Proceedings of
The Radio Club of America, Inc.

Founded 1909

Volume 27, No. 2

1950

TRAFFIC HANDLING CAPACITY OF 100 CHANNEL DISTANCE-MEASURING-EQUIPMENT (DME) STANDARDIZED BY RTCA SC-40 AND ICAO

by Charles J. Hirsch

Summary of:

"Television Receiving Antennas" by David C. Kleckner, A. E. Joust

"Loudspeakers" by H. C. Hardy

Report on IBCG Memorial by George E. Burghard

THE RADIO CLUB OF AMERICA

11 West 42nd Street ★ ★ ★ New York City

The Radio Club of America, Inc.

11 West 42nd Street, New York City **Telephone — LOngacre 5-6622**

Officers for 1950

President
O. James Morelock

Vice President
John H. Bose

Corresponding Secretary
Ralph R. Batcher

Treasurer
Joseph J. Stantley

Recording Secretary
Bernard D. Loughlin

Directors

- Robert M. Aiken, Jr.
- Ernest V. Amy
- Edwin H. Armstrong
- Salvatore Barone
- P. S. Christaldi
- Howard L. Cobb
- Donald Fink
- Alan Hazeltine
- Harry W. Houck
- F. A. Klingenschmitt
- Jerry B. Minter
- Harry Sadenwater
- Paul Ware

Committe Chairman

Advertising
Paul Ware

Affiliation
Ernest V. Amy

Budget
Joseph J. Stantley

Entertainment
F. A. Klingenschmitt

Medal
Lawrence C. F. Horle

Membership
Perce B. Collison

Year Book
George E. Burghard

Legal Counsellor
George E. Burghard

Papers
Frank H. Shepard

Publications
Jerry B. Minter

Publicity
Austin C. Lescarboura

MEETINGS

Technical meetings are held on the second Thursday evening each month from September through May at either Havemeyer or Pupin Hall, Columbia University, Broadway and 116th Street, New York. The public is invited.

MEMBERSHIP

Application blanks for membership are obtainable at the Club office. For the Member grade the invitation fee is one dollar and the annual dues are three dollars.

PUBLICATIONS

Subscription: Four dollars per year, or fifty cents per issue. Back numbers to members, twenty-five cents each.

PROCEEDINGS
OF THE
RADIO CLUB OF AMERICA

Volume 27 1950 No. 2

TRAFFIC HANDLING CAPACITY OF 100 CHANNEL DISTANCE-MEASURING-EQUIPMENT (DME) STANDARDIZED BY RTCA SC-40 AND ICAO

By

Charles J. Hirsch*

Part of this paper was presented
at the Fifth Annual National Elec-
tronics Conference, September 1949.

I. INTRODUCTION

The Distance-Measuring-Equipment, better known by the abbreviation DME, is an element of the Omni-Bearing-Distance Navigation System (OBD). In this system, also called Rho-theta (ρ, θ), an airplane locates itself, anywhere within the service area, from an origin by means of polar coordinates. The origin is defined by two radio facilities (see Fig. 1) consisting of a V-H-F omnirange station and a DME transpondor beacon. The polar coordinates consist of (1) the bearing angle "θ" which the V-H-F omnirange transmitter on the ground transmits to the airborne navigation receiver, and (2) the distance "ρ" of the aircraft from the DME transpondor beacon. These two facilities define any point (ρ, θ) within their service area. The pilot can therefore find his way to any other point (such as r, Φ) shown in Fig. 1. In addition, he can find his way to that point along any desired course "β". Automatic course-line computers have been developed which solve the trigonometry of this navigational problem. The data fed manually into the computer consists of (1) the location of the destination

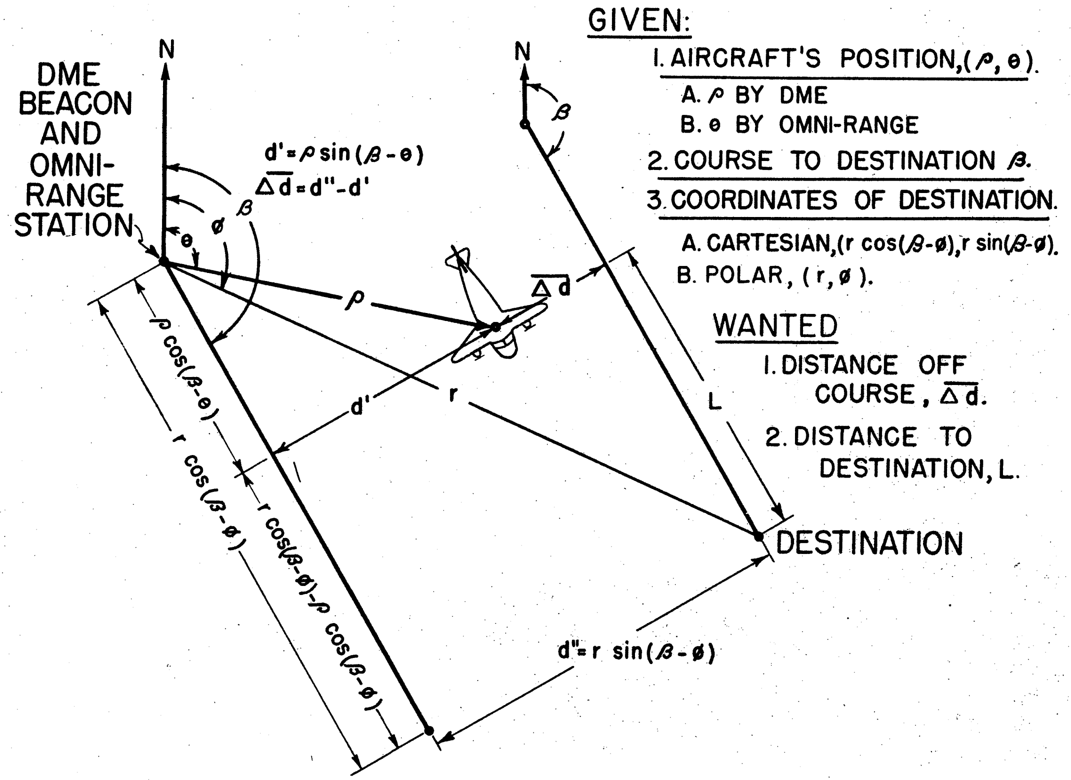


Figure 1
RHO-THETA SYSTEM OF NAVIGATION

---

* Chief Engineer, Research Division
Hazeltine Electronics Corporation

(r, Φ) from the origin, and (2) the desired course β on which it is desired to approach the destination. The airborne DME and navigational receiver automatically supply continuous information of the plane's position to the computer (ρ, θ). The computer then calculates (a) the distance Δd that the plane is off course, and (b) the distance L along the course to the destination. The necessary computation is shown in Fig. 1.

This paper describes only the DME part of the OBD system.

Each aircraft can determine its distance from 100 transpondors, assuming they are in range, and each transpondor can service as many aircraft as its power-handling capacity or condition of interference will permit.

These 100 channels are divided into 60 channels, which are associated with V-H-F omni-range, and 40 channels which are associated with V-H-F localizers of the Instrument Landing System (ILS). The DME channels associated with the V-H-F omni-range are to be divided in a pattern enclosed in a square whose side is 500 miles long. There is one V-H-F omni-distance-measuring equipment combination in every 100 mile square. This requires only twenty-five equipment to cover the 500 mile grid. The remaining 35 channels are to be used in special localities where extra coverage is desired.

It is the purpose of this paper to determine the performance that can be expected as the number of aircraft serviced by each transpondor increases.

II. DESCRIPTION OF THE DME

A. General

The operation of DME is based on the secondary radar principle illustrated in Fig. 2.

Like all secondary radars, it makes use of an interrogator responser (IR), in this case airborne, whose interrogation signals trigger a ground transpondor beacon into emitting reply signals. The time difference between the transmission of the interrogation and the receipt of the reply by the IR is proportional to the round-trip distance between the IR and the transpondor (after certain built-in delays are subtracted). The DME differs from other secondary-radars in that (1) the indication of distance is given by a d-c voltage, or angular position of a shaft, which can then be read on a meter and used to operate a navigational computer and an automatic pilot; and (2) it contains automatic search and tracking circuits whose function is to find the desired reply and to track it as the distance changes.

The operation is confined to two frequency bands. One of these, 960-990 Mc, is used by the airborne transmitter to interrogate the ground transpondor. The other band, 1185-1215 Mc, is used by the transpondor for reply.

