## Essentials of ELECTRICITY

## for RADIO and TELEVISION



SLURZBERG \& OSTERHELD

# ESSENTIALS of ELECTRICITY FOR 

## RADIO and TELEVISION

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Second Edition

## Essentials of Electricity for Radio and Television

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## PREFACE

In the history of the world, the various periods of time are referred to as the Stone Age, Coal Age, Iron Age, etc. The historians of the future might well refer to the present era as the Electronic Age. Radio and television are only two branches of the vast field of electronics. When this book was first prepared for its publication six years ago it was intended to: (1) present a text of electrical principles to be associated with radio rather than with power, and (2) written on a level between the elementary and engineering texts. The advent of commercial television in 1947 led to the current revision for the purpose of including both an introductory knowledge of television and the electrical principles associated with television and their applications. Because of the widespread acceptance of electronic principles, indicated to the authors by the favorable acceptance of the first edition of this text, all explanations of electrical principles, analysis of electric circuit action, and operation of circuit components are presented in this edition in terms of electron flow in place of the former reference to current flow.

This book represents the notes and supplementary material used by the authors for the past eighteen years in teaching electricity to high-school pupils and to men working in the radio, television, and communication industry. These notes have also been used by the authors in training men in the signal corps and in the enlisted signal corps reserve.

This book is written for the reader with a limited mathematical background who is interested in studying the principles of electricity as applied to radio and television. He may be a radio amateur; a person working in the radio, television, or communication industry; or a student attending a high school, a trade school, a technical school, a junior college, or a school sponsored by any branch of the armed services. This book provides the necessary background for the study of radio circuits as presented in the authors' book Essentials of Radio. The following features, not ordinarily found in a single book, have been incorporated in this text:

1. No previous knowledge of mathematics above that of simple addition, subtraction, multiplication, and division is required. All other principles are taken up as the need arises. These are then explained in a thorough manner and are followed by an illustrative example. Included
are the use of equations, square root, plotting and interpretation of curves, and the use of vectors.
2. Examples are used throughout the book to illustrate the applications of the equations. These examples follow the general procedure outlined in Chap. II for the solution of problems. Examples of complex as well as simple circuits are illustrated for both direct and alternating currents.
3. The principles of electricity are explained according to the electron theory. This eliminates the necessity of using vague and inadequate mechanical or hydraulic analogies.
4. In recognition of the value of visual instruction, drawings are used to illustrate each principle as it is presented. As many of the important features of the parts used in radio and television equipment cannot be shown by a diagram, numerous photographs of actual commercial products are distributed throughout the text.
5. Flectrical principles peculiar to communications, although generally considered to be intricate, are explained in a comparatively simple manner. This is particularly true of the chapters on a-c circuits, inductance, capacitance, resonance, and basic electronic circuits.
6. The chapter on electrical instruments, in addition to explaining the basic principle on which the various instruments are based, also explains how to select, connect, and read an instrument.
7. The thirteen appendixes have been selected to provide sufficient reference data so that all problems in this book may be solved without the necessity of using any additional reference. One appendix contains a hundred symbols used in electrical, radio, and television circuits, listed with a picture or diagram opposite each symbol to illustrate the part that it represents. Another appendix contains various forms of generally used electrical equations arranged according to the subject, as a-c circuits, resonance, and capacitance.
8. As an aid to the instructor and a challenge to the more interested student, there are numerous questions and problems at the end of each chapter. As an aid to both the teacher and pupil, Appendix XIII lists the answers to approximately every other problèm given at the end of each chapter.
9. The textbook has been brought up to date to include all new standards of units, symbols, and abbreviations.

Numerous industrial organizations have been of great assistance in providing illustrations and technical information regarding their products, and this service is gratefully acknowledged. These organizations are Aerovox Corporation; Allied Radio Corporation; American Telephone and Telegraph Company; Amperite Company; Bliley Electric Company; Browning Laboratories, Inc.; The Electric Storage Battery Company;

Electro Dynamic Works; Eicor, Inc.; General Electric Company; General Motors Corporation; The Hammarlund Manufacturing Company, Inc.; International Resistance Company; Jensen Radio Manufacturing Company; Philco Corporation; P. R. Mallory \& Company, Inc.; Meissner Manufacturing Division, Maguire Industries, Inc.; National Carbon Company, Inc.; Ohmite Manufacturing Company; Radio News; Radio Corporation of America; Shure Brothers, Inc.; Solar Manufacturing Corporation; Standard Transformer Corporation; Thordarson Flectric Manufacturing Division, Maguire Industries, Inc.; Trimm Manufacturing Company, Inc.; Westinghouse Electric Corporation; Weston Electrical Instrument Corporation.

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Morris Slurzberg
Wilidam Osterifeld

## CONTENTS

IREFACE ..... v
Chapter I. Communication ..... 1
Visual Means of Communication. Auditory Means of Communication.Early History of Radio Communication. The Vacuum Tube and RadioDevelopment. Development of the Radio Circuit. Early History of Tele-vision. Radio Waves. Wavelength, Frequency. Sound. Light. SimpleExplanation of Radio and Television Transmission and Reception. GeneralPicture of Radio Transmission and Reception. General Picture of Televi-sion Transmission and Reception. Need for a Knowledge of Electricity.
Chapter II. Basie Theory of Electricity ..... 45Electrostatics. Structure of Matter. The Electron Theory of Matter.Positive and Negative Charges. Charging and Discharging. ElectrostaticFields. Introduction to Dynamic or Current Electricity. Methods ofProducing an Electric Current. Effects of Electric Current. Kinds ofElectric Current. Ohm's Law. Fundamental Electrical Units. Work,Power, Energy. Other Units. Symbols and Abbreviations. How to SolveProblems.
Chapter III. Batteries ..... 87The Cell. Battery Terms. Action of the Voltaic Cell. Polarization andLocal Action. The Dry Cell. Sizes and Proper Use of Dry Cells. Combi-nation of Cells. Radio Batteries. Air Cell A Battery. Secondary Cellsand Storage Batteries. Action of the Lead-acid Cell. The CommercialStorage Battery. Charging the Storage Battery. Care of Batteries.
Chapter IV. Electric Circuits ..... 118Resistance of Conductors. Specific Resistance. Conductors. Insulators.Resistors. Electric Circuits. The Simple Circuit. The Series Circuit.The Parallel Circuit. Simple Combination Circuits. More AdvancedCombination Circuits. Rheostats and Potentiometers. The Voltage Di-vider. Use of Exponents in Calculations. Calculation of a Typical VoltageDivider.
Chapter V. Magnetism. ..... 162
Relation of Magnetism to Electricity. Magnetism, Magnets, Magnetic Materials. Natural Magnets and Artificial Magnets. Permanent and

Temporary Magnets. Poles of a Magnet. Theory of Magnetism. Laws of Magnetic Attraction and Repulsion, Pole Strength, Force of Attraction and Repulsion. Magnetic Fields, Lines of Force, Field Intensity, Flux Density. Magnetic Induction. Magnetic Properties and Classification of Materials. Magnetic Shapes. The Earth's Magnetism and the Compass. Magnetic Field about a Wire Carrying a Current. Relation of Magnetic Field and Electron Flow. Magnetic Field of a Coil, Left-hand Rule. Magnetic Circuits and Calculations.


#### Abstract

Chapter VI. Meters Electrical Instruments. Electrostatic Meters. Electrothermal Meters. Permanent-magnet Moving Coil. Iron Vane. Dynamometer. Rectifiertype Meters. Ammeters. Voltmeters. Meter Scales. Parallax. Shunts. Multipliers. Low-range Meters. Ohmmeters. Combination Meters. Wheatstone Bridge. A-C Bridge.


Chapter VII. Electrical Power Apparatus......................................... 234
Types of Power Supply and Equipment. Faraday's Discovery, Electromagnetic Induction. The Simple Generator, Value of Induced EMF. The A-C Generator. A-C Characteristics: Alternation, Cycle, Frequency. A-C Voltage and Current Characteristics. The D-C Generator. Commercial Generators. Transformers. Efficiency.

Chapter Vili. Inductance. . ................................................................... 267
Inductance, Lenz's Law. Self-inductance. Inductive Reactance, Angle of Lag. Mutual Inductance. Coefficient of Coupling. Series and Parallel Inductances. Low-frequency Inductance Coils. High-frequency Inductance Coils. Shielding. Resistance of Coils. Measuring Inductance. Noninductive Windings. Use of Inductors in Radio, Television, and Electronic Circuits.

Cifapter IX. Capacitance. ........................................................................ 306
Capacitance, Capacitor Action. Factors Affecting the Capacitance. Fixed Capacitors, Commercial Types. Variable Capacitors. Capacitive Reactance, Angle of Lead. Losses in a Capacitor. Voltage Ratings of Capacitors. Capacitors in Series and Parallel. Distributed Capacitance. Flectrolytic Capacitors. Wet and Dry Electrolytic Capacitors. Electrical Characteristics of Electrolytic Capacitors. Measurement of Capacitance. Use of Capacitors.

Chapter X. Alternating-current Circuits.......................................... 357
Circuit Characteristics. Fffects of Inductive and Capacitive Reactance. Series A-C Circuits Containing Resistance, Capacitance, and Inductance. Complex Series Circuits. Vectors: Voltage and Current. Vectors: Resistance, Reactance, and Impedance. Power in A-C Circuits. Series Circuit Problems. Parallel Circuits. Parallel-series Circuits. Series-parallel Circuits.
Chapter XI. Resonance ..... 302
Resonance. Graphs. Plotting, Use, and Interpretation of Curves. SeriesResonance. Resonance Curves. Circuit Q. LC Product. Voltage Ratiosin Series Resonant Circuits. Parallel Resonance. Currents in ParallelResonant Circuits. Comparison of Series and Parallel Resonant Circuits.Uses of Resonant Circuits.
Chapter XII. Basic Electronic Circuits ..... 427The Electric Circuit. Filter Action. Types of Filter Circuits. Multisec-tion Filter Circuits. Filter Circuits as a Whole. Other Filter Circuits.Coupling of Circuits. Characteristics of Mutual-inductive-coupled Cir-cuits. Band-pass Amplifier Circuits. Wide-band-pass Amplifier Circuits.Delayed-action Circuits.
APPENDIX
I. Drawing Symbols Used in Electronics ..... 477
II. Symbols and Abbreviations Used in Electronics ..... 487
III. Formtilas Commoniy Used in Electronics ..... 490
IV. Table of Specific Resistance and Temperature Coffficient of Variocs Metals at $20^{\circ}$ Centigrade. ..... 497
V. Bare Copper Wire Tables at $25^{\circ}$ Centighade, $77^{\circ}$ Fairenheit ..... 498
VI. Dielectric Constant (K) and Dielectric Strength (volts per 0.001 in.) of Various Materials ..... 499
VII. Standard Color Coding for Resistors ..... 500
VIII. Standard Color Coding for Mica Capacitors ..... 502
IX. Standard Color Codffor Transformer Leads ..... 504
X. Trigonometry ..... 506
XI. Sine and Cosine Tables ..... 507
XII. Sine and Cosine Valces for Angles Greater than 90 Degrees ..... 509
XIII. Answers to Problems ..... 510
INDEX ..... 517

## CHAPTER I

## COMMUNICATION

Radio and television are the two most modern means of communication. It is estimated that there are over 60 million radio receivers in use and that over 90 per cent of the homes in America have one or more radio receivers. In the first two years of large-scale commercial television (1947 and 1948), approximately one million television reccivers have been produced, and it is predicted that in a comparatively short time over 25 million homes will have television reccivers.

There are two methods of communication, namely, the auditory and the visual. The auditory method is used to carry on a conversation or even to utter the familiar "Ouch." We communicate our thoughts to all within hearing range by our conversation and make known that we have suffered some unusual sensation or pain by shouting "Ouch." The auditory method is also used by many animals for communication, as may readily be understood by a study of animal and bird life. The smoke signals used by the Indians are an example from the past of the visual means of communication, while the printed page, facsimile, and television are modern examples.

1-1. Visual Means of Communication. The contents of this page are being communicated to you by the visual method. The newspaper is a visual means that reaches a great number of people. Books offer another way of reaching a large number of people by the visual means of communication. Printing is a permanent method since it may be used by different persons at different periods of time. The visual method is a very old one, as is illustrated by the hieroglyphics of the ancient Egyptians found in the ruins of their old cities. The American Indians used the visual method of communication when they signaled messages by means of a blanket and a smoldering fire. The blanket was used to prevent smoke from rising for brief intervals of time, and by prearranged signals code messages could be transmitted through greater distances than by the auditory method.

The U. S. Signal Corps modernized the visual method by using the wigwag, semaphore, and heliograph systems for transmitting messages. In the wigwag system, one flag is used, and the operator moves it briskly in a to-and-fro motion according to the Morse code, shifting it to the left side for a dash and to the right for a dot. In the semaphore system, two flags are quickly shifted from one position to another, each position of the two
flags representing a letter of the alphabet, a number, or some other signal. The heliograph is a means of flashing a light on and off at definite periods of time to represent the Morse code, a long interval being a dash and a short one a dot. These three methods are still employed. The use of


Fig. 1-1.-A portion of the Rosetta stone illustrating hieroglyphics.


Fig. 1-2.-Communication by smoke signals.
the ielescope made it possible for these signals to be seen over fairly long distances. The development of new types of lenses and new kinds of light beams made possible the transmitting of signals without interception during the night and also increased the range of communication. The most recent method of visual communication is the transmission of pictures through the air as in television and facsimile.

1-2. Auditory Means of Communication. Voice. The auditory method is also a very old one, and until modern science gave us the telegraph, telephone, and radio, this method was limited because of the comparatively short distances for which it could be used.

This may be illustrated by the case of two persons in an open field. If one person stands still and shouts as loud as possible while the other walks away until he can no longer hear the shouting, the distance between the


Fig. 1-3.- Communication by wigwag and sound. (From the Special U.S. Army Signal Corps Issue of Radio News, photograph by U.S. Army Signal Corps.)
two will not be very great. If this experiment were repeated in the city, the distance would be decreased further, as the voice would be absorbed or diverted in direction by the various buildings, trees, etc., generally present in any city.

Telegraph. Samuel F. B. Morse, an American inventor, increased the distance through which sound could be heard by inventing and developing telegraphy. This is a means by which sound is sent and received through wires connecting the sending and receiving apparatus. He also developed a system of long and short sounds in different combinations to
represent numbers and the letters of the alphabet; by means of this system the sound is made and broken at definite intervals. The Morse code, which bears the inventor's name, is now transmitted according to the idea that he developed.

Telephone. Alexander Graham Bell, the inventor of the telephone, was a young professor of vocal physiology and a student of electrical


Fig. 1-4.-Sending a message by semaphore. (From the Special U.S. Army Signal Corps Issue of Radio News, photograph by U.S. Army Signal Corps.)
science when he left Scotland to come to Boston in 1871 as a teacher of deaf-mutes. While experimenting on his harmonic telegraph, which led to the invention of the telephone, Prof. Bell outlined the following idea to his associate, Thomas A. Watson: "If I could make a current of electricity vary in intensity precisely as the air varies in density during the production of sound, I should be able to transmit speech telegraphically."

On June 2, 1875, they were carrying on experiments based on this idea when Prof. Bell accidentally overturned a jar of battery acid, causing him to shout to his assistant in another room, "Mr. Watson, come here. I want
you!" Over an electrified wire Watson heard this cry for help and immediately responded.

By employing a diaphragm to produce an clectric current whose intensity varied the same as sound waves, Bell, just as he had foretold, was able to transmit speech. It was therefore no accident that on Mar. 10, 1876, he was able to transmit once again to his assistant the immortal


Fig. 1-5.-Signaling by means of a heliograph. (From the Special U.S. Army Signal Corps Issue of Radio News, photograph by U.S. Army Signal Corps.)
words, "Mr. Watson, come here. I want you!"-his time over the first practical experimental telephone line. In that same year the first telephone conversation between two cities was carried on from Boston to Cambridge, a distance of two miles.

By continuous research and experimentation, telephony has grown to be one of the world's greatest inventions. Distance is no longer an obstacle to communication, as the telephone makes it possible for two persons to talk to one another as if they were in the same room, no matter how great the distance separating them may be. All cities in a state, all states in


Fig 1-6.-Signaling with a tom-tom.


Fig. 1-7.- Alfred Vail and Samuel F. B. Morse working on the development of the telegraph. (American Telephone and Telegraph Company.)
the United States, all countries in the world are now brought together by the telephonc. It has developed from a luxury to a necessity, and modern living could not be carried on without it.

Wireless. Guglielmo Marconi, the inventor of wireless, made communication history when he successfully communicated between France and England across the English Channel in 1899 without having any wires connecting the sending and receiving apparatus in the two countries. On


Fig. 1-8. - A modern application of telegraphy. (From the Special U.S. Army Signal Corps Issue of Radio News, photograph by U.S. Army Signal Corps.)

Dec. 12, 1901, another great advance was made in the development of communications when Marconi received the first wireless telegraph signal ever transmitted across the Atlantic Ocean. This signal consisted only of the letter S, represented in Morse code by three dots. It had been sent out from a powerful transmitting antenna supported by two tall masts which Marconi had erected at Poldhu in Cornwall, England. At St. John's, Newfoundland, a vertical receiving antenna was used, held aloft by a kite.

From this simple beginning, wireless telegraphy has developed to its modern use in communication systems both in peace and in war. In
peace it is used for communication with ships at sea, with airplanes in the air, and with persons on land who are long distances away. In war, communications in code are maintained by wireless between the various units of each of our armed forces and between all the armed forces-Navy, Army, Marine Corps, and Air Force.



Fic. 1-9.-Morse code.
1-3. Early History of Radio Communication. Basic Electrical Principles. Through consistent research and experimentation, many scientists have contributed to the development of radio communication as we use it today. Credit for the invention of radio can go to no one person, as we credited Morse with the telegraph, Bell with the telephone, and Marconi with wireless telegraphy. Its development has taken years, and many men have made important contributions. A brief history of radio progress
can, therefore, be outlined by presenting the names of these scientists and their contributions.

In 1865, James Clerk Maxwell, utilizing the electrical and magnetic experiments developed by Michael Faraday and IIans Christian Oersted, proposed the following theories: (1) that light waves were electromagnetic in character; (2) that a charge of electricity moving through space constituted an electric current as well as a charge moving in the wires of an electric circuit; (3) that a magnet moving in space generated an electromotive force in the space around it.


Fig. 1-10.-A. G. Bell and T. A. Watson at work in the 109 Court Street, Boston, garret, 1875. (American Telephone and Telegraph Company.)

In 1888, Heinrich IIertz proved by direct experiments that the predictions made by Maxwell were true. Hertz made a very careful study of electric waves and found not only that they move with the same speed as light but that they behave in the same manner as do light waves in every way except that they cannot be seen by the human eye. While the waves of visible light are so short that 30,000 to 60,000 are required to equal the space of one inch, the electric waves were discovered by Hertz to have lengths ranging from several inches to several miles.

Wireless. In 1895, Marconi invented the aerial, and he was able to increase the distance by which the electric waves could be projected into space. To increase the energy of transmission, antenna structures were made very large, and high voltages were used. The early commercial
transmitters were of the spark type, utilizing the charge and discharge of a capacitor through an oscillator circuit containing a spark gap which was

(a)

(b)

Fig. 1-11.-Progress in telephone design. (a) Two views of a model of Bell's first telephone. (b) Internal construction of a modern hand set. (American Telephone and Telegraph Company.)
inductively coupled to the antenna circuit and in resonance with it. The principle of inductive coupling and the resonance between various parts
of the transmitting circuit were discovered by Sir Oliver Lodge. Following the spark system of transmission the continuous-wave method was used, and during this time the Poulsen are and the Alexanderson and Goldschmidt alternators came into use. For detection at the receiving end of the radio system, the coherer and the crystal detector were generally used.

1-4. The Vacuum Tube and Radio Development. Necessity of the Vacuum Tube. Radio development was thus far hampered by the fact that the amount of power radiated from the transmitter could not be in-


Fig. 1-12.-A simple telephone circuit. (American Telephone and Telegraph Company.)


Fig. 1-13.-Guglielmo Marconi and G. S. Kemp receiving the first radio-wireless signal transmitted across the Atlantic Ocean. (From the Telephone Almanac, American Telephone and Telegraph Company.)
creased sufficiently to be received by low-power apparatus situated any appreciable distance from the transmitter. Furthermore, the sensitivity of the detectors of radio waves, the crystal and coherer, was very low. The search for a sensitive detector to convert the received energy efficiently led to the development of the vacuum tube. The invention and perfection of the vacuum tube gave radio engineers the device they had been looking for. It greatly increased the sensitivity of reception and also made it possible to amplify a weak signal to any desired volume.

Edison Effect. In 1883, Thomas Edison, while conducting experiments with the incandescent lamp, noticed that if a second electrode, in the form of a wire or plate, was placed inside the lamp and this electrode made positive with respect to one end of the filament, a small current flowed
to this electrode when the filament was heated. This electronic effect is called the Edison effect.

Vacuum-tube Development. Professor J. A. Fleming of England learned of the Edison effect, and about 1896 he developed a crude form of radio detector tube which became known as the Fleming valve. About 1906, Dr. Lee De Forest added a third electrode, called a grid, to the vacuum


Fig. 1-14.-Evolution from the early De Forest tube to the modern vacuum tubes of more complex structure. (RCA Manufacturing Co., Inc.)
tube. The vacuum tube was improved by further study and experimentation carried on by Dr. Langmuir and other scientists, and we now have the vacuum tube of today.

1-5. Development of the Radio Circuit. Besides the improvement of the vacuum tube, many other changes, additions, and methods of connecting the various parts of radio transmitters and receivers have been introduced during the past 30 years. In 1914 Major E. H. Armstrong obtained a patent on his regenerative circuit, also known as a feed-back, or self-heterodyning circuit.

In 1924, Louis Alton Hazeltine gave the world his tuned radio-frequency method of amplification and the principle of neutralization of the capacitance of tubes. Hartley, Colpitts, and Meissner made variations in the oscillator circuit that is used in all superheterodyne receivers and transmitter circuits. The constant-current system of plate modulation as developed by Heising is the method most commonly used by transmitting stations. Major Armstrong, in seeking a way to get rid of static, decided that some method of modulating the signal must be used for which nature had no duplicate. His method of frequency modulation is the result, and


Fig. 1-15.-Dr. Irving Langmuir, Sir Joseph John Thomson, and Dr. William D. Coolidge at the General Electric laboratories when Thomson, the famous discoverer of the electron, visited America in 1923. (General Electric Company.)
it has revolutionized the field of radio communication. It is now possible to hear entertainment over the radio without being disturbed by the annoying noises made by static.

1-6. Early History of Television. Basic Principles. In television, just as in radio, credit for its discovery and development must be given to many men for their individual contributions to this new field of communications. One of the earliest discoveries that later had a part in the development of television, was that of selenium in 1817 by a Swedish chemist Baron Jöns Jakob Berzelius. Its early use was chiefly as a substance to provide resistance in electric circuits.

In 1873, an Irish telegrapher named Joseph May observed while work-
ing his instruments that the intensity of light shining on a piece of selenium had an effect upon the amount of electric current flowing through the circuit in which the selenium was connected. This discovery led to the conclusion that selenium could be used to vary the amount of current in a circuit for different amounts of light in-


Fig. 1-16.-W Walkie-talkie, an example of radio circuit development. (From the Special U.S. Army Signal Corps Issue of Radio News, photograph by U. S. Army Signal Corps.) tensity. Other scientists and experimenters took up this thought and expanded upon the idea. It was proposed that, since a picture is composed of light and dark areas, a circuit containing a selenium unit could be used to translate the variations of light and dark areas into changes in electric current. It was further proposed that if changes in light could produce changes in electric current, such changes in electric current could be made to reproduce a similar picture with light and dark areas. Actual experiments were suggested, one of which involved the use of a series of small selenium cells each independently connected to a separate lamp. It was believed that if the image of an object was projected against the group of selenium cells the varying amounts of current could be conducted by wires over a short distance and there cause a group of lamps to light with corresponding intensities to reproduce the image. This proved unsuccessful because it would be too cumbersome and its wiring was too complicated for that cra.

Mechanical Scanning. As early as 1880, it was proposed by Maurice Leblanc of France that the individual cells be replaced by a method called scanning. In this method the image would be broken down into a large number of parts each of which would be examined one at a time.

In 1884, Paul Nipkow of Germany introduced the idea of a perforated rotating disk to do the scanning of the image. The holes in the disk were arranged in a spiral pattern, and thus when the disk was rotated the image would be scanned line by line. The varying intensity of light being passed
through the holes as they scanned the image would cause a varying current to flow in a circuit containing a selenium cell.

Noah S. Amstutz, an American, is credited with sending the first successful halftone picture in 1890 when he sent a picture over a 25 -mile length of wire. This first wire photography process required eight minutes for complete transmission. In 1909, Ians Knudsen successfully transmitted a photograph by wireless.

In 1923, Dr. Vladimir K. Zworykin, of the United States, introduced a tube called the iconoscope which replaced the selenium cell and also made it possible to scan the image without the use of moving parts. Also in 1923, Philo Farnsworth, another American scientist, introduced a tube called the image dissector which was to replace the selenium cell. Both of these tubes picked up the image and projected it through a lens onto the light-sensitive plate in the tube.

In 1925, Capt. Richard Ranger of the United States armed forces transmitted war-game pictures and maps by radio from New York to Honolulu, a distance of over 5000 miles. The process employed a rotating glass cylinder around which a photographic film was placed. A powerful thin beam of light was passed across the cylinder and through the film as it was being rotated, and a photoelectric cell was used to transform the varying intensity of light into a varying electric current. This current was amplified and then transmitted by radio into space. At the receiver the radio signal was picked up and converted back to a varying electric current. This varying current operated a thin beam of light which reproduced the original picture on a sensitized paper wound around a cylinder revolving in step with the one at the transmitter.

In 1926, John L. Baird, a Scottish scientist, demonstrated in England the transmission of halftone pictures by television.

Early in 1928, Dr. E. F. W. Alexanderson gave a public demonstration in Schenectady, N. Y., of television transmission with a picture about three inches square. In September, 1928, the first full dramatic program was telecast. In 1929, experiments conducted in color television produced a picture about one inch square.

Many experiments in television employing mechanical scanning were carried on up until about 1930. While some actual transmission and reception of pictures were carried on in this era, the results were not satisfactory for commercial use.

Electronic Scanning. Although mechanical scanning served a useful purpose in the development of television, it imposed certain limitations, namely, (1) the scanning was limited to a low rate of speed, and (2) the area of the image transmitted was so small that it permitted a view of only the hands, face, etc.

The iconoscope, invented by Dr. V. K. Zworykin, and the image dissector, invented by Philo Farnsworth, made it possible to scan the image at the transmitter by use of electronic circuits in place of the older mechanical method of scanning. The kinescope, introduced to television in 1928, made it possible to have electronic scanning at the television receiver. In 1929, Dr. Zworykin gave a public demonstration of his first all-electronic television receiver made possible by the use of the kinescope. The all-


Fig. 1-17.-View of a television program being televised. (General Electric Company.)
electronic method of scanning made it possible to overcome the disadvantages of mechanical scanning.

In 1931, experimental television broadcasting was carried on with 120line pictures. In 1936, the transmission of pictures was accomplished with 343 -line pictures. In 1939, the first all-electronic receivers (with 441-line pictures) were commercially available to the public. During the war years immediately following this period, there was very little new development in commercial television.

In 1947, after the Federal Communications Commission ruled that color television was not yet ready for commercial application, the monochrome (black-and-white) television with the newly adopted standard of 525 lines
went ahead at a rapid pace. Transmitters were being set up in all parts of the country, and the production of receivers increased at a tremendous rate. By 1949, receivers were being produced at the rate of nearly a million per year, and it became evident that television was a new and large industry. The prediction of 25 million receivers was no longer considered fantastic.

1-7. Radio Waves. Radio and television transmitting stations convert sound waves and light waves to electrical impulses. The electrical impulses that represent the original sound and light waves are sent out by the use of high-frequency alternating currents. These currents produce magnetic and electric fields that radiate in all directions over long distances without losing much of their original strength. The magnetic and electric fields produced by this means are called radio waves. The strength and frequency of the radio wave is dependent on the high-frequency alternating current producing it; therefore, it will vary in the same manner as the alternating current.

An a-c (alternating-current) wave (see Fig. 1-18) reverses its direction at fixed intervals, and during each interval the current will rise from zero to its maximum value, then diminish to zero. By referring to this figure it can be seen that an a-c wave completes one cycle after it


Fig. 1-18.—An a-c wave. has made two alternations, one in the positive direction and one in the negative direction. The fixed interval required for each alternation is 180 degrees, and for one cycle or two alternations it would be 360 degrecs. $I$ is a symbol used to denote current and $m a x$ an abbreviation of the word maximum. $I_{\text {max }}$ would, therefore, mean the maximum amount of current flow; according to Fig. 1-18 this would occur at every 90 - and 270 -degree instant of an a-c cycle.

1-8. Wavelength, Frequency. Speed of Radio Waves. Radio waves travel at the same speed as light waves, 186,000 miles per second. In radio and television calculations the metric system is used, and it is desirable to express the speed of the waves in meters per second.

Example 1-1. If radio waves travel at the rate of 186,000 miles per second, what is their rate in meters per second?

Note: One meter is equal to 39.37 inches; also, one mile is equal to 5280 feet.

Given:

Miles per second $=186,000$ Find:

Feet per mile $=5280$
Inches per meter $=39.37$

Solution:

$$
\begin{aligned}
\text { Meters per second } & =\frac{\text { inches per second }}{39.37} \\
& =\frac{186,000 \times 5280 \times 12}{39.37} \\
& \cong 300,000,000
\end{aligned}
$$

Wavelength and Frequency Definitions. Wavelength. The distance that the radio wave travels in one cycle is called its wavelength; it is expressed in meters and is often represented by the symbol $\lambda$, a letter of the Greek alphabet pronounced lambda.

Frequency. The number of cycles per second of a radio wave is called its frequency and is gencrally represented by the letter $f$. In radio work it is common practice to refer to the frequency as the number of cycles instead of in terms of cycles per second. This is merely an abbreviation, and it should be remembered that a reference to the number of cycles really means cycles per second.

Wavelength and Frequency Calculations. Wavelength. If the frequency of a wave is known, it will be possible to calculate the distance traveled in one cycle by means of the equation

$$
\begin{equation*}
\lambda=\frac{300,000,000}{f} \tag{1-1}
\end{equation*}
$$

where $\lambda=$ wavelength, meters
$f=$ frequency, cycles per second
Example 1-2. What is the length of a radio wave whose frequency is 570,000 cycles?

Given:
$f=570,000$

Find:
$\lambda=$ ?

Solution:

$$
\begin{aligned}
\lambda & =\frac{300,000,000}{f} \\
& =\frac{300,000,000}{570,000} \\
& =526.3 \text { meters }
\end{aligned}
$$

Kilocycles. The frequencies of the common radio waves are of high values, that is, in the hundreds of thousands and in the millions. For convenience these frequencies are generally expressed in kilocycles. Kilois a prefix meaning thousand; hence a kilocycle is equal to 1000 cycles. Recalling the abbreviation referred to above, one kilocycle actually means 1000 cycles per second. As lost radio frequencies are expressed in kilocycles, Eq. (1-1) becomes

$$
\begin{equation*}
\lambda=\frac{300,000}{f} \tag{1-2}
\end{equation*}
$$

where $\lambda=$ wavelength, meters
$f=$ frequency, kilocycles
Example 1-3. What is the wavelength of radio station WMCA, which operates on a frequency of 570 kilocycles?

Given:
$f=570 \mathrm{kc}$
Solution:

$$
\begin{aligned}
\lambda & =\frac{300,000}{f} \\
& =\frac{300,000}{570} \\
& =526.3 \text { meters }
\end{aligned}
$$

## Find:

$\lambda=$ ?

Megacycles. The frequencies of the carrier waves used in frequency modulation and television are much higher than those used in ordinary radio broadcasting. The frequency of commercial $\mathrm{f}-\mathrm{m}$ and television transmitters ranges from 50 to 216 million cycles per second. For convenience these frequencies are generally expressed in megacycles. Mega- is a prefix meaning million; hence a megacycle is equal to $1,000,000$ cycles. As $\mathrm{f}-\mathrm{m}$ and television-transmitter frequencies are generally expressed in megacycles, Eq. (1-1) may be expressed as

$$
\begin{equation*}
\lambda=\frac{300}{f} \tag{1-3}
\end{equation*}
$$

where $\lambda=$ wavelength, meters
$f=$ frequency, megacycles
Example 1-4. What is the wavelength of a television video carrier wave whose frequency is 77.25 megacycles?

$$
\begin{array}{ll}
\text { Given: } & \text { Find: } \\
f=77.25 \mathrm{mc} & \lambda=?
\end{array}
$$

Solution:

$$
\begin{aligned}
\lambda & =\frac{300}{f} \\
& =\frac{300}{77.25} \\
& =3.88 \text { meters }
\end{aligned}
$$

Frequency. Equation (1-2) can be transposed to solve for frequency instead of wavelength by multiplying both sides of the equation by the frequency and dividing both sides by the wavelength, as

$$
\frac{\lambda \times f}{\lambda}=\frac{300,000 \times f}{\lambda \times f}
$$

Equation (1-2) then becomes

$$
\begin{equation*}
f=\frac{300,000}{\lambda} \tag{1-4}
\end{equation*}
$$

where $f=$ frequency, kilocycles
$\lambda=$ wavelength, meters
Equations (1-1) and (1-3) can be transposed in the same manner to solve for frequency instead of wavelength.

Example 1-5. If by definition a short-wave radio is one whose wavelength does not exceed 200 meters, what is the lowest frequency at which a short-wave radio may operate?

$$
\begin{array}{lr}
\text { Given: } & \text { Find: } \\
\lambda=200 \text { meters } & f=?
\end{array}
$$

Solution:

$$
\begin{aligned}
f & =\frac{300,000}{\lambda} \\
& =\frac{300,000}{200} \\
& =1500 \mathrm{kc}
\end{aligned}
$$

From Eq. (1-4) it can be seen that the greater the length of the radio wave the lower its frequency will be; conversely the shorter a radio wave is the higher its frequency will be. By applying this conclusion to Example $1-5$ it becomes evident that the frequency of short-wave radio transmitters will be 1500 kilocycles and higher.

In order to get an idea of the length of a radio wave, it is necessary only to change the wavelength to our common units of feet or miles.

Example 1-6. What is the length in feet of one radio wave of the broadeast station referred to in Example 1-3?

$$
\begin{array}{ll}
\text { Given: } & \text { Find: } \\
\lambda=526.3 \text { meters } & \text { Feet }=?
\end{array}
$$

Solution:

$$
\begin{aligned}
& 1 \text { meter }=39.37 \mathrm{in} \\
& \text { Feet }=\frac{\text { meters } \times 39.37}{12} \\
&=\frac{526.3 \times 39.37}{12} \\
&=1726 \mathrm{ft}
\end{aligned}
$$

The solution of Example 1-6 indicates that each wave transmitted by station WMCA is 1726 feet long, or approximately a third of a mile.


Fig. 1-19.-Relation of frequency and wavelength of various waves.

Since radio waves travel 186,000 miles per second, the time required for a radio wave to get from one place to another can be readily calculated.

Example 1-7. How long does it take a radio wave to travel from New York to San Francisco, a distance of approximately 2600 miles?

Given:

$$
\text { miles }=2600
$$

Miles per sceond $=186,000$

## Find:

$$
\text { Time }=\text { ? }
$$

Solution:

$$
\begin{aligned}
t & =\frac{\text { miles }}{186,000} \\
& =\frac{2600}{186,000} \\
& =0.0139 \mathrm{sec}
\end{aligned}
$$

The solution of Example 1-7 indicates that it takes only about fourteenthousandths of a second for a person's voice broadeast on a radio program to travel from New York to San Francisco.

1-9. Sound. Its Use in Radio. Radio is a means of sending information through space from one point to another. The information may be either the sound waves produced by the voice or some musical instrument or a wave so interrupted that it is broken into a combination of long and short groups corresponding to the characters of the Morse code. Therefore, radio is nothing more than the sending out and receiving of sound through space from one point to another, without any wires connecting the two points. It is therefore essential to know something about sound and sound waves before studying the principles of radio.

Characteristics of Sound. Sound is the sensation produced in the brain by sound waves. It makes use of one of our five fundamental senses, hearing. The air in a room in which no sound is present is in a static condition; in other words, it is motionless. If a sound is made by a person, by a musical instrument, or by any other means, the air about it is set into vibration. These vibrations are transmitted to adjacent layers of air and so on until all of the original energy is expended. Such air vibrations are called sound waves. When these vibrations strike the cardrum of a person, the cardrum too will vibrate in a similar manner. The auditory nerves will be stimulated and will communicate the sensation of sound to the brain.

Sound waves are produced by the mechanical vibration of any material in elastic media such as gases, liquids, and some solids, but they will not travel in a vacuum. Sound waves are longitudinal waves and travel outward in all directions from the source. A longitudinal wave may be defined as one in which the vibrating molecules or particles of the transmitting
medium move back and forth in the same direction in which the sound wave is traveling.

Intensity. The intensity, or loudness, of a sound depends upon the energy of motion imparted to the vibrating molecules of the medium transmitting the sound. A greater amount of energy causes more violent movement of the molecules, which in turn exerts a greater pressure upon the eardrum, thus causing the auditory nerves to send the sensation of a louder sound to the brain. Loudness is affected by the distance between the listener and the source of the sound. Actually, the intensity of a sound varies inversely with the square of the distance between the listener and the source of the sound. For example, if the distance between the listener and the source of the sound is doubled, the intensity is reduced to one-


Fig. 1-20.-Internal structure of the human ear. (From "Unified Physics" by G. L. Fletcher, I. Mosbacher, and S. Lehman, MfGraw-IIill Book Company, Inc., New York.)
quarter; if the distance is increased to three times the original amount, the intensity is reduced to one-ninth. Also, if the distance between the listener and the source of the sound is decreased to one-half the original amount, the intensity of the sound will be four times as great; for one-third the distance, it will be nine times as great.

Speed of Sound. The speed at which sound waves travel varies with the kind of material through which it is traveling. For air, the most common medium used for transmitting sound waves, the speed is 1130 feet per second at the normal room temperature of $68^{\circ}$ Fahrenheit.

