SUPPLEMENT

TO THE POST OFFICE ELECTRICAL ENGINEERS' JOURNAL

Vol. 73 Part 1 **April 1980**

ISSN 0309-2720

TEC & CGLI: GUIDANCE FOR STUDENTS

Contents

CITY AND GUILDS OF LONDON INSTITUTE

Questions and Answers

Answers are occasionally omitted or reference is made to earlier Supplements in which questions of substantially the same form, together with the answers, have been published. Some answers contain more detail than would be expected from candidates under examination conditions.

COMPUTERS A 1979

Students were expected to answer any 6 questions

Q1 (*a*) *Draw a truth table for a 3-input* NAND *gate.*

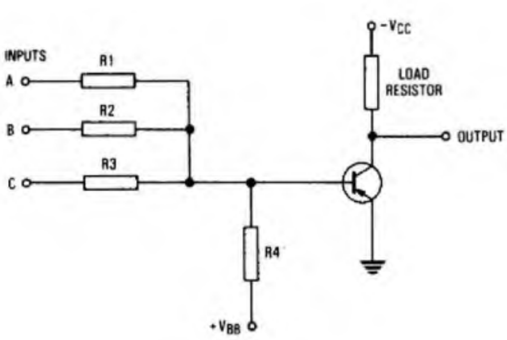
(*b*) *Draw a circuit diagram for the* NAND *gate in part* (*a*).

(*c*) *Explain the operation of the gate using a positive-logic convention.*

A1 (*a*) In positive logic, the logic value 1 is assigned to the more positive voltage (in this case 0 V), and logic value 0 to the more negative voltage ($-V_{cc}$ volts). The truth table for a 3-input NAND gate is as shown below.

Inputs			Output
A	B	C	
0	0	0	1
0	0	1	1
0	1	0	1
0	1	1	1
1	0	0	1
1	0	1	1
1	1	0	1
1	1	1	0

(*b*) The circuit diagram of a positive-logic resistor-transistor 3-input NAND gate is shown in the sketch.



(*c*) The operation of the gate using a positive-logic convention is as follows. The input resistors, R1, R2 and R3 have the same value. The value of the base-bias resistor, R4, is such that, when all 3 inputs A, B and C are at 0 V (or a very low negative voltage), the base of the transistor is at a positive potential sufficient to hold the transistor in the OFF state. The output voltage is then approximately equal to the supply voltage, $-V_{cc}$ volts. If one or more of the 3 inputs is at a large negative voltage (say $-V_{cc}$ volts), the base of the transistor is at a negative potential, and the transistor is in the ON state. The output voltage is then approximately zero. These conditions are shown in the following voltage table.

Inputs			Output (V)
A	B	C	
$-V_{cc}$	$-V_{cc}$	$-V_{cc}$	0
$-V_{cc}$	$-V_{cc}$	0	0
$-V_{cc}$	0	$-V_{cc}$	0
$-V_{cc}$	0	0	0
0	$-V_{cc}$	$-V_{cc}$	0
0	$-V_{cc}$	0	0
0	0	$-V_{cc}$	0
0	0	0	$-V_{cc}$

Q2 *Draw separate 3-element Venn diagrams to represent the following Boolean statements, shading the appropriate areas.*

(*a*) $A.B.\overline{C}$.

(*b*) $\overline{A}.\overline{B}.C$.

(*c*) $A_{\vee}B$.

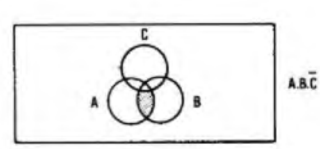
(*d*) $\overline{A.\overline{B}.C}$.

(*e*) $B.(C_{\vee}A)$.

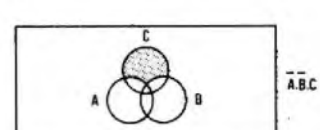
Key: *The sign* . *means logic* AND; *the sign* ¯ *means logic* NOT; *the sign* $_{\vee}$ *means logic* OR.

A2 The Venn diagrams for the Boolean statements are shown in sketches (*a*) to (*e*). The shaded areas represent where the statements are true.

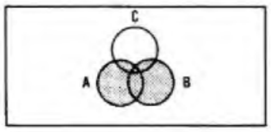
(*a*)



(*b*)

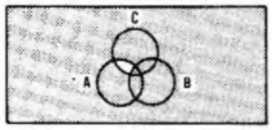


(*c*)



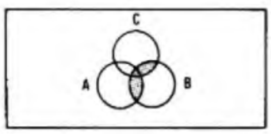
$A \vee B$

(*d*)



$\overline{A.\bar{B}.C}$

(*e*)



$B.(C \vee A)$

Q3 (*a*) (*i*) *Explain the term 2's complement of a number.*

(*ii*) *For which binary arithmatic operations is it usually needed?*

(*iii*) *Give the 2's complement of 01 101 010.*

(*b*) *Multiply binary numbers 0 110 110 and 0 010 110 and convert their product to denary form. Show all working clearly.*

A3 (*a*) (*i*) The 2's complement of a number is obtained as follows. To create the one's complement, each 1 is replaced by a 0, and each 0 is replaced by a 1. The 2's complement is created by adding 1 to the one's complement.

(*ii*) The 2's complement is usually needed as a means of performing the binary subtraction operation.

(*iii*) The 2's complement of 01 101 010 can be obtained as follows:

One's complement:	10 010 101
Add 1:	1+
	10 010 110

(*b*) Binary multiplication can be carried out using a successive shifting-and-adding technique, as shown below.

Multiplicand:	0 110 110
Multiplier:	0 010 110×
Copy multiplicand, shifted 1 place:	0 001 101 100
Copy multiplicand, shifted 2 places:	0 011 011 000+
Copy multiplicand, shifted 4 places:	1 101 100 000+
	10 010 100 100

The binary product is therefore 10 010 100 100.

This binary number may be converted to denary form by summing the individual terms where each term represents a multiple of a power of 2.

$$\therefore \quad 10\,010\,100\,100_2 = 0 \times 2^0 + 0 \times 2^1 + 1 \times 2^2 + 0 \times 2^3 + 0 \times 2^4 + 1 \times 2^5 + 0 \times 2^6 + 1 \times 2^7 + 0 \times 2^8 + 0 \times 2^9 + 1 \times 2^{10},$$
$$= 4 + 32 + 128 + 1024 = 1188_{10}.$$

Q4 (*a*) *What is meant by the term "backing store"?*

(*b*) *Compare briefly a backing store with a computer main store.*

(*c*) *Compare briefly the advantages and disadvantages of TWO types of magnetic backing store.*

A4 (*a*) The *backing store* of a computer system is used to hold information which is not required to be accessed immediately by the processor. Any type of store capable of containing bulk information at a relatively low cost can be used; for example, slow core, magnetic drum or magnetic disc.

(*b*) The following are the main differences between backing store and the computer main store:

(*i*) A backing store is capable of storing much more information than a main store.

(*ii*) The cost per bit of storing the information in backing store is significantly cheaper than storing the information in the main store.

(*iii*) The time to access the information in a backing store is not, and does not need to be, as fast as for a main store.

(*iv*) The information in a backing store is written and read as blocks of words; for example, files, data arrays. The information in a main store is written and read one word at a time.

(*c*) Moving-head magnetic discs and fixed-head magnetic drums are 2 types of computer backing store.

The moving-head magnetic-disc memory consists of a number of rotating discs, coated with a magnetic material, stacked one on top of the other with spaces in between. Information is recorded and read from the surface by magnetic heads which are positioned above and below the disc surfaces. The information is recorded in bands (tracks) and it is necessary to move the head to position it over the required track.

The fixed-head magnetic-drum memory consists of a single drum coated with a magnetic material and the information is recorded and read using magnetic heads. The information is recorded in tracks and there is one head provided for every track and therefore there is no need for movement of the heads.

The fixed-head magnetic-drum memory has the advantage of having a relatively fast access time (because there is no need to move the heads) and of being relatively reliable (because there are less moving parts than in the moving-head magnetic-disc memory). However, the fixed-head magnetic-drum memory is more expensive because of the need to provide heads and associated addressing and control logic for each track. The moving-head disc memory is cheaper on a cost-per-bit basis and it is possible to store larger quantities of information. Also, the magnetic-disc memory has the advantage that discs can be removed and exchanged.

Q5 (*a*) *With the aid of a carefully drawn diagram, describe how information is represented on punched paper tape.*

(*b*) *With the aid of sketches, explain how the information may be converted into signals suitable for use in a digital computer.*

Q6 *A computer is considered to comprise several essential basic parts.*

(*a*) *Name these parts.*

(*b*) *Draw a block diagram with suitable interconnexions.*

(*c*) *Write brief descriptions of EACH part and explain how they react with each other.*

Q7 *Write a program to calculate the value of $x^2 + y^2$, using a machine code of your own choice (with a key). The factors x and y are to be read from punched cards; the terminating condition is to be a card punched with a non-numeric code in column 1. Provide a means of listing answers to individual calculations and a final total.*

A7 A machine-code program which performs the required function is shown, together with a key to the operations. It is assumed that every character has a hexadecimal representation and that the hexadecimal representations for the figures 0 to 9 are F0 to F9.

Label	Program	Comments
	STA	Start the program
	LD 65	Zero the total
	ST 64	
L1	RDB 01	Read x card into buffer
	MVB	Put first character of buffer into accumulator
	ST 68	Store first character
	SUB 66	Subtract F0 from first character
	JLS L2	Jump to L2 if result is negative
	LD 68	Reload accumulator with first character
	SUB 76	Subtract F9 from accumulator
	JGT L2	Jump if result is positive; that is, character is not numeric
	CCB	Convert buffer string to binary number
	ST 60	Store x
	MULT 60	Multiply accumulator by x
	ST 61	Store x^2
	RDB 01	Read y card into buffer
	MVB	Put first character of buffer into accumulator
	ST 68	Store first character
	SUB 66	Subtract F0 from first character
	JLS L2	Jump to L2 if result negative
	LD 68	Reload accumulator with first character
	SUB 67	Subtract F9 from accumulator
	JGT L2	Jump if result is positive
	CCB	Convert buffer string to binary number
	ST 62	Store y
	MULT 62	Multiply accumulator by y
	ADD 61	Add x^2 to accumulator
	BCC	Convert binary number to character string in buffer

Label	Program	Comment
	WRB 02	Write buffer to line printer
	ADD 64	Add sum of squares to total
	ST 64	Store total
	JMP L1	Return to read another card
L2	LD 64	Load total into accumulator
	BCC	Convert binary number to character string in buffer
	WRB 02	Write buffer to line printer
	STP	Stop program

Key: Table 1 shows the values of store locations; Table 2 shows the codes used for devices and Table 3 shows the instruction set.

TABLE 1

Constant Store	Value
60	x
61	x^2
62	y
63	y^2
64	Total
65	0
66	F0
67	F9
68	Work space

TABLE 2

Device	Description
01	Card reader
02	Line printer

TABLE 3

Instruction	Function
STA	Start program
STP	Stop program
LD X	Set accumulator to contents of store location X
ST X	Store contents of accumulator in store location X
RDB N	Read from device N into buffer
WRB N	Write to device N from buffer
MVB	Set accumulator to first character in the buffer
CCB	Convert the character string in the buffer to a binary number and leave in accumulator
BCC	Convert the binary number in accumulator to a character string in the buffer
ADD X	Add contents of location X to accumulator
SUB X	Subtract contents of location X from accumulator
MLT X	Multiply accumulator by contents of location X
JGT L	If contents of accumulator positive, jump to location L
JLS L	If contents of accumulator negative, jump to location L
JMP L	Unconditionally jump to location L

Q8 *(a) Discuss the following factors as they apply to digital and analogue computers:*

(i) component tolerance, and
(ii) component reliability.

(b) State what values would be expected and give reasons for your answer.

Q9 *(a) Describe the essential differences between astable, monostable and bistable devices.*

(b) State which of the above devices would be used for the following:

(i) a mechanism to produce a pulse of definite duration,
(ii) a clock-pulse generator,
(iii) a mechanism to produce a short (3 μs) pulse from the back edge of a 10 ms positive pulse, and
(iv) a mechanism to remember a transient failure of a supply voltage.

A9 (*a*) Astable, monostable and bistable devices are all 2-state devices but differ from each other in the conditions under which they remain in each state.

An *astable device* is symmetrical in design; that is, both halves of the device are identical. The 2 halves are cross-coupled with components which have time-constants and are therefore unstable in time. This results in the astable device staying in one state for a fixed period of time before decaying from that state and automatically triggering the device into its other state. The device eventually returns to its previous state and so oscillates between the 2 states. The frequency of oscillation is determined by the time-constants of the cross-coupling components. The output from the device is a square wave and the mark : space ratio is unity. The mark : space ratio can be asymmetrical if the components which give rise to the time-constants are different and the device is therefore not symmetrical.

A *monostable device* is asymmetrical in design in that the halves of the device contain different components. The difference is in the cross-coupling where one set of components have a time-constant and the other set do not. This results in the monostable device staying in one state indefinitely until triggered by an external pulse into the other state. The duration of the second state is controlled by a time-constant and eventually decays until the device switches back to the first, stable state. Thus the device has one stable (monostable) state. The period of the unstable state is determined by the time-constant of the cross-coupling components.

A *bistable device* is symmetrical in design so that both halves of the device are the same. The 2 halves are cross-coupled with components which do not have time-constants so that the device can be in either of 2 stable (bistable) states. The device enters one state at random when switched on and remains in that state until triggered by an external signal into the other state. The device then remains in the second stable state until given an external trigger to return it to the first stable state. Thus the bistable device has 2 states, neither of which is dependent upon time.

(*b*) (*i*) A monostable device would be used to produce a pulse of definite duration. The duration of the pulse would depend on the time-constant of the unstable state of the monostable circuit.

(*ii*) An astable device would be used to produce clock pulses where the repetition rate and mark : space ratio of the pulses would depend on the time-constants of the unstable states of the astable circuit.

(*iii*) A negative-edge-triggered monstable can be used to produce a short duration pulse from the back edge of a 10 ms positive pulse. The leading (positive-going) edge of the 10 ms pulse would be suppressed by the trigger circuitry and thus would not trigger the monostable circuit.

(*iv*) An RS (reset/set) bistable can be used to remember a transient failure of a supply voltage. The bistable can be reset by a manual push-button, and be triggered on its SET input by a trigger circuit pulse from a supply rail of the monitored supply voltage. The bistable would be SET by a transient failure and would remain set until manually reset. The SET output could operate an indicator lamp or a warning alarm.

Q10 *(a) Explain how shift registers, whose outputs are connected to a serial adding unit, are used to add 2 numbers. Use a diagram to illustrate the answer.*

(b) What arrangement would be required if ONE *of the registers in part (a) is to be used to write an answer back to a core store?*

TELECOMMUNICATION PRINCIPLES A 1979

Students were expected to answer any 6 questions

Q1 *(a) On what factors does the capacitance of a capacitor depend?*

(b) An initially uncharged capacitor is charged for 1 s by a constant current of 0·05 mA which raises the potential difference across the capacitor by 20 V. Calculate the value of the capacitance.

(c) If the capacitor referred to in part (b) consists of air-spaced plates, calculate the rate of rise of the potential difference when a sheet of mica having a relative permittivity of 6·0 fills the gap between the plates.

(d) Sketch graphs of the rise of potential difference across the capacitor with respect to time for parts (b) and (c). Show the scale values on the axes and mark clearly the voltages at 0·5 s and 1·0 s.

A1 (*a*) The capacitance of a capacitor is proportional to the effective area of the plates and the relative permittivity of the dielectric. It is inversely proportional to the distance between the plates.

$$\text{Capacitance (farads)} = \frac{\text{effective area (metres}^2\text{)}}{\text{separation (metres)}} \times \text{permittivity},$$

where the permittivity (farads/metre) is the absolute permittivity of free space (ϵ_0) $\times$ the relative permittivity (ϵ_r).

Therefore, $$C = \frac{\epsilon_0 \epsilon_r A}{d}.$$

(*b*) The charge, Q coulombs, is related to a capacitance, C farads, charged to V volts by the expression

$$Q = CV.$$

A current is a flow of electrons, which is a transfer of charge in coulombs/second.

Hence, 0·05 mA flowing for 1 s = $0{\cdot}05 \times 10^{-3}$ C.

$$\therefore \quad C = \frac{0{\cdot}05 \times 10^{-3}}{20} = \underline{2{\cdot}5\ \mu\text{F.}}$$

(*c*) If the air gap is replaced with mica, the capacitance will increase by $\frac{\epsilon_m}{\epsilon_a} = 6$ where ϵ_m is the relative permittivity of mica and ϵ_a is the relative permittivity of air.

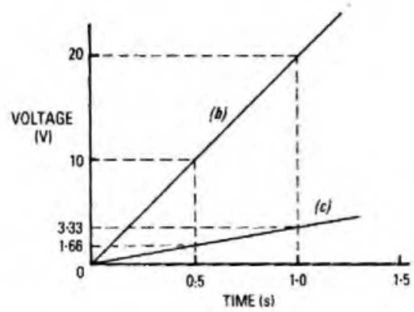
$$\therefore \quad C = 15\ \mu\text{F.}$$

The rate of change of charge is 0·05 mC/s.

Therefore, the rate of rise of the potential difference $= \frac{0{\cdot}05 \times 10^{-3}}{15 \times 10^{-6}}$,

$$= \underline{3{\cdot}33\ \text{V/s.}}$$

(*d*) The graphs of the rise of potential difference with time are shown in the sketch. Both are straight lines. For part (*b*) the slope is 20 V/s and for part (*c*) 3·33 V/s.



Q2 (a) The instantaneous value of an alternating voltage is given by $v = 20 \sin 200\pi t$ volts. Find

(i) the RMS value,
(ii) the frequency, and
(iii) the periodic time.

(b) Sketch ONE *cycle of the voltage waveform on scaled axes and mark the important values.*

(c) Find an expression for the instantaneous current that when flowing in a 10 Ω resistor would give the voltage of part (a).

(d) Sketch ONE *cycle of the waveform in part (c) on the same time axis as the voltage in part (b).*

A2 (*a*) The general expression for a sinusoidal alternating voltage is

$$v = V \sin 2\pi f t$$

where v is the instantaneous voltage at time t, V is the peak voltage and f is the frequency.

(*i*) The RMS value is the peak value $\div \sqrt{2} = 20/\sqrt{2} = \underline{14{\cdot}14\ \text{V.}}$

(*ii*) The frequency is $\frac{200}{2} = \underline{100\ \text{Hz.}}$

(*iii*) The periodic time is the reciprocal of the frequency

$$= \frac{1}{100} = \underline{10\ \text{ms.}}$$

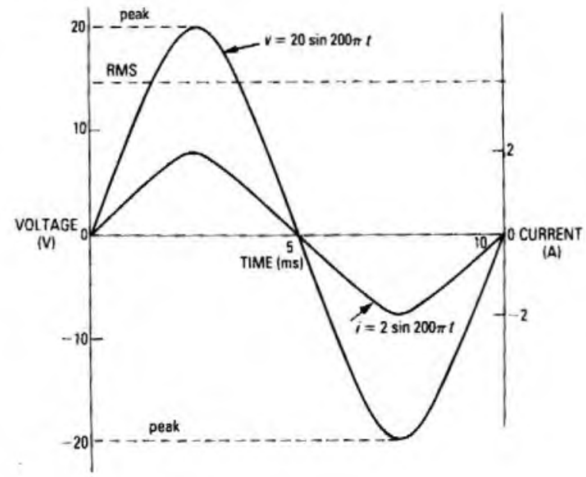
(*b*) The waveform is shown the sketch.

(*c*) From Ohm's law, when a current of i amperes flows in a resistor of r ohms, the voltage across the resistor is $v = ir$.

(It should be noted that Ohm's law applies to an instantaneous condition as well as a steady condition.)

$$\therefore \quad i = \frac{20 \sin 200\pi t}{10} = \underline{2 \sin 200\pi t.}$$

(*d*) The current waveform is shown in the sketch.



Q3 (a) Explain, with the aid of a circuit diagram, how a moving-coil milliammeter can be used as

(i) an ammeter, and
(ii) a voltmeter.

(b) A moving-coil instrument has a 25 Ω coil resistance and gives full-scale deflexion for 2·0 mA. Calculate the values of resistances needed to give ranges of 0–1·0 A and 0–2·0 V.

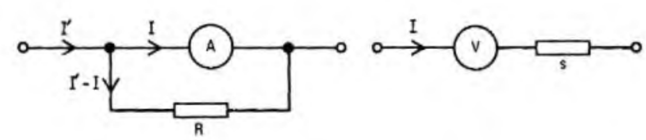
A3 (*a*) (*i*) The milliammeter can be used as an ammeter by bypassing a proportion of the current through a resistor in parallel with the meter movement. This is shown in sketch (*a*). If the resistance of the movement is r ohms, the value of current required to produce full-scale deflexion is I amperes and the current to be measured is I' amperes, the value of the shunt resistor, R ohms, is given by

$$R = \frac{rI}{I' - I}. \qquad \ldots\ldots (1)$$

(*ii*) The milliammeter can be used to measure a voltage by connecting a large-value resistor in series with the meter movement such that a known voltage appears across the external resistor at the current required to give full-scale deflexion. This is shown in sketch (*b*). If S ohms is the value of the series resistor, r ohms the resistance of the meter movement, I the current required for full-scale deflexion and V volts the maximum voltage to be measured

$$V = I(r + S).$$

$$\therefore \quad S = \frac{V}{I} - r. \qquad \ldots\ldots (2)$$



(*a*) (*b*)

(*b*) For full-scale deflexion of 2·0 mA, substituting in equation (1) gives a shunt resistor value of

$$\frac{25 \times 0{\cdot}002}{1 - 0{\cdot}002} = \underline{50\ \text{m}\Omega} \text{ to acceptable accuracy.}$$

For a full-scale deflexion of 2·0 V, substituting in equation (2) gives a series resistor value of

$$\frac{2{\cdot}0}{2{\cdot}0 \times 10^{-3}} - 25 = \underline{975\ \Omega.}$$

Q4 (a) Explain why the EMF of a battery and its terminal voltage are different in value when the battery is on load.

(b) For the circuit shown in Fig. 1 calculate

(i) the total resistance across the 12 V supply,
(ii) the current flowing in each resistor, and
(iii) the power dissipated in the 4 Ω resistor.

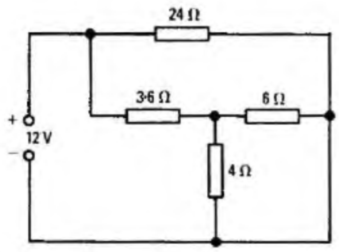


Fig. 1

A4 The difference between the EMF of a battery and the terminal voltage when the battery is on load is caused by the internal resistance of the battery. This internal resistance exists because the battery is a conductor that uses resistive materials; for example, lead and acid. There must therefore be an internal voltage drop whenever a current flows from, or into, the battery and this is independent of the EMF generated by chemical action within the battery. The on-load terminal voltage of the battery will be less than the EMF by the value of the internal voltage drop. If the internal resistance of the battery is r ohms, I is the load current and E the EMF of the battery, then

$$\text{terminal voltage} = E - rI.$$

(*b*) The circuit may be simplified as shown in sketch (*c*).

(*i*) The total resistance across the 12 V supply is 4·8 Ω.

(*ii*) Total current $= \frac{12}{4 \cdot 8}$ A = 2·5 A.

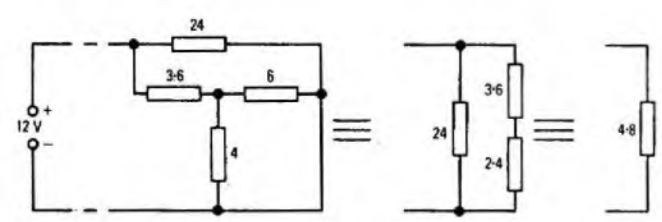
Refering to sketch (*b*), the total current will divide in the ratios $2 \cdot 5 \times \frac{6}{30}$ A through the 24 Ω resistor and $2 \cdot 5 \times \frac{24}{10}$ A through the 3·6 Ω and 2·4 Ω resistors.

Thus, the current in the 24 Ω resistor is 0·5 A; the current in the 3·6 Ω resistor is 2·0 A.

The 2·4 Ω resistor in sketch (*b*) is made up of the 6 Ω and 4·0 Ω resistors in parallel. Therefore, the current in the 2·4 Ω resistor will divide in the ratios $2 \times \frac{6}{10}$ A in the 4 Ω resistor and $2 \times \frac{4}{10}$ A in the 6 Ω resistor.

Thus, the current in the 4 Ω resistor is 1·2 A; the current in the 6 Ω resistor is 0·8 A.

(*iii*) The power dissipated in the 4 Ω resistor is $1 \cdot 2^2 \times 4$ = 5·76 W.



