T = \frac{1}{\frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} + \frac{1}{C_4}} = \frac{1}{\frac{1}{10} + \frac{1}{10} + \frac{1}{5} + \frac{1}{5}} = 1.67\ \mu f$$

(*b*)
$$X_{C\cdot 1} = X_{C\cdot 2} = \frac{10^6}{2\pi f C} = \frac{10^6}{6.28 \times 60 \times 10} = 265 \text{ ohms}$$

$$X_{C\cdot 3} = X_{C\cdot 4} = \frac{10^6}{2\pi f C} = \frac{10^6}{6.28 \times 60 \times 5} = 530 \text{ ohms}$$

(*c*)
$$X_{C\cdot T} = \frac{10^6}{2\pi f C_T} = \frac{10^6}{6.28 \times 60 \times 1.67} = 1{,}590 \text{ ohms}$$

also, $X_{C\cdot T} = X_{C\cdot 1} + X_{C\cdot 2} + X_{C\cdot 3} + X_{C\cdot 4} = 265 + 265 + 530 + 530 = 1{,}590$ ohms

(*d*) In a series circuit, the current at each element is the same as the line current:

$$I_{C\cdot 1} = I_{C\cdot 2} = I_{C\cdot 3} = I_{C\cdot 4} = I_L = \frac{E}{X_{C\cdot T}} = \frac{110}{1{,}590} = 0.069 \text{ amp}$$

(*e*)
$$E_{C\cdot 1} = I_{C\cdot 1} \times X_{C\cdot 1} = 0.069 \times 265 = 18.3 \text{ volts}$$
$$E_{C\cdot 2} = I_{C\cdot 2} \times X_{C\cdot 2} = 0.069 \times 265 = 18.3 \text{ volts}$$
$$E_{C\cdot 3} = I_{C\cdot 3} \times X_{C\cdot 3} = 0.069 \times 530 = 36.7 \text{ volts}$$
$$E_{C\cdot 4} = I_{C\cdot 4} \times X_{C\cdot 4} = 0.069 \times 530 = 36.7 \text{ volts}$$

Charge on a Series-connected Group. When two or more capacitors are connected in series, the charge on each capacitor is the same and the charge on the group is also the same; thus

$$Q_T = Q_1 = Q_2 = Q_3, \text{ etc.} \qquad (9\text{-}31)$$

EXAMPLE 9-21 (*a*) What is the charge in coulombs on the capacitors of Example 9-20? (*b*) Determine the voltage distribution among the capacitors by means of their charges and capacitances.

GIVEN: Example 9-20

FIND: (*a*) Q_T, Q_1, Q_2, Q_3, Q_4 (*b*) E_1, E_2, E_3, E_4

SOLUTION:

(*a*)
$$Q_T = C_T E_{\text{line}} = 1.67 \times 10^{-6} \times 110 = 183.7 \times 10^{-6} \text{ coulomb}$$
$$Q_1 = Q_2 = Q_3 = Q_4 = Q_T = 183.7 \times 10^{-6} \text{ coulomb}$$

(*b*)
$$E_1 = \frac{Q_1}{C_1} = \frac{183.7 \times 10^{-6}}{10 \times 10^{-6}} = 18.37 \text{ volts}$$
$$E_2 = \frac{Q_2}{C_2} = \frac{183.7 \times 10^{-6}}{10 \times 10^{-6}} = 18.37 \text{ volts}$$
$$E_3 = \frac{Q_3}{C_3} = \frac{183.7 \times 10^{-6}}{5 \times 10^{-6}} = 36.74 \text{ volts}$$
$$E_4 = \frac{Q_4}{C_4} = \frac{183.7 \times 10^{-6}}{5 \times 10^{-6}} = 36.74 \text{ volts}$$

9-31 Capacitors in Parallel

Current and Voltage Distribution. As in any parallel circuit, the voltage at each individual element in the circuit is the same as the line voltage. The voltage rating of each capacitor must therefore be sufficiently high to withstand the full line voltage. The current at each element will vary in proportion to its impedance when connected to an a-c power source and in proportion to its resistance when connected to a d-c power source. When several capacitors are connected in parallel, the line current divides among them in proportion to their capacitances. Conversely, the line current is equal to the sum of the individual currents; this is true if all the capacitors have about the same value of Q or of dissipation factor.

Capacitance of a Parallel-connected Group. When several capacitors are connected in parallel, the combined capacitance will be greater than the highest individual capacitance. This is true because connecting them in parallel has the same effect as increasing the area of the plates. If two capacitors are connected in parallel as in Fig. 9-21, the electrons from plates A and C are transferred through the external circuit to plates B and D. The effect is therefore the same as one capacitor whose plate area is equal to the total area of the plates of both capacitors.

The combined capacitance of capacitors connected in parallel may be found by

$$C_T = C_1 + C_2 + C_3, \text{ etc.} \tag{9-32}$$

Capacitive Reactance of a Parallel-connected Group. The combined capacitive reactance of capacitors connected in parallel may be found by

$$X_{C \cdot T} = \frac{10^6}{2\pi f C_T} \tag{9-29}$$

or

$$X_{C \cdot T} = \frac{1}{\dfrac{1}{X_{C \cdot 1}} + \dfrac{1}{X_{C \cdot 2}} + \dfrac{1}{X_{C \cdot 3}}, \text{ etc.}} \tag{9-33}$$

EXAMPLE 9-22 Two 10-μf and two 5-μf capacitors are connected in parallel across a 110-volt 60-cycle power source. Find (*a*) the capacitance of the group, (*b*) the capacitive reactance of each capacitor, (*c*) the capacitive reactance of the group, (*d*) the voltage at each capacitor, (*e*) the current at each capacitor, (*f*) the line current.

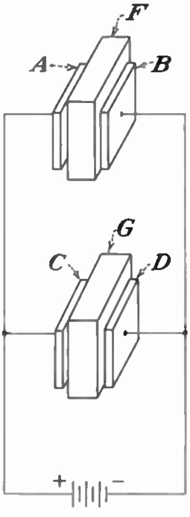


Fig. 9-21 Effect of two capacitors connected in parallel.

GIVEN: $C_1 = 10\ \mu f$ $C_2 = 10\ \mu f$ $C_3 = 5\ \mu f$ $C_4 = 5\ \mu f$ $E = 110$ volts
$f = 60$ cycles

FIND: (*a*) C_T (*b*) X_C at each capacitor (*c*) $X_{C\cdot T}$ (*d*) E at each capacitor
(*e*) I at each capacitor (*f*) I_{line}

SOLUTION:

(*a*)
$$C_T = C_1 + C_2 + C_3 + C_4 = 10 + 10 + 5 + 5 = 30\ \mu f$$

(*b*)
$$X_{C\cdot 1} = X_{C\cdot 2} = \frac{10^6}{2\pi fC} = \frac{10^6}{6.28 \times 60 \times 10} = 265 \text{ ohms}$$

$$X_{C\cdot 3} = X_{C\cdot 4} = \frac{10^6}{2\pi fC} = \frac{10^6}{6.28 \times 60 \times 5} = 530 \text{ ohms}$$

(*c*)
$$X_{C\cdot T} = \frac{10^6}{2\pi fC} = \frac{10^6}{6.28 \times 60 \times 30} = 88.3 \text{ ohms}$$

(*d*) In a parallel circuit, the voltage at each element is the same as the line voltage:

$$E_{C\cdot 1} = E_{C\cdot 2} = E_{C\cdot 3} = E_{C\cdot 4} = E_{\text{line}} = 110 \text{ volts}$$

(*e*)
$$I_{C\cdot 1} = \frac{E}{X_{C\cdot 1}} = \frac{110}{265} = 0.415 \text{ amp}$$

$$I_{C\cdot 2} = \frac{E}{X_{C\cdot 2}} = \frac{110}{265} = 0.415 \text{ amp}$$

$$I_{C\cdot 3} = \frac{E}{X_{C\cdot 3}} = \frac{110}{530} = 0.2075 \text{ amp}$$

$$I_{C\cdot 4} = \frac{E}{X_{C\cdot 4}} = \frac{110}{530} = 0.2075 \text{ amp}$$

(*f*)
$$I_{\text{line}} = \frac{E}{X_{C\cdot T}} = \frac{110}{88.3} = 1.245 \text{ amp}$$

also

$$I_{\text{line}} = I_{C\cdot 1} + I_{C\cdot 2} + I_{C\cdot 3} + I_{C\cdot 4} = 0.415 + 0.415 + 0.2075 + 0.2075 = 1.245 \text{ amp}$$

Charge on a Parallel-connected Group. When two or more capacitors are connected in parallel, the charge on the group is equal to the sum of the individual charges; thus

$$Q_T = Q_1 + Q_2 + Q_3, \text{ etc.} \tag{9-34}$$

9-32 Distributed Capacitance

Distributed Capacitance of a Coil. Coils used in radio, television, and other electronic applications are designed to have primarily a definite amount of inductance. All coils have some resistance and capacitance, and these factors increase the energy loss of a coil. It is therefore desirable to keep the resistance and capacitance of the coil at a minimum.

As two conductors separated by an insulator form a capacitor, a coil will have capacitances between (1) adjacent turns, (2) turns that are not adjacent, (3) terminal leads, and (4) ground and each turn (Fig. 9-22). The

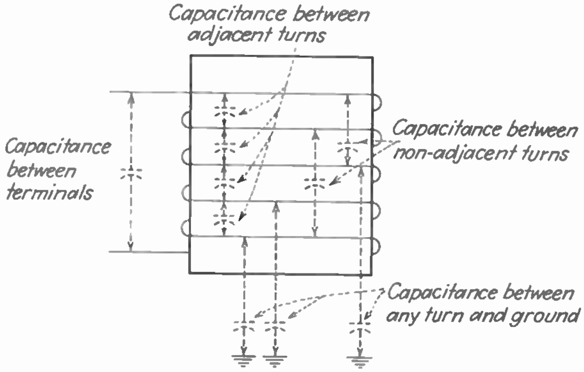


Fig. 9-22 Distributed capacitances of a coil.

amount of energy stored in each of these capacitances is proportional to its capacitance and the square of the voltage existing between the coil turns involved [see Eq. (9-26)]. When a coil is connected to a power source, the full voltage will exist between the two ends of the coil but there will also be a voltage between each adjacent turn or any two turns. For example, if 100 volts is impressed across a coil having 50 turns, the voltage between any two adjacent turns will be 2 volts and between every second turn 4 volts, etc. The amount of voltage between each turn and ground will depend on how the coil is connected in respect to ground. Each turn is separated from its adjacent turn by a small space which may be occupied by the insulation of the wire or air. As the capacitance between two conductors varies inversely with the distance between them, the greatest capacitance will be between two adjacent turns. The total effect produced by the individual capacitances can be represented by a single capacitance of an equivalent value, connected across the terminals of the coil (Fig. 9-23). This equivalent capacitance C_D is called the *distributed capacitance* of the coil.

Effects of Distributed Capacitance. One effect of distributed capacitance is to bypass a portion of the r-f currents. The amount of current that is bypassed varies directly with the frequency. At low frequencies, the effects of distributed capacitance are negligible and can be ignored. At very high frequencies, the distributed capacitance of a coil may become of greater importance than its inductance. The leakage of current through the distributed capacitance causes a loss of energy that is equivalent to a resistance loss, and the effect produced is the same as if the effective resistance of the coil was increased.

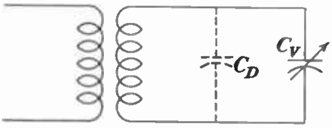


Fig. 9-23 The equivalent distributed capacitance of a coil considered as a capacitor C_D connected in parallel with the coil.

Distributed capacitance also affects tuning circuits. Figure 9-23 shows the distributed capacitance connected in parallel with a variable tuning capacitor, thus increasing the effective capacitance of the circuit. The increase in the effective capacitance decreases the value of frequency to which the circuit becomes tuned. The effect of the distributed capacitance is the same as that of a poor capacitor having a large amount of resistance. The increase in the equivalent resistance of the circuit will decrease the amount of current and increase the energy loss.

Distributed capacitance produces still another effect in that the distributed capacitance and the inductance of a coil form a parallel resonant circuit which will be resonant at a rather high frequency. At this frequency, oscillating currents will circulate in the winding and the distributed capacitances. This effect is explained in detail in Chap. 11.

Methods of Reducing Distributed Capacitance. The distributed capacitance of a coil may be reduced by using one or more of the following methods: (1) increasing the length of the coil, (2) decreasing the coil diameter, (3) using wire whose insulation has a low dielectric constant, (4) decreasing the size of wire, and (5) using a core with a low dielectric constant.

9-33 Measurement of Capacitance

Methods Used. Two basic methods are available for measuring the capacitance of a capacitor, namely, the comparison method and the impedance method.

Fig. 9-24 Capacitor analyzer. (*Sprague Electric Company*)

Comparison Method. The comparison method involves the use of a standard capacitor and some form of bridge circuit. There are a number of commercial bridges available for measuring capacitance, one of which is shown in Fig. 9-24. The use of such an instrument is not difficult, and quite accurate results can be obtained.

Impedance Method. The impedance method involves the taking of voltmeter, ammeter, frequency meter, and wattmeter readings of the capacitor when connected to an a-c power source. The capacitance may then be calculated by use of Eqs. (2-14), (9-4), (9-7), (9-8) as illustrated in the following example.

EXAMPLE 9-23 It is desired to determine the capacitance of a capacitor by the impedance method. The capacitor is connected to a circuit as shown in Fig. 9-25, and the following meter readings were obtained: 110 volts, 0.415 amp, 1.5 watts, and 60 cycles. (*a*) What is the capacitance of the capacitor? (*b*) What is its power factor? (*c*) What is its angle of lead?

GIVEN: $E = 110$ volts $I = 0.415$ amp $P = 1.5$ watts $f = 60$ cycles

FIND: (*a*) C (*b*) PF (*c*) θ

SOLUTION:

(*a*)

$$Z = \frac{E}{I} = \frac{110}{0.415} = 265 \text{ ohms}$$

$$R = \frac{P}{I^2} = \frac{1.5}{0.415 \times 0.415} = 8.72 \text{ ohms}$$

$$X_C = \sqrt{Z^2 - R^2} = \sqrt{265^2 - 8.72^2} = 264.86 \text{ ohms}$$

$$C = \frac{10^6}{2\pi f X_C} = \frac{10^6}{6.28 \times 60 \times 264.86} = 10\ \mu\text{f}$$

(*b*)

$$\text{PF} = \frac{R}{Z} = \frac{8.72}{265} = 0.0329$$

(*c*)

$$\theta = 88°$$

Voltmeter-ammeter Method. A quick yet practical method of determining the capacitance of a capacitor requires the use of only an ammeter and a voltmeter, assuming that the frequency of the power source is 60 cps. This simplified method (Fig. 9-26) does not take into consideration any

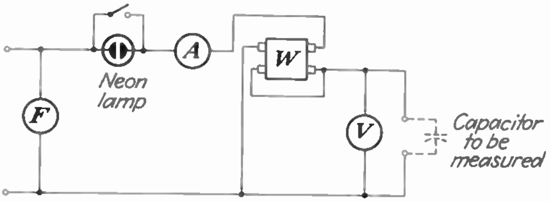


Fig. 9-25

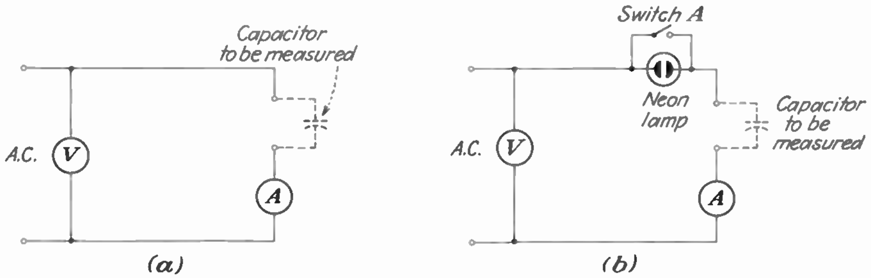


Fig. 9-26 Impedance method of determining capacitance. (a) Voltmeter-ammeter method. (b) Voltmeter-ammeter method including a neon lamp to test for a short-circuited capacitor.

series equivalent resistance that the capacitor may have, and it also assumes that the impedance of the ammeter is zero. For these conditions, Eq. (9-6) may be transposed to show that

$$C = \frac{10^6 I}{2\pi f E} \tag{9-35}$$

The error involved due to ignoring the resistance of the capacitor and the impedance of the ammeter is generally very small, and therefore this method can be used for a quick check on the capacitance of a capacitor.

When capacitors are being tested, care should be taken to determine whether the capacitor is short-circuited before connecting it in series with the ammeter and the power source. A short-circuited capacitor connected to a circuit similar to Fig. 9-26*a* would probably cause the ammeter to be damaged. A test for a short-circuited capacitor can be included by connecting a neon lamp and a single-pole switch in the circuit, as shown in Fig. 9-26*b*. The switch *A* is left open, and if the neon lamp lights, the capacitor is shorted and no further tests should be made. If the lamp does not light, switch *A* may be closed and the capacitor may then be tested for its capacitance.

EXAMPLE 9-24 It is desired to determine the capacitance of a capacitor using the voltmeter-ammeter method and the circuit shown in Fig. 9-26. The meter readings are 3 volts and 9 ma. What is the capacitance if the frequency is 60 cycles?

GIVEN: $E = 3$ volts $I = 9$ ma $f = 60$ cycles

FIND: C

SOLUTION:

$$C = \frac{10^6 I}{2\pi f E} = \frac{10^6 \times 0.009}{6.28 \times 60 \times 3} = 7.96\ \mu\text{f}$$

Testing Electrolytic Capacitors. The impedance methods just described can be used to obtain an approximate value of the capacitance of an electrolytic capacitor. However, it is preferable to test electrolytic capacitors by using an alternating voltage of either 60 or 120 cycles, not to be in excess of the maximum rated ripple voltage, plus a direct polarizing voltage equal to the rated operating voltage. Such a circuit is shown in Fig. 9-27. The capacitor C_1 is used to bypass the d-c power supply and should be as large as possible. The capacitor C_2 should be of such a value as to provide a reactance which is small compared with the resistance of the voltmeter. Readings should be taken on all three meters simultaneously. The capacitance, equivalent series resistance, and power factor may then be found by

$$C_x = \frac{10^6}{2\pi f \sqrt{\left(\frac{E}{I}\right)^2 - \left(\frac{P}{I^2}\right)^2}} \tag{9-36}$$

$$R_x = \frac{P}{I^2} \tag{2-14}$$

$$\mathrm{PF} = \frac{P}{E \times I} \tag{7-19}$$

The simplified formula for capacitance [Eq. (9-35)] illustrated in Example 9-24 can be used if neglecting the equivalent series resistance is permissible.

9-34 Uses of Capacitors

Basic Uses. From a practical standpoint, a capacitor has the ability to store electrical energy in the form of an electrostatic charge. The capacitor can be connected in a circuit so that this stored energy can be made to flow in a desired circuit in such a manner that it will perform any one of a number of specific functions. Eight important basic functions that can be performed by a capacitor are as follows:

1. A capacitor can store a charge of electrical energy and release it as desired.

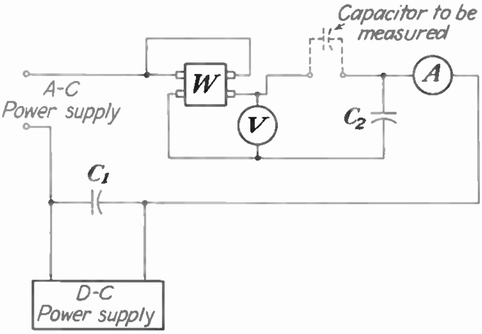


Fig. 9-27 Impedance method of determining capacitance of electrolytic capacitors using a wattmeter, a voltmeter, and an ammeter.

2. A capacitor can block the flow of direct current and at the same time permit the flow of alternating current.

3. Because the reactance of a capacitor varies with the frequency, it can be used in a suitable circuit to permit the flow of a high percentage of current at one frequency and practically eliminate the flow of current at another value of frequency.

4. When a capacitor is used in conjunction with an inductor, certain circuits can be made to pass the currents of a relatively narrow band of frequencies and practically eliminate the flow of current for all frequencies above and below this narrow band.

5. When a capacitor is used in conjunction with an inductor, certain circuits can be made to prevent the flow of current for a relatively narrow band of frequencies and permit the flow of current for all frequencies above and below this band of frequencies.

6. When a capacitor is used in conjunction with a combination of inductors, resistors, vacuum tubes, and/or transistors, oscillator circuits can be made to provide alternating currents over a very broad range of frequencies.

7. When a capacitor is used in conjunction with resistors and/or inductors, certain circuits can be used to produce a controlled time delay for action in a desired circuit.

8. When a capacitor is used in conjunction with resistors and/or inductors, certain circuits can be used to control the phase relation between the voltage and current of a circuit.

Radio Circuit Applications. The type, size, and number of capacitors used will depend on the size and type of application. For example, a simple five-tube a-m receiver, a high-fidelity f-m receiver, a more complex a-m/f-m/f-m-stereo receiver, or a broadcast transmitter all have different sizes, types, and numbers of capacitors. Typical applications and the function number (corresponding to the listing in the preceding paragraph) of capacitors in radio circuits are (*a*) coupling capacitors in a-f, i-f, or r-f circuits (2); (*b*) tuning capacitors in r-f, i-f, and oscillator circuits (4); (*c*) bypass capacitors in a-f, i-f, and r-f circuits (3); (*d*) decoupling capacitors (3); (*e*) power-supply filter-circuit capacitors (1).

Television Circuit Applications. Although a television receiver contains a greater number of circuits and uses a greater number of capacitors than a radio receiver, the types of functions performed are similar to those in a radio receiver. These basic functions include tuning, coupling, decoupling, bypassing, and filtering.

Industrial Electronic Circuit Applications. The field of industrial electronics is so vast that all the basic functions will be found in one application or another. Many industrial electronic circuits are quite similar in principle to those used in radio and television applications. Capacitors are used in these circuits to serve the same purposes as in radio and television circuits.

QUESTIONS

1. Define (*a*) capacitance, (*b*) capacitor, (*c*) condenser.
2. (*a*) Describe the action of a simple capacitor, in terms of the electron theory, during its period of charge. (*b*) Explain two important characteristics of capacitor action.
3. Describe the action of a simple capacitor, in terms of the electron theory, during its period of discharge.
4. Describe the action of a simple capacitor when alternating current is applied.
5. (*a*) What factors affect the amount of charge on a capacitor? (*b*) In what unit is the amount of charge expressed?
6. Define (*a*) the basic unit of capacitance, (*b*) mircofarad, (*c*) micromicrofarad, (*d*) picofarad.
7. Name and explain how the various factors affect the capacitance of a capacitor.
8. (*a*) Define capacitive reactance. (*b*) What factors affect its magnitude? (*c*) In what unit is it expressed? (*d*) What is its symbol?
9. (*a*) How does the magnitude of the resistance of a capacitor compare with its reactance? (*b*) With what regard is the resistance of a capacitor frequently treated?
10. (*a*) What is meant by the impedance of a capacitor? (*b*) What factors affect its magnitude? (*c*) In what unit is it expressed? (*d*) What is its symbol?
11. (*a*) To what factors is the power factor of a capacitor proportional? (*b*) What is the relationship between the angle of lead and the power factor of a capacitor?
12. (*a*) What is meant by the time constant of a capacitor? (*b*) What factors affect the time constant and in what manner?
13. (*a*) What is meant by a fixed capacitor? (*b*) Name six types of fixed capacitors. (*c*) Name six characteristics to be considered when selecting a capacitor for a specific application.
14. Describe the following capacitor ratings: (*a*) d-c working voltage, (*b*) a-c working voltage, (*c*) peak voltage, (*d*) surge voltage, (*e*) breakdown voltage.
15. (*a*) What factors affect the voltage rating of a capacitor? (*b*) What is meant by voltage derating?
16. Describe the following capacitor terms: (*a*) dielectric leakage loss, (*b*) dielectric absorption loss, (*c*) resistance loss, (*d*) combined loss.
17. (*a*) What is meant by the equivalent series resistance of a capacitor? (*b*) What does it represent? (*c*) How can its magnitude be determined?
18. Describe the following capacitor ratings: (*a*) tolerance, (*b*) operating-temperature range, (*c*) capacitance-temperature stability, (*d*) temperature coefficient.
19. Describe the following capacitor ratings: (*a*) power factor, (*b*) dissipation factor, (*c*) *Q*, (*d*) insulation resistance.
20. Describe the following capacitor characteristics: (*a*) direct leakage current, (*b*) normal leakage current, (*c*) initial leakage current.
21. (*a*) What are the advantages of mica capacitors? (*b*) What range of capacitance values is generally available? (*c*) For what types of circuit applications are they adaptable?
22. (*a*) What are the advantages of silvered-mica capacitors? (*b*) How do they differ in construction from the plain mica capacitors?
23. Explain the significance of the three body colors used to identify molded mica capacitors.

24. (*a*) Describe the construction of the paper-dielectric capacitor. (*b*) How is its inductance kept at a reasonably low value? (*c*) What are its advantages? (*d*) What range of capacitance and voltage ratings is generally available?
25. (*a*) Name three general types of synthetic-film materials used with capacitors. (*b*) Name two specific types of polyester dielectrics. (*c*) What two advantages of synthetic-film capacitors are of outstanding importance? (*d*) Name five additional advantages.
26. (*a*) What is a dual-dielectric capacitor? (*b*) What are its advantages?
27. (*a*) What are the construction features of the metallized capacitor? (*b*) Describe its self-healing ability. (*c*) What are its other advantages?
28. (*a*) Describe the general construction features of the ceramic capacitor. (*b*) What are its advantages? (*c*) What range of ratings is available?
29. What is the outstanding characteristic of the variable-voltage ceramic capacitor?
30. (*a*) Describe the chief purpose of the temperature-compensating capacitor. What is meant by (*b*) ppm? (*c*) NP0? (*d*) P100? (*e*) N750?
31. (*a*) What is the construction feature of the oil-dielectric capacitor? (*b*) What are its chief applications?
32. What are the advantages of glass-dielectric capacitors?
33. Describe two methods used in marking a capacitor to indicate its ratings.
34. (*a*) Explain the principle of the electrolytic capacitor. (*b*) What factors affect its capacitance? What constitutes (*c*) the dielectric? (*d*) The negative electrode? (*e*) The positive electrode?
35. (*a*) What is meant by the forming process in the construction of an electrolytic capacitor? How does the forming process affect (*b*) the thickness of the dielectric? (*c*) The capacitance of the unit? (*d*) The voltage rating?
36. Describe the self-healing characteristic of the electrolytic capacitor.
37. Describe the construction of (*a*) an aluminum electrolytic capacitor, (*b*) the electrolytic multiple-capacitor block.
38. (*a*) Name three basic types of tantalum capacitors. (*b*) What are the advantages of tantalum capacitors?
39. Describe the construction features of the following types of tantalum capacitors: (*a*) foil, (*b*) wet anode, (*c*) solid.
40. Compare the construction and characteristics of the niobium capacitor with those of the tantalum capacitor.
41. (*a*) What is meant by a polarized capacitor? (*b*) What is meant by a nonpolarized capacitor? (*c*) How do they differ in construction?
42. (*a*) What is the principle of energy-storage-capacitor applications? (*b*) How do the ratings of these capacitors differ from those of the general-purpose capacitors? (*c*) Name some applications of energy-storage capacitors.
43. (*a*) What are the general requirements of variable-capacitor applications? (*b*) Describe the constructional features of the variable capacitor.
44. Name three classifications of variable capacitors in terms of (*a*) the frequency range of their applications, (*b*) their physical size. (*c*) What is the general capacitance range of variable capacitors?
45. How does a variable capacitor used in a transmitter differ from those used in receiving equipment?
46. (*a*) What is meant by a split-stator capacitor? (*b*) Why is this type of capacitor used?
47. (*a*) What is meant by a multiple capacitor? (*b*) What is its advantage?

48. (*a*) Describe the construction of adjustable capacitors. (*b*) Where are they used?
49. (*a*) How does the capacitance of a series-connected group of capacitors compare with the individual capacitances in the group? (*b*) How does the capacitive reactance of a series-connected group of capacitors compare with the individual reactances in the group? (*c*) How does the charge of a series-connected group of capacitors compare with the charges of the individual capacitors?
50. (*a*) Describe the voltage distribution in a series-connected group of capacitors. (*b*) Describe the current distribution in a series-connected group of capacitors.
51. (*a*) How does the capacitance of a parallel-connected group of capacitors compare with the individual capacitances in the group? (*b*) How does the capacitive reactance of a parallel-connected group of capacitors compare with the individual reactances in the group? (*c*) How does the charge of a parallel-connected group of capacitors compare with the charges of the individual capacitors?
52. (*a*) Describe the voltage distribution in a parallel-connected group of capacitors. (*b*) Describe the current distribution in a parallel-connected group of capacitors.
53. (*a*) What is meant by the distributed capacitance of a coil? (*b*) What effects are produced by the distributed capacitance of a coil? (*c*) How may the distributed capacitance of a coil be reduced?
54. (*a*) Name two methods of determining the capacitance of a capacitor. (*b*) Describe each method.
55. What are the assumptions on which the voltmeter-ammeter method of determining capacitance is based?
56. Why is it necessary to use a direct polarizing voltage in addition to the alternating voltage in determining the capacitance of electrolytic capacitors?
57. Name eight basic functions that can be performed by a capacitor.
58. Name five or more applications of capacitors in radio circuits.
59. Name five or more applications of capacitors in television circuits.
60. Name five or more applications of capacitors in industrial electronic circuits.

PROBLEMS

1. (*a*) How much charge will a 0.5-μf capacitor attain when connected to a 50-volt d-c power source? (*b*) What is the average charging current if the capacitor becomes charged in 0.0005 sec?
2. (*a*) How much charge is placed on a capacitor by an average current of 2 amp flowing for 0.003 sec? (*b*) What is the capacitance if a 150-volt charge is attained?
3. (*a*) What is the capacitance of a capacitor made of two plates of aluminum foil, each 1 inch square, separated by a piece of mica 0.01 inch thick? (*b*) What is the capacitance if paraffined paper is substituted for the mica?
4. If it is desired to double the capacitance of the capacitor of Prob. 3*a*, find (*a*) the length of the plates if their width is kept at 1 inch, (*b*) the thickness of the dielectric if the plates are to remain 1 inch square.
5. What is the capacitance of a capacitor made of 25 plates of aluminum foil, each ½ by 1 inch, separated by layers of mica 0.01 inch thick?
6. What is the capacitance of a capacitor made of 76 plates of aluminum foil, each 1½ by 2 inches, separated by paraffined paper 0.005 inch thick?
7. How many plates must be used to make a capacitor of 0.0125 μf if they are to be 2 inches square and the dielectric is to be mica sheets 0.012 inch thick?

8. How many plates must be used to make a capacitor of 500 pf if they are to be ¾ inch square and the dielectric is to be mica sheets 0.007 inch thick?
9. What is the capacitance of a capacitor consisting of two plates, each 1 inch wide and 81 inches long, separated by sheets of paraffined paper 0.005 inch thick?
10. It is desired to construct a capacitor with two aluminum-foil sheets 2 inches wide and separated by paraffined paper 0.001 inch thick. The capacitor is to be rolled into a compact cylindrical form. What is the length of each aluminum-foil plate if the capacitance is to be 0.25 μf?
11. Find the reactance of a 0.1-μf capacitor when it is connected in a circuit whose frequency is (*a*) 60 cycles, (*b*) 1 kc, (*c*) 500 kc, (*d*) 2 mc.
12. Find the reactance of a 100-pf capacitor to a frequency of (*a*) 50 cycles, (*b*) 50 kc, (*c*) 1 mc, (*d*) 500 mc.
13. How much current will a 1,000-μμf capacitor take from a 12-volt power source if the frequency is (*a*) 60 cycles, (*b*) 5 kc, (*c*) 1 mc, (*d*) 60 mc?
14. How much current will a 50-pf capacitor take from a 5-volt power source if its frequency is (*a*) 100 kc, (*b*) 20 mc, (*c*) 100 mc, (*d*) 1,000 mc?
15. An 8-μf capacitor that has a resistance of 10 ohms is connected to a 110-volt 60-cycle power source. Find (*a*) the capacitive reactance, (*b*) the impedance, (*c*) the current, (*d*) the power factor, (*e*) the phase angle.
16. A 20-μf capacitor that has an equivalent series resistance of 8 ohms is connected to a 300-volt 60-cycle power source. Find (*a*) the capacitive reactance, (*b*) the impedance, (*c*) the current, (*d*) the power factor, (*e*) the phase angle.
17. (*a*) What is the time constant of a circuit that has a 25-μf capacitor connected across a 270-ohm resistor? (*b*) How long would it take for the capacitor to discharge practically completely through the resistor?
18. What value of resistance must be used with a 0.0022-μf capacitor in order to produce a time constant corresponding to the period of a 5-kc signal?
19. A certain 40-μf capacitor has a 3,000-ohm resistance connected in series with it while being charged from a 300-volt d-c power source, and it is then discharged through a circuit having a resistance of 1,000 ohms. Find (*a*) the time constant of the charging circuit, (*b*) the time required for full charge, (*c*) the voltage at the capacitor after 0.06 sec of charging time, (*d*) the current at the start of charging from zero volts, (*e*) the current after 0.06 sec of charging time, (*f*) the maximum discharge current, (*g*) the voltage after 0.04 sec of discharge from a 300-volt charge, (*h*) the current after 0.04 sec of discharge from a 300-volt charge. (Use Tables 12-1 and 12-2 for values at various portions of the time constant.)
20. What is the per cent dielectric absorption of a capacitor if, after being discharged from 150 volts to zero, it recovers a charge of 2 volts?
21. What is the per cent dielectric absorption of a capacitor if, after being discharged from 450 volts to zero, it recovers a charge of 1 volt?
22. If the dielectric absorption of a capacitor is rated at 0.05 per cent, what is the recovery voltage after it has been discharged from its rated voltage of 1,000 volts?
23. A certain 10-μf 350-volt electrolytic capacitor has a normal direct leakage current of 0.55 ma, its plate and lead resistance is 1 ohm, and the resistance effect of dielectric absorption and the electrolyte is 23 ohms. (*a*) What is the parallel resistance effect of the leakage loss? (*b*) What is the series equivalent of the parallel resistance effect at a reference frequency of 120 cycles? (*c*) What is the series equivalent resistance of the capacitor?

24. A certain 100-μf 150-volt electrolytic capacitor has a normal direct leakage current of 2.3 ma, its plate and lead resistance is 0.2 ohm, and the resistance effect of dielectric absorption and electrolyte is 2.3 ohms. (*a*) What is the parallel resistance effect of the leakage loss? (*b*) What is the series equivalent of the parallel resistance effect at a reference frequency of 120 cycles? (*c*) What is the series equivalent resistance of the capacitor?
25. What is the equivalent series resistance of a 10,000-μf 40-volt electrolytic capacitor that has a dissipation factor of 50 per cent at 120 cycles?
26. What is the equivalent series resistance of a 0.1-μf paper-dielectric capacitor that has a dissipation factor of 1.5 per cent at 1,000 cycles?
27. What is the equivalent series resistance of a 0.1-μf metallized dual-dielectric capacitor that has a dissipation factor of 0.5 per cent at 1,000 cycles?
28. What is the equivalent series resistance of a 5,000-pf mica capacitor that has a dissipation factor of 0.1 per cent at 1 mc?
29. What is the dissipation factor of a 40-μf 150-volt electrolytic capacitor if its equivalent series resistance is 7.5 ohms at a reference frequency of 120 cycles?
30. What is the dissipation factor of a 200-μf 300-volt electrolytic capacitor if its equivalent series resistance is 1.3 ohms at a reference frequency of 120 cycles?
31. A certain capacitor rated at 100 μf and 300 volts has a dissipation factor of 0.15 when measured at 120 cycles. Find (*a*) the equivalent series resistance, (*b*) the capacitive reactance, (*c*) the impedance, (*d*) the power factor.
32. A certain capacitor rated at 5,000 μf and 50 volts has a dissipation factor of 38 per cent at the 120-cycle reference frequency. Find (*a*) the equivalent series resistance, (*b*) the capacitive reactance, (*c*) the impedance, (*d*) the power factor.
33. What is the Q of the capacitor in Prob. 31?
34. What is the Q of the capacitor in Prob. 32?
35. What is the Q of the capacitor in Prob. 15?
36. What is the Q of the capacitor in Prob. 16?
37. (*a*) What is the leakage current of an 80-μf 150-volt aluminum-foil electrolytic capacitor at 25°C if the value of K is 0.005? (*b*) What is the approximate leakage current at 85°C?
38. (*a*) What is the leakage current of a 60-μf 100-volt computer-grade aluminum electrolytic capacitor at 25°C if the value of K is 0.002? (*b*) What is the approximate leakage current at 85°C?
39. What is the leakage current of a 100-μf 15-volt solid electrolyte tantalum capacitor at (*a*) 25°C if the value of K is 0.02? (*b*) 85°C if the value of K is 0.2?
40. What is the leakage current of a 40-μf 25-volt niobium-foil electrolytic capacitor at (*a*) 25°C if the value of K is 0.01? (*b*) 85°C if the value of K is 0.06?
41. What ratings of N750 and NP0 capacitors are required to produce 340 pf with a temperature coefficient of N225?
42. What ratings of N330 and NP0 capacitors are required to produce 280 pf with a temperature coefficient of N150?
43. What is the energy rating of a 2.5-μf 12,000-volt energy-storage capacitor?
44. What is the energy rating of a 40-μf 8-kv energy-storage capacitor?
45. What is the energy rating of a 0.39-f 20-kv bank of energy storage capacitors?
46. What is the voltage rating of a 480-μf capacitor bank if its energy storage rating is 10,000 joules?
47. What is the capacitance of a 27-plate micro variable capacitor if the area of each plate is 0.35 sq in. and the air gap is 0.0205 inch?

48. What is the capacitance of a 14-plate micro variable capacitor if the area of each plate is 0.35 sq in. and the air gap is 0.0205 inch?
49. A certain midget variable capacitor has 43 plates, and its maximum capacitance is 320 pf. The air gap between adjacent plates is 0.025 inch. What is the approximate area of each rotor plate?
50. What is the capacitance of a 19-plate double-spaced midget variable capacitor if the area of each plate is 0.89 sq in. and the air gap is 0.0715 inch?
51. A certain circuit contains two 8-μf and one 4-μf capacitors connected in series across a 250-volt 60-cycle power source. Find (*a*) the capacitance of the group, (*b*) the reactance of each capacitor, (*c*) the reactance of the group, (*d*) the current at each capacitor, (*e*) the voltage at each capacitor.
52. A certain circuit contains a 20-, a 10-, a 10-, and a 4-μf capacitor connected in series across a 250-volt 60-cycle power source. Find (*a*) the capacitance of the group, (*b*) the reactance of each capacitor, (*c*) the reactance of the group, (*d*) the current at each capacitor, (*e*) the voltage at each capacitor.
53. (*a*) What is the charge in coulombs on the capacitors of Prob. 51? (*b*) Determine the voltage distribution among the capacitors by means of their charges and capacitances.
54. (*a*) What is the charge in coulombs on the capacitors of Prob. 52? (*b*) Determine the voltage distribution among the capacitors by means of their charges and capacitances.
55. A certain circuit contains an 8-, a 4-, and a 2-μf capacitor connected in parallel across a 110-volt 60-cycle power source. Find (*a*) the capacitance of the group, (*b*) the reactance of each capacitor, (*c*) the reactance of the group, (*d*) the voltage at each capacitor, (*e*) the current at each capacitor, (*f*) the line current.
56. A certain circuit contains a 20-, a 10-, and a 5-μf capacitor connected in parallel across a 110-volt 60-cycle power source. Find (*a*) the capacitance of the group, (*b*) the reactance of each capacitor, (*c*) the reactance of the group, (*d*) the voltage at each capacitor, (*e*) the current at each capacitor, (*f*) the line current.
57. What value of capacitance must be connected in series with a 10-μf capacitor in order to obtain a capacitance of 6.66 μf?
58. What value of capacitance must be connected in series with a 10-μf capacitor in order to obtain a capacitance of 3.33 μf?
59. What value of capacitance must be connected in parallel with a 10-μf capacitor in order to obtain a capacitance of 15 μf?
60. What is the capacitance of the circuit shown in Fig. 9-28?
61. What is the capacitance of the circuit shown in Fig. 9-29?
62. What is the capacitance of the circuit shown in Fig. 9-30?
63. What is the capacitance of the circuit shown in Fig. 9-31?

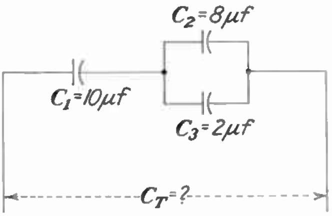


Fig. 9-28

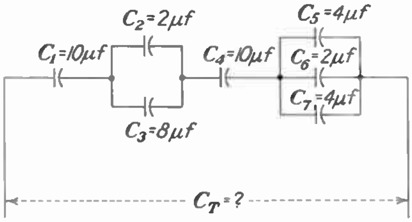


Fig. 9-29

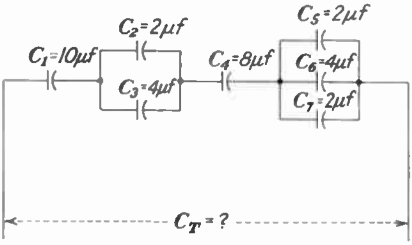


Fig. 9-30

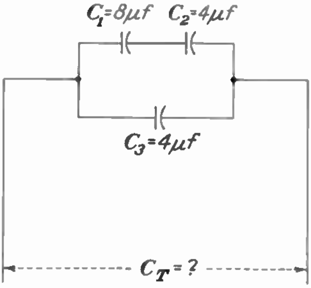


Fig. 9-31

64. It is desired to obtain a capacitance of 5 μf by using a combination of 2-, 4-, and/or 8-μf capacitors. What circuit arrangement will provide 5 μf with a minimum number of capacitors?
65. It is desired to determine the capacitance of a capacitor by the impedance method using the circuit shown in Fig. 9-25. The meter readings obtained are voltmeter 220 volts, ammeter 0.55 amp, wattmeter 10 watts, frequency meter 60 cycles. Find *(a)* the capacitance, *(b)* the power factor, *(c)* the phase angle.
66. It is desired to determine the capacitance of a capacitor with the voltmeter-ammeter method and the circuit shown in Fig. 9-26. The voltmeter indicates 3 volts and the ammeter 23 ma. What is the capacitance if the frequency is 60 cycles?
67. A 0.1-μf capacitor is tested by the method and circuit used in Prob. 66. The line voltage is 100 volts, and the frequency is 60 cycles. What is the ammeter reading?

Chapter 10

Alternating-current Circuits

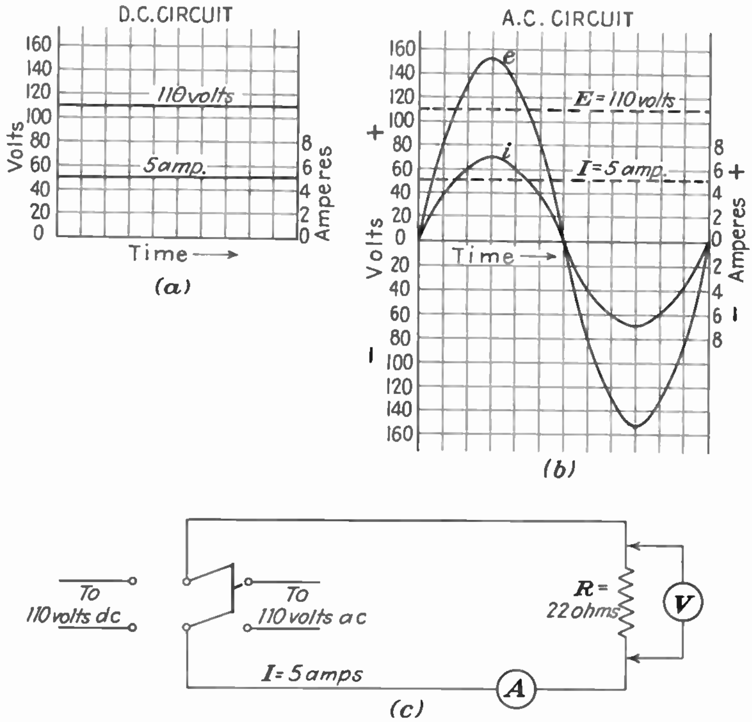
The relation of voltage, current, and power in electric circuits was presented in Chap. 4 on the basis of a continuous or direct current flow. The study of a-c generators and transformers in Chap. 7 has shown that there are advantages of alternating current over direct current. Because over 90 per cent of the electrical power is generated as alternating current, it is necessary to know the circuit characteristics with alternating current flow.

10-1 Circuit Characteristics

D-C Circuits. Because of the difference in the voltage and current characteristics of alternating current and direct current, it is necessary to introduce some new terms in the study of a-c circuits. The difference in the voltage and current characteristics is shown in Fig. 10-1. The voltage and current, as indicated by a voltmeter and an ammeter, are the same for each circuit, namely, 110 volts and 5 amp.

The voltage for the d-c circuit is 110 volts at all instants of time, and the current is 5 amp at all instants of time as indicated in Fig. 10-1*a*. All the opposition to the flow of current is accounted for in the resistance of the circuit, and Ohm's law, $I = E/R$, $E = IR$, and $R = E/I$, applies to all d-c circuits. The power consumed by the d-c circuit may be calculated by $P = EI$, $P = I^2R$, or $P = E^2/R$.

A-C Circuits. The a-c circuit conditions shown in Fig. 10-1*b* indicate that the effective voltage and effective current are the same as for the d-c circuit shown in Fig. 10-1*a*. While this is so, it is also evident that the alternating voltage and current are continually changing in magnitude. It should be noticed that the current goes through its cycle at the same time that the voltage is going through its cycle. They are said to be of the same time phase, or *in phase*. This condition of being in phase is more fully described by the statement: The two waves start from zero and rise together, each reaching its maximum at the same instant; they descend together, go through zero, and alternate in polarity at the same time; they



(a) (b) (c)

Fig. 10-1 Direct current versus alternating current characteristics. (a) Current and voltage when the circuit is connected to a d-c power source. (b) Current and voltage when the circuit is connected to an a-c power source. (c) The circuit diagram.

reach their maximum negative values at the same time; and then they decrease to zero to complete their cycles together.

With the current and voltage in phase, the circuit follows the same laws that apply to the d-c circuit, namely, that $I = E/R$, $E = IR$, $R = E/I$, $P = EI$, $P = I^2R$, and $P = E^2/R$.

The study of inductance and capacitance has shown that it is possible for conditions to exist whereby the current wave may not be in phase with the voltage wave. Such conditions exist in a-c circuits more often than do in-phase currents. To study a-c circuits, it is necessary to account for these conditions. When the current and voltage are not in phase, only one of the laws mentioned above will apply; that is, $P = I^2R$; the others cannot be used.

Reactance. In d-c circuits, the only opposition to the flow of current is resistance. In a-c circuits, two additional factors must be considered, namely, inductive reactance and capacitive reactance. When an a-c circuit contains resistance only, the current and voltage will be in phase. If an a-c circuit contains inductive reactance, capacitive reactance, or both, the current and voltage will be out of phase except in the case of resonance which is described in Chap. 11.

10-2 Effects of Inductive and Capacitive Reactances

Inductive Reactance. The effect of inductive reactance is twofold: (1) It offers an opposition to the flow of current; (2) it causes the current to lag behind the voltage. With a perfect inductor, that is, one having no resistance, the current would lag the voltage by 90 electrical degrees, and no power would be taken from the line because the coil stores up energy in the magnetic field during one half-cycle and returns this energy to the line in the following half-cycle. The current taken by a perfect inductor would be equal to its voltage divided by its inductive reactance, or

$$I_L = \frac{E_L}{X_L} \tag{8-7}$$

Figure 10-2 shows the current and voltage waves for an inductive reactance of 22 ohms connected across a 110-volt a-c power source.

Capacitive Reactance. The effect of capacitive reactance is twofold: (1) It offers an opposition to the flow of current; (2) it causes the current to lead the voltage. With a perfect capacitor, that is, one having no resistance, the current would lead the voltage by 90 electrical degrees, and no power would be taken from the line because the capacitor stores up energy in the electrostatic field during one half-cycle and returns this energy to the line in the following half-cycle. The current taken by a perfect capacitor would be equal to its voltage divided by its capacitive reactance, or

$$I_C = \frac{E_C}{X_C} \tag{9-5}$$

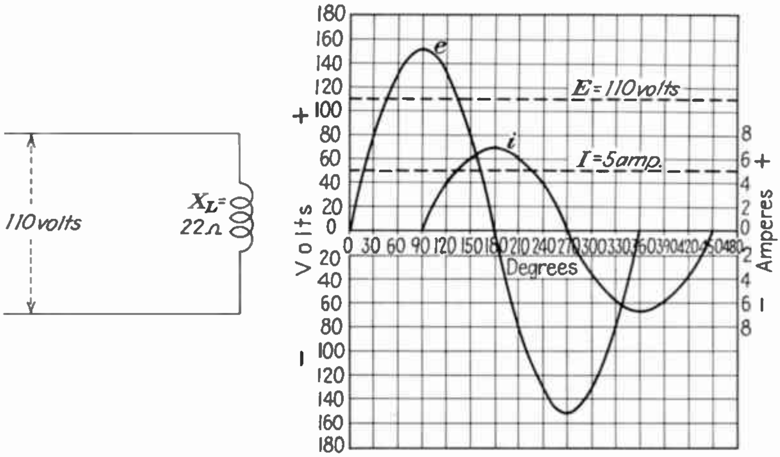


Fig. 10-2 Voltage and current characteristics if a perfect inductor were connected to an a-c power source.

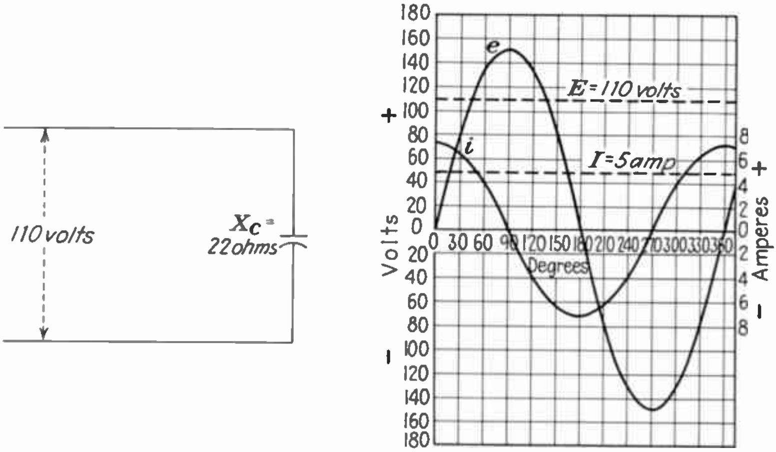


Fig. 10-3 Voltage and current characteristics if a perfect capacitor were connected to an a-c power source.

Figure 10-3 shows the current and voltage waves for a capacitive reactance of 22 ohms connected across a 110-volt a-c power source.

The conditions just described are for perfect inductors and capacitors, which require that they have no resistance. While it is impossible to manufacture such an inductor or capacitor, these conditions are used to explain the effect of each when considered alone.

10-3 Series A-C Circuits Containing Resistance, Inductance, and Capacitance

Impedance. Alternating-current circuits may consist of any combination of resistances, inductive reactances, and capacitive reactances. The combined effect of resistance and reactance is called *impedance*. It, too, is expressed in ohms, and its symbol is Z. When a circuit contains resistances and reactances, it is necessary to combine their ohmic effect by a different method from that used for resistances only.

Calculation of Impedance. The reactance of any circuit will always be 90° out of phase with its resistance. Whether it leads or lags the resistance will depend on whether the reactance is capacitive or inductive (Fig. 10-4). Observation of the diagrams in this figure will show that the resistance, reactance, and impedance form a right-angle triangle. Their relations to one another when connected in series can be expressed by the following equations. The impedance of a circuit containing resistance and inductive reactance is

$$Z = \sqrt{R^2 + X_L^2} \tag{8-8}$$

The impedance of a circuit containing resistance and capacitive reactance is

$$Z = \sqrt{R^2 + X_C^2} \tag{9-7}$$

EXAMPLE 10-1 What is the impedance of a circuit having a resistance of 30 ohms and a capacitive reactance of 40 ohms?

GIVEN: $R = 30$ ohms $\quad X_C = 40$ ohms

FIND: Z

SOLUTION:

$$Z = \sqrt{R^2 + X_C^2} = \sqrt{30^2 + 40^2} = \sqrt{2{,}500} = 50 \text{ ohms}$$

EXAMPLE 10-2 An inductance coil has a resistance of 5 ohms and a reactance of 10 ohms. What is the impedance of the coil?

GIVEN: $R = 5$ ohms $\quad X_L = 10$ ohms

FIND: Z

SOLUTION:

$$Z = \sqrt{R^2 + X_L^2} = \sqrt{5^2 + 10^2} = \sqrt{125} = 11.18 \text{ ohms}$$

The impedance of a circuit containing resistance, inductive reactance, and capacitive reactance is

$$Z = \sqrt{R^2 + (X_L - X_C)^2} \qquad (10\text{-}1)$$

EXAMPLE 10-3 What is the impedance of a series circuit containing 40 ohms resistance, 40 ohms inductive reactance, and 70 ohms capacitive reactance?

GIVEN: $R = 40$ ohms $\quad X_L = 40$ ohms $\quad X_C = 70$ ohms

FIND: Z

SOLUTION:

$$Z = \sqrt{R^2 + (X_L - X_C)^2} = \sqrt{40^2 + (40 - 70)^2} = \sqrt{1{,}600 + 900} = 50 \text{ ohms}$$

Current. The current flowing in an a-c circuit is equal to the voltage applied to the circuit divided by the impedance of the circuit, or

$$I = \frac{E}{Z} \qquad (8\text{-}9)$$

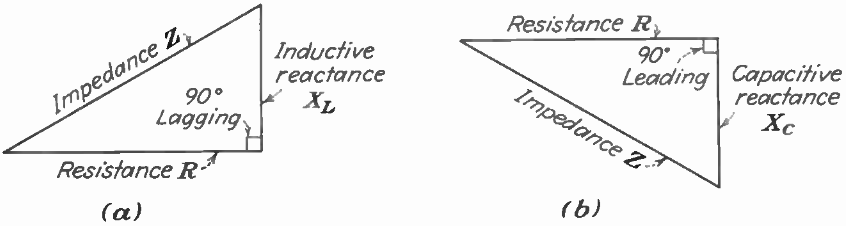


Fig. 10-4 Relation of resistance, reactance, and impedance. (a) Resistance, inductive reactance, and impedance. (b) Resistance, capacitive reactance, and impedance.

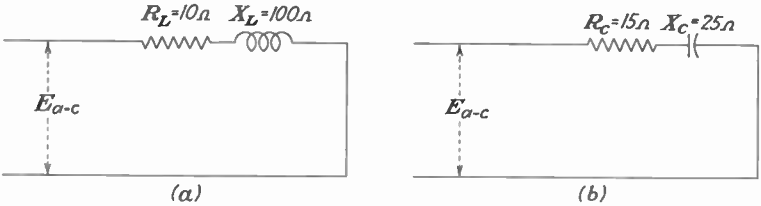


Fig. 10-5 (a) Circuit diagram for a coil showing its resistance and its inductive reactance. (b) Circuit diagram for a capacitor showing its resistance and its capacitive reactance.

EXAMPLE 10-4 How much current flows through the coil in Example 10-2 when 10 volts is impressed across its terminals?

GIVEN: $E = 10$ volts $Z = 11.18$ ohms

FIND: I_T

SOLUTION:

$$I_T = \frac{E}{Z} = \frac{10}{11.18} = 0.895 \text{ amp}$$

10-4 Multielement Series Circuits

A series a-c circuit is formed whenever one or more resistances are connected in series with one or more reactances. Since all coils or capacitors contain some resistance in addition to their reactance, a series circuit is formed whenever a coil or a capacitor is connected across an a-c power supply (see Fig. 10-5). The resistance and reactance are actually part of the coil and capacitor and not two separate devices as the circuit seems to indicate. However, in calculating a-c circuits, the reactance and resistance are always treated as separate units.

If the coil and capacitor in Fig. 10-5 are connected in series, the circuit diagram will be as shown in Fig. 10-6. As all resistances produce currents that are in phase with one another, the total resistance will be equal to the sum of the two resistances. The two reactances are 180° out of phase with each other, and therefore the resultant reactance will be equal to the difference between the two. The inductive reactance in this case is larger; therefore the resultant reactance must be inductive.

If two or more inductive reactances are connected in series, the total

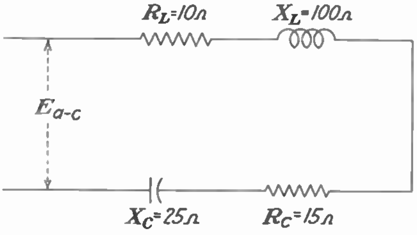


Fig. 10-6 Circuit diagram representing a coil and a capacitor connected in series.

inductive reactance will be equal to the sum of the individual inductive reactances, as they are all in phase with one another. In a like manner, the total capacitive reactance of a series circuit will be equal to the sum of the individual capacitive reactances.

From the foregoing statements, the following equation can be used to solve for the impedance of any series circuit:

$$Z = \sqrt{(R_1 + R_2 + R_3, \text{etc.})^2 + (X_{L1} + X_{L2} + X_{L3}, \text{etc.} - X_{C1} - X_{C2} - X_{C3}, \text{etc.})^2} \tag{10-2}$$

This one equation may be used for all series circuits by placing all the values of R in the first parentheses and all the reactances in the second. When the reactances in the second parentheses are added, all inductive reactances are given plus signs and all capacitive reactances minus signs.

EXAMPLE 10-5 What is the impedance of the circuit shown in Fig. 10-6?

GIVEN: $R_L = 10$ ohms $R_C = 15$ ohms $X_L = 100$ ohms $X_C = 25$ ohms

FIND: Z

SOLUTION:

$$\begin{aligned} Z &= \sqrt{(R_L + R_C)^2 + (X_L - X_C)^2} = \sqrt{(10 + 15)^2 + (100 - 25)^2} \\ &= \sqrt{25^2 + 75^2} = \sqrt{625 + 5{,}625} = \sqrt{6{,}250} = 79.05 \text{ ohms} \end{aligned}$$

EXAMPLE 10-6 The series circuit shown in Fig. 10-7 is connected to a 150-volt 60-cycle power line. (*a*) What is the impedance of the circuit? (*b*) How much current will the circuit draw from the line?

GIVEN: $E = 150$ volts $R_1 = 28$ ohms $X_{L1} = 10$ ohms $X_{L2} = 20$ ohms
$X_{C1} = 100$ ohms $R_2 = 2$ ohms $X_{L3} = 60$ ohms $X_{C2} = 50$ ohms
$R_3 = 10$ ohms

FIND: (*a*) Z (*b*) I

SOLUTION:

(*a*)

$$\begin{aligned} Z &= \sqrt{(R_1 + R_2 + R_3)^2 + (X_{L1} + X_{L2} + X_{L3} - X_{C1} - X_{C2})^2} \\ &= \sqrt{(28 + 2 + 10)^2 + (10 + 20 + 60 - 100 - 50)^2} \\ &= \sqrt{(40)^2 + (-60)^2} = \sqrt{1{,}600 + 3{,}600} = \sqrt{5{,}200} = 72.1 \text{ ohms} \end{aligned}$$

(*b*)

$$I = \frac{E}{Z} = \frac{150}{72.1} = 2.08 \text{ amp}$$

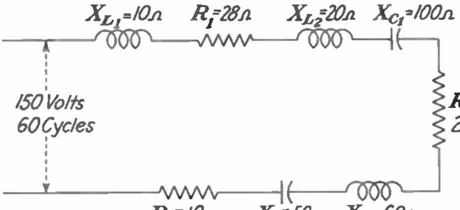


Fig. 10-7

10-5 Vectors—Voltage and Current

Vector Representation of Voltage. The sine-wave diagrams provide excellent illustrations of a-c voltages and currents, but they require a great deal of time to prepare, especially if accurate drawings are necessary. A simpler method, used extensively because of its timesaving feature, is the vector representation. By definition, a vector is a line that has both magnitude and direction and is used to represent an alternating voltage or current.

By the vector method, the sine-wave voltage illustrated in Fig. 7-5*b* is represented by the single line shown in Fig. 10-8. Vectors are usually drawn to a suitable scale as is indicated in the figure where 1 inch represents 100 volts.

The wheel diagram of Figs. 7-6 and 10-9*a* might be analyzed in terms of vectors, since each "spoke" is really a vector used to obtain an instantaneous value as is shown in Fig. 10-9. Four vector diagrams are included in this figure to illustrate that a vector has both magnitude and direction. The first vector diagram (Fig. 10-9*b*) corresponds to position 1 of the wheel diagram, which is for zero degrees. This is the starting point, and it is generally laid off in a horizontal position and to the right of the point of origin *O*. The second vector (Fig. 10-9*c*) corresponds to position 2, or 30°. As the standard direction of vector rotation about its origin is counterclockwise, the vector $E_{\max}$ has been advanced 30° from the starting position and in that direction. A vertical line projected from the end of the vector to the horizontal plane will be equal to the instantaneous value of the voltage, *e*. The line *e*, when measured, is found to be five units, and its voltage therefore will be 5×10, or 50 volts. The third vector diagram (Fig. 10-9*d*) corresponds to position 3, and the vector is drawn 60° from the horizontal. The vertical projection *e* represents the instantaneous voltage which is found to be approximately 87.5 volts (actual value is 86.6 volts). The fourth vector diagram (Fig. 10-9*e*) is for position 9, or 240°. The vector is now below the horizontal line, and its instantaneous value will be negative. The value of *e* found by the vertical projection is approximately -87.5 volts. For any position, the instantaneous value may be found in a similar manner. This method of solving for *e* by constructing a diagram to scale, measuring the length of the line *e*, and then converting its length from inches to volts is called the *graphical method* of solution. The values of *e* in Fig. 10-9*d* and *e* solved by this method have been found to be approximately 87.5 and -87.5 volts, while their accurate values are 86.6 and -86.6 volts. The accuracy of the graphical method of solution depends upon the size of the diagram and the accuracy of drawing.

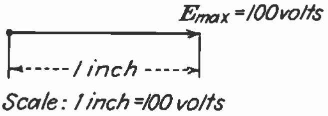


Fig. 10-8 Vector representation of a sine-wave voltage.

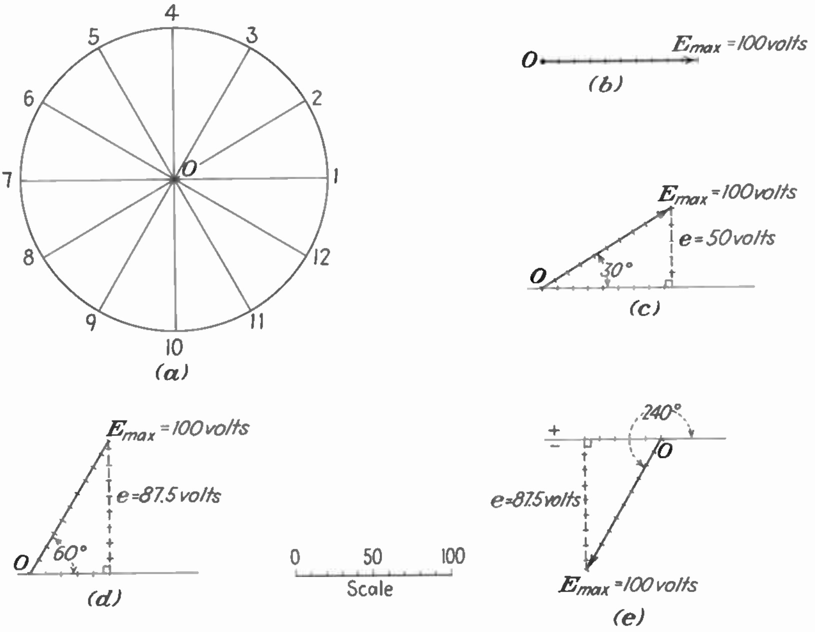


Fig. 10-9 Vector representation of the wheel diagram. (a) The wheel diagram showing 12 equally spaced points. (b) Vector representing point 1. (c) Vector representing point 2. (d) Vector representing point 3. (e) Vector representing point 9.

A more accurate method of solution is the mathematical method. In this method, the value of e may be found by Eq. (7-4). Using this method to check the graphical solution,

$$\begin{aligned} e_{0^\circ} &= E_{\max} \times \sin 0^\circ = 100 \times 0.000 = 0 \text{ volts} \\ e_{30^\circ} &= E_{\max} \times \sin 30^\circ = 100 \times 0.500 = 50 \text{ volts} \\ e_{60^\circ} &= E_{\max} \times \sin 60^\circ = 100 \times 0.866 = 86.6 \text{ volts} \\ e_{240^\circ} &= E_{\max} \times \sin 240^\circ = 100 \times (-0.866) = -86.6 \text{ volts} \end{aligned}$$

Vector Representation of Voltage and Current. When several sine waves are plotted together as in Fig. 10-1*b*, the vector representation requires two lines, one for the voltage and one for the current. The vector diagram for this condition is given in Fig. 10-10. As the current and voltage are in phase with each other, the two vectors must be drawn on the same line. Because of the great difference in the numerical value of the current and voltage, it is permissible and also common practice to use separate scales as in Figs. 10-10 and 10-11. By careful observation, it may also be noted that

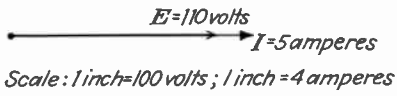


Fig. 10-10 Vector representation of current and voltage when the current is in phase with the voltage.

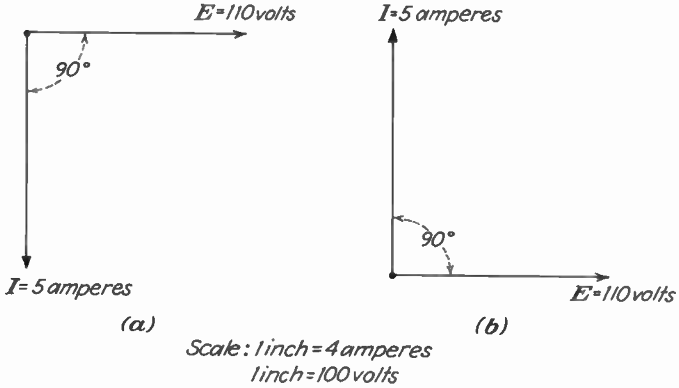


Fig. 10-11 Vector representation of current and voltage. (a) Current lagging the voltage by 90°. (b) Current leading the voltage by 90°.

these vectors are for effective values, whereas in Fig. 10-9 maximum values were used; either values may be used as long as they are properly marked. Figure 10-11*a* shows the vector diagram corresponding to Fig. 10-2 in which the current lags the voltage by 90°. As vector rotation is counterclockwise, the current is downward, or lags the voltage by 90°. Figure 10-11*b* is the vector diagram corresponding to Fig. 10-3 in which the current leads the voltage by 90°. The arrows on the current vectors are closed, while on the voltage vectors they are left open in order to distinguish currents from voltages.

10-6 Vectors—Resistance, Reactance, and Impedance

Vectors are also used to show the relation among resistance, inductive reactance, capacitive reactance, and impedance. This was taken up in Art. 10-3, but the values were not considered as vectors at that time. If the reactance and resistance vectors are drawn to scale, the impedance of the circuit can be determined graphically.

EXAMPLE 10-7 By means of vectors, determine the impedance of a series circuit containing a resistance of 40 ohms and an inductive reactance of 30 ohms. Check the answer by solving for impedance mathematically.

GIVEN: $R = 40$ ohms $X_L = 30$ ohms

FIND: Z

SOLUTION: Since the resistance causes an in-phase current and the inductive reactance causes a 90° lagging current, the inductive reactance is drawn 90° from the resistance. The combination of the two as determined graphically is shown in Fig. 10-12 and is equal to 50 ohms.

Checking mathematically:

$$Z = \sqrt{R^2 + X_L{}^2} = \sqrt{40^2 + 30^2} = \sqrt{2{,}500} = 50 \text{ ohms}$$

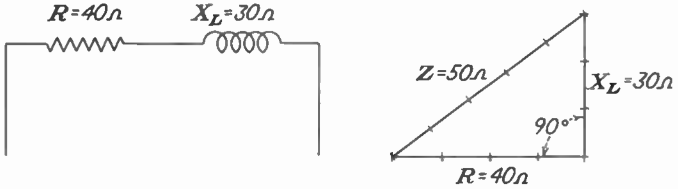


Fig. 10-12

EXAMPLE 10-8 A circuit has a resistance of 30 ohms connected in series with a capacitive reactance of 40 ohms. Determine its impedance graphically and mathematically.

GIVEN: $R = 30$ ohms $\quad X_C = 40$ ohms

FIND: Z

SOLUTION: The capacitive reactance causes a 90° leading current and is therefore drawn 90° from the resistance. The combination of the two as determined graphically is shown in Fig. 10-13 and is equal to 50 ohms.

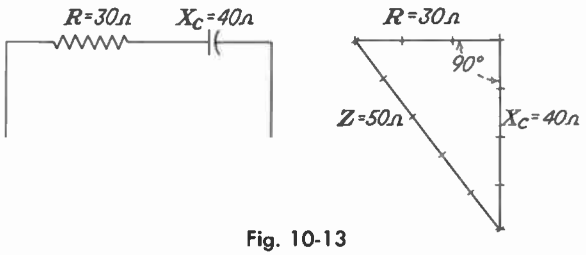


Fig. 10-13

Checking mathematically:

$$Z = \sqrt{R^2 + X_C^2} = \sqrt{30^2 + 40^2} = \sqrt{2{,}500} = 50 \text{ ohms}$$

Series circuits may contain resistance, inductance, and capacitance. In such circuits, the total impedance is determined graphically by combining the resultant resistance vector with the resultant reactance vector.

EXAMPLE 10-9 A series circuit contains a resistance of 40 ohms, an inductive reactance of 30 ohms, and a capacitive reactance of 60 ohms. Determine the impedance of this circuit, using vectors. Check the answer mathematically.

GIVEN: $R = 40$ ohms $\quad X_L = 30$ ohms $\quad X_C = 60$ ohms

FIND: Z

SOLUTION: Referring to Fig. 10-14, the inductive reactance and the capacitive reactance are first combined into the single value $(X_L - X_C)$, which is then added to the resistance to form the triangle for finding the impedance. Using this method, $Z = 50$ ohms.

Checking the answer mathematically:

$$Z = \sqrt{R^2 + (X_L - X_C)^2} = \sqrt{40^2 + (30 - 60)^2} = \sqrt{2{,}500} = 50 \text{ ohms}$$

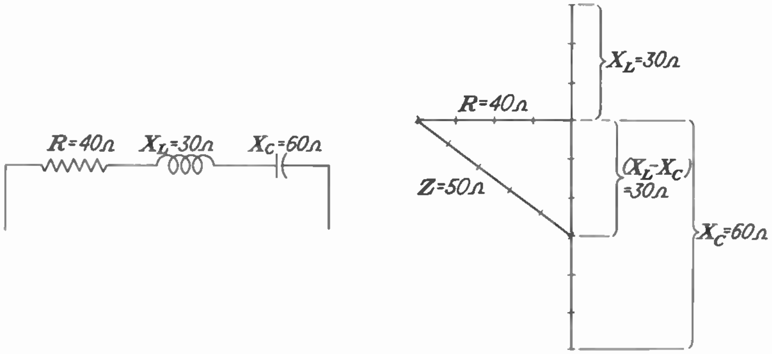


Fig. 10-14

10-7 Power in A-C Circuits

Apparent Power. The power consumed by a d-c circuit was given in Art. 2-13 as $P = E \times I$ [Eq. (2-12)]. In a-c circuits this will be true only when the current is in phase with the voltage as is the case for circuits containing resistance alone and for resonant circuits to be studied later. In cases where the current is not in phase with the voltage, the product of the volts and amperes will not be equal to the power actually consumed by the circuit. Instead, the product of the volts and amperes is called the *apparent power* and is expressed in volt-amperes. To avoid confusion, it is preferable always to call the product of the volts and amperes the apparent power.

Actual Power. The actual power consumed by an a-c circuit is best determined by the use of a wattmeter, which always indicates the true power, regardless of whether the current is in phase with the voltage or not. The power may, however, be determined without the use of a wattmeter if the angle between the current and voltage is known. Power may then be calculated by the equation

$$P = E \times I \times \cos \theta \tag{10-3}$$

where P = power, watts
E = voltage of the circuit, volts
I = current of the circuit, amp
θ = angle between current and voltage, deg
$\cos \theta$ = cosine of the angle θ (from tables in Appendix XI)

The power may also be found by Eq. (2-14), $P = I^2R$, but care must be exercised that only the resistance is used and that it is not confused with inductive reactance, capacitive reactance, or impedance.

Power Factor. The value of $\cos \theta$ is often referred to as the *power factor.* When the current is in phase with the voltage, the angle θ is zero and the power factor, $\cos 0°$, is 1.000. When the current is out of phase by 90°, either leading or lagging, the power factor, $\cos 90°$, is zero. For lagging or leading currents with angles between zero and 90°, the cosine value, and hence the power factor, varies from zero to 1.0 as indicated in the cosine table. Substituting the apparent power VA for $E \times I$ and PF for $\cos\theta$ in Eq. (10-3), it becomes

$$P = VA \times \text{PF} \tag{10-4}$$

or

$$\text{Actual power} = \text{apparent power} \times \text{power factor}$$

A definition for power factor may be derived from this equation, namely, power factor is a factor or number whose value varies from zero to 1.0, by which the apparent power must be multiplied in order to determine the actual power. From the power equations, it may be seen that, when the power factor is 1.0, all the volt-amperes become actual power, while if the power factor is less than 1.0, only part of the volt-amperes becomes actual power. Since only the actual power does useful work, it is evident that it is desirable to have the power factor as near to unity as possible. Combining two facts (1) that only resistance takes power and (2) that impedance is the combined effect of resistance and the reactances, it follows that the power factor can also be expressed in terms of the resistance and impedance of a circuit. Hence

$$\text{PF} = \frac{R}{Z} \tag{10-5}$$

In practical work, the power factor of a circuit is generally determined from meter readings. When the volts, amperes, and watts of a circuit are known, the power factor may be calculated by the equation

$$\text{Power factor} = \frac{\text{watts}}{\text{volts} \times \text{amperes}} \qquad \text{or} \qquad \text{PF} = \frac{P}{EI} \tag{10-6}$$

This equation shows that the power factor may also be defined as the ratio of the actual power to the apparent power.

A power-factor meter that gives a direct indication of the power factor of a circuit is also available, but this method of determining the power factor is not used so much as the voltmeter, ammeter, wattmeter method.

Phase Angle. The phase angle, represented by the Greek letter θ, is the angle between two a-c quantities, most commonly between the current and voltage, although it is often used with two voltages, two currents, resistance and impedance, etc. When it is used with current and voltage, especially as the power-factor angle, it is general practice also to state whether the current lags or leads the voltage by the angle θ.

10-8 Series Circuit Problems

As most of the principles presented in Arts. 10-1 to 10-7 are closely related, the following examples will illustrate several of them in each example.

EXAMPLE 10-10 A 10-henry choke coil is connected to a 110-volt 60-cycle power source. It is assumed that the resistance of the coil is zero. Find (*a*) the current, (*b*) the power, (*c*) the power factor, (*d*) the phase angle. (*e*) Draw a vector diagram showing the current and voltage relations.

GIVEN: $E = 110$ volts $f = 60$ cycles $L = 10$ henrys $R = 0$

FIND: (*a*) I (*b*) P (*c*) PF (*d*) θ (*e*) Vector diagram

SOLUTION:

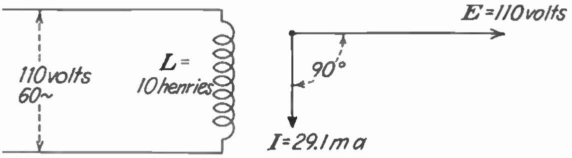
(*a*) $$I_L = \frac{E_L}{2\pi fL} = \frac{110}{6.28 \times 60 \times 10} = \frac{110}{3{,}768} = 0.0291 \text{ amp}$$

(*b*) $$P = I^2R = 0.0291^2 \times 0 = 0$$

(*c*) $$\text{PF} = \frac{R}{Z} = \frac{0}{3{,}768} = 0$$

(*d*) $\theta = 90°$ (lagging current)

(*e*) Vector diagram, Fig. 10-15*b*



(*a*)- Circuit diagram (*b*)- Vector diagram Fig. 10-15

EXAMPLE 10-11 A 10-μf capacitor is connected to a 250-volt 60-cycle circuit. It is assumed that the resistance of the capacitor is zero. Find (*a*) the current, (*b*) the power, (*c*) the volt-amperes, (*d*) the power factor, (*e*) the phase angle. (*f*) Draw a vector diagram of the current and voltage relations.

GIVEN: $E = 250$ volts $f = 60$ cycles $C = 10\ \mu\text{f}$ $R = 0$

FIND: (*a*) I (*b*) P (*c*) VA (*d*) PF (*e*) θ (*f*) Vector diagram

SOLUTION:

(*a*) $$I = 2\pi fCE10^{-6} = 6.28 \times 60 \times 10 \times 250 \times 10^{-6} = 0.943 \text{ amp}$$

(*b*) $$P = I^2R = 0.943^2 \times 0 = 0$$

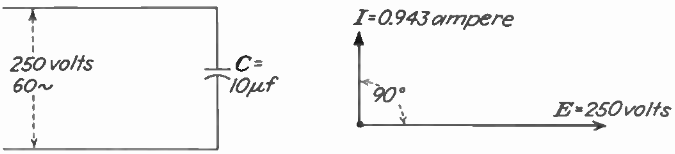
(*c*) $$VA = EI = 250 \times 0.943 = 235.7$$

(*d*) $$\text{PF} = \frac{P}{VA} = \frac{0}{235.7} = 0$$

(*e*) $\theta = 90°$ (leading current)

(*f*) Vector diagram, Fig. 10-16*b*

EXAMPLE 10-12 A series circuit containing a resistor of 40 ohms, an inductor of 80 mh, and a capacitor of 40 μf is connected across a 250-volt 60-cycle line as shown in Fig. 10-17*a*. The resistances of the inductor and the capacitor are assumed to



(a)-Circuit diagram (b)-Vector diagram

Fig. 10-16

be zero. Find (*a*) the inductive reactance, (*b*) the capacitive reactance, (*c*) the impedance, (*d*) the current, (*e*) the power, (*f*) the apparent power, (*g*) the power factor, (*h*) the phase angle, (*i*) the voltage drops across the resistor, inductor, and capacitor. (*j*) Draw a vector diagram.

GIVEN: $E = 250$ volts $f = 60$ cycles $R = 40$ ohms $L = 80$ mh
$C = 40\ \mu\text{f}$

FIND: (*a*) X_L (*b*) X_C (*c*) Z (*d*) I (*e*) P (*f*) AP (*g*) PF (*h*) θ
(*i*) E_R, E_L, E_C (*j*) Vector diagram

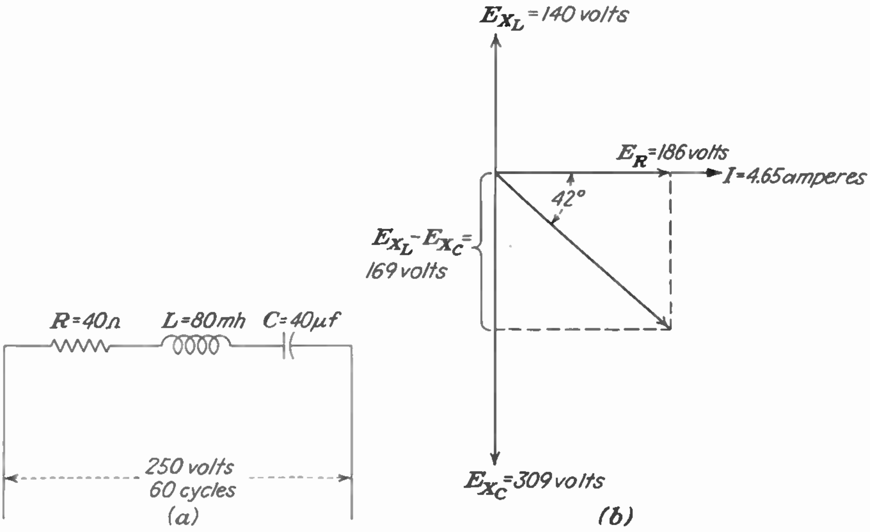


Fig. 10-17

SOLUTION:

(*a*) $$X_L = 2\pi fL = 6.28 \times 60 \times 80 \times 10^{-3} = 30.1 \text{ ohms}$$

(*b*) $$X_C = \frac{10^6}{2\pi fC} = \frac{10^6}{6.28 \times 60 \times 40} = 66.2 \text{ ohms}$$

(*c*) $$Z = \sqrt{R^2 + (X_L - X_C)^2} = \sqrt{40^2 + (30.1 - 66.2)^2}$$
$$= \sqrt{1{,}600 + 1{,}303} = \sqrt{2{,}903} = 53.8 \text{ ohms}$$

(*d*) $$I = \frac{E}{Z} = \frac{250}{53.8} = 4.65 \text{ amp}$$

(*e*) $$P = I^2R = 4.65^2 \times 40 = 865 \text{ watts}$$

(*f*) $$\text{AP} = EI = 250 \times 4.65 = 1{,}162 \text{ va}$$

(*g*) $$\text{PF} = \frac{P}{VA} = \frac{865}{1{,}162} = 0.744$$

(*h*) $$\theta = 42° \text{ (leading current)}$$

(*i*) $$E_R = IR = 4.65 \times 40 = 186 \text{ volts}$$

$$E_L = IX_L = 4.65 \times 30.1 = 140 \text{ volts}$$

$$E_C = IX_C = 4.65 \times 66.2 = 309 \text{ volts}$$

(*j*) Vector diagram, Fig. 10-17*b*

Note: The voltage drop across the capacitor is greater than the line voltage. This is not in error, for this can occur in a series circuit that contains resistance, inductance, and capacitance. Whether or not it occurs depends upon the relative values of resistance, inductive reactance, and capacitive reactance.

EXAMPLE 10-13 An inductor is connected in a circuit as shown in Fig. 10-18*a*. The readings taken show 110 volts, 60 cycles, 3 amp, and 66 watts. Find (*a*) the impedance, (*b*) the resistance, (*c*) the apparent power, (*d*) the power factor, (*e*) the phase angle, (*f*) the inductive reactance, and (*g*) the inductance of the coil. (*h*) Draw a vector diagram.

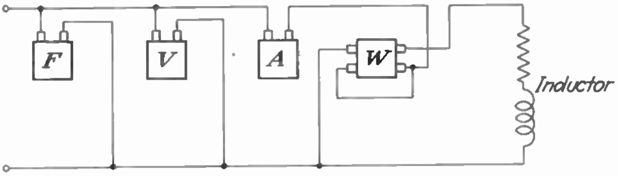


Fig. 10-18a

GIVEN: $E = 110$ volts $I = 3$ amp $P = 66$ watts $f = 60$ cycles

FIND: (*a*) Z (*b*) R (*c*) AP (*d*) PF (*e*) θ (*f*) X_L (*g*) L
(*h*) Vector diagram

SOLUTION:

(*a*) $$Z = \frac{E}{I} = \frac{110}{3} = 36.7 \text{ ohms}$$

(*b*) $$R = \frac{P}{I^2} = \frac{66}{3^2} = 7.33 \text{ ohms}$$

(*c*) $$\text{AP} = EI = 110 \times 3 = 330 \text{ va}$$

(*d*) $$\text{PF} = \frac{P}{VA} = \frac{66}{330} = 0.200$$

(*e*) $$\theta = 78.5° \text{ (lagging current)}$$

(*f*) $$X_L = \sqrt{Z^2 - R^2} = \sqrt{36.7^2 - 7.33^2}$$
$$= \sqrt{1{,}292.2} = 35.9 \text{ ohms}$$

(*g*) $$L = \frac{X_L}{2\pi f} = \frac{35.9}{377} = 0.0952 \text{ henry}$$

(*h*) Vector diagram, Fig. 10-18*b*

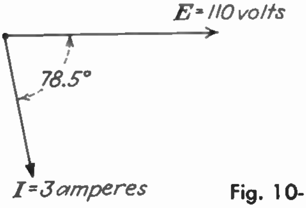


Fig. 10-18b

10-9 Parallel Circuits

Parallel circuits are used more frequently in electrical systems than are series circuits. In electronic equipment, series, parallel, and combination circuits are used. Because all inductors and capacitors have some resistance, it is not possible to set up a circuit containing pure reactances connected in parallel. However, in some inductors and capacitors, especially capacitors, the resistance is so low in comparison with the reactance that the resistance is assumed to be zero. Under these conditions, a circuit may be considered as if it would contain any combination of pure resistances and pure reactances connected in parallel. The characteristics of such a parallel circuit can best be illustrated by solving for the impedance, currents, power, etc., of a typical parallel circuit.

The various factors of a parallel circuit can be determined with the least difficulty by observing the following order:

1. Find the impedance of each branch.
2. Find the current of each branch.
3. Draw a vector diagram of the currents.
4. Find the total in-phase or resistance current.
5. Find the total reactance current.
6. Find the line current.
7. Find the impedance of the circuit.
8. Find the line power.
9. Find the line volt-amperes.
10. Find the line power factor.

This order is recommended but is not essential; in fact it may have to be altered to meet the needs of the problem being considered. If the impedance of a parallel circuit is desired but no line voltage is specified, any convenient value may be assumed for calculating purposes.

The equation for the line current in a parallel circuit containing only pure resistances and pure reactances (see Fig. 10-19*a*) is obtained by combining steps 1 to 6 and is

$$I_{\text{line}} = \sqrt{(I_{R1} + I_{R2} + I_{R3})^2 + (I_{XC1} + I_{XC2} - I_{XL1} - I_{XL2})^2} \qquad (10\text{-}7)$$

In the second parentheses, all leading currents (caused by capacitive reactance) are plus and all lagging currents (caused by inductive reactance) are minus. When the sum of the values in the second parentheses is a plus, it indicates that the line current is leading the voltage, and if negative it indicates that the line current is lagging the voltage.

EXAMPLE 10-14 The parallel circuit shown in Fig. 10-19*a* consists of only pure resistances and pure reactances. Find (*a*) the current of each branch, (*b*) the line current, (*c*) the impedance of the circuit, (*d*) the power consumed by each branch, (*e*) the power taken by the complete circuit, (*f*) the volt-amperes of the complete circuit, (*g*) the line power factor, (*h*) the phase angle between line current and line voltage. (*i*) Draw a vector diagram of the voltage and all the currents.

GIVEN:

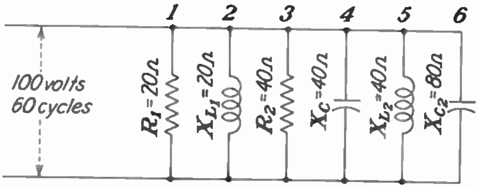


Fig. 10-19a

SOLUTION:

(*a*) Use steps 1 and 2.

$$I_1 = \frac{E_1}{Z_1} = \frac{100}{20} = 5 \text{ amp}$$

$$I_2 = \frac{E_2}{Z_2} = \frac{100}{20} = 5 \text{ amp}$$

$$I_3 = \frac{E_3}{Z_3} = \frac{100}{40} = 2.5 \text{ amp}$$

$$I_4 = \frac{E_4}{Z_4} = \frac{100}{40} = 2.5 \text{ amp}$$

$$I_5 = \frac{E_5}{Z_5} = \frac{100}{40} = 2.5 \text{ amp}$$

$$I_6 = \frac{E_6}{Z_6} = \frac{100}{80} = 1.25 \text{ amp}$$

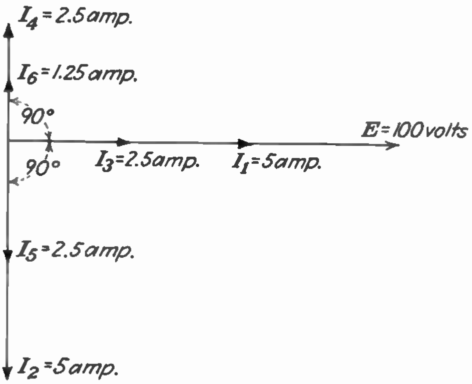


Fig. 10-19b

(*b*) Use steps 3, 4, 5, 6.
Step 3, vector diagram (Fig. 10-19*b*).

Steps 4, 5, 6 are combined in Eq. (10-7).

$$\begin{aligned} I_{\text{line}} &= \sqrt{(I_{R1} + I_{R2})^2 + (I_{XC} + I_{XC2} - I_{XL1} - I_{XL2})^2} \\ &= \sqrt{(5 + 2.5)^2 + (2.5 + 1.25 - 5 - 2.5)^2} \\ &= \sqrt{(7.5)^2 + (-3.75)^2} = \sqrt{70.31} = 8.38 \text{ amp} \end{aligned}$$

(*c*) Use step 7.

$$Z_{\text{cct}} = \frac{E_{\text{line}}}{I_{\text{line}}} = \frac{100}{8.38} = 11.93 \text{ ohms}$$

(*d*) Power may be calculated by Eq. (2-14); note that circuits containing no resistance take no power.

$$\begin{aligned} P_1 &= I_1{}^2R_1 = 5^2 \times 20 = 500 \text{ watts} \\ P_2 &= \text{no resistance in circuit} = \text{zero} \\ P_3 &= I_3{}^2R_2 = 2.5^2 \times 40 = 250 \text{ watts} \\ P_4 &= \text{no resistance in circuit} = \text{zero} \\ P_5 &= \text{no resistance in circuit} = \text{zero} \\ P_6 &= \text{no resistance in circuit} = \text{zero} \end{aligned}$$

(*e*) Use step 8.

$$\begin{aligned} P_{\text{line}} &= P_1 + P_2 + P_3 + P_4 + P_5 + P_6 \\ &= 500 + 0 + 250 + 0 + 0 + 0 = 750 \text{ watts} \end{aligned}$$

(*f*) Use step 9.

$$VA_{\text{line}} = E_{\text{line}} \times I_{\text{line}} = 100 \times 8.38 = 838$$

(*g*) Use step 10.

$$\text{PF}_{\text{line}} = \frac{P_{\text{line}}}{VA_{\text{line}}} = \frac{750}{838} = 0.895$$

(*h*) $\theta_{\text{line}} = 26.5°$ lagging (from Appendix XI)

(*i*) Vector diagram, Fig. 10-19*c*

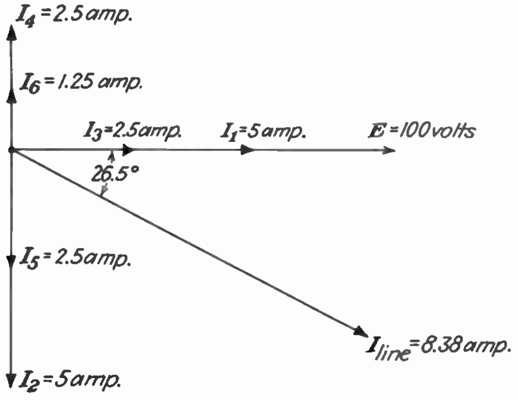


Fig. 10-19c

10-10 Parallel-series Circuits

When one or more series groups become part of a parallel system, the combined circuit is called a *parallel-series circuit.* The following procedure is suggested for solving such circuits:

1. Find the impedance, current, power, cos θ, and sin θ for each branch.
2. Resolve each current into its in-phase and quadrature (90° angle) components. This is done in the following manner. For example, a current of 5 amp lags its voltage by 30°, as shown in Fig. 10-20. As a similar current could be caused by a resistance current I_R and a reactance current I_X, we may resolve the current I into two such components. By trigonometry (see Appendix X)

$$I_R = I \times \cos\theta = 5 \times \cos 30^\circ = 5 \times 0.866 = 4.33 \text{ amp}$$
$$I_X = I \times \sin\theta = 5 \times \sin 30^\circ = 5 \times 0.500 = 2.5 \text{ amp}$$

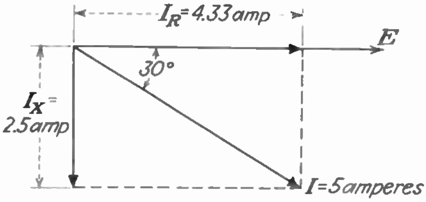


Fig. 10-20 The current I lagging the voltage by 30° resolved into its in-phase or resistance component and its quadrature or reactance component. This procedure may also be applied to any leading current.

3. Calculate the line current by combining the in-phase components and the quadrature components of all the branch currents. This may be done in the form of the following equation:

$$I_{\text{line}} = \sqrt{(I_1 \cos\theta_1 + I_2 \cos\theta_2, \text{etc.})^2 + (\pm I_1 \sin\theta_1 \pm I_2 \sin\theta_2, \text{etc.})^2} \quad (10\text{-}8)$$

Note: $\pm I_1 \sin\theta_1$, etc.: Use + for leading currents and − for lagging currents. When the sum of the second parentheses is +, it indicates that I_{line} is a leading current, and if the sum is −, it indicates that I_{line} is a lagging current.

4. Find the impedance of the circuit.
5. Find the power taken by the circuit.
6. Find the volt-amperes of the circuit.
7. Find the line power factor.

This order is recommended but is not essential. It may even have to be altered to meet the needs of the problem being considered.

EXAMPLE 10-15 A parallel-series circuit is shown in Fig. 10-21*a*. (*a*) Find the current, power, power factor, and phase angle for each branch. (*b*) Find the current, power, apparent power, power factor, and phase angle for the complete circuit. (*c*) Draw a vector diagram.

GIVEN:

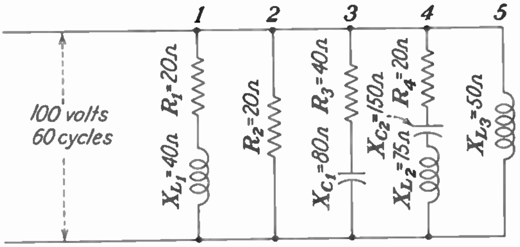


Fig. 10-21a

SOLUTION:

(*a*) $Z_1 = \sqrt{R^2 + X_L^2} = \sqrt{20^2 + 40^2} = \sqrt{2{,}000} = 44.7$ ohms

$$I_1 = \frac{E_1}{Z_1} = \frac{100}{44.7} = 2.23 \text{ amp}$$

$$P_1 = I_1^2R_1 = 2.23^2 \times 20 = 99.5 \text{ watts}$$

$$\text{PF}_1 = \cos\theta_1 = \frac{R_1}{Z_1} = \frac{20}{44.7} = 0.447$$

$$\theta_1 = 63.5° \text{ lagging}$$

$$I_2 = \frac{E_2}{Z_2} = \frac{100}{20} = 5 \text{ amp}$$

$$P_2 = I_2^2R_2 = 5^2 \times 20 = 500 \text{ watts}$$

$$\text{PF}_2 = \frac{R_2}{Z_2} = \frac{20}{20} = 1.00$$

$$\theta_2 = 0°$$

$$Z_3 = \sqrt{R^2 + X_C^2} = \sqrt{40^2 + 80^2} = \sqrt{8{,}000} = 89.4 \text{ ohms}$$

$$I_3 = \frac{E_3}{Z_3} = \frac{100}{89.4} = 1.12 \text{ amp}$$

$$P_3 = I_3^2R_3 = 1.12^2 \times 40 = 50 \text{ watts}$$

$$\text{PF}_3 = \frac{R_3}{Z_3} = \frac{40}{89.4} = 0.447$$

$$\theta_3 = 63.5° \text{ leading}$$

$$Z_4 = \sqrt{R^2 + (X_L - X_C)^2} = \sqrt{20^2 + (75 - 150)^2} = \sqrt{6{,}025} = 77.6 \text{ ohms}$$

$$I_4 = \frac{E_4}{Z_4} = \frac{100}{77.6} = 1.29 \text{ amp}$$

$$P_4 = I_4^2R_4 = 1.29^2 \times 20 = 33.3 \text{ watts}$$

$$\text{PF}_4 = \frac{R_4}{Z_4} = \frac{20}{77.6} = 0.257$$

$$\theta_4 = 75° \text{ leading}$$

$$I_5 = \frac{E_5}{Z_5} = \frac{100}{50} = 2 \text{ amp}$$

$$P_5 = I_5{}^2R_5 = 2^2 \times 0 = 0 \text{ watts}$$

$$\text{PF}_5 = \frac{R_5}{Z_5} = \frac{0}{50} = 0$$

$$\theta_5 = 90° \text{ lagging}$$

(*b*)
$$\begin{aligned} I_{\text{line}} &= \sqrt{(I_1 \cos\theta_1 + I_2 \cos\theta_2 + I_3 \cos\theta_3 + I_4 \cos\theta_4 + I_5 \cos\theta_5)^2 + (\pm I_1 \sin\theta_1 \pm I_2 \sin\theta_2 \pm I_3 \sin\theta_3 \pm I_4 \sin\theta_4 \pm I_5 \sin\theta_5)^2} \\ &= \sqrt{(2.23 \times 0.447 + 5 \times 1.0 + 1.12 \times 0.447 + 1.29 \times 0.257 + 2 \times 0.0)^2 + (-2.23 \times 0.895 + 5 \times 0.0 + 1.12 \times 0.895 + 1.29 \times 0.966 - 2 \times 1.0)^2} \\ &= \sqrt{(1.00 + 5 + 0.50 + 0.331 + 0)^2 + (-2.0 + 0 + 1.0 + 1.25 - 2.0)^2} \\ &= \sqrt{6.831^2 + (-1.75)^2} = \sqrt{49.72} = 7.05 \text{ amp} \end{aligned}$$

$$P_{\text{line}} = P_1 + P_2 + P_3 + P_4 + P_5 = 99.5 + 500 + 50 + 33.3 + 0 = 682.8 \text{ watts}$$

$$\text{AP}_{\text{line}} = E_{\text{line}} \times I_{\text{line}} = 100 \times 7.05 = 705 \text{ va}$$

$$\text{PF}_{\text{line}} = \frac{P_{\text{line}}}{VA_{\text{line}}} = \frac{682.8}{705} = 0.968$$

$$\theta_{\text{line}} = 14.5° \text{ lagging}$$

(*c*) Vector diagram, Fig. 10-21*b*

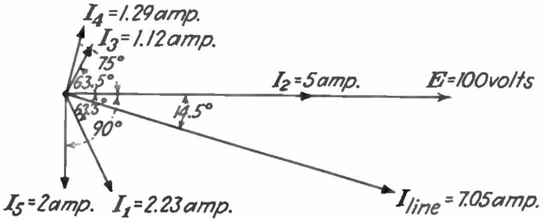


Fig. 10-21b

10-11 Series-parallel Circuits

When one or more parallel groups become part [illegible] a series circuit, the combined circuit is called a *series-parallel circuit.* The following procedure is suggested for solving such circuits:

1. Reduce each parallel group to a single value of resistance and reactance (either inductive or capacitive) so that when connected in series they will produce the same effect as the original parallel group. These values are called the *equivalent resistance* and the *equivalent reactance.* This may be done by (*a*) assigning a convenient assumed voltage to the group which is to be used only for finding the impedance and phase angle of the group, (*b*) finding the equivalent resistance by $R_{\text{eq}} = Z \times \cos\theta$, (*c*) finding the equivalent reactance by $X_{\text{eq}} = Z \times \sin\theta$.

