JOURNAL OF
The British Institution of Radio Engineers
(FOUNDED IN 1925 - INCORPORATED IN 1932)
"To promote the general advancement of and to facilitate the exchange of information and ideas on Radio Science."
Vol. IX (New Series) No. 5 MAY 1949

CIRCULATION OF SCIENTIFIC INFORMATION

One of the principal objects of The Royal Society Scientific Information Conference (July 1948) was to secure improvement of library and information services. During the discussions the following recommendation was made and action invited :

> "Science rests upon its published record, and ready access to public scientific and technical information is a fundamental need of scientists everywhere. All bars which prevent access to scientific and technical publications hinder the progress of science and should be removed.
>
> "Making single copies of extracts from books or periodicals is essential to research workers, and the production of such single extract copies, by or on behalf of scientists, is necessary for scientific practice."

The difficulties of obtaining new technical information have frequently been discussed, but even after publication there are many obstacles which prevent wide dissemination and useful discussion. For example, a certain amount of copying is permissible under the Fair Usage clause of the Copyright Act, but legal opinion is not unanimous on the question of where the dividing line between what is permissible and what is not permissible should fall.

After carefully considering all aspects of the subject, and particularly taking into account the interests of scientists in the Commonwealth, the Royal Society now suggests that the problem could be solved without loss to any organization or person, if scientific societies and other publishers of scientific periodicals would state that they have no objection to the making of *single* copies of parts of their publications *under certain limited and stated conditions.* To enable this to be done, a Declaration on fair dealing in regard to copying has been prepared by the Royal Society with the help and co-operation of a number of interested organizations.

The suggested Declaration is not a blank cheque to use published material in any way desired. It only applies to making, *without profit* and within stated conditions, a single copy ; it does not give permission to use such copy for any subsequent reproduction by any means ; and it does not include permission to quote or reprint the whole or a part. The author will not lose any rights conferred by the Copyright Act, and the Declaration would not prevent publishers from taking legal action against an offender.

The Royal Society accepts the fact that scientific societies and their members wish to place no obstacle in the way of disseminating scientific knowledge and has, therefore, invited the Council of the Institution to subscribe to a "Fair Copy Declaration." If generally accepted the proposal should benefit both Scientists and Engineers and it is proposed to review the situation again at the end of two years.

Meanwhile the first report of the Committee on Industrial Productivity emphasises the fact that an early increase in productivity could be achieved by the rapid application of existing knowledge. This Committee was appointed by the Lord President of the Council in December 1947 and from the outset established a panel of technical information services and contributed to the work of the Commonwealth Conference on scientific information. The panel has to consider under its terms of reference what improvements can be made to ensure a rapid and wide dissemination of information to industry.

FOURIER ANALYSIS IN RELATION TO THE ELECTROCARDIOGRAM*

by

W. E. Benham, B.Sc., D.Sc. (Hon.) †

SUMMARY

The bio-electric potentials arising in the Purkinje fibres of the heart may by suitable apparatus be displayed and recorded. The resulting trace, known as an electrocardiogram, has characteristics designated by the letters P, Q, R, S, T, as indicated on Fig. 1. Both P and T " waves " are seen to be rounded as compared with the QRS " complex." The latter is comparatively rich in " high " frequencies (say, greater than 50 c/s), but to what extent, and as to which frequencies, can only be accurately assessed by Fourier analysis.

Experimentally, using a Wave Analyser, this is a relatively simple matter, but to make a Fourier analysis of the electrocardiogram would be prohibitively time-consuming if carried out for individual patients. There is, however, an advantage in studying mathematically some of the more outstanding types of QRS impulse (or simple approximations thereto), as in this way theoretical support can be adduced for the contention that the results of experimental wave analysis really do refer, at least mainly, to impulses arising in the Purkinje fibres of the human heart and not to potentials having their origin in the general body muscles. The latter appear as a sort of roughness superposed on the trace, and when analysed definitely cause effects at the higher frequencies. Mathematical analysis of the kind given in the present paper helps in separating out the various bio-electric phenomena.

An initial account in terms of elementary Fourier Analysis (leading to the well-known line spectrum of harmonic components) helps to establish the somewhat surprising fact that the impulses from the human heart do not require this form of treatment. The experimental frequency analysis corresponds, on the contrary, to isolated impulses, successive " kicks " of the needle varying somewhat according to the variation in shape from impulse to impulse. The mathematical development proceeds with this fact in mind, and a large number of variants is provided with continuous " spectra," by application of Fourier's Integral Theorem.

In cases of patients giving particularly large P or T waves it may be necessary to take them into account, particularly at the lower frequencies. Suitable analyses are given. Experimentally, a technique is always available for separate study of P and T waves.

Introduction

In a recent paper by Dr. W. E. Boyd, Hon. Director of these Laboratories, entitled " Wave Analysis of the Low Frequency Potentials of the Human Body " the outputs from human subjects in the usual electrocardiograph connections were studied over the frequency range 10-1,000 c/s by means of a wave-analyser. The subject to be analysed is placed in a cage in order to minimize pick-up of unwanted static charges or of spurious potentials of A.C. Mains frequency. The leads from the subject are passed through a hole in the cage to the input of a high gain amplifier, itself partly screened, whence the amplified body potentials are taken to the input of the wave-analyser. The design of the amplifier is such that its frequency response is level to about $\pm\frac{1}{2}$db over the frequency range of 0·1 to 1,000 c/s, most of the error being due to the frequency error of a transformer at the input of the analyser. The analyser has a frequency error (mainly due to the input transformer) which is known by accurate calibration and for which allowance can be made.

For the interpretation of these curves a Fourier Analysis of a repeating Λ shaped impulse was performed by Mr. N. H. Langton at the suggestion of Dr. Eric Fairley. This analysis, with minor changes, is given in Section 1. The envelope of the lines (each representing a harmonic) showed some similarity to part of the experimental gain-frequency curves, and it was decided to take the matter farther. See Figs. 2 and 3.

During further experimental work with the electrocardiograph in these laboratories it was found that assumed forms of PQRST wave form approximating more closely to those actually obtained from the subject afforded a greater

* Manuscript Received July 1948.
U.D.C. No. 578.088.7 : 517.512.2.

† Consultant to Boyd Medical Research Trust Laboratories.

measure of agreement, and generally provided information of interest and possible value in relation to the experimental curves obtained. A typical electrocardiogram, as taken with a drum camera off an oscilloscope,[3] is shown in Fig, 1, and the experimental output of the subject of Fig. 1 as a function of analyser frequency setting is given in Fig. 2.

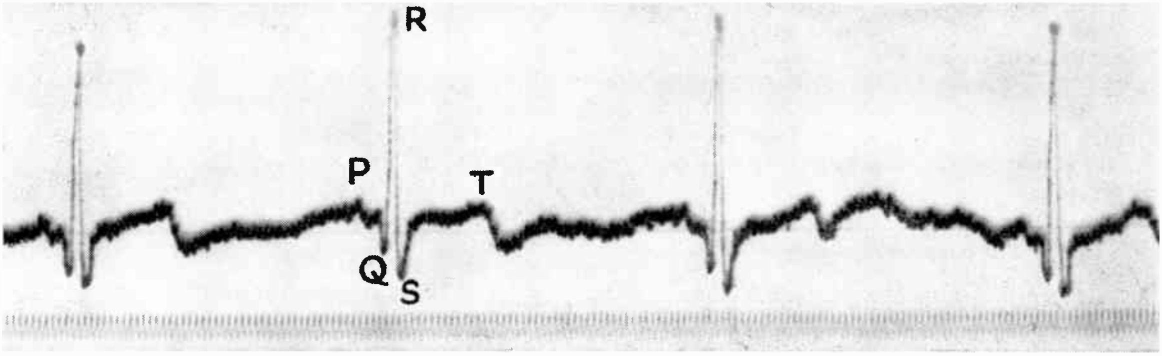

Fig. 1.—Oscillocardiogram (with 50 c/s timing wave and iso-electric line).

Comments on the Fourier Analysis

Mathematicians sometimes explicitly deny that the Fourier expansion of Section 1 represents the function over the interval between impulses.[3] The

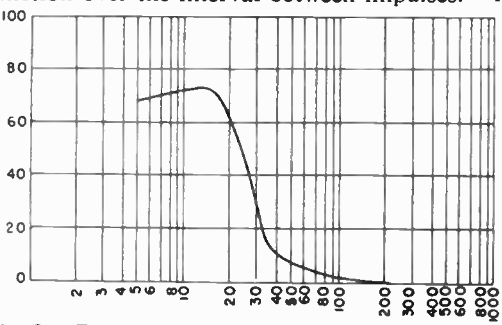

Fig. 2.—Experimental Spectrum for subject of Fig. 1. Electrodes to forearms (left, earthed). Subject prone.

developments outlined in Sections 2, 3 and 4 largely remove this *a priori* limitation (see p. 74 of ref. 4, $\varphi(x)$ being now defined for all x).

In case, however, it should be thought that the mathematical restriction to the effect that the assumed expansion represents a given function only over the interval during which an impulse occurs, except this interval coincide with the impulse repetition period, is necessarily of pedagogic interest only and may be ignored by practical engineers, it may be as well to point out that in Section 8 we disclose a measure of support for the mathematicians, in that the analyser registers over a time interval barely longer than the actual duration of the impulse under examination. Were the function represented between impulses we should expect the selected frequency to be found at all instants. It should, however, be mentioned here that, on the assumption that we are, in reality, dealing with heavily damped waveforms, and not with the undamped harmonics of elementary Fourier analysis, the experimental results could perhaps receive explanation on the basis of die-away terms of the right time constant. In the present paper we make no attempt to deal in detail with damped impulses, but the very simple treatment for an exponential impulse is given in Appendix I. It is supposed to be possible to represent even a damped function using undamped waves. It should, however, be pointed out that on analogy with Lerch's theorem relating to Laplace transforms,[8] a continuous spectrum should uniquely represent the function, and vice versa.

Now in Sections 2, 3 and 4 we show, firstly, how, assuming the spectrum of an isolated impulse to be known, that of a train of N impulses may be arrived at. We use the word "impulse" rather than "pulse" here to avoid conflict with the medical use of the term. Next it is indicated how the infinite wave train may be obtained by application of suitable analysis to the finite train. In Section 4 the analysis of a single impulse is given, and in subsequent sections, further single impulses (made up, maybe, of simpler constituents). In Section 8 evidence is presented that the Fourier spectrum of the single impulse, in conjunction with the bandwidth of the analyser, is what is needed, rather than that of a repeating train.

The present paper is intended as suggestive of promising lines of research rather than as a complete treatment of the subject, and no special originality is claimed. Quite possibly at least some of the waveforms have been analysed by several workers.

No attempt has been made to supply a complete bibliography, but for those wishing to study the mathematics of Operational Calculus and Fourier Integrals, a few general references are given.[4,5,6,7]

Section I. Fourier Analysis of Triangular Wave for Different Repetition Rates

Let us first consider the case of period $2c$, the indefinitely recurring, equally spaced pulses being on base $c/10$ and of height 1·4 mV (1400 µV; the individual Fourier terms will eventually be expressed in µV). The function of time which gives the assumed triangular wave train is

$$\varphi(t) = b_o + \sum_{n=1}^{\infty} b_n \cos\frac{2n\pi}{2c}t + \sum_{n=1}^{\infty} a_n \sin\frac{2n\pi}{2c}t$$

where b_o b_n and a_n are coefficients to be determined. We shall take $2c = 0{\cdot}8$ secs. as corresponds to a pulse rate of 1·25 c/s or 75 per minute, which is close to the normal value for a healthy subject, as is the 1400 µV voltage output when taken in Electrocardiograph connection I. We have to integrate over a period and then divide by the period to obtain b_o

$$b_o = \frac{1}{2c}\int_o^{2c} \varphi(t)\,dt \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots(1)$$

Since the function is zero at all times within any one period $2c$ except over a time interval $c/10$, we have, dividing this interval into two equal parts to correspond with the rising and falling portion of the triangle,

$$b_o = \frac{1}{2c}\left[\int_o^{\frac{c}{20}} 70t\,dt + \int_{\frac{c}{20}}^{\frac{c}{10}} (2{\cdot}8 - 70t)\,dt\right] \quad \ldots.(1a)$$

in which the slope of the lines is $m = 1{\cdot}4/(c/20) = 28/c = 28/0{\cdot}4 = 70$ mV sec^{-1}

Inserting limits we have

$$b_o = \frac{1}{2c}\left\{\Big[35t^2\Big]_o^{\frac{c}{20}} + \Big[2{\cdot}8t\Big]_{\frac{c}{20}}^{\frac{c}{10}} - \Big[35t^2\Big]_{\frac{c}{20}}^{\frac{c}{10}}\right\}$$

$$= \frac{-35c}{400} + \frac{1{\cdot}4}{20} = 0{\cdot}035 \text{ mV} \ldots\ldots\ldots(2)$$

If we multiply $\varphi(t)$ by $\cos(n\pi t/c)$ and integrate over a period $2c$ the only term of the summations which does not vanish is the one in b_n.

We obtain $\left(\text{after cancellation of factor } \frac{1}{2}\right)$

$$b_n = \frac{1}{c}\int_o^{2c} \varphi(t)\cos(n\pi t/c)\,dt \quad \ldots\ldots\ldots\ldots\ldots(3)$$

Applying similar considerations to those used for b_o, we have

$$b_n = \frac{1}{c}\int_o^{\frac{c}{20}} 70t\cos(n\pi t/c)\,dt + \int_{\frac{c}{20}}^{\frac{c}{10}} 2{\cdot}8\cos(n\pi t/c)\,dt$$

$$- \int_{\frac{c}{20}}^{\frac{c}{10}} 70t\cos(n\pi t/c)\,dt$$

$$= \frac{70c}{n^2\pi^2}\left[\Big[x\sin x + \cos x\Big]_o^{\frac{n\pi}{20}} - \Big[x\sin x + \cos x\Big]_{\frac{n\pi}{20}}^{\frac{n\pi}{10}} + \frac{2{\cdot}8}{\pi}\Big[\sin x\Big]_{\frac{n\pi}{20}}^{\frac{n\pi}{10}}\right],$$

$$= \frac{28}{n^2\pi^2}\left[\left(\frac{n\pi}{20}\sin\frac{n\pi}{20} + \cos\frac{n\pi}{20} - 1\right) + \left(\frac{n\pi}{20}\sin\frac{n\pi}{20} + \cos\frac{n\pi}{20} - \frac{n\pi}{10}\sin\frac{n\pi}{10} - \cos\frac{n\pi}{10}\right)\right]$$

$$+ \frac{2{\cdot}8}{n\pi}\left(\sin\frac{n\pi}{10} - \sin\frac{n\pi}{20}\right).$$

Inspection reveals that the sin terms of the final expression cancel those in the square brackets. Writing $1 + \cos(n\pi/10) = 2\cos^2(n\pi/20)$ we obtain

$$b_n = \frac{56}{n^2\pi^2}\cos\frac{n\pi}{20}\left(1 - \cos\frac{n\pi}{20}\right)$$

$$= \frac{56}{n^2\pi^2}\cos 9^\circ n\,(1 - \cos 9n^\circ) \ldots\ldots\ldots\ldots(4)$$

Similarly

$$a_n = \frac{56}{n^2\pi^2}\sin\frac{n\pi}{20}\left((1 - \cos\frac{n\pi}{20}\right)$$

$$= \frac{56}{n^2\pi^2}\sin 9n^\circ\,(1 - \cos 9n^\circ) \ldots\ldots\ldots\ldots(5)$$

Hence

$$\sqrt{b_n^2 + a_n^2} = \frac{56}{n^2\pi^2}\left(1 - \cos\frac{n\pi}{20}\right) = \frac{56}{n^2\pi^2}(1 - \cos 9n^\circ) \qquad \ldots\ldots\ldots\ldots\ldots\ldots(6)$$

When n is very small, $1 - \cos(n\pi/20) \rightarrow n^2\pi^2/800$,

so that for $n \rightarrow o$, $\sqrt{a_n^2 + b_n^2} = \frac{56}{800} = \cdot 07$ mV

$= 70\ \mu V$ ....(6a)

The following table gives sufficient details for present purposes. In view of (6) we do not require to evaluate a_n and b_n separately, the amplitude $(a^2_n + b^2_n)^{\frac{1}{2}}$ gives all that is needed. However, we give these separate values in order to indicate the phase, in case at some time this should prove useful in electrocardiogram studies.