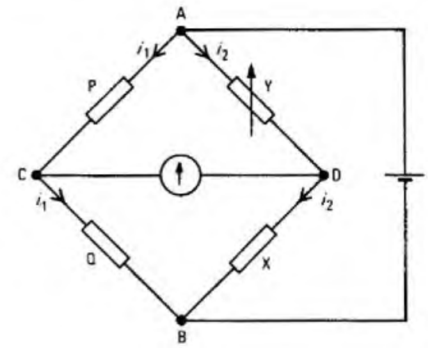
(*a*) (*b*) (*c*)

Q5 (a) Describe the principle of the Wheatstone bridge and derive the balance equation.

(b) Explain

(i) the advantages of using a null method of resistance measurement,
(ii) why small voltage variations in the supply voltage to the Wheatstone bridge are unimportant, and
(iii) why large voltage variations are undesirable.

A5 The circuit of the Wheatstone bridge is based on 2 potentiometers connected in parallel, as shown in the sketch. Resistors P and Q, known as the ratio arms, make one potentiometer. Resistors X and Y, X being the resistance to be measured and Y being adjustable and calibrated, form the other potentiometer. A source of EMF (for example, a 6 V battery) is connected across points A and B. A centre-zero sensitive galvanometer is connected across points C and D. Resistor Y is adjusted until the galvanometer reads zero (that is, there is no potential difference between points C and D). At this stage the bridge is said to be *balanced*. Balance can be achieved quickly by inserting a resistor in series with the galvanometer to limit the current until balance is nearly achieved, at which stage the resistor can be short-circuited by a key while final balance is achieved.



The balance equation is derived as follows. Since no current flows in the galvanometer at balance, the current in Q equals the current in P; let this be i_1. Also, the current in X equals the current in Y; let this be i_2.

Therefore, the potential drop across Q and X must be equal; that is

$$i_1 Q = i_2 X.$$

$$\therefore \quad \frac{Q}{X} = \frac{i_2}{i_1}.$$

Similarly,

$$\frac{P}{Y} = \frac{i_2}{i_1}.$$

$$\therefore \quad \frac{Q}{X} = \frac{P}{Y}.$$

$$\therefore \quad X = \frac{Q}{P} \times Y.$$

(*b*) (*i*) A null method (that is, one in which the meter reads zero at the condition of balance) has the advantage that the balance meter requires no calibration, except that the zero must correspond to no current flowing in the meter. For this reason, supply variations do not interfere with the balance condition.

(*ii*) The current in the bridge arms cancelled out in the calculation of the balance equation. Therefore, balance does not depend on these currents and voltage variations do not affect the balance condition.

(*iii*) If supply voltage variations are large, however, they can give rise to variations in the resistor values due to heating effects.

Q6 Describe ONE *of the following experiments and comment on any likely sources of error.*

(a) Determination of the static characteristic of a vacuum diode, OR

(b) illustration of the relationship between induced EMF, number of turns and flux in a mutually-inductive circuit.

Set out the answer as a laboratory report. Include a circuit diagram, details of equipment and procedure, and typical results with conclusions.

Q7 (a) For an iron-cored inductor, use a B/H curve to explain the meaning of

(i) magnetic saturation, and
(ii) relative permeability.

Hence explain why the relative permeability of iron is not constant.

(b) Table 1 gives the relation between B teslas and H amperes/metre, for an iron-cored inductor. Plot the curve of B against H.

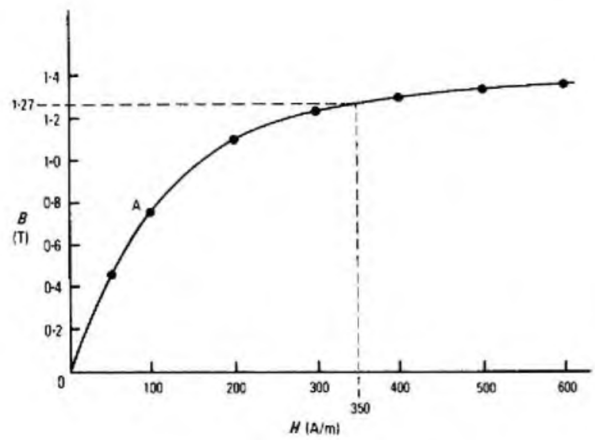
(c) Use the curve drawn in part (b) to estimate the value of relative permeability when B = 1·27 T.

Table 1

H	50	100	200	300	400	500	600
B	0·45	0·76	1·1	1·23	1·3	1·34	1·37

A7 (*a*) Refering to the sketch. Over the range of values from points O to A, the ratio of B/H is reasonably constant. However, beyond point A the slope starts to decrease and approaches zero beyond the knee. When an increase in H produces no further increase in B, the iron is said to be *saturated*.

The ratio of B/H is the magnetic permeability of the iron. The magnetic permeability (μ) = the absolute permeability of free space (μ_0) × the relative permeability of the material (μ_r). Since μ_0 is constant ($4\pi \times 10^{-7}$ H/m in the SI system of units), the slope of the B/H curve represents the relative permeability of the iron. Since this slope is not constant, the relative permeability is not constant and falls as



saturation is approached.

(*b*) The curve is shown in the sketch.

(*c*) From the sketch, when B = 1·27 T, H = 350 A/m.

Therefore, $\mu_r = \dfrac{B}{H\mu_0} = \dfrac{1 \cdot 27}{350 \times 4\pi \times 10^{-7}} = \underline{2888 \text{ H/m}}.$

Q8 (a) Name TWO *different types of rectifying device and give a practical application of each.*

(b) Draw a circuit diagram of

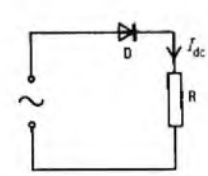
(i) a half-wave rectifier unit, and
(ii) a full-wave rectifier unit.

(c) With the aid of input and output waveforms for current and voltage explain the operation of each of the circuits in part (b) over one cycle.

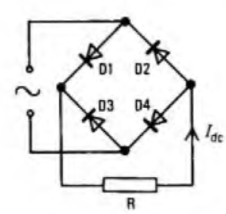
(d) Show how a smoothing circuit may be added after the rectifier circuit and explain briefly how it operates.

A8 (*a*) Two commonly used rectifiers are the p n junction and the thermionic diode. The p n diode is mostly used for small-signal rectification such as in demodulators. The thermionic diode is commonly used as a means of rectifying the mains supply to provide high-tension supplies for equipment. However, recent developments in semiconductor devices have largely made the use of the thermionic diode obsolescent because of the larger power handling capacity of the former.

(*b*) Circuit diagrams are shown in sketches (*a*) and (*b*).



(*a*)



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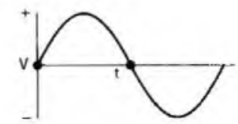

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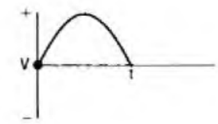
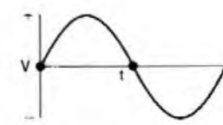
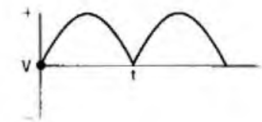


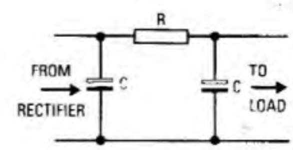
(*d*)



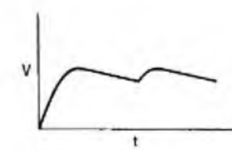
(*e*)



(*f*)



(*g*)



(*h*)

(*c*) (*i*) The input and output waveforms for a half-wave rectifier are shown in sketches (*c*) and (*d*). Because the load is resistive, the voltage and current waveforms will be in phase with one another. The diode of sketch (*a*) passes only the positive half cycle of the waveform and blocks the negative half cycle, thus producing the waveform shown in sketch (*d*).

(*ii*) The input and output waveforms for a full-wave rectifier are shown in sketch (*e*) and (*f*). Referring to sketch (*b*), during one half cycle current flows from the supply, through rectifier D2, through the load, through rectifier D4 and back to the supply. During the other half-cycle, current flows from the supply, through rectifier D3, through the load, through rectifier D1 and back to the supply.

(*d*) A typical smoothing circuit is shown in sketch (*g*). A large capacitor, usually an electrolytic type, acts as a reservoir which is kept charged by the successive half-wave pulses from the rectifier. The load draws current from the capacitor through a series resistor. The effective voltage at the output is shown in sketch (*h*) and comprises a large DC component with a ripple voltage at twice the supply frequency superimposed on it.

Q9 (a) Describe briefly an experiment to give the anode current/anode voltage characteristics of a thermionic triode. Draw a circuit diagram, list the equipment and outline the procedure. Sketch a set of curves resulting from the experiment, showing typical values.

(b) Select a suitable anode load resistance and HT voltage and draw a load line on the I_a/V_a *curves.*

Q10 (a) A microphone converts sound energy into a varying electric current. Explain, with the aid of diagrams how this is done EITHER

(i) in a carbon-granule microphone, OR
(ii) by a moving-coil microphone.

(b) Describe briefly the principle of operation of a telephone earpiece that converts the electrical signals back into sound waves.

LINE PLANT PRACTICE A 1979

Students were expected to answer any 6 questions

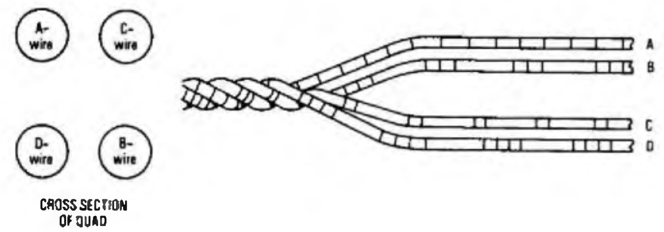
Q1 Describe, with a diagram, the main features of a national telephone network. Name the types of switching centres, the parts of the network, and typical underground plant that a customer-to-customer circuit would pass through on a long-distance call.

A1 See A10, Line Plant Practice A 1976, *Supplement*, Vol. 70, p. 19, Apr. 1977.

Q2 Describe a paper-core quad-trunk (PCQT) cable, with particular reference to use, size, construction, marking, balancing and anti-interference measures.

A2 PCQT cable is used for long junction and trunk circuits.

Each conductor is of annealed copper, and is insulated with an open lapping of paper string and an overlapped lapping of paper. Four



insulated conductors, all from the same reel of wire, are stranded around a central paper string to form one quad. Each quad is whipped with coloured cotton, and a number of quads are stranded in layers around a central core of either 1 or 4 quads to form a compact and symmetrical cable core. The core is wrapped with at least two thicknesses of insulating paper, completely covered with polyethylene-coated aluminium foil, and sheathed with polyethylene. Cables are available in sizes up to 1040 pairs.

A quad is illustrated in the sketch. The individual conductors of a quad are marked with rings on the paper insulation. Alternate quads have red and blue markings. Marker and reference quads are whipped with cotton strands coloured white and orange in centre and even layers, and black and orange in odd layers. All other quads are whipped with white cotton in centre and even layers, and black in odd layers.

To maintain correct capacitance balance, air-spacing between conductors is usually provided by the open lapping of paper string. This produces a dielectric between the conductors with a permittivity close to unity. The coloured marking rings about the conductors are spaced such that all conductors have the same amount of ink per unit length.

To keep mutual interference to acceptable values, the direction of stranding alternates in successive layers, and different twist lengths and stranding lays are used.

Q3 *(a) Explain how a sound pole can be recovered without the need to excavate the butt. Under what circumstances would this method not be used?*

(b) Explain how a decayed pole should be replaced by a new pole alongside, describing the precautions that should be taken, particularly those necessary before the overhead wires are changed-over from the old pole to the new.

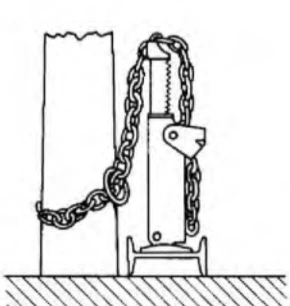
(c) If a decayed pole is to be replaced by a new one at some distance from the old, explain how the overhead wires are changed-over, and describe the precautions that are necessary if the decayed pole cannot be climbed.

(d) If a pole is decayed at ground level, but it is essential to climb it, what must be done to make it safe?

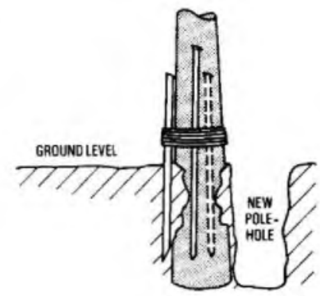
(e) If the pole is decayed at a height in excess of 1·5 m above the ground, but the pole must be climbed, what precaution is necessary?

A3 (*a*) Recovery of a sound pole, without excavation, is carried out with the aid of the pole-lifting jack. The arrangements are illustrated in sketch (*a*). A stout *choke* chain is placed around the base of the pole and over the top of the lifting jack. Operation of the jack-lever gradually lifts the pole out of the ground by a direct vertical pull, thus saving the labour of excavation. The pole is stayed during the operation, and is then either lowered gradually or permitted to fall, according to the situation. This method cannot be used in cases where blocks are fixed to the butt of the pole.

(*b*) Where the new pole can be erected before the decayed pole is recovered, it is usual to set the new pole close to the one to be replaced. Precautions have to be taken to prevent the old pole from collapsing into the new pole-hole when the ground is disturbed. Additional



(*a*)



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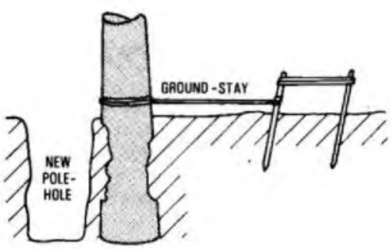


(*c*)

(*d*)

temporary stays may be fitted, and crow-bars should be driven into the ground around the pole and lashed to it to form a splint to prevent it from shearing off. This method is shown in sketch (*b*). Alternatively, a ground-stay may be provided by roping the pole to a couple of crow-bars driven into the ground on the side of the pole remote from the new pole-hole, as shown in sketch (*c*).

When the new pole has been erected, additional support for the decayed pole can be provided by lashing the two poles together in several places before the work to change the wires over begins. This is often necessary because the overhead wires on the old pole afford it some measure of support before they are changed-over.

(*c*) If the new pole is erected some distance from the decayed pole and it is unsafe to climb the old pole because of extensive decay, then new wires are run from adjacent poles to the new pole; the old wires are cut off one at a time on either side of the old pole to prevent undue straining of the old pole. Before cutting the wires, however, the old pole should be temporarily stayed. A wire-stringing tool (used for the erection of overhead wires) or an elevating platform should be used to avoid the need to climb the pole.

(*d*) If it is essential to climb a pole dangerously decayed near or below ground level, the pole must be made safe by erecting a *bolster* pole close to it, and lashing the two poles together.

(*e*) If the dangerous decay is more than 1·5 m above ground level, the pole can be bolstered with two stout ladders lashed together and to the pole, as shown in sketch (*d*). In certain situations, scaffolding can be used.

Q4 *(a) Describe the method of manually erecting a medium or light pole (up to 9 m).*

(b) Describe a method of erecting an extra tall pole using a pole-derrick.

Q5 *(a) Give four reasons why it is necessary to have underground jointing boxes and manholes.*

(b) Describe a manhole, making particular reference to its structure, materials of construction, facilities and fittings.

(c) Describe the building of a surface-entry brick-built footway jointing box.

A5 (*a*) See A10, Line Plant Practice A 1977, *Supplement*, Vol. 71, p. 16, Apr. 1978.

(*b*) See A6, Line Plant Practice B 1973, *Supplement*, Vol. 67, p. 52, Oct. 1974, and A7, Line Plant Practice A 1974, *Supplement*, Vol. 68, p. 23, Apr. 1975.

(*c*) See A3, Line Plant Practice A 1978, *Supplement*, Vol. 72, p. 11, Apr. 1979, and A4, Line Plant Practice A 1975, *Supplement*, Vol. 69, p. 50, Oct. 1976.

Q6 Describe four different methods of providing a telephone line from a distribution point to a customer's premises, stating particular advantages and disadvantages of each method. Indicate the cheapest method.

A6 All the various methods of providing a telephone line from a distribution point to a subscriber's premises are described below in note form. The standard method is used wherever possible but, if the distribution scheme is already ducted, or if overhead distribution is unsuitable, underground service is provided. On new housing estates, when a financial agreement is made with the developer, the service is run underground.

(*a*) *Standard Method* Overhead, with insulated drop wire from pole to premises. Drop-wire continuously run to convenient point in building. Radial distribution in suburban areas. Running distribution from aerial cable in rural areas.

(*b*) Underground radial distribution.

(*c*) Underfloor (for terraced properties).

(*d*) Frontage tee-joint at each dwelling, using buried cables.

(*e*) Continuous cabling on surface of building (terraced properties).

The tables compare aspects of the various methods.

Method	Comparative Cost	Reliability	Accessibility for Maintenance
Standard (Overhead)	100%	Good	Good
Underground Radial	170%	Good	Good
Underfloor	130%	Very Good	Good
Front Tee	200%	Fair	Poor
Surface	150%	Good	Fair

Method	Advantages	Disadvantages
Standard (Overhead)	Cheapness; ease of erection	May be regarded as unsightly; dropwire lengths restricted by transmission and signalling limits
Underground Radial	Hidden; can be mole-ploughed in before houses are built	Prone to damage by building operations
Underfloor	Hidden	Provision for installation must be made during building; access to premises other than those affected may be needed when a fault occurs
Front Tee	Hidden	Large number of joints increases fault liability
Surface	Unobtrusive	Large number of joints increases fault liability

Q7 (a) Describe an auger borer and explain how it is used to install ducts.

(b) Describe briefly a percussion borer and explain how it is used.

(c) Why is it necessary to use borers for duct installation? State three typical examples of where a borer could be used to advantage.

(d) Why would an auger borer be used in preference to a thrust borer?

A7 (*a*) The auger of an auger borer is hydraulically rotated, but is moved forward along a drive-frame by means of a chain and sprocket driven manually. The auger shaft is made up in lengths so that when one length has been driven forward, the drive can be moved back along the frame and another length coupled. When the receiving pit is reached, duct or cable is attached to be drawn back along the bore, each length of auger being uncoupled in turn. The accuracy of the direction of the bore depends on careful setting up of the drive-frame in the driving pit, alignments with the receiving pit being made with the aid of a spirit level and sighting rods.

(*b*) Another type of borer, available commercially, consists of a head unit about 1·5 m long driven by compressed air. Effectively, it is a long, slim version of a pneumatic road-breaking drill, and progresses through the ground by rapid hammer-like blows, trailing its air line behind it. It has quite a high rate of progression, the speed depending on soil conditions but, like the previous method described, accuracy of alignment relies on accurate setting-up and assumes that obstacles such as large stones and tree roots do not deflect the borer.

(*c*) Boring is used when excavation of the ground for the purpose of laying a conduit (or cable) is undesirable or difficult. Cases generally suitable for boring are:

(*i*) road crossings (other than electric tramway crossings),
(*ii*) short lengths of conduit under expensive pavings,
(*iii*) level crossings and embankments (other than on electrified railways), and
(*iv*) cases where road or other material is stacked, and the removal of the material would be expensive.

(*d*) An auger borer is used in preference to a thrust borer in soils that cannot be compressed.

Q8 (a) With the aid of a diagram, show how a Wheatstone bridge is used to measure the resistance of a single wire in a cable.

(b) With a diagram, show how a Wheatstone bridge can be used to determine the distance to an earth fault. Derive the formula.

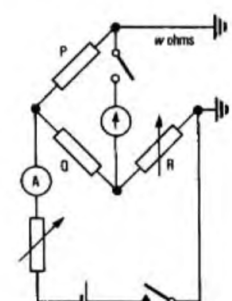
(c) Explain how the distance to a contact fault between two wires can be determined using the same bridge as in part (b) above.

(d) The single-wire resistance to an earth fault is measured as 136 Ω. It is known that the earth fault is in a length of 0·32 mm diameter copper conductor and that between that length and the point of measurement are two lengths of conductor, one of 300 m of 0·5 mm copper, and one of 500 m of 0·5 mm aluminium. Using the ohm/metre constants for various gauges of conductor given in the table, find the distance to the earth fault.

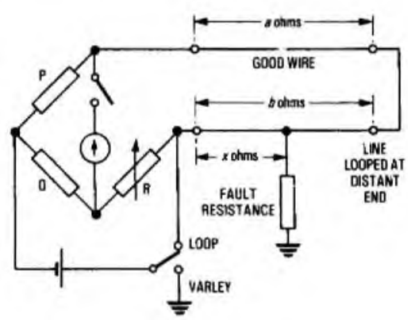
Conductor and Size	*Ohm/metre Constant of Single Wire*
0·32 mm copper	*0·21*
0·5 mm copper	*0·08*
0·5 mm aluminium	*0·14*

A8 (*a*) When measuring the resistance of a single wire, the distant end of the wire is connected to earth and the near end is connected to one line terminal of the Wheatstone bridge; the other line terminal is connected to earth, as shown in sketch (*a*). At balance, $P/Q = w/R$, whence $w = PR/Q$.

The test should be repeated after the direction of the battery current through the wire has been reversed; the value of the wire's resistance is then found by taking the mean of the two results. (Errors due to stray currents are minimized by this method.) The value obtained includes the resistance of the earth circuit.



(*a*)



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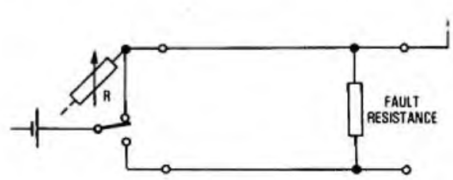


(*c*)

(*b*) The connexions necessary for finding the distance to an earth fault are shown in sketch (*b*). With the battery switch in the LOOP position, the loop resistance, $a + b$, is measured. The battery switch is then turned to the VARLEY position and the bridge is rebalanced. (This second test is known as the *Varley test*, and the value of resistor R to balance the bridge is known as the *Varley reading*.)

At balance,

$$\frac{P}{Q} = \frac{a + b - x}{R + x}.$$

$$\therefore\ Q(a + b - x) = P(R + x).$$

$$\therefore\ Q(a + b) - Qx = PR + Px.$$

$$\therefore\ x(P + Q) = Q(a + b) - PR.$$

$$\therefore\ x = \frac{Q(a + b) - PR}{P + Q}.$$

If the value of x is divided by the ohms/metre constant for the cable, the distance to the fault is obtained.

(*c*) The procedure for finding the distance to a contact fault is the same as for the earth fault, except that the Varley test is made with the battery connected to one of the two wires in contact, instead of to earth, as shown in sketch (*c*).

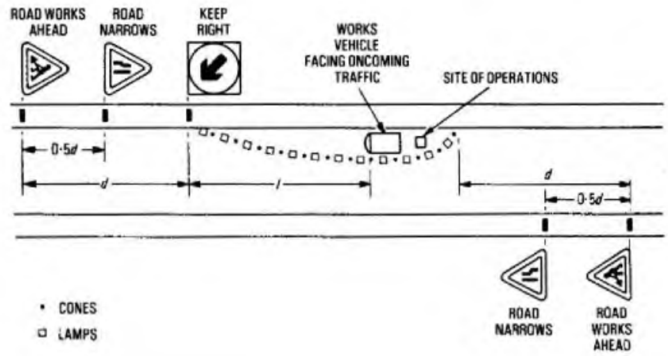
(*d*) The resistance of the 0·5 mm copper cable is $300 \times 0{\cdot}08 = 24\ \Omega$, and the resistance of the 0·5 mm aluminium cable is $500 \times 0{\cdot}14 = 70\ \Omega$, giving a total resistance of 94 Ω.

Thus, the resistance to the fault along the 0·32 mm copper cable is $136 - 94 = 42\ \Omega$. Hence the length of the 0·32 mm copper cable to the fault is $42/0{\cdot}21 = 200$ m.

Therefore, the total distance to the fault
$= 300 + 500 + 200\ \text{m} = \underline{1\ \text{km}}.$

Q9 A distribution pole is situated in a grass verge alongside a carriageway manhole in a country road. It is necessary to connect a spur cable from the manhole to a terminal block at the top of the pole. List and describe briefly the safety precautions necessary in undertaking this work.

A9 *Roadworks Guarding*
The site of the operations is guarded as illustrated in the sketch.



d and *l* are dependent on traffic speed

Pole Testing
There are three tests used to detect decay in wooden poles: the hammer test, the prodding test and the boring test.

In the hammer test, the pole is tapped with a light hammer, the sound obtained indicating the condition of the pole at the point struck.

The prodding test consists of prodding the pole with a sharp-pointed tool. If the timber is in good condition, it will resist penetration and grip the point.

The boring test is used in exceptional cases where the hammer test raises doubts about the internal condition of the pole. A special auger is used, and decay is indicated if little resistance is felt during boring.

Ladders
The safe working angle for a ladder is attained when the spread from its support is one quarter of the height of the ladder in use. The base of the ladder should rest firmly on the ground and its upper end should rest against a strong and rigid support. The top end of the ladder should be securely lashed to its support when the surface of the ground is uneven, soft, sloping or slippery, or where there is a risk of pedestrians knocking against it, or in any other conditions where the ladder is likely to be disturbed.