Frequency, Pitch, and Wavelength. The vibration of the reeds in a harmonica, of the skin on a drum, of the strings on a violin, or of the cone of a radio loudspeaker will all send out various sound waves. These waves will produce different sounds, depending on the number of vibrations that the wave makes per second. The number of complete waves or vibrations created per second is known as the frequency of the sound and is generally
expressed as the number of cycles per second. For example, a sound wave that is making 2000 vibrations per second is the same as a sound whose frequency is 2000 cycles per second; this is also commonly referred to as a 2000 -cycle sound or a 2000 -cycle note.

If the sound is loud enough to be heard by the human car, it is said to be audible. Its pitch will vary with the frequency. High frequencies produce sounds having a high pitch, and low frequencies produce sounds of low pitch.

Sound waves may also be referred to in terms of the length of a wave. Knowing that sound waves travel 1130 feet per second in air, the length of one wave can be calculated by dividing the number 1130 by the frequency


Fig. 1-21.-Propagation of sound waves.
of the sound. Figure 1-21 illustrates a tuning fork producing sound waves whose frequency is 256 cycles per second and whose wavelength is 4.41 feet.

Frequency Ranges of Sound Waves. The range of frequencies that the human ear is capable of hearing will vary with the individual, the lower limit being approximately 20 cycles and the upper limit 20,000 cycles. Some persons are able to hear the low-pitch sounds but cannot hear those of high pitch, while others can hear the high-pitch sounds but cannot hear those of low pitch. However, there are people able to hear sounds covering a wide range of frequencies.

A few of the common audible sounds and their approximate frequency ranges are listed in Table I-I.

Code signals may employ any audio frequency, but experience has shown that a signal with a frequency of 1000 cycles will produce a pleasing sound for continual, easy listening; it will permit each dit, dah, or space to be quickly distinguished.

The frequency range of sound waves, commonly taken as 20 to 20,000 cycles, is at the lower end of the wave spectrum (see Fig. 1-19). Sound waves are capable of traveling only comparatively short distances and usually travel at the rate of 1130 feet per second. In order for sounds to be carried through the air over long distances, the sound waves are converted into electrical waves of corresponding frequencies and applied to a high-frequency carrier wave by modern radio and television transmitting stations.

Table I-I
Sound Cycles
Human voice. ............................................................. . . . . $75-3000$
Male (average).............................................................. . . . 128
Female (average)............................................................. 256
Piano. . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 25-8000
Violin. . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 200-3000
Trombone. . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 100- 500
Clarinet. . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 150-1500
Flute. . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . $250-2300$
Piccolo. . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 500-4500
Frequency Range of Radio Receivers. The sounds produced by a symphony orchestra contain practically all the frequencies likely to be produced by any type of radio program. To obtain perfect fidelity of reproduction of the music produced by such an orchestra, sounds from 20 to 20,000 cycles may have to be reproduced. For the average receiver such accuracy of reproduction is neither obtainable nor necessary. The frequency ranges of the various units used in radio transmitters and receivers limit the frequency reproduction for the average high quality a-m (amplitudemodulation) receiver to a practical range of 100 to 5000 cycles. The popular low-cost five-tube a-m receivers for a-c/d-c operation do not provide such high-quality reproduction, and some of these may cover only a range of 160 to 3600 cycles. High-fidelity $\mathrm{f}-\mathrm{m}$ transmitters and receivers extend the practical range of frequencies to 30 and 15,000 cycles.

Musical Sounds and Noise. When sound waves are produced repeatedly at regular intervals, the result is a musical sound at some definite pitch which is more or less pleasant to the ear. The orderly repctition produces rhythm which is also a requirement to obtain a musical note. When sound waves of constant or varying frequencies are produced at haphazard irregular intervals, the result is an unpleasant sound called noise.

Quality, F'undamentals, and Overtones. The middle C of a piano has a
frequency of 256 cycles per second. A corresponding note of 256 cycles can also be produced on other musical instruments such as a violin, clarinet, or harmonica. Yet most persons can identify various instruments from hearing each produce a note of the same frequency. The notes from the various instruments differ in quality which depends upon the number and relative amplitude of the overtones blended with the fundamental.

The fundamental note is the lowest tone produced, which in the above example would be the 256 -cycle note. The overtones, which are higher pitched notes, blend with the fundamental and give each instrument (and each human voice) its individuality. The overtones are vibrations whose frequencies are multiples, namely, $2,3,4,5$, etc., of the fundamental. In the above example the frequencies of the overtones would be 512,768 , 1024,1280 , etc., cycles. The overtones are also often referred to as harmonics.

Reflection of Sound. When sound waves strike a solid object such as a wall of a building, the side of a cliff, or the wall of a room, the sound will be reflected and may cause an echo. An echo is the effect produced when a reflected sound returns to the ear a fraction of a second after the original source of the sound has ceased. If the interval between the original sound and the reflected sound is one-tenth of a second or greater, an echo is likely to result. Echoes do not appear in small rooms because the reflected sounds return to the ear too soon after the original sound to be distinguished. Large rooms or auditoriums, where the reflecting surfaces are more than 50 feet away from the source of the sound, often produce echoes. In such cases the walls and ceilings may be decorated or padded with tapestries or soft materials to eliminate or reduce the production of echocs.

Sympathetic Vibration. The sound waves set up by one sound-producing object can cause a nearby object to start vibrating and thereby also produce sound waves if both objects have the same natural frequency. The vibrations of the second object are said to be the result of sympathetic vibration. An example of this phenomenon may sometimes be observed when the music from a radio receiver causes a metal vase, picture frame, or other object to start vibrating and give off sound waves. Sympathetic vibration may also cause a loose part in a receiver itself to start vibrating.

Sympathetic vibration may be explained in the following manner: The original sound wave strikes all nearby objects and sets them in motion, even though the amount of movement is very slight. If the second wave strikes at the precise instant which causes it to add its motion in perfect unison to that of the preceding wave, the movement will be increased. If all the succeeding waves strike at the corresponding precise instants, the movement will be cumulative and the object will vibrate at the frequency of the source of the sound. If, however, successive waves do not strike
the object at precisely the proper instant, some of the waves will neutralizo the motion imparted to the object by some preceding waves and no sympathetic vibration will result.

Forced Vibrations. An object in the area near a source of sound may be set into vibration by the sound waves of frequencies other than the natural frequency of the object if the intensity of the sound is great enough. This phenomenon is called forced vibration. Examples illustrating forced vibrations are the sounding board of a piano, sound reflectors of musical instruments, and the loudspeaker of a radio or television recciver.

Resonance. When a sound wave sent out by an object is reflected in such a manner that it returns to the object at the proper instant, it will produce sympathetic vibration within that object and thereby increase the intensity of the sound. This phenomenon is called resonance. An example of resonance can be observed by causing air waves to flow through pipes as in a pipe organ. The resonance of a pipe (for a fixed value of frequency) depends upon the length of the pipe and whether it is open or closed. A pipe closed at one end will produce resonance when the length of the pipe is one-quarter of the wavelength of the sound. A pipe open at both ends will produce resonance when the length of the pipe is one-half of the wavelength of the sound.

Beats. In the preceding paragraphs on Sympathetic Vibration and Resonance it was shown that two sounds of the same frequency would reinforce each other. If two sounds of different frequency are considered, it can be shown that the two sounds when started at the same instant of time will at first reinforce each other and then, after a number of cycles have been completed, will be out of step and will neutralize each other. Over a relatively long span of time there will be periods of reinforcements producing a strong sound and periods of neutralizing effect during which no sound will be produced. The sounds produced in this manner are called beats. If the number of beats appears frequently and at regular intervals, a new sound or beat note will be produced. The frequency of the beat note will be equal to the difference between the frequencies of the two sounds; thus, for example, sounds of 256 and 200 cycles will produce a beat note of 56 cycles.

1-10. Light. Its Use in Television. Television is a means of transmitting and receiving a visible image or scene between two points. At the transmitter the light rays of an image or a scene are converted into electrical impulses which are then transmitted to one or more receivers by radio waves or over a wire. At the receiver the electrical impulses are converted back into light rays to provide a reproduction of the original image or scene. It is therefore essential to know something about light and light waves before studying the principles of television.

Characteristics of Light. Light is the sensation produced in the brain by light rays. It makes use of one of our five fundamental senses, sight. In the study of physics light is considered as a form of energy that may be derived from mechanical, electrical, chemical, heat, or light energy. Examples of light encrgy obtained from other forms are as follows: (1) The sparks seen when flint is hammered by a piece of steel illustrates light obtained from mechanical energy. (2) The light of a neon lamp is derived from electrical energy. (3) The light from the glow of phosphorus is derived from chemical energy. (4) The light from an incandescent lamp is obtained from heat energy. (5) The light from the cathode-ray tube of a television receiver (or the luminous dial of a watch) is obtained by fluorescence, which is a form of light energy.

Objects are visible because light rays from them reach the eyes, which in turn stimulate the optic nerves and send the sensation of sight to the brain. When the visible object is the source of the light encrgy, it is said to be luminous. The electric light,


Fig. 1-22.-Internal structure of the human eye. (From "Unified Physics" by G. L. Fletcher, I. Mosbacher, and S. Lehman, Mfc-Graw-Hill Book Company, Inc., New York.) the sun, and the picture tube of a television receiver are examples of luminous objects. Most objects with which we come in daily contact are not luminous, and the light rays that reach the eyes from such objects are actually the reflected light from some luminous body. Objects visible because of the light they reflect are called illuminated bodies.

Several theories have been presented to explain light energy. Sir Isaac Newton (1642-1727), the famous English physicist, introduced the corpuscular theory; this assumed that a luminous body emitted tiny particles (corpuscles) which moved through the air and bounced off objects to produce reflected light rays. Christian Huygens (1629-1695), a Dutch scientist, introduced the theory that light was a form of wave motion. In about 1900 Max Planck, a German physicist, introduced the theory that light was given off in quanta (which are units or particles of light); hence this is referred to as the quantum theory. A modern version of light energy is that light should be explained by both particles and wave motion; this is sometimes expressed as particles guided by wave motion.

Light is said to be transmitted by transverse waves, that is, waves in which the motion is transmitted in a direction at right angles to the vibrations. Light waves can be transmitted only by a transparent medium. including vacuum. In terms of the ability to transmit light, matter may
be divided into three classifications: transparent, translucent, and opaque materials. Materials that transmit light so well that objects can be seen clearly through the material are called transparent. Materials that transmit light so poorly that objects cannot be seen clearly through the material are called translucent. Materials through which light cannot pass are called opaque.

Brightness and Intensity of Illumination. The first consideration in regard to the measure of light is usually its brightness. The brightness of a source of light is expressed in candle power, and a light source of one candle power produces the same amount of light as that emitted by a candle of standard dimensions. A more useful measure of light is the intensity of illumination, which expresses the rate of flow of light energy upon a unit of surface. The unit of the intensity of illumination is the foot-candle, which is the amount of illumination produced on a surface that is one foot away from a standard candle of one candle power.

The intensity of illumination varies inversely as the square of the distance between the light source and the surface to be illuminated. For example, a 100 -candle-power lamp will provide an intensity of illumination of 100 foot-candles at a distance of one foot, 25 foot-candles at two feet, 6.25 foot-candles at four feet, and 1 foot-candle at ten fect. In practice, the intensity of illumination is generally determined by use of a photometer, usually a photoelectric device used frequently by photographers.

Speed of Light. At one time scientists believed that light traveled instantancously from its source to the observer. In 1675, Olaus Römer, a Danish astronomer, calculated that the speed of light through air was approximately 186,000 miles per second. By more accurate means Albert A. Michelson (1852-1931), a noted American physicist, determined the speed of light to be 186,284 miles per second. For general purposes, the speed of light through air is taken as 186,000 miles per second.

The speed of light varies with the medium through which it travels, such as air, water, vacuum, or glass. Its speed through various media depends upon the density of the medium to light rays, as is indicated by the fact that light will travel faster through vacuum than through "air, water, glass, etc.

Frequency, Wavelength, and Color. If light is considered as wave motion, it can be expressed in terms of frequency and wavelength the same as sound waves. The color of light varies with its frequency just as the pitch of a sound varies with the frequency of the sound waves. Also as in the case of sound, when the velocity and frequency of the light waves are known, it is possible to calculate the wavelength of the light waves. The values listed in Table I-II indicate the average frequency and wavelength.s of the various colors of light that make white light when combined.

From this table it can be observed that the frequency of light waves is very high compared with sound waves and that the wavelength of light waves is much shorter than sound waves. Furthermore, examination of the frequency spectrum chart of Fig. 1-19 reveals that the frequency of light waves is also much higher than radio waves. It may be observed that white is not included in Table I-II; it is omitted because white light contains all seven colors listed in the table. The spectrum of visible light consists of a band of colors changing gradually from violet at one end to deep red at the other, just as in a rainbow.

The phenomenon of color is explained by the fact that different materials may transmit (or reflect) lights of different colors. For example, a red piece of glass appears red because it transmits only the red and absorbs all of the other colors contained in white light. If a material transmits

Table I-II

| Color | Frequency, cycles per second | Wavelength |  |  |
| :---: | :---: | :---: | :---: | :---: |
|  |  | Centimeters | Inches | Microinches |
| Red. | 423,000,000,000,000 | 0.000071 | 0.000028 | 28 |
| Orange. | 483,000, 000,000,000 | 0.000062 | 0.000024 | 24 |
| Yellow. | 525,000,000,000,000 | 0.000057 | 0.000022 | 22 |
| Green | 576,000,000,000,000 | 0.000052 | 0.000020 | 20 |
| Blue. | 639,000,000,000,000 | 0.000047 | 0.000018 | 18 |
| Indigo. | 682,000,000,000,000 | 0.000044 | 0.000017 | 17 |
| Violet. | 732,000,000,000,000 | 0.000041 | 0.000016 | 16 |

two or more colors, a new color results. When a material transmits all seven colors, it appears colorless, as illustrated by ordinary window glass.

Frequencies just below those of visible red light are classed as infrared, and frequencies just above those of violet are classed as ultraviolet. These comparatively new classifications are being used in a rapidly increasing number of applications that include electronic equipment.

Propagation of Light. As in radio- and sound-wave motion, propagation refers to the transmitting or spreading out of the wave motion. The outstanding factor concerning the propagation of light is that light rays travel in straight lines when the medium transmitting the light is homogeneous (which means that the transmitting medium must be uniform). That light travels in a straight line may be observed when rays of sunlight enter a darkened room through a small opening. The straight path of the light rays becomes plainly visible due to the illumination of the dust particles in the air.

Reflection of Light. When light energy from a source strikes the surface of an object, some of the light energy is reflected. The amount and the color of the light reflected will depend on the condition of the surface and the color of the reflecting body. Smooth bodies reflect light better than irregular ones; also light bodies reflect more light than dark ones. This further explains the theory of color as illustrated by the fact that an opaque red body appears red because it reflects only the red light and absorbs all others. In the case of black, an object appears black because it absorbs all colors and hence reflects none.

When the reflecting surface is smooth and flat, the reflected rays will be reproductions of the original and the reflection is said to be regular. When the reflecting surface is irregular, the reflected rays will not have the same relation to one another as the original and the reflection is said to be diffused.

Refraction of Light. Refraction is the name used to describe the effect that causes rays of light to bend when they pass from one medium to another of different optical density. For example, air and water have different optical densities, which is another way of saying that light travels through air and water at different rates of speed. (In air the speed of light is 186,000 miles per second, but in water it is only approximately 140,000 miles per second.) Thus if an object such as a pencil is placed in a glass of water, the refraction of the light causes the pencil or object to appear bent or broken.

Lenses. A lens is a piece of transparent substance denser than the surrounding medium and with at least one of the two surfaces ground to conform to a definite curvature. Lenses are classified in terms of the curvature being convex or concave. Convex lenses are thicker at the center than at the edges, and concave lenses are thinner at the center than at the edges. Although lenses are usually made of glass, they can also be made of quartz crystals, water or other clear liquids, and a variety of other materials.

Fundamentally, the function of a lens is to change the direction of rays of light. Convex lenses cause the light rays to converge, that is, to come to a common point from different directions; the image may be enlarged or decreased in size depending upon the distances between the lens, the object, and the image. Concave lenses cause the light rays to diverge, that is, to extend from a common point in different directions; the size of the image is always decreased.

There are many applications of lenses in everyday life such as eyeglasses, magnifying glasses, cameras, motion-picture projectors, microscopes, and telescopes. In television, lenses are used in the cameras at
the transmitters and also in projection-type receivers. In some televisionreceiver installations, a magnifying lens, often of the type employing a glass chamber filled with liquid, is placed directly in front of the picture tube to enlarge the picture.

Persistence of Vision. An important property of the human eye that makes television and motion pictures possible is persistence of vision. The eye cannot observe or follow any sequence of changes that occur at a rate of 10 or more times per sccond. Anything in excess of this rate produces the effect of a continuous picture. This phenomenon is called persistence of vision.

In a television receiver the image appearing on the picture tube is not a steady picture but is actually a sequence of 60 individual pictures per second, each separated from the preceding one by a short interval of time during which the screen of the picture tube is dark. The viewing area of the picture tube is coated with a fluorescent substance, usually a phosphor, that will glow for only a very short period of time after the electron beam strikes the screen; hence the tube is said to have low persistency. Because the persistence of vision of the human eye is much greater than the persistence of the picture tube, the image at the television receiver appears as a continuous picture that also reproduces the movements of the objects being viewed.

1-11. Simple Explanation of Radio and Television Transmission and Reception. Have you ever asked the question: "How is it possible for a person to sing, talk, or play a musical instrument, in fact to make any audible sound, and be heard almost instantly by people thousands of miles away"? Or have you asked, "How can pictures be transmitted through the air"? To answer these questions, the sending and receiving of a radio or television program will be compared with the delivery and recciving of a ton of coal.

If a person orders a ton of coal from a coalyard, the coal is loaded on a truck which carries it to his home. The driver stops the truck at the person's home because he ordered the coal. The buyer does not want the truck-he wants the coal; so the driver and his helpers unload the coal into the bin and drive away with the truck to deliver the rest of the load.

In a similar manner, an audible sound wave made at a radio transmitting station or an object being viewed at a television transmitting station can get to the listener's home only if a means of carrying it there is provided. In place of a coal truck, transmitting stations use a carrier wave. Just as the coal had to be put on the truck, the sound wave or the picture image must be put on the carrier wave. A modulator is used for this purpose. At a radio transmitting station the modulator takes the audible sound
wave that has been changed to electrical impulses by the microphone and superimposes it on the carrier wave. At a television transmitting station the modulator takes the picture image that has been changed to electrical impulses by the camera tube (iconoscope) and superimposes it on the carrier wave. In either case the resultant is called a modulated carrier wave.

This modulated carrier wave is now sent out by a transmitting antenna, just as the coal truck was sent from the coalyard. During the day any number of coal trucks may pass the door of the buyer, but the only truck that stops is the one delivering the coal to his home. In the same manner any number of modulated carrier waves pass the antenna of the listener's receiver. He turns a dial on his receiver and selects the station he wants. This is actually selecting the desired modulated carrier wave.

Next the ton of coal was separated from the truck and placed in the bin, and the truck then continued on its way to make other deliveries. In radio or television, part of the energy of the modulated carrier wave enters the receiver; the remainder is available for other receivers. At the receiver the electrical impulses corresponding to the sound wave or the picture image are separated from the modulated carricr wave by the detector, which may also be called the demodulator.

No heat is obtained from the coal unless it is burned; similarly, no sound is obtained from the audio wave unless it causes some material to vibrate, and no picture is obtained from the picture-image wave unless it causes light and dark impulses on a fluorescent material. The amount of energy delivered by the detector of a radio receiver is so small that it is sufficient only to operate a set of earphones. The amount of energy delivered by the detector of a television receiver is so small that it is sufficient only to produce a small, dimly lighted picture or no picture at all. In order to obtain sufficient energy to operate a loudspeaker and to produce a reasonably large, bright picture, it is necessary to include amplifiers in radio and television receivers.

Just as there are numerous trucks carrying coal, so too there are numerous carrier waves carrying radio and television programs. Also, just as the trucks must be controlled to prevent interference between them, so too the modulated carrier waves must be controlled. To prevent interference among modulated carrier waves, the Federal Communications Commission (FCC) assigns definite carrier frequencies to the transmitting or broadcasting stations. For example, the frequency of the carrier wave of station KFI, Los Angeles, is 640 kilocycles; WLAC, Nashville, 1510 kilocycles; WENR, Chicago, 890 kilocycles; and WNBT, New York, 66 to 72 megacycles (channel 4). By setting the dial of a radio recciver to 710 kilocycles, a person in New York will he able to hear the program being


ALIDIYLDATG do STVILNGSSG

Fig. 1-23.-Comparison of delivery, receiving, and use of coal (top) with sending, receiving, and use of sound waves (below).
broadcast by station WOR. Similarly by setting the dial of a television receiver to channel 4 ( 66 to 72 megacycles), a person in New York will receive the program of station WNBT.

1-12. General Picture of Radio Transmission and Reception. The chart shown as Fig. 1-24 presents a simple picture of the various operations required to send a sound wave out into space and to have it received many miles away.

The top line is a block diagram illustrating the essential portions of a radio transmittor and receiver. It is called a block diagram because each section is represented by merely drawing a block and labeling it to conform with the portion it represents. The first unit is the microphone where the audible sound waves are picked up and changed into electrical impulses. The electrical impulses from the microphone are too weak to be sent through space on the carrier wave and, therefore, must be amplified. This is accomplished by sending the wave from the microphone to the speech amplifier. The next block is called the oscillator, which is the portion that sets up the carrier wave of the transmitter, in this example, 550 kilocycles. This is followed by the modulator, which receives energy from both the oscillator and the speech amplifier. At the modulator the audio waves of the speech amplifier are superimposed on the carrier wave, and this modulated carrier wave is then sent out into space by the transmitting antenna.

The receiving antenna is affected by the magnetic and electric fields set up in space by the transmitting antenna, and if the selector or tuning portion of the receiver is set for the proper frequency (in this example 550 kilocycles), a workable amount of electrical energy enters the receiver. The amount of energy is small and must be increased in strength at this point by the r-f (radio-frequency) amplifier. The selector and the r-f amplifier are shown in a single block because these two operations are generally combined. The next block, labeled detector, might also be called the demodulator because at this point the audio waves are separated from the carrier wave. The audio waves coming from the detector are too weak to operate a loudspeaker and therefore must be sent through an a-f (audio-frequency) amplifier before going on to the loudspeaker.

The second line in the chart indicates the frequency of the wave as it enters and leaves the various parts of the transmitter and recciver operated at a frequency assumed to be 550 kilocycles. The third line is a diagrammatic representation of these frequencies. A careful examination of the figure will show that every step performed in the transmitter is also performed in the receiver but in reversed order, starting with the sound waves entering the microphone at the transmitter and ending with similar sound waves leaving the loudspeaker of the receiver.


Fig. 1-24.-Radiobroadcasting operations from microphone to loudspeaker.

The fourth line summarizes the function of each part of the radio transmitter and receiver.

1-13. General Picture of Television Transmission and Reception. The broadcasting of a television program requires the transmission of both the sound and picture portions of the program. At the transmitter these two functions are performed by two separate transmitters: one for the sound, or audio, portion as illustrated by Fig. 1-24 and described in the preceding article, and another for the picture, or video transmission, as illustrated by Fig. 1-25 and described in the following paragraphs. At the receiver a single antenna and selector unit are used to receive and tune in the sound and picture carrier waves of the desired station. Following the selector circuit, the sound carrier wave is separated from the picture carrier wave and passed through a circuit similar (but not necessarily identical) to that illustrated in Fig. 1-24 and described in the preceding article. The picture carrier wave, after being separated from the sound carrier wave, is passed through the circuit shown in Fig. 1-25 and described in the following paragraphs.

A study of Figs. 1-24 and 1-25 will show that the transmission and reception of the picture portion of a television program are in many ways similar to the transmission and reception of a radio program. The chart of Fig. 1-25 presents a simple picture of the principal functions required to send and receive the picture portion of a television program.

Referring to the block diagram of Fig. 1-25, the first unit is the camera tube, which picks up the light rays reflected from the person or object to be televised and converts these light rays into electrical impulses; this corresponds to the microphone of the radio transmitter of Fig. 1-24. The electrical impulses from the camera tube are very weak and therefore are passed through the picture-signal amplifier; this corresponds to the speech amplifier of the radio transmitter. The next block, labeled oscillator, sets up the carrier wave of the transmitter. It should be noted that the television carrier-wave frequency of 77.25 megacycles (corresponding to television channel 5) is much higher than the carrier-wave frequency of the radio transmitter of Fig. 1-24, which was 550 kilocycles, or 0.55 megacycle. At the next block, labeled modulated $r$-f amplifier, the picture signal-wave is superimposed on the carrier wave and amplified so that it becomes a powerful signal. This powerful modulated carrier wave is then sent out into space by the transmitting antenna.

The television system requires an additional function represented on Fig. 1-25 by the block labeled synchronizing and sweep circuits. The sweep circuit sets up a narrow moving path of electrons, called an electron beam, that is made to move across the light-sensitive part of the camera tube. This beam of electrons is moved across the tube from left to right at the

TRANSMITTER
RECEIVER

top of the tube, brought back to the left and moved down a short distance, then again moved from left to right. This process is repeated again and again, so that the beam scans across the tube 525 times in going from the top to the bottom. Upon reaching the bottom, the beam is returned to the top and the process is repeated. In modern television systems the tubes, picture area is scanned 30 times each second. In order to reproduce the picture at a receiver, the picture tube in the receiver must be scanned by an electron beam in the same manner and at exactly the same rate as at the picture tube of the transmitter. This is accomplished by having the synchronizing circuit superimpose pulses of very short duration on the carrier wave each time a new scanning line is started in motion.

When the receiving antenna is within the effective range of the carrier waves of the transmitter, and if the selector or tuning unit of the receiver is set for the proper frequency (in this example 76 to 82 megacycles, representing the frequency span of channel 5), a workable amount of energy enters the receiver. It should be noted here that both the sound carrier wave of 81.75 megacycles and the picture carrier wave of 77.25 megacycles are received, selected, and given a certain amount of amplification by the single antenna and tuning unit. After this, the sound carrier wave is usually separated from the picture carrier wave. The sound carrier wave is then passed through a circuit similar (but not necessarily identical) to that illustrated in Fig. 1-24 and described in the preceding article. The picture carrier wave is amplified at the $i-f$ (intermediate-frequency) amplifier and passed on to the detector, where the picture signial is removed from the modulated picture carrier wave. Because a strong signal is needed at the picture tube, the signal is amplified at the video (or picture) amplifier before being passed on to the picture tube of the receiver. The synchronizing and sweep circuits enable the scanning beam at the receiver's picture tube to operate at the proper time and rate so that it corresponds with that of the transmitter. The proper correlation of the picture signal and the scanning electron beam produce the picture on the light-sensitive area of the cathode-ray tube of the receiver.

The second line of lig. 1-25 indicates the frequency of the various waves as they enter and leave the various parts of the transmitter and receiver operating on television channel 5 . The third line is a diagrammatic representation of these frequencies. A careful examination of the figure will show that the various steps performed in the transmitter are also performed in the receiver but in reversed order, starting with the light rays being picked up by the camera tube and ending with similar light rays leaving the picture tube at the receiver. The fourth line summarizes the function of each part of the television transmitter and receiver.

1-14. Need for a Knowledge of Electricity. In the block diagrams of Figs. 1-24 and 1-25, each function is represented merely as a square and called an oscillator, modulator, amplifier, detector, etc. Each one of these parts is made up of various clectrical devices such as resistors, inductors, capacitors, and tubes, all properly connected in order to perform the function desired.

There have been a number of changes made in the simple radio circuits used years ago to give us the modern radio and television receivers and transmitters. Where the radio broadcast band formerly extended from 500 to 1500 kilocycles, its range now extends to 108 megacycles for f-m broadcasting and the television band extends from 50 to 216 megacycles. Of the various circuits used in the development of radio receivers only two, the tuned-radio-frequency and the superheterodyne, are still in general use; the superheterodyne principle is used in the majority of the modern radio receivers and in all the television receivers. In order to improve the operation of radio and television receivers, numerous additions and changes have been made to the original radio circuits. Many such circuits and their parts no longer resemble the original except for the fundamental principle involved.

It has been previously stated that radio and television circuits consist of various kinds and types of electrical devices properly connected. This also applies to modern industrial electronic equipment, which usually employs circuits and principles that often are only applications or variations of the ordinary electrical, radio, and television circuits. In order to understand these circuits, one must have a broad and thorough knowledge of electrical and radio theory. It takes time to master these fundamental principles, and one should not be discouraged because he is not working on or studying radio and television circuits at the beginning. Once these fundamental principles are mastered one will be able to understand all kinds of sets, both present and future.

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## QUESTIONS

1. Name three applications of (a) the visual method of communication, (b) the auditory method of communication.
2. What contribution did Samuel Morse make toward the advancement of the auditory method of communication?
3. What was Alexander Graham Bell's idea that led to the invention of the telephone?
4. How was Bell able to transmit speech?
5. Who invented the aerial, and what is its importance in regard to communication?
6. What contributions did James Clerk Maxwell make toward the development of radio?
7. Who proved that the theories advanced by Maxwell were true? What further contribution did he make?
8. What principles of electricity were discovered by Sir Oliver Lodge?
9. What were the difficulties encountered in radio transmission and reception previous to the development of the vacuum tube?
10. What two important factors did the vacuum tube add to radio communication?
11. What did each of the following men add to the development of vacuum tubes: (a) Thomas A. Edison? (b) J. A. Fleming? (c) Dr. Lee De Forest?
12. What did each of the following men add to the development of radio circuits: (a) Armstrong? (b) Hazeltine? (c) Hartley? (d) Colpitts? (e) Meissner?
13. Name two important features of the f-m system of radio communication.
14. (a) In what manner is selenium associated with the early development of television? (b) Name two men connected with this work.
15. (a) What is meant by mechanical scanning? (b) What did Leblanc and Nipkow contribute to the development of television?
16. Describe the contribution made by Capt. Richard Ranger to the development of television.
17. (a) What is the important feature of the iconoscope and the image dissector? (b) Name the inventors of these two devices.
18. (a) Name five or more men associated with the development of television. (b) What was the contribution of each to the science of television?
19. (a) What were the disadvantages of mechanical scanning? (b) What new method of scanning replaced the mechanical system?
20. Prepare a chronological list of developments in television.
21. How are radio waves produced?
22. What two factors make up radio waves?
23. How does the speed of radio and television waves compare with (a) light? (b) Sound?
24. Define (a) wavelength; (b) frequency; (c) cycle; (d) kilocycle; (e) megacycle.
25. Using the chart of Fig. 1-19, what is the frequency band assigned for (a) commercial a-m broadcasting? (b) Commercial f-m broadcasting? (c) Television?
26. What is sound?
27. Explain what occurs when sound waves strike the human ear and produce the sensation of sound.
28. (a) How are sound waves produced? (b) What factors affect the intensity at which a sound is heard?
29. Define the following terms as used with sound: (a) frequency, (b) pitch, (c) wavelength, (d) audible.
30. What frequency is usually used for the audio wave of code signals? Why?
31. What is the approximate frequency range of a receiver having (a) low fidelity? (b) Modium fidelity? (c) High fidelity?
32. Define the following terms as used with sound: (a) quality, (b) fundamental, (c) overtone, (d) harmonic.
33. Explain what is meant by (a) reflection of sound waves, (b) echoes, (c) sympathetic vibrations, (d) forced vibrations.
34. What is meant by resonance?
35. How are beat notes produced?
36. What is light?
37. Explain how objects are made visible.
38. Name and describe four theories of light energy.
39. Define the following terms: (a) transparent, (b) translucent, (c) opaque.
40. (a) What is meant by the intensity of illumination? (b) What is its unit of measurement? (c) What is its relation to distance and the source of light?
41. (a) What is the relation between the color of light and its frequency? (b) Why is white omitted from the frequency-color-spectrum chart?
42. Explain the phenomenon of color.
43. What is meant by (a) propagation of light? (b) Reflection of light?
(c) Refraction of light?
44. (a) What is a lens? (b) How are lenses usually classified? (c) Describe each of the classifications named in (b).
45. (a) What materials are used in making lenses? (b) Name six applications of lenses.
46. (a) What is meant by persistence of vision? (b) Explain two commercial applications of this phenomena.
47. What are the essential functions of a radio transmitter?
48. What is the purpose of each function given in the answer to Question 47?
49. What are the essential functions of a radio receiver?
50. What is the purpose of each function given in the answer to Question 49?
51. What are the essential functions of a television transmitter?
52. What is the purpose of each function given in the answer to Question 51?
53. What are the essential functions of a television receiver?
54. What is the purpose of each function given in the answer to Question 53?
55. To what extent is a television system similar to radio?
56. Name two types of radio circuits used in present-day radio and television receivers.
57. Why is a knowledge of electricity necessary in order to study radio, television, and industrial electronics?

## PROBLEMS

1. What is the wavelength of a carrier wave of a transmitter whose frequency is 1200 kc ?
2. What is the wavelength of a carrier wave of a transmitter whose frequency is 660 kc ?
3. What is the frequency of the radio waves from a transmitter operating on a 10 -meter wavelength?
4. What is the frequency of the radio waves of a transmitter if its wavelength is 75 meters?
5. A certain radio station located in New York operates on a carrier frequency of 30 mc . How long does it take for an audio signal being transmitted to reach (a) Honolulu, Hawaii (approximately 5000 miles)? (b) Melbourne, Australia (approximately 10,000 miles)?
6. A certain radio station located in Chicago operates on a carrier frequency of 1210 kc . How long does it take for an audio signal being transmitted to reach (a) New York (approximately 800 miles)? (b) San Francisco (approximately 2200 miles)?
7. If the frequency of middle C on a piano is 256 cycles, what is its wavelength in (a) meters? (b) Fect?
8. If the frequency of high C on a piano is 4096 cycles, what is its wavelength in (a) meters? (b) Feet?
9. If an a-f wave of 256 cycles is superimposed on a carrier wave of 1200 kc , how many cycles does the carrier wave make during the time it takes the a-f wave to complete one cycle?
10. If an a-f wave of 4096 cycles is superimposed on a carrier wave of 660 kc , how many cycles does the carrier wave make during the time it takes the a-f wave to complete one cycle?
11. The creaking of a door makes a sound of approximately 15,000 cycles. What is its wavelength?
12. What is the wavelength of the sound waves being produced by an insect if the frequency of the sound is 12,000 cycles?
13. How many cycles will a carrier wave of 1200 kc make during the time required for (a) one cycle of a 256 -cycle a-f wave? (b) One cycle of a 15,000 -cycle sound wave?
14. How many cycles will a carrier wave of 100 mc make during the time required for (a) one cycle of a 256 -cycle a-f wave? (b) One cycle of a 12,000 -cycle sound wave?
15. Radio programs are often presented to studio audiences as well as to the radio audience. (a) How long does it take the sound waves to reach a listener in the studio audience seated 100 feet away? (b) How long does it take for the program to reach a listener at the loudspeaker of a radio receiver 200 miles away? (c) Which listener hears the program first?
16. How far would a radio wave travel in the time it takes for a sound wave to travel 10 feet?
17. How far would a sound wave travel in the time it takes for a radio wave to travel around the earth (approximately 25,000 miles)?
18. A pipe of an organ is measured and found to be 0.565 foot long. At what frequency is it resonant if the pipe is (a) closed at one end? (b) Open at both ends?
19. What length of organ pipe, open at both ends, is required to produce resonance for a frequency of (a) 5000 cycles? (b) 500 cycles? (c) 50 cycles?
20. What is the frequency of the beat note produced when a sound having a frequency of 400 cycles is combined with one of (a) 300 cycles? (b) 350 cycles? (c) 450 cycles? (d) 500 cycles?
21. If it is desired to produce a beat note of 250 cycles, what frequency sound wave must be added if the original sound has a frequency of (a) 100 cycles? (b) 200 cycles? (c) 500 cycles?
22. What is the intensity of illumination from a 200 -candle-power lamp at a distance of (a) 2 ft ? (b) 5 ft ? (c) 20 ft ?
23. What size lamp must be used to produce an intensity of illumination of 10 foot-candles: (a) 10 ft from the lamp? (b) 5 ft from the lamp? (c) 2 ft from the lamp?
24. A source of light having a color between red and orange has a frequency of $450,000,000 \mathrm{mc}$. What is its wavelength in (a) centimeters? (b) Inches? (c) Microinches?

25．A source of blue light has a wavelength of 17.5 microinches．What is its wavelength in（a）megacycles？（b）Cycles？

26．What is the frequency of a television carrier wave if its wavelength is 5 meters？

27．What is the wavelength of a television transmitter＇s sound carrier wave if its frequency is 81.75 mc ？

28．A certain f－m radio station operates on an assigned frequency of 101.1 mc ． （a）What is its wavelength in meters？（b）What is its wavelength in feet？

29．Television channel 11 is assigned frequencies of 198 to 204 mc ．What are the corresponding wavelengths？

30．Television channel 4 is assigned frequencies of 66 to 72 mc．What are the corresponding wavelengths？

31．Complete the table of frequency and wavelength ratings of the stations listed as follows：

| Station | Location | Kilo－ cycles | Me－ <br> ters | Station | Location | Mega－ cycles | Me－ <br> ters |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| WOR | New York | 710 |  | WRUL | Boston |  | 19 |
| WCRW | Chicago | 1210 |  | W9XPD | St．Louis |  | 11 |
| KRLD | Dallas | 1040 |  | WLAP | Louisville | 1.2 |  |
| WWI， | New Orleans | 850 |  | WLWO | Cincinnati |  | 49 |
| WMBC | Detroit | 1420 |  | WPIT | Pittsburgh |  | 25 |
| KPCB | Seattle | 650 |  | KGEI | San Francisco |  | 31 |

32．Add four of your favorite stations to the list of Prob．31，and fill in their frequencies and wavelengths．

33．Decipher the following code message：
－ーー ．．．．•－－$\cdots \cdot$－－$\cdot \cdots$ ーー・ ーーー ー・•
－—— •—• —－－．．－——．．．．．－
34．Complete the following code message：


35．Write the following in Morse code：（a）your name，（b）name of your school， （c）city in which you live，（d）state in which you live．

## CHAPTER II

## BASIC THEORY OF ELECTRICITY

Electricity cannot be perceived by any one of our five senses, as we cannot see, hear, taste, smell, or feel it. You may say electricity can be seen--look at the electric light. This statement is false, because what is seen is not electricity but one of the effects of electron flow. The same may be said about the heat from an electric iron or the shock one gets from coming in contact with an electric current; in both eases it is the effects of electron flow that is felt and not electricity. In the former the wire in the iron gets hot and we feel the heat given off, and in the latter it is the burn on the body that is felt.

