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"To promote the advancement of radio, electronics and kindred subjects by the exchange of information in these branches of engineering."

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On Being Understood

ONE reason why we report on page 173 of this issue a speech made by Lord Mountbatten is because it highlights a modern problem, that of effective communication between engineers.

It is fifty years or more since an engineer could claim a reasonable understanding of all engineering disciplines other than his own. It is therefore a fair assumption that it is now unreasonable for an electronic engineer to take more than an intelligent layman's interest in, for instance, the complexities of mining engineering or hydrofoil design. It is *not*, however, unreasonable to expect that a radio engineer should be able to understand, in broad general terms at least, what is being done in, say, computer engineering or indeed in the many other fields of electronics.

Accelerating developments in all branches of electronics means that a few years' specialization may lead to technical isolation from specialists in other fields because of the jungle of jargon which every specialization seems to be unable to avoid. Consequently, few engineers specializing in, say, television may accurately describe what 'software' is or what a 'simultaneous set-reset binary trigger' does: similarly the computer engineer may be just as hard pressed to give even a broad definition of 'phase alternation, line', or identify a 'colour killer'.

Efforts at rationalizing terminology are made by the British Standards Institution through its Committees concerned with definitions and nomenclature. The resulting Glossaries frequently cite (and deprecate) some of the more extreme examples of inexact and over-colloquial terminology which provide language barriers. Agreed definitions are however subject to inevitable, even desirable, delay since terms which have not yet passed the test of general usage and acceptance cannot be accepted into the language.

About twenty years ago the principles of setting-out circuit diagrams in a manner which makes their function clearly apparent were put forward by the late L. H. Bainbridge-Bell. Today, circuit diagrams in most branches of electronic engineering fall into recognizable configurations. Just how revolutionary the work of Bainbridge-Bell really was may be seen from examples of circuit diagrams 'before' and 'after' re-drawing in a paper which he contributed to this Institution's *Journal* in July 1953. The absence of 'tram-lines' from complicated diagrams is but one instance of the great improvement over the years. Could not some similar guiding principles be evolved for technical language?

Resolution of the language problem is a cardinal factor in tackling the information explosion. Indeed, it was for this reason that much effort was given, and international co-operation sought, in establishing an agreed thesaurus in the first phases of the Selective Dissemination of Information project which the Institution initiated.

It should not be impossible to lay down rules for achieving clarity in technical terms right from the earlier stages of development of new techniques rather than allow the engineer to follow the easy way of obscure jargon. The abandonment of inexact and parochial jargon, picturesque though it may be, need not mould all creative writing into stereotyped form. The pronouncements by the Académie Française in coining new words stultify neither the elegance of the French language nor its precision. If technical terms are rationally conceived their linking together will lead naturally, and in fact more concisely, to understanding.

G. D. C.

INSTITUTION NOTICES

Annual General Meeting

The Seventh Annual General Meeting of the Institution since incorporation by Royal Charter will be held on Wednesday, 13th November, at 6 p.m., at the London School of Hygiene and Tropical Medicine, Keppel Street, Gower Street, London, W.C.1. The Chair will be taken by the President, Sir Leonard Atkinson, K.B.E.

The meeting will be followed by an address on 'Electronics in the Nation's Economy', given by I. Maddock, C.B., O.B.E., F.R.S., C.Eng., F.I.E.R.E.

Nominations for Election to the 1968–69 Council were printed on page 51 of the May–June 1968 issue of the Institution's *Proceedings*; the Agenda appeared on page 75 of the July–August issue; and the Annual Report to the Council will be published in the September–October *Proceedings*. Members overseas may obtain a copy of these issues free of charge, on application to the Secretary.

Semiconductor Device Research

A second Conference on Semiconductor Device Research will be held in Munich in April 1969. This conference is sponsored by the German Section of the I.E.E.E., the German Physical Society (DPG), the German Society of Electrical Engineers (VDE) and its Communications Engineering Division (NTG).

Invited papers will be presented at the sessions planned to cover the following topics:

Effects using majority-carriers (Gunn-effect, ATT-diode, hot electrons, plasma effects); field effect and thin film transistors, including interface and surface problems; high frequency junction devices (novel transistor systems, tunnel diodes, Schottky diodes, varactors); opto-electronic devices (luminescence-laser and photo-diodes, coupling elements, radiation detectors); galvanomagnetic devices; piezo-electric semiconductor devices, including phonon interactions; semiconductor problems in power electronics (thyristor, thermoelectric phenomena, sensors).

Included in the scope of the programme are materials problems, special technologies, semiconductor effects for microwaves and memories. (Applications and circuitry problems corresponding to the state of the art, phosphorescence and related phenomena will *not* be dealt with.)

Short papers of about 15 minutes' presentation time are invited and authors are asked to submit ten copies of abstracts to Dr. W. Heywang, Siemens AG, Research Laboratory, Balanstrasse 73, 8 Munich 80, Germany, before 15th December 1968. The conference languages are English, French and German (no simultaneous translation facilities will be available).

Joint Conferences in 1969

A Joint Conference on **Computer Science and Technology,** organized by the I.E.E. (Computer Design Professional Group), the I.E.R.E. (Computer Group), the Institute of Mathematics and its Applications, and the Institute of Physics and the Physical Society, will be held at the University of Manchester Institute of Science and Technology from 30th June to 3rd July, 1969.

The computer design problems can be represented as an interaction between new developments in technology and the requirements of the user. The Conference will emphasize this relationship with sessions on the following:

Impact of high level language, etc.; store hierarchies: future requirements, engineering developments; multi-computer systems; requirements of particular applications, such as large scientific problems, information banks, multi-access, etc.; special purpose machines and systems; modern solutions of logic design problems; cellular arrays, LSI, design automation; education of computer scientists and engineers; storage technology; display methods.

The Organizing Committee invite contributions of up to 2,500 words; completed texts will be required by 24th February, 1969, but contributors are asked to submit 250-word synopses by 18th November, 1968.

A Conference on **Industrial Applications of Dynamic Modelling** will be held from 16th to 18th September, 1969. It is being organized jointly by the Institution of Electrical Engineers (Control and Automation Division) and the Institute of Measurement and Control, in association with the Institute of Mathematics and its Applications and the I.E.R.E.

The Conference will be concerned with the formulation and application of those dynamic models of plant, processes and systems having industrial significance. The scope will include models derived off-line from equations governing inter-relationships between system variables as well as those devised by observation of the system under normal and perturbed conditions of operation. The application of such models to the control of industrial scale plants and to the increased understanding of process and system behaviour will be discussed.

Offers of contributions (not exceeding 3,000 words) are invited and synopses of about 300 words should be sent, before 1st November, 1968, to the I.E.E. Conference Department. Completed papers will be required by 1st March, 1969.

Further details for both the above Conferences may be obtained from the Conference Secretariat, Institution of Electrical Engineers, Savoy Place, London, W.C.2.

U.D.C. 621.372.57

Synthesis of Active Filters with Optimum Sensitivity

By

A. ANTONIOU,

B.Sc., Ph.D.†

Summary: Two general RC-active-network synthesis procedures previously described by the author are applied to the synthesis of filters. A third synthesis procedure is now described in which Guillemin's parallel-ladder procedure is used to realize the passive part of the network. This and the previous two procedures use the Horowitz decomposition which optimizes the sensitivity. A method for reducing the number of passive elements is explained. The three procedures are used to realize all the possible second-order filter transfer functions. Design tables are included in which the element values are given in terms of the transfer function coefficients. Finally a low-pass, fifth-order, elliptic filter is designed using the tables.

List of Principal Symbols

$A(s)$, $B(s)$, $Q(s)$, $q(s)$	polynomials in s
a, b, c, d	transfer function coefficients
d_0, H_0, h, H_a, H_b	constant multipliers of polynomials
$D(s)$, $d_1(s)$, $d_3(s)$, $d'_3(s)$	denominator polynomials in s
G	amplifier voltage gain
K, K_1, K_2, K_3	multiplier constants of short-circuit transfer admittances
k	conversion factor of n.i.c. $(= G-1)$
$N(s)$, $N'(s)$, $n_1(s)$, $n_2(s)$	numerator polynomials in s
Q	Q-factor
s	complex frequency variable $(= \sigma+j\omega)$
S_k^Q	Q-factor sensitivity to variations in k
$T(s)$	transfer function
V_o/V_i	voltage ratio
Y_L	load admittance
λ_1	admittance level
ζ, λ, μ, ν, ξ	positive constants

1. Introduction

Three RC-active-network synthesis procedures have been described in Ref. 1. The configuration used consists of two active elements (a negative-gain amplifier and a negative-impedance convertor), two RC 2-port networks and one RC 1-port network. The three procedures differ in the method used to realize the passive networks. Thus the first method uses RC-tree networks, the second uses RC-ladder networks and the last uses inverse-L networks. By using a positive-gain amplifier as a negative-impedance convertor (n.i.c.) it was found possible to synthesize sections which can be cascaded without isolating amplifiers. Furthermore, the sensitivity can be optimized by using the Horowitz decomposition[2] which minimizes the sensitivity to active and passive element variations. The passive networks are synthesized using well-known RC synthesis procedures such as those of Foster,[3] Cauer,[4] Guillemin,[5] Fialkow and Gerst[6] and Dasher.[7]

The procedures described in Ref. 1 are aimed at realizing a completely general transfer-function and consequently are somewhat involved. In general, filters have transfer functions with zeros on the imaginary axis or in the left-half s plane. It follows that the numerator of the transfer function is a Hurwitz polynomial and has positive coefficients. It has been shown[1] that for such a transfer function one of the RC 2-port networks and the negative-gain amplifier are unnecessary. Furthermore, it was claimed that for second-order transfer functions the RC 1-port can always be eliminated by converting the 2-port into a 3-port network; this reduces the number of passive elements. Then configurations result which resemble those described by Sallen and Key.[8]

In this paper another RC-active synthesis procedure is described in which the passive 2-port is realized using Guillemin's[5] parallel-ladder procedure. This and the active synthesis procedures described in Ref. 1 will be applied to the synthesis of active filters. General second-order filter transfer functions are realized and formulae for the component values in terms of the transfer function coefficients are given in tables. These should prove useful to the network designer.

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2. RC-active-network Synthesis Procedures

It has been shown[1] that for transfer functions with positive numerator coefficients the inverse-L procedure needs more active elements than the RC-tree or RC-ladder procedure. In addition the latter two procedures give less sensitive networks and hence are more suitable for the realization of active filters. The only difference between the RC-tree and the RC-ladder procedure is in the synthesis method used to realize the passive 2-port network. The simplified configuration is shown in Fig. 1; when $G = 2$ it has a voltage ratio

$$\frac{V_o}{V_i} = \frac{G(-y_{12})}{y_{22}-(G-1)Y_L} = \frac{G(-y_{12})}{y_{22}-Y_L} \qquad ......(1)$$

The transfer function to be realized is

$$T(s) = \frac{N(s)}{D(s)} = \frac{H_0 N'(s)}{D(s)} \qquad ......(2)$$

where H_0 is the multiplier constant of $N(s)$. The polynomials $N(s)$ and $N'(s)$ have positive coefficients.

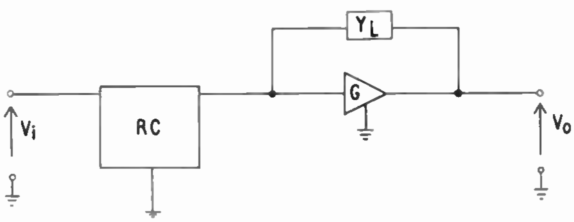


Fig. 1. Configuration for the synthesis of active filters.

The denominator polynomial is of even degree n and has complex zeros (transfer functions with real poles can be realized using passive synthesis procedures).

Let $D(s)$ be expressed as a difference of polynomials

$$D(s) = d_1(s) - d_3(s) \qquad ......(3)$$

where $d_1(s)$ and $d_3(s)$ have n and $n-1$ distinct negative real zeros respectively. The polynomial $d_3(s)$ can be written as

$$d_3(s) = d_0 d_3'(s) \qquad ......(4)$$

where d_0 is the multiplier constant and hence $d_3'(s)$ is a monic polynomial (a monic polynomial is one whose highest power coefficient is equal to 1).

For the RC-tree procedure, from eqns. (2) and (3) we write

$$hT(s) = \frac{N(s)h}{d_1(s)-d_3(s)} \qquad ......(5)$$

where h is a positive constant small enough to make the coefficients of $hN(s)$ equal to or less than the corresponding coefficients of $d_1(s)$. Select a polynomial $q(s)$ having $n-1$ distinct negative real zeros interlacing with those of $d_1(s)$ and $d_3(s)$ so that $d_1(s)/q(s)$ and $d_3(s)/q(s)$ are RC driving-point admittance functions (an RC driving-point admittance function has poles and zeros which interlace on the negative real axis so that the nearest critical point to the origin is a zero and the nearest critical point at infinity is a pole). On dividing the numerator and denominator of the right-hand side of eqn. (5) by $q(s)$ and equating the resulting expressions to the numerator and denominator of the right-hand side of eqn. (1) the following assignments can be made

$$\left.\begin{aligned} -y_{12} &= hN(s)/q(s) \\ y_{22} &= d_1(s)/q(s) \\ Y_L &= d_3(s)/q(s) \end{aligned}\right\} \qquad ......(6)$$

Then

$$\frac{V_o}{V_i} = GhT(s) \qquad ......(7)$$

In the RC-ladder procedure let

$$y_{12} = KY_{12} \qquad ......(8)$$

where Y_{12} has unit multiplier constant, i.e. K is the multiplier constant of y_{12}. From eqns. (1) and (8) we have

$$\frac{V_o}{GKV_i} = \frac{-Y_{12}}{y_{22}-Y_L} \qquad ......(9)$$

Equation (2) can be written as

$$\frac{T(s)}{H_0} = \frac{N'(s)}{D(s)} \qquad ......(10)$$

The following assignments can now be made:

$$\left.\begin{aligned} -Y_{12} &= N'(s)/q(s) \\ y_{22} &= d_1(s)/q(s) \\ Y_L &= d_3(s)/q(s) \end{aligned}\right\} \qquad ......(11)$$

Hence

$$\frac{V_o}{V_i} = \frac{GK}{H_0}T(s) = \frac{GKN'(s)}{D(s)} \qquad ......(12)$$

Since no control can be exercised over the multiplier constant K (see Ref. 5) the transfer function is realized to within a multiplier constant.

An alternative RC-active synthesis procedure will now be described which has not been given in Ref. 1. The passive 2-port is synthesized using Guillemin's parallel-ladder synthesis.

On considering the passive 2-port to be made up of two parallel RC 2-ports, A and B, the voltage ratio is found to be

$$\begin{aligned} \frac{V_o}{V_i} &= \frac{G\{(-y_{12}^A)+(-y_{12}^B)\}}{y_{22}^A+y_{22}^B-Y_L} \\ &= \frac{G\{(-K_1 Y_{12}^A)+(-K_2 Y_{12}^B)\}}{y_{22}^A+y_{22}^B-Y_L} \qquad ......(13) \end{aligned}$$

where Y_{12}^A and Y_{12}^B have unit multiplier constants and K_1 and K_2 are positive multiplier constants whose magnitude cannot be controlled. Let

$$y_{22}^{A} = y_{22}^{B} = \tfrac{1}{2}y_{22} \qquad ......(14)$$

On changing the admittance level of network A by a factor λ_1 so that $y_{22}^{A'} = \lambda_1 y_{22}^{A}$ and $Y_{12}^{A'} = \lambda_1 Y_{12}^{A}$ eqns. (13) and (14) give

$$\frac{(\lambda_1+1)}{2GK_2}\frac{V_o}{V_i} = \frac{\left(\frac{-\lambda_1 K_1}{K_2} Y_{12}^{A}\right)+(-Y_{12}^{B})}{y_{22}-\frac{2Y_L}{\lambda_1+1}} \qquad ......(15)$$

The numerator polynomial of the transfer function is written as

$$N(s) = H_a n_1(s)+H_b n_2(s) \qquad ......(16)$$

where $n_1(s)$ and $n_2(s)$ are monic polynomials which have zeros on the negative real axis. The denominator polynomial $D(s)$ is expressed as a difference of polynomials as shown by eqns. (3) and (4). The transfer function may then be written as

$$\frac{T(s)}{H_b} = \frac{\frac{H_a}{H_b}n_1(s)+n_2(s)}{d_1(s)-d_3(s)} \qquad ......(17)$$

On comparing eqns. (15) and (17) the following assignments can be made:

$$\left.\begin{aligned} -Y_{12}^{A} &= n_1(s)/q(s)\\ -Y_{12}^{B} &= n_2(s)/q(s)\\ y_{22} &= d_1(s)/q(s)\\ Y_L &= (\lambda_1+1)d_3(s)/2q(s)\end{aligned}\right\} \qquad ......(18)$$

Hence

$$y_{22}^{A'} = \lambda_1 d_1(s)/2q(s) \quad \text{and} \quad y_{22}^{B} = d_1(s)/2q(s)$$

The passive 2-port network can now be synthesized using Guillemin's procedure and the resulting multiplier constants K_1 and K_2 can be calculated. The admittance level λ_1 is chosen to satisfy the equation

$$\lambda_1 = \frac{K_2 H_a}{K_1 H_b} \qquad ......(19)$$

Y_L can then be synthesized as a driving-point admittance. Equations (15), (17) and (18) give

$$\frac{V_o}{V_i} = \frac{2GK_2}{(\lambda_1+1)H_b}T(s) = GHT(s) \qquad ......(20)$$

Again the transfer function is realized to within a positive multiplier constant.

3. Sensitivity Optimization

Some flexibility exists in the choice of decomposition given by eqn. (3); this can be used to optimize the coefficient sensitivity.[1] A particular decomposition giving minimum coefficient sensitivity is that due to Horowitz.[2] It has been shown by Horowitz that such a decomposition minimizes the sensitivity to variations in the active and passive elements.

For a second-order transfer function

$$D(s) = s^2+ds+c \qquad ......(21)$$

the Horowitz decomposition is given by

$$\begin{aligned} D(s) &= (s+\sqrt{c})^2-(2\sqrt{c}-d)s\\ &= A(s)-B_0 B(s) \end{aligned} \qquad ......(22)$$

It is seen that $A(s)$ has n roots (two) which are not distinct, and $B_0 B(s)$ has $(n-1)$ roots (one). Equation (22) cannot be identified with eqn. (3) since $d_1(s)$ is required to have *distinct* negative real zeros. On adding a term ξs ($\xi > 0$) to the positive and negative parts of eqn. (22) we have

$$D(s) = \{A(s)+\xi s\}-\{B_0 B(s)+\xi s\} \qquad ......(23)$$

The positive and negative parts of eqn. (23) have n and $n-1$ distinct negative real zeros respectively as required. The following assignments may then be made:

$$\left.\begin{aligned} d_1(s) &= A(s)+\xi s = s^2+(2\sqrt{c}+\xi)s+c\\ d_3(s) &= d_0 d_3'(s) = B_0 B(s)+\xi s\\ &= (2\sqrt{c}+\xi-d)s \end{aligned}\right\} \qquad ......(24)$$

where

$$d_0 = (2\sqrt{c}+\xi-d)$$

It is observed that increasing the size of ξ moves the zeros of $d_1(s)$ further away from $s=-\sqrt{c}$: one zero moves towards the origin and the other towards infinity.

On choosing

$$q(s) = s+\sqrt{c} \qquad ......(25)$$

$d_1(s)/q(s)$ and $d_3(s)/q(s)$ are driving-point admittance functions. Furthermore, the poles and zeros of $d_1(s)/q(s)$ are evenly distributed on the negative real axis.

It has been shown[1] that the parameter ξ should be chosen as small as possible to give minimum sensitivity. As $\xi \to 0$ the zeros of $d_1(s)/q(s)$ tend to coincide with the pole; then the pole-zero distribution becomes lopsided. This has the effect of increasing the range of element values. *The choice of ξ is, therefore, a compromise between low sensitivity and permissible range of element values.*

Equation (1) can be written as

$$\frac{V_o}{V_i} = \frac{G(-y_{12})}{y_{22}-kY_L} \qquad ......(26)$$

where k is the conversion factor of the n.i.c. ($k = G-1$ and G is nominally equal to 2). For the RC-tree (or RC-ladder) procedure eqns. (6) (or (11)) and (24)–(25) give

$$\frac{V_o}{V_i} = \frac{G(-y_{12})(s+\sqrt{c})}{s^2+d's+c} \qquad ......(27)$$

where

$$d' = (2\sqrt{c}+\xi)-k(2\sqrt{c}+\xi-d) \qquad ......(28)$$

The Q-factor and the Q-factor sensitivity to the active element are defined as

$$Q = \sqrt{c}/d' \qquad ......(29)$$

$$S_k^Q = \frac{k}{Q}\frac{\partial Q}{\partial k} \qquad ......(30)$$

Equations (28)–(30) give

$$S_k^Q = 2Q-1+\frac{\xi}{d} \qquad ......(31)$$

The above equation supports the suggestion made by Bown[9] where $S_k^Q > 2Q-1$. The parameter ξ cannot be made equal to zero as this would make the range of element values infinite; it should, however, be chosen as small as possible. It can easily be shown that the Q-factor sensitivity for the RC-parallel-ladder procedure is also given by eqn. (31).

4. Reduction of Passive Elements

The number of elements can be reduced by eliminating Y_L in Fig. 1. If a new port (port 3) is created in the RC 2-port network by cutting an earth lead, the short-circuit admittances y_{12} and y_{22} remain unchanged (this follows from the definition of the y-parameters). Using the residue condition it can be shown that y_{23} cannot have poles not possessed by y_{22} (Ref. 10, p. 381). It is thus possible to find an earth lead in which y_{23} has poles given by $q(s)$. If, in addition, the zeros of y_{23} are identical to those of $d_3(s)$, Y_L can be eliminated as shown in Fig. 2. It follows that

$$-y_{23} = K_3 d_3'(s)/q(s) \qquad ......(32)$$

where K_3 is a multiplier constant whose magnitude cannot be controlled. The voltage ratio of Fig. 2 is

$$\frac{V_o}{V_i} = \frac{G(-y_{12})}{y_{22}-(-Gy_{23})} \qquad ......(33)$$

For the RC-tree (or RC-ladder) procedure, on adjusting the gain G so that

$$G = d_0/K_3 \qquad ......(34)$$

eqns. (32) and (34) give

$$-Gy_{23} = \frac{d_0 d_3'(s)}{q(s)} = \frac{d_3(s)}{q(s)} = Y_L \qquad ......(35)$$

Hence Y_L can be eliminated, as the voltage ratio remains unchanged to within a constant multiplier.

Y_L may also be eliminated in the parallel-ladder synthesis procedure. Now let

$$G = d_0(\lambda_1+1)/2K_3 \qquad ......(36)$$

then

$$-Gy_{23} = \frac{(\lambda_1+1)}{2}\frac{d_3(s)}{q(s)} = Y_L \qquad ......(37)$$

It will be shown that Y_L can be eliminated for all second-order filter transfer functions.

In some cases two leads can be found in which y_{23} has the required poles and zeros (some twin-T or parallel-ladder networks). In such a case either or both leads can be used as port 3. On using both leads

$$y_{23} = y_{23}^A+y_{23}^B = -K_3 d_3'(s)/q(s) \qquad ......(38)$$

where

$$K_3 = K_3^A+K_3^B$$

(K_3^A and K_3^B are the respective multiplier constants of y_{23}^A and y_{23}^B).

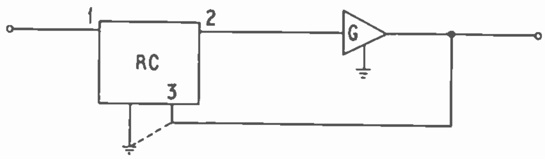


Fig. 2. Modified configuration to reduce the number of passive elements.

The Q-factor sensitivity to variations in G with Y_L eliminated is given by

$$S_G^Q = 2Q-1+\frac{\xi}{d} \qquad ......(39)$$

On comparing eqns. (31) and (39) it is observed that eliminating Y_L does not alter the sensitivity.

5. Realizations of Optimum-sensitivity Active Filters

The remaining part of this paper is concerned with the synthesis of optimum-sensitivity active filters. Since, in general, high-order realizations are very sensitive to element variations, a high-order filter transfer function is expressed as a product of second-order transfer functions, each being realized as a single section. These are connected in cascade to give the required transfer function. Hence the second-order transfer function section may be considered as the basic building block. All the possible second-order filter transfer functions are realized using the procedures described. Formulae giving the element values in terms of the coefficients are given in Tables 1–4. The realizations given in Table 1 were obtained by using the RC-ladder procedure, whereas the realizations of Tables 2, 3 and 4 were obtained by using the RC-tree and RC-parallel-ladder procedures respectively.

All second-order transfer functions have a denominator polynomial given by eqn. (21). The numerator can assume any one of the following forms:

$$N_1(s) = c \qquad \ldots\ldots(40a)$$

$$N_2(s) = as^2 \qquad \ldots\ldots(40b)$$

$$N_3(s) = bs \qquad \ldots\ldots(40c)$$

$$N_4(s) = bs+c \qquad \ldots\ldots(40d)$$

$$N_5(s) = as^2+bs \qquad \ldots\ldots(40e)$$

$$N_6(s) = as^2+c \qquad \ldots\ldots(40f)$$

$$N_7(s) = as^2+bs+c \qquad \ldots\ldots(40g)$$

The type of suitable synthesis procedure depends on $N(s)$. For transfer functions having a numerator given by eqns. (40a)–(40c) only the RC-ladder procedure can be used. On the other hand for complex transmission zeros either the RC-tree or the RC-parallel-ladder procedure has to be used.

For transmission zeros on the negative real axis, from eqns. (11), (24) and (25), we have

$$\left.\begin{aligned} -Y_{12} &= \frac{N'(s)}{s+\sqrt{c}} \\ y_{22} &= \frac{s^2+(2\sqrt{c}+\xi)s+c}{s+\sqrt{c}} \\ Y_L &= \frac{(2\sqrt{c}+\xi-d)s}{s+\sqrt{c}} \end{aligned}\right\} \qquad \ldots\ldots(41)$$

It is evident that the passive 2-port can be synthesized using Cauer's procedure. The short-circuit admittance y_{22} is expanded into a ladder development so that series elements have zero admittance or shunt elements have zero impedance at the zeros of $N'(s)$. For example when $N(s) = c$ both transmission zeros occur at infinity. The realization is obtained by performing a continued fraction expansion on y_{22} with polynomials $d_1(s)$ and $q(s)$ arranged in descending powers of s. Thus circuit 1 in Table 1 results. The remaining transfer functions can be realized in a similar fashion. On eliminating Y_L as explained in Section 4 circuits 1–6 are obtained.

In the RC-tree procedure the passive 2-port can be realized using either Dasher's or Fialkow and Gerst's synthesis procedure (the RC-tree procedure was so called because it was originally[1] used with the Fialkow and Gerst synthesis which in general yields a tree-like structure[11]). On using Dasher's procedure from eqns. (6), (25) and (40g), we have

$$-y_{12} = \frac{hN_7(s)}{s+\sqrt{c}} \qquad \ldots\ldots(42)$$

where h is a constant chosen small enough to make the numerator coefficients of $-y_{12}$ equal to or less than the corresponding numerator coefficients of y_{22} (Fialkow and Gerst Condition[11]); hence $0 < h \leqslant 1$. Using partial fractions the above equation can be written as

$$-y_{12} = k\left(s+k_0 - \frac{k_{12}s}{s+\sqrt{c}}\right) \qquad \ldots\ldots(43)$$

where

$$k = ah$$

$$k_0 = \sqrt{c}/a$$

$$k_{12} = \{(a+1)\sqrt{c}-b\}/a$$

Now we remove a shunt admittance

$$Y = (1-ah)s+\sqrt{c}(1-h) \qquad \ldots\ldots(44)$$

from y_{22} as given by eqn. (41), so that

$$\begin{aligned} y'_{22} &= y_{22}-Y \\ &= k\left(s+k_0+\frac{1}{\alpha}\frac{k_{12}s}{s+\sqrt{c}}\right) \end{aligned} \qquad \ldots\ldots(45)$$

where

$$\alpha = kk_{12}/\xi$$

The remaining part of the network can now be realized as a twin-T or bridged-T section as shown in Ref. 11 (page 306). Table 2 gives five realizations of the general biquadratic transfer function. It is seen that certain restrictions are imposed on the relative sizes of the numerator coefficients.

On using Fialkow and Gerst's procedure, which is described in the Appendix, alternative realizations are obtained. The passive 2-port whose y_{22} and y_{12} are given by eqns. (41) and (42) respectively is synthesized in the Appendix. Table 3 gives three realizations; a fourth realization can be obtained [$N(s) = as^2+c$] by letting $b \to 0$ in the third one. Fialkow and Gerst's procedure is invariably associated with a large number of elements. Although this is true for higher-order transfer functions it is not so for second-order transfer functions, as seen by comparing Tables 2, 3 and 4.

In Tables 2 and 3 the choice of h is arbitrary as long as the Fialkow and Gerst condition is satisfied. The realizations show that h can always be chosen to eliminate at least one passive element; this value of h is given in the tables in the form

$$h = \min(x, y) \qquad \ldots\ldots(46)$$

which implies that h is equal to the smaller of parameters x, y.