Testing for Gas
The manhole cover should be raised about 50 mm and the air in the upper part of chamber tested with a gas indicator. If the test is negative, the cover can be removed and a test made at the bottom, or just above the water level if the chamber contains water. If this proves negative, any water can be pumped out and the test repeated. The air should be tested for asphyxiating gas by lowering a safety lamp to the floor of the manhole. If this test is negative, a further mid-point gas-indicator test should be made. If all tests are negative, the manhole can be entered but, before work is begun or a flame is used, tests using the gas indicator should be made in the mouths of selected ducts in each nest entering the manhole.

Manhole-Cover Lifting
Ideally, to raise a carriageway cover, a mechanical cover-lifter should be used. Otherwise, two men should lift the cover together using the correct lifting technique.

Observer
While a man is in the manhole, it is necessary that another man be in attendance above ground.

Q10 (a) Explain why it is sometimes necessary to use a temporary method of closure for a joint.

(b) Describe the modern method of effecting a temporary closure.

(c) Explain how it is used.

(d) Describe the method of repairing cable-sheath damage using "wraparound" repair sleeving.

A10 See A1, Line Plant Practice A 1978, *Supplement*, Vol. 72, p. 11, Apr. 1979.

COMPUTERS B 1979

Students were expected to answer any 6 questions

Q1 (a) Convert the following numbers into binary, showing all working.
(i) $1278{\cdot}45_{10}$
(ii) $(473521)_8$
(iii) $(A9F3CD)_{16}$.

(b) Explain how the following binary patterns could be said to represent the same denary value
(i) 01010011
(ii) 00110101.

(c) Explain what is meant by binary-coded decimal (BCD) notation. How would the denary number 348 be represented in BCD, weighted
(i) 8421
(ii) 2421
(iii) 8421?

A1 (*a*) (*i*) The binary representation of $1278{\cdot}45_{10}$ is obtained by repeated division by 2 of the integral part, and repeated multiplication by 2 of the fractional part, as shown in the tables.

Integral Part	
Quotient	Remainder
2)1278	
639	0
319	1
159	1
79	1
39	1
19	1
9	1
4	1
2	0
1	0
0	1

Fractional Part	
Result	Product
	0·45 × 2
0	0·9
1	0·8
1	0·6
1	0·2
0	0·4
0	0·8
1	0·6
1	0·2

For the integral part, the remainder is noted in reverse order (the final remainder being the most significant digit). For the fractional part, the result is noted in correct order.

$\therefore \quad 1278{\cdot}45_{10} = \underline{10\,011\,111\,110{\cdot}011\,100\,11}$

(*ii*) The radix 8 gives a numbering system known as the *octal system*. Because 8 is a power expansion of 2 (that is, 2^3), each octal digit represents the denary value of a 3 digit binary number, as shown in the table.

Octal	Binary
1	001
2	010
3	011
4	100
5	101
6	110
7	111
0	000

$\therefore \quad (473521)_8 = \underline{100\,111\,011\,101\,010\,001_2}$

(*iii*) The radix 16 gives a numbering system known as the *hexadecimal system*. Alphabetic characters are used so that the numbers 10–15 can be represented in single-character form. Since $16 = 2^4$, each hexadecimal digit (including the alphabetic characters) represents a 4-digit binary number, as shown in the table.

Hexadecimal	Binary
1	0001
2	0010
3	0011
4	0100
5	0101
6	0110
7	0111
8	1000
9	1001
A	1010
B	1011
C	1100
D	1101
E	1110
F	1111
0	0000

$\therefore \quad (A9F3CD)_{16} = \underline{1010\,1001\,1111\,0011\,1100\,1101_2}$

(*b*) The two binary patterns can represent the same denary value if the first pattern is taken to be binary-coded decimal (BCD) and the second pattern taken as a binary number.

Thus, 01 010 001 is split into two binary coded decimal numbers 0101 and 0011.

Assuming that the BCD weighting is 8421,

$0101 = 5_{10}$ and $0011 = 3_{10}$.

$\therefore$ 01 010 011 represents 53_{10}.

The pattern 0 0110 101 taken as a binary number

$= 2^0 + 2^2 + 2^4 + 2^5$
$= 1 + 4 + 16 + 32$
$= 53_{10}$.

(*c*) Binary-coded decimal notation is a method by which each digit of the decimal number is replaced by its binary equivalent. A commonly used weighting for a 4-digit binary code gives the least significant bit the value 1, the next the value 2, the next the value 4 and the most significant the value 8. Such a code is said to be weighted 8421.

(*i*) 348_{10} with BCD 8421

3 4 8
0011 0100 1000

$348_{10} = \underline{001\,101\,001\,000_{BCD8421}}$.

(*ii*) 348_{10} with BCD 2421

3 4 8
0011 0100 1110

$348_{10} = \underline{001\,101\,001\,110_{BCD2421}}$.

(*iii*) 348_{10} with BCD 8421

With BCD system weighted $8\bar{4}21$ the second digit takes the value −4.

3 4 8
0011 1100 1000

$348_{10} = \underline{001\,111\,001\,000_{BCD8\bar{4}21}}$.

Q2 (a) Give the main reason why the majority of digital computers perform subtraction by complementary addition.

(b) Convert 1475_{10} and 1368_{10} to binary and then subtract 1475 from 1368 in binary using two's complement notation. Finally, convert the answer from binary to denary. Show all working.

A2 (*a*) The main reason why the majority of digital computers perform subtraction by complementary addition is that it simplifies the process. The addition circuits in the arithmetic unit can also be used for subtraction, so removing the requirement for special subtraction circuits.

(*b*) 1475_{10} may be converted to binary by repeated division by 2, and noting the remainder.

Quotient	Remainder
2)1475	
737	1
368	1
184	0
92	0
46	0
23	0
11	1
5	1
2	1
1	0
0	1

The binary number is obtained by writing down the remainder in reverse order

$\therefore \quad 1475_{10} = 10\,111\,000\,011_2$.

Similarly, for 1368_{10}

Quotient	Remainder
2)1368	
684	0
342	0
171	0
85	1
42	1
21	0
10	1
5	0
2	1
1	0
0	1

$\therefore \quad 1368_{10} = 10\,101\,011\,000_2$.

The two's complement of a binary number is that number which, when added to the original number, will result in an all zeros answer and a CARRY from the left-most bit. The two's complement is obtained by finding the one's complement and adding 1. The one's complement is obtained by inverting each bit of the original number. The two's complement is a convention used for representing negative numbers.

In two's complement, each number is assigned a sign bit: 0 for positive and 1 for negative. The sign bit is the left-most bit of the binary number and once assigned is treated as part of the number so that during calculations it is not necessary to keep a check of the sign of partial results.

To represent the negative of a number, the positive number including the sign bit is two's complemented. If the answer to a calculation is negative (that is, the sign bit is 1) its two's complement is taken to obtain the magnitude of the negative number.

	Sign bit	
1475_{10}	0	10 111 000 011
Two's complement of 1475_{10}	1	01 000 111 101
Add 1368_{10}	0	10 101 011 000
	1	11 110 010 101

The 1 in the sign bit indicates a negative number and, hence, the result has to be two's complemented again.

The two's-complement answer = 0 00 001 101 011.

$\underline{1368 - 1475 = -1\,101\,011_2}$.

To convert to denary:

$1\ 101\ 011_2 = 1\times2^0+1\times2^1+0\times2^2+1\times2^3+0\times2^4+1\times2^5+1\times2^6$
$= 1 + 2 + 8 + 32 + 64$
$= 107_{10}$.

$1368 - 1475 = -107_{10}$.

Q3 *(a) Draw a logic diagram of a full adder circuit.*

(b) Give truth tables for the inputs and output of each logic element in the full adder drawn in part (a) in terms of the initial inputs.

A3 *(a)* A full adder is a device that adds together 2 binary numbers, one digit at a time, together with the carry digit from the previous stage of addition. The output is a SUM digit and a CARRY digit.

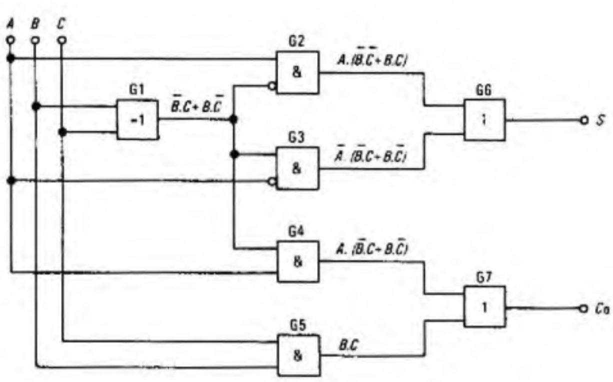
In the truth table below, *A* and *B* are the 2 binary numbers to be added, *C* is the CARRY digit from the previous stage, *S* is the SUM output and C_0 is the CARRY output.

A	*B*	*C*	*S*	C_0
0	0	0	0	0
0	0	1	1	0
0	1	0	1	0
0	1	1	0	1
1	0	0	1	0
1	0	1	0	1
1	1	0	0	1
1	1	1	1	1

From the above truth table the Boolean expressions for the sum and carry outputs are:

$$S = \bar{A}.\bar{B}.C + \bar{A}.B.\bar{C} + A.\bar{B}.\bar{C} + A.B.C,$$
$$= A.(\bar{B}.\bar{C} + B.C) + \bar{A}.(\bar{B}.C + B.\bar{C})$$
$$C_0 = \bar{A}.B.C + A.\bar{B}.C + A.B.\bar{C} + A.B.C,$$
$$= B.C\,(\bar{A} + A) + A.(\bar{B}.C + B.\bar{C}),$$
$$= B.C + A.(\bar{B}.C + B.\bar{C})$$

A circuit to implement these functions is shown in the sketch.



(b) The truth tables for the logic elements G1–G7 are given below. The initial inputs are *A*, *B* and *C* and the upper and lower inputs to each gate are denoted by *X* and *Y* respectively and the output is denoted by *Z*. Where the input to a logic element is via an inverter, the values of *X* and *Y* are the values before inversion.

Gate G1 The inputs to the gate are the original inputs *B* and *C*. The gate performs the EXCLUSIVE-OR function:

B	*C*	*Z*
0	0	0
0	1	1
1	0	1
1	1	0

Gate G2 The gate is an AND gate with an inverter on one of the inputs.

A	*B*	*C*	*X*	*Y*	*Z*
0	0	0	0	0	0
0	0	1	0	1	0
0	1	0	0	1	0
0	1	1	0	0	0
1	0	0	1	0	1
1	0	1	1	1	0
1	1	0	1	1	0
1	1	1	1	0	1

Gate G3 The gate is an AND gate with an inverter on one of the inputs.

A	*B*	*C*	*X*	*Y*	*Z*
0	0	0	0	0	0
0	0	1	1	0	1
0	1	0	1	0	1
0	1	1	0	0	0
1	0	0	0	1	0
1	0	1	1	1	0
1	1	0	1	1	0
1	1	1	0	1	0

Gate G4 The gate is an AND gate.

A	*B*	*C*	*X*	*Y*	*Z*
0	0	0	0	0	0
0	0	1	1	0	0
0	1	0	1	0	0
0	1	1	0	0	0
1	0	0	0	1	0
1	0	1	1	1	1
1	1	0	1	1	1
1	1	1	0	1	0

Gate G5 The gate is an AND gate and the inputs are the original inputs *B* and *C*.

B	*C*	*Z*
0	0	0
0	1	0
1	0	0
1	1	1

Gate G6 The gate is an OR gate.

A	*B*	*C*	*X*	*Y*	*Z*
0	0	0	0	0	0
0	0	1	0	1	1
0	1	0	0	1	1
0	1	1	0	0	0
1	0	0	1	0	1
1	0	1	0	0	0
1	1	0	0	0	0
1	1	1	1	0	1

Gate G7 The gate is an OR gate.

A	*B*	*C*	*X*	*Y*	*Z*
0	0	0	0	0	0
0	0	1	0	0	0
0	1	0	0	0	0
0	1	1	0	1	1
1	0	0	0	0	0
1	0	1	1	0	1
1	1	0	1	0	1
1	1	1	0	1	1

Q4 *(a) Explain the difference between a monostable device and a bistable device. Give a typical use for each of them.*

(b) Draw a circuit diagram of a practical bistable using n p n transistors and negative logic, and explain how it functions. Show clearly the SET input, the RESET input, and the outputs.

Q5 (a) Explain the difference between positive and negative logic, giving examples of both.

(b) Using the circuit shown in Fig. 1 below, draw TWO *truth tables incorporating inputs A, B and C, and output D to show the functioning of the circuit for*

(i) positive logic, and
(ii) negative logic.

What are these logic elements called?

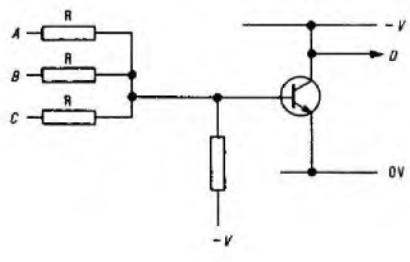
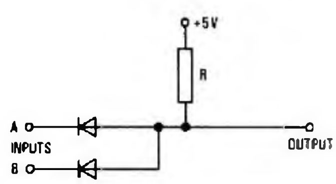


Fig. 1

A5 (*a*) Positive logic is defined as the convention that the more positive of the two voltage levels represents logic 1 and the more negative of the two voltage levels represents logic 0. Negative logic is the opposite convention; that is, the more negative of the two voltage levels represents logic 1 and the more positive represents logic 0. The actual polarity of the voltages used is immaterial to the logic convention. For example, both voltage levels may be less than 0 V or one voltage may be above 0 V and the other below 0 V.

Sketch (*a*) shows a diode input gate which can be used as an example of either positive or negative logic. The voltage table for this gate is shown below.



(*a*)

A	*B*	Output
0 V	0 V	0 V
0 V	+5 V	0 V
+5 V	0 V	0 V
+5 V	+5 V	+5 V

Using positive logic, the diode gate is an AND gate as shown in the following truth table.

A	*B*	Output
0	0	0
0	1	0
1	0	0
1	1	1

The application of negative logic to the voltage table shows that the same circuit obeys the OR function as shown in the following truth table.

A	*B*	Output
1	1	1
1	0	1
0	1	1
0	0	0

(*b*) For the circuit shown in Fig. 1 the voltage table is as follows.

A	*B*	*C*	*D*
−	−	−	+V
−	−	+	0 V
−	+	−	0 V
−	+	+	0 V
+	−	−	0 V
+	−	+	0 V
+	+	−	0 V
+	+	+	0 V

Table 1

A	*B*	*C*	*D*
0	0	0	1
0	0	1	0
0	1	0	0
0	1	1	0
1	0	0	0
1	0	1	0
1	1	0	0
1	1	1	0

Table 2

A	*B*	*C*	*D*
1	1	1	0
1	1	0	1
1	0	1	1
1	0	0	1
0	1	1	1
0	1	0	1
0	0	1	1
0	0	0	1

The value of the base-bias resistor relative to the input resistors is assumed to be such that when all three inputs *A*, *B* and *C* are at the negative (or low voltage level) the base of the n p n transistor is held at a negative potential. Thus the transistor is switched OFF and the output voltage rises to $+V$. If one or more of the inputs goes to the positive or high-voltage state the base potential goes positive and the transistor is switched ON, causing the output voltage to fall to the low voltage level.

The application of positive and then negative logic conventions to the truth table shows the gate is a positive logic NOR gate (Table 1) and a negative logic NAND gate (Table 2).

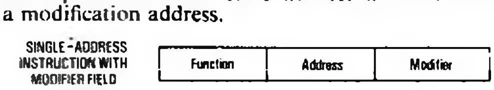
Q6 (a) Explain, using suitable examples, what is meant by the terms single, two and three-address instructions.

(b) Explain, using a suitable example, what is meant by address modification.

(c) What is the main advantage of address modification compared with direct addressing?

A6 (*a*) See Computers B 1977, *Supplement*, Vol. 71, p. 29, Apr. 1978.

(*b*) *Address modification* is the process of changing the address part of an instruction in a program while the program is in operation. One method of performing address modification is by splitting the address part of the instruction into two parts as shown in the sketch. The address field specifies a main store address and the modifier field specifies a modification address.



For example, suppose the location 8 contains the constant +10. The instruction ADD 50, 8 is obeyed as follows:

Add the contents of address 8 (+10) to the main-store address 50 to give new address 60. Obey instruction, which is now interpreted as ADD 60.

(*c*) The main advantage of address modification compared with direct addressing is that it facilitates the changing of the address part of an instruction so that the instruction can be made to operate on a different operand each time the routine containing the instruction is performed.

Q7 Write a program, using a machine code of your own choice, to generate and print out a series of 20 pseudo-random numbers, the value of the random numbering being in the range 1 to 55. As a start to this program, it is suggested that you select any number n where $0 < n \leq 1$*, multiply it by* $(1099)_{10}$ *and divide by* $(20)_{10}$*. Give a key to the machine code used.*

A7 To generate a series of random numbers the following algorithm is used:

(*a*) A number *n*, where $0 < n \leq 1$, is multipled by 1099 and divided by 20.

(*b*) The result is split into the integral and fractional parts.

(*c*) The integral part is incremented by one and this is the first of the series of numbers.

(*d*) The fractional part is used as the starting point for generating the next number in the series.

Consider a processor which has an accumulator of twice the length of a single addressable storage area.

Fixed-point arithmetic is used throughout and, thus, the left-hand-side (LHS) of the accumulator is the integral part and the right-hand-side (RHS) of the accumulator is the fractional part of the real number.

Table 1 gives the machine code that could be used to write a program to perform the algorithm, and the program using these instructions is shown in Table 2.

Table 1

Instruction	Function
LDL N	Load content of location N into LHS of accumulator.
LDR N	Load content of location N into RHS of accumulator.
STL N	Store content of LSH of accumulator in location N.
STR N	Store content of RHS of accumulator in location N.
INL	Increment LHS of accumulator by one.
DCL	Decrement RHS of accumulator by one.
ZRO	Zero accumulator.
MLT N	Multiply content of whole accumulator by content of location N, leaving result in accumulator.
DIV N	Divide content of whole accumulator by content of location N, leaving result in accumulator.
JIZ N	Jump to address N if content of accumulator is equal to zero.
JMP N	Jump unconditionally to address N.
READ	Read next number from input device into whole accumulator.
PRINT	Print content of accumulator via output device.
START	Start program.
STOP	Stop program.

Table 2

Label	Program	Comment
		Let location 60 hold the value 20
		Let location 61 hold the counter
		Let location 62 hold the value 1099
		Let location 63 be temporary storage area
		Let location 64 be temporary storage area
	START	
	LDL 60	Load counter
	STL 61	Store counter
	READ	Read value of n $(0 < n < 1)$
L1	MLT 62	Multiply by 1099
	DIV 60	Divide by 20
	INL	Increment integral part
	STR 63	Store fractional part
	STL 64	Store integral part
	ZRO	Zero accumulator
	LDL 64	Reload integral part
	PRINT	Print the integral part
	LDL 61	Load counter
	DCL	Decrement counter
	JIZ L2	If counter zero jump to L2
	STL 61	Store counter
	LDR 63	Load fractional part
	JMP L1	Jump to L1
L2	STOP	

Q8 Draw a schematic diagram of a total core store. Explain fully how a binary digit would be written to a core, and how it would subsequently be read out of store.

Q9 (a) Explain the meaning of "the representation of physical quantities by the magnitude of electrical signals".

(b) In what type of computer would this principle be used?

(c) Using a suitable diagram, prove that ONE *signal may be subtracted from another of the same scale by means of a passive resistor network.*

Q10 (a) List the principal components of an analogue computer and, using these, sketch diagrams of arrangements suitable for the following mathematical operations:

(i) summation,
(ii) sign changing, and
(iii) integration.

In each case, state the relationship between input, output and component values.

(b) Given that $y = 2x + 4$, show how y could be found using operational amplifiers, given that the following items are available:

(i) ± 10 V,
(ii) the variable x,
(iii) 1 MΩ and 10 MΩ resistors, and
(iv) two potentiometers.

MATHEMATICS B 1979

Students were expected to answer any 6 questions

Q1 In a network the mesh currents x, y, z amperes are given by the simultaneous equations

$$x + 6y + 4z = 0,$$
$$x + 2(x - y) + 5(2 - z) = 20,$$
$$4(y - x) + 12y + 5(y - z) = 0.$$

Calculate EACH *of these currents to 2 significant figures.*

A1

$$x + 6y + 4z = 0. \quad \ldots\ldots (1)$$
$$x + 2(x - y) + 5(2 - z) = 20. \quad \ldots\ldots (2)$$
$$4(y - x) + 12y + 5(y - z) = 0. \quad \ldots\ldots (3)$$

Rearranging equations (2) and (3):

$$3x - 2y - 5z = 10. \quad \ldots\ldots (4)$$
$$-4x + 21y - 5z = 0. \quad \ldots\ldots (5)$$

Subtracting equation (5) from equation (4):

$$7x - 23y = 10.$$
$$\therefore \quad x = \frac{10 + 23y}{7}.$$

Substituting for x in equation (1):

$$\frac{10 + 23y}{7} + 6y + 4z = 0.$$
$$\therefore \quad 10 + 23y + 42y + 28z = 0.$$
$$\therefore \quad 65y + 28z = -10. \quad \ldots\ldots (6)$$

Substituting for x in equation (4):

$$\frac{3(10 + 23y)}{7} - 2y - 5z = 10.$$
$$\therefore \quad 30 + 69y - 14y - 35z = 70.$$
$$\therefore \quad 55y - 35z = 40.$$
$$\therefore \quad 11y - 7z = 8. \quad \ldots\ldots (7)$$

Multiplying equation (7) by 4:

$$44y - 28z = 32. \quad \ldots\ldots (8)$$

Adding equations (6) and (8):

$$109y = 22.$$
$$\therefore \quad y = 0{\cdot}2018.$$

From equation (7):

$$7z = 11y - 8,$$
$$= 11 \times 0{\cdot}2018 - 8.$$
$$\therefore \quad z = \frac{2{\cdot}2198 - 8}{7},$$
$$= -0{\cdot}8257.$$

Substituting for y and z in equation (1):

$$x = -6 \times 0{\cdot}2018 + 4 \times 0{\cdot}8257,$$
$$= -1{\cdot}2108 + 3{\cdot}3028,$$
$$= 2{\cdot}0920.$$

Hence, to 2 significant figures,

$$\underline{x = 2{\cdot}1 \text{ A}, y = 0{\cdot}20 \text{ A and } z = -0{\cdot}83 \text{ A}.}$$

Note: It is important to check the results in each of the original equations.

Q2 If a circuit involving inductance L henrys, resistance R ohms and capacitance C farads is to oscillate, then the roots of the quadratic equation

$$Lz^2 + Rz + \frac{1}{C} = 0$$

must be complex.

(a) Derive the condition, in terms of L, R and C, for this to be true.
(b) Calculate the TWO *values of z when $L = 1{\cdot}2 \times 10^{-3}$, $R = 30$ and $C = 4 \times 10^{-6}$. Display these values in the complex plane.*
(c) The frequency f hertz of the circuit's natural oscillations is given by

$$4\pi^2 f^2 = \frac{1}{LC} - \frac{R^2}{4L^2}.$$

Calculate this frequency to the nearest 10 Hz.

A2 (*a*) From the general solution to a quadratic equation

$$z = \frac{-R \pm \sqrt{(R^2 - 4L/C)}}{2L}.$$

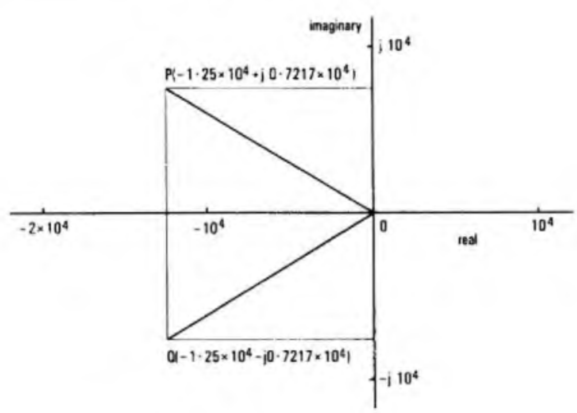
For z to be complex, the expression under the square-root sign must be negative; that is,

$$4L/C > R^2 \text{ or } \underline{L > CR^2/4.}$$

(*b*) When $L = 1{\cdot}2 \times 10^{-3}$, $R = 30$ and $C = 4 \times 10^{-6}$,

$$z = \frac{-30 \pm \sqrt{\{900 - (4 \times 1{\cdot}2 \times 10^{-3})/(4 \times 10^{-6})\}}}{2 \times 1{\cdot}2 \times 10^{-3}},$$
$$= -\frac{30 \times 10^3}{2{\cdot}4} \pm \frac{10^3}{2{\cdot}4}\sqrt{(900 - 1200)},$$
$$= -12\,500 \pm \mathrm{j}416{\cdot}6\sqrt{300},$$
$$= -12\,500 \pm \mathrm{j}416{\cdot}6 \times 17{\cdot}3205,$$
$$= \underline{-12\,500 \pm \mathrm{j}7217.}$$

These two values are shown in the sketch, represented by points P and Q or by the vectors $\overrightarrow{OP}$ and $\overrightarrow{OQ}$.