The exact nature of electricity is not known. However, recent investigations indicate that it consists of small negative charges called electrons. When the electrons are standing still, we have electricity at rest, or static electricity, and when they are forced to travel, a movement of electrons results and we have electricity in motion, or dynamic electricity. We are more concerned with dynamic electricity as it is the kind of electricity used in the home, school, factory, and any other place where light, heat, ventilation, or some kind of work must be performed. It can be used to run a motor that will operate a machine, lift heavy articles, or run a locomotive, as well as to supply energy for lighting homes and operating radio and television receivers.

2-1. Electrostatics. Flectrostatics may be defined as the study of electricity at rest, or static electricity. Static clectricity is produced by friction, and there are many examples of this frictional electricity in normal daily occurrences. After combing one's hair, the static electricity in the comb will cause it to attract bits of paper. After a person walks across a rug, very often a spark caused by static electricity will appear if a radiator is touched. Trucks carrying a load of gasoline accumulate a charge of static electricity; here the spark that may result can cause serious consequences, and to prevent trouble a chain is allowed to drag along the ground and pass the static electricity to the earth. Sometimes sparks are obtained from leather belts that are operating machinery. Friction of clouds produces static electricity which we know as lightning.

Electrostatics is one of the first subjects in which man became inter-
ested. It had its beginning when man first began to wonder about lightning. The start of the science of static electricity, however, is usually credited to Thales, a Greek mathematician and philosopher. In about the year 600 b.c., Thales discovered that if amber was rubbed it would attract light objects. Nothing much was done to further this science until about the year 1600 when William Gilbert, an English physicist, conducted numerous experiments and found that many other bodies besides amber had the same property of attraction after being rubbed. Gilbert is also credited with giving electricity its name, for he called the attraction electric force and the bodies that possessed the power electrics-after the Greek word for amber, which is elektron.

Static electricity is of little practical use; in fact, its presence is generally unwanted. It has, however, served a very useful purpose in the study of electricity. Many of the advancements made in the field of dynamic or current electricity owe their development to knowledge scientists obtained from electrostatics. The most useful outcomes of static electricity are the development of the lightning rod and the capacitor. The modern explanation of electricity is based upon the electron theory, which tells us that all matter is definitely related to electricity.

2-2. Structure of Matter. All matter is made up of either a single element or a combination of two or more elements which is then called a compound.

Elements. An element is a substance which cannot be decomposed by ordinary chemical means. Recently, however, some elements have been decomposed by electrical means. At present there are more than 90 elements known to science, and the electronic structure of each is known. All the elements now known may be divided into the six types listed below. With each type there are listed several of its elements that are used in the manufacture of various electrical and radio parts.

1. Inert gases: helium, neon, argon, krypton, xenon
2. Ligit metals: sodium, magnesium, barium, calcium, potassium, strontium, radium
3. Rare earths: lanthanum, cerium, holmium, erbium
4. ITeavy metals: iron, copper, silver, aluminum, nickel, cobalt, zinc, lead, bismuth, cadmium, thorium, germanium
5. Nonmetals: boron, carbon, nitrogen, oxygen, chlorine, silicon, phosphorus, sulphur, iodine, arsenic, selenium
6. Hydrogen: Note. This resembles no other element and is therefore not considered as being in any of the above divisions.

Compounds. Although many substances (forms of matter), such as copper, silver, and iron, that we see and use in our daily life are composed of a single element, a far greater number of substances are compounds.

Paper, wood, glass, clothing, food, stone, fiber, Bakelite, our bodies, ete., are examples of compounds. The infinite number of possible combinations that can be made by using any two or more elements, and also by varying the proportions of each element used, explains why it is possible to have so many different substances.


Gases Used in © Manufacture
neon - hydrogen - carbon dioxide - llluminating gas
helum - argon - natural gas - nitrogen - oxygen
Elements Ensering into the © Manufacture
ARGOM - ALUMINUM - BORON - BARILM - CAESIUM - CALCIUM - COPPER - CARBON - CHROMIUM - CHLORINE COBALT - hYDROGEN - hELUM - IRIDIUM - IRON - LEAD - MAGNESIUM - MERCURY - MOLYbDENUM NICKEL - NEON - NTTROGEN - OXYGEN - POTASSIUM - PHOSPHORUS - PLATINUM - SODIUM - SILVER SILCON - - STRONTIUM - TUMGSTEN - THORWM - TANTALUM - TTANIUM - TIN - ZINC - RARE EARTHS

Fig. 2-1.-Materials used in the construction of radio tubes. (RCA Manufacturing Co., Inc.)
The smallest particle imaginable into which an element can be broken down and still retain all the properties of the original element is called an atom. The smallest particle into which a compound can be broken down and still retain the original properties of the compound is called a molecule. Two or more atoms of the various elements, therefore, combine to form a
molecule of a compound. The resulting combination may have no physical or chemical properties resembling the atoms of the elements of which it is made. For example, when two atoms of hydrogen gas combine properly with one atom of oxygen gas they form one molecule of water. Chemically this is generally expressed as $\mathrm{H}_{2} \mathrm{O}$, where $\mathrm{H}_{2}$ indicates that there are two atoms of hydrogen and $O$ indicates one atom of oxygen. Another example of a familiar compound is the combination of one atom of sodium, a very active heavy metal, and one atom of chlorine, a heavy greenish-yellow poisonous gas, which results in common table salt. Chemically this is known as sodium chloride and is generally expressed as NaCl , where Na is the symbol for sodium and Cl is the symbol for chlorine.

Size of Molecules and Atoms. A molecule is so small that it cannot be seen with the naked eye or even with the use of lenses, because both the eye and the lenses are themselves made of a vast number of molecules. They would have to become molecular in size in order for the molecules to become visible. Molecules are so small that a drop of water contains millions of molecules of water. As small as molecules are, they are relatively large compared with an atom, as a molecule may be made up of a number of atoms.

2-3. The Electron Theory of Matter. Research work conducted by scientists like Sir William Crookes, J. J. Thomson, Robert A. Millikan, Sir Ernest Rutherford, and Niels Bohr has contributed much to the study of matter and has led to the electron theory. According to the electron theory of matter it is believed that every atom consists of one or more positively charged particles called protons and one or more negatively charged particles called electrons and that an atom in its normal or neutral state has an equal number of protons and electrons.

Structure of the Atom. The protons, which are about 1845 times the weight of an electron, form the heavier central part of the atom called the nucleus. The nucleus contains the protons and some other particles of which science is not yet certain and which, in our study, are not of great importance. We do know, however, that the nucleus is positively charged and that some of the atom's electrons move about the nucleus in orbits much as the earth and the other planets move about the sun. These electrons that move about the nucleus are referred to as orbital or planetary electrons.

All electrons are alike whether from an atom of copper, oxygen, or any of the various elements. The difference in the elements lies solely in the number and the arrangenent of its protons and electrons. Figure 2-2 shows atoms of carbon, copper, and aluminum. Each orbit can hold only a limited number of electrons, namely, $2,8,8,18,18,32,32$, respectively. Thus in the case of carbon, which has 12 protons and 6 planetary electrons,
the first (inner) orbit will have its full quota of two planetary electrons and the remaining four electrons must move in a different orbit. Since the second orbit can accommodate eight electrons, it will readily take care of these four.

The atom of copper has 64 protons and 29 orbital electrons. These electrons will require four orbits, which will have $2,8,8,11$ electrons, respectively.

The atom of aluminum has 27 protons and 13 orbital electrons. The aluminum atom will therefore have three orbits containing $2,8,3$ electrons, respectively.

Electrons and Current Flow. Electric current has been described as electrons in motion or as a flow of electrons. The copper described above is well known as a conductor of electricity; that is, it is a substance that offers very little resistance to the flow of electric current. Its ability to


CARBON 12 Protons 6 Orbital electrons


ALUMINUM 27 Protons 13 Orbital electrons

Fig. 2-2.-Structure of carbon, copper, and aluminum atoms.
provide a good path for the movement of electrons is explained in the following manner: The electric current is a flow of free electrons set in motion by some source such as a battery. These electrons can change their position, moving from one atom on to another. Since the outer orbit of the copper atom can accommodate more electrons than are present, it is likely that the frce electrons may travel along a path of copper.

It is believed that the electrical action in a conductor carrying current involves only electrons in the outermost orbit. Figure 2-3 shows what is believed to be the path of electrons along a conductor of carbon. Because the carbon atom contains only six orbital electrons, it is easier to show a conductor of carbon than one of copper. A conductor of copper could be shown in a similar manner, but because it has a greater number of planetary electrons it would be more difficult to illustrate.

2-4. Positive and Negative Charges. According to the electron theory, all matter consists of positive charges (protons) and negative charges (elec-
trons). This comparatively new theory also corresponds to the charges produced by friction as discovered by Thales.

When a hard-rubber rod is rubbed briskly with cat's fur or flannel, bits of paper and other light objects are attracted by it. The fur or flannel also has an attraction for these bodies. When these conditions exist, we say that the rod and the wool are elcetrically charged. When


Fig. 2-3.-Path of a free electron in a carbon conductor.


Fig. 2-4.-Repulsion of two similarly charged rods.
a glass rod is rubbed briskly with silk, it too will possess the power to attract bits of paper and other light objects.

Law of Charges. If a glass rod $A$, suspended on a string, is rubbed with silk so that it becomes charged and another glass rod $B$, also charged by rubbing with silk, is brought near to the rod $A$, it will be found that the two rods will repel each other as indicated in Fig. 2-4. If two hard-rubber rods, charged by rubbing with fur, were tested in a like manner, it would be found that they too would repel each other. But if a charged glass rod
is suspended and a charged hard-rubber rod is brought near to it, the two rods will attract each other as shown in Fig. 2-5. These tests indicate two things: (1) there are two kinds of charges; (2) like charges repel each other, and unlike charges attract each other.

Positive and Negative Charges. Benjamin Franklin contributed much to electrical science, and he is credited with introducing the idea that electricity existed in two states, calling the charge on the glass rod positive and that of the rubber rod negative. The cause of these charges is explained by the electron theory in the following manner. When the hard-rubber rod is rubbed with the cat's fur, the friction between them causes some of the planetary electrons of the fur to become detached from their atoms and pass to the rubber rod. The rod has gained electrons and is therefore no


Fig. 2-5.-Attraction of two dissimilarly charged rods.
longer in a neutral state but now possesses an excess of electrons and is said to have a negative charge. The fur has lost electrons, and it too is no longer in a neutral state but now has more protons than electrons (because it has lost electrons) and is said to have a positive charge.

When the glass rod is rubbed with the silk, the friction between some of the planetary clectrons of the glass causes them to leave their atoms and transfer to the silk. The glass rod, having lost electrons to the silk, is no longer in a neutral state but now has more protons than electrons and hence is said to have a positive charge. The silk now has an excess of electrons and is a negatively charged body.

2-5. Charging and Discharging. Charging by Contact. A charged body, such as a rubber rod, can transmit some of its charge to a neutral
body in either of two ways, namely, by contact or by induction. If the rubber rod (negatively charged body) is placed so that it makes contact with the neutral body as shown in Fig. 2-6b, some clectrons will pass from the rod to the body and thereby charge it. Figure $2-6 a$ shows the body with an equal number of protons and electrons, or in a neutral state. In Fig. 2-6b the negatively charged body is in contact with the neutral body, and electrons pass from the charged body to the neutral body. When the negatively charged rod is removed (Fig. 2-6c), the body that had been


Fig. 2-6.-Charging by contact.
neutral now has an excess of electrons and is therefore negatively charged. If a glass rod (positively charged) had been used, the neutral body would have lost some electrons to the glass rod and would, therefore, have become positively charged. In either case the neutral body becomes charged by contact, and it takes on a charge of the same polarity as the charged body.

Charging by Induction. Figure 2-7a shows a neutral body that is freely suspended. If a negatively charged body is brought near, though not in actual contact with it as shown in Fig. 2-7b. the negative charge will pro-
duce a repelling action on the electrons of the free body. If the free body is connected with the ground as shown in Fig. 2-7b, then the repelling action would send some of the electrons from the free body to the ground. If the charged rod and the ground are removed as shown in Fig. 2-7c, the free body will now have more protons than electrons and hence will be positively charged. If a charged glass rod had been used, the free body would have taken on electrons (from the ground) and would, therefore, have become negatively charged. In either case no electrons pass between the free body


Fig. 2-7.-Charging by induction.
and the charged body, and the free body is said to be charged by induction. It should be noticed that in this case the polarity of the charge on the free body is opposite to that of the charging body.

Discharging. The rate at which a charge passes from a body is dependent largely upon the shape of the body. If it is pointed, the charge will pass off very rapidly because the electrons become concentrated in a small area and hence build up a considerable pressure. If the body from which a discharge takes place has a large area, such as a sphere or ball,
the electrons are distributed over a large area and the pressure causing the discharge is very small. Therefore, if it is desired to build up a large charge, it is necessary to have a large area; this is actually done in building capacitors (discussed in Chap. IX).

Lightning. Lightning is the discharge that occurs between clouds of unlike charges or between a cloud and the earth. Benjamin Iranklin proved with his famous kite-and-key experiment that lightning and electricity were really the same thing. Man has long wondered about lightning and sought to understand its cause. It is now believed that during the uprush of warm moist air from the earth the friction between the air and the tiny particles of water causes the building up of charges. When drops of water are formed, the larger drops become positively charged and the smaller drops become negatively charged. When the drops of water accumulate, they form clouds, and hence clouds may possess either a positive or negative charge, depending upon the charge of the drops of water they contain. The charge on a cloud may become so great that it will discharge to another cloud or to the earth, and we call this discharge lightning. The thunder which accompanies lightning is caused by the lightning suddenly heating the air, thereby causing it to expand. The surrounding air pushes the expanded air back and forth causing the wave motion of the air which we recognize as thunder.

Lightning Rods. The purpose of the lightning rod is to offer protection from lightning discharges by discharging small electrical charges as rapidly as they accumulate. Because pointed objects are relieved of charges more rapidly than any other shaped bodies, the lightning rod is made with a point.

In Fig. 2-8, the cloud has a positive charge, and it induces a negative charge on the lightning rods on the house. The molecules surrounding the points of the rods become negatively charged (by induction) and repel one another as they accumulate. Also, the cloud, being of opposite charge, attracts some of these charges. Each charged molecule that reaches the cloud neutralizes some of the charge on the cloud. Usually this action completely discharges the cloud. However, if the cloud has a very strong charge, it will attract electrons with a terrific force. These electrons are drawn up from the earth through the lightning rod, which is intentionally made of a good conductor to provide a safe path. If the house did not have the lightning rod, the electrons would rush through the material of which the house is made. If the house were made of wood (a poor conductor), the rush of electrons would cause much heat and would quite likely set the house afire.

It is evident that lightning rods do not prevent lightning but rather prevent charges from accumulating on the buildings to which they are
attached. The lower end of the rod should be attached to a conductor embedded deep enough in the ground so that it is always in moist earth. If it is not properly grounded, the lightning rod will prove more of a menace than a protection because it will invite cloud discharges.

Single tall objects in open spaces such as a farmhouse or a single tree in a large field will accumulate charges rapidly and are therefore very likely to be struck by lightning. Farm buildings, church steeples, high brick chimneys, etc., are frequently protected by lightning rods. Steel buildings in a city have numerous metal parts that extend into the ground and therefore have a good deal of protection.


Fig. 2-8.-Application of the lightning rod.
2-6. Electrostatic Fields. Electrostatic Field. Figure 2-4 indicates that, when the positively charged glass rod $B$ is brought close to the positively charged glass rod $A$, they will repel each other and cause the freely suspended $\operatorname{rod} A$ to move away from $B$. The fact that this action can take place without the rods making actual contact indicates that the air about the rods has been affected by the charges. The area about a charged body that is influenced by that charge is called its electrostatic field. As a force of repulsion (or attraction in the case of unlike charges) exists in this area, it is also known as the field of force.

Electrostatic Lines of Force. The field about a charged body is generally represented by lines which are referred to as electrostatic lines of force.

These lines are only imaginary lines and are used merely to represent the direction and strength of the field. To avoid confusion, the lines about a positive charge are always shown leaving the charge, and for a negative they are shown as entering the charge. Figure 2-9 illustrates the use of lines to represent the field about a positive charge and a negative charge,


Fig. 2-9.-Electrostatic fields: (a) positive charge, (b) negative charge.

(b)

Fig. 2-10.-Electrostatic fields: (a) like charges, (b) unlike charges.
while Fig. 2-10 illustrates the electrostatic field due to like charges and to unlike charges.

Force between Charges. Figure 2-11 shows two negative charges placed one centimeter apart. The charges repel each other with a force of one dyne. This illustrates the unit of electrostatic charge, which, by defini-
tion, is a charge that repels another equal and like charge one centimeter away with a force of one dyne. The electrostatic unit is generally abbreviated esu.

Coulomb proved (about the year 1785) that the force between two charges is directly proportional to the product of the charges and inversely proportional to the square of the distance between the charges. This is expressed mathematically as

$$
\begin{equation*}
F=\frac{q_{1} q_{2}}{k d^{2}} \tag{2-1}
\end{equation*}
$$

where $F=$ force, dynes
$q_{1}=$ strength of charge 1 , esu
$q_{2}=$ strength of charge 2 , esu
$d=$ distance between charges, centimeters
$k=$ dielectric constant of the medium through which the force is exerted.
Note. $k$ for vacuum and air is 1 ; for other materials see Appendix VI.


Fig. 2-11.-Unit electrostatic charge.
Example 2-1. What force is exerted between two negative charges of 10 and 20 electrostatic units, respectively, when placed two centimeters apart in air?

$$
\begin{array}{lr}
\text { Given: } & \text { Find: } \\
q_{1}=10 & F= \\
q_{2}=20 & \\
k=1 & \\
d=2 &
\end{array}
$$

Solution:

$$
\begin{aligned}
F & =\frac{q_{1} q_{2}}{k d^{2}} \\
& =\frac{10 \times 20}{1 \times 2 \times 2} \\
& =50 \text { dynes (repulsion) }
\end{aligned}
$$

Example 2-2. What force is exerted between two charges, one negative and one positive, each with a strength of 50 electrostatic units? They are placed in air four centimeters apart.

$$
\begin{aligned}
& \text { Given: } \\
& q_{1}=50 \\
& q_{2}=50 \\
& k=1 \\
& d=4
\end{aligned}
$$

Solution:

$$
\begin{aligned}
F & =\frac{q_{1} q_{2}}{k d^{2}} \\
& =\frac{50 \times 50}{1 \times 4 \times 4} \\
& =156 \text { dynes (attraction) }
\end{aligned}
$$

2-7. Introduction to Dynamic or Current Electricity. Potential. Figure 2-12 shows two metal spheres of unequal size, each mounted on an insulated stand. It is assumed that each is charged with a similar amount of negative electricity. The charges on each


Fig. 2-12.-Spheres of unequal size used to demonstrate the concentration of charges. sphere will be evenly distributed over its own surface because of the mutual repulsion between the individual like charges. Since $A$ is smaller, yet has the same amount of charge, the repulsion on sphere $A$ is greater than on sphere $B$ because the distance between charges is smaller [see Eq. (2-1)]. There is more force tending to rid sphere $A$ of its charge than tending to rid $B$. We say that the pressure or potential of sphere $A$ is greater than that of sphere $B$. If the two spheres are connected by a wire, electrons will flow from $A$ to $B$ and will flow until the potentials of the two bodies are the same. The flow of electrons in the wire is called an electric current.

Difference of Potential. In current electricity the difference of potential is of great importance, for in order to maintain a flow of electrons there must be a continual difference of potential between the two points in which the electron flow is desired.

Potential is measured relative to that of the earth, the potential of the earth being considered as zero. Notice that this is only a level of potential. Positively charged bodies have a higher potential than that of the earth and negatively charged bodies lower than that of the earth. The unit of potential is the volt.

Current Flow. Current flow has already been described as a movement of electrons. Such a movement occurs whenever a wire (conductor) is connected between two points of different potential. If one end of a wire is connected to a negative potential and the other end is connected to the
positive, electrons will flow from the negative to the positive. If both ends of the wire are connected to positive potential but of different levels, the electrons will flow from the lower positive potential to the higher. If both ends of the wire are connected to negative potential but of different levels, the electrons will flow from the higher negative potential to the lower.

According to the electron theory, when an electric current flows through a circuit, electrons are continually flowing and no matter how great the flow or how long it is maintained, no part of the circuit ever gains any weight. It is evident then that electricity is never made, but, rather, that part of the amount already existing is set into motion, thus producing an electron flow.

2-8. Methods of Producing an Electric Current. It has been shown that in order to obtain an electron flow it is necessary to establish and maintain a difference of potential between the two points in which the electron flow is desired. There are three methods that may be used to set up a potential difference, namely, chemical, thermal, and magnctic.

Chemical Method. In this method, two dissimilar metals, such as copper and zinc, called elements, are immersed in a salt solution called the electrolyte. When the elements are connected at their terminals by a wire or any other form of conductor, a chemical action takes place, causing an electric current to flow through the conductor. A unit such as this is called a cell, and a combination of two or more cells properly connected is known as a battery. Unless the battery is very large, the amount of electricity that can be obtained by this method is very small. Because of this cumbersome feature and also because of the initial high cost and expensive upkeep, batteries are used only (1) where a small electric current is needed, (2) where a portable source of electricity is required, (3) for a reserve supply in case of an emergency. The first use is illustrated by the dry cells employed in our homes for ringing doorbells; the second use, by the cells in flashlights, portable radios, and the storage batteries in automobiles; and the third use, by the batteries kept in hospitals to supply light and power during the interval when the power lines fail and the hospital's own generating system is started.

Thermal Method. An electric current can be produced by heating two dissimilar metals at their junction point as illustrated in lig. 2-13. This is called a thermocouple. The amount of electricity that can be obtained by this method will depend upon the heat applied and the metals used, different metals giving different amounts of electricity. Practical combinations are made from antimony and bismuth, German silver and copper sulphide, copper and constantan, iron and constantan. The amount of electricity produced at any one junction point is very small, and therefore
its uses are limited. Three important uses of a thermocouple are (1) to measure the heat inside large furnaces or wherever a thermometer is not practical; (2) to measure high-frequency currents; (3) as a protective device. This thermoelectric effect sometimes causes trouble in radio sets when a soldered joint composed of copper, tin, and lead is heated by a nearby tube, resistor, or transformer. The voltage produced is very small. If amplified, it may make disturbing noises in the loudspeaker. In public-address systems or wherever high amplification is used, both the stationary and movable contacts of all adjustable elements must be made of the same


Fig. 2-13.-Three methods of obtaining an electric current: (a) chemical, (b) thermal, (c) magnetic.
material; otherwise the voltage introduced may cause noises to be produced in the loudspeaker.

Magnetic Method. Electricity may be produced in a conductor whenever it is cut by or is itself cutting through lines of force coming from a magnet or from another conductor. The act of forcing the electrons to move in a conductor by either of these means is called electromagnetic induction. The amount of electricity obtained will depend upon the number of magnetic lines to be cut, the number of conductors cutting them, and the speed with which it is done. This is the basic principle of the electric generator and the alternator, the former producing a direct current and the
latter an alternating current. Its flexibility as to the amount of electricity obtained, its ease of production, and its low cost make it the most practical method of obtaining electricity. Because of this it is the means by which practically all the electricity used in the world is obtained.

To rotate the conductors so as to cut the magnetic lines, we may use a water wheel, steam turbine, gas engine, or Diesel engine. Which one of these methods is used depends upon the locality and the amount of electricity required.


Fig. 2-14.-Modern applications of the three methods of producing an electric current: (a) chernical, (b) thermal, (c) magnetic.

2-9. Effects of Electric Current. When an electric current flows through a conductor, it produces a number of effects, the three most important being (1) thermal, (2) chemical, (3) magnetic. The operation of all electrical appliances, devices, instruments, or machinery is based on one or more of these effects.

Thermal Effect. Any conductor through which an electric current flows will become heated owing to the fact that energy is used in forcing the current through the resistance offered by the conductor. The rise in temperature may be small or large, according to the current flowing and the resistance offered to the flow, but some heat is always produced. Because of this effect we are able to have the various electrical heating devices such as
soldering irons, baking ovens, irons, toasters, and heating pads. The current flowing through an electric lamp heats the filament to incandescence causing the lamp to give off light. Vacuum tubes use this effect as their basic principle of operation, as any material will emit electrons when heated. This heat is obtained by haviing an electric current flow through a wire inside the tube.

Chemical Effect. Chemically, an clectric current is capable of decomposing water, that is, breaking it up into its elements of hydrogen and oxygen. This effect, the decomposition of a chemical compound by an electric current, is called electrolysis and is used in all applications of the chemical effect of electron flow. Battery cells, electroplating, electrotyping, and therapeutics are a few illustrations of the many applications of electrolysis.


Fig. 2-15. -Uses of each of the three effects of electric current: (a) chemical, (b) thermal, (c) magnetic.

Magnetic Effect. An electric current flowing in a conductor causes it to be surrounded by a magnetic field consisting of lines of force encireling the conductor at all points. This effect is the basis of operation of all motors, generators, induction coils, and transformers; in fact, practically all electrical machinery and many electrical devices use this effect.

2-10. Kinds of Electric Current. All electrical currents are essentially the same in nature, but they may vary in their method of flow, direction, current strength, or a combination of these. There are six different kinds of electric currents: (1) continuous, (2) pulsating, (3) direct, (4) alternating, (5) oscillatory, (6) interrupted.

Continuous Current. A continuous current is one in which the direction and amount of current flow does not vary with time. Referring to Fig. $2-16$, it can be seen that the direction of flow does not change and that the
intensity of the current is constant at all times as $I_{1}=I_{2}=I_{3}$, etc. Continuous currents are obtained from battery cells.

Pulsating Current. The direction of flow of a pulsating current is constant, but its strength rises and falls at fixed intervals. In Fig. 2-16, $t_{1}=t_{\underline{2}}=t_{3}$, etc. Pulsating currents are obtained from any rectifier of alternating currents. The difference between the maximum and minimum values will depend on the rectifier and its filter circuit; the better the rectifier and its filter circuit, the smaller this difference becomes and the nearer the pulsating current approaches being a continuous current. There are numerous types of electric current of various wave forms used in radio and television that are variations of pulsating current.

Direct Current. A direct current is a pulsating current whose flow varies so little that it is almost equivalent to a continuous current. Because the current flows in only one direction, it is a unidirectional current. This has been abbreviated by common usage to simply direct current. Direct-current generators actually generate an alternating current, buti the commutator rectifies it to a direct current. Increasing the number of commutator segments will reduce the difference between the value of the maximum and minimum currents.

Alternating Current. An alternating current reverses its direction of flow at fixed intervals. During each interval the current rises from zero to maximum, then diminishes from maximum to zero. Referring to Fig. $2-16$, it can be seen that (1) the current flows in a positive direction during the intervals $t_{1}, t_{3}$, etc., and in a negative direction during the intervals $t_{2}, t_{4}$, etc.; (2) the intervals are all equal as $t_{1}=t_{2}=t_{3}$, etc.; (3) the maximum current in the positive dircetion is equal to the maximum current in the negative direction. Alternating currents are produced by a-c generators commonly called alternators. Because an alternating current can be readily transformed from low to high voltages, and vice versa, it is possible to send large amounts of power at a low current through conductors having a comparatively small diameter. It is, therefore, cheaper to transmit an alternating current over great distances than a direct current, and for this reason it is the type of current usually found in homes, offices, and any other building.

Oscillatory Current. An oscillatory current is one which reverses its direction at fixed intervals and decreases in magnitude with each reversal until all current flow ceases. An example of an oscillatory current occurs when a capacitor is discharged through an inductor and a resistor. The energy stored in the capacitor will be transferred to the inductor, which will then in turn return its energy to the capacitor. This process repeats itself, but during each transfer some of the energy is dissipated (given off as heat) at the resistor. The time required for the capacitor to discharge
completely and the maximum value of the current will depend on various factors in the circuit, such as the voltage applied and the size of the capacitor, inductor, and resistor.

Interrupted Current. An interrupted current is one in which the circuit is made and broken at fixed intervals. It may be an alternating, direct, pulsating, or continuous current. Interrupted currents can be obtained


Fig. 2-16.-Six kinds of electric current.
by either mechanical or electrical means. An example of the mechanical means of interrupting a current is the rotating drum which is used to break an electric circuit at definite intervals. The rotating-drum method may be used to control the lights on the various parts of an electric flashing sign. By this means it is possible to control the number of lights in various circuits and also the intervals at which these lights are turned on and off.

Interrupted currents can be obtained electrically by several methods. One method, used in some types of portable radio receivers and electronic devices, employs a vibrating coil in its power-supply circuit to increase the comparatively low battery voltage to a higher voltage required by the radio receiver or electronic device. Another method of interrupting a current by electronic means is to apply (at the desired intervals of time)


Fig. 2-17.-Complex waves.
an amount of voltage to the control grid of a vacuum tube sufficient to interrupt the flow of current in its plate circuit.

Complex Currents. Combinations of two or more of the various fundamental types of current described in the preceding paragraphs will result in a current of complex wave form. Currents having a complex wave form will be encountered in radio and television circuits, especially in television receivers. Figure 2-17 illustrates a number of wave forms that are either variations of the fundamental waves or are complex waves.

2-11. Ohm's Law. The flow of electrons through a circuit resembles in many ways the flow of water through pipes. Therefore, the action of an
electric current can be understood by comparing its flow with the flow of water.

Hydraulic Analogy of Current Flow. In hydraulics, which is the scientific study of the flow of water in pipes, there are three fundamental considerations, namely, (1) the cause of the flow, (2) the amount of flow in a given time, or rate of flow, (3) the factors regulating the flow.

The rate of flow is controlled by hydraulic pressure obtained by means of pumps. If the pressure is increased, the amount of water flowing will be increased; if the pressure is decreased, the rate of water flow will decrease; and if there is no pressure, the water will stand still. The rate of water flow is, therefore, directly proportional to the pressure causing it to flow. If the resistance (friction, bends, elbows) is increased, the flow of water will be decreased, and if the resistance is decreased, the flow will increase. The rate of water flow is, therefore, inversely proportional to the resistance of the pipes through which it must flow.

The foregoing conclusions can be expressed by the simple mathematical equation

$$
\text { Rate of water flow }=\frac{\text { hydraulic pressure }}{\text { resistance of the pipes }}
$$

In the study of electricity there are three fundamental terms that compare with the three factors in hydraulics:

Electric pressure is called voltage.
Rate of electron flow is called current.
Electric resistance is called resistance.
Substituting the comparative electrical terms in the above equation,

$$
\begin{equation*}
\text { Current }=\frac{\text { voltage }}{\text { resistance }} \tag{2-2}
\end{equation*}
$$

Ohm's Law. The mathematical relation between voltage, current, and resistance was discovered by Georg Simon Ohm and is therefore called Ohm's law. Ohm's law is the foundation upon which the study of all branches of electricity is based. It is essential that this law be fully understood before a study of any one of the branches of electricity is attempted. Ohm's law may be stated in three forms, the first as in Eq. (2-2) and the others as in Eqs. (2-3) and (2-4).

Equation (2-2) can be used to find the current that will flow in a circuit when the resistance and voltage are known. This equation can be transposed by algebra so that it will be possible to solve for the voltage when the current and resistance are known. This may be done by multiplying
both sides of Eq. (2-2) by resistance and then simplifying the equation, which will produce the new equation

$$
\begin{equation*}
\text { Voltage }=\text { current } \times \text { resistance } \tag{2-3}
\end{equation*}
$$

An equation to solve for resistance when the voltage and current are known can be obtained by dividing both sides of Eq. (2-3) by current and then simplifying the equation; this will produce the equation

$$
\begin{equation*}
\text { Resistance }=\frac{\text { voltage }}{\text { current }} \tag{2-4}
\end{equation*}
$$

A study of these three equations will lead to the following conclusions:

1. From Eq. (2-2) it can be seen that the current flowing in a circuit having a fixed amount of resistance will increase with an increase in the voltage and decrease with a decrease in voltage. Also, the current flowing in a circuit with a fixed amount of voltage will decrease with an increase in resistance and increase with a decrease in resistance. This may be stated more briefly: the current flowing in a circuit varies directly with the voltage and inversely with the resistance.
2. From Eq. (2-3) it can be seen that the voltage at any circuit or circuit element is directly proportional to the current and resistance of the circuit or circuit element.
3. From Eq. (2-4) it can be seen that the resistance requirements of a circuit or circuit element vary directly with the voltage and inversely with the current.

2-12. Fundamental Electrical Units. In every scientific field there are definite and precise units of measurement. In electrical work the basic units are the volt, ampere, and ohm. In practical work the numerical values of voltage and current of a circuit are obtained by use of a voltmeter and an ammeter. The resistance of a circuit may be obtained by the calculations of Ohm's law when the current and voltage are known, or it may be measured directly with a bridge or with an ohmmeter as is common in radio and television work. For voltmeters, ammeters, and ohmmeters to provide accurate indications of voltage, current, and ohms, they are calibrated to conform to standard meters of these quantities. Standard instruments are maintained by meter manufacturers and precision measuring laboratories, who in turn periodically have their instruments checked with United States standards at the National Bureau of Standards at Washington.

Standard Units. So that the volt, ampere, and ohm might represent the same value all over the world, standards were set up by an international commission in 1881 and made legal by the governments of all civilized

| Hydraclic Analogy |  |  | Electrical Terms and Units |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Term | Units of measure | How controlled | Term | Units of measure | How controlled | How measured | Symbols and abbreviations | Ohm's law relation |
| Pressure | Weight per unit area, lb/sq in., grams/sq cm | Pump | Electromotive force | Volt | D-C: batteries, generator A-C: alternator, transformer | Voltmeter | $\begin{aligned} & E=\text { impressed } \\ & \text { voltage } \\ & e=\text { voltage drop } \end{aligned}$ | $\begin{aligned} & \text { Current } \times \text { re- } \\ & \text { sistance } \\ & I \times R \end{aligned}$ |
| Water | Quantity per unit time, gal/ min, cu ft/ sec | Valves | Current | Ampere | Rheostat, switch | Ammeter | $\begin{aligned} i & =\text { branch current }, \\ I & =\text { line current } \end{aligned}$ | Voltage Resistance $\frac{E}{R}$ |
| Resistance | $k$, coefficient of friction | Turns, bends, friction | Resistance | Ohm | Length, cross-section area, material | Ohmmeter, bridge | $\begin{aligned} & r=\text { branch } \\ & \text { resistance } \\ & R=\text { line resistance } \\ & \Omega \text { or } \omega=\text { ohm } \end{aligned}$ | Voltage <br> Cuirent <br> E <br> $I$ |
| Power | Horsepower-foot-pounds | Pressure or amount of water | Power | Horsepower, watt, kilowatt | Voltage or current | Wattmeter | ```P= power W= watt kw = kilowatt kva = kilovolt- ampere``` | $\begin{aligned} & E \times I \\ & I^{2} \times R \\ & \frac{E^{2}}{R} \end{aligned}$ |
| Energy | Horsepower-hour | Power, time | Energy | Kilowatt-hour, watt-second | Power, time | Kilowatt-hour meter | ```\(W=\) energy \(\mathrm{kwh}=\) kilowatt- hour ws \(=\) watt-second``` | $\begin{aligned} & \text { Power } \times \text { time } \\ & P \times T \end{aligned}$ |

Fig. 2-18.-Comparison of electrical and hydraulic terms and units.
countries. In 1893 the following standards were adopted by the International Flectrical Congress at Chicago: The international ampere was defined as the amount of current which, when passed through a specified standard solution of nitrate of silver and water, deposits silver at the rate of 0.001118 gram per second. The international ohm was defined as the resistance of a column of mercury of uniform cross section having a length of 106.3 centimeters and a mass of 14.4521 grams at $0^{\circ}$ centigrade. The international volt was defined as the output of the Clark standard cell. In 1912, the international volt was redefined as the output of the Weston normal cell and was later changed to $1 / 1.0183$ of the voltage of the Weston normal cell.

In 1948 the units of volts, amperes, and ohms in the United States were changed from the international units to absolute units. The absolute units are based on electromagnetic principles instead of the rate of depositing silver for current measurements and the resistance of a column of mercury for the standard ohm. The new standards are now expressed in the centimeter-gram-second system of measurements, which is desirable because it is the system used in the fundamental mechanical units of length, mass, and time.

The old international units can be converted to the new absolute units by multiplying currents by 1.000495 and voltages by 1.00033 . These conversion factors indicate that the variation between the two systems is very small, in fact so small that they have no effect on ordinary measurements made in practical work on electrical, radio, and television circuits.

The Ampere in T'erms of the Electron Theory. In an electrical conductor there are some electrons which are rather loosely bound to the atoms and are able to travel more or less freely from one atom to another; these are called free electrons. Under ordinary conditions the free electrons do not travel in any particular direction, but when a conductor is connected to the terminals of a source of electrical energy, such as a battery or a generator, there will be a movement of free electrons from the negative terminal toward the positive terminal of the power source. If the voltage of the power source is increased, the number of electrons moving from the negative toward the positive terminals increases.

The ampere is used to express the rate of flow of electrons, but as the electron represents a very small quantity it is necessary to have 6,280 ,$000,000,000,000,000$ ( 6.28 quintillion) electrons flowing past a point to constitute one ampere. Because such a large number is difficult to use, it is replaced by the coulomb, which represents a charge of $6.28 \times 10^{8}$ (see Art. 4-14) electrons. Thus the ampere, named after André Marie Ampere, is the quantity of electricity equivalent to one coulomb passing a given point per second.

The Volt. It has been shown that the free electrons in a conductor can be made to flow in a definite direction by applying a positive-charge terminal to one end of a conductor and a negative-charge terminal to the other end. These charges exert a force on the electrons and cause them to move in the direction from negative toward positive. An electric force therefore exists between any two bodies having different polarities of charge and is called the electromotive force, which is generally abbreviated as emf or EMF. The practical unit of emf is the volt, named in honor of Alessandro Volta, and is equivalent to the electric pressure required to force one ampere through a resistance whose value is one ohm. Through common usage, the cmf of various sources of electric energy is often referred to as voltage rather than electromotive force. Various amounts of voltage can be doveloped by any one of the three methods of producing electron flow. These devices produce a continuous difference of potential between their terminals.

