INTERNATIONAL TEXTBOOKS IN ELECTRICAL ENGINEERING
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Electrical Engineering

ELECTRICAL ENGINEERING

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Preface for Third Edition

This book is written especially for engineering college students who are not majoring in electrical engineering and also for those who are majoring in electrical communication.

The beginning portion contains enough single-phase, polyphase, and transient circuit analysis to enable the student to understand the more common types of industrial electrical machinery as described in the middle half and also to understand the more common elements of electronic industrial control described in the last portion. It is believed that the contents are sufficient to give a "non-electrical" engineer a good foundation or working knowledge of all electrical apparatus that he is likely to use. It should enable him to speak and understand the language of the electrical engineer.

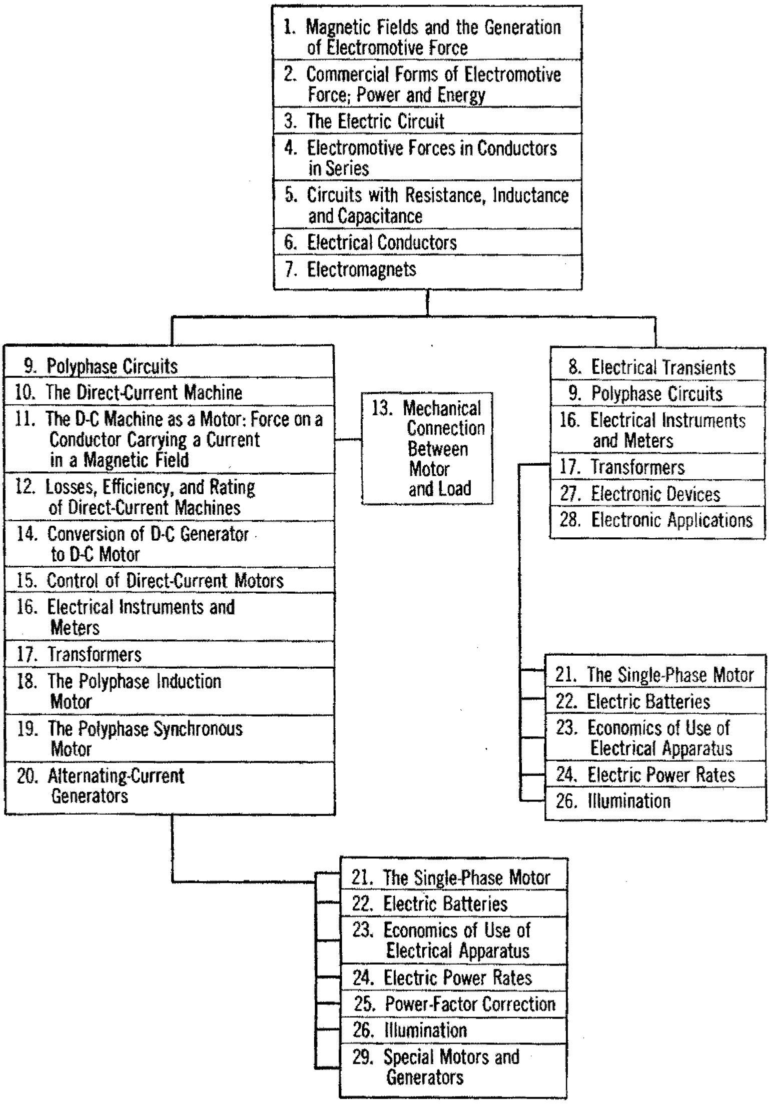
Furthermore it is believed that an electrical engineer whose major interest is communication will find about all of the working knowledge of power machinery that he is likely to need in his chosen specialty.

The choice of material is also intended to provide an introductory course for electrical engineers where that course will be followed by more detailed courses. The pace is set to enable the instructor to start laboratory assignments almost immediately after the first lecture.

Inasmuch as it may be desirable to omit some parts of the content to accommodate a foreshortened course, a flow sheet has been added to show chapters that must be studied in sequence and others that may be interspersed as laboratory progress permits.

E. E. KIMBERLY

FLOW SHEET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Introduction

The Structure of Matter.—All matter is believed to be made up of *atoms*. Nothing was known about the structure of atoms until Thomson discovered in 1895 that they contain both positively and negatively charged electrical particles. The positively charged particle is called a *proton* and has a mass of approximately 1 atomic-weight unit. The negatively charged particle is called an *electron* and has a mass about $\frac{1}{1840}$ times that of a proton. The hydrogen atom consists of one proton and one electron. The electron rotates in an orbit about the proton as if in a miniature solar system. The negative charge on the electron has the same magnitude as the positive charge on the proton, and so a combination of the two exhibits no electrical manifestations.

An atom may contain more than one proton and usually contains as many electrons as protons. The proton group, regardless of the number of protons it contains, is called the *nucleus* of the atom. Chemical elements are different one from the other because of the different numbers of protons and electrons contained in their atoms. The characteristics of atoms are not entirely accounted for by assuming only protons and electrons in their structures. As more is learned about the atom, additional particles—such as neutrons, positrons, and mesons—are imagined in order to explain the new characteristics that are discovered. In the present state of the science of engineering, only protons and electrons are useful in explaining electrical phenomena.

Static Electricity.—When glass is rubbed with silk in the classical experiment in physics, some of the electrons in atoms in the glass are induced to move over to the silk and so leave more protons than electrons in the rod. The rod then exhibits a *positive charge*, and the silk has a *negative charge*. Such charges are called *static charges* because they "stand on" or inhabit the bodies which exhibit the charges. Such manifestations have few uses in engineering and, indeed, are likely to be troublesome. Static charges appear on sheets of paper passing through a printing press and cause the sheets to cling to each other and to the press. They appear on the uniforms of nurses in hospital operating rooms and, by their discharge, explode vapors of anaesthetics which may be present. They appear on rapidly moving power-transmission belts and may produce sparks several feet long.

Probably the most useful application is that of removing dust, smoke, and vapor particles by precipitation from gases, from air in particular. The particles in passage through the precipitator are given a charge, which is usually positive, and are then caused to flow near plates or screens of opposite charge to which they are attracted. The plates or screens are shaken down periodically, and the residue is removed. No "static machine" is used, however, for these industrial applications. Instead, high-voltage electron tubes described in a subsequent chapter are used to obtain high potentials.

Dynamic Electricity.—Early scientists studying electricity recognized what seemed to be a migration phenomenon, but did not know whether the electrons or the protons flow from one place to another. Faced by the need of a convention, they chose to say that the flow was that of protons. Later discoveries proved their choice to be an unfortunate one. A flow of electrons constitutes an electric current, which is *dynamic electricity* or electricity in motion. Electrons will not flow from one place to another unless some force is applied to them. Any force which will cause such flow is called an *electromotive force* (abbreviated emf).

Conductors and Insulators.—An electromotive force applied to the ends of a copper rod will cause a flow of electrons from atom to atom in the rod, because the copper atom yields its electrons readily. The iron atom yields its electrons less readily; so, if the rod were of iron, less current would flow. If the rod were of wood or glass, still less current would flow because atoms of those substances yield their electrons very reluctantly. A substance that yields its electrons readily is called a *conductor*, and one that yields them reluctantly is called an *insulator*. Throughout the range of possible substances to which electromotive forces may be applied, there is no definite dividing point to distinguish conductors from insulators. No substance is a perfect conductor, and none is a perfect insulator. Silver is the best conductor known. Copper, while not so good a conductor as silver, is most commonly used because of its relatively low cost. Dry wood, porcelain, glass, and many synthetic materials are very good insulators.

Sources of Electromotive Force.—When a sheet of copper and a sheet of zinc are immersed in a dilute solution of sulfuric acid, an electromotive force is produced and a difference in electrical potential is found to exist between the metal sheets. When these two sheets of metal are connected by a conducting wire, electrons flow through the wire from the zinc to the copper while within the solution electrons flow from the copper to the zinc. Thus, the electrons are said to flow around the *circuit*. Many combinations of other metals and other solutions will produce the same phenomenon involving flow of electrons. Electromotive force is thus produced by chemical action. This is the principle of all *primary batteries*, including the so-called dry cells used in pocket flashlights.

When a rod of a metal such as bismuth is joined to another rod of a metal such as antimony and their joined ends are brought to a temperature different from that of their free ends, an electromotive force is produced and a difference in electrical potential appears between their free ends. Such a combination is called a *thermo-couple*. Similar use of many other metals produces similar results. Thermo-couples find use in the measurement of temperatures, particularly those for which mercury-in-glass thermometers would be too sluggish in response or where it would be inconvenient or impossible to place them satisfactorily.

The most common method of producing or *generating* an electromotive force is that of moving a conductor across a magnetic field or by causing the strength of a magnetic field to change in a coil of a conductor. These phenomena will be explained independently.

CHAPTER 1

Magnetic Fields and the Generation of Electromotive Force

Magnetic Field.—A bar of hard steel that has been magnetized is surrounded by a space within which other magnets are attracted or repelled by it. This space is called the *magnetic field* and is theoretically infinite in volume. The force with which a field acts upon another magnet at any point is dependent on the *intensity* of the field at that point. The intensity at any point in a field is dependent on its distance from the magnet which produces the field. The intensity of a magnetic field at any point is defined in terms of the force which it would exert upon a *unit pole* placed at that point. In the concept of unit pole, it is assumed that either an isolated north pole or an isolated south pole of a magnet can be produced in space. If two such isolated poles be placed 1 centimeter apart and their magnetic strengths be made equal and of such a value that they attract or repel one another with a force of 1 dyne, then each pole is said to have the strength of 1 unit pole.

By definition, a magnetic pole which is attracted toward the north by the earth's magnetic field is a *north pole*, and one which is attracted toward the south is a *south pole*. If a magnetic compass be brought into the proximity of a bar magnet, as in Fig. 1–1, the compass needle will assume a definite direction.

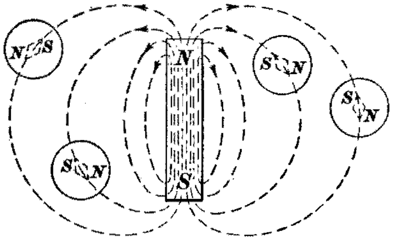

Fig. 1–1. Magnetic Field Around a Bar-Type Magnet

As the compass is moved about in the field, it will be seen that the direction of the force acting on the needle changes from point to point. At every point the magnetic field has two defining properties, *intensity* and *direction*.

If the compass be moved always on a line drawn longitudinally through the needle, it will describe a smooth curve in space between the north and south poles of the magnet. This curve is said to represent the path of a magnetic *line of force*. For quantitative purposes the strength of a magnetic field at any point is defined in terms of lines of force or *maxwells* per square centimeter. Thus, if a unit pole in a magnetic field be acted upon by a magnetic force of 1 dyne, the field intensity at that point is said to be 1 line of force per square centimeter or 1 *gauss*. All lines of force are said to "emerge" from the north pole and to "enter" at the south pole of a magnet and to close upon themselves within the magnet. Fig. 1–2 shows the magnetic field produced by a horseshoe magnet. Properties of the magnetic field are further discussed in Chapter 7.

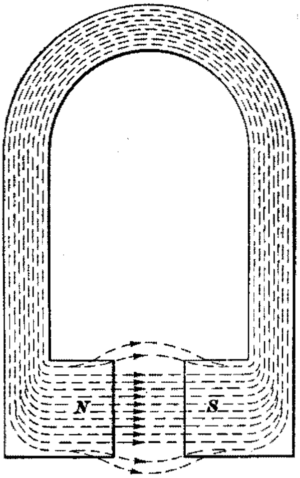

Fig. 1–2. Magnetic Field Around the Poles of a Horseshoe-Type Magnet

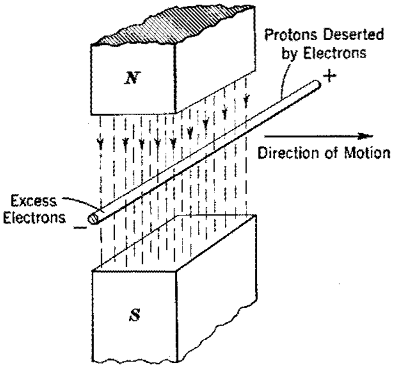

Fig. 1–3. Generation of Electromotive Force in a Conductor

Electromotive Force Produced by Motion.—If a rod of conducting material be caused to move in a magnetic field, as shown in Fig. 1–3, there will be a disturbance in the balance of protons and electrons within the atoms of the rod. Some electrons will migrate to the near end of the rod, leaving an excess of protons on the far end. This disturbance in balance is said to be caused by an electromotive force (emf) resulting from the motion of the conductor *across* the magnetic field. The electromotive force is said to produce a *difference of potential* between the two ends of the rod. In practical units, electromotive force is measured in *volts*. When the conductor is cutting a magnetic field or "magnetic flux" at the rate of *10^8 lines of force per second,* 1 volt is being generated.

Expressed as a formula, the foregoing statement may be written as follows:

$$e = \frac{N\,d\phi}{10^8\,dt} \tag{1–1}$$

where e = instantaneous generated emf;
N = number of conductors in series (1 in Fig. 1–3);
ϕ = number of lines of magnetic flux;
t = time, in seconds.

This equation is called Faraday's Equation in honor of the discoverer of these relationships. The phenomenon is known as *electromagnetic induction* and is the most common means used in the generation of voltages for commercial use.

If there be a difference in potential between the ends of a rod, as in Fig. 1–3, and the ends be joined by a metal wire, the electrons will flow from the near end of the rod through the wire to the far end of the rod and restore the electron-proton balance. Inasmuch as the excess electrons on the near end of the rod are attracted to the excess positive protons on the far end of the rod, the far end is said to be at positive potential or "positive."

If the direction of motion in Fig. 1–3 be reversed, the direction of the emf generated in the rod will be reversed also. Or, if the direction of motion remain as before and the polarity of the magnet be reversed, the direction of the emf generated in the rod will be reversed. The magnitude of the emf generated in a conductor depends directly on the rate at which that conductor cuts magnetic lines of force. Therefore, the emf is a direct function of the velocity of the conductor and of the strength of the magnetic field through which it moves.

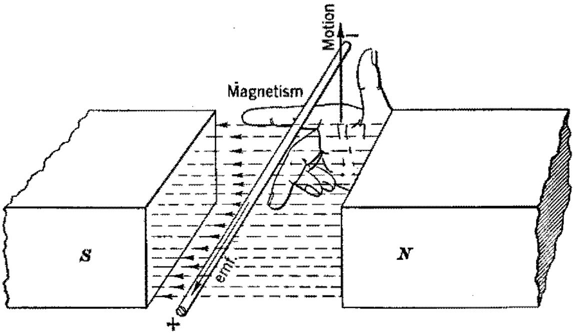

Fig. 1–4. Fleming's Rule of Generated emf

Fleming's Right-Hand Rule.—By a rule known as *Fleming's right-hand rule*, the direction of the emf generated in a moving conductor may be found if the direction of the magnetic field and the direction of motion of the conductor in the field are known. The rule is as follows: Extend the thumb, the first finger, and the second finger of the *right* hand mutually at right angles along the axes of an imaginary three-plane coordinate system. Point the thumb in the *direction of motion of the conductor relative to the field*, and point the forefinger in the *direction of the flux* (magnetic lines), as indicated in Fig. 1–4. The second finger will then point along the conductor in the direction of the positive end of the conductor.

If the conductor of Fig. 1–4 were a part of a complete circuit, the current which the electromotive force would cause to flow would be said by convention to flow from the positive end of the conductor. All rules of current flow are based on that premise, regardless of the fact that it is now known that the electrons actually flow in the opposite direction. It is not necessary to modify that point of view until the actual direction of electron flow must be considered in electron tubes.

Electromotive Force Produced by Change of Flux Linkages.—If the conductor of Fig. 1–4 were entirely above the magnetic field and its ends were connected by a loop of wire hanging below the field, the field would be entirely enclosed by the link or turn of conductor so formed. If then the link were dropped downward an inch, it would no longer encircle or link any flux. The flux in the conductor link would then have changed in the amount of ϕ, the flux of the field. The flux lines of the field itself are closed links, as shown in Fig. 1–2. Therefore, the linkages of the flux lines with the conductor link would have been changed; in this case, they would have been reduced to zero. The voltage generated in the conductor link would be proportional to the *rate of change of total flux linkages.* If the downward movement were accomplished in 1 second and the links of flux were 10^8 lines, the average voltage generated would be 1 volt because the flux linkage change per second would be $10^8 \times 1 = 10^8$. The equation of the voltage at any instant in the movement of the conductor link is then

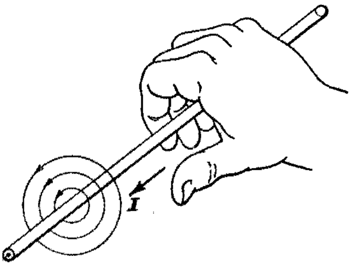

Fig. 1–5. Use of Rule of Thumb

$$e = N\frac{d\phi}{10^8\,dt}$$

This equation is the same as equation 1–1, but N is the number of conductor links or turns (1 in this case). The flux-linkage viewpoint then leads to the same result as the conductor-cutting-flux viewpoint. The flux-linkage theory will be enlarged under the subject of inductance in a subsequent chapter.

If—instead of being moved as described—the conductor had been held fixed while the flux was reduced to zero in 1 second, the rate of change of flux linkages and hence the average voltage generated would have been the same. That is the principle of the power distribution transformer.

Magnetic Field About a Conductor Carrying Current.—An electric current flowing in a conductor produces a magnetic field around that conductor. Fig. 1–5 shows the direction of this field when the current flow is toward the observer. A dot (suggesting the point of an arrow) on the cross-section of a conductor conventionally represents current or emf toward the observer. A cross or plus sign (suggesting tail feathers of an arrow) represents current or emf away from the observer. If the conductor be grasped by the right hand with the thumb pointing in the direction of current flow, then the fingers will point in the direction of the magnetic lines of flux around the conductor, as shown in Fig. 1–5. This is the *magnetic rule of thumb.*

The Ampere.—In the centimeter-gram-second (cgs) system of units, electric current is measured in *abamperes.* The *ampere* is the practical unit of current and is equal to one-tenth of an abampere. The ampere may also be defined as that constant current which, when passed through a standard solution of nitrate of silver in water, deposits silver at a rate of 0.001118 gram per second. Also the current is said to be 1 ampere when the rate of electron flow is 6.28×10^{18} electrons per second.

Electrical Resistance.—When water flows through a pipe, the walls of the pipe offer resistance to the flow. Similarly, when an electric current flows through a conductor, the substance of the conductor offers *resistance* to the flow. The *ohm*[1] is the unit of measurement of electrical resistance, and the conventional symbol for it is Ω. A current of 1 ampere will flow in a conductor having a resistance of 1 ohm when a difference of potential of 1 volt is applied. The flow of electric current through a conductor produces heat and increases the temperature of the conductor.

Ohm's Law.—If a constant difference of potential V be impressed between the ends of a conductor, the current which will flow will be directly proportional to the difference of potential and inversely proportional to the resistance of the conductor. That fact was discovered by Ohm and the law bears his name. The relation may be expressed by the following formulas:

$$I = \frac{V}{R} \tag{1–2}$$

$$R = \frac{V}{I} \tag{1–3}$$

$$V = IR \tag{1–4}$$

where I = the current, in amperes;
R = the resistance, in ohms;
V = difference of potential, in volts.

Thus, the resistance of a conductor appears as a ratio of the difference of potential to the current which flows.

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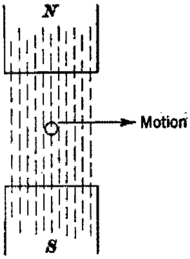
[1] A column of mercury having a mass of 14.4521 grams, a length of 106.3 centimeters, a temperature of 0 C, and a uniform cross-section has a resistance of 1 ohm.

PROBLEMS

1. In the diagram, a cross-section of a conductor is shown moving to the right in a magnetic field. Will the emf generated in the conductor be toward or away from the observer?

2. If the magnetic field of Problem 1 be of a uniform strength of 10,000 lines per sq cm, the immersed perpendicular length of conductor therein be 10 cm, and the velocity of motion of the conductor perpendicular to the field be 100 ft per sec, what is the magnitude of the generated emf? *Ans. 3.05 volts*



Problem 1–1

3. Show the flux in the magnetic circuit of the four-pole generator of Fig. 10–13. Indicate the directions of the lines of flux with arrowheads.

4. An electric heater whose resistance is 60 ohms is connected to a 115-volt circuit. How much current flows in the heater?

5. A magnetic field links a 1-turn coil or one turn of wire and varies in magnitude at the rate of 8×10^9 lines of flux per second. Calculate the generated voltage.

6. If the coil of wire in Problem 5 contained 10 turns in series, what would be the generated voltage?

7. If in Fig. 2–2 (*a*) the total flux is 1.27×10^8 lines and it is uniformly distributed, what would be the maximum value of the sinusoidal voltage generated in a conductor rotating 1 revolution per sec? *Ans. 4 volts*

8. A conductor 10 cm long is caused to move across a field of 80,000 lines per sq in., as in Fig. 2–1 (*a*), at a velocity of 100 ft per sec. Find the emf generated between the ends of the conductor.

9. Calculate the maximum emf generated in a 20-turn coil rotating on its axis 200 revolutions per sec. The maximum flux enclosed in the loop is 400,000 lines.

10. A coil of 100 turns is cut by a magnetic field which is rising at a rate of 1 million lines per minute. Calculate the emf generated. *Ans.* $\frac{1}{60}$ *volt*

11. When at working temperature the heating element of an electric smoothing iron has a resistance of 12 ohms. How much current will it draw from a 115-volt line?

12. A conductor 4 in. long is caused to oscillate linearly with sinusoidal velocity across a magnetic field whose strength is 80,000 lines per sq in. The amplitude of oscillation is 4 in. Calculate the maximum voltage generated in the conductor when its frequency of oscillation is 20 cycles per sec.

CHAPTER 2

Commercial Forms of Electromotive Force; Power and Energy

Instantaneous Values of Electromotive Force.—When a conductor moves with uniform velocity through a magnetic field of uniform intensity, there is generated in that conductor an emf of constant value. See Fig. 2–1 (*b*). This is called direct emf.[1] If, however, the field is not of uniform intensity, or for any other reason the rate of cutting flux lines is not constant, then the emf generated is not of constant value.

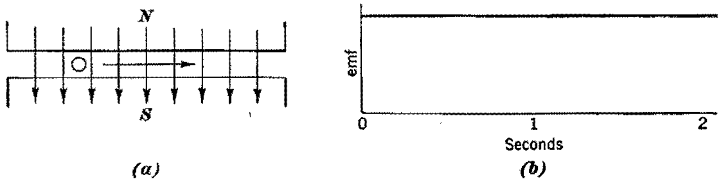


Fig. 2-1. Generation of Direct emf

Assume that in Fig. 2–2 (*a*) the flux in the airgap between the poles N and S is of uniform intensity and that the angular velocity of the conductor shown is 2π radians per second and is constant. Fig. 2–2 (*b*) is a plot of the *rate* at which the conductor cuts the magnetic field at every point in one revolution. The generated emf wave is, hence, of the same shape. In position A, Fig. 2–2 (*a*), the motion of the conductor is parallel to the lines of flux of the field; and, since no flux is being cut, no emf is being generated. This instant of time corresponds to A in Fig. 2–2 (*b*). In position C in (*a*) the conductor is moving at right angles to the lines of flux, and hence the rate of cutting these lines is maximum. This position corresponds to the instant C in Fig. 2–2 (*b*). Between positions A and C the rate of cutting the magnetic field is a sinusoidal function of position and, therefore, of time. After position E is reached, the conductor begins to cut across the field in the opposite direction, and hence its generated emf is reversed in direction and is shown as negative on the sinusoid.

In making one complete revolution in 1 second, the conductor has generated in it all possible values of emf at a fixed speed and with a fixed flux intensity; hence, the emf is said to have completed one *cycle*. The

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[1] While the terms direct emf and direct current are somewhat difficult to justify completely in describing such quantities, custom has firmly established such usage.

number of cycles completed in 1 second is called the *frequency*. Thus, the emf in Fig. 2–2 has a frequency of 1 cycle per second. The instantaneous value of a sinusoidal emf is frequently expressed as

$$e = E_m \sin \omega t \tag{2-1}$$

where e = instantaneous value of the emf;
E_m = maximum value of e;
$\omega = 2\pi f$ = angular velocity, in radians per second;
t = time, in seconds;
f = frequency, in cycles per second.

On the North American continent 25-cycle, 40-cycle, 50-cycle, and 60-cycle frequencies only are in commercial power use.

Current caused to flow in a circuit by sinusoidally varying voltage is called *alternating current.*

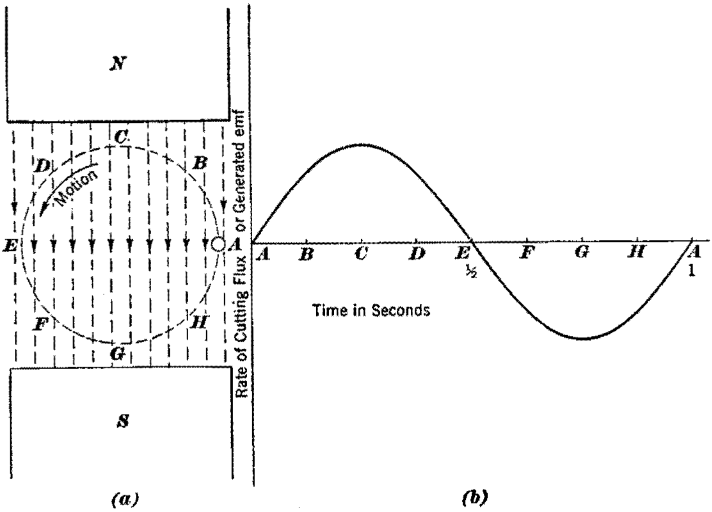

Fig. 2–2. Generation of Sinusoidal emf

If the magnetic field of Fig. 2–2 (*a*) were not of uniform intensity, the emf generated would, of course, be alternating, but would not be sinusoidal. In many cases in practical engineering the generated electromotive forces and currents are not sinusoidal. In almost all modern alternating-current generators the conductors are stationary and the magnetic poles are rotated past them at constant speed. However, the fluxes from the poles cannot effectively be distributed over the pole faces so as to produce a sinusoidal voltage under all conditions of loading. The voltage produced by any particular coil of the generator looks approximately like that shown in Fig. 10–5 (*a*). However, by using many coils and shaping them properly, the terminal voltage wave shape is made so nearly sinusoidal that it may be considered as such for practically all purposes.

Joule's Law.—When a continuous current flows in a resistance, the rate at which electrical energy is converted into heat energy is directly proportional to the resistance and to the square of the current. This law was discovered by Joule and bears his name.

Power.—By definition, rate of change of energy is *power.* Therefore, the power in a resistance is

$$P = RI^2 \tag{2-2}$$

where P = electrical power, in *watts*;
R = resistance, in ohms;
I = current, in amperes.

The abbreviation for watt is w.

Since $R = \frac{V}{I}$,

$$P = \frac{V}{I} \times I^2$$

or

$$P = VI \text{ watts} \tag{2-3}$$

where V = electromotive force, in volts.

Also

$$P = \frac{V^2}{R} \tag{2-4}$$

As explained on page 5 the ampere is defined in terms of the cgs system of units. If a conductor 1 cm in length be caused to move with a velocity of 10^8 cm per sec in and perpendicular to the lines of flux of a magnetic field having a uniform intensity of 1 gauss, an emf of 1 volt will be generated in the conductor (page 2). If at the same time a current of 1 ampere be caused to flow in the conductor, the reacting force against which the conductor must be moved will be $\frac{1}{10}$ dyne (page 128). The electrical power generated in the conductor will be

$$P = VI = 1 \times 1 = 1 \text{ watt}$$

The mechanical power input to the conductor will be

$$P = \frac{FD}{t} \tag{2-5}$$

where P = mechanical power, in dyne-centimeters per second;
F = force on the conductor, in dynes;
D = distance through which the conductor is moved, in centimeters;
t = time, in seconds.

Thus,

$$P=\frac{\frac{1}{10}\times 10^8}{1}=10^7 \text{ dyne-cm per sec}$$

By the law of conservation of power, the mechanical power must be equal to the electrical power. Hence,

$$1 \text{ watt}=10^7 \text{ dyne-cm per sec}$$

The most commonly used unit of mechanical power is the horsepower. Since 1 hp = 550 ft-lb per sec, 1 ft = 30.48 cm, and 1 lb = 445,072.3 dynes, it follows that

$$\begin{aligned}1 \text{ hp} &= 550\times 30.48\times 445{,}072.3 \text{ dyne-cm per sec}\\ &= 746\times 10^7 \text{ dyne-cm per sec}\\ &= 746 \text{ watts}\end{aligned}$$

Also, 1 kilowatt = 1000 watts.

Energy.—The product of power and time is energy. If the power in a resistance is 1 kilowatt and the time of use is 1 hour, the energy used is 1 kilowatt-hour (kw-hr). The heat energy required to raise the temperature of 1 lb of water 1 deg F is called the British Thermal Unit (Btu). The Btu is approximately equal to 778 ft-lb. Some useful conversion equivalents follow:

$$\begin{aligned}1 \text{ watt} &= 10^7 \text{ dyne-cm per sec} = 0.74 \text{ ft-lb per sec}\\ 1 \text{ kilowatt} &= 1000 \text{ watts}\\ 1 \text{ horsepower} &= 746 \text{ watts}\\ &= 550 \text{ ft-lb per sec}\\ &= 33{,}000 \text{ ft-lb per min}\\ 1 \text{ kilowatt-hour} &= 3413 \text{ Btu}\end{aligned}$$

PROBLEMS

1. How many horsepower are required to hoist 4000 lb at the rate of 50 ft per min?

2. How many kilowatts are required to supply the power in Problem 1?

3. What is the power in an electric iron whose resistance is 11 ohms, when the iron is connected to a 115-volt circuit?

4. A bill for electric energy is $20 for 150 hours. The price of energy is 7 cents per kilowatt-hour. What was the average power?

5. How much heat per hour is produced in the iron of Problem 3?

6. How many hours would be required for a heater of 10 ohms resistance connected to a 220-volt line to produce enough heat to melt 100 lb of ice? Latent heat of ice is 144 Btu per pound.

7. A baseball field is lighted by 40 lamps, each of which draws 2.3 amperes from a 115-volt line. If the field is lighted for 2½ hours and the cost of electric energy is 3 cents per kilowatt-hour, what is the energy cost? *Ans. $0.795*

8. If power for the field in Problem 7 is furnished by an individual engine-generator unit and the efficiency of the generator at the required load is 82%, what must be the horsepower output of the engine? *Ans. 17.3 hp*

9. A waterfall 72 ft high has a flow of 5000 tons of water per minute, 10 per cent of which is diverted through a water turbine whose efficiency at full load is 72%. The turbine drives an electric generator whose efficiency is 86% at full load. Calculate: (*a*) the rated horsepower of the turbine; (*b*) the rated power output of the generator; (*c*) the dollar value of the yearly energy output if the equipment is operated at full load continuously and the market price of the energy is 1.2 cents per kilowatt-hour.

10. An immersion heater used to maintain the temperature of a plating solution draws 15 amperes from a 120-volt line. How many Btu per hour are delivered to the solution?

11. An elevator with an unbalanced weight of 2000 lb is being driven at 250 ft per min by a motor whose efficiency is 86%. The hoisting-gear efficiency is 72%. Calculate the power input to the motor. How much energy is used in raising the elevator 80 ft?

12. If the voltage of Problem 10 were reduced 10%, what would be the reduction in power in the heater?

13. A 500-watt, 120-volt soldering iron will overheat if left connected to its supply without use. How much resistance must be connected in series with it to reduce its rate of heat input 20%.

14. In a magnetic field like that shown in Fig. 2–2 a coil of 500 turns is rotating. When the coil is in a horizontal position, it encloses 4×10^4 lines of flux. If the maximum voltage generated is 400 volts, what is the speed of rotation of the coil in revolutions per minute?

CHAPTER 3

The Electric Circuit

Application of Ohm's Law.—Ohm's Law states that the direct current in a circuit is equal to the impressed direct difference of potential divided by the circuit resistance. Thus, by equation (1–2),

$$I = \frac{V}{R}$$

Example 3–1.—In Fig. 3–1 let the 230-volt direct-current generator G be connected to the terminals of the resistor R whose resistance is 50 ohms. Find the current.

Solution.—The current is

$$I = \frac{V}{R} = \frac{230}{50} = 4.6 \text{ amp}$$

Voltage Rise and Voltage Drop.—When a voltage is generated in any device or in any part of a circuit, the point of positive potential is said to be at a higher potential than the point of negative potential. The means by which the voltage is generated is said to produce a *voltage rise*, and the rise is designated by the letter E. In some instances the instantaneous value of a changing voltage rise may be under consideration and it will be designated by e. When the circuit is completed, as in Fig. 3–1, the available voltage E forces a current through the remainder of the circuit and there is said to be a *voltage drop* through that part of the circuit. Drop in voltage is designated by the letter V. The sum of all voltage rises in the circuit must be equal to the sum of all voltage drops. It may seem that, as indicated in Fig. 3–1, there is a distinction without a difference because E and V appear across the same points in the circuit. However, in more complicated circuits the distinction will be found to be valuable.

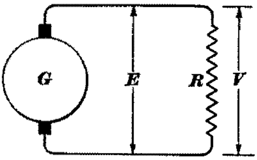

Fig. 3–1. Circuit in Example 3–1

Comparison of Direct and Alternating Current.—As explained on page 8, when an alternating voltage is impressed on a circuit an alternating current flows. An alternating current may be compared to a direct current on the basis of their respective abilities to produce heat in a resistance. The instantaneous rate at which an electric current is capable of producing heat is directly proportional to the square of that current, as indicated by equation (2–2). Hence, throughout the cycle of i (instantaneous value of alternating current), Fig. 3–2, the rate of heat production at any instant is proportional to the value for the corresponding instant on the curve of i^2. The average heating value, or *effective value*, of the current over a whole cycle is therefore proportional to the average value of the i^2 curve. The i^2 wave is seen to be a sinusoid with twice the frequency of the i wave and located above the axis of the i wave.

The average value of a sinusoid is zero about its own axis. Hence, the average value of i^2 about the axis of i is $\frac{I_m^2}{2}$[1] above the axis of i. That is,

$$\text{Average } i^2 = \frac{I_m^2}{2}$$

$$\text{Effective } I = \sqrt{\text{Average } i^2}$$

$$= \sqrt{\frac{I_m^2}{2}}$$

$$= \frac{I_m}{\sqrt{2}}$$

or

$$\text{Effective } I = 0.707\ I_m \tag{3-1}$$

Thus, the effective value of a sinusoidal current (or voltage) is $\frac{1}{\sqrt{2}}$ (or 0.707) times its maximum value.

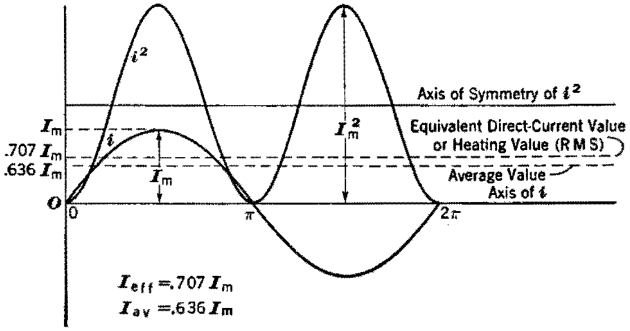

Fig. 3–2. Comparison of Direct and Alternating Current

Alternating Currents in Resistive Circuits.—When an alternating voltage is applied to a circuit, there are, in addition to the resistance of the circuit, other factors which affect the magnitude of the current. These factors are *inductance* and *capacitance*. In circuits in which inductance and capacitance effects are small, such as lamp circuits and heater circuits, Ohm's Law may be used for alternating currents as well as for direct

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[1] The effective value of current is the square root of the average i^2 value and so is sometimes called the root-mean-square (rms) value. An alternating current or voltage is always specified by its effective value unless it is definitely stated otherwise. Maximum values are indicated by the subscript m, as in I_m and E_m. Average values are indicated by the subscript av, as in I_{av} and E_{av}.

currents with acceptable accuracy. The effects of inductance and capacitance in a circuit will be considered in Chapter 5.

Resistors in Series.—If two or more resistors be connected in series, the current will be the same in all. The voltage drop across each resistor is the product of the resistance of that resistor and the current; and the voltage drop across the whole circuit is the sum of the voltage drops across the individual resistors. In the circuit in Fig. 3–3, for example,

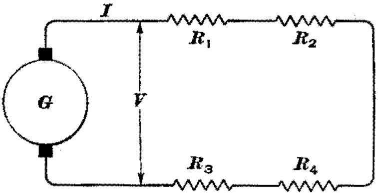


Fig. 3–3. Resistors in Series

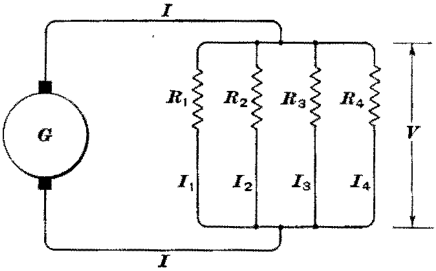


Fig. 3–4. Resistors in Parallel

$$V = R_1I + R_2I + R_3I + R_4I$$

or

$$V = I(R_1 + R_2 + R_3 + R_4) \tag{3–2}$$

In a series circuit the applied voltage is divided among the various resistance parts of the circuit in proportion to their respective resistances.

Example 3–2.—Connected in series to a 230-volt line are four resistors having the following resistances: $R_1 = 50$ ohms, $R_2 = 10$ ohms, $R_3 = 15$ ohms, $R_4 = 40$ ohms. Find the voltage drop across each resistor.

Solution.—The total resistance is

$$\begin{aligned} R &= R_1 + R_2 + R_3 + R_4 \\ &= 50 + 10 + 15 + 40 = 115 \text{ ohms} \end{aligned}$$

By equation (1–2),

$$I = \frac{V}{R} = \frac{230}{115} = 2.0 \text{ amp}$$

Then the voltage drops are:

$$\begin{aligned} R_1I &= 50 \times 2.0 = 100.0 \text{ volts} \\ R_2I &= 10 \times 2.0 = 20.0 \text{ volts} \\ R_3I &= 15 \times 2.0 = 30.0 \text{ volts} \\ R_4I &= 40 \times 2.0 = 80.0 \text{ volts} \end{aligned}$$

$$100.0 + 20.0 + 30.0 + 80.0 = 230 \text{ volts (check)}$$

Resistors in Parallel.—If two or more resistors be connected in parallel, the voltage drop across every one will be the same. The current in each resistor may be calculated by Ohm's Law. The total current in the combination will be the sum of the currents in all the resistors. Thus, in Fig. 3-4,

$$I = I_1 + I_2 + I_3 + I_4$$

$$= \frac{V}{R_1} + \frac{V}{R_2} + \frac{V}{R_3} + \frac{V}{R_4}$$

or

$$I = V\left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \frac{1}{R_4}\right) \tag{3-3}$$

The reciprocals of the resistances, or $\frac{1}{R_1}$, $\frac{1}{R_2}$, $\frac{1}{R_3}$, and $\frac{1}{R_4}$, are called the *conductances* of the branches 1, 2, 3, and 4, respectively.

In solving for the line current, the resistances of all resistors may be combined into an equivalent single resistance. The reciprocal of the equivalent single resistance is equal to the sum of the reciprocals of the resistances in parallel. Let R = the equivalent single resistance. Then,

$$I = \frac{V}{R}$$

$$= V \times \frac{1}{R}$$

Hence,

$$\frac{1}{R} = \left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \frac{1}{R_4}\right) \tag{3-4}$$

The value of $\frac{1}{R}$ is the conductance of the whole parallel group. Thus, the conductance of the group is equal to the sum of the conductances of the individual resistors.

Example 3-3.—Connected in parallel to a 230-volt line, as indicated in Fig. 3-4, are four resistors having the following resistances: $R_1 = 50$ ohms, $R_2 = 10$ ohms, $R_3 = 15$ ohms, $R_4 = 40$ ohms. Find the current I in the line.

First Solution.—In this case,

$$I = I_1 + I_2 + I_3 + I_4$$

Also,

$$I_1 = \frac{V}{R_1} = \frac{230}{50} = 4.6 \text{ amp}$$

$$I_2 = \frac{230}{10} = 23.0 \text{ amp}$$

$$I_3 = \frac{230}{15} = 15.33 \text{ amp}$$

$$I_4 = \frac{230}{40} = 5.75 \text{ amp}$$

Therefore,

$$I = 4.6 + 23.0 + 15.33 + 5.75 = 48.68 \text{ amp}$$

Second Solution.—The conductance of the entire group of resistors is

$$\frac{1}{R} = \frac{1}{50} + \frac{1}{10} + \frac{1}{15} + \frac{1}{40} = 0.2116$$

The line current is

$$I = V \times \frac{1}{R}$$
$$= 230 \times 0.2116 = 48.68 \text{ amp}$$

Three-Wire Circuits.—Almost all lighting is done with 115-volt or 120-volt lamps, whereas almost all motors of more than $\frac{1}{4}$ hp are operated at 230 volts or more. It is sometimes desirable to distribute power for both lamps and motors over the same line. For this purpose, three-wire circuits are used, as shown in Fig. 3–5. The two 115-volt generators G_1 and G_2 are connected in series. This arrangement permits motors to be operated at 230 volts across the two outside wires while lamps may be used at 115 volts between either outside wire and the middle wire at the same time.

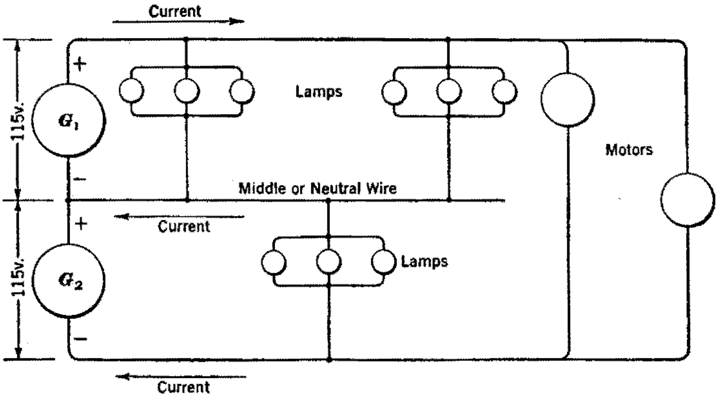


Fig. 3–5. Three-Wire Distribution System

Furthermore, the power to the lamps is supplied with less power loss in the line wires than would be incurred with two-wire, 115-volt distribution and the use of the same amount of line copper. If, as in Fig. 3–5, the currents taken by the loads on the two parts of the line are unequal, the middle wire carries the difference between the currents in the outside wires. An attempt is always made to balance the loads on the two parts of the line as far as practicable.

Example 3–4.—A load of 20 amperes at 115 volts is to be supplied by a two-wire line each wire of which has a resistance of 0.5 ohm. Calculate the power loss in the line and the power in the load.

Solution.—By equation (2–2), the power loss in the line is

$$P = RI^2$$
$$= (0.5 + 0.5) \times 20^2 = 400 \text{ watts}$$

By equation (2–3), the power in the load is

$$P = EI$$
$$= 115 \times 20 = 2{,}300 \text{ watts}$$

If the load in the foregoing example were supplied by a three-wire line, it should be divided approximately equally between the two parts to avoid neutral-wire current as far as possible. The neutral wire should, in general, be of the same size as either outside wire in order that, if the load be removed from one side only, the middle wire will be as capable of carrying the remaining load as is the remaining outside wire because both then carry the same current.

Example 3–5.—If another similar wire be added to the line of Example 3–4, making a three-wire, 115-230-volt circuit, how much power could be delivered at the load point without having the loss in the line exceed 400 watts?

Solution.—For a given outside-wire current, maximum power can be delivered only to a load which is equally balanced between the two sides of the line. Then, no current will flow in the middle wire. With the same loss of 400 watts in two wires as in the line in Example 3–4, the outside-wire current must be 20 amp. The maximum power permissible in the load is

$$230 \times 20 = 4{,}600 \text{ watts}$$

The preceding examples show that, *by adding 50 per cent more copper, the power transmissible by a three-wire circuit with the same power loss as in a two-wire circuit is increased 100 per cent.* This fact demonstrates the economic advantage of three-wire distribution over two-wire distribution for providing 115-volt service as ordinarily used in home-lighting circuits. The greater the load unbalance, the less advantageous is the three-wire system. The load is carried with minimum line loss when the load is balanced between the two sides of the line and there is no neutral-wire current.

Kirchhoff's Laws.—Kirchhoff has propounded two laws relating to electrical circuits, and they now bear his name. In his first law, Kirchhoff stated that there is always as much current flowing *away* from a point in a circuit as there is flowing *to* that point. For example, in Fig. 3–7 at the point M, the sum of the currents I_b and I_m flowing away from M is equal to the current I_a flowing to M. This law is sometimes called *Kirchhoff's law of currents.*

In his second law, Kirchhoff stated that the sum of all voltage drops around any one path of an electric circuit equals the sum of the voltages impressed on that same path. For example, in Fig. 3–7, the equations are:

$$\text{Path 1}\ldots \quad I_aR_a + I_aR_{L1} + I_mR_m = E_{g1}$$
$$\text{Path 2}\ldots \quad -I_mR_m + I_bR_{L2} + I_bR_b = E_{g2}$$
$$\text{Path 3}\ldots I_aR_a + I_aR_{L1} + I_bR_{L2} + I_bR_b = E_{g1} + E_{g2}$$

This law is sometimes called *Kirchhoff's law of voltages.*

Network Solutions.—A circuit containing other than components in simple series or simple parallel is called a *network.* An example of a network is shown in Fig. 3–6. The currents and voltages in them are determined by the simultaneous applications of Kirchhoff's voltage and current laws.

In writing the path or *loop* equations for a *network*, such as that in Fig. 3–6, it is necessary that the directions taken around all paths be the same. After all the voltages have been found for all parts of the circuit, the voltage loop equations can be written for any closed traverse of the circuit. At any junction point in the circuit the currents in the branches connected to that point can be calculated by Kirchhoff's current law by so-called *node equations.*

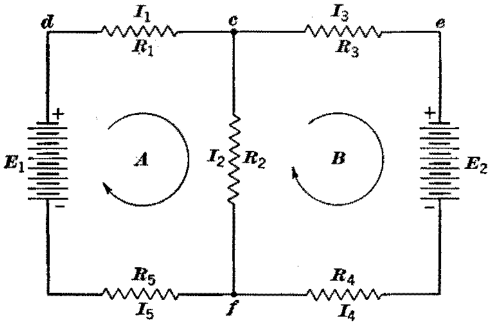


Fig. 3–6. Example of a Network

In Fig. 3–6 there are only three unknown currents designated as I_1, I_2, and I_3, since $I_5 = I_1$ and $I_4 = I_3$. Therefore, there must be three equations. These may consist of two voltage equations and one current equation and are as follows:

Loop A. . . . . $-E_1 + I_1R_1 + I_2R_2 + I_5R_5 = 0$

Loop B. . . . . $+E_2 + I_4R_4 - I_2R_2 + I_3R_3 = 0$

At junction C. . . . $I_{dc} + I_{ec} + I_{fc} = 0$ or $I_{dc} - I_{ce} - I_{cf} = 0$

or $I_1 - I_3 - I_2 = 0$

Also, $I_1 = I_5$ and $I_4 = I_3$

It should be noted that the resistance voltage drop in a circuit is in the same direction as the current in it.

Example 3–6.—In Fig. 3–6, let $E_1 = 100$, $E_2 = 90$, $R_1 = 10$, $R_2 = 8$, $R_3 = 15$, $R_4 = 20$, and $R_5 = 30$. Calculate the currents in and the voltage drops across all resistances.

Solution.—By Kirchhoff's voltage laws,

$$-100 + 10I_1 + 8I_2 + 30I_5 = 0$$
$$+\ 90 + 20I_4 - 8I_2 + 15I_3 = 0$$

After simplification by substituting equivalent currents, these equations become:

$$48I_1 - \ 8I_3 = 100$$
$$-8I_1 + 43I_3 = -90$$

By the solution of these simultaneous equations,

$$I_1 = +1.79 \text{ amp } (I_{dc})$$
$$I_3 = -1.76 \text{ amp } (I_{ce} \text{ or } -I_{ec})$$

Then, $$I_2 = I_{cf} = 1.79 - (-1.76) = +3.55 \text{ amp}$$

The negative sign before 1.76, the value of I_3, means that I_3 is flowing in R_3 in the direction opposite to that assumed when the loop B was traversed in a clockwise direction. Positive signs before the answers for I_1 and I_2 mean that the actual currents are in the same sense around the circuit as was assumed for loop A.

Voltage Drop of Two-Wire and Three-Wire Systems.—In the two-wire line in Example 3–4, there is a voltage drop equal to

$$I_L R_L = 20 \times 1 = 20 \text{ volts}$$

In order that 230 volts may be available at the load, the generator voltage must be raised to $230 + 20 = 250$ volts. The difference between the sending-end (generator) voltage and the receiving-end (load) voltage is called the *line voltage drop.*

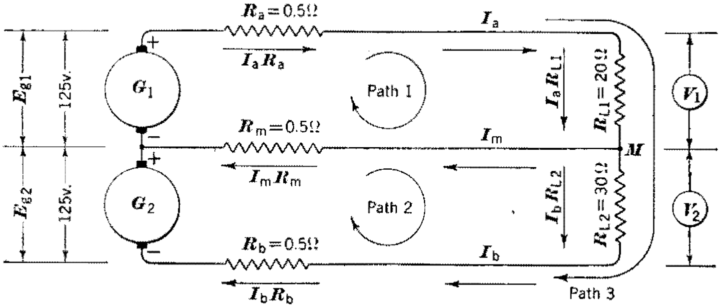


Fig. 3–7. Three-Wire Distribution

If, in a three-wire circuit, the load is not equally divided between the two parts of the circuit, the voltages across the two parts at the load will not be equal. This inequality is caused by resistance in the middle wire.

Example 3–7.—A constant-voltage double generator set supplies power over a three-wire line with a resistance of 0.5 ohm per wire to unbalanced-load resistance units of 20 ohms and 30 ohms, as in Fig. 3–7. Calculate the currents in all three lines and the voltage appearing at each section of load.

Solution.—The load resistor of lesser value will have a greater current. Hence, I_m will be in the direction shown, and

$$I_m = I_a - I_b$$

The sum of all voltage drops in a series circuit is equal to the applied voltage. In this case, the voltages applied to the top and bottom parts of the circuit are 125 volts. Therefore, by Kirchhoff's voltage laws, the equations are:

$$\text{Path 1} \ldots\ldots\ I_a R_a + I_a R_{L1} + I_m R_m = 125$$
$$\text{Path 2} \ldots\ -I_m R_m + I_b R_{L2} + I_b R_b = 125$$

It having been established that I_m is flowing toward the left, the I_mR_m drop is from right to left as indicated. Hence, in traversing the lower circuit in the direction of I_b, the established I_mR_m drop is taken as negative. Since $I_m = I_a - I_b$,

$$I_aR_a + I_aR_{L1} + I_aR_m - I_bR_m = 125$$

and

$$-I_aR_m + I_bR_m + I_bR_{L2} + I_bR_b = 125$$

By simultaneous solution

$$I_b = 4.12 \text{ amp}$$

and

$$I_a = 6.05 \text{ amp}$$

Then,

$$I_m = I_a - I_b = 1.93 \text{ amp}$$

Also,

$$I_aR_{L1} = 6.05 \times 20 = 121.0 \text{ volts across } R_{L1}$$

and

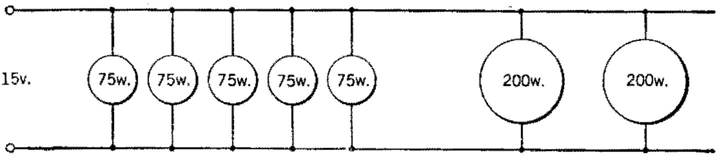
$$I_bR_{L2} = 4.12 \times 30 = 123.6 \text{ volts across } R_{L2}$$

Effect of Fuse or Circuit Breaker in Middle Wire.—A fuse or a circuit breaker should not be used in the middle wire of a three-wire single-phase circuit, because the opening of the middle wire with the outside wires closed would put the two parts of the load in series between the outside wires. If the loads were not well balanced, the difference in voltages resulting across them might be even greater than that found in Example 3–7 and might cause serious injury to load equipment.

PROBLEMS

1. Plot accurately on $\frac{1}{10}$-in. cross-section paper a sinusoidal current whose maximum value is 3 amperes. Plot on the same axes a curve of current squared. Demonstrate that the square root of the average ordinate of the i^2 wave is 0.707 times the maximum ordinate of the i wave.

2. Determine the average value and the maximum value of one half-cycle (starting at zero value) of a sinusoidal alternating current which will produce heat in a resistor at the same average rate as a direct current of 30 amperes.



Problem 3–6

3. A 100-turn coil is rotated in a uniform magnetic field and generates a voltage at a frequency of 25 cycles. A voltmeter connected across the coil reads 30 volts. What is the maximum flux enclosed by the coil?

4. A 200-turn coil in the form of a 6 in. × 6 in. rectangle rotates at 3600 rpm in a uniform field of 50,000 lines per sq in. Calculate the instantaneous emf when the coil is 60 deg past the point of zero voltage.

5. What is the maximum value of a sine wave of voltage which would produce heat in a resistor at the same average rate as 100 volts of direct current?

6. As indicated in the diagram, five lamps rated at 75 watts, 115 volts are connected in parallel with two lamps rated at 200 watts, 115 volts across a two-wire, 115-volt line. Calculate the line current. *Ans. 6.73 amp*

7. A 100-watt, 115-volt lamp is connected in series with a 500-watt, 115-volt smoothing iron across a two-wire, 230-volt circuit. Calculate the voltage applied to each. Calculate the power in each.

$$\textit{Ans.}\begin{cases}\textit{Lamp, 191.8 v.; Iron, 38.4 v.}\\ \textit{Lamp, 278 w.; Iron, 55.6 w.}\end{cases}$$

8. A 25-watt, 115-volt lamp is connected in series with a 200-watt, 115-volt lamp across a 230-volt supply line. How many 25-watt, 115-volt lamps placed in parallel with the first one will cause the 200-watt lamp to light to normal brilliance?

9. A 120-volt voltmeter has a resistance of 60,000 ohms. How much resistance must the series multiplier for this meter have to make it a 600-volt meter? How much current would the meter then draw from a 600-volt circuit?

10. The resistance of a resistor of about 4000 ohms is to be found by using a voltmeter having a resistance of 60,000 ohms. What per cent of error would be introduced by the current taken by the voltmeter? The voltmeter is connected directly across the resistor.

11. If the voltmeter of Problem 10 were connected to include the ammeter and the ammeter resistance is 0.001 ohm, what per cent of error in the measurement would be introduced by the ammeter resistance?

12. If all of the shunt field coils of the generator of Fig. 10–12 are connected in series across a 600-volt bus, what would be the voltage across each coil?

13. A series street-lighting circuit has 75 lamps rated at 6.6 amperes, 225 watts. The line resistance is 50 ohms. How many volts must be supplied by the supply transformer?

14. A compound generator whose full-load current is 50 amperes compounds its voltage too much. The series field, which has a resistance of 0.06 ohm, is to be relieved of 25 per cent of its current by means of a "diverter" or parallel resistor. What must be the resistance of the diverter?

15. Four shunt field coils as shown in Fig. 10–13 have a combined resistance of 100 ohms. If they were connected in parallel, what would be their combined resistance?

16. A lighting load of 100 lamps, each rated at 100 watts at 120 volts, is to be supplied over a two-wire system each wire of which has a resistance of 0.1 ohm. What must be the voltage at the generator to insure rated voltage at the lamps? *Ans. 136.6*

17. Calculate the voltages that would appear across R_{L1} and across R_{L2} of Example 3–7 if the middle wire were removed? *Ans.* $I_aR_{L1} = 98.0$; $I_bR_{L2} = 147$

18. Assume that R_{L1} of Fig. 3–7 is adjusted until I_a is 14 amperes and that at the same time R_{L2} is adjusted until I_b is 10 amperes. What will then be the voltages across R_{L1} and across R_{L2}?

19. Assume that in the circuit of Fig. 3–7 the resistor R_m is disconnected and the middle line is left open. What voltage will then appear between the mid-point between the generators and the point M of the circuit?

20. A three-wire, 120/240-volt feeder of 2/0 conductor to an isolated machine shop is 250 ft long. How many kilowatts can be drawn from the two outside conductors by the shop motors alone without causing a loss of more than 10 volts in the line? How many kilowatts can be drawn from one side of the system only without causing a line voltage drop in excess of 10 volts. See the Appendix for the resistance per 1000 feet of 2/0 conductor.

21. If in Fig. 3–7 the current in R_{L1} is 6.43 amperes and that through R_{L2} is 4.35 amperes and the resistance of each line is 0.4 ohm, what is the voltage across the terminals of each generator?

22. What is the effective voltage generated in the coil of Problem 14, Chapter 2?

CHAPTER 4

Electromotive Forces in Conductors in Series

Use of Vectors.—The action of a sinusoidal voltage applied to a circuit can best be analyzed by means of *vectors*. All physical quantities are either *scalars* or *vectors*. A quantity that can be described by one attribute only, such as weight, power, energy, length, emf, or current, is a *scalar* quantity. A directed quantity which cannot be described by less than two attributes, such as force, velocity, or acceleration, is a *vector* quantity. A vector quantity has *magnitude* and *direction.*

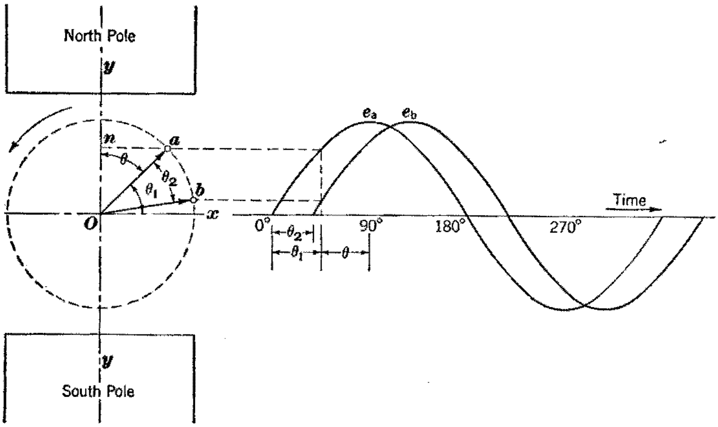


Fig. 4–1. Generation of Sinusoidal emf

Vectors were first used to define space relationships; when so used they may be called *space vectors.* The same devices may also be used to define time relationships of two or more events; when so used they are called *time vectors.* It has been recently proposed to call a time vector by the term *phasor* in order to retain the term *vector* for its original meaning applying to space only. The term vector is so firmly entrenched in the language of engineering, however, that no confusion seems to arise from its use in a time sense. Therefore, the term vector will be used in this book and will be construed to mean a *time vector* unless otherwise indicated.

In Fig. 2–2 it was shown that the emf generated in a conductor moving with constant velocity in a circular path in a uniform magnetic field may be represented by a sinusoid. In Fig. 4–1 the conductor *a* is shown located at the end of a radius space vector *Oa* revolving counter-clockwise with constant velocity about the origin O of a rectangular-coordinate system with axes Ox and Oy. With the x-axis as reference, the generated emf in a is a sine function of the angle θ_1 by which the space vector Oa is displaced from that axis. Thus,

$$On = Oa \sin \theta_1$$

The sinusoidally varying emf E_a in the conductor a is similar to the sinusoidally varying projection On of the space vector Oa on the y-axis. Thus, the sinusoid marked e_a is a plot of $Oa \sin \theta_1$. Hence, *although emf is a scalar quantity, it may be treated as a vector quantity.* Similarly, current may be treated as a vector and hereafter will be so used.

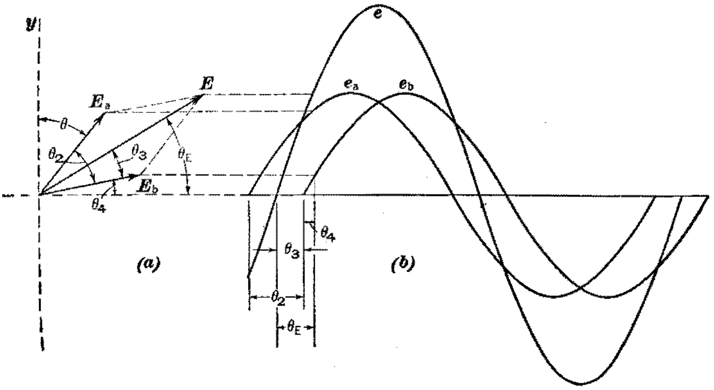


Fig. 4–2. Addition of Voltages in Conductors in Series

Phase Displacement.—If in Fig. 4–1 a second conductor b, which is separated from conductor a by the angle θ_2, be caused to move in the same path as a and with the same velocity and in the same direction as a, there will be induced in conductor b a sinusoidally varying emf E_b. The sinusoid marked e_b is a plot of the instantaneous values of E_b. The maximum value of E_b will occur later than that of E_a by a time interval corresponding to the angle θ_2 and the velocity of rotation. The angle θ_2 is the space angle between conductors a and b and is also called the *time phase displacement angle* between E_a and E_b. The emf E_b is said to lag E_a by θ_2 degrees, or E_a is said to lead E_b by θ_2 degrees.

Addition of Voltages With Phase Displacement.—In Fig. 4–2 let E_a and E_b be two electromotive forces generated in conductors a and b with a time phase displacement angle θ_2 similar to that in Fig. 4–1.

If in Fig. 4–1 the farther end of conductor a be connected to the nearer end of conductor b by a conductor which lies outside the magnetic field, the electromotive forces E_a and E_b will act in series to produce a difference of potential at their free ends which at every instant will be the sum of the instantaneous values of E_a and E_b. The resultant sum will also vary sinusoidally, as shown by the curve marked e in Fig. 4–2 (*b*). In Fig. 4–2 (*a*) the vector E is the vector sum of E_a and E_b and is seen to be the diagonal of a parallelogram which has E_a and E_b as adjacent sides. The instantaneous value e of E is always equal to the algebraic sum of e_a and e_b at that instant. The sinusoid e of Fig. 4–2 (*b*) is thus a plot of $E \sin \theta_E$.

In the application of vectors in computations, it is customary to use effective values which are maximum values times 0.707, as shown by equation (3–1). Vectors are assumed to rotate counter-clockwise.

Example 4–1.—A conductor with an effective generated emf E_a of 0.10 volt is connected in series with another conductor with an effective generated emf E_b of 0.20 volt. The angle θ_1 by which E_a leads E_b in time phase is 45°. What is the resultant emf E that would be shown by a voltmeter?

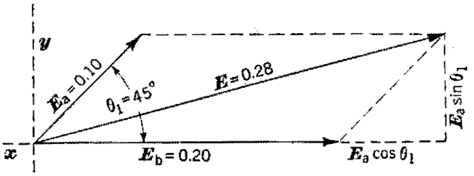


Fig. 4–3. Vector Sum of Two Voltages

Solution.—The resultant emf E is the vector sum of E_a and E_b, and it can be found by constructing a parallelogram of vectors, as shown in Fig. 4–3. This problem is one in simple trigonometry, as it is merely required to find the length of a diagonal of a parallelogram in which two sides and the included angle are known. The effective values may be used in the vector diagram. The computations follow:

$$\begin{aligned} E &= \sqrt{(E_b + E_a \cos \theta_1)^2 + (E_a \sin \theta_1)^2} \\ &= \sqrt{(0.20 + 0.10 \cos 45°)^2 + (0.10 \sin 45°)^2} \\ &= 0.28 \text{ volt} \end{aligned}$$

$$\theta_2 = \tan^{-1} \frac{E_a \sin \theta_1}{E_b + E_a \cos \theta_1} = \tan^{-1} \frac{0.10 \sin 45°}{0.20 + 0.10 \cos 45°} = 14°38'$$

In solving Example 4–1, one of the vectors E_b is laid out along the horizontal axis of a rectangular-coordinate system. Also E_a is resolved into its horizontal component $E_a \cos \theta_1$ and its vertical component $E_a \sin \theta_1$. Then, in the equation for E, the horizontal components E_b and $E_a \cos \theta_1$ are added to make the base of the triangle of which E is the hypotenuse. The value of E is then found to be the square root of the sum of the squares of the base and the perpendicular side of a right triangle.

If, instead of only two electromotive forces E_a and E_b, there had been a sheaf of emf vectors resulting from a large number of conductors moving in the circular path, the sum of all the electromotive forces in series could most easily be found as follows: Resolve each emf into its horizontal and vertical components; add algebraically all horizontal components and all vertical components; and combine the algebraic sums by taking the square root of the sum of their squares. If it is desired to find both the magnitude of the resultant emf and its phase displacement from a particular one of the given electromotive forces, it is convenient to locate the sheaf of vectors on a coordinate system the x-axis of which will be along that particular emf vector.

Example 4–2.—Five conductors connected in series have the following electromotive forces: $E_a = 20$ volts, $E_b = 40$ volts lagging, E_a by 45°, $E_c = 50$ volts lagging E_a by 80°, $E_d = 45$ volts leading E_a by 55°, and $E_e = 50$ volts leading E_a by 75°. Find the magnitude of the resultant voltage and its phase displacement from E_a.

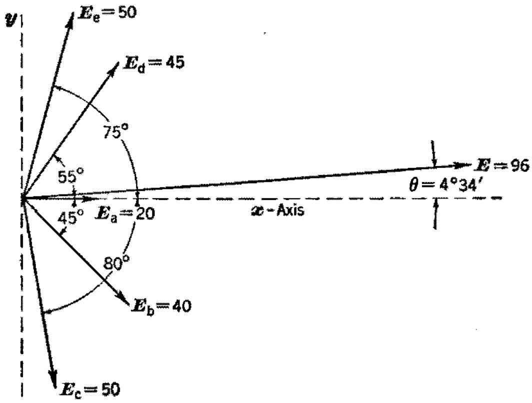


Fig. 4–4. Vector Addition of Several Voltages

Solution.—Fig. 4–4 shows graphically the arrangement of the vectors given. Their components are as follows:

Voltage	Horizontal Component	Vertical Component
$E_a = 20$ volts	$20 \cos 0° = +20.00$	$20 \sin 0° = 0.0$
$E_b = 40$ volts lagging 45°	$40 \cos 45° = +28.28$	$40 \sin 45° = -28.28$
$E_c = 50$ volts lagging 80°	$50 \cos 80° = +8.68$	$50 \sin 80° = -49.24$
$E_d = 45$ volts leading 55°	$45 \cos 55° = +25.81$	$45 \sin 55° = +36.86$
$E_e = 50$ volts leading 75°	$50 \cos 75° = +12.94$	$50 \sin 75° = +48.29$
	$+95.71$	$+7.63$

Hence,

$$E = \sqrt{95.71^2 + 7.63^2} = 96.0 \text{ volts}$$

and

$$\theta = \tan^{-1} \frac{7.63}{95.71} = 4°34'$$

The resultant emf is 96 volts and it leads E_a by 4°34′.

A More Convenient Notation.—The terms horizontal component and vertical component become burdensome. Also in some cases the axis of reference is neither vertical nor horizontal as seen by the observer. In mathematics, vectors are frequently expressed in what is called *complex notation* and the vector is called a *complex quantity* or *complex number.*

The quantity is not really complex in the usual sense, but it is called complex because it is composed of two parts corresponding to the *horizontal component* and the *vertical component* or, as they are sometimes called, the *in-phase component* and the *quadrature component*. The general mathematical form used is $a+jb$, in which a is the in-phase component and b is the quadrature component. These in-phase and quadrature components by definition correspond to the horizontal and vertical components, respectively, as previously used. The prefix j indicates that b is a component to be added to a at an angle of 90° counter-clockwise. The significance of j is illustrated in Figs. 4–5 and 4–6.

Fig. 4–5. Vector Addition $a+b$

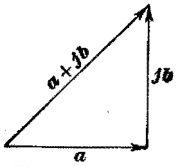


Fig. 4–6. Vector Addition $a+jb$

The prefix j is usually defined as an *operator* which rotates counterclockwise through 90° any vector by which it is multiplied. The operator j is the same as the mathematician's imaginary number i which is $\sqrt{-1}$. A function expressed in complex form is usually designated by a dot above it; thus, $\dot{E}=E_a+jE_b$. However, a generally accepted negligence has resulted in omission of the dot where such omission will not result in confusion.

Example 4–3.—Solve Example 4–2 by using complex notation.

Solution.—The given vectors are expressed as follows:

	a	jb
$\dot{E}_a = 20$ volts	$+20.00$	$+j0$
$\dot{E}_b = 40$ volts lagging 45°	$+28.28$	$-j28.28$
$\dot{E}_c = 40$ volts lagging 80°	$+\ 8.68$	$-j49.24$
$\dot{E}_d = 45$ volts leading 55°	$+25.81$	$+j36.86$
$\dot{E}_e = 50$ volts leading 75°	$+12.94$	$+j48.29$
	$+95.71$	$+j\ 7.63$

Therefore,

$$\dot{E} = +95.71 + j7.63$$

$$E = \sqrt{95.71^2 + 7.63^2}$$
$$= 96 \text{ volts (voltmeter reading)}$$

$$\theta = \tan^{-1}\frac{7.63}{95.71} = 4°34'$$

The Polar Form of Vector Representation.—The rectangular vector complex form $a+jb$ is convenient to use in addition and subtraction of vectors, but is somewhat less convenient for multiplication, division, and the finding of powers and roots. The *polar form* $A\underline{/\theta}$ describes a vector of magnitude A at an angle θ from the axis of reference. A vector which would be $\dot{E} = 8.66 + j5$ would be expressed in polar form as $E = 10\underline{/30^\circ}$. Also, the complete specification of the voltage E in Example 4–3 would be $E = 96\underline{/4^\circ 34'}$ volts. The vector length is called the *absolute value,* and the angle is called the *argument.*

Vectors may be multiplied in polar form by multiplying their absolute values and adding their arguments. Thus,

$$A_1\underline{/\theta_1} \times A_2\underline{/\theta_2} = A_1A_2\underline{/\theta_1 + \theta_2} \tag{4–1}$$

Vectors may be divided in polar form by dividing their absolute values and subtracting their arguments. Thus,

$$A_1\underline{/\theta_1} \div A_2\underline{/\theta_2} = \frac{A_1}{A_2}\underline{/\theta_1 - \theta_2} \tag{4–2}$$

Vectors may be raised to powers by raising the absolute value to the required power and multiplying the argument by the order of the power. Thus,

$$(A_1\underline{/\theta})^n = A_1^n\underline{/n\theta} \tag{4–3}$$

Also, roots may be obtained by taking the root of the absolute value and dividing the argument by the order of the root. Thus,

$$\sqrt[n]{A_1\underline{/\theta}} = \sqrt[n]{A_1}\,\underline{\Big/\frac{\theta}{n}} \tag{4–4}$$

If the sign of θ in the result of a calculation is positive, the absolute value lies θ degrees counter-clockwise from the positive axis of reference. If the sign of θ is negative, the absolute value lies θ degrees clockwise.

Exponential Form of Vector Representation.—From calculus,

$$\cos\theta = 1 - \frac{\theta^2}{\underline{|2}} + \frac{\theta^4}{\underline{|4}} - \frac{\theta^6}{\underline{|6}} + \cdots$$

and

$$\sin\theta = \theta - \frac{\theta^3}{\underline{|3}} + \frac{\theta^5}{\underline{|5}} - \frac{\theta^7}{\underline{|7}} + \cdots$$

where θ is expressed in radians.

Therefore,

$$\cos\theta + j\sin\theta = 1 + j\theta - \frac{\theta^2}{\underline{|2}} - j\frac{\theta^3}{\underline{|3}} + \frac{\theta^4}{\underline{|4}} + j\frac{\theta^5}{\underline{|5}} + \cdots$$

By McLaurin's Theorem, ϵ^θ may be expressed as an infinite series as follows:

$$\epsilon^\theta = 1 + \theta + \frac{\theta^2}{\underline{|2}} + \frac{\theta^3}{\underline{|3}} + \frac{\theta^4}{\underline{|4}} + \cdots$$

Therefore,
$$\epsilon^{j\theta} = 1 + j\theta - \frac{\theta^2}{\underline{|2}} - j\frac{\theta^3}{\underline{|3}} + \frac{\theta^4}{\underline{|4}} + j\frac{\theta^5}{\underline{|5}} + \cdots$$

which is the expression just derived for $\cos\theta + j\sin\theta$ and

$$\epsilon^{j\theta} = \cos\theta + j\sin\theta$$

A vector $\dot{A}$ can then be expressed as follows:

$$\dot{A} = A\epsilon^{j\theta} \tag{4-5}$$

PROBLEMS

1. A conductor in which the generated voltage is 0.80 volt (effective) is connected in series with a second conductor in which the generated voltage of the same frequency is 2.0 volts (effective), the second voltage being 30° out of phase with the first and lagging it in time. Calculate the resulting voltage appearing across the free ends of the two conductors. *Ans. 2.7 volts*

2. Express as a complex quantity the sum of the two voltages of Problem 1, assuming the 0.80-volt vector to lie along the axis of reference.

3. Construct the sinusoids to represent the voltages of Problem 1.

4. Assume that on the rotating member of Fig. 2–2 there are eight conductors in the positions A, B, C, D, E, F, G, and H, respectively, at a given instant. Let the ends of the conductors away from the observer be designated by primes. Assume that every conductor generates a voltage of 10 volts rms and that its ends are connected through sliding brush contacts to a stationary terminal board. If A′ were connected to C, what voltage would appear between A and C′? *Ans. 14.14 volts*

5. If, in Problem 4, A′ were connected to C′, what voltage would appear between A and C? *Ans. 14.14 volts*

6. If, in Problem 4, A′ were connected to D, what voltage would appear between A and D′? *Ans. 7.65 volts*

7. If, in Problem 4, A′ were connected to D′ and D were connected to G, what voltage would appear between A and G′? *Ans. 24.14 volts*

8. How could all the conductors in Problem 4 be connected in series to produce the highest possible voltage at two external terminals? How much would this voltage be?
Ans. Connect A′ to E′, E to B, B′ to F′, F to H, H′ to D′, D to C, C′ to G′. Read 52.66 volts between A and G. Several other equivalent combinations are possible.

9. Assuming the voltage of conductor AA′ in Problem 5 to lie along the axis of reference in a positive sense, show the addition of voltages in complex form. Use rms values. *Ans.* $(10+j0) + (0-j10) = 10 - j10 = 14.14$ *v.*
or $(10+j0) - (0+j10) = 10 - j10 = 14.14$ *v.*

10. If, in Problem 4, A′ were connected to D′ and the conductor DD′ were 120° ahead of conductor AA′, what voltage would appear between A and D? Show the complex addition of voltages. *Ans.* $(10+j0) - (-5+j8.66) = 15 - j8.66 = 17.32$ *v.*

11. Two generators on the same shaft generate voltages $E_1 = 80$ and $E_2 = 60$. There is a phase angle of 45 deg between the two voltages. Calculate the two voltages that could be obtained by connecting the two generators in series.

CHAPTER 5

Circuits With Resistance, Inductance, and Capacitance

Inductance.—As described on page 4 and shown in Fig. 1–5, a current in a conductor produces a magnetic field around that conductor. When the current is increased or decreased, the strength of the magnetic field also increases or decreases. Any change in the field strength causes an emf to be generated or *induced* in the conductor. This phenomenon is called *self-induction*. The emf of self-induction is always in such a direction along the conductor as to oppose the change in current which causes the change in field strength that produces the emf. Thus, if the current decreases, the induced emf tends to prevent the decrease.

The emf of self-induction is proportional to the rate of change of field strength. The field strength produced by a current in a conductor with no iron in its magnetic field is proportional to the current. If the conductor is formed into a coil of N turns which are so located that all the flux produced by the current in every turn cuts every turn, then the emf generated in each turn would be N times as great as if any turn were cut by only the flux produced by the current in that turn. The flux produced by the current in every turn generates an emf in every other turn also. In a coil, the entire inductance phenomenon within the coil produced by the current of the coil is called the self-inductance, or merely the *inductance*, of the coil. Such inductance is denoted by L.

As stated on page 2, if a magnetic flux through a coil changes magnitude at a rate of 10^8 flux linkages per second, 1 volt is generated in the coil. If the flux through the coil is produced by current in the coil and that current is changing at a rate of 1 ampere per second when an electromotive force of 1 volt is generated, the coil is said to have an inductance of 1 *henry*. This relation may be stated in equation form as follows:

$$L = \frac{N\,d\phi}{dI \times 10^8} \quad \text{or} \quad \frac{N\phi}{10^8 I} \tag{5–1}$$

where L is the inductance, in henrys; ϕ is the flux, in maxwells; I is the current, in amperes; and N is the number of turns in the coil. The constant 10^8 converts maxwells of the cgs system to the practical system of volts and amperes.

Example 5–1.—A coil is wound with 1000 turns. When a current of 4 amperes is passed through the coil, a flux of 3,000,000 maxwells is produced. (*a*) How many flux linkages are there in the coil? (*b*) What is the inductance L_1 in henrys? (*c*) If the number of turns is reduced to 500 and the current is kept the same as before, what is the new inductance L_2?

Solution.—(*a*) The number of flux linkages is

$$1{,}000 \times 3{,}000{,}000 = 30 \times 10^8 \text{ (cgs units)}$$

(*b*) The inductance is

$$L_1 = \frac{30 \times 10^8}{4 \times 10^8} = 7.5 \text{ henrys}$$

(*c*) The new flux is

$$3{,}000{,}000 \times \frac{500}{1{,}000} = 1{,}500{,}000 \text{ maxwells}$$

The corresponding number of flux linkages is

$$1{,}500{,}000 \times 500 = 7.5 \times 10^8$$

and the inductance is

$$L_2 = \frac{7.5 \times 10^8}{4 \times 10^8} = 1.875 \text{ henrys}$$

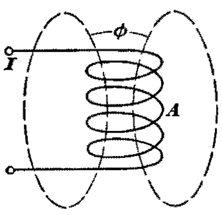


Fig. 5-1. Coil With Self Inductance

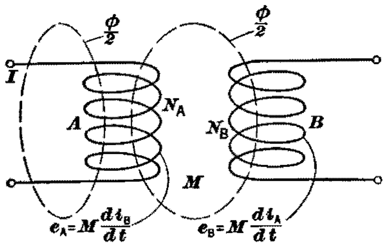


Fig. 5-2. Two Coils With Mutual Inductance

It should be noted that in the example the inductances were proportional to the squares of the numbers of turns in the coil. With constant current the flux is proportional to the number of turns; and, when the number of turns is reduced to half of the original number, the flux is also reduced to half its original value. Therefore, the flux linkages per ampere are proportional to the squares of the numbers of turns in the coil. This is an idealized case in which it is assumed that the coil does not change size or shape when the number of turns is changed and that the ability of the flux path material to carry flux does not change as the flux density changes. Refinements of these calculations which apply to more practical cases of coils having various ratios of core diameter, length, and coil diameter may be found in a Bulletin of the U.S. Bureau of Standards designated as Reprint 169.

Mutual Inductance.—In Fig. 5-1 is shown a coil A with self-inductance L_A carrying an alternating current I which produces a flux ϕ. All of the flux is assumed to link all the turns N_1. If a second coil B with $N_2(=N_1)$ turns is wound very closely over coil A so that all of the flux ϕ links also all turns of coil B, then the voltages e_A and e_B generated in the two coils by the flux ϕ will be equal. The coils are mutually linked by the same flux and are said to be mutually coupled. Inasmuch as the flux linkages per ampere are the same in both coils, their inductances are the same. However, the inductance of coil B is not a self-inductance because the flux ϕ which generates the voltage e_B in coil B is not produced by current in coil B. That type of inductance is called *mutual inductance,* to distinguish it from self-inductance, and its symbol is M.

If coil B instead of coil A carried the current, all flux linkages would be the same as when coil A carried the current. Therefore, M is the symbol for the coefficient of flux interaction between mutually coupled coils and the following relations may be written:

$$e_B = M \frac{di_A}{dt} \tag{5–2a}$$

and

$$e_A = M \frac{di_B}{dt} \tag{5–2b}$$

where e_A and i_A are associated with coil A; e_B and i_B are associated with coil B; and M is the coefficient of mutual inductance.

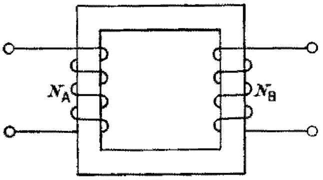


Fig. 5–3. Two Coils With Iron in the Path of Mutual Flux to Improve the Coefficient of Coupling

If, as in Fig. 5–2, coil B with $N_2(=N_1)$ turns is not intimately coupled with coil A but is located so that only half of the flux ϕ links it, then the mutual inductance is only half as much as in Fig. 5–1 and only half as much voltage is generated in coil B because of i_A. The effectiveness of coupling is then only half as great because half of the flux ϕ has *leaked* through paths that do not pass through coil B. When the coils are intimately associated, the leakage does not exist, the coupling is perfect, and the coefficient of coupling K_L is unity. Hence,

$$M = K_L\sqrt{L_A L_B} \tag{5–3}$$

where L_A and L_B are the coefficients of self-inductance of coils A and B, respectively.

The coefficient K_L expressed as a fraction simply shows how much of the flux ϕ produced by coil A is intercepted by coil B. The coefficient of coupling can be varied by moving coil B nearer to or farther from coil A, by rotating the axis of either coil with respect to the other, or by introducing in their mutual path some magnetic material, such as iron, which will increase the flux in that path. Fig. 5–3 shows two coils with a linking iron path which improves the coupling coefficient of the two coils. Mutual-inductance devices are very useful ones, and those called static transformers are discussed in Chapter 17.

Example 5-2.—A coil A has a self-inductance $L_1 = 220 \times 10^{-4}$ henry and has $N_A = 1000$ turns. Coil B has a self-inductance of 95×10^{-4} henry and has $N_B = 400$ turns. When coil A is energized by an alternating electromotive force of 220 volts and coil B is brought to within 1 in. of it, coil B has 25 volts generated in it. Calculate the mutual inductance M.

Solution.—If $K_L = 1$, the voltage generated per turn in coil B would be equal to that in coil A. For this condition,

$$\frac{E_A}{E_B} = \frac{N_A}{N_B}$$

and

$$E_B = \frac{E_A \times N_B}{N_A}$$

$$= \frac{220 \times 400}{1000} = 88 \text{ volts}$$

Since coil B has only 25 volts generated in it,

$$K_L = \frac{25}{88} = 0.284$$

Then,

$$M = 0.284\sqrt{220 \times 10^{-4} \times 95 \times 10^{-4}}$$

$$= 41 \times 10^{-4} \text{ henry}$$

Inductive Reactance.—According to equation 1-1, the instantaneous emf[1] generated in a coil by a varying current in that coil is

$$-e = N\frac{d\phi}{dt} \times 10^{-8}$$

Also, if the flux linkages per ampere are constant,

$$N\frac{d\phi}{dt} \times 10^{-8} = L\frac{di}{dt}$$

Therefore,

$$-e = L\frac{di}{dt}$$

If the current is $i = I_m \sin \omega t$,

$$-e_g = L\frac{d(I_m \sin \omega t)}{dt}$$

$$= L\omega I_m \cos \omega t$$

When $\cos \omega t = 1$,

$$E_m = L\omega I_m$$

Also,

$$\frac{E_m}{E_{\text{eff}}} = \frac{I_m}{I_{\text{eff}}}$$

Hence,

$$E_{\text{eff}} = L\omega I_{\text{eff}}$$

$$= 2\pi f L I_{\text{eff}}$$

or, as it is conventionally written,

$$E = 2\pi f L I = (2\pi f L)I \tag{5-4}$$

---

[1] In the equation $-e_L = N\frac{d\phi}{dt} \times 10^{-8}$, the minus sign is used because the voltage generated in the coil by changes in flux linkages is opposite in sense to the applied voltage causing the change in current which produces the change in flux.

The expression ($2\pi fL$) is called the *inductive reactance* of the circuit. The symbol for inductive reactance is X_L, and it is expressed in ohms, as is resistance. Inductance is a property of a device or circuit. Inductive reactance depends on the inductance and also on the frequency.

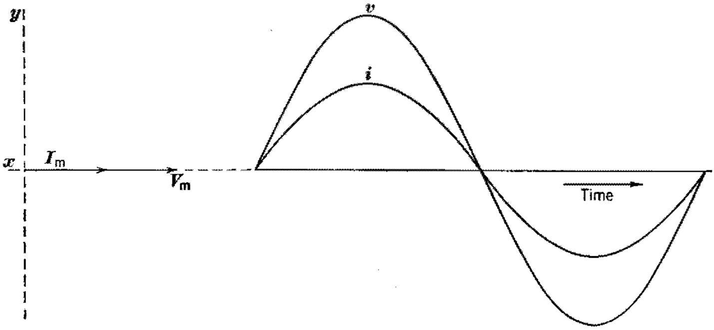


Fig. 5–4. Voltage and Current Relations in a Circuit With Resistance Only

Circuits With Resistance and Inductive Reactance.—In a circuit with resistance only, the current rises and falls in unison with the impressed voltage, according to Ohm's Law, as indicated in Fig. 5–4. There is no time-phase displacement between them.

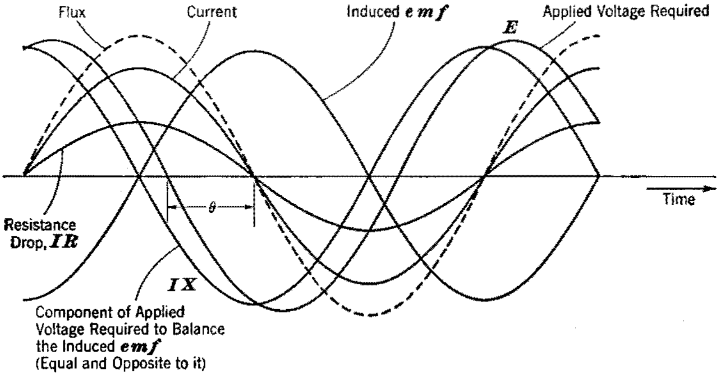


Fig. 5–5. Current, Flux and Voltages in a Coil in an A-C Circuit

If a circuit like that of Fig. 5–6 has induced in it an appreciable emf because of inductance, the impressed voltage V must be large enough to provide for the resistance voltage drop and also for the reactive voltage drop. The resistive voltage drop and the reactive voltage drop cannot be added algebraically because there is a time-phase displacement between them. Hence, they must be added vectorially.

The resistive voltage drop IR through the circuit is that component of the applied voltage required to force the current through the resistance of the circuit. The reactive voltage drop IX through the circuit is that component of the applied voltage required to balance the opposing counter-voltage generated by the change in flux linkages.

The magnetic field in a coil is in time phase with the coil current which produces it. The induced emf caused by the sinusoidal change in magnitude of flux is greatest when the flux is zero because the magnitude of a sinusoidal function is changing most rapidly when passing through zero value. Hence, the induced emf is greatest when the magnetic field and its parent current are passing through their zero values and is in such a sense as to oppose the change of current. But the resistance drop is greatest when the current is at its maximum. The resistive drop and the induced emf have a time-phase displacement of 90°, as indicated in Fig. 5–5.

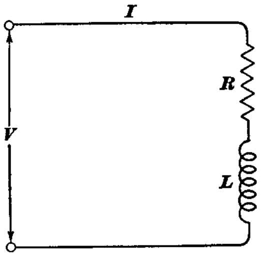
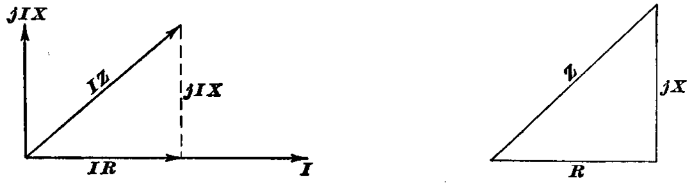


Fig. 5–6. Resistance and Inductance in Series

The applied voltage is the vector sum of the IR drop and the IX drop, and is in phase with no other variable. The current lags behind the applied voltage by the angle θ because of the inductance in the circuit.

For convenience, a circuit with resistance and inductive reactance, such as a coil of wire, may be shown schematically with the resistance and reactance portions separated as in Fig. 5–6. If it were possible actually to separate R and L in a circuit, it would be possible to measure IR across R and IX across L with a voltmeter.



(a) The Voltage Triangle

(b) The Impedance Triangle

Fig. 5–7. Relation of Resistance, Inductive Reactance, and Impedance

In Fig. 5–5, the voltage drop IX leads the voltage drop IR by 90°. The vector sum $IR+jIX$ is called the impedance drop of the circuit and is symbolized by IZ. The relation of IR, IX, and IZ is shown in Fig. 5–7 (*a*). The impedance drop IZ of a circuit is always equal to the applied voltage E. Thus,

$$V = IZ \tag{5–5}$$

Inasmuch as in a series circuit the *current* is common to all parts, it is convenient to use the current vector as a reference and to draw it horizontally. If all sides of Fig. 5-7 (*a*) be divided by I, the result is the impedance triangle of Fig. 5-7 (*b*). The impedance Z of the circuit is, therefore,

$$Z = R + jX \tag{5-6}$$

or

$$Z = \sqrt{R^2 + X^2} \tag{5-7}$$

Example 5-3.—A pure resistance $R = 50$ ohms is connected in series with a pure inductance $L = 0.106$ henry to a 220-volt line whose frequency is 60 cycles per second. Find the current and the angle θ by which it lags the applied voltage.

Solution.—In order to find the impedance, it is necessary first to compute the reactance, which is

$$X = 2\pi f L$$
$$= 2\pi \times 60 \times 0.106 = 40 \text{ ohms}$$

As shown in Fig. 5-8, the impedance is

$$Z = 50 + j40$$
$$= \sqrt{50^2 + 40^2} = 64.0 \text{ ohms}$$

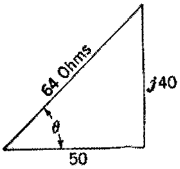


Fig. 5-8. Impedance Triangle for Example 5-3

Since $V = IZ$, the current is

$$I = \frac{V}{Z}$$
$$= \frac{220}{64.0} = 3.43 \text{ amp}$$

Also, the angle by which the current lags the voltage is

$$\theta = \tan^{-1} \frac{40}{50} = 38°40'$$

Therefore,

$$I = 3.43 \diagdown \overline{38°40'}$$

The angle θ, as calculated in Example 5-3, is actually the angle between the resistance line and the impedance line. But the impedance drop is the applied voltage and the current is always in phase with the resistance drop. Therefore, θ is the angle by which the current lags the applied voltage. The angle θ is called the *characteristic angle* of the circuit.

Inasmuch as a vector diagram is always associated with some circuit, a schematic diagram of the circuit should always be used with appropriate references clearly marked unless the circuit is so simple that lack of a circuit diagram will not lead to confusion.

Series Circuit of More Than One Part With Resistance and Inductance. When two or more devices with resistance and inductance are connected in series across an alternating voltage, the resistance of the whole is the arithmetical sum of the resistances of all the parts and the inductance of the whole is the arithmetical sum of the inductances of all the parts.

Example 5-4.—A coil having resistance $R_1 = 20$ ohms and inductance $L_1 = 0.10$ henry is connected in series with a second coil having resistance $R_2 = 50$ ohms and inductance $L_2 = 0.05$ henry. It is required to force 10 amperes with 60-cycle frequency through this circuit. What must be the applied voltage?

Solution.—The schematic circuit diagram is shown in Fig. 5–9. The total resistance is

$$R = R_1 + R_2$$
$$= 20 + 50 = 70 \text{ ohms}$$

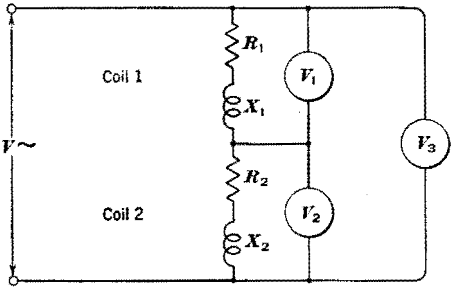


Fig. 5–9. Impedance Coils in Series

Also, the total inductance is

$$L = L_1 + L_2$$
$$= 0.10 + 0.05 = 0.15 \text{ henry}$$

The remainder of the solution follows:

$$X_1 = 2\pi f L_1 = 37.73 \text{ ohms}$$
$$X_2 = 2\pi f L_2 = 18.83 \text{ ohms}$$
$$X = 2\pi f L = 56.56 \text{ ohms}$$
$$Z = R + jX$$
$$= 70 + j56.56$$
$$= 90 \text{ ohms}$$
$$V = IZ$$
$$= 10 \times 90 = 900 \text{ volts}$$

Voltmeter V_1, Fig. 5–9, measures the voltage $I(R_1+jX_1)$ across the impedance R_1+jX_1. Voltmeter V_2 measures the voltage $I(R_2+jX_2)$ across the impedance R_2+jX_2. The readings of these voltmeters may be computed as follows:

$$V_1 = IZ_1$$
$$= I(R_1+jX_1)$$
$$= 10(20+j37.73)$$
$$= 427 \text{ volts}$$
$$V_2 = IZ_2$$
$$= I(R_2+jX_2)$$
$$= 10(50+j18.83)$$
$$= 534 \text{ volts}$$

The arithmetical sum of V_1 and V_2 is greater than 900, the required voltage, but this sum has no significance because V_1 and V_2 are not in phase. Their vector sum is 900. The impedance diagram is shown in Fig. 5–10.

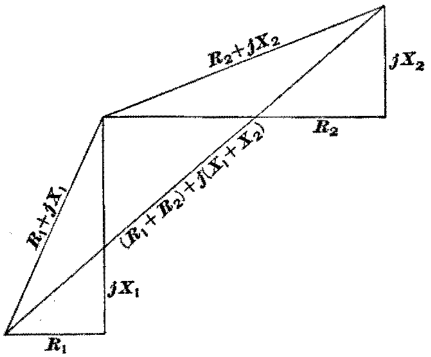


Fig. 5–10. Impedance Diagram for Example 5–4

Parallel Circuits With Resistance and Inductance.—When circuit parts with resistance and inductance are connected in parallel to a common source of alternating voltage, the current in each parallel part will have the same magnitude and phase relation to the applied voltage as if each circuit existed alone. The total current drawn from the source will be the vector sum of the currents in the parts that are in parallel. Inasmuch as the applied voltage is common to all the circuit branches, it is customary to lay out the voltage vector horizontally and to refer all current vectors to it.

Example 5–5.—In Fig. 5–11 are represented three branches of a circuit with resistances and 60-cycle reactances which are connected in parallel to a 220-volt, 60-cycle line. The resistances and reactances have the following magnitudes:

$R_1 = 40$ $X_1 = 10$
$R_2 = 10$ $X_2 = 50$
$R_3 = 5$ $X_3 = 100$

Find the current in each branch. Find the total current taken from the line and its phase displacement from the applied voltage.

Solution.—In branch 1,

$$I_1 = \frac{V}{Z_1}$$

$$\dot{I}_1 = \frac{220}{40 + j10}$$

$$= 5.33 \text{ amp}$$

$$\theta_1 = \tan^{-1}\frac{10}{40} = 14^\circ 3'$$

$$I_1 = 5.33\overline{\diagdown 14^\circ 3'}$$

$$I_1 \cos\theta_1 = 5.33 \times \cos 14^\circ 3'$$

$$= 5.33 \times 0.9600 = 5.110$$

$$I_1 \sin\theta_1 = 5.33 \times \sin 14^\circ 3'$$

$$= 5.33 \times 0.2427 = 1.29$$

$$\dot{I}_1 = 5.110 - j1.29 \qquad (1)$$

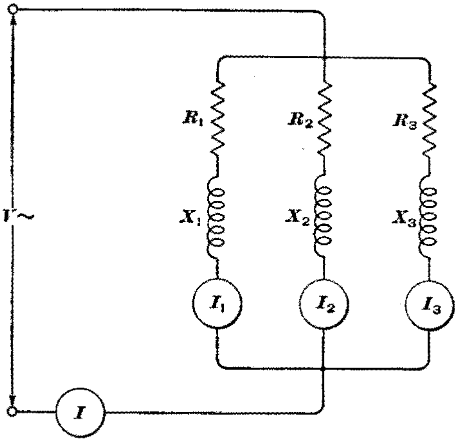


Fig. 5–11. Circuit With Resistances and Inductances in Parallel

In branch 2,

$$I_2 = \frac{V}{Z_2}$$

$$\dot{I}_2 = \frac{220}{10 + j50}$$

$$= 4.32 \text{ amp}$$

$$\theta_2 = \tan^{-1}\frac{50}{10} = 78^\circ 42'$$

$$I_2 = 4.32\overline{\diagdown 78^\circ 42'}$$

$$I_2 \cos\theta_2 = 4.32 \times \cos 78^\circ 42'$$

$$= 4.32 \times 0.1959 = 0.846$$

$$I_2 \sin\theta_2 = 4.32 \times \sin 78^\circ 42'$$

$$= 4.32 \times 0.9806 = 4.23$$

$$\dot{I}_2 = 0.846 - j4.23 \qquad (2)$$

In branch 3,

$$I_3 = \frac{V}{Z_3}$$

$$\dot{I}_3 = \frac{220}{5 + j100}$$

$$= 2.17 \text{ amp}$$

$$\theta_3 = \tan^{-1}\frac{100}{5} = 87°8'$$

$$I_3 = 2.17\overline{\backslash 87°8'}$$

$$I_3 \cos\theta_3 = 2.17 \times \cos 87°8'$$
$$= 2.17 \times 0.0500 = 0.108$$

$$I_3 \sin\theta_3 = 2.17 \times \sin 87°8'$$
$$= 2.17 \times 0.9987 = 2.16$$

$$\dot{I}_3 = 0.108 - j2.16 \qquad (3)$$

The current I taken from the line may be found by combining the components of the currents in the branches, as follows:

$$\begin{aligned} \dot{I}_1 &= 5.110 - j1.29 && (1) \\ \dot{I}_2 &= 0.846 - j4.23 && (2) \\ \dot{I}_3 &= \underline{0.108} - j\underline{2.16} && (3) \\ \dot{I} &= 6.064 - j7.68 \\ I &= 9.78 \text{ amp} \end{aligned}$$

$$\theta = \tan^{-1}\frac{7.68}{6.064} = 51°42' \text{ lagging}$$

$$I = 9.78\overline{\backslash 51°42'}$$

The vector diagram is shown in Fig. 5-12.

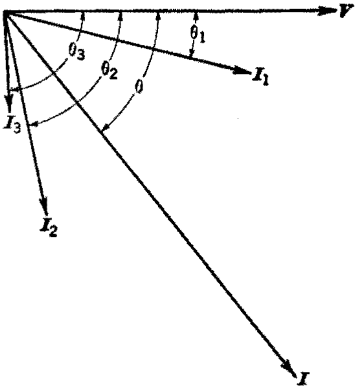


Fig. 5-12. Vector Diagram for Example 5-5

Conductance, Susceptance, and Admittance.—The in-phase and quadrature components of complex numbers may be found by a method differing somewhat from that explained on page 24. In a simple series circuit containing resistance and inductance,

$$I = \frac{V}{Z}$$

When complex-number notation is used,

$$\dot{V} = V + j0$$

$$\dot{Z} = r + jx$$

$$\dot{I} = \frac{V + j0}{r + jx}$$

If the right-hand member of the last equation be multiplied by $\frac{r-jx}{r-jx}$, its value will not be changed. Thus,

$$\dot{I}=\frac{V+j0}{r+jx}\times\frac{r-jx}{r-jx}$$

$$=\frac{V(r-jx)}{r^2-j^2x^2}$$

$$=\frac{V(r-jx)}{r^2+x^2}$$

$$=V\left(\frac{r}{r^2+x^2}\right)-jV\left(\frac{x}{r^2+x^2}\right)$$

or

$$\dot{I}=I_g-jI_b \tag{5-8}$$

in which I_g is the component of current *in phase* with V and I_b is the component of current *in quadrature* with V. The vector diagram for current is shown in Fig. 5–13.

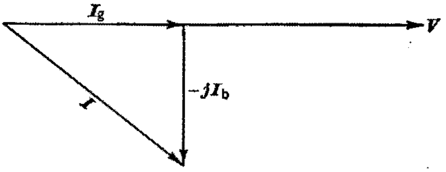


Fig. 5–13. Vector Diagram for Current

The quantity $\frac{r}{r^2+x^2}$ is a factor by which the voltage V may be multiplied to find the component of current in phase with it, and the quantity $\frac{x}{r^2+x^2}$ is a factor by which V may be multiplied to find the component of current in quadrature with it. The factor $\frac{r}{r^2+x^2}$ is called the *conductance*, and its symbol is g. The factor $\frac{x}{r^2+x^2}$ is called the *susceptance*, and its symbol is b. Thus,

$$g=\frac{r}{r^2+x^2} \tag{5-9}$$

$$b=\frac{x}{r^2+x^2} \tag{5-10}$$

Just as all the reactances of a series circuit may be added algebraically, so may all the susceptances of parallel circuits be added algebraically. Also, all conductances of several parallel circuits may be added algebraically.

The complex quantity whose components are g and b is called *admittance*, and its symbol is Y. Thus,

$$\dot{Y}=g-jb \tag{5-11}$$

The admittance is the reciprocal of the impedance; that is,

$$Y=\frac{1}{Z} \tag{5-12}$$

Also

$$I=VY \tag{5-13}$$

The unit of g, b, and Y is the reciprocal ohm or *mho*.

Example 5-6.—Solve Example 5-5 by considering conductance, susceptance, and admittance.

Solution.—The computations for determining the admittance Y follow:

$$\begin{array}{lll}
g_1=\dfrac{r_1}{r_1^2+x_1^2}=\dfrac{40}{1600+100}=0.02350 & & \\
b_1=\dfrac{x_1}{r_1^2+x_1^2}=\dfrac{10}{1600+100}= & & 0.00588 \\
g_2=\dfrac{r_2}{r_2^2+x_2^2}=\dfrac{10}{100+2500}=0.00384 & & \\
b_2=\dfrac{x_2}{r_2^2+x_2^2}=\dfrac{50}{100+2500}= & & 0.01920 \\
g_3=\dfrac{r_3}{r_3^2+x_3^2}=\dfrac{5}{25+10000}=0.00049 & & \\
b_3=\dfrac{x_3}{r_3^2+x_3^2}=\dfrac{100}{25+10000}= & & 0.00980 \\
\hline
\qquad\qquad\qquad\qquad\qquad 0.02783 & - & j0.03488
\end{array}$$

$$Y=0.0445$$

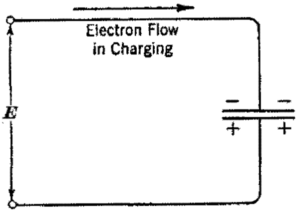


Fig. 5-14. Electron Flow in a Condenser

The current from the line is

$$\begin{aligned}
I&=VY\\
&=220\times0.0445=9.78 \text{ amp}\\
\theta&=\tan^{-1}\frac{0.03488}{0.02783}=51^\circ42'\\
I&=9.78\overline{\backslash 51^\circ42'}
\end{aligned}$$

Capacitance.—If a device consisting of two plates of a conducting material separated by an air-gap be connected to a source of voltage, as in Fig. 5-14, the excess protons of the positive plate will attract the excess electrons of the negative plate and the plates are said to be charged. The electrons will flow to the upper (−) plate until the attraction from the positive plate is balanced by the mutual repulsion of the electrons on the negative plate, and then the flow will cease. The plates are then said to be *fully* charged. Such a device is called a *condenser*, or *capacitor*.

If the plates of a condenser be moved closer together, the attraction between electrons and protons will increase, and more electrons will flow into the negative plate. If, furthermore, the opposing areas of the two plates be increased, the electrons will redistribute themselves, their number per unit of surface will be less, their mutual repulsion will be less, and more electrons will flow into the negative plate to restore the balance. If the applied voltage be increased, more electrons will flow into the negative plate. For any certain opposing (facing) plate area and distance between plates, the device will have a certain capacity for assuming a charge. This capacity is called *capacitance*. Its conventional symbol is C, and its unit of measurement is the *farad*. The farad is inconveniently large for most practical purposes, and hence the *microfarad* (*mf*), which is one-millionth of a farad, is commonly used.

A condenser is said to have a capacitance of 1 farad if its capacity is 1 *coulomb* of electricity per volt of applied voltage. A coulomb is that quantity of electricity which will pass a given point in a conductor in 1 second if the rate of current flow is 1 ampere.

Let Q denote the charge in the condenser. Then, the rate of current flow is

$$I = \frac{dQ}{dt}$$

But, since $Q = CE$,

$$I = C \frac{dE}{dt} 10^{-6} \tag{5-14}$$

where C = capacitance in microfarads.

Not all condensers have only air between their plates. Some have waxed paper, mica, or other suitable material between the plates; and this material, which is called the *dielectric*, makes the capacitance greater than it would be if air alone were present.

Alternating Voltage Applied to a Condenser.—A sinusoidal voltage applied to the terminals of a condenser produces a sinusoidal current. When the applied voltage is maximum, the voltage of the charge on the condenser is equal to it and opposite in sign, and therefore no current flows. Experiment shows that, as the applied voltage decreases, the potential of the charged condenser forces current out of the condenser, and that this current is maximum at the instant at which the applied sinusoidal voltage is zero. The current taken by a perfect condenser leads the applied voltage by 90° and is, therefore, a function of rate of change of voltage, or $\frac{dv}{dt}$. It is not possible to make a perfect condenser, but for all except the most exacting cases the angle may be taken as 90°.

Capacitive Reactance.—With any applied voltage, the current that will flow into a perfect condenser in which there is no initial charge is proportional to its capacitance. The condenser is said to have capacitive reactance. The symbol for capacitive reactance is X_c. The effective current in a condenser with an impressed sinusoidal voltage is

$$I_c = \frac{V_c}{X_c} \tag{5-15}$$

where I_c = condenser current;
V_c = effective voltage impressed on the condenser;
X_c = capacitive reactance.

The current at any instant is

$$\begin{aligned} i &= C\frac{dv}{dt} \\ &= C\frac{d(V_m \sin \omega t)}{dt} \\ &= CV_m\omega \cos \omega t \end{aligned}$$

The maximum current is

$$I_m = CV_m\omega$$

The effective current is

$$I_c = CV_c\omega$$

Thus, $I_c = \dfrac{V_c}{1/C\omega}$ and

$$X_c = \frac{1}{C\omega}$$

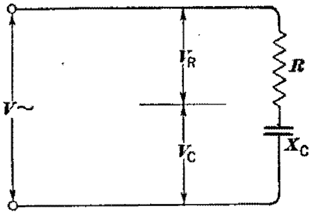


Fig. 5-15. Resistance and Capacitive Reactance in Series

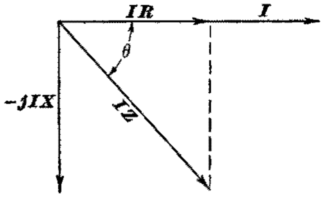


Fig. 5-16. Voltage Diagram for Circuit of Fig. 5-15

Since the current taken by a condenser is proportional to its capacitance and, by equation (5-15), it is inversely proportional to its reactance, then its reactance is inversely proportional to its capacitance. Hence,

$$X_c = K\frac{1}{C}$$

By a procedure similar to that used in the case of inductive reactance (page 32), it may be shown that

$$K = \frac{1}{2\pi f}$$

Therefore, $$X_c = \frac{1}{2\pi fC} = \frac{1}{C\omega} \qquad (5\text{–}16)$$

and $$I_c = 2\pi fCV_c = V_cC\omega \qquad (5\text{–}17)$$

Series Circuits With Resistance and Capacitance.—In the circuit of Fig. 5–15 the same current flows in both the resistance and the reactance. The resistance voltage drop is in phase with the current, and the capacitive reactance voltage drop lags the current by 90°, as indicated in Fig. 5–16. The angle θ is the characteristic angle.

Example 5–7.—A resistor with $R = 30$ ohms is connected in series with a condenser with $C = 50$ microfarads to a 220-volt line whose frequency is 60 cycles. How much current will flow?

Solution.—The capacitive reactance is

$$\begin{aligned} X_c &= \frac{1}{2\pi fC} \\ &= \frac{1}{2\pi \times 60 \times 0.00005} = 53.2 \text{ ohms} \end{aligned}$$

The current is found as follows:

$$\begin{aligned} \dot{I} &= \frac{V}{\dot{Z}} \\ &= \frac{V}{R - jX_c} \\ I &= \frac{V}{\sqrt{R^2 + X_c^2}} \\ &= \frac{220}{\sqrt{30^2 + 53.2^2}} = 3.6 \text{ amp} \end{aligned}$$

Example 5–8.—A resistor whose resistance is 10 ohms has a maximum safe current capacity of 5 amperes. It is to be used on a 220-volt, 60-cycle line in series with a condenser. What must be the capacitance of the condenser to limit the current to a safe value?

Solution.—The impedance is

$$\begin{aligned} Z &= \frac{V}{I} \\ &= \frac{220}{5} = 44 \text{ ohms} \end{aligned}$$

But $$\begin{aligned} Z &= \sqrt{R^2 + X_c^2} \\ &= \sqrt{R^2 + \left(\frac{1}{2\pi fC}\right)^2} \end{aligned}$$

Therefore, $$Z = \sqrt{10^2 + \left(\frac{1}{2\pi fC}\right)^2} = 44$$

from which $$C = 0.000061 \text{ farad or } 61 \text{ microfarads}$$

Parallel Circuits With Resistance and Capacitance.—When two or more circuits with resistance and capacitance are connected in parallel to a common source of voltage, each circuit acts as if it existed alone. The total current drawn by all the circuits may be calculated by the same method as was used for inductive circuits in Example 5–5.

Series Circuits With Resistance, Inductance, and Capacitance.—The characteristics of series circuits with resistance, inductance, and capacitance may best be studied by solving an example.

Example 5–9.—As shown in Fig. 5–17, a resistor with resistance $R_R = 10$ ohms, an inductive reactor with 60-cycle reactance $X_L = 50$ ohms and resistance $R_L = 5$ ohms, and a condenser with 60-cycle reactance $X_c = 45$ ohms are connected in series to a 60-cycle line of 220 volts. Find the current, the voltage across the inductive reactor, and the voltage across the condenser.

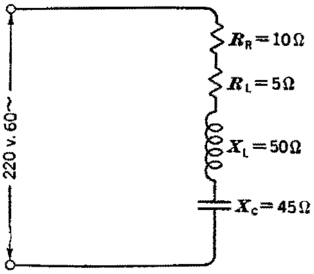


Fig. 5–17. Resistance, Inductance, and Capacitance in Series

Solution.—

$$
\begin{aligned}
\dot{Z}_R &= 10 + j0\\
\dot{Z}_L &= 5 + j50\\
\dot{Z}_c &= 0 - j45\\
\hline
\dot{Z} &= 15 + j5
\end{aligned}
$$

$$\dot{I} = \frac{V}{\dot{Z}}$$

$$= \frac{220}{15 + j5}$$

$$I = 13.9 \text{ amp}$$

$$\theta = \tan^{-1}\frac{5}{15} = 18^\circ 29'$$

$$I = 13.9\diagdown\!\underline{18^\circ 29'}$$

The voltage across the inductive reactor is

$$
\begin{aligned}
\dot{I}\dot{Z}_L &= 13.9(5 + j50)\\
&= 13.9 \times 50.2 = 697.7 \text{ volts}
\end{aligned}
$$

The voltage across the condenser is

$$
\begin{aligned}
\dot{I}\dot{Z}_c &= 13.9(0 - j45)\\
&= 13.9 \times 45 = 625.5 \text{ volts}
\end{aligned}
$$

Fig. 5–18 is the vector diagram for this example.