RC-parallel-ladder realizations of second-order transfer functions can be obtained from Table 1. For example if

$$N(s) = as^2+c, \quad n_1(s) = s^2 \quad \text{and} \quad n_2(s) = 1$$

Hence network A has two transmission zeros at $s = 0$ whereas network B has two transmission zeros at $s = \infty$. On using circuits 4 and 1 as networks A and B

Table 1
Realizations using Cauer's synthesis procedure

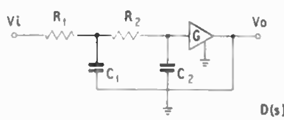
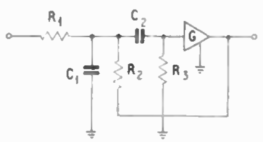
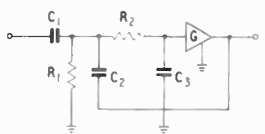
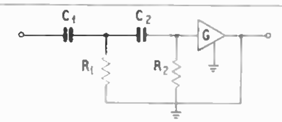
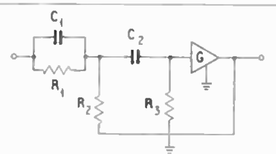
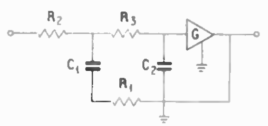
CIRCUIT	x	R_x	C_x
1  $\frac{V_o}{V_i}=\frac{GH}{D(s)}$	1	$\xi/\mu\sqrt{c}$	$\mu^2/\xi\sqrt{c}$
	2	$1/\mu$	1
$D(s)=s^2+ds+c$		$G=(\nu-d)/\mu$	$H=c$
2  $\frac{V_o}{V_i}=\frac{GHs}{D(s)}$	1	$2\xi/\mu^2$	μ/ξ
	2	$2\xi/\mu^2$	$\mu/\sqrt{c}$
	3	$1/\sqrt{c}$	—
$\mu\equiv\sqrt{c}+\xi$, $\nu\equiv2\sqrt{c}+\xi$		$G=2(\nu-d)/\mu$	$H=\mu/2$
3  $\frac{V_o}{V_i}=\frac{GHs}{D(s)}$	1	$\xi/\mu\sqrt{c}$	$\mu^2/2\xi\sqrt{c}$
	2	$1/\mu$	$\mu^2/2\xi\sqrt{c}$
	3	—	1
		$G=2(\nu-d)/\mu$	$H=\mu/2$
4  $\frac{V_o}{V_i}=\frac{GHs^2}{D(s)}$	1	ξ/μ^2	μ/ξ
	2	$1/\sqrt{c}$	$\mu/\sqrt{c}$
		$G=(\nu-d)/\mu$	$H=1$
5  $\frac{V_o}{V_i}=\frac{GHs(s+\frac{b}{a})}{D(s)}$	1	$\xi a/\mu b$	μ/ξ
$(\sqrt{c}+\xi)>\frac{b}{a}$	2	$\xi a/\mu(\mu a-b)$	$\mu/\sqrt{c}$
	3	$1/\sqrt{c}$	—
		$G=(\nu-d)a/(\mu a-b)$	$H=1$
6  $\frac{V_o}{V_i}=\frac{GH(s+\frac{c}{b})}{D(s)}$	1	$\xi b/(\mu-b)^2\sqrt{c}$	$(\mu-b)^2/\xi\sqrt{c}$
$\sqrt{c}>b$	2	$\xi/(\mu-b)\sqrt{c}$	1
	3	$(\sqrt{c}-b)/(\mu-b)\sqrt{c}$	—
		$G=(\nu-d)/(\mu-b)$	$H=b$

Table 2
Realizations of $T(s)=\dfrac{h(as^2+bs+c)}{s^2+ds+c}$ using Dasher's synthesis procedure

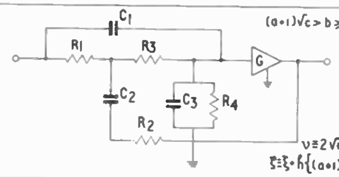
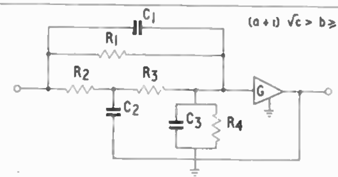
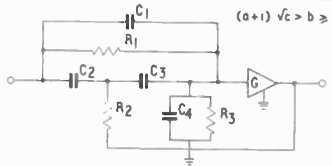
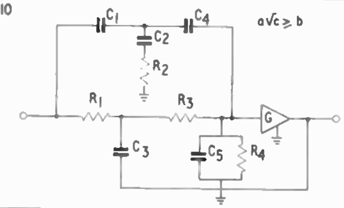
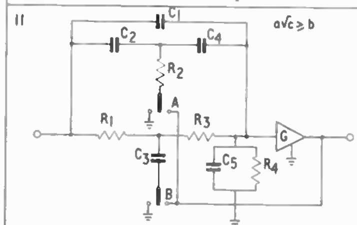
CIRCUIT	x	R_x	C_x
7  $(a+1)\sqrt{c}>b\geq a\sqrt{c}$	1	$\xi/\hbar\zeta\sqrt{c}$	$a\hbar$
	2	$\xi(b-a\sqrt{c})/\zeta^2\sqrt{c}$	$\zeta^2/\xi\sqrt{c}$
	3	$(\zeta-\xi)/\zeta\hbar\sqrt{c}$	$1-a\hbar$
$\nu\equiv2\sqrt{c}+\xi$	4	$1/(1-\hbar)\sqrt{c}$	—
$\zeta\equiv\xi+\hbar\{(a+1)\sqrt{c}-b\}$		$G=(\nu-d)/\zeta$	$\hbar=\min(1,1/a)$
8  $(a+1)\sqrt{c}>b\geq a\sqrt{c}$	1	$1/\hbar(b-a\sqrt{c})$	$a\hbar$
	2	$\xi/\zeta(\zeta-\xi)$	$\zeta^2/\xi\sqrt{c}$
	3	$1/\zeta$	$1-a\hbar$
	4	$1/(1-\hbar)\sqrt{c}$	—
		$G=(\nu-d)/\zeta$	$\hbar=\min(1,1/a)$
9  $(a+1)\sqrt{c}>b\geq\sqrt{c}$	1	$1/\hbar\sqrt{c}$	$\hbar(b-\sqrt{c})/\sqrt{c}$
	2	ξ/ζ^2	$\zeta(\zeta-\xi)/\xi\sqrt{c}$
	3	$1/(1-\hbar)\sqrt{c}$	$\zeta/\sqrt{c}$
	4	—	$1-a\hbar$
		$G=(\nu-d)/\zeta$	$\hbar=\min(1,1/a)$
10  $a\sqrt{c}\geq b$	1	$\xi/\hbar\zeta\sqrt{c}$	$a\hbar\zeta/\xi$
	2	$(\zeta-\xi)\xi/\hbar\zeta^2(a\sqrt{c}-b)$	$a\hbar\zeta^2(a\sqrt{c}-b)/\xi b(\zeta-\xi)$
	3	$(\zeta-\xi)/\hbar\zeta\sqrt{c}$	$\hbar\zeta^2/(\zeta-\xi)\xi$
	4	$1/(1-h)\sqrt{c}$	$a\hbar\zeta/(\zeta-\xi)$
	5	—	$1-a\hbar$
		$G=(\nu-d)(\zeta-\xi)/\hbar\zeta\sqrt{c}$	$\hbar=\min(1,1/a)$
11  $a\sqrt{c}\geq b$	1	$\xi/\hbar\zeta\sqrt{c}$	$b\hbar/\sqrt{c}$
	2	$\xi(\zeta-\xi)/(a\sqrt{c}-b)\hbar\zeta^2$	$\hbar\zeta(a\sqrt{c}-b)/\xi\sqrt{c}$
	3	$(\zeta-\xi)/\hbar\zeta\sqrt{c}$	$\hbar\zeta^2/(\zeta-\xi)\xi$
	4	$1/(1-\hbar)\sqrt{c}$	$\hbar\zeta(a\sqrt{c}-b)/(\zeta-\xi)\sqrt{c}$
	5	—	$1-ah$
		$G^A=(\nu-d)(\zeta-\xi)/(a\sqrt{c}-b)\hbar\zeta$	$\hbar=\min(1,1/a)$
		$G^B=(\nu-d)(\zeta-\xi)/\hbar\zeta\sqrt{c}$	$G^{AB}=(\nu-d)/\zeta$

Table 3

Realizations using Fialkow and Gerst's synthesis procedure

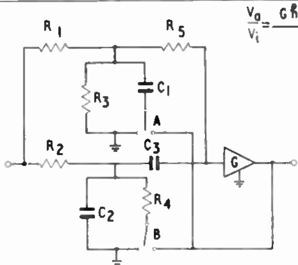
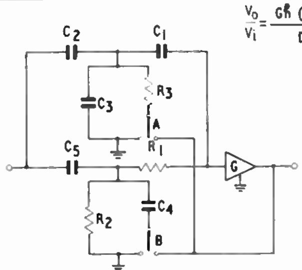
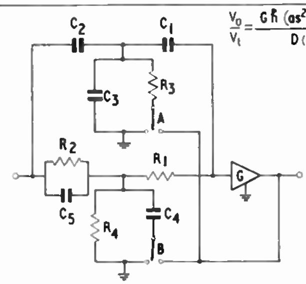
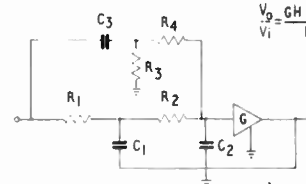
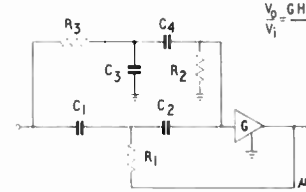
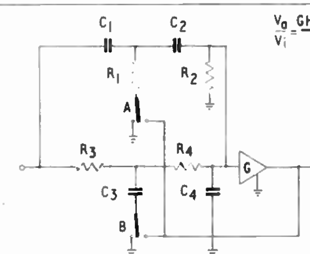
CIRCUIT	x	R_x	C_x
12  $\frac{V_o}{V_i}=\frac{G\hbar(bs+c)}{D(s)}$	1	$\xi/\nu\hbar\sqrt{c}$	$\nu^2/2\xi\sqrt{c}$
	2	$\xi/\nu b\hbar$	ν/ξ
	3	$\xi/\nu(1-\hbar)\sqrt{c}$	$\nu/2\sqrt{c}$
	4	$2\xi/\nu(\nu-2b\hbar)$	—
	5	$2/\nu$	—
		$G^A=2(\nu-d)/\nu$	$\hbar=\min(1,\nu/2b)$
$D(s)=s^2+ds+c$		$G^B=2(\nu-d)/(\nu-2b\hbar)$	$G^{AB}=(\nu-d)/(\nu-b\hbar)$
13  $\frac{V_o}{V_i}=\frac{G\hbar(as^2+bs)}{D(s)}$	1	$2/\nu$	$\nu/2\sqrt{c}$
	2	$\xi/\nu\sqrt{c}$	$\nu a\hbar/\xi$
	3	$2\xi/\nu^2$	$\nu(1-a\hbar)/\xi$
	4	—	$\nu(\nu-2b\hbar)/2\xi\sqrt{c}$
	5	—	$\nu b\hbar/\xi\sqrt{c}$
		$G^A=2(\nu-d)/\nu$	$\hbar=\min(1/a,\nu/2b)$
$\nu=2\sqrt{c}+\xi$		$G^B=2(\nu-d)/(\nu-2b\hbar)$	$G^{AB}=(\nu-d)/(\nu-\hbar b)$
14  $\frac{V_o}{V_i}=\frac{G\hbar(as^2+bs+c)}{D(s)}$	1	$2/\nu$	$\nu/2\sqrt{c}$
	2	$\xi/\nu h\sqrt{c}$	$a\nu\hbar/\xi$
	3	$2\xi/\nu^2$	$\nu(1-a\hbar)/\xi$
	4	$\xi/(1-\hbar)\nu\sqrt{c}$	$\nu(\nu-2\hbar b)/2\xi\sqrt{c}$
	5	—	$\nu hb/\xi\sqrt{c}$
		$G^A=2(\nu-d)/\nu$	$h=\min(1,1/a,\nu/2b)$
		$G^B=2(\nu-d)/(\nu-2\hbar b)$	$G^{AB}=(\nu-d)/(\nu-\hbar b)$

Table 4

Realizations using Guillemin's synthesis procedure

CIRCUIT	x	R_x	C_x
15  $\frac{V_o}{V_i}=\frac{GH(bs+c)}{D(s)}$	1	$2\xi/\mu\sqrt{c}$	$\mu^2/2\xi\sqrt{c}$
	2	$2/\mu$	$(\mu+b)/2\mu$
	3	$2\xi/b\sqrt{c}$	$\mu b/2\xi\sqrt{c}$
	4	$2/b$	—
$D(s)=s^2+ds+c$		$G=(b+\mu)(\nu-d)/\mu^2$	$H=\mu/(b+\mu)$
16  $\frac{V_o}{V_i}=\frac{GH(as^2+bs)}{D(s)}$	1	$2\xi b/a\mu^3$	$\mu^2 a/2\xi b$
	2	$2b/(b+\mu a)\sqrt{c}$	$\mu^2 a/2b\sqrt{c}$
	3	$2\xi/\mu^2$	$\mu/2\xi$
	4	—	$\mu/2\sqrt{c}$
$\mu=\sqrt{c}+\xi$ $\nu=2\sqrt{c}+\xi$		$G=(\mu a+b)(\nu-d)/\mu^2 a$	$H=\mu/(\mu a+b)$
17  $\frac{V_o}{V_i}=\frac{GH(as^2+c)}{D(s)}$	1	$2\xi/a\mu^2$	$\mu a/2\xi$
	2	$2/a\sqrt{c}$	$\mu a/2\sqrt{c}$
	3	$2\xi/\mu\sqrt{c}$	$\mu^2/2\xi\sqrt{c}$
	4	$2/\mu$	$1/2$
		$G^A=(a+1)(\nu-d)/a\mu$	$H=1/(a+1)$
		$G^B=(a+1)(\nu-d)/\mu$	$G^{AB}=(\nu-d)/\mu$

respectively and then adjusting the admittance level and also the amplifier gain, as explained in Sections 3 and 4, circuit 17 in Table 4 is obtained. The remaining circuits in Table 4 are obtained in a similar fashion.

The following identities are used in Tables 1–4.

$$\left.\begin{aligned} \mu &\equiv \sqrt{c+\xi} \\ \nu &\equiv 2\sqrt{c+\xi} \\ \zeta &\equiv \xi + h\{(a+1)\sqrt{c}-b\} \end{aligned}\right\} \qquad \ldots\ldots(47)$$

G^{A}, G^{B} and G^{AB} are the amplifier gains when either or both terminals A, B are used as port 3.

6. Design of Low-pass Filter

The tables will now be used to realize a low-pass filter having the following specifications:

Maximum pass-band attenuation:	1 dB
Minimum stop-band attenuation:	28 dB
Cut-off frequency:	3·6 kHz
Selectivity factor:	0·92

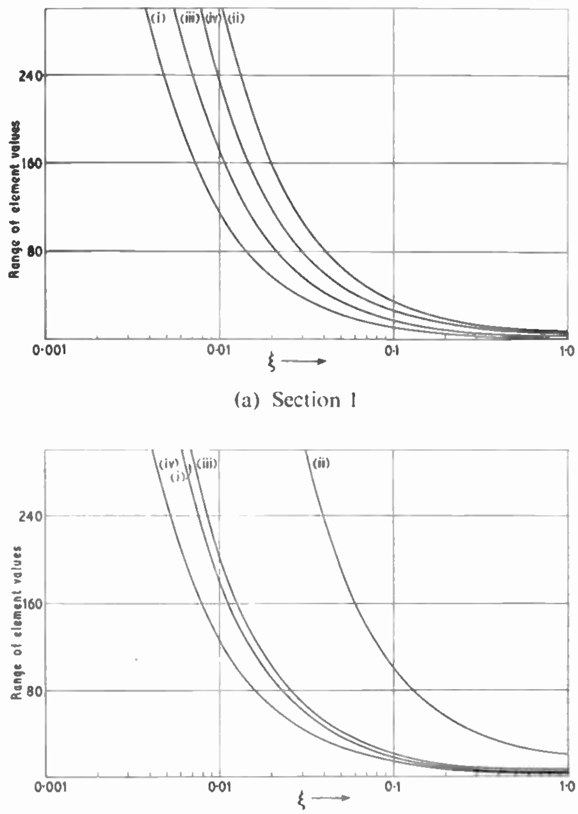
A suitable approximation is the elliptic transfer function

$$T(s) = \frac{(0{\cdot}3635s^2+0{\cdot}7128)(0{\cdot}8005s^2+1{\cdot}004)(0{\cdot}4776)}{(s^2+0{\cdot}4067s+0{\cdot}7128)\times(s^2+0{\cdot}0672s+1{\cdot}004)(s+0{\cdot}4776)}$$

$$= T_1(s)T_2(s)T_3(s) \qquad \ldots\ldots(48)$$

In a filter design the permissible range of element values is first decided by choosing the maximum and minimum permissible resistances and capacitances.

Each of the first two sections can be realized using Table 2, 3 or 4. The optimum realization will be the one in which ξ is smallest (sensitivity is optimum) for the given permissible range of element values. On using ξ as the independent parameter a number of realizations for each synthesis procedure can be obtained, using a simple computer program; graphs of R_{max}/R_{min} and C_{max}/C_{min} against ξ will give the required value for ξ and also the optimum realization. Such graphs are shown in Fig. 3. Several interesting points are evident. The Dasher procedure gives unequal capacitance and resistance ranges whereas the other two procedures give equal ranges. For Section 1 ($Q = 2{\cdot}08$) the Dasher procedure gives the largest capacitance range and also the smallest resistance range for a given ξ; if both resistance and capacitance ranges are of equal importance the Fialkow and Gerst procedure gives the smallest ξ for a given range of element values whereas the Guillemin procedure gives a marginally larger ξ. On the other hand, for Section 2 ($Q = 14{\cdot}9$), the Guillemin procedure gives a somewhat smaller ξ than the Fialkow and Gerst procedure; the Dasher procedure still gives the largest capacitance range for a given ξ.



(a) Section 1

(b) Section 2

(i) (ii) R_{max}/R_{min} and C_{max}/C_{min} for Dasher realization

(iii) $R_{max}/R_{min} = C_{max}/C_{min}$ for Fialkow and Gerst realization

(iv) $R_{max}/R_{min} = C_{max}/C_{min}$ for Guillemin realization.

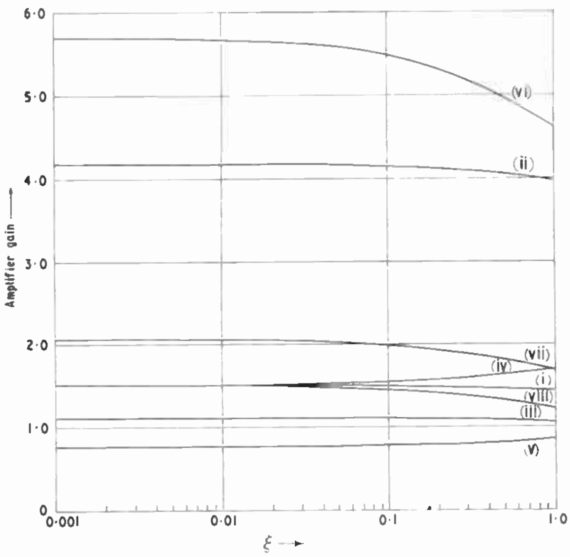
Fig. 3. Variation of range of element values with ξ.

Figure 4 shows the variation of amplifier gain with ξ for Sections 1 and 2. For each realization three different gains are possible, G^{A}, G^{B} and G^{AB}, depending on which leads are used as port 3 (see Tables).

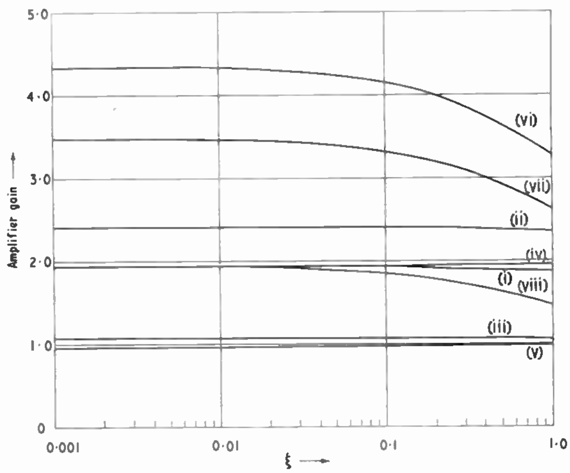
The parameter h can be chosen equal to unity. Then the Dasher and the Fialkow and Gerst realizations need seven elements per section whereas the Guillemin procedure needs eight.

The Fialkow and Gerst procedure was chosen to realize the low-pass filter. Assuming that

$$R_{max}/R_{min} = C_{max}/C_{min} < 45$$



(a) Section 1



(b) Section 2

(i) (ii) (iii) G^A, G^B, G^{AB} for Dasher realization

(iv) (v) G^A or G^B, G^{AB} for Fialkow and Gerst realization

(vi) (vii) (viii) G^A, G^B, G^{AB} for Guillemin realization.

Fig. 4. Variation of amplifier gain with ξ.

$\xi = 0{\cdot}04$ for Section 1 and $\xi = 0{\cdot}05$ for Section 2. The realizations of $T_1(s)$ and $T_2(s)$ are obtained from circuit 14 in Table 3 by letting $b \to 0$. $T_3(s)$ is realized using one resistor and one capacitor. The realization shown in Fig. 5 is obtained after frequency and impedance denormalization. The third amplifier is used to provide low output resistance. The filter was constructed using elements of tolerance $\pm 1\%$.

It is possible to adjust the position of the poles of each section after construction by adjusting the amplifier gain. For the RC-tree (or RC-ladder) procedure from eqns. (32)–(33), (6) (or (11)) and (24)–(25)

$$\frac{V_o}{V_i} = \frac{G(-y_{12})(s+\sqrt{c})}{s^2+d's+c}$$

where

$$d' = 2\sqrt{c}+\xi-GK_3$$

Since the negative real part of the pole is equal to $d'/2$ and d' is a linear function of G, increasing G will move the pole nearer to the j-axis whereas decreasing G will have the opposite effect. When $c \gg d'$ the pole movement is nearly parallel to the negative real axis. In practice the adjustment can be carried out by applying a signal having a frequency equal to the imaginary part of the pole and then adjusting the amplifier gain until the measured filter attenuation attains the theoretical value. It can be shown that pole adjustment is also possible in the RC-parallel-ladder procedure.

After adjustment the measured frequency response shown in Figs. 6(a) and (b) was obtained, which agrees

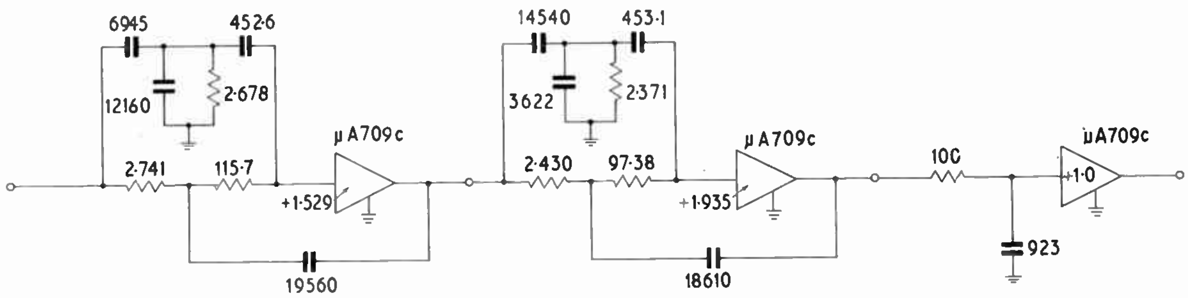
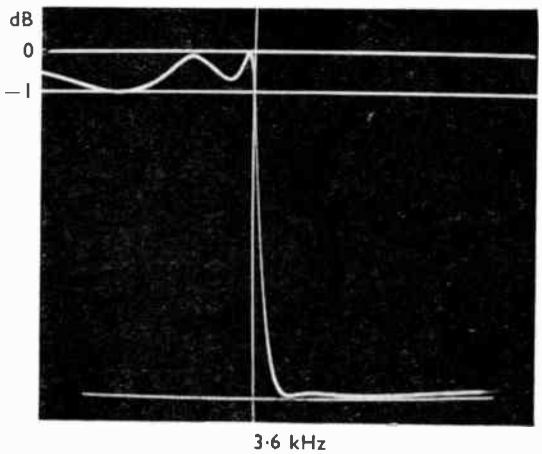
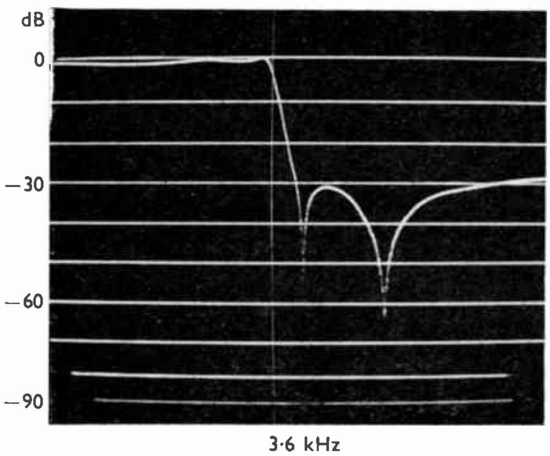


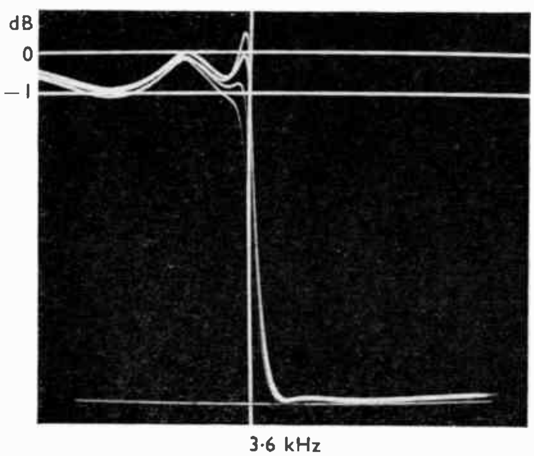
Fig. 5. Fifth-order active filter.



(a) Linear gain and frequency scales.



(b) Logarithmic gain scale and linear frequency scale.



(c) Response at temperatures −10°C, 20°C, 40°C and 70°C; gain near cut-off increases as the temperature is reduced.

Fig. 6. Frequency response of low-pass filter.

with the theoretical. The filter was found to be relatively insensitive to variations in the supply voltage and also temperature. Thus for the voltage range 5–15 V the deviation in the attenuation was less than $\pm 0{\cdot}1$ dB; for the voltage range 7–15 V the deviation was $\pm 0{\cdot}05$ dB. Figure 6(c) shows the variation of frequency response with temperature in the range −10°C to +70°C; the gain of the filter near cut-off increases as the temperature is reduced.

7. Conclusions

It has been shown that Y_L can be eliminated in all possible second-order filter realizations thus reducing the number of passive elements; then structures result which resemble those given by Sallen and Key. It is thus seen that the n.i.c. procedures, the positive-gain amplifier procedures and the Sallen and Key procedure are combined into a single class. The Q-factor sensitivity is a linear function of the Q-factor and also a parameter ξ upon which the range of element values depends. Different realizations of the same transfer function give different ranges of element values for a fixed value of ξ; conversely the value of ξ is different for each realization when the range of element values is fixed; consequently their sensitivities differ even though the sensitivity of each realization has been optimized.

8. Acknowledgments

Acknowledgment is made to the Senior Director of Engineering, Post Office Engineering Department, for his permission to publish the information contained in this paper. Thanks are due to J. M. Rollett for his comments; also to D. Knight for building and testing the active filter and D. R. Wise for checking the tables and carrying out the computer programming.

9. References

1. Antoniou, A., 'New RC-active-network synthesis procedures using negative-impedance convertors', *Proc. Instn Elect. Engrs*, **114**, No. 7, pp. 894–901, 1967.
2. Horowitz, I. M., 'Optimization of negative-impedance conversion methods of active RC synthesis', *Trans. Inst. Radio Engrs on Circuit Theory*, CT-7, pp. 296–303, 1959.
3. Foster, R. M., 'A reactance theorem', *Bell Syst. Tech. J.*, **3**, pp. 259–67, 1924.
4. Cauer, W., 'Synthesis of Linear Communication Networks' (McGraw-Hill, New York, 1958).
5. Guillemin, E. A., 'Synthesis of Passive Networks' (Wiley, New York, 1957).
6. Fialkow, A. and Gerst, I., 'The transfer function of general two terminal-pair networks', *Quart. Appl. Math.*, **10**, pp. 113–27, 1952.
7. Dasher, B. J., 'Synthesis of RC transfer functions as unbalanced two terminal-pair networks', *Trans. I.R.E.*, CT-1, pp. 20–34, 1952.
8. Sallen, R. P. and Key, E. L., 'A practical method of designing RC active filters', *Trans. I.R.E.*, CT-2, pp. 74–85, 1955.

9. Bown, G. C., 'Sensitivity in active RC-filters', *Electronics Letters*, 3, pp. 298–9, July 1967.

10. Seshu, S. and Balabanian, N., 'Linear Network Analysis' (Wiley, New York, 1959).

11. Balabanian, N., 'Network Synthesis' (Prentice-Hall, Englewood Cliffs, N.J., 1958).

10. Appendix

It is generally thought that the Fialkow and Gerst procedure requires an excessive number of passive elements. Although this is true for third- or higher-order transfer functions it is not so for second-order transfer functions where, in effect, twin-T structures result. In this Appendix the Fialkow and Gerst synthesis procedure is presented in a slightly modified form compared to that given in the original paper. It is then used to realize a biquadratic transfer function.

Consider an nth-order transfer function

$$T(s) = N(s)/D(s) \qquad \ldots\ldots(49)$$

which has positive numerator coefficients equal to or less than the corresponding denominator coefficients. The denominator polynomial has distinct, negative real zeros. Such a transfer function is called a ' realizable function '.*

Let

$$D(s) = sd_1(s)+d_2(s) \qquad \ldots\ldots(50)$$

where $d_1(s)$ and $d_2(s)$ have $n-1$ distinct negative real zeros. Similarly let

$$N(s) = sn_1(s)+n_2(s) \qquad \ldots\ldots(51)$$

where $n_1(s)$ and $n_2(s)$ are of degree $n-1$. The coefficients of $n_1(s)$ and $n_2(s)$ should be equal to or less than the corresponding coefficients of $d_1(s)$ and $d_2(s)$ respectively. (This requirement can always be satisfied when the coefficients of $N(s)$ are equal to or less than the corresponding coefficients of $D(s)$.)

A monic polynomial $Q(s)$ is chosen so that $d_1(s)/Q(s)$ and $d_2(s)/Q(s)$ are RC driving-point admittance functions. On dividing the numerator and the denominator of the right-hand side of eqn. (49) by $Q(s)$ eqns. (49)–(51) give

$$T(s) = \frac{\{sn_1(s)/Q(s)\}+\{n_2(s)/Q(s)\}}{\{sd_1(s)/Q(s)+d_2(s)/Q(s)\}} \qquad \ldots\ldots(52)$$

Now consider a 2-port made up of two parallel 2-ports (A and B). This network has a voltage ratio

$$\frac{V_o}{V_i} = \frac{(-y^A_{12})+(-y^B_{12})}{y^A_{22}+y^B_{22}} \qquad \ldots\ldots(53)$$

On equating the right-hand side of eqn. (52) to the right-hand side of eqn. (53) the following assignments can be made

$$\left.\begin{aligned} -y^A_{12} &= sn_1(s)/Q(s) \\ y^A_{22} &= sd_1(s)/Q(s) \\ -y^B_{12} &= n_2(s)/Q(s) \\ y^B_{22} &= d_2(s)/Q(s) \end{aligned}\right\} \qquad \ldots\ldots(54)$$

Then networks A and B have open-circuit transfer functions

$$\left.\begin{aligned} T^A(s) &= \frac{(-y^A_{12})}{y^A_{22}} = \frac{n_1(s)}{d_1(s)} \\ T^B(s) &= \frac{(-y^B_{12})}{y^B_{22}} = \frac{n_2(s)}{d_2(s)} \end{aligned}\right\} \qquad \ldots\ldots(55)$$

The above equations show that the problem of realizing an nth-order transfer function is resolved to that of realizing two transfer functions of order $n-1$. To complete a cycle of events the driving-point admittances y^A_{22} and y^B_{22} should be reduced in order too. The poles of y^A_{22}, now given by $Q(s)$, should be reduced by one. Then the resulting $y^{A'}_{22}$ will have $n-2$ poles, given by $q_1(s)$, say. Similarly the poles of y^B_{22} should be reduced by one. Then the resulting $y^{B'}_{22}$ will have $n-2$ poles given by $q_2(s)$, say. If this can be done, $q_1(s)$ and $q_2(s)$ are of the right order relative to the transfer functions $T^A(s)$ and $T^B(s)$. Then the cycle can be repeated on each of the two parallel networks and so on, until first-order transfer functions result.