2. Draw a new circuit diagram, substituting the equivalent resistances

and reactances in place of the parallel groups. This circuit will be a series circuit similar to Example 10-6.

3. Solve for the impedance, current, power, apparent power, power factor, and phase angle of the circuit as in Example 10-6.

4. If it is required to get the division of current in the parallel parts of the circuit, this may be done by first finding the voltage drop (IZ) of that part and then getting the separate currents.

EXAMPLE 10-16 A series-parallel circuit is shown in Fig. 10-22*a*. (*a*) Find the current, power, apparent power, power factor, and phase angle of the line. (*b*) Find the voltage and current of each part of the circuit. (*c*) Draw a vector diagram.

GIVEN:

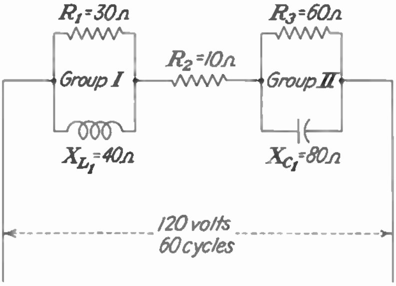


Fig. 10-22a

SOLUTION:

Step 1*a*. Equivalent series circuit of group I. (Assume 120 volts.)

$$I_R = \frac{120}{30} = 4 \text{ amp}$$

$$I_{XL} = \frac{120}{40} = 3 \text{ amp}$$

$$I_{\text{line}} = \sqrt{4^2 + 3^2} = \sqrt{25} = 5 \text{ amp}$$

$$Z_{\text{eq}} = \frac{120}{5} = 24 \text{ ohms}$$

$$\cos\theta = \frac{I_R}{I_{\text{line}}} = \frac{4}{5} = 0.800$$

$$\sin\theta = \frac{I_X}{I_{\text{line}}} = \frac{3}{5} = 0.600$$

$$\theta = 37^\circ \text{ lagging}$$

$$R_{\text{eq}} = Z_{\text{eq}} \times \cos\theta = 24 \times 0.800 = 19.2 \text{ ohms}$$

$$X_{L\text{eq}} = Z_{\text{eq}} \times \sin\theta = 24 \times 0.600 = 14.4 \text{ ohms}$$

Step 1*b*. Equivalent series circuit of group II. (Assume 240 volts.)

$$I_R = \frac{240}{60} = 4 \text{ amp}$$

$$I_{XC} = \frac{240}{80} = 3 \text{ amp}$$

$$I_{\text{line}} = \sqrt{4^2 + 3^2} = \sqrt{25} = 5 \text{ amp}$$

$$Z_{\text{eq}} = \frac{240}{5} = 48 \text{ ohms}$$

$$\cos\theta = \frac{I_R}{I_{\text{line}}} = \frac{4}{5} = 0.800$$

$$\sin\theta = \frac{I_{XC}}{I_{\text{line}}} = \frac{3}{5} = 0.600$$

$$\theta = 37° \text{ leading}$$

$$R_{\text{eq}} = Z_{\text{eq}} \times \cos\theta = 48 \times 0.800 = 38.4 \text{ ohms}$$

$$X_{C\text{eq}} = Z_{\text{eq}} \times \sin\theta = 48 \times 0.600 = 28.8 \text{ ohms}$$

Step 2. Equivalent-series-circuit diagram (Fig. 10-22*b*)

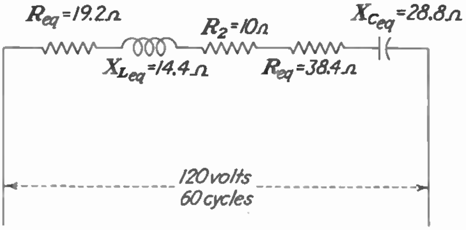


Fig. 10-22b

Step 3.

$$Z_{\text{cct}} = \sqrt{(19.2 + 10 + 38.4)^2 + (14.4 - 28.8)^2}$$
$$= \sqrt{67.6^2 + (-14.4)^2} = \sqrt{4{,}777} = 69.1 \text{ ohms}$$

(*a*)

$$I_{\text{line}} = \frac{E_{\text{line}}}{Z_{\text{cct}}} = \frac{120}{69.1} = 1.73 \text{ amp}$$

$$P_{\text{cct}} = I^2R_{\text{cct}} = 1.73^2 \times 67.6 = 203 \text{ watts}$$

$$\text{AP}_{\text{cct}} = E_{\text{line}} \times I_{\text{line}} = 120 \times 1.73 = 208 \text{ va}$$

$$\text{PF}_{\text{cct}} = \frac{\text{P}_{\text{cct}}}{\text{AP}_{\text{cct}}} = \frac{203}{208} = 0.976$$

$$\theta_{\text{cct}} = 12.5° \text{ leading}$$

(*b*)

$$\text{Voltage at group I} = I_{\text{line}} \times Z_{\text{eq}} = 1.73 \times 24 = 41.5 \text{ volts}$$

$$\text{Voltage at } R_2 = I_2R_2 = 1.73 \times 10 = 17.3 \text{ volts}$$

Voltage at group II $= I_{GR \cdot II} \times Z_{\text{eq}} = 1.73 \times 48 = 83$ volts

$$I_{R1} = \frac{E_{GR \cdot I}}{R_1} = \frac{41.5}{30} = 1.38 \text{ amp}$$

$$I_{XL1} = \frac{E_{GR \cdot I}}{X_{L1}} = \frac{41.5}{40} = 1.04 \text{ amp}$$

Check: $\sqrt{1.38^2 + 1.04^2}$ should equal $1.73 = \sqrt{1.91 + 1.08} = \sqrt{2.99} = 1.73$.

$$I_{R2} = I_{\text{line}} = 1.73 \text{ amp}$$

$$I_{R3} = \frac{E_{GR \cdot II}}{R_3} = \frac{83}{60} = 1.38 \text{ amp}$$

$$I_{XC} = \frac{E_{GR \cdot II}}{X_C} = \frac{83}{80} = 1.04 \text{ amp}$$

The voltage and current distribution for this circuit is shown in Fig. 10-22*c*.

(*c*) Vector diagram, Fig. 10-22*d*

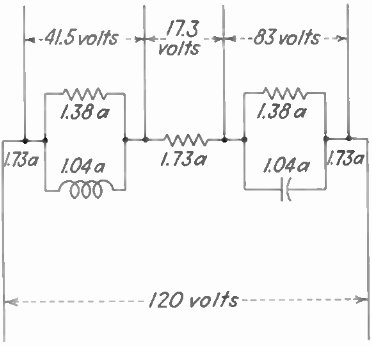


Fig. 10-22c

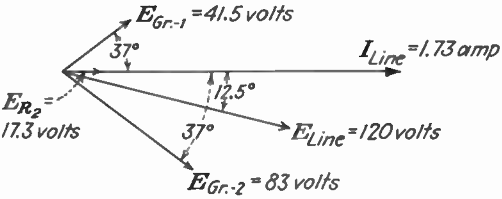


Fig. 10-22d

10-12 Equivalent Circuits

In solving a-c circuit problems, it is sometimes desirable to substitute an equivalent series circuit for a parallel combination or an equivalent parallel circuit for a series combination.

Series Equivalent for a Parallel Circuit. When a circuit has a resistance and a reactance connected in parallel, a certain resistance value $R_{S'}$ and reactance value $X_{S'}$ when connected in series will produce the same effect as the original resistance and reactance connected in parallel.

$$R_{S'} = \frac{R_P X_P^2}{R_P^2 + X_P^2} \tag{10-9}$$

$$X_{S'} = \frac{X_P R_P^2}{R_P^2 + X_P^2} \tag{10-10}$$

EXAMPLE 10-17 What are the resistance and reactance values of a series equivalent circuit that may be substituted for a parallel circuit that has a 20-ohm resistance and a 40-ohm reactance?

GIVEN: $R_P = 20$ ohms $X_P = 40$ ohms

FIND: $R_{S'}$ $X_{S'}$

SOLUTION:

$$R_{S'} = \frac{R_P X_P^2}{R_P^2 + X_P^2} = \frac{20 \times 40^2}{20^2 + 40^2} = \frac{32{,}000}{2{,}000} = 16 \text{ ohms}$$

$$X_{S'} = \frac{X_P R_P^2}{R_P^2 + X_P^2} = \frac{40 \times 20^2}{20^2 + 40^2} = \frac{16{,}000}{2{,}000} = 8 \text{ ohms}$$

Parallel Equivalent for a Series Circuit. When a circuit has a resistance and a reactance connected in series, a certain resistance value $R_{P'}$ and reactance value $X_{P'}$ when connected in parallel will produce the same effect as the original resistance and reactance connected in series.

$$R_{P'} = \frac{R_S^2 + X_S^2}{R_S} \qquad (10\text{-}11)$$

$$X_{P'} = \frac{R_S^2 + X_S^2}{X_S} \qquad (10\text{-}12)$$

EXAMPLE 10-18 What are the resistance and reactance values of a parallel equivalent circuit that may be substituted for a series circuit that has a 16-ohm resistance and an 8-ohm reactance?

GIVEN: $R_S = 16$ ohms $X_S = 8$ ohms

FIND: $R_{P'}$ $X_{P'}$

SOLUTION:

$$R_{P'} = \frac{R_S^2 + X_S^2}{R_S} = \frac{16^2 + 8^2}{16} = \frac{320}{16} = 20 \text{ ohms}$$

$$X_{P'} = \frac{R_S^2 + X_S^2}{X_S} = \frac{16^2 + 8^2}{8} = \frac{320}{8} = 40 \text{ ohms}$$

10-13 Solving A-C Problems with Vector Algebra

Purpose. The use of vector algebra makes it possible to solve problems that would be very difficult to solve by the methods previously presented and also saves time when a large number of simple problems are to be solved. Understanding the principles about to be presented requires a more extensive background in algebra than was necessary for the preceding methods of solving a-c circuit problems.

Imaginary Numbers, the j Operator, and Complex Numbers. In mathematics, the square root of a negative quantity is designated as an *imaginary* *number.* For example, there is no single answer for $\sqrt{-4}$, as there is no number that when multiplied by itself will become -4. In algebra, this is treated as $\sqrt{-4} = \sqrt{-1} \times \sqrt{4}$ or $i\sqrt{4} = i \times 2$, where i indicates that it is an imaginary quantity.

When working with electrical problems, the letter j, commonly called the *j operator,* is used in place of i to avoid confusion with the general use of the letter i for current.

A *complex number* is defined as being the sum or difference of a real quantity and an imaginary quantity.

Graphical Representation. The a-c quantities of volts, amperes, and ohms are frequently represented graphically by vector diagrams. Understanding the solution of a-c circuit problems by use of vector algebra is greatly aided by understanding the graphical representation of the quantities involved. The basic graphic representation (Fig. 10-23*a*) has two axes: (1) the horizontal or X-X axis and (2) the vertical or Y-Y axis. With vector algebra, the X-X axis is called the axis of the reals and represents real positive numbers from 0 to $+X$ and real negative numbers from 0 to $-X$. The Y-Y axis is called the axis of the imaginaries and represents $+j$ values from 0 to $+Y$ and $-j$ values from 0 to $-Y$. The basic graph is divided into four sections called the first, second, third, and fourth quadrants (Fig. 10-23*b*). The quadrant in which a vector is located is dependent on the signs of its R and X components, as is shown in Fig. 10-23*b*.

Rectangular Notation. When a vector does not lie along either the X-X axis or the Y-Y axis, it can be represented by two substitute vectors, one on the X-X axis and the other on the Y-Y axis. The magnitudes of these substitute vectors must be of such values that their resultant effect will be

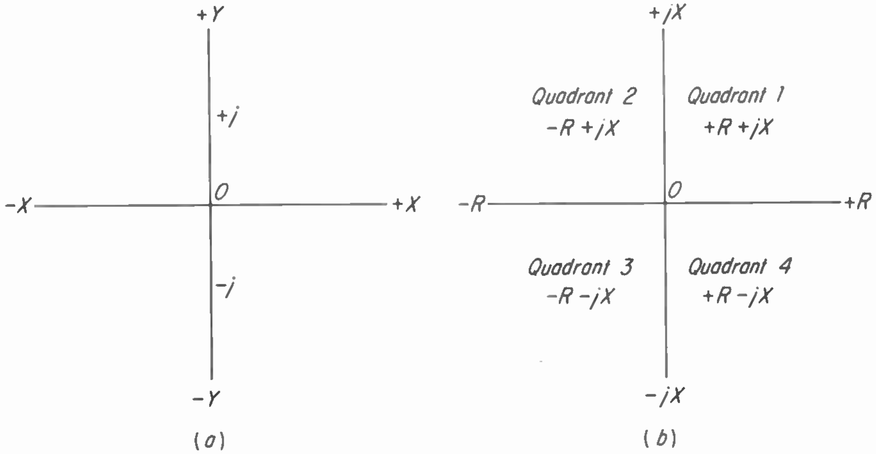


Fig. 10-23 (a) Basic graphical representation for alternating current quantities. (b) Location of vectors in terms of their resistance and reactance values.

the same in magnitude and direction as the original vector (see Art. 10-10). When vectors are treated in this manner, their resultant equations are said to be expressed in rectangular notation. For example, in Fig. 10-24 the vector for the current I_1 can be represented in rectangular notation as

$$\text{Vector } I_1 = I_1 \cos\theta_1 + I_1 \sin\theta_1 \tag{10-13}$$

or

$$\text{Vector } I_1 = I_1 \cos\theta_1 + jI_1 \sin\theta_1 \tag{10-14}$$

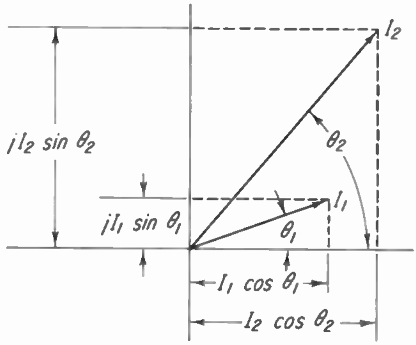


Fig. 10-24 Converting vectors into their equivalent rectangular components.

Vector-algebra equations can easily be set up to represent simple a-c series circuits. Because the resistance and reactance effects differ in phase by 90°, they may be represented on the X and the j axes if it is observed that (1) resistances have a $+X$ value, (2) inductive reactances have a $+j$ value, and (3) capacitive reactances have a $-j$ value. For example, the impedances of the circuits of Figs. 10-5 and 10-6 may be stated as

$Z = 10 + j100$ for Fig. 10-5*a*
$Z = 15 - j25$ for Fig. 10-5*b*
$Z = 10 + j100 + 15 - j25$ for Fig. 10-6

Addition, Subtraction, Multiplication, and Division of Vector Quantities. All four basic kinds of mathematical operations are required in the solution of one or another of the types of a-c circuits that might be encountered.

Addition of Vector Quantities. As the impedance of a series circuit is equal to the sum of its individual impedances, the total impedance can be obtained by adding the rectangular components in the manner used to add binomials in algebra.

EXAMPLE **10-19** The circuit of Fig. 10-6 represents a coil and a capacitor connected in series. Set up the equations in rectangular notation form for (*a*) the coil, (*b*) the capacitor, (*c*) the line. (*d*) Calculate the numerical value of the line impedance.

GIVEN: $R_{\text{coil}} = 10$ ohms $X_L = 100$ ohms $R_{\text{cap}} = 15$ ohms $X_C = 25$ ohms

FIND: (*a*) Equation for coil (*b*) Equation for capacitor (*c*) Equation for Z_{line} (*d*) Z_{line}

SOLUTION:

(*a*) $$Z_{\text{coil}} = 10 + j100$$

(*b*) $$Z_{\text{capacitor}} = \underline{15 - j25}$$

(*c*) $$Z_{\text{line}} = 25 + j75$$

(*d*) $$Z_{\text{line}} = \sqrt{25^2 + 75^2} = \sqrt{6{,}250} = 79 \text{ ohms}$$

This same principle may be applied to adding any type of vector by resolving each vector into its equivalent in-phase and quadrature components (Art. 10-10). For example, the two current vectors I_1 and I_2 of Fig. 10-24 may be added as,

$$I_1 = I_1 \cos \theta_1 + jI \sin \theta_1$$
$$I_2 = I_2 \cos \theta_2 + jI_2 \sin \theta_2$$
$$\underset{\cdot}{I}_T = \underset{\cdot}{I}_1 + \underset{\cdot}{I}_2 = I_1 \cos \theta_1 + I_2 \cos \theta_2 + jI_1 \sin \theta_1 + jI_2 \sin \theta_2$$

Note: The · under the *I* indicates vector addition, not arithmetic.

EXAMPLE 10-20 In Fig. 10-24, I_1 and I_2 have values of 10 and 20 amp, respectively, and the values of θ_1 and θ_2 are 20 and 50°, respectively. Write the equations for (*a*) I_1, (*b*) I_2, (*c*) $\underset{\cdot}{I}_1 + \underset{\cdot}{I}_2$. (*d*) What is the phase angle of $\underset{\cdot}{I}_1 + \underset{\cdot}{I}_2$ with respect to the *X* axis? (*e*) What is the numerical value of $\underset{\cdot}{I}_1 + \underset{\cdot}{I}_2$?

GIVEN: $I_1 = 10$ amp $\quad I_2 = 20$ amp $\quad \theta_1 = 20°$ $\quad \theta_2 = 50°$

FIND: (*a*) Equation of I_1 (*b*) Equation of I_2 (*c*) Equation of $\underset{\cdot}{I}_1 + \underset{\cdot}{I}_2$
(*d*) θ_T (*e*) I_T

SOLUTION:

(*a*) $$I_1 = 10 \cos 20° + j10 \sin 20° = 10 \times 0.940 + j10 \times 0.342 = 9.40 + j3.42$$

(*b*) $$I_2 = 20 \cos 50° + j20 \sin 50° = 20 \times 0.643 + j20 \times 0.766 = \underline{12.86 + j15.32}$$

(*c*) $$\underset{\cdot}{I}_1 + \underset{\cdot}{I}_2 = 22.26 + j18.74$$

(*d*) $$\tan \theta_T = \frac{18.74}{22.26} = 0.842$$

$$\theta_T = 40° \text{ leading}$$

(*e*) $$I_T = \frac{22.26}{\cos \theta_T} = \frac{22.26}{0.766} = 29 \text{ amp}$$

Subtraction of Vector Quantities. Subtraction of vector quantities is performed in a manner similar to addition.

EXAMPLE 10-21 Adjacent coils of a certain alternator each have an induced emf of 20 volts and a phase displacement of 10° (Fig. 10-25). What are the resultant voltage and its phase angle with respect to e_1 when the coils are connected in series so that they are (*a*) aiding, (*b*) opposing?

GIVEN: $e_1 = 20$ volts $\quad e_2 = 20$ volts $\quad \theta = 10°$

FIND: (*a*) $\underset{\cdot}{e}_1 + \underset{\cdot}{e}_2, \theta$ (*b*) $\underset{\cdot}{e}_1 - \underset{\cdot}{e}_2, \theta$

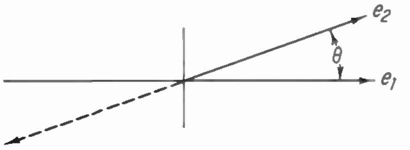


Fig. 10-25

SOLUTION:

(*a*)

$$\begin{aligned} e_1 &= 20\cos 0^\circ + j20\sin 0^\circ = 20\times 1.000 + j20\times 0.000 = 20 \quad + j0.000\\ e_2 &= 20\cos 10^\circ + j20\sin 10^\circ = 20\times 0.985 + j20\times 0.173 = \underline{19.7 + j3.46}\\ &\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad \underset{\cdot}{e}_1 + \underset{\cdot}{e}_2 = 39.7 + j3.46 \end{aligned}$$

$$\tan\theta = \frac{3.46}{39.7} = 0.087 \qquad \theta = 5^\circ \text{ leading}$$

$$\underset{\cdot}{e}_1 + \underset{\cdot}{e}_2 = \frac{39.7}{\cos 5^\circ} = \frac{39.7}{0.996} = 39.8 \text{ volts}$$

(*b*)

$$\begin{aligned} e_1 &= 20\cos 0^\circ + j20\sin 0^\circ = 20\times 1.000 + j20\times 0.000 = 20 \quad + j0.00\\ e_2 &= 20\cos 10^\circ + j20\sin 10^\circ = 20\times 0.985 + j20\times 0.173 = \underline{19.7 + j3.46}\\ &\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad \underset{\cdot}{e}_1 - \underset{\cdot}{e}_2 = \ \ 0.3 - j3.46 \end{aligned}$$

$$\tan\theta = \frac{-3.46}{0.3} = -11.5 \qquad \theta = 85^\circ \text{ lagging}$$

$$\underset{\cdot}{e}_1 - \underset{\cdot}{e}_2 = \frac{0.3}{-\cos 85^\circ} = \frac{0.3}{-0.087} = -3.45 \text{ volts}$$

The dash-line vector in Fig. 10-25 is not needed for the solution of Example 10-21 but has been added to help visualize the magnitude and position of the resultant vector for the case of $\underset{\cdot}{e}_1 - \underset{\cdot}{e}_2$.

Multiplication of Vector Quantities. The need for multiplication and division of vector quantities occurs when solving for the impedance of two circuits connected in parallel by use of the equation

$$Z_T = \frac{Z_1Z_2}{Z_1 + Z_2} \tag{10-15}$$

When multiplying vector quantities, ordinary algebraic procedure is used, except that wherever j^2 occurs in the final result, its equivalent of -1 is to be substituted.

EXAMPLE 10-22 Find the product of $(12 + j36)$ and $(4 + j6)$.

SOLUTION:

$$\begin{array}{l} 12 + j36\\ \underline{\ \ 4 + j6\qquad\ }\\ 48 + j144\\ \underline{\qquad + j72 + j^2216}\\ 48 + j216 + j^2216 = 48 + j216 + (-1)216 = -168 + j216 \end{array}$$

Division of Vector Quantities. The procedure used to divide one vector quantity by another is (1) rationalize the denominator by multiplying both the numerator and the denominator by the conjugate of the denominator, (2) perform the multiplication operations indicated after step 1 is completed, (3) divide the final numerator by the rationalized number found for the denominator in step 2. The conjugate of a vector quantity is obtained by merely changing the sign between the two members; for example, the conjugate of $(4 + j6)$ is $(4 - j6)$.

EXAMPLE **10-23** Divide the vector quantity $(12 + j36)$ by $(4 - j6)$.

SOLUTION:

$$\frac{12 + j36}{4 - j6} \times \frac{4 + j6}{4 + j6} = \frac{48 + j72 + j144 + j^2 216}{16 - j^2 36} = \frac{48 + j216 + (-1)216}{16 - (-1)36}$$

$$\frac{-168 + j216}{52} = -3.23 + j4.15$$

10-14 Polar Vectors

Use of Polar Vectors. The mathematical operations of addition, subtraction, multiplication, division, powers and roots of vectors are sometimes required in solving advanced electrical problems. Multiplication and division can be accomplished more rapidly with polar vectors than with their equivalent rectangular vectors. Solving for powers and roots of vectors becomes a simple operation when polar vectors are used. With addition and subtraction, the polar vectors are first converted to their rectangular equivalents and then follows the same procedure as is used with addition and subtraction of rectangular vectors.

Polar Notation. When a vector is defined by both its magnitude and direction it is called a *polar vector.* The vectors I_1, I_2, I_3, and I_4 of Fig. 10-26 are expressed in polar notation as $4\underline{/25°}$ amp, $3\underline{/140°}$ amp, $3\underline{/-60°}$ amp, and $2\underline{/-147°}$ amp. In this manner, the information provided includes (1) the magnitude and the type of quantity represented by the vector, (2) its angle with respect to the positive portion of the X axis, (3) whether the angle is measured in a counterclockwise rotation (+ angles) or a clockwise rotation (− angles), and (4) the quadrant in which it is located. Angles between 0 and 90°, indicated as $\underline{/0 \text{ to } 90°}$, are in the first quadrant; angles between 90 and 180°, indicated as $\underline{/90 \text{ to } 180°}$, are in the second quadrant; angles between 0 and −90°, indicated as $\underline{/0 \text{ to } -90°}$, are in the fourth quadrant; angles between −90 and −180°, indicated as $\underline{/-90 \text{ to } -180°}$, are in the third quadrant.

Converting Rectangular Vectors to Polar Vectors. Any vector expressed in rectangular form, such as $a \pm jb$, can be converted to its polar equivalent by

$$a \pm jb = C\underline{/\theta} = \sqrt{a^2 + b^2}\,\underline{/\tan^{-1}\frac{jb}{a}} \qquad (10\text{-}16)$$

EXAMPLE 10-24 What is the polar equivalent of $8 + j6$ amp?

GIVEN: $8 + j6$ amp

FIND: $I\underline{/\theta}$

SOLUTION:

$$I = \sqrt{8^2 + 6^2} = \sqrt{64 + 36} = \sqrt{100} = 10 \text{ amp}$$

$$\tan\theta = \frac{6}{8} = 0.75 \qquad \text{and} \qquad \theta = 37°$$

therefore,

$$8 + j6 = 10\underline{/37°} \text{ amp}$$

Converting Polar Vectors to Rectangular Vectors. Any polar vector, such as $C\underline{/\theta}$, can be converted to its rectangular equivalent by

$$C\underline{/\theta} = a \pm jb = C\cos\theta \pm C\sin\theta \tag{10-17}$$

EXAMPLE 10-25 (*a*) What is the rectangular vector equation for the polar vector $50\underline{/-60°}$? (*b*) If the vector quantity is ohms, what kind of circuit does this represent?

GIVEN: (*a*) $50\underline{/-60°}$ (*b*) Ohms

FIND: (*a*) $a \pm jb$ (*b*) Circuit

SOLUTION:

(*a*) $a \pm jb = 50\cos 60° - j50\sin 60° = 50 \times 0.5 - j50 \times 0.866 = 25 - j43.3$

(*b*) This represents a circuit that has an equivalent series resistance of 25 ohms and an equivalent series capacitive reactance of 43.3 ohms.

Addition and Subtraction of Polar Vectors. Polar vectors may be added or subtracted by converting them to their equivalent rectangular notation and then proceding in the same manner as with rectangular vectors.

EXAMPLE 10-26 Find the sum of I_1 and I_2 of Fig. 10-26.

GIVEN: $I_1 = 4\underline{/25°}$ amp $\qquad I_2 = 3\underline{/140°}$ amp

FIND: $I_1 + I_2$

SOLUTION:

$$\begin{aligned} I_1 &= 4\cos 25° + j4\sin 25° = 4 \times 0.906 + j4 \times 0.422 = && 3.62 + j1.69 \\ I_2 &= -3\cos 40° + j3\sin 40° = -3 \times 0.766 + j3 \times 0.643 = && \underline{-2.3 \;\; + j1.93} \\ & && I_1 + I_2 = \;\; 1.32 + j3.62 \end{aligned}$$

As both members of the resultant equation are positive, the resultant vector will be in the first quadrant.

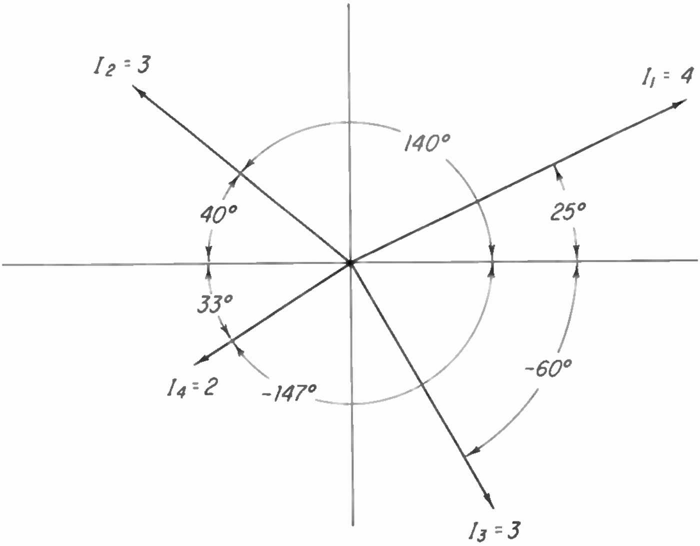


Fig. 10-26 Current vectors in the four quadrants.

EXAMPLE 10-27 What is the result if I_4 is subtracted from I_3 of Fig. 10-26?

GIVEN: $I_3 = 3\underline{/-60^\circ}$ $I_4 = 2\underline{/-147^\circ}$

FIND: $\dot{I}_3 - \dot{I}_4$

SOLUTION:

$$\begin{aligned} I_3 &= 3\cos 60^\circ - j3\sin 60^\circ = 3 \times 0.5 - j3 \times 0.866 = &1.5 - j2.6 \\ I_4 &= -2\cos 33^\circ - j2\sin 33^\circ = -2 \times 0.839 - j2 \times 0.544 = &\underline{-1.68 - j1.09} \\ &\qquad\qquad\qquad\qquad\qquad\qquad\qquad \dot{I}_3 - \dot{I}_4 = &3.18 - j1.51 \end{aligned}$$

The plus and minus signs, respectively, of the two members of the resultant equation indicate that the resultant vector will be in the fourth quadrant.

Multiplication of Polar Vectors. The product of two polar vectors and the resultant angle is obtained by multiplying their magnitudes and adding their angles.

$$A\underline{/\theta_a} \times B\underline{/\theta_b} = AB\underline{/\theta_a + \theta_b} \qquad (10\text{-}18)$$

EXAMPLE 10-28 (*a*) Using polar vectors, find the product and the resultant angle of I_1 and I_2 of Fig. 10-26. (*b*) Find the product of I_1 and I_2 by multiplying their rectangular vectors. *Note:* The rectangular vectors are available in Example 10-26. (*c*) Convert the results of solution (*b*) to a polar vector and compare this answer with the result of solution (*a*).

GIVEN: $I_1 = 4\underline{/25^\circ}$ amp $I_2 = 3\underline{/140^\circ}$ amp

FIND: (*a*) $I_1 \times I_2$ (polar) (*b*) $I_1 \times I_2$ (rect.) (*c*) $I_1 \times I_2$ (polar)

SOLUTION:

(a) $I_1 \times I_2 = 4\underline{/25^\circ} \times 3\underline{/140^\circ} = 4 \times 3\underline{/25^\circ + 140^\circ} = 12\underline{/165^\circ}$ amp

(b)

$$\begin{array}{l} 3.62 + j1.69 \\ \underline{-2.3 + j1.93} \\ -8.33 - j3.88 \\ \underline{ + j6.99 + j^2 3.26} \\ -8.33 + j3.11 + j^2 3.26 = -8.33 + j3.11 - 3.26 = -11.59 + j3.11 \end{array}$$

(c)

$$\sqrt{-11.59^2 + 3.11^2} = \sqrt{144} = 12 \text{ amp}$$

$$\tan\theta = \frac{3.11}{-11.59} = -0.27 \qquad \theta = 180^\circ - 15^\circ = 165^\circ$$

therefore, $I_1 \times I_2 = 12\underline{/165^\circ}$ amp

Example 10-28 illustrates the advantage of using polar vectors over rectangular vectors in multiplication operations.

Division of Polar Vectors. The quotient of two polar vectors and the resultant angle is obtained by taking the quotient of their magnitudes and subtracting their angles.

$$\frac{A\underline{/\theta_a}}{B\underline{/\theta_b}} = \frac{A}{B}\underline{/\theta_a - \theta_b} \tag{10-19}$$

EXAMPLE 10-29 For the vectors of Fig. 10-26, find the quotient and the resulting angle for (a) I_2 divided by I_1, (b) I_1 divided by I_3.

GIVEN: $I_1 = 4\underline{/25^\circ} \qquad I_2 = 3\underline{/140^\circ} \qquad I_3 = 3\underline{/-60^\circ}$

FIND: (a) $\dfrac{I_2}{I_1}$ (b) $\dfrac{I_1}{I_3}$

SOLUTION:

(a)

$$\frac{I_2}{I_1} = \frac{3\underline{/140^\circ}}{4\underline{/25^\circ}} = \frac{3}{4}\underline{/140^\circ - 25^\circ} = 0.75\underline{/115^\circ}$$

(b)

$$\frac{I_1}{I_3} = \frac{4\underline{/25^\circ}}{3\underline{/-60^\circ}} = \frac{4}{3}\underline{/25^\circ - (-60^\circ)} = 1.33\underline{/85^\circ}$$

In the same manner as was done with the multiplication problem of Example 10-28, it can be shown that it is much easier to divide with polar vectors than to divide with their equivalent rectangular vectors.

Powers of Polar Vectors. Polar vectors may be raised to any power by (1) increasing the magnitude of the vector by the power and (2) multiplying the angle by the power.

$$(A\underline{/\theta})^n = A^n\underline{/n\theta} \tag{10-20}$$

EXAMPLE 10-30 Perform the following operations: (*a*) $(4\angle 28^\circ)^2$, (*b*) $(0.5\angle 70^\circ)^2$, (*c*) $(3\angle 32^\circ)^3$, (*d*) $(2\angle 35^\circ)^4$.

SOLUTION:

(*a*) $$(4\angle 28^\circ)^2 = 4^2\angle 2 \times 28^\circ = 16\angle 56^\circ$$

(*b*) $$(0.5\angle 70^\circ)^2 = 0.5^2\angle 2 \times 70^\circ = 0.25\angle 140^\circ$$

(*c*) $$(3\angle 32^\circ)^3 = 3^3\angle 3 \times 32^\circ = 27\angle 96^\circ$$

(*d*) $$(2\angle 35^\circ)^4 = 2^4\angle 4 \times 35^\circ = 16\angle 140^\circ$$

Roots of Polar Vectors. Any root of a polar vector may be found by (1) taking the root of the magnitude of the vector and (2) dividing the angle by the root.

$$\sqrt[n]{A\angle\theta} = \sqrt[n]{A}\,\angle\frac{\theta}{n} \tag{10-21}$$

EXAMPLE 10-31 Perform the following operations: (*a*) $\sqrt{64\angle -60^\circ}$, (*b*) $\sqrt{0.25\angle 140^\circ}$, (*c*) $\sqrt[3]{64\angle 120^\circ}$, (*d*) $\sqrt[4]{81\angle 110^\circ}$.

SOLUTION:

(*a*) $$\sqrt{64\angle -60^\circ} = \sqrt{64}\,\angle -60^\circ/2 = 8\angle -30^\circ$$

(*b*) $$\sqrt{0.25\angle 140^\circ} = \sqrt{0.25}\,\angle 140^\circ/2 = 0.5\angle 70^\circ$$

(*c*) $$\sqrt[3]{64\angle 120^\circ} = \sqrt[3]{64}\,\angle 120^\circ/3 = 4\angle 40^\circ$$

(*d*) $$\sqrt[4]{81\angle 110^\circ} = \sqrt[4]{81}\,\angle 110^\circ/4 = 3\angle 27.5^\circ$$

Reciprocal of a Polar Vector. The reciprocal of a polar vector may be obtained by (1) taking the reciprocal of its magnitude and (2) transferring the angle from the denominator to the numerator and reversing its sign.

$$\frac{1}{A\angle\theta} = \frac{1}{A}\angle -\theta \tag{10-22}$$

EXAMPLE 10-32 What is the reciprocal of $5\angle 40^\circ$?

SOLUTION:

$$\frac{1}{5\angle 40^\circ} = \frac{1}{5}\angle -40^\circ = 0.2\angle -40^\circ$$

10-15 Applications of Vector Algebra

Series Circuits. The impedance of a series circuit containing a number of resistances, inductances, and/or capacitances can readily be found by vector algebra. Caution must be observed to designate the inductive reactances as $+jX$ and the capacitive reactances as $-jX$.

EXAMPLE 10-33 (*a*) What is the rectangular vector equation for the circuit of Example 10-6? (*b*) What circuit characteristics does this equation indicate? (*c*) What is the polar vector representation of this circuit? (*d*) What circuit characteristics does this polar vector indicate?

GIVEN: Circuit of Example 10-6

FIND: (*a*) $R + jX$ (*b*) cct. char. (*c*) $Z\underline{/\theta}$ (*d*) cct. char.

SOLUTION:

(*a*) $$Z = j10 + 28 + j20 - j100 + 2 + j60 - j50 + 10 = 40 - j60$$

(*b*) The above equation indicates a series circuit that has an equivalent resistance of 40 ohms and an equivalent capacitive reactance of 60 ohms. The series circuit represented by this equation could have (1) one resistor of 40 ohms and one capacitor whose reactance is 60 ohms or (2) a number of resistances totaling 40 ohms and a number of capacitors or inductors and capacitors totaling 60 ohms equivalent capacitive reactance.

(*c*) When $Z = 40 - j60$, $\tan\theta = \frac{-60}{40} = -1.5$ and $\theta = 56.5°$ (lead)

$$Z = \frac{40}{\cos 56.5°} = \frac{40}{0.552} = 72.4 \text{ ohms}$$

therefore, $Z = 72.4\underline{/-56.5°}$ ohms

(*d*) The polar vector $72.4\underline{/-56.5°}$ ohms indicates an impedance of 72.4 ohms that when connected to a power source of its rated frequency will result in a current that leads the voltage by 56.5°.

Any further calculations for quantities such as the current, power, power factor, apparent power, and the voltages at the individual circuit elements can be performed in the same manner as in Example 10-12.

Parallel Circuits. With vector algebra, it becomes possible to calculate the total impedance of a circuit directly, that is, without first finding currents and then solving for the impedance by use of Ohm's law. Also, with vector quantities, it becomes possible to use the same basic formulas for solving a-c circuits as were used for solving d-c circuits. For example, in a d-c circuit

$$R_T = \frac{1}{\frac{1}{r_1} + \frac{1}{r_2} + \frac{1}{r_3}, \text{ etc.}} \tag{4-15}$$

and

$$R_T = \frac{r_1 r_2}{r_1 + r_2} \tag{4-15a}$$

For corresponding a-c circuits,

$$Z_T = \frac{1}{\dfrac{1}{Z_1} + \dfrac{1}{Z_2} + \dfrac{1}{Z_3}, \text{ etc.}} \tag{10-23}$$

and

$$Z_T = \frac{Z_1 Z_2}{Z_1 + Z_2} \tag{10-15}$$

EXAMPLE 10-34 (*a*) Calculate the impedance and phase angle of the circuit given in Example 10-14 by vector algebra. (*b*) What is its simple equivalent circuit?

GIVEN: Circuit of Example 10-14

FIND: (*a*) Z_T, θ_T (*b*) Equiv. cct.

SOLUTION:

(*a*)

$$Z_T = \frac{1}{\dfrac{1}{Z_1} + \dfrac{1}{Z_2} + \dfrac{1}{Z_3}, \text{ etc.}}$$

$$Z_T = \frac{1}{\dfrac{1}{20} + \dfrac{1}{j20} + \dfrac{1}{40} + \dfrac{1}{-j40} + \dfrac{1}{j40} + \dfrac{1}{-j80}}$$

Using 80 as the lowest common denominator and changing the signs of j when transferring it from the denominator to the numerator,

$$Z_T = \frac{1}{\dfrac{4 - j4 + 2 + j2 - j2 + j}{80}} = \frac{1}{\dfrac{6 - j3}{80}} = \frac{80}{6 - j3}$$

then rationalizing,

$$Z_T = \frac{80}{6 - j3} \times \frac{6 + j3}{6 + j3} = \frac{480 + j240}{36 + 9} = 10.66 + j5.33$$

and

$$Z_T = \sqrt{10.66^2 + 5.33^2} \cong \sqrt{143} \cong 11.92 \text{ ohms}$$

$$\tan\theta = \frac{5.33}{10.66} = 0.5 \qquad \theta = 26.5° \text{ lag}$$

(*b*) A 10.66-ohm resistance connected in series with a 5.33-ohm inductive reactance.

EXAMPLE 10-35 A certain circuit consists of a 20-ohm inductive reactance and a 40-ohm resistance connected in parallel. Find its impedance by (*a*) Eq. (10-15), (*b*) Eq. (10-23), (*c*) the method used in Art. 10-9.

GIVEN: $X_L = 20$ ohms $R = 40$ ohms

FIND: (*a*) Z_T (*b*) Z_T (*c*) Z_T

SOLUTION:

(*a*)

$$Z_T = \frac{Z_1 Z_2}{Z_1 + Z_2} = \frac{40 \times j20}{40 + j20} = \frac{j800}{40 + j20}$$

rationalizing,

$$Z_T = \frac{j800}{40 + j20} \times \frac{40 - j20}{40 - j20} = \frac{j32{,}000 - j^2 16{,}000}{1{,}600 + 400} = \frac{16{,}000 + j32{,}000}{2{,}000} = 8 + j16$$

then

$$Z_T = \sqrt{8^2 + 16^2} = \sqrt{320} = 17.9 \text{ ohms}$$

(*b*)

$$Z_T = \frac{1}{\frac{1}{Z_1} + \frac{1}{Z_2}} = \frac{1}{\frac{1}{40} + \frac{1}{j20}} = \frac{1}{\frac{1 - j2}{40}}$$

rationalizing,

$$Z_T = \frac{40}{1 - j2} \times \frac{1 + j2}{1 + j2} = \frac{40 + j80}{1 + 4} = 8 + j16$$

$$\tan \theta = \frac{16}{8} = 2 \qquad \text{and} \qquad \theta = 63.5^\circ \text{ lag}$$

$$Z_T = \frac{8}{\cos 63.5^\circ} = \frac{8}{0.446} = 17.9 \text{ ohms}$$

(*c*) Using an assumed line voltage of 200 volts, then

$$I_R = \frac{E}{R} = \frac{200}{40} = 5 \text{ amp} \qquad I_{X_L} = \frac{E}{X_L} = \frac{200}{20} = 10 \text{ amp}$$

$$I_T = \sqrt{I_R{}^2 + I_{X_L}{}^2} = \sqrt{5^2 + 10^2} = \sqrt{125} = 11.2 \text{ amp}$$

$$Z_T = \frac{E_T}{I_T} = \frac{200}{11.2} = 17.9 \text{ ohms}$$

EXAMPLE 10-36 Solve Example 10-35 by multiplication and division of polar vectors using Eqs. (10-15), (10-18), and (10-19).

GIVEN: $Z_1 = 40\underline{/0^\circ}$ $\qquad Z_2 = 20\underline{/90^\circ}$

FIND: $Z_T\underline{/\theta_T}$

SOLUTION:

$$Z_1 + Z_2 = 40 + j20 = \sqrt{40^2 + 20^2} = 44.7 \qquad \theta = \tan^{-1}\frac{20}{40} = 26.5^\circ$$

then

$$\frac{Z_1 Z_2}{Z_1 + Z_2} = \frac{40\underline{/0^\circ} \times 20\underline{/90^\circ}}{44.7\underline{/26.5^\circ}} = \frac{800\underline{/90^\circ}}{44.7\underline{/26.5^\circ}} = 17.9\underline{/90^\circ - 26.5^\circ} = 17.9\underline{/63.5^\circ}$$

Parallel-series Circuits. From the preceding text and examples, it should be conceivable that a parallel-series circuit problem may be solved by any one of several methods of approach. By practice, one learns to recognize which method to use that will shorten the work involved in solving such problems.

EXAMPLE 10-37 A parallel-series circuit is shown in Fig. 10-27. (*a*) Find the impedance, current, power, power factor, and phase angle for each branch. (*b*) Find the total impedance, the line current, the line phase angle, and the line power factor. (*c*) Give the equation for its equivalent series circuit.

GIVEN: Fig. 10-27

FIND: See example.

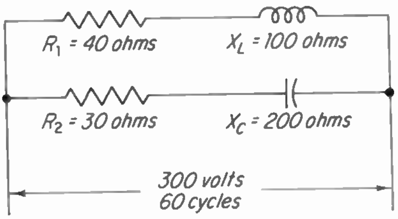


Fig. 10-27

SOLUTION:

(*a*) $Z_1 = 40 + j100$ also, $\sqrt{40^2 + 100^2}\,\underline{/\tan^{-1}\left(\frac{100}{40}\right)} = 107.6\underline{/68^\circ}$ ohms

$$I_1 = \frac{E}{Z_1} = \frac{300}{107.6} = 2.79 \text{ amp}$$

$$P_1 = I_1{}^2R_1 = 2.79^2 \times 40 = 312 \text{ watts}$$

$$\text{PF}_1 = \cos\theta_1 = \cos 68^\circ = 0.375$$

$$\theta_1 = 68^\circ \text{ lag}$$

$Z_2 = 30 - j200$ also, $\sqrt{30^2 + 200^2}\,\underline{/\tan^{-1}\left(\frac{-200}{30}\right)} = 202\underline{/-81.5^\circ}$ ohms

$$I_2 = \frac{E}{Z_2} = \frac{300}{202} = 1.485 \text{ amp}$$

$$P_2 = I_2{}^2R_2 = 1.485^2 \times 30 = 66 \text{ watts}$$

$$\text{PF}_2 = \cos\theta_2 = \cos -81.5^\circ = 0.148$$

$$\theta_2 = 81.5^\circ \text{ lead}$$

(*b*) where $Z_1 = 107.6\underline{/68^\circ}$ $Z_2 = 202\underline{/-81.5^\circ}$

and $Z_1 + Z_2 = 40 + j100 + 30 - j200 = 70 - j100$

$$= \sqrt{70^2 + 100^2}\,\underline{/\tan^{-1}\left(\frac{-100}{70}\right)} = 122\underline{/-55^\circ} \text{ ohms}$$

then
$$Z_T = \frac{Z_1Z_2}{Z_1 + Z_2} = \frac{107.6\underline{/68^\circ} \times 202\underline{/-81.5^\circ}}{122\underline{/-55^\circ}} = \frac{21{,}735\underline{/-13.5^\circ}}{122\underline{/-55^\circ}}$$

$$= 178\underline{/-13.5^\circ - (-55^\circ)} = 178\underline{/41.5^\circ} \text{ ohms}$$

$$I_{\text{line}} = \frac{E}{Z_T} = \frac{300}{178} = 1.685 \text{ amp}$$

$$\theta_{\text{line}} = 41.5^\circ \text{ lag}$$

$$\text{PF}_{\text{line}} = \cos\theta_{\text{line}} = \cos 41.5^\circ = 0.749$$

(*c*)
$$Z_{s\text{-}eq} = Z\cos\theta \pm jZ\sin\theta = 178\cos 41.5^\circ + j178\sin 41.5^\circ$$
$$= 178 \times 0.749 + j178 \times 0.663 = 133 + j118$$

EXAMPLE 10-38 The circuit of Example 10-37 is altered by adding a third parallel member as shown in Fig. 10-28. Find (*a*) the line impedance, (*b*) the line current, (*c*) the line power factor.

GIVEN: Fig. 10-28 and Example 10-37

FIND: (*a*) Z_{line} (*b*) I_{line} (*c*) PF_{line}

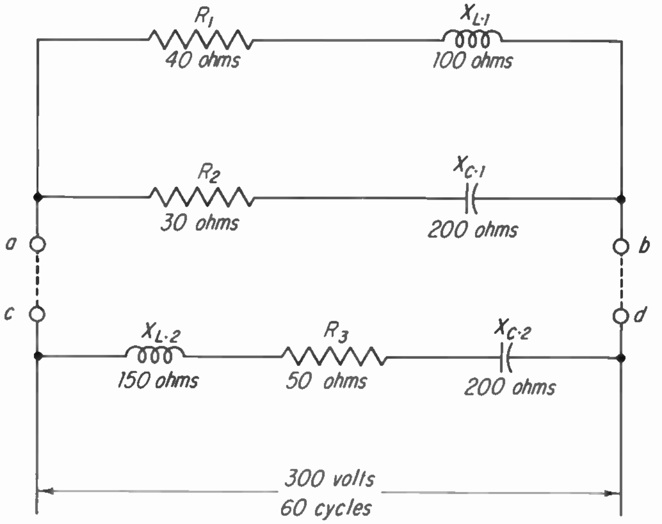


Fig. 10-28

SOLUTION: A simple method of finding the impedance of this circuit is to use the series equivalent circuit of Example 10-37 and combine it with the newly added member.

(*a*) From Example 10-37 $Z_1 = Z_{a\text{-}b} = 133 + j118$ also, $178\underline{/41.5^\circ}$

$Z_2 = Z_{c\text{-}d} = j150 + 50 - j200 = 50 - j50$ also, $70.7\underline{/-45^\circ}$

$Z_1 + Z_2 = 133 + j118 + 50 - j50 = 183 + j68$ also, $195\underline{/20.5^\circ}$

$$Z_{\text{line}} = \frac{Z_1 Z_2}{Z_1 + Z_2} = \frac{178\underline{/41.5^\circ} \times 70.7\underline{/-45^\circ}}{195\underline{/20.5^\circ}} = \frac{12{,}600\underline{/41.5^\circ + (-45^\circ)}}{195\underline{/20.5^\circ}}$$

$$= 64.6\underline{/-3.5^\circ - 20.5^\circ} = 64.6\underline{/-24^\circ} \text{ ohms}$$

(*b*) $$I_{\text{line}} = \frac{E}{Z_{\text{line}}} = \frac{300}{64.6} = 4.64 \text{ amp}$$

(*c*) $$\text{PF}_{\text{line}} = \cos\theta_{\text{line}} = \cos 24^\circ = 0.913$$

Series-parallel Circuits. The basic procedure in solving series-parallel circuits is (1) reduce each parallel member to its equivalent series circuit; (2) draw a new circuit diagram, substituting the equivalent resistances and reactances in place of the parallel groups; (3) solve for the impedance, current, phase angle, etc.

EXAMPLE 10-39 A series-parallel circuit is shown in Fig. 10-29. (*a*) Find the impedance, phase angle, current, power factor, and power of the line. (*b*) Find the voltage and current of each part of the circuit. (This is similar to Example 10-16.)

GIVEN: Fig. 10-29*a*

FIND: See example.

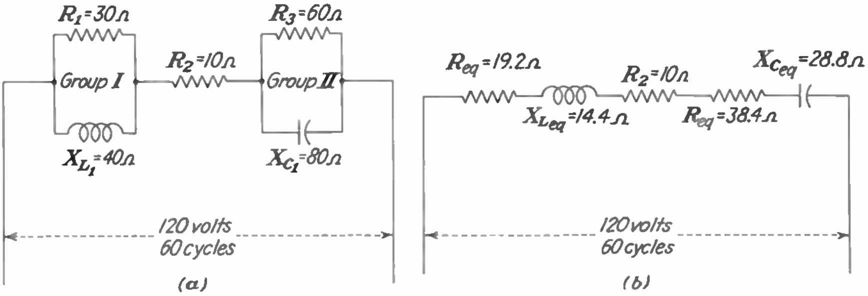


Fig. 10-29

SOLUTION:

(*a*)
$$Z_{G\cdot I} = \frac{Z_1 Z_2}{Z_1 + Z_2} = \frac{30 \times j40}{30 + j40} \times \frac{30 - j40}{30 - j40} = \frac{j1{,}200\,(30 - j40)}{900 + 1{,}600} = 19.2 + j14.4$$
$$= 24\underline{/37^\circ} \text{ ohms}$$

$$Z_{G\cdot II} = \frac{Z_1 Z_2}{Z_1 + Z_2} = \frac{60 \times (-j80)}{60 - j80} \times \frac{60 + j80}{60 + j80} = \frac{-j4{,}800\,(60 + j80)}{3{,}600 + 6{,}400}$$
$$= 38.4 - j28.8 = 48\underline{/-37^\circ} \text{ ohms}$$

The equivalent series circuit may now be drawn as shown in Fig. 10-29*b*.

$$Z_{\text{line}} = Z_{G\cdot 1} + R_2 + Z_{G\cdot II} = 19.2 + j14.4 + 10 + 38.4 - j28.8 = 67.6 - j14.4$$
$$= 69\underline{/-12^\circ} \text{ ohms}$$

$$I_{\text{line}} = \frac{E}{Z_{\text{line}}} = \frac{120}{69} = 1.74 \text{ amp}$$

$$\text{PF}_{\text{line}} = \cos \theta_{\text{line}} = \cos 12^\circ = 0.978$$

$$P_{\text{line}} = EI \text{ PF} = 120 \times 1.74 \times 0.978 = 204 \text{ watts}$$

(*b*)
$$E_{R\cdot 1} = E_{X_L\cdot 1} = E_{G\cdot I} = I_{\text{line}}\, Z_{G\cdot I} = 1.74 \times 24 = 41.8 \text{ volts}$$

$$I_{R\cdot 1} = \frac{E_{G\cdot I}}{R_1} = \frac{41.8}{30} = 1.39 \text{ amp}$$

$$I_{X_L\cdot 1} = \frac{E_{G\cdot I}}{X_{L\cdot 1}} = \frac{41.8}{40} = 1.04 \text{ amp}$$

$$E_{R\cdot 2} = I_{\text{line}} R_2 = 1.74 \times 10 = 17.4 \text{ volts}$$

$$I_{R\cdot 2} = I_{\text{line}} = 1.74 \text{ amp}$$

$$E_{R\cdot 3} = E_{X_{C\cdot 1}} = E_{G\cdot II} = I_{\text{line}} Z_{G\cdot II} = 1.74 \times 48 = 83.6 \text{ volts}$$

$$I_{R\cdot 3} = \frac{E_{G\cdot II}}{R_3} = \frac{83.6}{60} = 1.39 \text{ amp}$$

$$I_{X_{C\cdot 1}} = \frac{E_{G\cdot II}}{X_{C\cdot 1}} = \frac{83.6}{80} = 1.04 \text{ amp}$$

In comparison with Example 10-16, the same results are achieved with fewer mathematical operations.

10-16 Thévenin's Theorem

The solution of many complex circuit problems, both d-c and a-c, can be simplified by the use of Thévenin's theorem or a variation thereof.

Thévenin's theorem, which is rather lengthy, may be divided into the following five statements:

1. Any two-terminal linear network of power sources and impedances involved in supplying current to a given impedance (Fig. 10-30*a*) may be replaced by a single power source and a single impedance (Fig. 10-30*b*).

2. The voltage E_{eq} of the substituted power source is equal to the voltage at the terminals *a-b* (Fig. 10-30*a*) when the load Z_4 is disconnected.

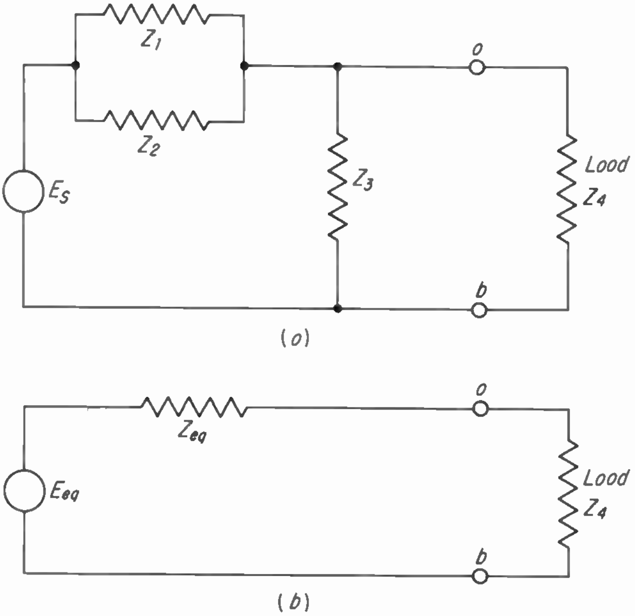


Fig. 10-30 (a) A series-parallel a-c circuit. (b) Thévenin's theorem equivalent series circuit.

3. The equivalent impedance Z_{eq} substituted for the source network of impedances is equal to the impedance at terminals *a-b* of Fig. 10-30*a* with the load Z_4 disconnected and the power source E_S replaced by an impedance equal to the internal impedance of the power source. When the impedance of the power source is assumed to be negligible, the power source is replaced by an impedance of zero ohms.

4. The current and voltage for the load impedance Z_4 in the equivalent circuit of Fig. 10-30*b* will be the same as the current and voltage for the load impedance Z_4 in the original circuit of Fig. 10-30*a*.

5. Thévenin's theorem applies to voltages and currents and to the power at only the load impedance.

EXAMPLE **10-40** Find the voltage, the current, and the power at each circuit element in Fig. 10-31*a*.

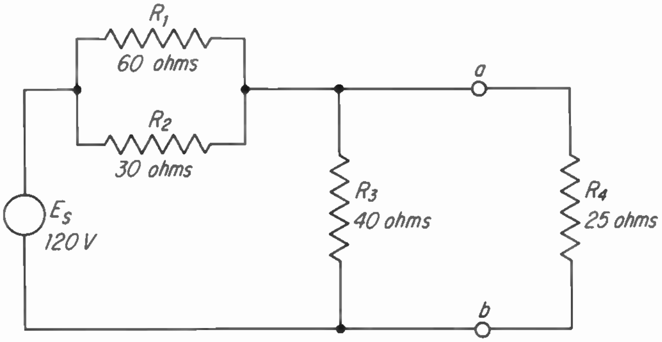


Fig. 10-31a

GIVEN: Fig. 10-31*a*

FIND: $E_{R\cdot1}$, $I_{R\cdot1}$, $P_{R\cdot1}$ $E_{R\cdot2}$, $I_{R\cdot2}$, $P_{R\cdot2}$ $E_{R\cdot3}$, $I_{R\cdot3}$, $P_{R\cdot3}$ $E_{R\cdot4}$, $I_{R\cdot4}$, $P_{R\cdot4}$

SOLUTION: Solving by Thévenin's theorem,

With R_4 disconnected,

$$E_{a-b} = \frac{E_S R_3}{R_3 + \dfrac{R_1 R_2}{R_1 + R_2}} = \frac{120 \times 40}{40 + \dfrac{60 \times 30}{60 + 30}} = 80 \text{ volts}$$

With R_4 disconnected and $R_{\text{source}} = 0$,

$$R_{a-b} = \frac{R_3 \times \dfrac{R_1 R_2}{R_1 + R_2}}{R_3 + \dfrac{R_1 R_2}{R_1 + R_2}} = \frac{40 \times \dfrac{60 \times 30}{60 + 30}}{40 + \dfrac{60 \times 30}{60 + 30}} = 13.33 \text{ ohms}$$

The equivalent circuit may now be drawn as shown in Fig. 10-31*b*.

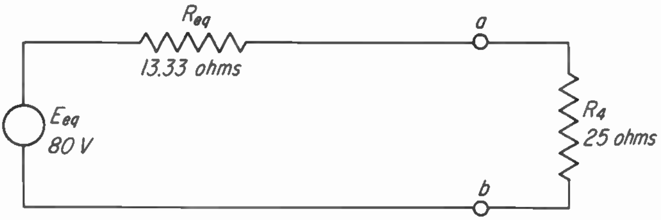


Fig. 10-31b

$$I_{R\cdot 4} = \frac{E_{eq}}{R_{eq} + R_4} = \frac{80}{13.33 + 25} = 2.085 \text{ amp}$$

$$E_{R\cdot 4} = I_4R_4 = 2.085 \times 25 = 52.2 \text{ volts}$$

From Fig. 10-31*a*

$$E_{R\cdot 3} = E_{R\cdot 4} = 52.2 \text{ volts}$$

$$I_{R\cdot 3} = \frac{E_{R\cdot 3}}{R_3} = \frac{52.2}{40} = 1.305 \text{ amp}$$

$$E_{R\cdot 2} = E_{R\cdot 1} = E_S - E_{R\cdot 3} = 120 - 52.2 = 67.8 \text{ volts}$$

$$I_{R\cdot 2} = \frac{E_{R\cdot 2}}{R_2} = \frac{67.8}{30} = 2.26 \text{ amp}$$

$$E_{R\cdot 1} = E_{R\cdot 2} = 67.8 \text{ volts}$$

$$I_{R\cdot 1} = \frac{E_{R\cdot 1}}{R_1} = \frac{67.8}{60} = 1.13 \text{ amp}$$

$$P_{R\cdot 1} = E_{R\cdot 1}I_{R\cdot 1} = 67.8 \times 1.13 = 76.5 \text{ watts}$$

$$P_{R\cdot 2} = E_{R\cdot 2}I_{R\cdot 2} = 67.8 \times 2.26 = 153 \text{ watts}$$

$$P_{R\cdot 3} = E_{R\cdot 3}I_{R\cdot 3} = 52.2 \times 1.305 = 68 \text{ watts}$$

$$P_{R\cdot 4} = E_{R\cdot 4}I_{R\cdot 4} = 52.2 \times 2.085 = 109 \text{ watts}$$

To illustrate the advantage of using Thévenin's theorem in solving problems, Examples 4-13 and 4-14 will now be solved by means of Thévenin's theorem.

EXAMPLE 10-41 Find the voltage and current at each circuit element for Fig. 10-32. (This is the same as Example 4-13.)

GIVEN: Fig. 10-32*a*

FIND: E and I at each resistor

SOLUTION:

With terminals *c-d* opened,

$$E_{c-d} = E_{a-b} = \frac{E_S(R_5 + R_6)}{R_1 + R_5 + R_6} = \frac{360 \times (300 + 300)}{20 + 300 + 300} = 348.4 \text{ volts}$$

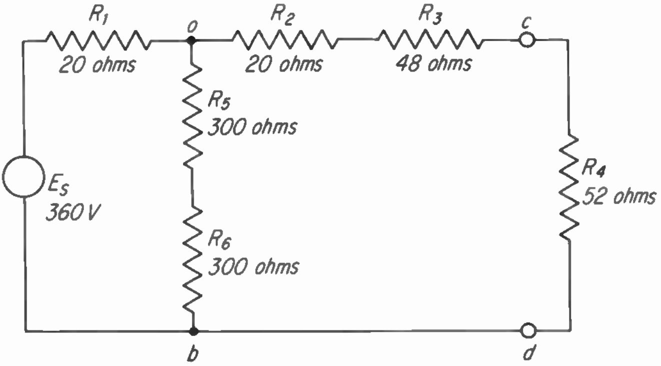


Fig. 10-32a

With terminals *c-d* opened and E_S replaced by $R_S = 0$,

$$R_{c-d} = (R_3 + R_2) + \frac{R_1(R_5 + R_6)}{R_1 + (R_5 + R_6)} = 48 + 20 + \frac{20 \times (300 + 300)}{20 + (300 + 300)}$$
$$= 48 + 20 + 19.35 = 87.35 \text{ ohms}$$

The equivalent circuit may now be drawn as shown in Fig. 10-32*b*.

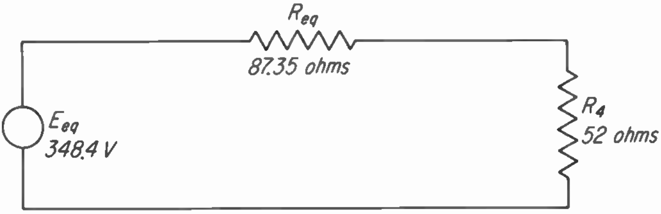


Fig. 10-32b

$$I_{R\cdot 4} = \frac{E_{eq}}{R_{eq} + R_4} = \frac{348.4}{87.35 + 52} = 2.50 \text{ amp}$$

$$E_{R\cdot 4} = I_{R\cdot 4}R_4 = 2.50 \times 52 = 130 \text{ volts}$$

From Fig. 10-32*a*,

$$I_{R\cdot 3} = I_{R\cdot 4} = 2.50 \text{ amp}$$

$$E_{R\cdot 3} = I_{R\cdot 3}R_3 = 2.50 \times 48 = 120 \text{ volts}$$

$$I_{R\cdot 2} = I_{R\cdot 4} = 2.50 \text{ amp}$$

$$E_{R\cdot 2} = I_{R\cdot 2}R_2 = 2.50 \times 20 = 50 \text{ volts}$$

$$E_{a-b} = E_{R\cdot 2} + E_{R\cdot 3} + E_{R\cdot 4} = 50 + 120 + 130 = 300 \text{ volts}$$

$$I_{R\cdot 5} = \frac{E_{a-b}}{R_5 + R_6} = \frac{300}{300 + 300} = 0.50 \text{ amp}$$

$$E_{R\cdot 5} = I_{R\cdot 5}R_5 = 0.50 \times 300 = 150 \text{ volts}$$

$$I_{R\cdot 6} = I_{R\cdot 5} = 0.50 \text{ amp}$$

$$E_{R\cdot 6} = I_{R\cdot 6}R_6 = 0.50 \times 300 = 150 \text{ volts}$$

$$E_{R\cdot 1} = E_S - (E_{R\cdot 5} + E_{R\cdot 6}) = 360 - (150 + 150) = 60 \text{ volts}$$

$$I_{R\cdot 1} = \frac{E_{R\cdot 1}}{R_1} = \frac{60}{20} = 3 \text{ amp}$$

EXAMPLE 10-42 Find the voltage and current at each circuit element for Fig. 10-33. (This is the same as Example 4-14.)

GIVEN: Fig. 10-33*a*

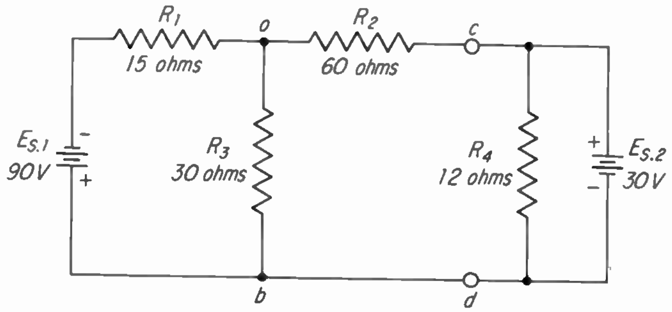


Fig. 10-33a

FIND: *E* and *I* at each resistor

SOLUTION:

With terminals *c-d* opened,

$$E_{c-d} = E_{a-b} = \frac{E_{S\cdot 1}R_3}{R_1 + R_3} = \frac{90 \times 30}{15 + 30} = 60 \text{ volts}$$

With terminals *c-d* opened and $E_{S\cdot 1}$ replaced by $R_{S\cdot 1} = 0$,

$$R_{c-d} = R_2 + \frac{R_1R_3}{R_1 + R_3} = 60 + \frac{15 \times 30}{15 + 30} = 60 + 10 = 70 \text{ ohms}$$

The equivalent circuit may now be drawn as shown in Fig. 10-33*b*.

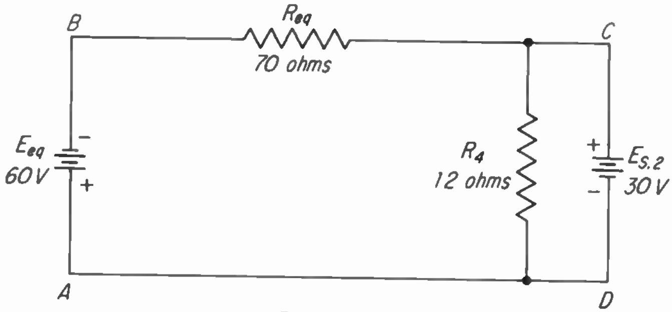


Fig. 10-33b

Using Kirchhoff's voltage law,

$$E_{ABCD} = -60 + I_{eq}R_{eq} - 30 = 0 \qquad \text{or} \qquad -60 + 70I_{eq} - 30 = 0$$

and $$I_{eq} = \frac{90}{70} = 1.285 \text{ amp}$$

From Fig. 10-33*a*,

$$E_{R\cdot 4} = E_{S\cdot 2} = 30 \text{ volts}$$

$$I_{R\cdot 4} = \frac{E_{R\cdot 4}}{R_4} = \frac{30}{12} = 2.50 \text{ amp}$$

$$I_{R\cdot 2} = I_{eq} = 1.285 \text{ amp}$$

$$E_{R\cdot 2} = I_{R\cdot 2}R_2 = 1.285 \times 60 = 77.1 \text{ volts}$$

Using Kirchhoff's voltage law,

$$E_{bacdb} = -30I_{R\cdot 3} + 60I_{R\cdot 2} - 30 = 0 \qquad \text{or} \qquad 30I_{R\cdot 3} = 77.1 - 30$$

$$I_{R\cdot 3} = \frac{77.1 - 30}{30} = 1.57 \text{ amp}$$

$$E_{R\cdot 3} = I_{R\cdot 3}R_3 = 1.57 \times 30 = 47.1 \text{ volts}$$

$$E_{R\cdot 1} = E_{S\cdot 1} - E_{R\cdot 3} = 90 - 47.1 = 42.9 \text{ volts}$$

$$I_{R\cdot 1} = \frac{E_{R\cdot 1}}{R_1} = \frac{42.9}{15} = 2.86 \text{ amp}$$

Comparing the solutions of Examples 10-41 and 10-42 with Examples 4-13 and 4-14, respectively, shows that the use of Thévenin's theorem eliminates the need of solving with simultaneous equations and thereby shortens the amount of work involved.

EXAMPLE 10-43 The circuit of Fig. 10-34*a* represents a low-pass filter circuit designed to pass currents of frequencies below 10 kc and to prevent currents above 10 kc from reaching the load resistance R_2. (*a*) What is the value of L if the inductive reactance is 4,000 ohms at the cutoff frequency of 10 kc? (*b*) What is the value of C if the capacitive reactance is 2,000 ohms at 10 kc? (*c*) Check the accuracy of the L and C values by use of the equation $f_0 = 1/\pi\sqrt{LC}$. What is the magnitude of the output

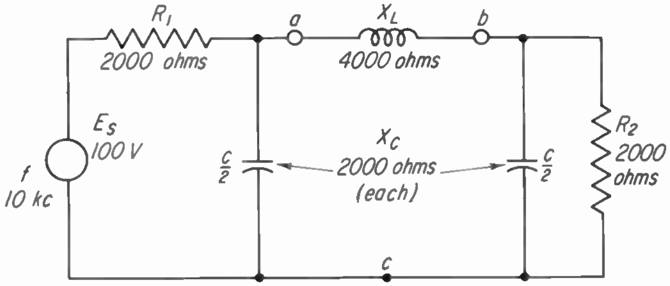


Fig. 10-34a

voltage E_o (Fig. 10-34*b*) at a frequency of (*d*) zero or d-c, (*e*) 1 kc, (*f*) 5 kc, (*g*) 10 kc, (*h*) 20 kc?

GIVEN: Fig. 10-34*a*

FIND: (*a*) L (*b*) C (*c*) f_0 (*d*) to (*h*) E_o

SOLUTION:

(*a*) $$L = \frac{X_L}{2\pi f} = \frac{4{,}000}{6.28 \times 10^4} = 63.7 \text{ mh}$$

(*b*) $$\frac{C}{2} = \frac{10^6}{2\pi f X_C} = \frac{10^6}{6.28 \times 10^4 \times 2{,}000} = 0.00795\ \mu\text{f}$$

$$C = 2 \times 0.00795 = 0.0159\ \mu\text{f}$$

(*c*) $$f_0 = \frac{1}{\pi\sqrt{LC}} = \frac{1}{3.14\sqrt{63.7 \times 10^{-3} \times 0.0159 \times 10^{-6}}} \cong 10^4 = 10 \text{ kc}$$

(*d*) When $f = 0$ cps, this is equivalent to direct current, and with the resistance of the inductor considered to be zero, then

$$E_o = E_{R\cdot 2} = \frac{E_S R_2}{R_1 + R_2} = \frac{100 \times 2{,}000}{2{,}000 + 2{,}000} = 50 \text{ volts}$$

(*e*) When $f = 1$ kc,

$$R_1 = 2{,}000 \text{ ohms} \qquad X_L = 400 \text{ ohms} \qquad X_C = 20{,}000 \text{ ohms}$$

With the circuit opened at *b*,

$$E_{eq} = E_{a-c} = \frac{E_S X_C}{\sqrt{R_1{}^2 + X_C{}^2}} = \frac{100 \times 20{,}000}{\sqrt{2{,}000^2 \times 20{,}000^2}} = \frac{2{,}000{,}000}{20{,}100} = 99.5 \text{ volts}$$

The Thévenin theorem equivalent circuit may now be drawn as shown in Fig. 10-34*b*

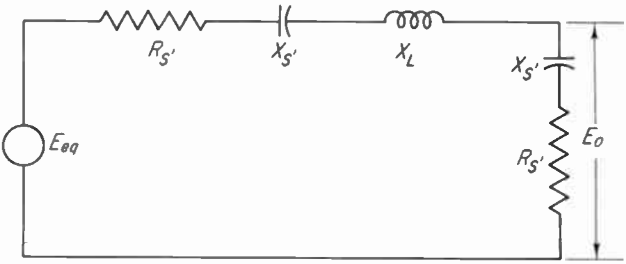


Fig. 10-34b

where $$R_{S'} = \frac{R_P X_P{}^2}{R_P{}^2 + X_P{}^2} = \frac{2{,}000 \times 20{,}000^2}{2{,}000^2 + 20{,}000^2} = 1{,}980 \text{ ohms}$$

$$X_{S'} = \frac{X_P R_P{}^2}{R_P{}^2 + X_P{}^2} = \frac{20{,}000 \times 2{,}000^2}{2{,}000^2 + 20{,}000^2} = 198 \text{ ohms}$$

$$E_o = \frac{E_{eq}(R_{S'} - jX_{S'})}{2R_{S'} + jX_L - j2X_{S'}} = \frac{99.5\sqrt{1{,}980^2 + 198^2}}{\sqrt{3{,}960^2 + (400 - 396)^2}} = 50 \text{ volts}$$

(*f*) When $f = 5$ kc,

$$R_1 = 2{,}000 \text{ ohms} \qquad X_L = 2{,}000 \text{ ohms} \qquad X_C = 4{,}000 \text{ ohms}$$

$$E_{eq} = \frac{100 \times 4{,}000}{\sqrt{2{,}000^2 + 4{,}000^2}} = 89.4 \text{ volts}$$

$$R_{S'} = \frac{2{,}000 \times 4{,}000^2}{2{,}000^2 + 4{,}000^2} = 1{,}600 \text{ ohms}$$

$$X_{S'} = \frac{4{,}000 \times 2{,}000^2}{2{,}000^2 + 4{,}000^2} = 800 \text{ ohms}$$

$$E_o = \frac{89.4\sqrt{1{,}600^2 + 800^2}}{\sqrt{3{,}200^2 + (2{,}000 - 1{,}600)^2}} = 49.6 \text{ volts}$$

(*g*) When $f = 10$ kc,

$$R_1 = 2{,}000 \text{ ohms} \qquad X_L = 4{,}000 \text{ ohms} \qquad X_C = 2{,}000 \text{ ohms}$$

$$E_{eq} = \frac{100 \times 2{,}000}{\sqrt{2{,}000^2 + 2{,}000^2}} = 70.7 \text{ volts}$$

$$R_{S'} = \frac{2{,}000 \times 2{,}000^2}{2{,}000^2 + 2{,}000^2} = 1{,}000 \text{ ohms}$$

$$X_{S'} = \frac{2{,}000 \times 2{,}000^2}{2{,}000^2 + 2{,}000^2} = 1{,}000 \text{ ohms}$$

$$E_o = \frac{70.7\sqrt{1{,}000^2 + 1{,}000^2}}{\sqrt{2{,}000^2 + (4{,}000 - 2{,}000)^2}} = 35.3 \text{ volts}$$

(*h*) When $f = 20$ kc,

$$R_1 = 2{,}000 \text{ ohms} \qquad X_L = 8{,}000 \text{ ohms} \qquad X_C = 1{,}000 \text{ ohms}$$

$$E_{eq} = \frac{100 \times 1{,}000}{\sqrt{2{,}000^2 + 1{,}000^2}} = 44.7 \text{ volts}$$

$$R_{S'} = \frac{2{,}000 \times 1{,}000^2}{2{,}000^2 + 1{,}000^2} = 400 \text{ ohms}$$

$$X_{S'} = \frac{1{,}000 \times 2{,}000^2}{2{,}000^2 + 1{,}000^2} = 800 \text{ ohms}$$

$$E_o = \frac{44.7\sqrt{400^2 + 800^2}}{\sqrt{800^2 + (8{,}000 - 1{,}600)^2}} = 6.2 \text{ volts}$$

QUESTIONS

1. In what manner do the voltage and current characteristics of alternating current differ from those of direct current?
2. What is meant by the expression that the current and voltage are in phase?
3. What opposition is offered to the flow of current in a d-c circuit?
4. What opposition is offered to the flow of current in an a-c circuit?
5. Describe two effects of inductive reactance upon the current in an a-c circuit.
6. Describe two effects of capacitive reactance upon the current in an a-c circuit.

7. What is meant by (*a*) a perfect inductor? (*b*) A perfect capacitor?
8. How much power would be consumed by (*a*) a perfect inductor? (*b*) A perfect capacitor?
9. (*a*) Define impedance. (*b*) What symbol is used to represent impedance? (*c*) What unit is used to express the amount of impedance?
10. (*a*) What is an impedance triangle? (*b*) How is an impedance triangle drawn for a circuit containing resistance and inductive reactance?
11. How are the resistance and inductive reactance of a practical inductor considered in calculating a-c circuits?
12. How are the resistance and capacitive reactance of a practical capacitor considered in calculating a-c circuits?
13. In solving multielement series circuits, how are the following individual factors considered: (*a*) resistance, (*b*) inductive reactance, (*c*) capacitive reactance?
14. (*a*) What is a vector? (*b*) What quantities may be represented by vectors? (*c*) What is the standard direction of vector rotation?
15. (*a*) Describe the graphical method of solving vector values. (*b*) What factors determine the accuracy of this method of solution?
16. Explain the use of vectors when both current and voltage are to be represented.
17. Explain what is meant by the expression the current and voltage are out of phase.
18. What is meant by the expression the current lags the voltage?
19. What is meant by the expression the current leads the voltage?
20. Explain the construction of an impedance triangle for a circuit containing (*a*) resistance and inductance; (*b*) resistance and capacitance; (*c*) resistance, inductance, and capacitance.
21. Describe the vector method of solving a-c circuits.
22. (*a*) What is meant by apparent power? (*b*) How is its value obtained?
23. (*a*) What is meant by the volt-amperes of a circuit? (*b*) How is its value obtained?
24. (*a*) What is meant by the actual power of a circuit? (*b*) How may its value be obtained?
25. (*a*) What is meant by the power factor of a circuit? (*b*) How may its value be obtained?
26. Under what conditions will the power factor of a circuit be (*a*) unity (one)? (*b*) Zero?
27. (*a*) What is the most desired value for the power factor of a circuit? (*b*) Why?
28. Describe two methods of obtaining the power factor of a circuit by meter readings.
29. What is meant by phase angle?
30. How is a leading or lagging angle expressed in terms of the voltage and current?
31. Compare the line current in a series a-c circuit with the current in its component parts.
32. Compare the impedance of a series circuit with the impedance of its component parts when the circuit contains (*a*) resistance and inductive reactance; (*b*) resistance and capacitive reactance; (*c*) resistance, inductive reactance, and capacitive reactance.
33. Compare the line voltage of a series circuit with the voltage of its component parts when the circuit contains (*a*) resistance and inductive reactance; (*b*) resistance and capacitive reactance; (*c*) resistance, inductive reactance, and capacitive reactance.

34. Compare the line voltage of a parallel a-c circuit with the voltage of its component parts.
35. Compare the impedance of a parallel circuit with the impedance of its component parts when the circuit contains (*a*) resistance and inductive reactance; (*b*) resistance and capacitive reactance; (*c*) resistance, inductive reactance, and capacitive reactance. *Note:* Assume the inductor and the capacitor to be perfect.
36. Compare the line current of a parallel circuit with the current in its component parts when the circuit contains (*a*) resistance and inductive reactance; (*b*) resistance and capacitive reactance; (*c*) resistance, inductive reactance, and capacitive reactance. *Note:* Assume the inductor and the capacitor to be perfect.
37. What procedure is recommended in solving parallel a-c circuits?
38. What is meant by a parallel-series a-c circuit?
39. What procedure is recommended in solving parallel-series a-c circuits?
40. What is meant by a series-parallel a-c circuit?
41. What procedure is recommended in solving series-parallel a-c circuits?
42. What is meant by the series equivalent for a parallel circuit?
43. What is meant by the parallel equivalent for a series circuit?
44. What is the purpose of vector algebra?
45. Define (*a*) imaginary number, (*b*) j operator, (*c*) complex number.
46. In graphical representation of a-c quantities, what is meant by (*a*) X-X axis? (*b*) Y-Y axis? (*c*) The four quadrants?
47. Describe rectangular notation of a vector.
48. What basic types of mathematical operations can be performed with vector quantities?
49. What is meant by a polar vector?
50. What types of mathematical operations can be performed with polar vectors?
51. Name four types of information given by the polar notation of a vector.
52. What purpose does Thévenin's theorem serve?
53. Describe the basic premise on which Thévenin's theorem is based.
54. To what quantities may Thévenin's theorem be applied?

PROBLEMS

1. A 100-henry inductor having a resistance of 500 ohms is used in the parallel feed plate circuit of a vacuum tube. Find (*a*) the inductive reactance at 50 cycles, (*b*) the impedance at 50 cycles, (*c*) the inductive reactance at 5,000 cycles, (*d*) the impedance at 5,000 cycles.
2. A 10-henry inductor has a d-c resistance of 475 ohms. (*a*) What is its inductive reactance at 60 cycles? (*b*) What is its impedance at 60 cycles? (*c*) How much current will flow when it is connected to a 150-volt 60-cycle power source?
3. An 8-μf filter capacitor used in a power-supply circuit has an effective series resistance of 8 ohms. (*a*) What is its capacitive reactance at 60 cycles? (*b*) What is its impedance?
4. A 20-henry filter choke has a d-c resistance of 400 ohms. Find (*a*) the inductive reactance at 120 cycles, (*b*) the impedance at 120 cycles, (*c*) the d-c voltage drop through the coil when a direct current of 85 ma is flowing through the coil.
5. A coil having a resistance of 5 ohms and an inductive reactance of 6,280 ohms is connected in series with a capacitor having a reactance of 3,200 ohms. (*a*) What

is the impedance of the circuit? (*b*) How much current will flow when the impressed voltage is 20 volts?

6. How much current will flow in an a-c series circuit having a resistance of 5 ohms, an inductive reactance of 80 ohms, and a capacitive reactance of 140 ohms when the impressed voltage is 120 volts?
7. Two inductors are connected in series with each other across a 300-volt power source. One inductor has a resistance of 160 ohms and an inductive reactance of 750 ohms; the second inductor has a resistance of 230 ohms and an inductive reactance of 2,250 ohms. Find (*a*) the impedance of the circuit, (*b*) the line current, (*c*) the voltage across each inductor.
8. (*a*) What is the impedance of a series circuit consisting of 34 ohms capacitive reactance, 276 ohms inductive reactance, and 10 ohms resistance? (*b*) How much current will flow through the circuit when 80 volts is impressed across it? (*c*) What are the voltages across the resistor, capacitor, and inductor?
9. The antenna circuit of a radio receiver may be represented by Fig. 10-35. (*a*) What impedance will this circuit offer to a 1,200-kc signal if $L_A = 120\ \mu$h, $L_P = 50\ \mu$h, and $C = 100$ pf? (Assume the resistance to be so low that it may be ignored.) (*b*) How much current will flow when a 1,200-kc 120-μv signal is impressed across the circuit? (*c*) What is the amount of voltage across L_P?
10. The circuit of Fig. 10-36 has the following values: $E = 180$ volts; $X_{C\cdot 1} = 1{,}590$ ohms; $R_{C\cdot 1} = 10$ ohms; $X_{C\cdot 2} = 784$ ohms; $R_{C\cdot 2} = 5$ ohms; $X_{L\cdot 1} = 90{,}000$ ohms; $R_{L\cdot 1} = 8$ ohms; $X_{L\cdot 2} = 10{,}000$ ohms; $R_{L\cdot 2} = 2$ ohms. Find (*a*) the impedance of the circuit, (*b*) the line current, (*c*) the voltages e_1, e_2, e_3, e_4.
11. The circuit of Fig. 10-36 has the following values: $E = 41$ volts; $X_{C\cdot 1} = 1{,}600$ ohms; $R_{C\cdot 1} = 400$ ohms; $X_{C\cdot 2} = 400$ ohms; $R_{C\cdot 2} = 200$ ohms; $X_{L\cdot 1} = 8{,}000$ ohms; $R_{L\cdot 1} = 800$ ohms; $X_{L\cdot 2} = 2{,}000$ ohms; $R_{L\cdot 2} = 400$ ohms. Find (*a*) the impedance of the circuit, (*b*) the line current, (*c*) the voltages e_1, e_2, e_3, e_4.
12. By means of vectors, determine the impedance of a series circuit containing a resistance of 12 ohms and an inductive reactance of 32 ohms. Check the answer by solving for the impedance mathematically.
13. A series circuit has a resistance of 50 ohms and a capacitive reactance of 100 ohms. Determine its impedance graphically and mathematically.
14. A series circuit contains a resistance of 250 ohms, an inductive reactance of 3,000 ohms, and a capacitive reactance of 1,500 ohms. Determine the impedance of the circuit by vectors. Check the answer mathematically.
15. By means of vectors, determine the impedance of the circuit used in Prob. 4. Check this answer with the value obtained mathematically.
16. A series circuit contains a resistance of 150 ohms, a capacitive reactance of 850

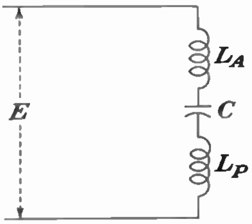


Fig. 10-35

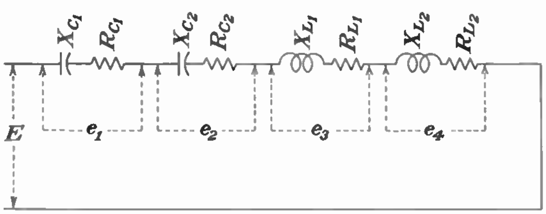


Fig. 10-36

ohms, and an inductive reactance of 600 ohms. Determine the impedance of this circuit by means of vectors. Check the answer mathematically.

17. An 8-henry choke coil is to be tested by connecting it to a 120-volt 60-cycle power source. Assuming that the resistance of the coil is zero, determine (*a*) the current, (*b*) power, (*c*) volt-amperes, (*d*) power factor, (*e*) phase angle. (*f*) Draw a vector diagram showing the voltage and current relations.
18. If the d-c resistance of the inductor of Prob. 17 is 290 ohms, find (*a*) the current, (*b*) power, (*c*) volt-amperes, (*d*) power factor, (*e*) phase angle. (*f*) Draw a vector diagram showing the voltage and current relations.
19. A 30-μf capacitor has a capacitive reactance of 88.4 ohms when connected to a 300-volt 60-cycle power source. The resistance of the capacitor is assumed to be zero. Find (*a*) the current, (*b*) power, (*c*) volt-amperes, (*d*) power factor, (*e*) phase angle. (*f*) Draw a vector diagram showing the relation between the current and the voltage.
20. If the capacitor of Prob. 19 has an equivalent series resistance of 8 ohms, find (*a*) the impedance, (*b*) current, (*c*) power, (*d*) apparent power, (*e*) power factor, (*f*) phase angle. (*g*) Draw a vector diagram showing the relation between the current and voltage.
21. A series circuit containing 30 ohms resistance, 60 ohms inductive reactance, and 40 ohms capacitive reactance is connected to a 220-volt 60-cycle power source. Find (*a*) the impedance, (*b*) current, (*c*) power, (*d*) apparent power, (*e*) power factor, (*f*) phase angle. (*g*) Draw a vector diagram showing the relation of the voltages and current for this circuit.
22. A series circuit containing a 5,000-ohm resistor, a 0.5-μf capacitor, and a 10-henry inductor is connected to a 250-volt 100-cycle power source. Find (*a*) the capacitive reactance, (*b*) inductive reactance, (*c*) impedance, (*d*) current, (*e*) power, (*f*) apparent power, (*g*) power factor, (*h*) phase angle, (*i*) voltage across the resistor, (*j*) voltage across the capacitor, (*k*) voltage across the inductor. (*l*) Draw a vector diagram showing the voltages and current for this circuit.
23. An inductor is connected in a circuit as shown in Fig. 10-18*a*. The readings taken are 150 volts, 60 cycles, 40 ma, 0.75 watt. Find (*a*) the impedance, (*b*) resistance, (*c*) apparent power, (*d*) power factor, (*e*) phase angle, (*f*) inductive reactance, (*g*) inductance. (*h*) Draw a vector diagram showing the current and voltage relations.
24. A capacitor is connected in a circuit as shown in Fig. 10-18*a*. The readings taken are 150 volts, 60 cycles, 150 ma, and 0.5 watt. Find (*a*) the impedance, (*b*) resistance, (*c*) apparent power, (*d*) power factor, (*e*) phase angle, (*f*) capacitive reactance, (*g*) capacitance. (*h*) Draw a vector diagram showing the current and voltage relations.
25. The parallel circuit shown in Fig. 10-37 consists of only pure resistances and pure reactances. Find (*a*) the current of each branch, (*b*) line current, (*c*) impedance of the circuit, (*d*) power consumed by each branch, (*e*) power taken by the complete circuit, (*f*) volt-amperes of the complete circuit, (*g*) line power factor, (*h*) phase angle between the line current and line voltage. (*i*) Draw a vector diagram of the voltage and all currents.
26. Repeat Prob. 25 for the circuit shown in Fig. 10-38.
27. Repeat Prob. 25 for the circuit shown in Fig. 10-39.
28. Repeat Prob. 25 for the circuit shown in Fig. 10-40.

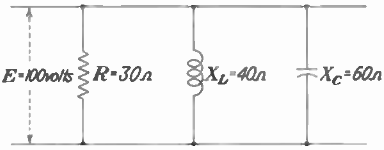


Fig. 10-37

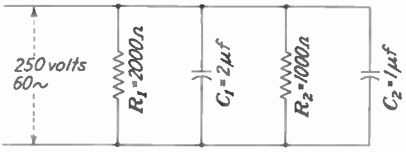


Fig. 10-38

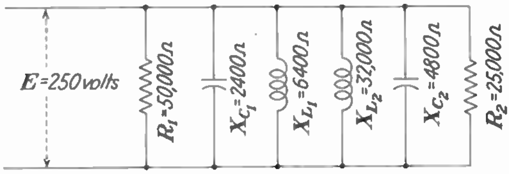


Fig. 10-39

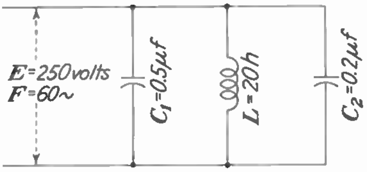


Fig. 10-40

29. A parallel-series circuit is shown in Fig. 10-41. (*a*) Find the current, power, power factor, and phase angle for each branch. (*b*) Find the current, power, apparent power, power factor, and phase angle for the complete circuit. (*c*) Draw a vector diagram.
30. Repeat Prob. 29 for the circuit shown in Fig. 10-42.
31. A series-parallel circuit is shown in Fig. 10-43. (*a*) Find the current, power, apparent power, power factor, and phase angle of the line. (*b*) Find the voltage and current of each part of the circuit. (*c*) Draw a vector diagram.
32. Repeat Prob. 31 for the circuit shown in Fig. 10-44.
33. The circuit shown in Fig. 10-45 is sometimes called a *low-pass filter*. Find (*a*) the impedance of the capacitor to a 3,000-cycle a-f current, (*b*) impedance of the capacitor to a 750-kc r-f current, (*c*) per cent of a-f current that flows through the

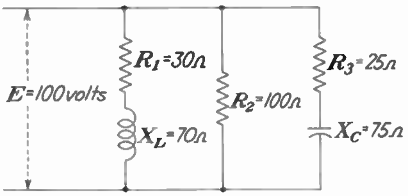


Fig. 10-41

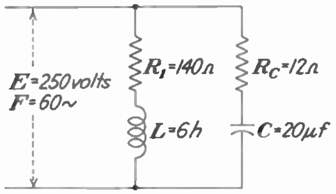


Fig. 10-42

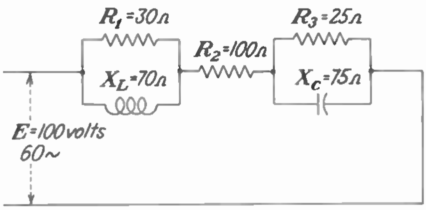


Fig. 10-43

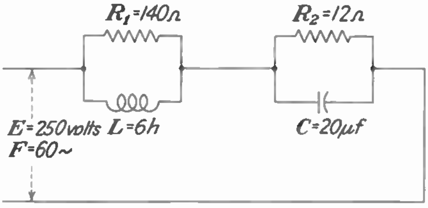


Fig. 10-44

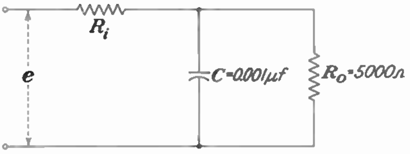


Fig. 10-45

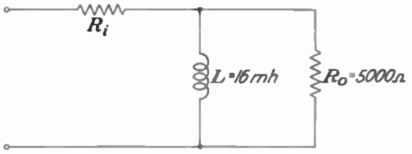


Fig. 10-46

capacitor, (*d*) per cent of a-f current that flows through R_o, (*e*) per cent of r-f current that flows through the capacitor, (*f*) per cent of r-f current that flows through R_o. (*g*) From the percentage of a-f and r-f currents that flow through the output resistor R_o, justify the designation of the circuit as a low-pass filter.

34. The circuit shown in Fig. 10-46 is sometimes called a *high-pass filter.* Find (*a*) the impedance of the inductor to a 3,000-cycle a-f current, (*b*) impedance of the inductor to a 750-kc r-f current, (*c*) per cent of a-f current that flows through the inductor, (*d*) per cent of a-f current that flows through R_o, (*e*) per cent of r-f current that flows through the inductor, (*f*) per cent of r-f current that flows through R_o. (*g*) From the percentage of a-f and r-f currents that flow through the output resistor R_o, justify the designation of the circuit as a high-pass filter.
35. Figure 10-47 shows a combination of capacitors and inductors. (*a*) What is the impedance of the capacitors to an a-f current of 2,000 cycles? An r-f current of 1,500 kc? (*b*) What is the impedance of the inductors to an a-f current of 2,000 cycles? An r-f current of 1,500 kc? (*c*) Can this circuit be classed as either a high-pass or a low-pass filter? Explain.
36. Figure 10-48 shows a combination circuit of capacitors and inductors. (*a*) What is the impedance of the inductors to a 500-cycle a-f current? A 550-kc r-f current? (*b*) What is the impedance of the capacitors to a 500-cycle a-f current? A 550-kc r-f current? (*c*) Can this circuit be classed as either a high-pass or a low-pass filter? Explain.
37. What are the resistance and reactance values of a series equivalent circuit that may be substituted for a parallel circuit that has a 500-ohm resistance and a 200-ohm inductive reactance?
38. What are the resistance and reactance values of a series equivalent circuit that may be substituted for a parallel circuit that has a 20,000-ohm resistance and a 5,000-ohm capacitive reactance?
39. What are the resistance and reactance values of a parallel equivalent circuit that may be substituted for a series circuit that has a 200-ohm resistance and a 400-ohm capacitive reactance?
40. What are the resistance and reactance values of a parallel equivalent circuit that

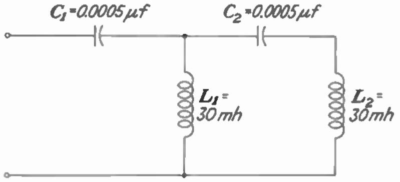


Fig. 10-47

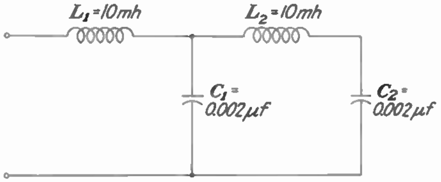


Fig. 10-48

may be substituted for a series circuit that has a 50,000-ohm resistance and a 100,000-ohm inductive reactance?

