# SUPPLEMENT

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GUIDANCE FOR STUDENTS

## Contents



# CITY AND GUILDS OF LONDON INSTITUTE

## Questions and Answers

**Answers are occasionally omitted or reference is made to earlier Supplements in which questions of substantially the same form, together with the answers, have been published. Some answers contain more detail than would be expected from candidates under examination conditions.**

### MATHEMATICS C 1978

**Students were expected to answer any 6 questions**

*Q1 (a) If R, L and C are constant, derive an expression for $\omega_0$, the value of $\omega$ for which the function*

$$\frac{1}{R^2 + (\omega L - 1/\omega C)^2}$$

*is greatest. Sketch the graph of this function for positive values of $\omega$.*

*(b) When $R = 40$, $L = 1{\cdot}8 \times 10^{-3}$ and $C = 2 \times 10^{-6}$, calculate*

*(i) the value of $\omega_0$, and*

*(ii) the smaller value of $\omega$ at which the function attains half its maximum value.*

**A1** (*a*) Let $f(\omega) = \dfrac{1}{R^2 + (\omega L - 1/\omega C)^2}$

$f(\omega)$ will be greatest when the denominator $R^2 + (\omega L - 1/\omega C)^2$ is least. Since $R^2$ is constant, this will occur when $(\omega L - 1/\omega C)^2$ is least; that is, when this term is zero, since, being a square, it cannot be negative.

Hence, for $f(\omega)$ to be a maximum, $\omega_0 L - 1/\omega_0 C = 0$, or

$$\omega_0{}^2 = \frac{1}{LC}.$$

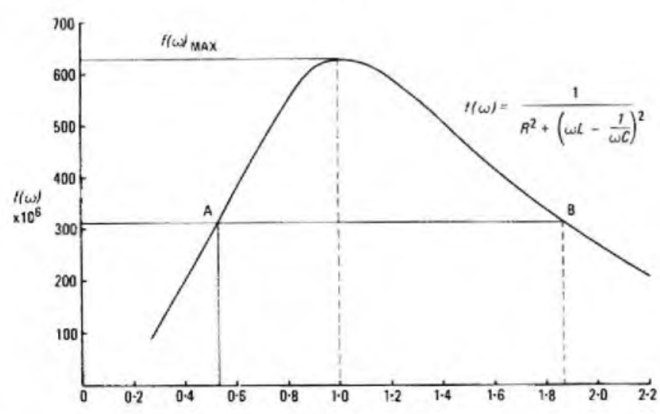
$$\therefore \quad \omega_0 = \underline{\sqrt{(1/LC)}}.$$

The graph of the function is shown in the sketch.

*Note:* The values used for the sketch were those given in part (*b*) of the question, but a typical freehand sketch approximating to the shape shown would be sufficient.

(*b*) (*i*) $\omega_0 = 1/\sqrt{(1{\cdot}8 \times 10^{-3} \times 2 \times 10^{-6})}$,

$$= \sqrt{\left(\frac{10^{10}}{36}\right)} = \frac{10^5}{6} = \underline{16\,666{\cdot}\dot{6}}.$$




(*ii*) When $f(\omega) = \frac{1}{2} \times f(\omega)_{max}$, $\dfrac{f(\omega)}{f(\omega)_{max}} = 0{\cdot}5$,

$$= \{R^2 + (\omega_0 L - 1/\omega_0 C)^2\}\,\{R^2 + (\omega L - 1/\omega C)^2\}$$

But, $\quad \omega_0 L - 1/\omega_0 C = 0.$

$\therefore \quad 0{\cdot}5\{R^2 + (\omega L - 1/\omega C)\}^2 = R^2,$

or $\quad 0{\cdot}5(\omega L - 1/\omega C)^2 = 0{\cdot}5R^2.$

$\therefore \quad \pm(\omega L - 1/\omega C) = R$, since $R$ is positive.

Substituting the given values of $R$, $L$ and $C$ gives

$\pm\{1{\cdot}8 \times 10^{-3}\omega - 1/(2 \times 10^{-6}\omega)\} = 40.$

$$\therefore \quad \pm\left(1{\cdot}8 \times 10^{-3}\omega^2 - \frac{10^6}{2}\right) = 40\omega,$$

$$\text{or } \pm 1{\cdot}8 \times 10^{-3}\omega^2 - 40\omega \mp \frac{10^6}{2} = 0.$$

$$\therefore \quad \omega = \frac{40 \pm \sqrt{(1600 + 3{\cdot}6 \times 10^3)}}{\pm 3{\cdot}6 \times 10^{-3}},$$

$$= \frac{40 \pm \sqrt{5200}}{\pm 3{\cdot}6 \times 10^{-3}},$$

$$= \frac{40 \pm 72{\cdot}111}{\pm 3{\cdot}6 \times 10^{-3}},$$

$$= 31{\cdot}142 \times 10^3 \text{ or } 8{\cdot}920 \times 10^3.$$

Thus, the smaller value of $\omega$ at which the function attains half its maximum value is <u>8920.</u>

*Note:* The smaller and larger values of $\omega/\omega_0$ corresponding to $\omega = 8920$ and $\omega = 31\,142$ are $0{\cdot}5352$ and $1{\cdot}8685$ respectively. These are shown in the sketch at points A and B.

*Q2 (a) Use a binomial expansion to calculate the cube root of 8·04 to four decimal places.*

*(b) The probability of throwing three sixes with eight dice is the term involving $q^5p^3$ in the binomial expansion of $(q + p)^8$, where $p = \frac{1}{6}$ and $q = \frac{5}{6}$. Show that this probability means approximately one chance in 9·6.*

**A2** (*a*) $\quad \sqrt[3]{8{\cdot}04} = 8^{1/3}\left(1 + \dfrac{0{\cdot}04}{8}\right)^{1/3},$

$$= 2(1 + 0{\cdot}005)^{1/3}.$$

The binomial expansion may be written as

$$(1 + x)^n = 1 + nx + \frac{n(n-1)}{2!}x^2 + \frac{n(n-1)(n-2)}{3!}x^3 + \ldots$$

When $x < 1$ and $n$ is not a positive integer, the series is absolutely convergent.

Hence, $(1 + 0\cdot005)^{1/3} = 1 + \frac{1}{3} \times 0\cdot005 + \frac{\frac{1}{3}(\frac{1}{3} - 1)}{1 \times 2} \times (0\cdot005)^2 +$

$$\frac{\frac{1}{3}(\frac{1}{3} - 1)(\frac{1}{3} - 2)}{1 \times 2 \times 3} \times (0\cdot005)^3 + \dots$$

$$= 1 + 0\cdot0016\dot{6} - \frac{1 \times 2}{3^2 \times 2} \times 0\cdot000025 +$$

$$\frac{1 \times 2 \times 5}{3^3 \times 6} \times 0\cdot000000125 - \dots$$

$$= 1\cdot00166\dot{6} - 0\cdot000002\dot{7} + 0\cdot000000007716 - \dots$$

For an accuracy of four decimal places, it is clear that the term in $(0\cdot005)^3$ and all subsequent terms may be discarded.

$\therefore \quad \sqrt[3]{8\cdot04} \approx 2 \times 1\cdot0016638$

$= \underline{2\cdot0033}$ correct to four decimal places.

(*b*) Using the binomial expansion,

$$(q + p)^8 = q^8 + 8q^7p + \frac{8 \times 7}{1 \times 2}q^6p^2 + \frac{8 \times 7 \times 6}{1 \times 2 \times 3}q^5p^3 + \dots$$

When $p = \frac{1}{6}$ and $q = \frac{5}{6}$, the term involving $q^5p^3$ is

$$\frac{8 \times 7 \times 6}{1 \times 2 \times 3} \times \left(\frac{5}{6}\right)^5\left(\frac{1}{6}\right)^3,$$

$$= 56 \times \frac{5^5}{6^8} = 56 \times \frac{3125}{1679616} = 0\cdot10419.$$

The probability is therefore $\frac{1}{0\cdot10419} = 9\cdot59785$ or one chance in approximately 9·6 throws.

*Q3* (*a*) *Sketch the graph of* $y = e^{-2t} \cos 3t$ *from* $t = 0$ *to* $t = \frac{2\pi}{3}$.

(*b*) *Write down the series expansion of* $e^x$ *as far as the term in* $x^4$, *and hence calculate the value of* $\frac{1}{e^{0\cdot2}}$ *to four significant figures.*

(*c*) *Evaluate* $\int_0^{0\cdot1} e^{-2x}\,dx$ *to three significant figures.*

**A3** (*a*) $\qquad y = e^{-2t} \cos 3t$

Within the range $t = 0$ to $t = \frac{2\pi}{3}$, the angle of the cosine function varies from 0 to $2\pi$ rad and hence there will be one complete cycle of this function; the values of $y$ will decrease rapidly from unity at $t = 0$ and will become negative between $t = \frac{\pi}{6}$ to $t = \frac{\pi}{2}$ (the corresponding values of $t$ being $\frac{\pi}{2}$ and $\frac{3\pi}{2}$ or 90° and 270°). At $t = \frac{\pi}{6}$ and $\frac{\pi}{2}$, cos $3t$ is is zero and hence $y$ is zero.

Additional values of $y$ to enable the graph to be drawn are derived in the table.

The graph is shown in the sketch.

(*b*) $\qquad e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \frac{x^4}{4!} + \dots$

$$\frac{1}{e^{0\cdot2}} = e^{-0\cdot2},$$

$$= 1 - 0\cdot2 + \frac{0\cdot2^2}{2} - \frac{0\cdot2^3}{3 \times 2} + \frac{0\cdot2^4}{4 \times 3 \times 2} - \dots$$

$$= 0\cdot8 + 0\cdot02 - 0\cdot00133\dot{3} + 0\cdot00006\dot{6} - \dots$$

It is clear from the expansion that the term in $x^5$ will affect the sixth decimal place to a small extent only and hence that and all subsequent terms may be ignored.

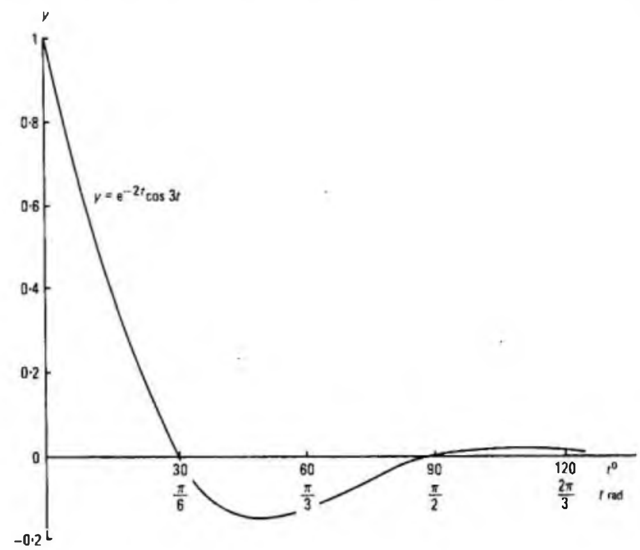
Hence, $\frac{1}{e^{0\cdot2}} \approx 0\cdot820067 - 0\cdot00133\dot{3}$,

$= 0\cdot818734$,

$= \underline{0\cdot8187}$ to four significant figures.

| $t$ rad | 0 | $\frac{\pi}{12}$ | $\frac{\pi}{6}$ | $\frac{\pi}{4}$ | $\frac{\pi}{3}$ |
|---|---|---|---|---|---|
| $t$° | 0 | 15 | 30 | 45 | 60 |
| $-2t$ | 0 | −0·524 | −1·047 | −1·571 | −2·094 |
| $e^{-2t}$ | 1 | 0·592 | 0·351 | 0·208 | 0·123 |
| $3t$° | 0 | 45 | 90 | 135 | 180 |
| cos $3t$ | 1 | 0·707 | 0 | −0·707 | −1 |
| $y$ | 1 | 0·419 | 0 | −0·147 | −0·123 |

| $t$ rad | 0 | $\frac{5\pi}{12}$ | $\frac{\pi}{2}$ | $\frac{7\pi}{12}$ | $\frac{2\pi}{3}$ |
|---|---|---|---|---|---|
| $t$° | 0 | 75 | 90 | 105 | 120 |
| $-2t$ | 0 | −2·618 | −3·142 | −3·665 | −4·189 |
| $e^{-2t}$ | 1 | 0·0729 | 0·0432 | 0·0256 | 0·0152 |
| $3t$° | 0 | 225 | 270 | 315 | 360 |
| cos $3t$ | 1 | −0·707 | 0 | 0·707 | 1 |
| $y$ | 1 | −0·0515 | 0 | 0·0181 | 0·0152 |




(*c*) $\int_0^{0\cdot1} e^{-2x}\,dx = \left[\frac{e^{-2x}}{-2}\right]_0^{0\cdot1}$,

$= -\frac{1}{2}(e^{-0\cdot2} - e^0)$,

$= -\frac{1}{2}(0\cdot81873 - 1)$,

$= \frac{1}{2} \times 0\cdot18127$,

$= 0\cdot090635$,

$= \underline{0\cdot0906}$ correct to 3 significant figures.

*Note:* The value of $e^{-0\cdot2}$ may be obtained from part (*b*) but, because of the division by 2, it is necessary to use 0·81873 as the value. Otherwise, the value of the integral is obtained as 0·09065 which would correct to 0·0907.

*Q4* (*a*) *If* $\frac{P}{W} = \frac{\sin(\alpha + \beta)}{\cos \alpha}$, *calculate* $\alpha$ *(between 0–180°) given* $P = 367$, $W = 1000$, $\beta = 29°$.

(*b*) *Solve for* $0 < \phi < 360°$ *the trigonometrical equation* $3 \sin \phi = 1 + 2 \cos \phi$.

**A4** (*a*) $$\frac{P}{W} = \frac{\sin(\alpha + \beta)}{\cos\alpha},$$

$$= \frac{\sin\alpha\cos\beta + \cos\alpha\sin\beta}{\cos\alpha},$$

$$= \tan\alpha\cos\beta + \sin\beta.$$

When $P = 367$, $W = 1000$ and $\beta = 29°$,

$$\frac{367}{1000} = \tan\alpha\cos 29° + \sin 29°.$$

$$\therefore \quad 0{\cdot}367 = 0{\cdot}87462\tan\alpha + 0{\cdot}48481.$$

$$\therefore \tan\alpha = \frac{-0{\cdot}11781}{0{\cdot}87462} = -0{\cdot}13469.$$

$\therefore$ $\alpha = \underline{172°\ 20'}$ to the nearest minute.