The order of y^A_{22} and y^B_{22} can be reduced by removing series impedances from networks A and B as this has no effect on the open-circuit transfer function. To reduce the number of poles y^A_{22} and hence its order a series capacitance $C = y^A_{22}(0)/s$ must be removed from network A as shown in Fig. 7. Since

$$\frac{1}{y^A_{22}} = \frac{1}{sC}+\frac{1}{y^{A'}_{22}} \qquad \ldots\ldots(56)$$

and

$$C = d_1(0)/Q(0) \qquad \ldots\ldots(57)$$

we have

$$y^{A'}_{22} = \frac{d_1(s)}{\dfrac{1}{s}\left\{Q(s)-\dfrac{Q(0)d_1(s)}{d_1(0)}\right\}} = \frac{d_1(s)}{q_1(s)} \qquad \ldots\ldots(58)$$

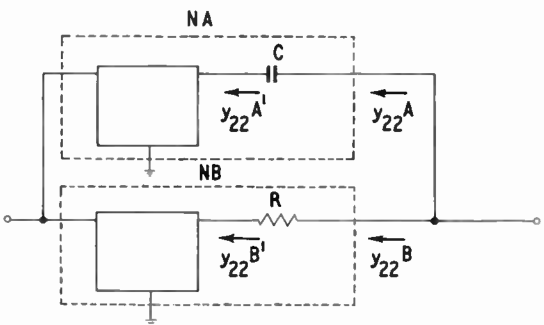


Fig. 7. Fialkow and Gerst's configuration.

It can be shown that $q_1(s)$ is of degree $n-2$ as required. To reduce the number of poles of y_{22}^{B} and hence its order, a series resistance $R = 1/y_{22}^{B}(\infty)$ must be removed from network B as shown in Fig. 7. Since

$$R = Q(\infty)/d_2(\infty) \qquad \ldots\ldots(59)$$

and

$$\frac{1}{y_{22}^{B}} = R + \frac{1}{y_{22}^{B'}} \qquad \ldots\ldots(60)$$

we have

$$y_{22}^{B'} = \frac{d_2(s)}{\left\{Q(s) - \dfrac{Q(\infty)d_2(s)}{d_2(\infty)}\right\}} = \frac{d_2(s)}{q_2(s)} \qquad \ldots\ldots(61)$$

It can be shown that $q_2(s)$ is of degree $n-2$ as required.

At this point the cycle is complete. The same procedure can be repeated on the resulting sub-networks. Note that the polynomials $q_1(s)$ and $q_2(s)$ are not monic, in general, in contrast to $Q(s)$. Finally first-order transfer functions result which can be realized using inverse-L networks. The degree of the final polynomials $q_1(s)$ and $q_2(s)$ is one less than the numerator or denominator of the first-order transfer functions. Hence $q_1(s)$ and $q_2(s)$ degenerate into constants. After dividing the numerator and denominator of the first-order transfer functions by the respective constants, transfer functions of the form

$$T(s) = \frac{a_0 s + a_1}{b_0 s + b_1} \qquad \ldots\ldots(62)$$

result. Then y_{22} is given by

$$y_{22} = b_0 s + b_1 \qquad \ldots\ldots(63)$$

The above transfer function can be realized using the inverse-L network given in Fig. 8.

Example:

The ‘ realizable function ’

$$T(s) = \frac{N(s)}{D(s)} = \frac{ahs^2 + bhs + ch}{s^2 + vs + c} \quad \text{where } v = 2\sqrt{c} + \xi$$

will now be synthesized. The parameters a, b, c, h, v are all positive constants; h is chosen to make the numerator coefficients equal to or less than the corresponding denominator coefficients; it follows that $0 < h \leqslant 1$.

The active synthesis procedure imposes that $Q = s + \sqrt{c}$. Equation (50) can be written as

$$D(s) = s(s + v - \lambda) + (\lambda s + c)$$

where λ is a positive constant. Hence

$$d_1(s) = s + v - \lambda$$

and

$$d_2(s) = \lambda s + c$$

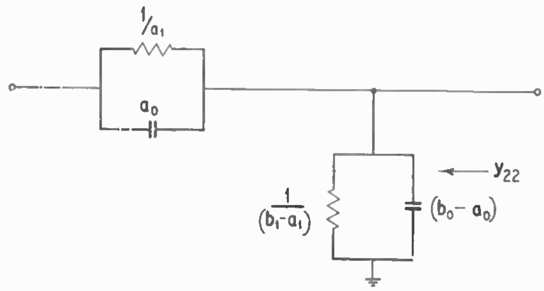


Fig. 8. Inverse-L realization of first-order transfer function.

Also from eqn. (51),

$$n_1(s) = ahs$$

$$n_2(s) = bhs + ch$$

Equations (54) and (55) give

$$T^{A}(s) = \frac{ahs}{s + v - \lambda}$$

$$y_{22}^{A}(s) = \frac{s(s + v - \lambda)}{s + \sqrt{c}}$$

$$T^{B}(s) = \frac{bhs + ch}{\lambda s + c}$$

$$y_{22}^{B}(s) = \frac{\lambda s + c}{s + \sqrt{c}}$$

The expressions assigned to y_{22}^{A} and y_{22}^{B} are admittance functions provided that λ is chosen to satisfy the condition

$$\sqrt{c} < \lambda < (v - \sqrt{c})$$

As $\lambda \to \sqrt{c}$ the zero of y_{22}^{B} tends to coincide with the pole of y_{22}^{B}. Likewise as $\lambda \to (v - \sqrt{c})$ a zero of y_{22}^{A} tends to coincide with the pole of y_{22}^{A}. Thus when λ has a value near its permissible limits the pole-zero distribution of y_{22}^{A} or y_{22}^{B} becomes lopsided. The effect of this will be to give networks with a large range of element values. For best evenness of pole-zero distributions and hence minimum range of element values λ is chosen to have a value in the centre of the permissible range, that is

$$\lambda = \tfrac{1}{2}v$$

On removing a series capacitance C_1 equal to $y_{22}^{A}(0)/s$ from network A eqn. (57) gives

$$C_1 = v/2\sqrt{c}$$

Equation (58) gives

$$y_{22}^{A'} = \frac{(s + \frac{1}{2}v)}{\xi/v}$$

since

$\xi = v - 2\sqrt{c}$. Thus $q_1(s) = \xi/v$ (i.e. it is a constant).

Similarly a series resistance R_1 equal to $1/y^{B}_{22}(\infty)$ is removed from y^{B}_{22}. Equation (59) gives

$$R_1 = 2/v$$

From eqn. (61) we have

$$y^{B'}_{22} = \frac{\frac{v}{2}s + c}{\sqrt{c\xi}/v}$$

and

$$q_2(s) = \sqrt{c\xi}/v$$

On dividing the numerator and denominator of $T^{A}(s)$ (or $T^{B}(s)$) by $q_1(s)$ (or $q_2(s)$) we have

$$T^{A}(s) = \frac{[avhs/\xi]}{[(vs/\xi)+(v^2/2\xi)]}$$

$$T^{B}(s) = \frac{[(vhbs/\sqrt{c\xi})+(h\sqrt{cv/\xi})]}{[(v^2 s/2\sqrt{c\xi})+(v\sqrt{c/\xi})]}$$

Using eqn. (62) and Fig. 8, the appropriate inverse-L networks are obtained as shown in circuit 14, Table 3. It is evident that for transfer functions in which a, b and $c \leqslant 1$ the constant h should be chosen equal to unity since R_4 becomes infinite. When $b = 0$ (the transfer function has j-axis transmission zeros) $C_5 = 0$. If, in addition, $a = 1$ $C_3 = 0$.

Circuits 12 and 13 in Table 3 can be obtained in a similar fashion.

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Of Current Interest . . .

Black Arrow Satellite Experiments

The Space and Instrumentation Group of British Aircraft Corporation's Guided Weapons Division has been awarded a contract by the Electron Physics Department, The University of Birmingham, for a space experiment. A micrometeoroid counter will be flown in the British technological satellite *Black Arrow X-3* to be launched in 1971. The purpose of this counter is to investigate the flux of small extra terrestrial dust particles (micrometeoroids) entering the Earth's atmosphere. The University of Birmingham system detects the charge released when micrometeoroids collide with a solid surface and by this means it is possible to detect particles as small as one millionth of an inch in diameter. This is in orders of magnitude smaller than has been possible using other techniques; the weight of 100,000 of these particles would be less than a billionth of an ounce (1 ounce 528 grammes).

The Space and Instrumentation Group is already engaged in the design development and manufacture of four *Black Arrow X-3* satellite structures, together with associated handling equipment, under contract to the Ministry of Technology.

X-3 is one of the series of technological satellites to be launched in 1971 by the all-British *Black Arrow* launching system. The satellite will be placed in a near polar orbit and rotate at some 200 rev/min. It will weigh about 75 kg and be approximately 70 cm high by 1·4 cm in equatorial diameter.

Other space activities of the Guided Weapons Division include production of the attitude sensing system for *HEOS-A*, the first Highly Eccentric Orbiting Satellite, programmed for launch in October 1968 on behalf of the European Space Research Organization, and the supply and pay-load preparation of the *Skylark* Upper Atmosphere Sounding Rocket for the British space research programme and for the European Space Research Organization.

Instrument Collaboration with Soviet Russia

The Scientific Instrument Manufacturers' Association has signed an agreement with the U.S.S.R. Ministry of Instrument Making, Automation and Control Systems, to promote direct co-operation and collaboration in the field of instrumentation. This follows an inter-governmental agreement between the Ministry of Technology and the U.S.S.R. Council of Ministers for Science and Technology on collaboration in the technological field.

The main areas covered by the S.I.M.A. agreement are: instruments for spectral and chemical analysis; precise electrical measuring laboratory instruments; instruments for data processing of the results of scientific investigations; and glass and laboratory ware.

Emphasis has been placed on the necessity to achieve specific results and projects between United Kingdom industry, represented by individual firms, and the various organizations concerned in the U.S.S.R. This series of discussions forms part of an overall project for collaboration between the United Kingdom and the Soviet Union which is being co-ordinated by the Confederation of British Industry.

European Passive Electronic Component Manufacturers Associations

At a meeting of the Committee of European Passive Electronic Component Manufacturers Associations (C.E.P.E.C.), held in Paris on 28th June, Mr. A. C. Bentley (Companion), who is Secretary of the United Kingdom Radio and Electronic Component Manufacturers' Federation, was nominated President-elect to take office on 1st January, 1969. He will succeed M. Y. Simmler of S.C.F.C.E.F.-S.I.P.A.R.E. (France). The Vice-President-elect is Mr. Van der Weiden of the F.A.P.E.L. organization in the Netherlands.

The members of C.E.P.E.C. are the appropriate trade associations of Belgium, France, Germany, Italy, the Netherlands and the United Kingdom.

European Physical Society

As reported in the March issue of *The Radio and Electronic Engineer*, progress has been made in the formation of a European Physical Society. At a meeting of the Steering Committee of the Society, held in Prague on 3rd and 4th May, agreement was reached on a provisional constitution and on a preliminary budget. The formal foundation of the Society will take place in Geneva on 26th and 27th September 1968, when an interim Executive Committee will be elected.

The inaugural Scientific Meeting will be held from 8th to 11th April 1969 in Florence. The programme will include, in addition to plenary sessions, parallel sessions on: astronomy, astrophysics, cosmology and relativity; nuclear and elementary particle physics; physics of condensed materials; atomic, molecular and plasma physics; and quantum electronics and optics. Emphasis will be placed on the growth points in these fields.

Further information on the Society and its activities may be obtained from: Dr. L. Cohen, Secretary of the Institute of Physics and The Physical Society, 47 Belgrave Square, London, S.W.1; or from Mrs. L. Etienne Amberg, Ecole de Physique de l'Université, 32 Boulevard d'Yvoy, 1211 Geneva 4, Switzerland.

International Scientific Radio Union

Every three years the International Scientific Radio Union (U.R.S.I.) holds a General Assembly in order to promote international co-operation in the scientific study of radio. The 16th General Assembly of U.R.S.I. will be held from 18th to 28th August 1969 at the Carleton University, Ottawa, and all arrangements are being made by the National Research Council of Canada.

Subjects to be discussed in the scientific sessions include probing of the atmosphere by electromagnetic waves, communications at millimetre wavelengths, and computers in radio science.

Further details may be obtained from the Chairman of the General Arrangements Committee for the Assembly: Dr. R. S. Rettie, Chief of Space Research Facilities Branch, National Research Council, Ottawa 7, Canada.

U.D.C. 621.396.969.33:656.6.05

Operational Aspects of V.H.F. Communication and Radar Surveillance by Port Operations Centres

By

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Presented at the Aerospace, Maritime and Military Systems Group Symposium on 'Integrated Harbour Surveillance Systems', held in London on 21st February 1968.

Summary: The Port Operations Centre passes information from the shore-based radar station to the ship through the Pilot of the ship by v.h.f. communication. The advantages of such rapid, clear and direct communication between a ship underway within a port and a shore authority are described. The operation of this system and present and future developments are also discussed.

1. Introduction

In view of the vast capital cost of ships, it will be evident that it is of vital importance that the time they spend in port in discharging and loading cargoes should be as short as possible. Such developments as bulk carriage and containers have been and are being introduced to this end.

In a busy major port, there is inevitably a measure of traffic congestion; this of course applies especially in tidal ports where the bulk of inward and outward movements has to be concentrated into a short period on either side of high water. For example, in the London River, there are some 1000 movements a week which represents about 70 movements per tide.

Moreover, many major ports and especially those in North West Europe are estuarial and thus have to be approached through long and at times tortuous and narrow channels; also there are many river ports which require ships to proceed for many miles in the river where from time to time bends are encountered.

Taking into account the increase in the size of certain ships, notably tankers, and the higher speeds now usually available in many ships, it follows that problems concerning navigation are intensifying and this is especially so in low visibility. These problems are, of course, being offset by improvements in various directions and in particular, by electronic navigational aids.

The advantages of rapid, clear and direct communication between a ship underway within a port and a shore authority, i.e. the Harbour Master and his staff, who have a complete and up to date knowledge of all current activities, are obvious. It was this requirement which brought about the marriage of shore-based radar and v.h.f. aids which provide the main material pillars of a Port Operations Centre.

In fog, especially if it persists, unless special measures are taken, the working of a port may well come to a standstill with all the undesirable economic effects of that situation.

2. Operation

The Mersey Docks and Harbour Board and the Dock and Harbour Authorities of the Port of Liverpool, exemplifying the old adage of 'what Lancashire does today the rest of the world does tomorrow', introduced some twenty years ago the concept of a port radar station fitted with v.h.f. radio telephony passing navigational information to ships whose Pilots had taken on-board with them portable a.m.–v.h.f. transmitter receivers.

While, of course, ship-borne radar is invaluable as an aid to navigation, the additional information that a well-sited shore-based radar can provide—especially in a river with a number of bends—is highly useful.

Such a radar station confers the following advantages:

Good siting of the aerial or aerials ensures an uninterrupted view over the whole area and with a tailor-made aerial a very high degree of range and bearing discrimination is practicable. Moreover, a large-scale comprehensive display can be readily provided consisting of a number of displays with overlapping views of each section, in addition to a central display covering the whole area, of the approaches and channel within the coverage of the set. The displays may be the conventional type of p.p.i. or the photo plot. Each fixed display of a particular section normally would have a reproduction of the corresponding section of the chart showing the position of buoys etc., which can be illuminated

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Table 1

Transmitting frequencies for the band 156–174 MHz for radiotelephony in the International Maritime Mobile Service. (From International Regulations for Maritime Radio, Article 35, Appendix 18.)

Channel designators	Transmitting frequencies (MHz) Ship Stations	Transmitting frequencies (MHz) Coast Stations	Intership	Port Operations Single frequency	Port Operations Two frequency	Public Corre-spondence
1	156·05‡	160·65			10	8
2	156·10	160·70			8	10
3	156·15‡	160·75			9	9
4	156·20	160·80			11	7
5	156·25	160·85			6	12
6	156·30		1			
7	156·35	160·95			7	11
8	156·40		2			
9	156·45	156·45	5	5		
10	156·50		3	§		
11	156·55	156·55		3		
12	156·60	156·60		1		
13	156·65	156·65	4	4		
14	156·70	156·70		2		
15	156·75	Guard band 156·725 - 156·775 MHz				
16	156·80	156·80		Calling and Safety		
17	156·85	Guard band 156·825 - 156·875 MHz				
18	156·90	161·50			3	
19	156·95	161·55			4	
20	157·00	161·60			1	
21	157·05	156·05‡ *or* 161·65			5	
22	157·10	161·70			2	
23	157·15	156·15‡ *or* 161·75				5
24	157·20	161·80				4
25	157·25	161·85				3
26	157·30	161·90				1
27	157·35	161·95				2
28	157·40	162·00				6

‡ See note (e). § See note (f)

Notes

(a) The figures in the column headed 'Intership' indicate the normal sequence in which channels should be taken into use by mobile stations.

(b) The figures in the columns headed 'Port Operations' and 'Public Correspondence' indicate the normal sequence in which channels should be taken into use by each coast station. However, in some cases, it may be necessary to omit channels in order to avoid harmful interference between the services of neighbouring coast stations.

(c) During ice seasons, ship stations shall avoid harmful interference to communications on 156·30 MHz (Channel 6) between icebreakers and assisted ships.

(d) Administrations should, as far as possible, arrange that ship stations fitted with the channels corresponding to the figures in a circle can obtain a reasonably adequate use of available services.

(e) The frequencies 156·05 and 156·15 MHz marked ‡ are used as ship station frequencies in Channels 1 and 3 respectively and as coast station frequencies in Channels 21 and 23 respectively when these latter are used in the special semi-duplex public correspondence systems employed by France and Belgium, with 1 MHz separation between transmit and receive frequencies.

(f) Channel 10 marked § is also available for port operations in Region 2.

(g) In the United States of America, the frequencies 156·35, 156·90, 156·95, 157·05, 157·10, 157·15 and 157·20 MHz are not available for use in accordance with this Table. These frequencies will be used for other functions in the maritime mobile service.

as necessary to enable the current position of any ship echo to be rapidly related to that of a buoy or landmark or to that of another ship.

It is important to recognize that the basic function of the Port Operations Centre system, initially developed in Liverpool and subsequently adopted in many other ports throughout the world, particularly in North West Europe where the incidence of fog is high, is *advisory*. That is to say, the shore-station primarily gives the ship information and advice but not orders. It is possible that, at times, some of the advice is couched in fairly positive terms but it is freely accepted by the shore authorities that subject to what is described as general regulation, the Master as advised by the Pilot remains fully responsible for the actual handling and manoeuvring of his ship. This is of course different from the corresponding situation in air traffic control and this difference is due to the fundamental difference between the two environments.

In the early and mid-1950s, it became evident that v.h.f. channels for use by ships in conjunction with Port Operation Centres should be allocated on an international basis. However, there was then raging a considerable controversy as to whether an f.m. or a.m. system should be adopted. At a regional conference, at Hague in 1957, the decision for f.m. was taken which was subsequently endorsed by the World Radio Conference at Geneva in 1959. This provided for 28 channels spaced 50 kHz apart in the international maritime band, 156–174 MHz. These allocations were embodied in Appendix 18 of the International Regulations, and are shown in Table 1.

These provide a Safety and Calling Channel which provides the initial means of contact and either single or double frequency Port Operations Channels.

The single channels are normally used for the broadcasting by the shore-station of general information of the state of traffic and weather, whereas the double frequency channels are used, for example, in giving radar advice to an individual ship and thus the shore-station can continue to communicate without interference or delay. Allocations are also made for public correspondence purposes.

The adoption of this international maritime v.h.f. allocation enabled a fairly rapid build-up of Port Operations Centres. These included Southampton and the Thames Navigational Service and a number of other ports in the United Kingdom. Similarly, such centres were established in Europe, notably at Rotterdam, where the whole system on the New Waterway embraces some seven sections each with its own surveillance radar and v.h.f. Much the same applies on the Elbe.

The build-up of these systems has been much slower in the U.S.A. than in Europe. This is very surprising as there is quite a high incidence of fog on both the Atlantic and Pacific seaboards where there are a number of large ports and approaches by lengthy channels. The reasons for this delay were, of course, political and administrative rather than technical or financial.

However, a development did occur in the U.S.A. which while not directly related to the main theme has a bearing on it.

Following the disastrous collision between the *Andrea Doria* and the *Stockholm* off the Nantucket Light Vessel in 1956 a Congressional Enquiry took place. As a result the U.S. Government adopted a policy commonly referred to as 'bridge-to-bridge', the underlying assumption being that had the *Andrea Doria* and *Stockholm* been able to communicate with each other the collision would not have occurred. The implications of such a policy would require all ships to be fitted with v.h.f. and to keep constant watch on a specified navigational channel for the purposes of collision avoidance.

This U.S. policy has not received international support; the serious objections to it are that in areas of traffic density such as the Straits of Dover, the volume of shipping and the corresponding volume of inter-communication needed rapidly and clearly could not be attained. Language differences would accentuate this problem and the upshot would be delay and/or confusion allied to dangerous distraction from other tasks.

A v.h.f. system is not a compulsory fitting in ships although there is international provision to make it so in certain areas where this would contribute to the safety of navigation and movement and this already applies in the Suez Canal and in the St. Lawrence including the New Seaway.

However, although the v.h.f. system is not a mandatory requirement, over 80% of British and main European maritime countries' ships are so fitted. The shipowners who have had their ships fitted regard the use of v.h.f. primarily as providing a means of communication with Port Operations Centre to expedite entering and leaving harbour.

Of course, many harbours in the world, especially those in the Southern hemisphere where by and large there is not much fog, do not have Port Operation Centres of the type described and ships trading in such areas would thus not require v.h.f. installation.

3. Present-day and Future Developments

It is of interest to note that a few years ago the U.S. introduced for trial purposes a system known as RATAN in New York. The basis of this system was that the display of the harbour radar station was transmitted by television and could thus be received in ships equipped with a normal commercial television receiver. This trial system was designed primarily as an aid to the very large number of recreational craft—power boats—in which many U.S. citizens spend their leisure. There was not general acceptance of the system and it has been discontinued. A practical problem which could arise with such a system, especially in crowded areas, would be the difficulty of positively identifying one's own ship among the many echoes that would be visible. In addition, trials in the United Kingdom have shown that on frequencies which are available for allocation for this purpose, the picture is stable only in those areas where the direct line-of-sight signal is much greater than any reflected signal; even in this region the near passage of another ship can upset the picture. This, of course, is a serious limitation and disadvantage.

The question of transmitting the Port Operations Centre's radar display by data link so that it could be re-displayed on a shipborne radar set has also been considered. This of course is technically feasible but as yet there is no operational requirement and again the problem of identifying the echo of one's own ship could arise.

An interesting trial which is essentially long-term is being conducted by the Southampton Harbour Authorities in conjunction with selected Shipping Companies in a technique known as 'blind pilotage' which is defined in the Admiralty Manual of Navigation as 'conducting the passage of a ship in pilotage waters using all means not denied the navigator by low visibility'.

The technique of blind pilotage has been well developed in H.M. Ships over the years which, of course, have fairly elaborate facilities both in terms of radar and plotting equipment and relatively large numbers of officers and ratings are available.

For the trials at Southampton which are closely related to the navigational problem engendered by the S-shaped bend in the main approach channel off Calshot, two pile beacons with radar reflectors have been established in carefully selected positions. The radar range and bearing of ships underway in the channel is measured by interscan from the appropriate pile by the Port Operations Centre and the resulting information, having accuracy of a very high order, is frequently passed by v.h.f. system to the ship concerned. The ships are provided with special plotting sheets and are thus able to detect very rapidly whether they are adhering to, departing from or regaining their pre-planned track in the channel. This is an interesting and promising experiment although it is a little too early to draw firm conclusions.

4. Conclusions

It seems likely that the steady growth in the number of Port Operations Centres in the world will continue. As a result of the *Torrey Canyon* incident a tremendous impetus has been given in I.M.C.O. to improving the safety of navigation both in the open sea and in port approaches.

In anticipation of such a growth and additional frequencies that will be required in the International Maritime Frequency band, the World Administrative Radio Conference at their meeting in Geneva in the autumn of 1967 decided to adopt a carefully-phased programme to reduce the channel spacing from 50 to 25 kHz. This programme would be finally completed in 1982.

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Microwave-link Characteristics for a Harbour Surveillance System

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Presented at the Aerospace, Maritime and Military Systems Group Symposium on 'Integrated Harbour Surveillance Systems', held in London on 21st February 1968.

Summary: The paper outlines the technical requirements for a broadband, two-way single hop microwave link with hot stand-by facilities and automatic changeover. The basic design of the various units is described. The method of multiplexing the various signals, namely, radar video, radar sync, aerial bearing, two-state controls and supervisory data, is given in outline and also the techniques employed in order to provide continuous 24-hour operation by means of hot stand-by facilities with automatic changeover and sub-unit duplication are described.

1. The Justification for a Microwave Link

The previous papers[1, 2] have established the need for two-way communication between the remote radar stations and central control—incoming to transfer the radar data to the integrated display and outgoing to control the radar and other facilities at the out-station.

Study of the technical, economic and environmental factors in precision harbour radar systems shows that at present a wideband microwave link, substituting on a 1 : 1 basis for the normal cables between radar and display, is the simplest, cheapest and most flexible means of sending the extra wideband radar data to the control centre. Direct cable connection is neither practicable nor technically feasible for this bandwidth at the ranges involved. Again, data processing and conversion techniques for reducing bandwidth (including direct television transmission of a p.p.i. picture) are certainly practicable and today many types are in use or under development. All of these, however, are more complex, more costly, inherently less reliable and probably less accurate than those which simply transmit the raw data as is done in the case of a microwave link to be discussed.

For remote control of the radar out-stations, any one of a number of standard communication links (direct telephone line, v.h.f., u.h.f. or microwave radio, etc.) can be used depending on the circumstances. Often a direct line is too inflexible, insecure and limited in bandwidth to be acceptable and the natural choice when all factors are taken into account is to integrate the communication links by adding a conventional low or medium capacity microwave link in the reverse direction.

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2. The Link Specification in More Detail

2.1. *Remote Station to Control Centre*

The information to be transmitted is typically as follows:

2.1.1. Radar video signal

For normal complex radar receiver output waveform of varying echo pulses in clutter, noise, etc., the pulse characteristics are as follows:

Amplitude: +5 V maximum in 75 ohm

Rise-time: 20 ns minimum

Width: 60 ns to 640 μs (corresponding to 48 nautical mile radar-range)

Shoulder noise: +0·5 V.

The effective (i.e. baseband) bandwidth is to be not less than 150 Hz to 10 MHz (subject to approval by Licensing Authority) in order to preserve sufficient picture detail.

2.1.2. Radar sync. pulses

Amplitude: −5 V in 75 ohm

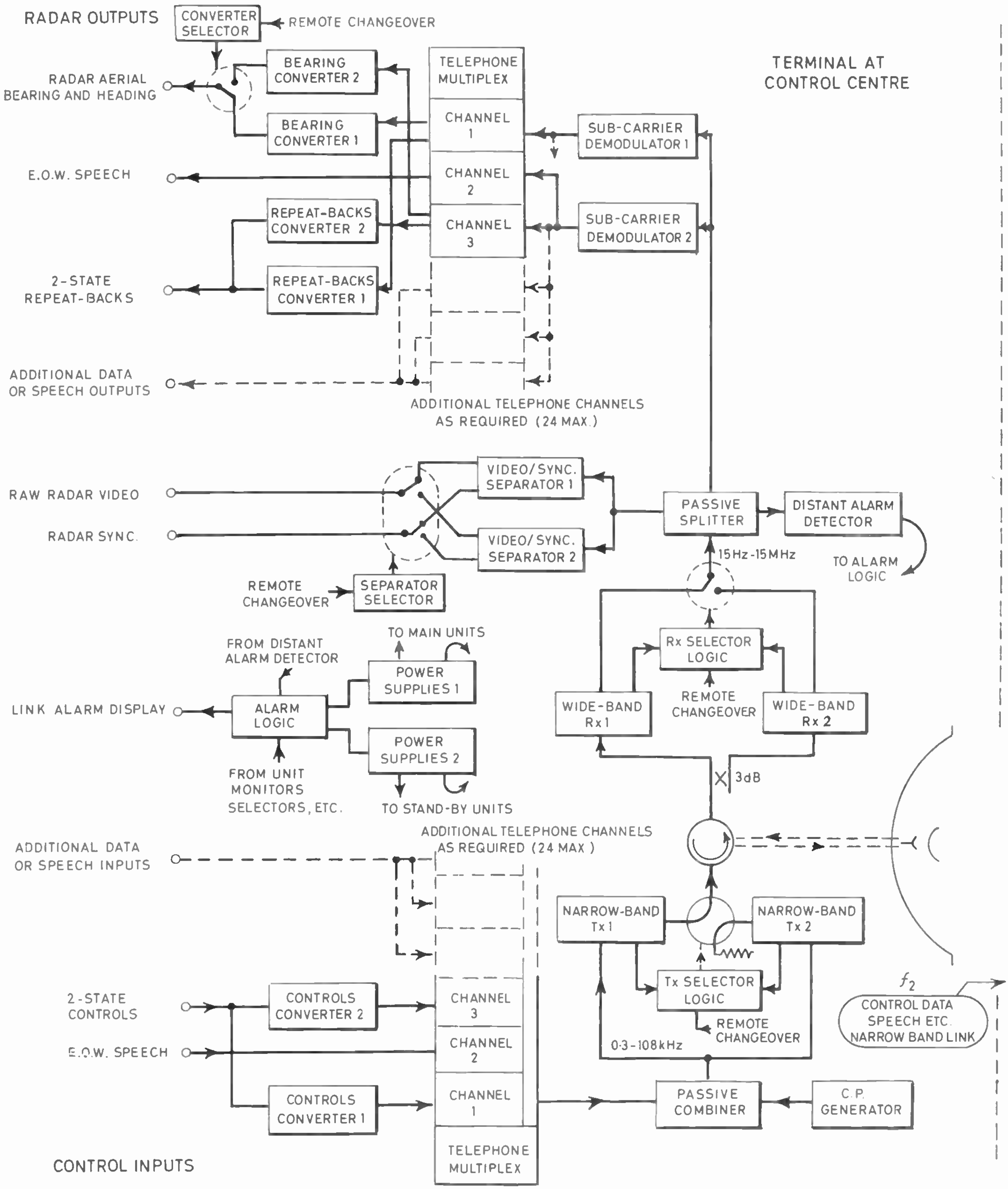
Rise-time: 20 ns

Width: 100 ns

The bandwidth to be as for radar video; relative timing of sync. and video to be unaffected by the link.

2.1.3. Radar aerial bearing and heading

The bearing waveform is a nominal 46 Hz sine-wave 5 V peak-to-peak in 75 ohm, the instantaneous frequency of which can vary continuously due to wind forces. The final following accuracy demanded is ±0·1°. The heading marker is a 5 V pulse occurring once per revolution whose timing relative to the bearing signal is to be preserved by the link.