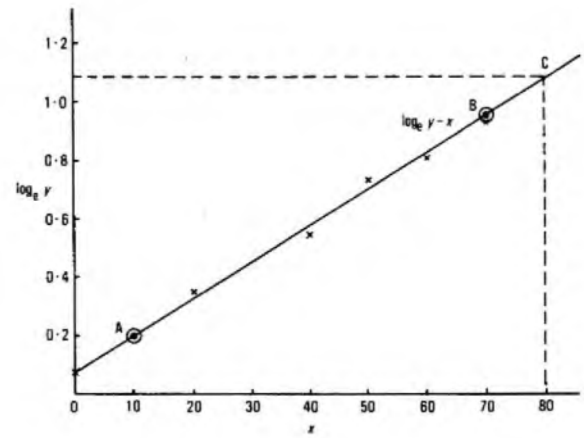
(*c*)
$$4\pi^2 f^2 = \frac{1}{1{\cdot}2 \times 10^{-3} \times 4 \times 10^{-6}} - \frac{900}{4 \times 1{\cdot}2^2 \times 10^{-6}},$$
$$= \frac{10^9}{4{\cdot}8} - \frac{9 \times 10^8}{5{\cdot}76},$$
$$= (2{\cdot}08\dot{3} - 1{\cdot}5625) \times 10^8.$$
$$\therefore \quad f^2 = \frac{0{\cdot}5208\dot{3} \times 10^8}{4\pi^2},$$
$$= 1{\cdot}3193 \times 10^6.$$
$$\therefore \quad f = \underline{1150 \text{ Hz}} \text{ to the nearest 10 Hz.}$$

Q3 The growth in population of a city since 1900 is shown in the table.

x (years after 1900)	*0*	*20*	*40*	*50*	*60*	*70*
y (population in millions)	*1·08*	*1·41*	*1·76*	*2·07*	*2·28*	*2·61*

(a) By plotting suitable variables to obtain a straight-line graph, show that the population is proportional to e^{kx}, where k is a positive constant.
(b) Calculate a value for k from the graph.
(c) Estimate what the population will be in 1980.

A3 (*a*) If $y \propto e^{kx}$, let $y = ae^{kx}$, where a is a constant.



Taking logarithms to base e,

$$\log_e y = \log_e a + \log_e e^{kx},$$
$$= \log_e a + kx.$$

Since $\log_e a$ and k are constants, this equation is of the form $y = mx + c$; that is, it is a linear equation.

The appropriate values of $\log_e y$ are shown in the table.

x	0	20	40	50	60	70
y	1·08	1·41	1·76	2·07	2·28	2·61
$\log_e y$	0·077	0·344	0·565	0·728	0·824	0·959

The values of $\log_e y$ plotted against x are shown in the sketch, from which it is seen that it is possible to draw a straight line reasonably close to all the plotted points. Hence, the linear law $\log_e y = \log_e a + kx$ can be assumed to be valid for the data given, and therefore y is proportional to e^{kx}.

(*b*) k is the gradient of the straight-line graph and is obtained from the co-ordinates of two widely separated points on the graph. Taking points A (10, 0·2) and B (70, 0·965), the gradient is given by

$$k = \frac{0{\cdot}965 - 0{\cdot}2}{70 - 10},$$
$$= \frac{0{\cdot}765}{60} = \underline{0{\cdot}0128.}$$

(*c*) From the graph, when $x = 80$ (at point C)

$$\log_e y = 1{\cdot}095.$$
$$\therefore \quad y = 2{\cdot}989.$$

Thus, the population in 1980 will be approximately <u>3 million.</u>

Q4 (a) From the definition of a logarithm prove that for any positive numbers a and b
(i) $\log_a x - \log_a y = \log_a (x/y)$, and
(ii) $\log_a b \times \log_b a = 1$.

Check that this is true when $a = 10$ and $b = e$.

(b) The market price of a car now six years old is quoted as £350. Its original price was £1350.

(i) Calculate the approximate rate per cent per annum depreciation (assumed compound interest) in the car's value.
(ii) In how many years would its quoted value be below £150?

A4 (*a*) (*i*) Let $\log_a x = p$ and $\log_a y = q$. Then, by definition,

$$x = a^p \text{ and } y = a^q.$$
$$\therefore \quad \frac{x}{y} = \frac{a^p}{a^q} = a^{p-q}.$$
$$\therefore \quad \log_a \frac{x}{y} = p - q,$$
$$= \underline{\log_a x - \log_a y.} \qquad \text{QED}$$

(*ii*) Let $\log_a b = u$ and $\log_b a = v$. Then, by definition,

$$b = a^u \text{ and } a = b^v.$$
$$\therefore \quad b = (b^v)^u \quad (\text{since } a = b^v).$$
$$\therefore \quad \log_b b = \log_b (b^{uv}),$$
$$= uv = 1.$$
$$\therefore \quad \underline{\log_a b \times \log_b a = 1.} \qquad \text{QED}$$

When $a = 10$ and $b = e$,

$$\log_a b \times \log_b a = \log_{10} e \times \log_e 10,$$
$$= 0{\cdot}4343 \times 2{\cdot}3026,$$
$$= 1{\cdot}0000.$$

Thus the identity is confirmed by the tables.

(*b*) (*i*) Let $r\%$ be the annual rate of depreciation. After 1 year, the car's value will be

$$£\left(1350 - \frac{r}{100} \times 1350\right),$$
$$= £1350\left(1 - \frac{r}{100}\right).$$

After 2 years, the value becomes

$$= £\left\{1350\left(1 - \frac{r}{100}\right) - \frac{r}{100} \times 1350\left(1 - \frac{r}{100}\right)\right\},$$
$$= £1350\left(1 - \frac{r}{100}\right)\left(1 - \frac{r}{100}\right),$$
$$= £1350\left(1 - \frac{r}{100}\right)^2.$$

In a similar manner, it may be shown that after n years the value becomes

$$£1350\left(1 - \frac{r}{100}\right)^n.$$

Hence, when $n = 6$

$$350 = 1350\left(1 - \frac{r}{100}\right)^6,$$

or $\left(1 - \frac{r}{100}\right)^6 = \frac{35}{135} = 0{\cdot}25926.$

$\therefore \quad 1 - \frac{r}{100} = \sqrt[6]{0{\cdot}25926},$

$= 0{\cdot}7985.$

No.	log
0·25926	$\bar{1}{\cdot}4137 \div 6$
0·7985	$\bar{1}{\cdot}9023$

$\therefore \quad \frac{r}{100} = 1 - 0{\cdot}7985 = 0{\cdot}2015,$

or $r \approx$ 20 %

(*ii*) Let x be the number of years in which the value falls below £150

Then, $150 = 1350(1 - 0{\cdot}2014)^x$,

or $0{\cdot}7986^x = \frac{150}{1350} = 0{\cdot}\dot{1}.$

$\therefore \quad x \log_{10} 0{\cdot}7986 = \log_{10} 0{\cdot}\dot{1},$

or $x = \frac{\bar{1}{\cdot}0457}{\bar{1}{\cdot}9023},$

$= \frac{-0{\cdot}9543}{-0{\cdot}0977},$

$= 9{\cdot}77.$

Thus, the value would be less than £150 after 10 years.

Q5 (*a*) *Assuming the compound-angle formulae for* $\sin(A + B)$ *and* $\cos(A + B)$ *and without using trigonometrical tables or a calculator, evaluate*

(*i*) *sin 15°, and*
(*ii*) *cos 225°.*

(*b*) *Derive expressions for*

(*i*) *sin 2θ in terms of sin θ and cos θ, and*
(*ii*) *sin² θ in terms of cos 2θ.*

(*c*) *If* $\frac{\cos(\beta - \alpha)}{\sin\beta} = 2{\cdot}5$, *express* $\tan\beta$ *in terms of* α *and hence calculate* β *to the nearest degree (between −180° and +180°) when* $\alpha = 52°$.

A5 (*a*) $\sin(A + B) = \sin A \cos B + \cos A \sin B$, and
$\cos(A + B) = \cos A \cos B - \sin A \sin B$.

(*i*) $\sin 15° = \sin(60° - 45°),$
$= \sin 60° \cos(-45°) + \cos 60° \sin(-45°),$
$= \frac{\sqrt{3}}{2} \times \frac{1}{\sqrt{2}} + \frac{1}{2} \times \left(-\frac{1}{\sqrt{2}}\right),$
$= \frac{1}{2\sqrt{2}}(\sqrt{3} - 1),$
$= \frac{\sqrt{2}}{4} \times 0{\cdot}7321,$
$= \frac{1{\cdot}4142}{4} \times 0{\cdot}7321,$
$= 0{\cdot}2588.$

(*ii*) $\cos 225° = \cos(180° + 45°),$
$= \cos 180° \cos 45° - \sin 180° \sin 45°,$
$= -1 \times \frac{1}{\sqrt{2}} - 0 \times \frac{1}{\sqrt{2}},$
$= -0{\cdot}7071.$

(*b*) (*i*) $\sin 2\theta = \sin(\theta + \theta),$
$= \sin\theta\cos\theta + \cos\theta\sin\theta,$
$= 2\sin\theta\cos\theta.$

(*ii*) $\cos 2\theta = \cos(\theta + \theta),$
$= \cos\theta\cos\theta - \sin\theta\sin\theta,$
$= \cos^2\theta - \sin^2\theta \quad \ldots\ldots (1)$

But, $\sin^2\theta + \cos^2\theta = 1.$

Substituting for $\cos^2\theta$ in equation (1),

$\cos 2\theta = 1 - 2\sin^2\theta.$

$\therefore \quad 2\sin^2\theta = 1 - \cos 2\theta.$

$\therefore \quad \sin^2\theta = \frac{1 - \cos 2\theta}{2}.$

(*c*) $2{\cdot}5 = \frac{\cos(\beta - \alpha)}{\sin\beta},$
$= \frac{\cos\beta\cos(-\alpha) - \sin\beta\sin(-\alpha)}{\sin\beta},$
$= \frac{\cos\alpha}{\tan\beta} + \sin\alpha.$

$\therefore \quad 2{\cdot}5\tan\beta = \cos\alpha + \sin\alpha\tan\beta.$

$\therefore \tan\beta(2{\cdot}5 - \sin\alpha) = \cos\alpha.$

$\therefore \quad \tan\beta = \frac{\cos\alpha}{2{\cdot}5 - \sin\alpha}.$

When $\alpha = 52°$, $\cos\alpha = 0{\cdot}6157$ and $\sin\alpha = 0{\cdot}7880$.

$\therefore \quad \tan\beta = \frac{0{\cdot}6157}{2{\cdot}5 - 0{\cdot}7880}.$

$\therefore \quad \beta = 20°$ or $(20° - 180°)$ to the nearest degree,
$= 20°$ or $-160°$ to the nearest degree.

Q6 (*a*) *Sketch the graph of the sinusoid*

$$y = 5\cos(100\pi t - \pi/4)$$

from $t = 0$ *to* $t = 0{\cdot}02$.

(*b*) *Use the graph drawn in part (a) to obtain approximate values of t for which, within this range,*

$$5\cos(100\pi t - \pi/4) = 4.$$

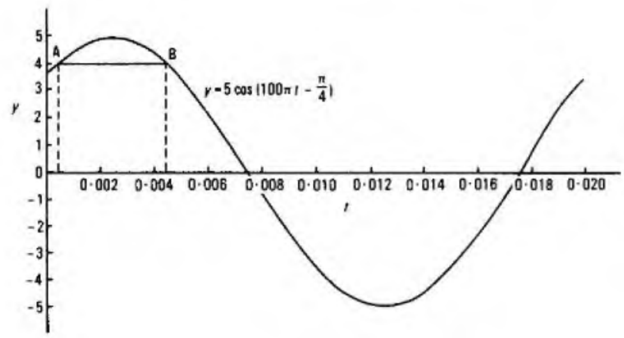
(*c*) *By calculation derive these values of t to 3 significant figures.*

A6 (*a*) The graph of $y = 5\cos(100\pi t - \pi/4)$, plotted from the following table of values, is shown in the sketch.

t	0	0·002	0·004
$100\pi t$	0	$0{\cdot}2\pi$	$0{\cdot}4\pi$
$100\pi t - \pi/4$	$-0{\cdot}25\pi$	$-0{\cdot}05\pi$	$0{\cdot}15\pi$
$\cos(100\pi t - \pi/4)$	0·707	0·988	0·891
y	3·54	4·94	4·46

t	0·006	0·008	0·01	0·012π
$100\pi t$	$0{\cdot}6\pi$	$0{\cdot}8\pi$	$1{\cdot}0\pi$	$1{\cdot}2\pi$
$100\pi t - \pi/4$	$0{\cdot}35\pi$	$0{\cdot}55\pi$	$0{\cdot}75\pi$	$0{\cdot}95\pi$
$\cos(100\pi t - \pi/4)$	0·454	−0·156	−0·707	−0·988
y	2·27	−0·78	−3·54	−4·94

t	0·014	0·016	0·018	0·020
$100\pi t$	$1·4\pi$	$1·6\pi$	$1·8\pi$	$2·0\pi$
$100\pi t - \pi/4$	$1·15\pi$	$1·35\pi$	$1·55\pi$	$1·75\pi$
$\cos(100\pi t - \pi/4)$	−0·891	−0·454	0·156	0·707
y	−4·46	−2·27	0·78	3·54



(*b*) When $y = 4$, reading from the graph at points A and B,

$t = \underline{0·00045 \text{ and } 0·00455}.$

(*c*) At $y = 4$,

$$\cos(100\pi t - \pi/4) = 0·8.$$

$$\therefore \quad 100\pi t - \pi/4 = \pm 0·6435.$$

$$\therefore \quad t = \frac{\pi/4 \pm 0·6435}{100\pi},$$

$$= 0·0025 \pm 0·0020483,$$

$= \underline{0·000452 \text{ and } 0·00455}$ to 3 significant figures.

Q7 (a) Differentiate from first principles the function $x^3 - 6x^2 + 5$.

(b) Using the result in part (a), find the turning points (maximum and minimum) in the graph of $y = x^3 - 6x^2 + 5$.

(c) Sketch this graph from $x = -2$ to $x = +6$.

(d) From the graph obtain (to 2 significant figures) the roots of $x^3 - 6x^2 + 5 = 0$.

A7 (*a*) Let $y = x^3 - 6x^2 + 5$.

Suppose x increases by a small amount δx and the corresponding increase in y is δy.

Then, $y + \delta y = (x + \delta x)^3 - 6(x + \delta x)^2 + 5,$

$= x^3 + 3x^2\,\delta x + 3x\,\delta x^2 + \delta x^3 - 6(x^2 + 2x\,\delta x + \delta x^2) + 5.$

$\therefore \quad \delta y = x^3 + 3x^2\,\delta x + 3x\,\delta x^2 + \delta x^3 - 6x^2 - 12x\,\delta x - 6\delta x^2 + 5 - x^3 + 6x^2 - 5,$

$= 3x^2\,\delta x + 3x\,\delta x^2 + \delta x^3 - 12x\,\delta x - 6\,\delta x^2.$

$\therefore \quad \dfrac{\delta y}{\delta x} = 3x^2 + 3x\,\delta x + \delta x^2 - 12x - 6\delta x,$

$= 3x^2 - 12x + 3x\,\delta x - 6\delta x + \delta x^2.$

But, in the limit as $\delta x \to 0$,

$$\frac{\delta y}{\delta x} \to \frac{dy}{dx}.$$

Hence, $\dfrac{dy}{dx} = \lim\limits_{\delta x \to 0} \dfrac{\delta y}{\delta x},$

$= \underline{3x^2 - 12x}.$

(*b*) For maximum and minimum values of y, $\dfrac{dy}{dx} = 0$.

$\therefore \quad 3x^2 - 12x = 0.$

$\therefore \quad 3x(x - 4) = 0.$

$\therefore \quad x = 0$ or 4.

Also, $\dfrac{d^2y}{dx^2} = \dfrac{d}{dx}(3x^2 - 12x),$

$= 6x - 12.$

When $x = 0$, $\dfrac{d^2y}{dx^2} = -12$, that is, it is negative and hence a maximum value must occur at $x = 0$.

When $x = 4$, $\dfrac{d^2y}{dx^2} = +12$, which is positive and hence a minimum must occur at this point.

$\therefore \quad \underline{y_{max} = 5 \text{ at } x = 0},$ and

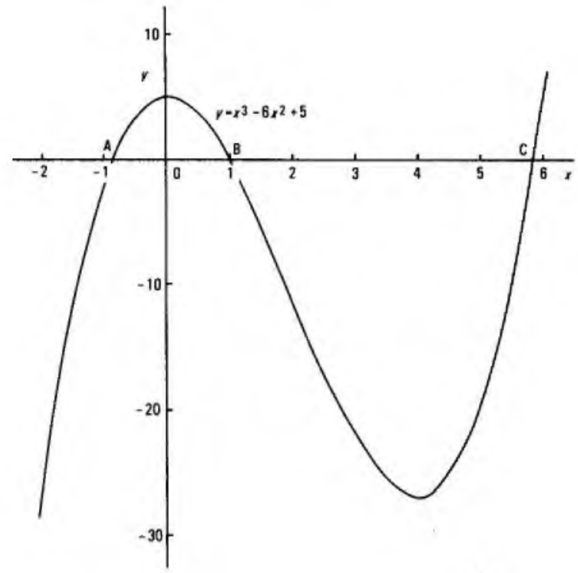
$\underline{y_{min} = -27 \text{ at } x = 4}.$

(*c*) The graph, shown in the sketch, was drawn from the following table of values.

x	−2	−1	0	1	2	3	4	5	6
x^3	−8	−1	0	1	8	27	64	125	216
$-6x^2$	−24	−6	0	−6	−24	−54	−96	−150	−216
5	5	5	5	5	5	5	5	5	5
x	−27	−2	5	0	−11	−22	−27	−20	5

(*d*) From the graph, $y = 0$ at the three points A, B and C, where $x = -0·85$, 1 and 5·85 respectively.

Hence, the three roots of the equation $x^3 - 6x^2 + 5 = 0$, to 2 significant figures are $\underline{-0·85,\ 1·0 \text{ and } 5·9}$.



Q8 A lift moves from rest at ground-floor level to rest at the sixth floor. It has moved for t seconds when its velocity v metres/second is given by $v = 0·24(8t - t^2)$. Calculate

(a) the initial acceleration of the lift,

(b) the maximum velocity,

(c) the total time in motion between these 2 floors, and

(d) the total height ascended.

A8 (*a*) Acceleration $= \dfrac{dv}{dt},$

$= 0·24(8 - 2t).$

The initial acceleration occurs at $t = 0$ and is therefore $0·24 \times 8 = \underline{1·92 \text{ m/s}^2}$.

(*b*) For a maximum velocity, $\dfrac{dv}{dt} = 0.$

$\therefore \quad 0·24(8 - 2t) = 0$ or $t = 4$ s.

For a maximum to occur at $t = 4$,

$\dfrac{d}{dt}\left(\dfrac{dv}{dt}\right)$ must be negative.

$$\frac{d}{dt}\left(\frac{dv}{dt}\right) = 0·24 \times -2 = -0·48.$$

This is negative and confirms that the velocity is a maximum at $t = 4$ s.

$$\therefore \quad v_{max} = 0\cdot24(8 \times 4 - 4^2),$$
$$= 0\cdot24(32 - 16)$$
$$= \underline{3\cdot84 \text{ m/s.}}$$

(*c*) The lift comes to rest at the sixth floor when $v = 0$; that is

$$0\cdot24(8t - t^2) = 0$$
$$t(8 - t) = 0.$$
$$\therefore \quad t = 0 \text{ or } 8 \text{ s.}$$

Hence, the total time in motion is $\underline{8 \text{ s.}}$

(*d*) If x metres is the height ascended after time t, then,

$$v = \frac{dx}{dt} = 0\cdot24(8t - t^2).$$

$$\therefore \quad x = \int \frac{dx}{dt}\,dt,$$
$$= \int 0\cdot24(8t - t^2)\,dt.$$

The total height is ascended after 8 s.

$$\therefore \quad \text{Total height} = \int_0^8 0\cdot24(8t - t^2)\,dt,$$
$$= 0\cdot24\left[\frac{8t^2}{2} - \frac{t^3}{3}\right]_0^8,$$
$$= 0\cdot24\left[4 \times 8^2 - \frac{8^3}{3} - 0\right],$$
$$= 0\cdot24\left[256 - \frac{512}{3}\right],$$
$$= 0\cdot08 \times 256,$$
$$= \underline{20\cdot48 \text{ m.}}$$

Q9 (*a*) *Evaluate* $\int_0^2 (x^2 - 4)^2\,dx$.

(*b*) (*i*) *Calculate the area enclosed between the curve* $y = 4 + 3x - x^2$ *and the x-axis.*

(*ii*) *Sketch the curve and shade in the area calculated in part* (*i*).

(*iii*) *Derive the mean height of this shaded area.*

A9 (*a*)
$$\int_0^2 (x^2 - 4)^2\,dx = \int_0^2 (x^4 - 8x^2 + 16)\,dx,$$
$$= \left[\frac{x^5}{5} - \frac{8x^3}{3} + 16x\right]_0^2,$$
$$= \left(\frac{2^5}{5} - \frac{8 \times 2^3}{3} + 32\right) - 0,$$
$$= \frac{32}{5} - \frac{64}{3} + 32 = \underline{17\cdot0\dot{6}.}$$

(*b*) (*i*) $\qquad y = 4 + 3x - x^2.$

The curve will cross the x-axis where $y = 0$; that is, when

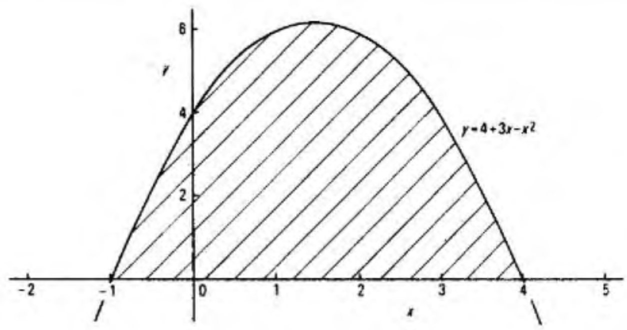
$$4 + 3x - x^2 = 0 = (4 - x)(1 + x).$$
$$\therefore \quad x = -1 \text{ or } 4.$$

The area enclosed between the curve and the x-axis is given by

$$\int_{-1}^4 y\,dx,$$
$$= \int_{-1}^4 (4 + 3x - x^2)\,dx,$$
$$= \left[4x + \frac{3x^2}{2} - \frac{x^3}{3}\right]_{-1}^4,$$
$$= \left(16 + \frac{3 \times 16}{2} - \frac{64}{3}\right) - \left(-4 + \frac{3}{2} + \frac{1}{3}\right),$$
$$= 16 + 24 + 4 - \frac{65}{3} - \frac{3}{2},$$
$$= 44 - 23\tfrac{1}{6},$$
$$= \underline{20\tfrac{5}{6} \text{ or } 20\cdot8\dot{3}.}$$

(*ii*) The curve of y, plotted from the tabulated values, is shown in the sketch.

x	-1	0	1	2	3	4	5
$-x^2$	-1	0	-1	-4	-9	-16	-25
$3x$	-3	0	3	6	9	12	15
4	4	4	4	4	4	4	4
y	0	4	6	6	4	0	-6



The calculated area of part (*i*) is shown shaded in the sketch.

(*iii*) The mean height of the shaded area =

$$\frac{\text{area under the curve}}{4 - (-1)},$$
$$= 20\cdot8\dot{3}/5 = \underline{4\cdot1\dot{6}.}$$

Q10 (*a*) *If* $z = 3 - j7$, *evaluate in the form* $a + jb$ *to 3 significant figures*

(*i*) $(3z + 5)(z - j2)$, *and*

(*ii*) $\dfrac{z + j2}{z - j2}$.

(*b*) *Two impedances* Z_1 *and* Z_2 *connected in parallel have the combined impedance* Z *given by*

$$\frac{1}{Z} = \frac{1}{Z_1} + \frac{1}{Z_2}.$$

Calculate Z *when* $Z_1 = 12 - j16$ *and* $Z_2 = 4 + j3$.