(*b*) $3\sin\phi = 1 + 2\cos\phi$ or $3\sin\phi - 2\cos\phi = 1$.

$$\therefore \sqrt{(3^2 + 2^2)}\left\{\frac{3}{\sqrt{(3^2 + 2^2)}}\sin\phi - \frac{2}{\sqrt{(3^2 + 2^2)}}\cos\phi\right\} = 1.$$

$$\therefore \frac{3}{\sqrt{13}}\sin\phi - \frac{2}{\sqrt{13}}\cos\phi = \frac{1}{\sqrt{13}}.$$

$$\therefore \sin\phi\cos\theta - \cos\phi\sin\theta = \frac{1}{\sqrt{13}}, \text{ where } \theta = \sin^{-1}\frac{2}{\sqrt{13}},$$

$$\text{or } \sin(\phi - \theta) = 0{\cdot}27735.$$

$$\therefore \quad \phi - \theta = 16°\ 6' \text{ or } 180° - 16°\ 6',$$

$$= 16°\ 6' \text{ or } 163°\ 54'.$$

But $\theta = \sin^{-1} 0{\cdot}5547 = 33°\ 41'$.

$$\therefore \quad \phi = 33°\ 41' + 16°\ 6' \text{ or } 33°\ 41' + 163°\ 54',$$

$$= \underline{49°\ 47' \text{ or } 197°\ 35' \text{ for } 0 < \phi < 360°.}$$

*Q5* (*a*) *Draw the polar curve given in polar coordinates as*

$$r = ae^{-k\theta}, \textit{ from } \theta = 0 \textit{ to } \theta = 2\pi$$

$\left(\textit{in steps of } \frac{\pi}{4}\right)$ *when* $a = 5$ *and* $k = \frac{1}{\pi}$.

(*b*) *By transforming to cartesian coordinates, with the pole as origin and the line* $\theta = 0$ *as x-axis, show that the polar equation*

$$r\cos(\theta - \alpha) = 2,$$

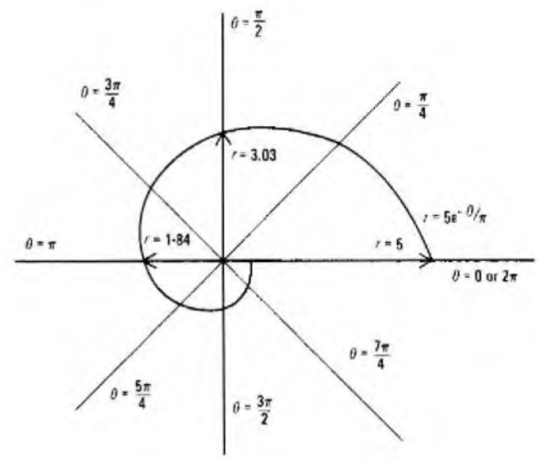
*in which the acute angle* $\alpha$ *is* $\tan^{-1}\left(\frac{4}{3}\right)$, *represents a straight line.*

**A5** (*a*) $$r = ae^{-k\theta} = 5e^{-\theta/\pi}$$

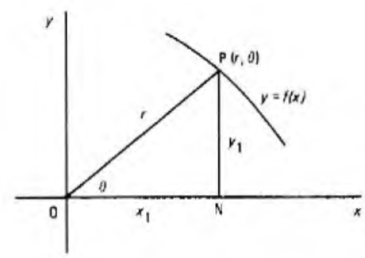
The values of $r$, for $\theta$ in steps of $\frac{\pi}{4}$ from $\theta = 0$ to $\theta = 2$, are derived from the following table.

| $\theta$ rad | 0 | $\frac{\pi}{4}$ | $\frac{\pi}{2}$ | $\frac{3\pi}{4}$ | $\pi$ |
|---|---|---|---|---|---|
| $-\frac{\theta}{\pi}$ | 0 | $-\frac{1}{4}$ | $-\frac{1}{2}$ | $-\frac{3}{4}$ | $-1$ |
| $e^{-\theta/\pi}$ | 1 | 0·779 | 0·6065 | 0·472 | 0·368 |
| $r = 5e^{-\theta/\pi}$ | 5 | 3·89 | 3·03 | 2·36 | 1·84 |

| $\theta$ rad | $\frac{5\pi}{4}$ | $\frac{3\pi}{2}$ | $\frac{7\pi}{4}$ | $2\pi$ |
|---|---|---|---|---|
| $-\frac{\theta}{\pi}$ | $-\frac{5}{4}$ | $-\frac{3}{2}$ | $-\frac{7}{4}$ | $-2$ |
| $e^{-\theta/\pi}$ | 0·2865 | 0·223 | 0·174 | 0·135 |
| $r = 5e^{-\theta/\pi}$ | 1·43 | 1·12 | 0·87 | 0·68 |




(*a*)




(*b*)

The curve is shown in sketch (*a*). (It is known as an *equiangular spiral*, since the angle $\phi$ made between the radius and the tangent to the curve is constant.)

(*b*) $$r\cos(\theta - \alpha) = 2$$

$$\therefore \quad r\cos\theta\cos\alpha + r\sin\theta\sin\alpha = 2. \quad \ldots\ldots (1)$$

Since $\alpha = \tan^{-1}\frac{4}{3}$, $\sin\alpha = \frac{4}{5}$ and $\cos\alpha = \frac{3}{5}$.

Hence, $\frac{3}{5}r\cos\theta + \frac{4}{5}r\sin\theta = 2$ (from equation 1),

or $$3r\cos\theta + 4r\sin\theta = 10. \quad \ldots\ldots (2)$$

If $P(r, \theta)$ is any point on a curve $y = f(x)$, as shown in sketch (*b*), and PN is the perpendicular from P to the $x$-axis, then, from triangle PON,

$$x = r\cos\theta \text{ and } y = r\sin\theta.$$

Substituting for $r\cos\theta$ and $r\sin\theta$ in equation (2),

$$3x + 4y = 10 \text{ or},$$

$$y = -\frac{3x}{4} + \frac{5}{2}.$$

This equation is of the form $y = mx + c$ and therefore represents a straight line.

*Q6* (*a*) *Obtain and simplify expressions for* $\frac{dy}{dx}$ *for* EACH *of the functions*

(*i*) $y = 3xe^{2x}$, (*ii*) $y = \log_e(\cos 3x)$, *and* (*iii*) $y = \frac{x^2 - 4}{x^2 + 4}$.

(*b*) *Verify that only* TWO *of the functions in part* (*a*) *have a minimum value.*

(*c*) *Sketch the graph of any* ONE *of the functions in part* (*a*).

**A6** (*a*) (*i*) $$y = 3xe^{2x}.$$

$$\frac{dy}{dx} = 3x\frac{d}{dx}(e^{2x}) + e^{2x}\frac{d}{dx}(3x),$$

$= 6xe^{2x} + 3e^{2x},$

$= \underline{3e^{2x}(2x + 1).}$

(*ii*) $y = \log_e(\cos 3x).$

$$\frac{dy}{dx} = \frac{1}{\cos 3x}\frac{d}{dx}(\cos 3x),$$

$$= \frac{1}{\cos 3x} \times 3(-\sin 3x),$$

$$= \underline{-3\tan 3x.}$$

(*iii*) $y = \dfrac{x^2 - 4}{x^2 + 4}.$

$$\frac{dy}{dx} = \frac{(x^2 + 4)\,2x - (x^2 - 4)\,2x}{(x^2 + 4)^2}$$

$$= \frac{2x^3 + 8x - 2x^3 + 8x}{(x^2 + 4)^2},$$

$$= \underline{\frac{16x}{(x^2 + 4)^2}}.$$

(*b*) (*i*) From part (*a*), the first derivative of $y = 3xe^{2x}$ is $3e^{2x}(2x + 1)$. For a maximum or minimum value of $y$, $\dfrac{dy}{dx} = 0$.

$$\therefore\ 3e^{2x}(2x + 1) = 0.$$

$$\therefore\ e^{2x} = 0 \text{ or } (2x + 1) = 0.$$

The first condition occurs at an infinite discontinuity ($2x = \log_e 0 = \infty$) and is inadmissible. From the second condition, $x = -\frac{1}{2}$.

Now, $\dfrac{d^2y}{dx^2} = 3e^{2x} \times 2 + 6e^{2x}(2x + 1),$

$$= 6e^{2x} + 12xe^{2x} + 6e^{2x},$$

$$= 12e^{2x}(1 + x).$$

When $x = -\frac{1}{2}$, $\dfrac{d^2y}{dx^2} = 12e^{-1} \times \dfrac{1}{2} = \dfrac{6}{e}$, which is positive and hence a minimum value of $y$ occurs at this point.

(*ii*) From part (*a*), $\dfrac{dy}{dx} = -3\tan 3x.$

$$\frac{d^2y}{dx^2} = -3 \times 3\sec^2 3x = -9\sec^2 3x.$$

When $\dfrac{dy}{dx} = 0$, $\tan 3x = 0$ or $x = 0$.

Hence, $\dfrac{d^2y}{dx^2} = -9\sec^2 0 = -9$, which is negative, and therefore there is no minimum value of $y$.

(*iii*) From part (*a*), $\dfrac{dy}{dx} = \dfrac{16x}{(x^2 + 4)^2}.$

$$\therefore\ \frac{d^2y}{dx^2} = \frac{(x^2 + 4)^2 \times 16 - 16x \times 2(x^2 + 4) \times 2x}{(x^2 + 4)^4},$$

$$= \frac{16(x^2 + 4) - 64x^2}{(x^2 + 4)^3},$$

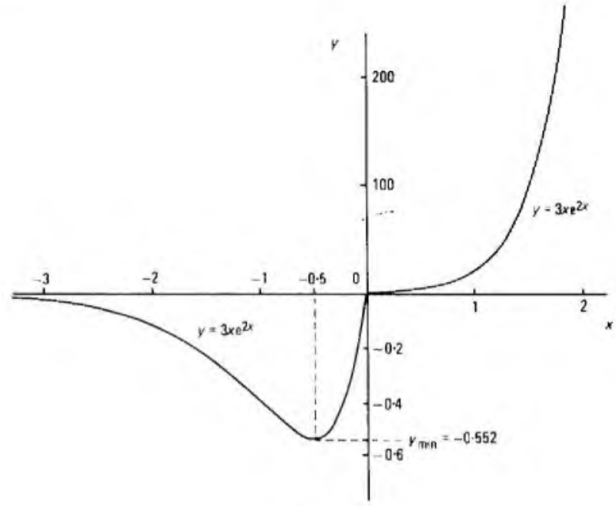
$$= \frac{16x^2 + 64 - 64x^2}{(x^2 + 4)^3},$$

$$= \frac{-48x^2 + 64}{(x^2 + 4)^3}.$$

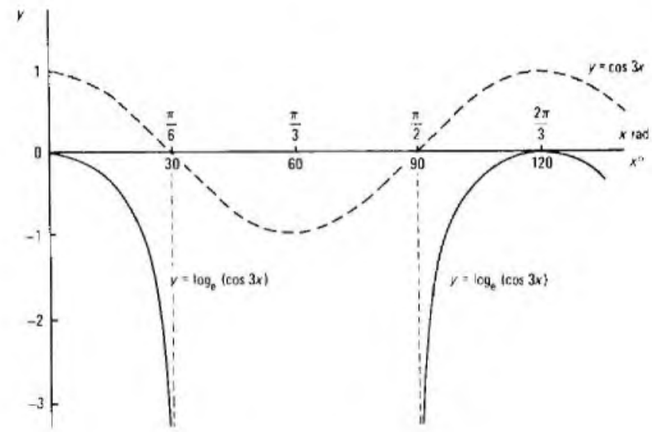
When $\dfrac{dy}{dx} = 0$, $\dfrac{16x}{(x^2 + 4)^2} = 0$ or $x = 0$.

Hence, at this point $\dfrac{d^2y}{dx^2} = \dfrac{64}{64} = 1$ and, since this is positive, a minimum value occurs.

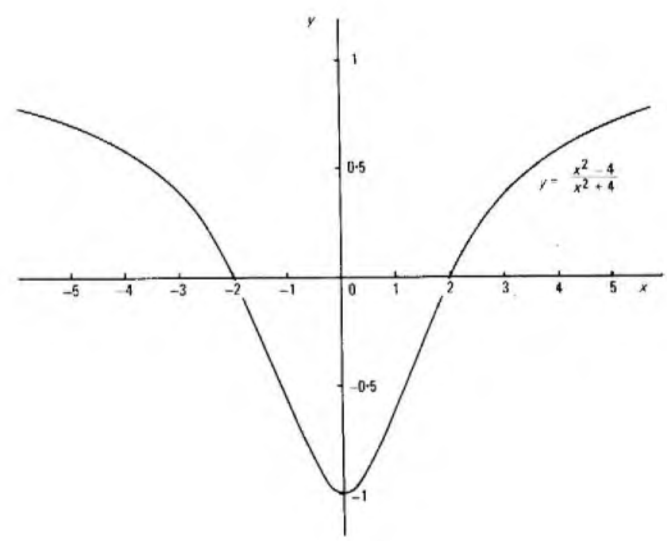
(*c*) Graphs of the functions in parts (*i*), (*ii*) and (*iii*) are shown in sketches (*a*), (*b*) and (*c*) respectively.




(*a*)




(*b*)




(*c*)

*Notes:* Only one sketch was required from students, but all 3 are given here for information. In sketch (*a*), a much larger scale has been used for negative values of $x$ because of the small values of $y$. As $x \to -\infty$, $y \to 0$. The minimum value of $-0{\cdot}552$ occurring at $x = -0{\cdot}5$ is shown in the sketch.

Sketch (*b*) includes the graph of $y = \cos 3x$ (shown dotted), one complete cycle of which occurs between $x = 0$ and $x = \frac{2\pi}{3}$ rad. The graph of $y = \log_e(\cos 3x)$ is therefore also cyclic. At $x = \frac{\pi}{6}$ and $\frac{\pi}{2}$, $y \to -\infty$ and the curve becomes asymptotic to the ordinates at these points, but between these points, since $\cos 3x$ is negative, there are no real values of $\log_e(\cos 3x)$. It is seen that there are no minima, but maxima occur at $x = 0, \frac{2\pi}{3}, \frac{4\pi}{3}, \ldots$

The minimum value of $-1$ at $x = 0$ is shown for the function $y = \frac{x^2 - 4}{x^2 + 4}$ in sketch (*c*). As $x \to \pm\infty$, $y \to 1$ and both ends of the graph become asymptotic to the line $y = 1$.