RADAR OUTPUTS
CONVERTER SELECTOR
REMOTE CHANGEOVER
RADAR AERIAL BEARING AND HEADING
BEARING CONVERTER 2
BEARING CONVERTER 1
TELEPHONE MULTIPLEX
CHANNEL 1
CHANNEL 2
CHANNEL 3
SUB-CARRIER DEMODULATOR 1
SUB-CARRIER DEMODULATOR 2
TERMINAL AT CONTROL CENTRE
E.O.W. SPEECH
REPEAT-BACKS CONVERTER 2
REPEAT-BACKS CONVERTER 1
2-STATE REPEAT-BACKS
ADDITIONAL DATA OR SPEECH OUTPUTS
ADDITIONAL TELEPHONE CHANNELS AS REQUIRED (24 MAX.)
RAW RADAR VIDEO
RADAR SYNC.
VIDEO/SYNC. SEPARATOR 1
VIDEO/SYNC. SEPARATOR 2
PASSIVE SPLITTER
DISTANT ALARM DETECTOR
TO ALARM LOGIC
15 Hz-15 MHz
REMOTE CHANGEOVER
SEPARATOR SELECTOR
TO MAIN UNITS
FROM DISTANT ALARM DETECTOR
POWER SUPPLIES 1
POWER SUPPLIES 2
LINK ALARM DISPLAY
ALARM LOGIC
FROM UNIT MONITORS SELECTORS, ETC.
TO STAND-BY UNITS
Rx SELECTOR LOGIC
REMOTE CHANGEOVER
WIDE-BAND Rx 1
WIDE-BAND Rx 2
3 dB
ADDITIONAL TELEPHONE CHANNELS AS REQUIRED (24 MAX)
ADDITIONAL DATA OR SPEECH INPUTS
NARROW-BAND Tx 1
NARROW-BAND Tx 2
Tx SELECTOR LOGIC
REMOTE CHANGEOVER
0·3-108 kHz
f_2
CONTROL DATA SPEECH ETC. NARROW BAND LINK
2-STATE CONTROLS
CONTROLS CONVERTER 2
CONTROLS CONVERTER 1
E.O.W. SPEECH
CHANNEL 3
CHANNEL 2
CHANNEL 1
TELEPHONE MULTIPLEX
PASSIVE COMBINER
C.P. GENERATOR
CONTROL INPUTS

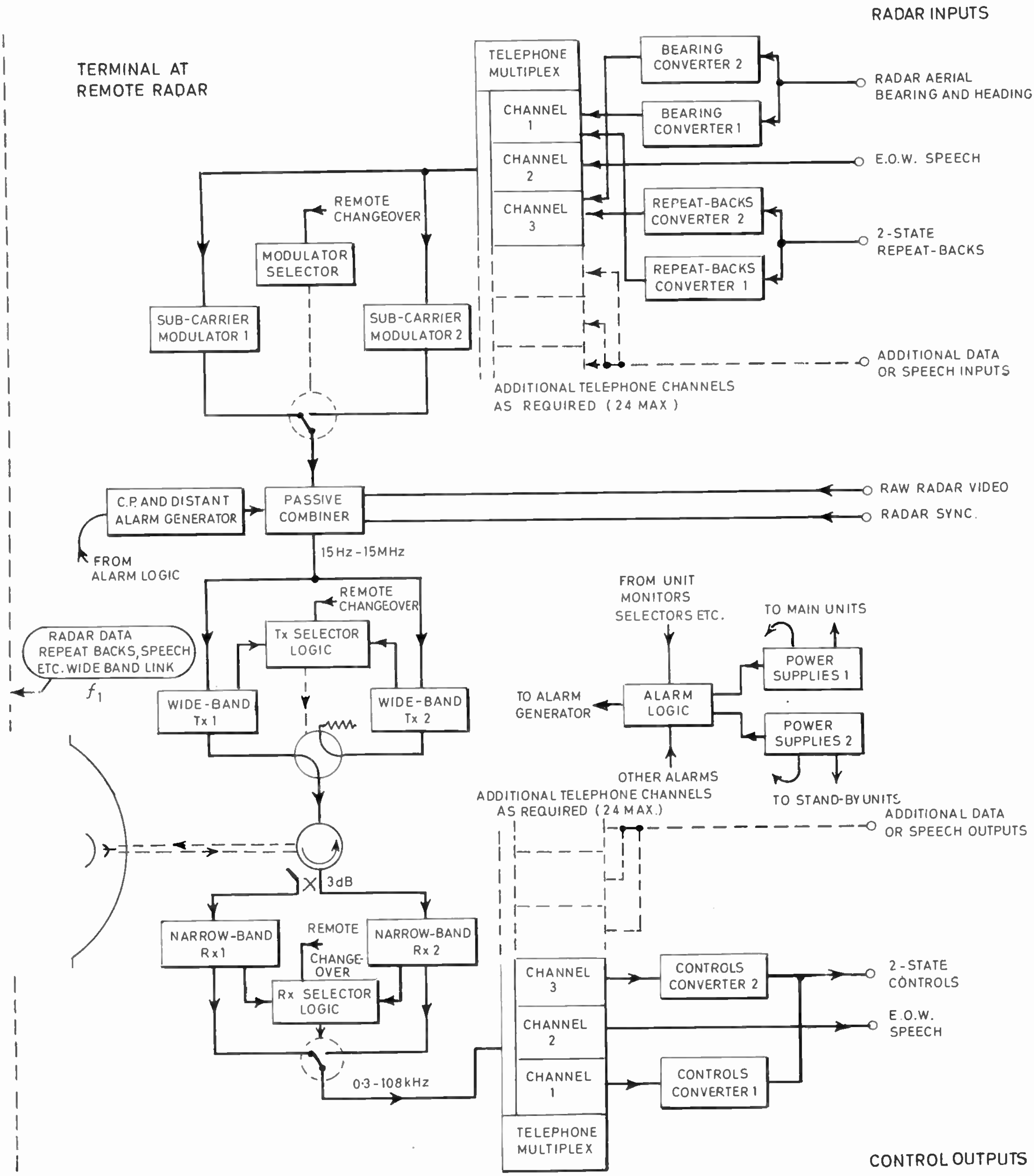


Fig. 1. Simplified functional diagram of two-way multiplex and microwave-link equipment with hot stand-by facility.

2.1.4. Radar and other remote station repeat-backs

For supervisory purposes a number of on/off back-indications, typically up to 24, are required.

2.1.5. Speech channels

A conventional omnibus speech circuit for maintenance work in the presence of traffic, plus a varying number of standard 3·1 kHz speech channels for such out-station facilities as remote ship-to-shore v.h.f., etc., are needed.

2.2. *Control Centre to Remote Station*

2.2.1. Radar and other remote station controls

A number of on/off control ways, typically up to 24, are required for remote control purposes.

2.2.2. Speech channels

As for section 2.1.5 above.

2.3. *Continuity of Service*

All transmission channels in both directions, except engineer's order wire and the less important controls and tell-backs, require maximum possible reliability.

2.4. *Environment*

2.4.1. Electrical

The link must be unaffected by the strong interference likely in industrial marine surroundings and in particular must be immune both to the high-power radar equipment it serves and to equipment of ships in the vicinity.

Propagation conditions will often be poor. Over-water paths are, as usual, prone to obstruction by ships, cranes, etc. Ranges may be up to many tens of miles in some cases, so needing unattended repeaters.

Power supply may be secure public mains or intermittent local generators. Battery stand-by working is therefore implied.

2.4.2. Mechanical

Industrial marine atmosphere is the typical situation. Where protective buildings are available the equipment must meet Class B specifications set by the Ministry of Transport. Otherwise, full Class X capability is necessary.

Maintenance will usually be in the hands of Port Authority staff who have no time to spare in repairing the electronic equipment. Simplified routines, suitable for a single trained man and the minimum of test gear, are therefore essential. Unit replacement on site, followed by repair only at base workshop, is the method to be used.

3. The Equipment

The dominant features in the link operational requirement are 24-hour uninterrupted service and sufficient bandwidth to avoid any significant deterioration of the radar picture. It cannot be stressed too strongly that failure on either count could create a hazard to shipping worse perhaps than no control system at all.

These two features, taken together with the desire for a self-contained 'package' fitting naturally in place of the normal direct connections between radar transceiver and display, determine all the principal aspects of the equipment as developed. With the exception of a single klystron in each transmitter (the most economical means of obtaining the 40 MHz or so of r.f. bandwidth needed) the system uses solid-state devices throughout. Special extra wideband microwave transmitters and receivers had to be developed and these, combined with comprehensive hot stand-by, automatic switching and fault reporting facilities built-in, enable the specification to be met in full.

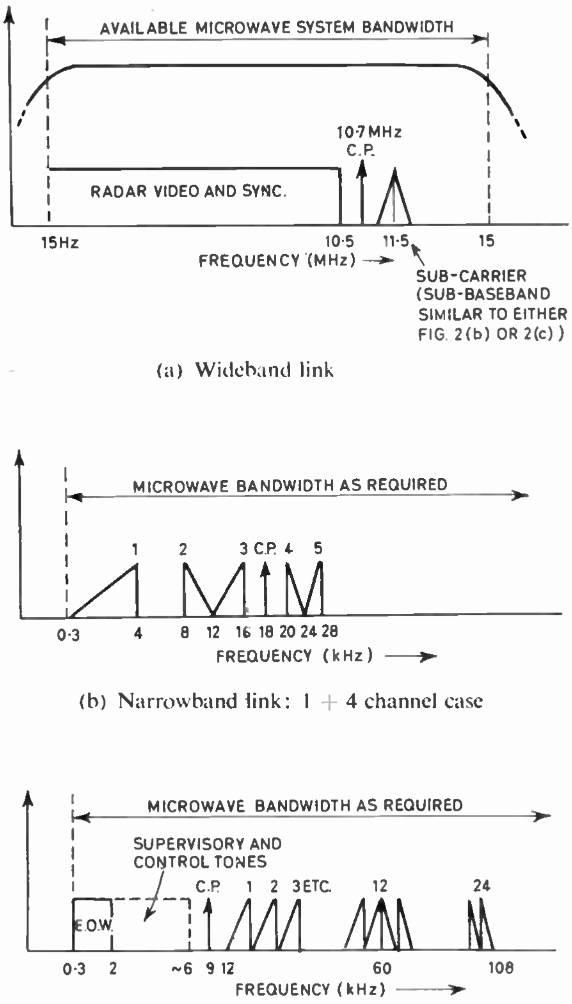
3.1. *Wideband Link*

The paths taken by the 'inwards' signals through the link are shown in the upper part of the simplified functional diagram (Fig. 1). This should be read together with the baseband spectral distribution (Fig. 2(a)).

3.1.1. Transmit (remote) terminal

Input MODEM's (*Modulator-Demodulator*). Formation of the composite baseband input to the microwave transmitter uses both time and frequency division multiplexing. The radar sync. and video inputs occur in time sequence. By adding these algebraically in the passive combiner only one wideband channel is needed. This typically occupies the band from about 150 Hz to 10·5 MHz, and the input is band-limited by a filter to fit this slot. Insufficient space is left at the bottom of the band for the auxiliary data which are therefore taken by a sub-carrier, usually at 11·5 MHz.

The sub-baseband input to the sub-carrier modulator is first channelled as required to fit standard telephone multiplex equipment. Normal maximum capacity at this point is twenty-four 3·1 kHz channels or equivalent. For the radar aerial bearing and heading signals, this means frequency modulation on twin variable frequency tones passing through the same telephone channel to stabilize relative delay. For the two-state repeat-backs it means one or more sub-sub-systems using standard t.d.m. or f.d.m. methods to multiplex the two-state data within one telephone channel. For the engineer's order wire and any other speech inputs, however, it means simply direct feed into their



(a) Wideband link

(b) Narrowband link: 1 + 4 channel case

(c) Narrowband link: 12 or 24 channel case

Fig. 2. Baseband spectrum arrangement.

respective channels. The combined multiplexed signal then frequency-modulates the sub-carrier and is passed through to the passive combiner.

The final addition to the microwave transmitter input at this point is the continuity pilot, usually chosen to be 10·7 MHz. This is used for automatic monitoring and switching of the microwave units and also carries modulation corresponding to the equipment state at the out-station.

Two features are worth noting here. The composite output signal from the combiner is set to conform to C.C.I.R. recommendations for television (i.e. +0·7 V video and −0·3 V sync. in 75Ω, plus standard sub-carrier and pilot levels). Secondly, the full baseband capability of the microwave system in this direction is 15 Hz to 15 MHz. Considerable scope exists therefore for changing the service provided, subject to the Licensing Authority's agreement. For instance, video bandwidth can be increased by about 30% over 10·5 MHz by using different video band-limiting, sub-carrier and pilot parameters. Alternatively, a full 625-line colour television picture can be substituted for raw radar video and sync. should the application demand it.

Microwave units. The microwave transmitter is a special development using exceptionally wideband frequency modulation of a 1-watt long-life low-noise klystron. This has liquid/vapour phase cooling and constant voltage supplies so that no a.f.c. is needed. The operating frequency is a spot in the band 5·8 to 8·2 GHz as allocated. C.C.I.R.-type pre-emphasis is built in along with continuity pilot and r.f. power monitors.

The r.f. output feeds via a waveguide change-over switch and duplexer to a Cassegrain-type paraboloid aerial of exceptionally low v.s.w.r. over a broad band. Aerial gain is chosen to suit hop length and 4, 6 or 10 ft diameters are typical. Where the equipment is at mast-foot, a dried and pressurized waveguide-run, statically charged with nitrogen, is used to give long-life corrosion-free operation.

3.1.2. Receive (local) terminal

Microwave units. Aerial, waveguide and duplexer arrangements here are similar to those at the remote terminal. Like the transmitter, the microwave receiver is a specially developed, extra wideband all solid-state device which faithfully recovers the original composite baseband signal at C.C.I.R. level, as applied to the transmitter. Continuity pilot and a.g.c. monitors are built-in and de-emphasis, as recommended by C.C.I.R., is incorporated.

Output MODEM's. The baseband signal is separated into its three parts by filtering in the passive splitter. The 'video/sync. separator' discriminates between the opposite signs of the video and sync. waveforms and provides amplified outputs virtually identical to those fed into the remote link terminal. The chain through the sub-carrier demodulator, telephone channelling and output converter units similarly recovers all the other input signals in their original forms. Thirdly, the 'distant alarm detector' deduces the equipment state as reported by the continuity pilot, and feeds to the 'alarm logic' and 'link alarm display' system.

3.2. *Narrowband Link*

The paths taken by the 'outwards' signals through the link are shown in the lower part of Fig. 1 and the baseband distribution in Fig. 2(b) or (c) as appropriate.

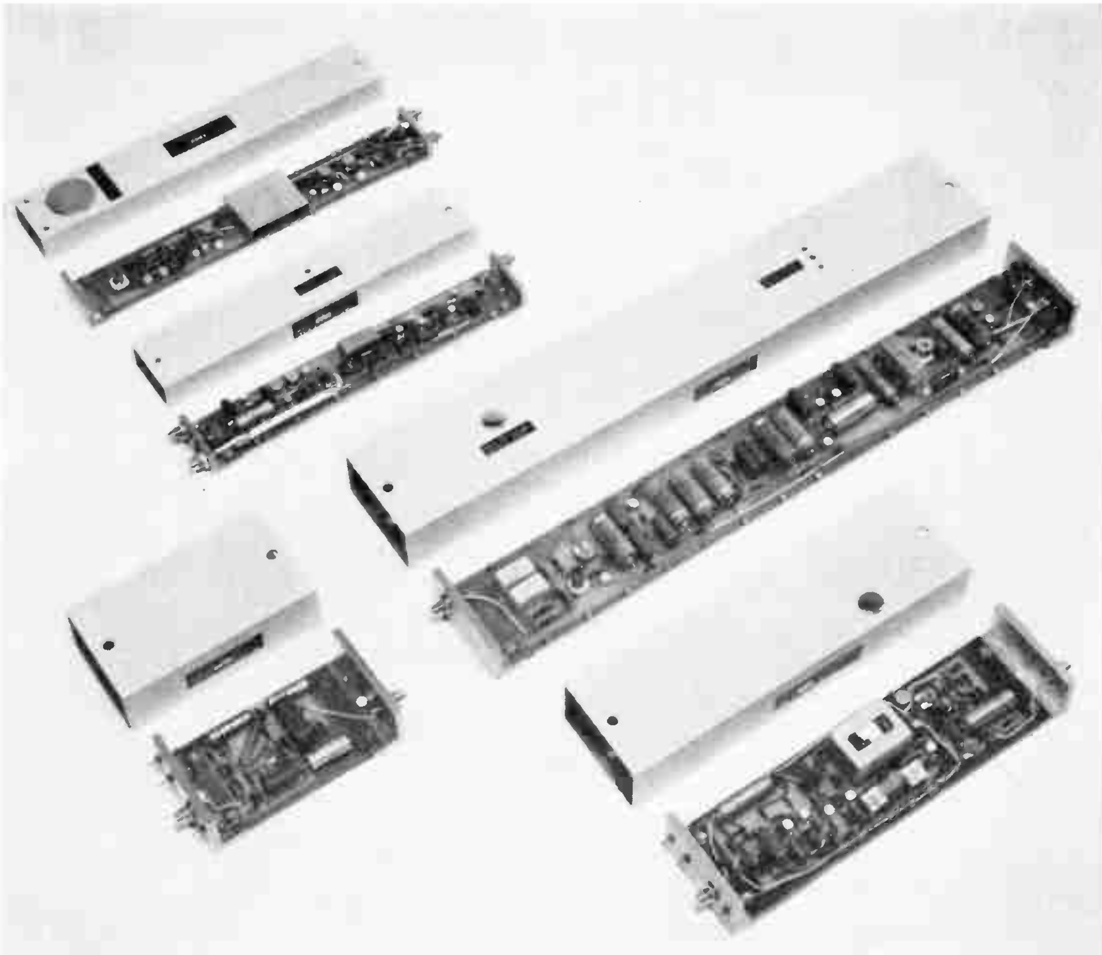


Fig. 3. Typical electronic sub-units.

Fig. 4. Typical duplicated multiplex and microwave link terminal.

The equipment involved in this direction is, of course, conventional multi-channel solid-state microwave equipment. Its traffic capacity is matched to the system requirement and is generally the same as that of the sub-carrier sub-system in the wideband direction.

3.3. *Hot stand-by, Automatic Switching and Fault-reporting Facilities*

So far, we have covered the main steps taken to ensure that the quality of the remote radar picture passed by the link is indistinguishable from that of the local picture displayed on the integrated photoplot, and in the process have shown how the link acts essentially as a 1 : 1 substitute for direct connections.

On the subject of 24-hour reliability, however, it has to be accepted that even the best equipment will fail sometimes, and further elaboration is necessary. For this reason, complete hot stand-by is provided for all active units and sub-units in the multiplex and microwave-link system.

Figure 1 shows the duplicated paths which are provided for all top-priority signals. Careful planning keeps the number of switches to a minimum. All the microwave and most of the multiplex stages either work permanently in parallel or are switched automatically from their respective monitors when required. Where the signals handled are too complex for automatic monitoring to be feasible, as with the 'video/sync. separator', the monitoring is done subjectively by the harbour controller. Two controls are fitted to the console, one for local and one for remote operation, to enable the controller to change to stand-by on these units if he is dissatisfied with their performance.

The same monitors which automatically control main to stand-by switching also report faults automatically as part of the built-in supervisory system. The 'link alarm display' warns the controller of a fault occurrence. From the convenient test points on the local microwave cabinet the maintenance man

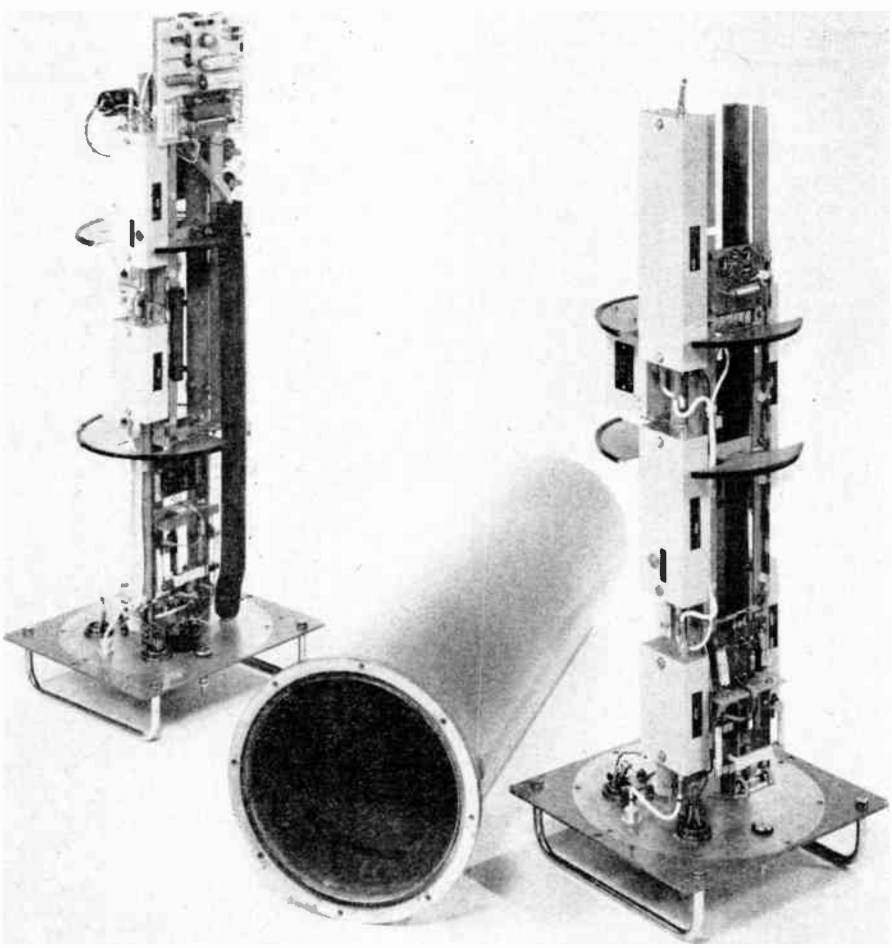


Fig. 5. Hermetically-sealed microwave transmitter and receiver.

identifies the location and type of fault and proceeds with repair work without interrupting traffic.

3.4. *Mechanical Aspects*

Basic sub-units and units in the system are all in modular form (Fig. 3). The physical arrangement of the equipment can therefore readily take different forms to suit the circumstances. The installation illustrated in Fig. 4 is a terminal intended for installation in a protective building. The whole of the fully-duplicated multiplex and microwave equipment for the terminal is housed in a single cabinet.

The microwave transmitters and receivers are built into individual containers for easy transport back to base when servicing is needed. These containers are designed for hermetic sealing should unprotected operation in an unfavourable environment be required, and sealed variants of each of the others are also available (Fig. 5).

4. Conclusions

It has not been possible in this brief paper to deal other than superficially with the many design considerations involved in making this equipment behave satisfactorily. Wherever possible, standard microwave-link practice has been followed and many features are in fact common to a normal range of such units. By the same token, advantage can be taken of established designs to build up different link configurations from this range of basic modules. For example, the single-hop system described, which caters for line-of-sight ranges up to 30 or 40 miles, can be extended over several hops in series by adding unattended repeaters. Then again, for 24-hour service on long hops over sea, diversity working becomes essential and the same basic units serve for this equally as for hot stand-by. As a final example, for the case where several remote radars have to feed a single display and control centre, the design allows for the extra r.f. channels to be added on either common or separate aerials as determined by the geography. Only the Licensing Authority can possibly object to such proliferation!

5. Acknowledgment

The author wishes to thank Ferranti Ltd., for permission to publish this paper.

6. References

1. Graham, P. W. W., 'Operational aspects of v.h.f. communication and radar surveillance by port operation centre', *The Radio and Electronic Engineer*, **36**, pp. 149–52, September 1968.
2. Harrison, A., 'A display centre for harbour surveillance and control', *ibid.*, pp. 161–9.

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A Display Centre for Harbour Surveillance and Control

By

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Summary: The operational requirements of a harbour control centre are dominated by the problems of radar surveillance. Narrow channels require a large-scale presentation, which leads to multiple displays, and the difficulties which arise in tracking a target from one display to another. The photographic projector display presents numerous advantages in this respect, as well as in preserving a permanent record.

Consideration is given to the design requirements of the different units of the system arising specifically in this application, and the procedure by which a system is designed to suit a particular site, and then integrated with the non-radar equipment which is equally necessary for efficient control of traffic in the harbour and its surroundings.

1. The Operational Requirements

In a harbour control centre the controller requires information on the distribution of traffic in the area under surveillance, which can be provided by radar. He also needs the following facilities to be at his disposal:

(i) ability to talk to the crews of ships and tugs by v.h.f. radio, or by signalling lamp, and to shore installations by telephone;

(ii) knowledge of the depth of water, and the state of the tide at different points, which can be provided by remote indicating gauges;

(iii) information of the wind-speed and direction, and perhaps of the visibility at remote points;

(iv) knowledge of the state of efficiency and readiness of the equipment available to him.

In addition he may require remote control of certain navigational aids such as lights and sound signals and some facilities to record events and perhaps complete v.h.f. communications on magnetic tape, written or other records of ship movements or incidents, such as the displacement or non-operation of navigational aids, etc.

These functions will only be executed efficiently when the control station is designed ergonomically to make the task possible. The greatest problem arises from the radar requirements, so that is the logical starting point, around which the rest of the facilities must be assembled.

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2. Radar Surveillance of a Harbour

In a harbour installation, the area to be covered is usually long and narrow, and winding between obstructing hills or buildings. Some typical channels are Milford Haven, which is 5 miles (8 km) long with one right-angle bend, 600 yards (550 m) wide opposite the tanker jetties and at the entrance, but the latter is not visible from inside the harbour; or Southampton, which has several bends in its 20 mile (32 km) length, is 600 yards wide at the entrance, narrowing to 300 yards in the last few miles; or London, where 30 miles (48 km) from the docks, off Southend Pier, the dredged channel is only 1000 ft (330 m) wide.

Two problems arise. First, it is often not possible to cover the whole area from a single radar, so remote radars must be used, controlled from the central site where the information is collected and displayed. This implies at least one individual display for each radar; but multiple displays are in any case made necessary by the requirement to distinguish ship targets in a narrow channel, which fixes a limit to the scale of the display. The channel should be displayed perhaps $\frac{1}{2}$ inch ($1\frac{1}{4}$ cm) wide, preferably more, for easy viewing. Taking the case of Southampton, this fixes the display scale at about 3 inches to the mile (1 : 20000), on which scale the whole channel is 45 inches (1·15 m) long, and requires three display c.r.t.s, even though only one radar at Calshot is used to cover the whole area.

An operational problem of maintaining the identification of one ship target moving from one c.r.t. to the adjacent one, perhaps with other ships in company or passing, immediately arises. This is

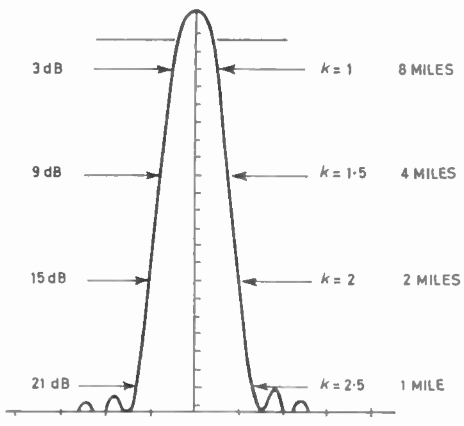


Fig. 1. Antenna radiation pattern (not actual).

difficult enough when both displays are connected to the same radar, but if the ship is passing to an area covered by different radars, the change of aspect and range of the ship cause big changes in the appearance of the target on the display.

The displayed size of a target depends on the radar pulse length and beam-width, and the c.r.t. spot-size, The latter is small enough to neglect in practice. The pulse length determines the radial dimension of the displayed target, while the beam-width defines the circumferential dimension. With increase in range, one would expect a constant radial dimension and a linearly increasing circumferential dimension; but signal strength decreases with range on an inverse fourth-power law at first, which has the effect of moving the signal to a different level on the aerial radiation pattern. This can be considered as the introduction of a further multiplying factor k, where

$$\text{effective beam-width} = k \times (\text{beam-width at 3 dB points})$$

and in a typical example, k will have the values shown in Fig. 1. Over operational ranges, the circumferential dimension of a target increases very approximately with the square-root of range.

Thus when a target moves to a different display where it is observed by a different radar, the apparent width of the target may change. What is more disturbing however is that when the target is observed by the second radar, it will have a different position and shape. Consider first a point target like a buoy. In Fig. 2 the spot is the actual target, while the shaded area is its appearance seen from different directions. A ship can be considered to be made up of a number of point targets distributed over an area comparable with the channel width, some of these points being invisible at certain aspects. For example, the ship outlined in Fig. 3 appears a completely different shape according to whether it is viewed from astern, when the bow is invisible, or abeam when it may break up into two separate targets. When several ships are in this area, where the observer must transfer his attention from one c.r.t. to another, there is every chance of confusion.

Many of these difficulties can be overcome by the use of the photographic projector type of radar display,† which offers a number of advantages in this application. It is interesting to note that the first suggestion of this technique appears in the Ministry of Transport Report of the International Meeting on Radio Aids to Marine Navigation in 1946, and many of the recommendations of this Report are still valid.

3. Photographic Projector Display

3.1. *The Display and its Advantages*

This display uses a small c.r.t., which is photographed on 16 mm cine film. The film is developed, fixed, washed and dried in a minimum of 3¾ seconds, then projected on a screen to produce a picture which may be 24 inches (60 cm) diameter and approach the brilliance of a domestic television set. Once set up, the photographic processing is completely automatic, and requires only simple daily maintenance and replacement of chemicals and film as required. At a picture renewal rate of 15 seconds, one roll of film lasts for a day, and one charge of chemicals suffices for several days.

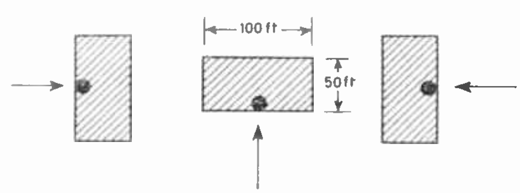


Fig. 2. Displayed signal from buoy 10 ft diameter. Range 1 mile, radar beam-width 1 degree, radar pulse length 0·1 μs.

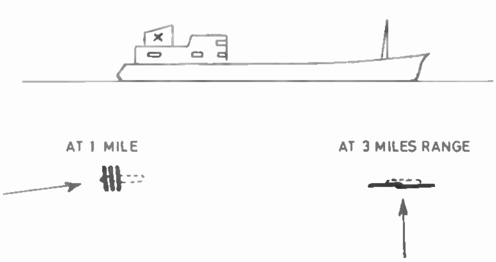


Fig. 3. Displayed signal from ship 600 ft long. (Scale 1½ inches = 1 mile.)

† Parsons, S. R., 'The application of rapid access photographic techniques to radar display', *J. Brit. Instn Radio Engrs*, **24**, No. 3, pp. 213–20, September 1962.

Harrison, A., 'Photoplot Display for Harbour Surveillance'. I.A.L.A. Bulletin, No. 33, July 1967.

The advantages of the display can be listed as follows:

(i) The displayed picture is 24 inches in diameter, i.e. half as big again as on the commonly used (16-inch) 40 cm c.r.t. Hence fewer displays are required to present a given area on a fixed scale.

(ii) The picture is bright enough to be viewed without screening or visors, in artificial light or subdued daylight, by several people at once. The observer does not require a dark-room, and avoids the consequent difficulties of restricted illumination for log-writing, observation of other instruments such as tide gauges and clocks, and use of v.h.f. and telephones.

