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by the exchange of information in these branches of engineering."*

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## 1957 CONVENTION PAPERS

The selection of subjects for publication in the *Journal* is designed to achieve a balanced output of papers covering the whole field of radio and electronics.

For this reason, papers read at this year's Convention are spread over a number of *Journals*. It is, however, hoped to complete the printing of all Convention papers within the next three months.

## AUTOMATIC INSPECTION AS THE KEY CONTROL ELEMENT IN FULL AUTOMATION\*

by

John A. Sargrove (Member)† and Denis L. Johnston, B.Sc.(Eng.)†

*A paper presented at the Convention on "Electronics in Automation" in Cambridge on 28th June 1957. In the Chair: Mr. E. E. Webster (Member).*

### SUMMARY

A fully automatic process is essentially an exercise in "system design", and in the simple application of the techniques of analogue or digital computers. The feedback signals to control and stabilize the system will be originated by inspection or measuring devices, dependent on the human senses or automatic instrumentation. The characteristics of a number of classes of such systems are distinguished, as well as the special requirement of the inspection equipment, which must have a higher order of reliability than is usually associated with laboratory measuring instruments. A good visual display of the information derived, plus some degree of manual control, may often be a more economic solution than an attempt to achieve 100 per cent. automation. Transistor circuits will probably be adopted rapidly because of their inherent advantages of economy in size and power, and high reliability. Recent work in ergonomics has shown that automatic inspection can be consistently superior to human performance.

### 1. What is Automation?

The electronics engineer is fortunate to be trained in a discipline of thought having much wider applications than circuit design. The circuit analysis methods introduced by Nyquist, Hazen and Black in the mid-1930's are, of course, now used in other fields as diverse as hydraulic systems and economic studies.

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Automation has been defined by John Diebold briefly as "the philosophy of organizing any work as a system." This concept of "system design" has been pioneered by electronics and servo engineers, and the block diagram of Fig. 1 will be quite familiar as a feedback circuit, although it represents the essentials of a fully automated production process, such as machining or grinding.

Automated production usually comprises a whole series of work-stations, through which the articles proceed. In the pioneer machine,

E.C.M.E., first described before this Institution in 1947, each processing station was provided with means for automatic inspection and corrective action.‡ This is still not found in the usual application of what is known as "Detroit Automation," typified by the transfer machining line.

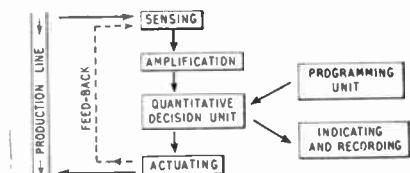


Fig. 1. Fundamental elements of an Automation System. The "decision unit" forms the heart of the equipment, constituting a significant difference from mechanization.

A process, such as an oil refinery with a continuous flow, comprises a complex network of motive and feedback elements. If some of these feedback functions are controlled by human operators we can define the degree of automation as something less than 100 per cent. In the case of the catalytic cracker of the oil refinery, the process only becomes practicable under fully automatic control, and the staff on duty are limited to supervision and monitoring of the recorded information channelled into the control room.

The feedback signals to control and stabilize such systems will be originated by automatic inspection or measuring devices, which are the subject of the present paper.

## 2. Characteristics of Processes

It may be helpful to distinguish four classes of inspection system, as each has a somewhat different objective, as follows:—

### 2.1. Stable, Self-Monitoring Processes

Continuous inspection is unnecessary, and only a failure alarm is needed, preferably of the type that itself "fails safe": typical examples are thermostatic and feed controls. When large numbers are used, as in a continuous oven, failures may occur relatively frequently, and a centralized logical display of fault-indications is highly desirable.

‡ J. A. Sargrove, "New methods of radio production," *J.Brit.I.R.E.*, 7, pp. 2-33, January 1947.

### 2.2. Automatic Correction of Errors and Faulty Parts

This type of system was not possible under manual control, and is probably best known in oil refining and fractionating towers, where a reflex process occurs, the out-of-limits fraction of production being continually recirculated and reprocessed until it comes within the "pass" limits, or is rejected. This method is applicable to centre-less grinding machines, where oversize parts can be passed back again.

### 2.3. Data and Information Handling

Our definition of "organizing work as a system" covers also those instances where the main problem is to obtain and digest a great bulk of information in a short time: this would usually be impracticable by direct human effort. Examples of this are in commercial applications of digital computers for accounting, banking and production control, and analogue process and flight simulators. The end result here is processed or computed information, and the inspection function comprises reading the information and detecting errors and faults.

### 2.4. Complex Manufacturing Processes with Critical Reject Rates

When a number of processes are carried out in series, each must be controlled to a low scrap rate if the overall cumulative wastage is to be within economic limits. In critical manufacturing processes, such as valve-making, careful inspection is essential at every stage, but in typical transfer machining the life of cutting tools and grinding wheels is relatively long, and it is not usual to provide automatic inspection and feedback control. It can be argued that it would be more economical to run the machine tools much nearer their limit of capacity, and obtain a greater rate of output at the expense of a slight increase in reject percentage and the cost of installing instrumental warning devices.

## 3. Inspection and Measuring Devices

The sensing devices applicable to inspection and control in automated systems are drawn from the wide field of measuring and control instruments, as set out in the chart of Fig. 2, which is an expansion of Fig. 1.

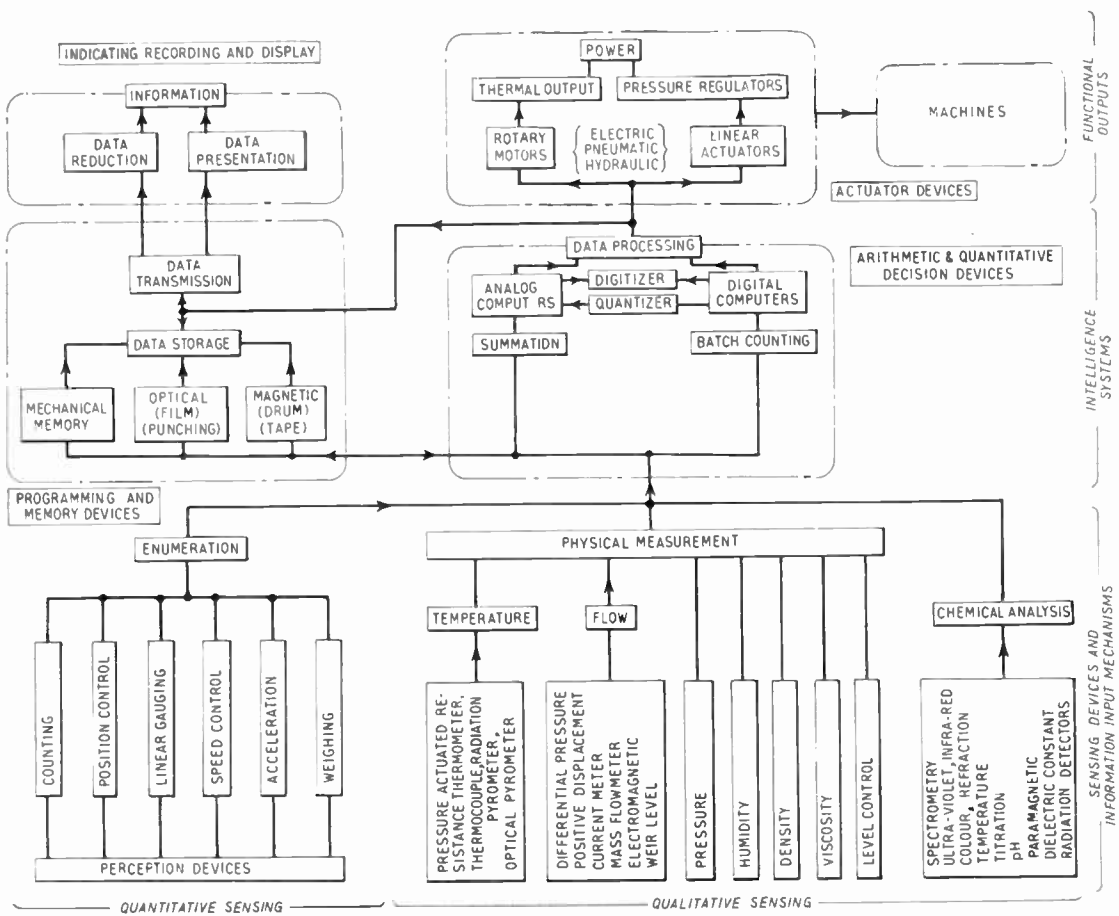


Fig. 2. An expansion of Fig. 1 showing the whole range of techniques and their adaptations which can be covered under the general heading "Automation."

Although the whole range of classical measuring techniques is available for adaptation, the special conditions we are considering determine the following requirements:—

- (a) *Robustness*: of a much higher order than laboratory instruments, preferably with fully sealed housings, which are essential when exposed to climatic or explosion hazards.
- (b) *Reliability*: average failure rates of once per few thousand hours are not nearly good enough when many units are employed together: the usual control instrument, adequate when used alone, may not be acceptable.
- (c) *Stability*: there should be no significant drift, as the need for zero-setting and

calibrating adjustments is almost as troublesome as outright failure.

- (d) *Speed of response*: this may have to be considerably faster than for laboratory purposes, but where changes occur very slowly it may be possible to simplify the control instrument accordingly.
- (e) *Compatibility*: signal outputs should be in a form suitable for direct connection to other devices, to facilitate and simplify system design. The control instrument industry has not taken very active steps to bring this about.

It is clear that the standard type of laboratory bench instrument, intended for intermittent use, is inappropriate in an automation system. The

stability performance is generally not adequate, nor the reliability, over a long period, and the construction is not usually sufficiently robust.

#### 4. Design of Inspection and Control Systems

The measuring or sensing devices are connected into the control system for the actuating devices. Some elementary form of "brain" can then be provided, to confer the ability to decide what pre-determined action shall occur in particular circumstances. This conscious element in the system may be mechanical or electro-mechanical, and is more often electronic or a hybrid system.

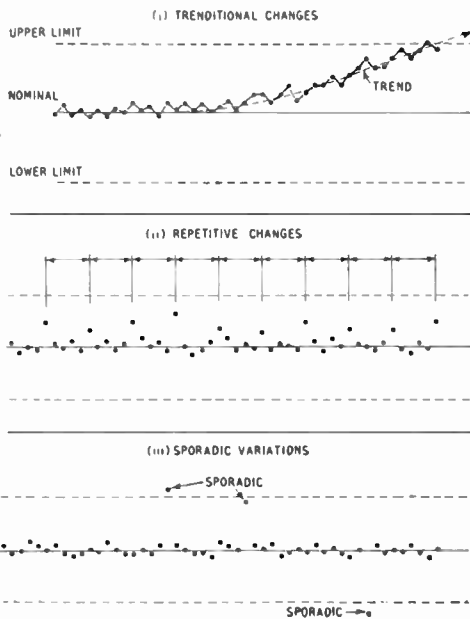


Fig. 3. Classes of statistical variation in production processes.

The degree of sophistication of this "brain" can be developed as far as the technical complication can be justified. It is, of course, a computer, but in most practical cases in industrial processes it can be a very slow machine indeed, by comparison with the high-speed digital machines and large analogue simulators. Because it can be slow, it may be made relatively simple. For example, a very slow speed magnetic drum can usually be employed, having much more in common with the gramophone turntable than the computer

magnetic memory whose cost is in the four-figure region.

An important point is that the quantities to be measured are the span of the acceptance limits, rather than the total magnitude, and so a comparator method can be used instead of an absolute one. For limits of the order of 5 per cent. to be held to  $\frac{1}{2}$  per cent., the measurement problem is discrimination to one part in 10, rather than the one part in 200 represented by  $\frac{1}{2}$  per cent. of the total magnitude. It becomes possible to use analogue methods, such as magnetic recording, with adequate accuracy.

The recording function is an important one, because a time displacement occurs in continuous processes, and means of storing information for a short time is necessary. For example, if several dimensions are to be gauged, it is more convenient to do this at successive stations along a conveyor than to do it simultaneously. A slowly revolving magnetic drum enables this information, derived at successive time intervals, to be made available simultaneously to a decision limit circuit, at a later time when the test pieces reach an accept/reject gate.

#### 5. Statistical Variations in Automatic Inspection

It must be recognized that there will be a statistical characteristic peculiar to the process we are considering, and we must deal with this rather as we would deal with noise in a communication system, in order to realize the optimum signal-to-noise ratio.

The different characteristics can be classified as follows:—

- (i) *Trenditional changes* represent a tendency towards "out-of-limit" conditions, which can be corrected by feedback into the process. If the practical limits of feedback are approached some warning or action may be necessary: for example, wheel wear on an automatically controlled centre-less grinder. Obviously the original cause of the trend should be investigated and minimized as far as possible.
- (ii) *Repetitive changes* point to the continual recurrence of a fault, in one station of a

multi-position machine; beyond certain limits a warning can be given by the control system, or alternatively a reserve unit changed for the faulty one.

(iii) *Sporadic variations*: the conscious system may be required to distinguish between chance faults and signs of incipient breakdown.

Action will be taken automatically by the control system, and in addition it is desirable to present visually the existing situation and immediate history for the information of the superintending engineer.

A good visual display of the information available with suitable controls, designed on ergonomic principles, may be a more economic solution than a system in which a somewhat higher degree of automation is attempted.

A continuous quality-control chart can be plotted by standard-type recorders, and the characteristics of the above classes will be recognized as they appear in Fig. 3.

When limiting conditions are approached, a clear warning indication must be provided for the supervisor, who can then take special action to deal with the situation. Special circumstances that recur, such as the starting-up and shutting-down routine, can be "programmed," the simplest methods being by a bank of cams, or by coding on teleprinter tape. Fig. 4 shows a machine programme unit of this type.

### 6. Example of Automatic Inspection for Density

Some of the features already discussed are incorporated in the system for inspection of density shown in Fig. 5.

Round flat objects are passed along a conveyor by intermittent motion at a rate of, say, one per second. They pass under displacement transducer heads, which detect height ( $H$ ), diameter ( $D$ ) and weight ( $W$ ). For the small percentage limits set, the density  $\delta = \frac{\text{weight}}{\text{volume}}$  can be expressed sufficiently accurately in differential form. Thus, if the three primary fractional errors are  $\delta H$ ,  $\delta D$  and  $\delta W$ , the fractional error in density is given by

$$\delta d = \delta W - \delta H - 2\delta D$$

The computation can thus be carried out by a simple summation circuit, without the necessity for multiplication or division.

In the diagram it will be seen that each transducer is followed by an amplifier, and the signal proportional to deflection is recorded as an a.c. tone on the slowly rotating magnetic drum. The three measurements are recorded at intervals one-third second apart, as the test object proceeds along the conveyor. An angular displacement of the read-out heads enables the signals to be extracted simultaneously and fed to limit circuits for height, diameter and weight.

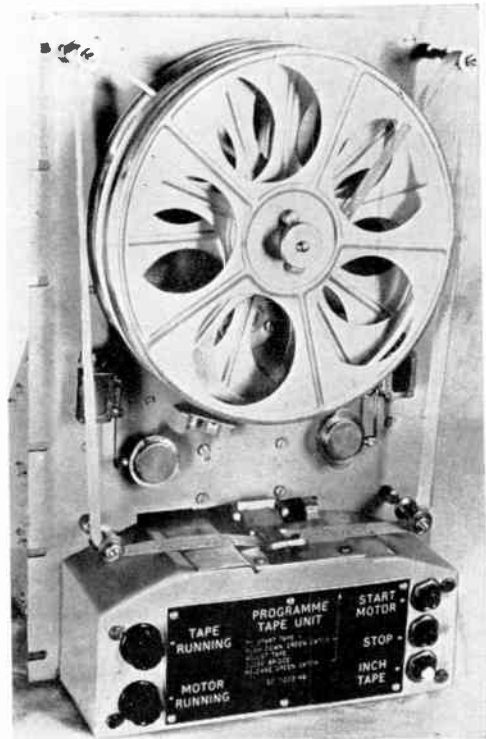


Fig. 4. Typical punched-tape programming unit. From a 5-hole code 32 types of commands can be produced. In certain machines two sequence codes are used simultaneously for one command enabling the same type of tape to produce 1,024 different types of command.

The signals are also fed to a summation amplifier and the signal proportional to density is fed to further limit circuits. The outputs of the limit circuits are connected so that density is the primary quantity, but if weight is low, rejection will also occur.

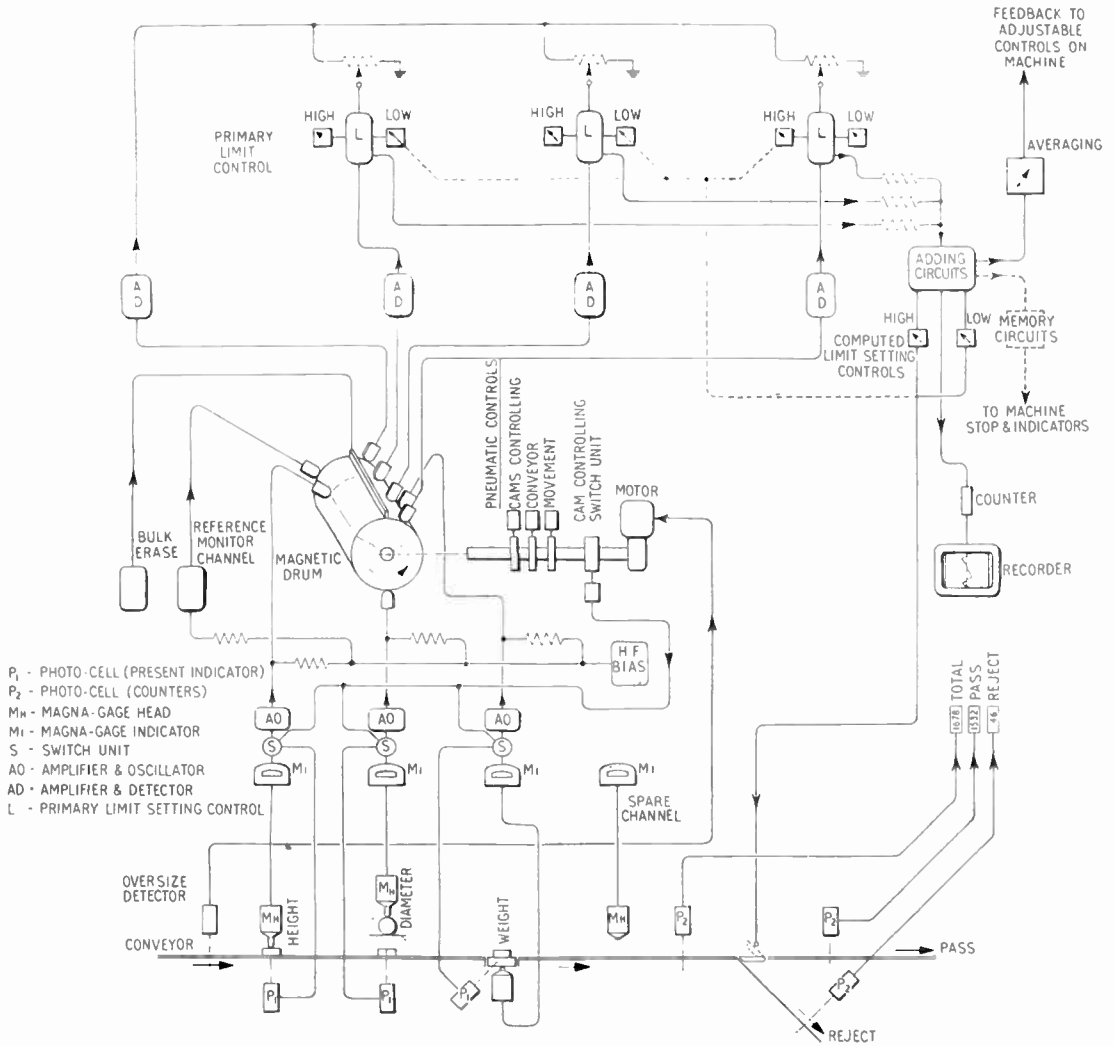


Fig. 5. Schematic circuit diagram of a monitor gauging system for the determination of density. The system can operate processes occurring at 200 per minute.

**7. The Future Trends in Automatic Inspection**

Recent work in the field of ergonomics, notably by Dr. R. M. Belbin, has established conclusively that a human inspector is incapable of absolute judgment and can only sustain comparative observations for a short period before fatigue and hypnosis lowers efficiency very seriously.

Now that this is recognized it can be said that instrumental methods of inspection should be adopted wherever possible, and it is very

desirable that the process be automatic and not dependent on the attention of an observer, except in an emergency. This can be described as "management by exception."

If this automatic inspection is used as a means of controlling the process, a more closely uniform product will result.

The system for automatic inspection shown in Fig. 5 is designed entirely with junction transistors instead of thermionic valves. This represents an important trend in equipment of

this sort, as these transistors are entirely suitable in their characteristics for such relatively low-frequency signals.

In use such transistors have already proved more reliable than valves, over a period of several years, and they now have a sufficient "pedigree" to be adopted with confidence. The all-transistor counter-batcher shown in Fig. 6 (a) is unitized into a number of sub-sections (Fig.

6 (b)) to facilitate servicing by non-technical staff, and for convenience in assembling it for different numbers of decades of count.

For the future it is probable that such equipment will be designed as plug-in bricks for ease of servicing, and that encapsulated construction and a high degree of miniaturization will be achieved, as in the photo-head amplifier of Fig. 7.

Fig. 6 (a) (left). All-transistor counter-batcher designed for industrial use with facilities for servicing by non-technical staff.

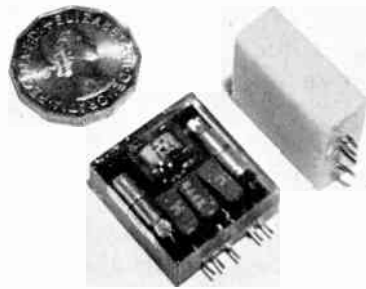
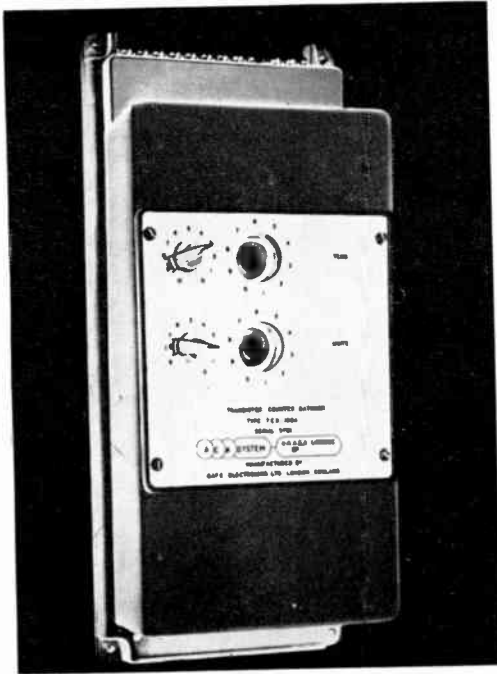


Fig. 7 (above). Typical encapsulated construction of a plug-in unit. This high degree of miniaturization is sometimes used in photocell amplifiers and other sensing elements placed in inaccessible positions inside machines.

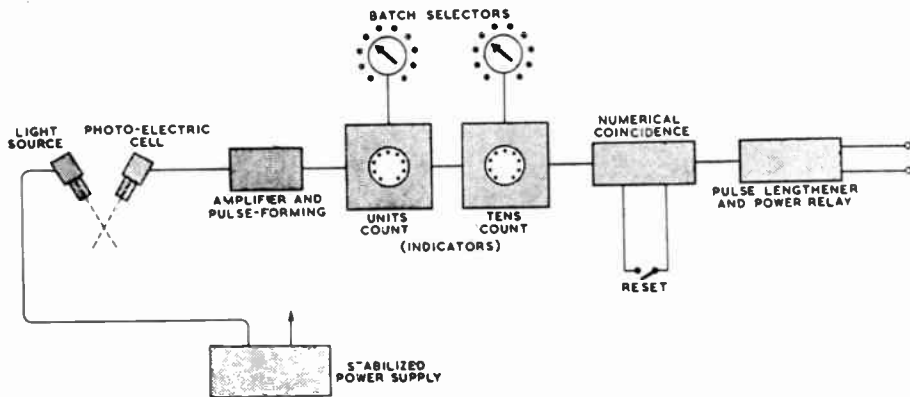


Fig. 6 (b). Skeleton diagram of the all-transistor counter-batcher shown in Fig. 6 (a)

# 31st ANNUAL REPORT OF THE COUNCIL OF THE INSTITUTION

*The Thirty-Second Annual General Meeting (the twenty-fourth since Incorporation) will be held on Wednesday, November 27th, 1957, at 6 p.m., at the London School of Hygiene and Tropical Medicine, Keppel Street, Gower Street, London, W.C.1.*

The Council has pleasure in submitting the following Report which covers the activities of the Institution for the twelve months ended 31st March, 1957. The Accounts and Balance Sheet made up to the 31st March, 1957, are appended, together with the Auditor's report.

Detailed reports of the various standing Committees will be published in the November 1957 *Journal*.

In December, 1956, proposals were laid before the membership for some revision of the Articles of Association. Apart from alteration to the rates of subscriptions, as recommended in the last Annual Report, the proposals mainly concerned alterations to the regulations for admission to membership, and to some of the Articles governing Council procedure.

These changes were approved at an Extraordinary General Meeting of the Institution held on January 31st, 1957. One particular benefit has been to enable Council members to give more time to the work of Standing Committees.

As the office of Chairman of Council no longer exists, certificates of membership are now signed by the President, one Vice-President, and the General Secretary. It was necessary to make changes in the layout of the membership certificate, and a new certificate has been designed which now incorporates the Institution's Armorial Bearings.

**The Fourth Post-war Convention.**—Outstanding in the year's work were arrangements for the 1957 Convention, and the Council wishes to place on record its great appreciation of the work of the Convention Committee which comprised:—

L. H. Bedford, C.B.E., M.A. (Chairman)  
A. D. Booth, D.Sc., Ph.D.  
J. E. Rhys-Jones, M.B.E.  
Denis Taylor, Ph.D., M.Sc.  
Professor D. G. Tucker, D.Sc.  
E. E. Webster  
Professor E. E. Zepler, Ph.D.

Plans were also made for advance and separate publication of the Convention programme. Publishing arrangements enabled a copy to be sent free of charge to every corporate member of the Institution.

By the end of the year under review (March, 1957) it was obvious that the theme agreed by Council—*Electronics in Automation*—met with the approval of the membership, and that the Convention would be the largest ever held by the Institution.

The success of all the Conventions arranged by the Institution in the past has again raised the question of holding symposiums concurrently with specialized exhibitions organized by Industry and Trade Associations. Members will recall that during the 1951 Radio Show the Institution held a symposium on Audio Frequency Engineering. There are, however, a number of factors which prove the undesirability of holding symposiums simultaneously with what are, in the main, trade exhibitions. One difficulty is the increasing number of Fairs or Shows with specialized radio and electronic interest.

Save in exceptional circumstances the Council believes that special meetings of the Institution, such as residential Conventions, are more convenient and more valuable to the membership as a whole, than the extra meetings which might be held during the period of trade shows.

**Standards and Codes of Practice.**—As indicated later in this Report, the Institution continues its support of the work of the British Standards Institution, and welcomes the opportunity of representation on the various Committees of the B.S.I. Before a standard is drafted, however, much initial work is done, not only by Institutions, but by trade bodies, and the constituent associations of the Radio Industry Council have done a great deal of such work in recent years.



Much benefit could probably be derived from a wider discussion of proposed recommendations, both in regard to new standards and codes of practice. The Council is, therefore, examining ways in which the Institution might collaborate with all who are concerned in initiating new standards and codes of practice.

**External Relationship.**—Under this heading regular reports have been made in pursuance of the Council's declared policy of wishing to develop collaboration with other Institutions in all matters affecting the welfare of the professional engineer.

The general subject of collaboration between professional engineering bodies was, in fact, discussed at a meeting of the Parliamentary and Scientific Committee in the Spring of 1956. That Committee has been concerned to secure discussions with the Minister of Labour, whose main impact on the work of professional bodies is in the compilation and administration of the Technical and Scientific Register.