There is now a change under the integral. We now have

$$b_n' = \frac{c}{2}\left[\int_o^{\frac{c}{20}} - \int_{\frac{c}{20}}^{\frac{c}{10}} 70t \cos\frac{2\pi nt}{c}\,dt + \int_{\frac{c}{20}}^{\frac{c}{10}} 2\cdot 8 \cos\frac{2\pi nt}{c}\,dt\right]$$

$$= 2\,\frac{70c}{(2n)^2\pi^2}\left\{\Big[x \sin x + \cos x\Big]_o^{\frac{2n\pi}{20}} - \Big[x \sin x + \cos x\Big]_{\frac{2n\pi}{20}}^{\frac{2n\pi}{10}}\right\} + \frac{2\cdot 8}{2n\pi}\Big[\sin x\Big]_{\frac{2n\pi}{20}}^{\frac{2n\pi}{10}}$$

$$= 2b_{2n} \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(9)$$

Thus the rule for arriving at the coefficients is to take twice the double order harmonic corresponding to the original repetition rate. The following consideration makes it unnecessary to tabulate any figures in this connection. The double order harmonic corresponding to the original repetition rate of period $2c$ is of the same frequency as the single order harmonic (fundamental) corresponding to the period c. The rule is thus transformed into: 'Double the amplitude for double the repetition rate—leaving the frequency unchanged.' We may thence conclude that changes in the patient's pulse should be without effect on the position of the maxima and minima along the frequency axis.

Table 1.

n	$\frac{56}{n^2\pi^2}$	cos $9n^\circ$	b_n mV	sin $9n^\circ$ mV	a_n	$(a_n^2+b_n^2)$ µV	f (= $1\cdot 25n$)c/s
1	5·673	·9877	·06890	·1564	·01091	69·76	1·25
2	1·4183	·9511	·06594	·3090	·02143	69·35	2·5
3	·6303	·8910	·06109	·4540	·03119	68·76	3·75
4	·3546	·8090	·0548	·5878	·0398	67·72	5
5	·2269	·7071	·0470	·7071	·0470	66·48	6·25
6	·1576	·5878	—	·8090	—	64·95	7·5
7	·1158	·4540	—	·8910	—	63·24	8·75
8	·08865	·3090	—	·9511	—	61·25	10
10	·05673	0	0	1	·05673	56·73	12·5
20	·01418	−1	−·0284	0	0	28·37	25
30 -	·006303	0	0	−1	−·00630	6·30	37·5
40, 80, 120	·003545, etc.	1	0	0	0	0	50, 100, 150
60	·001576	−1	−·00315	0	0	3·15	75
100	·000567	−1	−·001135	0	0	1·135	125
140	000289	−1	− 000578	0	0	·58	175

The figures from the last two columns of Table I give the line spectrum of Fig. 3.

We now enquire what happens when the same pulses are separated by interval c instead of interval $2c$. Since there are pulses twice as often, the mean value b_0 will simply be doubled. Thus instead of (1) we have

$$b_0' = \frac{1}{c}\int_o^c \varphi(t)\,dt = 70\ \mu V \qquad \ldots\ldots\ldots\ldots\ldots\ldots(7)$$

The limits of the integrals are here unchanged, the only change being that of the factor outside the square brackets. For b_n' we have instead of (3)

$$b_n' = \frac{2}{c}\int_o^c \varphi(t)\cos\left(\frac{2n\pi}{c}\,t\right)dt \qquad \ldots\ldots\ldots\ldots(8)$$

The above analysis does, however, predict that the heights of the lines (shown on Fig. 3 for pulse rate of 75 per minute) would alter with the pulse rate of the subject. If the analyser be tuned for maximum response, it may be fairly held that, on the assumption of a line spectrum, the centre of the passband of the analyser coincides with the strongest line. The selectivity of the analyser will then not matter provided it is adequate to exclude the neighbouring lines. In other words, the response would be doubled on account of the halving of the repetition period, and there is no compensating reduction on account of the increased line spacing, under the particular conditions obtaining. It cannot be said that this result is borne out in practice. A crucial experiment in this connection

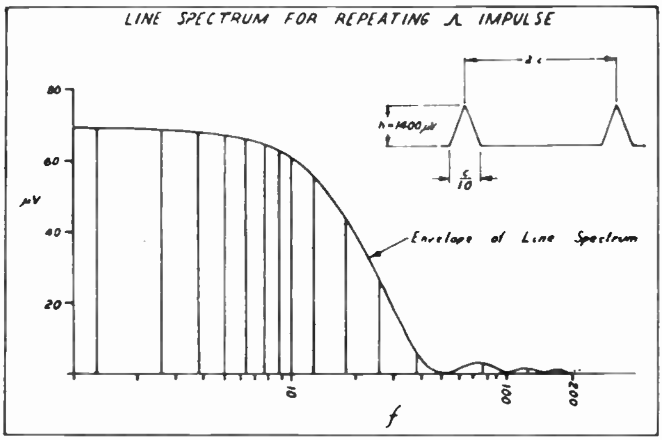

Fig. 3.—Line Spectrum of repeating V impulse. Every line shown up to 10 c/s, representative lines thereafter.

is described in Section 8. For the moment the following will suffice. The experimental curve of Fig. 2 strongly suggests that we are dealing with a continuous rather than with a line spectrum, but, waiving this point for the moment, we would expect the maximum of Fig. 2, assuming it to correspond with the strongest line of the Fourier spectrum, to increase in height as the subject's pulse increases, the value of h and of τ remaining the same. The fact that this increase does not happen in practice means either that the foregoing analysis is irrelevant or that the analyser is allowing through so many lines at once that what is gained in respect of increased line height is lost in respect of the number of lines let through the analyser at anything approaching maximum strength, this number, be it noted, decreasing as the repetition frequency is raised. Now this is not the case, for it can be shown that the height of the maximum in Fig. 2 happens to agree fairly closely with what theory (for a diphasic impulse, q.v.) would require for a single line only being passed by the analyser. Hence the first of the alternatives, that the analysis of 1 is irrelevant, is apparently forced upon us. We shall, however, see that the expressions obtained in 1 are closely related to expressions corresponding to continuous spectra, which will be derived in the sequel. Now a continuous spectrum may be regarded as an extreme case of a line spectrum in which the spacing of the lines is made to approach zero. But this is just what is obtained if the repetition period T is made to approach infinity, for the line spacing on the frequency axis is proportional to $1/\mathrm{T}$.

The sequel will thus be concerned largely with the examination of isolated impulses, but before going on to consider the spectra of these, a brief account of finite wave trains will be given. An experiment designed to establish that the analyser readings do correspond to a single impulse, and not more, in the range of repetition frequencies of interest in Heart Analysis, is described in Section 8.

Section 2. Frequency Distribution corresponding to a Finite Train of Impulses of any form

In Section 1 we dealt with an infinite wave train. For a finite train the procedure is along totally different lines. We begin with a single impulse.

The distribution in f, or (complex) spectrum, for an isolated impulse starting at $t=o$ and ending at $t=\tau$ is most simply represented by employing the cisoidal form of Fourier's Integral Theorem. We then require to evaluate the Fourier transform

$$\mathrm{S}(f, o) = \int_{o}^{\tau} \varphi(t) e^{-2\pi i f t}\, dt \quad \ldots\ldots\ldots\ldots\ldots (10)$$

For a similar impulse starting at the later instant $t=\mathrm{T}$, we have

$$\begin{aligned}\mathrm{S}(f,\mathrm{T}) &= \int_{\mathrm{T}}^{\tau+\mathrm{T}} \varphi(t-\mathrm{T}) e^{-2\pi i f t}\, dt \\ &= \int_{o}^{\tau} \varphi(t) e^{-2\pi i f(t+\mathrm{T})}\, dt \\ &= e^{-2\pi i f \mathrm{T}}\, \mathrm{S}(f,o) \quad \ldots\ldots\ldots\ldots\ldots (11)\end{aligned}$$

The exponential factor thus gives the change in the spectrum which corresponds to the shift of the impulse along the axis of t. For N pulses we have the frequency distribution :

$$\sum_{r=0}^{N-1} S(f, rT) = S(f,o)\frac{1-e^{-N.2ifT}}{1-e^{-2\pi ifT}}$$

$$= S(f,o)e^{-\pi if(N-1)T}\frac{\sin\pi fNT}{\sin\pi fT} \quad ..(12)$$

which has the spectrum

$$2|S(f,o)|\left|\frac{\sin\pi fNT}{\sin\pi fT}\right| \quad ....(13)$$

In this way we readily derive expressions for the spectra of particular finite wave trains, of which a few examples are listed below

$N = 2,\ 4|S(f,o)\cos\pi fT|,$

$N = 3,\ 6\left|S(f,o)\frac{1+2\cos 2\pi fT}{3}\right|,$

$N = 5,\ 10\left|S(f,o)\times\frac{(1+2\cos 2\pi fT)\cos 2\pi fT + 2\cos\pi fT\cos 3\pi fT}{5}\right|$

Values of $\frac{N}{1}\sin\pi fNT/\sin\pi fT$ for the first four even N values are indicated on Fig. 4, which shows clearly the tendency for the spectrum to resolve itself into lines as N is increased. Had we plotted the absolute values, thus showing lines everywhere positive going, the curves would have been less generally useful.

Section 3. Infinite train of impulses

To proceed from (13) to an infinite train involves limiting processes, which involve several pages of analysis to establish rigorously. It may be shown thereby, that when integrated over a line width the coefficient of $2S_o$ tends to $1/T$.

We shall in Section 4 see that in this way the spectrum for an infinite sequence of pulses, as derived from an individual pulse repeating at equal intervals, is reconciled with the elementary Fourier analysis of Section 1. It now remains to specify the frequency distributions for a single impulse of various forms.

4. Single Inverted V Impulse

By Fourier's Integral Theorem

$$S(f,o) = \int_{o}^{\frac{\tau}{2}}\frac{h}{(\tau/2)}te^{-i\omega t}dt + \int_{\frac{\tau}{2}}^{\tau}\left[2h-\frac{ht}{(\tau/2)}\right]e^{-i\omega t}dt =$$

$$\frac{2h}{(i\omega)^2\tau}\left\{\left[xe^{-x}+e^{-x}\right]_{\frac{i\omega\tau}{2}}^{i\omega\tau} - \left[xe^{-x}+e^{-x}\right]_{o}^{\frac{i\omega\tau}{2}} + i\omega\tau\left[-e^{-x}\right]_{\frac{i\omega\tau}{2}}^{i\omega\tau}\right\} =$$

$$\frac{2h}{(i\omega^2\tau)}\left\{e^{-i\omega\tau} - 2e^{-\frac{i\omega\tau}{2}} + 1\right\} =$$

$$-\frac{2h\tau}{(i\omega\tau)^2}e^{-\frac{i\omega\tau}{2}}.2\left(1-\cos\frac{\omega\tau}{2}\right) \quad ....(14)$$

$$2|S(f,o)| = 2h\tau\frac{1-\cos\frac{1}{2}\omega\tau}{(\frac{1}{2}\omega\tau)^2} \quad ....(15)$$

Division by T, as explained in Section 3, gives results in agreement with section 1. Thus, writing $T = 2c = 0{\cdot}8$ sec., $h\tau/T = 70\ \mu V$. Also $\omega\tau/2 = n\pi\tau/T = n\pi/20$. Equation (24) for the isolated

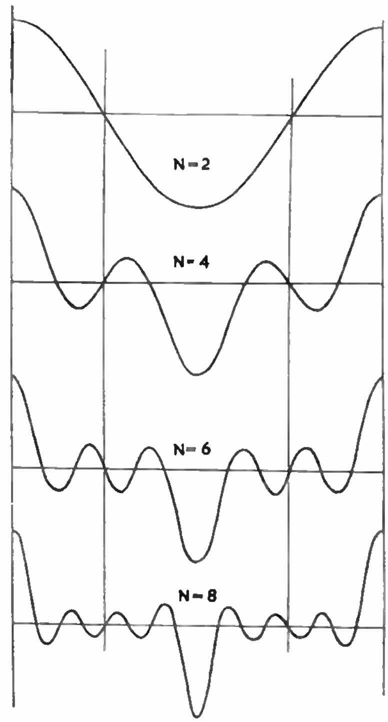

Fig. 4.—Curves of $\sin\pi fNT/\sin\pi fT$ for N = 2, 4, 6, 8 : the ordinates of single pulse spectra must be multiplied by the modulus of this factor to give spectrum for train of N waves.

pulse corresponds to $T = \infty$, which means a continuous spectrum, the lines (now of height approaching zero) being so closely packed (interval $1/T \rightarrow 0$) on the frequency axis that they are indistinguishable.

For $h = 1400\ \mu V$ and $\tau = \cdot 04$ secs., equation (15) gives

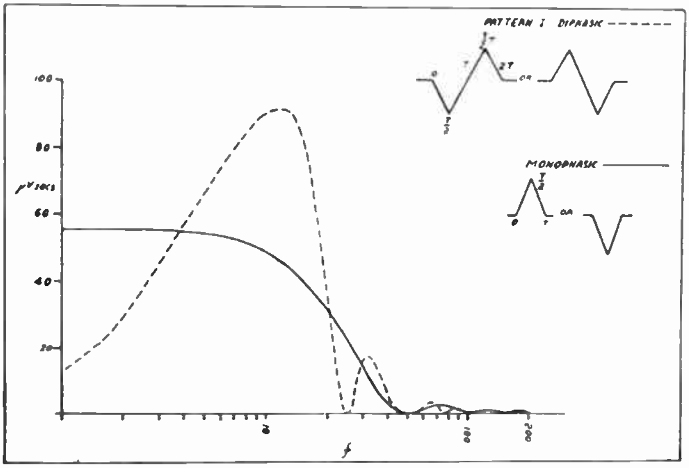

Fig. 5.—Continuous Spectra of single V (monophasic, isosceles) impulse and single pattern I diphasic (VI- or N-type) impulse.

$$2|S(f, o)| = 56\left(\frac{\sin \frac{1}{4}\omega\tau}{\frac{1}{4}\omega\tau}\right)^2 \mu V \text{ sec.} \quad \ldots\ldots(16)$$

This curve is plotted on Fig. 5 (full line).

If we wish to plot the amplitude corresponding to a selected frequency band we multiply by Δf (= bandwidth), to give

$$2|S(f, 0)| \,.\, \Delta f = 56\left(\frac{\sin \frac{1}{4}\omega\tau}{\frac{1}{4}\omega\tau}\right)^2 \mu V \quad \ldots\ldots(16a)$$

which, if, and only if, $\Delta f = T^{-1}$ gives, for $T = 0{\cdot}8$ secs.,

$$\frac{2|S(f, 0)|}{T} = 70\left(\frac{\sin \frac{1}{4}\omega\tau}{\frac{1}{4}\omega\tau}\right)^2 \mu V \quad \ldots\ldots\ldots\ldots(17)$$

which is seen to agree with equations (6, 6a) and Fig. 3. It is considered (see Section 8) that in heart applications we require $2|S(f, o)|\Delta f$ rather than $2|S(f, o)|/T$.

A train of two such impulses separated by time interval τ is clearly represented by $S(f, o)(1 + e^{-i\omega\tau})$, which has the spectrum

$$2|S(f, o) + S(f, \tau)| = 2h\tau\left(\frac{\sin \frac{1}{4}\omega\tau}{\frac{1}{4}\omega\tau}\right)^2 \left|\cos \frac{\omega\tau}{2}\right| \quad \ldots(18)$$

While this two-impulse train occasionally approximates to the form of a QRS of the M type, the centre part of the experimental curve usually dips below the isoelectric line.*

Section 5a. Diphasic QRS of the N types—" Pattern 1"

We are, however, more interested in the case where the second impulse is inverted. We then obtain a waveform which is sometimes a very fair approximation to a QRS impulse of the diphasic variety (for equal excursions above and below the line the term equiphasic is also applicable, though it is to be remembered that this term need not apply to a

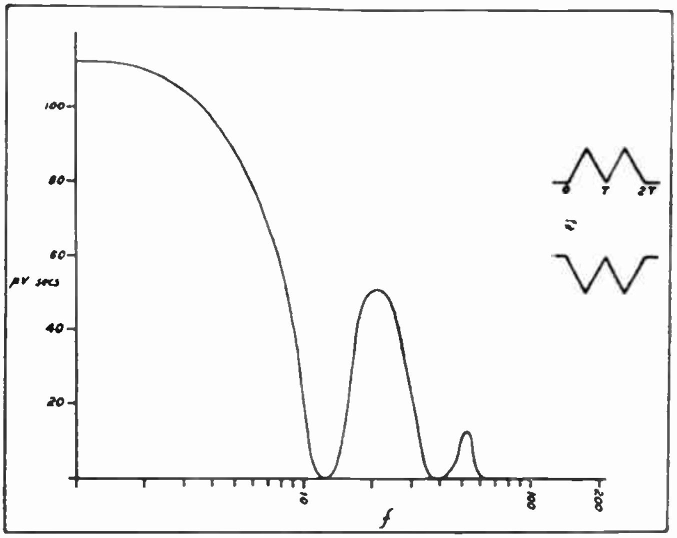

Fig. 6.—Continuous Spectrum of single pattern I triphasic (M- or W-type) impulse.