*Q7* (*a*) *If $y = e^{-x} \sin 2x$, prove that*

$$\frac{d^2y}{dx^2} + 2\frac{dy}{dx} + 5y = 0.$$

(*b*) *Show, with the aid of a phasor diagram, that voltages $V_1$ and $V_2$ which differ in phase by $\theta$ give a resultant voltage $V$, where*

$$V^2 = V_1^2 + V_2^2 + 2V_1V_2 \cos\theta.$$

(*c*) *If $V_1$ and $V_2$ are constant but $\theta$ is varied, write expressions for the maximum and minimum values of $V$.*

**A7** (*a*) $y = e^{-x} \sin 2x$

$$\therefore \quad \frac{dy}{dx} = e^{-x}\, 2\cos 2x - e^{-x} \sin 2x,$$

$$= e^{-x}(2\cos 2x - \sin 2x).$$

$$\frac{dx^2}{d^2y} = e^{-x}(-4\sin 2x - 2\cos 2x) - e^{-x}(2\cos 2x - \sin 2x)$$

$$= e^{-x}\sin 2x(-4 + 1) - e^{-x}\cos 2x(2 + 2),$$

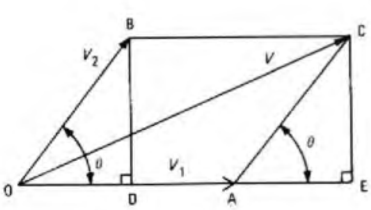
$$= -3e^{-x}\sin 2x - 4e^{-x}\cos 2x.$$

$$\text{Hence, } \frac{d^2y}{dx^2} + 2\frac{dy}{dx} + 5y = -3e^{-x}\sin 2x - 4e^{-x}\cos 2x + 2e^{-x}(2\cos 2x - \sin 2x) + 5e^{-x}\sin 2x,$$

$$= e^{-x}\sin 2x(-3 - 2 + 5) + e^{-x}\cos 2x(-4 + 4),$$

$$= 0.$$

(*b*) In the sketch, OA represents the voltage $V_1$ and OB the voltage $V_2$, advanced in phase by the angle $\theta$. Then, by the normal rules of vector addition, OC, the diagonal of the parallelogram OACD, will represent the resultant, $V$, of the other 2 voltages, $V_1$ and $V_2$.




If the perpendiculars BD and CE are dropped from B and C respectively on to OA, it is clear that triangles BOD and CAE are congruent.

Hence, in triangle COE, $OC^2 = OE^2 + EC^2$,

$$= (OA + AE)^2 + EC^2.$$

But, $AE = CA\cos\theta$ and $EC = CA\sin\theta$.

$$OC^2 = (OA + CA\cos\theta)^2 + (CA\sin\theta)^2,$$

$$= (OA + OB\cos\theta)^2 + (OB\sin\theta)^2,$$

or $\quad V^2 = (V_1 + V_2\cos\theta)^2 + V_2^2\sin^2\theta,$

$$= V_1^2 + 2V_1V_2\cos\theta + V^2(\cos^2\theta + \sin^2\theta),$$

$$= V_1^2 + V_2^2 + 2V_1V_2\cos\theta.$$

(*c*) Since $V_1$ and $V_2$ are constant, the maximum and minimum values of $V$ depend only on the maximum and minimum values of $\cos\theta$, which are $+1$ and $-1$ respectively.

Hence, $\underline{V_{max} = \sqrt{(V_1^2 + V_2^2 + 2V_1V_2)}}$, and

$$\underline{V_{min} = \sqrt{(V_1^2 + V_2^2 - 2V_1V_2)}}.$$

*Q8* (*a*) *Evaluate*

(*i*) $\int_{-1}^{2} (3x - 2)^2\, dx$, and (*ii*) $\int_4^6 \frac{dx}{2x - 7}$

(*b*) *Calculate*

(*i*) *the mean value, and* (*ii*) *the RMS value of the function $y = 5e^{-3x}$ from $x = 0$ to $x = 0{\cdot}5$.*

**A8** (*a*) (*i*) $\int_{-1}^{2} (3x - 2)^2\, dx = \int_{-1}^{2} (9x^2 - 12x + 4)\, dx$

$$= \left[\frac{9x^3}{3} - \frac{12x^2}{2} + 4x\right]_{-1}^{2}$$

$$= (3\times 2^3 - 6\times 2^2 + 8) - \{3\times(-1)^3 - 6\times(-1)^2 - 4\},$$

$$= 24 - 24 + 8 - (-3 - 6 - 4),$$

$$= 8 + 13 = \underline{21.}$$

(*ii*) $$\int_4^6 \frac{dx}{2x - 7} = \left[\frac{\log_e(2x - 7)}{2}\right]_4^6,$$

$$= \tfrac{1}{2}(\log_e 5 - \log_e 1),$$

$$= \tfrac{1}{2}(1{\cdot}60944 - 0),$$

$$= \underline{0{\cdot}80472.}$$

(*b*) (*i*) The mean value, $y_{mean}$, is given by

$$y_{mean} = \frac{\int_0^{0\cdot5} 5e^{-3x}\, dx}{0{\cdot}5},$$

$$= 10\left[\frac{e^{-3x}}{-3}\right]_0^{0\cdot5},$$

$$= -\frac{10}{3}(e^{-1\cdot5} - e^0),$$

$$= -\frac{10}{3}(0{\cdot}2231 - 1),$$

$$= \frac{7{\cdot}769}{3} = 2{\cdot}589\dot{6} \approx \underline{2{\cdot}59.}$$

(*ii*) The RMS value of $y$ is the square root of the mean value of $y^2$.

$$y^2 = (5e^{-3x})^2,$$

$$= 25e^{-6x}.$$

$$\therefore \quad y^2_{mean} = \frac{\int_0^{0\cdot5} 25e^{-6x}\, dx}{0{\cdot}5},$$

$$= 50\left[\frac{e^{-6x}}{-6}\right]_0^{0\cdot5},$$

$$= -\frac{25}{3}(e^{-3} - e^0),$$

$$= -\frac{25}{3}(0{\cdot}04979 - 1),$$

$$= \frac{25}{3} \times 0{\cdot}95021,$$

$$= 7{\cdot}9184.$$

$$\therefore \quad y_{RMS} = \sqrt{7{\cdot}9184} \approx \underline{2{\cdot}814.}$$

***Q9*** *(a) Use Simpson's rule with 7 ordinates to evaluate the integral*

$$\int_0^3 \frac{1+2x}{1+x^2}\,dx.$$

*Give the answer to 3 significant figures.*

*(b) Calculate the volume V generated by the rotation of $y = \cos x$ from $x = -\frac{\pi}{2}$ to $x = \frac{\pi}{2}$ about the x-axis.*

**A9** *(a)* The ordinates at $x = 0, 0{\cdot}5, 1 \ldots 3$ will total 7 and these are deduced in the following table:

| $x$ | 0 | 0·5 | 1·0 | 1·5 | 2·0 | 2·5 | 3·0 |
|---|---|---|---|---|---|---|---|
| $1+2x$ | 1 | 2 | 3 | 4 | 5 | 6 | 7 |
| $1+x^2$ | 1 | 1·25 | 2 | 3·25 | 5 | 7·25 | 10 |
| $\frac{1+2x}{1+x^2}$ | 1 | 1·6 | 1·5 | 1·2308 | 1 | 0·8276 | 0·7 |

Simpson's rule for the approximate evaluation of an area, $A$, under a curve states that, if the area is divided into an even number, $2n$, of strips of equal width, $h$, the area is given by

$$A = \frac{h}{3}\{y_1 + y_{2n+1} + 2(y_3 + y_5 + \ldots) + 4(y_2 + y_4 \ldots)\}$$

where $y_1, y_2 \ldots y_{2n+1}$ are the ordinates erected at the points of division of the strips.

Thus, referring to the table, $h = 0{\cdot}5$, $y_1 = 1$ and $y_{2n+1} = 0{\cdot}7$; the odd ordinates at $x = 1{\cdot}0$ and $x = 2{\cdot}0$ are $1{\cdot}5$ and 1 respectively and the remaining even ordinates are those at $x = 0{\cdot}5$, $1{\cdot}5$ and $2{\cdot}5$.

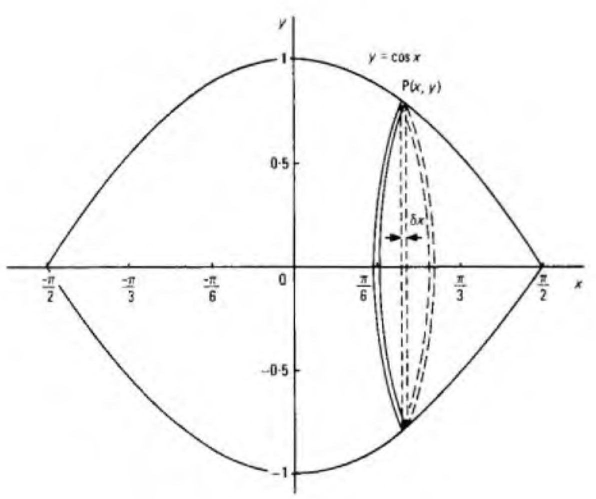
Hence, $\int_0^3 \frac{1+2x}{1+x^2}\,dx \approx \frac{0{\cdot}5}{3}\{1 + 0{\cdot}7 + 2(1{\cdot}5 + 1{\cdot}0) + 4(1{\cdot}6 + 1{\cdot}2308 + 0{\cdot}8276)\}$,

$= \frac{1}{6}(1{\cdot}7 + 5{\cdot}0 + 4 \times 3{\cdot}6584)$,

$= \frac{1}{6}(6{\cdot}7 + 14{\cdot}6336) = 3{\cdot}5556$,

$= \underline{3{\cdot}56}$ correct to 3 significant figures.

*(b)* The curve of $y = \cos x$ from $x = -\frac{\pi}{2}$ to $x = +\frac{\pi}{2}$ is shown in the sketch.




When the curve is rotated through 360° about the $x$-axis, it will generate a solid figure as depicted in the sketch. Any point $P(x, y)$ on the curve will generate a circle of radius $y$.

Consider a second point very close to $P$, with an abscissa of $(x + \delta x)$. The 2 points together generate a thin circular lamina whose volume is approximately $\pi y^2 \delta x$.

The total volume generated from $x = -\frac{\pi}{2}$ to $x = +\frac{\pi}{2}$ is therefore

$\sum_{x=-\pi/2}^{x=\pi/2} \pi y^2 \delta x$ and this is given by

$\int_{-\pi/2}^{\pi/2} \pi y^2\,dx$ in the limit as $\delta x \to 0$.

$\therefore$ the volume generated $= \pi \int_{-\pi/2}^{\pi/2} \cos^2 x\,dx$,

$= \pi \int_{-\pi/2}^{\pi/2} \frac{1+\cos 2x}{2}\,dx$ (since $\cos^2 2x = 2\cos x - 1$),

$= \frac{\pi}{2}\left[x + \frac{\sin 2x}{2}\right]_{-\pi/2}^{\pi/2}$,

$= \frac{\pi}{2}\left\{\frac{\pi}{2} + \frac{\sin \pi}{2} - \left[-\frac{\pi}{2} + \frac{\sin(-\pi)}{2}\right]\right\}$,

$= \frac{\pi}{2}(\pi) = \underline{4{\cdot}935.}$

***Q10*** *(a) If $z = \frac{3 - j4}{1 + j2}$*

*(i) express z in the polar form $r \angle \theta$, and*
*(ii) display in a complex plane diagram the complex numbers z, j(3z), and $z^2$ as phasors.*

*(b) Evaluate in the form $X + jY$ the phasor*

$$Z = R + \frac{1}{(j\omega C + 1/j\omega L)},$$

*where $R = 600$, $C = 2{\cdot}4 \times 10^{-8}$, $L = 8 \times 10^{-3}$ and $\omega = 10^5$.*
*What is the phase angle of this phasor?*

**A10** *(a) (i)*

$z = \frac{(3 - j4)(1 - j2)}{(1 + j2)(1 - j2)}$,

$= \frac{3 - j6 - j4 - 8}{1 + 4}$,

$= \frac{-5 - j10}{5}$,

$= -1 - j2$,

$= \sqrt{\{(-1)^2 + (-2)^2\}} \angle \tan^{-1}\frac{-2}{-1}$,

$= \sqrt{5} \angle (180° + 63° 26')$,

$= \underline{2{\cdot}236 \angle 243° 26'.}$

*(ii)* $j(3z) = j3(2{\cdot}236 \angle 243° 26')$,

$= 6{\cdot}708 \angle (243° 26' + 90°)$,

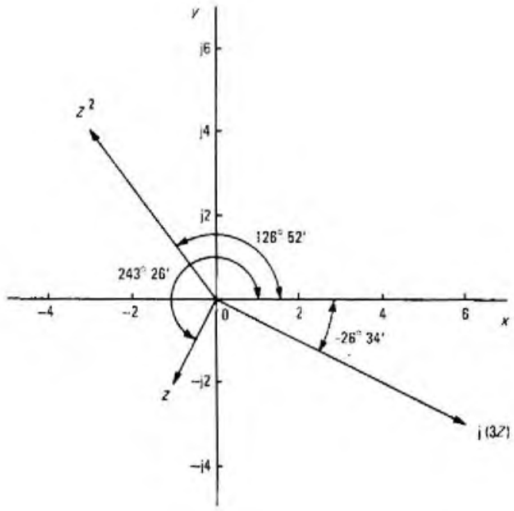
$= 6{\cdot}708 \angle 333° 26'$ or $6{\cdot}708 \angle -26° 34'$.

$z^2 = (\sqrt{5} \angle 243° 26')^2$,

$= 5 \angle 486° 52'$,

$= 5 \angle (360° + 126° 52')$.

The 3 phasors are shown in the sketch.