As reported to the membership in 1954, the Register includes radio and electronics only under the general heading *Electrical Engineering*. This cannot wholly be a matter of convenience in classification since the Ministry of Labour itself recognizes the separate entity of the radio industry in its *Choice of Careers* pamphlets, e.g. in "The Royal Air Force."

One of the results of the present system of classification is that the Minister of Labour is deprived of the advice of the Institution as a result of having an Electrical Engineering Advisory Committee which includes no representation of the Institution and which has advised the Minister against such representation.

Reluctance to amend the classification of the Register is incomprehensible in the face of the increasing importance of radio-electronics. The industry is not only vitally essential to the armed services, but is a necessary concomitant to most other industries in providing equipment which helps automation and thereby assists in the economics of production. Radio and electronic engineering cannot, therefore, any longer be regarded as a mere entertainment industry subjected to variation because of changing economic conditions, but as a basic industry employing a considerable proportion.

and needing more of the world's engineers and scientific workers.

Indeed, the size of the industry and its diversified products make it rank as one of the most important in Great Britain, U.S.A. and U.S.S.R. In the absence of official willingness to give ranking figures, it is not possible accurately to appraise its place in industrial life; probably it is the fifth or sixth largest industry in Great Britain, with a total employment figure exceeding the 100,000 mentioned by a Past President of the Institution.\* This would indicate that the industry, broadcasting, and other services absorb about 20,000 engineers of professional standing.†

Figures published by the Radio Industry Council on the export of capital equipment and components also provide testimony to the economic, as well as the strategic importance of the industry.

The status of the radio and electronics engineer and of the industry is well exemplified by the life of its Institutions; the American I.R.E. has been in existence some 44 years, the Brit.I.R.E. for 32 years, and many other countries also now have their own Institutions devoted to the science of radio and electronics. Throughout the world, but particularly in Great Britain and America, there are also very many commercial publications catering for the professional radio and electronics engineer.

There is, moreover, the need to encourage the recruitment of young engineers, a fact particularly appreciated by the Ministry of Education which is providing more facilities and schemes of training with special application to the needs of radio and electronics.

Whilst it is appreciated that the Technical and Scientific Register may not now be credited with the same importance as in time of war, it is difficult to understand why, in the face of present evidence, there should not be some revision of the compilation of the Register. It is on the principle of recognizing the particular

\* W. E. Miller, *J.Brit.I.R.E.*, 13, page 6, January 1953.

† The American Institute of Radio Engineers estimates that 100,000 radio and electronics engineers are employed directly by the American radio industry, but that number falls very short of actual requirements.

place of the radio and electronics engineer that the Council continues its representations to the Minister.

**Appointments Register.**—In the light of the foregoing it is of particular interest to record that the Institution's Appointments Service is being increasingly used by manufacturers and all who wish to engage qualified radio and electronics engineers. All over the world, there is a shortage of such engineers and in general the Institution has been able to provide individual members seeking new engagements with introductions to a fairly wide selection of prospective appointments.

**Membership outside Great Britain.**—The Committee wishes to record appreciation of the help given by many members overseas, particularly to the Membership Committee.

It is, perhaps, too early to comment on the results of the establishment of an Institution office in Bangalore, details of which were given in the last Annual Report. The members who have assisted in the operation of that office have, however, been of considerable help in dealing with local enquiries and in ensuring that entrance to the examination is confined to candidates who have undertaken adequate preparation.

In Canada there has been an increase in enquiries regarding membership; members who have settled in Canada are naturally anxious to enjoy sectional activities and the Council is prepared to take the necessary steps towards forming official Section(s) in Canada when the membership reaches sufficient strength. The establishment of Sections will be dependent upon some measure of co-operation with existing Canadian associations; in the interests of members resident and contemplating residence in Canada, the Council continues negotiations with the Association of Professional Engineers of Ontario.

Particular mention must be made of the small but enthusiastic membership in New Zealand; it can hardly be expected that membership in that country will grow very large, but it is encouraging to note that the members in New Zealand are serving their own interests as well as those of the Institution by organizing regular meetings.

**General Administration.**—A joint meeting with the Finance Committee resulted in some discussion as to whether, in the light of revised postal charges and the increasing cost of operating centres overseas, it will be possible to continue the present practice of giving members resident outside Great Britain the concession of paying a slightly lower subscription. The Committee still has this matter under consideration.

Views were exchanged with another professional and Chartered body on the possibility of a case being presented to the Chancellor of the Exchequer, possibly through the Parliamentary and Scientific Committee, to permit of Institutions being able to claim tax rebate on the annual subscriptions paid by members to their professional body. The idea is not new, and there are several reasons which can be advanced against such a proposal. In general, however, professional bodies are having to reconsider such proposals in the light of the *decrease in value* of subscriptions, and there is a feeling that where subscriptions are paid by the individual member, some measure of tax rebate would be helpful.

**Graduateship Examination.**—May, 1956, was the last time that the Institution held its Graduateship Examination under the syllabus which broadly had been in force since 1943.

Naturally, very many candidates were anxious to take this last opportunity of being examined under the old syllabus and 722 entries were accepted—the highest figure ever recorded by the Institution.

The new syllabus, with its associated change in entry requirements, started with the November, 1956, examination, and it was also to be expected that there would be fewer entries; the figure of 467 entries finally approved was, however, higher than originally anticipated. Over the year, therefore, the Institution had 1,189 entries for its examinations.

**Examination results.**—In the last Annual Report a table was given showing the entries and results in the Graduateship Examination of a period of five years. This showed that only 10 per cent. of candidates entering succeeded in passing in all subjects.

Possibly increased efforts were made by candidates entering for the May, 1956 examination, but the results still cannot be regarded as satisfactory, since only 231 candidates did, in fact, obtain pass marks for the subjects they attempted. Similarly, on a subject for subject basis, only 118 of the November candidates justified their entries by obtaining pass marks in the subjects written.

**Prize Winners.**—Council awarded Examination Prizes to the following candidates who were successful in the 1956 examinations:—

President's Prize ..... E. Senior (Graduate)  
S. R. Walker Prize ..... L. Williams (Graduate)  
Audio Frequency Engineering Prize .....  
J. D. Smith (Associate Member)  
Mountbatten Medal ... J. H. Masters (Graduate)

**Syllabus and regulations.**—Under the new examination scheme candidates are no longer required to succeed in five parts, but as the regulations show, the examination has now been divided into two sections—A and B—each comprising three subjects with an alternative choice of specialist subject in Section B.

New entry regulations have also been approved by the Council. After 1958 candidates will be required to take all parts of each section *at one sitting* and no entry will be permitted for Section B until the candidate has satisfied all the requirements of Section A. Secondly, evidence of course work or suitable industrial training will be insisted upon before an examination entry is accepted. The introduction of this last regulation is to encourage more laboratory work in technical colleges or, alternatively, more industrial experience such as would be obtained by an apprenticeship or trainee course.

The Council attaches much importance to a satisfactory standard of knowledge in both Physics and Mathematics. The latter subject is, of course, compulsory in Section A, and the Council has under consideration a proposal to divide the syllabus into two pages to provide for the inclusion of Mathematics in both Sections A and B.

Whilst it is one of the functions of the Institution to encourage education and training, it must be recognized that the cost of holding examinations is by no means covered by examination fees. Stricter entry regulations will,

therefore, reduce the considerable work involved by limiting the number of candidates who do not appear for the examinations for which they have entered, or who withdraw after only attempting part of the examination. Revised entry conditions will also help candidates in ensuring that they have undertaken adequate training before incurring the cost of entry.

Before the close of the year over 500 candidates had submitted entries for the May, 1957 examination; thus it seems likely that the Institution will record for the seventh successive year over 1,000 entries for its Graduateship Examination, notwithstanding the material changes to the syllabus and scheme.

**Exempting Qualifications.**—For the present, the exempting qualifications detailed in the 23rd edition of the examination regulations remain unchanged. It will be recalled that in past Annual Reports it has been suggested that there may be fewer candidates wishing to write the Institution's examination because of increased facilities for obtaining technical training up to university level, or such other equivalent qualifications as may be recognized for exemption. As this report shows, however, interest in qualifying by means of the Institution's Graduateship examination has increased rather than diminished.

There has nevertheless been an increase in the number of applications for exemption submitted by candidates holding Higher National Certificates with suitable endorsements. This fact, coupled with more general appreciation of the standards required by the Institution, has led to an improvement in the quality of applications for the whole or part exemption from the examination. Thus, during the year, 468 applications for exemption from the Graduateship Examination were considered by the Committee; 108 of those applicants possessed the necessary qualifications for exemption from the entire examination, and 327 were granted exemption from part of the examination.

**Theses.**—In addition to the normal method of gaining exemption from the Graduateship Examination, more enquiries are being made as to the possibility of securing exemption by the submission of a thesis. Too few candidates

realize, however, that to be acceptable a thesis must describe original work and embody the results of research *personally undertaken* by the candidate.

Five candidates qualified for exemption in this way during the year under review.

**Approval of Courses.**—In many respects, the Institution is now finding reward for its efforts over the last thirty years in pressing for improvement in the scope and standard of courses designed for the training of the young radio and electronic engineer. An increasing number of colleges now include Radio or Electronics or Telecommunications in the A1 and A2 years, with appropriate endorsement subjects in order to satisfy the Institution's requirements.

Correspondingly, more applications are being made to the Institution from technical colleges for the approval of their courses. Details of exemptions granted in respect of approved courses are given in the Examination Regulations.

Before approval is given to any course, a Committee appointed by the Council visits the College concerned. Such visits also serve the useful purpose of enabling the views of Council to be exchanged with the staff of the technical college departments concerned, and are particularly valuable where the opinion of the Institution has been sought on the suitability of new courses, particularly Sandwich Courses.

**Extension of Training Schemes.**—It is now twelve years since the Institution first lodged proposals with the Ministry of Education on the extension of the National Certificate Scheme so as to give better opportunity to specialize in radio and electronics.

The proposals put forward by the Institution were discussed at a meeting convened by the Ministry of Education\* and attended by representatives of the Institution, the Institution of Electrical Engineers, and the Institute of Physics. It was then felt that the syllabus proposed by the Institution was too comprehensive to be taught thoroughly in the available time.

\* 11th September, 1945. See 20th Annual Report, *J.Brit.I.R.E.*, 6, page 131, August 1946.

There has nevertheless been a marked change in courses designed to cover the Higher National Certificate and an increasing number of colleges are now providing a more flexible syllabus permitting more time for electronics subjects in the A1 and A2 years.

Arrangements made by individual colleges are, of course, very much governed by the requirements of industry, but overall there is now in Great Britain far wider and better opportunity for specialization in the field represented by the Institution.

**Membership.**—Increase in Institution membership is still partly dependent upon success in the Graduateship Examination; as indicated in the last Annual Report, however, it is reasonable to expect that increased facilities for taking University Degrees in radio subjects, or otherwise qualifying by means of approved Higher National Certificate examinations, will materially influence increased membership of the Institution. This has been the experience of older Institutions, although it is doubtful whether even 50 per cent. of those who obtain Higher National Certificates and Diplomas (estimated to be about 8,000 a year) do, in fact, proceed to obtain professional status through their appropriate Institution.

Graduateship of the Institution, whether by success in the Brit.I.R.E. examination, or by exemption therefrom because of a degree or other suitable alternative, is evidence of a high standard of theoretical knowledge in radio and electronic engineering. Election to Associate Membership or Membership is dependent on subsequent practical and administrative ability—factors which are the principal concern of the Membership Committee. There is, therefore, the problem of securing the interest of the younger man in his professional body, and so far as our own Institution is concerned there is

Table 1

Membership Growth over the Last Five Years				
<i>As at</i>		<i>Total</i>		<i>Annual</i>
<i>March 31st</i>		<i>Membership</i>		<i>Increase</i>
1953	...	4,383	...	365
1954	..	4,750	...	367
1955	...	5,082	...	332
1956	...	5,392	...	310
1957	...	5,568	...	176

still the problem of the man of 40 or more who may lack the academic qualifications which are required.

In the latter connection, the Council has endeavoured to be sympathetic by admitting election to Associateship; thus, the new Graduate or Associate, whilst not immediately being eligible for election to corporate membership with its implied professional status, is nevertheless not debarred from the activities of his professional body and the opportunity for service.

Any reduction in standards either of theoretical knowledge or practical and administrative ability in order to augment membership is of course impossible. All the various factors affecting membership are constantly before the Council. Table 1 shows that membership of the Institution has continued to grow during the past year.

**Direct Elections.**—Although the total number of proposals received during the year was

less than in the preceding 12 months, there were more proposals for direct election to graduate and corporate membership. The decrease in the total number was mainly because of a drop of over 70 in the number of applications for Studentship registration—due to the insistence of success in the Common Preliminary Examination or the possession of a recognized equivalent qualification.

Table 2 summarizes elections and transfers and the losses sustained during the year.

**Membership Enquiries.**—During the year 1,617 enquiries were dealt with at the Institution's headquarters and the distribution of regulations and forms of application by the local secretaries has been encouraged. In addition the office in Bangalore is now printing forms and regulations locally for distribution in India. These arrangements lead to earlier contact between prospective members and local Sections and are therefore felt to be especially helpful.

**Table 2**  
**Statement of Elections and Transfers for the Year Ended March 31st, 1957**

	Total considered	Elections and transfers approved							
		Honorary Member	Member	Associate Member	Companion	Associate	Graduate	Student	Total
Direct elections	622	1	5	66	—	38	115	359	584
Proposals for Transfer	283	—	11	78	—	15	95	—	199
Proposals for reinstatement	17	—	—	1	—	2	4	8	15
<i>Totals</i>	922	1	16	145	—	55	214	367	798

**Losses During the Year**

Loss by resignation, removal and decease	—	10	24	1	32	17	339	423	
Transfer to other grades	—	—	11	—	21	47	120	199	
<i>Totals</i>	—	10	35	1	53	64	459	622	
<b>Net gain in membership</b>		1	6	110	—1	2	150	—92	176

**Publication of Papers.**—The decision to adopt a larger format for the *Journal* was implemented in January, 1957. As explained in the last Annual Report, the changes were made in order to improve the presentation of diagrams and mathematical formulae, as well as readability. The cover was also redesigned to show the Institution's Armorial Bearings.

Papers published during the year under review covered a wide range of subjects to meet the general and specific interest of almost every member. The *Journal* fulfills many useful functions in developing the work of the Institution. It has also become a publication to which other organizations interested in the development of the profession and industry turn for information, and plans for its expansion are mainly limited by financial consideration. Constantly increasing paper, printing and postal charges tend to limit expansion and can only be offset by increased income, mainly from subscriptions and advertising revenue.

**Award of Premiums.**—The Council has pleasure in announcing that the following authors have been awarded Premiums for papers published in the *Journal* during 1956:—

**Clerk Maxwell Premium:** K. D. Froome, Ph.D. "Microwave Determinations of the Velocity of Light." (Published September, 1956).

**Heinrich Hertz Premium:** A. G. Edwards, Ph.D. "The Effect of Atmospherics on Tuned Circuits." (January).

**Brabazon Premium:** K. E. Harris, B.Sc. "Some Problems of Secondary Surveillance Radar Systems." (July).

**Louis Sterling Premium:** A. van Weel, Dr.Tech.Sc. "Some Remarks on the Radio-frequency Phase and Amplitude Characteristics of Television Receivers." (May).

**Marconi Premium:** P. M. Honnell, Ph.D. "Prescribed-Function Vibration Generator." (April).

**Norman Partridge Memorial Award:** H. J. Leak (Member). "High Fidelity Loudspeakers: The Performance of Moving Coil and Electrostatic Transducers." (December).

**Circulation of the Journal.**—The circulation of the *Journal* steadily increased during the year and the figure certified by the Audit Bureau of Circulations for the period ended December, 1956, was 6,504—an increase of 400 over the 1955 figure.

Advertising revenue has continued to rise and the *Journal* for January, 1957, the largest monthly issue ever published, attracted a record number of pages of advertising.

**Acknowledgments.**—The Council wishes to thank the authorities of universities, colleges, etc., for facilities granted to the Institution for holding meetings. Thanks are also due to authors of papers, and to their firms.

**Income and Expenditure.**—The Revenue Account given with this Report does not, of course, reflect the increase in subscription rates which only became operative on the 1st April, 1957. The unanimous approval of members to these increases encouraged the hope that it would be possible for the Institution to build up a strong capital reserve out of which it would be possible to finance future expansion of the Institution's activities.

As the Income and Expenditure Account shows, the small excess of income may easily be absorbed by ever increasing costs, and the resolution taken at the last Extraordinary General Meeting was, in the light of events, most timely.

Similarly, improved advertising revenue has assisted in keeping the main printing costs of the *Journal* to about the same level as in the previous year. It has, however, meant some restriction of stocks and this has meant that the Institution is frequently unable to meet requests for past issues of the *Journal* and reports.

The Council again wishes to express appreciation of the support being given by members and industry to the Institution's Building Appeal. Many of these contributions are received under a deed of covenant and, as the Balance Sheet shows, donations are reserved, through investment, solely for the purpose of purchasing a building. The need for further accommodation increases every year, and the help given toward the acquisition of suitable premises is therefore especially appreciated.

**Balance Sheet.**—Some capital expenditure has been required during the year, and although there has been a slight increase in the Reserve Account, the Council still has the concern of carrying forward the accumulated capital expenditure which is shown under current liabilities. Reduction of this figure can only be achieved by securing a large excess of income in future years, or by donations. The Finance Committee has, therefore, launched a scheme to secure redemption of this accrued liability.

## GENERAL ACCOUNT

### INCOME AND EXPENDITURE ACCOUNT FOR THE YEAR ENDED 31ST MARCH, 1957

1956		£	s.	d.	£	s.	d.	1956		£	s.	d.
£								£				
	Administration Expenses:											
8,824	<i>Salaries and State Insurance</i> ...	9,282	9	3				15,377	Subscriptions including Arrears ...	15,898	14	4
378	<i>Pension Scheme</i> ...	435	2	0				189	Life Subscriptions ...	126	0	0
1,130	<i>Postage and Telephone</i> ...	1,170	12	10				4,034	Building Appeal Donations ...	2,289	4	3
860	<i>Printing and Stationery</i> ...	1,501	16	8				2,733	Examination and Exemption Fees ...	2,225	8	2
	<i>Travelling and Entertaining Expenses</i> ...	581	15	9				1,686	Entrance and Transfer Fees ...	1,265	3	1
545	<i>Delegates' Expenses</i> ...	421	1	5				4,843	Sale of Examination Papers and Journals ...	5,222	17	0
393	<i>Council and Committee Expenses</i> ...	501	10	3				232	Interest on Investments (Gross) ...	336	11	1
448	<i>Audit Fees</i> ...	115	10	0					Radio Trades Examination Board: Secretarial			
115	<i>Bank Interest and Charges</i> ...	652	16	0				1,058	Charges ...	1,333	6	8
530	<i>Legal Expenses</i> ...	246	17	8								
107	<i>Sundry Expenses</i> ...	14,909	11	10								
209												
	Institution Premises:											
1,517	<i>Rent, Rates and Insurance (Net)</i> ...	1,592	2	8								
423	<i>Lighting and Heating</i> ...	334	13	9								
598	<i>Office Expenses and Cleaning</i> ...	612	3	11								
404	<i>Repairs</i> ...	327	2	1	2,866	2	5					
	Institution's Journal and Reports:											
6,258	<i>Printing and Publishing less Advertising Receipts</i> ...	5,685	15	10								
734	<i>Postage</i> ...	968	10	8								
295	<i>Envelopes and Wrappers</i> ...	463	5	11	7,117	12	5					
	Examination Expenses:											
369	<i>Printing of Papers and Regulations Examiners' and Invigilators' Fees and Expenses</i> ...	554	11	0								
488	<i>Hire of Accommodation</i> ...	517	11	7								
122		117	14	0	1,189	16	7					
	Section Expenses:											
530	<i>Printing, Stationery and Postage</i> ...	458	5	1								
574	<i>Hire of Accommodation, etc.</i> ...	570	1	1								
456	<i>Travelling Expenses and Subsistence</i> ...	556	10	10	1,584	17	0					
50	Expenses for Coat of Arms ...				41	14	6					
320	Grants to other Institutions ...				193	8	0					
193	Premiums and Awards ...				76	1	6					
	Depreciation:											
281	<i>Office Furniture and Fittings</i> ...	357	14	11								
73	<i>Library</i> ...	88	7	10	446	2	9					
	Excess of Income over Expenditure carried to Reserve Account ...				271	17	7					
2,928					£28,697	4	7			£30,152	4	7
<u>£30,152</u>								<u>£30,152</u>				

**THE BRITISH INSTITUTION  
GENERAL**

**BALANCE SHEET**

1956 £		£	s.	d.	£	s.	d.
	<i>RESERVE ACCOUNT</i>						
	Excess of Income over Expenditure: Balance 1.4.1956 .. ..	204	10	7			
204	<i>Add</i> Surplus for the year .. .. .	271	17	7			
					476	8	2
	<i>CURRENT LIABILITIES</i>						
4,172	Sundry Creditors .. .. .	7,349	0	10			
2,601	Subscriptions and Examination Fees in Advance .. ..	2,268	1	10			
16,416	Bank Overdraft .. .. .	17,314	4	8			
					26,931	7	4

*NOTE:* (i) The Balance on Reserve Account, viz. £476 8s. 2d., is after credit has been taken for Building Appeal Donations and Interest on Investments.

{ G. A. MARRIOTT (*President*)  
*Signed* { G. A. TAYLOR (*Honorary Treasurer*)  
           { G. D. CLIFFORD (*General Secretary*)

£23,393

£27,407 15 6

**REPORT OF THE AUDITORS TO THE MEMBERS OF**

We have obtained all the information and explanations which to the best of our knowledge and belief were necessary for the purposes of our audit. In our opinion proper books of account have been kept by the Institution so far as appears from our examination of those books and proper Returns adequate for the purposes of our audit have been received from the Sections Overseas.

We have examined the above Balance Sheet and annexed Income and Expenditure Account which are in agree-

42 Bedford Avenue, London, W.C.1.

19th July, 1957.



**OF RADIO ENGINEERS  
ACCOUNT**

AS AT 31ST MARCH, 1957

1956 £		£ s. d.	£ s. d.
	<i>FIXED ASSETS</i>		
	Office Furniture and Fittings at cost .. .. .	5,954 15 9	
	Less Depreciation to date .. .. .	2,735 15 9	
2,529		3,219 0 0	
	The Louis Sterling Library at cost .. .. .	1,473 7 4	
	Less Depreciation to date .. .. .	688 7 4	
654		785 0 0	
	<i>INVESTMENTS AT COST</i>		
	£200 3% Savings Bonds 1960/70 .. .. .	200 0 0	
	£800 4% Consolidated Stock .. .. .	712 15 6	
	£400 British Transport 4% Guaranteed Stock 1972/77 .. .. .	383 16 10	
1,297	(Market Value 31st March, 1957, £1,143 0s. 0d.)	1,296 12 4	
	<i>Building Appeal :</i>		
	£1,400 4% Consolidated Stock .. .. .	1,219 3 0	
	£1,700 3½% War Loan .. .. .	1,274 7 3	
	£300 3% British Electricity Guaranteed Stock .. .. .	233 10 9	
	£2,000 British Transport 4% Guaranteed Stock 1972/77 .. .. .	1,921 0 11	
	(Market Value 31st March, 1957, £4,375 0s. 0d.)	4,648 1 11	
6,270	Halifax Building Society .. .. .	2,600 0 0	7,248 1 11
	<i>CURRENT ASSETS</i>		
	<i>General :</i>		
3,228	Stock of Stationery, Journals and Examination Papers at valuation .. .. .	3,855 13 5	
714	Income Tax Repayment Claim .. .. .	875 18 9	
2,037	Sundry Debtors and Subscriptions in Arrear—less Reserve .. .. .	2,900 14 6	
447	Sections—Balances at Bank and in Hand .. .. .	479 12 2	
4	Cash in Hand .. .. .	2 6 3	
	<i>Building Appeal :</i>		
6,213	Cash at Bank .. .. .	6,744 16 2	14,859 1 3
£23,393			£27,407 15 6

**THE BRITISH INSTITUTION OF RADIO ENGINEERS**

ment with the books of account. In our opinion and to the best of our information and according to the explanations given to us, the said accounts give the information required by the Companies Act, 1948, in the manner so required. The Balance Sheet gives a true and fair view of the state of the Institution's affairs as at 31st March, 1957, and the Income and Expenditure Account gives a true and fair view of the Excess of Income over Expenditure for the year ended on that date.

GLADSTONE, JENKINS & CO.,  
*Chartered Accountants, Auditors.*

## **Brit.I.R.E. BENEVOLENT FUND**

### **NOTICE OF ANNUAL GENERAL MEETING OF SUBSCRIBERS**

NOTICE IS HEREBY GIVEN that in accordance with the Rules the Annual General Meeting of Subscribers to the Institution's Benevolent Fund will be held on WEDNESDAY, NOVEMBER 27th, 1957, at the London School of Hygiene and Tropical Medicine, Keppel Street, Gower Street, London, W.C.1. The meeting will commence at 6.45 p.m. (immediately after the close of the Annual General Meeting of the Institution).

#### **AGENDA**

1. To confirm the Minutes of the Annual General Meeting of Subscribers held on 31st October, 1956. (Reported on page 590, Volume 16 of the *Journal*, November, 1956.)
2. To receive the Annual Report of the Trustees. (Published on pages 547-549 of this *Journal*.)
3. To receive the Income and Expenditure Account and Balance Sheet of the Benevolent Fund for the year ended 31st March, 1957. (Published on page 548.)
4. To elect the Trustees for the year 1957-58.

*Rules 5 and 6 state :—*

5. The Trustees of the Fund shall consist of not more than five and not less than three members of the Institution who have been elected at an Annual General Meeting of Subscribers to the Benevolent Fund.

6. The Trustees shall be elected at the Annual General Meeting by all members *who have subscribed to the Fund* during the preceding twelve months, ended March 31st in each year, and the Trustees shall hold office until their successors are appointed.

*The present Trustees, who offer themselves for re-election, are :—*

G. A. Marriott, B.A. (President).

Rear Admiral Sir Philip Clarke, K.B.E., C.B., D.S.O. (Past President).

A. H. Whiteley, M.B.E. (Companion).

G. A. Taylor (Member) (*Honorary Treasurer*).

The death of Mr. E. J. Emery has caused a vacancy for a fifth Trustee and Subscribers are invited to approve the election of Mr. A. A. Dyson, O.B.E. (Member).

5. To appoint Honorary Solicitors.

*The Trustees recommend the re-appointment of :—*

Messrs. Braund & Hill, 6 Gray's Inn Square, London, W.C.1.

6. To appoint the Honorary Accountant.

*The Trustees recommend the re-appointment of :—*

Mr. R. H. Jenkins, F.C.A., 42 Bedford Avenue, London, W.C.1.

7. Any other business.

*By Order of the Trustees,*

(Signed) G. D. CLIFFORD

(*Honorary Secretary*)

*(The Rules governing the operation of the Benevolent Fund are published in the List of Members.)*

# Brit.I.R.E. BENEVOLENT FUND

## Annual Report of the Trustees for the year 1956-57

The Trustees of the Benevolent Fund submit the following report on the working of the Fund during the year ended 31st March, 1957. Whilst total income was a little less than in 1956, the Trustees have been able to meet all applications for assistance and again express thanks to all those who have supported the Fund.

As shown in the Revenue Account, there has been a decrease in subscriptions, partly due to the fact that only 874 members contributed—25 fewer than in 1955. The number of subscribers represents a very low percentage of the total membership of the Institution, and every member is urged to give some financial support, however small, to this important Fund.

The majority of subscribers to the Fund give annual donations; the support of such members makes it possible to meet claims and commitments, such as donations to various schools, without realizing some of the Fund's investments.

As the Institution grows, it is reasonable to expect that there will be more claims made upon the Fund; the Trustees hope, therefore, that very many more members will add some amount to their annual subscriptions for the credit of the Benevolent Fund.