**See for example, L. N. Katz, Electrocardiography (Kimpton, 1941), Fig. 48M, p. 87, and Fig. 65E, p. 108.*
rted" N.

diphasic waveform but is equally applicable to triphasic or polyphasic forms). We shall refer to this diphasic as "Pattern I," the features being that the slope of the long portion is assumed equal (in absolute magnitude) to the slope of either side portion of the impulse.

We have for this case

$$S(f,o)-S(f,\tau) = S(f,o)(1 - e^{-i\omega\tau})$$

$$= e^{-2i\theta} \, . \, h\tau \left(\frac{\sin \frac{1}{2}\theta}{\frac{1}{2}\theta}\right)^2 i \sin \theta \qquad \ldots\ldots\ldots\ldots\ldots\ldots(19)$$

where θ has been written for $\omega\tau/2$. We have for the modulus of this expression

$$2|S(f,o) - S(f,\tau)|$$

$$= 2h\tau \, . \, \frac{2(1 - \cos \theta)}{\theta^2} \, . \, |\sin \theta| \, . \, .(20)$$

This is everywhere less than twice the corresponding quantity for the single V impluse (equation 15). This is due to the tendency of the two halves of the diphasic to interfere at some frequency. Thus, even at the frequency of maximum reinforcement the residual phase angle between the two components prevents a doubling of the amplitude. The inverted N type of QRS has the same spectrum as the N type, namely as given by (29), which is plotted for $h\tau = 56$ μV secs. on Fig. 5 (dotted curve).

Section 5b. Diphasic QRS of the N and inverted N types—" Pattern II "

The feature of this pattern lies in the fact that the curve QRSC has the portion RS of double the steepness of either of the rising portions QR, SC. It may be broken down into constituents QRB, ASC as indicated (Fig. 7). These two are correctly summed as to sign and time displacement by the following expression, in which τ is the time that would be taken for QRB, that is, τ is two thirds of the time that is taken for QRSC.

$$S(f,o) - S\left(f, \frac{\tau}{2}\right)$$

$$= h\tau \, . \, e^{-\frac{1}{4}i\omega\tau}\left(\frac{\sin \frac{1}{4}h\tau}{\frac{1}{4}h\tau}\right)^2 (1 - e^{-i\omega\tau})$$

$$= h\tau \, . \, e^{-\frac{3}{2}i\theta}\left(\frac{\sin \frac{1}{2}\theta}{\frac{1}{2}\theta}\right)^2 . \sin \tfrac{1}{2}\theta \,\ldots\ldots\ldots(21)$$

$$2\left|S(f,o) - S\left(f, \frac{\tau}{2}\right)\right|$$

$$= 2h\tau \, . \, \frac{2}{\theta^2} (1 - \cos \theta) \, . \, |\sin \tfrac{1}{2}\theta| \,\ldots\ldots\ldots(22)$$

which are seen to be identical with (19), (20) but with sin ($\theta/2$) replacing sin θ and (21) leads (19) in phase by the angle $\theta/2$.

Section 5c. Diphasic QRS of the N and inverted N types—" Pattern III "

Taking, *as usual*, τ to apply to the base of an isosceles triangle, the "chip" shown will have the base $\tau/2$. For the "chip" we have the spectrum

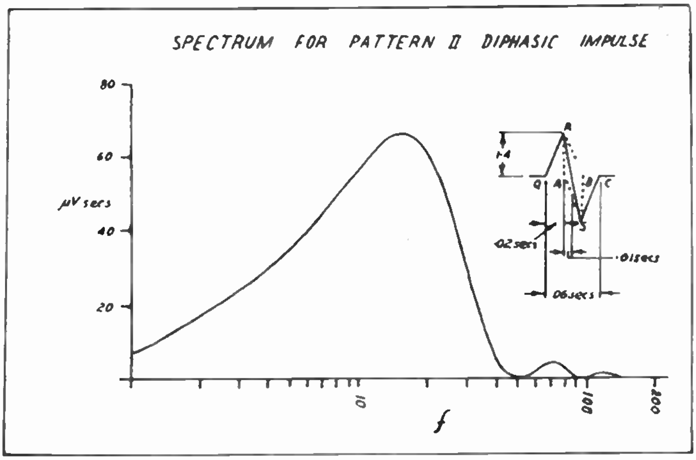

Fig. 7.—Continuous Spectrum of single pattern II diphasic (VI- or N type) impulse.

$$S(f,o) = \int_o^{\frac{1}{2}\tau} \frac{2ht}{\tau} e^{-i\omega\tau} dt \,\ldots\ldots\ldots\ldots\ldots\ldots(23)$$

$$= \frac{2h}{(i\omega)^2\tau}\Big[(1 + x)e^{-x}\Big]^{o}_{i\omega\frac{\tau}{2}}$$

$$2|S(f,o)| = \frac{h\tau}{2}\left|\frac{2}{\theta^2}(1 - e^{-i\theta} - i\theta e^{-i\theta}\right| \,\ldots\ldots(24)$$

The function specified by equation (24) is plotted on Fig. 8 (broken line). For the diphasic we have to add in an inverted pulse in negative τ. We thus obtain (for diphasic centred on zero)

$$[S(f,o) + [-[S(f,o)^*] = \frac{h\tau}{4}\frac{4i}{\theta^2}(\theta \cos \theta - \sin \theta) \qquad \ldots\ldots\ldots\ldots\ldots\ldots(25)$$

$$2|S(f,o) - S(f,o)^*| = 2h\tau\left|\frac{\sin\theta - \theta\cos\theta}{\theta^2}\right| \quad (26)$$

Equation (26) is plotted (full line) on Fig. 8 for the case $\tau = \cdot 08$.

It is noteworthy that in the spectrum of the chip (or single sawtooth) waveform we have for the first time in this paper a curve which has no zeros. The explanation lies in the symmetry of the remaining waveforms assumed, which enables destructive interference to take place between the harmonics corresponding to either half of the wave.

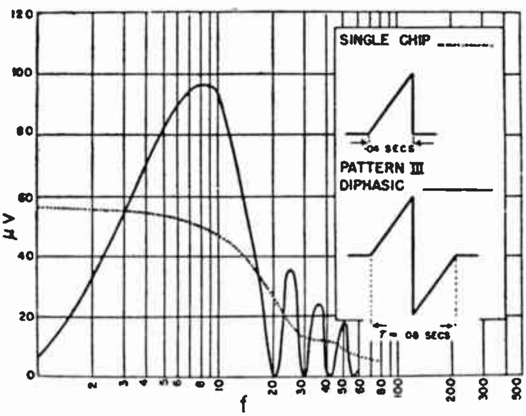

Fig. 8.—Continuous Spectra of single chip-shaped (sawtooth) impulse and single pattern III diphasic (VI- or N-type) impulse.

For the Λ pattern the two symmetrical halves are two "chips" back to back. However we try to subdivide a single chip, however, we fail to obtain any degree of likeness between the two parts. For all assumed diphasic patterns there is perfect symmetry between the two halves of the impulse. In the waveform of Section 6 we do not have this perfect symmetry, but the two parts on either side of $-\tau/2$ are sufficiently alike, and so constituted, as nevertheless to cause marked zeros.

Section 6. Triphasic Impulse—Jagged Outline

The triangular waveform is, of course, an idealization frequently departed from in practice, and one reason for not always noticing such departure is the narrowness of the QRS impulse as recorded by international convention using a string electrocardiograph camera. On a cathode ray instrument it will frequently be found that the steepness of the sides of the pulse can be seen to vary from top to bottom quite a lot.

A hypothetical waveform which brings out this feature is that shown here in which some of the sides are actually made vertical. It may be pointed out that the same distribution in ω will be obtained if we regard the trace as executed from right to left.* In the diagram,† the impulse of which $S(f,-\tau)$ is the distribution, is that between $-\tau$ and 0. Now for the part between 0 and τ,

$$S(f,o) = \int_o^{\frac{\tau}{2}} \frac{2h}{\tau} te^{-i\omega t}\,dt - \int_{\frac{\tau}{2}}^{\tau}\left(2h - \frac{2h}{\tau}t\right)e^{-i\omega t}\,dt = \frac{2h\tau}{(i\omega\tau)^2}\,.\,e^{-\frac{1}{2}i\omega\tau}\,.\,2(1-\cos\tfrac{1}{2}\omega\tau) \quad (27)$$

as would be expected (c.f. Section 4), while

$$S(f,-\tau) = \int_{-\tau}^{-\frac{\tau}{2}} \frac{2h}{\tau} te^{-i\omega t}\,dt + \int_{-\frac{\tau}{2}}^{o}\left(2h - \frac{2h}{\tau}t\right)e^{-i\omega t}\,dt =$$

$$\frac{2h\tau}{(i\omega\tau)^2}\left\{\Big[xe^{-x}+e^{-x}\Big]_{\frac{-i\omega t}{2}}^{o} - \Big[xe^{-x}+e^{-x}\Big]_{-i\omega\tau}^{-\frac{i\omega\tau}{2}} + i\omega\tau\Big[-e^{-x}\Big]_{-\frac{i\omega\tau}{2}}^{o}\right\} =$$

$$\frac{2h\tau}{(i\omega\tau)^2}(1-i\omega\tau)\left\{1-2e^{\frac{i\omega\tau}{2}}+e^{i\omega\tau}\right\} - \frac{2h\tau}{(i\omega\tau)^2}e^{\frac{i\omega\tau}{2}}(1-i\omega\tau)\,.\,2\left(1-\cos\frac{\omega\tau}{2}\right) \quad (28)$$

$$S(f,-\tau) + S(f,o) = 2h\tau\,.\,e^{\frac{i\omega\tau}{2}}\left(1-\cos\frac{\omega\tau}{2}\right)\,.\,\frac{2}{(i\omega\tau)^2}(i\omega\tau - 1 + e^{-i\omega\tau}) \quad (29)$$

**Assuming the modulus to be taken.*

†Fig. 9. The waveform there drawn is actually shown τ seconds later.

$$2|S(f, -\tau) + S(f, o)| = 4h\tau(1 - \cos\theta) . \left|\frac{2}{(2i\theta)^2}(2i\theta - 1 + e^{-i\theta})\right| \quad \text{.................(30)}$$

Fig. 9 gives the spectrum according to (30), taking $h = 800\ \mu V$.

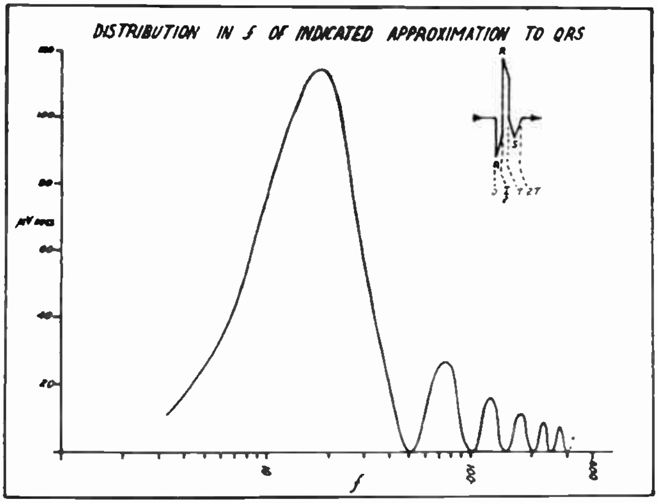

Fig. 9.—Continuous Spectrum of single triphasic impulse having a certain jagged outline.

Section 7. Depressed Monophasic Type

We have for the spectrum of this triphasic waveform, starting with result (25), and placing two of these waveforms back to back,

$$S(f, -\tau) = h\tau\frac{i}{\theta^2}(\theta\cos\theta - \sin\theta)(e^{i\theta} - e^{-i\theta}) \quad \text{.................(31)}$$

$$= h\tau\frac{2\sin\theta}{\theta^2}(\theta\cos\theta - \sin\theta)$$

$$2|S(f, -\tau)| = \frac{4h\tau}{\theta^2}\left|\sin\theta(\theta\cos\theta - \sin\theta)\right| \quad \text{..(32)}$$

The spectrum (32) is given on Fig. 10.

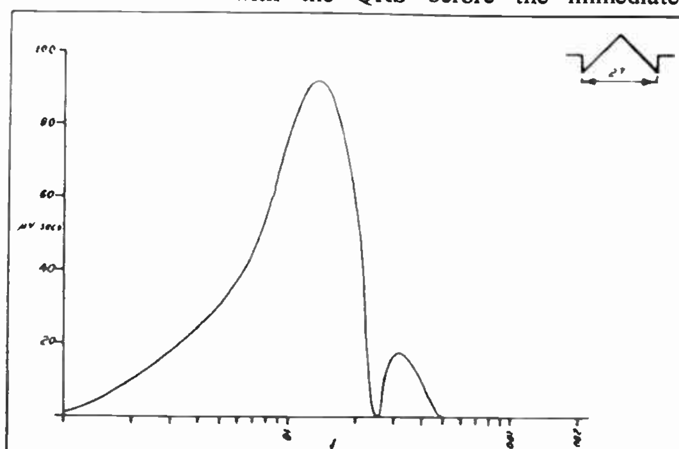

Fig. 10.—Continuous Spectrum of single triphasic impulse derived by depressing isosceles impulse below iso-electric line.

Section 8. Justification of use of Continuous Spectrum in Heart Analysis

In considering the general validity of the foregoing analysis, the chief evidence from the experimental side is as follows:

(1) The analyser, when read during an interval of time sufficient to contain a single heart beat only, gave substantially the same spectrum as was obtained when the amplified heart impulses were being passed *continuously* to the analyser. This admission of a single impulse was effected by means of a single switch (hand operated). The amplifier was left on during the off periods of the switch in order that the occurrence of a heart beat could be observed on the output meter, and the operation of the switch thereby correctly timed.

(2) If the response curve obtained really did correspond to an appreciable train of impulses rather than to a single impulse, there would be found a change in the ordinates of the spectrum when the pulse rate was made to vary. As a result of exercise the pulse rate of subject E. G. was caused to rise from 73 to 82 per minute. The response curve, instead of being some 15 per cent higher, was actually found to be, if anything, lower. Changes with exercises of the height and duration of the QRS impulse were insufficient to explain the departure from the theory for a line spectrum.

(3) In the case of a subject with an abnormally large T wave the analyser gave separate kicks for QRS and T wave in the region of 5 c/s. This indicates that the analyser has, so to speak, finished with the QRS before the immediately following T wave occurs. This observation is borne out by photographic traces taken of representative frequencies, which traces indicate that the die-away of all the QRS is complete well before the end of the S.T. interval.

9. Acknowledgments

It is a pleasure to record my indebtedness to correspondence with Dr. R. A. Smith, of Telecommunications Research Establishment, Malvern, on sundry mathematical points connected with the limiting processes involved in passing from a finite to an infinite wave train, and to Dr. Eric Fairley, of Royal Technical College, Glasgow, for helpful discussion. I would also thank Dr. Boyd and the Boyd Medical Research Trust for facilities in connection with the preparation of this paper which is published by their permission.

10. References

[1]Boyd, W. E., *J.Brit.I.R.E.* pp. 1-13 Mar/Apr. 1948.

[2]The advantages of Cathode Ray technique (freedom from inertia and from susceptibility to overload) are now unaccompanied by the drawbacks common to early forms of tube (see Pumphrey's article in Electronics (Ed. B. Lovell, Pilot Press, Ltd.), p. 489.

[3]" The expression (3) does not represent $f(x)$ at points beyond the interval—l to l unless $f(x)$ has period $2l$ ", *Encl. Britt. XIth Edition*, Vol. 10, p. 754 (Dr. E. W. Hobson). See also ibid Vol. 11, p. 308 (Prof. A. E. H. Love).