The most important point to be noted in Example 5–9 is that, while the whole circuit has only 220 volts applied to it, the condenser part of the circuit suffers a potential of 625.5 volts. If the condenser were designed for only 220 volts, there would almost certainly be a puncture of its insulation. In general, an excessive voltage of 697.7 volts applied to an inductance coil designed for only 220 volts would not endanger the coil unless it were the cause of excessive current which might cause overheating. When an inductance is connected in series with a capacitance, care should be taken to avoid voltages across the capacitor greater than that for which it was designed.

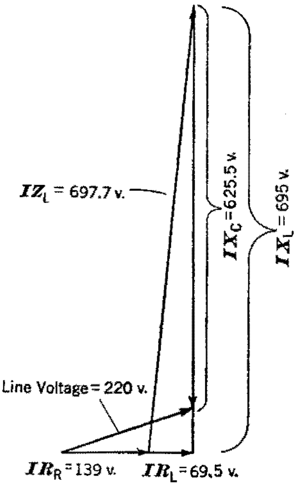


Fig. 5–18. Vector Diagram for Example 5–9

If the capacitance or inductance of a series circuit be so adjusted that the voltage across the capacitor is just equal to the inductive voltage in the reactor at some frequency, the circuit will act as if neither inductance nor capacitance were present at that frequency and the resistance alone will limit the current. This condition is called *series resonance.* One practical use of series resonance is found in some types of radio receivers in which a feeble voltage change between the antenna and the ground is made to provide a relatively large voltage change for use on the grid of the first amplifying tube.

Parallel Circuits With Resistance, Inductance, and Capacitance.—Each of a number of parallel circuits connected to a common source of voltage acts as if it exists alone. The characteristics of a combination of parallel circuits may best be studied by solution of an example.

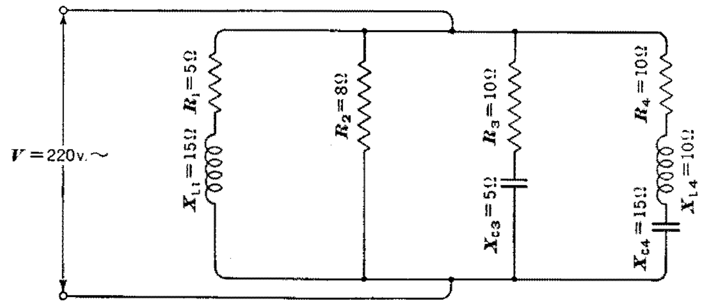


Fig. 5–19. **Parallel Circuits With Resistance, Inductive Reactance, and Capacitive Reactance**

Example 5–10.—Four circuits with constants as given in Fig. 5–19 are connected in parallel across a 220-volt, 60-cycle line. Find the total current and its phase-angle displacement from the applied voltage.

Solution.—Since this is a parallel circuit, V is common to all branches and will be taken as the reference. For circuit 1,

$$\dot{I}_1 = \frac{V}{\dot{Z}_1}$$

$$= \frac{220}{5+j15}$$

$$I_1 = 13.9 \text{ amp (lagging } V)$$

$$\theta_1 = \tan^{-1}\frac{15}{5} = 71°34'$$

$$\dot{I}_1 = 4.39 - j13.18 \qquad (1)$$

For circuit 2,

$$\dot{I}_2 = \frac{V}{\dot{Z}_2}$$

$$= \frac{220}{8+j0}$$

$$I_2 = 27.5 \text{ amp (in phase with } V)$$

$$\theta_2 = \tan^{-1}\frac{0}{8} = 0$$

$$\dot{I}_2 = 27.5 + j0 \tag{2}$$

For circuit 3,

$$\dot{I}_3 = \frac{V}{\dot{Z}_3}$$

$$= \frac{220}{10-j5}$$

$$I_3 = 19.61 \text{ amp (leading } V)$$

$$\theta_3 = \tan^{-1}\frac{5}{10} = 26°34'$$

$$\dot{I}_3 = 17.57 + j8.78 \tag{3}$$

For circuit 4,

$$\dot{I}_4 = \frac{V}{\dot{Z}_4}$$

$$= \frac{220}{10+j10-j15} = \frac{220}{10-j5}$$

$$I_4 = 19.61 \text{ amp (leading } V)$$

$$\theta_4 = \tan^{-1}\frac{5}{10} = 26°34'$$

$$\dot{I}_4 = 17.57 + j8.78 \tag{4}$$

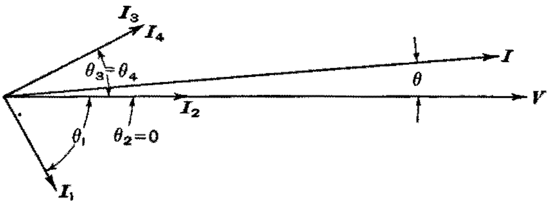


Fig. 5-20. Vector Diagram for Example 5-10

The vector diagram is shown in Fig. 5-20. The total current is found as follows:

$$\begin{aligned}
\dot{I}_1 &= 4.39 - j13.18 && (1)\\
\dot{I}_2 &= 27.50 + j0 && (2)\\
\dot{I}_3 &= 17.57 + j8.78 && (3)\\
\dot{I}_4 &= \underline{17.57 + j8.78} && (4)\\
\dot{I} &= 67.03 + j4.38
\end{aligned}$$

$$\theta = \tan^{-1}\frac{4.38}{67.03} = 3°45'$$

$$\dot{I} = 67.2\underline{/3°45'}$$

The most significant point in Example 5–10 is that, while the arithmetical sum of all the branch currents is 80.6 amp, the actual line current is only 67.2 amp. If there were added another parallel circuit with pure inductance which would take a current $0-j4.38$ amp, the sum of all leading and lagging components would be zero and the line current would be $67.03+j0$ and would be in phase with V. This would be a case of *anti-resonance.* A practical use for anti-resonance is found in industrial-power distribution. The current used by mills and factories usually lags the line voltage. By adding a condenser in parallel with the load at the factory, anti-resonance may be approached; thus, there will be a reduction in the line current and in the cost of conductors to provide the necessary power.

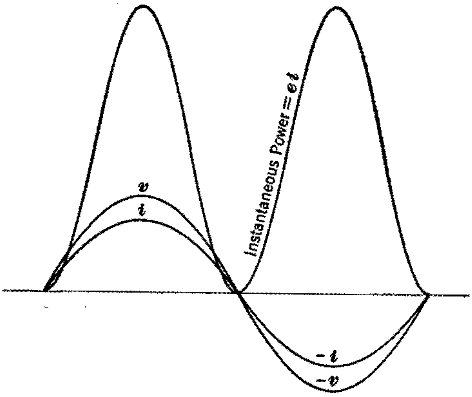


Fig. 5–21. Power in a Circuit With Resistance Only

The instantaneous power is the product of the instantaneous voltage and instantaneous current.

Instantaneous and Average Power in a Circuit With Resistance Only. The instantaneous power in a circuit is always the product of the instantaneous current and instantaneous voltage. Fig. 5–21 shows the power wave as a locus of successive products of v and i when the voltage and current are in phase. The power passes through two cycles while the emf passes through one. All the power values are positive, and all of the energy represented by the area between the power wave and the axis is energy flowing in one direction in the circuit. If $\dot{V}$ is the voltage applied to a resistor and $\dot{I}$ is the resulting current, then the energy is that transformed into heat in the resistor in every cycle multiplied by the number of cycles. The power in the resistor is

$$P = VI \tag{5–18}$$

in which P = average power;
V = rms volts;
I = rms amperes.

Power in a Circuit With Resistance and Inductance.—In Fig. 5–22 are shown in sinusoids the conditions in Example 5–3. In such a case the instantaneous product of v and i is not always positive. The part of the power wave below the axis is negative; therefore, the energy represented by the shaded areas is negative energy, that is, flowing from the circuit back into the line. The net energy converted, per cycle, into heat in the circuit is then proportional to the area above the axis less the area below the axis.

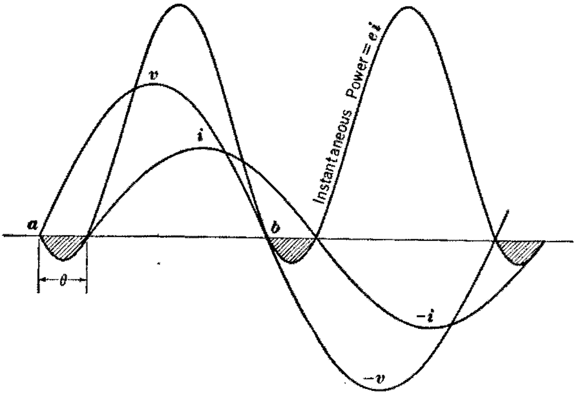


Fig. 5–22. Power in a Circuit With Resistance and Inductance in Series

$P = VI \cos\theta$

In Fig. 5–22 the power wave completes one cycle of 2π radians in going from a to b, and the power may be expressed as

$$p = V_m \sin\omega t \times I_m \sin(\omega t - \theta)$$
$$= V_m I_m \sin\omega t \sin(\omega t - \theta)$$

$$P_{av} = \frac{\int_0^{2\pi} V_m I_m \sin\omega t \sin(\omega t - \theta)\, d(\omega t)}{2\pi}$$

$$P_{av} = \frac{V_m I_m}{2\pi}\int_0^{2\pi} \sin\omega t \sin(\omega t - \theta)\, d(\omega t)$$

$$= \frac{\sqrt{2} I_{eff}\,\sqrt{2} V_{eff}}{2\pi}\int_0^{2\pi} (\sin\omega t\cos\theta - \cos\omega t\sin\theta)\sin\omega t\, d(\omega t)$$

$$= \frac{VI}{\pi}\int_0^{2\pi} (\sin^2\omega t\cos\theta - \sin\omega t\cos\omega t\sin\theta)\, d(\omega t)$$

$$= \frac{VI}{\pi}(\pi\cos\theta)$$

Hence,

$$P_{av} = VI\cos\theta \tag{5–19}$$

The vectors of Fig. 5–23 show V and I and their relationship. It may be said that the power is equal to the product of the voltage vector and the projection of the current vector on it. The projection is $I \cos \theta$. The product of V and I then is not equal to the power in a circuit with inductance or capacitance; and equation (5–18), or $P = VI$, is a special case in which $\cos \theta = 1$. Thus, $\cos \theta$ is a factor by which the product VI of a circuit must be multiplied to obtain the true power, and this factor is called the *power factor*. The theory applied here in the inductive circuit applies to capacitive circuits as well. In Example 5–10, cos 3°45′ is the power factor.

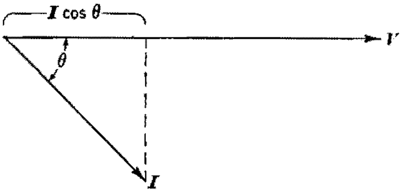


Fig. 5–23. Vectors for Power

The following terms are used to identify the various power components:

$$VI = \textit{apparent power}, \text{ in volt-amperes;}$$
$$P = VI \cos \theta = \textit{active power}, \text{ in watts;}$$
$$P_r = VI \sin \theta = \textit{reactive power}, \text{ in vars;}$$
$$\cos \theta = \text{power factor, which is } \frac{P}{VI}.$$

The abbreviation for volt-ampere is v-a.

Measurement of Power and Power Factor.—An a-c ammeter indicates effective current in amperes. An a-c voltmeter indicates effective voltage. An a-c wattmeter indicates average power in watts or kilowatts. The proper location of instruments for measuring V, I, and P in Example 5–3 is shown in Fig. 5–24.

The ammeter A should be read while neither the voltmeter wires nor the potential-coil wires of the wattmeter are connected. The wattmeter should be read when the voltmeter wires are not connected. Some wattmeters are provided with two terminals on the "zero" side of the potential coil, one marked "C" and the other "D". Connections for such wattmeters are shown in Fig. 5–25. Connection to post "C" provides for automatic compensation in indication for the power loss in the voltage coil of the wattmeter. Connection to the "D" post is used when the wattmeter is used in the secondary of a current transformer, as in Fig. 16–10.

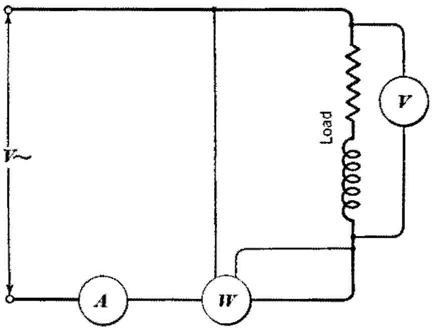
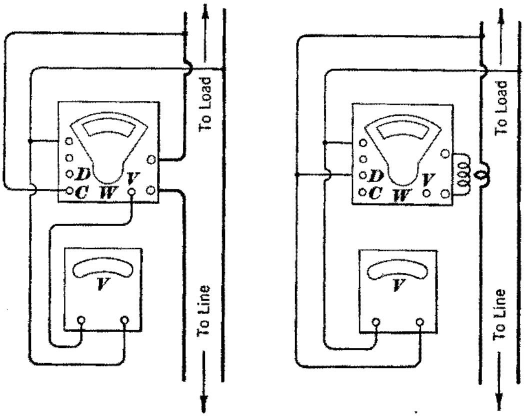


Fig. 5–24. Meter Arrangement for Example 5–3

The Kv-a Vector Diagram.—If the voltage triangle of a circuit be multiplied by the current of the circuit, a volt-ampere triangle is the result. By dividing each side of this triangle by 1000, a kilovolt-ampere (kv-a) diagram may be obtained. Such a triangle is shown in Fig. 5-26.



Compensating Non-compensating

Fig. 5-25. Wattmeter Connections

A motor or other device which takes its power at a low power factor requires generating and transmitting equipment of a size proportional to the total kv-a. For that reason the reactive kv-a of industrial power apparatus is kept to the lowest economic minimum, as described in Chapter 25 relating to power-factor correction.

Effective Resistance.—By equation (2-2), which is based on Joule's Law,

$$R = \frac{P}{I^2} \tag{5-20}$$

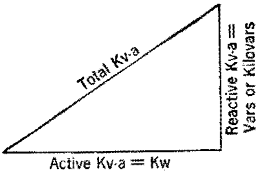


Fig. 5-26. Kv-a Triangle

If the resistance of a solenoid containing an iron core be computed by equation (5-20), and if the voltage be alternating and P be measured by a wattmeter, the power indicated by the wattmeter will include that converted into heat in the copper of the coil and also that caused by eddy currents and hysteresis loss in the iron of the core. The reason for these iron losses is discussed in Chapter 7. The resistance so calculated would be greater, of course, than that which would have been found if direct current had been used, because direct current can produce power loss only in the copper of the coil. The value of the resistance found by a direct-current test is called the "ohmic" resistance. The resistance found by using alternating current is called "effective" resistance and is denoted by R_e. The difference between measured ohmic resistance and effective resistance is determined largely by the quality and structure of the iron core. Effective resistance is also somewhat different from ohmic resistance because the alternating current does not distribute itself over the cross-section of its conductor so effectively as does direct current, and the alternating current sets up localized eddy currents in the conductor which contribute further to the power loss and hence to the increase in the effective resistance. For this reason, when a large conductor must be used, it is usually made up of several smaller conductors grouped together and periodically transposed among themselves in the group so that in the total length of the conductor every one of the small component conductors has occupied every position in the group at least once, as shown in Fig. 5-27.

Fig. 5-27. Transposition of Grouped Conductors to Mitigate Eddy-Current Losses

PROBLEMS

1. An impedance of $5+j8$ ohms at 60 cycles is connected in series with a second impedance of $8-j0$ ohms at 60 cycles across an alternating voltage of 230 volts at 60 cycles. Calculate the current. *Ans. 15.1 amp*

2. Calculate the current that would flow if the two impedances of Problem 1 were connected in series across a line whose direct voltage is 230 volts.

3. Three single-phase motors whose impedances at standstill are $2+j3$, $4+j2$, and $1+j2$ ohms, respectively, are connected in parallel through the same starting switch to a 230-volt alternating-current line. Calculate the current in the line at start. *Ans. 210 amp*

4. Calculate the voltage applied to each impedance in Problem 1.

5. How many ohms of pure capacitive reactance must be connected in parallel with the load of Problem 3 to correct its power factor to 100%?

6. A reactor, whose 60-cycle impedance is $5+j8$, is connected in series with a capacitor, whose 60-cycle rating at 230 volts is 20 kv-a, across a 230-volt, 60-cycle a-c line. Calculate the voltage across the capacitor.

7. Calculate the active power, the apparent power, and the reactive power in Problem 3.

8. A condenser with a capacitance of 200 mf is connected in series with a non-inductive 20-ohm resistor across a 220-volt line. Determine the current that will flow at each of the following frequencies: 30, 40, 50, and 60 cycles. What is the voltage drop across each part of the circuit at 60 cycles? What is the power factor of the circuit in each case?

9. In starting a certain automobile, the starting motor draws 140 amperes (average) for 4 seconds. How many coulombs of electricity pass through the motor?

10. A condenser has a reactance of 10 ohms at 25 cycles. Calculate its capacitance.

11. A condenser has a capacitance of 0.0002 farad. What voltage at 40 cycles would produce 5 amperes of current in it?

12. A condenser of 0.00212 farad takes 4 amperes from a 120-volt line. What is the line frequency?

13. In Problem 11, how much energy per cycle is exchanged between the condenser and the supply?

14. How many farads of capacitance would be required in a condenser to produce the same reactance in ohms on 60 cycles as does a pure inductance of 0.1 henry on 40 cycles? *Ans. 0.000106 farad*

15. An air-core inductance coil draws 6 amperes with a power loss of 400 watts when it is connected to a 115-volt, 60-cycle supply. If the frequency were reduced to 25 cycles, how many volts would be required to produce the same power loss?

16. A pure resistance of 5 ohms and a pure reactance of 10 ohms are connected in parallel to a voltage of $100+j0$ volts. Write the complex expression for the current.

17. A circuit containing 60-cycle impedances of $12+j0$, $0-j12$, and $0+j6$ connected in series draws 10 amperes from a 60-cycle line of 134 volts. How much current would the circuit draw from a 25-cycle line of 220 volts?

18. An inductive 60-cycle susceptance of 0.10 mho is connected in parallel with a conductance of 0.15 mho across a 230-volt supply. What are the line current and the angle by which it lags the line voltage?

19. What is the angle of displacement between two voltages whose complex expressions are $0+j50$ and $5+j80$?

20. A voltage of 125 volts is applied to a circuit containing three impedances of $4+j5$, $3-j6$, and $4+j0$ in series. Write the complex expression for the voltage across the reactor.

21. Five condensers of 0.0012 farad each are connected in series across a 60-cycle, 440-volt a-c line. Calculate the current.

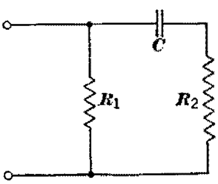
22. Calculate the current that would be taken from a 220-volt, 25-cycle line by the five condensers of Problem 21 connected in parallel.

23. A coil of 1000 turns of wire is wound on a wooden ring. When a voltage of 240 volts of 60-cycle frequency is applied, the current is 20 amperes. If one-half of the turns are removed and a voltage of 120 volts of 120-cycle frequency is applied, how much current will flow? Assume negligible resistance. *Ans. 20 amp*

24. A wire solenoid has a power loss of 200 watts when a voltage of 240 volts of 60-cycle frequency is applied and a current of 2 amperes flows. What voltage at a frequency of 25 cycles must be applied to produce the same power loss? *Ans. 135 volts*

25. Two resistors, $R_1 = 1000$ ohms and $R_2 = 20{,}000$ ohms, are connected in series. How many microfarads of capacitance must be connected across R_1 to reduce the impedance across R_1 to 100 ohms, if the frequency f is 60 cycles per second? How many, if $f = 400{,}000$?

26. In the circuit shown in the diagram, $C = 0.1$ mf, $R_1 = 20{,}000$ ohms, and $R_2 = 60{,}000$ ohms. Calculate the impedance of the circuit: (*a*) at 80 cycles per second; (*b*) at 8000 cycles per second.



Problem 5–26

CHAPTER 6

Electrical Conductors

Resistance of Conductor.—Commercial copper wire for electrical use may be had in many sizes and cross-sectional shapes. Round wire is usually used in winding small magnet coils, motors, and generators. Wire of rectangular section is more economical in space and is used in large equipment. Conductors that may be bent frequently are usually built up of a number of small wires and are called *stranded* conductors.

As may be shown by test and demonstrated by Ohm's Law, the resistance of a conductor of uniform cross-section is directly proportional to its length and inversely proportional to its cross-sectional area. This statement may be expressed by the formula

$$R = \rho \frac{l}{A} \tag{6–1}$$

in which R = resistance of a conductor, in ohms;
l = length of the conductor, in feet;
A = cross-sectional area of the conductor, in circular mils;
ρ = a constant depending on the resistive properties of the substance of the conductor.

For any conducting substance, ρ may conveniently be taken as the resistance of a round bar of that substance 1 foot long and 0.001 inch or 1 mil in diameter. The cross-sectional area of such a bar is called 1 circular mil and the bar is said to be one circular mil-foot in size. The resistance of 1 circular mil-foot of a substance is called the *resistivity* of the substance. The resistivity of a substance is the reciprocal of its conductivity. At 25 C the resistivity ρ of annealed copper of 100 per cent conductivity is 10.6 ohms. The resistivity of aluminum is about 2 times that of copper, and the resistivity of iron is about 7 times that of copper.

The areas of similar plane figures are proportional to the squares of their like dimensions; so a circle with a diameter of M mils has an area A of M^2 circular mils, or $A = M^2$.

Example 6–1.—A two-wire line whose copper wires have diameters of 0.25 in. is 1 mile long. Find the resistance of the line at 25 C.

Solution.—The diameter of the wire is 250 mils. Then, the resistance of the line is

$$\begin{aligned} R &= \rho \frac{l}{A} \\ &= 10.6 \times \frac{5{,}280 \times 2}{250^2} = 1.79 \text{ ohms} \end{aligned}$$

Wire Gages.—Although it is convenient to express all wire sizes in circular mils, round wires are frequently specified by American Wire Gage numbers.[1] By remembering the progressive law followed in the table, it is easy to convert gage numbers to circular mils or vice versa. A No. 10 copper wire has a diameter of approximately 100 mils and a resistance of 1 ohm per 1000 feet at 25 C.

As the gage number of the wire decreases, the cross-sectional area doubles at every three numbers and the diameter doubles at every six numbers. As the gage number increases, the cross-sectional area is halved at every three numbers and the diameter is halved at every six numbers. The area of any round wire is therefore $\sqrt[3]{2}$ times as great as that of the next smaller integral size. Sizes of conductors larger than 0000 are specified in circular mils instead of by numbers.

Change in Resistance With Change in Temperature.—Almost all metals and alloys show an increase in resistance with an increase in temperature. Some alloys properly proportioned show practically no such change in resistance with change in temperature. When the temperature of a pure copper wire above 0 C increases by 1 deg C, its resistance rises 0.427 per cent of its resistance at 0 C. This law is expressed by the formula

$$R_t = R_0(1 + \alpha_0 t) \tag{6–2}$$

in which R_t = resistance at temperature t;
R_0 = resistance at 0 C;
$\alpha_0 = 0.00427$, for 0 C as reference;
t = temperature of conductor, in deg C.

It is only when 0 C is used as the base temperature that $\alpha_0 = 0.00427$. For every other base temperature, there is another coefficient. To avoid the need of a table of coefficients, it is best to remember only α_0, which is 0.00427.

Example 6–2.—At 75 C a conductor has a resistance of 400 ohms. What is its resistance at 35 C?

First Solution.—First, find the resistance of the conductor at 0 C. Thus,

$$R_{75} = R_0(1 + \alpha_0 t)$$

and

$$R_0 = \frac{400}{1.32} = 303.3 \text{ ohms}$$

Then, the desired resistance at 35 C is

$$\begin{aligned} R_{35} &= R_0(1 + \alpha_0 t) \\ &= 303.3 \times 1.149 = 348.5 \text{ ohms} \end{aligned}$$

Second Solution.—In the temperature range from 0 C to 100 C, in which copper conductors are usually used, the resistance of pure copper varies linearly with the temperature. If this linear relationship continued in the lower temperatures, the

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[1] See Appendix for American Wire Gage table.

resistance of pure copper would be zero at -234.5 C. It is therefore permissible to say that the resistance of pure copper is directly proportional to the temperature above -234.5 C. The variation for this example is represented in Fig. 6–1. The resistance at 35 C may then be found as follows:

$$\frac{R_{35}}{R_{75}} = \frac{234.5+35}{234.5+75}$$

$$R_{35} = 348.5 \text{ ohms}$$

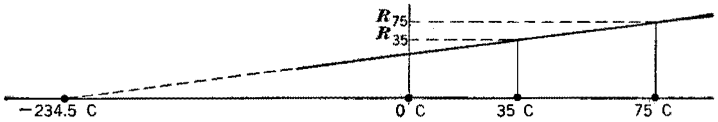

Fig. 6–1. **Variation of Resistance With Temperature**

Economical Size of Wire.—If the cross-sectional area of the wire used be doubled, the power loss would be only half as much for the same current. The choice of size of conductor is determined by four major considerations:

1. The mechanical strength must be sufficient to withstand stresses of suspension. For interior wiring, except fixture cord, the minimum size is No. 14. Fixture cord may be as small as No. 18.
2. The voltage drop in the line should not exceed 3% for lamp circuits and 10% for motor circuits.
3. The temperature rise must not exceed the maximum permissible temperature of the insulation used. See Tables A and B of the National Electric Code in the Appendix.
4. The annual cost of the energy lost in the conductor should be approximately equal to the annual investment costs of those parts of the distribution system which are proportional to the weight of the copper. This balancing of fixed charges against the cost of energy lost establishes a relation such that the sum of the annual investment costs and the cost of wasted line energy for a year will be a minimum.

The problem involved in choosing the size of conductor will be further discussed in Chapter 23 under the subject, Most Economical Size of Cable.

PROBLEMS

1. A distribution system has an average power loss of 8 kw at a summer temperature of 86 F. What would be the average power loss at -20 F with the same current loading?

2. The copper field circuit of a direct-current generator has a resistance of 120 ohms at a room temperature of 22 C. After 4 hours of operation, its resistance is found to be 132 ohms. What is its temperature?

3. A rectangular bus conductor has a cross-sectional area of $\frac{1}{2}$ sq in. What is its area in circular mils?

4. What is the resistance at 25 C of a coil of commercial copper wire containing 1200 ft of No. 18 conductor?

5. At what temperature would the coil of Problem 4 have no resistance?

6. A load of 50 kw is located 200 ft from a generator. The generator terminal voltage is 248 volts, and the voltage at the load is 230 volts. Calculate the area, in circular mils, of the distribution conductor. Would the National Electric Code permit the use of this size of conductor (rubber-covered) indoors? (See Table B in the Appendix.)

7. What is the power loss in 1500 ft of No. 4 copper conductor carrying 100 amp at 60 C?

8. A coil whose mean diameter is 6 in. contains 4000 turns of No. 22 copper wire. What is its resistance at 50 C?

9. A 6-ft instrument lead of No. 18 copper wire is accidentally connected across a 115-volt a-c supply. Calculate the maximum rate at which heat is generated in the wire.

10. A transformer winding has a resistance of 0.2 ohm at 21 C. What will be its temperature when its resistance measures 0.31 ohm?

11. A tungsten lamp on a 120-volt circuit draws 0.68 amp at a temperature of 2700 C. Calculate the current inrush at the instant of switching when the filament temperature is 25 C. Assume a constant temperature coefficient of 0.0045. *Ans. 8.02 amp*

12. Aluminum has a resistance 1.6 times that of copper and the weight of a unit volume is one-third as great. If the conductor of Problem 7 were of aluminum instead of copper, what would be its weight in pounds to produce the same heat loss at the same current of 100 amp?

13. A 10-ft length of alloy resistance wire has a resistance of 2 ohms. Its diameter is 0.020 in. What is the resistance per circular mil-foot of this alloy? *Ans. 80 ohms*

CHAPTER 7

Electromagnets

Magnetic Circuit.—Magnetism that depends on a flow of electric current is called *electromagnetism.* Fig. 7–1 shows a typical magnetic pattern produced by a loosely wound coil or *solenoid* with air core. While most of the magnetic flux links all the turns of the solenoid, some of it leaks through, as at a, and does not link the end turns. This is called *flux leakage* with respect to the end turns. It may be shown by calculation[1] that, if a solenoid whose length is many times its diameter be wound with N turns uniformly distributed over its length l, the field intensity at its center will be

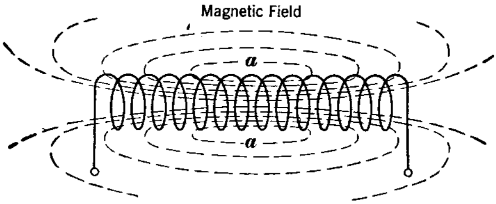

Fig. 7–1. Solenoid With Air Core

$$H = \frac{4\pi NI}{10l} \qquad (7\text{–}1)$$

and

$$NI = \frac{Hl}{0.4\pi} \qquad (7\text{–}2)$$

where I = current flowing, in amperes;
l = length of solenoid, in centimeters;
N = number of turns in solenoid;
H = field intensity, in gausses.

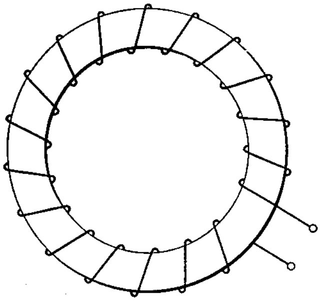

Fig. 7–2. Closed-Loop Solenoid

If the solenoid were bent so that its axis formed a closed loop, as in Fig. 7–2, and there were no flux leakage, the amperes required to produce a field intensity H with N turns would be

$$I = \frac{10Hl}{4\pi N}$$

The flux leakage from such a solenoid or coil may be reduced to an amount negligible for most purposes by substituting for the air core a core of

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[1] *Principles of Direct-Current Machines,* by A. S. Langsdorf. McGraw-Hill Book Co.

some other material such as iron in which a magnetic field may be established more easily than in air. Thus, for a magnetic circuit all or most of which is in iron, equation (7–1) may be used except as modified in equation (7–5). See the solution of Example 7–1. It is much easier to magnetize iron than a vacuum, or air.

The ease with which a material may be made to carry magnetic lines of induction, compared to the ease with which they may be established in a vacuum, is called its *permeability.*

The following comparison may be made between the electric circuit and the magnetic circuit.

Electric Circuit	*Magnetic Circuit*
Electromotive Force, emf or E.	*Magnetomotive Force*, mmf or M.
Produced by any one of several means. Unit is the volt.	Produced only by ampere-turns. Unit is the gilbert, which equals $0.4\pi NI$.
Conductivity, K.	*Permeability*, μ.
Varies for different materials, from practically zero to high values. Has its highest values for a few pure metals, notably silver and copper. Is independent of current density, but varies with temperature, the change being different for different metals.	Is unity for a vacuum, air, and many other materials. Varies from very slightly below 1 for a few diamagnetic materials to more than 1000 for iron and some alloys. Varies greatly with flux density B, but is not materially affected by moderate temperature. The amount of change with B varies greatly for different alloys.
Resistivity, ρ.	*Reluctivity.*
Reciprocal of conductivity.	Reciprocal of permeability.

If equation (7–1) were used to find the flux density in the core of a solenoid like that in Fig. 7–1, which is provided with an iron core of uniform cross-section and of permeability μ, then

$$B = \mu H = \frac{4\pi NI}{10l}\,\mu \tag{7–3}$$

Also,

$$\phi = BA \tag{7–4}$$

or

$$\phi = \frac{4\pi NI\mu A}{10l} \tag{7–5}$$

where B = flux density in the iron, in lines per square centimeter;
ϕ = total flux in the iron;
A = cross-sectional area of the iron, in square centimeters;
l = length of the iron, in centimeters.

The term $\frac{l}{\mu A}$ is called *reluctance*, and its symbol is Ρ (rho). Reluctance in a magnetic circuit corresponds to resistance in an electric circuit. Thus,

$$\mathrm{P}=\frac{M}{\phi} \tag{7-6}$$

or

$$\phi=\frac{M}{\mathrm{P}} \tag{7-7}$$

in which M = magnetomotive force.

Equation (7–6) or equation (7–7) is called Ohm's Law of the magnetic circuit because of its similarity to the equation $R=\frac{V}{I}$ or $I=\frac{V}{R}$ of the electric circuit.

The resistance of air to low electrical potentials is infinite, and so it is possible to "open" an electric circuit. The reluctivity of air, however, is 1; so it is not possible to have a magnetic path of infinite reluctance and hence, it may be said that a magnetic circuit cannot be opened.

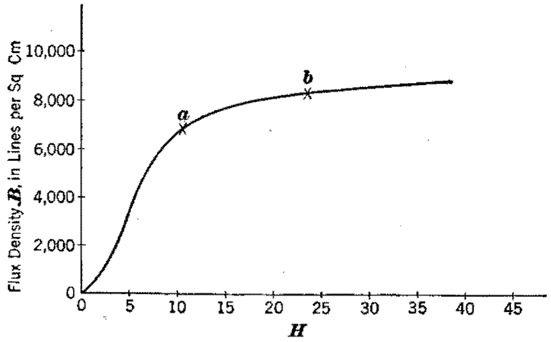


Fig. 7–3. Magnetization Curve of Iron

Magnetomotive Force.—If a unit magnetic pole be placed in the magnetic circuit of Fig. 7–2 and be carried along the complete path of a magnetic line of force, the work done will be the *magnetomotive force* of the coil. Its symbol is M or mmf. In equation (7–5) the term $\frac{4\pi NI}{10}$ is the magnetomotive force, upon which the flux density depends directly.

Permeability and Saturation of Iron.—The permeability of a vacuum, air, or any one of many other media is a constant, regardless of the flux

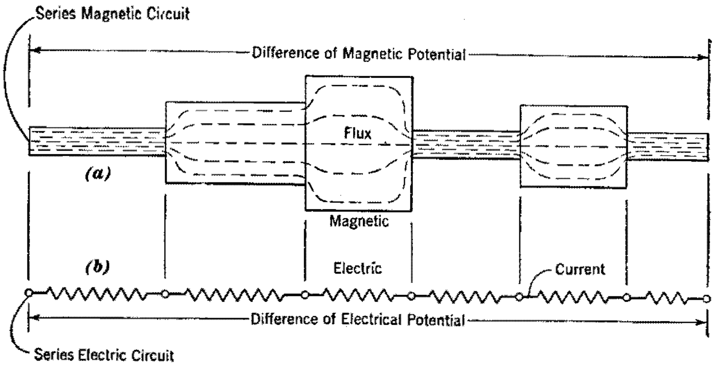
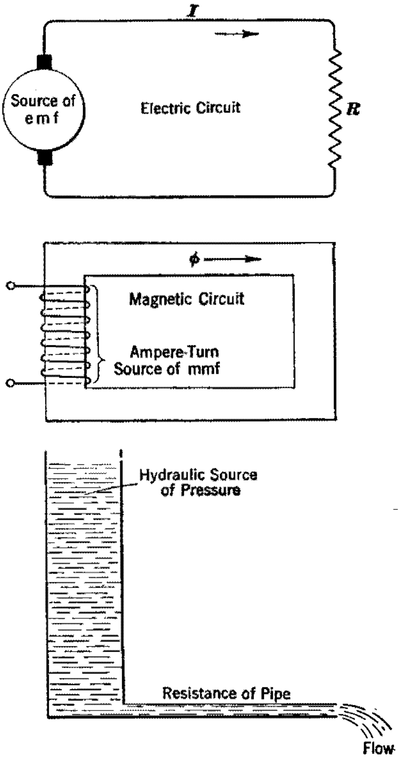


Fig. 7–4. **Comparison of Series Electric and Magnetic Circuits**



In an electric circuit containing resistance only at constant temperature, the resistance is always the same regardless of the amount of current flowing.

In a magnetic circuit without iron (or high-permeance alloy), the reluctance is always the same regardless of the amount of flux present. In a magnetic circuit containing iron, the reluctance varies as a complicated function of flux density, according to the magnetic characteristics of the iron. When only the mmf applied to the circuit is known, the flux cannot be calculated directly because the reluctance is not known until the flux is known.

In a hydraulic circuit, the resistance to fluid flow varies as a complicated function of the velocity of flow. When only the pressure in pounds per square inch is known, the water flow cannot be calculated directly because pipe friction is not a constant but is a function of velocity of fluid flow.

Fig. 7–5. **Comparison of Electric, Magnetic, and Hydraulic Circuits** density in it. The permeability of iron, nickel, cobalt, or any one of several of their alloys is a variable function of the flux density in it. Fig. 7–3 is a typical magnetization curve of flux density B in iron plotted against H which, from equation (7–1), is $\frac{4\pi}{10}$ times the ampere-turns (NI) used for each centimeter of length. For convenience, the magnetization curves for commercial iron are usually given as gausses plotted against ampere-turns per centimeter of length of path. Beyond the point a, Fig. 7–3, called the *knee* of the curve, the increase in B for a given increase in H becomes very small. It is quite possible, but seldom practicable, to carry the magnetization beyond the point b.

In calculating the number of ampere-turns required to magnetize a piece of iron to a certain flux density, the permeability of the iron *at that flux density* must be known, or a curve of NI per unit length of path plotted against flux density must be available for the specific magnetic material being used.

Series Magnetic Circuits.—A magnetic circuit may be made up of sections of different reluctances in series, just as an electric circuit may be made up with different resistances in series.

Fig. 7–4 (a) shows a rod of iron of uniform quality, but of various cross-sections in series. The analogous electric circuit is shown in (b). The different component lengths of the magnetic circuit may be of different materials, and therefore may have different permeabilities, just as the electric circuit may be made up of sections of iron, copper, aluminum, etc., every one of which has different resistivity. There is, however, an important difference. The permeability of any section (being a complex function of flux density in that section) cannot be known unless the flux density is known. Since the resistance of a metallic conductor does not change (constant temperature being assumed) when the current density in it changes, Ohm's Law may be applied readily regardless of current magnitude. A comparison of the electric, magnetic, and hydraulic circuits is shown in Fig. 7–5.

Although the sum of the differences of magnetic potential in a magnetic circuit is equal to the total applied difference in magnetic potential, the actual percentage division of difference of potential among the several sections of Fig. 7–4 (a) will change as the flux density is changed. Therefore, it is not possible to calculate directly the flux density in any section of Fig. 7–4 (a) when a known difference of magnetic potential is applied to the whole. Such a problem can be solved by trial and error only. However, it is possible to calculate accurately the total mmf necessary to produce any specific flux density in any part, as is demonstrated in Example 7–1, which follows.

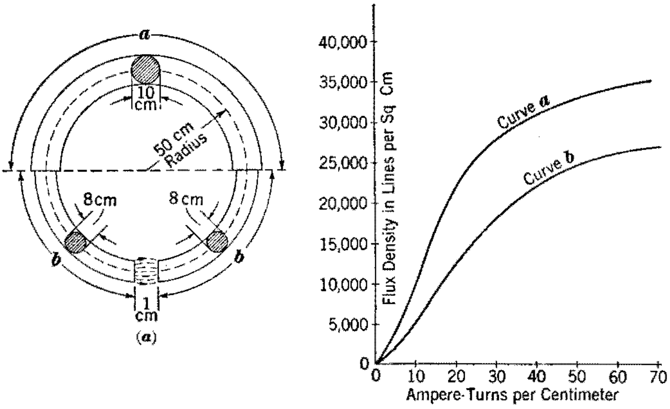
Example 7-1.—Determine the number of ampere-turns necessary to produce a magnetic field of 1,000,000 lines of flux in the structure of Fig. 7-6 (*a*).

Solution.—The first step is to find the number of ampere-turns for part *a* of the magnetic circuit and the number for part *b*. For part *a*,

$$\frac{\phi}{A_a}=\frac{1{,}000{,}000}{\pi\times\left(\frac{10}{2}\right)^2}=12{,}730 \text{ lines per sq cm}$$

$$l_a=\frac{2\pi\times\text{Radius}}{2}$$

$$=\frac{2\pi\times 50}{2}=157 \text{ cm}$$



(*a*) (*b*)

Fig. 7-6. Magnetic Circuit for Example 7-1

From curve *a* in Fig. 7-6 (*b*) corresponding to the kind of steel in section *a* in (*a*), a flux density of 12,730 lines per sq cm requires 11.5 ampere-turns per centimeter.

$$NI_a=\text{amp-turns per centimeter}\times\text{centimeters in part } a$$
$$NI_a=11.5\times 157=1805 \text{ amp-turns for part } a$$

For part *b*,

$$\frac{\phi}{A_b}=\frac{1{,}000{,}000}{\pi\times\left(\frac{8}{2}\right)^2}=19{,}890 \text{ lines per sq cm}$$

$$l_b=\left(\frac{2\pi\times\text{Radius}}{2}\right)-1$$

$$=156 \text{ cm}$$

From curve *b* in (*b*) corresponding to the kind of steel in section *b* in (*a*), a flux density of 19,890 lines per sq cm requires 33 ampere-turns per centimeter.

$$NI_b=\text{amp-turns per centimeter}\times\text{centimeters in part } b$$
$$NI_b=33\times 156=5148 \text{ amp-turns for part } b$$

For the air-gap,

$$H=\frac{4\pi NI}{10l}$$

The flux density in the air-gap (fringing being ignored) is the same as that in part *b*, or 19,890 lines per sq cm. Hence,

$$B = 19{,}890 \text{ lines per sq cm}$$

$$H = B = \frac{4\pi N I_g}{10l}$$

and

$$N I_g = \frac{10 \times 1 \times 19{,}890}{4\pi}$$

$$= 15{,}830 \text{ amp-turns}$$

The total number of ampere-turns is

$$\begin{aligned} N I_a + N I_b + N I_g &= 1805 + 5148 + 15{,}830 \\ &= 22{,}783 \text{ amp-turns} \end{aligned}$$

When it is required to find the number of ampere-turns necessary to produce a given flux density in air and the dimensions are in inches instead of centimeters, it is convenient to use the converted equation for ampere-turns. Thus,

$$NI = 0.313Bl \tag{7–8}$$

where B = flux density, in lines per square inch;
l = length of gap, in inches.

Parallel Magnetic Circuits.—Magnetic circuits in parallel under the influence of a common magnetomotive force carry flux in proportion to their cross-sections and permeabilities and in inverse proportion to their lengths. Fig. 10–13 shows a good example of magnetic circuits in parallel. It should be remembered that any magnetic circuit has in parallel with it a number of so-called leakage paths which carry useless flux. In some types of apparatus, such as control magnets, the leakage flux may be neglected; but in other apparatus, such as motors, generators, and transformers (see Chapter 17), the design must include an allowance for leakage flux.

Hysteresis.—The theory advanced by Weber is probably the best explanation of the magnetic phenomenon known as hysteresis. He supposes that the molecules of all matter are small magnets. The molecules of iron are strong magnets, and those of all other kinds of matter are weak magnets. When a piece of iron is not magnetized, the molecules lie in no orderly position, and the north poles and south poles are in such disorder that no appreciable lines of force leave the mass. However, under the urge of a magnetomotive force, the molecules tend to arrange themselves parallel to the lines of force acting upon them. In soft iron many molecules are thus rearranged against the stresses of their original positions; but, when the magnetizing force is removed, almost all return to their original positions and there is again disorder. The few molecules that retain their new positions produce a faint remanent magnetic field around the iron. In very hard steel, a greater portion of the total number of molecules that are rearranged adopt the new positions permanently; and, when the magnetizing force is removed, they produce a strong permanent magnetic field around the iron. To rearrange a given number of molecules requires a much stronger magnetizing force in hard steel than in soft steel or iron.

If a bar of magnetically inert steel be surrounded by a solenoid, the magnetomotive force produced may be varied at will by varying the current in the solenoid. In Fig. 7–7 (*a*) is shown a plot of flux ϕ in a given circuit of hard steel as a function of magnetizing ampere-turns.

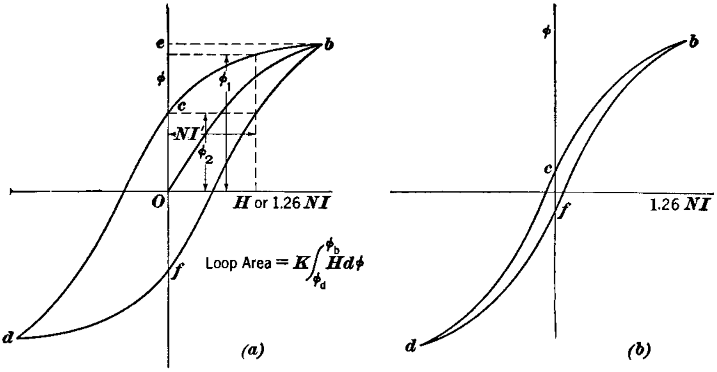


Fig. 7–7. The Hysteresis Loop

The molecules cannot be rearranged without work being done on them. If the magnetization be carried to *b*, the relation of ϕ to NI will follow the curve *Ob* and the work done will be proportional to the area *Obe*. If now the magnetizing force be removed slowly, the ratio of ϕ to NI will follow the curve *bc*. The remaining flux *cO* when $NI = 0$ is called the remanent flux and is a measure of degree of *permanent magnetism*. An amount of work proportional to area *ebc* will be returned to the solenoid circuit by induction, while work proportional to area *Obc* will have been used in overcoming the molecular friction of the process and will appear as heat in the iron. Upon reversal of current in the solenoid, the magnetizing force may be carried to *d* and to *f*. After a second reversal of current, the magnetizing force may be carried to *b*. For any value, say NI', there may be either of the two values ϕ_1 or ϕ_2 after the iron has a magnetic history of at least one cycle. The loop *bcdfb* is called the *hysteresis loop*[2] and its area is a measure of the hysteresis energy per cycle converted into heat.

Iron subjected to the magnetizing force of an alternating current passes through one hysteresis cycle for every cycle of the current. By experiment

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[2] *Electric and Magnetic Measurements*, by C. M. Smith, The Macmillan Co., New York.

it has been found that hysteresis loss in iron approximately is proportional to the 1.6 power of the maximum flux density. Fig. 7–7 (*b*) shows a typical hysteresis loop for soft iron as used in most electrical machinery.

The hysteresis loss per pound is different for different kinds of iron and steel, and is an important item to be included in purchase specifications. Producers of magnetic sheet steel usually provide this information in the form of a curve such as that in Fig. 7–8.

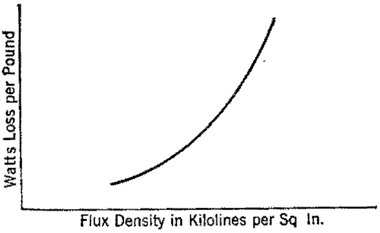


Fig. 7–8. Hysteresis Loss in Iron (for one frequency)

The hysteresis loss is proportional to the frequency and to the volume of iron, but is not affected by lamination, as is the eddy-current loss. The magnetic quality of iron is affected by heat-treating and by mechanical working.

Permanent Magnets.—If an annular ring of hardened steel or special alloy is magnetized to the point *b* in Fig. 7–7 (*a*) and the mmf is then removed, the magnetic strength decreases to point *c*. Such a structure would have no practical value because all of the flux would be inside the metal and would not be available for use. To make the flux available it is necessary to cut a portion of the metal out to provide an air-gap, as in Fig. 7–6. The air-gap causes an increase in the reluctance of the circuit and, hence, a reduction of remanent flux. The result is the same as though a reversed mmf had been applied to the ring and had driven the flux farther down along the hysteresis loop to a point such as *m* in Fig. 7–9, which shows only the demagnetization portion of Fig. 7–7 (*a*); permanent-magnet operation is confined to that portion. The distance to *m* from *c* is determined by the length of the air-gap cut in the ring. If the metal of the air-gap were to be replaced, the flux density would not return to point *c* but would return to point *n* along what is called a minor hysteresis loop. If the air-gap metal were again removed, the density would return to *m* along the upper portion of the minor hysteresis loop.

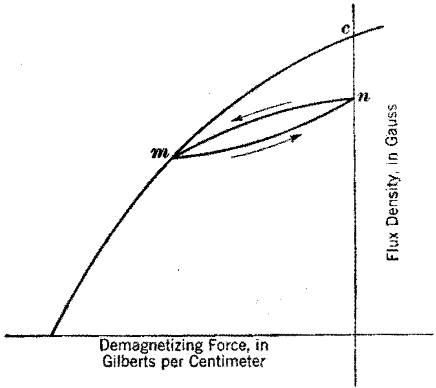


Fig. 7–9. Demagnetization Curve and Minor Hysteresis Loop

If a permanent magnet is to be used in a device, such as a direct-current measuring instrument, from which it may be removed at intervals to facilitate repair of the device, its magnetism must be stabilized at some value below that at which it is expected to work after it is assembled. This stabilization is usually accomplished by "knocking down" the magnetism by applying a small amount of demagnetizing mmf.

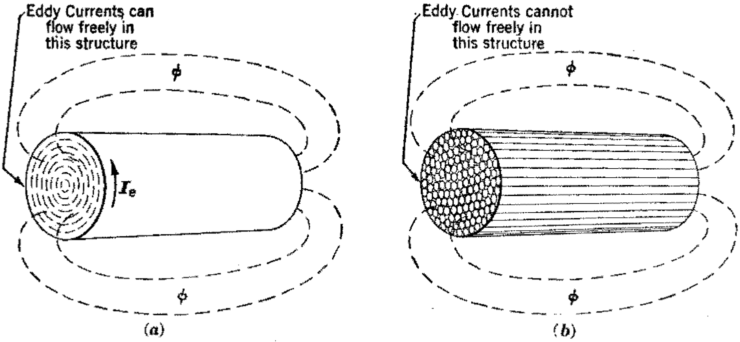
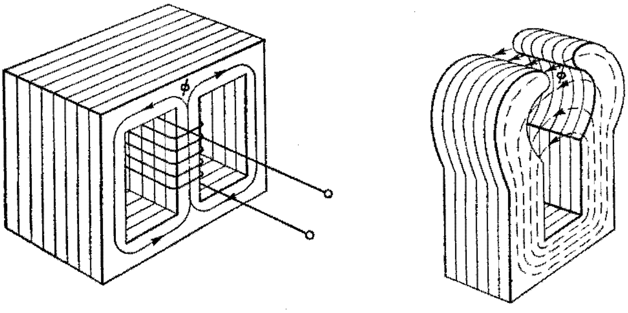


Fig. 7–10. Eddy Currents and Their Paths

Eddy Currents.—A rod of solid metal, when subjected to an alternating magnetic field, has electromotive forces set up in it. The rod may be considered as being made up of concentric laminating rings, as in Fig. 7–10 (*a*). The alternating field cuts the rings and generates alternating electromotive forces in them. These electromotive forces cause alternating currents, called *eddy currents*, to flow through the resistance of the rings. The magnitude of an eddy current is designated as I_e. A power loss and a rise of temperature result from eddy currents.



Transformer Core Toy Motor Frame

Fig. 7–11. Laminated Magnetic Paths

If the rod of Fig. 7–10 (*a*) be divided into many slender rods insulated from one another, as in (*b*), the eddy-current paths will be broken up and the power loss caused by these currents will be very greatly reduced. Such a subdivision of an iron path is effective in mitigating eddy currents but is so poor mechanically that it is unsatisfactory for most practical purposes.

In electrical machinery, eddy-current losses are mitigated by making the magnetic paths of sheet-steel laminations. A sheet thickness of about 0.014 in. is commonly used. Fig. 7–11 shows the laminated structure of a transformer and one of a toy a-c motor. In alternating-current machines, the eddy-current loss is usually much less than the hysteresis loss. Lamination increases the resistance per volt induced in the eddy-current paths, and so greatly mitigates the power loss caused by such current.

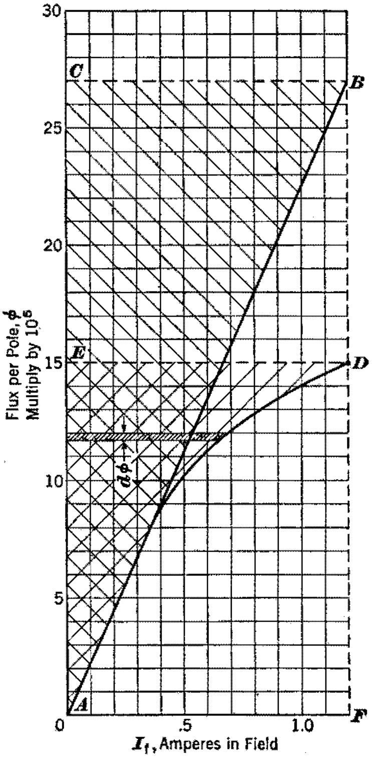


Fig. 7–12. Magnetization Curve for Example 7–2

Inasmuch as eddy currents are caused by voltages generated in the iron, they are proportional to the rate of change of flux. In the iron core of a coil, therefore, they are proportional to the maximum flux density and to the frequency. In the armature of a d-c motor or generator they are proportional to the flux density and to the speed. Therefore, the losses caused by them are proportional to the square of the maximum flux density and to the square of the frequency or speed.

Energy Stored in a Magnetic Field. When a solenoid is energized by a current, part of the energy input is expended in heating the conductor while the other part is expended in building up the magnetic field and is stored in it. After a voltage v is applied to the solenoid and produces a current i, the work done in a time dt is

$$dW = vi\,dt$$

This work may be divided into two parts, as follows:

$$dW = i^2R\,dt + L\frac{di}{dt}i\,dt$$

The last term of this equation represents the energy stored in the magnetic field in a time dt. By integrating that term between the limits zero and I, the energy stored in a given time may be found. Thus,

$$W_L = \int_0^I L\frac{di}{dt}i\,dt$$

or

$$W_L = \tfrac{1}{2}LI^2 \text{ watt-seconds} \tag{7–9}$$

This equation of energy is correct only if the inductance L of the circuit is constant.

When the magnetic circuit contains iron, the inductance is not constant and the stored energy is not so easily found. To find the energy in such a field, the saturation curve like that given in Fig. 7–6 (*b*) must be available.

Example 7–2.—The field winding of a two-pole d-c generator has 1000 turns per pole. With a field current of 1.2 amp, the flux per pole is 1,500,000 lines. Calculate the energy stored. The magnetization curve is as shown in Fig. 7–12.

Solution.—Draw a line through the origin tangent to the curve AD, and let it intersect FD extended at B. If the inductance of the circuit had remained constant as the current was increased from zero, the inductance L would have been

$$L = \frac{N\phi}{10^8 I}$$

$$= \frac{2 \times 1000 \times 27 \times 10^5}{10^8 \times 1.2} = 45 \text{ henrys}$$

and the stored energy would have been

$$W = \frac{LI^2}{2}$$

$$= \frac{45 \times 1.2^2}{2} = 32.4 \text{ watt-seconds}$$

This energy is represented by (proportional to) the area ABC. However, with a current $I_f = 1.2$ amp, the flux obtained was AE instead of AC; hence, the area ADE represents the actual work done. The following is a proof of this statement.

$$P = v_L i$$

$$= L\frac{di}{dt}i = Li\frac{di}{dt}$$

$$= N\frac{d\phi}{di}i\frac{di}{dt}$$

$$= Ni\frac{d\phi}{dt}$$

or

$$P\,dt = Ni\,d\phi$$

But

$$W = \int P\,dt$$

and

$$W = N\int_0^{\phi} i\,d\phi$$

Therefore, the energy W is proportional to $\int_0^{\phi} i\,d\phi$, which is area ADE. This area may be measured with a planimeter. The energy then is

$$32.4 \times \frac{\text{Area } ADE}{\text{Area } ABC} = 32.4 \times \frac{49.6}{162} = 9.94 \text{ watt-seconds}$$

Stored energy in a magnetic circuit is used in one method of welding thin sheets of metal by impulse. Stored energy presents a problem when interrupting an inductive circuit carrying direct current, because the energy must be dissipated in the arc and the heat produced by the arc is destructive to the opening contact points. In case the amount of energy is relatively small, it is feasible to provide a condenser across the opening points (as at the breaker points of an automobile distributor) to receive the energy instead of permitting it to burn the points.

Application of Magnets.—Lifting magnets of large diameters suspended from cranes are widely used in the handling of steel stock and scrap iron. Magnets for this service are usually energized by direct current. The same type of magnet is used in removing tramp iron from grains before milling and from more valuable machine-tool cuttings of copper, brass, etc.

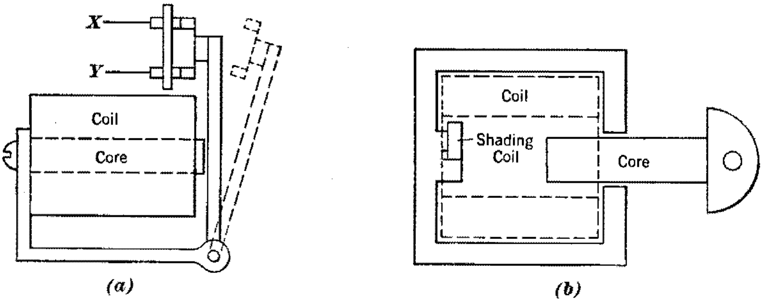


Fig. 7–13. Examples of One Type of Magnet and One Type of Solenoid

Many mechanical operations, such as opening and closing switches and valves, are performed by use of magnets. Alternating-current magnets have a more violent action than direct-current magnets and are generally less satisfactory. The action of an a-c magnet is more violent than that of a d-c magnet because the flux is almost as great in the open position of the magnet as in the closed position. The flux in a d-c magnet is determined largely by the armature air-gap, and so is relatively small when the armature is at the starting part of its closing stroke. The closing force is therefore small when the air-gap is large and increases as the stroke proceeds and closes the air-gap.

In the a-c magnet almost all of the impedance is reactance, which is low when the air-gap is large as in the beginning of the closing stroke. The low impedance permits a high inrush of current which magnetizes the core almost as strongly with the long air-gap as the reduced current does with a short air-gap at the end of the closing stroke. The average force then in the closure of an a-c magnet is a greater per cent of the maximum force than is that in a d-c magnet. In some applications in which alternating current only is available, but the violence of the a-c magnet is intolerable, a *torque motor* is used. The torque motor does not rotate far, but merely turns through a portion of one revolution to accomplish its purpose and then remains stationary though energized. In some cases, particularly on motor control panels, the alternating current is rectified (see Chapter 27) and direct-current magnets are then used. This method gives less violent action and much quieter operation.

It is not difficult to make a satisfactory d-c magnet for moving mechanical parts. An a-c magnet must, however, have a laminated magnetic circuit unless the core is made of special high-resistance iron and must also be fitted with a shading coil to reduce magnetic hum to a satisfactory minimum. These items of construction are not easily achieved in make-shift construction. It is therefore better to buy an a-c magnet for a special use on the open market from an experienced builder than to attempt to make one from materials at hand.

Pull of Magnets and Solenoids.—In order to make the most effective use of the ampere-turns on an electromagnet, it is necessary that a complete iron path be provided for the flux when the movable part or armature is in the closed position. Fig. 7–13 (*a*) shows the elements of a d-c magnet used in a d-c motor starter to close the contacts X and Y of a portion of the starter circuit. The force with which the armature is held may be expressed by the equation

$$F = \frac{B^2 A}{72{,}130{,}000} \tag{7–10}$$

in which F is the force, in pounds; B is the flux density at the pole face, in lines per square inch; and A is the area of the pole face, in square inches.

When the armature is in its open position, the force available to start its closure is less than the holding force after it is closed, because the reluctance of the air-gap reduces the flux that reaches the armature. The forces at the open and intermediate positions can be calculated only after a rather complicated flux-density plot has been made to ascertain how much flux actually reaches the armature. In practice the air-gap is made relatively small; and, if greater motion is desired, it is obtained by the mechanical advantage of a lever.

A solenoid is actually a coil of wire, but the term is frequently used to specify a type of electromagnet in which the air-gap is placed inside the coil where flux leakage can occur less readily. Fig. 7–13 (*b*) shows the elements of a solenoid. A solenoid has a longer effective movement than a magnet of the type in (*a*). When a solenoid is magnetized with alternating current from a constant-voltage source, the flux from the high current with the core at the open position is not wasted so much in leakage as in the type of magnet in (*a*). If equation (7–10) for the force F is to be used to calculate the force exerted by an a-c magnet or solenoid, B must be the root-mean-square value of the flux density.

The Shading Coil.—If no provision were made to prevent it, the pulsating pull of an a-c magnet or solenoid would fall to zero twice every cycle and the armature would be pulled away slightly by its restoring spring, only to be pulled back again on the next rise of flux. Such action causes chattering of the armature. This chattering can be greatly reduced or even eliminated by surrounding a portion of the iron at the air-gap by a closed loop or ring of copper called a *shading coil.* A shading coil is shown in Fig. 7–13 (*b*). Because of the inductance of the coil, the flux through the shading coil does not pass through zero when the rest of the flux does. Therefore, there is always some flux in the gap to produce a closing force.

PROBLEMS

1. Calculate the flux density in part *a* of the magnetic path in Fig. 7–6 (*a*) when the total ampere-turns are 15,000.

2. Why is the toy motor frame of Fig. 7–11 open at the top?

3. A coil of 1360 turns of No. 18 copper wire (see Appendix) is wound in one layer on a wooden ring whose circular cross-section is 4 sq in. and whose mean radius is 10 in. What will be the current when the coil is connected to a source of 20 volts and the current is direct? What will be the total flux in the core? *Ans. 1045 lines*

4. If the wooden ring in Problem 3 is replaced by a cast-steel ring, how many volts would be needed at the terminals of the coil to produce 200,000 lines of flux? See page 389 in the Appendix for the magnetization curve.

5. What will be the flux density in lines per square centimeter in Problem 4, if the voltage is halved?

6. A change of 4 amp in a certain 1000-turn coil causes a flux change of 1,000,000 flux lines. Calculate the inductance of the coil.

7. A direct current of 4 amp flowing through 500 turns of wire is required to maintain a flux of 4×10^6 lines in an air-core reactor. What is the inductance of the reactor?

8. An air-core reactor of negligible resistance has an inductance of 0.2 henry. What current will flow when a 60-cycle voltage of 230 volts is applied?

9. In Problem 8, what is the maximum instantaneous rate at which energy is stored in the magnetic field?

10. A magnetic circuit of cast steel is magnetized to a flux density of 45,000 lines per square inch. What percentage change in current will be necessary to double this flux density? See the magnetization curve on page 389.

11. What percentage change would be required in Problem 10 if the circuit were of 1% silicon steel instead of cast steel?

12. When the armature iron of a d-c motor is magnetized to a density of 95,000 lines per square inch and the speed is 1800 rpm, the hysteresis loss is 420 watts. What would be the hysteresis loss if the flux density were reduced to 90,000 lines and the speed were increased to 2500 rpm? Assume that all parts of the armature have the same flux density.

13. When the speed and flux density of the motor of Problem 12 are changed as specified, what will be the per cent of change in the armature eddy-current loss?

14. A magnetic contactor, when closed, requires a sustained current to hold it so. What becomes of the power used?

15. A direct-current motor running at 2000 rpm is found to have a hysteresis loss of 200 watts and an eddy-current loss of 300 watts in the armature iron. What will be the total power loss in the armature iron if the speed is increased to 2500 rpm while the field flux remains constant? The eddy-current loss is proportional to the square of the frequency of reversal of magnetic flux. *Ans. 718.7 watts*

CHAPTER 8

Electrical Transients*

In most electrical circuits, only the steady-state currents and voltages are of importance to the engineer, inasmuch as any temporary deviation therefrom caused by closing or opening a circuit lasts but a short time. Currents or voltages in a state of non-repetitive change are called *transients*. Transients are of practical importance because, even though they exist for a short time only, they may cause the blowing of fuses or the puncturing of insulation. Furthermore, repetitive transients are of great importance and highly useful in electronic and allied circuits. Transients are likely to appear in a circuit at the time of starting or stopping of current flow because of inductance or capacitance, if they are present in the circuit. Resistance in a circuit tends to limit both the magnitude and duration of a transient.

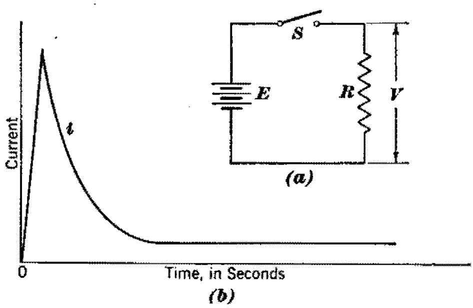


Fig. 8-1. Current Transient Caused by Temperature Change in an Incandescent Lamp Filament When the Circuit Is Closed

Direct-Current Transients.—If a switch is caused to close in a hypothetical circuit containing a battery, a switch, and only unvarying resistance, as in Fig. 8-1 (*a*), the current will rise instantly to its ultimate or steady-state value, which is $\frac{V}{R}$. There will be no transient when the switch is closed or when it is opened. If, however, the resistance is not constant but varies with its temperature, there will be a current transient which will subside only when the temperature has reached its ultimate value. Fig. 8-1 (*b*) shows such a transient current in a lamp which has a filament with a perceptible inductance.

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* For further analysis of transients see "Alternating Current Circuits," by K. Y. Tang, International Textbook Company.

Current Transient in a Purely Inductive Circuit.—If the pure resistance of Fig. 8–1 be replaced by a hypothetical pure inductance, as in Fig. 8–2 (*a*), and the switch be closed, the only deterrent to current rise is the induced counter electromotive force in the inductance caused by the rise of current and hence rise of flux therein. The induced voltage must be constant, and the rise of current therefore must be constant. The induced voltage is $e = -L\frac{di}{dt}$. This voltage is negative because it is opposite in direction to the current in the circuit. The resulting current would rise as shown in Fig. 8–2 (*b*). The rate of current rise is inversely proportional to the inductance L.

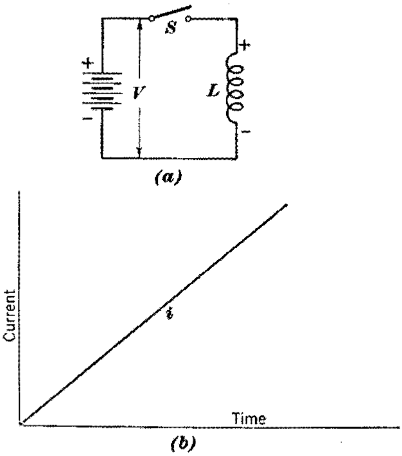

Fig. 8–2. Rise of Current in an Inductive Circuit

Current Transient When RL Circuit Is Closed.—If the resistance and inductance of Figs. 8–1 and 8–2 be connected in series, as in Fig. 8–3 (*a*), the rate of rise of current i will be affected by both circuit impedance elements simultaneously; and the result will be similar to that shown in Fig. 8–3 (*b*) when the switch is closed. At any instant after the switch is closed the voltage V must accomplish two things. It must provide one component, which is the iR voltage drop across R, and a second component, which is equal to the induced voltage $-L\frac{di}{dt}$ of the inductance L. Therefore,

$$V = Ri + L\frac{di}{dt} \tag{8–1}$$

To derive the equation for the current at any instant after the circuit is closed, it is convenient first to rewrite the voltage equation as follows:

$$\frac{V}{R} = i + \frac{L}{R}\frac{di}{dt}$$

Multiplying by $\frac{R}{L}dt$ and dividing by $\frac{V}{R} - i$, we obtain

$$-\frac{R}{L}dt = -\frac{di}{\frac{V}{R} - i}$$

$$-\frac{R}{L}\int_0^t dt = \int_0^i -\frac{di}{\frac{V}{R}-i}$$

$$-\left[\frac{Rt}{L}\right]_0^t = \left[\log_e\left(\frac{V}{R}-i\right)\right]_0^i$$

This reduces to

$$i = \frac{V}{R} - \frac{V}{R}\epsilon^{-\frac{Rt}{L}} \tag{8–2}$$

The current then at any time after the switch is closed is the steady-state value $i_{ss} = \frac{V}{R}$ less the inductive transient component $i_t = \frac{V}{R}\epsilon^{-\frac{Rt}{L}}$, as shown in Fig. 8–3 (*b*). The transient component of current approaches zero exponentially with time. Theoretically, the circuit current does not reach its final value of $\frac{V}{R}$ until time is infinite. In most instances, however, it will attain practically its steady-state value in a second or less.

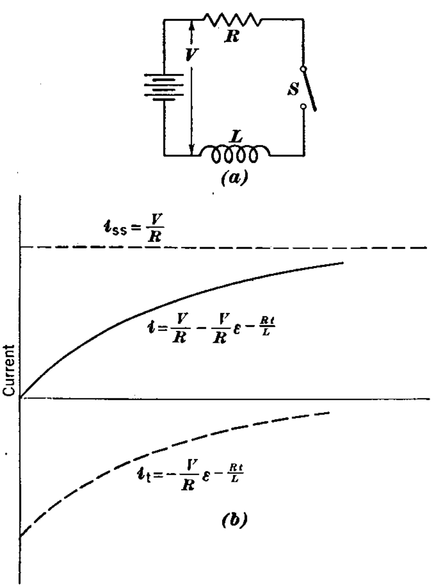

Fig. 8–3. Transient Rise of Current in an LR Circuit

Example 8–1.—A constant emf of 10 volts is applied to a coil having a resistance of 5 ohms and an inductance of 0.01 henry. How much time will elapse after the switch is closed until one-half of the steady-state value of current is reached?

Solution.—Steady-state current $i_{ss} = \frac{10}{5} = 2$ amp. By equation (8–2),

$$2 \times \frac{1}{2} = \frac{10}{5} - \frac{10}{5}\epsilon^{-\frac{5t}{0.01}}$$

Hence, $\epsilon^{-\frac{5t}{0.01}} = 0.5$

and $t = 0.00139$ sec

Time Constant of the RL Circuit.—An examination of equation (8–2) will reveal that the time required for the current in an *RL* circuit to rise to a certain percentage of its final value is proportional to *L* and inversely proportional to *R*. The larger the ratio of the inductance to the resistance, the more time will be taken for the current to rise and the longer will be the time of the effective transient.

The ratio $\frac{L}{R}$ is called the *time constant* of the circuit. When *t* equals $\frac{L}{R}$, equation (8–2) reduces to

$$i = \frac{V}{R}(1 - \epsilon^{-1})$$

or

$$i = 0.632\,\frac{V}{R} \qquad \textbf{(8–3)}$$

Therefore, after a time $t = \frac{L}{R}$ (beginning when $t = 0$), the current will have risen to 63.2 per cent of its final value.

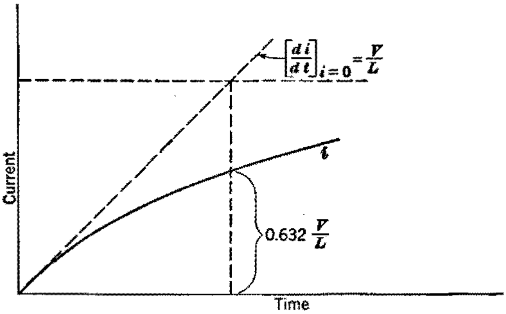

Fig. 8–4. Time Constant in LR Circuit

The initial rate of rise of current after the circuit is closed may be found by differentiating equation (8–2) with respect to t. Thus,

$$\frac{di}{dt} = \frac{V}{R}\frac{d}{dt}(1 - \epsilon^{-\frac{Rt}{L}})$$

$$= \frac{V}{L}\epsilon^{-\frac{Rt}{L}} \text{ amperes per second}$$

When $t = 0$, $\frac{di}{dt} = \frac{V}{L}$ amperes per second.

A current rising at the rate of $\frac{V}{L}$ amperes per second will require t_1 seconds to reach the steady-state value of $\frac{V}{R}$. This statement may be expressed by the relation

$$\frac{V}{L}t_1 = \frac{V}{R}$$

from which

$$t_1 = \frac{V}{R} \times \frac{L}{V} = \frac{L}{R}$$

It is true, therefore, that the time constant $\frac{L}{R}$ of a circuit is equal to the time that would be required for the current i to reach its final value of $\frac{V}{R}$ if it continued to rise at its initial rate when $t = 0$. Fig. 8–4 shows that relationship.

In the circuit of Fig. 8–3 the voltage Ri across the resistance, being proportional to i, will have a rising characteristic similar to that of i in

Fig. 8–4; this is shown in Fig. 8–5. The difference between the constant voltage V and the rising voltage drop Ri is the voltage $e_L = L\frac{di}{dt}$ across the inductance. At the instant at which the circuit is closed, the applied voltage V appears across L. After steady-state current is achieved, the voltage V appears across R.

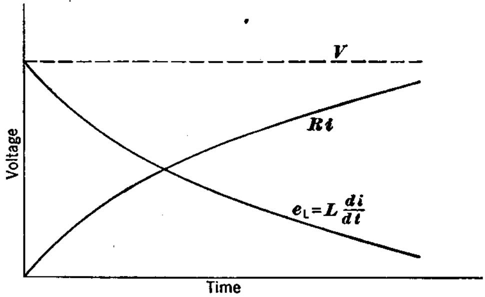

Fig. 8–5. Rate of Change of Voltage Across L and R With Transient Rise of Current

Storage of Electromagnetic Energy.—Switching transients in RL circuits are fundamentally caused by the storage of energy in the magnetic field of the circuit or by the release of such energy previously stored. When the circuit shown in Fig. 8–3 is closed, the power at any instant thereafter is Vi. Of that power a component i^2R is converted to heat in the resistance, and the remainder P_L is stored in the magnetic field of the inductance L. The value of P_L is

$$P_L = e_L i = Li\frac{di}{dt}$$

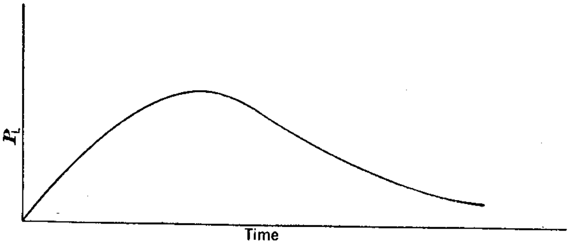

Fig. 8–6. Rate of Storage of Energy by Direct Voltage Applied to an RL Circuit

By substituting in this equation the value of i from equation (8–2), we get:

$$P_L = L\left(\frac{V}{R} - \frac{V}{R}\epsilon^{-\frac{Rt}{L}}\right)\frac{V}{L}\epsilon^{-\frac{Rt}{L}}$$

or

$$P_L = \frac{V^2}{R}\left(\epsilon^{-\frac{Rt}{L}} - \epsilon^{-\frac{2Rt}{L}}\right) \tag{8–4}$$

A plot of values of this equation is shown in Fig. 8-6. The area under this curve represents the energy stored in the magnetic field of the inductance and is equal to

$$W = \int_0^\infty P_L \, dt$$

$$= \int_0^I Li \, di$$

or

$$W = \tfrac{1}{2}LI^2 \text{ joules} \tag{8-5}$$

If the circuit is broken after the field has grown for any length of time t, the energy stored up to that time must be dissipated in the form of heat in the circuit resistance. The circuit resistance R_c now includes the resistance R_a (arc resistance) at the opened contacts in addition to the resistance R of the original circuit element. The energy dissipated will be divided between R_a and R in proportion to their magnitudes. That portion dissipated at the circuit break will cause an arc. If the breaking contact points are separated by a wide gap quickly, R_a will be relatively large; and proportionately more of the energy W will appear there in the arc as heat. The faster the contact points are separated, the higher will be the voltage across the arc and so the longer will be the arc. Also, the longer the arc, the higher will be the voltage across it. For that reason the circuits of inductive devices, such as field circuits of motors and generators having large stored energy, should be opened slowly lest the insulation of the device be ruptured by high voltage induced in the winding.

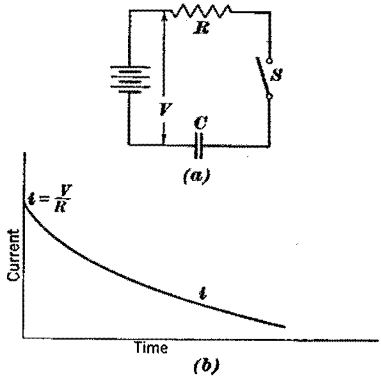

Fig. 8-7. Current Transient in RC Circuit

Transient in an RC Circuit.—Assume that a constant voltage V, a resistance R, and an uncharged condenser of capacitance C are connected as in Fig. 8–7 (*a*). Assume also that the inductance of the circuit is zero. At the instant at which the circuit is closed, the condenser—having no charge—has no emf across it and so offers no restriction to flow of current. The circuit then acts as one of pure resistance, and the initial current is $i=\frac{V}{R}$. As current flows, however, the condenser receives a charge and there is produced a counter-voltage which tends to reduce the current flow. The emf across the condenser is

$$e_C=\frac{1}{C}\int i\,dt$$

and

$$V=Ri+\frac{1}{C}\int i\,dt \tag{8–6}$$

To find the expression for current at any time after the circuit is closed, it is only necessary to differentiate equation (8–6). Thus,

$$0=R\frac{di}{dt}+\frac{i}{C}$$

or

$$i=K\epsilon^{-\frac{t}{RC}}$$

Inasmuch as $i=\frac{V}{R}$ when $t=0$, $K=\frac{V}{R}$. Therefore,

$$i=\frac{V}{R}\epsilon^{-\frac{t}{RC}} \tag{8–7}$$

The graph of Fig. 8–7 (*b*) is a plot of this equation.

The voltage across the resistor R at any time is

$$e_R=Ri=R\times\frac{V}{R}\epsilon^{-\frac{t}{RC}}$$

or

$$e_R=V\epsilon^{-\frac{t}{RC}}\text{ volts} \tag{8–8}$$

The voltage across the condenser at any time, when there is a charge q, is

$$e_C=\frac{q}{C}=V-V\epsilon^{-\frac{t}{RC}}$$

or

$$e_C=V(1-\epsilon^{-\frac{t}{RC}}) \tag{8–9}$$

Example 8–2.—A constant voltage of 100 volts is suddenly impressed on a series circuit of $R=20$ ohms and $C=0.000004$ farad. How many seconds after the circuit is closed will be required for the voltage across the condenser to rise to 50 volts?

Solution.—By equation (8–9),

$$50=100(1-\epsilon^{-\frac{t}{0.00008}})$$

from which

$$\epsilon^{-\frac{t}{0.00008}}=0.5$$

and

$$t=0.0000554\text{ sec}$$

Energy Stored in a Condenser.—The instantaneous power in the circuit of Fig. 8–7 (*a*) is

$$p=Vi=\frac{V^2}{R}\epsilon^{-\frac{t}{RC}} \tag{8–10}$$

The power in the resistance only is

$$p_R=Ri^2=\frac{V^2}{R}\epsilon^{-\frac{2t}{RC}} \tag{8–11}$$

The power or rate of storage of energy in the electric field of the condenser is

$$p_C=p-p_R=\frac{V^2}{R}\epsilon^{-\frac{t}{RC}}-\frac{V^2}{R}\epsilon^{-\frac{2t}{RC}}$$

or

$$p_C=\frac{V^2}{R}(\epsilon^{-\frac{t}{RC}}-\epsilon^{-\frac{2t}{RC}}) \tag{8–12}$$

The energy stored in the condenser at any time t after the circuit is closed is proportional to the area under a graph plotted from equation (8–12) from $t=0$ to time t. The final steady-state energy in the condenser is

$$w=\int e_C i\,dt=\int_0^Q-\frac{q}{C}\,dq$$

or

$$w=\tfrac{1}{2}\frac{Q^2}{C}=\tfrac{1}{2}CV^2 \tag{8–13}$$

When the condenser is fully charged and the circuit is opened, the energy remains in storage to do useful work.

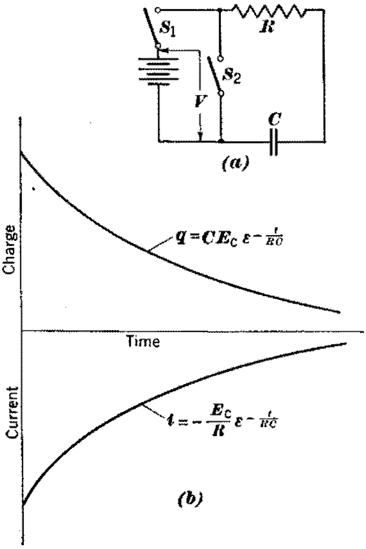

Fig. 8–8. Discharge of Current in an RC Circuit

Discharge of a Condenser in an RC Circuit.—When a condenser is charged to a voltage E, the charge will produce a voltage E at its terminals after the circuit is opened. If then, as shown in Fig. 8–8 (*a*), the condenser and resistance circuit is closed at S_2, there will be a rush of current from C through R. As the condenser thus discharges, the voltage across its terminals decreases and the decrease in discharge current is an exponential function of time, as shown in Fig. 8–8 (*b*).

By Kirchhoff's Law, when the circuit of Fig. 8–8 (*a*) is closed, equation (8–6) becomes

$$Ri+\frac{1}{C}\int i\,dt=0$$

Hence,

$$R\frac{di}{dt}+\frac{i}{C}=0$$

and
$$i = K\epsilon^{-\frac{t}{RC}}$$

When $t = 0$, $i = -\frac{E_C}{R}$. Therefore, $K = -\frac{E_C}{R}$ and the discharge current is

$$i_C = -\frac{E_C}{R}\epsilon^{-\frac{t}{RC}} \tag{8-14}$$

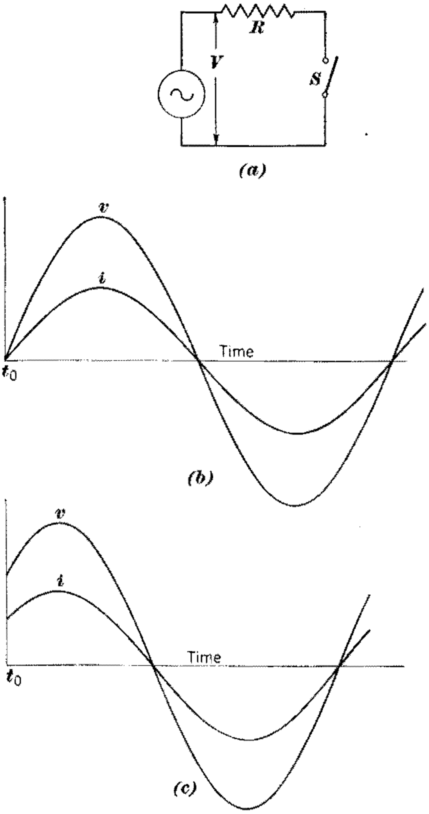

Fig. 8-9. Current and Voltage Relationships in R Circuit

Example 8-3.—A condenser having a capacitance of 0.000004 farad and charged to a potential of 100 volts is connected across a resistance of 20 ohms. How much time will elapse before the voltage across the condenser reaches 50 volts?

Solution.—In this case, $e_C = 50$, $e_R = -50$, and $i_C = \frac{E}{R} = 2.5$ amp. From equation (8-14),

$$-2.5 = -\frac{100}{20}\epsilon^{-\frac{t}{0.00008}}$$

and
$$t = 0.0000554 \text{ sec}$$

If the resistance in the preceding example were 20 megohms, the time required for the condenser voltage to fall to 50 volts would be 55.4 seconds. This circuit then can be used as a time delay device in electronic control circuits, inasmuch as the charge may be "leaked" off the condenser at any desired rate. The accuracy of control is not good, however, for delays of more than a few seconds because of the low rate of change of voltage and current after about half the charge has been spent.

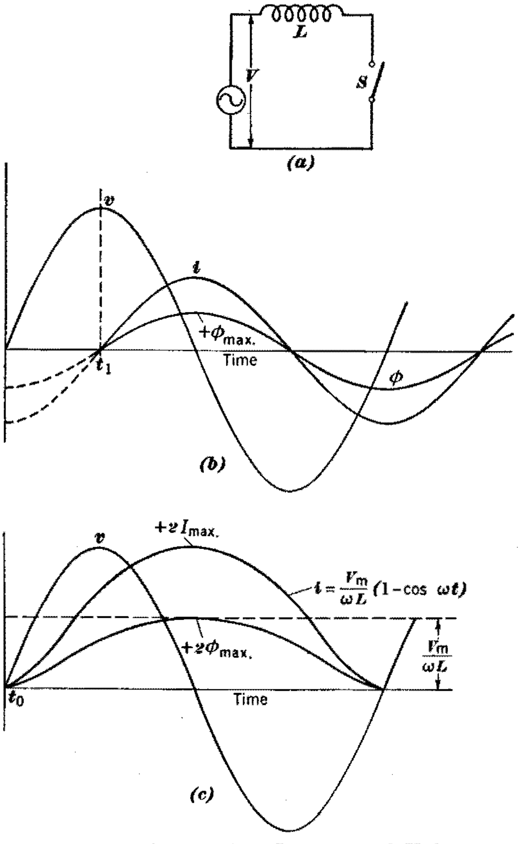

Fig. 8–10. Alternating Current and Voltage Transients in an L Circuit

Transients in Alternating-Current Circuits.—Transient currents and voltages are likely to occur in circuits when those circuits are suddenly connected to or disconnected from a source of alternating voltage. The transients are caused by the inductance and capacitance of the circuit and are mitigated by the resistance of the circuit in much the same manner as in the direct-current circuit. It will be assumed in all cases, unless otherwise stated, that the applied voltage is sinusoidal and that the impedances of all circuit elements are constant.

Circuit With Resistance Only.—In Fig. 8–9 is shown a circuit containing only a resistance R. If the circuit is closed when the voltage is zero, as at t_0, the current will be zero at that instant and will then rise sinusoidally in phase with v in accordance with Ohm's Law. Inasmuch as the only storage of energy is that in the resistor in the form of heat, there will be no current transient. In fact, there will be no transient, regardless of the time of closure, because there is no inductive or capacitive energy storage.

Circuit Containing Inductance Only.—When a circuit containing only inductance L, such as that of Fig. 8–10 (*a*), is closed, the current must produce a voltage drop in the circuit that is equal at every instant (including $t=0$) to the applied voltage v. The counter emf must be sinusoidal to match the applied voltage. To produce a sinusoidal emf, the rate of change of flux linkages in the inductance must be a sinusoidal function of time from the instant at which the circuit is closed. If the circuit is closed at time t_1 when the voltage is at its maximum value, as indicated in Fig. 8–10 (*b*), the conditions of $i=0$ and $\phi=0$ are the same as they would be if the circuit had been closed many cycles earlier with no transient persisting. The current and flux then start from zero and vary sinusoidally without a transient component. In the first half-cycle, ϕ makes a total change of $2\phi_{\text{max.}}$ from $\phi=0$ to $\phi=+\phi_{\text{max.}}$ to $\phi=0$.

If, however, the circuit is closed at t_0, as in Fig. 8–10 (*c*), $-\phi_{\text{max.}}$ does not exist at that instant and a sinusoidal change of $2\phi_{\text{max.}}$ must occur in the first half-cycle beginning with $\phi=0$. Therefore, in the first half-cycle, the flux must rise to $2\phi_{\text{max.}}$. Inasmuch as the flux is proportional to the current i which produces it, i must also rise to $2I_{\text{max.}}$ in the first half-cycle. Under these ideal conditions i would vary sinusoidally between zero and $2I_{\text{max.}}$ indefinitely, and the current could not be called a transient. In practice, however, there is always some resistance in the circuit which causes the current to diminish and there is established a steady-state variation which is symmetrical about the time axis.

When the circuit is closed at any instant,

$$L\frac{di}{dt}=V_m \sin\,(\omega t+\Phi)$$

The angle Φ is that corresponding to the instant at which the circuit is closed after $v=0$. From this equation

$$di=\frac{V_m}{L}\sin\,(\omega t+\Phi)\;dt$$

By integration,

$$i = -\frac{V_m}{\omega L}\cos(\omega t + \Phi) + K$$

When $t=0$, $i=0$ and the constant of integration is

$$K = \frac{V_m}{\omega L}\cos\Phi$$

Therefore,

$$i = \frac{V_m}{\omega L}\cos\Phi - \frac{V_m}{\omega L}\cos(\omega t + \Phi) \tag{8-15}$$

The term $\frac{V_m}{\omega L}\cos\Phi$ is the transient component of the current, and the term $\frac{V_m}{\omega L}\cos(\omega t + \Phi)$ is the steady-state component.

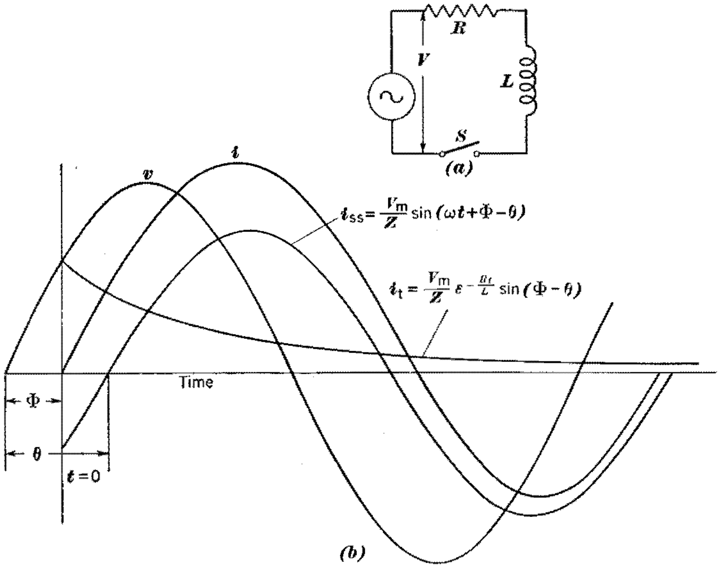

Fig. 8-11. Current Transient in RL Circuit

In Fig. 8-10 (*c*) the circuit was closed when $t=0$ and $v=0$ with the slope $\frac{dv}{dt}$ positive. Therefore, $\Phi=0$ and

$$i = \frac{V_m}{\omega L}(1 - \cos\omega t) \tag{8-16}$$

Circuit With Inductance and Resistance Only.—In a circuit containing only R and L, as in Fig. 8-11 (*a*), let the circuit be closed at any time t, as in Fig. 8-11 (*b*), corresponding to the angle Φ after the voltage v is zero and about to rise in a positive sense. The initial current will be zero, inasmuch as the circuit contains inductance which will prevent an instantaneous rise of current. The initial current may be thought of as being composed of a steady-state component i_{ss} and a transient component i_t which are equal and opposite immediately after the circuit is closed. Thus,

$$I_{ss(m)} = \frac{V_m}{Z}$$

Therefore, $i_{ss} = \frac{V_m}{Z} \sin (\omega t + \Phi - \theta)$ and is negative at $t = 0$. The angle θ is $\tan^{-1} \frac{X_L}{R}$. The transient current i_t is of the same magnitude but is positive in sign when $t = 0$.

The condition for dynamic stability is

$$L \frac{di}{dt} + Ri = V_m \sin (\omega t - \Phi)$$

from which

$$i_t = K\epsilon^{-\frac{RT}{L}}$$

The total current is

$$i = i_{ss} + i_t = \frac{V_m}{Z} \sin (\omega t + \Phi - \theta) + K\epsilon^{-\frac{RT}{L}}$$

When $t = 0$, $i = 0$ and the equation becomes

$$0 = \frac{V_m}{Z} \sin (\Phi - \theta) + K$$

and

$$K = -\frac{V_m}{Z} \sin (\Phi - \theta)$$

Therefore,

$$i = \frac{V_m}{Z} [\sin (\omega t + \Phi - \theta) - \epsilon^{-\frac{RT}{L}} \sin (\Phi - \theta)] \qquad (8\text{–}17)$$

Example 8–4.—A circuit having the constants $R = 100$ ohms and $L = 0.5$ henry is switched to a 60-cycle supply of 220 volts at the instant when the voltage is zero $\left(\frac{dv}{dt} \text{ positive}\right)$. What will be the magnitude of the current 0.00417 second after the circuit is closed?

Solution.—In equation (8–17), $V_m = 310$, $Z = 213.6$, $\omega = 377$, $\Phi = 0$, $\theta = 62°$, and $t = 0.00417$. Hence,

$$i = \frac{310}{213.6} [\sin 28° - \epsilon^{-\frac{100 \times 0.00417}{0.5}} \sin (-62°)] = 1.24 \text{ amp}$$

Circuit With Capacitance Only.—If a circuit consisting of a condenser with no charge is connected to an alternating voltage at the instant of zero voltage, there will be no transient component of current. If the circuit is closed after the voltage has any value whatsoever, the current inrush will theoretically be infinite, inasmuch as the condenser will act as a short-circuit until it has accumulated some charge. Practically, the inrush is controlled by the impedance of the source of voltage, and that impedance cannot be zero.

Circuit With Resistance and Capacitance Only.—In Fig. 8–7 (*a*) is shown a circuit containing only resistance and an uncharged condenser. If this circuit is connected to a source of alternating voltage instead of direct voltage, the current that will flow to establish dynamic equilibrium must satisfy the condition that

$$Ri+\frac{1}{C}\int i\,dt=V_m \sin\,(\omega t+\Phi)$$

The angle Φ is the angle at which the circuit is closed after $v=0$ when $\frac{dv}{dt}$ is positive.

If the condenser is initially uncharged, there is no voltage across it and it acts with no impedance. The initial current then is $i=\frac{v}{R}$. As a result of initial current flow, the condenser acquires a charge which produces a counter voltage. That voltage is

$$e_C=\frac{q}{C}=\frac{1}{C}\int i\,dt$$

The steady-state current is

$$i_{ss}=\frac{V_m}{Z}\sin\,(\omega t+\Phi-\theta)$$

where $\theta=\tan^{-1}\frac{X_C}{R}$.

The transient current is

$$i_t=K\epsilon^{-\frac{t}{RC}}$$

Therefore, $$i=\frac{V_m}{Z}\sin\,(\omega t+\Phi-\theta)+K\epsilon^{-\frac{t}{RC}}$$

When $t=0$, $i=\frac{V_m}{R}\sin\,(\Phi-\theta)$ and

$$K=\frac{V_m \sin\Phi}{R}-\frac{V_m}{Z}\sin\,(\Phi-\theta)$$

$$=\frac{V_m}{R}\sin\theta\cos\,(\Phi-\theta)$$

By substitution of this quantity for K, we obtain:

$$i=\frac{V_m}{Z}\sin\,(\omega t+\Phi-\theta)+\frac{V_m}{R}\sin\theta\cos\,(\Phi-\theta)\epsilon^{-\frac{t}{RC}} \tag{8-18}$$

When $\theta=0$ or $(\Phi-\theta)=\frac{\pi}{2},\frac{3\pi}{2}$, etc., there will be no transient component of current. Therefore, when the circuit is closed at the instant at which i_{ss} would be a maximum, there is no transient component.

When the condenser has an initial charge, the residual condenser voltage is $E_C = \frac{Q}{C}$. This voltage may be either positive or negative in the circuit loop and is, therefore, added algebraically to the transient voltage. From equation (8–6),

$$Ri + \frac{1}{C}\int i\,dt = V_m \sin(\omega t + \Phi) + \frac{Q}{C}$$

The constant of integration K for this equation is

$$K = \frac{V_m}{R}\sin\theta\cos(\Phi - \theta) + \frac{Q}{RC}$$

and

$$i_t = \left[\frac{V_m}{R}\sin\theta\cos(\Phi - \theta) + \frac{Q}{RC}\right]\epsilon^{-\frac{t}{RC}} \tag{8–19}$$

The current $\frac{Q}{RC}\epsilon^{-\frac{t}{RC}}$ is that caused by the original charge Q and is positive if the flow is around the series loop in the positive direction. It is apparent that there will be no transient current when the condenser initially is negatively charged. Hence,

$$\frac{V_m}{R}\sin\theta\cos(\Phi - \theta) = -\frac{Q}{RC}$$

When the condenser is initially charged, the general expression for the current is

$$i = \frac{V_m}{Z}\sin(\omega t + \Phi - \theta) + \left[\frac{V_m}{R}\sin\theta\cos(\Phi - \theta) + \frac{Q}{RC}\right]\epsilon^{-\frac{t}{RC}} \tag{8–20}$$

Example 8–5.—A circuit containing a resistance $R = 5$ ohms and uncharged capacitance $C = 200$ microfarad is connected to a 60-cycle voltage source of 70.0 volts at the instant when the voltage has reached 86.6 per cent of its maximum value in a positive sense. Write the equation for the current.

Solution.—From equation (8–20),

$$i = \frac{100}{14.14}\sin(377t + 129.35°) + 11.87\epsilon^{-1000t}$$

CHAPTER 9

Polyphase Circuits

Classes of Circuits.—Circuits energized by one alternating emf are called single-phase circuits. Circuits of more than two lines energized by two or more electromotive forces not all of which are in time phase with each other are called polyphase circuits. The use of polyphase circuits is justifiable for economic reasons and also because of the superior performance of machines operating on them. Industrial polyphase circuits are usually three-phase.

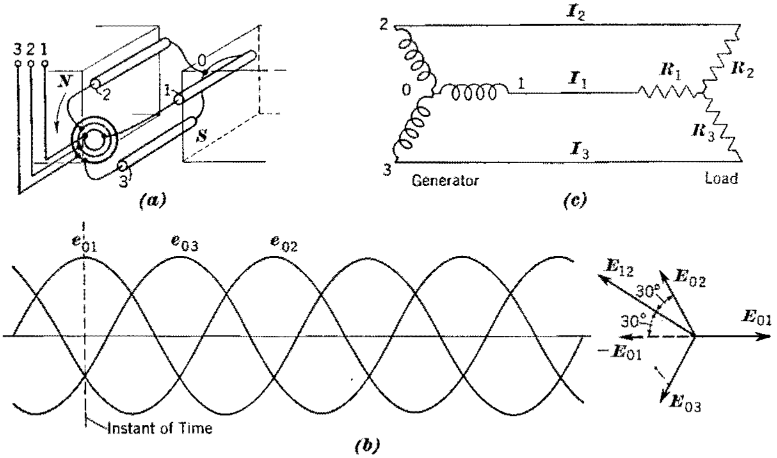

Fig. 9–1. Relationships of Three-Phase emf's

The Use of the Double Subscript.—In the analysis of single-phase circuits the use of the single subscript, as in E_1 or E_2, is sufficient and simple for designating sinusoidal and vector quantities. In polyphase circuits it is advantageous to use a double-subscript notation which aids in describing such quantities. For example, in Fig. 9–1 the vector E_{02} represents the voltage generated in conductor 2 with point 0 as reference. Likewise, a vector E_{20} would represent the same voltage but with reference to the near end of the conductor. Therefore, $E_{20} = -E_{02}$. The double subscript is then derived from the diagram of connections; and the vector point of origin is indicated by the first letter or figure of the subscript, and the termination of the vector is indicated by the second letter or figure of the subscript.

Three-Phase Circuits.—Fig. 9–1 (*a*) is a three-dimensioned representation of a magnetic field in which there are three revolving conductors displaced 120 degrees from one another on their common circular path.[1] The voltages generated in them will then be displaced in time by 120 degrees, as shown in Fig. 9–1 (*b*) by the sinusoids e_{01}, e_{02}, and e_{03} and their corresponding vectors. The conductors will be said to have a positive sense from the back, or zero, end toward the nearer end, as indicated by the sequence of the numbers in the subscripts identifying the vectors which represent the voltages generated in them. It is common practice to connect the conductors or phases inside the machine, as at 0, so that the terminals 1, 2, and 3 only are necessary to connect the machine to its load. The generator is then said to be Y-connected or wye-connected.

A voltmeter connected to the terminals 1 and 2 in Fig. 9–1 (*a*) would be affected by the voltages E_{01} and E_{02}. However, if the circuit be traced through the two conductors from terminal 1 to terminal 2, the course would be in a negative sense through conductor 1 and in a positive sense through conductor 2. Therefore, the voltage appearing between terminals 1 and 2 will be the vector difference of E_{01} and E_{02}, and not their vector sum. It is not important whether this vector difference be taken as $E_{01}-E_{02}$ or $E_{02}-E_{01}$. If, however, the voltage between terminals 1 and 2 is to be called E_{12}, and the double subscript is to be consistent in denoting the vector direction, it is necessary that $E_{12}=E_{02}-E_{01}$. Likewise, $E_{23}=E_{03}-E_{02}$ and $E_{31}=E_{01}-E_{03}$. The voltages E_{01}, E_{02}, and E_{03} are called the phase voltages; and E_{12}, E_{23}, and E_{31} are called the line-to-line voltages or simply the *line voltages.* The voltage at which a generator is rated is always the line voltage, unless otherwise stated.

By the trigonometry of Fig. 9–1 (*b*),

$$E_{12}=E_{02}\cos 30^\circ+(-E_{01})\cos 30^\circ$$

Since $E_{01}=E_{02}$ (in magnitude),

$$E_{12}=2E_{02}\cos 30^\circ$$

$$=1.732\ E_{02}$$

or

$$E_{12}=\sqrt{3}\ E_{02} \tag{9–1}$$

Therefore, in a Y-connected, three-phase system of balanced voltages, *the line voltage equals $\sqrt{3}$ times the phase voltage or line-to-neutral voltage.*

Phase Sequence.—In Fig. 9–1 (*a*), the emf in conductor 1 is at its maximum value. After 120 degrees more of rotation, the emf in conductor 3 will be maximum in the same sense. After an additional 120 degrees

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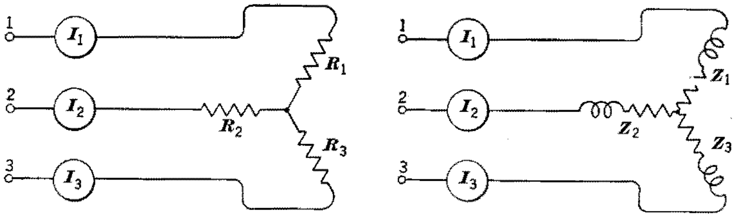
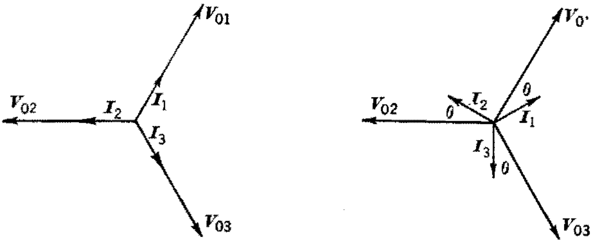
[1] In practical generators many conductors are used in every phase. They are arranged in many ways not described in this book. For further study of windings, see "Theory of Alternating Current Machinery," by A. S. Langsdorf, McGraw-Hill Book Co.

the emf in conductor 2 will be maximum in like sense. The emf's in these conductors are said to have a *phase sequence* of 1, 3, 2. If the direction of rotation were reversed, the phase sequence would be 1, 2, 3. The phase sequence of a three-phase power source is important in the application of three-phase induction and synchronous motors described in Chapters 18 and 19 because the direction of rotation of such motors is determined by the phase sequence of the applied voltages.



(*a*) Non-Inductive Load (*b*) Inductive Load



(*c*) (*d*)

Fig. 9–2. Voltage and Current Relationships in Balanced Y-Connected Systems

Electromotive Forces in a Delta-Connected System.—If three conductors like those in Fig. 9–1 (*a*) be connected in series (by proper collector-ring arrangement) by connecting the near end of conductor 1 to the far end of conductor 2, the near end of conductor 2 to the far end of conductor 3, and the near end of conductor 3 to the far end of conductor 1, then every conductor would be traversed in a positive sense and the corresponding voltage vectors would be found to add. The generator is then said to be Δ-connected or delta-connected. For the Δ-connected generator,

$$\dot{E}_{01}=E_{01}\cos 0^\circ \pm jE_{01}\sin 0^\circ$$
$$\dot{E}_{02}=E_{02}\cos 120^\circ + jE_{02}\sin 120^\circ$$
$$\dot{E}_{03}=E_{03}\cos 240^\circ + jE_{03}\sin 240^\circ$$

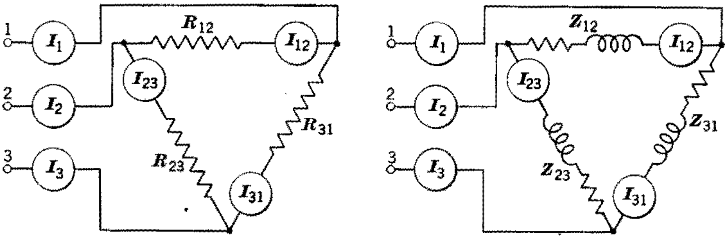
Since $\dot{E}_{01}=\dot{E}_{02}=\dot{E}_{03}$ (in magnitude),

$$\dot{E}_{01}+\dot{E}_{02}+\dot{E}_{03}=3\dot{E}_{01}(\cos 0^\circ+\cos 120^\circ+\cos 240^\circ)+ j3\dot{E}_{01}(\sin 0^\circ+\sin 120^\circ+\sin 240^\circ)$$

or

$$\dot{E}_{01}+\dot{E}_{02}+\dot{E}_{03}=0 \qquad (9\text{–}2)$$

Therefore, the net voltage tending to cause a current to flow around the series delta (Δ) circuit is zero, and the connection is permissible because no current will flow around the closed circuit. Three wires or *leads* may be connected to the three junction points of the phases and may be used to supply either single-phase or three-phase loads.



(a) Non-Inductive 3-Phase Load *(b)* Inductive 3-Phase Load

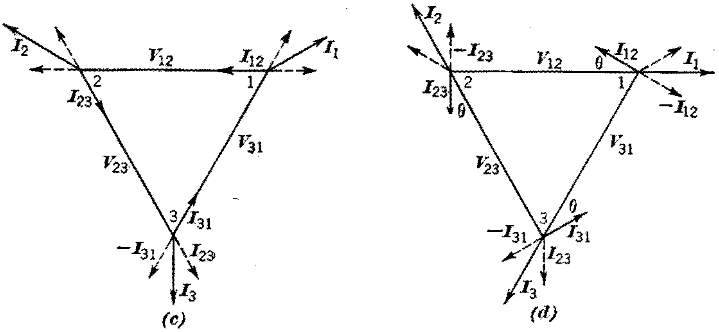
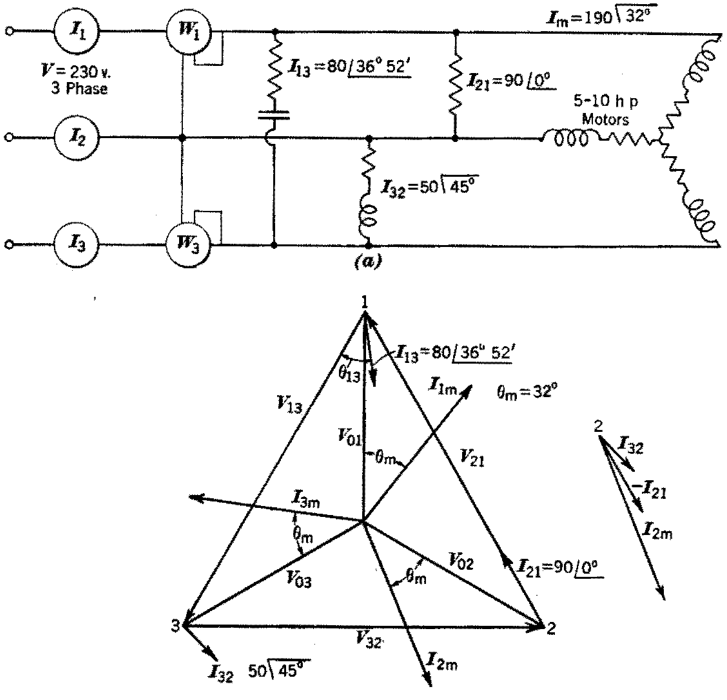


Fig. 9–3. Voltage and Current Relationships in Δ-Connected Systems

The Balanced Three-Phase Wye-Connected Load.—If three identical resistances R_1, R_2, and R_3 or three identical impedances Z_1, Z_2, and Z_3 are connected in wye, as in Fig. 9–2 (*a*) or 9–2 (*b*), and are then connected to a source of three-phase voltages, three equal currents will flow in the branches of the load circuit thus constructed. Voltage drops $V_{01} = I_1Z_1$, $V_{02} = I_2Z_2$, and $V_{03} = I_3Z_3$ will appear across the impedances. If the impedances are purely resistive, as in (*a*), the currents will be in phase with the voltage drops across the respective impedances, as in (*c*). If the impedances contain both resistances and inductances, as in (*b*), the currents will lag the respective voltage drops by the angle θ, as in (*d*), where θ is the characteristic impedance angle as has previously been explained for the usual single-phase circuit. The phase currents in the impedances are, of course, the line currents I_1, I_2, and I_3.

The Balanced Three-Phase Delta-Connected Load.—If a group of three identical resistances R_{12}, R_{23}, and R_{31} or three identical impedances Z_{12}, Z_{23}, and Z_{31} are connected in delta, as in Fig. 9–3 (*a*) or Fig. 9–3 (*b*), and are then connected to a source of three-phase voltages, three equal currents will flow in the three phase or branch impedances. If the three line-to-line voltages have the phase sequence V_{31}, V_{23}, and V_{12}, they will produce currents I_{31}, I_{23}, and I_{12}, respectively, in the three impedances. If the impedances are purely resistive, the currents are in phase with the voltage drops across the respective impedances, as in (*c*). The angle θ in (*d*) between any current and the voltage which produces it will be the characteristic angle of the impedance through which the current flows.



In every case in this diagram, the angle shown after a current is the angle by which that current leads or lags the voltage which produces it.

Fig. 9-4. Line Currents in Unbalanced Three-Phase Load

At a corner of the voltage delta, such as 3, I_{23} is positive to the corner and I_{31} is negative to the corner. The corner 3 of the voltage delta corresponds to line 3 on the associated circuit diagram. The line current at terminal 3 is made up of the currents of the two phase branches connected to it. Therefore, $I_3 = I_{23} + (-I_{31})$ or $I_{23} - I_{31}$. Inasmuch as the phases have been assumed to be of identical impedances, the angle between I_{23} and $-I_{31}$ is 60°. Therefore,

$$I_3 = 1.732\ I_{23}$$

or

$$I_3 = \sqrt{3}\ I_{23} \qquad (9\text{–}3)$$

Combined Delta and Wye Loads.—At its worst, a problem of calculating line currents may involve one or more delta loads, one or more wye loads, and one or more single-phase loads connected between one line and any other line, including the neutral line if there is one. Unbalanced wye-connected loads are extremely rare, and their solutions are relatively so complicated that they will be neglected entirely in the general case of unbalance in line currents. Every problem involving unbalanced delta-connected loads and miscellaneous single-phase loadings can be solved by the methods used with the balanced delta-connected load; but, inasmuch as currents at a delta point will almost never be just 60° apart in phase, the factor $\sqrt{3}$ cannot be used.

Example 9–1.—As indicated in Fig. 9–4 (*a*), a factory load consists of the following:

One department with seven 10-hp motors, 3-phase, 60-cycle, 230-volt, all of which together draw 190 amp per line lagging their respective line-to-neutral voltages by 58°.

One department with a lighting load of 90 amp at 100 per cent power factor between lines 1 and 2.

One department with a total load of 50 amp at 70.7 per cent power factor, lagging, connected between lines 2 and 3.

One department with a total load of 80 amp at 80 per cent power factor, leading, connected between lines 3 and 1.

Calculate the line currents I_1, I_2, and I_3 in lines 1, 2, and 3 with a phase sequence of E_{32}, E_{21}, E_{13}.