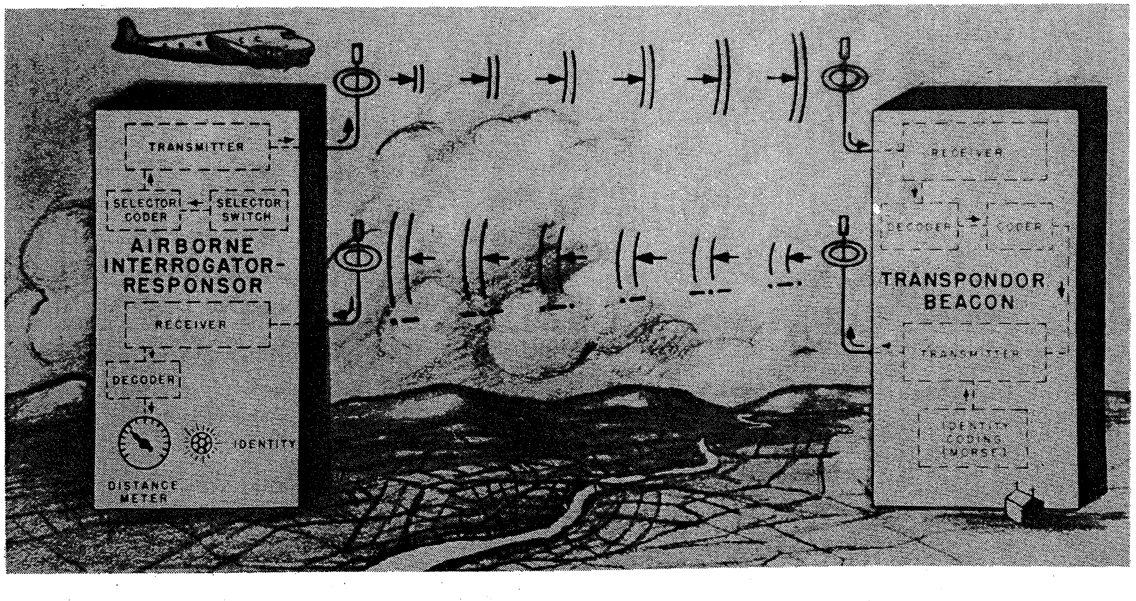


Figure 2
BLOCK DIAGRAM SHOWING OPERATION
OF PULSE MULTIPLEX DME

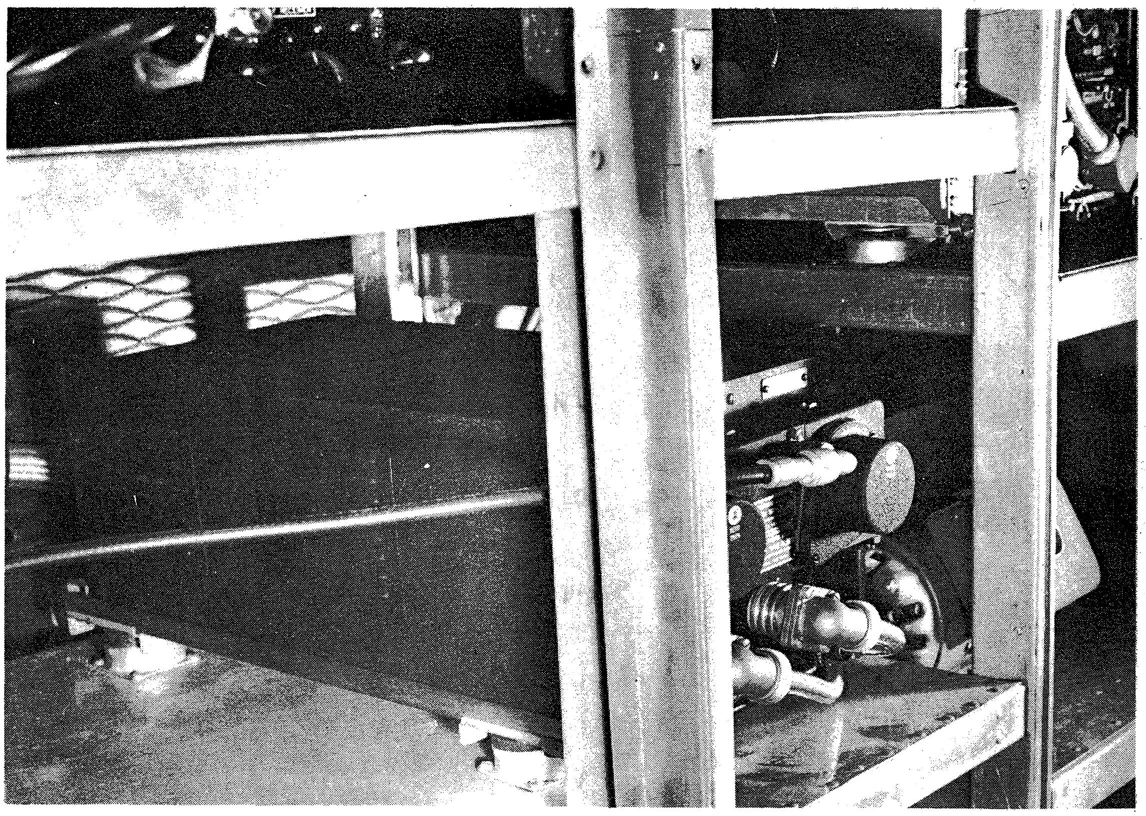


Figure 3
EARLY FORM OF DME INSTALLED ON AIRCRAFT

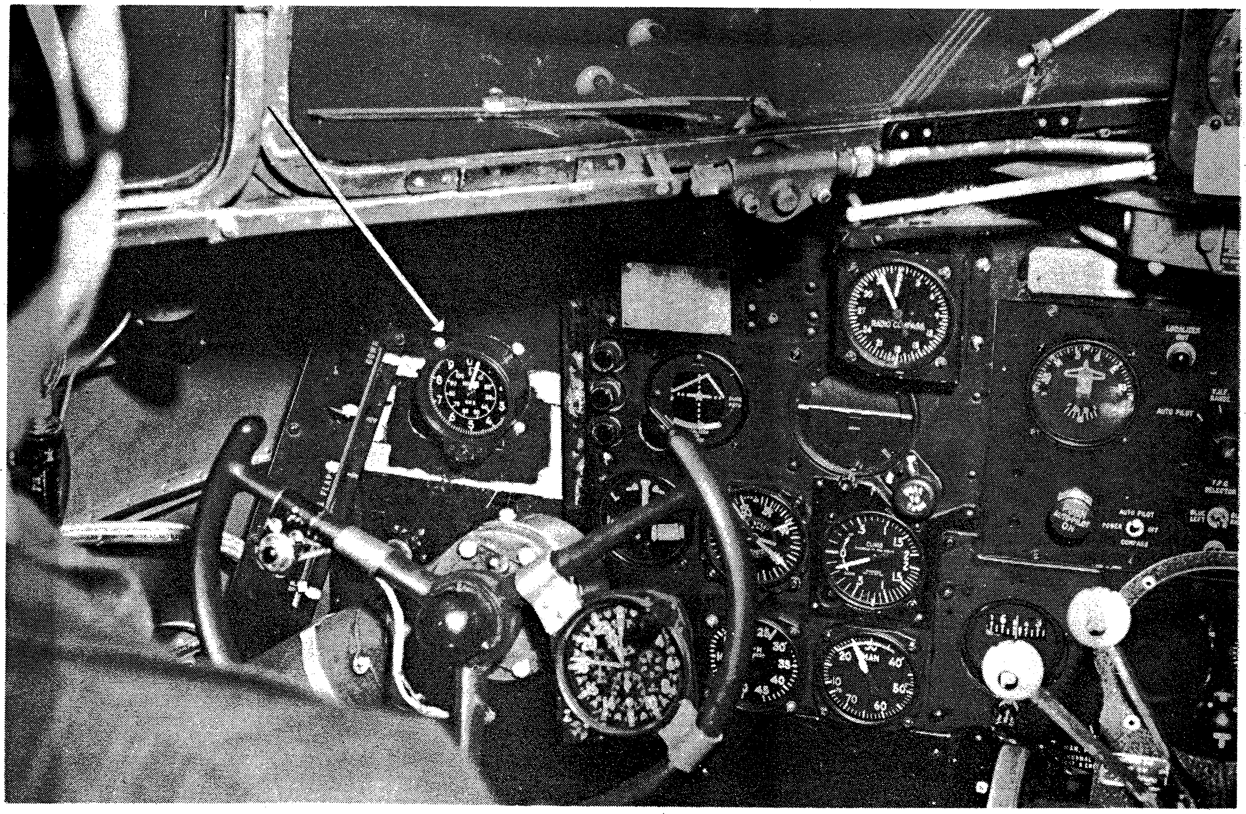


Figure 4
DME INSTALLATION ABOARD DC-3. INDICATOR IS IN UPPER LEFT HAND CORNER OF INSTRUMENT PANEL

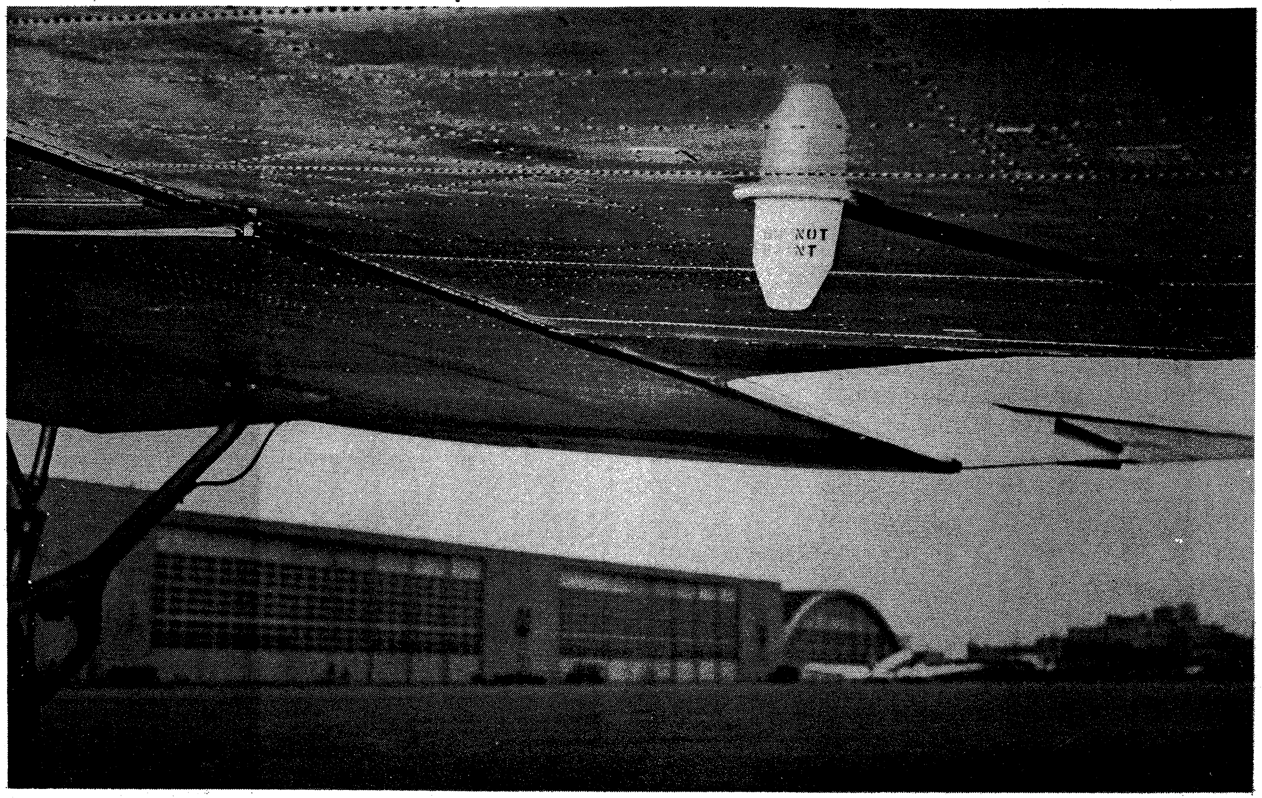


Figure 5
DME ANTENNA ON DC-3

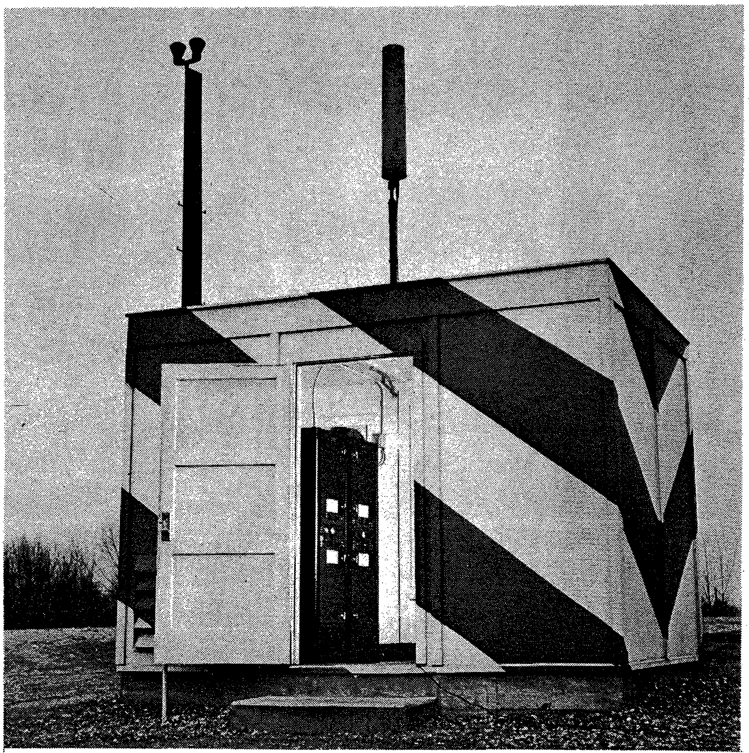


Figure 6
DME TRANSPONDOR INSTALLATION

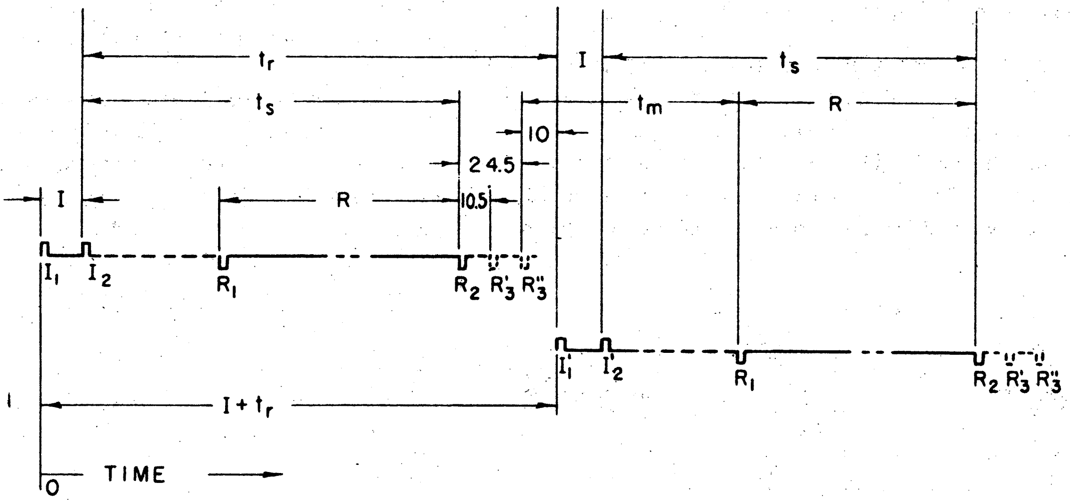
In order to provide adequate means of channelling, each interrogation and reply signal is made to consist of a pair of r-f pulses of distinctive time separation so that an aircraft will receive a reply from any one of 100 ground transpondors only if (1) the interrogation and reply frequencies are correct, and (2) the interrogation and reply pulse spacings also are correct.

The airborne IR consists of (1) a coded transmitter which emits interrogations, having the proper frequency and pulse spacing, at a random rate which averages 30 pulse pairs per second; (2) a responser which selects the frequency of the reply signal; (3) a decoder which examines the reply pulse pairs for the proper spacing; and (4) a timer which measures the time interval between the interrogation and reply, allows for built-in delays, and displays the distance in the preferred form. This airborne equipment is shown, in one of its early forms, installed on an aircraft in Fig. 3. The distance indicator is shown in Fig. 4 in the upper left side of the instrument panel. The airborne vertically polarized half-wave antenna is shown in Fig. 5.

The ground transpondor consists of (1) a receiver to receive the interrogation, (2) a decoder which examines the pulse spacing, (3) a coder which generates the reply pulse pair and introduces the specified delays, (4) a modulator which raises the reply to the necessary power level to modulate, (5) the transmitter which emits the reply on the assigned frequency, and (6) the automatic frequency control circuits which keep the transmitter and receiver on frequency. A typical ground installation, with a 4 λ vertically polarized antenna, is shown in Fig. 6.

B. Channelling (See Table I)

Each airborne DME interrogates the selected transpondor on a unique "interrogation channel".



I-DEFINITIONS

a-I_1 and I_2 are interrogation pulse-pairs

b-R_1 and R_2 are reply pulse-pairs

C-R_3 is an identifying pulse which alternates between R_3' and R_3''

d-I = interrogation pulse spacing = $14 + 7\ [C - 1]$ microseconds [where C = 1,2,3···10]

e-R = reply " " = $77 - 7\ [C - 1]$ "

I + R = total time for coding = 91 "

f-t_s = suppressed time = 115 "

g-t_r = transponder dead time = $t_s + [R_3'' - R_2] + 10 = 150$ "

II-DEDUCTIONS FROM FIGURE

a. Interrogations cannot be accepted unless they are separated by at least $[I - t_r]$ where $227 < [I + t_r] > 129$ microseconds

b. Last pulse of a reply cannot be followed by the first pulse of the next reply by less than $t_m = 10 + I + t_s - R = 34 - 2I$ microseconds [i.e. $188 > t_m < 62$ microseconds]

Figure 7
TIME RELATIONS BETWEEN SUCCESSIVE REPLIES FROM ONE TRANSPONDER

The transpondor replies on a unique "reply channel". Together these form an "operating channel" of which there are 100. Each interrogation signal consists of a pair of r-f pulses having a characteristic time spacing (known as an interrogation code) and frequency. There are ten such spacings, differing from each other in steps of 7μsec (14,21,... 70,77μsec) and ten frequencies separated by 2.5 Mc (963.5,966.0,... 986 Mc) so that 100 combinations of pulse spacing and frequency are available.

Each reply signal is also characterized by paired pulses (known as a reply code) having the same general characteristics as the interrogation spacings and ten frequencies (1185.5,1191.0,... 1211.0 Mc) so that 100 reply combinations of pulse spacing and frequency are available.

The pulse spacings are used in a complementary manner in the interrogation and reply channels so that a pulse separation of 14μsec in one channel is paired with one of 77μsec in the other.

The reply signal also carries a third pulse to identify the transpondor. This third pulse is only used while identity is actually being transmitted. This is two-thirds of the time for the longest identity code. This third pulse follows the second reply pulse by 10.5 or 24.5μsec depending on whether it is coding "mark" or "space".

The nature of the interrogation and reply signals is shown in Fig. 7.

C. Recognition of Pulse Spacings

Pulse spacings are recognized by a "decoder", an example of which is shown in Fig. 8. In this particular case the desired pulse spacing is 14μsec. The pulse pair to be examined is impressed directly on the inner grid of tube V-2 and through a delay line, which delays each pulse by 14μsec, on the outer grid of the same tube. The plate current is cut off by biases on both grids. Current will not flow unless both biases are relieved. This occurs when the second pulse, which is undelayed, coincides in time with the first pulse which is delayed by 14 μsec. The coincidence occurs only when the pulse-spacing and the delay are the same. Fig. 9 shows the tolerance of an actual decoder set to recognize spacings of approximately 14μsec.

In actual equipment, a magnetostriction delay line is used with 10 delays of 14 - 21 - 28, etc. μsec. selected by means of a switch. A similar delay line is used to create the interrogation spacing.