(*b*)
$$Z = R + \frac{1}{j\omega C + 1/j\omega L},$$
$$= R + \frac{j\omega L}{1 - \omega^2 LC}.$$
$$\omega L = 10^5 \times 8 \times 10^{-3},$$
$$= 800.$$
$$1 - \omega^2 LC = 1 - 10^{10} \times 8 \times 10^{-3} \times 2{\cdot}4 \times 10^{-8},$$
$$= 1 - 1{\cdot}92 = -0{\cdot}92.$$
$$\therefore \quad Z = 600 + \frac{j800}{-0{\cdot}92},$$
$$= 600 - j869{\cdot}565.$$

The angle of the phasor is given by

$$\tan^{-1}\frac{-869{\cdot}565}{600} = \tan^{-1}(-1{\cdot}4493),$$
$$= -55° \, 24'.$$

Thus, $Z = \underline{600 - j869{\cdot}6}$ and its phase angle is $\underline{-55° \, 24'}$.

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## LINE TRANSMISSION C 1978

**Students were expected to answer any 6 questions**

***Q1*** (*a*) *Write down the expression for the characteristic impedance of a uniform transmission line in terms of its primary coefficients.*

(*b*) *Calculate the characteristic impedance of a uniform line at $\omega = 5000$ rad/s given that $R = 40\ \Omega$/km, $L = 1{\cdot}5$ mH/km, $C = 0{\cdot}1\ \mu$F/km and $G = 1\ \mu$S/km.*

(*c*) *Show how the characteristic impedance of a line can be obtained from two separate impedance measurements.*

**A1** (*a*) For a uniform transmission line, the characteristic impedance, $Z_0$, is given by the expression

$$Z_0 = \sqrt{\left(\frac{R + j\omega L}{G + j\omega C}\right)} \text{ ohms},$$

where $R$ is the loop resistance in ohms/kilometre, $L$ is the loop inductance in henrys/kilometre, $G$ is the loop leakance in siemens/kilometre, $C$ is the capacitance in farads/kilometre and $\omega$ is the angular velocity in radians/second.

(*b*) Substituting the given values,

$$R + j\omega L = 40 + j7{\cdot}5 = 40{\cdot}70 \angle 10{\cdot}6°, \text{ and}$$
$$G + j\omega C = 10^{-6}(1 + j500) = 500 \times 10^{-6} \angle 89{\cdot}9°.$$
$$\therefore \quad |Z_0| = \sqrt{\left(\frac{40{\cdot}70}{500 \times 10^{-6}}\right)} = 285{\cdot}3\ \Omega, \text{ and}$$
$$\arg Z_0 = \frac{10{\cdot}6 - 89{\cdot}9}{2} = -39{\cdot}6°.$$
$$\therefore \quad Z_0 = \underline{285{\cdot}3 \angle -39{\cdot}6°}.$$

(*c*) The characteristic impedance of a transmission line can be calculated from bridge measurements of the input impedance when the far end is (*i*) short-circuited ($Z_{SC}$) and (*ii*) open-circuited ($Z_{OC}$). Under these circumstances,

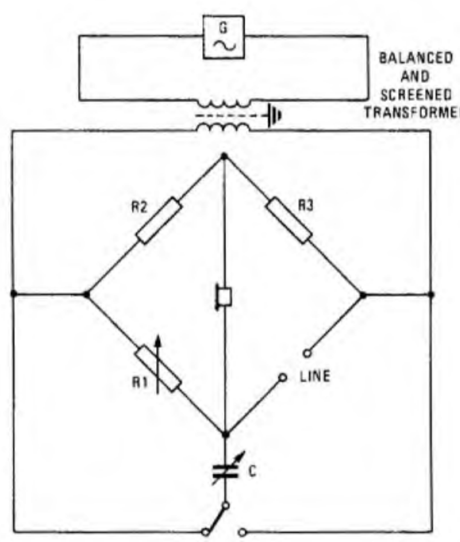
$$Z_0 = \sqrt{(Z_{SC} Z_{OC})} \text{ ohms}.$$

***Q2*** (*a*) *Draw the circuit of a bridge suitable for measuring the impedance of an audio-frequency cable pair.*

(*b*) *Derive an expression for the modulus and angle of the measured impedance in terms of the bridge components.*

(*c*) *Say what precautions are necessary to ensure a reasonable degree of accuracy.*

**A2** (*a*) The sketch shows the circuit of a bridge suitable for measuring the impedence of an audio-frequency cable pair. R2 and R3 are fixed ratio arms (typically 100 Ω each) and C a variable capacitor which can be switched across the variable resistance R1 when the line impedance being measured has a small negative angle, or across the line itself when its impedance has a small positive angle.




A high-impedence telephone receiver acts as the detector, and the frequency source is connected to the bridge through a balanced and screened transformer.

(*b*) Assume the bridge to be balanced with C switched as shown in the sketch and, if $Z_L$ is the impedance of the line and $Y$ is the impedance of $C$ in parallel with $R_1$

$$R_2 Z_L = R_3 Y.$$
$$\therefore \quad Z_L = \frac{R_3}{R_2} Y.$$

But
$$Y = \frac{\dfrac{R_1}{j\omega C}}{R_1 + \dfrac{1}{j\omega C}} = \frac{R_1}{1 + j\omega C R_1}.$$

Rationalization gives

$$Y = \frac{R_1 - j\omega C R_1^2}{1 + \omega^2 C^2 R_1^2} = \frac{R_1}{1 + \omega^2 C^2 R_1^2} - \frac{j\omega C R_1^2}{1 + \omega^2 C^2 R_1^2}.$$
$$\therefore \quad Z_L = \frac{R_3}{R_2}\left(\frac{R_1}{1 - \omega^2 C^2 R_1^2} - \frac{j\omega C R_1^2}{1 + \omega^2 C^2 R_1^2}\right) \text{ ohms},$$

or in polar form $|Z_L| = \frac{R_3}{R_2}\frac{R_1}{\sqrt{(1+\omega^2C^2R_1^2)}}$, and

$$\arg Z_L = \tan^{-1}(-\omega CR_1).$$

If however, C is switched across the line it can be shown that

$$Z_L = \frac{R_1R_2R_3}{R_2^2+\omega^2C^2R_3^2R_1^2} - \frac{j\omega CR_3^2R_1^2}{R_2^2+\omega^2C^2R_3^2R_1^2},$$

or in polar from $|Z_L| = \frac{R_1R_3}{\sqrt{(R_2^2+\omega^2C^2R_3^2R_1^2)}}$, and

$$\arg Z_L = \tan^{-1}\frac{R_3}{R_2}\omega CR_1.$$

(*c*) In order to ensure the best degree of accuracy the following precautions are necessary.

(*i*) Each component must be screened and all the screens connected to a common earth point.

(*ii*) The oscillator must be connected to the bridge through a balanced and screened transformer.

(*iii*) The oscillator must be free from harmonics.

(*iv*) The telephone receiver used as the detector must be of high quality.

(*v*) All variable components must be properly calibrated.

*Q3* (*a*) *Define balance return-loss.*

(*b*) *Explain how balance return-loss can be measured.*

(*c*) *Calculate the balance return-loss between a line having a characteristic impedance of* $(500 - j100)$ Ω *and a balancing network having an impedance of* $600 \angle 30°$ Ω.

**A3** (*a*) Balance return-loss is a measure of the accuracy with which the impedance of the balancing network of a 2-wire–4-wire termination (hybrid transformer) matches the impedance of the 2-wire line.

If $Z_0$ is the impedance of the line in ohms, and $Z_B$ is the impedance of the balancing network in ohms, then the balance return-loss is

$$20\log_{10}\left|\frac{Z_0+Z_B}{Z_0-Z_B}\right| \text{ decibels.} \qquad \ldots\ldots (1)$$

This shows that if $Z_0 = Z_B$, then the balance return-loss is infinite and therefore no reflection occurs.

(*b*) The sketch shows how a hybrid transformer can be arranged to measure balance return-loss.

A voltmeter reading ($V_1$) is first obtained with the balancing network disconnected. A further reading ($V_2$) is then obtained with the balancing network connected. The balance return-loss is then given by

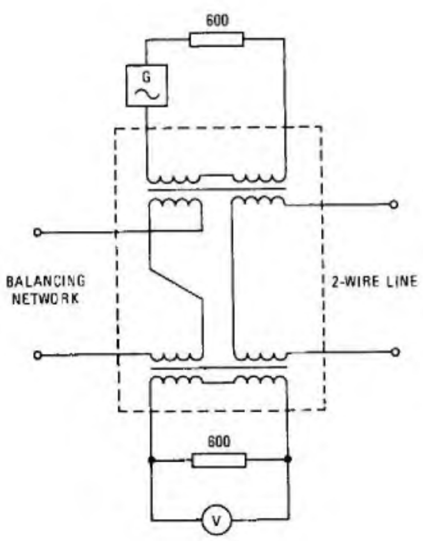
$$20\log_{10}\frac{V_1}{V_2} \text{ decibels.}$$

(*c*) $600\angle 30° = 600(\cos 30° + j\sin 30°)$,

$= 520 + j300$ Ω.

From equation (1), the balance return-loss is therefore

$$20\log_{10}\left|\frac{500 - j100 + 520 + j300}{500 - j100 - 520 - j300}\right|,$$




$$= 20\log_{10}\left|\frac{1020 + j200}{-20 - j400}\right|,$$

$$= 20\log_{10}\sqrt{\left(\frac{1020^2+200^2}{20^2+400^2}\right)},$$

$= 20\log_{10} 2\cdot595 = \underline{8\cdot28 \text{ dB.}}$

*Q4* *A cable pair has the following primary coefficients:* $R = 30$ Ω/km, $L = 1$ mH/km, $C = 0\cdot1$ μF/km. *G can be neglected. The pair is loaded with 88 mH coils at 2 km intervals.*

(*a*) *Calculate the attenuation of the loaded pair at* $\omega = 5000$ *rad/s, assuming the resistance of the coils to be zero.*

(*b*) *Calculate the theoretical cut-off frequency.*

(*c*) *Estimate the upper usable frequency.*

**A4** (*a*) The propagation coefficient $\gamma = \sqrt{\{(R + j\omega L)(G + j\omega C)\}} = \alpha + j\beta$, where $R$ is the loop resistance in ohms/kilometre, $\omega$ is the angular velocity in radians/second, $L$ is the loop inductance in henrys/kilometre, $G$ is the loop leakance in siemens/kilometre, $C$ is the loop capacitance in farads/kilometre, $\alpha$ is the attenuation coefficient in nepers/kilometre and $\beta$ is the phase-change coefficient in radians/kilometre.

$R + j\omega L = 30 + j5000(1\times10^{-3} + 44\times10^{-3}) = 30 + j225$,

$= 227\angle\tan^{-1}7\cdot5 = 227\angle 82°\,24'$ Ω.

$G + j\omega C = 0 + j5000\times0\cdot1\times10^{-6} = 0 + j5\times10^{-4}$,

$= 5\times10^{-4}\tan^{-1}\infty = 5\times10^{-4}\angle 90°$ S.

$\therefore\ \gamma = \sqrt{\{(227\angle 82°\,24')(5\times10^{-4})\angle 90°)\}}$

$= \sqrt{(11\cdot35\times10^{-2}\angle 172°\,24')}$,

$= 0\cdot337\angle 86°\,12'$.

$\therefore\ \alpha = 0\cdot337\cos 86°\,12' = \underline{0\cdot02 \text{ Np/km.}}$

(*b*) A coil-loaded line acts as a low-pass filter with series-connected inductances and shunt capacitances. The cut-off frequency of such a filter ($f_c$ hertz) is given by

$f_c = \frac{1}{\pi d\sqrt{LC}}$ hertz, where $d$ is the distance between the coils.

$$\therefore\ f_c = \frac{1}{\pi 2\sqrt{\{(1\times10^{-3}+44\times10^{-3})(0\cdot1\times10^{-6})\}}},$$

$$= \frac{1}{42\cdot15\times10^{-5}} = \underline{2\cdot372 \text{ kHz.}}$$

(*c*) The cut-off is not sharp and therefore the upper usable frequency will be approximately 80% of the theoretical cut-off frequency; that is, $\underline{1\cdot898 \text{ kHz.}}$

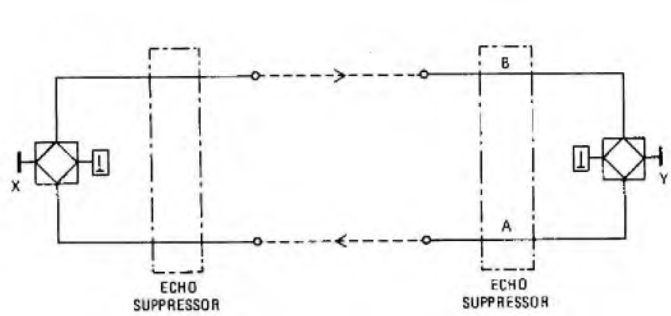
*Q5* (*a*) *Outline the function of an echo suppressor.*

(*b*) *Explain the basic principle of its operation.*

(*c*) *On what kind of circuit is an echo suppressor needed?*

**A5** (*a*) The sketch shows a 4-wire connexion between 2 subscribers, X and Y. If the balance network perfectly matched the 2-wire termination at the terminating set, the loss across the terminating set would be infinite. However, this ideal is impossible to meet in practice and therefore part of any signal originating at X passes from the RECEIVE line at Y and is transmitted back to X.

If the connexion is electrically short, and therefore the delay time is small, X hears the reflected signal as enhanced sidetone. However, if the connexion is electrically long, X hears the reflected signal as an echo of his own voice delayed by the loop-delay time of the circuit. Depending on the delay and the level of the reflected signal, the echo can make conversation very difficult. Therefore echo suppressors are inserted in the line to reduce the level of the reflected signal to negligible proportions.



(*b*) When subscriber X is talking and the signal at Y exceeds a preset value, a 60 dB attenuator is inserted in the return line from Y (shown as point A). However, this would prevent subscriber Y interrupting subscriber X. Therefore, if subscriber Y attempts to interrupt, the 60 dB attenuator is replaced by 6 dB attenuators at points A and B. Thus, under these conditions, each speech path is attenuated by 6 dB but the echo path by 12 dB. This condition remains until only one subscriber is talking, when the 60 dB attenuator is re-inserted in the appropriate speech path. Similar conditions apply to speech from Y to X.