Solution.—The vector diagram is shown in Fig. 9–4 (*b*). The currents in line 1 may be combined as follows:

$$I_1 = I_{21}\underline{\backslash 120^\circ} + (-I_{13}\underline{\backslash 96^\circ 52'}) + I_{1m}\underline{/90^\circ - 32^\circ}$$

$$\begin{aligned} I_{21} &= 90 \cos 120^\circ + j90 \sin 120^\circ = -\ 45.00 + j77.94 \\ -I_{13} &= 80 \cos 96^\circ 52' + j80 \sin 96^\circ 52' = -\ 9.57 + j79.42 \\ I_{1m} &= 190 \cos 58^\circ + j190 \sin 58^\circ = \underline{+100.68 + j161.12} \\ & \qquad\qquad\qquad\qquad\qquad\qquad\qquad + 46.11 + j318.48 \end{aligned}$$

$$I_1 = 321.8\underline{/81^\circ 46'} \text{ amp}$$

For line 2, the computations follow:

$$I_2 = I_{32}\overline{\backslash 45^\circ} + (-I_{21}\overline{\backslash 60^\circ}) + I_{2m}\overline{\backslash 32^\circ + 30^\circ}$$

$$\begin{aligned} I_{32} &= 50 \cos(-45^\circ) - j50 \sin(-45^\circ) = +\ 35.35 - j35.35 \\ -I_{21} &= 90 \cos(-60^\circ) - j90 \sin(-60^\circ) = +\ 45.00 - j77.94 \\ I_{2m} &= 190 \cos(-62^\circ) - j190 \sin(-62^\circ) = \underline{+\ 89.18 - j167.75} \\ & \qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad + 170.33 - j281.04 \end{aligned}$$

$$I_2 = 328.6\overline{\backslash 58^\circ 47'} \text{ amp}$$

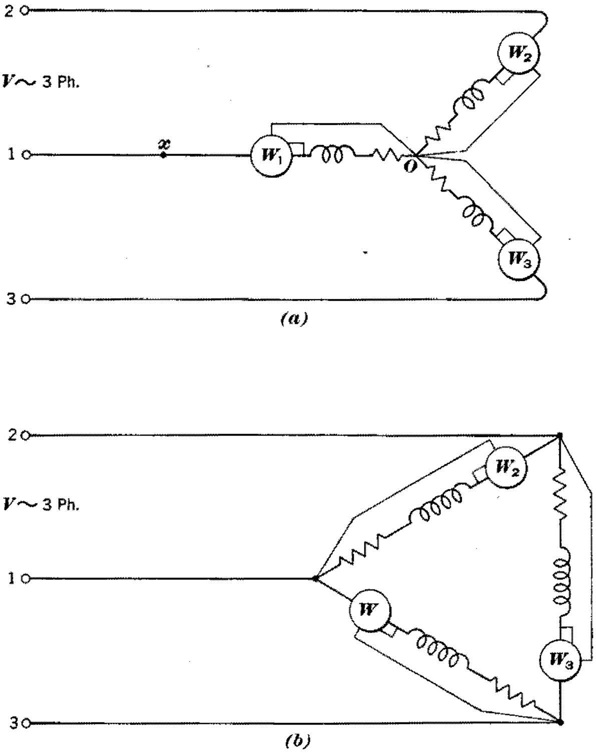
For line 3, the values are:

$$I_3 = I_{13}\overline{\backslash 83^\circ 8'} + (-I_{32}\underline{\backslash 135^\circ}) + I_{3m}\underline{\backslash 178^\circ}$$

$$\begin{aligned} I_{13} &= 80 \cos(-83^\circ 8') - j80 \sin(-83^\circ 8') = +\ 9.57 - j77.94 \\ -I_{32} &= 50 \cos 135^\circ + j50 \sin 135^\circ = -\ 35.35 + j35.35 \\ I_{3m} &= 190 \cos 178^\circ + j190 \sin 178^\circ = \underline{-189.88 + j6.63} \\ & \qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad -215.66 - j35.96 \end{aligned}$$

$$I_3 = 218.6\overline{/170^\circ 32'} \text{ amp}$$

If the phase sequence of the voltages applied to the load is reversed by reversing any two of the leads, then the currents in those lines will be reversed. In assembling the vectors for addition at any corner of the voltage triangle to find the corresponding line current, all current vectors associated with voltage vectors positive to the point must be taken as positive and all current vectors associated with voltage vectors negative to the point must be taken as negative (reversed).



(a)

(b)

Fig. 9–5. Measurement of 3-Phase Power by Three-Wattmeter Method

Power in Three-Phase Circuit by Three-Wattmeter Method.—The power in a three-phase system may be measured by placing a single-phase wattmeter in each phase and adding the three readings. This method is possible in a Δ system only if the phase units can be separated and in a Y-system only if the neutral or wye point can be reached. Fig. 9–5 shows the proper arrangement of wattmeters. In either (*a*) or (*b*),

$$P = P_1 + P_2 + P_3$$

$$P = V_{01} I_1 \cos \theta_1 + V_{02} I_2 \cos \theta_2 + V_{03} I_3 \cos \theta_3$$

In (*a*), when $\theta_1 = \theta_2 = \theta_3 = \theta$,

$$P = 3 V_{01} I_1 \cos \theta$$

By equation (9–1), in which V_{01} is used instead of V_{02},

$$V_{01} = \frac{V_{21}}{\sqrt{3}}$$

Therefore,

$$P = \frac{3V_{21}I_1 \cos\theta}{\sqrt{3}}$$

$$= \sqrt{3}VI \cos\theta$$

and

$$\cos\theta = \frac{P}{\sqrt{3}VI} \qquad (9\text{–}4)$$

This value is the power factor of the balanced system whether Δ-connected or Y-connected.

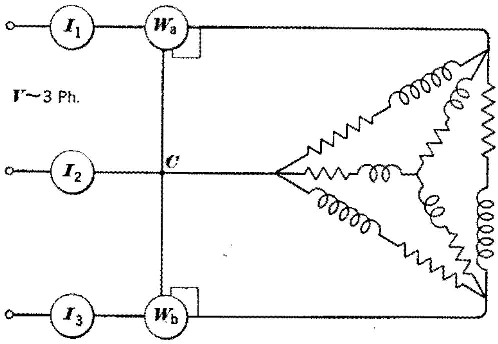


Fig. 9–6. Measurement of 3-Phase Power by Two-Wattmeter Method

Measurement of Three-Phase Power by Two-Wattmeter Method.—If in Fig. 9–5 (*a*) the three meter potential coil terminals at O be kept joined, but be removed from the neutral of the system, the readings of all wattmeters will be unchanged, because the wattmeter potential coils themselves form a balanced Y-connected circuit and so the voltage across every potential coil remains unchanged. This method of measurement is called the "floating neutral" method and is accurate on a three-phase three-wire or four-wire system regardless of power factor or load unbalance.

If, then, the junction of the potential leads be moved and connected to one of the line wires, as at x on line 1, the sum $P_1 + P_2 + P_3$ will be unchanged, although the power read from wattmeter W_1 will be zero. Thus, it is possible and feasible to measure three-wire, three-phase power in the circuit in Fig. 9–4 (*a*) by using only the two wattmeters W_1 and W_3. This is called the two-wattmeter method and may be used with convenience on any three-wire system, whether Y-connected or Δ-connected and whether balanced or unbalanced, as in Fig. 9–6.

A proof of the correctness of the two-wattmeter method in measuring balanced three-phase loads is as follows. For convenience Fig. 9–7 is drawn for a Y-connected circuit. In the vector diagram,

V_{01}, V_{02}, and V_{03} are phase voltages;
V_{12}, V_{23}, and V_{31} are line voltages;
θ = displacement angle between a current and its respective phase-to-neutral voltage.

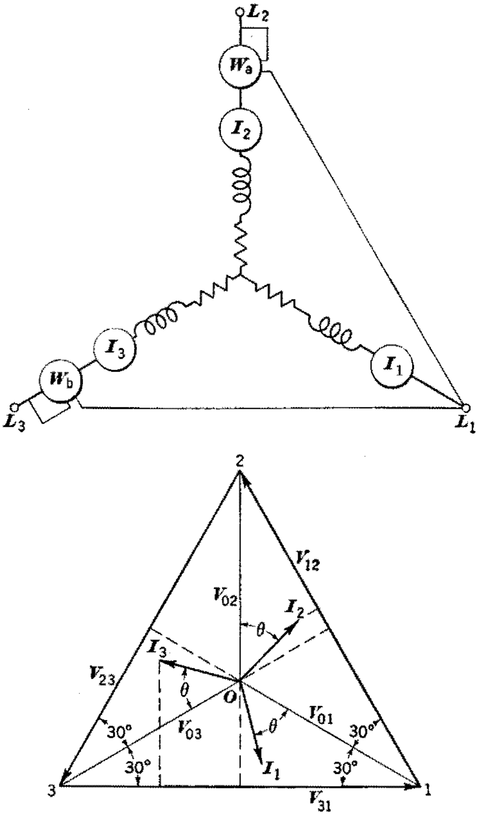


Fig. 9–7. Vector Diagram of Two-Wattmeter Method

By the three-wattmeter method,

$$P = P_1 + P_2 + P_3$$

or

$$P = V_{01} I_1 \cos\theta + V_{02} I_2 \cos\theta + V_{03} I_3 \cos\theta \qquad (a)$$

By the two-wattmeter method,

$$P = P_a + P_b$$

or

$$P = V_{12} I_2 \cos(30° + \theta) + V_{13} I_3 \cos(30° - \theta) \qquad (b)$$

Since the right-hand members of equations (*a*) and (*b*) must be equal, if both methods are to give the same results,

$$\begin{aligned} V_{01}I_1 \cos\theta + V_{02}I_2 \cos\theta + V_{03}I_3 \cos\theta = \\ V_{12}I_2 \cos(30^\circ + \theta) + V_{13}I_3 \cos(30^\circ - \theta) \end{aligned}$$

But $V_{01} = V_{02} = V_{03}$ and $I_1 = I_2 = I_3$; and $V_{12} = \sqrt{3}V_{01} = V_{31}$. Therefore,

$$3V_{01}I_1 \cos\theta = \sqrt{3}V_{01}I_1 [\cos(30^\circ + \theta) + \cos(30^\circ - \theta)]$$

Since this equation is an identity, it follows that

$$P_a + P_b = P_1 + P_2 + P_3$$

Interpretation of Wattmeter Readings in Two-Wattmeter Method.—If θ in Fig. 9–7 were greater than 60°, then the angle $(30^\circ + \theta)$ would be greater than 90° and its cosine would be negative. Hence, the reading of wattmeter W_a would be negative and the power P_a would be negative.

It should be noted that, for all cases when the current is lagging and when θ is greater than 0° and less than 90°, W_b will give a higher reading than W_a. If the currents were leading—rather than lagging—their respective phase voltages, W_a would be greater than W_b for values of θ between 0° and 90°.

A good method of determining whether the low meter reading on a balanced load should be taken as positive or negative is as follows: From the common line (at *C*, Fig. 9–6) remove the potential coil wire of the lower-reading wattmeter and touch it on the line containing the other wattmeter. If, then, the needle of the lower-reading wattmeter reverses, the reading is to be taken as negative. If the needle does not reverse, then the reading is to be taken as positive. Return the moved wire to its original position.

Example 9–2.—The meters in the line to a three-phase motor being tested indicate the following values:

$$\begin{array}{ll} V = 220 \text{ volts} & I = 17.6 \text{ amp (per line)} \\ W_a = 1288 & W_b = 3780 \end{array}$$

What are the power and the power factor?

Solution.—The power is

$$\begin{aligned} P &= W_a + W_b \\ &= 1288 + 3780 = 5068 \text{ watts} \end{aligned}$$

The power factor is, by equation (9–4),

$$\begin{aligned} \cos\theta &= \frac{P}{\sqrt{3}VI} \\ &= \frac{5068}{\sqrt{3} \times 220 + 17.6} = 0.758 \end{aligned}$$

Thus, Power factor = 75.8%

The power factor for a balanced load may also be found from the equation

$$\text{PF} = \cos \tan^{-1} \frac{W_b - W_a}{W_b + W_a} \sqrt{3} \tag{9–5}$$

Example 9–3.—Calculate the readings of wattmeters W_1 and W_3 in Fig. 9–4 (*a*) and the total power.

First Solution.—The wattmeter readings are:

$$\begin{aligned} W_1 &= \dot{V}_{21} \times \dot{I}_1 \\ &= 230\backslash\underline{120^\circ} \times 321.8\underline{/81^\circ 46'} \\ &= 74{,}014 \cos (120^\circ - 81^\circ 46') \\ &= 58{,}138 \\ W_3 &= \dot{V}_{23} \times \dot{I}_3 \\ &= 230\underline{/180^\circ} \times 218.6/\overline{170^\circ 32'} \\ &= -230 \times (-218.6 \cos 9^\circ 28') \\ &= 49{,}602 \end{aligned}$$

The total power is

$$W_1 + W_2 = 107{,}733 \text{ watts}$$

As a check,

$$\begin{aligned} P_{13} &= 230 \times 80 \cos 36^\circ 52' &&= 14{,}720 \\ P_{32} &= 230 \times 50 \cos 45^\circ &&= 8{,}130 \\ P_{21} &= 230 \times 90 \cos 0^\circ &&= 20{,}700 \\ P_3 &= 3 \times \frac{230}{\sqrt{3}} \times 190 \cos 32^\circ &&= \underline{64{,}183} \\ P_{\text{total}} &= && 107{,}733 \text{ watts} \end{aligned}$$

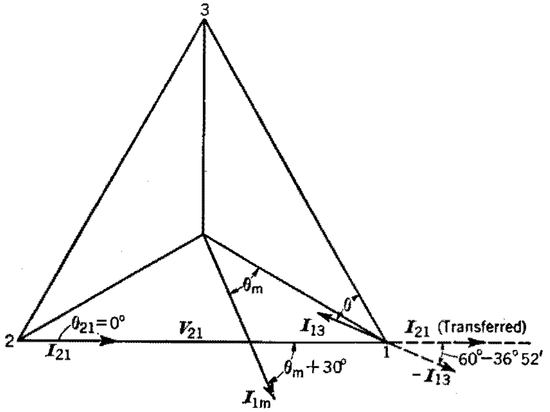


Fig. 9–8. Rotated Vector Diagram for Computing Reading of Wattmeter W_1 in Example 9–3

Second Solution.—In the calculation of power in problems of this type, when the actual values of line currents are not needed, some students run less risk of error in sign or position of current components if the power indicated by each meter is computed on a different reference axis, instead of being computed on one common axis. For example, the reading of wattmeter W_1 in Fig. 9–4 is the product of V_{21} (the voltage on the potential coil) and $I_1 \cos \alpha$, where α is the angular displacement of I_1 from V_{21}. There is an advantage, therefore, in taking the x axis of reference through V_{21} and also in rotating the vector diagram so that V_{21} is horizontal and runs from left to right as viewed by the student. The diagram will then appear as in Fig. 9–8. To solve for the reading of wattmeter W_3, the diagram would be used as in Fig. 9–4, with V_{23} along the x axis of reference. The simplified vector diagram for computing the reading of W_3 is shown in Fig. 9–9.

With the axis taken through V_{21}, as in Fig. 9–8, the reading of wattmeter W_1 can be found by computing only the in-phase or cosine components of current and then multiplying their sum by V_{21}. Thus,

$$I_1 = I_{21} + I_{1m} - I_{13}$$

$$\begin{array}{lr} I_{21} \cos \theta_{21} & = \quad 90.00 \\ I_{1m} \cos (\theta_m + 30^\circ) & = \quad 89.20 \\ -I_{13} \cos (60^\circ - 36^\circ 52') & = \quad \underline{73.57} \\ I_1 \cos \alpha & = 252.77 \end{array}$$

$$W_1 = V_{21} I_1 \cos \alpha = 230 \times 252.77 = 58{,}138$$

To calculate the reading of wattmeter W_3, the work follows:

$$I_3 = I_{13} + I_{3m} - I_{32}$$

$$\begin{array}{lr} I_{13} \cos (120^\circ - 36^\circ 52') & = + \quad 9.57 \\ I_{3m} \cos (180^\circ + 30^\circ - 32^\circ) & = -189.88 \\ -I_{32} \cos (180^\circ - 45^\circ) & = \underline{- \quad 35.35} \\ I_3 \cos \alpha & = -215.66 \end{array}$$

$$W_3 = -V_{32} \times I_3 \cos \alpha = 230 \times (-215.66) = 49{,}602$$

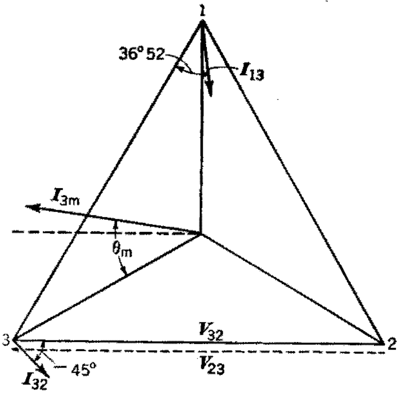


Fig. 9–9. Simplified Vector Diagram for Computing Reading of Wattmeter W_3 in Example 9–3

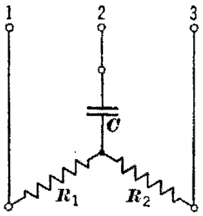


Fig. 9–10. Circuit for Phase Sequence Indicator.

It is important to note that the reading of the wattmeter W_3 is positive because its potential coil is connected across lines 3 and 2 in such a way that the vector $-V_{32}$ is used in the vector diagram. It must also be recognized that wattmeters W_1 and W_3 do not actually measure power, but produce two readings which, when added algebraically, give a sum which is equal to the power. The mathematical proof of this fact is given on page 96.

Phase Sequence Indicator.—The direction of rotation of a polyphase motor depends on the phase sequence of the voltage supply to which it is connected. It is sometimes necessary to know in advance the phase sequence of the power supply before the motor is started because reversed rotation might damage the load which the motor is to drive. Fig. 9–10 shows a wye-connected network which will indicate phase sequence. Resistors R_1 and R_2 are small neon-filled glow lamps described in Chapter 27. If lamp R_1 glows when the network is connected to the three-phase power lines 1, 2, and 3, the phase sequence is 3–2–1. If lamp R_2 glows, the phase sequence is 1–2–3.

Three-Phase Kv-a and Power Factor.—Power in a three-phase load is the sum of the powers in the three component parts of the load. Where the load is Y-connected, as in Fig. 9–5 (*a*), the power is the same whether the phases are balanced or unbalanced. Thus,

$$\begin{aligned} P &= W_1 + W_2 + W_3 \\ &= V_{01} I_1 \cos\theta_1 + V_{02} I_2 \cos\theta_2 + V_{03} I_3 \cos\theta_3 \end{aligned}$$

When the load is balanced,

$$V_{01} = V_{02} = V_{03}$$

$$I_1 = I_2 = I_3$$

and

$$\theta_1 = \theta_2 = \theta_3$$

Also,

$$V_{01} = \frac{V_{21}}{\sqrt{3}}$$

Therefore,

$$P = 3\left(\frac{V_{21}}{\sqrt{3}} I_1 \cos\theta_1\right)$$

or

$$P = \sqrt{3} VI \cos\theta \qquad (9\text{–}6)$$

Where the load is Δ-connected, as in Fig. 9–5 (*b*), the following relation holds whether the phases are balanced or unbalanced:

$$\begin{aligned} P &= W_1 + W_2 + W_3 \\ &= V_{13} I_{13} \cos\theta_{13} + V_{21} I_{21} \cos\theta_{21} + V_{32} I_{32} \cos\theta_{32} \end{aligned}$$

When the load is balanced,

$$V_{13} = V_{21} = V_{32}$$

$$I_{13} = I_{21} = I_{32}$$

and

$$\cos\theta_{13} = \cos\theta_{21} = \cos\theta_{32}$$

Also,

$$I_{13} = \frac{I_1}{\sqrt{3}}$$

Therefore,

$$P = 3\left(V_{13} \frac{I_1}{\sqrt{3}} \cos\theta_{13}\right)$$

$$P = \sqrt{3} VI \cos\theta \qquad (9\text{–}7)$$

Similarly, it may be shown that the number of reactive volt-amperes (vars, reactive power, fictitious power) in a three-phase load is the algebraic sum of the reactive volt-amperes of the component parts of the load.

Volt-amperes of a capacitive portion of a load must be taken opposite in sign to volt-amperes of an inductive portion in the algebraic sum. When the load is balanced, the number of reactive volt-amperes is

$$va_r = \sqrt{3} VI \sin \theta \tag{9-8}$$

The total number of volt-amperes (apparent power) in a three-phase load is the square root of the sum of the squares of the power and the reactive volt-amperes. The method of combining the several kinds of volt-amperes graphically is shown in Fig. 9–11.

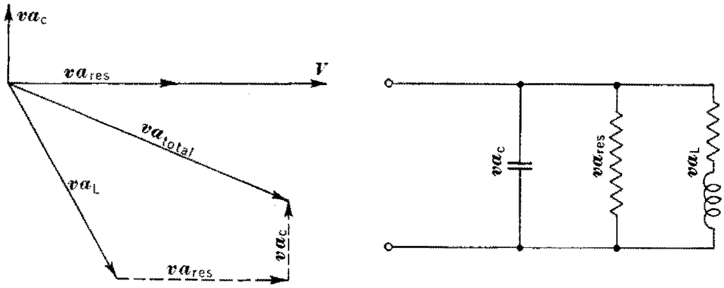


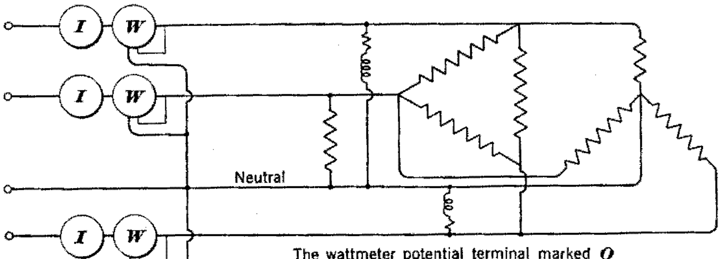
Fig. 9–11. **Volt-Ampere Diagram**

The power factor of a three-phase load is the ratio of the power to the total volt-amperes. The foregoing definitions are predicated on the assumption of sinusoidal currents and voltages only.

For some purposes, such as the calculation of condenser capacity required for power-factor correction (see Chapter 25), the three-phase quantities are used in an equivalent single-phase diagram.

The Three-Phase, Four-Wire System.—When both single-phase load and three-phase load must be taken from the same power system with probability of considerable load unbalance, it is advantageous to add a fourth wire connected to the neutral of the power supply. The polyphase loads are connected to the three line wires, and the single-phase loads are connected between line and neutral on the various phases. This system is particularly advantageous in business sections of large cities where the load per square mile is very high. This type of distribution is called a grid network, because the layout of the system somewhat resembles a gridiron.

In low-voltage grid networks the most common power voltage is 208 volts and the most common lamp voltage is 120 volts. If the line-to-line voltage in the system were 220 volts, the line-to-neutral voltage would be $\frac{220}{\sqrt{3}} = 127$ volts. Lamps are not commonly available for 127 volts, and a compromise voltage must be used. It is common practice to reduce the more usual line-to-line voltage of 220 volts to 208 volts, which gives 120 volts line-to-neutral for lamps.



The wattmeter potential terminal marked *O* should be the one that is connected to the line which passes through the meter's current coil

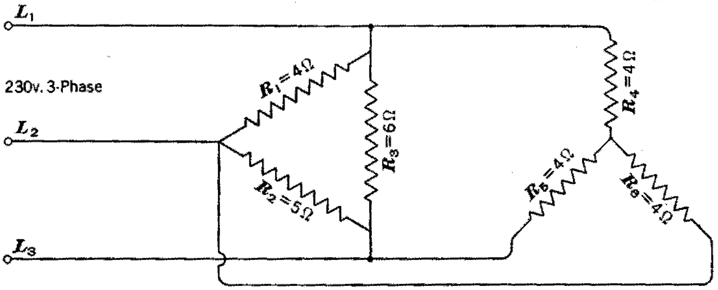
Fig. 9–12. Arrangement of Meters in Four-Wire System

Choice of Methods of Measuring Three-Phase Power.—In any three-phase, three-wire circuit, either the three-wattmeter method or the two-wattmeter method may be used, regardless of the degree of load unbalance. If the system has a fourth wire leading to the neutral point of the generator, the system is a four-wire system. Single-phase loads may be connected at random between the neutral wire and the other wires of the line. Three wattmeters connected as in Fig. 9–12 are required to measure the power in a four-wire system. *The number of wattmeters required to measure the power in a system is always one less than the number of wires in the system.*

PROBLEMS

1. A group of Y-connected resistors is connected in parallel with a group of Δ-connected resistors to a 230-volt line, as shown in the accompanying diagram. Calculate the line currents.

Ans. $I_{L1} = 116.5$ *amp*
$I_{L2} = 123.6$ *amp*
$I_{L3} = 106.3$ *amp*



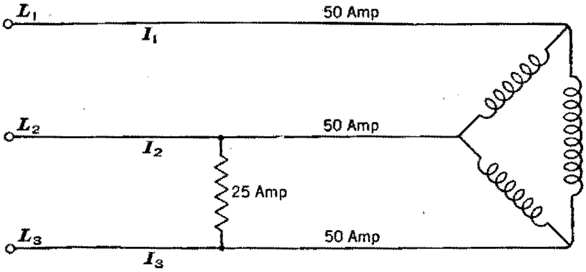
Problem 9–1

2. As indicated in the diagram on page 103, an induction motor drawing 50 amp at 220 volts with 50% power factor, lagging, from a three-phase line has connected in parallel with it between two line wires a single-phase electric furnace which draws from

the line 25 amp at 100% power factor. Calculate the total active power. Calculate the current in each line. *Ans.* $I_{L1} = 50$ *amp* $I_{L2} = 72$ *amp* $I_{L3} = 56$ *amp*

3. If, in Problem 2, the motor power only be measured by the two-wattmeter method, what would be the reading of each wattmeter?

4. By use of a given size of wire, a load of 1000 kilowatts can be delivered over a two-wire single-phase line with a loss of 750 watts per wire. How much power could be delivered over a three-phase line through the same distance with the same weight of copper and with the same power loss in the line? *Ans. 1155 kw*



Problem 9–2

5. A balanced three-phase, Y-connected load of three single-phase impedances each of which is $5+j6$ ohms is connected to a 230-volt line. Determine the line current and the power in the load. Draw the vector diagram.

6. Three impedances $Z_{AB} = 4+j5$, $Z_{BC} = 5-j6$, and $Z_{CA} = 10+j0$ are connected in delta across a 230-volt three-phase line whose voltage sequence is V_{AB}, V_{CA}, V_{BC}. Calculate the line currents and the total power in the load. Draw the vector diagram of voltages and currents.

7. Calculate the line currents in Problem 6 for the condition of reversed phase rotation.

8. A balanced three-phase load at 60% power factor is being metered by the two-wattmeter method. The total power is 4000 watts. What is the power indicated by each meter?

9. Calculate the reactive volt-amperes in Problem 8.

10. A balanced load of three hundred 75-watt lamps on a three-phase system is measured by the two-wattmeter method. What will be the reading on each wattmeter?

11. An induction motor draws from a three-phase line 5000 watts at 70% power factor. If a single-phase wattmeter is connected with its current coil in line *A* and with its potential coil across lines *B* and *C*, how much will be the reading of the wattmeter and what will it measure?

12. A three-wire line of No. 1 conductors 200 ft long with Type-SN insulation (see Appendix) supplies a 120/240-volt single-phase load with maximum permissible current. What per cent of load increase would be permissible if the system voltage were changed to three-phase 240 volts at the load? *Ans. 73.2%*

13. If three 5-ohm resistors connected in wye draw 10 amp per phase from a three-phase line, how much power will they draw from the same line if they are connected in delta? *Ans. 4500 watts*

CHAPTER 10

The Direct-Current Machine

Types of Direct-Current Machines.—Direct current is produced commercially by application of the principle pertaining to generation of electromotive force that was discovered by Faraday and is described on pages 3 and 4. A machine that is used for production of direct current is called a direct-current generator or dynamo.

Direct-current motors, which utilize direct current to drive other machines, operate on the principle laid down by Lenz in his law of force on a conductor carrying current in a magnetic field, as described in Chapter 11. The direct-current motor and the direct-current generator are alike in construction and differ only slightly in their design proportions. The use of the direct-current machine as a generator will first be discussed, and then its use as a motor will be considered.

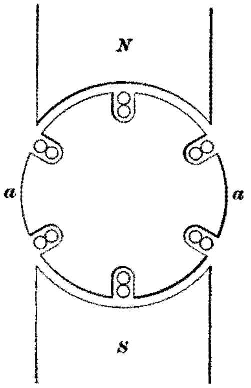
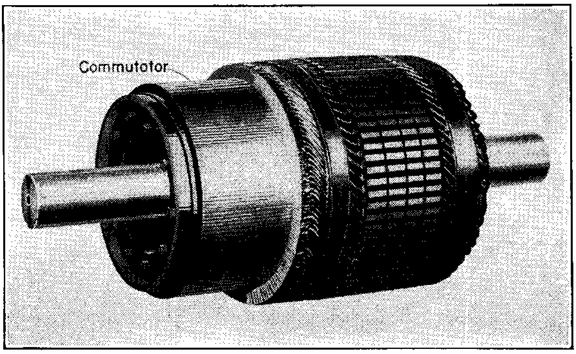


Fig. 10–1. Rudimentary Field and Armature Elements

Armature of Direct-Current Generator.—In direct-current generators, the conductors are laid in slots on the periphery of a laminated soft-steel cylinder called the armature core. Fig. 10–1 shows the rudimentary magnetic field and the armature elements, and Fig. 10–2 shows an armature. In all except very large generators or those designed for low voltage, each slot holds many conductors wound in long narrow coils. For simplicity, all coil sides in wiring diagrams will be shown as single conductors. See the elemental coils on the armatures of Figs. 10–6 and 10–8.

In modern generators, all conductors are not located on the same circle as in Fig. 9–1 (*a*), but the conductors are equally divided between two concentric circles, forming a double-layer winding as in Fig. 10–1. Inasmuch as the emf generated in a conductor of a direct-current generator need not be sinusoidal, the surfaces of the poles are made to conform almost to the circular contour of the armature, in order that the magnetic circuit may be more easily magnetized.

The Commutator.—In order that the alternating emf in each conductor on a rotating armature core may appear at the terminals as unidirectional voltage, the connections of every conductor to the external circuit must be reversed at every point in the revolution where its generated emf is zero, as at *a* in Fig. 10–1. To accomplish this reversal, connecting leads are brought out to the bars of a *commutator*. The commutator consists of a bundle of wedge-shaped copper segments assembled in cylindrical form and mounted on the armature shaft, as shown in Fig. 10–2, but insulated from one another. The details of construction of the commutator are indicated in Fig. 10–3.



(*Courtesy Fairbanks, Morse & Co.*)

Fig. 10–2. Direct-Current Motor or Generator Armature

The Elementary Generator.—Fig. 10–4 shows an elemental generator in which two conductors with a back connector form a loop terminating at the two segments of a reversing switch or commutator. When the conductors reach a position *a* midway between the pole tips, they will cut no flux and hence generate no electromotive force. At that same instant, each of the brushes which connect the loop to the outside circuit will change contact from one commutator segment to the opposite segment, thus reversing the connection of the coil to the outside circuit. Inasmuch as the direction of the electromotive force generated by each conductor as it re-enters the magnetic field will be opposite to the direction it had before

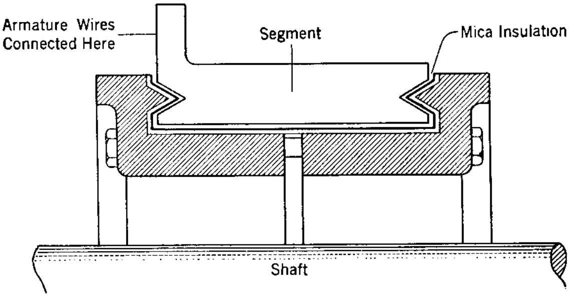


Fig. 10–3. Details of Commutator

the midpoint was reached, the direction of the electromotive force at the brushes relative to the outside circuit will be the same as before. Hence, the voltage appearing at the terminals of the generator will be approximately as shown in Fig. 10–5.

Brushes.—The brushes of a generator ride on the commutator and connect the armature to the external circuit. They are so located as to connect directly to conductors which are approximately midway between poles.

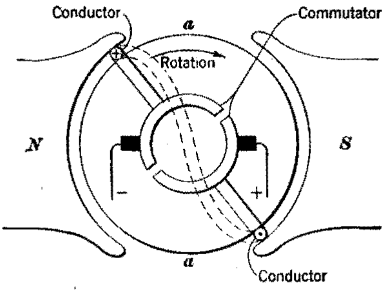
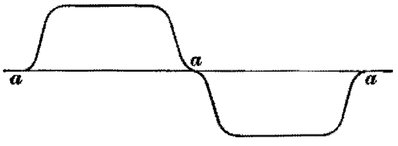
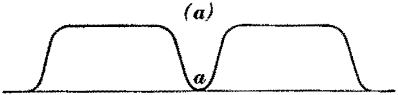


Fig. 10–4. Elementary Generator

Windings.—A direct-current generator may have any even number of poles. The number of poles used is a matter of design economics and depends chiefly on the speed and power output for which the generator is intended. A wiring diagram of a four-pole generator is shown in Fig. 10–6. In a two-pole generator the poles are 180 degrees apart and the emf of a conductor passes through one cycle of 360 degrees in every revolution or for every two poles passed. In a four-pole generator two cycles of emf are generated in every revolution; and, therefore, 720 *electrical degrees* are passed through in every 360 mechanical degrees. In any generator there are $\frac{P}{2}$ electrical degrees in a mechanical degree, where P is the number of field poles. A conductor is said to have passed through 360 electrical degrees when it has experienced one complete cycle of generated emf.



Electromotive Force Generated in Conductors
(a)

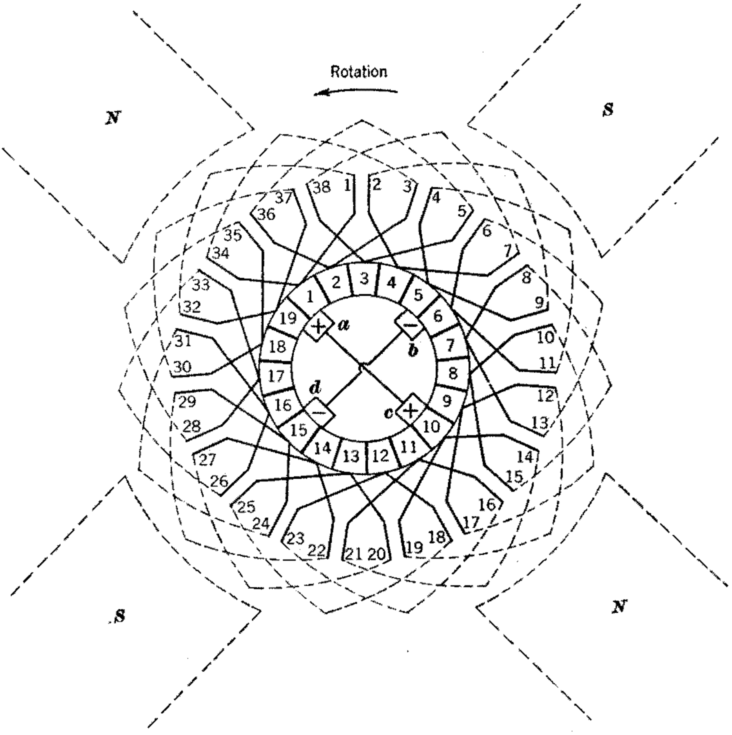


Voltage at Terminals of Generator
(b)

Fig. 10–5. Voltage in Direct-Current Generator

In Fig. 10–6 the active parts of the conductors lying in the armature slots, as 1, 2, 3, 4, etc., are shown radially for convenience. The end connections shown dotted are on the back end of the armature. Conductors 1 and 2, 3 and 4, etc., lie in the bottom and top, respectively, of one slot. The brushes *a*, *b*, *c*, and *d* are shown on the inside of the commutator for convenience.

Lap Winding.—The winding in Fig. 10–6 is completely closed with all conductors in series, as in the Δ connection described on page 90. In this case of many conductors distributed around the periphery of the armature, it may be shown that, as in the Δ connection, the instantaneous algebraic sum of all emf's is zero. Hence, no current will flow *around* the winding. By Fleming's right-hand rule, the emf generated in each conductor under a south pole will be *down*, while the emf in any conductor under a north pole will be *up*. Brush *a* is said to be positive because current would leave the commutator at that point if an external circuit were available. Similarly, it may be shown that brush *c* is positive and at the same potential as brush *a*. Brushes *a* and *c* may be connected together and be used as the positive terminal of the generator. Brushes *b* and *d* are negative and may likewise be connected together to form the negative terminal of the generator. The poles alternate in polarity in sequence around the armature, and the brushes likewise alternate in sign around the commutator. If the winding in Fig. 10–6 is followed, it will be seen that the path laps back upon itself repeatedly. The armature is therefore said to be *lap wound*. Fig.

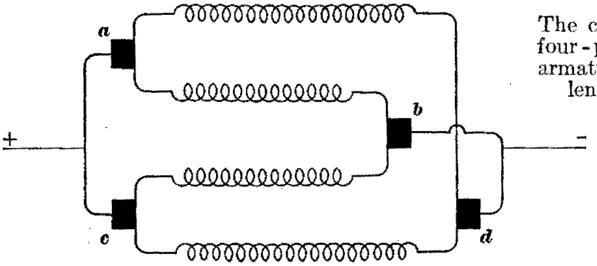


There are as many paths through the winding as there are poles.

Fig. 10–6. Lap Winding of Four-Pole Generator

10–7 shows a resistive circuit equivalent to that of a lap-wound armature for a four-pole generator.

Each brush must be wide enough to make contact with at least two segments at all times. Therefore, at least one coil is always short-circuited by every brush. These coils are said to be *under commutation* because, when the generator is loaded, the current in them is reversed in the interpolar spaces. The brushes are always so placed that the conductors of the commutated coil are in the interpolar regions. They therefore cut no main pole flux and generate no emf, except as described under the subject of interpoles on page 122. Conductors under commutation contribute nothing to the generator's terminal voltage. Fig. 10–10 is a *developed* lap winding of a four-pole generator with 25 armature slots.



The circuit through a four-pole lap-wound armature is equivalent to this one.

Fig. 10–7. Resistive Circuit Equivalent to That of a Four-Pole Lap-Wound Armature

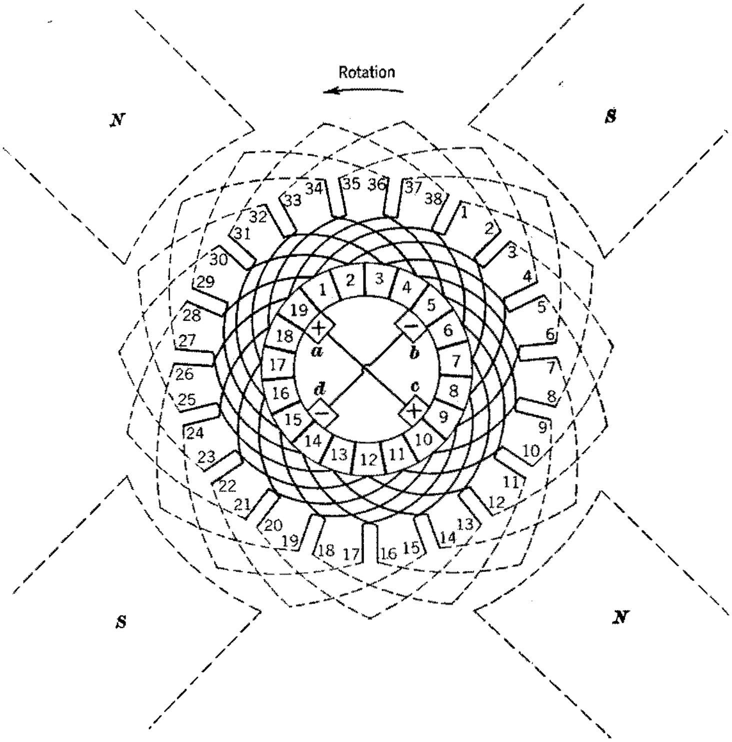
Wave Winding.—In Fig. 10–8 is shown another type of winding on an armature core similar to that of Fig. 10–6. This winding proceeds in the same direction many times around the armature and closes on itself. This type of winding is called a *wave* winding. In Fig. 10–9 is shown a resistive circuit equivalent to that of a wave-wound armature for a four-pole generator. Fig. 10–11 shows a developed wave winding on an armature similar to that used in Fig. 10–10. The designer's choice between the wave winding and the lap winding is determined by the voltage, speed, and kilowatt capacity of the generator. For a given speed, number of poles, and number of armature conductors, the wave winding gives a higher voltage than a lap winding for the following reason: In a wave winding (simplex)[1] half of the conductors are in series between brushes, whereas in a lap winding (simplex) the number of conductors in series between brushes is equal to the total number of conductors divided by the number of poles.

In a lap-wound armature, two paths in parallel deliver current to each positive brush. There are as many brushes as there are poles, and so a lap winding always has as many paths in parallel as there are poles. With

---

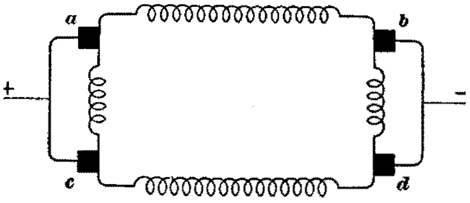
[1] For complex and multiple re-entrant windings and special windings, see *Principles of Direct-Current Machines*, by A. S. Langsdorf, McGraw-Hill Book Company.

a wave-wound armature there are usually as many brushes as there are poles, although there are *only two paths* through the armature irrespective of the number of poles. Inspection of Fig. 10–8 will verify the foregoing statement for the four-pole generator. In Fig. 10–8 segment 5 touching negative brush *b* is directly connected to segment 14 touching negative brush *d* through conductors 9 and 16. The coil 9–16 is then under commutation at the point shown and is contributing nothing to the generator's terminal voltage.



There are only two paths, regardless of the number of poles.

Fig. 10–8. Wave Winding of Four-Pole Generator



The circuit through a wave-wound armature is equivalent to this one, irrespective of the number of poles.

Fig. 10–9. Resistive Circuit Equivalent to That of a Four-Pole Wave-Wound Armature

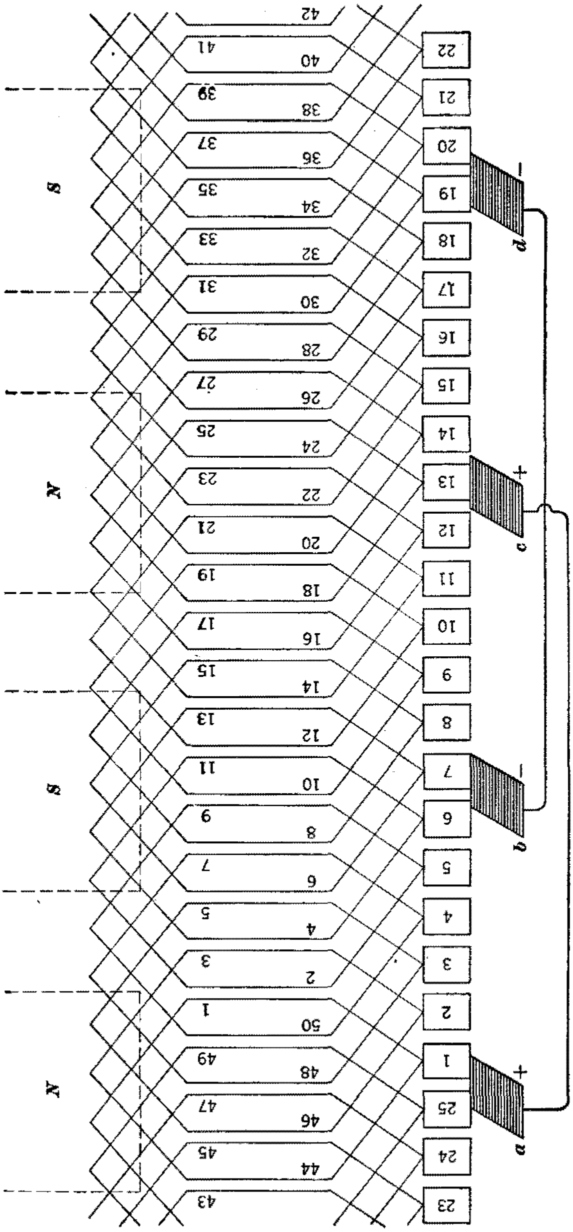


Fig. 10–10. Developed Lap Winding

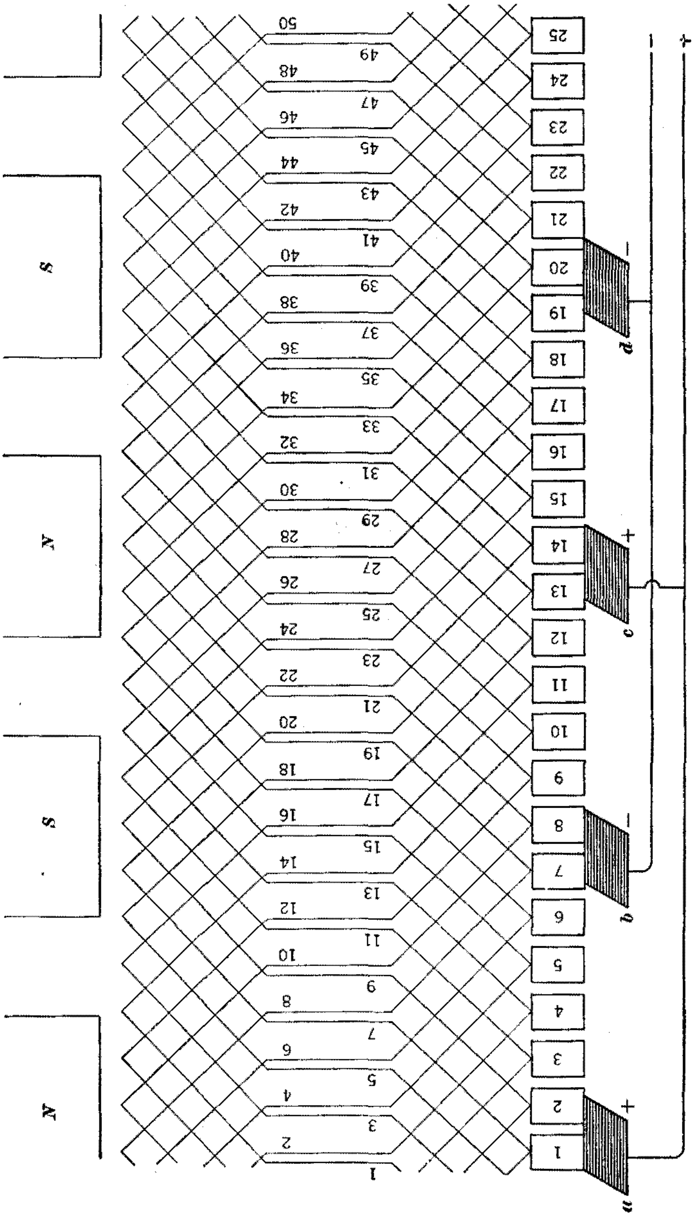
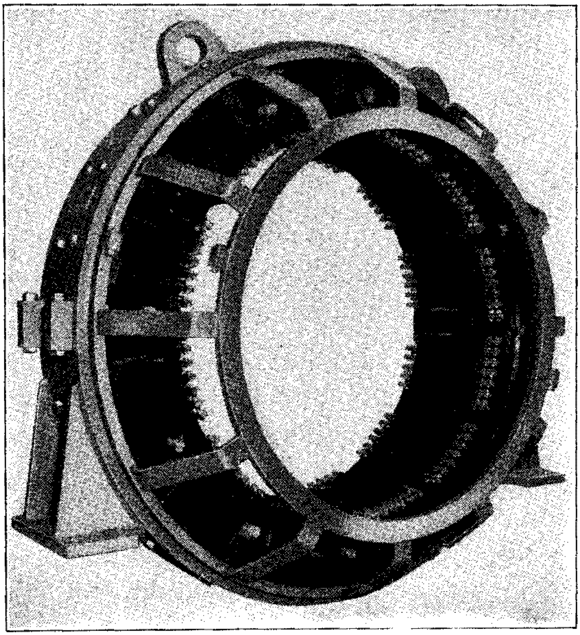


Fig. 10–11. Developed Wave Winding

Field Structure.—In most direct-current generators the frame is used as part of the magnetic circuit. Fig. 10–12 shows a common type of construction. The pole pieces are usually built up of rather thick ($\frac{1}{16}''$) laminations and are either cast in the frame or bolted to it. The bolted type of construction permits removal, for repair, of a pole piece with its exciting coil without disturbing other parts of the generator. The exciting coils on the pole pieces are connected in series in such manner that the current in them produces north and south poles alternately around the frame, as in Figs. 10–6 and 10–8.



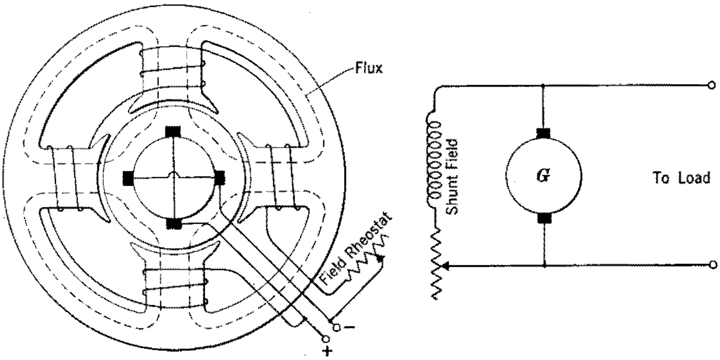
(Courtesy General Electric Co.)

Note the arc-welded construction, the interpoles, and the compensating windings embedded in the pole faces.

Fig. 10–12. Direct-Current Generator Frame

Field Excitation of Shunt Generator.—The field poles may be magnetized or *excited* by current from any suitable d-c source, but the current is usually taken from the generator itself. A generator whose field-winding current is obtained from its own armature is said to be *self-excited.* This may be done by using field coils of many turns of small wire connected in series with a control rheostat across the generator terminals (called shunt connected). The series type of connection in which the field coils are in series with the armature is unsuited to most applications and will be discussed later. The shunt connection is so called because the armature supplies current to its field circuit in *shunt* with the load circuit if there is one.

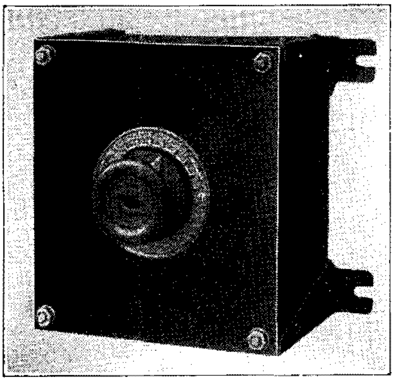
The circuit diagram of Fig. 10–13 shows the connections of a self-excited shunt generator with field rheostat. A field rheostat is illustrated in Fig. 10–14. The field rheostat is used to vary the terminal voltage within limits. The field circuit has many turns of small wire, and its resistance is high enough to permit its safe connection to the generator at rated voltage without protective resistance.



The field circuit is connected directly across the armature.

Fig. 10–13. Diagram of Self-Excited Shunt Generator

Building up Voltage of a Self-Excited Generator.—When a generator is driven at constant speed without its shunt circuit being closed, a very small *residual* voltage (1 to 5 volts) appears at its terminals because of the residual magnetism remaining in the iron. When the field circuit is properly connected, the residual voltage forces a small exciting current through the field circuit and thereby increases the field strength. Because of the increased field strength, the generated voltage increases. This reciprocal action continues until a point of stability is reached at which the flux produced by the current in the field is just sufficient to generate the voltage required to produce the field current. Most modern generators are so designed that, with no resistance in series with the field,



(Courtesy Cutler-Hammer, Inc.)

Fig. 10–14. Shunt Field Rheostat

the voltage will rise to about 125 per cent of rated value. This condition is represented by the point *a* in Fig. 10–15. The straight line *Oa* is called a *field resistance line* because its slope $\frac{V}{I_f}$ is equal to R_f, the resistance of the field circuit including the field rheostat. After the voltage has built up, it may be adjusted to any desired value between *a* and *c* by increasing the resistance of the field circuit by means of its rheostat. Generators are usually designed so that the rated voltage is generated at the point *b* or at somewhat higher field current. For every value of field-circuit resistance, the resistance line will have a particular slope, such as *Od* or *Ob*. If the resistance is made so high that the slope of the resistance line, as *Oe*, is equal to or greater than that of the lower straight part of the magnetization curve, the voltage of the generator will collapse and will drop to the residual value *O′*. A generator may fail to build up for any of the following reasons:

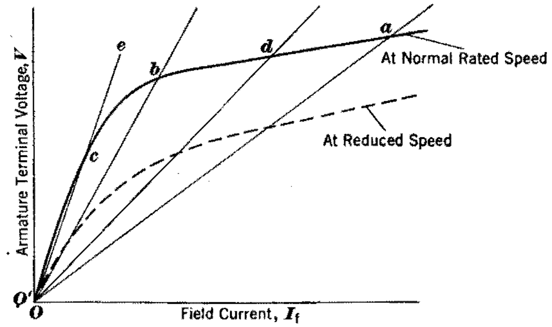


Fig. 10–15. Generator Self-Excitation Curve

1. Field-circuit resistance too high; including open circuit.
2. Speed too low.
3. Residual magnetism lost.
4. Direction of rotation incorrect.
5. Generator terminals connected to external circuit of too low resistance.
6. Shunt-field terminals reversed.

If the generator speed be reduced, the magnetization curve will be reduced in height proportionately, as shown by the dash curve of Fig. 10–15. Whereas at rated speed the generator would build up when the field resistance line is *Ob*, this line is too steep to permit build-up at the reduced speed. If the direction of rotation is incorrect, the residual voltage will force current through the field coils in a direction to demagnetize the field, and build-up cannot occur. If the generator is connected to a load circuit of too low resistance, the load-circuit current due to residual voltage may, by its magnetizing action in the armature, prevent build-up. Residual magnetism may be restored by connecting the field circuit to any suitable source of direct current.

Calculation of Electromotive Force in a Generator.—Whether an armature winding is lap or wave, the active conductors lie in two or more parallel paths. By active conductors are meant those which are not under commutation but are actively cutting flux and so contributing to the useful emf of the generator. The conductors in every path are in series, and the sum of the electromotive forces generated in them determines the emf appearing at the generator terminals at no load.

Let ϕ = the total lines of flux per pole;
Z = the total number of active conductors.

Then the flux cut per second by one conductor is

$$\phi \times \text{poles} \times \frac{\text{rpm}}{60}$$

The average emf generated in one conductor is

$$\phi \times \text{poles} \times \frac{\text{rpm}}{60} \times 10^{-8} \text{ volts}$$

The number of active conductors per path is Z divided by the number of parallel paths, and the total emf generated by the machine is

$$E_g = \phi \times \text{poles} \times \frac{\text{rpm}}{60} \times 10^{-8} \times \frac{Z}{\text{parallel paths}} \tag{10–1}$$

Example 10–1.—An 8-pole, lap-wound generator with 800 active conductors is excited to 8×10^6 lines of flux per pole and is driven at 600 rpm. Calculate its generated emf.

Solution.—This generator, being lap wound, has 8 parallel paths in its armature. Hence, the total emf generated is

$$E_g = \frac{8 \times 10^6 \times 8 \times 600 \times 800}{60 \times 10^8 \times 8}$$
$$= 640 \text{ volts}$$

In a generator the conductors lie in slots which present a high reluctance to the main flux, and the main flux appears almost entirely in the teeth. It might seem then that the conductors shielded from flux by the teeth would cut flux at a low or ineffective rate. One theory proposed is that the flux lines "snap" across the slot from tooth to tooth as the armature rotates. It is properly more logical to consider the turn in the coil rather than a single conductor of that turn. When the turn has its sides in the neutral zones between poles, it encloses all of the flux of a pole and generates no voltage because there is no rate of change of flux. When the turn has its sides under the middles of adjacent poles, it encloses no flux but the rate of change of flux is high and the voltage generated is $e = \frac{N}{10^8}\frac{d\phi}{dt}$.

Performance of a Shunt Generator.—Not all of the voltage generated in a loaded generator is available at the terminals because of the potential drop in the resistance of the armature, brushes, leads, and contacts between the brushes and the commutator. The total armature current I_a is the sum of the load current and the field current, and

$$V_1 = E_g - I_aR_a \qquad (10\text{–}2)$$

where V_1 = terminal voltage;
E_g = generated voltage;
I_aR_a = voltage drop through the resistance of the armature circuit.

If the exciting current of the generator is obtained from any source other than its own armature, it is said to be *separately excited.* Separate excitation is necessary when the armature voltage is not suitable for excitation, as in high-voltage generators and arc-welding generators. Curve (*a*) of Fig. 10–16 shows a typical *external-voltage* characteristic curve of a separately excited generator. Curve (*c*) shows the external characteristic of the same generator plotted against external or load current, when self-excited. For curve (*b*) and *total* characteristic is plotted against I_a, which includes both the field current and the load current.

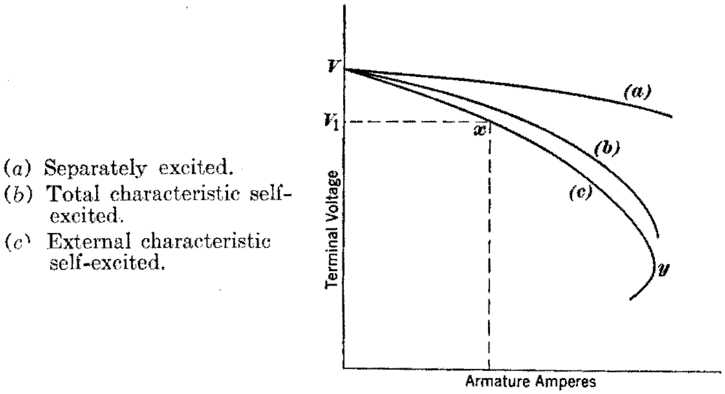


Fig. 10–16. **Shunt-Generator Characteristics**

Curve (*a*) is not a straight line for the following reason: In addition to the I_aR_a voltage drop in the armature circuit, there is some voltage decrease because the magnetomotive force of the armature caused by its own current weakens somewhat the main field flux. Curves (*b*) and (*c*) are much lower than (*a*) because, as the terminal voltage drops because of I_aR_a drop, the voltage applied to the field decreases and reduces the excitation. Because of this cumulative action, a point is reached, as at y in Fig. 10–16, beyond which a further attempt to increase the load current will result in collapse of the terminal voltage. This action determines one of the limits of maximum *momentary* power output of any self-excited generator.

Other Limits of Power Output.—In addition to collapse of terminal voltage, the permissible momentary power output of a generator is limited by destructive sparking between the commutator and the brushes. The maximum *continuous* power output is determined by the maximum permissible temperature rise. That load on a generator which, when carried continuously, produces the maximum permissible temperature rise is called *full load.*

Voltage Regulation.—The change in the terminal voltage of a generator when its load is changed from full load to no load while the speed remains constant is called its *voltage regulation.* Regulation is usually expressed as a per cent of full-load voltage. For example, if a generator has a terminal voltage of 230 volts at full load and 250 volts at no load, its regulation is

$$\frac{250-230}{230}=0.087 \text{ or } 8.7 \text{ per cent}$$

Operation at Voltages Above or Below Rated Voltage.—The voltage at which a generator is designed to operate is always indicated on the name plate. At less than rated voltage, but at rated speed, the generator operates nearer the knee of the saturation curve and has a regulation greater (poorer) than normal. The armature reaction described on page 120 affects the weaker main field more than it would affect a field of normal strength and tends to cause still greater voltage regulation. Because the current is limited to the rated value to avoid excessive heating, the full rated power output cannot be obtained at voltages much below normal.

When a generator is operated at a voltage above the rated value, but at rated speed, the regulation is less (better) than normal. The higher voltage permits a power output somewhat greater than normal. Increased heating because of increased iron losses in the armature core usually limits the permissible armature current to a value less than the rated value, and hence the possible advantage is slight. The shunt field coils of most generators will not overheat when carrying the maximum current possible with self-excitation. Most generators are guaranteed to deliver their rated kilowatt output at any voltage between 90 and 110 per cent of the rated voltage.

Operation of Speeds Above or Below Rated Speed.—When a shunt generator is operated at reduced speed, but with rated voltage, the regulation is better than normal because of the greater saturation required. If self-excited, a generator may not be capable of producing rated voltage at speeds less than 90 per cent of rated speed. At reduced speed the voltage generated per ampere of field excitation is less, and the effect on the maximum voltage is almost the same as an increase in shunt field resistance. Because of reduced ventilation at reduced speed, the temperature rise for a given armature current will be greater.

When a shunt generator is driven at higher than rated speed, but at rated voltage, the regulation is somewhat poorer than at rated speed because of operation lower down on the saturation curve. Also a change in the speed of the prime mover affects the terminal voltage more at higher speeds. It is not advisable to operate a generator at more than 15 per cent above rated speed because of dangerous centrifugal stresses, especially where the armature coils are held in their slots by band wires only.

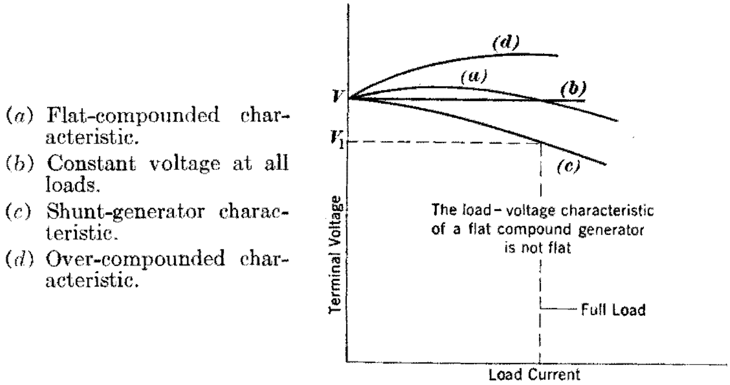


Fig. 10–17. Compound-Generator Characteristic Curves

The Compound Generator.—Curve (*c*) of Fig. 10–16 corresponds to the condition of fixed field-circuit resistance in a shunt generator adjusted to give rated voltage V_1 at rated load. If the no-load voltage had been set at V, as in Fig. 10–17, the full-load voltage for operation indicated by curve (*c*) would have been V_1, which is much too low. The voltage may be kept constant at all loads, as indicated by curve (*b*), by adjustments of the field rheostat. The tendency for the voltage to fall when the load current is increased is then just balanced by the increased excitation accomplished by adjustment of the rheostat.

A performance somewhat similar to that indicated by curve (*b*) may be obtained automatically by causing the load current to flow through a few turns of heavy wire on each field pole. The *series-field* exciting ampere-turns thus added to the shunt-field exciting ampere-turns increase the total ampere-turns in direct proportion to the load current. If there were no magnetic leakage and no armature reaction (see page 120), and the saturation curve of iron were a straight line, the curve (*b*) could be almost exactly realized. However, the actual performance will be more like that indicated by curve (*a*). Such a generator with both shunt-field and series-field windings is called a *compound generator*. A wiring diagram for a compound generator is shown in Fig. 10–18.

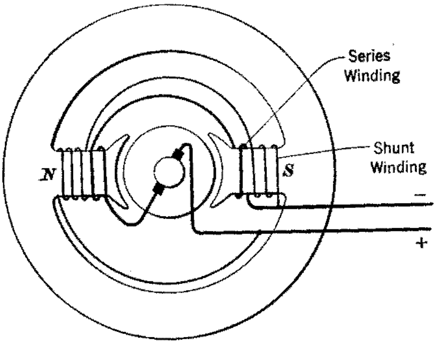
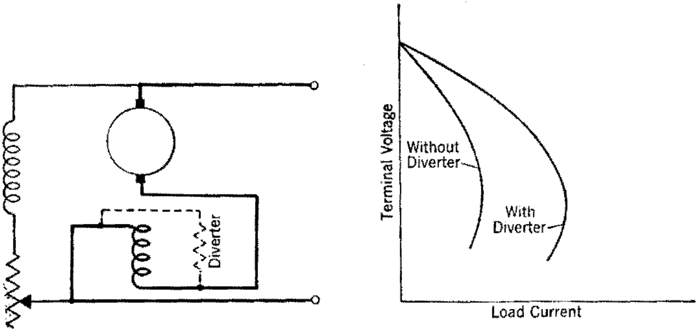


Fig. 10–18. Wiring Diagram of Compound Generator

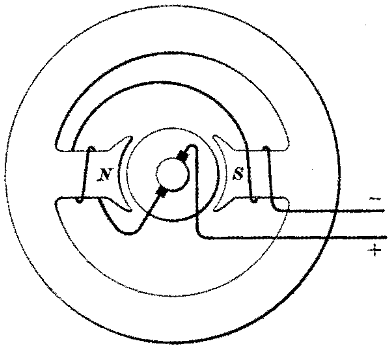
When the series-field ampere-turns *aid* the shunt-field ampere-turns, as just described, the machine is called a *cumulative compound generator.* Such generators are used to supply most d-c distribution systems. They may be *over-compounded* to compensate for voltage lost in IR drop in feeders. If the series-field ampere-turns tend to produce flux in a direction *opposite* to that of the flux produced by the shunt-field ampere-turns, the machine is called a *differential compound generator.* Such generators have very few practical applications, but are ideally suited to supplying power to electrically powered excavators, inasmuch as the drooping voltage characteristic prevents stalling of the driving engine when the excavating bucket is accidentally stalled.



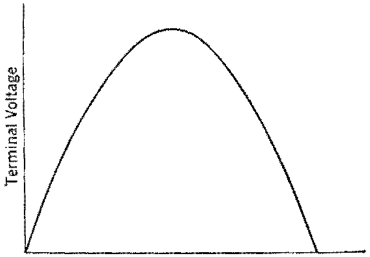
Connection Diagram Characteristic Curves

Fig. 10–19. Differential Compound Generator

A cumulative compound generator whose series-field ampere-turns at full load are just sufficient to produce the terminal voltage that the generator has at no load, as indicated by curve (*a*) of Fig. 10–17, is said to be *flat-compounded.* It should be noted that in a flat-compounded generator the terminal voltage is not constant between no load and full load, as the name *flat* implies. If the full-load voltage is greater than the no-load voltage, the generator is said to be *over-compounded.* The degree of compounding of a finished generator may be adjusted by placing a suitable low-resistance, high-current-capacity resistor in parallel with the series field and thus by-passing more or less of the load current. Such by-pass resistors are called *diverters.* Diverters are usually made of nickel-silver strip. The arrangement of a diverter on a differential compound generator and the effect of the diverter on the characteristic curve are shown in Fig. 10–19.



Wiring Diagram of Series Generator



Characteristic Curve

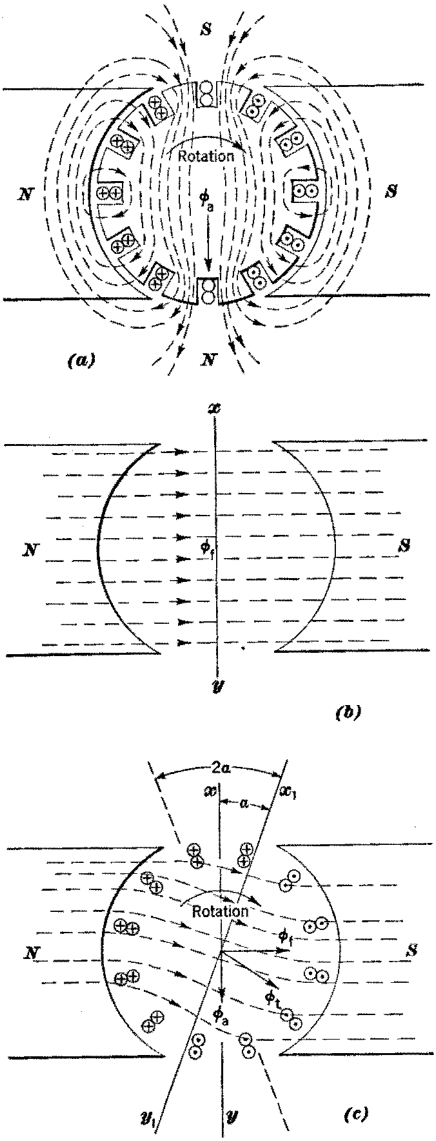
Fig. 10–20. Series Generator

The Series Generator.—A generator which derives all of its excitation from a series-field winding is called a *series generator.* A wiring diagram and a typical load characteristic curve of a series generator are shown in Fig. 10–20. Such a characteristic is suited to but few industrial uses. At one time, series generators were used extensively in street lighting. They are still used to some extent as series voltage boosters in d-c transmission systems.

Armature Reaction.—When a generator is carrying a load, the current in its armature produces a magnetic pole at the armature surface at every interpolar space. See Fig. 10–21 (*a*). When the flux ϕ_a of the field produced by the loaded armature exists jointly with the flux ϕ_f of the main field, shown in (*b*), the resultant flux has the direction ϕ_t in (*c*). As a result, the *commutating axis*, that is, the axis on which commutation should occur, would shift from xy in (*b*) at no load through the angle α to x_1y_1 in (*c*) at some particular load. The influence of armature flux in distorting the main-field pattern is called *armature reaction.* In order that destructive sparking between the brushes and the commutator may be avoided, it is necessary that the brushes be moved forward in the direction of rotation to the axis x_1y_1 corresponding to the particular load. Since the angle α varies as the load varies, the brushes must be moved to a new position corresponding to every value of load current. Furthermore, if the direction of rotation of the generator armature be reversed, the commutating axis will shift to the opposite side of xy and it will be necessary to provide for a possible brush shift of angle 2α from the position x_1y_1 shown in (*c*).

Calculation of Armature Reaction.—In a generator without interpoles, the armature-reaction ampere-turns may be considered as being divided into two parts, namely, those produced by the conductors in the interpolar space included in angle 2α in Fig. 10–21 (*c*) and those produced by all other armature conductors.

The axis of the reaction mmf produced by the conductors and current in the angle 2α coincides with the axis of the main-field mmf; but, by Lenz's Law, the reaction mmf has a sense opposite to that of the main-field mmf, and so tends to demagnetize the main field. This component of the armature reaction is called the *demagnetizing* component. The mmf produced by armature current in the conductors outside the angle 2α has an axis perpendicular to that of the main field. The armature-reaction component produced by that mmf is called the *cross-magnetizing* component.



Armature reaction changes the main field flux density distribution.

Fig. 10–21. Effect of Armature Reaction in a Generator

In a generator having interpoles (see the next article), the brushes are set at points midway between the poles; and so there can be no demagnetizing armature reaction. There is only a cross-magnetizing reaction.

Example 10–2.—A 4-pole generator has 720 armature conductors. The conductor current at full load is 25 amp. If the brushes are shifted forward in the direction of rotation 10 mechanical degrees, what will be the number of demagnetizing ampere-turns per pole?

Solution.—The number of conductors per pole is $\frac{720}{4} = 180$. In a 4-pole generator, 10 mechanical degrees are equal to 20 electrical degrees. Hence, the number of demagnetizing conductors per pole is

$$180 \times \frac{2 \times 20^\circ}{180^\circ} = 40$$

Since 40 conductors are equivalent to 20 turns, the number of demagnetizing ampere-turns pei pole is

$$25 \times 20 = 500 \text{ amp-turns}$$

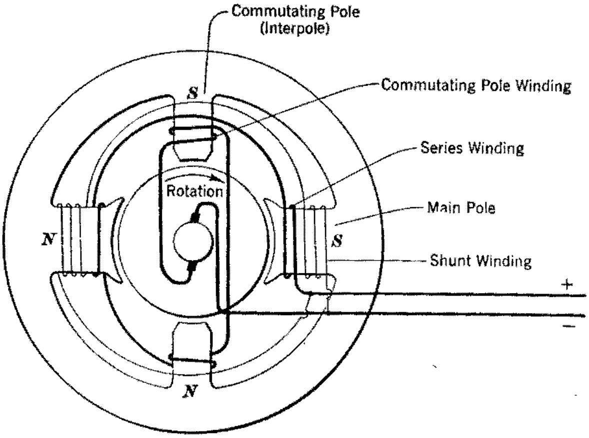


Fig. 10–22. Interpole Arrangement and Polarity in a Generator

Interpoles.—To avoid the commutation difficulty actually encountered in old-style generators, narrow poles called *interpoles* or commutating poles are used, as indicated in Fig. 10–22. These interpoles, being midway between main poles, are just opposite the armature poles shown in Fig. 10–21 (*a*). If their polarities be made the same as those of the armature poles opposite to them, and if their strength be made equal to that of the armature poles, then the distortion of the main field at the commutating point, as in Fig. 10–21 (*c*), may be almost entirely prevented. In practice the interpoles are made stronger than just sufficient to neutralize the armature flux, in order to aid in the reversal of current in the coils being commutated. The inductance of the coil tends to retard the reversal of the coil current by generating a prolonging voltage $-e = L\frac{di}{dt}$, and the interpoles must have sufficient additional strength to neutralize this inductive voltage. The interpole windings are connected in series with the armature windings, and therefore their correction ampere-turns increase or decrease in the same proportion as do the distorting ampere-turns of the armature.

In a generator *the interpoles must be of the same polarity as the main poles which they precede in the direction of rotation.*

It is very difficult for a designer to calculate the exact number of turns to be placed on the interpoles, and it is sometimes necessary to connect an adjustable diverter around the interpole windings to permit shunting of a part of the total armature current around those coils. If the ordinary resistance diverter were used, a sudden surge of current caused by a sharp increase in load would not divide between the interpole path and the diverter path in inverse proportion to their respective resistances; but, because of the high reactance of the interpole path, a disproportionately large portion of the surge current would flow through the diverter. Such an improper division would cause the interpole field to fail to rise in proportion to the armature current to be commutated, and would probably cause the commutator to flash over at the brushes. The proper division of surge current can be obtained by inserting, in series with the interpole shunt, a conductor wound around an iron core to form an inductance just equal to that of the interpole coils. Such a diverter is called an *inductive shunt.*

Change in Characteristics.—A generator may be used successfully at much less than rated voltage if separately excited, the desired voltage being obtained by excitation adjustment, by speed adjustment, or by both. If the ventilation were equally good at all speeds, the current that could be taken from the armature without overheating would be the same at all speeds. At reduced speed and correspondingly reduced generated voltage, the *voltage regulation is poorer than normal* because the I_aR_a voltage loss per ampere in the armature is as great as before and the armature reaction is also as great as before.

If the generator be driven at rated speed but be under-excited to obtain reduced voltage, the *voltage regulation will be poor* because the field-distorting ampere-turns of the armature for a given load are as great as ever and have more effect on the weakened field than upon a normally strong field. However, when the generator is so operated, full-load rated current may be drawn from the armature without excessive heating.

As the speed of a self-excited generator is reduced, the maximum terminal voltage to which it will build up is also reduced, for reasons discussed on page 114. A generator cannot be operated at half of its rated voltage when self-excited, for reasons discussed on page 114. In order that a self-excited generator may maintain its voltage within reasonable limits from no load to full load, the magnetic circuit must be excited *above* the knee of its saturation curve. If a generator is separately excited, its terminal voltage for any fixed current output is proportional to its speed. An example will serve to demonstrate the possibility of altering the characteristics of a self-excited generator to fit a definite need, without actually altering the windings.

Example 10-3.—A compound-wound interpole generator rated 25 kw, 115 volts at 900 rpm is needed. There is on hand a generator rated 60 kw, 230 volts at 1800 rpm. Can the generator at hand be satisfactorily converted without rewinding? Give instructions.

Solution.—The 115-volt requirement will be met when the speed is established at 900 rpm, instead of 1800 rpm. The shunt-field circuit must be broken in the middle, and the two halves must be connected in parallel. Each field coil will then receive full voltage when connected across the armature for self-excitation. The armature cooling will be less effective at the lower speed, and so the permissible continuous current output will be less. The number of ampere-turns in the series field per ampere of load will be the same as before. The compounding percentage will be the same, therefore, and may be satisfactorily altered by means of an adjustable low-resistance diverter connected across the series field. The commutating voltage needed will not be quite so great at the low speed, and the strength of the interpole field should be reduced by means of a diverter.

The generator is therefore capable of delivering somewhat less than its original normal current at 115 volts and 900 rpm, and should be satisfactorily convertible.

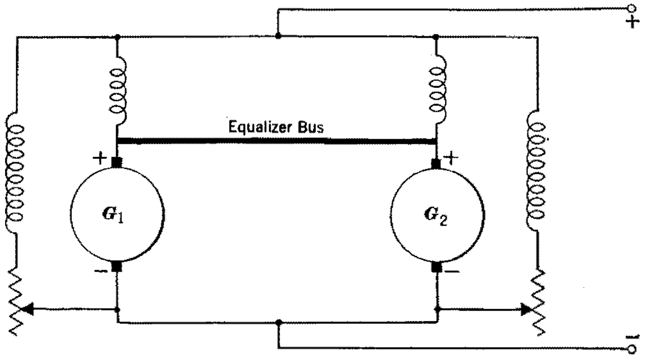


Fig. 10-23. **Cumulative Compound Generators in Parallel With Equalizer Bus**

Parallel Operation of Compound Generators.—The economic generation of direct current sometimes involves the operation of two or more generators supplying current to the same bus bars. The parallel operation of shunt generators is accomplished with no particular difficulty, inasmuch as the reduction of load on one generator for any reason permits a rise of its terminal voltage and it therefore tends to recover the lost load. For this reason, parallel operation of shunt generators is said to be "stable." However, if the load on an over-compounded generator be reduced, the generated voltage will *decrease* and, therefore, the ability of the generator to carry load decreases. When such a generator is connected in parallel with a similar one, the second generator assumes any load dropped by the first and the additional load causes its voltage to rise still higher. Thus, the loss of load and voltage by one generator and the subsequent gain of load and voltage by the other results in an unstable state of operation, and the underloaded generator will quickly be driven as a differential motor by the other generator, which has also assumed all of the load. This unstable condition can be remedied by connecting a low-resistance conductor, or "equalizer," between the series fields of the two generators at the points where they connect to their respective armatures, as indicated in Fig. 10–23. When such an equalizer is used, the current in either series field is not determined by the armature current of its own generator alone but depends on the division of the load current between the parallel paths of the two fields. In that way the compounding of the two generators is interdependent, and complete stability of operation may be achieved.

In order that two over-compounded generators may share a common load in proportion to their respective capacities, their characteristics must be identical and the resistances of their series fields, including the equalizer conductor, must be inversely proportional to their respective capacities.

Calculation of Series-Field Turns for Compounding a Shunt Generator. If space permits, a shunt generator may be converted into a compound generator by the addition of a series-field winding. In order to calculate the correct number of turns to produce flat compounding, it is necessary to know the number of shunt-field turns per pole and to make a simple test. With the generator loaded to full-load current, the field rheostat should be adjusted until the required terminal voltage is obtained, and the field current should be recorded. The difference between this field current and the field current required for no-load voltage will be the necessary increase in the field current. This current increase multiplied by the number of shunt-field turns per pole gives the increase in the number of ampere-turns per pole required to accomplish compounding. The ampere-turn increase divided by the full-load current gives the required number of series turns per pole.

Example 10–4.—A 230-volt shunt generator requires 1 ampere of field current at no load. At full load of 100 amperes the field current required for 230 volts is 1.4 amperes. The shunt-field winding has 3000 turns per pole. How many series-field turns per pole must be added to flat-compound this generator?

Solution.—The required increase in the field current for a change from no load to full load is

$$1.4 - 1 = 0.4 \text{ amp}$$

and the increase in the number of ampere-turns in the shunt field is

$$3000 \times 0.4 = 1200 \text{ amp-turns per pole}$$

The series field should contain

$$\frac{1200}{100} = 12 \text{ turns per pole}$$

Effect of Brush Shift.—In a commutating-pole generator the brushes are usually adjusted very accurately to commutate coils whose sides are *exactly* under the interpoles. If the brushes were shifted slightly *in the direction of rotation*, some of the interpole flux would enter the armature *back* of the conductors under commutation. Since the polarity of an interpole in a generator is opposite to that of the main pole which precedes it in the direction of rotation, the net flux cut by active conductors between brushes is the *difference* between the main-pole flux and the interpole flux. When the generator is loaded, the interpole flux produces an effect similar to a differentially connected series field and causes the terminal voltage to drop abnormally.

A slight cumulative compounding may be achieved by shifting the brushes slightly opposite to the direction of rotation; and such shifting is good accepted practice, provided the commutation is not impaired.

Commutator Wear.—As the commutator bars move under the brushes of a generator carrying load, minute sparks are formed and hasten the erosion of the two surfaces. If brushes are properly fitted to a new commutator of a properly designed generator, the minute sparks will gradually become invisible to the eye and will all but disappear. The commutator should then assume a chocolate-colored polish and should operate for months without appreciable loss of metal.

Unless the mica insulation between bars is *undercut* below the commutator surface, the unequal wear of mica and copper will cause the brushes finally to rest largely on the mica edges and there will be acute arcing and erosion of the copper. The commutator is then said to have *high mica*. It is important, therefore, to use a brush of the proper abrasive properties to wear away the mica at the rate at which the copper is vaporized. Only brushes recommended by the generator manufacturer or a carbon-brush specialist should be used.

When high mica has developed, the commutator should be turned to a true surface and the brushes should be refitted. Undercutting the mica from $\frac{3}{64}$ to $\frac{1}{16}$ in. is often to be recommended. Slight roughness may be removed with sand-paper. Emery cloth should never be used on a commutator, because emery is a conductor of electricity and, if allowed to lodge between bars, may cause the short-circuiting of one or more coils.

PROBLEMS

1. Construct a diagram similar to Fig. 10-21 (*a*) and (*c*) for a 4-pole generator.

2. If the armature of a 50-kw, 230-volt, 4-pole, 1200-rpm lap-wound generator were reconnected to form a wave winding, what would be its new rated voltage at the same speed?

3. If the lap-wound generator of Problem 2 were 6-pole, at what speed would it have to be driven to generate 230 volts when reconnected to form a wave winding?
Ans. 400 rpm

4. Would it be necessary to alter the connections of the shunt-field coil of Problem 3 to properly self-excite the generator? Explain.

5. What would be the new kilowatt rating of the generator when reconnected as in Problem 3?

6. If a self-excited generator failed to build up, and you were required to test for the cause, in what order would you investigate the six causes in the list on page 114?

7. A certain 4-pole lap-wound generator, rated 50 kw, 125 volts at 1800 rpm, has 89 commutator bars. If each brush covers three bars exactly and a bar is 10 times as wide as a mica strip between bars, what is the average time rate of change of current at full load during its commutating interval?

8. What must be the per cent of over-compounding of a 220-volt generator to produce 220 volts across a 30-kw load located 500 feet from the generator and supplied through 2/0 conductor?

9. A certain shunt generator has a no-load voltage of 230 volts, with a field current of 2.75 amperes. In order to maintain that voltage when the load current is 120 amperes, the field current must be increased 20 per cent. How many turns of series-field winding must be added to accomplish the same result by compounding, if each shunt-field coil contains 1200 turns?

10. At rated speed a generator has a no-load voltage of 230 volts. What will be its no-load voltage, if its field flux is increased 5 per cent and its speed is increased 10 per cent?

11. A separately excited shunt generator has a no-load voltage of 230 volts and a full-load voltage of 215 volts. Its full-load current is 50 amperes. The armature-circuit resistance at full load is 0.2 ohm. How much of the terminal-voltage loss is caused by armature reaction?

12. Two 600-volt shunt generators with ratings of 250 kw and 400 kw, respectively, are operating in parallel to carry a 500-kw, 600-volt load divided between them in proportion to their ratings. The voltage regulation of the first generator is 10 per cent, and that of the second is 7.5 per cent. With a declining load, at what ampere load will the two generators deliver equal power? *Ans. 370 amp, 622 volts*

13. What is the full-load rated current of a 1200-kw, 600-volt generator?

14. What is the load current per conductor at full load in Problem 13, if the armature is lap wound and the generator has 12 poles?

15. If the shunt-field coils of a lap-wound generator were connected in two or more parallel paths, instead of in series, what would be the effect of unequal resistances of the paths?

16. A 4-pole, 900-rpm, wave-wound generator has 89 armature coils of 4 turns each. The flux per pole is 10^7 lines, and 5 per cent of the coils are always under commutation. Calculate the generated emf. *Ans. 2029 volts*

17. If the armature of Problem 16 were connected lap-wound, and all other conditions remained the same, what would be its terminal voltage? *Ans. 1015 volts*

18. If the generator of Problem 16 is rated at 200 kw, what will be the kw rating when the armature winding is changed as suggested in Problem 17?

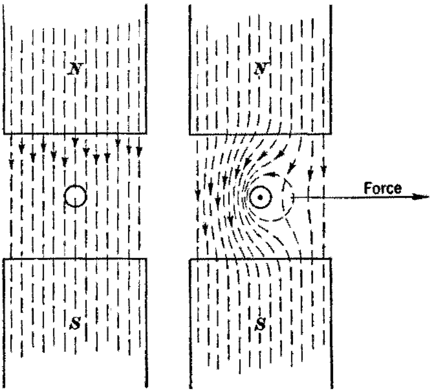
19. A 1200-kw generator and a 750-kw generator share a 1500-kw load. How many kilowatts should be carried by each machine?

20. For a temporary emergency requirement, 25 amperes at 115 volts are needed. A 50-kw, 230-volt, 1200-rpm generator is available. How may this generator be used?
Ans. Drive at 600 rpm. Reconnect field coils into two parallel circuits, and self-excite as originally.

CHAPTER 11

The D-C Machine as a Motor: Force on a Conductor Carrying a Current in a Magnetic Field

Lenz's Law.—In Fig. 11–1 (*a*) is shown an endwise view of a conductor immersed in a magnetic field, but carrying no current. No force is exerted on that conductor by the field. In (*b*) is shown the effect of passing an electric current through the conductor.



(*a*) (*b*)

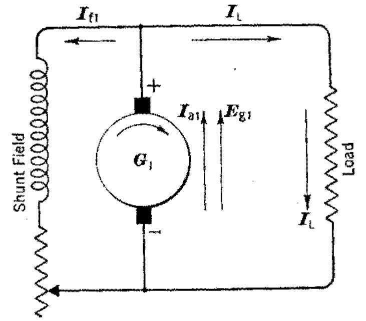
Fig. 11–1. Force on a Conductor Carrying Current in a Magnetic Field

The magnetic rule of thumb stated on page 5 will be found useful in studying the action in Fig. 11–1 (*b*). With the current flowing toward the observer, the magnetic flux produced by it encircles the conductor in a counter-clockwise direction, thus strengthening the main field on the left of the conductor and weakening it on the right of the conductor. This distortion of the main field produces a force to the right on the conductor. By experiment, it has been found that the force on the conductor is directly proportional to the current in the conductor and to the strength of the main magnetic field. It should be noted that, if the current is flowing toward the observer, as in Fig. 11–1 (*b*), and the conductor were forced to move to the left against the magnetic force on it, the voltage generated in it would, according to Faraday's Law given on page 3, be in a direction toward the observer. Lenz's Law summarizes the foregoing facts by stating that a current induced in a conductor always produces a force which opposes the action that induces the current.

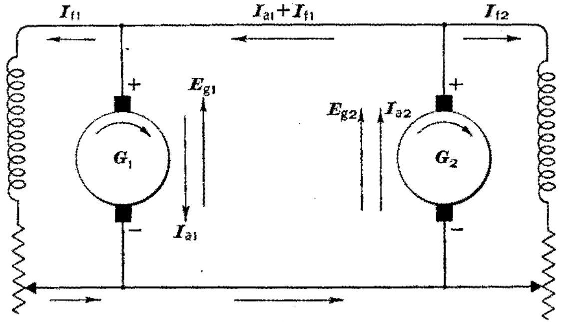
A current of 1 abampere, or 10 amperes, in a conductor located in a magnetic field with a strength of 1 gauss in a direction perpendicular to the conductor will cause each centimeter of length of that conductor to be acted upon by a force of 1 dyne. It therefore follows that, under the same conditions, 1 ampere will cause a force of $\frac{1}{10}$ dyne per centimeter of immersed length of conductor. Hence, the force on a conductor may be expressed in terms of the intensity of the field in which it is immersed and the conductor length normal to the flux lines. The relation is:

$$F = \frac{BIl}{10} \tag{11-1}$$

in which F = force, in dynes;
B = field intensity, in lines per square centimeter;
I = current, in amperes;
l = length of conductor normal to the field, in centimeters.



(a)
G_1 as Generator



(b)
G_1 as Motor being run by Generator G_2

Fig. 11-2. Driving a Generator as a Motor

Simultaneous Application of Faraday's Law and Lenz's Law.—When a generator is driven, but no current is delivered by its conductors, a voltage appears at its terminals in accordance with Faraday's Law. When a load current is produced by the generator, that current flowing in the armature conductors immersed in the main magnetic field produces a reactive force on the conductors in accordance with Lenz's Law, and this force accounts for the increase in driving torque when the current output increases. Thus, in a loaded generator these laws apply simultaneously.

When a motor is operated at no load, it follows from Lenz's Law that just sufficient current flows in the armature conductors to produce the torque required to maintain its no-load speed. Simultaneously, according to Faraday's Law, the movement of the armature conductors through the field generates an electromotive force opposite in sense to the applied voltage. When a mechanical load is demanded at the motor shaft, the no-load torque is not sufficient to maintain the speed, and the speed drops.

When the speed drops, the generated voltage also drops in proportion, and more current can then flow to supply the new demand for torque. Thus, in a motor both Faraday's Law and Lenz's Law always apply, regardless of the amount of the load; whereas, in the generator both laws apply only when load current is being supplied by the generator.

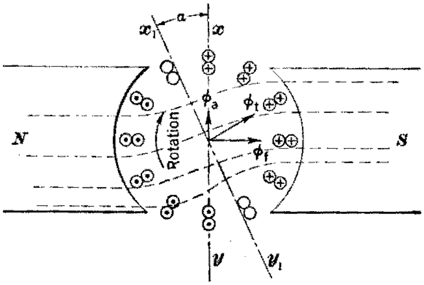


Fig. 11-3. Effect of Armature Reaction in a Motor

There is no essential difference in construction between the direct-current generator and the direct-current motor. There are, however, two minor differences. First, the motor shunt-field coils are somewhat smaller; secondly, the series-field coils (if there are any) and the interpole windings are inter-connected with the armature and the shunt field differently in the two machines.

The Shunt Motor.—Let Fig. 11-2 (*a*) represent a shunt generator. If the top brush is positive, all currents will be in the directions shown. By definition, the positive terminal of a generator is the one from which the current is said to emerge, and the positive terminal of a motor is the one at which the current is said to enter. If the load be replaced by an electric battery or by another shunt generator G_2, with the same polarity as G_1 but with slightly greater emf, all currents will be as shown in (*b*). Then, the currents are related as follows:

$$I_{a2} = I_{a1} + I_{f1} + I_{f2}$$

The direction of I_{f1} is unchanged because the voltage applied to the field of G_1 is not reversed, and so the field polarity of G_1 is unchanged. However, the direction of I_{a1} is reversed because of the higher voltage of G_2, and hence the armature torque is reversed. When the machine G_1 operated as a generator, the torque with which the armature resisted the clockwise rotation by the prime mover was counter-clockwise; while the torque on a motor with the same brush polarity is clockwise, and hence the *machine runs as a motor in the same direction as when driven as a self-excited generator.*

Interpole Connections in a Motor.—When the machine G_1 in Fig. 11–2 (*b*) is used as a motor, the directions of the currents in all its armature conductors are reversed from those of Fig. 10–21 (*a*). The effect of this condition on the total field is shown in Fig. 11–3.

Whereas the north pole of armature flux is at the bottom in Fig. 10–21 (*a*), where the machine is assumed to operate as a generator, the north pole is at the top in Fig. 11–3, where the machine is treated as a motor. Therefore, the interpole polarity in a d-c machine used as a motor must be opposite to that of the machine when it is used as a generator with the same direction of rotation and with the same magnetic-field polarity. *In a direct-current motor the polarity of any interpole must be opposite to that of the main pole which it just precedes in the direction of rotation.*

Interpoles in either a generator or a motor maintain the commutating zones midway between the main poles. This condition permits operation of the machine in either direction of rotation without shifting the brushes.

Calculation of Torque in a Motor.—As shown by equation (11–1), the force on a conductor carrying a current is

$$\frac{BIl}{10} \text{ dynes}$$

The torque on one conductor at a radius of r centimeters is

$$\frac{BIl}{10} \times r \text{ dyne-centimeters}$$

Then the total torque, in pound-feet, on all active conductors (those under poles and so capable of producing torque) is

$$T = 7.37 \times 10^{-9} \times ZBIlr \qquad \textbf{(11–2)}$$

in which T = torque, in pound-feet;
Z = total number of active conductors;
B = air-gap flux density, in lines per square inch;
I = armature-conductor current, in amperes;
l = active length of one conductor, in inches;
r = average lever arm, or radius, in inches.

Example 11–1.—A 4-pole 230-volt shunt motor has a lap-wound armature 8 in. long with 800 conductors. The air-gap density is 50,000 lines per sq in. The mean radius from the armature center to the conductors is 6 in. The pole faces cover 65 per cent of the conductors. Calculate the gross torque when the armature draws 100 amp from the line.

Solution.—For purposes of substitution in equation (11–2),

$Z = 800 \times 0.65 = 520$ active conductors;
$B = 50{,}000$ lines per sq in.;
$I =$ current divided by number of paths $= \frac{100}{4} = 25$ amp;
$l = 8$ in.;
$r = 6$ in.

Hence, the torque is

$$\begin{aligned} T &= 7.37 \times 10^{-9} \times 520 \times 50{,}000 \times 25 \times 8 \times 6 \\ &= 230 \text{ lb-ft} \end{aligned}$$

The method described for calculating the torque of a motor is correct and is one that is generally used, but the theory of the method includes the assumption that the conductors lie in a field of average intensity under the pole faces. Actually, most of the field flux follows the lower-reluctance paths through the armature teeth and so leaves the conductors in a relatively weak field in the higher-reluctance paths through the slots of the armature. However, the ampere-turns of the armature produce poles on the armature midway between the field poles. It is the mutual attraction and repulsion between field-pole fluxes and armature-pole fluxes that produces the torque of the motor. Although this theory of torque production is correct in fact, its application in calculations is very difficult.

Characteristics of Shunt Motors.—Shunt motors are called *constant-speed motors,* although the speed may change as much as 12 per cent from full load to no load on motors developing $\frac{3}{4}$ to 5 hp, and 10 per cent on motors of larger size. When a shunt motor runs without load, its armature current is just large enough to produce the torque required to drive the armature against the frictional resistance of air, bearings, and brushes and the retarding drag (load) of armature-iron loss. The current is limited by the armature-circuit resistance and by the *counter electromotive force* in the armature conductors. The counter electromotive force of a motor is the generated emf which would appear as terminal voltage if the motor were being operated as a self-excited generator in the same direction, at the same speed, and with the same field excitation. Then,

$$V = I_aR_a + E_g \tag{11–3}$$

where $V =$ voltage applied to the motor;
$I_aR_a =$ resistance voltage drop in armature circuit;
$E_g =$ counter electromotive force.

In an unloaded motor the relative magnitudes of quantities in equation (11–3) may be about as follows:

$$230 = 2 + 228$$

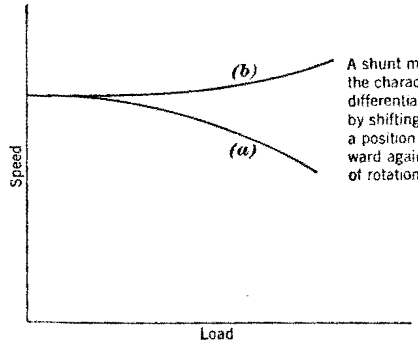
The motor power equation may be obtained by multiplying each member of equation (11–3) by I_a. Thus,

$$VI_a = I_a^2 R_a + E_g I_a \tag{11–4}$$

The term $I_a^2 R_a$ represents the power lost in the armature because of its resistance. The term $E_g I_a$ represents the remainder of the power delivered to the armature, and is the sum of the iron loss, the windage and friction loss, and the power output at the shaft if there is a load on the motor. Inasmuch as E_g is a function of speed, its magnitude is a fairly good indication of the motor speed for any given load. Hence,

$$\frac{V}{E_g} = \frac{\text{Speed at no load}}{\text{Speed under load}}$$

Speed Control of a Shunt Motor by Field Adjustment.—Just as the voltage of a shunt generator may be controlled by varying the resistance of its field circuit, so may the speed of a shunt motor be controlled by the same means. Refer again to equation (11–3), and let it be assumed that a shunt motor is running at 2000 rpm. If the field-circuit resistance be increased, the main-field flux will be decreased and E_g will be proportionately decreased. To balance the equation, I_a must increase. An increase in I_a produces an increase in torque. The increase in torque causes the speed to increase to a new value, which just requires the new torque to maintain it. Thus, *an increase in the shunt-field resistance of a shunt motor causes its speed to increase.*



A shunt motor may be given the characteristic of a differential compound motor by shifting the brushes to a position slightly backward against the direction of rotation.

Fig. 11–4. **Shunt-Motor Speed Regulation**

Because of excess mechanical stresses and also because of commutation limitations, it is not advisable to operate a constant-speed motor at more than 15 per cent above its rated speed. With the field-circuit resistance at its minimum, it is usually possible to obtain a speed as low as 90 per cent of the rated speed at full load. However, the speed of many shunt motors cannot be reduced to rated value at no load, even by removing the external rheostat resistance entirely.

Effect of Shifting Brushes From Neutral.—If the brushes be shifted slightly backward from their neutral position against the direction of rotation, a curve like (*b*) in Fig. 11–4 may be obtained. When the brushes are shifted backward, the conductors between them cut both the main flux and the flux of one interpole. Since the interpoles are opposite in polarity to the main poles paired with them between brushes, their fluxes have the effect of weakening the main-pole fluxes. Furthermore, the conductors lying in the angles as 2α in Fig. 10–21 (*c*) produce demagnetizing ampere-turns that weaken the main field.

As explained on page 133, a weakening of the main-field flux causes the speed to rise, and so the differential action of the interpoles and the demagnetizing mmf causes the usual droop of curve (*a*) to be approximately compensated as shown by (*b*). Too much backward shift of the brushes may over-accentuate the characteristic of curve (*b*) and may cause unstable speeds at loads greater than one-third or one-half of the rated load. Any improvement in speed regulation obtained by this method is gained at the expense of good commutation.

Speed Regulation.—When a shunt motor is loaded, the speed decreases because the no-load current can produce only that torque required at no load. Since E_g is a function of rate of cutting of a constant magnetic field, its value decreases when the speed decreases. In equation (11–3) both V and R_a are constant; and, therefore, to maintain a balanced equation with a decreased value for E_g, the value of I_a must increase. The torque is proportional to I_a, and so increases as I_a increases. This related action of speed decrease and torque increase continues until a stable condition is reached, when the current corresponding to a certain speed is just sufficient to drive the motor and its load at that speed. The change in speed corresponding to a change in load from zero to full load is called *speed regulation*. Speed regulation is usually expressed as a per cent of full-load speed. Thus,

$$\text{Regulation, in per cent} = \frac{\text{No-load speed} - \text{full-load speed}}{\text{Full-load speed}} \times 100$$

Stabilizing Windings.—To insure sparkless commutation, many motors are designed with their interpole fluxes quite strong in comparison with the main-pole fluxes. When such motors are operated at their maximum speeds, and hence with their main poles only weakly excited, the distortion of the main flux and the resulting saturation of the tips of the trailing poles decrease the total flux and the net counter emf so seriously that unstable operation results. To prevent such instability, a series winding composed

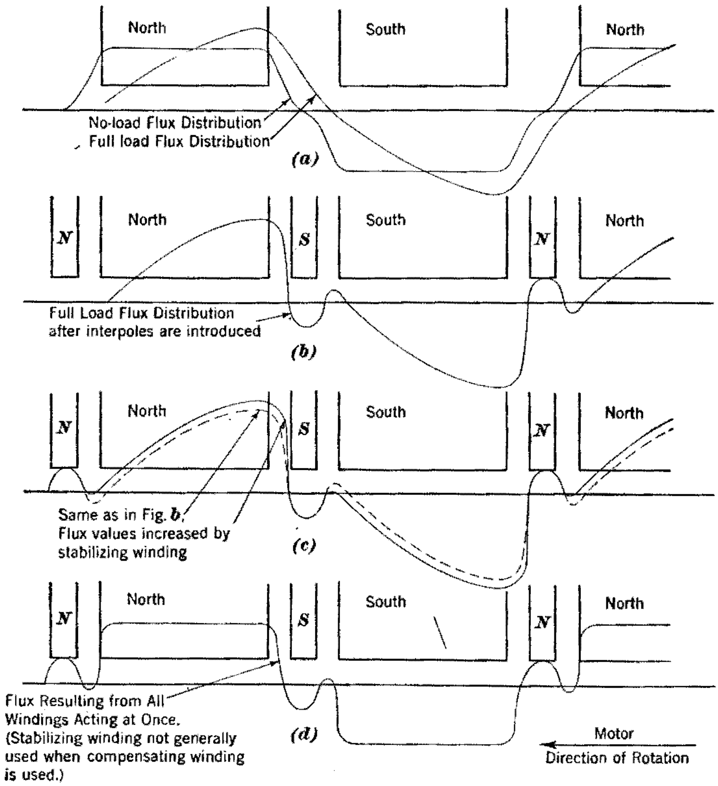


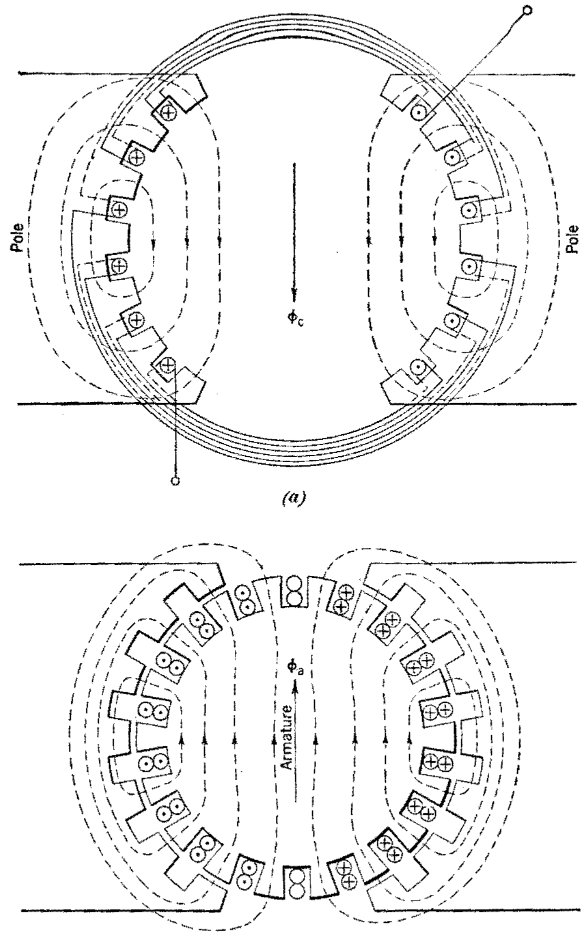
Fig. (*a*) shows the distortion of main field flux caused by loading (no interpoles).

Fig. (*b*) shows the effect of interpoles in not only correcting field distortion at the commutating point but also providing reversed flux at that point to aid in the reversal of the conductor current.

Fig. (*c*) shows the effect of the addition of a stabilizing winding which restores the total flux per main pole to the original no-load value by adding the flux which was lost by saturation caused by crowding of flux to one pole tip.

Fig. (*d*) shows the effect of the compensating winding in restoring the flux density distribution under load to the same form which it had at no load (under main poles).

Fig. 11-5. Effect of Stabilizing Winding



(a)

(b)

The compensating winding produces an mmf opposite in direction to that of the armature conductors

Fig. 11–6. Effect of Compensating Winding

of a very few turns of large conductor is added to each main pole and is so connected that its mmf is in the same direction as the mmf of the shunt-field winding. Such a winding is called a *stabilizing winding* because its flux, which is added in the same relative amount as that lost because of pole-tip saturation due to "crowding," prevents field weakening and stabilizes the speed. Fig. 11–5 (*c*) shows the effect of a stabilizing winding. Many so-called shunt motors are provided with stabilizing windings.

Compensating Windings.—Stabilizing windings do not prevent crowding of the main-field flux to one side of the pole. Commutating poles maintain the commutating zone at a point midway between the main poles, but they do not correct distortion of the main flux at the main poles. Under extremely severe operating conditions of reversal of rotation and wide range of speed variation, it is necessary to add a *compensating winding.* If distortion of the main-field flux is not controlled by a compensating winding, the violent shifting of the flux across the pole faces and the armature conductors may induce high voltages in the armature coils and may cause flashover between commutator bars when the motor is reversed quickly. Compensating windings are embedded in longitudinal slots in the pole faces and are connected in series with the armature. Fig. 11–6 (*a*) shows the arrangement of compensating conductors and the flux ϕ_c that would be produced by them if they acted alone. By making the compensating ampere-turns just equal to the armature ampere-turns, the main-field distortion is practically eliminated, as indicated in Fig. 11–5 (*d*).

Compensating windings require a costly construction and are justified only under extreme conditions, such as those encountered in steel-mill motors.

The Compound Motor.—The compound motor is simply a shunt motor with a series winding added to the main poles. This series winding is similar to that in the compound generator. When the load current flowing through the series winding produces an mmf having the same direction as the mmf of the shunt-field winding, the motor is called a *cumulative compound motor.* When the two windings tend to produce fluxes in opposite directions, the motor is called a *differential compound motor.*

When a cumulative compound motor is loaded, its main field is stronger than when the motor is not loaded because of the added excitation of the series winding. Equation (11–3) applies to the compound motor as well as to the shunt motor. However, because of its greater field strength, the compound motor does not need so much speed to generate the counter emf E_g as does the shunt motor, and it therefore runs at a somewhat lower speed and has greater speed regulation when all other conditions are the same.

The cumulative compound motor has somewhat higher starting torque than a shunt motor of the same rating. In some applications, such as punch-press service, shears, and other apparatus having large fly-wheels, it is essential that the motor speed decrease more than could be accomplished with a shunt motor when the heavy part of the load occurs, in order that part of the fly-wheel energy may be delivered up to smooth out the fluctuations of power from the power line. The cumulative compound motor is well adapted to applications of this kind. A typical compound-motor characteristic is shown in Fig. 11–7. Curve (*a*) is for a shunt motor, and curve (*b*) is for the same motor after a series winding has been added.

In the differential compound motor, the series-field mmf is opposite in direction to the shunt-field mmf, and so the resultant main flux is less than it would be if the series field were not used. Inasmuch as the field is weakened by increase in load, the speed rises with increase in load. This is an unstable condition at loads greater than about one-third of the rated load (the limit depends on the relative strengths of the shunt-field and series-field mmf's), and the motor may reach an unsafe uncontrollable speed or "run away" very quickly. If the series field is exceptionally strong, the motor may even stop and reverse its direction before running away. There are extremely few practical applications for the differential compound motor.

When the connections of a compound motor must be made the first time without the benefit of a connection diagram, a simple test should be made to assure proper connection of the series field. With the shunt field fully excited, the motor should be started[1] slowly. If the motor over-speeds or otherwise performs erratically, it should be stopped and the series-field connections should be reversed. Even if the motor reaches full speed uneventfully, it cannot yet be assumed that the series-field connections are correct. A slight load should be applied. If the speed decreases, the series-field connections are correct. If the speed increases when the motor is loaded, the series field is differentially connected and must be reversed.

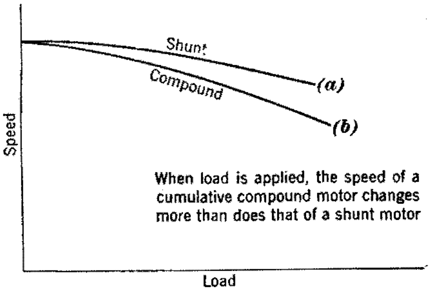


Fig. 11-7. **Shunt-Motor and Compound-Motor Characteristics**

Another method, which is possibly more convenient, consists of starting the motor slowly and noting the direction of rotation; and then again starting it with the shunt field opened. If the direction of rotation is the same in both trial starts, then the compounding is cumulative.

The Series Motor.—The general construction of a series motor is identical with that of a shunt or compound motor, but its field flux is derived solely from a series-field winding. The field flux is great at start, and the starting torque is very high. As the speed increases, the back emf E_g increases and therefore the armature current decreases. The decrease of armature current also flowing in the field causes the field strength to decrease and the motor speed to increase still further. This cumulative action continues until the load, the armature-iron loss, the windage and

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[1] The starting handle should be moved slowly, about 15 seconds being allowed for full operation. See Chapter 15.

other friction are just great enough to require the torque available at some particular speed. When the load is varied, the speed varies greatly; and the series motor is therefore unsuited to service requiring relatively constant speed. Fig. 11–8 shows a typical series-motor speed-torque curve. The torque is almost inversely proportional to the speed.

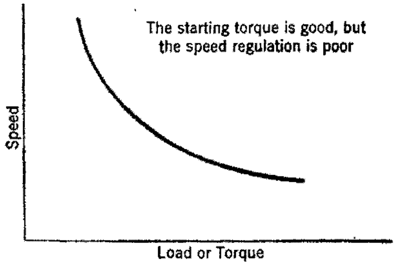

Fig. 11–8. Series-Motor Characteristic

If the load of a series motor be too greatly reduced, the speed may become excessive and cause damage to the windings. Therefore, it is not safe to connect a series motor to a load by a belt or other insecure means which might fail and allow a dangerous speed. Series motors are used for tractive purposes on locomotives, street cars, ore bridges, and cranes, and for small household devices.

Fig. 11–9 shows superposed the characteristic speed-load curves of a shunt motor, a compound motor, and a series motor, all rated at the same horsepower and speed.

Mathematical Comparison of Characteristics.—For a shunt motor the following equation may be written to relate speed and load defined in terms of armature current:

$$n = \frac{V - I_a R_a}{\phi Z'} \times k \qquad \textbf{(11–5)}$$

in which n = speed, in rpm;
V = applied voltage;
$I_a R_a$ = resistive voltage drop in the armature;
ϕ = main-field flux;
Z' = number of active armature conductors;
k = a constant of the machine.

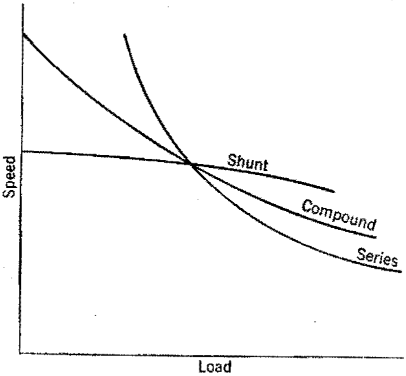

Fig. 11–9. Characteristics of Shunt, Cumulative Compound, and Series Motors, All Referred to the Same Rated Speed and Power

Inasmuch as $\phi Z'$ is a constant k_1 with constant excitation,

$$n = k_1(V - I_a R_a)$$

or

$$n = k_1 E_g \qquad \textbf{(11–6)}$$

This is the equation of a drooping straight line to which the shunt-motor characteristic of Fig. 11–9 is an approximation. Armature reaction pre-

TABLE 11-1[1]

CHARACTERISTICS* AND APPLICATIONS OF D-C MOTORS, 1 TO 300 HP

Type	Starting Duty	Maximum Momentary Running Torque	Speed Regulation	Speed Control†	Applications
Shunt-wound, constant-speed	Medium starting torque. Varies with voltage supplied to armature, and is limited by starting resistor to 125 to 200 per cent of full-load torque	125 to 200 per cent. Limited by commutation	8 to 12 per cent	Basic speed to 200 per cent of basic speed by field control	Drives where starting requirements are not severe. Use constant-speed or adjustable-speed, depending on speed required. Centrifugal pumps, fans, blowers, conveyors, elevators, wood- and metal-working machines
Shunt-wound, adjustable-speed			10 to 20 per cent. Increases with weak fields	Basic speed to 600 per cent of basic speed (lower for some ratings) by field control	
Compound-wound, constant-speed	Heavy starting torque. Limited by starting resistor to 130 to 260 per cent of full-load torque	130 to 260 per cent. Limited by commutation	Standard compounding 25 per cent. Depends on amount of series winding	Basic speed to 125 per cent of basic speed by field control	Drives requiring high starting torque and fairly constant speed. Pulsating loads. Shears, bending rolls, plunger pumps, conveyor crushers, etc.
Series-wound, varying-speed	Very heavy starting torque. Limited to 300 to 350 per cent of full-load torque	300 to 350 per cent. Limited by commutation	Very high. Infinite no-load speed	From zero to maximum speed, depending on control and load	Drives where very high starting torque is required and speed can be regulated. Cranes, hoists, gates, bridges, car dumpers, etc.