A10 (*a*) (*i*)
$$(3z + 5)(z - j2)$$
$$= \{3(3 - j7) + 5\}\{3 - j7 - j2\},$$
$$= (14 - j21)(3 - j9),$$
$$= 42 - j63 - j126 - 189,$$
$$= \underline{-147 - j189.}$$

(*ii*)
$$\frac{z + j2}{z - j2} = \frac{3 - j7 + j2}{3 - j7 - j2},$$
$$= \frac{3 - j5}{3 - j9},$$
$$= \frac{(3 - j5)(3 + j9)}{(3 - j9)(3 + j9)},$$
$$= \frac{9 - j15 + j27 + 45}{9 + 81},$$
$$= \frac{54 + j12}{90},$$
$$= \underline{0\cdot600 + j0\cdot133} \text{ to 3 significant figures.}$$

(*b*)
$$\frac{1}{Z} = \frac{1}{Z_2} + \frac{1}{Z_2},$$
$$= \frac{Z_1 + Z_2}{Z_1Z_2}.$$
$$\therefore \quad Z = \frac{Z_1Z_2}{Z_1 + Z_2}.$$

Substituting the given values of Z_1 and Z_2:

$$Z = \frac{(12 - j16)(4 + j3)}{12 - j16 + 4 + j3},$$
$$= \frac{48 - j64 + j36 + 48}{16 - j13},$$
$$= \frac{(96 - j28)(16 + j13)}{(16 - j13)(16 + j13)},$$
$$= \frac{1536 - j448 + j1248 + 364}{256 + 169},$$
$$= \frac{1900 + j800}{425},$$
$$= \underline{4\cdot47 + j1\cdot88} \text{ to 3 significant figures.}$$

TELECOMMUNICATION PRINCIPLES B 1979

Students were expected to answer any 6 questions

Q1 When connected in parallel, two impedances have an equivalent impedance, Z, of 4 − j2 Ω.

(a) If one of the two is 2 + j3 Ω, calculate the second impedance.

(b) Find the components corresponding to the equivalent impedance when the operating frequency is 500 Hz.

A1 (*a*) Let the second impedance be $a + jb$ ohms.

Then
$$\frac{1}{a+jb} + \frac{1}{2+j3} = \frac{1}{4-j2}.$$
$$\therefore \quad \frac{1}{a+jb} = \frac{(2+j3)-(4-j2)}{(2+j3)(4-j2)}.$$
$$\therefore \quad a+jb = \frac{14+j8}{-(2-j5)} = \frac{(14+j8)(2+j5)}{-(4+25)},$$
$$= \frac{12-j86}{29} = \underline{0{\cdot}414 - j2{\cdot}97\ \Omega.}$$

(*b*) The equivalent impedance, given as 4 − j2 Ω, is a resistance of 4 Ω in series with a capacitance of reactance 2 Ω. At 500 Hz, the value of this capacitance is
$$\frac{1}{2\pi \times 500 \times 2}\ \text{F} = \underline{159\ \mu\text{F}.}$$

Q2 (a) (i) Give the circuit of an AC bridge suitable for measuring inductance at 100 kHz.

(ii) Explain the principle of operation of the circuit, and indicate the features that make possible the measurement of resistive components.

(b) Derive balance equations from which the unknowns can be calculated.

(c) Show how screening should be applied to prevent unwanted interference voltages entering the bridge.

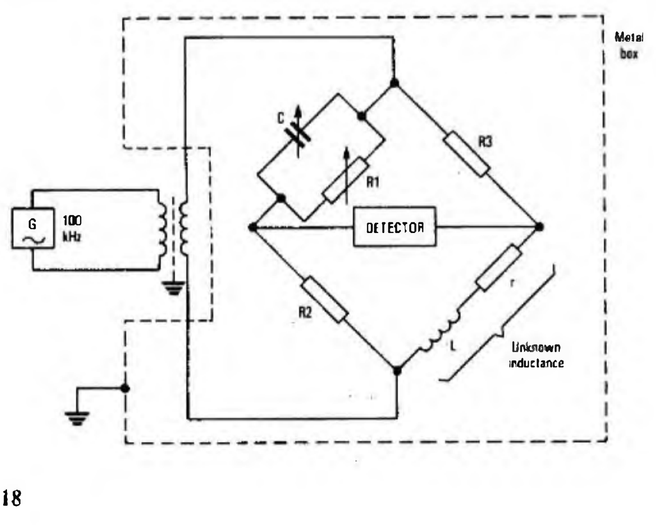
A2 (*a*) Maxwell's inductance bridge is shown in the sketch. R2 and R3 are known fixed resistors. The unknown inductance, L, with its resistive component, r, is balanced by adjusting a calibrated capacitor, C, and a resistor R1. The detector is a 100 kHz amplifier followed by headphones, or an electronic voltmeter.

At balance, the instantaneous potential each side of the detector must be the same, so that both amplitude and phase must be corrected. The arm opposite the unknown inductance thus consists of resistance and reactance, both being adjustable to allow balancing.

(*b*) At balance,
$$R_2R_3 = (r + j\omega L) \times \frac{1}{\dfrac{1}{R_1} + j\omega C},$$
where $\omega = 2\pi f$ radians/second and f is the frequency.
$$\therefore \quad \frac{R_2R_3}{R_1} + j\omega R_2R_3C = r + j\omega L.$$

Equating real and imaginary parts gives
$$\underline{r = R_2R_3/R_1} \text{ and } \underline{L = R_2R_3C.}$$

(*c*) Interference voltages can be induced into the components and wiring of the bridge from external fields, but they can be prevented by enclosing the whole unit inside an iron box. This shunts magnetic fields, and acts as an electrostatic screen if earthed. Longitudinal currents tend to enter along the power-supply leads unless the bridge is fed through an isolating transformer that has an earthed metal screen between windings. The detector must also be adequately screened, with screened connecting leads, all carefully taken back to a common earthing point.



Q3 (a) (i) Explain the principle of operation of an electronic AC voltmeter.

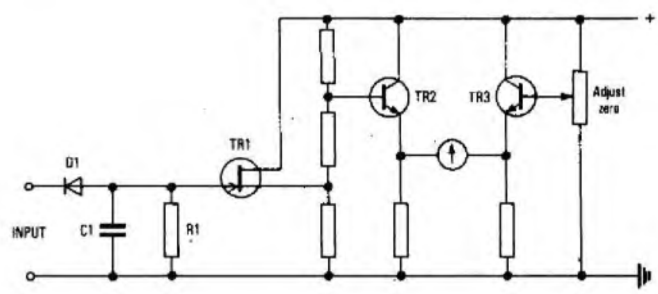
(ii) Describe an input circuit suitable for use with this voltmeter.

(b) Give a brief account of an experiment in which an electronic voltmeter is used to measure power at radio frequencies.

A3 (*a*) (*i*) An electronic AC voltmeter is a sensitive measuring device which can have a high input impedance, making it suitable for measuring voltages without disturbing the electrical conditions in a circuit.

A low-capacity test-probe connects the circuit under test, via a screened lead, to an AC amplifier which provides sensitivity by amplifying at the test frequency. The amplifier feeds a DC meter via a rectifier.

Alternatively, to obtain a very high input impedance, the probe can be connected via a rectifier to the gate of a field-effect transistor, which can feed DC (on a long lead, if necessary) to a balanced transistor pair with a centre-zero meter connected between their emitters. Such a circuit is shown in the sketch. A test voltage is rectified at D1, and charges capacitor C1. This charge is applied to the gate of transistor TR1, causing the source current to reduce and the source voltage to fall. The base current of TR2 is reduced, and its emitter current falls. The circuit conditions of TR3 remain constant, so that the meter deflects in response to the emitter-current imbalance between TR2 and TR3.



(*ii*) The input circuit shown is a field-effect transistor connected as a source-follower. This has an input impedance of several megohms, so that shunt resistor R1 can itself have a high value, say 10 MΩ. Short connexions must be used, especially at high frequencies. This circuit tends to register peak input voltage, and it is usual to calibrate the meter to show RMS voltage for sinusoidal input waveforms.

(*b*) Current in a circuit can be determined by measuring the voltage across a small-value resistance inserted in series with the circuit. (The current is then given by the voltage divided by the resistance.) The voltage across the whole circuit can be measured directly. The power in the circuit is then the current multiplied by the voltage (assuming the power factor approximates to unity).

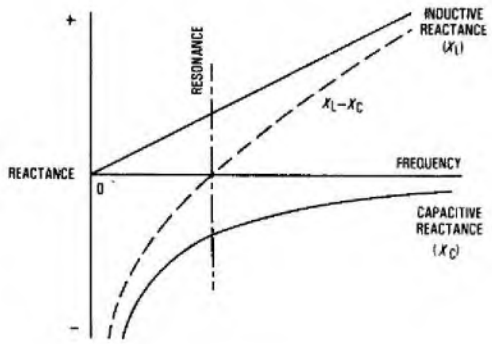
Q4 (a) Explain the meaning of reactance. Why is capacitive reactance regarded as negative?

(b) Sketch on the same axes the reactance/frequency graphs for a capacitor and an inductor, starting from zero frequency. Hence, explain how series resonance can occur.

(c) Deduce an expression for the frequency of resonance of a series circuit.

A4 (*a*) Reactance is the property of a capacitance or inductance which opposes the flow of an alternating current without dissipating any power. Reactance is frequency-dependent. Capacitive reactance is considered negative because the alternating voltage across the component lags by 90° the current flowing through it, whereas inductive reactance is considered positive because the voltage leads the current by 90° (assuming in each case that there is no resistance in the circuit).

(*b*) The graphs are shown in the sketch. Inductive reactance, X_L, is proportional to frequency, so that the graph is a straight line originating at zero frequency. The graph of capacitive reactance, X_C, is a reciprocal curve wholly within the fourth (negative) quadrant. For a capacitor in series with an inductor, the behaviour of the circuit is



explained by adding algebriacally the two reactances, shown by the graph of $X_L - X_C$. By definition, the resonant frequency is the point of zero reactance; that is, where the positive inductive reactance is offset exactly by the negative capacitive reactance.

(*c*) At resonance, therefore, $X_L = X_C$. Also $X_L = 2\pi fL$ and $X_C = 1/2\pi fC$, where f is the frequency, L is the inductance, and C is the capacitance.

Thus $$2\pi fL = \frac{1}{2\pi fC},$$

or $$f^2 = \frac{1}{4\pi^2 LC}.$$

$$\therefore \quad f_{\text{resonance}} = \frac{1}{2\pi\sqrt{(LC)}} \text{ hertz.}$$

Q5 (*a*) *Use a typical B/H curve to show why incremental permeability (AC permeability) usually differs from the static value at the same flux density.*

(*b*) *The inductor core in Fig. 1 has a 1 mm air gap. Calculate the magnetomotive force needed to produce a flux density of 0·60 T in the air gap. The relative permeability (μ_r) of the iron is 500 and there is a 20% loss of flux due to leakage.*

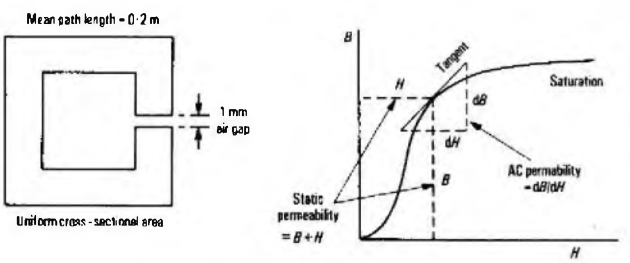


Fig. 1 (*a*)

A5 (*a*) A typical flux-density/field-strength (*B/H*) curve for iron is shown in sketch (*a*). The relative, or *static* permeability is the ratio of *B* to *H*. Clearly, this ratio diminishes as *H* increases. The *incremental* permeability is defined as d*B*/d*H*; that is, the slope of the tangent to the *B/H* curve. This changes markedly as *H* increases, becoming zero at saturation.

(*b*) The flux density in the air gap is 0·6 T. The magnetomotive force (MMF) needed to produce this is given by Bl/μ_0, where l is the length and μ_0 is the permeability of free space.

Thus, $\text{MMF}_{\text{air gap}} = 0{\cdot}6 \times 10^{-3}/4\pi \times 10^{-7} = 477{\cdot}5$ A.

The flux density in the iron is $\frac{5}{4} \times 0{\cdot}6 = 0{\cdot}75$ T.

$\text{MMF}_{\text{iron}} = Bl/\mu_0\mu_r = 0{\cdot}75 \times 0{\cdot}2/4\pi \times 10^{-7} \times 500,$
$= 238{\cdot}7$ A.

Total MMF $= 477{\cdot}5 + 238{\cdot}7 = 716{\cdot}2$ A.

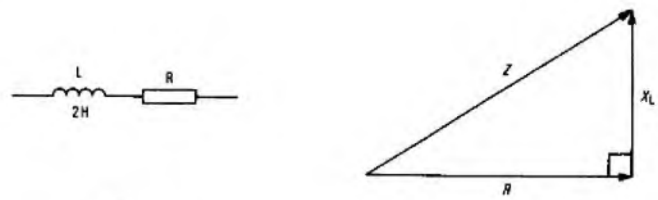
Q6 (*a*) *What is the meaning of the phrase "power factor of a circuit"?*

(*b*) *A 10 V, 50 Hz supply, when connected across a resistive inductor of 2 H, gives a current of 10 mA. By means of a phasor diagram, or otherwise, calculate the resistance of the inductor.*

(*c*) *For the circuit in (b), find*

(*i*) *the power factor, and*
(*ii*) *the power dissipated in the resistance.*

A6 (*a*) The power factor of a circuit is the ratio of the resistance to the impedance.



(*a*) (*b*)

(*b*) The equivalent circuit of the inductor is shown in sketch (*a*). The corresponding impedance triangle is shown in sketch (*b*). Vector *R* represents the unknown resistance, vector X_L represents the inductive reactance, and vector *Z* represents the impedance of the circuit. $X_L = 2\pi fL = 2\pi \times 50 \times 2 = 628{\cdot}3\ \Omega$, where f is the frequency, and L is the inductance.

Z is given by the voltage divided by the current; that is,

$10/10 \times 10^{-3} = 1000\ \Omega.$

Thus $R = \sqrt{(Z^2 - X_L^2)},$
$= \sqrt{(1000^2 - 628{\cdot}3^2)} = 778\ \Omega.$

(*c*) (*i*) Power factor $= 778/1000 = 0{\cdot}778.$

(*ii*) Power = current² × resistance,
$= (10 \times 10^{-3})^2 \times 778$ W $= 77{\cdot}8$ mW.

Q7 (*a*) *Describe an experiment to determine curves relating collector-current/collector-emitter voltage for a common-emitter connected transistor. Give a circuit showing the test equipment. Outline the procedure and sketch typical curves.*

(*b*) *The transistor is to be used as an amplifier with a resistive load.*

(*i*) *Explain the construction of a load line on the family of curves in (a).*
(*ii*) *Explain how the current gain of the amplifier can then be deduced.*

A7 See A4, Telecommunication Principles B 1977, *Supplement*, Vol. 71, p. 56, Oct. 1978, and A4, Telecommunication Principles B 1978, *Supplement*, Vol. 72, p. 17, Apr. 1979.

Q8 (*a*) *Explain, with diagrams, the principle of operation of the field-effect transistor (FET).*

(*b*) *Sketch a circuit of the FET as an amplifier. Why is this type of amplifier proving valuable in telecommunications?*

(*c*) *State any one precaution that must be taken to ensure the reliability of an FET when in use. Give a reason for your answer.*

A8 See A9, Telecommunication Principles B 1976, *Supplement*, Vol. 70, p. 31, Apr. 1977.

Q9 (*a*) *Give briefly, with sketches, the principle of the simple DC generator. Explain the purpose of the commutator.*

(*b*) *Derive an expression in terms of speed and magnetic flux for the half-wave EMF generated.*

(*c*) *State the effect of armature resistance on the terminal voltage when the generator is on load.*

Q10 (*a*) *A relay coil of 1000 Ω resistance operates from a constant 50 V source. When the relay is not operated, the inductance of the coil is 20 H. When it is operated, the inductance is 80 H.*

(*i*) *Explain why the inductance changes when the relay operates.*
(*ii*) *Find the time constant for each circuit condition.*

(*b*) *Suggest a modification of the circuit which would make both time constants equal. Calculate the value of any component needed.*

A10 (*a*) (*i*) Flux is given by magnetomotive force divided by reluctance. Any change in the reluctance of the flux path thus alters the flux produced per unit of current. Further, inductance depends on the amount of flux cutting the turns of the coil, so that inductance alters if the flux changes.

When a relay operates, the air gap between the armature and the pole-face closes, thus greatly reducing the reluctance of the flux path. Hence, the flux increases, and so does the inductance.

(*ii*) The time constant of a series inductive-resistive circuit is *L/R* seconds, where *L* is the inductance and *R* is the resistance.

For the unoperated state, $L/R = 20/1000$ s $= 20$ ms.

For the operated state, $L/R = 80/1000$ s $= 80$ ms.

(*b*) The time constants could be made equal by altering the series resistance in the circuit in the second case. If 3000 Ω were switched into circuit by a contact on the relay itself, the time constant would become 80/4000 s = 20 ms.

Students were expected to answer any 6 questions

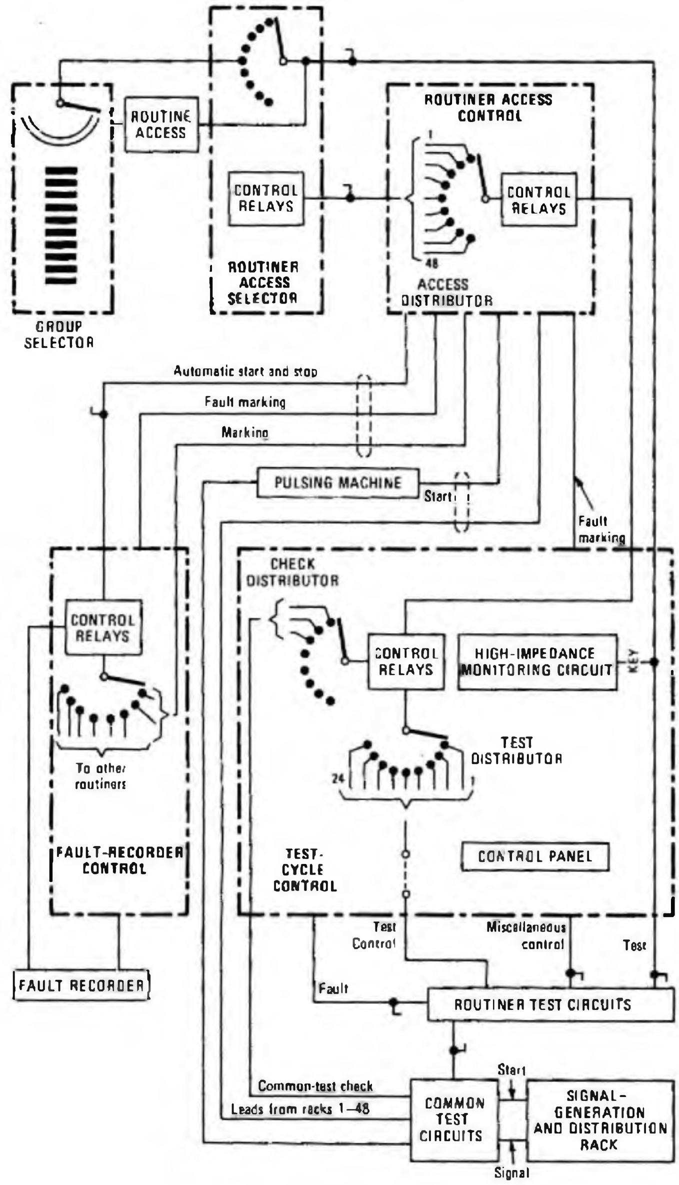
Q1 (*a*) *With the aid of a block diagram, describe the operation of an automatic routiner suitable for testing group-selectors in an electro-mechanical automatic Telex exchange.*

(*b*) *How is access gained to the switch to be tested?*

(*c*) *What factors must be taken into account when deciding whether to provide an automatic routiner or a semi-automatic routine tester?*

A1 (*a*) (*b*) The sketch shows the block diagram of an automatic routiner for group-selectors, together with the access equipment and fault recorder. The routiner consists of a rack of equipment containing the access control, the test-cycle control, the routiner's test circuits and the common test circuits. The access equipment comprises uni-selectors mounted either on the group-selector rack, or on a separate adjacent rack to minimize cabling. The routiner is controlled either manually from a key-and-lamp control panel or automatically from the fault-recorder rack.

The routiner is prepared by operating a key on the control panel. The routiner is normally under the control of the fault-recorder, which starts it at a predetermined time. The access distributor provides connexion to a large number of equipment racks by selecting each group of access uniselectors in turn and ensuring the progression of testing in rack sequence. The distributor can select up to 48 racks of access selectors, stepping over contacts for unequipped racks. The bank contacts of the access uniselectors are cabled to the routiner U-points of the group selector to be tested on the basis of one arc for each test lead. Before testing begins, the routiner checks that all uniselectors are on the home position. The access selector of the selected rack then moves to the first equipped position on the rack and a test is made that each common test circuit is working correctly and is within limits. (Common test circuits are provided for the more complicated or repeated tests such as relay timing.) If these are satisfactory, tests are applied to the group selector using the routiner test circuits, which are connected to the test leads one at a time by the test distributor; each test depends on the successful completion of the previous test. If any of the tests is not satisfactory, or if the routiner has to wait more than a specified time for a test (for example, 3-6 min for a forced-release test), the fault recorder is seized and a docket printed showing the equipment and the nature of the fault. If no fault is found, the access uniselector steps to the next group selector and the tests are repeated. The progress of the tests is shown by lamps on the control panel, which also offers facilities for manual control. The pulsing machine supplies the dial pulses at limiting values to test the operation of the group-selector stepping mechanisms.



(*c*) A semi-automatic routine tester consists of trolley-mounted equipment which has to be placed in position near to the group selectors and plugged-in to rack supplies. The tester has also to be plugged into the group selector under test and started manually. If a fault is detected, the tester stops until the operator has made a note of the fault and re-started the tester. The tester therefore requires constant attendance at what is repetitive and uninteresting work. The tester does, however, provide the facility of repeating tests at leisure, and of testing the switch when the fault has been cleared; for an automatic routiner, this would require the operator to walk between the selector and the control panel.

The tester is relatively inexpensive to provide, does not require rack space nor space for access selectors, and may be stored in a room away from the apparatus floor; also, it does not require permanent wiring between routiner and access and group-selector racks.

The advantages of a routiner are in operator economy, night and weekend testing, increased frequency of testing, testing before the exchange becomes busy, regular tests, tests of greater complexity and the facility of maintenance control by numbered and dated fault dockets. The routiner is more costly to design and purchase and occupies valuable floor space in an exchange. When all factors have been considered, it is normally a question of the number of group selectors an exchange will contain. Above a certain number, an automatic routiner is more economic and effective than a semi-automatic routine tester.

Q2 *For an automatic Telex exchange, describe with the aid of diagrams*

(*a*) *how a called subscriber's line is tested for the engaged condition,*

(*b*) *how the possibility of two switches seizing the same disengaged circuit is minimized, and*

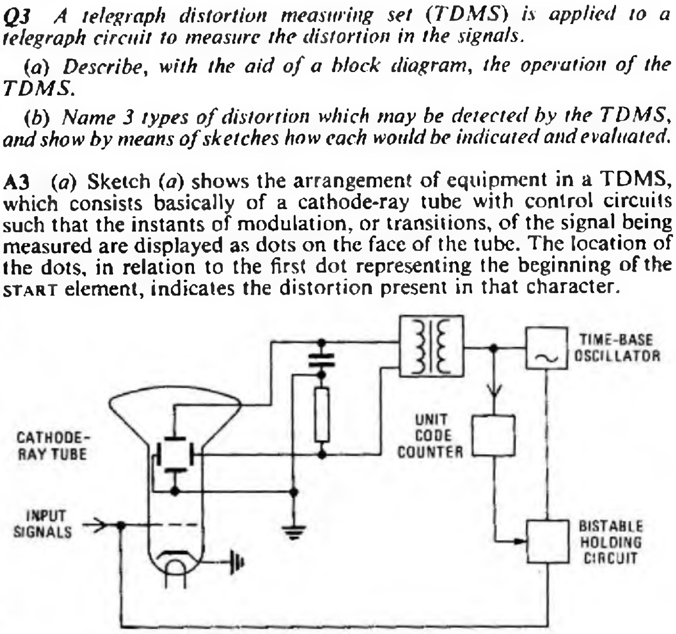
(*c*) *the sequence of the signals which are exchanged with the subscriber's station if the circuit is disengaged.*

Q3 *A telegraph distortion measuring set (TDMS) is applied to a telegraph circuit to measure the distortion in the signals.*

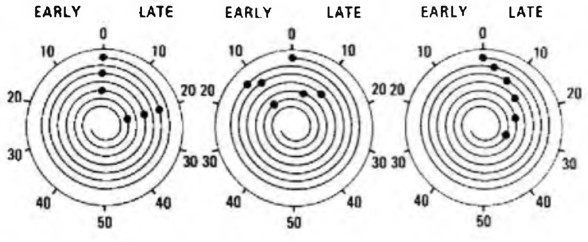
(*a*) *Describe, with the aid of a block diagram, the operation of the TDMS.*

(*b*) *Name 3 types of distortion which may be detected by the TDMS, and show by means of sketches how each would be indicated and evaluated.*

A3 (*a*) Sketch (*a*) shows the arrangement of equipment in a TDMS, which consists basically of a cathode-ray tube with control circuits such that the instants of modulation, or transitions, of the signal being measured are displayed as dots on the face of the tube. The location of the dots, in relation to the first dot representing the beginning of the START element, indicates the distortion present in that character.



(*a*)



(*b*) (*c*) (*d*)

(*b*) (*i*) Bias distortion is characterized by the dots appearing in two groupings on the TDMS display. Sketch (*b*) shows this for the spiral type of display with a START polarity bias.