After publication of the last Annual Report, many members requested information on deeds of covenant and have since undertaken to make an annual donation in this way. For the benefit of new members of the Institution, therefore, the table showing the tax recoverable on contributions made by deed of covenant is reproduced below. The Trustees wish to point out that the

forms of covenant are obtainable from the offices of the Institution; all that is necessary is for the subscriber to insert the annual amount which he wishes to donate and the subscriber is *not* involved in any legal or additional costs.

**Non-member Donations.**—For some years Electric and Musical Industries Limited, and the Radio Industries Clubs of London and Manchester have made generous donations to the Fund. The Trustees wish to record their appreciation of this continuing support.

**General Policy.**—Meetings of the subscribers to the Fund over the past 17 years have always endorsed the recommendation that the education of fatherless or orphaned children should receive prior consideration in administering the Fund.

In this connection, the Trustees recall with appreciation the help given by the Governors of Reed's School in the early days of the Benevolent Fund when it was not possible to make very large donations to the School.

Four children of deceased members have been educated at Reed's Schools. It was deeply regretted that sufficient voluntary support could not be obtained for the maintenance of the high

Amount Covenanted and to be paid annually	Tax Reclaimed Annually <small>(Based on Tax at 8s. 6d.)</small>	Gross amount for one year	Estimated Gross amount over 7 years
1 0 0	14 9	1 14 9	12 3 3
1 1 0	15 6	1 16 6	12 15 6
1 10 0	1 2 2	2 12 2	18 5 2
1 11 6	1 3 3	2 14 9	19 3 3
2 0 0	1 9 7	3 9 7	24 7 1
2 2 0	1 11 0	3 13 0	25 11 0
3 0 0	2 4 4	5 4 4	36 10 4
3 3 0	2 6 7	5 9 7	38 7 1
4 4 0	3 2 1	7 6 1	51 2 7
5 5 0	3 17 7	9 2 7	63 18 1

*The subscription actually paid by the subscriber is the amount stated in the first column and this is the net sum which should be inserted in the deed of covenant.*

**THE BRITISH INSTITUTION OF RADIO ENGINEERS  
BENEVOLENT FUND**

**INCOME AND EXPENDITURE ACCOUNT FOR THE YEAR ENDED 31ST MARCH, 1957**

1956	1956
£	£
559 Grants ... ..	877 Subscriptions and Donations ... ..
200 Purchase of Bursaries at Reed's School ... ..	269 Interest Received (Gross) ... ..
5 Postage and Stationery ... ..	
16 Sundry Expenses ... ..	
366 Balance being Surplus for the year carried to Reserve Account ... ..	
<b>£1,146</b>	<b>£1,146</b>
£ s. d. 438 0 10 150 0 0 5 9 7 — — — 497 1 7 <u>£1,090 12 0</u>	£ s. d. 794 2 0 296 10 0 <u>£1,090 12 0</u>

**BALANCE SHEET AS AT 31ST MARCH, 1957**

1956	1956
£	£
<i>RESERVE ACCOUNT</i>	<i>FIXED ASSETS</i>
6,576 Balance as at 1st April, 1956 ... ..	<i>Investments at Cost:—</i>
497 Add Surplus for Year ... ..	£200 3% Savings Bonds 1960/70 ...
7,073 5 10	£200 3% Savings Bonds 1965/75 ...
<i>CURRENT LIABILITIES</i>	£1,200 3½% War Loan ...
114 Amount due to The British Institution of Radio Engineers General Fund ...	£200 British Electricity 3% Guaranteed Stock 1968/73 ...
38 1 2	£1,300 British Transport 4% Guaranteed Stock 1972/77 ...
<i>For Trustees:—</i>	£4,000 4% Consolidated Stock ...
( G. A. MARRIOTT (Chairman)	Jays & Campbells (Holdings) Limited—
Signed G. A. TAYLOR (Honorary Treasurer)	500 5½% Cumulative Preference Shares of £1 each ...
( G. D. CLIFFORD (Honorary Secretary)	Associated Newspapers Limited—
	200 Deferred Shares of 5s. each ...
	(Market value as at 31st March, 1956, £6,175 10 0)
<b>£6,690</b>	6,430
£ s. d. 6,576 4 3 497 1 7 <u>£7,111 7 0</u>	<i>CURRENT ASSETS</i>
	117 Income Tax Repayments Claim ...
	143 Cash at Bank ... ..
	309 6 2
	<b>£7,111 7 0</b>

I have audited the above written Balance Sheet dated 31st March, 1957, in respect of the Benevolent Fund.

I have received all the information and explanations I have required and in my opinion the Balance Sheet represents the true and accurate state of the Benevolent Fund.

42 Bedford Avenue, London, W.C.1.  
19th July, 1957

R. H. JENKINS, Chartered Accountant,  
Honorary Auditor.

standards at both the Boys' and the Girls' Schools, and the Girls' School at Dogmersfield Park had to be closed. The Governors have, however, been able to concentrate on much needed improvements to the Boys' School at Cobham, including a new Science Block. Members may be especially interested to note that boys wishing to specialize in Science may now continue at Reed's School after taking the G.C.E. examination at "O" level, and have facilities for qualifying for University entrance on the Science side as well as the Arts.

The existence of a Science Sixth in the School is thus an incentive to boys to stay on at school to study science, as well as an encouragement to the science staff who will no longer be losing their most promising pupils at a time when they would most like them to remain.

The Trustees wish to record that the President personally, as well as the Institution, donated equipment to the School for the new Science Block.

The Trustees are pleased to be associated with many professional benevolent funds, and the Royal Air Force Benevolent Fund, in helping to maintain the work of Reed's School.

Consequent upon the closing of Reed's Girls' School, the Trustees arranged for two girls of a deceased member to be placed in the Royal Wolverhampton School. More recently, the Governors of the School were helpful in the case of the son of a member of the Institution whose long illness has prevented him from returning to normal work. In this case, the Trustees believed that the best help which could be given to the member, who still requires considerable medical treatment, was to give the child the opportunity of a boarding school education. This arrangement has also eased very difficult accommodation problems.

The Royal Wolverhampton School provides education up to the G.C.E. examination at "O" level and has facilities for boarding some 360 pupils in the Junior and Senior Schools. Like Reed's School, it is entirely dependent upon voluntary contributions, and the Institution's Benevolent Fund has given support to the School for the past three years.

Finally, the Trustees have also made grants to the Royal Wanstead School, which caters for the younger child.

**Other Grants.**—The Trustees have also dealt with other applications, besides maintaining the help given in the two particular cases referred to in previous reports.

The case of the widow left with three young children has now been the concern of the Trustees for the past eight years. The eldest child is now in employment but the other two children are still receiving education at the Royal Wolverhampton School. The Trustees believe that with the exception of help in respect of the two younger children, this case will no longer require intensive assistance after the current year in view of the full-time employment of the mother and eldest child.

Another case which has been the concern of the Trustees for the last four years is that of the member who will unfortunately always require medical attention. The member's son is now receiving education at the Royal Wolverhampton School. Regular grants have been made to the member to supplement his very small income, and in this connection the Trustees are grateful to the Electrical Industries Benevolent Fund who have also assisted by making grants.

Towards the close of the year applications were being considered, and some help given, to two other members, one of whom would be prevented by illness from returning to his normal occupation for some months. The other case concerned a disabled member of the Services, for whom the Royal Air Force Benevolent Fund had already undertaken some responsibility, but the member is in difficulty with the maintenance and education of his three young children. The Trustees will comment further on these two cases in their next report.

Having thus met all reasonable claims upon the Fund, the Trustees are pleased to report that it was possible to carry a small surplus to the Reserve Asset of the Fund, slightly in excess of last year's figure.

**Acknowledgments.**—An annual report of this character would not be complete without thanking two people whose advice is so often sought and so willingly given, namely the Fund's Solicitor and Auditor. Mr. Charles Hill and Mr. Hugh Jenkins have given their services voluntarily and the Trustees are much indebted to them for their help.

# INSTITUTION NOTICES

## THE IMMEDIATE PAST PRESIDENT

Rear Admiral Sir Philip Clarke, K.B.E., C.B., D.S.O., and Lady Clarke were the guests of honour at a dinner given by members of Council and Committees at the Savoy Hotel on October 11.

The President, Mr. George A. Marriott, proposed the toast of his predecessor, and particularly referred to the Institution's strong association with the Royal Navy. Mr. Marriott said that membership in the Royal Navy, as well as the whole of the Institution, had taken particular pride in the fact that Admiral Clarke had achieved the office of Director of the Naval Electrical Department after serving in the office of Director of Manning where he had been particularly concerned with the Royal Navy's utilization of electronics.

Reviewing Admiral Clarke's work for the Institution, the President particularly mentioned the importance of the decision taken to increase still further the examination requirements. Inevitably resulting in some reduction in new membership, the policy has been justified by making even more worth while the qualification of membership. On behalf of the Institution, Mr. Marriott presented to Admiral Clarke a silver cigarette box bearing the Institution's Coat of Arms, and after referring to the work of the Institution's Benevolent Fund, presented Lady Clarke with dressing-table lamps.

In his response, Admiral Clarke reiterated views he had expressed at Council and Committee meetings on the need to stimulate recruitment of properly trained young men to the profession of radio and electronic engineering. He expressed the belief that his contribution to the development of the Institution was best served by looking to the future.

The toast of the guests was most humorously and informatively proposed by Mr. L. H. Bedford, a Past President of the Institution, who associated with the words "Guests and Visitors" the problems of launching a satellite!

For the first time, the Council had the opportunity of entertaining Mr. Eric K. Cole, who responded to the toast of the guests and who expressed the belief that the Institution was indispensable to the industry. Mr. Cole particu-

larly mentioned the Institution's recent Convention on "Electronics in Automation" and gave other examples of the importance of electronics in industry, as well as such professions as medicine, and the increasing importance of electronics in conserving manpower—especially in the Armed Forces. Mr. Cole concluded by emphasizing that the future of the nation, and perhaps of the world, depended upon scientific development and, in the field of electronics, "... the Brit.I.R.E. had a most important part to play."

## Further Courses for the Diploma in Technology

The National Council of Technological Awards has announced the recognition of three further courses which include radio and electronics. All are existing sandwich courses: Electrical Engineering at Loughborough College of Technology, and the courses in Applied Physics and Electrical Engineering at the Northampton College of Advanced Technology, London.

## Forthcoming International Publications

The journals *Optica Acta* and *Acta Electronica* have undertaken the joint publication of the full proceedings (invited lectures and communications, mostly in English), of the International Symposium on the Physical Problems of Colour Television, held in Paris last July. The advance subscription is £4 and orders should be sent to *Optica Acta*, 3 boulevard Pasteur, Paris 15—or *Acta Electronica*, 23 rue du Retrait, Paris 20—France.

The central records of the International Geophysical Year are being published for the Special Committee of the I.G.Y. under the title of *Annals of the International Geophysical Year*. The first volume (Volume III) is an instruction manual entitled "The Ionosphere" and it consists of four parts—ionospheric vertical soundings, the measurement of ionospheric absorption, the measurement of ionospheric drifts, and miscellaneous radio measurements. It is anticipated that four to six volumes of the *Annals*, of approximately 400 pages each, will be published during 1957 and 1958. The subscription for each volume is £6 and orders should be placed with the publishers, Pergamon Press, 4 Fitzroy Square, London, W.1.

# SOME ASPECTS OF PROCESS CONTROL INSTRUMENTATION\*

by

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## SUMMARY

This paper discusses some of the instrumentation aspects of process control. It considers the electrical methods of measuring temperature and includes a description of the low-level d.c. amplifiers used in connection with thermo-couple measurements based on the contact modulator, galvanometer-photocell and galvanometer-oscillator methods.

The application of vane controlled r.f. oscillators to direct-current telemetering systems and electrical force balance measurements are also described and discussed.

## 1. Introduction

Although industrial electronic techniques are relatively new, their influence in the automatic process control field hitherto largely dominated by pneumatic and hydraulic systems is already considerable and steadily increasing.

If the rate of acceptance of electronic equipment in the process industries seems slow it is as well to remember that for precision measurement and control, electronics must compete with several other forms of instrumentation of mature experience and with a long record of dependability. In contrast to radio, television and radar where electronic techniques offer the only solution, electronic systems of process control are used only where they offer advantages in improved performance, flexibility, cost and reliability over competing systems.

In general, the successful application of electronics to industrial measurement and control requires special techniques not often encountered in other branches of electronics, in particular, the inherent advantages of mechanical, pneumatic, hydraulic and electronic techniques are often combined in hybrid systems.

This paper includes a description of some

new electronic circuits<sup>1</sup> which were developed to meet a need which frequently arises in direct-current telemetering and electrical force balance measurement for a simple circuit capable of providing a bi-directional direct current output of the order of +15 mA–0–15 mA. The principle employed in these circuits enables such an output to be obtained from an extremely simple and economic circuit arrangement.

The basic method of operation was first developed to meet the need for a phase-sensitive rectifier capable of providing a bi-directional current output, and the method has since been extended to the design of vane-controlled oscillators and transducers.

Needs of this sort continually arise in process control for new instruments and methods, some of which could be met by electronic techniques. A necessary prerequisite however to the successful application of electronics in process control is a thorough understanding of the process industries' needs and problems by the electronics engineer.

Before describing the new circuits mentioned above some aspects of electronics in process control illustrative of present-day techniques will therefore be considered.

Defining automatic process control as the measurement and control of process variables, it is at once obvious that measurement is the most important part of any control system, since

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it is axiomatic that before a process variable can be controlled a change in that variable must first be measured. It is in measurement moreover that the inherent properties of electronics are most immediately applicable, and where electronics has found its greatest application, notably in temperature measurement and electro-chemical instruments and more recently in pressure, fluid flow and liquid level measurements.

In order to examine typical design requirements for the electronic sections of industrial measuring instruments it is useful to reconsider briefly the prominent characteristics of the familiar resistance thermometers and thermocouples.

## 2. Temperature-sensitive Elements

Temperature has a wider effect than that of any other physical variable: few physical properties of materials are unaffected by temperature. Unlike pressure and flow which can be measured directly by dead-weight reference and volumetric methods, temperature must be measured indirectly by utilizing one of the physical properties affected by it. For electronic instrumentation there are several temperature-sensitive elements which produce changes which can be measured electrically, and of these the thermocouple and the resistance thermometer are the most extensively used in the range  $-200$  to  $+1400^{\circ}\text{C}$  and  $-200$  to  $+500^{\circ}\text{C}$  respectively.

Resistance thermometers of platinum or nickel have the characteristics of high accuracy and stability, and large electrical outputs can be obtained without serious self heating. They can be connected in bridge networks of purely passive elements. Typically resistance thermometers operate at maximum currents of 10–20 mA, have resistance values of 100–130 ohms at  $0^{\circ}\text{C}$  and produce resistance changes of about  $\frac{1}{2}$  ohm per  $^{\circ}\text{C}$ . A bridge out-of-balance voltage of the order of  $2.5\text{ mV}/^{\circ}\text{C}$  is therefore readily obtainable. Where temperature must be held within narrow limits with a high degree of stability the resistance thermometer is the obvious choice. Resistance temperature characteristics for typical industrial resistance thermometers are shown in Fig. 1.

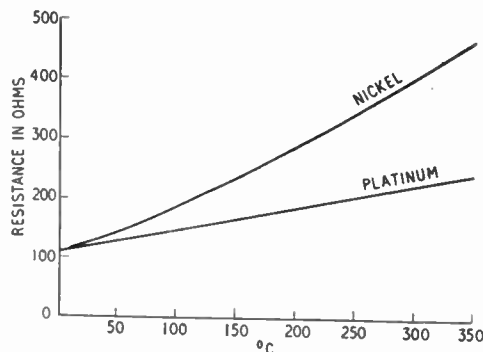


Fig. 1. Resistance thermometer characteristics.

Thermocouples formed from junctions of dissimilar metals such as iron-constantan, chromel-alumel and platinum-platinum rhodium have the advantages of low cost, fast response, wide range of application, and are the most widely used sensitive elements for temperature measurement. The combinations of metals commonly used, yield substantially linear relationships between e.m.f. and the temperature difference between the two effective junctions necessarily formed in a closed circuit of dissimilar metals. High accuracy is attainable providing temperature changes at the reference junction are compensated and current is not drawn from the thermocouple. Characteristics of thermocouples are low voltage output and low resistance. A typical iron-constantan thermocouple installation with 100 yards of compensating cable would give an output of  $5.3\text{ mV}/100^{\circ}\text{C}$  with a total resistance of 25–75 ohms. E.m.f. temperature characteristics for various industrial thermocouples are shown in Fig. 2.

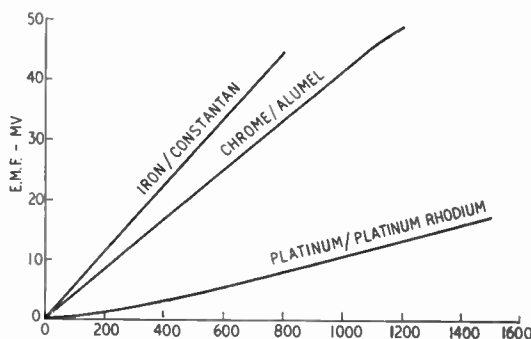


Fig. 2. Thermocouple characteristics.



### 3. Self-balancing Bridges and Potentiometers

Based on the well known Wheatstone bridge and potentiometer null balance methods of measurement widely used for precision laboratory measurement, self-balancing electronic recorders carry out the re-balancing operation continuously and automatically.<sup>2</sup>

The requirements of potentiometric measurement of thermocouple e.m.f.s and bridge measurement of resistance thermometer values have led to the development of the present wide range of electronic temperature recording and controlling instruments.

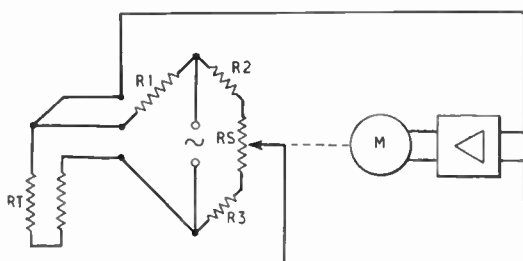


Fig. 3. Self-balancing Wheatstone bridge.

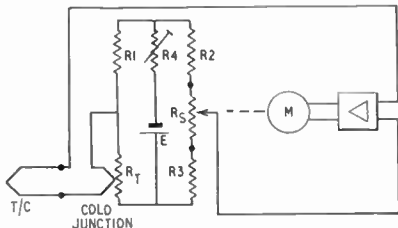


Fig. 4. Self-balancing potentiometer.

Figure 3 shows a typical self-balancing bridge arrangement, and Fig. 4 a typical potentiometric instrument.

Instruments of this class are among the most accurate instruments in everyday use, accuracies of  $\pm \frac{1}{4}$  per cent. being readily obtainable down to spans of  $10^{\circ}\text{C}$  for bridge instruments and  $50^{\circ}\text{C}$  for potentiometer instruments.

The circuits in Figs. 3 and 4 are basically similar in that electronic amplifiers are used to energize the re-balancing motor from the out-of-balance voltage of the bridge or potentiometer. Resistance bridges are usually a.c. operated, phase sensitivity being obtained by

the use of 2-phase induction motors with quadrature-energized reference windings. Unequal windings are generally used to reduce the output requirements of the amplifier to a maximum of about 8 watts at 50 volts.

Potentiometers being essentially d.c. instruments some form of d.c./a.c. conversion is necessary if 2-phase balancing motors are to be used. Potentiometer circuits are complicated by the necessity for standardizing facilities and cold junction compensation. The combination of a temperature sensitive resistor in close thermal contact with the reference junction as shown in Fig. 4 is commonly used to provide cold junction compensation.

Many reliable instruments of these types are commercially available in this country and abroad, mechanical arrangements and operational features differing widely from instrument to instrument. It is with the design of the amplifiers that the electronics engineer is most concerned.

In the case of resistance bridge instruments this is not difficult. A.c. excitation can be used and for  $\pm \frac{1}{4}$  per cent. accuracy at  $100^{\circ}\text{C}$  span, an a.c. error signal of the order of  $500 \mu\text{V}$  is available at the amplifier input. By comparison the amplifier requirements for potentiometer type instruments are much more stringent. For the same accuracy and span using iron/constantan thermocouples, movement of the balancing motor would be required from d.c. signals of the order of  $10 \mu\text{V}$  at the amplifier input. This sensitivity of discrimination must moreover be maintained over long periods of continuous unattended operation under adverse industrial conditions.

### 4. Current Feedback Amplifiers

Self-balancing instruments incorporating a motor-driven slide wire or reactance are essentially mechanical output devices, the output being in the form of a mechanical displacement of pen pointer or control mechanism. Where an electrical signal is required to feed directly into an electronic computer or controller, and where a higher speed of response than can be obtained using a motor driven instrument is required, thermocouple e.m.f.s can be amplified potentiometrically by means of current feedback amplifiers.

Figure 5 shows a block diagram of current feedback amplifier arrangement. Use is made of current forcing output stages to obtain overall conductances ( $G$ ) of from  $1 \text{ mA}/\mu\text{V}$  to  $0.05 \text{ mA}/\mu\text{V}$ . With current feedback the effective input impedance ( $Z_{in}$ ) can be made a thousand times the value of the grid circuit resistance ( $R_g$ ) in a practical amplifier. Current feedback amplifiers have advantages in electrical force balance measurement and current telemetering systems.

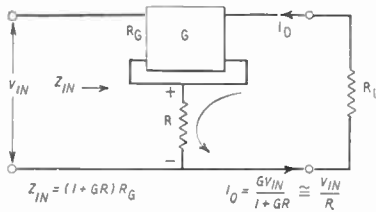


Fig. 5. Current feedback amplifier.

The design requirements of current feedback amplifiers are similar to those for self-balancing potentiometers at least for the input stage. More careful control of the gain/phase characteristic is necessary due to the faster response and absence of the balancing motor

which acts as the dominant time constant in a self-balancing instrument. By obtaining the required gain from the minimum number of high gain stages the stability problem is eased. Special arrangements for concentrating the gain in a single valve stage are commonly used.

Since both input and output are d.c. signals, d.c./a.c. conversion of the input signal is not essential but may be chosen from other considerations, in this case phase-sensitive rectification is needed to provide a d.c. output of correct polarity.

5. Low-level D.C. Amplifiers

At the present stage of development neither valves nor transistors are sufficiently drift or noise free to permit direct amplification of d.c. signals at microvolt level. The additional difficulty of maintaining levels in multi-stage high gain direct-coupled amplifiers precludes their practical employment. Two broad lines of approach have emerged in practice. The first is the conversion of the direct current to alternating current by means of a modulator that is not subject to drift, followed by a.c. amplification and phase-sensitive rectification. The second is the use of a single stage d.c. amplifier preceded by a device capable of

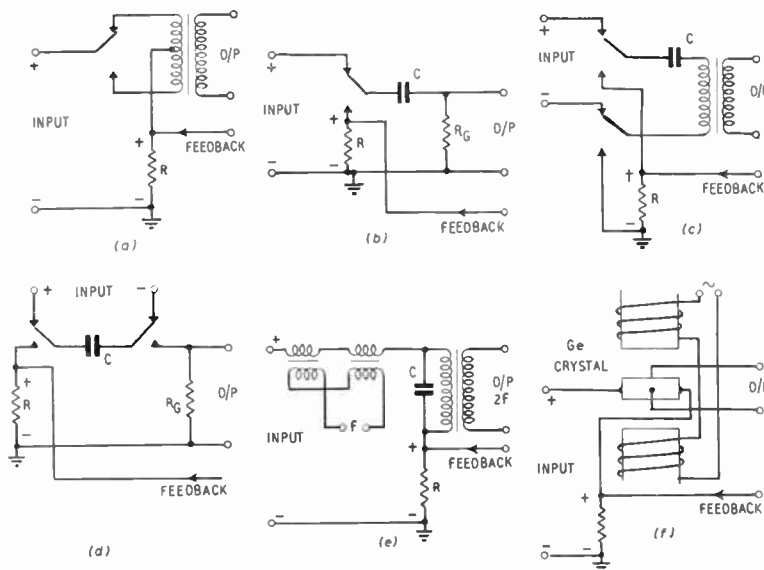


Fig. 6. D.c./a.c. converters.

increasing the amplitude of the input signal without drift.

The same considerations that prevent valves being used for low-level d.c. inputs precludes the use of valve modulators. Modulators based on precision electro-mechanical vibrators<sup>3</sup> are used extensively, others, based on the Hall effect in single crystal germanium<sup>4</sup> and second harmonic magnetic modulators<sup>5</sup> have also been used.

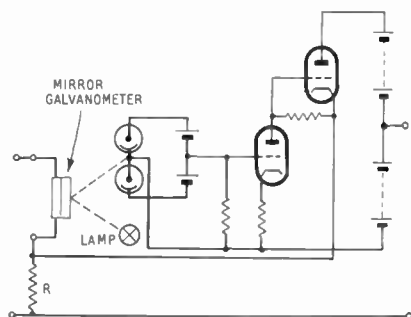


Fig. 7. Galvanometer/photocell amplifier.

Figure 6 shows various types of modulator arranged for current feedback.

For low impedance circuits the centre-tapped transformer arrangement of Fig. 6 (a) can provide voltage step up and low noise using m.b.b. contacts. The arrangement of Fig. 6 (b) is useful for high impedance circuits. Where multiple earthing and d.c. level problems occur the arrangements of Figs. 6 (c) and (d) can be used to provide complete isolation between input and output circuits. From noise and pick-up considerations (d) is superior to (c).

A second harmonic magnetic modulator is shown in Fig. 6 (e) and a Hall effect modulator in Fig. 6 (f).

All these circuits are suitable for microvolt level signals, the contact modulators having the highest conversion efficiency.

For the second technique sensitive galvanometer input stages are used. Movement of the galvanometer is used to modify the light distribution of a balanced photocell bridge. Alternatively the galvanometer movement may be used to detune a r.f. oscillator by means of a light metal vane or link as described in a later section.

A typical circuit of a galvanometer/photocell amplifier is shown in simplified form in Fig. 7. The use of a series amplifier output stage is noteworthy. The centre-tapped power supply is commonly of the well-known "telegraph rectifier circuit" type.

Well-designed amplifiers of this type can provide open loop gains of the order of  $10^6$  or more. Despite the use of a galvanometer adequate speed of response can be obtained. It can be shown<sup>6</sup> that a large feedback factor has the effect of considerably reducing the natural undamped period of the galvanometer and of increasing the effective stiffness and thus reducing the effects of external vibration.

## 6. Phase-sensitive Rectifiers

Phase sensitive rectifiers are used in modulated type d.c. amplifiers to provide a d.c. output of correct polarity. Conventional phase-sensitive detectors normally provide voltage output signals which require subsequent amplification if a current output is required for feedback purposes.

The circuit and waveforms of a new type of phase sensitive rectifier capable of giving a current output directly are shown in Figs. 8 (a) and (b).

The action of the circuit is as follows:—

During the half cycle of the mains voltages, when C is positive with respect to D, a positive charging current flows through V1A and MR2 into capacitor C1 via points C, E, J, H and K. Reverse voltages are developed during this period across V1B and MR1, which therefore remain non-conducting.

During the following half cycle, when C is negative with respect to D, a negative charging current flows through MR1 and V1B into capacitor C1, via points C, G, J, F and K; V1A and MR2 remain non-conducting during this period.

In the absence of an input signal across AB, the capacitor receives equal positive and negative charging currents, resulting in zero charge, and zero current in the external load RL, as illustrated in column (1) of Fig. 8 (b).

If an input signal is applied to AB in phase with the voltage across CD, the grid of V1A

will be driven positively when the positive charging current is flowing into capacitor C1, and the grid of V1B will be driven negatively when the negative charging current is flowing into capacitor C1. The positive pulses of charging current will therefore be greater than the negative pulses of charging current, the fluctuations being smoothed out by the capacitor to produce a steady positive direct current in the external load RL, as illustrated in column (2) of Fig. 8(b).

Similarly, if the phase of the input signal applied to AB is reversed, the negative charging current will be greater than the positive charging current, and a steady negative direct current will flow through RL as shown in column (3) of Fig. 8(b).

Figure 8(a) represents the simplest form of this circuit when used as a half-wave phase-sensitive detector; two such circuits can be combined to provide full-wave operation.