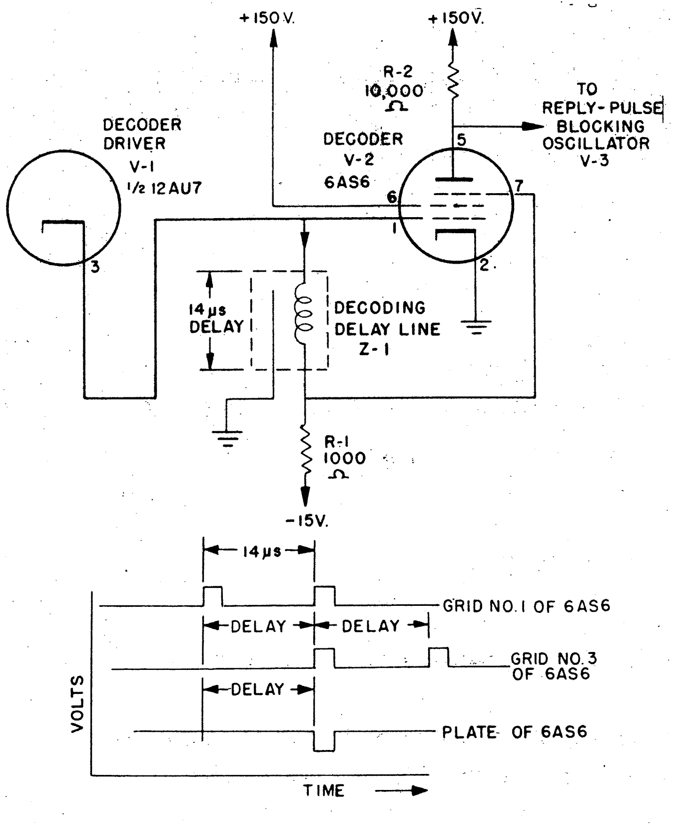


Figure 8

OPERATION OF DECODER

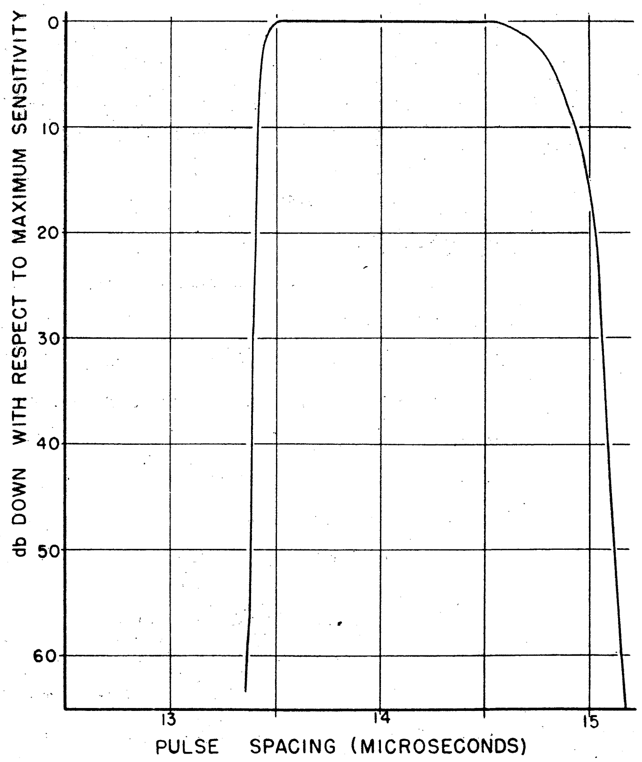


Figure 9

DECODER PERFORMANCE CURVE:
TOLERANCE IN PULSE SPACING OF A TYPICAL DECODER SET TO ACCEPT 14-MICROSECOND SPACING

D. Overall System Timing

The timing of the DME system is shown in Fig. 10 where the abscissa shows elapsed time. The interrogation consists of a pair of pulses separated by time interval d_1 (line 1 of Fig. 10). It is received, detected, and decoded by the transpondor after transmission time ℓ (lines 2, 3 and 4). Since the signal is recognized after receipt of the second pulse, all timing is referred to it. The transpondor replies, after a delay equal to δ with a pair of consecutive pulses spaced by a time interval d_2 (line 5). This reply is received, detected, and decoded by the aircraft receiver (lines 6, 7 and 8). Meanwhile a time-measuring sawtooth-voltage was started at the time of emission of the second interrogation pulse (line 9) but this voltage was not effective until after the delay time δ because it was negative until that time. The time sweep generates a narrow and a wide gate (approximately 10 and 20 μsec. respectively, which correspond to one and two miles) as shown on line 9, which straddle the received signal. The time of generation of the gates is determined by memory of their previous location (in time) from preceding interrogations and replies. There is an average of 30 interrogations per second.

Zero distance is computed after the fixed time delay δ. This delay allows the transpondor beacon to be located away from the point of desired indicated zero distance, such as a runway touchdown point, by subtracting the time corresponding to that distance from the built-in delay (thereby keeping the total effective delay constant).

E. Tracking and Searching

An individual plane segregates the replies of its own interrogations, from replies by the same

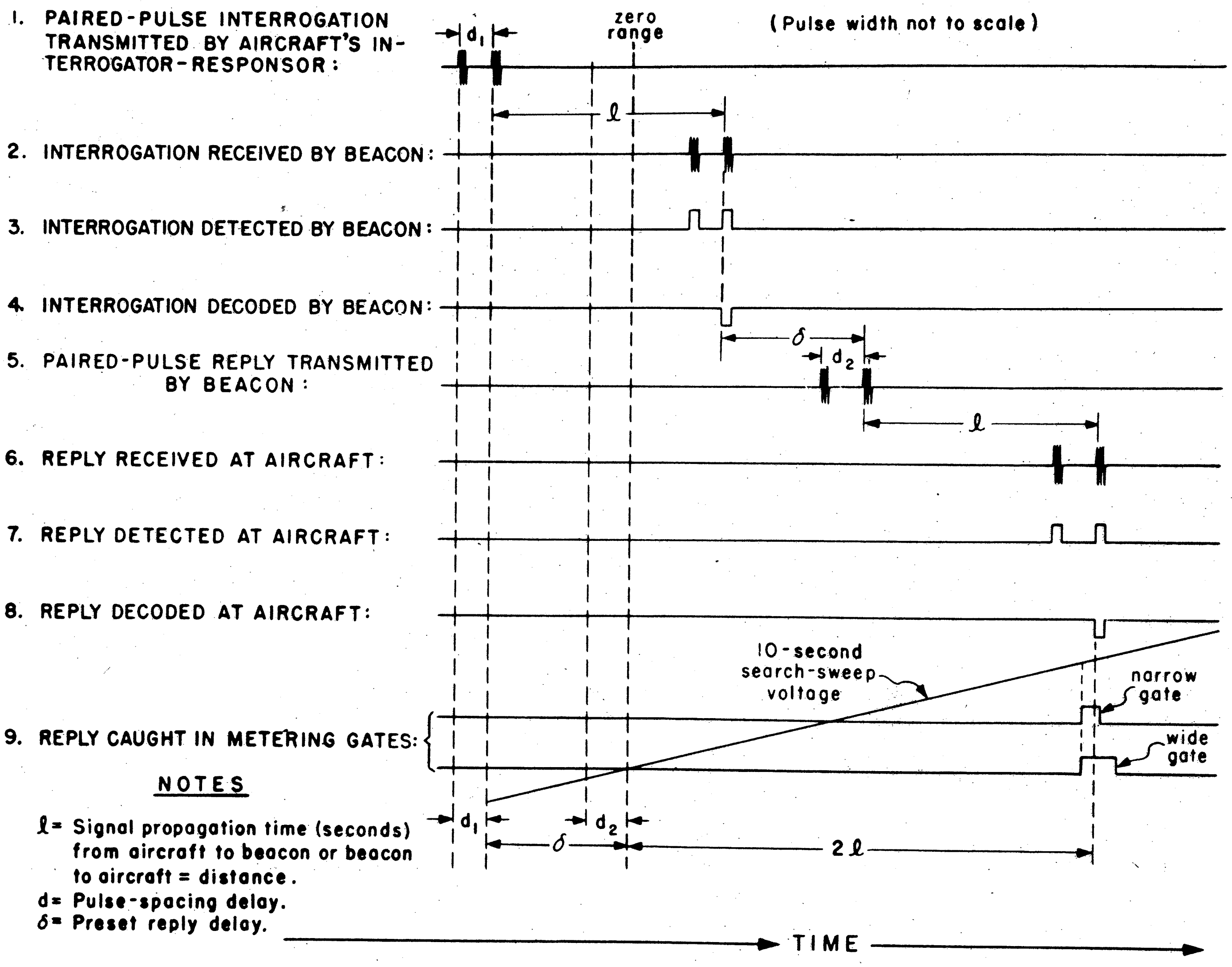


Figure 10
SYSTEM-TIME RELATIONS

transpondor to the interrogations of other planes, by the fact that its own replies occur after a fixed (or slowly changing) time (i.e. distance) after its interrogation, while the replies to other planes (also called "fruit" occur at a random time. Following the interrogation by its own transmitter, the airborne receiver is "gated-on" for a very short period, at a time equal to the round trip time. The reply to its own interrogation is received during this period because it is the only time that it can arrive at the receiver. A reply to the interrogation of another aircraft occurs at random so that the probability that it will be received when the receiver is "gated-on" is very small. It is equal to the time that the receiver is "gated-on" divided by the time that it is "gated-off". If the gate duration is 20μsec. and there are 30 interrogations per second, the probability is $20 \text{x} 10^{-6} \text{x} 30 = 0.0006$ that the reply to the interrogation of the second aircraft occurs when the receiver of the first aircraft is "gated-on" to receive its own reply.

When the IR is first turned on to interrogate a particular transpondor, there is no knowledge of the distance between the two. The DME does not know how soon after the interrogation the receiver should be "gated-on". The process of finding this time, and therefore the distance, is called "searching".

While searching for the transpondor reply, the gate is made to travel slowly over the total distance, advancing a fraction of its duration after each interrogation. It is as if the gate asked the question on one interrogation, "Is there a reply between 20 and 22 miles?", then after the next interrogation it asked, "Is there a reply between 20.5 and 22.5 miles?", etc. If the gate finds enough consecutive replies at any one distance, the search is terminated and tracking begins, that is, the receiver thereafter holds the signal which has been found.

The gate which is normally 20μsec., (i.e. 2 miles) is divided into an "early-gate" and a "late-gate". Let us assume that the center of the two gates is at 20 miles so that the distance from 19 to 20 miles is covered by the early-gate and 20 to 21 miles by the late-gate. If the signal occurs in the early-gate, the time delay of the gate is decreased for later interrogations (it is gated on at 19 instead of 20 miles). Likewise, if the signal occurs in the late-gate, the time delay is increased for later interrogations (it is gated on at 21 instead of 20 miles). This system of sub-gates allows automatic following of the signal. The time between the interrogation and the "Gate" is translated into volts by a measuring sawtooth-voltage which defines the volt-time (i.e. volt-distance) relationship.

F. Transpondor Dead Time (Fig. 7)

The transpondor is disabled for a short time after replying. This time is called "Transpondor Recovery Time", or "Transpondor Dead Time". While this "Recovery Time" serves many useful purposes, such as preventing over-interrogation, it also results in failure to reply to those interrogations which occur while the transpondor is recovering from a previous interrogation. This failure results in lowered Transpondor Reply Efficiency which is expressed as the ratio of $\frac{\text{number of replies}}{\text{number of interrogations}}$.

III. INCREASE IN EFFECTIVE TRAFFIC DUE TO "BUNCHING" OF PULSE-PAIRS.

A. Description of "Bunching" Process

A spacing acceptable to the decoder can be created by chance from two single pulses, which may each belong to separate pulse-pairs having spacings which are not accepted by the decoder. This occurs when the two pairs are received by chance in such a time sequence that one pulse of each pair is followed by a pulse of the other pair after the time required by the decoder. This is shown in Fig. 11. In this figure, a decoder is set to recognize pairs having a spacing of 21μsec. It receives and ignores pulses A and B which are separated by 14μsec, and pair C D which are separated by 35μsec. However, pulses B and C are separated by 21μsec. and form an acceptable, but spurious, pair which is accepted by the decoder. This creation of spurious pairs is called "bunching".

The increase in the number of interrogating pairs accepted by the decoder caused by "bunching", causes the system to behave as if it were servicing

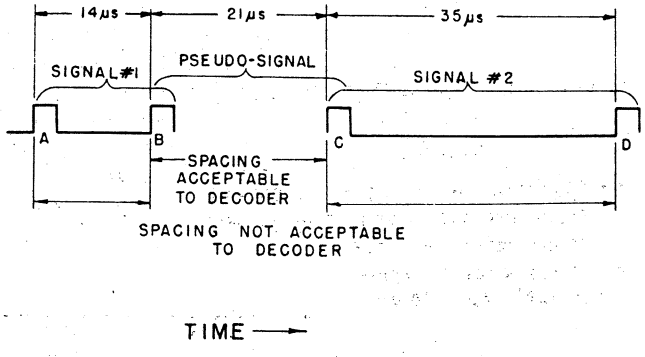


Figure 11
CREATION OF FALSE SIGNALS BY BUNCHING

more aircraft than is actually the case. Thus, the increase in the number of interrogations results in 1) a greater demand on the transpondor, 2) lower transpondor reply efficiency, and 3) greater number of "fruit" replies (i.e. interference).

The further increase in the number of "fruit" replies caused by "bunching" usually results in 1) increasing the search time (i.e. the time required to find the distance at which the desired reply is located, when the equipment is first turned on after the signal is lost); and 2) delaying, or even preventing, the resumption of search when the signal is lost beyond the desired "memory" time, because "fruit" may be mistaken for the desired reply.

The errors introduced in the measurement of distance by high "fruit density" can be shown to be small. This is because they occur approximately equally in both gates and their effects tend to cancel.

B. Scope of Analysis

The remainder of this study provides quantitative relations, in the form of equations and curves, to determine how the search time and "memory" time are affected by the number of interrogations on each transpondor. Before these relations can be established it is necessary to make a preliminary analysis to determine

1) the increase in interrogation rate due to "bunching".
2) the effect of the interrogation rate on the transpondor reply efficiency.
3) the number of "fruit" replies accepted by each gate and therefore having the appearance of the desired reply.

While these calculations are made for several values of traffic, examples are worked out for two extreme operating conditions specified by ICAO - Doc. 6580 (see reference 2 in Bibliography). These are:

1. For the transpondor - see paragraph 2.3.1.13 which states that

   "The system shall be capable of including a transpondor providing reliable service to at least fifty equipped aircraft in the presence of 1950 other aircraft distributed over the remaining ninety-nine operating channels.

2. For the Interrogator Responser - see paragraph 2.3.2.8 which states that

   "The interrogator shall be capable of successful search with transpondor efficiencies as low as 67 per cent, and through random pulse levels up to 1,600 random pairs per second on each of the ten reply codes on the same reply frequency channel."

We shall refer to these two conditions in the rest of the paper as 1) ICAO transpondor specification, and 2) ICAO I-R specification. These are not, and need not be, mutually consistent in that the traffic called for on 1) will not result in the interference stated in 2). While the transpondor specification represents more traffic than need be expected, it will result in an interference level much lower than the I-R specification requires. However, it is desirable that the I-R be overdesigned in this respect in order to allow for deterioration of the equipment and momentarily extreme bunching of traffic.

IV. COMPUTATION OF ADDITIONAL SIGNALS CREATED BY "BUNCHING" AND EFFECT OF "BUNCHING" ON PERFORMANCE

A. Increase in Interrogation Rate Due to Bunching.

All pulse-pairs on the common interrogation frequency are received by the receiver of the transpondor at random times. The decoder sorts out those pulse pairs which have the acceptable spacing and rejects the others. However, the randomness of the order in which these pulse-pairs are received results in the creation of new pairs, from the existing pulses, which have the acceptable spacing and are passed by the decoder. The following paragraphs calculate the number of these extra pairs.