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* Table shows average values for standard motors.

† Minimum speed below basic speed by armature control limited by heating.

[1] Courtesy General Electric Co.

TABLE 11-2[1]

GENERAL EFFECT OF VOLTAGE VARIATION ON D-C MOTOR CHARACTERISTICS

■ = Increase ▲ = Decrease

Voltage Variation	Starting and Max. Running Torque	Full-Load Speed	Efficiency			Full-Load Current	Temperature Rise, Full Load	Maximum Overload Capacity	Magnetic Noise
			Full Load	¾ Load	½ Load				
				SHUNT-WOUND					
120% Voltage	■ 30%	110%	■ Slight	No change	▲ Slight	▲ 17%	■ Main field. ▲ Commutator field and armature	■ 30%	■ Slight
110% Voltage	■ 15%	105%	■ Slight	No change	▲ Slight	▲ 8.5%	■ Main field. ▲ Commutator field and armature	■ 15%	■ Slight
90% Voltage	▲ 16%	95%	▲ Slight	No change	■ Slight	■ 11.5%	▲ Main field. ■ Commutator field and armature	▲ 16%	▲ Slight
				COMPOUND-WOUND					
120% Voltage	■ 30%	112%	■ Slight	No change	▲ Slight	▲ 17%	■ Main field. ▲ Commutator field and armature	■ 30%	■ Slight
110% Voltage	■ 15%	106%	■ Slight	No change	▲ Slight	▲ 8.5%	■ Main field. ▲ Commutator field and armature	■ 15%	■ Slight
90% Voltage	▲ 16%	94%	▲ Slight	No change	■ Slight	■ 11.5%	▲ Main field. ■ Commutator field and armature	▲ 16%	▲ Slight

Notes: Starting current is controlled by starting resistor.
This table shows general effects, which will vary somewhat for specific ratings.

[1] Courtesy General Electric Co.

vents exact similarity between the actual and the theoretical characteristics.

In the cumulative compound motor, ϕ is not constant but increases with load. As ϕ of equation (11–5) increases, the value of the fraction decreases; and there is produced a curve with increasing droop, which approximates the cumulative-compound characteristic of Fig. 11–9. The amount of the droop depends on the number of ampere-turns of the shunt field relative to those of the series field when loaded.

In the series motor,

$$T = k\phi I \tag{11–7}$$

If the saturation of the iron is low, ϕ is approximately proportional to I, and

$$T = k_1 I^2 \tag{11–8}$$

The speed of a series motor is proportional to $\dfrac{(V - I_a R_a)}{\phi}$ and may be controlled by altering either V or ϕ. The applied voltage may be reduced by inserting a resistance in series with the armature, and ϕ may be reduced by shunting part of the current around the field coils. The interaction of ϕ and I_a produces an approximately hyperbolic speed-load characteristic within the operating range of the motor. Tables 11–1 and 11–2 give data relating to characteristics and applications of d-c motors.

PROBLEMS

1. A certain 230-volt shunt motor with an armature current of 80 amp produces a full-load running torque of 100 lb-ft. The armature-circuit resistance is 0.1 ohm. If field flux is decreased 2 per cent and the speed is restrained from rising by external means such as a large constant-speed motor, what will be the new torque?

2. A 230-volt, 25-hp, 1200-rpm shunt motor has a full-load armature-circuit resistance of 0.12 ohm and a full-load efficiency of 88%. Calculate the counter emf at full load.

3. Why are interpoles desirable even when compensating windings are used?

4. A conductor carrying 1200 amp is placed in and perpendicular to a field whose intensity is 50,000 lines per sq in. Calculate the force on the conductor, in pounds per foot of length.

5. A 20-hp, 230-volt, 1200-rpm motor has a full-load current of 74 amp. Its armature resistance is 0.15 ohm. How many ohms must be provided in its starting rheostat to limit the starting current to 150 per cent of the rated full-load current? Under these conditions, how many pound-feet of starting torque would be produced?
Ans. 1.92 ohms; 131 lb-ft

6. A 50-hp, compound-wound, 230-volt motor takes an armature current of 180 amp at full load. Its armature-circuit resistance with 180 amp flowing is 0.06 ohm. Its no-load speed with minimum shunt-field-circuit resistance is 820 rpm, and its full-load speed is 780 rpm, approximately. How much resistance must be placed in series with the armature to produce full-load torque at 500 rpm? *Ans. 0.43 ohm*

7. If a motor has a torque of 20 lb-ft when the armature current is 15 amp, what will be the torque when the armature current is reduced to 12 amp and the field flux is reduced 5 per cent?

8. A shunt motor is operated with its field excited from a 230-volt supply. Variable voltage is available for the armature. The armature resistance is 0.102 ohm, exclusive of brush contact. With 230 volts across the armature, the full-load speed is 600 rpm and the armature current is 90 amp. Calculate: (*a*) the speed when the armature applied voltage is 100 volts and the current is 80 amp; (*b*) the horsepower output in (*a*) if 5 per cent of the armature input power is spent in windage, friction, and iron loss.

CHAPTER 12

Losses, Efficiency, and Rating of Direct-Current Machines

Types of Losses.—Only a part of the mechanical power required to drive a generator is available as useful electric power at the generator terminals. The difference between the power input and the power output is lost in the generator in the following four ways:

1. $I_f^2R_f$ loss in the shunt-field winding.
2. $I_a^2R_a$ loss in the armature circuit.
3. Friction loss (bearings, brushes, windage).
4. Hysteresis and eddy-current loss in armature iron.

Loss in Shunt-Field Circuit of D-C Generator.—The shunt-field rheostat is necessary for control of the generator voltage, and so its power loss must be included as chargeable to the generator along with the loss in the field winding itself. Thus, the total field loss is VI_f.

Loss in the Armature Circuit.—The armature-winding power loss is proportional to the square of the armature current. The armature-circuit resistance is the sum of the component resistances of the armature winding, commutator, brushes, brush holders, leads, and contact between the brushes and the commutator. At any fixed temperature, the resistances of all the component parts are constant except those of the brushes and brush contacts. The phenomenon of brush-commutator contact resistance is not fully understood. Contact resistance between copper and carbon decreases as the current density increases. With current densities commonly used, the contact-resistance voltage drop between carbon brushes and the commutator is approximately 2 volts (1 volt at the positive brush set and 1 volt at the negative brush set) and is substantially constant at common operating current densities.

Copper leaf brushes are commonly used on generators designed for low voltage (below 40 volts). In some machines brushes molded or sawed from blocks of a mixture of carbon and metallic powder are used. However, in most machines, brushes of carbon or carbon and graphite are used.

When brushes of carbon are used, their commutator-contact resistances are found to decrease with an increase in current density. Because of the inverse resistance characteristic of carbon-brush contacts, the resistance of an armature circuit may be much less at full load than at no load. Therefore, in measuring the armature-circuit resistance of a direct-current generator to be used in calculating its losses at full load, the rated full-load current should be used.

Losses by Windage and Other Friction (Mechanical Losses).—Power losses caused by air friction depend to a large extent on the shape of the armature (projecting parts, etc.), and are approximately proportional to the cube of the speed. Some directed blower action for cooling the generator is frequently provided by fan blades attached to the armature near the windings. The additional windage loss is more than compensated for economically by the greater output possible because of the added cooling.

Friction loss in sleeve bearings with oil rings is independent of the load, and is proportional to the three-halves power of the speed. Brush friction loss is proportional to the speed and is independent of the load.

Stray Load Loss.—The classification *stray load loss* includes all losses which vary with the load but which are not accurately determinable. Such losses are caused by:

1. Eddy currents in armature conductors.
2. Short-circuit currents in coils under commutation.
3. Eddy currents in bolts and other solid parts of the armature.
4. Flux pulsations produced by change in the reluctance of the magnetic path at teeth and slots; and flux pulsations produced by currents in coils under commutation.
5. Distortion of the flux in the armature produced by the armature reaction.

Inasmuch as the stray load losses are not accurately determinable, but do actually exist, the recommendation of the American Institute of Electrical Engineers is that they be assumed as 1 per cent of the power output when calculating conventional efficiency of machines having an output of 30 or more hp.

Armature-Iron Loss.—The reversal of flux in the armature iron because of its turning produces a hysteresis loss, as explained on page 64. Eddy currents also are set up in the iron, as explained on page 66. Both hysteresis and eddy currents cause power loss, and therefore affect the efficiency of the machine. Hysteresis losses are approximately proportional to the 1.6 power of the maximum flux density, and eddy-current losses are proportional to the square of the maximum flux density.

Efficiency.—The power efficiency of a generator is the ratio of the power output to the power input. It may be expressed as follows:

$$\text{Efficiency, in per cent} = \frac{\text{Power output}}{\text{Power input}} \times 100$$

Inasmuch as power input equals the sum of the power output and the power losses, the efficiency may also be expressed by the following relation:

$$\text{Efficiency, in per cent} = \frac{\text{Power output}}{\text{Power output} + \text{Power losses}} \times 100$$

This efficiency is called the *conventional efficiency*. The efficiency of a generator may be measured by loading, if a calibrated prime mover and a load of sufficient capacity are available. This efficiency is called the *direct efficiency*.

No-Load Losses From "Running Light" Test.—The no-load losses may be found by running the generator as a motor at rated speed and voltage, and measuring the power input. The power input of the armature less the small amount of armature resistance loss is the friction and iron loss. The "running light" test for determining the so-called constant losses is particularly convenient when it is not feasible to drive the generator by a calibrated motor.

Efficiency of D-C Motors.—The losses in a direct-current motor are of the same type as those in a direct-current generator and may be found in the same manner. For a given power output the shunt-field loss and the armature-iron loss of a direct-current machine used as a motor are *less* than when the machine is used as a generator because of the lower flux required in the motor.

Example 12–1.—The efficiency of a direct-current, self-excited, flat compound generator of 500 kw, 250 volts is to be determined. A 15-hp calibrated motor is available. Give instructions for obtaining the test data required for calculating the conventional efficiency at any load.

Solution.—Drive the generator at rated speed with the calibrated motor. Excite it separately (if separate d-c supply is available), and plot a curve of no-load terminal voltage vs. power from the driving motor. The generated voltage is proportional to the field flux. Therefore, the curve so drawn will show the values of fixed losses (windage, friction, and iron), as they vary with the generated voltage. Measure the armature-circuit resistance (terminal to terminal), using 125, 100, 75, 50, and 25 per cent of the rated full-load current. Draw a curve of armature-circuit resistance vs. load current. Measure the shunt-field current.

Example 12–2.—The no-load losses of the generator in Example 12–1, including shunt-field circuit loss, amount to 15 kw. The armature-circuit resistance at 31.6 C is as follows:

% Full-load current	Res. in Ohms	% Full-load current	Res. in Ohms
125	0.0035	50	0.0042
100	0.0037	25	0.0046
75	0.0039		

Calculate the efficiency of this generator at rated load at 75 C.

Solution.—The full-load current is

$$\frac{\text{kw} \times 1000}{V} = \frac{500{,}000}{250} = 2000 \text{ amp}$$

At full-load current and 75 C, the armature-circuit resistance is 0.0043 ohm. The generated voltage is then

$$250 + (0.0043 \times 2000) = 258.6 \text{ volts}$$

Assume that, from the curve of generated voltage vs. armature power input, the power input at 258.6 volts is 15,000 watts; and assume that the stray load loss is $0.01 \times 500{,}000 = 5000$ watts. Then,

$$\text{Conventional efficiency} = \frac{\text{Output}}{\text{Output} + \text{Losses}}$$

$$= \frac{2000 \times 250}{(2000 \times 250) + 15{,}000 + (2000^2 \times 0.0043) + 5000}$$

$$= 0.930 \text{ or } 93.0\%$$

The foregoing solution includes numerous small losses, such as pole-face losses and eddy-current losses in the armature conductors, and also the increased shunt-field current necessary to obtain the rated voltage when the generator is loaded.

When it is not practicable to measure the armature-circuit resistance at the requisite percentage of rated current, it is recommended that the resistance of only the copper portion of the armature circuit be used in calculating the $I_a^2R_a$ loss. To this loss then must be added $2I_a$, which is the brush-contact loss discussed on page 144.

Ratings and Standards.—All motors and generators are guaranteed to perform satisfactorily when operated within certain arbitrary limits. The temperature limits are usually specified on the name plate. The guarantees usually allow a departure in speed or voltage of 10 per cent of the name-plate value in the case of constant-speed machines.

Series motors are used frequently for varying-speed duty in which the power demand changes rapidly through a wide range. Motors for such service have an *intermittent* rating for perhaps ½ hour or 2 hours. Their rating in horsepower indicates their ability to develop the required torque at rated speed without commutating distress, but the time specified indicates the limits of duration of rated load beyond which excessive temperature will occur.

Permissible Temperatures for Insulation.—The full-load rating of a motor or generator is that load which it can deliver without exceeding the safe temperature of the insulation on the windings. Many insulating materials are used in various combinations, and the temperature permissible for any combination is that permissible for the material in it which has the lowest safe rating. The National Electrical Manufacturers Association (NEMA), which is an organization of electrical manufacturers formed for the purpose of setting manufacturing standards for all types of electrical apparatus, and the American Institute of Electrical Engineers have grouped the various materials into classes according to permissible operating temperatures. These classes are *O*, *A*, *B*, *C*, and *H*.

Class O: This class of insulation consists of cotton, silk, paper, and similar organic materials when neither impregnated nor immersed in liquid dielectric. The term organic is defined as a chemical compound containing carbon in some form. The maximum safe temperature for this material is 90 C. The 90 degrees are generally thought of as being composed of 40 C ambient, 40 degrees average rise over ambient, and 10 degrees hot spot over average rise. The standard ambient temperature of 40 C has been selected because it is near the maximum encountered in most parts of the United States. Should the ambient be at some temperature lower than 40 C, the average rise may be increased by the difference between the actual ambient temperature and 40 C. Machines with Class-O insulation are called 40-degree machines.

Class A: This classification includes: (1) all insulation materials listed under Class O when impregnated or immersed in a liquid dielectric; (2) molded and laminated materials with cellulose filler, phenolic resins, and other resins with similar properties; (3) films and sheets of cellulose acetate and other cellulose derivatives having similar properties; (4) varnishes (enamel) as applied to conductors. By a "rule of thumb" it is considered that the life of this type of insulation is halved with every increase in temperature of 8 degrees C. The limiting hot-spot temperature for this class is 105 C. Machines with Class-A insulation are called 55-degree machines.

Class B: Class-B insulation consists of inorganic materials with organic substances as binders and a small percentage of Class-A materials for structural purposes. Mica, asbestos, fiberglass, and similar inorganic materials are included in this class. The binder is generally recognized as being the temperature-limiting part in most Class-B insulation. The limiting hot-spot temperature for this class is 105 C. The average life of this class is said to be halved for every increase in temperature of 10 degrees C. Machines with Class-B insulation are called 80-degree machines.

Class C: This class of insulation includes only inorganic materials, such as porcelain, quartz, glass, and mica. The temperature stability of these materials is so great that no limiting hot-spot temperature is specified. The performance of the machine, rather than its temperature, usually serves to limit the maximum operating load.

Class H: This class includes almost all Class-B materials, but the binding substance is composed of silicone compounds or materials of equivalent properties. The recommended limiting hot-spot temperature for this class is 180 C.

The temperature specified on the name plate of power machinery is the temperature rise over ambient permissible for the class of insulation used. The temperature is measured on the outside of the insulation by a mercury-in-glass thermometer.

A number of books known as the *NEMA Standards* are published by the National Electrical Manufacturers Association. In addition to the permissible temperature rises, *NEMA* has established horsepower ratings of motors and many other standards of performance and design.

Rating of Enclosed Motors.—Motors having no restriction to ventilation other than that necessitated by good mechanical construction are called *open motors.* When the ventilating openings in the frame of a motor are covered with wire screen, expanded metal, or perforated plates, the openings in which do not exceed $\frac{1}{2}$ sq in. in area and are of such shape as not to permit the passage of a rod larger than $\frac{1}{2}$ in. in diameter (except in special cases), the motor is called a *semi-enclosed motor.* A semi-enclosed motor is illustrated in Fig. 12–1. Motors so enclosed as to prevent circulation of air between the inside and the outside of the case, but not covered sufficiently to be termed air-tight, are called *totally enclosed* motors.[1]

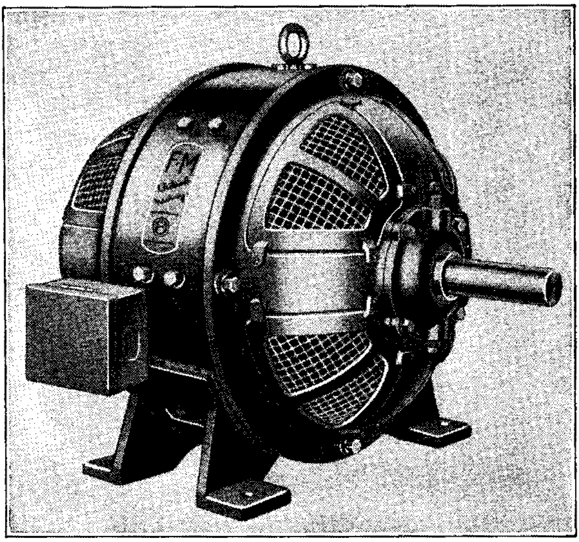
(Courtesy Fairbanks, Morse & Co.)

Fig. 12–1. Semi-Enclosed Motor

Semi-enclosed motors are used where it is necessary to protect the internal parts against injury from foreign objects or to prevent personal injury to workmen. Totally enclosed motors are used where it is necessary to protect the internal parts completely against dust or corrosive vapors.

Explosion-Resisting Motors.—Totally enclosed motors are not necessarily gas-tight. For operation in atmospheres of petroleum derivatives (gasoline, etc.), lacquer-solvent vapor, etc., specially enclosed motors,

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[1] For further classification of motors, see Standards of the American Institute of Electrical Engineers, No. 5.

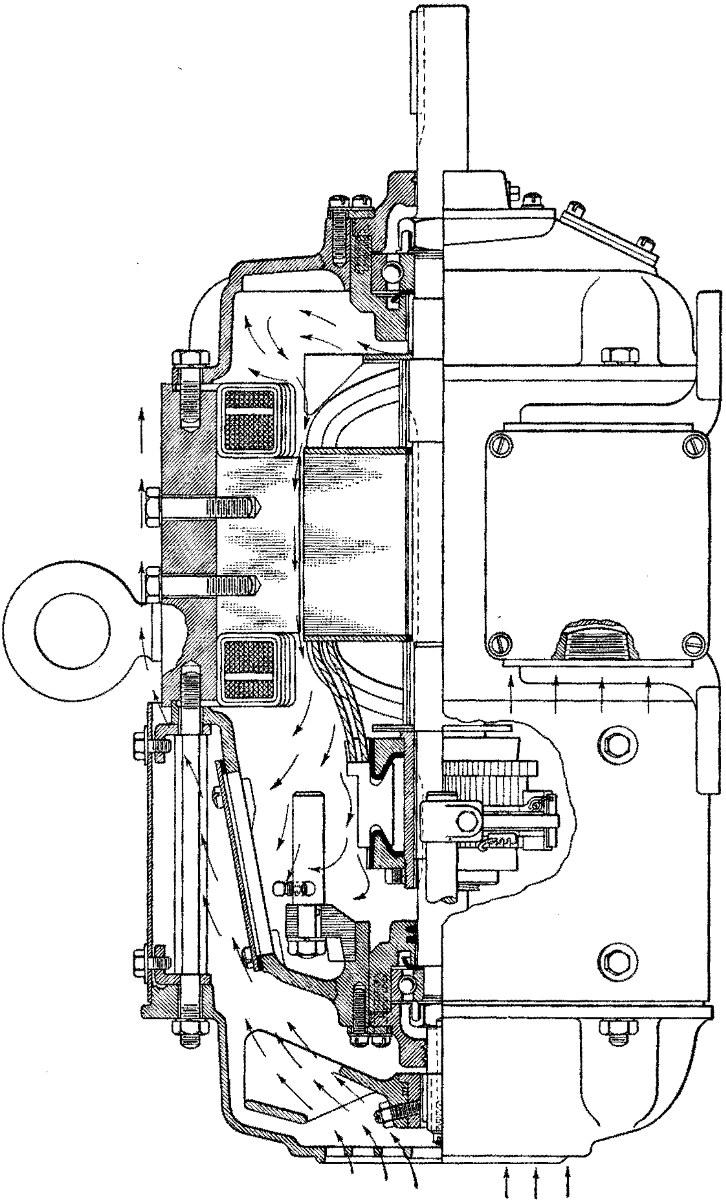


Fig. 12-2. Totally Enclosed Fan-Cooled Motor

known as *explosion-resisting motors*, are required by the National Board of Fire Underwriters.

Totally Enclosed Fan-Cooled Motors.—The horsepower rating of a totally enclosed motor may be restored almost to its "open" rating by means of a fan arranged to blow air over the outside of the frame. Another fan inside the motor is used to agitate and circulate the enclosed air to bring the heat from the windings to the cooling surfaces. A totally enclosed fan-cooled motor is shown in the cut-away drawing in Fig. 12–2.

Full-Load Currents.—See the Appendix for full-load currents of direct-current motors.

The continuous rating of a totally enclosed motor is about 60 per cent of its rating when open. If an open type motor be semi-enclosed, its rating will be reduced somewhat, the amount depending on the degree of enclosure.

PROBLEMS

1. A separately excited generator requires 69.7 hp to drive it at full load of 100 amp and 500 volts. Its armature resistance is 0.1 ohm. What would be the continuous rated horsepower of this generator when run as a motor on a 550-volt line?

2. Give instructions for calibrating a 230-volt d-c motor.

3. A 1-hp, 220-volt, 1800-rpm shunt motor has an armature resistance of 2.2 ohms and a field resistance of 320 ohms. When running without load, but at rated voltage, the armature current is 2.0 amp. What is the conventional efficiency when the load is such that the armature current is 6 amp.?

4. A 25-kw, 230-volt shunt generator has a full-load speed of 1200 rpm. Its friction and windage and iron loss is 700 watts. The resistance of the armature circuit at full load is 0.15 ohm. The field current is 2.5 amp. Calculate the full-load efficiency.

5. A 75-kw, 250-volt shunt generator has a windage, friction, and iron loss of 4750 watts at rated speed. The armature-circuit resistance at full load is 0.05 ohm. The field-circuit resistance is 50 ohms. Calculate the full-load efficiency.

6. A centrifugal pump with an efficiency of 59% is designed to deliver 3000 gal of water per minute to an overhead tank against an effective pressure head of 50 ft and a suction lift of 10 ft. Calculate the horsepower required of the driving motor, and select the proper standard rating from the table in the Appendix.

7. What horsepower would be required of a motor to operate a hoist that is to lift 10 tons at the rate of 50 ft per min? The efficiency of the hoist is 72%.

8. A 30-hp, 230-volt shunt motor has a full-load armature current of 110 amp. The armature-circuit resistance, not including brushes, is 0.125 ohm. The no-load armature current is 4.2 amp. Calculate the horsepower output when the armature current is 90 amp.

9. A 230-volt shunt motor, rated 50 hp, has an armature current of 180 amp at full load. When the motor is run at no load at rated speed and field current, the armature current is 8 amp and the voltage across the armature is 110 volts. The field current is 2.4 amp. The armature-circuit resistance, exclusive of brush contact, is 0.1 ohm. Calculate the efficiency at full load. *Ans. 86.2%*

CHAPTER 13

Mechanical Connection Between Motor and Load

Direct Drive and Gear-Head Motors.—Direct connection of a motor to its load through a rigid shaft coupling or a cushioning flexible coupling is most desirable if the rated motor speed is not too much higher than that of the shaft to be driven. Specially designed motors are available for unit assembly with the driven machine. If the rated speed of the motor relative to the driven shaft is too high, a *geared-head* unit with motor and gear reduction assembled together may be had.

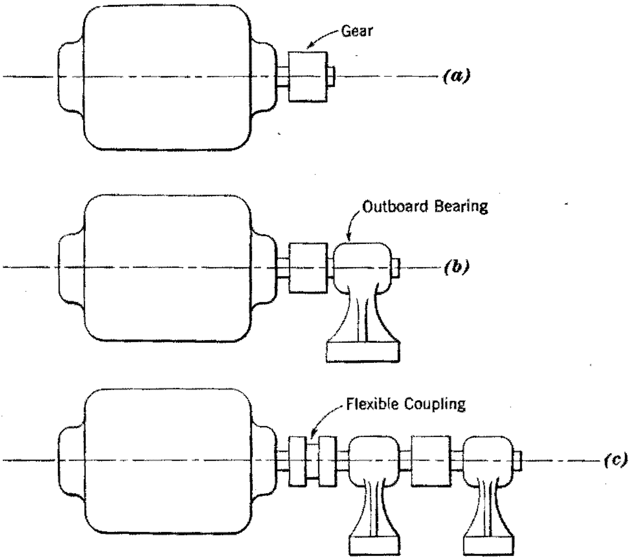
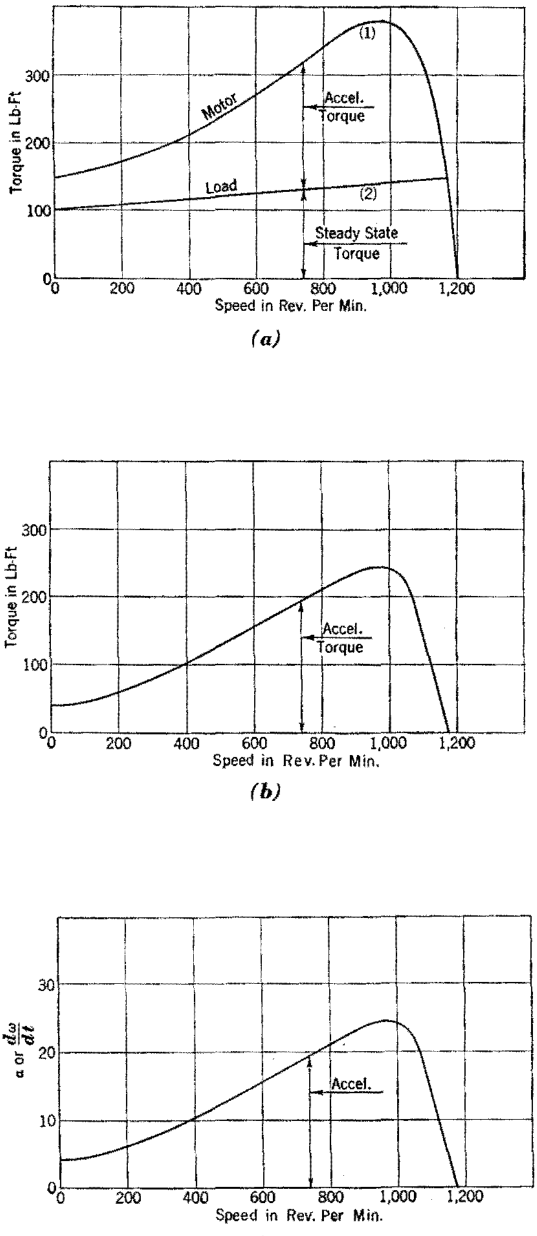


Fig. 13-1. **Methods of Gear Mounting for Motor Drive**

Belt Drives.—A series motor should not be connected to its load by a belt because failure of the belt would permit the motor to reach a dangerous speed. A shunt or compound motor may be used with belt drive. Where a great speed reduction is obtained by using a very small driving pulley, excessive belt tension must be avoided in any attempt to prevent intolerable slipping. Such tension may cause the adjacent motor bearing to be overloaded. Also, a pulley which is too long may allow the center of belt tension to fall too far from the motor bearing and thus cause excessive



(a)

(b)

(c)

$$T = \mathfrak{I}\alpha$$

$$\mathfrak{I} = \frac{WR^2}{32.2}$$

$$\text{Time} = \int_{\omega_2}^{\omega_1} \frac{dt}{d\omega}\, d\omega$$

W is in pounds

R is in feet

Time is in seconds

Fig. 13–2. Acceleration Curves for Motor and Load shaft deflection. The latter difficulty may be avoided by providing an *outboard* bearing on the motor shaft. Because of the evils of excessive belt tension and dangerous shaft moments, a minimum diameter and a maximum width of pulley for any motor will be specified by the manufacturer on request.

The economic aspects of group drive of machinery by belts vs. individual drive will be discussed in Chapter 23.

Gear Drives.—Because of vibration and shaft deflection, a pinion on the unsupported end of a motor shaft, as indicated in Fig. 13–1 (*a*), should not be used to drive a load greater than 75 hp. In applications requiring more than 75 hp, it is advisable to use either an outboard bearing, as Fig. 13–1 (*b*), or two outboard bearings and a flexible coupling, as Fig. 13–1 (*c*).

Calculation of Acclerating Time.—It is sometimes necessary to predict the time required to accelerate a load by means of a motor of known speed-torque characteristics, or it may be necessary to produce a given speed in a specified time. The total energy required to accelerate a load to full speed may be allotted to two classifications, namely, the kinetic energy of inertia of the motor and load, and the friction loss or its equivalent in the driven machine. If the power required to drive the machine at any constant speed equal to or less than full speed is not known, it may be determined approximately by analysis of the load or may be determined accurately by test. It is also necessary to know the equivalent moment of inertia of the driven machine as well as that of the driving motor.

Let the torque required to drive a given load at a series of constant speeds be given by the load curve (2) in Fig. 13–2 (*a*). For this purpose the relatively simple speed-torque curve of an induction motor (described in Chapter 18) has been used rather than the stepped curve of a direct-current motor, which would be more difficult to use. Let the speed-torque characteristic of the driving motor also be as shown in Fig. 13–2 (*a*) by curve (1). At any speed, the torque of the motor performs two functions, namely, provides the power necessary for steady state at that speed, and provides the power of acceleration. The torque available for acceleration, as obtained by taking the difference between the ordinates to curves (1) and (2) in Fig. 13–2 (*a*), is plotted in (*b*). The acceleration, which is determined by dividing the accelerating torque by the moment of inertia, is plotted in (*c*). Here, the acceleration α, or $\frac{d\omega}{dt}$, is in radians per second per second.

In general,

$$T = I\alpha \tag{13–1}$$

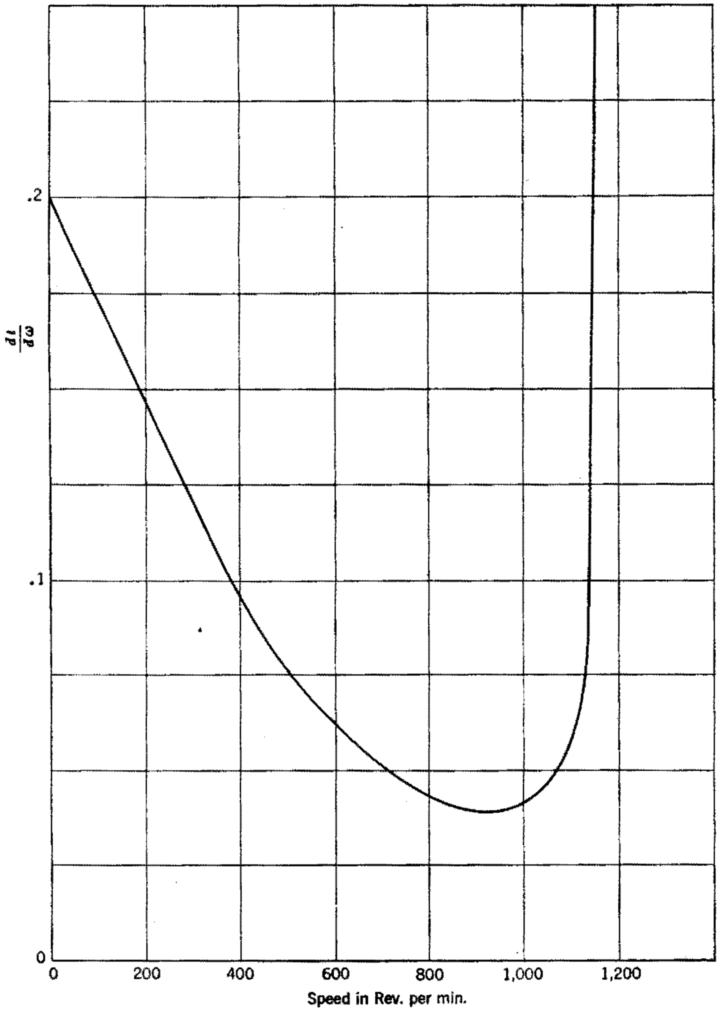


Fig. 13–3. Reciprocal of Acceleration

in which T = torque, in pound-feet;
$\mathbf{I}$ = moment of inertia;
α = motor acceleration, in radians per second per second.

The curve of Fig. 13–3 is a reciprocal of the curve of Fig. 13–2 (*c*). The total time required to attain any speed from any initial speed is the integral of the function $\frac{dt}{d\omega}$ of Fig. 13–3, with respect to ω, between the initial and desired speeds. The curve of $\frac{dt}{d\omega}$ vs. speed is asymptotic to a vertical line at the steady-state ultimate speed. Theoretically the motor would continue to accelerate indefinitely. However, the time required to attain a speed of, say, 98 per cent of the ultimate speed may be readily calculated and gives results sufficiently accurate for most purposes.

Example 13–1.—A centrifuge has a steady-state torque requirement as a function of speed, as shown by curve (2) of Fig. 13–2 (*a*). The moment of inertia of the motor and all connected parts is 10. Calculate the time required for the motor with the speed-torque characteristic shown by curve (1) in Fig. 13–2 (*a*) to accelerate the centrifuge to 98 per cent of its ultimate speed.

Solution.—Fig. 13–2 (*b*) is a plot of the difference between curves (1) and (2) of Fig. 13–2 (*a*), and represents the accelerating torque. The acceleration is

$$\alpha = \frac{T}{I}$$

Fig. 13–2 (*c*) is a plot of the acceleration α in radians per second per second against speed. Fig. 13–3 is a plot of reciprocal values from Fig. 13–2 (*c*). Synchronous speed of 1200 rpm is a speed ω of $\frac{2\pi \times 1200}{60} = 125.8$ radians per sec. In Fig. 13–3 the ordinates $\frac{dt}{d\omega}$ are plotted to a scale of 1 in. = 0.05; and the abscissas are plotted to a scale of 1 in. = 400 rpm, or 1 in. = 41.9 radians per sec. Hence, 1 sq in. under the curve of Fig. 13–3 equals $41.9 \times 0.05 = 2.095$ sec. There are 5.3 sq in. under that curve to 98% speed. The time required to accelerate to 98% speed is $2.095 \times 5.3 = 11$ sec.

CHAPTER 14

Conversion of D-C Generator to D-C Motor

Similarity of Motor to Generator.—Inasmuch as the principal parts, except the shunt-field winding, of a machine built for use as a motor are identical with those intended for use in a generator, a d-c generator may be used as a d-c motor with only small sacrifice in performance. The shunt field of a machine designed for generator use is stronger than if the machine is intended for motor use, and therefore the possible operating-speed *range* by shunt-field control as a motor is somewhat greater than normal. Also a machine with field coils designed for a motor will, when used as a self-excited generator, have a maximum voltage lower than may reasonably be expected of a machine designed for use as a generator. In converting a motor to a generator, or vice versa, it is usually necessary to change the connection of the armature and the various field windings relative to each other.

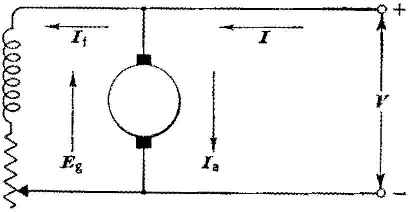
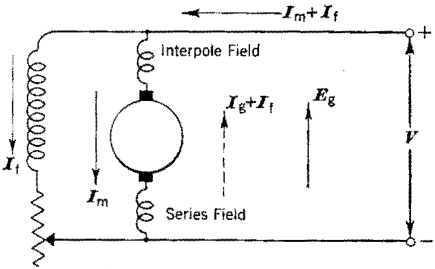


Fig. 14–1. Direction of Currents in a Shunt Motor

Conversion of Shunt Machine.—A shunt machine used as a motor with line polarity as shown in Fig. 14–1 will have currents in its armature and field windings as indicated. If the motor be disconnected from the power supply, but its armature speed be maintained by an external driver, the back emf E_g will maintain I_f in the same direction and almost as great as before. Hence, a d-c shunt motor will operate as a self-excited generator in the same direction of rotation without changing the direction of the field flux and without changing any internal connections. If it were necessary to reverse the direction of rotation from that when used as a motor, it would also be necessary to reverse the field-coil connections to the armature in order that the voltage might build up as explained on page 113.

Conversion of Compound Interpole Type Machine.—A compound interpole machine used as a motor with line polarity as shown in Fig. 14–2 will have currents in its armature and field windings as indicated. If this motor be disconnected from the power supply, but its speed be maintained by an external driver, the motor will become a self-excited generator in the same manner as that of Fig. 14–1. If then the generator be connected to a load, the load current I_g (together with the shunt-field current) will flow through the armature in a direction opposite to the direction when used as a motor.

If the motor in Fig. 14–2 was connected cumulative compound (in which the series field aided the shunt field), the reversed direction of current in the series field of the generator would weaken the total field and so cause the generator to be differential compound. The current through the interpole winding would also be reversed. However (page 131), when the machine is used in the same direction of rotation and with the same main-pole polarity, the interpole polarity for motor use must be opposite to that for generator use, and hence no change is necessary in the interpole connections. In the case of Fig. 14–2 the series-field connections must be reversed if the generator is to be cumulative compound. Current is said to *enter* a motor at the positive terminal and to *leave* a generator at the positive terminal.



I_m = current as a motor
I_g = current as a generator

Fig. 14–2. Direction of Currents in a Compound Motor

The maximum voltage obtainable by self-excitation of a motor used as a generator is seldom more than 5 per cent greater than the rated voltage of the machine as a motor.

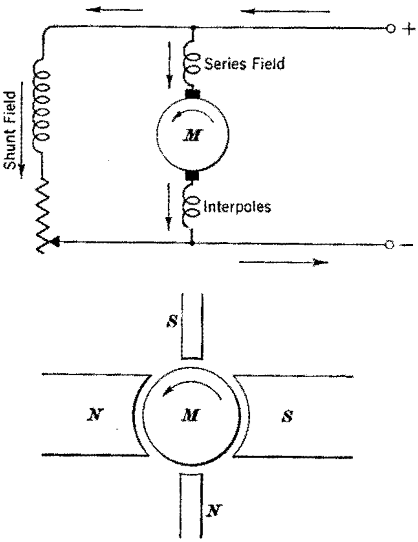


Fig. 14–3. Machine of Example 14–1 Used as a Motor

Example 14–1.—A cumulative compound interpole motor is to be operated as a cumulative compound generator in the same direction of rotation, but the terminal which was positive as a motor is to be negative as a generator. Give instructions for accomplishing the change by reversal of remanent magnetism.

Solution.—Let the two diagrams of Fig. 14–3 show the magnetic polarities, current directions, and direction of rotation in the machine used as a motor. Let the two diagrams of Fig. 14–4 show the magnetic polarities, current directions, and direction of rotation of the machine after all the changes have been made. The conditions of Fig. 14–4 will be determined as the solution progresses.

The armature-generated emf in the motor is toward the positive terminal (always opposite to the direction of the armature current). With the same direction of rotation as before, the reversal of remanent magnetism as specified in the problem produces the required terminal polarity by reversal of the generated emf. The shunt-field current will hence be reversed, as in Fig. 14–4, and the generator will proceed to build up its voltage from the reversed remanent magnetism. Current in the series field must flow in the direction opposite to that it had for the original magnetic polarity. This it does not do; and so the series field must be reversed, as in Fig. 14–4. Each interpole in a generator must be of the same polarity as the main pole which it precedes in the direction of rotation. Since the current through the interpoles has not been reversed, their polarities are correct as shown, and their coil connections need not be changed.

The instructions for converting the motor to a generator are as follows:

1. Reverse the remanent magnetism of the field by separate excitation of the shunt winding only.

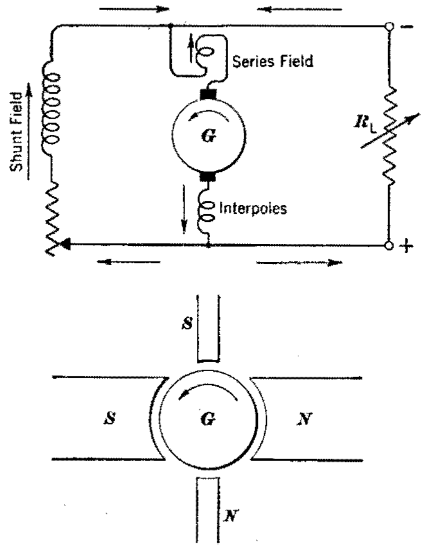


Fig. 14–4. Machine of Example 14–1 Used as a Generator

2. Reverse the series-field coil connections.

A self-excited generator cannot reverse its own remanent magnetism because its ability to produce demagnetizing current in its own shunt-field winding is dependent on the presence of some flux. After the interpoles of a direct-current machine have been properly connected in series with the armature, no subsequent conversion will require changing that connection.

PROBLEMS

1. A cumulative compound interpole motor connected for operation in a clockwise direction is to be driven as a cumulative compound generator in a counter-clockwise direction with reversed polarity. Give instructions for making necessary changes without reversing the remanent magnetism.

2. A cumulative compound interpole, self-excited generator is to be used as a cumulative compound motor in the same direction of rotation. Give instructions for making necessary changes in connections.

3. Two compound generators in parallel are used to energize a factory power line. The driving engine of one generator is stopped before the generator is disconnected from the line. Analyze the probable results.

CHAPTER 15

Control of Direct-Current Motors

Starting[1] of D-C Motors.—A very small direct-current motor with low armature inertia and relatively high armature-circuit resistance may be started by connecting it without intervening resistance directly to the power supply. Any other type of d-c motor must be started with the armature in series with a rheostat of sufficient size to limit the starting current to a safe value, or the armature must be connected to a variable-voltage power source which will accomplish the same result. Such a device is called a *starting rheostat.*

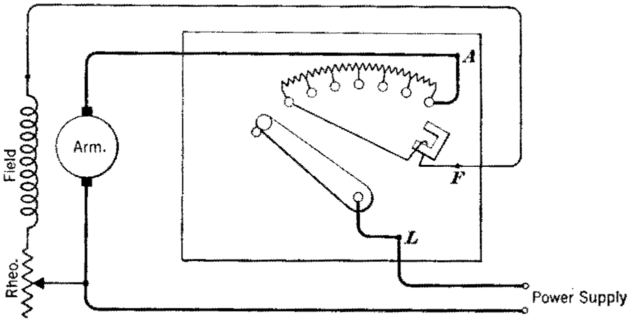


Fig. 15-1. **Three-Point Starting Rheostat**

It is a common practice to limit the starting current for a d-c motor to about 1½ times the full-load rated current. Inasmuch as no back emf is present in the motor at the instant of start, the starting rheostat must contain such resistance that, when it is added to the resistance of the armature circuit, the initial current will be limited to a safe value. These starters are rated according to the size and voltage of the motor which they are intended to start. The resistance of a 10-hp starter would be too small for starting a 5-hp motor, and such a starter might cause distress at the commutator. The resistance of a 10-hp starter would be too large for starting a 20-hp motor, and such a starter might be burned out before the motor reached full speed.

Starting rheostats are intended only for the intermittent duty of starting. No more than 15 seconds should be consumed in moving the control handle from the first point of contact to the final position. When the

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[1] For further study of motor controllers, see *Control of Electric Motors,* by P. B. Harwood. John Wiley and Sons.

starting cycle must be repeated many times in rapid succession, as in crane or dredge service, a special resistance unit adapted to continuous service must be used.

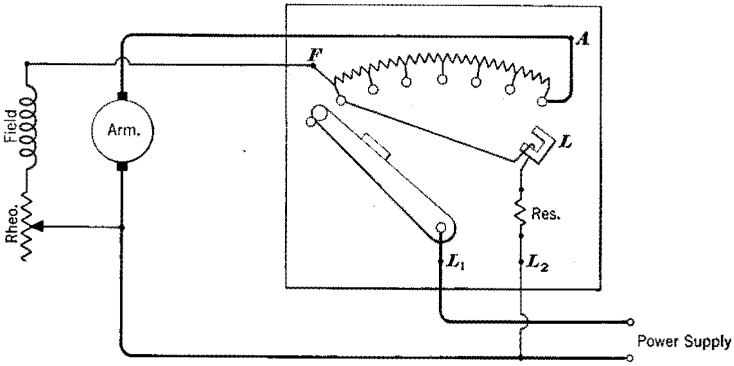
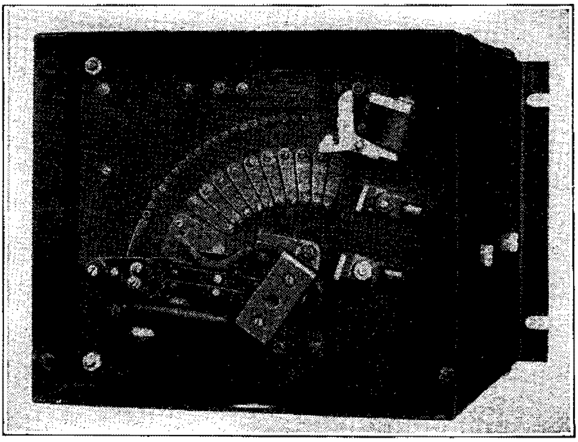


Fig. 15–2. **Four-Point Starting Rheostat**

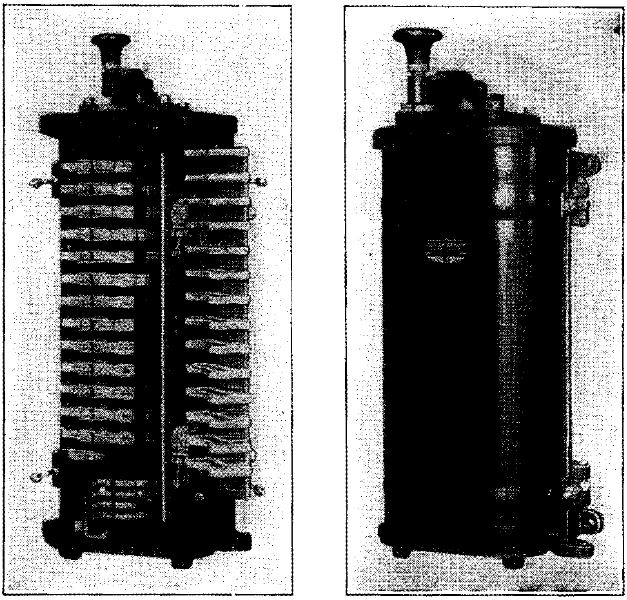
In Fig. 15–1 is shown a *three-point* starting rheostat, to which only three connections are required. The solenoid or holding coil, which holds the starting handle in the running position, is connected in series with the motor field. By this connection a break in the shunt-field circuit also opens the holding-coil circuit and permits the starting handle to return to the "off" position; and over-speeding of the motor is thus prevented.



(Courtesy Cutler-Hammer, Inc.)

Fig. 15–3. **Starting and Speed-Regulating Rheostat**

In Fig. 15–2 is shown a *four-point* starting rheostat, to which four connections are required. This type of rheostat is required by adjustable-speed motors in which the shunt-field current may be so greatly reduced at high speeds that a series holding coil would release the starter handle. In the four-point starter the holding-coil current is independent of motor operating conditions. The four-point starter is suitable for starting a series motor, whereas the three-point starter obviously is not.



(*Courtesy Cutler-Hammer, Inc.*)

Note the arc barriers in the opened position to permit inspection and repair.

Fig. 15–4. Drum Controller

In both the three-point starter and the four-point starter, the handle is returned to the "off" position by a spring if the line voltage becomes too low.

Some starters or controllers, like that illustrated in Fig. 15–3, are intended for adjustable-speed duty and are designed so generously that the starting handle may be left on any control point indefinitely without causing overheating. Such controllers cost more than simple starters and, when such equipment is purchased, it is advisable to specify the type of duty—whether simple starting or combined starting and speed control. By permitting some of the starting resistance to remain in the circuit, it is possible to obtain motor speeds lower than those which may be had by shunt-field control. The efficiency of such control is poor because of the loss of power in the rheostat. Also, the speed regulation of a motor controlled by series resistance in the armature circuit is high (great), and this type of control is thus rendered unsuitable for many purposes.

Drum Controllers.—Drum controllers, like that in Fig. 15–4, are used on electric passenger coaches, cranes, hoists, dredges, etc., where extreme ruggedness must be combined with great current-carrying capacity. They are frequently provided with vertical operating handles geared to the drum, so that the movements of the operator's hand may more naturally be in the direction in which the controlled device, such as a dredge, is intended to move.

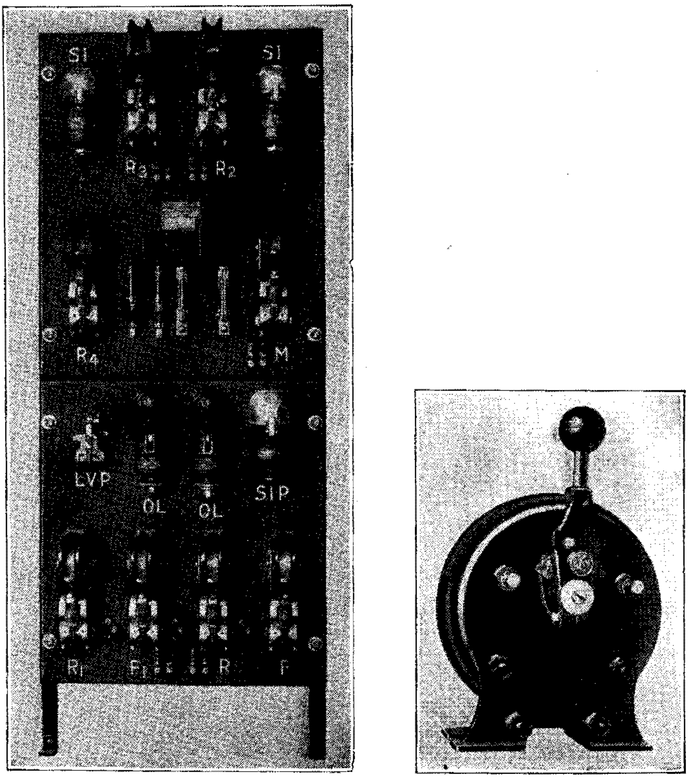


Fig. 15–5. Automatic D-C Motor Starter

Automatic Starters.—When hand-operated starters are used by careless or unskilled operators, the controlled device may be damaged by too rapid starting, or the starter itself may be overheated by too slow starting. Automatic starters operated by push-buttons eliminate the personal element of hand-starters and assure proper control of the starting time. Another advantage of automatic starters is the ease with which they may be operated from one or more distant points. Fig. 15–5 shows a typical automatic starter or controller.

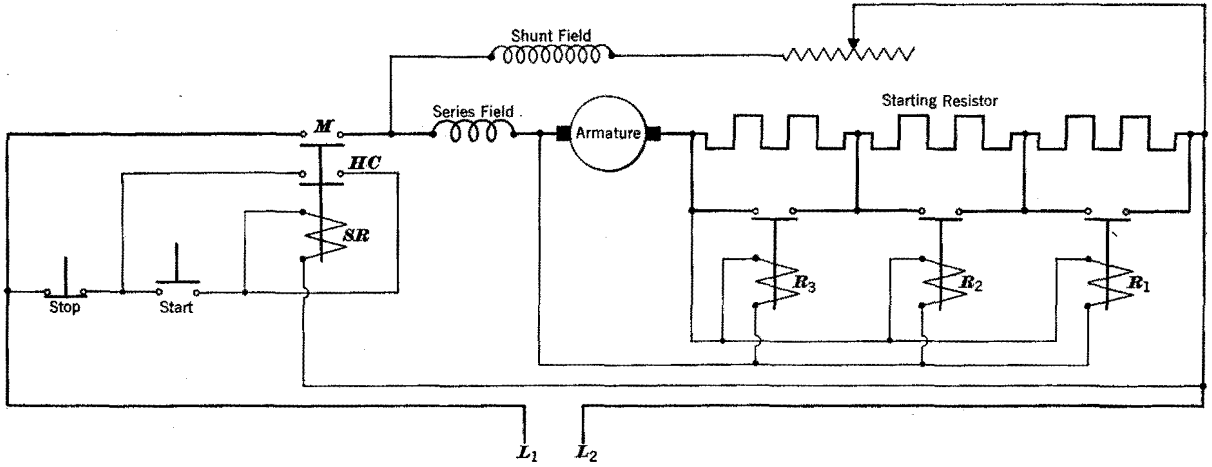


Fig. 15-6. Wiring Diagram of Counter-emf Automatic D-C Motor Starter

All types of automatic starters for d-c motors are provided with one or more solenoids which close contacts in order to short-circuit sections of the starting series resistance. Small motors rated at only a few horsepower may be started rapidly, and their starters may short-circuit the starting resistance in only one or two steps. Larger motors require smoother acceleration, and their starters must have many solenoids to short-circuit only small sections of resistance successively. The reversing starter shown in Fig. 15–5 short-circuits the starting resistance in three steps by closing solenoid-operated switches R_2, R_3, and R_4 consecutively.

Types of Automatic Motor Starters.—There are four general types of direct-current automatic motor starters, which differ mainly in the means used to initiate the short-circuiting of the sections of the starting resistor. The principal types of starters are the following:

Counter-emf starters
Series-relay starters
Series-lockout starters
Definite-time-interval starters

Counter-emf Starters.—As a motor accelerates, its counter emf increases and the voltage E_a across the armature increases almost in proportion to the increase in motor speed. In Fig. 15–6 is shown the wiring diagram of a counter-emf type of motor starter. The starting relay *SR* closes the main starting switch *M* when the starting contacts are closed. The holding contacts *HC* short-circuit the starting-button contacts and hold the circuit closed through the coil of *SR* after the starting button is released. The motor starts and accelerates until the voltage E_a across the armature rises to the voltage at which the relay R_1 is designed to close. Relay R_1 is usually designed to close at about one-half the rated voltage of the motor. This voltage occurs at about one-half the rated speed. Relays R_2 and R_3 are designed to close at about 80 per cent and 90 per cent, respectively, of rated voltage and rated speed. The relays R_1, R_2, and R_3 are usually made adjustable in order that the motor speeds at which they close may be selectively chosen for the particular characteristics of the service for which the motor is installed.

Counter-emf starters are unsatisfactory in sizes above about 3 hp because the closing pressures on the contacts of the relays are small and the large currents taken by larger motors are likely to cause the contacts to weld together so that they cannot be opened.

Series-Relay Starters.—As a motor accelerates, its counter emf increases and causes the armature current to decrease. In Fig. 15–7 is shown the wiring diagram of a series-relay type of starter which depends on decreasing armature current for its operation. When the starting

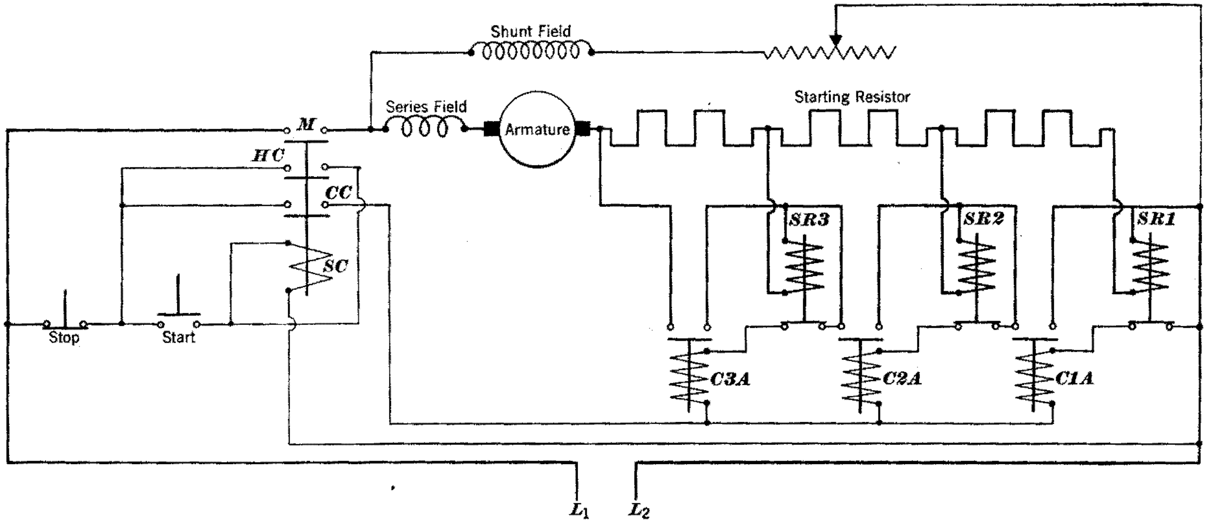


Fig. 15-7. Wiring Diagram of Series-Relay Automatic D-C Motor Starter

button is pushed, the relay *SC* closes its contacts. Contacts *M* close the armature circuit and cause the motor to start. Contacts *HC* are holding contacts which short-circuit the starting-button contacts. Contacts *CC* close the circuit on one side of the coils of relays *C1A*, *C2A*, and *C3A*. Relays *C1A*, *C2A*, and *C3A* are high-inductance relays, which act slowly; and so the quick-acting relay *SR1* in series with the armature opens its contacts before relay *C1A* can close. As the motor accelerates, its armature current decreases to the point at which relay *SR1* is no longer held open and its contacts close and energize *C1A*. Relay *C1A* closes its contacts and thereby short-circuits *SR1* and also the first section of the starting resistor. The whole armature current then flows in *SR2* and causes it to open its contacts before slow-acting relay *C2A* can act. The initial inrush of current in *SR2* then subsides as the motor further accelerates, until the armature current is no longer large enough to hold *SR2* closed. This performance is repeated by the other relays, *SR3*, *C2A*, and *C3A*, until all of the starting resistance has been short-circuited and the motor is connected directly to the line.

The closure of the contacts of relays *C1A*, *C2A*, and *C3A* is firm, and the danger of contact welding is much less than it is with the counter-emf type of starter. The series-relay type of starter has the disadvantage of having a multitude of relays, which require an unusual amount of attention in their maintenance.

Series-Lockout Starters.—Some of the undesirable features of the series-relay type of starter are avoided in the series-lockout type shown by diagram in Fig. 15–8. This type of starter makes use of a special type of relay containing two coils in series. Each coil is on a separate magnetic frame and produces a separate magnet. One magnet used, as *HC1*, *HC2*, or *HC3* of Fig. 15–8, is called the holding magnet; and, when energized, it acts to close the relay contacts. The other magnet used, as *LO1*, *LO2*, or *LO3*, is called the lockout magnet, and acts simultaneously to open the relay contacts. The holding magnet is designed for high magnetic leakage and is magnetized by the armature current to a point high above the knee on its magnetization curve. The lockout magnet is designed so as to provide a large air-gap in its magnetic circuit. It is magnetized by the motor inrush current only to a point just below the knee on its magnetization curve. When the magnets are magnetized simultaneously by the motor inrush current, the lockout magnet is the stronger of the two; and the contacts are held open, and all of the starting resistor is left in the circuit. As the inrush current decreases, the lockout magnet loses strength more rapidly than does the holding magnet, until a current value is reached at which the holding (closing) magnet strength predominates and closes the relay contacts.

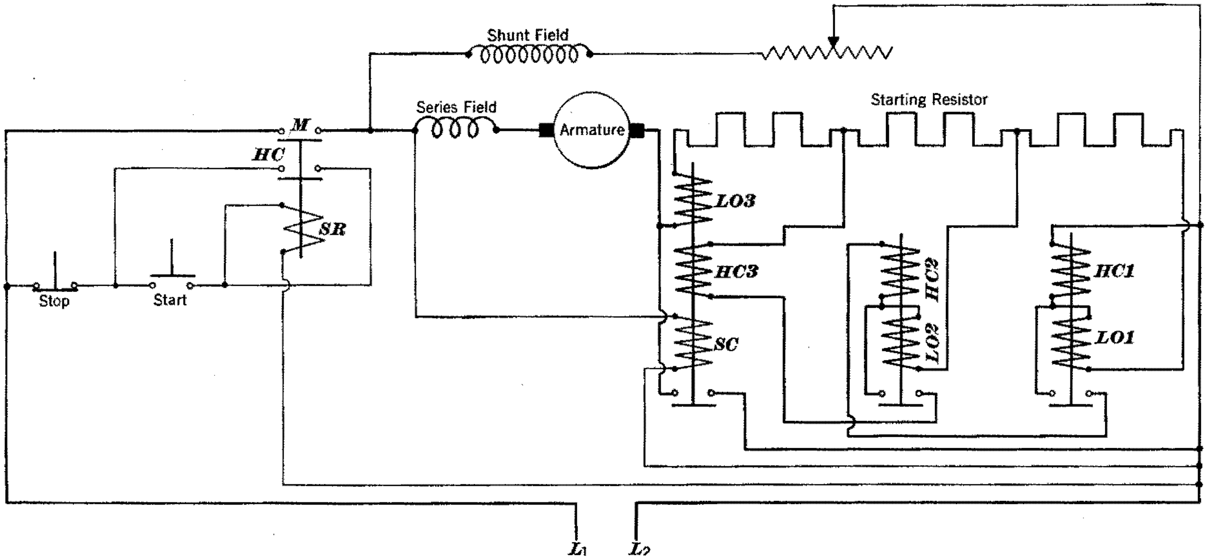


Fig. 15–8. Wiring Diagram of Series-Lockout Automatic D-C Motor Starter

When the starting button is pushed, the starting relay *SR* in Fig. 158–closes the main motor contact switch *M* and also the usual holding contacts *HC*. The starting inrush current flows through *LO3*, the starting resistor, *LO1*, and *HC1*. When the starting current has decreased sufficiently, *HC1* closes the relay contacts and almost completely short-circuits the first section of the starting resistor. The armature current then flows through *LO3*, two-thirds of the starting resistor, *LO2*, *HC2*, and *HC1*. When the inrush current decreases again sufficiently, *HC2* closes its relay contacts. The cycle is repeated by *LO3* and *HC3*. When *HC3* closes its contacts, all series coils are de-energized and the *HC3* contacts are held closed by a small shunt coil *SC*.

The series-lockout starter is simpler than the series-relay starter and requires no interlock relays or fingers. The series-lockout type of starter is not well adapted to the control of variable-speed motors.

Definite-Time-Interval Starters.—The three types of automatic motor starters previously described have one common fault under some starting conditions. If the motor should fail to accelerate at any point in the sequence of starting because of overload or because of low line voltage, the changes in voltage or in current which initiate the successive steps of operation of the starter would not take place and the starting cycle would not be completed. In the definite-time-interval starter, the time-delay between the successive closings of the contactors which short-circuit the starting-resistor sections is a fixed interval, regardless of the magnitude of current or of voltage changes in the motor. In Fig. 15–9 is shown a wiring diagram of a definite-time-delay (interval) starter. This type of starter will cause a motor to accelerate to its proper speed or to be disconnected from the power line by overload protective devices.

When the starter button is pushed, the relay *SC* closes its contacts. Contacts *M* close the armature circuit and cause the motor to start. Contacts *HC* are the usual holding contacts which short-circuit the starting-button contacts. At the same time, contacts *PM1* close and start the pilot motor, which rotates the contact drum slowly away from position 1 toward position 2. In position 2, contact 3 closes and energizes relay R_1, which short-circuits the first one-third of the starting resistor. In positions 3 and 4 of the contact drum, relays R_2 and R_3 are caused to short-circuit the remaining sections of the starting resistor. The pilot motor is stopped by the opening of its circuit in drum position 1.

Automatic starters of this type are suited to applications in which more than one motor is to be started at the same time. Corresponding additional contact fingers and starting resistors must be provided on the contact drum when multiple starting must be accomplished.

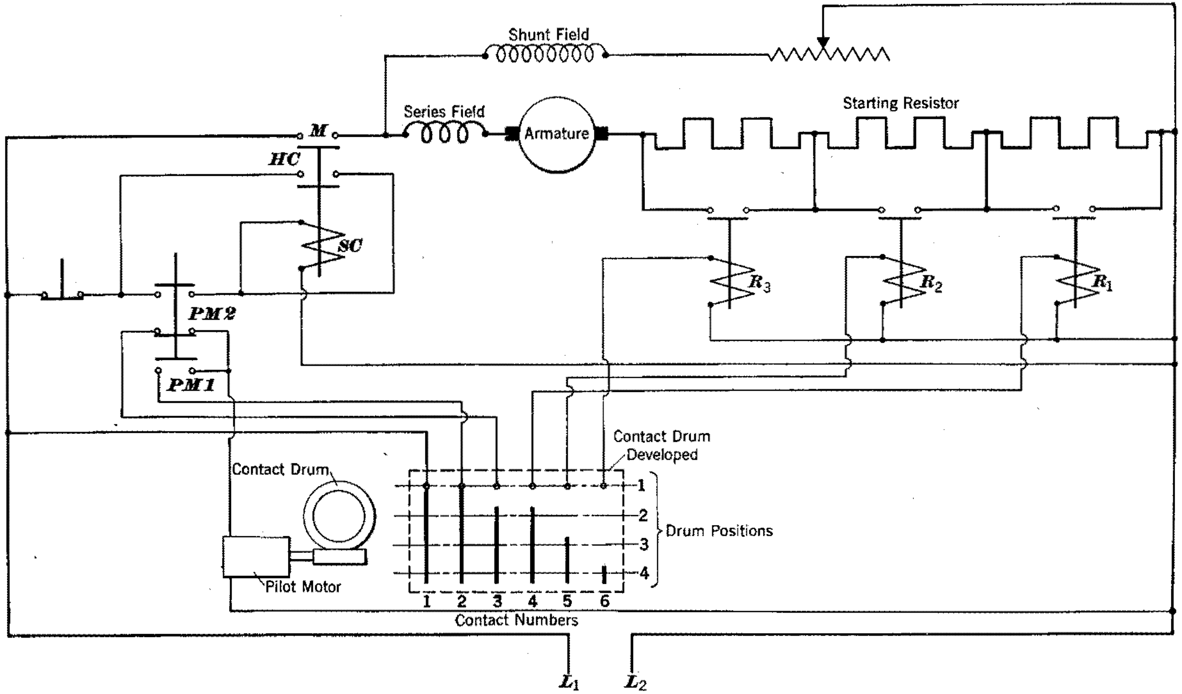


Fig. 15-9. Wiring Diagram of Definite-Time-Delay Automatic D-C Motor Starter

Overload Protection of D-C Motors.—Overload protection of a motor may be obtained by the use of simple fuses in series with the power line. However, fuses are generally unsatisfactory because they may open the circuit on a short-time overload which would not injure the motor. For the specific duty of motor protection there have been developed thermal overload devices with heating characteristics as nearly as possible like those of the motor to be protected. Such a device will disconnect the motor only after an overload has been sustained for a dangerous period of time. It is relatively difficult to make a heater-type relay with the same heating characteristics as the motor which it is intended to protect. In some cases, and particularly in protecting very small and totally enclosed motors, the relay is therefore placed on or in the motor, so that it may also be operated by heat received by convection or by radiation directly from the hottest part of the motor.

Low-Voltage Release and Low-Voltage Protection.—Any automatic starter of a type described, which uses a shunt coil on the final contact-holding solenoid, will automatically disconnect the motor from the line if the line voltage decreases to an abnormally low value. If such a starter will restart the motor without attention after such a voltage failure, it is said to provide *low-voltage release.* With such an arrangement it is certain that the starting resistance will be used when the motor restarts after line-voltage recovery provides resumption of service; and this condition is desirable for some fan and pump applications. Low-voltage release is undesirable and dangerous, however, when used in control of motors in some types of service, because of the hazard involved in the unsupervised starting of some kinds of machinery. Starters that are so designed as to require supervised starting after every stop are said to provide *low-voltage protection.* Automatic control equipment costs much more than manually operated control for comparable service.

Miscellaneous Starting Devices.—In addition to the starting devices described, many special starters have been produced for special service. In one type, the field of the motor is constantly excited from one source of power while the armature is supplied by a variable-voltage generator. This method of multi-voltage control, called the Ward-Leonard system, while higher in cost than most others, provides extremely smooth acceleration and is used extensively in the control of high-speed elevators, mine hoists, etc. In another type of starter for small d-c motors, the series resistance consists of a stack of carbon discs the resistance of which is smoothly varied by pressure.

Reversal of Direct-Current Motors.—The torque, and hence the direction of rotation, of a d-c motor may be reversed by reversing *either* the field flux or the direction of armature current. If the reversal of rotation is accomplished by field reversal, both the shunt field and the series field, if there is one, must be reversed. The direction of current through the interpole windings need not be reversed in order to maintain the proper polarity sequence, as discussed on page 131.

If reversal of rotation is accomplished by reversal of armature current, the direction of current through the interpole windings must also be reversed. In "plugging" service, such as found in some steel-mill applications in which the motor is reversed frequently, reversal is accomplished by armature and interpole reversal, rather than by field-flux reversal, because of the simpler control required. Automatic reversing switches controlled from a master switch are used in such service.

PROBLEMS

1. A 40-hp, 230-volt shunt-wound motor has an armature-circuit resistance of 0.11 ohm. Calculate the series starting-box resistance necessary to limit the starting current to 1.5 times the rated running current. See Appendix for full-load current.

2. A motor name plate gives the following information: 7.6/9 hp, 115 volts, 56/68 amp, 900/1200 rpm, 100% load, 24 hours 50 C.
   (*a*) What is the rated speed at 9 hp, full load?
   (*b*) What is the significance of the double speed specification?
   (*c*) What is the significance of the double current rating?
   (*d*) What is the significance of the temperature rating?

CHAPTER 16

Electrical Instruments and Meters

Types of Instruments.—Indicating instruments, such as ammeters, voltmeters, wattmeters, power-factor meters, and reactive-factor meters, provide continuous indications of the respective values which they are expected to measure. Integrating meters, such as watt-hour meters, ampere-hour meters, volt-ampere-hour meters and watt-hour-demand meters, operate at rates corresponding, respectively, to the power, current, volt-ampere product, etc.; and, over any period of time, such meters indicate an *integrated quantity* accrued in that time.

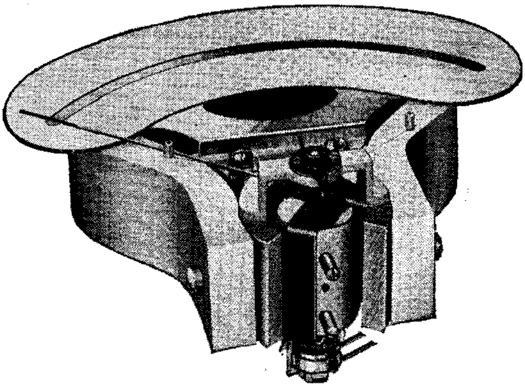


Fig. 16–1. Essential Parts of Permanent-Magnet Moving-Coil Instrument

Direct-Current Voltmeters (D'Arsonval Type).—As explained on page 128, a conductor carrying a current and having a direction perpendicular to a magnetic field is acted upon by a force which is proportional to the current. In a d-c voltmeter, the essential parts of which are indicated in Figs. 16–1 and 16–2, a constant field is produced by a permanent magnet. The very small current taken by the instrument flows through a coil of a few turns of wire mounted in the constant field in such manner that the force causes the coil to tend to rotate against the torque of two restraining spiral springs. Electrical connections to the coil are made through the two springs. A resistor in series with the coil limits the current to a few milliamperes. The current is proportional to the voltage applied to the instrument, and hence a scale swept by a pointer attached to the pivoted coil may be calibrated in volts. The voltage range to which a voltmeter is adapted is determined by the resistance of the series resistor. A voltmeter is connected in a circuit in the manner indicated in Fig. 16–3.

Direct-Current Ammeter.—It is feasible to use the springs of a voltmeter element like that shown in Figs. 16–1 and 16–2 in meters intended to measure 5 amperes or less. Therefore, such an element without series resistance may be calibrated and used as an ammeter. In measuring direct currents greater than 5 amperes, a definite known fraction of the current is diverted around the meter, but the meter is calibrated in terms of the total current.

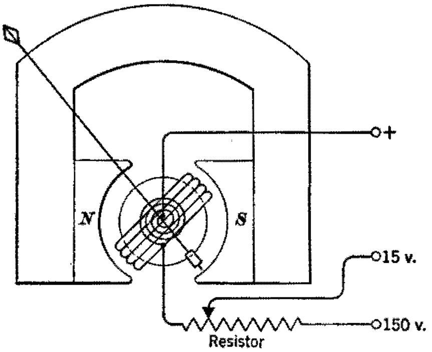


Fig. 16–2. Direct-Current Voltmeter

A voltmeter connected across a resistor measures the voltage drop, which is proportional to the current in the resistor. Hence, a voltmeter connected across a known unvarying resistance may be calibrated in amperes. Such a resistor is called a *shunt*. The voltmeter so used has an extremely small resistance and is called a *millivoltmeter*. The combination of a millivoltmeter and a shunt is called an ammeter. For small currents the shunt is usually enclosed in the millivoltmeter case. For large currents the heat liberated by the shunt is too great to be tolerated in the meter case, and so the shunt is separately mounted. In switchboard installations the shunt is frequently placed several feet away from the meter. The meter must be calibrated with connecting leads having the same resistance as those to be used between it and the shunt, because the lead resistance is a large portion of the total millivoltmeter-circuit resistance and an indis-

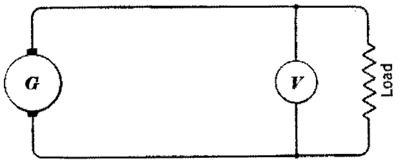


Fig. 16–3. Direct-Current Voltmeter Connection

criminate use of long or short leads may introduce intolerable errors in calibration. Fig. 16–4 shows proper connections of the two forms of d-c ammeters.

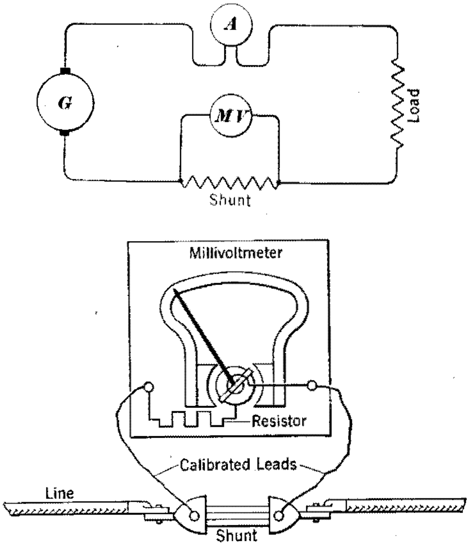


Fig. 16–4. Direct-Current Ammeter Connections

Alternating-Current Dynamometer-Type Voltmeters and Ammeters. If a D'Arsonval-type direct-current voltmeter be connected to a source of alternating voltage, the torque is alternately in opposite directions and produces no readable deflection. If, however, the constant field of the permanent magnet be replaced by the alternating field of a coil connected in series with the movable coil, then the fields of the two coils reverse simul-

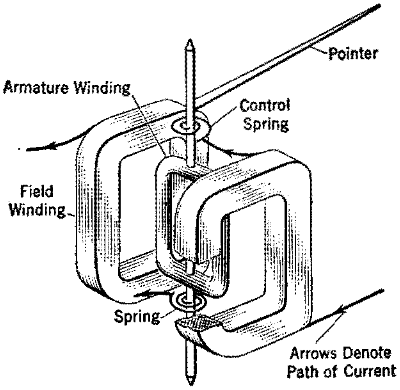
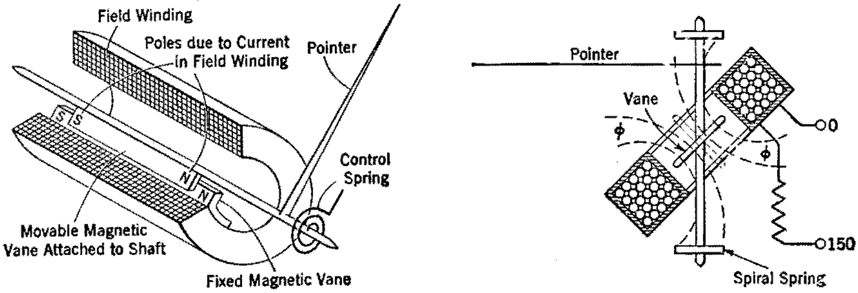


Fig. 16–5. Elements of Dynamometer Type Instrument

taneously, the successive torque pulses are in the same direction, and a deflection of the pointer results. Such an instrument, the elements of which are shown in Fig. 16–5, is called a *dynamometer-type* meter. The dynamometer-type voltmeter may be used with alternating or direct voltage. The scale divisions are badly crowded in the lower part, and readings in the lowest 20 per cent of the scale are not reliable.

Ammeters of the dynamometer type are available only in low ranges because of the difficulty of carrying large current in the springs of the moving coil. This type of meter measures *effective* values.



Repulsion-Type Instrument
(a)

Inclined-Coil Attraction-Type Instrument
(b)

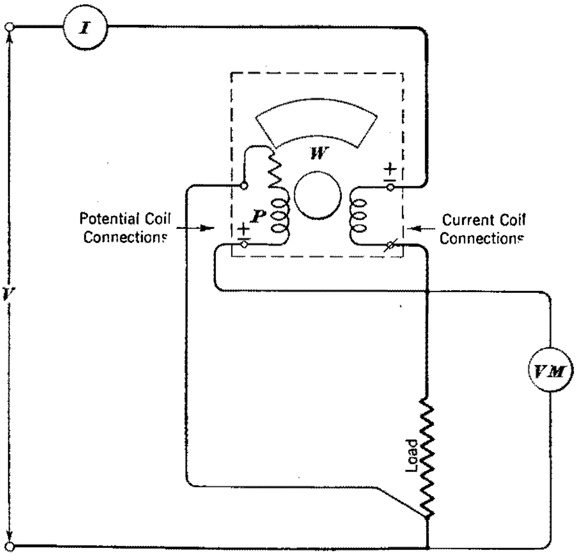
Fig. 16–6. Magnetic-Vane Voltmeters and Ammeters

Iron-Vane Voltmeters and Ammeters.—In the most popular and most rugged type of portable iron-vane ammeter, the elemental parts of which are shown in Fig. 16–6 (*a*), two iron vanes are mounted coaxially in a coil. Current passing through the coil produces a magnetic flux, which is parallel to the axis of the two vanes. One vane is fixed, and the other is free to rotate against the restraining torque of a spiral spring. When flux appears, the two vanes are magnetized with like poles. They then repel each other, and the action produces a deflection of a pointer attached to the movable vane. The torque is proportional to the square of the flux and, hence, to the square of the current in the coil. It is also a function of the angle turned through. The scale therefore is not uniform, and either end, or both ends, of it may be too badly crowded for accurate reading. Readings in the lowest 20 per cent of the scale are not reliable.

The iron-vane voltmeter is similar to the ammeter, but many turns of small wire are used in the coil instead of a few turns of large wire.

Another type of iron-vane instrument, called the inclined-coil attraction-type voltmeter and ammeter, is shown in Fig. 16–6 (*b*). In this instrument the axis of the coil is inclined 45° in one direction while the iron vane is inclined 45° in the same direction. When flux appears in the coil, the vane attempts to align itself with the flux against the action of a restraining spiral spring. By this ingenious arrangement a scale with a span of almost 180° is possible. Because of the crowding of the scale at the ends, the instrument is usually designed to be used in only the most open part of the possible scale.

Voltmeters and ammeters of the iron-vane and dynamometer types, when calibrated at a frequency of 60 cycles, will have a degree of accuracy ordinarily acceptable up to about 130 cycles. At frequencies above 130 cycles the accuracy becomes noticeably poorer with increase in frequency, and so meters for high frequencies should be recalibrated at the frequencies at which they are to be used.



I = Ammeter
VM = Voltmeter
W = Wattmeter

Fig. 16–7. Single-Phase Wattmeter Connections

Iron-vane instruments are suitable for both alternating-current and direct-current measurements. However, when such a meter is used in direct-current measurement, readings should be taken with reversed connections also, to eliminate the effect of possible stray fields and of magnetic retentivity of the iron vane. The true reading is then the average of that taken with normal connections and that taken with reversal of connections. This type of meter measures *effective* values.

Electronic Voltmeters.—The electronic voltmeter is particularly useful in measuring alternating voltages of only a few volts or a fraction of a volt, because it draws no current and so will not disturb the most delicate circuit.

It consists of a D'Arsonval-type voltmeter energized through an electron-tube circuit, and has the disadvantage of requiring a source of a-c power for its operation. See Chapter 28.

The Single-Phase Wattmeter.—The most common type of wattmeter operates on the same general principle as does the dynamometer-type voltmeter. The fixed coil contains a few turns of large wire and is connected in *series* with the power circuit, just as is an ammeter. The movable coil with its series resistor is connected *across* that part of a circuit in which the power is to be measured. The divisions of the scale are somewhat crowded in the lowest part, and readings obtained in the lowest 10 per cent of the range should be regarded as unreliable. A wattmeter measures average power.

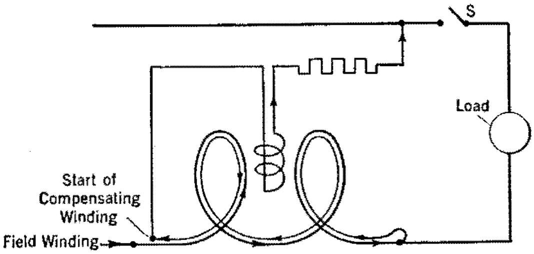
Use of Wattmeter.—A wattmeter should be connected in the circuit to be metered in the manner shown in Fig. 16–7. The connecting terminals of the potential coil at *P* are always insulated to the hand, and the connecting terminals of the series or current coil are almost never so insulated. The coil terminals may usually be identified by that difference.

It is important that the potential-coil terminal marked zero or ± be the one connected to the line passing through the current coil and on the *load* side of that coil, as shown in Fig. 16–7. If the meter reading is negative, it should be corrected by reversing the *current-coil* connections. If the potential-coil connections were reversed instead, the moving-coil end of the potential-coil circuit would be at full line potential with respect to the closely adjacent current coil, and electrostatic forces would tend to influence the reading and introduce errors.

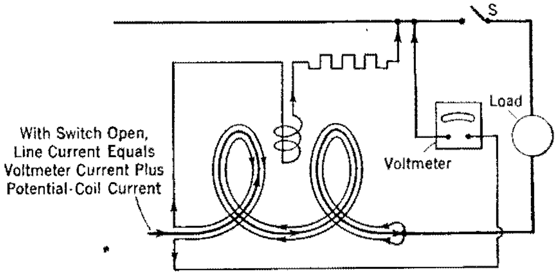
Inasmuch as the reading of a wattmeter is a function of the cosine of the load phase angle, as well as of the current in the circuit, a moderately high reading is no assurance that the current coil is not seriously overloaded unless the power factor is known to be comparatively high. To protect the current coil against such accidental abuse, an ammeter of the same current rating as the current coil of the wattmeter should always be used in the circuit, although readings of power only may be important as data.

With few exceptions, wattmeters may be used in either direct-current or alternating-current circuits. The rare induction type of wattmeter (Westinghouse) may be used on alternating-current circuits only.

The Compensated Wattmeter.—Inasmuch as in Fig. 16–7 the potential-coil current flows through the current coil, the wattmeter so connected will read too high by the amount of the power lost in the potential coil. When the power factor of the load is specially low, the wattmeter reading will be relatively low, and the error introduced by the potential-coil power may be too great to be ignored. In this case a compensated wattmeter is convenient. In the compensated wattmeter, a compensating coil having the same number of turns as the current coil is wound on the current coil and is connected in series with the potential coil. The current in the compensating coil flows in a direction opposite to that of the current in the current coil, and therefore its effect on the wattmeter field is nullified. In some cases it is convenient also to have the effect of the voltmeter current nullified in the wattmeter, so that the voltmeter need not be disconnected when the wattmeter is read. Such a connection is shown in Fig. 5–25. Fig. 16–8 shows schematically a wattmeter compensated for its own potential-coil error, and also one compensated for both its own potential-coil error and that which would be introduced by an accompanying voltmeter.



Action of Compensating Wattmeter



Action of Compensating Wattmeter having Voltmeter Winding

Fig. 16–8. Schematic Diagram of Compensated Wattmeter

Polyphase Wattmeters.—As explained on page 96, two wattmeters may be used to measure the total power in a three-wire, three-phase circuit. A polyphase wattmeter, in which two wattmeter elements are connected to a common pointer, costs less and requires less space than would two single-phase wattmeters with the same current and voltage range. In Fig. 16–9 is shown a polyphase wattmeter connected for use.

Potential Transformers.—It is common practice to use wattmeters and voltmeters with a nominal voltage range of 0 to 150 volts on all a-c circuits of 100 volts or more. When such a meter is to be used on a circuit whose voltage is greater than 150 volts, a *potential transformer* or *voltage transformer* is used between the meter potential coil and the circuit. The potential transformer is relatively small because its secondary burden is only a few volt-amperes taken by one, two, or three meters. Fig. 16–10 shows two potential transformers properly connected for the measurement of polyphase power by the two-wattmeter method. All instruments used in the secondary of a potential transformer must be *connected in parallel*, as are the voltmeter and the potential coil of W_1 in Fig. 16–10.

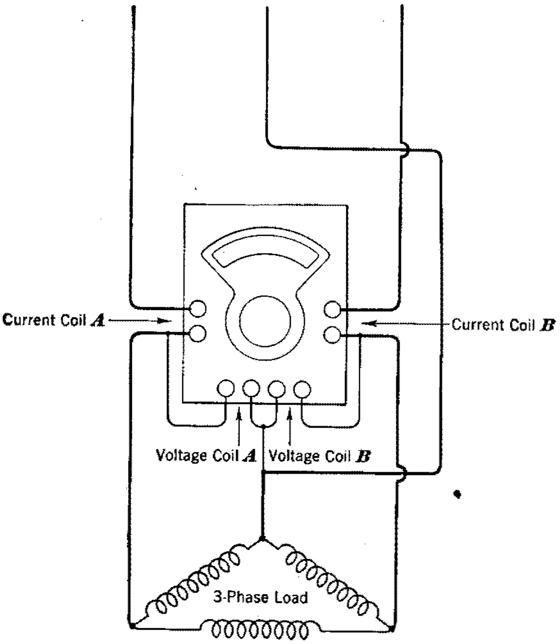


Fig. 16–9. Polyphase Wattmeter Connections

Current Transformers.—*Current transformers* are used in a-c circuits, either when large currents are involved or when the line voltage is dangerously high and it is advisable to insulate the meter from the power circuit. Current transformers usually "step down" the current to 5 amperes or less, and for this reason meters and other devices connected in the secondary circuit should be suitable for only 5 amperes. All instruments or other devices, such as protective relay trip coils, should be *connected in series* when used in current-transformer secondary circuits.

If the secondary circuit of a current transformer be open when the primary circuit is energized, all of the primary current becomes magnetizing current, and a high voltage dangerous to life may appear at the secondary terminals. The high flux density may also overheat the iron core seriously enough to damage the insulation. To prevent this condition, the secondary winding of a current transformer should be short-circuited when its circuit is not otherwise closed. The secondary circuit of a potential transformer must never be short-circuited.

The theory of current transformers and potential transformers is the same as that of all other transformers described in Chapter 17. With the few precautions already described these transformers may be used advantageously, even though the theory of their operation is not fully understood.

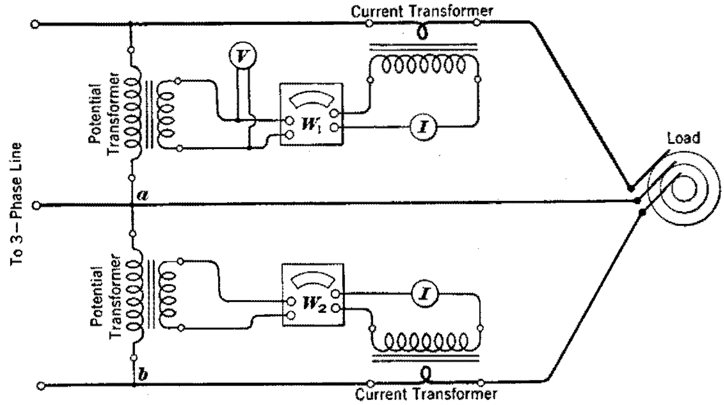


Fig. 16–10. Three-Phase Power Measurement by Two-Wattmeter Method Using Two Potential Transformers and Two Current Transformers

Proof of Connections When Instrument Transformers Are Used. When instrument transformers are used in measuring three-phase power by the two-wattmeter method, verification of the connection of the low-reading element may be made by removing the potential-transformer primary connection at the common line (it is assumed that the upper meter is the one of lower reading), as at *a* in Fig. 16–10, and touching it at *b*. If W_1 reverses, its reading is to be taken as negative. If it does not reverse, then its reading is to be taken as positive. The potential-transformer connection must be restored to its original position at *a*.

The connections of a polyphase wattmeter may be checked in the same manner as are those of two separate wattmeters. The wattmeter should first be connected so that each element alone produces a positive reading. With the potential coil of the higher-reading element disconnected, the algebraic sign of the lower-reading element is then checked in the usual way. If this test shows a negative reading, then that element must be reconnected so that its torque on the needle is opposed to the torque of the higher-reading element.

The same procedure should be used in proving the correctness of connections of a polyphase wattmeter when potential transformers are used. Fig. 16–11 shows a circuit diagram in which both current transformers and potential transformers are used with a polyphase wattmeter to measure power in a three-phase circuit.

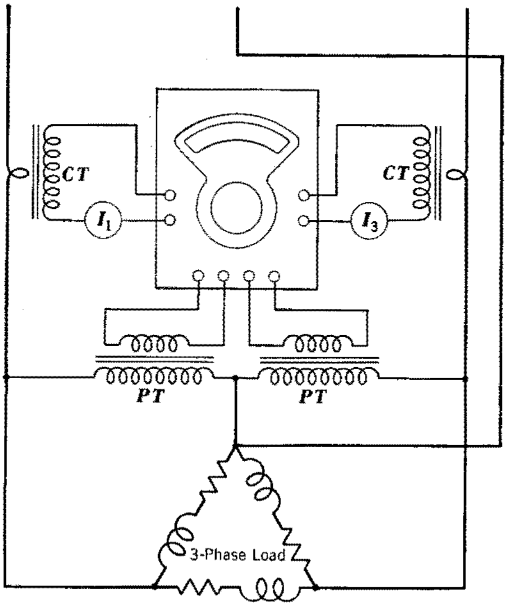


Fig. 16–11. Polyphase Wattmeter Connections Using Current Transformers and Potential Transformers to Measure Polyphase Power

Burdens of Current and Potential Transformers.—The load on a current transformer or a potential transformer is called the *burden*. Potential transformers are usually designed to operate with commercially acceptable accuracy with a burden of two potential instruments, such as a voltmeter and a wattmeter. Greater burden should not be imposed without specific knowledge of the ultimate safe burden and the resultant ratio and phase-angle errors of the transformer when so burdened.

Current transformers are usually designed to operate with commercially acceptable accuracy with burdens consisting of an ammeter, a wattmeter, and an overload relay trip coil of a circuit breaker. Some current transformers of inferior quality are intended for trip-coil burden only or ammeter burden only, and are not acceptably accurate for additional burden or for measurement of power. Transformers of inferior quality, or others of excellent quality that are over-burdened, permit excessive phase-angle errors when used with wattmeters, and should not be so used.

Power-Factor Meters.—Single-phase and polyphase power-factor meters provide direct continuous readings of circuit power factor. The accuracy of some types is poor in the scale range between 90 per cent lagging and 90 per cent leading. The pointer is without spring restraint and assumes a definite position only when the instrument is energized.

Reactive-Factor Meters.—The reactive power changes more with a given change of load phase angle near zero than does the active power, and hence a reactive-factor meter is more accurate than some types of power-factor meters near unity power factor of load. Reactive-factor meters are preferable for use with synchronous converters (see Chapter 29) because of the delicate adjustment of phase angle desirable in the operation of these machines.

Frequency Meters.—Commercial frequency meters are made in several forms. The most common form employs vibrator reeds of different lengths arranged consecutively in a row on a common mounting. The mounting is caused to vibrate by a magnet whose exciting winding is connected across the circuit in the same manner as a voltmeter. The reed weights are adjusted so that, when the common mounting is vibrated within the frequency range of the reeds, the reed tuned to that frequency vibrates. A scale adjacent to the row of reeds is calibrated directly in frequency.

Another type of frequency meter employs the dynamometer principle. The accuracy of ordinary frequency meters of the non-precision type seldom exceeds 99 per cent.

Rectifier-Type Meters.—Alternating-current voltmeters of any particular type require a definite volt-ampere input for a given pointer deflection, regardless of the range of voltage of the instrument. Hence, a low-voltage meter must draw from the circuit more current than a high-voltage meter. If the current is small through the device of low impedance across which the voltage is to be measured, the low impedance of the low-reading voltmeter may cause sufficient current to be by-passed around the device to seriously disturb the measurement.

The rectifier-type voltmeter consists of a sensitive D'Arsonval movement in conjunction with a rectifier unit of copper oxide. The impedance of such an instrument may be made much higher than that of an iron-vane or dynamometer-type instrument, but the readings cannot be relied on within 5 per cent of full-scale value. Such voltmeters are calibrated for sinusoidal wave shape only. Low-range low-impedance a-c ammeters are also available in this type. A circuit for a rectifier-type instrument is shown in Fig. 16–12.

Thermocouple Meters.—A sensitive D'Arsonval instrument connected to a thermocouple described on page xv measures the current caused by heating of the couple junction. If the couple be heated by a small resistor, the meter may be calibrated in terms of current in the resistor. That is the principle of the thermocouple ammeter extensively used for measurement of high-frequency currents such as those used in radio communication.

The voltage of a thermocouple is proportional to the difference between the temperature of its heated junction and the temperature of the other ends of the couple metals. If no temperature compensation were provided, the accuracy of the instrument would decrease as the ambient temperature in which it is used changed from that at which it was calibrated. To avoid such errors, the ends of the heater wires are commonly connected to rather massive metal blocks to which are also connected the instrument leads. This is called "cold junction compensation" and helps to minimize temperature errors. A circuit for a thermocouple-type instrument is shown in Fig. 16–13.

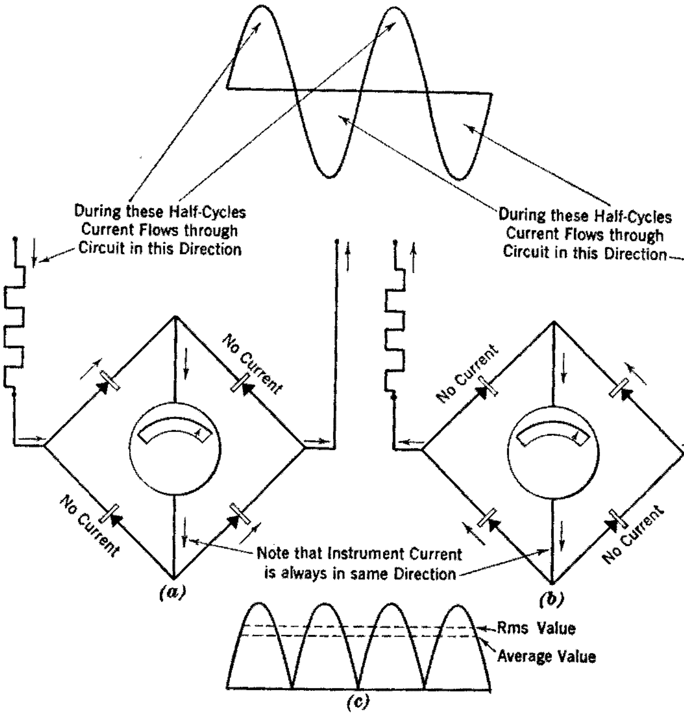


Fig. 16–12. Rectifier-Type Instrument Circuit

Graphic Instruments.—Graphic instruments draw curves for a permanent record of the variation of current, voltage, power, power factor, etc. with time. For every electrical indicating instrument, there is a corresponding graphic or curve-drawing instrument. These instruments are highly useful in analyzing factory loads and loads of individual machines. A graphic voltmeter and ammeter may show that a motor overheats because the line voltage is allowed to become too low and the current therefore becomes too great. A graphic wattmeter may show that the load on a motor reaches the full-load value of the motor rating for only 10 minutes in every hour and does not exceed half-load in the other 50 minutes. An induction motor so loaded runs at a lower power factor and efficiency, and should be replaced with one that is nearer the correct size and represents less invested capital.

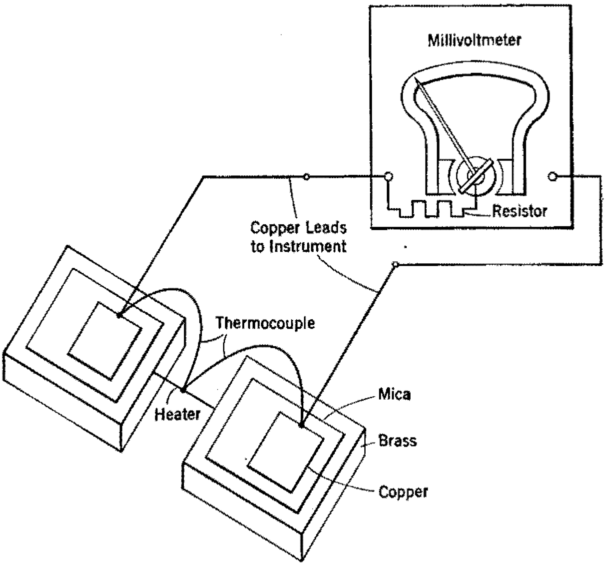


Fig. 16–13. Circuit for Heater Thermocouple-Type Instrument With Temperature Compensation

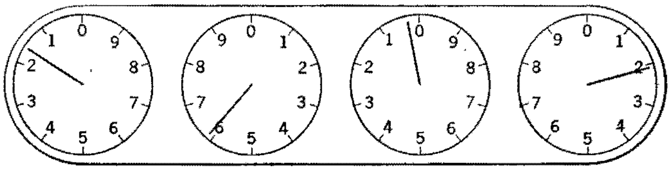
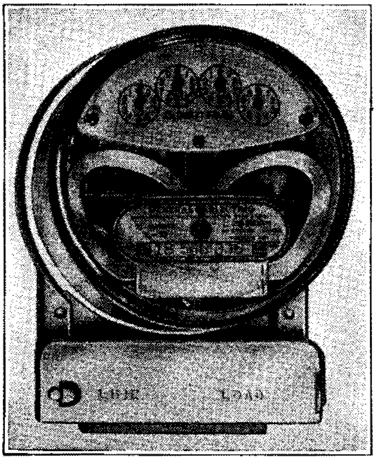


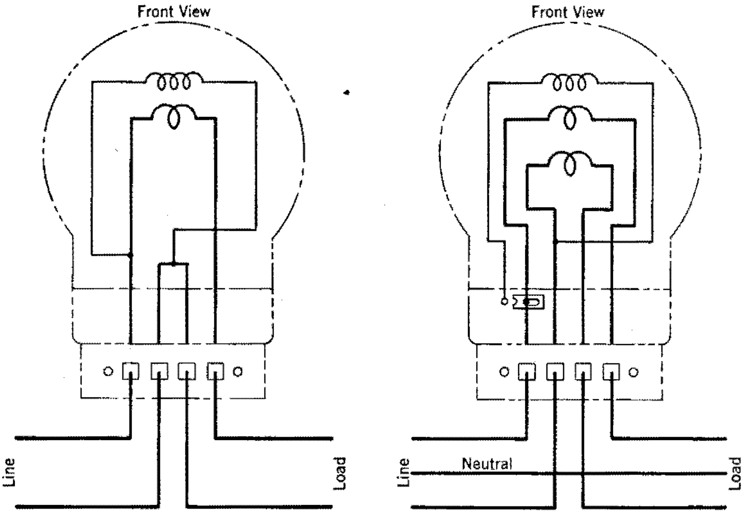
Fig. 16–14. Watt-Hour Meter Register

Watt-Hour Meters.—Watt-hour meters are essentially motors so designed that the speed is proportional to the power taken through them by the load. The motor is geared to a gear-train register, and hence the register reading is a product of power and time and indicates directly the



(*Courtesy General Electric Co.*)

Fig. 16–15. Single-Phase Watt-Hour Meter



(*a*) Two-Wire Connection (*b*) Three-Wire Connection

Fig. 16–16. Single-Phase Watt-Hour Meter Connections

work done in kilowatt-hours. The dials of the register are read from left to right. The figure read on each dial is the one which the pointer points to or pointed to last. In Fig. 16–14 the reading is 1602 kw-hr.

Direct-current watt-hour meters are of the commutator-motor type or of the mercury type. The single-phase watt-hour meter, an example of which is shown in Fig. 16–15, operates on the induction-motor principle.

The speed of the disc armature is proportional to the power of the load. In Fig. 16–16 (*a*) is shown the proper method of connecting a two-wire single-phase meter, and in (*b*) is shown the proper method of connecting a three-wire meter for measuring power in a single-phase, three-wire circuit as supplied by the transformer of Fig. 17–7. The two-element polyphase watt-hour meter contains two motor elements driving the same register, and is connected in the same manner as a polyphase indicating wattmeter, as indicated in Fig. 16–17. However, the third wire is also connected to the meter in order that potential-coil connections may be made permanently when the meter is assembled.

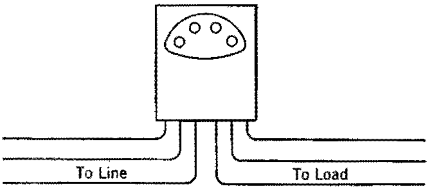


Fig. 16–17. Three-Phase, Three-Wire Watt-Hour Meter Connections

Watt-hour meters for use on four-wire, three-phase circuits contain three motor elements and are connected essentially in the manner shown for the three meters in Fig. 9–12. A diagram of connections is usually furnished with each meter.

When the load being metered by a two-element polyphase watt-hour meter is reasonably steady, it is sometimes convenient to be able to determine the power, the current, and the power factor of the load without the use of additional meters if the watt-hour meter is not sealed. It must first be assumed that the voltage of the line is at its rated value. The potential coils of the meter are provided with small links or other means for disconnecting the coils for calibration purposes. To carry out the calibration, disconnect one element and count the revolutions of the disc for a minute; call this number S_1. Reconnect the element, and repeat the counting procedure with the other element disconnected; call this number S_2. If it is assumed that S_1 is larger than S_2, the power factor is

$$\mathrm{PF} = \cos \tan^{-1} \frac{S_1 - S_2}{S_1 + S_2}\sqrt{3}$$

and

$$P = \frac{K_h(S_1 + S_2)}{60} \tag{16–1}$$

where K_h = disc constant = number of watt-hours per revolution.

Also,

$$I = \frac{P}{\sqrt{3}V \cos \theta} \tag{16–2}$$

The Oscillograph.—Because of the inertia of the moving parts of the more common types of electrical measuring instruments, their indications cannot follow accurately any rapid fluctuations of torque. Hence, they can only show readings proportional to average torque, and so are of no value in determining instantaneous values.

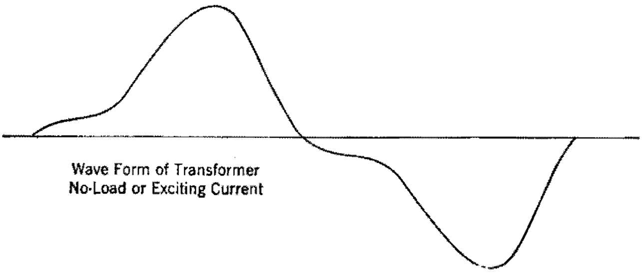


Fig. 16–18. Typical Oscillogram

The Duddell type of *oscillograph*[1] operates on the principle of the D'Arsonval-type voltmeter, but the weight of the moving element is so small that its time lag because of inertia is negligible. In order to reduce the weight, the coil is made of only one loop of very small span. A mirror cemented to the coil provides an optic lever by which the coil deflection is indicated on a viewing screen. Between the mirror and the viewing screen is an oscillating or rotating mirror which deflects the beam at an angle to the direction imparted to it by the mirror. The result of the two motions of the light beam is a pattern in two dimensions such as that in Fig. 16–18, which shows the wave form of the exciting current of a transformer. Such instruments are useful in determining wave forms of currents or voltages and the transient starting current inrushes of motors, transformers, etc.

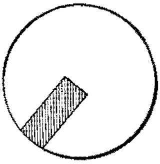


Fig. 16–19. End of Motor Shaft With Stroboscopic Spot

Another type of oscillograph uses a cathode-ray electron tube, and the two-dimensional image is made to appear on the flattened end of the tube. This type of instrument is especially useful in the analysis of ultra-high speed transients to which the Duddell type of instrument is too sluggish to respond. The electromagnetic or Duddell type of oscillograph is useful for studying phenomena which require 50 micro-seconds or more. For phenomena of shorter duration the cathode-ray type of instrument is required. Both types of oscillograph are adapted to visual uses and to the making of permanent photographic records.

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[1] *Principles and Practice of Electrical Engineering*, by Gray and Wallace. McGraw-Hill Book Co.
Elements of Electrical Engineering, by Arthur L. Cook. John Wiley & Sons, Inc.

The Stroboscope.—The electric thermionic stroboscope is an instrument which emits accurately timed light flashes of extremely short duration. If the light flashes are caused to illuminate a line on the end of a motor shaft, as in Fig. 16–19, and the flashes are so synchronized with the motor speed that the interval between successive flashes is exactly one revolution or a whole number of revolutions, the line will appear to be stationary. Thus, if 3600 flashes occur every minute to illuminate a shaft running 3600 rpm, the line will appear as a radius as in Fig. 16–19. If the motor were running only 1800 rpm, the shaft would be illuminated two times in every revolution, and the line would appear as a diameter. If the speed were 1200 rpm, three radial lines displaced 120° from each other would appear. When the shaft is in a two-pole, 60-cycle induction motor, the spot fails to make a complete revolution between flashes because of the motor's slip; and it therefore appears to drift in a direction opposite to the direction of shaft rotation with a speed equal to the slip of the motor. The speed of an induction motor may be found very accurately by subtracting the speed of slip from the synchronous speed.

(Courtesy General Radio Co.)

Fig. 16–20. Electric Stroboscope

Sometimes, an electronic stroboscope is provided with a self-contained calibrated oscillator by which the flash frequency may be controlled throughout a wide range. Such an instrument is shown in Fig. 16–20. If a vibrating shaft be illuminated by such an instrument, the flash frequency may be adjusted to that of the shaft, and the shaft will be made to appear stationary. Such a procedure permits analysis of critical speeds, vibration amplitudes, and other phenomena of high-speed machines.

PROBLEMS

1. A millivoltmeter with a range of 0 to 50 is to be used with a 400-amp, 50-millivolt shunt. What will be the constant for this combination?

2. What is the resistance of a 50-millivolt shunt rated at 10 amp?

3. A direct-reading 5-amp, 100-volt wattmeter is used with a potential transformer of 20:1 ratio and a current transformer of ,50:5 ratio. By what constant must the reading be multiplied?

4. Give complete instructions for proving the correctness of connections of a polyphase wattmeter, using both current and potential transformers, when measuring a balanced three-phase three-wire load.

5. A 50-millivolt meter draws 1 milliampere for every 5 millivolts impressed on it. How many ohms in series must be added to cause the meter to read full scale on 150 volts?

6. A voltmeter with a resistance of 15,000 ohms draws 10 milliamperes with full scale deflection. How could this meter be converted to a satisfactory milliammeter with 10 milliamperes full scale?

7. It is required to measure a d-c voltage of about 700 volts. A 600-volt meter and a 150-volt meter are available. Is it permissible to use these meters in series and to add their readings? Explain.

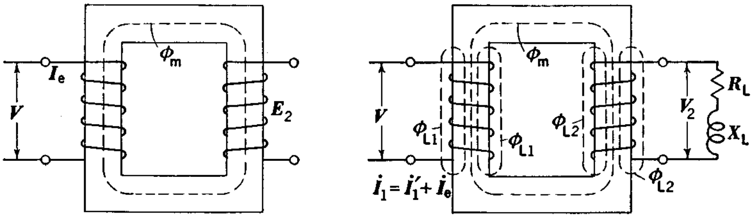
8. In using an ammeter and a wattmeter in the secondary circuit of a current transformer, why must the current coils be connected in series?

9. In using a voltmeter and a wattmeter in the secondary circuit of a potential transformer, why must the potential coils be connected in parallel?

CHAPTER 17

Transformers

Uses of Transformers.—One of the many advantages of alternating-current electricity is the ease with which it may be transformed from high voltage to low voltage and vice versa. Economy of transmission is greatest at high voltage, but lower voltage is better suited to generation, distribution, and use. Voltages of 13,500 or fewer volts are commonly used in generation; 13,500 to 250,000 volts are common for transmission over great distances; and 13,500 or fewer volts are best for distribution and use among customers. Transformers are used to "step" up or "step" down the voltage of a system to meet a particular need. A transformer is, in general, the electrical analog to the gear reduction box of a mechanical-power transmission system.



(*a*) Unloaded (*b*) Loaded

Fig. 17–1. Schematic Transformer Diagram

Theory of Operation.—Transformers operate on the principle of mutual inductance described in Chapter 5. Those used for unusual purposes sometimes have air cores, but almost all have iron cores because of the advantages of smaller size, lower cost, and higher efficiency.

Fig. 17–1 (*a*) shows a simple transformer having two coils on an iron core. An alternating voltage V applied to one coil, called the *primary*, will cause a current I_e, called the exciting current, to flow as in an ordinary inductance coil. The exciting current produces the mutual flux ϕ_m and also produces in the air around the primary coil a very small leakage flux which is neligibly small compared to ϕ_m. The flux ϕ_m produces a counter electromotive force E_1, which would be equal and opposite to V if there were no resistance and no other flux linking the primary coil. If the primary coil has N_1 turns and the secondary coil has N_2 turns and $N_1 = N_2$, then ϕ_m will also generate a voltage E_2 in the secondary coil; E_2 will equal E_1 because both coils have the same number of turns. The number of volts generated per turn of N_1 and N_2 will be the same because ϕ_m is common to both coils. The voltage E_2 is then proportional to N_2 and can be made any desired value by proper choice of secondary turns N_2. The ratio of input voltage to output voltage is then

$$\frac{V}{E_2} = \frac{N_1}{N_2} \tag{17–1}$$

When a load is connected to the secondary coil, the resistances and leakage fluxes of both coils modify the actual ratio of primary voltage to secondary terminal voltage because they cause a *leakage impedance* voltage drop. When a current I_2 flows in N_2 turns and a load circuit, as shown in Fig. 17–1 (*b*), a magnetomotive force mmf$_2$, which is proportional to N_2I_2, is produced in the secondary coil. By Lenz's Law, mmf$_2$ is in opposition to ϕ_m and tends to reduce ϕ_m. Also, mmf$_2$ produces a flux ϕ_{L2} in the air paths around the secondary coil, and ϕ_{L2} generates a voltage $-e_2 = L_2 \frac{di_2}{dt}$ which is subtractive from E_2. Furthermore, the slight reduction of ϕ_m reduces E_1, and so an additional primary component of current I_1' appears in the primary coil turns N_1. A magnetomotive force mmf$_1$ is produced in the primary coil by N_1I_1 and produces a leakage flux ϕ_{L1} in the air paths around the primary coil. This flux in turn generates a voltage $-e_1 = L_1 \frac{di_1}{dt}$ in the primary coil.