(*ii*) Fortuitous distortion is characterized by the dots having a random distribution about their ideal positions. This type of distortion is shown in sketch (*c*).

(*iii*) A speed discrepancy between the transmitter and TDMS time-base is shown by a cumulative increase in distortion throughout each character. In the case of the spiral type of display, the mean position of the dots is no longer radial. Sketch (*d*) shows this type of display.

Q4 (*a*) *What modifications and tests must be made to a long-distance speech circuit before it may be used for a multi-channel voice-frequency (MCVF) telegraph system?*

(*b*) *What factors govern the maximum amount of power which may be transmitted to line for each channel of an MCVF system?*

A4 See A3, Telegraphy C 1973, *Supplement*, Vol. 67, p. 78, Jan. 1975, and A3, Telegraphy C 1976, *Supplement*, Vol. 70, p. 65, Oct. 1977.

Q5 (*a*) *Describe 3 types of signalling used for international Telex calls.*

(*b*) *Under what circumstances would each type be used?*

(*c*) *What are the advantages and disadvantages of each type of signalling?*

A5 (*a*) See A7, Telegraphy C 1974, *Supplement*, Vol. 68, p. 70, Oct. 1975.

(*b*) The main problem in Telex interworking between countries concerns the methods of selection and signalling, which differ widely between one national network and another. Some countries use direct dialling, others use register systems; some networks use dial selection while others prefer keyboard selection. Interworking problems have been eased by standardizing all Telex networks within the two basic systems of signalling known as Type-A and Type-B. Within these broad classifications there are variations which must be agreed between the two countries concerned, but a basic requirement of the CCITT Recommendation is that, where two systems using signalling techniques based on the two different standards are to be interconnected, the equipment at the outgoing (calling) end of an international trunk circuit should conform to the signalling requirements of the system at the incoming end.

Type-C signalling is used throughout the transit network, where international calls may be routed through several countries. The calling and selection signals are transmitted as a single group of signal combinations to each transit exchange on the connexion in turn, and are extended to each succeeding transit exchange with the minimum delay until the final transit exchange is reached.

(*c*) The main differences between Type-A and Type-B signalling are on the backward signalling path during selection, and apply to the *call-confirmation* and *call-connect* signals. The advantage of Type-B working is that, on bothway circuits, locking-up caused by line interruptions which simulate call signals are prevented by the operation of the signalling system, rather than by modifications to the switching equipment. In addition, if a connexion is partially released by a line interruption, Type-B signalling prevents teleprinter signals operating Strowger selectors and switching the call to a different destination. For these reasons, dial-selection systems normally use Type-B signalling. To overcome these difficulties with Type-A systems, modifications are required within the switching equipment, and these facilities can more easily be incorporated in a register system. Hence, Type-A has been more generally adopted for keyboard-selection systems, as a register is necessary to operate the step-by-step switches.

Type-C signalling has the advantage of being used for all transit traffic irrespective of the country of origin or of destination; no code-converters are required. The signals consist of teleprinter characters; this overcomes the problem of sending dial pulses over synchronous multiplexed radio and cable systems. Using Type-C signalling, calls are connected faster, with a reduction in expensive holding time on high-cost circuits; 5-unit signals can be stored with ease, if required, and register-holding times are reduced.

Q6 (*a*) *Describe the operation of a semi-automatic message-relay system*

(*i*) *for a single-address message, and*
(*ii*) *for a multi-address message.*

(*b*) *What safeguards are provided for messages which may be lost or mutilated during transmission?*

(*c*) *What are the advantages and disadvantages of replacing the system with fully automatic equipment?*

Q7 *Describe how a Telex subscriber on an automatic exchange is charged for*

(*a*) *a local call,*

(*b*) *a manually-connected international radio call, and*

(*c*) *an international call using automatic ticketing equipment.*

A7 (*a*) See A5, Telegraphy C 1978, *Supplement*, Vol. 72, p. 45, July 1979.

(*b*) See A8, Telegraphy C 1973, *Supplement*, Vol. 67, p. 80, Jan. 1975.

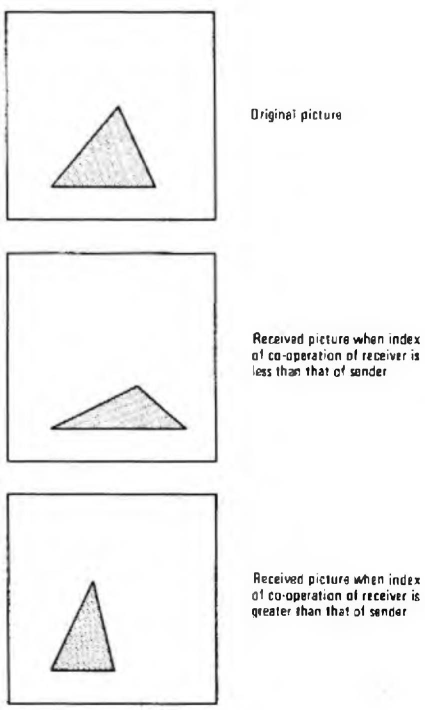
(*c*) See A10, Telegraphy C 1976, *Supplement*, Vol. 70, p. 68, Oct. 1977.

Q8 (*a*) *What is meant by the term "index of co-operation" as applied to facsimile telegraphy?*

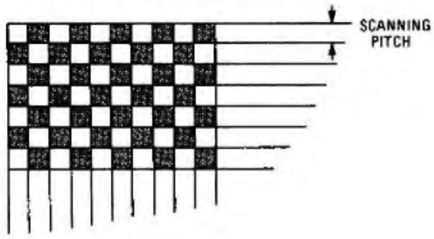
(*b*) *What is the effect of receiving a facsimile document on a machine with an index of co-operation different from that of the transmitter?*

(*c*) *Derive an expression, in terms of the index of co-operation (M), for the bandwidth required for the transmission of facsimile signals.*

A8 (*a*) (*b*) To build a received picture as a faithful representation of a transmitted picture using facsimile telegraphy, it is essential that the transmitter and receiver operate in synchronism. The first requirement is that the drum speeds must be the same, otherwise the received picture will demonstrate a skew. The second requirement is that the two machines be in phase (that is, the edge of the paper must be in the same position relative to the light spot in both machines), otherwise the received picture will be split. The third characteristic is the index



(*a*)



(*b*)

of co-operation. This is the ratio of the length to the breadth of the picture and, whatever the relative size of the received picture, this ratio must be the same at the transmitter and receiver if the received picture is to have the same proportions as the original. Should the indices of co-operation differ, the received picture will either be broader and flatter or taller and thinner than the picture being transmitted, as illustrated in sketch (*a*). If D is the drum diameter, P the helix pitch and F the scanning density in lines per unit length, the index of co-operation is given by

$$M = \frac{D}{P} = DF.$$

(*c*) The output of the photoelectric cell at the transmitter consists of alternating signals superimposed on a DC signal, the frequency being determined by the speed of scanning and the definition required. The definition in one direction is fixed by the scanning pitch and, as it is assumed that the definition is required to be the same in the other direction, the subject may be considered to consist of squares of side equal to the scanning pitch; this is shown in sketch (*b*). The most onerous condition arises when alternate squares are black and white, the number of black and white elements per line being $\pi D/P$. Hence, the number of squares scanned per second is $\pi DN/P$, where N is the speed of the drum in revolutions/s. If one black and one white square are considered to constitute one cycle, the maximum frequency transmitted

$$= \frac{\pi DN}{2P} \text{ hertz.}$$

But $M = D/P$. Therefore the maximum frequency

$$= \frac{\pi MN}{2} \text{ hertz.}$$

Q9 A DC teleprinter signal is to be transmitted over a cable circuit.

(*a*) *Explain the effects of the primary coefficients of the line on the signal as transmitted and received.*

(*b*) *Describe the operation of equipment which would be used*

(*i*) *if the received signal were too weak to operate the teleprinter receive magnet, and*

(*ii*) *if the distortion were such as to cause mutilation of the signal.*

A9 (*a*) The primary coefficients of a circuit in an underground cable are capacitance, resistance, inductance and leakance. Of these, the inductance and leakance of the cable have negligible effect on a telegraph signal. The cable may be represented by the network shown in sketch (*a*), with resistance (R) in the line and capacitance (C) to earth. The resistance attenuates the signal according to Ohm's law so that, if resistance only were present, the received and transmitted currents (I_r and I_s respectively) would be the same; that is,

$$I_r = I_s = V/R,$$

where V is the sending voltage.

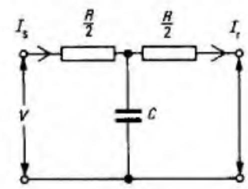
If the resistance were of high value, the current would be insufficient to operate the receiver and the resistance would have to be decreased, the voltage increased or a repeating relay placed in the line to apply a refreshed signal. The effect of line capacitance is shown in sketch (*b*). The capacitance initially acts as a short-circuit so that the initial transmitted current is high and is equal to

$$\frac{V}{R/2} = \frac{2V}{R}.$$

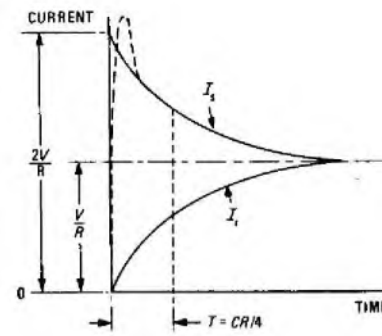
The transmitted current gradually charges the capacitor which eventually acts as a disconnexion, and the current reduces to V/R. At the receiver, the current rises exponentially to a value of about two-thirds of its final value in a time, T, equal to

$$C \times \frac{R/2 \times R/2}{R/2 + R/2} = C \times \frac{R^2/4}{R} = \frac{CR}{4}.$$

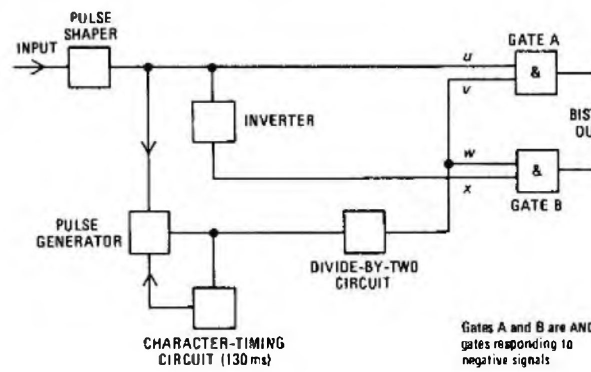
The current increases at a rate of two-thirds of the remaining value per time interval T until, after a time of about $5T$, the received current is equal to the transmitted current, V/R.



(*a*)



(*b*)



(*c*)

(*b*) (*i*) If the received signal were too weak to operate the teleprinter, a repeating relay would be used.

(*ii*) Sketch (*c*) shows a block diagram of a regenerative repeater. When the line is idle, the input is at negative potential, so that input *u* to gate A is negative, and input *x* to gate B, fed via the inverter, is positive. When a telegraph signal is received, the START element, which is of positive polarity, is applied to the pulse generator, and this generates reversals at a precise frequency of 100 Hz. The divide-by-two circuit reduces the reversals to accurate narrow pulses of negative potential at 20 ms intervals. The pulses are applied to gates A and B at inputs *v* and *w* 10 ms after the START element has been detected, and are used to examine the incoming signal at the theoretical centre of each element. If the element is of negative potential (MARK signal), gate A conducts, as inputs *u* and *v* are at negative potential when the timing pulse is applied, and gate B does not conduct, as input *x* is at positive potential and input *w* is at negative potential. The bistable output transmits negative potential to line. After 20 ms, the input is again examined by the timing pulse and, if the line potential is positive (SPACE signal), gate B conducts, as inputs *w* and *x* are negative, and gate A does not conduct. The bistable output then transmits a positive signal for the ensuing 20 ms.

Therefore, incoming signals are repeated to the outgoing circuit and, as the duration of the timing pulses is very short (a few microseconds) and the pulse generator is stable, the incoming signals may bear almost 50% early or late distortion and still be regenerated as perfect signals.

Q10 A Telex message is to be transmitted over a high-frequency radio circuit.

(*a*) *What factors could cause the signals to be mutilated during transmission?*

(*b*) *Explain what arrangements could be made at the radio transmitter and receiver to minimize the distortion and mutilation.*

(*c*) *Describe briefly a system which will virtually eliminate mutilation.*

A10 See A5, Telegraphy C 1977, *Supplement*, Vol. 71, p. 66, Oct. 1978.

TECHNICIAN EDUCATION COUNCIL

Certificate Programme in Telecommunications

Sets of model questions and answers for TEC units are given below. They have been designed following analysis of assessment test papers actually set during the 1978–79 session by a number of colleges all over the country. The model questions and answers reflect the types and standard of question set and answer expected, and include the styles of both in-course and end-of-unit assessments.

The model questions and answers therefore illustrate the assessment procedures that students will encounter, and are useful as practice material for the skills learned during the course.

The use of calculators is permitted except where otherwise indicated.

Representative time limits or proportion of marks are shown for each question (or group of questions), and care has been taken to give model answers that reflect these limits. Where additional text is given for educational purposes, it is shown within square brackets [ ] to distinguish it from the information expected of students under examination conditions.

We would like to emphasize that, because the model questions are based on work at a number of colleges, they are not representative of questions set by any particular college.

As a general rule, questions are given in italic type and answers in upright type. Answers are sometimes shown in bold upright type; this is because, for some objective questions, it is convenient to place the questions and answers side by side, and bold type enhances the distinction in such cases. Where possible, answers have been positioned such that they may be covered up if desired.

MATHEMATICS 1 1978–79

Students are advised to read the notes above

Q1–15 are multiple-choice objective questions. Students would normally be expected to tick or ring the correct answer. In the model answers below, the designations of the correct answer are printed to the right of the question in such a way that students may, if they wish, cover them up in order to check their answers after working through the questions. Q1–15 should be completed within 30 min.

Q1 *Which of the following fractions is greatest?*

(*a*) $\frac{5}{8}$ (*b*) $\frac{9}{16}$ (*c*) $\frac{23}{32}$ (*d*) $\frac{37}{64}$ **(c)**

Q2 *The value of* $\sqrt[3]{\left(\frac{3^3 \times 3^{12}}{3^9}\right)}$ *is*

(*a*) 3^3 (*b*) 3^2 (*c*) 3 (*d*) $3^{4/3}$ **(b)**

Q3 *Which of the following expresses 0·65 as a vulgar fraction in its lowest terms?*

(*a*) $\frac{650}{1000}$ (*b*) $\frac{65}{100}$ (*c*) $\frac{13}{20}$ **(c)**

Q4 *If 2·001504 is to be corrected to 3 decimal places, the answer is*

(*a*) 2·001 (*b*) 2·002
(*c*) 2·00150 (*d*) 2·00 **(b)**

Q5 $\log\left(\frac{a}{b}\right)$ *can be expressed as*

(*a*) $\log(a - b)$ (*b*) $\log a - \log b$
(*c*) $\log 10^{(a-b)}$ (*d*) $\log 10^{a/b}$ **(b)**

Q6 *Which of the following is greater then* $\frac{5}{8}$*?*

(*a*) 0·6 (*b*) 0·62 (*c*) $\frac{31}{50}$ (*d*) 0·6275 **(d)**

Q7 *The binary number 1111 expressed in denary form is*

(*a*) 30 (*b*) 15 (*c*) 5 (*d*) 35 **(b)**

Q8 *The simplest form of* $1\frac{1}{5} + 6\frac{2}{15} - 3\frac{1}{6}$ *is*

(*a*) $4\frac{1}{30}$ (*b*) $4\frac{1}{6}$ (*c*) 4 (*d*) $10\frac{1}{2}$ **(b)**

Q9 *3·5 expressed as a percentage of 70 is*

(*a*) 5% (*b*) 10% (*c*) 20% (*d*) 7% **(a)**

Q10 *The highest common factor (HCF) of 420 and 450 is*

(*a*) 5 (*b*) 6 (*c*) 10 (*d*) 30 **(d)**

Q11 *The lowest common multiple (LCM) of 2, 8, 12 and 18 is*

(*a*) 36 (*b*) 216 (*c*) 72 (*d*) 144 **(c)**

Q12 *0·0100513 expressed to 3 significant figures is*

(*a*) 0·01 (*b*) 0·0101 (*c*) 0·01005 (*d*) 0·010 **(b)**

Q13 *The denary number 17 expressed in binary form is*

(*a*) 10 001 (*b*) 1001 (*c*) 10 010 (*d*) 10 011 **(a)**

Q14 $27^{-2/3}$ *is equal to*

(*a*) $\frac{1}{18}$ (*b*) $\frac{1}{9}$ (*c*) 9 (*d*) −18 **(b)**

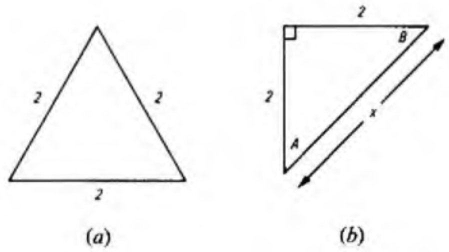
Q15 $2[5 - 3(5 - 2) + 4\{2(3 + 1) - 7\}]$ *is equal to*

(*a*) 0 (*b*) −24 (*c*) 42 (*d*) −12 **(a)**

Q16–35 are constructed-response objective questions. Again, the answers are so arranged that they may be covered up. This set of questions should be answered in 40 min.

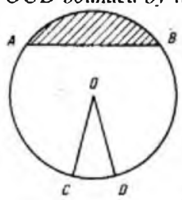
Q16 *In triangle (a) give the radian measure of each angle and in triangle (b) determine x and the angles A and B in degrees*

(a) each angle = π/3 rad
(b) x = 2·828
∠A = ∠B = 45°



Q17 *In the circle with centre O, name (a) AB: (b) the shaded part above AB, and (c) the portion OCD bounded by the radii and arc CD*

(a) chord
(b) minor segment
(c) sector



Q18 *If a = 1, b = 2, and c = 3, then* $3a + 5b(c^2 - 7) =$ **23**

Q19 *If* $x - y = 1$ *and* $x^2 - y^2 = 5$, *find x and y* **x = 3**, **y = 2**

Q20 *Using tables or a calculator find*
(*a*) $\log_{10} 495{\cdot}3$ (*b*) $\log_{10} 0{\cdot}00513$

(a) 2·695
(b) $\bar{3}$**·7101**

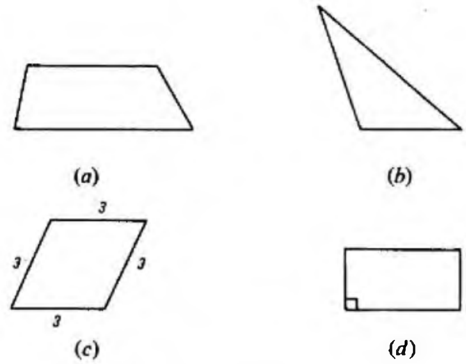
Q21 *If* $i = \frac{V}{\sqrt{(R^2 + X^2)}}$, *rearrange the formula to give an equation for X and determine X when V = 100, i = 5 and R = 16.*

$\mathbf{X} = \sqrt{\left(\frac{V^2}{i^2} - R^2\right)}$

X = 12

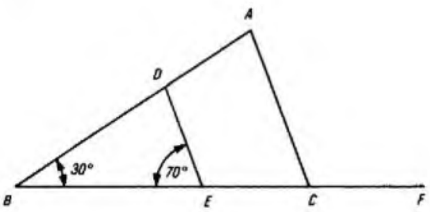
Q22 *Name each figure shown below.*

(a) trapezium
(b) scalene triangle (also an obtuse-angled triangle)
(c) rhombus
(d) rectangle



Q23 *In the figure shown, in which AC and DE are parallel, state the values of the following angles* (*a*) *BDE* (*b*) *DAC* (*c*) *DEC* (*d*) *ACE* (*e*) *ACF*

(a) 80°
(b) 80°
(c) 110°
(d) 70°
(e) 110°



Q24 *Using logarithmic tables, evaluate*

$$\frac{45\cdot 1}{\sqrt{97\cdot 22}}$$

4·574

Q25 *Factorize* $2xy + 2ay - bx - ab$ **(x + a)(2y − b)**

Q26 *Simplify the expression*

$$\frac{9a^3b^{-1}c^{-2}}{(3a)^2b^{-2}c^{-3}}$$

abc

Q27 *Add the binary numbers 1000 and 100 and give the answer in denary form* **12**

Q28 *The denary number 31 expressed in binary form is* **11 111**

Q29 *The binary number 10 101 expressed in denary form is* **21**

Q30 *A rectangular garden is drawn to a scale of 1 cm representing 1 m. If the sides of the scale drawing measure 270 mm and 720 mm, what is the area of the garden in hectares? (1 hectare =* 10^4 *m*^{2}*)* **0·1944**

Q31 *A pie chart is to be divided into percentage sectors of 10, 20, 30 and 40%. What is the angle in degrees of the sector representing 20%?* **72°**

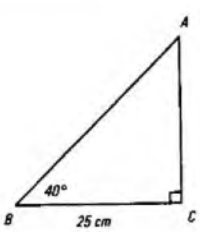
Q32 *An electric motor rotates at 3000 rev/min. What is its angular speed in radians/second?* **100π = 314·16**

Q33 *If* $\sin A = 0\cdot 312$, $\cos B = 0\cdot 312$ *and* $\tan C = 0\cdot 312$, *what are the acute angles A, B and C?*

A = 18° 11′
B = 71° 49′
C = 17° 20′

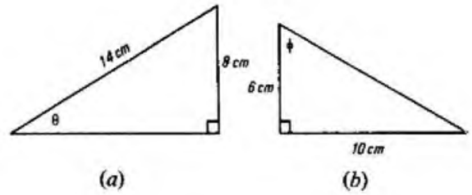
Q34 *In the triangle shown in the sketch, determine AB and AC*

AB = 32·64 cm
AC = 20·98 cm



Q35 *Find the angles θ and ϕ shown in sketches (a) and (b)*

θ = 34° 51′
ϕ = 59° 2′



Q36–45 are questions requiring longer answers; the total time for this set of questions is 3 hours. The percentage allocation of marks is shown at the end of each question.

Q36 (*a*) *Express the following in standard form*

(*i*) *2 430 000, and* (*ii*) *0·00569*

(*b*) *With the aid of tables, evaluate the following*

(*i*) $\dfrac{7\cdot 31^2 \times 0\cdot 000541}{\sqrt{0\cdot 002931}}$, *and*

(*ii*) $5\cdot 1\{(3\cdot 6 \times 71)^2 - 2\sqrt{69\cdot 12}\}$ (*10%*)

A36 (*a*) (*i*) $\underline{2\cdot 43 \times 10^6}$ (*ii*) $\underline{5\cdot 69 \times 10^{-3}}$

(*b*) (*i*)

No.	log
7·31	0·8639 (×2)
$7\cdot 31^2$	1·7278
0·000541	$\bar{4}\cdot 7332$ (+)
Numerator	$\bar{2}\cdot 4610$
Denominator	$\bar{2}\cdot 7335$ (−)
0·5339	$\bar{1}\cdot 7275$

No.	log
0·002931	$\bar{3}\cdot 4670$ (÷2)
$\sqrt{0\cdot 002931}$	$\bar{2}\cdot 7335$

Answer = <u>05339.</u>

(*ii*)

No.	log
3·6	0·5563
71	1·8513 (+)
3·6 × 71	2·4076 (×2)
$(3\cdot 6 \times 71)^2$ = 65340	4·8152

No.	log
69·12	1·8396 (÷2)
$\sqrt{69\cdot 12}$	0·9198
2	0·3010 (+)
16·63	1·2208

Hence $5\cdot 1\{(3\cdot 6 \times 71)^2 - 2\sqrt{69\cdot 12}\}$,
$= 5\cdot 1(65340 - 16\cdot 6)$,
$= 5\cdot 1 \times 65323\cdot 4$,
$= \underline{333\ 200}$

No	log
5·1	0·7076
65323	4·8151 (+)
333 200	5·5227

Q37 (*a*) *Find x from the following equation*

$$\frac{4x - 3}{2} - 3 = \frac{5}{6}(2 - 5x).$$

(*b*) *Solve the simultaneous equations*

$$5x - 4y = 28, \text{ and}$$
$$14x + 8y = 40. \qquad (8\%)$$

A37 (*a*) Multiplying throughout by 6 gives

$$3(4x - 3) - 18 = 5(2 - 5x).$$
$$\therefore \quad 12x - 9 - 18 = 10 - 25x.$$
$$\therefore \quad 37x = 10 + 27 = 37.$$
$$\therefore \quad \underline{x = 1.}$$

(*b*)

$$5x - 4y = 28 \quad \ldots\ldots (1)$$
$$14x + 8y = 40 \quad \ldots\ldots (2)$$

Divide equation (2) by 2.

$$7x + 4y = 20 \quad \ldots\ldots (3)$$

Add equations (1) and (3).

$$12x = 48.$$
$$x = 4.$$

Substitute for x in equation (3).

$$28 + 4y = 20.$$
$$\therefore \quad 4y = -8.$$
$$\therefore \quad y = -2.$$

Thus, $\underline{x = 4}$ and $\underline{y = -2.}$

Q38 (*a*) *Plot the graph of* $3x - 5y = 2$ *over the range of* $x = -3$ *to* $x = +3$.