The decoder of transpondor A, which is one of $c = 10$ similar transpondors sharing the same interrogation frequency, receives one pulse within an interval of one second. The probability p that a second pulse, randomly located with respect to the first pulse, will precede or follow the first by an interval acceptable to the decoder is $2b/1$, where "b" is the acceptance time interval of the decoder (i.e., the total time interval during which coincidence can exist) and is usually equal to twice the duration "d" in seconds of the pulses presented to the decoder, then

$$p = 2b/1 = 4d/1 = 4d \qquad \text{(A-1)}$$

("d" is not necessarily the same as the duration of the transmitted pulse. In fact, it is usually recreated from it and is often shorter.)

A pulse of a pair, belonging to the spacing accepted by transpondor A, will not increase the number of replies by combining with pulses of the same or (c-1) other spacings. The reason for this is that, if the transpondor is receptive when this interrogation arrives, it replies normally and becomes paralyzed for a time "t_r", whose duration is longer than the longest spacing, and is incapable of replying to real or accidentally created spacings. Therefore, only those interrogations having spacings unacceptable to the decoder can create additional acceptable interrogations by combining with each other.

Let there be a total of N such interrogations in each second, i.e., 2N interrogation pulses. These pulses can create 1/2 (2N) (2N-1) pairs, or since

$$2N-1 \cong 2N \text{ approximately}$$

the actual approximate possible number of pairs is $2N^2$.

Since the probability that any two pulses will result in the accepted spacing is given by equation (A-1), page 11, the average number of false interrogations (N_E) to be expected is equal to this probability multiplied by the number of pairs or

$$N_E = 8dN^2 \text{ per second} \qquad \text{(A-2)}$$

N_E is the increased demand on the transpondor caused by N interrogations of unacceptable spacing per second. The total effective number n_t of interrogations accepted by the transponder's decoder is therefore

$$n_t = n + N_E \qquad \text{(A-3)}$$

$$n_t = n + 8dN^2 \text{ per second}$$

(where n is the number of interrogations intended for the transpondor).

N is the total number of interrogations intended for (c-1) transpondors and, if we assume that all

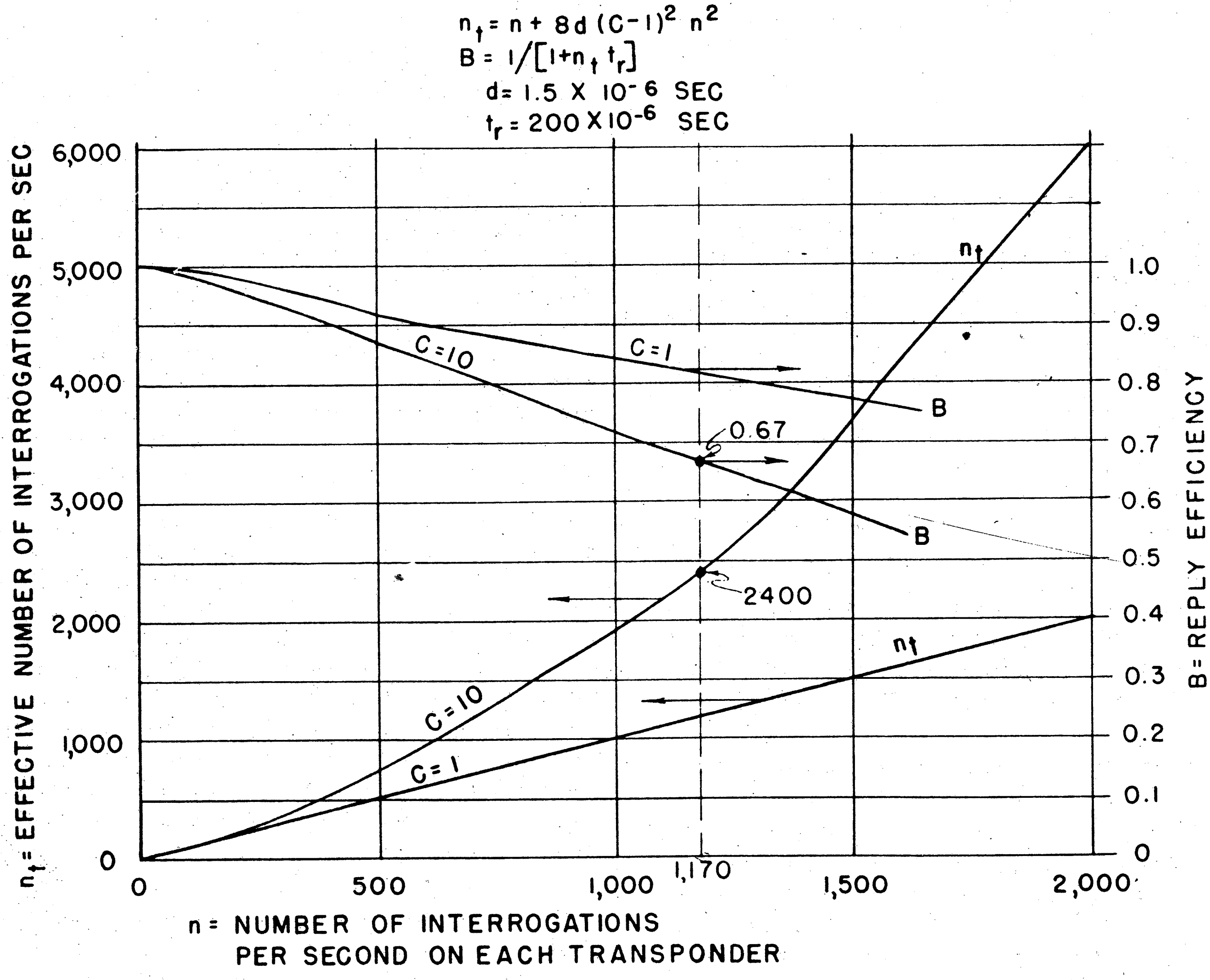


Figure 12

EFFECT OF TRAFFIC DENSITY (n) INCLUDING "BUNCHING EFFECT" ON
(1) EFFECTIVE NUMBER OF INTERROGATIONS (n_t) AND
(2) REPLY EFFICIENCY (B)

transpondors are interrogated at the same rate, n, as transpondor A, then eq. (A-2) becomes

$$N_E = 8d\,(c-1)^2 n^2 \tag{A-2a}$$

The proportional increase is

$$N_E/n = 8d\,(c-1)^2\,n \tag{A-4}$$

The effective number of interrogations is

$$n_t = n + 8d\,(c-1)^2 n^2 \tag{A-3a}$$

$$= (1 + 8d\,(c-1)^2 n)n = An \tag{A-3b}$$

$$\text{where } A = 1 + 8d\,(c-1)^2 n \tag{A-3c}$$

= effective number of interrogations per intended interrogation.

The total number (n_t) of interrogations accepted by the transpondor decoder, regardless of the distribution of their origin is given by eq. (A-3). The case of equally loaded transpondors is given by eq. (A-3a) and is plotted in Fig. 12.

The actual values of n_t will lie between the curves labelled $c = 10$ and $c = 1$ respectively. The values for $c = 10$ correspond to the case where the transpondor is within range of all aircraft interrogating the ten transpondors sharing the common interrogating frequency. The values for $c = 1$ correspond to the case where only the aircrafts demanding service of a transpondor are within range of that transpondor.

Example A - The ICAO specifications for DME require that each transpondor-beacon be capable of operating when interrogated by 50 aircraft and with an average of 20 aircraft on each of the remaining 9 channels on the same frequency. The average interrogation rate is 30 per second. The maximum average effective interrogation rate is wanted for the two types of transpondors. (d is assumed to be 1.5×10^{-6} seconds).

A - 50 aircraft transpondor

N (in eq. (A-2) is produced by the interrogations from the aircraft on the 9 lightly loaded transpondors.

1- $N = 30\text{x}20\text{x}9 = 5{,}400$ interrogations/sec.

2- Extra interrogations $= N_E = 8dN^2 = 350$/sec.

3- Direct interrogations $= n = 30\text{x}50 = 1{,}500$ per sec.

4- Total number of effective interrogations: $n_t = n + 8dN^2$ (eq.A-3) $= 1{,}850$ per sec.

5- $n_t/n = A = 1{,}850/1{,}500 = 1.23$ per sec.

B - 20 aircraft transpondor

1- $N' = 30\text{x}20\text{x}8 + 30\text{x}50 = 6{,}300$ interro/sec.

2- Extra interrogations $= N'_E = 8dN'^2 = 480$ per sec.

3- Direct interrogations $= n' = 30\text{x}20 = 600$ per sec.

4- Total number of effective interrogations $= n'_t = n' + 8dN'^2 = 1{,}080$ per sec.

5- $n'_t/n' = A' = 1{,}080/600 = 1.80$ per sec.

B. Determination of Transpondor Reply Efficiency.

Transpondor reply efficiency is the ratio of the number of replies to the number of interrogations. It is designated here by B, which is a number less than unity. The total number of replies per second is Bn_t. The average number of replies to n interrogations is Bn.

Assume the transpondor to be inoperative for a time "t_r" (which is a fraction of a second) after each interrogation is decoded. Since Bn_t is the number of replies per second then the total transpondor dead-time per second is $Bn_t t_r$. Only these interrogations which occur during the "Live Time" $1-Bn_t t_r$ will be accepted and will result in replies. The ratio of replies per second to the number of interrogations per second "n_t" is equal to the ratio of the live time to the total time or:

$$\frac{Bn_t}{n_t} = \frac{1 - Bn_t t_r}{1} \tag{B-1}$$

Solving this relation for B gives

$$B = 1/\,(1 + n_t t_r) \tag{B-2}$$

$$= 1/\,(1 + Ant_r) \tag{B-2a}$$

where n_t is given by eq. (A-3)

From (B-2) we can say that the probability "p" that a single interrogation will elicit a reply from the transpondor in the presence of "n_t" interrogations per second is

$$p = B = \frac{1}{1 + n_t t_r} = \frac{1}{1 + Ant_r} \tag{B-3}$$

(which is also the average number of replies for a single interrogation). The average number of replies "s" to "m" interrogations in the presence of a total of "n_t" interrogations per second is

$$s = Bm = \frac{m}{1 + n_t t_r} \tag{B-4}$$

The number of usable replies for one aircraft per second is

$$N_d = Bq = \frac{q}{1 + n_t t_r} = \frac{q}{1 + Ant_r} \tag{B-5}$$

where q is the interrogation rate for a single aircraft.

The total number of replies per second for each transponder is

$$Bn_t = \frac{n_t}{1 \quad n_t t_r} \qquad \text{(B-6)}$$

Equation (B-6) shows the total number of replies per transponder including those due to increased interrogation. Equation (B-3) gives the average number (a fraction less than unity) of usable replies elicited by a single interrogation in the presence of "n" interrogations.

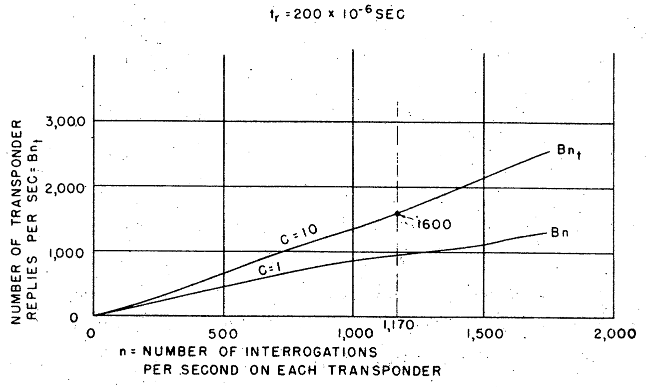


Figure 13

EFFECT OF TRAFFIC DENSITY (n) INCLUDING "BUNCHING EFFECT" ON THE REPLY RATE OF A TRANSPONDOR (Bn_t).

HAZELTINE ELECTRONICS CORPORATION

The manner in which the transpondor reply efficiency B and the total number of replies per second Bn_t vary with the actual interrogation rate per transpondor is shown in Fig. 12 and Fig. 13, respectively. The actual values of B and Bn_t will lie between the curves labeled c = 10 and c = 1 respectively for the reasons previously given for n_t.

Example B - What is, A) the transpondor reply efficiency; B) the number of replies for each of the conditions of Example A. Assume the transpondor dead-time $t_r = 200 \times 10^{-6}$ secs.

A - 50 aircraft transpondor

1- Transpondor reply efficiency, B $= 1/(1 + n_t t_r)$

($n_t = 1850$/sec) (See Example A) $= 1/(1 + 1850 \times 200 \times 10^{-6})$

$= 1/1.37 = 0.73$

2- Total number of replies per transpondor,

$Bn_t = n_t/(1 + n_t t_r)$

$= 1850 \times 0.73 = 1{,}350$ per sec.

B - 20 Aircraft Transpondor

1- Transpondor reply efficiency

$B' = 1/(1 + n_t t_r)$

$= 1/(1 + 1080 \times 200 \times 10^{-6}$

$= 0.82$

2- Total number of replies,

$B'n_t' = 1{,}080 \times 0.82 = 890$ per sec.

C - Total Number of Replies on Common Reply Frequencies

1- From nine 20-aircraft transpondors, $F = 9B'n_t' = 9 \times 890 = 8{,}010$ per sec.

2- From one 50-aircraft transpondors, Bn_t $= 1{,}350$ per sec.

3- From 10-transpondors, $F_t = 9B'n_t' + Bn_t$ $= 9{,}360$ per sec.

C. Determination of the Number of "Fruit" Replies.

The number of fruit replies per second accepted by the decoder of a given plane A is equal to the sum of

1. The number of replies to interrogations of the other aircraft which are serviced by the same transpondor as A. This is equal to

$$Bn_t = \frac{A(n-q)}{1 + A\,(n-q)\,t_r}$$

and since $n \gg q$

$$Bn_t \cong \frac{An}{1 + Ant_r} \quad \text{- approx.} \qquad \text{(B-2)}$$

and

2. Combinations of replies from transpondors replying on the same frequency, whose random arrival at the plane's decoder result in the acceptable spacing.

We shall evaluate the number of these random combinations.

The probability that successive replies of one transpondor combine into the accepted pulse-spacing is very small for those cases where it is possible. Likewise, the probability that a reply from one transpondor locates itself between two successive replies of another to create acceptable pulse-pairs with both is very small and can be neglected.

However, a pulse from one transpondor may combine with a pulse from any other transpondor to form an acceptable pair. Each of the three* pulses of a transpondor's reply can form three pairs with each of the three* pulses of another transpondor's

---

*The actual number is somewhat smaller than 2-2/3 for the ICAO specification.

reply. These two replies can therefore form nine pairs of pulses.