The current I_1 is the vector sum of I_e and I_1'. The counter voltages $-e_1$ and $-e_2$ cause reactance drops I_1X_1 and I_2X_2 in the primary coil and secondary coil, respectively. The fluxes ϕ_{L1} and ϕ_{L2} are called leakage fluxes because each links only the coil that produces it and leaks around the other instead of linking it as does ϕ_m. The leakage fluxes are not actually distinguishable one from the other in the transformer but are separated in theory for the purpose of analysis.

The transformer serves not only to transform voltages from one level to another but also to isolate one circuit from all others when necessary. The voltage and power losses are so small in most transformers that, for most practical purposes, it can be said that the volt-ampere input is equal to the volt-ampere output. Unless the leakage impedances are unusually large, the power factor on the primary side is about the same as that on the loaded secondary side.

Voltage Transformation Ratio.—A transformer with an equal number of turns on its primary and secondary windings will have a secondary voltage only slightly less than the primary applied voltage, and its *voltage ratio* is said to be 1:1. If, however, the secondary winding have only one-half as many turns as the primary winding, the secondary voltage will be only one-half as great as the primary voltage. The voltage ratio then will be 2:1. *The primary and secondary generated electromotive forces are proportional to the primary and secondary turns, respectively.* Thus,

$$\frac{E_1}{E_2} = \frac{N_1}{N_2} \tag{17-2}$$

A transformer with a turns ratio of 13,500 to 110,000 would be used to step up the voltage at the terminals of a 13,500-volt generator to 110,000 volts for long-distance transmission. At the receiving end of the transmission line, another transformer with a turns ratio of 110,000 to 13,500 would be used to step down the voltage to 13,500 volts for local distribution. On the distribution *feeders*, transformers with turns ratios of 13,500 to 440 or 13,500 to 220 would be used to reduce the distribution voltage to a safe and convenient value for general power application.

Current Transformation Ratio.—In a transformer of 1:1 ratio, 1 load ampere flows in the primary for every load ampere in the secondary because the corrective or cancellation ampere-turns added in the primary must be equal to the demagnetizing ampere-turns N_2I_2. The current-carrying capacity, and hence the size of wire, in the primary winding should be the same as that in the secondary winding. The exciting current is so small as to be negligible and has been ignored in the foregoing statement. The weight of wire in the secondary should be the same as that in the primary because the volt-amperes are the same in each. If the number of secondary turns be only one-tenth as great as the number of primary turns, the current-carrying capacity of the secondary conductors should be 10 times as great as that of the primary turns. For every 10 load amperes in the secondary, 1 load ampere will flow in the primary. Thus, the current transformation ratio of a transformer that is normally designed is inversely proportional to the turns ratio. The transformation is accomplished with but small power loss in the transformer.

The Equivalent Circuit of a Transformer.—The leakage impedances of a transformer cause a voltage loss between line and load exactly the same as if they were not in the transformer but in the lines to or from the transformer. The exciting current I_e remains substantially the same for all loadings. If a circuit be made up of two branches in parallel, as in Fig. 17–2 (*a*), the voltage at the load and the current in the supply line will be the same as actually would occur if a transformer with a 1:1 ratio and the same values of X and R were used. Such a circuit is called the *equivalent circuit* of the transformer because its effect between the supply line and the load would be the same.

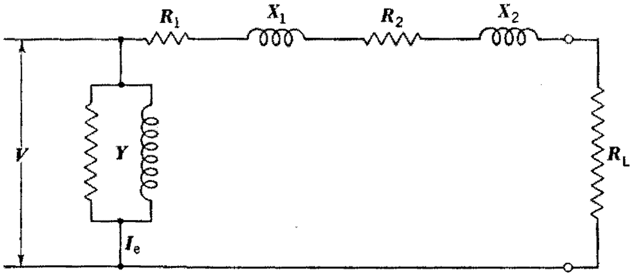
If the turns ratio is not 1:1, a modification in the circuit elements must be made to suit the turns ratio. For any definite secondary load, the primary current will be the same regardless of the turns ratio because N_2I_2

merely produces a magnetomotive force mmf$_2$ which must be balanced by the force mmf$_1$ at the available supply voltage. If the transformer had a turns ratio of 4:1 and both windings had the same size of wire, their resistances would be in the same ratio and R_1 would equal $4R_2$. However, if the volt-ampere input and the volt-ampere output are to be equal, then $I_2 = 4I_1$ and the current-carrying capacity of the secondary coil conductor must be four times that of the primary coil conductor. As a result, the secondary conductor must have four times the cross-sectional area and, therefore, one-fourth of the resistance per turn. Hence,

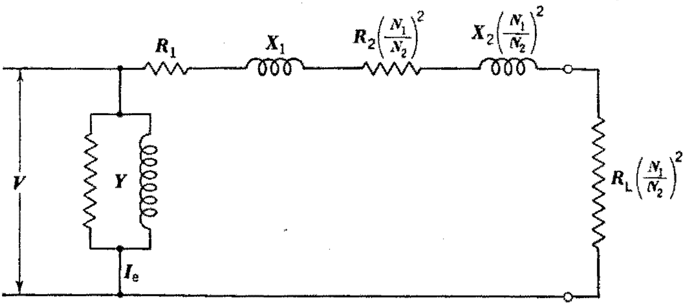
$$R_2 = R_1 \times \frac{1}{4} \times \frac{1}{4} = \frac{R_1}{16}$$

Since $\left(\frac{N_2}{N_1}\right)^2 = \frac{1}{16}$, it follows that

$$\frac{R_2}{R_1} = \left(\frac{N_2}{N_1}\right)^2 \tag{17-3}$$



(*a*) 1:1 Turn Ratio



(*b*) Any Ratio $\frac{N_1}{N_2} = a$

Fig. 17-2. Transformer Equivalent Circuits

Regardless of the value of the ratio $\frac{N_2}{N_1}$, the secondary resistance is equal to the primary resistance divided by the square of the turns ratio, provided the mean length of turn is the same in the two windings and the number of circular mils per ampere is also the same. If R_2 is to have the same effect on the line current with only I_1' flowing in it, then its magnitude must be modified by $\left(\frac{N_1}{N_2}\right)^2$ in the equivalent circuit. Similar reasoning may be applied to X_2, and its magnitude in the equivalent circuit must be $X_2\left(\frac{N_1}{N_2}\right)^2$. The secondary constants of the transformer as used in the equivalent circuit are called "reflected" constants. For use in the equivalent circuit the load constants R_L and X_L must be modified in the same way by multiplying them by $\left(\frac{N_1}{N_2}\right)^2$. An equivalent circuit is shown in Fig. 17-2 (*b*) with reflected constants.

The equivalent circuit is useful for calculating the voltage change on the secondary of a transformer when its load is varied or for calculating the voltage necessary at the primary terminals to provide rated voltage at the secondary terminals when the secondary is loaded.

Example 17-1.—A 5-kv-a transformer with a 4:1 turns ratio is to be used to supply 110 volts to a resistive load which draws full rated current from the transformer. The transformer constants are: $R_1 = 2.75$ ohms, $R_2 = 0.172$ ohm, $X_1 = 4.0$ ohms, and $X_2 = 0.25$ ohm; and the load resistance is $R_L = 2.5$ ohms. Calculate the necessary primary applied voltage.

Solution.—The rated currents are:

Primary.........5000 ÷ 440 = 11.38 amp
Secondary.......5000 ÷ 110 = 45.45 amp

Also,

$$R_2\left(\frac{4}{1}\right)^2 = 2.75 \text{ ohms}$$

$$X_2\left(\frac{4}{1}\right)^2 = 4.0 \text{ ohms}$$

$$R_L\left(\frac{4}{1}\right)^2 = 40 \text{ ohms}$$

Then,

$$Z = R_1 + jX_1 + R_2\left(\frac{N_1}{N_2}\right)^2 + jX_2\left(\frac{N_1}{N_2}\right)^2 + R_L\left(\frac{N_1}{N_2}\right)^2$$

$$= 2.75 + j4.0 + 2.75 + j4.0 + 40 = 46.19 \text{ ohms}$$

and

$$V = I_1 Z = 11.38 \times 46.19 = 525.64 \text{ volts}$$

Determination of Equivalent R, X, and Z of a Transformer by Test. If the secondary coil of a transformer is short-circuited and just enough voltage is applied to the primary to cause rated full-load currents to flow in the primary and secondary coils, the mutual flux required is so small that the exciting current may be neglected. Also, because of the low flux density in the iron, the iron losses may be neglected. However, the effective resistance and reactance are unchanged and may be determined by measurements. With reduced voltage applied to the transformer, readings of P, V, and I_1 are the only ones necessary for calculating the constants. Thus,

$$R_1 + R_2\left(\frac{N_1}{N_2}\right)^2 = \frac{P}{I_1^2} \tag{17-4}$$

$$Z_1 + Z_2\left(\frac{N_1}{N_2}\right)^2 = \frac{V}{I_1} \tag{17-5}$$

$$X_1 + X_2\left(\frac{N_1}{N_2}\right)^2 = \sqrt{\left(\frac{V}{I_1}\right)^2 - \left(\frac{P}{I_1^2}\right)^2} \tag{17-6}$$

Inasmuch as the test is made with low voltage, it is advantageous to short-circuit the low side of the transformer because the voltage then needed on the high side will be generally better suited to the available instruments. If the test were made with the high side short-circuited, the reflection constant would be $\frac{N_2}{N_1}$.

Example 17-2.—A 5-kv-a, 440/110-volt transformer is short-circuited on its low side, and reduced voltage is applied to the high side. The meter readings are as follows: $P = 110$ watts, $V = 13$ volts, and $I_1 = 11.38$ amp. Calculate the equivalent resistance R and inductance X of the transformer.

Solution.—By equations (17-4), (17-5), and (17-6),

$$R_1 + R_2\left(\frac{4}{1}\right)^2 = \frac{110}{11.38^2} = 0.851 \text{ ohm}$$

$$Z_1 + Z_2\left(\frac{4}{1}\right)^2 = \frac{13}{11.38} = 1.141 \text{ ohms}$$

$$X_1 + X_2\left(\frac{4}{1}\right)^2 = \sqrt{1.141^2 - 0.851^2} = 0.759 \text{ ohm}$$

Unless the transformer is of unusual design, X_1 may be taken as equal to $X_2\left(\frac{4}{1}\right)^2$ and R_1 may be taken as equal to $R_2\left(\frac{4}{1}\right)^2$. Then,

$R_1 = 0.425$ ohm $\qquad X_1 = 0.379$ ohm
$R_2 = 0.0265$ ohm $\qquad X_2 = 0.0237$ ohm

Vector Diagrams and Voltage Regulation.[1]—Fig. 17-3 (*a*) is the vector diagram of an actual transformer and shows how the leakage impedances exact their vectorial toll of the voltage and leave only a substantial residue which varies with change in either the power factor or the current in the load. The secondary vectors are drawn to the same scales as the primary ones and are then multiplied by the turns ratio $\frac{N_1}{N_1} = a$ or by the reciprocal $\frac{N_2}{N_1} = \frac{1}{a}$, the factor depending on the reflected quantities needed.

The exciting current I_e which produces the flux ϕ_m would be in phase with ϕ_m if there were no copper or iron losses. The flux ϕ_m generates the

---

[1] For further study of the transformer vector diagram and voltage regulation, see *Elements of Electrical Engineering*, by A. L. Cook, John Wiley & Sons, Inc., or *Alternating Current Machinery*, by R. R. Lawrence, McGraw-Hill Book Co.

electromotive force E_1 in the primary coil and E_2 in the secondary coil. The voltage E_2 forces current I_2 through the secondary leakage impedance R_2+jX_2 and the load impedance. The current I_1' is the added or "cancellation" current that appears in the primary coil to balance the magnetomotive force mmf_2 caused by N_2I_2. The total primary current I_1 causes an impedance drop $I_1(R_1+jX_1)$ in the primary coil. Voltage V_1 then has a component $-E_1$ to balance E_1 and another to provide the primary voltage drop I_1Z_1. The secondary generated voltage aE_2 then fails to equal V_1 because of the combined effect of both primary and secondary impedances.

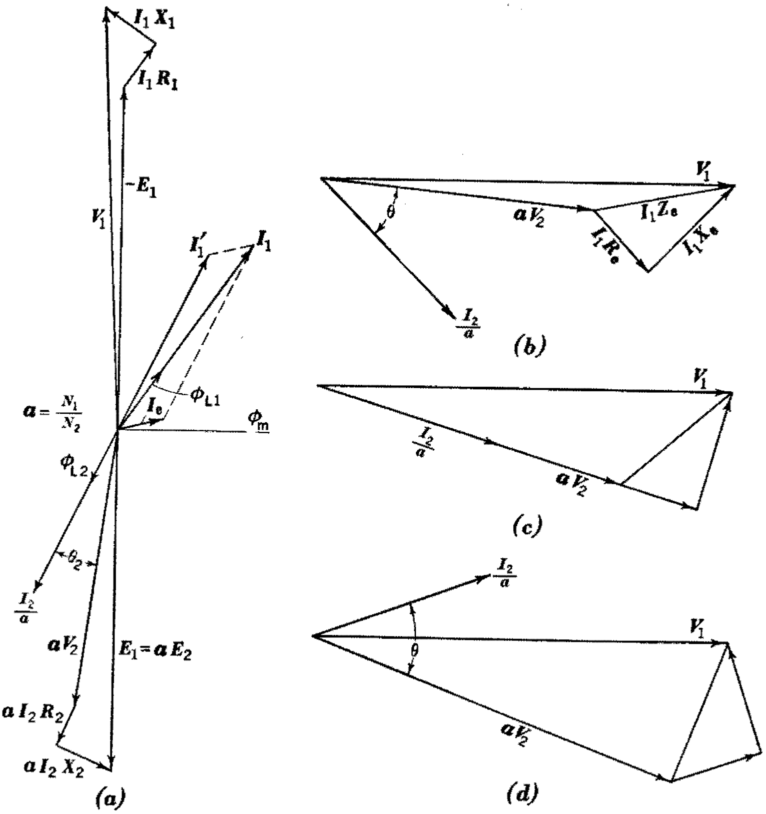


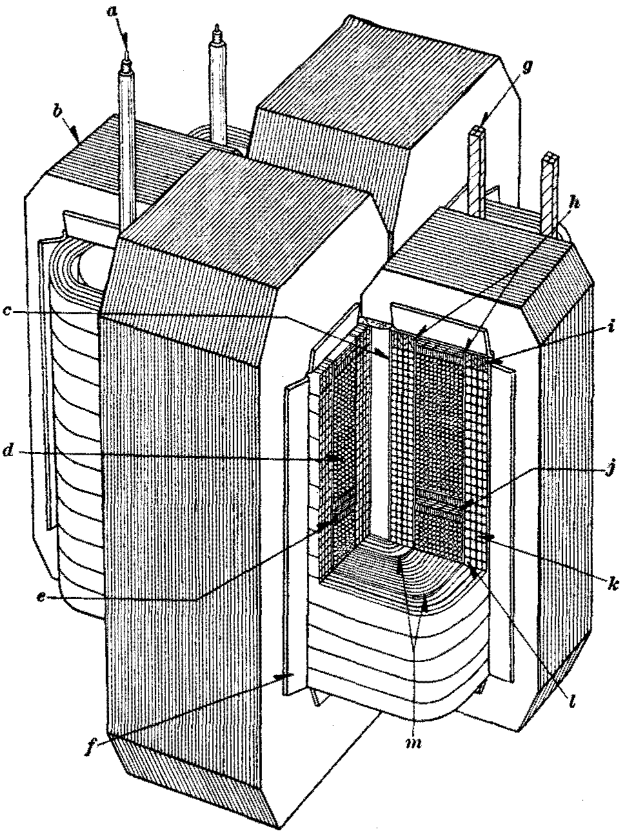
Fig. 17-3. Transformer Vector Diagrams

The impedances Z_1 and Z_2 may be combined to make a total effective impedance Z_e. Thus,

$$\begin{aligned} Z_e &= Z_1+Z_2 \\ &= (R_1+jX_1)+a^2(R_2+jX_2) \\ &= (R_1+a^2R_2)+j(X_1+a^2X_2) \end{aligned}$$

or

$$Z_e = R_e+jX_e \tag{17-7}$$



(*Courtesy Westinghouse Electric Corp.*)

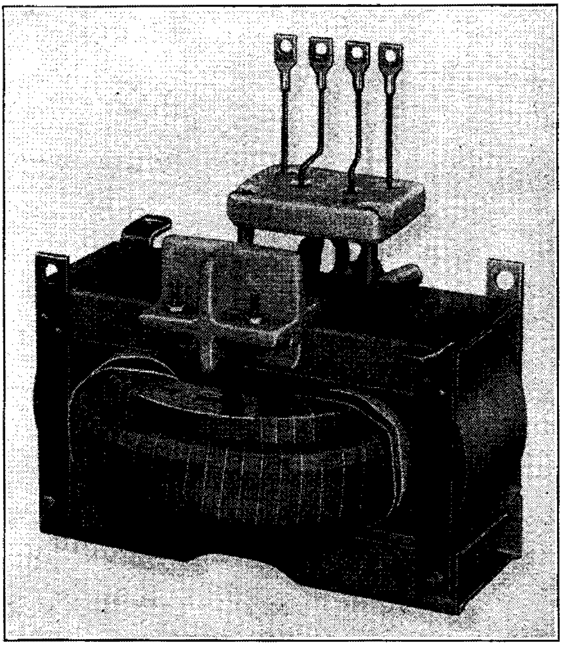
Fig. 17-4. Transformer Construction Details

a	High-Voltage Leads
b	Distributed Shell Form of Magnetic Circuit
c	Barrier Between Coils and Iron
d	High-Voltage Windings of Cotton-Covered Enameled Copper Wire
e	Layer Material of Micarta Paper Crimped at Ends
f	Fullerboard Channels Between Coils and Iron
g	Low-Voltage Leads
h	Barrier Between High-Voltage and Low-Voltage Coils
i	Fullerboard Collars to Provide Creepage Distance and Give Mechanical Strength
j	Fullerboard Washers
k	Cloth Insulation Between Coil Sections
l	Low-Voltage Winding of Copper Ribbon Insulated With Paper and Cambric Covering
m	Cooling-Oil Ducts

where Z_e is the effective transformer impedance of the equivalent circuit of Fig. 17-3 (*b*) exclusive of the exciting admittance Y which is negligibly small. The voltage regulation of the transformer, in per cent, is

$$\text{Regulation} = \frac{V_1 - aV_2}{aV_2} \qquad (17\text{-}8)$$

In Fig. 17-3 (*b*), (*c*), and (*d*) are shown equivalent circuit vector diagrams drawn for constant values of V_1 and I_2 but for three different load power factors, to show how the secondary terminal voltage is dependent on the power factor of the load as well as on the magnitude of I_2.



(*Courtesy General Electric Co.*)

Fig. 17-5. Wound-Core Type Transformer

Construction.—A common type of magnetic circuit for a transformer is shown in Fig. 17-4. If the primary and secondary coils are "interleaved" or assembled very closely together, the leakage flux between coils and the voltage regulation are reduced to a minimum. Transformers designed for arc-furnace service are purposely arranged for high flux leakage, in order to produce large voltage regulation and thus protect the connected power line from excessive voltage disturbances when the arcs are struck.

Fig. 17–5 shows another type of core and coil assembly in which the primary and secondary coils are form-wound and are nested together before the iron core is added. The core is in two parts, each consisting of a coil of core steel wound around one side of the O-shaped coil assembly. This construction has several advantages compared to the construction using punchings. The magnetic flux is carried in a path in which the grain direction in the iron is parallel to the flux direction and the maximum permeability of the iron is therefore utilized. The air-gap reluctance is a minimum because the flux in its whole path crosses but one gap; and that gap is only as long as the distance between adjacent convolutions of the iron strip, and its area is equal to that of one side of the strip. The weight is less than with older types of construction and, because the core can be applied by a special machine, the cost and the selling price are advantageous.

Losses and Efficiency.—A transformer energized from a power line and supplying no load, but in constant readiness to do so, draws an exciting current which is necessary to maintain the core flux, and this flux in turn maintains the secondary voltage. The exciting current has a power component which supplies the hysteresis and eddy-current losses in the iron and also a very small amount of resistive power loss in the primary winding. These losses are almost constant and are independent of the load, and constitute a cost to the power system which must be passed on to the consumer as a part of the *readiness-to-serve* charge in power contracts, as discussed in Chapter 24.

When a transformer is loaded, the secondary current and the added primary current cause an additional resistive loss which is proportional to the square of the load current. The losses may be summarized as follows:

$$\text{1. Core loss (hysteresis and eddy-current)}\left\{\begin{array}{l}\text{Fixed or constant}\\ \text{losses}\end{array}\right.$$

2. Copper loss

$$\left.\begin{array}{l}(a)\ I_p^2R_p \text{ (resistive loss in primary winding)}\\ (b)\ I_s^2R_s \text{ (resistive loss in secondary winding)}\end{array}\right\}\text{Variable losses}$$

The *power efficiency*, in per cent, is:

$$\text{Power efficiency} = \frac{\text{Input power} - \text{Losses}}{\text{Input power}} \times 100 \qquad \textbf{(17–9)}$$

or

$$\text{Power efficiency} = \left(1 - \frac{\text{Losses}}{\text{Input power}}\right) \times 100 \qquad \textbf{(17–10)}$$

The power efficiency is maximum when the load is great enough to make the fixed losses equal to those which are variable functions of the load.

The *energy efficiency* is of great importance in the case of any device, such as a distribution transformer, which must be kept energized and ready for use without notice but is actually used only a small part of the time. The energy efficiency is:

$$\text{Energy efficiency} = \frac{\text{Energy output in kw-hr}}{\text{Energy input in kw-hr}} \times 100 \qquad (17\text{–}11)$$

Example 17–2.—A 100-kv-a, 2300/230-volt, single-phase transformer, when excited at normal rated voltage with no load, has a power input of 1000 watts. The resistance of the primary coil is 0.243 ohm and that of the secondary coil is 0.00243 ohm. Calculate the power efficiency when the secondary current is 200 amp and the load power factor is 100%.

Solution.—Core loss = 1000 watts; secondary copper loss $= I_s^2R_s = 200^2 \times 0.00243 =$ 97.2 watts; primary current $= \frac{1}{10} \times 200 = 20$ amp; primary copper loss $= I_p^2R_p = 20^2 \times 0.243$ = 97.2 watts. Hence,

$$\text{Efficiency} = \left(1 - \frac{\text{Losses}}{\text{Input power}}\right) \times 100$$

$$= \left(1 - \frac{1000 + 97.2 + 97.2}{(200 \times 230) + 1000 + 97.2 + 97.2}\right) \times 100 = 97.4\%$$

Voltage Regulation.—The voltage regulation of a transformer depends on the magnitudes of the resistances and the reactances of the windings. The impedance of a transformer is usually specified in per cent. This designation means that, with either the primary or the secondary winding short-circuited, the specified per cent of the rated voltage applied to the other winding will cause full-load current to flow in both windings. Distribution transformers are usually designed with impedances of 2 to 4 per cent. The per cent of voltage regulation with loads of rated current or less cannot exceed the per cent of impedance. The impedance specified on the name plate of a transformer is therefore an indication of the *maximum* voltage regulation that may be expected of it. The voltage regulation of a transformer is based on the change in secondary voltage from full load to no load with constant primary applied voltage. Thus,

$$\text{Regulation} = \frac{V_{\text{no load}} - V_{\text{full load}}}{V_{\text{full load}}} \times 100 \qquad (17\text{–}12)$$

Operation at Other Than Rated Voltages and Frequencies.—Transformers may be operated at voltages or frequencies within 10 per cent of the rated values. If a 60-cycle transformer be connected to a 25-cycle line of its rated voltage, it will draw excessive exciting current, the core will overheat, and the result will be generally unsatisfactory. A 25-cycle transformer may be used on a 60-cycle line of its rated voltage, but with much higher voltage regulation than normal. It may be stated as a general rule that, within the breakdown limits of its insulation, *a transformer may be operated at any frequency satisfactorily if the ratio of the applied voltage to the frequency for which it was designed is maintained approximately.*

Additive and Subtractive Polarity.—When transformers are to be connected in parallel or in groups for three-phase service, identification of their terminal polarities is imperative. A transformer with *additive polarity* is connected internally so that, if its primary coil be excited and one primary terminal be connected to the secondary terminal *nearest* it, a voltmeter connected between the other two terminals of the two windings will indicate the *sum* of the primary and secondary voltages. If, when the transformer is so connected and excited, the voltmeter indicates the *difference* between the primary and secondary voltages, the transformer is said to be of *subtractive polarity*. Fig. 17–6 indicates the two types of internal connections.

A standard plan of marking the terminals of power transformers has been adopted by the National Electrical Manufacturers Association so that interconnection of units may readily be made without polarity tests. When an observer faces the high-voltage side of the transformer, the extreme right high-voltage lead will be H_1. Other high-voltage leads reading right to left will be H_2, H_3, H_4, etc. Facing the low-voltage side of the transformer, the observer will see the low-voltage leads marked X_1, X_2, X_3, etc., reading from either right or left. If terminal X_1 is diagonally across the transformer from H_1, the transformer is of additive polarity. When H_1 and X_1 are adjacent, the transformer is of subtractive polarity.

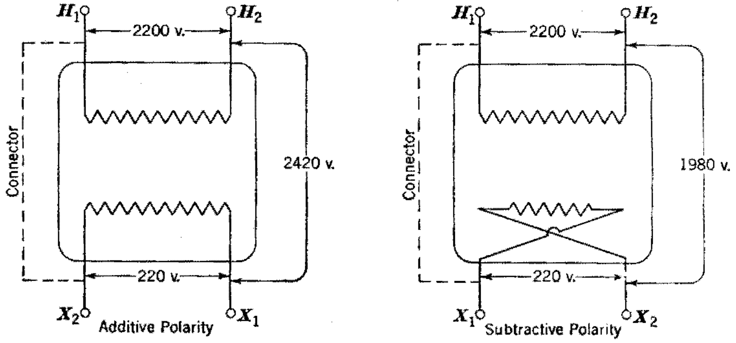
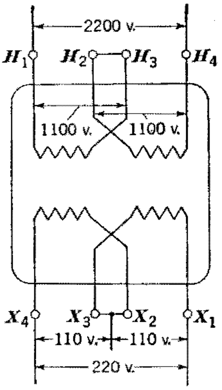


Fig. 17–6. Additive and Subtractive Polarity Connections

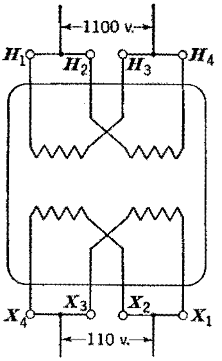
Transformer Single-Phase Connections.—Transformers are sometimes provided with divided coils in both primary and secondary windings. The transformer in Fig. 17–7 has four coils connected to provide 110-volt or 220-volt service from a single-phase 2200-volt feeder. Fig. 17–8 shows the same transformer connected to provide only 110-volt service from a 1100-volt feeder, but its effective capacity is then unchanged.

Parallel Operation.—Two or more transformers may be operated in parallel from the same power line, if their primary and secondary voltages and impedances are the same. If their resistances and reactances are not similar, circulating currents will flow between them and their effective capacities and efficiencies will be reduced. Transformers intended for parallel operation should have the same per cent of impedance. If their impedances are not the same, they will share the load in approximately inverse proportion to their respective impedances. Thus, a transformer with a small internal impedance will assume a greater part of the load than will a transformer with a higher internal impedance.



Arranged for either 110 volts or 220 volts.

Fig. 17–7. House-Lighting Transformer



Arranged for 110 volts only.

Fig. 17–8. House-Lighting Transformer

Grouping of Single-Phase Transformers for Three-Phase Operation. Single-phase transformers may be connected in groups in many ways for three-phase transformation. Fig. 17–9 shows a Δ-Δ group and Fig. 17–10 shows a Y-Δ group. In general, groups with Y-Δ connection, but without use of a fourth connection as a neutral, are to be avoided where unbalanced loads must be carried, because an unbalance in load will cause a circulating current of fundamental frequency to flow around the delta and will cause increased heating. If, however, in a Y-Δ bank with ungrounded neutral a primary conductor be opened, as by blowing of a fuse, while the secondary is carrying a three-phase motor load, the bank will no longer attempt to maintain three-phase voltage at its secondary terminals and neither the primary nor the secondary winding will be overloaded. Under these conditions a three-phase motor on the secondary line will still operate single-phase and will attempt to carry its assigned load. The current in one motor line will be twice as large as that in either of the other two lines; and, if the motor is protected by overload relays in only two of its lines, then there is one chance in three that the motor will not be adequately protected against excessive current.

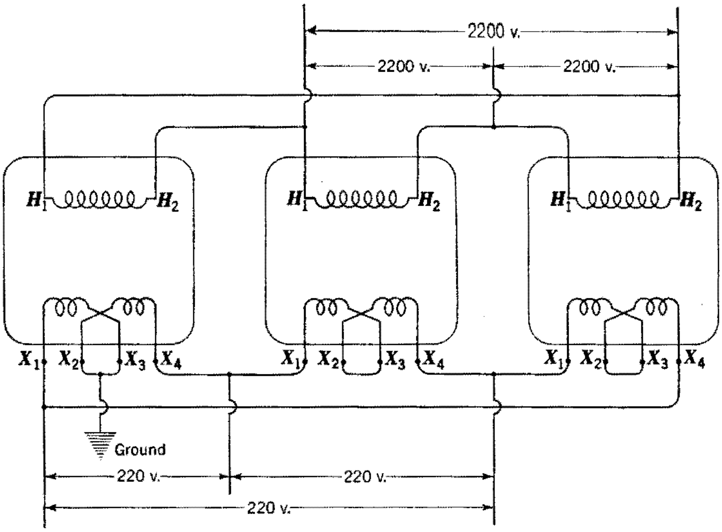


Fig. 17–9. Transformer Connections. 2200/220-Volt, 3-Phase, Δ-Δ

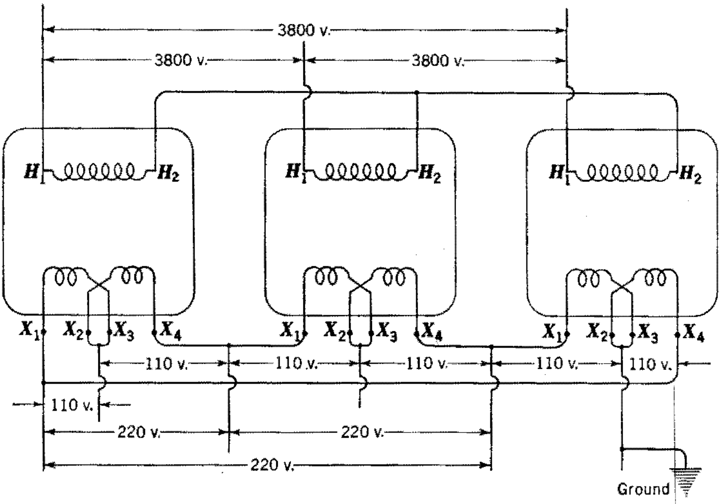
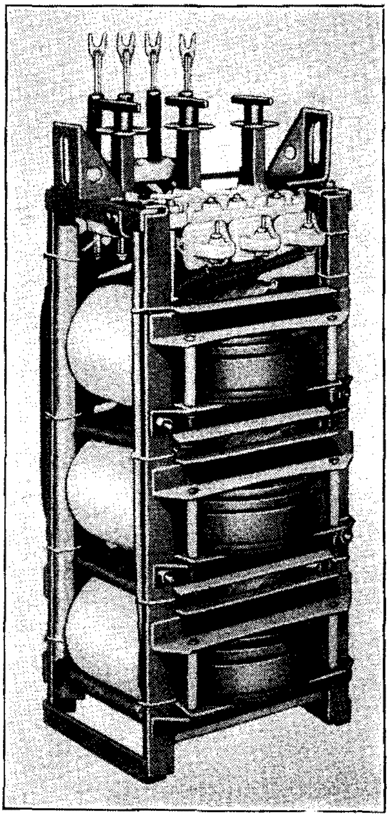


Fig. 17–10. Transformer Connections. 3800/220-110-v, 3-Phase, Y-Δ

Fig. 17–11 shows three single-phase transformers stacked in a common frame for immersion in a single tank for pole mounting.

The Δ-Δ connection has many advantages. If the load to be carried may be expected to increase materially in a few years, the transformers may be installed Δ-Δ, but with one unit omitted as in Fig. 17–12. Such a group is called an *open delta.*



(*Courtesy Line Material Company*)

Fig. 17–11. Three "Round-Wound" Transformers Stacked for Immersion in One Tank for Three-Phase Service

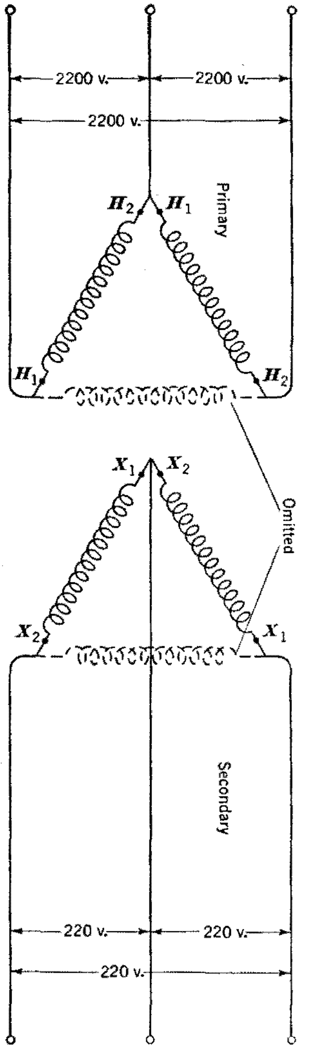


Fig. 17–12. Transformer Connections. 2200/220-Volts, Open-Delta, 3-Phase

The open delta eliminates the fixed charges and depreciation on the third transformer until the growing load requires the addition of the third. However, the two transformers so connected have only approximately 58 per cent as much capacity as the complete group of three.[1] The reason

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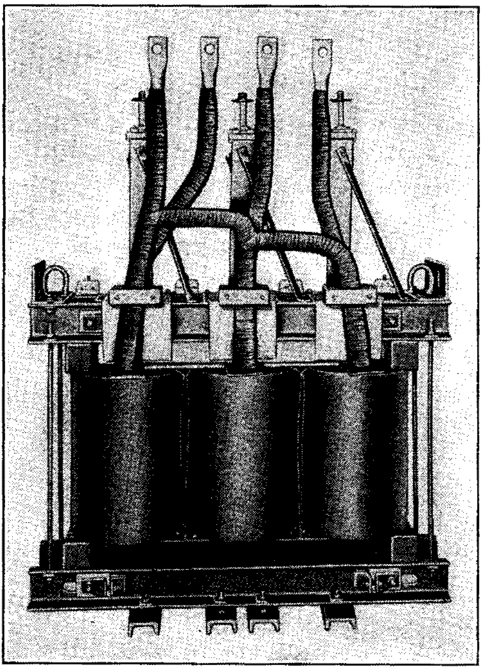
[1] *Principles and Practice of Electrical Engineering*, by A. Gray and G. A. Wallace. McGraw-Hill Book Co.

for this is as follows: In a group of three the line current to the load is $\sqrt{3}$ times the single transformer current; whereas, in an open-delta bank each transformer must carry the same current as each of the two lines connected to their "outside" terminals. Therefore, the line current must not exceed the current capacity of one transformer, which is $\frac{1}{\sqrt{3}}$, or 0.58, times the three-transformer capacity.

If one of the group of a Δ-Δ bank of transformers fails, it may be removed and the service may be continued with open-delta connections until repairs can be made, although the system will have only 58 per cent of the original capacity.



(Courtesy Line Material Company)

Fig. 17–13. Elements of Y-Connected, Three-Phase Transformer With Three Magnetically Interconnected Legs

It is sometimes desirable to increase the distribution voltage of a power system to accommodate a growing load. If the transformers connected to the system are Δ-Δ connected, it is feasible to reconnect them Y-Δ and thus permit raising of the line voltage 73 per cent without replacing transformers. Before deciding upon such a change, however, the adequacy of the insulation in the transformers must be investigated.

Three-Phase Transformers.—Three-phase transformers, instead of groups of three single-phase units, may sometimes be used with convenience and economy. These transformers consist of three separate windings mounted on a three-legged or four-legged core. They are lighter and lower in cost, require less floor space, and are more efficient than three single-phase transformers of equivalent capacity. However, their greater cost of repairs, the greater cost of spare units, and the greater derangement of service in case of failure weigh so heavily against their installation that groups of three single-phase units are more commonly used. Fig. 17–13 shows a three-phase transformer on which the coils are assembled on three magnetically interconnecting cores.

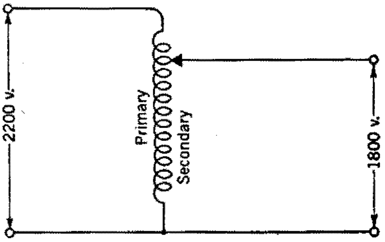
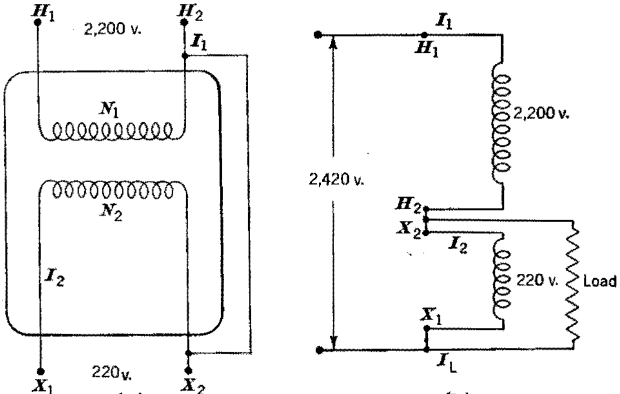


Fig. 17–14. **Auto-Transformer**



(*a*) (*b*)

Fig. 17–15. **Two-Winding Transformer Connected as an Auto-Transformer**

Auto-Transformers.—An auto-transformer has but one winding from which two or more tap leads are made available, as in Fig. 17–14. It is not economically adapted to high-ratio transformation, and the voltage difference between the high-voltage and low-voltage sides seldom exceeds 50 per cent of the higher value. Auto-transformers are commonly used in thermionic battery-charging circuits and transformer-type starters for induction and synchronous motors. The secondary circuit is not insulated from the primary circuit, as in the two-winding transformer, and the auto-transformer is therefore dangerous to life for some purposes.

If the losses in an auto-transformer are ignored, it may be said that the volt-amperes input is equal to the volt-amperes output. An ordinary two-winding transformer, such as the one shown in Fig. 17–6 with a permanent connector as in Fig. 17–15 (*a*), may be used as a step-up or step-down auto-transformer. By connecting terminals H_1 and H_2 to a 2200-volt line, 2420 volts may be obtained at terminals H_1 and X_1. Also, by connecting terminals X_2 and X_1 to a 220-volt line, 2420 volts may again be obtained from terminals H_1 and X_1.

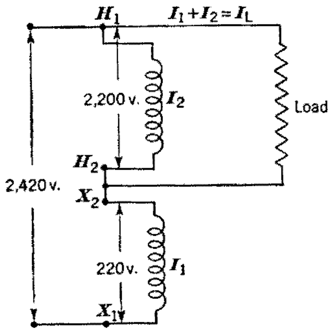


Fig. 17–16. Transformer in Example 17–3

It should be noted that, when the secondary is loaded as in the simplified diagram of Fig. 17–15 (*b*), part of the load current comes to the load "conductively" from the line downward toward the load tap. The remainder of the load current comes to the load "inductively" from the secondary coil upward toward the load tap. The portion of the load current in the secondary winding produces a counter magnetomotive force which tends to reduce the mutual flux. There must then be a corresponding rise in primary magnetomotive force to counteract that of the secondary, if the mutual flux is to be kept constant. The rise in primary amperes flowing through the primary coil is said to produce "cancellation" ampere-turns. For this arrangement,

$$N_1I_1=N_2I_2 \tag{17–13}$$

and

$$I_L=I_1+I_2 \tag{17–14}$$

Example 17–3.—If the transformer of Fig. 17–16 is rated 100 kv-a as a two-winding transformer, how many amperes may be delivered to a 2200-volt load connected to terminals H_2 and H_1 without overloading either winding when the applied voltage at H_1 and X_1 is 2420?

Solution.—In a two-winding transformer the full-load current of the 2200-volt winding is

$$\frac{100{,}000}{2200}=45.5 \text{ amp}$$

This then is the maximum permissible value of I_2. Hence,

$$N_1I_1=N_2I_2$$

or

$$N_1I_1=10N_1\times 45.5$$

and

$$I_1=455 \text{ amp}$$

Also,

$$I_L = I_1 + I_2$$
$$= 455 + 45.5 = 500.5 \text{ amp}$$

With this loading, neither the primary nor the secondary winding is overloaded

The Scott, or T, Connection.—Remnants of two-phase power systems in this country and the adherence of certain foreign manufacturers to two-phase special equipment sometimes make it desirable to convert power from three-phase to two-phase or vice versa. This transformation may be accomplished by the Scott, or T, connection of two transformers, as in Fig. 17–17.

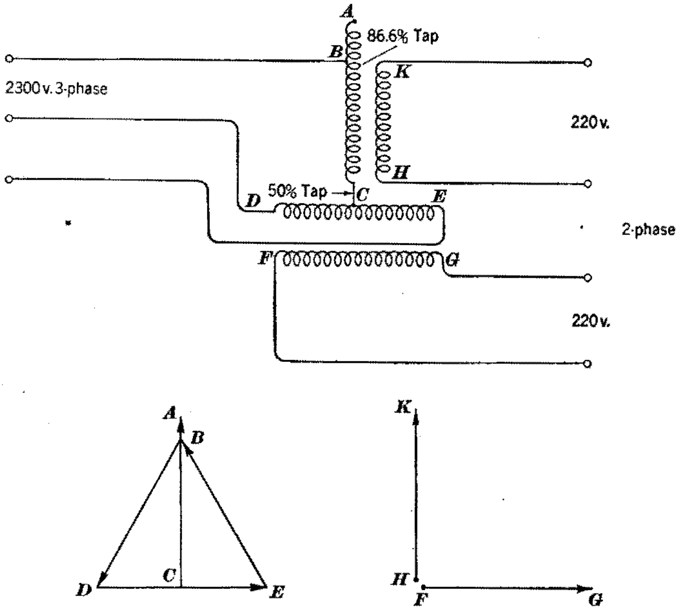


Fig. 17–17. Scott, or T, Transformer Connections

Two identical transformers may be used, but a tap point B in one of them must be available where 86.6 per cent of the turns have been traversed from C, and a tap at the mid-point must be available in the other. When the line is loaded, the secondary voltages become somewhat unbalanced, but not sufficiently so to cause undesirable performance for most industrial uses.

PROBLEMS

1. The efficiency of a single-phase, 10-kv-a, 2200/220-volt, 60-cycle transformer is to be measured by loading. Show by a diagram the proper locations of all necessary meters, and specify the required ranges of all instruments.

2. The single-phase transformer of Problem 1 has a primary resistance of 3.0 ohms, a secondary resistance of 0.03 ohm, and a no-load power input of 100 watts. Calculate the energy efficiency in a period of 24 hours on the following load schedule:

4 hours full load at 90% power factor;
2 hours ½ rated load at 100% power factor;
18 hours no load.

The exciting current may be neglected.

3. Show by a diagram a three-phase group of single-phase transformers connected Y-Δ. Let the transformers be similar, except that one is of additive polarity and two are of subtractive polarity.

4. Specify the voltage ratio for each of a group of three single-phase transformers to be used to "step down" Y-Δ from a three-phase 2200-volt line to 220 volts for factory service.

5. Specify the voltage ratios and kv-a capacities of two single-phase transformers to "step down" from 2200 volts to 220 volts, three-phase, by open-delta connection to supply a 175-kv-a load.

6. An auto-transformer is to be used to "step down" a voltage of 220 volts to 120 volts. The load current is to be 50 amp. Neglecting the exciting current, find the current in each part of the transformer.

7. A 2300/230-volt, 10-kv-a, single-phase transformer has an effective resistance of 2% and a leakage reactance of 1%. Calculate the regulation at full load at 60% power factor leading.

8. Calculate the regulation of the transformer of Problem 7 with 80 per cent of full load at 60% power factor lagging.

9. A 50-kv-a, 60-cycle, 2300/230-volt transformer is to be used on a 25-cycle line as a temporary expedient. What would be the approximate maximum voltage permissible on the 2300-volt winding?

10. An anthracite mine has the following 3-phase, 60-cycle motors: two 100-hp induction motors with estimated efficiency of 90% and power factor of 91% lagging; five 50-hp induction motors with estimated efficiency of 82% and power factor of 78% lagging; three 100-hp synchronous motors with estimated efficiency of 88% and power factor of 80% leading. The available transmission line is 3100-volt, 60-cycle, 3-phase. Write purchase specifications for three single-phase transformers to step down the voltage to 440 volts by a Y-Δ connection for the motors. Assume a load diversity factor of 69%.

11. A transformer has a full-load voltage of 108 volts and a no-load voltage of 115. What is the regulation in per cent? Assume constant primary voltage.

12. A balanced three-phase load of 1740 kv-a is to be supplied by two transformers connected open delta. What must be the single-phase kv-a rating of each transformer?

13. An auto-transformer for starting an induction motor steps down the line voltage from 230 volts to 84 volts per phase line-to-neutral for the motor. If the transformer furnishes 20 kv-a per phase at 50% power factor, calculate the primary and the secondary currents in the auto-transformer. Neglect the exciting current.

14. What would be the maximum permissible continuous kw output at unity power factor, three-phase, from two similar 100-kv-a single-phase transformers connected in T to a two-phase line?

15. Two 100-kv-a, 2400/480-volt transformers are connected in open delta for a three-phase load of unity power factor. How many kilowatts of power can be delivered to the load without overloading the transformers? *Ans. 174 kw*

16. A welding transformer operating from a 240-volt line has 500 turns in its primary winding and 1 turn in its secondary winding. When the transformer during a welding cycle draws 20 amp from the line, approximately how many amperes are flowing in the weld? *Ans. 10,000 amp*

17. A 100-kv-a, single-phase, 2400/240-volt, 60-cycle transformer has an iron loss of 1200 watts at rated voltage and a copper loss of 1500 watts at rated current. If it is assumed that 80 per cent of the iron loss is caused by eddy currents, what will be the approximate efficiency at rated-load current when supplying a resistance load from a 25-cycle power line of 1000 watts? *Ans. 95.8%*

18. Under the conditions of Problem **17**, the iron losses are abnormally low but the transformer's heat-dissipative capacity is as great as ever. What per cent of increase in rated-load current is permissible, if the operating temperature is not to exceed that for which the transformer was designed? *Ans. 24.8%*

CHAPTER 18

The Polyphase Induction Motor

Characteristics of Polyphase Induction Motor.—The polyphase induction motor is the simplest, cheapest, sturdiest, and generally most reliable industrial motor in general use in sizes greater than $\frac{1}{2}$ hp. The squirrel-cage rotor is extremely simple and has no electrical connections to any other part of the motor or to the power supply. With proper design proportions the polyphase motor may be satisfactorily started by connecting it directly to the power supply through a manually operated or automatic starting switch of simple construction and low cost. However, it does have certain disadvantages, such as low efficiency and low power factor at less than half-load; and, being essentially a constant-speed motor, it cannot be adapted to variable-speed duty without serious loss in efficiency. In the variable-speed form of polyphase motor, the rotor must be provided with collector rings to allow connection to an external resistor, and this form is not so simple and rugged as the constant-speed form.

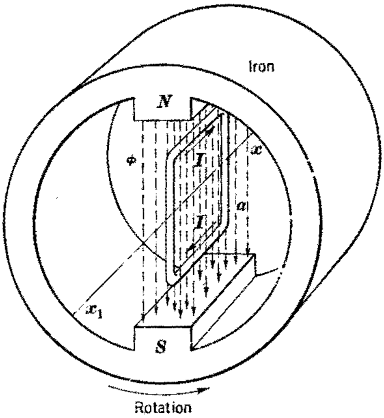


Fig. 18-1. Fundamental Elements of Induction Motor

Principle of Operation.—If the magnetized iron field structure of Fig. 18-1 be caused to rotate counter-clockwise on its axis xx_1, there will be, at the instant shown, electromotive forces induced in the closed loop of wire a; and these electromotive forces will cause a current to flow in the direction shown, according to Faraday's Law. As explained on page 128, the current in the upper horizontal conductor will produce a force toward the left, and the same current in the lower horizontal conductor will produce a force on it toward the right. These forces produce a couple tending to rotate the loop counter-clockwise about the axis xx_1. *The loop of wire will tend to rotate in the same direction as does the magnetic field, and it may be called a rotor.*

If free to rotate, the rotor would tend to rotate at a speed equal to that of the main field ϕ. If it should succeed, it would be said to *run in synchronism with* the main field. But the rotor could not attain synchronous speed because, for its driving forces, it depends on relative motion between itself and the field. Hence, the rotor must always run at a speed somewhat less than that of the main field, in order that some driving torque may exist. The numerical difference between the speed of the main field (called *synchronous speed*) and the speed of the rotor is called *slip*. Slip is usually expressed as a per cent of synchronous speed.

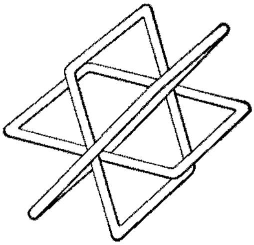


Fig. 18–2. Step in Development of Squirrel-Cage Rotor

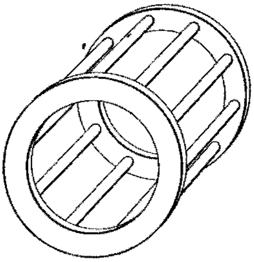
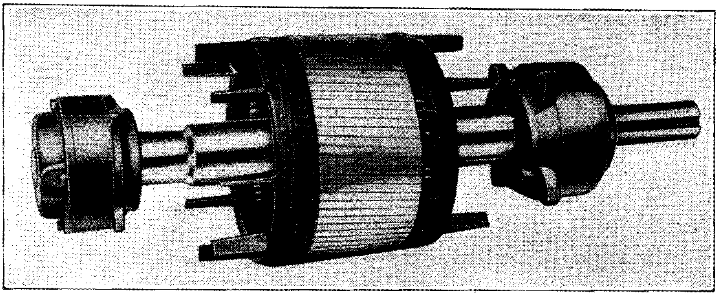


Fig. 18–3. Winding on Squirrel-Cage Rotor

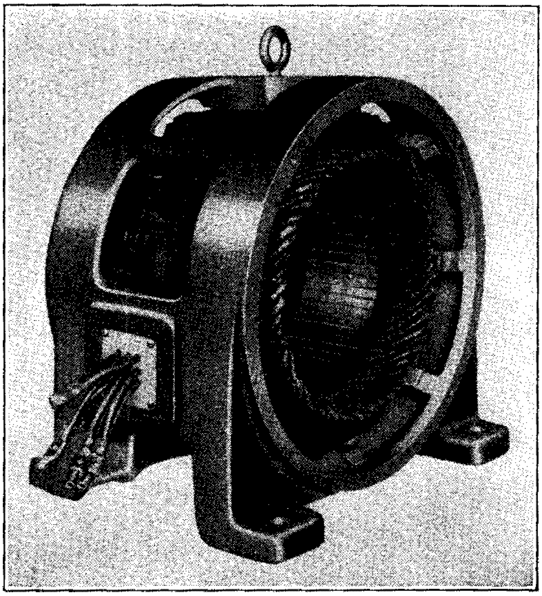
The Squirrel-Cage Rotor.—If the single loop of wire of Fig. 18–1 be replaced by several similar loops uniformly displaced circumferentially about the same axis, as in Fig. 18–2, the main field would never be without immersed bars and the turning effort on the whole would be less pulsating than with the single loop. In industrial induction motors many bars are used and, instead of being connected in loops as in Fig. 18–2, they are connected to *end rings* as in Fig. 18–3. A rotor with such an electrical construction is called a *squirrel-cage rotor*. The squirrel cage itself is built up of copper or brass bars welded to end rings of similar material, or is cast of aluminum alloy as one piece in place without insulation on a laminated steel cylindrical core. A finished rotor of squirrel-cage construction is



(Courtesy Fairbanks, Morse & Co.)

Fig. 18–4. Squirrel-Cage Induction-Motor Rotor shown in Fig. 18–4. The extreme simplicity and ruggedness of construction and the freedom from moving electrical contacts makes the squirrel-cage motor the most popular in general industrial use.

The Rotating Magnetic Field.—The polyphase induction motor does not have a rotating field structure, as suggested in Fig. 18–1; but, instead, has a field of rotating flux density produced by a distributed winding on a frame structure without moving parts. Such a frame, together with its winding, is called a *stator.* The stator winding is sometimes called the *primary,* and the rotor cage is the *secondary.* A completed stator is shown in Fig. 18–5.



(*Courtesy Fairbanks, Morse & Co.*)

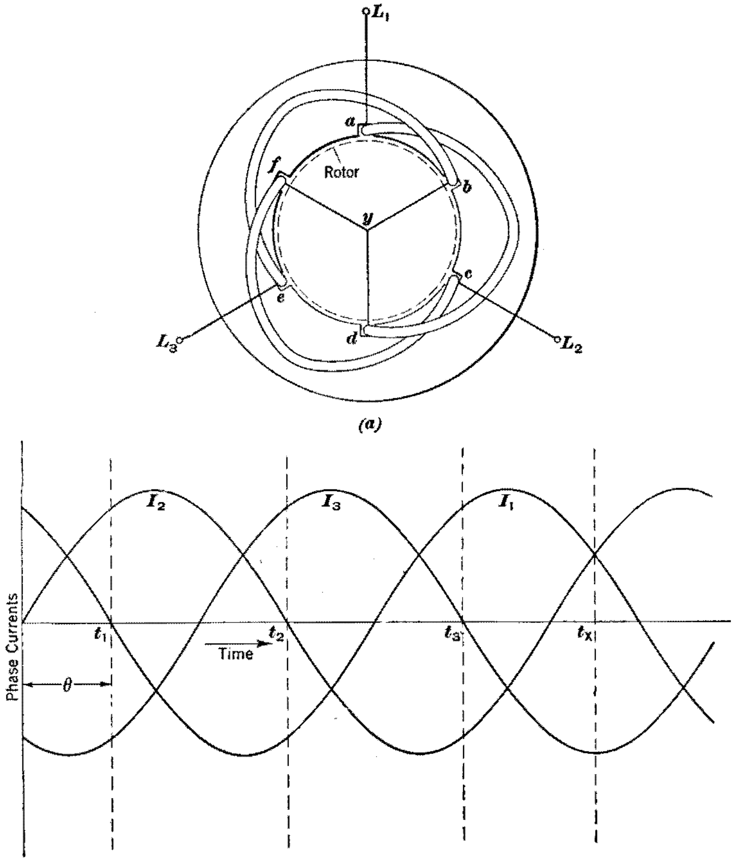
Fig. 18–5. Induction-Motor Stator

Fig. 18–6 (*a*) represents a simple three-phase, two-pole, Y-connected stator with one coil per phase. The winding terminals L_1, L_2, and L_3 connect to the three wires of the three-phase line. A convention will be used in Fig. 18–6 (*b*), in which instantaneous values of current above the axis of sinusoids represent currents entering the stator winding and going down in the slots and toward the Y-point of the winding. Three instants of time will be chosen, and it will be demonstrated that the point of maximum flux density moves around the inner periphery of the stator.

If an instant t_1 be chosen in Fig. 18–6 (*b*), I_1 is passing through zero and phase 1 contributes nothing to the magnetic field. However, the direction of I_2 is down in slot *c* and this current produces an mmf which would produce a flux density

$$B_2 = B_{2\max} \sin \theta$$

(if not affected by other magnetomotive forces)



(a)

(b)

Fig. 18-6. Windings and Currents in an Elemental Induction-Motor Stator

This flux goes *into* the rotor along *c, d, e, f,* and *out of* the rotor along *f, a, b, c*. At the same instant, the direction of I_3 is up in slot *e* and this current produces an mmf which would produce a flux density

$$B_3 = B_{3\max} \sin (\theta - 120°)$$

(if not affected by other magnetomotive forces)

If superposed, these magnetomotive forces cancel each other between *f* and *e* and between *b* and *c*, but add between *f* and *b* and between *c* and *e*. Since the superposed magnetomotive forces are equal, the total flux density (it being assumed that there is no iron saturation between f and b and between c and e) is

$$B_{t1} = 2B_{2\max} \sin \theta$$

The magnetic axis lies on a line through a and d, and the north pole is at the top at a, as shown in the diagram in Fig. 18–7 for time t_1.

By a similar procedure it may be shown that at a time t_2, 120° later than t_1, when I_2 is zero, the currents I_1 and I_3 produce a field whose density is

$$B_{t2} = 2B_{2\max} \sin \theta$$

The axis of this field is on a line through f and c, and the north pole is at c, as shown in the diagram in Fig. 18–7 for time t_2.

Similarly, it may be demonstrated that at time t_3 the axis of magnetism will be as shown in the diagram in Fig. 18–7 for time t_3. Also, it may be demonstrated that the value of flux density is practically constant,[1] irrespective of time or the position of the axis of polarity. A magnetic field, the strength of which is constant and the points of maximum intensity of which rotate with constant velocity, is called a *perfect rotating field.* In commercial polyphase motors with the common drum type of stator winding, as shown in Fig. 18–5, a great number of coils evenly distributed in slots around the inner stator bore are used. These coils are grouped in *phase belts*, in which all coils (two or more adjacent coils) of a belt are connected in series.[2] A phase belt occupying its portion of the total peripheral space is actually used instead of a single coil per phase, as shown in the elemental diagram of Fig. 18–6 (a). The direction of rotation of the field, and hence of the motor, may be reversed by interchanging any two of the three line leads.

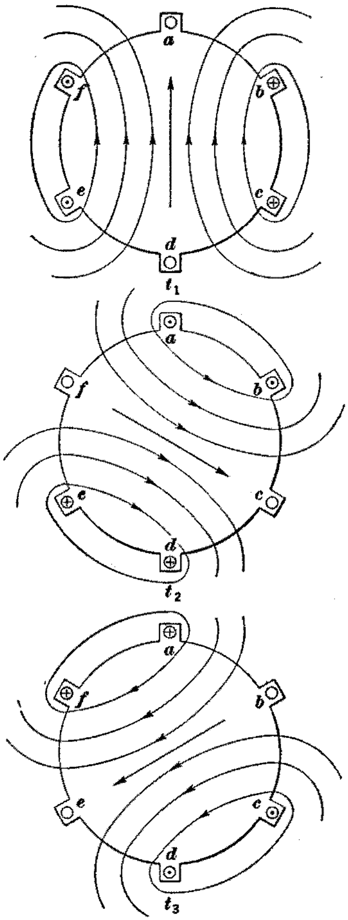


Fig. 18–7. Field Positions Corresponding to Times t_1, t_2, t_3 of Fig. 18–6 (b)

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[1] *Principles of Alternating Current Machinery*, by R. R. Lawrence. McGraw-Hill Book Co.

[2] *Connecting Induction Motors*, by A. M. Dudley. McGraw-Hill Book Co.

Speed of an Induction Motor.—It has just been shown that in two-thirds of one cycle the magnetic field rotated through two-thirds of one revolution. With a frequency of 60 cycles, the field would rotate 60 times per second and 3600 times per minute. In one cycle the field moves through 360 electrical degrees. Since there are 360 electrical degrees for every two poles around the winding, a four-pole motor contains 720 electrical degrees; so two cycles are required for every revolution of the field. Hence, a four-pole motor runs only one-half as fast as a two-pole motor. A few common synchronous speeds for 60-cycle motors are given in the following table.

Number of Poles	Speed in rpm
2	3600
4	1800
6	1200
8	900
10	720
12	600

Rotating fields for motors with two, four, and six poles are shown in Fig. 18–8.

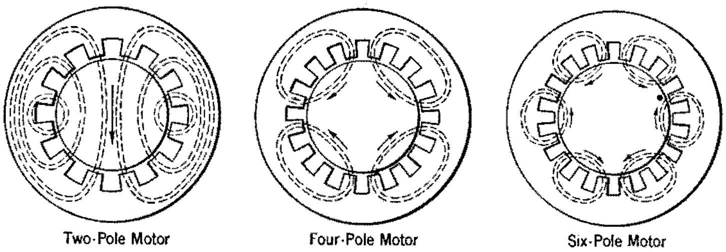

Fig. 18–8. Induction-Motor Rotating Fields

Torque at Standstill.—The torque of a squirrel-cage motor is produced by the mutual reaction of the main rotating field and the rotor field which is produced by the rotor currents. The rotor bars and the end rings are both resistive and inductive, and therefore the bar currents lag the induced electromotive forces which produce them. When the motor is at standstill, the frequency of the rotor currents is the same as that of the stator current. Between standstill and synchronous speed the rotor-current frequency is proportional to the slip. Therefore, the rotor reactance is $\frac{2\pi f L s}{100}$, in which s is the slip in per cent of synchronous speed, and this reactance varies in proportion to the slip.

It may be shown[3] that the maximum torque is developed in a rotor when the rotor reactance is approximately equal to the rotor resistance. If a motor be designed so that the rotor resistance and reactance are equal at standstill, the maximum torque will occur at standstill and the speed-torque curve will be similar to curve (*a*) of Fig. 18–9.

If the rotor resistance be made somewhat less, but the inductance be kept as before, the resistance and reactance will become equal at some speed other than zero and the maximum torque will occur at that speed, as indicated in Fig. 18–9 by curve (*b*). If then the resistance be still further reduced as far as practicable, a characteristic curve similar to curve (*c*) may be obtained. It is significant that the *maximum values* of curves (*a*), (*b*), and (*c*) are *all equal*; and that the change in rotor resistance produces only a change in the speed at which the maximum torque occurs, and also a change in torque at standstill or starting torque. This phenomenon is of great industrial importance in adapting motor characteristics to special applications.

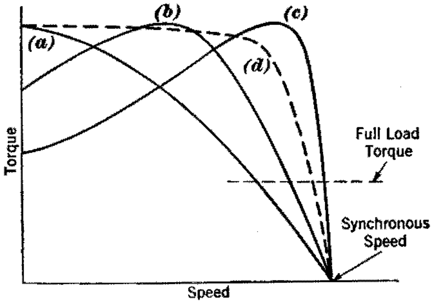

Fig. 18–9. Induction Motor Characteristic Speed-Torque Curves

The Constant-Speed Induction Motor.—The American Institute of Electrical Engineers has defined the *constant-speed* motor as one the speed of which is either constant or does not materially vary; such motors are the synchronous motor, the induction motor with small slip, and the direct-current shunt motor. Therefore, a motor with characteristic like curve (*c*) of Fig. 18–9 is a constant-speed motor and is widely used for general-purpose duty. This type of motor is especially adapted to driving loom spindles, d-c generators, circular saws, printing presses, and other constant-speed equipment, which does not require in starting more than about 150 per cent of full-load torque. Such a motor is called a *normal-torque* motor. In order to have a basis for normal starting torques, the electrical industry has established minimum values for motors having two to sixteen poles.

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[3] *Principles of Alternating Current Machinery*, by R. R. Lawrence. McGraw-Hill Book Co.

With full rated voltage applied at the instant of starting, the starting torques will not be less than the following per cents of full-load torque:

2-pole, 150 per cent of full-load torque
4-pole, 150 per cent of full-load torque
6-pole, 135 per cent of full-load torque
8-pole, 125 per cent of full-load torque
10-pole, 120 per cent of full-load torque
12-pole, 115 per cent of full-load torque
14-pole, 110 per cent of full-load torque
16-pole, 105 per cent of full-load torque

The constant-speed induction motor cannot be adapted to varying-speed service, such as is required on stokers and some forced-draft blowers. If it is desirable to change the characteristic of an induction motor from that of curve (*c*) in Fig. 18-9 to that of curve (*b*), the rotor resistance must be increased. The resistance may sometimes be increased by the requisite amount by turning some of the metal off the end rings with a lathe. Rotors with cast-on cooling fans cannot be effectively altered much in this way.

By further increasing the rotor resistance it is possible to obtain characteristic (*a*) which produces maximum torque at the instant of starting. Motors having many variations of characteristics between those of (*a*) and (*c*) are manufactured. The most popular one in general use has characteristic (*c*) because the starting torque is sufficient for most applications and its starting current is low enough to permit starting it directly on full line voltage without using the more expensive starter required to provide reduced starting voltage. The starting torque and current of an induction motor are controlled in its design mainly by the resistance and reactance of the rotor. The resistance is determined by the size of rotor bar and the resistance of the metal used. The reactance is determined by the length of air-gap, shape of slot, depth of slot below the rotor surface, and amount of overhang of the bars beyond the rotor iron. Characteristic (*d*) is obtained by using two cages of bars, one inside the other. The outer cage has high resistance and low reactance, while the inner cage has low resistance and high reactance. This combination produces a high starting torque with low starting current and also good speed regulation. The differences in rotor design are seldom discernible by mere inspection. By means of a code letter, adopted by the National Electrical Manufacturers Association, on the name plate it is possible to calculate the starting current of any induction motor except one of fractional horsepower. The code letters are shown in the accompanying tabulation.

By use of the code letter, it is possible to calculate the starting inrush current of a motor and so to decide whether the starting of the motor would cause excessive voltage disturbance on the power supply to which it is

Code Letter	Kilovolt-Amperes per Horsepower, with Locked Rotor	Code Letter	Kilovolt-Amperes per Horsepower, with Locked Rotor
A	0–3.14	L	9.0– 9.99
B	3.15–3.54	M	10.0–11.19
C	3.55–3.99	N	11.2–12.49
D	4.0 –4.49	P	12.5–13.99
E	4.5 –4.99	R	14.0–15.99
F	5.0 –5.59	S	16.0–17.99
G	5.6 –6.29	T	18.0–19.99
H	6.3 –7.09	U	20.0–22.39
J	7.1 –7.99	V	22.4 and up
K	8.0 –8.99		

intended to be connected. For example, a 100-hp, 440-volt motor with code letter *C* would have a starting inrush current of

$$\frac{100 \times 3.99 \times 1000}{\sqrt{3} \times 440} = 523 \text{ amp}$$

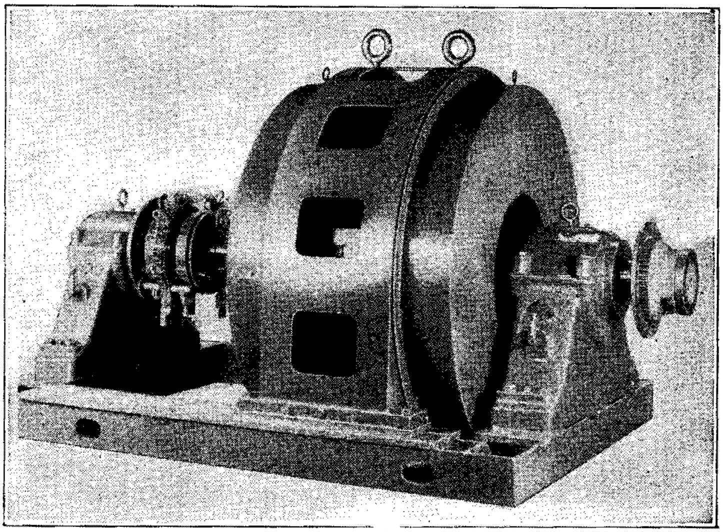
and a 100-hp, 440-volt motor with code letter *H* would have a starting inrush of

$$\frac{100 \times 7.09 \times 1000}{\sqrt{3} \times 440} = 929 \text{ amp}$$

The motor with the code letter *H* would require reduced-voltage starting on most power systems.

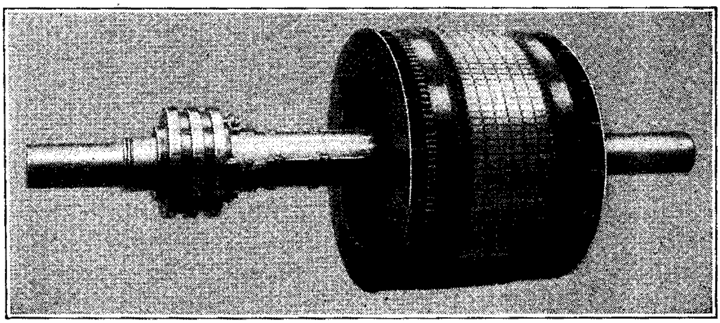
Multi-Speed Motors.—Some constant-speed motors are provided with leads from the pole groups brought out to an external switch by which the number of poles and hence the synchronous speed may be changed quickly even while the motor is running. Such a motor is defined as a *multi-speed* motor. For example, the pole leads may be so arranged as to provide quick changes from 4 to 6 to 8 poles with corresponding synchronous speeds of 1800, 1200, and 900 rpm, respectively, on 60-cycle supply. At each speed the motor will run as a constant-speed motor, and no intermediate speeds are available. Two-speed motors are sometimes built with two separate windings to accomplish the change in speed.

Wound-Rotor (Slip-Ring) Motors.—For varying-speed duty it is necessary that the resistance of the rotor winding be under control of the operator. For this purpose the rotor is provided with a polyphase winding that is similar to the stator winding and is connected to an external polyphase rheostat through collector, or slip, rings and brushes. A slip-ring motor is illustrated in Fig. 18–10, and its wound rotor is shown in Fig. 18–11. By proper control of the rotor rheostat, the operator is able to maintain maximum torque at all speeds in the starting period if desired. Rotor rheostats are usually designed to permit speed control also between 50 and 100 per cent of full-load speed. Such a method of speed control results in reduced efficiency, however, because of the $I_r^2R_r$ losses in the rheostat. Slip-ring motors are well adapted to printing machinery, reciprocating pumps, ammonia compressors, and other applications requiring speed control or large sustained starting torque. Such motors may be started by direct connection to the power line through a line-starter (described on page 228).



(*Courtesy Westinghouse Electric Corp.*)

Fig. 18–10. Slip-Ring Motor



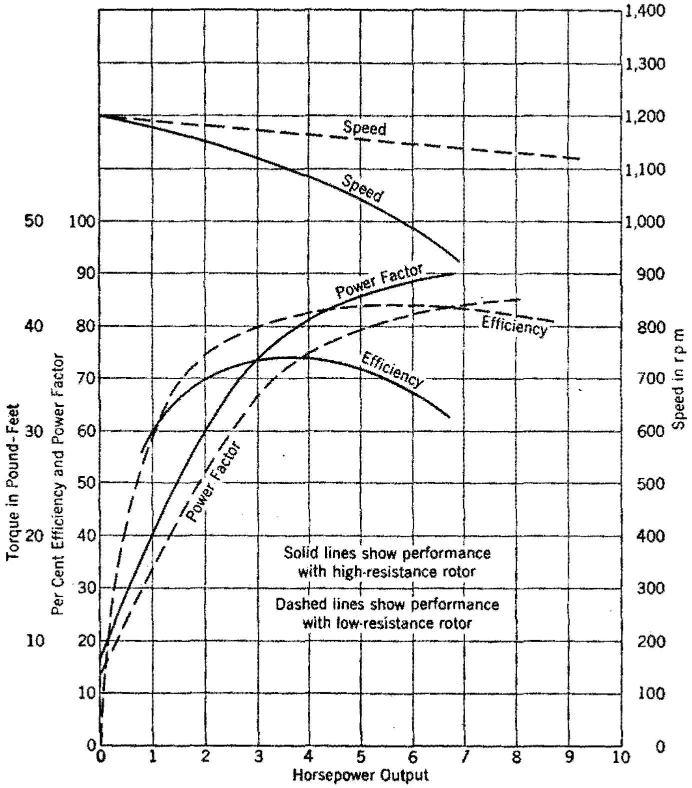
(*Courtesy Westinghouse Electric Corp.*)

Note the oil-throwing grooves in the shaft, the cooling fans, and the wire bands around the end connections.

Fig. 18–11. Wound Rotor

The following points of economic importance should be noted.

1. With the high-resistance rotor in Fig. 18–12:
   (*a*) The motor efficiency is lower because of higher rotor I^2R loss.
   (*b*) The motor will not attain a maximum power output so great as that with a normal-torque rotor, mainly because of the greater speed regulation.

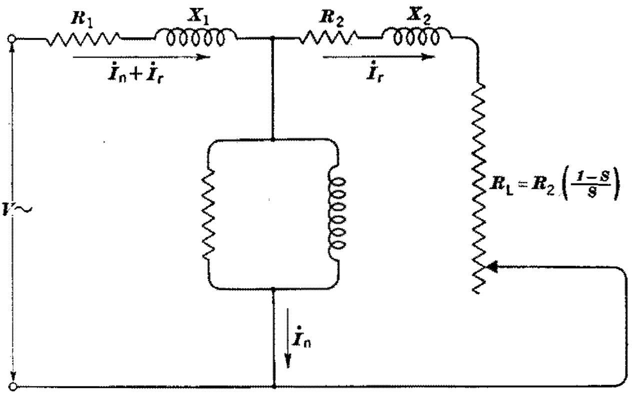


7½ hp, 3-phase, 60-cycle, 220-volt, 1200 rpm

Fig. 18–12. Performance Curves of Three-Phase Induction Motor

   (*c*) The power factor is higher than with a normal-torque motor because of higher rotor loss.
   (*d*) The maximum efficiency occurs with a much lower output than with a normal-torque rotor because rotor I^2R losses are greater.

2. With either rotor in Fig. 18–12:
   (*a*) The efficiency is poor at loads less than 25 per cent of rated full load, and neither motor should be applied to a load requiring much less than its rated output.



$\dot{I}_n$ = equivalent of no-load current.
$\dot{I}_r$ = equivalent of rotor current.
$\dot{I}_n+\dot{I}_r$ = equivalent of stator current at any load.
R_1 = resistance of stator.
R_2 = equivalent resistance of rotor.
R_L = resistance equivalent to shaft loading.
S = rotor slip, expressed as fraction of synchronous speed.

Fig. 18–13. **Three-Phase Induction-Motor Equivalent Circuit**

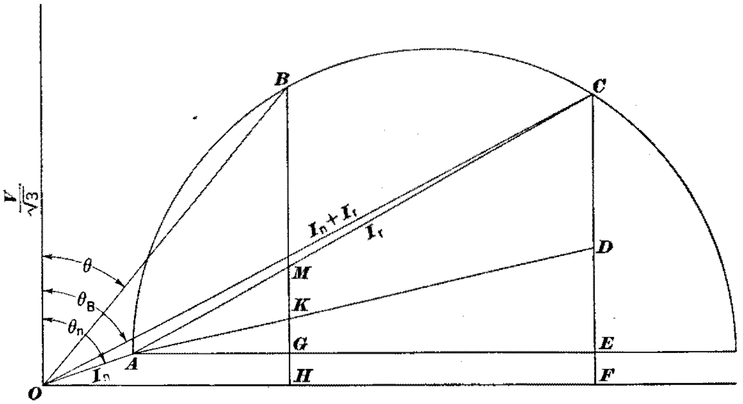


Fig. 18–14. **Simplified Circle Diagram of Motor With Low-Resistance Rotor**

(*b*) The power factor is very low at all loads less than about half of rated full load. See Chapter 25 on power-factor correction.

Determination of Performance of Polyphase Induction Motor Without Loading.—The approximate characteristics of a polyphase induction motor may be found without the necessity of loading. It is possible to construct a circuit of resistances and reactances in a series and parallel combination such that, when one of the elements is varied, the current variations simulate those in an induction motor. Such a circuit, which is shown in

Fig. 18–13, is known as an *equivalent circuit*. This circuit is not actually set up, but is shown to justify the vector diagrams of Figs. 18–14 and 18–15.

The quantities R_1, X_1, R_2, and X_2 of the motor are also in its equivalent circuit; but R_L is a *variable resistance* in the circuit whose variation may be made to simulate a variable load on the motor. The vector diagram and locus resulting from changes in the motor load is called the *circle diagram* of the motor.[4]

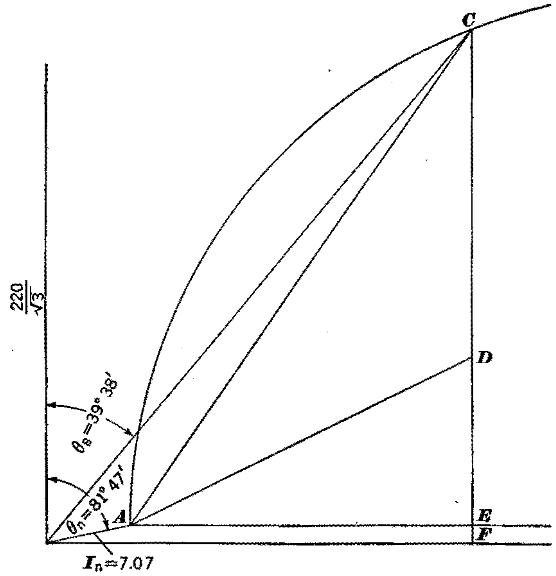


Fig. 18–15. Circle Diagram for Example 18–2

The Simplified Circle Diagram.—In Fig. 18–13 the power in the resistor R_L corresponds to the power output of the motor shaft. The powers in R_1 and R_2 correspond to the power losses in the stator and rotor, respectively, which cause their temperatures to rise. The reactances X_1 and X_2 correspond to the leakage reactances of the stator and rotor, respectively. The parallel circuit carrying current $\dot{I}_n$ carries the equivalent no-load current of the motor, and the losses in it corresponding to the windage, friction, and iron losses (and some stator I^2R loss) are assumed constant regardless of speed. The current $\dot{I}_n$ is the no-load current and lags the applied voltage $\frac{V}{\sqrt{3}}$ per phase by the constant angle θ_n, as shown in Fig. 18–14.

The current $\dot{I}_r$ corresponds to the rotor current, and in Fig. 18–14 it is shown added vectorially to $\dot{I}_n$ to complete the line current $\dot{I}_n + \dot{I}_r$. It is

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[4] *Principles of Alternating Current Machinery*, by R. R. Lawrence. McGraw-Hill Book Co.

assumed that the effective resistance of the rotor (and therefore of R_2) is constant. However, the rotor-circuit reactance is constant (as its effect is viewed through the line terminals), and the effective resistance R_2+R_L decreases as the load increases. The current $\dot{I}_r$ describes a locus which is a circle whose diameter lies along *AGE*. Point *C* on the locus corresponds to the condition when R_L is reduced to zero in Fig. 18–13, and corresponds to the condition of blocked rotor or standstill in the motor.

The most obvious usefulness of the circle diagram is its help in determining the speed-torque characteristics of a motor without the necessity of loading it. The diagram cannot be used with accuracy for a rotor (particularly the double-deck rotor) whose effective resistance changes with change in speed. The circle diagram is drawn with "per phase" values. The motor may always be assumed to be Y-connected. As the motor load changes, the point *B* of the circle diagram in Fig. 18–14 moves along the circular locus between *A* and *C*.

Before the circle diagram can be drawn, three tests must be made and the following data obtained:

1. Resistance of stator (hot) between line and neutral.
2. Current, voltage, and power per phase when running at no load.
3. Current, voltage, and power per phase when rotor is at standstill with applied voltage reduced.

From the circle diagram, the line current, speed, power factor, power input, power output, efficiency, and torque may be found and identified as follows:

I_n = no-load current per phase;

OC = current per phase with rotor blocked and full voltage on stator;

OB = line current in amperes for any chosen load;

$\frac{MK}{BK}$ = slip as a fraction of synchronous speed;

Speed = synchronous speed minus the slip, in rpm;

$\cos\theta$ = power factor;

$\frac{BH\times V}{\sqrt{3}}$ = power input per phase;

$\frac{BM\times V}{\sqrt{3}}$ = power output per phase at shaft;

$\frac{BM}{BH}\times 100$ = efficiency in per cent;

$$\text{Internal torque} = \frac{BK\times\frac{V}{\sqrt{3}}\times 33{,}000}{746\times 2\pi\times\text{synch. speed in rpm}}\times 3 \qquad \textbf{(18–1)}$$

This torque is the total torque produced in the rotor, and is measurable directly in pound-feet only at standstill.

The horsepower output per phase is

$$P = \frac{BM \times \frac{V}{\sqrt{3}}}{746} \tag{18-2}$$

Example 18–1.—Give instructions for obtaining necessary data for construction of the circle diagram and for determination of the speed-torque curve of a three-phase squirrel-cage induction motor 7½ hp, 1200 rpm, 220 volts, 60 cycle.

Solution.—1. Lock the rotor to prevent turning, and apply about one-fourth of the rated voltage to the stator terminals until the temperature of the stator frame rises 40 deg C by thermometer reading.

2. Measure the terminal-to-terminal d-c resistance between any two stator terminals. As a check, measure the resistance in the other two possible circuits of the stator also.

3. Apply a 3-phase voltage of about one-half of the rated voltage to the stator with the rotor locked. Measure quickly the current per terminal, the applied voltage, and the power input.

4. Release the rotor, and apply the rated voltage. Measure the current, voltage, and power with the motor running at no load.

Example 18–2.—Construct the circle diagram and determine the starting torque of the motor of Example 18–1, on the basis of the following data:

1. Temperature of room, 22 C.
   Temperature of motor, 62 C.
2. $I = 26.5$ amp, $V = 22.1$ volts by d-c between terminals.
3. $I = 28.0$ amp, $V = 109$ volts, $P = 4066$ watts with rotor locked.
4. $I = 7.2$ amp, $V = 224$ volts, $P = 400$ watts at no load.

Solution.—From the data in item No. 4 the no-load power factor is

$$\text{PF} = \frac{400}{\sqrt{3} \times 224 \times 7.2} = 0.143 = 14.3\%$$

In Fig. 18–15 lay out 7.07 amp at an angle θ lagging the phase voltage $\frac{220}{\sqrt{3}}$ and to some convenient scale. Note that the circle diagram is to be constructed on the basis of the rated motor voltage of 220 volts, although the actual test voltage was 224. A scale of $\frac{1}{16}$ in. = 1 amp is used in Fig. 18–15. In this case,

$$\theta_n = \cos^{-1} 0.143 = 81°47'$$

The locked-rotor current from the data in item No. 3 is 28 amp at 109 volts. At 220 volts it is

$$28 \times \frac{220}{109} = 56.5 \text{ amp}$$

The power factor of the blocked-rotor current is

$$\text{PF} = \frac{4066}{\sqrt{3} \times 109 \times 28} = 0.77 \text{ or } 77.0\%$$

Next, lay out 56.5 amp at $\theta_B = \cos^{-1} 0.77 = 39°38'$.

Draw the arc of a circle that passes through points A and C and has its center on AE or AE extended.

The line CF represents the component of the phase current which is in phase with the phase voltage when the rotor is locked with the rated voltage applied. Its length is therefore indicative of the power input per phase. Of the power input per phase, the portion DE is indicative of the added power I^2R per phase of the stator.

The line-to-line resistance is

$$R_{L-L} = \frac{V}{I} = \frac{22.1}{26.5} = 0.834 \text{ ohm}$$

Since the stator is assumed to be Y-connected, the resistance per phase is

$$R_p = \frac{0.834}{2} = 0.417 \text{ ohm}$$

The resistance of a stator to alternating current is greater than that to direct current, and the d-c resistance should be multiplied by a conversion factor. The a-c resistance is greater than the d-c resistance because, when alternating current flows in the stator conductors, the current distribution across the conductor section is not uniform and the accompanying iron losses *appear* to be caused by an added series resistance. Furthermore, this factor helps to correct for some of the approximations permitted in the simplified circle diagram. Judgment in choice of this factor is based on performance of similar motors; and, depending largely on the type of rotor used, the factor may have any value between about 1.1 and 1.4.

A conversion factor of 1.4 should be satisfactory for the purpose of this example. Hence, the corrected resistance per phase is

$$0.417 \times 1.4 = 0.584 \text{ ohm}$$

When the rotor is locked,

$$I^2R = 56.5^2 \times 0.584 = 1860 \text{ watts per phase}$$

$$I = \frac{1860}{\frac{220}{\sqrt{3}}} = 14.6 \text{ amp}$$

This is the component DE of CE, and so the point D is determined. The component CD is, by measurement on Fig. 18–15, 27.1 amp.

The power per phase corresponding to CD is

$$27.1 \times \frac{220}{\sqrt{3}} = 3450 \text{ watts}$$

Since $\text{hp} = \frac{2\pi NT}{33{,}000}$,

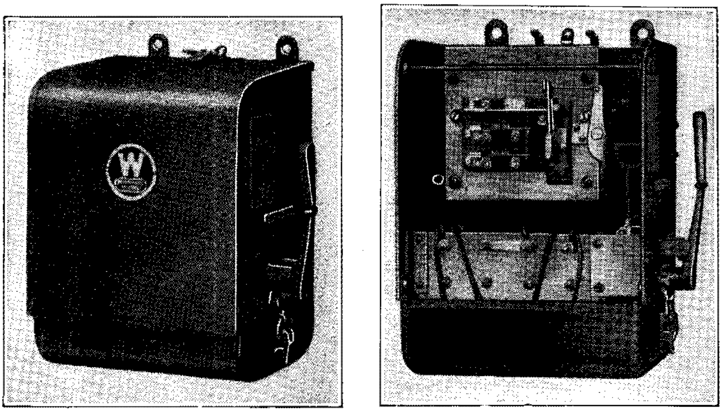
$$T = \frac{\text{hp} \times 33{,}000}{2\pi \times 1200} = \frac{\frac{3450}{746} \times 33{,}000}{2\pi \times 1200} = 20.3 \text{ lb-ft per phase}$$

$$T = 20.3 \times 3 = 60.9 \text{ lb-ft for three phases}$$

From Fig. 18–15 it is apparent that under no condition of speed will CD, and hence the torque of this motor, be larger than at standstill. Therefore, the motor has a high-resistance rotor.

Starting of Polyphase Induction Motors.—When a polyphase squirrel-cage induction motor with a low-resistance rotor is connected directly to a power line of its rated voltage and frequency, it draws from that line 5 to 7 times its rated full-load current. Such starting is called *full-voltage* starting. If the impedance of the power line is small, and the voltage regulation and frequency regulation of the generator are small and the rapid acceleration of the load is not objectionable, then full-voltage starting is satisfactory. If the voltage regulation and frequency regulation of the power system are not unusually good, the starting current of a motor with a low-resistance rotor that develops $7\frac{1}{2}$ or more hp causes intolerable line disturbances and may seriously affect the performance of other apparatus on the same line. For these reasons such a motor should be started by a reduced-voltage starter. Voltage reduction is usually accomplished by "step-down" auto-transformers, by series line resistors, or by smoothly-variable voltage regulators.

The auto-transformer type of starter, one of which is shown in Fig. 18–16, is usually arranged for a choice of two or three starting voltages ranging from 50 to 80 per cent of full line voltage. The choice of starting voltage is determined by the starting torque required by the load.



(Courtesy Westinghouse Electric Corp.)

Note the thermal overload relays on the enclosed panel.

Fig. 18–16. Auto-Transformer Type of Induction-Motor Starter

Motors with line-start rotors, high-torque high-resistance rotors, or phase-wound rotors may be started on full voltage. Such starting requires merely a starting switch that provides overload and low-voltage protection. A starter of this type is called a "line-starter." The only advantage of full-voltage starting over reduced-voltage starting lies in the difference in cost of starting equipment. A full-voltage automatic line-starter is shown in Fig. 18–17. The National Electric Code[5] requires that a safety switch like that shown in Fig. 18–18 be installed between the starter and the line to insure safety from shock when the starter is being repaired or adjusted.

A three-phase motor with all six phase leads available may be started on reduced voltage in another manner, if it is designed for delta connection. The phase leads are connected to a multi-blade double-throw switch, by which the phases may be connected in Y for starting and in delta for run-

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[5] *National Electric Code Handbook*, by A. L. Abbott. McGraw-Hill Book Co.

ning. This arrangement provides $\frac{1}{\sqrt{3}}$ times the normal phase voltage for starting and is satisfactory when the starting duty is not severe.

Regenerative Braking of Induction Motors.—The speed-torque curves of Fig. 18–9 are only motor characteristics of much more extensive general curves such as those shown in Fig. 18–19. If a rotor with characteristic (*a*), (*b*), or (*c*) is driven above its synchronous speed S by external means in a stator normally excited, its torque becomes negative and the motor becomes an *induction generator* delivering electrical power back into the system to which it is connected. The power factor and the power output of an induction generator are determined by its design and the speed at which it is driven, and are independent of the character of the load. An

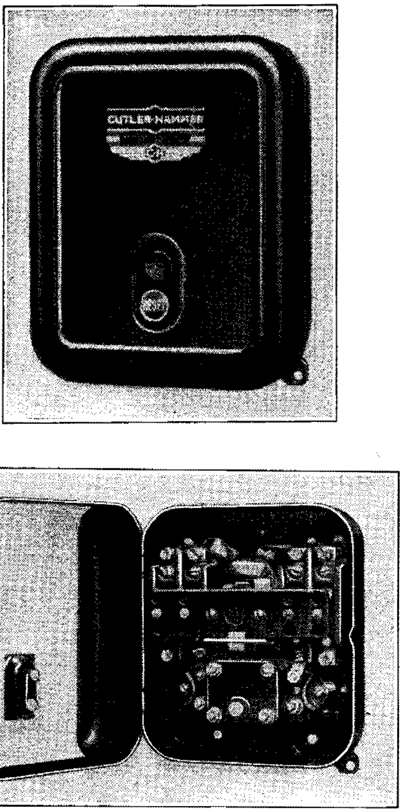
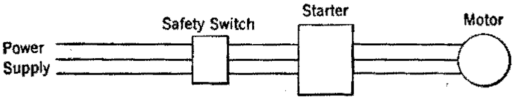
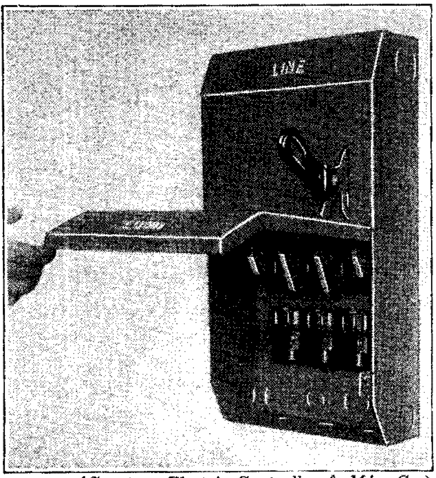

(*Courtesy Cutler-Hammer, Inc.*)

Fig. 18–17. Automatic Full-Voltage Induction-Motor Starter

induction generator cannot supply magnetizing current to inductive reactive loads and must be operated in parallel with a synchronous generator which will supply its magnetizing current and also establish and control its frequency.





(*Courtesy Electric Controller & Mfg. Co.*)

Fig. 18–18. Safety Switch

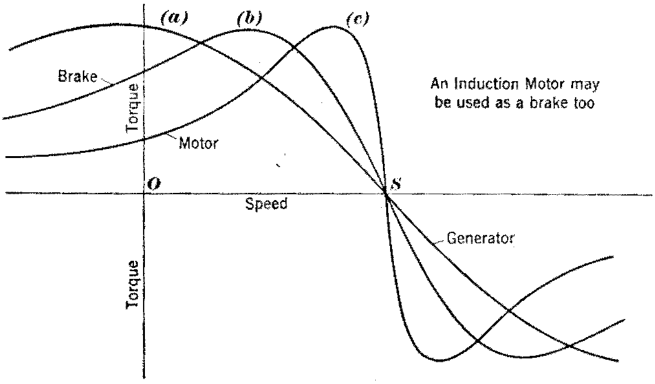


Fig. 18–19. Generalized Induction-Motor Speed-Torque Curves

Furthermore, if an induction motor (while normally excited) be driven in a direction opposite to that of its torque, as in the region to the left of point *O* in Fig. 18–19, it will convert both mechanical energy and electrical energy into heat and will act as a brake. Thus, an induction motor will act as a brake if driven backward or if driven above its synchronous speed while connected to its power line. Such means are sometimes used to control the rate of stopping a connected device such as a small hoist.

A multi-speed motor running at its high speed will, when changed to a reduced synchronous speed, decelerate to that speed very quickly because of regenerative braking at all speeds higher than the reduced synchronous speed. The low speed is convenient in "inching" a load, as when adjusting a machine tool or when leveling an elevator car that is to be stopped at a floor.

Single-Phase Operation of a Polyphase Induction Motor.—If one line wire of a three-phase induction motor be opened while the motor is running at full load, the motor will continue to run. However, the current per terminal and the slip will be greater than normal, and the motor will be quickly overheated. For this reason, overload protection should be provided in at least two lines in order that unsupervised, unintentional, single-phase operation cannot continue if it should occur.

A polyphase motor connected to a single-phase power source has no starting torque unless provided with special *phase-splitting* starting equipment. If started by such a starter or by some external means, the motor will carry about 60 per cent of its normal rated load with about normal slip and normal temperature rise. The maximum torque will be only about 30 per cent of normal.

Operation at Other Than Rated Voltage and Frequency.—The torque of an induction motor is proportional to the square of the applied voltage. If the voltage applied is 10 per cent less than the rated value, the torque at any speed will be only $0.90 \times 0.90 = 0.81$ or 81 per cent of normal. If the starting requirements are particularly severe, a reduction of 10 per cent in voltage may cause failure to start.

Synchronous speed is directly dependent on frequency, and will consequently vary as the frequency is varied. Motors designed for 60-cycle frequency may usually be operated on 50-cycle frequency at rated load with a temperature rise of not more than 55 deg C. A 60-cycle motor connected to a 25-cycle line will sometimes overheat without load because of excessive exciting current. In general, a polyphase induction motor may be operated at rated voltage at rated frequency or as much as 20 per cent above rated frequency without injury, but with somewhat impaired characteristics.

TABLE 18–1

OPERATION OF 60-CYCLE INDUCTION MOTOR UNDER VARIOUS LINE AND LOAD CONDITIONS

CONDITIONS				RESULTS*			
Load	Time	Voltage	Frequency	Current	Speed	Square of Torque	Temperature Rise
Normal	Continuous	Normal	Normal	Normal	Normal	Normal	Normal
Normal	Continuous	10% High	Normal	Lower	Normal	20% Higher	Normal or Higher
Normal	Continuous	10% Low	Normal	Higher	Normal	20% Lower	Normal or Higher
Normal	Continuous	Normal	5% High	Small Change	5% Higher	7% Lower	Small Change
Normal	Continuous	Normal	5% Low	Higher	5% Lower	7% Higher	Higher
Normal	Continuous	10% Low	5% High	Higher	5% High	Too Low (¾)	Higher
15% High	Continuous	Normal	Normal	+15%	Small Change	Normal	50°
35% High	Intermittent	Normal	Normal	+35%	Slower	Normal	50°
35% High	Continuous	Normal	Normal	+35%	Slower	Normal	Too High (70°)
Normal	Continuous	Normal	50 Cycles	Higher	17% Lower	35% Higher	50°
15% High	Continuous	Normal	50 Cycles	Too High	17% Lower	35% Higher	Too High (60°)
35% High	Intermittent	Normal	50 Cycles	Too High	17% Lower	35% Higher	Too High (80°)

* All comparisons based on motors at normal load, voltage, and frequency.

PROBLEMS

1. Construct accurately the circle diagram for Example 18-2 on page 226. Calculate the torque and the power output at one-half of rated speed. Calculate the starting torque with 200 volts applied (line-to-line).

2. At what continuous power output may a 50-hp, 3-phase, 60-cycle, 220-volt, polyphase, open-type, 40-deg C induction motor be operated if it be totally enclosed and connected to a single-phase, 208-volt, 50-cycle line?

3. The full-load speed of an induction motor is given as 960 rpm. What is the frequency for which it was probably designed? If the windings are open for inspection, how may the number of poles be determined?

4. A 3-phase, 220-volt, 6-pole, 25-hp, 25-cycle induction motor has a power input of 575 watts with 19.5 amp at no load. With the rotor blocked and 51.5 volts applied to the stator, the power input is 2050 watts with a line current of 66 amp. The effective resistance of the motor is 0.085 ohm per phase. Draw the approximate circle diagram, and determine the starting torque and the maximum running torque in pound-feet.

5. A 3-phase, 4-pole, 60-cycle, 220-volt, polyphase induction motor is to be used as a frequency converter by driving the open-circuited rotor backward against the direction of the rotating field. If the rotor has half as many conductors in series per phase as the stator, what will be the voltage and frequency available at the slip-rings when the rotor is driven backward at 900 rpm?

6. A 6-pole, 50-hp, 25-cycle induction motor runs at 5% slip at full load. What is the frequency of the rotor currents?

7. At full load, what is the speed of the motor of Problem 6?

8. A 40-hp, 440-volt, 1200-rpm induction motor with 5% slip at full load has a starting torque 1.5 times its full-load running torque. How many pound-feet of starting torque would it develop with 230 volts applied?

9. How many pound-feet of starting torque would the 40-hp motor of Problem 8 develop if started on the 80% tap of its starting auto-transformer?

10. When tested for the purpose of finding the characteristics by circle diagram, a certain two-phase induction motor yields the following data:

	Phase 1			Phase 2			
	Volts	Amp	Watts	Volts	Amp	Watts	Speed
No load	440	5.5	440	440	5.5	440	1800 rpm
Blocked	113	11.4	575	113	11.4	575	0

Resistance = 1.56 ohms per phase.

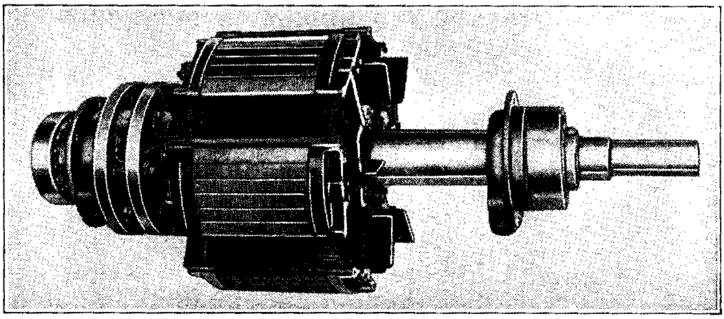
Draw the circle diagram and calculate:

(a) Starting torque, in pound-feet, considering starting torque as 1.5 times full-load torque.
(b) Torque at half of rated speed.
(c) Efficiency at 1.5 times full-load hp.

CHAPTER 19

The Polyphase Synchronous Motor

General Characteristics of the Synchronous Motor.—In contrast to the induction motor, the polyphase synchronous motor runs at a constant speed at any load for which the torque is less than its "pullout" or "breakdown" torque. Such a motor has the advantage also of usefulness in power-factor correction when operated in conjunction with other load of low power factor characteristics. Prior to 1922, the poor starting torque obtainable from the starting amortisseur winding, as developed to that time, discouraged widespread adoption of the synchronous motor. However, improvements in design have vastly broadened its field of application until it has become even more popular than the polyphase induction motor in the larger sizes.



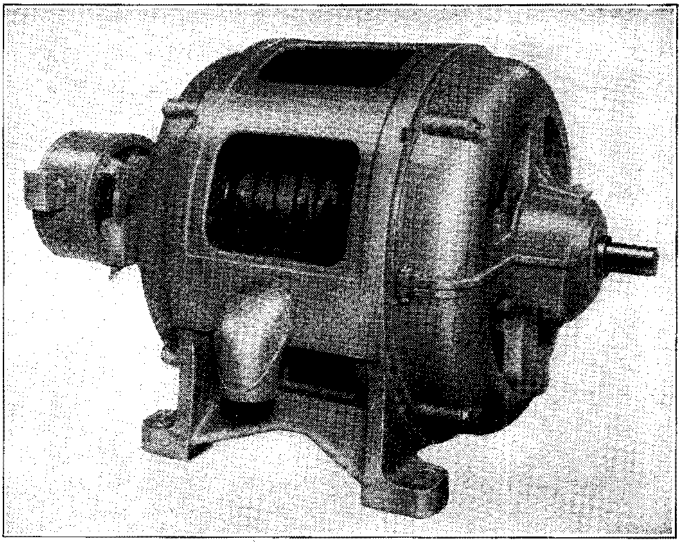
(Courtesy Fairbanks, Morse & Co.)

Fig. 19–1. Salient-Pole Rotor for Synchronous Motor

In the polyphase induction motor the rotor field, which reacts upon the stator field to produce torque, is dependent on slip for its existence. In the synchronous motor the rotor field is produced by electromagnets mounted radially on the shaft, as shown in Fig. 19–1, and excited (magnetized) by *direct current.* The direct current is supplied from some external source through slip-rings similar to those of a slip-ring induction motor. In Fig. 19–2 is shown a small direct-current exciting generator directly connected to the motor.

The stator of the polyphase synchronous motor is exactly similar to that of the polyphase induction motor. The mutual attraction of the rotating fields of the stator and the fields of the magnetized salient-pole revolving rotor causes the two sets of fields to "lock together" and to rotate in synchronism after they have been synchronized.

Starting of Synchronous Motors.—In order that a synchronous motor may be self-starting as such, the rotor must be so light and its inertia must be so small that it will accelerate to synchronism and its field poles will lock in step with the rotating stator-produced fields as they pass by at synchronous speed. In any but the very smallest synchronous motors used in timing devices, the rotor inertia is too great for such rapid acceleration, and the resulting alternate torque in opposite directions produces no rotation.

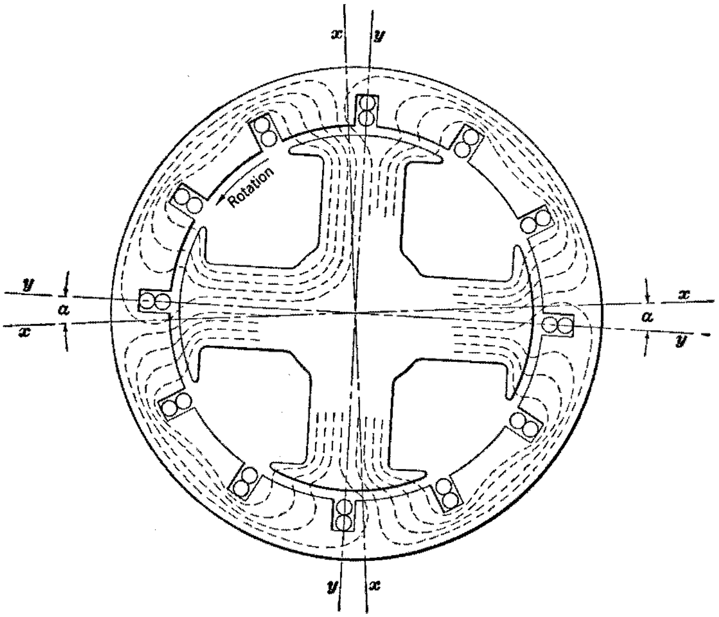


(Courtesy Westinghouse Electric Corp.)

Fig. 19–2. **Synchronous Motor With Direct-Connected Exciter**

In starting an industrial synchronous motor, it must be driven to some speed within about 5 per cent of synchronous speed by some means before the revolving-field structure is magnetized. When the field is then magnetized, it pulls into synchronism with the rotating field produced by the stator winding, and the original driving torque may be discontinued. The rotor may be brought to proper speed by a belted or direct-connected motor or by the more usual method of providing the pole faces with a squirrel-cage winding similar to that of an induction motor. This winding, called an *amortisseur winding,* is shown occupying the longitudinal slots in the faces of the poles in Fig. 19–1. After the rotor has attained near-synchronous speed by amortisseur-winding drive, the field structure is excited by direct current, and the rotor locks with and runs synchronously with the rotating field.

Synchronous motors are available in both line-start and reduced-voltage-start types. The type chosen is usually determined by the starting torque, synchronizing torque and horsepower required by the load, and also by the capacity of the power system on which it is to be used.



To produce torque the rotor must lag slightly behind the flux which it follows.

Fig. 19-3. Elements of a Synchronous Motor

Running Conditions.—In Fig. 19-3 a synchronous motor with a four-pole field is shown at a definite instant while rotating counter-clockwise. At the instant chosen in Fig. 19-3, the stator fields are at the axis x-x and the rotor and its fields are at the axis y-y. The angle α between the two axes, or the angle by which the rotor lags the stator fields, is called the *torque angle.* If the axes x-x and y-y were coincident, the flux in the air-gap would be radial and would produce no turning effort on the rotor. As the load on the motor increases, the rotor lags behind the rotating field by a greater angle, to produce the necessary torque. Thus, for a definite constant load the rotor runs in perfect synchronism with, and at a definite torque angle behind, the rotating field. With an increase in load, the rotor drops back to a new position, to meet the new demand for torque. The action is similar to that of dragging the rotor around by springs in tension.

The amortisseur winding, which is a rudimentary squirrel-cage winding, was first used on alternating-current generators driven by reciprocating engines to damp out fluctuations in speed caused by the torque impulses of the engine. "Amortisseur" means "damper."

If the load be too greatly increased, the magnetic coupling between the rotor and the stator fields will be over-stressed and broken. Having *slipped a pole*, the rotor will come to rest very quickly unless supported by a strong amortisseur-winding torque. At synchronous speed the amortisseur winding is electrically inert, because it has no motion relative to the rotating field. Momentary changes in torque angle caused by changes in load or slipping of poles are opposed by the amortisseur damping action.

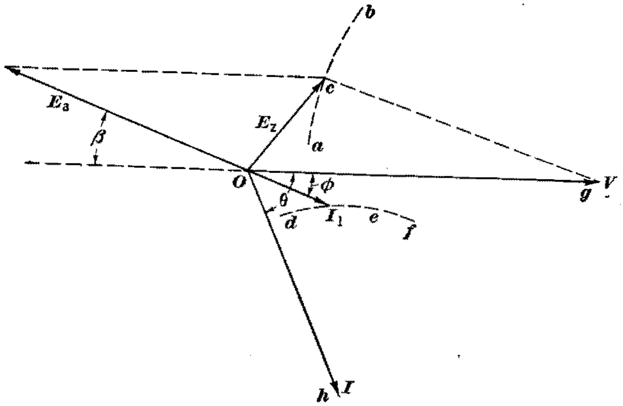


Fig. 19–4. Synchronous-Motor Vector Diagram

A salient-pole synchronous motor, after having been synchronized, can carry a small fraction of its rated load even without field excitation. Induction motors are available with slots cut longitudinally in the rotors to produce rudimentary salient poles, in order to simulate unexcited synchronous motors. Their low ratio of rated power to weight seriously limits the use of such motors. They are called "reluctance" motors because their torque derives from the attempt of the flux to decrease the reluctance of the path through which it must pass.

Vector Diagram of the Synchronous Motor.—A synchronous motor with its polyphase winding energized and producing a rotating set of field poles will usually have enough reluctance torque to run without load if the field structure is rotated to synchronous speed by some external driving force so that the salient poles can lock in with the rotating set of field poles as in Fig. 19–3. Such a motor when running would have a vector diagram for every phase similar to that of a simple reactance coil.

In Fig. 19–4, V is the phase voltage applied and I is the phase current for that condition of operation. The current I is Oh on the diagram and it lags V by the angle θ which is the characteristic impedance angle of the unexcited motor. The rotor salient poles lag the stator flux poles by α mechanical degrees, as in Fig. 19–3, to produce enough torque to maintain the speed.

If the windings on the rotor poles are then energized by direct current passed through the rotor winding in proper direction to add to the flux already in the poles, there will be a voltage E_a generated in the stator coils by the new rotor flux sweeping past them. The voltage E_a is not quite 180 deg out of time phase with V because the rotor is lagging α mechanical degrees behind the stator-produced poles. Since α mechanical degrees correspond to 2α electrical degrees (in a four-pole motor), let $2\alpha=\beta$, in which β is the lag angle in electrical degrees. The net voltage available for forcing current through the stator is $E_z=V+E_a$; and I is reduced to I_1 in the ratio $\frac{I_1}{I}=\frac{E_z}{V}$. But I_1 still lags E_z which produces it by θ deg.

If the voltage E_a is kept constant by keeping the direct current in the field structure constant, and the motor is then loaded, the angle β will increase to meet the new load demand. The voltage E_z will then lie on the locus acb which has $gc=E_a$ as a radius. If E_z follows a circular locus as the shaft load varies, I_1 must also follow a circular locus def with radius hd. The motor power factor is $\cos\phi$. It is then apparent that the power factor of a synchronous motor with constant field excitation is no more constant regardless of load than is that of an induction motor, because ϕ changes as the changing load causes I_1 to move to the right or left along its locus.

Synchronous Impedance.—The voltage E_z which forces the current I_1 through the impedance of the motor windings produces a voltage drop I_1Z_s. The impedance Z_s is called the synchronous impedance and is composed of two parts. One part is R_a+jX_a, which is the resistance and leakage reactance of the winding. The other part is X_r, which is not a reactance in the usual sense but is the result of the effect of the stator ampere-turns on the total flux of the motor. This effect is called armature reaction and its magnitude depends almost altogether on the angle by which the rotor poles lag the stator poles in space. It is apparent then that X_r depends on both the magnitude of the motor load and the motor power factor. If the mmf of the armature reaction tends to decrease the total flux, its effect is the same as additional reactance drop; and, therefore, X_r and X_a are commonly combined and the result is called X_s. The value of Z_s is composed of both parts. Thus,

$$Z_s=R_a+jX_a+jX_r \tag{19–1}$$

To determine Z_s experimentally, the machine is driven at synchronous speed without excitation. The polyphase winding is short-circuited through three ammeters, one in each lead, to measure the current I_p per phase. A small direct current is then used to magnetize the field poles to the point where the three ammeters show full-load rated current. From the open-circuit saturation curve of the machine, the voltage E_p per phase is found corresponding to the direct current used. The synchronous impedance, in ohms, is then $Z_s = \frac{E_p}{I_p}$. With the polyphase winding short-circuited, all of E_p is used to force I_p through the total impedance Z_s of the machine. Refinements of this procedure are necessary if Z_s must be determined accurately for some particular condition of loading.

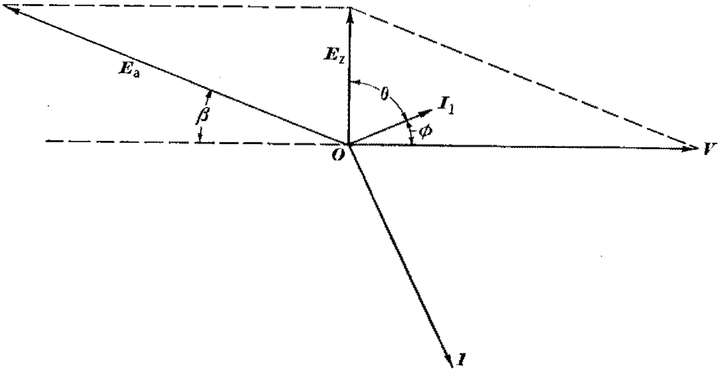


Fig. 19–5. Vector Diagram of Over-Excited Synchronous Motor.

Effect of Varying Field Excitation When the Load Is Constant.—The total power input per phase in Fig. 19–5 is

$$P_i = VI_1 \cos \phi \tag{19–2}$$

The power P_i includes the shaft power output, if any, and the windage, friction, iron, and copper losses (all per phase). Let it be assumed that, while the motor is running with the vector conditions of Fig. 19–4, the field excitation of the rotor is increased. That increase in rotor field strength will increase E_a and cause E_z to swing counter-clockwise, carrying with it I_1 at angle θ lagging. If neither the shaft load nor the motor losses changed during the field adjustment, $I_1 \cos \phi$ would remain constant but I_1 would be made to lie in a new position *leading* V by some phase angle ϕ_1, as in Fig. 19–5. If $\phi_1 = \phi$, the power input per phase would actually be the same as before because both load and losses would be unchanged.

The motor under this new condition of field excitation takes from the power source not only $VI_1 \cos \phi$ as a power component but also

$VI_1 \sin \phi$ in leading reactive volt-amperes. This is a valuable property of a synchronous motor because, if such a motor is run "over-excited" on a power system which has a preponderance of lagging power factor loads, it acts partly as a condenser and improves the over-all power factor of the system. This use is considered further in Chapter 25.