(iii) The observed picture is projected on a screen. It is therefore possible to arrange two or more displays so that the edges of the projected pictures meet or overlap in their correct relative positions and alignment. A target at the crossover point therefore appears in the same position on the screen from both displays. This helps to overcome the difficulties of maintaining the identification of a target crossing this division. A photograph such as Fig. 4 can show the effect, but such a static picture, which is no more than a mosaic of p.p.i. photographs such as has been reproduced many times before this, cannot convey the advantages of a live dynamic presentation of this type.

(iv) The small diameter of the c.r.t. which is photographed makes it possible to arrange for photographic superimposition of a chart on the radar picture. This allows the navigational channel to be shown on the radar picture, together with buoys, leading lines and other marks. As well as providing the essential co-ordinates system for position reporting to ships, this is also invaluable for revealing movement of a buoy from its moorings.

(v) A further advantage arises from the photographic process. This provides a permanent record which can be of considerable value not only in preserving the facts leading up to an incident, but in the training of operators, the analysis of traffic, and the control of port operations such as dredging, spoil disposal, hydrographic survey, and so on.

4. Design and Engineering of the Radar Equipment

4.1. *Antenna*

The operational requirement to define a ship in a narrow channel at a distance from the radar calls for an antenna of narrow horizontal beam-width, i.e. wide aperture, which also exhibits low side-lobe responses. The slotted waveguide antenna permits close control of the aperture illumination to achieve side-lobe levels about 30 dB below the main beam. The requirement for flatness of the radiated wavefront, which is essential for a narrow beam, requires mechanical tolerances of the order of $\lambda/16$, i.e. rather less than 2·5 mm. This is also relatively easy to obtain with the slotted waveguide construction.

There would be no mechanical difficulty in manufacturing a long antenna of this type, but a limit is set by the transmitted pulse length. Taking typical figures of 0·05 μs pulse length and a 25 ft (7·5 m) aperture, the pulse would enter the radiating section and travel along it, its leading edge reaching the end of the waveguide just as the trailing edge of the pulse enters. The whole length of slotted guide would be radiating for an infinitesimal time only, preceded and followed by time intervals during which only part of the radiating aperture is operative. The effect would be to broaden the main beam, increase the side-lobe level, and drag out the leading and trailing edges of the pulse. The practical limit appears to be a guide length not exceeding about half the equivalent pulse length, i.e. 15 ft (4·6 m) for a 0·05 μs pulse, corresponding to just under half a degree beam-width.

This has important repercussions on the cost of the antenna. Other things being equal, the mechanical power required to rotate an antenna in still air rises with the fourth power of its length, while the wind-drag rises linearly with the length. In addition, the maintenance of the same mechanical tolerance over a greater length requires stiffer and heavier construction. This leads to a heavier turning mechanism, and a stronger supporting tower, so that the cost of civil engineering associated with the scanner overtakes the cost of the scanner, which is itself rising rapidly as the aperture is increased to obtain narrower beam-widths.

Apertures above 15 ft (4·6 m) have to date been obtained by the use of reflector type antennae. These exhibit even larger peak wind loads and require heavier driving motors, further increasing the cost of supporting towers. Apertures of 30 ft (9 m) giving beam-widths of about $\frac{1}{4}°$ have been used. It is important to appreciate how rapidly overall costs escalate when the minimum horizontal beam-width is demanded. This point is of importance in connection with system engineering problems discussed in Section 5.

Reduction of the vertical beam-width would allow the use of lower transmitted power, or alternatively would increase the range sensitivity of the system. There appears to be no difficulty in achieving the required performance with wide vertical beam-width.

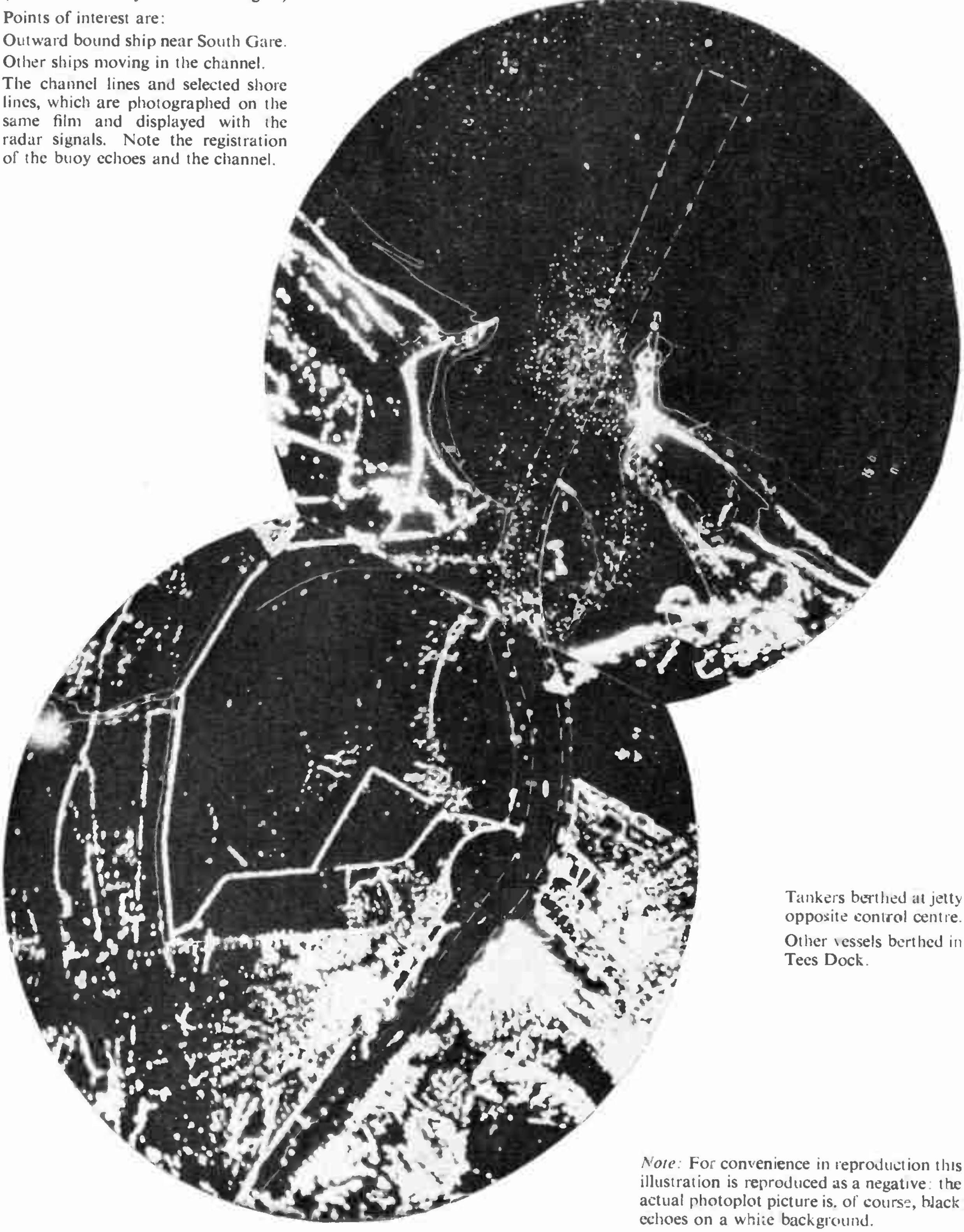
Fig. 4. The radar displays, Teesport. (To be read in conjunction with Fig. 5.)

Points of interest are:

Outward bound ship near South Gare.

Other ships moving in the channel.

The channel lines and selected shore lines, which are photographed on the same film and displayed with the radar signals. Note the registration of the buoy echoes and the channel.



Tankers berthed at jetty opposite control centre.

Other vessels berthed in Tees Dock.

Note: For convenience in reproduction this illustration is reproduced as a negative: the actual photoplot picture is, of course, black echoes on a white background.

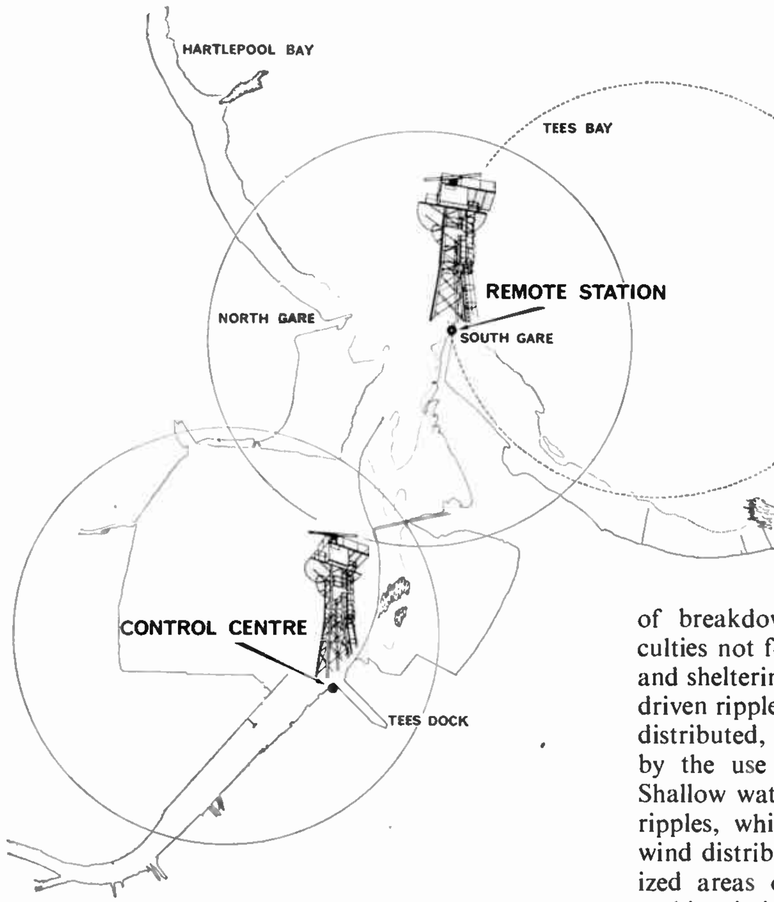


Fig. 5. Radar sites at Teesport.

On the other hand, to narrow the beam-width requires a greater vertical aperture and incurs penalties of windage and weight which cannot be justified. Hence most harbour radar aerials use excessive vertical beam-width, and attempts to narrow or shape the vertical beam do not appear worthwhile.

4.2. *Transmitter/receiver*

Apart from the requirement to use a slightly different frequency band, harbour radar transmitter/receiver systems are very similar to civil marine radar units. There is no requirement for high power, due to the aerial gain. Short pulses down to 50 ns are normally used, with the corresponding 20 or 30 MHz bandwidths. The limiting factor here is the magnetron design. Remote radars sometimes do not require the highest definition, and these figures may be relaxed, particularly since this eases the problems of the associated radio link. Controls such as tuning have to be mechanized for remote operation via the link, as well as the obvious ones such as on/off switching, but this presents no serious problems. Duplication and interswitching, also remotely operable, are usual, to provide continuity of service in case of breakdown. Anti-clutter circuits present difficulties not found in the open sea, due to shoal water and sheltering by the land. In the open sea, the wind-driven ripples which produce sea clutter are uniformly distributed, and the clutter signal may be suppressed by the use of swept gain circuits in the receiver. Shallow water upsets the uniform distribution of the ripples, while buildings and land masses upset the wind distribution which produces the ripples. Localized areas of strong or weak clutter signals exist, making it impossible to set the receiver controls to suppress all the clutter without oversuppressing some areas, at the risk of losing signals. Hence the control setting must be a compromise.

4.3. *Displays*

If the displays are on separate c.r.t.s accuracies of the same order as marine radar appear sufficient. The introduction of chart matching on any individual display requires the improvement of linearity and stability to better than 1% of the radius. The accurate matching of the edges of two projected pictures introduces considerably tougher requirements on stability of e.h.t. supply, off-centering, linearity, scale, and angular alignment. To illustrate this, consider a channel which appears $\frac{1}{2}$ inch (1·3 cm) wide at the edge of a display of 12 inches (31 cm) radius. An error of $\frac{1}{8}$ inch (3 mm) from one display to the other is as much as can be tolerated. Sharing this between the several contributory errors leads, for example, to the conclusion that the orientation of each display must be stable and accurate to better than $\frac{1}{8}°$ from antenna to display. A servo system to achieve this accuracy, and the transmission of the scanner azimuth data through the radio link, present considerable difficulty.

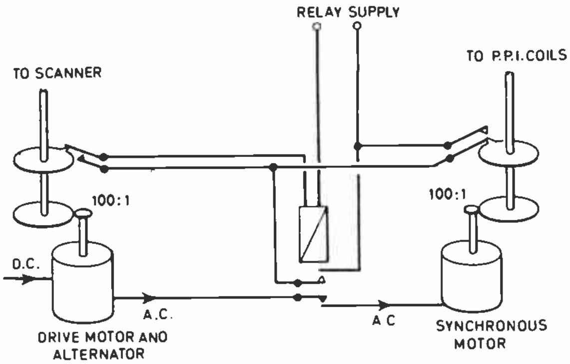


Fig. 6. Simplified diagram of contactor system.

The approach we have used is to generate a single-phase a.c. waveform in the scanner turning mechanism, at a ratio of 100 : 1 to scanner speed, and identify one of these cycles by a pulse at a particular scanner azimuth. These two signals are carried by the link. At the display, a second orthogonal phase of the a.c. waveform is generated, the two phases then driving a synchronous motor geared 100 : 1 to the rotating deflector coils. A contact on these coils must close in synchronism with the identifying pulse from the scanner—if it does not, as may occur on starting-up, the coils stop and re-start in the correct phase. Figure 6 shows a simplified diagram of the contactor system. Driving the nominally constant load of the rotating coils, the synchronous motor does not vary its angular position by more than about 10 degrees with respect to the phase of the a.c. waveform, so the rotating coil, which is geared 100 : 1 to the motor, does not vary by more than about one-tenth of a degree. This system is considerably cheaper, more reliable, and more efficient than a servo system for this particular application. It requires careful choice of the synchronous motor to achieve the required stopping and re-starting characteristics for initial alignment.

5. System Engineering

The control centre and the area to be covered by radar surveillance are usually specified for a particular installation. A survey of the area must be made to determine the requirement for remote radars. This can be started on charts and Ordnance Survey maps, but often requires verification by field trials with a transportable radar. The scale of display and the number of display units can then be considered. This is a most critical stage, since it largely determines the overall cost of the installation. The shape of the channel, its bends and obstructing hills and buildings, will determine the minimum number of radars required to observe the whole area. In the smaller sections no problems of definition will arise, but in the larger sections, where a ship must be clearly distinguished from the edges of a narrow channel, at a considerable range, very careful consideration must be given to the alternatives of:

(i) one radar, centrally-sited, which may require a large narrow beam aerial,

(ii) two or more radars, suitably placed in the area, where the cost of the extra electronic equipment may be offset by the reduced price of each antenna, supporting tower, etc.

An important advantage of a multiple installation is the increased operational reliability it provides. Failure of one radar can only lose coverage of part of the area, and indeed it may be possible to give coverage of the section, at a lower level of definition from the other radar, until the fault is cleared.

By the use of single displays on some radars, and multiple off-centre displays from others, it is possible to arrange the display scale to be constant over the whole area. An exception may be made in the port approaches, where it is useful to use the same radar to cover the local channel on the uniform scale adopted, and to operate a long range display at a reduced scale. Additional survey work may be necessary to determine line-of-sight paths for microwave links, and sites for antenna towers, where existing buildings cannot be used. The provision of ancillaries such as v.h.f. aerials, tide and wind gauges, etc. must also be considered at this stage, and the links between them and the control-centre if the microwave links are not available. This work should lead to a detailed specification of the equipment and buildings to meet the requirement.

Figure 5 shows the layout finally agreed for Teesport with one radar covering the docks and inner half of the channel, sited at the control centre, and a remote radar on a tower at South Gare, covering the outer half of the channel and the sea approaches, with a microwave link to the control-station.

Figure 7 shows the control-station with the local radar tower also carrying the dish for the microwave link. A servicing platform gives access to the transmitter hut, and also serves to isolate the link aerial from the radar aerial. Crosstalk here can raise problems, but the shielding action of this platform appears to have assisted in eliminating it. The control room is on the front of the building, with big windows giving a clear view of the channel in all directions. In the right-hand corner of the picture is one of the transportable radars used to assess the value of different possible sites in the initial survey.

The control room is organized around the radar displays since these are the most complicated units. The required number of displays are engineered into a console as shown in Fig. 8, individually built up

Fig. 7.
Control station and antenna mast, Teesport.

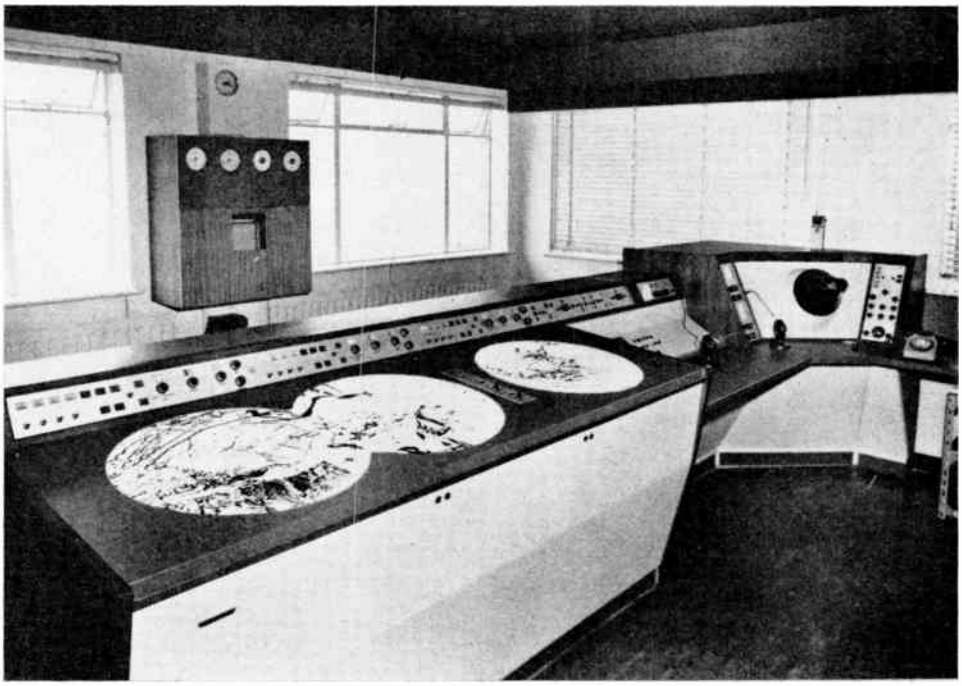
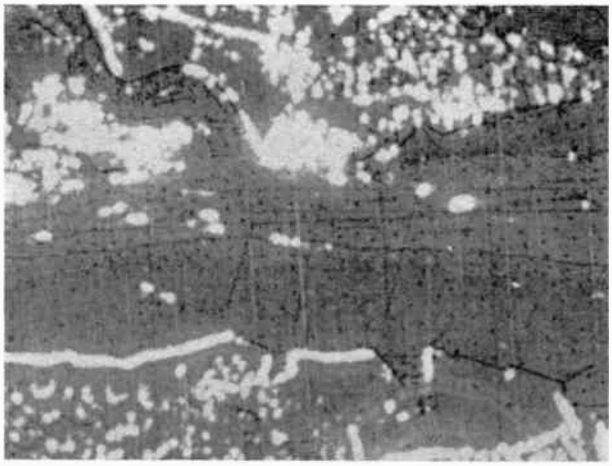
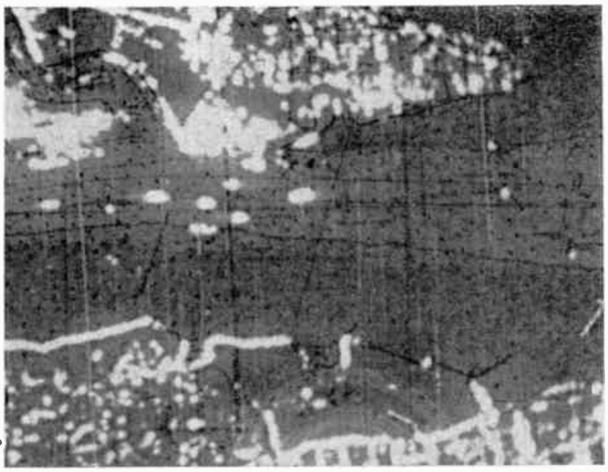


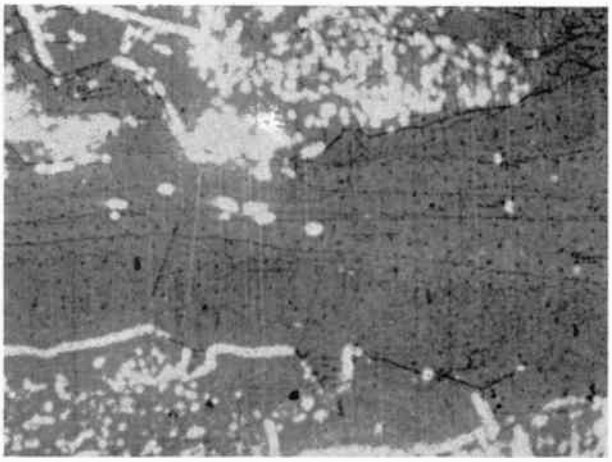
Fig. 8.
Control room, Teesport.



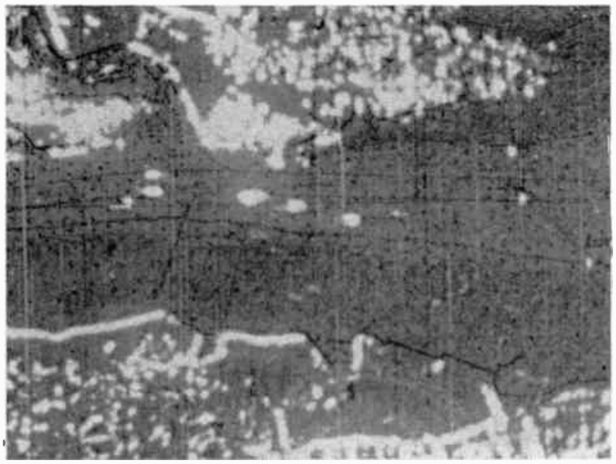
(a) Time −8 minutes: The initial situation. Tanker inbound off Canvey Island.



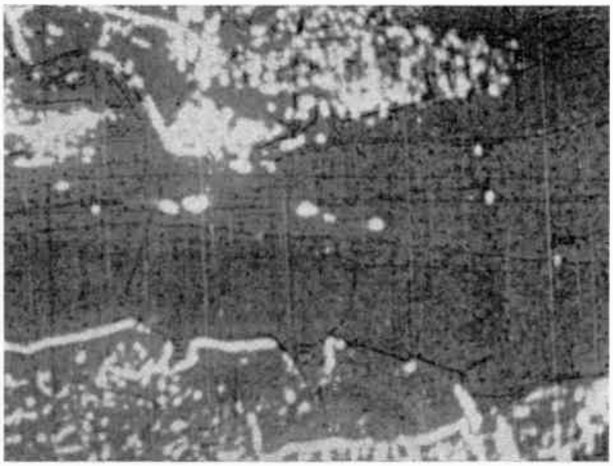
(b) Time −4 minutes: Collision inevitable with one of four outbound ships.



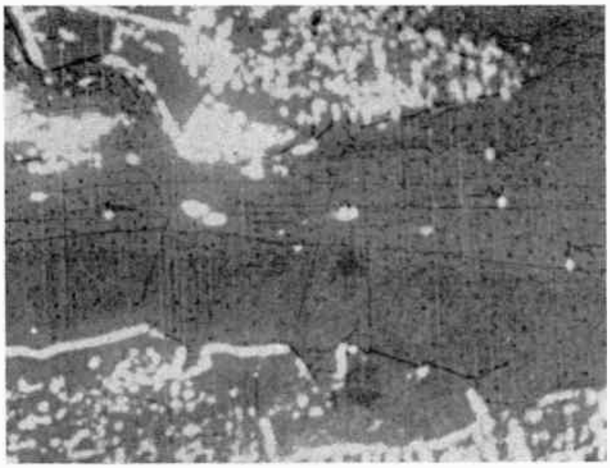
(c) Time −1 minute: First and second outbound ships have managed to avoid, but—



(d) Time '0'—collision occurs with third ship.



(e) Time +2 minutes: Fourth ship manoeuvres wildly to avoid interlocked ships, bearing down out of control—



(f) Time +4 minutes: —and succeeds in clearing. Note two tugs leaving north bank for collision site.

Fig. 9. Record of collision in river Thames. The deep water channel (faint black lines) is 1000 ft wide. Visibility ¼ mile, tide about 1 knot to eastward.

form a standardized plan as far as possible. While providing a clear working area around the displays, the console must also accommodate the following:

(i) the radar controls for local and remote stations, with confirmatory signals to indicate the state of readiness or operation of units, and warning devices for failure, etc.,

(ii) the microwave link controls, state indicators, and warnings,

(iii) communication equipment, both line and v.h.f.,

(iv) space for writing of logs, recording messages, etc.

(v) controls for other facilities, such as a tape recorder.

Thus everything the operator needs to see closely or handle is centralized into one display console. The back of this should be completely clear to allow space for interchange or maintenance of all units from the back, since this may have to be undertaken with the system in full operation.

Other indicators such as clock, calendar, tide and current gauges, anemometer, etc., are grouped on a wall, where they can readily be seen by any operator in front of the radar. The use of the bright display obviates the difficulties which would arise in the adequate illumination of these instruments in an otherwise darkened room, and permits the use of windows for a clear view of the channel, light signalling, etc.

The details of the layout will depend on the operational requirements of each specific system, the density of traffic to be handled, etc.

6. Conclusion

The quality of radar information presented can be judged from Fig. 4, which is made from frames of the actual film used in the projector display.

The value of the system as a historical record of an incident is well illustrated by the series of photographs in Fig. 9 showing a collision which occurred in the river Thames.

7. Acknowledgment

The author wishes to express his thanks to the directors of Kelvin Hughes for permission to publish this paper.

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Discussion on
'Integrated Harbour Surveillance Systems'

Under the Chairmanship of Captain F. J. Wylie

Captain W. R. Colbeck†: I agree with Admiral Graham that communications are all-important in any Port Information System. At the present time there is still a comparatively large number of vessels that are not fitted with v.h.f. as such fitting is still voluntary and this can produce a serious problem in low visibility. Many cases occur where vessels which are not fitted with either v.h.f. or radar and are caught in fog anchor and, when several vessels anchor, the channel may be virtually blocked and so a number of vessels which would be able to enter or leave the port, with radar assistance, are unable to do so.

As regards microwave links, I will be interested to learn whether there has been any interference with the quality of the transmissions from external sources. I have in mind a visit to Germany a few years ago to see the radar installation at Cuxhaven and I was told that the transmission of information from the remote transmitter at Belum had occasionally been interrupted for short periods which they had attributed to flocks of seagulls.

Mr. G. Wanless (*in reply*): We have no evidence of our microwave link signals being interrupted by birds. In link system design, a large signal strength margin is deliberately built-in to ensure that even under the worst fading or obstruction conditions to be expected over a given path, an acceptable signal is still received. The possible attenuation due to flocks of seagulls would be too small to have any significant effect in a typical case.

Mr. E. Jamieson‡: I would like to add to the reply given by Mr. Wanless to the question about 'seagull obstruction'.

† Mersey Docks and Harbour Board.

‡ Ferranti Ltd.

The energy from the microwave transmitting aerial is normally in the form of a circular beam. The effective diameter of this beam is known as the Fresnel zone diameter. A paper by Megaw† written some twenty years ago indicates that if one half of this beam is obstructed by, say, a ship's superstructure, the loss in signal will be approximately 12 dB. An obstruction of up to 1/10th of the beam cross-section will cause virtually no loss, due to diffraction round the edge. A representative diameter might be 100 ft and consequently any loss of signal due to obstruction by seagulls is most unlikely.

It should be noted that from a radar point of view, different rules apply and unwanted echoes from seagulls are possible.

Captain Colbeck: In considering the three displays at the Teesport installation, I would be interested to learn whether the centre display is photographed separately from the display in the approaches or whether it is an enlarged portion of the photograph of the p.p.i. of the outer area.

Mr. A. Harrison (*in reply*): The radar information from the South Gare radar at Teesport is not assembled into a radar picture at the remote station. It is transmitted as 'raw radar' on the microwave link, and used at Tees Dock to form two independent pictures; namely,

(a) A short-range p.p.i. covering about two miles of the channel above and below South Gare. This picture is displayed in the middle position, matched at its inward edge to the local picture from Tees Dock, displayed on the left.

(b) A long-range p.p.i. covering the approaches to the port. This is displayed on the right-hand side of the console, completely separate from the two integrated pictures.

Thus the two displays are selected from the same radar information, but are separately photographed and projected. Switching is provided to permit the right-hand (long-range) display to 'double up' on either of the other displays immediately in case of malfunction.

Captain Colbeck's earlier point about the channel being blocked by ships which have attempted a passage in bad visibility, and failed to clear the channel, raises three questions.

(i) Is the Master conducting his vessel in safety by attempting the passage in bad conditions without the assistance of aids to navigation such as v.h.f. and radar, in the absence of which he has small chance of success?

(ii) Is he not endangering other vessels in doing so?

(iii) Does the Port Authority carry some responsibility for permitting this hazard to develop in their area, to the detriment of other port users?

These questions have deliberately been put in a provoking form, to highlight different aspects of the arguments for advice, warning, or control.

† Megaw, E. C. S., 'Some effects of obstacles on the propagation of very short radio waves', *J.I.E.E.*, **95**, Part III, No. 28, pp. 97–105, March 1948.

Lt. Cdr. R. B. Richardson‡: We use most of the things the authors have been discussing in the Thames Navigation Service and it is interesting to hear the manufacturers' views.

But there is one feature which never comes out in such papers. I suppose it is really a problem of 'education' of a sort. I would say there is a considerable gulf between the manufacturer's understanding of the way his equipment should be used, and the actual user's distinction and interpretation of the very limited parameters under which he is forced to use it under the law.

For example, Mr. Harrison referred to a 'controller' and, as I understood him, implied that such installations can be used to control events or movements in the area.

Admiral Graham emphasized that the essential basis of the system is that it is advisory only in its uses. This is all very well, but I would like to ask him how he would propose to deal with situations where some act on advice, and others don't, in the same locality? I would like to see our Chamber of Shipping seeking, through the Government and international agencies concerned, a much more rational set of rules under which ships could benefit, far more than they are able at present, from the use of these excellent equipments.

Rear Admiral P. W. W. Graham (*in reply*): With regards to Commander Richardson's point, my firm impression is that Masters who, I am sure everyone will agree, are highly responsible, can normally be entirely relied upon to act on advice which in general they welcome. Of course, where the Master considers that he has information in his possession which is not available to the shore station, he inevitably uses his judgment. I suggest however that such incidents are extremely rare.

Mr. Wanless (*in reply*): It is worth noting that experience at Teesport has shown that consistently good advice (based on the integrated system) is appreciated by both ship's Masters and Pilots alike. Increasingly, this advice is being accepted, resulting at least in full co-operation if not in actual control.