41. Using the principle of addition of vector quantities, determine the impedance equations for the following circuits: (*a*) an inductor having 150 ohms reactance and 20 ohms resistance connected in series with a capacitor having 450 ohms reactance and 30 ohms resistance, (*b*) an inductor having 1,000 ohms reactance and 65 ohms resistance connected in series with another inductor having 250 ohms reactance and 20 ohms resistance, (*c*) a capacitor having a reactance of 650 ohms and a resistance of 15 ohms connected in series with an inductor having a reactance of 1,000 ohms and a resistance of 75 ohms, (*d*) a series circuit containing the six components described in parts (*a*), (*b*), and (*c*).
42. Applying the principle of vector addition illustrated in Example 10-20 to the circuit described in Example 10-15, find the sum of the following currents: (*a*) $\dot{I}_1 + \dot{I}_2$, (*b*) $\dot{I}_3 + \dot{I}_4$, (*c*) $\dot{I}_1 + \dot{I}_2 + \dot{I}_3 + \dot{I}_4$ by adding the answers to parts (*a*) and (*b*); (*d*) add I_5 to the answer of part (*c*).
43. Using the principle of vector subtraction, find the resultant equations for (*a*) $(40 + j1{,}200) - (20 + j300)$, (*b*) $(30 - j600) - (50 + j500)$, (*c*) $(80 - j2{,}500) - (40 - j1{,}000)$, (*d*) $(20 + j650) - (20 - j400)$.
44. Find the product of the following vector quantities: (*a*) $(30 + j50) \times (20 + j60)$, (*b*) $(30 + j50) \times (20 - j60)$, (*c*) $(40 - j100) \times (20 - j50)$, (*d*) $(-30 - j50) \times (20 + j60)$.
45. Find the quotients of the following vector quantities: (*a*) $(30 + j50) \div (20 + j60)$, (*b*) $(30 + j50) \div (20 - j60)$, (*c*) $(40 - j100) \div (20 - j50)$, (*d*) $(-30 - j50) \div (20 + j60)$.
46. What is the equivalent polar vector for (*a*) $20 + j60$? (*b*) $30 - j50$? (*c*) $40 - j100$? (*d*) $100 + j40$?
47. What is the rectangular vector equivalent of (*a*) $40\underline{/60^\circ}$ ohms? (*b*) $100\underline{/-45^\circ}$ ohms? (*c*) $2{,}000\underline{/-20^\circ}$ ohms? (*d*) $500{,}000\underline{/75^\circ}$ ohms?
48. Find the sums of the following vector quantities: (*a*) $40\underline{/60^\circ}$ ohms + $100\underline{/-45^\circ}$ ohms, (*b*) $2{,}000\underline{/-20^\circ}$ ohms + $1{,}000\underline{/75^\circ}$ ohms, (*c*) $1{,}000\underline{/60^\circ}$ ohms + $500\underline{/30^\circ}$ ohms, (*d*) $200\underline{/37.5^\circ}$ ohms + $400\underline{/-62.5^\circ}$ ohms.
49. Find the differences of the following vector quantities: (*a*) $100\underline{/50^\circ}$ ohms − $60\underline{/30^\circ}$ ohms, (*b*) $200\underline{/41.5^\circ}$ ohms − $100\underline{/-53.5^\circ}$ ohms, (*c*) $8\underline{/45^\circ}$ ohms − $8\underline{/-45^\circ}$ ohms, (*d*) $10\underline{/18.5^\circ}$ ohms − $20\underline{/71.5^\circ}$ ohms.
50. Find the products of the following polar vectors: (*a*) $10\underline{/36^\circ} \times 17.5\underline{/42^\circ}$, (*b*) $100\underline{/48^\circ} \times 22\underline{/-5.5^\circ}$, (*c*) $25\underline{/-25^\circ} \times 6\underline{/52^\circ}$, (*d*) $4\underline{/-60^\circ} \times 3\underline{/138^\circ}$.
51. Find the quotients of the following polar vectors: (*a*) $40\underline{/63^\circ} \div 8\underline{/21^\circ}$, (*b*) $150\underline{/-18^\circ} \div 25\underline{/-58^\circ}$, (*c*) $8\underline{/21^\circ} \div 40\underline{/63^\circ}$, (*d*) $60\underline{/26.5^\circ} \div 15\underline{/-27.5^\circ}$.
52. Find the powers indicated for the following polar vectors: (*a*) $(15\underline{/26.5^\circ})^2$, (*b*) $(10\underline{/15^\circ})^3$, (*c*) $(4\underline{/23^\circ})^4$, (*d*) $(2\underline{/12^\circ})^5$.
53. Find the roots indicated for the following polar vectors: (*a*) $\sqrt{625\underline{/68^\circ}}$, (*b*) $\sqrt[3]{125\underline{/75^\circ}}$, (*c*) $\sqrt[3]{512\underline{/135^\circ}}$, (*d*) $\sqrt{4{,}225\underline{/63^\circ}}$.
54. Find the reciprocals of the following polar vectors: (*a*) $8\underline{/36.5^\circ}$, (*b*) $0.2\underline{/-43^\circ}$, (*c*) $0.4\underline{/-16^\circ}$, (*d*) $25\underline{/82^\circ}$.
55. (*a*) Determine the rectangular vector equation for the circuit of Prob. 10. (*b*) Calculate the impedance of the circuit from the answer to part (*a*).
56. (*a*) Determine the rectangular vector equation for the circuit of Prob. 11. (*b*) Calculate the impedance of the circuit from the answer to part (*a*).

57. Using the principle illustrated in Example 10-34, find the impedance of the circuit given in Prob. 25.

58. Using the principle illustrated in Example 10-34, find the impedance of the circuit given in Prob. 26.

59. A certain circuit consists of a 300-ohm resistance and a 400-ohm capacitive reactance connected in parallel. Find the circuit impedance by use of (*a*) Eq. (10-15), (*b*) Eq. (10-23).

60. A certain circuit consists of a 10-ohm resistance and a 100-ohm inductive reactance connected in parallel. Find the circuit impedance by use of (*a*) Eq. (10-15), (*b*) Eq. (10-23).

61. Solve Prob. 59 by multiplication and division of polar vectors using Eqs. (10-15), (10-18), and (10-19).

62. Solve Prob. 60 by multiplication and division of polar vectors using Eqs. (10-15), (10-18), and (10-19).

63. Find the impedance and the phase angle of the circuit of Prob. 29 by use of polar vectors.

64. Find the impedance and the phase angle of the circuit of Prob. 30 by use of polar vectors.

65. Find the impedance, the current, and the phase angle for the circuit of Prob. 31 by means of vector algebra.

66. Find the impedance, the current, and the phase angle for the circuit of Prob. 32 by means of vector algebra.

67. By use of Thévenin's theorem, find the current and voltage at each circuit element in the circuit of Fig. 10-45 when $e = 10$ volts, $f = 318$ kc, $X_C = 500$ ohms, and $R_i = 1{,}000$ ohms.

68. By use of Thévenin's theorem, find the current and voltage at each circuit element in the circuit of Fig. 10-46 when $e = 10$ volts, $f = 100$ kc, $X_L = 10{,}000$ ohms, and $R_i = 5{,}000$ ohms.

69. By use of Thévenin's theorem, find the current and voltage at each circuit element in the circuit of Fig. 10-49. *Note:* This circuit is the same as that of Prob. 47 in Chap. 4.

70. By use of Thévenin's theorem, find the current and voltage at each circuit element in the circuit of Fig. 10-50. *Note:* This circuit is the same as that of Prob. 50 in Chap. 4.

71. For the circuit of Example 10-43, find the output voltage for an input of 100 volts and (*a*) 2.5 kc, (*b*) 7.5 kc.

72. For the circuit of Example 10-43, find the output voltage for an input of 100 volts and (*a*) 15 kc, (*b*) 50 kc.

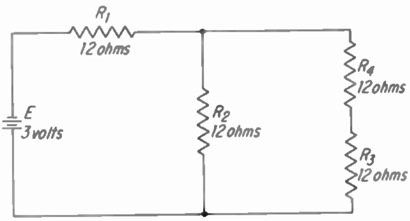


Fig. 10-49

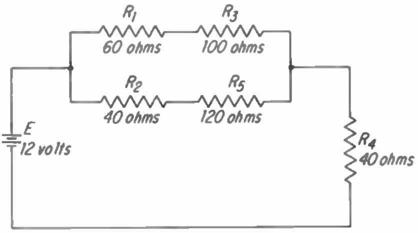


Fig. 10-50

Chapter 11

Resonance

A knowledge of resonant circuits is important in the study of electronics. In the operation of a radio or television receiver, the desired program is obtained by tuning the receiver. Actually, when a desired station is being selected, the tuning circuit of the receiver is being adjusted so that it is in resonance with the carrier frequency of the station transmitting the desired program. Tuned circuits are also used in all other forms of communications, such as radar, sonar, telemetry, etc. In other fields of electronics resonant circuits have many applications, some of which are quality control, thickness control, capacitor-relay alarms, data processing, etc.

11-1 Resonance

Resonance is a condition that exists when the inductive reactance and the capacitive reactance of a circuit are equal. Under this condition, the effective or total reactance of a series resonant circuit will be zero and its impedance will be equal to the resistance of the circuit [see Eq. (10-1)]. The impedance will therefore be at its minimum value, and the current will be at its maximum value. As the current is at its maximum value, the power of the circuit will also be at its maximum value.

Resistance is the only opposition that a direct current has when it flows through an electric circuit. When an alternating current flows through a circuit, its flow is opposed by the combined effects of resistance and reactance. The opposition to alternating current offered by a resistance is the same as that offered to a direct current. In Chap. 8, it was shown that the current due to inductive reactance lags the voltage by 90°. In Chap. 9, it was shown that the current due to capacitive reactance leads the voltage by 90°. Therefore the effects of inductive reactance and capacitive reactance are 180° out of phase with each other, and the resultant reactance of the circuit is equal to the algebraic sum of its inductive and capacitive reactances.

If either the capacitor or inductor in a series circuit is adjusted so that their individual reactances are equal, the total reactance of the circuit will be zero and the impedance of the circuit will be equal to its resistance. Such a circuit is called a *series resonant circuit.* The action of a resonant circuit is best explained by reference to graphs illustrating the variation in the circuit conditions at or near resonance.

11-2 Graphs

Use of Graphs. A graph is a pictorial representation illustrating the manner in which one factor varies with changes in any other factor on which it is dependent. Graphs are used in all branches of electronics to illustrate the operating conditions of the various circuits and parts used. Electronic manufacturers use graphs to illustrate the operating characteristics of their products. Many of the answers to radio, television, and electronics problems can be explained better if the results of the problem are plotted in the form of a graph. The characteristics of a circuit or a circuit component for various operating conditions can be compared with one another more easily and understood much better when illustrated on a graph than from using the comparison of a number of figures representing the results for the same operating conditions. It is therefore desirable to know how to plot and interpret graphs.

Simple Graphs. The simplest type of graph is one illustrating the variation of two quantities with each other. This type of variation is generally plotted on cross-section paper, called *graph paper.* This paper consists of a series of equally spaced vertical and horizontal lines (Fig. 11-1). One of the horizontal lines is used as a reference line to represent one of the quantities and is marked with a suitable scale of values as 0, 1, 2, 3, shown on Fig. 11-1; all distances in a horizontal direction are called *abscissas* (or abscissae). One of the vertical lines is used as a reference line to represent the other quantity and is also marked with a scale (though not necessarily the same scale), such as 0, 5, 10, 15, as shown on Fig. 11-1; all distances in a vertical direction are called *ordinates.* The point where the two reference lines meet is called the *point of origin* and represents zero value for both quantities. For ease in plotting and reading graphs, each square is made to represent either one, two, or five units or some multiple thereof. The square farthest from the point of origin should represent a value equal to or slightly greater than the maximum value to be plotted, and each square between these points represents a definite fraction of this value.

Independent and Dependent Variables. When graphs are plotted, the independent variable is plotted as the abscissa and the dependent variable as the ordinate. For example, in Fig. 11-1 curve *A* was plotted to show the values of voltage required to force currents of 2, 4, 6, 8, and 10 amp through a 5-ohm resistance. Because current is the unit to which values have been assigned, it is the *independent variable* and hence is plotted as the abscissa. The value of the voltage is dependent on the assigned value of the current, $E = IR$, and is called the *dependent variable;* hence it is plotted as the ordinate. Values of the ordinates must be obtained for defi-

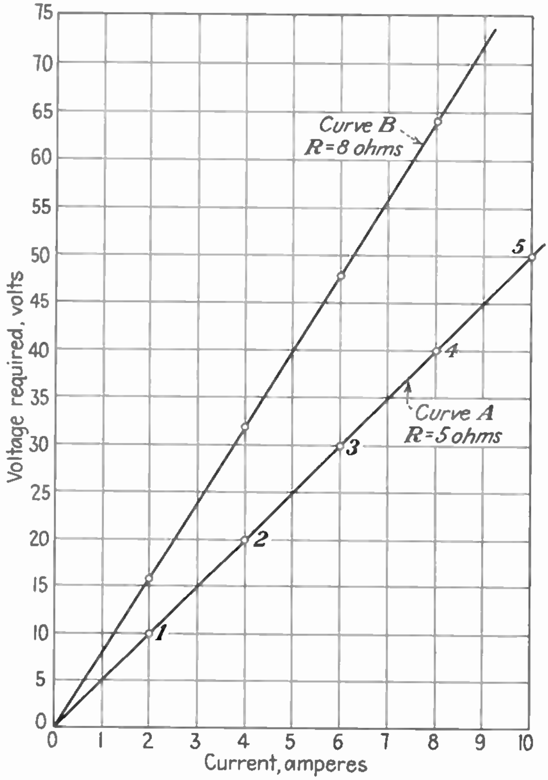


Fig. 11-1 Curves showing the variation of voltage with current in a circuit: curve A for a circuit whose resistance is 5 ohms and curve B for a circuit whose resistance is 8 ohms.

nite values of the abscissas. These values are then plotted on the graph paper by making points along the ordinate scale at values corresponding to the amount that was obtained for the definite values of abscissa. A line is drawn through this series of points to form a continuous straight or curved line. This line represents the variation of the two quantities with respect to each other and is useful for finding the value of the variable quantity for any value of the quantity being varied.

11-3 Plotting, Use, and Interpretation of Curves

By substituting a series of values in any mathematical equation, the results obtained will show how one quantity is affected by any change in another. If these results are plotted in graph form for each of the values used, a curve can be obtained that will illustrate how a factor of this equation varies with the values substituted. A curve can be plotted for any electrical equation to illustrate how the variable quantity changes with the value being varied.

EXAMPLE 11-1 Plot a curve showing how the voltage required to force a current through a 5-ohm resistor varies with currents of 1 to 10 amp.

Table 11-1

R, OHMS	I, AMP	E, VOLTS
5	2	10
5	4	20
5	6	30
5	8	40
5	10	50

Using Eq. (2-6), $E = I \times R$, five points can be obtained for the curve by substituting 2, 4, 6, 8, and 10 amp for the current, the resistance remaining constant at 5 ohms. These values should be listed as shown in Table 11-1.

For this graph, the current is the quantity being varied and is the abscissa. The voltage is the dependent variable and is the ordinate. The reference abscissa is divided in equal divisions representing current in amperes, and the reference ordinate is divided into equal divisions representing the voltage required in volts. Referring to Fig. 11-1, a line is drawn parallel to the ordinates at the 2-amp mark on the reference abscissa; another line is drawn parallel to the abscissas at the 10-volt mark on the reference ordinate. The point where these two lines meet gives one point for the curve to be drawn. Repeating this process for the results obtained for 4, 6, 8, and 10 amp, four more points are obtained for the curve. A line is drawn through points 1, 2, 3, 4, and 5. This line illustrates the manner in which the voltage in a circuit varies with the amount of current flowing.

Graphs Containing a Series of Curves. If necessary, more than one curve can be plotted on the same graph. If the resistance in Example 11-1 is increased to 8 ohms, five points can be obtained for a new curve by substituting 8 ohms for the resistance value in Eq. (2-6). This curve is shown in Fig. 11-1 as curve B. Graphs having a series of curves are quite common in electronics, as they are very useful in illustrating more than one variable condition. How the voltage varies with increases in current and resistance is illustrated in Fig. 11-1.

Nonlinear Curves. The curves in Fig. 11-1 are straight lines. An illustration of how a curved-line graph is obtained is shown by the following example.

EXAMPLE 11-2 By means of a graph, illustrate how the power in a circuit having a resistance of 10 ohms changes as the current is varied from 0 to 10 amp.

Using Eq. (2-14), $P = I^2R$, points can be obtained for the curve by substituting 1, 2, 3, 4, 5, 6, 7, 8, 9, and 10 for the current I. The results in tabular form are shown in Table 11-2.

For this graph, the current is the quantity being varied and is the abscissa. The power is the dependent variable and is the ordinate. The abscissas and ordinates are divided in equal divisions and marked to represent current and power, respectively.

Table 11-2

R, OHMS	*I*, AMP	*P*, WATTS
10	1	10
10	2	40
10	3	90
10	4	160
10	5	250
10	6	360
10	7	490
10	8	640
10	9	810
10	10	1,000

The 10 points for the curve are now located in the same manner as in the previous example. This graph is shown in Fig. 11-2. The line joining these points is a curved line and should be drawn with the aid of a French curve. In order to obtain an accurate representation of the variation of two quantities when the variation produces a curved line, it is necessary to plot as many points as practical.

Point of Origin. The point of origin is not always drawn at the extreme ends of the abscissa and ordinate. When negative values are used, the point of origin may be at the center of, or any point along, either or both of the reference lines. Figure 11-3 illustrates the variation in plate current of a vacuum tube with various values of grid voltage. As the potential on the grid varies from high negative values to lower positive values, the point of origin is drawn a small distance to the right of the center of the abscissa.

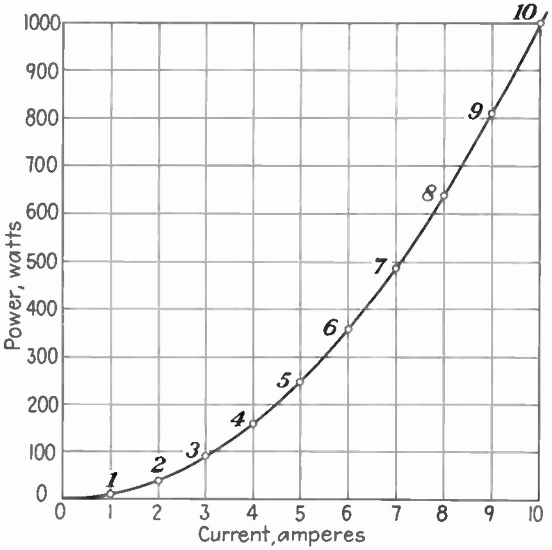


Fig. 11-2 Curve showing the variation of power with current in a circuit whose resistance is 10 ohms.

Interpretation of Curves. Curves are useful in determining the value of any quantity for any condition shown on the graph. An ordinate may be drawn from any point on the abscissa, and the corresponding value of the variable quantity is read from the point where this line crosses the curve. Referring to Fig. 11-1, 25 volts is required to force 5 amp through a resistance of 5 ohms, and 40 volts will be required if the resistance of the circuit is increased to 8 ohms. Referring to Fig. 11-2, the power in the circuit with 3.5 amp flowing is approximately 120 watts. In Fig. 11-3, the plate current output for the tube is approximately 3.3 ma when its grid voltage is adjusted to -0.5 volt.

A graph is a visual illustration of the relation between two factors. When one factor is increased, the other will either increase or decrease. If joining the series of points plotted on the graph paper forms a straight line, the two factors are proportional to each other. If one increases when the other is increased, the two factors are directly proportional to each other. If one increases when the other is decreased, the two factors are inversely proportional to each other. When the series of points forms a curved line, the two factors may be varying with each other according to the square law, as in Fig. 11-2, or according to some complex equation.

Curves are also useful in indicating whether or not the numerical values obtained in an experiment or test operation are correct. For example, the curve shown in Fig. 11-3 was plotted from results obtained from a laboratory experiment; points A and B do not fall on the curve that goes through the other points. Therefore, it is assumed that these points were incorrectly

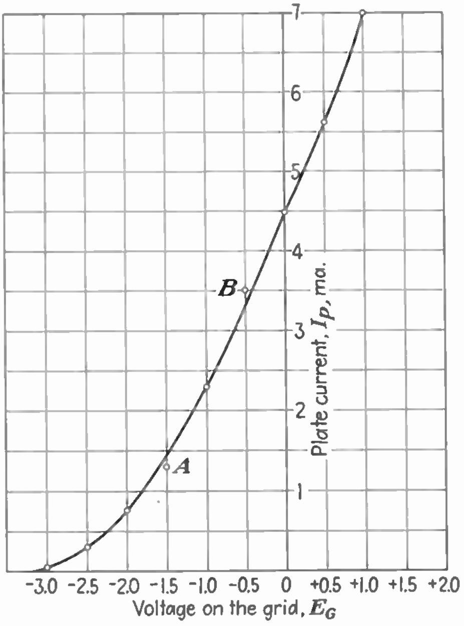


Fig. 11-3 Curve showing the variation in the plate current of a tube with changes in the grid voltage. Commonly called an E_GI_P curve.

taken, and the adjustments that produced these values should be repeated in order to check the results. Other interpretations of curves will be taken up as the need arises.

11-4 Series Resonant Circuit

Impedance and Current Characteristics. When the inductance and the capacitance of a series circuit are of such values that the inductive reactance is equal to the capacitive reactance at the frequency of the applied voltage, the circuit is called a *series resonant circuit* (Fig. 11-4*a*). The impedance of a series circuit is expressed by Eq. (10-1):

$$Z = \sqrt{R^2 + (X_L - X_C)^2} \tag{10-1}$$

However, at resonance $(X_L - X_C) = 0$ and the impedance of the resonant series circuit becomes equal to the resistance of the circuit, or

$$Z_{s\cdot r} = R \tag{11-1}$$

The current of a series resonant circuit may then be expressed as

$$I_{s\cdot r} = \frac{E}{R} \tag{11-2}$$

If the frequency is either increased or decreased from its resonant value, the quantity $(X_L - X_C)$ will no longer be zero. Also, if the frequency is kept constant and either the inductance or the capacitance is changed, the quantity $(X_L - X_C)$ will no longer be zero. As $(X_L - X_C)$ is zero only when the circuit is at resonance, any change in frequency, inductance, or capacitance will result in a significant value for $(X_L - X_C)$ and the impedance of the circuit must increase. Therefore, *in a series resonant circuit, the impedance will be at its minimum value and the current will be at its maximum value.*

Relation among f, L, and C at Resonance. At resonance, the inductive reactance is equal to the capacitive reactance; therefore from Eqs. (8-6) and (9-3)

$$2\pi f_r L = \frac{1}{2\pi f_r C} \tag{11-3}$$

where f_r = resonant frequency, cps
L = inductance, henrys
C = capacitance, farads

Multiplying both sides of the equation by f_r and dividing both sides by $2\pi L$,

$$f_r^2 = \frac{1}{4\pi^2 LC} \tag{11-4}$$

Taking the square root of both sides of the equation,

$$f_r = \frac{1}{2\pi\sqrt{LC}} \tag{11-5}$$

This is a very important equation in electronics. With this equation as a basis, the equations used for calculating tuned circuits, filters, oscillators, etc., are derived. In some circuits, the inductance may be fixed in amount, and it is then necessary to determine the amount of capacitance needed to meet the specified conditions of the circuit, while in others, the capacitance may be fixed and the amount of inductance needed must be calculated. The value of inductance or capacitance required can be found by transposing Eq. (11-4) to solve for the inductance in terms of capacitance and frequency or to solve for the capacitance in terms of inductance and frequency. Multiplying both sides of Eq. (11-4) by L and dividing both sides by f_r^2, the equation for the inductance of a resonant circuit becomes

$$L = \frac{1}{4\pi^2 f_r^2 C} \tag{11-6}$$

Multiplying both sides of Eq. (11-4) by C and dividing both sides by f_r^2, the equation for the capacitance of a resonant circuit becomes

$$C = \frac{1}{4\pi^2 f_r^2 L} \tag{11-7}$$

In many instances, the frequency is expressed in kilocycles, the inductance in microhenrys, and the capacitance in microfarads. Therefore, the more practical forms of Eqs. (11-5), (11-6), and (11-7) are

$$f_r = \frac{159}{\sqrt{LC}} \tag{11-8}$$

$$L = \frac{25{,}300}{f_r^2 C} \tag{11-9}$$

$$C = \frac{25{,}300}{f_r^2 L} \tag{11-10}$$

where f_r = resonant frequency, kc
L = inductance, μh
C = capacitance, μf

EXAMPLE 11-3 What is the resonant frequency of a series circuit having an inductance of 250 μh if the capacitor is adjusted to 350 pf?

GIVEN: $C = 350$ pf $\quad L = 250\ \mu$h

FIND: f_r

SOLUTION:

$$f_r = \frac{159}{\sqrt{LC}} = \frac{159}{\sqrt{250 \times 350 \times 10^{-6}}} = \frac{159}{0.296} = 538 \text{ kc}$$

EXAMPLE 11-4 How much inductance is required in a series circuit having a capacitance of 250 $\mu\mu$f to produce resonance with a 500-kc signal input?

GIVEN: $C = 250\ \mu\mu\text{f}$ $\quad f_r = 500$ kc

FIND: L

SOLUTION:

$$L = \frac{25{,}300}{f_r^2C} = \frac{25{,}300}{500 \times 500 \times 250 \times 10^{-6}} = 405\ \mu\text{h}$$

EXAMPLE 11-5 To what value of capacitance must the variable capacitor of a series circuit be adjusted to produce resonance at 600 kc if the inductance of the circuit is 300 μh?

GIVEN: $L = 300\ \mu\text{h}$ $\quad f_r = 600$ kc

FIND: C

SOLUTION:

$$C = \frac{25{,}300}{f_r^2L} = \frac{25{,}300}{600 \times 600 \times 300} = 0.000234\ \mu\text{f} = 234 \text{ pf}$$

11-5 Resonance Curves

If a voltage of constant frequency is applied to a series circuit containing a fixed inductance and a variable capacitance, the circuit may be adjusted so that it will be resonant at this frequency. If the frequency of the applied voltage is then varied from a value starting below the frequency of resonance and gradually increased to a value above the frequency of resonance, the current flowing through the circuit will vary from a very low value at a frequency below resonance, increasing until the frequency of resonance is reached. At this point the current will be at its maximum value and will decrease to a very low value again when the frequency is increased above the frequency of resonance. This variation in signal current with frequency change is shown by a resonance curve.

Method of Obtaining Resonance Curves. Figure 11-4*b* shows two resonance curves for the series circuit of Fig. 11-4*a* with two values of R. The values used to plot these resonance curves are given in Table 11-3. The impedance values were obtained by substituting the values of R, X_L, and X_C in Eq. (10-1) for a number of values of frequency above and below the frequency of resonance. The current values were obtained by substituting these values of impedance in Eq. (8-9) for a constant value of voltage of 0.5 volt. As the circuit is resonant at 1,500 kc, values were obtained for frequencies between 1,470 and 1,530 kc. Examination of the

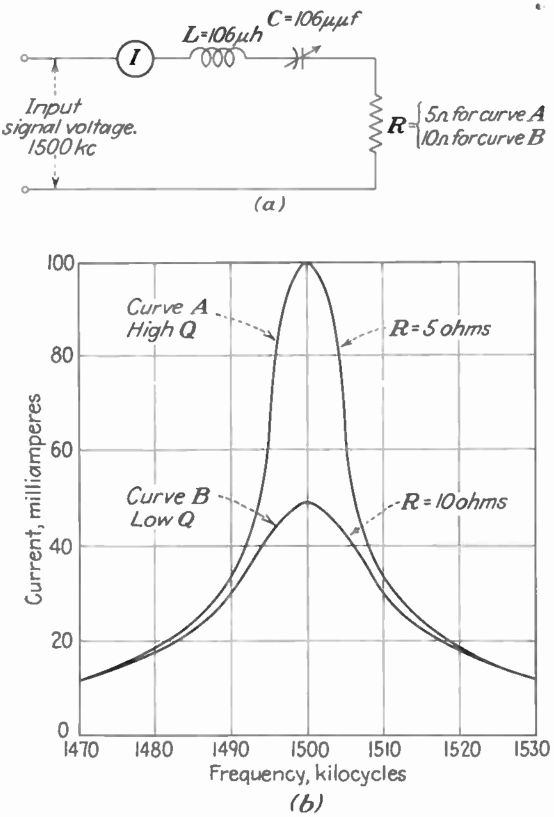


Fig. 11-4 Series resonance. (a) A series resonant circuit. (b) Resonance curves of the circuit shown in (a).

curves shows that no appreciable current flows until the frequency is approximately 1,470 kc. The current increases slowly until the frequency is very close to the resonant frequency. Then the current increases very rapidly until the maximum current is reached at the frequency of resonance. As the frequency is increased above resonance, the current decreases very rapidly at first and then decreases more slowly until no appreciable current flows at approximately 1,530 kc. A circuit that is or can be adjusted so that it is resonant for a definite frequency is referred to as a *tuned circuit.*

Effect of Resistance in Series Resonant Circuits. At resonance, the current in a series tuned circuit (with a constant voltage) is dependent entirely upon the resistance of the circuit. Figure 11-4*b* shows the resonance curves for two circuits tuned to the same frequency, one having a resistance of 5 ohms and the other a resistance of 10 ohms. At resonance, the current flowing through the circuit having 5 ohms resistance is twice the amount flowing through the circuit having 10 ohms resistance. Therefore, to produce maximum current, it is best that the resistance of a series tuned circuit be as low as possible.

Table 11-3

FREQUENCY, KC	X_L, OHMS	X_C, OHMS	R, OHMS	Z, OHMS	I, MA $E = 0.5$ VOLT
1,470	980	1,020	5	40.31	12.4
1,475	983.5	1,016.5	5	33.37	14.98
1,480	987	1,013	5	26.47	18.8
1,485	990	1,010	5	20.61	24.2
1,490	993	1,007	5	14.86	33.6
1,495	997	1,003	5	7.81	64.02
1,500	1,000	1,000	5	5.0	100
1,505	1,003	997	5	7.81	64.02
1,510	1,007	993	5	14.86	33.6
1,515	1,010	990	5	20.61	24.2
1,520	1,013	987	5	26.47	18.8
1,525	1,016.5	983.5	5	33.37	14.98
1,530	1,020	980	5	40.31	12.4
1,470	980	1,020	10	41.23	12.1
1,475	983.5	1,016.5	10	34.48	14.5
1,480	987	1,013	10	27.85	17.9
1,485	990	1,010	10	22.36	22.3
1,490	993	1,007	10	17.20	29.1
1,495	997	1,003	10	11.68	42.8
1,500	1,000	1,000	10	10.0	50.0
1,505	1,003	997	10	11.68	42.8
1,510	1,007	993	10	17.20	29.1
1,515	1,010	990	10	22.36	22.3
1,520	1,013	987	10	27.85	17.9
1,525	1,016.5	983.5	10	34.48	14.5
1,530	1,020	980	10	41.23	12.1

11-6 Circuit Q

Definition of Circuit Q. The ratio of the inductive reactance of a tuned circuit to the resistance of the circuit is referred to as the *Q of the circuit.* Expressed mathematically

$$Q = \frac{X_L}{R} \qquad (11\text{-}11)$$

As R is the resistance of the entire series circuit, its value will be higher, though only slightly higher, than the resistance of the coil. The circuit Q will therefore be less than the coil Q. The difference between the values of the circuit Q and the coil Q is normally small because the resistance of the capacitor and the connecting wires of the circuit is small compared with the resistance of the coil.

Relation between Circuit Q and the Slope of the Resonance Curves. The slope of the resonance curve is determined by the Q of the resonant circuit. This is shown by the relative slopes of the two curves in Fig. 11-4*b* and is explained in the following manner.

At resonance, the impedance of the series circuit becomes equal to R, and therefore the current is inversely proportional to the resistance as shown by

$$I_r = \frac{E}{R} \tag{11-12}$$

also,

$$R = \frac{X_L}{Q} \tag{11-13}$$

therefore

$$I_r = \frac{E}{\dfrac{X_L}{Q}} = \frac{EQ}{X_L} \tag{11-14}$$

Equation (11-14) shows that the current at resonance is directly proportional to the circuit Q; therefore, the higher the resistance, the lower will be the current and circuit Q, while the lower the resistance, the higher will be the current and circuit Q.

The impedance of the circuit for frequencies differing appreciably from the resonant frequency is practically equal to the reactance of the circuit, because at these frequencies the reactance is much greater than the resistance and the impedance becomes practically equal to the reactance of the circuit. The current at these frequencies is therefore practically independent of the circuit resistance. Increasing the circuit resistance, which also lowers the circuit Q, will decrease the current at resonance without affecting the current at frequencies differing appreciably from resonance. This action can be seen by observation of the two curves in Fig. 11-4*b*. The slope of the curve having a high circuit Q is steep. When the resistance of the circuit is increased, the current at resonance will decrease, thus decreasing the slope of the curve. This slope is very important in tuning and filter circuits and is discussed in more detail in subsequent articles.

Relation between the Width of the Resonance Curve and the Circuit Q. The width of the resonance curve can be used as a measure of the ability of a circuit to select the signal at a desired frequency and to eliminate the signals of all other frequencies. This characteristic is very important in tuning and filter circuits. It is general practice to measure the width of resonance curves at the point where the current is 0.707 times the current at resonant frequency.

This condition of $I = 0.707I_r$ will occur when the effective reactance $X_L - X_C$ is equal to the resistance R. This is shown mathematically in the following steps:

$$I = \frac{E}{\sqrt{R^2 + (X_L - X_C)^2}} \tag{11-15}$$

Substituting R for $X_L - X_C$, then

$$I = \frac{E}{\sqrt{2R^2}} = 0.707\frac{E}{R} \tag{11-16}$$

Substituting I_r for E/R,

$$I = 0.707I_r \tag{11-17}$$

If the frequency of the impressed voltage is varied both above and below the frequency of resonance, two values of frequency f_1 and f_2 can be obtained that will produce currents I_1 and I_2 each equal to $0.707I_r$, as shown in Fig. 11-5. The width of the resonance curve at this point is $f_2 - f_1$ and is generally referred to as the width of the band. The circuit Q can be expressed in terms of these frequencies and is equal to the resonant frequency divided by the difference between the two frequencies required to obtain a current of $0.707I_r$. This is derived mathematically in the following steps:

$$Q = \frac{X_L}{R} = \frac{2\pi f_r L}{R} \tag{11-18}$$

Substituting $X_L - X_C$ for R, then

$$Q = \frac{2\pi f_r L}{X_L - X_C} \tag{11-19}$$

However, as the condition of $X_L - X_C = R$ occurs at f_1 and f_2, then $X_L - X_C$ will also be equal to $2\pi f_2 L - 2\pi f_1 L$, and substituting this in the above equation, it becomes

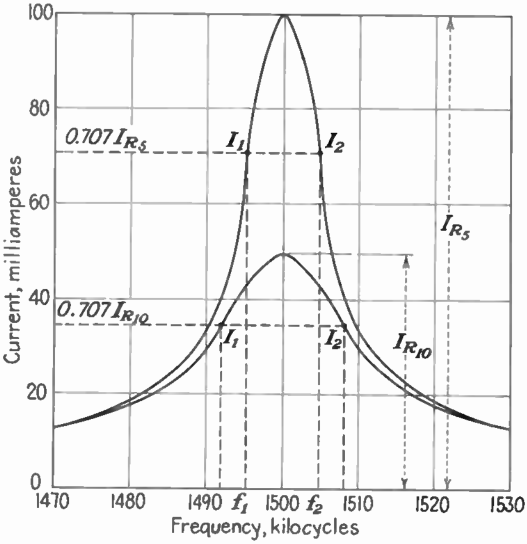


Fig. 11-5 Resonance curves with the width of the band ($f_2 - f_1$) indicated.

$$Q = \frac{2\pi f_r L}{2\pi f_2 L - 2\pi f_1 L} = \frac{f_r}{f_2 - f_1} \tag{11-20}$$

The quantity $f_2 - f_1$ is referred to as the width of the frequency band and may be expressed mathematically as

$$f_2 - f_1 = \frac{f_r}{Q} \tag{11-21}$$

Substituting X_L/R or $2\pi f_r L/R$ for Q in the above equation, it becomes

$$f_2 - f_1 = \frac{f_r R}{2\pi f_r L} = \frac{R}{2\pi L} \tag{11-22}$$

This equation indicates that the width of the resonance curve is directly proportional to its circuit resistance and inversely proportional to its inductance. In order to obtain high selectivity in tuning circuits it is essential that the width of the resonance curve be as narrow as practically possible. This means that the resistance of the circuit must be small and the inductance of the coil high; in other words, the circuit Q must be high.

EXAMPLE 11-6 A series tuned circuit has a resistance of 5 ohms and an inductance of 225 μh. (*a*) What is the width in cycles of its resonance curve at a point where the current in the circuit is equal to 0.707 times the current at resonance? (*b*) If the resistance of the circuit is increased to 10 ohms, how does the change affect the width of the resonance curve?

GIVEN: (*a*) $R = 5$ ohms $L = 225\ \mu$h (*b*) $R = 10$ ohms

FIND: (*a*) $f_2 - f_1$ (*b*) $f_2 - f_1$

SOLUTION:

(*a*) $$f_2 - f_1 = \frac{R}{2\pi L} = \frac{5}{6.28 \times 225 \times 10^{-6}} = 3{,}538 \text{ cycles}$$

(*b*) $$f_2 - f_1 = \frac{R}{2\pi L} = \frac{10}{6.28 \times 225 \times 10^{-6}} = 7{,}076 \text{ cycles}$$

11-7 *LC* Product

Relation of f_r, L, and C. Equation (11-5) shows that there is a definite relation among the frequency of resonance, the inductance, and the capacitance of a circuit. If the values of any two of these quantities are known, the third can be found by substituting the known values in this equation and solving for the unknown quantity. Solving by use of this equation is not always practicable; therefore other means have been devised.

LC Product. Equation (11-5) also shows that the frequency of resonance is dependent on the product of the inductance and capacitance of the circuit. For each value of L times C there can be only one frequency at which resonance occurs and, conversely, there can be only one LC

product for each resonant frequency. If the LC product for a desired frequency is known, the capacitance required can be found by dividing this product by the value of inductance used, or the inductance required can be found by dividing the LC product by the capacitance used. Values of L times C for commonly used frequencies are often listed in tabular or graphical form. These charts or tables can be found in reference books.

LC Ratio. As the frequency of resonance of a circuit is dependent only on its LC product, any number of combinations of L and C can be used to obtain the same resonant frequency. The numerical value of inductance can be made greater than, less than, or equal to that of the capacitance. The resonant-circuit application determines to a large extent the LC ratio that should be used.

When resonant circuits are used in the tuning circuit of a radio receiver, it is desirable to have high values of inductance. This is due to the fact that increasing the inductance increases the circuit Q, thereby increasing the slope of the response curve. The steeper this slope, the greater will be the degree of selectivity that can be obtained. However, increasing the selectivity beyond a certain point should be avoided, as it may decrease the fidelity of the receiver. This is but one example of the many applications of resonance. Its application to filter circuits is taken up in Chap. 12.

11-8 Voltage Ratios in Series Resonant Circuits

In a series resonant circuit, the impedance of the circuit at resonance is very low, thus allowing a comparatively high current to flow. This high current flowing through the capacitor and inductor causes a voltage to be developed across these reactances which is higher than the voltage applied to the circuit. The amount of voltage developed across either reactance is equal to the product of the current flowing through the circuit and the value of the reactance. Expressed mathematically,

$$E_{X_L} = I_r X_L \tag{11-23}$$

$$E_{X_C} = I_r X_C \tag{11-24}$$

where E_{X_L} = voltage developed across the inductor at resonance, volts
E_{X_C} = voltage developed across the capacitor at resonance, volts
I_r = current in the circuit at resonance, amp
X_L = inductive reactance at resonance, ohms
X_C = capacitive reactance at resonance, ohms

The current flowing through the circuit at resonance is equal to the voltage applied divided by the resistance of the circuit, or

$$I_r = \frac{E}{R} \tag{11-12}$$

Substituting E/R for I_r in Eq. (11-23),

$$E_{X_L} = \frac{EX_L}{R} \qquad (11\text{-}25)$$

Substituting Q for X_L/R,

$$E_{X_L} = EQ \qquad (11\text{-}26)$$

Relation between the Circuit Q and the Reactance Voltages. The voltages across the reactances have the same value because at resonance the reactances are equal. From this fact and from Eq. (11-26), it can be seen that the voltage across either the inductor or the capacitor at resonance is dependent on the circuit Q. The value of Q for many of the series resonant circuits used in electronics is greater than 100, and a series resonant circuit will develop high reactive voltages with a low applied voltage. This is possible because the voltages across the inductor and capacitor are equal and opposite. This leaves the resultant voltage across the circuit equal to the voltage across the resistor, which is equal to the voltage applied to the circuit. This ratio of voltages is illustrated in Fig. 11-6 and is explained in the following example.

EXAMPLE 11-7 The resistance of a series resonant circuit is 10 ohms, and its inductive reactance is equal to 500 ohms at resonant frequency. What voltage is developed across the inductor, capacitor, and resistor when the applied voltage is 5 volts?

GIVEN: $E = 5$ volts $\quad R = 10$ ohms $\quad X_L = 500$ ohms

FIND: $E_{X_L} \quad E_{X_C} \quad E_R$

SOLUTION:

$$I_r = \frac{E}{R} = \frac{5}{10} = 0.5 \text{ amp}$$

$$E_{X_L} = I_r X_L = 0.5 \times 500 = 250 \text{ volts}$$

At resonance, $X_L = X_C$

Therefore $\quad E_{XC} = E_{X_L} = 250$ volts

$$E_R = I_r R = 0.5 \times 10 = 5 \text{ volts}$$

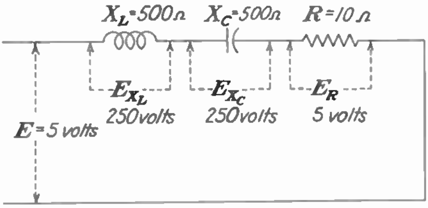


Fig. 11-6 Voltages across the components of a series resonant circuit at resonant frequency.

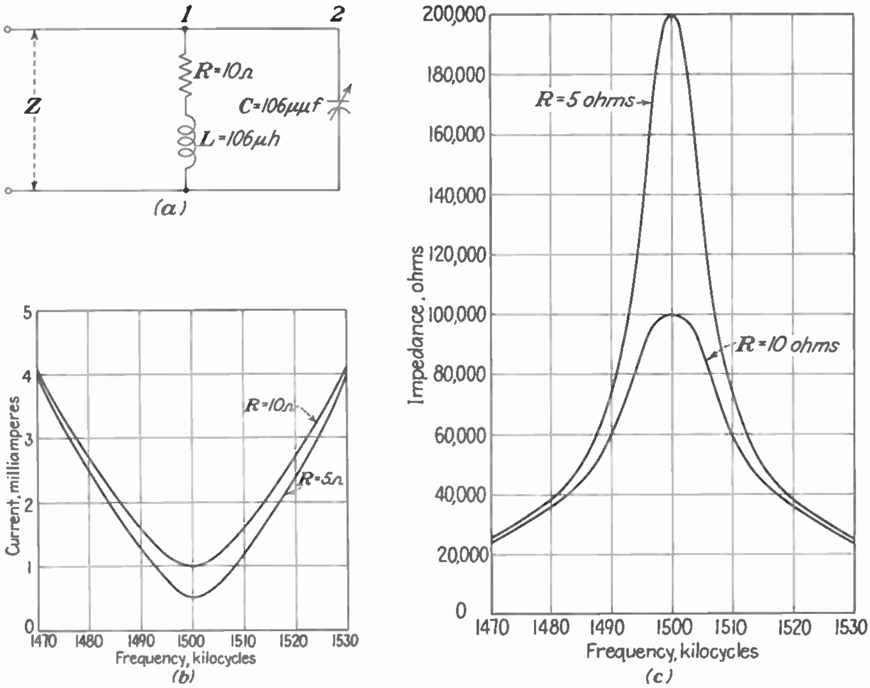


Fig. 11-7 Parallel resonance. (a) A parallel resonant circuit. (b) Current resonance curves. (c) Impedance resonance curves.

11-9 Parallel Resonant Circuit

When the inductance and capacitance of a parallel circuit are of such values that the inductive reactance is equal to the capacitive reactance at the frequency of the applied voltage, the circuit is called a *parallel resonant circuit.* The resistance of the circuit cannot be treated as a single value of R as was the case in series resonant circuits. It is generally true that most of the resistance of the circuit is in the coil and that the resistance of the capacitor is so low that it may be neglected. On this basis, the circuit is considered as an inductor with a small amount of resistance in series in one branch and a perfect capacitor (resistance neglected) in the other branch; such a circuit is shown in Fig. 11-7*a*. This circuit is actually a parallel-series circuit, and to understand its action, it should be treated as such. This may be done by assigning values to R, L, and C and solving for the reactances and impedances at a number of frequency values.

EXAMPLE 11-8 For the circuit of Fig. 11-7*a*, find: (*a*) the resonant frequency, (*b*) the inductive reactance, (*c*) the capacitive reactance, and (*d*) the circuit impedance.

GIVEN: $R = 10$ ohms $L = 106\ \mu\text{h}$ $C = 106$ pf

FIND: (*a*) f_r (*b*) X_L (*c*) X_C (*d*) Z_{cct}

SOLUTION:

(*a*) $$f_r = \frac{159}{\sqrt{LC}} = \frac{159}{\sqrt{106 \times 106 \times 10^{-6}}} = 1{,}500 \text{ kc}$$

(*b*) $$X_L = 2\pi f_r L = 6.28 \times 1{,}500 \times 10^3 \times 106 \times 10^{-3} = 1{,}000 \text{ ohms}$$

(*c*) $$X_C = \frac{1}{2\pi f_r C} = \frac{1}{6.28 \times 1{,}500 \times 10^3 \times 106 \times 10^{-12}} = 1{,}000 \text{ ohms}$$

(*d*) Z_{cct} may be found by using any one of the methods presented in Articles 10-10, 10-13, and 10-15. Using Eq. (10-15) and the method employing the *j* operator,

$$Z_{cct} = \frac{Z_1 Z_2}{Z_1 + Z_2} = \frac{(10 + j1{,}000)(-j1{,}000)}{10 + j1{,}000 - j1{,}000} = \frac{-j10{,}000 - j^2 1{,}000{,}000}{10}$$
$$= -j1{,}000 + 100{,}000 = \sqrt{1{,}000^2 + 100{,}000^2} \cong 100{,}000 \text{ ohms}$$

Parallel Resonance Curves. If Example 11-8 is repeated for a number of different frequencies, a set of values may be obtained for the plotting of a resonance curve. A parallel resonance curve is generally plotted with frequency as the abscissa and impedance as the ordinate, although occasionally curves may be plotted with current as the ordinate. In Table 11-4 a set of values for the circuit of Fig. 11-7*a* is shown, and resonant curves plotted from these values are shown in Fig. 11-7*b* and *c*.

EXAMPLE 11-9 Calculate the impedance of the parallel resonant circuit of Fig. 11-7*a* for a frequency of (*a*) 1,485 kc, (*b*) 1,500 kc, (*c*) 1,530 kc.

Note 1: As the circuit *Q* is high, Z_1Z_2 may be taken as X_LX_C. These values appear in Table 11-4 and will be taken from the table.

Note 2: $Z_1 + Z_2$ is equal to the series impedance of the circuit, and at points near resonance it must include the resistance whether it is high or low. Therefore, $Z_1 + Z_2 = \sqrt{R^2 + (X_L - X_C)^2}$.

GIVEN: (*a*) $f = 1{,}485$ kc (*b*) $f = 1{,}500$ kc (*c*) $f = 1{,}530$ kc (Fig. 11-7*a*)

FIND: (*a*) Z_p (*b*) Z_p (*c*) Z_p

SOLUTION:

(*a*) From Table 11-4, $R = 10$ ohms, $X_L = 990$ ohms, $X_C = 1{,}010$ ohms

$$Z_p = \frac{Z_1 Z_2}{Z_1 + Z_2} = \frac{990 \times 1{,}010}{\sqrt{10^2 + (990 - 1{,}010)^2}} = 44{,}718 \text{ ohms}$$

(*b*) From Table 11-4, $R = 10$ ohms, $X_L = 1{,}000$ ohms, $X_C = 1{,}000$ ohms

$$Z_p = \frac{Z_1 Z_2}{Z_1 + Z_2} = \frac{1{,}000 \times 1{,}000}{\sqrt{10^2 + (1{,}000 - 1{,}000)^2}} = 100{,}000 \text{ ohms}$$

(*c*) From Table 11-4, $R = 10$ ohms, $X_L = 1{,}020$ ohms, $X_C = 980$ ohms

$$Z_p = \frac{Z_1 Z_2}{Z_1 + Z_2} = \frac{1{,}020 \times 980}{\sqrt{10^2 + (1{,}020 - 980)^2}} = 24{,}244 \text{ ohms}$$

Table 11-4

FREQUENCY, KC	X_L, OHMS	X_C, OHMS	R, OHMS	Z (approx), OHMS	I, MA $E = 100$ VOLTS
1,470	980	1,020	5	25,000	4.00
1,475	983.5	1,016.5	5	30,000	3.33
1,480	987	1,013	5	38,000	2.63
1,485	990	1,010	5	48,000	2.08
1,490	993	1,007	5	67,000	1.49
1,495	997	1,003	5	128,000	0.78
1,500	1,000	1,000	5	200,000	0.50
1,505	1,003	997	5	128,000	0.78
1,510	1,007	993	5	67,000	1.49
1,515	1,010	990	5	48,000	2.08
1,520	1,013	987	5	38,000	2.63
1,525	1,016.5	983.5	5	30,000	3.33
1,530	1,020	980	5	25,000	4.00
1,470	980	1,020	10	24,250	4.12
1,475	983.5	1,016.5	10	29,000	3.44
1,480	987	1,013	10	36,000	2.78
1,485	990	1,010	10	44,500	2.25
1,490	993	1,007	10	58,000	1.72
1,495	997	1,003	10	85,500	1.17
1,500	1,000	1,000	10	100,000	1.00
1,505	1,003	997	10	85,500	1.17
1,510	1,007	993	10	58,000	1.72
1,515	1,010	990	10	44,500	2.25
1,520	1,013	987	10	36,000	2.78
1,525	1,016.5	983.5	10	29,000	3.44
1,530	1,020	980	10	24,250	4.12

Relation among Z, X, and Q. The impedance at resonance can be found by a simpler method as is shown in the following explanation. In the solutions for obtaining values for impedance for Table 11-4 the circuit impedance was obtained by dividing the voltage of the line by the current in the line, or

$$Z_{pr} = \frac{E_{\text{line}}}{I_{\text{line}}} \tag{11-27}$$

Examining the step where Eq. (10-8) was used to find I_{line}, it will be observed that the value of the second parentheses, which represents the total reactance current, is zero and also that the value of $I_2 \cos \theta_2$, which represents the in-phase or resistance component of the capacitor current, is zero. This is always true when there is resonance and the resistance of the capacitor is disregarded; hence at resonance I_{line} becomes equal to

$I_1 \cos \theta_1$. However, $E_{\text{line}} = E_C = E_L = E$, $I_1 = E/X_L$, and $\cos \theta_1 = R/X_L$; therefore

$$I_{\text{line}} = I_1 \cos \theta_1 = \frac{E}{X_L} \times \frac{R}{X_L} = \frac{ER}{X_L^2} \tag{11-28}$$

Substituting these values in Eq. (11-27),

$$Z_{pr} = \frac{E}{I_{\text{line}}} = \frac{E}{\dfrac{ER}{X_L^2}} = \frac{X_L^2}{R} \tag{11-29}$$

At resonance $X_L = X_C$; therefore this equation may be expressed in any of the following forms:

$$Z_{pr} = \frac{X_L^2}{R} \text{ or } \frac{X_C^2}{R} \text{ or } \frac{X_C X_L}{R} \text{ or } \frac{X_L X_L}{R} \text{ or } \frac{X_C X_C}{R} \tag{11-30}$$

Using next to the last form and substituting Q for X_L/R, the equation can be stated as

$$Z_{pr} = X_L Q \tag{11-31}$$

EXAMPLE 11-10 What is the circuit Q and the impedance at resonance for the circuit shown in Fig. 11-7*a*? The frequency of resonance and the value of X_L at resonance, taken from Table 11-4, are 1,500 kc and 1,000 ohms, respectively.

GIVEN: $R = 10$ ohms $X_L = 1{,}000$ ohms

FIND: Q Z_{pr}

SOLUTION:

$$Q = \frac{X_L}{R} = \frac{1{,}000}{10} = 100$$

$$Z_{pr} = X_L Q = 1{,}000 \times 100 = 100{,}000 \text{ ohms}$$

The error introduced in Eqs. (11-29) and (11-31) by disregarding the resistance of the capacitor will depend on the amount of the capacitor resistance and the circuit Q. This error is negligible for ordinary calculations and can be ignored if the circuit Q is not extremely low. Examination of Figs. 11-7*c* and 11-4*b* will show that the resonance curve for the impedance of a parallel circuit varies in the same manner as the resonance curve for current in a series circuit. The rules for the interpretation of the series resonance curve in regard to the slope and width of the curve will apply to the parallel circuit as well, and the frequency of resonance can be determined by the same equations used for the series resonant circuit.

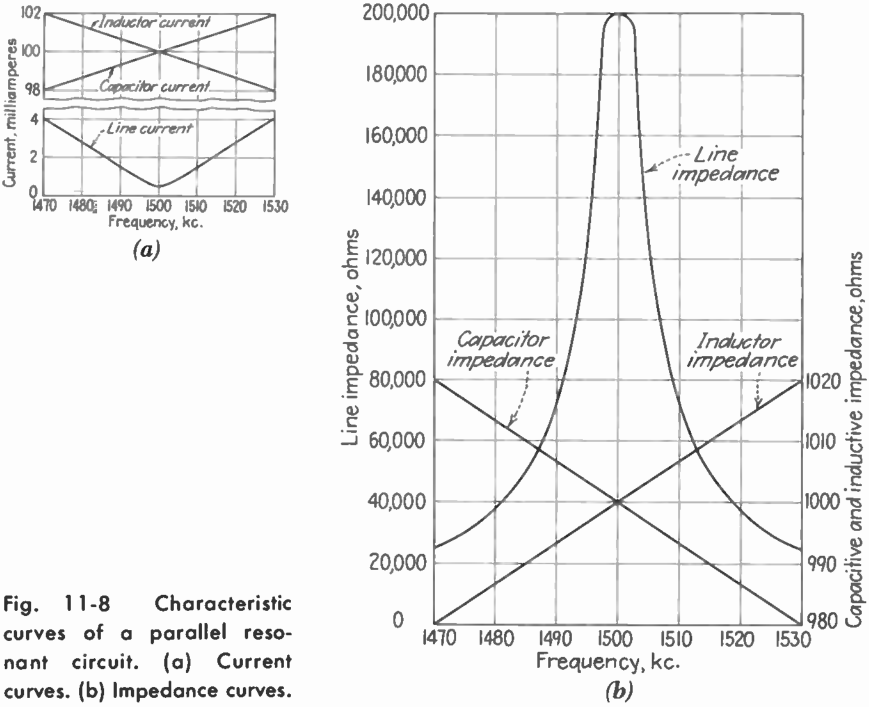
At resonance, the impedance of the parallel circuit is at its maximum value. Equation (11-31) shows that the impedance at resonance is equal to the product of Q and the reactance of either branch and is therefore much higher than the impedance of either branch. As the circuit Q is generally higher than 100, the impedance of the circuit will be more than 100 times the impedance of either branch.

If the resistance of the circuit of Fig. 11-7*a* were reduced to 5 ohms, it would result in a higher value of circuit Q as indicated by Eq. (11-11) and also a higher circuit impedance as indicated by Eq. (11-29). The effect of a resistance change upon the impedance and current can also be seen by examination of the resonance curves of Fig. 11-7*b* and *c*. This property of a parallel resonant circuit has many applications in electronic circuits, some of which are discussed in another part of this chapter.

11-10 Currents in Parallel Resonant Circuits

Line Current. The line current in a parallel resonant circuit is equal to the applied voltage divided by the impedance of the circuit. At resonance, the impedance of the circuit is maximum; therefore the line current is at its minimum. Because the impedance at resonance is very high (more than 100 times either reactance), the amount of current flowing in the line is very small. As the frequency changes from resonance, the circuit impedance decreases very rapidly and the line current will increase in the same manner.

Branch Currents. The current in each branch of a parallel resonant circuit is equal to the voltage applied to the circuit divided by the imped-



(a)

(b)

Fig. 11-8 Characteristic curves of a parallel resonant circuit. (a) Current curves. (b) Impedance curves.

ance of that branch. The impedance of each branch varies slowly with frequency change, and therefore the currents in the branch circuits do not go through resonant action. At resonance, the impedance of each branch is relatively low compared with the line impedance; therefore the currents flowing through the capacitor and inductor are much higher than the line current. This resonant action is due to the fact that the currents in each branch of a parallel resonant circuit are largely reactive and approximately 180° out of phase with one another; therefore the resultant current will be practically zero. At resonance, a high current will keep circulating between the inductance and the capacitance. The resistance of resonant circuits is generally very low, and hence the energy losses will be low. The current flowing in the line is only the amount needed to supply the circuit losses, and as the circuit losses are low the line current is relatively low.

Currents at Frequencies off Resonance. For frequencies other than resonance, the reactive currents are not equal. The resultant reactive current must be supplied by the line. As the difference in the reactive currents in each circuit will increase with the amount of deviation from resonant frequency, the line current will increase in the same manner. The relation between the line and branch currents is shown in the graph of Fig. 11-8*a*, while the graph in Fig. 11-8*b* shows the relation between the branch impedances and the line impedance.

11-11 Comparison of Series and Parallel Resonant Circuits

Resonance Curves and Frequency. In series resonant circuits, the resistance, inductance, capacitance, and the supply voltage are connected in series with one another. In parallel resonant circuits, an inductance containing a small amount of resistance is connected in parallel with a capacitance of negligible resistance. In both circuits, resonance occurs when the capacitive reactance equals the inductive reactance. The resonant frequency of both circuits can be calculated by Eqs. (11-5) and (11-8); both have similar resonance curves. Increasing the circuit Q will increase the slope of the resonance curves in both circuits, thus decreasing the width of their resonance curves.

The characteristics of series resonant circuits and parallel resonant circuits may be readily compared with each other by reference to the tabular listing of their circuit characteristics in Table 11-5.

Characteristics of Series Resonant Circuits. At resonance, the impedance of a series resonant circuit is at its minimum, and its value is equal to the resistance of the circuit. The circuit acts the same as a resistor. The current through all parts of the circuit is the same and is equal to the line current; the current is at its maximum value and is in phase with the applied voltage. The power factor of the circuit will therefore be unity. The voltages across the reactances are approximately equal and nearly 180° out of phase with each other, and the voltage across the resistance is equal to

Table 11-5

QUANTITY	SERIES CIRCUIT	PARALLEL CIRCUIT
At Resonance:		
Reactance; $(X_L - X_C)$	Zero; (because $X_L = X_C$)	Zero; (because $X_L = X_C$)
Frequency of resonance	$\frac{159}{\sqrt{LC}}$	$\frac{159}{\sqrt{LC}}$
Impedance	Minimum value; $Z = R$	Maximum value; $Z = QX_L$
I_{line}	Maximum value	Minimum value
I_L	I_{line}	Q times I_{line}
I_C	I_{line}	Q times I_{line}
E_L	Q times E_{line}	E_{line}
E_C	Q times E_{line}	E_{line}
Phase angle between I_{line} and E_{line}	0°	0°
Angle between E_L and E_C	180°	0°
Angle between I_L and I_C	0°	180°
Desired value of Q	High	High
Desired value of R	Low	Low
Highest selectivity	High Q; low R; high LC ratio	High Q; low R; high CL ratio
When f is greater than f_r		
Reactance; $(X_L - X_C)$	Inductive	Capacitive
Phase angle between I_{line} and E_{line}	Lagging current	Leading current
When f is less than f_r		
Reactance; $(X_L - X_C)$	Capacitive	Inductive
Phase angle between I_{line} and E_{line}	Leading current	Lagging current

the applied voltage. Increasing the value of the resistance will decrease both the current in the line and the voltage across each reactance. For frequencies below resonance, the capacitive reactance is greatest and the current is leading. For frequencies above resonance, the inductive reactance is greatest and the current is lagging.

Characteristics of Parallel Resonant Circuits. At resonance, the impedance of parallel resonant circuits is at its maximum, and its value is equal to the product of either reactance and the circuit Q. The circuit acts the same as a resistor, and the current and voltage are in phase; the power factor of the circuit will therefore be unity. The line current is at a minimum and is equal to the applied voltage divided by the impedance of the circuit. The voltages across the inductance and the capacitance are the same and are equal to the applied voltage. At resonance, the currents in the inductor and capacitor are approximately equal and nearly 180° out of phase with each other. Increasing the resistance of the circuit decreases the circuit impedance, thereby increasing the line current. For frequencies below resonance, the current in the inductor increases and the line current is lagging. For frequencies above resonance, the current in the capacitor increases and the line current is leading.

11-12 Uses of Resonant Circuits

Fundamental Uses. In radio, television, and other electronic circuits, the principles of resonance are used to increase the strength of a desired signal and to decrease to a minimum the strength of undesired signals. Series resonant circuits are used wherever maximum current is desired for a definite frequency or band of frequencies. Parallel resonant circuits are used wherever the signal strength of any one frequency or band of frequencies is to be reduced to a minimum.

Classification of Circuits as Series or Parallel. Frequently resonant circuits form only a part of the complete circuit. Because of this, it may be difficult to ascertain whether the resonant circuit is of the series or parallel type. In some instances, circuits that have all the appearances of a parallel resonant circuit are actually classed as series resonant circuits. This is usually the case when the inductor is the secondary of a transformer and the voltage of the circuit is an induced voltage rather than a voltage applied from an entirely separate source. For example, Fig. 11-9*c* shows two resonant circuits (drawn in heavy lines), each of which has the general appearance of a parallel circuit, but in analyzing the complete circuit the one at the left is considered a parallel resonant circuit and the one at the right is considered a series resonant circuit. The circuit at the left is con-

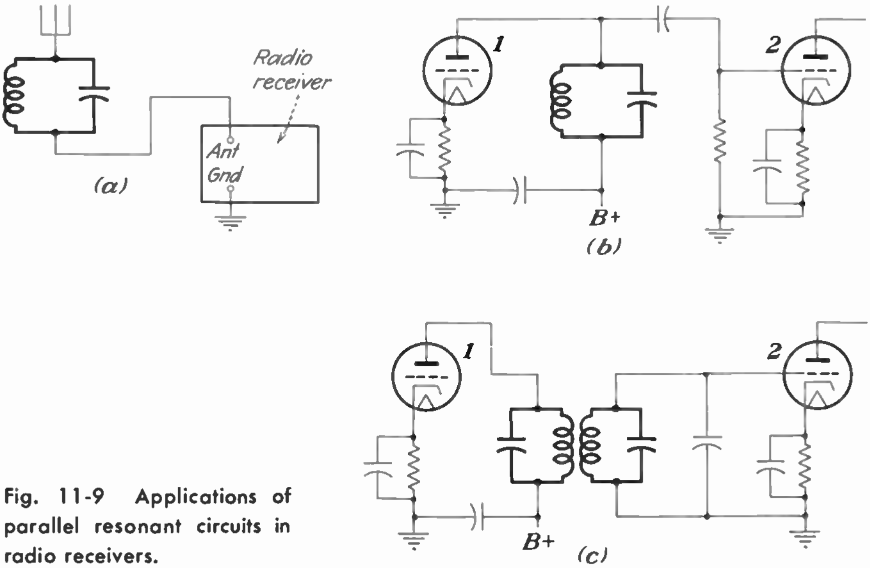


Fig. 11-9 Applications of parallel resonant circuits in radio receivers.

sidered a parallel resonant circuit because it receives its electrical energy from the plate circuit of tube 1. The circuit at the right is considered a series resonant circuit because no separate voltage is applied to the inductor and capacitor, but instead a voltage is induced in the inductor (secondary of an r-f transformer), which is considered as a voltage connected in series with the inductor and the capacitor. The following procedure will help to determine whether a circuit should be classed as parallel or series: (1) locate the inductive and capacitive components forming the resonant circuit; (2) locate the source of a-c voltage for these components; (3) determine whether the components are in series or parallel with the source of a-c voltage. In electronic circuits, the source of a-c voltage may be derived from any one of the following circuits: antenna, output of a vacuum tube, or the induced voltages from other circuits. Some of the applications of resonant circuits as applied to radio receivers are illustrated in Figs. 11-9 and 11-10. The resonant circuit in each illustration is shown in heavy lines.

Application of Resonant Circuits in the Antenna Circuit of a Receiver. A simple illustration of the use of the principle of resonance is given in the circuit shown in Fig. 11-9*a*. Here a parallel resonant circuit is connected in series with the antenna circuit of a receiver. This circuit will offer a very high impedance to any signal current at the frequency for which it is resonant and will therefore reduce such a signal current to a minimum. It thereby hinders currents of this frequency from entering the receiver but does not prevent currents of any other frequency from entering the receiver. Such a circuit is used to eliminate the signals of a station causing troublesome interference and is referred to as a *wave trap.*

A series resonant circuit also may be used as an antenna wave trap as illustrated in Fig. 11-10*a*. Being a series resonant circuit, it will offer a very low impedance to any signal current at the frequency for which it is resonant and a high impedance to currents of all other frequencies. It provides a low-impedance path to the ground for any signal current of the resonant frequency to which it is adjusted. It thereby keeps signal currents of the undesired frequency from entering the receiver and acts as a wave trap.

Wave traps may be used in the antenna circuit of all types of communication equipment to eliminate the signals from a particular station or from

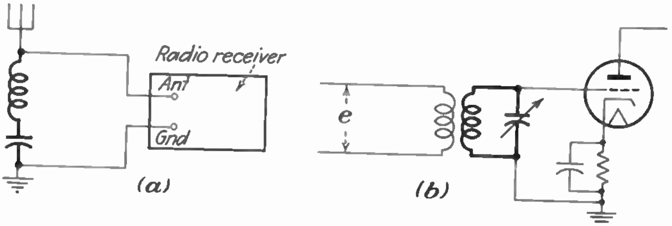
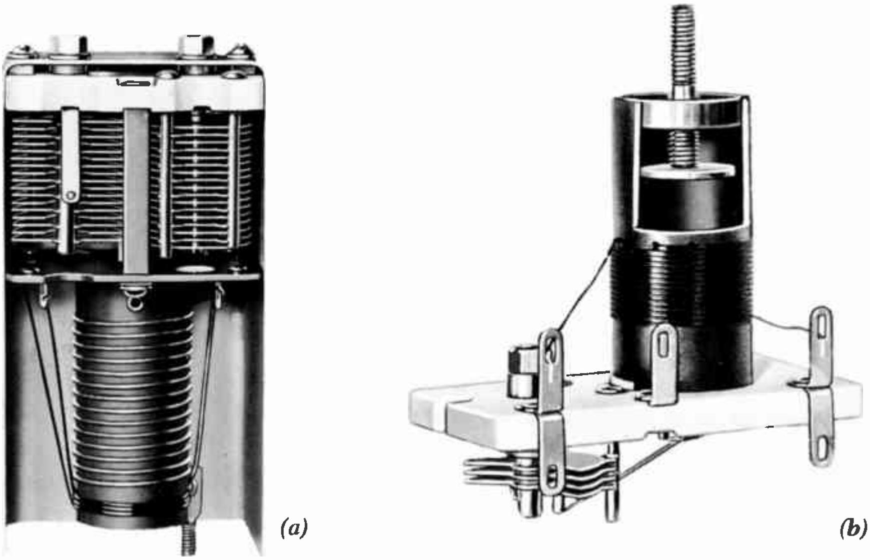


Fig. 11-10 Applications of series resonant circuits in radio receivers.



(a) (b)

Fig. 11-11 Applications of resonant circuits. (a) Tuning unit with an adjustable capacitor. (b) Tuning unit with an adjustable capacitor and adjustable inductor. (*Hammarlund Manufacturing Company, Inc.*)

any other source of interference. Resonant circuits are used to separate the currents of different frequencies in the various sections of all types of electronic equipment. As such they may be used to separate audio-frequency signals from intermediate-frequency signals, audio-frequency signals from video-frequency signals, direct currents from all signal currents, etc.

Use of Series Resonant Circuits in the Tuning Circuit of a Receiver. In order to listen to a signal being transmitted, the tuning circuit of the receiver must be adjusted so that its capacitive and inductive components produce resonance at the frequency of the carrier wave of the station transmitting the signal desired. The series resonant circuit shown in Fig. 11-10*b* is typical of many tuning circuits. The input signal voltage e may come from the antenna circuit or from the output of a previous tuning stage. The amount of signal voltage across the secondary of the transformer will be much greater because of the high reactive voltages developed in a series circuit at its resonant frequency. The value of either reactive voltage is more than that which could have been obtained by direct transformer ratio. This increase in voltage represents a gain in signal strength obtained by means of coupling. This voltage is now applied to the input of the following stage.

Use of Parallel Resonant Circuits to Obtain Selectivity and High Impedance. Parallel resonant circuits are used as the output load of a tube where it is necessary that this circuit, in addition to having a high selectivity, must

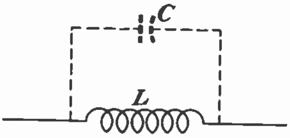


Fig. 11-12 Theoretical parallel resonant circuit of an inductance coil.

also have a high impedance. Such a circuit is shown in Fig. 11-9*b*. The advantages obtained by proper use of series and parallel resonant circuits is further illustrated by the manner in which they are used in the intermediate stages of superheterodyne receivers. Such a circuit is shown in Fig. 11-9*c*. The resonant circuit connected to the input of tube 2 must have a high selectivity but a low impedance in order to obtain a high reactive voltage; a series resonant circuit is therefore used. The resonant circuit connected to the output of tube 1 must have a high selectivity and a high impedance; a parallel circuit is therefore used. This circuit arrangement has excellent selective characteristics and is used in practically all types of communication receivers.

Resonant Frequency of Choke Coils. In Chap. 9, it has been shown that the distributed capacitance existing between the turns of a coil may be considered theoretically as a lumped capacitance placed across the terminals of the coil. If a coil is considered in this manner (Fig. 11-12), it is equivalent to a parallel resonant circuit and therefore has a resonant frequency of its own. Choke coils are designed to have a comparatively high inductive reactance for a wide band of frequencies below its resonant frequency when considered as a parallel resonant circuit. At resonance its impedance is very high, and for frequencies above resonance its reactance becomes capacitive and the coil acts as a capacitor. If there is no disadvantage in having the reactance capacitive instead of inductive, a coil with high distributed capacitance is superior for frequencies lower than its resonant frequency. Above this frequency, the actual impedance of the coil is less than it would be for a coil having a negligible amount of distributed capacitance.

QUESTIONS

1. Why are resonant circuits considered important to the study of electronics?
2. What is meant by the term *resonance* as applied to electric circuits?
3. (*a*) What is a graph? (*b*) Where are graphs used in electronics? (*c*) Why is the study of the plotting and interpretation of graphs essential to understanding radio, television, and other electronic circuits?
4. What is meant by (*a*) the abscissa? (*b*) The ordinate? (*c*) The point of origin?
5. (*a*) In plotting a graph, which variable is plotted as the ordinate? (*b*) What is the other variable called?
6. What type of curve will a graph of the following equations illustrate: (*a*) Plotting P versus E for the equation $P = E^2/R$? (*b*) Plotting X_L versus f for the equation $X_L = 2\pi fL$? (*c*) Plotting f_r versus LC for the equation $f_r = 1/2\pi\sqrt{LC}$?

7. Why is it necessary to obtain more points when plotting a graph having a curved line than for one having a straight line?
8. If a graph is plotted from results obtained from a laboratory experiment, is the curve drawn through all the points obtained? Explain.
9. In a series circuit containing inductance and capacitance, how do the inductive and capacitive reactances vary when the frequency of the input voltage is (*a*) increased? (*b*) Decreased?
10. Why should the resistance of series resonant circuits be kept at a minimum?
11. What quantities are generally used to plot (*a*) a series resonance curve? (*b*) A parallel resonance curve?
12. What is meant by a tuned circuit?
13. How does the shape of the resonance curve vary when its resistance is (*a*) increased? (*b*) Decreased?
14. How does the circuit Q of a tuned circuit compare with the coil Q?
15. How does the slope of the resonance curve for a tuned circuit vary when its circuit Q is (*a*) increased? (*b*) Decreased?
16. When the following factors are increased, how is the circuit Q of a tuned circuit affected: (*a*) resistance, (*b*) inductance, (*c*) frequency?
17. In order to obtain high selectivity in tuning circuits, should the circuit Q be large or small? Explain your answer.
18. Explain how the voltage across either reactance in a series tuned circuit can be greater than the applied voltage.
19. Will the reactive voltages in a series resonant circuit increase or decrease when the circuit Q is increased? Explain.
20. (*a*) When solving parallel resonant circuits, why must they be treated as parallel-series circuits? (*b*) Why is the resistance of the capacitor generally ignored?
21. Under what conditions may the impedance of a parallel resonant circuit be taken as the product of the inductive reactance and the circuit Q?
22. How do parallel resonant circuits differ from series resonant circuits?
23. How do the rules for the interpretation of parallel resonance curves compare with those for a series circuit? Why?
24. How does a decrease in the circuit resistance affect the impedance of a parallel resonant circuit?
25. How does the impedance of each branch of a parallel resonant circuit compare with the impedance of the circuit (*a*) at the resonant frequency? (*b*) For frequencies below resonance? (*c*) For frequencies above resonance?
26. How does the current flowing in each branch of a parallel resonant circuit compare with the line current (*a*) at the resonant frequency? (*b*) For frequencies below resonance? (*c*) For frequencies above resonance?
27. How does the power factor of the inductance and capacitance of parallel and series resonant circuits compare with the power factor of the line (*a*) at the resonant frequency? (*b*) For frequencies below resonance? (*c*) For frequencies above resonance?
28. In what respects are parallel resonant circuits similar to series resonant circuits?
29. In what respects do parallel resonant circuits differ from series resonant circuits?
30. What factor determines whether a resonant circuit should be classed as a series or parallel type?
31. What procedure should be followed in determining whether a resonant circuit is of the series or parallel type?

32. Name four applications of series resonant circuits to radio.
33. Name four applications of parallel resonant circuits to radio.
34. What is meant by the distributed capacitance of a coil?
35. Why must the distributed capacitance of a choke coil be taken into consideration?

PROBLEMS

1. Plot a curve showing how the power varies with current changes from zero to 10 amp if the voltage is kept constant at 125 volts.
2. Plot a curve showing how the power varies with voltage changes from zero to 125 volts for a circuit whose resistance is 0.25 megohm.
3. The following data were taken for plotting a curve for a certain tube characteristic. The grid voltage of the tube was kept constant at -5 volts. The following readings of plate current (milliamperes) were obtained for the values of plate voltage listed below. Plot an E_pI_p curve.

E_p	I_p
50	0.35
75	1.2
100	2.5
125	4.0
150	5.5
175	7.2
200	9.0

4. The following data were taken for plotting a curve for a certain tube characteristic. The plate voltage of the tube was kept constant at 200 volts. The following readings of plate current (milliamperes) were obtained for the values of grid voltage listed below. Plot an E_gI_p curve.

E_g	I_p
-14	0
-12	0.5
-10	1.5
-8	3.5
-6	7.0
-4	11.0
-2	16.0
0	21.0

5. A resonant circuit has an inductance of 316 μh and a capacitance of 80 pf. (*a*) Plot a curve showing how the inductive reactance changes when the frequency is varied from 975 to 1,025 kc. (*b*) On the same paper, using the same reference abscissa, plot another curve showing how the capacitive reactance changes for the same frequency range. In plotting these curves, obtain values for every 5 kc.
6. A resonant circuit has an inductance of 0.775 μh and a capacitance of 7.5 pf. (*a*) Plot a curve showing how the inductive reactance changes when the frequency is varied from 56 to 76 mc. (*b*) On the same paper, using the same reference abscissa, plot another curve showing how the capacitive reactance

changes for the same frequency range. In plotting these curves, obtain values for every 2 mc.

7. Using the curves obtained in Prob. 5, determine the resultant circuit reactance at the following frequencies: (*a*) 980, (*b*) 990, (*c*) 1,000, (*d*) 1,010, (*e*) 1,020 kc.
8. Using the curves obtained in Prob. 6, determine the resultant circuit reactance at the following frequencies: (*a*) 57, (*b*) 63, (*c*) 66, (*d*) 69, (*e*) 75 mc.
9. Determine the resonant frequency of the circuit used in Prob. 5 (*a*) by referring to the curves obtained in Prob. 5, (*b*) by substituting the values of inductance and capacitance in the equation for finding the frequency of resonance.
10. Determine the resonant frequency of the circuit used in Prob. 6 (*a*) by referring to the curves obtained in Prob. 6, (*b*) by substituting the values of inductance and capacitance in the equation for finding the frequency of resonance.
11. A series tuned circuit has an inductance of 316 μh. To what value of capacitance must its variable capacitor be adjusted in order to obtain resonance for the following frequencies: (*a*) 500 kc? (*b*) 1,500 kc?
12. A series tuned circuit has a capacitance of 9.2 pf. Determine the value of inductance required to obtain resonance for the following frequencies: (*a*) 54 mc, (*b*) 198 mc.
13. A variable capacitor having a maximum capacitance of 350 $\mu\mu$f is used for tuning a broadcast radio receiver. (*a*) What inductance is required to make the circuit resonant at the lowest frequency 500 kc? (*b*) If the minimum capacitance of the capacitor is 15 pf, what is the highest frequency that can be obtained with the inductance determined in part (*a*)?
14. Determine the value of capacitance required for a tuned circuit having the following values: (*a*) $L = 0.0755$ μh; $f_r = 473$ mc; (*b*) $L = 0.08$ μh; $f_r = 749$ mc.
15. The coil used in Prob. 13 plus the circuit wiring has a distributed capacitance of 15 $\mu\mu$f thus increasing the circuit capacitance by this amount. What is the frequency range of the circuit?
16. Determine the frequency of the signal that is attenuated by a parallel resonant circuit having the following circuit values: (*a*) $C = 40$ pf and $L = 0.335$ μh; (*b*) $C = 37$ $\mu\mu$f and $L = 0.4$ μh.
17. It is desired to tune a frequency band whose lowest frequency is 1,700 kc by connecting a different coil to the capacitor in Prob. 13. (*a*) Find the inductance of the coil. What is the highest frequency that can be tuned (*b*) if the distributed circuit capacitance (15 pf) is to be taken into consideration and the inductance coil is used as determined in part (*a*)? (*c*) If the distributed capacitance of the circuit is ignored? (*d*) If the distributed circuit capacitance is ignored and the minimum value of the variable capacitor is 10 pf?
18. A variable capacitor having a maximum capacitance of 140 $\mu\mu$f and a minimum capacitance of 10 $\mu\mu$f is used to tune a frequency band whose lowest frequency is 1,700 kc. (*a*) Find the inductance of the coil. What is the highest frequency to which the circuit can be tuned (*b*) if the distributed circuit capacitance (10 pf) is to be taken into consideration? (*c*) If the distributed circuit capacitance is ignored? (*d*) If the distributed circuit capacitance is ignored and the minimum value of the variable capacitor is 5 $\mu\mu$f?
19. The inductance of a tuned circuit is 0.0304 μh, the capacitance is 20 pf, and its effective distributed capacitance is 2 $\mu\mu$f. Determine its resonant frequency when the distributed capacitance (*a*) is ignored, (*b*) is taken into consideration.

20. The capacitor in Prob. 18 is to be used to tune a frequency band whose lowest frequency is 6.5 mc. In this problem the distributed capacitance is to be ignored. (*a*) Find the inductance of the coil. (*b*) What is the highest frequency that can be tuned?

21. It is desired to tune a frequency band whose lowest frequency is 1,700 kc by the use of a fixed capacitor C_S connected in series with the tuning capacitor C_T, as shown in Fig. 11-13. The maximum and minimum capacitances of the tuning capacitor are 350 and 15 pf, respectively; the inductance of the secondary is 290 μh; and the distributed capacitance is 15 pf. (*a*) Find the capacitance of the series capacitor C_S. What is the highest frequency to which the circuit may be tuned (*b*) if the distributed circuit capacitance is to be taken into consideration and the series capacitor is used as determined in part (*a*)? (*c*) If the distributed circuit capacitance is ignored? (*d*) If the distributed circuit capacitance is ignored and the minimum value of the variable capacitor is 10 μμf?

22. A variable capacitor having a maximum capacitance of 140 pf and a minimum capacitance of 10 pf is used to tune a frequency band whose lowest frequency is 4.63 mc. The distributed capacitance is 5 μμf, and the inductance of the secondary winding is 58.5 μh. (*a*) What value of capacitance must be connected in series with the tuning capacitor in order to obtain this minimum frequency when all its plates are in mesh? What is the highest frequency to which the circuit may be tuned (*b*) if the distributed circuit capacitance is to be taken into consideration and the series capacitor is used as determined in part (*a*)? (*c*) If the distributed circuit capacitance is ignored? (*d*) If the distributed circuit capacitance is ignored and the minimum value of the variable capacitor is 2.5 pf?

23. A series resonant circuit is to be used as a wave trap to eliminate the effect of a 1,200-kc signal. What value of capacitance must be used if the coil has an inductance of 80 μh and a distributed capacitance of 10 pf?

24. A coil having a distributed capacitance of 10 μμf is connected in series with a 350-pf capacitor in order to pass a 750-kc signal. Find the inductance of this coil.

25. A 120-μμf adjustable capacitor and an inductance coil are connected in parallel to form the primary side of an i-f transformer whose resonant frequency is 460 kc (Fig. 11-14). (*a*) What is the inductance of the primary winding? (*b*) What is the Q of the primary winding if its resistance is 9.85 ohms?

26. A 4.5-pf capacitor and an adjustable inductance coil are connected in parallel to form the primary side of an i-f transformer whose resonant frequency is 41.25 mc. (*a*) What is the inductance of the primary winding? (*b*) What is the Q of the primary winding if its resistance is 4.62 ohms?

27. A 2.5-mh coil and an adjustable capacitor are connected to form the secondary

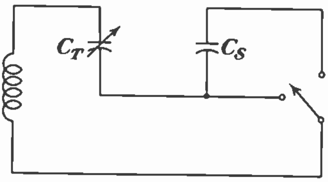


Fig. 11-13

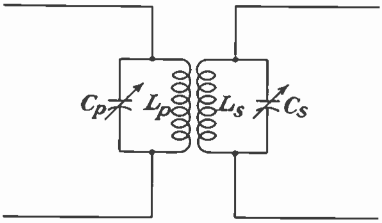


Fig. 11-14

side of an i-f transformer whose resonant frequency is 460 kc (Fig. 11-14). (*a*) What is the capacitance of the adjustable capacitor? (*b*) What is the *Q* of the secondary winding if its resistance is 13.75 ohms?

28. An adjustable inductance coil that is set at 10 μh and a fixed capacitor are connected to form the secondary side of an i-f transformer whose resonant frequency is 10.7 mc. (*a*) What is the capacitance of the fixed capacitor? (*b*) What is the *Q* of the secondary winding if its resistance is 4.1 ohms?

29. A 10-mv signal is applied to a series resonant circuit having an inductance of 316 μh, a capacitance of 80 pf (same as Prob. 5), and a resistance of 10 ohms. Plot the resonance curve for this circuit.

30. A 10-mv signal is applied to a series resonant circuit having an inductance of 0.775 μh, a capacitance of 7.5 pf (same as Prob. 6), and a resistance of 9.72 ohms. Plot the resonance curve for this circuit.

31. Determine the width of the frequency band of the circuit used in Prob. 29 (*a*) from the resonance curve as plotted, (*b*) substituting the values of inductance and resistance in the equation for finding the width of the frequency band.

32. Determine the width of the frequency band of the circuit used in Prob. 30 (*a*) from the resonance curve as plotted, (*b*) substituting the values of inductance and resistance in the equation for finding the width of the frequency band.

33. (*a*) What voltage is developed across the resistor, inductor, and capacitor of the series resonant circuit used in Prob. 29? (*b*) What is the value of the circuit *Q*?

34. (*a*) What voltage is developed across the resistor, inductor, and capacitor of the series resonant circuit used in Prob. 30? (*b*) What is the value of the circuit *Q*?

35. What is the impedance of the parallel resonant circuit used in Prob. 25?

36. What is the impedance of the parallel resonant circuit used in Prob. 26?

37. A parallel resonant circuit is to be used as a wave trap to eliminate the effects of a 1,300-kc signal. The circuit has a resistance of 1.5 ohms and a capacitance (distributed and wiring) of 10 pf. (*a*) What value of inductance must be used with a capacitor whose value is 65 μμf? (*b*) What is the circuit *Q*? (*c*) What is the width of the band being eliminated?

38. A parallel resonant circuit is to be used as a wave trap to eliminate the effects of a 100-mc signal. The circuit has a resistance of 1.25 ohms and a capacitance (distributed and wiring) of 5.3 μμf. (*a*) What value of inductance must be used with a capacitor of 20 pf? (*b*) What is the circuit *Q*? (*c*) What is the width of the band being eliminated?

39. A coil having an inductance of 320 μh is connected in parallel with an adjustable capacitor in order to bypass a band of frequencies between 999.25 and 1,000.75 kc. The distributed capacitance of the coil and circuit is 9 pf. (*a*) What is the value of the adjustable capacitor? (*b*) What is the resistance of the circuit?

40. A coil having an inductance of 0.9 μh is connected in parallel with an adjustable capacitor in order to bypass a band of frequencies between 39.75 and 40.25 mc. The distributed capacitance of the coil and circuit is equal to 5.5 μμf. (*a*) What is the value of the adjustable capacitor? (*b*) What is the resistance of the circuit?

41. Using the method outlined in the article on parallel resonance, find the impedance for the following points for the 5-ohm curve Fig. 11-7*c*: (*a*) $f = 1492.5$ kc; (*b*) $f = 1502.5$ kc.

42. Using the method outlined in the article on parallel resonance, find the impedance for the following points for the 10-ohm curve Fig. 11-7*c*: (*a*) $f = 1{,}497.5$ kc; (*b*) $f = 1{,}507.5$ kc.

43. Plot a parallel resonance curve for a circuit having a capacitance of 80 $\mu\mu$f, an inductance of 316 μh, and a resistance of 10 ohms [use Eq. (10-15)]. *Note:* These are the same values used in Prob. 5.
44. Plot a parallel resonance curve for a circuit having a capacitance of 7.5 pf, an inductance of 0.775 μh, and a resistance of 0.1 ohm [use Eq. (10-15)]. *Note:* These are the same values used in Prob. 6.

Chapter 12

Basic Electronic Circuits

A circuit diagram of a radio receiver, a television receiver, or a piece of industrial electronic equipment is generally quite complex and when examined as a whole unit may be very confusing. Every electronic circuit is made up of a number of individual circuits, and to understand the complete circuit, each individual circuit must be analyzed separately.

The purpose of this chapter is to study the electrical effects of each circuit and its relation to other circuits. The applications of these circuits in producing specific actions such as amplification, detection, or oscillation are taken up in the authors' text *Essentials of Radio—Electronics.*

12-1 The Electric Circuit

Circuit Elements. Every electric circuit must contain at least one conducting element. This element may be a resistor, inductor, or capacitor. The purpose of each type of element and its effects on the electric circuit have been discussed in previous chapters. In general, a resistor in a circuit will limit the amount of current flowing in that circuit. Inductance in a circuit will oppose any change in the amount of current flow. Capacitance in a circuit will oppose any change in the amount of voltage impressed across the circuit.

Stray Resistance, Capacitance, and Inductance. Every wire-wound resistor will have, in addition to its resistance, an inductance effect due to the turns of wire and a capacitance effect due to the space (insulation) between each turn. Every coil will have, in addition to its inductance, a resistance whose value will depend on the size, length, and kind of wire used and a capacitance effect due to the insulation and spacing between each turn. The current at a capacitor must flow through a path consisting of its leads and plates. There will therefore be some resistance in this path. In order to obtain the maximum capacitance in the minimum amount of space, the plates of fixed capacitors are wound in the form of a coil. This construction causes an inductance effect to exist in this type of capacitor.

Stray resistance, inductance, and capacitance effects will therefore exist in all resistors, inductors, and capacitors. Many of these units are designed in such a way as to minimize these effects. At audio frequencies and the lower radio frequencies, stray effects can usually be ignored, but at the

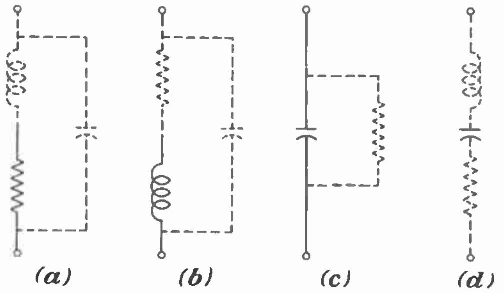


Fig. 12-1 Equivalent circuits for the three kinds of circuit elements. (a) A resistor. (b) An inductor. (c) A capacitor used on direct current. (d) A capacitor used on alternating current.

higher frequencies, these effects are troublesome and have to be taken into consideration when selecting the units to be used. It is therefore appropriate at this time to sum up the action of these stray effects on each of the circuit elements.

Equivalent Circuits. The inductance of a resistor may be considered as an inductor connected in series with the resistance. The capacitance of the resistor may be considered as a capacitor connected in parallel with the resistance. The equivalent circuit for the resistor is then as shown in Fig. 12-1*a*.

The resistance of a coil may be considered as a resistor connected in series with the inductance. The distributed capacitance between adjacent turns and nonadjacent turns and between each turn and the ground may be considered as a single lumped capacitance connected in parallel with the coil. The equivalent circuit for the inductance coil is then as shown in Fig. 12-1*b*.

Capacitors are used in both d-c and a-c circuits. When a capacitor is used in a d-c circuit, there is no inductive effect and the resistance of the plates and leads is considered as a resistor connected in parallel with the capacitor (Fig. 12-1*c*). When a capacitor is used in a-c circuits, the equivalent circuit may be considered as either series or parallel. The series equivalent circuit is generally more useful and is shown in Fig. 12-1*d*. Often the inductance is negligible, and for capacitors with low losses the equivalent series resistance approaches zero.

EXAMPLE 12-1 The stray inductance of a 5,000-ohm resistor is equal to 11 μh. If the distributed capacitance is ignored, what is the equivalent impedance of this resistor at (*a*) 500 kc? (*b*) 100 mc?

GIVEN: $R = 5{,}000$ ohms $L = 11\ \mu\text{h}$ (*a*) $f = 500$ kc (*b*) $f = 100$ mc

FIND: (*a*) Z_{500} (*b*) Z_{100}

SOLUTION:

(*a*)

$$X_L = 2\pi fL = 6.28 \times 500 \times 10^3 \times 11 \times 10^{-6} = 34.54 \text{ ohms}$$

$$Z_{500} = \sqrt{R^2 + X_L^2} = \sqrt{(5{,}000)^2 + (34.54)^2} = 5{,}000.11 \text{ ohms}$$

(*b*)
$$X_L = 2\pi fL = 6.28 \times 100 \times 10^6 \times 11 \times 10^{-6} = 6{,}908 \text{ ohms}$$
$$Z_{100} = \sqrt{R^2 + X_L{}^2} = \sqrt{(5{,}000)^2 + (6{,}908)^2} = 8{,}527 \text{ ohms}$$

The solution of Example 12-1 indicates that for low r-f signals, such as those used in commercial a-m broadcast receivers, the effects of the stray inductance are negligible. However, for higher frequency signals, such as those used in short-wave, f-m, television, and radar receivers, the inductive reactance may be equal to or many times greater than the ohmic resistance of the resistor. In these instances the resistor acts more like an inductor than a resistor.

EXAMPLE 12-2 An r-f plate-choke coil has an inductance of 200 μh and a distributed capacitance of 15 pf. If the ohmic resistance is ignored, find the inductive and capacitive reactances of the coil at (*a*) 500 kc, (*b*) 100 mc.

GIVEN: $L = 200$ μh $C_D = 15$ pf (*a*) $f = 500$ kc (*b*) $f = 100$ mc

FIND: (*a*) X_L, X_C at 500 kc (*b*) X_L, X_C at 100 mc

SOLUTION:

(*a*)
$$X_L = 2\pi fL = 6.28 \times 500 \times 10^3 \times 200 \times 10^{-6} = 628 \text{ ohms}$$
$$X_C = \frac{1}{2\pi fC_D} = \frac{1}{6.28 \times 500 \times 10^3 \times 15 \times 10^{-12}} \cong 21{,}200 \text{ ohms}$$

(*b*)
$$X_L = 2\pi fL = 6.28 \times 100 \times 10^6 \times 200 \times 10^{-6} \cong 125{,}600 \text{ ohms}$$
$$X_C = \frac{1}{2\pi fC_D} = \frac{1}{6.28 \times 100 \times 10^6 \times 15 \times 10^{-12}} \cong 106 \text{ ohms}$$

The solution of Example 12-2 indicates that for low r-f signals the effects of the distributed capacitance can usually be ignored, as X_C is many times greater than X_L and hence practically all the signal current will flow through the inductance. However, for higher frequency signals X_L is many times greater than X_C and practically all the signal current will flow through the distributed capacitance. In these instances the inductor acts more like a capacitor than an inductor.

EXAMPLE 12-3 (*a*) Determine the frequency of resonance of the inductor used in Example 12-2. (*b*) Determine the impedance of the inductor at its resonant frequency if its ohmic resistance is 4 ohms.

GIVEN: $L = 200$ μh $C_D = 15$ pf $R = 4$ ohms

FIND: (*a*) f_r (*b*) Z_r

SOLUTION:

(*a*)
$$f_r = \frac{159}{\sqrt{LC}} = \frac{159}{\sqrt{200 \times 15 \times 10^{-6}}} \cong 2{,}900 \text{ kc}$$

(*b*)
$$X_L = 2\pi fL = 6.28 \times 2.9 \times 10^6 \times 200 \times 10^{-6} \cong 3{,}642 \text{ ohms}$$
$$Z_L = QX_L = \frac{3{,}642}{4} \times 3{,}642 \cong 3{,}316{,}000 \text{ ohms}$$

From the solution of Example 12-3 it is evident that the impedance of the inductor is so great at resonance that it will completely block all signals of this frequency from passing through the inductor.

The equivalent series resistance of a capacitor may be obtained by the equation

$$\text{ESR} = X_C\,\text{DF} \tag{12-1}$$

The equivalent parallel resistance of a capacitor may be obtained by the equation

$$\text{EPR} = \frac{X_C}{\text{DF}} \tag{12-2}$$

where DF is the dissipation factor of the dielectric in decimals.

EXAMPLE 12-4 Determine the equivalent series resistance of a 0.002-μf mica capacitor at (*a*) 1 kc, (*b*) 1 mc.

GIVEN: $C = 0.002\ \mu\text{f}$ di = mica   (*a*) $f = 1$ kc   (*b*) $f = 1$ mc

FIND: (*a*) ESR at 1 kc   (*b*) ESR at 1 mc

SOLUTION:

From Table 9-1,

$$\text{DF} = 0.1\% \text{ at 1 kc} \qquad \text{DF} = 0.1\% \text{ at 1 mc}$$

(*a*)

$$X_C = \frac{1}{2\pi fC} = \frac{1}{6.28 \times 10^3 \times 2{,}000 \times 10^{-12}} = 79{,}500 \text{ ohms}$$

$$\text{ESR} = X_C\,\text{DF} = 79{,}500 \times 0.001 = 79.5 \text{ ohms}$$

(*b*)

$$X_C = \frac{1}{2\pi fC} = \frac{1}{6.28 \times 10^6 \times 2{,}000 \times 10^{-12}} = 79.5 \text{ ohms}$$

$$\text{ESR} = X_C\,\text{DF} = 79.5 \times 0.001 = 0.0795 \text{ ohm}$$

EXAMPLE 12-5 Determine the equivalent parallel resistance of a 0.002-μf mica capacitor at (*a*) 1 kc, (*b*) 1 mc.

GIVEN: $C = 0.002\ \mu\text{f}$ di = mica   (*a*) $f = 1$ kc   (*b*) $f = 1$ mc

FIND: (*a*) EPR at 1 kc   (*b*) EPR at 1 mc

SOLUTION:

(*a*)

$$X_C = 79{,}500 \text{ ohms} \qquad \text{from Example 12-4}$$

$$\text{EPR} = \frac{X_C}{\text{DF}} = \frac{79{,}500}{0.001} = 79.5 \text{ megohms}$$

(*b*)

$$X_C = 79.5 \text{ ohms} \qquad \text{from Example 12-4}$$

$$\text{EPR} = \frac{X_C}{\text{DF}} = \frac{79.5}{0.001} = 79{,}500 \text{ ohms}$$

Examples 12-4 and 12-5 indicate that the effects of either the equivalent series resistance or equivalent parallel resistance of low-value mica capacitors are negligible for practically all frequencies.

Individual Circuits. Resistors, inductors, or capacitors can be connected individually, in multiple, or in combination with one another. The electric circuit resulting from the interconnection of the circuit elements may be a simple, series, parallel, or complex circuit. The analysis of these circuits for both direct and alternating currents has been previously discussed.

Combined Circuit. Electronic circuits consist of a number of individual circuits, each designed to accomplish a definite purpose. It is the combined effect of each of its individual circuits which produces the desired performance of any type of electronic equipment. In combining two or more individual circuits, two factors must be taken into consideration: (1) the means used to transfer energy from one circuit to another, (2) the separation of the different types of current so that they will flow through the proper circuit.

Coupling. In order to transfer electrical energy from one circuit element to another, a conducting material, usually a wire, is used. In order to transfer electrical energy from one circuit to another, a common impedance must exist between the two circuits. The common impedance is called the *coupling element*, and the two circuits connected by means of a coupling element are said to be *coupled*. There are various ways in which two circuits may be coupled with each other, and each method has a definite use and produces different effects. Various types of coupling are discussed later in this chapter.

Filters. When two or more individual electronic circuits are combined, they form a complex circuit through which the following kinds of currents may flow: (1) direct, (2) power frequency (60 cycles), (3) audio frequencies (20 to 20,000 cycles), and (4) radio frequencies (100 kc to 30,000 mc). One side of any number of these circuits may be completed through a common wire or through the chassis. It is the purpose of a filter to separate these currents at any desired point and to direct each of them into the conductor or circuit through which it is desired to have them flow.

12-2 Filter Action

A filter circuit consists of a combination of capacitors, inductors, and resistors connected so that it will separate alternating currents from direct currents or alternating currents within a band of frequencies from those alternating currents outside of this band. Filter circuits may range from a very simple circuit to a very intricate circuit, depending upon their application. However, no matter how simple or involved a filter circuit may be, its action must depend upon the following principles of a-c circuits:

1. The opposition offered to the flow of alternating currents by a circuit containing only inductance will increase with frequency increase. Such a

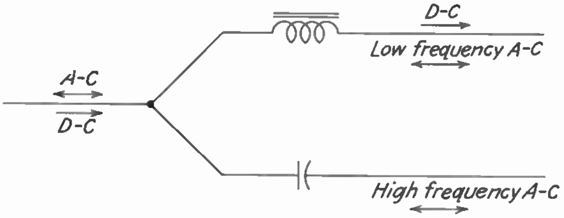


Fig. 12-2 A capacitor and an inductor used to separate alternating current from direct current and low-frequency currents from high-frequency currents.

circuit will offer comparatively little opposition to the flow of direct, pulsating, or low-frequency alternating currents. The opposition offered to r-f currents by such a circuit will be comparatively high (Figs. 12-2 and 12-3).