If a synchronous motor had no copper losses within itself, the power input with constant shaft load would be constant regardless of any change in field excitation, as long as synchronism was maintained. In that case, if the field excitation were changed, I_1 would change but would follow a vertical locus XY shown in Fig. 19–6 for three states of field excitation. In a large motor E_z is so large compared to the IR drop in the polyphase winding that the IR drop may be neglected without introducing serious error. All the synchronous-motor vector diagrams have been drawn with the assumption that the synchronous impedance is the same at all power factors. In a salient-pole motor which has been under consideration, the impedance actually is different for different power factors but the differences within the normal or usual range of use are negligible.

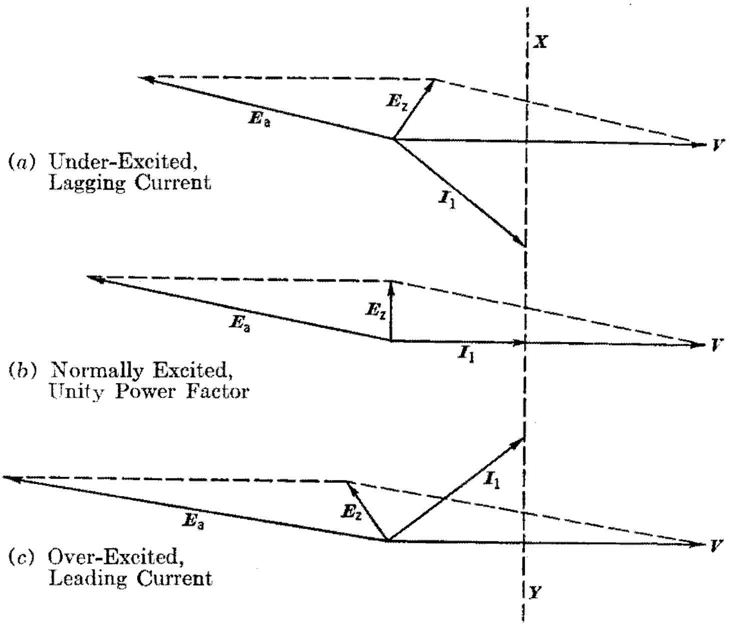


Fig. 19–6. **Vector Diagrams of a Synchronous Motor With Different Degrees of Field Excitation (Polyphase Winding Assumed to Be Without Copper Loss, in Accordance With Common Approximation)**

The over-excited synchronous motor provides leading vars for power-factor correction at remarkably low cost per var. A motor built to furnish no leading vars but to run at unity power factor has only enough copper in its polyphase winding to enable it to take the necessary minimum current to produce rated shaft horsepower. Its field winding has only enough copper to provide enough E_a for unity power factor. Thus, a motor rated at 100 per cent power factor is built at minimum cost and is desirable when no leading vars are needed. However, by a small increase in the amount of copper used in both windings, the motor may be over-excited to produce vars at much less cost than that of the equivalent vars in static condensers. The power factor specified on the name plate of a synchronous motor is the minimum power factor *leading* at which the motor can be run while carrying its rated horsepower load. Fig. 19–7 shows some power-factor and current curves of a typical synchronous motor with and without load and with a wide range of field excitation. There is almost never a need to run such a motor with a lagging power factor.

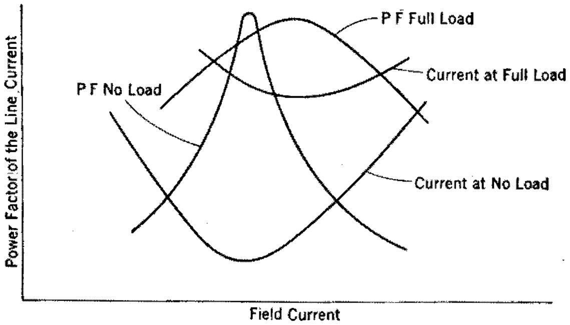


Fig. 19–7. Synchronous-Motor Curves

The Synchronous Condenser.—Some synchronous motors are built without shaft extensions but with great capacity for over-excitation. Such machines are intended only to produce leading vars and are called *synchronous condensers.* They are usually built in large sizes only (1200 kv-a or more) because, when the whole cost of the machine must be charged against its var-producing ability, it does not compare favorably economically with static condensers in smaller sizes. It is commonly used in electric power transmission systems to aid in controlling the voltage at points in the system far distant from the generating station. The over-excited synchronous motor that can drive machinery is preferred in industrial plants. A vector diagram of a synchronous condenser is shown in Fig. 19–8.

Starting Procedure for Synchronous Motors.—As explained on page 235, synchronous motors are usually started by the induction-motor action of amortisseur windings. The rotating fields sweeping past the rotor poles induce an alternating voltage of possibly 1500 volts or more in the exciting windings on the poles. If not controlled in some manner, this high voltage may be a hazard to life or it may cause failure of the field-coil insulation. If the required starting torque of the motor is small, the exciting winding may be short-circuited at start and thus relieved of the high-potential stress. A more common practice, however, is to connect a resistor, called a *field discharge resistor*, across the open field terminals. This resistor is usually a part of the control equipment provided for the motor. After the motor has attained nearly synchronous speed, the dangerous field voltage no longer exists and the discharge resistor is disconnected. The field circuit is then connected through a control rheostat to its source of direct-current excitation.

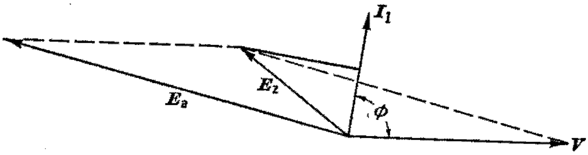


Fig. 19–8. Vector Diagram of a Synchronous Condenser

In order that the amortisseur winding may react vigorously with the rotating field produced by the stator, that field must find a ready passage through the rotor pole structure. If the field windings are short-circuited, they become highly reactive. As a result, the rotating-field fluxes lose much of their strength by spilling over into the leakage paths in the air-gap and between the poles, instead of passing normally through the poles where they would react with the amortisseur winding and produce starting torque. This loss of flux in the amortisseur winding reduces the starting torque to some value considerably less than that which would be obtainable with the field winding open. The starting torque may therefore be adjusted between narrow limits by changing the value of the resistance in the discharge resistor.

Synchronous-Motor Applications.—Synchronous motors are used to excellent advantage on ball and tube mills; paper-pulp beaters; centrifugal blowers; centrifugal and reciprocating compressors; crushers of cone, gyratory jaw, or roll type; pulp grinders; steel rod mills; rolling mills; rubber mills; and many other heavy drives.

PROBLEMS

1. A factory load of 1200 kv-a, 440 volts, 62% power factor lagging is to be increased with improvement in power factor. With an efficiency of 90%, how many horsepower in synchronous motors operating at 80% power factor leading would be required to correct the over-all power factor to 85% lagging?

2. A 3-phase synchronous motor rated 100 hp, 440 volts, 0.8 power factor, 140 amp, 60 cycles has a synchronous impedance per phase $Z_s = 0.0931 - j0.3477$ ohm, which is

assumed to be constant. When the motor is running at its rated load and power factor, what is the generated voltage E_a per phase? Graphical solution is suggested.
Ans. 276 volts

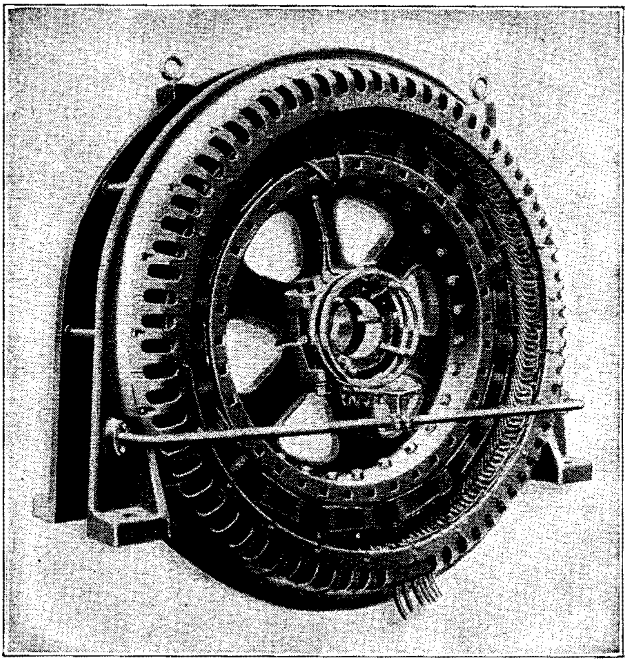
3. Calculate the windage, friction, and iron losses of the motor running as described in Problem 2. *Ans. 4926 watts*

4. If the loss in the d-c field of the motor of Problem 2 is 4000 watts, what is the efficiency when operating as described?

CHAPTER 20

Alternating-Current Generators

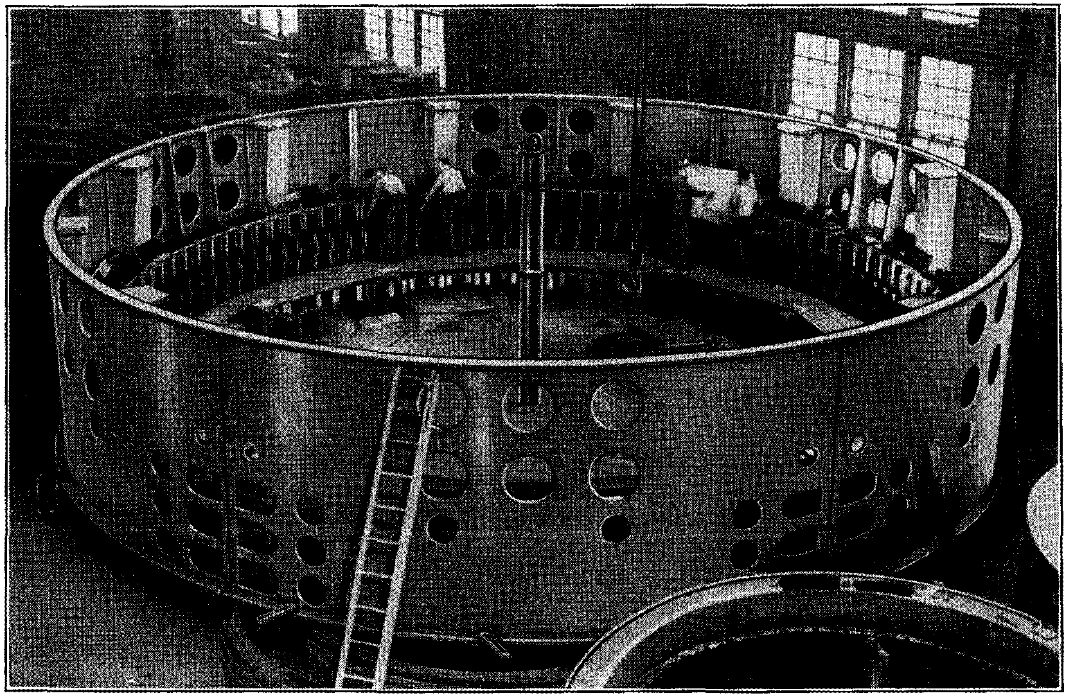
Types of Alternating-Current Generators.—Alternating-current generators are of two general types, namely, induction and synchronous. The induction generator was discussed in Chapter 18 as much as is justified by its relative importance. The synchronous generator is so named because, in contrast to the induction generator, it is capable of establishing a frequency proportional to its speed and independent of its load. Synchronous generators may be classified, according to their rotor construction, as *smooth-core generators* and *salient-pole generators.*



(Courtesy Fairbanks, Morse & Co.)

Fig. 20–1. Engine-Type Synchronous Generator

Smooth-core rotors are of cylindrical form with exciting windings embedded in longitudinal slots. They are used in high-speed generators of large size (750 kv-a and greater at 1800 rpm or more). Generators with salient-pole rotors are not essentially different in construction from

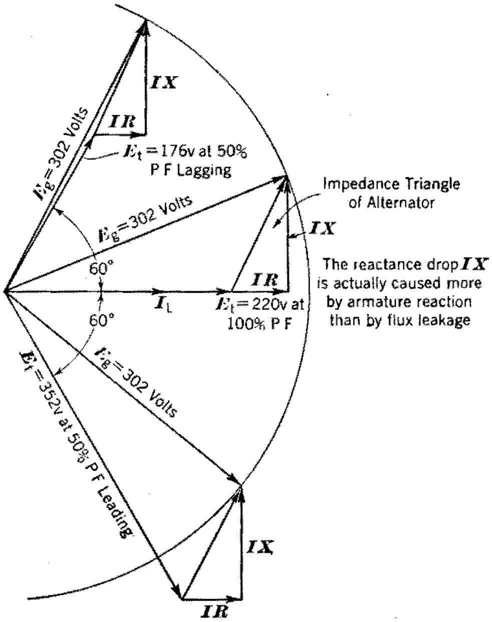


(*Courtesy General Electric Co.*)

Fig. 20–2. Waterwheel-Driven Vertical Alternating-Current Generator Under Construction

synchronous motors described in Chapter 19. A salient-pole generator is shown in Fig. 20–1. The terminal voltage at no load is simply that produced by the sweeping of the rotor flux across the stator conductors.

Further than mitigating excessively high voltage transients caused by line-to-line system faults, amortisseur windings serve no useful purpose if the generator is driven by a turbine. If the generator is driven by a reciprocating engine, however, the amortisseur serves to smooth out the pulsations in angular velocity characteristic of such engines. Fig. 20–2 shows the stator of a large waterwheel-driven alternator under construction.



Impedance Triangle of Alternator

The reactance drop IX is actually caused more by armature reaction than by flux leakage

For any specified load current, more exciting current is required with a lagging current than with a leading current.

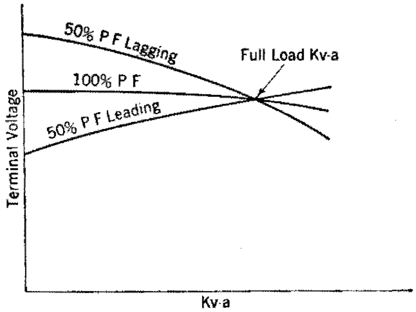
Fig. 20–3. Terminal Voltage of A-C Generator

Voltage Regulation.—The terminal voltage of an alternating-current generator changes with change in load in a manner similar to that of a direct-current shunt generator. Thus,

$$\text{Regulation} = \frac{\text{No-load voltage} - \text{Full-load voltage}}{\text{Full-load voltage}} \times 100 \qquad \textbf{(20–1)}$$

However, under some conditions of load the terminal voltage of an alternating-current generator will *rise above* that at no load. Fig. 20–3 shows how this phenomenon can occur. When the terminal voltage is adjusted to normal 220 volts with a load of 100% power factor, the necessary generated voltage is 302 volts. When the power factor of the load is changed to 50% lagging without change of load current or of field excitation, the terminal voltage becomes only 176 volts. Under similar conditions, but with a load power factor of 50% leading, the terminal voltage becomes 352 volts, although the voltage produced by the d-c field winding is only 302 volts. The rest of the 352 volts are produced by the magnetizing effect of the leading component of current in the stator.

Fig. 20–4 shows representative terminal-voltage curves as a function of load in amperes for power factors of 100%, 50% leading, and 50% lagging.



With fixed excitation, the terminal voltage depends on both the current and the power factor of the load.

Fig. 20–4. A-C Generator Voltage Regulation

Effects of Armature Reaction.—It may be somewhat surprising to learn that a load that draws leading vars from a synchronous generator may cause the terminal voltage to be greater than the voltage without load. It is evident that a magnetizing effect is being produced in the generator by something other than the mmf of the direct-current winding. That other source is the mmf of the armature reaction. As the term armature is used here, it means the part of the generator that contains the polyphase winding.

In Fig. 20–5 is shown one phase of a three-phase generator on a developed diagram with rotor poles. When the rotor poles are in the position in (*a*) with respect to the stator coil sides, the voltage generated in the phase is of maximum magnitude. If the load is purely resistive, the current will also be at maximum value at the instant shown in (*a*). The current in the stator winding will produce the magnetomotive force mmf_s in the directions shown. The direct current in the windings on the field poles will produce the magnetomotive force mmf_r in the directions shown. Examination of the diagram in (*a*) shows that on every pole one half of the

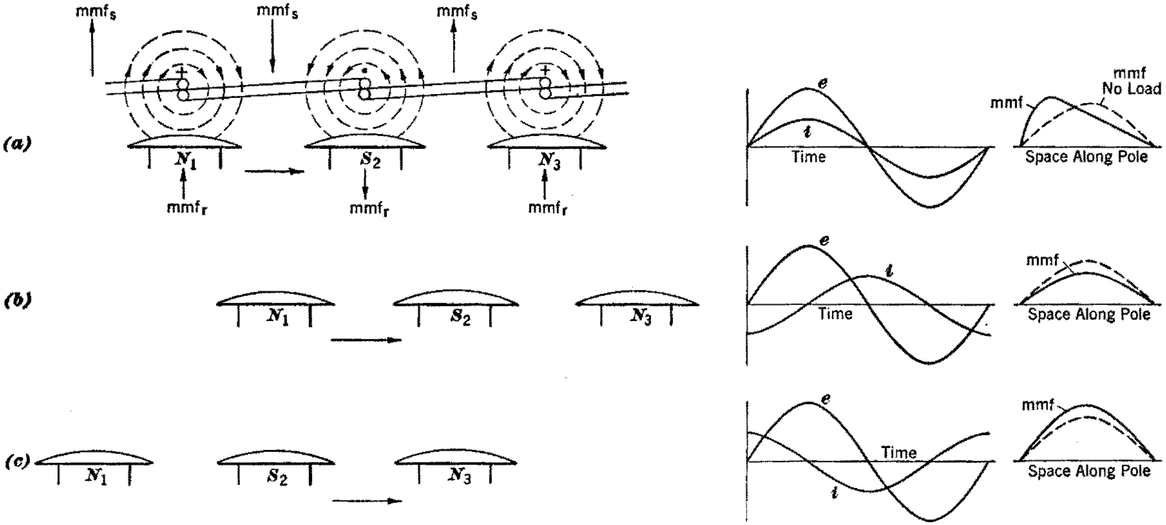


Fig. 20–5. Effects of Armature Reaction in a Synchronous Generator

pole has applied to it a resultant mmf equal to mmf$_r$+mmf$_s$ and the other half has applied to it a resultant mmf equal to mmf$_r$−mmf$_s$. The result is a distorted mmf pattern across the pole. Half of the pole will be over-saturated, and half will be under-saturated. Because of the saturation characteristics of iron, more flux will be lost on the weakened side of the pole than is gained on the strengthened side. The machine then incurs a net loss of flux, and the terminal voltage will be less because of that action. This is one cause of loss of terminal voltage when the generator supplies a load with unity power factor.

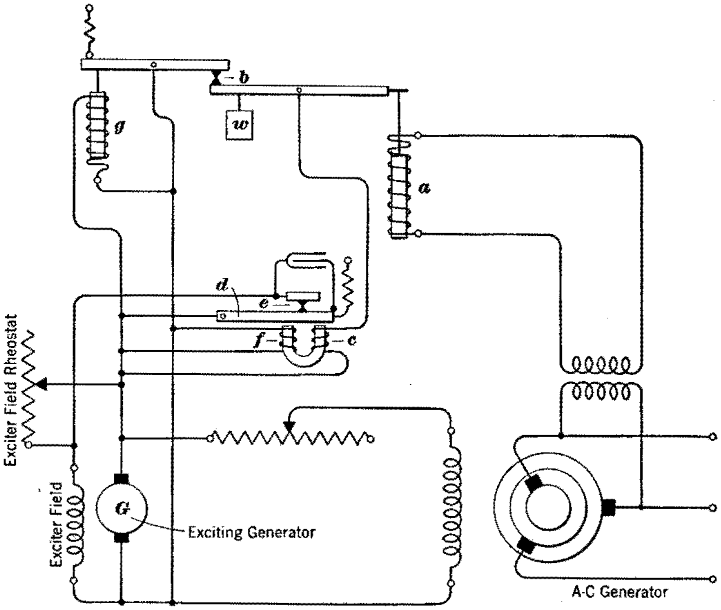


Fig. 20–6. Circuit Diagram of A-C Generator With Exciter and Tirrill Voltage Regulator

If the load were entirely inductive, as in Fig. 20–5 (*b*), the current would be at its maximum value 90 electrical degrees later than the instant at which the voltage is maximum. In that time lapse the poles would have moved 90 electrical degrees from their positions in (*a*) and would be in direct line with mmf$_s$, the direction of which is opposite to that of mmf$_r$. Therefore, the mmf over the whole pole would be equal to mmf$_r$−mmf$_s$, and the generator would suffer a loss of flux. For any load of *I* amperes of purely inductive load, the terminal voltage of a generator will be less than that for the same current *I* of resistive load.

If the load were purely capacitive, the rotor poles would be in the positions shown in (*c*), or not yet up to the positions in (*a*) by 90 electrical degrees. Therefore, the mmf over the whole pole would be equal to $\mathrm{mmf}_r + \mathrm{mmf}_s$ and the generator would gain flux and have a terminal voltage probably greater than that at no load. Loads having intermediate power factors would have intermediate effects.

If the generator has three phases, the other two phases contribute to the magnetomotive force mmf_s shown for one phase and produce a wave of armature reaction mmf that rotates at synchronous speed always in fixed position relative to the rotor poles under any particular condition of shaft load and power factor.

Voltage Control of A-C Generators.—The voltage regulation of alternating-current generators is greater than that of direct-current shunt generators of comparable size. Correction of terminal voltage is usually accomplished by automatic voltage control. A device known as a voltage regulator is commonly used for this purpose.

The action of the Tirrill regulator is as follows: In Fig. 20–6 a transformer, with its primary connected across the terminals of the alternator whose voltage is to be regulated, has its secondary connected to a solenoid *a*. Assume that the alternator voltage is at the desired value and that the regulator parts are therefore at rest. Assume next that additional load is connected to the alternator so that its terminal voltage falls. The decreased pull of solenoid *a* will permit the weight *w* to open the contacts *b*. This action de-energizes coil *c* and releases the armature *d*, which closes the contacts at *e*. These contacts short-circuit the portion of the exciter field rheostat which is in use; and the exciting generator voltage rises and boosts the field of the alternator, so that there is a rise in its terminal voltage. The increased alternator voltage causes solenoid *a* to be strengthened, and so the contacts at *b* are closed again. The reclosing of contacts *b* restores the circuit through coil *c*, causing the contacts *e* to open and the exciter field rheostat to be inserted again in the exciter field; and the excitation of the alternator is again decreased. This cycle of events takes place so rapidly that the rise and fall of the alternator voltage back and forth across the normal or average is not perceptible. Coil *f* is almost, but not quite, strong enough to hold the contacts *e* open, so that only a small change in current through coil *c* is required to move armature *d*. This arrangement increases sensitivity and speed of response. Coil *g* tends to prevent overshooting of the voltage and to eliminate the action known as "hunting."

Such a regulator will control the voltage of the a-c generator within close limits, even with large sudden changes in load. It is also possible to cause the terminal voltage to rise with increase in load, so as to compensate for loss of voltage at the load because of impedance voltage drop in the intervening feeders.

Fig. 20–7 shows a direct-acting voltage regulator of a simpler type, as built by the General Electric Company. In series with the shunt field of the exciter is a resistance unit consisting of resistance plates in series. The ends of the plates, with contact tips forming a resistance stack, are held together by a spring; and, when not in use, the plates are short-circuited and offer no resistance. When the alternator is started and its voltage approaches the rated voltage, it energizes a torque motor through a potential transformer and rectifier; and the torque motor, acting against the spring of the resistance stack, causes the stack contacts to open one by one and thus inserts added resistance into the exciter field circuit. This action lowers the exciter voltage, and so lowers the generator voltage and

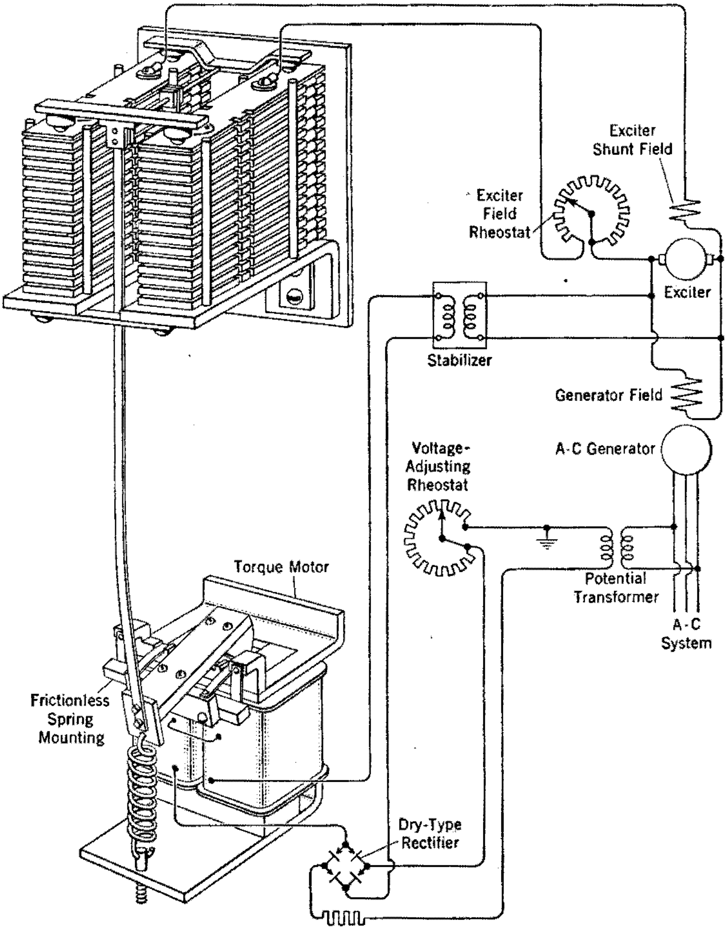


Fig. 20–7. Resistance-Stack Type of Voltage Regulator

thus causes some of the contacts to reclose as the torque motor relaxes. These operations would be repetitive, and the generator voltage would tend to drift or "hunt" above and below the desired normal voltage. To prevent this hunting, a stabilizing transformer is connected across the exciter-armature terminals. As the exciter voltage rises, a voltage is generated in the secondary of the stabilizing transformer. This generated voltage, acting through the rectifier, tends to nullify the rise in voltage on the torque motor and so to stabilize the action and prevent "overshooting."

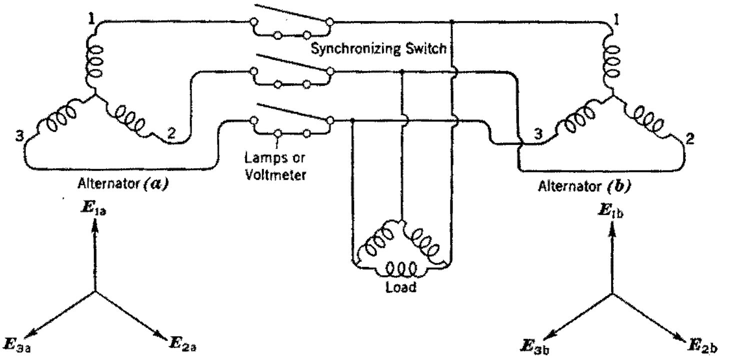


Fig. 20–8. Circuit Connections for Alternators to Be Operated in Parallel

Rating.—An alternator is usually rated at the load in kv-a which it is capable of carrying continuously at rated speed and voltage without exceeding the allowable temperature rise. Also, it is usually rated at a power factor of 80 per cent or 100 per cent. The 80% rating means that the field is sufficiently strong to provide the rated terminal voltage at the rated kv-a, even though the load power factor be as low as 80 per cent lagging.

Parallel Operation.—The economic generation of alternating current frequently involves parallel operation of two or more alternators. Direct-current generators excited to the same terminal voltage may be connected in parallel merely by connecting together terminals of like polarity. However, before two or more alternators can safely be connected in parallel, their frequencies must be almost the same and their terminal voltages must be almost the same. Furthermore, the terminal voltages of the machines must be almost in phase. Fig. 20–8 shows a circuit diagram of two three-phase alternators arranged to be operated in parallel, and shows also the ideal vector relationship desirable at the instant of closing the synchronizing switch. If the speed of the incoming alternator (*a*) is slightly greater than that of alternator (*b*), there will be alternate instants when the respective phase voltages of the two machines will be in phase, as in Fig. 20–8, and 180° out of phase, as in Fig. 20–9.

When the respective phase voltages of the two generators are exactly out of phase, the lamps of Fig. 20–8 will receive the combined voltage of both alternators in series and will be fully lighted. If the synchronizing switch were closed at that instant, the combined voltages of the alternators would be applied to their combined impedances only, and there would result a tremendous short-circuit current which would possibly damage the alternators. If, however, the switch be closed when the voltages of the two alternators are in phase, as in Fig. 20–8, the resultant voltage across the lamps will be zero, the lamps will be dark, and no circulating current will flow. This method is called "dark-lamp" synchronization.

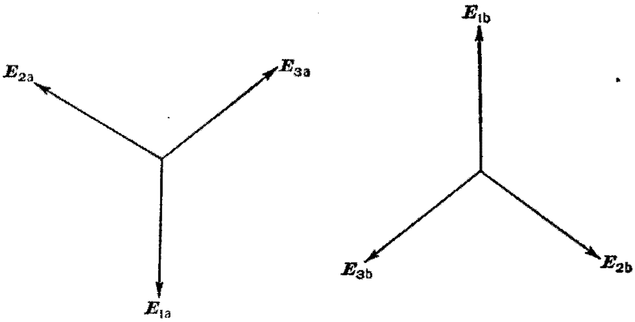


Fig. 20–9. Alternator Voltages Exactly out of Phase

Fig. 20–10 shows the vectors of Fig. 20–8 superposed at an instant when alternator (*a*) has drifted somewhat past the proper synchronizing point and resultant voltages $E_{1a}E_{1b}$, $E_{2a}E_{2b}$, and $E_{3a}E_{3b}$ have appeared across the synchronizing lamps. The shock to the alternators resulting from careless timing of the synchronizing switch is proportional to these resultant voltages.

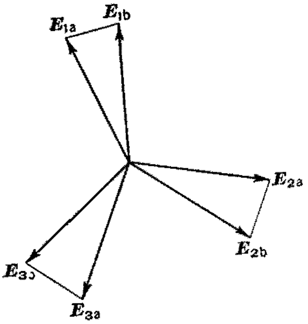


Fig. 20–10. Vectors When the Voltages Are Somewhat out of Phase

Significance of Phase Sequence.—If the phase sequences of the alternators in Fig. 20–8, as seen at the synchronizing switch, are the same, all the lamps will brighten and darken in unison as synchronism is approached. If they do not so brighten and darken in unison, but the brilliance seems to "rotate," then the phase sequences are not the same and the synchronizing switch must not be closed. The phase sequence may be corrected by reversing only two leads on either side of the synchronizing switch.

Synchronizing Current.—After the synchronizing switch in Fig. 20–8 has been closed at the proper instant, the drifting of alternator (*a*) does not instantly cease but continues until the resultant voltages $E_{1a}E_{1b}$, $E_{2a}E_{2b}$, and $E_{3a}E_{3b}$ become large enough to enable the motorizing current circulating between the alternators to hold them exactly in synchronism. This current is called *synchronizing current.* If the prime mover driving alternator (*a*) were then disconnected, the synchronizing current would rise just enough to drive machine (*a*) as a synchronous motor.

Division of Load Between Two Alternators.—If the power of the driver of alternator (*a*) in Fig. 20–8 be increased by changing its governor setting, then machine (*a*) will carry a larger portion of the load and alternator (*b*) will be partly unloaded. The proportion of the total load taken by each alternator will depend entirely on the prime-mover performance and not at all on the relative field excitations of the alternators.

Effect of Change in Field Excitation.—If alternators (*a*) and (*b*) in Fig. 20–8 are supplying a load jointly and the field excitation of machine (*a*) is reduced, the division of power load will not be affected. However, a circulating current will be caused to flow around the load circuit formed by the two alternators. This circulating current will be *leading* with respect to the voltage of alternator (*a*) and will thus supply the excitation deficiency by armature reaction. Furthermore, the circulating current will be *lagging* with respect to the voltage of alternator (*b*) and so will detract from its excitation. The line voltage will decrease somewhat because of the increased impedance drops in the two alternators, but the division of power load will remain constant.

Synchronizing Devices.—When lamps are used to indicate synchronism, it is not possible to determine exactly when the resultant voltages, as $E_{1a}E_{1b}$, $E_{2a}E_{2b}$, and $E_{3a}E_{3b}$ in Fig. 20–10, across the lamps are zero, because these resultant voltages must have some considerable magnitude before the lamps will light at all. The indication of approximation to synchronism afforded by lamps is sufficiently good for small alternators, but large alternators of high speed or large inertia must be synchronized by more accurate devices. For this duty *synchronoscopes* or *synchroscopes* have been developed. The synchroscope is essentially a two-phase motor which rotates at a speed equal to the difference in frequencies of the two alternators. The direction of rotation indicates whether the incoming alternator is running too fast or too slow. An indicating pointer attached to the rotating shaft shows when the correct condition for synchronizing is reached. An attempt to synchronize two large alternators with too great displacement error may cause a broken shaft or loosened stator laminations.

Hunting of Alternators.—The torque of a reciprocating engine is pulsating, and hence the power delivered to an alternator driven by it is pulsating. If such a power unit is operated in parallel with another power unit, an unstable condition may result as follows. When the torque of the engine is greatest, its alternator takes momentarily more than its average load and the other alternator takes less than its average load. When the engine torque is least, its alternator takes less than its average load. This oscillation of power, if too violent, may be cumulative; and, if the period of oscillation is too near the natural oscillating frequency of the rotor, the alternators may be thrown out of synchronism. Such oscillation is called "hunting." Destructive oscillation is prevented in alternators driven by internal-combustion engines by heavy amortisseur windings similar to the starting squirrel-cage windings of synchronous motors or by heavy flywheels or their equivalents.

PROBLEMS

1. What is the full-load current per terminal of a 2300-volt, 3-phase, 2000-kv-a alternator?

2. A 2300-volt, Y-connected, 100-amp, 0.8-PF alternator has internal impedance of $1+j5$ ohms per phase. A full load of 100% PF is being supplied at the rated terminal voltage. If the load is removed, but the field-rheostat setting is kept the same, what will be the terminal voltage? If, then, a full load with 70% PF leading is applied, what will be the terminal voltage? Assume the internal impedance to remain constant.

3. A 40,000-kv-a, 13,200-volt, 3-phase, 60-cycle, Y-connected alternator has a stray power loss equal to 1.5% and a field loss equal to 0.4% of the rated output kv-a at 85% power factor lagging. The effective stator resistance is 0.10 ohm per phase. Calculate the efficiency at full load of 85% power factor, lagging.

4. A 50-kv-a, Y-connected, 3-phase, 220-volt, 1200-rpm, 0.8-power factor, 60-cycle alternator requires 2.5 hp to drive it at rated speed at no load with field excitation. The stator resistance is 0.05 ohm per phase. The field-circuit power input at full load is 2 kw. Calculate the efficiency at full load of 80% power factor lagging.

5. Two similar 3-phase, 2000-kv-a alternators in parallel supply a 3500-kw load of 80% power factor lagging. If the output of the first alternator is 1600 kw at 70% power factor, what are the kw and kv-a outputs of the other?

6. A 96-pole alternator is to be used to supply a 50-cycle power system. At what speed must it be run?

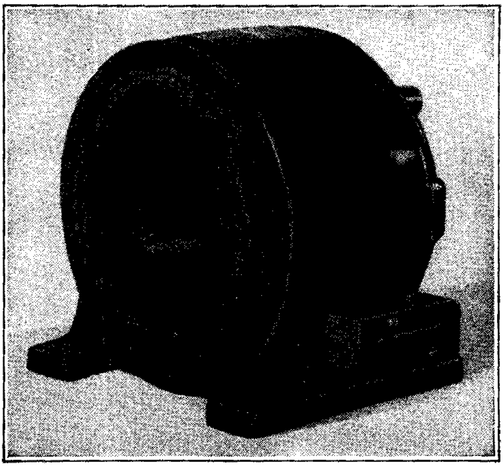
7. An alternator running at 375 rpm generates 25 cycles per second. How many poles does it have?

8. The stator of a 1000-kv-a alternator designed to be driven by a steam engine is much larger and shorter than one of the same electrical size designed for direct connection to a steam turbine. Explain.

CHAPTER 21

The Single-Phase Motor

Need for Small Motors.—A man can produce about 100 watts of power continuously throughout a working day. In an 8-hour working day his energy output in useful work would be 800 watt-hours. If he were to sell this energy at prevailing rates which are charged for household consumption, or at 6 cents per kilowatt-hour, his day of labor would be valued at 4.8 cents. His efforts would not be sufficient to operate the family washing machine for a whole day.



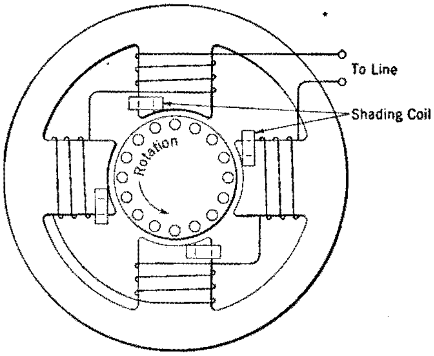
(Courtesy The Peerless Electric Co.)

Fig. 21-1. Single-Phase Motor Stator

Large motors turn the factory wheels, but fractional-horsepower motors in particular save the backs of men. Fractional-horsepower motors are almost altogether direct-current or single-phase alternating-current machines. Because of the greater cost of providing polyphase power, few small polyphase motors are in use. The small direct-current motor is not essentially different from the large one. This chapter will be devoted to the single-phase motor only. Except for railway service, the single-phase motor is built in sizes up to 50 hp only.

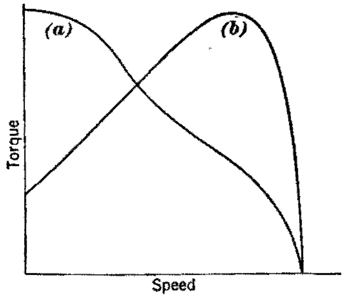
The many types of single-phase motors are distinguished by the means by which their starting is achieved, and they will be described in the approximate ascending order of their merits.

The Single-Phase Induction Motor.—A polyphase squirrel-cage induction motor connected to a single-phase line of its rated voltage and frequency will develop no starting torque; but, having been started in either direction by some external means, such a motor will perform approximately with polyphase-motor characteristics. The single-phase induction motor is seldom wound with the distributed lap type of winding, as in Fig. 18–5, but usually has a concentric type, as in Fig. 21–1.



The Cost is Low; also the Efficiency

Fig. 21–2. **Four-Pole Shaded-Pole Motor**



(*a*) With high-resistance rotor
(*b*) With low-resistance rotor

Fig. 21–3. Shaded-Pole Motor Characteristics

The Shaded-Pole Induction Motor.—Of the many devices used for obtaining starting torque in a single-phase motor, the *shaded pole* is the simplest and lowest in cost, but it is otherwise the least meritorious. Fig. 21–2 shows schematically the electrical features of a four-pole shaded-pole motor. The name arises from the so-called *shading coil* of one or more short-circuited turns of conductor surrounding a portion of each pole on the air-gap end. The function of that coil is to delay the time of rise of flux in the shaded portion of the pole, and also to delay the time of fall of the same flux relative to the flux in the unshaded portion. This action simulates very imperfectly the rotating fields of a polyphase motor and produces a torque which tends to rotate the squirrel-cage rotor from the unshaded to the shaded portion of the pole.

Fig. 21–3 shows some typical speed-torque curves of shaded-pole motors with low-resistance and high-resistance rotors. These motors are of relatively low efficiency and are unsatisfactory for any use requiring more than a small fraction of 1 hp. The efficiency could be improved by opening the shading-coil circuits after a starting; but, inasmuch as simplicity and low cost are the chief merits of this kind of motor, the added mechanism for opening the coils is not justified. This type of motor is used in small desk fans, domestic-furnace control, and other services in which its inferior characteristics may be tolerated. Fig. 21–4 shows a two-pole, so-called core type of shaded-pole motor commonly used in small fans and phonograph-record players.

The Split-Phase Induction Motor.—As the name implies, the split-phase motor splits the line current into two parts at start. One part flows in the *running* winding at a great angle of lag behind the line voltage, because the winding is of low resistance and has high reactance as it is deeply embedded in the slots and is closely surrounded by iron. See Fig. 21–5 (*a*). The other part of the line current flows in the *starting* winding at a much smaller angle of lag behind the line voltage, as this winding is of relatively high resistance because smaller wire is used and is of low reactance because it is in the tops of the slots. The phase is said to be split by resistance. The starting and running windings are located 90 electrical degrees apart in the stator frame, and the fluxes produced by the currents I_s and I_R are approximately 30 degrees displaced in time. The resulting field is a closer approach to the perfect rotating field than is that of the shaded-pole motor. Details of construction of a split-phase induction motor are shown in Fig. 21–6.

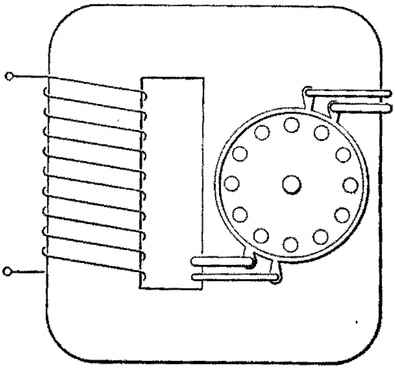
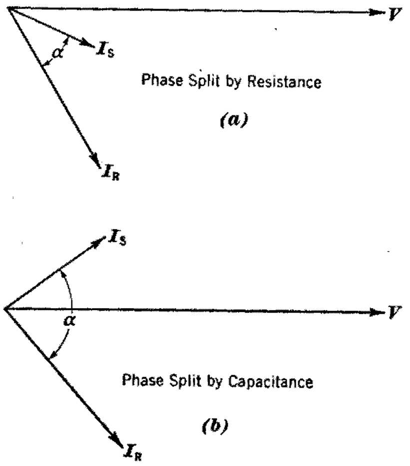


Fig. 21–4. Two-Pole Core-Type Shaded-Pole Motor With Two Shading Coils per Pole (More Than One Shading Coil per Pole Improves Performance)

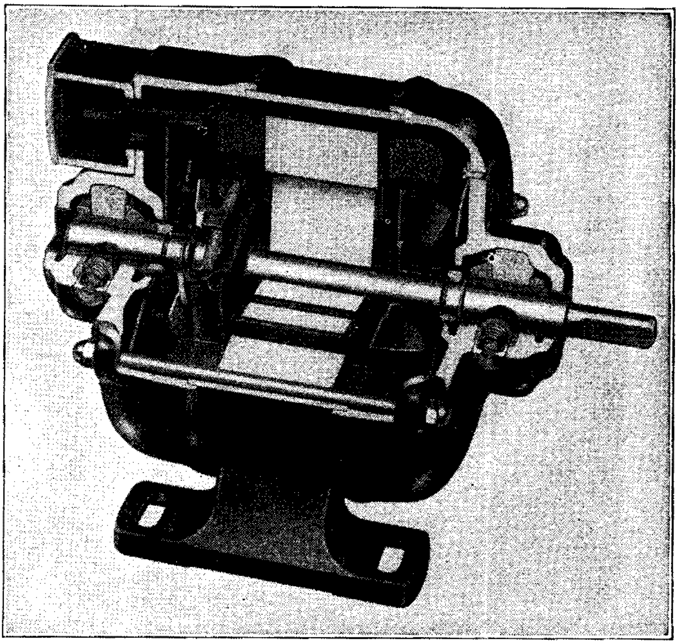


I_s = current in starting winding;
I_R = current in running winding;
α = time phase angle between I_s and I_R.

Fig. 21–5. Vector Diagram of Split-Phase Motor

The starting torque of the split-phase motor is usually about $2\frac{1}{2}$ times the full-load running torque. However, the starting current may be 7 to 9 times as great as the rated full-load current, and for that reason this type of motor is almost never built in sizes greater than $\frac{1}{2}$ hp. Because of voltage disturbance caused by the high inrush of starting current, most power-generating companies prohibit the use of split-phase motors greater than $\frac{1}{4}$ hp on domestic service lines.

The starting winding is almost always designed for intermittent duty only, and is disconnected from the line by a centrifugal switch when the motor approaches synchronous speed. Failure of the switch to open the circuit because of a defect or because of mounting the motor vertically when it is designed for horizontal operation will cause the starting winding to be overheated.



(Courtesy Robbins & Myers, Inc.)

Fig. 21-6. Split-Phase Induction Motor

Split-phase motors may be reversed by reversing the connections of either the starting winding or the running winding. A typical speed-torque curve is shown in Fig. 21-7. This type of motor is used for driving washing machines, mangles, and other small devices which require only a moderate starting torque.

The Capacitor Motor.—A split-phase motor may be provided with a capacitor (condenser), whereby the phase-splitting is accomplished more perfectly than is done by resistance and the starting characteristics are improved. See Fig. 21–5 (*b*). Such a motor is called a *capacitor* motor. Its construction is illustrated in Fig. 21–8. The starting torque obtained is 3 to 4 times the full-load running torque, and is usually about 30 per cent higher than the maximum running torque. The starting-current inrush is 6 to 7 times the rated full-load current; but, because of the superior power factor provided by the capacitor, the inrush current is only about 60 per cent of that of a resistance split-phase motor of comparable rating. Any device which requires a single-phase motor of high starting torque, high efficiency, small slip, and quiet operation will be satisfactorily driven by this type of motor.

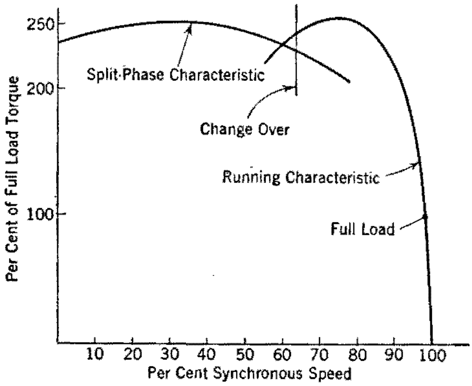
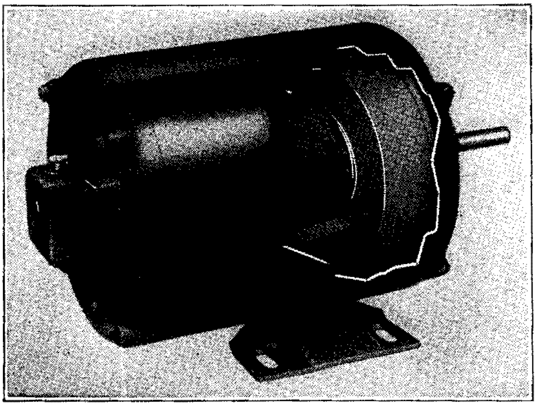


Fig. 21-7. Split-Phase Motor Characteristics



(*Courtesy Howell Electric Motors Co.*)

Note the squirrel-cage rotor and the enclosed capacitor in the end of the frame.

Fig. 21-8. Capacitor Motor

The Repulsion Motor.—The repulsion motor has operating characteristics similar to those of the series direct-current motor. It has a stator with a winding similar to that of a split-phase motor without the starting winding. The rotor is similar to the armature of a direct-current motor, but it has no electrical connection to the supply line. The brushes are connected together in a common short-circuit. The principle of operation will be described with the aid of Fig. 21–9.

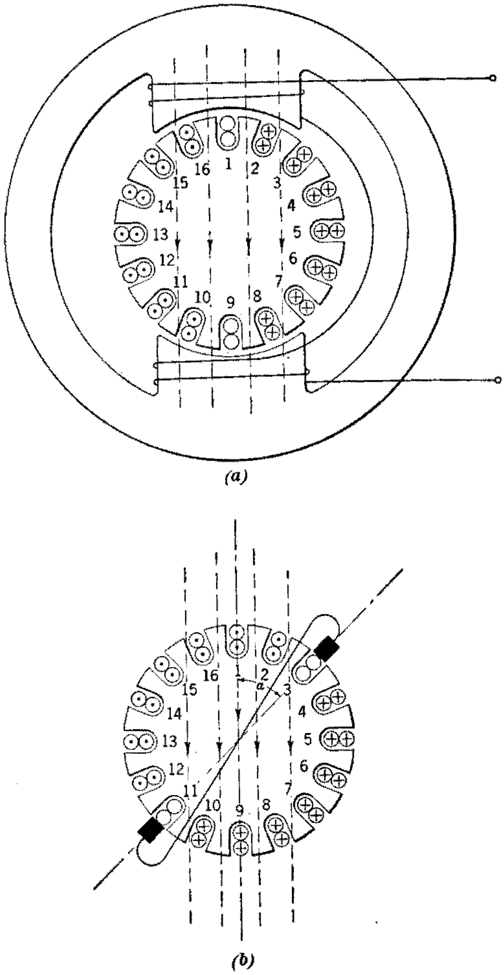


Fig. 21–9. Elements of Repulsion Motor

For simplicity the motor is shown with a salient-pole stator and with a rotor having full-pitch winding in which a conductor followed down in any slot would be found to return in a diametrically opposite slot. Consider the rotor to be stationary and the flux to be rising in the direction shown in Fig. 21–9 (*a*). No flux threads the loop 1-9, and hence no voltage is generated therein. In all other loops, by Faraday's Law, voltages are generated in the directions shown on the respective conductors. If a set of short-circuited brushes be placed on the horizontal axis connecting the winding at the two points 5 and 13, no current will flow because on either the top half or the bottom half the sum of the voltages is zero. If the brushes be rotated to the vertical axis, a large current will flow in the brushes and in all conductors in the directions shown in (*a*). By Lenz's Law, conductors 2, 3, 10, and 11 will produce a torque in a counter-clockwise direction, while conductors 7, 8, 15, and 16 will produce an equal torque in a clockwise direction. Hence, the motor will not tend to rotate.

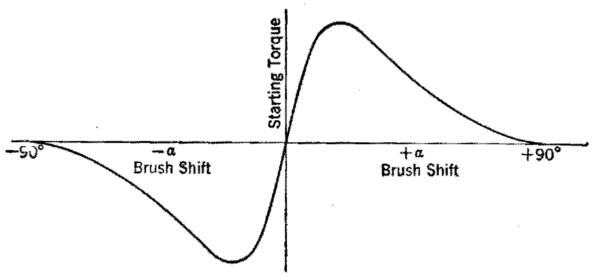


Fig. 21–10. Repulsion-Motor Starting Characteristics

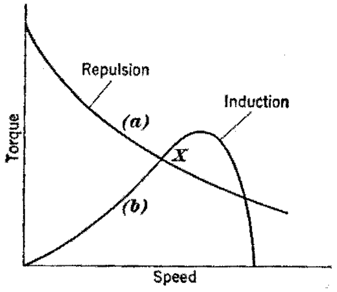


Fig. 21–11. Repulsion and Induction Characteristics

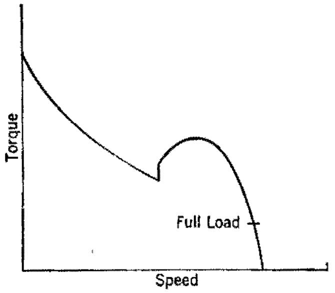
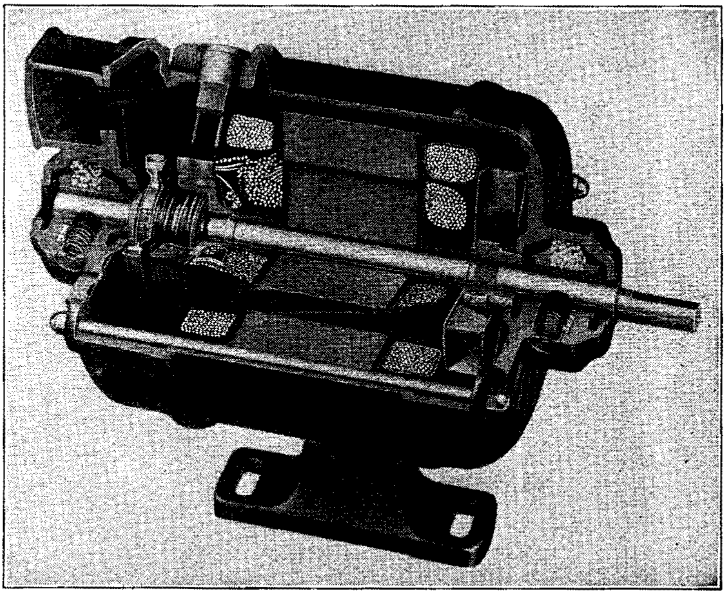


Fig. 21–12. Repulsion-Induction Characteristic

If, however, the brushes be rotated through the angle α to the positions shown in Fig. 21–9 (*b*), a new pattern of currents will appear, as shown. By Lenz's Law, conductors 16, 1, 2, 8, 9, and 10 will produce a clockwise torque without an opposing torque, and the motor will tend to rotate. If the brushes be displaced counter-clockwise from the vertical axis through a similar angle α, the torque will be of the same magnitude as before, but will be in a counter-clockwise direction. *The direction of the torque is the same as the direction of brush displacement.* Fig. 21–10 shows a typical curve of torque vs. α. The speed-torque characteristic is shown by curve (*a*) of Fig. 21–11.

The repulsion motor is well adapted to small-crane service or other applications in which large starting torque is essential and great speed regulation is not objectionable.



(*Courtesy Robbins & Myers, Inc.*)

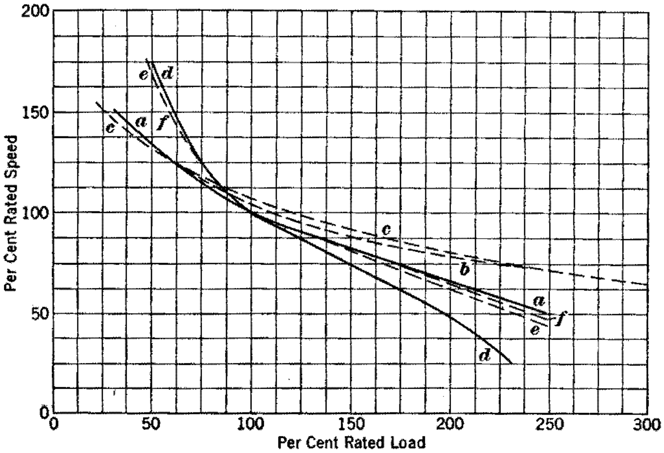
Fig. 21–13. **Cut-Away Model of Repulsion-Induction Motor**

The Repulsion-Induction Motor.—If the commutator of a repulsion motor be short-circuited, the rotor becomes merely a type of squirrel-cage rotor with a characteristic as shown by curve (*b*) of Fig. 21–11. The repulsion-induction motor starts by its repulsion characteristics; and then by the operation of a centrifugal switch the commutator is short-circuited at about the speed marked X, and the motor continues its acceleration on the induction characteristic (*b*) which provides good speed regulation in the normal-load region of speed. See Fig. 21–12.

Details of a repulsion-induction motor are shown in Fig. 21–13. These motors are usually designed for a starting torque of 4 to 6 times the full-load running torque. The starting-current inrush is from 2 to 4 times the full-load current. The starting current may be adjusted to a lower value by increasing the angle α of brush shift, but there will be a sacrifice in starting torque. Reversal is accomplished by shifting the brushes from one side of the neutral axis to the other, just as in the repulsion motor.

This type of motor costs more than a polyphase motor of comparable rating and is somewhat larger. It is well adapted to refrigerator compressors, milling machines, floor finishers, machine tools, hoists, etc.

The Series A-C Motor (Universal Motor).—This motor is called a *universal motor* because it can operate equally well on direct current or alternating current. Its construction is similar to that of the direct-current series motor with salient poles or to that of the repulsion motor with distributed stator windings. However, the field and armature are connected in series. All of the magnetic circuit must be laminated to minimize eddy-current loss. The operating characteristics are similar to those of the series direct-current motor in that both the starting torque and the speed regulation are high. Curves for various conditions are shown in Fig. 21–14.



Without Compensating Windings	With Compensating Windings
a—60 cycles	*d*—60 cycles
b—direct-current	*e*—direct-current
c—25 cycles	*f*—25 cycles

Fig. 21–14. Universal-Motor Characteristics

With the exception of those built for railway-traction service, series a-c motors are built in fractional-horsepower sizes only. They are inherently best adapted to high-speed service (3000 to 10,000 rpm), and are commonly used in floor sanders, vacuum sweepers, food mixers, and other household and office appliances. To avoid the evils of poor commutation, it is customary to shift the brushes in these motors a few degrees against the direction of rotation, or to place the brushes on the neutral axis with respect to the physical poles but to rotate the commutator on the shaft a few degrees in the direction of rotation with respect to the armature winding. Such a motor is suitable for operation in only one direction of rotation. Motors provided with compensating windings like those described on page 137 will have identical characteristics in both directions of rotation. Universal motors of the cheaper construction are not provided with compensating windings.

CHAPTER 22

Electric Batteries

Primary Batteries.—Batteries are of two general types, namely, *primary* and *secondary*. When a primary battery such as a "dry" flashlight battery or a "wet" battery of zinc and copper sulfate (obsolete) is constructed, all of the energy which the battery will ever deliver as electrical energy is stored in the constituents as chemical energy. When a circuit is provided between the terminals of such a battery, the current flow is produced by the reaction and decomposition of the chemical reagents. This decomposition and loss of energy is accompanied by an increase in internal resistance.

The open-circuit terminal voltage of a dry battery within the useful life of the battery is practically constant, and hence such voltage cannot be used in determining the physical condition of the constituents. When an attempt is made to draw current from an old used dry battery, the terminal voltage decreases greatly because of internal resistance and the battery is found to be worthless. Recharging is impractical and economically unsound.

When a dry battery produces electric current, the chemical reagent in contact with the electrode surfaces is used up and hydrogen gas collects on the surface of the positive electrode. The collection of hydrogen gas is called "polarization." Because of the slow infiltration of fresh reagent and the slow dispersion of hydrogen, the battery must be allowed to recuperate after a short period of use. For this reason primary batteries of the dry type are fit for only intermittent full duty.

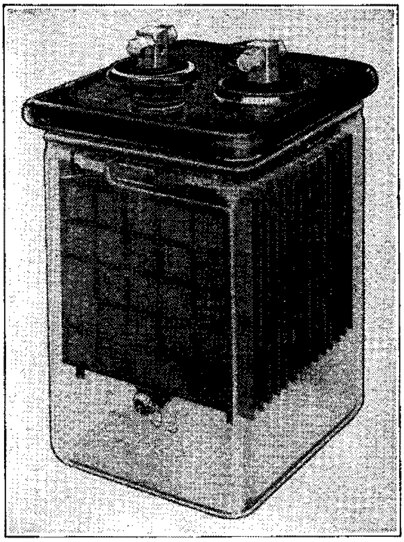
Secondary (Storage) Batteries.—A secondary or storage battery is a device of one or more cells for storing energy at one time for use at another time. Energy is put into it in the form of direct-current electrical energy and is delivered from it in the same form, but is stored in the form of chemical energy. After the battery is assembled, the energy is put in by "charging" with direct current. When the battery is delivering energy, it is said to be "discharging." A battery may be charged and discharged many times before it is worn out. In charging, a direct current is forced through the battery in opposition to its electromotive force.

The Lead-Acid Storage Battery.—The lead-acid battery consists of one or more cells in which one or more "positive" plates of lead peroxide on a lead frame are interleaved with two or more "negative" plates of spongy lead and the whole is immersed in a solution of sulfuric acid and water. See Fig. 22–1.

When the battery is discharging, the lead peroxide is combining with the sulfate (SO_4) of the electrolyte to form lead sulfate ($PbSO_4$). The lead sulfate collects on the positive plates. Lead sulfate is formed on the negative plates and collects on them. This may be written as follows:

$$\begin{matrix} & \text{Charge} & & \text{Discharge} & \\ PbO_2 + 2H_2SO_4 + & Pb & \underset{\longleftarrow}{\overset{\longrightarrow}{=}} & PbSO_4 + PbSO_4 + 2H_2O \end{matrix}$$

$$\begin{matrix}\text{lead}\\ \text{peroxide}\end{matrix} + \begin{matrix}\text{sulfuric}\\ \text{acid}\end{matrix} + \text{lead} = \begin{matrix}\text{lead}\\ \text{sulfate}\end{matrix} + \begin{matrix}\text{lead}\\ \text{sulfate}\end{matrix} + \text{water}$$



(*Courtesy The Electric Storage Battery Co.*)

Fig. 22–1. Lead-Acid Storage Battery

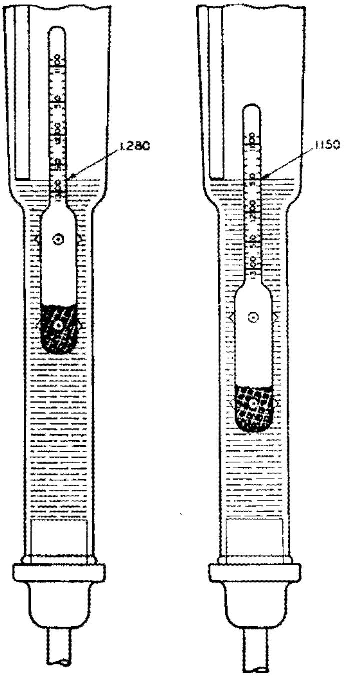


Fig. 22–2. Hydrometer Used in Testing Lead-Acid Batteries

Thus, when the battery is fully charged, the SO_4 is in the electrolyte; and, when the battery is discharged, the SO_4 is on the plates in the form of lead sulfate and the electrolyte is greatly diluted with water. Hence, the condition of charge may be determined by measuring the specific gravity of the electrolyte. Fig. 22–2 shows a hydrometer reading of 1.280 indicating full charge and one reading 1.150 indicating complete practical discharge.

Water lost by evaporation and electrolysis must be replaced frequently by distilled water or other water of satisfactory purity. Batteries designed for portable service and for high momentary power output such as is required in automotive service usually have a specific gravity of 1.280 when fully charged. Batteries designed for stationary service and continuous duty usually have a specific gravity of about 1.220. It is therefore necessary to know the specific gravity for which the battery was designed before a hydrometer can be used intelligently to determine the condition of charge.

The automotive type of battery has the Fauré type of plate, which consists of active material pasted into recesses or pockets of a lead frame. This type of plate affords a maximum energy capacity per pound of weight and per cubic foot of space. The life of the Fauré plate seldom exceeds 2 or 3 years in active service, largely because of shedding from the positive plates.

The stationary type of battery has the Planté type of plate, which consists of active material formed electrolytically on the lead frame. This type of plate affords a maximum useful life, but is too heavy and bulky for portable use.

Change in Voltage During Charge and Discharge.—The normal open-circuit voltage of a lead-acid battery is approximately 2 volts per cell. The battery should not be discharged further after its terminal discharge voltage falls to $1\frac{3}{4}$ volts per cell. Over a discharge period of 6 to 8 hours, the average voltage is roughly 1.95 volts per cell.

The maximum charging voltage required is about 2.6 volts per cell.

Charging Rates.—The rate at which a battery may be safely charged depends on the rate at which lead sulfate may be reduced on the plates and hence depends on the rating and condition of the battery. If an attempt is made to charge a battery at an excessive rate, part of the current is used in reducing the lead sulfate and the excess current causes decomposition of the water of the electrolyte. The liberated hydrogen rises to the surface. That action is called "gassing." The rate at which sulfate reduction may be accomplished decreases as the charging progresses, and hence the charging rate should be "tapered" to a relatively small value at the end of the charging period. Excessive gassing should be avoided because the scouring action of the rising bubbles tends to wear away the active material of the plates, particularly the positive plate. The scoured-off material falls to the bottom of the jar, and may eventually fill the jar up to the bottoms of the plates and short-circuit them. Furthermore, gassing indicates a waste of energy and creates a fire and explosion hazard.

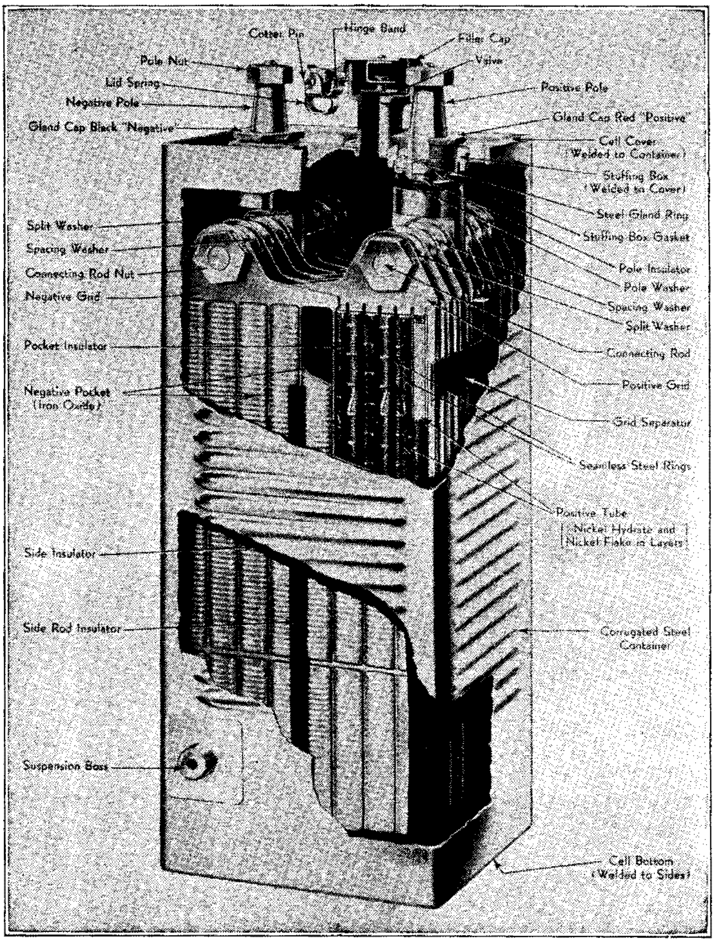
The maximum safe ampere rate of charge for any lead-acid battery, regardless of its size or condition, is equal to the number of ampere-hours required to bring it to full charge. The finishing rate, however, does not need to conform to this rule, but may be as great as recommended by the manufacturer. Quick charging can be achieved by following this rule, but the operation requires constant skilled attention. A compromise consists in charging at such a rate that 63 per cent of the desired charge is received by the battery in the first hour, and 85 per cent is received in the first two hours. Such rapid rates of charging should be used only in emergency because of the attention required to prevent damage to the battery.

Charging Methods.—Charging methods must be chosen to fit the type of service. Batteries used for electric vehicle propulsion are discharged during the travel hours of the vehicle and are then recharged in off-service hours. They are sometimes given a "booster" charge during the noon hour, if the vehicle is near the charging station at that time. The booster charge is kept as high as possible without objectionable gassing, in order to store a maximum amount of energy in the short time allowable. A short-time booster charge not only replaces some used-up energy but it also accelerates the circulation of the electrolyte and makes the remaining portion of the original charge more readily available for high discharge rates. When a battery is charged in off-service hours, a moderate rate with the usual tapered "finish charge" is preferable. Equipment designed for charging the batteries of a fleet of vehicles will automatically provide a tapered charge.

Batteries used for central telephone or power station service are usually charged continuously at a low "trickle charge" rate. A constant voltage of appropriate value impressed across the battery terminals is sufficient for proper charging. In no case should the battery temperature be permitted to exceed 110 F.

Equalizing Charge.—In the usual cycle of charging and discharging, all of the sulfate is not reduced in all the cells. If sulfate is permitted to remain unreduced, its reduction becomes increasingly more difficult. The hardened sulfate clogs the pores of the plates and reduces the capacity of the battery. To prevent this accumulation, an "equalizing charge" at a rate no higher than the "finish charge" rate should be given periodically. A battery which is charged no oftener than once every week should have every charge continued into an equalizing charge until all cells gas freely and three consecutive half-hourly hydrometer readings show no increase in specific gravity. A battery charged oftener than once every week may be kept in best condition by making every fifth or sixth charge an equalizing charge.

Discharge Rates.—Most lead-acid storage batteries made by reputable manufacturers may safely be discharged at any current which they are capable of producing. However, the ampere-hours obtainable from a battery are greater at a low rate of discharge for a long time than at a high rate for a short time. This difference exists because the voltage drops faster at the higher rate. The ampere-hour capacity rating of a lead-acid battery is usually referred to an 8-hour discharge time.



(Courtesy Thomas A. Edison, Inc.)

Fig. 22–3. **Nickel-Iron-Alkaline (Edison) Battery**

Effect of High and Low Temperature.—In some types of batteries, the effect of high temperature is primarily to cause the sulfuric acid of the electrolyte to attack and carbonize the wood used as separators between the positive and negative plates. Temperatures of 110 F or more tend greatly to accelerate the destruction.

Impurities in the plate substance and in the electrolyte cause localized disintegration of the reagents and loss of charge. Batteries will hold a charge much better if kept in a cool place.

Low temperature decreases the immediate effective capacity of a cell, but causes no permanent injury unless the cell is in a low state of charge or water has been added without subsequent charging.

Gas.—The gas given off by a lead-acid battery is hydrogen and is highly explosive when properly mixed with air. Flames or glowing cigarettes or cigars should not be brought near the battery during the charging period or shortly afterward. Care should be taken to avoid disrupting the charging circuit near the charging battery, lest the escaping gas be ignited by the spark.

Care of Lead-Acid Battery.—When a battery is in use, water should be added frequently. The top and the case should be kept clean to prevent leakage paths which slowly discharge the battery. The battery should not be permitted to stand in a discharged condition. It should be given an equalizing charge at well-chosen intervals.

When not in use the battery should be fully charged and stored in a cool dry place. If the electrolyte be drained off and stored separately, local disintegration is minimized. After such draining the cells should be filled with distilled water, unless there is a possibility of freezing temperature.

The Nickel-Iron-Alkaline Storage Battery.—The nickel-iron-alkaline storage battery was invented by Thomas A. Edison in an attempt to develop a battery in which the chemical reaction is completely reversible. The details of such a battery are shown in Fig. 22–3. The positive plates consist of perforated steel tubes containing nickel hydrate, and the negative plates are steel sheets with perforated pockets containing iron oxide. The electrolyte is an aqueous solution of potassium and lithium hydroxide.

The chemical equation of charge and discharge is as follows:

$$3Fe + 8KOH + 4H_2O + 6NIO_2 \overset{\longrightarrow}{\underset{\longleftarrow}{=}} Fe_3O_4 + 8KOH + 4H_2O + 2Ni_3O_4$$

(Charged) (Discharged)

In this equation the hydroxide concentration ($8KOH$) is the same whether the battery is charged or discharged. Hence, the state of charge of a nickel-iron-alkaline battery cannot be determined by hydrometer readings. The most convenient method of determining the state of charge at any time is to charge and discharge the battery through an ampere-hour meter arranged to compensate for the additional ampere-hours required for charge.

Charge and Discharge Characteristics.—The nickel-iron-alkaline battery should be charged at such a rate as to complete the charge in about 7 hours. The ampere discharge rate is usually given on the basis of a 5-hour discharge time. An accidental excess charge or charge in the reverse direction does not injure the battery. An occasional short-circuit is not harmful.

The maximum voltage per cell when the battery is fully charged is about 1.4 volts on open circuit. The average voltage per cell throughout the discharge period is about 1.2 volts.

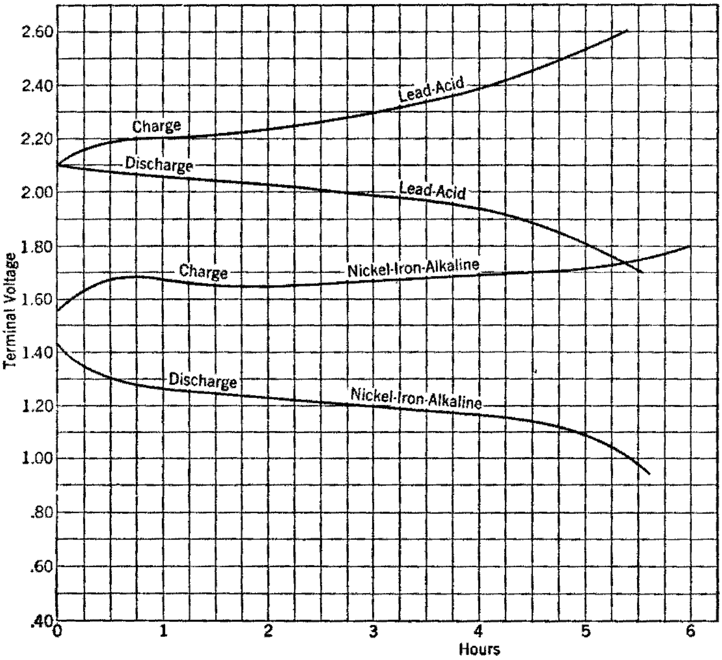


Fig. 22–4. Comparison of Performances of Lead-Acid and Nickel-Iron-Alkaline Battery

Care of Nickel-Iron-Alkaline Storage Battery.—No permanent injury results when a nickel-iron-alkaline battery is over-discharged or is allowed to stand in a discharged condition. Water must be added occasionally to replace that lost by "gassing." The gases produced are inflammable, and charging requires the same safety precautions as described for lead-acid batteries. The fumes are not corrosive to wood or metal, as are those of the lead-acid battery, and need not be guarded against. The electrolyte should be renewed at intervals of 3 to 6 years. When the battery is to be stored, it should be fully discharged and short-circuited.

Comparison of Lead-Acid and Nickel-Iron-Alkaline Batteries. Advantages of the lead-acid battery are as follows:

1. It has much higher momentary overload capacity.
2. It has less change in voltage between full charge and maximum practical discharge.
3. It costs less for comparable capacity.
4. Its efficiency is higher than that of the nickel-iron-alkaline battery.

Advantages of the nickel-iron-alkaline battery are:

1. Its weight is less for comparable capacity.
2. It requires less attention.
3. It does not give off obnoxious fumes.
4. It has a longer life.

CHAPTER 23

Economics of Use of Electrical Apparatus

Types of Problems Involved.—The economical use of electrical apparatus demands the intelligent analysis of many problems of the type common to all types of engineering, and also of some problems peculiar to electrical engineering alone. There are problems involving economic life, efficiency, and other items of operating cost. These items are common to all kinds of machinery and other structures. Then there are problems of energy rates, problems of uneconomic use of invested capital because of low power factor, and problems of personal safety, which are encountered only in the application and use of electrical devices.

Purchase of Industrial Machinery.—There is little danger of buying shoddy new electrical apparatus in the United States of America, unless it is in the field of fractional-horsepower motors and very small transformers. In these markets there are offered items of low cost and, of course, of inferior construction. However, the fact that inferior construction and possibly low efficiency may be found does not condemn the manufacturers. It merely requires an accurate understanding of the financial engineering of purchase to guide the buyer in his choice between high and low first cost, when the operating costs are different.

There are motors, generators, transformers, protective equipment, etc., with various relative first costs, efficiencies, performance factors, and life expectancies. First cost, efficiency, and performance are *reducible* items and may be readily compared, but maintenance and life expectancy are *irreducible* items and can be judged only by the history of the apparatus if such is available. Furthermore, the reputation of the manufacturer is the only assurance that unsatisfactory performance, if encountered, will be rectified. It is common practice for all manufacturers to refuse responsibility for defects arising more than 1 year or 2 years after the date of purchase. This is felt to be necessary for their protection against the results of abuse of the apparatus in service.

A motor, for example, is bought with the expectancy of about 20 years of service. Where the need for a motor continues, a reinvestment would be required every 20 years. If the purchaser could contract with a party of the second part to furnish this motor service henceforth, the contract price paid that party at the beginning of the service would be the *present worth* of the permanent service. For the present worth of the investments and reinvestments, let the symbol PW_D be used. Furthermore, if the user could contract with a party of the third part for the *cost of operation* henceforth, the contract price paid would be the present worth of that permanent service. For this present worth let the symbol PW_o be used.

The Calculation of PW_D.—In the following analysis,

C_c = the purchase price;
n = the life expectancy in years;
r = the interest rate.

The first motor requires an investment of C_c dollars immediately. Thus,

$$PW_{D1} = C_c$$

To provide another C_c dollars with which to buy the second motor at the end of n years, it is necessary to invest at interest *now* the amount

$$PW_{D2} = \frac{C_c}{(1+r)^n}$$

Similarly, to provide money for purchase of the third motor, it is necessary to invest at interest *now* the amount

$$PW_{D3} = \frac{C_c}{(1+r)^{2n}}$$

Then, the present worth PW_D of the investment and reinvestment for the life of three machines would be

$$PW_D = PW_{D1} + PW_{D2} + PW_{D3} \tag{23-1}$$

If it were likely that the service would be required indefinitely, it would be necessary to sum up the preceding series to $(xn+1)$ terms. However, in dealing with a rapidly progressing science such as the applications of electricity to industry, it is pure folly to assume that obsolescence will not retire any piece of apparatus in less than 100 years.

Calculation of PW_o.—The yearly operating cost of a motor includes interest, taxes, insurance, repair cost, and energy cost. This total operating cost per year divided by the interest rate gives the capital investment necessary to yield an amount of money equal to the annual operating cost every year forever. The annual cost of operation is then said to be *capitalized*. This capitalized amount is the item PW_o.

Let C_o = the annual cost of operation;
r = the interest rate;
m = number of machines to be bought consecutively.

An amount $PW_o = \frac{C_o}{r}$ invested now would yield an operating fund forever, whereas a need is usually anticipated for an estimated term of years only. Therefore, for the years after the termination of the anticipated need at the end of n years, there should be no investment made now to provide for operating funds. The present worth of this unnecessary sum is

$$\frac{C_o}{r}\times\frac{1}{(1+r)^{mn}}$$

and this amount must be subtracted from $\frac{C_o}{r}$ to obtain PW_{op}, the partial present worth corresponding to the years of anticipated need. Thus,

$$PW_o=\frac{C_o}{r}-\frac{C_o}{r}\times\frac{1}{(1+r)^{mn}}$$

or

$$PW_o=\frac{C_o}{r}\left[1-\frac{1}{(1+r)^{mn}}\right] \tag{23-2}$$

Therefore, PW_o is capital set aside for the operation of the motor throughout its life and the lives of its intended successors.

Calculation of PW_t.—The item PW_t is the total present worth of the service desired over the period of n years. Hence,

$$PW_t=PW_D+PW_o \tag{23-3}$$

Example 23–1.—A motor is to be bought to drive an air compressor. The motor will run continuously 8 hours every day. The load is intermittent and of uncertain duration, but is 50 hp for an average of 6 hours per day and 10 hp for 2 hours per day. Power is available at 440 volts, 3 phase, 60 cycles.

Salesman No. 1 offers the following synchronous motor:

3 phase, 60 cycles, 440 volts, 50 hp, 1.0 power factor, 40 deg C temperature rise, 1200 rpm
Life expectancy, 20 years
Price complete with exciter and automatic starter and base plate \$1400
Cost of installation 100

Total cost (C_c) \$1500
Efficiency at full load, 89%
Efficiency at 1/5 full load, 35%
Cost of maintenance, \$25 per year

Salesman No. 2 offers the following wound-rotor induction motor:

3 phase, 60 cycles, 440 volts, 50 hp, 40 deg C temperature rise, 1150 rpm
Life expectancy, 20 years
Price complete with base plate and automatic starter \$1068
Cost of installation 100

Total cost (C_c) \$1168
Efficiency at full load, 88.5%
Efficiency at 1/5 full load, 34.0%
Power factor at full load, 88.5%
Power factor at 1/5 full load, 52.0%
Cost of maintenance, \$20 per year

The cost of energy is \$0.025 per kw-hr
The cost of taxes and insurance is 9 per cent of the purchase price
The interest rate is 6%
It is required to determine the choice of motor on the assumption that service will be discontinued after 40 years.

Solution.—The present worth PW_t of proposal No. 1 will be found first, as follows:

$$PW_{D1} = C_c = \$1500$$

$$PW_{D2} = \frac{C_c}{(1+r)^n} = \frac{1500}{(1+0.06)^{20}} = \$467.87$$

$$PW_D = PW_{D1} + PW_{D2} = 1500.00 + 467.87 = \$1967.87$$

The annual cost of operation is the sum of the separate costs enumerated as follows:

$$\left(\frac{\text{Full-load hp} \times 746 \times \text{hr per year of full-load hp}}{1000 \times \text{Eff at full load}} + \frac{\frac{1}{5}\text{ full-load hp} \times 746 \times \text{hr per year of } \frac{1}{5}\text{ full-load hp}}{1000 \times \text{Eff at } \frac{1}{5}\text{ full load}}\right) \times \$0.025$$

$$= \left(\frac{50 \times 746 \times (6 \times 365)}{1000 \times 0.89} + \frac{10 \times 746 \times (2 \times 365)}{1000 \times 0.35}\right) \times \$0.025 = \$2683.50$$

The cost of taxes and insurance is

$$\$1400 \times 0.09 = \$126.00$$

The total annual operating cost is then

$$C_o = \$2683.50 + \$126.00 + \$25.00 = \$2834.50$$

Therefore,

$$PW_o = \frac{C_o}{r}\left[1 - \frac{1}{(1+r)^{mn}}\right] = \frac{2834.50}{0.06}\left[1 - \frac{1}{(1+0.06)^{2 \times 20}}\right] = \$42{,}648.74$$

$$PW_t = PW_D + PW_o = \$1967.87 + \$42{,}648.74 = \$44{,}616.61$$

The present worth of proposal No. 2 will now be found as follows:

$$PW_{D1} = C_c = \$1168.00$$

$$PW_{D2} = \frac{C_c}{(1+r)^n} = \frac{1168}{(1+0.06)^{20}} = \$386.67$$

$$PW_D = PW_{D1} + PW_{D2} = 1168.00 + 386.67 = \$1554.67$$

The annual cost of operation is the sum of the separate costs enumerated as follows:

$$\left(\frac{\text{Full-load hp} \times 746 \times \text{hr per year of full-load hp}}{1000 \times \text{Eff at full load}} + \frac{\frac{1}{5}\text{ full-load hp} \times 746 \times \text{hr per year of } \frac{1}{5}\text{ full-load hp}}{1000 \times \text{Eff at } \frac{1}{5}\text{ full load}}\right) \times \$0.025$$

$$= \left(\frac{50 \times 746 \times (6 \times 365)}{1000 \times 0.885} + \frac{10 \times 746 \times (2 \times 365)}{1000 \times 0.34}\right) \times \$0.025 = \$2707.95$$

The cost of taxes and insurance is

$$\$1068 \times 0.09 = \$96.12$$

The total operating cost is then

$$C_o = \$2707.95 + \$96.12 + \$20.00 = \$2824.07$$

Therefore,

$$PW_o = \frac{2824.07}{0.06}\left[1 - \frac{1}{(1+0.06)^{2\times 20}}\right] = \$42{,}491.80$$

$$PW_t = PW_D + PW_o = \$1554.67 + \$42{,}491.80 = \$44{,}046.47$$

The difference between the costs of the two proposals is

$$\$44{,}616.61 - \$44{,}046.47 = \$570.14$$

Based on the present worth of the first proposal, the second one is better by 1.28%.

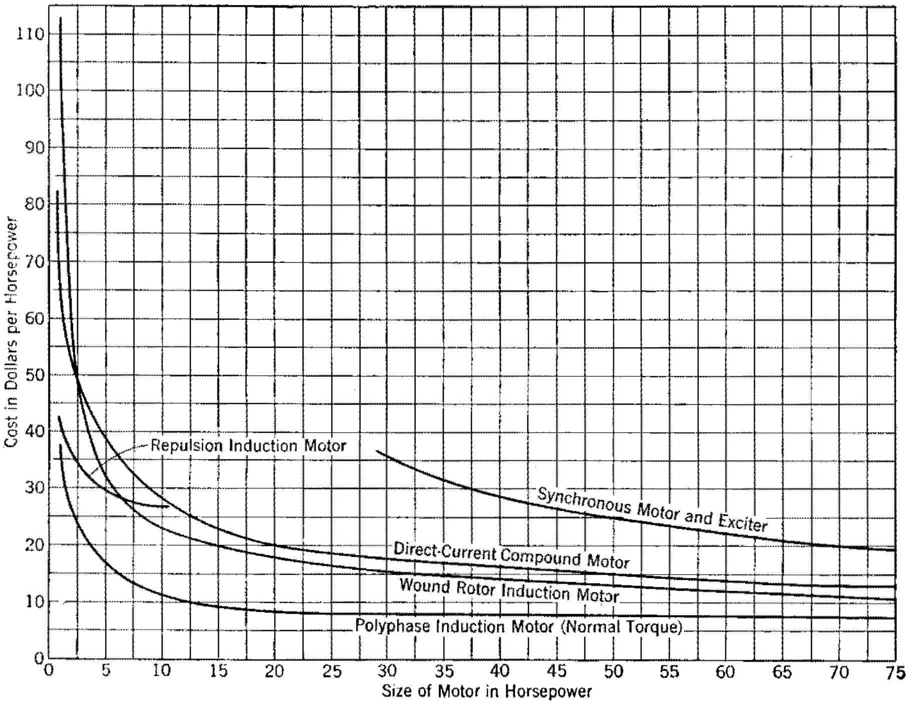


Fig. 23-1. **Graphical Representation of Cost of Motors as a Function of Size**

The power-factor correction possibilities of the synchronous motor (see Chapters 19 and 25) have not been credited to it. Furthermore, if the load should decrease by 40 or more per cent, the decreased power factor of the induction motor would bring a penalty through the power-factor penalty clause, if any, in the power contract, whereas the power factor of the synchronous motor could be held above the penalty point.

If the number of hours of use should increase above that given, the slightly superior efficiency of the synchronous motor would cause the foregoing calculations to favor that machine even more.

Choice of Size of Apparatus.—It obviously is economically unsound to to use a 25-hp motor to drive a machine requiring only 10 hp. Not only is the capital investment unjustified, but at such a small fraction of full load the efficiency of the motor is poor. If it is an induction motor, the power factor at small fractional loads will be low. The same conditions exist in an under-loaded transformer; but, when a growing load is to be supplied, it is sometimes more economical to install transformer capacity in excess of the immediate need instead of later replacing a small unit with a larger one.

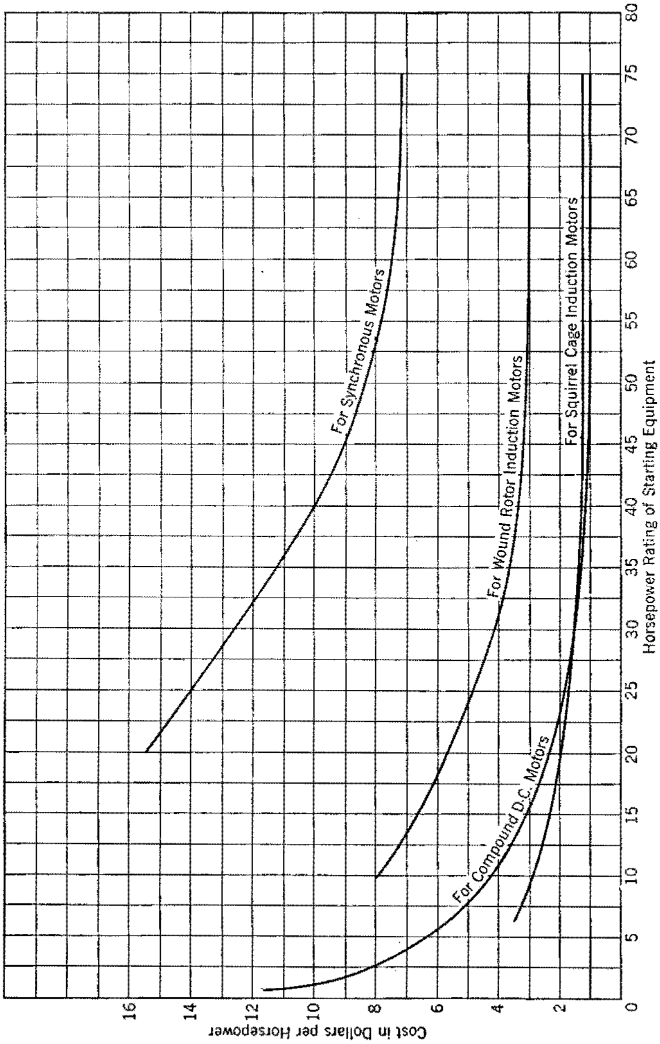


Fig. 23–2. Graphic Representation of Cost of Motor Starting Equipment as Function of Size

The curves of Fig. 23–1 show the relative purchase costs per horsepower of several types of motors, and the curves of Fig. 23–2 show the costs of their starting control equipment in a range of sizes from 1 to 75 hp. It is obvious from the curve of polyphase-induction-motor costs, for example, that the cost of 10 hp in 1-hp motors is \$39.00 × 10 = \$390, whereas the cost of the same horsepower in one 10-hp motor is but \$120.00. This difference in cost would suggest group drive by the 10-hp motor driving a line shaft.

Group Drive vs. Individual Drive.—No hard-drawn rules favoring group drive or individual drive for any application can be made. The factors involved in any application are peculiar to that application and are not common to all applications. Each case must be decided after the advantages and disadvantages of both methods have been weighed. In general, a saving will be realized with group drive under the following conditions:

1. Where there are compact groups of constant-speed machines which are to be run continuously or simultaneously.
2. Where there are compact groups of machines which, because of their diversity of load, may be driven by a single motor of much smaller rating than the combined capacities of the motors required for individual drives.
3. Where groups of constant-speed machines with heavy peak-load demands might require individual motors of a size much in excess of the average running load.
4. Where the motors required for individual drive are small.
5. Where, in changing over an existing installation, the old system of line-shafting may be used as already installed.

The initial cost will usually be less with individual drive under the following conditions:

1. Where the machines are isolated and line-shafting is impracticable.
2. Where the roof construction will not safely support line-shafting and the floor structure does not admit of hanging the line-shafting beneath.
3. Where the speed of the machines must be independently variable.
4. Where it is necessary to move the machines from one location to another frequently.

In addition to the foregoing considerations it should be remembered that group drive with open belting presents some personal hazard. Furthermore, where good appearance and working conditions are of great importance, the individual drive is preferable because of the absence of belts and line-shafting.

Operating Costs of Group Drive and Individual Drive.—The higher efficiency of the group-drive motor is offset at least in part by the losses in belting and shaft bearings. There may, however, be a saving in power costs with a group drive when the maximum demand for billing purposes is based on "connected horsepower" (see Chapter 24). Thus, it might be necessary because of peak loads to provide a 10-hp motor to drive a given machine. Ten such machines would require ten motors for individual drive, and the demand would be based on 100 connected horsepower. It might be possible to arrange these machines for group drive and use only a 40-hp motor, if the peak loads did not seriously overlap in the times of their occurrences. Thus, the demand charge for group drive would be only 0.4 times that for individual drive. Such economies in first cost and demand charges (see Chapter 24) can sometimes be effected. In the case of overtime work the group-drive motor must run badly under-loaded; at times it may be necessary to drive the whole line-shaft system to operate possibly only 10 per cent of the machines of the group.

Most Economical Size of Cable.—In planning the layout of a factory distribution system, the four limitations stated on page 55 relative to conductor size must be considered. The required mechanical strength and the maximum allowable current-carrying capacity are both specified in the National Electric Code of the National Board of Fire Underwriters. The voltage drop allowable is fixed by the voltage variation which may be tolerated at the load. However, these three requirements may be satisfied by a conductor that is too small to meet the requirement of Lord Kelvin's Law, which is: *For maximum economy the yearly cost of energy dissipated in the conductor must be equal to the yearly charges (depreciation, interest, taxes, and repairs) against the investment in the conductor.*

Example 23–2.—In a large cement plant it is necessary to provide three-phase power at 500 amp to a 440-volt motor located 500 ft from the transformer substation. The cost of energy is $0.02 per kw-hr. The load is to be carried 12 hours every day of the year. Determine the proper size of conductor to be used.

Solution.—If rubber-covered cable is used on brackets in open air inside the building, the National Electric Code requires that a 700,000 cir mil cable be used to carry 500 amp.

The resistance of 700,000 cir mil cable at 75 C is 0.0197 ohm per 1000 ft

Resistance of 500 ft = 0.00985 ohm

IR loss in each cable = 500 × 0.00985 = 4.92 volts

Voltage from line to neutral of a 440-volt motor is $\frac{440}{\sqrt{3}} = 254$ volts

Maximum tolerable voltage reduction at the motor terminals is 10%, and the tolerable reduction in each conductor is

$$254 \times 0.1 = 25.4 \text{ volts}$$

Phase displacement between line voltage drop and motor voltage may be neglected. Hence, an actual drop of 4.92 volts is well within the allowable limit. The mechanical strength is certainly sufficient.

Kelvin's Law should next be applied.

The cost of energy lost per conductor per year is I^2Rt times the cost per kilowatt-hour. Thus,

$$\text{Energy cost} = \frac{500^2 \times 0.00985}{1000} \times (12 \times 365) \times \$0.02 = \$215.71$$

The cost of 700,000 cir mil cable is \$527.80 per 1000 ft. For 500 ft, the cost is

$$\frac{527.80}{2} = \$263.90$$

This is the cost of one conductor. The fixed charges against such an investment with an unknown salvage value may be taken at 12%. The life expectancy is 20 years. The annual fixed charge is

$$\$263.90 \times 0.12 = \$31.67$$

Since the energy loss costs \$215.71 per year, a larger cable will be more economical. The cable loss will be inversely proportional to the area of the cross-section. The cost of cable within a small range may be assumed to be directly proportional to the area of the cross-section.

Let X = the new cross-section.

Then the new annual energy cost in terms of the new cross-section is

$$\$215.71 \times \frac{700{,}000}{X} = \frac{150{,}997{,}000}{X} \qquad \text{(a)}$$

Also, the annual fixed charge on the new cable is

$$\$31.67 \times \frac{X}{700{,}000} = 0.000045X \qquad \text{(b)}$$

By Kelvin's Law, cost (*a*) must equal charge (*b*). Therefore,

$$\frac{150{,}997{,}000}{X} = 0.000045X$$

or

$$X^2 = \frac{150{,}997{,}000}{0.000045}$$

and

$$X = 1{,}830{,}000 \text{ cir mils}$$

The nearest commercial size is 1,800,000 cir mils. This, then, would be the most economical size of conductor.

The assumption of linear variation of cost of conductor with conductor cross-section cannot be justified exactly, but the error from such assumption is negligibly small. Because of difficulty in handling very large conductors it is sometimes advisable to substitute for each conductor two or more conductors in parallel.[1]

PROBLEMS

1. A 10-hp motor whose full-load efficiency is 87.5 per cent cost \$160. What must be the selling price of a second motor having a full-load efficiency of 89 per cent, if full-load operation only is considered and the two motors are to be exactly comparable economically? The cost of energy is 2 cents per hp-hr. The time of operation is 3000 hours per year. Assume the life of either motor to be 25 years, and the interest rate to be 6 per cent. The duration of the service is 75 years. *Ans. \$308*

2. A 150-hp, 220-volt motor that costs \$1200 has a guaranteed efficiency of 91 per cent at full load. The cost of energy is 2 cents per hp-hr. The motor has a life of 25 years and is to be used 2500 hours per year. The duration of the service is 50 years. If, instead, a 440-volt motor with an efficiency of 90 per cent were bought at the same price, what would be the increase in the amount of the present worth of the service?

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[1] See *National Electric Code Handbook*, by Abbott. McGraw-Hill Book Co.

3. If in Problem 1 the hours of use per year were 500, what price must be paid for the motor of higher efficiency to cause the present worth of the service to be the same when performed by either motor?

4. A factory load of 500 kv-a of 460-volt, 3-phase motors is to be supplied by a three-wire line from a bank of transformers 300 ft away. What size of soft copper Type W insulated conductors must be used to satisfy the code requirements for outdoor overhead distribution? (See Appendix for current-carrying capacities of conductors.) *Ans. 630 amp, 600,000 cir mils*

5. If the available voltage at the transformers in Problem 4 is 460 volts, line-to-line, what will be the line-to-line voltage available at the motors when the line current is 630 amp? *Ans. 454 volts*

6. If the motors of Problem 5 are guaranteed to perform satisfactorily at a voltage 10 per cent less than their rated voltage and the current drawn by them when fully loaded at 414 volts is 680 amp, what would be the maximum length of line for which the 600,000 cir mil conductor would be acceptable? *Ans. 1368 ft*

7. A city has a bank of three single-phase transformers, rated 200 kv-a each, supplying a sewage-disposal plant. Each transformer has a core loss of 1100 watts and a full-load copper loss of 1300 watts. In accord with the trend in standardization of voltages, the power company from which energy is purchased plans to raise its distribution voltage from 2200 to 2460 volts. New transformers have a core loss of 800 watts each. What will be the yearly saving in energy cost? The cost of energy is 1 cent per kw-hr. Assume as an approximation that the total iron loss is proportional to the square of the maximum flux density. *Ans. $65.26*

CHAPTER 24

Electric Power Rates

Power Charges.—All the charges involved in the purchase of electric service are popularly called "power charges." However, of the total payment made to the power utility, only a small portion is actually a function of rate of change of energy. The monthly power bill actually includes the cost of fuel, the fixed charges on the equipment theoretically reserved for the customer, and other costs of operation, in addition to the profit for the utility stockholders and for the retirement of bonds.

When a prospective passenger purchases a railroad ticket, he pays a simple price which includes the cost of the foregoing items, but he does not know how his money is to be divided among them. Similarly, when a customer pays for electric service in his home, he seldom knows even approximately how his money is allocated among the many items which make up the cost of that service. The expense involved in analyzing the cost for every customer would be prohibitive. In the bill presented to the domestic user, energy only is mentioned. The other items of cost are said to be "hidden." However, the size of most industrial installations and the monthly energy bill involved justify the expense of periodic or continuous analyses of the various items of cost in the billings, and they are charged for individually.

Energy Charge.—It is obvious that there should be an item of charge which is proportional to the energy used. This charge is fixed by the cost of fuel and other items of operation which are functions of the kilowatt-hours of energy used. It is called the *energy charge* and is based on the monthly reading of the kilowatt-hour meter.

Demand Charge.—The *load factor* of all consumers is not the same. That is, one consumer may use 5000 kw-hr of energy per month with a nearly continuous load, which never exceeds 10 kw. The physical plant equipment necessary to provide the maximum demand service for this consumer will be only that required for 10 kw, and so the fixed charges to be passed on to him should be only those associated with 10 kw of equipment capacity. Another consumer may also use only 5000 kw-hr of energy per month, but may demand as much as 500 kw through only a few hours of use. In this case the physical plant equipment to be allotted to him is 500 kw, and so the fixed charges associated with his service would be 50 times as great as those for the other consumer. For this reason there is a "demand charge" based on the maximum demand for power made by the consumer.

The demand charge is sometimes based on the horsepower of connected load, which is considered to be the consumer's continuous threat of demand for service. Another method of determining the demand for billing purposes for moderately large consumers is to place a demand meter on the premises two or three times in a year, and to use the reading taken at a certain time as a basis for charge until the next reading is made, regardless of possible changes in operating conditions in the interim. In the case of very large consumers a demand meter is permanently installed and is read every month at the time the kilowatt-hour meter is read. The demand meter consists of a graphic wattmeter which produces a continuous record to be analyzed monthly, or of a kilowatt-hour meter with a high-speed register which is automatically reset to zero at the end of every time interval of 10, 15, 30, or 60 minutes, according to the choice of the utility. An indicator, which can be reset to zero by the meter reader only, shows the maximum demand since the last resetting.

Power-Factor Charges.—A consumer having a maximum demand of 100 kw with 100% power factor would pay the same demand charge as if he had a power factor of say 50%. But the kilovolt-ampere capacity in generating and distributing equipment required by a 100-kw load with 50% power factor is approximately two times as great as that required for 100 kw with 100% power factor. Therefore, in addition to the demand charge, there must be a penalty charge against low power factor to distribute equitably the fixed charges of the capital investment.

The actual methods used in applying the penalty for low power factor are quite diverse, and may best be studied through reference to the actual examples of power contracts shown in the latter part of this chapter. Inasmuch as most industrial electrical power apparatus draws from the power line some lagging reactive volt-amperes, the electric power utilities welcome loads which take leading reactive volt-amperes in any amount up to total compensation. It is sometimes possible to arrange a power contract whereby the consumer is given a premium for maintaining a leading power factor within certain specified limits, in order to compensate for some of the lagging reactive kv-a of other nearby consumers. The economics of power-factor correction will be further dealt with in Chapter 25.

The demand meter is read directly in kilowatts. In some instances the demand is determined in kilovolt-amperes, instead of kilowatts, in the belief that the former is a more accurate measure of the investment in physical equipment than is the latter.

Minimum Charges.—Although a consumer may use no energy within a billing period, the threat to do so and the overhead costs of keeping records on the service connection demand that a minimum monthly charge be made. Some of the forms of the minimum charge may be seen in the examples of power contracts shown.

Restricted Service Rates.—Because of the high peak demand on the systems of some power utilities and a substantial margin of unused capacity provided for emergency service, a restricted service is sometimes offered. The use of such service is permitted at certain hours only, when the generating-station load is normally low. An attractive rate is possible for such service, because of the curtailed investment required. Ideal loads for restricted service are water pumping, water heating, and other uses of electricity which are not necessarily synchronized with peak load operations.

SAMPLE RATE STRUCTURES

Schedule A

Alternating and Direct Electric Current Power Service. The Service furnished under this schedule is Alternating Electric Current, Single or Three Phase, 60 cycles, 120–240 or 220–440 volts, and Direct Electric Current at 120–240 volts.

A minimum monthly charge of $1.25 per kw or fraction thereof, of maximum demand, plus the following kw-hr charges for energy consumed.

First 2,500 kw-hr per month at 2½ cents per kw-hr.
Next 2,500 kw-hr per month at 2¼ cents per kw-hr.
Next 2,500 kw-hr per month at 2 cents per kw-hr.
Next 2,500 kw-hr per month at 1¾ cents per kw-hr.
Next 90,000 kw-hr per month at 1 cent per kw-hr.
Next 100,000 kw-hr per month at .9 cent per kw-hr.

Schedule B

Character of Service:

115–230–460 volts alternating current.

Lighting will be included and will be billed at the power rate where the normal monthly power bill amounts to not less than $125.00 per month, provided the lighting demand is not more than 25% of the power demand.

115–230 volts direct current.

All direct current power service shall be supplied under this schedule and the lighting will be billed at the power rate under the following conditions:

A—where the normal monthly demand is 150 kilowatts or more.

B—where the normal monthly power bill is not less than $125.00 and the lighting demand is not more than 25% of the power demand.

Minimum charge: Two dollars and fifty cents ($2.50) net per month on all installations up to and including 5 hp of connected load. Above 5 hp of connected load, fifty cents ($0.50) net per horsepower per month.

Demand: In determining the maximum demand of an installation under power schedule B, the following rules shall apply:

If an installation is less than 5 hp, its demand will be arbitrarily fixed at 5 hp. In installations of 5 hp to 25 hp, inclusive, the connected load in horsepower will be multiplied by 0.6 and the product will be considered the maximum demand in kilowatts. In installations of 25 hp to 50 hp, inclusive, the connected load in horsepower will be multiplied by 0.55 and the product will be considered the maximum demand in kilowatts. In installations of 51 hp to 200 hp, inclusive, the connected load in horsepower will be multiplied by 0.5 and the product will be considered the maximum demand in kilowatts. In installations of 201 hp and above, the connected load in horsepower will be multiplied by 0.45 and the product will be considered the maximum demand in kilowatts.

RATES

Hours use of Demand*	Rate	Hours use of Demand*	Rate	Hours use of Demand*	Rate
1 to 50	$0.0660	156 to 160	$0.0345	266 to 270	$0.0272
51 to 55	.0610	161 to 165	.0340	271 to 275	.0270
56 to 60	.0576	166 to 170	.0335	276 to 280	.0269
61 to 65	.0544	171 to 175	.0330	281 to 285	.0268
66 to 70	.0512	176 to 180	.0325	286 to 290	.0267
71 to 75	.0480	181 to 185	.0320	291 to 295	.0266
76 to 80	.0465	186 to 190	.0315	296 to 300	.0265
81 to 85	.0450	191 to 195	.0310	301 to 310	.0264
86 to 90	.0435	196 to 200	.0305	311 to 320	.0263
91 to 95	.0420	201 to 205	.0302	321 to 330	.0262
96 to 100	.0415	206 to 210	.0299	331 to 340	.0261
101 to 105	.0400	211 to 215	.0296	341 to 350	.0260
106 to 110	.0395	216 to 220	.0293	351 to 360	.0259
111 to 115	.0390	221 to 225	.0290	361 to 370	.0258
116 to 120	.0385	226 to 230	.0288	371 to 380	.0257
121 to 125	.0380	231 to 235	.0286	381 to 390	.0256
126 to 130	.0375	236 to 240	.0284	391 to 400	.0255
131 to 135	.0370	241 to 245	.0282	401 to 425	.0254
136 to 140	.0365	246 to 250	.0280	426 to 450	.0253
141 to 145	.0360	251 to 255	.0278	451 to 475	.0252
146 to 150	.0355	256 to 260	.0276	476 to 500	.0251
151 to 155	.0350	261 to 265	.0274	Above 500	.0250

* See page 290.

The company reserves the right to make tests at any time on the premises of any customer, and may, at its option, fix the demand by test, and the demand so fixed shall remain in force until the company makes other tests to determine the maximum demand. In fixing the demand of any installation, if there is a considerable portion of the load which is not used during the company's load peak, and if the customer will agree not to use this portion of the load during such hours as the company may specify, then the company may take this feature into consideration in fixing the maximum demand.

Discounts: Quantity discounts in accordance with the following schedule are applicable to Power Rate B and are determined by the gross bill, viz:

$100.01 to	$125.00	4%
125.01 to	150.00	6%
150.01 to	175.00	8%
175.01 to	200.00	10%
200.01 to	225.00	11%
225.01 to	250.00	12%
250.01 to	275.00	13%
275.01 to	300.00	14%
300.01 to	325.00	15%
325.01 to	350.00	16%
350.01 to	375.00	17%
375.01 to	400.00	18%
400.01 to	450.00	19%
450.01 to	500.00	20%
500.01 to	550.00	22%
550.01 to	600.00	23%
600.01 to	650.00	24%
650.01 to	700.00	25%
700.01 to	750.00	26%
750.01 to	800.00	27%
800.01 to	850.00	28%
850.01 to	900.00	29%
900.01 to	950.00	30%
950.01 to	1000.00	32%
1000.01 to	1050.00	33%
1050.01 to	1100.00	34%
1100.01 to	1200.00	35%
1200.01 to	1300.00	35½%
1300.01 to	1400.00	36%
1400.01 to	1500.00	36½%
1500.01 to	1750.00	37%
1750.01 to	2000.00	37½%
2000.01 to	2250.00	38%
2250.01 to	2500.00	38½%
2500.01 to	2750.00	39%
2750.01 to	3000.00	39½%
3000.01 to	3250.00	40%
3250.01 to	3500.00	40½%
3500.01 to	3750.00	41%
3750.01 to	4000.00	41½%
4000.01 to	4250.00	42%
4250.01 to	4500.00	42½%
4500.01 to	4750.00	43%
4750.01 to	5000.00	43½%
5000.01 and above		44%

Schedule C

Rate: The consumer's monthly bill shall be the sum of the demand charge and the energy charge.

Demand Charge:

First 10 kw of billing demand at $30.00 per kw per year payable at the rate of $2.50 per month.
Next 30 kw of billing demand at $25.00 per kw per year payable at the rate of $2.10 per month.
Next 160 kw of billing demand at $21.60 per kw per year payable at the rate of $1.80 per month.
Next 200 kw of billing demand at $19.20 per kw per year payable at the rate of $1.60 per month.
All over 400 kw of billing demand at $18.00 per kw per year payable at the rate of $1.50 per month.

Energy Charge: Energy used each month will be billed at the following rates:

All kw-hrs in first quantity block at—38 mills per kw-hr.
All kw-hrs in second quantity block at—19 mills per kw-hr.
All kw-hrs in third quantity block at—16 mills per kw-hr.
All kw-hrs in excess of the sum of the three quantity blocks at 13 mills per kw-hr.

Energy Blocks

First Quantity Block: The kilowatt-hours in the first quantity block shall be as follows:

For corrected demands of:

From 5 kw to 25 kw—350 kw-hr.

From 25 kw to 100 kw—350 kw-hr plus 6 kw-hr for each kw of demand in excess of 25 kw.

From 100 kw to 400 kw—800 kw-hr plus 4 kw-hr for each kw of demand in excess of 100 kw.

From 400 kw to 1600 kw—2000 kw-hr plus 3 kw-hr for each kw of demand in excess of 400 kw.

From 1600 kw to 5600 kw—5600 kw-hr plus $2\frac{1}{2}$ kw-hr for each kw of demand in excess of 1600 kw.

Over 5600 kw—15,600 kw-hr plus 2 kw-hr for each kw of demand in excess of 5600 kw.

Second Quantity Block: The kilowatt-hours in the second quantity block shall be four times the kilowatt-hours in the first quantity block.

Third Quantity Block: The kilowatt-hours in the third quantity block shall be four times the kilowatt-hours in the second quantity block.

Measured Demand: The measured demand shall be the average kilowatt capacity used by the consumer during the fifteen-minute period of maximum consumption during the month.

Power Factor: The company reserves the right to test or meter the power factor of the consumer's load and, if greater than 85% lagging, then the measured monthly demand shall be decreased for billing in the ratio that 85% bears to the actual power factor (in per cent) as determined.

The correction for all leading power factors shall be the same as for 100% power factor.

If the power factor of the consumer's load is less than 75% lagging, then the measured monthly demand shall be increased for billing in the ratio that 75% bears to the actual power factor (in per cent) as determined.

Corrected Demand: The measured demand in kilowatts corrected for power factor shall be the corrected demand. In no event, however, shall the corrected demand be less than 5 kw.

Billing Demand: The billing demand shall be the corrected demand in kilowatts of the current month, but not less than 50% of the highest billing demand of the preceding months.

Minimum Charge: The minimum bill in any month shall be a sum equivalent to the charge for 50% of the highest billing demand of the preceding eleven months.

Character of Service: This rate is available only to consumers having demands of 5 kilowatts or over.

Example 24–1.—A consumer uses 6400 kw-hr of energy in a month. The maximum demand as measured is 50.6 kw. Calculate the power bill under Schedule A.

Solution.—The total energy charge is found as follows:

$$\begin{aligned} 2500 \times 0.025 &= \$\ 62.50 \\ 2500 \times 0.0225 &= \ \ 56.25 \\ 1400 \times 0.020 &= \ \ 28.00 \\ \hline \text{Total} &= \$146.75 \end{aligned}$$

Demand charge = 50.6 kw at $1.25 = $63.75
Total bill = $146.75 + $63.75 = $210.50
If the bill is paid when due, the charge is

$$\$210.50 \times 0.95 = \$199.97$$

Hours Use of Demand.—The product of the kilowatts of demand and the hours of use of that demand is a quantity of kilowatt-hours known as a block of energy.

Example 14–2.—Solve Example 14–1, using the rates of Schedule B and taking the demand as 50.6 hp.