Captain J. Andrew§: In considering terms to describe certain functions in port radar and communications systems, there are two pairs of terms sometimes used to describe the same activity; they are:

(i) Control of Shipping Movements and Advice.

(ii) Co-ordination of Shipping Movements and Warning.

I prefer the second pair of terms as the word co-ordination cannot be misconstrued to the same degree as control which is invariably likened to the word 'control' as used in the 'control of aircraft'. Navigation information is passed to those in charge of a vessel to assist them to navigate. Should they be standing into danger they will be 'warned' by the port radar station.

‡ Port of London Authority.

§ Southampton Harbour Board.

Admiral Graham: It seems to me that Captain Andrew's concept of 'advice' and 'warning' puts the matter admirably and I agree with him whole-heartedly.

Mr. Jamieson: Would Admiral Graham please say what are the technical differences between a harbour surveillance requirement and an air traffic control requirement, which make it *obvious* that, although aircraft should be controlled, ships need not be.

Admiral Graham (*in reply*): The essential difference between the air and marine environments is surely that ships can stop, go astern or even anchor while proceeding to their berth. None of these manœuvres are open to fixed-wing aircraft, and moreover towards the end of their flight the latter normally have very little fuel remaining, while ships are not normally embarrassed in this respect.

Mr. D. Barnett†: The comparison between air traffic control and the control of shipping is of interest to me as one who, in the past, has had to study the problem of rapid landing of aircraft at crowded airports.

There is little doubt that the principles of mandatory control are at present accepted much more fully by aircraft operators than appears to be the case with their marine counterparts. Perhaps the reason for this difference is the pressure of physical necessity in the case of air traffic; the speeds of aircraft are much higher and the relative density of air traffic in terminal areas is much greater than the corresponding parameters for shipping.

However, looking to the future, it is quite obvious that with the increasing speed of ships and the increasing numbers of their operations, this difference will become less and less noticeable, and sheer technical pressure will in due course force the acceptance of mandatory systems of control for shipping. If this is not done, it is likely that there will be a mounting loss due to collision or other hazards.

It would appear that one of the ways to overcome prejudice or political opposition to more sophisticated control systems for shipping would be for the appropriate authorities to support a large-scale experiment based on ideas such as we have heard this evening, to be followed by a careful analysis of the results.

There is very little doubt in my mind that, if the technical claims made by their sponsors are true, this type of equipment would lead to considerable improvement in ship traffic control. At least, by making some such experimental trial as this there would be objective proof or otherwise that this was the case. It is probable that if it is technically successful, much of the opposition to a more positive control of shipping would disappear.

Mr. Harrison (*in reply*): Different harbour authorities have expressed wide variations of opinion on the question of how control of shipping should be exercised—indeed Captain Parmiter and Lt. Cdr. Richardson of the P.L.A. have written very interesting papers proposing rigid control of ship movements by the shore authority.‡ The comparison of ships with aircraft is not entirely valid—for example, all aircraft landing at an airport must carry communication and blind flying equipment, so that they can be fitted into a sequence, and can arrive at a single point—the runway entrance—in succession. This is not so with ships. Their equipment is not standardized in the same way, and they can and sometimes must change their sequence during transit of a channel. The channel does not terminate at the same point for all ships. Again, an aircraft once committed must continue its landing procedure, while a ship can anchor, or even go backwards, if circumstances demand it. For these and similar reasons, the control of aircraft will always be more rigid than the control of ships. Nevertheless, an aircraft Captain may, if he considers that the circumstances demand it, override a controller's instructions, and in the ultimate, the controller will admit the Captain's authority and co-operate with him, but the Captain must be able to justify his action.

The safe passage of ships through a channel may not be possible without shore control (e.g. Suez Canal). With the increase in size of ships many authorities will find it necessary to introduce some measure of control, beginning perhaps as advice, proceeding to warning if suggestions and information are disregarded, and logically culminating in the Master's obedience to shore instructions, subject to his overriding responsibility for the safety of his ship.

It will no doubt take many years before this state of affairs is universally accepted, but I cannot help feeling that the safe commercial operation of large ships will not be possible if other large ships are free to occupy the channel in an unrestricted manner. It may be that the shore authority will allocate times at which a ship may occupy specified portions of a channel, the Captain being free to determine how he fits his ship into this plan, but not to depart from it without grave reason. The admission of a large ship to a port might be made conditional on the acceptance of such shore control, and the mandatory fitting of equipment such as v.h.f. for communication from ship to ship, and from ship to shore, and radar for channel guidance and clearance of other ships.

Mr. J. M. Miller§: I would like to ask what, in view of the most elaborate stand-by facilities provided by the link manufacturer, are the measures provided to guard against failure of the radar transceiver and display unit?

Mention has been made of the use of a slotted waveguide type scanner which results in excessive vertical beam-width. I should like to know how the parameters compare with our own installation consisting of a hog-horn fed, 25 ft by 2 ft part parabola, utilizing a magnetron of 10 kW peak power, with a p.r.f. of 1840 and a pulse duration of 0·05 microsecond. This installation's parameters are:

† S. G. Brown Ltd.

‡ Parmiter, G. V., 'Automation in Sea Transport', Quadripartite Navigation Conference, Paris, April 1967. Richardson, R. B., 'Profile for an Era in Coastal and Port Approaches', Lecture at Thurrock Technical College, October 1967.

§ Medway Conservancy Board.

Vertical beam-width to −3 dB points: 4·0°
Horizontal beam-width to −3 dB points: 0·3°
Gain (one way) above an isotropic radiator: 41·5 dB

An important feature of this type of scanner is that it does allow tailoring to suit the site. A further short-section part parabola is mounted above the main mirror which is angled to give short-range coverage of targets under the main beam.

Further to this, I do feel that it is not wise to carry v.h.f. information over the same link as the radar. For however good the m.t.b.f. in the possible event of failure, it would be far more serious if the port operations centre were unable to inform shipping of this fact.

Mr. Harrison (*in reply*): For the reasons stated in the paper there is no equivalent slotted waveguide scanner to compare with the above figures. For the 15 ft slotted waveguide scanner:

Vertical beam-width to −3 dB points: 15°
Horizontal beam-width to −3 dB points: 0·45°
Gain (one way) above isotropic radiator: 35 dB

The lower horizontal aperture times beam-width product of the slotted waveguide scanner, 15 × 0·45, as against 25 × 0·3, indicates the more efficient use of the available aperture which results from the better control of aperture illumination possible with the slotted waveguide design.

The higher gain of the 25 ft reflector serves little useful purpose, since both equipments have adequate gain and sensitivity margins.

The use of part-parabola above the main mirror may be regarded as 'tailoring to suit the site'. Alternatively, it may be regarded as evidence of the facts that:

(a) The vertical beam formed by the main mirror is too narrow.

(b) Energy from the primary radiator is spilling past the edges of the main mirror. This could be the source of unwanted 'back echoes' and side-lobes.

Very roughly, the reflector has five times the weight, and twice the wind-resistance of the slotted waveguide, and requires five times the mechanical driving power. The overall cost of the reflector and its supporting tower is obviously many times greater than the slotted guide. Nevertheless, as the paper states, there will be occasions on which these disadvantages have to be accepted to gain the narrower beam-width.

While I agree that failure of any v.h.f. link can upset port operations, I do not see any objection to the carriage of v.h.f. information on the same link as the radar. If a remote v.h.f. repeater station is required to achieve the coverage, and the remote radar provides a suitable site, then a link of some kind is essential. The duplicated microwave link is more reliable than any other type, and the expense of a separate v.h.f. link of equal reliability cannot be justified. Radar transceivers are duplicated and all equipment is interswitchable.

Mr. Wanless (*in reply*): Regarding the point about simultaneous loss of radar and v.h.f. data, Mr. Miller is perhaps overlooking the fully-duplicated cross-connected nature of the microwave link and modern equipment. The v.h.f. would have to be similarly in full hot-stand-by to be even comparable in reliability. Further, the v.h.f. would of course only be remoted as indicated in the general case if some real system advantage resulted (e.g. improved coverage over an area of otherwise poor reception). A local back-up v.h.f. set would also be provided in any case at the port operation centre.

Mr. A. P. Tuthill†: Mr. Harrison briefly referred to the position of the working platform on the aerial mast, so that it also acted as a radiation screen between the radar and the microwave link aerials. By inference I assume that interference had been suffered at some stage and I would ask either Mr. Harrison or Mr. Wanless if they would enlarge on this point and indicate the effects observed.

Mr. Wanless (*in reply*): The screening platforms are a fine example of 'better safe than sorry' engineering. Easily built-in during installation, with negligible increase in first cost of the whole system, but so awkward and embarrassing if they had had to be added later, these screens were only one of a number of design steps taken to prevent interference difficulties. At Teesport very sensitive microwave link receivers are in very close proximity to the high-powered radar equipment. Isolation upwards of about 150 dB is needed, no simple task.

† Decca Radar Ltd.

Computers and Information Processing

The 4th Congress of the International Federation for Information Processing (I.F.I.P.) took place in Edinburgh from 5th to 10th August. The Congress was formally opened by Admiral of the Fleet The Earl Mountbatten of Burma, Charter President of the Institution of Electronic and Radio Engineers and Immediate Past President of the British Computer Society, two of the bodies concerned with the organization of this notable international meeting. Attended by over 3850 computer specialists and engineers from all over the world, the Congress was supported by a comprehensive exhibition.

In the course of his remarks Lord Mountbatten recalled that in October 1946, in his first Presidential Address to the Institution of Electronic and Radio Engineers, he discussed the development and use of an electronic machine for the storage and retrieval of information, and described it as the reference library of the future. The practical realization of such a project depended, of course, upon the enterprise of electronic engineers in developing and creating a machine to meet an obvious need. Electronic digital stored program computers were developed simultaneously in several countries in 1949, using thermionic valves for logic and storage techniques based on delay lines or cathode-ray tubes.

Development of new techniques posed problems which challenged the Electronics Industry to evolve new concepts of engineering, first in microelectronics and then large-scale integration of components and circuits.

Lord Mountbatten continued: 'With the aid of these revolutionary new techniques it has become possible to apply the stored program approach to the *control* of a number of real time situations such as industrial process control, air defence, fire control, civil air traffic control; in fact, to any operation which is amenable to solution by the stored program method.

'One of the most important new applications which arises out of these new powers for computer storage is in information handling; not information in general, but information in the sense of the printed word. This makes new systems for the storage, retrieval and dissemination of information practicable.

'The British project in which I have the greatest personal interest is called S.D.I.—Selective Dissemination of Information—and was initiated by the National Electronics Council under my Chairmanship. After a two-year feasibility study run by N.E.C. I was able to arrange for the S.D.I. Project to be taken over by the Institution of Electrical Engineers. This involved their developing a comprehensive service based on the computer manipulation of data obtained from all published literature in the fields of physics, electronics, electrical engineering, control and computer science.

'By 1972 the total significant world knowledge in physics, electronics, computers and control should be on computer data file. All this information will be capable of quick retrieval in any form required, through magnetic tapes, computer printout, S.D.I. profiles, etc. This British enterprise will be worked by the Institution of Electrical Engineers in co-operation with the American Institute of Physics, the American Institute of Electrical and Electronics Engineers, the German Physikalische Berichte and, of course, our own Institute of Physics and Institution of Electronic and Radio Engineers.

'This is only one example of how a need expressed many years ago for information processing, coupled with other needs, stimulated engineering development. It is very valuable that this Congress can exchange views on how best to utilize available scientific and engineering knowledge on a wide front of human activity in industry and commerce.

'Since man developed the wheel, much of his ingenuity has been applied to saving labour and thought. Regrettably we frequently find that human intellect at all levels is not adequately using the devices which are already available.

'Why is this? Looking at the programme of this Congress, I wonder whether we are not in danger of inventing a complicated new language when surely we should be aiming to express ourselves more simply and be sure thereby to achieve one of the purposes of the Congress—which is to communicate with each other in easily understandable terms.

'It was for this very reason that in the S.D.I. Project we started by making sure that the standard words used in the system were incorporated in a Controlled List or Thesaurus of Unambiguous Terms in which synonyms had been removed, each word had a specific meaning, and the relationship between the words was clearly defined. Thus, as I explained in the lecture that I gave before the Royal Institution in January 1966†, the terms used by the physicist, the electronic engineer and the electrical engineer can be equally well understood by all three. It is equally important that the terms you use should be well understood by the systems analyst and the programmer, but above all, by the user of the data processing system.

'There is another reason why I hope you will study your terminology and use, so far as possible, what I would call easily understood everyday language. We may well ask whether by complicated descriptions of our work in unusual language we frighten away the very people who might best benefit from our labours or indeed the young people we wish to recruit.

'Personally, I am not sufficiently up in this new language to be able to understand what the following example, taken at random from the Congress papers, really means:

> "Partial Isomorphisms in Graphs and Structural Similarities in Tree-like Organic Modules".

'I am much in sympathy with the Czechoslovakian paper entitled:

> "Unambiguity and Ambiguity of Context-Free Grammars and Languages".

† 'Controlling the Information Explosion', *The Radio and Electronic Engineer*, **31**, No. 4, pp. 195–208, April 1966.

'Information processing, whether it be in landing aircraft, accountancy, or manufacturing processes, depends greatly upon the instant feedback principle to be able to provide a service which can be improved by user participation. It is not enough in future to produce expensive standard machines or "software" equipment; the end product must give adequate results in terms of lower cost, good reliability and robustness, and, above all, human satisfaction with the job. The last I regard as most important for, as I have already stated, one of man's continual endeavours has been and must be to reduce the tedium of labour. We must see to it that new jobs and responsibility do not involve mere machine minding—that would be condemning work people to a duller existence than many already have.

'Some of the sessions of this Congress are devoted to the important question of reliability and I hope that in your deliberations you will bear in mind the need to promote improvement in process and quality control. Mass production has often lowered quality, and this must be guarded against.

'In the United Kingdom, which I believe has a higher proportion of large machines in its total of computer installations than any other country, the development of data transmission facilities is a top priority. I think it is fair to say that this country has done more than any other —in relation to its size—to try and bring computer operations and awareness into the daily life of its people, via many central institutions and public utilities, banks, airlines, the National Computing Centre, and the industrial training boards. I know what a particularly valuable role the British Computer Society plays in this from first hand experience as I was the President during 1967.

'Medical needs provide an exciting new application for data processing. A system is now being installed in the Department of Pathology of the Royal Infirmary, Edinburgh, to process automatically data on a large number of patients. Interpretation of such data normally requires the services of highly skilled staff who are in short supply. The computer system now being installed will allow the highly skilled technician to get through many times his previous work load. The computer will record and store the information obtained on patients on magnetic tape which can be retrieved at a later date. Alternatively the magnetic tape can be passed on to a data processing installation for merging with other data to establish a true record of the patients.

'As information processing becomes more widely understood at all levels so will demand increase for new types of "hardware". After only two decades of computer designing, we know that the engineer is able to meet demands for more specialized and smaller machines with increasing efficiency. This technical process should be helped by a greater awareness and understanding of the "software" demands which you will be discussing during this Congress.

'I hope that attention will be given to the demands of the smaller industries and especially teaching aids which feature in some of your papers. With reliability it is also necessary to think of price. It may well be that we are striving too often in this field to achieve over-sophisticated performance at consequent high cost when something much less sophisticated and elaborate and so much less expensive would serve just as well, particularly in the first stages of introducing computer systems into small firms.

'User education must obviously be closely geared to the pace of new developments and all you experts in information processing can help a great deal towards stimulating the provision of the necessary education. Backed by the Department of Education and Science, the National Electronics Council, and the British Computer Society, much is being done in Great Britain to attract young people to new careers so that as a people we may exploit the benefits of modern technology.

'This is not, however, purely a national problem. It is an international problem which we must solve if the peoples of the world are to derive the maximum benefit from modern inventions. Because the whole problem of education and information processing is an international problem, I especially welcome overseas delegates and hope that you will all have a rewarding and happy Conference.'

U.D.C. 621.395.623.45

Some Aspects of Electrostatic Transducers

By

Professor
JOSEPH MERHAUT,
Dr. Sc.†

Presented at a Meeting of the Institution held in London on 10th January 1968.

Summary: Usually an electrostatic transducer is represented theoretically as an ideal electromechanical transformer followed by a piston-like diaphragm. This is not exact if a thin diaphragm is used. It is shown that by mathematical treatment of known physical relations the four-pole equations (or matrixes) for the relations between acoustical values and electrical ones may be derived. Further it is shown that it is useful to introduce the term of acoustical impedance of the thin diaphragm as a whole which may be represented by an analogue network, having damped poles and zeros at frequencies of the respective modes. A new analogue network of the complete transducer is then given. Curves are shown of normalized impedances of thin diaphragms as a function of relative frequency for different parameters calculated by a computer.

List of Symbols

C_0	capacitance of the transducer
c_n	negative mechanical compliance
Z_{md}	mechanical impedance of the diaphragm
F	mechanical force
v	velocity
k_{em}	electromechanical transducer constant
U_0	polarizing direct voltage
d	static distance between the electrodes in the transducer
Q_0	static charge due to the polarizing voltage
S	surface-area of the diaphragm
ε_0	permittivity of the vacuum
u	signal alternating voltage on the transducer
η	excursion of a part of the diaphragm
p	acoustic pressure over the diaphragm
Y	total volume displacement of the diaphragm
Y_0	total volume between the electrodes
c_{na}	negative acoustical compliance
k_{ea}	electro-acoustical constant of the transducer
ω	angular frequency ($=2\pi f$, f being frequency)
V	volume velocity to the whole transducer
q	alternating component of charge on the transducer
i	alternating current component in the transducer
Z_{ad}	acoustical impedance of the diaphragm
ρ_m	mass of the diaphragm per unit square
z_s	specific (acoustical) impedance over the diaphragm per unit area
ν	mechanical tension per unit length
r	polar co-ordinate
c_0	velocity of the wave-propagation over the diaphragm
$k = \dfrac{\omega}{c_0}$,	wave number
z	normalized value of the specific impedance over the diaphragm per unit area
R	radius of the diaphragm
J_i	Bessel function of ith order
$\beta = R\sqrt{k^2 - jkz}$	
a_i, b_i, λ_i	constants
m_i	acoustical mass in the equivalent acoustical network
r_i	acoustical resistance in the equivalent acoustical network
c_{di}	acoustical compliance due to the diaphragm in the equivalent acoustical network
c_{ai}	acoustical compliance due to the air loading in the equivalent acoustical network

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1. Introduction

In all known papers dealing with the theory of electrostatic transducers it is assumed that the device may be represented by an ideal electromechanical transducer with a capacitance C_0, negative mechanical compliance c_n and the mechanical impedance of the diaphragm Z_{md}, followed by a mechano-acoustic transformer. This is shown in Fig. 1. The ideal

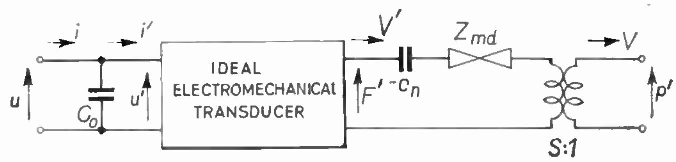


Fig. 1. Analogue network or equivalent circuit of an ideal electromechanical transducer.

transducer in this figure is an analogue of a transformer, for which the cascade matrix equation may be written

$$\begin{bmatrix} F' \\ v' \end{bmatrix} = \begin{bmatrix} k_{em} & , & 0 \\ 0 & , & k_{em}^{-1} \end{bmatrix} \cdot \begin{bmatrix} u' \\ i' \end{bmatrix}$$

where the electromechanical transducer constant is

$$k_{em} = \frac{C_0 U_0}{d} = \frac{Q_0}{d}$$

when U_0 is the polarizing direct voltage, Q_0 the static charge and d the distance between the electrodes. The mechano-acoustic transformer has the ratio $S : 1$, if S is the effective area of the diaphragm and it transforms the mechanical impedances to acoustical impedances in the ratio $1 : S^2$.

This way of representation has two great disadvantages. Firstly, one must introduce an effective area S of an equivalent piston replacing the actual thin diaphragm, the parts of which do not move in the same phase. There may be several ways of defining the equivalent area S. The second difficulty comes when we want to express the mechanical impedance of Z_{md}. A rigid piston has necessarily one mass, one compliance and one resistance. Such a system has only one single resonant frequency, though the real diaphragm has an infinite number of resonances and anti-resonances (or nodes and anti-nodes) some of these being important. So as a result of this the representation in Fig. 1 cannot be precise, especially above the first node of the diaphragm, where we usually at present operate.

2. Electro-acoustical Transducer

It will be shown that a more precise representation which is closer to reality may be derived, using an ideal direct electro-acoustical transducer.†

The elementary attractive force dF on a differential part of the area of diaphragm dS is given by the formula

$$dF = \frac{\varepsilon_0 (U_0 + u)^2}{2(d-\eta)^2} dS \qquad \ldots\ldots(1)$$

where ε_0 is the permittivity of vacuum and η the excursion of that part of the diaphragm from original static distance d. From (1) the force per unit area (pressure) is

$$p = \frac{dF}{dS} = \frac{\varepsilon_0 (U_0 + u)^2}{2(d-\eta)^2} \qquad \ldots\ldots(2)$$

The total differential of p may be written

$$dp = \frac{\partial p}{\partial u} du + \frac{\partial p}{\partial \eta} d\eta \qquad \ldots\ldots(3)$$

From eqn. (2)

$$\frac{\partial p}{\partial u} = \frac{\varepsilon_0 (U_0 + u)}{(d-\eta)^2} \qquad \ldots\ldots(4a)$$

If $u \ll U_0$ and $\eta \ll d$, as it usually is, equation (4a) may be simplified to

$$\frac{\partial p}{\partial u} = \frac{\varepsilon_0 U_0}{d^2} \qquad \ldots\ldots(4b)$$

From eqn. (2) a similar relationship may also be obtained

$$\frac{\partial p}{\partial \eta} = \frac{\varepsilon_0 (U_0 + u)^2}{(d-\eta)^3} \qquad \ldots\ldots(5a)$$

This may be written, under the above conditions as

$$\frac{\partial p}{\partial \eta} = \frac{\varepsilon_0 U_0^2}{d^3} \qquad \ldots\ldots(5b)$$

Inserting eqns. (4b) and (5b) into eqn. (3) and integrating with respect to time t, we obtain

$$p = \frac{\varepsilon_0 U_0}{d^2} u + \frac{\varepsilon_0 U_0^2}{d^3} \eta \qquad \ldots\ldots(6)$$

If we multiply this equation by dS and integrate over the area of the diaphragm S we get

$$p \iint_S dS = \frac{\varepsilon_0 U_0}{d^2} u \iint_S dS + \frac{\varepsilon_0 U_0^2}{d^3} \iint_S \eta \, dS \qquad \ldots\ldots(7)$$

In eqn. (7) put

$$Y = \iint_S \eta \, dS \quad \text{which}$$

is the total volume displacement of the diaphragm, no matter how complicated its deformation is. So after dividing eqn. (7) by S we have

$$p = \frac{\varepsilon_0 U_0}{d^2} u + \frac{\varepsilon_0 U_0^2}{S \cdot d^3} Y \qquad \ldots\ldots(8)$$

The coefficient of u is the electro-acoustical constant of the transducer, expressing the relationship between

† Merhaut, J. 'A contribution to the theory of electro-acoustic transducers based on electrostatic principles', *Acustica*, **19**, p. 283, May 1968.

p and u. It may be written

$$k_{ea} = \frac{\varepsilon_0 U_0}{d^2} = \frac{C_0 U_0}{Y_0} = \frac{Q_0}{Y_0} \qquad \ldots\ldots(9)$$

where Y_0 is the total static volume between the back electrode and diaphragm.

The coefficient of Y in eqn. (8) represents the acoustic negative stiffness. It is given by:

$$\frac{1}{c_{na}} = \frac{\varepsilon_0 U_0^2}{S d^3} = \frac{\varepsilon_0^2 U_0^2}{d^4} \cdot \frac{d}{\varepsilon_0 S} = k_{ea}^2 \frac{1}{C_0} \qquad \ldots\ldots(10)$$

The negative acoustic compliance c_{na} is therefore equal to the capacitance C_0 transferred to the acoustical side of the transducer. Thus

$$c_{na} = \frac{C_0}{k_{ea}^2} \qquad \ldots\ldots(11)$$

For harmonic motion with the angular frequency ω, we can write

$$Y = \frac{V}{j\omega}$$

where V is the total acoustic flow (volume velocity) to the transducer. Therefore eqn. (8) may be written as follows:

$$p = k_{ea} u + \frac{1}{j\omega c_{na}} V \qquad \ldots\ldots(12)$$

This is one of fundamental four-pole equations of the electrostatic transducer with a thin diaphragm.

The second four-pole equation may be derived in the following way:

The elementary charge on the surface element dS is

$$dq = (U_0 + u)\frac{\varepsilon_0 \, dS}{d - \eta} \qquad \ldots\ldots(13)$$

The total charge on the whole diaphragm is

$$q = (U_0 + u)\varepsilon_0 \iint_S \frac{dS}{d - \eta} \qquad \ldots\ldots(14a)$$

If $\eta \ll d$, eqn. (14a) may be written

$$q = \frac{\varepsilon_0 (U_0 + u)}{d} \iint_S \left(1 + \frac{\eta}{d}\right) dS \qquad \ldots\ldots(14b)$$

The derivation of (14b) with respect to time is

$$\frac{\partial q}{\partial t} = \frac{\partial u}{\partial t}\frac{\varepsilon_0}{d} \iint_S \left(1 + \frac{\eta}{d}\right) dS + \frac{\varepsilon_0 (U_0 + u)}{d^2} \iint_S \frac{\partial \eta}{\partial t} dS \qquad \ldots\ldots(15)$$

The first integral in eqn. (15) is equal to S for $\eta \ll d$. The second integral is the volume velocity V, because $\partial\eta/\partial t = v$. Further $\partial q/\partial t = i$ and $\partial u/\partial t$ for harmonic signal may be written as $j\omega u$.

Thus for $u \ll U_0$ eqn. (15) gives

$$i = j\omega C_0 u + k_{ea} V \qquad \ldots\ldots(16)$$

From eqn. (12) and (16) a new analogue network for electrostatic transducers may be drawn as shown in Fig. 2.

For the ideal electro-acoustic transducer itself the signal values are u', i', p' and V'. The four-pole equations for these may be written

$$p' = k_{ea} u' + 0 . i' \qquad \ldots\ldots(17)$$

$$V' = 0 . u' + k_{ea}^{-1} i' \qquad \ldots\ldots(18)$$

The cascade matrix of this part of the transducer is

$$\begin{bmatrix} p' \\ V' \end{bmatrix} = \begin{bmatrix} k_{ea} & , & 0 \\ 0 & , & k_{ea}^{-1} \end{bmatrix} \cdot \begin{bmatrix} u' \\ i' \end{bmatrix} \qquad \ldots\ldots(19)$$

This confirms that the complete electro-acoustic transducer may be represented as it is in Fig. 2.

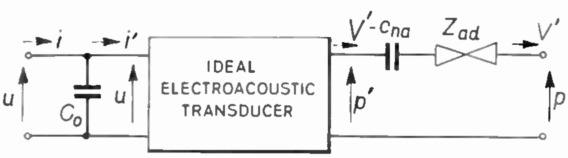


Fig. 2. Analogue network, or equivalent circuit for an electrostatic transducer.

3. The Acoustical Impedance of the Diaphragm

The acoustical impedance of the diaphragm Z_{ad} may be defined as

$$Z_{ad} = \frac{p}{V} = \frac{p}{j\omega \iint_S \eta \, dS} \qquad \ldots\ldots(20)$$

where p denotes the acoustic pressure over the diaphragm. The definition of Z_{ad} is valid only for the case of the uniform distribution of p.

In the standard textbooks the expression for deflection η of a thin membrane is derived. The solution in classical form, however, cannot be used here. In electro-acoustical transducers the diaphragm is always situated extremely close to the rigid back electrode, which is perforated or has small cavities. For this reason, the diaphragm of a transducer does not vibrate in free space, as is supposed in the classical theory, but is usually loaded with a specific impedance z_s (per square unit). This impedance consists mostly of the viscous damping resistance and also the acoustic stiffness of the cavities.

In this case the following equation for the motion for an element of area of circular diaphragm may be written:

$$\rho_m \frac{\partial^2 \eta}{\partial t^2} + z_s \frac{\partial \eta}{\partial t} - \nu \left(\frac{\partial^2 \eta}{\partial r^2} + \frac{1}{r}\frac{\partial \eta}{\partial r}\right) = p \qquad \ldots\ldots(21)$$

where ρ_m is the mass of the diaphragm per unit area, ν the mechanical tension per unit length at the margin of the diaphragm and r the polar co-ordinate.