The Ohm. Certain materials allow the free electrons to travel through them more easily than do other materials. These materials offer a comparatively small amount of resistance to the flow of electron current and are called conductors. The unit of electrical resistance is the ohm, named in honor of Georg Simon Ohm, and is equal to the amount of opposition offered by a conductor to the flow of one ampere of current when a pressure of one volt is applied across its terminals.

Practical Units. The practical units used in electricity, radio, television, and electronics have been adopted from the standards described above. These units are: (1) the ampere, for current flow; (2) the volt, for electrical pressure; (3) the ohm, for the resistance offered to the flow of current.

The mathematical relation between these terms is expressed by Ohm's law and has been stated as Eqs. (2-2), (2-3), and (2-4). It is common practice to express Ohm's law by using symbols for the words ampere, volt, resistance (see Art. 2-15). The symbol for ampere is $I$, for volt $E$, and for resistance $R$. Equations (2-2), (2-3), and (2-4) then become

$$
\begin{align*}
\text { Amperes } & =\frac{\text { volts }}{\text { ohms }} & \text { or } & I
\end{aligned}=\frac{E}{R}, ~ \begin{aligned}
\text { Volts } & =\text { amperes } \times \text { ohms } & \text { or } & E \tag{2-5}
\end{align*}=I R,
$$

Example 2-3. How much current flows through an electric circuit that has a resistance of 20 ohms and is connected to a power supply whose pressure is 110 volts?

Given:
$R=20$ $E=110$

Find:
$I=$ ?

Solution:

$$
\begin{aligned}
I & =\frac{E}{R} \\
& =\frac{110}{20} \\
& =5.5 \mathrm{amp}
\end{aligned}
$$

Example 2-4. What is the resistance of a circuit that has a current of 2.5 amperes flowing through it when the electrical pressure is 125 volts?
Given:

$$
\begin{aligned}
& I=2.5 \\
& E=125
\end{aligned}
$$

Find:
$R=$ ?

Solution:

$$
\begin{aligned}
R & =\frac{E}{I} \\
& =\frac{125}{2.5} \\
& =50 \text { olms }
\end{aligned}
$$

Example 2-5. What pressure is required to cause 1.75 amperes to flow through a circuit whose resistance is 60 ohms?

| Given: | Find: |
| :--- | ---: |
| $I=1.75$ | $E=?$ |
| $R=60$ |  |

Solution:

$$
\begin{aligned}
E & =I R \\
& =1.75 \times 60 \\
& =105 \text { volts }
\end{aligned}
$$

2-13. Work, Power, Energy. Work. Work is the accomplishment of motion against the action of a force tending to resist it. Work may be (1) useful, as drilling holes in a large sheet of steel; (2) destructive, as tearing one's clothes; (3) pleasant, as hitting a baseball for a home run; (4) accidental, as falling down a flight of stairs. In all cases, work is accomplished by the action of some force through a distance. Therefore, the work done upon any body would be equal to the product of the force exerted upon it and the distance through which it acts. Expressed mathematically,

$$
\begin{equation*}
\text { Work }=\text { force } \times \text { distance } \tag{2-8}
\end{equation*}
$$

If the distance is expressed in feet and the force in pounds, then the work done will be expressed in foot-pounds.

Time is not a factor in the consideration of work done. For example, one person, being fairly strong, could move a 100 -pound bag of sand across
a room 10 feet long in one trip taking one minute, and another person, not being so strong, might have to do it in 10 trips, carrying 10 pounds in each trip and taking one minute per trip. In both cases the work accomplished would be the same.

$$
\text { Work done first case }=100 \mathrm{lb} \times 10 \mathrm{ft}=1000 \mathrm{ft}-\mathrm{lb}
$$

Work done second case $=10 \mathrm{lb} \times 10 \mathrm{ft} \times 10$ trips $=1000 \mathrm{ft}-\mathrm{lb}$
Power. Power is the rate of doing work per unit of time. Therefore, it may be said that the first person possessed more power than the second because he did the job at the rate of 1000 foot-pounds per minute, while the second one worked at the rate of 100 foot-pounds per minute to do the same job. Expressed mathematically,

$$
\begin{equation*}
\text { Power }=\frac{\text { work done }}{\text { time }} \tag{2-9}
\end{equation*}
$$

It has been established that the average work horse could work at the rate of 33,000 foot-pounds per minute, or 550 foot-pounds per second. Thus the horsepower is a larger unit of power and is equal to

$$
\begin{align*}
& \text { Horsepower }=\frac{\text { foot-pounds of work per minute }}{33,000}  \tag{2-10}\\
& \text { Horsepower }=\frac{\text { foot-pounds of work per sccond }}{550} \tag{2-11}
\end{align*}
$$

The work done by the flow of an electric current may be the illumination of a room, the running of a motor for almost any kind of mechanical work, the operation of a radio receiver or transmitter, etc. The unit of electrical power is the watt, and it is equivalent to the work done in one second by a steady current flow of one ampere flowing under a pressure of one volt. Mathematically this is expressed as

$$
1 \text { watt }=1 \text { volt } \times 1 \text { ampere }
$$

or

$$
\begin{equation*}
\text { Watts }=\text { volts } \times \text { amperes } \quad P=E \times I \tag{2-12}
\end{equation*}
$$

If $E / R$, which is the equivalent of $I$ as indicated in Eq. (2-5), is substituted for $I$ in Eq. (2-12), it becomes $P=E \times E / R$, or

$$
\begin{equation*}
\text { Watts }=\frac{(\text { volts })^{2}}{\text { ohms }} \quad P=\frac{E^{2}}{R} \tag{2-13}
\end{equation*}
$$

Also, if $I R$, which is the equivalent of $E$ as indicated in Eq. (2-6), is substituted for $E$ in Eq. (2-12), it becomes $P=I R \times I$, or

$$
\begin{equation*}
\text { Watts }=(\text { amperes })^{2} \times \text { ohms } \quad P=I^{2} R \tag{2-14}
\end{equation*}
$$

The above equations provide three means of calculating the power of a circuit. The equation to be used is best determined by first identifying the known quantities and then choosing the equation which best fits the conditions. A careful examination of the above equations will show that an equation has been provided for each combination of two known quantities of the Ohm's law group, namely, $E, I$, and $R$.

Example 2-6. What power is consumed by a circuit that has a current of 2.5 amperes flowing when connected to a source whose pressure is 120 volts?

| Given: | Find: |
| :--- | ---: |
| $I=2.5$ | $P=?$ |
| $E=120$ |  |

Solution:

$$
\begin{aligned}
P & =E \times I \\
& =120 \times 2.5 \\
& =300 \text { watts }
\end{aligned}
$$

Example 2-7. What power is consumed by a circuit whose resistance is 80 ohms and which has a current of 1.5 amperes flowing?

$$
\begin{array}{lr}
\text { Given: } & \text { Find: } \\
R=80 & P=? \\
I=1.5 &
\end{array}
$$

Solution:

$$
\begin{aligned}
P & =I^{2} R \\
& =1.5 \times 1.5 \times 80 \\
& =180 \mathrm{watts}
\end{aligned}
$$

Example 2-8. What power is consumed by a circuit whose resistance is $\mathbf{7 5}$ ohms and which is connected to a source whose pressure is 15 volts?

$$
\begin{array}{cr}
\text { Given: } & \text { Find: } \\
R=75 & P=? \\
E=15 &
\end{array}
$$

Solution:

$$
\begin{aligned}
P & =\frac{E^{2}}{R} \\
& =\frac{15 \times 15}{75} \\
& =3 \mathrm{watts}
\end{aligned}
$$

Energy. Energy is the capacity to do work. The units for energy are, therefore, the same as those for work. The amount of energy in the universe is always constant, for, according to the law of conservation of energy, it can neither be created nor destroyed but is merely changed from one form to another. A battery changes chemical energy into electrical
energy, and a generator changes mechanical energy into electrical energy. The energy of an electric current will be the energy of the moving electrons.

If a 100 -watt lamp was placed in a room and no current flowed through its filament, the lamp would radiate no light. No work would be done, and therefore no energy would be used. If an clectric current was caused to flow through its filament for one hour, the lamp would radiate light for one hour and would perform 100 watt-hours of work, thereby using 100 watt-hours of electrical energy. Expressed mathematically,

$$
\begin{equation*}
\text { Encrgy (watt-hours) }=\text { power }(\text { watts }) \times \text { time (hours) } \tag{2-15}
\end{equation*}
$$

$\mathbf{2 - 1 4}$. Other Units. The units of electricity such as the volt, ampere, ohm, and watt are basic quantities for the smaller and larger units that are necessary to use from time to time. The names of these units are formed by using the base name and a prefix to designate the quantity.

The prefix milli-means one-thousandth $\left(\frac{1}{1000}\right)$ and micro-, one-millionth $\left(-\frac{1}{1000000}\right)$. Both these prefixes are used to designate the small amounts of voltage and current in electrical instruments and various radio, television, and electronic circuits.

$$
\begin{aligned}
1 \text { millivolt } & =\frac{1}{1000} \text { of a volt } \\
1 \text { milliampere } & =\frac{1}{1000} \text { of an ampere } \\
1 \text { microvolt } & =\frac{1}{1,000,000} \text { of a volt } \\
1 \text { microampere } & =\frac{1}{1,000,000} \text { of an ampere }
\end{aligned}
$$

The prefix kilo- means one thousand (1000) and is used to designate large amounts of voltage, power, and energy.

$$
\begin{aligned}
1 \text { kilovolt } & =1000 \text { volts } \\
1 \text { kilowatt } & =1000 \text { watts } \\
1 \text { kilowatt-hour } & =1000 \text { watt-hours }
\end{aligned}
$$

Another unit of electrical power is the horsepower:

$$
1 \text { horsepower }=746 \text { watts }
$$

The prefix meg-means one million ( $1,000,000$ ) and is used to designate high resistances such as the grid leak and insulation resistances.

$$
1 \text { megohm }=1,000,000 \mathrm{ohms}
$$

2-15. Symbols and Abbreviations. Symbols. Symbols may be defined as abbreviations for any item or classification and may consist of letters, figures, emblems, or signs. The purpose of symbols is (1) to indicate definitely a particular item to the exclusion of all others and (2) to obtain brevity in the writing of equations, specifications, reports, drawings, etc. Symbols promote exactness, because each symbol is definitely associated with only one item. A description of an article or an electrical circuit may be vague, even though the words are used in the true sense of their dictionary definitions, for the reason that all persons do not ascribe the same meaning to words. It is easier to write $\$ 123.45$ than "one hundred


Fig. 2-19.-Examples of symbols used in everyday life.
and twenty-three dollars and forty-five cents," and it would be quite a problem to add 20 or more different amounts of money if each amount had to be written out instead of using symbols.

Symbols used for the diagrammatic representation of electrical, radio, television, and electronic apparatus have not all been standardized, and various modifications will be found. Those listed in Appendix I are the forms easily understood and easily made and are therefore the ones most commonly used. There is likewise no definite standard of size, and those listed are drawn to a scale used in general practice. At times, it may be necessary to enlarge or reduce the size of the symbols listed. When this is done, care must be taken so that all the symbols used are enlarged or reduced in the same proportion.

Abbreviations. The terms voltage, current, resistance, power, etc., are very seldom written out in the solution of problems or in an explanation
of any electronic or radio circuit. It is common practice to use the following abbreviations for the few terms that have been mentioned. A complete list will be found in Appendix II. In general, capital letters are used to denote the total current, resistance, power, etc., in a circuit and small letters to denote the resistance, current, power, etc., in any branch or part of a circuit.

| $E$ | The first letter of the abbreviation for electromotive force, <br> used to designate the voltage applied to any circuit from a <br> power supply. The subscript method of notation is used to <br> denote the source of the applied emf as: |
| :--- | :--- |
| $E_{B}$ | Voltage supplied from a battery |
| $E_{G \cdot A}$ | Voltage supplied from generator $A$ |
| $E_{L}$ | Voltage supplied from a power line |
| $e$ | The first letter of the abbreviation for electromotive force <br> is used to designate voltage drops and all applied or output |
| voltages other than those from power supplies. For ex- |  |

## 2-16. How to Solve Problems:

1. Read the problem very carefully.
2. Note the values that are given and the values to be found.
3. Draw a circuit diagram wherever possible.
4. Using the various equations that are known, find the one that involves the known and unknown values.
5. If none of the equations available will meet the needs, then find one that involves most of the knowns and that, when solved, will produce a value that can be used in another equation to solve for the desired unknown.
6. Write down the equation.
7. Substitute the known values.
8. Solve for the unknown, performing all necessary arithmetic on another sheet of paper.
9. Keep the work neat.
10. Underline the answer.

The following examples will illustrate the above rules:
Example 2-9. If a 40 -watt lamp operates from a 110 -volt line, what current does it draw?

In this problem power and voltage are the known values and current the unknown value. This is recorded in the following manner, and a circuit diagram is drawn:
$E=110$ volts
$P=40$ watts
$I=$ ?


Fig. 2-20.
As Eq. (2-12) contains just these three quantities, it may be used to solve this problem. The quantities are transposed in order to solve for the current, as

$$
I=\frac{P}{E}
$$

substituting the known values,

$$
I=\frac{40}{110}
$$

Dividing 40 by 110 on a separate piece of paper,

$$
I=\underline{0.363 \mathrm{amp}}
$$

The above problem should appear in the following form:
Given:

$$
\begin{aligned}
& E=110 \text { volts } \\
& P=40 \text { watts }
\end{aligned}
$$

Find: $I=$ ?


Fig. 2-20.
Solution:

$$
\begin{aligned}
I & =\frac{P}{E} \\
& =\frac{40}{110} \\
& =0.363 \mathrm{amp}
\end{aligned}
$$

The above example is very simple and requires the use of only one formula. When a problem involves the application of more than one formula, it is necessary to use step 5 . This is illustrated by the following problem.

Example 2-10. If a fatiron draws five amperes from a 110 -volt line, how much energy does it consume in eight hours?

In this problem, current, voltage, and time are the known values and energy the unknown value.

The energy consumed can be found by Eq. (2-15), but this requires knowing the value of the power. It is therefore necessary to determine the power first, which can be found by use of Eq. (2-12). The solution of the problem will then appear in the following form.


Fig. 2-21.
Solution:

$$
\begin{aligned}
P & =E \times I \\
& =110 \times 5 \\
& =550 \mathrm{watts} \\
W & =P \times T \\
& =550 \times 8 \\
& =4400 \mathrm{watt}-\text { hours } \\
& =4.4 \mathrm{kwh}
\end{aligned}
$$

If the cost of operation of any electrical appliance is desired, it is necessary to multiply the energy consumed by the rate at which electricity is sold. This is expressed mathematically as

$$
\begin{equation*}
\text { Cost }=\text { energy } \times \text { rate } \tag{2-16}
\end{equation*}
$$

Example 2 -11. If electricity sells for five cents per kilowatt-hour, how much does it cost to operate the flatiron used in Example 2-10?

The solution of Example 2-10 shows that 4.4 kilowatt-hours is consumed, and the following step must be added:

$$
\begin{aligned}
\text { Cost } & =W \times \text { rate } \\
& =4.4 \times 0.05 \\
& =0.22 \\
& =22 \text { cents }
\end{aligned}
$$

These examples illustrate the general rules to be followed in solving problems. As there are any number of different kinds of problems, no one set of rules can hold for all of them; the above form may be varied somewhat, but the general thought always is kept intact.

Square Root. Solving problems involving Eqs. (2-13) and (2-14) may require extracting the square root of a number. For example, if in Eq. (2-13) the watts and ohms are known and it is required to determine the voltage, then

$$
E^{2}=P \times R \quad \text { and } \quad E=\sqrt{P \times R}
$$

The process of deriving the square root can best be explained by working out an example.

Example 2-12. If the value of $E^{2}$ was found to be $52,805.367$, what is the value of $E$ ?

$$
\begin{array}{lr}
\text { Given: } & \text { Find: } \\
E^{2}=52,805.367 & E=?
\end{array}
$$

Solution:

$$
E=\sqrt{52,805.367}
$$

The following procedure is recommended in working for the square root of a number.

1. Set down the number, and divide the digits on each side of the decimal point into groups of two; start at the decimal point, and work toward the left, and then again from the decimal point work toward the right. It is possible for the extreme left-hand group to have only one digit. If the extreme right-hand group contains only one digit, a zero is added so that it will have two digits.

$$
\sqrt{52805.3670}
$$

2. Find the largest number that when multiplied by itself will go into the first group. In this example, the group is 5 and the largest number that can be used is 2 . Now write the 2 above the 5 . Next square the root number 2 , and place it under the 5 ; its value is 4 . Then subtract the 4 from 5 , leaving a value of 1 .

3. Now bring down the next group of numbers alongside the remainder and draw a $\qquad$ to the left of this complete number. Double that part of the square root already found, and place it in the $\qquad$ I.
2
$\sqrt{52805.3670}$
$4 \frac{4}{1128}$
4. The next step is to add a number alongside that part of the square root already found and also place this same number alongside the 4 in the $\qquad$ |. This number $A$ should be the largest that when multiplied by $\underline{4 A}$ will go into the remainder 128 . This step is done by trial-anderror procedure; therefore let us try the number 3 .
23
$\sqrt{52805.3670}$
4
$43[128$
$\underline{129}$

At this point it is necessary to subtract again, but unfortunately 129 is just a bit too large. Therefore the number 2 must be used.
22
$\sqrt{52805.3670}$
4
$42 \mid 128$
$\frac{84}{44}$
5. Bring down the next group (05), and repeat the procedure. Double the root already found, and find the largest multiplier that can be used.
229
$\sqrt{52805.3670}$
4
$42 \mid 128$
4494405
$\frac{4041}{364}$
6. Continue until all places are filled above each group in the original number.

| $\begin{array}{rrr} 2 & 29.79 \\ \sqrt{52805.3670} \end{array}$ |
| :---: |
|  |  |
|  |
| 84 |
| $449 \sqrt{4405}$ |
| 4041 |
| $4587 \overline{36436}$ |
| 32109 |
| $4 5 9 4 9 \longdiv { 4 3 2 7 7 0 }$ |
| 41354 |
| 19229 |

7. Notice that the decimal point in the root is placed directly above the decimal point in the original number. If desired, the solution can be carried to additional decimal places by adding groups of two zeros to the right of the existing digits, then proceeding as before.

$$
\begin{array}{r}
229.794 \\
\sqrt{52805.367000}
\end{array}
$$

Therefore the square root of $52,805.367$ when worked to three decimal places is 229.794 .
8. Should any of the divisors be too large to go into the remainder, a zero is placed in the root above that group and the next pair of numbers is brought down. This step is illustrated in the solution of the following example.

Example 2-13. What is the square root of 1620 ?

$$
\begin{array}{r}
4 \quad 1 \\
\sqrt{1620 .} \\
16 \\
81520 \\
81
\end{array}
$$

Note that 1 is too large; therefore

| $4 \quad 0.2$ |
| ---: |
| $\sqrt{1620.00}$ |
| 16 |
| 8021 |
| $\frac{2000}{1604}$ |
| 396 |

The square root of 1620 when worked to one decimal point is 40.2 .

Example 2-14. What is the voltage of an electrical circuit that takes 50 watts and whose resistance is 25 C ohms?

```
Given:
\(P=50\)
Find:
    \(E=\) ?
\(R=250\)
```

Solution:
Using Eq. (2-13),
then

$$
P=\frac{E^{2}}{R}
$$

and

$$
E^{2}=P R
$$

$$
\begin{aligned}
E^{\prime} & =\sqrt{\overline{P R}} \\
& =\sqrt{50 \times 250} \\
& =\sqrt{12,500} \\
& =111.8 \text { volts }
\end{aligned}
$$

Example 2-15. What current is flowing through a 25 -ohm resistance if the power being consumed is 500 watts?

| Given: | Find: |
| :--- | ---: |
| $R=25$ | $I=?$ |
| $P=500$ |  |

Solution:
Using Eq. (2-14),
then

$$
\begin{aligned}
& P=I^{2} R \\
& I^{2}=\frac{P}{R}
\end{aligned}
$$

and

$$
\begin{aligned}
I & =\sqrt{\frac{P}{R}} \\
& =\sqrt{\frac{500}{25}} \\
& =\sqrt{20} \\
& =4.47 \mathrm{amp}
\end{aligned}
$$

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## QUESTIONS

1. (a) Name the two kinds of electricity. (b) Which is used more widely?
2. How is static electricity produced?
3. List all the examples of static electricity that you know.
4. Define the following terms: (a) matter, (b) element, (c) compound, (d) atom, (e) molecule.
5. Describe the composition of a compound.
6. What importance does the electron theory have in the study of (a) electricity?
(b) Radio and television? (c) Electronics?
7. What are (a) electrons? (b) Protons? (c) Neutral bodics?
8. Describe an atom of copper in terms of the electron theory.
9. Describe the theory of electron flow along a conductor.
10. Name five men who have contributed to the development of the electron theory.
11. Describe how electricity may be produced by friction.
12. State the law of charges.
13. When does a body have (a) a positive charge? (b) A negative charge?
14. What did Benjamin Franklin contribute to the study of electrical charges?
15. (a) In what manner may charges be transferred from one body to another?
(b) Describe each way.
16. Compare discharging from a pointed body with discharging from a sphere.
17. (a) What is lightning? (b) What is its cause?
18. Describe the use of the lightning rod.
19. What is meant by an electrostatic field?
20. What means is used to represent the electrostatic field?
21. Illustrate the field about (a) a positive charge, (b) a negative charge, (c) adjacent positive and negative charges.
22. Describe Coulomb's law.
23. What is meant by (a) potential? (b) Potential difference?
24. Describe current flow in relation to potential difference.
25. Does electron flow affect the weight of a conductor? Explain.
26. Name and explain the three methods of producing an electric current.
27. (a) Name and explain the three important effects of electron flow.

Name five applications for each of these three effects.
28. Name and explain the six kinds of electric current.
29. (a) What are complex currents? (b) Where are complex currents likely to be found?
30. Explain where each of the above-mentioned currents may be found in (a) electrical apparatus, (b) radio receivers, (c) television receivers, (d) electronic equipment.
31. What are the three ways of expressing Ohm's law using the following electrical terms: voltage, current, and resistance?
32. Why is Ohm's law essential to the study of electricity, television, radio and electronics?
33. What is the difference between a coulomb and an ampere?
34. Explain what is meant by the following: (a) potential difference, (b) electromotive foree, (c) voltage, (d) volt.
35. (a) What is meant by electrical resistance? (b) What is its unit of measure?
36. Express Ohm's law in three ways, using the practical units of electricity.
37. Is there any difference between the electrical units of measure in this country and any other country? Explain.
38. Explain the difference between the following terms: (a) work, (b) power, (c) energy.
39. What are the basic units of electrical power and electrical energy?
40. What are the electrical and mechanical equivalents of a horsepower?
41. What are the three equations that can be used to solve for power?
42. What do the following prefixes mean: (a) milli-, (b) micro-, (c) kilo-, (d) meg-, (e) mega-?
43. (a) What are symbols?
(b) What is their purpose?
44. Have all electrical or radio symbols been standardized? Explain.
45. What is the purpose of the subscript method of notation?
46. Name the 10 steps suggested for the proper solution of problems.

## PROBLEMS

1. What is the magnitude and direction of the force between two positive charges of 200 and 300 esu, respectively, when placed in air 1.5 cm apart?
2. What is the magnitude and direction of the force between a positive and negative charge, each of 300 esu, when placed in air 2.5 cm apart?
3. What is the magnitude and direction of the force between two negative charges of 400 esu each when separated by a piece of fiber 2 cm thick?
4. If the fiber separator of Prob. 3 is replaced by one of common glass, what force would be exerted?
5. If the fiber separator of Prob. 3 is removed and not replaced by anything other than air, what force would be exerted?
6. What is the hot resistance of the filament of a 1 A 6 tube if it draws 0.06 amp from a 2 -volt power supply?
7. How much current does the heater of a 955 acorn tube draw if it has a resistance of 42 ohms and is connected across a 6.3 -volt source of power?
8. The resistance of the heater of a 6A7 tube is 21 ohms. What voltage is required to force 0.3 amp , its rated current, through it?
9. What is the resistance of a circuit drawing 2.5 amp when 120 volts is impressed across its terminals?
10. The voltage drop across a rheostat is 3.6 volts when 1.8 amp is flowing through it. What is the resistance of the rheostat?
11. If a 2250 -ohm resistor causes a voltage drop of 1.5 volts, how much current is flowing through the resistor?
12. How much signal voltage is lost in a 2 -megohm grid leak when $0.1 \mu$ a flows through it?
13. What is the resistance of an ammeter whose full-scale reading is 10 amp and this maximum allowable current causes a voltage drop of 50 mv ?
14. What is the rated maximum voltage of a voltmeter having a resistance of 150,000 ohms if the maximum rated current is 1 ma?
15. When an $8000-\mathrm{ohm}$ resistance is connected in the plate circuit of a 6 A 4 tube, the plate current flow is 22 ma . What is the voltage drop across the 8000 -ohm resistance?
16. An electric lamp whose hot resistance is 240 ohms is to be used on a 120 -volt circuit. How much current does it draw?
17. Which resistance is greater, one that requires 250 volts to force a current of
2.5 ma through it or one that requires 2.5 volts to force a current of 0.25 amp through it?
18. Through which resistor does the greater current flow, a 2 -megohm resistor having a 250 -mv drop or a 20 -ohm resistor having a 25 -volt drop?
19. Which resistance causes the greater voltage drop, a 250,000 -ohm resistor having a current flow of 25 ma or a 250 -ohm resistor having a current flow of 2.5 amp?
20. What is the rate of power loss in a 75 -ohm resistor when 150 ma flows through it?
21. What is the power rating of an electric lamp operating from a 110 -volt supply and drawing 0.909 amp ?
22. If the total resistance of a radio set is 240 ohms and it operates from a 120 volt line, what is its rated power?
23. If the input of a d-c motor is 5 hp , how much current does it draw when operated from a 110 -volt line?
24. If the input of a d-c motor is 5 hp , how much current does it draw when operated from a 220 -volt line?
25. Which motor has the greater resistance, a $10-\mathrm{hp}$ motor operated from a 220 volt line or a $2-\mathrm{hp}$ motor operated from a 110 -volt line?
26. How much does it cost to operate a flatiron that draws 10 amp from a $110-$ volt line if the iron is used 4 hr per day for 10 days and electricity sells for 4 cents per kilowatt-hour?
27. How much energy does a $7 \frac{1}{2}-\mathrm{hp}$ (input) motor consume in 1 month of 30 days if it is operated 2 hr each day?
28. What is the resistance of the output circuit of a 6 A 3 tube if it delivers 15 watts and has a current flow of 40 ma ?
29. If a radio set draws 0.85 amp from a 110 -volt line, what is the cost of operation for 1 month of 30 days if it is used 3 hr each day and electricity costs 3 cents per kilowatt-hour?
30. A 600 -watt toaster is used every day for a half hour. What does it cost to operate this toaster for one month of 30 days if electricity costs 3 cents per kilowatthour?
31. How much energy is used by an electric clock that runs every day for a month of 30 days if it operates from a 115 -volt line and has a resistance of 4000 ohms?
32. Which costs more to operate, a 5 -hp (input) motor used 1 hr every other day for 1 month ( 30 days) or a 500 -watt lamp used 4 hr each day for 1 month ( 30 days)?
33. The power output of a tube is 6.5 watts, and the current flow is 20 ma . What is the voltage developed across the circuit?
34. If the rated output power of a 6 L 6 tube is 6.5 watts and the load resistance is 2500 ohms, what voltage is developed across the output circuit?
35. A 16,000 -ohm resistor has a power rating of 10 watts. What is the voltage across its terminals when it is delivering its rated power?
36. What current flows through the heater of a 2575 tube which has a resistance of 83.3 ohms and consumes 7.5 watts of power?
37. What is the current rating of a 10 -watt 500 -ohm resistor?
38. What current must be flowing in a $7500-\mathrm{ohm}$ resistor if the power consumed is 25 watts?
39. What is the distance between two positive charges each of 250 esu if the repelling force is 750 dynes? The charges are separated only by the surrounding air.
40. What thickness of fiber separator would cause the force between the two charges of Prob. 39 to be 500 dynes?
41. (a) What is the resistance of the heater circuit of a 5TP4 projection-type cathode-ray tube if its rated voltage is 6.3 volts and its rated current is 0.6 amp ? (b) What is the power rating of the heater circuit of this tube?
42. (a) What is the resistance of the heater circuit of a 12 AP 4 direct-view-type cathode-ray tube if its rated voltage is 2.5 volts and its rated current is 2.1 amp ? (b) What is the power rating of the heater circuit of this tube?
43. A six-tube a-m superheterodyne radio receiver has a power rating of 40 watts. What is the cost of operation for 30 days if the receiver is used on an average of 10 hr per day and the cost of electricity is 3 cents per kilowatt-hour?
44. A 12 -tube a-m and f-m radio receiver has a power rating of 85 watts. What is the cost of operation for 30 days if the receiver is used on an average of 10 hr per day and the cost of electricity is 3 cents per kilowatt-hour?
45. A 32 -tube $75-\mathrm{sq}$.-in. television receiver that also provides $\mathrm{f}-\mathrm{m}$ radio reception has a power rating of 150 watts as an $\mathrm{f}-\mathrm{m}$ radio receiver and 400 watts as a television receiver. (a) What is the cost of operation as an $f$-m receiver for 30 days at 10 hr per day if the cost of electricity is 3 cents per kilowatt-hour? (b) What is the cost of operation as a television receiver for 30 days at 10 hr per day if the cost of electricity is 3 cents per kilowatt-hour?
46. A combination television receiver with a $20-\mathrm{in}$. cathode-ray tube which also provides $a-m / f-m$ radio reception and phonograph has the following power ratings: a-m radio 160 watts, f-m radio 223 watts, phonograph 152 watts, and television 543 watts. (a) What is the current rating for each type of operation if the rated voltage is 110 volts? (b) What is the cost of operation for each type of service for 30 days at 8 hr per day if the cost of electricity is 4 cents per kilowatt-hour?
47. A projection-type television receiver has a power supply that delivers 300 ma at 325 volts (d-c) and $80 \mu$ at 20,000 volts (d-c). (a) What is the power rating of the low-voltage supply? (b) What is the power rating of the high-voltage supply? (c) What is the apparent resistance of the low-voltage load? (d) What is the apparent resistance of the high-voltage load?

## CHAPTER III

## BATTERIES

The battery has played a very important part in the development of radio and is still used to operate some radio equipment. The radio receivers of the early broadcast cra were operated from batteries, and except for the need to replace or recharge run-down batteries, often at inopportune times, we might still be using battery-operated receivers in our homes. Many of the disturbing noises that appear in power-operated receivers are not present in battery-operated sets. Radios used in automobiles are essentially battery-operated receivers, and the portable radio receivers and transmitters, including the walkie-talkies used by the armed forces (see Fig. 1-16), also depend upon the battery as a source of energy.

It is estimated that there are more than five million homes in rural and isolated areas where ordinary electric power is not yet available, and to meet this situation most radio manufacturers still make battery-operated receivers for use in these homes. Although power may be available from farm-lighting sets, it is not steady enough for operating radios, and batteryoperated receivers are necessary.

3-1. The Cell. A cell is a device that transforms chemical energy into electrical energy. A battery consists of a number of cells assembled in a common container and connected to each other.

The fact that chemical action could produce electric current was accidentally discovered in 1786 by Luigi Galvani, but it remained for Alessandro Volta to understand this chemical action, and in 1800 he constructed the first cell. This simplest form of cell, known as either the galvanic or voltaic cell, is shown in Fig. 3-2 and consists of a piece of copper, Cu , and a piece of zinc, Zn , placed in a jar of water to which a little sulphuric acid has been added.

Using various metals and methods of construction, a large variety of cells has been made since Volta constructed the first cell. Among them are the galvanic, voltaic, Daniell, Leclanché, Edison-Lelande, Weston, Planté, Faure, gravity, dry, lead-acid, and nickel-iron-alkali types. As most of these either have become obsolete or are rarely used, further discussion will be limited to the commonly used types, namely, the dry cell and the lead-acid storage cell.

3-2. Battery Terms. Explanations of the terms used in the further description of batteries is given at this point in order that the student may better understand the description as it progresses.

The cell has already been defined as a device that transforms chemical energy into electrical energy. It may be further described as being the fundamental unit of the battery. It consists of two strips, or clectrodes, placed in a container that also holds the electrolyte which may be in the form of cither a liquid or a paste.


Fig. 3-1.-Battery-operated portable receiver. (Allied Radio Corporation, Chicago.)
The battery consists of two or more cells usually placed in a common box or container. The cells are connected to each other in series, in parallel, or in some combination of series and parallel, depending upon the amount of voltage and current required of the battery. For example, a 45 -volt B battery used for radio work may consist of 30 cells of $1 \frac{1}{2}$ volts each connected in series, and the six-volt battery used in the motorcars of today consists of three cells of two volts each, connected in series.

The electrodes are the conductors by which the current leaves or returns to the elcetrolyte. In the simple cell, they are the copper and zinc strips that were placed in the electrolyte, while in the dry cell they are the carbon
rod in the center and the zinc container or shell in which the cell is assembled.

The electrolyte is the solution that acts upon the electrodes which are placed in it. The electrolyte may be a salt, an acid, or an alkaline solution. In the simple galvanic cell and in the automobile storage battery, the electrolyte is in a liquid form, while in the dry cell it is a paste.

A primary cell is one in which the chemical action eats away one of the electrodes, usually the negative, and when this happens it has to be replaced or the cell must be thrown away. In the galvanic type, the zinc electrode and the liquid solution are usually replaced when this happens; in the case of the dry cell it is usually cheaper to buy a new cell.


Fig. 3-2.-Simple voltaic cell.
A secondary cell is one in which the electrodes and the electrolyte are altered by the chemical action that takes place when the cell delivers current but may be restored to their original condition by sending an electric current through them in the opposite direction. The motorcar storage battery is the common example of the secondary cell; charging of the storage battery after it is run down represents restoring the cell to its original state.

3-3. Action of the Voltaic Cell. Factors Affecting Cell Construction. If two electrodes of copper are placed in a weak sulphuric acid solution, no electrical energy will result, but if one electrode of copper and one of zinc are placed in the solution, the chemical action will cause electrical energy to be available at the open ends of the electrodes. By changing the kind of material, size, and spacing of the electrodes used or by changing the kind of solution used as the electrolyte, the following facts about cells have been established:

1. The two electrodes must be of different materials.
2. The electrodes must be conductors of electricity.
3. The electrolyte must contain an acid, alkali, or salt so that it will conduct the current.
4. The voltage of the cell will vary with the materials used as electrodes and the electrolyte but will not exceed approximately two volts.
5. The voltage of the cell is not affected by the size or spacing of its electrodes.
6. The current capacity of a cell may be raised by increasing the surface area of that part of its clectrodes actually making contact with the electrolyte.

Chemical Action of a Simple Cell. As a great number of combinations of materials can be used to make a cell and as the actions follow a similar pattern in each case, only one will be described


Fig. 3-3.-Simple cell delivering electrical energy to a lamp. here. If a cell as shown in Fig. 3-5 is made by placing a strip of zinc and one of carbon into a jar containing a pint of water to which three ounces of ammonium chloride (sal ammoniac) has been added, certain chemical actions will result. When the ammonium chloride, which has the symbols $\mathrm{NH}_{4} \mathrm{Cl}$, is placed in the water, many of its molecules become separated into their atoms $\mathrm{NH}_{4}$ (ammonia) and Cl (chlorine). Another action that takes place during the process of separating the molecules into their atoms is the unbalancing of the number of electrons and protons in each atom. When the atom is unbalanced, the number of electrons and protons no longer equal one another and it is then called an ion. If the protons are greater in number, it is a positive ( + ) ion (or a positively charged atom) ; if the electrons are greater in number, it is a negative ( - ) ion (or a negatively charged atom). This action is refcrred to as the ionization of the electrolyte.

Polarity of the Electrodes. In this cell, the ammonia takes on the positive charge and is now referred to as $\mathrm{NH}_{4}{ }^{+}$, while the chlorine becomes the negative ion $\mathrm{Cl}^{-}$. The zinc electrode gives off positive ions $\mathrm{Zn}^{++}$to the electrolyte, and as a result the zinc strip has an excess of electrons and is, therefore, said to be negatively charged. The zine thus becomes the negative terminal of the cell. The positive ions given off by the zinc, $\mathrm{Zn}^{++}$, repel the positive ions of the solution $\mathrm{NH}_{4}^{+}$and push them toward the carbon electrode. Upon reaching the carbon, the positive ions attract
some negative ions from it. The carbon then has an excess of protons and is, therefore, said to be positively charged and becomes the positive terminal of the cell. This excess of electrons at the zinc electrode and the positive charge at the carbon electrode are responsible for the voltage or emf available at the terminals of the cell. When the emf is established, the chemical action ceases until such time as a path for the electrons is provided by connecting some device to the terminals of the cell.

Current Flow. The ability of the cell to deliver electrical energy may also be demonstrated by connecting a low-voltage flashlight bulb to its terminals. The bulb will light, thus indicating that current is flowing. The electron theory explained in Chap. II presented the idea that circulation of electrons around a closed circuit constitutes a flow of current. The excess electrons on the zinc strip repel one another and push cach other around the external circuit where they are attracted by the positive carbon clectrode. Chemical action in the cell maintains a constant flow of electrons as long as there is an external circuit.

Erosion of the Negative Electrode. While this chemical action is going on in the cell, the zinc ions unite with the chlorine ions to form zinc chloride, $\mathrm{ZnCl}_{2}$, which remains in the solution. The fact that some of the zinc is continually being given off to the solution indicates that this electrode is being reduced or eaten away, and it is, therefore, a primary cell.

Electron Flow vs. Current Flow. This study of the action of a cell indicates


Fig. 3-4.-Electron flow is. current flow. that the electrons flow from the zinc electrode through the external path and return to the cell at the carbon electrode. In Chap. II, the flow of electrons has been described as the electric current; hence from these two facts we may conclude that the electrons, and therefore the current, flow from the zine or negative to the carbon or positive. This description of current flow comes from the electron theory, which is substantiated by scientifie research. In the days before the scientists developed and introduced the electron theory, the early experimenters chose the direction of current flow as being from positive to negative, which is opposite to the direction of flow of electrons. This has caused and will continue for some time to cause confusion. As this text concerns the study of electricity for radio, television,
and electronics (which are relatively new fields), all study and references will be based on the electron flow, namely, from negative to positive.