Solution.—The number of hours of use of maximum demand is

$$6400 \div 50.6 = 126.5 \text{ hr}$$

Under Schedule B the rate is $0.0375 per kw-hr, and the charge is

$$6400 \times 0.0375 = \$240.00$$

With a gross bill of $250.00 the consumer is entitled to a discount of 12%. Hence, the net bill is

$$\$240.00 \times 0.88 = \$211.20$$

PROBLEMS

1. A consumer has an average monthly energy consumption of 140,000 kw-hr. The maximum demand is 670 kw. The average power factor is 0.60. The plant works 576 hours per month. Calculate the average monthly power bill under Schedule C. *Ans. $3493.56*

2. Calculate the monthly bill of Problem 1 under Schedule A.

3. Calculate the monthly bill of Problem 1 under Schedule B, if the connected load is 1000 hp. *Ans. $2172.38*

CHAPTER 25

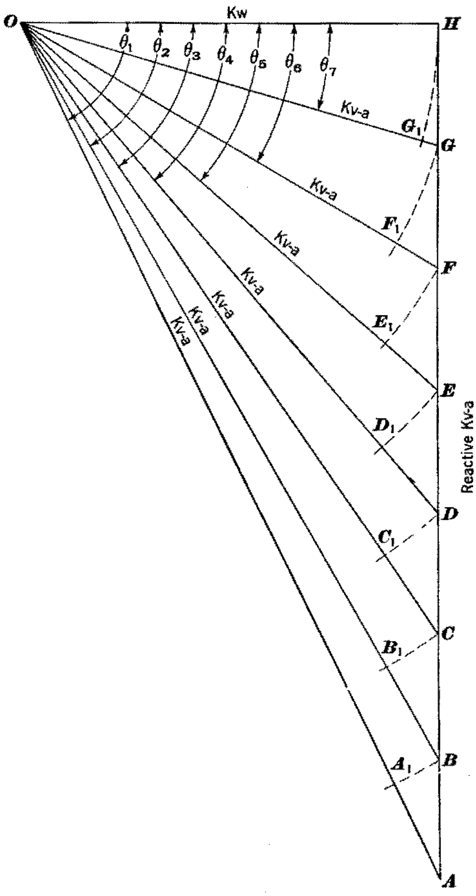
Power-Factor Correction

Importance of Power Factor.—The number of volt-amperes necessary to supply power to a consumer is proportional to the power supplied and is inversely proportional to the power factor at which it is supplied. Therefore, at power factors other than 1.0, the number of volt-amperes necessary to supply a given active power is greater than the active power itself. Inasmuch as the capital investment, the fixed charges of which the demand charge is intended to defray, is a function of volt-amperes, a demand charge based on watts alone is insufficient to allocate properly the fixed charges.

Economic Limits of Power-Factor Correction.—If no cost were involved in power-factor correction, the power factors of all industrial loads would be regularly corrected to 1.0. In Schedule C of page 288 the power company imposes a penalty on power factors less than 0.75. In some contracts a penalty is imposed for power factors less than 0.85. The theoretical criterion that determines the choice of power factor below which a penalty will be imposed is the cost of correction relative to the value of released apparatus. Obviously, if it would cost $1000 in installed corrective apparatus to release $1000 worth of generating station and distribution apparatus, the venture would yield a mere trading of dollars and would be economically unsound where such release is the sole consideration. However, low power factor causes poor voltage regulation in many cases, and the correction of such regulation may justify a power-factor correction in excess of that dictated by other considerations. The I^2R losses in the generator and the transmission apparatus caused by the increase of current made necessary by low power factor must also be charged to this evil of low power factor.

Power-factor correction is strictly a problem of the power company, but may affect the consumer through a penalty, if one is imposed by the power company in its contract with the consumer. When the consumer must pay a penalty for low power factor, power-factor correction becomes one of his problems too.

In Fig. 25–1 is shown a kv-a diagram of a load, the power of which is OH and the kv-a of which is OA. Let it be assumed that seven equivalent static condensers, of no loss and of AB kv-a each, are available to be connected one by one in parallel with the load. By connecting one condenser the angle of lag is decreased from θ_1 to θ_2, and the kv-a drawn from the line is decreased by the amount AA_1. By the installation of AB kv-a in a static condenser, AA_1 kv-a in generating station and transmitting equipment is released for other service. As successive condensers are connected, the line kv-a is reduced successively, but not proportionately. When the last condenser, which costs as much as any other, is added to reduce the line reactive kv-a to zero, it releases only GG_1 kv-a in generating and distributing equipment. Thus, the effectiveness per unit of corrective kv-a becomes less and less as the power factor to be improved approaches 1.0. With the present cost of corrective apparatus, it is seldom sound economy to correct the power factor to any value greater than 0.85.



This diagram demonstrates the economic effectiveness of devices for power-factor correction when used in circuits with low power factor.

Fig. 25–1. Graphic Representation of Power-Factor Correction by Static Condenser Units

Power-Factor Correction by Resistive Load.—A resistive load, such as a furnace, added to a system with low power factor will effect some improvement. Thus, the power factor of the load in Fig. 25–2 (*a*) may be improved to that of (*b*) by the addition of the load kw_2. As a result, the power factor $\cos \theta_1$ of (*a*) is improved to $\cos \theta_2$ of (*b*). It is seldom, however, that the need for power-factor correction will occur simultaneously with the need for addition of apparatus with a high power factor.

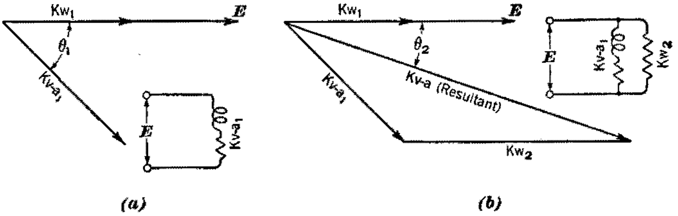


Fig. 25–2. Power-Factor Correction by Addition of Active Power Load

Power-Factor Correction by Static Condensers.—The current taken by a static condenser is practically 90° ahead of the voltage applied to it. Therefore, the condenser kv-a may be treated as wholely reactive and without loss component. Thus, if the power factor of the load of Fig. 25–2 (*a*) were corrected to $\cos \theta_2$ by a static condenser, the vector diagram would be as shown in Fig. 25–3.

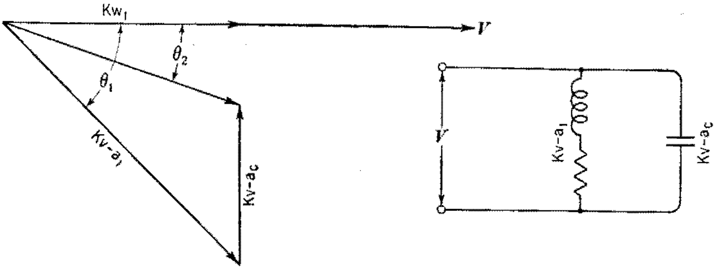


Fig. 25–3. Power-Factor Correction (One Phase) by Static Condenser

When the power factor of a three-phase system is to be corrected by static condensers, three condenser units are usually connected in delta, as near as practicable to the source of the low power factor. See Fig. 25–4.

Synchronous condensers are almost never used for power-factor correction by manufacturing industries.

Power-Factor Correction by Synchronous Motors.—As explained in Chapter 19, the current taken by a synchronous motor may be made to lead the applied voltage by over-exciting the field. Thus, a synchronous motor may be made to provide a load having a component that simulates a condenser and provides some capacitive correction to any associated load of lagging power factor. Fig. 25–5 shows the power factor of the load of Fig. 25–2 (*a*) corrected to cos θ_2 by a synchronous motor.

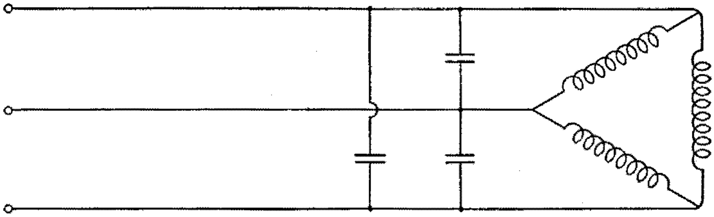


Fig. 25–4. Power-Factor Correction by Condensers (Three Phase)

Most synchronous motors are provided with a field winding of sufficient size to permit excitation to a power factor of 0.8 leading, when carrying the full rated shaft load. Thus, if a motor is rated at 100 hp with an efficiency of 92% at full load, the power input at 100% power factor would be

$$\frac{100 \times 746}{0.92 \times 1000} = 81 \text{ kw}$$

When the motor is excited to a leading power factor of 0.8, the total kv-a input would be

$$81 \div 0.8 = 101 \text{ kv-a}$$

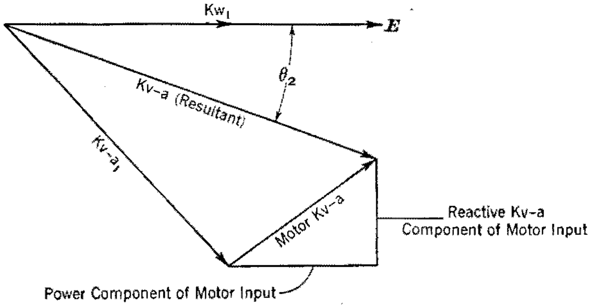


Fig. 25–5. Power-Factor Correction by Synchronous Motor

and the reactive kv-a would be

$$\sqrt{101^2 - 81^2} = 60 \text{ kv-a}$$

Power-factor correction obtained by over-excitation of synchronous motors is very low in cost and is highly attractive.

Example 25–1.—A consumer has an average monthly energy consumption of 140,000 kw-hr. The average maximum demand is 670 kw. The average power factor is 0.60.

The plant works 576 hours per month. Calculate the kv-a of static condensers required to avoid a power-factor penalty under Schedule C of Chapter 24.

Solution.—The average hourly values are:

$$140{,}000 \div 576 = 243 \text{ kw}$$

$$243 \div 0.60 = 405 \text{ kv-a}$$

$$\sqrt{405^2 - 243^2} = 324 \text{ reactive kv-a}$$

To avoid penalty the power factor must be raised to 0.75. The allowable kv-a values are:

$$243 \div 0.75 = 324 \text{ kv-a}$$

$$\sqrt{324^2 - 243^2} = 214.3 \text{ reactive kv-a}$$

The number of kv-a in static condensers required to avoid a power-factor penalty is

$$324 - 214.3 = 109.7 \text{ say } 110 \text{ kv-a}$$

The vector diagram is shown in Fig. 25-6.

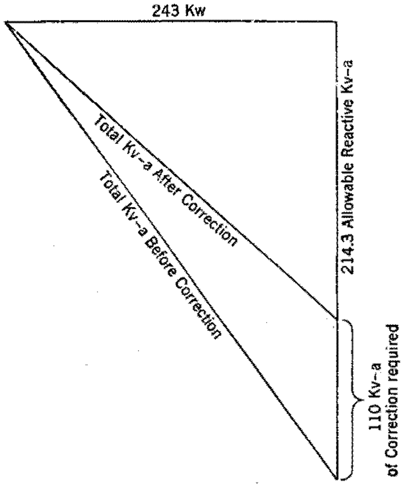


Fig. 25-6. Vector Diagram of Example 25-1

Example 25-2.—The static condenser of the preceding example would cost $3000 installed. Under Schedule C of Chapter 24, what would be the saving in the power bill in the first year?

Solution.—Inasmuch as the power factor is to be corrected to the minimum of 0.75 allowable without penalty, the billing demand will be equal to the measured demand of 670 kw. The charges are as follows:

Demand Charges:

$$\begin{array}{rr} 10 \times \$2.50 = \$ & 25.00 \\ 30 \times \ \ 2.10 = & 63.00 \\ 160 \times \ \ 1.80 = & 288.00 \\ 200 \times \ \ 1.60 = & 320.00 \\ (670 - 400) \times \ \ 1.50 = & 405.00 \\ \hline \text{Total} = & \$1101.00 \end{array}$$

Energy Charges:

1st quantity block $= 2000 + [3 \times (670 - 400)] = 2810$ kw-hr
2nd quantity block $= 4 \times 2810 = 11{,}240$ kw-hr
3rd quantity block $= 4 \times 11{,}240 = 44{,}960$ kw-hr
4th quantity block $= 80{,}990$ kw-hr

1st quantity block cost $= 2810 \times 0.038 = \$\ 106.78$
2nd quantity block cost $= 11{,}240 \times 0.019 = 213.56$
3rd quantity block cost $= 44{,}960 \times 0.016 = 718.86$
4th quantity block cost $= 80{,}990 \times 0.013 = 1052.87$

Total energy charge $= \$2092.07$

Demand plus energy charge $= \$1101.00 + \$2092.07$
$= \$3193.07$ per month

As calculated in Problem 1 on page 290, the monthly charge without power-factor correction would be $3493.56.

The amount saved per month because of power-factor correction is

$$\$3493.56 - \$3193.07 = \$300.49$$

and the amount saved in the first year is

$$\$300.49 \times 12 = \$3605.88$$

This is an excellent return in the first year on an investment of $3000.

PROBLEMS

1. In the plant of Example 25-1, a 300-hp synchronous motor is installed to run at full load 576 hours per month with an efficiency of 92% and at a power factor of 0.8 leading. Calculate the new demand charge.

2. The motor of Problem 1 cost $1850.00. A similar motor capable of only 1.0 power factor could have been bought for $1775.00. What price per leading reactive kv-a at full load and 0.80 power factor was paid to install the motor with a power factor of 0.8 in preference to the motor with a power factor of 1.0? *Ans. $0.41*

3. A factory load consists of 400 kw of load with a power factor of 71% lagging. Because of the power-factor penalty clause provided in the power and energy contract when the power factor is less than 85% lagging, it is desirable to improve the over-all power factor just enough to avoid the penalty. How many kv-a must be provided by static condensers to effect the required correction? *Ans. 150 kv-a*

4. If in Problem 3 the correction were accomplished by adding synchronous motors at 0.80 power factor, how many horsepower of motors running at full load and 88% efficiency must be added to the existing load to avoid the penalty imposed for low power factor? *Ans. 120 hp. Probably a 125-hp motor*

CHAPTER 26

Illumination

Factors Affecting Vision.—The ability of the human eye to see depends on the size of the object to be seen, the brightness to which it is illuminated, the time of its exposure to the eye, and the brightness of the background against which it is seen. The brightness of an object depends on the amount of light falling upon its surface. It also depends on the ability of that surface to reflect the light. An object upon which light falls is said to be illuminated.

Intensity of Illumination; The Foot-Candle.—If a hollow sphere with an inside radius of 1 foot be illuminated by the flame of one standard international candle placed at the center of the sphere, the intensity of the illumination on the inner surface is said to be 1 *foot-candle.* The foot-candle is the standard unit of illumination.

Light Flux.—The light flux falling upon 1 square foot of the interior sphere surface illuminated as just described is called a *lumen.* The area of a sphere of 1-foot radius is 4π square feet. Therefore, if a hollow sphere 1 foot in internal radius be illuminated from its center by a point light source of 1 standard candle, the total light flux emitted will be 4π lumens. The ability of a lamp to produce light is measured, not by its ability to illuminate an object in some particular direction from it, but by its total output in lumens. Hence, lamps are usually rated in lumens rather than in candle-power.

Out-of-doors on a very bright day the illumination may be as great as 10,000 foot-candles, while on a dark day it is sometimes as low as 200 foot-candles. The eye readily adapts itself to this wide range of intensity by contraction or dilation of the pupil to admit less or more light. In buildings without skylights the illumination seldom exceeds 100 foot-candles. Near windows the illumination may be as great as 50 to 100 foot-candles. This intensity is satisfactory for fine assembly work, inspection of small parts, etc. Drafting boards should have about 50 foot-candles. Office work and general reading requires 20 to 50 foot-candles. Illumination intensities of less than 10 foot-candles are unsatisfactory for any except general lighting such as used in hallways, auditoriums and large manufacturing assemblies. Local high-intensity lighting should be accompanied by some degree of general illumination.

Scientific research has shown that, while intensities somewhat less than those just suggested may be used with good results, the fatigue of workmen and their tendencies toward accidents increase rapidly with decrease of intensities below those recommended.

Light Sources.—The general popularity of the common incandescent gas-filled lamp is due chiefly to its pleasing light and relatively low cost. Lamps, such as the mercury-vapor, sodium-vapor, and neon-tube lamps, in which a luminous vapor or gas is used as the light source, give a light of marked characteristic color which is undesirable for most purposes. The mercury-vapor lamp gives off no red rays, and red objects appear black under its illumination. The arc lamp in various forms is used for production of ultra-violet rays, photography, and projection purposes.

(*Courtesy Sylvania Electric Products, Inc.*)

Fig. 26–1. Fluorescent Lamp

In an attempt to produce light approximating daylight in quality, some lighting units are provided with both incandescent lamps and mercury-vapor lamps. This is called *mixed* lighting. Mixed-lighting units cost much more than incandescent units with comparable lumen capacity, and cannot be economically justified for most purposes, except where the color of the light is highly important. Next to the incandescent lamp, the fluorescent lamp is the most popular for general illumination.

Fluorescent Lamps.—The fluorescent lamp, shown in Fig. 26–1, consists of a tubular bulb with an electrode sealed in each end. A small amount of mercury sealed in the bulb produces a vapor, which is a conductor of electricity. When a sufficient voltage is applied to the electrodes, the column of vapor glows weakly. However, the inside of the tube is coated with a phosphor powder which, when excited by the mercury-vapor glow, responds by glowing brightly and is said to be "luminescent." The various dominant colors available from these lamps are obtained by proper choice of phosphor used in the coating, and the shades are affected by the vapor pressure.

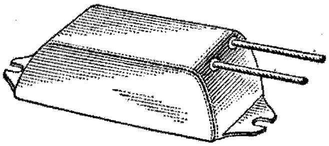
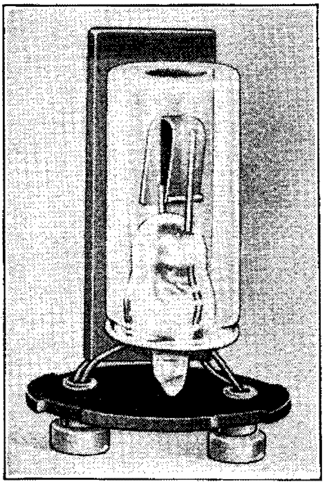


Fig. 26–2. Choke Coil for Fluorescent Lamp

Fluorescent lamps, in common with all other electric-discharge sources of light, require auxiliary control devices. One of these devices, which is shown in Fig. 26–2, is usually an iron-core choke coil or impedance coil, called a ballast, in series with the lamp. When the lamp is turned on, the choke coil permits a voltage which will start the discharge; but, by its impedance, it reduces the voltage across the lamp after the discharge current begins to flow. Another of these control devices, shown in Fig. 26–3, is a starting switch which momentarily closes the circuit through a heater coil in one end of the tube and then opens it after the glow has started. The low power factor caused by the ballast reactor is improved by the addition of a shunt condenser or by using the lamps in pairs, one lamp of each pair having a condenser element in its ballast.



(Courtesy Sylvania Electric Products, Inc.)

Fig. 26–3. Starter for Fluorescent Lamp

A tube with a heater in only one end will glow during only one half of every cycle, and would produce stroboscopic effects that are undesirable for most uses. For this reason, most fluorescent tubes are provided with a heater filament in each end, so that they glow during both halves of every cycle. Fig. 26–4 shows such a tube with its ballast and starter. Fig. 26–5 shows a circuit diagram similar to that of Fig. 26–4 but including also a condenser which has been added to improve the power factor. Fig. 26–6 shows a circuit in which two lamps are used with a ballast having a high power factor.

Fluorescent lamps have a spectrum which approaches daylight more closely than does that of incandescent lamps, and fluorescent lamps are sometimes preferred for that reason. In general, they lose their usefulness because of decrease in light output caused by darkening before they fail to operate.

Lamp Life.—Incandescent lamps, except those used in series street-lighting circuits, are rated in watts at a certain rated voltage. They are also designed for a certain length of life, such as 500, 750, 1000, 2000, and 3000 hours. The life of a lamp is dependent largely on the operating temperature of its filament. Its efficiency as a light producer also depends on its temperature. An economic balance between maximum life and maximum efficiency is attained by the manufacturer, so that the cost of the lumen-hours from the lamp will be a minimum. An incandescent lamp

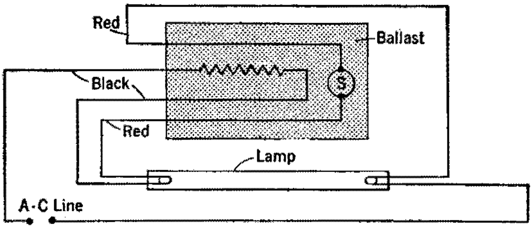


Fig. 26–4. Fluorescent-Lamp Circuit With Ballast and Starter Switch

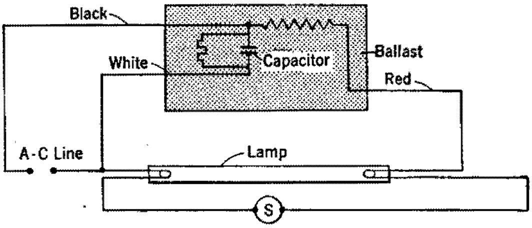


Fig. 26–5. Circuit for Single Fluorescent Lamp With Ballast Having High Power Factor

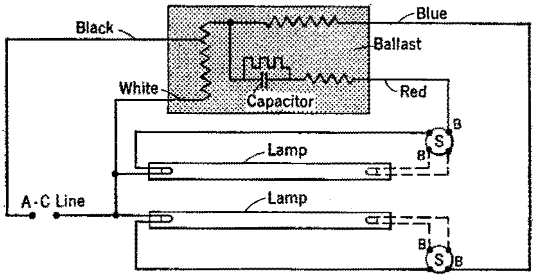


Fig. 26–6. Circuit for Two Fluorescent Lamps With Ballast Having High Power Factor

operated at low filament temperature will have a long life, but its lumen output per watt of power input will be low. An incandescent lamp operated at very high filament temperature will have a high lumen output per watt of input, but its life will be short. The rated voltage of a lamp is that voltage at which the best economic balance is obtained between efficiency and life. However, the inconvenience and cost of replacing a highly efficient 500-hour lamp frequently in an inaccessible location may be so great that a less efficient 2000-hour or 3000-hour lamp would be preferred because of less frequent replacements.

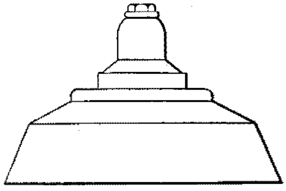
When the filament of an incandescent lamp is lighted, particles of filament are driven off. These particles collect on the inside of the bulb, and cause darkening and hence a reduction in efficiency of light production. This darkening may so reduce the efficiency that it would be more economical to replace the lamp, even though the filament has not burned out. In practice, however, this is seldom done. The voltage at a lamp should not depart more than 3 per cent from the rated voltage.

Frequent starting of fluorescent lamps may take more life from them than long hours of burning because electrode evaporation is much more violent in the starting period.

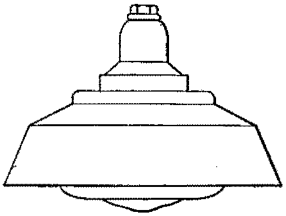
Comparison of Efficiency.—In an incandescent gas-filled lamp, 60 to 75 per cent of the energy is radiated as heat. In a fluorescent lamp, only about 25 to 35 per cent of the energy is radiated as heat. Because the efficiency of light production of fluorescent lamps is about double that of filament lamps, and because the per cent of radiated heat is only about half as much, the sensation of heat from a fluorescent lamp is only about one-fourth that of a filament lamp for the same light output. Fluorescent lamps may be operated economically on any voltage within 6 per cent above or below rated voltage, but their efficiency decreases at higher voltages. With present types of auxiliary equipments, they are not suited to flashing or dimming applications. They operate most efficiently in temperatures between 70 and 80 F, and are likely to develop starting difficulties in refrigerated show cases or in outdoor applications in winter.

In general it may be stated that, if the color of light is not important, filament lamps are more satisfactory than fluorescent lamps where the illumination needed is low or moderate and the use is intermittent. The fluorescent lamp is more satisfactory where the intensity must be high and the lamp is burned many hours with few starts.

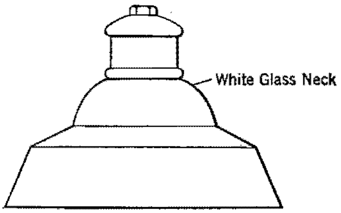
Light Control.—The eye must be protected from light that comes directly from a high-intensity source. This is done by redirecting some or all of the light upon the surface to be illuminated, or by enclosing the lamp in translucent glassware. Redirection is accomplished by reflectors, which not only salvage the light directed in non-useful angles but also diffuse that light to produce a soft illumination. Such light control produces *direct* lighting with a maximum efficiency of use of the light flux. Direct-lighting units must be so arranged that the eye in the usual working positions will receive no glare, either directly from the lamp itself or from reflecting polished surfaces. Light that is directed upward and reflected from a light-colored ceiling produces a more pleasing effect than does direct lighting, because of more effective diffusion. This type of lighting is said to be *indirect.* Indirect lighting, while most pleasing of all, is also least effective in its use of light flux. *Semi-indirect* lighting is accomplished with a luminaire which directs most of its light toward the ceiling but permits some to pass directly into the room.



Dome without Diffuser



Dome with Diffusing Globe



Dome with Diffusing White Glass Neck

Fig. 26–7. RLM Dome Reflectors

Selection of Industrial Lighting Units. General overhead industrial lighting is usually accomplished by the use of incandescent lamps with the RLM (Reflector and Lamp Manufacturers') Standard Dome Reflectors shown in Fig. 26–7, or by multiple units of fluorescent lamps. The selection and placing of incandescent units may be accomplished with the aid of Tables 26–1 and 26–2, as follows:

Step 1. Determine the mounting height as the highest point above the floor at which the reflectors can be suspended without interference with belting, shafting, etc.

Step 2. Refer to Table 26–1 and, with the mounting height of step 1, ascertain the proper spacing distance. If spacing so obtained does not fit well in the building bays, a smaller spacing with a lower mounting height should be used.

Step 3. Select from Table 26–2 the foot-candle illumination required by the class of work, and find the nearest corresponding value in Table 26–1 under the selected spacing distance.

Step 4. Having found the proper foot-candle intensity, read, from the first column in Table 26–1 on the left, the lamp size required. Reflectors must be bought to fit the lamp size.

TABLE 26-1

ARRANGEMENT OF INCANDESCENT UNITS FOR LOW-LEVEL GENERAL LIGHTING

Outlet spacing in feet		8½×8½	10×10	11½×11½	13×13	15×15	17×17	18×18	19×19	20½×20½	23×23	25×25
Area lighted per outlet (sq ft)		75	100	135	170	225	290	325	360	420	520	625
Mounting height (feet)		8½	9½	10½	11½	12½	13½	14½	15½	16½	17½	18½
Lamp Size (Watts)		**Average Foot-Candles Illumination on Plane of Work**										
100	*a*	6–7	3.5–4	3.0–3.5	2.5–3	2.0–2.5						
150		10–13	5–8	5–6	4–5	3.0–3.5	2.0–2.5					
200		13–10	10–12	7–9	6–7	4.5–5.5	3.5–4.0	3.0–3.5	2.5–3.0	2.0–2.5	1.5–3.0	
300				12–15	10–13	7–10	6–7	5–6	4–5	3.5–4.0	3.5–2.5	1.4–2.5
500						13–17	10–13	9–12	7–10	7–9	5.0–7.0	3.0–5.0
750								14–17	11–14	11–13	8.0–10.0	4.8–8.0
1000										14.5–17	11.0–15.5	6.8–11.0
100	*b*	5–6	3–4	2.5–3.0	2.0–2.5							
150		8–11	5–7	4–5	3–4	2.8–3.5	2.0–3.5					
200		11–15	8–11	6–8	5–6	3.5–4.0	3.0–3.5	2.5–3.0	2.0–2.5	1.5–2.0		
300				10–13	9–12	5.6–8.0	5–6	4–5	3.5–4.0	2.5–3.5	2.5–1.5	1.0–1.5
500					16–21	11–15	9–12	8–10	7–9	4.0–7.0	4.0–7.0	1.5–2.0

a—RLM Reflector without diffuser

b—RLM Reflector with diffuser

TABLE 26–2*

FOOT-CANDLE LEVELS RECOMMENDED FOR VARIOUS LIGHTING FIELDS

Offices Schools	Conference Reception Auditorium Cafeterias	Classrooms Casual Desk Work Filing Libraries	Bookkeeping Accounting Stenographic Sight Saving Critical Desk Work	Drafting Business Machines	Color Work	
Stores	Stockrooms	Circulation Areas	General Merchandising	Show Windows Open Counter Display Showcases Wall Cases	Show Windows Feature Displays	Show Windows Spotlighting Feature Displays
Industrial	Loading Packing Shipping Washrooms	Rough Work Pressing Shearing General Processing Woodworking Foundry Work	General Fabrication and Assembly Sewing Spray Painting Typesetting Automatic Machining	Proofreading Machining Inspection Fine Assembly	Color Inspection Fine Inspection Extra Fine Assembly	Color Identification
	10-20 **10**	15-35 **20**	35-70 **50**	70-150 **100**	150-300 **200**	300-750 **500**

General Lighting →

← Supplementary Lighting

*Courtesy General Electric Co.

TABLE 26-3*

CPID 260 TWO-LAMP FIXTURE FOR 100-WATT LAMPS

Mounting Height—Ft		8-10					10-13					14-17			
Area Per Unit—Sq Ft		60	70	80	90	100	60	70	80	90	100	60	70	80	90
Room		FOOT-CANDLES													
Width—Ft	Length—Ft														
8-12	8-14	53-47	45-41	39-35	35-31	31-28	49-40	42-34	37-30	33-27	30-24	41-40	35-34	31-30	27-27
	14-20	57-47	49-41	42-35	37-31	34-28	49-40	42-34	37-30	33-27	30-24	41-40	35-34	31-30	27-27
	20-42	58-53	50-45	43-39	38-35	34-31	52-40	45-34	39-30	35-27	32-24	41-40	35-34	31-30	27-27
	42-UP	62-55	53-47	46-41	41-36	37-33	57-47	49-40	42-35	38-32	34-29	47-40	40-34	35-30	32-27
12-20	14-20	62-53	53-45	46-39	41-35	37-31	57-40	49-34	42-30	38-27	34-24	41-40	35-34	31-30	27-27
	20-42	62-53	53-45	46-39	41-35	37-31	58-40	50-34	43-30	39-27	35-24	47-40	40-34	35-30	32-27
	42-90	65-55	56-47	49-41	44-36	39-33	62-47	53-40	46-35	41-32	37-29	57-40	49-34	43-30	38-27
	90-UP	65-58	56-50	49-43	44-38	39-34	62-52	53-45	46-39	41-35	37-32	58-47	49-40	43-35	39-32
20-50	30-60	67-62	58-53	50-46	45-41	40-37	66-58	57-49	50-43	44-39	40-35	62-53	53-45	46-39	41-35
	60-90	67-62	58-53	50-46	45-41	40-37	66-58	57-49	50-43	44-39	40-35	65-55	56-47	49-41	43-36
	90-150	67-63	58-54	50-47	45-42	40-38	66-58	57-49	50-43	44-39	40-35	65-58	56-49	49-43	43-39
	150-UP	67-63	58-54	50-47	45-42	40-38	66-58	57-49	50-43	44-39	40-35	65-58	56-49	49-43	43-39
50-UP	60-90	67-65	58-56	50-49	45-44	40-39	67-63	58-54	50-47	45-42	41-38	67-63	58-54	50-47	45-42
	90-140	67-65	58-56	50-49	45-44	40-39	67-63	58-54	50-47	45-42	41-38	67-63	58-54	50-47	45-42
	140-200	67-65	58-56	50-49	45-44	40-39	67-63	58-54	50-47	45-42	41-38	67-63	58-54	50-47	45-42
	200-UP	67-65	58-56	50-49	45-44	40-39	67-63	58-54	50-47	45-42	41-38	67-63	58-54	50-47	45-42

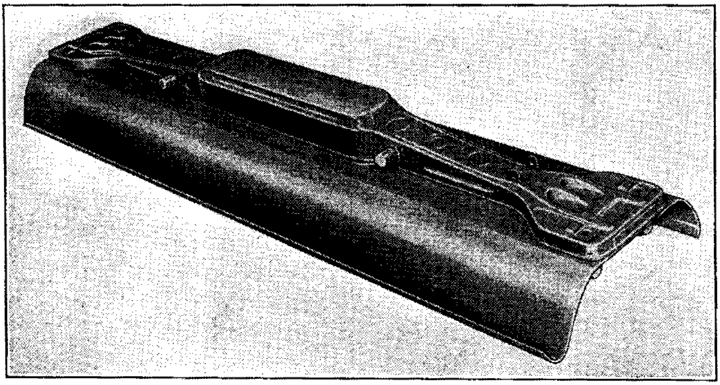
FOR INSTALLATION ON EXISTING OUTLETS, the spacing of outlets, area per outlet and room dimensions are already fixed. The foot-candle value provided is indicated in the columns corresponding to these fixed conditions. For uniform general illumination, select a mounting height above the working plane that is at least two-thirds of the maximum outlet spacing.

FOR INSTALLATION ON NEW OUTLETS, room dimensions and ceiling height are fixed. Locate desired foot-candle value under the mounting height selected. The area per unit will be indicated. Select spacing of outlet that will not exceed this area per unit. For uniform general illumination, the maximum spacing should not exceed 1½ times the mounting height above the working plane.

* Courtesy The Miller Company.

The procedure just outlined does not allow for reflection from walls, and takes no account of the height of room in relation to its width.[1] Table 26–1 is only for low-level general lighting.

Selection and spacing of other types of units should be carried out according to the recommendations of the manufacturer. In some installations, building vibration is so severe as to reduce seriously the life of lamps. Special shock-absorbing hangers are recommended for such service.



(Courtesy Sylvania Electric Products, Inc.)

Fig. 26–8. Industrial Type of Fluorescent-Lamp Fixture

Fig. 26–8 shows an industrial type of fluorescent-lamp fixture for two 100-watt lamps. Table 26–3 shows the proper spacing and mounting height to obtain any normally used intensity of lighting with such a fixture.

PROBLEM

1. A rough carpentry shop 50 ft by 100 ft, with no usable natural lighting, is to be illuminated by incandescent units with RLM dome reflectors. The suspension height is $14\frac{1}{2}$ ft. Specify the location of outlets and the size of lamp to be used in each.

Ans. *3 rows, 18-ft spacing, 18 outlets, 300 or 500 watts each. Use diffusing domes.*

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[1] For more accurate calculations, the reader is referred to booklets published by the General Electric Company, Nela Park, Cleveland, Ohio.

CHAPTER 27

Electronic Devices

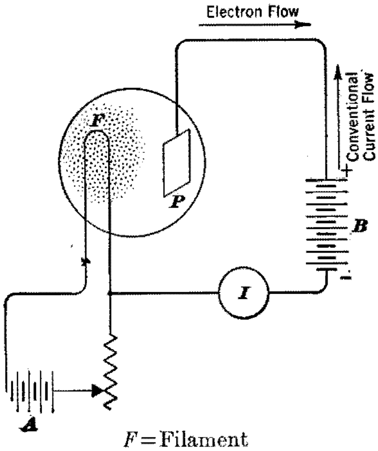
Thermionic Emission.—If the plates of the condenser of Fig. 5–14 be maintained at normal room temperature, say 22 C, and be separated by air at normal pressure, the attraction of the electrons of the charged negative plate toward the positive charge of the other plate will result in no migration between plates unless the potential difference be made so great as to cause breakdown of the resistance of the intervening air and permit a spark discharge. If the plates be sealed in a highly evacuated cell, the same phenomenon would be observed. If, however, the negative plate (or electrode) be heated to incandescence, the movements of the electrons therein will be greatly accelerated and some of the electrons themselves will leave the electrode and cluster in an invisible cloud around it. This cloud of electrons is said to constitute a "space charge," and the phenomenon is called "thermionic emission." Some of these electrons in space, having been freed from the negative electrode by high temperature, will migrate to the positive plate or electrode under the urge of the applied potential. This migration of electrons and their passage around the circuit constitutes an electric current, and its strength may be measured by a sensitive ammeter. A simple circuit carrying such a current is shown in Fig. 27–1. If no potential were applied, the cloud would form nevertheless; but no migration would occur.

It should be noted that the direction of flow of electrons in the circuit is opposite to that in which electricity is conventionally said to flow. This contradiction occurs because the convention was established before the facts were known and it cannot now conveniently be reversed.

Electron Affinity.—In an electron tube the element from which it is intended to draw electrons is called the *cathode*. The element to which the electrons are drawn is called the *anode*. Electrons are reluctant to leave a cathode at ordinary room temperatures, and high voltage is required to release them. If the temperature of the cathode is raised, the electrons acquire added velocities which; if the temperature is high enough, enable them to leave the cathode; however, they tend to fall back again. In the state of separation from the cathode the restraining attraction of the electron toward the cathode is greatly decreased, and the electron may be attracted to the anode by a voltage much less than that which would be necessary with a cold cathode. When an electron is acted upon by a potential, as from battery B in Fig. 27–1, it leaves the cathode at a velocity that depends on the plate potential and the initial velocity attributable to temperature. The electron mass acted upon by the potential, called the anode or plate potential, is accelerated until it reaches the anode. The velocity v of an electron "falling through" a voltage E is

$$v = 5.93 \times 10^7 \times \sqrt{E} \tag{27–1}$$

The velocity v is in centimeters per second, and is expressed in "electron volts." The initial velocity required for the electron to leave the cathode depends largely on the material of the cathode, and may vary between 1 and 6 electron volts. This voltage is called the electron affinity of the metal.



F = Filament
P = Plate

Fig. 27–1. **Diode and Simple Circuit**

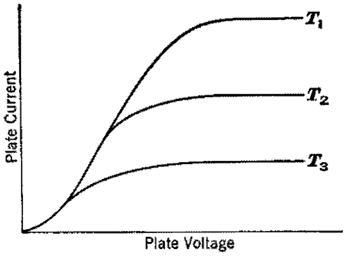


Fig. 27–2. **Saturation Curves for Cathode Temperatures T_1, T_2 and T_3**

Every metal when heated will emit electrons, and the rate of emission increases with rise in temperature. Pure metals have a high affinity and must be heated to very high temperatures. Only tungsten and tantalum will withstand the necessary high temperature.

Oxide-Coated Cathodes.—It has been discovered that a cathode coated with an oxide of barium, calcium, or strontium will emit many times as well as uncoated metal. The temperatures of such coated cathodes need not be so high as those of uncoated cathodes, and a less expensive metal may be used. Lower plate voltages may also be used. Uncoated cathodes are still used for tubes with very high voltage.

Direct and Indirect Heating of Cathodes.—If a cathode is heated by passing a current through it, it is said to be directly heated. The cathode is then a filament of ribbon or wire. Defects of this construction are: The filament is fragile, and the two ends of the filament are at different potentials with respect to the anode. This difference of potential leads to difficulties when the filament is heated by alternating current.

An alternate construction is one in which the coated cathode is in the form of a thin-walled coated tube. Inside the tube is the heating filament, and the cathode is heated by radiation from it. These cathodes require more time to become heated to operating temperature but have the virtues of being rugged and of having no direct connection to the heating element.

The Diode.—An electron tube with two elements only, as described, is called a *diode*. Fig. 27–1 shows a schematic diagram of a simple diode circuit. The filament or cathode F is heated by the battery A. The battery B produces the negative charge on the filament and the positive charge on the plate. The ammeter I indicates the flow of electrons constituting the electric current, which may be put to useful service. The maximum rate at which electrons may be drawn from the cathode cloud at high plate voltages is dependent on the number of electrons available. The rate at which electrons are "boiled off" the hot cathode and made available is determined by the cathode temperature. A curve of plate current vs. plate voltage like each of those shown in Fig. 27–2 is called the saturation curve of the electron tube.

Electrons of the space charge, by their repelling action on electrons at the surface of the cathode and by their proximity thereto, tend to limit the number of electrons emitted. For this reason, at the rated plate voltage or any lower voltage, the plate current of any particular electron tube is limited chiefly by the space charge and is said to be "space-charge limited." The characteristic curve flattens off because, for any particular cathode temperature, only a limited number of electrons can be emitted. Tubes operated in the flattened portion of the curve are said to be "temperature limited." Radio tubes are normally operated in the space-charge-limited range or the range below the knee of the curve in Fig. 27–2. This becomes important where high peak currents are required from the tube.

For a flat anode and cathode the characteristic curves of Fig. 27–2 for any cathode temperature may be expressed by Child's Law as long as the cathode is emitting freely and the current is therefore not space-charge limited. Under such conditions,

$$I = 2.34 \times 10^{-6} \times \frac{E^{\frac{3}{2}}}{d^2} \qquad \textbf{(27–2)}$$

where d is the spacing between anode and cathode, in centimeters.

The Diode as a Rectifier.—If for the battery B of Fig. 27–1 there be substituted a source of alternating emf, a current will flow as long as that emf charges the cathode negatively. When in the other half-cycle the *plate* is charged negatively, no current will flow because there are no free electrons around the cold plate to migrate to the filament. Hence, the tube will conduct in one direction only and may be used as a rectifier of alternating current. For rectification of all but very small currents, the gas-filled diode is more satisfactory because of its lower effective impedance. The high-vacuum diode is suited to high-voltage (50,000 volts or more) low-current service, such as for smoke and dust precipitation and radio transmission.

The Gas-Filled Diode.—In the high-vacuum type of diode, the voltage drop between elements is high. That high voltage drop may be greatly reduced by the introduction of a small amount of gas.

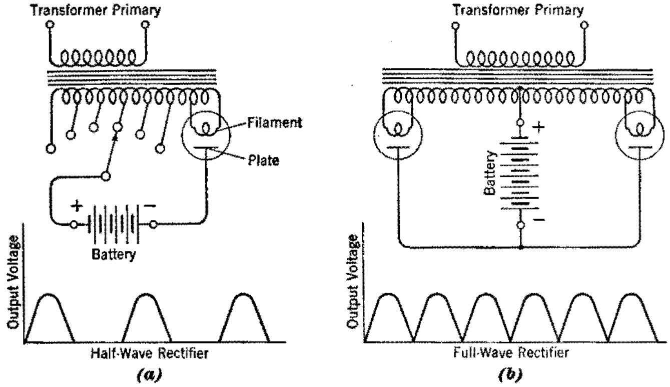
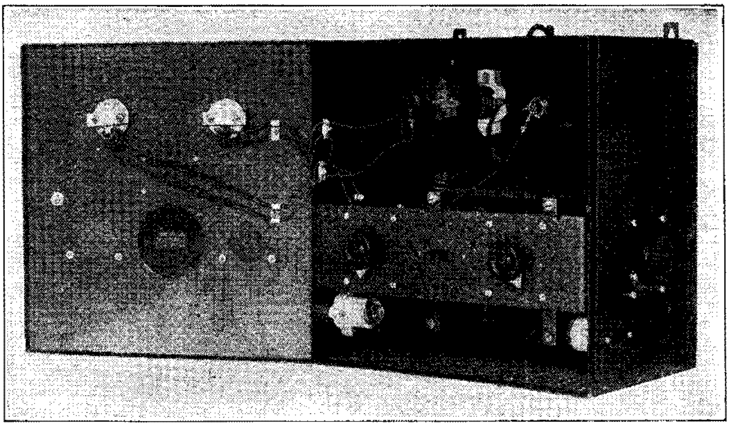
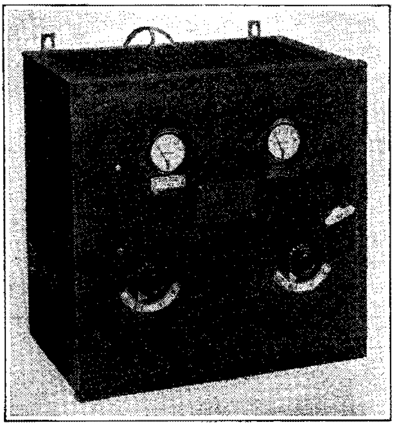


Fig. 27–3. **Elemental Circuit of *Tungar* Battery Charger**

If an electron, in its journey from the cathode to the plate, collide with a gas atom with sufficient velocity, one or more electrons will be detached from that atom and will be caused to join those in migration. The gas atom, having been robbed of one or more electrons, exhibits a positive charge and is said to be *ionized*. Thus, the gas atoms contribute electrons over and above those starting their migration from the cathode because of plate potential, and the resulting current flow is increased in that way because of the presence of gas and because the slow-moving ions tend to neutralize the space charge. Diodes of the gas type are used extensively in battery-charging service. In Fig. 27–3 (*a*) and (*b*) are shown simple circuits for battery charging by half-wave and by full-wave rectification. Fig. 27–4 shows an industrial type of diode charger. Overloads are liable to destroy the active surface of the cathode by bombardment from gas atoms which have lost one or more electrons by ionization and, being positively charged, are attracted to the cathode.





(*Courtesy Westinghouse Electric Corp.*)

Fig. 27–4. Diode Battery-Charger

Metering of Rectified Current.—The *average* value of one-half of a sine wave of current (beginning at a time when the current is zero) whose maximum value is I_m is

$$i_{\text{avg}} = \frac{1}{\pi}\int_0^{\pi} I_m \sin \omega t \, d(\omega t)$$

$$= 0.636 I_m$$

For the whole cycle of current, only one-half of which is used,

$$i_{\text{avg}} = \frac{0.636 I_m}{2} = 0.318 I_m \tag{27–3}$$

This is the value read on a direct-current ammeter.

The *effective* value of one-half of a sine wave of current (beginning at a time when the current is zero) whose maximum value is I_m is

$$i_{\text{eff}} = \sqrt{\frac{1}{\pi}\int_0^{\pi} I_m^2 \sin^2 \omega t \, d(\omega t)}$$

For the whole cycle of current, only one-half of which is used, the effective value is

$$i_{\text{eff}} = \sqrt{\frac{(0.707 I_m)^2}{2}} = 0.50 I_m \qquad (27\text{–}4)$$

Hence, an a-c ammeter in series with a d-c ammeter in a resistive circuit carrying half-wave rectified current would read higher than the d-c meter in the ratio of

$$\frac{0.50}{0.318} = 1.57$$

A direct-current ammeter is provided on a rectifier intended for battery charging because the charging value of any current is proportional to its *average* value, and not to its effective value. The effective value as read by an a-c ammeter is always the *heating* value.

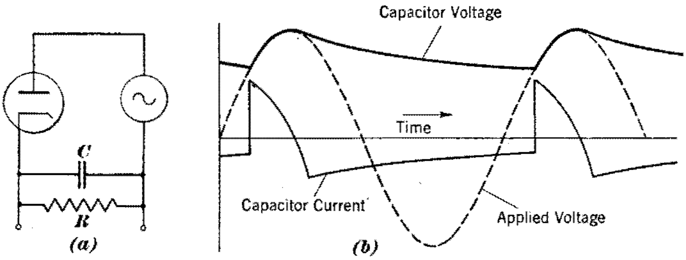


Fig. 27–5. Voltage Characteristic of a Condenser-Loaded Half-Wave Rectifier When a Resistive Load Is Added in Parallel

The ratio of the reading of an a-c ammeter to the reading of a d-c ammeter in a resistive circuit carrying *full-wave* rectified current is $\frac{0.707}{0.636} = 1.11$.

The ratio of the readings of a-c and d-c ammeters in a battery-charging circuit carrying half-wave or full-wave rectified current depends on the relative magnitudes of the battery voltage and the maximum value of the charging voltage; and so the foregoing ratios do not apply accurately in that case. Before a charging current can flow, it is necessary that the voltage of the charging device be greater than the emf of the battery. Hence, the charging-current impulse is not a complete half-cycle of a sinusoid.

The Half-Wave Rectifier With Capacitance Load.—If in the circuit of Fig. 27–3 (*a*) the battery is replaced by a condenser, the condenser will be charged to the peak plate voltage on the first half-cycle in which the plate is positive. Successive half-cycles of positive plate voltage will then have no effect on the potential of the condenser. If the condenser is used as a source of very small current, however, its charge will be drained off somewhat between positive half-cycles and will be restored successively by following half-cycles of positive voltage impulses. Fig. 27–5 shows the phenomenon of drain and recharge when the load is a high-resistance resistor. The rate of drain between charging impulses follows the exponential law of discharge of a condenser through a resistor and is therefore a function of *C* and *R*.

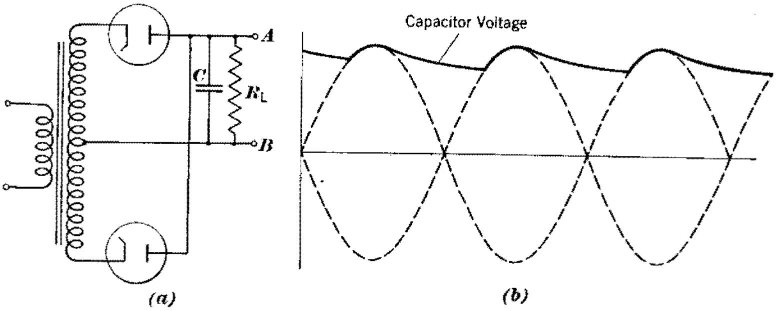


Fig. 27–6. Voltage Characteristic of a Condenser-Loaded Full-Wave Rectifier When a Resistive Load Is Added in Parallel

In applications where loss of condenser voltage because of current drain is within acceptable limits, the half-wave rectifier is satisfactory and is desirable because of its simplicity. If the current drain through the load is unusually heavy, it is advisable to use a full-wave rectifier, like that shown in Fig. 27–3 (*b*), which recharges the condenser to full voltage twice as often as does a half-wave rectifier. This voltage characteristic is shown in Fig. 27–6.

The Half-Wave Rectifier With Inductive Load.—If a purely ideal inductive load were connected to a half-wave rectifier, the voltage applied to the load would be switched on every conducting half-cycle when the voltage was zero and about to become positive. It is to be expected then that the current would rise as shown in Fig. 8–10. When the voltage reached zero at the end of the first conducting half-cycle, the current would be maximum instead of zero as it would be if the load were purely resistive. The tube would then continue to conduct during the following negative half-cycle because the decreasing flux of the inductance would simulate a generator with positive voltage applied to the plate. The current would then be completely sinusoidal but always positive.

If the load consists of both inductance and resistance in series, the self-induced voltage of the inductance cannot maintain the current throughout the entire negative half-cycle of generator voltage, and the current varies as shown in Fig. 27-7.

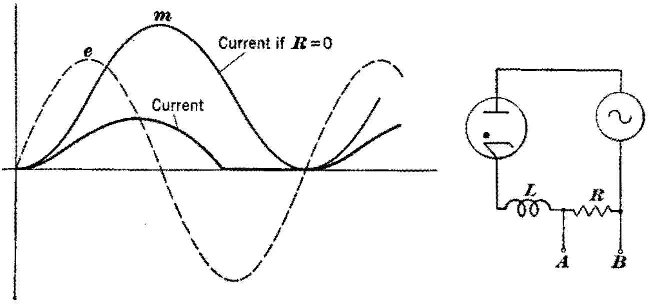


Fig. 27-7. **Current Characteristic of Half-Wave Rectifier Connected to an RL Load**

The Full-Wave Rectifier With Inductive Load.—If the load were purely inductive on a full-wave rectifier, the current in the inductance would rise on the first conducting half-cycle on one tube to point *m* in Fig. 27-7 but would not fall again because it would be augmented by the current from the

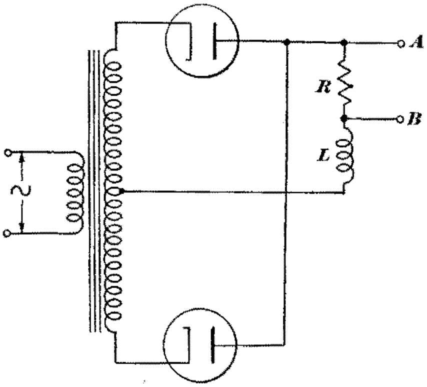


Fig. 27-8. **Full-Wave Rectifier With RL Load**

second tube on the succeeding half-cycle. As a result the load current would continue to rise until some element of the circuit was destroyed or the rise was limited by some element other than the inductance.

If, however, the load contains resistance in series with the inductance, as in Fig. 27-8, the current will rise only until the voltage drop across the resistance is equal to the d-c component of the load voltage. The d-c or average value of a half-wave of a sinusoid starting at zero is 0.636 times the maximum value, as derived on page 311. If the inductance of the load is large, the current through the load resistor and hence the voltage available between *A* and *B* will be nearly constant. A condenser connected across *R* will reduce still further whatever ripple remains. The foregoing statements are made on the assumption that the voltage drop through the tube is negligible.

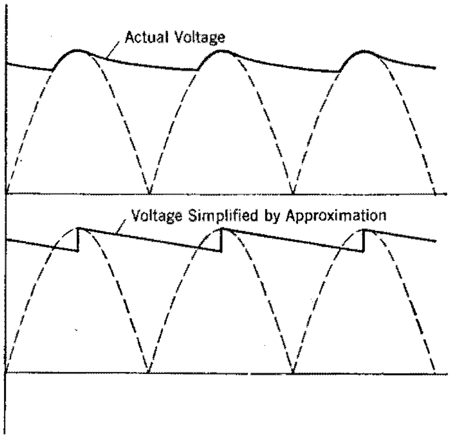


Fig. 27–9. Simplified Ripple Voltage (Steady State) Across RC Load Connected to Full-Wave Rectifier

Comparison of *RC* and *RL* Loads.—When a full-wave rectifier load consists of resistance and capacitance in parallel, the direct voltage available across the load is the maximum value of the applied voltage less that which occurs because of loss of condenser charge between charging impulses. When the load consists of resistance and inductance in series, the direct voltage available (across *R*) is only 0.636 times the maximum value of applied voltage.

The *RC* load, while making a relatively large voltage available when no current is drawn from it, does lose voltage when a current is taken from it because of draining of charge from the condenser. The *RL* load, in comparison, maintains its no-load voltage value across *R* more closely when current is drawn from terminals *A* and *B*.

Magnitude of Ripple in the Output Voltage of *RC* Load.—The voltage variation across the terminals of an *RC* load on a rectifier depends on the loss of charge on the condenser between charging impulses, and so depends on the magnitude of current drain to the follower circuit and also on whether the rectifier is half-wave or full-wave. With either half-wave or full-wave rectifier, sufficient accuracy may be had by using the simplifying assumption that the ripple is saw-toothed instead of having sinusoidal and exponential components. This comparison is shown in Fig. 27–9 for a full-wave rectifier.

Example 27–1.—Suppose that a condenser in an *RC* load on a full-wave rectifier is required to deliver an average current of 100 milliamperes at 1000 volts (average) with a deviation therefrom of only 10 volts. How many microfarads of capacitance are required? The frequency is 60 cycles per second.

Solution.—Under the simplifying assumptions the condenser discharges from 1010 to 990 volts in one half-cycle or $\frac{1}{120}$ sec. If the discharge continued at that rate, the time t required for the condenser to discharge would be

$$t = \frac{1010}{20} \times \frac{1}{120} = 0.421 \text{ sec}$$

The initial discharge current is

$$i_1 = 0.100 \times \frac{1010}{1000} = 0.101 \text{ amp}$$

The initial power of discharge is then

$$p_i = e \times i_1 = 1010 \times 0.101 = 102.01 \text{ watts}$$

The condenser voltage and current decrease together and their product is inversely proportional to the square of the time of discharge. The average value of this square-law function is one-third of the initial discharge rate. Therefore,

$$p_{\text{avg}} = \frac{p_i}{3} = \frac{102.01}{3} = 34 \text{ watts}$$

If the initial charge is Q_1 and the charge remaining $\frac{1}{120}$ sec later is Q_2, then

$$Q_1 = 34 \times 0.421 = 14.31 \text{ watt-sec}$$

and

$$Q_2 = Q_1 \times \frac{990}{1010} = 14.02 \text{ watt-sec}$$

In a condenser, $Q = EC$. Therefore,

$$Q_1 = 14.31 = 1010\,C$$
$$Q_2 = 14.02 = 990\,C$$

By solution of either of these equations, $C = 0.014115$ farad or 14,115 mfd.

Peak Inverse Voltage.—In the full-wave rectifier of Fig. 27–3 (*b*) either tube when not conducting will have impressed upon it a voltage equal to the peak value of the whole transformer. That voltage, which is called the *peak inverse voltage,* is attempting to force an electron current from anode to cathode (backward) through the tube. Therefore, when a diode is used in a full-wave rectifier, it should be chosen with proper regard for its rated peak inverse voltage.

The Triode.—If a mesh screen or grid be placed between the cathode and the plate of a diode, its presence will not materially affect the migration of electrons. However, if the grid be charged negatively with respect to the cathode, some of the electrons migrating from the hot filament will be driven back in accordance with the law of repulsion of similar charges, and the current flow in the plate circuit will be diminished. Conversely, a

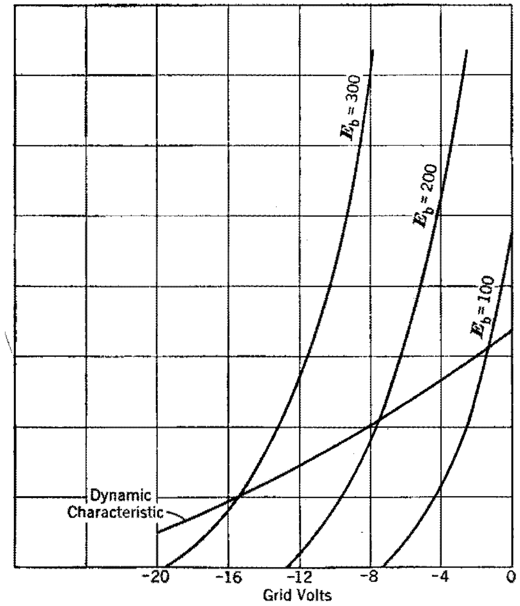


Fig. 27–10. **Mutual Characteristics of Type 6C5**

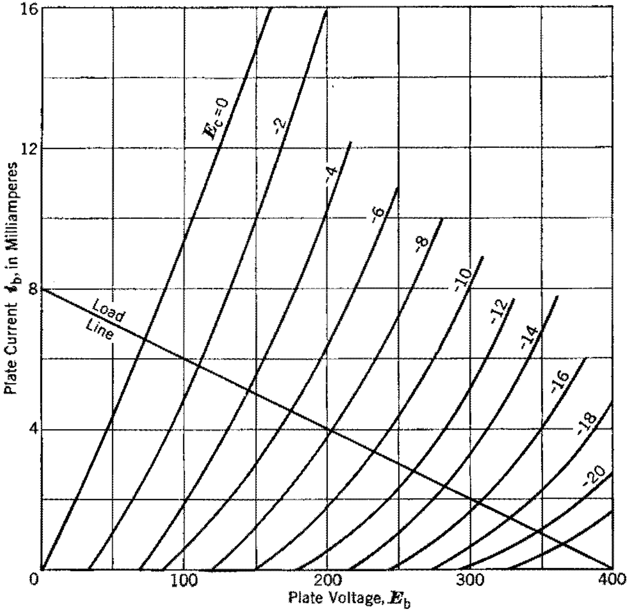
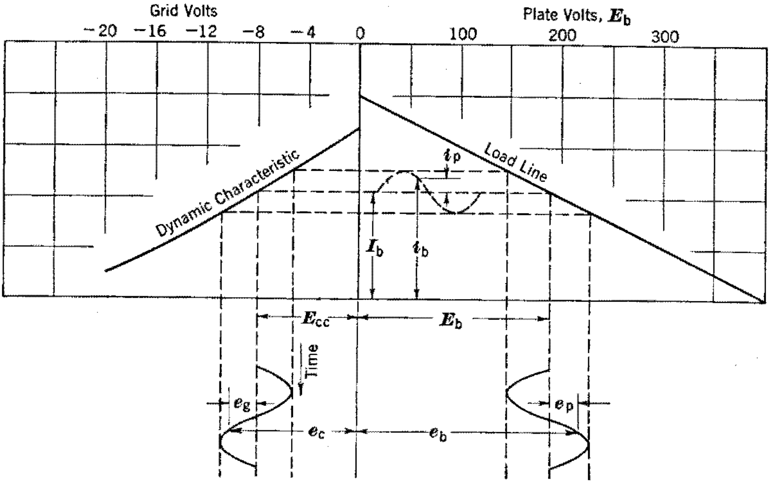


Fig. 27–11. **Plate Characteristics of Type 6C5**

positive charge on the grid will accelerate the migrating electrons and will increase the plate current. Because the grid is much nearer to the cathode than it is to the plate, a small change in grid potential may be made to cause a great change in plate current. This action is called *amplification*. The three-electrode tube is called a *triode*. By use of the triode a small amount of power may be made to control a large amount of power. Such a high-vacuum tube is known as a *pliotron*.



E_{cc} = negative grid bias
E_b = average value of plate voltage
e_b = instantaneous total plate voltage
e_c = instantaneous total grid voltage
e_g = instantaneous value of a-c component of grid voltage
e_p = instantaneous value of a-c component of plate voltage
I_b = average value of plate current
i_b = instantaneous total plate current
i_p = instantaneous value of a-c component of plate current

Fig. 27–12. Amplification of Alternating Voltage

5.5 volts peak-to-peak grid input
110 volts peak-to-peak plate output

Transfer and Plate Characteristics of a Triode.—In Fig. 27–10 is shown a family of curves, known as mutual or transfer characteristics, drawn for various plate voltages on a 6C5 triode. Inasmuch as the negative grid portions of the characteristics are of importance, only those portions are shown. Another set of characteristics, known as plate characteristics, is shown in Fig. 27–11 for a 6C5 triode. Fig. 27–10 and Fig. 27–11 are placed close to each other for convenience in further development. From these curves it is possible to predict the performance of a triode in a circuit. In Fig. 27–12 it is seen how a small negative grid bias voltage can be used to control the plate current produced by a large plate voltage.

The Load Line and Dynamic Characteristic.—Figs. 27–10 and 27–11 represent the static characteristics of a 6C5 which will now be used to develop the dynamic characteristic in conjunction with a circuit. In Fig. 27–13 is shown the circuit to be used as an amplifying device for alternating voltage. By Kirchhoff's Law,

$$E_{bb} = i_b R_L + e_b$$

or

$$e_b = E_{bb} - i_b R_L \tag{27–5}$$

This is the equation of a straight line called the *load line*. If e_b across the triode were zero, all of the voltage E_{bb} would appear across R_L; and the current i_b would be

$$i_b = \frac{E_{bb}}{R_L}$$

$$= \frac{400}{50{,}000} = 0.008 \text{ amp}$$

If the voltage e_b were 400 volts, no voltage would be left for R_L; so i_b would be zero. With these intercepts known, the load line is drawn in Fig. 27–11.

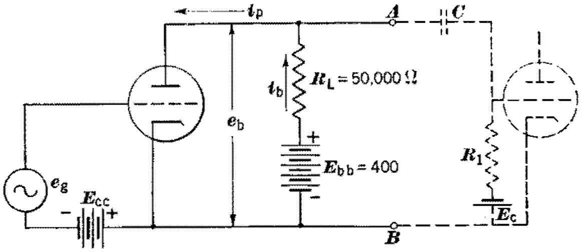


Fig. 27–13. Triode Amplifier Circuit

Intersections of the load line and the plate characteristics of Fig. 27–11 represent points for which the plate current is such that the sum of the voltage across the triode and the voltage drop across R_L is equal to the plate battery voltage E_{bb}. If all such points are transferred to the mutual characteristic graph of Fig. 27–10, the resulting locus is a tube-and-circuit characteristic called the *dynamic* characteristic.

Plate Resistance.—When the plate current flows in a tube, a portion of the plate circuit voltage is required to force the electrons from cathode to plate. That voltage acts as an ordinary *ri* drop in the circuit; however, inasmuch as the plate characteristic is not linear, the voltage drop is not proportional to the plate current. The resistance which causes the voltage drop is called the *plate resistance*. The plate resistance, being a dynamic function of plate current, must be specified for some particular value of grid voltage to have any meaning.

For any quiescent point on the tube characteristic, the plate resistance is equal to a small change of plate voltage divided by the resulting change in plate current. Thus,

$$r_p = \frac{\Delta e_b}{\Delta i_b} \tag{27-6}$$

The plate resistance may be found most easily by drawing a straight line tangent to the plate characteristic at the quiescent point chosen and then calculating the resistance from the slope of the tangent line. Most manufacturers of tubes give the plate resistance at several quiescent points.

Voltage Gain of a Tube.—The amplification factor μ of a tube cannot be considered as all profit because only the change of voltage across the load resistance R_L is available for use. The voltage gain v_g must be calculated by comparing the change in voltage across R_L only with change in e_c. The voltage change across R_L is only proportional to the total change in e_p because the circuit contains r_p. Therefore,

$$v_g = \frac{R_L}{R_L + r_p} \tag{27-7}$$

Amplification Factor, Plate Resistance, and Transconductance.—The plate current of a triode in its circuit may be made to change by changing either the plate voltage or the grid potential. The amplification factor relates these two functions. If, from a given operating point on the mutual characteristic, a change of grid voltage Δe_c results in a change of plate current Δi_b and a subsequent change of plate voltage $-\Delta e_b$ will restore the plate current to its original value, then the amplification factor is

$$\mu = -\frac{de_b}{de_c} \tag{27-8}$$

For example, if a change of 40 volts on the plate will restore a plate-current change caused by a change of +2 volts on the grid, the amplification factor is 20.

The plate resistance r_p is found by dividing a small change in plate voltage by the accompanying change in plate current (in amperes).

The grid-plate transconductance or mutual conductance g_m is found for any fixed plate voltage by dividing a small change in plate current Δi_b by the change in grid voltage Δe_c that produced it. Therefore,

$$g_m = \frac{di_b}{de_c}$$

and

$$\mu = g_m r_p \tag{27-9}$$

where g_m is in mhos and r_p is in ohms.

Amplification of Alternating Voltage.—In the circuit of Fig. 27-13 let it be assumed that an a-c generator impresses a voltage e_g between the grid and the cathode. This voltage is superposed on the bias voltage E_{cc} and therefore adds to and subtracts from E_{cc} on the dynamic characteristic. This action is shown in Fig. 27–12. A positive increase in grid voltage (reducing the grid bias) causes an increase in i_b. An increase in i_b causes an increase in i_bR_L across the load resistor, thus leaving a smaller voltage e_b across the triode. The voltage e_p is therefore reversed in phase from e_c. The instantaneous plate current is

$$i_b = I_b + i_{p(\text{max.})} \sin \omega t \tag{27–10}$$

The instantaneous component of voltage across the triode is

$$e_p = -i_pR_L = -i_pR_{L(\text{max.})} \sin \omega t \tag{27–11}$$

This is the output voltage available between terminals A and B of Fig. 27–13.

If the dynamic characteristic were linear, the output voltage would have the same wave form as the grid input signal. Only a portion of the dynamic characteristic is acceptably straight, however, and it is important that the bias voltage E_{cc} be so chosen that the swings of e_g will not overreach the straight portion. Regardless of the bias point chosen, excessive grid signal voltage will cause more than the straight part of the dynamic characteristic to be used and distortion of the output voltage across R_L will occur.

The Alternating-Current Equivalent Circuit.—For the circuit of Fig. 27–13, the equation for plate current change is

$$\Delta i_p = g_m \, \Delta e_g + \frac{\Delta e_p}{r_p}$$

Also, $$\Delta e_p = -R_L \, \Delta i_p$$

Therefore, $$\Delta i_p = g_m \, \Delta e_g + \frac{-R_L \, \Delta i_p}{r_p}$$

By simplifying the equation, we obtain

$$\Delta i_p = \frac{g_m r_p \, \Delta e_g}{r_p + R_L}$$

Since $g_m r_p = \mu$,

$$\Delta i_p = \frac{\mu \, \Delta e_g}{r_p + R_L} \tag{27–12}$$

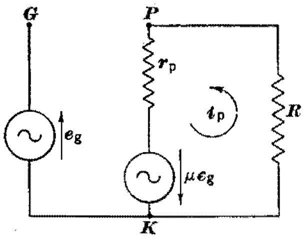


Fig. 27–14. Equivalent Circuit Corresponding to the Circuit of Fig. 27–13

It follows, therefore, that a generator having a voltage $\mu \, \Delta e_g$ and connected in series with the anode would produce the same change Δi_p in the plate current as Δe_g in the grid circuit actually does. It is therefore feasible to replace the actual circuit with an equivalent circuit such as that shown in Fig. 27–14. The battery voltage E_{bb} is not shown because Δi_p is only the usable incremental part of the total plate current. If the simple equivalent circuit is to be used, it is necessary that the triode be operated on the linear portion of its characteristic.

If e_g is a sinusoid, then Equation (27–12) may be written as a vector equation as follows:

$$E_p = \frac{\mu E_g}{r_p + R_L} \tag{27–13}$$

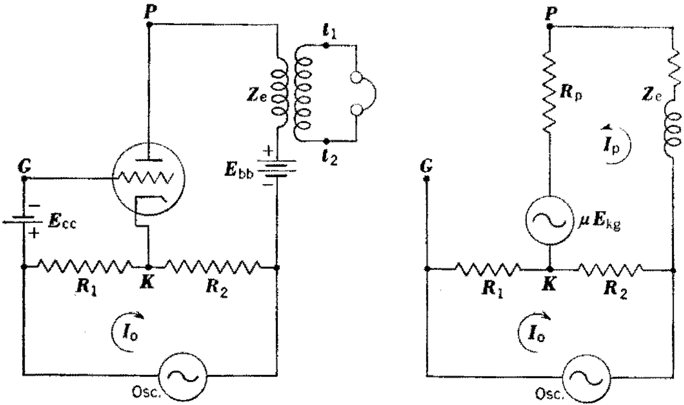
If the load contains inductance also, then Equation (27–13) becomes

$$E_p = \frac{\mu E_g}{(r_p + R_L) + jX_L} \tag{27–14}$$

where $Z_L = R_L + jX_L$.

The alternating component of plate voltage is

$$E_p = \frac{\mu E_g Z_L}{r_p + Z_L} \tag{27–15}$$



(a) Actual Circuit *(b)* Equivalent Circuit

Fig. 27–15. Circuit for Measuring μ

The gain of the amplifier of Fig. 27–13 is then expressed as follows:

$$\text{gain} = \frac{I_p Z_L}{-E_g} = -\frac{\mu E_L}{R_p + Z_L} \tag{27–16}$$

Application of the Equivalent Circuit in Measuring μ.—The circuit shown in Fig. 27–15 may be used in measuring μ. The impedance Z_e is the impedance of the coupling transformer as measured from terminals t_1 and t_2. The batteries do not appear in the equivalent circuit because only the incremental voltage is to be used and the battery resistances are negligible. The resistance R_p is the equivalent internal plate resistance of the triode. The voltage drop between cathode and grid is $V_{kg} = -IR_1$.

The voltage rise is $E_{kg} = -V_{kg} = IR_1$. When Kirchhoff's Law is applied around the anode circuit, we obtain:

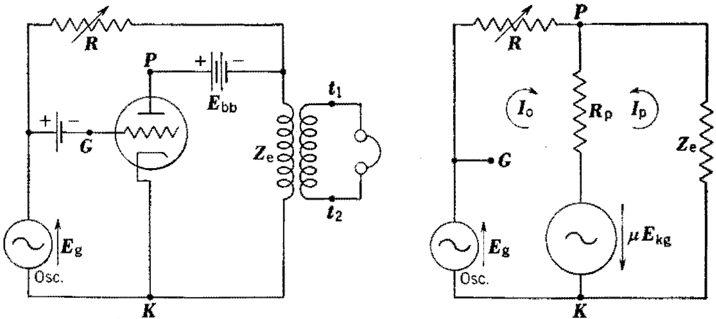
$$\mu E_{kg} = \mu I_o R_1 = (I_o + I_p)R_2 + I_p(Z_e + R_p)$$

from which

$$I_p = \frac{(\mu R_1 - R_2)I_o}{R_2 + Z_e + R_p} \tag{27–17}$$

If the values of R_1 and R_2 are so adjusted that I_p does not exist, there will be no signal at t_1 and t_2 and no signal voltage across Z_e. The two loop circuits are then said to be balanced. When $I_p = 0$, $\mu R_1 - R_2 = 0$ in Equation (27–17) and

$$\mu = \frac{R_2}{R_1} \tag{27–18}$$



(*a*) Actual Circuit (*b*) Equivalent Circuit

Fig. 27–16. Circuit for Measuring g_m

Application of the Equivalent Circuit in Measuring g_m.—The grid-plate transconductance g_m may be measured by means of the circuit shown in Fig. 27–16. The circuit is balanced by varying R until $I_p = 0$. For this circuit,

$$E_g = I_o R + (I_o + I_p)R - \mu E_g \tag{27–19}$$

$$\mu E_g = I_p Z_e + (I_p + I_o)R_p \tag{27–20}$$

From Equations (27–19) and (27–20),

$$I_p = \frac{E_g(\mu R - R_p)}{(R + R_p)(R_p + Z_e) - R_p^2}$$

When $I_p = 0$, $R = \frac{R_p}{\mu} = \frac{1}{g_m}$. Therefore,

$$g_m = \frac{1}{R} \tag{27–21}$$

The Resistance-Capacitance-Coupled Amplifier.—If the amplifier circuit of Fig. 27–13 is to be used to excite the grid of a succeeding stage of amplification as indicated, a condenser C must be interposed to prevent flow of positive current to that grid from E_{bb}. To be certain that the grid of the succeeding stage remains negative, a grid leak resistor R_1 and a negative bias voltage E_c must be added. The ratio of the grid voltage of the succeeding stage to the grid voltage on the tube shown in Fig. 27–13 is

$$\frac{e_{g2}}{e_{g1}}=\frac{\mu R_L}{R_L+r_p}=K_a \tag{27–22}$$

The ratio K_a is the amplification factor of the tube and circuit and it obviously can never be quite so large as μ. It is important therefore that a tube having high μ be used in this type of circuit.

This type of amplifier has the merit that the coupling is independent of frequency as long as the condenser reactance is not so high as to be comparable with R_1. If it is comparably great, the remaining voltage across R_1 that is available to the following grid will be noticeably reduced below that across R_L. At very high frequencies the grid-to-filament capacitance will provide a reactance that is small compared to R_1 and the amplification factor will be reduced because the alternating-current plate circuit impedance is reduced.

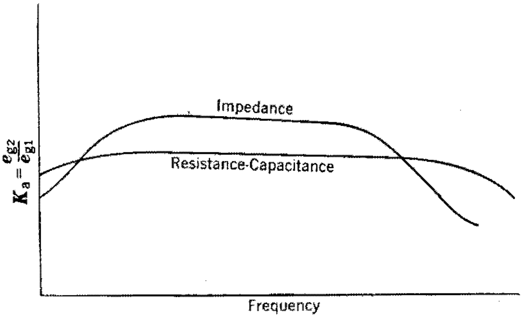


Fig. 27–17. Comparative Characteristics of Impedance-Coupled and Resistance-Capacitance-Coupled Amplifiers

The Impedance-Coupled Amplifier.—In the circuit of Fig. 27–13, E_{bb} must be great enough to cause the direct plate current to flow through the combined resistance of r_p and R_L. If R_L be replaced by an inductance L_p, less voltage E_{bb} is needed for the same direct plate current; and at the same time the plate circuit impedance ωL_p can be kept large. Such an amplifier is really an inductance-capacitance amplifier, but the term impedance-coupled is generally accepted. The amplification constant is

$$K_a=\frac{e_{g2}}{e_{g1}}=\frac{\mu\omega L_p}{r_p+jX_p} \tag{27–23}$$

At lower frequencies K_a for an impedance-coupled amplifier will decrease worse than for a resistance-capacitance-coupled amplifier because for a particular value of alternating-current plate current the reactance of

L_p is proportional to frequency. At very high frequencies also K_a will decrease worse than for a resistance-capacitance-coupled amplifier because the distributed capacity inherent in a coil used for L_p acts like a condenser in parallel with L_p and lowers the available voltage across L_p.

A comparison of characteristics of impedance-coupled amplifiers and resistance-capacitance-coupled amplifiers is shown in Fig. 27–17.

The Transformer-Coupled Amplifier.—In Fig. 27–13 the output voltage at AB for any particular plate-circuit alternating current is

$$E_{AB}=\frac{e_g R_L}{R_L+r_p} \tag{27–24}$$

If a transformer is substituted for R_L, the secondary output voltage will be higher than the primary voltage by the factor $a=\frac{N_2}{N_1}$. For effective use of plate signal current it is essential that the primary input impedance be larger than r_p. Since r_p is always several thousand ohms, the primary coil must have many turns and the core must be of high-permeability iron. Since the secondary coil has about the same volume of copper as the primary coil, the maximum number of secondary turns is limited by the size of the smallest commercially available wire and the step-up ratio is limited accordingly. Ratios between 3:1 and 10:1 have been commonly used in the audio range of frequencies. Any attempt to increase the turns ratio by decreasing the number of primary turns causes a decrease in input impedance and, when carried too far, results in less rather than more secondary voltage.

The voltage across the primary of the transformer is

$$E_{\text{pri}}=\mu e_{g1}\frac{X_p}{r_p+jX_p} \tag{27–25}$$

The amplification factor K_a is

$$K_a=\frac{e_{g2}}{e_{g1}}=\frac{N_2}{N_1}\frac{\mu X_p}{r_p+jX_p} \tag{27–26}$$

Tubes with high μ have high r_p and are generally unsatisfactory for transformer coupling.

The amplification characteristic of a transformer-coupled circuit is much the same as that of Fig. 27–17, but at higher frequencies there is a peaking of K_a because of resonance between the transformer leakage reactance and the distributed capacitance of its windings. The higher amplification factor is obtained at the expense of fidelity of reproduction of the input signal.

The Triode as a Detector.—When a triode is used as an amplifier, it should be operated in the straight portion of its characteristic curve to insure fidelity of its output. If the grid bias is so great that the grid potential swings are in the curved portion of the characteristic, as shown in Fig. 27–18, then a positive signal potential will cause a greater change in plate current than will a negative signal potential of the same strength. This phenomenon results in partial rectification and is popularly known as *demodulation* or *detection*. Detection is necessary in radio receivers to convert the alternating high-frequency incoming signals to audio-frequency signals to which a telephone receiver or similar instrument will respond.

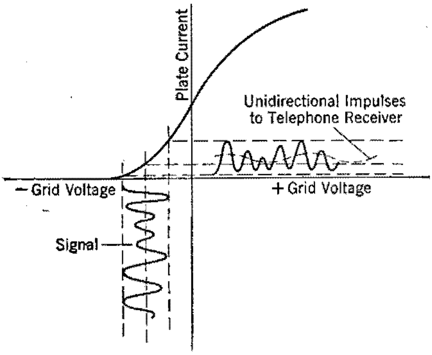


Fig. 27–18. The Triode Used as a Detector

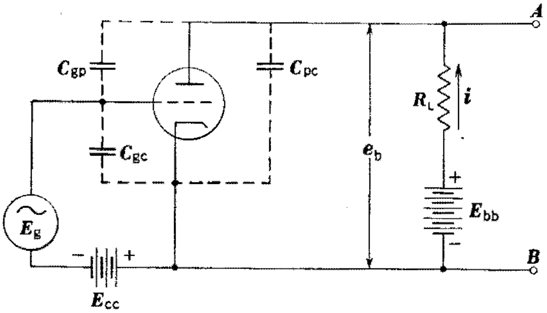


Fig. 27–19. Triode Amplifier Circuit Including Interelectrode Capacitances

Tetrodes.—In a triode every one of the three electrodes forms the plate of a small condenser. In a 6C5 triode these interelectrode capacitances are of the order of 3 to 4 micromicrofarads. When the plate frequency is very high, those capacitances produce reactances sufficiently low to alter substantially the performance of the triode. The refined circuit of Fig. 27–13 complete with interelectrode capacitance is shown in Fig. 27–19.

The grid-plate capacitance C_{gp} is most important because it constitutes a coupling circuit element between input and output which may cause the grid to lose control. Capacitance C_{gp} is magnified in importance also by the tube amplification. To mitigate the effect of C_{gp} a second grid, called the *screen grid*, is added between the plate and the control grid and is kept positive with respect to the cathode. Most of the electrostatic lines of force from the plate terminate on the screen grid instead of on the control grid or the cathode, giving the effect of greater distance between the plate and the cathode-grid group. The effect is the same as that of lowering the capacitance C_{gp}.

As a result of interposing the screen grid, the plate voltage has much less effect on the plate current beyond the region of instability, as shown in Fig. 27–20. These characteristics produce high amplification factors and high plate resistance.

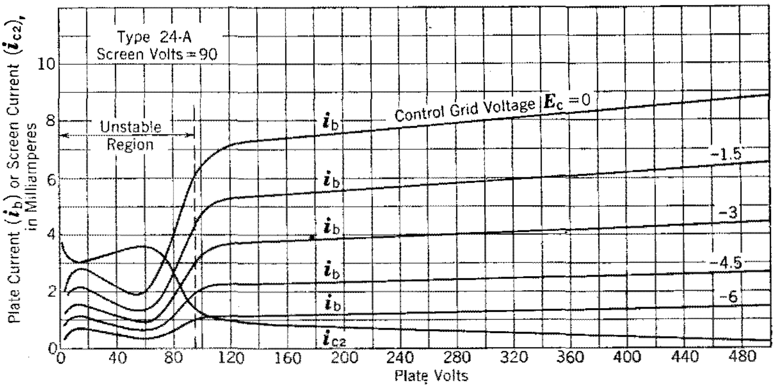


Fig. 27–20. Average Plate Characteristics for Type 24-A Screen Grid Tetrode

The unstable region is unusable for most purposes because of the non-linearity of the characteristics. Secondary emission at the plate, caused by bombardment of the plate by electrons from the cathode, causes the dip in characteristics in that region. In a triode the secondary emission electrons fall back to the plate; but, when a screen grid is added and its potential is higher than that of the plate, those electrons are drawn to the screen and produce a current in it. Increase in plate voltage in the unstable region causes greater secondary emission, and therefore less net plate current, because one electron striking the plate may release more than one electron from it. The non-linearity in the unstable region disappears when the plate potential is made equal to or greater than the screen potential, because the secondary electrons remain at the plate.

Although the amplification factor of the tetrode is many times as great as that of a triode, its plate current is much less. These qualities determine the choice of tube for a particular use.

Pentodes.—While some characteristics of a tetrode are superior to those of a triode, the unstable region is undesirable for most uses. This region may be made usable, however, by the addition of still another grid, called a *suppressor grid,* interposed between the screen grid and the plate. The suppressor grid is connected directly to the cathode and is therefore at cathode potential. Being at cathode potential, the suppressor grid repels the electrons of secondary emission back to the plate and prevents loss of plate current at low plate voltages. Furthermore, the suppressor shields the plate from the control grid and cathode, and gives an amplification factor even higher than that of the tetrode. Fig. 27–21 shows a pentode in a simple circuit; and Fig. 27–22 shows the characteristics of a 6SJ7 pentode.

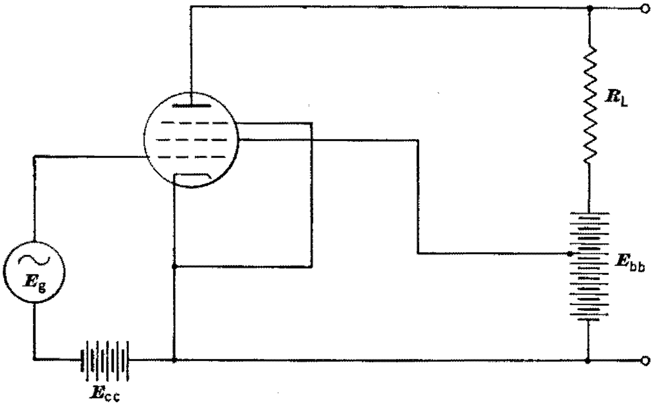


Fig. 27–21. The Pentode Used as an Amplifier

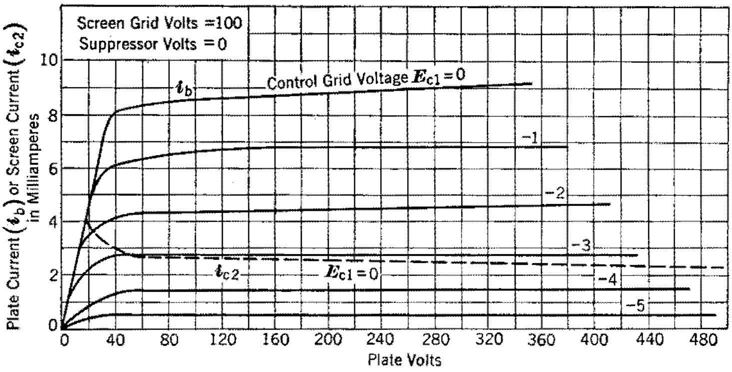


Fig. 27–22. Average Plate Characteristics of Type 6SJ7 Pentode

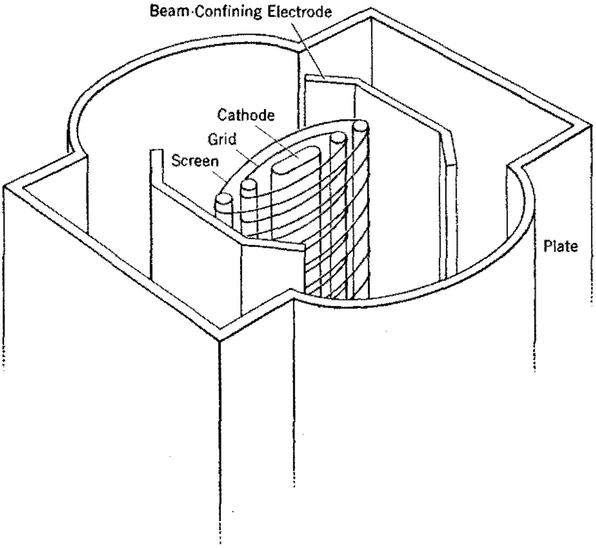


Fig. 27–23. Elements of the Beam Power Tube

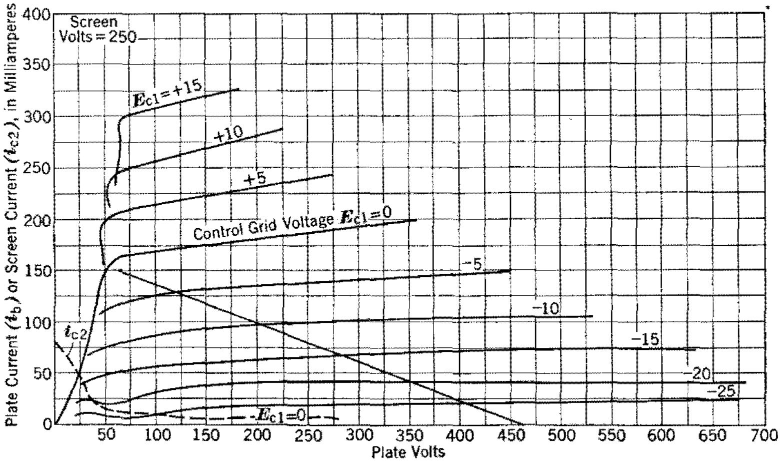


Fig. 27–24. Average Plate Characteristics for Type 6L6 Beam Power Tube

The Beam Power Tube.—The effects of secondary emission may be eliminated by means other than a suppressor grid. In the beam power tube shown in Fig. 27–23, the beam-confining plates or shields are at cathode potential and by their repelling action confine the cathode electron flow in two concentrated beams. These beams, while still producing secondary emission at the plate, act like a space charge and repel the emitted electrons back to the plate. In the resulting characteristics of

Fig. 27–24 the linear region extends over a wider range of plate voltage and the knee is more sharply defined than in those of a pentode.

The beam power tube is specially desirable where a maximum of power with a minimum of distortion is needed. If the screen grid of a beam power tube is connected to the plate instead of to the cathode, the plate characteristics become those in Fig. 27–25.

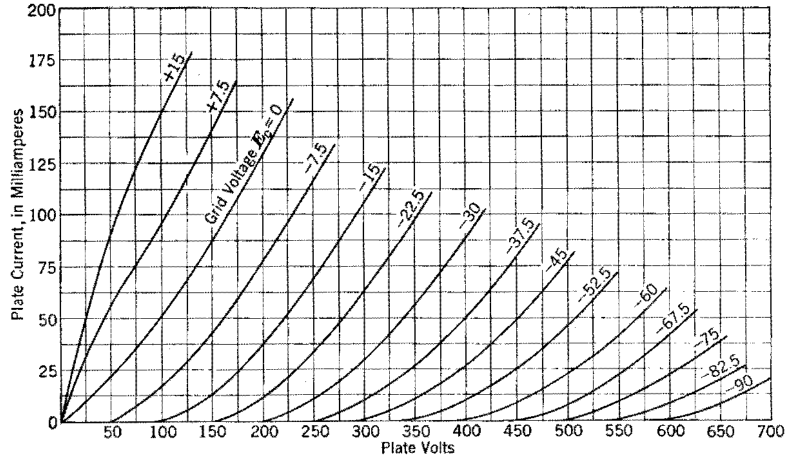


Fig. 27–25. Average Plate Characteristics for Type 6L6 Beam Power Tube Connected as a Triode (Screen Is Connected to Plate Instead of to Cathode)

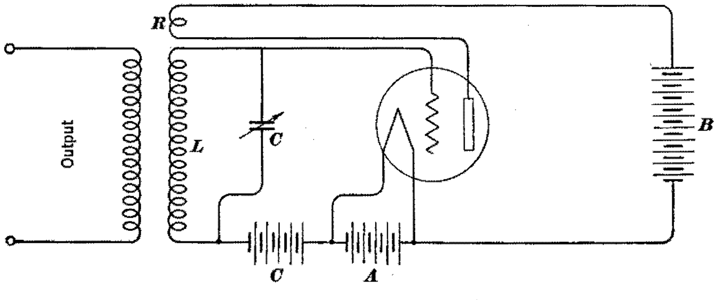
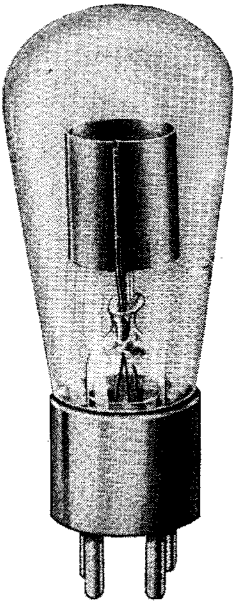


Fig. 27–26. The Triode as an A-C Generator

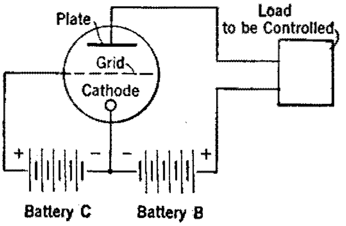
The Triode as a High-Frequency Generator.—A circuit such as shown in Fig. 27–26 may be used to provide variable-frequency a-c voltages for such purposes as radio transmission and operation of stroboscopic lamps. In Fig. 27–26 the triode acts as an amplifier of any slight signal applied to it. The amplified signal passes through the coil R which, if properly related inductively to L, will induce an additional emf in L; and this will add to the original signal. This action is called *regeneration*. The parallel circuit LC will then oscillate at its own natural frequency (resonance), and an a-c output voltage will appear at the transformer terminals. The frequency of the output voltage may be controlled by the adjustment of L or C or both. The output energy is derived from the battery B.

Gas-Filled Triodes.—The gas in a gas-filled diode will not ionize until the plate potential exceeds a certain minimum value because, in order that a migrating electron may dislodge another electron from a gas atom by collision, it must have attained a considerable velocity before it collides with the atom. The velocity attained by a free body is a direct function of the force applied and depends also on the duration of application of the force. Hence, if the gas pressure is high (there are many atoms per unit volume), an electron leaving the filament cloud will not travel far toward the plate before it collides with a gas atom. Therefore, to accelerate the electron to an ionizing velocity in such a short distance, a higher plate potential (force) must be used than if the gas were less dense.



(Courtesy Westinghouse Electric Corp.)

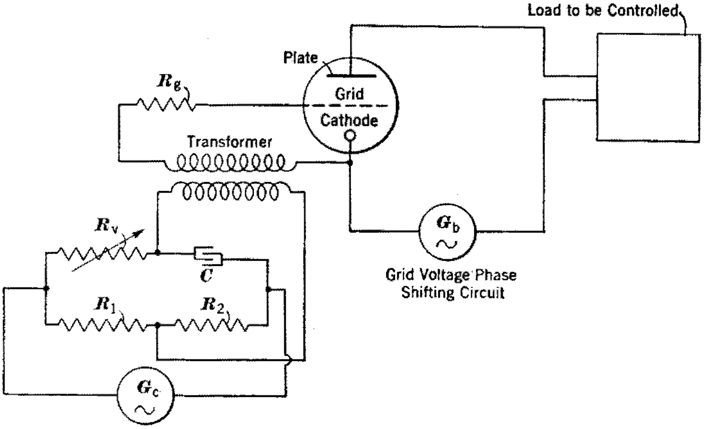
Fig. 27–27. Grid-Glow Tube



70 to 90 v from Battery C to start
60 v from Battery C to operate
110 v from Battery B minimum to start
70 v from Battery B to operate

Fig. 27–28. Grid-Glow Tube and Simple Circuit

A grid placed near the cathode and between the cathode and the plate may be charged and used to control the electron velocity and, hence, the plate voltage necessary for ionization of the gas. The ionization potential having been reached, the tube becomes highly conducting, and the current flow is limited almost entirely by the resistance of the external plate circuit. The grid potential, having started the current flow, is powerless to stop it, even though the grid be made highly negative to the cathode. This is called a "trigger" action. The grid voltage which just fails to produce ionization is called the *critical voltage.* If the plate current be interrupted, the grid will regain control. Thus, even though the gas-filled triode is similar in construction to the vacuum-type triode, its characteristics are much different. The cold-cathode tube of this type is called a grid-glow tube, and the hot-cathode type is a *thyratron.*



The circuit containing R_1, R_2, R_v and C is for phase-shift control.
Control is effected by varying R_v.
R_g permits grid-to-cathode potential but restricts flow of grid current.

Fig. 27–29. Grid-Glow Tube With Alternating Grid and Plate Voltages

The Grid-Glow Tube.—The grid-glow tube, as shown in Fig. 27–27, is a cold-cathode tube with two electrodes and a grid. One of the electrodes is specially designed to act better as a cathode by making it larger than the other and coating it with some substance, such as oxide of barium, strontium, or calcium, to improve its emission. If enough voltage is impressed between the cathode and the plate, with no voltage applied between the cathode and the grid, ionization of the gas will occur and a current will flow. When the tube conducts current in normal operation, all or part of the cathode glows with a pink greenish glow; hence, the name grid-glow is used. If, however, a voltage from battery C in Fig. 27–28 be impressed between the grid and the cathode with the polarity shown, the voltage required of battery B to start the glow and current flow will be changed. If the grid is made positive with respect to the cathode, the breakdown voltage required of battery *B* will be less. If the grid is made negative with respect to the cathode, the voltage required of battery *B* for breakdown will be greater.

After conduction through the tube in Fig. 27–28 has started between the cathode and the plate, the grid-cathode voltage is powerless to stop it, and current will continue to flow until the voltage between the plate and the cathode has been reduced below the cut-off voltage of about 50 volts in small tubes and 200 volts in large tubes. Meanwhile, of course, the grid-cathode voltage must have been reduced below the voltage required to start the flow. The grid then regains control, and retains it even though the plate-cathode voltage be raised again.

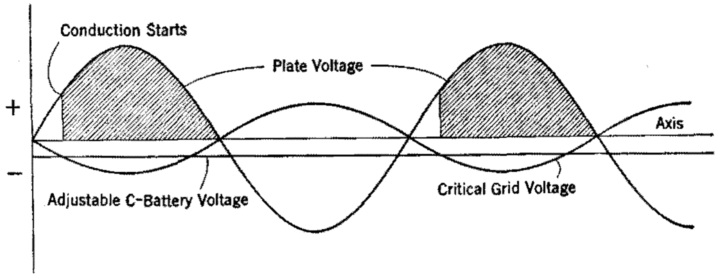


Fig. 27–30. **Current Flow Through a Grid-Glow Tube With Uni-Directional Grid Voltage and Alternating Plate Voltage**

If the *B* battery of Fig. 27–28 be replaced with a source of alternating voltage, as in Fig. 27–29, the plate voltage will pass through zero twice every cycle, and so the grid voltage may regain control once every cycle. Inasmuch as the tube is not a conductor when the cathode is positive, the plate-current impulses will be in one direction only, and one pulse will occur every cycle, as shown in Fig. 27–30.

The instant in the plate-voltage cycle at which conduction starts may be adjusted to any point in the first positive quarter of every cycle by adjusting the voltage received on the grid from the battery *C*. Obviously, the start cannot be delayed to any point in the second quarter of the cycle because all starting values in the second quarter are duplicated in the first. In order to start conduction in the second quarter of the cycle, it is necessary to apply alternating voltage to the grid as well as to the plate. In order to make the starting point adjustable, it is necessary to interpose a phase-shifting network in the grid-voltage circuit, as shown in Fig. 27–29. The critical grid voltage is that minimum grid voltage which will start conduction in the tube for any given plate voltage. If the grid is negative with respect to the cathode, then the critical grid voltage rises as the plate voltage rises.

In Fig. 27–31 is shown the effect of phase shift of grid voltage as it controls the time of start of tube conduction in every cycle. As the phase of the grid voltage is shifted, the average value of the current permitted to flow in the plate circuit and load is varied.

The grid voltage actually applied has a greater maximum value than the critical grid voltage. As long as the grid voltage exceeds the critical voltage at all points on the curve, the plate current is blocked. If, however, by a phase-shifting circuit such as shown in Fig. 27–29, the grid voltage be caused to lead in time phase its position shown in Fig. 27–31 (*a*) and shift to the position in (*b*), there will be a point *X* in the plate-voltage cycle at which the critical voltage will be slightly greater than the grid voltage and ionization and hence conduction will occur. After this breakdown, current will flow throughout the remaining portion (shaded in Fig. 27–31) of

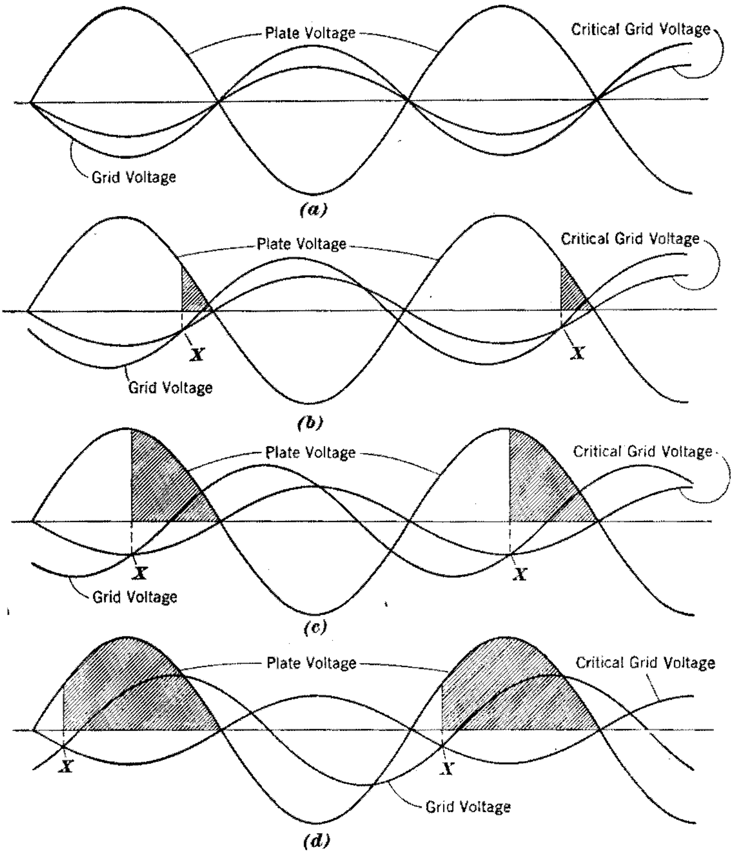


Fig. 27–31. Control of Plate-Circuit Current by Phase Shift of Grid Voltage

the positive half of the cycle. Because of the rectifying action previously described, no current will flow during the negative half-cycle. Also none will flow in the following positive half-cycle until the grid voltage again becomes less than the critical voltage. In this manner the effective value of the current permitted to flow in the plate circuit containing the load may be controlled by simply shifting the time phase of the grid potential.

The grid-glow tube starts almost instantaneously and requires no heating-up period. It is particularly useful in service where it must be ready for operation without notice and where instant starting is required. It does not deteriorate with age when not operating, and requires no power to keep it at operating temperature. It has a disadvantage, however, in that only a few milliamperes of current may be drawn in its plate circuit. Also, because its gas pressure is high and both electrodes are cold, the voltage at which breakdown will occur in the reverse direction from the anode to the cathode is not much higher than its normal cathode-to-anode breakdown voltage. For this reason, if a grid-glow tube is operated near its peak plate voltage, a momentary small increase in that voltage on alternating voltage may cause the tube to conduct in both directions.

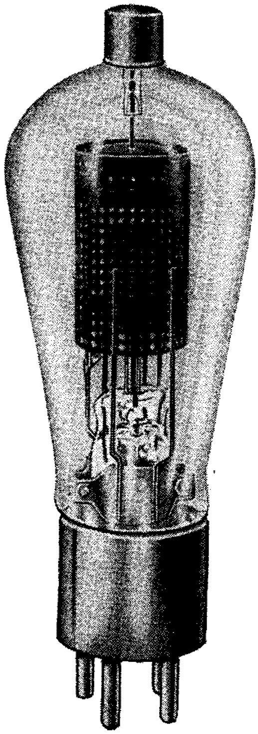

(Courtesy Westinghouse Electric Corp.)

Fig. 27–32. Thyratron

The Thyratron.—The thyratron, shown in Fig. 27–32, is similar in many respects to the grid-glow tube, but its dissimilar characteristics have made it a more generally useful tube. Its higher permissible plate current makes it readily applicable to motor control, welder control, sequence control in manufacturing processes, and innumerable other services. Its cathode is a filament or an electrode heated by radiation from a filament. The tube is filled with a gas, mercury vapor, or a mixture of the two.

The characteristics of thyratrons are similar to those of grid-glow tubes; but their electron emitters, being heated, will pass much greater current safely. The grid-to-cathode potential difference which will just start ionization depends largely on the voltage applied between the plate and the cathode. In Fig. 27–33 is a typical curve showing the relationship between the plate voltage and the grid-to-cathode potential or "bias." From this curve, for example, it is seen that a plate voltage slightly greater than 60 volts will start ionization and conduction through the tube, if there is no

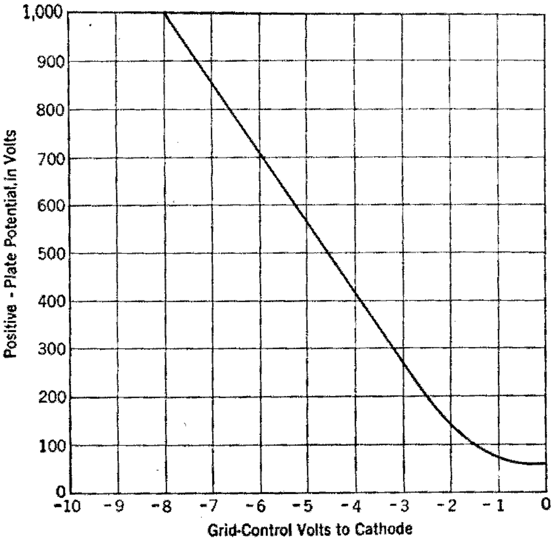


Fig. 27-33. Grid-Control Characteristics of a Thyratron

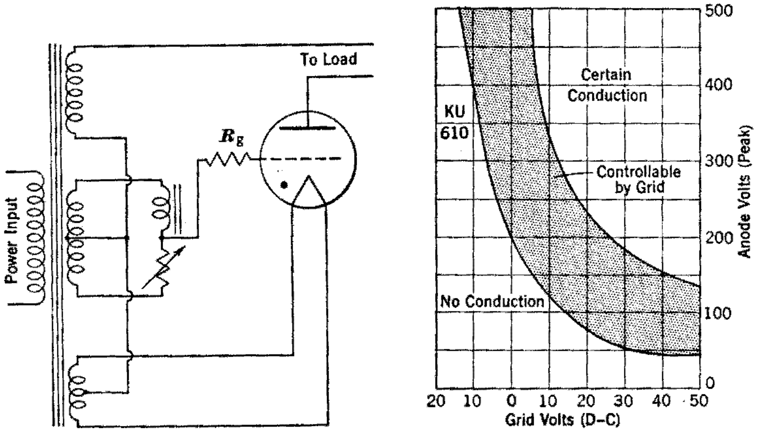


Fig. 27-34. Thyratron With Grid-Voltage Phase-Shift Control, Using Fixed Reactor and Variable Resistor

(Note Dot in Symbol Denoting a Gas or Vapor Tube)

grid-to-cathode bias. However, a grid voltage of only 6 volts will prevent ionization at any voltage below 700 volts. The grid voltage is so effective in preventing ionization because the grid is so close to the hot cathode, where the ionizing electrons are emitted.

After the grid voltage has triggered the thyratron, the grid is immersed in the avalanche of electrons passing from cathode to anode. The voltage drop between grid and cathode is then very low in that low-resistance path because of the plate current flow and is much lower than existed while the grid was below the triggering potential. To protect the grid circuit against overload current after firing, it is necessary to add a current-limiting resistor R_g in series with the grid.

Fig. 27–34 shows a thyratron and its phase-shifting circuit. In conjunction with the symbol for the tube in that circuit, there is shown a dot, which indicates gas in the tube. The effect of grid-voltage phase shift in a thyratron is essentially the same as shown in Fig. 27–31 for a grid-glow tube. Fig. 27–35 shows two thyratrons arranged in a circuit for full-wave rectification.

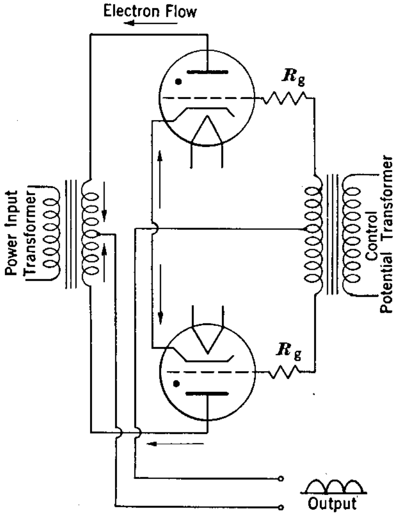
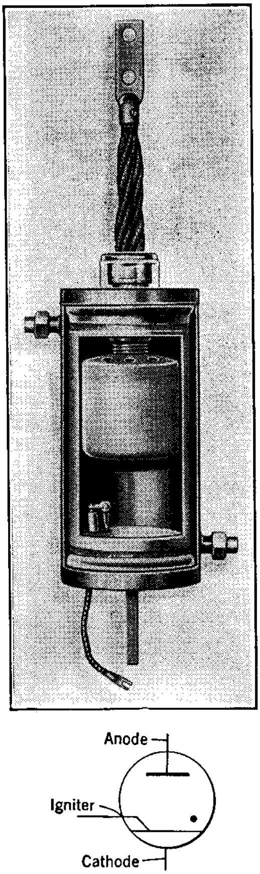


Fig. 27–35. Two Thyratrons Arranged in Circuit for Full-Wave Rectification



(*Courtesy General Electric Co.*)

Fig. 27–36. Cut-away View of an Ignitron and Also its Circuit Symbol

The Ignitron.—Of all gas-filled tubes the ignitron is capable of passing the highest plate current without injury, its physical dimensions being considered. Its construction is shown in Fig. 27–36. It consists of a double-walled cylindrical steel jacket enclosing a graphite anode, a mercury-pool cathode, and an "igniter" finger of silicon carbide or other substance of high resistivity which dips into the mercury pool. Water is circulated between the two walls of the jacket to keep the mercury vapor in the inside space at the proper pressure for the best results. The mercury, being an almost indestructible source of electrons, is capable of passing unusually high currents of electrons to the graphite anode. The anode is made of graphite to enable it to withstand the intense bombardment of these electrons. Because of these features the ignitron is capable of passing unbelievably large currents for heavy-power duty such as is found in electric spot-welding and the electrolytic reduction of ores.

The characteristics of operation of the ignitron are similar to those of the thyratron, but the control of ionization is accomplished differently. When an "igniting" current of a few amperes is passed between the igniter and the mercury cathode, a small arc appears at the area of contact and ionizes a portion of the vapor at that point. If at that time a sufficient voltage is present between the anode and the cathode, complete ionization of the vapor space occurs and conduction begins. Just as in the grid-glow tube and the thyratron, the igniter has no further control over the action of the tube until such conduction has been stopped by opening the plate circuit or by reducing the plate voltage below that required for maintaining the conduction.

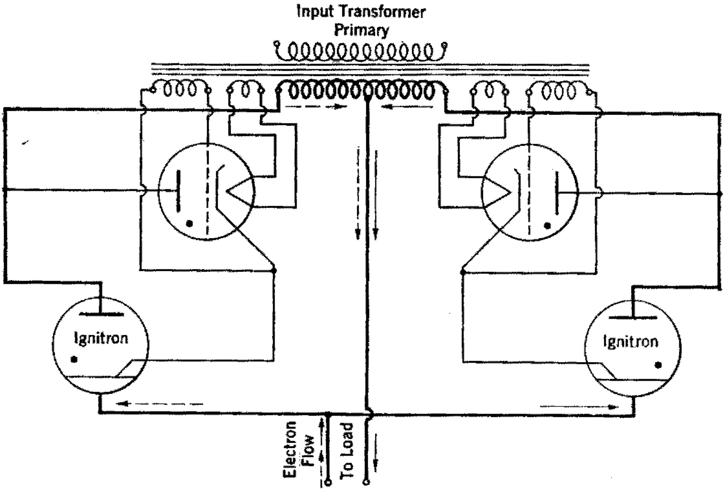


Fig. 27–37. **Two Ignitrons for Full-Wave Conduction With Thyratron-Controlled Igniter Circuits**

Unlike the thyratron, the ignitron does not require a warming-up period, and so it is always ready for instant use. It does, however, require much greater firing power in its igniter circuit than the thyratron does in its grid circuit. The power required for its firing is somewhat variable, and so the power in the igniter circuit must be comfortably higher than that required on the average in order to insure certainty of firing. If an ignitron fails to fire on an impulse from a thyratron, the igniter-to-cathode voltage will be permitted to rise far above normal, the igniter current will be excessive, and the protecting fuses in the igniter circuit will be blown. In the thyratron the grid-to-cathode voltage is held negative to prevent firing before the desired instant; whereas in the ignitron the conduction is started by positive action by the arcing between the igniter and the mercury-pool cathode.

Ignitron Circuits.—In Fig. 27–37 is shown a full-wave heavy-duty rectifier circuit with two ignitrons and two thyratrons controlling their igniter circuits. If current is allowed to flow backward in the igniter circuit, that is, from the cathode to the igniter, the igniter will be damaged. If a thyratron is not used to control the igniter current, some other type of rectifier, such as the copper-oxide rectifier described on page 342, must be used. The grid-control transformer is sometimes designed to give a sharp-peaked voltage output, so that the instant of firing of the thyratron may be more accurately controlled. The ignitron plate circuit and the ignitron are in parallel. When the ignitron fires, the thyratron is almost relieved of plate voltage and ceases to conduct. If, however, the ignitron fails to fire, the full peak of plate voltage is permitted on the thyratron; and damage will certainly occur unless the circuit is protected by a fuse or a current-limiting resistor.

The Mercury-Vapor Rectifier.—Mercury vapor may be used as the conducting medium in a diode of the gas-filled type, and is widely used in heavy-duty rectifiers. Just as in other types of diodes, the cathode of the mercury-vapor rectifier must be heated before conduction can take place. The use of mercury as the cathode permits the rectifier to withstand heavy momentary overloads without damage. During operation the mercury vapor is formed in the evacuated tube over a pool of mercury, which is the cathode. By tipping the tube momentarily, the mercury is caused to touch the anode and to produce an arc and vapor. The heat of the arc produces the necessary ionization of the mercury vapor, and conduction takes place through the external anode circuit if complete. However, after the first half-cycle of conduction is completed, ionization ceases and the conduction cannot restart until the tube is tipped again. Because of the practical difficulty of maintaining the current over the negative half-cycle of plate voltage, the mercury-vapor rectifying tube must be provided with a holding or "keeper" circuit when used for half-wave rectification.