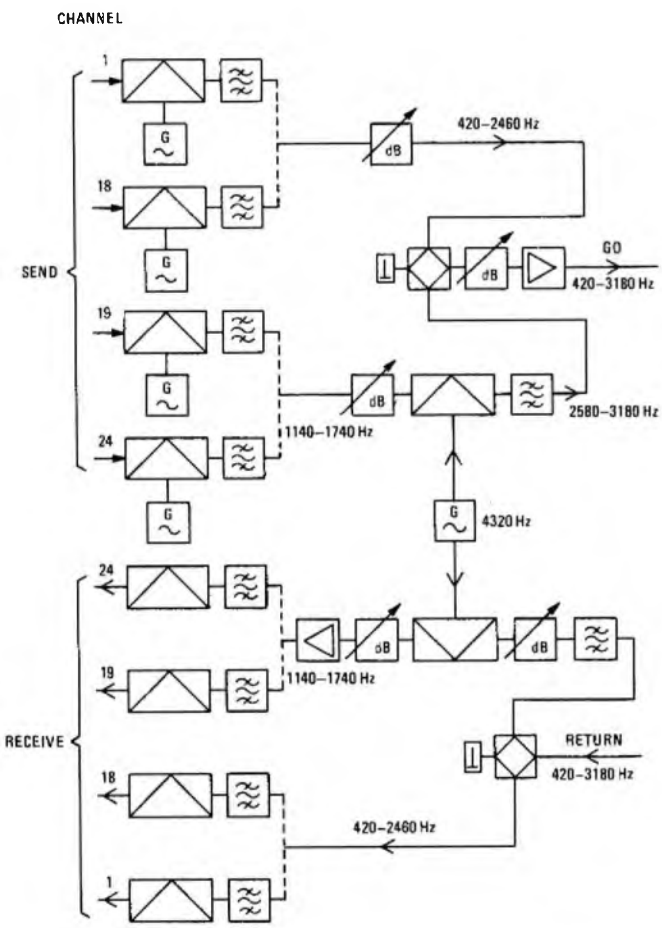
(*c*) Originally, echo suppressors were used on long 4-wire audio circuits. Since these have now been largely replaced by high-velocity carrier circuits, the necessity for echo suppressors on circuits within the UK has disappeared. In the UK their use is limited to international circuits. However, in countries where propagation times are significant, echo suppressors are used on inland circuits.

*Q6* (*a*) *Draw the block schematic diagram for one end of a 24-channel voice-frequency telegraph system.*

(*b*) *Explain the function of each block.*

(*c*) *State how and why the requirements for the channel filters differ from those of a multi-channel carrier-telephone system.*

**A6** (*a*) The block diagram is shown in the sketch.




(*b*) A 24-channel voice-frequency telegraph system consists of an 18-channel system in the frequency range 420–2460 Hz together with a 6-channel system in the range 1140–1740 Hz which is then modulated into the frequency range 2580-3180 Hz to give line frequencies in the range 420–3180 Hz.

**Transmit** Each channel consists of a modulator (or static relay) where the channel carrier frequency is either shunted by a low impedance when the sending teleprinter is sending a SPACE element, or allowed to pass to line when a MARK element is being transmitted. A band-pass filter in each TRANSMIT channel provides separation between the channels.

For the 18-channel system, an attenuator then reduces the input to the terminating unit to a suitable level. For the 6-channel system, an attenuator reduces the level to a suitable value prior to it modulating a 4320 Hz carrier to produce a band in the range 2580–3180 Hz. The 6-channel system then passes through a band-pass filter which provides separation from the 18-channel system. The two systems are then combined in the terminating unit. The combined 24 channels then pass through an attenuator and the SEND amplifier to line.

**Receive** The received signals are separated by band-pass filters into the individual channels for channels 1–18 and fed through individual amplifiers for transmission to the respective teleprinter. However for channels 19–24 the signals are first demodulated as a block then attenuated and passed into their individual channels via band-pass filters and amplified to the receiving teleprinter.

(*c*) In any carrier system, it is essential to make the best use of the available bandwidth, and the cost of channel filters is an important consideration. For a telegraph system, the required capacity can be obtained by using fairly inexpensive channel filters with a gradual cut-off characteristic, because the bandwidth needed for each channel is comparatively small. For a carrier-telephony system, however, a much larger bandwidth is required for each channel, and it becomes economic to provide more expensive filters with a sharper cut-off characteristic, in order to make the best use of the frequency spectrum available.

*Q7* (*a*) *Describe the various tests which would be needed before a new design of telephone set could be accepted for installation in an existing network. Distinguish between subjective tests and objective tests.*

(*b*) *State what is meant by side tone and explain its effect.*

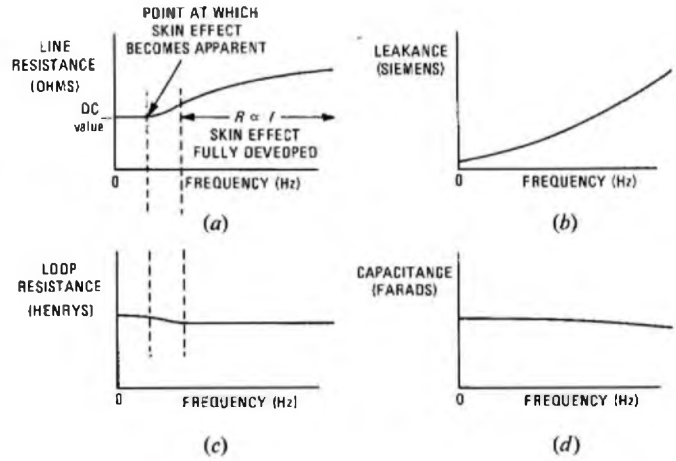
*Q8* *A coaxial cable designed for use up to 60 MHz has a solid copper inner conductor and an outer conductor fabricated from copper tape. The inner conductor is located centrally within the outer conductor by polyethylene discs attached to the centre conductor at regular intervals.*

(*a*) *Make sketches to show how the primary coefficients of this cable would vary over the frequency range 0–60 MHz.*

(*b*) *Explain why these variations occur.*

(*c*) *Sketch the attenuation/frequency characteristic of the cable.*

**A8** (*a*) The primary coefficients of a coaxial cable of the kind described vary with frequency as shown in sketches (*a*)–(*d*).



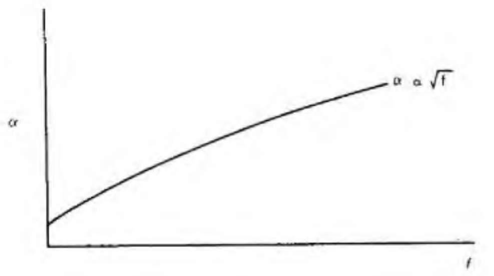

(*b*) **Loop Resistance** (*R*) This is the resistance of the inner conductor and outer conductor in series. At zero frequency the current is distributed uniformly over the cross-section of each conductor, but, as the frequency is increased, the skin effect becomes apparent and the effective cross-sectional area is reduced. The current shows an increasing tendency to confine itself to the outer surface of the inner conductor and the inner surface of the outer conductor. For high-frequency working it is thus important that the conductor surfaces should be smooth and uncontaminated.

**Loop Inductance** (*L*) Each conductor of a coaxial pair has self-inductance because the magnetic field produced by the current in any conductor links with the conductor itself. At high frequencies, the skin effect tends to reduce the effective cross-sectional area and the number of flux linkages falls, thus reducing the self-inductance.

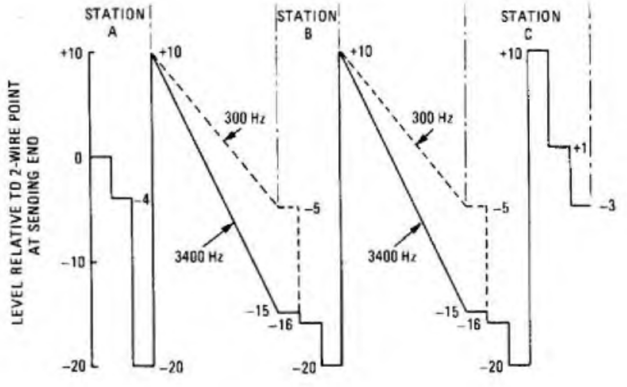
**Loop Leakance** (*G*) At zero frequency the leakance is the reciprocal of the insulation resistance. As the operating frequency rises, an increasing amount of energy is absorbed in the dielectric and the increase in dielectric loss causes an increase in the effective loop leakance.

**Loop Capacitance** (*C*) The capacitance between the inner and outer conductor tends to remain substantially independent of frequency. For a coaxial pair with polyethylene discs to locate the centre conductor, the capacitance would be influenced by the nature and quality of the polyethylene itself.

(*c*) The attenuation/frequency ($\alpha/f$) characteristic is shown in sketch (*e*).




(*e*)




(*b*)

*Q9 A 4-wire audio circuit is to be provided between 2 terminal repeater stations with one intermediate repeater. The overall loss is to be 3 dB between 2-wire points. The cable pairs have a total attenuation between terminal stations of 50 dB at 3400 Hz and 30 dB at 300 Hz.*

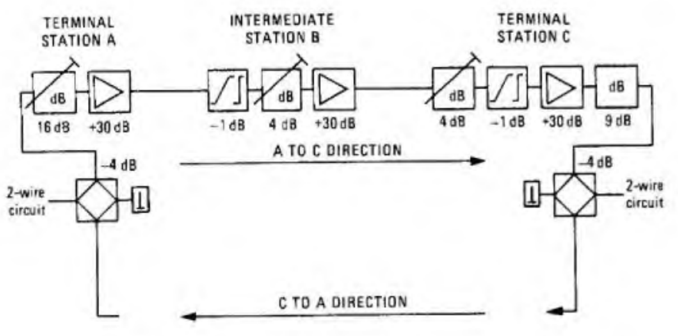
(*a*) *Draw a block schematic diagram to show the various items of equipment which would be needed.*

(*b*) *Explain the function of* EACH *item.*

(*c*) *Draw a level diagram for* ONE *direction of transmission.*

(*d*) *Explain the need for imposing upper and lower limits on planning levels.*

**A9** (*a*) Sketch (*a*) shows the block diagram of the arrangement of the various items of equipment needed to provide the circuit in one direction. The other direction is similar, but is omitted for simplicity.




(*a*)

(*b*) The 2-wire points are connected to the 4-wire circuit by means of hybrid transformers. These combine the unidirectional paths of the 4-wire section with the bidirectional paths of the 2-wire sections. Each hybrid transformer has a loss of approximately 4 dB from 2-wire to 4-wire.

Each repeater comprises a 30 dB fixed-gain amplifier and an adjustable attenuator which is used to vary the gain of the repeater. These are adjusted to the values shown. At the intermediate and receiving stations (stations B and C respectively), line equalizers are provided to compensate for the sloping attenuation/frequency characteristic of the preceding section of cable, thus making the attenuation of the circuit substantially independent of the frequency. Each equalizer introduces a basic loss of approximately 1 dB. A fixed 9 dB attenuator is inserted between the amplifier and hybrid transformer at the receiving station in order to reduce the signal level at the 2-wire point to the specified value.

(*c*) Sketch (*b*) shows the level diagram for one direction of transmission.

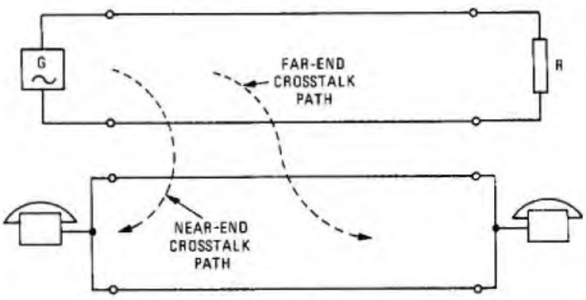
(*d*) It is necessary to impose limits on the signal level because, if the level is too high, crosstalk can arise between adjacent circuits and, if it is too low, the signal-to-noise ratio becomes so low that the signal is distorted.

A typical range of signal levels is from +10 dB to −20 dB relative to the sending 2-wire point.

*Q10* (*a*) *Explain what is meant by near-end crosstalk and far-end crosstalk.*

(*b*) *Describe the testing and balancing procedures required for a star-quad junction cable which is to be loaded and used for audio transmission.*

**A10** (*a*) Sketch (*a*) shows 2 circuits in a transmission system. One of these is active while the other is not. If there is any form of electrical coupling between the 2 circuits, part of the signal in the active circuit appears in the quiescent circuit.

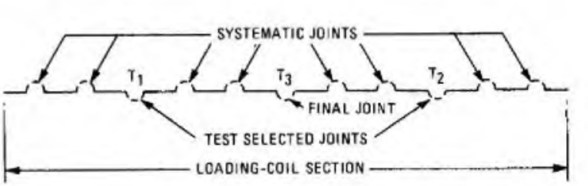



(*a*)

The induced signal which appears in the disturbed circuit in a direction opposite to the direction of propagation in the disturbing circuit is known as *near-end crosstalk*. When crosstalk is propagated in a disturbed channel in the same direction as the direction of propagation in the disturbing channel it is called *far-end crosstalk*.

The main sources of crosstalk are capacitance unbalance, resistance unbalance, inductive coupling, wire-to-wire contacts and low insulation resistance. Sound design and the use of good manufacturing techniques to ensure uniformity minimize some of these factors, but careful installation and good maintenance minimize the others.

(*b*) For audio cables, a loading-coil section forms the basic unit line balancing length and is composed of 12 manufactured lengths. Referring to sketch (*b*), 3 joints are systematically jointed; that is, they are jointed according to a schedule whereby adjacent quads are joined to remote quads in the next length. The 4 cable lengths thus formed are then jointed together by means of test-selected joints, T1 and T2. At these jointing positions, measurements are made of the capacitance unbalances (in magnitude and sign) between pairs in each quad, and the quads jointed to other quads having similar unbalances such that the unbalances cancel out. The final joint, T3, is made at the centre of the loading-coil section which is a test selected joint.




(*b*)

**Students were expected to answer any 6 questions**

*Q1* (*a*) *What is frequency modulation?*

(*b*) *A 50 V peak, sinusoidal carrier having a frequency of 2 MHz is frequency modulated by a signal $e_s = 2 \cdot 5 \sin(2\pi\, 500t)$ volts. The modulated wave has a peak frequency deviation of 12·5 kHz.*

*Determine*

(*i*) *the modulator sensitivity in hertz/volt of signal,*
(*ii*) *the peak phase deviation,*
(*iii*) *an expression for the instantaneous voltage of the modulated wave, and*
(*iv*) *an estimate of the bandwidth required for transmission.*

(*c*) *The signal is changed to $e_s = 6 \cos(2\pi\, 100t)$ volts.*
*Determine the consequent expression for the instantaneous voltage of the modulated wave.*

**A1** (*a*) In frequency modulation, the instantaneous frequency deviation of a carrier wave is made proportional to the instantaneous magnitude of the modulating signal.