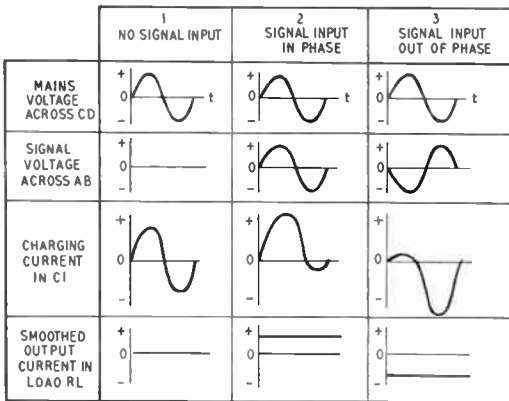
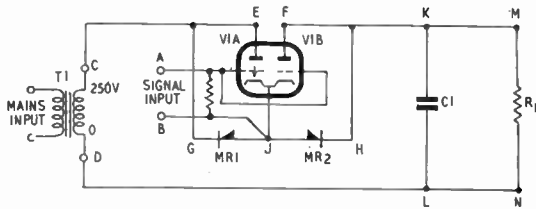


Fig. 8. Phase-sensitive rectifier and its operating waveforms.

As an example of the application of this type of circuit a practical 50 c/s amplifier and

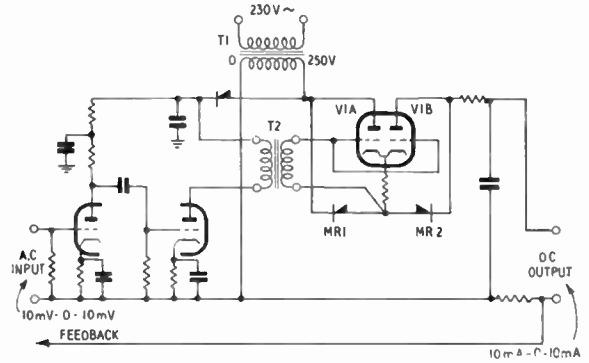


Fig. 9. Half-wave phase-sensitive detector circuit.

half-wave phase-sensitive detector circuit is shown in Fig. 9. In this circuit, which has been used as part of a d.c. chopper type amplifier system with overall negative feedback, a single 250 V a.c. supply provides both h.t. for the amplifier stages, and anode voltage to operate the phase-sensitive detector.

The open loop conductance of this circuit using 12AX7 and 6J6 valve types, is about 1 mA d.c. per 1 mV a.c., with a maximum output current of approximately 10 mA-0-10 mA.

The new phase-sensitive detector combines amplification with phase-sensitive detection, and since it provides a direct-current output, is readily applicable to force balance current-operated devices, in contrast with conventional phase-sensitive detectors which are normally only suitable for producing voltage output signals.

### 7. Vane-controlled Oscillators (General)

The use of r.f. oscillator circuits to detect the position of a light metal vane or link is well known. Successful circuits, in which a sharp and almost discontinuous transfer from the oscillate to non-oscillate condition occurs when movement of the vane causes the sense of the feedback from anode to grid to change from positive to negative, have been described in the literature.<sup>7-10</sup>

When oscillating, the valve draws grid current, and a negative d.c. bias is stored in the grid capacitor, resulting in a low value of anode current. In the non-oscillate condition, the negative grid bias disappears and the anode

current rises to its maximum value. In one typical commercial design<sup>11</sup> a vane movement of 0.007 in. produces an anode current change of from 2 to 12 mA.

Since the reaction upon the vane due to the r.f. field is negligibly small<sup>11</sup>, a controlled oscillator can form the basis of a stable high-gain d.c. amplifier<sup>12</sup>, using as the input a sensitive galvanometer to control the movement of the vane, and providing overall negative feedback.

Similar arrangements have been used to convert process measurements of pressure, flow, displacement and liquid level into direct current for remote electrical transmission.<sup>13</sup> For pressure transducing, the mechanical motion of a bourdon tube or bellows is applied to a calibrated spring to produce a force loading on a pivoted beam. The oscillator is used to detect the beam deflection, and a rebalancing force is provided by means of a permanent magnet moving-coil system, operated from the oscillator output. Equality of forces being automatically established, the current flowing through the moving coil is a direct measure of the originating force and pressure.

Owing to the fact that no matter how strongly a valve is oscillating, the self bias can never reduce the anode current below a certain minimum value, the design of a transducer of the above type, to give zero output current for zero input signal, normally involves the provision of an additional power supply and "backing-off" circuit. Considerable simplification is, however, possible if suitable oscillator circuits are combined with the basic circuit of the phase-sensitive rectifier circuit described previously.

Some of the circuits which have been designed on this basis are described below; all these circuits are capable of giving either a unidirectional current output extending from zero to about 18 mA maximum, or a bi-directional current output of about 15 mA-0-15 mA.

### 8. Capacitance Bridge Oscillator Type 1

This circuit, which is shown in Fig. 10(a), represents the simplest application of the oscillator principle to the basic phase sensitive

rectifier circuit of Fig. 8. The method of obtaining a direct-current output by smoothing out the inequality between the positive current pulses through V1A and MR2, and the negative current pulses through V1B and MR1, is retained. Triode sections V1A and V1B are, however, connected as two separate capacitance bridge oscillators, of the type described by Shepard.<sup>9</sup> A simple analysis shows that the voltage  $V_{g1}$  fed back from cathode to grid of V1A, assuming perfect coupling between the two halves of the centre tapped coil L1, is given by the approximate expression:—

$$V_{g1} = A \cdot V_{g1} \frac{(C1A - C2A)}{(C1A + C2A)} \dots\dots(1)$$

Since oscillation can only take place when the feedback from cathode to grid is positive, the critical condition for oscillation is  $C1A \geq C2A$ . Similarly for the identical circuit of V1B oscillation is maintained when  $C1B \geq C2B$ .

Similar conditions can be derived in the case where the cathode tapping point divides the coil into two unequal sections.

A differential capacitor is used for C2A and C2B so that an increase in the capacitance of C2A is accompanied by a decrease in the capacitance of C2B. C1A and C1B are adjusted so that the condition for oscillation is only satisfied for one triode section at a time. Under these conditions, a very small differential change in capacitances C2A and C2B will simultaneously switch V1A from the oscillate to non-oscillate condition, and V1B from the non-oscillate to oscillate condition, or vice versa.

With suitable component values, using a tightly coupled, centre-tapped coil of about 3 mH inductance, a differential change in capacitance of approximately 0.5 pF produces a change in output current through a 3.3 kilohm load resistance of from +18 mA to -18 mA.

### 9. Capacitance Bridge Oscillator Type 2

Since only one half of the twin triode valve used in the previous circuit is oscillating at a time, both oscillators can be made to operate from a single coil, as shown in Fig. 10(b). This circuit employs a 6J6 twin triode with common cathodes and a single centre-tapped

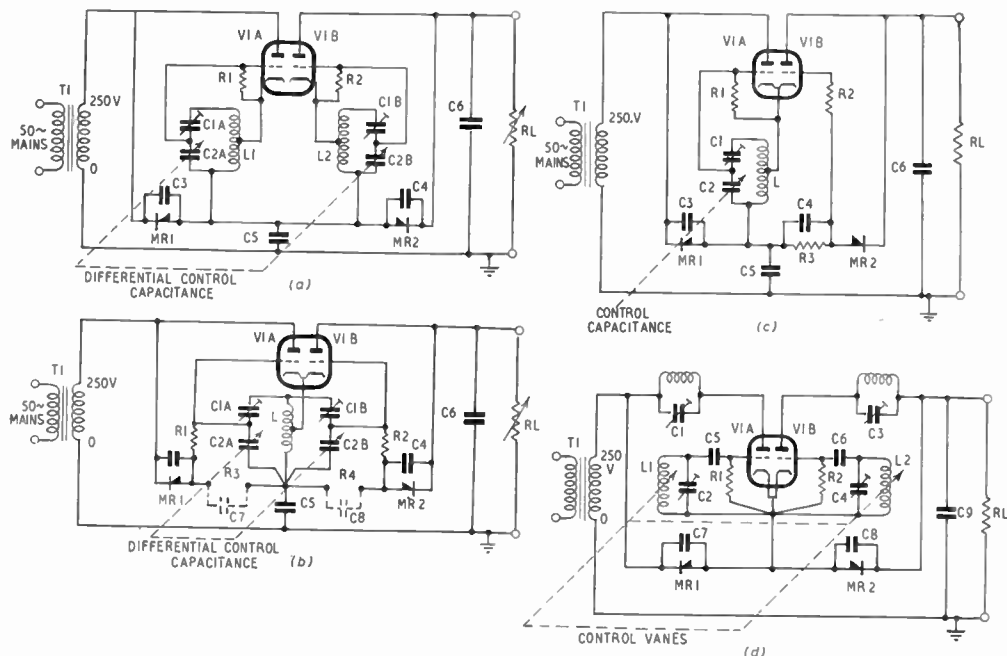


Fig. 10 (a), (b) and (c). Three types of capacitance bridge oscillator. (d) Tuned-anode tuned-grid oscillator.

coil of about 3 mH inductance. Positive feedback may be introduced by including networks R3, C7, and R4, C8. Unless the two halves of the centre-tapped coil are very tightly coupled, the adjustments of C1A and C1B are not independent, which makes the adjustment of this circuit for optimum performance much more difficult than in the case of the previous circuit. This difficulty can be overcome by expert coil design.

The circuit requires few components, especially in applications where positive feedback is not required and a centre tapped rectifier can be used in place of the two single ones, as in Fig. 10 (a).

### 10. Capacitance Bridge Oscillator Type 3

The circuits described so far have all contained two oscillators and, consequently, two sets of adjustments. It is possible, at the expense of a slight decrease in sensitivity, to use only one oscillator to control the bias on both triode sections, as shown in the circuit of Fig. 10 (c).

The anode current of V1A controls the bias on the grid of V1B by means of R3, C4. When

oscillating, V1A is biased negatively and its anode current is low; little bias is therefore applied to V1B, so that the anode current of V1B is large. When C2 is increased above the critical value, rendering V1A non-oscillating, the anode current of V1A rises to its maximum value, thereby increasing the bias on V1B, and reducing the anode current of V1B to a very low value. The smoothed output current through the external load RL consequently reverses.

With suitable component values an output current swing of about 12 mA–0–12 mA can be obtained. Feedback from V1B to the grid of V1A can be provided by a suitable network, as in the previous circuits, if greater sensitivity is required.

### 11. Tuned-Anode Tuned-Grid Oscillator

Many other applications of the oscillator principle to the basic phase-sensitive rectifier circuit are possible. Among those tried are the simple tuned-anode inductive feedback oscillator, and the tuned-anode tuned-grid inter-electrode capacitance feedback oscillator.

The tuned-anode tuned-grid circuit is capable

of high sensitivity if the grid circuit is detuned by means of a control vane, owing to the sudden reversal of the feedback from anode to grid, due to the well known Miller effect.

A practical circuit is shown in Fig. 10 (d), in which the anode and grid circuits are both tuned to approximately 40 Mc/s.

The simple tuned-anode feedback oscillator is normally only capable of low sensitivity, unless a subsidiary capacitive feedback path is added, preferably between anode and grid.

**12. Comparison between Capacitance Bridge and Tuned-anode Tuned-grid Oscillator Circuits**

All the capacitance bridge circuits gave full sensitivity for a differential change in control capacitance of 0.5 pF or less.

A 16-turn, single-layer plane spiral grid coil of  $\frac{3}{4}$  in. diameter with wide spacing, and a copper vane  $\frac{7}{8}$  in. wide and 0.04 in. thick, were used as the control elements of the tuned-anode tuned-grid oscillator. Oscillation ceased when the vane was brought within 0.001 in. of the coil. A current change of approximately 1 mA per 0.0006 in. was observed as the vane was withdrawn from the coil. At separations beyond 0.01 in. the vane had virtually no effect upon the anode current.

The results cannot be regarded as conclusive without further experimental work, but the above figures seem to indicate that the tuned-anode tuned-grid circuit is the more sensitive. Against this, however, is the difficulty of mechanical alignment of the light metal vane to operate as close as 0.001 in. to the coil without twisting or short circuiting.

The capacitance bridge circuit is probably simpler to construct and the sensitivity is more than adequate for most applications, but precautions have to be taken to guard against stray capacitance.

Both types of circuit are simple to adjust, and do not require close tolerance components, or special selection of valves.

**13. Applications to Force Balance Measurement and Control**

To illustrate the use of the circuits described in Figs. 8, 9 and 10, some possible applications are shown in Figs. 11, 12 and 13.

Figure 11 (a) is an example of the type of force balance transmitter described in connection with pressure transducing in Section 7. Any primary element capable of producing a mechanical displacement can be used to provide the initial force loading on the beam by means of the calibrated spring. The beam is linked

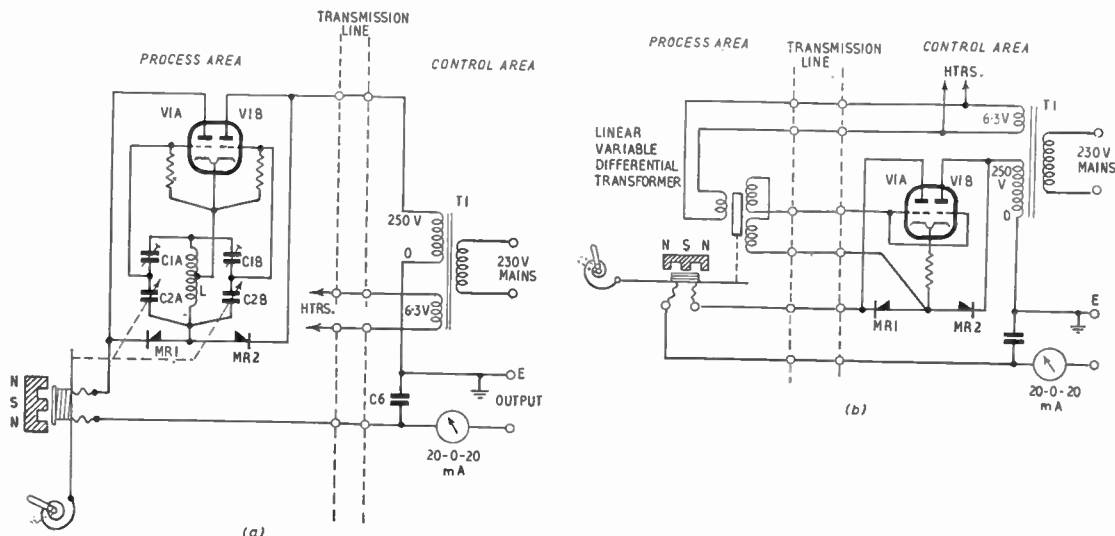


Fig. 11. Force balance transmitters. In (a) the oscillator valve is in the process area, while in (b) remote operation is achieved using a variable differential transformer.

to the differential control capacitances of the oscillator, and the oscillator output current is fed back through the moving coil to provide the rebalancing force. The output current may be transmitted to a distant location to provide remote indication and control. Since the system is inherently a closed loop servo-mechanism, the transducer is largely self-compensating for variations in mains voltage, frequency and line resistance.

With the above type of transducer the oscillator valve must be located at the point of measurement, since for obvious reasons the high frequency detector circuit cannot be mounted remotely from the beam. From the point of view of ease of maintenance, and operation in explosive atmospheres, the location of the valve in the process area is clearly undesirable. One method by which this disadvantage may be overcome is shown in Fig. 11 (b).

The circuit of Fig. 11 (b) uses a 50 c/s linear variable differential transformer to detect the beam deflection, and the basic phase sensitive detector circuit of Fig. 8 to provide the rebalancing current. The use of a 50 c/s error signal makes transmission over lines feasible and enables the electronic circuitry to be located remotely from the beam.

The arrangement of Fig. 11(b) was designed to operate with a special rotary form of differential transformer giving an output of 60 V per 12° rotation. This output is much greater than that claimed for 50 c/s differential transformers described in the literature<sup>14</sup>, but no difficulty arises in using conventional types of differential transformer in conjunction with the circuit of Fig. 9.

Figure 12 shows a proportioning amplifier based on the oscillator principle, which incorporates a differential galvanometer with two equal windings. Deflection of the galvanometer due to an input current  $I_1$  is detected by the oscillator, and opposed by an equal current  $I_2$  in the second coil. By means of a suitable shunt  $R_s$  across the second coil, the output current of the oscillator  $I_o$  can be made a multiple of the input current  $I_1$ . The system functions as an extremely stable current amplifier, suitable for analogue computer applications.

If the zero setting of the galvanometer hair spring is made adjustable, the system can be used to combine the functions of an error detector and proportional amplifier in a process control system. The input current derived from a suitable transducer represents the "measured value" and the setting of the hair spring the "desired value." The output current  $I_o$ , therefore, becomes the deviation multiplied by the proportional control factor ( $K^{\theta}$ ).

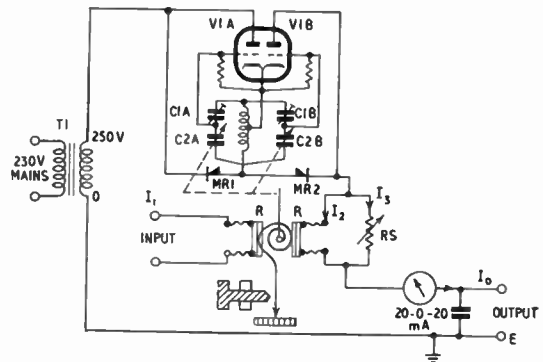


Fig. 12. Proportioning amplifier incorporating a differential galvanometer.

Figure 13 shows a circuit for feeding the current outputs of several computing amplifiers through a common load. Overall negative feedback is provided from the output rectifier to the input of each amplifier without interaction. A common earth line links the input signal source, the amplifier input and output circuits, and the output load RL. The output phase-sensitive rectifiers are all fed from a single 250 V a.c. supply.

In order to stress the economy of power supply requirements of these new circuits, it is pointed out that all the circuits described in Figs. 8 to 13 could be built on a single chassis and simultaneously operated from a single 250 V a.c. anode supply, and a single 6.3 V a.c. heater supply, and that these requirements would not increase if the outputs of the circuits of Figs. 11 (a) and (b) were fed into the circuit of Fig. 12, etc.

#### 14. Conclusions

Many problems in process control can be solved advantageously by electronic techniques, providing the basic problems and operational



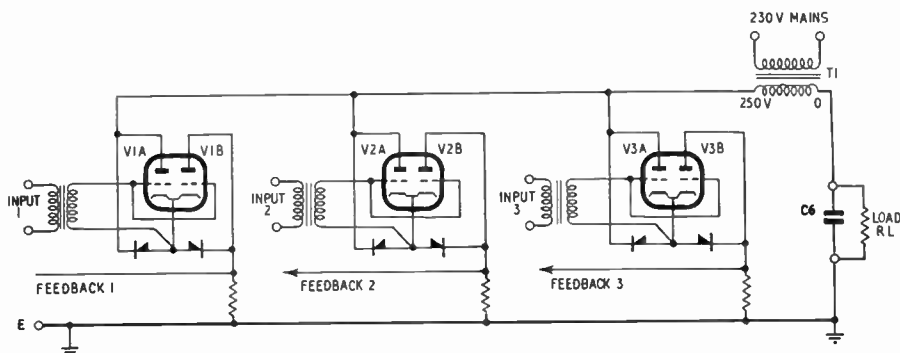


Fig. 13. Circuit for feeding the current outputs of several computing amplifiers through a common load.

requirements are thoroughly understood by the electronics engineer.

A brief description of some successful applications of electronics in industrial measurement has been given.

Some new phase-sensitive rectifier and vane controlled oscillator circuits and their applications have been described.

#### 15. References

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## APPLICANTS FOR ELECTION AND TRANSFER

As a result of its September meeting the Membership Committee recommended the following elections and transfers to the Council.

In accordance with a resolution of Council and in the absence of any objections, the election or transfer of the candidates to the class indicated will be confirmed fourteen days after the date of circulation of this list. Any objections or communications concerning these elections should be addressed to the General Secretary for submission to the Council.

### Direct Election to Member

HART, Air Marshal Sir Raymond George, K.B.E., C.B., M.C., A.R.C.S., R.A.F. *Aston Rowant.*  
WYLIE, Captain Francis James, R.N. (Retd.). *Northwood.*

### Transfer from Associate Member to Member

HUTTON, James Alan, B.Sc. *Buntingford.*

### Direct Election to Associate Member

GWYTHER, Noel Ralph. *Hayes, Middlesex.*  
KHAN, Lieut. Mohammad Idris, Pakistan Navy. *Karachi.*  
LEVENTHALL, Philip Eric, B.Sc.(Hons.). *Bexleyheath.*  
SCOLLAY, Charles Ian Chalmers, B.Sc., B.E. *Singapore.*  
TATNALL, Dennis Sidney. *Uppminster.*  
WHEELER, Leonard Gage. *Lossiemouth.*  
WILLIAMSON, John Irvine, B.Sc. *Toowoomba, Queensland.*

### Transfer from Associate to Associate Member

LANE, Albert. *Wembley.*  
MARSDEN, Ernest Wilson. *Westcliff-on-Sea.*  
PORTER, Ernest George. *Rickmansworth.*  
WILKINS, Peter Granville. *Elgin.*

### Transfer from Graduate to Associate Member

APPLEYARD, Albert Henry. *I.ford.*  
BAILEY, Douglas John. *Westcliff-on-Sea.*  
BARNET, Peter Alan. *London, W.C.1.*  
BROWN, Leonard Denis, B.Sc. *Romford.*  
CHISHOLM, Robert Oswald Ralph. *Bristol.*  
DOWSETT-MARSH, Julian Caryl. *Wiltwyn Garden City.*  
FULLERTON, Sqdn. Ldr. John Raymond Edward, D.F.C. D.F.M., R.A.F. *Peterborough.*  
GARDINER, Flt. Lt. Frederick Henry, R.A.F. *Woodley.*  
GREEN, Norman George. *Reading.*  
JARVIS, John Walter. *Basingstoke.*  
KUNDU, Mukul Ranjan, M.Sc. *Paris.*  
LAW, Flt. Lt. Peter Anthony, R.A.F. *Northampton.*  
MOYCE, Flt. Lt. Dennis Alfred, R.A.F. *Anglesey.*  
NELSON-JONES, Laurence. *Brentwood.*  
NILAKANTAN, Kadayam Ganesh. *Copenhagen.*  
PAVEY, Arthur Robert Brian. *Warrington.*  
SHANKARNARAYAN, Flt. Lt. Subramany Sastry, B.Sc.(Hons.) M.Sc., I.A.F. *New Delhi.*  
SHANNON, John Daniel. *Kampala, Uganda.*  
WILLETTS, Flt. Lt. Walter John, R.A.F. *Sheffield.*  
WOODWARD, Grieg Roger. *Stourbridge.*

### Transfer from Student to Associate Member

DOUGLAS, Walter Harry Brookes, B.Sc. *Portsmouth.*

### Direct Election to Associate

AUNG MYINT, Flt. Lt., Burma Air Force. *Rangoon.*  
CHEN, Michael Khin Onn. *Johore Bahru.*

CUNNINGHAM, Albert Roy. *London, N.9.*  
DICKSON, Major Denis Ayton, R. Sigs. *B.A.O.R.*  
DYER, Robert Berkeley. *London, W.7.*  
EASTLAND, Horace William. *Arborfield.*  
HENSHAW, William Charles, M.Sc. *Bristol.*  
HOLDEN, Jack. *Lusaka.*  
PATCHETT, Lloyd. *Timperley.*  
PICHAL, Henri Thomas. *Westcliff-on-Sea.*  
PULL, Edward Lionel. *London, S.E.21.*  
ROSKROW, Capt. Brian, R.Sigs. *Knockholt.*  
SAVEL, Josef. *Prague.*  
TILLET, George Walter. *London, N.7.*  
WILKINSON, Harry. *Bexley.*

### Transfer from Student to Associate

MATTHIAS, Ralph Boneventure. *Enugu, Nigeria.*  
WORDSWORTH, Geoffrey. *Biggleswade.*

### Transfer from Associate to Graduate

NEIGHBOUR, Kenneth John. *Christchurch.*

### Direct Election to Graduate

BISHOP, Donald Edwin. *London, N.13.*  
DUBIEL, Franciszek. *Wolverhampton.*  
DWIVEDI, Lt. Munesh Bal, M.Sc., Indian Corps of Signals, *Utnao.*  
HARDING, Jocelyn Victor. *Grays.*  
HATHAWAY, Raymond William. *Sunbury-on-Thames.*  
LEONG, Kwok Onn, B.Sc.(Eng.). *Brighton.*  
LE WARNE, John Arthur. *Maidenhead.*  
PUGH, Alan. *Cardiff.*  
SCOTT, Victor William. *Sevenoaks.*  
TOPPING, Douglas Randall Peter. *Worthing.*  
WHITEHEAD, Alan Peter. *Weston-super-Mare.*

### Transfer from Student to Graduate

ARCISZEWSKI, Henry, B.Sc.(Eng.). *London, S.W.18.*  
BENNETT, Wilfred Denis. *Stockport.*  
BURKE, Alan. *London, N.19.*  
BUTA SINGH. *Simla.*  
DIVECHA, Gautamrai Amritlal. *London, N.4.*  
DOCKERTY, William Harold. *Farnborough, Hants.*  
LUTHRA, Suraj Parkash, B.A. *London, E.1.*  
MCKENZIE, John George. *Mitcham.*  
MEHTA, Gunvant Jagjivandas. *Porbander.*  
MORAWSKI, Wojciech. *London, S.W.4.*  
NALBENTIAN, Benjamin. *Sao Paulo.*  
RAHEJA, Udhavlal Topandas. *Bombay.*  
RAJENDRA NATH. *Jorhat.*  
RAYBONE, Norman Edward. *Sutton Coldfield.*  
REID, William Lawrence. *London, W.13.*  
SHORT, Thomas. *Crossmaglan, Co. Armagh.*  
WALES, Sidney Alfred. *Hornchurch.*  
WALKER, Robert Anthony. *London, N.W.11.*  
WILKINSON, David. *Lagos.*  
WOOLFORD, Alan John. *Harrow.*

## STUDENTSHIP REGISTRATIONS

In addition to the above elections to Graduateship and higher grades of membership, the Committee has recommended the registration of 35 Students.

# A "BASIC" SYSTEM OF POSITION CONTROL FOR THE TRAVERSING TABLES OF MACHINE TOOLS\*

by

K. J. Coppin, B.Sc., A.R.C.S. (Associate Member)†

*A paper presented at the Convention on "Electronics in Automation" in Cambridge on June 27th 1957. In the Chair: Mr. E. E. Webster.*

## SUMMARY

The processes carried out by machine tools are divided into two types—"progressive" e.g. profile milling, grinding, turning, in which cutting and traversing operations occur simultaneously, and "non-progressive"—e.g. drilling, boring, punching—in which they alternate. The requirements for automatic control of each type of process are considered in detail, and a specification worked out for a "basic" system of position control for application to "non-progressive" processes, to which may later be added facilities for carrying out some "progressive" processes.

The design of a driving and measuring system for attachment to existing machines is derived, and from it are inferred the requirements of the control system, the major design features of which are then discussed. Finally, the possible use of the system in conjunction with punched card or similar programming devices is examined, and extension of its use to some "progressive" operations considered. The avoidance of complex electronic circuitry is made the governing factor in deciding the methods to be employed throughout.

## 1. Introduction

In the last decade or so, a considerable amount of attention has been directed to the problem of providing servo-controlled power operation of the traversing systems of the various types of machine tool—lathes, milling machines, jig-borers, etc. Since we are here concerned with electronics—within which may presumably be included pure "electrics"—it is not intended to discuss the application of pneumatic or hydraulic servos in this field, although, in fact, some very useful work has been done by various concerns on the use of such techniques.

The equipment which has been developed to date can be divided into two classes. On the one hand, there are the specialized devices developed by particular industries to assist their own production, but not made commercially available, and, indeed, often unsuited to more general use. On the other hand, there are the extremely versatile, electronically controlled systems which are capable of carrying out the most intricate operations encountered in

machine shop practice, to an accuracy at least as great as a skilled craftsman could achieve, and at a speed which he could not hope to rival. Most particularly, these systems are applied to profile milling in three dimensions, where their computer organized programme makes it possible for them to machine elaborate forms without attention (often from skeleton data used on an interpolation basis).