Fig. 27–38 shows a type of full-wave rectifier, together with its circuit, as used in battery charging. The electrodes *A* and *D* of the rectifier tube are anodes. The electrode *C* is a starting anode only. To start the operation, the tube is tilted until part of the mercury from reservoir B spills into contact with C and starts an arc. The arc produces mercury vapor and ionizes that vapor. If, in the half-cycle in which contact is made, the anode D is positive to B, electrons will pass from B to D and a charging current will flow through the battery. On the next half-cycle, the anode A will be positive to B and the electron flow will be from B to A, causing a current in the battery in the same direction as before.

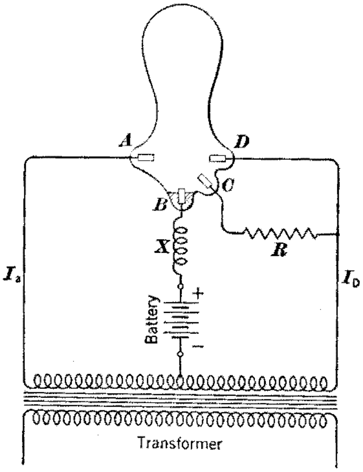
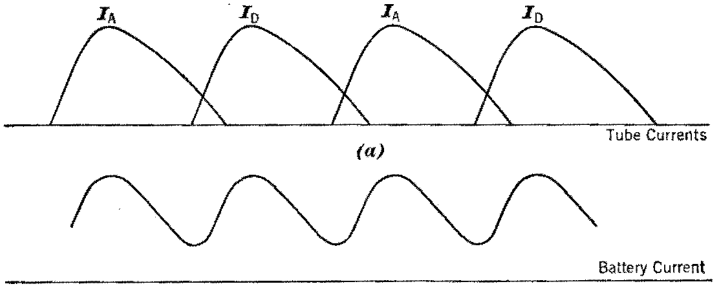


Fig. 27–38. **Mercury-Vapor Rectifier**



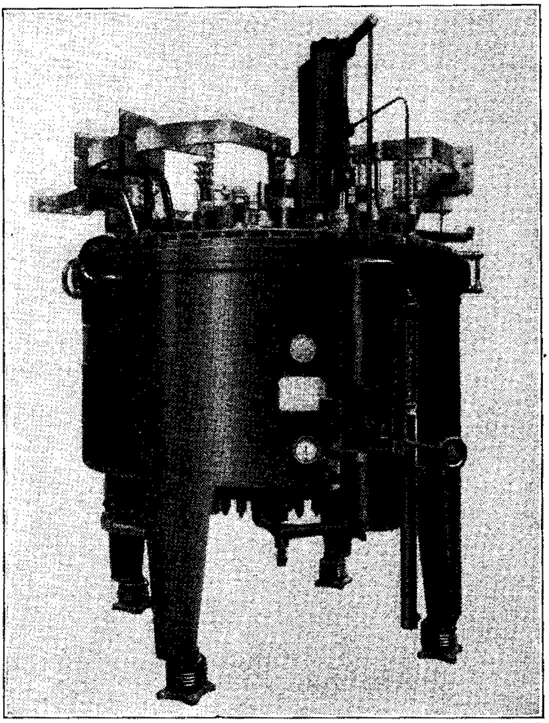
(a)

(b)

Fig. 27–39. **Mercury-Vapor Rectifier Output Current**

If the current of one half-cycle were allowed to fall to zero before the next half-cycle started, ionization would cease and rectification would be discontinued, as in the half-wave rectifier without keeper circuit. An inductance X inserted in the cathode lead causes the current flow to persist for more than 180 degrees of time, and so causes overlapping in time with the following half-cycle, as shown in Fig. 27–39 (a). Inasmuch as the resultant current of Fig. 27–39 (*b*) never falls to zero, the ionization and hence the tube conductance is never zero, and rectification continues from cycle to cycle.

Mercury-vapor rectifiers of steel-clad construction, like that shown in Fig. 27–40, are in use in street-railway and general-transportation service. They are usually built for operation on 3-phase supply. By proper transformer connections they may be operated as 6-phase or even 12-phase rectifiers. The ripple in the output of a 6-phase unit is less than that from a 3-phase unit, and the ripple from a 12-phase unit is even less than that from a 6-phase unit. The ripple sometimes causes objectionable noise in telephone systems whose lines parallel those of power circuits supplied by such rectifiers. The noise can be reduced to an acceptable minimum, however, by the use of filter circuits.

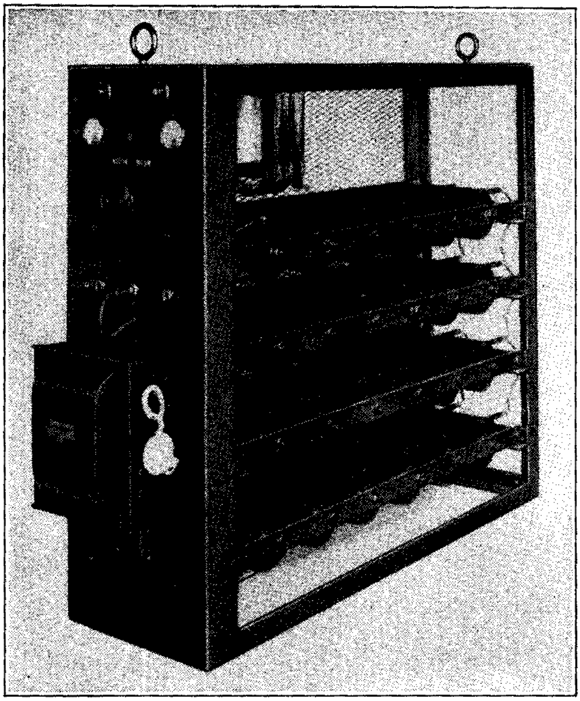


(Courtesy General Electric Co.)

Fig. 27–40. Mercury-Arc Rectifier

The Copper-Oxide Rectifier.—Some minerals and combinations of minerals and oxides demonstrate valve action when placed in an electric circuit. An outstanding example of this phenomenon is that of copper and its red oxide. A copper disc coated on one side with a thin layer of cuprous oxide exhibits a much higher electrical resistance when a current is passed through it in a direction from copper to oxide than when the direction is from oxide to copper.

Commercial rectifiers employing this principle may be bought complete or may be built up of the oxide-coated discs to suit a definite purpose. See Fig. 27–41. Contact is made on the bare side of the copper disc and also on the oxide-coated side by lead or foil washers in the tightly-bolted stack. Figs. 27–42 and 27–43 show typical circuits and arrangements of parts for copper-oxide rectifiers. Such rectifiers are widely used in radio receivers; in small battery chargers; in rectifier-type instruments described on page 183; and in many other services, such as in electroplating, industrial truck-battery charging, electrolysis prevention, welder controls, airplane-engine starting, motion-picture projection arcs, "quick-chargers" for batteries, and other high-current loads. Generally speaking, this rectifier



(Courtesy Westinghouse Electric Corp.)

Fig. 27–41. Copper-Oxide Rectifier

(Note the heat-radiating fins)

is competitive with motor-generator sets up to 12 volts at any current. Above 12 volts the choice of a rectifier would depend on other factors, such as reliability, freedom from moving parts, negligible amount of maintenance required, and ease of control from the a-c side.

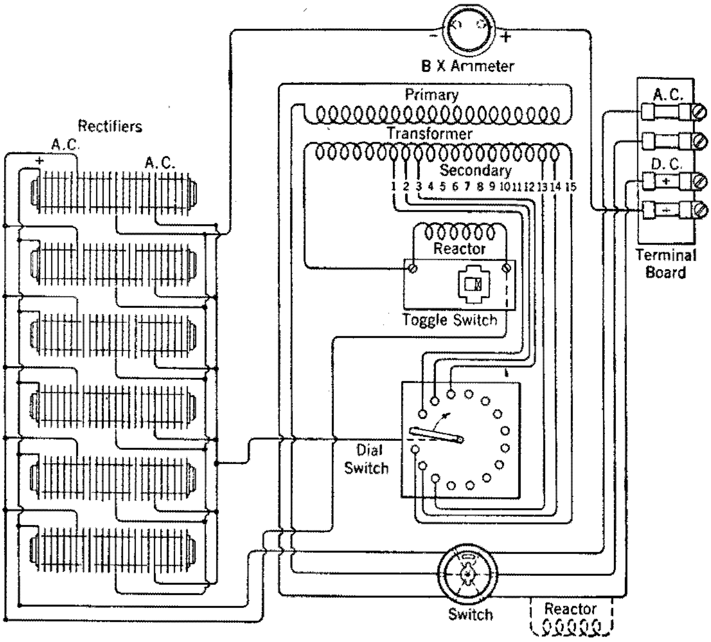


Fig. 27–42. Circuit Diagram of Fig. 27–41

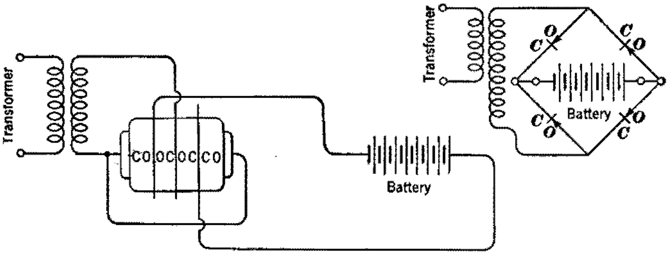


Fig. 27–43. Circuit of Full-Wave Copper-Oxide Rectifier

Photoelectric Cells.—The photoelectric cell, or phototube, is a diode in which the cathode is usually of half-cylindrical shape and covered inside with a material such as lithium, potassium, rubidium, sodium, or cesium which is capable of emitting electrons when it is struck by light. The anode is a small metal rod located at the axis of the cathode, and is made slender so as to cut off as little light as possible. When light strikes the emitting material, electrons are emitted as from a hot cathode but in relatively much smaller numbers. The resulting current is very small, and the plate voltage must be high to force that current through a resistance large enough to produce an IR drop that is usable as output voltage to the grid circuit of an amplifier. Fig. 27–44 shows a simple circuit for a phototube.

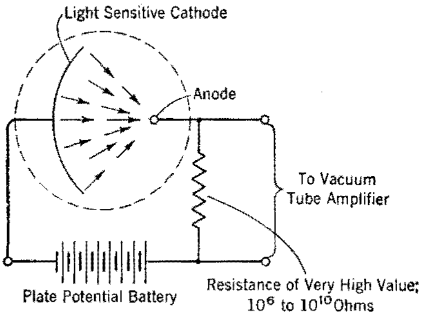


Fig. 27–44. Simple Photoelectric-Cell Circuit

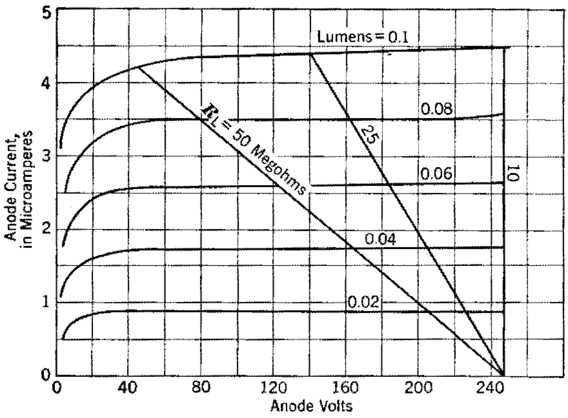


Fig. 27–45. Sample Characteristics of Vacuum Phototube

The phototube is not equally responsive to all colors or wavelengths of light. Lithium is most responsive to the blue region of the spectrum, while cesium is most responsive to the red region. By combining different metals the spectral response may be extended over a wide range. Phototubes are made in both vacuum and gas-filled types. The characteristics of the vacuum type are shown in Fig. 27–45. The vacuum phototube has almost linear response to variations of light intensity if the color quality of the light remains unchanged. It is therefore adaptable where reproducible results are necessary. The load line may be used on phototube characteristics the same as on those of a triode.

The Gas-Type Phototube.—When a suitable gas is introduced in a phototube, the passage of electrons causes ionization by collision the same as in a gas-filled triode, nullifies the space charge, and produces greater current than it would if the gas were not present. The gas-filled phototube is therefore not space-charge limited as the vacuum type is, and the plate current can be increased by increasing the plate voltage even above that which will draw all emitted electrons from the cathode. The increase in plate current with increase in plate voltage occurs because, in the migration of an electron from cathode to anode, the higher the plate voltage the greater

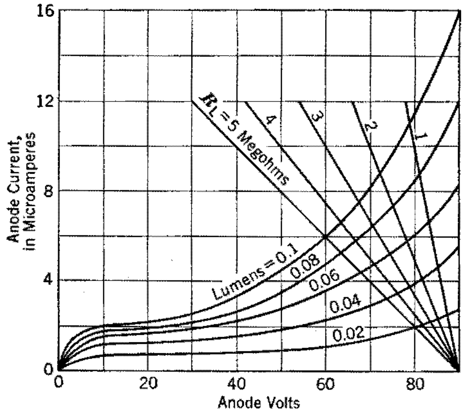
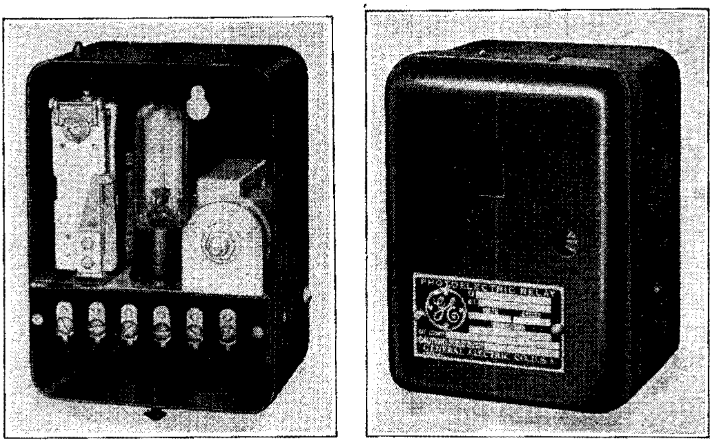


Fig. 27–46. Sample Characteristics of Gas-Type Phototube



(*Courtesy General Electric Co.*)

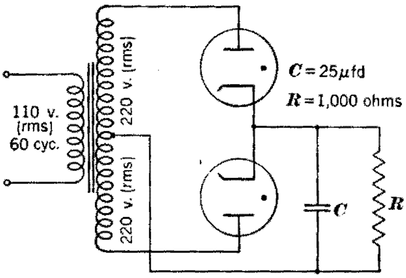
Fig. 27–47. Photoelectric Relay

the acceleration of the electron after a collision and so the more ionizing collisions it can have before reaching the anode. Fig. 27-46 shows a family of characteristic curves of the type 918 gas-filled phototube for different intensities of illumination. In applications where linearity of response is unimportant, such as "on" and "off" relay operation, the gas-filled phototube is preferred over the vacuum type because of its greater current and sensitivity.

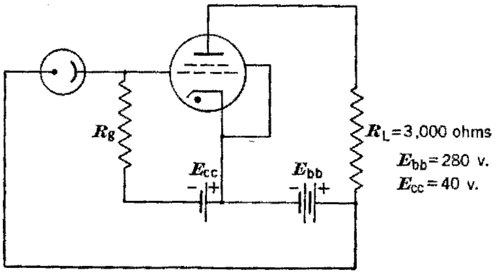
Anode voltages of gas-filled phototubes are generally limited to 90 volts or less because higher voltages are likely to lead to a sustained uncontrolled discharge. Fig. 27-47 shows a photoelectric relay using a gas-filled phototube.

PROBLEMS

1. Calculate the ripple voltage amplitude of a half-wave rectifier with a load resistor of $R = 5000$ ohms in parallel with a condenser of $C = 100$ mfd. The frequency of supply is 20 cycles. Use the simplifying assumptions relating to wave form.



Problem 27-3



Problem 27-4

2. Calculate the amplification factor of the 6C5 triode whose plate characteristics are shown in Fig. 27-11.

3. What is the average direct voltage across the load in the accompanying circuit? Assume that the tubes are perfect rectifiers.

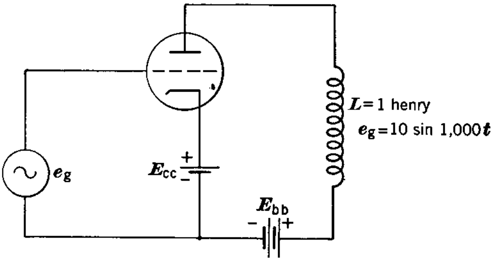
4. In the accompanying circuit it is necessary to make the gas tube fire by increasing the light on the phototube from 0 to 0.03 lumen. Assume that the gas tube fires when its

grid is at zero volts. What must be the value of R_g to just fire the gas tube? Use the characteristic curves of Fig. 27-45 for a vacuum-type phototube.

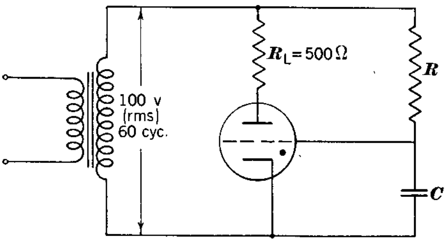
5. The following data are found in the tube manual for the triode in the accompanying circuit:

Plate voltage	250 v.	Heater current	0.3 amp
Grid bias	−20 v.	Amplification factor	8
Plate resistance	1700 ohms	Transconductance (g_m)	4700 micromhos
Heater voltage	6.3 v.	Plate current	40 ma

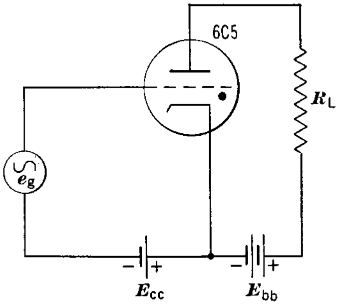
Find the values of E_{bb} and E_{cc} to give the specified operating conditions.
Find the effective value of alternating voltage across L.
Find the total power input to the triode with zero a-c grid voltage.



Problem 27-5



Problem 27-6



Problem 27-7

6. Assume that the thyratron in the accompanying circuit fires when the grid voltage is zero, and that there is no tube drop. If $C = 1$ microfarad and $R = 5000$ ohms, at what angle will the triode fire? If R is adjusted so that the thyratron fires at 45 deg, what is the average current through R_L?

7. In the accompanying circuit is shown a 6C5 triode connected as an amplifier. $E_{cc} = -8$, $R_L = 50{,}000$, and $E_{bb} = 300$. Draw the load line and determine the d-c component of the plate current. What are the maximum and minimum voltages across R_L, if e_g is a sine wave with an amplitude of 2 volts? Find r_p for this operating point.

CHAPTER 28

Electronic Applications

Circuits for Electronic Applications.—In electronic industrial control, there are a fairly well defined set of phenomena which are most frequently used. It is the purpose of this chapter to describe some of the most common fundamental circuits and available commercial devices in which they are used. These devices use one or more of the fundamental circuits in combinations to perform the desired functions. In addition to the common elements, fundamental circuits for these devices usually contain resistors, condensers, or inductances, strategically dispersed according to the experience of the designer, to improve performance for a definite or restricted purpose. Use is also made of some "trick" circuits which are too numerous and varied to include here.

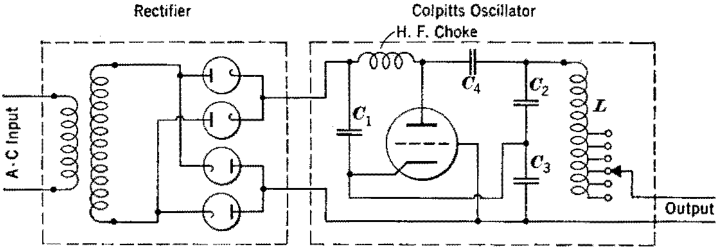


Fig. 28–1. **Fundamental Circuit With Colpitts Oscillator for Converting Low-Frequency Power to High-Frequency Power**

High-Frequency Oscillators.—High frequencies are here defined as those above 50,000 cycles per second. For industrial power supply the Colpitts circuit, Fig. 28–1, or the coupled-grid self-excited oscillator circuit, Fig. 28–2, is most commonly used. In either circuit, the alternating supply voltage is stepped up through a power transformer to some voltage between about 7500 and 15,000 volts and is then rectified by a suitable bank of mercury-vapor rectifier tubes. In Fig. 28–1 the condensers C_2 and C_3 form an anti-resonant circuit with *L* and the inductance of the load which may be an induction heating coil fed at the output terminals. The inductance *L* is tapped so that, even though different heating coils may be used, the total inductance of the circuit can be kept at proper value for anti-resonance at the desired frequency. Condenser C_4 is a "blocking" condenser and prevents the flow of direct current into the resonant circuit. The high-frequency choke coil passes the direct current to the anode of the oscillator tube, but presents a high limiting impedance to the high-frequency currents which would otherwise flow back into the rectifier. The condenser C_1 by-passes back to the cathode any high-frequency current which does pass through the choke.

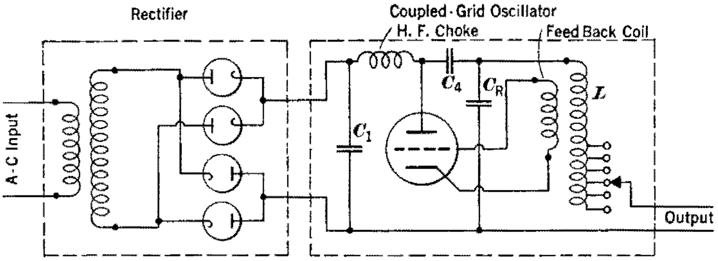


Fig. 28–2. Fundamental Circuit With Coupled-Grid Oscillator for Converting Low-Frequency Power to High-Frequency Power

The voltage across condenser C_3, being applied between the grid and the cathode of the oscillator tube, feeds back part of the tube's anode output, and so causes the tube to be an amplifier and hence a self-regenerator at the frequency established by the constants of the anti-resonant circuit. The oscillator tube is a high-voltage vacuum tube sometimes known as a *pliotron.*

In Fig. 28–2 the grid voltage is supplied by a feed-back coil inductively coupled to the variable inductance L. The condenser C_R replaces the condensers C_2 and C_3 of Fig. 28–1.

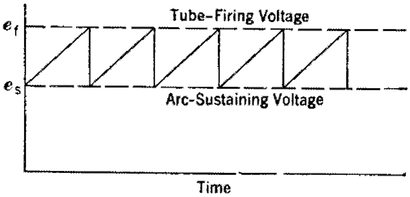


Fig. 28–3. Saw-Toothed Voltage Output of a Relaxation Oscillator

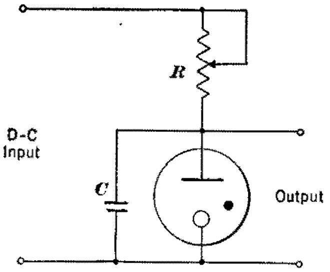


Fig. 28–4. Relaxation-Oscillator Circuit Using Gas-Filled Diode

Relaxation Oscillators.—The oscillator known as a relaxation oscillator works on the principle that the rate of charging a condenser from a d-c supply can be controlled by a series resistor, much as the rate of filling a tank with water at constant pressure can be controlled by throttling the discharge pipe. The chief use of this type of oscillator is in the production of periodic voltage pulses which rise from one value to a higher value linearly in time as shown in Fig. 28–3. When voltage is applied to the circuit represented in Fig. 28–4,

$$V = Ri + \frac{1}{C}\int i\,dt \qquad (28\text{–}1)$$

The term $\frac{1}{C}\int i\,dt$ is the voltage across the condenser C and is a function of time. The voltage e_c across the condenser rises, as in Fig. 28–3, until it reaches the diode firing voltage. The diode then fires, and the condenser discharges through its arc until e_c falls below the arc-sustaining value. The condenser then proceeds to recharge, and the cycle is repeated. The voltage e_c appears at the output terminals, as shown in Fig. 28–4. The lower the resistance R, the faster will the condenser charge, and the faster will the cycle be repeated. Thus, the frequency is readily controlled by adjusting the resistance R. This type of oscillator is used in the cathode-ray oscilloscope to sweep the beam along the reference axis at a constant speed and then to return it very quickly to its starting point for repetition. Another circuit, using a gaseous triode, is shown in Fig. 28–5. Resistors

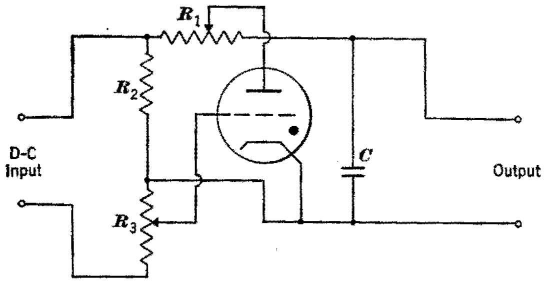


Fig. 28–5. Relaxation-Oscillator Circuit Using Gas-Filled Triode

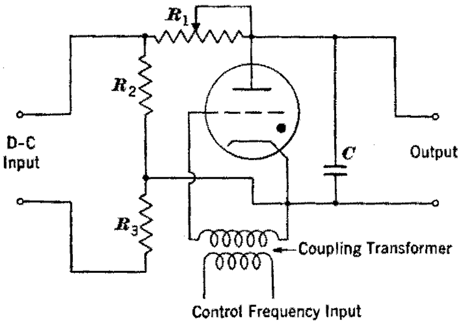


Fig. 28–6. Relaxation-Oscillator Circuit With Grid Synchronized With External Frequency

R_2 and R_3 form a voltage divider by which the grid voltage may be adjusted so that the firing voltage of the tube condenser, and hence of the tube, may be controlled. The frequency of discharge is controlled by the resistance R_1.

It is sometimes desirable to synchronize exactly the frequency of a relaxation oscillator with that of some other frequency source, such as a 60-cycle power line. This synchronizing may be accomplished by using the circuit of Fig. 28–5 modified as in Fig. 28–6. In operation, the synchronizer acts to fire the tube at one definite point on the control frequency cycle a brief instant before the oscillator condenser alone would have fired it. To accomplish this, the oscillator frequency is adjusted by resistor R_1 to a value just below that at which it is to be synchronized, as shown in Fig. 28–7. Let it be assumed that, on the first positive half-cycle of control frequency shown, the oscillator tube would have fired at exactly the correct time without grid-voltage urge. On the next positive half-cycle, it would have fired slightly too late without the urge of the grid-control voltage. Every successive cycle of the oscillator then is caused to start at the same instant on its corresponding grid-control voltage, and the oscillator-output frequency is the same as the grid-voltage frequency regardless of its tendency to lag. It is obvious that synchronizing cannot occur if the natural oscillator frequency is greater than that desired, because the tube could fire when the necessary plate voltage was reached irrespective of the urge or control by the grid voltage. Cathode-ray oscilloscopes use this method for producing stationary screen patterns.

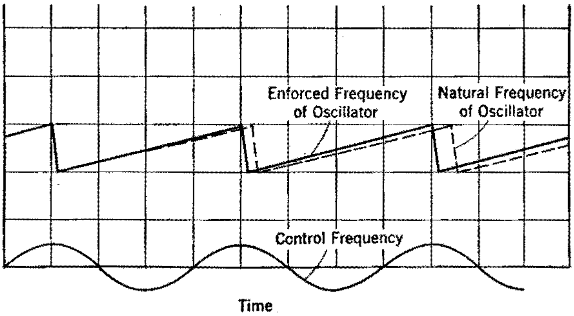


Fig. 28–7. Forced (Synchronized) Frequency of a Relaxation Oscillator Produced by Control-Frequency Excitation of Grid

Filter Circuits.—The output of a rectifier, as shown in Fig. 27–3, is satisfactory for charging a battery or for operating a relay, but for most purposes it must be smoothed out until practically pure direct current remains with very little "ripple." This smoothing is accomplished by means of filters consisting of networks of R, L, and C circuit components arranged in a unique fashion. Fig. 28–8 shows a very simple filter consisting of one L unit or "choke" and one condenser interposed between the rectifier and the load. For only moderate smoothing of the current, this simple filter may be sufficient; but, for more complete purification of current output, more units must be added as in Fig. 28–9 and Fig. 28–10.

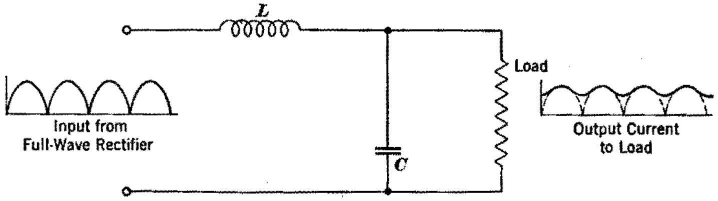


Fig. 28–8. Simple Filter

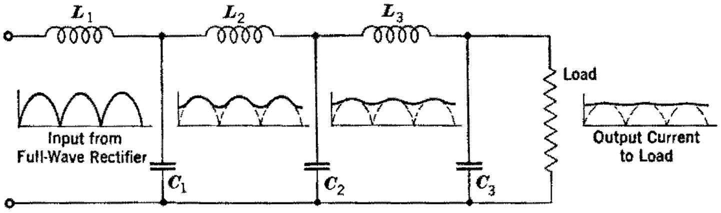


Fig. 28–9. Inductance-Input Filter With Three Sections

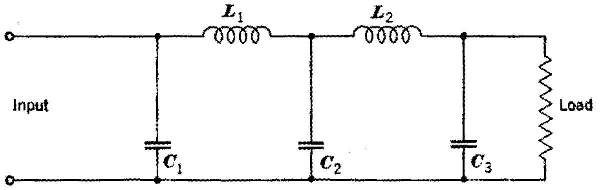


Fig. 28–10. Condenser-Input Filter

In Fig. 28–9 the current encounters the choke L_1 first, and so the filter is called an *inductance-input filter*. This choke offers high impedance to the high-frequency components of currents and reduces them, but gives the fundamental-frequency and d-c components little opposition. The fundamental component divides; part of it passes on through choke L_2 and part of it charges condenser C_1. As the component passing through L_2 starts to decrease, C_1 discharges and tends to maintain the current in L_2. Thus, the current in L_2 never falls to zero but varies about as shown in the graph at L_2. Successive filter units L_1C_1, L_2C_2, etc. may be added until the output is sufficiently pure for the purpose intended.

In Fig. 28–10 the current encounters the condenser C_1 first and the filter is called a *condenser-input filter*. The filtering action is essentially the same as that in an inductance-input filter.

In comparing the two types of filters, it is seen that in the condenser-input filter the last condenser is a reservoir of energy from which load energy may be drawn almost steadily while the replenishment is intermittent. If the load current be increased, it must be drawn at a reduced voltage because the intermittent input increments are not sufficient to maintain the originally higher state of charge. For this reason the condenser-input filter has a poor voltage regulation and is poorly suited for varying load.

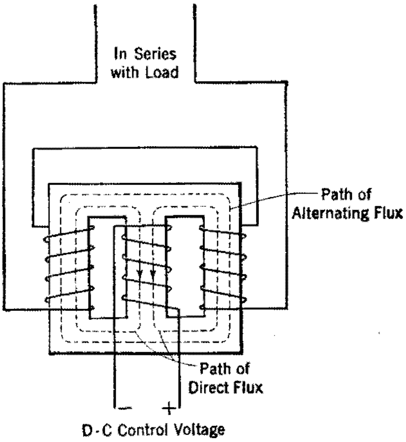


Fig. 28–11. Circuit With Saturable Reactor

An inductance-input filter must have a certain minimum current in its inductances to be effective. It therefore follows that with light load the filtering action is less satisfactory than at loads nearer rated load. The voltage regulation of this type of filter is better than that of a condenser-input filter, but the filter should always be at least partly loaded for best results. The inductance-input filter is best suited to varying power loads.

Saturable Reactors.—Because of the non-linear shape of the saturation curve of iron, the flux per ampere, and hence the inductance of an iron-core reactor, is a complex function of the flux density in the core. As the flux density increases beyond the first approximately straight portion of the magnetization curve, the reactance decreases rapidly. By superposing a uni-directional but variable mmf on the core, a variable reactor without moving parts may be obtained. Fig. 28–11 shows one of the simpler methods of doing this. The a-c coils produce simultaneous mmf's, and hence flux, around the long outer magnetic circuit but none of appreciable magnitude in the middle leg. In this way, no a-c voltage is induced in the d-c control coil on the middle leg. Flux produced by the d-c control coil divides between the outside two legs and produces a variable "pre-saturation" in them. The core is said to be "biased" by the d-c ampere-turns. The effect of bias on the reactance of the coil is evident in Fig. 28–12. If the core is biased by 2 ampere-turns per inch and an alternating current producing $NI_m = 0.5$ is passed through the a-c coil, the reactance of the coil will swing from 27.5 ohms when the alternating current is negative to only 13 ohms when the current is positive. The average reactance is about 20.2 (from the graph). However, the average reactance affecting the negative part of the a-c cycle is about 14.5 ohms and that affecting the positive half is about 16.5 ohms. Therefore, when the core is *biased* to a point on the bend of its magnetization curve, the reactance is not the same on both halves of the current cycle and the current will be distorted.

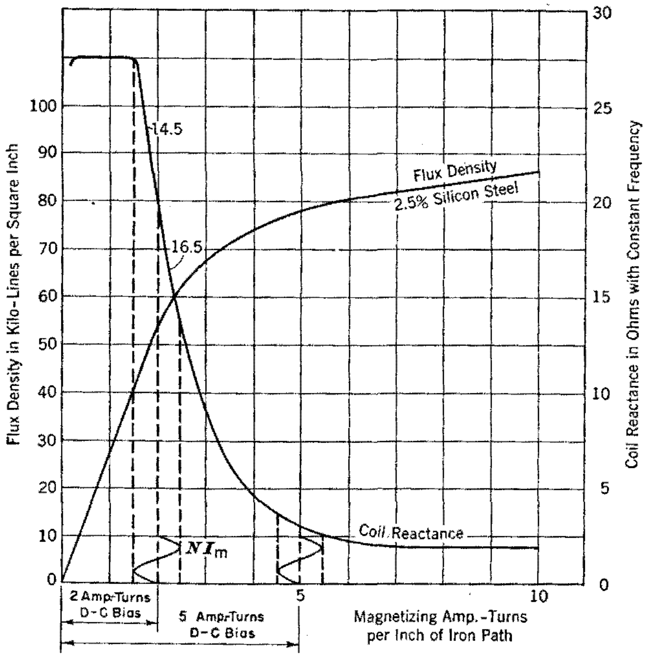


Fig. 28–12. **Reactance Variation in a Saturable Reactor**

If the bias were 5 ampere-turns per inch, the average reactance would be only 3 ohms and the current would be distorted less than with a 2-ampere-turn bias. With a bias of only 1 ampere-turn, the maximum reactance would be 27.5 ohms and both halves of the current would be equally affected. In Fig. 28–12 a change in reactance in the ratio of 10 to 1 is accomplished by a change in direct control current of less than 4 to 1. For some applications the distortion of current caused by biasing the core to a point near the curved part of the magnetization curve is not objectionable. In other cases such distortion is objectionable and the use of the saturable reactor must be restricted to the more nearly linear parts of the curve.

Peaking Transformers.—It is sometimes desirable to have the output voltage of a transformer sharply peaked instead of sinusoidal in form. One use for the peaking transformer is found in the grid-firing circuit of a thyratron, where the thyratron must fire at exactly the same instant in every cycle under varying load conditions or with changing characteristics of the thyratron caused by aging. In the usual type of transformer, as the flux density rises into the curved part of the magnetizing curve, more magnetizing current is drawn from the line to maintain the sinusoidal form of the flux. As a result, the normal magnetizing current is of the form shown as i_e in Fig. 28–13 (a).

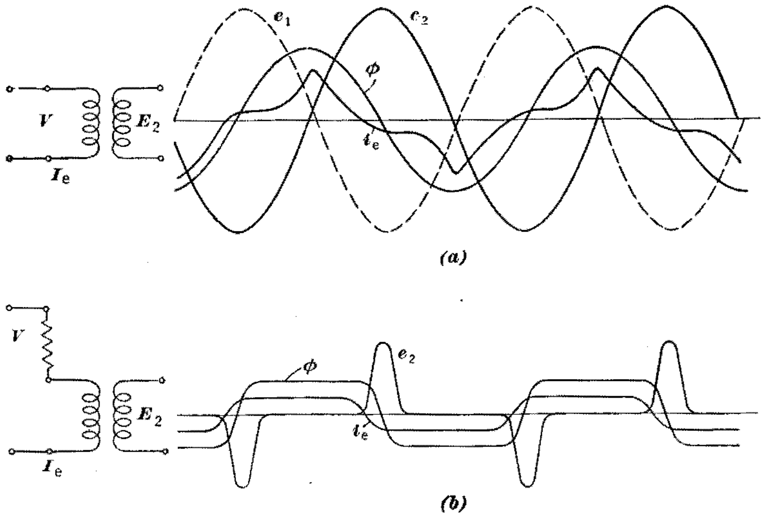


Fig. 28–13. **Wave Forms in a Peaking Transformer**

If a resistance is placed in series with the transformer primary, it will tend to dominate the impedance and hence to subdue the peak of current which would flow if the resistance were not present. The failure of the current to reach its normal peak causes the flux wave to be "starved" where its peak would normally be, and so the flux wave tends to be flat-topped. As a result the voltage is practically zero while the flux is approximately constant at its maximum, and sharp peaks of voltage are produced near the instant when the flux is zero, as shown in Fig. 28–13 (b).

Phase-Shifting Devices.—In the application of many electronic devices it is necessary to shift the phase of the grid voltage relative to the anode voltage, in order to adjust the tube firing point in the anode-voltage cycle. The circuits of Fig. 28–14 are suitable for such phase control. By proper choice of the circuit elements in (*a*), (*b*), and (*c*), the angle of voltage phase shift, θ, may be made as great as 165°. Any of these circuits may be found incorporated in control circuits requiring phase shift.

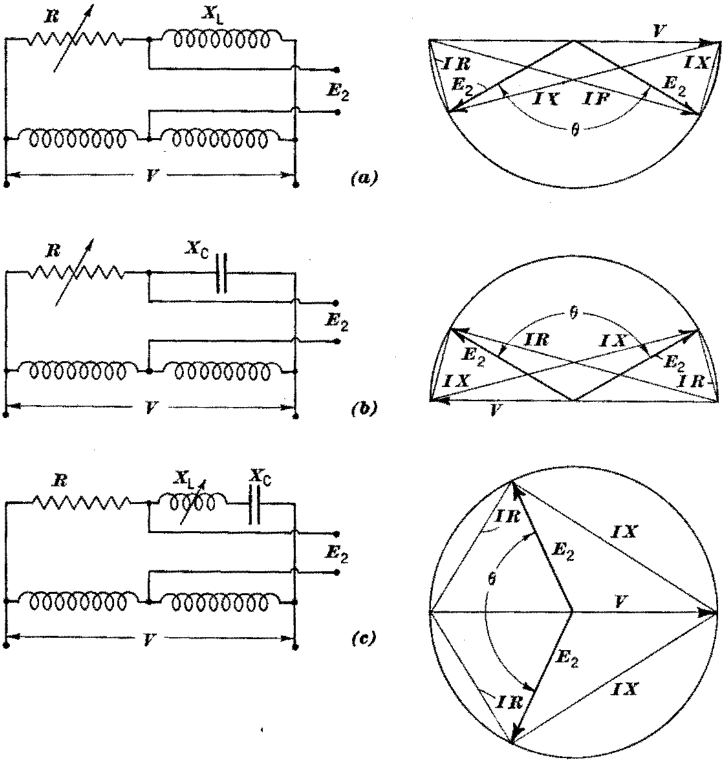


Fig. 28–14. Voltage Phase-Shifting Circuits

Phase of Voltage Obtained From *RL* Phase-Shifter.—It is necessary when using a simple phase-shifter of limited range, such as that shown in Fig. 28–14 (*a*), to connect the phase-shifting elements *R* and *L* in the proper relation to the applied voltage in order that the resulting control voltage will be variable in the proper phase range of plate voltage. This proper relationship will be demonstrated by means of Fig. 28–15.

Let it be assumed that e_c (negative from cathode to grid) is large enough to block conduction in the tube when it lags 180 degrees behind the voltage e_b, as in (*b*). That condition is shown vectorially in (*d*) and by the vector E_c in position qp in (c). To permit the tube to conduct late in its positive half-cycle, it is necessary for e_c to lag less than 180 degrees behind e_b in (b), in order that e_c will lose control of e_b before e_b falls to zero as in (e). The phase change of E_c in (c) that can be accomplished by changing R is a phase lag α with ideal R and L. In any inductance coil there is resistance, and E_c must always lead the position qp in (c). With an RL phase-shifter, therefore, conduction cannot be started in the first few degrees of the half-cycle because of the resistance remaining in the inductance coil. The RC phase-shifter allows control to start practically at the beginning of the conducting half-cycle because a condenser of high quality has negligible series resistance.

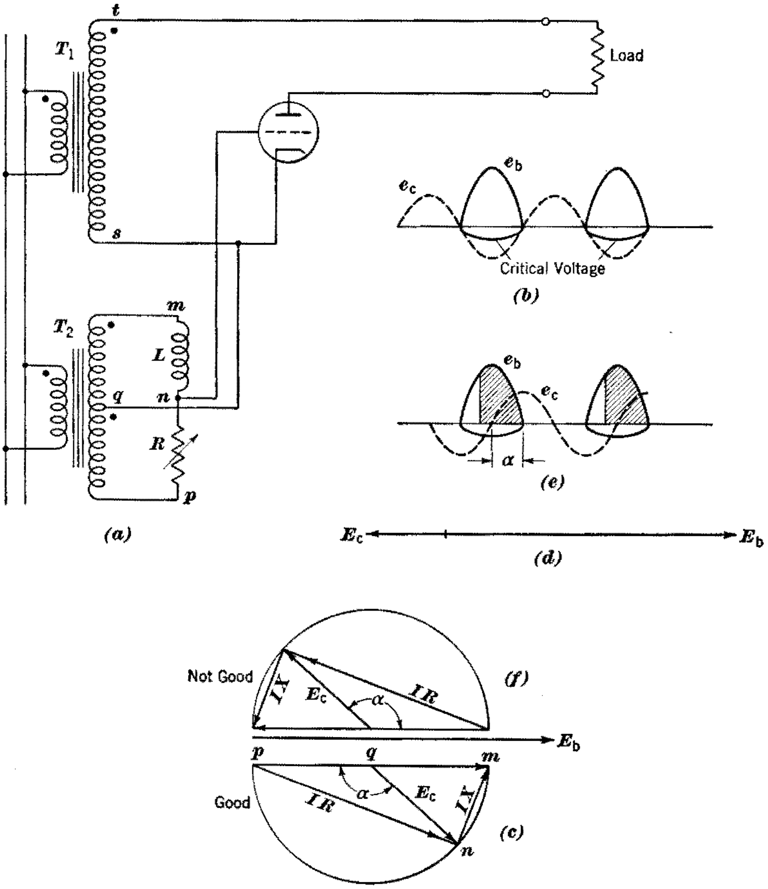


Fig. 28-15. Triode Controlled by RL Phase-Shifter

If the connections of the phase-shifter to the transformer T_2 were reversed, the locus of E_c would be as in (*f*). In that case the tube grid would always be positive at the beginning of the conducting half-cycle and could effect no control of the plate current. The same lack of control would result if the connections to T_2 were not reversed but R and L were merely interchanged in the circuit.

In Fig. 28–15 (*a*) a convention known as the *dot convention* is used to indicate relative instantaneous polarities in different parts of the circuit.

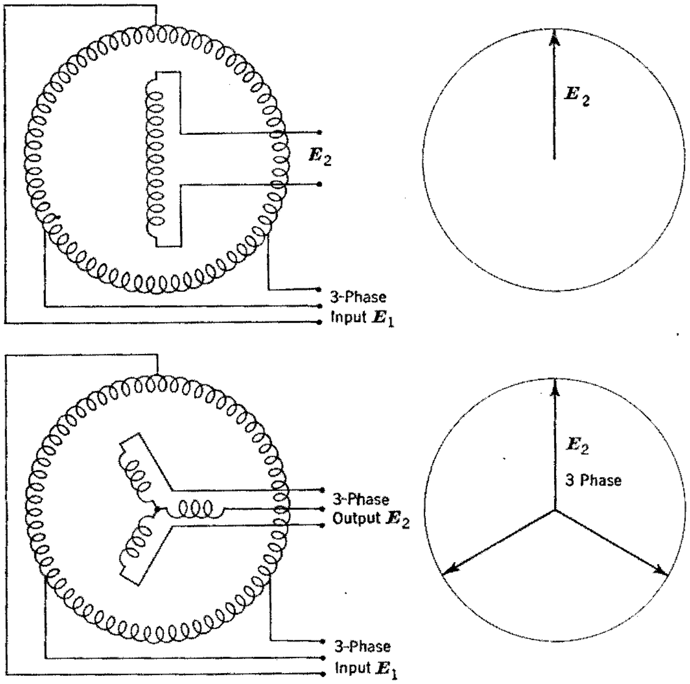
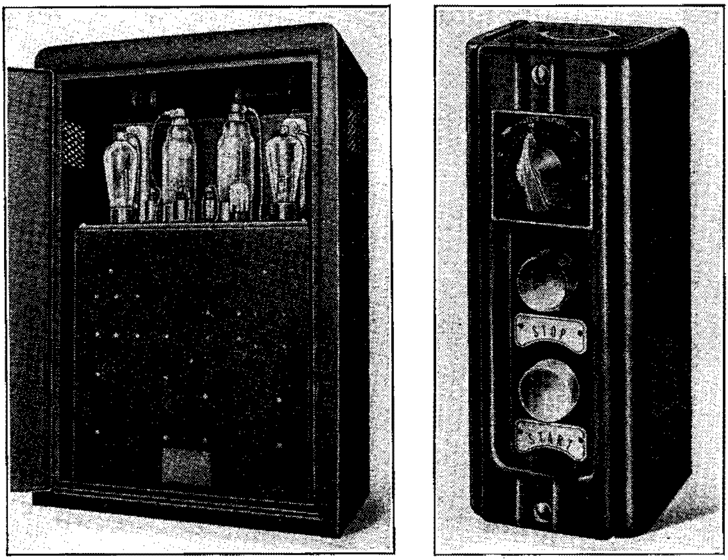


Fig. 28–16. **Phase Shift Using Variable-Position Rotor in a Three-Phase Stator**

A dot at the terminal of a transformer such as T_1 means that when this terminal is instantaneously positive any other dotted transformer terminal in the circuit is also positive with respect to the other end of its coil. In Fig. 28–15 (*a*) when the plate is made positive by the dot terminal of transformer T_1, point m of the RL circuit connected to transformer T_2 is also positive. All points in R and L between m and p are negative to m and therefore negative to the plate of the tube. The degree to which point n and the grid are negative to m depends on the value of the variable resistance R which controls the phase relation of the voltage E_c between q and n with respect to the voltages pm and st.

The grid voltage E_c would be constant in magnitude, regardless of its position in its range of shift, if the inductance L contained no resistance. That resistance, however, causes E_c to decrease somewhat as R in the phase-shifter is increased.

Fig. 28–16 shows the use of rotatable elements, known as "selsyns" or "auto-syns," for obtaining the full 360 degrees of phase shift either single-phase or polyphase, as used in multi-electrode tank-type power rectifiers.

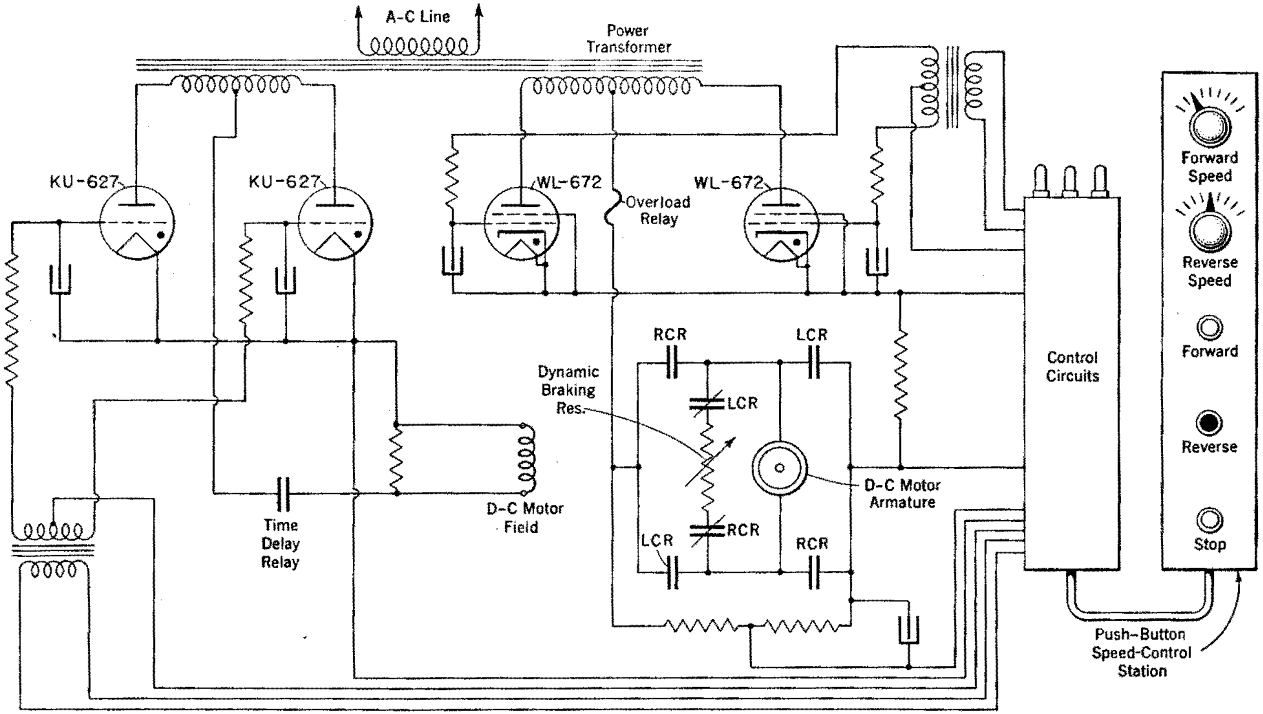


(*Courtesy Westinghouse Electric Corp.*)

Fig. 28–17. Mot-O-Trol Cabinet With Control Station

Direct-Current-Motor Control.—In Chapter 11 it was shown that the speed of a shunt or compound motor can be controlled within a limited range by shunt-field current adjustment, but that for speeds below this range a reduced voltage across the armature becomes necessary. By using electron tubes both of these means become available, and the power is taken from single-phase or polyphase a-c power supply. Several circuits for accomplishing this control have been developed but, inasmuch as all have the same essential principles, the description of a typical one will suffice.

The Mot-O-Trol (Westinghouse).—Fig. 28–17 shows the Mot-O-Trol unit complete with control station. The circuit for this controller is shown in Fig. 28–18. The armature power is obtained from the power transformer, and the current is rectified by the two thyratrons WL-672 producing full-wave rectification. The WL-672 tube is a gas-filled tetrode.

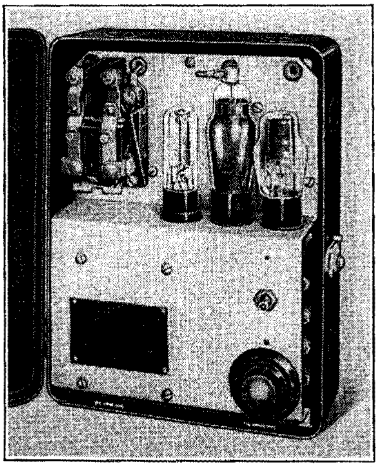


(*Courtesy Westinghouse Electric Corp.*)

Fig. 28-18. Simplified Control Circuit for Mot-O-Trol

The KU-627 tubes are quite similar to the triode mercury-vapor thyratrons previously described. They act as full-wave rectifiers to provide the direct current for the motor shunt field. The time-delay relay in the field circuit prevents current flow in the plate circuits of the two KU-627 tubes until their filaments have reached a proper operating temperature in starting. Dynamic braking is accomplished by closing the armature circuit through LCR and RCR and the variable resistor between them.

The effective current through the armature, and hence the voltage across the armature and in turn the motor speed, is controlled by a phase-shift circuit applied to the WL-672 grid voltage. A speed change of as much as 50 to 1 may be accomplished in this way, with the field excitation held constant. Additional speed change from 2 to 1 up to 4 to 1 (special motors) may be obtained by field-current control accomplished in the same manner by phase shift of grid voltage on the KU-627 tubes. In Fig. 28–17 both phase-shift potentiometers are controlled manually by the speed-control knob at the top of the operating control station.



(Courtesy Westinghouse Electric Corp.)

Fig. 28–19. Inside View of Photo-Electric Relay

Speed regulation of a motor whose armature is fed through electron tubes is poor, if no compensation is provided. This compensation is accomplished by means of the two-part (tapped) resistor just below the motor armature of Fig. 28–18. As the motor is loaded, it takes more armature current; and, so, less voltage appears across the armature and the speed-compensating two-part resistor in parallel with it. The voltage across the right-hand portion of this resistor is applied to the grid of a control tube in the control-circuit cabinet. As this voltage decreases with increase in load, the control tube acts to advance the firing angle of the WL-672 tubes and so to increase the armature voltage.

The Electronic (Light-Sensitive) Relay.—The light-sensitive relay has a wide field of applications, a few of which are as follows:

Opening doors
Counting moving parts on a conveyor belt
Selecting objects according to color
Counting vehicles
Operating burglar alarms
Leveling an elevator at any floor
Lighting lamps on approaching darkness

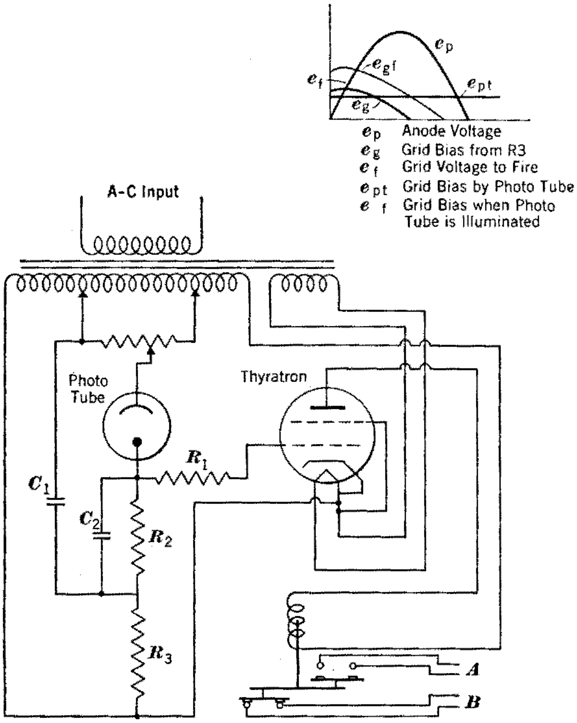
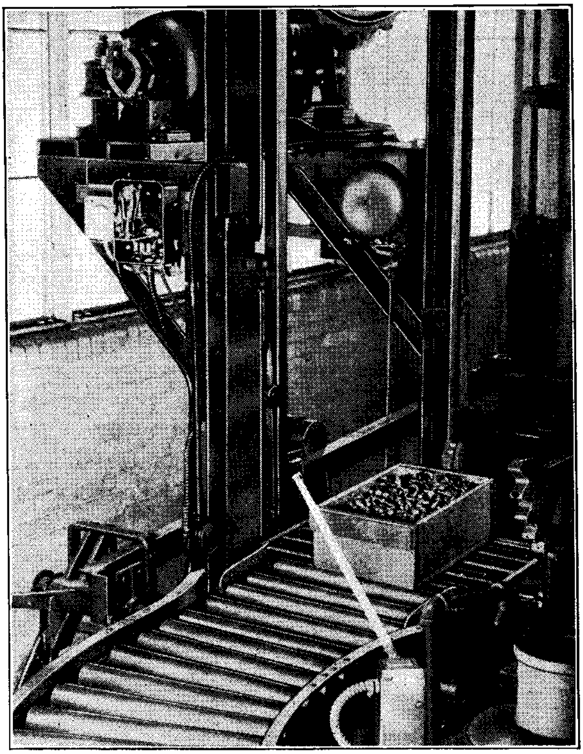


Fig. 28–20. Electronic-Relay Circuit for Control by Light Beam

These devices operate when a light beam is broken or dimmed below a minimum intensity. Fig. 28–19 is an inside view of such a device, and Fig. 28–20 is a sample of the circuit which is sometimes used.

In Fig. 28–20 the contacts to be opened or closed when the phototube is darkened are at *A* and *B*. The normal at-rest condition obtains when the thyratron is conducting and so holding the relay contacts up. The thyratron conducts only when the phototube is illuminated, and so the device operates on interruption of either the light source or the power source.

In operation, the phototube current, which is uni-directional, passes through resistor R_2, condenser C_1, and the potentiometer from which it receives its voltage. Condenser C_2, in parallel with R_2, is charged also; and discharges through R_2 when the phototube half-cycle current starts to decrease. This condenser therefore helps to maintain a positive grid bias on the thyratron, and insures its conducting. The circuit consisting of C_1, R_3, and a portion of the transformer winding carries a current which produces a voltage drop across R_3. This voltage biases the thyratron grid, but is shifted in phase from the thyratron's anode-circuit voltage and is not quite sufficient to fire the thyratron. It is obvious that the anode voltage e_p, even at its maximum, must not be great enough to fire the thyratron without the aid of the phototube-augmented positive grid voltage. How-



(Courtesy Westinghouse Electric Corp.)

Fig. 28–21. Application of Photo-Electric Tube and Light Beam for Counting Objects on a Conveyor

ever, when aided by the phototube-produced positive voltage across R_2, the voltage across R_3 will fire the thyratron on the positive voltage cycle. If the phototube current is interrupted because of light failure, the remaining bias is insufficient to fire the thyratron and the relay relaxes. The resistor R_1 is inserted to limit the thyratron grid current when the tube fires.

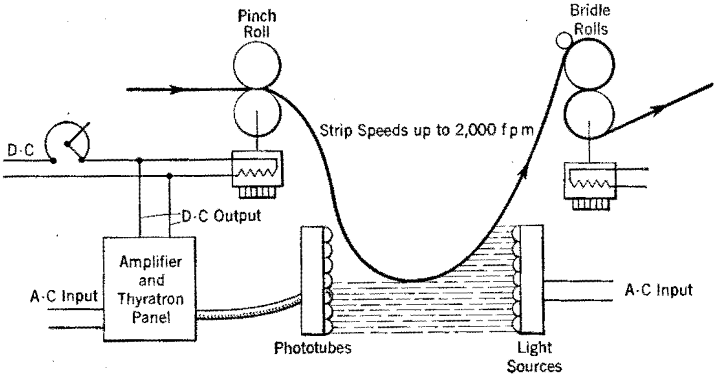
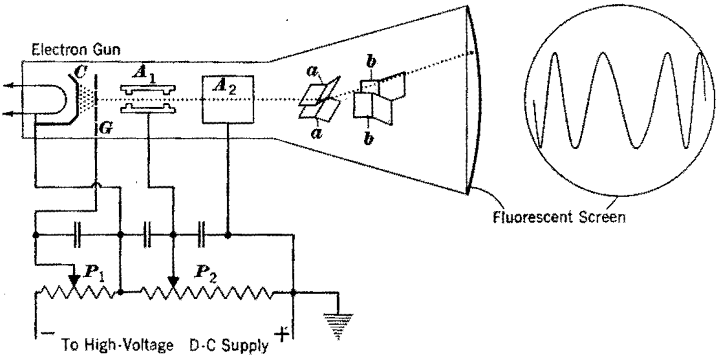


Fig. 28–22. **Industrial Application of Photo-Electric Relay to Control Slack in Rolling Strip Steel**



Sweep voltage assumed on deflectors *b*
Sinusoidal voltage on deflectors *a*

Fig. 28–23. **Fundamental Circuit of Cathode-Ray Oscilloscope**

Figs. 28–21 and 28–22 show two industrial control applications of the photo-electric relay.

The Cathode-Ray Oscilloscope.—The cathode-ray oscillograph or oscilloscope is a device for observing high-speed phenomena in electrical circuits, and operates very much like the magnetic Duddell type described on page 188. In the cathode-ray type, however, the visible trace is produced on the large end of a funnel-shaped tube, the end of which is coated with a fluorescent material. Fig. 28–23 shows the construction, schematically, of a cathode-ray tube and its accompanying control circuits.

In Fig. 28–23 the electrons are boiled off into a cloud around the cathode C, which is indirectly heated from the filament. High potentials on anodes A_1 and A_2 accelerate the freed electrons and focus some of them into a thin beam or ray as they pass through the anodes to bombard the fluorescent screen. If deflecting electrodes a and b are not energized, the spot of light on the fluorescent screen caused by the striking of the electron ray will be in the center of the screen. If, however, a saw-toothed voltage function, such as shown in Fig. 28–3, be impressed on deflecting plates b, the beam will be deflected by the electrostatic charge on the plates and will be caused to sweep horizontally at uniform velocity across the screen and then to snap back quickly. The frequency with which the sweeps are repeated is determined by the setting of the relaxation oscillator or an equivalent device. Such a sweep of the spot on the screen provides a time axis on which to superpose the magnitude function of any voltage wave to be investigated as to shape. The voltage to be analyzed is impressed across deflection plates a, which deflect the ray vertically in proportion to the magnitude of the voltage.

In Fig. 28–23 is shown a screen trace that would be obtained by energizing deflection plates a by a sinusoidal voltage. It is important to note that the wave trace is accurate only in the center portion of the screen and is distorted on both ends. In order to make use of the full width of the screen, there is usually maintained on the horizontal deflectors a constant voltage called a "pedestal" voltage which is just great enough to deflect the static spot to one side of the screen. In Fig. 28–23 the grid G is used to control the beam or ray density, and hence the brightness of the luminous spot is controlled by changing P_1. The structure of the grid and the aperture in anode A_1 are so designed as to prevent passage of all electrons which tend to stray from the desired beam path. The focusing of the beam is controlled by P_2. Most of the electron acceleration is accomplished by the potential on anode A_2.

Deflection of the beam is sometimes accomplished magnetically instead of electrostatically. Because of greater simplicity in the construction of magnetic-deflection tubes, they are cheaper than electrostatic-deflection tubes. However, the magnetic-deflection principle, because of the lag of current caused by inductance in the coils, is not suitable for voltage measurements, but is confined to the sweep axis. Electrostatic deflection has the advantage that practically no power is required, and the auxiliary circuits are simpler than those required for electromagnetic deflection.

The Duddell type of oscillograph is satisfactory for frequencies up to about 5000 cycles per second, but the cathode-ray oscilloscope is applicable to all frequencies up to the high-range radio frequencies at which the time of one cycle is comparable to the transit time of the electron between the cathode and the fluorescent screen.

Cathode-ray oscilloscopes are usually equipped with suitable preamplifiers for the deflector circuits, so that input voltages of low magnitude may be analyzed satisfactorily.

Industrial X-Rays.—X-rays are a form of invisible radiation similar to heat rays or light rays, but their wave lengths are shorter (frequencies higher). Because of their short wave lengths, X-rays are able to penetrate substances which are opaque to light rays, and so may be used to photograph in silhouette substances such as animal tissue, wood, and metals. Fig. 28–24 shows the range which X-rays occupy in the radiation spectrum.

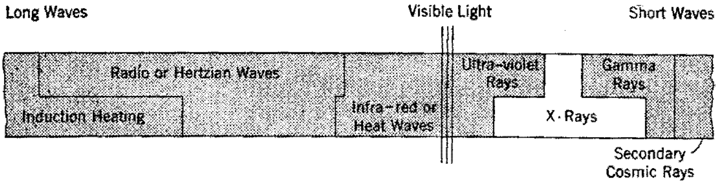


Fig. 28–24. Position of X-Rays in the Radiation Spectrum

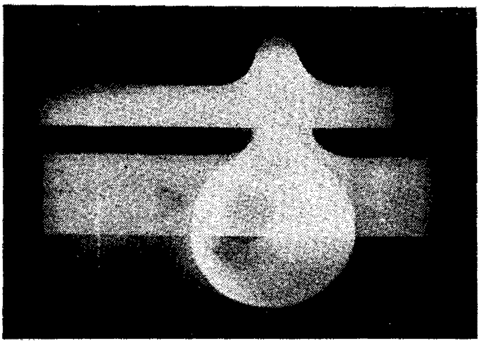


Fig. 28–25. Radiograph of Casting, With Blowhole Shown as Shadow near Center

Radiography is somewhat similar to photography, but the rays must be made to pass through the specimen to be studied and to strike a photographic plate after the passage. The number of X-rays which reach and affect the plate depends on the thickness and density of the specimen and, of course, on the time of exposure. The greater the thickness or density of the specimen to be X-rayed, the more powerful must be the X-ray beam directed at it, in order to obtain an acceptable exposure of the plate in a reasonable length of time. Brass is almost eight times, and steel is about two and one-half times, as opaque to X-rays as is aluminum. Fig. 28–25 shows a typical radiograph of a casting in which there is a gas pocket or blowhole that is invisible from the outside.

X-rays may be produced in any of several ways, but available industrial X-ray apparatus produces them by directing electrons against a metallic target. The electrons are produced by an electron gun similar to that shown in Fig. 28–23; but, instead of striking a fluorescent screen, they strike a slanted metallic anode as shown in Fig. 28–26. The X-rays produced by this collision are deflected at a right angle to the electron beam and, unhampered by the glass of the tube side, emerge as a narrow-angle beam directed toward the specimen, as shown in Fig. 28–26.

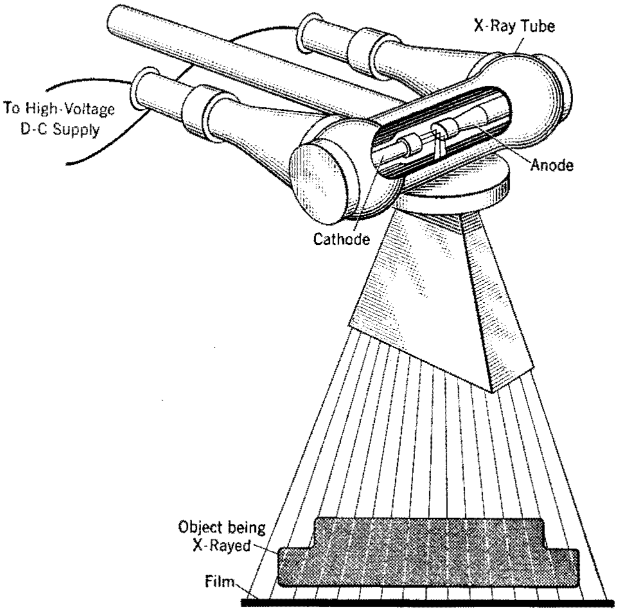
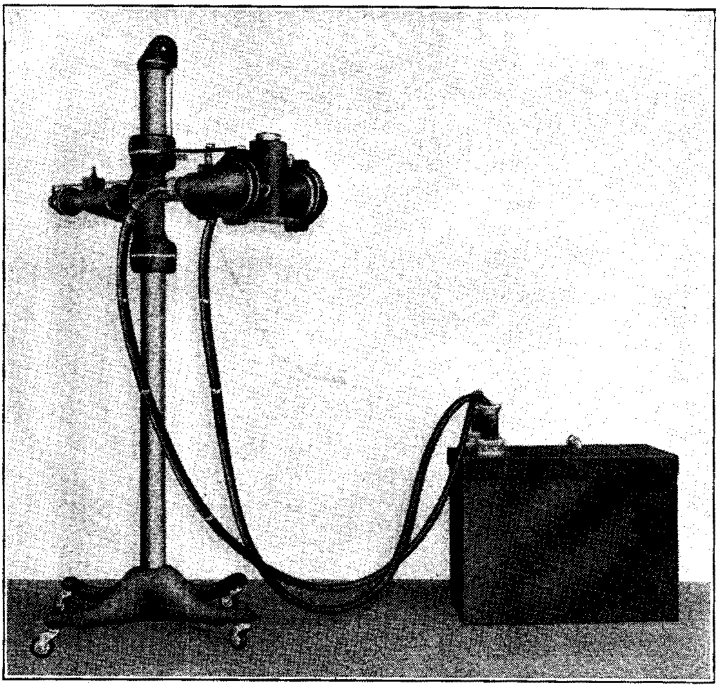


Fig. 28–26. Schematic Diagram of X-Ray Tube and Object to Be X-Rayed

X-ray equipment is rated according to the maximum voltage which can safely be applied to the X-ray tube. In the present state of development, industrial X-ray equipment is available from 140 kv to 10,000 kv; the voltage is selected according to the thickness and density of the object to be X-rayed. Fig. 28–27 shows a typical X-ray machine with power supply for 150 kv.

The use of industrial X-ray equipment involves a certain amount of personal hazard, if it is not properly shielded. Stray emission while the tube is in operation may reach near-by persons and, by its cumulative



(*Courtesy Westinghouse Electric Corp.*)

Fig. 28-27. Industrial X-Ray Machine With Transformer

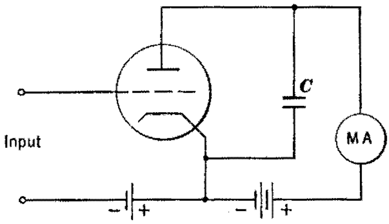


Fig. 28-28. Simplified Circuit Diagram for Electronic Voltmeter

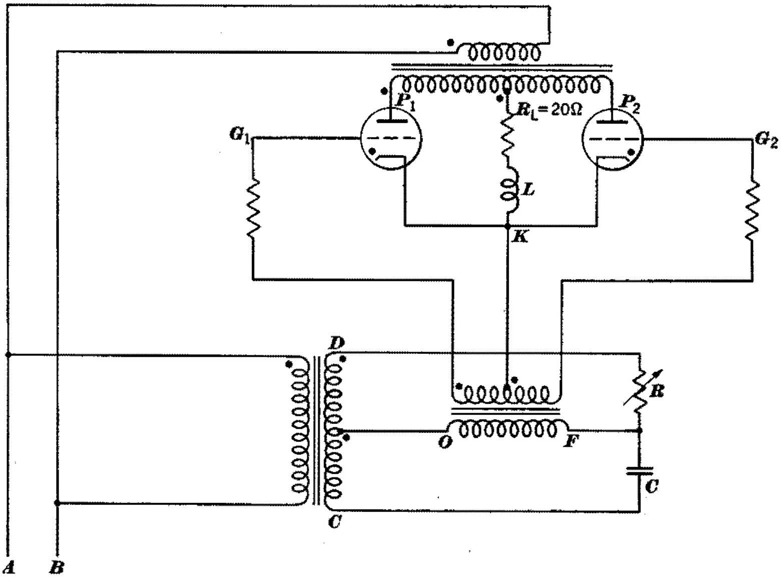
effect, may cause ray "burns." To prevent such injury to the operator and others, it is necessary to enclose the radiating portions of the equipment in walls of lead sheet or equivalent thickness of other substances, such as earth or water. The required thickness of the protecting wall depends on the power of the X-ray, and recommendations are offered by makers of the equipment.

Vacuum-Tube Voltmeters.—Vacuum-tube or electronic voltmeters make use of some of the unique characteristics of electron tubes, and so are available in many types. They are available for measuring direct, rms, average or peak voltages. Of the many types available, only the simplest and possibly the most commonly used type for measuring rms values of alternating voltages will be described here.

The chief advantages of the electronic voltmeter are its high input impedance, wide frequency range, and sensitivity. Because of its high impedance it does not divert enough current to disturb the circuit to which it is connected. Fig. 28–28 shows a simplified diagram of one type of electronic voltmeter. It uses a simple evacuated triode as a detector, as described in Chapter 27. The milliammeter in the plate circuit is calibrated in terms of rms volts applied to the grid. This type of voltmeter is available with a built-in rectifier and filter, so that it may be connected to a standard 120-volt, 60-cycle supply.

PROBLEMS

1. A 60-cycle, 120/60-volt phase-shifter is to be built by using a center-tapped transformer and an inductance coil of 3 henrys and 300 ohms resistance. Calculate the magnitude of the control voltage and its phase angle with respect to the secondary terminal voltage if the series resistor of the phase-shifter has a resistance of 2000 ohms. Calculate the phase angle for alternate positions of the phase-shifter elements.



Problem 28–2

2. The accompanying circuit is set up and found to be correct. On which end of the transformer primary OF should the dot be placed? If the load inductance L is large enough to insure constant tube current during the conduction period, what is the average load current for a firing angle of 30 degrees? The plate-to-plate voltage is 282.8 volts rms and R_L is 20 ohms.

CHAPTER 29

Special Motors and Generators

It is the purpose of this chapter to describe some of the more commonly used motors and generators designed with special characteristics to meet the requirements of special duties.

The Arc-Welding Generator.—The characteristics of an arc-welding generator are similar to those of an ordinary direct-current, compound-wound generator with the series field differentially connected, or one in which the armature reaction is very great. A typical circuit diagram for an arc-welding generator is shown in Fig. 29–1. The open-circuit voltage is adjustable between about 40 and 100 volts. This voltage is desirable for striking the arc, but the voltage across the arc when welding must be of the order of 25 to 40 volts at the chosen welding current. The welding current for a particular weld is chosen by the operator in accord with his best judgment. The welding current is sometimes controlled by a multi-blade switch, by which successive portions of the series-field winding may be short-circuited and the drop of the characteristic curve may be controlled. See Fig. 29–2 for typical characteristic curves of generators of one type.

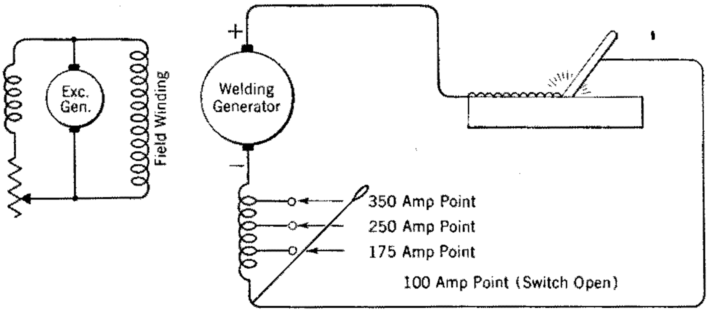


Fig. 29–1. Schematic Diagram of Separately Excited Arc-Welding Generator

With some types of welding generators it is necessary to connect a smoothing reactor in series with the line, to aid in stabilizing the arc. With others, stability is achieved by the design of the generator alone. Every builder of arc-welding generators uses a special circuit scheme and welder design.

The Third-Brush Generator.—The generator used on automobiles for battery charging must operate over a wide range of speed with a reasonably uniform current output. This characteristic may be achieved in several ways, the most popular of which is by use of the "third brush" method of exciting.

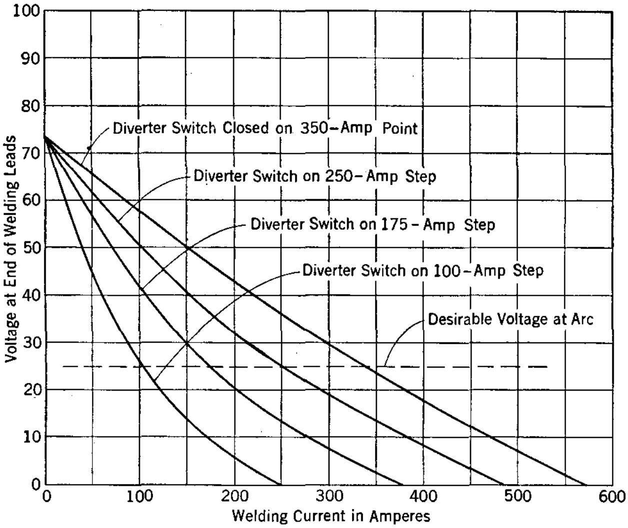


Fig. 29–2. **Characteristics of Welding Generators of One Type With Diverter Switch on Various Steps**

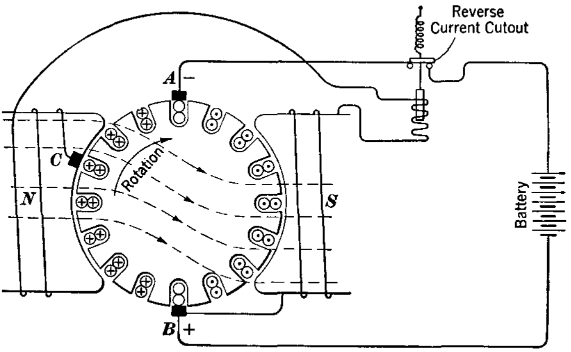


Fig. 29–3. **Third-Brush Generator**

In the third-brush generator, a schematic diagram of which is shown in Fig. 29–3, the shunt field is not excited directly from the main brushes, but is connected between one main brush and the third brush, which may be adjusted at will within limits around the periphery of the commutator between *A* and *B*. The voltage available for excitation is only that generated in the armature conductors producing voltage on the commutator between B and C. When the generator is started and begins to build up, the growing flux density is practically uniform at the pole faces. When the voltage rises to a value somewhat above that of the battery, the reverse-current cutout closes the charging circuit. There being no interpoles or compensating windings, the armature current produces a severe distortion of the field flux, as shown in Fig. 29–3. Much of the flux, which at first cuts conductors between B and C to produce excitation voltage, is thus forced into the leading pole tip nearer A and is lost for the purpose of excitation. Furthermore, there is a net loss of flux because of saturation of the pole tips. As the speed is increased above a minimum speed determined by design, the losses of voltage by armature reaction equal the gain in voltage by gain in speed, and the charging current can rise no higher.

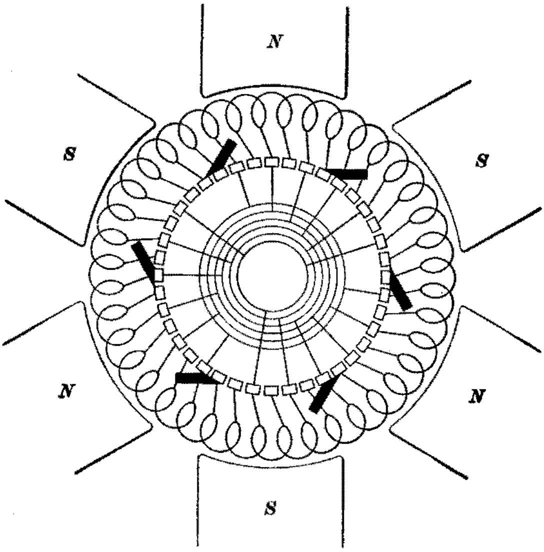


Fig. 29–4. Diagram of 6-Phase Synchronous Converter

In addition to the field distortion by the armature reaction, the strength of the field is further reduced by the demagnetizing action of the short-circuit current in the conductors being commutated by the third brush. The ultimate result is a reduction of charging current as the speed is increased beyond a limit determined by the design proportions of the generator.

The Synchronous Converter (Rotary).—The synchronous converter is a machine which may be used as a synchronous motor and a d-c generator simultaneously. It is essentially a d-c generator with armature taps connected through slip-rings to external terminals. Through these taps the machine may be run from a polyphase line as a synchronous motor. See Fig. 29–4 for the construction of such a machine.

In order to produce the usual d-c voltages, such as 230, 250, 550, and 600, it is necessary that the a-c voltages at the slip-rings have special values which are different from those in standard commercial use. For this reason, transformers of special secondary voltages are always supplied as regular equipment with the converter. The following table gives the required ring-to-ring a-c voltages necessary to produce 250 volts d-c at the commutator with 1-, 3-, 4- and 6-phase supply voltages.

Number of Supply Phases	Ring-to-Ring A-C Voltages
1	176.7
3	153
4	125
6	88.5

Operating Characteristics.—The voltage at the d-c terminals bears a fixed ratio to the a-c voltage at the slip-rings, and can be varied only by varying the a-c voltage. The a-c voltage may be varied by changing the transformer taps or, indirectly, by changing the d-c excitation as in a synchronous motor. Over-excitation causes the converter to take a leading current. The leading current flowing through the reactance of the transformers causes an increase in their secondary voltages and, hence, an automatic compounding of voltage. Because of excess heating of armature conductors at and near the tap points[1] when the power factor departs more than 10 per cent or so from unity, this method of voltage control by field rheostat can be used only within small confines near unity power factor. The same heating characteristic which limits the range of compounding also limits to almost negligible proportions the capacity of the converter for correcting the power factor of a power system, as may be done so readily with a synchronous motor. The relative merits of the synchronous converter and the synchronous motor and d-c generator set are as follows:

Advantages of converter:

1. Cost is less per kilowatt of capacity.
2. Efficiency is higher.
3. Weight is less, and foundation may be smaller.
4. Requires minimum of space.

Advantages of synchronous motor and generator set:

1. Wider range of d-c voltage is available.
2. Motor may be used to correct power factor.

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[1] *Principles of Alternating Current Machinery*, Chapter 27, by R. R. Lawrence, McGraw-Hill Book Co.

Balancer Sets.—Three-wire direct-current distribution systems are usually energized from a two-wire generator connected to the two outside lines. The approximately balanced voltages between the outside lines and the middle line are maintained in some cases by a *balancer set* connected as shown in Fig. 29-5.

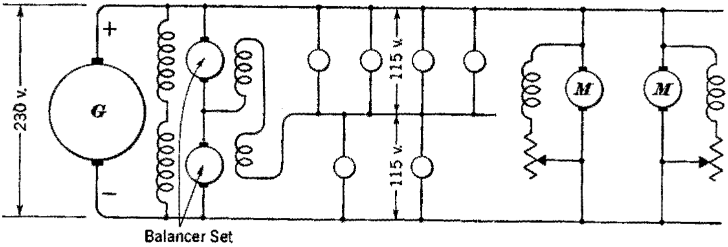

Fig. 29-5. Three-Wire System With Balancer Set

The balancer set consists of two identical shunt or compound direct-current machines with their shafts rigidly coupled together and with their armature circuits connected in series across the outside lines of the three-wire system. The middle wire of the system is connected to a point between the armatures of the two machines. The series fields of the two machines are so connected that, when a current flows therein because of load unbalance, the machine on the more heavily loaded side becomes a cumulative-compound generator while the machine on the more lightly loaded side becomes a differential-compound motor. Because the balancing action is dependent on *current difference* in the two sides of the system, it is not necessary that voltage unbalance actually occur before the restoring action is available. When the load is balanced, the two units run as

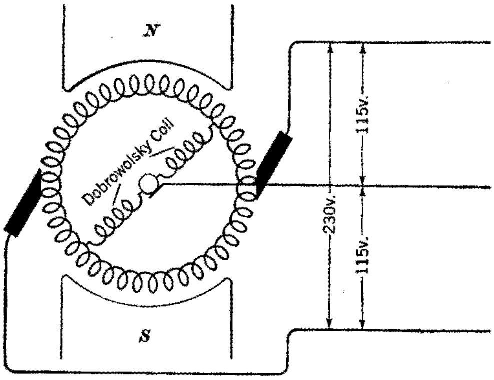

Fig. 29-6. Three-Wire System by Dobrowolsky Coils

motors in series without load. Very close voltage regulation is possible by this method of compounding. Balancer sets are made of such size as to compensate for load unbalance of 25 or less per cent of the full capacity of the main generator.

The Three-Wire Generator.—The use of the balancer set to provide voltage balance on a three-wire system is open to the objection that the balancer requires service and attendance and requires power for its operation in addition to a considerable capital investment. These objections have been almost entirely overcome by the use of a *balancer coil* due to Dobrowolsky. As shown in Fig. 29–6, the balancer coil is simply a pair of reactor coils connecting points 180 electrical degrees apart on the armature winding and being themselves connected. The point at the reactor-coil connection is connected to the middle line of the three-wire system.

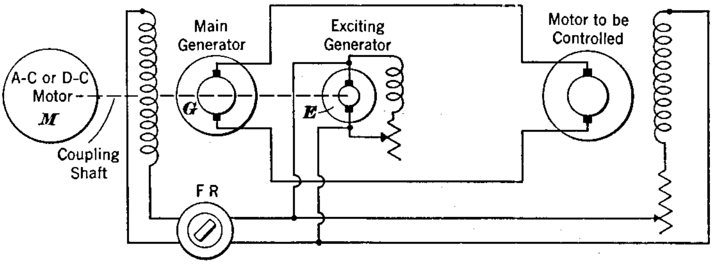

Fig. 29–7. Ward-Leonard Motor Control

Inasmuch as any pair of reactor coils connects two points on the armature 180 electrical degrees apart, the voltage across a pair of coils is alternating and the alternating current in the coils is small and similar to the exciting current of a transformer. However, the direct current of load unbalance flows freely in the coils. The coils are wound on separate iron cores mounted on the armature, or are wound on separate cores outside the machine. With coils mounted on the armature, only one slip-ring is necessary; whereas, with separate cores outside the machine, two or more slip-rings are necessary.

Adjustable-Speed Drives.—Many motor applications require adjustable speed throughout a wide range of speeds. Mechanical speed changers are sometimes satisfactory if the power involved is not very great or if the necessary rate of speed change is low. A speed change of 3 to 1 may be obtained economically by shunt-field control in a d-c motor, if the motor is specially built for adjustable-speed service. When the low speed must be very low compared to the high speed, the speed may best be changed by raising or lowering the voltage applied to the armature while the field strength is held constant. The armature voltage may be varied by simply inserting a control resistor in series with it. This method is used in controlling very small motors, such as sewing-machine and small-fan motors, but the power loss in the control resistor makes the method uneconomical with large motors. Furthermore, the speed regulation is poor and its amount is intolerable in most cases when the load is variable.

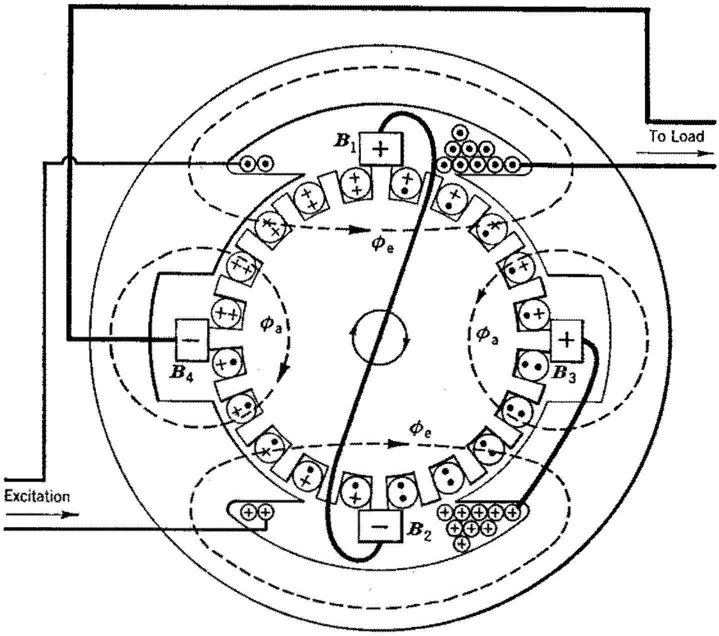

Fig. 29-8. Amplidyne Generator

One method of obtaining adjustable voltage consists of using a generator driven at constant speed but provided with variable separate excitation. The Ward-Leonard system used for this purpose is illustrated in Fig. 29-7. The main generator *G* and the exciting generator *E* are driven by a constant-speed motor *M* (a-c or d-c). The generator-armature leads are connected directly to the motor-armature leads, without an intervening resistor. The field of the main generator is increased, decreased, or reversed by the control rheostat *FR*. The resulting increase, decrease, or reversal of the generator voltage causes the corresponding desired changes in motor speed and direction of rotation. The cost of this type of speed control is prohibitive for all but special applications, such as large hoists, elevators, and rolling mills.

Alternating-current motors do not serve adjustable-speed applications so well as do direct-current motors. As mentioned on page 220, the speed of a wound-rotor induction motor may be varied by varying the rotor-circuit resistance, but the efficiency is low and the speed regulation is poor. Several motors of the brush-shifting type have been developed, in which a counter voltage is generated by a special winding on the motor itself. This counter voltage is applied to the rotor in such a way as to limit the rotor currents without the power loss that would occur if a rotor resistance were used. The magnitude of the counter voltage, and hence of the speed, is controlled by shifting the brushes around the commutator. The speed regulation of this type of motor is good at all speeds, but the cost is relatively great; and so its use is limited to spinning-frame drive, forced-draft fans in power houses, and similar applications which require accurate speed control.

The Amplidyne (General Electric Company).—The shunt-field circuit of the generator in the Ward-Leonard system of Fig. 29–7 is highly inductive, and so the current in it changes slowly after the field-rheostat setting is changed. For this reason the voltage change at the generator terminals is sluggish and not satisfactorily responsive for some purposes. The amplidyne generator, shown in Fig. 29–8, has a very rapid response to changes in exciting voltage, and permits the accurate control of great power by small changes in little power.

In Fig. 29–8 the exciting or control field flux ϕ_e is produced by a few ampere-turns only in the small exciting-field coil. The armature, driven at constant speed in this field flux ϕ_e, generates voltages in the usual manner about the vertical axis. Brushes B_1 and B_2 on this axis are short-circuited and so permit currents to flow in all armature conductors. These currents produce the usual armature reaction field ϕ_a, which is vertical in the figure and across the excitation field ϕ_e. The armature rotation in the field ϕ_a, which its own currents have produced, generates a new set of voltages around the horizontal axis. Brushes B_3 and B_4 on the horizontal axis make these voltages available at the load terminals. The load currents and the short-circuit currents obviously cannot exist simultaneously in the armature conductors, but combine algebraically to produce the true current. The excitation power is extremely small compared to the full-load output watts, and so may be supplied by a small electronic control circuit. The response in generator voltage change is very rapid and is especially useful in such applications as steel mills, excavators, and automatically-directed artillery.

APPENDIX

TABLE A—ALLOWABLE CURRENT-CARRYING CAPACITIES OF CONDUCTORS IN AMPERES

Not More Than Three Conductors in Raceway or Cable

(Based on Room Temperature of 30 C. 86 F.)

Size AWG MCM	Rubber Type R Type RW Type RU (14–6) Thermoplastic Type T (14–4/0) Type TW (14–4/0)	Rubber Type RH	Paper Thermoplastic Asbestos Type TA Var-Cam Type V Asbestos Var-Cam Type AVB	Asbestos Var-Cam Type AVA Type AVL	Impregnated Asbestos Type AI (14–8) Type AIA	Asbestos Type A (14–8) Type AA
14	15	15	25	30	30	30
12	20	20	30	35	40	40
10	30	30	40	45	50	55
8	40	45	50	60	65	70
6	55	65	70	80	85	95
4	70	85	90	105	115	120
3	80	100	105	120	130	145
2	95	115	120	135	145	165
1	110	130	140	160	170	190
0	125	150	155	190	200	225
00	145	175	185	215	230	250
000	165	200	210	245	265	285
0000	195	230	235	275	310	340
250	215	255	270	315	335	...
300	240	285	300	345	380	...
350	260	310	325	390	420	...
400	280	335	360	420	450	...
500	320	380	405	470	500	...
600	355	420	455	525	545	...
700	385	460	490	560	600	...
750	400	475	500	580	620	...
800	410	490	515	600	640	...
900	435	520	555	...	...	...
1,000	455	545	585	680	730	...
1,250	495	590	645	...	...	...
1,500	520	625	700	785	...	...
1,750	545	650	735	...	...	...
2,000	560	665	775	840	...	...

CORRECTION FACTOR FOR ROOM TEMPERATURES OVER 30 C. 86 F.

C	F						
40	104	.82	.88	.90	.94	.95	...
45	113	.71	.82	.85	.90	.92	...
50	122	.58	.75	.80	.87	.89	...
55	131	.41	.67	.74	.83	.86	...
60	140	...	.58	.67	.79	.83	.91
70	158	...	.35	.52	.71	.76	.87
75	167	...	...	.43	.66	.72	.86
80	176	...	...	.30	.61	.69	.84
90	194	...	...	...	.50	.61	.80
100	212	...	...	...	...	.51	.77
120	248	...	...	...	...	...	.69
140	284	...	...	...	...	...	.59

TABLE B—ALLOWABLE CURRENT-CARRYING CAPACITIES OF CONDUCTORS IN AMPERES

Single Conductor in Free Air

(Based on Room Temperature of 30 C. 86 F.)

Size AWG MCM	Rubber Type R Type RW Type RU (14–6) Thermoplastic Type T Type TW	Rubber Type RH	Thermoplastic Asbestos Type TA Var-Cam Type V Asbestos Var-Cam Type AVB	Asbestos Var-Cam Type AVA Type AVL	Impregnated Asbestos Type AI (14–8) Type AIA	Asbestos Type A (14–8) Type AA	Slow-Burning Type SB Weatherproof Type WP Type SBW
14	20	20	30	40	40	45	30
12	25	25	40	50	50	55	40
10	40	40	55	65	70	75	55
8	55	65	70	85	90	100	70
6	80	95	100	120	125	135	100
4	105	125	135	160	170	180	130
3	120	145	155	180	195	210	150
2	140	170	180	210	225	240	175
1	165	195	210	245	265	280	205
0	195	230	245	285	305	325	235
00	225	265	285	330	355	370	275
000	260	310	330	385	410	430	320
0000	300	360	385	445	475	510	370
250	340	405	425	495	530	...	410
300	375	445	480	555	590	...	460
350	420	505	530	610	655	...	510
400	455	545	575	665	710	...	555
500	515	620	660	765	815	...	630
600	575	690	740	855	910	...	710
700	630	755	815	940	1005	...	780
750	655	785	845	980	1045	...	810
800	680	815	880	1020	1085	...	845
900	730	870	940	....	....	...	905
1,000	780	935	1000	1165	1240	...	965
1,250	890	1065	1130	....	....	...	....
1,500	980	1175	1260	1450	....	...	1215
1,750	1070	1280	1370	....	....	...	....
2,000	1155	1385	1470	1715	....	...	1405
CORRECTION FACTOR FOR ROOM TEMPERATURES OVER 30 C. 86 F.							
C F							
40 104	.82	.88	.90	.94	.95	...	....
45 113	.71	.82	.85	.90	.92	...	....
50 122	.58	.75	.80	.87	.89	...	....
55 131	.41	.67	.74	.83	.86	...	....
60 140	....	.58	.67	.79	.83	.91	....
70 158	....	.35	.52	.71	.76	.87	....
75 167	....	....	.43	.66	.72	.86	....
80 176	....	....	.30	.61	.69	.84	....
90 194	....	....	....	.50	.61	.80	....
100 212	....	....	....	....	.51	.77	....
120 248	....	....	....	....	....	.69	....
140 284	....	....	....	....	....	.59	....

FULL LOAD CURRENTS OF MOTORS

The following data are approximate full-load currents for motors of various types, frequencies, and speeds. They have been compiled from average values for representative motors of their respective classes. Variations of 10 per cent above or below the values given may be expected.

Hp of Motor	Amperes—Full-load Current																								
	Direct-current Motors			Alternating-current Motors																					
				Single-phase Motors		Squirrel-cage Induction Motors										Slip-ring Induction Motors									
						Two-phase					Three-phase					Two-phase					Three-phase				
	115-volt	230-volt	550-volt	110-volt	220-volt	110-volt	220-volt	440-volt	550-volt	2200-volt	110-volt	220-volt	440-volt	550-volt	2200-volt	110-volt	220-volt	440-volt	550-volt	2200-volt	110-volt	220-volt	440-volt	550-volt	2200-volt
¼	....	....	....	7.6	3.8	....	....	....	....	....	....	....	....	....	....	....	....	....	....	....	....	....	....	....	....
½	4.5	2.3	....	8.0	4.0	4.1	2.1	1.0	0.9	....	5.0	2.5	1.3	1.0	....	....	....	....	....	....	....	....	....	....	....
¾	6.5	3.3	1.4	9.6	4.8	4.8	2.4	1.2	1.0	....	5.4	2.8	1.4	1.1	....	6.2	3.1	1.6	1.3	....	7.2	3.6	1.8	1.5	....
1	8.4	4.2	1.7	11.0	5.5	5.6	2.8	1.4	1.1	....	6.6	3.3	1.7	1.3	....	6.7	3.4	1.7	1.4	....	7.8	3.9	2.0	1.6	....
1½	12.5	6.3	2.6	15.0	7.5	7.8	3.9	2.0	1.7	....	9.4	4.7	2.4	2.0	....	11.7	5.9	3.0	2.3	....	....	....	....	....	....
2	16.1	8.3	3.4	20.0	10.0	9.7	4.9	2.4	2.0	....	12.0	6	3.0	2.4	....	12.5	6.3	3.1	2.5	....	14.4	7.2	3.6	2.9	....
3	23	12.3	5.0	27.8	13.9	....	7.8	3.9	3.1	....	....	9	4.5	4.0	....	....	8.7	4.3	3.5	....	20.2	10	5.0	4	....
5	40	19.8	8.2	46.6	23.3	....	13	6.5	5.2	....	....	15	7.5	6.0	....	....	13.0	6.5	5.2	....	....	15	7.5	6	....
7½	58	28.6	12	67.6	33.8	....	19	9.5	7.6	....	....	22	11	9.0	....	....	20.0	10.0	7.6	....	....	25	13	10	....
10	75	38	16	86.0	43.0	....	24	11.3	9.4	....	....	27	14	11	....	....	24.3	12.1	10.0	....	....	28	14	11	....
15	112	56	23	....	....	....	33	16.5	13.2	....	....	38	19	15	....	....	39	19.5	15.6	....	....	45	23	18	....
20	149	74	31	....	....	....	45	22.5	18.2	5.0	....	52	26	21	5.7	....	49	24.7	19.8	....	....	56	28	22	....
25	185	92	38	....	....	....	56	27.7	22.2	6.4	....	64	32	26	7	....	60	30.0	24.0	6.4	....	67	34	27	7.5
30	220	110	45	....	....	....	67	33.5	27.0	7.4	....	77	39	31	8	....	72	36.0	28.8	7.8	....	82	41	33	9
40	294	146	61	....	....	....	88	44	35.5	9.5	....	101	51	40	10	....	93	46.5	37.3	9.5	....	106	53	42	11
50	364	180	75	....	....	....	109	54	43.5	12.1	....	125	63	50	13	....	113	57	45	12.1	....	128	64	51	14
60	436	215	90	....	....	....	129	65	52.0	13.5	....	149	75	60	15	....	135	68	54	14.0	....	150	75	60	16
75	540	268	111	....	....	....	156	78	62.5	16.7	....	180	90	72	19	....	164	82	65	17.3	....	188	94	75	19
100	....	357	146	....	....	....	213	107	87	21.7	....	246	123	98	25	....	214	108	87	21.7	....	246	123	99	25
125	....	440	184	....	....	....	270	134	109	27.0	....	310	155	124	32	....	267	134	108	27	....	310	155	124	31
150	....	....	220	....	....	....	....	156	125	31.2	....	360	180	144	36	....	315	158	127	32	....	364	182	145	37
175	....	....	....	....	....	....	....	....	....	....	....	....	....	....	....	....	....	....	....	....	....	....	....	....	....
200	....	....	....	....	....	....	....	217	173	43.3	....	480	240	195	49	....	430	216	173	44	....	490	245	196	52

CONDUCTOR SIZES AND OVERCURRENT PROTECTION FOR MOTORS

Full load current rating of motor amperes	Minimum size conductor in raceways For conductors in air or for other insulations see Tables A and B AWG AND MCM			For Running Protection of Motors		Maximum Allowable Rating of Branch Circuit Fuses			
	Type R	Type RP	Type RH	Maximum rating of N.E.C. fuses Amperes	Maximum setting of time-limit protective device Amperes	**With Code Letters** Single-phase and squirrel cage and synchronous. Full voltage, resistor and reactor starting, Code letters F to R inc. **Without Code Letters** same as above.	**With Code Letters** Single-phase and squirrel cage and synchronous. Full voltage, resistor or reactor starting, Code letters B to E inc. Auto-transformer starting, Code letters F to R inc. **Without Code Letters** Squirrel cage and synchronous, auto-transformer starting, High reactance squirrel cage.*** Both not more than 30 amperes	**With Code Letters** Squirrel cage and synchronous Auto-transformer starting, Code letters B to E inc. **Without Code Letters** Squirrel cage and synchronous, auto-transformer starting, High reactance squirrel cage.*** Both more than 30 amperes.	**With Code Letters** All motors Code letter A. **Without Code Letters** DC and wound-rotor motors.
Col. No. 1	2	3	4	5	6	7	8	9	10
1**	14	14	14	2*	1.25*	15	15	15	15
2**	14	14	14	3*	2.50*	15	15	15	15
3**	14	14	14	4*	3.75*	15	15	15	15
4**	14	14	14	6*	5.0 *	15	15	15	15
5**	14	14	14	8*	6.25*	15	15	15	15
6**	14	14	14	8*	7.50*	20	15	15	15
7	14	14	14	10*	8.75*	25	20	15	15
8	14	14	14	10*	10.0 *	25	20	20	15
9	14	14	14	12*	11.25*	30	25	20	15
10	14	14	14	15*	12.50*	30	25	20	15
11	14	14	14	15*	13.75*	35	30	25	20
12	14	14	14	15	15.00	40	30	25	20
13	12	14	14	20	16.25	40	35	30	20
14	12	14	14	20	17.50	45	35	30	25
15	12	12	14	20	18.75	45	40	30	25
16	12	12	14	20	20.00	50	40	35	25

17	10	12	14	25	21.25	60	45	35	30
18	10	12	12	25	22.50	60	45	40	30
19	10	10	12	25	23.75	60	50	40	30
20	10	10	12	25	25.0	60	50	40	30
22	8	10	10	30	27.50	70	60	45	35
24	8	10	10	30	30.00	80	60	50	40
26	8	8	10	35	32.50	80	70	60	40
28	8	8	10	35	35.00	90	70	60	45
30	6	8	8	40	37.50	90	70	60	45
32	6	8	8	40	40.00	100	80	70	50
34	6	6	8	45	42.50	110	90	70	60
36	6	6	8	45	45.00	110	90	80	60
38	5	6	8	50	47.50	125	100	80	60
40	5	6	6	50	50.00	125	100	80	60
42	5	6	6	50	52.50	125	110	90	70
44	4	5	6	60	55.0	125	110	90	70
46	4	5	6	60	57.50	150	125	100	70
48	4	5	6	60	60.0	150	125	100	80
50	3	5	6	60	62.50	150	125	100	80
52	3	4	6	70	65.0	175	150	110	80
54	3	4	5	70	67.50	175	150	110	90
56	2	4	5	70	70.00	175	150	120	90
58	2	3	5	70	72.50	175	150	120	90
60	2	3	5	80	75.00	200	150	120	90
62	2	3	4	80	77.50	200	175	125	100
64	2	3	4	80	80.00	200	175	150	100
66	1	3	4	80	82.50	200	175	150	100
68	1	2	4	90	85.00	225	175	150	110
70	1	2	3	90	87.50	225	175	150	110
72	1	2	3	90	90.00	225	200	150	110
74	0	2	3	90	92.50	225	200	150	125
76	0	2	3	100	95.00	250	200	175	125
78	0	1	3	100	97.50	250	200	175	125

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* For running protection of motors of 1 horsepower or less, see section 4322 of National Electric Code.

** For the grouping of small motors under the protection of a single set of fuses, see section 4343 of National Electric Code.

*** High-reactance squirrel-cage motors are those designed to limit the starting current by means of deep-slot secondaries or double-wound secondaries and are generally started on full voltage.

CONDUCTOR SIZES AND OVERCURRENT PROTECTION FOR MOTORS—*Continued*

Full load current rating of motor amperes	Minimum size conductor in raceways For conductors in air or for other insulations see Tables A and B AWG AND MCM			For Running Protection of Motors		Maximum Allowable Rating of Branch Circuit Fuses			
	Type R	Type RP	Type RH	Maximum rating of N.E.C. fuses Amperes	Maximum setting of time-limit protective device Amperes	**With Code Letters** Single-phase and squirrel cage and synchronous. Full voltage, resistor and reactor starting, Code letters F to R inc. **Without Code Letters** same as above.	**With Code Letters** Single-phase and squirrel cage and synchronous. Full voltage, resistor or reactor starting, Code letters B to E inc. Auto-transformer starting, Code letters F to R inc. **Without Code Letters** Squirrel cage and synchronous, auto-transformer starting, High reactance squirrel cage.*** Both not more than 30 amperes	**With Code Letters** Squirrel cage and synchronous Auto-transformer starting, Code letters B to E inc. **Without Code Letters** Squirrel cage and synchronous, auto-transformer starting, High reactance squirrel cage.*** Both more than 30 amperes.	**With Code Letters** All motors Code letter A. **Without Code Letters** DC and wound-rotor motors.
Col. No. 1	2	3	4	5	6	7	8	9	10
80	0	1	2	100	100.00	250	200	175	125
82	0	1	2	110	102.50	250	225	175	125
84	0	1	2	110	105.00	250	225	175	150
86	00	1	2	110	107.50	300	225	175	150
88	00	1	2	110	110.00	300	225	200	150
90	00	0	2	110	112.50	300	225	200	150
92	00	0	2	125	115.00	300	250	200	150
94	00	0	1	125	117.50	300	250	200	150
96	00	0	1	125	120.00	300	250	200	150
98	000	0	1	125	122.50	300	250	200	150
100	000	0	1	125	125.00	300	250	200	150
105	000	00	1	150	131.5	350	300	225	175
110	000	00	0	150	137.5	350	300	225	175
115	0000	00	0	150	144.0	350	300	250	175
120	0000	000	0	150	150.0	400	300	250	200
125	0000	000	00	175	156.5	400	350	250	200

130	250	000	00	175	162.5	400	350	300	200
135	250	0000	00	175	169.0	450	350	300	225
140	250	0000	000	175	175.0	450	350	300	225
145	300	0000	000	200	181.5	450	400	300	225
150	300	0000	000	200	187.5	450	400	300	225
155	300	0000	000	200	194.0	500	400	350	250
160	350	250	000	200	200.0	500	400	350	250
165	350	250	0000	225	206.	500	450	350	250
170	350	250	0000	225	213.	500	450	350	300
175	400	300	0000	225	219.	600	450	350	300
180	400	300	0000	225	225.	600	450	400	300
185	400	300	0000	250	231.	600	500	400	300
190	500	300	250	250	238.	600	500	400	300
195	500	350	250	250	244.	600	500	400	300
200	500	350	250	250	250.	600	500	400	300
210	500	400	300	250	263.	. . .	600	450	350
220	600	400	300	300	275.	. . .	600	450	350
230	600	500	300	300	288.	. . .	600	500	350
240	700	500	350	300	300.	. . .	600	500	400
250	700	500	350	300	313.	. . .	. . .	500	400
260	750	600	400	350	325.	. . .	. . .	600	400
270	800	600	400	350	338.	. . .	. . .	600	450
280	900	600	500	350	350.	. . .	. . .	600	450
290	900	700	500	350	363.	. . .	. . .	600	450
300	1000	700	500	400	375.	. . .	. . .	600	450
320	1250	750	600	400	400.	. . .	. . .	. . .	500
340	1500	900	600	450	425.	. . .	. . .	. . .	600
360	1750	1000	700	450	450.	. . .	. . .	. . .	600
380	. . . .	1250	750	500	475.	. . .	. . .	. . .	600
400	. . . .	1500	900	500	500.	. . .	. . .	. . .	600
420	. . . .	1500	1000	600	525.	. . .	. . .	. . .	. . .
440	. . . .	1750	1250	600	550.	. . .	. . .	. . .	. . .
460	. . . .	. . . .	1250	600	575.	. . .	. . .	. . .	. . .
480	. . . .	. . . .	1500	600	600.	. . .	. . .	. . .	. . .
500	. . . .	. . . .	1500	. . .	625.	. . .	. . .	. . .	. . .

ROUND COPPER WIRE

Gage No.	Bare Diameter	Circular Mils	Square Inch	Ohms per 1000 feet 25° C.	Weight in lb per 1000 ft.
27	.0142	202	.000158	52.6	.609
26	.0159	253	.000199	41.7	.767
25	.0179	320	.000252	33.0	.971
24	.0201	404	.000317	26.2	1.22
23	.0226	511	.000401	20.7	1.54
22	.0254	645	.000507	16.4	1.95
21	.0285	812	.000638	13.0	2.46
20	.032	1020	.000804	10.3	3.10
19	.036	1300	.00102	8.14	3.93
18	.040	1600	.00126	6.59	4.85
17	.045	2030	.00159	5.22	6.12
16	.051	2600	.00204	4.07	7.86
15	.057	3250	.00255	3.26	9.82
14	.064	4100	.00322	2.58	12.4
13	.072	5180	.00407	2.04	15.7
12	.081	6560	.00515	1.61	19.8
11	.091	8280	.00650	1.28	25.0
10	.102	10400	.00817	1.02	31.5
9	.114	13000	.0102	.814	39.3
8	.129	16600	.0131	.634	50.5
7	.144	20700	.0163	.510	62.8
6	.162	26200	.0206	.403	79.4
5	.182	33100	.0260	.319	100.
4	.204	41600	.0327	.254	126.
3	.229	52400	.0412	.202	159.
2	.258	66600	.0523	.159	201.
0	.325	106000	.0830	.100	320.
2/0	.365	133000	.105	.0791	404.
4/0	.460	212000	.166	.0500	650.

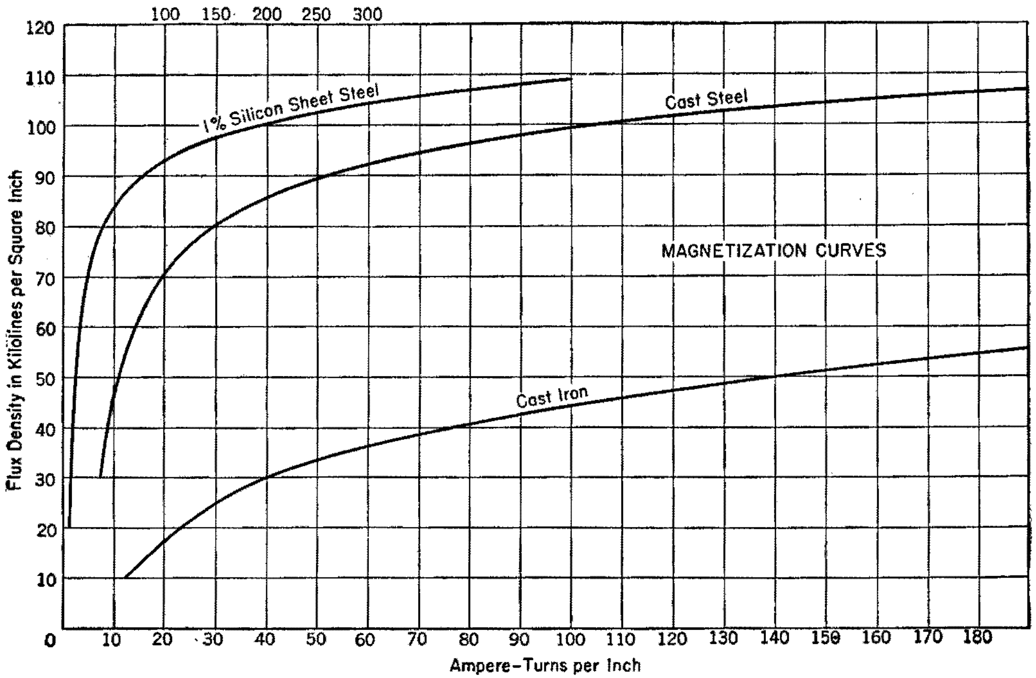

Fig. A-1. Magnetization Curves

MULTIPLYING FACTORS FOR CONVERTING D-C RESISTANCE OF LARGE CONDUCTORS AND CABLES TO A-C RESISTANCE

Conductor Size in Circular Mils	Multiplying Factor	
	25 Cycles	60 Cycles
250,000	....	1.005
300,000	....	1.006
350,000	....	1.009
400,000	....	1.011
500,000	....	1.018
600,000	1.005	1.025
700,000	1.006	1.034
750,000	1.007	1.039
800,000	1.008	1.044
900,000	1.010	1.055
1,000,000	1.012	1.067
1,250,000	1.019	1.102
1,500,000	1.027	1.142
1,750,000	1.037	1.185
2,000,000	1.048	1.233

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