For harmonic motion eqn. (21) may be written

$$-\nu\left(\frac{\partial^2\eta}{\partial r^2}+\frac{1}{r}\frac{\partial\eta}{\partial r}\right)-\omega^2\eta\rho_{\mathrm{m}}+\mathrm{j}\omega\eta z_{\mathrm{s}}=p \qquad \text{......(22)}$$

If we substitute

$$c_0=\sqrt{\frac{\nu}{\rho_{\mathrm{m}}}},\quad k=\frac{\omega}{c_0}\quad \text{and}\quad \frac{z_{\mathrm{s}}}{\sqrt{\nu\rho_{\mathrm{m}}}}=z$$

where z is a normalized value of z_{s}, eqn. (22) becomes

$$\frac{\partial^2\eta}{\partial r^2}+\frac{1}{r}\frac{\partial\eta}{\partial r}+(k^2-\mathrm{j}kz)\eta=-\frac{p}{\nu} \qquad \text{......(23)}$$

For the condition $\eta=0$ for $r=R$, the solution of this equation is

$$\eta=\frac{p}{\nu(k^2-\mathrm{j}kz)}\left[\frac{\mathrm{J}_0(\sqrt{k^2-\mathrm{j}kz}\,.\,r)}{\mathrm{J}_0(\sqrt{k^2-\mathrm{j}kz}\,.\,R)}-1\right] \qquad \text{......(24)}$$

From this and eqn. (20) we get for the acoustical impedance of the whole diaphragm

$$\frac{1}{Z_{\mathrm{ad}}}=\frac{\mathrm{j}\omega 2\pi}{p}\int_0^R \eta r\,\mathrm{d}r \qquad \text{......(25)}$$

This will give, after substituting the expression (24) for η

$$\frac{1}{Z_{\mathrm{ad}}}=\frac{\mathrm{j}\omega 2\pi R^4}{\nu}\,\frac{1}{\beta^3}\left[\frac{\mathrm{J}_1(\beta)}{\mathrm{J}_0(\beta)}-\frac{\beta}{2}\right] \qquad \text{......(26)}$$

where

$$\beta \doteq R\sqrt{k^2-\mathrm{j}kz} \qquad \text{......(27)}$$

If we expand the Bessel functions in eqn. (26) into series we finally obtain

$$Z_{\mathrm{ad}}=\frac{8\nu}{\mathrm{j}\omega\pi R^4}\times$$

$$\times\frac{1-\frac{1}{2^2}\beta^2+\frac{1}{2^2.4^2}\beta^4-\frac{1}{2^2.4^2.6^2}\beta^6+\dots}{1-\frac{2}{4.6}\beta^2+\frac{3}{4.6^2.8}\beta^4-\frac{4}{4.6^2.8^2.10}\beta^6+\dots} \qquad \text{......(28)}$$

This may be written

$$Z_{\mathrm{ad}}=\frac{8\nu}{\mathrm{j}\omega\pi R^4}\cdot\frac{\left(1-\frac{\beta^2}{a_1^2}\right)\left(1-\frac{\beta^2}{a_2^2}\right)\left(1-\frac{\beta^2}{a_3^2}\right)\dots}{\left(1-\frac{\beta^2}{b_1^2}\right)\left(1-\frac{\beta^2}{b_2^2}\right)\left(1-\frac{\beta^2}{b_3^2}\right)\dots} \qquad \text{......(29)}$$

where a_i is given by $\mathrm{J}_0(\beta)=0$ and b_i are the roots of the equation

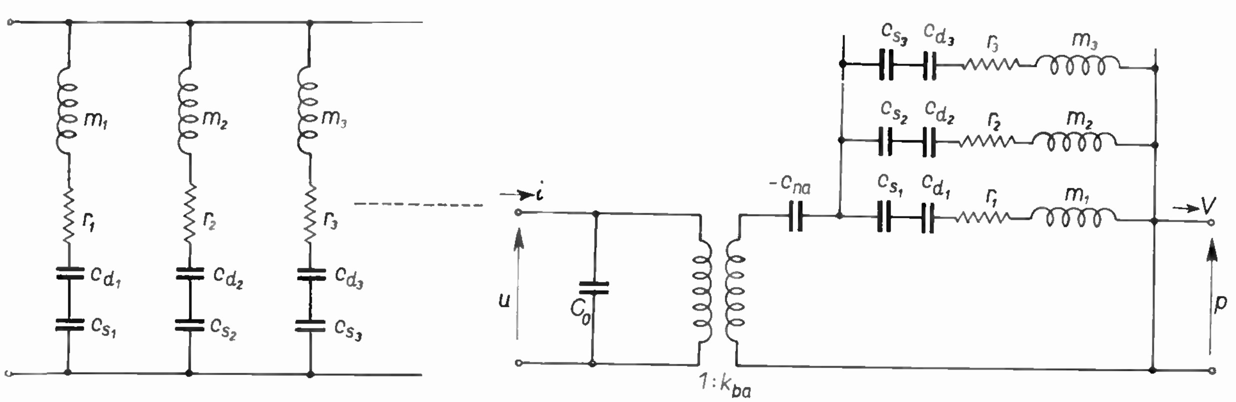
$$\frac{\mathrm{J}_1(\beta)}{\mathrm{J}_0(\beta)}=\frac{\beta}{2} \qquad \text{......(30)}$$

When we substitute

$$z_{\mathrm{s}}=r_{\mathrm{s}}+\frac{1}{\mathrm{j}\omega c_{\mathrm{s}}} \qquad \text{......(31)}$$

and other symbols we get from the last equation as a final result

$$Z_{\mathrm{ad}}=\frac{8\rho_{\mathrm{m}}}{\mathrm{j}\omega\pi R^2}\cdot\frac{b_1^2.b_2^2\dots}{a_1^2.a_2^2.a_3^2\dots}\cdot\frac{\left(\frac{a_1^2\nu}{\rho_{\mathrm{m}}R^2}+\frac{1}{\rho_{\mathrm{m}}c_{\mathrm{s}}}+\mathrm{j}\omega\frac{r_{\mathrm{s}}}{\rho_{\mathrm{m}}}-\omega^2\right)\cdot\left(\frac{a_2^2\nu}{\rho_{\mathrm{m}}R^2}+\frac{1}{\rho_{\mathrm{m}}c_{\mathrm{s}}}+\mathrm{j}\omega\frac{r_{\mathrm{s}}}{\rho_{\mathrm{m}}}-\omega^2\right)\dots}{\left(\frac{b_1^2\nu}{\rho_{\mathrm{m}}R^2}+\frac{1}{\rho_{\mathrm{m}}c_{\mathrm{s}}}+\mathrm{j}\omega\frac{r_{\mathrm{s}}}{\rho_{\mathrm{m}}}-\omega^2\right)\dots} \qquad \text{......(32)}$$



$$k_{ba}=\frac{C_0U_0}{Y_0}=\frac{Q_0}{Y_0}$$

$$C_{na}=\frac{C_0}{k_{ba}^2}$$

(a) Analogue network for acoustical impedance of thin diaphragm of electrostatic transducer.

(b) Network for complete transducer.

Fig. 3.

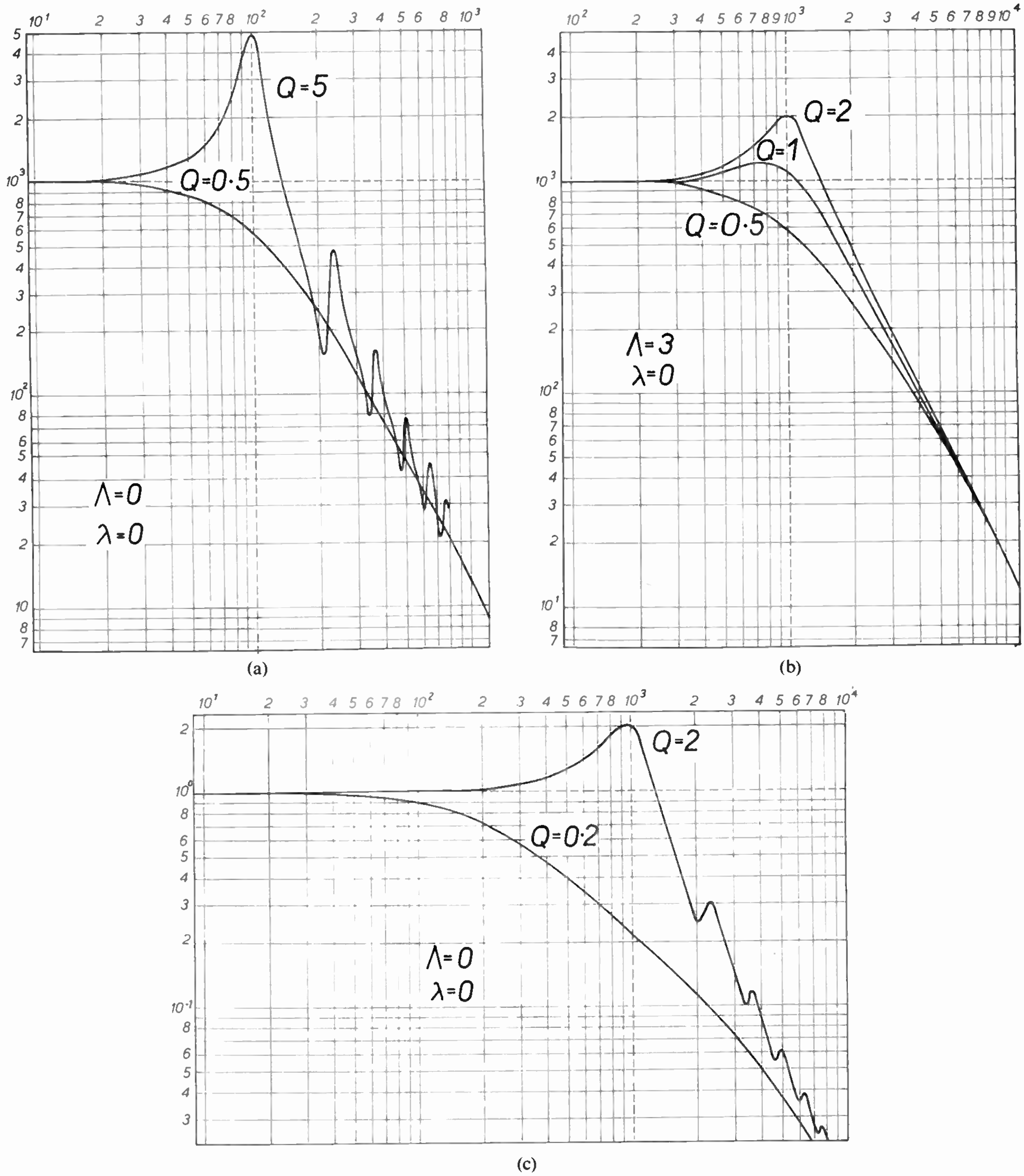


Fig. 4. Frequency-response curves for complex compliance.

This equation shows that Z_{ad} has an infinite number of zeroes and poles, with a damping caused by r_s.

On the basis of the last equation an analogue acoustical network for Z_{ad} may be drawn. This network is given in Fig. 3(a), where m_i are acoustic masses, r_i acoustic resistances, c_{di} acoustic compliances due to the diaphragm itself, and c_{si} acoustical compliances caused by air loading of c_s due to the cavities in the back electrode. (If the holes in the solid electrode go through, the c_{si} are infinite and vanish.) For individual components of the circuit in Fig. 3(b) the following expressions may be obtained:

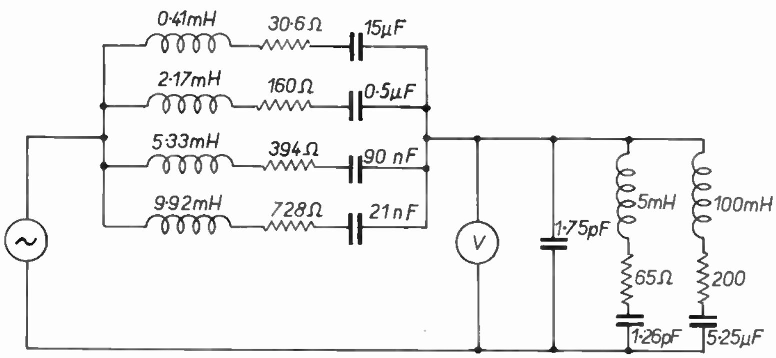


Fig. 5. Analogue network for an actual electrostatic earphone for measurement purposes.

$$m_i = \frac{\lambda_i}{S}\rho_m \qquad \ldots\ldots(33)$$

$$r_i = \frac{\lambda_i}{S} r_s \qquad \ldots\ldots(34)$$

$$c_{di} = \frac{S}{\lambda_i}\frac{R^2}{a_i^2 v} \qquad \ldots\ldots(35)$$

$$c_{si} = \frac{S}{\lambda_i} c_s \qquad \ldots\ldots(36)$$

when

$$S = \pi R^2$$

and

$$a_1^2 = 5{\cdot}78 \quad a_2^2 = 30.47 \quad a_3^2 = 74{\cdot}89 \quad a_4^2 = 139{\cdot}04$$

$$\lambda_1 = 1{\cdot}446 \quad \lambda_2 = 7{\cdot}59 \quad \lambda_3 = 18{\cdot}62 \quad \lambda_4 = 34{\cdot}52$$

The analogue network of the whole transducer is shown in Fig. 3(b).

4. Examples of the Use of the Analogue Network

As an example of one use of this theory curves of $1/j\omega Z_{ad}$ (the complex compliance Y/p) have been plotted. (This represents practically the frequency response of the e.m.f. of a pressure capacitor microphone.)

Curves are plotted as a function of relative frequency $\Omega = f/f_{01}$ where f_{01} is the frequency of the first symmetrical mode.

$Q = 2\pi f_{01}\rho_m/r_s$ and Λ is the ratio of the specific compliance of the diaphragm $c_d = R^2/5{\cdot}78v$ to the specific acoustic compliance (per unit area) of the air in the holes in the back-electrode.

The variation of the complex compliance, $1/j\omega Z_{ad}$, against frequency is shown in Figs. 4(a–c).

Another example of practical use of this theory may be the design of a transducer by means of an analogue network. This is shown in Fig. 5 in which the object was to design an electrostatic earphone for measurement purposes. For a given electret foil for diaphragm and chosen diameter and distance d an analogue electrical network was made. This network was connected with another electrical network representing the I.E.C. artificial ear for audiometry. Measurements have been done using a tone generator and level recorder. By this procedure it is easy to check the design and show what influence the changes of the ear impedance for the frequency response and level have.

5. Conclusion

It has been shown that an electro-acoustic transducer based on the electrostatic principle and having a thin diaphragm may be represented by an analogue network containing the discrete acoustical elements. This representation enables us to deal with them much more precisely using digital computers or analogue electric networks.

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A Contribution to the Theory of Multivibrators employing Distributed Circuits

By

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AND

B. S. WESTCOTT, B.Sc., Ph.D.†

Summary: The equations relating to the theory of multivibrators employing uniformly distributed circuits as cross-coupling elements are examined. By using integral transform methods, analytic solutions for each state of the multivibrator cycle are obtained. The solutions are expressed in terms of orthogonal functions and are readily adapted to numerical analysis. Advantages of the method over methods based on finite difference techniques are emphasized particularly when stored base charge effects are studied. Numerous results for both symmetric and asymmetric multivibrator configurations are presented.

1. Introduction

In a paper in 1965, Nichols[1] investigated the possibility of using distributed capacitance-resistance networks as the cross-coupling elements for multivibrator circuits, which may find application in thin-film microcircuits and integrated solid circuits.

Nichols restricted his considerations to uniform CR-lines, that is, to lines having a constant resistance and capacitance per unit length. The differential equation governing the voltage across the line at any point was set up together with the differential boundary conditions which exist at the transitions between the various states in the multivibrator cycle. The equations were solved numerically using a standard finite difference method and effects such as that due to stored base charge on the multivibrator waveform were only determined approximately due to computational difficulties.

It is the purpose of this paper to re-examine Nichols' equations and to show that analytical solutions in each state of the multivibrator cycle can be obtained. Thus the voltage across the line can be expressed in terms of orthogonal functions within each state enabling a simple match between solutions to be made at the transitions. This procedure is faster and more accurate than a purely numerical technique and enables a more accurate assessment of stored base charge effects to be made.

2. Operation of the Multivibrator

The uniformly distributed CR-line used by Nichols is shown in Fig. 1. The total resistances of the upper and lower arms are NR and R respectively and the total capacitance between them is C. The line is assumed to have unit length.

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The connections of the lines in a typical transistor multivibrator circuit are shown in Fig. 2, where the resistance R forms the collector load of transistor Tr1 and resistance NR is the base leak of transistor Tr2. The capacitance C cross-couples the collector of Tr1 to the base of Tr2.

The collector of Tr2 and the base of Tr1 are also cross-coupled by a second distributed line. The multivibrator is symmetrical if the values of N and CR coincide with those of the first line and is asymmetrical otherwise. v_{BB} and v_{CC} are supply voltages.

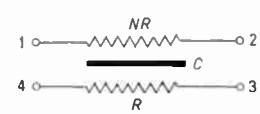

Fig. 1. Uniform, distributed CR line.

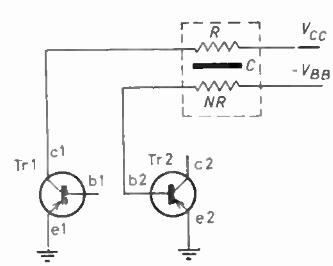

Fig. 2. The line as a cross-coupling network in a multivibrator circuit.

Under normal operating conditions only one transistor of the pair conducts at any instant, the other transistor being cut off by a positive voltage on its base. There are two normal states therefore of the multivibrator circuit in which (i) Tr1 is conducting with Tr2 cut off and (ii) Tr2 is conducting with Tr1 cut off.

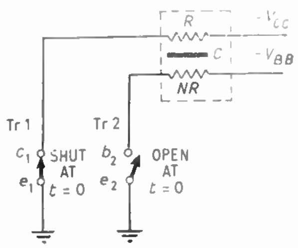

Fig. 3. Equivalent circuit A; Tr1 conducting, Tr2 cut-off. $v_{b2} > 0$, decaying towards $-v_{BB}$.

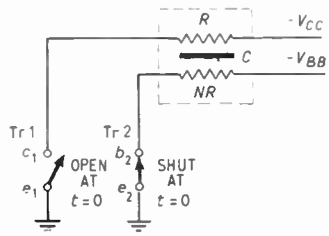

Fig. 4. Equivalent circuit B; Tr2 conducting, Tr1 cut-off. v_{C1} decaying towards $-v_{CC}$.

In the former state the positive voltage on b_2 decays to zero at which time Tr2 conducts and Tr1 is cut off by regenerative action. The equivalent circuit (case A) used by Nichols to determine the base waveform of the cut-off transistor is reproduced in Fig. 3.

In the latter state with Tr2 conducting and Tr1 cut off, the voltage on collector terminal c_1 decays from zero to approach $-v_{CC}$, its final value depending upon the time taken by the positive voltage on the base b_1 to decay to zero. The equivalent circuit (case B) used by Nichols in investigating the collector waveform is given in Fig. 4.

Ideally the switching time between the two normal states is negligible and there is no loss of energy in the cross-coupling circuit so that the final energy distribution on the line in one state is also the initial energy distribution along the line in the next state. Nichols investigates one condition, however, in which the switching time may be appreciable. This is the case in which there is an appreciable stored base charge Q in the conducting transistor which must be neutralized before the base voltage can be made positive to switch it off. This can be done only by drawing charge from the capacitance of the distributed line and may involve an appreciable delay due to the line resistance. Nichols' equivalent circuit (case C) for examining the neutralization process is reproduced in Fig. 5.

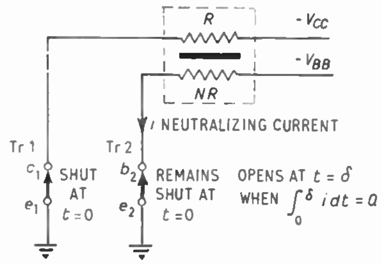

Fig. 5. Equivalent circuit C; Tr1 and Tr2 conducting while stored base charge of Tr2 is neutralized.

3. The Line Equations

The differential equation governing the line voltage $v(x, t)$ together with the boundary and initial conditions for circuit states A, B and C were set up, subject to certain assumptions by Nichols. His assumptions were:

(a) All switching transients other than those resulting from neutralization of stored base charge are negligible.

(b) Transistors are switched off immediately after neutralization of base charge is complete.

(c) Transistors conduct in the saturated state with v_{CE}, v_{BE} zero.

(d) Desaturation of a transistor occurs only in response to the start of conduction in the other transistor.

For the circuit coupling b_2 and c_1 the line equation is

$$\frac{\partial^2 v}{\partial x^2} = (N+1)CR\frac{\partial v}{\partial t} \qquad \text{......(1)}$$

The boundary and initial conditions are as follows.

3.1. *Case A* (*Fig.* 3)

At the left-hand end of the line $x = 0$ and

$$\frac{\partial v}{\partial x}(0, t) - \frac{v(0, t)}{N} = -v_{CC} - v_{BB}/N, \quad t > 0 \qquad \text{......(2)}$$

At the right-hand end of the line $x = 1$ and

$$v(1, t) = v_{BB} - v_{CC} \quad t > 0 \qquad \text{......(3)}$$

The initial condition $v(x, 0)$ is given by the final voltage distribution along the line in the preceding state of the multivibrator circuit. The initial distribution in case A is therefore given by the final distribution in case B or, if stored base charge is taken into account, by the final distribution in case C.

For entry from B we write

$$v(x, 0) = v_{BB}x - v_{CC} + v_{CC}k(x) \quad 0 \leqslant x \leqslant 1 \qquad \text{......(4)}$$

The functional form of eqn. (4) is chosen by the following considerations. If the circuit has been in the form of case B for a sufficiently long time before transforming to case A then Fig. 4 shows that the collector terminal of Tr1 will be at potential $-v_{CC}$ and hence the voltage distribution along the line will be linear and given by $v_{BB}x-v_{CC}$. In general, the additional function $v_{CC}k(x)$ represents the deviation from a linear distribution.

For entry from C we write, for convenience,

$$v(x,0) = (v_{BB}-v_{CC})x+v_{CC}g(x) \quad 0 \leqslant x \leqslant 1 \quad \ldots\ldots(5)$$

The circuit remains in state A until a time τ_A has elapsed, at which time the base voltage of Tr2 becomes zero, Tr2 conducts and the voltage on collector terminal c_1 of Tr1 can be determined using case B.

3.2. *Case B (Fig. 4)*

The boundary condition at the ends of the line are

$$\frac{\partial v}{\partial x}(0,t)-Nv(0,t) = Nv_{CC}+v_{BB} \quad \ldots\ldots(6)$$

$$v(1,t) = v_{BB}-v_{CC} \quad \ldots\ldots(7)$$

Entry to state B is always made from state A so that the initial line voltage distribution for B is given by the final distribution in state A. For convenience we write,

$$v(x,0) = v_{BB}-v_{CC}x+v_{CC}h(x), \quad 0 \leqslant x \leqslant 1 \quad \ldots\ldots(8)$$

The circuit remains in state B for the period τ_B taken by the positive voltage on b_1 to decay to zero. This period is clearly equal to τ_A if the multivibrator is symmetrical; otherwise it must be determined by a case A analysis in the coupling circuit connecting b_1 and c_2.

3.3. *Case C (Fig. 5)*

Both Tr1, Tr2 conduct so that

$$v(0,t) = 0 \quad \ldots\ldots(9)$$

$$v(1,t) = v_{BB}-v_{CC} \quad \ldots\ldots(10)$$

The initial voltage distribution for case C is given by the final distribution in case B and we write this in the form (4)

$$v(x,0) = v_{BB}x-v_{CC}+v_{CC}k(x)$$

Case C operates for a time τ_C during which the stored base charge is neutralized. The relationship between the amount of stored base charge q and τ_C is given by Nichols in the form

$$q = -\int_0^{\tau_C} \frac{1}{(N+1)CR}\left[\frac{1}{v_{CC}}\frac{\partial v}{\partial x}(0,t)+\frac{N+S}{N}\right]dt \quad \ldots\ldots(11)$$

where $S = v_{BB}/v_{CC}$ and q is written in normalized form as the equivalent length of line charged to the voltage v_{CC}, i.e. actual base charge, $Q = qCv_{CC}$.

4. Analytic Solutions

Analytic solutions for all three cases are derived in the Appendix.

4.1. *Case A*

The voltage distribution is given by either eqn. (42) or eqn. (43), both of which may be written in the form

$$\frac{v(x,t)}{v_{CC}} = S-x+2\sum_{n=0}^{\infty}\alpha_n H_n(x).\exp\left[-y_n^2 t/\lambda^2\right] \quad \ldots\ldots(12)$$

where

$$\lambda^2 = (N+1)CR$$

y_n is the root of

$$\tan y_n = -Ny_n \quad \ldots\ldots(13)$$

$$\text{with} \quad (2n+1)\frac{\pi}{2} < y_n < (n+1)\pi$$

$$H_n(x) = \frac{\sin y_n(1-x)}{\sin y_n}$$

and, if there is no stored base charge, i.e. the initial distribution is given by eqn. (4), α_n is defined by

$$\alpha_n = \frac{(Ny_n)^2\int_0^1 H_n(u)k(u)\,du-(S+1)N(N+1)}{N+1+(Ny_n)^2} \quad \ldots\ldots(14)$$

whereas, if the effects of stored base charge are included, i.e. the initial distribution is given by eqn. (5), α_n is defined by

$$\alpha_n = \frac{(Ny_n)^2\int_0^1 H_n(u)g(u)\,du-SN(N+1)}{N+1+(Ny_n)^2} \quad \ldots\ldots(15)$$

We write the final voltage distribution in the form

$$\frac{v(x,\tau_A)}{v_{CC}} = S-x+2\sum_{n=0}^{\infty}\tilde{\alpha}_n H_n(x) \quad \ldots\ldots(16)$$

where

$$\tilde{\alpha}_n = \alpha_n \exp(-y_n^2\tau_A/\lambda^2) \quad \ldots\ldots(17)$$

4.2. *Case B*

The voltage distribution shown in eqn. (48) may be written in the form

$$\frac{v(x,t)}{v_{CC}} = Sx-1+2\sum_{n=0}^{\infty}\beta_n K_n(x)\exp(-z_n^2 t/\lambda^2) \quad \ldots\ldots(18)$$

where z_n is the root of

$$\tan z_n = -z_n/N \quad \ldots\ldots(19)$$

$$\text{with} \quad (2n+1)\frac{\pi}{2} < z_n < (n+1)\pi$$

$$K_n(x) = \frac{\sin z_n(x-1)}{\sin z_n}$$

and

$$\beta_n = \frac{z_n^2\int_0^1 K_n(u)h(u)\,du+(S+1)(N+1)}{N(N+1)+z_n^2} \quad \ldots\ldots(20)$$

The final distribution is

$$\frac{v(x,\tau_B)}{v_{CC}} = S - x + 2\sum_{n=0}^{\infty} \tilde{\beta}_n K_n(x) \qquad \ldots\ldots(21)$$

where

$$\tilde{\beta}_n = \beta_n \exp(-z_n^2 \tau_B/\lambda^2) \qquad \ldots\ldots(22)$$

4.3. *Case C*

The voltage distribution shown in eqn. (51) may be written in the form

$$\frac{v(x,t)}{v_{CC}} = (S-1)x + 2\sum_{n=1}^{\infty} \gamma_n \sin n\pi x \exp(-n^2\pi^2 t/\lambda^2) \qquad \ldots\ldots(23)$$

where

$$\gamma_n = \int_0^1 k(u) \sin n\pi u \, du - \frac{1}{n\pi} \qquad \ldots\ldots(24)$$

The final distribution is

$$\frac{v(x,\tau_C)}{v_{CC}} = (S-1)x + 2\sum_{n=1}^{\infty} \tilde{\gamma}_n \sin n\pi x \qquad \ldots\ldots(25)$$

where

$$\tilde{\gamma}_n = \gamma_n \exp(-n^2\pi^2 \tau_C/\lambda^2) \qquad \ldots\ldots(26)$$

and τ_C is given by eqn. (11) which becomes, on substituting eqn. (23),

$$q\lambda^2 = -S\left(1+\frac{1}{N}\right)\tau_C - 2\lambda^2 \sum_{n=1}^{\infty} \frac{\gamma_n}{n\pi} \times \left[1 - \exp(-n^2\pi^2\tau_C/\lambda^2)\right] \qquad \ldots\ldots(27)$$

5. Matching of Solutions

5.1. *Entry to Case A from Case B*

Comparing eqn. (4) with eqn. (21) we see that

$$k(x) = 2\sum_{m=0}^{\infty} \tilde{\beta}_m K_m(x) \qquad \ldots\ldots(28)$$

The evaluation of α_n using eqn. (14) requires the integral

$$\int_0^1 H_n(u) k(u)\, du = 2\sum_{m=0}^{\infty} \tilde{\beta}_m \int_0^1 H_n(u) K_m(u)\, du$$

Now, using the definitions of y_n and z_n, we deduce that

$$\int_0^1 H_n(u) K_m(u)\, du = \frac{1}{\sin y_n \sin z_m} \times \int_0^1 \sin y_n(1-u) \sin z_m(1-u)\, du = \frac{N - \dfrac{1}{N}}{z_m^2 - y_n^2} \qquad \ldots\ldots(29)$$

Thus eqn. (14) becomes

$$\alpha_n = \frac{N(N+1)}{N+1+(Ny_n)^2} \times \left[2(N-1)y_n^2 \sum_{m=0}^{\infty} \frac{\tilde{\beta}_m}{z_m^2 - y_n^2} - (S+1)\right] \qquad \ldots\ldots(30)$$

5.2. *Entry to Case A from Case C*

Comparing eqn. (5) with eqn. (25) we deduce

$$g(x) = 2\sum_{m=1}^{\infty} \tilde{\gamma}_m \sin m\pi x$$

and, as

$$\int_0^1 H_n(u) \sin m\pi u\, du = \frac{m\pi}{m^2\pi^2 - y_n^2}$$

eqn. (15) becomes

$$\alpha_n = \frac{2(Ny_n)^2 \displaystyle\sum_{m=1}^{\infty} \tilde{\gamma}_m \frac{m\pi}{m^2\pi^2 - y_n^2} - SN(N+1)}{N+1+(Ny_n)^2} \qquad \ldots\ldots(31)$$

5.3. *Entry to Case B from Case A*

Comparing eqn. (8) with eqn. (16) we deduce

$$h(x) = 2\sum_{m=0}^{\infty} \tilde{\alpha}_n H_n(x)$$

and, on using eqn. (29), we replace eqn. (20) by

$$\beta_n = \frac{N+1}{N(N+1)+z_n^2} \times \left[\frac{2(N-1)z_n^2}{N} \sum_{m=0}^{\infty} \frac{\tilde{\alpha}_m}{z_n^2 - y_m^2} + S + 1\right] \qquad \ldots\ldots(32)$$

5.4. *Entry to Case C from Case B*

Again $k(x)$ is given by eqn. (28) and, as

$$\int_0^1 k_m(u) \sin n\pi u\, du = \frac{n\pi}{n^2\pi^2 - z_m^2}$$

eqn. (24) becomes

$$\gamma_n = 2\sum_{m=0}^{\infty} \tilde{\beta}_m \frac{n\pi}{n^2\pi^2 - z_m^2} - \frac{1}{n\pi} \qquad \ldots\ldots(33)$$

6. Numerical Method

6.1. *Overall Iteration Process*

We seek values of the coefficients α_n, β_n and γ_n and the times τ_A and τ_C such that the voltage distribution is periodic with time. Such values are calculated using the same overall iteration procedure as that used by Nichols.[1] Figure 6 shows the procedure if stored base charge is omitted and Fig. 7 the procedure if stored base charge is included. For an asymmetrical multivibrator $\tau_{A(1)}$ and $\tau_{A(2)}$ are the decay times for case A for lines 1 and 2 respectively. Since the time for which case B occurs for one line is determined by the time required for the case A base voltage on the other line to become zero, we have

$$\tau_{B(1)} = \tau_{A(2)}$$
$$\tau_{B(2)} = \tau_{A(1)}$$

where $\tau_{B(1)}$ and $\tau_{B(2)}$ are the case B times for lines 1 and 2 respectively.

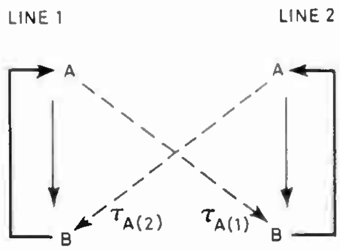

Fig. 6. Iteration loop for asymmetrical multivibrator without stored base charge.

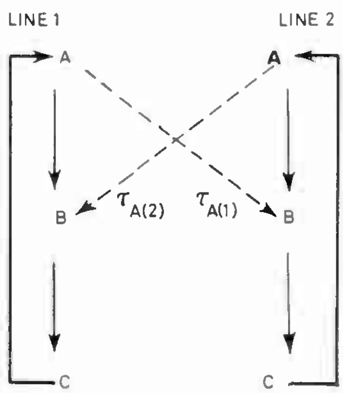

Fig. 7. Iteration loop for asymmetrical multivibrator with stored base charge.

The iterative procedure is started by assuming that

$$\tilde{\beta}_{n(1)} = \tilde{\beta}_{n(2)} = 0 \quad \text{for} \quad n = 0, 1 \ldots$$

that is, the output from case B for both lines is linear. If stored base charge is neglected, the next step is to find coefficients $\alpha_{n(1)}$ and $\alpha_{n(2)}$ for case A using eqn. (30). If stored base charge effects are included, the next step is to find coefficients $\gamma_{n(1)}$ and $\gamma_{n(2)}$ for case C using eqn. (33).