(*b*) *From the graph, state the value of x when y = 1 and the value of y* *when $x = -1{\cdot}5$.*

(*c*) *What is the gradient of the graph?* (*10%*)

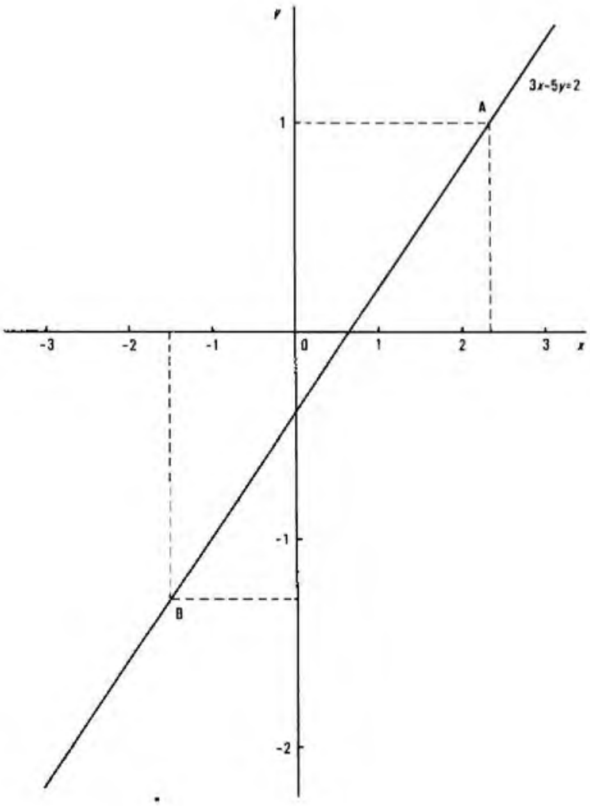
A38 (*a*)

$$3x - 5y = 2.$$
$$\therefore \quad 5y = 3x - 2.$$
$$\therefore \quad y = \frac{3x - 2}{5}.$$

The values of x and y required to plot the graph are tabulated.

x	-3	-2	-1	0	1	2	3
$3x - 2$	-11	-8	-5	-2	1	4	7
y	$-2{\cdot}2$	$-1{\cdot}6$	-1	$-0{\cdot}4$	$0{\cdot}2$	$0{\cdot}8$	$1{\cdot}4$

The graph is shown in the sketch.



(*b*) When $y = 1$, $\underline{x = 2{\cdot}34}$ (point A on the graph)

[*Note:* the accurate value of x is $2{\cdot}3$]

When $x = -1{\cdot}5$, $\underline{y = -1{\cdot}3}$ (point B on the graph).

(*c*) The gradient is $\underline{0{\cdot}6}$.

Q39 (*a*) (*i*) *Express the binary numbers 100, 1000, 10 000 and 100 000 in denary form.*

(*ii*) *Express the denary numbers 5, 10, 50 and 100 in binary form.*

(*b*) (*i*) *Add the binary numbers 1010 and 1101 and express the answer in both binary and denary form.*

(*ii*) *What percentage of the binary number 1001 is the binary number 101?* (*8%*)

A39 (*a*) (*i*)

$$100 = 2^2 = \underline{4}.$$
$$1\,000 = 2^3 = \underline{8}.$$
$$10\,000 = 2^4 = \underline{16}.$$
$$100\,000 = 2^5 = \underline{32}.$$

(*ii*)

$$5 = 4 + 1 = 2^2 + 2^0 = \underline{101}.$$
$$10 = 8 + 2 = 2^3 + 2^1 = \underline{1010}$$
$$50 = 32 + 16 + 2 = 2^5 + 2^4 + 2^1 = \underline{110\,010}.$$
$$100 = 64 + 32 + 4 = 2^6 + 2^5 + 2^2 = \underline{1\,100\,100}.$$

(*b*) (*i*)

$$\begin{array}{r} 1\,010 \\ 1\,101\ (+) \\ \hline \underline{10\,111} \end{array}$$

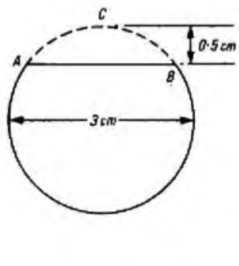
$$10\,111 = 2^4 + 2^2 + 2^1 + 2^0,$$
$$= 16 + 4 + 2 + 1,$$
$$= \underline{23}.$$

(*ii*)

$$101 = 2^2 + 2^0 = 4 + 1 = 5.$$
$$1001 = 2^3 + 2^0 = 8 + 1 = 9.$$
$$\therefore \quad \text{percentage} = \tfrac{5}{9} \times 100 = \underline{55{\cdot}5\%}.$$

Q40 (*a*) *The shaft of a motor running at constant speed rotates through 60° in 3⅓ ms. What is the speed in revolutions/minute?*

(*b*) *The figure shows the end of a rod on which a flat has been formed. The diameter of the rod is 3 cm and the depth of the flat is 0·5 cm. What is the width (AB) of the flat? Given that the area of a sector of a circle is given by $\frac{1}{2}r^2\theta$ (θ in radians), determine the area, in square millimeters, of the minor segment ACB.* (*12%*)

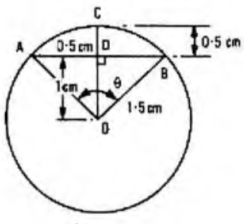


A40 (*a*) The shaft rotates through 60° or $\pi/3$ rad in $0{\cdot}00\dot{3}$ s and, hence, rotates through 180° or π rad in $0{\cdot}01$ s.

Therefore, one revolution or 2π rad takes $0{\cdot}02$ s to complete.

Hence, the speed of the shaft $= 50$ rev/s,
$= \underline{3000 \text{ rev/min.}}$

(*b*) In the sketch, OD is the perpendicular from the centre of the circle (O) on to AB. Since triangles ADO and DBO are congruent, AD = DB.



$$OC = 1\tfrac{1}{2} \text{ cm (being a radius).}$$
$$CD = 0{\cdot}5 \text{ cm.}$$
$$\therefore \quad OD = 1 \text{ cm.}$$
$$AO^2 = AD^2 + DO^2 \text{ (by Pythagoras' theorem).}$$
$$\therefore \quad AD^2 = (\tfrac{3}{2})^2 - 1^2,$$
$$= 1{\cdot}25.$$
$$\therefore \quad AD = \underline{1{\cdot}118 \text{ cm.}}$$

Therefore, the width of the flat AB $= \underline{2{\cdot}236 \text{ cm.}}$

Area of sector ACBO $= \frac{1}{2}r^2\theta$,
$= \frac{1}{2}(\frac{3}{2})^2 \times \theta$.

In triangle ADO:

$$\tan\frac{\theta}{2} = \frac{AD}{DO} = \frac{1{\cdot}118}{1} = 1{\cdot}118.$$
$$\therefore \quad \frac{\theta}{2} = 48°\,11' \text{ or } 0{\cdot}841 \text{ rad.}$$
$$\therefore \quad \theta = 1{\cdot}682 \text{ rad.}$$

Therefore, the area of sector ACBO $= \frac{1}{2} \times \frac{9}{4} \times 1{\cdot}682$,
$= 1{\cdot}892 \text{ cm}^2$.

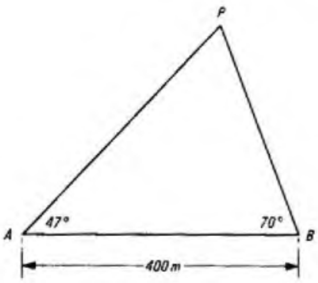
Area of triangle ABO $= \frac{1}{2} \times$ AB $\times$ OD,
$= 1{\cdot}118 \times 1$
$= 1{\cdot}118 \text{ cm}^2$

Therefore the area of the minor segment ACB
= area of sector ACBO − area of triangle ABO,
= 1·892 − 1·118,
= 0·774 cm² = 77·4 mm².

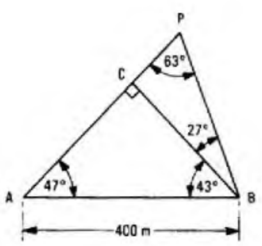
Q41 In a field survey, a pole at point P is observed from 2 fixed points A and B, which are 400 m apart, and angles of observation with respect to AB are as shown in the figure.

(a) Find the distance of P from A and from B, correct to 3 significant figures.

(b) Find the area of triangle ABP in square kilometres. (12%)



A41 In the triangle APB shown in the sketch, BC is the perpendicular from B to AP.



(a) In triangle ABC, ∠ACB = 90° and ∠CAB = 47°.

∴ ∠ABC = 180° − (90° + 47°) = 43°.

∠CBP = 70° − 43° = 27°.

In triangle ABC,

$$\frac{AC}{AB} = \cos 47°.$$

∴ AC = 400 × 0·68199 = 272·8 m.

Also, $$\frac{BC}{AB} = \sin 47°.$$

∴ BC = 400 × 0·73135 = 292·5 m.

In triangle BCP, $$\frac{CP}{BC} = \tan 27°.$$

∴ CP = 292·5 × 0·50953 = 149·0 m.

Also, $$\frac{CP}{BP} = \sin 27°.$$

∴ $$BP = \frac{149{\cdot}0}{0{\cdot}45399} = 328{\cdot}2 \text{ m}.$$

Hence, AP = AC + CP,
= 272·8 + 149·0 = 421·8 m.
= 422 m correct o 3 significant figures,

and BP = 328 m correct to 3 significant figures.

(b) The area of triangle APB is given by

½ × base × perpendicular height,
= ½ × AP × BC,
= ½ × 421·8 × 292·5,
= 61 688 m²,
= 0·0617 km².

Q42 In testing a batch of electrical heating elements, each nominally rated at 1 kW, the following results were obtained.

Actual dissipation (kW)	*0·941–0·96*	*0·961–0·98*	*0·981–1·00*	*1·001–1·02*	*1·021–1·04*	*1·041–1·06*
Number of elements	*3*	*19*	*41*	*37*	*18*	*2*

(a) Determine the relative frequencies of the dissipation variate as percentages and also determine the approximate mean dissipation of the whole batch.

(b) Represent the data in the form of a fully labelled histogram (10%)

A42 (a) The total number of elements in the batch is

3 + 19 + 41 + 37 + 18 + 2,
= 120.

The frequency of occurrence of each class is derived in the table below, both in fractional form and as a percentage.

Class (x)	0·941–0·96	0·961–0·98	0·981–1·00	1·001–1·02	1·021–1·04	1·041–1·06
Number in class (f)	3	19	41	37	18	2
Relative frequency	0·025	0·158	0·342	0·308	0·15	0·017
Relative frequency %	2·5	15·8	34·2	30·8	15	1·7

If $n = 120$ denotes the total number of elements, the approximate mean dissipation is given by

$$\frac{\Sigma (x \times f)}{n}.$$

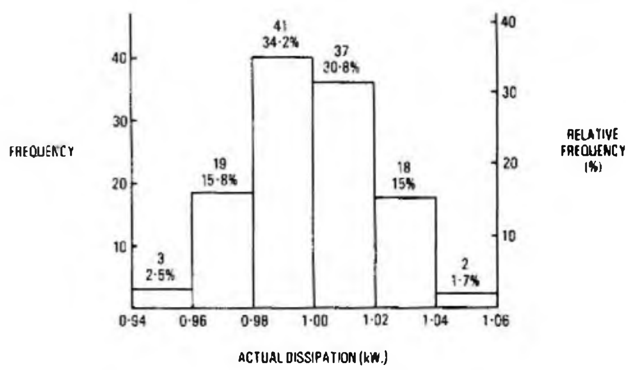
[*Tutorial note:* Since each class covers a range of dissipation values equal to 0·02 kW, it is necessary to take the mid-point of each class as the value of x.]

$$\frac{\Sigma(x\times f)}{n} = \frac{(0{\cdot}95\times3)+(0{\cdot}97\times19)+\ldots+(1{\cdot}03\times18)+(1{\cdot}05\times2)}{120},$$

$$= \frac{2{\cdot}85 + 18{\cdot}43 + 40{\cdot}59 + 37{\cdot}37 + 18{\cdot}54 + 2{\cdot}10}{120},$$

$$= \frac{119{\cdot}88}{120} = 0{\cdot}999 \text{ kW}.$$

(b) The histogram is shown in the sketch.



Q43 (a) A cylinder and a cone have equal volumes of 3600 cm³. If the height of the cone is 29 cm and the height of the cylinder is three times the radius of the cone, determine the radius of the cylinder to one decimal place.

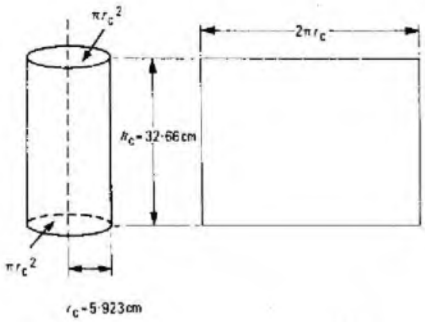
(b) Assuming the cylinder in part (a) to be totally enclosed, what is its total surface area? (10%)

A43 (a) The volume of a cone = $\frac{1}{3}\pi r^2 h$, where r is its base radius and h is its height.

∴ $3600 = \frac{1}{3}\pi r^2 \times 29.$

∴ $$r^2 = \frac{10\,800}{\pi \times 29} = 118{\cdot}54.$$

∴ $r = 10{\cdot}888$ cm.



(*a*) (*b*)

Hence, the height of the cylinder (h_c)

$= 3r = 32 \cdot 664$ cm.

But, the volume of a cylinder

$= \pi r_c^2 h_c$, where r_c is the radius of the cylinder.

$\therefore \quad r_c^2 = \dfrac{3600}{\pi \times 32 \cdot 664}$.

$= 35 \cdot 082$.

$\therefore \quad r_c = 5 \cdot 923$ cm.

Therefore, the radius of the cylinder is 5·9 cm to 1 decimal place.

(*b*) The total surface area of the cylinder comprises the areas of its 2 circular end-faces and the curved area of height h_c. This is shown in sketches (*a*) and (*b*), the latter being the development of the curved surface area.

The total surface area of the cylinder

$= 2 \times$ area of end-face + curved surface area,

$= 2\pi r_c^2 + 2\pi r_c h_c$,

$= 2\pi(5 \cdot 923^2 + 5 \cdot 923 \times 32 \cdot 664)$,

$= 1436\ \text{cm}^2$.

Q44 (*a*) *Find x in each of the following equations:*

(*i*) $10^x = 59$,
(*ii*) $3 \log x = 0 \cdot 927$, *and*
(*iii*) $x \log 3 = \log 9$.

(*b*) *Find an approximate value of the following expression, without use of tables or a calculator:*

$$\frac{79 \cdot 2\pi^2 \sin 29^\circ}{0 \cdot 608 \times 21\,300}.$$

(*c*) *Simplify the following, giving answers with positive indices only:*

(*i*) $\sqrt[3]{\left(\dfrac{8p^6}{q^{-6}}\right)}$ and (*ii*) $(4x^2y^{-3})^2$. (*10%*)

A44 (*a*) (*i*) $10^x = 59$.

Taking logarithms gives

$x \log 10 = \log 59$.

$\therefore \quad x = 1 \cdot 77085$.

(*ii*) $3 \log x = 0 \cdot 927$.

$\therefore \quad \log x = 0 \cdot 309$.

$\therefore \quad x = 2 \cdot 037$.

(*iii*) $x \log 3 = \log 9$.

$x = \dfrac{\log 3^2}{\log 3}$,

$= \dfrac{2 \log 3}{\log 3}$,

$= 2$.

(*b*) $\dfrac{79 \cdot 2\pi^2 \sin 29^\circ}{0 \cdot 608 \times 21\,300} \approx \dfrac{80 \times 10 \times \sin 30^\circ}{0 \cdot 6 \times 20\,000}$,

$= \dfrac{8 \times \frac{1}{2}}{\frac{3}{5} \times 200}$,

$= \dfrac{20}{600}$,

$= \dfrac{1}{30}$ or $0 \cdot 03$.

(*c*) (*i*) $\sqrt[3]{\left(\dfrac{8p^6}{q^{-6}}\right)} = 2 \times \sqrt[3]{(p^6q^6)} = 2p^2q^2$.

(*ii*) $(4x^2y^{-3})^2 = 16\left(\dfrac{x^2}{y^3}\right)^2 = \dfrac{16x^4}{y^6}$.

Q45 (*a*) *Draw accurately the graphs of* $y = 3 \sin \theta$ *and* $y = 4 \cos \theta$ *over one complete cycle from* $\theta = 0^\circ$ *to* $\theta = 360^\circ$.

(*b*) (*i*) *State, from your graphs, the solution of the equation*

$$3 \sin \theta - 4 \cos \theta = 0.$$

(*ii*) *Show, for values of θ of 24° and 69°, that the relationship*

$\cos \theta = \sin (90^\circ - \theta)$ *is true.* (*10%*)

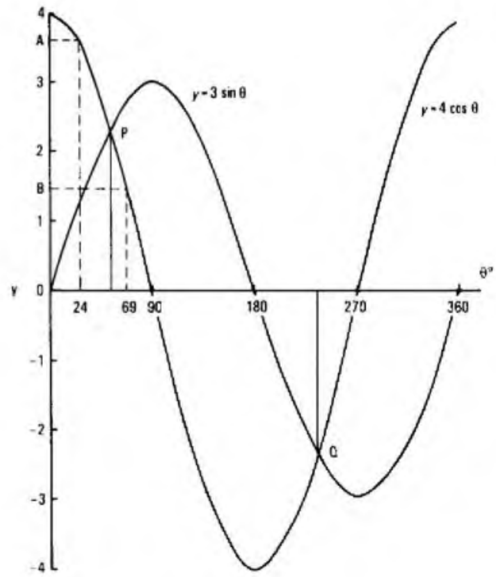
A45 (*a*) [*Tutorial Note:* To draw graphs of the sine and cosine functions reasonably accurately, it is necessary to take sub-divisions of θ at 15° intervals; the required values of $y = 3 \sin \theta$ and $y = 4 \cos \theta$ are derived in the table.]

θ°	0	15	30	45
sin θ	0	0·259	0·5	0·707
3 sin θ	0	0·777	1·5	2·121
cos θ	1	0·966	0·866	0·707
4 cos θ	4	3·864	3·464	2·828

θ°	60	75	90
sin θ	0·866	0·966	1
3 sin θ	2·598	2·898	3
cos θ	0·5	0·259	0
4 cos θ	2	1·036	0

[*Tutorial Note:* For sin θ, the values from 90° to 180° are the same as those from 0° to 90° but in reverse order. From 180° to 360°, the values have the same magnitude at those from 0° to 180°, but are negative instead of positive. For cos θ, the values from 90° to 180° repeat those from 0° to 90° in reverse order but are negative. From 180° to 360°, the values have the same magnitude as those from 0° to 180° but are of opposite sign.]

The graphs are shown in the sketch.



(*b*) (*i*) Since $3 \sin \theta - 4 \cos \theta = 0$, $3 \sin \theta = 4 \cos \theta$.
The solution to this equation obtains where the 2 graphs intersect and thus have the same value. From the sketch, this occurs at points P and Q where

$$\theta = \underline{53° \text{ and } 233°}.$$

[*Note:* Accurate values (to the nearest minute) are 53° 8′ and 233° 8′. Answers within 2° could reasonably be expected from good graphs.]

(*ii*) When $\theta = 24°$, the value of $y = 4 \cos \theta$ read from the graph is 3·65 (point A).

$\therefore \quad \cos 24° = 3{\cdot}65/4 = 0{\cdot}9125.$

$\sin (90° - 24°) = \sin 66°$
From the graph, $3 \sin 66° = 2{\cdot}74$.
$\therefore \quad \sin 66° = 2{\cdot}74/3 = 0{\cdot}91\dot{3}$, which checks closely with the value obtained for cos 24°.

When $\theta = 69°$, $\quad y = 4 \cos 69° = 1{\cdot}43$ (point B).
$\therefore \quad \cos 69° = 0{\cdot}3575.$
$\sin (90° - 69°) = 21°.$
From the graph, $3 \sin 21° = 1{\cdot}07$.
$\therefore \quad \sin 21° = 0{\cdot}35\dot{6}$, which checks closely with the value obtained for cos 69°.

Thus, the relationship $\cos \theta = \sin (90° - \theta)$ is shown to be true.

MATHEMATICS 2 1978–79

Students are advised to read the notes on p. 23

Q1–21 are multiple-choice objective questions. Students would normally be expected to ring or tick the correct answer. In the model answers below, the designations of the correct choices are printed to the right of the question in such a way that students may, if they wish, cover them in order to check their answers after working through the questions. Q1–21 should be completed within 30 min.

Q1 In order to eliminate the y term from the simultaneous equations

$$4x + 3y = 17 \quad \ldots\ldots (1)$$
$$4x - 3y = -1 \quad \ldots\ldots (2)$$

would you

(*a*) *add equation (1) to equation (2)*
(*b*) *subtract equation (1) from equation (2)*
(*c*) *subtract equation (2) from equation (1)* **(a)**

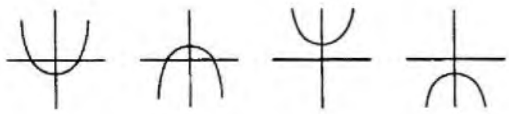
Q2 The quadratic equation which has the roots s = 4 and s = −6 is

(*a*) $s^2 + 2s + 24$ (*b*) $s^2 - 2s - 24$
(*c*) $s^2 - 2s + 24$ (*d*) $s^2 + 2s - 24$ **(d)**

Q3 The values of p and q which satisfy the equations 3p + 5q = −34 and p + 3q = −18 are

(*a*) $p = 3, q = 5$ (*b*) $p = 5, q = 3$
(*c*) $p = -3, q = -5$ (*d*) $p = 1, q = -3$ **(c)**

Q4 The shape of the graph $y = x^2 - 4$ is



(*a*) (*c*) (*d*) **(a)**

Q5 I and V are related by the law $I = aV^b$, where a and b are constants. This can be verified by plotting a graph of

(*a*) *I vertically against V horizontally*
(*b*) *V vertically against I horizontally*
(*c*) *log I vertically against log V horizontally*
(*d*) *log V vertically against log I horizontally* **(c)**

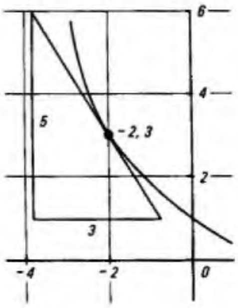
Q6 The values in the table obey the law $y = ab^x$, where a and b are constants. The type of graph paper required to test the law is

(*a*) *linear × log 4 cycles*
(*b*) *log 2 cycles × log 4 cycles*
(*c*) *log 4 cycles × log 2 cycles*
(*d*) *log 4 cycles × linear* **(d)**

x	*0·1*	*0·4*	*0·9*	*1·4*	*1·9*	*2·3*
y	*5·1*	*10·9*	*38·3*	*134·3*	*470·8*	*1285*

Q7 The slope of the graph at point (−2, 3) is

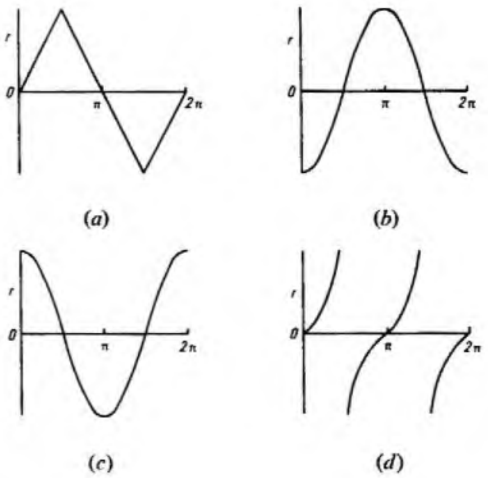
(*a*) *−1·67* (*b*) *−0·6*
(*c*) *+0·6* (*d*) *+1·67* **(a)**



Q8 If $y = 4x^3 + 3$ then $\frac{dy}{dx}$ =

(*a*) $\frac{4}{3}x^2$ (*b*) $4x^2 + 3$
(*c*) $12x^2 + 3$ (*d*) $12x^2$ **(d)**

Q9 Which of the following graphs demonstrates the rate of change (r) of sin θ?