(*b*) The equation of a frequency-modulated wave when the modulating signal is a sine wave is derived in the following manner.

Let $e_s = \hat{E}_s \sin\theta$, where $e_s$ is the instantaneous magnitude of the signal, $\hat{E}_s$ is the peak value of the signal and $\theta$ is proportional to time. Then the frequency deviation is given by $k_f \hat{E}_s \sin\theta$, where $k_f$ is the modulator sensitivity. Hence, the instantaneous frequency, $f_i$, can be written

$$f_i = f_c + k_f \hat{E}_s \sin\theta$$

where $f_c$ is the carrier frequency.

The instantaneous phase angle, $\omega_i$, is twice the integral, with respect to time, of this expression.

Thus,
$$\omega_i = 2\pi f_c t + 2\pi \int k_f \hat{E}_s \sin\theta \, dt,$$
$$= \omega_c t + \frac{k_f \hat{E}_s}{f_s} \cos \omega_s t$$

where $\theta = 2\pi f_s t$ and $f_s$ is the modulating frequency in hertz. The value of the frequency modulated wave at any instant would be

$$E_c \sin\left(\omega_c t + \frac{k_f \hat{E}_s}{f_s} \cos \omega_s t\right).$$

(*i*) Now, $k_f \hat{E}_s$ is the peak frequency deviation and is equal to $12 \cdot 5 \times 10^3$ Hz. Also, $\hat{E}_s = 2 \cdot 5$ V, and thus

$$k_f = \frac{12 \cdot 5 \times 10^3}{2 \cdot 5} = \underline{5 \text{ kHz/V}}.$$

(*ii*) Peak phase deviation $= \dfrac{k_f \hat{E}_s}{f_s} = \dfrac{5000 \times 2 \cdot 5}{500} = \underline{25 \text{ rad}}.$

(*iii*) The expression for the modulated wave, using the values given is

$$E_{mod} = 50 \sin\{4\pi \times 10^6 t + 25 \cos(2\pi\, 500t)\}.$$

(*iv*) The modulation index is large, and thus the bandwidth required is obtained from the expression $B = 2(m + 1) f_s$, where $B$ is the bandwidth and $m$ is the modulation index. Thus, substituting the values given,

$$B = 2(25 + 1) \times 500 \approx \underline{25 \text{ kHz}}.$$

(*c*) For the changed modulating signal, the modulated wave becomes

$$\underline{50 \sin\{4\pi \times 10^6 t + 300 \cos(2\pi\, 100t)\}}$$

since the new modulation index is $\dfrac{5000 \times 6}{100} = 300$.

*Q2* (*a*) *A fraction of the output voltage from an amplifier is connected as negative feedback, in series with the input. Write down expressions which relate*

(*i*) *the voltage gain,*
(*ii*) *the input impedence, and*
(*iii*) *the output impedence*

*to the corresponding parameters for the amplifier without feedback.*

*Define the symbols used in your expressions.*

(*b*) *An amplifier has an open-circuit gain of 4000 and an output resistance of 1·5 kΩ without feedback. Two per cent of the output voltage can be fed back, as negative feedback, in series with the amplifier input. A 1·5 mV signal is applied to the amplifier input and its output is connected to a 100 Ω load. Calculate the voltage across the load*

(*i*) *without feedback, and*
(*ii*) *with feedback.*

**A2** (*a*) (*i*) $$A' = \frac{1}{1 + \beta A}.$$

(*ii*) $$Z'_{in} = Z_{in}(1 + \beta A).$$

(*iii*) $$Z'_{out} = \frac{1}{1 + \beta A}.$$

The symbols used are

$A$ represents the gain without feedback,
$Z_{in}$ represents the input impedance without feedback,
$Z_{out}$ represents the output impedance without feedback,
$A'$ represents the gain with feedback,
$Z'_{in}$ represents the input impedance with feedback,
$Z'_{out}$ represents the output impedance with feedback, and
$\beta$ represents the feedback factor.

The particular relationships given for $Z_{in}$ and $Z_{out}$ refer to the type of feedback specified in the question.

(*b*) (*i*) With no feedback, $V_{in} = 1 \cdot 5$ mV.

$$\therefore \quad V_{out} = \frac{4000 \times 1 \cdot 5}{1000} \times \frac{100}{1600} = \underline{375 \text{ mV}}.$$

(*ii*) With feedback, $V_{in} = 1 \cdot 5 \times 10^{-3} - 0 \cdot 02 V_{out}$, and

$$V_{out} = 4000 V_{in} \times \frac{100}{1600},$$
$$= \frac{4000 \times 100}{1600}(1 \cdot 5 \times 10^{-3} - 0 \cdot 02 V_{out}).$$

$$\therefore \quad V_{out} = \underline{27 \cdot 7 \text{ mV}}.$$

*Note:* Alternatively, the gain and output impedance with feedback could have been calculated and the equivalent circuit derived.

*Q3* (*a*) *Explain the meaning of the following terms applied to uniform transmission lines:*

(*i*) *characteristic impedance,*
(*ii*) *wavelength, and*
(*iii*) *velocity of propagation.*

(*b*) *A 20 m length of coaxial cable is supplied at one end from a variable-frequency source, and the other end is terminated in the characteristic impedance. With the source frequency set to 100 MHz, the input and output voltages of the cable are in phase. The source frequency is then increased and the next frequency at which the output and input voltages are again in phase is 112·5 MHz. Assuming the velocity of propagation to be invariable with frequency, determine*

(*i*) *the wavelength at 100 MHz,*
(*ii*) *the wavelength at 112·5 MHz,*
(*iii*) *the velocity of propagation, and*
(*iv*) *the frequency between 100 MHz and 112·5 MHz at which the input and output voltages are in antiphase.*

**A3** (*a*) (*i*) The characteristic impedance of a transmission line is the same as the input impedance of an infinitely long piece of the same transmission line.

(*ii*) The wavelength of a transmission line is the length of the line necessary to produce a phase shift of $2\pi$ rad.

(*iii*) The velocity of propagation is the rate at which energy is transmitted.

(*b*) Let the 20 m length correspond to $n$ complete wavelengths at 100 MHz. Then, at 112·5 MHz the length corresponds to $(n + 1)$ wavelengths.

The velocity of propagation, $v$, is given by the expression $v = f\lambda$, where $f$ is the frequency in hertz and $\lambda$ is the wavelength in metres.

Since the velocity of propagation is independent of the frequency,

$$100 \times 10^6 \times \frac{20}{n} = 112 \cdot 5 \times 10^6 \times \frac{20}{n + 1}.$$

$$\therefore \quad n = 8.$$

(*i*) The wavelength at 100 MHz $= \dfrac{20}{8} = \underline{2 \cdot 5 \text{ m}}.$

(*ii*) The wavelength at 112·5 MHz $= \dfrac{20}{9} = \underline{2 \cdot 22 \text{ m}}.$

(*iii*) The velocity of propagation $= 100 \times 10^6 \times 2 \cdot 5,$
$= \underline{250 \times 10^6 \text{ m/s}}.$

(*iv*) If $f_a$ is the frequency between 100 MHz and 112·5 MHz at which the input and output voltages are in antiphase, and $\lambda_a$ is the wavelength,

$$\lambda_a = \frac{20}{8 + \frac{1}{2}}.$$

$$\therefore \quad f_a = 250 \times 10^6 \times \frac{8\cdot 5}{20} = \underline{106\cdot 25\ \text{MHz.}}$$

**Q4** *Write a short account of inductors and their requirements at various frequencies from thousands of hertz to tens of megahertz. Include discussion of Q-factor and the means by which acceptable Q-factors are obtained. Explain the factors which cause losses and the techniques used to keep the losses low.*

**Q5** (*a*) *Describe a laboratory method for determining the Thévénin equivalent circuit for a source with a resistive internal impedence.*

(*b*) *Obtain an equivalent circuit of the network shown in Fig. 1, referred to terminals AB. Hence, or otherwise, determine the magnitude of the current in an impedence of (30 + j180) Ω connected across AB.*

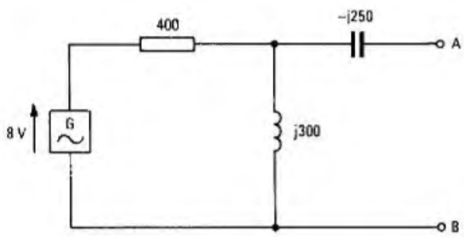
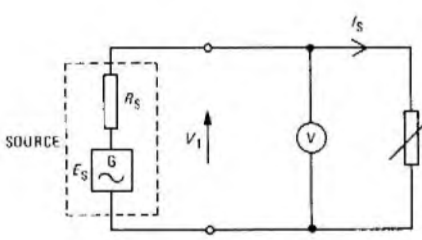



*Fig. 1*

**A5** (*a*) The Thévénin equivalent of a source can be determined using the circuit shown in sketch (*a*). Readings of the source output voltage, $V_1$, are taken for various values of load resistance, $R_1$. A graph of $1/V_1$ against $1/R_1$ is then plotted and the required values determined in the following manner.




(*a*)

$$V_1 = E_s - I_s R_s = E_s - \frac{V_1}{R_1} R_s.$$

$$E_s = V_1 \left(\frac{R_1 + R_s}{R_1}\right).$$

$$\therefore \quad V_1 = \frac{R_1}{R_1 + R_s} E_s.$$

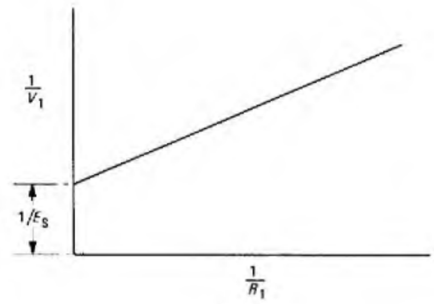
$$\therefore \quad \frac{1}{V_1} = \frac{1}{E_s} + \frac{R_s}{R_1 E_s}. \qquad \ldots\ldots(1)$$

From equation (1), the graph of $\frac{1}{V_1}$ against $\frac{1}{R_1}$ will be of the form

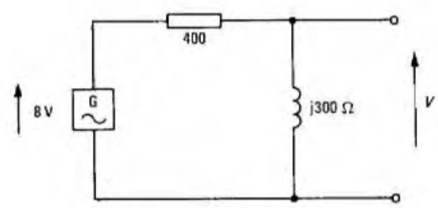
$$y = mx + c$$

and is shown in sketch (*b*). Thus, the slope of the graph is $\frac{R_s}{E_s}$ and the intercept is $\frac{1}{E_s}$ and from these the values required for the Thévénin equivalent circuit can be determined.

(*b*) Consider the circuit formed by the resistor and the inductor as shown in sketch (*c*).




(*b*)




(*c*)

$V$ is the open-circuit voltage and is also the Thévénin source voltage.

$$V = 8 \times \frac{j300}{400 + j300} = \frac{8(j300)(400 - j300)}{2500},$$

$$= \frac{72 + j96}{25} = 2\cdot 88 + j3\cdot 84\ \text{V}. \qquad \ldots\ldots(2)$$

$Z$, the source impedance seen looking back into the circuit, is the Thévénin source impedance for the resistor and inductor.

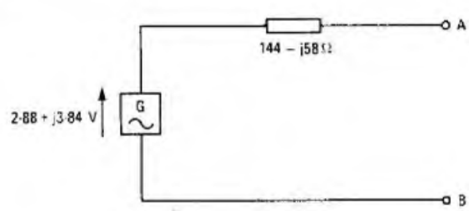
$$Z = \frac{400 \times j300}{400 + j300} = \frac{400 \times j300(400 - j300)}{400^2 + 300^2},$$

$$= 144 + j192\ \Omega. \qquad \ldots\ldots (3)$$

The total Thévénin source impedance including the capacitor is

$$144 + j192 - j250 = 144 - j58\ \Omega. \qquad \ldots\ldots (4)$$

The Thévénin equivalent circuit is shown in sketch (*d*).




(*d*)

The current in the impedance connected across AB is

$$\frac{V}{Z_{AB}} = \frac{2\cdot 88 + j3\cdot 84}{(144 - j58) + (30 + j180)} = \frac{2\cdot 88 + j3\cdot 84}{174 + j122}.$$

Therefore, the magnitude of the current is

$$\left(\frac{2\cdot 88^2 + 3\cdot 84^2}{174^2 + 122^2}\right)^{\frac{1}{2}} = \frac{4\cdot 8}{212\cdot 5} = \underline{22\cdot 6\ \text{mA}}.$$

**Q6** (*a*) *A source has a complex impedance* $R_s + jX_s$.

*State the conditions for a load impedance to take maximum power from the source, if*

(*i*) *the load resistance and reactance can both be varied, and*

(*ii*) *the load impedance can be varied in magnitude but not in phase.*

(*b*) *A 5 V, 100 Hz source has an internal impedance which is equivalent to a 600 Ω resistance in series with a 1·59 μF capacitance. Determine the load components required for maximum load power when the load is*

(*i*) *variable in resistance and reactance, and*

(*ii*) *a variable resistance only.*

(*c*) *Calculate the power developed in the load in parts* (*b*)(*i*) *and* (*b*)(*ii*).

**A6** (*a*) (*i*) Maximum power transfer takes place when the load is equal to $R_s - jX_s$.

(*ii*) Maximum power transfer occurs when the load impedance is equal in magnitude to the modulus of $R_s + jX_s$; that is, $\sqrt{(R_s^2+X_s^2)}$.

(*b*) The source impedance, $Z_s$, is equal to

$$600 + \frac{10^6}{j2\pi \times 100 \times 1{\cdot}59} = 600 - j1000\ \Omega.$$

(*i*) The load components required are a resistor of 600 Ω in series with an inductor of impedance $+j1000\ \Omega$. The inductor thus has an inductance of

$$\frac{1000}{2\pi \times 100} = \underline{1{\cdot}59\ \text{H}}.$$

(*ii*) The source impedance has a magnitude of

$$\sqrt{(600^2 + 1000^2)} = 1166\ \Omega.$$

Thus, the required resistance for the termination is 1166 Ω.