2. The opposition offered to the flow of alternating currents by a circuit containing only capacitance will decrease with frequency increase. Such a circuit will offer a comparatively high opposition to low-frequency currents, little opposition to r-f currents, and will block the flow of direct currents (Figs. 12-2 and 12-4).

3. A series resonant circuit has a low impedance at resonance and will offer little opposition to the flow of all currents whose frequencies lie within a narrow band above and below the resonant frequency. Such a circuit will offer a comparatively high opposition to the flow of currents of all other frequencies (Fig. 12-5*a*).

4. A parallel resonant circuit has a high impedance at resonance and will offer a comparatively high opposition to the flow of all currents whose frequencies lie within a narrow band above and below the resonant frequency. Such a circuit will offer little opposition to the flow of currents of all other frequencies (Fig. 12-5*b*).

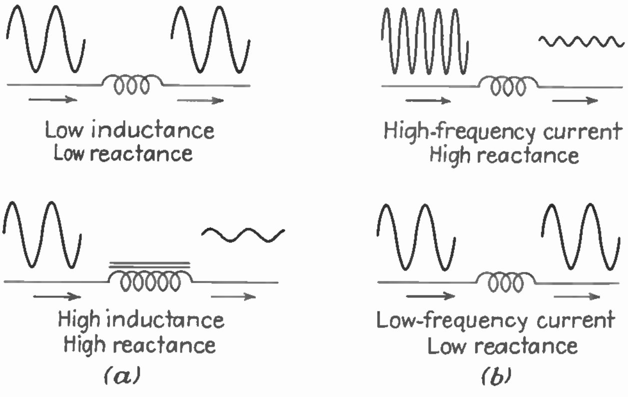


Fig. 12-3 Effects of inductance and frequency on current flow. (a) Variable inductance, constant frequency. (b) Variable frequency, constant inductance.

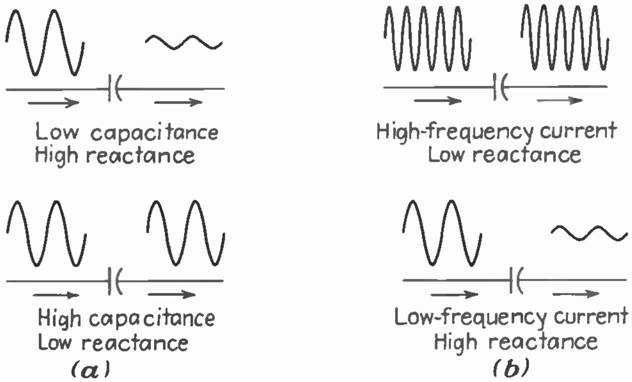


Fig. 12-4 Effects of capacitance and frequency on current flow. (a) Variable capacitance, constant frequency. (b) Variable frequency, constant capacitance.

5. Resistors do not provide any filtering action when used alone, as they oppose the flow of all currents regardless of their frequency. When connected in series with a capacitor or inductor or both, it increases the impedance of the circuit, thus decreasing the sharpness of the filter circuit. Increasing the resistance of the series resonant circuit decreases the current at the resonant frequency, thus decreasing the slope of the curve on either side of the resonant frequency (Fig. 11-4*b*).

EXAMPLE 12-6 A 10-henry filter choke has a resistance of 475 ohms. Determine its opposition (*a*) to direct current, (*b*) to 60-cycle alternating current.

GIVEN: $R = 475$ ohms $\quad L = 10$ henrys

FIND: (*a*) R $\quad$ (*b*) Z

SOLUTION:

(*a*) $R = 475$ ohms

(*b*) $X_L = 2\pi fL = 6.28 \times 60 \times 10 = 3{,}768$ ohms

$$Z = \sqrt{R^2 + X_L{}^2} = \sqrt{475^2 + 3{,}768^2} = 3{,}797 \text{ ohms}$$

Example 12-6 indicates that the opposition offered to the flow of 60-cycle alternating current is approximately eight times that offered to the d-c flow.

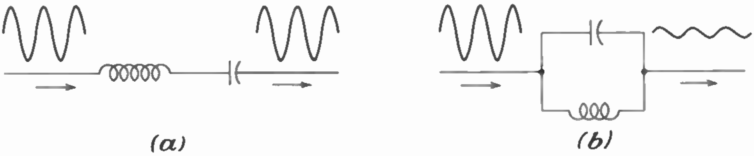


Fig. 12-5 Effect of resonant circuits on the amount of current flow at or near the resonant frequency. (a) Series resonant circuit. (b) Parallel resonant circuit.

This type of coil can therefore be used to pass direct current and block the flow of low-frequency alternating currents.

EXAMPLE 12-7 (*a*) To which type of current will a 0.2-μf capacitor offer the greater opposition, a 5,000-cycle audio signal or a 5,000-kc r-f signal? (*b*) How many times greater is the larger impedance than the smaller impedance?

GIVEN: $C = 0.2\ \mu f$ $f_{af} = 5$ kc $f_{rf} = 5{,}000$ kc

FIND: (*a*) $X_{C\cdot af}$ or $X_{C\cdot rf}$ (*b*) Ratio $X_{C\cdot af} - X_{C\cdot rf}$

SOLUTION:

(*a*) At 5,000 cycles

$$X_C = \frac{159{,}000}{f_{af}C} = \frac{159{,}000}{5 \times 10^3 \times 2 \times 10^{-1}} = 159 \text{ ohms}$$

At 5,000 kc

$$X_C = \frac{159{,}000}{f_{rf}C} = \frac{159{,}000}{5 \times 10^6 \times 2 \times 10^{-1}} = 0.159 \text{ ohm}$$

Greater opposition is offered to the a-f current.

(*b*) $$\text{Ratio of opposition} = \frac{X_{C\cdot af}}{X_{C\cdot rf}} = \frac{159}{0.159} = 1{,}000 \text{ to } 1$$

12-3 Types of Filter Circuits

Filter circuits are composed of inductors and capacitors having losses as low as commercially and economically practical. In the elementary consideration of filter circuits, it is assumed that the inductors and capacitors have no internal effective resistance. There are four general types of filter circuits, namely, low-pass filter, high-pass filter, bandpass filter, bandstop filter.

Low-pass Filter. A low-pass filter circuit allows all currents having a frequency below a certain value to pass into a desired circuit and diverts the flow of all currents having a frequency above this value (Fig. 12-6). An inductance coil inserted in the line (Fig. 12-7*a*) will offer little opposition to the flow of low-frequency currents and a large amount of opposition to

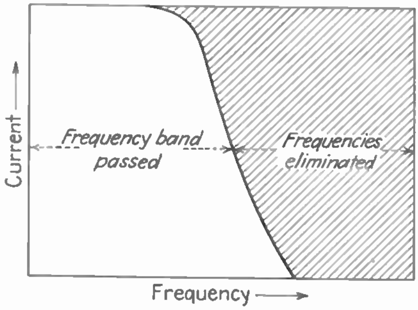


Fig. 12-6 Characteristic curve for a simple low-pass filter circuit.

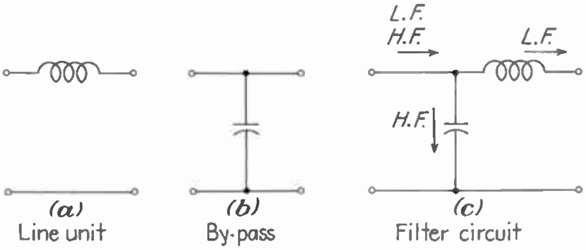


Fig. 12-7 Basic units of a low-pass filter circuit.

the flow of high-frequency currents. In order to divert the undesired high-frequency currents back to the source, a capacitor is used as a bypass path (Fig. 12-7*b*). The value of this bypass capacitor should be such that it will offer little opposition to the flow of current for all frequencies above a definite value and greatly oppose the flow of current for all frequencies below this value. When the coil and capacitor are connected as shown in Fig. 12-7*c*, the simplest type of low-pass filter circuit is obtained.

EXAMPLE 12-8 A 5-mh choke coil and a 0.001-μf capacitor are connected as shown in Fig. 12-7*c* to form a low-pass filter circuit. Determine the opposition offered to a 5-kc a-f signal (*a*) by the capacitor, (*b*) by the inductor. Determine the opposition offered to a 500-kc r-f signal (*c*) by the capacitor, (*d*) by the inductor.

GIVEN: $C = 0.001\ \mu f \qquad L = 5 \text{ mh} \qquad f_{af} = 5 \text{ kc} \qquad f_{rf} = 500 \text{ kc}$

FIND: (*a*) X_C at 5 kc (*b*) X_L at 5 kc (*c*) X_C at 500 kc (*d*) X_L at 500 kc

SOLUTION:

(*a*) $$X_C = \frac{159{,}000}{f_{af}C} = \frac{159{,}000}{5 \times 10^3 \times 1 \times 10^{-3}} = 31{,}800 \text{ ohms}$$

(*b*) $$X_L = 2\pi f_{af}L = 6.28 \times 5 \times 10^3 \times 5 \times 10^{-3} = 157 \text{ ohms}$$

(*c*) $$X_C = \frac{159{,}000}{f_{rf}C} = \frac{159{,}000}{500 \times 10^3 \times 1 \times 10^{-3}} = 318 \text{ ohms}$$

(*d*) $$X_L = 2\pi f_{rf}L = 6.28 \times 5 \times 10^5 \times 5 \times 10^{-3} = 15{,}700 \text{ ohms}$$

Analyzing the values obtained in this example, it can be seen that the r-f currents will find the path of least opposition through the capacitor and hence will take that path. The a-f currents will find the path of least opposition through the inductor and hence will take that path. A measure of the ability of a filter circuit to bypass undesired currents may be expressed in terms of the ratio of the impedances at the frequency limits.

High-pass Filter. A high-pass filter circuit allows all currents having a frequency above a certain value to pass into a desired circuit and diverts the flow of all currents having a frequency below this value (Fig. 12-8). A capacitor inserted in the line (Fig. 12-9*a*) will offer little opposition to the

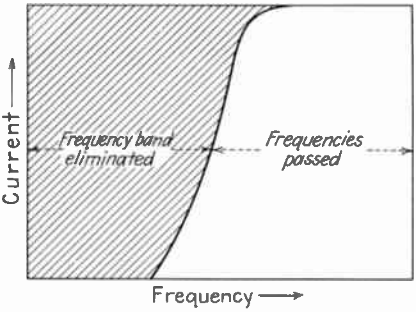


Fig. 12-8 Characteristic curve for a simple high-pass filter circuit.

flow of high-frequency currents, a large amount of opposition to the flow of low-frequency currents, and will block the flow of direct currents. The capacitor used should be of such a value that it will allow the passage of all currents whose frequencies are above a definite value and greatly oppose the flow of current for all frequencies below this value. In order to divert the undesired low-frequency currents back to the source, an inductance coil is used as a bypass path (Fig. 12-9*b*). The inductance of this coil should be of such a value that it will carry off the currents whose frequencies are below the cutoff point and reject the currents whose frequencies are above this value, thus forcing them to pass on through the circuit. When the coil and capacitor are connected as shown in Fig. 12-9*c*, the simplest type of high-pass filter circuit is obtained.

EXAMPLE 12-9 A 2-henry choke coil and a 0.5-μf capacitor are connected as shown in Fig. 12-9*c* to form a high-pass filter circuit. Determine the opposition offered to a 60-cycle power interference signal (*a*) by the capacitor, (*b*) by the inductor. Determine the opposition offered to a 1,200-cycle a-f signal (*c*) by the capacitor, (*d*) by the inductor.

GIVEN: $C = 0.5\ \mu f$ $L = 2$ henrys $f_{pf} = 60$ cycles $f_{af} = 1{,}200$ cycles

FIND: (*a*) X_C at 60 cycles (*b*) X_L at 60 cycles (*c*) X_C at 1,200 cycles
(*d*) X_L at 1,200 cycles

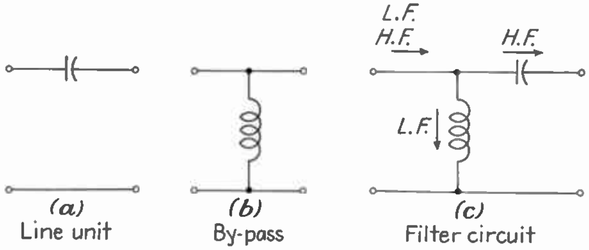


Fig. 12-9 Basic units of a high-pass filter circuit.

SOLUTION:

(*a*) $$X_C = \frac{159{,}000}{f_{pf}C} = \frac{159{,}000}{60 \times 0.5} = 5{,}300 \text{ ohms}$$

(*b*) $$X_L = 2\pi f_{pf}L = 6.28 \times 60 \times 2 = 753.6 \text{ ohms}$$

(*c*) $$X_C = \frac{159{,}000}{f_{af}C} = \frac{159{,}000}{1{,}200 \times 0.5} = 265 \text{ ohms}$$

(*d*) $$X_L = 2\pi f_{af}L = 6.28 \times 1{,}200 \times 2 = 15{,}072 \text{ ohms}$$

Analyzing the values obtained in this example, it can be seen that the high-frequency currents will take the path of the capacitor and the low-frequency currents will be bypassed through the inductor.

Bandpass Filter. A bandpass filter allows the current of a narrow band of frequencies to pass through a circuit and excludes all currents whose frequencies are either greater or less than the extreme limits of the band (Fig. 12-10).

Resonant circuits can serve as filters in a manner similar to the action of individual capacitors and inductors. The series resonant circuit (Fig. 12-11*a*) replacing the inductor of Fig. 12-7*a* would act as a bandpass filter, passing currents whose frequencies are at or near the resonant frequency and blocking the passage of all currents whose frequencies are outside this narrow band. The parallel resonant circuit (Fig. 12-11*b*) replacing the capacitor of Fig. 12-7*b*, if tuned to the same frequency as the series resonant circuit, will provide a path for all currents whose frequencies are outside the limits of the frequency band passed by the series resonant circuit. When the two resonant circuits are connected as shown in Fig. 12-11*c*, the simplest type of bandpass filter circuit is obtained.

Bandstop Filter. A bandstop filter opposes the flow of current for a narrow band of frequencies while allowing the current to flow for all frequencies above or below this band (Fig. 12-12). Bandstop filters are also known as *band-suppression* and *band-elimination filters.* Their purpose is

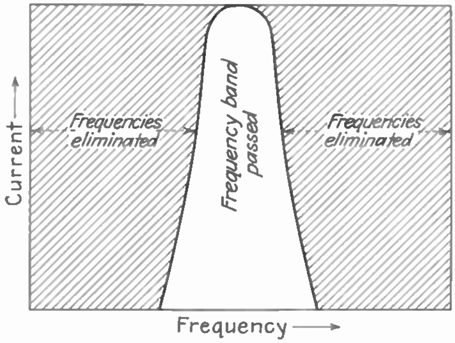
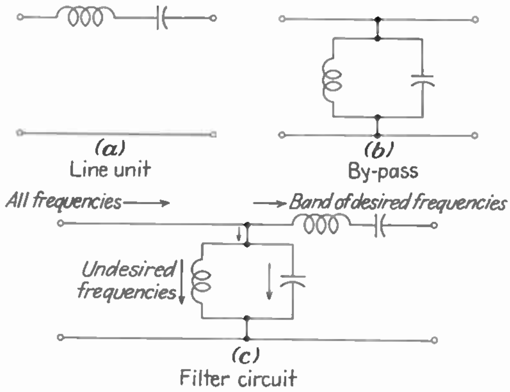


Fig. 12-10 Characteristic curve for a simple bandpass filter circuit.



(a) Line unit

(b) By-pass

(c) Filter circuit

Fig. 12-11 Basic units of a bandpass filter circuit.

opposite to that of a bandpass filter, and the relative position of the resonant circuits in the filter circuit are interchanged.

The parallel resonant circuit of Fig. 12-13*a*, which replaces the capacitor of Fig. 12-9*a*, acts as a bandstop filter circuit and blocks the passage of all currents whose frequencies are at or near its resonant frequency, passing all currents whose frequencies are outside this band. The series resonant circuit of Fig. 12-13*b*, which replaces the inductor of Fig. 12-9*b*, when tuned to the same resonant frequency as the parallel resonant circuit pro-

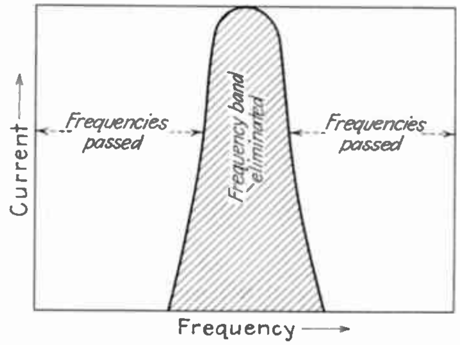
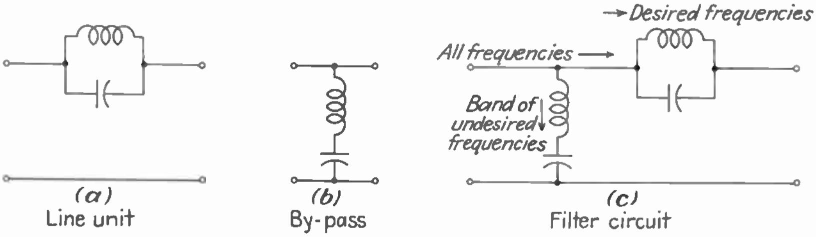


Fig. 12-12 Characteristic curve for a simple bandstop filter circuit.



(a) Line unit

(b) By-pass

(c) Filter circuit

Fig. 12-13 Basic units of a bandstop filter circuit.

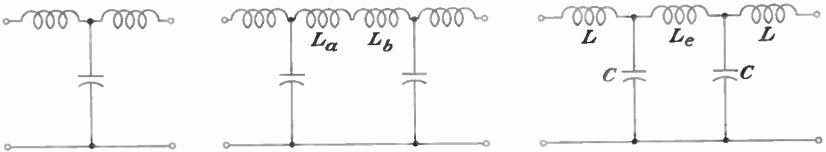
vides a bypass path for the undesired band of frequencies. When the two resonant circuits are connected as shown in Fig. 12-13*c*, the simplest type of bandstop filter circuit is obtained.

12-4 Multisection Filter Circuits

Need for Multisection Filter Circuits. All the filter circuits explained in the previous article have only one section. None of these single-section filter circuits provide a sharp reduction of current at the cutoff frequency. Adding a capacitor, inductor, or resonant circuit in series or parallel (depending on the circuit) with the filter circuit will improve its filtering action, thus sharpening the reduction of current at the desired frequency. When an additional unit is added to a single-section filter circuit, the form of the resulting circuit will resemble the letter T or the symbol π. They are therefore called T- or π-type filters, depending on which symbol they resemble. Two or more sections of either the T- or π-type filters may be joined to produce a sharper cutoff.

T-type Filter Circuits. When an inductor is connected in series with the simple low-pass filter of Fig. 12-7*c*, a T-type low-pass filter circuit is formed (Fig. 12-14*a*). When two of these filter circuits are connected as shown in Fig. 12-14*b*, the inductors L_a and L_b can be replaced by a single inductor L_e (Fig. 12-14*c*) whose value is equal to $L_a + L_b$. As all the inductors originally are of the same size, the center inductor L_e should have a value of twice the inductance of either of the end inductors.

When a capacitor is connected in series with the simple high-pass filter of Fig. 12-9*c*, a T-type high-pass filter circuit is formed (Fig. 12-15*a*). When two of these filter circuits are connected as shown in Fig. 12-15*b*,

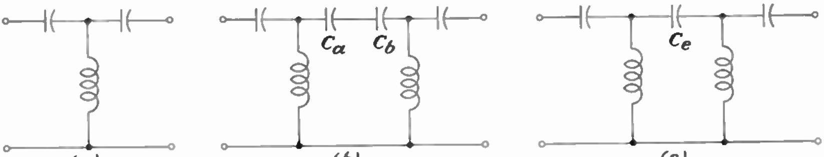


(a) Single-section T-type low-pass filter circuit

(b) A two-section T-type low-pass filter circuit

(c) Practical two-section low-pass filter circuit

Fig. 12-14 T-type low-pass filter circuits.



(a) Single-section T-type high-pass filter circuit

(b) A two section T-type high-pass filter circuit

(c) Practical two-section high-pass filter circuit

Fig. 12-15 T-type high-pass filter circuits.

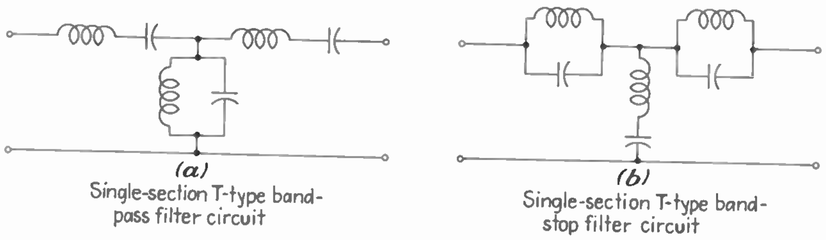
the capacitors C_a and C_b can be replaced by a single capacitor C_e (Fig. 12-15*c*). As the capacitors C_a and C_b are of equal value and connected in series, the value of C_e should be one-half the value of either C_a or C_b.

When a series resonant circuit is connected in series with the simple bandpass filter circuit of Fig. 12-11*c*, a T-type bandpass filter is formed (Fig. 12-16*a*). When a parallel resonant circuit is connected in series with the simple bandstop filter circuit of Fig. 12-13*c*, a T-type bandstop filter is formed (Fig. 12-16*b*).

Pi-type Filter Circuits. When a capacitor is connected to the simple low-pass filter of Fig. 12-7*c*, so that the line is shunted at both ends of the inductor by a capacitor, a π-type low-pass filter is formed (Fig. 12-17*a*). When two of these filter circuits are connected as shown in Fig. 12-17*b*, the capacitors C_a and C_b can be replaced by a single capacitor C_e (Fig. 12-17*c*) whose value is equal to $C_a + C_b$. As all the capacitors originally are of the same size, the center capacitor should have a value of twice the capacitance of either of the end capacitors.

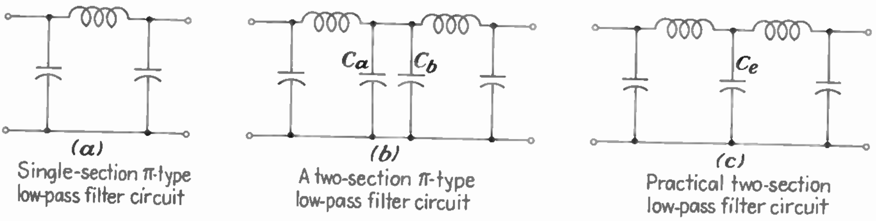
When an inductor is connected to the simple high-pass filter of Fig. 12-9*c*, so that the line is shunted at both ends of the capacitor by an inductor, a π-type high-pass filter is formed (Fig. 12-18*a*). When two of these filter circuits are connected as shown in Fig. 12-18*b*, the inductors L_a and L_b can be replaced by a single inductor L_e (Fig. 12-18*c*). As the inductors L_a and L_b are of equal value and connected in parallel, the value of L_e should be one-half the value of either L_a or L_b.

When a parallel resonant circuit is connected to the simple bandpass filter circuit of Fig. 12-11*c*, so that the line is shunted at both ends of the


(a) Single-section T-type bandpass filter circuit

(b) Single-section T-type bandstop filter circuit

Fig. 12-16 T-type bandpass and bandstop filter circuits.

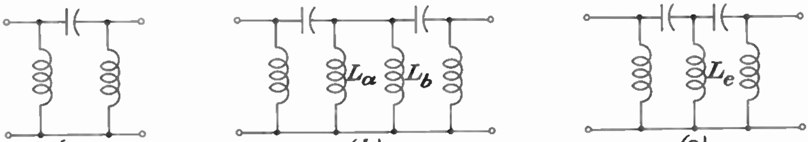


(a) Single-section π-type low-pass filter circuit

(b) A two-section π-type low-pass filter circuit

(c) Practical two-section low-pass filter circuit

Fig. 12-17 π-type low-pass filter circuits.

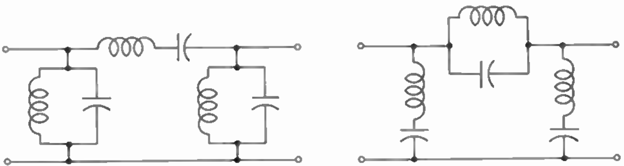


(a) Single-section π-type high-pass filter circuit

(b) A two-section π-type high-pass filter circuit

(c) Practical two-section high-pass filter circuit

Fig. 12-18 π-type high-pass filter circuits.



(a) Single-section π-type band-pass filter circuit

(b) Single-section π-type band-stop filter circuit

Fig. 12-19 π-type bandpass and bandstop filter circuits.

series resonant circuit by a parallel resonant circuit, a π-type bandpass filter circuit is formed (Fig. 12-19*a*). When a series resonant circuit is connected to the simple bandstop filter circuit of Fig. 12-13*c*, so that the line is shunted at both ends of the parallel resonant circuit by a series resonant circuit, a π-type bandstop filter circuit is formed (Fig. 12-19*b*).

12-5 Filter Circuits as a Whole

The design of filter circuits is a specialized field of electronics, and the calculations for the component parts is left to specialized texts. The choice of the type of circuit to be used is a matter for the designing engineer to decide. However, the following terms, generally used in connection with filter circuits, should be understood.

Source Impedance. The impedance of the circuit leading into the filter circuit is called the *source impedance*. This may be the plate circuit of a tube, a high resistance, etc.

Load Impedance. The impedance of the circuit into which the filter circuit feeds is called the *load impedance*. This may be a voltage divider, the plate resistor of a resistance coupled amplifier unit, etc.

Image Impedances. These are the impedances at each end of the filter. In order that there will be no reflection loss, the image impedance at the load end should equal the load impedance and the image impedance at the source end should equal the source impedance.

Characteristic Impedance. This is sometimes called the *iterative impedance* and is equal to the impedance that the filter circuit offers the source. Filter circuits are generally designed so that the load impedance equals the input impedance. The image impedances for this condition are equal and also equal to the characteristic impedance. The image impedance and characteristic impedance generally vary with frequency; therefore the two measurements must be taken at the same frequency.

Constant-k Filter. The filter circuits described up to now are of the *constant-k* type. This means that the product of the impedance of the series arm or arms and the impedance of the shunt arm or arms is constant for all frequencies and is equal to k^2. This value k is also equal to the characteristic impedance of the filter circuit over the greater portion of the passband, and

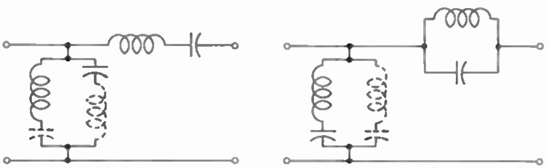
$$k = \sqrt{Z_{se}Z_{sh}} = Z_{ch} \tag{12-3}$$

where Z_{ch} = characteristic impedance, ohms
Z_{se} = impedance of series arm, ohms
Z_{sh} = impedance of shunt arm, ohms

The ideal properly terminated constant-k filter circuit acts as a resistance load throughout the passband. At the cutoff frequency the load becomes either zero or infinite, and thereafter it is imaginary. For example, in the attenuation band the ideal filter acts as a reactive load, does not take any energy from the source, and does not transmit any energy to its terminating impedance. In the ideal filter, (1) the frequencies within the passband would have zero attenuation, (2) the frequencies outside the passband would have infinite attenuation, and (3) the frequency band between passing and attenuation would be very narrow, thereby producing very sharp cutoff. Because of the resistance present in the circuit components, and because the impedances do not remain constant over the frequency range of the filter, the characteristics of the constant-k type of filter may vary considerably from the ideal filter (see results of Example 10-43).

(a) Low-pass

(b) High-pass



(c) Band-pass

(d) Band-stop

Fig. 12-20 Series-derived m-type filter circuits. The impedances shown in broken line are those added to the shunt unit to form the m-derived filter circuit.

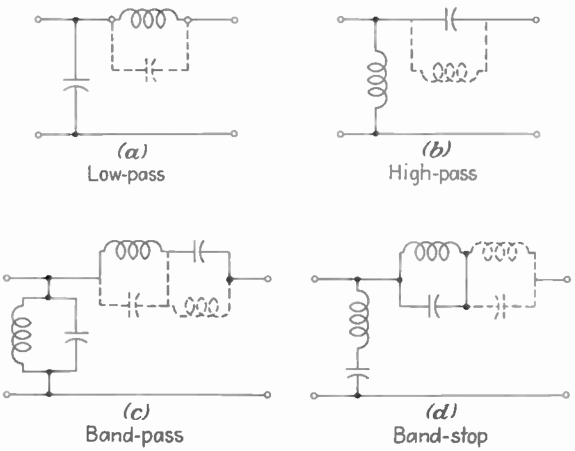


Fig. 12-21 Shunt-derived m-type filter circuits. The impedances shown in broken line are those added to the series unit to form the m-derived filter circuit.

When the performance of the constant-*k* type of filter does not fulfill the requirements of a particular application, the constant-*k* filter circuit may be modified by the use of additional inductors and/or capacitors in order to produce the desired operating characteristics. If the arrangement of the circuit components is varied, an infinite number of possible types of filters may be obtained; the *m*-derived filter is a commonly used example. Because the constant-*k* type of filter is the basic or elementary filter circuit, it is often referred to as being the *prototype filter.*

12-6 Other Filter Circuits

***m*-Derived Filter.** The *m*-derived filter circuits are variations of the basic prototype filters, and their behavior depends on a factor that is a function of a constant, *m*. Additional impedances are inserted into the basic circuit to form either a shunt-derived or a series-derived type of filter. If the additional impedances are added to the *shunt arm* of the section, the filter circuit is *series-derived* (Fig. 12-20). If the additional impedances are added to the *series arm* of the section, the filter circuit is *shunt-derived* (Fig. 12-21).

The addition of an inductor or a capacitor in the shunt arm of the *m*-derived filter (Fig. 12-20*a* and *b*) introduces a series resonant circuit in the bypass path which results in higher attenuation at the resonant frequency of the shunt arm and produces a sharper cutoff characteristic for the filter. The extent of the change in the characteristics of the filter circuit depends on the value of *m*, which may range between zero and unity; a commonly used value is in the order of 0.6. The value of *m* is determined by the components of the filter circuit and is expressed by the following equations:

For low-pass filters:

$$m = \sqrt{1 - \left(\frac{f_0}{f_{rs}}\right)^2} \tag{12-4}$$

For high-pass filters:

$$m = \sqrt{1 - \left(\frac{f_{rs}}{f_0}\right)^2} \tag{12-5}$$

where f_0 = cutoff frequency of the prototype filter
f_{rs} = resonant frequency of the shunt arm

From these equations, it can be seen that the lower values of m can be obtained by making the values of f_0 and f_{rs} closer to each other.

The addition of an inductor or a capacitor in the series arm of the m-derived filter (Fig. 12-21*a* and *b*) introduces a parallel resonant circuit in the line path which results in higher attenuation at the resonant frequency of the series arm and produces a sharper cutoff characteristic for the filter. The value of m may be determined by using the equations for obtaining m for series-derived filter circuits [Eqs. (12-4) and (12-5)].

Resistor-Capacitor Circuits. Many electronic circuits carry both alternating and direct current. A vacuum-tube circuit may carry direct current for the plate supply and an a-c signal at the same time. It is often necessary to separate the direct current and alternating signal current and to provide a path for the signal currents so that the voltage produced may be applied only to certain portions of the circuit.

One method of accomplishing this separation is to use a capacitor to provide a path for the alternating current and a resistor to provide a path for the direct current (Fig. 12-22). The circuit of Fig. 12-22*a* uses a capacitor to allow the passage of the alternating signal current from the screen-grid circuit of a tube to ground. The resistor keeps the alternating signal current from getting into the B supply, where it might cause trouble.

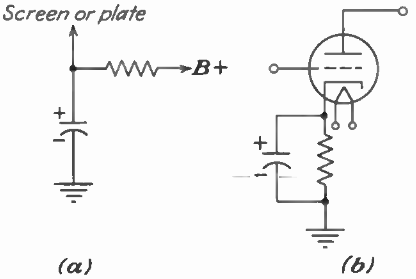


Fig. 12-22 Resistor-capacitor filter circuits. (a) Filter action in the plate or screen-grid circuit. (b) Cathode grid-bias circuit.

The resistor is also used to provide the correct voltage for the screen grid by acting as a dropping resistor.

In Fig. 12-22*b*, the resistor connected between cathode and ground is used to make the cathode positive with respect to ground, which in turn makes the grid negative with respect to the cathode. This resistor offers an impedance to the signal current and may reduce it to a critical value. When this reduction in signal current is large, it may introduce degeneration, an action that should be avoided. If a capacitor is connected across the resistor as shown in Fig. 12-22*b*, it will provide a path for the alternating signal current. The diversion of the signal current from the resistor will aid in fulfilling the purpose of the resistor, namely, to provide a fixed amount of bias for the grid of the tube.

12-7 Attenuators

Types of Attenuators. A resistance network that is used to reduce voltage, current, or power in controllable and known amounts that are independent of frequency is called an *attenuator.* The resistors in an attenuator circuit may be connected in various ways: (1) so that the input and output impedances are equal, (2) so that the input and output impedances are unequal, and (3) to provide different amounts of attenuation. Some of the ways in which resistors are connected to form an attenuator are the L, T, H, π, O, ladder, and bridged-T types of circuits (Fig. 12-23). When only fixed resistors are used in the network, the attenuation is a fixed amount that is determined by the type of network and the values of the resistors used. This type of network is sometimes called a *fixed attenuator pad.* When some or all of the resistors used in a network are variable, the amount of attenuation can be controlled by varying one or more of the variable resistors. This type of network is sometimes called a *variable attenuator pad.*

The amount of attenuation produced by a network may be expressed by

$$\alpha = \frac{I_L}{I_{L\cdot a}} \qquad (12\text{-}6)$$

where α = image transfer constant
I_L = load current without attenuator
$I_{L\cdot a}$ = load current with attenuator

L Attenuator. The L attenuator is an unbalanced network that consists of two resistors, one placed in either of the series arms and the other in the shunt arm (Fig. 12-23*a* and *b*). These resistors have such values that one of the image impedances of the attenuator remains constant regardless of the amount of attenuation. The L attenuator will therefore maintain an impedance that is independent of attenuation at either the input or output terminals. Because of this characteristic, the L attenuator is generally used when several different loads are connected to a common power source and the amount of power delivered to each load must be varied without affecting the impedance offered to the signal source.

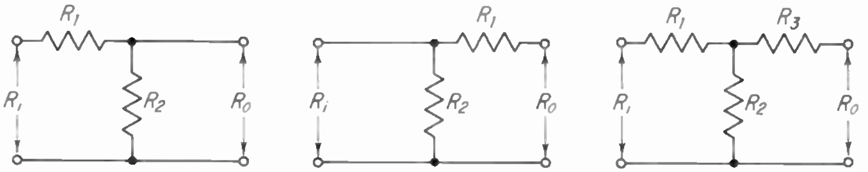
T Attenuator. The T attenuator is an unbalanced network in which three resistors are arranged to form the letter T (Fig. 12-23*c*). In this type of attenuator, resistors in the series arms, R_1 and R_3, are generally identical in value in order to make the input image impedance R_i equal to the output image impedance R_o. The T attenuator is generally used where the amount of attenuation or the presence of the network has no effect upon the impedance relations of the circuit. This is accomplished by making the image impedance of the attenuator equal to R_i or R_o. This type of attenuator will therefore maintain a constant impedance at both the input and output terminals irrespective of the amount of attenuation.

H Attenuator. The H attenuator is a balanced T network in which five resistors are arranged to form the letter H when viewed sidewise (Fig. 12-23*d*). The balanced H network can be made to produce the same operating characteristics as the unbalanced T network. This is accomplished by making the resistance of all the resistors in the series arms equal to each other and also equal to one-half the value required for a resistor to be used in the series arm of a T attenuator. In the circuit shown in Fig. 12-23*d*, $R_1 = R_3 = R_4 = R_5 =$ one-half the resistance of either R_1 or R_3 of Fig. 12-23*c*.

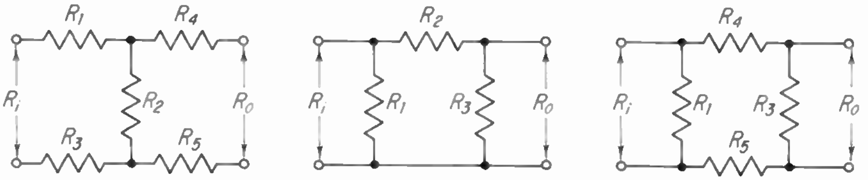
π Attenuator. The π attenuator is an unbalanced network containing three resistors connected to form the Greek letter π (Fig. 12-23*e*). In this type of attenuator, the resistors in the shunt arms, R_1 and R_3, are usually identical in value in order to make the input image impedance equal to the output image impedance. The π attenuator can be designed to produce the same operating characteristics as the T attenuator, namely, that the impedance relations of the circuit are not affected by any changes in attenuation or the presence of the network.

O Attenuator. The O attenuator is a balanced π network containing four resistors connected to form the letter O (Fig. 12-23*f*). The balanced O network can be made to produce the same operating characteristics as the unbalanced π network. This is accomplished by making the two resistors in the series arms identical in value and equal to one-half the value required of a resistor to be used in the series arm of a π attenuator. In the circuit shown in Fig. 12-23*f*, $R_4 = R_5 =$ one-half the resistance of R_2 of Fig. 12-23*e*.

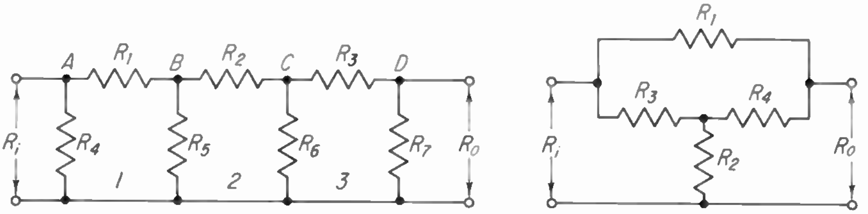
Ladder Attenuator. A ladder attenuator consists of a series of two or more symmetrical π-section networks. The arrangement shown in Fig. 12-23g consists of three cascaded π sections. The values of resistance used for the shunt and series arms of each section are such that they will produce image impedances at each end that are equal to the input and output image impedances. It should be noted that R_1, R_2, and R_3 represent the series arms of π sections 1, 2, and 3, respectively; R_4 and R_7 the shunt



(a) L type (b) L type (c) T type



(d) H type (e) π type (f) O type



(g) Ladder type (h) Bridged-T type

Fig. 12-23 Attenuator networks.

arms of sections 1 and 3, respectively; R_5 and R_6 the equivalent resistance of the two adjacent shunt arms for sections 2 and 3, respectively. The impedance between any junction point and the common side of a ladder attenuator is equal to one-half the image impedance.

The total attenuation of a ladder network is equal to the product of the attenuation of each section. Thus, if the image transfer constant of each section is 10, then each section reduces the load current by $\frac{1}{10}$ and the total attenuation of the three-section attenuator would be $\frac{1}{10} \times \frac{1}{10} \times \frac{1}{10}$, or $\frac{1}{1{,}000}$. Ladder networks are generally used in electronic equipment, such as signal generators, where it is required that the voltage and current be reduced in known ratios. When the various junction terminals of the ladder attenuator of Fig. 12-23*g* are connected to the terminals of a rotary deck switch and values are used to produce an α of 10, attenuations of 0, $\frac{1}{10}$, $\frac{1}{100}$, and $\frac{1}{1{,}000}$ can be obtained.

Bridged-T Attenuator. A bridged-T attenuator consists of four resistors connected as shown in Fig. 12-23*h*. To obtain a constant value of image impedance that is independent of attenuation, the four resistors should have values of $R_3 = R_4 = R_i$ and $R_1R_2 = R_i^2$. This type of attenuator is equivalent to the simple T attenuator. However, to obtain variable attenuation with constant values of image impedance the bridged-T network requires the variation of only two resistors R_1 and R_2 while the simple T network requires that all three resistors be varied.

Attenuator Circuit Calculations. The design of attenuators is a specialized area in electronics. The following equations are for only the L-, T-, and π-type attenuators; data for the other types of attenuators may be obtained from specialized reference books featuring attenuators.

For the simple resistor-type L attenuator circuit of Fig. 12-23*a*, supplying power to a load of R_L,

$$R_1 = \frac{R_L(K-1)}{K} \tag{12-7}$$

$$R_2 = \frac{R_L}{K-1} \tag{12-8}$$

$$K = \frac{e_i}{e_o} \tag{12-9}$$

EXAMPLE 12-10 A certain 1,000-ohm load is supplied by a 100-volt d-c power source. It is desired to reduce the load voltage to 10 volts by means of an L-type attenuator similar to Fig. 12-23*a* and still retain a 1,000-ohm load condition at the power source. (*a*) What are the values of K, R_1, and R_2? Using the values obtained in part (*a*), (*b*) calculate the resistance at the power source terminals, (*c*) check the output voltage and the value of K.

GIVEN: $R_L = 1{,}000$ ohms $\quad e_i = 100$ volts $\quad e_o = 10$ volts

FIND: (*a*) K, R_1, R_2 $\quad$ (*b*) R_i $\quad$ (*c*) e_o, K

SOLUTION:

(*a*)

$$K = \frac{e_i}{e_o} = \frac{100}{10} = 10$$

$$R_1 = \frac{R_L(K-1)}{K} = \frac{1{,}000\,(10-1)}{10} = 900 \text{ ohms}$$

$$R_2 = \frac{R_L}{K-1} = \frac{1{,}000}{10-1} = 111.1 \text{ ohms}$$

(*b*)

$$R_i = R_1 + R = 900 + 100 = 1{,}000 \text{ ohms}$$

where

$$R = \frac{R_2 R_L}{R_2 + R_L} = \frac{111.1 \times 1{,}000}{111.1 + 1{,}000} = 100 \text{ ohms}$$

(*c*)

$$e_o = \frac{e_i R}{R_1 + R} = \frac{100 \times 100}{900 + 100} = 10 \text{ volts}$$

$$K = \frac{e_i}{e_o} = \frac{100}{10} = 10$$

For the simple resistor-type T attenuator circuit of Fig. 12-23*c*, supplying power to a load of R_L,

$$R_1 = R_3 = R_L \frac{K-1}{K+1} \tag{12-10}$$

$$R_2 = \frac{2R_L K}{K^2 - 1} \tag{12-11}$$

$$K = \frac{e_i}{e_o} \tag{12-9}$$

EXAMPLE 12-11 A certain 2,000-ohm load is supplied by a 50-volt d-c power source. It is desired to reduce the load voltage to 25 volts by means of a T-type attenuator similar to Fig. 12-23*c* and still retain a 2,000-ohm load condition at the power source. (*a*) What are the values of K, R_1, R_2, and R_3? (*b*) By using the values obtained in part (*a*) calculate the resistance at the power source terminals.

GIVEN: $R_L = 2{,}000$ ohms $e_i = 50$ volts $e_o = 25$ volts

FIND: (*a*) K, R_1, R_2, R_3 (*b*) R_i

SOLUTION:

(*a*)
$$K = \frac{e_i}{e_o} = \frac{50}{25} = 2$$

$$R_1 = R_3 = R_L \frac{K-1}{K+1} = 2{,}000 \times \frac{2-1}{2+1} = 666.6 \text{ ohms}$$

$$R_2 = \frac{2R_L K}{K^2 - 1} = \frac{2 \times 2{,}000 \times 2}{(2 \times 2) - 1} = 2{,}666.6 \text{ ohms}$$

(*b*)
$$R_i = R_1 + \frac{R_2(R_3 + R_L)}{R_2 + (R_3 + R_L)} = 666.6 + \frac{2{,}666.6\,(666.6 + 2{,}000)}{2{,}666.6 + 666.6 + 2{,}000}$$

$$= 666.6 + 1333.3 \cong 2{,}000 \text{ ohms}$$

For the simple resistor-type π attenuator circuit of Fig. 12-23*e*, supplying power to a load of R_L,

$$R_1 = R_3 = R_L \frac{K+1}{K-1} \tag{12-12}$$

$$R_2 = R_L \frac{K^2 - 1}{2K} \tag{12-13}$$

$$K = \frac{e_i}{e_o} \tag{12-9}$$

EXAMPLE 12-12 A certain 12,000-ohm load is supplied by a 250-volt d-c power source. It is desired to reduce the load voltage to 50 volts by means of a π-type attenuator similar to Fig. 12-23*e* and still retain a 12,000-ohm load condition at the power source. (*a*) What are the values of K, R_1, R_2, and R_3? (*b*) By using the values obtained in part (*a*) calculate the resistance at the power source terminals.

GIVEN: $R_L = 12{,}000$ ohms $\quad e_i = 250$ volts $\quad e_o = 50$ volts

FIND: (*a*) K, R_1, R_2, R_3 $\quad$ (*b*) R_i

SOLUTION:

$$(a)\quad K = \frac{e_i}{e_o} = \frac{250}{50} = 5$$

$$R_1 = R_3 = R_L\frac{K+1}{K-1} = 12{,}000 \times \frac{5+1}{5-1} = 18{,}000 \text{ ohms}$$

$$R_2 = R_L\frac{K^2-1}{2K} = 12{,}000 \times \frac{(5\times 5)-1}{2\times 5} = 12{,}000 \times 2.4 = 28{,}800 \text{ ohms}$$

$$(b)\quad R_i = \frac{R_1(R_2+R)}{R_1+(R_2+R)} = \frac{18{,}000\,(28{,}800+7{,}200)}{18{,}000+28{,}800+7{,}200} = 12{,}000 \text{ ohms}$$

where $$R = \frac{R_LR_3}{R_L+R_3} = \frac{12{,}000\times 18{,}000}{12{,}000+18{,}000} = 7{,}200 \text{ ohms}$$

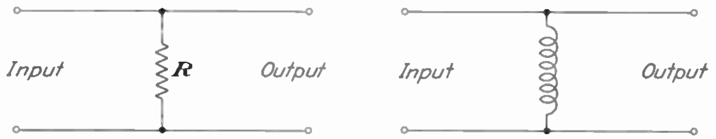
12-8 Coupling of Circuits

Principles of Coupling. Two circuits are *coupled* when they have a common impedance that permits the transfer of electrical energy from one circuit to another. This common impedance, called the *coupling element*, may be a resistor, an inductor, a capacitor, a transformer, or a combination of two or more of these elements.

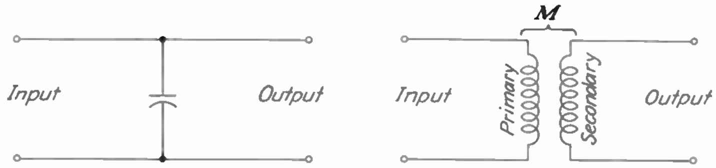
Coupling elements are usually required to perform some filter action in addition to transferring energy from one circuit to another. Conversely, every filter circuit contains a section that acts as a coupling device. Coupling circuits and filter circuits are so much alike that it is sometimes difficult to state whether they should be called filters or coupling units. The choice of name may be governed by that function which is considered of major importance. The type of impedance used is determined by the kinds of currents flowing in the input circuit and the kind desired in the output circuit. The characteristics of each type of impedance have already been presented in the study of filters.

Simple-coupled Circuits. A simple-coupled circuit is one in which the common impedance consists of a single element. A group of simple-coupled circuits is shown in Fig. 12-24.

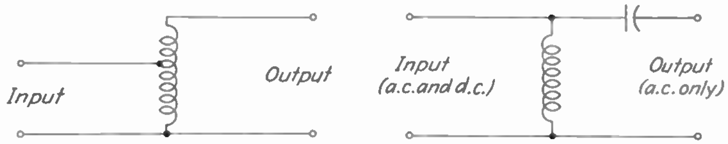
The resistance-, inductive-, and capacitive-coupled circuits are also called *direct-coupled* circuits. In these circuits the coupling is accomplished by having the current of the input circuit flow through the common impedance where it produces a voltage drop. This voltage is applied to the output circuit, thus resulting in a transfer of electrical energy from the input to the output circuit. The output voltage is equal to the product of the current in the coupling element and its impedance.



(a)-Resistance coupling (b)-Inductive coupling



(c)-Capacitive coupling (d)-Transformer coupling



(e)-Auto-transformer coupling; combined self and mutual inductance (f)-Inductive coupling. Capacitor added to block the direct current

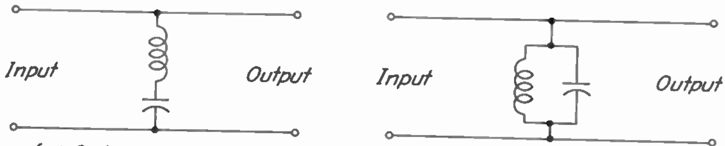
Fig. 12-24 Types of simple-coupled circuits.

The transformer-coupled circuit shown in Fig. 12-24*d* is also referred to as *indirect* coupling, *magnetic* coupling, or *mutual-inductive* coupling. In this type of coupling, the transfer of energy is accomplished by having the alternating current of the input circuit flow through the primary winding and setting up an alternating magnetic field. The magnetic lines of this field link the turns of the secondary winding and induce the voltage that supplies the energy for the output circuit.

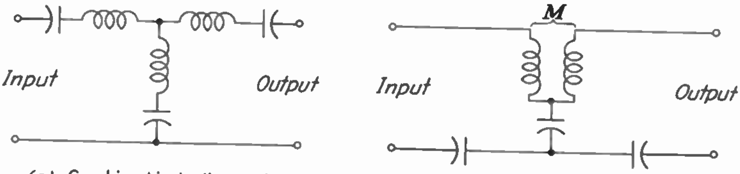
In some applications of coupling devices the input circuit may have both alternating and direct current flowing, and it is desired that the coupling unit transfer only the alternating current to the output circuit. The transformer-coupled unit will serve this purpose satisfactorily, as it will pass only the alternating current. The other simple-coupled circuits (Fig. 12-24*a*, *b*, and *c*) can be modified so that no direct current can reach the load. This is accomplished by placing a capacitor in the output side of the coupling element as shown in Fig. 12-24*f*.

Complex-coupled Circuits. A complex-coupled circuit is one in which the common impedance consists of two or more circuit elements. A few of the numerous types of complex coupling are shown in Fig. 12-25.

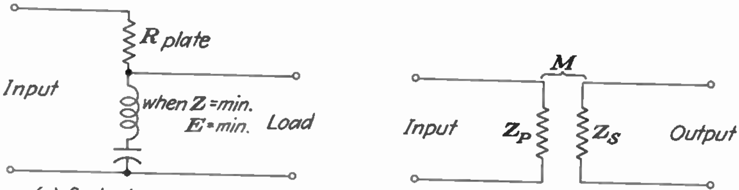
The proportion of energy transferred in a simple inductive-coupled circuit increases as the frequency increases, while with simple capacitive coupling, the proportion of energy transferred decreases as the frequency increases. Using combinations of two or more elements in the coupling unit makes it possible to obtain various proportions of energy transfer for inputs



(a)-Series tuned coupling (b)-Parallel tuned coupling



(c)-Combined inductive and capacitive coupling (d)-Combined mutual inductance and capacitive coupling



(e)-Series tuned coupling (f)-Equivalent circuit of any complex coupled circuit

Fig. 12-25 Types of complex-coupled circuits.

of varying frequency. For example, the coupling element of Fig. 12-25*a* is a series tuned circuit and hence will have a minimum impedance at its resonant frequency. The proportion of energy transferred will be lowest when the frequency of the input circuit is equal to the resonant frequency of the coupling unit. At frequencies above resonance the proportion of energy transfer will increase and will be inductive. At frequencies below resonance the proportion of energy transfer will also increase but will be capacitive. The fact that the energy transfer is minimum at the resonant frequency may be more clearly understood when the input side of the filter is shown to be a part of a series circuit, for example, the plate circuit of a tube (Fig. 12-25*e*). At resonance, when the impedance of the coupling unit is minimum, its voltage drop will be at its minimum and the proportion of energy transfer must also be at its minimum.

In general, the amount of energy transferred will be proportional to the current flowing through the coupling unit and to the impedance of the unit. For purpose of analysis, complex-coupled circuits may be represented by a simple equivalent circuit, as shown in Fig. 12-25*f*.

Coefficient of Coupling. The ratio of the energy of the output circuit to the energy of the input circuit is called the *coefficient of coupling. Critical, tight,* and *loose coupling* are terms used to express the relative value of the coefficient of coupling for mutual-inductive-coupled circuits.

The response curves for tight, critical, and loose coupling are shown in Fig. 12-26. When the maximum amount of energy is transferred from one circuit to another, the circuits possess *critical coupling*, also referred to as *optimum coupling*. If the coefficient of coupling is higher than that required to produce critical coupling, the coils are *tightly coupled;* if it is less than that required for critical coupling, the coils are *loosely coupled.*

The effect of varying the coupling between two circuits may be seen from the response curves of Figs. 12-26 and 12-31. When two circuits are very tightly coupled, resonance will be obtained at two new frequencies, one below and the other above the normal frequency of resonance for the capacitor and inductor used. As the coupling is decreased, the two peaks come closer together until critical coupling is reached and a single peak of maximum height is obtained. If the coupling is decreased below the critical value, a single peak of reduced height is obtained.

Air-core transformers illustrate the importance of the amount of coupling between the primary and the secondary windings. As it is difficult to design an air-core transformer in which a large portion of the magnetic lines set up by the primary winding will link the turns of the secondary winding, the coefficient of coupling is generally low. A low value for the coefficient of coupling is not objectionable in some circuits, as it provides certain desirable characteristics which are presented in the following article.

12-9 Characteristics of Mutual-inductive-coupled Circuits

Mutual-inductive coupling, as provided by a transformer, is a means commonly used to transfer energy from one circuit to another. The characteristics of these circuits depend upon (1) the type of circuit, that is, whether a capacitor is connected to the primary, to the secondary, or to both, and (2) the amount of coupling between the two circuits.

Coupled Impedance. The primary and the secondary circuits of a transformer are separate electrical circuits that are magnetically coupled. Each circuit has an impedance of its own generally designated as Z_p and Z_s. The

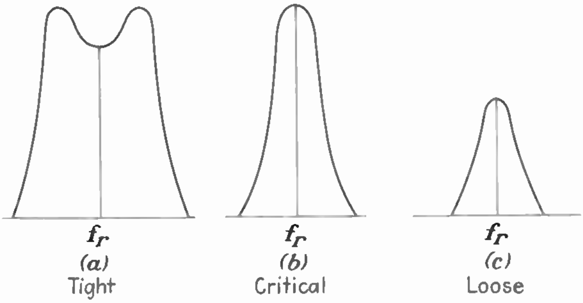


Fig. 12-26 Response curves showing the effect of various amounts of coupling.

impedance of the primary winding, when no load is applied to the secondary, consists of the resistance and inductance of the primary winding. The impedance of the secondary circuit consists of the resistance and inductance of the secondary winding plus the impedance of any load connected to the circuit. When the secondary circuit is left open, that is, when no load is applied to its terminals, the impedance of the secondary will be infinity or so large that it is immeasurable. Under this condition the presence of the secondary will have no effect upon the primary circuit.

When a load is applied to the secondary, the impedance will have a significant value and a current will flow in the secondary circuit. The amount of energy in this circuit will depend upon the secondary voltage and impedance. The secondary voltage, however, is dependent upon the number of magnetic lines linking the two circuits. The number of linkages is proportional to the coefficient of coupling; therefore the amount of energy transferred is also dependent upon the coefficient of coupling. As the energy in the secondary circuit must come from the primary, it is evident that the primary impedance will be affected by the impedance of the secondary circuit. The effect of the secondary circuit upon the primary is equivalent to adding an impedance in series with the primary. This added impedance is generally referred to as the *coupled impedance.*

The numerical value of the coupled impedance of a mutual-inductive-coupled circuit may be found by the equation

$$Z_{p-s} = \frac{(2\pi f M)^2}{Z_s} \tag{12-14}$$

where Z_{p-s} = impedance coupled into the primary by the secondary, ohms
f = frequency of the power source, cps
M = mutual inductance, henrys
Z_s = secondary impedance, ohms

The derivation of this equation is explained in the following steps:

1. From the definition, two circuits have a mutual inductance of 1 henry when a current in one circuit, changing at the rate of 1 amp per sec, induces an average emf of 1 volt in the second circuit; the induced voltage in the second circuit may be expressed as

$$e_s = M\frac{I_{p2} - I_{p1}}{t_2 - t_1} \tag{12-15}$$

This equation indicates that, when the mutual inductance M is 1 henry and the rate of current change in the primary $\frac{I_{p2} - I_{p1}}{t_2 - t_1}$ is 1 amp per sec, the average value of the voltage e_s induced in the secondary will be 1 volt. Thus, this equation is derived from the definition of the unit of mutual inductance.

2. When an alternating current I_p is flowing, the current is continually changing from a maximum value to zero in a positive and negative direction and at a rate proportional to the frequency. As the alternating current I_p is an effective value, the maximum current will be I_p divided by 0.707. Also, a change in current from the maximum value to zero occurs in a period of time corresponding to one-quarter of a cycle. Therefore

$$\frac{I_{p2} - I_{p1}}{t_2 - t_1} = \frac{I_{\max} - I_o}{\dfrac{1}{4f}} = \frac{I_{\max}}{\dfrac{1}{4f}} = \frac{\dfrac{I_p}{0.707}}{\dfrac{1}{4f}} = \frac{4fI_p}{0.707} \tag{12-16}$$

Substituting Eq. (12-16) in Eq. (12-15),

$$e_s = M\frac{4fI_p}{0.707} \tag{12-17}$$

3. The induced secondary voltage e_s is expressed as an average value, and in practical work it is desired to have it expressed as the effective value E. As the average value is equal to $2/\pi$ (or 0.637) times the maximum value and the effective value is equal to 0.707 times the maximum value, then the effective value may be expressed as

$$E_s = \frac{e_s}{\dfrac{2}{\pi}} \times 0.707 = \frac{0.707\pi e_s}{2} \tag{12-18}$$

or

$$e_s = \frac{2E_s}{0.707\pi} \tag{12-19}$$

Substituting Eq. (12-19) in Eq. (12-17),

$$\frac{2E_s}{0.707\pi} = M\frac{4fI_p}{0.707} \tag{12-20}$$

or

$$E_s = 2\pi fMI_p \tag{12-21}$$

4. The secondary current I_s will, therefore, be

$$I_s = \frac{E_s}{Z_s} = \frac{2\pi fMI_p}{Z_s} \tag{12-22}$$

5. This secondary current upon flowing through the secondary winding sets up a magnetic field of its own that induces a voltage in the primary. This induced voltage will be 180° out of phase with the primary impressed voltage and is referred to as a counter, or back, voltage. By the same reasoning as was used to derive the secondary induced voltage, it may be shown that this counter voltage induced in the primary will be

$$E_{\text{counter}} = 2\pi fMI_s \tag{12-23}$$

Substituting Eq. (12-22) for I_s in Eq. (12-23),

$$E_{\text{counter}} = (2\pi fM)\frac{(2\pi fM)I_p}{Z_s} \tag{12-24}$$

$$E_{\text{counter}} = \frac{(2\pi fM)^2}{Z_s}I_p \tag{12-25}$$

6. As this voltage represents the effect that the secondary has upon the primary and as an alternating voltage is equal to the product of impedance and current, it may be stated [from Eq. (12-25)] that the effect of the secondary impedance upon the primary is

$$Z_{p-s} = \frac{(2\pi fM)^2}{Z_s} \tag{12-14}$$

The coupled impedance expressed by Eq. (12-14) may be represented by an equivalent resistance and an equivalent reactance connected in series with the primary circuit. The numerical values of the equivalent resistance and equivalent reactance are expressed by the following equations:

$$R_{p-s} = \frac{(2\pi fM)^2R_s}{Z_s^2} \tag{12-26}$$

$$X_{p-s} = -\frac{(2\pi fM)^2X_s}{Z_s^2} \tag{12-27}$$

where R_{p-s} = resistance coupled into the primary by the secondary, ohms
X_{p-s} = reactance coupled into the primary by the secondary, ohms
f = frequency of the power source, cps
M = mutual inductance, henrys
R_s = resistance of the secondary circuit, ohms
Z_s = impedance of the secondary circuit, ohms
X_s = reactance of the secondary circuit, ohms

Note: When X_s is inductive, then X_{p-s} has a negative sign, and when X_s is capacitive, X_{p-s} has a positive sign.

EXAMPLE 12-13 A mutual-inductance-coupled circuit is shown in Fig. 12-27 together with the circuit values. Find (*a*) the inductive reactance of the primary, (*b*) the inductive reactance of the secondary, (*c*) the impedance coupled into the primary by the secondary, (*d*) the equivalent resistance component of the coupled impedance, (*e*) the equivalent reactance component of the coupled impedance, (*f*) the equivalent circuit diagram, (*g*) the effective impedance of the primary, (*h*) the primary current, (*i*) the secondary voltage, (*j*) the secondary current.

GIVEN: Fig. 12-27

FIND: (*a*) X_{LP} (*b*) X_{LS} (*c*) Z_{p-s} (*d*) R_{p-s} (*e*) X_{p-s} (*f*) diagram
(*g*) Z_{pT} (*h*) I_p (*i*) E_s (*j*) I_s

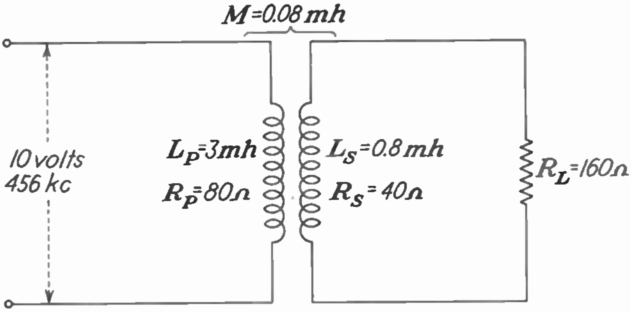


Fig. 12-27

SOLUTION:

(a) $X_{LP} = 2\pi fL_P = 6.28 \times 456 \times 10^3 \times 3 \times 10^{-3} = 8{,}590$ ohms

(b) $X_{LS} = 2\pi fL_S = 6.28 \times 456 \times 10^3 \times 0.8 \times 10^{-3} = 2{,}290$ ohms

(c) $$Z_{p-s} = \frac{(2\pi fM)^2}{Z_s} = \frac{(2\pi fM)^2}{\sqrt{(R_s + R_L)^2 + X_{LS}^2}}$$

$$= \frac{(6.28 \times 456 \times 10^3 \times 0.08 \times 10^{-3})^2}{\sqrt{(40 + 160)^2 + (2{,}290)^2}} = \frac{52{,}441}{2{,}298} = 22.8 \text{ ohms}$$

(d) $$R_{p-s} = \frac{(2\pi fM)^2(R_s + R_L)}{Z_s^2} = \frac{52{,}441 \times 200}{(2{,}298)^2} = 1.98 \text{ ohms}$$

(e) $$X_{p-s} = \frac{(2\pi fM)^2 X_s}{Z_s^2} = \frac{52{,}441 \times 2{,}290}{(2{,}298)^2} = 22.7 \text{ ohms}$$

(f)

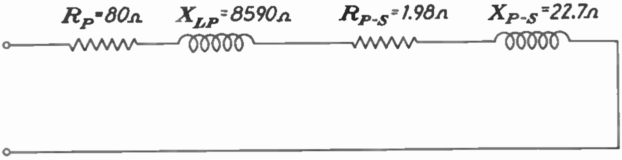


Fig. 12-28

(g) $$Z_{pT} = \sqrt{(R_p + R_{p-s})^2 + (X_{LP} - X_{p-s})^2}$$

$$= \sqrt{(80 + 1.98)^2 + (8{,}590 - 22.7)^2} = 8{,}567 \text{ ohms}$$

(h) $$I_p = \frac{E}{Z_{pT}} = \frac{10}{8{,}567} = 0.00116 \text{ amp} = 1.16 \text{ ma}$$

(i) $$E_s = 2\pi fMI_p = 6.28 \times 456 \times 10^3 \times 0.08 \times 10^{-3} \times 1.16 \times 10^{-3} = 0.266 \text{ volt}$$

(j) $$I_s = \frac{E_s}{Z_s} = \frac{0.266}{2{,}298} = 0.000115 \text{ amp} = 115\ \mu\text{a}$$

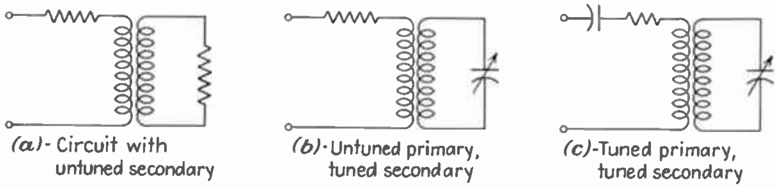
The results of this example indicate that the effect of the coupled equivalent resistance is to increase the effective resistance of the primary circuit. The equivalent reactance that is coupled into the primary by a secondary whose reactance is inductive is opposite in phase to the primary reactance and hence reduces the effective reactance of the primary circuit. The net result is a reduction in the effective primary impedance, more current thereby being allowed to flow in the primary circuit, thus making possible the transfer of more energy to the secondary circuit.

Many of the important characteristics of coupled circuits are explained by the effects of coupled impedance. Examination of Eq. (12-14) indicates that the coupled impedance will be low when the coefficient of coupling is low because the value of M decreases when the coefficient of coupling is decreased. Also, the coupled impedance will be low when the secondary impedance is high. Thus, when the coefficient of coupling is low or when very little load is applied to the secondary (high secondary impedance), the coupled impedance will be low and the effect of the secondary upon the primary will be negligible. However, when the coefficient of coupling is high or when the secondary carries considerable amounts of load (low secondary impedance), the coupled impedance will be high and the secondary will produce considerable effect upon the primary circuit.

Circuit with Untuned Primary and Untuned Secondary. The simplest type of transformer coupling is a circuit having an untuned primary and an untuned secondary with a resistance or inductance load. Such a circuit is shown in Fig. 12-29*a*. This circuit is often used as an equivalent circuit to represent the effects produced by a shield, metal panel, or other metal object located near a coil. The effect of the shield or panel upon the coil would be the same as that of a secondary winding consisting of inductance and resistance in series. The coupled impedance of such a circuit will increase the effective resistance of the primary and reduce its effective reactance. It also indicates that losses of the coil circuit are increased by an amount proportional to the resistance coupled into the primary by the secondary, which is actually the shield or nearby metal panel.

Circuit with Untuned Primary and Tuned Secondary. The circuit shown in Fig. 12-29*b* differs from Fig. 12-29*a* in that a capacitor is used in place of the resistor in the secondary circuit. The tuned r-f amplifier circuit is similar to this fundamental circuit.

The secondary circuit is similar to the series tuned circuit, and its characteristics will be the same as those of the series tuned circuit. At the



(a)- Circuit with untuned secondary

(b)- Untuned primary, tuned secondary

(c)-Tuned primary, tuned secondary

Fig. 12-29 Fundamental transformer-coupled circuits.

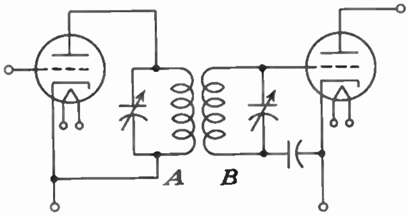


Fig. 12-30 Bandpass amplifier circuit.

resonant frequency the impedance will be at its minimum and the current at its maximum. The impedance coupled into the primary will be high and will have a critical effect upon the primary current. At frequencies above or below resonance the secondary impedance increases and its current decreases. The impedance coupled into the primary decreases, and its effect on the primary circuit is decreased.

Circuit with Tuned Primary and Tuned Secondary. This type of circuit (Fig. 12-29*c*) is used extensively in radio and television receivers. A common application of this circuit is the i-f amplifier of the superheterodyne receiver. This type of coupling is very useful for amplifiers because it can be designed to provide an approximately uniform secondary current response over the band of frequencies that are normally applied to the primary circuit.

12-10 Bandpass Amplifier Circuits

Ideal Response Curve. In order to obtain high fidelity and high selectivity, the ideal response curve should have a flat top and straight sides. This ideal can be closely approximated by using two resonant circuits tuned to the same frequency and coupled to each other (Fig. 12-30). This circuit is called a *bandpass amplifier, bandpass filter,* or *bandpass circuit.*

The width of the frequency band that will be passed depends upon the circuit application. In a-m radio receivers this band is usually 10 kc wide and represents a 5-kc side band above and below the resonant frequency. These side bands are part of an a-m wave and represent the amount that the frequency varies from the resonant frequency. The frequency deviation will depend upon the frequency of the audio signal that is being received. Although the frequency of an audio signal may vary from 20 to 20,000 cycles, most commercial a-m receivers reproduce sounds only up to 5,000 cycles. Amplifiers are designed to provide specific amounts of bandwidth depending upon the application. For example, (1) a 200-kc bandwidth is used in the i-f amplifier of an f-m radio receiver, (2) a 50-kc bandwidth is used in the sound section of a television receiver, and (3) a 4.5-mc bandwidth is used in the picture section of a television receiver.

The important characteristic of the bandpass amplifier circuit is the manner in which the current in the secondary circuit varies with the frequency when a constant voltage is applied to the primary circuit. The amount of current in the secondary is directly proportional to the amount of coupling between the two windings. The shape of the resonance curve will, therefore, be dependent on the coefficient of coupling and may be a narrow peaked curve, a flat-top curve, or a curve having two separate peaks with a valley between them.

Figure 12-31 illustrates the manner in which the current in the secondary of such a circuit varies with the amount of coupling for frequencies above and below its resonant frequency. When the coefficient of coupling is low, $K = 0.01$ for resonance at 500 kc, the secondary current will be low and the curve will be peaked. As the amount of coupling between the two circuits is increased, the amount of secondary current will increase and there will be a reduction in the sharpness of the peak, $K = 0.015$. With critical coupling $K = 0.02$, the maximum amount of secondary current is obtained and the resonance curve will be comparatively flat at the top and will have steep sides. With tight coupling, the coupled impedance at resonance is high, thus reducing the primary current, which in turn reduces the amount of voltage induced into the secondary, thus reducing the secondary current. This accounts for the decrease in secondary currents at resonance for coefficients of coupling greater than the critical value as indicated by the curves in Fig. 12-31. The reactance coupled into the primary is inductive for frequencies below resonance and capacitive for frequencies above resonance. This reactance is opposite to that of the primary circuit and will therefore reduce the equivalent impedance offered to the applied voltage. The primary current, and therefore the voltage induced into the secondary, will increase for frequencies off resonance. When the coupling is tight, this action will introduce new resonant frequencies above and below resonance corresponding to the amount of secondary current. This

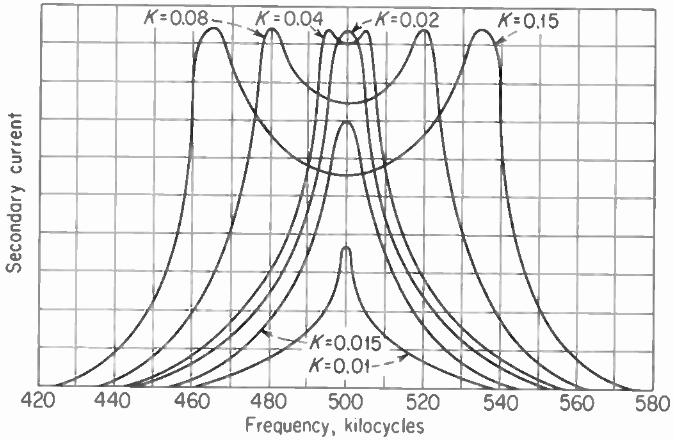


Fig. 12-31 Curves for a bandpass amplifier circuit showing the variation in secondary current with frequency.

accounts for the peaks in the resonance curves when the coupling is greater than the critical value. The current at these peaks is practically the same as the peak current with critical coupling. The spacing between these peaks is directly proportional to the coefficient of coupling.

Width of Bandpass. The maximum possible transfer of energy to the secondary at the resonant frequency is obtained with critical coupling. With tight coupling, a fairly constant secondary current and voltage can be obtained for a narrow band of frequencies. The width of this band, measured at 0.707 of the maximum response, is directly proportional to the coefficient of coupling and the resonant frequency of the tuned circuits. An approximate value of the bandwidth can be obtained by

$$\text{Width of bandpass} = K \times f_r \qquad (12\text{-}28)$$

where K = coefficient of coupling
f_r = resonant frequency of the tuned circuits

This equation indicates that the larger the value of K, the wider the bandpass will be. The coupled impedance will also increase with an increase in the amount of coupling, thus causing a decrease in the output current (Fig. 12-31).

EXAMPLE 12-14 What is the approximate bandwidth of a bandpass filter circuit having a resonant frequency of 456 kc and a coefficient of coupling of 0.02?

GIVEN: $K = 0.02$ $f_r = 456$ kc

FIND: Width of bandpass

SOLUTION:

$$\text{Width of bandpass} = K \times f_r = 0.02 \times 456 = 9.12 \text{ kc} = 9{,}120 \text{ cycles}$$

The most important properties of a bandpass circuit are the width of the band of frequencies it allows to pass and the uniformity of response within this band. Referring to Fig. 12-31 it can be seen that, when $K = 0.02$, the secondary current is fairly constant for a band of frequencies between 495 and 505 kc. The response will, therefore, be uniform for this band of frequencies with a coefficient of coupling of 0.02 at 500 kc. As the coefficient of coupling is increased ($K = 0.04$; $K = 0.08$; $K = 0.15$), the band becomes wider and less uniform.

The coefficient of coupling of bandpass circuits is usually of such a value that uniform response is obtained for the band of frequencies to be passed. The response should decrease rapidly for frequencies beyond these limits (Fig. 12-31).

The uniformity of response within the band of frequencies to be passed is dependent on the circuit Q and the value of the coefficient of coupling. The equation for finding the amount of coupling required to produce maximum transfer of energy is usually expressed in terms of the primary and secondary Q as follows:

$$K_c = \frac{1}{\sqrt{Q_pQ_s}} \tag{12-29}$$

where K_c = critical coupling
Q_p = Q of the primary circuit
Q_s = Q of the secondary circuit
$\sqrt{Q_pQ_s}$ = Q of the complete circuit

Figure 12-32 shows that, if the circuit Q is too high, pronounced double peaks occur and, if too low, the response curve is round instead of flat. Experiments have shown that the best value of Q is approximately 50 per cent more than that required to produce critical coupling. Substituting this value in Eq. (12-29) and solving for the circuit Q,

$$\sqrt{Q_pQ_s} = \frac{1.5}{K_c} \tag{12-30}$$

or

$$Q_pQ_s = \frac{2.25}{K_c^2} \tag{12-31}$$

The coefficient of coupling required may be obtained by transposing Eq. (12-28), as

$$K = \frac{\text{width of bandpass}}{f_r} \tag{12-32}$$

The resonant frequency of the i-f bandpass amplifier is approximately 460 kc for a-m radio receivers, 10.7 mc for f-m radio receivers, and 44 mc for television receivers.

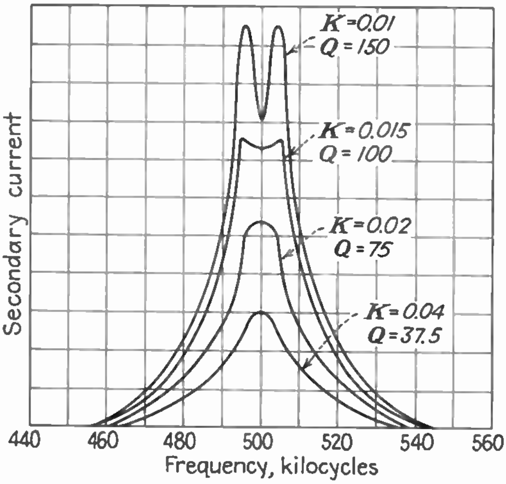


Fig. 12-32 Characteristics of a bandpass amplifier circuit showing the effect of circuit Q on uniformity of response within the band being passed.

EXAMPLE 12-15 A bandpass amplifier circuit used in an a-m radio receiver is tuned to a resonant frequency of 465 kc. What values of circuit Qs are necessary to produce uniform response for a 10-kc band?

GIVEN: Width of bandpass = 10 kc $f_r = 465$ kc

FIND: Q_p Q_s

SOLUTION:

$$K = \frac{\text{width of bandpass}}{f_r} = \frac{10}{465} = 0.0215$$

$$Q_pQ_s = \frac{2.25}{K_c^2} = \frac{2.25}{(0.0215)^2} = 4{,}870$$

If $Q_p = Q_s$, then

$$Q_p = \sqrt{4{,}870} \cong 70$$

Substituting values of 200 kc for the width of the bandpass and 10.7 mc for the frequency of resonance, it will be evident that the coupling required for the i-f bandpass amplifiers of f-m radio receivers is approximately 0.0187 and the Q of each coil is approximately 80.

12-11 Wide-bandpass Amplifier Circuits

Methods Used to Obtain a Wide Bandpass. The width of the band of frequencies to be passed in the r-f and i-f amplifier circuits of f-m radio receivers and television receivers is much greater than that which can normally be obtained by the use of a tuned circuit consisting only of a capacitor and an inductor. In order to obtain a wide bandpass, one or more of the following methods are generally used: (1) decreasing the value of the circuit Q, (2) increasing the value of the coefficient of coupling, (3) stagger tuning.

The value of circuit Q can be decreased either by adding resistance to the external circuit or by increasing the L/C ratio. In the first method a resistor is connected either in series or in parallel with the tuned circuit. As the addition of resistance to a tuned circuit will lower the value of circuit Q, the desired value can be obtained by adding the proper amount of resistance. The second method is based on the fact that the Q of a tuned circuit varies directly with the amount of capacitance in the circuit. Thus, by the use of a higher value of inductance and a lower value of capacitance, a lower value of Q is obtained. The effect of various values of Q on the width of bandpass was discussed in Art. 11-6 and is illustrated in Fig. 11-5.

Obtaining a wide bandpass by increasing the value of the coefficient of coupling has been discussed in the previous article. The relation between the coefficient of coupling and the width of the bandpass is illustrated in Fig. 12-31 and can be seen by a study of Eq. (12-28).

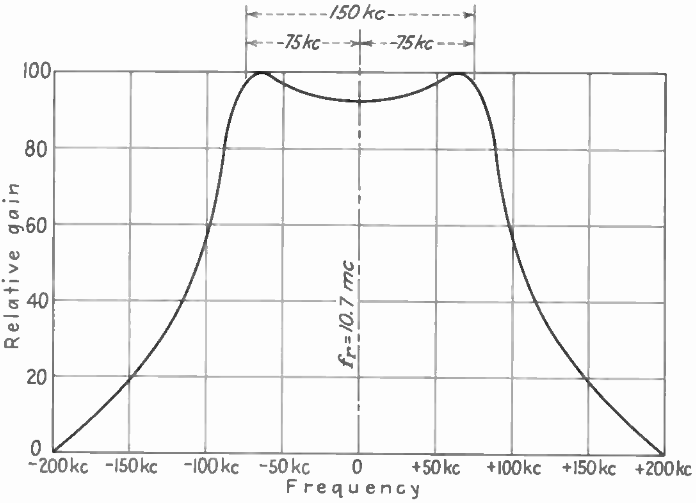


Fig. 12-33 A double peak i-f response curve for an f-m radio receiver produced by overcoupling.

A wide bandpass can also be obtained by stagger-tuning several stages of tuned amplifier circuits. In this method, each tuned circuit in the various stages of amplification involved is tuned to a different value of frequency, but all are within the desired range of the band to be passed. In order to obtain the overall characteristics desired, the Q of each circuit is adjusted to produce a certain amount of overlap.

Bandpass in F-M Receivers. A wide bandpass (Fig. 12-33) may be obtained by using either stagger tuning or overcoupled i-f transformers. When overcoupling is employed to obtain a wide bandpass, three i-f transformers tuned to the same frequency are generally used. The first and third transformers are single-peaked and operate just under critical coupling. The second transformer is overcoupled to produce a double-peaked response curve (Fig. 12-35).

Bandpass in Television Receivers. The overall bandpass of the i-f section of a television receiver is not made symmetrical because of the vestigial-sideband method of transmission. The ideal response curve for reception of this type of transmission is shown in Fig. 12-36. The operating frequency of i-f transformers is approximately 40 mc, and the width of its unsymmetrical bandpass approximates 4 mc. The ideal response curve is difficult to obtain; however, if various combinations of stagger tuning are used, a close approximation can be achieved. The combination used will vary with the designer. In general, three or more stages of tuned amplification are used. Each circuit may be tuned to a single frequency or may be overcoupled to produce peaks at two (or more) frequencies within the desired range of the bandpass. It is beyond the scope of this text to analyze each

Fig. 12-34 Variable-coupling i-f transformer used to adjust the width of bandpass. (*Hammarlund Manufacturing Company, Inc.*)

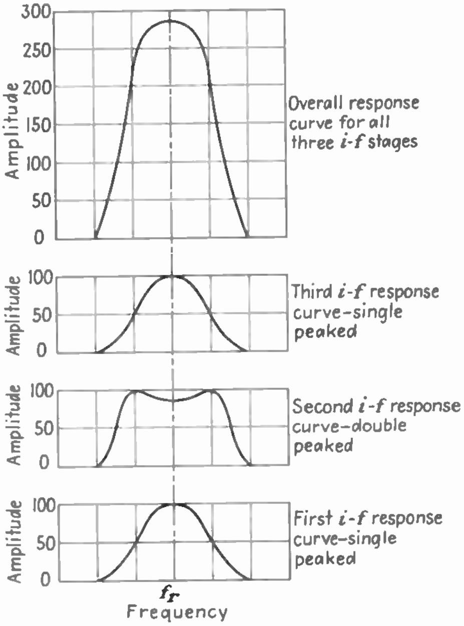


Fig. 12-35 Wide passband for the i-f response curve in an f-m radio receiver obtained by use of three stages with different values of coupling.

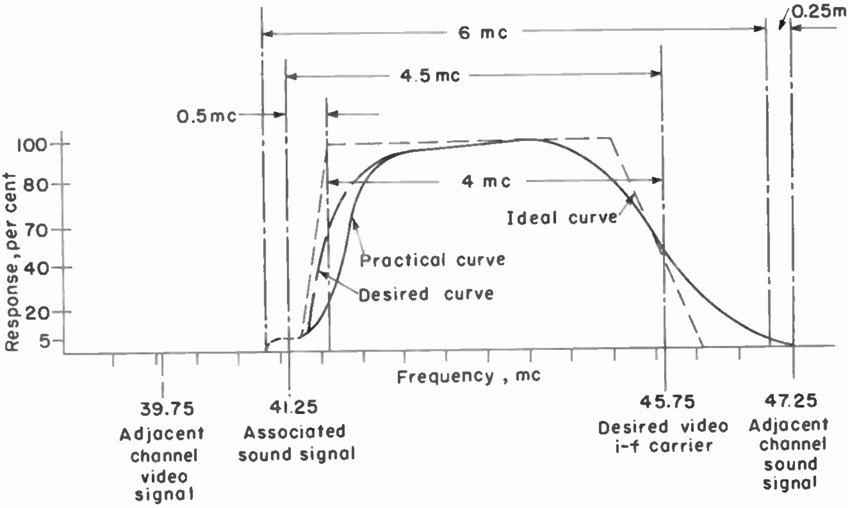


Fig. 12-36 The ideal, desired, and practical i-f response curves for the reception of vestigial-side-band transmission of television signals.

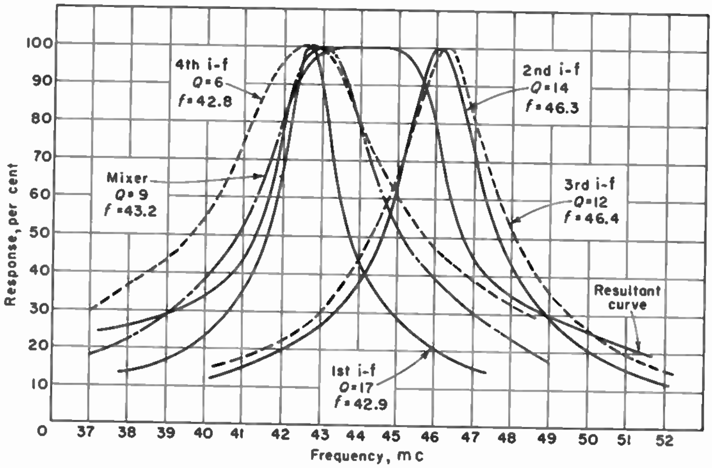


Fig. 12-37 Wide passband for the i-f response curve of a television receiver obtained by use of stagger tuning.

of the variations that may be used. An example of one of the combinations that may be used is illustrated by the response curves for a four-stage stagger-tuned i-f amplifier circuit shown in Fig. 12-37. The second and third i-f stages are tuned to the upper end of the desired passband. The first and fourth i-f stages and the mixer are tuned to the lower end of the desired passband. The resultant overall response curve approximates the desired curve of Fig. 12-36.

12-12 Delayed-action Circuits

Inductors or capacitors may be used in electric and electronic circuits to control the time required for the current or voltage to reach a pre-

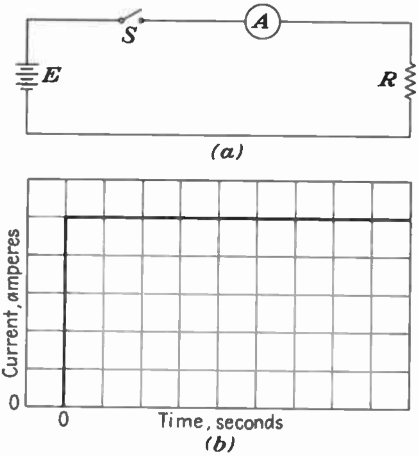


Fig. 12-38 Characteristics of current versus time for a circuit containing only resistance. (a) The circuit. (b) Current versus time characteristics.

determined value. The operation of these circuits is based on the time constant of the resistance-inductance or the resistance-capacitance combination. These circuits are generally referred to as *R-L* and *R-C* circuits.

Time Constant of Resistance-inductance Circuits. If an inductor, which may be considered as a resistance and inductance in series, is connected to a d-c power source, a current will flow in the circuit. The amount of current that will flow will be its Ohm's law value, namely, the voltage applied to the circuit divided by the resistance of the circuit. In a circuit having only resistance (Fig. 12-38*a*), the current will rise to its Ohm's law value practically instantaneously, as indicated in Fig. 12-38*b*. However, as the inductor has the effect of a resistance and inductance connected in series (Fig. 12-39*a*), the current will require an appreciable amount of time to reach its Ohm's law value, as is shown in Fig. 12-39*b*. This is explained by the fact that, for the current to reach its final value of 5 amp, it must progressively pass through its lesser values such as 1, 2, 3, and 4 amp. Under these conditions, the current is changing in amount, and the circuit

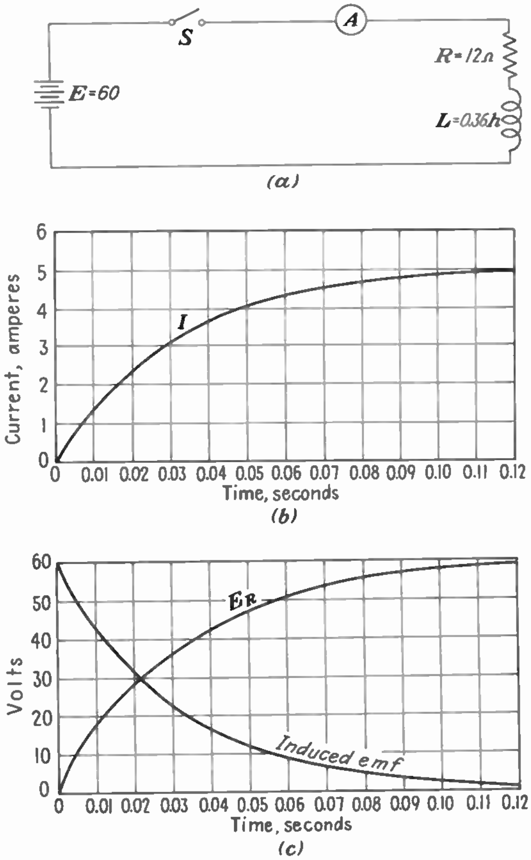


Fig. 12-39 Characteristics of current and voltage versus time for a circuit containing resistance and inductance. (a) The circuit. (b) Current versus time characteristics immediately after closing switch S. (c) Voltage versus time characteristics immediately after closing switch S.

will have an emf induced in it owing to the self-inductance of the circuit. This induced emf will oppose the impressed voltage and thus will prevent the current from reaching its Ohm's law value as long as the induced emf is present. The current will, however, eventually reach its Ohm's law value, the time required to accomplish this depending upon the relative values of the inductance and resistance. The current increases in a manner indicated by the graph shown in Fig. 12-39*b* and will rise to 63.2 per cent of its final value in a period of time, expressed in seconds, equal to the inductance of the circuit divided by the resistance of the circuit. This time interval is called the *time constant* of the circuit and is expressed mathematically as

$$t = \frac{L}{R} \tag{8-10}$$

EXAMPLE 12-16 An *R-L* circuit is used to control the time of closing a relay. The relay closes when the current reaches 63.2 per cent of its final value and the circuit resistance and inductance are 12 ohms and 2.4 henrys, respectively. What is the time interval between the closing of the line switch and the operation of the relay?

GIVEN: $R = 12$ ohms $L = 2.4$ henrys

FIND: t

SOLUTION:

$$t = \frac{L}{R} = \frac{2.4}{12} = 0.2 \text{ sec}$$

The time required for the current to reach values other than 63.2 per cent of the final value follows a curve known mathematically as an *exponential curve*. The universal time-constant curves of Fig. 12-43 provide a simple means of finding the current at any instant of time.

Further analysis of the *R-L* circuit will show that, when the current is increasing, the voltage drop across the resistance will increase at the same time rate as the current. This is so because the voltage drop across the resistance at any instant of time is equal to the product of the current and the resistance. Furthermore, as the sum of the voltages around the circuit must be equal to the applied voltage, the induced emf at any instant of time due to the inductance must be equal to the applied voltage less the *IR* drop. Figure 12-39*c* shows the voltage characteristics of the circuit when the current is building up.

The circuit shown in Fig. 12-40*a* is arranged so that the *R-L* circuit may either be connected to the d-c power source or be connected so that the inductance will be short-circuited through the resistance *R*. If the circuit is connected to the d-c power source, a current will flow in the circuit and will rise to its Ohm's law value according to the current-time curve previously described. When a current is flowing in the circuit, energy is transferred to the magnetic field. If the switch *S* is changed from position 1 to position 2, so that the inductance is disconnected from the power source and then instantaneously short-circuited across the resistance, the energy in the collapsing magnetic field will induce a voltage in the turns of the coil and cause a current to flow in the circuit. The current will decrease as the energy is dissipated in the resistance. The rate at which the current decreases will depend upon the relative values of the inductance and the resistance. The current-time changes will also follow an exponential curve but will be a descending curve. As the inductance is now actually in parallel with the resistance, the resistance voltage drop and the induced emf will be equal in value and will decrease according to an exponential curve. The current-time characteristics are shown in Fig. 12-40*b*, and the voltage-time characteristics are shown in Fig. 12-40*c*. The time in seconds as determined by dividing *L* by *R* represents the time in which the current (and voltage) decreases 63.2 per cent; hence the current and voltage will drop to 36.8 per cent of their maximum values in

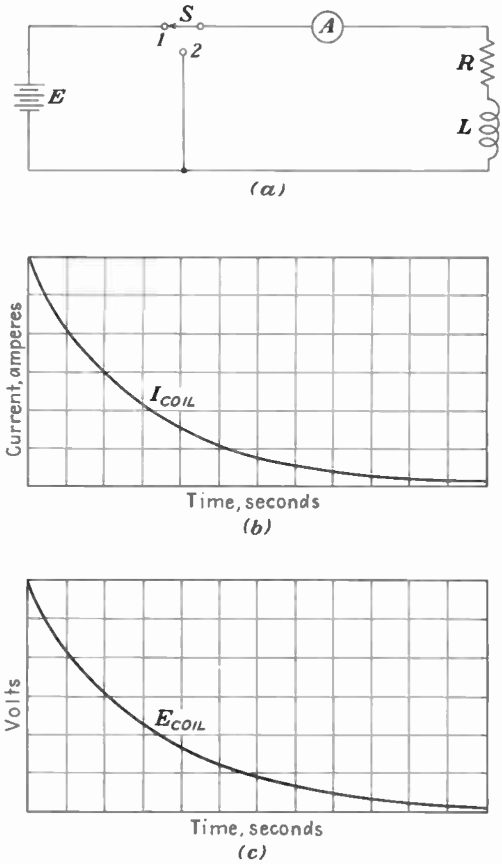


Fig. 12-40 Characteristics of current and voltage versus time. (a) The circuit. (b) Current versus time characteristics immediately after switch S is placed in position 2. (c) Voltage versus time characteristics immediately after switch S is placed in position 2.

L/R sec. The time required for the current and voltage to decrease to values other than 36.8 per cent of their maximum values can be found by use of the universal exponential curves presented at the end of this article.

Time Constant of Resistance-capacitance Circuits. If a perfect capacitor, that is, one having no resistance, is connected to a d-c power source, a high current surge will flow instantly and will charge the capacitor. As the capacitor becomes charged almost instantaneously, the amount of current flow will decrease rapidly. The capacitor will charge to the value of the impressed voltage, and the current flow will diminish to zero practically instantaneously.

If the circuit contains resistance in addition to the capacitance (Fig. 12-41*a*), the capacitor will become charged to the same value of voltage but will require a longer period of time to reach its final value. The voltage increases in a manner indicated by the graph shown in Fig. 12-41*b* and will rise to 63.2 per cent of its final value in a period of time, expressed

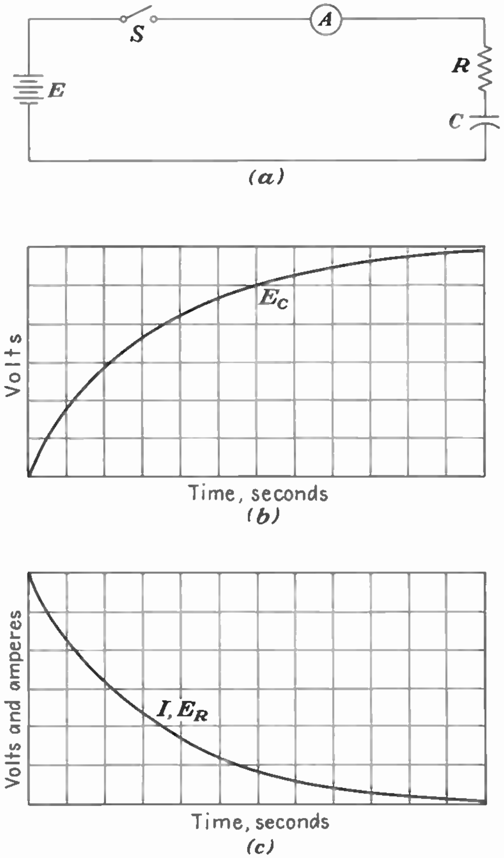


Fig. 12-41 Characteristics of current and voltage versus time for a circuit containing resistance and capacitance. (a) The circuit. (b) Capacitor volts versus time characteristics immediately after closing switch S. (c) Current and resistor volts versus time characteristics immediately after closing switch S.

in seconds, equal to the product of the capacitance and resistance of the circuit. This is called the *time constant* of the circuit and is expressed mathematically as

$$t = CR \tag{9-11}$$

EXAMPLE 12-17 What is the time constant of an automatic-volume-control filter circuit that uses a 1.25-megohm resistor and a 0.25-μf capacitor?

GIVEN: $R = 1.25$ megohms $\quad C = 0.25\ \mu\text{f}$

FIND: t

SOLUTION:

$$t = CR = 0.25 \times 10^{-6} \times 1.25 \times 10^{6} = 0.3125 \text{ sec}$$

The time required for the voltage to reach values other than 63.2 per cent of the final value follows an exponential curve. The universal time-constant curves of Fig. 12-43 provide a simple means of finding the voltage at any instant of time.

If the switch S of Fig. 12-42*a* is closed to position 1, the voltage and current characteristics of the circuit will conform to the voltage-time and current-time curves shown in Fig. 12-41. While a current is flowing in the circuit, energy is being stored in the capacitor. If the switch S is changed from position 1 to position 2, the energy stored in the capacitor will cause a current to flow and the capacitor will discharge through the resistor. At the instant of closing the switch the current will be at its highest value (Ohm's law value) and will decrease exponentially, as shown in Fig. 12-42*b*. The voltage across the capacitor and resistor will be equal in amount and will also decrease exponentially with time, as shown in Fig. 12-42*c*.

Universal Time-constant Curves. The time required for the current of an *R-L* circuit or the voltage across the capacitor of an *R-C* circuit to reach values other than 63.2 per cent of their final values may be determined mathematically by use of suitable equations. The mathematics involved is beyond the scope of this text. A shorter and more convenient method of determining the time required to atttain any percentage of the final value is by use of time-constant curves. As all the current-time and voltage-time relations vary exponentially, it is possible to represent these variations by the two general exponential curves shown in Fig. 12-43. These curves are plotted from values obtained mathematically and listed in Tables 12-1 and 12-2.

EXAMPLE 12-18 An *R-L* circuit used to control the action of a switch has a resistance of 12 ohms and an inductance of 0.5 henry and is connected to a 6-volt battery. (*a*) If the switch operates when the current attains 63.2 per cent of its final value, what time is required to operate the switch? (*b*) If the switch requires 400 ma to operate, what is the time between the start of current flow and the closing of the switch?