3-4. Polarization and Local Action. While some of the chemical actions in the cell transform chemical energy to electrical energy, other actions prove a detriment to the operation of the cell. Two such undesirable actions are polarization and local action.

Polarization. In the operation of the cell, positive hydrogen ions $\mathrm{H}^{+}$ are obtained from the water in the electrolyte, and when they come in contact with the carbon strip these positive ions take on electrons from the carbon and form neutral hydrogen bubbles along the carbon strip. In a


Fig. 3-5.-Chemical action of a simple cell.
similar manner, the positive ammonia ions also take on electrons from the carbon, become neutral ammonia atoms, and appear in the form of bubbles at the carbon electrode. The effect of the neutral ammonia and hydrogen bubbles at the carbon electrode is to reduce the active area of that electrode and hence lower the current capacity of the cell. A further effect of these bubbles is to set up a voltage which opposes the normal voltage of the cell and thereby reduces its effective voltage. This effect, called polarization, may be counteracted in several ways. A logical method is to introduce some substance that will readily combine with hydrogen and remove it, or better, prevent it from attaching to the carbon strip. A substance that has a large amount of oxygen which might unite with the hydrogen to form water would be highly desirable. It has been found that manganese dioxide, $\mathrm{MnO}_{2}$, may be used with reasonable success.

Local Action. If a cell is to be efficient, the chemical action that eats
away the zinc electrode should take place only when the cell is delivering electrical energy. As commercial zinc contains some impurities such as carbon or iron, many small cells are formed on the electrode itself. The effect of these small cells is to eat away the zinc whether the cell is delivering electrical energy or not, and this effect is called local action. Using pure zinc would remedy this condition, but it has been found to be far too expensive. The commercial practice is to apply mercury to the surface of the zinc so that the impurities become covered with mercury. This process is called amalgamation. As the zinc is eaten away, the mercury works its way on to combine with the remaining zinc.

3-5. The Dry Cell. The dry cell is so called because its electrolyte is not in a liquid state. Actually the electrolyte is a moist paste, and if it should lose its moisture and really become dry it would no longer be able to transform chemical energy to electrical energy. The name $d r y$ cell is, therefore, not strictly correct in a technical sense.

Construction of the Dry Cell. The construction of a common type of dry cell is shown in Fig. 3-6. The cell is built up in a cylindrical zinc container which also serves as its negative electrode. This zinc container is lined with a nonconducting material, such as blotting paper, to insulate the zinc from the paste. A carbon electrode is set up in the center, and it serves as the positive terminal of the cell. The paste is a mixture of several substances, and its composition may be different for the various battery manufacturers. In general, the paste will contain some combination of the following substances: ammonium chloride (sal ammoniac), powdered coke, ground carbon, manganese dioxide, zinc chloride, graphite, water. This paste, which is packed in the space between the carbon and the blotting paper, also serves to hold the carbon rigid in the center of the cell. When packing the paste in the cell, a small expansion space is left at the top and it is then sealed with layers of corrugated cardboard and a tar compound, respectively. Binding posts are attached to the electrodes so that wires may be conveniently connected to the cell. As the zine container is one of the electrodes and should be protected with some insulating material, it is common practice for the manufacturers to set the cells in cardboard containers.

Action of the Dry Cell. This dry cell is fundamentally the same as the simple cell described in Art. 3-1, and its action, therefore, will be the same. The action of the water and the ammonium chloride in the paste, together with the zinc and carbon electrodes, produces the electric current. The manganese dioxide is added to reduce the polarization due to hydrogen and the zinc chloride to reduce the polarization due to the ammonia. The blotting paper serves two purposes, one being to keep the paste from making actual contact with zinc and the other being to permit the electrolyte
to filter through to the zinc slowly. The cell is sealed at the top to keep the air from entering and drying up the electrolyte. Care should be exercised in handling to prevent breaking this seal.

3-6. Sizes and Proper Use of Dry Cells. The annual production of dry cells extends into the hundred millions, and despite the low replacement cost it is deemed advisable to give them proper care so that their maximum life may be obtained.


Fig. 3-6.-Cross-sectional view of the general-purpose dry cell. (National Carbon Company.)

Rating of the Standard-sized Cell. One of the most popular sizes for general use is the standard, or No. 6, dry cell; it is approximately $2 \frac{1}{2}$ inches in diameter and 6 inches high. The voltage is about $1 \frac{1}{2}$ volts when new and will decrease with the age of the cell. When the voltage on open circuit falls below 0.75 to 1.2 volts (depending upon the circuit design), the cell is usually discarded. The amount of current that the cell can deliver and yet give satisfactory service depends upon the length of time that the current is flowing. If a cell is to be used in a portable radio, it is likely that it may be expected to supply current constantly for several hours at a time and under these conditions the current should not exceed
one-eighth ampere. If a cell is required to supply current only occasionally and then for only short periods of time, it would be possible to supply currents of several amperes. The shorter the duration of current flow and the greater the interval of time between demands for current, the higher the amount of current available, that is, up to the amount that the cell will deliver on short circuit. This short-circuit current incidentally is another means of testing the condition of a dry cell. A new cell, when short-circuited through an ammeter, should show a reading of not less than 25 amperes, and a cell that has been in service should show a reading of at least 10 amperes if it is to render satisfactory service. When testing for the short-circuit current, never leave the meter connected to the cell longer than is absolutely necessary. When fairly large currents are taken from a dry cell, the polarization effect will rapidly exhaust the cell; however, if the cell is then left idle for a while, the depolarizer's action will restore the cell to practically normal. With constant use, the battery will run down because the zinc has been used up or the electrolyte consumed, or both. It is sometimes possible to revive a dead cell by puncturing the zinc container and then injecting some fresh solution or by setting the cell in a salt-water solution for several hours. After this procedure, the


Fig. 3-7.-Cross-sectional view of a size D unit cell. (National Carbon Company.) puncture holes should be sealed. As the cost of new cells is so low, revival of dead cells is suggested only as a last resort in an emergency.

Rating of the Unit-size Cell. Another popular size of dry cell is the size D ( $1_{8}^{3}$ by $2_{8}^{\frac{3}{8}} \mathrm{in}$.), or unit cell, that is used in standard sized flashlights, in radio-testing instruments, and in the construction of radio B and C batteries. In Art. 3-3 it was stated that only the kinds of materials used for electrodes and electrolyte, and not their size, affected the voltage of a cell; therefore, as this smaller cell is made of the same kinds of materials, it too will have an emf of 1.5 volts when new. The short-circuit current test of a new cell should read five or six amperes. Recently, a new highcurrent cell has been introduced, mainly for intermittent use as with photoflash bulbs. This new cell has a short-circuit rating of 10 to 12 amperes but is admittedly of shorter life. The corrosion and swelling of old cells
sometimes ruins the cell holders of flashlights, instruments, etc.; to prevent this corrosion and swelling, some manufacturers place a steel jacket around the zine container.

Shelf Life. A cell that is not put into use will gradually deteriorate because of unavoidable slow chemical reactions and changes in moisture content which take place in the cell. This deterioration is usually very slow. High-grade cells of the larger sizes, such as the standard, or No. 6, cell, should have a shelf life of a year or more. Smaller sized cells have a proportionately shorter shelf life, the very small sizes being good for only a few months. If unused cells are stored in a cool place, the shelf life will be greatly increased.

3-7. Combination of Cells. Need for Extra Cells. A standard, or No. 6 , dry cell has an emf of 1.5 volts and can deliver about $\frac{1}{8}$ ampere continuously. Such a cell can therfore supply electrical energy to a circuit


Fig. 3-8.-Dry cells connected in series: (a) pictorial representation, (b) diagrammatic representation.
requiring 1.5 volts and not more than $\frac{1}{8}$ ampere. Many occasions arise where higher voltage or higher current or both are required. To meet these needs and maintain satisfactory service, a number of cells are joined into a series-connected group, a parallel-connected group, or a series-parallel-connected group, depending upon the voltage and current requirements.

Cells Connected in Series. Whenever the voltage required exceeds 1.5 volts, it becomes necessary to use more than one cell and the cells must be connected in series. The cells are connected in series as shown in Fig. 3-8a by connecting the zinc or negative of the first cell to the carbon or positive of the second cell, the zinc of the second to the carbon of the third, etc., the zinc of each cell being connected to the carbon of the next. It will be noticed that the carbon of the first and the zinc of the last cell are free and they become the terminals of the battery. Figure $3-8 b$ shows the same connection in the schematic diagram form, a long, thin line being used to represent the + terminal and a shorter, heavier line for the - terminal. If the battery consists of only a few cells, the number of pairs of long
and short lines is made to equal the number of cells. In a series circuit, the battery voltage will equal the sum of the separate cell voltages.

$$
\begin{equation*}
E_{B}=E_{C 1}+E_{C 2}+H_{C 3}, \text { etc. } \tag{3-1}
\end{equation*}
$$

If each cell has the same voltage, then

$$
\begin{equation*}
E_{B}=\text { number of cells } \times \text { volts per cell } \tag{3-2}
\end{equation*}
$$

also Number of cells required $=\frac{\text { voltage of battery }}{\text { volts per cell }}$
In the series arrangement of cells, the current rating of the battery will be the same as that of an individual cell; hence this arrangement can be used only where the continuous current requirement is one-eighth ampere or less.


Fig. 3-9.-Dry cells connected in parallel: (a) pictorial representation, (b) diagrammatic representation.

Cells Connected in Parallel. Whenever a continuous current greater than one-eighth ampere is required, it becomes necessary to use more than one cell and the cells must be connected in parallel. The parallel arrangement of cells is shown in $a$ and $b$ of Fig. 3-9. Notice that now all the zincs $(-)$ are connected together and a terminal is brought off one end and that all the carbons $(+)$ are connected together and a terminal is brought off one end. In the parallel circuit, the current rating of the battery is equal to the sum of the currents of the separate cells.

$$
\begin{equation*}
I_{B}=I_{C 1}+I_{c 2}+I_{c 3}, \text { etc. } \tag{3-4}
\end{equation*}
$$

If each cell has the same current rating, then
also Number of cells required $=\frac{\text { current of battery }}{\text { current per cell }}$
In the parallel arrangement of cells, the voltage of the battery will be the same as that of a single cell; hence this arrangement can be used only where the voltage requirement is 1.5 volts.

Cells Connected in Series Parallel. When both the voltage and current required exceed the rated voltage and current of a single cell, it becomes necessary to use four or more cells connected in a series-parallel combination. To get the higher voltage, a number of cells must be connected in series, and to get the higher current rating, a number of scries-connected groups must be connected in parallel.
Number of cells in each series-connected group $=$

$$
\begin{align*}
& \text { Number of parallel groups }=\frac{\text { voltage of battery }}{\text { volts per cell }}  \tag{3-7}\\
& \text { current of battery } \tag{3-8}
\end{align*}
$$

Number of colls required $=$ number of cells in a series group $\times$ number of parallel groups

For example, if a current of $\frac{1}{2}$ ampere at an emf of $4 \frac{1}{2}$ volts is required, then the number of cells in each series-connected group will be $4 \frac{1}{2}$ divided by $1 \frac{1}{2}$, or 3 , and the number of parallel groups will be $\frac{1}{2}$ divided by $\frac{1}{8}$, or 4 .


Fig. 3-10.-Cells connected in series parallel.
The total number of cells required will be 3 times 4 , or 12 cells. The connections for this requirement will be four groups in parallel, each group consisting of three cells connected in series. This arrangement of the cells is shown in lig. 3-10.

3-8. Radio Batteries. At one time, radio sets required three separate batteries designated as A, B, and C batteries, each with a different voltage and current requirement. Fach of these types had a specific use, and while not used so widely today as in the past, there are many of these types still in use.

A Batteries. The A battery has the lowest voltage of the three and also the highest current requirement. The voltage required varies from $\frac{7}{10}$ to five volts depending upon the type of tubes used. For the five-volt
service, the storage battery (see Art. 3-12) is commonly used, while for lower voltages, dry-cell batteries are used. The dry cells do not have a long life for this service, and they also have a considerable drop in voltage as the cells become exhausted. These two features make the use of dry cells rather unpopular. A battery called the air-cell A battery (see Art. $3-9)$ has been developed, which offers the advantage of steady voltage and greatly increased life.


Fig. 3-11.-Applications of primary cells. (National Carbon Company.)
$B$ Batteries. The B battery has the highest voltage of the three but is required to supply only small amounts of current. The amount of current required varies with the number and type of tubes used in the radio set and usually is under 50 milliamperes. B batteries are made in two voltage ratings, namely, $22 \frac{1}{2}$ and 45 volts. The $22 \frac{1}{2}$-volt battery consists of 15 cells connected in series. The 45 -volt battery has 30 cells connected in series and is usually provided with an extra terminal that is tapped off at $22 \frac{1}{2}$ volts.

The original type of 45 -volt B battery consists of 30 (size D) unit cells. assembled in a rectangular container. The cells are connected in series by means of soldered wire connectors. A disadvantage of this type of battery is that it has a great deal of inactive space, resulting in a large battery. The construction of this type of battery is shown in Fig. 3-12.


Fig. 3-12.-Arrangement of cells in cylindrical-cell 13 battery. (National Carbon Company.)
The objections raised to the unproductive space in the round-cell type battery led to the development of a new type of construction used in the Layer-Bilt B battery. In this type of battery, the cells are made in thin squares, and 15 such cells are stacked one upon the other as shown in Fig. 3-13b. The 45 -volt battery contains two stacks connected in series. Figure $3-13 a$ is a cross section of a cell showing the arrangement of the material entering into its construction and indicating the efficient use of the space. The advantages of this type of battery are reduction in size, more efficient use of the materials of which it is made, longer life, and elimination of soldered connections between cells.

The introduction of the portable radio further emphasized the need of a smaller battery, more compact and lighter in weight. The Mini-

Max battery was developed to answer this need. This battery uses the principle of the Layer-Bilt type, but by reducing the thickness of the carbon electrode and increasing the relative quantity and the effective area of the mix materials a smaller, more efficient battery has been ob-


Fig. 3-13.-Construction of Layer-Bilt battery: (a) cross-sectional view of a cell, (b) stack of 15 cells. (National Carbon Company.)


Fig. 3-14.-A miniature $\mathbf{B}$ battery: (a) arrangement of the parts of a cell, (b) 30 cells assembled in two stacks of 15 each, (c) the completed battery. (National Carbon Company.)
tained. Figure 3-14a shows the arrangement of the various parts of the cell, and Fig. 3-14b shows two stacks of 15 cells each, as they are arranged in a 45 -volt battery.

When higher voltages such as 90,135 , or 180 volts are required, they are obtained by connecting two, three, or four 45 -volt batteries in scries.
$C$ Batteries. The C battery is a medium-voltage unit and is required
to supply only very small currents, usually so small that the battery has exceptionally long life. C batteries are available in $4 \frac{1}{2}$-, 9 -, and $22 \frac{1}{2}$-volt units. A popular unit has a maximum voltage of $22 \frac{1}{2}$ volts and is provided with extra taps for $4 \frac{1}{2}, 9$, and $13 \frac{1}{2}$ volts.

3-9. Air Cell A Battery. The air-cell battery provides a very good source of electrical energy for battery-operated radio receivers and transmitters.

This battery differs from the regular dry cell in that it uses a liquid. The battery is shipped dry and may be stored that way for years. When it is to be put into service, the user merely remores a thin rubber covering at the water-filler opening and a thin cellophane covering over the special breather electrodes and fills the water compartments with any water suitable for drinking purposes.

ritg. 3-15.-Cross-sectional view of an air-cell battery, (Fational Carbon Company.)
The carbon electrode is made porous so that it can absorb oxygen from the air, and it is therefore referred to as the lung or breather. This lung action supplies the orygen needed for depolarizing, and as the oxygen is obtained from the air it eliminates the necessity of providing a chemical depolarizing material. The electrolyte is formed when water is added to the cell. The material used for the electrolyte is a solid cake of caustic soda which dissolves very easily when the water is added, thereby forming the electrolyte.

The advantages of the air-cell battery are that it provides a relatively
constant voltage, has a longer life, and is lower in cost over long periods of time. It can be used only where the current drain does not exceed 0.65 ampere. This type of battery is arailable only in single-cell and twocell units and is very satisfactory for $A$ battery purposes.

3-10. Secondary Cells and Storage Batteries. Secondary cells operate on the same principles as primary cells but differ in the method in which they may be renowed. Some of the materials of a primary cell are used up in the process of changing chemical energy to electrical energy, and they must be replaced to renew the cell. In the secondary cell, the materials are merely transferred from one electrode to the other, and they may be restored to their original status by sending an electric current from some other source through the cell in the opposite direction.

The storage battery consists of a number of secondary cells connected in series, the most common battery having three cells. Properly speaking, this battery does not store electrical energy, but it does store chemical energy which it can transform to electrical energy. There are two types of storage cells, the lead-acid type, which has an emf of 2.2 volts per cell, and the nickel-iron-alkali type, which has an emf of 1.2 volts per cell. Of these two types, the lead-acid is the more popular and will be described here.

3-11. Action of the Lead-acid Cell. Simple Cell and Its Action. A simple secondary cell may be constructed by placing two strips of lead that have been thoroughly cleaned into a jar containing a weak solution of sulphuric acid. If a voltmeter or a low-voltage flashlight bulb is connected across the two lead strips, it is shown that no electrical energy is available because two electrodes of similar material are present. If these two electrodes are connected to a source of electricity, such as two dry cells connected in series, bubbles will appear at each plate; a greater number of bubbles will appear at the plate connected to the negative terminal of the dry cell. This indicates that a current is passing through the electrolyte from one plate to the other, and if this current is kept flowing for some time, the positive plate will take on a dark brown color because the action of the current has changed this electrode into lead peroxide. The negative elcctrode will show no such marked change, but upon careful inspection it may be seen that it is changing from solid lead to a spongy lead.

Discharge of a Simple Cell. If the dry cells are disconnected and the secondary cell is again tested by the voltmeter or by the flashlight bulb, it will be seen that electrical encrgy is now available from the cell and that its emf is approximately two volts. If the bulb is left connected to the cell, the light will gradually get dimmer and eventually go out. When this happens, it will be noticed that the positive electrode has lost most of its dark color and will more nearly resemble its original state. The lead
peroxide of the positive electrode has changed to lead sulphate, and the spongy lead of the negative electrode also becomes lead sulphate. As both electrodes are now similar, no electrical energy will be available.


Fig. 3-16.-A simple cell of similar plates. The lamp and voltmeter show that no electrical energy is available.


Fig. 3-17.-Forming the plates of a simple cell by passing a current through the cell.
Recharge of a Simple Cell. If the dry cells are again connected to this cell, the positive electrode will take on a brown color because of the lead peroxide forming there and the negative electrode will become spongy and porous. From the foregoing actions, it should be evident that the secondary cell stores up chemical energy and that the process is reversible.

Chemical Action. During the first process, when the dry cells are connected to the two lead strips in the dilute sulphuric acid solution, the following actions take place. The sulphuric acid, $\mathrm{H}_{2} \mathrm{SO}_{4}$, separates into positive hydrogen ions, $\mathrm{H}_{2}(+)$, and negative sulphate ions, $\mathrm{SO}_{4}(-)$. The positive hydrogen ions are attracted to the negative lead strip, where they form bubbles that pass off into the air, and during this action the lead strip changes to spongy lead. The negative sulphate ions are attracted to the positive strip and there react with the water, $\mathrm{H}_{2}(+)$ ions and $\mathrm{O}(-)$ ions, to form sulphuric acid and lead peroxide, $\mathrm{PbO}_{2}$. This process is known as forming the plates.


Fig. 3-18.-A simple cell delivering electrical energy to a lamp.
After the plates are formed, the electrodes are of chemically dissimilar materials and the cell functions as a primary cell. As such, it can transform its chemical energy into electrical energy. If a low-voltage light or bell is connected to it, a current will flow and the cell will discharge. During the discharge, the sulphuric acid is broken into hydrogen ions, $\mathrm{H}(+)$, and sulphate ions, $\mathrm{SO}_{4}(-)$. Any sulphate ions, $\mathrm{SO}_{4}$, near the spongy lead, Pb , strip will combine with lead and form lead sulphate, $\mathrm{PbSO}_{4}$, at this electrode. The sulphate ions, $\mathrm{SO}_{4}$, near the lead peroxide, $\mathrm{PbO}_{2}$, strip will combine with lead, Pb , to form lead sulphate, $\mathrm{PbSO}_{4}$, at this electrode. All the hydrogen ions, H , are attracted to the lead peroxide, $\mathrm{PbO}_{2}$, electrode and unite there with the oxygen, O , to form $\mathrm{H}_{2} \mathrm{O}$, or water. Electrons are given to the lead strip by the $\mathrm{SO}_{4}(-)$ ions making it the negative electrode, while the $H(+)$ ions take electrons from the lead peroxide strip making it the positive electrode; this accounts for the emf (or difference of
potential) of the cell. When all the lead peroxide is changed to lead sulphate and all the spongy lead is changed to lead sulphate, the two electrodes are again chemically similar materials and the cell is no longer capable of delivering electrical energy. It is said to be discharged. Note, too, that sulphuric acid has been changed to water, for this becomes a means of determining the charge of a battery as will be explained later.

It has been previously stated that in a secondary cell the process could be reversed, or it may be said that electrical energy can be transformed into chemical energy. It has been shown that to reverse the process a current must be sent through the cell in the direction opposite to that during discharge. The current flowing through the electrolyte divides the water, $\mathrm{H}_{2} \mathrm{O}$, into hydrogen, H , ions and oxygen, O , ions. The hydrogen ions, II , near the negative strip, $\mathrm{PbSO}_{4}$, combine with the sulphate ions, $\mathrm{SO}_{4}$, and produce sulphuric acid, $\mathrm{H}_{2} \mathrm{SO}_{4}$. The negative strip is changed from lead sulphate, $\mathrm{PbSO}_{4}$, to spongy lead, Pb . The hydrogen ions, H , near the positive strip, $\mathrm{PbSO}_{4}$, combine with the sulphate ions, $\mathrm{SO}_{4}$, and also produce sulphuric acid, $\mathrm{H}_{2} \mathrm{SO}_{4}$. All the oxygen ions, O , are attracted to the positive strip and combine with the lead, Pb , there to make this strip lead peroxide, $\mathrm{PbO}_{2}$. The electrodes have been restored to their previous state, that is, two chemically dissimilar materials, and the cell again has chemical energy that can be transformed to electrical energy. Notice that during this process water, $\mathrm{H}_{2} \mathrm{O}$, has been changed to sulphuric acid, $\mathrm{H}_{2} \mathrm{SO}_{4}$.

3-12. The Commercial Storage Battery. The most commonly used storage battery is the three-cell lead-acid type. The voltage of a fully charged battery is 6.6 volts, but it is commonly referred to as a 6 -volt battery.

Commercial Cells. The simple cell shown in Fig. 3-18 has no practical value because its capacity is too small. The commercial cell is much larger and incorporates many refinements in its construction that make it possible to increase the capacity a great deal. The capacity of a battery is expressed as the ampere-hour capacity and is proportional to the amount of active material in the electrodes or plates, as they are commonly called. The area of the plates is thercfore an important factor in the rating of a battery. By placing a positive plate between two negatives, both sides of the positive plate become active and the cell can be made smaller. In order to make batteries in convenient sizes, the total plate area is obtained by using a number of smaller plates set side by side and connected in parallel. To have both sides of each positive plate facing a negative, it is necessary to have one more negative plate than the number of positives. This accounts for the fact that baticries always have an odd number of plates, such as 11, 13,15 , and 17. A further advantage of using both surfaces of the plates
is that it prevents the buekling which takes place when only one side of a plate is active.

Construction of Plates. Another limitation of the simple cell is that any attempt to increase the cell capacity by increasing the thickness of the lead peroxide eoating usually causes the lead peroxide to flake and fall off the plate. The construction of the plates used in commercial cells is of improved design made by the Planté process, the Faure process, or some modification of these processes.

The Planté type of plate is made of a sheet of lead and has its active material formed from the plate itself by means of an electrochemical process. By special design and method of manufacture, the active area of this type of plate is 6 to 10 times greater than the apparent surface area.

The Faure type of plate is constructed by making the activematerial in a paste form and pasting it into a grid framework. Generally, a grid framework of harder material is provided, and the active material, that is, the spongy lead and the lead peroxide, is applied to it by one of various processes. The framework is usually made of a lead-antimony alloy


Fig. 3-19.-A single-cell glassjar battery. (Electric Slorage Battery Co.) which provides strength and in which very little chemical action takes place. This type of plate is commonly called the pasted plate.

Separators. In order to conserve space, the plates are placed close to one another, and insulators are placed between them to prevent a positive plate from touching its adjacent negative. These insulators, or separators as they are also called, have been of various kinds of materials, but those most commonly used are wood, rubber, and plastics.

Containers. Each cell is placed in a container made of hard rubber, glass, or wood lined with lead. The type of container depends upon the manner in which the battery is used, whether it is stationary or portable. Automobile batteries and most radio storage batteries use hard-rubber jars. In many types, all three jars are of hard rubber and are molded into a single unit. The containers are always made deeper than the space actually required by the plates, to allow the material that drops off to settle at the bottom without touching the plates and causing a short-circuited cell. After the plates are placed in the container, a hard-rubber cover
fitted with a filler tube and vent cap is set over the cell and sealed with a compound. The cells are connected in series by heavy lead connectors, and the positive terminal is marked by a large + sign or with a red color.

The Electrolyte. The electrolyte is a dilute sulphuric acid solution mixed in such a proportion so that with a fully charged battery its specific


Fig. 3-20.-Construction of plates: (a) grid partly filled with active material, (b) negative group, (c) positive group. (Electric Storage Battery Co.)
gravity will be approximately 1.280 . During the normal life of a battery that is properly cared for, the electrolyte loses none of the acid, and it is therefore necessary only occasionally to replace the water that has evaporated. To ensure normal battery life, only pure water should be added.


Fig. 3-21.-Separators: (a) wood separator, (b) Fiberglas separator. (Electric Storage Battery Co.)

Battery Ratings. It has already been stated that the capacity of a battery is generally expressed in ampere-hours and that the capacity is proportional to the active material on the plates. The ampere-hour rating

(a)

(b)

(c)

Fig. 3-22.-Battery containers: (a) wood case, (b) rubber container, (c) cross section of the lower part of a container. (Electric Storage Battery Co.)
is based upon the number of amperes that a battery can deliver for a specified length of time. Stationary batteries are rated at eight hours; for example, a battery that can deliver 12.5 amperes for 8 hours would be rated at $12.5 \times 8$, or 100 ampere-hours.


Fig. 3-23.-A complete cell cut away to show details of construction. (Electric Storage Battery Co.)


Fig. 3-24.-The hydrometer.


Fig. 3-25.--Reading the condition of a battery charge by means of a hydrometer.

The commercial storage battery is fundamentally the same as the simple storage cell described in Art. 3-11. It is modified in construction to provide ample rating and increased length of life. The action within the battery during charge and discharge is also fundamentally the same as that of the simple cell and has been already deseribed.

3-13. Charging the Storage Battery. Need of Charging. One of the most important requirements in storage-battery operation is that the battery be maintained in a charged condition. This is important for two reasons: (1) to be sure of a supply of electrical encrgy when needed and (2) to ensure a normal life for the battery.


Fig. 3-26.-Charging a storage battery from a d-c power line.
When to Charge a Battery. In studying the action of a simple cell, it was shown that during discharge some sulphuric acid is transferred from the electrolyte to the plates and that during charging sulphuric acid is returned to the electrolyte. As sulphuric acid is heavier than water, it becomes apparent that there will be a change in the weight of the electrolyte when the battery charge increases or decreases. The relative weight of the electrolyte to that of pure water may be determined by the use of an instrument called the hydrometer.

The hydrometer, shown in Fig. 3-24, consists of a long glass tube fitted with a thin rubber hose at its lower end, a rubber bulb at the upper end, and a hydrometer float inside the glass tube. By inserting the hose into a cell and then operating the bulb, a quantity of the electrolyte may be drawn into the glass tube. The float will rise with the liquid, and the
condition of the battery may be read. Most hydrometer floats are marked with a scale starting with 1000 and extending to 1300 , while some also have marks labeled charged, half-charged, and discharged.

When a battery is fully charged, the electrolyte is heavier because it contains a greater amount of acid and the hydrometer reading approaches the 1300 mark. When completely discharged, the electrolyte contains less acid, and the hydrometer will read about 1100. Hydrometer readings should be taken at regularly scheduled intervals, and when the reading drops to about 1225 the battery should be recharged.


Fig. 3-27.-A Tungar battery charger. (General Eleciric Co.)
Charging Rate. There are several methods of charging a battery, and the choice of the method will depend largely upon the kind of current available, that is, whether it is direct or alternating, and the kind of charging equipment at hand. Regardless of the method used, two precautions must be observed, (1) that the polarity is correct and (2) that the charging rate is not too high. The polarity is correct when the positive terminal of the battery is connected to the positive side of the charging source. The charging rate recommended as best for the battery is such that full charge can be attained in eight hours. The 100 -ampere-hour battery therefore should be charged at the rate of 100 divided by 8 , or 12.5 amperes, to obtain the maximum life of the battery. It is possible to start the charge
at a higher rate, but this should be gradually reduced to prevent damage to the cell. At no time should the rate be so high that it causes excessive temperature and gassing at the battery. To avoid accidents, do not go near the batteries with an open flame or lighted cigarettes, etc., while they are being charged.

Charging Apparatus. A battery may be charged from a lighting circuit without special apparatus only if that circuit is direct current. Even then it is necessary to connect some resistance in series with the circuit to keep the charging rate at a safe amount. It is common practice to connect ordinary lamp bulbs in the circuit for this purpose, one 100 -watt lamp providing a one-ampere rate; for each additional ampere of charging rate desired an additional 100 -watt lamp must be connected in parallel. The circuit shown in Fig. 3-26 will provide approximately a six-ampere charging rate.

When only alternating current is available, it is then necessary to have special equipment to charge a battery. The Tungar bulb rectifier is the most common and is popular where a low rate of charge is desired for a single battery. This type of charger can be purchased in sizes of 5 - and 10 -ampere charging rates. For larger amounts of current and where many batteries are to be charged at the same time, a copper oxide rectifier or a motor-generator


Fig. 3-28.-A copper oxide battery charger. (P. R. Mallory \& Co., Inc.) set is used.

3-14. Care of Batteries. The average life of a battery is two to four years, depending upon its quality and the kind of care exercised in its use. To get the maximum life out of a battery, the following procedures are recommended: Keep the top of the battery clean and dry at all times to prevent corrosion and leakage of current. If corrosion has started, the battery may be cleaned by the use of a stiff brush and then wiped with a rag moistened in a solution of household ammonia. When cleaned, the terminals and other metal surfaces should be covered with a pure mineral grease such as Vaseline to minimize further corroding.

Keep the electrolyte at the proper level. It should always be high enough to cover the plates and separators as this prevents decay of the
separators. This should be checked at regular intervals of one or two weeks and when found to be too low should be brought to the proper level by adding water free from impurities. Distilled water or clear rain water


Fig. 3-29.-A motor-generator set. (Electro Dunamic Works.)


Fig. 3-30.-Cross section of a cell showing the electrolyte at the proper level.
is excellent for this purpose. When adding water, the level should not be brought more than three-eighths inch above the top of the separators. As only the water evaporates, it is never necessary to add acid during normal overation of a battery.' Be sure to replace and tighten the vent caps.

Take frequent hydrometer readings to ascertain the charge of the battery. It is advisable to take these readings at the same time that the level of the eleetrolyte is being checked. When the hydrometer is down to 1225 , the battery shouk be recharged. Allowing the battery to remain in a low eharged condition will reduce its length of life.

Do not charge the battery at a rate higher than its normal current unless in an emergency. Charging at too high a rate causes excessive boiling and gassing and also a permanent loss in some of the active material of the plates. This will result in a shorter life for the battery. Nothing can be accomplished by charging a battery after it is fully charged; therefore, overcharging should also be avoided.

Do not leave a battery in a discharged condition for a long period of time even though it is not being used. If left in a discharged condition, a coating of hard sulphate may form on the plates. This sulphate coating cannot be converted to active materials, and the cell is permanently injured. This action is commonly called sulphation and may also be caused by high battery temperatures and too much sulphuric acid in the electrolyte.

If a battery is to be taken out of service and stored away for several months, it should be fully charged before storing and it should be inspected periodically to see that the level of the electrolyte does not get too low. If stored for longer periods of time, it should be charged occasionally so that it will be kept fully charged at all times.

Batteries used in subfreezing temperatures should be kept fully charged, for the freezing point of the electrolyte decreases as the hydrometer indication increases. The freezing point of the electrolyte at 1100 is approximately $20^{\circ}$ F'ahrenheit above zero; at 1200 it is approximately $15^{\circ}$ Fahrenheit below zero; and at 1250 it is approximately $60^{\circ}$ Fahrenheit below zero. Freezing of the electrolyte will cause the plates to be damaged and may also burst the container.

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## QUESTIONS

1. Why is it important to have a knowledge of batteries?
2. Describe the construction of a simple cell.
3. Define (a) electrode, (b) electrolyte, (c) primary cell, (d) secondary cell.
4. (a) What determines the voltage of a cell? (b) What is the approximate maximum value?
5. What determines the current rating of a cell?
6. Why does the zinc become the negative terminal of a primary cell?
7. Why does the carbon become the positive terminal of a primary cell?
8. (a) How is the difference in direction of electron flow and current flow explained? (b) Which is correct? (c) Which is used more and why?
9. (a) What is meant by polarization? (b) What is done to reduce it?
10. (a) What is meant by local action? (b) What is done to reduce it?
11. Describe the construction of a dry cell.
12. Why is the service date stamped on dry cells?
13. What is the maximum amount of current that a standard dry cell can furnish for a 3 -hr period without impairing its normal life?
14. Draw a diagram showing (a) 6 cells connected in series, (b) 6 cells connected in parallel, (c) 12 cells connected in a parallel-series combination so that the voltage will be 6 volts.
15. (a) Describe the construction of B batteries and C batteries. (b) What are the usual voltage ratings of these batteries?
16. (a) State two important advantages of the air cell over the dry cell. (b) What must be done to put an air-cell battery into service? (c) What is the objection to drawing large currents from an air-cell battery, even for short periods of time?
17. How do secondary eells differ from primary cells?
18. Describe the construction of a simple secondary cell.
19. What is meant by forming of the plates?
20. Does the battery store electrical or chemical energy? Explain your answer.
21. What is meant by the expression "the action of a secondary cell is reversible'"?
22. Describe the construction of a commercial storage battery.
23. What is the function of the (a) grid framework, (b) separators, (c) container, ( $d$ ) cover, ( $e$ ) connectors, $(f)$ vent caps?
24. (a) How is a storage battery rated? (b) Upon what number of hours is the normal rate based?
25. (a) What is meant by charging a storage battery? (b) Discharging?
26. (a) What is a hydrometer? (b) What is its use?
27. Name four means of charging a storage battery.
28. (a) What charging rate is recommended? (b) What is the maximum rate? (c) What precautions should be observed in the so-called one-hour charge?
29. Draw a diagram showing how the + and - terminals of a battery should be connected to the charging source.
30. What ordinary care does a battery require?
31. What care should be taken of a battery that is to be out of service for several months?
32. Do temperatures below freezing have the same effect on fully charged batteries as on half-charged batteries? Explain your answer.

## PROBLEMS

1. The filament circuit of a certain radio set requires 10.5 volts and draws 0.060 amp. If dry cells are to be used as the source of power, how many are required and how should they be connected?
2. A special battery is constructed of 1.5 -volt cells and has an emf of 37.5 volts at its terminals. How many cells are there in the battery?
3. A battery is made of 15 cells connected in series. Each cell has an emf of 2.2 volts. What is the voltage of the battery?
4. If two 45 -volt and one $22 \frac{1}{2}$-volt $B$ batteries are connected in series, what voltage will the combination supply?
5. The filament circuit of a certain radio set requires 0.48 amp at 1.5 volts. If dry cells are to be used as the source of power, how many are required and how should they be connected?
6. If a particular size and make of dry cell can safely deliver $\frac{1}{8}$ amp, how many cells are required to deliver 1.5 amp ? How should they be connected?
7. The filament circuit of a certain radio receiver requires 0.25 amp at 6.0 volts. How many dry cells would be required for this circuit, and how should they be corlnected?
8. A filament circuit requires 0.50 amp at 7.5 volts. How many dry cells would be required for this circuit, and how should they be connected?
9. An air cell A battery can supply 0.75 amp at 3.0 volts. How many dry cells would be required to do the same work if each cell is rated at 0.125 amp and 1.5 volts?
10. The air cell A battery of Prob. 9 has a life of 1000 hr and costs $\$ 5.50$. (a) If the dry cells have a life of $333 \frac{1}{3} \mathrm{hr}$, how many would be required to do the same work for the same length of time? (b) If the dry cells cost $\$ 0.35$ each, what would be their total cost? (c) Which is cheaper, the air cell A battery or the group of dry cells required for the same total rating?
11. The capacity of a cell increases as the area of the zinc container increases. The regular unit cell (size D) is $1 \frac{3}{8}$ in. in diameter and $2_{8}^{3}$ in. high. The "baby" unit cell is 1 in . in diameter and $1 \frac{7}{8} \mathrm{in}$. high. How much greater is the capacity of the regular size than the "baby" size?
12. A certain radio set requires 135 volts for the plate or B circuit. (a) If 45volt B batteries are available, how many would be required? (b) How many 1.5 -volt cells are there in each battery? (c) What is the total number of cells in the complete $B$ battery circuit?
13. One popular type of $C$ battery has terminals marked $+,-3,-4 \frac{1}{2},-16 \frac{1}{2}$, $-22 \frac{1}{2}$ volts. If the battery is constructed of 1.5 -volt cells, how many cells are required for the maximum voltage and at what points are taps taken off?
14. A battery manufacturer lists a Standard B battery at $\$ 2.35$ and a Super B at $\$ 2.85$. (a) If the standard is rated at $1500 \mathrm{ma}-\mathrm{hr}$ and the Super at $4000 \mathrm{ma}-\mathrm{hr}$, how many hours will each type battery supply a set requiring 50 ma? (b) What is the cost per hour of service for each type? (c) Which is the more economical and why?
15. A 100 -amp-hr storage battery is used to supply the 5 -volt filament circuit of a battery-operated recciver. How many hours can the battery operate the receiver if the current required is 1.5 amp ?
16. If the storage battery of Prob. 15 is run down, what ampere rate of charge is recommended on the basis of its normal rating?
17. If the storage battery of Prob. 15 is charged by means of a 5 -amp Tungar charger, how many hours are required to charge the battery fully?
18. A storage battery cell has 11 plates (five positive, six negative) each $5 \frac{1}{2}$ by $6 \frac{1}{2} \mathrm{in}$. The plates are of the pasted type and will give a capacity rating of $1 \mathrm{amp}-\mathrm{hr}$ for every 3 sq . in. of positive plate area. (a) What is the active area of the positive plates? (b) What is the ampere-hour capacity of the battery?
19. What is the ampere-hour capacity of a battery similar to that of Prob. 18 but with 17 plates per cell?
20. A storage battery using Planté-type plates has 15 plates per cell each 5 by 6 in . The active area is eight times the apparent surface area, and the battery will have an output of $1 \mathrm{amp}-\mathrm{hr}$ for each 20 sq . in. of active area. What is the ampere. hour rating of the battery?