(*c*) (*i*) The power developed in the load of part (*b*)(*i*) is

$$\frac{5^2}{4 \times 600} = \underline{10{\cdot}4\ \text{mW}}.$$

(*ii*) The power dissipated when the termination is 1166 Ω is

$$\frac{5^2}{(1766^2 + 1000^2)} \times 1166 = \underline{7{\cdot}08\ \text{mW}}.$$

*Q7* (*a*) *Give estimates for the bandwidths required, before modulation, for the effective transmission of*

(*i*) *telephone speech,*
(*ii*) *music, and*
(*iii*) *625-line monochrome television.*

*Explain, for* EACH *case, the basis upon which the bandwidth is determined.*

(*b*) *A modulated carrier is given by*

$$e_m = 12 \sin 10^4 t + 3 \cos (9{\cdot}5 \times 10^3 t) - 3 \cos (10{\cdot}5 \times 10^3 t) \text{ volts.}$$

*Identify the type of modulation and obtain*

(*i*) *the carrier frequency,*
(*ii*) *the modulating signal frequency, and*
(*iii*) *the RMS voltage of the modulated wave.*

*Q8* (*a*) *Compare briefly the properties of amplifiers using field-effect transistors with those using junction transistors.*

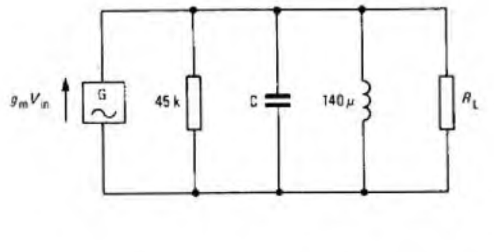
(*b*) *A tuned amplifier uses a field-effect transistor having* $g_m = 5$ *mA/V and* $r_d = 45$ *kΩ. The tuned circuit in series with the drain comprises a 140 μH coil in parallel with a capacitor. The amplifier has a maximum voltage gain of 185 at 800 kHz. Calculate*

(*i*) *the capacitance,*
(*ii*) *the Q-factor of the tuned circuit, and*
(*iii*) *the voltage gain at 800 Hz when a 100 kΩ resistor is connected across the tuned circuit.*

**A8** (*a*) The basic difference between field-effect transistors and junction transistors can be seen by studying their small-signal equivalent circuits. A junction transistor is a current-controlled device, while a field-effect transistor is voltage controlled. The internal feedback of a field-effect transistor is small compared with that of a junction transistor. Junction-transistor amplifiers have low input impedance and limited bandwidth unless special design techniques are used, while field-effect-transistor amplifiers can have high input impedance and have wide bandwidth.

(b) The equivalent circuit for the output of the amplifier is shown in the sketch.

(*i*) The resonant frequency is 800 kHz and is equal to $\dfrac{1}{2\pi\sqrt{(LC)}}$.




Thus $C = \dfrac{1}{4\pi^2 L f_0^2}$, where $f_0$ is the resonant frequency,

$$= \frac{10^6 \times 10^{-6}}{4\pi^2 \times 140 \times 800^2},$$

$$= \underline{282\ \text{pF}}.$$

(*ii*) To determine the *Q*-factor of the tuned circuit it is necessary to calculate the effect of the loss component of the load, $R_L$, and of the output resistance of the amplifier, $r_d$.

The voltage gain $= \dfrac{g_m V_{in} R_{eq}}{V_{in}} = 185$, where $R_{eq}$ is the parallel combination of $R_L$ and $r_d$.

Thus, $R_{eq} = 37$ kΩ.

The *Q*-factor of the tuned circuit $= \dfrac{R_{eq}}{\omega L}$

$$= \frac{37 \times 10^3}{2\pi \times 800 \times 10^3 \times 140 \times 10^{-6}}$$

$$= \underline{52{\cdot}6}.$$

(*iii*) With 100 kΩ connected across the tuned circuit, the equivalent load is

$$\frac{37 \times 100}{137} = 27\ \text{k}\Omega.$$

The new voltage gain is $5 \times 27 = \underline{135}$.

*Q9* (*a*) *Explain the factors which cause power loss in capacitors in AC circuits.*

(*b*) *A 3 μF capacitor has a loss angle of* $4 \times 10^{-4}$ *rad which may be assumed to be invariable with frequency. Calculate the effective series and parallel loss resistances at*

(*i*) *10 kHz, and*
(*ii*) *20 kHz.*

(*c*) *The capacitor in part* (*b*) *is connected in parallel with a 6 kΩ resistor and the combination connected to a 10 V, 20 kHz supply of negligible internal impedance. Evaluate the current component*

(*i*) *in phase with the supply voltage, and*
(*ii*) *in quadrature with the supply voltage.*

*Q10* (*a*) *Explain why the output stage of a power amplifier is often transformer-coupled to the load.*

(*b*) *A single-stage, class-A power amplifier uses a power transistor with its collector connected to a 15 V power supply through the primary winding of a transformer. The secondary winding is connected to a 3 Ω load resistance. The primary to secondary turns ratio is 9 to 1. If the average collector current is 60 mA, the power in the load is 300 mW and the load current is sinusoidal, calculate*

(*i*) *the collector efficiency,*
(*ii*) *the collector dissipation,*
(*iii*) *the peak-to-peak voltage swing at the collector, and*
(*iv*) *the maximum instantaneous collector current.*

**A10** (*a*) One of the prime considerations in the design of a power amplifier is the distortion introduced by the amplifier. This can be minimized if the amplifier load is optimum. The optimum load is rarely the system load or transistor impedance. One way of matching these is to use an impedance-matching transformer.

(*b*) (*i*) The average input power = 15 × 60 mW = 900 mW.

Therefore, the collector efficiency is 33%.

(*ii*) The collector dissipation is 600 mW.

(*iii*) Let the output voltage be $V_0$. Then $\dfrac{V_0^2}{3} = 300$ mW.

From this expression, $V_0 = 0{\cdot}95$ V. Since the transformer ratio is 9 to 1, the RMS collector swing is 8·538 V. Thus the peak-to-peak collector swing is $2(8{\cdot}538 \times \sqrt{2}) = \underline{24{\cdot}2\ \text{V}}$.

(*iv*) The maximum collector current occurs when the collector-emitter voltage, $V_{CE}$, is a minimum.

But, $I_{c\,max} \times R_L = 12{\cdot}1$,

where $I_c$ is the collector current and $R_L$ is the load.

Thus, $I_{c\,max} = \dfrac{12{\cdot}1}{9^2 \times 3} = \underline{49{\cdot}7\ \text{mA}}$.

## TELEGRAPHY B 1978

**Students were expected to answer any 6 questions**

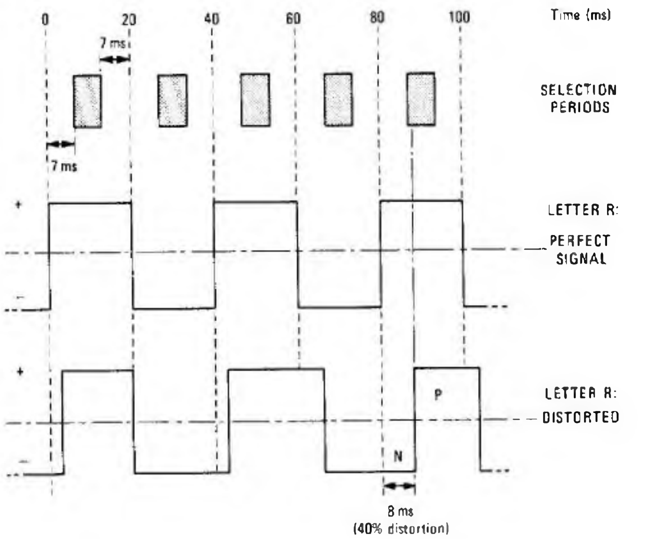
*Q1* (*a*) *With the aid of timing diagrams, describe how a teleprinter character, transmitted at 50 bauds and suffering 40% late distortion, could be mutilated when received by a teleprinter having 35% margin.*

(*b*) (*i*) *State 4 examples of equipment on the circuit, or characteristics of the line, which could introduce the distortion.*
(*ii*) *Give typical values for the amount of distortion due to each.*

**A1** (*a*) A teleprinter having a receive margin of 35% is capable of receiving teleprinter signals distorted by that amount; that is, the received signal may be $\frac{35}{100} \times 20$ ms = 7 ms early or late. If the signal is outside these limits, the transition will not be recognized because the teleprinter mechanism will have detected the polarity at the 7 ms point and will decode the character using incorrect information. From the sketch, the transition between the fourth and fifth elements is 40% late and occurs at $80 + (\frac{40}{100} \times 20) = 88$ ms, after the ideal instant. By this time, the RECEIVE mechanism has begun its SELECT period (at 87 ms) and will not recognize the positive signal P; the teleprinter will instead register the negative signal N. The incoming character will be incorrectly printed as G (SMSMM) in place of R (SMSMS), S denoting SPACE and M MARK.

(*b*) Examples of equipment or characteristics of the circuit which could introduce distortion are given below, together with typical values.

| | |
|---|---|
| Originating teleprinter transmitter mechanism: | 0–5% |
| Low-pass filter (capacitance and inductance): | 10% |
| Physical line (capacitance and resistance): | 0–10% |
| Multi-channel voice-frequency circuit: | 0–8% |
| Repeating relay: | 0–5% |
| Receiving relay (bias): | 0–5% |

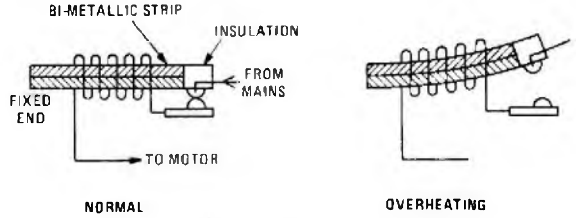



*Q2* (*a*) *With the aid of a sketch, describe how a private-circuit teleprinter motor is started* (*i*) *by incoming signals, and* (*ii*) *for an outgoing message.*

(*b*) (*i*) *What is the effect if the motor does not start when the current is applied?*
(*ii*) *What equipment is provided to safeguard the motor under these circumstances, and how does it operate?*

**A2** (*a*) See A5, Telegraphy B 1974, *Supplement*, Vol. 68, p. 34, July 1975.
(*b*) (*i*) If the motor stalls (that is, does not start) when power is connected, the starting current will flow through the motor windings until the power is disconnected. As the value of the starting current is higher than that of the running current, the motor may overheat, with consequent damage to the windings and a possible fire risk.

(*ii*) Any protective device must allow the starting current to flow for a reasonable time before disconnecting the circuit and, for this reason, a fuse is not suitable because it operates too quickly. A circuit-breaker using a bi-metallic strip is preferred, with the mechanical and electrical arrangements shown in the sketch. The circuit-breaker consists essentially of a bi-metallic strip surrounded by a coil carrying the motor current. If the high starting current persists, the bi-metallic strip is heated, the two metals expand at different rates, and the strip bends. The flexure of the strip operates contacts which break the circuit.




*Q3* (*a*) *Describe how a telegraph message is passed by each of the following services:*

(*i*) *Phonogram,*
(*ii*) *Telex,*
(*iii*) *Gentex, and*
(*iv*) *Printergram.*

(*b*) *How is the customer charged for each of the services provided?*

**A3** (*a*) (*i*)–(*iii*) For a description of the transmission of telegraph messages by Phonogram, Telex and Gentex, see A7, Telegraphy B 1975, *Supplement*, Vol. 69, p. 47, July 1976.

(*iv*) *Printergram* is the name given to the service whereby Telex subscribers may gain access to a Printergram position in the telegram centre by dialling a special code, normally 01. The substance of the telegram is received on the Printergram teleprinter and distributed to the TRANSMIT positions in the same way as a public telegram; for example, by a moving belt.

In the reverse direction, telegrams may be received in the centre for onward transmission to a Telex subscriber. The distant *teleprinter automatic switching system* operator dials a code giving access to the appointed office Printergram centre serving the required Telex subscriber. The telegram is received on the tape teleprinter, and the operator gums the message tape onto a blank form which is forwarded to the Printergram position. The Printergram operator dials the required Telex number and manually transmits the message to the teleprinter at that address.

(*b*) (*i*) A Phonogram is a telegram sent by a customer from a private telephone or a public call-office. The customer passes the message to the Phonogram operator who charges the customer by ticket (if a private telephone is used) or by requiring the customer to insert the necessary coins if a call-office is used. The charge for the telegram depends on the number of words (with a minimum fixed charge) and the destination of the message.

(*ii*) Each Telex line has a totalling meter at the exchange. Calls which are completed by subscriber-dialling are charged by periodic metering at a rate depending on the location of the called number; the pulsing rate is slower for local calls than for trunk calls. One pulse corresponds to one unit-fee charge. The charging rate is controlled by time-zone equipment in each local exchange which fixes the rate according to the digits dialled. For calls made via the international switchboard and connected by the operator, a ticket is made out by the operator to indicate the destination and duration of the call. Tickets are processed and added to the Telex subscriber's quarterly bill. International calls routed through automatic ticketing equipment are listed separately on the subscriber's bill.

(*iii*) The Gentex system carries telegrams between major European cities, and costs are incorporated in the charge for the telegram at its origin.

(*iv*) Printergram calls are originated by Telex subscribers to a Printergram teleprinter at a telegram centre. The Printergram operator charges the Telex subscriber by ticket, the charge being added to the subscriber's quarterly Telex account.

*Q4* (*a*) *Sketch the layout of equipment on a group-selector rack in an automatic Telex exchange.*

(*b*) *Draw a block diagram to show how a number of group-selector racks would be arranged together in one part of the exchange. Mark cable runs, power feeds and service-signal supplies to the racks.*

(*c*) *List 4 features of the layout which facilitate maintenance work on the equipment.*

*Q5* (*a*) *Calculate the maximum number of characters per second which may be received on a 50 baud $7\frac{1}{2}$-unit teleprinter.*

(*b*) *What are the advantages and disadvantages of using a teleprinter at a higher speed?*

(*c*) *Describe the operation of a teleprinter printing mechanism suitable for the reception of signals up to 30 characters/s.*

**A5** (*a*) For a 50 baud teleprinter, the duration of one character element is 1000/50 = 20 ms. The duration of one character is $7\frac{1}{2} \times 20 = 150$ ms. The maximum possible number of characters per second is therefore $1000/150 = 6\frac{2}{3}$.