GIVEN: $R = 12$ ohms $\quad L = 0.5$ henry $\quad I = 400$ ma $\quad E = 6$ volts

FIND: (*a*) t $\quad$ (*b*) t

SOLUTION:

(*a*)

$$t = \frac{L}{R} = \frac{0.5}{12} = 0.0416 \text{ sec}$$

(*b*)

$$I_{\max} = \frac{E}{R} = \frac{6}{12} = 0.5 \text{ amp}$$

$$\% \text{ of } I_{\max} \text{ required to operate the switch} = \frac{I}{I_{\max}} \times 100 = \frac{400}{500} \times 100 = 80$$

From Fig. 12-43 or Table 12-1, $k = 1.6$

$$t = k\frac{L}{R} = \frac{1.6 \times 0.5}{12} = 0.0666 \text{ sec}$$

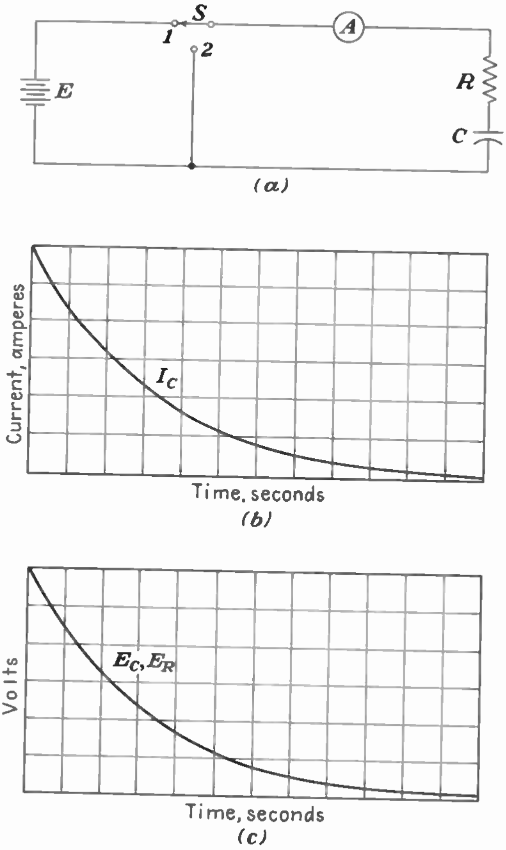


Fig. 12-42 Characteristics of current and voltage versus time. (a) The circuit. (b) Current versus time characteristics immediately after switch S is placed in position 2. (c) Voltage versus time characteristics immediately after switch S is placed in position 2.

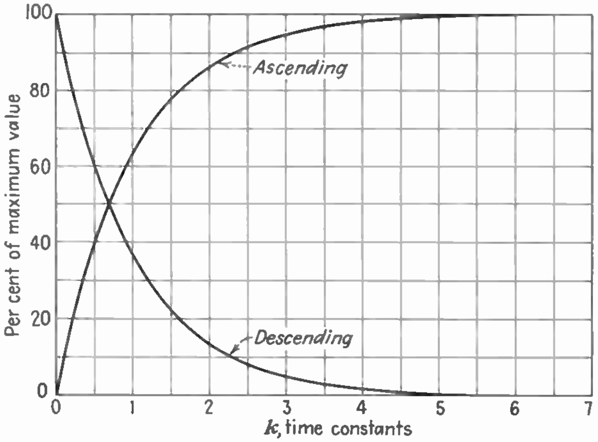


Fig. 12-43 Universal time-constant curves.

Table 12-1 Ascending Curve

k TIME CONSTANTS	PER CENT OF MAXIMUM VALUE	*k* TIME CONSTANTS	PER CENT OF MAXIMUM VALUE	*k* TIME CONSTANTS	PER CENT OF MAXIMUM VALUE
0.00	0.000	0.70	50.3	2.50	91.8
0.05	4.9	0.80	55.1	3.00	95.0
0.10	9.5	0.90	59.3	3.50	97.0
0.15	14.0	1.00	63.3	4.00	98.2
0.20	18.1	1.20	69.9	4.50	98.9
0.30	25.9	1.40	75.3	5.00	99.3
0.40	33.0	1.60	79.8	5.50	99.6
0.50	39.3	1.80	83.5	6.00	99.8
0.60	45.1	2.00	86.5	7.00	99.9

Table 12-2 Descending Curve

k TIME CONSTANTS	PER CENT OF MAXIMUM VALUE	*k* TIME CONSTANTS	PER CENT OF MAXIMUM VALUE	*k* TIME CONSTANTS	PER CENT OF MAXIMUM VALUE
0.00	100	0.70	49.7	2.50	8.2
0.05	95.1	0.80	44.9	3.00	5.0
0.10	90.5	0.90	40.7	3.50	3.0
0.15	86.0	1.00	36.8	4.00	1.8
0.20	81.9	1.20	30.1	4.50	1.1
0.30	74.1	1.40	24.7	5.00	0.7
0.40	67.0	1.60	20.2	5.50	0.4
0.50	60.7	1.80	16.5	6.00	0.2
0.60	54.9	2.00	13.5	7.00	0.1

EXAMPLE 12-19 A 0.005-μf capacitor and a 2-megohm resistor are connected to form an *R-C* circuit. If the *R-C* combination is connected to a 300-volt source of d-c power, what time is required for the voltage across the capacitor to reach (*a*) 100 volts? (*b*) 200 volts? (*c*) 270 volts? If the capacitor becomes fully charged (300 volts) and is then discharged through the 2-megohm resistor, what time is required to discharge the capacitor to (*d*) 250 volts? (*e*) 200 volts? (*f*) 50 volts?

GIVEN: $R = 2$ megohms $\quad C = 0.005\ \mu\text{f} \quad E_{\text{max}} = 300$ volts

FIND: t

SOLUTION:

(*a*) $$\%\text{ of }E_{\text{max}} = \frac{100}{300} \times 100 = 33.3$$

$$k = 0.40 \qquad \text{from Fig. 12-43}$$

$$t = kCR = 0.40 \times 0.005 \times 10^{-6} \times 2 \times 10^{6} = 0.004\text{ sec}$$

(*b*) $$\%\text{ of }E_{\text{max}} = \frac{200}{300} \times 100 = 66.6$$

$$k = 1.1 \qquad \text{from Fig. 12-43}$$

$$t = kCR = 1.1 \times 0.005 \times 10^{-6} \times 2 \times 10^{6} = 0.011\text{ sec}$$

(*c*) $$\%\text{ of }E_{\text{max}} = \frac{270}{300} \times 100 = 90$$

$$k = 2.27 \qquad \text{from Fig. 12-43}$$

$$t = kCR = 2.27 \times 0.005 \times 10^{-6} \times 2 \times 10^{6} = 0.0227\text{ sec}$$

(*d*) $$\%\text{ of }E_{\text{max}} = \frac{250}{300} \times 100 = 83.3$$

$$k = 0.19 \qquad \text{from Fig. 12-43}$$

$$t = kCR = 0.19 \times 0.005 \times 10^{-6} \times 2 \times 10^{6} = 0.0019\text{ sec}$$

(*e*) $$\%\text{ of }E_{\text{max}} = \frac{200}{300} \times 100 = 66.6$$

$$k = 0.40 \qquad \text{from Fig. 12-43}$$

$$t = kCR = 0.40 \times 0.005 \times 10^{-6} \times 2 \times 10^{6} = 0.004\text{ sec}$$

(*f*) $$\%\text{ of }E_{\text{max}} = \frac{50}{300} \times 100 = 16.6$$

$$k = 1.8 \qquad \text{from Fig. 12-43}$$

$$t = kCR = 1.8 \times 0.005 \times 10^{-6} \times 2 \times 10^{6} = 0.018\text{ sec}$$

Uses of Delayed-Action Circuits. There are numerous applications of *R-C* and *R-L* circuits in all branches of electronics. A few of the many applications are as follows.

One type of detector circuit, found in radio receivers, uses a capacitor and a resistor connected in parallel in the plate circuit of a diode. Actually this *R-C* combination is a time-constant circuit, and the values of *R* and *C* are chosen to produce a time constant of sufficient duration so that the charge on the capacitor gained during the positive half-cycles does not have time to dissipate any appreciable amount through the resistor during the negative half-cycles. Other examples of *R-C* circuits in radio apparatus include grid-leak detection, automatic-volume-control circuits, automatic-frequency-control circuits, and relaxation oscillators.

R-C and/or *R-L* circuits are used in television receivers and transmitters for producing timing, pulsing, triggering, and synchronizing circuits. The names of some of these circuits are differentiator, integrator, horizontal sweep, horizontal blanking, vertical blanking, automatic gain control, and automatic frequency control.

Industrial applications of time-constant circuits include controlling the length of time for a specific manufacturing operation, timing of electric welders, timing the exposures of photoflashes and photofinishing processes, timing of instruments, producing repeated action for life tests, timing and triggering circuits for computers and data-process equipment, and motor control.

QUESTIONS

1. (*a*) What is meant by the stray effects of inductors, capacitors, and resistors? (*b*) How is each of the circuit elements affected by the stray effects?
2. How is electrical energy transferred from (*a*) one circuit element to another? (*b*) One circuit to another?
3. Why is it necessary to use filters in electronic circuits?
4. (*a*) What are the essential parts of a filter circuit? (*b*) Explain the action of each part.
5. Explain the purpose of using each of the following units in a filter circuit: (*a*) capacitor, (*b*) inductor, (*c*) resistor, (*d*) series resonant circuit, (*e*) parallel resonant circuit.
6. (*a*) What is meant by a low-pass filter circuit? (*b*) Draw a simple low-pass circuit, and explain its action.
7. (*a*) What is meant by a high-pass filter circuit? (*b*) Draw a simple high-pass circuit, and explain its action.
8. (*a*) What is meant by a bandpass filter circuit? (*b*) Draw a simple bandpass circuit, and explain its action.
9. (*a*) What is meant by a bandstop filter circuit? (*b*) Draw a simple bandstop circuit, and explain its action.
10. Why is it necessary to use multisection filter circuits?
11. What is meant by (*a*) T-type filter? (*b*) π-type filter?
12. Draw a circuit diagram of one section of a T-type filter circuit for (*a*) low pass, (*b*) high pass, (*c*) bandpass, (*d*) bandstop.
13. Draw a circuit diagram of a two-section T-type filter circuit for (*a*) low pass, (*b*) high pass.
14. When two or more T-type low-pass filter circuits are joined, why should the connecting inductor have a value of twice the inductance of either of the end inductors?

15. When two or more T-type high-pass filter circuits are joined, why should the connecting capacitor have a value of one-half the capacitance of either of the end capacitors?
16. Draw a circuit diagram of one section of a π-type filter circuit for (*a*) low pass, (*b*) high pass, (*c*) bandpass, (*d*) bandstop.
17. Draw a circuit diagram of a two-section π-type filter circuit for (*a*) low pass, (*b*) high pass.
18. When two or more π-type low-pass filter circuits are joined, why should the connecting capacitor have a value of twice the capacitance of either end capacitor?
19. When two or more π-type high-pass filter circuits are joined, why should the connecting inductor have a value of one-half the inductance of either end inductor?
20. What is meant by the following terms: (*a*) Source impedance? (*b*) Load impedance? (*c*) Image impedance? (*d*) Characteristic impedance?
21. What is meant by (*a*) constant-*k* filter? (*b*) Imaginary load?
22. Describe the frequency requirements of an ideal filter.
23. What is meant by (*a*) *m*-derived filter? (*b*) Shunt-derived? (*c*) Series-derived?
24. Describe the circuit characteristics of (*a*) a shunt-derived low-pass filter, (*b*) a series-derived high-pass filter.
25. Explain how a resistor and a capacitor combine to form a filter circuit.
26. Define (*a*) attenuator, (*b*) fixed-attenuator pad, (*c*) variable-attenuator pad.
27. (*a*) Describe an L-attenuator circuit. (*b*) What are its circuit characteristics?
28. (*a*) Describe a T-attenuator circuit. (*b*) What are its circuit characteristics?
29. Describe (*a*) an H-attenuator circuit, (*b*) an O-attenuator circuit.
30. (*a*) Describe a π-attenuator circuit. (*b*) What are its circuit characteristics?
31. (*a*) Describe a ladder-attenuator circuit. (*b*) What are its circuit characteristics?
32. (*a*) Describe a bridged-T attenuator circuit. (*b*) How does this circuit compare with the simple-T attenuator circuit?
33. (*a*) Why is it necessary to couple circuits? (*b*) What is meant by the coupling element?
34. What is meant by (*a*) simple-coupled circuit? (*b*) Complex-coupled circuit?
35. Draw the circuit diagram and explain the action of four simple coupling circuits using a different coupling element for each.
36. Draw the circuit diagram and explain the action of three complex coupling circuits using a different coupling element for each.
37. What is the general purpose for using complex-coupled circuits?
38. Explain what is meant by (*a*) critical coupling, (*b*) loose coupling, (*c*) tight coupling.
39. What type of coupling element is commonly used to provide mutual-inductive coupling in electronic circuits?
40. Explain what is meant by coupled impedance.
41. How does the amount of coupled impedance affect the primary impedance?
42. How does the coupled impedance vary with the amount of coupling?
43. What is the bandwidth of the bandpass circuit used in (*a*) a-m radio receivers? (*b*) F-m radio receivers? (*c*) The picture section of television receivers?
44. Explain the relation between the width of band passed and the amount of coupling.
45. Explain the effect on secondary current of two resonant circuits coupled by means of a transformer with (*a*) critical coupling, (*b*) tight coupling, (*c*) loose coupling.
46. What are the important properties of a bandpass circuit?
47. How does the amount of circuit *Q* affect the uniformity of response?

48. Name and explain three methods of obtaining a wide bandpass.
49. Describe a method used for obtaining a wide bandpass in the i-f section of an f-m receiver.
50. Describe how the unsymmetrical wide bandpass required by the i-f video section of a television receiver is obtained.
51. What is the essential purpose of *R-L* and *R-C* circuits?
52. Explain the operation of a delayed-action (*a*) *R-L* circuit, (*b*) *R-C* circuit.
53. What is meant by the time constant of a delayed-action (*a*) *R-L* circuit? (*b*) *R-C* circuit?
54. (*a*) What is the purpose of the universal time-constant curve? (*b*) How is it used in the solution of time-delay circuits?
55. (*a*) Name four applications of time-delay circuits. (*b*) Explain the circuit actions of one of these applications.

PROBLEMS

1. A 2,500-ohm resistor has a stray inductance of 3 μh. What is the impedance of this resistor at the frequencies of (*a*) 1,500 kc? (*b*) 200 mc?
2. The stray inductance of a 12,000-ohm resistor is 4 μh. Determine its impedance, if the distributed capacitance is ignored, at (*a*) 15 mc, (*b*) 750 mc.
3. What is the impedance of the resistor in Prob. 1 to a frequency of (*a*) 4 mc? (*b*) 15 mc?
4. Determine the impedance of the resistor used in Prob. 2 if it is used in an electronic circuit whose frequency is (*a*) 30 mc, (*b*) 450 mc.
5. An r-f choke coil has an inductance of 150 μh and a distributed capacitance of 100 pf. Determine the inductive reactance, capacitive reactance, and impedance of the coil at (*a*) 1,500 kc, (*b*) 45 mc.
6. A coil has an inductance of 80 mh and a distributed capacitance of 20 μμf. Determine the inductive reactance, capacitive reactance, and impedance of the coil at (*a*) 15,750 cps, (*b*) 200 mc.
7. (*a*) Determine the frequency of resonance of the inductor used in Prob. 5. (*b*) Determine the impedance of the inductor at its resonant frequency if its ohmic resistance is 1.5 ohms.
8. (*a*) Determine the frequency of resonance of the inductor used in Prob. 6. (*b*) Determine the impedance of the inductor at its resonant frequency if its ohmic resistance is 4.75 ohms.
9. Determine the equivalent series resistance of a 12-μf paper-Mylar (dual dielectric) capacitor at (*a*) 60 cycles, (*b*) 1 kc.
10. Determine the equivalent series resistance of a 0.02-μf Teflon capacitor at (*a*) 1 kc, (*b*) 1 mc. *Note:* Use the lowest value of DF given in Table 9-1.
11. Determine the equivalent parallel resistance of the capacitor used in Prob. 9 at (*a*) 60 cycles, (*b*) 1 kc.
12. Determine the equivalent parallel resistance of the capacitor used in Prob. 10 at (*a*) 1 kc, (*b*) 1 mc.
13. It is desired that a filter choke, having a resistance of 45 ohms, oppose the flow of a 60-cycle current with ten times the opposition that it offers to direct current. What is the inductance of the coil?
14. A 15-henry filter choke has a resistance of 400 ohms. Determine (*a*) its opposition

to direct current, (*b*) its opposition to 60-cycle alternating current, (*c*) its ratio of opposition, alternating to direct current.

15. (*a*) To which type of current will a 0.04-μf capacitor offer the greater opposition, a 4,000-cycle a-f signal or a 1,500-kc r-f signal? (*b*) How many times greater is the larger impedance than the smaller impedance? (*c*) Which type of current is blocked by this capacitor?
16. (*a*) To which type of current will a 33-pf capacitor offer the greater opposition, a 4.5-mc signal or a 45.75-mc signal? (*b*) How many times greater is the larger impedance than the smaller impedance? (*c*) Which type of current is blocked by this capacitor?
17. A 10-mh inductor and a 250-pf capacitor are connected as shown in Fig. 12-7*c* to form a low-pass filter circuit. Determine the opposition offered to a 10-kc signal (*a*) by the capacitor, (*b*) by the inductor. Determine the opposition offered to a 1,500-kc signal (*c*) by the capacitor, (*d*) by the inductor.
18. A 1.8-μh inductor and a 68-pf capacitor are connected as shown in Fig. 12-7*c* to form a low-pass filter circuit. Determine the opposition offered to a 4.5-mc signal (*a*) by the capacitor, (*b*) by the inductor. Determine the opposition offered to a 45.75-mc signal (*c*) by the capacitor, (*d*) by the inductor.
19. A 4-henry inductor and a 0.4-μf capacitor are connected as shown in Fig. 12-9*c* to form a high-pass filter circuit. Determine the opposition offered to a 60-cycle power disturbance (*a*) by the capacitor, (*b*) by the inductor. Determine the opposition offered to a 1,200-cycle a-f signal (*c*) by the capacitor, (*d*) by the inductor.
20. A 1.2-μh inductor and a 20-pf capacitor are connected as shown in Fig. 12-9*c* to form a high-pass filter circuit. Determine the opposition offered to a 10.7-mc i-f signal (*a*) by the capacitor, (*b*) by the inductor. Determine the opposition offered to an 88-mc f-m signal (*c*) by the capacitor, (*d*) by the inductor.
21. Determine the value of m for a low-pass filter circuit whose resonant frequency is 40 mc and whose cutoff frequency is 32 mc.
22. Determine the value of m for a high-pass filter circuit whose resonant frequency is 4.5 mc and whose cutoff frequency is 5.0 mc.
23. A circuit similar to the one shown in Fig. 12-22*a* is to be used in the r-f stage of a receiver. It is desired that the capacitor offer an impedance of 100 times that of the resistor whose value is 6,500 ohms. What size capacitor is required if the signal is 1,500 kc?
24. A circuit similar to the one shown in Fig. 12-22*b* has the following values: The resistor has 7,200 ohms, and the capacitor has a value of 20 μf. (*a*) What impedance does the capacitor offer to a 5,000-cycle current? (*b*) Will the a-f signal flow through the resistor or the capacitor?
25. A certain 1,800-ohm load is supplied by a 60-volt d-c power source. It is desired to reduce the load voltage to 10 volts by means of an L-type attenuator similar to Fig. 12-23*a* and still retain an 1,800-ohm load condition at the power source. (*a*) What are the values of K, R_1, and R_2? Using the values obtained in part (*a*), (*b*) calculate the resistance at the power source terminals, (*c*) check the output voltage and the value of K.
26. A certain 48,000-ohm load is supplied by a 4.8-volt signal. It is desired that the signal be reduced to one-eighth its input value by an L-type attenuator similar to Fig. 12-23*a* and still retain a 48,000-ohm load condition at the signal source. (*a*) What are the values of K, R_1, and R_2? Using the values obtained in part (*a*),

(*b*) calculate the resistance at the signal-input terminals, (*c*) check the output voltage and the value of K.

27. A certain 120,000-ohm load is supplied by a 150-volt d-c power source. It is desired to reduce the load voltage by means of a T-type attenuator similar to Fig. 12-23*c* with an attenuation factor of 5 and still retain a 120,000-ohm load condition at the power source. (*a*) What are the values of e_o, R_1, R_2, and R_3? (*b*) By using the values obtained in part (*a*) calculate the resistance at the power source terminals.
28. A certain 8,000-ohm load is supplied by a 3.0-volt signal. It is desired that the signal be reduced to 0.2 volt by using a T-type attenuator similar to Fig. 12-23*c* and still retain an 8,000-ohm load condition at the signal source. (*a*) What are the values of K, R_1, R_2, and R_3? (*b*) By using the values obtained in part (*a*) calculate the resistance at the signal-input terminals.
29. A certain 12,000-ohm load is supplied by an 18-volt signal. It is desired that the signal be reduced to 12 volts by using a π-type attenuator similar to Fig. 12-23*e* and still retain a 12,000-ohm load condition at the signal source. (*a*) What are the values of K, R_1, R_2, and R_3? (*b*) By using the values obtained in part (*a*) calculate the resistance at the signal-input terminals.
30. A certain 60,000-ohm load is supplied by a 200-volt d-c power source. It is desired to reduce the load voltage to 150 volts by means of a π-type attenuator similar to Fig. 12-23*e* and still retain a 60,000-ohm load condition at the power source. (*a*) What are the values of K, R_1, R_2, and R_3? (*b*) By using the values obtained in part (*a*) calculate the resistance at the power source terminals.
31. An audio transformer with a 3-to-1 step-up ratio has an inductance of 80 henrys and a resistance of 100 ohms on its primary side. The inductance of the secondary winding is 720 henrys, and its resistance is 1,000 ohms. (*a*) What value of inductance is reflected to the primary by the secondary? (*b*) What value of resistance is reflected to the primary by the secondary? (*c*) If a load impedance of 7,200 ohms is connected across the secondary, what value of impedance is reflected to the primary by the secondary? (*d*) What value of inductance does the primary reflect to the secondary? (*e*) What value of resistance does the primary reflect to the secondary?
32. An audio transformer with a 2.5-to-1 step-up ratio has an inductance of 50 henrys and a resistance of 80 ohms on its primary side. The inductance of the secondary winding is 312.5 henrys, and its resistance is 500 ohms. (*a*) What value of inductance is reflected by the primary to the secondary? (*b*) What value of resistance is reflected by the primary to the secondary? (*c*) What value of inductance is reflected by the secondary to the primary? (*d*) What value of resistance is reflected by the secondary to the primary? (*e*) If the load on the secondary has a capacitance of 200 pf, what is the reflected capacitance on the primary side?
33. What is the approximate width of the frequency band passed by a bandpass amplifier circuit having a resonant frequency of 465 kc and a coefficient of coupling of 0.02?
34. What coefficient of coupling is required to produce a 150-kc bandpass at an operating frequency of (*a*) 10.7 mc? (*b*) 9 mc?
35. It is necessary that the critical coupling of an inductance-coupled bandpass circuit be equal to 0.008. If the circuit Q of the primary and secondary circuit are equal to each other, what is their value?

36. It is necessary that the critical coupling of an inductance-coupled bandpass circuit be equal to 0.02. If the circuit Q of the primary and secondary circuit are equal to each other, what is their value?
37. It is desired that a circuit pass a band 10-kc wide. The circuit Q is equal to 68.5. (*a*) What value of coefficient of coupling is required? (*b*) What are the extreme limits of the frequency band passed?
38. It is desired that a circuit pass a band 10 kc wide. The circuit Q is equal to 40. (*a*) What value of coefficient of coupling is required? (*b*) What are the extreme limits of the frequency band passed?
39. A low-current d-c relay that has an inductance of 25 henrys is connected in series with a 1,000-ohm resistor to form an R-L time-delay control circuit operated on a 110-volt d-c circuit. (*a*) What is the time constant of the circuit? (*b*) If the relay closes when the current is 88 ma, what time elapses between closing the line switch and operation of the relay?
40. A low-current d-c relay having an inductance of 10 henrys is to close 0.02 sec after the line switch is closed. What value of resistor should be connected in series with the relay if it closes when the current reaches (*a*) 63.2 per cent of its final value? (*b*) 80 per cent of its final value?
41. A radio receiver is to have an R-C circuit with a time constant of 0.2 sec for its automatic-volume-control circuit. (*a*) What value of resistor is required if a 0.1-μf capacitor is used? (*b*) What value of resistor is required if a 0.15-μf capacitor is used? (*c*) What value of capacitor is required if a 1-megohm resistor is used?
42. A 1-megohm grid resistor is shunted by a 250-pf capacitor. (*a*) What is the time constant of this circuit? (*b*) If the highest a-f signal to be applied to the circuit is 5,000 cycles, what is the time required to complete one of these cycles? (*c*) Under the conditions of (*a*) and (*b*) will the capacitor ever become completely discharged? (*d*) Explain your answer to (*c*).
43. A 0.05-μf capacitor and a 0.5-megohm resistor are connected to form an R-C circuit. The R-C combination is connected to a 250-volt d-c source. (*a*) What time is required for the voltage across the capacitor to reach 50, 100, 200 volts? (*b*) What current flows when the switch is closed? (*c*) What current flows when the voltage across the capacitor reaches 200 volts? (*d*) If the capacitor is fully charged and is then discharged through the 0.5-megohm resistor, what is the current at the instant it starts to discharge? (*e*) What is the value of the voltage RC sec after the capacitor starts to discharge? (*f*) At what time will the capacitor be discharged to half voltage?
44. What is the time constant of an R-C circuit as used in a television receiver when (*a*) $R = 2.2$ megohms, $C = 0.05$ μf? (*b*) $R = 470{,}000$ ohms, $C = 0.1$ μf? (*c*) $R = 680{,}000$ ohms, $C = 820$ pf?

Chapter 13

Motors and Generators

The basic purpose of a motor is to convert electrical energy to mechanical energy. The basic purpose of the generator is to convert mechanical energy to electrical energy. Modern applications of electric motors and generators include such fields as industry, transportation, automation, electronics, business, and data processing. This chapter presents the principles of operation and some of the more important characteristics of the most commonly used types of d-c and a-c motors and d-c generators.

13-1 Motor Ratings

Horsepower. A basic rating of motors, both d-c and a-c types, is the horsepower output available at the shaft of the motor. The ratings of motors range from a small fraction of 1 hp to several thousand horsepower. Motors of less than 1 hp are classed as fractional-horsepower motors, and those of one or more horsepower are classed as integral-horsepower motors.

Voltage. The voltage rating of d-c motors ranges from 1.5 to 1,500 volts; frequently used values are 6, 12, 28, 110, 220, and 440 volts. The higher voltages are used on only large motors. The voltage rating of a-c motors ranges from 1.5 to several thousand volts; frequently used values are 6, 24, 115, 208, 230, and 440 volts for small- and medium-size motors. Voltage ratings of 4,000 and 6,600 volts are sometimes used for very large-size motors.

Frequency. With a-c motors, the frequency rating is also important. For industrial motors, 60 cycles is almost universally used; other values of frequency sometimes used include 25, 40, 50, and 400 cycles.

Speed. Motors are also rated at the speed of rotation of the motor shaft; it is expressed in *revolutions per minute* and is abbreviated as *rpm.* The rated full-load speed for which d-c motors can be designed extends over a wide range. The speed of most types of a-c motors, when operated from 60-cycle power sources, is limited to such basic values as 3,600, 1,800, 1,200, 900, and 600 rpm [see Eq. (7-2)].

Because of the effect of centrifugal force, large-size motors are limited to lower speeds than are small-size motors. When very low speeds are required, it is generally advantageous to use a higher speed motor equipped with a reduction-gear system.

Torque. The turning effort produced at the shaft of a motor is called the *torque*. The torque of a motor is proportional to the force produced on the conductors of the rotating member and to the radial distance through which the force acts. For medium- and large-size motors, the force is expressed in pounds and the reference turning radius is 1 ft, thus

$$T = FR \tag{13-1}$$

where T = torque, lb-ft
F = force, lb
R = radius, ft

Very small motors are rated in ounce-inches of torque; it takes 192 oz-in. to equal 1 lb-ft.

Horsepower. Horsepower represents the rate of doing work, and one horsepower is equivalent to doing 33,000 foot-pounds of work in one minute. When the effects of torque and speed are combined, the horsepower output of a motor may be calculated by

$$HP = \frac{TS}{5{,}250} \tag{13-2}$$

where HP = horsepower
T = torque, lb-ft
S = speed, rpm

Equation (13-2) may be transposed to solve for torque, as

$$T = \frac{HP\ 5{,}250}{S} \tag{13-3}$$

EXAMPLE 13-1 What is the rated full-load torque of a 2-hp 1,800-rpm motor?

GIVEN: $HP = 2 \qquad S = 1{,}800$ rpm

FIND: T

SOLUTION:

$$T = \frac{HP\ 5{,}250}{S} = \frac{2 \times 5{,}250}{1{,}800} = 5.83 \text{ lb-ft}$$

EXAMPLE 13-2 What is the horsepower rating of a motor that delivers 4.2 oz-in. of torque at 2,900 rpm?

GIVEN: $T = 4.2$ oz-in. $\qquad S = 2{,}900$ rpm

FIND: HP

SOLUTION:

$$HP = \frac{TS}{5{,}250} = \frac{4.2 \times 2{,}900}{192 \times 5{,}250} = 0.012$$

Duty Cycle. The length of time that a motor is required to operate a load and its off-duty time, if any, affect the physical size of the motor. Motors may be rated for continuous duty or for intermittent duty. A continuous-duty motor must be capable of supplying power to a load continuously for 24 hr per day and not exceed a safe operating temperature. Intermittent-duty motor ratings may be for (*a*) 2 hr, (*b*) 1 hr, (*c*) ½ hr, (*d*) a specified number of on and off cycles per hour, or (*e*) the requirements for a particular application.

Temperature Rise. The rated temperature rise of a motor is the maximum permissible increase in temperature above a specified ambient temperature at any motor part after the completion of its duty cycle. Continuous-duty motors are generally rated for 40 or 50°C temperature rise above an ambient temperature of 25°C. Intermittent-duty motors are generally rated for 50 or 55°C temperature rise.

In addition to the duty cycle and the maximum allowable temperature rise, the physical size of a motor is also affected by the amount of ventilation available to the motor parts. A completely enclosed motor will be rated at a lower output than a motor of equal size and whose parts can freely dissipate heat to the surrounding air.

Efficiency. Although efficiency may not strictly be considered a motor rating, it is of importance to the user in that it affects the cost of operating the motor. The efficiency of motors varies greatly with their size as is indicated by the following approximate values: (1) ¼ hp, 62 per cent; (2) 1 hp, 75 per cent; (3) 50 hp, 90 per cent; and (4) 5,000 hp, 97 per cent. The efficiency of a motor may be calculated by

$$\text{Per cent efficiency} = \frac{\text{output}}{\text{input}} \times 100 \tag{13-4}$$

or

$$\text{Per cent efficiency} = \frac{\text{output}}{\text{output} + \text{losses}} \times 100 \tag{13-5}$$

EXAMPLE 13-3 What is the per cent efficiency of a ¼-hp a-c motor that takes 300 watts from the power source?

GIVEN: $P_o = 1/4$ hp $P_i = 300$ watts

FIND: Per cent eff

SOLUTION:

$$\text{Per cent eff} = \frac{\text{output}}{\text{input}} \times 100 = \frac{0.25 \times 746}{300} \times 100 = 62$$

EXAMPLE 13-4 What is the per cent efficiency of a 5-hp d-c motor if it has the following losses: (1) 500 watts in the iron core, (2) 50 watts in the shunt field copper, (3) 200 watts in the armature copper, and (4) 220 watts in windage and friction?

GIVEN: $P_o = 5$ hp Losses: Core = 500 watts Field = 50 watts
Arm = 200 watts Friction = 220 watts

FIND: Per cent eff

SOLUTION:

$$\text{Per cent eff} = \frac{\text{output}}{\text{output} + \text{losses}} \times 100$$

$$= \frac{5 \times 746}{(5 \times 746) + 500 + 50 + 200 + 220} \times 100$$

$$= \frac{3{,}730}{4{,}700} \times 100 = 79.3$$

Power Factor. Power factor, although not considered a motor rating, is an important characteristic of a-c motors. Some power companies allow a reduction in the cost of power to those customers who have an appreciable motor load if they maintain a load power factor above a specified minimum value. The power factor of a-c motors varies over a wide range, approximately 50 to 90 per cent, depending on the type of motor, its horsepower, and its speed. Small-size motors and low-speed motors have low power-factor values.

13-2 The D-C Motor

Principle of Operation. The structure of the simple d-c motor is similar to that of the simple d-c generator presented in Chap. 7. Operation of the generator requires that the armature be rotated by means of an external mechanical force, and electrical energy will then be available at the brushes of the generator. In the case of the motor, the process is reversed, and the electrical energy, which is applied to the brushes of the motor, causes the armature to rotate and produce a source of mechanical energy that is available at the shaft of the motor.

The simple motor (Fig. 13-1*a*) consists of (1) a field structure to provide a strong magnetic field between the poles *N* and *S*, (2) a coil *C* whose coil sides *aa'* and *bb'* are connected to the commutator segments K_1 and K_2, and (3) the brushes B_1 and B_2. The coil *C* and the commutator *K* are mounted on a common shaft S so that they are free to rotate. A d-c power source supplies electrical energy to the motor. When switch S_1 is closed, current will flow through the armature coil *C*, the path being from the negative terminal of the power source, through S_1, into the motor at the brush B_2, through the coil side *b* to *b'*, through the coil side *a'* to *a*, and out at brush B_1 to return to the power source at its positive terminal. The current flowing in the sides *aa'* and *bb'* will produce magnetic fields around these conductors as shown in Fig. 13-1*b*.

When a coil carrying a current is placed in a magnetic field, the interactions of the magnetic fields surrounding the coil sides with the magnetic field of

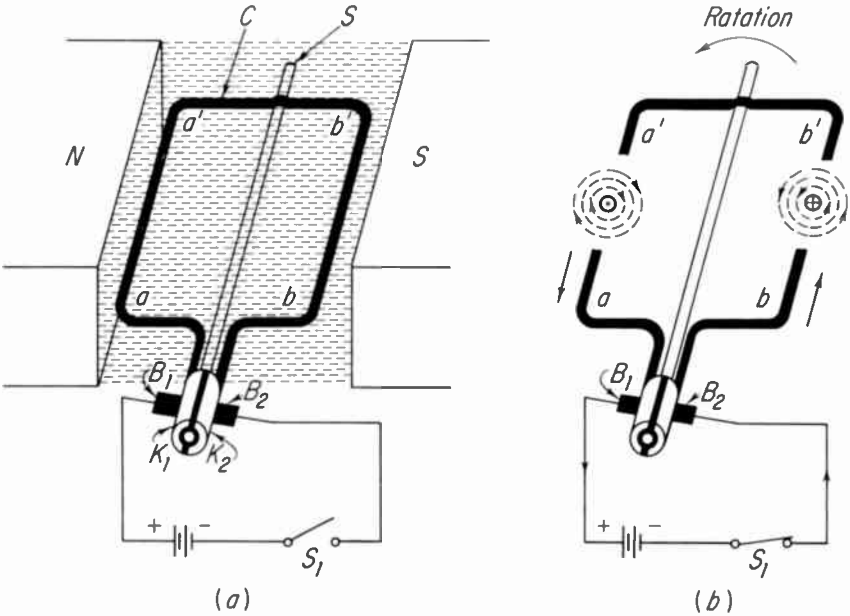


Fig. 13-1. (a) A simple d-c motor. (b) Magnetic fields produced around the armature conductors when a current is flowing.

the motor field structure will produce a torque at the coil sides and cause the coil to rotate (Fig. 13-2*b*). Figure 13-2*a* shows that the circular magnetic field around the conductor and the magnetic field between the *N* and S poles are in the same direction at the bottom of the conductor but are in opposite directions at the top of the conductor. The magnetic fields become distorted and produce a strong field at the bottom of the conductor and a weak field at the top. The magnetic lines thereby exert a force on the conductor that will move it in an upward direction. When this principle is applied to the simple motor, the conductors shown in Fig. 13-1 will cause both conductors *aa′* and *bb′* to produce a rotational force in a counterclockwise direction; this is also illustrated in Fig. 13-2*b*.

The direction of rotation of a motor can be determined by the right-hand rule which states: Hold the thumb, forefinger, and middle finger at right angles to one another; set the forefinger in the direction of the magnetic field, the middle finger in the direction of the electron flow in the conductor, and the thumb will then point in the direction of motion of the conductor.

Counter EMF. The amount of current that will flow in the armature of a motor at the instant when switch S_1 of Fig. 13-1 is closed will be its Ohm's law value, namely, the line voltage divided by the resistance of the armature. When the armature starts to rotate, a voltage will be induced in the armature coils owing to their cutting the magnetic field set up by the poles

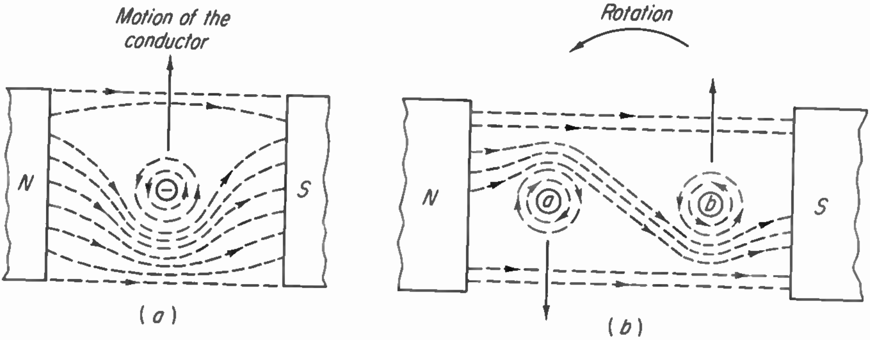


Fig. 13-2 Effect of placing current-carrying conductors in a magnetic field. (a) A single conductor with electrons flowing in toward the page as in conductor bb′ of Fig. 13-1b. (b) Two conductors of an armature coil similar to the conditions of Fig. 13-1b.

N and S. The polarity of the induced emf is opposite to the impressed voltage (Lenz's law) and is called the *counter emf.* The counter emf of the motor is produced by the same kind of action that produces the induced emf of a generator and is proportional to the same factors expressed by Eq. (7-1); thus

$$cemf = \frac{2\phi CS}{60 \times 10^8} \tag{13-6}$$

When a motor is operating at its normal speed, the counter emf is slightly lower than the impressed voltage. Actually, the counter emf is lower than the impressed voltage by the amount of the *IR* drop in the armature, or

$$cemf = E_i - I_A R_A \tag{13-7}$$

where $cemf$ = counter emf, volts
E_i = impressed voltage, volts
I_A = armature current, amp
R_A = armature resistance, ohms

The armature current at any time is, therefore,

$$I = \frac{E_i - cemf}{R_A} \tag{13-8}$$

Effect of Motor Load on the Armature Current. When the load applied to the shaft of a motor is increased, the following actions occur: (1) The motor speed decreases, (2) the reduction in the motor speed results in a decrease in the counter emf, (3) the decrease in the counter emf results in an increase in armature current, and (4) the increase in the armature current causes more power to be taken from the power source and thereby enables the motor to deliver an increased amount of load at its shaft.

EXAMPLE 13-5 The armature of a certain 120-volt d-c motor has a resistance of 1 ohm, its no-load current is 1 amp, and its no-load speed is 1,850 rpm. (*a*) What is the counter emf and the motor speed for a load current of 10 amp? (*b*) What is the armature current and the motor speed if the motor load causes the counter emf to drop to 105 volts? *Note:* On the basis of Eq. (13-6), assume that the motor speed varies directly with the counter emf.

GIVEN: $E_i = 120$ volts $R_A = 1$ ohm $I_{NL} = 1$ amp $S_{NL} = 1{,}850$ rpm
(*a*) $I_L = 10$ amp (*b*) $cemf = 105$ volts

FIND: (*a*) *cemf*, speed (*b*) I_A, speed

SOLUTION:

(*a*)

$$cemf = E_i - I_A R_A = 120 - (10 \times 1) = 110 \text{ volts}$$

$$\text{Speed} = S_{NL} \frac{cemf_L}{cemf_{NL}} = 1{,}850 \times \frac{110}{119} = 1{,}710 \text{ rpm}$$

where

$$cemf_{NL} = E_i - I_{NL} R_A = 120 - (1 \times 1) = 119 \text{ volts}$$

(*b*)

$$I_A = \frac{E_i - cemf}{R_A} = \frac{120 - 105}{1} = 15 \text{ amp}$$

$$\text{Speed} = S_{NL} \frac{cemf_L}{cemf_{NL}} = 1{,}850 \times \frac{105}{119} = 1{,}632 \text{ rpm}$$

Speed Regulation. The speed regulation of a motor expresses the amount of variation between the speed of a motor with zero load at its shaft and the speed with full load at the shaft. This is expressed by

$$\% \text{ regulation} = \frac{S_{NL} - S_{FL}}{S_{FL}} \times 100 \tag{13-9}$$

where S_{NL} = speed at no load, rpm
S_{FL} = speed at full load, rpm

Speed Control. The equation for the speed of a motor can be obtained by transposing Eq. (13-6), thus

$$S = \frac{cemf \times 60 \times 10^8}{2\phi C} \tag{13-10}$$

This equation shows that the speed of a motor is directly proportional to the counter emf and inversely proportional to the strength of the magnetic field. Thus, there are two possible methods of controlling or varying the speed of a d-c motor.

One method used to vary the speed of a motor is to vary the voltage applied to the armature, consequently also varying the counter emf. When a reduction in speed is desired, it can be achieved by (1) keeping the field at full strength and (2) connecting an external resistance in series with the armature; this will reduce the voltage at the armature terminals and thereby cause a reduction in the motor speed. This method of speed con-

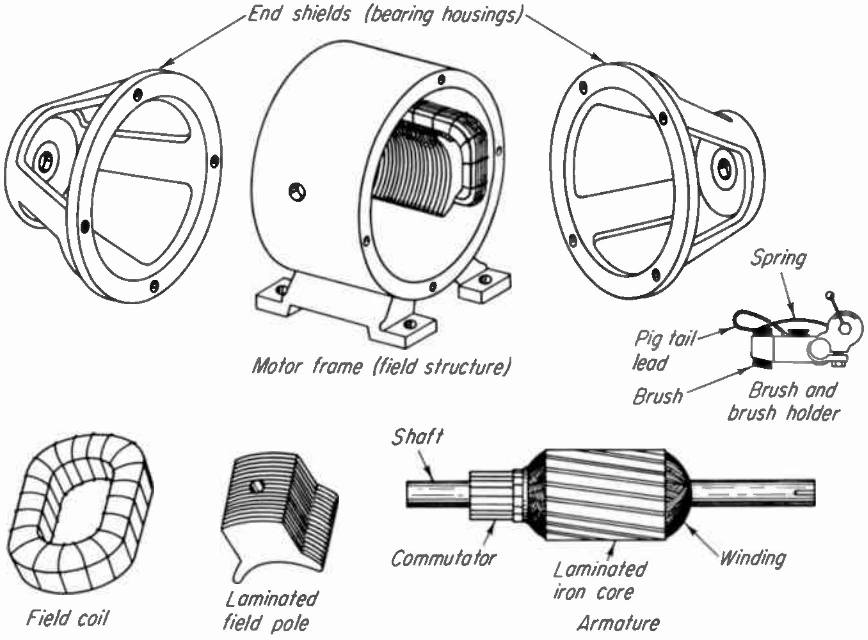


Fig. 13-3 Basic parts of the d-c motor.

trol has a disadvantage in that the efficiency of the system drops in direct proportion to the reduction in speed; it is used chiefly where operation at a reduced speed is for only short periods of time. The efficiency of speed control by varying the armature voltage can be maintained at a high level by using a variable voltage source such as a separate motor-generator set. This method, or variations thereof, is known as the Ward Leonard system.

Another method of varying the speed of a motor is to vary the strength of the magnetic field. In this method, the armature voltage is kept at its normal value and the current applied to the coils of the electromagnets is decreased by inserting resistance in series with the coils. The result is an increase in speed without decreasing the efficiency of the system. This is a commonly used method of speed control for ranges of 2 to 1 or 3 to 1. Specially designed motors can provide speed ranges up to 6 to 1.

Electronic motor-control systems have been devised to provide a wide range of precision control characteristics, such as (1) fixed speed, (2) variable speeds, (3) fixed torque, (4) variable torque, (5) reversing cycles, (6) electric braking. Thyratron tubes are used extensively in these control systems.

Starting Requirements. When a d-c motor is started from rest and its counter emf is zero, the armature current will tend to be equal to the impressed voltage divided by the armature resistance. As the armature resistance is normally very low, the motor will draw an excessive amount of instantaneous starting current. Although this current decreases rapidly with the building up of the counter emf, it is generally too high an amount for the motor and/or the power source supplying it to withstand without damage.

With the exception of very small motors, some type of starting and/or control equipment is used with a d-c motor. The type of control varies with the size of the motor and the requirements of the particular application and may be (1) a resistance temporarily connected in series with the armature, (2) a manually controlled starting box, (3) an automatic push-button-controlled starting box, or (4) an electronic control system.

13-3 Types of D-C Motors

Direct-current motors may be classed according to their type of field structure as (1) permanent magnet and (2) electromagnet. The use of permanent magnets for field poles is generally limited to very small motors. All medium- and large-size motors use electromagnets for the field poles. When electromagnets are used, the motors are further classified in terms of the manner in which the field windings are connected with respect to the armature. These classifications are (1) shunt, (2) series, and (3) compound.

The construction features of the shunt, series, and compound motors are quite similar. For a given horsepower and speed rating, the same basic armature might be used for all three types. The strength of the magnetic field, expressed in ampere-turns, for full-load condition is the same for all three types. The chief difference in construction is in the number of turns and the size of wire used in the field coils to obtain the required ampere-turns.

The operating characteristics of the three types of motors differ considerably and are described separately in the following articles.

13-4 The Shunt Motor

Field Connection. A shunt motor has its field windings connected directly across the power source and hence in parallel with, or *shunted* across, the armature.

Field Windings. Because the field coils are connected directly across the power source, their resistance must be relatively high in order to have a low value of field current. The required number of ampere-turns is obtained by using a large number of turns of small-size wire.

EXAMPLE 13-6 The field windings of a certain 115-volt motor have 960 turns per coil, and the resistance of the field circuit is 230 ohms. (*a*) What is the field current? (*b*) What is the magnetizing force per pole?

GIVEN: $E_f = 115$ volts $\quad N_{\text{coil}} = 960$ turns $\quad R_{fc} = 230$ ohms

FIND: (*a*) I_f $\quad$ (*b*) NI per pole

SOLUTION:

(a) $$I_f = \frac{E_f}{R_{fc}} = \frac{115}{230} = 0.5 \text{ amp}$$

(b) $$NI_{\text{pole}} = N_{\text{coil}}\, I_f = 960 \times 0.5 = 480 \text{ amp-turns}$$

Torque. The torque of a motor is proportional to (1) the armature current and (2) the strength of the magnetic field. As the field strength of a shunt motor is essentially constant, the torque will vary directly with the armature current (Fig. 13-5).

Speed Regulation. Because the field strength is essentially constant, the variation of speed with varying load conditions is small (Fig. 13-6). A shunt motor therefore has good speed regulation.

EXAMPLE 13-7 What is the speed regulation of a shunt motor whose full-load speed is 1,750 rpm and no-load speed 1,900 rpm?

GIVEN: $S_{FL} = 1{,}750$ rpm $S_{NL} = 1{,}900$ rpm

FIND: Regulation

SOLUTION:

$$\text{Regulation} = \frac{S_{NL} - S_{FL}}{S_{FL}} \times 100 = \frac{1{,}900 - 1{,}750}{1{,}750} \times 100 = 8.57\%$$

Speed Control. The shunt motor can provide a wide range of speed control by adding a variable resistance in its field circuit (Fig. 13-4). This method is used when it is desired to obtain speeds higher than its rated full-field speed.

Hazard of Loss of Field Current. Equation (13-10) indicates that the speed of a d-c motor varies inversely with the flux. If the field circuit of a shunt motor is broken, so that the value of ϕ approaches zero, the motor speed will increase to a dangerously high amount. The resultant centrifugal forces can cause physical damage to the motor and its surroundings.

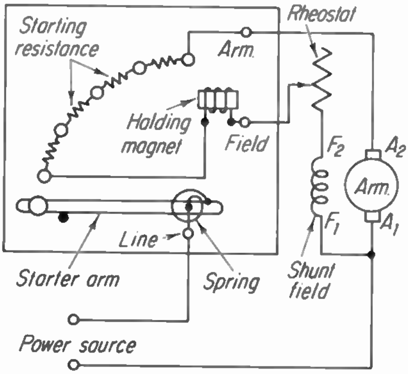


Fig. 13-4 Shunt motor with a three-point starter and a field rheostat.

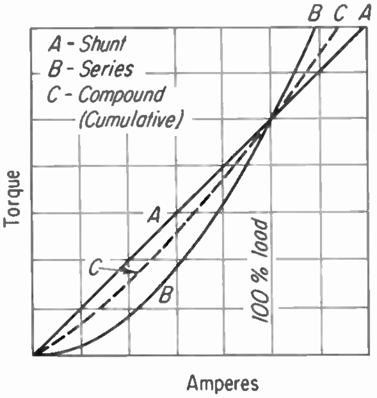


Fig. 13-5 Torque versus load characteristics of shunt, series, and compound motors.

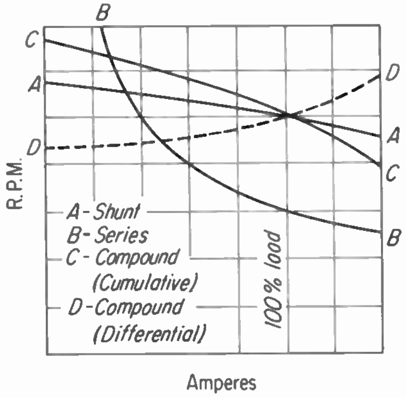


Fig. 13-6 Speed versus load characteristics of shunt, series, and compound motors.

Starting Requirements. The basic starting and speed-control equipment for the shunt motor is shown in Fig. 13-4. The starting box, called a *three-point starter* because it has three terminals, inserts resistance into the armature circuit to limit the current to a safe amount during the starting period. As the arm of the starting box is advanced toward the full running position, the resistance inserted in series with the armature is reduced to zero. The holding magnet keeps the starter arm in the full running position. It should be observed that the coil of the holding magnet is part of the shunt field circuit. If an open circuit should occur in the field circuit, the holding magnet would no longer hold the starter arm in the running position and the spring S would return the starter arm to the OFF position. Because of this action, the three-point starter is also called a *no-field release starter.*

Direction of Rotation. The right-hand rule for determining the direction of motion of an armature conductor was presented in Art. 13-2. This rule shows that the direction of rotation of a d-c motor can be changed by (1) reversing the polarity of the field poles or (2) reversing the direction of electron flow in armature conductors. Translated into practical terms, the direction of rotation can be reversed by interchanging either (1) the two leads of the field winding or (2) the two armature leads. Interchanging the line leads reverses the current in both the armature and field windings and hence does not cause a reversal in the direction of rotation of the motor.

Applications. The shunt motor is used where the basic requirements (1) are very small or moderate amounts of torque during the starting periods, (2) are nearly constant speed over a wide range of load conditions, (3) have the ability to provide considerable increases in speed above the normal full-field speed. Some of the many applications are the operation of (1) motor-generator units, (2) some types of machine tools, (3) business machines, (4) flexible shaft equipment, (5) engraving machines.

13-5 The Series Motor

Field Connection. A series motor has its field windings connected in series with the armature and hence must carry the same amount of current as the armature.

Field Windings. Because the field current is much higher than for the shunt motor, the field coils of the series motor have relatively few turns and are wound with a much larger size of wire.

EXAMPLE 13-8 It is desired that the motor of Example 13-6 be of the series type. How many turns per coil are required for the field winding if the full-load armature current is 8 amp?

GIVEN: $NI = 480 \qquad I = 8$ amp

FIND: N

SOLUTION:

$$N = \frac{NI}{I} = \frac{480}{8} = 60 \text{ turns}$$

Torque. As the load on a series motor increases, the current in both the armature and the field coils increases. The torque, being proportional to both the armature current and the field strength, will vary as the square of the change in current. This is approximately true up to the point where the magnetic field reaches saturation. Beyond saturation, the torque varies in nearly direct proportion to the armature current (Fig. 13-5).

Speed Regulation. Because the field strength varies almost directly with the load, the variation of speed with load is very great (Fig. 13-6). Therefore, a series motor has poor speed regulation.

Speed Control. When speed control is used with a series motor, it is in terms of reducing the speed. Speed control is then provided by inserting resistance into the armature circuit.

Hazard of Loss of Load. If the load should become disengaged from the shaft of a series motor, the motor speed will increase to a dangerously high amount. For this reason, the load on a series motor should be applied directly through gears or other means rather than indirectly through a pulley and belt.

Starting Requirements. Two forms of basic starters for use with series motors are shown in Fig. 13-7. Figure 13-7*a* shows a series motor and a two-point starter. Because the load current passes through the holding magnet, the two-point starter provides protection against low-load conditions. Figure 13-7*b* shows a series motor and a three-point starter. In this case, the holding magnet is connected across the power source. The three-point

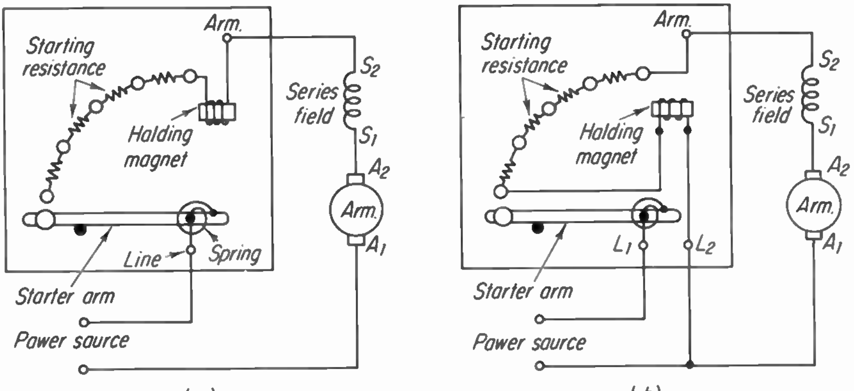


Fig. 13-7 Series motor with starter. (a) Motor and two-point starter. (b) Motor and three-point starter.

starter provides ***undervoltage protection*** but does not provide low-load protection.

Direction of Rotation. The direction of rotation of a series motor can be reversed by interchanging either (1) the two leads of the field winding or (2) the two armature leads. Interchanging the line leads reverses the current in both the armature and the field windings and hence does not cause a reversal in the direction of rotation of the motor.

Applications. The series motor is used where the basic requirements are (1) very high torque during the starting periods, (2) poor speed regulation is not objectionable, (3) operation at reduced speed is desired for short periods of time. Some applications of series motors are the operation of (1) hoists, (2) cranes, (3) conveyor systems, (4) mixing machines.

13-6 The Compound Motor

Field Connections. The compound motor has both a shunt field winding and a series field winding. When the shunt and series field windings produce the same magnetic polarity at the poles of the motor, they aid each other and the motor is called a *cumulative compound motor.* When the two field windings are connected to produce opposite polarities, they oppose each other and the motor is called a *differential compound motor.* When the shunt field winding is connected so that it receives full voltage, it is called a ***long shunt*** connection. When the shunt field winding is connected across the armature terminals, the shunt field voltage is lower than the line voltage by the amount of the series field IR drop and it is called a ***short shunt*** connection. The motor in Fig. 13-8 is connected as long shunt.

Field Windings. The shunt portion of the winding has a large number of turns of small-size wire, and the series portion has a small number of turns of large-size wire. The resistance of the shunt portion is high, and that of the series portion is low.

EXAMPLE 13-9 It is desired that the motor of Example 13-6 be made 25 per cent cumulative compound. (*a*) How many ampere-turns are required for the shunt winding? (*b*) How many ampere-turns are required for the series winding? (*c*) If the shunt field current is 0.5 amp, how many turns are required? (*d*) If the armature current is 8 amp, how many turns are required for the series field?

GIVEN: $NI_{\text{total}} = 480$ Compound = 25% $I_{\text{shunt}} = 0.5$ amp $I_{\text{series}} = 8$ amp

FIND: (*a*) NI_{shunt} (*b*) NI_{series} (*c*) N_{shunt} (*d*) N_{series}

SOLUTION:

(*a*)
$$NI_{\text{shunt}} = NI_{\text{total}} \times \frac{\%}{100} = 480 \times \frac{75}{100} = 360 \text{ amp-turns}$$

(*b*)
$$NI_{\text{series}} = NI_{\text{total}} \times \frac{\%}{100} = 480 \times \frac{25}{100} = 120 \text{ amp-turns}$$

(*c*)
$$N_{\text{shunt}} = \frac{NI_{\text{shunt}}}{I_{\text{shunt}}} = \frac{360}{0.5} = 720 \text{ turns}$$

(*d*)
$$N_{\text{series}} = \frac{NI_{\text{series}}}{I_{\text{series}}} = \frac{120}{8} = 15 \text{ turns}$$

Torque. The torque characteristics of the compound motor lie between those of the shunt and series motors (Fig. 13-5). Whether the characteristics are closer to those of the shunt or the series motor will depend on the percentage of compounding and whether the connection is for cumulative or differential operation.

Speed Regulation. The speed regulation of the cumulative compound motor lies between that of the shunt and series motors (Fig. 13-6). The speed regulation of the differential compound motor can be made even better than that of the shunt motor (Fig. 13-6).

When the speed of a motor increases in unpredictable amounts with variations in load, it is called *racing*. This condition occurs with the loss of load in the case of the series motor and in some cases may occur with an increase in load on a shunt motor. The cumulative compound motor eliminates or reduces to a safe amount the effect of racing. Any tendency of a motor toward racing will be accentuated with a differential compound motor.

Speed Control. Speed control can be obtained with armature resistance to provide speeds lower than the full-field full-armature-voltage value and with a field rheostat for speeds higher than the full-field full-armature-voltage value.

Hazard of Loss of Load Current. The danger of overspeed due to the loss of load current that is characteristic of the series motor can be overcome by providing a small percentage of shunt field ampere-turns.

The amount of shunt winding added should be sufficient to keep the no-load speed of the motor just under the safe maximum speed. Although there will be some reduction in the maximum starting torque, this may not be objectionable for some applications.

Hazard of Loss of Field Current. As most compound motors have 75 per cent or more of the total ampere-turns in the shunt field winding, the danger of overspeed in the event of the loss of field current is almost the same as with the shunt motor.

Starting Requirements. A four-point starter is generally used with a compound motor (Fig. 13-8). A rheostat may be included in the shunt field circuit to provide some amount of speed control. In the four-point starter, the holding magnet is connected across the line and the starter arm will be released in the event of line voltage failure. This type of starter is also called a *no-voltage-release starter.* The four-point starter is often used in preference to the three-point starter when speed control is to be obtained with a field rheostat. This is because the reduced field current may weaken the holding magnet to the extent that the starter arm may be unintentionally released.

Direction of Rotation. The direction of rotation of the compound motor can be reversed by (1) interchanging the two armature leads or (2) interchanging both the two shunt field leads and the two series field leads. In method 2, caution must be observed that the current is reversed in both field windings. Reversing the current in only one field winding will change the motor from cumulative to differential operation, or vice versa, and can result in unsatisfactory motor operation.

Applications. The compound motor is adaptable to a wide range of applications because its flexibility in design makes it possible to produce a wide range of torque and speed characteristics. The cumulative compound motor produces a higher starting torque than the shunt motor and is used for a great variety of applications. The differential compound motor has

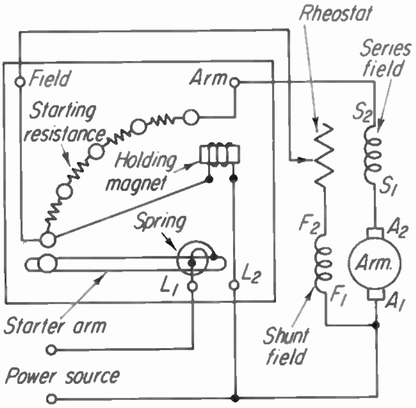


Fig. 13-8 A compound motor with a four-point starter and a field rheostat.

very limited use. The main advantage of the differential compound motor is that it can be designed to produce a slightly higher speed at full load than at no load.

13-7 The A-C Motor

Types. Alternating-current motors are made in a wide variety of types and are divided into a number of classifications. A broad general classification is in terms of their power requirements, namely, (1) polyphase and (2) single phase. They are further classified in terms of operating principles, such as (1) induction, (2) synchronous, (3) split phase, (4) shaded pole, (5) repulsion, (6) universal, (7) hysteresis, and (8) selsyn.

Principle of Operation. The basic principle of operation of the a-c motor requires (1) establishing a rotating magnetic field within the motor and (2) having the rotating magnetic field cut the conductors of its associated member so that a torque will be developed between the two members.

Producing a Rotating Magnetic Field. The principle of generating a rotating magnetic field can readily be shown for a two-phase or a three-phase motor. For simplicity of illustration, the two-phase rotating magnetic field will be used. Although practical a-c motors use slotted magnetic field structures (Fig. 13-9*a*) that produce nonsalient poles, for ease of illustration a salient-pole field structure (Fig. 13-9*b*) will be used.

When the currents of the two-phase power source represented by Fig. 13-10*a* flow through the windings of the two-phase motor represented by Fig. 13-10*b*, the rotating magnetic field shown in Fig. 13-11 will be produced. At the time of the cycle marked 1 in Fig. 13-10*a*, the current of phase *A* is at its maximum positive value and the current of phase *B* is zero. If at this time the direction of the electron flow is in at A_1 and out at A_2, the magnetic polarities as determined by the left-hand rule (Art. 5-14)

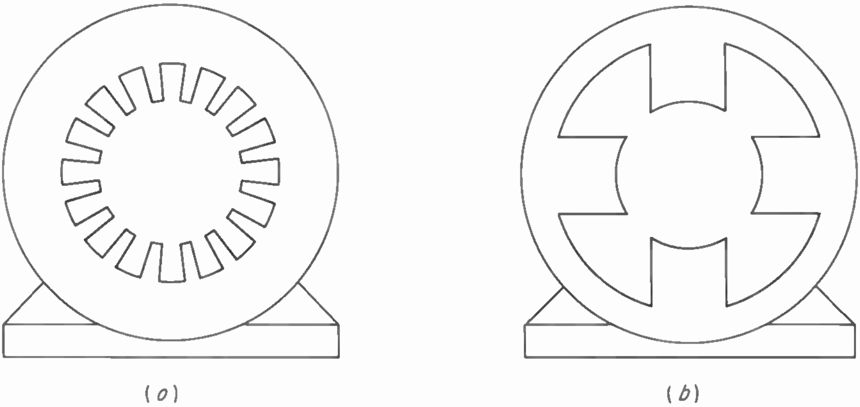


Fig. 13-9 Field structure of an a-c motor. (a) Slotted core used to produce nonsalient poles. (b) Salient pole field frame.

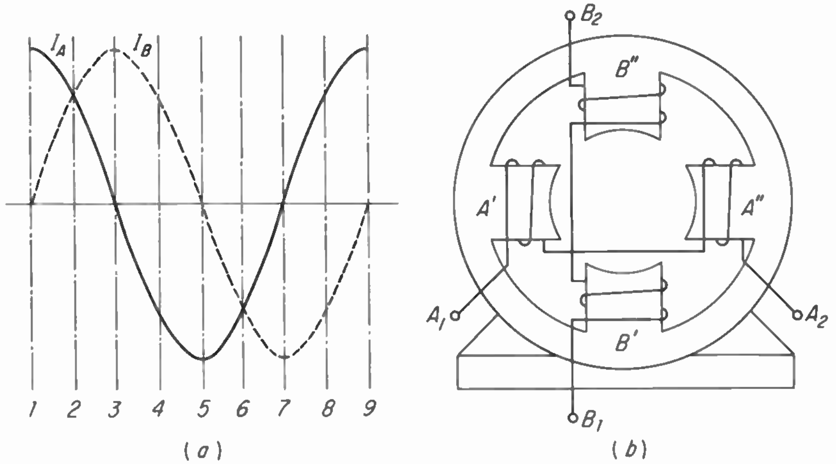


Fig. 13-10 Generating a rotating magnetic field in an a-c motor. (a) Currents of a two-phase power source. (b) Windings of a fundamental two-phase two-pole motor.

will be *N* for pole *A′* and *S* for pole *A″*. At this same instant of time, poles *B′* and *B″* are unmagnetized and the magnetic fields of the motor will be as shown at 1 of Fig. 13-11. At the time of the cycle marked 2, both current cycles have advanced 45° from time 1 and the currents I_A and I_B will each be at 70.7 per cent of their maximum values. With the current I_A entering the *A*-phase winding at A_1 and the current I_B entering the *B*-phase winding at B_1, the polarities of the magnetic poles will be as

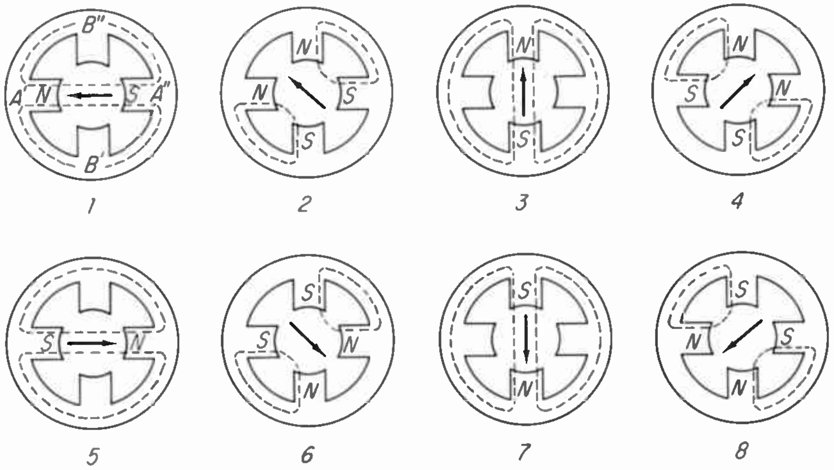


Fig. 13-11 Rotating magnetic field generated in the field structure of Fig. 13-10b when the electron flow enters at terminal A_1 for positive values of I_A and enters at terminal B_1 for positive values of I_B.

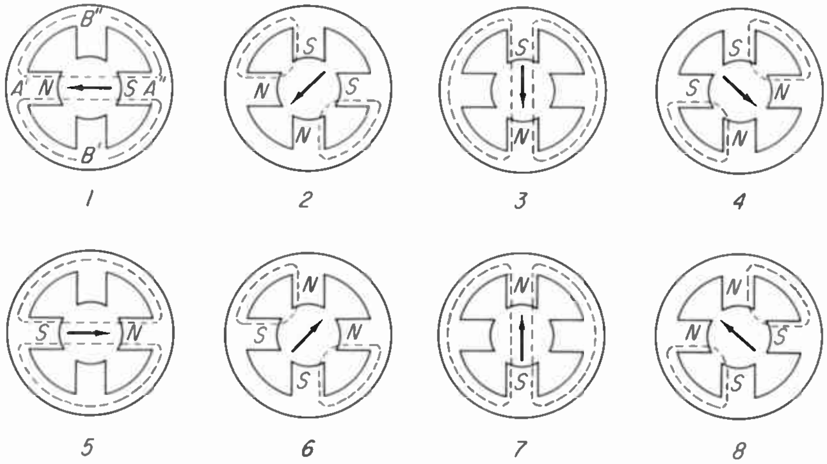


Fig. 13-12 Rotating magnetic field generated in the field structure of Fig. 13-10b when the electron flow enters at terminal A_1 for positive values of I_A and enters at terminal B_2 for positive values of I_B.

shown at 2 of Fig. 13-11. It should be observed that the position of the resultant magnetic field has advanced 45° in a clockwise direction during the change in the cycle from position 1 to position 2. At the time of the cycle marked 3, I_A will be zero and I_B will be at its maximum positive value. The resultant magnetic field, shown at 3 of Fig. 13-11, will have advanced another 45° in the clockwise direction. At the time of the cycle marked 4, it should be observed that I_A has reversed in polarity. With the electron flow now entering the A-phase winding at terminal A_2 and leaving at A_1, the magnetic polarities are N at pole A'' and S at pole A'; this is opposite to the magnetic polarities for positions 1 and 2. With the currents I_A and I_B each at 70.7 per cent of their maximum values, the resultant magnetic field, shown as 4 of Fig. 13-11, will have advanced another 45°. Continuing this analysis will show that for one complete cycle of 360 electrical degrees the magnetic field will complete one revolution of rotation.

Direction of Rotation of the Magnetic Field. If the power-source leads connected to terminals B_1 and B_2 are interchanged, the electron flow for the positive values of I_B will enter the winding at B_2 and leave at B_1. Repeating the step-by-step analysis of the magnetic field conditions will show that the magnetic field now rotates in the counterclockwise direction (Fig. 13-12). Consequently, the direction of rotation for a two-phase motor can be reversed by interchanging one pair of the line leads.

Speed of the Rotating Magnetic Field. The preceding explanation indicated that the magnetic field of a two-pole two-phase motor completes one revolution in the time of 1 cycle of the input power source. Thus the magnetic field of a two-pole two-phase motor operated from a 60-cycle power source will rotate at a speed of 60 × 60, or 3,600 rpm.

Figure 13-13 represents the field structure of a four-pole two-phase motor; for simplicity of drawing, only the A-phase windings are shown. At the time of the cycle marked 1 in Fig. 13-10*a*, the four poles of the A phase will be at their maximum strength and the four poles of the B phase will be at zero value. At the time of the cycle marked 3, the poles of the A phase will be at zero value and the strength of the poles of the B phase will be at their maximum values. Thus, it is shown that for an advance of 90 electrical degrees the magnetic field will advance only 45°. Continuing this analysis will show that for 360 electrical degrees the magnetic field will make only one-half of a revolution around the field structure. Thus, the magnetic field of a four-pole two-phase motor will rotate at one-half the speed of a two-pole motor, or 1,800 rpm. The speed of the magnetic field is therefore dependent on (1) the number of poles and (2) the frequency of the power source; it may be calculated by

$$S = \frac{120f}{P} \tag{13-11}$$

Action of the Rotor. The rotating member of the a-c motor is commonly called the *rotor.* It consists basically of a number of conductors placed in slots located on the outer periphery of an iron core which is mounted on the shaft of the motor. When a-c power is applied to the two-phase winding previously described, a rotating magnetic field is produced in the stationary member, which is commonly called the *stator.* When the rotor is at rest, the rotating magnetic field of the stator cuts the rotor conductors and induces an alternating voltage in these conductors. As the conductors are part of a low-resistance closed path, a high current will flow in these conductors and set up magnetic fields in the iron core of the rotor. The poles of the rotating field of the stator will exert a force on the magnetic poles of the rotor and thus pull the rotor around with the rotating magnetic field of the stator.

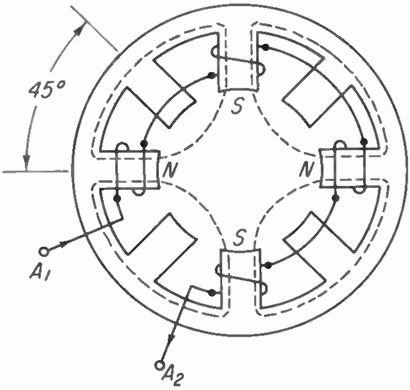


Fig. 13-13 Fundamental field structure for a four-pole two-phase motor; B-phase winding omitted for ease of reading the drawing.

Synchronous Speed. When the rotor is turning at the same rate of speed as the rotating magnetic field of the stator, the two rotating actions will be operating in synchronism; this speed is called the *synchronous speed.*

Rotor Slip. In the preceding explanation of the action of the rotor, the magnetic poles of the rotor were shown to be induced by the action of the stator field cutting the rotor conductors. If the rotor should reach the synchronous speed, there would no longer be any relative motion between the stator field and the rotor conductors. Consequently, the rotor current would cease and the rotor speed would decrease owing to the bearing and windage friction at the rotor. As soon as the rotor speed decreases enough for current to be induced again in the rotor conductors, the pulling action of the stator field would again become effective. The difference between the speed of the rotating field of the stator and the running speed of the rotor is called the *rotor slip* and is expressed as

$$\text{Per cent rotor slip} = \frac{S_s - S_r}{S_s} \times 100 \tag{13-12}$$

where S_s = speed of the stator field, rpm
S_r = speed of rotor rotation, rpm

EXAMPLE 13-10 A certain four-pole two-phase 60-cycle motor has a no-load speed of 1,795 rpm and a full-load speed of 1,710 rpm. Find (*a*) the synchronous speed, (*b*) the per cent rotor slip at no load, (*c*) the per cent rotor slip at full load.

GIVEN: $P = 4$ poles $f = 60$ cycles $S_{NL} = 1{,}795$ rpm $S_{FL} = 1{,}710$ rpm

FIND: (*a*) Synchronous speed (*b*) Per cent slip at *N-L* (*c*) Per cent slip at *F-L*

SOLUTION:

(*a*)
$$S_{\text{syn}} = \frac{120f}{P} = \frac{120 \times 60}{4} = 1{,}800 \text{ rpm}$$

(*b*)
$$\text{Per cent slip } N\text{-}L = \frac{S_s - S_r}{S_s} \times 100 = \frac{1{,}800 - 1{,}795}{1{,}800} \times 100 = 0.277$$

(*c*)
$$\text{Per cent slip } F\text{-}L = \frac{S_s - S_r}{S_s} \times 100 = \frac{1{,}800 - 1{,}710}{1{,}800} \times 100 = 5$$

13-8 Polyphase Induction Motors

Types. For integral-horsepower applications, three-phase induction motors are generally used. There are two types of rotor construction used for induction motors, and each serves a particular type of application. The motors using the two types of rotors are called (1) the squirrel-cage induction motor and (2) the wound-rotor induction motor.

The Squirrel-cage Induction Motor. The basic electrical components of the squirrel-cage induction motor are (1) the stator and (2) the rotor. The stator frame contains a core made of electrical-sheet-steel laminations, each having a large number of slots (Fig. 13-9*a*). Coils of copper wire are placed

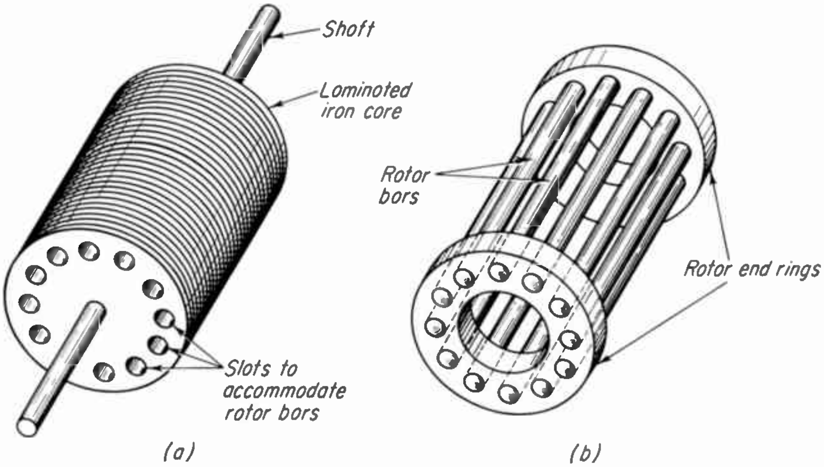


Fig. 13-14 Parts of a squirrel-cage rotor.

in these slots and then interconnected to form the stator winding. When the coils are connected in the proper manner, the stator winding can be made to produce a rotating magnetic field having the desired number of poles and phases. The rotor core is also made of electrical-sheet-steel laminations and contains a large number of slots (Fig. 13-14*a*). The rotor winding consists of a number of relatively large rods or bars of conducting material, such as copper or brass, and a short-circuiting ring at each end (Fig. 13-14*b*). The motor gets its name from the fact that the rotor winding resembles a rotary squirrel cage.

When the rotor bars and end rings are made of copper, the rotor resistance is quite low. A motor using this type of rotor has good general operating characteristics, such as (1) low rotor slip at full load and hence excellent speed regulation, (2) high efficiency, and (3) moderate starting torque and hence the ability to start light loads from rest.

When higher starting torque is required in order to start heavier loads from rest, a high-resistance rotor is used. The higher resistance for the rotor can be obtained by using brass or some other high-resistance material for the rotor bars and end rings. A motor using a high-resistance rotor will produce a higher starting torque but also has a higher slip and lower efficiency than a similarly rated motor with a low-resistance rotor.

Double Squirrel-cage Induction Motor. In order to obtain a high starting torque without too great an increase in slip at full load and also with good efficiency, a rotor having two squirrel-cage windings is used. This is accomplished by placing two windings in one slot or sometimes in separate slots. The two windings are referred to as the outer and inner windings. One winding is of a low-resistance material, and the other is of a high-resistance material. The high-resistance winding is most effective during the starting period and produces good starting torque. The low-resistance winding is most effective at the full running condition and produces characteristics similar to the low-resistance single-squirrel-cage motor.

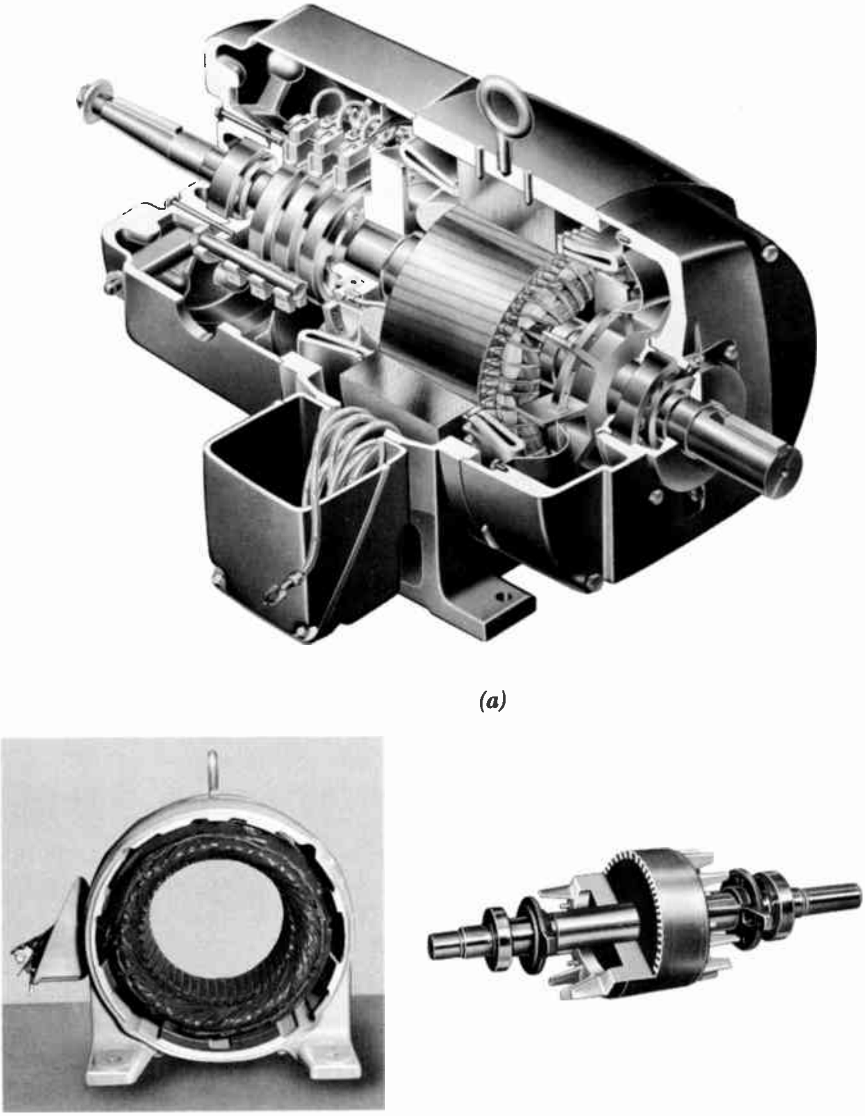
Wound-rotor Induction Motor. The squirrel-cage rotor winding can be replaced by a winding consisting of multiturn coils placed in the rotor slots. These coils are interconnected to form a three-phase winding with its leads connected to three slip rings. The rotor winding can then be connected to an external circuit through the slip rings and brushes. Higher starting torque can be obtained with a wound-rotor induction motor by connecting external resistance into the rotor circuit. Another use of the wound-rotor induction motor is for speed control. When external resistance is connected into the rotor circuit, the motor will operate at reduced speed. When the motor speed is controlled by the addition of various amounts of external resistance, the speed regulation of the motor increases considerably; this is accompanied by a reduction in the operating efficiency.

Multispeed Motors. Polyphase induction motors can be made to operate at two speeds by making special provisions in the stator winding. By bringing out the proper additional terminal leads from a single stator winding, it is possible to connect these leads in two different ways and thereby produce connections for two different numbers of poles; this will provide dual speed operation. The number of poles will be a multiple of 2, such as 4- and 8-pole operation, 6- and 12-pole operation, etc. For other combinations of numbers of poles, two separate windings are placed in the stator frame.

Polyphase Synchronous Motor. With a synchronous motor, the speed at all normal operating load conditions is the same as the speed of the rotating magnetic field. This is achieved by using a rotor having salient field poles that are provided with coils of wire which are energized from a d-c power source (Fig. 7-24). The rotor acts like a number of bar magnets which, when brought up to full speed, will be pulled around by the rotating magnetic field of the stator. Because the basic synchronous motor is not self-starting, a squirrel-cage winding is generally built into the surface of the rotor field poles. The squirrel-cage winding enables the motor to become self-starting.

Advantages of the polyphase synchronous motor are (1) constant-speed operation, (2) the power factor of the motor can be varied by changing the d-c field excitation, (3) the synchronous motor can be operated with leading current and thereby compensate for low overall power-factor conditions due to other loads. Disadvantages of the synchronous motor are (1) very low starting torque, (2) requires a source of d-c power, (3) available only in moderate and large sizes, (4) may have a tendency to drop out of synchronism and then stall under conditions of sharp load variations.

The general use of synchronous motors is for driving d-c generators, operating blowers and compressors, and such other applications requiring



(a)

(b) (c)

Fig. 13-15 Polyphase induction motors. (a) Cutaway view of a wound-rotor motor. (b) Wound stator core and frame of a 10-hp motor. (c) Cutaway view of a squirrel-cage rotor. *(General Electric Company)*

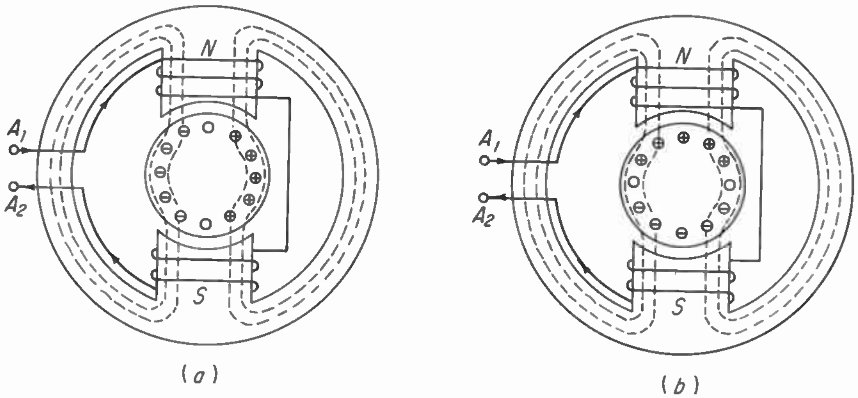


Fig. 13-16 Fundamental single-phase induction motor illustrating (a) stator field and rotor currents when the rotor is at rest, (b) stator field and rotor currents when the motor is in motion.

very little starting torque. When a large portion of the motor current is used to provide leading current for power-factor correction purposes, the synchronous motor is sometimes called a *synchronous capacitor.*

High-frequency Motors. When high motor speeds are desirable or are required, they can be obtained by operating the motors from a high-frequency power source. Some military and aerospace applications use a 400-cycle power system which permits the design and application of very high speed motors. At the higher speeds it becomes possible to achieve a reduction in the size and weight of a motor for a particular application.

Motor Controls. All medium- and large-size motors require the use of a starting and/or speed-control device; space limitations do not permit elaboration on control equipment. Basically, controllers are used to limit the starting currents to safe amounts either by adding resistance to the circuit or by reducing the input voltage by means of some type of transformer action.

13-9 Single-phase Motors

Types. Fractional-horsepower a-c motors are usually operated from single-phase power systems. Single-phase a-c motors may be divided into two classes: (1) induction motors and (2) series motors. A variety of types of single-phase motors is available, such as (1) split-phase induction, (2) capacitor induction, (3) shaded-pole induction, (4) series, (5) repulsion, (6) repulsion-induction, (7) universal series, (8) hysteresis or synchronous.

13-10 Single-phase Induction Motors

Principle of Operation. The single-phase induction motor uses the same type of stator frame and the same type of rotor as the polyphase squirrel-cage induction motor. For ease of illustration and explanation, a salient-pole type of field structure is used (Fig. 13-16). When terminals A_1 and A_2 are connected to an a-c power source, the magnetic fields at the poles of the stator will vary in strength in a sine-wave manner and will reverse in polarity every 180° of the input cycle. The magnetic field produced in the stator is therefore a pulsating field that reverses polarity at fixed intervals; it is not a rotating field as is produced in the polyphase motor. At standstill, the directions of the currents in the rotor conductors, caused by the expansion and contraction of the stator field, are illustrated in Fig. 13-16*a*. Under this condition, two equal but opposite torques are developed at the rotor, and consequently no rotor movement is produced. However, if by means of some externally applied force the rotor is started turning, the emf produced by the rotor conductors cutting the magnetic field will set up rotor currents in the directions shown in Fig. 13-16*b*. Because the rotor circuit is highly inductive, the rotor current lags the induced emf by almost 90° and sets up a pulsating flux in the rotor which is nearly 90° out of phase with the stator flux. The conditions produced thus simulate the actions of the two-phase rotating magnetic field. It should be apparent that the basic single-phase induction motor is not self-starting, but if it is brought up to speed by some other means, it will keep running. Several methods are used to make the single-phase induction motor self-starting.

Compared with polyphase motors of the same rating, single-phase induction motors are larger in size, have about the same speed-load characteristics, and have lower values of power factor and efficiency.

Split-phase Induction Motor. One method of making the single-phase induction motor self-starting is to put two windings on the stator core and place them 90 electrical degrees apart. A squirrel-cage rotor is used, and a switch that is actuated by centrifugal force is also provided (Fig. 13-17). The *running winding*, placed on poles 1 and 3, has a low resistance and a high inductance. The *starting winding*, placed on poles 2 and 4, has a high resistance and a low inductance. The relative values of resistance and inductance of the two windings cause their currents to differ in phase by an angle somewhat less than 90°. The result is that a rotating magnetic

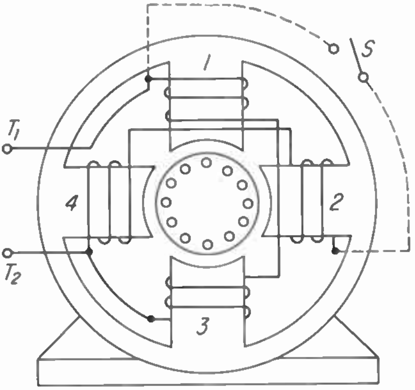


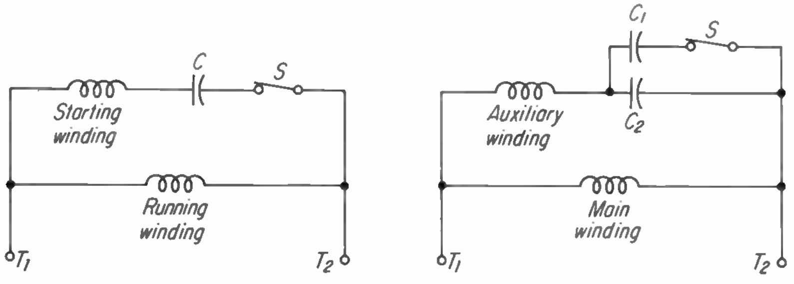
Fig. 13-17 Split-phase induction motor. Switch S and its connecting wires, shown in broken lines, are placed inside the motor. Switch S is closed when the motor is at rest and opens by centrifugal action when the motor is running.

field is set up in the stator core and the motor becomes self-starting. When the rotor is at rest, switch S is closed and both the running and starting windings will be connected to the power source. When the rotor speed reaches approximately 75 per cent of the synchronous speed, the switch S is opened by the action of centrifugal force and the starting winding is disconnected from the power source.

The direction of rotation may be changed by reversing the direction of current flow in either one of the two stator windings. If the motor has only two external leads, it will be necessary to disassemble the motor and make the change internally.

Capacitor-start Induction Motor. This motor is another form of the split-phase induction motor; its construction is quite similar to the motor described in the preceding paragraph. With a capacitor of suitable rating connected in the starting winding circuit (Fig. 13-18*a*), the current in this circuit is made to lead the voltage by about 45°. If the current in the running winding lags the voltage by about 45°, the resultant phase difference between the two currents will be approximately 90° and the motor will have a rotating magnetic field similar to that of a two-phase motor. The function of the centrifugal switch S is to connect the starting winding to the power source during the starting period and to disconnect this winding automatically from the power source at a predetermined speed. The direction of rotation may be changed by reversing the direction of current flow in either one of the two stator windings. If the motor has only two external leads, it will be necessary to disassemble the motor and make the change internally.

Capacitor-start Capacitor-run Induction Motor. It is sometimes desirable to keep the auxiliary or starting winding permanently connected in the circuit. One arrangement uses two capacitors connected in parallel for the starting condition (Fig. 13-18*b*) in order to provide high values of capacitance and torque for the starting interval. At about 75 per cent of synchro-



(*a*) (*b*)

Fig. 13-18 Capacitor motors. (a) Capacitor-start motor. (b) Capacitor-start capacitor-run motor. Switch S is closed when the motor is at rest and opens by centrifugal action when the motor is running.

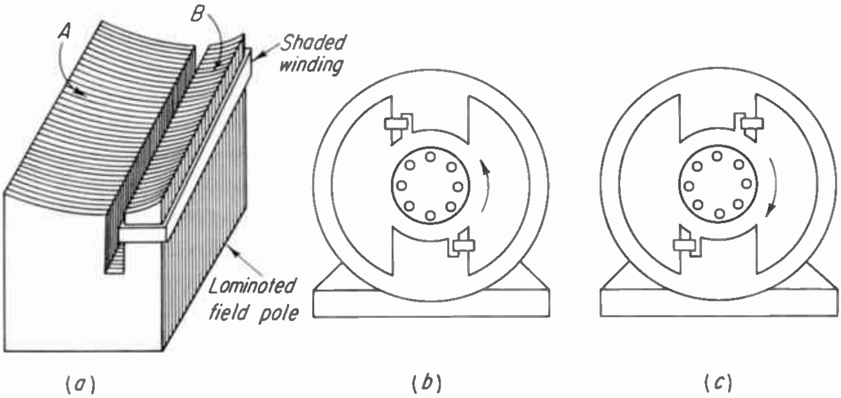


Fig. 13-19 Shaded-pole induction motor. (a) Field pole and shaded-pole winding. (b) and (c) Effect of the location of the shaded-pole on the direction of rotation.

nous speed, the centrifugal switch S disconnects one capacitor and permits the motor to operate continuously with the remaining capacitor and the auxiliary winding. Several other switching arrangements can also be used. These motors can provide higher starting torque and better full-running power factor and efficiency characteristics. The method of reversing the direction of rotation is the same as for the capacitor-start induction motor.

Shaded-pole Induction Motor. The shaded-pole motor may use salient field poles whose surfaces have been modified to accommodate the starting winding (Fig. 13-19*a*), or it may use a slotted iron core and place the shaded winding in appropriate slots. The shaded winding frequently consists of only a single turn of large-size copper wire which forms a closed or short circuit (Fig. 13-19). When the magnetic field due to the main field winding is increasing in strength, some of its magnetic lines cut the shaded winding and induce a voltage therein. The current caused by this voltage sets up its own magnetic field which opposes the main field, increasing the magnetic strength of the pole area *A* and decreasing the magnetic strength of the pole area *B*. When the main magnetic field reaches its maximum value, magnetic lines no longer cut the shaded winding and the induced voltage, current, and magnetic field of the shaded coil are zero. The main magnetic field is then evenly distributed over the pole surface areas *A* and *B*. When the main magnetic field is decreasing, the decrease in the induced voltage, current, and magnetic field is delayed so that the magnetic field is stronger in pole face area *B* and weaker in pole face area *A*. These actions indicate that the magnetic pole moves across the face of the field pole. This moving magnetic field causes reactions in the rotor that produce enough torque to make the motor become self-starting. Shaded-pole motors have low starting torque and are used chiefly for small motor applications.

Most shaded-pole motors have no provision for reversing the direction of rotation by electrical means. Reversal of the direction of rotation is then accomplished in the following manner: (1) disassemble the motor, (2) turn the stator around end for end, and (3) reassemble the motor. There are several methods available for making this type of motor reversible electrically. As the construction procedures of each method result in higher motor cost, the electrically reversible type of motor is not commonly used.

13-11 Series-type A-C Motors

Principle of Operation. In the study of the d-c shunt and series motors, it was observed that reversing the line leads of the motor did not affect the direction of rotation. Thus, it should be possible to operate a shunt or series motor from a single-phase a-c power source. The high inductance and the resultant high reactance of the field winding of a shunt motor make the use of this type of motor impractical for a-c operation; it is used only rarely for very small-size motors. The series motor is more adaptable to a-c operation.

The armature of the series a-c motor is similar to the armature of the d-c motor. A laminated iron core is used for the field structure (Fig. 13-21) in place of the solid field poles of the small d-c motor. The laminated core is necessary in order to keep the eddy-current losses in the iron core at a reasonable value.

The Series A-C Motor. The characteristics of the series a-c motor are similar to those of the d-c series motor, including the hazard of overspeed for no-load condition. These motors are made mostly in fractional-horsepower sizes and are usually an integral part of the appliance they are to drive. Common applications are drills, vacuum cleaners, food mixers, etc. The direction of rotation can be changed by reversing the direction of current flow in either the field or the armature winding.

The Universal Motor. A motor which can be operated from either an a-c or a d-c power source is called a *universal motor.* The a-c series motor is a modification of the d-c series motor and can be operated from either a-c or

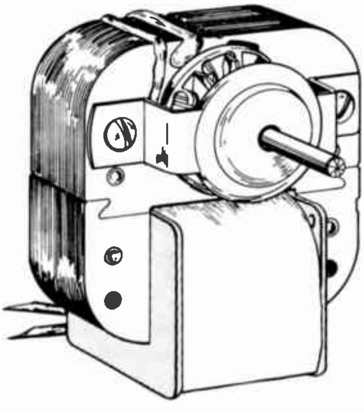


Fig. 13-20 Shaded-pole single-phase induction motor.

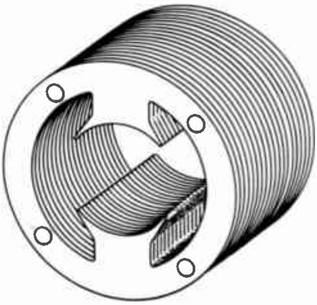


Fig. 13-21 Laminated field structure of universal and a-c series types of motors.

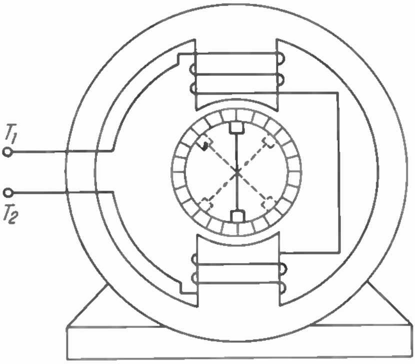


Fig. 13-22 Principle of the repulsion motor. Brushes are in an electrical neutral position when in a vertical plane. Rotation is produced by shifting the brushes away from the vertical plane.

d-c power sources. Although it might be necessary to make minor design variations for d-c, a-c, and universal motors, the principle of operation is the same for each. Universal motors are limited in use to fractional-horsepower sizes. The direction of rotation of the universal motor can be changed in the same manner as for the series motor. If changing the direction of rotation of a universal motor results in severe sparking at the commutator, it might be corrected by shifting the position of the brushes.

13-12 Repulsion-type Motors

The Repulsion Motor. The repulsion motor has an armature and a field structure similar to the series motor. It differs from the series motor in that (1) the power is applied directly to the field windings, (2) the motor brushes are connected together to short-circuit the armature, and (3) some mechanical arrangement is made so that the position of the brushes may be altered with respect to the electrical neutral. When the brushes are set under the centers of the two poles of the stator winding, they are in a neutral position (Fig. 13-22). With the brushes in this neutral plane and power applied to the stator winding, the currents induced in the armature will produce two equal but opposite amounts of torque and no rotation will occur. If the brushes are shifted a small amount in a clockwise direction, torque will be developed by the armature and the motor will run in a clockwise direction. Shifting the brushes counterclockwise from their neutral position will produce counterclockwise rotation. This type of motor produces a high starting torque and has a wide range of speed variation with changes in load conditions. In some repulsion motors, an additional stator winding, called a *compensating winding*, is used to improve the speed regulation and the power factor of the motor.

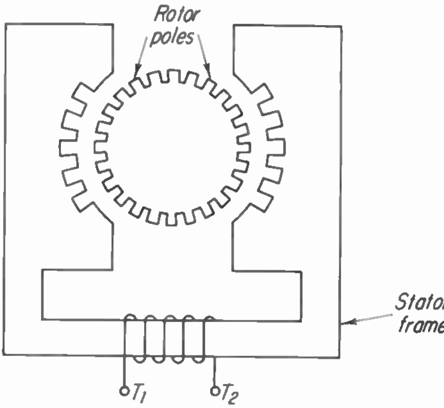


Fig. 13-23 Fractional-horsepower 24-pole synchronous motor.

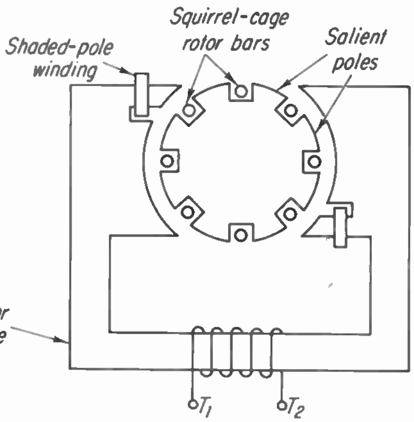


Fig. 13-24 Shaded-pole eight-pole fractional-horsepower synchronous motor.