Some smaller and simpler systems of more limited scope have also made their appearance, but even in these the tendency is to use rather elaborate electronic control arrangements in conjunction with specially contrived measuring scales which have to be to some degree "tailored" to the machine to which they are applied. It is the purpose of the paper, therefore, to examine the various functions which an automatic traversing system may be called upon to carry out, and, having classified these into two distinct groups, to lay down a basic specification in the case of the simpler—and, incidentally, larger—of these two groups. Finally, details will be given of a system which has been designed to meet this specification with, it is believed, the minimum of cost and complexity, and the maximum flexibility of installation.

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U.D.C. No. 621.9:621-52

### 1.1. *Classification*

Machine tool operations are naturally divided into two main categories: firstly, those in which the cutting tool is required to remain in contact with the work while traversing operations are carried out, and secondly, those in which the tool is completely withdrawn from the work during these operations. Into the first group fall profile milling, slotting, turning and grinding operations. In the second group are contained drilling, boring, punching, and spot-facing. The first group may be subdivided according as the outline of the work, or its section in a plane parallel to the plane containing the directions of traversing motion is, or is not, rectilinear. It will be shown later that, in the former case, the methods of control which are required are susceptible to considerable simplification. Whatever the application, the aim of the control system is the same, namely, to enable the finished part to be produced to the designed dimensions—within acceptable limits—with the minimum of intervention on the part of the machine operator.

### 1.2. *“Progressive” Operations*

In the discussion which follows, the term “progressive” will be used to describe processes in which cutting and traversing functions must be carried out simultaneously, whilst the term “non-progressive” will be applied to processes in which traversing and cutting operations occur sequentially. In order to obtain a clear picture of the most exacting requirements which a system of machine tool control may be called upon to meet, we shall consider briefly the design requirements of an arrangement for the servo control of profile milling in three dimensions. In order that such an operation can be carried out, it is necessary that the traversing system shall be able to produce relative movement between work and cutting tool according to a pre-arranged three-dimensional pattern. Except in cases in which the motive power used for traversing is pneumatic or hydraulic, the motion of the prime mover for the traverse is fundamentally rotary, and this rotation is converted into rectilinear movement of the traverse by means of a lead-screw or rack and pinion arrangement. It is from the backlash associated with the conversion arrangements that one of the major

problems of servo control arises. With the exception of grinding wheels, rotary cutting tools in ordinary use have discontinuous cutting edges, and consequently, even when a rectilinear cut is being made, the magnitude and direction of the cutter reaction do not remain constant. Where the profile being cut is not rectilinear, its curvature causes an additional variation of the reaction to be superimposed, so that in general if the cutter reaction is resolved into three components, one parallel to each of the traversing directions, these components will each be subject to variations both in magnitude and direction (“positive” or “negative”)\* as the work proceeds, and will tend to displace the work-piece to an extent which is governed by the amount of backlash in the traversing system. Consequently, it is virtually impossible to use even calibrated lead-screws for purposes of measurement in such applications, and it is necessary to provide measuring scales which are rigidly attached to the table. At present, optical or magnetic scales are favoured for this purpose.

Even the use of these independent scales does not, however, entirely overcome the effects of backlash in the traversing system: it merely ensures that displacement of the work by cutter reaction shall not occur undetected, so that the problem becomes one of correcting the displacement, and this is far from simple. Since the motion of the work with respect to the cutter is the resultant of rectilinear movements in three mutually perpendicular directions, it follows that a cut which is not in a direction parallel to one of these requires a velocity correlation between all three motions, and it would be difficult, if not impossible, to maintain such correlation if, superimposed upon the planned movement of the work, there were a random velocity variation due to varying cutter reaction. It therefore becomes a requirement for such systems that backlash between the controlling and measuring points should be, if not entirely eliminated, at least reduced to an amount less than the resolution of the servo system. When the resolution asked for is 1/10,000th inch or better, not only does this

\* “Positive” if aiding the traversing motion, “negative” if opposing.

represent a formidable problem for the electronics engineer designing the servo units which provide traversing power, but it also means that the machine tool designer is called upon to produce traversing arrangements at least one order better than has hitherto been regarded as acceptable.

It will be apparent from these considerations alone that, except in the case of a very limited class of operations in which the form required is rectilinear and the work can be set up in such a way that the cut is carried out in a direction parallel to one of the traversing motions, some form of computer will be required to co-ordinate the motions of the traverses. Where non-linear profiles are to be produced, the computer must be designed to generate the information needed to produce the correct curvature, or an acceptable approximation. Two methods of generating this information—each of which has its counterpart in mathematical techniques—have been employed for this purpose. One consists in employing such high speeds of information feed to the machine that the curve is in fact generated as a series of extremely short straight lines connecting a very closely plotted series of points. The other consists in feeding in fresh data only at points where changes of curvature occur, or at a series of widely spaced points on an extended curve, and providing within the computer means of interpolation, or of generating a set of “standard” curves, one of which is selected as being the best approximation to the form required. The latter method is often acceptable for curves lying entirely in a plane parallel to that containing two of the traversing motions, but is difficult to apply in the case of curvature in three dimensions.

It has already been indicated that the means utilized to effect the traverse movements call for considerable resourcefulness on the part of both the machine tool designer and the electronics engineer: there can in general be no possibility of just “tacking something on” to an existing machine. Unless very great care is taken, for example, in the design of lead-screws and table slides, “stiction” effects will tend to convert a smooth progressive motion of the prime mover into impulsive movements of the work, especially when the traverse speed

is very low. On the electronic side, the choice of possible servo arrangements is governed by the necessity of providing accurate control over an exceedingly large range of operating velocities. In general, it is necessary to provide a considerable amount of feedback around the complete servo loop (measuring instrument to prime mover). Because of this feedback, there will normally be a tendency to instability if there is any mechanical or electrical backlash in the loop.

Even when all these problems have been successfully solved, two vital questions remain to be answered. Can we, in fact, realize the “designed” accuracy, and what safeguards can be incorporated against failure of the system. Taking the second question first, the answer is that computer-controlled machines can be made self-checking to a considerable degree, but it is almost impossible to design a system in which the failure of a single component will not give rise to spoiled work. Good design merely makes such failure an increasingly remote possibility.

On the final question of realizable accuracy, certain problems remain to be solved before the accuracy of the finished work can be guaranteed to be as high as that of traverse positioning. These are the problems associated with the wear of the cutting tool, and the rigidity of its supports and of other parts of the machine which are assumed to be fixed. It would be inappropriate here to include a detailed analysis of the effects of the deficiencies mentioned; suffice it to say that as a broad generalization they combine to make the accuracy of the finished work one order worse than that of the traverse positioning system in the case of the best systems at present in use, that is to say, even though the system can position the traverses within “tenths of a thou.” or better, errors of a few “thou.” can still occur in the finished work unless extreme precautions are taken over such matters as cutter sharpening, tool rigidity, feed rates, and operating temperature.

### 1.3. “Non-progressive” Operations

There are, however, a considerable number of processes normally carried out on machines fitted with screw-driven traversing movements

for which the requirements are much less exacting, either in respect of the complexity of the process, or the accuracy of the finished work, or both. The simplest of these are pierce drilling and punching, which are strictly "non-progressive," requiring no control over relative velocities of the positioning traverses, and entailing motion of these only while the tool is completely withdrawn from the work. There are also what might be termed "transition" operations—boring, slotting, turning, surface milling and grinding, in which relative movement between tool and work is required while the two remain in contact, but this movement is rectilinear and restricted to one plane. Here relative velocities require control, but remain constant over any one operation and can therefore be pre-set, and cutter reactions, though variable in magnitude, can be arranged to preserve the same sign with respect to the feed motions, acting ideally as forces of pure compression, although deflecting components cannot always be avoided.

Let us now proceed to examine the requirements for the simplest possible case, that of pierce drilling or punching, and consider what is the simplest possible method of control, bearing in mind the requirements that it should be capable of fitting to existing machines, and should preferably be capable with little modification of working under some form of "programme" control, and of being developed to carry out operations in the "transition" class. Additionally, we will stipulate that it should be inexpensive, and simple enough not to require an army of technicians to maintain and service it.

The basic requirements appear to be simple enough: the device must be capable of moving the work by pre-arranged amounts in two dimensions from a given datum, in an acceptable time and to an acceptable order of accuracy. It must then either signal to the operator that the work is in position, so that he may feed in the tool, or it must carry out this operation itself before passing on to the next station. The difficulties are concealed in the words "acceptable time" and "acceptable order of accuracy," because the answers given will depend on whether the potential user is concerned with higher accuracy, higher rates

of production, or simple re-distribution of his labour force by making it possible for less-skilled labour to carry out operations which have hitherto required skilled men.

Considering first what constitutes an "acceptable time" it may be stated that an operator can move a traverse table by hand at about 12 inches per minute when he is not attempting to read the lead-screw thimbles. With power operated traverses, considerably higher speeds are possible. In general, except in the case of very small machines, the operator cannot move both traverses simultaneously, even when he is not reading the thimbles, and in any case an appreciable part of the total time taken in making any particular setting is associated with adjusting the last few "thou." on the thimbles. It would appear, therefore, that time will be saved in moving the table into position, if the *average* traverse speed exceeds 12 inches per minute, especially if both traverses can operate simultaneously, but it must be observed that for large batch production which justifies the making of jigs or other special tools, these are still likely to give the highest production rate, however high the traverse speed is made. For this reason, the design should take into account the possibility of its use to expedite the production of jigs and templates.

On the score of "acceptable accuracy" it is necessary to consider two distinct classes of work: that for which tolerances of  $\pm 0.003$  in. or more may be permitted, and the precision class, in which tolerances are of the order of 0.0005 in. or less. In general, for the former class, the work may be moved into position with sufficient accuracy by using the lead screws with their thimbles as the measuring elements. For the precision class of work, it is normally necessary to supplement the lead screws by slip gauges and "clock" gauges or micrometers, or by optical scales, or alternatively to calibrate the lead-screw or apply correcting systems (e.g. Genevoise jig borer). At least one firm has felt it worth while to produce an electronic scale to simplify the setting-up operation in this class of work.\*

\* C. R. Borley, C. H. Braybrook and L. Coates, "Automatic positioning systems for machine tools." *J.Brit.I.R.E.*, 17, pp. 513-521, September 1957.

Broadly speaking, attention should be given in any proposed system to simplifying the problem of converting a designed dimension into the appropriate movement of the work, because at least it reduces fatigue (and therefore liability to error) on the part of the skilled operator, and at best may enable industry to make better use of its skilled labour by relieving it of some of its less-rewarding tasks.

The requirements of a basic system of traverse control would therefore seem to be:

- (1) Suitability for attachment to existing machines with minimum of modification.
- (2) Traverse rate as much in excess of 12"/min as is reasonably possible.
- (3) Accuracy better than  $\pm 0.003''$  without special precautions, and better than  $\pm 0.0005''$  with reasonable additional precautions.
- (4) Displacement of work from datum to be directly presented in easily read decimal notation.
- (5) Datum-shifting facilities desirable.
- (6) Facilities for hand operation to be retained if possible.
- (7) Equipment should require the minimum of technical skill for maintenance.
- (8) Design to be flexible enough to permit of later development beyond these minimum requirements.

## 2. Implementing the Design

### 2.1. Use of the Lead-screw as the Measuring Element

The first requirement, that of the widest possible adaptability to existing machines, renders the use of the lead-screw as the measuring element almost inevitable, as there can be no certainty that any given design of separate measuring system will be suitable for more than a very few types of machine. The decision to use the lead-screw for measuring purposes has a direct bearing on the next two requirements of the specification, in the first place because excessive wear would be likely to occur to screws and nuts if very high traverse rates were attempted in installations not specially designed for them. It was therefore decided on the advice of machine tool

makers to aim at an average traverse speed of 24 inches per minute. It also means that the absolute accuracy obtainable is limited to that of the lead-screw unless special refinements such as calibration or built-in correction systems are employed, but the *repeat* accuracy can still be as great as the resolution of the servo-system will permit, and this should therefore be as high as is reasonably possible.

The errors which occur when the machine lead-screw is used as the measuring element arise from irregularities of pitch in the screw itself, and from irregular wear. Wear in the nut and in thrust bearings, etc., in general only increases the amount of backlash in the system, and the effects of this can be entirely eliminated by ensuring that the final approach to any desired position is always made in the same direction. Even a new lead-screw may have both cyclic and progressive errors of pitch, both of which can be corrected for by providing for the storage of corrective information in the control system. Errors due to wear are less easily handled, because they tend to increase to a maximum in the most used central part of the traverse, and reduce to minima at each extreme. A little thought will show that a screw which is so worn will produce errors of opposite sign according as movement is towards, or away from, the region of maximum wear. The resultant of these errors, even if no corrections are applied, is, however, unlikely to exceed  $\pm 0.003$  in. over a reasonable life if the machine is used solely for operations in which the screw does no work against cutter forces.

It cannot be too strongly emphasized that these defects of the screw affect the absolute accuracy (i.e. correspondence between "observed" and "predicted" dimension) only. The *repeat* accuracy is controlled by the resolution of the servo system and the general rigidity of the machine, so that it is possible to achieve an absolute accuracy of the same order as the repeat accuracy by using calibrated or precision lead-screws, with or without the addition of correction facilities in the information store. From the point of view of the normal decimal representation of dimensions, a 10 turns/in. lead-screw is the most convenient, but pitches of 4, 5, 6 and 8 turns/in. are used, so that a general purpose system should cater for these

as well. It was decided, therefore, to design a device which would be directly applicable to 10 turns/in. screws, and to couple to the screw through appropriate reduction gearing in other cases. Such gearing may in itself introduce backlash and cyclic errors, but correction for these can be fairly readily made. The basic requirements of the servo system now become clear: it must be capable of counting and identifying revolutions of its output shaft up to a number equal to at least ten times the maximum distance to be traversed, and of locating itself within a selected revolution to an accuracy determined by the projected tolerance limitation. For the device under consideration, a figure of  $\pm 0.0002$  in. was proposed which, under the conditions already described, corresponds to a resolution of 1/500th turn at the output shaft. This order of accuracy at once implies that it will be necessary to consider the effects of torsional deflection of shafts, of "stiction," and of table "creep." All these factors have considerable relevance to the choice of servo system.

## 2.2. *The Servo System*

The major design requirements which have so far emerged are that the servo system must be able to count revolutions and to resolve any given revolution to 1 part in 500. Many servo systems exist which are capable of this order of resolution or better, but in view of the requirement that the system should be capable of fitting to the widest possible range of machines (implying wide variation in mechanical power requirements) there is an obvious preference for one which employs conventional motors rather than special servo-motors. Additionally, wide variations of inertia and mechanical backlash can be anticipated, so that an "open loop" system, which does not contain these variables within its feedback path (if any), is likely to have advantages over the "closed loop" type. From earlier experience, the author was aware that a d.c. Wheatstone bridge servo system without amplifiers, working from a 25–30 volt d.c. supply, was capable of the desired resolution if backlash correction arrangements were provided, and this was therefore chosen as the basis of the design.

Consideration of the planned traverse speed of 24 in./min. in conjunction with the torque

required to rotate the lead-screw of a number of typical machines examined made it apparent that a motor of at least  $\frac{1}{4}$  h.p. would be required, and, allowing a safety margin for possible transmission losses,  $\frac{1}{2}$  h.p. would probably be more realistic. It was immediately apparent that it would not be possible to stop such a motor rapidly enough for a system which had an average speed of 24 in./min to have also a resolution of 0.0002 in. This would be true even of a proportional servo, in which the velocity decreases as the error decreases: it is even more true of a "bang-bang" servo. It was therefore necessary to provide either an elaborate speed control arrangement—probably involving a specially wound motor—or to introduce a dramatic—and discontinuous—reduction of speed as the designed rest position was approached. The latter method was chosen, and arrangements were made for the signal for the change of speed to be given when the system reached the particular lead-screw revolution in which the selected position lay. Early light duty models employed a magnetically operated two-speed epicyclic gearbox to effect the speed change, but this was regarded as likely to become troublesome in heavy duty applications, and replaced by an arrangement which employs an auxiliary motor.

Because of the ease with which regenerative braking can be applied, a permanent magnet, low-voltage d.c. motor was selected as the auxiliary, and the relatively low torque output made it necessary to employ a considerable reduction ratio between it and the output shaft which, in fact, it drives at 3 to 5 rev/min. It was considered undesirable to allow the main motor to drive the auxiliary motor back through its reduction gear train, but at the same time it was desired to avoid the use of clutches, and a differential system was therefore designed which permits either motor to drive the output shaft without driving the other. The signal for change of motor is derived as before, from the circuits which identify the lead-screw revolution in which the selected position lies.

Even with this arrangement, and taking full advantage of the facilities for regenerative braking, it was found impossible to achieve a resolution of 0.0002 in. without instability in applications of the system to light machines,



and arrangements were introduced to cause the motor to change from "continuous" to "impulsive" operation over the last few "thou." of its travel. An incidental advantage accrues from this, namely that the impulsive operation is valuable in breaking down "stiction" effects which can give rise to errors when surfaces between which considerable friction exists are given a relative motion of low velocity. The system was in fact found to be capable of making position changes as small as 0.0001 in. directly, and not only as part of a larger excursion.

Although the "impulse" arrangement is retained, however, the present system is designed to make a larger excursion in reverse as a preliminary to any small programmed excursion forward. The reason for this is concerned partly with the mode of operation of the counting or "turn identification" system employed in the control unit, and partly with the avoidance of errors due to "creep." Because of the existence of backlash in the traverses, there is always a possibility that the work will be displaced from its programmed position by forces produced by the discontinuous cutting faces of the drill or other tool, and the system therefore provides for the slides to be locked by electrically-operated clamps during the piercing operation. In general, this means that torsion and compression stresses exist in the lead-screw and other moving parts at the moment when the clamps operate. On releasing the clamps, these stresses tend to release spontaneously, and can produce a shift of 0.0003 in. to 0.0005 in. For consistent results, therefore, these stresses must be re-introduced in any subsequent displacement, and circuits are provided to ensure that, unless the "programme" calls for a displacement exceeding 0.002 in., the drive will start up in reverse on being switched on, and then run forward again to restore the normal stresses.

The drive units required for the system are simple and compact, each measuring only 9 in. × 9 in. × 4½ in., inclusive of motors. The servo elements are directly coupled to the drive unit output shaft, and consist of the high-resolution wire-wound potentiometers used as the slave pair of arms for the Wheatstone bridge servo employed to position the output

shaft accurately within the selected turn, and the counter mechanism utilized to identify that turn. The high resolution potentiometer is wound over 360 deg., leaving only a gap equal in width to one turn of wire, to permit of the necessary insulation between the two ends of the winding. A specially designed roller wiper is employed to minimize wear, but preserve a small degree of skidding so that self-cleaning action is not entirely lost. To the wiper is connected a dial calibrated from 0 to 100 thousands of an inch, and this is arranged to operate the final dial of a conventional mechanical three digit decade counter once in each revolution.

The dials of this counter therefore indicate a total of 999 revolutions of the output shaft, or 99.9 in. of movement. Each dial is mechanically coupled to the wiper of a 360 deg. potentiometer of low resolution, and these are combined with decade potentiometers in the Control Unit to form the additional Wheatstone bridge systems used to set up and identify the desired revolution. A lower supply voltage (5 volts), and a reduced detector sensitivity, is used for these bridges, which require a resolution of only 1 part in 20 to ensure that the system is definitely unbalanced if the dials are half way between two digits.

Refinements which have been incorporated include a special arrangement for uncoupling the measuring system from the drive, setting it to zero, and re-coupling, so that any position of the table may be chosen as datum, and for reversing the sense of the coupling, so that either direction of traverse movement may be chosen as "positive." Push-button control over the motors, both at high and low speed, is incorporated to facilitate the initial setting up of the work to a datum position, and these arrangements are supplemented by the provision of a manual "fine setting" control giving 0.002 in. displacement per revolution. Alternatively, a hand-wheel may be used for setting up. Insertion of the hand-wheel automatically renders the driving motors inoperative, to prevent danger to the operator if the wheel should be left in position when automatic operation is selected.

The motors are also interlocked with the tool feed, so that they cannot be accidentally

started while the tool is in the work, and a torque limiting clutch is provided which is designed to stop the motors and disconnect the drive if the motion of the table is arrested by any accidental obstruction.

### 2.3. *The Control Unit*

Because motion of the work table is required in two dimensions, necessitating the use of two drive units, the normal control unit provides the elements necessary to control these two drive units, together with the necessary power supply arrangements, and contactors for the main motors, in a single case.

The case, which measures 15 in.  $\times$  12 in.  $\times$  12 in., is arranged for fitting directly to the machine or mounting on an independent stand as preferred, and the electrical connections between the control unit and the drive units are made in conventional conduit, flexible sections being used where movement is required. The standard arrangement provides for traverses up to 100 in., the control information for each traverse being set up on a six-decade dial system, which is direct reading in inches and decimals to the nearest 0.0001 in.

It has already been stated that the accuracy requirement laid down can be met by a Wheatstone bridge servo system without amplifiers, in association with a pre-set revolution counter. Since this counter is itself operated on the Wheatstone bridge principle, it follows that no amplifiers are required, and therefore no valves are employed. The control unit in fact contains only relays, potentiometers, indicating lamps and power supply arrangements. Although the dimension required is set up as six digits, there are in fact only four Wheatstone bridges, three of which handle one digit each, while the final high-resolution bridge handles the remaining three digits, treating them as one composite quantity. This arrangement was adopted because it was considered undesirable to attempt to run any of the potentiometers in the measuring unit at a speed greater than that of the output shaft.

For reasons associated with the ambiguities which can occur in conventional mechanical counters around the "carry" position—that is, when any one dial is moving from "9" to "0" or vice versa, the first three digits are examined sequentially, and this makes it possible to

employ the same potentiometer in the control unit, in association with three independent tapping switches, to provide the master ratio arms for the first three digits of each dimension. The high power motors are employed to carry out the table traversing movements associated with the first three digits of the desired displacement from the datum. Control is then transferred to the auxiliary motor which carries out the movements dictated by the remaining three digits. In order to provide for decade presentation, each of these last three digits is associated with an independent ten step potentiometer; the voltages from these are correctly "weighted" and added in series for comparison with the voltage at the moving contact of the high resolution potentiometer coupled to the output shaft of the drive unit. As the method of "weighting" employed is somewhat unusual, a more detailed account of it will be given later, in the section devoted to punched card control.

The relays associated with each traverse are divided into two groups. The first group consists of the sealed medium duty relays used for sequencing purposes, which are mounted, together with the sensitive centre stable polarized relay used in the "detector" arm of the Wheatstone bridge, on a detachable sub-chassis to facilitate servicing by replacement. The remaining, heavy duty, unsealed relays and contactors used for the control of the motors, and for interlocking purposes, are grouped together on the main chassis, together with the power transformer and metal rectifier units. The main supply switch, "Auto-Manual" selection switch, indicator lamps, starting push button, and fuses, are grouped conveniently on the front panel below the two rows of six decade dials used for setting up. Each dial is separately illuminated from behind and viewed through a window which displays only the digit selected. A block diagram for one traverse is given in Fig. 1.

The method of operation will be described for simplicity as if only one traverse were operating, although in practice, both traverses operate simultaneously. The system is switched on and set to "Auto," and the dimension required is set up on the six dials appropriate to the particular traverse, all of which are illuminated while the system is at

## POSITION CONTROL FOR TRAVERSING TABLES

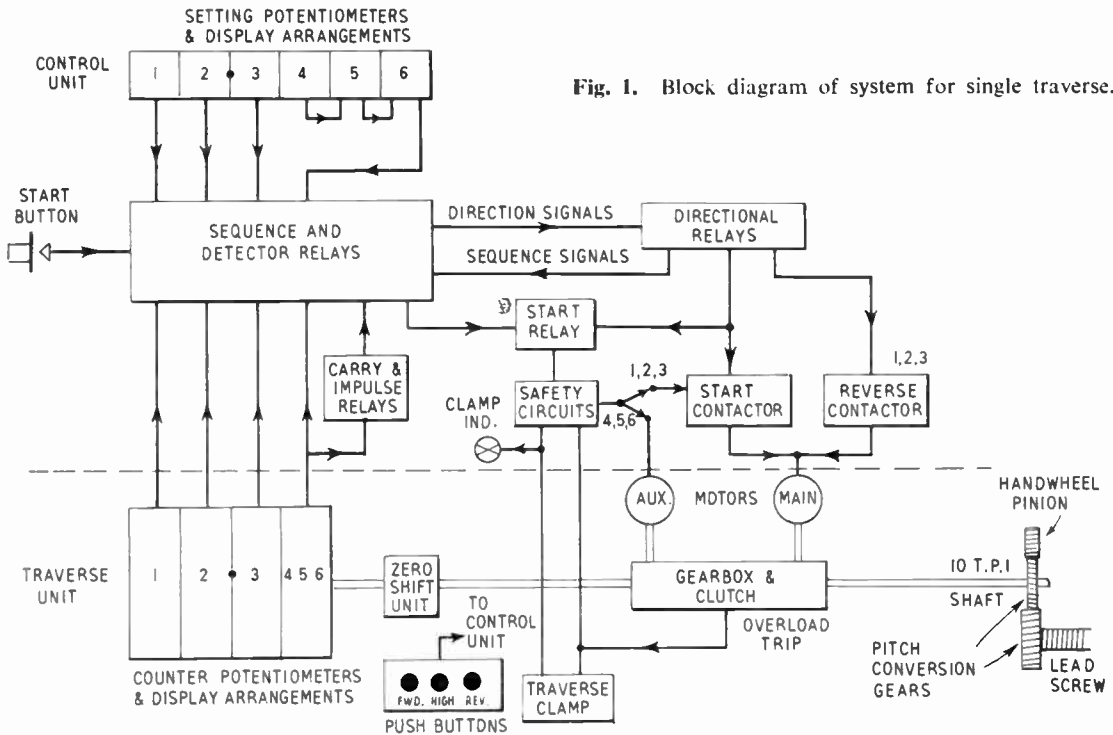


Fig. 1. Block diagram of system for single traverse.

rest. On pressing the "Start" button, the sequencing circuits operate to select the first digit, i.e. "tens of inches." If, and only if, these circuits have functioned correctly, the "start" relay operates to release the table clamps and, provided proving circuits verify that these are "off," the main motor contactor circuit is made. If the dimension called for involves a change in the first digit from that corresponding to the existing drive unit position, the main motor will start in the appropriate direction to correct the discrepancy. During this operation, the lights behind all dials but the first are extinguished to show that this section of the system is in control.

When this first Wheatstone bridge reaches its balance condition and the detector relay drops out, a signal is passed to the sequencing circuit, which selects the next digit ("inches") which in turn is illuminated alone. The main motor continues to run, under control of this second bridge, until it in turn balances, when the drop-out of the detector relay is again utilized to provide the signal for the sequence circuits to transfer control to the third digit, i.e. "tenths of an inch." Again, this digit alone is

illuminated while the main motor operates to set the associated element in the drive unit to correspond.

When the third bridge balances, the sequencing circuit receives a signal which causes it to switch off the main motor and apply heavy regenerative braking by injecting d.c. into the field and short-circuiting the armature. This is necessary to ensure that inertia cannot cause a run-on in excess of a tenth of an inch, which would be undetected since the associated bridge is disconnected as soon as it balances, to enable the final high resolution system to take over. Apart from the economy in sensitive relays which results from the process of switching the same relay from bridge to bridge which the system employs, it would be quite impracticable, for reasons associated with the ambiguity of the counters in the "carry" position which has already been mentioned, to allow all the bridges to remain connected until the end of the operation.

After the balancing of the third bridge, and disconnection of the main motor, the auxiliary motor is selected, and runs to complete the balancing of the *sum* of the voltages selected by

the last three dials against that of the wiper of the final potentiometer in the measuring head. Since the settings of these dials are in fact treated as a composite quantity, it is arranged that all three are illuminated during this operation. As the system nears the balance position a "backing-off" potential, derived from the d.c. power supply used for relay operation, is injected into the detector arm to cause a premature apparent balance. After this apparent balance point is reached, the small d.c. motor is operated impulsively until true balance is reached. This arrangement increases the effective resolution, making it dependent only on the constancy of the "drop-out" current of the detector relay, and also helps to break down "stiction" effects.