## CIIAPTER IV

## ELECTRIC CIRCUITS

An electric circuit is the path taken by an electric current from its source, through the conductors, and back to its starting point. From this definition, it can readily be seen that an electric circuit must be a closed path in order that the electrons leaving the starting point can return to that point upon completing the circuit.

The essential parts of any clectric circuit are the source of power, the conductors used to transmit the electric current, and the appliance or appliances to be supplied with electrical energy.

In Chap. II it was shown that the controlled movement of the free electrons in a conductor forms an electric current. Certain materials emit free electrons more casily than do others and offer very little resistance to the flow of electrons. These materials are good conductors of electric currents. Other materials emit very few electrons and greatly oppose their flow. These materials are poor conductors (or insulators) of electric currents.

4-1. Resistance of Conductors. The resistance of any conductor will vary with its length, cross-sectional area, the material of which it is made, and its temperature.

Length. A conductor of any given material and cross-sectional area will offer a definite amount of resistance to the flow of electrons per unit length of the conductor. If the length of the conductor is increased, the distance the electrons must travel is increased and therefore the resistance of the conductor is increased. In a similar manner, the resistance of the conductor will decrease if the length of the conductor is decreased. We can therefore conclude that the resistance of any conductor will vary directly with its length.

Cross-sectional Area. A conductor of any given material, length, and cross-sectional area will offer a definite amount of resistance to the flow of electrons. If this area is increased and the same flow of electrons is maintained, the resistance offered to its flow decreases as the area of the path for the electronic flow is increased. In a similar manner, it may be seen that the resistance would increase if the area is decreased. The resistance of a conductor will therefore vary inversely with its cross-sectional area.

Material. The resistance of any conductor depends upon the material
of which it is made, as some materials offer greater or less resistance to the flow of an electric current than others. For example, a piece of steel wire offers more resistance to the flow of an electric current than a piece of copper wire of the same length and cross-sectional area. The resistance of a conductor will therefore vary directly with a basic unit resistance depending on the material. This value is usually expressed by the letter $K$ and is called the specific resistivity of the material.

Temperature. The resistance of most materials increases if their temperature is increased, and a few materials show a decrease in resistance with a temperature rise. There are also a few materials whose resistance is not affected by any temperature change. The resistance of a conductor will therefore vary directly with its temperature coefficient. In general, the change in resistance due to temperature variation is very slight and for most practical purposes may be disregarded. For work where accurate results are desired, the following formula can be used. $A$ list of temperature coofficients for some of the more common materials used will be found in Appendix IV.

$$
\begin{equation*}
R_{F}=R_{i}+\left[R_{i} \times T_{c} \times\left(t_{f}-t_{i}\right)\right] \tag{4-1}
\end{equation*}
$$

where $R_{F}=$ final resistance
$R_{i}=$ initial resistance
$T_{c}=$ temperature coefficient
$t_{i}=$ initial temperature, degrees centigrade
$t_{f}=$ final temperature, degrees centigrade
Example 4-1. A resistor made of advance wire has a resistance of 10,000 ohms at $20^{\circ} \mathrm{C}$. What is its resistance at $40^{\circ} \mathrm{C}$ ?

| Given: | Find: |  |
| :--- | ---: | :--- |
| Material | $=$ advance | $R_{F}=?$ |
| $t_{i}$ | $=20^{\circ} \mathrm{C}$ |  |
| $t_{f}$ | $=40^{\circ} \mathrm{C}$ |  |
| $R_{i}$ | $=10,000$ ohms |  |

Solution:

$$
\begin{aligned}
T_{c} & \text { for advance wire }=0.000018 \text { (Appendix IV) } \\
R_{F} & =R_{i}+\left[R_{i} \times T_{c} \times\left(t_{f}-t_{i}\right)\right] \\
& =10,000+[10,000 \times 0.000018 \times(40-20)] \\
& =10,000+3.6 \\
& =10,003.6 \mathrm{ohms}
\end{aligned}
$$

Example 4-2. A carbon resistor has a resistance of 250,000 ohms at $20^{\circ} \mathrm{C}$ What is its resistance at $60^{\circ} \mathrm{C}$ ?

| Given: | Find: |  |
| ---: | :--- | ---: |
| Material | $=$ carbon | $R_{F}=?$ |
| $t_{i}$ | $=20^{\circ} \mathrm{C}$ |  |
| $t_{f}$ | $=60^{\circ} \mathrm{C}$ |  |
| $R_{i}$ | $=250,000 \mathrm{ohms}$ |  |

Solution:

$$
\begin{aligned}
& T_{c} \text { for carbon }=-0.0003 \text { (Appendix IV) } \\
R_{F} & =R_{i}+\left[R_{i} \times T_{c} \times\left(t_{f}-t_{i}\right)\right] \\
& =250,090+[250,000 \times(-0.0003) \times(60-20)] \\
& =250,000-3,000 \\
& =247,000 \mathrm{ohms}
\end{aligned}
$$

Relation of Material, Length, and Area to Resistance. From the foregoing explanations of the various factors affecting the resistance of conductors (disregarding temperature), the following mathematical expression has been derived:

$$
\begin{equation*}
\text { Resistance }=\frac{K \times \text { length }}{\text { area }} \text { or } \quad R=\frac{K L}{\Lambda} \tag{4-2}
\end{equation*}
$$

where $R=$ resistance, ohms
$K=$ specific resistance (Appendix IV)
$L=$ length, feet
$A=$ area, circular mils
Circular Mil Area. A mil is equal to one-thousandth of an inch. A square mil is equal to the area of a square whose sides are all one mil in length. A circular mil is an amount that is equal to the area of a circle whose diameter is one mil. To find the area in circular mils, simply square the diameter in mils. The circular mil is a smaller area than the square mil, and for convenience in arithmetic it is used to express the area of wire sizes.

Example 4-3. Find the area in circular mils of a wire 0.25 inch in diameter.

$$
\begin{array}{ll}
\text { Given : } & \text { Find: } \\
d=0.25 \text { in. } & \text { Area in cir mils }=?
\end{array}
$$

Solution:

$$
\begin{aligned}
0.25 \text { in. } & =250 \mathrm{mils} \\
\text { Area } & =250 \times 250=62,500 \mathrm{cir} \mathrm{mils}
\end{aligned}
$$

4-2. Specific Resistance. Definition. It has been stated that the resistance of a conductor will depend on the specific resistance of the material of which it is made. The specific resistance of a wire is the resistance of a circular mil-foot of that wirc. In other words, it is the resistance of a wire whose diameter is one mil and whose length is one foot.

Shapes of Wire and Conducting Materials. Wires manufactured for the transmission and distribution of electric currents are usually drawn round. Bus bars, switch blades, etc., are made rectangular but seldom enter into radio calculations; therefore they will be disregarded. The formula for finding the resistance of a wire then becomes

$$
\begin{equation*}
R=\frac{K L}{d^{2}} \tag{4-3}
\end{equation*}
$$

where $R=$ resistance, ohms
$K=$ specific resistance
$L=$ length, feet
$d=$ diameter, mils
Example 4-4. Find the resistance of a coil of wire, the average diameter of which is two inches and which consists of 126 turns of copper wire 20 mils in diameter.

| Given: |  | Find: |  |
| ---: | :--- | ---: | :--- |
| Diameter of coil | $=2 \mathrm{in}$. |  | Resistance $=?$ |
| Turns | $=126$ |  |  |
| Diameter of wire | $=20 \mathrm{mils}$ |  |  |
| Material | $=$ copper |  |  |

Solution:

$$
\begin{aligned}
\text { Length } & =\pi \times \text { diameter } \times \text { number of turns } \\
& =\frac{3.14 \times 2 \times 126}{12} \\
& =65.94 \mathrm{f} \\
\text { For copper, } K & =10.4(\text { Appendix IV) } \\
R & =\frac{K L}{d^{2}} \\
& =\frac{10.4 \times 65.94}{20 \times 20} \\
& =1.714 \mathrm{ohms}
\end{aligned}
$$

Wire Gauge. All wire is designated according to definite gauge sizes or numbers. Each number represents a wire of a certain diameter. In the United States, the American Wire Gauge is used for designating the size of any kind of wire. It is based on a constant ratio of cross-sectional areas between wires of successive gauge numbers. An increase in the gauge number will correspond to a decrease in the cross-sectional area but will also result in an increase in the resistance. A wire table showing the resistance per 1000 feet of copper wire (at $25^{\circ}$ centigrade) is given in Appendix V .

Example 4-5. What is the resistance of a coil having an average diameter of 1.5 inches and consisting of 320 turns of No. 28 copper wire?

Given:
Diameter of coil $=1.5 \mathrm{in} . \quad$ Resistance $=$ ?
Turns $=320$

$$
\text { Wire }=\text { No. } 28 \text { copper }
$$

Solution:

$$
\begin{aligned}
\text { Length of wire } & =\text { diameter } \times \pi \times \text { number of turns } \\
& =1.5 \times 3.14 \times 320 \\
& =1507 \mathrm{in} . \\
& =125.58 \mathrm{ft}
\end{aligned}
$$

Resistance of 1000 ft of No. 28 wire $=66.17$ ohms (Appendix $\mathrm{V}^{\text {( }}$

$$
\text { Resistance of } 125.58 \mathrm{ft}=\frac{125.58}{1000} \times 66.17=8.31 \mathrm{ohms}
$$

Conductance. Conductance is a term used to express the ease with which a material allows an electric current to flow through it. This is the opposite of resistance, and the unit of conductance, the mho, is obtained by spelling the unit of resistance, ohm, backward. By definition, conductance is the reciprocal of resistance. The reciprocal of any number is equal to one divided by that number. For example, the reciprocal of $\overline{5}$ is onefifth; the reciprocal of 10 is one-tenth, etc.

$$
\begin{equation*}
\text { Conductance }=\frac{1}{\text { resistance }} \text { or } G=\frac{1}{R} \tag{4-4}
\end{equation*}
$$

From this formula, it can be seen that a material having a high specific resistance would have a low specific conductance, and vice versa.

Example 4-6. What is the specific conductance of copper wire?

> Given:
> Material = copper

Find:
Specific conductance $=$ ?
Solution:

$$
\begin{aligned}
\text { Specific resistance } & =10.4 \text { (Appendix IV) } \\
\text { Specific conductance } & =\frac{1}{10.4} \\
& =0.0961 \mathrm{mho}
\end{aligned}
$$

4-3. Conductors. Conducting Material. A substance through which an electric current can flow easily is called a conductor. Erery substance is a conductor of electricity, at least to some slight extent, but some substances are far better conductors than others. Theoretically, the best material to be used as a conductor would be the one having the highest specific conductance; this would be the one having the lowest specific resistance.

Factors Determining Choice of a Conducting Material. The use of conducting materials for practical applications requires consideration of a number of other factors. These factors are (1) cost, (2) specific resistance, (3) ability to be fused, (4) ability to withstand nature's elements, (5) flexibility, (6) melting point, (7) weight, (8) elasticity, (9) tensile strength. For example, if a conductor is to be used for instrument work where the losses due to resistance should be as low as possible, the cost would be disregarded and the material having the lowest specific resistance would be used.

If a material is desired for general wiring purposes, other factors must be considered. The cost cannot be too high because of the large quantity to be used; the specific resistance must be low in order to minimize power ( $I^{2} R$ ) losses; the material should be easily joined whether it be soldered, brazed, or welded; the elements of nature such as heat, cold, and dampness should have no effect on it. It should be flexible enough to be handled and shaped into the various necessary bends that are peculiar to all wiring.

Finally, its melting point must be high enough to withstand abnormal temperature rises. In general, copper is used for most wiring purposes because it meets the necessary requirements best.

Sometimes a material that is elastic is needed for such uses as spring contacts and circuit breakers. Phosphor bronze is generally used for this purpose.

Whenever a sliding or wiping contact is required, the material used must be soft enough to prevent excessive wear on the surface with which it makes contact. Carbon, graphite, and soft copper, used individually or in combination with one of the others, are the materials used for making brushes (sliding contacts).

If the material desired is to be used as a resistor to limit the flow of electric current as in the case of rheostats, heaters, and resistors, a material having a high specific resistance is used. The melting point must also be high as this type of apparatus is constantly subjected to high temperatures.

The operation of a fuse depends on the ability of the material of which it is made to melt easily and thus break the circuit when more than its rated current is flowing. Therefore, materials used for making fuses must have a low melting point.

4-4. Insulators. Insulating Material. A perfect insulator is a material through which no electric current can pass. All materials emit a certain number of free electrons, no matter how few; therefore no such substance as a perfect insulator exists. However, very poor conductors are approximately such and are therefore used whenever an insulating material is required.

Breakdown Voltage of an Insulator. All materials offer some resistance to the flow of electric current. Insulators, or poor conductors, offer a high resistance to the flow of electricity; the higher the resistance, the better the insulator. If the voltage across an insulator is increased, the attractive force acting on the free electrons in the insulator will increase. This causes the free clectrons to move at an increased speed. If the voltage is high enough to increase the velocity of the electrons to a point that causes them to collide against the atoms with sufficient force to detach other electrons from the neutral atoms, a steady stream of free electrons will rapidly form through the insulating material and the insulator is now a conductor. The voltage required to cause this clectron flow, thus changing the insulator to a conductor, is called the breakdown voltage of the material.

Effect of Breakdown of the Insulation. This breakdown of the insulation between conductors is often accompanied by the passage of an electric spark. Sufficient heat is produced by this spark to burn a path through such insulating materials as paper, cloth, wood, or mica. Hard materials such as porcelain or glass will crack or allow a small path to be melted through them.

Dielectric Strength. Increasing the thickness of the insulator will increase the distance through which the voltage must force the electrons. This is the same as increasing the resistance of the insulator, for the resistance of any material will vary directly with its length. Therefore the breakdown voltage of a material will vary directly with its thickness. The dielectric strength of a material is expressed in volts per unit thickness of the material. In Appendix VI are listed some of the common materials used as insulators for radio and television apparatus and their corresponding dielectrie strength expressed in volts per thousandths of an inch thickness of the material.


Fig. 4-1.-Radio applications of insulating materials. (P. R. Mallory \& Co., Inc.)
Materials used for electrical insulation are generally manufactured compounds. The dielectric strength of a material will therefore vary with the manufacture and the proportion in which the required elements are combined. The physical structure of a material will vary with changes in weather, heat, cold, and moisture; this in turn will affect its dielectric strength.

Appendix VI indicates that a piece of cotton 0.001 inch thick has a dielectric strength of 300 volts. Increasing the thickness of the cotton to 0.005 inch will increase its dielectric strength five times, or to 1500 volts.

Effect of Alternating Current on Insulating Properties. The insulating property of a material is different for direct current than for an alternating
current. As the frequency of an alternating current is increased, the dielectric strength of a material will decrease. This decrease in dielectric strength not only varies inversely with the frequency but also decreases very rapidly with increases in frequency. The variation of the breakdown voltage of a material with a change in the frequency of the applied current must be taken into consideration in choosing a material for the dielectric of a capacitor. The various types of capacitors, their construction, and their use in radio circuits are discussed in Chap. IX.

Classification and Uses of Insulating Materials. Materials used as insulation for electrical apparatus may be classified as follows:

| 1. Vitreous | Glass, enamel |
| :---: | :---: |
| 2. Stony | Slate, marble, mica, porcelain, asbestos, ceramics |
| 3. Resinous | Shellac, resins, gums |
| 4. Bituminous | Asphalt, pitch |
| 5. Waxy | Beeswax, paraffin |
| 6. Elastic | Rubber, ebonite |
| 7. Oily | Oils of vegetable, animal, or mineral origin |
| 8. Cellulose | Paper, wood, cotton, linen, cambric, celluloid, cellophane, Bakelite, lucite |
| 9. Animal tiss | Silk, fiber, catgut |

Materials used for electrical insulation have a dual purpose, namely, to provide mechanical protection and to serve as electrical insulation. In selection of insulating material its use is the important factor in determining its electrical, physical, and meehanical characteristics. For example, if the insulator is to be subjected to great heat, as in soldering irons or toasters, mica must be used; if it must be fireproof, as in the power cord in small radios having a dropping resistor, asbestos is used; if space, flexibility, and a fair dielectric strength are to be the deciding factors, as in the dielectric for small fixed capacitors, cellulose and animal tissue materials are used; if the material to be insulated is of intricate design, so that it can be insulated only with a liquid that will harden when dry, as on the coils in audio and small power transformers, resinous, bituminous, and waxy materials are used. Where a high dielectric strength is desired, as in the case of radio transmitters and high-frequency or high-voltage transformers, glass and porcelain are used; where the insulation must remain liquid, like that used in large switches and circuit breakers to quench the are when the circuit is opened, the various oils are used.

4-5. Resistors. It has been stated that the conductors in any circuit are used to transmit the power to the appliance or devices used. In d-c circuits, all appliances and devices are considered as having only resistance. In a-c circuits, two new factors, inductance and capacitance, must be taken into consideration. How these factors affect the electric circuit will be
taken up in a later chapter on alternating currents. For the present, only d-c circuits and the corresponding resistance effect will be considered.

The number of resistors used in radio, television, and electronic circuits is quite large. It would be impossible to describe in detail all the resistors used. Improvements are constantly being made; new types are being developed and are continually replacing some of the older forms, thereby increasing the already vast number of types of resistors in use.

Classification of Resistors According to Material. There are two general types of construction employed in the manufacture of resistors, and the resistors are referred to as being either metallic or nonmetallic.

The material used in the metallic resistors is generally in the form of a wire or a ribbon, and these resistors are commonly called wire-wound resistors. The wire or ribbon is wound around a supporting form made of insulating material. The wire is generally an alloy containing two or more elements such as copper, iron, nickel, chromium, zinc, and manganese.

The material used in the nonmetallic resistors is carbon or graphite, both of which have a very high specific resistance. Because of the high specific resistance, the resistors can be made smaller than wire-wound resistors. As both carbon and graphite exist in the form of fine powder, it is necessary to add a substance, usually called a binder, that will hold the fine particles of carbon together. It is then formed into rods which are next cut into short pieces to make up the resistor. Nach piece is generally enclosed by some insulating material, and leads are attached to the ends. Resistors of very high values, as 10 megohms, etc., are sometimes made by applying a thin coating of carbon or graphite on a thin glass filament placed within a protective tube or on the inside wall of a glass tube. Metal caps attached at each end of the tube serve as connections to the deposit. Nonmetallic resistors are used quite extensively in radio, television, and electronic circuits because of the ease with which high resistance can be obtained and their low cost of manufacture.

Classification of Resistors According to Control. Resistors may be further classified as (1) fixed, (2) variable, (3) adjustable, (4) tapped, (5) automatic resistance control.

A fixed resistor is one whose value cannot be changed by any mechanical means. Fixed resistors may be made of carbon or of a metallic wire. The nonmetallic resistor made of carbon or graphite powder held together by a suitable binding substance is a widely used type of fixed resistor. Lowpower wire-wound fixed resistors are made by winding the wire on a Bakelite or fiber strip and attaching lugs to each of the ends in order to make connections to the resistor. A flexible low-power wire-wound resistor is made by winding a fine nichrome wire on a specially treated silk cord and then completely covering it with an impregnated fiber. High-power wire-
wound resistors are made by winding the wire on large threaded porcelain tubes and attaching copper terminals at each end; the entire unit is then dipped in enamel or a porcelain solution and baked.


A variable resistor is one whose value of resistance at its terminals may be varied, and it is commonly called a rheostat. The rheostat has a sliding contact arm that may be moved to any position along the resistor and has


Fig. 4-3.-Some of the various types of resistors used in radio. (Ohmite Manufacturing Company.)
one of its terminals attached to the contact arm and the other to one end of the resistance. As the position of the contact arm is varied, the value of the resistance between the two terminals will vary (see Fig. 4-26).

An adjustable resistor is one that may be adjusted to a desired value and then set at that value. It differs from the variable resistor in that once it is adjusted to the desired value it is kept at that value. Adjustable
resistors are always of the metallic type and are generally wound on porcelain forms. They are provided with one or more movable collars that may be clamped in a definite position after they have been adjusted to the desired value.

A tapped resistor is one that provides two or more definite values of resistance on a single unit. Tapped resistors are similar to adjustable resistors except that the collars are not made movable but are set at fixed positions along the resistor to give definite values of resistance. Tapped resistors are always of the metallic type; they may be of the low-power wire-wound varicty that uses a fiber or Bakelite


Fig. 4-4.-An automatic ballast-regulating tube that may be used as an automatic resistance-control resistor. (Amperite Company.) form or of the high-power wire-wound type that uses a porcelain form.

An automatic resistance control resistor is one whose resistance value changes automatically with a change in current or temperature. In the study of effects of temperature on the resistance of certain materials, it was found that as the temperature of certain metals increased the resistance increased. This principle is used in automatic line controls or ballast resistors, as they are usually called. $\Lambda$ nickel or iron wire is placed inside a glass tube filled with an inert gas such as hydrogen or inside an air-cooled metal case. When the current flowing through this unitincreases, it causes an increase in the temperature of the wire. The increase in temperature causes an increase in resistance which regulates the current and prevents it from rising excessively.

Use of Resistors. Resistors and rheostats are used in a number of ways to adjust the current and voltage of electrical circuits. In radio and television circuits they are used as voltage dividers, loads for the output of vacuum tubes, resistors to provide the proper grid bias, current regulators in filament circuits, filter networks, grid leaks, etc. The type of resistor to be used is determined by its application and the tolerance permitted in its value of resistance.

Commercial resistors of the wire-wound type may be obtained with a tolerance as low as $\pm 1$ per cent, which means that their values of resistance are accurate within one per cent of their rated values. Such resistors are used as multipliers and shunts for converting low-range voltmeters and ammeters to higher range voltmeters and ammeters. They are sometimes used in high-quality radio and television receivers and in electronic equipment when exact resistance values are of importance.

In selecting a resistor, its power rating as well as its resistance valuc must be taken into consideration. For example, a resistor rated at one watt and 10,000 ohms should not be used to carry a current of more than 10 milliamperes, as

$$
I=\sqrt{\frac{P}{R}}=\sqrt{\frac{1}{10,000}}=0.01 \text { ampere, or } 10 \text { milliamperes }
$$

Standard Color Code for Resistors. A color code has been established to indicate the resistance value of fixed resistors. This color code is very useful for identifying the values of the carbon resistors used so extensively in radio and television work, especially since many resistors of different ohmic values have the same appearance. Appendix VII lists the standard color coding for resistors and illustrates the method used to determine the resistance value from the color markings.

4-6. Electric Circuits. Resistance of Electric Circuits. The total resistance of a circuit is equal to the total pressure applied to that circuit divided by the current flowing in the circuit; expressed mathematically

$$
\begin{equation*}
R_{T}=\frac{E_{T}}{I_{T}} \tag{4-5}
\end{equation*}
$$

where $R_{T}=$ total line resistance, ohms
$E_{T}=$ total line voltage, volts
$I_{T}=$ total line current, amperes
The resistance of any particular part of a circuit is equal to the potential difference between the terminals of that part of the circuit divided by the current flowing through that part of the circuit; expressed mathematically

$$
\begin{equation*}
r=\frac{e}{i} \tag{4-6}
\end{equation*}
$$

where $r=$ branch resistance, ohms
$e=$ branch voltage, volts
$i=$ branch current, amperes
Four Kinds of Circuits. There are four ways in which electrical appliances may be connected: (1) simple circuit, (2) series circuit, (3) parallel circuit, (4) combination circuit.

4-7. The Simple Circuit. A circuit


Fig. 4-5.-A simple circuit. is said to be a simple circuit when one and only one resistance is connected directly across the source of power (see Fig. 4-5).

In this circuit

$$
\begin{aligned}
& R_{T}=\frac{E_{T}}{I_{T}} \\
& P_{T}=E_{T} \times I_{T} \\
& W=P_{T} \times{ }_{T}
\end{aligned}
$$

4-8. The Series Circuit. When two or more resistances are connected end to end so that the current passes in turn from one to another, it is a series circuit. In Fig. 4-6, the current leaves the negative side of the generator and returns through the positive side, thus completing the electric circuit.

Currents in a Series Circuit. As there is only one path through which the current may flow and as all the current that leaves the generator must return to it, the amount of current flowing in all parts of the circuit must be the same. Therefore


Voltages of a Series Circuit. The voltmeters $V_{1}, V_{2}$, and $V_{3}$ are connected to indicate the pressure required to force the current through the resistors $r_{1}, r_{2}$, and $r_{3}$, respectively. These voltage readings are also called the voltage drops at the resistors. As $E_{\theta}$ represents the total voltage required to force the current through the complete circuit, the voltage supplied by the generator must be equal to the sum of the voltage drops of the circuit, or

$$
\begin{equation*}
E_{G}=e_{1}+e_{2}+e_{3} \tag{4-8}
\end{equation*}
$$

Resistance of a Series Circuit. The current in its flow through this circuit must pass through all the resistors before it can return to the starting point. The total resistance offered to flow of current will therefore be the sum of all the resistances, or

$$
\begin{equation*}
R_{T}=r_{1}+r_{2}+r_{3} \tag{4-9}
\end{equation*}
$$

Power of the Serics Circuit. Each of the resistors consumes power, and as all the power must come from the generator, the total power taken by the series circuit must be equal to the sum of the separate powers, or

$$
\begin{equation*}
P_{T}=p_{1}+p_{2}+p_{3} \tag{4-10}
\end{equation*}
$$

Energy of the Series Circuit. As electrical energy is transformed to heat energy at each resistor and as all the energy must be supplied by the generator, the total energy of the series circuit must be equal to the sum of the separate amounts of energy, or

$$
\begin{equation*}
W=\mathrm{en}_{1}+\mathrm{en}_{2}+\mathrm{en}_{3} \tag{4-11}
\end{equation*}
$$

Characteristics of the Series Circuit. The characteristics of the series circuit may be summarized as follows:

1. The current in all parts of the circuit is the same.


Fig. 4-7.-..-Power sources connected in series.
2. The voltage applied, or the line voltage, is equal to the sum of the separate voltage drops of the circuit.
3. The resistance of the complete circuit is equal to the sum of the separate resistances of the circuit.
4. The total power is equal to the sum of the powers of the separate resistors.
5. The total energy is equal to the sum of the energies of the separate resistors.

Uses of the Series Circuit. Series circuits are used in radio and television receivers and may be found in some of the following circuits: (1) the plate circuit of vacuum tubes, (2) the filament (or heater) circuit of tubes in small sets, (3) in a circuit to which a dropping resistor is connected to limit the voltage across a certain part of the circuit.

Power supplies are connected in series when a high voltage is to be obtained from several lower voltage units. For example, two 110 -volt generators may be connected in series in order to obtain a 220 -volt power line, or three six-volt storage batteries may be connected in series in order
to obtain a total of 18 volts across the combination. When power supplies are connected in series, the terminals of opposite polarities must be connected together. This is illustrated in Fig. 4-7.

Disadvantages of the Series Circuit. If the wire should break at the point $X$ (Fig. 4-6) or at any other point, the circuit would be broken and no current would be able to flow through any part of the circuit. This is a serious disadvantage of the series circuit, for if a break occurs in any part of the circuit the entire circuit becomes useless.

If a pressure of 110 volts is desired for each resistor of Fig. 4-6, the line voltage would have to be 330 volts. If four more resistors were added to the circuit and each required a pressure of 110 volts, the line voltage would then have to be 770 volts. It can readily be seen that a high line voltage would be required if a number of 110 -volt lamps (in place of resistors) were to be connected in series; thercfore the series circuit is not practical for lighting circuits.

Solution of Series-circuit Problems. Using the rules for series circuits as outlined and the fundamental principles of Ohm's law, it is possible to solve any problem involving series circuits. This can best be illustrated by the following example.

Example 4-7. A 10 -, a 15 -, and a 30 -ohm resistor are connected in series across a 110 -volt line. (a) What is the resistance of the circuit? (b) If the circuit is used for 10 hours, how much energy is consumed by each resistor? (c) What is the total amount of energy consumed?

| Given: | Find: |  |
| ---: | ---: | :--- |
| $E_{T}$ | $=110$ volts | $R_{T}=?$ |
| $r_{1}$ | $=10$ ohms | $\mathrm{en}_{1}=?$ |
| $r_{2}$ | $=15 \mathrm{ohms}$ | $\mathrm{en}_{2}=?$ |
| $r_{3}$ | $=30 \mathrm{ohms}$ | $\mathrm{en}_{3}=?$ |
| $T$ | $=10 \mathrm{hr}$ | $W_{T}=?$ |

Solution:
In solving circuit problems it is always best to draw a circuit diagram. The circuit diagram for this problem would be drawn as shown in Fig. 4-8.


Fig. 4-8.
From the rules for series circuits
(a)

$$
R_{T}=r_{1}+r_{2}+r_{3}=10+15+30=55 \mathrm{ohms}
$$

(b) In order to find the energy, it is necessary to know the value of the power and the hours. The power of each resistor may best be found by $i^{2} r$, and since only the value of $r$ is known it is now necessary to solve for the current,

$$
I_{T}=\frac{E_{T}}{R_{T}}=\frac{110}{55}=2 \mathrm{amp}
$$

as $\quad I_{T}=i_{1}=i_{2}=i_{3}$
then $p_{1}=I_{T}^{2} \times r_{1}=2 \times 2 \times 10=40$ watts
$p_{2}=I_{T}^{2} \times r_{2}=2 \times 2 \times 15=60$ watts
$p_{3}=I_{T}{ }^{2} \times r_{3}=2 \times 2 \times 30=120$ watts
$I_{T}=p_{1}+p_{2}+p_{3}=40+60+120=220$ watts
It is good practice continually to check the problem in order to locate any mathematical errors. In this problem, the total power can be checked by using the formula

$$
P_{T}=E_{T} \times I_{T}=110 \times 2=220 \text { watts }
$$

This checks with the value obtained before by using a different method, and therefore the solution is mathematically correct. Continuing the solution of the problem

$$
\begin{align*}
& \mathrm{en}_{1}=p_{1} \times T=40 \times 10=400 \text { watt-hours } \\
& \mathrm{en}_{2}=p_{2} \times T=60 \times 10=600 \text { watt-hours } \\
& \mathrm{en}_{3}=p_{3} \times T=120 \times 10=1200 \text { watt-hours } \\
& W_{T}=\mathrm{en}_{1}+\mathrm{en}_{2}+\mathrm{en}_{3}=400+600+1200=2200 \text { watt-hours } \tag{c}
\end{align*}
$$

Checking the total energy by using the formula

$$
W_{T}=P_{T} \times T=220 \times 10=2200 \text { watt-hours, or } 2.2 \mathrm{kwh}
$$

This is the same answer as the above. Therefore the problem is mathematically correct.

4-9. The Parallel Circuit. When two or more resistances are connected so that the current will have two or more paths in which it may flow, it is a parallel circuit. In Fig. 4-9, the current leaves the generator at $A$ and when it reaches $B$ it divides into two paths, part of the current going toward $G$ and the remainder toward $C$. The current going toward $C$ upon reaching that point has two paths, and it will divide, part of this current going toward $F$ and the remainder toward $D$. At $F$, the current from $E$ joins with the current from $C$ and flows toward $G$. At $G$, the current from $F$ joins with the current from $B$ and returns to the generator at $I I$.

Currents in a Parallel Circuit. Figure 4-10 shows the distribution of the current flow in the circuit of Fig. 4-9. The values of current shown throughout the circuit indicate the amount of current flowing in those parts of the circuit. It can be seen from this diagram that the line current is equal to the sum of the branch currents, as 4 amperes +6 amperes +2 amperes $=12$ amperes.

$$
\begin{equation*}
I_{T}=i_{1}+i_{2}+i_{3} \tag{4-12}
\end{equation*}
$$

Voltages of a Parallel Circuit. The current in flowing from the gencrator to $r_{1}$ does not pass through any appreciable amount of resistance; therefore the voltage drop is negligible and $e_{1}=E_{G}$. Likewise as there is no appreciable resistance offered to the current going from $r_{1}$ to $r_{2}$ or from $r_{2}$ to $r_{3}$, the voltage across $r_{2}$ and $r_{3}$ is equal to that across $r_{1}$ and

$$
\begin{equation*}
E_{G}=e_{1}=e_{2}=e_{3} \tag{4-13}
\end{equation*}
$$



Fig. 4-9.-A parallel circuit.
These conclusions are based on the assumption that the connecting wires have no resistance, which actually is not truc; however, the resistance of these wires is ordinarily so low that the resulting voltage drop can be neglected. If more accurate values of the branch voltages are desired, the drop in the connecting wires can be calculated and subtracted from the line voltage, the remainder being the voltage at the resistor.


Fig. 4-10.-Current distribution of the parallel circuit shown in Fig. 4-9.
Resistance of a Parallel Circuit. The resistance of a parallel circuit cannot be calculated so easily as for the series circuit. It is calculated by the conductance method. The resistance of a parallel circuit may be obtained by using the formula

$$
\begin{equation*}
R_{\boldsymbol{T}}=\frac{1}{\frac{1}{r_{1}}+\frac{1}{r_{2}}+\frac{1}{r_{3}}} \tag{4-14}
\end{equation*}
$$

where $R_{T}$ is the total resistance of the circuit and $r_{1}, r_{2}$, and $r_{3}$ are the resistances of the individual branches.

Substituting the values from Fig. 4-10 in the above formula

$$
R_{T}=\frac{1}{\frac{1}{30}+\frac{1}{20}+\frac{1}{60}}=\frac{1}{\frac{2+3+1}{60}}=\frac{1}{\frac{6}{60}}=\frac{60}{6}=10 \mathrm{ohms}
$$

The resistance of the line is found to be 10 ohms, and it should be noticed that this is lower than the lowest individual resistance in the circuit which is 20 ohms. This is characteristic of all parallel circuits.

When the parallel circuit contains only two circuit clements, Fq. (4-14) may be expressed as,

$$
\begin{align*}
R_{T} & =\frac{r_{1} r_{2}}{r_{1}+r_{2}}  \tag{4-14a}\\
r_{2} & =\frac{R_{T} r_{1}}{r_{1}-R_{T}} \tag{4-14b}
\end{align*}
$$

Power of the Parallel Circuit. Each of the resistors consumes power, and as all the power must come from the generator, the total power taken by the parallel circuit must be equal to the sum of the separate powers (note that this is the same as for the series circuit), or

$$
\begin{equation*}
P_{T}=p_{1}+p_{2}+p_{3} \tag{4-10}
\end{equation*}
$$

Energy of the Parallel Circuit. As electrical energy is transformed to heat energy at each resistor and as all the energy must be supplied by the gencrator, the total energy of the parallel circuit must be equal to the sum of the separate amounts of energy (note that this is the same as for the series circuit), or

$$
\begin{equation*}
W=\mathrm{en}_{1}+\mathrm{en}_{2}+\mathrm{en}_{3} \tag{4-11}
\end{equation*}
$$

Characteristics of the Parallel Circuit. The characteristics of the parallel circuit may be summarized as follows:

1. The line current is equal to the sum of the currents in the separate branches.
2. The voltage across each branch is the same and is cqual to the line voltage.
3. The resistance of a parallel circuit is equal to the reciprocal of the sum of the reciprocals of each branch resistance. It is always less than the lowest branch resistance.
4. The total power is equal to the sum of the powers of the separate branches.
5. The total energy is equal to the sum of the energies of the separate branches.

Uses of the Parallel Circuit. Parallel circuits are used wherever a con-stant-voltage power-supply system is needed. IIouse wiring is a good example of the use of this type of circuit.

Power supplies are connected in parallel when a higher current is needed and the voltage is to remain constant. For example, tiwo 110 -volt generators may be connected in parallel in order to obtain an increased current output (and consequently power) at 110 volts. Only one of them need be used if the current required is equal to or is less than the current rating of one of the generators. Storage batteries may be connected in parallel if the current rating is to be increased without an increase in voltage. When power supplies are connected in parallel, all terminals of similar polarity must be connected together. The voltage across the combination is the same as the voltage of an individual unit, and the current rating becomes equal to the sum of the individual current ratings. Parallel circuits are illustrated in Fig. 4-11.


Fig. 4-11.-Power sources connected in parallel.
Disadvantage of the Parallel Circuit. A disadvantage of the parallel circuit is that as additional appliances or loads are added to the circuit the line current increases. If the increased amount raises the current above the safe carrying capacity of the wiring used, it becomes necessary to rewire the circuit with a larger size wire or to install additional feeder circuits.

Advantage of the Parallel Circuit. An advantage of the parallel circuit is that, if a break occurs in any one of the branch circuits ( $B G, C F$, or $D E$ in Fig. 4-9 or 4-10), it will have no effect on the other circuits. In house wiring, the use of parallel circuits makes it possible to switch any light or appliance on or off without affecting the other lights and appliances.

Solution of Parallel-circuit Problems. Using the rules for parallel circuits as outlined and the fundamental principles of Ohm's law, it is possible to solve any problem involving parallel circuits. This can best be illustrated by the following example.