Repulsion-start Induction-run Motor. This motor is similar to the repulsion motor except for a mechanism that is operated by centrifugal force. This mechanism short-circuits all the commutator bars at a predetermined speed, usually about 75 per cent of the synchronous speed. The motor starts from standstill in the same manner as the repulsion motor. At about 75 per cent of the synchronous speed, the centrifugal mechanism short-circuits all the armature coils and the motor operates as a single-phase induction motor with a squirrel-cage rotor. In some motor designs, an auxiliary centrifugal mechanism is used to raise the brushes from the commutator while the armature is operating as a squirrel-cage rotor. The method of reversing the direction of rotation is the same as for the repulsion motor.

Repulsion-induction Motor. The rotating member of the repulsion-induction motor has two separate windings: (1) an armature winding and (2) a squirrel-cage rotor winding. This motor has the starting characteristics of a repulsion motor and the full-running characteristics of an induction motor. The method of reversing the direction of rotation is the same as for the repulsion motor. Advantages of the repulsion-induction motor are (1) high starting torque, (2) good speed regulation, and (3) elimination of the centrifugal mechanism used with the repulsion-start induction-run motor.

13-13 Small Synchronous Motors

Fractional-horsepower synchronous motors can be made to operate without d-c excitation for their rotor as is required with integral-horsepower synchronous motors. Because some small synchronous motors utilize the magnetic property of residual magnetism in their principle of operation, they are also referred to as *hysteresis motors.*

One type of small synchronous motor is shown in Fig. 13-23. The rotor has 24 poles and will run at 300 rpm when operated on a 60-cycle power source. The stator core appears to be bipolar, but it has a number of salient poles of the same size as the rotor poles cut into the bipolar pole faces. This motor is not self-starting, but if it is brought up to speed by an externally applied rotational force, the rotor poles will be attracted to the stator poles and lock in step with the pulsations at the stator poles.

The shaded-pole principle can be applied to the small synchronous motor (Fig. 13-24) to achieve the self-starting feature. The shaded-pole construction of the stator field introduces motion to the magnetic field. The rotor has a squirrel-cage winding and also has salient poles. When power is applied, a rotating magnetic field cuts the conductors of the squirrel-cage winding and starts the rotor turning. The salient rotor poles become polarized by the stator field and lock in step with that field. The motor will run at the synchronous speed determined by the number of rotor poles and the stator input frequency. The eight-pole motor (Fig. 13-24) will run at 900 rpm on a 60-cycle power source.

The Telechron (General Electric trade name) motor is a hysteresis-type motor which uses the principle of residual magnetism to make the motor self-starting and to operate at synchronous speed. This motor has a bipolar field excited by a single coil, and each pole is split to accommodate one or more copper shading poles. The rotor consists of a number of thin, hardened, magnet-steel, ring-shaped elements that have high residual magnetism characteristics. When power is applied to the coil of the electromagnet, the rotating magnetic field of the stator induces poles in the rotor rings. Because of the residual magnetism characteristics of the rotor rings, the polarity of the rotor poles does not change instantly with the change in polarity of the stator field. Because the rotor poles persist locally after the inducing magnetism has advanced to the next alternation of the power

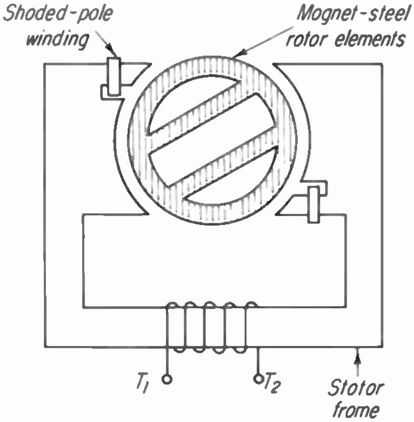


Fig. 13-25 Telechron (General Electric registered trade name) synchronous timing motor.

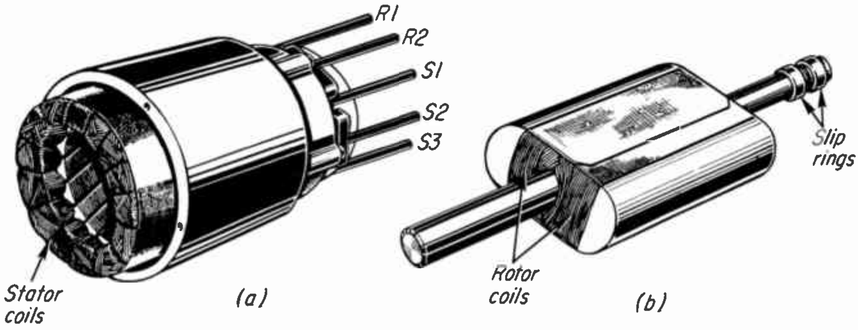


Fig. 13-26 Parts of the simple synchro. (a) The stator. (b) The rotor.

input cycle, the rotor poles tend to follow the stator field. The new polarity of the stator field eventually overcomes these local poles in the rotor rings and sets up new ones. The rotor rings, in the meantime, have advanced in the direction of the new poles. A high starting torque is maintained until synchronous speed is attained. A train of gears is generally used in conjunction with the motor to reduce the speed to the desired value.

13-14 Synchros or Selsyns

Purpose and Principle. A synchro is an electrical device used to transmit rotational information in the form of electrical signals. The most common type of synchro unit is a small rotating motor that has a three-phase winding similar to an induction motor and a salient-pole rotor (Fig. 13-26). It differs from a synchronous motor in that the rotor winding is excited by alternating current instead of direct current. Synchros are not used as motors and are therefore rated in torque rather than horsepower. They are also known by other names, such as *selsyn*, *autosyn*, and *synchrotie;* the name selsyn is derived from *self-synchronous*. Synchros are used in combinations of two or more similar units as data-transmission systems to perform such functions as (1) remote position indicator, (2) remote signaling, and (3) remote control.

Simple Synchro System. The operation of a simple synchro system is illustrated by Figs. 13-27 and 13-28. The shaft of one unit is connected to the device whose position is to be indicated; this unit is usually called the *generator* or the *transmitter*. The function of the second unit is to indicate, at some remote location, the position of the device under consideration; this unit is usually called the *motor* or the *receiver*. When the generator and the motor are both at the zero-degree position (Fig. 13-27), the voltages induced in each stator winding are alike and no current will flow between the two units. Also, the torque at each rotor is zero. If the object to which the synchro generator is attached is rotated through 30° and the synchro motor unit is forcibly held in its zero-degree position, unbalanced

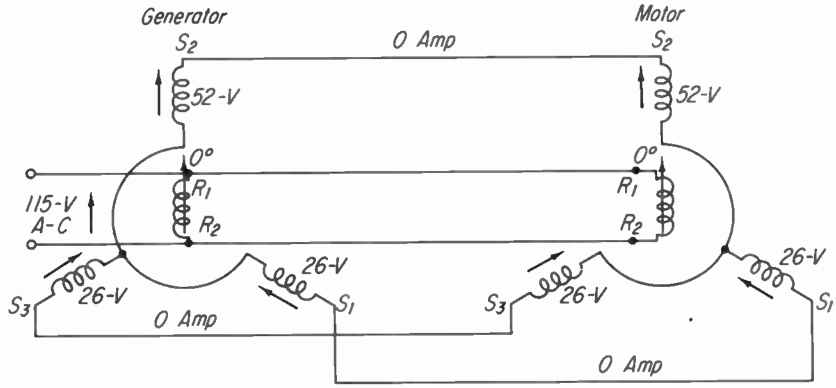


Fig. 13-27 Voltage and current distribution in a simple synchro system when the generator and the motor are at 0° positions.

voltage and current distributions will occur as shown in Fig. 13-28. The result is that a torque will be developed at the rotor of each unit. If the assumed restraining force at the synchro motor unit is released, its rotor will move through 30°, at which point the voltages of the units will again be in balance and the currents and torques will again become zero.

Synchros as Part of Servomechanisms. The common a-c synchro described here is capable of efficiently transmitting only small amounts of torque, thereby restricting its use largely to indicating information. When larger amounts of torque are involved, such as required for controlling remote apparatus, amplifying units must be added, and the complete system is then called a *servomechanism* or *servo system*. A complete study of synchros and servo systems covers a broad specialized field and is beyond the scope of this text.

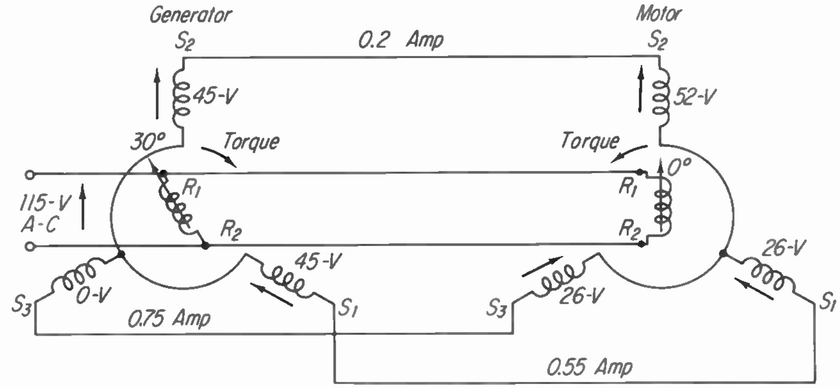


Fig. 13-28 Voltage and current distribution in a simple synchro system when the generator is in the 30° position and the motor is in the 0° position.

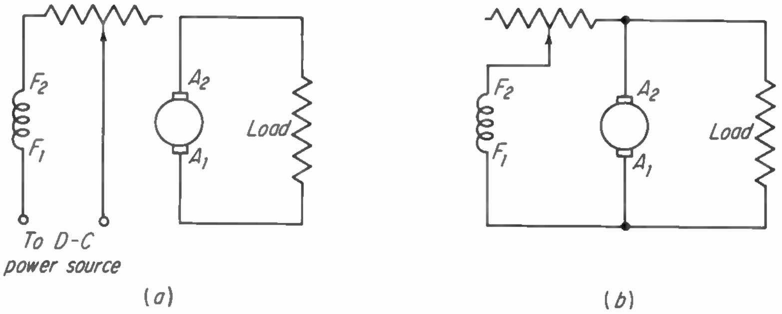


Fig. 13-29 Schematic diagrams of basic d-c shunt generators. (a) Separately excited, (b) self-excited.

13-15 D-C Generators

Construction. The construction of the d-c generator is the same as that of the d-c motor and has been described in Arts. 7-12, 7-13, and 13-2.

Generator Ratings. The basic ratings of a d-c generator are (1) the output in watts or kilowatts, which may range from a few watts to thousands of kilowatts; (2) output or terminal voltage, which may range from 1.5 to 1,500 volts with commonly used values in the order of 6, 12, 28, 115, 230, and 440 volts; (3) amperes that the generator can safely deliver—this is dependent upon its kilowatt and voltage ratings; (4) speed at which it is to be driven; (5) duty cycle—this is similar to the corresponding d-c motor rating; (6) temperature rating—this is similar to the corresponding d-c motor rating.

Classifications. Direct-current generators are classified in two groups according to (1) the manner in which the magnetic field is energized and (2) the type of field winding and the manner of connecting the field windings. When the energy for the field circuit is obtained from a power source other than the generator itself, the generator is called *separately excited*. When the energy for the field circuit is obtained from its own armature the generator is called *self-excited* (Fig. 13-29). Self-excited generators are further classified as (1) shunt, (2) series, or (3) compound. Compound generators may be further classified as (1) cumulative, (2) differential, (3) long shunt, and (4) short shunt.

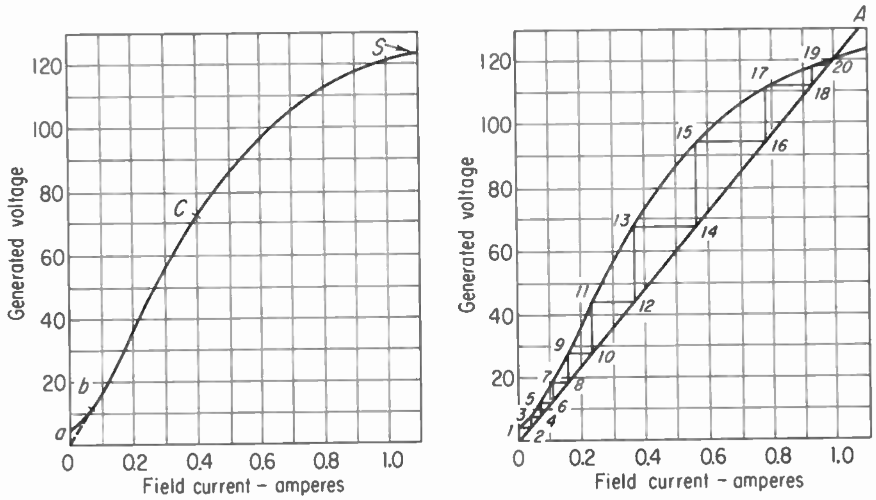
Principle of Operation. The basic principle of operation of the d-c generator has been described in Arts. 7-2, 7-3, and 7-12.

13-16 No-load Characteristics of the Generator

Voltage Buildup of the Self-excited Generator. When the frame and the field poles of a generator have some residual magnetism, which is generally true, the field windings may be excited from the armature. The small amount of voltage generated by the rotation of the armature conductors through the magnetic field due to residual magnetism is applied to the field coils and causes a small amount of current to flow. If the current flow is in the proper direction, it will increase the strength of the magnetic field in the field structure and thereby increase the generated voltage. The field current is then further increased, and this process is repeated until a stable point of operation is reached.

The Saturation Curve. If a rheostat is connected in the field circuit of the generator and the resistance of the field circuit is progressively decreased in suitable amounts, a useful series of armature voltage readings can be obtained. Plotting the armature voltage against the field current produces a ***saturation curve*** (Fig. 13-30*a*). According to Eq. (13-6), if the speed is kept constant, the generated voltage should vary directly with the field current and thus produce a straight line for Fig. 13-30*a*. Because of the residual magnetism, the curve starts at point *a* instead of at 0. Also, when the field current reaches an amount corresponding to point *C*, the strength of the magnetic field is such that a further increase in field strength causes a disproportionate change in the reluctance of the magnetic circuit and the generated voltage no longer follows a straight-line variation. Point S is called the ***saturation point,*** as beyond this point a large increase in field current is required to produce any appreciable increase in the generated voltage.

Voltage Buildup of the Self-excited Generator. The manner in which the voltage of a self-excited generator builds up is illustrated by Fig. 13-30*b*, which includes a field-resistance line in addition to the saturation curve. For the generator being considered, the resistance of the field winding is



(*a*) (*b*)

Fig. 13-30 (a) Saturation curve of a d-c generator. (b) Illustration of the manner in which the voltage of a self-excited d-c generator builds up.

120 ohms and is assumed to remain constant. For a constant resistance value, the voltage versus current characteristics will produce a straight line and two points will be sufficient to construct the line. By Ohm's law, the current at zero volts will be zero amperes and at 120 volts the current will be 1 amp. The resulting line 0-*A* is called the *field resistance line.*

When the generator is operating at rated speed, the residual voltage, indicated at point 1, is 4.8 volts. When this voltage is applied to the field winding, it will cause 0.04 amp to flow as is indicated at point 2. The saturation curve shows that at 0.04 amp the generated voltage will rise to 8 volts. With 8 volts applied to the field winding, the current will rise to 0.067 amp, which in turn causes the generated voltage to rise to 12.5 volts. This sequence of actions continues until the point is reached where the field resistance line and the saturation curve intersect, namely, at 120 volts and 1 amp.

Effect of Increasing the Resistance of the Field Circuit. If a rheostat is added to the field circuit and is set so that it adds 30 ohms to the resistance of the circuit, the field resistance line for 120 + 30 or 150 ohms (Fig. 13-31) shows that the generator voltage can no longer rise to 120 volts and will now reach only 104 volts. If the rheostat is set to add 120 ohms to the circuit, the resultant 240-ohm line shows that the final generated voltage cannot exceed 8 volts. From these illustrations, it can now be concluded that the generated voltage ceases to build up when the field resistance line is either to the left of or above the saturation curve. A field resistance line that is tangent to the straight portion of the saturation curve indicates the maximum field circuit resistance that will still permit the generator to build up voltage; this resistance value is called the *critical field resistance.*

EXAMPLE 13-11 At what resistance value should the rheostat be set in order to have the generator of Fig. 13-30 produce 100 volts?

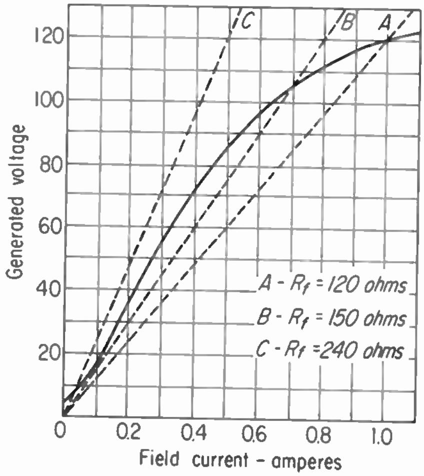


Fig. 13-31 Effect of increasing the resistance of the field circuit.

GIVEN: $E = 100$ volts Fig. 13-30

FIND: R_{rheo}

SOLUTION: From Fig. 13-30, when $E = 100$ volts, $I = 0.63$ amp.

$$R_{\text{rheo}} = R_{fc} - R_f = \frac{100}{0.63} - 120 = 159 - 120 = 39 \text{ ohms}$$

EXAMPLE 13-12 (*a*) What is the critical field resistance of the generator of Fig. 13-30? (*b*) What value of rheostat resistance will produce this condition?

GIVEN: Fig. 13-30

FIND: (*a*) Critical resistance (*b*) Rheostat resistance

SOLUTION: From Fig. 13-30, the tangent line ends at approximately 60 volts and the current at 60 volts is 0.33 amp.

(*a*) $$\text{Critical resistance} = \frac{60}{0.33} = 182 \text{ ohms}$$

(*b*) $$\text{Rheostat resistance} = 182 - 120 = 62 \text{ ohms}$$

13-17 Load Characteristics of the Generator

Effect of Load on the Output Voltage. When load is applied to a self-excited d-c generator, the output or terminal voltage will decrease owing to three effects: (1) voltage drop due to resistance, (2) voltage drop due to a decrease in the field current, (3) armature reaction.

Voltage Drop Due to Resistance. The load current flowing through the generator causes a voltage drop due to the resistance of the armature, the brushes, and the brush contacts with the commutator. In some types of generators there may be additional voltage drops due to the resistances of (1) a series field winding, (2) a commutating field winding, and/or (3) a compensating winding.

Voltage Drop Caused by a Decrease in Field Current. The decrease in the output voltage with the addition of load results in a reduction of the voltage applied to the field winding. The corresponding reduction in the field current causes an additional reduction in the generator output voltage.

Voltage Drop Caused by Armature Reaction. When load current flows through the armature conductors, it sets up magnetic fields around the conductors. The interaction of the resultant fields around the armature conductors with the main field causes a reduction in the strength of the main field and consequently results in a decrease in the output voltage of the generator.

Armature Reaction. The cause of armature reaction is explained by Fig. 13-32. Part *a* shows the magnetic field produced by the current-carrying turns of wire wound around the field poles *N* and *S*. When the conductors *A* and *B* of an armature coil are rotated through the magnetic field, a voltage will be induced in these conductors. Applying Fleming's right-hand rule (Art. 7-2) will show that the voltage induced is positive at *A* and nega-

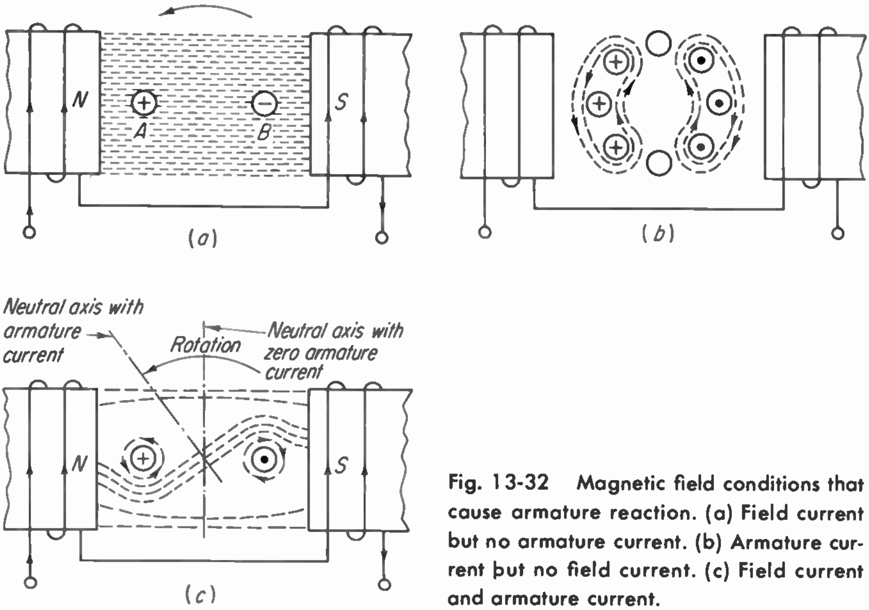


Fig. 13-32 Magnetic field conditions that cause armature reaction. (a) Field current but no armature current. (b) Armature current but no field current. (c) Field current and armature current.

tive at *B*. If the generator armature is not connected to an external load, no current will flow in the armature conductors. When a load is connected to the generator, electrons leave conductor *B*, flow through the external load, and return to the generator at conductor *A*. The current flowing in the armature conductors sets up magnetic fields around these conductors. Applying the left-hand rule for a wire carrying a current (Art. 5-13) and extending this to include a number of adjacent conductors will show the magnetic field to be as indicated in Fig. 13-32*b*. For convenience of illustration, the magnetic field of poles *N* and *S* is omitted. Figure 13-32*c* illustrates the effect produced by the interaction between the two magnetic fields shown separately in parts *a* and *b* of the figure.

Figure 13-32*c* shows that, in addition to weakening the main field, the field produced by the current flowing in the armature conductors also distorts the main field and causes a shift in the position of the neutral axis. These two effects are generally called *armature reaction.*

Commutation. At the portion of a revolution of rotation when an armature coil is virtually moving parallel to the magnetic field, the voltage induced in the coil is zero. At this instant of time, the commutator segments to which the coil is connected should be passing under the brushes. The coil and the brushes are then in the *neutral position*, and the coil is undergoing *commutation.* When a coil is undergoing commutation, it is also being short-circuited by the brushes. If the coil voltage is not zero during commutation, current will flow and cause a spark to occur as the commutator segments of that coil cease making contact with the brushes.

Figure 13-32*c* indicates that for perfect commutation the position of the brushes should be different for no-load and for full-load conditions and that the brushes must be shifted in the direction of rotation of the armature. Furthermore, intermediate load conditions will require various other brush positions. To correct such an awkward condition, compensating windings and/or commutating windings are used in medium- and large-size generators (and motors). Compensating windings are embedded in slots provided in the surface of the pole faces. They are connected in series with the armature, and the current is made to flow in such a direction that their magnetic fields counteract the armature field. Compensating windings are generally used only with large-size generators.

Commutating windings are wound on small poles mounted between the main field poles; these poles are called *interpoles.* The coils placed on these poles are connected in series with the armature, and the current is made to flow in such a direction that the magnetic fields produced counteract the field of the armature coils. Interpoles are used with medium- and large-size generators.

Armature Reaction in D-C Motors. Armature reaction also occurs in d-c motors, where it may cause (1) instability of speed with load variations and (2) sparking at the brushes if they are not in the proper position. When load is applied to a motor, the neutral position shifts in the opposite direction to that of a generator, and consequently, in order to counteract the effects of armature reaction with increased loads, the brushes of a motor must be shifted opposite to the direction of armature rotation. Compensating windings and interpoles are also used with d-c motors to provide automatic compensation for varying load conditions.

13-18 Types of Generators

Shunt Generator. A self-excited shunt generator has its field windings connected in parallel with, or *shunted* across, the armature terminals

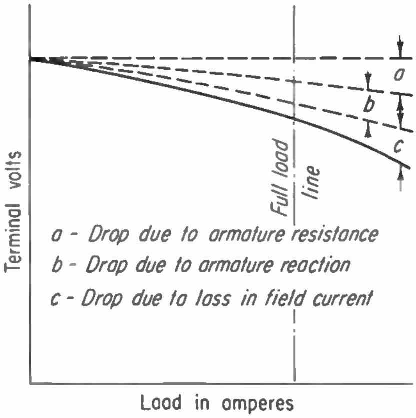


Fig. 13-33 Typical voltage versus load characteristics of a self-excited shunt generator.

(Fig. 13-29*b*). The field windings have (1) a large number of turns of small size wire, (2) a high resistance, and (3) a low current requirement. Because the output voltage decreases with an increase of load, the shunt generator cannot be used for most general-purpose loads such as lamps, motors, and ordinary appliances. Typical voltage versus load characteristics are shown in Fig. 13-33.

Series Generator. A series generator has its field winding connected in series with the armature (Fig. 13-34*a*) and hence must carry the same amount of current as the armature. The field windings have (1) a small number of turns of large-size wire, (2) a low resistance, and (3) a high current-carrying capacity. The typical voltage versus load characteristics of the series generator are shown in Fig. 13-34*b*. The output voltage is very low at no load and increases rapidly with load to a peak value and then decreases rapidly with a further increase in load. The series generator is usually operated on the portion of the curve indicated as *A*-*B* in Fig. 13-34*b*. Under this operating condition, relatively large changes in load resistance will produce only small changes in load current, and the generator is therefore capable of maintaining an approximately constant load current. Series generators were used with the series circuit arc-lamp system which required a constant-current power source.

Compound Generator. The compound generator has both a shunt field winding and a series field winding (Fig. 13-35). When the shunt field and series field windings produce the same magnetic polarity at the field poles, the generator is called *cumulative compound.* When the two field windings are connected to produce opposite polarities, the generator is called *differential compound.* If the shunt field winding is connected across only the armature terminals, it is called a *short shunt* field, and if it is con-

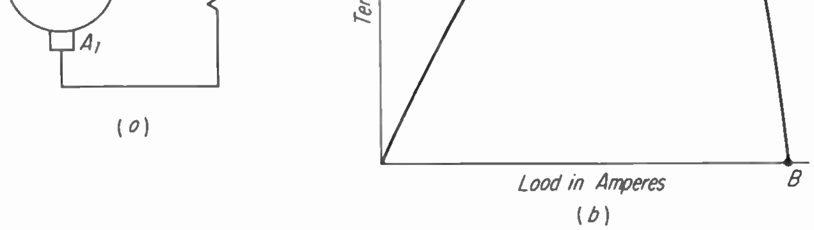


Fig. 13-34 Series generator. (a) Schematic diagram. (b) Typical voltage versus load characteristics.

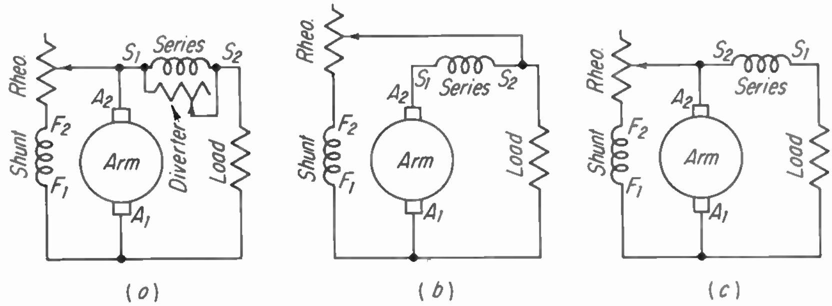


Fig. 13-35 Schematic diagrams of types of connections used with compound generators. (a) Cumulative-compound, short shunt, including a series field current diverter. (b) Cumulative-compound, long-shunt. (c) Differential-compound, short-shunt.

nected across both the armature and the series field, it is called a *long shunt* field.

Most d-c generators used to supply power for general-purpose loads are of the cumulative compound type in order to obtain either flat compounding or overcompounding (Fig. 13-36). A *flat-compounded* generator produces the same amount of voltage at full load as at no load. An *overcompounded* generator produces a higher voltage at full load than at no load. The degree of compounding is sometimes made variable by use of a diverter, which is a variable resistance connected across the series field terminals.

13-19 Other Generator Applications

Although generators are primarily thought of as a source of electric power, they are also used for a variety of other purposes.

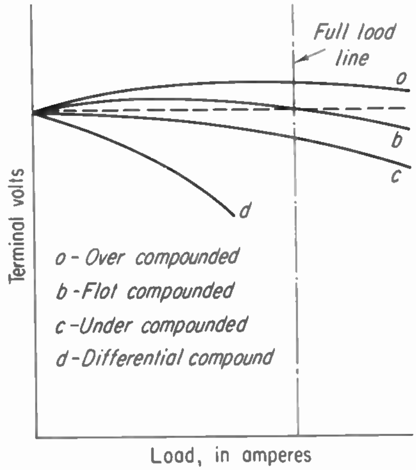


Fig. 13-36 Typical voltage versus load characteristics of compound generators.

Separately-excited Generator as a Rotary Amplifier. Three types of applications possible with the separately-excited generator are (1) amplification, (2) sensitive control, (3) automatic-motor-control system. Because changes of small amounts of current in the field circuit can produce much larger changes in the armature current, amplifying action is achieved. Gains in the order of 10 to 100 times can be obtained, and for this type of application the generator is sometimes called a *rotary amplifier.* Because of the high ratio of changes in the armature current to changes in the field current, it is possible to obtain sensitive control of the generator output. Automatic motor control can be achieved by connecting the motor to a generator armature circuit and controlling the generator output by means of its field circuit. All these types of applications require that the generator be driven by a prime mover, usually an electric motor.

Self-excited Generator as a Rotary Amplifier. When the field windings are constructed in two or more sections, amplifier action may be obtained with a self-excited generator. The main section of the field winding, which provides the major portion of the field strength, is connected to the generator armature terminals; a rheostat is usually also included in this circuit. One or more auxiliary winding sections provide additional field strength when it is desired. By means of the rheostat, the resistance of the main field circuit is made slightly higher than the critical field resistance value so that the generator voltage does not build up. Small amounts of current in the auxiliary field windings can then cause the generator voltage to build up and have the generator develop output power. Because small amounts of current in the auxiliary windings can cause the generator to deliver large amounts of output power, amplifying action is achieved. This type of generator is also called a *Regulex* generator or a *Rototrol* generator, trade names of Allis-Chalmers Mfg. Corp. and Westinghouse Electric Corp., respectively.

Two-stage Rotary Amplifier. If the amplification provided by a single generator is not sufficient to meet the needs of a particular application, a second generator may be added. The two generators are used in cascade; that is, the output of the first becomes the input of the second. A small signal current excites the field of the first unit and controls its amplified output, which is then used to excite the field of the second unit. The amplified output of the second unit supplies power to the load. The overall amplification becomes the product of the two individual amplifications; thus, if two cascaded rotary amplifiers produce gains of 40 and 25 times, respectively, the overall amplification is 1,000.

The Amplidyne. The amplidyne (General Electric Company) combines the two-stage rotary amplifier action in a single unit by using the effect of armature reaction. A weak signal input to the control field winding of a rotary amplifier (Fig. 13-37*a*) sets up a magnetic field ϕ_1 (Fig. 13-37*b*) which is sufficient to build up the generator voltage and make the gener-

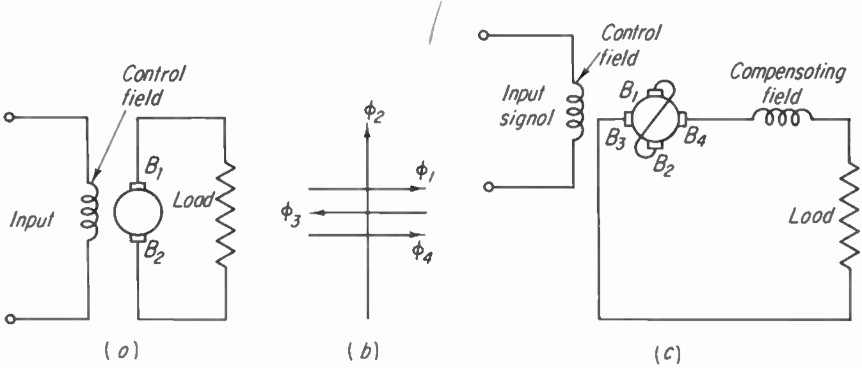


Fig. 13-37 (a) A single rotary amplifier. (b) Magnetic field representations of the amplidyne. (c) Schematic diagram of the amplidyne.

ator capable of delivering an amplified current to the load (Fig. 13-37*a*). However, instead of supplying current to a load, the brushes B_1 and B_2 are short-circuited (Fig. 13-37*c*), thereby causing a high armature current to flow. Because of armature reaction, the high armature current sets up a strong quadrature magnetic field ϕ_2 (Fig. 13-37*b*). The armature flux ϕ_2 is very strong compared with the flux ϕ_1, and the net flux is almost in line with ϕ_2. The armature conductors upon cutting the strong field ϕ_2 produce a high generated voltage at a second set of brushes B_3 and B_4. These brushes then supply a large amount of output power to the load (Fig. 13-37*c*). Owing to armature reaction, the output load current produces a new strong magnetic flux ϕ_3 which will be in quadrature with ϕ_2 and opposite in direction to ϕ_1 (Fig. 13-37*b*). The flux ϕ_3 produces two undesirable effects; namely, (1) it weakens the control flux ϕ_1, and (2) it produces a resultant field made up of ϕ_2 and ϕ_3 that shifts the neutral position of both sets of brushes. These two effects are counteracted by adding a compensating winding placed so that its flux ϕ_4 prevents the weakening of the control field flux and prevents the shifting of the neutral plane. When the compensating field winding is connected in series with the load and the armature, the amount of correction will automatically vary with the load conditions.

Ward Leonard Drive. The Ward Leonard drive system is an early development of a method used to control the operation of a d-c motor by means of controlling a d-c generator that supplies power to the motor. There are many variations of the basic Ward Leonard drive, each intended to meet the needs of a particular application. Figure 13-38 illustrates the principle of the Ward Leonard drive. A large d-c motor is used to drive a mechanical load, and it is desired to control its direction of rotation and to provide a wide range of speed control. A three-unit motor-generator set provides power for this drive system. The output of the large d-c generator

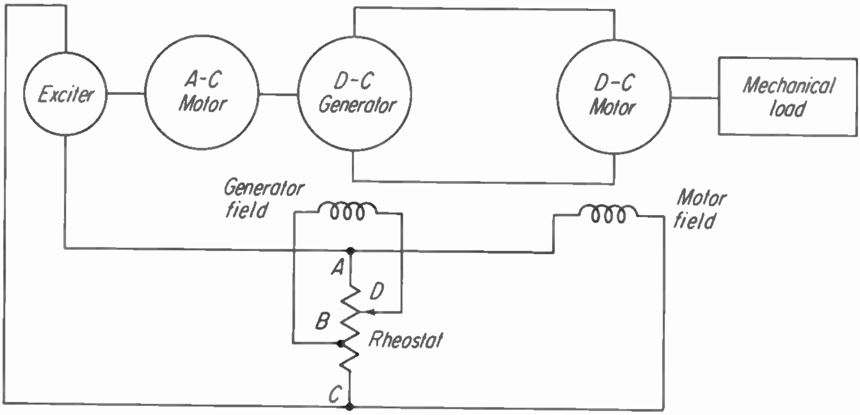


Fig. 13-38 Example of a Ward Leonard drive system.

is fed to the armature of the d-c motor. The output of the smaller generator, called the *exciter*, supplies energy to the field windings of the d-c motor and the large d-c generator. A large a-c motor is used as the prime mover for the two d-c generators and indirectly for the d-c motor. The field of the d-c motor receives constant excitation, while the excitation of the large d-c generator is variable in magnitude and polarity by means of the generator field rheostat. When the rheostat arm *D* is at position *B*, the generator field current is zero, the generator output is zero, and the d-c motor remains at standstill. When the rheostat arm is moved toward *A*, the generator field is excited, power is supplied to the armature of the d-c motor, and the mechanical load is set into motion. The speed of rotation of the load may be controlled by varying the position of the rheostat arm between points *A* and *B*. When the rheostat arm is moved from *B* toward *C*, the polarity of the excitation of the d-c generator is reversed and the d-c motor will rotate in the direction opposite that when the rheostat arm is between positions *A* and *B*. Speed control is obtained by varying the position of the rheostat arm between points *B* and *C*.

QUESTIONS

1. Name seven characteristics generally considered as motor ratings.
2. What voltage ratings are frequently used with (*a*) d-c motors? (*b*) A-c motors?
3. (*a*) What basic speed values are used with a-c motors? (*b*) What limitations are there to the rated speeds of d-c motors? (*c*) How does the size of a motor affect its maximum permissible speed?
4. (*a*) What is meant by torque? In what units are the torque expressed for (*a*) medium- and large-size motors? (*b*) Small-size motors?
5. (*a*) Define horsepower. (*b*) What is its mechanical equivalent? How does it vary with (*c*) torque? (*d*) Speed?

6. Define (*a*) duty cycle, (*b*) continuous duty, (*c*) intermittent duty.
7. (*a*) What is meant by the temperature rating of a motor? (*b*) What temperature values are generally used? (*c*) How does completely enclosing a motor affect its rating?
8. (*a*) What ratio does the efficiency of a motor represent? (*b*) How does the efficiency of a motor affect its operating cost?
9. (*a*) What is the range of power-factor values for a-c motors? (*b*) What motor rating characteristics affect the power factor? (*c*) How does the power factor affect the operating cost of a motor?
10. (*a*) Compare the principle of operation of the d-c motor with that of the d-c generator. (*b*) Describe the construction of the basic d-c motor.
11. Explain how torque is produced in the d-c motor.
12. (*a*) What is the counter emf of a motor? (*b*) How does its magnitude at the rated motor speed compare with the impressed emf? (*c*) How does the counter emf affect the magnitude of the armature current?
13. Explain how the amount of load on a motor shaft affects the armature current.
14. What is meant by speed regulation?
15. Describe the method of varying the speed of a d-c motor by varying the armature voltage.
16. Describe the method of varying the speed of a d-c motor by varying the field current.
17. Why is some type of starting equipment generally used with medium- and large-size d-c motors?
18. How are d-c motors classified in terms of (*a*) their type of field structure? (*b*) Their type of field windings?
19. (*a*) How is the name of the shunt motor associated with the manner of its field connections? What are the characteristics of the shunt motor in terms of its (*b*) field windings? (*c*) Torque? (*d*) Speed regulation? (*e*) Speed control?
20. Describe the construction and operation of the three-point starter.
21. (*a*) How would the loss of field current affect the operation of a shunt motor? (*b*) How does the three-point starter prevent this condition?
22. How may the direction of rotation of the shunt motor be reversed?
23. Describe some applications of the shunt motor.
24. (*a*) How is the name of the series motor associated with the manner of its field connections? What are the characteristics of the series motor in terms of its (*b*) field windings? (*c*) Torque? (*d*) Speed regulation? (*e*) Speed control?
25. (*a*) How would the loss of load at the motor shaft affect the operation of a series motor? (*b*) What precautions are taken to prevent this condition?
26. Explain the use of the two-point and three-point starters with a series motor.
27. How may the direction of rotation of the series motor be reversed?
28. Describe some applications of the series motor.
29. (*a*) How is the name of the compound motor associated with the manner of its field connections? What are the characteristics of the compound motor in terms of its (*b*) field windings? (*c*) Torque? (*d*) Speed regulation? (*e*) Speed control?
30. Define (*a*) cumulative compound, (*b*) differential compound, (*c*) long shunt, (*d*) short shunt, (*e*) racing.
31. What would be the effect on the operation of a compound motor in case of the loss of its (*a*) load current? (*b*) Shunt field current?

32. Describe the operating characteristics of the four-point starter.
33. (*a*) How may the direction of rotation of the compound motor be reversed? (*b*) What precautions must be observed with regard to reversing the currents in the shunt and series field windings?
34. Describe some applications of the compound motor.
35. How are a-c motors classified in terms of their (*a*) power requirements? (*b*) Principle of operation?
36. What are the two basic requirements for the operation of an a-c motor?
37. Describe how a rotating magnetic field is produced by a two-phase motor winding.
38. (*a*) How may the direction of rotation of the magnetic field of a polyphase a-c motor be reversed? (*b*) What factors affect the speed of the rotating magnetic field?
39. Define (*a*) rotor, (*b*) stator.
40. (*a*) Describe the construction of the rotor. (*b*) Explain the action of the rotor in the operation of the a-c motor.
41. Define (*a*) synchronous speed, (*b*) rotor slip.
42. (*a*) Describe the construction of the polyphase squirrel-cage induction motor. (*b*) How is the construction varied to obtain higher starting torque for the motor?
43. Describe the construction and operating features of the double-squirrel-cage induction motor.
44. Describe the construction and operating features of the wound-rotor induction motor.
45. Describe the construction and operating features of the multispeed induction motor.
46. (*a*) Describe the construction and operating features of the polyphase synchronous motor. (*b*) What are its advantages? (*c*) What are its disadvantages?
47. How are speeds above 3,600 rpm obtained with induction motors?
48. Explain the need for control devices with a-c motors.
49. (*a*) Name two classifications of single-phase motors. (*b*) Name eight types of single-phase motors.
50. Explain the principle of operation of the single-phase induction motor.
51. Explain the construction and operation of the split-phase induction motor.
52. Explain the construction and operation of the capacitor-start induction motor.
53. (*a*) How does the capacitor-start capacitor-run induction motor differ from the capacitor-start induction motor? (*b*) What are its advantages?
54. Explain the construction and operation of the shaded-pole induction motor.
55. Describe the construction and operation of the series-type a-c motor.
56. (*a*) Describe the characteristics of the series a-c motor. (*b*) Name some applications.
57. (*a*) What is meant by a universal motor? (*b*) Compare the universal motor with the series a-c motor.
58. (*a*) Describe the construction and operation of the repulsion motor. (*b*) Why is a compensating winding sometimes added to a repulsion motor?
59. Describe the construction and operation of the repulsion-start induction-run motor.
60. (*a*) Describe the construction and operation of the repulsion-induction motor. (*b*) What are some of its advantages?
61. (*a*) Describe the construction and operation of small synchronous motors. (*b*) What method is used to make these motors become self-starting?

62. What is meant by a hysteresis motor?
63. (*a*) What is meant by a synchro? (*b*) What other names are sometimes used for synchros?
64. (*a*) Describe a common type of synchro. (*b*) Name some uses of synchros.
65. Describe the operation of a simple synchro system.
66. Define (*a*) separately-excited generator, (*b*) self-excited generator.
67. Define (*a*) saturation curve, (*b*) field-resistance line, (*c*) critical field resistance.
68. (*a*) Explain how a self-excited generator builds up its voltage. (*b*) Describe three reasons why a self-excited generator might fail to build up a voltage.
69. Explain three reasons why the terminal voltage of a self-excited shunt generator decreases when the load increases.
70. Define (*a*) armature reaction, (*b*) neutral brush position, (*c*) commutation.
71. Describe three ways of counteracting the effects of armature reaction.
72. Define (*a*) shunt generator, (*b*) series generator, (*c*) compound generator.
73. Define (*a*) cumulative compound, (*b*) differential compound, (*c*) short shunt, (*d*) long shunt.
74. Explain the use of a separately-excited generator as a rotary amplifier.
75. Explain the use of a self-excited generator as a rotary amplifier.
76. (*a*) What is meant by a two-stage rotary amplifier? (*b*) What is its advantage over a single rotary amplifier?
77. Explain the operation of the amplidyne.
78. Explain the purpose and operation of the Ward Leonard drive system.

PROBLEMS

1. (*a*) How much torque is delivered by a 1/50-hp 3,000-rpm motor? (*b*) How much force is developed at the surface of the rotating member if its diameter is 1½ inches?
2. (*a*) How much torque is developed by a ½-hp 10,000-rpm motor? (*b*) How much force is developed at the surface of the rotating member if its diameter is 2½ inches?
3. (*a*) What is the horsepower output of a motor when it is delivering 150 lb-ft of torque at 1,750 rpm? (*b*) How much force is developed at the surface of the rotating member if its diameter is 18 inches?
4. (*a*) What is the torque of a motor that exerts a force of 72 lb at the surface of its rotating member whose diameter is 9 inches? (*b*) What is the horsepower output of the motor if its speed is 975 rpm?
5. What is the efficiency of a 1/45-hp 1,550-rpm motor that takes 67.5 watts from the power source?
6. What is the efficiency of a 25-hp 1,200-rpm d-c motor if it takes 100 amp at 220 volts when delivering its rated horsepower output?
7. What is the efficiency of a 1-hp motor that has the following losses: (1) 130 watts in the iron, (2) 70 watts in the copper, (3) 80 watts in friction?
8. How much current is taken by a 1/25-hp 28-volt d-c motor if its efficiency is 50 per cent?
9. What is the power factor of the motor of Prob. 5 if it takes 1 amp at 115 volts and 60 cycles?
10. A certain small motor that is delivering 3.6 oz-in. of torque at 2,900 rpm takes 0.7 amp and 38 watts from a 117-volt 60-cycle power source. Find (*a*) the horsepower output, (*b*) the efficiency, (*c*) the power factor.

11. If the armature of a $\frac{1}{16}$-hp 115-volt d-c motor has a resistance of 8 ohms and the armature current is 0.35 amp at no load and 0.78 amp at full load, what is the counter emf at (*a*) no load? (*b*) Full load?
12. If the armature of a $\frac{1}{25}$-hp 28-volt d-c motor has a resistance of 1 ohm and the armature current is 0.9 amp at no load and 2.2 amp at full load, what is the counter emf at (*a*) no load? (*b*) Full load?
13. If the motor of Prob. 11 has a no-load speed of 3,000 rpm, what is its approximate full-load speed?
14. If the motor of Prob. 12 has a no-load speed of 5,000 rpm, what is its approximate full-load speed?
15. What is the speed regulation of the motor of Prob. 13?
16. What is the speed regulation of the motor of Prob. 14?
17. A certain 5-hp 120-volt d-c shunt motor requires a magnetizing field strength of 1,500 amp-turns per pole, and the resistance of its field winding is 100 ohms. Find (*a*) the field current, (*b*) the number of turns required for each field coil.
18. A certain 25-hp 220-volt d-c shunt motor requires a magnetizing field strength of 8,000 amp-turns per pair of poles, and the resistance of its field winding is 88 ohms. Find (*a*) the field current, (*b*) the number of turns required for each field coil.
19. (*a*) If the armature resistance of a 2-hp 115-volt d-c motor is 0.25 ohm, how much current would the motor take at the instant of starting when connected directly across the line? How much resistance must a starting box add to the armature circuit if the starting current is to be limited to (*b*) 20 amp? (*c*) 30 amp?
20. (*a*) If the armature resistance of a 20-hp 220-volt d-c motor is 0.2 ohm, how much current would the motor take at the instant of starting when connected directly across the line? How much resistance must a starting box add to the armature circuit if the starting current is to be limited to (*b*) 120 amp? (*c*) 200 amp?
21. If the motor of Prob. 17 is to be made a series motor, how many turns are needed on each field coil if the full-load current is 37.5 amp?
22. If the motor of Prob. 18 is to be made a series motor, how many turns are needed on each field coil if the full-load current is 100 amp?
23. It is desired that the motor of Prob. 17 be made 20 per cent cumulative compound. Find (*a*) the shunt field ampere-turns, (*b*) the series field ampere-turns, (*c*) the shunt field turns if the field current is 1 amp, (*d*) the series field turns for a full-load current of 37.5 amp.
24. It is desired that the motor of Prob. 18 be made 75 per cent cumulative compound. Find (*a*) the shunt field ampere-turns, (*b*) the series field ampere-turns, (*c*) the shunt field turns if the field current is 1.25 amp, (*d*) the series field turns for a full-load current of 100 amp.
25. What is the speed of the rotating field for a 60-cycle motor winding if its number of poles is (*a*) 2? (*b*) 4? (*c*) 8? (*d*) 16?
26. What is the speed of the rotating field for a 60-cycle motor winding if its number of poles is (*a*) 6? (*b*) 10? (*c*) 12? (*d*) 20?
27. (*a*) What is the synchronous speed of a six-pole 60-cycle motor? (*b*) What is the per cent slip if the full-load rotor speed is 1,150 rpm?
28. (*a*) What is the synchronous speed of a 10-pole 60-cycle motor? (*b*) What is the per cent slip if the full-load rotor speed is 685 rpm?
29. What is the full-load speed of a two-pole 60-cycle motor if its slip is 4 per cent?
30. What is the full-load speed of an eight-pole 60-cycle motor if its slip is 5 per cent?

31. What is the current rating of a 10-kw 125-volt generator?
32. What is the current rating of a 300-volt generator that delivers 60 watts at full load?
33. What is the power-output rating of a generator that delivers 250 ma at 600 volts?
34. What is the power-output rating of a generator that delivers 30 amp at 250 volts?
35. What value of rheostat resistance is required in order that the voltage of the generator of Fig. 13-30 will build up to only 110 volts?
36. What value of rheostat resistance is required in order that the voltage of the generator of Fig. 13-30 will build up to only 80 volts?
37. A certain d-c self-excited shunt generator produces 125 volts at no load. What is its terminal voltage at full load if the voltage drops due to armature resistance, armature reaction, and the decrease in field current are 4, 5, and 6 volts, respectively?
38. What is the terminal voltage of the generator of Prob. 37 at half load if the three voltage drops each vary in direct proportion to the load?
39. What gain is indicated for a rotary amplifier by the following pair of readings: (1) $I_{fld} = 0.10$ amp; $I_{\text{arm}} = 10$ amp? (2) $I_{fld} = 0.15$ amp; $I_{\text{arm}} = 14$ amp?
40. A certain rotary amplifier produces a gain of 75. One load condition produces readings of $I_{fld} = 50$ ma, $I_{\text{arm}} = 4$ amp. What is the armature current if the field current increases to 80 ma?
41. What is the overall gain of a two-stage rotary amplifier if each stage has a gain of 75?
42. It is desired that the overall gain of a two-stage rotary amplifier be 850. What gain is required of the second stage if the gain of the first stage is 25?

Chapter 14

Electron Tubes

The importance of the electron tube lies in its ability to operate efficiently over a wide range of frequencies and to control almost instantaneously the flow of millions of electrons. Electron tubes are used in various types of electronic equipment, such as communication, industrial, scientific, therapeutic, computer, data processing, etc. These tubes may be either of the vacuum type, as used in radio and television receivers, or of the gas type, as used in some types of industrial electronic equipment. Electron tubes may be considered as a circuit element in the same manner as are resistors, inductors, and capacitors.

14-1 The Cathode

Purpose of the Cathode. When any substance is heated, the speed of the electrons revolving about their nucleus is increased and some electrons acquire sufficient speed to break away from the surface of the material and go off into space. This action, which is accelerated when the substance is heated in a vacuum, is utilized in vacuum tubes to produce the necessary electron supply. When used for this purpose, the substance is called the *cathode.* All vacuum tubes contain a cathode and one or more electrodes mounted in an evacuated envelope, which may be a glass bulb or a compact metal shell.

Purpose of the Heater. The cathode is an essential part of a vacuum tube because it supplies the electrons necessary for operation of the tube. The electrons are generally released by heating the cathode. The purpose of a heater in a vacuum tube is to radiate heat when an electric current flows through it. The amount of heat that is radiated is dependent on the material of which the conductor is made and the amount of current flowing in the conductor. The source of power used to supply current for heating the cathode is called the *heater power supply.*

Directly Heated Cathodes. A directly heated cathode, called the *filament type,* is one in which the heater is also the cathode. Materials that are good conductors are found to be poor electron emitters; therefore directly heated cathodes must be operated at high temperatures in order to emit a sufficient number of electrons. Directly heated cathodes require a comparatively small amount of heating power and are used in almost all the tubes

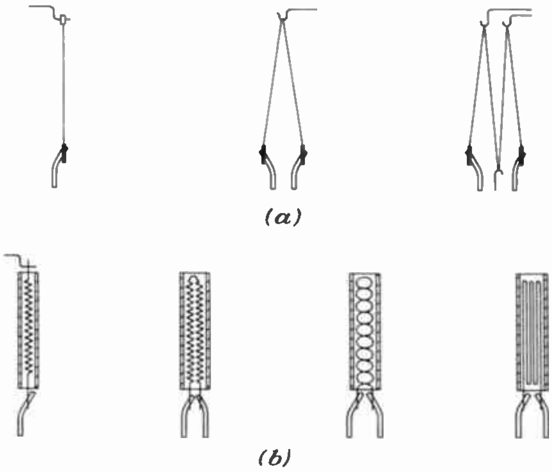


Fig. 14-1 Types of cathodes. (a) Directly heated cathodes. (b) Indirectly heated cathodes.

designed for battery operation. Alternating-current-operated tubes seldom use the filament-type cathode.

Indirectly Heated Cathodes. An *indirectly heated cathode* is one in which the emitting material is coated on a thin sleeve which is heated by radiation from a heater placed inside the sleeve and insulated from it. It is thus possible to use a material for the heater that will radiate the maximum amount of heat with the minimum amount of current and to use a material for the cathode that is a good electron emitter.

14-2 Diodes

The Plate. A vacuum tube having a cathode and one other electrode is called a *diode*. The second electrode is called the *plate* or *anode*. If a positive voltage is applied to the plate, the electrons emitted from the cathode, being negative, will be attracted to the plate. These electrons will flow through the external plate battery circuit as indicated in Fig. 14-2. This flow of electrons is called the *plate current*. If the polarity of the plate is made negative, the electrons will be forced back to the cathode and no current will flow in the plate circuit.

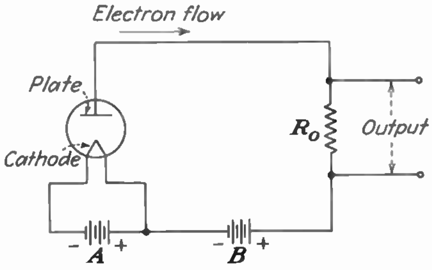


Fig. 14-2 Circuit diagram for a diode showing the electron flow and the proper connections for the filament and B power supplies.

When an alternating current is applied to the plate, its polarity will be positive during every other half-cycle. As electrons will flow to the plate only when it is positive, the current in the plate circuit will flow in only one direction, and the current is called a *rectified current* (Fig. 14-3).

Diodes are used in radio receivers as detectors and in a-c operated receivers and transmitters to convert alternating current to direct current. Rectifier tubes having one plate and one cathode are called *half-wave rectifiers* because the rectified current only flows during one-half of the cycle.

Duodiodes. Rectifier tubes having two plates and one or two cathodes have their plates connected into the external circuit so that each plate will be positive for opposite halves of the cycle. As one or the other of the plates will always be positive, current will flow in the external circuit during each half-cycle. These tubes are called *full-wave rectifiers* because current flows during both half-cycles.

Space Charge. The number of electrons drawn to the plate depends on the number given off by the cathode and also on the plate voltage. If the plate voltage is not high enough to draw off all the electrons emitted by the cathode, those not drawn off will remain in space. These electrons form a repelling force to the other electrons being given off by the cathode,

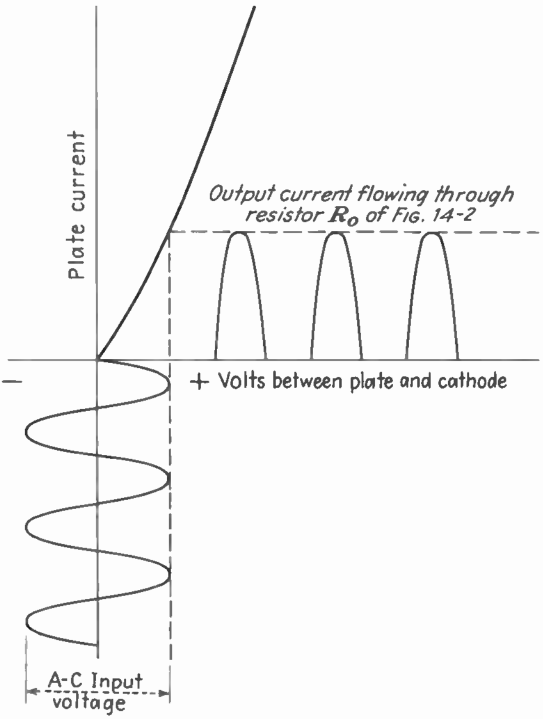


Fig. 14-3 The diode as a rectifier.

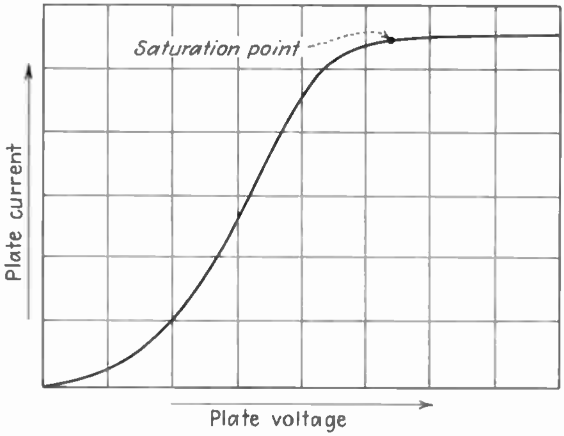


Fig. 14-4 Variation of plate current with changes in plate voltage.

thus impeding their flow to the plate. This repelling action is called the *space charge.* Space charge can be reduced by decreasing the space between the plate and cathode.

Saturation Current. Increasing the plate voltage will increase the plate current until all the electrons given off by the cathode are drawn to the plate. Further increase in plate voltage will not increase the plate current, as no more electrons can be drawn off to it (Fig. 14-4). The point on the curve at which the current has reached its highest value is called the *saturation point,* and the plate current for this condition is called the *saturation current* or *emission current.*

14-3 The Triode

Action of the Control Grid. When a third electrode is used, the tube is called a *triode.* The third element is called the *control grid* and consists of a spiral winding or a fine-mesh screen extending the length of the cathode and placed between the cathode and plate. The circuit connections and the direction of electron flow for a triode are shown in Fig. 14-5. If the grid is made more negative than the cathode, some of the electrons going toward the plate will be repelled by the grid, thus reducing the plate current. By making the grid still more negative, it is possible to reduce the plate current to zero. This condition is called *cutoff.*

Grid Bias. The third electrode is called the *control grid* because it controls the number of electrons allowed to flow from the cathode to the plate. The amount of voltage used to keep the grid negative with respect to the cathode is called the *grid bias.*

Relation between the Grid Bias and the Plate Current. If a varying signal voltage is applied to the grid, the number of electrons flowing in the plate circuit will vary in the same manner as the signal voltage. If the triode in the circuit of Fig. 14-5 operates with a grid bias of 3 volts

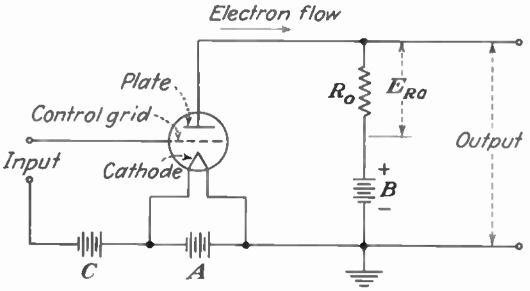


Fig. 14-5 Circuit diagram for a triode showing the electron flow and the proper connections for the cathode, plate, and grid.

(Fig. 14-6*b*), a steady plate current I_b will flow (Fig. 14-6*d*). If an alternating signal voltage of 1 volt (Fig. 14-6*a*) is impressed across the input terminals of the grid circuit, the voltage on the grid of the tube will vary from -2 to -4 volts as shown in Fig. 14-6*c*. As the number of electrons flowing to the plate is controlled by the negative potential on the grid, the plate current will vary in the same manner as the signal voltage. This is illustrated by the varying plate current i_b shown in Fig. 14-6*d*, maximum plate current flowing when the grid is least negative and minimum plate current flowing when it is most negative.

Phase Relation among the Varying Grid Voltage, Varying Plate Current, and Varying Plate Voltage. Since the plate current of a tube varies in the same manner as the signal voltage that is applied to its grid, the plate-current variations are in phase with the grid-voltage variations. The output of the tube circuit is taken off at the plate, or top of resistor R_o, and the ground. The voltage available between these two points is equal to the B supply voltage minus the voltage drop at the resistor R_o. If the tube has a grid bias of 3 volts, a steady plate current I_b will flow and the output voltage will have a value indicated as the steady voltage E_b (Fig. 14-6*e*). When a signal is applied to the grid, the positive portions of the signal voltage will cause an increase in the plate current, which will increase the voltage drop across the output resistor R_o and thus will decrease the output voltage e_b (Fig. 14-6*e*). The negative portions of the signal voltage will decrease the plate current, reduce the voltage drop across R_o, and increase the output voltage e_b. Thus an increase in signal voltage will produce a decrease in the output voltage, or a decrease in signal voltage will produce an increase in the output voltage. The output-voltage variations are therefore 180° out of phase with the signal-voltage variations.

Action of the Grid with a Positive Voltage. If the value of the signal voltage is such that it will make the grid positive, the grid will act in the same manner as a plate and will draw electrons to it, thus causing a current to flow in the grid circuit. This condition can be avoided by using a grid bias whose value is higher than the maximum amount of input signal voltage that is applied.

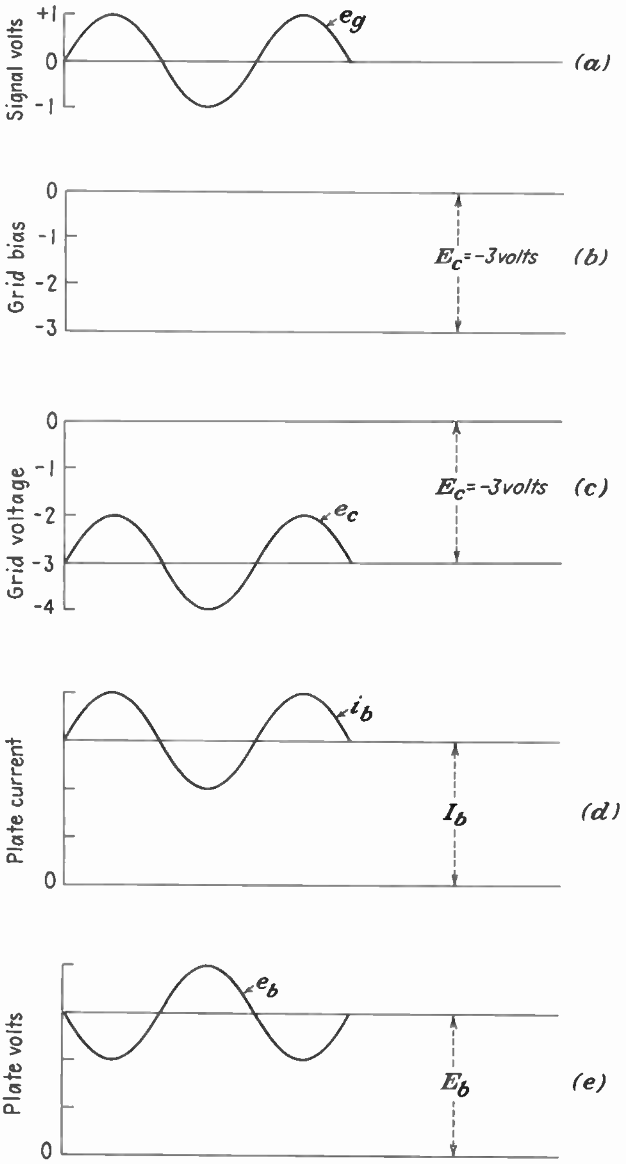


Fig. 14-6 Illustration of the effect of the signal voltage upon the grid voltage and plate current. (a) Signal voltage (instantaneous values of the varying component of the grid voltage). (b) Grid bias (average or quiescent value of the grid voltage). (c) Grid voltage (instantaneous total grid voltage). (d) Plate current I_b (average or quiescent value of plate current) and plate current i_b (instantaneous total plate current). (e) Plate voltage E_b (average or quiescent value of plate voltage) and plate voltage e_b, also output voltage (instantaneous total plate voltage).

14-4 Vacuum-tube Characteristics

Tube Constants. The major operating characteristics of a vacuum tube are used to identify the electrical features and operating values of the tube. These values may be listed in tabular form or plotted on graph paper to form a curve. When given in curve form, they are called *characteristic curves.* These curves are used to determine the performance of a tube under any operating condition. The constants of the tube can also be calculated from these curves. Tabular-form listings as found in most tube manuals are usually limited to one or two of the operating conditions commonly used for that particular tube.

Vacuum-tube characteristics are often referred to as the constants of the tube. The constants most commonly used are the *amplification factor* μ (mu), the *plate resistance* r_p, and the *control-grid-to-plate transconductance* g_m. Control-grid-to-plate transconductance is usually referred to as just *transconductance.* Transconductance is also known as *mutual conductance.*

Amplification Factor. The amplification factor of a tube may be defined as a measure of the relative ability of the grid and the plate to produce an equal change in the plate current. Mathematically it is defined as the ratio of the change in plate voltage to a change in control-grid voltage for a constant value of plate current, with the voltages applied to all other electrodes maintained constant, or

$$\mu = \frac{de_b}{de_c} \qquad i_b\text{—constant} \tag{14-1}$$

where μ = amplification factor
d = change or variation in value
de_b = change in instantaneous total plate voltage, volts
de_c = change in instantaneous total control-grid voltage necessary to produce the same effect upon the plate current as would be produced by de_b, volts
i_b = instantaneous total plate current, amp

EXAMPLE 14-1 An electron tube is operated at its normal heater voltage, a grid bias of 4 volts, and with a plate voltage of 160 volts. Determine the amplification factor if the characteristics of this tube are such that (1) the plate current is 7.5 ma and (2) if the plate voltage is increased to 200 volts, the grid bias will have to be increased to 6 volts in order to maintain the plate current at 7.5 ma.

GIVEN: $i_b = 7.5$ ma $\qquad de_b = 160$ to 200 volts $\qquad de_c = 4$ to 6 volts

FIND: μ

SOLUTION:

$$\mu = \frac{de_b}{de_c} = \frac{200 - 160}{6 - 4} = \frac{40}{2} = 20$$

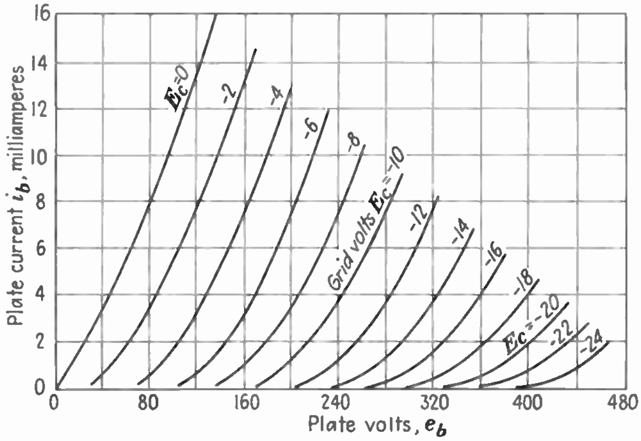


Fig. 14-7 Plate characteristic curves of a triode showing the variation in plate current for changes in plate voltage.

Plate Resistance. The plate resistance of a tube may be defined as the resistance to the flow of alternating current offered by the path between the cathode and the plate. This is sometimes called the *dynamic plate resistance* or the *a-c plate resistance.* The value of the plate resistance will depend upon the values of the grid and plate voltages being applied to the tube. Mathematically it is defined as the ratio of a change in plate voltage to the corresponding change produced in the plate current with the grid voltage maintained constant, or

$$r_p = \frac{de_b}{di_b} \qquad e_c\text{—constant} \tag{14-2}$$

where r_p = dynamic plate resistance, ohms
di_b = change in instantaneous total plate current, amp
e_c = instantaneous total grid voltage, volts

EXAMPLE 14-2 An electron tube is operated at its normal heater voltage, a grid bias of 4 volts, and with a plate voltage of 160 volts. Determine the plate resistance if the characteristics of this tube are such that (1) if the plate voltage is decreased to 140 volts, the plate current will be 5 ma; (2) if the plate voltage is increased to 180 volts, the plate current will increase to 10.5 ma; and (3) the grid bias is maintained at 4 volts.

GIVEN: $e_c = -4$ volts $\quad de_b$ = 140 to 180 volts $\quad di_b$ = 5 to 10.5 ma

FIND: r_p

SOLUTION:

$$r_p = \frac{de_b}{di_b} = \frac{180 - 140}{0.0105 - 0.005} = \frac{40}{0.0055} = 7{,}272 \text{ ohms}$$

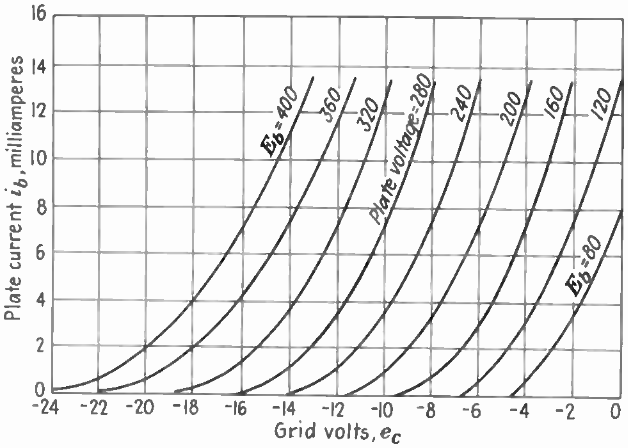


Fig. 14-8 Transfer characteristic curves of a triode showing the variations in plate current for changes in grid voltage. These curves have been cross-plotted from Fig. 14-7.

The solution of Example 14-2 gives the dynamic or a-c plate resistance of the tube. It should be noted that this is not the same value as would be obtained by dividing the operating plate voltage by its corresponding plate current. Such a value is known as the *static resistance* or the *d-c resistance* of the tube, but it is seldom used in the study of tubes and their circuits. The plate resistance is sometimes referred to as the *impedance* of the tube, the *internal resistance* of the tube, the *dynamic plate resistance*, or the *a-c resistance* of the tube.

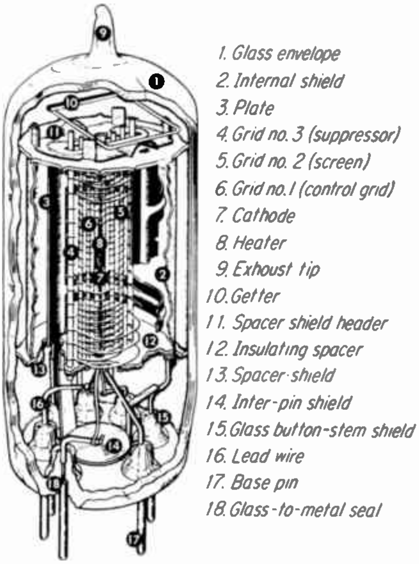


Fig. 14-9 Miniature-type tube with a button base. (RCA)

Transconductance. The transconductance of a tube may be defined as the ratio of the change in plate current to a change in the control-grid voltage when all other tube-element voltages are kept constant. Expressed mathematically,

$$g_m = \frac{di_b}{de_c} \qquad e_b\text{—constant} \tag{14-3}$$

where g_m = transconductance, mhos
e_b = instantaneous total plate voltage, volts

EXAMPLE 14-3 An electron tube is operated at its normal heater voltage, a grid bias of 4 volts, and with a plate voltage of 160 volts. Determine the transconductance if the characteristics of this tube are such that (1) if the grid bias is decreased to 3.5 volts, the plate current will be 8.5 ma; (2) if the grid bias is increased to 4.5 volts, the plate current will decrease to 6 ma; and (3) the plate voltage is maintained at 160 volts.

GIVEN: $e_b = 160$ volts $\quad di_b = 6$ to 8.5 ma $\quad de_c = -3.5$ to -4.5 volts

FIND: g_m

SOLUTION:

$$g_m = \frac{di_b}{de_c} = \frac{0.0085 - 0.006}{4.5 - 3.5} = \frac{0.0025}{1} = 0.0025 \text{ mho}$$

The unit of conductance is the mho, and its name is obtained by spelling ohm backward. For convenience of numbers, the transconductance is often expressed in micromhos. Thus, the answer to Example 14-3 may be expressed as 2,500 micromhos (abbreviated as 2,500 μmhos).

Equation (14-3) shows that a tube that produces a relatively large change in plate current for a relatively small change in grid voltage will have a relatively high value of transconductance, and since such conditions are generally desired, tubes having a high value of transconductance are preferred.

Relation among Amplification Factor, Plate Resistance, and Transconductance. A definite relation exists among the three tube constants, as

$$\mu = g_m r_p \tag{14-4}$$

$$g_m = \frac{\mu}{r_p} \tag{14-5}$$

$$r_p = \frac{\mu}{g_m} \tag{14-6}$$

EXAMPLE 14-4 What is the transconductance of the triode section of a 12SQ7 duodiode-triode tube when operated at such values of voltage that its amplification factor is 100 and its plate resistance is 91,000 ohms?

GIVEN: Tube = 12SQ7 $\mu = 100$ $r_p = 91{,}000$ ohms

FIND: g_m

SOLUTION:

$$g_m = \frac{\mu}{r_p} = \frac{100}{91{,}000} \cong 0.0011 \text{ mho, or } 1{,}100\ \mu\text{mhos}$$

EXAMPLE 14-5 What is the plate resistance of a 6AF4 high-frequency triode tube when operated at such values of voltage that its amplification factor is 15 and its transconductance is 6,600 μmhos?

GIVEN: Tube = 6AF4 $\mu = 15$ $g_m = 6{,}600\ \mu$mhos

FIND: r_p

SOLUTION:

$$r_p = \frac{\mu}{g_m} = \frac{15}{6{,}600 \times 10^{-6}} = 2{,}272 \text{ ohms}$$

Relation between the Transconductance and the Operating Performance of a Tube. Vacuum-tube amplifiers are ordinarily operated so that small variations in voltage of the grid or input circuit will produce large current variations in the plate or output circuit. Since it is usually desired to keep the plate resistance low, the transconductance must be high if a high value of amplification factor is desired. The transconductance is very useful when comparing the relative merits and performance capabilities of tubes designed for the same service. A comparison of the transconductance of a power-output tube with that of a tube used as a converter would, however, have no practical value. Generally, the value of transconductance is accepted as the best single figure of merit for vacuum-tube performance.

14-5 The Tetrode

The Screen Grid. When a fourth electrode is used, the tube is called a *tetrode;* it is also referred to as a *screen-grid tube* or a *four-electrode tube.* The fourth electrode is known as the *screen grid* and consists of a spiral-wound wire or a screen, slightly coarser than that used for the control grid, placed between the plate and the control grid of the tube. The circuit connections and the direction of electron flow for a tetrode are shown in Fig. 14-10.

Interelectrode Capacitance and Its Effects. In any tube, the electrodes act as conductors and the space between the electrodes acts as an insulator; therefore a capacitance will exist between each pair of electrodes. These capacitances, known as *interelectrode capacitances,* may form undesired paths through which current can flow (Fig. 14-11).

The capacitance between the grid and plate is generally the most troublesome. This capacitance causes some of the energy of the plate circuit to be

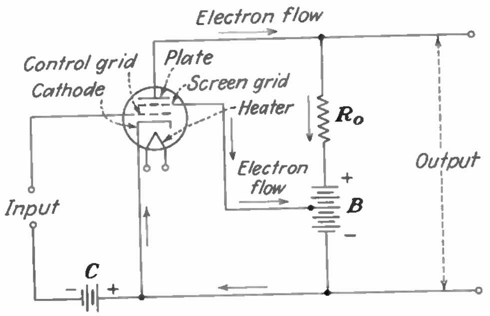


Fig. 14-10 Circuit diagram for a tetrode showing the electron flow and the proper connection for the screen grid.

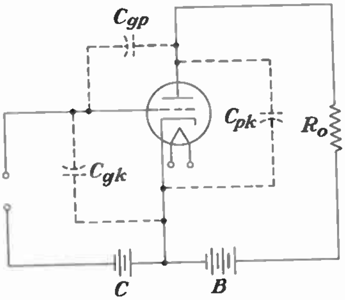


Fig. 14-11 Interelectrode capacitances of a triode.

applied to the grid circuit. The energy transferred from the output to the input circuit is called *feedback*. If the output circuit contains resistance load, the feedback voltage will be 180° out of phase with the input voltage and therefore will reduce the effect of the input signal; this is called *degeneration* or *negative feedback*. When the output circuit load is inductive, the feedback voltage will be in phase with the input voltage and therefore will increase the effect of the input signal; this is called *regeneration* or *positive feedback*.

The amount of feedback caused by the interelectrode capacitance is usually very small at audio frequencies because the capacitive reactance at these frequencies is very high. At radio frequencies, however, the capacitive reactance becomes much lower and the feedback may reach an amount sufficient to cause trouble in the operation of the circuit.

Positive feedback has the advantage of increasing the gain of a circuit but has the disadvantage of causing distortion. Negative feedback has the advantage of reducing distortion but has the disadvantage of causing a reduction in the voltage gain of the circuit. If either regeneration or degeneration gets out of control, the tube no longer operates successfully as an amplifier.

Elimination of Feedback by Use of a Screen Grid. The capacitance between the control grid and the plate of a triode can be reduced to a negligible amount by adding a fourth electrode or *screen grid*. The screen grid is mounted between the control grid and the plate and acts as an electrostatic shield between the two, thus reducing the control-grid-to-plate capacitance. Connecting a bypass capacitor between the screen grid and the cathode will increase the effectiveness of the shielding action.

The screen grid also reduces the space charge. Since the screen grid is situated between the control grid and the plate and has a positive potential, the electrons coming from the cathode receive added acceleration on their way to the plate and the tendency to form a space charge is reduced.

14-6 The Pentode

The Suppressor Grid. A tube with five electrodes—namely, a cathode, three grids, and a plate—is called a *pentode* or *five-electrode tube.* The fifth electrode is an extra grid called the *suppressor grid.* The electrodes of the pentode are arranged with the cathode at the center and surrounded by the control grid, the screen grid, the suppressor grid, and the plate, in the order named. The circuit connections and the direction of electron flow for the pentode are shown in Fig. 14-12.

The suppressor grid consists of a spiral-wound wire or a coarse-mesh screen placed between the screen grid and the plate. When the various grids of the tube are in the form of a screen, the control grid is of a very fine mesh so that small changes in control-grid voltage will produce relatively large changes in plate current and consequently will produce a high value of transconductance for the tube. The screen grid is of a somewhat coarser mesh so that it will not appreciably affect the flow of electrons to the plate, its purpose being largely to reduce the control-grid-to-plate capacitance. The suppressor grid is of a coarser mesh so that it will not retard the flow of electrons to the plate while serving its function of preventing the secondary emission from reaching the screen grid. In some pentodes the suppressor grid is internally connected to the cathode, while in others the suppressor grid is brought out as a separate terminal.

Action of the Suppressor Grid. In pentodes, the suppressor grid is added to prevent the secondary electrons from traveling to the screen grid. In order to accomplish this the suppressor grid is connected directly to the cathode. Being at cathode potential, the suppressor grid is negative with respect to the plate, and because it is close to the plate, it will repel the secondary electrons and drive them back into the plate.

Uses of Pentodes. Pentode tubes can be used as either voltage or power amplifiers. In power pentodes, a higher power output is obtained with lower grid voltages. Pentodes used as r-f amplifiers give a high voltage amplification when used with moderate values of plate voltage.

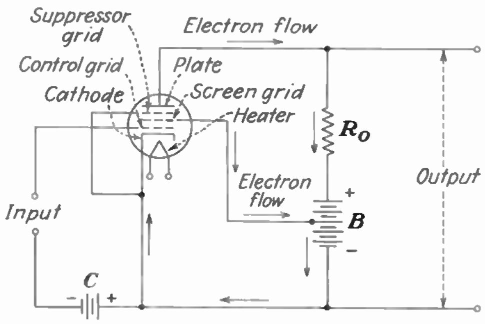


Fig. 14-12 Circuit diagram for a pentode showing the electron flow and the proper connection for the screen grid and the suppressor grid.

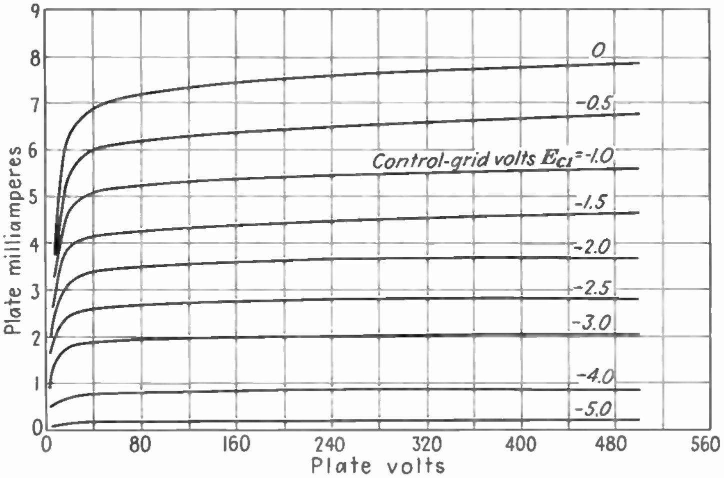


Fig. 14-13 Family of plate characteristic curves for a pentode.

14-7 Multiunit Tubes

When a tube contains within one envelope two or more groups of electrodes associated with independent electron streams, it is called a *multiunit tube*. In general, the combinations are easily identified by the names given to the tubes, such as duplex-diode, twin-triode, duplex-diode-triode, diode-triode-pentode, and rectifier-beam power amplifier (Fig. 14-14). In most cases a single cathode is used for all the units in a tube, although in some types separate cathodes are provided for each unit.

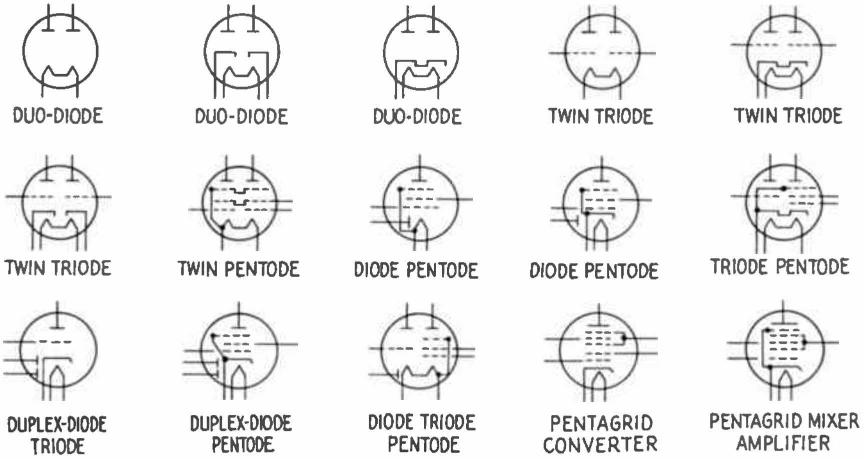


Fig. 14-14 Base diagrams for various types of multiunit tubes.

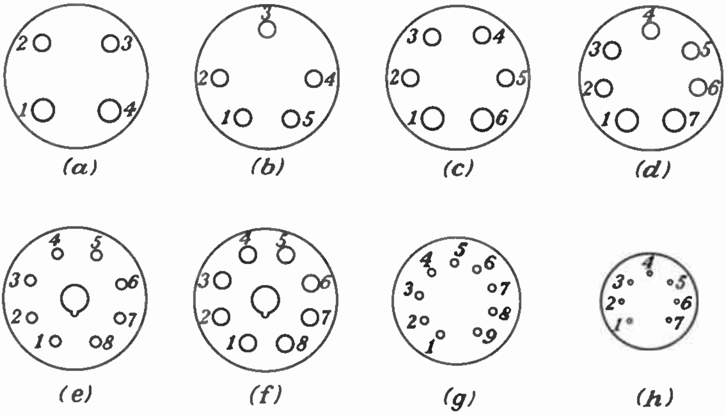


Fig. 14-15 Tube base and socket connections. (a) Standard four-pin base. (b) Standard five-pin base. (c) Standard six-pin base. (d) Standard seven-pin base. (e) Eight-pin loktal base. (f) Eight-pin octal base. (g) Nine-pin miniature base. (h) Seven-pin miniature base.

14-8 Tube Bases and Socket Connections

Methods of Identifying Socket Connections. The method of numbering the socket or tube-base connections for the early tube types is shown in Fig. 14-15. In this system, the filament or heater pins of the tube and the corresponding holes in the socket are larger in diameter than the others and are generally shown at the bottom of the diagram. The lower left-hand pin is designated as number 1, and the remaining pins are numbered consecutively in a clockwise direction. The order in which the tube elements are arranged varies with the tube types and may be obtained from a tube manual.

Metal and other octal-base tubes all use the same eight-pin socket. The *octal socket,* as it is commonly called, has eight equally spaced holes arranged in a circle. All the holes in the socket are of the same size, and in order to ensure correct placement of the tube in the socket, a large center hole with an extra notch is provided. The socket connection to the left of the centering notch is designated as number 1, and the remaining connections are numbered consecutively up to 8. The pins in the base of the tube are all of equal size and are arranged in a circle. A large insulated pin, provided with a centering key to fit the notch on the socket, is located in the center of the tube base. Some tubes using octal sockets have only six or seven pins, while others have eight. The six or seven pin bases merely omit one or two of the eight pins according to the number used. This, however, does not alter the numbering system for the socket connections. The order in which the tube elements are arranged varies with tube types and may be obtained from a tube manual. The loktal socket has an eight-pin base similar to the octal socket with the following differences: (1) the holes are smaller, and (2) provision is made for locking the tube in the socket. Miniature tubes are made with either a seven- or nine-pin base (Fig. 14-15). The miniature tube (Fig. 14-9) has a thin glass base, called a ***button base,*** and its socket is called a ***button socket.***

14-9 Voltage Amplification per Stage

Equivalent Amplifier Circuits Using Triodes. It has been shown that the grid of a tube is μ times as effective in controlling the plate current as is the plate. When an alternating signal voltage e_g is applied to the grid of a tube, the plate circuit may be considered as containing a generator of $-\mu e_g$ volts in series with the plate resistance r_p and the output impedance Z_o. This principle is a basis for calculating the operating characteristics of an amplifier circuit. The output voltage $-\mu e_g$ is negative because of the phase reversal (Art. 14-3).

The vacuum tube used in an amplifier circuit can therefore be considered as a generator whose output voltage is equal to $-\mu e_g$. An equivalent electrical circuit can be drawn for an amplifier circuit by substituting a generator for the vacuum tube. The equivalent electrical circuit for the elementary amplifier circuit of Fig. 14-16 is shown in Fig. 14-17*a*. This type of equivalent circuit is referred to as the *constant-voltage-generator* form and is convenient for studying the operating characteristics of amplifier circuits using triodes. The a-c component of the plate current flowing in this circuit is

$$i_p = \frac{\mu e_g}{Z_o + r_p} \qquad (14\text{-}7)$$

The voltage developed across the load impedance by this current will then be

$$e_p = i_p Z_o \qquad (14\text{-}8)$$

Substituting Eq. (14-7) in Eq. (14-8),

$$e_p = \frac{\mu e_g Z_o}{Z_o + r_p} \qquad (14\text{-}9)$$

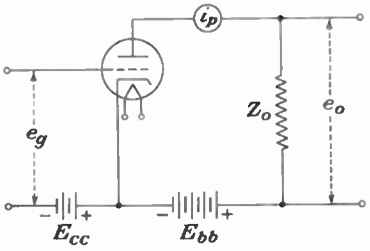


Fig. 14-16 Elementary voltage-amplifier circuit.

The voltage amplification of the circuit then becomes

$$\mathrm{VA} = \frac{e_p}{e_g} = \frac{\dfrac{\mu e_g Z_o}{Z_o + r_p}}{e_g} \tag{14-10}$$

and

$$\mathrm{VA} = \frac{\mu Z_o}{Z_o + r_p} \tag{14-11}$$

When the output impedance consists only of resistance, resistance-capacitance (R-C) coupled amplifier circuits, or a tuned amplifier circuit in which the reactive effects cancel one another so that the resultant impedance is only resistance, the output impedance Z_o will be equal to R_o. Under these conditions, Eqs. (14-7), (14-9), and (14-11) become

$$i_p = \frac{\mu e_g}{R_o + r_p} \tag{14-12}$$

$$e_p = \frac{\mu e_g R_o}{R_o + r_p} \tag{14-13}$$

$$\mathrm{VA} = \frac{\mu R_o}{R_o + r_p} \tag{14-14}$$

Relation among r_p, Z_o, and the Voltage Amplification. The maximum theoretical value of voltage amplification per amplifier stage is indicated by the amplification factor of the tube. However, this theoretical maximum value cannot be obtained in practical amplifier circuits, as the voltage amplification is limited by the plate resistance of the tube and the impedance of the load.

In order to obtain the maximum voltage amplification per stage with tubes having approximately the same amplification factor, a tube having the lowest plate resistance should be used. This is shown by Eqs. (14-11) and (14-14), which indicate that for a definite plate load the voltage amplification will increase as the plate resistance is decreased. This should not be confused with the maximum power output, which occurs when Z_o or R_o is equal to r_p. These two equations also show that for given values of

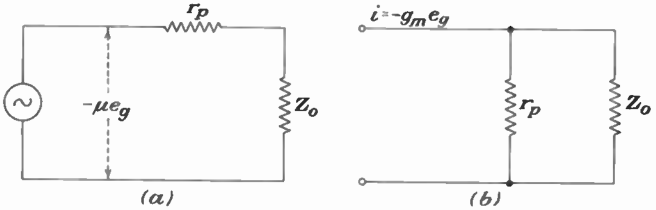


Fig. 14-17 Equivalent circuit for the elementary amplifier circuit of Fig. 14-16. (a) Constant-voltage-generator form. (b) Constant-current-generator form.

plate resistance and amplification factor of a tube, the voltage amplification of a circuit will increase if the value of the load impedance is increased.

Equivalent Amplifier Circuits Using Pentodes. The maximum voltage amplification of a circuit using a low-mu triode can be made almost equal to its amplification factor because of the comparatively low values of plate resistance obtainable in low-mu triode-amplifier tubes. In amplifier circuits using high-mu triodes, tetrodes, and pentodes the values of plate resistance are so high that the amount of voltage amplification obtainable is but a fraction of the amplification factor of the tube used. The large difference between the voltage amplification of the circuit and the amplification factor of these tubes is due to the high values of the plate resistance and the comparatively low values of plate-load impedance that must be used for practical amplifier circuits. When the ratio between the plate resistance and the load impedance becomes great, the transconductance of the output circuit approaches the value obtained when the load impedance is practically zero. The voltage amplification of the circuit will then be dependent on the value of the transconductance of the tube rather than on its amplification factor. This can be seen if Eqs. (14-7), (14-9), and (14-11) are expressed in terms of the transconductance instead of the amplification factor. Substituting Eq. (14-4) in Eq. (14-7) and regrouping the terms,

$$i_p = g_m e_g \frac{r_p}{Z_o + r_p} \tag{14-15}$$

Substituting Eq. (14-4) in Eq. (14-9) and regrouping, the voltage across the load impedance then becomes

$$e_p = g_m e_g \frac{r_p Z_o}{Z_o + r_p} \tag{14-16}$$

Substituting Eq. (14-4) in Eq. (14-11) and regrouping, the voltage amplification of the circuit then becomes

$$\text{VA} = g_m \frac{r_p Z_o}{Z_o + r_p} \tag{14-17}$$

When the terms in Eq. (14-17) are rearranged, the output impedance required to produce a definite value of voltage amplification may be found by

$$Z_o = \frac{\text{VA} r_p}{g_m r_p - \text{VA}} \tag{14-18}$$

Equation (14-16) shows that applying a signal voltage e_g to the input circuit of a tube may be considered the same as though the tube generated a current equal to $-g_m e_g$ that is made to flow through a parallel circuit formed by the plate resistance r_p and the load impedance Z_o. Thus an amplifier circuit may also be considered in the form shown in Fig. 14-17*b*, generally called the *constant-current-generator* form. This type of circuit is convenient for studying the operating characteristics of amplifier circuits using high-mu tubes.

The impedance of a tuned circuit at its resonant frequency has the effect of only resistance. When the load in the plate circuit of an r-f amplifier tube is a tuned circuit, the effective output impedance Z_o at resonance will have the effect of only resistance and can therefore be added arithmetically to the plate resistance r_p in Eqs. (14-15) to (14-17).

Voltage Amplification Calculations. All the equations presented in this article are interrelated, and the following examples illustrate their applications.

EXAMPLE 14-6 A certain triode, used as a voltage amplifier, has a plate resistance of 85,000 ohms and an amplification factor of 100 when operated with 100 volts on its plate and a grid bias of 1 volt. What is the voltage amplification of an amplifier stage using such a tube under these operating conditions when the load resistance is 65,000 ohms?

GIVEN: $r_p = 85{,}000$ ohms $\quad R_o = 65{,}000$ ohms $\quad \mu = 100$

FIND: VA

SOLUTION:

$$\text{VA} = \frac{\mu R_o}{R_o + r_p} = \frac{100 \times 65{,}000}{65{,}000 + 85{,}000} = 43.3$$

EXAMPLE 14-7 A 1-volt a-c signal (maximum value) is applied to the input side of the tube and amplifier circuit of Example 14-6. (*a*) What is the a-c component of the plate current? (*b*) What amount of voltage will be developed across the output load resistor?

GIVEN: $e_g = 1$ volt (max) $\quad \mu = 100 \quad r_p = 85{,}000$ ohms $\quad R_o = 65{,}000$ ohms

FIND: (*a*) i_p (*b*) e_p

SOLUTION:

(*a*)
$$i_p = \frac{\mu e_g}{R_o + r_p} = \frac{100 \times 1}{65{,}000 + 85{,}000} = 0.000666 \text{ amp} = 0.666 \text{ ma (max)}$$

(*b*)
$$e_p = \text{VA} \times e_g = 43.3 \times 1 = 43.3 \text{ volts (max)}$$

or
$$e_p = i_p R_o = 0.666 \times 10^{-3} \times 65{,}000 = 43.3 \text{ volts (max)}$$

EXAMPLE 14-8 A certain pentode is operated as a voltage amplifier. The transconductance of the tube is 1,575 μmhos and the plate resistance is 700,000 ohms when the tube is operated with 100 volts on the plate, 100 volts on the screen grid, and with a grid bias of 3 volts. (*a*) If the plate load is a tuned circuit, what must its effective impedance be in order to obtain a voltage amplification per stage of 90? (*b*) What is the a-c component of the plate current when a 20-mv a-c signal is applied to the grid circuit of the tube? (*c*) What amount of voltage will be developed across the output load resistor?

GIVEN: $g_m = 1{,}575\ \mu\text{mhos}$ $r_p = 700{,}000$ ohms $\text{VA} = 90$ $e_g = 20$ mv

FIND: (*a*) Z_o (*b*) i_p (*c*) e_p

SOLUTION:

(*a*) $$Z_o = \frac{\text{VA}\ r_p}{g_m r_p - \text{VA}} = \frac{90 \times 700{,}000}{(1{,}575 \times 10^{-6} \times 700{,}000) - 90} = 62{,}222 \text{ ohms}$$

(*b*) $$i_p = g_m e_g \frac{r_p}{Z_o + r_p} = \frac{1{,}575 \times 10^{-6} \times 20 \times 10^{-3} \times 7 \times 10^5}{62{,}222 + 700{,}000} = 28.9\ \mu\text{a}$$

(*c*) $$e_p = i_p Z_o = 28.9 \times 10^{-6} \times 62{,}222 = 1.798 \text{ volts}$$

or $$e_p = \text{VA}\ e_g = 90 \times 20 \times 10^{-3} = 1.8 \text{ volts}$$

14-10 Gas Tubes

Types of Cathodes. In the manufacture of high-vacuum tubes, as much of the air as possible is removed from inside the envelope. In the manufacture of gas-filled tubes, a high-vacuum tube is first produced and then gas is added under low pressure. The gases most frequently used are argon, neon, xeon, and mercury vapor.

Two basic types of gas-filled tubes are used: (1) the *hot-cathode type* and (2) the *cold-cathode type*. In the hot-cathode type, the cathode is heated in the same manner as thermionic vacuum tubes; in the cold-cathode type, the cathode is not purposely heated but may become hot as it is bombarded by moving particles. The symbolic representation of several types of gas-filled tubes is shown in Fig. 14-18. The small dot within the circle indicates that the tube is gas filled.

Electrical Conduction in Gas-filled Tubes. In a gas-filled tube, the electron stream from the cathode will encounter gas molecules on its way to the plate. When an electron collides with a gas molecule, the energy transmitted by the collision may cause the molecule to release an electron. This free electron may then join the original stream of electrons moving toward the plate and thus is capable of releasing other electrons through

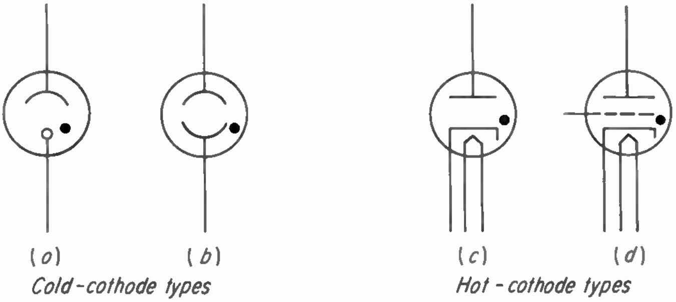


Fig. 14-18 Symbolic representation of several types of gas-filled tubes. (a) Voltage-regulator. (b) Neon-glow. (c) Gas-diode. (d) Thyratron.

collision with other gas molecules. This process, which is cumulative, is called *ionization*. The molecule that has lost an electron is called a *positive ion*. The tube in its ionized state contains gas molecules, positive ions, and free electrons within its envelope. The heavier positive ions are attracted toward the negative cathode, and while moving toward it they attract electrons from the space charge. The positive gas ions in the vicinity of the cathode also neutralize a portion of the space charge. Electrons will therefore flow from cathode to plate in a gas tube with less opposition than in a high vacuum tube.