[4]Carslaw, H. S., and Jaeger, J. C., " Operational Methods in Applied Mathematics " (Oxford 1941, 3 and 5), p. 74.

[5]McLachlan, N. W. " Modern Operational Calculus," (MacMillan, 1948).

[6]Wiener, Norbert, " The Fourier Integral and Certain of its Applications " (Cambridge, 1933).

[7]Titchmarsh, E. C., " Introduction to the Theory of Fourier Integrals " (Oxford, 1937).

[8]See Appendix I, p. 243 of ref. 4, and p. 5 of ref. 5.

APPENDIX I

Impulse with Sharp Leading Edge and Trailing Tail

If we take the steepness of the leading edge as infinite, and the tail as exponential, the impulse is manifestly such that

$$\varphi(t) = 0 \quad (t < 0)$$

$$\varphi(t) = he^{-t/t_1} \quad (t > 0)$$

$$S(f, o) = \int_o^\infty he^{-\left(\frac{1}{t_1} + i\omega\right)t}\, dt$$

$$= \frac{ht_1}{1 + i\omega t_1}$$

$$S|S(f, o)| = \frac{2ht_1}{\sqrt{1 + \omega^2 t_1^2}}$$

Taking $t_1 = 0{\cdot}1$ secs., and $h = 1000$ µV, we obtain the curve of Fig. A. The block inset of this diagram indicates the impulse.

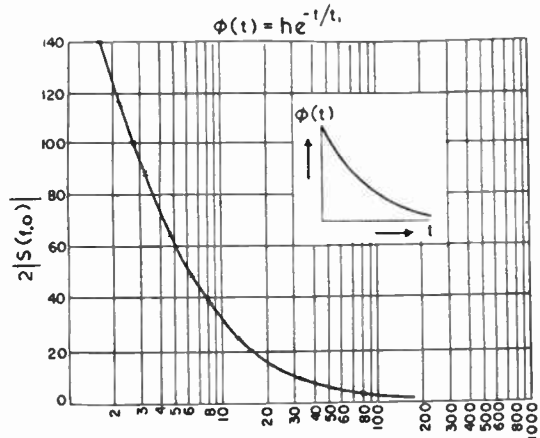

Fig. A.

APPENDIX II

Representation Applicable to P- or T- Waves

Reference to Fig. 1 shows a small wave, the P-wave, to the left of each QRS, and a larger wave the T-wave about 0·2 secs. to the right of S.

Ordinarily the spectrum contributed by the P-wave or T-wave is unimportant in comparison with that due to the QRS. We have mentioned, however, in Section 8, a case for which the T-wave is abnormally large. Since, moreover, it is possible to study this wave separately by suppressing the QRS, the spectrum of approximations to the T-wave becomes of interest.

Frequently the T-wave approximates to a half sine wave. We shall give the spectrum of this, and also of the square of this.

$$(a) \quad \varphi(t) = a \int_o^T e^{-2\pi i f t} \sin \frac{\pi t}{T}\, dt$$

$$S(f,o) = \frac{a}{2i}\int_o^{\tau}\left\{e^{i\left(\frac{\pi}{\tau}-2\pi f\right)t} - e^{-i\left(\frac{\pi}{\tau}+2\pi f\right)t}\right\}dt$$

$$= \frac{2a\frac{\tau}{\pi}}{1-4f^2\tau^2}e^{-i\pi f\tau}\cos\pi f\tau$$

$$2|S(f,o)| = \frac{4a\tau\,.\,\cos\pi f\tau}{\pi(1-4f^2\tau^2)}$$

When $f\tau = \frac{1}{2}$ this tends to the limit $a\tau$, which is $\pi/4$ times the value for $f\tau = 0$.

For repeating half sine waves the coefficients of the Fourier expansion are obtained by dividing by T, and placing $f = \frac{r}{T}$, $r = 1, 2, 3$.

(*b*) $\varphi(t) = a\sin^2\frac{\pi t}{\tau}$

Following similar lines, we obtain

$$S(f, o) = \frac{ae^{-i\pi\tau}}{2\pi f}\cdot\frac{\sin\pi f\tau}{1-f^2\tau^2}$$

$$2|S(f, o)| = \frac{a}{\pi f}\left|\frac{\sin\pi f\tau}{1-f^2\tau^2}\right|$$

When $f\tau = 1$ this tends to the limit $\frac{a}{2f} = \frac{a\tau}{2}$, which is one-half the zero frequency value.

The spectra for (*a*) and (*b*) are plotted on Fig. B. In each case $a = 200$ µV, $\tau = 0{\cdot}2$ sec., values which are the normal for the T-wave. For ready comparison with the contribution from the various QRS patterns the same scale has been chosen, which explains why the curves look rather small. We infer that at low frequencies such as $f = 5$ c/s a noticeable contribution from the T-wave is to be expected, but as to whether this will be in such a phase as to add or subtract from the QRS contribution must be considered in the light of Section 8, from which it appears that such contribution as is given by the T-wave does not affect the needle kicks due to the QRS, owing to the die-away previously mentioned. Thus Fig. B would be of value mainly in application to a T-wave separated out by suppression of the PQRS portion of the wave.

APPENDIX III

The Fourier Summation for a Repeating Half Sine Wave (Alternative Procedure)

The expression for $\sin\omega t$ for one half-cycle is obtained by causing (Fig. C) a wave train of equal amplitude and frequency to commence at $t = \frac{\pi}{\omega}$. The mathematical expression for this is (subject to the argument never having negative values)

$$y = \sin\omega t + \sin(\omega t - \pi) \quad \ldots\ldots\ldots\ldots(1)$$

which gives $y = 0$ except between 0 and $\frac{\pi}{\omega}$, during which time we must not employ the second term. Now the operational form of $\sin\omega t$ is $\frac{p\omega}{(p^2+\omega^2)}$, that of the wave train $\sin(\omega t - \pi)$, starting at $t = \frac{\pi}{\omega}$ being obtained by the application of the shifting theorem

$$\varphi_h(p) = e^{-ph}\varphi_0(p) \quad \ldots\ldots\ldots\ldots(2)$$

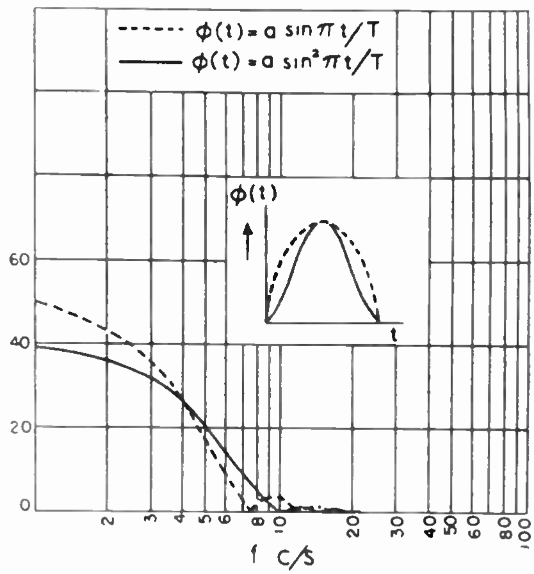

Fig. B.

where $\varphi_h(p)$ denotes the operational form of $f(t-h)$, φ_0 being that of $f(t)$. Thus the operational form of $\sin(\omega t - \pi)$ is obtained by writing $h = \frac{\pi}{\omega}$ and is

$$\varphi_{\frac{\pi}{\omega}}(p) = e^{-\frac{p\pi}{\omega}}\frac{p\omega}{p^2+\omega^2} \quad \ldots\ldots\ldots\ldots(3)$$

The operational form of a single half sine wave is thus (for the zeroth period)

$$\varphi_0(p) = \frac{p\omega\left[1+e^{-\frac{p\pi}{\omega}}\right]}{p^2+\omega^2} \quad \ldots\ldots\ldots\ldots(4)$$

We now apply the shifting theorem to (4) in the form

$$\varphi_{\frac{2\pi}{n}}(p) = e^{-\frac{2\pi p}{n}}\varphi_0(p) \quad \ldots\ldots\ldots\ldots (5)$$

in order to obtain the operational form of the single half sine wave commencing at $t = \frac{2\pi}{n}$ where $\frac{n}{2\pi}$ is the recurrence frequency of the pulses, and sum to infinity all such pulses in order to obtain the operational form of the recurrent pulses

$$\varphi(p) = \varphi_0(p) + \varphi_{\frac{2\pi}{n}}(p) + \varphi_{\frac{4\pi}{n}}(p) + \ldots \text{ to infinity}$$

$$= \frac{\varphi_0(p)}{1 - e^{-\frac{2\pi p}{n}}} = \frac{p\omega\left[1 + e^{-\frac{p\pi}{\omega}}\right]}{(p^2+\omega^2)\left[1 - e^{-\frac{2\pi p}{n}}\right]} \quad \ldots\ldots\ldots\ldots (6)$$

The Fourier expansion corresponding to (6) is, by the Fourier Mellin inversion theorem,

$$f(t) = \frac{1}{2\pi i}\int_{Br1} \frac{\omega e^{zt}}{z^2+\omega^2}\left(\frac{1 + e^{-\frac{z\pi}{\omega}}}{1 - e^{-\frac{2z\pi}{n}}}\right)dz \quad \ldots\ldots\ldots\ldots (7)$$

Owing to the factor $1 + e^{-\frac{\pi z}{\omega}}$ there is no contribution from the poles at $z = \pm i\omega$, so there is no component of fundamental period $\frac{2\pi}{\omega}$ in the expansion. The zeros of $1 - e^{-\frac{2\pi z}{n}}$ occur when $z = nri$, $r = -\infty$ *to* ∞. For the pole at $z = 0$ the contribution is

$$\left[\frac{\omega e^{zt}\left(1 + e^{-z\frac{\pi}{\omega}}\right)}{(z^2+\omega^2)\frac{d}{dz}\left(1 - e^{-z\left(\frac{2\pi}{n}\right)}\right)}\right]_{z=0}$$

$$= \frac{n\omega}{2\pi}\left[\frac{1 + e^{-z\frac{\pi}{\omega}}}{z^2+\omega^2}\right]_{z=0} = \frac{n}{\omega\pi} \quad \ldots (8)$$

The contribution from the poles corresponding to r between $\pm\infty$ and ± 1 is obtained by putting $z = nri$ in the second member of (8) and multiplying by 2:

$$2 \cdot \frac{n\omega}{2\pi}\sum_{r=1}^{\infty}\frac{\left\{1 + \cos\left(\frac{n}{\omega}\right)r\pi - i\sin\left(\frac{n}{\omega}\right)\right\}r\pi e^{nrit}}{\omega^2 - n^2r^2} \quad \ldots\ldots\ldots\ldots (9)$$

Hence by (8) and (9) the expansion of the recurrent half sine wave pulse is

$$f(t) = \frac{n\omega}{\pi} \times \left[\frac{1}{\omega^2} + \sum_{r=1}^{\infty}\frac{\left\{1 + \cos\left(\frac{r\pi n}{\omega}\right)\right\}\cos nrt + \sin\left(\frac{r\pi n}{\omega}\sin nrt\right)}{\omega^2 - n^2r^2}\right] \quad \ldots\ldots\ldots\ldots (10)$$

$$= \frac{\omega n}{\pi} \times \left[\frac{1}{\omega^2} + \sum_{r=1}^{\infty}\frac{1}{\omega^2 - n^2r^2}\left\{\cos r(nt) + \cos r\left(nt - \frac{n\pi}{\omega}\right)\right\}\right] \quad \ldots\ldots\ldots\ldots (11)$$

In the particular case when $n = 2\omega$, this reduces to $|\sin\omega t|$ given by the known expansion

$$|\sin\omega t| = \frac{2}{\pi}\left[1 - 2\sum_{r=1}^{\infty}\frac{\cos 2r\omega t}{(4r^2-1)}\right] \quad \ldots (11a)$$

If we write the pulse width time $\rho(2\pi/n)$, i.e., ρ is the fraction of the repetition period $2\pi/n$ occupied by pulses, we have

$$\pi/\omega = \rho(2\pi/n) \text{ or } n/\omega = 2\rho \quad \ldots\ldots\ldots (12)$$

Summing the two cosine terms by elementary trigonometry and using (12), (11) becomes

$$f(t) = \frac{2\rho}{\pi}\left[1 + \sum_{r=1}^{\infty}\frac{2\cos(r\rho\pi)}{1 - 4r^2\rho^2}\cos r(nt - \rho\pi)\right] \quad \ldots\ldots\ldots\ldots (13)$$

If we shift the pulses $\pi/2\omega$ to the left we have an even function having no term in sin rnt. Thus remembering (12),

$$f(t) = \frac{2\rho}{\pi}\left[1 + \sum_{r=1}^{\infty}\frac{2\cos r\rho\pi}{1 - 4r^2\rho^2}\cos rn\left(t + \frac{\pi}{2\omega} - \frac{\rho\pi}{n}\right)\right]$$

$$= \frac{2\rho}{\pi}\left[1 + \sum_{r=1}^{\infty}\frac{2\cos r\rho\pi}{1 - 4r^2\rho^2}\cos rnt\right] \quad \ldots\ldots\ldots (14)$$

In (14) we now place, for comparison with Appendix II (*a*),

$$\pi/\omega = \tau,\ 2\pi/n = T,\ \rho = \tau/T \quad \ldots\ldots\ldots\ldots (15)$$

so that

$$f(t) = \frac{2\tau}{\pi T}\left[1 + \sum_{r=1}^{\infty} \frac{2\cos(r\pi\tau/T)\cos rn^t}{1-4r^2\tau^2/T^2}\right] \quad ..(16)$$

Now $r = fT$, so that

$rnt = fTnt = 2\pi ft$

The coefficients in the expansion of (16) are thus

$4\tau \cos \pi f\tau/\pi T\ (1-4f^2\tau^2)$ ....(18)

as agrees with Appendix II (a). We thus note that the result using two entirely different modes of representation and analysis are identical.

Finally it should be pointed out that the use of the elegant Fourier-Mellin inversion theorem is unnecessary in many cases. For example, writing $p = i\omega$ in (6), and $\pi = \omega\tau$, we obtain $\{i\omega\pi/\tau)\ e^{-\omega\tau/2}.\ 2\cos(\omega\tau/2\}/(-\omega^2 + \pi^2/\tau^2)$ which quickly leads to the result of Appendix II (a). We have thus obtained this result by three methods.

LIST OF SYMBOLS

Symbol	First found in text at page	Meaning
a_n, b_n	172	Coefficient in Fourier's Series.
$2c, (c)$	172	Impulse repetition period for infinite wave train.
h	172	Height of impulse (in mV or µV).
n	172	Order of harmonic.
t	172	Time.
$\varphi(t)$	172	Function of time representing impulse (isolated, or recurring at equally spaced intervals).
f	173	Frequency.
T	174	Impulse repetition period for finite wave train.
τ	174	Duration of impulse (Λ shape), or half this duration (Section 8).
$S(f, o)$	174	Complex spectrum of isolated impulse starting at $t = o$ and finishing at $t = (2)\tau$.
N	175	No. of impulses in finite wave train.
$S(f, (N-1)T)$	175	Complex spectrum of Nth impulse of wave train.
$\sum_{r=0} S(f, rT)$	175	Complex spectrum of train of N waves.
$2\|S\|$	175	Spectral density of complex spectrum S.
ω	175	$2\pi f$
θ	177	$\omega\tau/2$

CERAMIC CAPACITORS*

by

W. G. Roberts, B.Sc.(Eng.)

A paper read before the Merseyside Section on January 5th, before the West Midlands Section on January 26th, before the South Midlands Section on February 24th, and before the North-Western Section on May 5th, 1949.

This paper gives a general survey of Ceramic Capacitors from the point of view of construction, manufacture and utilization, both as regards the material of which they are made and the form in which they are assembled.

1.0 Introduction

Ceramic capacitors have attained a considerable popularity over the last 15 years, having several distinct advantages over other types in their various fields of application. Some types of ceramics give very high dielectric constants, thus enabling a large capacitance to be obtained in a small space. Many have a high dielectric strength, which enables single units to be used at high voltages, particularly at radio frequencies, thus avoiding the difficulties encountered in ensuring an even distribution of voltage in large series arrangements. Again, extremely rigid attachment of electrodes and their connections, combined with the homogeneity of the dielectric, give completely cyclic changes of capacitance with temperature, and negligible changes with pressure. Another characteristic of great importance that is encountered in many ceramic dielectrics is that a predetermined temperature coefficient can be held within close limits.