The period of impulsive operation of the motor is produced by impulsive operation of the motor controlling relays, and therefore precautions have to be taken to ensure that a false "operation complete" signal is not given before their final drop-out. This is done by providing a one or two second delay of release of the "start" relay, which remains operated until both motor control relays have released and remained released for a period greater than this, and this condition is, of course, not fulfilled until final balance is reached.

Once this occurs, the "start" relay releases, applies the traverse clamps, cuts the motor power supplies as a safety precaution, restores illumination to all the dials, and switches on an indicator lamp to indicate that the desired position has been reached and that the tool can be brought down into the work. Alternatively, if power feed of the tool is fitted, it can directly operate this.

Once the system has reached this state, the operator can, if he wishes, commence to set the dials to the dimensions required for the next operation, as the motor circuits remain locked out until the tool has returned to its rest position, even if the "start" button is accidentally operated—although in fact a guard is fitted to prevent this.

In the description so far it has been assumed that the displacement required was in the "reference" direction, i.e. distance from the datum increasing. The operation in the opposite case (distance decreasing) is identical

except for the final stage, in which a voltage derived from the main d.c. supply is injected into the detector circuit in such a sense as to delay balance, so that the drive continues beyond the designed rest position. When the "delayed" balance point is reached, this voltage is removed, and the system then returns to the "designed" rest position in the normal way. Provided that the overshoot is greater than the total backlash of the system, errors due to that cause are therefore entirely eliminated. Arrangements are provided for pre-setting the injected voltage to a value which ensures that this requirement is met.

Mention should also, perhaps, be made of two important branch circuits, namely the "carry" and "impulse" circuits. As has already been stated, the potentiometers in the measuring section of the drive unit which form the "slave" arms of the Wheatstone bridge systems are wound over 360 deg., and their high and low potential ends are therefore very close together—so close, in fact, that they are short-circuited by the wiper once in each revolution. The point at which this occurs is arranged to lie between "9" and "0" of the associated counter dial, because this is the "carry" state: i.e. the counter corresponding to the next more significant digit is in a position midway between two digits, and its associated bridge is therefore necessarily unbalanced. This arrangement, in conjunction with the method of sequencing from the most significant digit towards the least, ensures that the wiper of each potentiometer must be clear of the shorting position before it is allowed to come into operation.

There is, however, a difficulty in the case of the final (high resolution) potentiometer. Once it has entered the revolution selected by the counter system, it must not be able to leave it, as a result of either accidental or designed overshoot. The small gap between the high and low potential ends of the winding, in fact, represents the boundary of its permitted excursion. To ensure that it does not exceed the limit when running in the direction of increasing dimension is merely a question of servo system design, but, if the overshoot method of backlash correction is to be effective under all conditions, the wiper will certainly cross the boundary "in reverse" whenever the

last three significant figures of the dimension represent a displacement less than the backlash correction.

The function of the "carry" circuit is to note that the wiper has, in fact, crossed the boundary, to allow it to run far enough beyond to provide adequate backlash correction in respect of the designed rest position, and then to bring it forward again to the designed position in the normal way. The "carry" circuit is brought into operation, and released, by the abrupt change of potential (the sign of which depends on the direction of motion) which occurs when the wiper crosses the boundary. The circuit employs two coils in the detector relay: to one is applied a voltage just sufficient to operate it, while to the other (in parallel aiding) is applied a voltage proportional to the last three digits. This arrangement ensures that the overshoot is regulated by the designed setting in this as in all other circumstances.

The "impulse" circuit is necessary because of the methods used to enhance the sensitivity and stability of the system beyond the limits which are normally imposed by the detector relay. In normal use, a Wheatstone bridge servo system of this nature has a "dead arc" whose magnitude is controlled by the minimum operating current of the detector relay: for the particular components used this dead arc normally represents about 0.0008 in., but varies from relay to relay. But by arranging that the driving system, having approached smoothly to within about 0.002 in. of the designed position, commences to impulse towards it in a series of steps of 0.0001 in. or less, the rest position is determined by the current at which the detector relay just drops out. Consequently, resolution of the order of 0.0002 in. is achieved, and the calibration errors which would otherwise result from different relay characteristics can be removed by introducing a small adjustable biasing potential.

The effect would, however, be lost on any occasion when the wiper chanced to be in the "dead arc" at the moment of switching on the final bridge, and the "impulse" circuit is used to produce an artificial displacement under such circumstances. A relay, operated by capacitor discharge, is momentarily energized when the final bridge is selected, and injects

into the detector relay circuit a small reverse voltage. Therefore, unless a larger forward voltage is already being offered by the normal control circuits, the device makes a reverse excursion large enough to take up any backlash, and then selects the rest position in the normal manner. The arrangement also has the effect, already noted, of ensuring that mechanical parts are normally stressed at the commencement of each operation.

So has been evolved an equipment which certainly meets the first six requirements laid down in Section 1.3. The seventh requirement concerns maintenance. As to how successfully that has been met the user must be final judge. How much skill, over and above that which a factory electrician would normally be expected to possess is, in fact, required to maintain a box of relays? Wheatstone bridges he may fairly be expected to understand, and facilities for checking whether these, and the sequencing circuits, are functioning correctly, have been built into the equipment. The sealed relays, which cannot be checked by visual inspection, are mounted together on detachable decks so that substitution checks can be made (by exchanging decks within the unit if a spare is not available). Such complication as does exist—in contradistinction to much electronic equipment—arises from the number of circuits rather than their nature. There is, in fact, scarcely a fault possible in the system which cannot be located by conscientious and systematic continuity checking, once the method of operation is understood. Finally, although it cannot be claimed that no faults can arise and remain undetected until revealed by spoilt work, it is considered that in a purely electrical system such faults will be rare, and their possibility does not justify the introduction of the very complex monitoring system which would be necessary to exclude it completely.

### 3. Future Development

#### 3.1. Punched Card Control

The eighth requirement of the specification was that the system should be sufficiently flexible to permit further development, and it is certainly envisaged that facilities for carrying out a work programme will be required almost immediately. Since the information required for the system is static in form, a card type

of store appears more logical than film or tape, although the latter would be capable of containing more data. Moreover, unless some form of automatic tool changing mechanism is designed, it will be necessary for an operator to be present to carry out tool changing operations, although he may very well be able to tend several machines. Consequently, there is little point in having a vast amount of information available in continuous form if its flow has to be interrupted while tool changes take place, and it is felt that an arrangement based on the use of one card per hole size would be appropriate. The operator would insert the required tool and the card associated with it, and set the system in motion, whereupon it would run until the information on the card was exhausted and then stop.

Since it would be quite practicable to accommodate the data for up to 12 holes on a card, the number of occasions when one card is insufficient to contain the data for all the holes of one size is likely to be small, and in most cases the stopping of the machine would be the signal for both tool and card to be changed. Where more than 12 holes of one size were required in the same work piece, additional cards could obviously be used, with arrangements for hopper feeding if necessary. Conversely, more than one hole size could be dealt with by a single card, up to a total of 12 holes, simply by using code perforations in the margins which would stop the machine for tool changing at the appropriate positions.

Since the positional information required by the system is presented to it on a digit-by-digit basis, and takes the form of d.c. potentials whose magnitude is proportional to the number represented, all that is needed in a card reader unit is a group of tapped voltage sources. In the first three decades, ten voltage steps are required, one for each of the digits 0 to 9, and these can be obtained from a common source, the punched hole in the card serving, by its position, to allow the correct one of a group of ten feelers to pass through and make contact with the appropriate tap. The last three digits, however, are treated by the servo system as a single group, representing as many parts in one thousand of the bridge supply voltage, and special treatment is necessary if single-hole

representation of each digit is to be preserved.

The problem of producing a potential difference which is divided accurately into a thousand or more equal steps without recourse to such unpractical means as the use of 1,000 or more equal resistors, or a smaller number arranged in groups whose ratios must be closely controlled, has been studied by many workers, and, when the problem of designing a punched card reader for the system was first considered, the proposal was made to deal with the last three digits on the Kelvin-Varley potentiometer principle. This arrangement has the disadvantage, however, of requiring two moving contacts for every digit except the last, and so would have necessitated double punching for two of the three digits here involved. Further, the resistance of the moving contacts is partially contained in the ratio arms, so that any variation alters the calibration, and it was considered unlikely that contacts sufficiently light to be operated by a punched card could be made to have resistance low and constant enough for such an application.

The intention to use punched card control in the future was therefore directly responsible for the design arrangements chosen for the last three decades in the manually-operated control unit, which operates as follows. A d.c. source of about 25 volts supplies the "slave" potentiometer and a ten-step potentiometer used in the control unit to set up the first of this group of three digits. The ten steps are obtained from pre-settable taps on a single winding, which can be adjusted to equality, so avoiding the use of close-toleranced resistors. In series between the tapping point of this potentiometer and the detector relay is connected the voltage derived from a conventional two-decade Crompton potentiometer, fed from a separate d.c. source. This is used to provide the voltage representing the remaining two digits, the resistances of the "coarse" and "fine" settings being in the ratio 10:1 to provide the correct decimal "weighting." Again the resistances are adjusted to the correct values and ratios by pre-settable sliders to avoid the necessity for closely toleranced resistors, and the conventional rheostat is used to adjust each major voltage step to one-tenth of that corresponding to a shift of one digit in the previous decade.

Thus an arrangement is provided in which three single-pole ten-way switches can select respectively tenths, hundreds and thousands of the voltage applied to the "slave" potentiometer, and apply the sum of the selected voltages to the detector relay. Further, as in standard potentiometer practice, any contact resistance appears in series with the detector arm, and so does not affect calibration, a fact which is of major importance in a card reader tap selector.

Calibration will, however, be affected by variation of the relative magnitude of the two d.c. voltages the arrangement employs. Since the d.c. source used to supply the potentiometer controlling the first of the three digits is also used for the "slave," variations in its voltage affect sensitivity only, provided the voltage of the other source varies in the same manner. For this reason, the two voltages are derived from full-wave bridge rectifiers fed from separate secondaries on the same transformer. The rectifiers are of the same type, and the resistance values are chosen to ensure that they are equally loaded, so that supply voltage variations may be expected to change both d.c. outputs in the same proportion. Checking and correction facilities are built in so that the user can compensate (by an adjustment which can be made in a few minutes without any other equipment) for any slight calibration drift due to differential "ageing" of the rectifiers.

Virtually, therefore, the punched card control system merely substitutes cards and feelers for rotary switches as a means of inserting information; the remaining circuitry remains unchanged, apart from the additions necessary to arrange for the step-by-step feeding of the card through the reader, and the stopping of the machine when all the information on a particular card has been used.

### 3.2. "Progressive" Operations

It will be apparent from what has already been said that the system as it stands is quite unsuitable for profiling work. In fact, no device which does not employ a scale effectively fixed to the work could be used for such purposes except on a machine which was absolutely free of backlash. No doubt a system working on the same basic principles as have already been described could be designed to

measure the actual work position, but would inevitably involve a number of additional fittings, and in any case, the method of information presentation is not well suited to this class of work.

There is, however, a type of operation associated with such processes as milling, slotting, grinding, turning, and drilling or boring to controlled depth, in which it is required only to make a direct linear traverse from point A to point B. For this only start, stop, and feed-rate information is required. If the work can be so arranged that the component(s) of cutter reaction in the feed direction(s) do not change sign, so that play in the traversing arrangements is of no importance, the system described can be modified to enable it to carry out these processes (the "transition" class) also, either under operator or punched card control.

In the basic system so far described, the method by which the final rest position is reached is of no consequence: in particular, "overshoot and return" methods are acceptable and are in fact used both to eliminate the effects of backlash and to resolve the ambiguities associated with the "carry" states of the counter. Modifications are necessary before the system can be used for the "transition" class of operation simply because overshoot now becomes inadmissible. It would be inappropriate to discuss the modifications in detail, but it may be stated broadly that a changed counter design becomes necessary to eliminate the "carry" ambiguities in the first three decades, and the domain of the high resolution system used for the last three digits, needs to be extended beyond 360 deg. to overcome the rather special "carry" difficulties already described in connection with it. These changes, and the necessary introduction of feed-rate control, inevitably add somewhat to the complexity of the system, but in no way alter the basic principles, and leave it still vastly simpler to instal and maintain than its computer-operated counterparts.

### 4. Acknowledgments

The author is indebted to the Directors of E. K. Cole Ltd. for permission to publish this paper.

621.317.382.029.6  
**On precision measurement of the noise temperature of two-terminal networks at decimetre wavelengths by substitution.** L. MOLLWO. *Archiv der Elek'rischen Übertragung*, 11, pp. 295-306, July 1957.

With a connection of different two-terminal networks, the indication at the output of a noise measuring setup is determined not by noise temperatures alone, but by the matching to the input and its noise temperature. General formulae are given for the measuring errors caused by impedance mismatch; they contain separately the matching data between two-terminal network and line system and those between input and line system, as well as the line lengths. The measuring errors can be considerably reduced by averaging two measurements taken with line lengths differing by one quarter wave. The discussion relates in particular to radio-astronomical measuring equipment. The transforming effects in a noise generator with a saturated diode are similarly considered for the general case of a mismatched load. For measuring the series resonance of the diode system a new method is given where no disturbing action is required inside the generator.

621.372.622:621.397.61  
**Broad band diplexer for television transmitters.** R. CHESNAU. *Onde Electrique*, 37, pp. 640-645, July 1957.

The author presents a study of a diplexer for television broadcasting, which makes it possible to use a single feeder and a common aerial for sound and vision transmitters. The reasons for the choice of wide band elements are given. The diplexer constructed is described and its characteristics are given.

621.373.421:621.372.54  
**Study of RC-oscillators with T selective circuits and the problems involved in their design.** R. STERE. *Automatica si electronica (Bucharest)*, 1, pp. 3-14, February 1957.

An RC type oscillator with a T selective circuit is analysed to determine theoretically the oscillation condition, the amplitude stability, the frequency stability, the distortion and the frequency band to be covered. From the analysis presented indications are given concerning the optimal working conditions, accompanied by a designing method illustrated with a practical example. Precautions to be taken in constructing the oscillator are suggested and the results obtained in the experimental tests given.

621.383.2:535.376  
**Electroluminescence and image intensification.** G. DIEMER, H. A. KLASSENS and P. ZALM. *Philips Technical Review*, 19, pp. 1-11, No. 1, July 1957.

The effect of electroluminescence, exhibited by layers of specially prepared ZnS powder activated by copper, may be usefully applied for dial illumination and the like. Electroluminescent panels are not very promising for general lighting purposes: high efficiency and high emittance can at present be obtained only with an electrical supply in the kilocycle range and using layers emitting green light. A mechanism accounting for the more important details of the complicated phenomenon of electroluminescence is presented in this article. A brief account is given of the employment of the phenomena in solid-state image intensifiers (amplificons) which may find important applications in radar, X-ray fluoroscopy, etc.

*A selection of abstracts from European and Commonwealth journals received in the Library of the Institution. All papers are in the language of the country of origin of the journal unless otherwise stated. The Institution regrets that translations cannot be supplied.*

621.395.34  
**The international position in the technique of electronic telephone exchanges.** K. STEINBUCH. *Nachrichtentechnische Zeitschrift*, 10, pp. 225-343, July 1957.

The most important methods used in the engineering of electronically operated exchanges (purely electronic low-frequency systems, time multiplex systems and electro-mechanically operated systems), are briefly described. With the present day knowledge, each of these systems permits the design of large exchange installations with technical advantages (e.g. reduction in volume, low maintenance costs, fast switching of connections). It is however doubtful whether these systems will be economically competitive within the next few years.

621.396.677:621.397.61  
**The directional panel antenna and its application to television transmitters.** S. DRABOWITZ. *Onde Electrique*, 37, pp. 625-639, July 1957.

The author studies new radiating structures and the associated feeder equipment, with special reference to the use of the directional panel with slots for producing an antenna with a specified polar diagram. The feeder elements include a variable power dividing network, and a broad band junction by means of which the shape of the polar diagram of a television antenna can be continuously varied according to actual reception results.

621.396.933.4:681.84.083.8  
**Multiple-track tape recorders in air traffic control (ATC).** K. HEIDELAU. *Nachrichtentechnische Zeitschrift*, 10, pp. 344-348, July 1957.

Radio-telephone and telephone conservations in ATC should be registered for documentation. Tape recording and play-back equipment can be used for this purpose. The fundamental aspects for the technical design of the equipment are described by way of example of a multiple-track magnetic tape recorder used in ATC for this purpose.

621.397.5  
**Industrial colour television system.** J. PERILHOU. *Onde Electrique*, 37, pp. 679-687, July 1957.

The author first reviews technical and economic factors that arise in the problem of colour television. Systems so far proposed and developed lead to receivers that are considerably dearer than black and white receivers. A system of colour television is then put forward with mixed line-frame sequence, where only two colours per frame are transmitted. Green is present permanently, red and blue appearing in alternate frames; the two colours per frame are transmitted by line sequence, producing a video signal analogous to that in black and white practice. This allows receivers to be made with only two or even one projection tube, under very simple and economical conditions. Two practical examples are then described. Some technical and economic observations allow future applications to be estimated.



# THEORY OF THE HELICAL WAVEGUIDE OF RECTANGULAR CROSS-SECTION\*

by

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## SUMMARY

The helical waveguide may be regarded as being formed by rotating a rectangle about a line, at the same time moving it parallel to the line. If the motion parallel to the line is omitted, the figure obtained is circular in form, but points which differ in azimuth by  $2\pi$  are not equivalent, and infinite azimuthal angles become possible. This figure is called the infinite circular guide; it cannot exist physically, but for purposes of calculation may be taken as an approximation to the helical guide. An exact treatment of the infinite circular guide is given; this treatment is also relevant to the problem of the circular waveguide bend. Perturbation theory is then used to find the correction that must be made to the value of the propagation constant for the infinite circular guide to give the value appropriate to the helical guide. It is found, in fact, that the correction is zero.

## LIST OF PRINCIPAL SYMBOLS

$r, \theta, z$	Cylindrical co-ordinate system.	$V_0$	Volume of infinite circular guide between two azimuthal planes having angular separation $2\pi/\beta$ .
$\mathbf{l}, \mathbf{m}, \mathbf{n}$	Unit vectors parallel to the co-ordinate axes.	$V_1$	Volume lost from the "floor" of the guide on shearing to the helical form.
$\omega$	Working frequency.	$V_2$	Volume gained at the "ceiling" of the guide on shearing to the helical form.
$\mathbf{E}_0, \mathbf{D}_0, \mathbf{H}_0, \mathbf{B}_0$	Electric and magnetic fields and inductions in the infinite circular guide.	$\epsilon_0, \mu_0$	Permittivity and permeability, respectively, of free space.
$\mathbf{E}_1, \mathbf{D}_1, \mathbf{H}_1, \mathbf{B}_1$	Fields and inductions which, when added to $\mathbf{E}_0, \mathbf{D}_0, \mathbf{H}_0$ , and $\mathbf{B}_0$ , give the fields in the helical guide.	$\kappa$ or $\kappa$	Constant giving $z$ -dependence of electromagnetic fields.
$\mathbf{E}_r, \mathbf{E}_\theta, \mathbf{E}_z$	Electric field components in the infinite circular guide.	$k$	$= \omega^2 \epsilon_0 \mu_0 - \kappa^2$ .
$\mathbf{H}_r, \mathbf{H}_\theta, \mathbf{H}_z$	Magnetic field components in the infinite circular guide.	$\beta$	Angular propagation, or more properly phase, constant.
$a, b$	External and internal radii, respectively, of the infinite circular and helical guides.	$\lambda_0$	Free-space wavelength.
$c$	Dimension of the infinite circular and helical guides parallel to the $z$ -axis.	$X_\beta(x) = J_\beta(x) \cdot Y_\beta(ka) - J_\beta(ka) \cdot Y_\beta(x)$	
		$Z_\beta(x) = J_\beta(x) \cdot Y'_\beta(ka) - J'_\beta(ka) \cdot Y_\beta(x)$	
$p$	Pitch of the helical guide.	$\nu, i$	Integers designating orders of modes.

## 1. Introduction

The helical waveguide to be discussed in the present paper is illustrated in Fig. 1. It is a guide of rectangular cross-section such that the locus of any point in this cross-section is a helix. The geometry is described accurately in

Section 2. An exact treatment of such a guide would be a formidable task, but if the pitch is small a good approximation can be made by considering the infinite circular guide, which is a guide whose axis is uniformly curved, but in

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which we adopt the mathematical fiction that points at angular separations of  $2\pi$  are not equivalent, so that azimuthal angles of any value between  $-\infty$  and  $+\infty$  become possible. The geometry of this guide is also described accurately in Section 2.

Where the pitch of the helical guide is not so small, it may be regarded as a distorted form of the infinite circular guide, and perturbation theory is used in Section 4 to determine the correction which must be applied to the propagation constant for the infinite circular guide to give the value proper to the helical guide. It will be found, in fact, that the correction is zero, i.e. the propagation constant is independent of the pitch.

Very little attention has been paid in the past to the theory of curved waveguides, although curved bends are widely applied. Jouguet<sup>1</sup> has treated the case of a curved guide in the form of a circular arc of large radius of curvature by regarding it as a perturbation of a straight guide. This treatment applies essentially to the case of small curvature, whereas in the present paper we are concerned with curvatures which may be arbitrarily small or large. Lewin<sup>2</sup> gives a method of calculation of the propagation constant in a circular bend by a series expansion in terms of the mean radius of curvature of the bend. This treatment is not exact, but by taking enough terms of the series a value for the propagation constant can be found to any desired accuracy. Formulae are also given for the  $z$  components (according to the notation of the present paper) of electric field in the case of an  $H$ -plane bend, and of magnetic field in the case of an  $E$ -plane bend; however, no formulae are given for the other field components, and this treatment does not allow a detailed study of the modes. There is also the disadvantage, from the point of view of the present paper, that it is not possible to derive a correction, by Lewin's method, which can be applied to the propagation constant for the infinite circular guide to give that for the helical guide.

Buchholtz<sup>3</sup> has also given a treatment of waveguide bends. He gives the characteristic equation in the exact form, as in the present paper, and then solves it by a series expansion, finally arriving at an approximate explicit

formula for the propagation constant which holds only if the radius of curvature is large compared with the dimensions of the guide cross-section. He also discusses in detail the relationship of the modes in the curved waveguide with the Längsschnittwellen (longitudinal-section waves) in a straight waveguide.

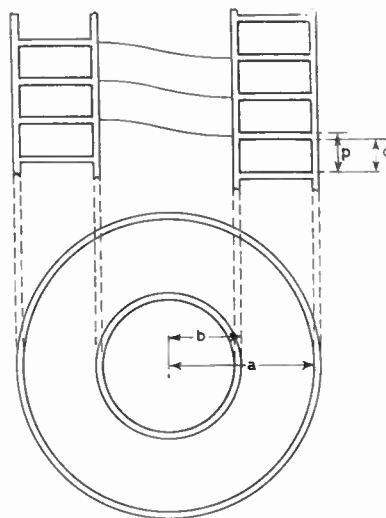


Fig. 1. The helical waveguide.

In the present paper we shall be concerned with small radii of curvature, to which Jouguet's and Buchholtz's treatments are not applicable. It would be possible to use Lewin's method to calculate the propagation constant, but as the curvature became sharper more terms of the series would have to be taken, and the calculation would become tedious; even for small curvatures the method given in the present paper is simpler to use. This method is exact in principle, but the tables required for the calculation have only been computed to a limited accuracy which will, however, be sufficient for most practical purposes.

## 2. Geometry of the Helical and Infinite Circular Waveguides

The form of helical guide to be considered in the present paper may be described as follows: take a rectangle lying in a plane  $\Pi$ , and a line  $L$  in  $\Pi$ , parallel to one pair of sides of the rectangle, and not cutting the rectangle. Cause the plane  $\Pi$  to rotate about  $L$  as axis, with constant angular velocity, at the same time

allowing the rectangle to move, always in II, in a direction parallel to L with constant velocity relative to II. This velocity must be sufficiently large that while II makes a complete revolution of  $2\pi$  the rectangle moves a distance greater than the length of either of its sides parallel to L. The figure traced out by the rectangle is the form of helical waveguide with which we shall be concerned in the present paper. It is characterized by four dimensions, the internal radius  $b$ , the external radius  $a$ , the height  $c$ , and the pitch  $p$  (see Fig. 1), with the proviso that  $p$  is always greater than  $c$ .

The infinite circular guide to be used as an approximation to the helical guide may be described as follows: consider a section of helical guide contained between two planes which intersect in the axis L of the helix, the angle between the planes being  $\alpha$  (the planes are two positions of II). This section may be regarded as having been made by taking a section of guide in the form of a circular arc and suitably shearing it. The circular guide would be generated as above if the rectangle were kept fixed in II instead of moving parallel to L. If  $\alpha$  is greater than  $2\pi$  it is, of course, impossible for the circular arc to exist physically, but it may still be taken as a mathematical approximation to the helix. With a physical circle it would be necessary to impose the condition that whatever physical conditions held in a given plane  $\theta = \theta_0$  must also hold in the planes  $\theta = \theta_0 + 2n\pi$ ,  $n$  being an integer. All these planes would, in fact, coincide. In our case this restriction is not applied, and the planes given by  $\theta = \theta_0 + 2n\pi$  are not coincident and not equivalent. We shall consider  $\alpha$  as being doubly infinite, and refer to this guide always as the "infinite circular guide" to distinguish it from the physically possible circular guide with  $\alpha = 2\pi$ .

**3. Theory of the Infinite Circular Waveguide of Rectangular Cross-Section**

**3.1. Derivation of the Characteristic and Field Equations**

For the fields in the guide, we write

$$\begin{aligned} \mathbf{E} &= \{ \mathbf{1}E_r + \mathbf{m}E_\theta + \mathbf{n}E_z \} \{ e^{jkz} + Pe^{-jkz} \} \cdot e^{-j\beta\theta} \cdot e^{j\omega t} \\ \mathbf{H} &= \{ \mathbf{1}H_r + \mathbf{m}H_\theta + \mathbf{n}H_z \} \{ e^{jkz} + Qe^{-jkz} \} \cdot e^{-j\beta\theta} \cdot e^{j\omega t} \end{aligned} \quad \dots\dots\dots(1)$$

where  $\mathbf{l}$ ,  $\mathbf{m}$ ,  $\mathbf{n}$ , are unit vectors in the radial, tangential, and  $z$  directions respectively,  $z$  being the direction of the axis of the helix or infinite circle, i.e. the line L of Section 2.  $\beta$  is the angular propagation constant; this means that if in a given intersection of the plane II with the guide the field components have certain values, they take the same values at any other intersection which is reached from the first one by changing  $\theta$  an integral number times  $2\pi/\beta$ . The quantity  $2\pi/\beta$  is an angular "wavelength." Substituting from equation (1) into Maxwell's equations and eliminating the radial and tangential field components, we obtain the wave equations

$$\begin{aligned} (\nabla_0^2 + k^2)E_z &= 0 \\ (\nabla_0^2 + k^2)H_z &= 0 \end{aligned} \quad \dots\dots\dots(2)$$

where  $\nabla_0 = \nabla - n \frac{\partial}{\partial z}$ ,  $\nabla^2$  being the Laplacian operator, and  $k^2 = \omega^2 \epsilon_0 \mu_0 - \alpha^2$ ,  $\epsilon_0$  and  $\mu_0$  being the permittivity and permeability, respectively, of free space (or other material which may be in the guide, provided this is isotropic and fills the guide completely).

We can now distinguish two sets of normal modes, the  $E$ -modes, characterized by  $H_z = 0$ , and the  $H$ -modes, characterized by  $E_z = 0$ . It should be noted that these are not analogous to the  $E$ - and  $H$ -modes in a straight guide, which are characterized by the vanishing of a longitudinal field component, i.e. one parallel to the direction of propagation. Rather, they are related to the Längsschnittwellen, but this relation is only of interest in the case of a large radius of curvature. It is better not to think in terms of a straight waveguide deformed into a circle, but in terms of co-axial waveguide with parallel conducting planes inserted perpendicular to its axis.