Let us assume, for the moment, that all transpondors are equally loaded. There are "c" such transpondors, each of which emits an average of Bn_t replies per second, and these c Bn_t replies can combine with the (c-1) Bn_t replies from the remaining transpondors to create

$$1/2\ (c\ Bn_t)\ \ (c-1)\ Bn_t = 1/2\ \ c\ (c-1)\ B^2n_t^2$$

reply combinations per second. Each of these combinations creates nine pulse-pairs. The probability that any one pair is acceptable to the airborne decoder is given by

$$p = 4d \qquad \text{(A-1)}$$

so that the average number (ΔF) of spurious pulse-pairs accepted by the aircraft is equal to the total number of pairs multiplied by the probability that each pair is acceptable. This is

$$\Delta F = 4d \times 9 \times 1/2\ \ c\ (c-1)\ B^2n_t^2 \qquad \text{(C-1)}$$

$$= 18d\ F_tF \qquad \text{(C-2)}$$

where $F_t = c\ Bn_t$ = total number of replies by all transpondors

and $F = (c-1)\ Bn_t$ = total number of replies by the (c-1) transponders which are not directly interrogated by aircraft A

The total number of replies having the proper spacing is

$$R = Bn_t + \Delta F \qquad \text{(C-3)}$$

$$= Bn_t + 18dc\ (c-1)\ B^2n_t^2 \qquad \text{(C-3a)}$$

$$= Bn_t + 18d\ \ F_tF \qquad \text{(C-3b)}$$

If we make the simplifying assumption that c-1 = c approximately, (this increases the calculated amount of interference by about 10% of the actual value, then

$$\Delta F = 18dc^2\ \ B^2n_t^2 = 18d\ F_t^2 \qquad \text{(C-2a)}$$

If, for any reason, the signals from transpondor A are not received, as may occur when the plane's antenna is shielded by the wings when banking, then only the fruit from (c-1) transpondors can combine to create the acceptable spacing. The number of such acceptable replies can be obtained by substituting (c-1) for c and eliminating the first term in equation C-3a which becomes

$$R' = 18d\ (c-1)\ (c-2)\ B^2n_t^2 \qquad \text{(C-3c)}$$

If all transpondors are interrogated by the same number of aircraft then

$$Bn_t = n_t/\ \ (1 + n_tt_r) \qquad \text{(B-2)}$$

$$= A_n/\ \ (1 + Ant_r) \qquad \text{(B-2a)}$$

If all the transpondors are interrogated by different numbers of aircraft than Bn_t must be calculated from a summation of eq. B-2 for all transpondors.

The condition imposed by ICAO for the I-R (that successful search be completed with each of the ten transpondors on the reply frequency emitting 1,600 replies with a reply efficiency of 0.67) can be seen to correspond to an effective interrogation rate of n_t = 1600/0.67 = 2,400 effective interrogations per second. These are caused by a direct interrogation rate, calculated from equation (A-3a) and plotted in Fig. 12 of n = 1170 actual interrogations per second (.i.e. 1170/30 = 39 aircraft) on each of 10 transpondors. The transpondor dead time can be calculated from eq. B-2 to be about 200 microseconds.

D. Determination of Number of Fruit Pulses Accepted by the Gate in One Second.

Definition - Effective gate duration "g" (Fig. 10, line 9)

The function of the gate is to sample the time after each interrogation for replies. There is one gate per interrogation. The gate accepts those pulses which coincide, fully or partially, with it in time. There will be at least partial coincidence if the leading edge of the pulse occurs not earlier than its own duration, "d", ahead of the leading edge of the gate and no later than the lagging edge of the gate. This is a time interval equal to the sum of the pulse and the gate duration and is equal to d + g'. To simplify our notation we shall assume an effective gate duration g = d + g' acting on a pulse of zero duration; i.e., the total time is regarding as constituting the width of the gate.

The gate will pass all the decoded pulses which occur whenever it is opened. Since there are

$$R = Bn_t + 18\ F_tF \text{ decoded pulses per second} \qquad \text{(C-3b)}$$

and the gate is opened for "g" seconds, the gate will pass

$$N_F = gR = g\ (Bn_t + 18d\ F_tF) \text{ pulses per interrogation} \qquad \text{(D-1)}$$

$$= g\ \left[Bn_t + 18dc\ (c-1)\ B^2n_t^2\right] \text{ pulses per interrogation} \qquad \text{(D-1a)}$$

$$\cong g\ (Bn_t + 18d\ F_t^2)\ \text{(approx.) pulses per interrogation} \qquad \text{(D-1b)}$$

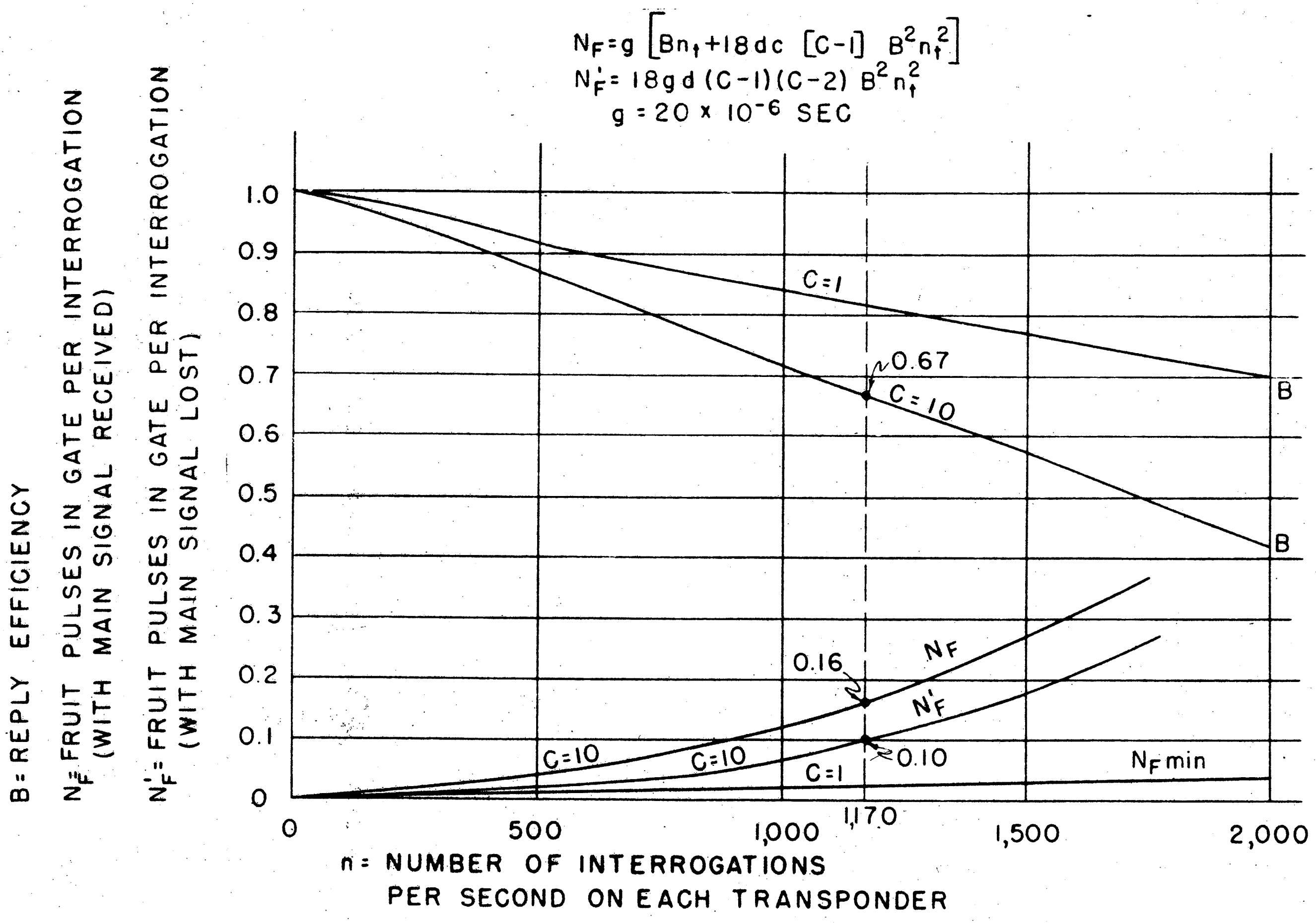


Figure 14

EFFECT OF TRAFFIC DENSITY (n) ON
(1) NUMBER OF "FRUIT" PULSES CAPTURED BY GATE WITH AND WITHOUT RECEPTION OF THE MAIN SIGNAL (N_F AND N'_F AND
(2) REPLY EFFICIENCY (B)

When the desired transpondor is lost, the number of pulses passed by the gate is given by substituting R', given by equation (C-3b), for R in equation (D-1a) which then becomes

$$N'_F = gR' = 18\ dg\ (c-1)\ \ (c-2)\ \ B^2 n_t^2 \quad \text{(D-2)}$$

Equations (D-1a) and (D-2) are plotted in Fig. 14 against "n", the actual number of interrogations on each transpondor for c = 10 and c = 1. The actual values of N_F and N'_F will lie between these limits depending on the number of transpondors, on both the interrogation and reply frequencies, whose aircraft are within range.

Example C - What is the average number of fruit pulses accepted by each 20 microsecond gate of an aircraft interrogating the 50-aircraft transpondor of Example A (i.e., ICAO Transpondor Specification).
1) Substituting in equation (D-1) from values found in Example B

$Bn_t = 1{,}350$	From Example B
$F_t = 9{,}360$	From Example B
$F = 8{,}010$	From Example B
$d = 1.5 \times 10^{-6}$	From Example A

$N_F = 20 \times 10^{-6}\ (1{,}350 + 18 \times 1.5 \times 10^{-6} \times 9{,}360 \times 8{,}010)$

= 0.067 fruit pulses per interrogation (i.e., an average of one fruit pulse is accepted by the gate every 1/0.067 = 15 interrogations).

Example D - What is this number when the replies from the 50-aircraft transpondor are not received?

Substituting the following values in equation D-2

$B'n'_t$ = 890 (from example B)

c = 10

d = 1.5×10^{-6} secs.

g = 20×10^{-6} secs.

N'_F = 0.031 fruit pulses per interrogation (i.e. an average of one fruit pulse is received every $\frac{1}{0.031} = 32$ interrogations.

Example E - What is the average number of fruit pulses accepted by each 20 microsecond gate of an aircraft operating under the conditions set by ICAO I-R Specification?

In this case, formulas (D-1a) and (D-2) apply

$$N_F = g\ (Bn_t + 18dc\ (c-1)\ B^2 n_t^2) \qquad \text{(D-1a)}$$

$$= 20 \times 10^{-6}\ (1600 + 18 \times 1.5 \times 10^{-6} \times 10 \times 9 \times 1600^2)$$

= 0.16 fruit pulses in gate per interrogation when desired signal is received

$$N'_F = 18dg\ (c-1)\ (c-2)\ B^2 n_t^2 \qquad \text{(D-2)}$$

$$= 18 \times 1.5 \times 10^{-6} \times 20 \times 10^{-6} \times 9 \times 8 \times 1{,}600^2$$

= 0.10 fruit pulses in gate per interrogation when desired signal is lost.

E. Relation between desired and "fruit" replies captured by gate of airborne DME.

Examination of Curve B (c = 10) of Fig. 14 shows that when each transpondor is interrogated 1170 times per second (i.e. 39 aircraft on each channel), it replies with an average of 67 replies to each 100 interrogations of one airplane while these 100 interrogations also cause an average of 17 fruit replies to be received by the gate of the airborne unit (Curve N_F). All the desired 67 replies follow the interrogations by the same time interval (corresponding to a fixed distance from the aircraft), while the 17 fruit replies are distributed at random. The preponderance of replies identifies the distance of the aircraft from the transpondor and, when searching, cause the searching gate of the aircraft to stop at the proper distance.

Number of Answers Available at the Proper Distance

The number of replies obtainable at the proper distance, and therefore available to end search or for tracking, is made up of 1) the replies to the plane's own interrogations (B), and 2) the replies of other transpondors which occur when the gate is open. The probability of receiving the plane's own reply is B. The probability of receiving a "fruit" reply is N_F. Therefore, the probability of receiving no reply is (1-B) $(1-N_F)$ and the probability of receiving at least one reply is

$$P\ (1) = 1-\ (1-B)\ (1-N_F)$$

$$= B + N_F - BN_F \qquad \text{(E-1)}$$

P(1) is seen to be greater than B. In other words, end of search and tracking are facilitated by the presence of a small amount of "fruit".

F. Factors Affecting the Choice of Gate Duration

If the gate is made very short, it will capture few "fruit" pulses, and the amount of traffic can be increased. However, this will result in 1) increased search time which will be discussed in the following sections, and 2) losing the signal more often because of shielding as when banking, flying between lobes, or interference taking over the gate. The table shows the relation between the gate duration and the maximum time for which a signal may be lost without going out of the control of the gate.

TABLE

Gate Duration	5	10	15	20 microseconds
Gate Control Time for a 300 Knot Airplane	2.5	5	7.5	10 seconds

In this paper a gate duration of 20 microseconds is assumed.

G. Automatic Search

1. Time Required to Find Desired Reply in Absence of "Fruit"

To end the search successfully, the gate must capture at least "s" replies to the interrogations of its transmitter. These replies are identified from "fruit" replies by the fact that they are concentrated at one distance.

To find these, the gate must not move a distance which is greater than its own length "g" during the "m" interrogations which are required to produce "s" replies and which take mT seconds to complete. (Where T = time between successive interrogations.) The maximum velocity of search V_s is therefore $V_s = g/mT = gq/m$ sec/sec where q is the interrogation frequency.

The minimum time "T_s" required to search the distance "L" is

$$T_s = L/V_s = Lm/gq \text{ seconds} \qquad \text{(G-2)}$$

$$= Lm/g \text{ interrogations} \qquad \text{(G-3)}$$

"g", "m", and "q" in equation G-2 are the factors which may be chosen in order to arrive at a satisfactory value of search time. "g" cannot be made too large as the gate will capture too many fruit replies as discussed in the preceding sections. The value of "m" can be chosen to reduce the frequency with which the search will end on fruit as described in the next section.

Equation G-3 states that Lm/g interrogations are necessary to complete the search. A higher interrogating repetition frequency (i.e. a higher value of q) will allow this to be accomplished in a shorter time.

Actual airborne DME equipment AN/APN-34 (XA-7) searches 100 nautical miles (L=1200 x 10^{-6} secs) in 15 seconds while interrogating at a rate of q=30 times per second, and exploring with a gate having a duration of 20 microseconds. This gate, therefore, explores its own duration in

$$\frac{20 \times 10^{-6} \times 15 \times 30}{1200 \times 10^{-6}} = 7.5 \text{ interrogations}$$

This equipment ends search whenever m = 3 interrogations result in s = 2 replies.

However, the main cause of long search time is the time lost in resuming the search whenever it ends on fruit so that these equations apply only if this is infrequent.

2. Frequency with which Search will be interrupted by Fruit.

a. Computation.

As the gate searches for the desired replies it captures occasional fruit before it enters the range at which they are located. It may encounter enough fruit to interrupt the search.

To decrease the chance of the search ending on fruit, we shall take advantage of the concentration of the desired replies at one range, and require that the "s" replies be captured in no more than "m" interrogations.

The probability that the search will end on fruit is the same as the probability, P_m ($\geqq$s), that the gate will capture at least "s" fruit pulses in "m" interrogations when the probability of catching a fruit pulse in a single interrogation is $p=N_F$ = gR and is given by equation (D-1) and plotted in Fig. 14.