Naturally, not all ceramics are suitable for dielectric applications. Many, such as ordinary porcelain, exhibit excessive electrical losses, others are porous, some are too weak mechanically, impracticable firing conditions may be imposed, the material may be too expensive, or it may react with the usual electrode surfaces. There are thus only six basic types of ceramic in common use: clinoenstatite or magnesium silicate, titanium dioxide or rutile, and four Group II*a* titanates, those of magnesium, calcium, strontium and barium. Of the remaining Group II*a* elements, beryllium does not form a satisfactory ceramic, and the use of radium titanate as a dielectric would not be suitable for obvious reasons.

* Manuscript received December, 1948. U.D.C. No. 621.319.4.

2.0 Basic Types of Ceramic Dielectrics

2.1 *Clinoenstatite*

Lowest in dielectric constant, and the first ceramic dielectric to be used, is clinoenstatite. This gives a dielectric constant of 6, that is, approximately the same as that of mica. With careful preparation it shows a low loss angle at all frequencies, and it is possessed of very high dielectric and mechanical strength. With the discovery of high permittivity bodies, it has rather fallen out of favour for capacitor use in this country, being commonly found only in capacitors of values of 1 pF and below. Nevertheless, it is still extensively used for high frequency insulators, bushings, coil formers, and the like, although a newly discovered zirconium silicate body, having higher mechanical strength, is tending to replace it in the U.S.A.

2.2. *Magnesium Titanate*

Next in order of permittivity comes magnesium titanate. This has a dielectric constant of 14, a temperature coefficient of + 100 pts./10^{-6}/°C, an exceedingly low dielectric loss if properly prepared (loss angle approx. 1×10^{-4}), and a specific resistance of 10^{14} ohms per cm^3. It is extensively used as a dielectric both for low power and high power applications. (1)

2.3. *Titanium Dioxide*

Rutile, or titanium dioxide, in its polycrystalline form as found in a ceramic body, gives a dielectric constant of about 90, with a negative temperature coefficient of 750 pts./10^{-6}/°C. Normally, a loss angle of 40×10^{-4} at a frequency of 100 cycles per second and 5×10^{-4} at 1 megacycle/sec. is obtained, but by

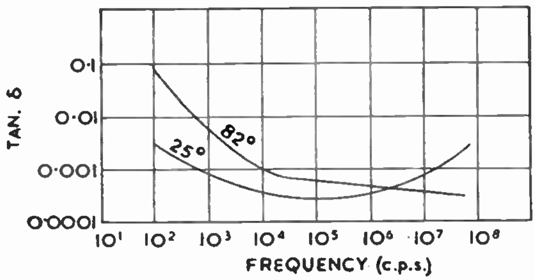

Fig. 1.—Power Factor versus Frequency. $CaTiO_3$.

carefully controlled firing conditions and preparation of the raw material, these values are reduced by over 50 per cent. Breakdown voltage can be as high as 600 volts per mil. (2 and 3)

2.4. *Calcium Titanate*

Calcium titanate gives a dielectric constant of 160 and a negative temperature coefficient of $-1{,}400$ pts./10^{-6}/°C. (3) Permittivity is practically constant with frequency, but power factor depends on frequency to a considerable extent, as shown in Fig. 1. As will be seen, this characteristic is modified at elevated temperatures. Fig. 2 shows that the power factor remains sensibly constant up to about 50°C,

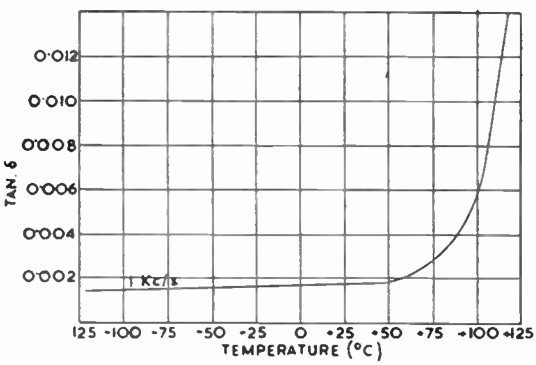

Fig. 2.—Power Factor versus Temperature. $CaTiO_3$.

and then rises considerably. This would appear to indicate some change in crystalline configuration, but this aspect has not yet been fully investigated. The immediate result is to restrict the use of this dielectric to applications where no appreciable radio frequency power is supplied. Obviously, once the temperature of the capacitor rises above 50°C, losses increase and a vicious spiral ensues.

2.5. *Strontium Titanate*

Strontium titanate has a dielectric constant of about 240 and a strongly negative temperature coefficient of 3,000 pts./10^{-6}/°C. This is reasonably constant at normal working temperatures, but some workers found that the

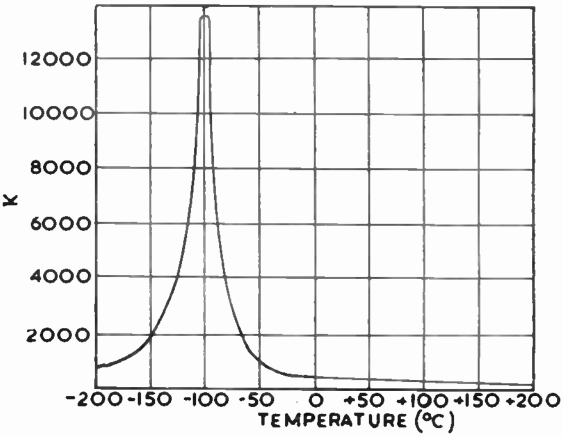

Fig. 3.—Dielectric Constant versus Temperature. $SrTiO_3$ (per Johns Hopkins University).

dielectric constant rises rapidly to a peak approaching 14,000 at about -100°C (Fig. 3). (3) This behaviour, we shall see, is comparable with barium titanate. In this body again the loss angle increases considerably with temperature, and thus large radio frequency loads cannot be applied. The variation of power factor with frequency is similar to calcium titanate, as shown in Fig. 4.

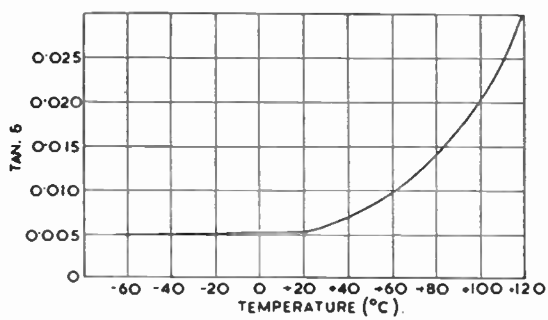

Fig. 4.—Power Factor versus Temperature. $SrTiO_3$.

2.6. *Barium Titanate*

Barium titanate has the highest dielectric constant of the group of basic dielectrics, amounting to 1,200 at room temperature. (3)

Its properties are, however, most unusual, and in a paper of this nature only a brief sketch of some of the more salient characteristics can be outlined. Briefly then, barium titanate, in common with the other ceramics we have been reviewing, is a polycrystalline agglomerate.

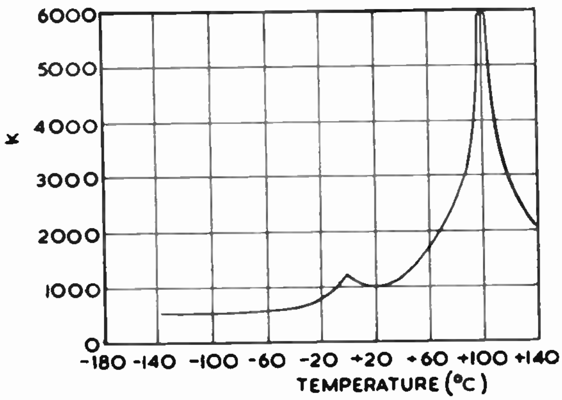

Fig. 5.—Dielectric Constant versus Temperature. $BaTiO_3$.

The individual crystals change from a tetragonal form to a cubic form at about 100°C. Figure 5 shows a typical curve of dielectric constant against temperature, and the high peak obtained at 100°C is very noticeable. Figure 6 gives the dielectric constant against frequency and Fig. 7 shows this parameter against field strength. (4)

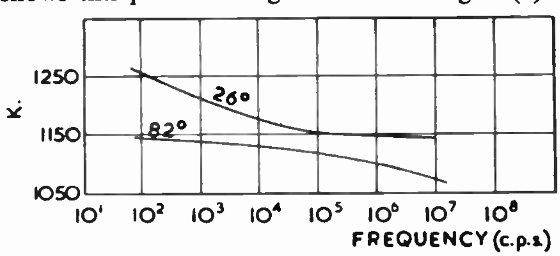

Fig. 6.—Dielectric Constant versus Frequency. $BaTiO_3$.

It is thus obvious that the capacitance of a capacitor made in barium titanate must be expressed with respect to the conditions of frequency and voltage under which it is measured.

Barium titanate is a ferroelectric substance. Ferroelectrics are defined as substances which possess the property of spontaneous polarization, and it may be said that the dielectric properties of ferroelectrics are to a certain extent similar to the magnetic properties of ferromagnetics. The temperature at which spontaneous polarization disappears and the substance ceases to be a ferroelectric is called the "Curie Point" in analogy with the ferromagnetic phenomena. In barium titanate this takes place at about 115°C. Figure 8 shows

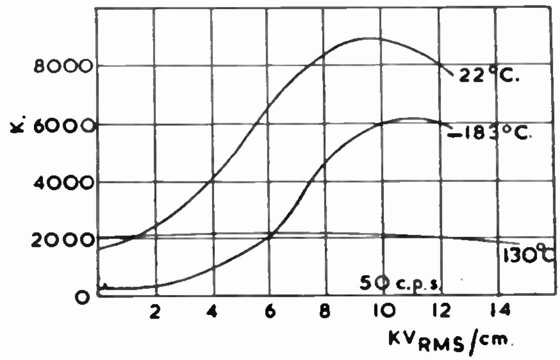

Fig. 7.—Dielectric Constant versus Field Strength. $BaTiO_3$.

some hysteresis curves taken on barium titanate, by an oscillographic method. (5)

From the practical point of view this ferroelectric property restricts the use of this dielectric to applications where non-linearity is

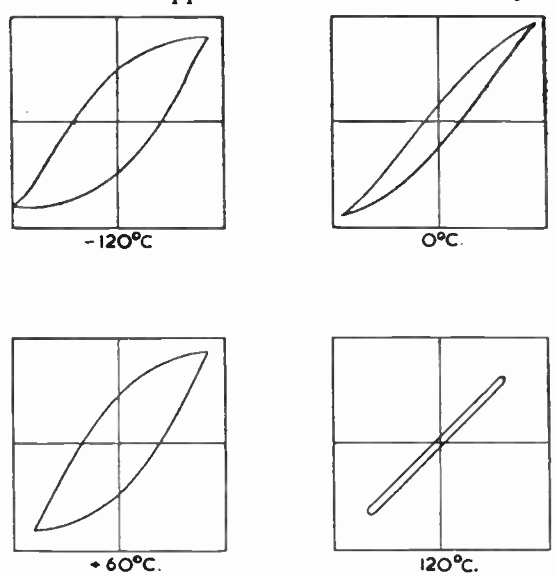

Fig. 8.—Oscillograph Hysteresis Curves versus Temperature. $BaTiO_3$.

unimportant. For example, apart from considerations of temperature coefficient barium titanate capacitors could not be used in carrier filter networks, as harmonics would be introduced. They could not be used in R.F. power circuits, as, apart from their normally high power factor (Fig. 9), the energy represented by the hysteresis loop would have to be supplied.

On the other hand, the very high dielectric constant of barium titanate renders it most attractive for making by-pass capacitors of small size, and it thus is of some importance in the manufacture of what are termed " Hi-K " capacitors.

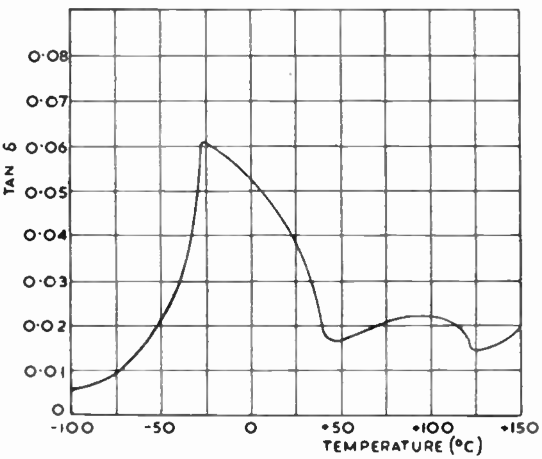

Fig. 9.—Power Factor versus Temperature. $BaTiO_3$.

The non-linear characteristics of barium titanate are obviously capable of use in special circuit elements for harmonic generation, frequency modulation and similar applications, but none of these has reached a practical stage.

After polarization by application of a high voltage, barium titanate exhibits piezoelectric properties, and pick-ups and microphones are already produced in the U.S.A. with plates of this material as the piezoelectric element. They have the advantages of cheapness and greater ease of soldering compared with Rochelle salt. (6)

3.0. Practical Types of Ceramic Dielectrics

3.1. *Additions to Facilitate Manufacture*

The basic materials outlined above are seldom used *per se*. In order to enable convenient working of the ceramic body before firing, a small addition of a clay type of material is usually made, and often some gum is added to hold the body together before firing. However, these additions must be carefully controlled so that the dielectric properties desired are not affected, and the whole process must be carefully laid out with this end in view.

3.2. *Mixtures to Obtain Desired Characteristics*

Magnesium titanate gives a dielectric constant of 14 with a temperature coefficient of + 100 pts./10^{-6}/°C, and titanium dioxide a permittivity of 90 with a temperature coefficient of −750 pts./10^{-6}/°C, and it is therefore logical to combine the two dielectrics to obtain intermediate values. It is, for example, useful to have a zero temperature coefficient capacitor, and this can be obtained with a dielectric constant of about 36. The Lichtenecker equation,

$$\text{Log}E = M_1\text{Log}E_1 + M_2\text{Log}E_2$$

where E_1, E_2, and E are the dielectric constants of the respective components and the resultant mixture, and M_1 and M_2 are the proportions of the constituents, can be applied to such a mix in most cases. (7)

In actual practice, this mixture is not the only one to give the desired result, and several others have been proposed and used. It is, however, quite possible to obtain a body with any temperature coefficient between + 100 pts./10^{-6}/°C and —750 pts./10^{-6}/°C with a close tolerance.

3.3. *Mixtures with Barium Titanate*

A larger number of experimental mixtures based on barium titanate has been made. Various percentages of the other titanates, particularly strontium titanate have been added together with the oxides, carbonates, silicates, zirconates, fluorides and other inorganic compounds of Group II*a* or the rare earth metals. The effects have been various, some of the more interesting moving the peak of the barium titanate down the temperature scale, or moving that of strontium titanate up the temperature scale giving dielectric constants of 12,000 or even more at around room temperatures. (8) Such dielectrics are, however, rather restricted in use, as, for example, one may have a permittivity of 10,000 at 20°C reduced to 1,000 at 45°C, which is of little practical value as compared with the properties of ordinary barium titanate. On the other hand, by careful flattening of the peak by suitable additions and moving it to the centre of the working range, one may obtain a characteristic as shown in Fig. 10. This, it will be seen, is quite useful for by-pass capacitor work, although quite unsuitable for use as a tuning capacitor.

Another reason for additions is to increase the insulation resistance of barium titanate, which may be as low as 10^9 ohms/cm^3, against 10^{14} ohms/cm^3 for rutile. In addition, pure barium titanate tends to have a low breakdown

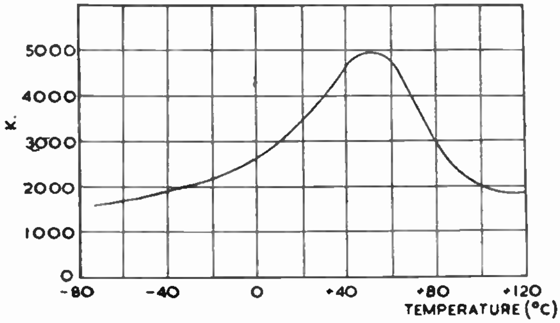

Fig. 10.—Dielectric Constant versus Temperature. Unilator K.3000/1.

voltage, particularly after a few hundred hours on voltage. As a result of research on the effects of additions to barium titanates, high initial insulation resistances unaffected by prolonged application of voltage are now obtainable, and have rendered the miniature by-pass capacitor in ceramic dielectric a far more attractive proposition.