The solutions of the wave equations are:

$E$ -modes

$$\begin{aligned} E_z &= \{ Fe^{jkz} + Ge^{-jkz} \} \{ J_\beta(kr) + RY_\beta(kr) \} \cdot e^{-j\beta\theta} \cdot e^{j\omega t} \\ H_z &= 0 \end{aligned} \quad \dots\dots\dots(3)$$

H-modes

$$\left. \begin{aligned} E_z &= 0 \\ H_z &= \\ \{ F' e^{jkz} + G' e^{-jkz} \} \{ J_\beta(kr) + R' Y_\beta(kr) \} \cdot e^{-j\beta\theta} \cdot e^{j\omega t} \end{aligned} \right\} \dots\dots(4)$$

where  $F, G, R, F', G'$  and  $R'$ , are arbitrary constants.

The other field components, for both  $E$ - and  $H$ -modes, are given by

$$\left. \begin{aligned} H_r &= \frac{1}{k^2} \left[ \frac{\partial}{\partial z} \left( \frac{\partial H_z}{\partial r} \right) + \frac{j\omega\epsilon_0}{r} \frac{\partial E_z}{\partial \theta} \right] \\ H_\theta &= \frac{1}{k^2} \left[ \frac{1}{r} \frac{\partial}{\partial z} \left( \frac{\partial H_z}{\partial \theta} \right) - j\omega\epsilon_0 \frac{\partial E_z}{\partial r} \right] \\ E_r &= \frac{1}{k^2} \left[ -\frac{j\omega\mu_0}{r} \frac{\partial H_z}{\partial \theta} + \frac{\partial}{\partial z} \left( \frac{\partial E_z}{\partial r} \right) \right] \\ E_\theta &= \frac{1}{k^2} \left[ j\omega\mu_0 \frac{\partial H_z}{\partial r} + \frac{1}{r} \frac{\partial}{\partial z} \left( \frac{\partial E_z}{\partial \theta} \right) \right] \end{aligned} \right\} \dots\dots(5)$$

We now apply the boundary conditions, treating  $E$ -modes and  $H$ -modes separately.

3.1.1. E-modes

The boundary conditions are that at  $z=0$  and  $z=c$ ,  $E_r = E_\theta = 0$ . Hence from (3) and (5),  $F = G$  and  $z = v\pi/c$ ,  $v$  being an integer. Also at  $r=a$  and  $r=b$ ,  $E_\theta = E_z = 0$ , so that

$$\left. \begin{aligned} J_\beta(ka) + RY_\beta(ka) &= 0 \\ \text{and } J_\beta(kb) + RY_\beta(kb) &= 0 \end{aligned} \right\} \dots\dots(6)$$

Since these two equations must give the same value of  $R$ , we must have

$$\begin{vmatrix} J_\beta(ka) & Y_\beta(ka) \\ J_\beta(kb) & Y_\beta(kb) \end{vmatrix} = 0 \dots\dots(7)$$

which is the characteristic equation. It is also the characteristic equation for  $E$ -modes in the co-axial cylindrical waveguide, which justifies our choice of the terms  $E$ -modes and  $H$ -modes. In the co-axial guide, however, it is the quantity that we have called  $z$  that is the propagation constant, so that equation (7) has to be solved for  $k$  instead of  $\beta$ . Now write

$$X_\beta(x) = J_\beta(x) \cdot Y_\beta(ka) - Y_\beta(x) \cdot J_\beta(ka) \dots\dots(8)$$

We could equally well write

$$X_\beta(x) = J_\beta(kb) \cdot Y_\beta(x) - Y_\beta(kb) \cdot J_\beta(x) \dots\dots(8a)$$

These two expressions for  $X_\beta(x)$  differ only by a constant multiplier, which can be taken up in the arbitrary constant  $F$  which has not yet been assigned. It is thus immaterial which definition is adopted. From equations (3), (5), (7), and (8), using the boundary conditions, we obtain for the field equations

$$\left. \begin{aligned} E_r &= \frac{-v\pi}{kc} \cdot X'_\beta(kr) \cdot \sin \frac{v\pi z}{c} \cdot e^{-j\beta\theta} \cdot e^{j\omega t} \\ E_\theta &= \frac{-jv\pi\beta}{k^2rc} \cdot X_\beta(kr) \cdot \sin \frac{v\pi z}{c} \cdot e^{-j\beta\theta} \cdot e^{j\omega t} \\ E_z &= X_\beta(kr) \cdot \cos \frac{v\pi z}{c} \cdot e^{-j\beta\theta} \cdot e^{j\omega t} \\ H_r &= \frac{-\omega\epsilon_0\beta}{k^2r} \cdot X_\beta(kr) \cdot \cos \frac{v\pi z}{c} \cdot e^{-j\beta\theta} \cdot e^{j\omega t} \\ H_\theta &= \frac{-j\omega\epsilon_0}{k} \cdot X'_\beta(kr) \cdot \cos \frac{v\pi z}{c} \cdot e^{-j\beta\theta} \cdot e^{j\omega t} \\ H &= 0 \end{aligned} \right\} \dots\dots(9)$$

where  $X'_\beta(kr) = \frac{dX_\beta}{d(kr)}$ . Rough sketches of the field patterns are shown in Fig. 2.

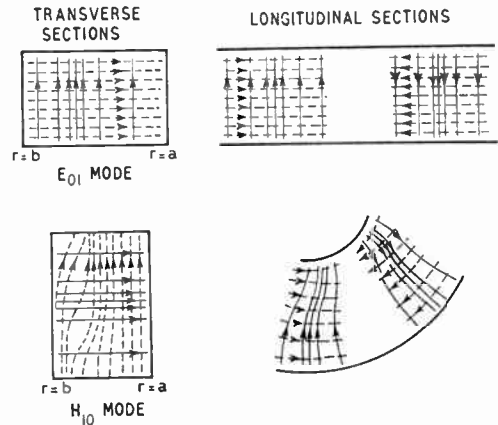


Fig. 2. Field patterns in the infinite circular guide. Full lines represent electric fields; broken lines represent magnetic fields.

3.1.2. H-modes

The boundary conditions are that at  $z=0$  and  $z=c$ ,  $E_r = E_\theta = 0$ , whence  $F' = -G'$  and  $z = v\pi/c$ . Also, at  $r=a$  and  $r=b$ , we have  $E_\theta = 0$ , whence

$$\left. \begin{aligned} J'_\beta(ka) + R'Y'_\beta(ka) &= 0 \\ \text{and } J'_\beta(kb) + R'Y'_\beta(kb) &= 0 \end{aligned} \right\} \dots\dots(10)$$

leading to the characteristic equation

**Table 1**  
 First Zeros of  $J_\beta(x)$ .  $Y_\beta(qx) - J_\beta(qx)$ .  $Y_\beta(x)$

$q$	$\beta = 0$	$\beta = 1$	$\beta = 2$	$\beta = 3$	$\beta = 4$	$\beta = 5$	$\beta = 6$	$\beta = 7$	$\beta = 8$	$\beta = 9$	$\beta = 10$
1.0	For purposes of interpolation, note that as $q$ approaches 1, $(q - 1)x$ approaches $\pi$ for all values of $\beta$										
1.2	15.702*	15.728*	15.806*	15.939	16.119	16.34(6)	16.62(3)	16.945	17.310	17.711	18.150
1.5	6.270*	6.322*	6.474*	6.720	7.050	7.452	7.899	8.427	8.977	9.566	10.179
2.0	3.123*	3.197*	3.407*	3.729	4.133	4.595	5.095	5.618	6.155	6.700	7.250
2.5	2.073*	2.157*	2.387	2.720	3.093	3.545	3.990	4.448	4.893	5.342	5.790
3.0	1.548*	1.636*	1.868	2.189	2.554	2.931	3.314	3.695	4.075	4.451	4.825
3.5	1.235	1.32(2)	1.54(8)	1.83(4)	2.175	2.508	2.839	3.167	3.493	3.815	4.136
4.0	1.024*	1.112*	1.335	1.606	1.900	2.193	2.484	2.772	3.056	3.338	3.619
4.5	0.87(5)	0.96(1)	1.16(9)	1.418	1.686	1.949	2.208	2.464	2.717	2.968	3.217
5.0	0.763*	0.847*	1.045	1.281	1.518	1.754	1.987	2.217	2.445	2.671	2.895
5.5	0.67(6)	0.75(8)	0.94(4)	1.162	1.380	1.595	1.807	2.016	2.223	2.428	2.632
6.0	0.607*	0.686*	0.864	1.064	1.265	1.462	1.656	1.848	2.038	2.226	2.412
6.5	0.55(0)	0.62(7)	0.79(4)	0.949	1.167	1.349	1.529	1.706	1.881	2.054	2.227
7.0	0.503	0.578	0.735	0.902	1.084	1.253	1.419	1.584	1.746	1.908	2.068
7.5	0.46(3)	0.53(6)	0.68(4)	0.846	1.012	1.170	1.325	1.478	1.630	1.781	1.930
8.0	0.429*	0.500*	0.641	0.798	0.949	1.096	1.242	1.386	1.528	1.669	1.809

$$\begin{vmatrix} J'_\beta(ka) & Y'_\beta(ka) \\ J'_\beta(kb) & Y'_\beta(kb) \end{vmatrix} = 0 \dots\dots(11)$$

which is also the characteristic equation for  $H$ -modes in the co-axial cylindrical waveguide. We now write

$$Z_\beta(x) = J'_\beta(x) \cdot Y'_\beta(ka) - Y'_\beta(x) \cdot J'_\beta(ka) \dots\dots(12)$$

which, with an alteration of  $F'$  may alternatively be written as

$$Z_\beta(x) = J'_\beta(kb) \cdot Y'_\beta(x) - Y'_\beta(kb) \cdot J'_\beta(x) \dots\dots(12a)$$

From equations (4), (5), (11), and (12), using the boundary conditions, we obtain for the field equations

$$\left. \begin{aligned} E_r &= \frac{\omega \mu_0 \beta}{k^2 r} \cdot Z_\beta(kr) \cdot \sin \frac{\nu \pi z}{c} \cdot e^{-j\beta \theta} \cdot e^{j\omega t} \\ E_\theta &= \frac{j\omega \mu_0}{k} \cdot Z'_\beta(kr) \cdot \sin \frac{\nu \pi z}{c} \cdot e^{-j\beta \theta} \cdot e^{j\omega t} \\ E_z &= 0 \\ H_r &= \frac{\nu \pi}{kc} \cdot Z'_\beta(kr) \cdot \cos \frac{\nu \pi z}{c} \cdot e^{-j\beta \theta} \cdot e^{j\omega t} \\ H_\theta &= \frac{j\nu \pi \beta}{k^2 rc} \cdot Z_\beta(kr) \cdot \cos \frac{\nu \pi z}{c} \cdot e^{-j\beta \theta} \cdot e^{j\omega t} \\ H_z &= Z_\beta(kr) \cdot \sin \frac{\nu \pi z}{c} \cdot e^{-j\beta \theta} \cdot e^{j\omega t} \end{aligned} \right\} \dots\dots(13)$$

where  $Z'_\beta(kr) = \frac{dZ_\beta}{d(kr)}$ . Field patterns for the  $H_{10}$  mode are sketched in Fig. 2.

3.2. Solution of the Characteristic Equation

Write  $kb = x$ ,  $a/b = q$ . Then equations (7) and (11) become, respectively,

$$\begin{vmatrix} J_\beta(qx) & Y_\beta(qx) \\ J_\beta(x) & Y_\beta(x) \end{vmatrix} = 0 \dots\dots\dots(14)$$

and

$$\begin{vmatrix} J'_\beta(qx) & Y'_\beta(qx) \\ J'_\beta(x) & Y'_\beta(x) \end{vmatrix} = 0 \dots\dots\dots(15)$$

The solutions of equations (14) and (15) are discussed by Kline<sup>4</sup>, and graphical solutions are given by Bondi and Kuhn<sup>5</sup>. Jahnke and Emde<sup>6</sup> and Dwight<sup>7</sup> give tables of solutions, but these are not extensive enough for our purpose, since these authors are concerned with the related problem of the co-axial cylindrical waveguide, and have been interested in different ranges of parameters. Accordingly, we have prepared tables for the present paper that cover ranges of parameters more suitable to our purpose. Some values in these tables have been

**Table 2**  
Second Zeros of  $J_\beta(x)$ ,  $Y_\beta(qx) - J_\beta(qx)$ ,  $Y_\beta(x)$

$q$	$\beta = 0$	$\beta = 1$	$\beta = 2$	$\beta = 3$	$\beta = 4$	$\beta = 5$	$\beta = 6$	$\beta = 7$	$\beta = 8$
1.0	For purposes of interpolation, note that as $q$ approaches 1, $(q-1)x$ approaches $2\pi$ for all $\beta$								
1.2	31.412*	31.426*	31.466*	—	—	—	—	—	—
1.5	12.560*	12.586*	12.665*	12.80	12.97	13.20	13.48	13.80	14.15
2.0	6.273*	6.312*	6.43*	6.62	6.87	7.19	7.56	7.97	8.42
2.5	4.177*	4.223*	4.36	4.57	4.86	5.21	5.60	6.02	6.46
3.0	3.129*	3.178*	3.32	3.55	3.84	4.19	4.57	4.96	5.35
4.0	2.081*	2.134*	2.29	2.52	2.79	3.09	3.40	3.71	4.01
5.0	1.557*	1.611*	1.76	1.98	2.22	2.47	2.72	2.96	3.21
6.0	1.243*	1.296*	1.44	1.64	1.84	2.06	2.26	2.47	2.67
7.0	1.04	1.09	1.23	1.39	1.58	1.76	1.94	2.12	2.29
8.0	0.884*	0.935*	1.07	1.22	1.38	1.54	1.70	1.85	2.00

**Table 3**  
 Zeroth Zeros of  $J'_\beta(x) \cdot Y'_\beta(qx) - J'_\beta(qx) \cdot Y'_\beta(x)$

$q=0$	$\beta = 0$	$\beta = 1$	$\beta = 2$	$\beta = 3$	$\beta = 4$	$\beta = 5$	$\beta = 6$	$\beta = 7$	$\beta = 8$	$\beta = 9$	$\beta = 10$	$\beta = 11$	$\beta = 12$
1-0	0	1*	2*	3*	4	5	6	7	8	9	10	11	12
1-2	0	0.910*	1.821*	2.731*	3.641	4.550	5.458	6.367	7.28	8.20	9.11	10.02	10.93
1-5	0	0.805*	1.608*	2.407*	3.208	3.981	4.760	5.525	6.280	7.03	7.76	8.49	9.21
2-0	0	0.677*	1.341*	1.979*	2.592	3.169	3.731	4.279	4.819	5.35(5)	5.88(5)	6.41(0)	6.94(0)
2.5	0	0.585*	1.137*	1.643*	2.118	2.561	2.999	3.431	3.858	4.28(4)	4.70(8)	5.12(8)	5.55(2)
3-0	0	0.514*	0.977*	1.388*	1.769	2.138	2.501	2.859	3.215	3.57(0)	3.92(3)	4.27(3)	4.62(7)
3.5	0	0.45(8)	0.85(2)	1.19(8)	1.52(2)	1.833	2.144	2.451	2.755	3.06(0)	3.36(3)	3.66(3)	3.96(6)
4-0	0	0.411*	0.752*	1.048*	1.329	1.604	1.876	2.144	2.411	2.67(8)	2.94(2)	3.20(5)	3.47(0)
4.5	0	0.37(3)	0.67(2)	0.93(2)	1.18(0)	1.426	1.667	1.906	2.143	2.38(0)	2.61(6)	2.84(9)	3.08(4)
5-0	0	0.341*	0.607	0.840	1.064	1.283	1.501	1.715	1.929	2.14(2)	2.35(4)	2.56(4)	2.77(6)
5.5	0	0.31(4)	0.55(4)	0.76(6)	0.967	1.167	1.364	1.559	1.753	1.94(7)	2.14(0)	2.33(1)	2.52(4)
6-0	0	0.290*	0.508	0.700	0.886	1.069	1.250	1.430	1.607	1.78(5)	1.96(2)	2.13(7)	2.31(3)
6.5	0	0.27(0)	0.470	0.647	0.818	0.987	1.154	1.320	1.484	1.64(8)	1.81(1)	1.97(2)	2.13(5)
7-0	0	0.253	0.436	0.601	0.759	0.917	1.072	1.225	1.378	1.53(0)	1.68(1)	1.83(1)	1.98(3)
7.5	0	0.23(8)	0.407	0.561	0.709	0.855	1.000	1.144	1.286	1.42(8)	1.56(9)	1.70(9)	1.85(1)
8-0	0	0.233*	0.382	0.525	0.664	0.802	0.938	1.072	1.206	1.33(9)	1.47(1)	1.60(2)	1.73(5)

obtained from Jahnke and Emde's and from Dwight's tables; these cases are indicated by an asterisk. Tables 1 and 2 give the first set and second set, respectively, of values of  $x$  which satisfy equation (14), for given values of  $q$  and  $\beta$ . Tables 3 and 4 give the zeroth and first set, respectively, of values of  $x$  which satisfy equation (15) for given values of  $q$  and  $\beta$ . It may be noted that a zero of equation (14) for  $\beta=1$  is a zero of equation (15) for  $\beta=0$ , and in this way the two sets of zeros for the  $E$ - and  $H$ -modes may be related. It is for this reason that the zeros given in Tables 3 and 4 are called zeroth and first, instead of first and second. This nomenclature seems more logical, although it conflicts with that of Dwight; apart from the one just given, there are other reasons for adopting the present nomenclature. One is given in the next paragraph but one, another in Section 3.3.3. A fourth reason is that whilst most solutions can be obtained by asymptotic expansions, those which we have called "zeroth" are not given in this way, and so appear to be fundamentally different in character from the rest.

It is apparent that as  $\beta$  or  $q$  becomes large, the solutions of equations (14) and (15) approach those of the equations  $J_\beta(qx)=0$  and  $J'_\beta(qx)=0$  respectively, and the regions of the tables in which this approximation may be made without reducing the accuracy are those below and to the right of the heavy lines. Thus the tables may be readily extended, if required, except near cut-off ( $\beta \sim 0$ ) or when the curvature is not very great ( $q \sim 1$ ). Table 5 gives first and second zeros of  $J_\beta(x)$  and  $J'_\beta(x)$ ; note that it is the  $(n+1)$ th zero of  $J'_\beta(x)$  that goes with the  $n$ th solution of equation (15).

We may now have one of two problems—to find the propagation constant  $\beta$  for a given waveguide, or to design a guide to have a desired propagation constant. In the first case, if  $q$  is not one of the values given in the tables, the appropriate values of  $x$  can be found by interpolation of the function  $(q-1)x$ , except in Table 3, where  $x$  can be interpolated directly. Values of  $q$  near unity may be dealt with in Tables 1, 2, and 4 by noting that

$$\lim_{q \rightarrow 1} (q-1)x = i\pi$$

where  $i$  is the order of the solution, i.e.  $i=1$

**Table 4**  
First Zeros of  $J'_\beta(x)$ .  $Y'_\beta(qx) - J'_\beta(qx)$ .  $Y'_\beta(x)$

$q$	$\beta = 0$	$\beta = 1$	$\beta = 2$	$\beta = 3$	$\beta = 4$	$\beta = 5$	$\beta = 6$	$\beta = 7$	$\beta = 8$
1.0	For purposes of interpolation, note that as $q$ approaches 1, $(q-1)x$ approaches $\pi$ for all $\beta$								
1.2	15.728*	15.754*	15.834*	15.96	16.14(5)	16.39	16.64	16.99	17.36
1.5	6.322*	6.376*	6.54	6.80	7.16	7.59	8.09	8.65	9.26
2.0	3.197*	3.282*	3.53	3.92	4.42	4.99	5.61	6.26	6.90
2.5	2.157*	2.264*	2.57	3.01(5)	3.54	4.08	4.65	5.16	5.64
3.0	1.636*	1.758*	2.09	2.54	3.02	3.48	3.91	4.31	4.71
4.0	1.112*	1.251*	1.59	1.97	2.31	2.63	2.93	3.23	3.53
5.0	0.847*	0.992*	1.30	1.59	1.86	2.10	2.35	2.59	2.82
6.0	0.686*	0.831*	1.10	1.34	1.55	1.75	1.96	2.16	2.35
7.0	0.578	0.77	0.96	1.15	1.33	1.50	1.68	1.85	2.02
8.0	0.500*	0.633*	0.84	1.00	1.16	1.32	1.47	1.62	1.76



for Tables 1 and 4,  $i=2$  for Table 2. In Table 3, values of  $q$  near unity may be dealt with by noting that  $\lim_{q \rightarrow 1} (x) = \beta$ . This behaviour of the zeroth solutions justifies our putting them in a different category from the others and assigning indices to the other solutions as if the zeroth solutions did not exist. By calling these different zeros "zeroth," we are not so much assigning to them the index 0 as standing them aside from the whole run of indices. We now have

$$k^2 = \omega^2 \epsilon_0 \mu_0 - \alpha^2 = \frac{4\pi^2}{\lambda_0^2} \left( 1 - \frac{v^2 \lambda_0^2}{4c^2} \right) \dots (16)$$

where  $\lambda_0$  is the free-space wavelength, so that  $kb = x$ , is easily found. It is evident that  $v$  should be chosen before seeking a solution.

Table 5

Zeros of Bessel Functions and Derivatives of Bessel Functions

$\beta$	Zeros of $J_\beta(x)$		Zeros of $J'_\beta(x)$	
	First Zeros	Second Zeros	First Zeros	Second Zeros
0	2.4048	5.5201	0	3.8317
1	3.8317	7.0156	1.8412	5.3314
2	5.1356	8.4172	3.054	6.7061
3	6.3802	9.7610	4.204	8.0152
4	7.5883	11.065	5.316	9.2824
5	8.7715	12.339	6.416	10.520
6	9.9361	13.589	7.503	11.735
7	11.086	14.821	8.577	12.932
8	12.225	16.038	9.644	14.116
9	13.354	—	10.71	—
10	14.475	—	11.77	—
11	—	—	12.82	—
12	—	—	13.88	—

For the appropriate value of  $q$ , the value of  $\beta$  can then be found from the tables either by linear interpolation or, less accurately, graphically.

For the second problem, interpolation will give values of  $x$  as a function of  $q$  for the desired value of  $\beta$ . Again, a graphical method may be used, with some loss of accuracy.  $q$  and  $x$  may then be chosen, having regard to the desired geometry of the system and the difficulties of manufacture.

3.2.1. Accuracy

The values of  $x$  given in Tables 1 and 3 are accurate to  $\pm 1$  in the last significant figure, except where a lower accuracy is indicated by brackets.  $\beta$  can be calculated to an accuracy of  $\pm .01$ , corresponding to a phase error of  $\pm .01 \times 360^\circ = 3.6^\circ$ , in the worst case, as the wave travels through an azimuthal angle of  $360^\circ$ . The values of  $x$  in Tables 2 and 4 are accurate to  $\pm 1$  in the last significant figure; the value of  $\beta$  found in these cases will be less accurate.

3.3. Discussion of the Modes

3.3.1. Nomenclature

It is convenient to designate the modes as  $E_{vi}, H_{vi}$ ,  $v$  being the same as in Section 3.1 and  $i$  being the order of the solution of the characteristic equation. Thus  $i=1, 2, 3, \dots$  for  $E$ -modes, while for  $H$ -modes  $i=0, 1, 2, \dots$ . Examination of equations (9) and (13) also shows that for  $E$ -modes  $v$  may be  $0, 1, 2, \dots, v_0$ , while for  $H$ -modes the value  $v=0$  is excluded, since this value causes all the field components to vanish, so that we have  $v=1, 2, 3, \dots, v_0$ .  $v_0$  is the highest value of  $v$  such that  $k$  is real.

These modes should not be confused with the modes of the co-axial waveguide. While the suffix  $i$  here is related to the second suffix of the co-axial guide modes, the suffix  $v$  here is concerned with the  $z$  dimension, unlike the first suffix of the co-axial guide modes which is concerned with the  $\theta$  dimension.

3.3.2. Cut-offs

It is evident from equations (2) that propagation can only take place if  $k$  is real. From equation (16) this imposes an upper limit  $v_0$  on  $v$ , and modes with a higher value of  $v$  will be cut off. But although  $k$  real is a necessary

condition for propagation to take place, it is not sufficient, since it is still necessary to find a solution for  $\beta$  from the tables. If no such solution can be found, the mode is cut off. Thus there are two kinds of cut-off—those due to imaginary  $k$ , which we shall call “cut-offs of the first kind,” and those for which  $k$  is real, but no solution can be found from the tables, which we shall call “cut-offs of the second kind.”

If  $\nu=0$ , which is possible only for  $E$ -modes, we see that  $k$  is real and independent of  $c$ . Thus for  $E_{0i}$  modes, only cut-offs of the second kind can occur. For  $H$ -modes, there is always a solution of the characteristic equation for  $i=0$ , provided  $k$  is real. Thus for the  $H_{\nu 3}$  modes, only cut-offs of the first kind can occur. Since  $\nu$  cannot be zero, there is always a sufficiently small value of  $c$  to make  $k$  imaginary, so there is a cut-off. When neither  $\nu$  nor  $i$  is zero, there will, of course, only be one cut-off for each mode, and whether this is of the first or second kind will depend, for a given mode, on the geometry of the system—more precisely, on the ratio of  $c$  to  $(a-b)$ .

For a given mode, the cut-off frequency may be found in the following manner. Firstly, find from the tables the value of  $x$  for which  $\beta=0$ . Then  $\lambda_0$  may be calculated from equation (16). Let us call the value so found  $\lambda_1$ . Secondly, put  $k=0$  and again solve equation (16) for  $\lambda_0$ , obtaining the value  $\lambda_2$ , where

$$\lambda_2 = \frac{2c}{\nu} \dots\dots\dots(17)$$

For propagation to take place it is necessary both for  $k$  to be real and a solution to be obtainable from the tables. Thus in general we must satisfy both conditions;  $\lambda_0 < \lambda_1$  and  $\lambda_0 < \lambda_2$  so that the smaller of  $\lambda_1$  and  $\lambda_2$  is the cut-off value. From this value of the free-space wavelength the cut-off frequency is easily determined. In practice, we shall be concerned mainly with the  $E_{01}$  and  $H_{10}$  modes. For the  $E_{01}$  mode, we need only consider  $\lambda_1$ ;  $\lambda_2$  does not occur. For the  $H_{10}$  mode, we need only consider  $\lambda_2$ ;  $\lambda_1$  does not occur.

### 3.3.3. The lowest modes

The lowest mode is that having the lowest

cut-off frequency, and so is the mode that can propagate in the absence of all others. Which mode is the lowest will depend on the relative magnitudes of the dimensions  $c$  and  $(a-b)$ . If  $c$  is sufficiently small,  $k$  is only real when  $\nu$  is zero; in this case, if  $(a-b)$  is sufficiently large, i.e. if  $q$  is sufficiently large, a solution can be found for equation (14). The lowest mode is thus the  $E_{01}$  mode. If  $c$  is larger,  $k$  may still be real when  $\nu=1$ . Then if  $q$  and  $x$  are sufficiently small, i.e. if  $b$  and  $a$  are sufficiently small, the  $E_{01}$  mode may be cut off. But for  $i=0$  a solution exists of equation (15), so that the lowest mode is the  $H_{10}$ .

The infinite circular guide may be regarded as a distorted form of the straight guide when the radius of curvature is large. The bend may take place in one of two ways, in a plane parallel either to the broad or the narrow surface of the guide, giving rise to the cases  $c < (a-b)$  and  $c > (a-b)$  respectively. Thus the  $H_{10}$  mode of the straight waveguide gives rise to the  $E_{01}$  and  $H_{10}$  modes, respectively, of the infinite circular guide, as we might have expected. Since there is no  $E_{10}$  or  $E_{01}$  mode in a straight waveguide, the  $H_{10}$  mode is a Längsschnittwellen, and so this relationship between the modes is not at variance with the statement made in Section 3.1 that the normal modes of the infinite circular guide are related not to the normal modes but to the Längsschnittwellen of the straight waveguide.