The probability P_m (h) that the gate will capture exactly "h" replies in "m" challenges, when the probability of catching one reply in one challenge is "p" is given by the Binomial Law as follows:

$$P_m (h) = \frac{m!}{h' \ (m-h) \ !} \ p^h \ (1-p)^{m-h} \qquad \text{(G-4)}$$

The probability of catching fewer than "s" replies is

$$P_m(<s) = P_m (o) + P_m(1) + P_m(2) + \ldots \ P_m(s-1) \qquad \text{(G-5)}$$

$$= \sum_{h=o}^{h=s-1} \ P_m (h) \qquad \text{(G-6)}$$

and the probability of catching "s" or more replies is

$$P_m (\leqq s) = 1 - P_m (<s)$$

$$= 1 - \sum_{h=o}^{h=s-1} \frac{m!}{h! \ (m-h) \ !} \ p^h (1-p)^{m-h} \qquad \text{(G-7)}$$

If then m = 3 and s = 2 then equation (G-7) becomes

$$P_m (\geqq s) = P_3 (\geqq 2) = 3p^2 \ (1 - \tfrac{2}{3}p) \qquad \text{(G-8)}$$

and since $p << 1$

then $P_3 (\geqq 2) = 3P^2$ approx. (G-9)

Circuits have been devised which examine the number of replies obtained from a set of exactly "m" interrogations. The equipment delivered to CAA ended its search whenever 3 interrogations (m) resulted in 2 or more replies (s). This agrees with the ICAO specifications which require successful search with a transpondor efficiency of 67%.

If the probability is P_m ($\geqq$ s) that "m" interrogations result in "s" or more replies, then the average number of sets of "m" interrogations which are required to result in one set of "s" or more replies is $1/P_m$ ($\geqq$ s), and m/P_m ($\geqq$ s) interrogations will be required, so that a set of "m" interrogations results in "s" or more replies. Since "s" refers to fruit pulses the search will end on fruit every m/P_m ($\geqq$ s) interrogations.

Since a complete search requires at least

$$T'_s = Lm/g \text{ interrogations} \qquad \text{(G-3)}$$

the search will end on fruit S_F times per complete search where

$$S_F = T'_s / \ m/P_m(\geqq s) = \frac{T'_s P_m \ (\geqq s)}{m} =$$

$$\frac{L \ P_m \ (\geqq s)}{g} \qquad \text{(G-10)}$$

(for minimum search time i.e. $T'_s = Lm/g$) and when L = 1200 x 10^{-6} secs. and g = 20 x 10^{-6} secs.

$$S_F = 60 \ P_m \ (\geqq s) \text{ per search} \qquad \text{(G-11)}$$

For the equipment described above, in which m = 3, and s = 2

$$P_m \ (\geqq s) = P_3 \ (\geqq 2) = 3p^2 \text{ approx.} \qquad \text{(G-9)}$$

and equation (G-11) becomes

$$S_F = 60 \times 3p^2 = 180 \ p^2 \text{ approx.} \qquad \text{(G-10)}$$

$$\text{since } p = N_F = g \ (Bn_t + 18d \ F_t F) \qquad \text{(D-1)}$$

$$= g \ (Bn_t + 18dc \ (c-1) \ B^2 n_t^2) \qquad \text{(D-1a)}$$

then equation (G-11) becomes

$$S_F = 180 \ g^2 \ (Bn_t + 18dc \ (c-1) \ B^2 n_t^2)^2 \qquad \text{(G-12)}$$

or

$$= 180\ g^2\ (Bn_t + 18d\ F_tF)^2 \qquad \text{(G-12a)}$$

Example C showed that $p = N_f = 0.067$ for the traffic condition set by the ICAO transpondor specifications. This traffic will then result in the search ending on fruit at least

$S_f = 180p^2 = 180\ (0.068)^2 = 0.8$ times per search on the average. In the ICAO Interrogator-Responser Specification, where each transpondor is interrogated 1170 times per second

$p = N_f = 0.16$ so that

$S_f = 180 \times (0.16)^2 = 4.6$ times per search.

b. Reduction in "lost-search-time" by the use of "delayed memory".

This high number of search interruptions would be excessive were it not for the use of delayed memory. Memory functions as follows:

When a signal has been tracked for 5 or more seconds, the loss of the signal (i.e. its reception at a lower rate than is required to maintain tracking) will not cause search to be resumed until a certain memory time (about 6 seconds) has elapsed to give the signal a chance to reappear. If, however, the signal has been tracked for less than 5 seconds, the memory is made shorter and if the signal has been tracked for less than 1/2 second there is zero memory so that inability to maintain tracking results in immediate resumption of search.

In equipment actually operating, searching is resumed immediately, after a signal is lost, unless an average of at least two replies from three interrogations is maintained. Memory is not restored at all unless this rate is maintained for at least 1/2 second, which must yield 10 or more replies from 15 interrogations. The probability $P_{15}\ (\geq 10)$ that 15 interrogations yield at least 10 replies is given by equation G-7. An approximation of this equation is given by Poisson's exponential summation in Fig. 16. Making use of this summation we see that, for the ICAO Interrogator-Responser Specification, in which the probability that one interrogation yields one fruit reply in the gate is

$p = N_F = 0.16,$

the probability that 15 interrogations (m) will yield 10 or more replies (s) is 0.0003.

Substituting this value in eq. G-11 shows that this will occur

$S_F = 60\ P_{15}\ (\geq 10) = 60 \times 0.0003 = 0.018$ times per search

or, on the average, once every 56 searches. In other words, the search will be delayed by the application of partial memory once every 56 searches on the average. However, search will still be interrupted 4.6 times per search, but will be resumed almost immediately.

H. Resumption of Search After Loss of Signal.

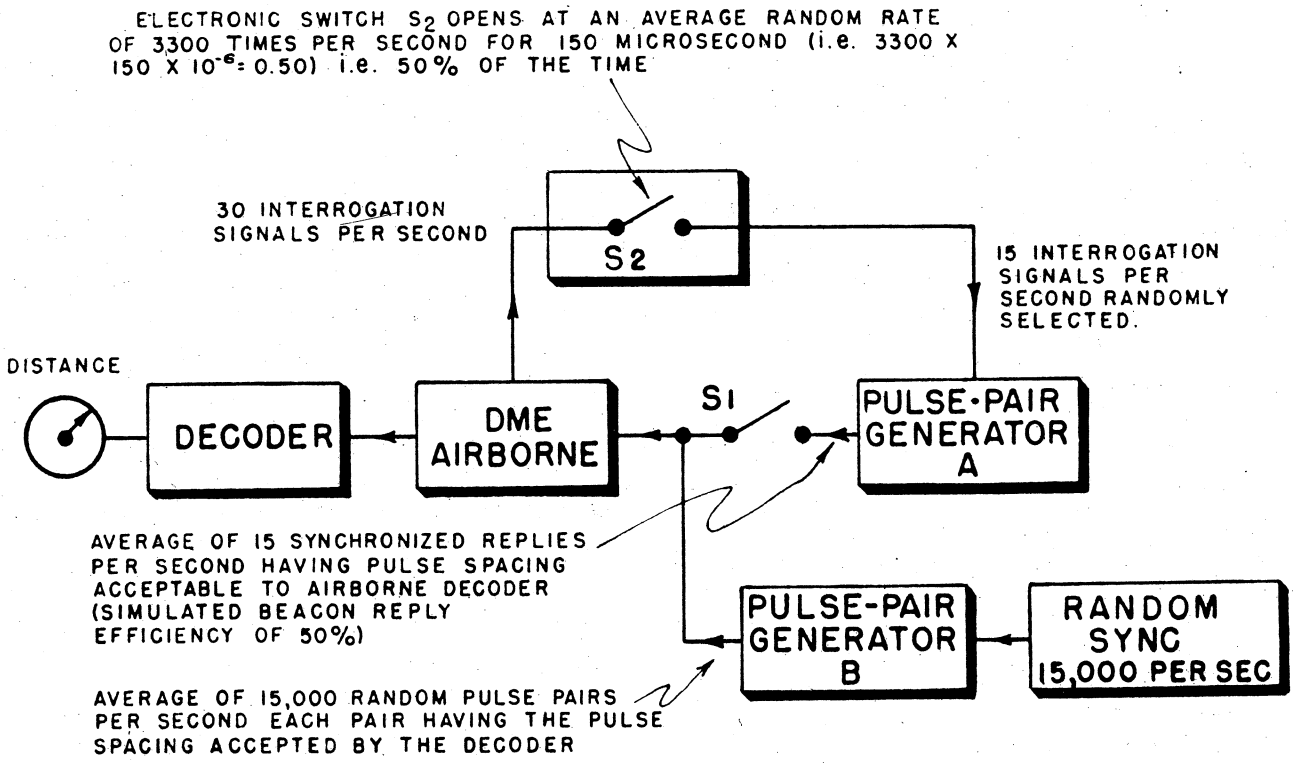
The signal may be lost for several reasons such as 1) the plane's surfaces may shield the antenna during a turn, 2) the plane may fly in a null in the transpondor's antenna pattern, 3) the transpondor may be beyond line of sight, 4) the transpondor may have had a temporary failure, etc. If the signal is lost for more than the maximum allowable "memory" time, search for the signal must be initiated.

A chance bunching of fruit replies may postpone the initiation of search. It is therefore important to determine the possibility of this occurrence.

Continued failure to receive the desired transpondor also results in failure to receive the fruit from that transpondor. The fruit which may prevent the initiation of search comes from the transpondors which share the reply frequency of the desired beacon. The number of such transpondors is (c-1).

The average number of fruit replies N'_F per interrogation which find their way into the gate is given by equation D-2 which is plotted in Fig. 14, for varying values of n, and is equal to 0.031 for the traffic specified by the ICAO Transpondor Specification (See Example D).

Even the very high traffic consisting of n = 1170 interrogations per second per transpondor specified by the ICAO I-R Specifications results in a value of only $N'_F = 0.10$ replies per interrogation when the desired signal is lost. When the desired signal is received, replies are received at the rate of 0.67 or higher (equation E-1). To operate in the presence of 1170 interrogations per second per transpondor the "search circuit" must operate when the number of pulses caught by each gate is less than 0.67 and more than 0.10. Let this number be 0.40. Then to start search after the signal has been lost for 6 seconds (i.e. 6 x 30 = 180 interrogations) the gate must capture fewer than 0.40 x 180 = 72 replies. The probability of capturing more than 72 replies in 180 interrogations (when the probability of one interrogation resulting in one reply is 0.10) is given as .000,001 in Fig. 16



TEST RESULTS

ON OPENING S_1 25 TIMES, SEARCH WAS INITIATED

6 TIMES IN NO MORE THAN 5 SECONDS
11 TIMES IN NO MORE THAN 6 SECONDS
8 TIMES IN NO MORE THAN 7 SECONDS

ON CLOSING S_1 25 TIMES, FULL 115 MILE SEARCH WAS SUCCESSFULLY COMPLETED

1 TIME IN NO MORE THAN 14 SECONDS
5 TIMES IN NO MORE THAN 15 SECONDS
6 TIMES IN NO MORE THAN 16 SECONDS
3 TIMES IN NO MORE THAN 17 SECONDS
4 TIMES IN NO MORE THAN 18 SECONDS
6 TIMES IN NO MORE THAN 19 SECONDS

Figure 15
TEST OF DME IMMUNITY TO INTERFERENCE

where mp = 180 x 0.10 = 18 and s = 72. In other words, on the average the resumption of search will be delayed less than once in 1,000,000 times that the signal is lost for more than 6 seconds.

V. Test of Performance in Presence of High Traffic.

In order to test the ability of the gate to see through a density of fruit pulses, the test shown in Fig. 15 was made. The IR interrogated pulse generator A, acting as a transpondor, 30 times per second, through switch S_2 which was open 50% of the time so that the transpondor replied with only 15 replies per second (reply efficiency = 0.50). These replies had the accepted spacing and were delayed, after the trigger, to simulate a known distance. The replies of pulse generator B were mixed with those of generator A and also had the accepted pulse spacing. Pulse generator B was triggered at random by a noise generator so as to supply 15,000 pairs of pulses per second.

Since the 15,000 random pairs per second had the proper spacing they were accepted by the airborne decoder and the number falling within each gate (whose duration was 20 x 10^{-6} sec.) was N_F =

$15{,}000 \times 20 \times 10^{-6} = 0.3$. In other words, when away from the proper distance, every three interrogations resulted, on the average, in one "fruit" reply falling within the gate. At the proper range an average of 0.65 ($0.5 + 0.3 - 0.15 = 0.65$) see equation E-1) replies were received. This corresponds to 1750 interrogations per second on each of the ten transpondors.

The memory and the time required to search 115 miles were set for 6 seconds and 14 seconds respectively in the absence of "fruit". Switch S_1 was opened and the time required to initiate search (i.e. memory) was noted. When search was initiated switch S_1 was closed and the time to search through 115 miles was noted.

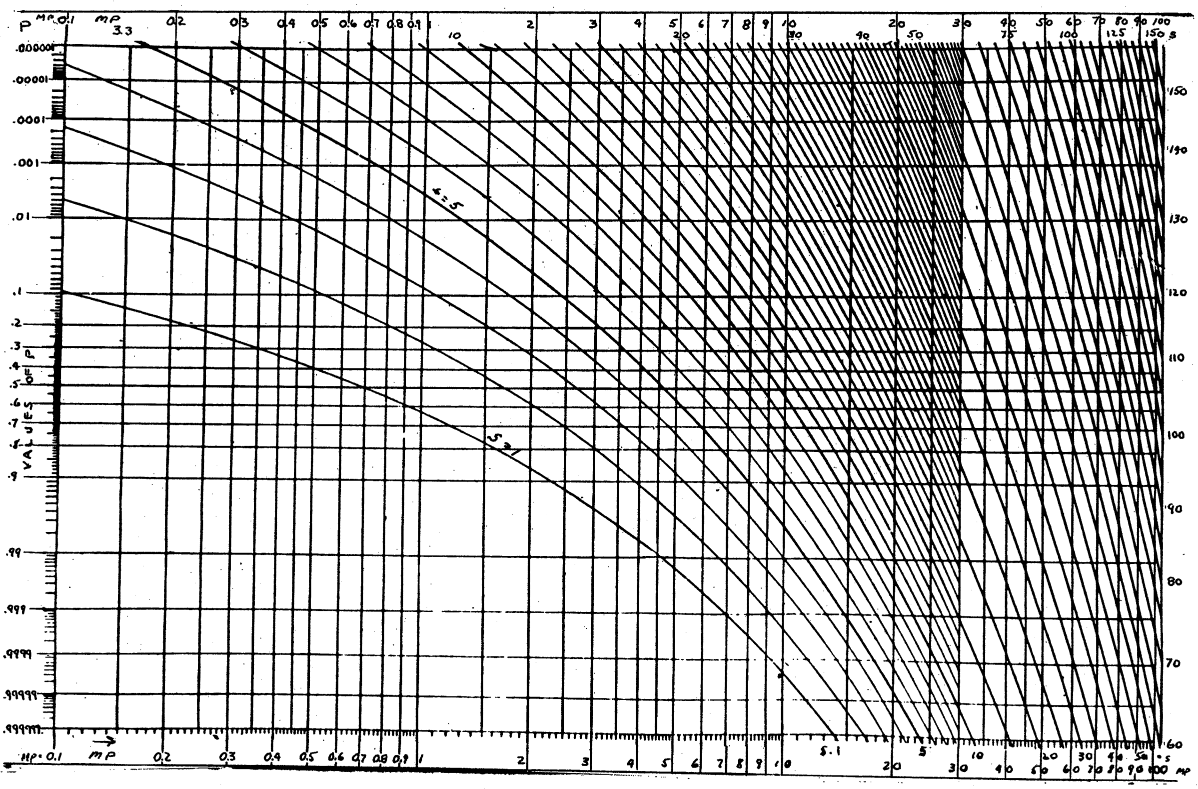
The average of twenty-five trials resulted in 1) an average and longest memory of 6.0 and 7 seconds respectively, 2) an average and longest search time of 17 and 19 seconds respectively.