4.0. Choice of Dielectric for Specific Applications

4.1. *Necessity for Choosing Suitable Dielectrics*

Discretion must be used by the equipment designer when selecting ceramic capacitors to ensure that the dielectric used is suitable for the application envisaged. Some of the more usual capacitor positions are described in this section.

4.2. *R.F. Power Applications*

For R.F. power use, such as in oscillator tank circuits, low power factor is important. The suitable dielectrics are therefore :—

- Clinoenstatite (also known as Calit, K6, Frequentite, Frequalex).
- Magnesum titanate (also known as Tempa S, K14, P120, Tempradex, Templex, HF115).
- Rutile (also known as Conda C, K90, N750, HF102, Faradex, Permalex).
- Mixtures intermediate between rutile and $MgTiO_3$ (such as Tempa T, K36, N30, HF530).

4.3. *Receiver Tuning and Padding Applications*

For receiver tuning and padding, the high inherent stability and cyclical symmetry of ceramic capacitors offer considerable advantages, and the low power factor and high insulation resistance of the dielectric employed enable good Q values to be obtained. Care must be taken, however, with temperature coefficient. Most inductances have a positive coefficient, and a dielectric with a negative coefficient should thus be chosen to offer a measure of compensation. The use of such dielectrics as rutile, calcium titanate, and strontium titanate would over compensate to a very considerable degree. For this application, then, one of the " mixture " class should be chosen ; Tempa T or N30, with a temperature coefficient of -30 pts./10^{-6}/°C is very suitable, but if the capacitance is low the permittivity of these materials may be too high and the slight positive coefficient of magnesium titanate or clinoenstatite must be tolerated.

4.4. *Temperature Coefficient Compensation*

When it is desired to tune a circuit with a variable capacitance, it is usually found that there is a positive temperature coefficient in the tuned circuit as a whole. The effects of this may be overcome by adding a small additional capacitance having a strongly negative temperature coefficient. The use of a small capacitance is dictated by the necessity of maintaining the maximum capacitance swing, so that, whilst rutile was formerly used for this application, small compensating capacitances in the titanates of strontium or calcium are now coming into use.

4.5. *By-Pass and Filter Applications*

Any of the dielectrics described may be used for R.F. by-passing or filtering. Naturally the actual dielectric used depends on the capacitance required, and the normal tendency will be to select the highest available dielectric constant. The manufacturers' recommendations as to working voltage should be followed closely, as on plain barium titanates, now falling out of use, there is a risk of breakdown should these be exceeded.

4.6. *Interstage Coupling Applications*

For interstage coupling use, ceramic dielectrics with high volume resistivity should be selected. This implies the avoidance of plain barium titanate, but not of special mixtures of this substance where high insulation resistance is guaranteed.

5.0. Methods of Manufacture

5.1. *Preparation of Ceramic*

The constituents of radio frequency ceramics must be chosen to be of a high degree of purity, as impurities may profoundly affect the dielectric properties. They should be milled and thoroughly admixed, to ensure uniformity throughout the batch ; this is of especial importance when an important ingredient may be only a small proportion of the bulk.

5.2. *Manufacture of Unfired Body*

After mixing, the body is made into a suitable consistency for the ensuing processes ; this may be anything from a dry powder to a soft clay. The actual process may be any of those used in the ceramic art, such as wet or dry pressing, slip casting, turning, throwing, casting, or extrusion. Commonly, the smaller sizes of receiver capacitor, excluding tubes, are pressed, larger transmitter capacitors are turned, and tubes are always extruded in long lengths and usually cut before firing. It should be recalled that ceramics are very easily handled with normal machine and hand tools whilst in the dry clay form, but after firing they can only be cut with carborundum or diamond. The ceramic body is therefore in its finished shape before firing but somewhat larger to allow for the shrinkage which occurs during that operation.

5.3. *Firing*

The kilning of radio frequency ceramics is an extremely critical process, both as regards the final temperature—which is usually rather higher than for ordinary ceramics—and the periods of heating, soaking and cooling. These have to be carefully laid down for each type of body if the desired characteristics are always to be obtained. Generally speaking, underfiring gives a porous ceramic, and overfiring a ceramic which is too grossly crystalline. If correctly fired the ceramic has a fine crystalline structure, and is completely vitreous. The kiln atmosphere must be carefully controlled, as titanium oxide in particular is prone to chemical reduction. Should this happen, it is at once revealed by a blue coloration, and the ceramic will be found to exhibit semi-conductor properties. Electrically fired kilns are preferred for firing radio frequency ceramics, as they enable a carefully controlled firing schedule to be maintained, and the kiln atmosphere is not contaminated by products of combustion.

5.4. *Inspection of Fired Body*

After firing, the ceramics require inspection to determine their suitability for further processing. A 100 per cent. inspection is made for obvious chips, cracks, deformation, and so forth, and a sample is then taken for further tests. A complete check of physical dimensions is made, which may involve destruction of the component, and a second destructive test is that for porosity. A number of bodies are taken and boiled in a fuschine solution, which may be under pressure. They are then broken, when any porosity cracks and so forth are at once rendered obvious by penetration of the dye. There are various other methods of crack detection, such as fluorescent solutions or by an electrostatic method similar to the electromagnetic crack detection method used on ferrous castings, but these only give an indication of what is happening on the surface.

Other samples are taken, and made up into actual capacitors and the various electrical parameters measured. The usual sampling considerations are followed to determine whether to release the batch or whether an uneconomical reject level on subsequent operations will arise.

5.5. *Glazing*

Certain types of power capacitor have glazed flash-over insulation sheds, and in this case glazing is performed at this stage. The glazing medium is in the form of a watery slip or paste, and this is brushed or sprayed on to the parts where glazing is required. The bodies are then fired to about 800°C-900°C, which is rather less than the original firing temperature. The glaze must be carefully chosen to match the coefficient of thermal expansion of the ceramic, otherwise "crazing" will occur. A transparent glaze is usually preferred, but it may be opaque or of any colour, with the limitation that it must be absolutely non-conducting. Some glazes are definitely conductive, a fact used to advantage in "grading" high voltage insulators.

5.6. *Application of Electrodes*

Silver electrodes are invariably used on ceramic capacitors, not only for the superior conductivity of this material, but because it lends itself to methods of application which key it firmly to the ceramic. The nature of ceramic dielectric material lends itself to the use of electrodes which become an integral part of the ceramic. This avoids all the difficulties involved in the use of foil electrodes, and enables full advantage to be taken of the permittivity of the ceramic. The usual method of application of the silver is to apply a mixture of silver or silver oxide with a suitable flux. This takes the form of a suspension in a suitable liquid, which can be painted or sprayed on to the surfaces that it is desired to metallize. When dry, the capacitors are fired to a temperature of around 700°C, as a result of which a continuous bright silver layer is obtained, firmly affixed to the ceramic. For most purposes this is sufficient, but for power capacitors where large currents may be carried the layer may be reinforced by a second fired layer or by electroplating or spraying silver or copper by any of the usual processes. It should be stressed that the first fired silver layer is the chief factor in determining the current carrying capacity of a power capacitor, as, regardless of how it is reinforced, the current must still flow from the silver layer into the ceramic.

5.7. *Connections*

The next stage in the manufacturing process is to provide connections to the electrode layers. These may take the form of wires, tags, screws, and so forth, and are invariably soldered to the silvered ceramic. Low melting point solders are frequently employed with non-corrosive activated resin fluxes. Little difficulty is found in soldering to ceramics, providing one avoids thermal shock on thick sections, and that the solder is not held in the molten state too long, so that it dissolves the silver layer. The adhesion, even of butt soldered joints can be surprisingly high if the silver layer is well keyed to the ceramic. Naturally, on power capacitors the connections are made over wide areas and some manufacturers use a wire gauze to spread the current.

It should be added that clamped connections to silvered ceramics are no more desirable than clamped connections to any other component, where soldering is a practicable procedure.

5.8. *Adjusting*

When a close tolerance of capacitance is required, it is necessary to adjust the capacitor by grinding away portions of the electrode layers with a carborundum or diamond wheel. With experience, this can be a very quick process. Care must be taken to maintain a clean electrode edge, and to avoid leaving small detached " islands " of silver. If this is not done, the loss angle of the capacitor may be considerably increased as, in effect, one parallels small capacitors of high loss to the main capacitor by high resistances. On application of voltage, these detached islands may be connected by minute arcs, giving a variation of capacitance known as " flutter " or " scintillation."

It is not possible to adjust power capacitors, for reasons which will be explained later.

5.9. *Protection*

As previously explained, radio frequency ceramics are vitreous, and do not absorb moisture. On the other hand, the gaps between the electrodes can become covered with moisture, and thus lower the insulation resistance of the capacitor. The silvered layers are liable to tarnish if any sulphur is in the atmosphere, and this will increase their resistance. It is therefore usual to protect the capacitor with a coat of some suitable material. The cheaper protection for normal use takes the form of a coat of enamel, and this enamel must be carefully chosen. It will be realized that any enamel in the gaps between the dielectric or any part of the electrostatic field forms part of the dielectric, and may contribute as much as 2 or 3 picofarads towards the capacitance of the whole unit. If this enamel has a bad power factor or a high temperature coefficient, it will obviously affect adversely the performance of the capacitor.

To raise the performance of such a capacitor to R.C.S.C. Category " C " (9) for tropical use up to 70°C, it is sufficient to give the capacitor a thick coating of a suitable wax. Wax without the enamel is not sufficient, as most waxes are slightly permeable to moisture. Another form of protection uses an outer ceramic tube, into which the dielectric tube is cemented and the whole vacuum impregnated. This has the advantage of good insulation, but the tropical performance is limited by the cement sealing the interior. Yet another protection uses an outer metallic tube with neoprene sealing bungs at each end ; this is not always completely effective as leakage may take place along the wire past the bung.

The problem of sealing the connecting wires eliminates the use of polyethylene and similar moulded coatings, which do not completely wet the wire. To reach the R.S.C.S. Category " A " (9) the only solution appears to be to use an outer ceramic tube with the connecting wires sealed to it by means of solder, but a difficulty exists in obtaining an insulated capacitor at reasonable cost.

Power capacitors usually confine their protection to enamelling the silver layers. The leakage paths of this type of capacitor are usually glazed so that losses due to enamel do not arise. The leakage paths themselves are sufficiently long to prevent losses due to moisture deposition.

5.10. *Final Inspection*

Before leaving the manufacturer, each capacitor is tested for capacitance, power factor, insulation resistance, and breakdown, to the limits given in the manufacturer's specification. In addition, samples are tested for temperature coefficient and tropical protection, when these factors are of importance. Further samples may be tested for strength of connections and strength of the ceramic, others may be put on an accelerated life test at high voltage and elevated temperature. The apparatus that has been developed expeditiously to perform tests on large numbers of capacitors is of great interest, but outside the scope of this paper.

6.0. Receiver Type Capacitors

6.1. *Choice of Shapes*

On looking through a manufacturer's catalogue, one may find one capacitor offered in a variety of shapes. The user will naturally select the shape which suits his layout the best, bearing in mind that some are cheaper than others. To the manufacturer however, the shapes form a logical sequence, for the aim is always to obtain the maximum capacitance in the smallest space for the cheapest cost, bearing in mind the limitations imposed by temperature coefficient. Whilst any capacitance can be obtained on the extruded tube, this is not necessarily the most convenient or the cheapest to manufacture, and, moreover, low capacitances in a high dielectric constant material cannot be obtained on tubular forms.

6.2. *Pressed Types*

The first group of bodies to be considered are those normally made by pressing, and are shown in Fig. 11. The pearl type is the smallest, being about 5 mm. diameter and 3-6 mm. long. A capacitance as low as 0·5 pF can be obtained in clinoenstatite. It has very low inductance. Owing to its small size, it cannot be adjusted. If we extend the centre part of the pearl radially, we obtain the disc capacitor, which can be adjusted, and gives up to 50 pF in titanium

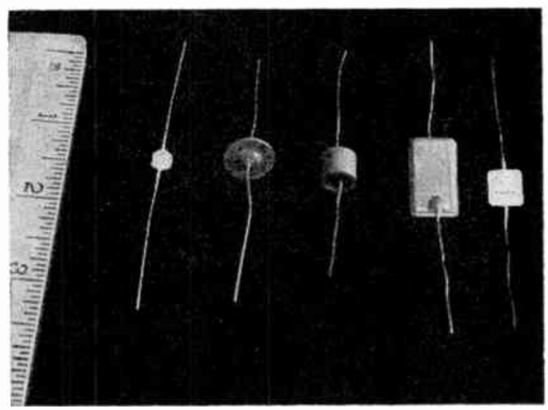

Fig. 11.

dioxide. Finally, if we turn the edges of the disc downwards, we obtain the hat capacitor, which gives us 220 pF in titanium dioxide, and is about 11 mm. diameter and 11 mm. long. A different approach is given by the wafer capacitor. This is a flat piece of ceramic with a projecting rim to increase the leakage path between the two electrode surfaces, and gives up to 100 pF in rutile. A wafer 11 mm. × 8 mm. will give 1,000 pF in modified barium titanate, and forms a useful by-pass capacitor.

6.3. *Thin Plate Types*

A form of ceramic capacitor made in the U.S.A. uses a very thin plate of barium titanate, some 0·3 to 0·5 mm. thick. This is made by spreading the ceramic material on to a moving belt, which is passed through a tunnel kiln. The plate so obtained is cut into convenient sizes.

Naturally, a very high capacitance per unit area is obtained, but the plate is very fragile, and must be reinforced mechanically. These plates are used for printed circuits, or may be moulded into an outer sheath for mechanical strength, or even bound round with layers of varnished paper. A promising use for these plates is in gramophone pick-ups and microphones using the piezoelectric property explained above.

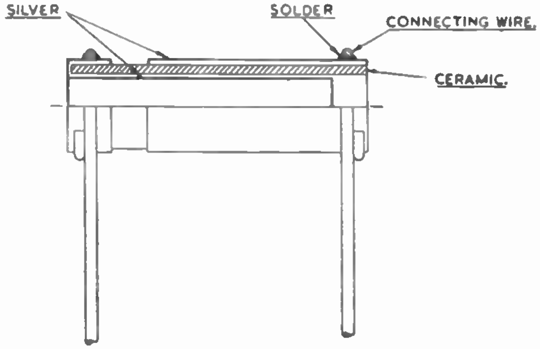

Fig. 12.—Tubular Ceramic Capacitor.

6.4. *Tubular Types*

The thin-walled ceramic tube forms one of the most popular types of ceramic capacitor, as it is compact and matches resistors in shape, thus facilitating the design of mounting panels. A typical construction is shown in Fig. 12. It will be seen that gaps are provided on the outside and inside of the tube, so that the connecting wires can be placed at its extremities.

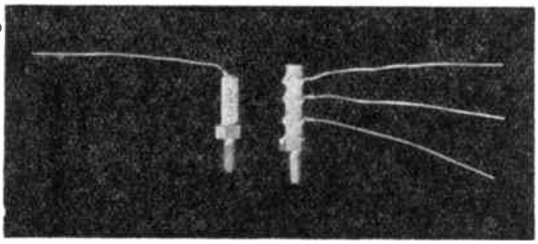

Fig. 13.

6.5. *Special Forms of Capacitor*

The tubular type lends itself to various special types of construction which are designed to facilitate manufacture and improve the efficiency of by-passing arrangements in receivers. Figure 13 shows two stand-off or post type capacitors. The larger type is under ¾ in. high, but gives three capacitances of 1,000 pF to a common ground. The method of manufacture involves silvering three bands on the outside of the tube with a gap between each. Figure 14 shows two double capacitors arranged to by-pass the usual terminals on a B7G valveholder. These capacitors are only 10 mm. long and have two outside silvered bands, the common inner electrode being connected by a wire soldered inside the tube. Figure 15 shows the lead-through type of capacitor. It will be seen that the inner electrode layer is connected to the centre tag at either end, and the outer layer is soldered directly into the bush. The capacitor shown in Fig. 16 gives a capacitance of 10,000 pF. The form of construction acts as a π-section low pass filter at high frequencies.

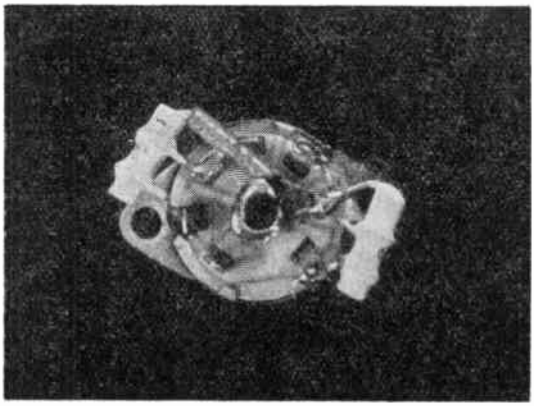

Fig. 14.