A voltage source between the plate and cathode supplies the energy required to dislodge electrons from their atomic orbits to produce ionization. There is a definite voltage value for each type of gas-filled tube at which ionization starts. At the instant ionization occurs, large currents will flow at relatively low values of plate voltage. The voltage at which ionization commences is called *ionization potential*, *striking potential*, or *firing point*. After ionization has started, the action is maintained even at a voltage considerably lower than the firing point. However, a minimum amount of voltage is required to maintain ionization. Should the voltage across the tube drop below this minimum value, the gas will deionize and conduction will stop. The voltage at which current ceases to flow is called the *deionizing potential* or the *extinction potential*.

A gas-filled tube has almost infinite resistance before ionization and a very low resistance after ionization. This characteristic makes possible its use as an electronic switch that closes at a certain value of voltage and permits current to flow and opens at some lower value of voltage and blocks the flow of current.

Cold-cathode Gas Diodes. In this type of tube, the structure of the electrodes may be such that (1) current is permitted to flow in only one direction, as in the voltage-regulator tube of Fig. 14-18*a*, or (2) the cathode and plate may have the same size and shape to permit the tube to conduct in either direction, as in the neon-glow lamp of Fig. 14-18*b*. In this type of tube, the cathode is not heated; hence no electrons are emitted to aid in the ionization process. However, some ions and free electrons are always present in gases in their normal state. When a positive potential is applied to the plate, the free electrons and negative ions will drift toward the plate while the positive ions will move toward the cathode. During the movement of these charged particles, numerous collisions will take place, resulting in the release of additional electrons and the production of additional ions. Since this process is cumulative, ionization eventually takes place. However, the firing potential for this type of tube will be much higher than for a hot-cathode gas tube.

Voltage Regulation. The characteristics of the cold-cathode gas diode may be used to obtain voltage regulation. The voltage-regulator tube of Fig. 14-19 will maintain the output voltage relatively constant regardless

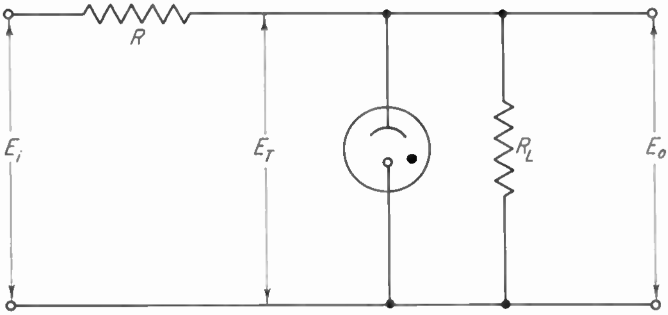


Fig. 14-19 Voltage-regulator circuit.

of variations in load current or input voltage. When the load current increases, the voltage drop across the limiting resistor R also increases and reduces the voltage across the tube. A reduction of the tube voltage will cause a decrease in the tube current, which, in turn, decreases the voltage drop across R. The voltage across the tube will therefore remain almost constant. When the input voltage varies, the degree of ionization and the tube resistance will change, which in turn maintains a constant voltage across the tube. Since the tube is across the output resistor R_L, the voltage of E_T will be equal to E_o.

Hot-cathode Diodes. Most high-vacuum diodes draw currents of less than 1 amp, while some gas-filled diodes are capable of handling currents of several hundred amperes. The gas-filled diode is therefore used whenever large quantities of rectified current are required. The gas used in this type of rectifier may be any one of the gases previously mentioned; however, mercury vapor is most commonly used. A small amount of liquid mercury is vaporized by the hot cathode, causing mercury vapor to form inside the tube. These tubes are not capable of supplying their rated output until all the mercury has been completely vaporized. Before the tube starts conducting, the relatively high voltage between the plate and cathode causes a large increase in the electron velocity. These high-velocity electrons cause the gas ions to acquire a higher positive charge, thus causing them to bombard the cathode with greater impact. If this action is permitted to continue for even a short period of time, the force of this bombardment is high enough to disintegrate the surface of the emitter. Therefore the plate voltage should not be applied until the tube has had sufficient time for the mercury to become completely vaporized.

Thyratrons. Adding a control grid to the hot-cathode gas diode produces a gas-filled triode, called a *thyratron.* The function of the grid in this tube is similar to that of the control grid in a vacuum tube, but the resultant control action is entirely different. The grid-control action of a typical thyratron is shown in Fig. 14-20. From this curve, it can be seen that, if the grid bias is -10 volts and 800 volts is applied to the plate, conduction will not start until the grid bias is reduced to approximately -8 volts, and if only 200 volts is applied to the plate, the grid bias must be reduced to approximately -3 volts. When conduction starts, (1) positive ions are formed as a result of the collisions between electrons and gas molecules, (2) some of these positive ions are attracted to the negative grid and form a positive-ion sheath around it, and (3) some of the positive ions move toward the cathode and neutralize the space charge. As a result of these three actions, the grid loses control and the current rises rapidly to a high value.

Because of its grid-control action, thyratrons have many practical applications in industry. They may be used as (1) controlled rectifiers to supply variable direct current or voltage from an a-c power source; (2) inverters to supply a-c power from a d-c power source; (3) electronic switches in counting, triggering, and control circuits; (4) frequency changers; (5) sawtooth generators.

14-11 Phototubes

Principle of Operation. A *photoelectric tube*, commonly called a *phototube*, is capable of converting light energy directly into electrical energy. When energy in the form of light strikes a photosensitive metal, electrons are emitted from this metal under the impact of the energy of the light rays. This method of producing electron emission is called *photoelectric emission* and is the basic principle of operation of phototubes.

Construction. The basic phototube consists of two electrodes that are placed in an evacuated glass envelope. One of the electrodes emits electrons when light rays strike its surface, and it is called the *cathode*. The other electrode attracts these electrons to it when a positive voltage is applied to its terminal, and it is called the *plate*. The plate usually consists of a small rod that is placed in the center of the tube. The cathode is a half cylinder that is placed in the tube so that it surrounds the plate. The inside surface

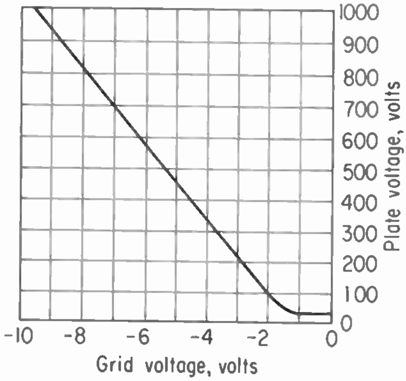


Fig. 14-20 Grid-control characteristics of a typical thyratron.

of the cathode is covered with multiple layers of a photosensitive metal such as cesium.

The sensitivity of a phototube is dependent on the frequency, or color, of the light used to excite the cathode. Phototubes are manufactured to provide sensitivity characteristics for different types of application. For example, some tubes are designed to be sensitive to blue light, some to red light, while others have response characteristics that are similar to those of the human eye.

Vacuum and Gas Phototubes. Phototubes may be obtained with a high vacuum or be gas filled. The amount of current that a phototube is capable of passing for a given amount of cathode illumination is higher for gas-filled tubes, because of ionization, than for vacuum tubes. The positive gas ions will also strike the cathode and thus produce an appreciable amount of secondary emission that will also increase the sensitivity of the tube. Gas phototubes are therefore generally used where high-sensitivity requirements are important, such as the reproduction of sound from sound-motion-picture film. Vacuum phototubes are more stable and less easily damaged, have a higher internal resistance, and are used for light-operated relay applications.

QUESTIONS

1. What are the important characteristics of electron tubes?
2. Define the following terms: (*a*) cathode, (*b*) filament, (*c*) heater, (*d*) directly heated cathode, (*e*) indirectly heated cathode.
3. Explain the purpose and theory of operation of (*a*) the cathode, (*b*) the heater.
4. (*a*) What are the advantages of directly heated cathodes? (*b*) Where are they generally used?
5. What are the advantages of indirectly heated cathodes?
6. Explain the following terms: (*a*) plate, (*b*) diode, (*c*) duodiode.
7. Explain the rectifier action of a diode.
8. Explain the following terms: (*a*) space charge, (*b*) emission current, (*c*) saturation current.
9. Explain the following terms: (*a*) control grid, (*b*) triode, (*c*) grid bias.
10. Draw a circuit diagram for a triode showing the connections for the heater, plate, and grid-bias power supplies. Indicate the direction of the electron flow on the diagram.
11. Explain the action of the triode.
12. Explain the action of a triode for the following conditions: (*a*) grid bias but no signal input, (*b*) grid bias and a varying signal input, (*c*) positive grid voltage.
13. In the circuit of Fig. 14-5, what is the phase relation between the following quantities: (*a*) e_g and i_b? (*b*) e_g and e_b? (*c*) e_b and i_b?
14. Why should the grid bias of a tube always be greater than the maximum value of the input signal voltage?
15. What are the advantages of characteristic curves over the tabular listing of tube characteristics?
16. (*a*) What is meant by the tube constants? (*b*) Name the three tube constants.

17. (*a*) What does the amplification factor of a tube represent? (*b*) What factors affect the amplification factor of a tube?
18. (*a*) What does the plate resistance of a tube represent? (*b*) What factors affect the plate resistance of a tube?
19. (*a*) What does the transconductance of a tube represent? (*b*) What factors affect the transconductance of a tube?
20. What is the relation between the transconductance of a tube and its operating characteristics?
21. (*a*) What is a tetrode? (*b*) Describe the location and construction of the screen grid.
22. (*a*) What is meant by interelectrode capacitance? (*b*) Why is it necessary to keep these capacitances at a minimum?
23. What is meant by (*a*) regeneration? (*b*) Degeneration?
24. How does the addition of a screen grid eliminate feedback?
25. (*a*) What is a pentode? (*b*) Describe the location and construction of the suppressor grid.
26. (*a*) What is meant by multiunit tubes? (*b*) What are their advantages?
27. Describe two systems of tube-pin and socket-connection numbering.
28. (*a*) Describe the constant-voltage-generator form of equivalent vacuum-tube circuit. (*b*) When is this type of equivalent circuit most useful?
29. (*a*) What is meant by the voltage amplification per stage? (*b*) How does the maximum possible voltage amplification of a triode amplifier stage compare with the amplification factor of the tube?
30. (*a*) How does a variation in the plate resistance affect the voltage amplification of the stage? (*b*) How does a variation in the load impedance affect the voltage amplification of the stage?
31. (*a*) Describe the constant-current-generator form of equivalent vacuum-tube circuit. (*b*) When is this type of equivalent circuit most useful?
32. Describe what is meant by (*a*) a hot-cathode gas tube, (*b*) a cold-cathode gas tube.
33. Explain the following terms: (*a*) ionization, (*b*) positive ion, (*c*) firing point, (*d*) extinction potential.
34. Describe the electric conduction in a cold-cathode gas diode.
35. Explain how a cold-cathode gas diode may be used as a voltage regulator.
36. Describe the electric conduction in a hot-cathode gas diode.
37. (*a*) Explain the operation of a thyratron. (*b*) Name five applications of the thyratron.
38. Explain what is meant by (*a*) a phototube, (*b*) photoelectric emission.
39. Describe the construction of a basic phototube.
40. What are the characteristics of (*a*) vacuum phototubes? (*b*) Gas-filled phototubes?

PROBLEMS

1. A high-mu triode is operated at its normal heater voltage, a grid bias of 1.75 volts, and a plate potential of 220 volts. Determine the amplification factor if the characteristics of this tube are such that (1) the plate current is 2 ma when $e_b = 200$ volts and $e_c = -1.5$ volts and (2) when the plate voltage is increased to 240 volts, the grid bias has to be increased to 2 volts in order to maintain the plate current at 2 ma.

2. A sharp-cutoff beam triode is operated at its normal heater voltage, a grid bias of 4.5 volts, and a plate potential of 22,500 volts. Determine the amplification factor if the characteristics of this tube are such that (1) the plate current is 2.0 amp when $e_b = 20{,}000$ volts and $e_c = -3.25$ volts and (2) when the plate voltage is increased to 25,000 volts, the grid bias has to be increased to 5.75 volts in order to maintain the plate current at 2 amp.
3. Determine the plate resistance of the electron tube used in Prob. 1 if the characteristics of this tube are such that (1) if the plate voltage is decreased to 200 volts, the plate current will be 1.6 ma; (2) if the plate voltage is increased to 240 volts, the plate current will increase to 2.4 ma; and (3) the grid bias is maintained at 1.75 volts.
4. Determine the plate resistance of the electron tube used in Prob. 2 if the characteristics of this tube are such that (1) if the plate voltage is decreased to 20,000 volts, the plate current will be 0.75 amp; (2) if the plate voltage is increased to 25,000 volts, the plate current will increase to 1.5 amp; and (3) the grid bias is maintained at 7.0 volts.
5. Determine the transconductance of the electron tube used in Prob. 1 if the characteristics of this tube are such that (1) if the grid bias is decreased to 1.5 volts, the plate current will be 2.2 ma; (2) if the grid bias is increased to 2.0 volts, the plate current will decrease to 1.5 ma; and (3) the plate voltage is maintained at 220 volts.
6. Determine the transconductance of the electron tube used in Prob. 2 if the characteristics of this tube are such that (1) if the grid bias is decreased to 3.75 volts, the plate current will be 2.12 amp; (2) if the grid bias is increased to 5.25 volts, the plate current will decrease to 1.62 amp; and (3) the plate voltage is maintained at 22,500 volts.
7. An electron tube when operated with 200 volts at its plate and with a grid bias of 2 volts has a plate resistance of 50,000 ohms and an amplification factor of 70. Determine the transconductance of this tube for these conditions.
8. What is the transconductance of each section of a twin triode when it is operated at such values of voltage that its amplification factor is 20 and its plate resistance is 6,450 ohms?
9. When the tube of Prob. 7 is operated with 100 volts at its plate and with a grid bias of 1 volt, its plate resistance is 53,000 ohms and its transconductance is 1,400 μmhos. Determine the amplification factor of this tube for these conditions.
10. What is the amplification factor of each section of a twin triode when it is operated at such values of voltage that its plate resistance is 15,000 ohms and its transconductance is 4,000 μmhos?
11. When the tube of Prob. 7 is operated with 200 volts at its plate and with a grid bias of 1 volt, its transconductance is 2,500 μmhos and its amplification factor is 80. Determine the plate resistance of this tube for these conditions.
12. What is the plate resistance of each section of a twin triode when it is operated at such values of voltage that its transconductance is 6,000 μmhos and its amplification factor is 36?
13. A certain triode, used as a voltage amplifier, has a plate resistance of 36,000 ohms and an amplification factor of 80 when operated with 150 volts on its plate and a grid bias of 1.5 volts. What is the voltage amplification of an amplifier stage using such a tube under these operating conditions when the load resistance is 64,000 ohms?

14. A certain triode, used as a voltage amplifier, has a plate resistance of 6,500 ohms and an amplification factor of 20 when operated with 250 volts on its plate and a grid bias of 2 volts. What is the voltage amplification of an amplifier stage using such a tube under these operating conditions when the load resistance is 13,500 ohms?
15. A 0.5-volt a-c signal (maximum value) is applied to the input side of the tube and amplifier circuit of Prob. 13. (*a*) What is the a-c component of the plate current? (*b*) What amount of voltage will be developed across the output load resistor?
16. A 1.5-volt a-c signal (maximum value) is applied to the input side of the tube and amplifier circuit of Prob. 14. (*a*) What is the a-c component of the plate current? (*b*) What amount of voltage will be developed across the output load resistor?
17. It is desired that the amplifier stage used in Prob. 13 produce a voltage amplification of 60. What value of load resistance is required if all other operating values are maintained the same as in Prob. 13?
18. It is desired that the amplifier stage used in Prob. 14 produce a voltage amplification of 15. What value of load resistance is required if all other operating values are maintained the same as in Prob. 14?
19. A 0.5-volt a-c signal (maximum value) is applied to the input side of the tube and amplifier circuit of Prob. 17. (*a*) What is the a-c component of the plate current? (*b*) What amount of voltage will be developed across the output load resistor?
20. A 1.5-volt a-c signal (maximum value) is applied to the input side of the tube and amplifier circuit of Prob. 18. (*a*) What is the a-c component of the plate current? (*b*) What amount of voltage will be developed across the output load resistor?
21. A certain pentode is operated as a voltage amplifier. The operating voltages at this tube are such that $g_m = 1{,}265$ μmhos and $r_p = 300{,}000$ ohms. (*a*) If the plate load is a tuned circuit, what must be its effective impedance in order to obtain a voltage amplification per stage of 80? (*b*) What is the a-c component of the plate current when a 100-mv a-c signal is applied to the grid circuit of the tube? (*c*) What amount of voltage will be developed across the output load impedance?
22. A certain pentode is operated as a voltage amplifier. The operating voltages at this tube are such that $g_m = 11{,}000$ μmhos and $r_p = 90{,}000$ ohms. (*a*) If the plate load is a tuned circuit, what must be its effective impedance in order to obtain a voltage amplification per stage of 70? (*b*) What is the a-c component of the plate current when a 50-mv a-c signal is applied to the grid circuit of the tube? (*c*) What amount of voltage will be developed across the output load impedance?
23. The load impedance of the pentode amplifier circuit of Prob. 21 is increased to 100,000 ohms. If the operating voltages are maintained the same as in Prob. 21, determine (*a*) the voltage amplification per stage, (*b*) the a-c component of the plate current when a 100-mv a-c signal is applied to the grid circuit of the tube, (*c*) the amount of voltage developed across the output load impedance.
24. The load impedance of the pentode amplifier circuit of Prob. 22 is increased to 7,500 ohms. If the operating voltages are maintained the same as in Prob. 22, determine (*a*) the voltage amplification per stage, (*b*) the a-c component of the plate current when a 50-mv a-c signal is applied to the grid circuit of the tube, (*c*) the amount of voltage developed across the output load impedance.

Chapter 15

Transistors

A transistor is basically a resistor that amplifies electrical impulses as they are transferred through it from its input to its output terminals. The name *transistor* is derived from the words *transfer* and *resistor*. The transistor has many advantages among which are its (1) low current requirement, (2) small size, (3) light weight, (4) long operating and shelf life, (5) elimination of warm-up time, (6) mechanical strength, and (7) photosensitivity. Transistors may be considered as a circuit element in the same manner as electron tubes.

15-1 Transistors

The transistor is a solid-state electronic device whose operation depends on the flow of electric charges carried within the solid. Two fundamental actions are involved: (1) generation of carriers within the solid and (2) control of these carriers within the solid.

A triode junction transistor consists of a single crystal, the leads, and an envelope. The crystal is essentially a three-layer unit. However, the layers are not separate pieces but are areas within the crystal which have slightly different electrical characteristics. Transistors are made from a *semiconductor* whose electrical properties are midway between that of a conductor such as silver and an insulator such as porcelain. A semiconductor under one condition may act as a conductor allowing an easy flow of current, while under another condition it may act as an insulator and virtually block the flow of current.

In a conductor, an electric current is thought of only as the movement of free electrons along the conductor. In a semiconductor, current consists of the movement of free electrons and holes. When an electron leaves an atom, the resulting charge on the atom is positive. The absence of an electron from the atomic structure in a crystal is referred to as a *hole*. The application of an electric field will cause electrons and holes to drift in opposite directions. The holes being positively charged will drift toward the negative terminal, while the electrons being negatively charged will drift toward the positive terminal.

15-2 Physical Concepts of Solids

Conductors and Insulators. Solids having a low electrical resistivity at room temperature are called *conductors*. The solids in this group include most of the common metals such as copper, aluminum, and silver. The resistivity of copper is of the order of 1.6×10^{-6} ohm per centimeter cube. Solids having a high electrical resistivity at room temperature are called *insulators*. Their resistivity is of the order of 10^9 to 10^{18} ohms per centimeter cube. Examples of solids in this group are porcelain, quartz, glass, and mica.

Semiconductors. Solids having a value of resistivity midway between that of conductors and insulators are called *semiconductors*. Their resistivity varies nonlinearly with temperature changes, and these solids may possess either a positive or negative temperature coefficient. The resistance characteristics depend largely upon the amount of impurities in the material. Examples of solids in this group are elements such as germanium and silicon and compounds such as zinc oxide and copper oxide.

When the particle concentration in a given semiconductor consists of both electrons and holes in approximately equal numbers, the material is called an *intrinsic semiconductor*. When the holes predominate, the material is called a *positive* or *P-type semiconductor*. When the electrons predominate, the material is called a *negative* or *N-type semiconductor*. An intrinsic semiconductor may be given P- or N-type characteristics by adding various substances called *impurities* to the base material.

Energy Levels. In order for conduction to take place, there must be a movement of electrons. In atomic theory, the electrical properties of the elements are explained by the concept of energy bands. The electrons in the outer orbit of an atom can be moved with the least amount of energy and are called *valence electrons*. Valence electrons have definite energy levels or bands (Fig. 15-1), and the conductivity of an element is determined by the energy required to move its valence electrons from their normal energy level, or *valence band*, to their highest energy level, the *conduction* or *energy band*. The distance a valence electron moves in its travel from

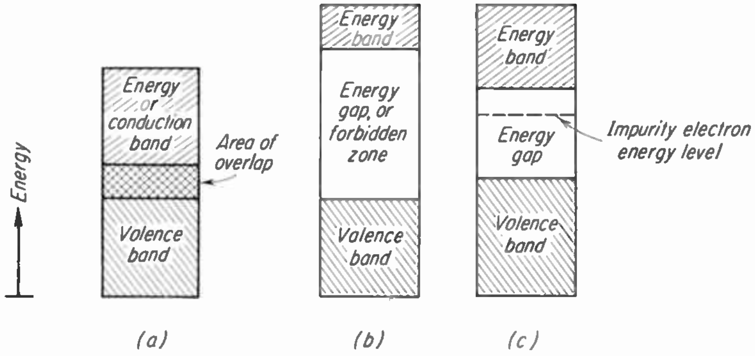


Fig. 15-1 Energy and valence bands. (a) Conductor. (b) Insulator. (c) Semiconductor.

the valence band to the energy band varies with each type of atom. The energy gap separating the valence and conduction bands in an insulator is extremely large, and it is very difficult for a valence electron to reach the energy band. In a conductor the valence and energy bands overlap, and the valence electrons are available for conduction. In a semiconductor the energy gap is very small, and the thermal energy of the valence electrons at room temperature is sufficient to permit an appreciable amount of conduction. Because an electron cannot remain in the space between the valence and energy bands, this region is sometimes referred to as the *forbidden zone.*

15-3 Conduction in Crystals

Conduction. In a semiconductor the atoms of the material form a lattice structure called a *crystal.* A crystal may be (1) intrinsic, that is, having approximately equal numbers of positive and negative charges, (2) N-type, that is, having an excess of negative charges or electrons, or (3) P-type, that is, having an excess of positive charges or holes (Fig. 15-2). Conduction may take place in a crystal by the movement of either electrons or holes. In negative conduction the electrons flow toward the positive terminal of the power source, while in positive conduction the holes flow toward the negative terminal of the power source.

Carbon as a Crystal. Carbon in its common (noncrystalline) form is a conductor; its atomic structure is shown in Fig. 2-2. Carbon is also found in crystallized form as the *diamond.* Although carbon and the diamond have the same atomic structure, carbon is a conductor and the diamond is an insulator. The difference in their electrical conductivity is explained by the actions of their valence electrons. In ordinary carbon the atom has four valence electrons that can move quite freely from one atom to another to constitute a flow of current. In a diamond the arrangement of the atoms forms a lattice-type structure and the valence electrons of neighboring atoms form covalent bonds (Figs. 15-2*a* and 15-3). Because of these characteristics, the valence electrons have a strong bond with their nuclei and cannot easily be dislodged from their orbits. Therefore, the diamond is a good insulator.

Covalent Bonds. In a crystalline structure each valence electron of an atom, orbiting around its nucleus, coordinates its motion with that of a valence electron from an adjacent atom (Figs. 15-2 to 15-5). The association of these electrons with each other is called a *covalent bond.* Figure 15-3 shows that valence electron *a* from atom *A* and valence electron *b* from atom *B* form a covalent bond between atoms *A* and *B*. In a similar manner covalent bonds are formed between atoms *A* and *C*, *A* and *D*, *A* and *E*, *F* and *C*, *F* and *G*, etc.

Lattice Structures. The atoms of a pure crystalline material form a lattice-type pattern as represented by Fig. 15-2*a*. Figure 15-3 is one of several methods of presenting an enlargement of a section of the lattice pattern of Fig. 15-2*a*. Although carbon is used here to illustrate the princi-

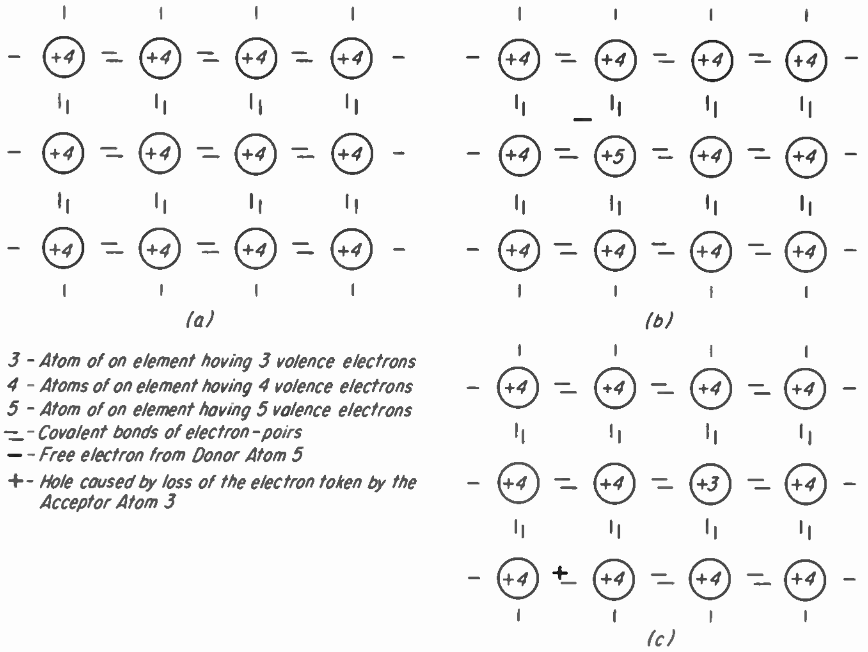


Fig. 15-2 Lattice structure of a crystal. (a) Pure crystal. (b) Crystal containing an impurity having five valence electrons. (c) Crystal containing an impurity having three valence electrons.

ples of the lattice-type structure and covalent bonding, there are certain other elements having atomic structures that will act in the same manner. Two elements frequently used in the construction of transistors, namely silicon and germanium, also have four valence electrons and can be produced in crystalline form. The preceding explanations also apply to these materials. The following discussion, although referring only to silicon, will also apply to germanium.

Conductivity of a Pure Crystalline Material. In a pure crystalline material all atoms have four valence electrons and form complete sets of valence-bond pairs (Figs. 15-2*a* and 15-3). Under this condition each atom behaves as though its outer ring contained eight electrons and all electrons had a strong bond with their nuclei. The resulting lattice structure is in equilibrium as there are no excess positive or negative charges and the material acts as an insulator.

Conductivity of an Impure Crystalline Material. If a suitable second material, hereafter called an *impurity*, is added to a pure crystalline material, it will produce a semiconductor material that will perform satisfactorily in a transistor. Adding an impurity to a pure material is also referred to as *doping* the material.

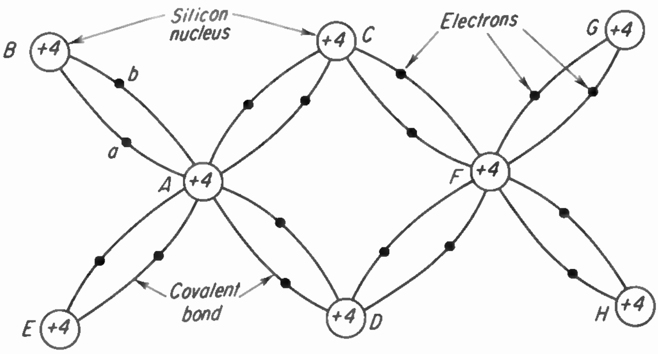


Fig. 15-3 Lattice structure of a pure silicon crystal illustrating the covalent bonds between adjacent atoms.

Silicon crystals are produced with an impurity ratio of less than one part per 100 million. The amount of impurity that is added is very small and also very critical. For example, if added in the ratio of one atom of impurity to every 100 million atoms of silicon, the resistivity drops from 60 to 3.8 ohms per centimeter cube. This value is satisfactory for transistor use. However, if the ratio is increased to 10 impurity atoms for every 100 million atoms of silicon, the resistivity drops to 0.38 ohm per centimeter cube. This value is too low for transistor use.

Donor Impurities. If an impurity material having five valence electrons (for example, arsenic or antimony) is added to pure silicon, four valence electrons of each impurity atom will form four covalent bonds with one valence electron from each of four nearby silicon atoms in order to keep the lattice structure intact. The fifth valence electron of the impurity atom is free to wander about the crystal in a manner much like the movement of free electrons in an ordinary metallic conductor (Figs. 15-2*b* and 15-4).

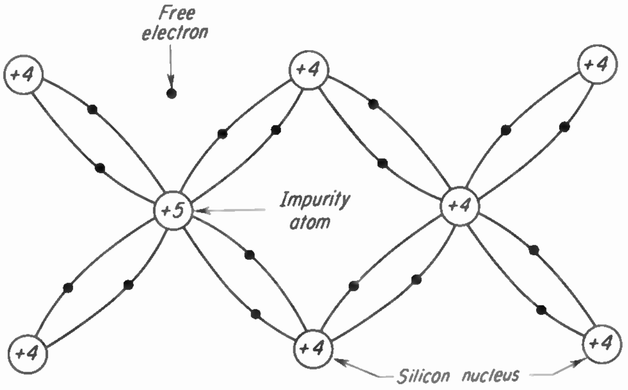


Fig. 15-4 Effect of adding a donor impurity to pure silicon.

As even a very small crystal will contain many billion atoms, an impurity ratio as low as one part per 100 million produces a sufficient number of free electrons to carry the amount of current required for transistor circuit applications. If the impurity ratio is increased, more electrons will be released to wander about the crystal; consequently, the conductivity of the silicon increases, and its resistivity decreases. Silicon having an excess of electrons is called *N-type silicon.* When the impurity donates electrons to the crystal conductivity, it is called a *donor impurity.* After contributing a free electron to the crystal structure, a donor impurity atom has a positive charge.

Acceptor Impurities. If an impurity material having three valence electrons (for example, gallium or indium) is added to pure silicon, the three valence electrons of each impurity atom will form three complete covalent bonds with one valence electron from each of three adjacent silicon atoms. If the crystal lattice structure is to remain intact, the impurity atom will have to borrow (or *accept*) a valence electron from a nearby silicon atom in order to form a fourth covalent bond with its fourth adjacent silicon atom (Figs. 15-2*c* and 15-5). The nearby silicon atom that provides the electron for the fourth covalent bond pair of the impurity atom now suffers a deficiency of one electron in one of its covalent bond pairs. This deficiency of an electron in a covalent bond group is called a *hole.* Holes have a positive charge, can move about from atom to atom, and can conduct a current just as negative electron charges do. If the impurity ratio is increased, more holes are formed and the conductivity of the crystal increases. Silicon having an excess of holes is called a *P-type silicon.* When the impurity accepts electrons in order to keep the lattice structure of the crystal intact, it is called an *acceptor impurity.* An acceptor impurity has a negative charge.

Negative and Positive Conduction. An N-type crystal has a large number of free electrons that can become current carriers. Current resulting

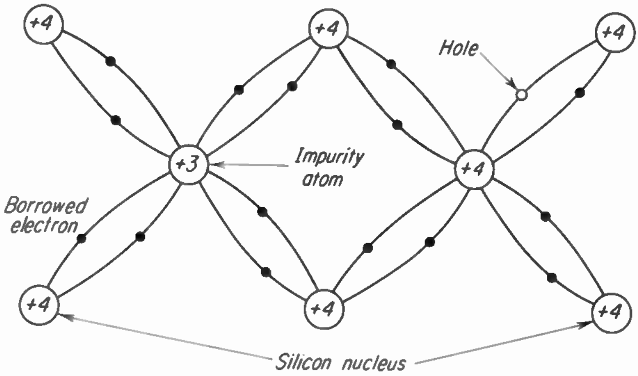


Fig. 15-5 Effect of adding an acceptor impurity to pure silicon.

from electron movement is called *negative conduction*. A P-type crystal has a large number of hole charges that also can become current carriers. Current resulting from hole movement is called *positive conduction*.

Majority and Minority Carriers. An N-type crystal will contain some stray hole charges and a P-type crystal will contain some stray electrons. When a crystal is connected in an electric circuit there will be some electron movement and some hole movement in both the N-type and P-type crystals. In the N-type crystal the electrons will constitute the *majority carriers* and the holes the *minority carriers*. In the P-type crystal the holes will constitute the *majority carriers* and the electrons the *minority carriers*.

Hole Injection. In the operation of certain transistors, a forward bias is applied to one portion of its crystal and a reverse bias to another portion. When an N-type material is involved, the forward bias causes free electrons to move from one portion of the transistor to another. The movement of an electron out of an N-type portion of the transistor temporarily creates a hole in that part. The creation of holes in this manner, called *hole injection*, plays an important part in the performance of the transistor. The holes will drift along the material, some as positive conduction current, others losing their identity owing to their recombining with available free electrons.

Solid-state Conduction. Units of N-type and P-type crystals have an excess of electrons and holes respectively, and hence are capable of transmitting electrons or holes through a solid material. The principles of conduction through solids are referred to as *solid-state physics*. The application of solid-state devices is referred to as *solid-state circuitry*.

15-4 The PN Junction or Diode

Characteristics. A PN junction is formed when sections of P-type and N-type silicon are joined together to produce a *silicon diode* (Fig. 15-6). The circles with negative symbols in the P-type silicon represent *acceptor atoms*. The holes, created by missing electrons in some covalent bond pairs, are positive charges and are represented by the positive symbols. The circles with positive symbols in the N-type silicon represent *donor atoms*. The free electrons, being negative charges, are represented by negative symbols.

Potential Barrier. Free electrons in the N section will be attracted by the holes in the P section, and vice versa. However, as soon as any electrons from the N section cross the junction they will be repelled by the negative charge of the acceptor atoms in the P section. Likewise, hole charges from the P section that cross the junction will be repelled by the positive charge of the donor atoms in the N section. As a result, the potential barrier soon established across the two surfaces of the junction prevents any large movement of either electrons or holes across the junction. The potential established at the junction depends on the amount of the impurity in the crystal and is in the order of 0.1 to 0.3 volt. This voltage is called a *poten-*

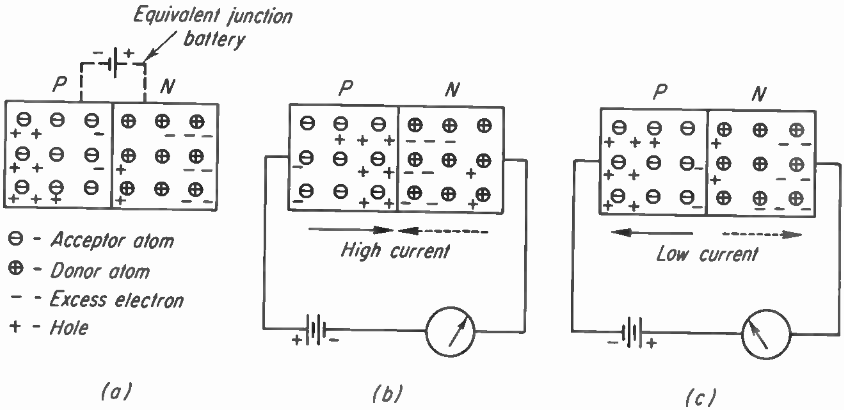


Fig. 15-6 Effect of junction bias voltage. (a) Neutral or zero bias. (b) Forward bias. (c) Reverse bias. The solid line arrows indicate the direction of hole current, and the broken line arrows indicate the direction of electron current.

tial barrier or a *potential hill,* and its effect is the same as if a battery was connected across the junction as shown in Fig. 15-6*a*.

Forward Bias. In order to transmit a current through the junction, the potential barrier across the junction must be neutralized. This may be done by connecting a power source across the two sections of silicon as shown in Fig. 15-6*b*. The voltage applied in this manner is called a *forward potential* or *forward bias.* Free electrons in the N section will be repelled by the negative force set up by the power source and will move toward the junction. At the same time the holes in the P section will be repelled by the positive force set up by the power source and will move toward the junction. The voltage of the power source imparts sufficient energy to these carriers to overcome the potential barrier at the junction and enables them to cross through the junction. Once the junction is crossed, free electrons from the N section will combine with holes in the P section, and holes from the P section will combine with free electrons in the N section. This action decreases the potential barrier at the junction. For each hole in the P section that combines with an electron from the N section, an electron from an electron-pair bond leaves the crystal and enters the positive terminal of the power source. This action creates a new hole that is forced to move toward the junction because of the electric field produced by the power source. For each electron in the N section that combines with a hole from the P section, an electron enters the crystal from the negative terminal of the power source. This constant movement of electrons toward the positive terminal and holes toward the negative terminal produces a high forward current I_f (Fig. 15-7*a*). It should be noted that the relatively high current in the external circuit is the sum of the positive and negative conduction currents.

Reverse Bias. If the polarity of the power source connected across the two sections of silicon is reversed, as shown in Fig. 15-6*c*, a *reverse potential* or *reverse bias* is obtained. The excess holes in the P section will be attracted by the negative force of the power source and will move away from the junction. At the same time the free electrons in the N section will be attracted by the positive force of the power source and will also move away from the junction. The net effect will be the same as increasing the voltage of the potential barrier and, as a result, theoretically the current through the junction will be zero. However, in actual practice a small amount of reverse current I_r will flow as is indicated by Fig. 15-7*a*.

Static Characteristics. The PN junction possesses the property of a rectifier, since the current resulting from a voltage applied in one direction across the junction is different from the current resulting from the same amount of voltage applied in the opposite direction across the junction. The static volt-ampere characteristic curve for a typical germanium diode is shown in Fig. 15-7*a*. The forward and reverse current curves are drawn to different voltage and current scales, and both are nonlinear over a considerable portion of their ranges. The forward-current curve has a sharp upward swing at a relatively low value of voltage. The reverse-current curve indicates that a relatively high voltage is required to produce even a very low value of current.

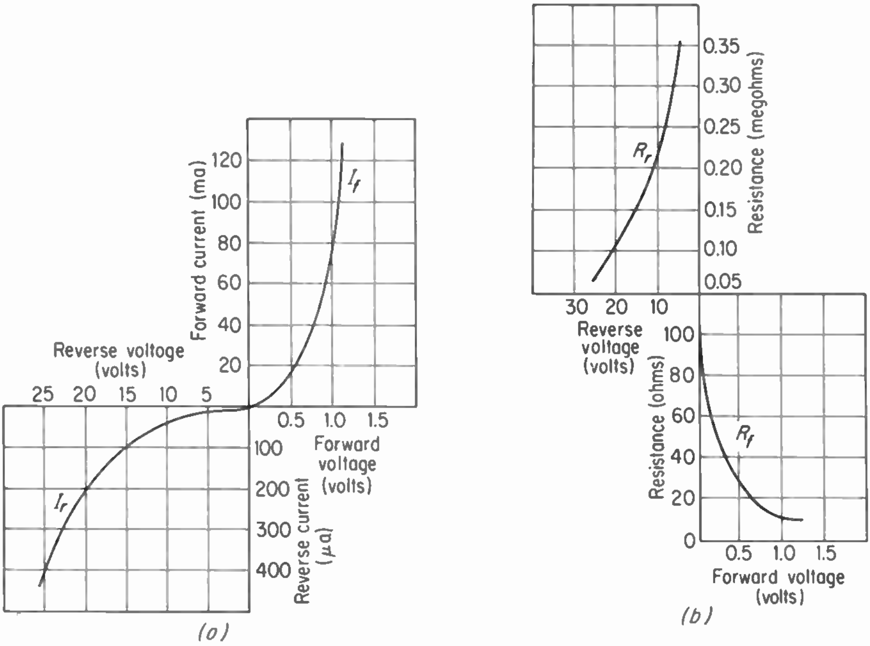


Fig. 15-7 Static characteristics of a PN junction diode. (a) Volt-ampere. (b) Volt-ohm.

The static voltage-resistance curve for a typical germanium diode is shown in Fig. 15-7*b*. This curve was obtained by dividing the voltage at a number of points on the curve of Fig. 15-7*a* by the current at these voltages. The forward and reverse resistance curves are drawn to different voltage and resistance scales. Increasing the forward voltage decreases the resistance to a low value, usually less than 100 ohms. Decreasing the forward voltage increases the resistance, and at zero voltage the resistance is in the order of hundreds of thousands of ohms.

15-5 Junction Transistors

Types of Junction. The characteristics of a semiconductor diode, triode, or tetrode will depend on the material and type of PN junction used. Germanium is usually better than silicon at high frequencies, while at high power levels silicon is generally better than germanium. The PN junction may be made in various ways; among these are (1) point contact, (2) grown, (3) diffused, (4) recrystallized, (5) alloyed, and (6) surface barrier (Fig. 15-8). There are many variations of these six methods, and in the commercial manufacture of transistors more than one method is generally used.

The Triode. When a layer of N-type germanium is joined to a PN junction, an NPN junction is formed (Fig. 15-9*a*), and when a layer of P-type germanium is joined to a PN junction, a PNP junction is formed (Fig. 15-10*a*). Either of these two junctions can be used as a triode junction transistor, and they are generally referred to as *NPN junction transistors* or *PNP junction transistors*. The center section is called the *base*, one of the outside sections the *emitter*, and the other outside section the *collector*.

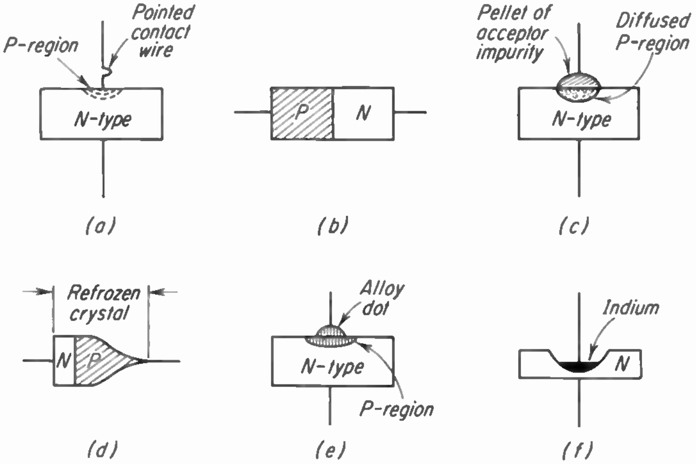
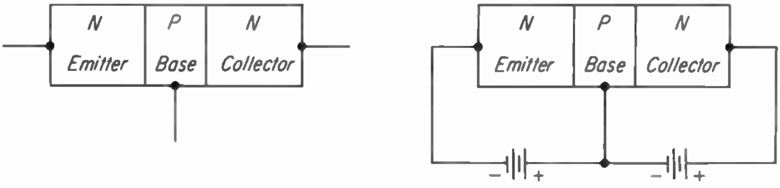


Fig. 15-8 PN junctions. (a) Point contact. (b) Grown. (c) Diffused. (d) Recrystallized. (e) Alloy technique. (f) Surface barrier.



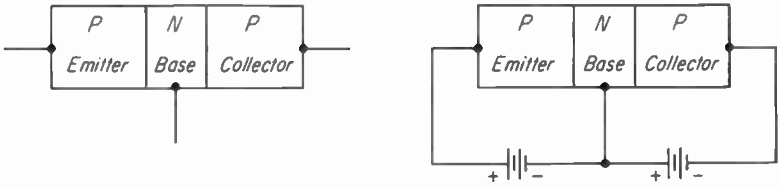
(a) (b)

Fig. 15-9 NPN junction transistor. (a) The junction and the name of each section. (b) Biasing potentials.

Transistor Biasing. In order to use the NPN transistor as an amplifier, the emitter-base junction is biased with a forward voltage and the base-collector junction with a reverse voltage (Fig. 15-9*b*). A relatively high current will flow through the emitter-base junction, and a relatively low current through the base terminal. The base current is very low, as only a small number of emitter electrons (usually less than 5 per cent) will combine with the base holes. Since the remaining electrons flow toward the collector, the emitter-collector current is high.

The operation of a PNP transistor is similar to the operation of an NPN transistor except that the bias voltage polarities are reversed and the current carriers are holes instead of electrons. The emitter-base junction is biased with a forward voltage, and the base-collector junction with a reverse voltage (Fig. 15-10*b*). The holes from the emitter will diffuse through the base to the base-collector junction, where the applied negative voltage will draw them to the collector terminal.

Symbol and Lead Identification. The symbols for triode transistors are shown in Fig. 15-11. Although there are several variations of symbols in use, these are frequently used and have been adopted by the IEEE. The horizontal line represents the base, and the two angular lines represent the emitter and collector. The arrowhead drawn on the emitter indicates the direction of current. In the PNP type, the arrow is drawn pointing toward the base (Fig. 15-11*a*), and in the NPN type, the arrow is drawn pointing away from the base (Fig. 15-11*b*).



(a) (b)

Fig. 15-10 PNP junction transistor. (a) The junction and the name of each section. (b) Biasing potentials.

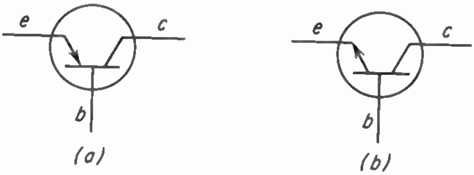


Fig. 15-11 Symbol identification for triode transistors. (a) PNP. (b) NPN.

The lead identification of a triode transistor varies with the manufacturer. Three systems in general use are shown in Fig. 15-12. When the leads of a transistor are in the same plane but unevenly spaced (Fig. 15-12*a*), they are identified by the position and spacing of the leads. The center lead is the base lead, and the emitter and collector leads are on either side of this lead. The collector lead is identified by the larger spacing existing between it and the base lead. When the leads on a transistor are in the same plane but evenly spaced (Fig. 15-12*b*), the center lead is the base, the lead identified by a dot the collector, and the remaining lead the emitter. When the leads are spaced around the circumference of a circle as shown in Fig. 15-12*c*, the center lead is the base lead. The collector and emitter leads are identified by their relative position to the base lead when viewed looking into the base of the transistor. Some manufacturers add a fourth lead that is electrically connected to the mounting base, or case, of the transistor. Other manufacturers connect the common electrode to the base, or case, and use only two leads.

15-6 Transistor Characteristics

Methods of Operation. The input and output impedances of a transistor vary with the manner in which it is connected into a circuit. Correct impedance match can be made by selecting the proper method of connection. The characteristics of a transistor will therefore depend upon its method of connection. Basically there are three methods of connection: (1) *common base*, (2) *common emitter*, (3) *common collector*. The transistor may be considered similar to a vacuum tube in the following respects: (1) The emitter and cathode both serve as the source of electron flow; (2) the base and control grid both serve to control the electron flow through the unit; (3) the collector and plate are normally part of the output circuit.

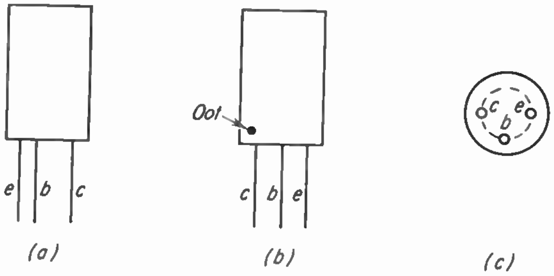


Fig. 15-12 Lead identification for triode transistors. (a) Unevenly spaced leads. (b) Evenly spaced leads. (c) Symmetrically spaced leads.

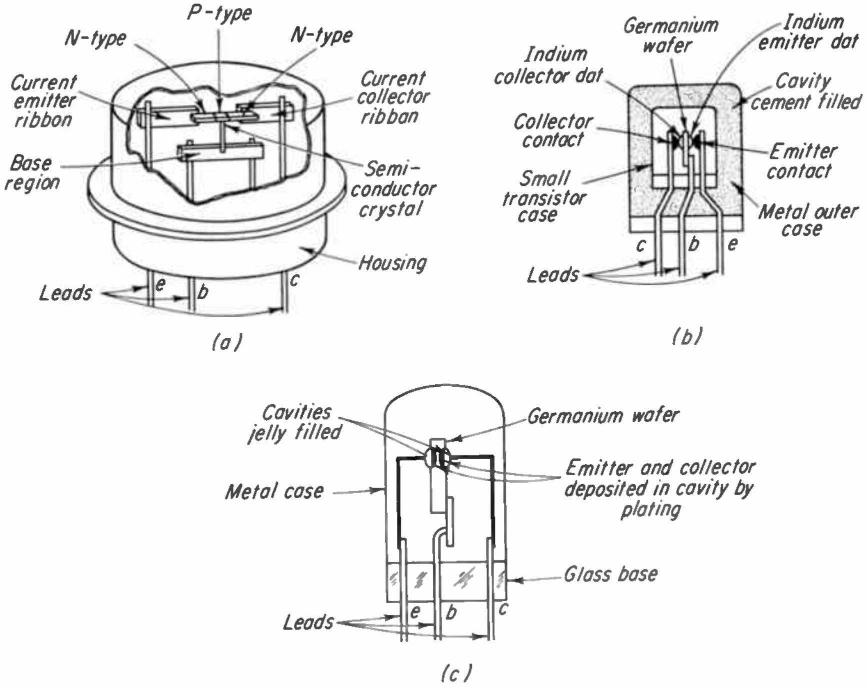


Fig. 15-13 Cutaway view of commercial transistors. (a) Grown junction. (b) Diffused junction. (c) Surface barrier.

Because of the comparable relation between the components of a transistor and those of a vacuum tube, the basic circuit operation of transistors may be compared with a comparable basic circuit of a vacuum tube (Fig. 15-14). However, in these comparable circuit operations it should be noted that (1) the transistor is current controlled (its collector characteristics are plotted with the control current I_b as the parameter, Fig. 15-16) and (2) the vacuum tube is voltage controlled (its plate characteristics are plotted with the control voltage E_c as the parameter, Fig. 14-13). In transistor operation, the term *common* is used to denote the electrode that is common to both the input and output circuits. Since the common electrode is usually grounded, it may also be referred to as a *grounded base, grounded emitter,* or *grounded collector.* The advantage of using the common-base connection is that it has a low input impedance and a high output impedance. The input impedance of the common-emitter connection is low to medium, and its output impedance is medium to high. The input impedance of the common-collector connection is high, and its output impedance is low.

Characteristic Curves. The major operating characteristics of a transistor are used to identify the electrical features and operating values of the transistor. These values may be listed in tabular form or plotted on graph

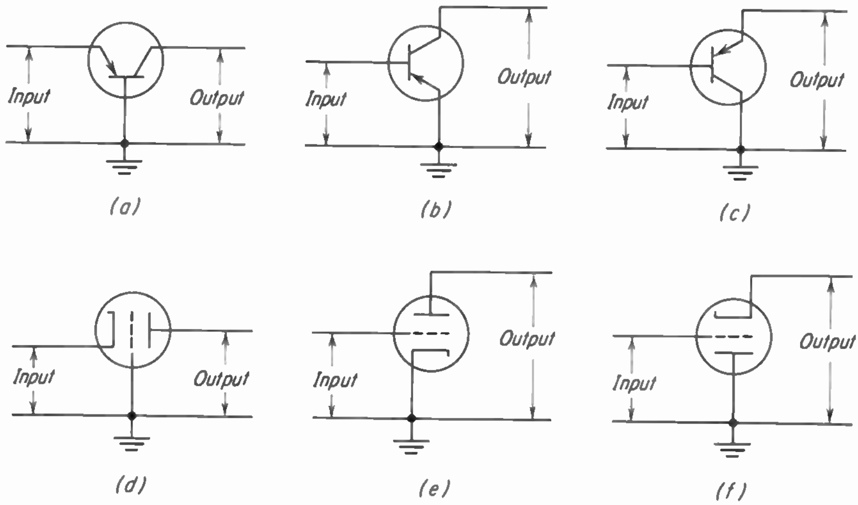


Fig. 15-14 Comparison of transistor and vacuum-tube operation. (a) Common base. (b) Common emitter. (c) Common collector. (d) Grounded grid. (e) Grounded cathode. (f) Grounded plate.

paper to form a curve. The two families of curves generally used are (1) collector current versus collector voltage for constant values of emitter current (Fig. 15-15) and (2) collector current versus collector voltage for constant values of base current (Fig. 15-16). The curves of Fig. 15-15 are generally used for grounded-base connected transistors, and those of Fig. 15-16 for grounded-emitter connected transistors. The shapes of the characteristic curves for transistors are similar to those for a pentode vacuum tube (Fig. 14-13). The plate current in a pentode is relatively independent of plate voltage and primarily dependent on the grid voltage. The collector current in a grounded-base connected transistor is relatively independent of collector voltage and primarily dependent on the emitter current. When the transistor is connected as a grounded emitter, the collector voltage has very little effect on the collector current for low values of base current. However, as the base current increases, the effect of the collector voltage on the collector current also increases.

Voltage Gain and Power Gain. The direct voltage gain in a transistor can be expressed as the product of the direct current gain and resistance gain, or

$$VG = IG \times RG \tag{15-1}$$

In a junction transistor, the collector current is equal to the difference between the emitter and base currents, and the resulting current represents a loss rather than a gain. The voltage gain is obtained because of the high ratio between the collector-base resistance and the emitter-base resistance.

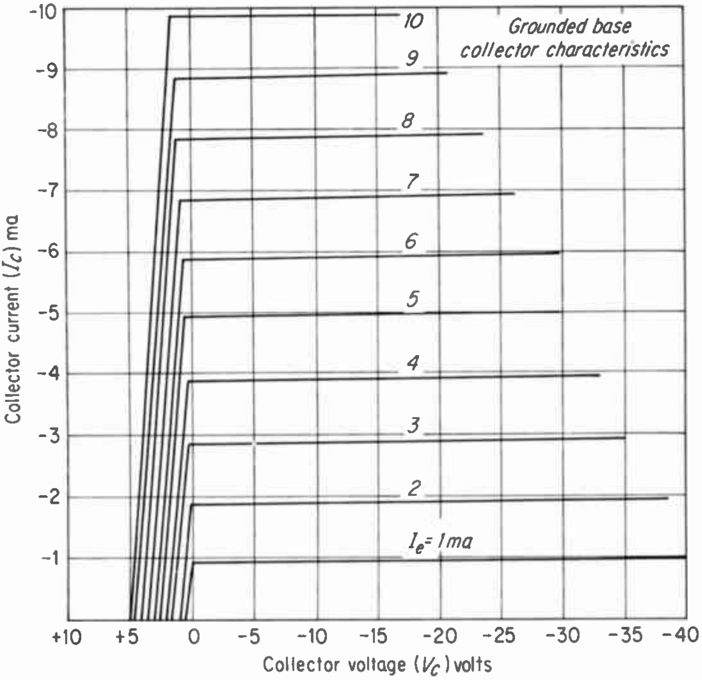


Fig. 15-15 Static common-base collector characteristics of a triode junction transistor.

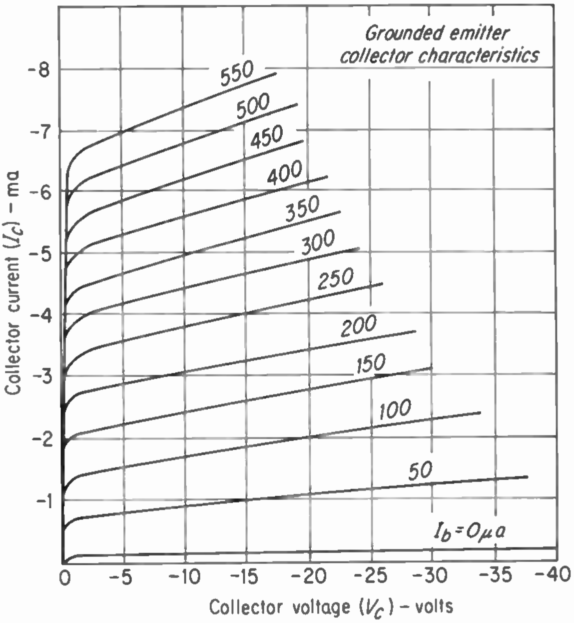


Fig. 15-16 Static common-emitter collector characteristics of a triode junction transistor.

This high ratio of resistance can also be used to produce a direct power gain. A small amount of power in the input circuit (emitter-base) controls a much larger amount of power in the output circuit (collector-base).

$$\mathrm{PG} = (\mathrm{IG})^2 \times \mathrm{RG} \tag{15-2}$$

EXAMPLE 15-1 The collector current in a junction transistor is 96 per cent of the emitter current, the collector-base resistance is 750,000 ohms, and the emitter-base resistance is 500 ohms. (*a*) What is the direct voltage gain of the transistor? (*b*) What is the direct power gain of the transistor?

GIVEN: IG = 0.96 $r_{cb} = 750{,}000$ ohms $r_{eb} = 500$ ohms

FIND: (*a*) VG (*b*) PG

SOLUTION:

(*a*) $$\mathrm{VG} = \mathrm{IG} \times \mathrm{RG} = 0.96 \times \frac{750{,}000}{500} = 1{,}440$$

(*b*) $$\mathrm{PG} = (\mathrm{IG})^2 \times \mathrm{RG} = 0.96 \times 0.96 \times 1{,}500 = 1{,}382$$

The voltage and power gains obtained in Example 15-1 can be obtained only if the transistor is operated into a very-high-impedance circuit. In practical amplifier circuits, the voltage and power gains obtained are much less than these values because of the impedances that must be used to match the high output impedance of a prior stage with the low input impedance of the following stage.

15-7 Transistor Specifications

Electrical Characteristics. Although many properties of a transistor may be specified, this section will describe only those specifications usually listed on manufacturers' transistor characteristic charts. The values are listed under three general headings: (1) general data, (2) maximum ratings, and (3) typical operation. Under general data are listed the following: (1) application or class of service, (2) type, (3) outline dimensions, and (4) lead arrangement. The maximum ratings are the direct voltage and current values that must not be exceeded in the operation of the unit. Maximum ratings usually include the direct (1) collector-base voltage, (2) emitter-base voltage, (3) collector current, (4) emitter current, and (5) collector power dissipation. The typical operating values are presented as a guide, since the values vary widely, as they are dependent on the operating voltages and which electrode is used as the common circuit element. The values listed may include the (1) common electrode, (2) collector-emitter volts, (3) collector current, (4) current-transfer ratio, (5) input resistance, (6) load resistance, (7) power gain, (8) noise factor, and (9) alpha cutoff frequency. Some manufacturers also list for the common emitter circuit the small-signal (1) hybrid-π parameters, (2) H parameters, and (3) T parameters.

Letter Symbols. Maximum, average (d-c), and effective (root-mean-square) values are represented by the uppercase letter of the proper symbol.

I = current V = voltage R = resistance P = power

Instantaneous values that vary with time are represented by the lowercase letter of the proper symbol.

i = current v = voltage r = resistance p = power

Direct-current values and instantaneous total values are indicated by uppercase subscripts.

$$I_C \quad V_{EB} \quad P_C \quad i_C \quad v_{EB} \quad p_C$$

Varying component values are indicated by lowercase subscripts.

$$I_c \quad V_{eb} \quad P_c \quad I_c \quad V_{eb} \quad P_c$$

To distinguish among maximum, average, and effective values the maximum value is represented by the addition of the subscript m or M and the average value by the addition of the subscript av or AV.

$$i_{c\cdot m} \quad I_{c\cdot m} \quad I_{C\cdot M} \quad I_{C\cdot \mathrm{AV}} \quad i_{c\cdot \mathrm{av}}$$

Other abbreviations used as subscripts are

E, e = emitter electrode
B, b = base electrode
C, c = collector electrode
J, j = electrode, general
X, x = circuit node
Q = average (d-c) value with signal applied

The first subscript designates the electrode at which the current is measured or where the electrode potential is measured with respect to the reference electrode or circuit node, which is designated by the second subscript. When the reference electrode or circuit node is understood, the second subscript may be omitted where its use is not required to preserve the meaning of the symbol.

Supply voltages may be indicated by repeating the electrode subscript. The reference electrode may then be designated by the third subscript.

$$V_{EE} \quad V_{CC} \quad V_{BB} \quad V_{EEB} \quad V_{CCB} \quad V_{BBC}$$

In devices having more than one electrode of the same type, the electrode subscripts are modified by adding a number following the subscript and on the same line.

$$V_{B1} \quad V_{b2} \quad I_{B2}$$

In multiple-unit devices, the electrode subscripts are modified by a number preceding the electrode subscript, as $1b$, $2B$, $1c$, $2C$.

Current Gain. When a transistor circuit uses a common base, its current gain is the ratio of the change in collector current to the change in emitter current for a constant value of collector voltage. When the current gain is specified for static or relatively large values of collector and emitter currents, it is called the *alpha direct-current gain.* It is represented by the symbol α or the *H*-parameter symbol h_{FB} and may be expressed as

$$\alpha = h_{FB} = \frac{\text{change in collector current}}{\text{change in emitter current}} = \frac{dI_C}{dI_E} \qquad V_C\text{—constant} \tag{15-3}$$

The value of alpha for junction transistors is always less than 1, as the collector current can never exceed the emitter current (Fig. 15-15). Its value is in the range of 0.94 to 0.99.

When a transistor circuit uses a common emitter, its current gain is the ratio of the change in collector current to the change in base current for a constant value of collector voltage. When the current gain is specified for static or relatively large values of collector and base currents, it is called the *beta direct-current gain.* It is represented by the symbol β or the *H*-parameter symbol h_{FE} and may be expressed as

$$\beta = h_{FE} = \frac{\text{change in collector current}}{\text{change in base current}} = \frac{dI_C}{dI_B} \qquad V_C\text{—constant} \tag{15-4}$$

The value of beta is always greater than unity and can be obtained by the use of the common-emitter $I_C V_{CE}$ characteristic curves of a transistor (Fig. 15-16).

EXAMPLE 15-2 A junction transistor having the characteristics shown in Fig. 15-16 is operated as a grounded emitter with a collector-emitter potential of -7 volts. What is the current-amplification factor when the base current is changed from 50 to 100 μa?

GIVEN: $V_{ce} = -7$ volts $\quad I_{B1} = 50\ \mu\text{a} \quad I_{B2} = 100\ \mu\text{a}$

FIND: β

SOLUTION: From Fig. 15-16 it can be noted that a change in base current from 50 to 100 μa with $V_{ce} = -7$ volts produces a change in the collector current from 0.8 to 1.6 ma.

$$\beta = \frac{dI_C}{dI_B} = \frac{1.6\text{ ma} - 0.8\text{ ma}}{100\ \mu\text{a} - 50\ \mu\text{a}} = \frac{0.8 \times 10^{-3}\text{ amp}}{50 \times 10^{-6}\text{ amp}} = 16$$

For a grounded emitter, the relation between β and α is

$$\beta = \frac{\alpha}{1 - \alpha} \tag{15-5}$$

This equation shows that β increases as α approaches unity.

EXAMPLE 15-3 A junction transistor operated as a grounded emitter is biased so that α is 0.941. What is β for these conditions?

GIVEN: $\alpha = 0.941$

FIND: β

SOLUTION:

$$\beta = \frac{\alpha}{1 - \alpha} = \frac{0.941}{1 - 0.941} \cong 16$$

The current gain is the most important property of a transistor in determining the gain of an amplifier. Since beta is partially dependent on frequency, its value is usually specified for a definite frequency, and for transistors used at audio and low radio frequencies, it is generally specified at 1 kc.

Current-transfer Ratio. When a transistor is connected with its base grounded, the current-transfer ratio is the ratio of the a-c variation in the collector current to the a-c variation in the emitter current for a constant value of collector voltage. This ratio is called the *forward-current-transfer ratio* and is represented by the symbol h_{fb}.

When a transistor is connected with its emitter grounded, the current-transfer ratio is the ratio of the a-c variation in the collector current to the a-c variation in the base current for a constant value of collector voltage. This ratio is called the *forward-current-transfer ratio* and is represented by the symbol h_{fe}.

H Parameters. A *parameter*, also called a *network constant*, is a constant that enters into a functional equation and corresponds to some characteristic of a circuit such as resistance, inductance, capacitance, or any other property value in a network. The small signal parameters of transistors are usually specified in terms of the H parameters. These parameters are defined for any network (Fig. 15-17) by the following equations:

$$v_i = h_i i_i + h_r v_o \tag{15-6}$$

$$i_o = h_o v_o + h_f i_i \tag{15-7}$$

where v_i = input voltage, volts
v_o = output voltage, volts
i_i = input current, amp
i_o = output current, amp
h_i = input impedance, ohms (output circuit shorted)
h_o = output admittance, μmhos (input circuit open)
h_f = forward-current transfer ratio (output circuit shorted)
h_r = reverse-voltage transfer ratio (input circuit open)

Fig. 15-17 Input and output voltage and current measurements for any network.

Table 15-1 Power Gain of a Transistor

COMMON ELECTRODE	INPUT RESISTANCE, OHMS	LOAD RESISTANCE, OHMS	POWER GAIN, DB
E	1,980	100,000	44.1
E	2,670	2,670	34.5
C	500,000	10,000	17.0
B	215	500,000	32.5

When H parameters are used for a transistor circuit, a second subscript is added to indicate the terminal that is grounded. For example, h_{FE} would be the forward-current transfer ratio of a grounded-emitter circuit and h_{IB} the input impedance of a grounded-base circuit.

Power Gain. The power gain P_g of a transistor is expressed in decibels. Since its value is dependent on the transistor (1) circuit, (2) input resistance, and (3) load resistance, these values are usually listed in the characteristic chart for the power gain specified. The variations of the power gain of a transistor for different circuits and values are listed in Table 15-1. This table shows (1) that the greatest power gain is obtained by using the transistor with a grounded emitter and a high load resistance, (2) that the least power gain is obtained with the transistor connected with a grounded collector, and (3) that a low load resistance with a grounded emitter produces a power gain less than that obtained with a high load resistance but greater than that obtained with either of the other two connections. These conditions are typical for all transistors.

15-8 Tetrodes

To improve the high-frequency operation of a transistor, a fourth lead is added to an NPN transistor. The additional lead b_2 is attached to the base region at a position that is on the side opposite to the original base connection b_1 (Fig. 15-18*a*). This type of transistor is called a *tetrode*, and its schematic symbol is shown in Fig. 15-18*b*. In the same manner as with

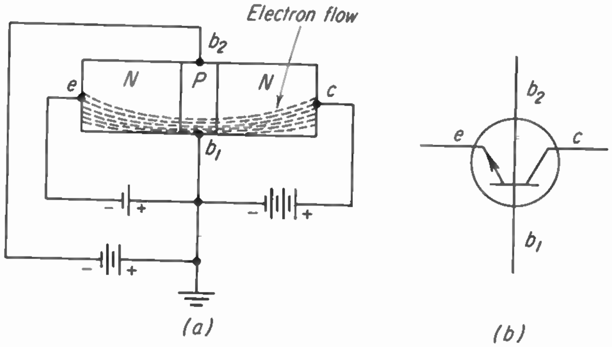


Fig. 15-18 Tetrode transistor. (a) D-c biasing voltages. (b) Schematic symbol.

an NPN triode transistor, the emitter is biased in the direction of greatest electron flow and the collector is biased in the direction of least electron flow. The additional lead is biased with a negative potential that is considerably greater than the normal emitter-base voltage. This negative potential restricts the electrons flowing through the base and causes them to flow through a relatively narrow area of the base region (Fig. 15-18*a*). The improvement in high-frequency operation is obtained because (1) the reduced effective area of each region adjacent to the base decreases the emitter and collector capacitances, (2) the shorter path taken by the base current decreases the base resistance. The base resistance of a tetrode may be decreased in the order of 10 to 1 over that of a triode.

When the electrons are forced through a narrow channel in the base region, the collector-current capabilities of the transistor are reduced, thus also reducing its power-handling capabilities. The tetrode is primarily intended for high-frequency use as an r-f amplifier, i-f amplifier, mixer, and oscillator. The second base connection may be used for automatic gain control, as this connection will cause very little detuning of the collector circuit.

15-9 Gain Per Stage

Equivalent Circuits. The transistor is an active resistance network that may be represented by three resistors connected in either a three- or four-terminal network. In solving for the current, voltage, or power amplification of a transistor-amplifier circuit, it is important to know the following: (1) input impedance Z_i, (2) output impedance Z_o, (3) current-transfer ratio α or β, and (4) active mutual characteristics of the network r_m. The equations used in solving for these parameters in either a three- or four-terminal network are rather complex. In addition, a change in the input impedance is reflected into the output circuit, and a change in the load impedance is reflected into the input circuit. For example, as the load impedance increases, the input impedance decreases, and vice versa. These equations also vary with frequency, signal input, and type of impedance used in the input and output circuits.

In this article a series of simplified equations will be presented that assume (1) that $r_e + r_b$ is much smaller than R_L, (2) that R_L is much smaller than r_m, and (3) that R_G is much smaller than r_c. These equations are for a small-signal, low-audio-frequency, four-terminal network having resistive input and output impedances, in which

$$r_m = \alpha r_c \tag{15-8}$$

where α is the current-transfer ratio when $R_L = 0$

$$\alpha_L = \frac{\beta}{1 + \beta} \tag{15-9}$$

where α_L is the current-transfer ratio when R_L has a significant value.

The current-transfer ratio α as listed in transistor manuals is for a short-circuited output, which is a usual mode of operation; thus $R_L = 0$. In solving equations that do not contain the term R_L, this value of α is used. In solving equations containing the term R_L or equations in which the term R_L has been dropped, the current-transfer ratio must be calculated to include the effects of this load. The results obtained from using these simplified equations are usually sufficiently accurate for many practical applications.

The bias supplies for the basic transistor circuits of Figs. 15-19*a*, 15-20*a*, and 15-21*a* have been omitted from their equivalent-circuit diagrams (Figs. 15-19*b*, 15-20*b*, and 15-21*b*) to avoid confusion. In these equivalent-circuit diagrams (1) V_G is the source generator having an internal resistance of R_G, (2) R_L is the load resistance, and (3) the arrows indicate the direction of electron flow in the input and output circuits due to the fixed-bias supplies. As the input signal is a varying voltage, the polarity of V_G has been arbitrarily chosen. The polarity of V_G should not be associated with the direction of electron flow indicated on these diagrams. The voltage of the output generator is equal to $r_m i_e$, which is analogous to μe_g in a

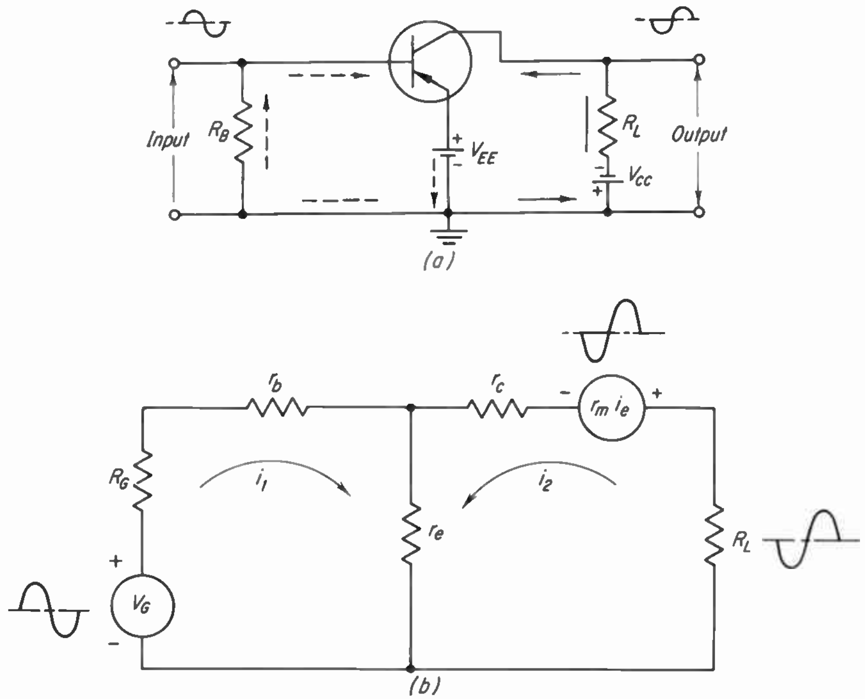


Fig. 15-19 Grounded-emitter PNP transistor amplifier. (a) Basic circuit. (b) Equivalent four-terminal network.

vacuum-tube circuit, and the polarity shown indicates the phase relationship between the input and output signals.

Current, Voltage, and Power Gain of a Grounded-emitter Amplifier. A four-terminal equivalent circuit for the grounded-emitter amplifier circuit of Fig. 15-19*a* is shown in Fig. 15-19*b*. The equations for determining the current gain, voltage gain, and power gain are

Current Gain:

$$I_g \cong \frac{r_m}{R_L + r_c - r_m} \tag{15-10}$$

Voltage Gain:

$$V_g \cong \frac{\alpha_L R_L}{r_e + r_b(1 - \alpha_L)} \tag{15-11}$$

Power Gain:

$$P_g \cong \frac{\alpha_L{}^2 R_L}{(1 - \alpha_L)[r_e + r_b(1 - \alpha_L)]} \tag{15-12}$$

EXAMPLE 15-4 A certain PNP junction transistor when connected as a grounded emitter has the following circuit parameters: $r_e = 23$ ohms; $r_b = 1{,}430$ ohms; $r_c = 3.93$ megohms; $R_L = 100{,}000$ ohms; $\alpha = 0.982$. Find (*a*) the current gain, (*b*) the voltage gain, (*c*) the power gain.

GIVEN: $r_e = 23$ ohms $\quad r_b = 1{,}430$ ohms $\quad r_c = 3.93$ megohms
$R_L = 100{,}000$ ohms $\quad \alpha = 0.982$

FIND: (*a*) I_g (*b*) V_g (*c*) P_g

SOLUTION:

(*a*) $r_m = \alpha r_c = 0.982 \times 3.93 = 3.86$ megohms

$$I_g \cong \frac{r_m}{R_L + r_c - r_m} \cong \frac{3.86 \times 10^6}{100{,}000 + 3.93 \times 10^6 - 3.86 \times 10^6} \cong 22.7$$

(*b*)
$$\alpha_L = \frac{\beta}{1 + \beta} = \frac{22.7}{1 + 22.7} = 0.957$$

$$V_g \cong \frac{\alpha_L R_L}{r_e + r_b(1 - \alpha_L)} \cong \frac{0.957 \times 100{,}000}{23 + 1{,}430(1 - 0.957)} \cong 1{,}135$$

(*c*)
$$P_g \cong \frac{\alpha_L{}^2 R_L}{(1 - \alpha_L)[r_e + r_b(1 - \alpha_L)]}$$

$$\cong \frac{0.957 \times 0.957 \times 100{,}000}{(1 - 0.957)[23 + 1{,}430(1 - 0.957)]} \cong 25{,}200$$

Current, Voltage, and Power Gain of a Grounded-base Amplifier. A four-terminal equivalent circuit for the grounded-base amplifier circuit of Fig. 15-20*a* is shown in Fig. 15-20*b*. The equations for determining the

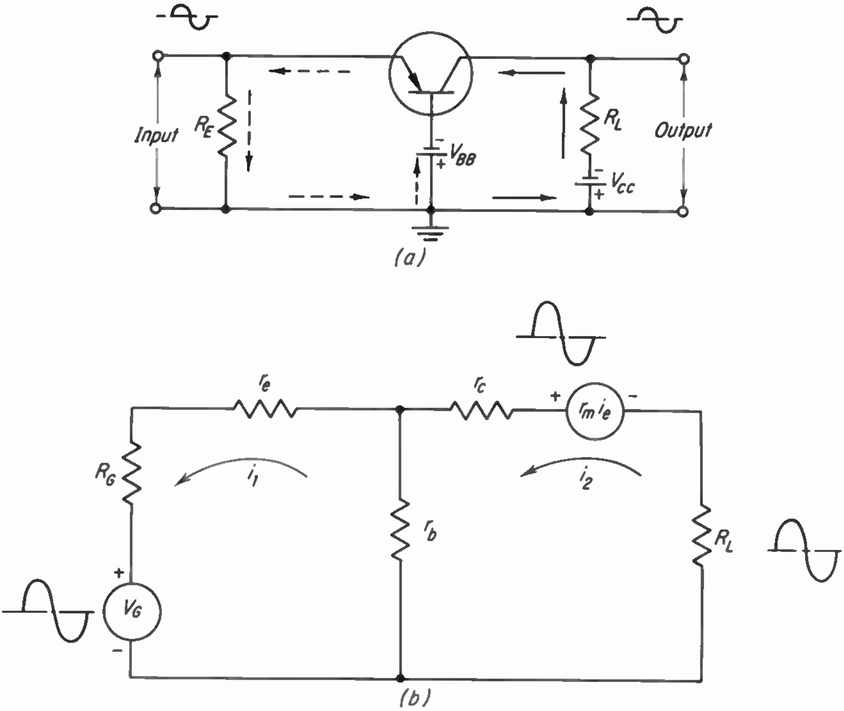


Fig. 15-20 Grounded-base PNP transistor amplifier. (a) Basic circuit. (b) Equivalent four-terminal network.

current gain, voltage gain, and power gain are

Current Gain:

$$I_g \cong \frac{r_m}{r_c + R_L} \tag{15-13}$$

Voltage Gain:

$$V_g \cong \frac{\alpha_L R_L}{R_G + r_e + r_b(1 - \alpha_L)} \tag{15-14}$$

Power Gain:

$$P_g \cong \frac{\alpha_L{}^2 R_L}{R_G + r_e + r_b(1 - \alpha_L)} \tag{15-15}$$

EXAMPLE 15-5 The PNP junction transistor used in Example 15-4 is connected with a grounded base and an output load resistance of 500,000 ohms. The internal resistance of the source generator is 50 ohms. Find (*a*) the current gain, (*b*) the voltage gain, (*c*) the power gain.

GIVEN: $R_L = 500{,}000$ ohms $\qquad R_G = 50$ ohms $\qquad$ Transistor of Example 15-4

FIND: (*a*) I_g $\qquad$ (*b*) V_g $\qquad$ (*c*) P_g

SOLUTION:

(*a*)
$$I_g \cong \frac{r_m}{r_c + R_L} \cong \frac{3.86 \times 10^6}{3.93 \times 10^6 + 500{,}000} \cong 0.871$$

(*b*)
$$V_g \cong \frac{\alpha_L R_L}{R_G + r_e + r_b(1 - \alpha_L)} \cong \frac{0.871 \times 500{,}000}{50 + 23 + 1{,}430(1 - 0.871)} \cong 1{,}690$$

(*c*)
$$P_g \cong \frac{\alpha_L{}^2 R_L}{R_G + r_e + r_b(1 - \alpha_L)} \cong \frac{0.871 \times 0.871 \times 500{,}000}{50 + 23 + 1{,}430(1 - 0.871)} \cong 1{,}470$$

Current, Voltage, and Power Gain of a Grounded-collector Amplifier. A four-terminal equivalent circuit for the grounded-collector amplifier circuit of Fig. 15-21*a* is shown in Fig. 15-21*b*. The equations for determining the current gain, voltage gain, and power gain are

Current Gain:

$$I_g \cong \frac{r_c}{R_L + r_c - r_m} \tag{15-16}$$

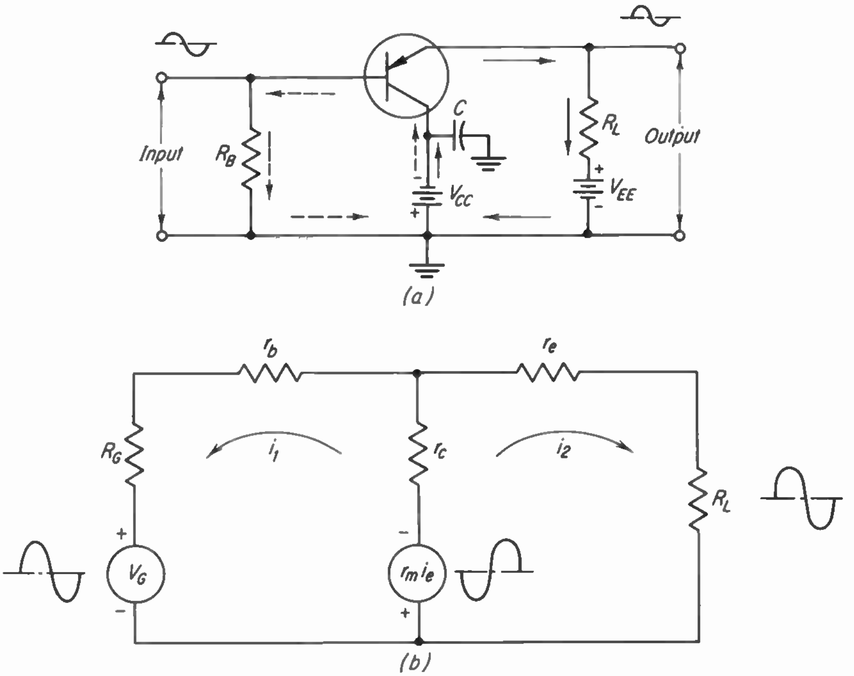


Fig. 15-21 Grounded-collector PNP transistor amplifier. (a) Basic circuit. (b) Equivalent four-terminal network.

Voltage Gain:

$$V_g \cong 1 \tag{15-17}$$

Power Gain:

$$P_g \cong \frac{1}{1 - \alpha_L} \tag{15-18}$$

EXAMPLE 15-6 The PNP junction transistor used in Example 15-4 is connected with a grounded collector and an output-load resistance of 10,000 ohms. Find (*a*) the current gain, (*b*) the voltage gain, (*c*) the power gain.

GIVEN: $R_L = 10{,}000$ ohms   Transistor of Example 15-4

FIND: (*a*) I_g (*b*) V_g (*c*) P_g

SOLUTION:

(*a*) $$I_g \cong \frac{r_c}{R_L + r_c - r_m} \cong \frac{3.93 \times 10^6}{10{,}000 + 3.93 \times 10^6 - 3.86 \times 10^6} \cong 49$$

(*b*) $$V_g \cong 1$$

(*c*) $$\alpha_L \cong \frac{\beta}{\beta + 1} \cong \frac{49}{49 + 1} \cong 0.98$$

$$P_g \cong \frac{1}{1 - \alpha_L} \cong \frac{1}{1 - 0.98} \cong 50$$

15-10 Solid-state Light-sensitive Devices

Basic Principles of the Photocell. The solid-state photocell is based on the principle that, when certain materials are exposed to light, light rays are absorbed by the material by liberating free-to-move electrons and holes. The direction of movement of these negative and positive charges can be controlled to produce an electric current. The strength of the current and voltage produced is dependent upon the material used, its physical size, and the intensity of the source of light. The material used in the construction of a photocell will depend on its application. Some of the materials used are silicon, germanium, selenium, cadmium sulfide, gallium arsenide, indium antimonide, and indium arsenic.

In addition to converting light energy to electrical energy, photocells may also be used as control devices. Photocells used for control purposes are usually of a high-voltage type, such as the cadmium-sulfide cell, with ratings up to 500 volts d-c. These cells may be operated directly from a 115-volt a-c (230-volt d-c) power source with a normal power dissipation of ½ watt and a maximum power dissipation of 15 watts. The illumination from a very small neon lamp can be used to switch as much as 40 watts of power, an action representing a power gain of 80.

One of the many applications of the photocell as a control device is shown in Fig. 15-22*a*. Smooth control of the speed of the 115-volt a-c

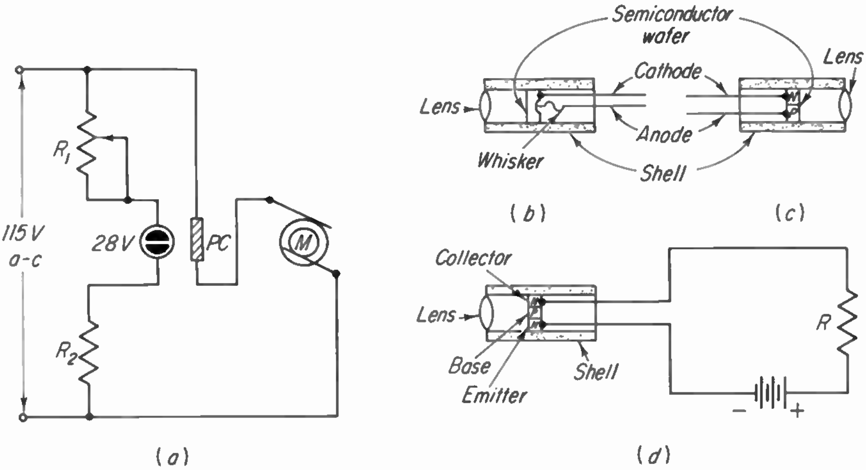


Fig. 15-22 Solid-state light-sensitive devices. (a) Photocell used to control the speed of a motor. (b) Point-contact type of photodiode. (c) Junction-type of photodiode. (d) A simple phototransistor circuit.

motor may be obtained by varying the resistance of the photocell. This action is accomplished by adjusting the resistance of R_1, which in turn varies the brilliance of the neon lamp, thus also varying the resistance of the photocell. Smooth speed control may also be obtained by operating the neon lamp at maximum brilliance and controlling the illumination at the cell by rotating a variable-opacity vane between the neon lamp and the cell.

Photodiodes. The basic construction of two types of photodiodes is shown in Fig. 15-22*b* and *c*. The semiconductor wafer, made of germanium or silicon, is illuminated through a very small lens mounted at one end of the enclosed unit. Illumination of the wafer causes (1) the resistance of the diode to change from a very high value to a comparatively low value and (2) with no applied voltage, the production of a direct voltage across its output terminals. Because of its photoconductive properties the photodiode has many applications where small size and fast operation are desirable. Some of these applications are relay, counting, alarm systems, sound on film, and punched-tape and punched-card reading. The solar cell, as explained in Art. 3-16, is one of the applications of the voltaic properties of the photodiode.

Phototransistor. The construction of a phototransistor is similar to that of a junction transistor. However, only two external leads are used, one connected to the collector and the other connected to the emitter, with no connection being made to the base (Fig. 15-22*d*). The transistor is positioned so that the base is illuminated through a small lens that is mounted at one end of the enclosing shell. A direct voltage is applied between the emitter and collector in the usual manner for a common-emitter amplifier. With no illumination the wafer is darkened and the collector current is extremely low, being the normal collector cutoff current for the applied voltage. Illumination of the wafer causes current carriers to be injected into the base region. A large collector current, which is proportional to the illumination and is equal to beta times the base current, will flow through the external circuit. The phototransistor has the advantage over the photodiode in that it also provides amplification, because a relatively low-light intensity will produce a comparatively high output current. Typical applications of the phototransistor are similar to those of the photodiode with the addition of optical coupling, light-beam reception, and electron-optical control service.

Miscellaneous Types of Light-sensitive Devices. The basic principles of the photocell, the photodiode, and the phototransistor are used to produce many other types of light-sensitive devices. Two of these are the light-activated switch and the Raysistor. The light-activated switch is a bistable component of the PNPN type. In a manner similar to a four-layer diode, once the unit is triggered to its ON state, it will continue to conduct current from its external power source until the power is interrupted. The Raysistor consists of a light source and a photoconductive cell that are enclosed in a lighttight housing. The light source may be either an incandescent or glow lamp. The operation of this unit is similar to that of a lamp-photocell combination. The dark (OFF) resistance is of the order of 1,800 megohms, while the light (ON) resistance may drop to values as low as 300 ohms.

QUESTIONS

1. Name seven favorable characteristics of transistors.
2. Explain the movement of electrons and holes in a semiconductor.
3. Define the following terms: (*a*) conductor, (*b*) insulator, (*c*) semiconductor.
4. What is meant by (*a*) intrinsic semiconductor? (*b*) P-type semiconductor? (*c*) N-type semiconductor?
5. Define the following terms: (*a*) valence electron, (*b*) valence band, (*c*) conduction band, (*d*) energy band, (*e*) forbidden zone.
6. Explain the lattice structure of a silicon or germanium crystal.
7. (*a*) What is meant by impurity conduction? (*b*) How does the relative amount of impurity addition affect the resistance of a semiconductor?
8. Explain the effect on a semiconductor of adding (*a*) a donor impurity, (*b*) an acceptor impurity.
9. Describe the characteristics of a PN junction.
10. Describe how a high forward current is obtained from a PN junction.
11. What is meant by (*a*) reverse bias? (*b*) Reverse current? (*c*) Potential hill?
12. Compare the forward- and reverse-current static volt-ampere characteristics of a semiconductor diode.
13. Describe the construction of the following types of transistors: (*a*) NPN, (*b*) PNP.

14. Describe the method used for biasing the following types of transistors for use as amplifiers: (*a*) NPN, (*b*) PNP.
15. Describe three systems used for identifying the leads of a triode transistor.
16. Name three basic methods of transistor connection.
17. Compare the fundamental actions involved in the operation of a vacuum tube with those involved in the operation of a transistor.
18. In a comparison of transistor and vacuum-tube circuit operation, what two important factors must be considered?
19. Describe the input and output circuit characteristics of a grounded-emitter transistor connection.
20. Describe the input and output circuit characteristics of a grounded-base transistor connection.
21. Describe the input and output circuit characteristics of a grounded-collector transistor connection.
22. Describe the static characteristics of transistors for (*a*) collector current versus collector voltage for a constant value of emitter current, (*b*) collector current versus collector voltage for a constant value of base current.
23. Explain the following terms: (*a*) direct-voltage gain, (*b*) direct-current gain, (*c*) direct-resistance gain, (*d*) direct-power gain.
24. What are the maximum ratings of a transistor that are generally included in manufacturers' specifications?
25. What are the operating values of a transistor that are generally included in manufacturers' specifications?
26. What is meant by (*a*) alpha direct-current gain? (*b*) Beta direct-current gain? (*c*) Current-transfer ratio $h_{f \cdot b}$? (*d*) Current-transfer ratio $h_{f \cdot e}$?
27. How are transistor small-signal parameters usually specified?
28. How is the power gain of a transistor affected by (*a*) its method of connection? (*b*) Its input resistance? (*c*) Its load resistance?
29. (*a*) Describe the construction and operation of a tetrode transistor. (*b*) What are the primary applications of a tetrode transistor?
30. Describe how the tetrode improves the high-frequency operation of a transistor.
31. What circuit parameters of a transistor amplifier circuit should be known in order to solve for its current, voltage, or power amplification?
32. Name six variable factors that affect the current, voltage, and power amplification of a transistor amplifier.
33. Name three conditions that are assumed in using the simplified equations presented in this chapter for solving small-signal low-audio-frequency four-terminal networks having resistive input and output impedances.
34. (*a*) Describe the difference between α and α_L. (*b*) For what circuit conditions is α used? (*c*) For what circuit conditions is α_L used?
35. Draw a four-terminal equivalent circuit for a common-emitter PNP transistor amplifier circuit showing the direction of electron flow in the input and output circuits.
36. Draw a four-terminal equivalent circuit for a common-base PNP transistor amplifier circuit showing the direction of electron flow in the input and output circuits.
37. Draw a four-terminal equivalent circuit for a common-collector PNP transistor amplifier circuit showing the direction of electron flow in the input and output circuits.

38. (*a*) What is the basic principle of the solid-state photocell? (*b*) What factors affect the strength of the voltage and current produced?
39. Describe the two basic applications of a photocell.
40. (*a*) What is meant by a photodiode? (*b*) Describe its photoconductive properties.
41. Describe (*a*) the construction of a phototransistor, (*b*) its circuit operation.
42. Describe the circuit operation of (*a*) a light-activated switch, (*b*) the Raysistor.

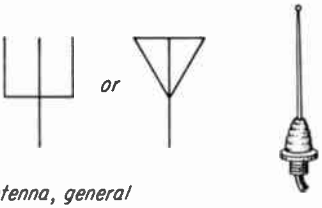
PROBLEMS

1. The collector current in a junction transistor is 95 per cent of the emitter current; the collector-base resistance is 1,200,000 ohms, and the emitter-base resistance is 600 ohms. (*a*) What is the direct voltage gain of the transistor? (*b*) What is the direct power gain of the transistor?
2. A junction transistor has an input resistance of 1,400 ohms, an output resistance of 3.5 megohms, and $\alpha = 0.97$. Find (*a*) the direct voltage gain of the transistor, (*b*) the direct power gain of the transistor.
3. A junction transistor having the characteristics shown in Fig. 15-15 is operated with a grounded base with a collector-base potential of -5 volts. What is the alpha direct-current gain when the emitter current changes from 1 to 2 ma?
4. A junction transistor having the characteristics shown in Fig. 15-15 is operated with a grounded base with a collector-base potential of -10 volts. What is the alpha direct-current gain when the emitter current changes from 2 to 3 ma?
5. A junction transistor having the characteristics shown in Fig. 15-16 is operated with a grounded emitter with a collector-emitter potential of -10 volts. What is the beta direct-current gain when the base current is changed from 100 to 150 μa?
6. A junction transistor having the characteristics shown in Fig. 15-16 is operated with a grounded emitter with a collector-emitter potential of -5 volts. What is the beta direct-current gain when the base current is changed from 50 to 100 μa?
7. A junction transistor operated with a grounded emitter is biased so that $\beta = 38$. What is α for these conditions?
8. A junction transistor operated with a grounded emitter is biased so that $\alpha = 0.97$. What is β for these conditions?
9. A certain PNP junction transistor when connected as a grounded emitter has the following circuit parameters: $r_e = 34$ ohms; $r_b = 976$ ohms; $r_c = 3.45$ megohms; $R_L = 20{,}000$ ohms; $\alpha = 0.982$. Find (*a*) the current gain, (*b*) the voltage gain, (*c*) the power gain.
10. A certain PNP junction transistor when connected as a grounded emitter has the following circuit parameters: $r_e = 21$ ohms; $r_b = 580$ ohms; $r_c = 1.82$ megohms; $R_L = 20{,}000$ ohms; $\alpha = 0.977$. Find (*a*) the current gain, (*b*) the voltage gain, (*c*) the power gain.
11. The PNP junction transistor used in Prob. 9 is connected with a grounded base and an output load resistance of 500,000 ohms. The internal resistance of the source generator is 50 ohms. Find (*a*) the current gain, (*b*) the voltage gain, (*c*) the power gain.
12. The PNP junction transistor used in Prob. 10 is connected with a grounded base and an output load resistance of 500,000 ohms. The internal resistance of the source generator is 100 ohms. Find (*a*) the current gain, (*b*) the voltage gain, (*c*) the power gain.

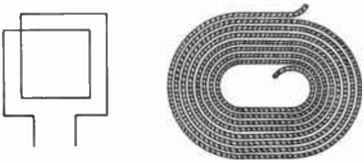
13. The PNP junction transistor used in Prob. 9 is connected with a grounded collector and an output load resistance of 10,000 ohms. Find (*a*) the current gain, (*b*) the voltage gain, (*c*) the power gain.
14. The PNP junction transistor used in Prob. 10 is connected with a grounded collector and an output load resistance of 18,000 ohms. Find (*a*) the current gain, (*b*) the voltage gain, (*c*) the power gain.