This test is seen to be much more severe than the ICAO I-R Specification in which each of 100 transpondors is interrogated 1170 times per second (i.e. by 39 aircraft, all of which are within range of all transpondors) because the reply efficiency ($B = 0.50$) and the "fruit" rate ($N'_F = 0.33$) in the absence of the desired signal are worse than in the case of the ICAO I-R Specifications (where $B = 0.67$ and $N'_F = 0.10$).

This test condition should result in 1) the resumption of search being delayed approximately once in 30 times (see Fig. 16 where $mp = 180 \times 0.3 = 54$ and $s = 180 \times 0.4 = 72$); 2) the average search being interrupted at least (eq. G-10) $180 \times (.3)^2 = 16.2$ times.

VI. CONCLUSIONS

For the traffic contemplated for many years on DME, paired-pulse coding may be safely used as a channelizing method. With ten paired-pulse codes, ten interrogation frequencies, and ten reply frequencies, unique interrogation and reply channels can be created for each of 100 transpondors. The amount of interference caused by the sharing of each frequency by ten transpondors is well below the level that would interfere with the automatic



ADAPTED FROM BSTJ Vol. 5, p. 612. $P = 1 - \left[1 + \frac{mp}{1!} + \frac{[mp]^2}{2!} + \ldots + \frac{[mp]^{s-1}}{(s-1)!}\right] e^{-mp}$ for the probability P that an event occur at least "s" times in a large groups of trials for which the average number of occurences is "mp". A scale proportional to the normal probability integral is used for P_1 a logarithmic scale for mp.

Figure 16

PROBABILITY CURVES SHOWING POISSON'S EXPONENTIAL SUMMATION functions of the airborne unit for the amount of traffic contemplated by CAA and ICAO for many years to come. Even the case of the ICAO I-R Specification, where each of 100 transpondors is interrogated by 39 aircraft (with all aircraft within range of all transpondors), results in negligible deterioration of these functions. An actual test, which simulates more severe conditions, corroborates the results. Comparisons of the ICAO Specifications with the test conditions are shown in Table II labeled "Performance Summary" under columns 1, 11, and 111. Thus the table shows that the average search time was increased from 14 seconds to 17 seconds by traffic which resulted in each gate capturing 0.30 "fruit" pulse reply per interrogation, while the ICAO traffic results in only 0.067 "fruit" pulses per interrogation. Successful search was accomplished with a reply efficiency of 50% while the ICAO traffic results in a reply efficiency of 79%. The average memory was not appreciably prolonged under test conditions resulting in 0.30 "fruit" pulses per interrogation while the ICAO traffic will result in only 0.031 "fruit" pulses per interrogation.

TABLE I

DEFINITION OF OPERATING CHANNEL

1. Each of 100 operating channels is unique and consists of one unique interrogation and one unique reply channel.

2. 100 unique interrogation (or reply) channels are made up of the combination of 10 interrogation (or reply) frequencies and 10 interrogation (or reply) pulse spacings.

2a. The 10 interrogation (and reply) frequencies are:

	1	2	3 through 10
Frequency in Mc Interrogation	963.5	966.0	968.5 ... 986.0
Reply Frequencies (Mc)	1188.5	1191.0	1193.5 ... 1211.0

2b. The 10 interrogation (and reply) pulse spacings are:

	1	2	3 through 10
Spacings in μsec Interrogation	14	21	28 ... 77
Reply	77	70	63 ... 14
Total Coding Time	91	91	91 ... 91

TABLE II

PERFORMANCE SUMMARY

	I ICAO-TRANSP. SPEC.	II ICAO-I.R SPEC.	III TEST
A - Transpondor	50 a/c on one transpondor. 20 a/c on all others	39 a/c on each transpondor	corresponds to 58 a/c on each transpondor
1 - Number of actual interrogations (n) on 50 a/c transpondor	1,500	1,170	1,750
2 - Number of effective interrogations (n_t)	1,850	2,400	4,600
3 - Transpondor reply efficiency (B)	0.79	0.67	0.50
4 - Number of replies per second by transpondors (Bn_t)	1,350	1,600	2,300 (eq. C-3a)
5 - Number of fruit pulses in gate per interrogation when desired transpondor is received (N_F)	0.067	0.16	0.30
6 - Number of fruit pulses in gate per interrogation when desired transpondor is not received (N'_F)	0.031	0.10	0.30
7 - Number of search interruptions per search (S_F)	0.8	4.6	16.2
8 - Number of search resumptions delayed	<0.000001	<0.000001	0.03
9 - Search time set for			14 seconds
9a - Actual search time (average of 25)			17 seconds
9b - Actual search time (longest of 25)			19 seconds
10- Average memory set for			6 seconds
10a- Actual memory (average of 25)			6 seconds
10b- Actual memory (longest of 25)			7 seconds

TABLE III

A. Ground Transpondor Beacon

1. Pulse-duration - 2.5μsec.
2. Pulse-spacing - 14, 21, 28, etc. μsec.
3. Reply delay - 115μsec.
4. Peak power-5.0 kw
5. Transmitter (reply) frequency-one of 10 frequencies in 1,188.5- to 1,215- Mc band
6. Receiver (interrogation) frequency-one of 10 frequencies in 960- to 986- Mc band
7. Reply sensitivity-100 db <1 volt open circuit
8. Selectivity
   - 3-db bandwidth 1.3 Mc
   - 30-db bandwidth 3.2 Mc
9. Image ratio-52 db
10. Intermediate frequency-60 Mc

B. Airborne Unit

1. Challenge repetition frequency-30 per second
2. Transmitter (interrogation) frequency-same as beacon receiver frequency
3. Peak power-3 kw
4. Receiver (reply) frequency-same as beacon transmitter frequency
5. Search sensitivity-100 db <1 volt open circuit
6. Selectivity
   - 3-db bandwidth 1.3 Mc
   - 30-db bandwidth 3.2 Mc
7. Image ratio-57 db
8. Intermediate frequency-60 Mc

C. Antenna

1. Airborne-nondirectional-one-half wavelength, vertically polarized
2. Ground-nondirectional-four wavelengths, vertically polarized

SYMBOLS

Moded-Operation - operation by pulse pairs separated by a specified time interval. Each specified time interval is one mode.

Fruit - a non-synchronous reply (to a foreign interrogation)

The following terms are used interchangeably:
transponder and beacon
interrogation and challenge.

a - average number of airplanes per beacon

A - ratio of effective challenges to actual challengers $= n_t/n$

b - total time interval during which the decoder recognizes coincidence between pulses

B - Transponder Reply efficiency $= \frac{\text{Number of Replies}}{\text{Number of Interrogations}}$

c - number of pulse spacings (modes) used on each frequency

d - duration of pulse in seconds

ΔF - additional fruit replies per second caused by the random arrival at decoder in proper sequence, of pulses from several beacons that are on the acceptable frequency but have unacceptable pulse spacing.

g - effective duration of gate pulse in seconds $= d + g'$

g' - duration of gate pulse in seconds (also called strobe)

L - maximum distance serviced in round trip time of propagation of radio wave - (seconds)

m - maximum number of challenges required to elicit "s" replies

n - total number of actual challenges per beacon per second $= aq$

n_t - total number of effective challenges per beacon per second $(n' = An)$

N - total number of interrogations having a pulse spacing unacceptable to the decoder

N_E - number of false challenges per second

N_F - number of fruit replies accepted by the gate per interrogation when the desired beacon is received by the aircraft

N'_F - number of fruit replies accepted by the gate per interrogation when the desired beacon is not received by the aircraft

p - probability of one event

$P_m(s)$ - probability of capturing exactly "s" replies in "m" challenges

$P_m(\geq s)$ - probability of capturing "s" or more replies in "m" challenges

$P_m(<s)$ - probability of capturing fewer than "s" replies in "m" challenges

q - average number of challenges per second per aircraft

R - total number of replies of the proper spacing when the desired beacon is received by the aircraft

R' - total number of replies of the proper spacing when the desired beacon is not received by the aircraft

s - minimum number of replies required to end search

S_F - number of times that the search ends on fruit while searching over the complete range of "L" seconds

t_r - recovery time of ground beacon

T - time between successive challenges in seconds

T_s - time required to search for desired replies over complete range of L (in seconds)

T'_s - search time in number of challenges $= q\ T_s$

V_s - searching velocity in seconds/seconds

BIBLIOGRAPHY

1. DME System Characteristics (Transition Period, RTCA, Paper 121-48/DO-24, December 15, 1948).

2. Docket 6580, Com/520, February 26, 1949, COM Final Report, International Civil Aviation Organization, "Development Specification for UHF Distance-Measuring-Equipment".

3. Docket 2553, COT/26 Final Report, First Session, Special Radio Technical Division, Provisional International Civil Aviation Organization, January 1947, Pages 36, 65, and 73.

4. Traffic Handling Capacity of Distance-Measuring-Equipment by Charles J. Hirsch, Hazeltine Electronics Corporation, Report 2059W-R-1, August 1947.

5. Pulse-Multiplex System for Distance-Measuring-Equipment (DME) Proceedings of I.R.E. November, 1949.

TELEVISION RECEIVING ANTENNAS

By

David C. Kleckner

and

A.E. Joust

EDITOR'S NOTE: Following is a summary of a Paper presented by Mr. Kleckner and Mr. Joust before the Radio Club of America, New York City on February 18, 1949.

The antenna described consists of a large corner reflector and broad band dipole. Curves were shown for the directivity which indicated a single major lobe on all 12 channels. Standing wave data indicated standing wave ratios less than two to one over most of the range. Exact means for matching the broad band dipole were not disclosed.

o

"LOUDSPEAKERS—WHAT GOES IN AND WHAT COMES OUT"

By

H.C. Hardy*

EDITOR'S NOTE: Following is a summary of a Paper presented by Mr. Hardy before a joint meeting of the Audio Society and the Radio Club of America, New York City on April 12, 1949.

A considerable number of tests indicated poor correlation between amplitude response curves on loudspeakers and their utility for listening to music. Rather good correlation was obtained between cone breakup and listener fatigue. Cone breakup is, of course, a function of power level and results in the mechanical production of spurious non-harmonically related acoustical outputs. In most cases it is not necessary to use more than one scanning test frequency to determine whether a speaker is free of cone breakup at a particular power level. Some data was presented which indicated extremely limited power handling capacity free of cone breakup for most commercial speakers. One might draw the conclusion from this paper that only a properly designed exponential horn and small diaphragm driver would be free of breakup at reasonable power levels.

o

REPORT ON IBCG MEMORIAL

By

George E. Burghard

The Town of Greenwich has given the Radio Club permission to erect the 1BCG monument at the intersection of Clapboard Ridge Road and North Street. This location is about 100 feet from the original site of the station but faces on a main thoroughfare. The marker will consist of a five ton solid granite block with suitable inscription commemorating the sending of the first short wave message across the Atlantic using low power.

The stone is now being cut at Barre, Vermont and should be completed by the end of March. The dedication ceremonies will be held at Greenwich, Conn. in late May or early June of this year.

---

* Supervision Acoustic and Vibration Section, Armour Research Foundation.

New Duo-Cone loudspeaker, developed at RCA Laboratories, achieves the illusion of "living presence."

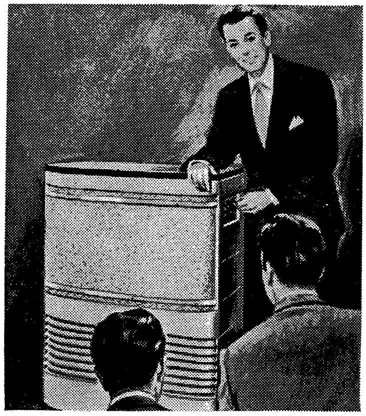
Music lovers' **"horn of plenty"**

For years, working toward the ultimate in sound reproduction, scientists have sought for *living presence*—the illusion that a real musician or speaker is talking, singing, playing in your home.

Now, with RCA's Duo-Cone loudspeaker, the goal is achieved. Two sound-cones in acoustical alignment reproduce sound without distortion, and give you every tonal value from a frequency of 30 on to 15,000 . . . and even 18,000 cycles. It is in the area above a frequency of 4,000—seldom touched by conventional speakers – that most of the all-important "overtones" lie. RCA's Duo-Cone loudspeaker faithfully reproduces every overtone—to the very peak of a violin's range—and just as faithfully gives you the deep low notes of a bass drum!

In addition, the RCA Duo-Cone loudspeaker's wide angle of sound pervades every corner of a room without sharply directed blast or blare. Its response to tones of every frequency is smooth, flowing, and even.

See the newest advances of radio, television, and electronics at RCA Exhibition Hall, 36 W. 49th St., New York. Admission is free. Radio Corporation of America, Radio City, N. Y.



Ideal for monitoring radio—AM-FM television programs—RCA's Duo-Cone loudspeaker gets, and passes, its toughest tests from engineers and experts in sound reproduction!

RADIO CORPORATION *of* AMERICA

World Leader in Radio — First in Television

WIRE *Made to Your Order*

- Precisely! Consider this your own wire-production department, because: (1) Here are engineering and laboratory facilities ready to collaborate on your particular problems and needs; (2) Production facilities geared to kind and quantity of wire you require; (3) Unsurpassed craftsmanship still available in a closely-knit organization; and so (4) You can always count on wire that's made right.

We invite your inquiries on standard or special wire and insulation. Samples promptly furnished on request.

Partial List of Products...