The different cases are then considered in the order shown in Figs. 6 and 7. Each time case A is entered, the coefficients α_n are found using eqns. (30) or (31) and on completion of case A, the coefficients $\tilde{\alpha}_n$ are calculated. Since the $\tilde{\alpha}_n$ rapidly become negligible as n increases, the summations involved are only extended over a finite number of terms. Similarly for cases B and C the coefficients β_n, $\tilde{\beta}_n$, γ_n and $\tilde{\gamma}_n$ may be calculated. In all calculations, less than five terms are required in the summations for cases A and B during the iteration process. If case C is included, the calculation of final values of $v(0, t)$ for case A for small values of t requires up to 20 terms in eqn. (12). For small stored base charge ($q \leqslant 0{\cdot}002$) it is necessary to take up to 200 terms in eqn. (23) for case C.

Usually, less than ten overall iterations are required to find the periodic time to four significant figures. For small values of N ($N < 10$) up to 30 iterations are required.

The constants y_n and z_n ($n = 0, 1, \ldots$) are determined in advance by solving eqns. (13) and (19) iteratively.

6.2. *Evaluation of* τ_A

On entry to case A the α_n are determined and then we seek τ_A such that

$$v(0, \tau_A) = 0$$

since

$$H_n(0) = 1$$

$t = \tau_A$ is the root of

$$S + 2\sum_{n=0}^{\infty} \alpha_n \exp(-y_n^2 t/\lambda^2) = 0 \qquad \ldots\ldots(34)$$

As y_n^2 increases like n^2, it is only necessary to include a few terms in the summation. In all the numerical cases considered in this paper less than five terms are required to determine τ_A to five significant figures. The *regula falsi* iterative process[2] is used to solve eqn. (34). This simple iterative method is considered adequate especially as a good estimate of τ_A is usually available from the previous overall iteration cycle.

6.3. *Evaluation of* τ_C

The time τ_C taken to neutralize the stored base charge is given by eqn. (27) and on substituting eqn. (33) we obtain

$$q\lambda^2 = -S\left(1 + \frac{1}{N}\right)\tau_C + \frac{\lambda^2}{3} - 2\lambda^2 \sum_{m=0}^{\infty}\left(\frac{1+N}{z_m^2}\right)\tilde{\beta}_m +$$
$$+ \lambda^2 \sum_{n=1}^{\infty} \frac{\gamma_n}{n\pi} \exp(-n^2\pi^2\tau_C/\lambda^2) \qquad \ldots\ldots(35)$$

where we have used the identities

$$\sum_{n=1}^{\infty} \frac{1}{n^2\pi^2} = \frac{1}{6}$$

$$\sum_{n=1}^{\infty} \frac{1}{n^2\pi^2 - z_m^2} = \frac{1}{2z_m}\left[\frac{1}{z_m} - \cot z_m\right] = \frac{N+1}{2z_m^2}$$

The *regula falsi* method is again used to solve eqn. (35) for τ_C.

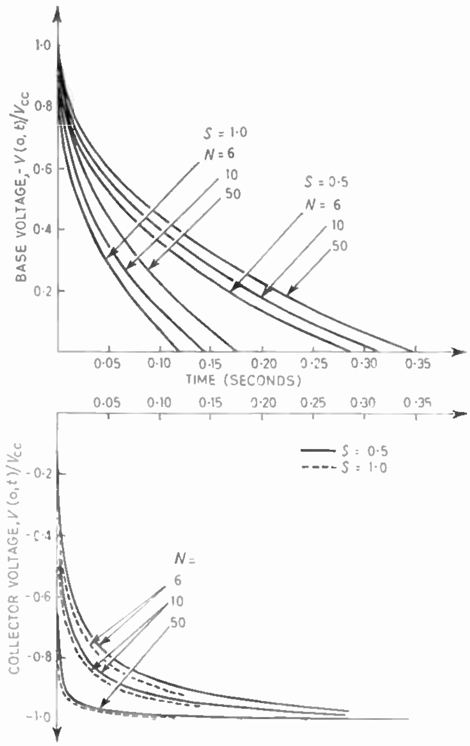

Fig. 8. Symmetrical multivibrator without stored base charge. For all cases shown $(N+1)CR = 1$.

7. Results

7.1. *Multivibrator without Stored Base Charge*

Computations were made for various symmetrical multivibrators and typical waveforms are shown in Fig. 8. The variation of periodic time τ_A against N is shown in Fig. 9. These results show that for a given supply voltage ratio S the variation of τ_A against N is greater than that obtained by Nichols for a univibrator. Waveforms of a typical asymmetrical multivibrator are shown in Fig. 10. In all cases the voltages for both the base and collector decay exponentially as is indicated by the form of the solution.

7.2. *Multivibrator with Stored Base Charge*

The waveform of the cut-off base is most affected by the neutralization of stored base charge and waveforms for a symmetrical multivibrator with

$$N = 10, \quad S = 1, \quad (N+1)CR = 1$$

are shown for various values of q in Fig. 11. For all values of q the case B collector voltage waveform is almost identical with its form for $q = 0$ (Fig. 10). The dependence of the periodic time on q is shown in Fig. 12, for $s = 0{\cdot}5$ and $1{\cdot}0$ and the dependence of τ_C on q in Fig. 13. The crosses indicate points obtained

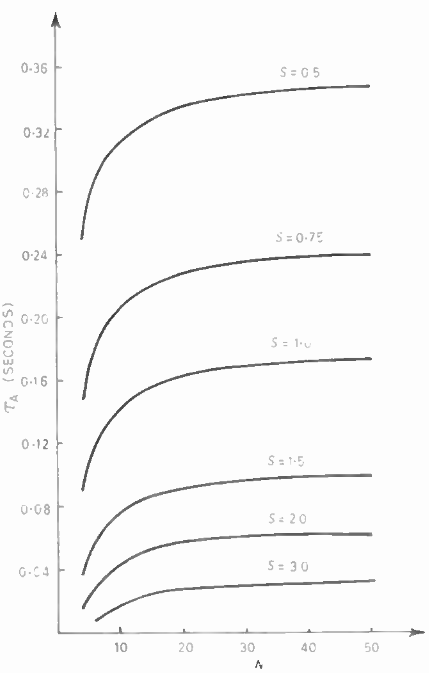

Fig. 9. Symmetrical multivibrator without stored base charge; variation of periodic time against voltage ratio, S, and resistance ratio, N.

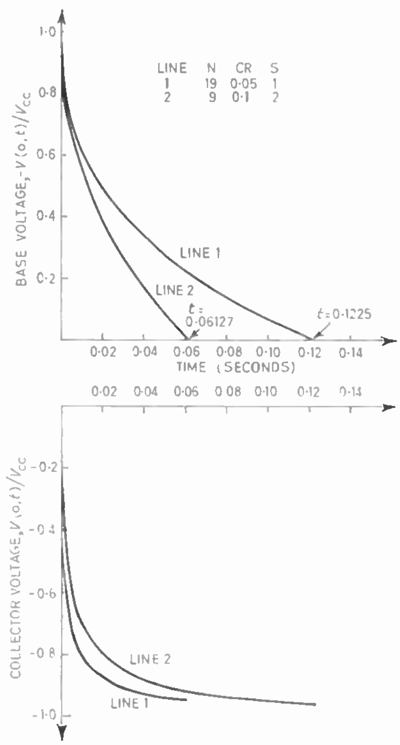

Fig. 10. Asymmetrical multivibrator without stored base charge.

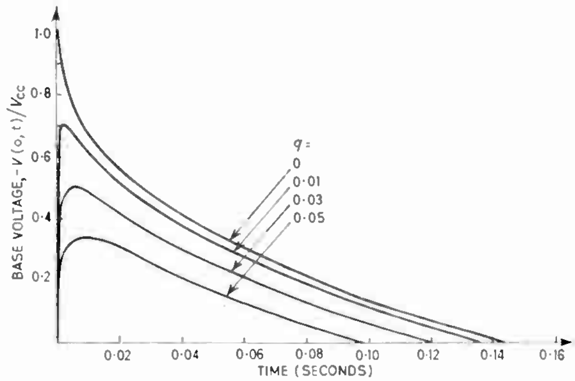

Fig. 11. Symmetrical multivibrator with stored base charge. Waveforms of voltage of cut-off base. For all cases shown $N = 10$, $S = 1$ and $(N + 1)CR = 1$.

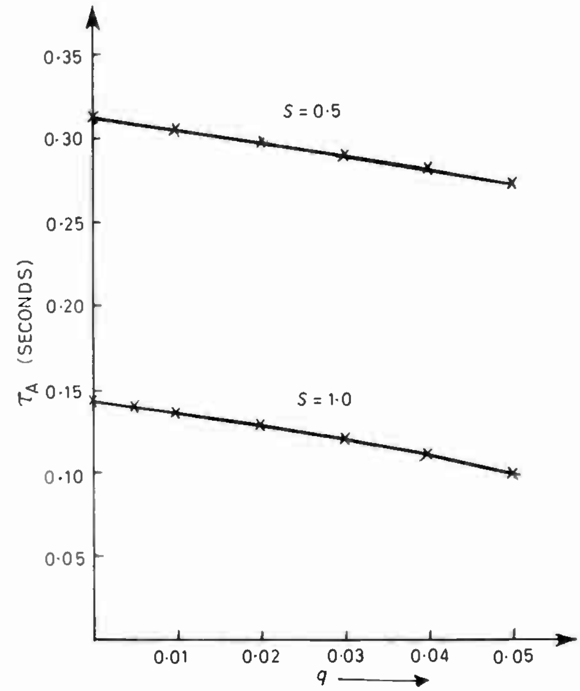

Fig. 12. Variation of periodic time τ_A with stored base charge q. $N = 10$, $(N + 1)CR = 1$ in both cases.

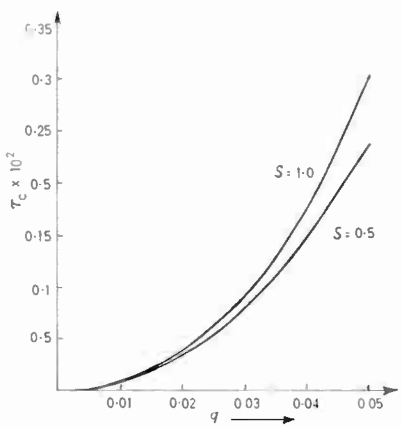

Fig. 13. Symmetrical multivibrator: variation of the time taken to neutralize the stored base charge against q. For both cases $N = 10$ and $(N + 1)CR = 1$.

in the calculation. Nichols found that, for the finite difference method, the periodic times obtained as q is decreased did not seem to approach the value for $q = 0$. Even at $q = 0{\cdot}001$ our solution appears accurate. At $q = 0{\cdot}01$ our value is larger than that of Nichols and even at $q = 0{\cdot}05$ the multivibrator appears to oscillate. The oscillations have definitely ceased, however, for $q = 0{\cdot}1$. The results presented here are considered more accurate for two reasons. Firstly, the points shown in Fig. 12 lie on smooth curves and secondly, there is a singularity in $\partial v/\partial x$ at $x = 0$ and $t = 0$ for case C and to obtain τ_C it is necessary to integrate $\partial v/\partial x$ with respect to time. Even though the integral exists, it is difficult to determine accurately using finite differences. The time steps would need to be very small as $\tau_C < 0{\cdot}003$ and the corresponding space interval would also need to be very small.

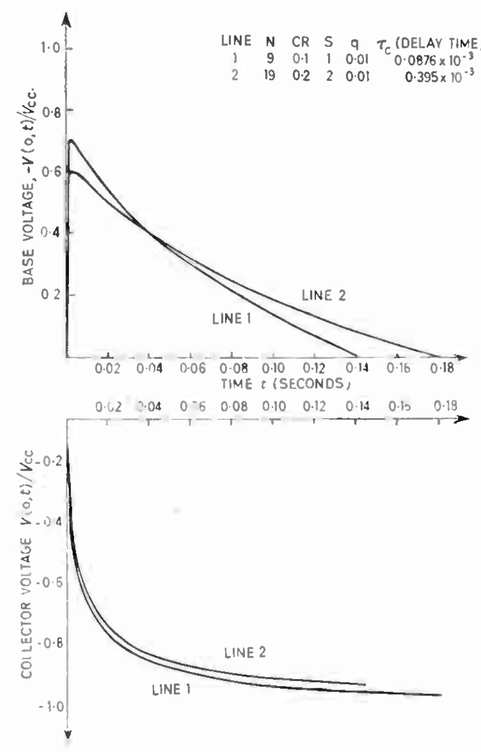

Fig. 14. Asymmetrical multivibrator with stored base charge.

Waveforms for an asymmetrical multivibrator are shown in Fig. 14. To illustrate the nature of the coefficients in the solution, some of these are shown for the multivibrator of Fig. 14 in Table 1.

Table 1

Coefficients in the solution of the problem of Fig. 14

	line 1		line 2	
n	α_n	β_n	α_n	β_n
1	−1·45996	0·13902	−2·10538	0·05958
2	−0·21589	0·16558	−0·32050	0·07718
3	−0·06228	0·13034	−0·13364	0·08507
4	−0·02156	0·09671	−0·05906	0·07981
5	−0·00505	0·07191	−0·02615	0·07058
6	+0·00320	0·05447	−0·01028	0·06115
7	+0·00787	0·04220	−0·00152	0·05259
8	+0·01072	0·03342	+0·00381	0·04518
9	+0·01256	0·02700	+0·00726	0·03890

8. Conclusions

Analytic solutions have been found for the differential equation, devised by Nichols, for a multivibrator circuit using uniform distributed CR lines as the cross-coupling elements. Three solutions have been obtained corresponding to boundary conditions for three different states of the multivibrator. Following Nichols, all switching transients, except those due to stored base charge, have been neglected and it has been assumed that, for a saturated transistor, the voltages v_{BE} and v_{CE} are zero and that desaturation of a transistor commences only when the other starts conducting.

Each analytic solution consists of a series expansion in orthogonal functions. The set of functions involved differs for each of the three states of the multivibrator. Series expansions of each of the orthogonal functions corresponding to one state may be obtained in terms of the orthogonal functions corresponding to the next state. It is possible, therefore, to proceed from a solution of one state to a solution of the next state. This process was used successfully to obtain a number of numerical solutions. An immediate advantage of the method, over the finite difference method used by Nichols, is that the exact solution is known for each state of the multivibrator and therefore the various transient (exponential-decay) terms may be inspected. Another advantage is that the effects of neutralization of stored base charge may be computed more accurately. The method also involves less calculation than finite differences and should be applicable to other problems involving distributed lines and switching devices causing discontinuous boundary conditions.

The results are similar to those obtained by Nichols except when neutralization of stored base charge is included. The effect of such neutralization on the periodic time and waveform was found to be less pronounced. Reasons have been given for believing these results to be more accurate.

9. Acknowledgments

The authors express their thanks to Mr. K. G. Nichols for the useful discussions they had with him and for permission to reproduce Figs. 1–5.

10. References

1. Nichols, K. G. 'Distributed CR cross-coupling circuits for multivibrators', *The Radio and Electronic Engineer*, **30**, No. 1, pp. 5–17, July 1965.
2. Hildebrand, F. B., 'Introduction to Numerical Analysis', pp. 446–7 (McGraw-Hill, New York, 1956).
3. Jaeger, J. C., 'An Introduction to the Laplace Transformation', Chapter IV (Methuen, London, 1949).

11. Appendix

Analytical solutions for multivibrator states

The line eqn. (1) transformed[3] becomes

$$V''(x,p)-(N+1)CRpV(x,p) = -(N+1)CRv(x,0) \qquad \ldots\ldots(36)$$

where $V(x,p)$ is the Laplace transform of $v(x,t)$ and the primes denote differentiation with respect to x.

The general solution of eqn. (36) may be written as

$$V(x,p) = V(0,p)\cosh\lambda\sqrt{p}x + V'(0,p)\frac{\sinh\lambda\sqrt{p}x}{\lambda\sqrt{p}} - \frac{\lambda}{\sqrt{p}}\int_0^x v(u,0)\sinh[\lambda\sqrt{p}(x-u)]\,du \qquad \ldots\ldots(37)$$

where

$$\lambda^2 = (N+1)CR$$

The functions $V(0,p)$, $V'(0,p)$ may be eliminated using the transformed boundary conditions which exist for each state of the multivibrator.

Case A

The transformation of the boundary conditions (2), (3) yields

$$V'(0,p)-V(0,p)/N = -v_{CC}(1+S/N)/p \qquad \ldots\ldots(38)$$

$$V(1,p) = (S-1)v_{CC}/p \qquad \ldots\ldots(39)$$

where

$$S = v_{BB}/v_{CC}$$

The initial condition $v(x,0)$ for case A is determined by the final voltage distribution in the state which precedes. This state can either be B or C so that $v(x,0)$ is given correspondingly by either eqn. (4) or eqn. (5). For entry to state A from state B the combination of eqns. (37), (38), (39) and (4) gives

$$\frac{V(x,p)}{v_{CC}} = \frac{Sx-1}{p} + \frac{(S+1)(N+1)\sinh(\lambda\sqrt{p}(1-x))}{p(N\lambda\sqrt{p}\cosh\lambda\sqrt{p}+\sinh\lambda\sqrt{p})} + \int_0^1 k(u)H(x,u,p)\,\mathrm{d}u \qquad \ldots\ldots(40)$$

where $H(x, u, p)$ is symmetric in x, u and defined by

$$H(x,u,p) = \begin{cases} \dfrac{\lambda\sinh(\lambda\sqrt{p}(1-x))(N\lambda\sqrt{p}\cosh\lambda\sqrt{p}u+\sinh\lambda\sqrt{p}u)}{\sqrt{p}(N\lambda\sqrt{p}\cosh\lambda\sqrt{p}+\sinh\lambda\sqrt{p})} & 1 \geqslant x \geqslant u \geqslant 0 \\ \dfrac{\lambda\sinh(\lambda\sqrt{p}(1-u))(N\lambda\sqrt{p}\cosh\lambda\sqrt{p}x+\sinh\lambda\sqrt{p}x)}{\sqrt{p}(N\lambda\sqrt{p}\cosh\lambda\sqrt{p}+\sinh\lambda\sqrt{p})} & 1 \geqslant u \geqslant x \geqslant 0 \end{cases}$$

Equation (40) shows that $V(x, p)$ contains simple poles at $p = 0$ and at $p = p_n$ where p_n satisfies the equation,

$$\frac{\tanh\lambda\sqrt{p_n}}{\lambda\sqrt{p_n}} = -N \qquad \ldots\ldots(41)$$

The solutions of eqn. (41) are given by

$$\lambda\sqrt{p_n} = \pm \mathrm{j}y_n \qquad n = 0, 1, 2\ldots$$

where

$$\frac{\tan y_n}{y_n} = -N$$

The values of y_n are such that

$$(2n+1)\frac{\pi}{2} < y_n < (n+1)\pi \quad \text{for} \quad n = 0, 1, 2\ldots$$

and for N large

$$y_n \simeq (2n+1)\frac{\pi}{2} \quad \text{with} \quad n = 0, 1, 2\ldots$$

The residues of $V(x, p)$ at the poles

$$p = 0, \qquad p = -y_n^2/\lambda^2, \qquad n = 0, 1, 2\ldots$$

can be obtained to give the total inversion of the eqn. (40) in the form

$$\frac{v(x,t)}{v_{CC}} = S-x+2\sum_{n=0}^{\infty}\left[(Ny_n)^2\int_0^1 H_n(u)k(u)\,\mathrm{d}u-(S+1)N(N+1)\right]\frac{H_n(x)\exp(-y_n^2 t/\lambda^2)}{N+1+(Ny_n)^2} \qquad \ldots\ldots(42)$$

where

$$H_n(x) = \frac{\sin y_n(1-x)}{\sin y_n}$$

For entry to state A from state C we can similarly obtain

$$\frac{V(x,p)}{v_{CC}} = \frac{(S-1)x}{p} + \frac{S(N+1)\sinh\lambda\sqrt{p}(1-x)}{p(N\lambda\sqrt{p}\cosh\lambda\sqrt{p}+\sinh\lambda\sqrt{p})} + \int_0^1 g(u)H(x,u,p)\,\mathrm{d}u$$

with an inverse

$$\frac{v(x,t)}{v_{CC}} = S-x+2\sum_{n=0}^{\infty}\left[(Ny_n)^2\int_0^1 H_n(u)g(u)\,\mathrm{d}u-SN(N+1)\right]\frac{H_n(x)\exp(-y_n^2 t/\lambda^2)}{N+1+(Ny_n)^2} \qquad \ldots\ldots(43)$$

Case B

The boundary eqns. (6), (7) transformed are

$$V'(0,p)-NV(0,p) = (N+S)v_{CC}/p \qquad \text{......(44)}$$

$$V(1,p) = (S-1)v_{CC}/p \qquad \text{......(45)}$$

The initial condition is

$$v(x,0) = v_{CC}(S-1+h(x)) \qquad \text{......(46)}$$

An analysis similar to that used for case A using eqns. (36) and (44–46) gives

$$\frac{V(x,p)}{v_{CC}} = \frac{S-x}{p} - \frac{(S+1)(N+1)\sinh(\lambda\sqrt{p}(1-x))}{p(\lambda\sqrt{p}\cosh\lambda\sqrt{p}+N\sinh\lambda\sqrt{p})} + \int_0^1 h(u)K(x,u,p)\,du \qquad \text{......(47)}$$

where $K(x,u,p)$ is symmetric in x, u and defined by

$$K(x,u,p) = \frac{\lambda\sinh(\lambda\sqrt{p}(1-x))}{\sqrt{p}}\frac{(\lambda\sqrt{p}\cosh\lambda\sqrt{pu}+N\sinh\lambda\sqrt{pu})}{(\lambda\sqrt{p}\cosh\lambda\sqrt{p}+N\sinh\lambda\sqrt{p})} \qquad 1\geqslant x\geqslant u\geqslant 0$$

$$= \frac{\lambda\sinh(\lambda\sqrt{p}(1-u))(\lambda\sqrt{p}\cosh\lambda\sqrt{px}+N\sinh\lambda\sqrt{px})}{\sqrt{p}(\lambda\sqrt{p}\cosh\lambda\sqrt{p}+N\sinh\lambda\sqrt{p})} \qquad 1\geqslant u\geqslant x\geqslant 0$$

The inversion of $V(x,p)$ gives

$$\frac{v(x,t)}{v_{CC}} = Sx-1+2\sum_{n=0}^{\infty}\left[z_n^2\int_0^1 K_n(u)h(u)\,du+(S+1)(N+1)\right]\frac{K_n(x)\exp(-z_n^2t/\lambda^2)}{N(N+1)+z_n^2} \qquad \text{......(48)}$$

where

$$K_n(x) = \frac{\sin z_n(1-x)}{\sin z_n} \qquad\qquad \frac{\tan z_n}{z_n} = -\frac{1}{N}$$

with

$$(2n+1)\frac{\pi}{2} < z_n < (n+1)\pi$$

It is noted that for N large then

$$z_n \simeq (n+1)\pi, \qquad n = 0,1,2\ldots$$

for at least the first few integer values of n.

Case C

The transformed boundary conditions are

$$V(0,p) = 0 \qquad \text{......(49)}$$

$$V(1,p) = v_{CC}(S-1)/p \qquad \text{......(50)}$$

The initial condition is

$$v(x,0) = v_{CC}(Sx-1)+v_{CC}k(x)$$

The analysis for case C gives after some simplification

$$\frac{V(x,p)}{v_{CC}} = \frac{Sx-1}{p} + \frac{\sinh(\lambda\sqrt{p}(1-x))}{p\sinh\lambda\sqrt{p}} + \int_0^1 k(u)J(x,u,p)\,du$$

with

$$J(x, u, p) = \frac{\lambda \sinh \lambda\sqrt{p(1-x)} \sinh \lambda\sqrt{pu}}{\sqrt{p} \sinh \lambda\sqrt{p}} \qquad 1 \geqslant x \geqslant u \geqslant 0$$

$$= \frac{\lambda \sinh \lambda\sqrt{p(1-u)} \sinh \lambda\sqrt{px}}{\sqrt{p} \sinh \lambda\sqrt{p}} \qquad 1 \geqslant u \geqslant x \geqslant 0$$

The inversion of $V(x, p)$ gives

$$\frac{v(x, t)}{v_{CC}} = (S-1)x + 2\sum_{n=1}^{\infty} \left[\int_0^1 k(u) \sin n\pi u \, du - \frac{1}{n\pi}\right] \sin n\pi x \exp(-n^2\pi^2 t/\lambda^2) \qquad \ldots\ldots(51)$$

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STANDARD FREQUENCY TRANSMISSIONS

(Communication from the National Physical Laboratory)

Deviations, in parts in 10^{10}, from nominal frequency for **August 1968**

August 1968	24-hour mean centred on 0300 U.T.			August 1968	24-hour mean centred on 0300 U.T.		
	GBR 16 kHz	MSF 60 kHz	Droitwich 200 kHz		GBR 16 kHz	MSF 60 kHz	Droitwich 200 kHz
1	− 300·0	0	+ 0·1	17	− 300·0	− 0·1	− 0·2
2	− 300·0	0	+ 0·2	18	− 300·0	0	− 0·4
3	− 300·0	0	− 0·1	19	− 300·0	0	− 0·2
4	− 300·0	0	− 0·1	20	− 300·0	− 0·1	− 0·3
5	− 300·0	0	− 0·1	21	− 300·0	− 0·1	− 0·3
6	− 300·1	0	− 0·2	22	− 300·3	0	− 0·2
7	− 300·0	0	− 0·2	23	− 300·5	+ 0·1	− 0·1
8	− 300·1	− 0·1	− 0·2	24	− 300·4	+ 0·1	—
9	− 300·0	− 0·1	− 0·2	25	− 300·4	0	—
10	− 300·1	− 0·1	− 0·2	26	− 300·4	+ 0·1	—
11	− 300·1	− 0·1	− 0·2	27	− 300·0	+ 0·1	− 0·1
12	− 300·0	− 0·1	− 0·2	28	− 300·1	0	− 0·1
13	− 300·0	0	− 0·1	29	− 300·1	0	− 0·2
14	—	—	− 0·2	30	− 300·1	− 0·1	− 0·2
15	− 300·0	− 0·1	− 0·2	31	− 300·1	− 0·1	− 0·1
16	− 300·1	0	− 0·2				

Nominal frequency corresponds to a value of 9 192 631 770·0 Hz for the caesium F,m (4,0)–F,m (3,0) transition at zero field.

Notes: (1) The radiated frequencies of the GBR and MSF transmissions were in excess of their normal tolerances over the period 0400 to 0500 U.T. on 14th August 1968, and were also subject to small phase changes during the day.

(2) The radiated frequency of the Droitwich transmission was in excess of the normal tolerance over the period 0345 to 1630 U.T. on 18th August 1968.

Radio Engineering Overseas . . .

The following abstracts are taken from Commonwealth, European and Asian journals received by the Institution's Library. Abstracts of papers published in American journals are not included because they are available in many other publications. Members who wish to consult any of the papers quoted should apply to the Librarian giving full bibliographical details, i.e. title, author, journal and date, of the paper required. All papers are in the language of the country of origin of the journal unless otherwise stated. Translations cannot be supplied.

PRINTED CIRCUIT RELIABILITY

Most multi-layer printed circuit boards employ the plated-through-hole connection as a method of connecting wiring patterns on each layer. This plated-through-hole connection can be applied to either double-sided or multi-layer printed boards and its production method may use a combination of photo-engraving and electro-deposition.

Reliability of plated-through-hole connections has been tested by the Fujitsu company of Japan mainly on double-sided printed boards, through various tests such as temperature cycle, humidity resistance, vibration, and combined tests of all these. Observations were made on the development of failure through these tests: it was found that most holes maintain favourable connection yet, depending on the material tested and/or type of tests, numerous connection failures have occurred. Observations of these malconnections have revealed that most of them were the initial failures developed during production, but some board material involved hazard of through-connection breakdown.

The paper investigates the reliability of plated-through-hole connection from the above phenomena and its related data such as strength, corrosivity, solderability, etc.

'Reliability of printed wiring boards with plated-through-hole connections', T. Obana, K. Takagi, K. Murakawa and H. Takanohashi, *Fujitsu Scientific and Technical Journal*, 3, No. 2, pp. 111–56, September 1967.

CORRELATED AERIAL ARRAYS

The analysis of the resolving power of the correlation arrays shows that such receiving antenna systems have a significantly better resolution (in the sense of Rayleigh and Sparrow criteria) than the summing arrays and that they can be successfully utilized for measurements of angular co-ordinates of point sources in radio astronomy, sonar, acoustics, etc.

The possibility of utilization of the high-resolution capabilities of correlation arrays (antennas) (for resolution of point sources) in radio astronomy, for example, may be severely limited by the significantly lower sensitivity of these arrays, in comparison to the additive arrays. This is due to the fact that the non-linear processing of a noise signal causes the variance of the output voltage of correlation arrays, which depends on the finite averaging time, to be many times larger than in the case of linear signal processing. For this reason an estimate of the sensitivity of correlation arrays is quite important.

In a Soviet paper the relationship between the input and the output signal/noise ratios in six- and four-element correlation arrays is derived for noise and sinusoidal signal sources. The sensitivity of the cross-shaped radio telescope (DKR-1000) is estimated for the case when the elements are connected to form a correlation array.

'Signal/noise ratio in correlation lattices', Y. P. Shitov, *Radio Engineering and Electronic Physics* (English language edition of *Radiotekhnika i Elektronika*), pp. 1969–74, No. 11, 1967.

REVERSE DIODES

Reverse diodes form a sub-group of tunnel diodes and feature a low peak current and a weak region of negative dynamic resistance. Another characteristic of a reverse diode is that its current-voltage characteristic is more non-linear than that for any other type of semiconductor diode. The low dynamic resistance is especially useful for the detection of weak signals.

In a Czechoslovak paper experimental work carried out to determine the non-linearity is described and a theoretical formula for the current-voltage relationship is derived. The factors which limit the validity of this formula are discussed.

The paper concludes with a comparison of a reverse diode as a detector of weak signals and clearly shows its superiority in this respect over a conventional germanium or silicon diode.

'Reverse diodes', J. Karlovský, *Slaboproudý Obzor*, 29, No. 7, pp. 373–9, 1968.

MOTIONAL FEEDBACK WITH LOUDSPEAKERS

One of the main causes of distortion in an electro-acoustical system consisting of an amplifier and a loudspeaker is to be found in the mechanism of the loudspeaker, particularly if the loudspeaker is mounted in a small, closed cabinet. This distortion is not affected by the electronic negative feedback conventionally used for reducing distortion. With motional feedback, the loudspeaker is included in the feedback loop; there are several forms of motional feedback and a method specially developed for feedback at low audio frequencies has been tested experimentally at the Philips Research Laboratories, Eindhoven. In this method an acceleration transducer containing a piezoelectric element is fixed to the moving coil of a normal loudspeaker: this gives a feedback signal proportional to the acceleration of coil and cone. A signal proportional to the velocity is also obtained by integration and a certain fraction of this is added to the first feedback signal. In this way an amplitude characteristic can be obtained which is flat down to below the mechanical resonant frequency of the loudspeaker, and the non-linear distortion in this frequency range is substantially reduced.

'Motional feedback with loudspeakers', J. A. Klaassen and S. H. de Koning, *Philips Technical Review*, 29, No. 5, pp. 148–57, 1968.