Example 4-8. A 10 -, a 15 -, and a 30 -ohm resistor are connected in parallel across a 110 -volt line. (a) What is the resistance of the circuit? (b) If the circuit is used for 10 hours, how much energy is consumed by each resistor? (c) What is the total amount of energy consumen?

| Given: |  |
| ---: | :--- |
| $E_{T}$ | $=110$ volts |
| $r_{1}$ | $=10 \mathrm{ohms}$ |
| $r_{2}$ | $=15 \mathrm{ohms}$ |
| $r_{3}$ | $=30 \mathrm{ohms}$ |
| $T$ | $=10 \mathrm{hr}$ |

$$
\begin{aligned}
E_{T} & =110 \text { volts } \\
r_{1} & =10 \mathrm{ohms} \\
r_{2} & =15 \mathrm{ohms} \\
r_{3} & =30 \mathrm{ohms} \\
T & =10 \mathrm{hr}
\end{aligned}
$$

Find:
$h_{T}=$ ?
$\mathrm{en}_{1}=$ ?
$\mathrm{en}_{2}=$ ?
$\mathrm{en}_{3}=$ ?
$W_{T}=$ ?

The circuit diagram for this problem would be as shown in Fig. 4-12,


Fig. 4-12.

## Solution:

From the rules for parallel circuits,
(a)

$$
R_{T}=\frac{1}{\frac{1}{r_{1}}+\frac{1}{r_{2}}+\frac{1}{r_{3}}}=\frac{1}{\frac{1}{10}+\frac{1}{15}+\frac{1}{30}}=\frac{1}{\frac{6}{30}}=\frac{30}{6}=5 \mathrm{ohms}
$$

(b)

$$
\begin{aligned}
i_{1} & =E_{T} \div r_{1}=110 \div 10=11 \mathrm{amp} \\
i_{2} & =E_{T} \div r_{2}=110 \div 15=7.33 \mathrm{amp} \\
i_{3} & =E_{T} \div r_{3}=110 \div 30=3.67 \mathrm{amp} \\
I_{T} & =i_{1}+i_{2}+i_{3}=11+7.33+3.67=22 \mathrm{amp}
\end{aligned}
$$

Checking the answer:

$$
\begin{gathered}
I_{T}=\frac{E_{T}}{R_{T}}=\frac{110}{5}=22 \mathrm{amp} \\
p_{1}=E_{T} \times i_{1}=110 \times 11=1210 \mathrm{watts} \\
p_{2}=E_{T} \times i_{2}=110 \times 7.33=806.3 \mathrm{watts} \\
p_{3}=E_{T} \times i_{3}=110 \times 3.67=403.7 \mathrm{watts} \\
P_{T}=p_{\mathrm{t}}+p_{2}+p_{3}=1210+806.3+403.7=2420 \mathrm{watts}
\end{gathered}
$$

Checking the total power:

$$
P_{T}=E_{T} \times I_{T}=110 \times 22=2420 \text { watts }
$$

$$
\mathrm{en}_{1}=p_{\mathrm{t}} \times T=1210 \times 10=12,100 \text { watt-hours, or } 12.1 \mathrm{kwh}
$$

$$
\mathrm{cn}_{2}=p_{2} \times T=806.3 \times 10=8063 \text { watt-hours, or } 8.063 \mathrm{kwh}
$$

$$
\mathrm{en}_{3}=p_{3} \times T=403.7 \times 10=4037 \text { watt-hours, or } 4.037 \mathrm{kwh}
$$

(c)

$$
W_{T}=\mathrm{en}_{1}+\mathrm{en}_{2}+\mathrm{en}_{3}=12.1+8.063+4.037=24.2 \mathrm{kwh}
$$

Checking this answer:

$$
W_{T}=P_{T} \times T=2420 \times 10=24,200 \text { watt-hours, or } 24.2 \mathrm{kwh}
$$

4-10. Simple Combination Circuits. When a circuit contains both series and parallel circuits, it is a combination circuit. Combination circuits may be connected in series-parallel or parallel-series.

The Series-parallel Circuit. When groups of parallel circuits are connected in series, it is called a series-parallel circuit. In Fig. 4-13, the current leaves the generator at $A$ and returns at $G$. The current divides at point $C$ and joins at $D$, dividing and joining a second time at points $E$ and $F$ as indicated by the arrows.

Expressed mathematically,

$$
\begin{equation*}
I_{T}=\left(i_{1}+i_{2}\right)=\left(i_{3}+i_{4}+i_{5}\right) \tag{4-15}
\end{equation*}
$$

Close observation of Fig. 4-13 will show that this circuit is fundamentally a series circuit; therefore the voltage drops should equal the line voltage, or

$$
\begin{equation*}
E_{G}=e_{1}+e_{2}+e_{3} \tag{4-16}
\end{equation*}
$$



Solution of Series-parallel Circuits. To solve series-parallel circuits, each group of parallel resistance values are first combined into an equivalent single resistance value by using the reciprocal method. The whole is then treated as a series circuit.

Example 4-9. Find the voltage and current of each resistor in the circuit of Fig. 4-14.

## Given:



Fig. 4-14.

Solution:

$$
\begin{aligned}
& R_{G R \cdot 1}=\frac{1}{\frac{1}{r_{2}}+\frac{1}{r_{3}}}=\frac{1}{\frac{1}{40}+\frac{1}{40}}=\frac{40}{2}=20 \mathrm{ohms} \\
& R_{G A \cdot 2}=\frac{1}{\frac{1}{r_{4}}+\frac{1}{r_{5}}+\frac{1}{r_{6}}}=\frac{1}{40}+\frac{1}{20}+\frac{1}{40}=\frac{40}{4}=10 \mathrm{ohms}
\end{aligned}
$$

Figure 4-14 then becomes a simple series circuit as illustrated in Fig. 4-15.


Fig. 4-15.
$R_{T}=r_{1}+R_{G R \cdot 1}+R_{G R \cdot 2}=20+20+10=50$ ohms
$I_{T}=E_{G} \div R_{T}=100 \div 50=2 \mathrm{amp}$
$e_{1}=I_{T} \times r_{1}=2 \times 20=40$ volts
$e_{2}=I_{T} \times R_{G R \cdot 1}=2 \times 20=40$ volts
$e_{3}=I_{T} \times R_{G R-2}=2 \times 10=20$ volts
Checking:

$$
\begin{aligned}
E_{G} & =e_{1}+e_{2}+e_{3}=40+40+20=100 \mathrm{volts} \\
i_{1} & =I_{T}=2 \mathrm{amp} \\
i_{2} & =E_{G R \cdot 1} \div r_{2}=40 \div 40=1 \mathrm{amp} \\
i_{3} & =E_{G R-1} \div r_{3}=40 \div 40=1 \mathrm{amp} \\
I_{G R \cdot 1} & =i_{2}+i_{3}=1+1=2 \mathrm{amp} \\
i_{4} & =E_{G R \cdot 2} \div r_{4}=20 \div 40=0.5 \mathrm{amp} \\
i_{5} & =E_{G R \cdot 2} \div r_{5}=20 \div 20=1.0 \mathrm{amp} \\
i_{6} & =E_{G R \cdot 2} \div r_{6}=20 \div 40=0.5 \mathrm{amp} \\
I_{G R-2} & =i_{4}+i_{5}+i_{6}=0.5+1.0+0.5=2 \mathrm{amp}
\end{aligned}
$$

The voltage across each resistance, each parallel group of resistances, and across the entire circuit is now known. The value of the current flowing in each resistance, in each group of resistances, and in the entire circuit is also known. If the power used by the entire circuit or any part of it is desired, it can easily be obtained by using the power formula.

The Parallel-series Circuit. When groups of series circuits are connected in parallel, it is called a parallel-series circuit. Such a circuit is shown in Fig. 4-16. The current leaves the generator at $A$ and returns at $H$. It divides into three paths, $B G, C F$, and $D E$. The directions of the currents are shown by the arrows.

$$
\begin{equation*}
I_{T}=I_{1}+I_{2}+I_{3} \tag{4-17}
\end{equation*}
$$

As each group is a series circuit, the sum of the voltage drops in each group should equal the line voltage.

$$
\begin{equation*}
E_{G}=E_{G R \cdot 1}=E_{G R \cdot 2}=E_{G R \cdot 3} \tag{4-18}
\end{equation*}
$$

Solution of Parallel-series Circuits. To solve parallel-series circuits, each group of series resistance values is first solved for its equivalent single

resistance by adding all resistances in that group. The whole is then treated as a parallel circuit.

Example 4-10. Find the total resistance of each branch, the total resistance of the circuit, the current in each resistor, and the voltage across each resistor of the circuit shown in Fig. 4-17.


Fig. 4-17.
Solution:

$$
\begin{aligned}
& R_{G R \cdot 1}=10+40+30=80 \mathrm{ohms} \\
& R_{G R \cdot 2}=60+20=80 \mathrm{ohms} \\
& R_{G R \cdot 3}=30+50=80 \mathrm{ohms}
\end{aligned}
$$

The circuit of Fig. 4-17 can now be simplified and becomes a simple parallel circuit as shown in Fig 4-18.


Fig. 4-18.

$$
\begin{aligned}
R_{T} & =\frac{1}{\frac{1}{R_{G R \cdot 1}}+\frac{1}{R_{G R \cdot 2}}+\frac{1}{R_{G R \cdot 3}}}=\frac{1}{\frac{1}{80}+\frac{1}{80}+\frac{1}{80}}=\frac{80}{3}=26.66 \mathrm{ohms} \\
I_{T} & =\frac{E_{G}}{R_{T}}=\frac{100}{26.66}=3.75 \mathrm{amp} \\
I_{G R \cdot 1} & =\frac{E_{G R \cdot 1}}{R_{G R \cdot 1}}=\frac{100}{80}=1.25 \mathrm{amp} \\
I_{G R \cdot 2} & =\frac{E_{G R \cdot 2}}{R_{G R \cdot 2}}=\frac{100}{80}=1.25 \mathrm{amp} \\
I_{G R \cdot 3} & =\frac{E_{G R \cdot 3}}{R_{G R \cdot 3}}=\frac{100}{80}=1.25 \mathrm{amp}
\end{aligned}
$$

Checking:

$$
\begin{aligned}
I_{T} & =I_{G R \cdot 1}+I_{G R \cdot 2}+I_{G R \cdot 3}=1.25+1.25+1.25=3.75 \mathrm{amp} \\
e_{1} & =I_{G R \cdot 1} \times r_{1}=1.25 \times 10=12.5 \mathrm{volts} \\
e_{2} & =I_{G R \cdot 1} \times r_{2}=1.25 \times 40=50 \mathrm{volts} \\
e_{3} & =I_{G R \cdot 1} \times r_{3}=1.25 \times 30=37.5 \mathrm{volts}
\end{aligned}
$$

Checking:

$$
\begin{aligned}
E_{G R \cdot 1} & =e_{1}+e_{2}+e_{3}=12.5+50+37.5=100 \text { volts } \\
e_{4} & =I_{G R \cdot 2} \times r_{4}=1.25 \times 60=75 \text { volts } \\
e_{5} & =I_{G R \cdot 2} \times r_{6}=1.25 \times 20=25 \text { volts }
\end{aligned}
$$

Checking:

$$
\begin{aligned}
E_{G R \cdot 2} & =e_{4}+e_{5}=75+25=100 \text { volts } \\
e_{6} & =I_{G R \cdot 3} \times r_{6}=1.25 \times 30=37.5 \text { volts } \\
e_{7} & =I_{G R \cdot 3} \times r_{7}=1.25 \times 50=62.5 \text { volts }
\end{aligned}
$$

Checking:

$$
E_{G R \cdot 3}=e_{6}+e_{7}=37.5+62.5=100 \text { volts }
$$

If the power used by any part of the circuit or by the entire circuit is desired, it can easily be obtained by using the power formulas.

Advantages of Combination Circuits. Combination circuits combine the advantages of both series and parallel circuits and minimize their disadvantages. Generally, less copper is required and a smaller size wire can be used. Also, if one resistor should burn out and thereby cause an open circuit, it may affect only one or two branch circuits or it may affect the entire circuit depending upon the kind of circuit and the location of the faulty resistor.

Uses of Combination Circuits. Combination circuits are used whenever various types of circuits must be fed from the same power supply. In an automobile the starting, lighting, and ignition circuits are all individual circuits joined to make a combination circuit drawing its power from one battery.

Radio and television receivers contain a number of separate circuits such as the tuning circuits, r-f amplifiers, oscillator, detector, a-f amplifier, and synchronizing and picture-tube circuits. Individually they may be simple series or parallel circuits or complex combination circuits. When a receiver is considered as a whole, the result is a complicated combination circuit. In solving radio and television circuit problems, each circuit is solved separately and then combined to obtain the final result.

Power supplies are connected in series to get a higher voltage and connected in parallel to obtain a higher current. Rows of series-connected battery cells may be so connceted that the rows themselves are grouped in parallel to form a parallel-series arrangement as shown in Fig. 4-19.


Fig. 4-19.-Parallel-series grouping of battery cells.


Fig. 4-20.-Series-parallel grouping of battery cells.

Rows of parallel-connected battery cells may be so connected that the rows themselves are grouped in series to form a series-parallel arrangement as shown in Fig. 4-20.

4-11. More Advanced Combination Circuits. Many combination circuits are neither simple series-parallel nor parallel-series circuits. The resistances combine to form complicated combination circuits.

To solve such circuits, it is necessary to perform the following steps in the order listed:

1. Combine the resistance values in each group to obtain one single equivalent resistance value for each section.
2. Combine the resistance values of all sections to obtain one single equivalent resistance value for the line.
3. Solve for the line current.
4. Find the current flowing in each resistor.
5. Find the voltage across each resistor.

In the solution of some circuits, steps 4 and 5 may have to be interchanged.

Example 4-11. Find the current distribution and the voltage drop across each resistor in the circuit shown in Fig. 4-21.


Fig. 4-21.

$$
\begin{aligned}
R_{G R \cdot 1} & =\frac{1}{\frac{1}{1000}+\frac{1}{1500}}=\frac{3000}{5}=6 \\
R_{G R \cdot 2}= & 500+700=1200 \mathrm{ohms}
\end{aligned}
$$

$$
R_{S e c \cdot 1}=\frac{1}{\frac{1}{R_{G R \cdot 1}}+\frac{1}{R_{G R \cdot 2}}}=\frac{1}{\frac{1}{600}+\frac{1}{1200}}=\frac{1200}{3}=400 \text { ohms }
$$

$$
R_{G R \cdot 3}=400+800=1200 \mathrm{ohms}
$$

$$
R_{S e c \cdot 2}=\frac{1}{\frac{1}{R_{800}}+\frac{1}{R_{G R .3}}}=\frac{1}{\frac{1}{600}+\frac{1}{1200}}=\frac{1200}{3}=400 \mathrm{ohms}
$$

$$
R_{G R \cdot 4}=100+20=120 \mathrm{ohms}
$$

$$
R_{G R \cdot 5}=\frac{1}{\frac{1}{100}+\frac{1}{200}+\frac{1}{600}}=\frac{600}{10}=60 \mathrm{ohms}
$$

$$
R_{S e c .3}=\frac{1}{\frac{1}{R_{G R .4}}+\frac{1}{R_{G R \cdot 5}}}=\frac{1}{\frac{1}{120}+\frac{1}{60}}=\frac{120}{3}=40 \mathrm{ohms}
$$

By substitution of values found for sections 1,2, and 3 the equivalent circuit may now be drawn as shown in Fig. 4-22.


Fig. 4-22.

Close observation of Fig. 4-22 will show that this circuit is now a simple parallelseries circuit having two groups of series resistors connected in parallel.

$$
\begin{aligned}
& R_{G R \cdot A}=R_{S e c \cdot 1}+R_{S e c \cdot 2}=400+400=800 \mathrm{ohms} \\
& R_{G R \cdot B}=R_{20}+R_{S e c \cdot 3}+R_{60}=20+40+60=120 \mathrm{ohms} \\
& R_{T}=\frac{1}{\frac{1}{R_{G R \cdot A}}+\frac{1}{R_{G R \cdot B}}}=\frac{1}{\frac{1}{800}+\frac{1}{120}}=\frac{1}{\frac{3+20}{2400}}=\frac{2400}{23}=104.3 \mathrm{ohms} \\
& I_{T}=\frac{E_{T}}{R_{T}}=\frac{240}{104.3}=2.30 \mathrm{amp} \\
& I_{G R \cdot A}=\frac{E_{T}}{R_{G R \cdot A}}=\frac{240}{800}=0.300 \mathrm{amp} \\
& I_{G R \cdot B}=\frac{E_{T}}{R_{G R \cdot B}}=\frac{240}{120}=2.00 \mathrm{amp} \\
& I_{T}=I_{G R \cdot A}+I_{G R \cdot B}=0.300+2.00=2.30 \mathrm{amp} \\
& E_{S e c \cdot 1}=I_{\text {GR } \cdot \mathrm{A}} \times R_{\text {Sec. } 1}=0.300 \times 400=120 \text { volts } \\
& E_{S e c \cdot 2}=I_{G R \cdot A} \times R_{\text {Sec. } 2}=0.300 \times 400=120 \text { volts } \\
& E_{G R \cdot A}=E_{S e c \cdot 1}+E_{S e c \cdot 2}=120+120=240 \text { volts } \\
& E_{20}=I_{\text {GK }-B} \times R_{20}=2 \times 20=40 \text { volts } \\
& E_{S e c \cdot 3}=I_{G R \cdot B} \times R_{S e c \cdot 3}=2 \times 40=80 \text { volts } \\
& E_{60}=I_{G R \cdot B} \times R_{60}=2 \times 60=120 \text { volts } \\
& E_{G R \cdot B}=E_{20}+E_{S e c \cdot 3}+E_{60}=40+80+120=240 \text { volts } \\
& i_{G R .2}=\frac{E_{S e c-1}}{R_{G R .2}}=\frac{120}{1200}=0.10 \mathrm{amp} \\
& i_{1000}=\frac{E_{\text {Sec } .1}}{R_{1000}}=\frac{120}{1000}=0.12 \mathrm{amp} \\
& i_{15 c 0}=\frac{E_{S e c-1}}{R_{1500}}=\frac{120}{1500}=0.080 \mathrm{amp} \\
& I_{S e C .1}=i_{G R \cdot 2}+i_{1000}+i_{1600}=0.10+0.12+0.080=0.300 \mathrm{amp} \\
& E_{500}=i_{G R .2} \times R_{\text {b00 }}=0.10 \times 500=50 \text { volts } \\
& E_{700}=i_{G R \cdot 2} \times R_{700}=0.10 \times 700=70 \text { volts } \\
& E_{\text {Sce }} \cdot=E_{500}+E_{700}=50+70=120 \text { volts }
\end{aligned}
$$

$$
\begin{aligned}
& i_{600}=\frac{E_{\text {Scc. } 2}}{R_{600}}=\frac{120}{600}=0.200 \mathrm{amp} \\
& i_{G R .3}=\frac{E_{S e c .2}}{R_{G R .3}}=\frac{120}{1200}=0100 \mathrm{amp} \\
& I_{S C C \cdot 2}=i_{600}+i_{G R \cdot 3}=0.200+0.100=0.300 \mathrm{amp} \\
& E_{400}=i_{G R \cdot 3} \times R_{400}=0.100 \times 400=40 \text { volts } \\
& E_{800}=i_{G R \cdot 3} \times R_{800}=0.100 \times 800=80 \text { volts } \\
& E_{S c c-2}=E_{400}+E_{500}=40+80=120 \text { volts } \\
& i_{G R .4}=\frac{E_{S e c .3}}{R_{G R .4}}=\frac{80}{120}=0.667 \mathrm{amp} \\
& i_{G R \cdot 5}=\frac{E_{S c c \cdot 3}}{R_{G H \cdot 5}}=\frac{80}{60}=1.333 \mathrm{amp} \\
& I_{S e c \cdot 3}=i_{G R \cdot 4}+i_{G R \cdot 6}=0.667+1.333=2.00 \mathrm{amp} \\
& E_{100}=i_{\text {GR. } 4} \times R_{100}=0.667 \times 100=66.7 \text { volts } \\
& E_{20}=i_{G R \cdot 4} \times R_{20}=0.667 \times 20=13.3 \text { volts } \\
& E_{S e c \cdot 3}=E_{100} \times E_{20}=66.7+13.3=80 \text { volis } \\
& \begin{aligned}
i_{100} & =\frac{E_{\text {Sec.3 }}}{R_{100}}=\frac{80}{100}=0.800 \mathrm{amp} \\
i_{200} & =\frac{E_{\text {Sec. } 3}}{R_{200}}=\frac{80}{200}=0.400 \mathrm{amp} \\
i_{600} & =\frac{E_{\text {Scc.3}}}{R_{600}}=\frac{80}{600}=0.133 \mathrm{amp} \\
i_{\text {GR } \cdot 5} & =i_{100}+i_{200}+i_{600}=0.800+0.400+0.133=1.333 \mathrm{amp}
\end{aligned}
\end{aligned}
$$

By solving each circuit separately and combining them whenever necessary, the current flowing through each resistor and the voltage drop across it have been obtained. Any eircuit, no matter how complicated, can be solved in a similar manner.

4-12. Rheostats and Potentiometers. Rheostats. A rheostat has been defined as a variable resistor constructed so that the value of its resistance may be varied by means of a sliding contact arm. Rheostats are generally used to control the amount of current flowing in the load to which it is connected. Two types of circuits illustrating the use of rheostats are shown in Fig. 4-23.

Figure 4-23a shows a rheostat connected in series with the load. When the sliding contact arm $B$ is moved toward $A$, the amount of resistance through which the current must flow, that is, section $A B$, is reduced and the voltage available at the load is increased, which in turn causes a greater amount of current to flow through the load. When the sliding contact arm is moved toward $C$, the resistance of section $A B$ is increased, the voltage available at the load is decreased, and the load current is thereby decreased.

Figure $4-23 b$ shows a circuit with a rheostat connected in parallel with the load and the power obtained between the terminals of an antenna and ground. When current from the antenna reaches $C$, it has two paths through which it may flow. The proportion of the current that will flow in either path depends upon the relative amounts of resistance offered by each path. When the sliding contact arm is moved toward $A$, the resistance in the path $B C$ is increased, thereby causing a greater proportion of the current to flow through the load. When the contact arm is moved toward $C$, the resistance in section $B C$ is decreased and it will take a greater portion of the current, thereby reducing the current flowing through the load.


Fig. 4-23.-Circuits illustrating uses of rheostats: (a) rheostat in series with a load, (b) rheostat in parallel with a load.

It should be observed that in the series circuit the load current is increased by decreasing the amount of rheostat resistance being used, while in the parallel circuit the current flowing through the load is increased by increasing the amount of rheostat resistance being used. It should also be noticed that only two of the three terminals (marked $A B C$ on the diagram) are used and that current flows through only that part of the resistance actually between the sliding contact arm and that end of the resistor being used as a terminal. Use of terminals $A$ and $B$ in the series circuit of Fig. 4-23a and terminals $B$ and $C$ in the parallel circuit of Fig. 4-23b results in obtaining an increase in current flow-


Fig. 4-24.-Circuit illustrating the use of a potentiometer. ing through the load by rotating the sliding contact arm of the rheostat in a clockwise direction, see Fig. 4-26.

Potentiometers. A potentiometer may be defined as a variable resistor connected so that it may be used for subdividing a voltage. Figure 4-24 shows how a potentiometer is connetted to the line and the load.

The points $A$ and $C$ of the potentiometer are connected to the line, and the load terminals are connected to the sliding contact arm and to terminal C. By varying the position of the sliding contact arm, it is possible to obtain any voltage from zero to full line voltage at the load. The voltage across the load will be equal to the voltage across section $B C$ because they are connected in parallel with each other. The amount of voltage across $B C$, and hence across the load, will depend upon the resistance between $B C$ and the current flowing through $B C$. It may be expressed as

$$
\begin{equation*}
E_{\text {lond }}=E_{B C}=I_{B C} \times R_{B C} \tag{4-19}
\end{equation*}
$$

When the sliding contact arm is moved toward $A$, the amount of resistance between $B$ and $C$ is increased and the voltage supplied to the load is increased. When the arm is moved toward $C$, the resistance of section $B C$ is decreased and the voltage supplied to the load is decreased.

The current flowing through section $A B$ will be equal to the sum of the currents flowing through section $B C$ and the load. This may be expressed mathematically as

$$
\begin{equation*}
I_{A B}=I_{B C}+I_{\text {load }} \tag{4-20}
\end{equation*}
$$

When selecting a potentiometer, be sure it is large enough to carry the current drawn by the load plus the amount drawn by the potentiometer itself. As the power consumed by a potentiometer is all lost, its current should be kept at a minimum. This can be accomplished by increasing the resistance between $A$ and $C$ to a very high value in order to keep the current in $B C$ at a minimum.

Uses of Rheostats and Potentiometers. Rheostats and potentiometers are used to control various types of circuits used in radio such as volume, tone, antenna, plate voltage, and audio. As the amount of current flowing in these circuits is very small, carbon resistors can be used (see Fig. 4-25).

When higher currents are required, metallic or wire-wound resistors are used (see Fig. 4-26). An objection to metallic resistors is that noisy operation of the receiver may result when the contact arm moves from one turn of wire to another. This occurs when there is an appreciable amount of voltage drop between adjacent turns of wire. Carbon controls do not present such conditions, as the resistance change progresses smoothly and not in steps as in the wire-wound controls.

Taper. Rheostats and potentiometers used for control circuits may vary in direct ratio, or they may taper. In a direct ratio potentiometer, the resistance value varies directly with the degree of rotation. That is, at quarter rotation the resistance value is one-quarter of the total resistance, and similarly at half rotation it is one-half of the total resistance. When a potentiometer is tapered, the resistance does not vary directly with the
rotation. The potentiometer shown in Fig. 4-25 has a total resistance of 500,000 ohms. At half rotation the resistance is only $50,000 \mathrm{ohms}$, and at quarter rotation it will be less than 25,000 ohms as the resistance is tapered and not uniform between the off position and the mid-point. In a similar manner, the resistance at three-quarters rotation would not be equal to one-half of 450,000 plus 50,000 (or 275,000 ), because the resistance between the mid-point and the on position is not uniform but is tapered.


Total resistance 500,000 ohms
Fig. 4-25.-A carbon-type potentiometer with a left-hand taper.


Fig. 4-26.-A wire-wound potentiometer with a left-hand taper.

It is necessary to taper the resistance of a control in order to obtain an apparent uniform control of the signal. When the control is turned to the halfway position, it is generally expected that the signal volume will be one-half that obtained at the full or on position of the control. In order to double a given volume of sound, an increase of approximately 10 times the original intensity is required. At one-half full volume, only one-tenth of the full volume voltage is required, and therefore one-tenth of the total resistance is all that is needed.

Potentiometers have either left-hand or right-hand tapers, depending on which side is tapered out. In Figs. 4-25 and 4-26, the left hand of the control is tapered out; therefore each is a left-hand taper.

Comparison of Wire-wound and Carbon Controls. Wire-wound and carbon controls have a number of advantages and disadvantages which, for purposes of comparison, are listed below. The choice of a control will depend on the use to whicb it is to be put.

## Advantages

Wire-wound<br>1. Absolute accuracy of the resistance value<br>2. High current-carrying ability<br>3. Low resistance values easily obtained ( $\frac{1}{2} \mathrm{ohm}$ )<br>Carbon<br>1. Ease of obtaining taper<br>2. Silent operation<br>3. Iligh resistance values easily obtained (two or more megohms)

## Disadvantages

1. More difficult to obtain a taper
2. Noisy operation
3. Limited high resistance value that can be obtained ( 150,000 ohms)
4. Resistance will vary with heat, humidity, wear, etc.
5. Low current-carrying ability
6. Limited low resistance value obtainable ( 500 ohms )

4-13. The Voltage Divider. By using the principle of the potentiometer, a high resistance may be connected across a power supply and a number of loads requiring different amounts of voltage can be connected to a series of taps along the body of the resistor. The voltage between any two points will be equal to the product of the current flowing through that part of the resistor and the value of the resistance between the two points (see Fig. 4-27). A resistor used in this manner is called a voltage divider.

Voltage dividers are used to divide the voltage of the power supply into such values of potential as are required by the various parts of the circuit. Voltage dividers also act as a safety load to protect the capacitors from having too high a voltage placed across their terminals. In order to obtain a uniform voltage output, the power supply should be worked as near as possible to the rated power output of its transformer. This can be controlled by the amount of current drawn by the voltage divider which is known as the bleeder current.

The calculation of the correct resistance values and the power rating of the voltage divider may be accomplished by the use of Ohm's law. The following procedure should be observed:

1. Determine the voltage required at each tap and the current to be drawn from it.
2. Determine the amount of bleeder current desired. 'This is the difference between the total current required by the tubes and the current necessary to operate the power supply at 90 per cent of its rated value.
3. Determine the current flow in each section of the divider.
4. Calculate the resistance of one section at a time by Ohm's law.
5. Determine the power rating of the voltage divider.

The power rating of the voltage divider may be calculated by the equation

$$
\begin{equation*}
P=\frac{I^{2} R}{10^{6}} \tag{4-21}
\end{equation*}
$$

where $P=$ power, watts
$I=$ current, milliamperes (the highest value of current in any section)
$R=$ total resistance of the voltage divider
4-14. Use of Exponents in Calculations. When circuit or problem calculations involve the use of very large or very small numbers, the method of expressing these numbers and performing arithmetic operations can be simplified by the use of exponents. This is really a shorthand method of mathematics.

Calculations involving the power when the current is in milliamperes as in Eq. (4-21) may be performed more easily by this method. The following table shows a list of numbers and the corresponding representations by the exponent method.

Table IV-I

| Number | Exponent method | Number | Exponent method |
| :---: | :---: | :---: | :---: |
| 100,000,000 | $10^{8}$ | 1 | $10^{0}$ |
| 10,000,000 | $10^{7}$ | $0.1=\frac{1}{10}$ | $10^{-1}$ |
| 1,000,000 | $10^{6}$ | $0.01={ }^{1} 0.0$ | $10^{-2}$ |
| 100,000 | $10^{5}$ | $0.001=$ T000 | $10^{-3}$ |
| 10,000 | $10^{4}$ | 0.0001 | $10^{-4}$ |
| 1,000 | $10^{3}$ | 0.00001 | $10^{-5}$ |
| 100 | $10^{2}$ | 0.000001 | $10^{-6}$ |
| 10 | $10^{1}$ | 0.0000001 | $10^{-7}$ |
| 1 | $10^{\circ}$ | 0.00000001 | $10^{-8}$ |

The following examples illustrate the use of the exponent method of expressing common numbers:

1. $5 \mathrm{ma}=0.005 \mathrm{amp}=5 \times 10^{-3} \mathrm{amp}$
2. $25 \mu \mathrm{a}=0.000025 \mathrm{amp}=25 \times 10^{-6} \mathrm{amp}$

3 . $3.9 \mathrm{mc}=3,900,000$ cycles $=3.9 \times 10^{\circ}$ cycles
4. $8,500,000=8.5 \times 10^{\circ}$
5. $0.0035=3.5 \times 10^{-3}$
6. $6.28 \times 10^{18}=6,280,000,000,000,000,000$

Note: This is the number of electrons corresponding to one ampere (Art. 2-12).
Numbers that have similar exponent characteristics may be added or substracted as indicated by the following illustrations.
7. $\left(4.5 \times 10^{6}\right)+\left(8.25 \times 10^{6}\right)+\left(0.25 \times 10^{6}\right)=13 \times 10^{6}$
8. $\left(8.5 \times 10^{3}\right)-\left(3.5 \times 10^{3}\right)=5 \times 10^{3}$

When numbers are multiplied, the exponents are added. The exponents do not have to be the same.

$$
\begin{aligned}
& \text { 9. } 650,000 \times 3000=\left(6.5 \times 10^{5}\right) \times\left(3 \times 10^{3}\right)=19.5 \times 10^{4} \\
& 10.2,500,000 \times 0.005=\left(2.5 \times 10^{8}\right) \times\left(5 \times 10^{-3}\right)=12.5 \times 10^{3} \\
& \text { 11. } 0.015 \times 0.0006=\left(1.5 \times 10^{-2}\right) \times\left(6 \times 10^{-4}\right)=9 \times 10^{-6}
\end{aligned}
$$

When numbers are divided, the exponents are subtracted. The exponents do not have to be the same.

```
12. \(750,000 \div 150=\left(7.5 \times 10^{5}\right) \div\left(1.5 \times 10^{2}\right)=5 \times 10^{3}\)
13. \(2,500 \div 50,000=\left(25 \times 10^{2}\right) \div\left(5 \times 10^{4}\right)=5 \times 10^{-2}\)
14. \(5,000 \div 0.025=\left(5 \times 10^{3}\right) \div\left(2.5 \times 10^{-2}\right)=2 \times 10^{5}\)
```

4-15. Calculation of a Typical Voltage Divider. The procedure to be followed in calculating the resistance and power values of a voltage divider was suggested in Art. 4-13. In order better to understand the procedure to be followed, a typical voltage divider will now be calculated.

Fxample 4-12. Determine the resistance and power values of a voltage divider for a small superheterodyne receiver that employs a 6A8 oscillator-mixer tube, a 6SK7 i-f amplifier tube, a 6SQ7 detector tube, and a 6 F 6 power tube in the output stage.

Figure 4-27 represents the voltage divider to be used with this receiver. All values given in the table below were obtained from a tube manual. This, however, does not mean that these are the only values that can be used. The design of the receiver itself will determine the proper voltages at which each tube should be operated in a particular receiver.

Table IV-II. Data for Tubes in Example 4-12.
(From a tube manual)

| Tube | 6 A 8 | 6SK7 | 6SQ7 | 6F6 |
| :---: | :---: | :---: | :---: | :---: |
| $E_{P}$ | 250 | 250 | 250 | 250 |
| $E_{S G}$ | 100 | 100 |  | 250 |
| $E_{G}$ | $-3.0$ | $-3.0$ | -2.0 | -16.5 |
| $I_{P}$ | 7.5 | 9.2 | 0.9 | 34 |
| $I_{S G}$ | 2.7 | 2.6 | . . | 6.5 |

Following the procedure suggested in Art. 4-13 for the solution of voltage-divider problems, the solution of Example $4-12$ becomes as follows:

1. Voltage and current at each tap. These values are obtained from Table IV-II and may be shown best by means of a circuit diagram as illustrated by lig. 4-27.

The total voltage required from the power supply will be equal to the sum of the highest amount of plate voltage and the highest amount of grid voltage. This is equal to $250+16.5=266.5$ volts.

The total current required at the 250 -volt tap will be the sum of all the plate currents and the screen-grid current of the 6F6 tube, or $7.5+$ $9.2+0.9+34+6.5=58.1$ milliamperes.

The total current required at the 100 -volt tap will be the sum of the screen-grid currents of the 6.18 and the 6 SK7 tubes, or $2.7+2.6=5.3$ milliamperes.

The current required at the negative taps, that is, for the control grids, is so small that it is ignored.

The total current required by the tubes $=58.1+5.3=63.4$ milliamperes.


Fig. 4-27.
2. The amount of bleeder current required. If a transformer rated at 100 milliamperes is used in the power supply, it must have a load of 90 milliamperes if it is to be operated at 90 per cent of its rated load. The bleeder current must therefore be $90-63.4=26.6$ milliamperes.
3. Current in each section of the voltage divider. From the values established in steps 1 and 2, and by carefully analyzing the circuit of the voltage divider, it should now be possible to determine the amount of current that will flow in each section of the voltage divider. Such an analysis reveals that sections 5,4 , and 3 , namely the path from $D$ to $C$, must carry the bleeder current and the current for all the plates and screen grids of the various tubes; this is a total of 90 milliamperes as is indicated on Fig. 4-27. At point $C$ the current divides into two paths, with 63.4 milliamperes taking the path through the plates and screen grids of the various tubes and the bleeder current of 26.6 milliamperes flowing through section

2 of the voltage divider. At point $B$ the 5.3 milliamperes from the screen grids of two tubes joins with the 20.6 milliamperes of section 2 of the voltage divider, and as a result their sum, or 31.9 milliamperes, must flow through section 1 of the voltage divider. At point $A$ the 58.1 milliamperes from the plates and one sereen grid unite with the 31.9 milliamperes from section 2 , which corresponds to the total of 90 milliamperes that is being furnished by the power supply.
4. Resistance of each section of the vollage divider.

$$
\begin{aligned}
& \text { Resistance of section } 1=\frac{e_{1}}{i_{1}}=\frac{250-100}{31.9 \times 10^{-3}}=4700 \mathrm{ohms} \\
& \text { Resistance of section } 2=\frac{e_{2}}{i_{2}}=\frac{100-0}{26.6 \times 10^{-3}}=3760 \mathrm{ohms} \\
& \text { Resistance of section } 3=\frac{e_{3}}{i_{3}}=\frac{2-0}{90 \times 10^{-3}}=22 \mathrm{ohms} \\
& \text { Resistance of section } 4=\frac{e_{4}}{i_{4}}=\frac{3-2}{90 \times 10^{-3}}=11 \mathrm{ohms} \\
& \text { Resistance of section } 5=\frac{e_{5}}{i_{5}}=\frac{16.5-3}{90 \times 10^{-3}}=150 \mathrm{ohms} \\
& \text { Total resistance of all sections }=\frac{8643 \mathrm{ohms}}{}
\end{aligned}
$$

5. The power rating of the voltage divider. Assuming that the divider is to have a uniform power rating, it is necessary to use the highest current in determining the power rating. Therefore

$$
\text { Power rating }=\frac{I^{2} R}{10^{6}}=\frac{90 \times 90 \times 8643}{10^{6}}=70 \text { watts }
$$

Close examination of the voltage divider will show that it really consists of a number of resistances connected in series. Therefore, five separate resistors could be connected in serics for the divider in Example 4-12, and the power rating of each would be equal to the product of its resistance and the square of the current flowing through it. For example,
Power of resistor $1=31.9 \times 31.9 \times 4700 \times 10^{-6}=4.78$ watts
Power of resistor $2=26.6 \times 26.6 \times 3760 \times 10^{-6}=2.66$ watts
Power of resistor $3=90 \times 90 \times 22 \times 10^{-6}=0.179 \mathrm{watt}$
Power of resistor $4=90 \times 90 \times 11 \times 10^{-6}=0.089 \mathrm{watt}$
Power of resistor $5=90 \times 90 \times 150 \times 10^{-6}=1.215$ watts
Total power lost in the voltage divider $\quad=\overline{8.923 \text { watts }}$
As the voltage divider is usually mounted under the chassis of most radio receivers and therefore does not have much ventilation, it is recom-
mended that its power rating be approximately double that of the load it is to carry. Thus the voltage divider of Example 4-12 should be rated at 140 watts if it is designed on the assumption that the maximum current flows through the entire unit. If it is made of five individual sections each designed for the current actually flowing through it, the power rating should be approximately 20 watts. $\lambda s$ the five-section divider has a much lower power rating, it is less expensive.