Bare Wire (Hudson Wire Division)
- Copper (Electrolytic and OFHC)
- High and Low Brass, Bronze
- Zinc
- Silver-plated
- Tin
- Phosphor-bronze
- Nickel-silver
- Cadmium
- Specialty Wires

Insulated Wire (Winsted Division)
- Enameled
- Multiplied-enameled
- Litz
- Cement-coated Enameled
- Cotton-covered
- Silk-covered
- Celanese-covered
- Celanese-covered Enameled
- Glass-fibre Insulated
- Voice-coil
- E-Z Sol
- Enameled Aluminum
- Formvar
- Loop Wire

One of the Oldest Fine Wire Companies in America

HUDSON WIRE COMPANY

HUDCO · BARE FINE WIRE MANUFACTURERS · Winco

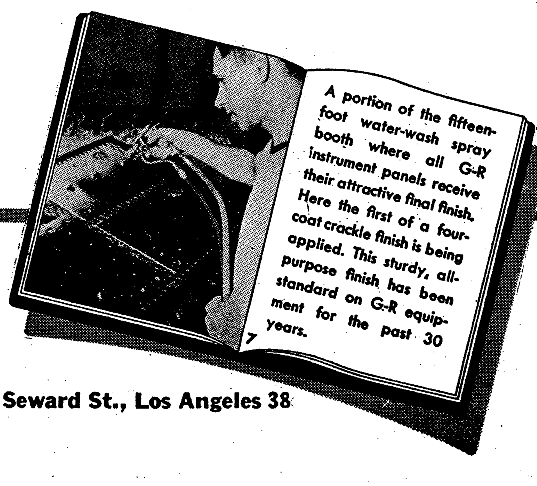
BARE WIRES GENERAL OFFICES: OSSINING, N.Y. • WINSTED DIVISION: WINSTED, CONN. INSULATED WIRES

for GREATER ACCURACY • LESS DISTORTION • IMPROVED ZERO ADJUSTMENT • MANY OPERATING CONVENIENCES

WITH many improvements over its predecessors . . . the Type 913 Series . . . the Type 1304-A Beat-Frequency Oscillator is the finest instrument of its type obtainable. For general laboratory use it has many features found in no other beat-frequency oscillator. High accuracy . . . low drift . . . low waveform error . . . these are only some of the features of this instrument. Others are:

- **Frequency Range:** 20 to 20,000 cycles.
- **Frequency Control:** main dial is engraved with logarithmic scale from 20 to 20,000 cycles; frequency increment dial calibrated from +50 to −50 cycles.
- **Frequency Calibration:** can be standardized to within 1 cycle at any time by setting instrument to line frequency or to zero beat. Frequency control dial calibration is accurate to within ±(1% + 0.5 cycle).
- **Low Distortion:** harmonic content approximately 0.25% between 100 and 7500 cycles with 0.3 watt maximum output; on HIGH OUTPUT of 1 watt, distortion is less than 1% between 100 and 7500 cycles. Below 100 and above 7500 cycles distortion is greater.
- **Frequency Stability:** drift is less than 7 cycles in first hour from cold start, and is completed in 2 hours.
- **Zero Beat Indicator:** neon lamp for zero beat at line frequency or zero scale.
- **Output Impedance:** 600 ohms, either grounded or balanced-to-ground, and essentially constant at all output voltages.
- **Output Voltage:** approximately 25 volts open circuit. For matched resistive load, voltage varies less than ±0.25 db between 20 and 20,000 cycles.
- **Output Control:** calibrated from +25 to −25 db referred to 1 milliwatt into 600 ohms.
- **A-C Hum:** for NORMAL output voltage a-c hum is less than 0.1% of output voltage

TYPE 1304-A BEAT-FREQUENCY OSCILLATOR . . . $450

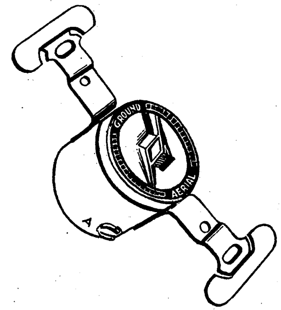

A portion of the fifteen-foot water-wash spray booth where all G-R instrument panels receive their attractive final finish. Here the first of a four-coat crackle finish is being applied. This sturdy, all-purpose finish has been standard on G-R equipment for the past 30 years.

7

GENERAL RADIO COMPANY

Cambridge 39, Massachusetts

90 West St., New York 6 920 S. Michigan Ave., Chicago 5 1000 N. Seward St., Los Angeles 38



Better AM, FM and SW for 17,062 Families . . .

The Multicoupler Antenna System is thoroughly covered by basic and detail patents owned by Amy, Aceves & King, Inc.

Licensed manufacturers are supplying multicoupler outlets and other components for the system.

The Arrow-Hart & Hegeman Electric Co., Hartford, Conn., offers all necessary equipment such as installed in the apartment-house developments herein referred to.

Leading electrical contractors handle the installation in various localities, in accordance with the rigid A.A.K. specifications.

The Multicoupler Antenna System for years past has been operating in Parkchester (12,273 radio outlets) and in other apartment colonies and individual apartment houses.

This system is the one tried-tested-perfected means of providing satisfactory reception in multi-family dwellings.

● **The Multicoupler Antenna System* now comes to the rescue of the 17,062 families who will reside in New York's five huge apartment-house developments under construction.**

Tenants of Clinton Hills (1560 apartments), Peter Cooper Village (2500 apartments), Stuyvesant Town (8761 apartments), Riverton (1236 apartments) and Fresh Meadows (3005 apartments) will soon be enjoying better AM, FM and SW reception merely by plugging their sets into convenient radio outlets. **The usual self-contained loop sets which cannot perform satisfactorily in large apartment houses because of the steel framework and metal lath, will be placed on a par with sets operating in the open country, thanks to the excellent aerial provided by the Multicoupler Antenna System.**

Thus five more mighty installations are added to the impressive roster of Multicoupler Antenna Systems providing individual-antenna performance in multi-family dwellings.

● **Designed, developed and engineered by Amy, Aceves & King, Inc., and manufactured by licensees. Literature on request.**

A. A. K.
NOISE - REDUCING
ANTENNA SYSTEMS

AMY, ACEVES & KING, INC.

Inventors, Engineers and Licensors
of the MULTICOUPLER ANTENNA SYSTEM

* Reg. U. S. Patent Office

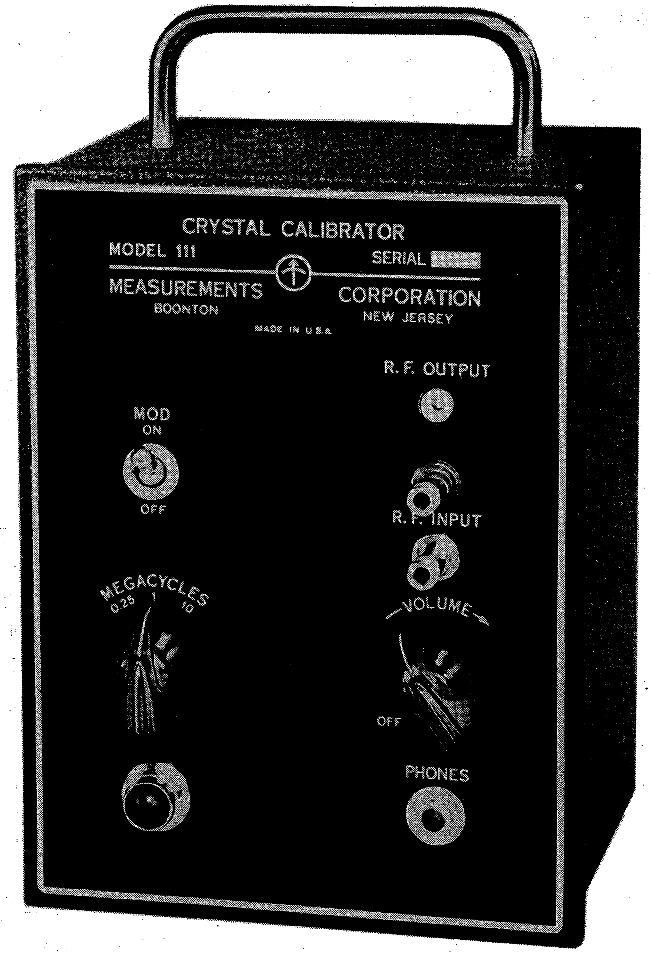
11 West 42nd Street New York City

CRYSTAL CALIBRATOR
Model 111

An Extremely Accurate Instrument
For The Frequency Calibration Of
Equipment In The Range Of—

250 Kc. to 1000 Mc.

FREQUENCY ACCURACY ±0.001%



The Model 111 was designed as a dual-purpose calibrator. It not only provides a test signal of crystal-controlled frequency, but also has a self-contained receiver with a sensitivity of 2 microwatts.

A new circuit arrangement utilizes the cross-modulation products of three separate oscillators operating at the fundamental frequencies of .25, 1.0 and 10 megacycles. This system extends the usable range of harmonic frequencies far beyond that of previously available equipment.

Form 100-10M 3-50

DESIGNED FOR THE CALIBRATION AND FREQUENCY CHECKING OF—

- Signal Generators
- Transmitters
- Receivers
- Grid-Dip Meters

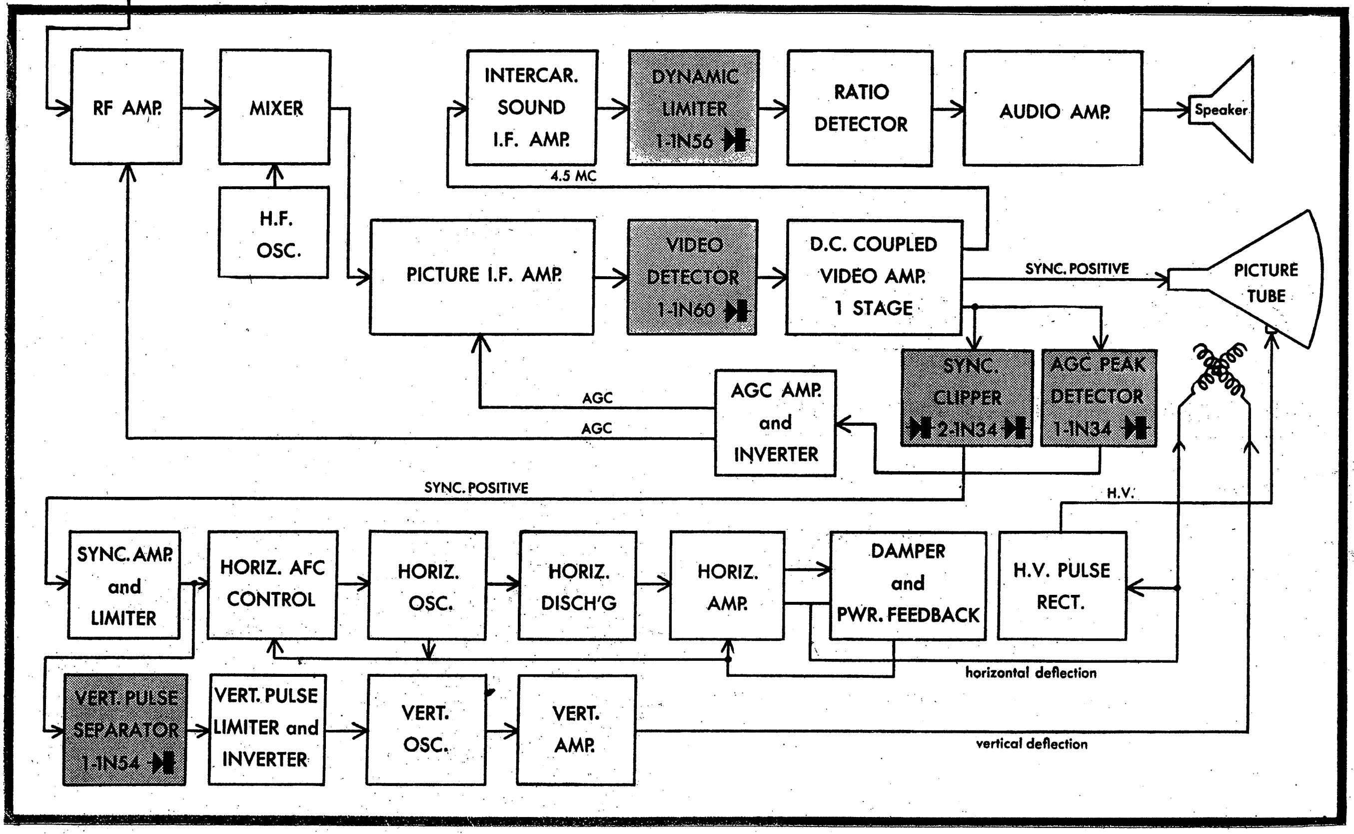
And other equipment where a high degree of frequency accuracy is required.

SPECIFICATIONS:

FREQUENCY RANGE:	.25 to 1000 Megacycles
HARMONIC RANGE:	.25 Mc. Oscillator: .25-450 Mc. 1 Mc. Oscillator: 1-600 Mc. 10 Mc. Oscillator: 10-1000 Mc.
FREQUENCY ACCURACY:	± 0.001%
POWER SUPPLY:	117 volts, 50/60 cycles; 18 watts.
DIMENSIONS:	6" wide, 8" high, 5" deep.
WEIGHT:	4 lbs.

MEASUREMENTS CORPORATION
BOONTON NEW JERSEY

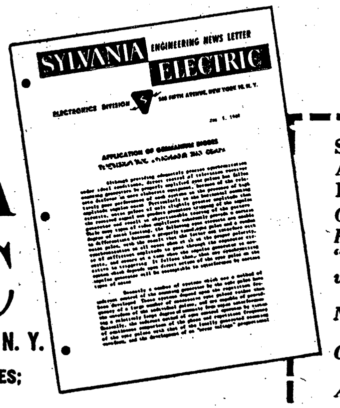
PUT BETTER PICTURES ON YOUR TV SCREENS...
BY PUTTING SYLVANIA GERMANIUM DIODES IN YOUR SETS!



Here are 5 television receiver applications especially suited to utilize to the full the inherent advantages of these unique circuit components

Designers of television receivers can substantially improve set performance through the use of Sylvania Germanium Diodes. Outstanding features of these elements in TV set design are freedom from hum, high efficiency, low capacity, no contact potential and exceptional linearity. Of course, in TV receivers as in all other applications, Sylvania's Germanium Diodes offer the advantages of small size and ease of mounting.

Find out just *how* and *why* Sylvania Germanium Diodes make television receivers better. Mail coupon for Electronic Engineering News Letter #8, which gives detailed circuit information on the 5 applications shown in the block diagram above.


SYLVANIA ELECTRIC
ENGINEERING NEWS LETTER

SYLVANIA ELECTRIC

Electronics Division, 500 Fifth Avenue, New York 18, N. Y.

ELECTRONIC DEVICES; RADIO TUBES; CATHODE RAY TUBES; PHOTOLAMPS; FLUORESCENT LAMPS, FIXTURES, WIRING DEVICES, SIGN TUBING; LIGHT BULBS

Sylvania Electric Products Inc.
Advertising Dept., Room E-1004
Emporium, Pa.

Gentlemen:

Please send me your Engineering News Letter #8, "Applications of Sylvania Germanium Diodes in Television Receivers."

*Name*____________________

*Company*____________________

*Address*____________________

*City*______________ *State*__________