In the case of the  $H_{10}$  mode of a straight waveguide, the significance of the suffix 0 is that there is no variation of field strength as one moves parallel to the  $y$ -dimension of the guide. In the case of the  $E_{01}$  mode of the infinite circular guide, the significance of the suffix 0 is that there is no variation of field strength as one moves parallel to the  $z$ -dimension of the guide. In the case of the lowest  $H$ -mode of the infinite circular guide, if  $q$  is not much greater than unity the field strengths are approximately constant as one moves in a radial direction, and this approximate constancy suggests a suffix zero, so that the mode which we have called  $H_{10}$  is properly so called. This is a further justification of our designating the solutions of equation (15) as zeroth-order, first-order, etc., instead of as first-order, second-order, etc.

3.3.4. Wave impedance

In a straight waveguide, if the propagation directions are labelled 1 and 2, the impedance direction is labelled 3, and the two transverse to a wave travelling along the guide is

$$Z = \frac{E_1}{H_2} = \frac{-E_2}{H_1} \dots\dots\dots(18)$$

These two formulae give the same result, which is a constant, and the sign of  $Z$  is unambiguous if the numbers 1, 2, 3, are applied to the axes in cyclical order. Thus in a rectangular guide 1, 2, 3  $\sim$   $x, y, z$ , and in a cylindrical guide 1, 2, 3  $\sim$   $r, \theta, z$ .

For the infinite circular guide, 1, 2, 3  $\sim$   $z, r, \theta$ , and we have, from equations (9) and (18)

$$Z = \frac{-k^2 r}{\omega \epsilon_0 \beta} = \infty$$

for  $E$ -modes, while for  $H$ -modes

$$Z = 0 = \frac{\omega \mu_0 \beta}{k^2 r}$$

We note first that the results are ambiguous. If we take account only of the finite expressions,  $Z$  is a function of  $r$ . Thus if a straight guide be coupled to a curved guide there can only be perfect matching, if at all, for one value of  $r$ ; elsewhere reflections will take place, the value of the reflection coefficient or standing wave ratio depending on the distance from this point of perfect matching. This trouble will be worse for sharp curvatures than for slight curvatures; if serious, some device such as an iris or post will have to be used to improve the match. It seems, therefore, that the best approach to the problem of joining a straight guide to a curved guide is an experimental one, and that the wave impedance is not a useful concept when the waves are not travelling along straight lines.

**4. The Helical Waveguide of Rectangular Cross-Section**

The field equations and solution of the characteristic equation given in Section 3 for the infinite circular guide apply approximately to the helical guide, but it is desirable to consider a correction to apply to the propagation constant to take account of the pitch. This correction can be calculated by a perturbation method. We shall suppose that as the infinite circular guide is deformed into the helical guide, a frequency shift  $\delta\omega$  is applied simultaneously

in such a way as to keep the phase constant constant. We shall find that this frequency shift is zero; it follows that the shift in phase constant, when the frequency is kept constant, is also zero.

We shall also consider in this section the question of wave velocity in both the infinite circular guide and the helical guide. It is convenient to treat both cases together, and this is why the wave velocity in the infinite circular guide was not dealt with in Section 4.

**4.1. Perturbation Theory of the Infinite Circular Guide**

Consider the region of helical or infinite circular guide contained between two planes which each contain the common axis of circle and helix, the angle between the planes being  $2\pi/\beta$ . The segment of circular guide is a volume bounded by the surfaces  $r=a, r=b, z=0, z=c, \theta=0, \theta=2\pi/\beta$ . We shall call this volume  $V_0$ , and note here that the fields in the planes  $\theta=0, \theta=2\pi/\beta$ , are identical. The total surface enclosing  $V_0$  we shall denote by  $S_0$  (see Fig. 3).

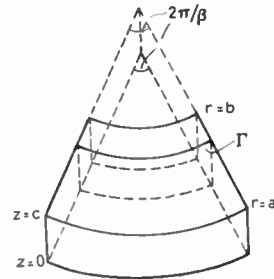


Fig. 3. The volume  $V_0$ , intersected by the cylinder  $\Gamma$ .

Now consider a cylindrical co-axial with the circular and helical guides, cutting them in a section  $\Gamma$  (Figs. 3 and 4). Take the region between the planes  $\theta=0, \theta=2\pi/\beta$ , and unroll it into a plane. The section with the circular guide projects into a rectangle  $\Sigma'_0$ , while that with the helical guide projects into the parallelogram  $\Sigma'_0 - \Sigma'_1 + \Sigma'_2$  (Fig. 5). The length of the rectangle is  $2\pi r/\beta$ , the height  $c$ . The length of the vertical side of the parallelogram is also  $c$ , and the angle between the sloping side of the parallelogram and the horizontal side of the rectangle is  $\psi = \tan^{-1}(p/2\pi r)$ .

Project back onto the cylindrical surface  $\Gamma$ .  $\psi$  is unchanged.  $\Sigma'_0$  projects back into a quasi-rectangular figure  $\Sigma_0$ ,  $\Sigma'_1$  and  $\Sigma'_2$  into the quasi-triangles  $\Sigma_1$  and  $\Sigma_2$ . Let  $r$  vary from

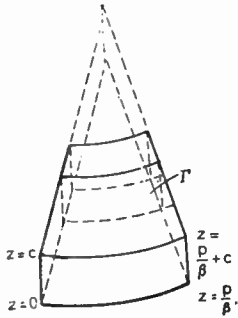


Fig. 4. Section of the helical guide, intersected by the cylinder  $\Gamma$ .

$b$  to  $a$ ;  $\Sigma_0$  traces out the volume  $V_0$ . The volumes similarly traced out by  $\Sigma_1$  and  $\Sigma_2$  we shall call  $V_1$  and  $V_2$  respectively.

We shall now use a modified form of the perturbation theory given by Casimir<sup>8</sup>. In the unperturbed state, i.e. in the infinite circular guide, let the fields be

$$\begin{aligned} \mathbf{E} &= \mathbf{E}_0 \cdot e^{-j\beta z} \cdot e^{j\omega t} \\ \mathbf{H} &= \mathbf{H}_0 \cdot e^{-j\beta z} \cdot e^{j\omega t} \end{aligned} \quad \dots\dots(19)$$

On distorting the guide into the helical form and suitably altering the frequency, the fields become

$$\begin{aligned} \mathbf{E}' &= (\mathbf{E}_0 + \mathbf{E}_1) \cdot e^{-\beta z} \cdot e^{j(\omega + \delta\omega)t} \\ \mathbf{H}' &= (\mathbf{H}_0 + \mathbf{H}_1) \cdot e^{-j\beta z} \cdot e^{j(\omega + \delta\omega)t} \end{aligned} \quad \dots\dots(20)$$

We also define the inductions

$$\mathbf{B}_0 = \mu_0 \mathbf{H}_0, \quad \mathbf{D}_0 = \epsilon_0 \mathbf{E}_0 \quad \dots\dots(21)$$

in the unperturbed state, and

$$\begin{aligned} \mathbf{B}_1 &= \mu_0 [\mu (\mathbf{H}_0 + \mathbf{H}_1) - \mathbf{H}_0] \\ \mathbf{D}_1 &= \epsilon_0 [\epsilon (\mathbf{E}_0 + \mathbf{E}_1) - \mathbf{E}_0] \end{aligned} \quad \dots\dots(22)$$

in the perturbed state. In a given region,  $\epsilon$  and  $\mu$  must be assigned the values appropriate to the material contained in that region. The region  $V_0 - V_1$  is always occupied by free space, for which  $\epsilon = \mu = 1$ , so that  $B_1 = \mu_0 H_1$  and  $D_1 = \epsilon_0 E_1$ .  $V_1$  and  $V_2$  are occupied by free space and metal, respectively, in the unperturbed state, and by metal and free space, respectively, in the perturbed state. For a region occupied

by metal, we assume that the metal is a perfect conductor, and shall subsequently take  $\epsilon$  to be  $\infty$  and  $\mu$  to be 0.

From Maxwell's equations,

$$\begin{aligned} \text{curl } \mathbf{E}_1 &= -j\delta\omega \mathbf{B}_0 - j(\omega + \delta\omega) \mathbf{B}_1 \\ \text{curl } \mathbf{H}_1 &= j\delta\omega \mathbf{D}_0 + j(\omega + \delta\omega) \mathbf{D}_1 \end{aligned}$$

On forming the scalar product of  $\mathbf{H}_0$  with the first of these, and of  $\mathbf{E}_0$  with the second, we obtain

$$\begin{aligned} j\delta\omega [(\mathbf{E}_0 \cdot \mathbf{D}_0 + \mathbf{E}_0 \cdot \mathbf{D}_1) - (\mathbf{H}_0 \cdot \mathbf{B}_0 + \mathbf{H}_0 \cdot \mathbf{B}_1)] \\ = (\mathbf{E}_0 \cdot \text{curl } \mathbf{H}_1 + \mathbf{H}_0 \cdot \text{curl } \mathbf{E}_1) + j\omega (\mathbf{H}_0 \cdot \mathbf{B}_1 - \mathbf{E}_0 \cdot \mathbf{D}_1) \end{aligned} \quad \dots\dots(23)$$

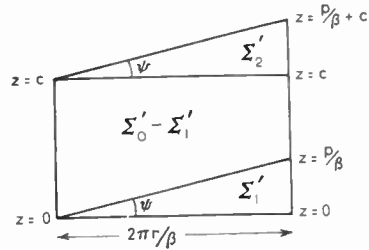


Fig. 5. Section of the cylinder  $\Gamma$  with the infinite circular and helical guides, projected on to a plane surface  $\Gamma'$ .

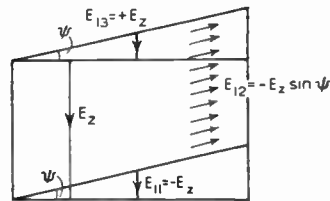


Fig. 6. Electric fields in the surface  $\Gamma'$ .

$$\begin{aligned} \text{Now, div } [(\mathbf{H}_0 \times \mathbf{E}_1) + (\mathbf{E}_0 \times \mathbf{H}_1)] \\ = -\mathbf{H}_0 \cdot \text{curl } \mathbf{E}_1 + \mathbf{E}_1 \cdot \text{curl } \mathbf{H}_0 - \mathbf{E}_0 \cdot \text{curl } \mathbf{H}_1 + \mathbf{H}_1 \cdot \text{curl } \mathbf{E}_0 \end{aligned}$$

so that equation (23) may be written

$$\begin{aligned} j\delta\omega [(\mathbf{E}_0 \cdot \mathbf{D}_0 + \mathbf{E}_0 \cdot \mathbf{D}_1) - (\mathbf{H}_0 \cdot \mathbf{B}_0 + \mathbf{H}_0 \cdot \mathbf{B}_1)] \\ = -\text{div } [(\mathbf{H}_0 \times \mathbf{E}_1) + (\mathbf{E}_0 \times \mathbf{H}_1)] + \\ + j\omega [(\mathbf{E}_1 \cdot \mathbf{D}_0 - \mathbf{E}_0 \cdot \mathbf{D}_1) + (\mathbf{H}_0 \cdot \mathbf{B}_1 - \mathbf{H}_1 \cdot \mathbf{B}_0)] \end{aligned}$$

where we have put  $\text{curl } \mathbf{H}_0 = j\omega \mathbf{D}_0$  and  $\text{curl } \mathbf{E}_0 = -j\omega \mathbf{B}_0$ .

We now integrate over the volume  $V_0 + V_2$  and obtain

$$j\delta\omega \iiint_{V_0+V_2} [\mathbf{E}_0 \cdot (\mathbf{D}_0 + \mathbf{D}_1) - \mathbf{H}_0 \cdot (\mathbf{B}_0 + \mathbf{B}_1)] dV$$

$$= j\omega \iiint_{V_0+V_2} [(\mathbf{E}_1 \cdot \mathbf{D}_0 - \mathbf{E}_0 \cdot \mathbf{D}_1) + (\mathbf{H}_0 \cdot \mathbf{B}_1 - \mathbf{H}_1 \cdot \mathbf{B}_0)] dV$$

$$- \iiint_{V_0+V_2} \text{div} [(\mathbf{H}_0 \times \mathbf{E}_1) + (\mathbf{E}_0 \times \mathbf{H}_1)] dV$$

The divergence integral vanishes. This is shown in the Appendix for the case of the  $E_{01}$  mode; the proof may be carried out in an analogous way for any other mode. We thus have

$$\frac{\delta\omega}{\omega} = \frac{\iiint_{V_0+V_2} [(\mathbf{E}_1 \cdot \mathbf{D}_0 - \mathbf{E}_0 \cdot \mathbf{D}_1) + (\mathbf{H}_0 \cdot \mathbf{B}_1 - \mathbf{H}_1 \cdot \mathbf{B}_0)] dV}{\iiint_{V_0+V_2} [\mathbf{E}_0 \cdot (\mathbf{D}_0 + \mathbf{D}_1) - \mathbf{H}_0 \cdot (\mathbf{B}_0 + \mathbf{B}_1)] dV}$$

In the region  $V_0 - V_1$ ,  $\mathbf{E}_1 \cdot \mathbf{D}_0 = \mathbf{E}_1 \cdot \epsilon_0 \mathbf{E}_0 = \mathbf{D}_1 \cdot \mathbf{E}_0$  and  $\mathbf{H}_1 \cdot \mathbf{B}_0 = \mathbf{H}_1 \cdot \mu_0 \mathbf{H}_0 = \mathbf{B}_1 \cdot \mathbf{H}_0$ , so that the numerator vanishes. Therefore

$$\frac{\delta\omega}{\omega} = \frac{\iiint_{V_1+V_2} [(\mathbf{E}_1 \cdot \mathbf{D}_0 - \mathbf{E}_0 \cdot \mathbf{D}_1) + (\mathbf{H}_0 \cdot \mathbf{B}_1 - \mathbf{H}_1 \cdot \mathbf{B}_0)] dV}{\iiint_{V_0+V_2} [\mathbf{E}_0 \cdot (\mathbf{D}_0 + \mathbf{D}_1) - \mathbf{H}_0 \cdot (\mathbf{B}_0 + \mathbf{B}_1)] dV} \dots\dots\dots(24)$$

This equation holds exactly for all modes of propagation.

4.2. Application of the Perturbation Formula to the  $E_{01}$  Mode

In this mode,  $\mathbf{E}_0 = \mathbf{E}_z = \mathbf{E}_z(r)$ , while  $\mathbf{H}_0$  consists of the two components  $\mathbf{H}_r = \mathbf{H}_r(r)$  and  $\mathbf{H}_\theta = \mathbf{H}_\theta(r)$ . We consider first the electric fields and inductions, and confine our attention to one of the cylindrical surfaces  $\Gamma$  (Fig. 5).

In the unperturbed state, the field  $\mathbf{E}_0$  is equal to  $\mathbf{E}_z$ , and exists only in the region  $0 < z < c$ , being zero outside this region (see Fig. 6). In the perturbed state, the resultant

field in  $\Sigma_1$  must be zero, since the region is now filled with metal. Hence in this region  $\mathbf{E}_1$  has a single component  $\mathbf{E}_{11} = -\mathbf{E}_z$ . In the region  $\Sigma_0 + \Sigma_2 - \Sigma_1$ , since  $\mathbf{E}_z$  has a component  $E_z \sin \psi$  parallel to the perturbed surface,  $\mathbf{E}_1$  must have a component  $E_{12} = -E_z \sin \psi$  to make the tangential fields vanish on the boundary surfaces. In the region  $\Sigma_2$ , since  $E_z$  must be continuous at the boundary between  $\Sigma_0$  and  $\Sigma_2$ ,  $\mathbf{E}_1$  must have a component  $\mathbf{E}_{13} = \mathbf{E}_z$ . The induction  $\mathbf{D}_0$  is not normal to the perturbed surfaces, and has a component  $\epsilon_0 E_z \sin \psi$  parallel to them. There must therefore be a component of  $\mathbf{D}_1 = -\epsilon_0 E_z \sin \psi$  to cancel this (Fig. 7). Then as far as electrical terms are

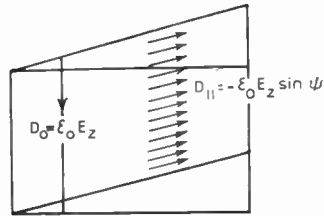


Fig. 7. Electric inductions in the surface  $\Gamma'$ .

concerned, the integrals in the numerator of equation (24) may be written

$$\iiint_{V_1} \mathbf{E}_1 \cdot \mathbf{D}_0 dV = -\epsilon_0 \iiint_{V_1} E_z^2 dV$$

$$\iiint_{V_1} \mathbf{E}_0 \cdot \mathbf{D}_1 dV = -\epsilon_0 \iiint_{V_1} E_z^2 \sin^2 \psi dV$$

$$\iiint_{V_2} \mathbf{E}_1 \cdot \mathbf{D}_0 dV = \epsilon_0 \iiint_{V_2} E_z^2 (1 - \sin^2 \psi) dV$$

$$\iiint_{V_2} \mathbf{E}_0 \cdot \mathbf{D}_1 dV = 0$$

Hence

$$\iiint_{V_1+V_2} (\mathbf{E}_1 \cdot \mathbf{D}_0 - \mathbf{E}_0 \cdot \mathbf{D}_1) dV$$

$$= \epsilon_0 \iiint_{V_2 - V_1} E_z^2 (1 - \sin^2 \psi) dV \dots\dots\dots(25)$$

We now consider the magnetic fields. Since the section of the guide by a plane  $\theta = \text{constant}$  is unchanged in shape and size by the perturbation, there is no change of the radial component  $H_r$  of  $H_0$ . For the perturbation of the tangential component  $H_\theta$  of  $H_0$ , we again consider the surface  $\Gamma$ .  $H_\theta$  exists both inside and outside the metal. In the perturbed state,  $H_\theta$  has a component  $H_\theta \sin \psi$  normal to the guide surfaces. This must be cancelled by the field  $H_1$ , which is therefore  $-H_\theta \sin \psi$  in the direction perpendicular to the helical guide surfaces (Fig. 8). This field exists in  $\Sigma_0$ ,  $\Sigma_1$  and  $\Sigma_2$ .

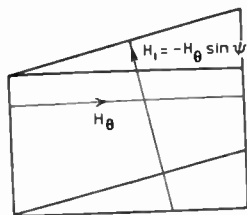


Fig. 8. Magnetic fields in the surface  $\Gamma$ .

In the unperturbed state,  $B_0 = \mu_0 H_\theta$ , existing in  $\Sigma_0$  but not in  $\Sigma_2$ . In the perturbed state, the metal in  $\Sigma_2$  is replaced by vacuum, so that a component  $B_{12}$  of  $B_1$  must exist there, equal to  $+\mu_0 H_\theta$ . In  $\Sigma_1$ , vacuum is replaced by metal, so that the resultant induction must vanish and  $B_1$  in  $\Sigma_1$  becomes  $B_{11} = -\mu_0 H_\theta$ . Also, in  $\Sigma_0 + \Sigma_2 - \Sigma_1$ , there is a component of  $H_\theta$  in the direction perpendicular to the sloping surfaces of the guide, and this gives rise to a component  $B_{13}$  of  $B_1$ , equal to  $-\mu_0 H_\theta \sin \psi$ . These components of induction are illustrated in Fig. 9.

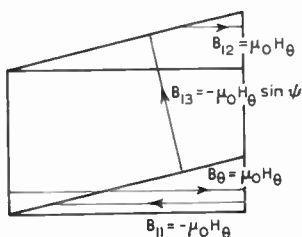


Fig. 9. Magnetic inductions in the surface  $\Gamma$

We can now express the magnetic terms of the integrals in the numerator of equation (24) in terms of  $H_\theta$ , bearing in mind that since  $H_r$  and

$B_r$  are perpendicular to  $\Gamma$ , their scalar products with  $H_\theta$ ,  $H_1$ ,  $B_\theta$ ,  $B_1$ , are zero. We thus have

$$\iiint_{V_1} H_0 \cdot B_1 dV = -\mu_0 \iiint_{V_1} H_\theta^2 dV$$

$$\iiint_{V_1} H_1 \cdot B_0 dV = -\mu_0 \iiint_{V_1} H_\theta^2 \sin^2 \psi dV$$

$$\iiint_{V_2} H_0 \cdot B_1 dV = \mu_0 \iiint_{V_2} H_\theta^2 (1 - \sin^2 \psi) dV$$

$$\iiint_{V_2} H_1 \cdot B_0 dV = 0$$

Hence

$$\begin{aligned} & \iiint_{V_1 + V_2} (H_0 \cdot B_1 - H_1 \cdot B_0) dV \\ &= \mu_0 \iiint_{V_2 - V_1} H_\theta^2 (1 - \sin^2 \psi) dV \end{aligned} \quad \dots\dots\dots(26)$$

It is easily seen that the integrals over  $V_1$  and  $V_2$  in equations (25) and (26) are equal, so that they cancel. Thus in equation (24),  $\delta\omega/\omega$  is zero. No approximations have been introduced in this perturbation treatment, so that the frequency shift is exactly zero.

The change in the propagation constant is given by

$$\frac{\delta\beta}{\beta} = -\frac{\delta\omega}{\omega} \cdot \frac{\omega}{\beta} \cdot \frac{\partial\beta}{\partial\lambda_0} \cdot \frac{\partial\lambda_0}{\partial\omega} \quad \dots\dots\dots(27)$$

Thus the propagation constant shift is also zero.

### 4.3. Other Modes

Other modes than the  $E_{01}$  may be treated in the same way; the descriptions of the perturbation fields are more complicated than those discussed above, but the method is essentially similar, and it is found that equations (25) and (26) apply equally to all modes. Thus for all modes the propagation constant in the helical guide is exactly the same as in the infinite circular guide.

### 4.4. Wave Velocity

In the infinite circular guide, the waves travel along a circular path, and if we choose the cylindrical surface  $\Gamma$  of radius  $r$ , the actual

phase velocity (i.e. the linear velocity, not the angular velocity) is

$$u_{\phi} = \frac{\omega r}{\beta} \quad \dots\dots\dots(28)$$

while the group velocity is

$$u_g = r \cdot \frac{d\omega}{d\beta} \quad \dots\dots\dots(29)$$

In the helical guide, a wave travels a distance  $p$  in the  $z$  direction at the same time as it travels through  $360^\circ$  in the  $\theta$  direction, so that in one revolution it suffers a phase change of  $2\pi(\beta + vp/2c)$ , whereas the wave in the infinite circular guide suffers a phase change of  $2\pi\beta$ . In the helical guide, the phase velocity in the cylindrical surface of radius  $r$  is evidently

$$v_{\phi} = \frac{\omega \sqrt{\{r^2 + p^2/4\pi^2\}}}{\beta + vp/2c} \quad \dots\dots\dots(30)$$

and the group velocity is

$$v_g = \sqrt{\{r^2 + p^2/4\pi^2\}} \cdot \frac{d\omega}{d(\beta + vp/2c)} \quad \dots\dots\dots(31)$$

When  $p=0$ , equations (30) and (31) reduce to equations (28) and (29) respectively. When  $p$  becomes indefinitely large, we have

$$\lim_{p \rightarrow \infty} v_{\phi} = \frac{\omega c}{v\pi} \quad \dots\dots\dots(32)$$

But  $2c/v$  is the wavelength in the  $z$  direction, so that equation (32) expresses the fact that as  $p$  approaches infinity, the phase velocity in the helical guide approaches the value proper to a co-axial cylindrical guide. Also, for large  $p$  we can write

$$v_g \cong \frac{p}{2\pi} \cdot \frac{d\omega}{d(\beta + vp/2c)}$$

In the limit, we neglect  $\beta$  in comparison with  $vp/2c$ , and put  $v/2c = 1/\lambda$ , where  $\lambda$  is the wavelength in the  $z$  direction. Then

$$\lim_{p \rightarrow \infty} v_g = \frac{p}{2\pi} \cdot \frac{d\omega}{d(p/\lambda)} = \frac{1}{2\pi} \cdot \frac{d\omega}{d(1/\lambda)} \quad \dots\dots\dots(33)$$

Equation (33) expresses the fact that as  $p$  approaches infinity, the group velocity in the helical guide approaches the value proper to a co-axial cylindrical guide.

If  $v=0$ ,  $\lambda$  is infinite, and the mode in the co-axial guide does not exist.

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**7. Appendix: Proof that the Divergence Integral Vanishes**

We shall prove here that the divergence integral of Section 4.2 vanishes for the  $E_{01}$  mode. The proof may be carried out in an analogous way for any mode, but is more tedious than the case we shall consider. The notation is the same as in Section 4.

First, by Gauss's theorem, we write

$$\int \int \int_{V_0+V_2} \text{div} \left\{ (\mathbf{H}_0 \times \mathbf{E}_1) + (\mathbf{E}_0 \times \mathbf{H}_1) \right\} dV = \int \int_{S_0+S_2} \left\{ (\mathbf{H}_0 \times \mathbf{E}_1) + (\mathbf{E}_0 \times \mathbf{H}_1) \right\} \cdot \mathbf{n} dS \tag{34}$$

where  $\mathbf{n}$  is a unit vector normal to the surface  $S_0+S_2$ , which encloses the volume  $V_0+V_2$ .  $\mathbf{n}$  is directed outwards from  $V_0+V_2$ . There are three pairs of faces of the volume  $V_0+V_2$  to be considered; we shall take each pair in turn, and show that its contribution to the integral is zero.

We take first the end-faces, lying in the planes  $\theta=0$  and  $\theta=2\pi/\beta$ . Write

$$\left\{ (\mathbf{H}_0 \times \mathbf{E}_1) + (\mathbf{E}_0 \times \mathbf{H}_1) \right\} \cdot \mathbf{n} = (\mathbf{E}_1 \times \mathbf{n}) \cdot \mathbf{H}_0 + (\mathbf{H}_1 \times \mathbf{n}) \cdot \mathbf{E}_0$$

The various components of  $\mathbf{E}$  and  $\mathbf{H}$  are derived in Section 4 and illustrated in Figs. 6 and 8. Evidently

$$(\mathbf{E}_1 \times \mathbf{n})_{s_0+s_2} = (\mathbf{E}_{13} \times \mathbf{n})_{s_2} + (\mathbf{E}_{12} \times \mathbf{n})_{s_0+s_2-s_1} + (\mathbf{E}_{11} \times \mathbf{n})_{s_1}$$

Since  $\mathbf{E}_{11} = -\mathbf{E}_{13}$  and  $S_1 = S_2$ , this becomes

$$(\mathbf{E}_1 \times \mathbf{n})_{s_0+s_2} = (\mathbf{E}_{12} \times \mathbf{n})_{s_0+s_2}$$

The field  $\mathbf{E}_{12}$  is the same at  $\theta=0$  and  $\theta=2\pi/\beta$ , but the vector  $\mathbf{n}$  is directed oppositely on the two surfaces. Thus the contributions to the integral cancel exactly. Further, we have explained in Section 4.2 that  $\mathbf{H}_1$  lies in a cylindrical surface  $\Gamma$ , so that  $(\mathbf{H}_1 \times \mathbf{n})$  lies in a radial direction, and its scalar product with  $\mathbf{E}_0$ , which consists of a single component in the  $z$  direction, is zero. There is thus no contribution to the divergence integral from the end faces of the volume  $V_0+V_2$ .

Secondly, we consider the cylindrical surfaces  $r=b$  and  $r=a$ . We note that  $\mathbf{E}_0$  and  $\mathbf{E}_1$  contain the common factor  $E_z \propto X_\beta(kr)$ . We have that  $X_\beta(kb) = X_\beta(ka) = 0$ , so that the integrand on the right-hand side of equation (34) is identically zero.

The third pair of faces to consider are those which are twisted in the helical guide, and on reducing the pitch to zero become two parallel planes in the infinite circular guide. At these surfaces,  $\mathbf{E}_{11}$  and  $\mathbf{E}_{13}$  are parallel to  $\mathbf{n}$ , so that their vector products with  $\mathbf{n}$  are zero.  $\mathbf{n}$  is oppositely directed on the two surfaces, while the values of  $\mathbf{E}_{12}$  are equal, so that the contributions due to  $\mathbf{E}_{12}$  cancel exactly. We have seen above that  $(\mathbf{H}_1 \times \mathbf{n}) \cdot \mathbf{E}_0$  is zero.

We have thus shown that all contributions to the divergence integral are either identically zero or cancel each other exactly.