(c)

Q10 *The value of* $e^{-2\cdot 1}$ *is*
(*a*) 0·1225 (*b*) 0·8166
(*c*) 1·225 (*d*) 8·166 **(a)**

Q11 $\log_x y$ *is equal to*
(*a*) $\log_x 10 \times \log_{10} y$ (*b*) $\log_x 10 + \log_{10} y$
(*c*) $\log_x 10 - \log_{10} y$ (*d*) $\log_{10} 10 \times \log_{10} y$ **(a)**

Q12 *The base of Naperian logarithms is*
(*a*) 0·4343 (*b*) 2·3026 (*c*) 2·7182 (*d*) 10 **(c)**

Q13 *Given that* $\log_3 81 = x$, *the value of x is*
(*a*) 3 (*b*) 4 (*c*) 9 (*d*) 27 **(b)**

Q14 *The value of* $\log_e 0\cdot 0216$ *is*
(*a*) $\bar{5}\cdot 3753$ (*b*) $\bar{4}\cdot 1649$ (*c*) $\bar{3}\cdot 835$ (*d*) $\bar{1}\cdot 4676$ **(b)**

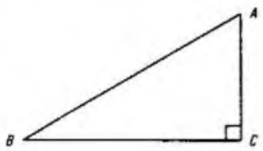
Q15 *If* $\log_e N = 3\cdot 6481$, *the value of N is*
(*a*) 1·2942 (*b*) 1·951 (*c*) 3·84 (*d*) 38·4 **(d)**

Q16 *The arithmetic mean of the set of numbers 63, 84, 27, 72, 68, 89, 17 is*
(*a*) 60 (*b*) 68 (*c*) 72 (*d*) 84 **(a)**

Q17 *A box contains 200 small lamps, 3% of which are faulty. The number of good lamps is*
(*a*) 180 (*b*) 184 (*c*) 194 (*d*) 197 **(c)**

Q18 *The probability of drawing a blue ball from a bag containing 8 white balls and 6 blue balls is*
(*a*) $\frac{1}{8}$ (*b*) $\frac{3}{7}$ (*c*) $\frac{4}{7}$ (*d*) $\frac{3}{4}$ **(b)**

Q19 *With reference to the figure, sec A is*
(*a*) $\frac{AB}{AC}$
(*b*) $\frac{BC}{AB}$
(*c*) $\frac{AC}{AB}$
(*d*) $\frac{BC}{AC}$ **(a)**



Q20 *The value of cosec 235° is*
(*a*) −1·7434 (*b*) −1·2208
(*c*) 1·2208 (*d*) 1·7434 **(b)**

Q21 *The truth table for the logic function* $C = A + \bar{B}$ *is*

(*a*)

A	B	C
0	0	1
1	0	1
0	1	0
1	1	1

(*b*)

A	B	C
0	0	0
1	0	1
0	1	1
1	1	1

(*c*)

A	B	C
0	0	0
1	0	1
0	1	0
1	1	0

(*d*)

A	B	C
0	0	0
1	0	1
0	1	1
1	1	0

(a)

Q22–41 are short-answer questions and should be completed within 40 min.

Q22 *What constant term must be added to the expression* $a^2 - 6a$ *to form a perfect square?*

A22 The constant term is $\left(\frac{-6}{2}\right)^2 = (-3)^2 = \underline{9}$.
[The expression becomes $a^2 - 6a + 9$, which equals $(a - 3)^2$]

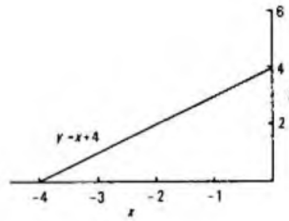
Q23 *Find the factors of* $6x^2 + 2x - 20$.

A23 $6x^2 + 2x - 20 = \underline{(3x - 5)(2x + 4)}$.

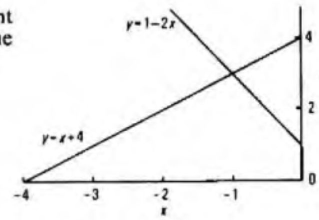
Q24 *For what purpose is a planimeter used?*

A24 A planimeter is used to measure the areas of irregular plane figures.

Q25 *Complete the graph and solve the simultaneous equations*
$y = 1 - 2x$
$y = x + 4$



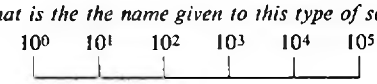
A25 The lines cross at the point (−1, 3). Hence, the solution to the equations is
$\underline{x = -1 \text{ and } y = 3}$.



Q26 *Given the formula* $a = pb^2 + qb$, *state which variables must be plotted to obtain a straight-line graph.*

A26 Plot $\frac{a}{b}$ vertically against b horizontally.

Q27 *What is the the name given to this type of scale?*

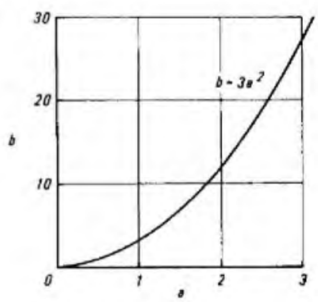


A27 Logarithmic.

Q28 *The following set of numbers is to be plotted on a logarithmic scale. How many cycles will the scale need to cover?*
0·031, 0·064, 0·82, 2·3, 6·7, 11·9, 64·3

A28 4 cycles.

Q29 *What is the gradient of the graph* $b = 3a^2$ *between a = 2 and a = 3?*



A29 Gradient $= \frac{27 - 12}{3 - 2} = \frac{15}{1} = \underline{15}$.

Q30 *Use exponential tables to find* e^x *when x is*
(*a*) 1·3 (*b*) 0·32 (*c*) −1·8 (*d*) −0·22.

A30 (*a*) $e^{1\cdot 3} = \underline{3\cdot 6693}.$

(*b*) $e^{0\cdot 32} = \underline{1\cdot 3771}.$

(*c*) $e^{-1\cdot 8} = \underline{0\cdot 1653}.$

(*d*) $e^{-0\cdot 22} = \underline{0\cdot 8025}.$

Q31 *Find the value of v when* $v = 50e^{-4\cdot 6}$.

A31
$$e^{-4\cdot 6} = 0\cdot 01.$$
$$\therefore \quad v = 50 \times 0\cdot 01 = \underline{0\cdot 5}.$$

Q32 *Using the tables of Naperian logarithms, find the value of* $\log_e 1\cdot 462$.

A32 $\underline{0\cdot 3798}.$

Q33 *Using the tables of Naperian logarithms, find the value of* $\log_e 234\cdot 6$.

A33 Converting 234·6 to standard form gives $2\cdot 346 \times 10^2$.
$$\therefore \quad \log_e 234\cdot 6 = \log_e 2\cdot 346 + \log_e 10^2,$$
$$= 0\cdot 8528 + 4\cdot 6052 = \underline{5\cdot 4580}.$$

Q34 *Using the tables of Naperian logarithms, find A if* $\log_e A = 2\cdot 1342$.

A34
$$A = \text{antilog}_e\, 2\cdot 1342.$$
$$\therefore \quad A = \underline{8\cdot 45}.$$

Q35 *Using tables of Naperian logarithms, find M if* $\log_e M = \bar{6}\cdot 381$.

A35
$$\log_e 0\cdot 001 = \bar{7}\cdot 0922.$$
$$\therefore \quad \log_e \bar{6}\cdot 381 = \bar{7}\cdot 0922 + (\bar{6}\cdot 381 - \bar{7}\cdot 0922),$$
$$= \bar{7}\cdot 0922 + 1\cdot 2887.$$
$$M = \text{antilog}_e\, \bar{7}\cdot 0922 \times \text{antilog}_e\, 1\cdot 2887,$$
$$= \underline{10^{-3} \times 3\cdot 628}.$$

Q36 *What is the median value of the set of numbers 8, 3, 7, 6, 6, 8, 9, 5?*

A36 [Arranging the numbers in rank order gives 3, 5, 6, 6, 7, 8, 8, 9]

$$\text{The median value} = \frac{6+7}{2} = \frac{13}{2} = \underline{6\cdot 5}.$$

Q37 *Twenty cards numbered 1–20 are laid face down. If 2 cards are selected at random, what is the probability that the numbers are both divisible by 2*

(*a*) *with replacement and* (*b*) *without replacement.*

A37 (*a*) $\text{Probability} = \frac{10}{20} \times \frac{10}{20} = \frac{100}{400} = \underline{\frac{1}{4}}.$

(*b*) If the first card drawn is even, probability
$$= \frac{10}{20} \times \frac{9}{19} = \frac{90}{380} = \underline{\frac{9}{38}}.$$

If the first card drawn is odd, probability
$$= \frac{10}{20} \times \frac{10}{19} = \frac{100}{380} = \underline{\frac{10}{38}}.$$

Q38 *Using trigonometrical tables, find*

(*a*) *sec 167°*	(*b*) *cosec 82°*	(*c*) *tan 337°*
(*d*) *sin 285° 28′*	(*e*) *cot 222° 43′*	(*f*) *cosec 266° 14′*

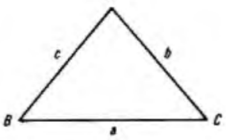
A38

(*a*) −1·0263	(*b*) 1·0098	(*c*) −0·4245
(*d*) −0·9638	(*e*) 1·0831	(*f*) −1·0022

Q39 *Referring to the figure, state*

(*a*) *the sine rule*

(*b*) *the cosine rule to find side b*



A39 (*a*) $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}.$

(*b*) $b^2 = a^2 + c^2 - 2ac \cos B.$

Q40 *Simplify the Boolean expression* $(A + B) . (\bar{A} + \bar{B})$.

A40
$$(A + B) . (\bar{A} + \bar{B}) = (A + B) . \bar{A} + (A + B) . \bar{B},$$
$$= A . \bar{A} + B . \bar{A} + A . B + B . \bar{B},$$
$$= 0 + B . \bar{A} + A . \bar{B} + 0$$
$$= \underline{B . \bar{A} + A . \bar{B}}.$$

Q41 *What is the value of the Boolean expressions* (*a*) $A + 0$ *and* (*b*) $A.0$*?*

A41 (*a*) *A* (*b*) *0.*

Q42–51 are long-answer questions. The approximate time for completion is given at the end of each question.

Q42 *Solve the simultaneous equations*
$$8x + 7y = 40$$
$$2x - 4y = -13$$
(*10 min*)

A42
$$8x + 7y = 40 \quad \ldots\ldots (1)$$
$$2x - 4y = -13 \quad \ldots\ldots (2)$$
Multiplying equation (2) by 4 gives
$$8x - 16y = -52 \quad \ldots\ldots (3)$$
Subtracting equation (3) from equation (1) gives
$$23y = 92.$$
$$\therefore \quad \underline{y = 4}.$$
Substituting for *y* in equation (1) gives
$$8x + 28 = 40.$$
$$\therefore \quad 8x = 12.$$
$$\therefore \quad \underline{x = 1\cdot 5}.$$

Q43 *Use the general formula for the solution of a quadratic equation to find the roots of the equation* $3m^2 + 3m - 36 = 0$. **(*8 min*)**

A43 The general formula for the solution of a quadratic equation of the form $ax^2 + bx + c$ is
$$x = \frac{-b \pm \sqrt{(b^2 - 4ac)}}{2a}.$$
Substituting in the formula,
$$m = \frac{-3 \pm \sqrt{\{3^2 - 4 \times 3 \times (-36)\}}}{2 \times 3},$$
$$= \frac{-3 \pm \sqrt{(9 + 432)}}{6},$$
$$= \frac{-3 \pm \sqrt{441}}{6},$$
$$= \frac{-3 \pm 21}{6},$$
$$= \underline{-4 \text{ or } +3}.$$

Q44 *The table illustrates the voltage developed across a capacitor, measured at 1 ms intervals over a period of 8 ms.*

Time	0	1	2	3	4	5	6	7	8
Voltage	0	1·0	1·2	1·8	3·9	5·2	4·4	2·4	0

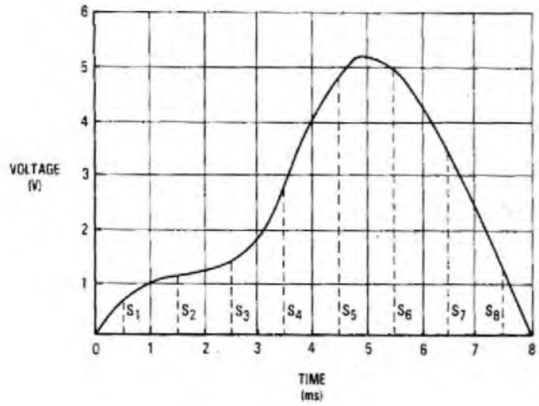
(*a*) *Plot the graph of voltage against time.*

(*b*) *Taking 8 strips, use the mid-ordinate rule to determine the area between the waveform and the horizontal axis.*

(*c*) *Determine the average value of the voltage over the period shown.* **(*30 min*)**

A44 (*a*) The graph is shown in the sketch.

(*b*) Let the width of each of the strip be 1 unit. Then,
area under the curve = width of interval × sum of mid-ordinates,
$$= 1 \times (s_1 + s_2 + s_3 + s_4 + s_5 + s_6 + s_7 + s_8),$$
$$= 1 \times (0\cdot 7 + 1\cdot 1 + 1\cdot 4 + 2\cdot 7 + 4\cdot 8 + 5\cdot 0 + 3\cdot 5 + 1\cdot 2),$$
$$= 1 \times 20\cdot 4,$$
$$= \underline{20\cdot 4 \text{ square units}}.$$



(*c*) The average value of the voltage

$$= \frac{\text{sum of mid-ordinates}}{\text{number of mid-ordinates}},$$

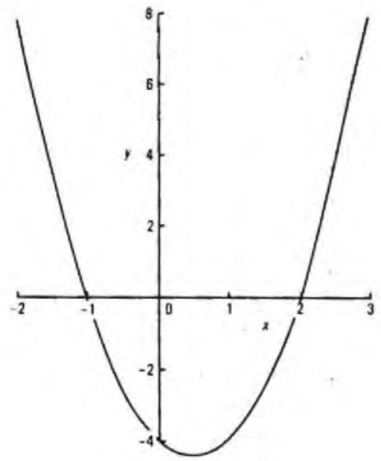
$$= \frac{20 \cdot 4}{8} = \underline{2 \cdot 55 \text{ V}}.$$

Q45 Plot the graph of $y = 2x^2 - 2x - 4$ over the range $x = -2$ to $x = 3$ and so find the roots of the quadratic equation $2x^2 - 2x - 4 = 0$. *(20 min)*

A45 Completing a table for $x = -2$ to $x = 3$.

x	−2	−1	0	1	2	3
$2x^2$	8	2	0	2	8	18
$-2x$	4	2	0	−2	−4	−6
−4	−4	−4	−4	−4	−4	−4
y	8	0	−4	−4	0	8

The graph of $y = 2x^2 - 2x - 4$ cuts the x-axis at the points $x = -1$ and $x = +2$. The roots of the equation are therefore $\underline{x = -1 \text{ and } x = 2}$.

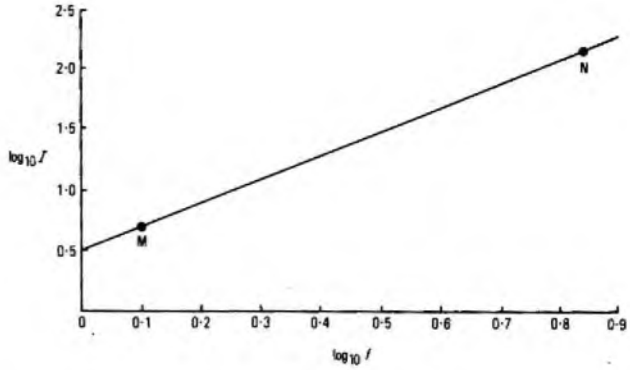


Q46 The signal current, I, flowing through an electrical filter circuit is related to the signal frequency, f, by the law $I = af^b$. Use the values in the table to plot a suitable straight-line graph and so obtain values for the constants a and b. *(30 min)*

f (kHz)	*2*	*3*	*4*	*5*	*6*	*7*
I (mA)	*12*	*27*	*48*	*75*	*108*	*147*

A46 Taking logarithms of both sides,

$$\log I = \log a + b \log f.$$



A straight-line graph is obtained if $\log I$ is plotted vertically against $\log f$ horizontally. A table is constructed and the graph drawn as shown in the sketch.

f	2	3	4	5	6	7
$\log f$	0·3010	0·4771	0·6021	0·6990	0·7782	0·8451
I	12	27	48	75	108	147
$\log I$	1·0792	1·4314	1·6812	1·8751	2·0334	2·1673

The graph of $\log I = \log a + b \log f$ may be compared with the straight line graph formula $y = mx + c$, where m is the slope of the line and c the intercept with the y-axis.

Selecting 2 points on the graph, M (0·1, 0·68) and N (0·8, 2·0 gives the slope as

$$b = \frac{2 \cdot 08 - 0 \cdot 68}{0 \cdot 8 - 0 \cdot 1} = \frac{1 \cdot 4}{0 \cdot 7} = \underline{2}.$$

When produced, the graph cuts the $\log I$ axis at 0·48.

$\therefore \quad \log a = 0 \cdot 48.$

$\therefore \quad a = 3 \cdot 02$, which may be rounded to $\underline{a = 3}$.

The law is, therefore, $I = 3f^2$.

Q47 Derive from first principles an expression for $\frac{ds}{dt}$ when

$$s = 2t^2 + 2t + 1.$$

(20 min)

A47 Let s increase by a small amount δs and t increase by a corresponding amount δt.

$$s + \delta s = 2(t + \delta t)^2 + 2(t + \delta t) + 1,$$
$$= 2\{t^2 + 2t\,\delta t + (\delta t)^2\} + 2t + 2\delta t + 1,$$
$$= 2t^2 + 4t\,\delta t + 2(\delta t)^2 + 2t + 2\delta t + 1.$$

But, $s = 2t^2 + 2t + 1$.

$\therefore \quad \delta s = 4t\,\delta t + 2(\delta t)^2 + 2\delta t$ (by subtraction).

$\therefore \quad \frac{\delta s}{\delta t} = 4t + 2\delta t + 2.$

As $\quad \delta t \to 0,\ 4t + 2\delta t + 2 \to 4t + 2.$

$$\therefore \quad \frac{ds}{dt} = \lim_{\delta t \to 0} \frac{\delta s}{\delta t} = \underline{4t + 2}.$$

Q48 (a) What is meant by the addition law of probability?

(b) What is meant by the multiplication rule of probability?

(c) A bag contains 6 red, 12 blue, 5 green and 7 orange balls. If 2 balls are taken in turn, with replacement, what is the probability of drawing

(i) 2 red balls,
(ii) a green ball and an orange ball,
(iii) a blue ball and a red ball, and
(iv) a green ball or a blue ball as the first ball? *(20 min)*

A48 (*a*) The addition law of probability applies to mutually exclusive events. If the probability of one event taking place is p_1 and the probability of a second event is p_2, the probability of *either* one *or* the other taking place is found by *adding* the probabilities of the 2 events; that is, $p_1 + p_2$.

(*b*) The multiplication law of probability applies to the probability of 2 separate events *both* happening, and is found by *multiplying* together the probabilities of the individual events. If the 2 probabilities are p_3 and p_4, the probability of *both* p_3 *and* p_4 happening is $p_3 \times p_4$.

(*c*) (*i*) The probability of drawing 2 red balls is

$$\frac{6}{30}\times\frac{6}{30}=\frac{36}{900}=\underline{\frac{1}{25}}.$$

(*ii*) The probability of drawing one green ball and one orange ball is

$$\frac{5}{30}\times\frac{7}{30}=\frac{35}{900}=\underline{\frac{7}{180}}.$$

(*iii*) The probability of drawing a blue ball and a red ball is

$$\frac{12}{30}\times\frac{6}{30}=\frac{72}{900}=\underline{\frac{2}{25}}.$$

(*iv*) The probability of drawing a green ball or a blue ball as the first ball is

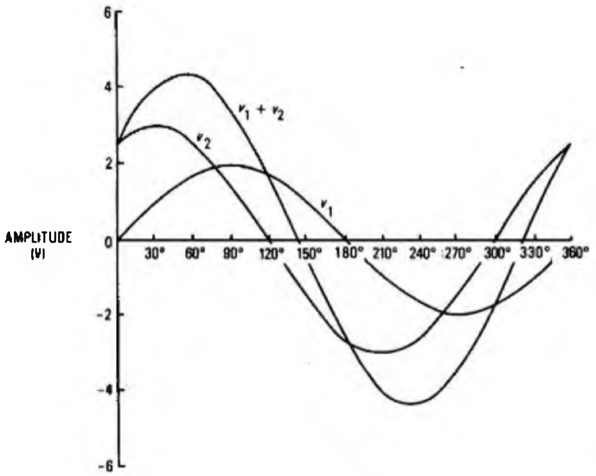
$$\frac{5}{30}+\frac{12}{30}=\underline{\frac{17}{30}}.$$

Q49 The instantaneous values, v_1 and v_2, of 2 sinusoidal voltages are given by $v_1 = 2\sin A$ and $v_2 = 3\sin(A + 60°)$. Plot, at 30° intervals, the graphs of the waveforms on the same axes over one cycle and obtain an expression for $v_1 + v_2$. (*30 min*)

A49 The values of $v_1 = 2\sin A$ and $v_2 = 3\sin(A + 60°)$ are given in the table.

A	0	30°	60°	90°	120°	150°	180°
$\sin A$	0	0·5	0·866	1	0·866	0·5	0
$2\sin A$	0	1·0	1·732	2	1·732	1·0	0
$\sin(A + 60°)$	0·866	1·0	0·866	0·5	0	−0·5	−0·866
$3\sin(A + 60°)$	2·598	3·0	2·598	1·5	0	−1·5	−2·598

A	210°	240°	270°	300°	330°	360°
$\sin A$	−0·5	−0·866	−1·0	−0·866	−0·5	0
$2\sin A$	−1·0	−1·732	−2·0	−1·732	−1·0	0
$\sin(A + 60°)$	−1·0	−0·866	−0·5	0	0·5	0·866
$3\sin(A + 60°)$	−3·0	−2·598	−1·5	0	1·5	2·598

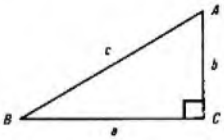


The graphs of v_1 and v_2 and the resultant $v_1 + v_2$ are shown in the sketch.

From the graph, the resultant waveform has a maximum value of 4·36 and is leading v_1 by 36°. The expression for the resultant waveform is

$$v_1 + v_2 = \underline{4{\cdot}36\sin(A + 36°)}.$$

Q50 (a) The figure shows a right-angled triangle ABC. Using the ratios of the sides, prove that $\operatorname{cosec}^2 A = 1 + \cot^2 A$.



(b) Using trigonometrical tables, prove that

$\sec^2 A = 1 + \tan^2 A$ for

(i) $A = 128°$, and
(ii) $A = 225°$. (*20 min*)

A50 (*a*) Referring to the figure,

$$\operatorname{cosec}^2 A = \frac{c^2}{a^2}.$$

By Pythagoras' theorem,

$$c^2 = a^2 + b^2.$$

$$\therefore \quad \operatorname{cosec}^2 A = \frac{a^2 + b^2}{a^2},$$

$$= \frac{a^2}{a^2} + \frac{b^2}{a^2},$$

$$= 1 + \cot^2 A. \qquad \text{QED}$$

(*b*) (*i*) $\sec^2 A = \sec^2 128° = (-\sec 52°)^2 = 2{\cdot}638.$

$1 + \tan^2 A = 1 + \tan^2 128° = 1 + (-\tan 52°)^2 = 1 + 1{\cdot}638 = 2{\cdot}638$

$\therefore \quad \sec^2 128° = 1 + \tan^2 128°.$ QED

(*ii*) $\sec^2 A = \sec^2 225° = (-\sec 45°)^2 = (-1{\cdot}4142)^2 = 2.$

$1 + \tan^2 A = 1 + \tan^2 225° = 1 + (-\tan 45°)^2 = 1 + 1 = 2$

$\therefore \quad \sec^2 225° = 1 + \tan^2 225°.$

Q51 Simplify the Boolean expression $(A + B).(A + C).\bar{A}$ and draw up a truth table to prove your answer. (*20 min*)

A51
$$\begin{aligned}(A + B).(A + C).\bar{A} &= (A.A + A.B + A.C + B.C).\bar{A},\\ &= (A + A.B + A.C + B.C).\bar{A},\\ &= \bar{A}.A + \bar{A}.A.B + \bar{A}.A.C + \bar{A}.B.C,\\ &= 0 + 0.B + 0.C + \bar{A}.B.C,\\ &= \underline{\bar{A}.B.C}.\end{aligned}$$

A	B	C	$A + B$	$A + C$	$\bar{A}$	$(A + B).(A + C).\bar{A}$	$\bar{A}.B.C$
0	0	0	0	0	1	0	0
1	0	0	1	1	0	0	0
0	1	0	1	0	1	0	0
1	1	0	1	1	0	0	0
0	0	1	0	1	1	0	0
1	0	1	1	1	0	0	0
0	1	1	1	1	1	1	1
1	1	1	1	1	0	0	0

The 2 columns $(A + B).(A + C).\bar{A}$ and $\bar{A}.B.C$ are identical.

$$\therefore \quad (A + B).(A + C).\bar{A} = \bar{A}.B.C.$$

Published by *The Post Office Electrical Engineers' Journal*, 2–12 Gresham Street, London EC2V 7AG
and printed in Great Britain by Unwin Brothers Limited, Old Woking, Surrey