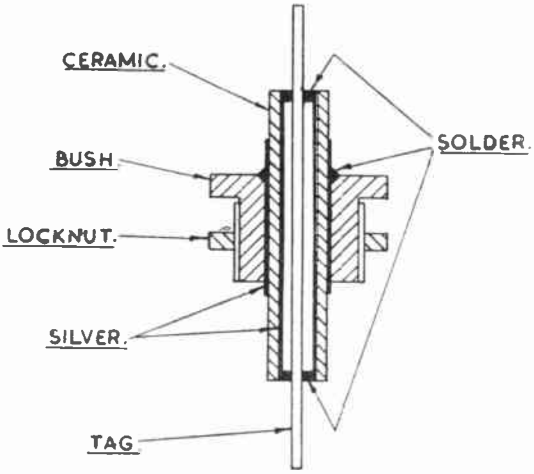

Fig. 15.—Lead-through Ceramic Capacitor.

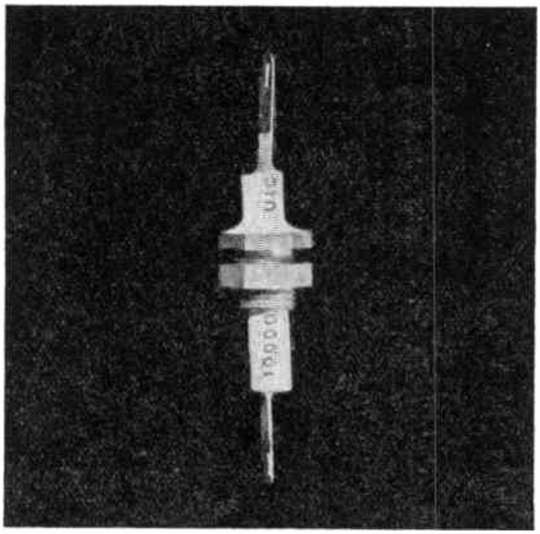

Fig. 16.

6.6. *Trimmer Capacitors*

The ceramic trimmer capacitor enables the advantages of ceramic capacitors, such as stability and controlled temperature coefficient, to be obtained in a variable form. The ceramic trimmer also has the advantage that it is mechanically strong, and not easily short-circuited by any dust or filings. It consists of a base made of a low permittivity material, such as clinoenstatite, which provides a good insulated mounting. It is provided with a top surface which is ground optically flat, and on it a silvered segment is fired, with a silver conductive path to which one of the tags is riveted, and, if required, soldered. A ceramic disc having a ground bottom surface, but with an electrode of the same shape as that in the base fired to its top surface, is arranged to rotate on this base, by means of a spindle keying to splines on the ceramic and pulled down by a spring in the base. This base spring is extended with a tag to form the second connection. The disc may be soldered to the spindle. The meshing of the two electrodes produces a linear capacitance variation over about 180°. The upper disc can be made in any type of dielectric ceramic to give the desired temperature coefficient or capacitance.

An interesting device with a similar construction was made in Germany before the war (Fig. 17). The rotor in this case was fully silvered, but was made of two dielectrics, one with positive, and the other with negative temperature coefficient, fastened together by glaze. The thickness of the dielectrics was chosen so that a constant capacitance was obtained for any position of the rotor, but the temperature coefficient of the unit depended on the proportion of each dielectric meshing with the stator. The unit could thus be used accurately to compensate for temperature drift, but had rather a high constant capacitance (about 30 pF).

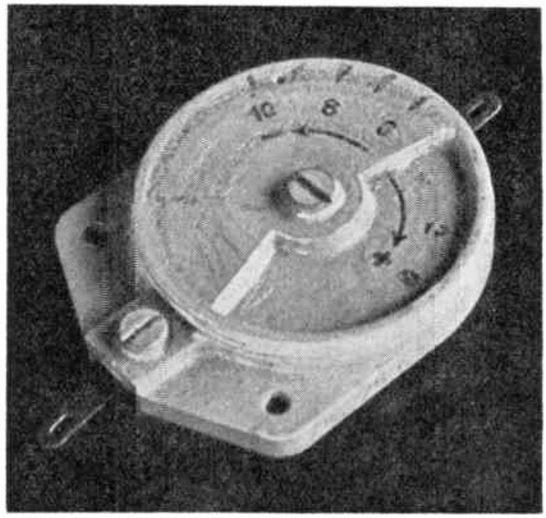

Fig. 17.

7.0. Transmitter Type Capacitors

7.1. *Essential Features of Transmitter Ceramic Capacitors*

As explained in Section 2, the choice of dielectrics for transmitter type capacitors is restricted to those which have a low power factor with no appreciable temperature coefficient of this parameter. This restricts the maximum dielectric constant to about 90, that is, a titanium dioxide dielectric. In addition to this, exceptional care has to be taken in the preparation of the ceramic bodies to obtain a low power factor ; whereas a loss angle of 5×10^{-4} is very satisfactory for a receiver dielectric, and is indeed better than most mica; 2×10^{-4} is the most that can be accepted for the transmitter types.

The majority of transmitter types have to withstand comparatively high voltages ; this implies a greater thickness of dielectric and, therefore, a larger size for the same capacitance. There is, however, another vital influence on shape. Figure 18 represents the lines of electrostatic force in a stressed dielectric. It will be seen that, however large the gaps between the two electrodes may be, there is a very high concentration of dielectric flux on the edge of one electrode. This would lead to corona at the electrode edge, local overheating and breakdown. It is therefore necessary to take the

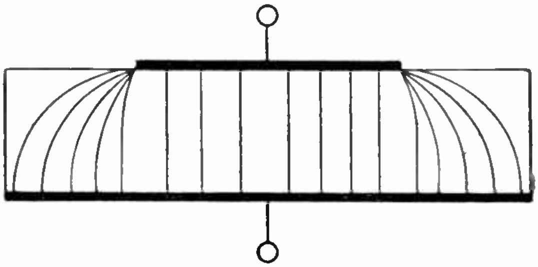

Fig. 18.—Lines of Electrostatic Force in a Stressed Dielectric.

dielectric edge out of the electrostatic field, as in Fig. 19, thus eliminating edge stresses.

The ceramic body is usually given a smooth curve to the electrode edge, in accordance with the usual principles of high voltage insulator design, and glazed to prevent accumulations of dirt on the surface. It will be realized that the usual principle of adjustment by grinding away the electrode is not possible on these types for obvious reasons, and thus close tolerance transmitter type capacitors can only be achieved by selection, and are correspondingly expensive. It is best, therefore, to adjust the resonance of a circuit by varying the inductance, and allowing

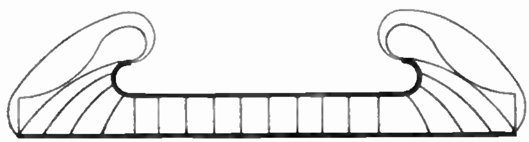

Fig. 19.—Reduction of Stress by Provision of Sheds.

the greatest possible tolerance on the ceramic capacitors.

The transmitter capacitor must also have electrodes with the lowest possible resistance, particularly at the electrode/ceramic interface and will usually have at least two coatings of fired silver. The electrode connections are large and strong, to support the weight of the capacitor and to enable appropriate connections to be made to them in turn.

7.2. *Separation of Losses and Load Rating*

The losses in a capacitor under R.F. load can be divided into two parts, the dielectric loss, due to loss in the dielectric itself, and the ohmic loss, due to losses in the dielectric layer. If we plot the loss in watts against the square of the current, in a capacitor loaded with constant kVA, we get the result shown in Fig. 20. It will be realized that, the kVA being constant, the voltage is increasing along the X axis towards the origin with decreasing frequency. We see that the curve is divisible into two parts, AB, BC. If we extrapolate BC to the axis, it is found that the value so obtained corresponds to the loss attributable to the static power factor of the dielectric. The triangle DCE represents the I^2R. loss in the electrodes. We are then left with the area between AB and BD, which corresponds to an increase of dielectric loss with voltage. The inception of this increase usually occurs quite sharply at about 600 volts

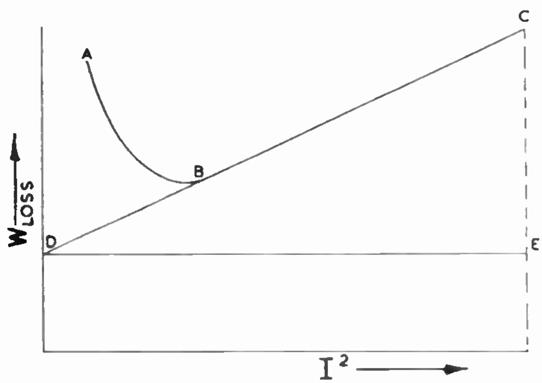

Fig. 20.—Loss VI^2—Constant kVA.

per mm. in titanium oxide. This is far below the voltage at which breakdown is initiated.

Hitherto, it has usually been thought sufficient to rate power capacitors for a maximum current, depending on the electrodes, a maximum voltage, depending on the dielectric, and a maximum kVA, depending on the capability of the body to dissipate heat. Figure 20 shows such a simple rating to be unrealistic, for at the frequency corresponding to B an optimum kVA rating can be obtained, falling off on either side, so that at neither maximum voltage

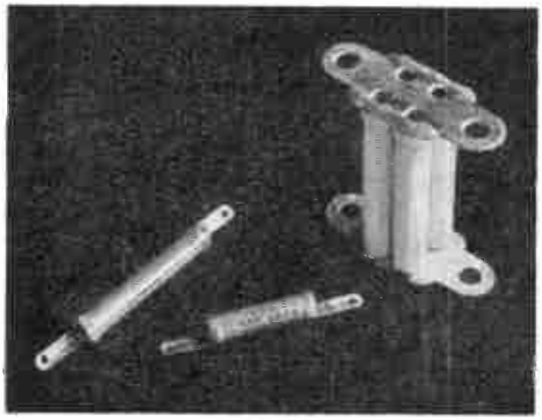

Fig. 21.

nor maximum current can the maximum kVA be obtained. It is, therefore, becoming the custom to issue load curves, giving the exact conditions, and a recent range of very large capacitors has been intentionally designed to give the maximum kVA rating at the induction heating frequencies of about 400 kc/s.

7.3. *Tubular Type Transmitter Capacitors*

The lower-rated type of transmitter capacitor consists of a tube exactly similar to a tubular receiver capacitor, but having larger inter-electrode gaps and stout terminals (Fig. 21). This type does not possess radiussed electrode edges, and thus is only suitable for voltages up to about 500V r.m.s. The rating is usually expressed in volt/amperes per pF, as the heat dissipation naturally depends on the area of active electrode. Adjustment to close tolerances is possible, and the tubular type is thus suitable for use in master oscillator circuits where

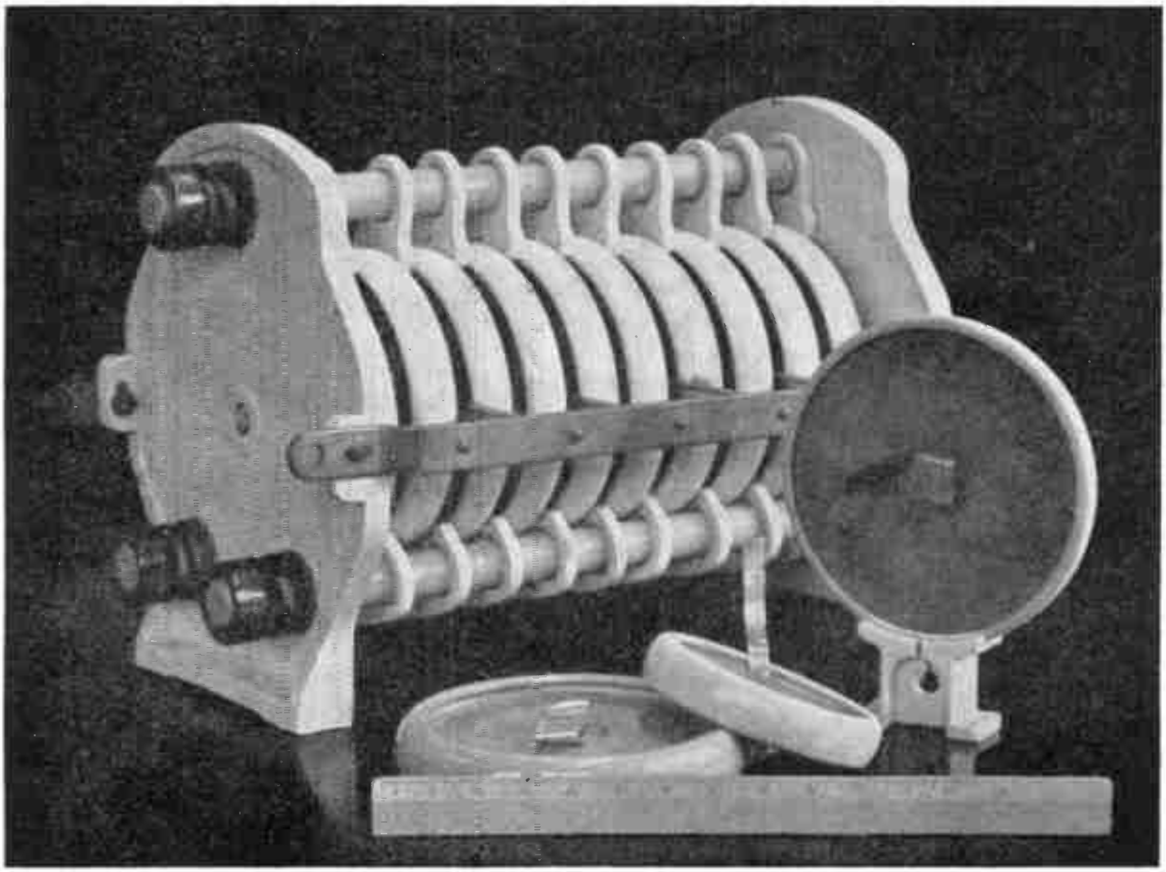

Fig. 23.

voltages are usually low. On the right is shown a continental type of capacitor consisting of a number of tubes with common end caps ; this

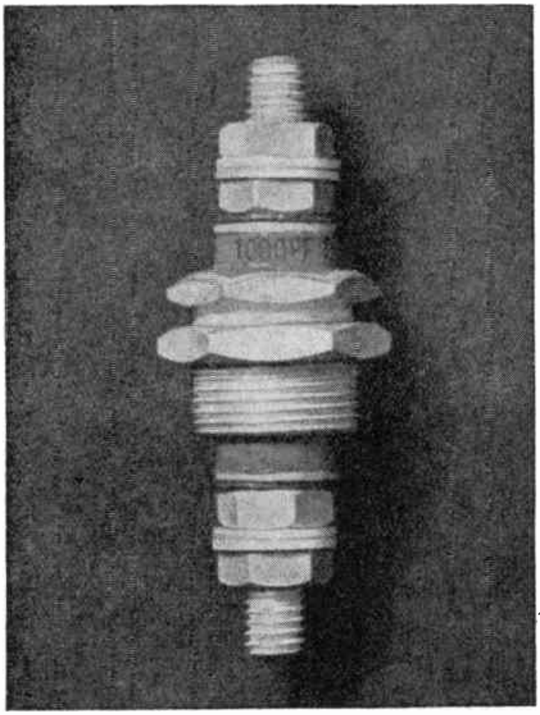

Fig. .

enables a large capacitance in a small space to be achieved with a reasonable volt/ampere rating, but naturally only a low voltage can be applied. Figure 22 shows an interesting type of tubular capacitor using a strontium titanate dielectric. This corresponds to the receiver type mentioned earlier in (6.5). With a permissible lead-through current of 200 amperes, it is

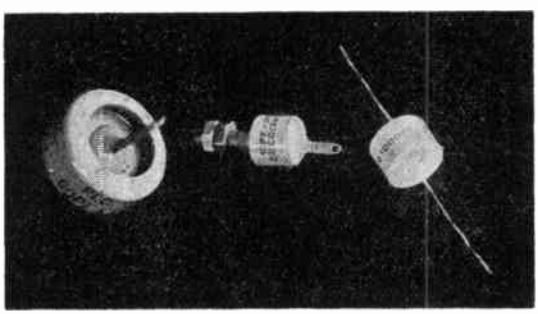

Fig. 24.

designed as a filament feed-through and by-pass capacitor for transmitting valves. The solid construction gives enough heat dissipation to allow the strontium titanate to take the R.F. current found in this part of the circuit.