(*b*) One advantage of using a teleprinter at a higher speed is that the traffic can be cleared at a faster rate. Also, a larger number of character elements can be used, permitting a larger selection of characters, including upper and lower case, and giving a greater choice of functional characters. Parity bits could also be included for use with error-detecting equipment.

A disadvantage of a high-speed teleprinter is that an operator could not sustain the speed of working and would probably have to use an automatic transmitter. The greater baud speed would require a more expensive circuit due to the increase in bandwidth required for satisfactory transmission at the higher speed. The machine would require more maintenance for faster-moving parts due to increased wear. The machine would also have to interwork with other machines at the same speed and with the same alphabet, and for this the present Telex network would not be suitable.

(*c*) Teleprinters operating at higher speeds than those used on the Telex network use a variety of methods of printing. Up to about 30 characters/s, percussion methods are used, with the type selected and brought to a stop before the type-hammer moves it into contact with the paper. Above this speed, printing "on the fly" is used, whereby the hammer strikes the type while the type is in motion past the paper. The type may be mounted on a continuously-running belt, with the instant of moving the hammer electronically-selected to coincide with the arrival of the correct character at the moment of impact. This type of teleprinter can operate at up to about 60 characters/min. Higher speeds may be obtained by matrix-type printing, whereby each character is constructed by, for example, the operation of needles in a 7 × 5 matrix to make a reasonable facsimile of a printed character. An example of a percussion teleprinter for use up to 30 characters/s is the *daisy-wheel* printer. As the name implies, the type-head is in the form of a flat disc with spoke-like arms, each holding on its end one type symbol. The wheel is moved by a stepping motor controlled by the receipt and storage of an incoming character. The motor makes the required number of steps from the previous printing position to bring the new character into place, and a hammer strikes to print the character. The wheels are made of plastic to give low inertia when starting and stopping, are cheap and are easy to change to give different typefaces. Electronic storage of incoming characters assists in providing faster working.

*Q6* (*a*) *Describe a method of automatic voltage control for a large central telegraph power supply.*

(*b*) *What are the causes of variation in voltage in the power supply?*

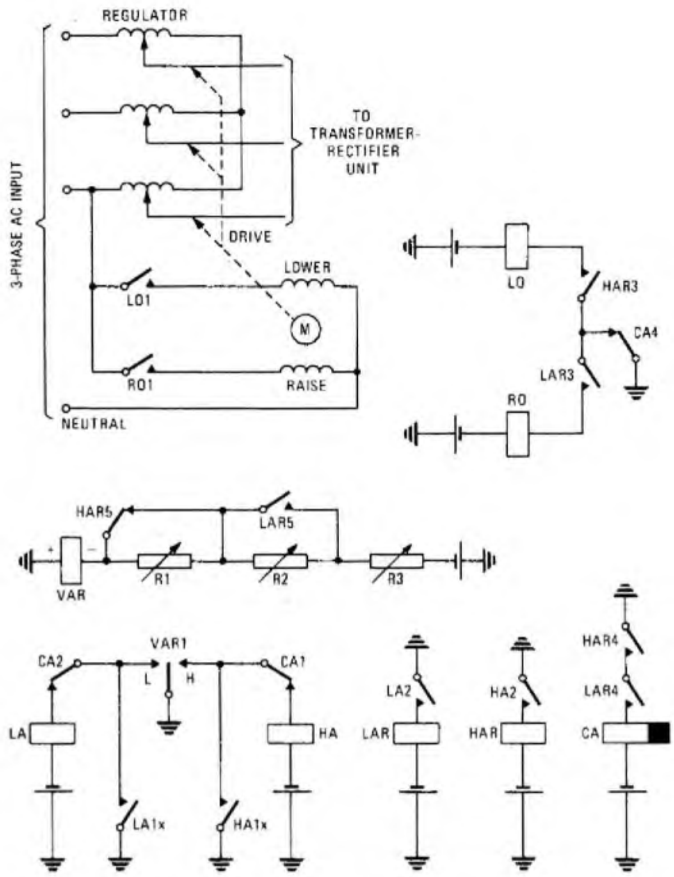
(*c*) (*i*) *Why is it necessary to control the voltage within close limits?*
(*ii*) *What are the normal limits?*

**A6** (*a*) The sketch shows a method of automatic voltage control of power supplies derived from a bank of rectifiers. The busbar voltage is monitored by a voltage-monitoring circuit which is common to the plant, but which is connected to only one rectifier at a time. Moving-coil relay VAR is connected to the busbar in series with variable resistors R1–3 such that, with the circuit normal, contact VAR1 makes with the low (L) and the high (H) sides when the busbar voltage is 51·2 V and 51·8 V respectively. This provides for a busbar voltage range of 51·5 ± 0·3 V.

When the exchange load increases, the busbar voltage falls and, at 51·2 V, contact VAR1(L) operates relay LA which locks at LA1x. Contact LA2 operates relay LAR; contact LAR3 operates relay RO, and contact RO1 completes a circuit to start the regulator motor. The motor drives to increase the input to the transformer-rectifier unit and the busbar voltage rises. As resistor R2 is now short-circuited, the sensitivity of relay VAR is changed so that an increase of 0·1 V closes contact VAR1(H), operating relays HA, HAR and CA. Contact CA4 disconnects the motor-drive circuit. The operation of contacts CA1 and CA2 disconnects relays HA and LA respectively, and the circuit restores to normal. If the busbar voltage rises by 0·1 V due to a decrease in the exchange load, contact VAR1(H) operates and the circuit functions in a similar way to reduce the voltage.

The voltage-control circuit described above is normally connected to the first rectifier set, with a current-operated moving-coil relay monitoring the output. When the rectifier reaches full-load, the combination of full-load-current detection and signals from the voltage-monitoring circuit transfer voltage control to the second rectifier set. Rectifier No. 1 works on full-load with rectifier No. 2 compensating for load variations.

As the second rectifier becomes fully loaded, the control changes to rectifier No. 3, and so on, the control moving up and down the bank of rectifiers depending on the load.




(*b*) Variations in the voltage supply to the exchange may be caused by:

(*i*) variations in the load due to increases or decreases in the traffic carried, an increase in the load current causing a decrease in the voltage (and vice versa),

(*ii*) variations in the mains supply voltage,

(*iii*) the condition of the battery cells,

(*iv*) fault conditions, and/or

(*v*) for 80 V circuits with earth return, variations in the earth currents.

(*c*) It is important to maintain −50 V supplies within limits to sustain the design performance of relays and selectors. The limits are 46–52 V at the equipment.

To maintain the correct current in teleprinter RECEIVE magnets and to maintain the speed of working, the +80 V and −80 V supplies must remain within ±4 V to prevent distortion due to unequal MARK and SPACE currents. The limits are 75–85 V.

*Q7* (*a*) *What are the merits of double-current (as compared with single-current) working for DC telegraph signalling?*

(*b*) *Under what circumstances would DC signals be converted to AC signals over part of the circuit?*

(*c*) *Describe, with the aid of a block diagram, how a number of circuits using AC signals could be combined on one telephone circuit using frequency-division techniques.*

**A7** (*a*) (*b*) See A1, Telegraphy B 1974, *Supplement*, Vol. 68, p. 33, July 1975.

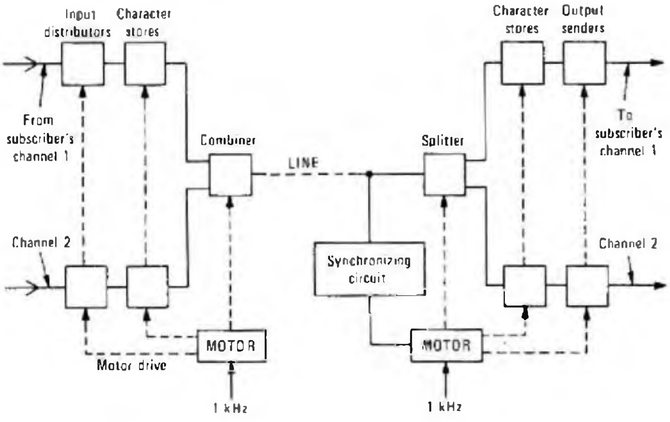
(*c*) See A2, Telegraphy B 1975, *Supplement*, Vol. 69, p. 45, July 1976.

*Q8* (*a*) *Describe, with the aid of a block diagram, the operation of a time-division multiplex system for use with telegraph signals.*

(*b*) *What are the advantages and disadvantages of this type of system as compared with a frequency-division (multi-channel voice-frequency) system?*

**A8** (*a*) A block diagram of a time-division multiplex system is shown in the sketch.

A time-division telegraph system allows each of a number of channels the exclusive use of the transmission path for a short fixed time. The SEND and RECEIVE ends of the transmission path operate in




synchronism to identify the signals proper to each channel. Signals from each channel are received on an input distributor and the characters are stored. Combiner equipment transmits characters or elements from each store at regular intervals and in cyclic order. At the distant termination, splitter equipment, operating in synchronism with the combiner equipment, selects the elements or characters proper to each channel, and passes them to the appropriate store. As the system is synchronous, the START and STOP elements for each character are not required to be transmitted over the synchronous path; this permits a more rapid rate of character transmission within the available bandwidth. The output sender transmits the signals, re-inserting the START and STOP elements, to the output channel.

(*b*) The frequency-division system is more suitable for a switched telegraph network because it is flexible and adaptable in application and does not require channels in tandem to operate in synchronism. Channels may be added or withdrawn, as required, and each channel may be worked independently of any other channel, provided that the transmitting apparatus is within the signalling-speed limit imposed by the bandwidth. This system may be provided more cheaply than the time-division system if the existing telephone network is used. Where high-cost circuits are involved, such as on radio or submarine-cable networks, the additional cost of time-division equipment may be economically justified.

***Q9*** *(a) Describe the operation of the test-and-plugging-up (T and PU) circuit on an engineering control board (ECB) in an automatic Telex exchange.*

*(b) Draw a diagram of the circuit-interception jacks for a station line and show how the line may be connected to the T and PU circuit.*

*(c) List 6 other facilities provided on the ECB.*

**A9** (*a*) (*b*) The T and PU circuit is used to extend a subscriber's faulty line to the test desk, to prevent a faulty line applying a continuous spurious calling condition to the exchange, and to indicate to an incoming caller that the called line is faulty. About 20 T and PU circuits are provided on each test desk, and each circuit is terminated on 2 jacks on the ECB. A subscriber's faulty line can be connected to a T and PU circuit by inserting a double-ended test-cord between the subscribers' interception jacks on the ECB and the T and PU access jacks. Sketch (*a*) shows the connecting circuits and sketch (*b*) shows the lamps and keys provided on the test-desk for each T and PU circuit.

The T and PU circuit extends the subscriber's line to the test desk. Upon operation of the TEST key, the line is connected to the test circuits and tested. When the TEST key is operated, a red lamp glows on the test desk to show that the testing circuits on the desk are connected to a T and PU circuit and cannot be used for any other purpose. If the subscriber's circuit ceases to be faulty while connected to a T and PU circuit, an alarm sounds and the green CLEAR lamp glows to give an indication to the testing officer that the line may be tested and possibly released for service. If the officer is otherwise engaged, the CALL-IN key may be operated to extinguish the CLEAR lamp and silence the alarm. Should the subscriber call, the white CALL lamp glows and an audible alarm operates; the testing officer may transmit the wait (MOM) signal to the calling station by operating the ACKNOWLEDGE (ACK) key.

If an incoming call is received for the faulty circuit, the *out-of-order* (DER) signal is transmitted to the calling station, and the call is cleared.

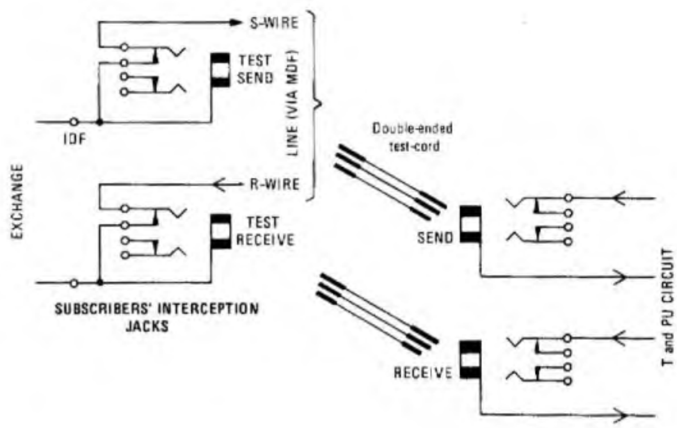
If the faulty line is one of a group of lines serving a subscriber, the operation of the AUXILIARY (AUX) key busies the faulty line only, and allows a calling final selector to step over the faulty line and search for the next free line.

(*c*) The following facilities are provided on an ECB in addition to the interception jacks and T and PU circuits. The ECB is a capacious jackfield normally adjacent to the test desk.

| | |
|---|---|
| Trunk circuits | Test-desk circuits |
| Test cord | Patching and interception |
| Milliammeter circuit | |

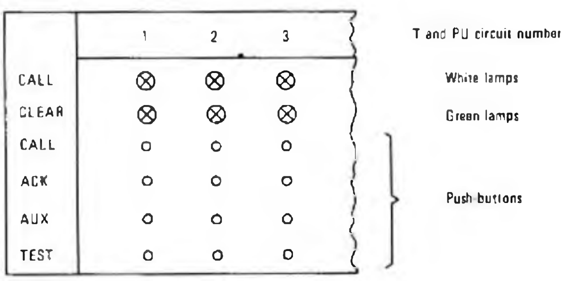
The following facilities are provided, terminating on access jacks.

| | |
|---|---|
| Temporarily out-of-service | Test messages with variable distortion |
| Service signals | Earth, +80 V, −80 V, −50 V, loop jacks |




MDF: Main distribution frame
R: RECEIVE
S: SEND
IDF: Intermediate distribution frame

(*a*)




(*b*)

***Q10*** *(a) Explain, with the aid of diagrams, how telegraph signals on a long DC circuit may be improved by the use of a repeating relay connected for:*

*(i) an earth-return single-wire circuit, and*
*(ii) a 2-wire loop circuit.*

*(b) Explain clearly in what circumstances each type of circuit would be used.*

*(c) State typical limiting distances for each circuit.*

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