Appendix I

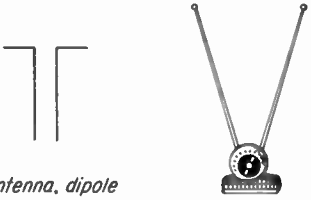
Drawing Symbols Used in Electronics



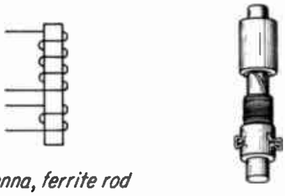
Antenna, general



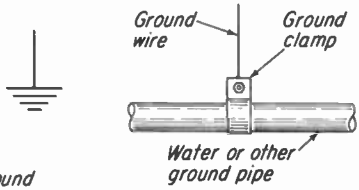
Antenna, loop



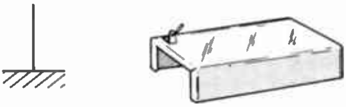
Antenna, dipole



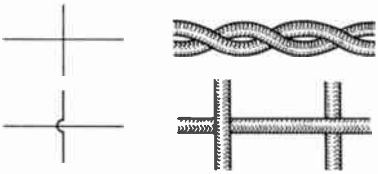
Antenna, ferrite rod



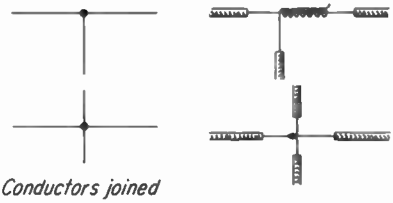
Ground



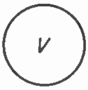
Chassis



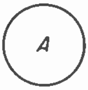
Conductors not joined



Conductors joined

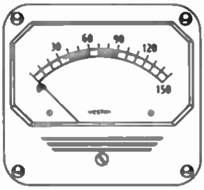


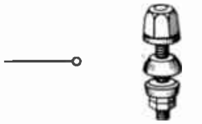
Voltmeter



Ammeter

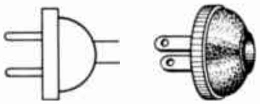
Milliammeter



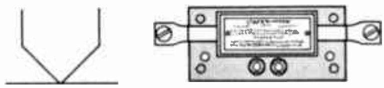


Terminal

Fuse

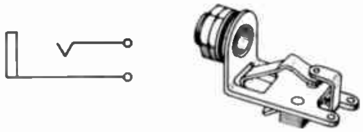


Line plug



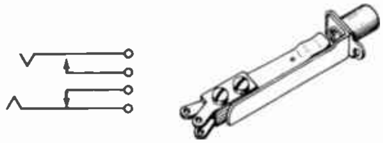
Thermocouple

Key

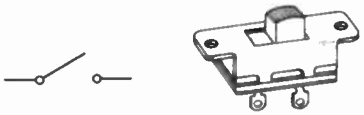


Two-terminal jack

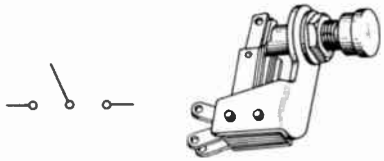
Three-terminal jack



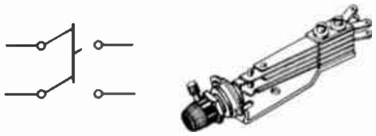
Multi-terminal jack



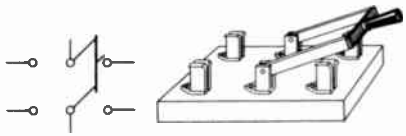
Single-pole single-throw switch



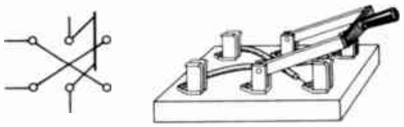
Single-pole double-throw switch



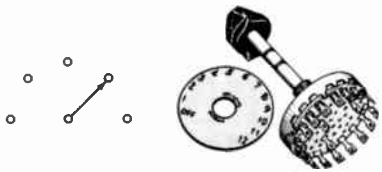
Double-pole single-throw switch



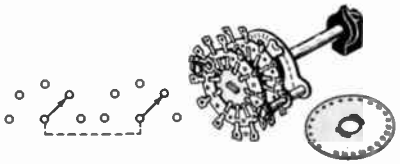
Double-pole double-throw switch



Double-pole double-throw reversing switch

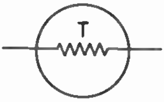


Single-deck circuit-selector switch

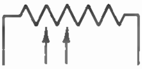


Multiple-deck circuit-selector switch

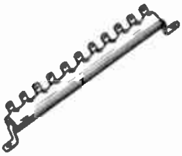
Fixed resistor



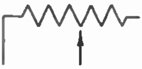
Thermistor



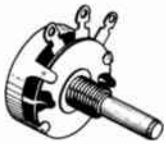
Adjustable resistor



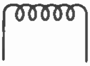
Tapped resistor



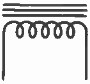
Rheostat

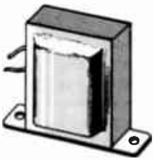


Potentiometer

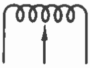


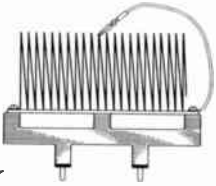
Inductor, air-core



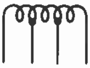


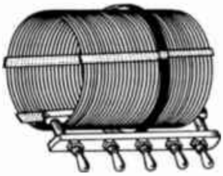
Inductor, iron-core



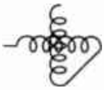


Adjustable inductor

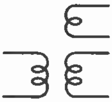


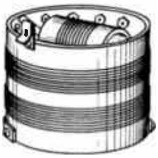


Tapped inductor

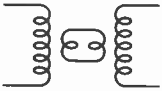


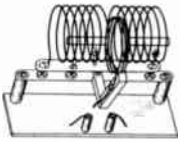
Variable inductor
(variometer)



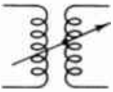


Three-circuit tuner

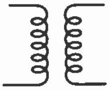




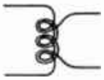
Link coupling



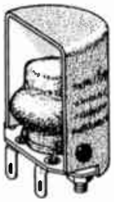
Transformer with variable coupling
(varicoupler with the moving coil indicated)

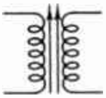


Transformer, air-core

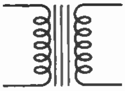


Transformer bifilar winding

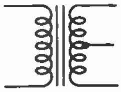




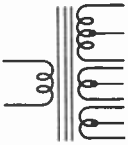
Variable-core transformer



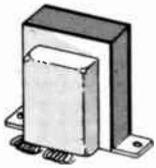
Transformer, iron-core

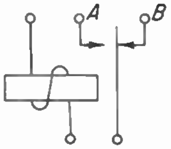


Push-pull transformer

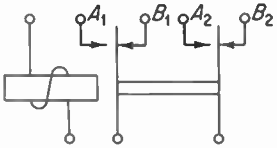


Power transformer

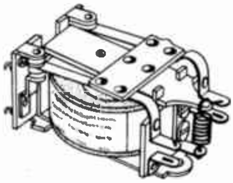




Relay; circuit A open when deenergized



Relay; circuits B_1 and B_2 closed when deenergized

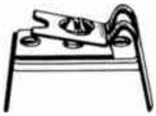


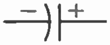
Fixed capacitor (Paper, mica, or ceramic)

(Padder)

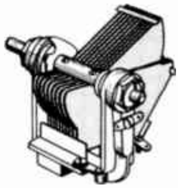
(Trimmer)

Adjustable capacitors

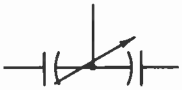


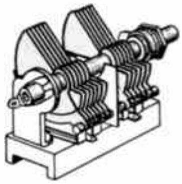


Electrolytic capacitor

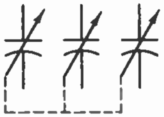


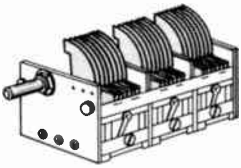
Variable capacitor



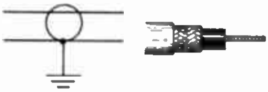


Split-stator variable capacitor

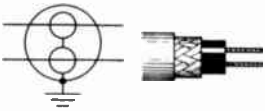




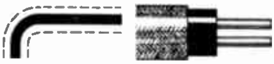
Ganged variable capacitors, mechanical linkage



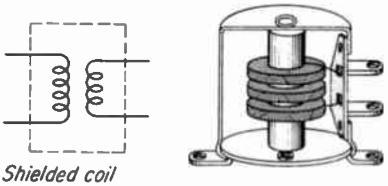
Coaxial cable, single



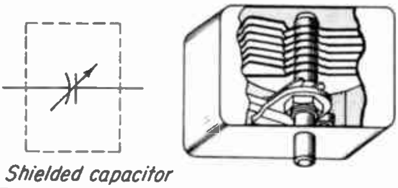
Coaxial cable, twin



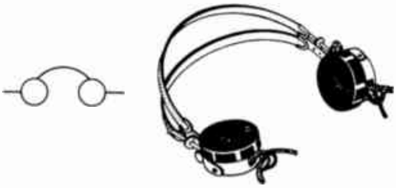
Shielded cable



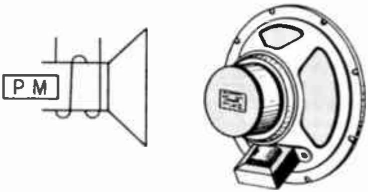
Shielded coil



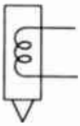
Shielded capacitor



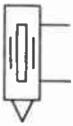
Earphones



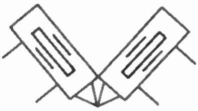
Permanent-magnet dynamic loudspeaker



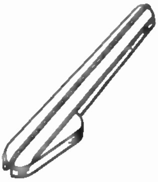
Magnetic phonograph-pickup

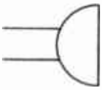


Crystal phonograph-pickup

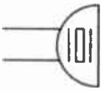


Stereophonic phonograph-pickup

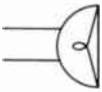




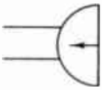
Carbon microphone



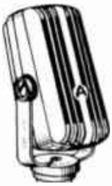
Crystal microphone

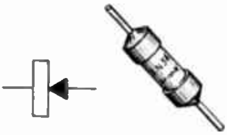


Dynamic microphone (moving coil)

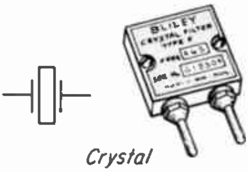


Velocity microphone (ribbon)

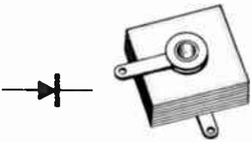




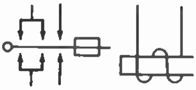
Crystal diode



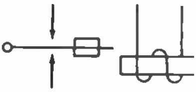
Crystal



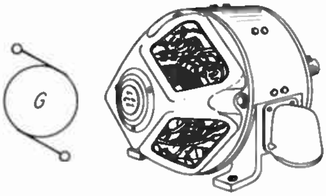
Contact rectifier, half-wave



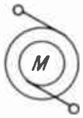
Vibrator, synchronous



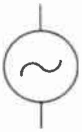
Vibrator, nonsynchronous



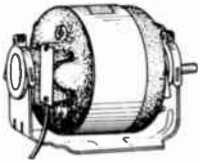
D-C motor or generator

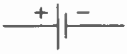


A-C motor or generator (single phase)

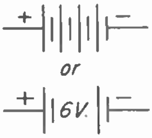


A-C voltage source



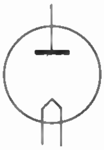


Single cell

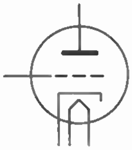


Battery

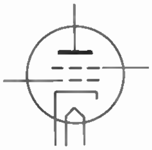
Vacuum and gas tubes



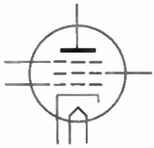
Diode



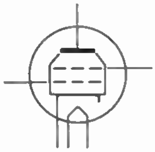
Triode



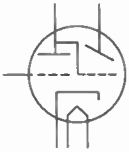
Tetrode



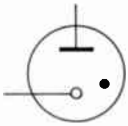
Pentode



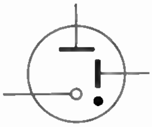
Beam-power amplifier



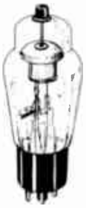
Electron-ray indicator tube

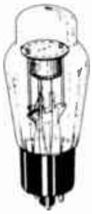


Cold cathode diode (gaseous)



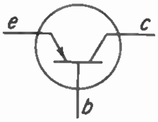
Cold cathode triode (gaseous)



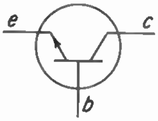


See also art 14-7 and figure 14-14

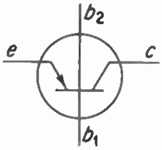
Transistors



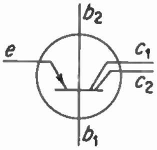
PNP triode



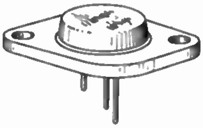
NPN triode

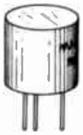


Tetrode

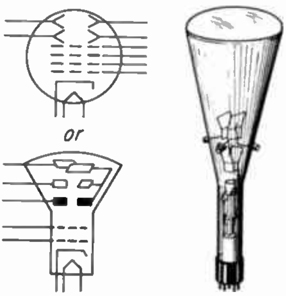


Pentode

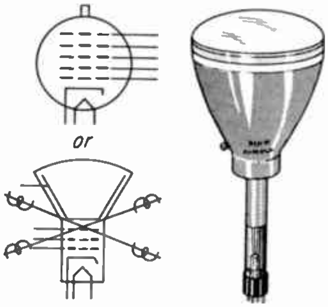




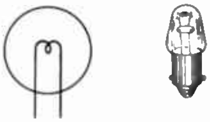
Cathode-ray tubes



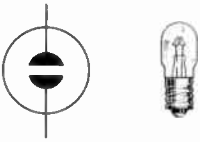
Electrostatic-deflection type



Electromagnetic-deflection type



Pilot or panel lamp



Neon lamp

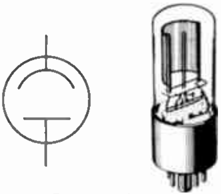


Photo tube

Appendix II

Symbols and Abbreviations Used in Electronics

TERM	SYMBOL	ABBREVIATION
Ampere	---	a or amp
Milliampere	---	ma
Microampere	---	μa
Ampere turn	*NI*	A-T
American Wire Gauge	---	AWG
Amplitude-modulated	---	a-m
Angle	∠	
Phase angle	θ	
Antenna	---	ant
Apparent power	---	AP
Area	---	A
Circular mils	---	cm or cir mils
Square centimeters	---	sq cm
Square inches	---	sq in.
Capacitance	---	*C*
Conductance	*G*	
Constant, dielectric	*K*	
Coulomb	*q or Q*	
Coupling, coefficient	*K*	
Current	*I*	
Alternating	---	a-c or A-C
Average value	I_{ave}	
Direct	---	d-c or D-C
Effective value	*I*	
Instantaneous value	i_e	
Maximum value	I_{max}	
Signal	*i*	
Cycles	~	c
Per second	~	cps, Hz
Kilocycle	---	kc, kHz
Megacycle	---	mc, mHz
Decibel	db	db
Diameter	*d*	d or diam
Distance	*d*	
Efficiency	---	eff
Electromotive force	---	emf or EMF
Counter electromotive force	---	cemf
Electrostatic unit	---	esu
Energy	---	en or En
Equivalent	---	eq
Farad	---	f
Microfarad	---	μf
Micromicrofarad	---	μμf
Picofarad	---	pf
Flux, magnetic	ϕ	
Density	*B*	
Force	*F*	
Magnetomotive	---	mmf
Frequency	*f*	
Audio	---	a-f
Intermediate	---	i-f
Modulation	---	f-m
Radio	---	r-f
Resonance	f_r	
Ultrahigh	---	uhf
Very high	---	vhf
Gausses	*B*	
Gilbert	*F*	
Ground	---	gnd
Henry	---	h
Millihenry	---	mh
Microhenry	---	μh
Hertz	---	Hz
Impedance	*Z*	
Inductance, self-	*L*	
Mutual	*M*	
Intensity, magnetic field	*H*	
Kilo	---	k
Length	*l*	
Mathematical symbols:		
Equals	=	

TERM	SYMBOL	ABBREVIATION
Is approximately equal to	$\cong$	
Does not equal	$\neq$	
Is greater than	$>$	
Is much greater than	$\gg$	
Is less than	$<$	
Is much less than	$\ll$	
Therefore	$\therefore$	
Multiplied by	X or ·	
Divided by	÷ or :	
Positive, or plus	$+$	
Negative, or minus	$-$	
Plus or minus	$\pm$	
Angle	$\angle$	
Cosine	---	cos
Sine	---	sin
Tangent	---	tan
Common logarithm	---	log
Antilogarithm	---	antilog
Cologarithm	---	colog
Megohm	MΩ	
Meter	---	m
Centimeter	---	cm
Millimeter	---	mm
Oersted	H	
Ohm	ω or Ω	
Permeability	μ	
Permeance	$\mathcal{P}$	
Pi (3.1416)	π	
Pole, north seeking	N	
South seeking	S	
Power	p or P	p or P
Power factor	---	PF
Primary	P	p or pri
Reactance	X	
Inductive	X_L	
Capacitive	X_C	
Reluctance	$\mathfrak{R}$	
Resistance	R or r	resis
Root mean square	---	rms
Secondary	S	s or sec
Switch	---	sw

TERM	SYMBOL	ABBREVIATION
Single-pole single-throw	---	spst
Single-pole double-throw	---	spdt
Double-pole single-throw	---	dpst
Double-pole double-throw	---	dpdt
Three-deck, four-circuit, eight positions	---	3D-4C-8P
Temperature, coefficient	T_C	
Degrees centigrade	C	°C
Degrees Fahrenheit	F	°F
Thickness	t	
Time	t or T	
Seconds	---	sec
Microseconds	---	μsec
Minutes	---	min
Time constant	t	
Transformer	T	
Turns, number of	N	
Vacuum-tube voltmeter	---	vtvm
Volt	---	v
Kilovolt	---	kv
Millivolt	---	mv
Microvolt	---	μv
Voltage	E	
Average value	E_{ave}	
Effective value	E	
Instantaneous value	e_e	
Maximum value	E_{max}	
Signal	e	
Volt-ampere	VA	va or VA
Kilovolt ampere	KVA	kva
Watt	W	w
Kilowatt	---	kw
Kilowatthours	---	kwh or kwhr
Milliwatt	---	mw
Microwatt	---	μw
Wave, continuous	---	c-w
Wavelength	λ	

Appendix III

Formulas Commonly Used in Electronics

Note: The numbers appearing opposite the equations correspond to the numbers of the same equations in the text or to the equations from which they were derived. These numbers are included to facilitate reference to figures, text, and nomenclature when such reference is desirable.

Direct Current

Ohm's Law

$$\text{Voltage} = IR = \frac{P}{I} = \sqrt{RP} \qquad \text{(2-6), (2-12), (2-13)}$$

$$\text{Current} = \frac{E}{R} = \frac{P}{E} = \sqrt{\frac{P}{R}} \qquad \text{(2-5), (2-12), (2-14)}$$

$$\text{Resistance} = \frac{E}{I} = \frac{P}{I^2} = \frac{E^2}{P} \qquad \text{(2-7), (2-14), (2-13)}$$

$$\text{Power} = EI = I^2R = \frac{E^2}{R} \qquad \text{(2-12), (2-14), (2-13)}$$

$$\text{Energy} = PT; \quad \text{power} = \frac{\text{En}}{T}; \quad \text{time} = \frac{\text{En}}{P} \qquad \text{(2-15)}$$

Series Circuit

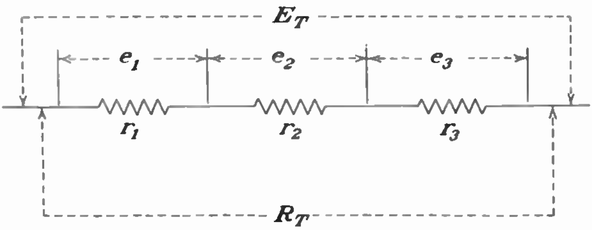


Fig. A-1

$$R_T = r_1 + r_2 + r_3, \text{ etc.} \qquad \text{(4-10)}$$

$$r_1 = R_T - (r_2 + r_3, \text{ etc.}) \qquad \text{(4-10)}$$

$$E_T = e_1 + e_2 + e_3, \text{ etc.} \qquad \text{(4-9)}$$

$$e_2 = E_T - (e_1 + e_3, \text{ etc.}) \qquad \text{(4-9)}$$

$$I_T = i_1 = i_2 = i_3, \text{ etc.} \qquad \text{(4-8)}$$

$$P_T = p_1 + p_2 + p_3, \text{ etc.} \qquad \text{(4-11)}$$

$$\text{En}_T = \text{en}_1 + \text{en}_2 + \text{en}_3, \text{ etc.} \qquad \text{(4-12)}$$

Parallel Circuits

Two resistors in parallel

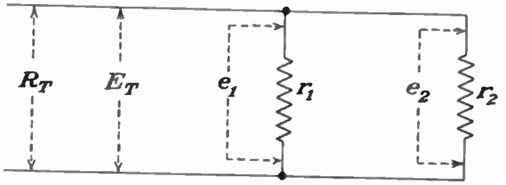


Fig. A-2

$$R_T = \frac{1}{\frac{1}{r_1} + \frac{1}{r_2}} = \frac{r_1 r_2}{r_1 + r_2} \tag{4-15a}$$

$$r_1 = \frac{R_T r_2}{r_2 - R_T} \tag{4-15b}$$

Any number of resistors in parallel

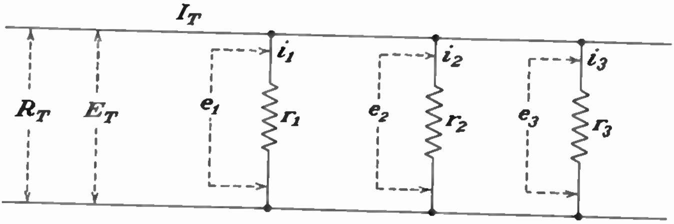


Fig. A-3

$$R_T = \frac{1}{\frac{1}{r_1} + \frac{1}{r_2} + \frac{1}{r_3}, \text{ etc.}} \tag{4-15}$$

$$E_T = e_1 = e_2 = e_3, \text{ etc.} \tag{4-14}$$

$$I_T = i_1 + i_2 + i_3, \text{ etc.} \tag{4-13}$$

$$I_3 = I_T - (i_1 + i_2, \text{ etc.}) \tag{4-13}$$

$$P_T = p_1 + p_2 + p_3, \text{ etc.} \tag{4-11}$$

$$\text{En}_T = \text{en}_1 + \text{en}_2 + \text{en}_3, \text{ etc.} \tag{4-12}$$

Alternating Current

Maximum, Effective, and Average Values of Sine Wave Currents and Voltages

$$\text{Maximum value} = \sqrt{2} \text{ effective value} = 1.414 \text{ effective value} \tag{7-17}$$

$$= \frac{\text{effective value}}{0.707} = 1.57 \text{ average value} \tag{7-16}$$

$$\text{Effective value} = \frac{\text{maximum value}}{\sqrt{2}} = \frac{\text{maximum value}}{1.414} \tag{7-17}$$

$$\text{Effective value} = 0.707 \text{ maximum value} = 1.11 \text{ average value} \qquad \text{(7-15), (7-16)}$$

$$\text{Average value} = 0.637 \text{ maximum value} = 0.9009 \text{ effective value} \qquad \text{(7-6), (7-8)}$$

Alternating Current

Ohm's Law

$$\text{Voltage} = IZ = \frac{P}{I\,\text{PF}} \qquad \text{(8-9), (10-3)}$$

$$\text{Current} = \frac{E}{Z} = \frac{P}{E\,\text{PF}} \qquad \text{(8-9), (10-3)}$$

$$\text{Impedance} = \frac{E}{I} = \frac{R}{\text{PF}} = \sqrt{R^2 + (X_L - X_C)^2} \qquad \text{(8-9), (10-5), (10-1)}$$

$$\text{Volt-amperes} = EI = \frac{\text{P}}{\text{PF}} \qquad \text{(7-18), (10-4)}$$

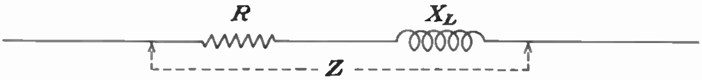
$$\text{Power} = EI\,\text{PF} = I^2R \qquad \text{(7-20), (10-3), (2-14)}$$

$$\text{Power factor} = \frac{P}{EI} = \frac{R}{Z} = \cos\theta \qquad \text{(7-19), (10-5)}$$

$$\text{Energy} = PT \qquad \text{(2-15)}$$

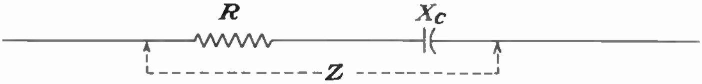
Series Circuits

SERIES CIRCUIT	RESISTANCE AND INDUCTANCE (FIG. A-4*a*)	RESISTANCE AND CAPACITANCE (FIG. A-4*b*)	RESISTANCE, INDUCTANCE, AND CAPACITANCE (FIG. A-4*c*)
$Z =$	$\sqrt{R^2 + X_L^2}$	$\sqrt{R^2 + X_C^2}$	$\sqrt{R^2 + (X_L - X_C)^2}$
$R =$	$\sqrt{Z^2 - X_L^2}$	$\sqrt{Z^2 - X_C^2}$	$\sqrt{Z^2 - (X_L - X_C)^2}$
$X_L =$	$\sqrt{Z^2 - R^2}$	-----------	$\sqrt{Z^2 - R^2} + X_C$
$X_C =$	-----------	$\sqrt{Z^2 - R^2}$	$X_L - \sqrt{Z^2 - R^2}$



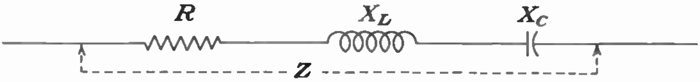
Resistance and inductive reactance

(*a*)



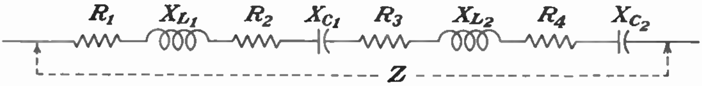
Resistance and capacitive reactance

(*b*)



Resistance, inductive reactance and capacitive reactance

(*c*)



Any combination of resistances, inductive reactances and capacitive reactances

(*d*)

Fig. A-4

For any combination of resistance, inductance, and capacitance (Fig. A-4*d*)

$$Z = \sqrt{(R_1 + R_2 + R_3, \text{etc.})^2 + (X_{L_1} + X_{L_2} + X_{L_3}, \text{etc.} - X_{C_1} - X_{C_2} - X_{C_3}, \text{etc.})^2} \quad (10\text{-}2)$$

$$E_T = e_1 + e_2 + e_3 + e_4, \text{ etc. (to be added vectorially)}$$
$$I_T = i_1 = i_2 = i_3 = i_4, \text{ etc.}$$
$$P_T = p_1 + p_2 + p_3 + p_4, \text{ etc.}$$
$$\mathrm{En}_T = \mathrm{en}_1 + \mathrm{en}_2 + \mathrm{en}_3, \text{ etc.}$$

Parallel Circuits

Two impedances in parallel

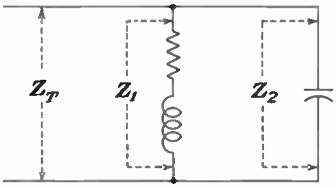


Fig. A-5

$$Z_T = \frac{Z_1Z_2}{Z_1 + Z_2} \quad (10\text{-}23)$$

$$Z_1 = \frac{Z_TZ_2}{Z_2 - Z_T} \quad (10\text{-}23)$$

Any number of impedances in parallel

Z_T: No single equation is available for this type of circuit. For solution see Arts. 10-9, 10-15, and 10-16.

$$E_T = e_1 = e_2 = e_3, \text{ etc.}$$
$$I_T = i_1 + i_2 + i_3, \text{ etc. (to be added vectorially)}$$
$$P_T = p_1 + p_2 + p_3, \text{ etc.}$$
$$\mathrm{En}_T = \mathrm{en}_1 + \mathrm{en}_2 + \mathrm{en}_3, \text{ etc.}$$

Equivalent Circuits

Series equivalents for a parallel circuit

$$R_{S'} = \frac{R_PX_P^2}{R_P^2 + X_P^2} \quad (10\text{-}9)$$

$$X_{S'} = \frac{X_PR_P^2}{R_P^2 + X_P^2} \quad (10\text{-}10)$$

Parallel equivalents for a series circuit

$$R_{P'} = \frac{R_S^2 + X_S^2}{R_S} \quad (10\text{-}11)$$

$$X_{P'} = \frac{R_S^2 + X_S^2}{X_S} \quad (10\text{-}12)$$

Polyphase Systems

Three-phase delta-connected system

$$E_{\text{line}} = E_{\text{phase}} \tag*{(7-21)}$$
$$I_{\text{line}} = 1.73 I_{\text{phase}} \tag*{(7-22)}$$
$$\text{Power} = 1.73 E_{\text{line}} I_{\text{line}}\ \text{PF} = 3 E_{\text{phase}} I_{\text{phase}}\ \text{PF} \tag*{(7-26), (7-27)}$$
$$\text{Volt-amperes} = 1.73 E_{\text{line}} I_{\text{line}} = 3 E_{\text{phase}} I_{\text{phase}} \tag*{(7-24), (7-25)}$$
$$\text{Power factor} = \frac{P}{1.73 E_{\text{line}} I_{\text{line}}} = \frac{P}{3 E_{\text{phase}} I_{\text{phase}}} \tag*{(7-28), (7-29)}$$

Three-phase wye-connected system

$$E_{\text{line}} = 1.73 E_{\text{phase}} \tag*{(7-30)}$$
$$I_{\text{line}} = I_{\text{phase}} \tag*{(7-31)}$$
$$\text{Power} = 1.73 E_{\text{line}} I_{\text{line}}\ \text{PF} = 3 E_{\text{phase}} I_{\text{phase}}\ \text{PF} \tag*{(7-26), (7-27)}$$
$$\text{Volt-amperes} = 1.73 E_{\text{line}} I_{\text{line}} = 3 E_{\text{phase}} I_{\text{phase}} \tag*{(7-24), (7-25)}$$
$$\text{Power factor} = \frac{P}{1.73 E_{\text{line}} I_{\text{line}}} = \frac{P}{3 E_{\text{phase}} I_{\text{phase}}} \tag*{(7-28), (7-29)}$$

Magnetism

$$B = \frac{\phi}{A} \tag*{(5-3)}$$
$$F = \phi \mathfrak{R} = 1.26 NI \tag*{(5-7), (5-8)}$$
$$\phi = \frac{B}{A} = \frac{F}{\mathfrak{R}} \tag*{(5-3), (5-7)}$$
$$\mathfrak{R} = \frac{F}{\phi} = \frac{\nu l}{A} \tag*{(5-7), (5-6)}$$
$$\mu = \frac{B}{H} = \frac{1}{\nu} \tag*{(5-9), (5-5)}$$
$$\nu = \frac{1}{\mu} \tag*{(5-5)}$$

Reluctances in series

$$\mathfrak{R}_T = \mathfrak{R}_1 + \mathfrak{R}_2 + \mathfrak{R}_3, \text{ etc.} \tag*{(5-10)}$$

Reluctances in parallel

$$\mathfrak{R}_T = \frac{1}{\dfrac{1}{\mathfrak{R}_1} + \dfrac{1}{\mathfrak{R}_2} + \dfrac{1}{\mathfrak{R}_3}, \text{ etc.}} \tag*{(5-11)}$$

Inductors

Inductance of a Coil

Solenoid with iron core

$$L = \frac{1.26 N^2 \mu A}{10^8 l} \tag*{(8-2)}$$

Solenoid with air core

$$L = \frac{a^2 N^2}{9a + 10b} \tag*{(8-5)}$$

Multilayer coil with air core

$$L = \frac{0.8a^2N^2}{6a + 9b + 10c} \qquad (8\text{-}3)$$

Flat coil with air core

$$L = \frac{a^2N^2}{8a + 11c} \qquad (8\text{-}4)$$

Inductance of a Coil or a Circuit

$$L = \frac{X_L}{2\pi f} \qquad (8\text{-}6)$$

Inductive Reactance

$$X_L = 2\pi fL = \sqrt{Z^2 - R^2} = \frac{E_{X\cdot L}}{I_{X\cdot L}} = X - X_C \qquad (8\text{-}6), (8\text{-}8), (8\text{-}7)$$

Impedance

$$Z_L = \sqrt{R_L{}^2 + X_L{}^2} = \frac{E_L}{I_L} \qquad (8\text{-}8), (8\text{-}9)$$

Coil Q

$$Q = \frac{X_L}{R} = \frac{2\pi fL}{R} \qquad (8\text{-}25)$$

Inductors in Series

With flux linkage between coils

$$L_T = L_1 + L_2 \pm 2K\sqrt{L_1L_2} \qquad (8\text{-}22)$$

No flux linkage between coils

$$L_T = L_1 + L_2 + L_3, \text{ etc.} \qquad (8\text{-}15)$$

$$X_{L\cdot T} = 2\pi fL_T = X_{L\cdot 1} + X_{L\cdot 2} + X_{L\cdot 3}, \text{ etc.} \qquad (8\text{-}17), (8\text{-}18)$$

Inductors in Parallel

Any number of inductors

$$L_T = \frac{1}{\dfrac{1}{L_1} + \dfrac{1}{L_2} + \dfrac{1}{L_3}, \text{ etc.}} = \frac{X_{L\cdot T}}{2\pi f} \qquad (8\text{-}16), (8\text{-}17)$$

$$X_{L\cdot T} = 2\pi fL_T = \frac{1}{\dfrac{1}{X_{L\cdot 1}} + \dfrac{1}{X_{L\cdot 2}} + \dfrac{1}{X_{L\cdot 3}}, \text{ etc.}} \qquad (8\text{-}17), (8\text{-}19)$$

Two inductors

$$L_T = \frac{L_1L_2}{L_1 + L_2} \qquad L_1 = \frac{L_TL_2}{L_2 - L_T} \qquad (8\text{-}16a)$$

$$X_{L\cdot T} = 2\pi fL_T = \frac{X_{L\cdot 1}X_{L\cdot 2}}{X_{L\cdot 1} + X_{L\cdot 2}} \qquad (8\text{-}17), (8\text{-}19a)$$

Mutual Inductance

$$M = K\sqrt{L_1L_2} \qquad L_1 = \frac{M^2}{K^2L_2} \tag{8-13}$$

Coefficient of Coupling

$$K = \frac{M}{\sqrt{L_1L_2}} \tag{8-13}$$

Capacitors

Charge

$$Q = It = CE \qquad (9\text{-}1), (9\text{-}1a)$$

Capacitance

$$C = \frac{22.45KA(N-1)}{10^8t} = \frac{10^6}{2\pi fX_C} \qquad (9\text{-}2a), (9\text{-}4)$$

Capacitive Reactance

$$X_C = \frac{10^6}{2\pi fC} = \frac{159{,}000}{fC} = \sqrt{Z^2 - R^2} = \frac{E_C}{X_C} = X - X_L \qquad (9\text{-}4), (9\text{-}7), (9\text{-}5)$$

Impedance

$$Z = \sqrt{R^2 + X_C^2} \tag{9-7}$$

Power Factor

$$\mathrm{PF} = \frac{R}{Z} = \frac{R}{\sqrt{R^2 + X_C^2}} = \cos\theta = \frac{\mathrm{ESR}}{Z} \qquad (9\text{-}9), (9\text{-}10), (9\text{-}18)$$

Equivalent Series Resistance

$$\mathrm{ESR} = \frac{159{,}000\,\mathrm{DF}}{fC} = Z\mathrm{PF} = X_C\mathrm{DF} \qquad (9\text{-}17), (9\text{-}18), (9\text{-}19)$$

Dissipation Factor

$$\mathrm{DF} = \frac{\mathrm{ESR}}{X_C} = \frac{1}{Q} \qquad (9\text{-}19), (9\text{-}21)$$

Q Factor

$$Q = \frac{X_C}{\mathrm{ESR}} = \frac{1}{\mathrm{DF}} \qquad (9\text{-}20), (9\text{-}21)$$

Energy

$$\mathrm{En} = \frac{CE^2}{2} = \frac{QE}{2} \qquad (9\text{-}26), (9\text{-}27)$$

Capacitors in Series

Any number of capacitors

$$C_T = \frac{1}{\frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}, \text{ etc.}} = \frac{10^6}{2\pi fX_{C\cdot T}} \qquad (9\text{-}28), (9\text{-}29)$$

Two capacitors

$$C_T = \frac{C_1C_2}{C_1 + C_2} \qquad C_1 = \frac{C_TC_2}{C_2 - C_T} \qquad \text{(9-28}a\text{), (9-28}b\text{)}$$

Reactance

$$X_{C\cdot T} = \frac{10^6}{2\pi fC_T} = X_{C\cdot 1} + X_{C\cdot 2} + X_{C\cdot 3}, \text{ etc.} \qquad \text{(9-29), (9-30)}$$

Charge

$$Q_T = Q_1 = Q_2 = Q_3, \text{ etc.} \qquad \text{(9-31)}$$

Capacitors in Parallel

$$C_T = C_1 + C_2 + C_3, \text{ etc.} = \frac{10^6}{2\pi fX_{C\cdot T}} \qquad \text{(9-32), (9-29)}$$

$$X_{C\cdot T} = \frac{10^6}{2\pi fC_T} = \frac{1}{\dfrac{1}{X_{C\cdot 1}} + \dfrac{1}{X_{C\cdot 2}} + \dfrac{1}{X_{C\cdot 3}}, \text{ etc.}} \qquad \text{(9-29), (9-33)}$$

$$Q_T = Q_1 + Q_2 + Q_3, \text{ etc.} \qquad \text{(9-34)}$$

Resonance

Series and Parallel Circuits

$$\left.\begin{aligned} f_r &= \frac{159}{\sqrt{LC}} \\ L &= \frac{25{,}300}{f_r^2C} \\ C &= \frac{25{,}300}{f_r^2L} \end{aligned}\right\} \begin{array}{l} f_r \text{ is expressed in kilocycles} \\ L \text{ is expressed in microhenrys} \\ C \text{ is expressed in microfarads} \end{array} \qquad \begin{array}{r} \text{(11-8)} \\ \text{(11-9)} \\ \text{(11-10)} \end{array}$$

Series Resonant Circuit

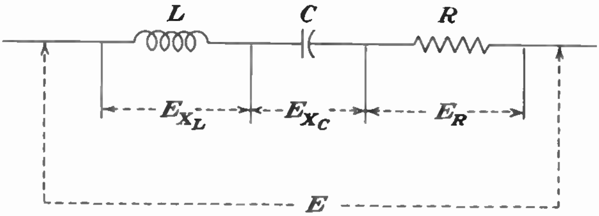


Fig. A-6

At any frequency

$$Z = \sqrt{R^2 + (X_L - X_C)^2} \qquad \text{(10-1)}$$

At resonance

$$Z = R \text{ (minimum value possible)} \qquad \text{(11-1)}$$

$$I = \frac{E}{R} \text{ (maximum value possible)} \qquad (11\text{-}12)$$

$$E_{X_L} = E_{X_C} = EQ \qquad (11\text{-}26)$$

Parallel Resonant Circuit

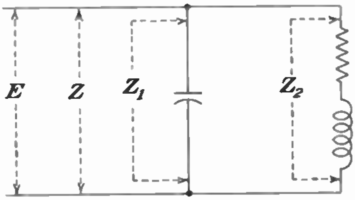


Fig. A-7

At any frequency

$$Z = \frac{Z_1 Z_2}{Z_1 + Z_2} \qquad (10\text{-}15)$$

At resonance

$$Z = X_L Q \text{ (maximum value obtainable)} \qquad (11\text{-}31)$$

$$I = \frac{E}{Z} \text{ (minimum value obtainable)} \qquad (11\text{-}27)$$

Width of Frequency Band for Single Resonant Circuit at 0.707 of the Maximum Response

$$f_2 - f_1 = \frac{f_r}{Q} = \frac{R}{2\pi L} \qquad (11\text{-}21), (11\text{-}22)$$

Coupled Resonant Circuits

Width of Bandpass

$$f_2 - f_1 = Kf_r \qquad (12\text{-}28)$$

Critical Coupling

$$K_c = \frac{1.5}{\sqrt{Q_p Q_s}} \qquad (12\text{-}30)$$

$$Q_p Q_s = \frac{2.25}{K_c^2} \qquad (12\text{-}31)$$

Relation between Wavelength and Frequency

Wavelength

$$\lambda = \frac{300{,}000{,}000}{f(\text{cps})} = \frac{300{,}000}{f(\text{kc})} = \frac{300}{f(\text{mc})} \qquad (1\text{-}1)$$

Frequency

$$f = \frac{300{,}000{,}000}{\lambda}\ (\text{cps}) = \frac{300{,}000}{\lambda}\ (\text{kc}) = \frac{300}{\lambda}\ (\text{mc}) \qquad (1\text{-}2)$$

Delayed-action Circuits

Time Constants

Resistance-inductance circuit

$$t = \frac{L}{R} \qquad (8\text{-}10)$$

Resistance-capacitance circuit

$$t = CR \qquad (9\text{-}11)$$

Electron Tubes

Tube Constants

$$\mu = g_m r_p \qquad (14\text{-}4)$$

$$g_m = \frac{\mu}{r_p} \qquad (14\text{-}5)$$

$$r_p = \frac{\mu}{g_m} \qquad (14\text{-}6)$$

Triode Amplifier

$$\text{VA} = \frac{\mu Z_o}{Z_o + r_p} \qquad (14\text{-}11)$$

Pentode Amplifier

$$\text{VA} = g_m \frac{r_p Z_o}{Z_o + r_p} \qquad (14\text{-}17)$$

Transistors

Transistor Constants

Alpha direct-current gain

When $R_L = 0$,

$$\alpha = h_{FB} = \frac{dI_C}{dI_E} \qquad V_C\text{—constant} \qquad (15\text{-}3)$$

When R_L has a significant value,

$$\alpha_L = \frac{\beta}{1 + \beta} \qquad (15\text{-}9)$$

Beta direct-current gain

$$\beta = h_{FE} = \frac{dI_C}{dI_B} \qquad V_C\text{—constant} \qquad (15\text{-}4)$$

Relation between β and α for grounded emitter

$$\beta = \frac{\alpha}{1 - \alpha} \tag{15-5}$$

Current Gain of Amplifiers

Grounded-emitter amplifier

$$I_g \cong \frac{r_m}{R_L + r_c - r_m} \tag{15-10}$$

Grounded-base amplifier

$$I_g \cong \frac{r_m}{r_c + R_L} \tag{15-13}$$

Grounded-collector amplifier

$$I_g \cong \frac{r_c}{R_L + r_c - r_m} \tag{15-16}$$

Appendix IV

Table of Specific Resistance and Temperature Coefficient of Various Metals at 20°C

MATERIAL	DESCRIPTION	SPECIFIC RESISTANCE OHMS PER CIR-MIL-FT	TEMPERATURE COEFFICIENT PER °C
Advance	Same as constantan	295	0.000018
Aluminum	Wire	17	0.0039
Antimony		252	0.0036
Bismuth		700	0.004
Brass	Copper, zinc	40	0.002
Cadmium		46	0.0038
Carbon	Graphite	2,600–7,500	−0.0003
Chromax	Chromium, nickel, iron	603	0.00031
Cobalt		58	0.0033
Constantan	Copper, nickel	295	0.000018
Copper	Hard drawn	10.4	0.004
German silver	Nickel, copper, zinc	200	0.00027
Gold		14.5	0.0034
Ideal	Same as constantan	295	0.000018
Indium		93.6	0.00498
Iron	Cast	450–570	0.006
Iron	Pure	58.2	0.0052–0.0062
Lead		130	0.0039
Magnesium		27.8	0.004
Manganin	Manganese, copper, nickel	265	0.00002
Mercury		565	0.00089
Molybdenum	Drawn	34.3	0.0045
Monel	Copper, nickel	265	0.002
Nichrome	Nickel, chromium	600–660	0.0004
Nickel		46	0.0047
Palladium		64.5	0.0033
Phosphor bronze		47	0.003
Platinum		60	0.003
Silver		9.8	0.0038
Steel	Soft	95.4	0.005
Steel	Hard	275	0.0016
Tin		69	0.0042
Tungsten		34	0.0045
Zinc		35.2	0.0037

The values given in the above table are approximate, since these values will depend upon the exact composition of the material.

Appendix V

Bare Copper Wire Tables at 25°C

AWG AND BROWN AND SHARPE GAUGE	DIAMETER, MILS	AREA, CIR MILS	OHMS PER 1,000 FT
1	289.3	83,690	0.1264
2	257.6	66,370	0.1593
3	229.4	52,640	0.2009
4	204.3	41,740	0.2533
5	181.9	33,100	0.3195
6	162.0	26,250	0.4028
7	144.3	20,820	0.5080
8	128.5	16,510	0.6405
9	114.4	13,090	0.8077
10	101.9	10,380	1.018
11	90.74	8,234	1.284
12	80.81	6,530	1.619
13	71.96	5,178	2.042
14	64.08	4,107	2.575
15	57.07	3,257	3.247
16	50.82	2,583	4.094
17	45.26	2,048	5.163
18	40.30	1,624	6.510
19	35.89	1,288	8.210
20	31.96	1,022	10.35
21	28.46	810.1	13.05
22	25.35	642.4	16.46
23	22.57	509.5	20.76
24	20.10	404.0	26.17
25	17.90	320.4	33.00
26	15.94	254.1	41.62
27	14.20	201.5	52.48
28	12.64	159.8	66.17
29	11.26	126.7	83.44
30	10.03	100.5	105.2
31	8.93	79.70	132.7
32	7.95	63.21	167.3
33	7.08	50.13	211.0
34	6.31	39.75	266.0
35	5.62	31.52	335.5
36	5.00	25.00	423.0
37	4.45	19.83	533.4
38	3.96	15.72	672.6
39	3.53	12.47	848.1
40	3.14	9.89	1069.0
41	2.80	7.84	1323
42	2.50	6.22	1667
43	2.22	4.93	2105
44	1.98	3.91	2655
45	1.75	3.06	3460

Appendix VI

Dielectric Constant (*K*) and Dielectric Strength (Volts per 0.001 Inch) of Various Materials

MATERIAL	DIELECTRIC CONSTANT K	DIELECTRIC STRENGTH, VOLTS PER 0.001 INCH
Air	1	80
Aluminum oxide layer	10	
Bakelite	6	500
Cambric, varnished	4.5	1,200
Ceramics	5–4,000	200–1,200
Mycalex	6–8	300–500
Steatite, commercial	5–6.5	
low loss	4.4	150–315
Titanium dioxide	90–170	100–200
Fiber	2.5–8	50–200
Average	6.5	
Glass	4–10	200–2,000
Common	4.2	200
Electrical	4–5	2,000
Mica	5.5	2,000
Clear India	7–7.3	600–1,500
Neoprene	6.7	300
Niobium oxide layer	50	
Nylon	3.6–3.7	300–400
Oil, castor	4.7	380
Pyranol	4.2	350
Transformer	2.2–2.6	250
Paper, beeswaxed	3.1	1,800
Kraft	3.5	
Paraffined	2.2	1,200
Polyethylene	2.25	1,000
Polystyrene	2.5–2.7	500–700
Porcelain	4.5–6.5	300–1,200
Quartz	3.5–4.5	200–400
Resins	2.5–4	
Rubber	2–4	300–500
Shellac	2.5–4	900
Tantalum oxide layer	28	
Teflon	2.1	1,000–2,000
Vacuum	1	
Water, pure	81	

Note: The values in the above table may vary considerably depending upon the quality and manufacture of the material. The values in the table are average values; for greater accuracy, values should be obtained from the manufacturer of the materials used.

Appendix VII

Standard Color Coding for Resistors

For the identification of resistance values of small carbon-type resistors, numbers are represented by the colors indicated in Table A-1. Three colors are used on each resistor to identify its value in ohms according to the methods indicated in Table A-2. Two systems are used for placing the color band or dot on a resistor, one for the axial-lead resistor (Fig. A-8) and the other for the radial-lead resistor (Fig. A-9). The tolerance is indicated by a fourth band or dot having the following color identification.

Table A-1 Color Code Used for the Identification of Carbon Resistors and Flat-paper, Mica, and Ceramic Capacitors

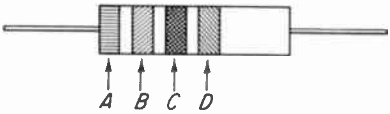
1—Brown	4—Yellow	7—Violet
2—Red	5—Green	8—Gray
3—Orange	6—Blue	9—White
	0—Black	

Gold, 5 per cent   Silver, 10 per cent   None, 20 per cent

Table A-2 Methods Used for Identifying the Color Bands or Dots and Their Functions as Used with Carbon Resistors and Flat-paper, Mica, and Ceramic Capacitors

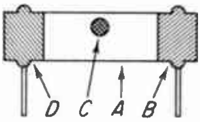
AXIAL LEADS	IDENTIFICATION	RADIAL LEADS
Band *A*	First significant figure	Body *A*
Band *B*	Second significant figure	End *B*
Band *C*	The number of zeros following the first two figures	Band *C* or dot *C*
Band *D*	Per cent tolerance above and below the nominal resistance or capacitance value	End *D*

For resistors having fractional values of more than 1 ohm but less than 10 ohms *band C is gold* indicating a multiplying factor of 0.1. For resistors having a value of less than 1 ohm *band C is silver* indicating a multiplying factor of 0.01.



Axial-lead resistor

Fig. A-8



Radial-lead resistor

Fig. A-9

Table A-3 Application of Resistor Color Code

RESISTANCE, OHMS	A	B	C	RESISTANCE, OHMS	A	B	C
0.27	Red	Violet	Silver	10 k	Brown	Black	Orange
0.56	Green	Blue	Silver	11 k	Brown	Brown	Orange
1.0	Black	Brown	Black	12 k	Brown	Red	Orange
1.8	Brown	Gray	Gold	18 k	Brown	Gray	Orange
3.3	Orange	Orange	Gold	22 k	Red	Red	Orange
10	Brown	Black	Black	27 k	Red	Violet	Orange
12	Brown	Red	Black	33 k	Orange	Orange	Orange
22	Red	Red	Black	39 k	Orange	White	Orange
39	Orange	White	Black	47 k	Yellow	Violet	Orange
47	Yellow	Violet	Black	56 k	Green	Blue	Orange
68	Blue	Gray	Black	68 k	Blue	Gray	Orange
82	Gray	Red	Black	82 k	Gray	Red	Orange
100	Brown	Black	Brown	100 k	Brown	Black	Yellow
120	Brown	Red	Brown	120 k	Brown	Red	Yellow
180	Brown	Gray	Brown	150 k	Brown	Green	Yellow
220	Red	Red	Brown	180 k	Brown	Gray	Yellow
270	Red	Violet	Brown	200 k	Red	Black	Yellow
330	Orange	Orange	Brown	220 k	Red	Red	Yellow
470	Yellow	Violet	Brown	390 k	Orange	White	Yellow
560	Green	Blue	Brown	470 k	Yellow	Violet	Yellow
820	Gray	Red	Brown	510 k	Green	Brown	Yellow
1,000	Brown	Black	Red	680 k	Blue	Gray	Yellow
1,200	Brown	Red	Red	1.5 meg	Brown	Green	Green
1,500	Brown	Green	Red	2.2 meg	Red	Red	Green
1,800	Brown	Gray	Red	3.9 meg	Orange	White	Green
2,700	Red	Violet	Red	5.6 meg	Green	Blue	Green
3,900	Orange	White	Red	6.8 meg	Blue	Gray	Green
4,700	Yellow	Violet	Red	12 meg	Brown	Red	Blue
5,600	Green	Blue	Red	18 meg	Brown	Gray	Blue
6,800	Blue	Gray	Red	22 meg	Red	Red	Blue

Appendix VIII

Standard Color Coding for Capacitors

The basic system used for color coding capacitors is the same as for resistors, and all capacitance values are in micromicrofarads. (1) The colors used for identifying the numbers are the same as those used for resistors (Table A-1). (2) The methods used for identifying the color bands or dots are also the same as those used for resistors (Table A-2). Capacitors may also use one or more of four additional identification bands or dots, namely,

E—third significant figure, used in some of the old systems
F—d-c working voltage
G—Classification or operating characteristic
H—Temperature coefficient

It is impossible to describe a standard method of identification that would be applicable to all of the many different types of capacitors manufactured. Typical examples of the identification systems for commonly used capacitors are illustrated in Fig. A-10, and the method of identifying the values of the multiplying factor, tolerance, and d-c working voltage in Tables A-4 and A-5.

Table A-4 Identification of Molded Paper and Molded Mica Capacitors

COLOR OF BAND OR DOT	*C*	*D*		*F*
	MULTIPLYING FACTOR	TOLERANCE PAPER, %	TOLERANCE MICA, %	WORKING VOLTAGE
Black	1	20	20	
Brown	10	- - -	- - -	100
Red	10^2	- - -	2	200
Orange	10^3	30	3	300
Yellow	10^4	40	- - -	400
Green	10^5	5	5 (EIA)	500
Blue	10^6	- - -	- - -	600
Violet	- - -	- - -	- - -	700
Gray	- - -	- - -	- - -	800
White	- - -	10 (EIA)	- - -	900
Gold	0.1	10 (MIL)	5 (MIL)	1,000
Silver	0.01	- - -	10	2,000

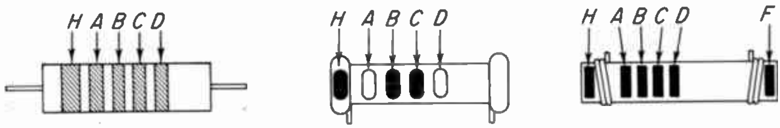
Table A-5 Identification of Ceramic Capacitors

COLOR OF BAND OR DOT	C	D		F
	MULTIPLYING FACTOR	TOLERANCE, CAPACITANCE LESS THAN 10$\mu\mu$f, $\mu\mu$f	TOLERANCE, CAPACITANCE MORE THAN 10$\mu\mu$f, %	WORKING VOLTAGE
Black	1	2	20	
Brown	10	0.1	1	150
Red	10^2	---	2	
Orange	10^3	---	2.5	350
Yellow	10^4			
Green	---	0.5	5	500
Blue				
Violet				
Gray	0.01	0.25		
White	0.1	1.0	10	

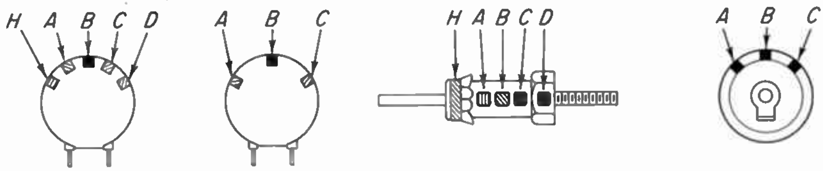
Table A-6 Application of Capacitor Color Code

SYSTEM FIG. A-10	CAPACITANCE, $\mu\mu$f	TOLERANCE, %	WORKING VOLTAGE	A	B	C	D	E	F
(a)	54	1	---	Green	Yellow	Black	Brown		
(a)	270	2.5	---	Red	Violet	Brown	Orange		
(b)	8	0.5 $\mu\mu$f	---	Black	Gray	Black	Green		
(b)	820	5	---	Gray	Red	Brown	Green		
(c)	1,200	10	150	Brown	Red	Red	White	---	Brown
(c)	6,800	20	350	Blue	Gray	Red	Black	---	Orange
(d)	33	0.25 $\mu\mu$f	---	Orange	Orange	White	Gray		
(d)	0.68	0.1 $\mu\mu$f	---	Blue	Gray	Blue	Brown		
(d)	91	2	---	White	Brown	Black	Red		
(e)	7.5	---	---	Violet	Green	White			
(e)	4,700	---	---	Yellow	Green	Red			
(f)	500	10	---	Green	Black	Brown	White		
(f)	2,300	20	---	Red	Orange	Red	Black		
(g)	560	5	---	Green	Blue	Brown	Green		
(g)	10,000	20	---	Brown	Black	Orange	Black		
(h)	62	2	---	Blue	Red	Black	Red		
(h)	7,500	10 (EIA)	---	Violet	Green	Red	White		
(h)	910	3	---	White	Brown	Brown	Orange		
(i)	5	2	---	Black	Green	Black	Red		
(j)	33	3	300	Orange	Orange	Black	Orange	---	Orange
(k)	2,200	10 (MIL)	600	Red	Red	Red	White	---	Blue
(l)	360	2	1,000	Orange	Blue	Brown	Red	---	Gold
(m)	8,200	20	400	Gray	Red	Red	Black	---	Yellow
(n)	430	5 (MIL)	2,000	Yellow	Orange	Black	Gold	Black	Silver

Ceramic



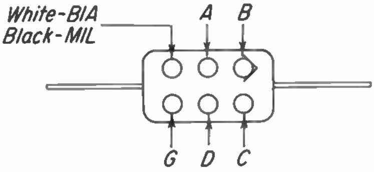
(a) Axial lead (b) 5-dot radial (c) 6-dot radial



(d) 5-dot disc (e) 3-dot disc (f) Standoff (g) Button

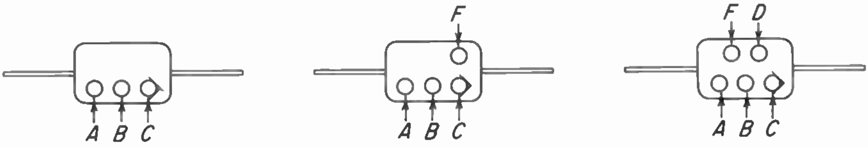
Molded mica or flat paper

Present system

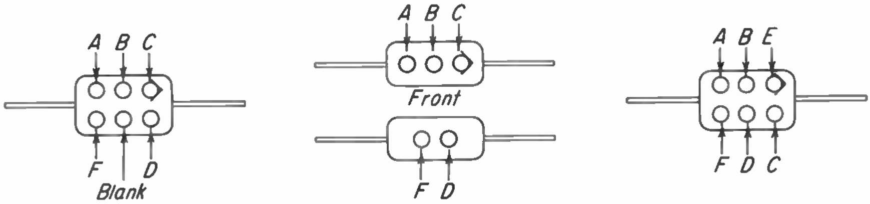


(h) 6-dot

Old systems



(i) 3-dot (j) 4-dot (k) 5-dot

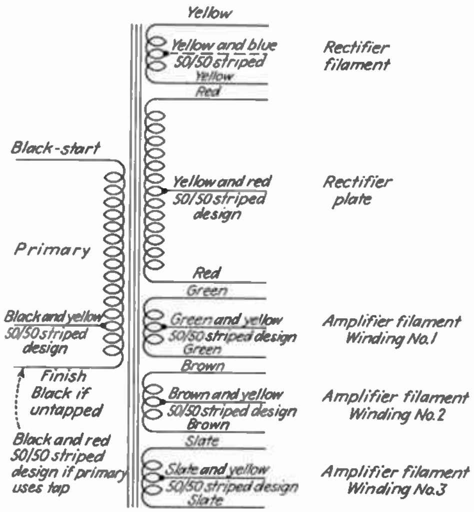


(l) 5-dot (m) 5-dot (n) 6-dot

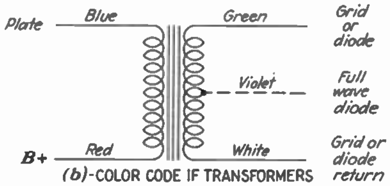
Fig. A-10 Identification systems used for ceramic, molded mica, and flat-paper capacitors.

Appendix IX

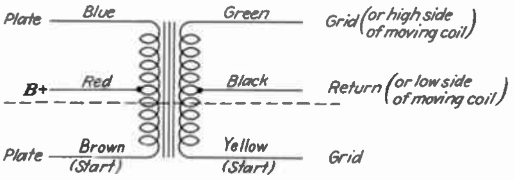
Standard Color Coding for Transformer Leads



(a)- POWER TRANSFORMER COLOR CODE



(b)-COLOR CODE IF TRANSFORMERS



The upper portion (that code above the dotted line) for single primary and/or secondary transformers

(c)-COLOR CODE-AUDIO TRANSFORMERS

Fig. A-11

Appendix X

Trigonometry

The solution of a-c problems frequently involves adding or subtracting quantities such as voltages, currents, and ohmages by means of vectors. The mathematical solution of these problems requires the use of trigonometry. The method of solution presented in the text makes it possible to solve all such problems by the use of right triangles. The following statements apply to any right triangle and are illustrated in Fig. A-12.

1. A right triangle is one in which one of the angles is a right angle (90°).
2. The hypotenuse is the side opposite the right angle.
3. The legs of a right triangle are the two sides that form the right angle.
4. The sine of any angle θ is equal to the side opposite that angle divided by the hypotenuse.
5. The cosine of any angle θ is equal to the side adjacent to that angle divided by the hypotenuse.
6. The tangent of any angle θ is equal to the side opposite to that angle divided by the side adjacent to the angle.
7. The square of the hypotenuse is equal to the sum of the squares of the two legs of the triangle.

$$\sin A = \frac{a}{c} \qquad a = c \sin A \qquad c = \frac{a}{\sin A}$$

$$\cos A = \frac{b}{c} \qquad b = c \cos A \qquad c = \frac{b}{\cos A}$$

$$\tan A = \frac{a}{b} \qquad a = b \tan A \qquad b = \frac{a}{\tan A}$$

$$\sin B = \frac{b}{c} \qquad b = c \sin B \qquad c = \frac{b}{\sin B}$$

$$\cos B = \frac{a}{c} \qquad a = c \cos B \qquad c = \frac{a}{\cos B}$$

$$\tan B = \frac{b}{a} \qquad b = a \tan B \qquad a = \frac{b}{\tan B}$$

$$c^2 = a^2 + b^2 \qquad a^2 = c^2 - b^2 \qquad b^2 = c^2 - a^2$$

The tables of Appendix XI list the values of sine and cosine for angles between 0 and 90°. In some instances, it is desired to obtain the sine of angles greater than 90°; they may be obtained in the following manner:

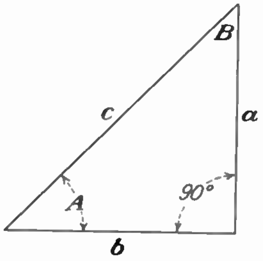


Fig. A-12

When θ is between 90 and 180°

$$\sin \theta = \cos (\theta - 90)$$

EXAMPLE: What is the sine of 137°?

$$\begin{aligned} \sin 137^\circ &= \cos (137 - 90) \\ &= \cos 47^\circ \\ &= 0.682 \end{aligned}$$

When θ is between 180 and 270°

$$\sin \theta = - \sin (\theta - 180)$$

EXAMPLE: What is the sine of 218°

$$\begin{aligned} \sin 218^\circ &= -\sin (218 - 180) \\ &= -\sin 38^\circ \\ &= -0.616 \end{aligned}$$

When θ is between 270 and 360°

$$\sin \theta = -\cos (\theta - 270)$$

EXAMPLE: What is the sine of 336°?

$$\begin{aligned} \sin 336^\circ &= -\cos (336 - 270) \\ &= -\cos 66^\circ \\ &= -0.407 \end{aligned}$$

Appendix XI

Sine, Cosine, and Tangent Tables

DEGREES	SIN	COS	TAN	DEGREES	SIN	COS	TAN
0.0	0.000	1.000	0.000	19.0	0.325	0.945	0.344
0.5	0.009	1.000	0.009	19.5	0.334	0.942	0.354
1.0	0.017	1.000	0.017	20.0	0.342	0.940	0.364
1.5	0.026	1.000	0.026	20.5	0.350	0.937	0.374
2.0	0.035	0.999	0.035	21.0	0.358	0.933	0.384
2.5	0.043	0.999	0.044	21.5	0.366	0.930	0.394
3.0	0.052	0.999	0.052	22.0	0.374	0.927	0.404
3.5	0.061	0.998	0.061	22.5	0.383	0.924	0.414
4.0	0.070	0.998	0.070	23.0	0.391	0.920	0.424
4.5	0.078	0.997	0.079	23.5	0.399	0.917	0.435
5.0	0.087	0.996	0.087	24.0	0.407	0.913	0.445
5.5	0.096	0.995	0.096	24.5	0.415	0.910	0.456
6.0	0.104	0.994	0.105	25.0	0.422	0.906	0.466
6.5	0.113	0.993	0.114	25.5	0.430	0.902	0.477
7.0	0.122	0.992	0.123	26.0	0.438	0.899	0.488
7.5	0.130	0.991	0.132	26.5	0.446	0.895	0.499
8.0	0.139	0.990	0.140	27.0	0.454	0.891	0.510
8.5	0.148	0.989	0.149	27.5	0.462	0.887	0.521
9.0	0.156	0.988	0.158	28.0	0.469	0.883	0.532
9.5	0.165	0.986	0.167	28.5	0.477	0.879	0.543
10.0	0.173	0.985	0.176	29.0	0.485	0.875	0.554
10.5	0.182	0.983	0.185	29.5	0.492	0.870	0.566
11.0	0.191	0.981	0.194	30.0	0.500	0.866	0.577
11.5	0.199	0.980	0.203	30.5	0.507	0.862	0.589
12.0	0.208	0.978	0.213	31.0	0.515	0.857	0.601
12.5	0.216	0.976	0.222	31.5	0.522	0.853	0.613
13.0	0.225	0.974	0.231	32.0	0.530	0.848	0.625
13.5	0.233	0.972	0.240	32.5	0.537	0.843	0.637
14.0	0.242	0.970	0.249	33.0	0.544	0.839	0.649
14.5	0.250	0.968	0.259	33.5	0.552	0.834	0.662
15.0	0.259	0.966	0.268	34.0	0.559	0.829	0.675
15.5	0.267	0.963	0.277	34.5	0.566	0.824	0.687
16.0	0.275	0.961	0.287	35.0	0.574	0.819	0.700
16.5	0.284	0.959	0.296	35.5	0.581	0.814	0.713
17.0	0.292	0.956	0.306	36.0	0.588	0.809	0.727
17.5	0.301	0.954	0.315	36.5	0.595	0.804	0.740
18.0	0.309	0.951	0.325	37.0	0.602	0.798	0.754
18.5	0.317	0.948	0.335	37.5	0.609	0.793	0.767

DEGREES	SIN	COS	TAN	DEGREES	SIN	COS	TAN
38.0	0.616	0.788	0.781	64.5	0.903	0.430	2.10
38.5	0.622	0.783	0.795	65.0	0.906	0.423	2.14
39.0	0.629	0.777	0.810	65.5	0.910	0.415	2.19
39.5	0.636	0.772	0.824	66.0	0.913	0.407	2.25
40.0	0.643	0.766	0.839	66.5	0.917	0.399	2.30
40.5	0.649	0.760	0.854	67.0	0.920	0.391	2.36
41.0	0.656	0.755	0.869	67.5	0.924	0.383	2.41
41.5	0.663	0.749	0.885	68.0	0.927	0.375	2.48
42.0	0.669	0.743	0.900	68.5	0.930	0.366	2.54
42.5	0.675	0.737	0.916	69.0	0.934	0.358	2.61
43.0	0.682	0.731	0.933	69.5	0.937	0.350	2.67
43.5	0.688	0.725	0.949	70.0	0.940	0.342	2.75
44.0	0.695	0.719	0.966	70.5	0.943	0.334	2.82
44.5	0.701	0.713	0.983	71.0	0.945	0.326	2.90
45.0	0.707	0.707	1.000	71.5	0.948	0.317	2.99
45.5	0.713	0.701	1.018	72.0	0.951	0.309	3.08
46.0	0.719	0.695	1.036	72.5	0.954	0.301	3.17
46.5	0.725	0.688	1.054	73.0	0.956	0.292	3.27
47.0	0.731	0.682	1.072	73.5	0.959	0.284	3.38
47.5	0.737	0.675	1.091	74.0	0.961	0.276	3.49
48.0	0.743	0.669	1.111	74.5	0.964	0.267	3.61
48.5	0.749	0.663	1.130	75.0	0.966	0.259	3.73
49.0	0.755	0.656	1.150	75.5	0.968	0.250	3.87
49.5	0.760	0.649	1.171	76.0	0.970	0.242	4.01
50.0	0.766	0.643	1.192	76.5	0.972	0.233	4.17
50.5	0.772	0.636	1.213	77.0	0.974	0.225	4.33
51.0	0.777	0.629	1.235	77.5	0.976	0.216	4.51
51.5	0.783	0.622	1.257	78.0	0.978	0.208	4.70
52.0	0.788	0.616	1.280	78.5	0.980	0.199	4.92
52.5	0.793	0.609	1.303	79.0	0.982	0.191	5.14
53.0	0.798	0.602	1.327	79.5	0.983	0.182	5.40
53.5	0.804	0.595	1.351	80.0	0.985	0.174	5.67
54.0	0.809	0.588	1.376	80.5	0.986	0.165	5.98
54.5	0.814	0.581	1.402	81.0	0.988	0.156	6.31
55.0	0.819	0.574	1.428	81.5	0.989	0.148	6.69
55.5	0.824	0.566	1.455	82.0	0.990	0.139	7.12
56.0	0.829	0.559	1.483	82.5	0.991	0.130	7.60
56.5	0.834	0.552	1.511	83.0	0.992	0.122	8.14
57.0	0.839	0.544	1.540	83.5	0.994	0.113	8.78
57.5	0.843	0.537	1.570	84.0	0.994	0.104	9.51
58.0	0.848	0.530	1.600	84.5	0.995	0.096	10.4
58.5	0.853	0.522	1.632	85.0	0.996	0.087	11.4
59.0	0.857	0.515	1.664	85.5	0.997	0.078	12.7
59.5	0.862	0.507	1.698	86.0	0.997	0.070	14.3
60.0	0.866	0.500	1.732	86.5	0.998	0.061	16.4
60.5	0.870	0.492	1.768	87.0	0.998	0.052	19.1
61.0	0.875	0.485	1.804	87.5	0.999	0.043	22.9
61.5	0.879	0.477	1.842	88.0	0.999	0.035	28.6
62.0	0.883	0.469	1.881	88.5	1.000	0.026	38.2
62.5	0.887	0.462	1.921	89.0	1.000	0.017	57.3
63.0	0.891	0.454	1.963	89.5	1.000	0.009	115.
63.5	0.895	0.446	2.01	90.0	1.000	0.000	Inf.
64.0	0.899	0.438	2.05				

Appendix XII

Answers to Odd-numbered Problems

Note 1: Answers are provided for approximately 50 per cent of the problems. Instructors using this text may obtain an answer book containing answers to even-numbered problems.

Note 2: As far as is practicable, all answers are accurate to three significant figures.

Note 3: Answers to problems involving values obtained from curves are generally difficult to check accurately because of variations in reading the curves. In preparing the answer book, enlarged drawings of the curve were used to aid in obtaining greater accuracy. In most cases the values obtained from the curves for use in solving the problems have been included with the answers.

CHAPTER 1

1. 250 m
3. 30 mc
5. (*a*) 0.469 m
   (*b*) 1.54 ft
   (*c*) 18.5 in.
7. 561.25 mc
9. (*a*) 3.14 m
   (*b*) 10.3 ft
   (*c*) 123.6 in.
11. 0.197 in.
13. (*a*) 0.0268 sec
    (*b*) 0.0537 sec
15. 0.000538 sec
17. (*a*) 1.345 m
    (*b*) 4.414 ft
19. 4,687.5 cycles
21. 0.0753 ft
23. (*a*) 4,687.5 cycles
    (*b*) 80 cycles
25. (*a*) 0.0885 sec
    (*b*) 0.00107 sec
    (*c*) The home listener
27. 1,646 miles
29. (*a*) 500 cycles
    (*b*) 1,000 cycles
31. (*a*) 100 cycles
    (*b*) 50 cycles
    (*c*) 50 cycles
    (*d*) 100 cycles
33. (*a*) 50 ft-c
    (*b*) 4 ft-c
    (*c*) 0.5 ft-c
35. (*a*) 0.0000666 cm
    (*b*) 0.0000262 in.
    (*c*) 26.2 μin.

CHAPTER 2

1. 26,667 dynes (repulsion)
3. 6,153 dynes (repulsion)
5. 40,000 dynes (repulsion)
7. 0.15 amp
9. 48 ohms
11. 0.667 ma
13. 0.005 ohm
15. 176 volts
17. The resistor that requires 250 volts for 2.5 ma
19. The 250,000-ohm resistor
21. 100 watts
23. 33.9 amp
25. (*a*) 3,730 watts
    (*b*) 2.01 hp
27. 335.7 kwhr
29. $0.25
31. 2.38 kwhr
33. $9.72
35. $11.73
37. 127.4 volts
39. 0.3 amp
41. 0.0577 amp
43. 4.38 cm
45. (*a*) 1.67 ohms
    (*b*) 15 watts
47. $0.77
49. $0.97

CHAPTER 4

1. 248,125 ohms
3. (*a*) 5 volts
   (*b*) 4.96 volts
5. 400 ft
7. 2,541 turns
9. (*a*) 1,200 ohms

(b) 0.2 amp
(c) 60 volts
100 volts
80 volts
(d) 12 watts
20 watts
16 watts
(e) 48 watts
11. *(a)* 35 volts
(b) 106.7 ohms
66.7 ohms
93.3 ohms
(c) 266.7 ohms
(d) 15 watts
9.375 watts
13.125 watts
(e) 37.5 watts
13. *(a)* 199.3 ohms
(b) 17.94 watts
15. 8.33 ohms
17. 84 ohms, 84 ohms
84 ohms
233.3 ohms
233.3 ohms
14.7 ohms
19. 2,980 ohms
7,480 ohms
21. *(a)* 127.6 ohms
(b) 0.8 amp
0.48 amp
0.6 amp
(c) 1.88 amp
(d) 192 watts
115.2 watts
144 watts
(e) 451.2 watts
23. *(a)* 0.909 amp,
5 amp
6 amp
11.909 amp
(b) 121 ohms
22 ohms
18.3 ohms
9.23 ohms
25. I_{coil}, 0.01 amp
I_{shunt}, 0.99 amp
27. *(a)* 200 ma
10 ma
550 ma
(b) 454.5 ohms
29. *(a)* Group 1
80 ohms
Group 2
60 ohms
Group 3
120 ohms
(b) 26.67 ohms
(c) 9 amp
(d) $i_{r\cdot1} = i_{r\cdot2} =$
$i_{r\cdot3} = 3$ amp
$i_{r\cdot4} = i_{r\cdot5}$
$= 4$ amp
$i_{r\cdot6} = i_{r\cdot7} =$
$i_{r\cdot8} = 2$ amp
(e) $e_{r\cdot1} = 30$ volts
$e_{r\cdot2} = 90$ volts
$e_{r\cdot3} = 120$ volts
$e_{r\cdot4} = 60$ volts
$e_{r\cdot5} = 180$ volts
$e_{r\cdot6} = 120$ volts
$e_{r\cdot7} = 90$ volts
$e_{r\cdot8} = 30$ volts
31. *(a)* Group 1
6.31 ohms
Group 2
11.25 ohms
Group 3
9.47 ohms
(b) 27.03 ohms
(c) 8.88 amp
(d) Group 1
56 volts
Group 2
100 volts
Group 3
84 volts
(e) $i_{r\cdot1} = 5.6$ amp
$i_{r\cdot2} = 1.87$ amp
$i_{r\cdot3} = 1.4$ amp
$i_{r\cdot4} = 6.66$ amp
$i_{r\cdot5} = 2.22$ amp
$i_{r\cdot6} = 1.4$ amp
$i_{r\cdot7} = 1.87$ amp
$i_{r\cdot8} = 5.6$ amp
33. 5 ohms
35. *(a)* 0.8 amp
(b) 0.4 amp
(c) 0.4 amp
(d) E_{AC}, 4 volts;
E_{AB}, 2.4 volts;
E_{BC}, 1.6 volts
(e) $i_{r\cdot2} = 0.0667$
amp
$i_{r\cdot3} = 0.200$
amp
$i_{r\cdot4} = 0.133$
amp
37. *(a)* 55 ohms
(b) $I_1 = 0.1818$
amp
$I_2 = 0.109$ amp
$I_3 = 0.0727$ amp
(c) $E_{AB} = 4.55$ volts
$E_{BC} = 5.45$ volts
$E_{BD} = 5.45$ volts
39. $R_{AB} = 20$ ohms
$R_{BC} = 18.5$ ohms
$R_{CD} = 106.5$ ohms
$R_{AD} = 145$ ohms
41. $i_{r\cdot1} = 2.07$ amp
$i_{r\cdot2} = 1.43$ amp
$i_{r\cdot3} = 0.96$ amp
$i_{r\cdot4} = 0.48$ amp
$i_{r\cdot5} = i_{r\cdot6}$
$= 0.636$ amp
$i_{r\cdot7} = i_{r\cdot10}$
$= 0.779$ amp
$i_{r\cdot8} = 0.518$ amp
$i_{r\cdot9} = 0.259$ amp
$i_{r\cdot11} = i_{r\cdot12} =$
$i_{r\cdot13} = 1.102$ amp
$i_{r\cdot14} = i_{r\cdot15}$
$= 0.184$ amp
43. $E_{AB} = 41.4$ volts
$E_{BC} = 38.2$ volts
$E_{CD} = 220.4$ volts
45. $P_{AB} = 85.7$ watts
$P_{BC} = 79$ watts
$P_{CD} = 456.3$ watts
$P_{\text{Total}} = 621$ watts
47. *(a)* $e_{r\cdot1} = 1.8$ volts
$e_{r\cdot2} = 1.2$ volts
$e_{r\cdot3} = e_{r\cdot4}$
$= 0.6$ volt

(b) $E_A = -0.6$ volt
$E_B = +0.6$ volt
49. $e_{r\cdot 1} = e_{r\cdot 2}$
$= 1.8$ volts
$e_{r\cdot 3} = e_{r\cdot 4}$
$= 1.2$ volts
$e_{r\cdot 5} = 0$
51. (a) $i_{r\cdot 1} = i_{r\cdot 4} =$
$i_{r\cdot 8} = 1$ ma
$i_{r\cdot 2} = i_{r\cdot 6} =$
$i_{r\cdot 9} = 3$ ma
$i_{r\cdot 3} = i_{r\cdot 7} =$
$i_{r\cdot 10} = 6$ ma
$i_{r\cdot 5} = 2$ ma
(b) $e_{r\cdot 1} = e_{r\cdot 4} =$
$e_{r\cdot 8} = 2$ volts
$e_{r\cdot 2} = e_{r\cdot 3} =$
$e_{r\cdot 9} = e_{r\cdot 10}$
$= 3$ volts
$e_{r\cdot 5} = e_{r\cdot 7}$
$= 6$ volts
$e_{r\cdot 6} = 12$ volts
53. (a) $i_{r\cdot 1} = i_{r\cdot 4} =$
1.25 amp
$i_{r\cdot 2} = i_{r\cdot 5} =$
3.75 amp
$i_{r\cdot 3} = 2.50$ amp
(b) $e_{r\cdot 1} = e_{r\cdot 4} =$
12.5 volts
$e_{r\cdot 2} = e_{r\cdot 5} =$
37.5 volts
$e_{r\cdot 3} = 25$ volts
55. $I_{BC} = 0.45$ ma
$I_{AB} = 5.45$ ma
57. $I_{BC} = 0.45$ ma
$I_{AB} = 3.78$ ma
59. (a) $I_{BC} = 136\ \mu$a
$I_{AB} = 1136\ \mu$a
(b) $R_{BC} = 367{,}500$ ohms
$R_{AB} = 132{,}500$ ohms
61. (a) Section 1
17,857 ohms
Section 2
10,000 ohms
Section 3
14,705 ohms
Section 4
44.4 ohms
Section 5
200 ohms
(b) 195 watts
(c) 2.79 watts

CHAPTER 5

9. (a) 555.5 dynes
(b) Repulsion
11. 4.56 cm
13. 100 unit poles
15. 7,500 dynes
17. 2,089 oersteds
19. 1,250 oersteds
21. 5 sq in.
23. 628.5 lines
27. (a) 4,000 gausses
(b) 9,000
(c) 0.000111
29. (a) 0.000333
(b) 3,000
31. (a) 10.66 gilberts
(b) 8.46 amp-turns
33. (a) 0.211 amp
(b) 151 turns
35. (a) 0.00102
(b) 14.32 amp-turns
37. (a) 0.0032
(b) 200 turns
39. (a) 0.00366
(b) 0.000277
(c) 0.00394
41. 300 turns
43. (a) 4.35 ohms
(b) 1.08 volts
45. (a) 0.0625
(b) 0.129
(c) 3,096 gilberts
47. (a) 1,635 turns
(b) No. 22
(c) 5.6 ohms
(d) 8.4 volts
49. (a) 0.0115
(b) 45.6 amp-turns
51. 0.2007
53. (a) 0.00136
(b) 16.18 amp-turns
55. (a) 0.0046
(b) 95 amp-turns
57. (a) 190 turns
(b) No. 25
(c) 0.915 volt
59. (a) 0.1296
(b) 2,675 amp-turns

CHAPTER 6

1. (a) 7.0 volts
(b) 113 volts
3. 99,800 ohms
5. (a) 15,000 ohms
(b) 450,000 ohms
7. (a) 100,000 ohms
(b) 500,000 ohms
(c) 1.0 megohm
(d) 2.5 megohms
(e) 10 megohms
9. 5,000 ohms per volt
11. 5,000 ohms per volt
13. (a) 50%
(b) 33.3%
15. (a) 40 volts
(b) 20%
17. (a) 49.3 volts
(b) 1.4%
19. (a) 13.89 ohms
(b) 5.208 ohms
21. (a) 2.777 ohms
(b) 1.315 ohms
(c) 0.641 ohm
(d) 0.0626 ohm
23. (a) 10
(b) 20
(c) 40
25. (a) $E = 5.97$ volts
$I = 2.9856$ amp
$R = 2$ ohms
% error = nil
(b) $E = 6$ volts
$I = 2.985$ amp
$R = 2.01$ ohms
Error = 0.5%
27. (a) $E = 0.9$ volt
$I = 0.3003$ amp
$R = 3$ ohms
% error = nil

(*b*) $E = 1.5$ volts
$I = 0.3$ amp
$R = 5$ ohms
Error = 66.7%
29. 300,000 ohms
31. (*a*) 13.5 megohms
(*b*) 1.5 megohms
(*c*) 1.0 megohm
33. 9,000 ohms
35. (*a*) 75,000 ohms
(*b*) 30,000 ohms
(*c*) 15,000 ohms
37. (*a*) Diagram
(*b*) Resistors for voltage measurements
10-volt scale, 9,970 ohms
50-volt scale, 49,970 ohms
100-volt scale, 99,970 ohms
250-volt scale, 249,970 ohms
500-volt scale, 499,970 ohms
Shunts for current measurements
1-ma scale, no shunt needed
10-ma scale, 3.33 ohms
50-ma scale, 0.612 ohm
100-ma scale, 0.303 ohm
1,000-ma scale, 0.0300 ohm
39. 0.3 ohm
41. 4 μf
43. 1,500 mh

CHAPTER 7

1. 249.6 volts
3. 6 turns per coil
5. 435,600 maxwells
7. (*a*) 60 cycles
(*b*) 40 cycles
(*c*) 25 cycles
9. (*a*) 40 poles
(*b*) 20 poles
11. (*a*) 25 cycles
(*b*) 60 cycles
(*c*) 500 cycles
13. (*a*) 13.365 amp
(*b*) −9.33 amp
(*c*) −15 amp
(*d*) −14.055 amp
(*e*) −3.885 amp
15. 36.4 volts
17. 159.25 volts
19. 127.4 volts
23. 141.4 volts
25. 409.4 volts
27. (*a*) 200 volts
(*b*) 127.4 volts
(*c*) 141.4 volts
29. (*a*) 990 va
(*b*) 0.833
31. (*a*) 11.76 amp
(*b*) 1,294 va
33. (*a*) 3,806 va
(*b*) 0.788
(*c*) 220 volts
(*d*) 5.78 amp
35. (*a*) 635 va
(*b*) 1,905 va
(*c*) 1,238 watts
(*d*) 0.866
37. (*a*) 300 amp
(*b*) 2,400 volts
(*c*) 173.4 amp
(*d*) 1,000 kw
39. 120 volts, 85 volts
41. 261.4 volts
241.4 volts
43. 1,750 turns
45. (*a*) 4 turns
(*b*) 10 turns
(*c*) 1,200 turns
(*d*) 600 turns
47. 4.375 amp
49. 1.43 amp
51. 87%
53. 78.3 watts
55. 1,489 hp

CHAPTER 8

1. 40 volts
3. 9.3 volts
5. 494 μh
7. 260 turns
9. (*a*) 0.265 henry
(*b*) 9 henrys
11. 39.3 mh
13. 4.92 mh
15. 12.75 mh
17. (*a*) 11,304 ohms
(*b*) 22,608 ohms
19. (*a*) 863.5 ohms
(*b*) 1,570 ohms
(*c*) 2,355 ohms
(*d*) 11,775 ohms
21. (*a*) 99.85 ohms
(*b*) 3,391 ohms
23. (*a*) 3,768 ohms
(*b*) 3,798 ohms
(*c*) 28.9 ma
25. 79.1 ma
27. 0.12 sec
29. (*a*) 11,304 ohms
(*b*) 11,307 ohms
(*c*) 88.5°
(*d*) 26.5 ma
31. 1.46 henrys
33. 0.0817
35. 9 volts
37. (*a*) 16 henrys
(*b*) 6,029 ohms
39. (*a*) 200 μh
(*b*) 3,768 ohms
41. (*a*) 900 μh
(*b*) 100 μh
(*c*) 500 μh
(*d*) 700 μh
43. 77.08 μh
45. 7.93
47. (*a*) 0.304 henry
(*b*) 22.96
49. 106 mh
51. (*a*) 29.191 henrys
(*b*) 91.66

CHAPTER 9

1. (*a*) 25×10^{-6} coul
(*b*) 50 ma
3. (*a*) 123.475 pf
(*b*) 49.39 pf
5. 0.00148 μf
7. 31 plates
9. 0.016 μf
11. (*a*) 26,500 ohms
(*b*) 1,590 ohms
(*c*) 3.18 ohms
(*d*) 0.795 ohm
13. (*a*) 4.52 μa
(*b*) 376.8 μa
(*c*) 75.36 ma
(*d*) 4.52 amp
15. (*a*) 331 ohms
(*b*) 331 ohms
(*c*) 0.332 amp
(*d*) 0.0302
(*e*) 88.5°
17. (*a*) 0.00675 sec
(*b*) 0.03375 sec
19. (*a*) 0.12 sec
(*b*) 0.6 sec
(*c*) 117.9 volts
(*d*) 100 ma
(*e*) 60.7 ma
(*f*) 300 ma
(*g*) 110.4 volts
(*h*) 110.4 ma
21. 0.22%
23. (*a*) 636,600 ohms
(*b*) 0.0276 ohm
(*c*) 24.0276 ohms
25. 0.0663 ohm
27. 7.95 ohms
29. 22.6%
31. (*a*) 1.99 ohms
(*b*) 13.25 ohms
(*c*) 13.4 ohms
(*d*) 14.8%
33. 6.65
35. 33.1
37. (*a*) 0.545 ma
(*b*) 2.18 ma
39. (*a*) 30 μa
(*b*) 300 μa
41. N750, 100 pf
NP0, 240 pf
43. 180 watt-sec
45. 78×10^{6} watt-sec
47. 99.65 pf
49. 0.848 sq in.
51. (*a*) 2 μf
(*b*) 331, 331,
662 ohms
(*c*) 1,324 ohms
(*d*) 189 ma (each)
(*e*) 62.5, 62.5,
125 volts
53. (*a*) 500×10^{-6} coul
(*b*) 62.5, 62.5,
125 volts
55. (*a*) 14 μf
(*b*) 331, 662,
1,324 ohms
(*c*) 189.2 ohms
(*d*) 110, 110,
110 volts
(*e*) 332, 166, 83 ma
(*f*) 581 ma
57. 20 μf
59. 5 μf
61. 2.5 μf
63. 6.67 μf
65. (*a*) 6.65 μf
(*b*) 0.0825
(*c*) 85°
67. 3.76 ma

CHAPTER 10

1. (*a*) 31,400 ohms
(*b*) 31,404 ohms
(*c*) 3,140,000 ohms
(*d*) 3,140,000 ohms
3. (*a*) 331.7 ohms
(*b*) 331.8 ohms
5. (*a*) 3,080 ohms
(*b*) 6.49 ma
7. (*a*) 3,025 ohms
(*b*) 99.17 ma
(*c*) 76 volts,
224.2 volts
9. (*a*) 45 ohms
(*b*) 2.67 μa
(*c*) 1 mv
11. (*a*) 8,200 ohms
(*b*) 5 ma
(*c*) e_1, 8.25 volts
e_2, 2.23 volts
e_3, 40.2 volts
e_4, 10.2 volts
13. 111.8 ohms
15. 15,074 ohms
17. (*a*) 39.8 ma
(*b*) Zero
(*c*) 4.776 va
(*d*) Zero
(*e*) 90° (lagging)
(*f*) Vector diagram
19. (*a*) 3.39 amp
(*b*) Zero
(*c*) 1,017 va
(*d*) Zero
(*e*) 90° (leading)
(*f*) Vector diagram
21. (*a*) 36 ohms
(*b*) 6.11 amp
(*c*) 1,120 watts
(*d*) 1,344 va
(*e*) 0.833
(*f*) 33.5° (lagging)
(*g*) Vector diagram
23. (*a*) 3,750 ohms
(*b*) 468.75 ohms
(*c*) 6 va
(*d*) 0.125
(*e*) 83° (lagging)
(*f*) 3,720.6 ohms
(*g*) 9.87 henrys
(*h*) Vector diagram
25. (*a*) I_1, 3.33 amp
I_2, 2.5 amp
I_3, 1.67 amp
(*b*) 3.43 amp
(*c*) 29.1 ohms
(*d*) P_1, 332.7 watts
P_2, zero
P_3, zero
(*e*) 332.7 watts
(*f*) 343 va
(*g*) 0.970

(*h*) 14° (lagging)
(*i*) Vector diagram
27. (*a*) I_1, 5 ma
I_2, 104.16 ma
I_3, 39.06 ma
I_4, 7.81 ma
I_5, 52.08 ma
I_6, 10 ma
(*b*) 110.3 ma
(*c*) 2,266 ohms
(*d*) P_1, 1.25 watts
P_2, zero
P_3, zero
P_4, zero
P_5, zero
P_6, 2.5 watts
(*e*) 3.75 watts
(*f*) 27.575 va
(*g*) 0.136
(*h*) 82° (leading)
(*i*) Vector diagram
29. (*a*) Z_1, 76.1 ohms
I_1, 1.31 amp
P_1, 51.48 watts
PF_1, 0.394
θ_1, 67° (lagging)
Z_2, 100 ohms
I_2, 1 amp
P_2, 100 watts
PF_2, 1.00
θ_2, 0°
Z_3, 79 ohms
I_3, 1.26 amp
P_3, 39.7 watts
PF_3, 0.316
θ_3, 71.5° (leading)
(*b*) I_{line}, 1.914 amp
P_{line}, 191.18 watts
$\mathrm{AP}_{\mathrm{line}}$, 191.4 va
$\mathrm{PF}_{\mathrm{line}}$, 0.9988
θ_{line}, 3° (lagging)
(*c*) Vector diagram
31. (*a*) I_{line}, 0.676 amp
P_{line}, 67.57 watts
$\mathrm{AP}_{\mathrm{line}}$, 67.6 va
$\mathrm{PF}_{\mathrm{line}}$, 0.9996
θ_{line}, 1.5° (lagging)
(*b*) E_{R1}, 18.65 volts
I_{R1}, 0.622 amp
E_{XL}, 18.65 volts
I_{XL}, 0.266 amp
E_{R2}, 67.6 volts
I_{R2}, 0.676 amp
E_{R3}, 16 volts
I_{R3}, 0.64 amp
E_{XC}, 16 volts
I_{XC}, 0.213 amp
(*c*) Vector diagram
33. (*a*) 53,078 ohms
(*b*) 212 ohms
(*c*) Approx 10%
(*d*) Approx 90%
(*e*) Approx 95%
(*f*) Approx 5%
35. (*a*) 159,235 ohms, 212 ohms
(*b*) 376.8 ohms, 282,600 ohms
(*c*) High pass
37. *R*, 68.9 ohms
X, 172 ohms
39. *R*, 1,000 ohms
X, 500 ohms
41. (*a*) $50 - j300$
(*c*) $90 + j350$
42. (*a*) 6.32 amp
(*c*) 6.83 amp
43. (*a*) $20 + j900$
(*c*) $40 - j1{,}500$
44. (*a*) $-2{,}400 + j2{,}800$
(*c*) $-4{,}200 - j4{,}000$
45. (*a*) $0.9 - j0.2$
(*c*) 2
46. (*a*) $63.2\underline{/71.5^\circ}$
(*c*) $107.7\underline{/-68^\circ}$
47. (*a*) $20 + j34.64$
(*c*) $1{,}880 - j684$
48. (*a*) $97.6\underline{/-21.5^\circ}$ ohms
(*c*) $1{,}454\underline{/50^\circ}$ ohms
49. (*a*) $48.2\underline{/75^\circ}$ ohms
(*c*) $11.32\underline{/90^\circ}$ ohms
50. (*a*) $175\underline{/78^\circ}$
(*c*) $150\underline{/27^\circ}$
51. (*a*) $5\underline{/42^\circ}$
(*c*) $0.2\underline{/-42^\circ}$
52. (*a*) $225\underline{/53^\circ}$
(*c*) $256\underline{/92^\circ}$
53. (*a*) $25\underline{/34^\circ}$
(*c*) $8\underline{/45^\circ}$
54. (*a*) $0.125\underline{/-36.5^\circ}$
(*c*) $2.5\underline{/16^\circ}$
55. (*a*) $25 + j97{,}626$
(*b*) 97,626 ohms
57. 29.1 ohms
59. (*a*) 240 ohms
(*b*) 240 ohms
61. $240\underline{/-37^\circ}$ ohms
63. $52\underline{/0.5^\circ}$ ohms
65. *Z*, 148 ohms
I, 0.675 amp
θ, 1.5° (lagging)
67. E_{Ri}, 8.63 volts
I_{Ri}, 8.63 ma
E_C, 4.3 volts
I_C, 8.6 ma
E_{Ro}, 4.3 volts
I_{Ro}, 0.86 ma
69. E_{R1}, 1.8 volts
I_{R1}, 150 ma
E_{R2}, 1.2 volts
I_{R2}, 100 ma
E_{R3}, 0.6 volt
I_{R3}, 50 ma
E_{R4}, 0.6 volt
I_{R4}, 50 ma
71. (*a*) Approx 50 volts
(*b*) 46 volts

CHAPTER 11

7. (*a*) 86 ohms (capacitive)
(*b*) 46 ohms (capacitive)
(*c*) 7 ohms (capacitive)
(*d*) 33 ohms (inductive)
(*e*) 73 ohms (inductive)

9. (*a*) 1,001.8 kc
   (*b*) 1,000 kc
11. (*a*) 320 pf
   (*b*) 35.5 pf
13. (*a*) 289 μh
   (*b*) 2,416 kc
15. 489 to 1,707 kc
17. (*a*) 25 μh
   (*b*) 5,807 kc
   (*c*) 8,212 kc
   (*d*) 10,056 kc
19. (*a*) 204 mc
   (*b*) 194 mc
21. (*a*) 15.86 pf
   (*b*) 1.96 mc
   (*c*) 3.36 mc
   (*d*) 3.77 mc
23. 209.6 pf
25. (*a*) 996 μh
   (*b*) 292
27. (*a*) 47.8 μμf
   (*b*) 525
29. (*a*) 5,000 cycles
   (*b*) 5,039 cycles
33. (*a*) $E_R = 10$ mv
   $E_L = 1.987$ mv
   $E_C = 1.987$ mv
   (*b*) 198.7
35. 840,457
37. (*a*) 199 μh
   (*b*) 1,083
   (*c*) 1.2 kc
39. (*a*) 70 pf
   (*b*) 3.014 ohms
41. (*a*) 89,443 ohms
   (*b*) 171,371 ohms

Note: Values of X_L and X_C were obtained from Table 11-4 by interpolation.

CHAPTER 12

1. (*a*) 2,500.15 ohms
   (*b*) 4,522 ohms
3. (*a*) 2,501 ohms
   (*b*) 2,515 ohms
5. (*a*) $X_L = 1{,}413$ ohms
   $X_C = 1{,}060$ ohms
   $Z = 353$ ohms (inductive)
   (*b*) $X_L = 42{,}390$ ohms
   $X_C = 35.3$ ohms
   $Z = 42{,}355$ ohms (inductive)
7. (*a*) 1,300 kc
   (*b*) 1 megohm
9. (*a*) 0.663 ohm
   (*b*) 0.06625 ohm
11. (*a*) 73,666 ohms
   (*b*) 2,650 ohms
13. 1.18 henrys
15. (*a*) 4,000-cycle audio frequency
   (*b*) 375
   (*c*) 4,000-cycle audio frequency
17. (*a*) 63,600 ohms
   (*b*) 628 ohms
   (*c*) 424 ohms
   (*d*) 94,200 ohms
19. (*a*) 6,625 ohms
   (*b*) 331.25 ohms
   (*c*) 1,507.2 ohms
   (*d*) 30.144 ohms
21. 0.6
23. 0.163 pf
25. (*a*) $K = 6$
   $R_1 = 1{,}500$ ohms
   $R_2 = 360$ ohms
   (*b*) $R_i = 1{,}800$ ohms
   (*c*) $e_o = 10$ volts
   $K = 6$
27. (*a*) $e_o = 30$ volts
   $R_1 = R_3 = 80{,}000$ ohms
   $R_2 = 50{,}000$ ohms
   (*b*) $R_i = 120{,}000$ ohms
29. (*a*) $K = 1.5$
   $R_1 = R_3 = 60{,}000$ ohms
   $R_2 = 5{,}000$ ohms
   (*b*) $R_i = 12{,}000$ ohms
31. (*a*) 80 henrys
   (*b*) 111 ohms
   (*c*) 800 ohms
   (*d*) 720 henrys
   (*e*) 900 ohms
33. 9.30 kc
35. 187.5
37. (*a*) 0.0219
   (*b*) 451 kc
   461 kc

Note: By use of Eqs. (12-28) and (12-30)

39. (*a*) 0.025 sec
   (*b*) 0.04 sec
   $(k - 1.60)$
41. (*a*) 2 megohms
   (*b*) 1.33 megohms
   (*c*) 0.2 μf
43. (*a*) 0.005 sec
   $(k - 0.2)$
   0.0125 sec
   $(k - 0.5)$
   0.04 sec
   $(k - 1.6)$
   (*b*) 500 μa
   (*c*) 100 μa
   (*d*) 500 μa
   (*e*) 92 volts
   (*f*) 0.0175 sec
   $(k - 0.7)$

CHAPTER 13

1. (*a*) 6.72 oz-in.
   (*b*) 8.96 oz
3. (*a*) 50 hp
   (*b*) 200 lb
5. 24.5%
7. 72.7%
9. 58.7%
11. (*a*) 112.2 volts
   (*b*) 108.76 volts
13. 2,900 rpm
15. 3.45%
17. (*a*) 1.2 amp
   (*b*) 1,250 turns

19. (*a*) 460 amp
    (*b*) 5.5 ohms
    (*c*) 3.58 ohms
21. 40 turns
23. (*a*) 1,200 amp-turns
    (*b*) 300 amp-turns
    (*c*) 1,200 turns
    (*d*) 8 turns
25. (*a*) 3,600 rpm
    (*b*) 1,800 rpm
    (*c*) 900 rpm
    (*d*) 450 rpm
27. (*a*) 1,200 rpm
    (*b*) 4.17%
29. 3,456 rpm
31. 80 amp
33. 150 watts
35. 23 ohms
    ($I_f = 0.77$ amp)
37. 110 volts
39. 80
41. 5,625

CHAPTER 14

1. 80
3. 50,000 ohms
5. 1,400 μmhos
7. 1,400 μmhos
9. 74.2
11. 32,000 ohms
13. 51.2
15. (*a*) 400 μa
    (*b*) 25.6 volts
17. 108,000 ohms
19. (*a*) 280 μa
    (*b*) 30 volts
21. (*a*) 80,000 ohms
    (*b*) 100 μa
    (*c*) 8 volts
23. (*a*) 95
    (*b*) 95 μa
    (*c*) 9.5 volts

CHAPTER 15

1. (*a*) 1,900
   (*b*) 1,805
3. 0.95 ($I_{c\cdot 1} = 0.95$,
   $I_{c\cdot 2} = 1.9$)
5. 14 ($I_{c\cdot 150} = 2.4$,
   $I_{c\cdot 100} = 1.7$)
7. 0.974
9. (*a*) 42.4
   (*b*) 347
   (*c*) 14,700
11. (*a*) 0.858
    (*b*) 1,925
    (*c*) 1,655
13. (*a*) 49.3
    (*b*) 1
    (*c*) 50

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