7.4. *Reinforced Disc Type Capacitors*

One of the earliest forms of transmitter capacitor is the reinforced disc type shown in Fig. 23. This consists of a flat disc with dished edges, thus taking the electrode edge completely out of the dielectric field. The disc shape does not, however, lend itself to easy cooling, and the type is now obsolete. A smaller version, known sometimes as the double cup capacitor (Fig 24) is still popular as a low capacitance, low kVA unit. In point of fact, the capacitor on the right is not a transmitter type, but a capacitor in a modified barium titanate giving 1,000 pF with a break-down voltage of about 30 kV, intended for use as a smoothing capacitor in television high tension circuits. Figure 25 shows an interesting experimental German trimmer capacitor. Effectively, it consists of two reinforced discs with a

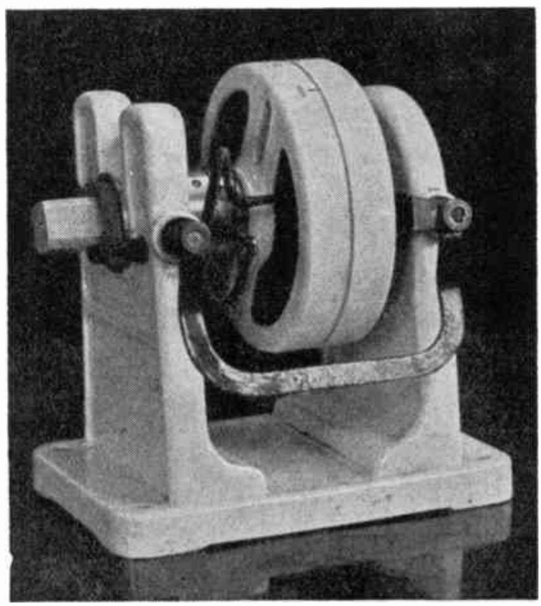

Fig. 25.

centre barrier on each. Both halves of each disc are silvered, and each is paralleled to half of the other disc, thus giving a capacitance variation over 180°. The contacting faces are optic-

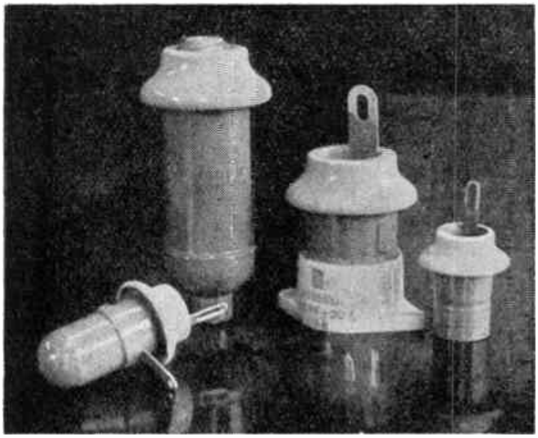

Fig. 26.

ally ground and the capacitor was rated at 60 kVA.

7.5. *Pot Type Transmitter Capacitors*

One of the most widely used type of transmitter capacitor is the pot type, shown in Fig. 26. This is a logical development from the disc, as it is in effect a disc with the centre pushed out. The outer electrode is cleared from the dielectric field by means of a radiussed undercut in the insulator shed, the centre electrode being cleared by extending its length. The shape is rather better for cooling than the disc, but the inside electrode, being enclosed, is not very well cooled.

7.6. *Shell Type Transmitter Capacitors*

It is logical to take the pot capacitor and open the other end, providing a flashover shed there, and thus obtaining the shell capacitor, as shown in Fig. 27. This type of capacitor has the advantage that both electrode surfaces are cooled by natural convection, and very high loadings are thus obtained. The large capacitor shown, actually the largest single ceramic capacitor unit, gives a loading of 150 kVA at 400 kc/s, and is made in titanium dioxide. The capacitance is 4,000 pF. The smaller capacitors shown

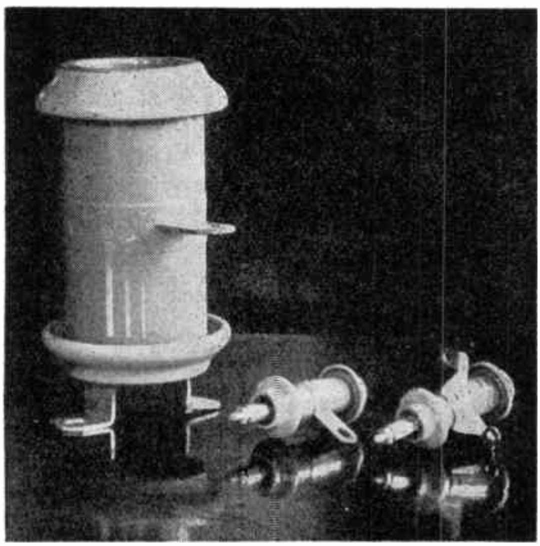

Fig. 27.

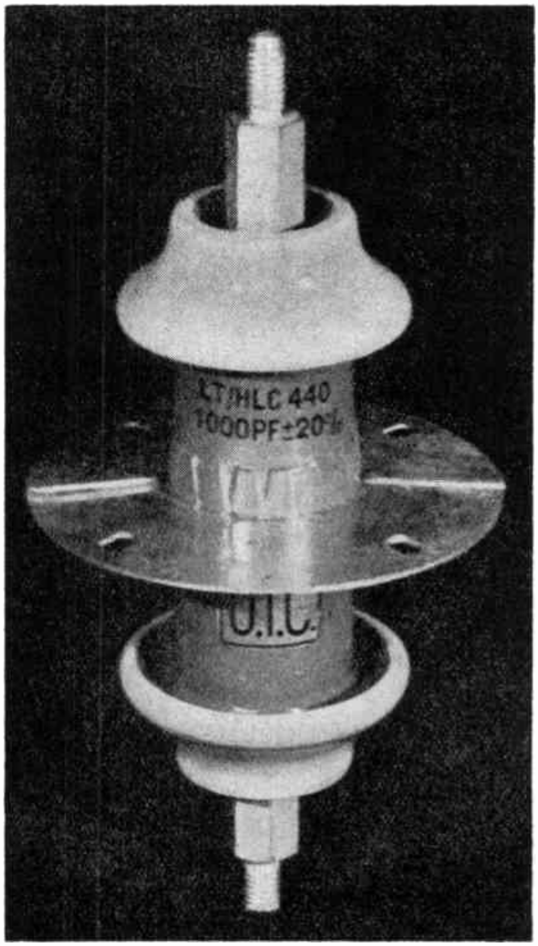

Fig. 28.

give a capacitance of 1,000 pF in titanium dioxide, with a loading of 25 kVA at 1 Mc/s, and 200 pF in magnesium titanate, with a loading of 75 kVA at that frequency. Various mounting electrodes are shown. It is expected that this type will replace the pot shape in time.

7.7. *Forced Cooling of Transmitter Capacitors*

The increase of loading obtained by allowing free natural convection to transmitter capacitors naturally leads one to consider forced cooling by some convenient method. One obvious method is to use forced convection, and, in actual fact, quite a gentle draught such as is obtained from an ordinary desk fan will increase the loading of the large capacitor in Fig. 25 from 150 to 300 kVA. This is an important point to remember in design, as usually on large oscillators or amplifiers a supply of cooling air is provided for the valves, and it is a simple matter to divert some to cool the capacitors.

The next step is to add a radiator. The capacitor in Fig. 28 was intended for use as a high voltage feed-through capacitor. It was found that, with the centre flange screwed to a metal plate 6 in. square, the radiation so obtained increased the loading from 25 kVA to 40 kVA. If, however, we fit a valve type radiator, as in Fig. 29, and provide definite forced cooling, extremely large ratings are obtained, exceeding ten times that in still air. This represents a very promising approach, as with modern air blast cooled transmitter valves it is easy to tap off a small quantity of the air blast by means of a canvas hose to the tank capacitor, the slightly increased cost of the fan being compensated by the use of one ceramic tank capacitor instead of ten.

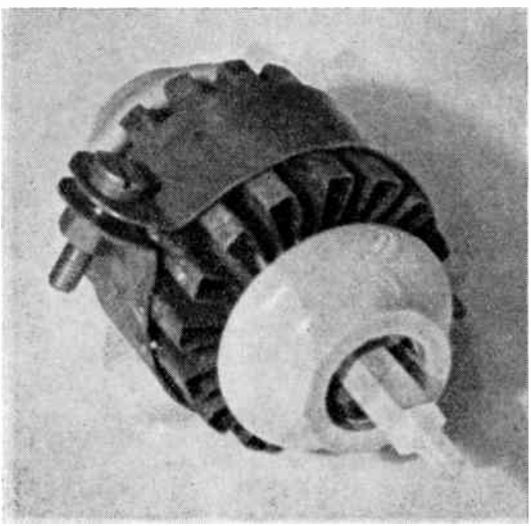

Fig. 29.

Sometimes ceramic power capacitors are immersed in oil for cooling purposes. This enables some increase in loading by virtue of the radiation area of the oil tank, but care must be taken to ensure good circulation of the oil to transfer heat from the capacitors to the tank wall. Generally, however, air cooling is to be preferred, as the modern tendency is to do away with oil owing to its tendency to seep and the fire risk. It should be stressed that the voltage rating cannot be increased by immersing a capacitor in oil as this is controlled by the thickness of the dielectric.

8.0. Trend of Future Development

8.1. *Improved Dielectrics*

This survey has shown that a steady advance in the field of ceramic capacitors has been made, and their development is far from finalized. In the last section, some reference has been made to the improvements made in load rating of ceramic capacitors by attention to the cooling problem, and it is to be expected that further developments on these lines will enable higher loads in smaller spaces to be obtained. However, the chief weight of development will fall on the dielectrics themselves. In the direction of power capacitors, attempts will be made to achieve lower loss angles with materials of high dielectric constants, and with receiver capacitors to obtain still greater permittivities. It should be possible in the not too distant future to replace electrolytic capacitors by ceramic capacitors, a point of particular importance in Service equipment.

8.2. *Complete Tropicalization*

Complete climatic protection over a range of −60°C to +110° C appears to be the minimum target for Service equipment which may have to work in any part of the earth or sky. As far as ceramic capacitors are concerned, the dielectrics can withstand the range, and their complete insulation is more a matter of economics and space. On the other hand, certain other components offer considerably greater problems, and one school of thought maintains that the complete equipment should be contained in a sealed container. This implies miniaturization, and the new high dielectric constant ceramics offer manifest advantages.

8.3. *Printed Circuits*

No paper on ceramic capacitors would be complete without reference to printed circuits. Very briefly, a printed circuit is a baseplate on which the wiring, resistors, inductances and so forth are printed rather than being separate components. Some units have been evolved where the whole circuit is printed on a high dielectric constant ceramic, but this construction, calling for the use of a thin plate, is rather fragile. The more usual form is the use of a ceramic or bakelite base on which the resistors or wiring are printed, and on to which thin flat discs of ceramic are affixed, either mechanically or by soldering, to provide the capacitors. This enables the use of a strong base of a cheaper material, and less trouble is found with stray capacitances. The use of multiple components is, however, a subject of controversy, as one defective item leads to scrapping the whole unit.

9.0. Conclusion

In conclusion, the author expresses his thanks to the United Insulator Company for permission to read and publish this paper.

References

1. " Ceramic and Dielectric Properties of TiO_2-MgO-ZrO." Bodes, Riecke and Ungewiss. *Berichte deut Keram. Ges.*, Vol. 17, No. 5, pp. 237-64 (1936), *Ceramic Abstracts*, Vol. 16, p. 26 (1937).
2. " The Dielectric Properties of the Rutile Form of TiO_2." By Berberich and Bell. *J. Applied Physics*, Vol. II, pp. 6891-92 (1940).
3. " High Dielectric Constant Ceramics." By A. von Hippel et al (MIT), *Ind. Eng. Chem.*, Vol. 38, pp. 1097-1109 (1946).
4. " Dielectric Constant of Barium Titanate as a Function of an Alternating Field." By Wul and Goldman. *Compt. rend. acad. sci.* (URSS), Vol. 49, pp. 117-80 (1940).
5. " Oscillographic Study of the Dielectric Properties of Barium Titanate." By A. de Breiteville. *J. Am. Cer. Soc.*, Vol. 29, pp. 303-7 (1946).
6. " Dielectric and Piezoelectric Properties of Barium Titanate." By S. Roberts. *Physical Review*, Vol. 71, No. 12, pp. 890-5 (1947).
7. " Dielectric Permeability of Rutile Mixtures." By Wul. *Compt Rend. (Doklady) acad. sci.* (URSS), Vol. 43, pp. 292-294 (1944).
8. U.S. 2,402,515-8. " High Dielectric Material and Method of Making Same." Wainer (1946).
Several other patents exist both here and in the U.S.
9. Radio Components Standardization Committee Specification RCS/1.

SOUTH MIDLANDS SECTION DISCUSSION

Mr. O. F. Gullick : In the case of the largest transmitter type ceramic capacitors very high kVA ratings have been achieved under test conditions with external radiators. It would appear that the power factor would remain sensibly constant at 5×10^{-4}, thus the power loss would be proportional to the kVA; for example at 3,000 kVA the loss would be 1,500 watts maximum against 150 watts as at present rated. There is no doubt that this extra heat can be removed by blowing through a suitable radiator, but in view of the bad thermal conductivity of the ceramic material I would expect a dangerous temperature gradient across the thickness of the ceramic material and consequent tendency to develop cracking due to differential expansion on repeated heat cycles.

(2) When the kVA rating is increased the high frequency currents through the condenser will be increased and this means that the losses in the condenser terminals and contacts to the silver coating will all increase. Does Mr. Roberts consider the contact area adequate to handle this increased current rating, and can he fix any limit of current capacity for these contacts ?

(3) When the condensers are air cooled with 10 ft. per second in accordance with the published technical data, I suspect that the voltage shrouds at the top and bottom of the condenser proper tend to deflect the air away from the outer surface of the condenser. Does Mr. Roberts recommend the use of an air deflector to produce adequate scrubbing on the outer surface, and if so would he indicate the shape of such a deflector ?

Mr. W. G. Roberts : It should be remembered that the power factor of a capacitor is contributed to by the electrode losses and the loss at the electrode/dielectric interface. Only a relatively small proportion of the loss will be developed in the interior of the dielectric, and thus, if the outer surfaces are kept cool, no serious thermal gradients will occur.

(2) With regard to the second point, maximum current and voltage ratings are specified for transmitter capacitors, and should not be exceeded whatever the kVA rating.

(3) It is useful to enclose the capacitor being cooled by an insulating cylinder spaced away from the sheds so as to leave an annular space about twice that of the inner bore of the capacitor.

Mr. I. S. Christie : We have carried out numerous measurements at frequencies up to 25 Mc/s on capacitors containing material with a very high value of K, and have found that the capacity, as measured at these frequencies, varies with both ambient temperature and applied D.C. potential. Large capacitors (4,700 pF) exhibit larger changes than small ones (470 pF). Is it possible to reduce these changes by variation of the dielectric while still maintaining small size since, in certain circumstances, they may be a disadvantage? Is it also possible to manufacture, for special purposes, condensers of approximately 50 pF which exhibit substantial changes of capacity with changes in the above parameters ?

Mr. W. G. Roberts : Certain high dielectric constant dielectrics show pronounced voltage coefficients of permittivity, and the effect on a capacitor made of such material will depend on the thickness of the dielectric. Recent work shows that this voltage coefficient is not present in certain admixtures of barium titanate with other compounds, which will shortly replace the older dielectrics. To obtain as low a capacitance as 50 pF with a high permittivity dielectric and a high voltage coefficient would necessitate the use of a thin plate with a very small electrode area.

Mr. L. E. McAllister : Mr. Roberts has stated that considerable increases in the kVA rating of transmitter capacitors can be obtained by using airblast. Can he give any definite figures relating the quantities of air blown with the permissible increase in kVA ratings ?

Mr. W. G. Roberts : If an air velocity of 10 ft. per second is maintained over the surface of a capacitor the kVA rating can be doubled. To achieve still higher ratings the use of radiators is recommended.