The voltage divider of Example 4-12 is designed to supply all the plate and screen-grid voltages (generally called the B voltages) and all the negative, or grid-bias, voltages (generally called the $C$ voltages) of the receiver. Many radio and television receivers obtain their $C$ voltages from sources other than the voltage divider and thereby eliminate the grid-bias voltage section of the voltage divider shown as section $C D$ in Fig. 4-27. In such cases the negative terminal of the power supply, the B - terminal of the voltage divider, and the ground are connected together.

If one or more sections of a voltage divider break down, it is not necessary to replace the entire divider. Resistors of the correct values may be substituted in place of the defective sections after they have been disconnected from the circuit.

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## QUESTIONS

1. What are the essential parts of any electric circuit?
2. Explain by use of the electron theory why the electric circuit has to be a closed circuit.
3. What are the factors that affect the resistance of an electrical conductor?
4. For practical purposes, why can the change in resistance due to variations in temperature be disregarded?
5. What is meant by a negative, positive, and zero temperature coefficient?
6. Name several materials that have (a) a relatively high temperature coefficient, (b) a relatively low temperature coefficient.
7. What is the mathematical relation between the factors, other than temperature, affecting the resistance of a conductor?
8. What is the relation between a mil, square mil, circular mil, inch, square inch?
9. (a) How is the diameter of a wire usually expressed? (b) How is the cross section of a wire usually expressed?
10. What is meant by specific resistance?
11. How is the size of a wire designated?
12. (a) What is meant by conductance? (b) What is its unit of measurement?
13. What is the difference between a conductor and an insulator?
14. What factors determine the material to be used (a) as a conductor? (b) As an insulator?
15. What is the difference between the dielectric strength of a material and its breakdown voltage?
16. Name five conductors used in radio, television, and electronic apparatus, and explain where and why they are used.
17. Name five insulators used in radio, television, and electronic apparatus, and explain where and why they are used.
18. Name five metals used in making alloys to be used as resistors.
19. What material is used to obtain high resistances?
20. (a) What are the advantages and disadvantages of a fixed metallic resistor? (b) Of a fixed carbon resistor?
21. Explain what is meant by the following terms used in conjunction with a resistor: fixed, variable, adjustable, tapped, automatic resistance control, rheostat.
22. What is meant by a low-power resistor?
23. What is meant by a high-power resistor?
24. What are the four ways of connecting electrical appliances?
25. What is meant by a series circuit?
26. How are the current, voltage, and resistance related in a series circuit (a) in any individual part of the circuit? (b) In the entire circuit?
27. What are the advantages and disadvantages of series circuits?
28. Name and explain three uses of the series circuit.
29. (a) Why are power supplies connected in series? (b) Name an application of the series connection of power supplies.
30. Why is it good practice continually to check the answers in solving circuit problems?
31. What is meant by a parallel circuit?
32. How are the current, voltage, and resistance related in a parallel circuit (a) in any individual part of the circuit? (b) In the entire circuit?
33. What are the advantages and disadvantages of parallel circuits?
34. Name and explain three uses of the parallel circuit.
35. (a) Why are power supplies connected in parallel?
(b) Name an application of the parallel connection of power supplies.
36. (a) What is meant by a combination circuit? (b) A series-parallel circuit? (c) A parallel-series circuit?
37. (a) Where are combination circuits used? (b) What are their advantages?
38. What is the general procedure to be used in solving (a) series-parallel circuits? (b) Parallel-series circuits? (c) Complicated combination circuits?
39. (a) What is meant by a rheostat?
(b) A potentiometer?
40. Name and explain some of the uses of rheostats and potentiometers.
41. (a) What is meant by taper? (b) Left-hand taper? (c) Right-hand taper?
42. Why is it necessary to use resistances that are tapered?
43. What are the advantages and disadvantages of carbon controls?
44. What are the advantages and disadvantages of wire-wound controls?
45. What is the main purpose of the voltage divider?
46. What other purposes does it perform?
47. Where are voltage dividers generally used?
48. How is the power rating of a voltage divider determined?
49. Why must voltage dividers be wire-wound?
50. If one section of a voltage divider goes bad, is it necessary to replace the entire divider? Explain.

## PROBLEMS

1. What is the operating resistance of a $\frac{1}{4}$-megohm carbon resistor if its temperature increases from $20^{\circ} \mathrm{C}$ when the set is not being used to $45^{\circ} \mathrm{C}$ when it is being operated? ( $T_{c}$ for carbon $=-0.0003$.)
2. What is the voltage drop across the resistor in the above problem when $20 \mu \mathrm{a}$ is flowing through it (a) at $20^{\circ} \mathrm{C}$ ? (b) At $45^{\circ} \mathrm{C}$ ?
3. How many turns of No. 28 copper wire are necessary to wind a coil having an average diameter of 2 in . and a resistance of 88 ohms ?
4. What is the average diameter of a coil wound with No. 24 copper wire having 150 turns and a resistance of 2.56 ohms ?
5. What is the resistance of a piece of aluminum wire 0.0126 in . in diameter and $2 t$ in. long?
6. A $300-$, a 500 -, and a $400-\mathrm{hm}$ resistor are connected to form a series circuit across a 240 -volt line. Find (a) the total resistance of the circuit, (b) the current flowing through the circuit, (c) the voltage drop across each resistor, (d) the power taken by each resistor, (e) the power taken by the circuit.
7. A $1500-$, a $2500-$, a 1000 -, and a 5000 -ohm resistor are connected inseries across a 250 -volt power supply. Find (a) the total resistance of the circuit, (b) the current flowing through the circuit, (c) the voltage drop across each resistor, (d) the power taken by each resistor, (e) the power taken by the circuit.
8. The current flow in a series circuit consisting of three resistances is 375 ma . The voltage drop across resistance $A$ is 40 volts and across $B 25$ volts. If the voltage across the line is 100 volts, find (a) the voltage drop across resistance $C$, (b) the resistance of $A, B, C,(c)$ the resistance of the entire circuit, (d) the power taken by each resistor, (e) the power taken by the circuit.
9. A resistor is connected between the cathode of a radio tube and the negative terminal of the B power supply to produce a negative voltage (grid bias) on the grid of the tube. When the plate current, which also passes through the bias resistor, is 24 ma, what must be the value of the resistor in order to produce a grid bias of 12 volts?
10. A four-tube radio set has its heaters connected in series. The rated voltages are $25,6.3,6.3$, and 12.6 volts, and they all draw 0.3 amp . (a) What value of resistance must be connected in series with these heaters in order to operate them directly from a 110 -volt line? (b) How much power is consumed by the dropping resistor?
11. A 30,000 -ohm resistor is connected in series with a 250 -volt B power supply and the plate circuit of a radio tube. If the plate current is 2.5 ma , what is the voltage across (a) the plate circuit of the tube? (b) The resistor?
12. What value of resistance must be connected in serics with the heater of a 1 C 6 tube rated at 2.0 volts and 0.12 amp , if it is to be operated from two 1.5 -volt cells connected in series with each other?
13. Two 2 A 3 tubes have their filaments connected in series with a 6.3 -volt battery and a filament resistor. If the filaments are rated at 2.5 volts and 2.5 amp , what is the value of the filament resistor?
14. A five-tube radio receiver uses the followng tubes: $12 \mathrm{~A} 8 \mathrm{GT}, 12 \mathrm{~K} 7 \mathrm{GT}, 12 \mathrm{Q}$ $7 \mathrm{GT}, 35 \mathrm{~L} 6 \mathrm{GT}$, and $35 / 4 \mathrm{GT}$. The heaters of these tubes are connected in serics and are rated at $12.6,12.6,12.6,35$, and 35 volts in the order listed. A 2.2 -volt pilot light is connected in series with the heaters and the 110 -volt line. What is the resistance of each of the heaters and the filament of the pilot light in order that they draw their rated current? The $35 \% 4 \mathrm{GT}$ rectifier heater is rated at 0.15 amp .
15. A voltmeter reading its maximum rated voltage of 150 volts draws 10 ma . What value of resistance is connected in series with the moving coil of the voltmeter if the resistance of the coil is 20 ohms (see Fig. 4-28)?
16. The voltmeter in the previous problem is to have a 30 - and 75 -volt terminal in addition to the 150 -volt terminal (see Fig. 4-28). If the current drawn is to be 10 ma , at what value of resistance must the resistor $R$ be tapped?
17. If 10 ma is drawn by the voltmeter in Prob. 15 and an external resistance connected in series with it, what is the value of this resistance when the voltmeter reads (a) 300 volts? (b) 450


Fig. 4-28. volts?
18. A 300 -, a 500 -, and a $400-\mathrm{ohm}$ resistor are connected in parallel across a 240 volt power line. Find (a) the total resistance of the circuit, (b) the current flowing in each resistor, (c) the line current, (d) the power taken by each resistor, (e) the power taken by the circuit.
19. Four resistors are connected in parallel across a 180 -volt source of power. The current flowing in each circuit is $20,250,180$, and 300 ma . Find the value of each resistor and the total resistance of the circuit.
20. An electric circuit in a home has a 100 -watt lamp, a 550 -watt toaster, and a 660 -watt broiler connected in parallel with a 110 -volt line. Find (a) the current drawn by each appliance and the total current flowing in the circuit, (b) the resistance of each appliance and the resistance of the circuit.
21. The resistance of the moving coil of an ammeter is 5 ohms. What value resistor must be connected in parallel with this coil in order that the voltage drop across the meter will be 50 mv when 1 amp is flowing through the line?
22. The resistor connected in parallel with an ammeter coil is called a shunt. How much current is flowing through the coil and through the shunt of Prob. 21 when the line current is 1 amp ?
23. If the ammeter in Prob. 21 is to have the same voltage drop across it, what value of shunt resistor must be connected in parallel with it in order that it read (a) 5 amp ? (b) 10 amp ?
24. Three resistors are connected in parallel across a 250 -volt source of power. The total current flowing in the line is 760 ma . Two of the resistors have a value of 1250 ohms and 25,000 ohms. (a) What is the current flowing in each resistor? (b) What is the value of the third resistor?
25. A radio receiver has four tubes whose heaters each draw 0.3 amp . The heaters are connected in parallel to a 2.5 -volt tap of the power transformer. (a) What is the resistance of each heater? (b) What is the combined resistance of all the heaters? (c) What is the total current taken from the power transformer?


Fig. 4-29.
26. Find the following quantities for the circuit shown in Fig. 4-29: (a) resistance of each group, (b) resistance of the entire circuit, (c) current taken by the entire circuit, (d) current in each resistance, (e) voltage drop across each resistance.


Fig. 4-30.
27. Find the following quantities for the circuit shown in Fig. 4-30: (a) resistance of each group, (b) resistance of the line, (c) current taken by the entire circuit, (d) voltage drop across each group, (e) current in each resistance.
28. Find the total resistance of the


Fig. 4-31. circuit shown in Fig. 4-31.
29. In the circuit used in Prob. 28, E' $=40$ volts. Find (a) current in each resistance, ( $b$ ) voltage across $A B$ and $B C$.
30. Find the following quantities for the circuit shown in Fig. 4-32: (a) voltage across $A B$ and $B C$, (b) current flow through $A C$ and $A D,(c)$ resistance of the entire circuit.
31. In the circuit shown in Fig. 4-33 find (a) resistance of the entire circuit, (b) $I_{1}, I_{2}$, and $I_{3},(c)$ voltage across $A B$, $B C$, and $B D$.
32. A Wheatstone bridge, shown in Fig. 4-34, is used to find the resistance of $R_{3}$. The two ratio arms $R_{1}$ and $R_{2}$ have a resistance of 200 ohms and 800 ohms. The variable resistance $R_{4}$ reads 292 ohms when no current flows between $C$ and $I$. Find (a) $I_{1}, I_{2}, I_{3}, I_{4}$, (b) $R_{3}$.


Fig. 4-32.
33. A 50,000 -ohm potentiometer connected to a 45 -volt battery supplies a 4000 -ohm load with 20 volts. The resistance of section $A B$ (Fig. 4-24) is 4595 ohms. Find the current in each part of the potentiometer.
34. If a $21,850-\mathrm{ohm}$ potentiometer with $R_{A B}$ sct at 4081 ohms is used in Prob. 33, what is the current in each part of the potentiometer?
35. The load in Prob. 33 is changed to 6000 ohms and $R_{A B}$ is set at 6650 ohms. What is the current in each part of the potentiometer?
36. $\Lambda 20,000$-ohm potentiometer connected to a 45 -volt battery supplies a 2000 ohm load with 20 volts. The resistance of section $A B$ (Fig. 4-24) is 2247 ohms. Find the current in each part of the potentiometer.
37. A radio receiver has five tubes, each having a plate voltage of 250 volts.


Fig. 4-33.


Fig. 4-34.

Their plates draw the following currents: 7, 3.5, 7, 1.1, and 34 ma. The screen grids of two tubes operate at 100 volts and 1.7 ma , one tube at 250 volts and 6.5 ma , and one tube at 50 volts and 1.6 ma . One of the tubes has no screen grid. The control grids of three tubes operate at -3 volts and one at -16.5 volts. One tube used as a diode detector has no control-grid bias. A transformer rated at 75 ma is used in the power supply. Find (a) the resistance of each section of the voltage divider if the transformer is to be operated at 90 per cent of its rated value, (b) the power rating of the voltage divider using the largest current flowing in any part of the resistor, (c) the power rating of the voltage divider if it is made of individual resistors cach rated according to the current actually flowing in the section.
38. A radio receiver has six tubes, each having a plate voltage of 250 volts. Their plates draw the following currents: 7, 7, 3.5, 1.1, 32, and 32 ma. The screen grids of four tubes operate at 100 volts, and their currents are 1.7, 1.7, 2.2, and 0.35 ma. The screen grids of the remaining.two tubes operate at 250 volts, and their currents are 5.5 ma each. A transformer rated at 120 ma is used in the power supply. Find (a) the resistance of each section of the voltage divider if the transformer is to be operated at 90 per cent of its rated value, (b) the power rating of the voltage divider using the largest current flowing in any part of the resistor, (c) the power rating of the voltage divider if it is made of individual resistors each rated according to the current actually flowing in the section.
39. Find the resistance between $A B, B C, C D$, and $A D$ of the circuit shown in Fig. 4-35.


Fig. 4-35.
40. The circuit in Prob. 39 is connected to a 300 -volt supply. Find the current flowing through each resistor.
41. In Prob. 40 what is the voltage between $A B, B C$, and $C D$ ?
42. Find the resistance between $A B, C D, D E$ and the total resistance of the circuit shown in Fig. 4-36.


Fig. 4-36.
43. In Prob. 42, $E=150$ volts; what is the current flowing in each resistor?
44. What is the voltage drop between the points $A F, F G, G B, C D$, and $D E$ of the circuit of Prob. 43?
45. In Prob. 40 what is the power used by sections $A B, B C, C D$, and the entire circuit?
46. In Prob. 43 what is the power used by sections $A F, F G, G B, C D, D E$, and the entire circuit?

## CHAPTER V

## MAGNETISM

Magnetism has been known to man for many centuries, and the Chinese are said to have been aware of some of its effects as early as 2600 b.c. Its first practical use, the magnetic compass, is credited to the Chinese and was introduced in Europe about a.d. 1200. Dr. William Gilbert (1540-1603), an English physician, made further discoveries about magnetism and is also credited with being the first to publish records of his work. Among other carly scientists who have contributed to the study of magnetism are IIans Christian Oersted (1771-1851) of Denmark, Karl liriedrich Gauss (1777-1855) of Germany, and James ClerkMaxwell (1831-1879) of Scotland. These men are mentioned here because they have been honored by having magnctic units named for them.


Fig. 5-1.-Harly Chinese compass. (General Motors Corporation.)
5-1. Relation of Magnetism to Electricity. Magnetism is so closely related and so important to electricity that the two are often called twins. In the study of magnetism, certain definite rules or laws have been established concerning the action of magnets, and in the study of electricity the laws concerning the flow of clectricity are similar to these in many ways. Then, too, electricity is so dependent upon magnetism that without it very few of our modern devices would be possible. Without the
aid of magnetism, it would be impossible to generate and transmit power in large enough quantities to meet the needs of our industrial and home use. Without the use of either magnetism or electricity, we should be deprived of such valuable assets as the radio, telephone, telegraph for communications, and the ignition systems for our cars, airplanes, trucks, etc., all of which are important to our welfare in times of peace as well as in times of war.

5-2. Magnetism, Magnets, Magnetic Materials. Magnetism is generally defined as the property or power of a material to attract and hold pieces of iron or steel. While this is true, it would be better to consider magnetism as the study of all the properties and actions of magnets and magnetic materials.

A magnet is defined as a body that has the property of polarity and the power of attracting iron and steel.

Magnetic materials are those which will be attracted to a magnet; they may or may not possess the property of polarity and may or may not have the power of attracting other magnetic materials.

A study of the definitions of magnets and magnetic materials leads to the conclusion that all magnets are magnetic materials but not all magnetic materials are magnets.

5-3. Natural Magnets and Artificial Magnets. Natural Magnets. Centuries ago it was discovered that certain stones taken from the earth had two peculiar properties. One was that they possessed the power to attract and hold to them other bits of similar stones or iron. The other was that when an elongated piece of this stone was suspended from a cord it would always come to rest with one end pointing north. The Chinese were the first to discover and use this stone to aid in determining directions. However, its later common use in navigation resulted in the name of lodestone, meaning leading stone. This substance taken from the earth is now called magnetite. The name magnet was given to the lodestone because large deposits of the stonc were found near the city of Magnesia in Asia Minor. These stones are called natural magnets because they possess magnetic power when taken from the earth. Natural magnets no longer have any practical value, as it is now possible to produce powerful magnets by the use of electricity.

Artificial Magnets. The lodestone possessed the property of being able to pick up bits of steel (see Fig. 5-2), and


Fig. 5-2.-Lodestone. though each bit of steel could be attracted to the lodestone, it was found
that there was no attraction between the various bits of steel themselves. It was found that if a bar or rod of steel was rubbed or stroked with a


FIRST STROKING


SECOND STROKING
Fig. 5-3.-Making a bar magnet with a lodestone.
piece of lodestone (Fig. 5-3), the steel bar would then have the same properties as the lodestone and would be able to attract some of the bits of steel to it. The bar would then be


Fig. 5-4.-Magnetizing a steel bar by means of an electric current. classed as an artificial magnet. The magnet produced by rubbing with a lodestone would be weak in terms of modern magnets; such magnets are now made by inserting the steel bar in a coil of wire that has an electric current flowing through it (sec Fig. 5-4).

5-4. Permanent and Temporary Magnets. Permanent Magnets. If a piece of steel is hardened by heattreatment and is then made an artificial magnet by rubbing with a lodestone or with another magnet, or by placing it in a coil of wire carrying an electric current, it will be found that the hardened steel will remain a magnet for a long time thereafter. It is then classed as a permanent magnet.

Temporary Magnets. If a piece of iron, soft steel, or nickel is made an artificial magnet in a similar way, it will be found that the iron, soft steel, or nickel will lose nearly all its magnetism almost immediately after it is taken away from the magnetizing force. Thus magnets of iron, soft steel, or nickel are classed as temporary magnets.

Uses of Magnets. Temporary magnets are used mostly where the magnet has a coil of wire wound around it and an electric current is flowing through the coil. Examples of this are generators, motors, trans-


Fig. 5-5.-Suspended bar magnet used as a compass. formers, electric bells, buzzers, telegraph sounders, relays, dynamic loud-speakers and microphones, magnetic phonograph pickups, and deflection and focusing coils used with cathoderay tubes of television receivers.

Permanent magnets are used in compasses, earphones, radio loudspeakers, electrical meters, magnetos, etc.


Fig. 5-6.-Illustrating the poles of a magnet by the use of iron filings.
5-5. Poles of a Magnet. If an elongated lodestone or bar magnet is suspended so that it can turn freely, it will come to rest in such a position that one end will point approximately to the earth's geographical north pole. If the magnet is turned and comes to rest of its own accord, it will settle in the same position as at first. The end of the magnet that points toward the earth's north geographical pole is called the north seeking pole, or in short the north pole. The other end of the magnet, which points toward the earth's south geographical pole, is called the south-seeling pole, or simply the south pole.

If a bar magnet is placed upon a flat surface and a large quantity of iron filings are sprinkled over it, most of the filings will accumulate at two areas. If the magnet is lifted and rotated gently, it will be seen that many of the iron filings in the middle portion will drop off, while those at the ends will cling to the magnet. This is but another way of describing the poles of a magnet; that is, the two points where the magnetic strength is greatest are called the poles of the magnet (see Fig. 5-6).

5-6. Theory of Magnetism. Weber's Theory. There have been various theories developed from time to time in the scientist's search for the explanation of magnetism. Weber's theory, which is also known as the molecular theory, is the most popular explanation. It is based on the assumption that the molecules of a magnetic substance are all individual minute magnets. If a magnetic substance lacks the property of polarity and the power of attraction, it is believed that the many tiny magnets are arranged in an utterly disorganized manner as shown in Fig. 5-7a. However, when a magnetic substance possesses polarity and power of attraction, it is believed that the

(a)-Unmagnetized bar

(b)-Magnetized bar

Fig. 5-7. -The molecular theory of magnetism. molecular magnets are arranged in orderly rows, each with its north pole in the same direction as shown in Fig. 5-7b. Also, according to this theory, the molecules in a magnetic substance such as steel, iron, cobalt, or nickel can readily rearrange themselves from a disorganized manner in orderly rows.

Explanation of Magnetic Actions. A lodestone when taken from the earth has a large majority of its molecular magnets lined up in even rows and possesses the powers of magnetism. As the many tiny magnets are lined up in orderly rows, they all aid in building up the strength of the magnet and in forming its poles.

In a piece of iron, however, the molecular magnets ordinarily are not lined up in an orderly fashion but instead settle in a haphazard manner. Because of this haphazard arrangement, the tiny magnets neutralize one another and the iron will not possess the powers of magnetism.

If the lodestone is brought near the iron, it will attract the iron to it; and if the piece of iron is not too large and heavy, the lodestone will be able to lift it. The explanation for this action is that the magnetism of the lodestone influences the tiny magnets of the iron and causes them to rearrange themselves in orderly rows. The iron then becomes a magnet.

If the lodestone is taken away from the iron, the molecular magnets of the iron will shift about and again fall into a haphazard position. The iron is no longer a magnet but is merely a magnetic material.

All the magnetic actions are explained by this theory. The natural magnet already has its molecules arranged in an orderly way when taken from the earth, while the artificial magnet has to have its molecules lined up by some artificial means. In the permanent magnet, once the molecules are set into orderly rows they will retain their positions, while in
the temporary magnet, the molecules stay in line only as long as there is some external magnetic influence to keep them in line.

5-7. Laws of Magnetic Attraction and Repulsion, Pole Strength, Force of Attraction and Repulsion. Lau of Attraction and Repulsion. If two bar magnets with their poles marked $N$ and $S$ are used, the laws of magnetic attraction and repulsion may be derived from them. If one of the magnets is suspended by means of a string so that it can move freely and the second magnet is brought near to it so that similarly marked poles (such as two norths or two souths) are brought close together, the suspended magnet will be repelled (see Fig. 5-8). When poles of unlike markings such as an $N$ and an $S$ are brought close together, the suspended magnet will be attracted by the other one. This action is commonly stated as the magnetic law: Like poles repel one another and unlike poles attract one another.


Fig. 5-8.-Repulsion between magnetic poles of like polarity.
Pole Strength. The force with which two poles will attract (or repel) one another depends upon the strength of the poles and the distance between them. The pole strength is measured in unit poles which is described as follows: A unit magnetic pole is one which, if placed in air one centimeter from a similar pole of the same strength, will repel it with a force of one dyne. ( 981 dynes $=1$ gram; 454 grams $=1$ pound.)

Force of Attraction and Repulsion. The force of attraction or repulsion between two poles varies inversely as the square of the distance between them. For example, if two poles four centimeters apart exert a force of two dynes, cutting the distance in half so that they are only two centimeters apart, the force will become four times $(2 \times 2)$ as great, or eight dynes. If the distance is reduced to one centimeter or one-quarter of the original amount, the force will become 16 times $(4 \times 4)$ as great, or 32 dynes. If, however, the distance is increased to eight centimeters, or
double the original, the force will become one-quarter $\left(\frac{1}{2} \times \frac{1}{2}\right)$ of the original amount, or one-half dyne. This relation among force, pole strength, and distance is expressed by Coulomb's law, which states that the force between two magnetic poles is directly proportional to the strengths of


Fig. 5-9.-Measuring the force of repulsion by the torsion balance.
the poles and inversely proportional to the square of the distance between the poles. This is shown mathematically by the equation

$$
\begin{equation*}
f=\frac{m_{1} m_{2}}{d^{2}} \tag{5-1}
\end{equation*}
$$

where $f=$ force in dynes between two poles in air
$m_{1}=$ strength in unit poles of first pole
$m_{2}=$ strength in unit poles of second pole
$d=$ distance in centimeters between the poles
Example 5-1. A north pole with a strength of 20 unit poles is placed five centimeters from a south pole whose strength is 30 unit poles (Fig. 5-10). What is the force acting between these poles?

Given:

$$
\begin{aligned}
m_{1} & =20 \\
m_{2} & =30 \\
d & =5
\end{aligned}
$$

Solution:


Fig. 5-10.

Find:

$$
f=?
$$

$$
\begin{aligned}
f & =\frac{m_{1} m_{2}}{d^{2}} \\
& =\frac{20 \times 30}{5 \times 5} \\
& =24 \text { dynes (attraction) }
\end{aligned}
$$

5-8. Magnetic Fields, Lines of Force, Field Intensity, Flux Density. Magnetic Fields. The properties of a magnet are not restricted to the magnet itself but also influence an area surrounding it. This may readily be shown by placing a piece of glass or paper over a bar magnet and then sprinkling iron filings over the glass or paper. The iron filings will take positions in a definite pattern similar to Fig. 5-11. The filings become


Fig. 5-11.-Magnetic field about a bar magnet illustrated by iron filings.
tiny magnets under the influence of the bar magnet, and the pattern therefore represents the bar's magnetism. The space surrounding the magnet in which this influence exists is called its magnetic field. The magnetic field can also be shown by a number of small compasses placed


Fig. 5-12.-Magnetic field about a bar magnet illustrated by small compasses.
about the magnet as shown in Fig. 5-12. The magnetic fields between like poles and unlike poles of two magnets are shown in Fig. 5-13.

Magnetic Lines. A careful examination of the magnetic fields shown in Figs. 5-11 to $5-13$ leads to the conclusion that the magnetic field takes the form of lines arranged in an orderly fashion. These lines are com-
monly referred to as lines of magnetism or lines of induction. One line is called a maxwell. The total number of lines leaving or entering a pole is called its magnetic flux and is usually represented by the Greek letter $\phi$, pronounced phi. While these lines are invisible and said to be imaginary, their effect or existence may readily be shown by the iron filings and the compass. The magnetic lines follow definite rules which are listed below:

1. Magnetic lines always form a closed loop. The lines leave the magnet at the north pole, travel along definite paths outside the magnet, enter the magnet at the south pole, and travel through the magnet to the starting point at the north pole. This can be seen in Fig. 5-12.
2. Magnetic lines never cross one another (see Figs. 5-12 and 5-14).


How the filings arrange thenselves when under the influence of like poles

(b)

How the filings arrange themselves when under the influence of unlike poles

Fig. 5-13.-Magnetic fields about the poles of two bar magnets.
3. Magnetic lines can pass through any material, but they will take the path that offers the least resistance. This is also shown in Fig. 5-14.
4. Magnetic lines act like rubber bands. They will stretch outward if a force is exerted upon them and will contract when that force is removed. Since magnetic lines do not cross one another, they push adjacent lines away from them.


Fig. 5-14.-Path taken by magnetic
lines when a magnetic substance is near
Fig. 5-14.-Path taken by magnetic
lines when a magnetic substance is near the magnet.

Lines of Force. If a pole of a second magnet is brought into the magnetic field, a force will be exerted upon it by that ficld. The force will be proportional to the lines per square centimeter acting at right angles to the field. These lines are called lines of force, and they extend only from one pole, along the external path, and end at the other pole. They differ in this respect from lines of induction, which are always closed loops. The lines of force and the lines of induction are the same when their path
is through air, but they will differ when the path is a magnetic substance.

Field Intensity. The strength of a magnetie field, which is also called the field intensity, is expressed in terms of the force it will exert upon a magnetic pole of unit strength. The unit of field intensity is the oersted, and a magnetic field is said to be of unit intensity when it is capable of exerting a force of exactly one dyne upon a unit pole. This may be expressed mathematically by the equation

$$
\begin{equation*}
f=m \times H \tag{5-2}
\end{equation*}
$$

where $f=$ force, dynes, acting upon a magnetic pole placed in a magnetic field
$m=$ strength of the pole expressed in unit poles
$H=$ field intensity expressed in dynes per unit pole
Nota: $I I$ is also expressed in oersteds or lines per square centimeter.
Example 5-2. What is the intensity of a magnetic field that exerts a force of 500 dynes upon a magnet of 40 -unit pole strength placed in this field?

Given:

$$
\begin{array}{rlrl}
f & =500 & H=? \\
m & =40 &
\end{array}
$$

Solution:

$$
\begin{aligned}
f & =m \times H \\
\text { Therefore, } H & =\frac{f}{m}=\frac{500}{40}=12.5 \text { units of field intensity or dynes per unit pole }
\end{aligned}
$$

The strength of the magnetic field, or field intensity, is also expressed as the number of lines per square centimeter in a plane at right angles to these lines.

The field at $A B C D$ of Fig. 5-15 is said to be of unit field intensity when one line per square centimeter passes through this section perpendicular to it. This unit of one line per square centimeter is called the oersted.

Example 5-3. The two parallel pole sides shown in Fig. 5-15 are each four by six centimeters, and the magnetic field consists of 72,000 lines uniformly distributed and passing from


Fig. 5-15.-Field intensity of a magnet. the north to the south pole. (a) What is the field intensity? (b) What force would be exerted upon a pole of 25 -unit pole strength placed in this field in dynes and in ounces?

Note: 1 ounce $=28.4$ grams; 981 dynes $=1$ gram.

Given:

$$
\begin{aligned}
\phi & =72,000 \\
\text { Area of pole } & =4 \times 6 \mathrm{sq} \mathrm{~cm} \\
m & =25
\end{aligned}
$$

Find:

$$
\begin{aligned}
H & =? \\
f & =?
\end{aligned}
$$

Solution:
(a) Field intensity $H$ equals oersteds or lines per square centimeter

$$
H=\frac{\phi}{\text { area }}=\frac{72,000}{4 \times 6}=3000 \text { oersteds }
$$

(b)

$$
\begin{gathered}
f=m \times H=25 \times 3000=75,000 \text { dynes } \\
f=\frac{75,000}{981 \times 28.4}=2.69 \mathrm{oz}
\end{gathered}
$$

Flux Density. The number of magnetic lines per square centimeter in a plane perpendicular to the direction of the magnetic field is commonly called the fux density and is designated by the symbol $B$. When the magnetic field is uniform, that is, each square centimeter contains the same number of lines, then the flux density may be expressed mathematically as

$$
\begin{equation*}
B=\frac{\phi}{A} \tag{5-3}
\end{equation*}
$$

where $B=$ flux density, oersteds
$\phi=$ total flux
$A=$ area, square centimeters
Example 5-4. A magnetic pole has a flux of 150,000 maxwells. If the field is uniformly distributed and the pole is five centimeters wide and 10 centimeters long, what is the flux density?

Given: Find:

$$
\begin{aligned}
& \phi=150,000 \\
& A=5 \times 10 \mathrm{sq} \mathrm{~cm}
\end{aligned}
$$

Solution:

$$
\begin{aligned}
B & =\frac{\phi}{A}=\frac{150,000}{50} \\
& =3000 \text { oersteds }
\end{aligned}
$$

Flux density is often expressed in lines per square inch to correspond with the English units in place of the metric units. In such cases, the flux density is still found by dividing the total flux by the area, but as the area is in square inches the flux density will be expressed in lines per square inch and not in oersteds.

Example 5-5. What is the flux density of a magnetic pole 1 by 3 inches that has an evenly distributed flux of 4500 maxwells?

Given:

$$
\begin{aligned}
& \phi=4500 \\
& A=1 \times 3 \mathrm{sq} \text { in. }
\end{aligned}
$$

Find:

$$
B=\text { ? }
$$

Solution:

$$
\begin{aligned}
B & =\frac{\phi}{A} \\
& =\frac{4500}{1 \times 3}=1500 \text { lines per square inch }
\end{aligned}
$$

The magnetic lines of the bar magnet of Fig. 5-11 represent its magnetic field in only one plane. If the bar were set on its edge and iron filings were again used, they would show a similar magnetic field extending out from the magnet. It can be shown by this means that the magnetic field extends in all directions from the pole of a magnet. If a unit magmetic pole is placed in the center of a sphere of one centimeter radius as shown in Fig. 5 -16, it will have a field of $4 \pi$ or $12.57(4 \times 3.1416=12.5664)$ lines. This is true because according to the definition of the unit pole an


Fig. 5-16.-Magnetic field of a unit pole in a sphere two centimeters in diameter.
equal and like pole placed anywhere on the sphere which would be one centimeter away, would be repelled with a force of one dyne. Also the field must be of unit intensity, one oersted or one line per square centimeter, in order to exert a force of one dyne upon the unit pole. As the area of a sphere is $4 \pi r^{2}$, there will be $4 \pi(1)^{2}$ or 12.57 square contimeters and therefore 12.57 lines. The number of lines from a pole whose strength is $m$ becomes

$$
\begin{equation*}
\phi=4 \pi m \tag{5-4}
\end{equation*}
$$

Example 5-6. How many magnetic lines are emitted bv a magnetic pole whose strength is 20 unit poles?

Given:

$$
\mathrm{m}=20
$$

Find:

$$
\phi=?
$$

Solution:

$$
\begin{aligned}
\phi & =4 \pi m=12.57 \times 20 \\
& =251.4 \text { maxwells }
\end{aligned}
$$

5-9. Magnetic Induction. It has been previously stated that a magnet has the power of attracting iron and steel to it. As the magnet is brought close to a piece of iron or steel, that piece becomes magnetized by induction. Figure $5-17 a$ shows an iron nail $A$ brought close to a magnet. Some of the magnetic lines leaving the north pole of the magnet find that their path of least resistance is through the nail and hence take such a path. The nail becomes magnetized by induction, and the lines entering at the head


Fig. 5-17.-Magnetic induction: (a) nail $A$ magnetized by induction, (b) nail $B$ magnetized by contact with nail $A$.
of the nail make it a south pole and upon leaving at the point make it a north pole. If a second nail is now placed in contact with the first (Fig. $5-17 b$ ), it will cling to it. The second nail has become magnetized by magnetic induction through its contact with the first nail. The head will be a south pole because the magnetic lines enter there, and the point will be a north pole because the lines leave the nail at the point. If the space between the magnet and the first nail is increased, the two nails will no longer hold together and the second one will fall off, thus showing that they were magnetized only while in the influence of the bar magnet.

Poles Produced by Magnetic Induction. Magnetic induction always causes a south pole to be produced in that part of a magnetic substance nearest to the north pole of the magnet and a north pole at that end nearest the south pole of the magnet. As the induced pole nearest to the magnet is always of a polarity opposite that of the inducing pole, there will be a
fore of attraction between the two poles. Furthermore, as the magnetic lines always tend to shorten themselves, they will try to pull the induced pole against the inducing pole. This explains why a magnet draws a piece of iron to it.

If a comparatively weak north pole of one magnet is brought near a strong north pole of another magnet, it is possible that there will be an attraction between the two poles instead of the repulsion that would be expected. This is explained by the fact that when the weak north pole is brought near the strong north pole the strong north induces a south in the first magnet that exceeds its own weak north and attraction results. The polarity of the weaker magnet becomes reversed under this condition. For this reason, a compass needle or another weak magnet should not be brought too close to a strong pole of similar polarity.

Magnetism may be induced in several ways. In Fig. 5-17a, the magnetism is induced in the nail $A$ by its mere presence in the magnetic field of the bar magnet. In Fig. $5-17 b$, the magnetism is induced in the nail $B$ by its contact with the magnet, nail $A$. In Fig. 5-3, magnetism is induced in the bar by stroking it with a lodestone.

5-10. Magnetic Properties and Classification of Materials. The properties of magnetic materials most commonly referred to are reluctance, reluctivity, permeance, permeability, and retentivity.

Reluctance. Reluctance is defined as the opposition offered by a material to the passage of magnetic lines. This corresponds to the term resistance in the electric circuit, which is the opposition to the flow of electric current. Reluctance, however, has the additional peculiar characteristic that its value for magnetic materials is not constant but varies if the flux density is changed. The reluctance of nonmagnetic materials is constant. This term is used in magnetic-circuit calculations. Reluctance is represented by the script letter $\mathcal{R}$, but no name is given to this quantity.

In some references and texts the name oersted is given as the unit of magnetic reluctance. In accordance with the agreement in 1930 by the International Electrotechnical Commission, the name oersted was established to designate the unit of magnetizing force and the name gauss to designate the.unit of magnetic induction.

Reluctivity. Reluctivity is the specific reluctance or the reluctance per centimeter cube. For nonmagnetic materials, its value is one, and for magnetic materials, its value varies with changes in flux density. Reluctivity corresponds to resistivity in the electric circuit. There is no name assigned for the unit of reluctivity, but its symbol is the Greck letter $\nu$, pronounced nu.

Permeance. Permeance is the ability of a material to carry magnetic lines. Its value is equal to the reciprocal of the reluctance. Permeance
corresponds to conductance in the electric circuit. Its symbol is $\mathcal{P}$, but no name has been adopted for its unit.

Permeability. Permeability is a measure of the ease with which magnetic lines can pass through a material. Numerically it is equal to the reciprocal of the reluctivity. The permeability of nonmagnetic materials is one, while for magnetic materials it is a variable depending upon the flux density. It may also be considered as the ratio of the lines of force passing through a material to the lines of force passing through air for the same conditions. Good magnetic materials will have a high value of permeability. Its symbol is the Greek letter $\mu$, pronounced mu. No name has been assigned to this unit.

Retentivity. Retentivity is the ability of a material to hold its magnetism after the magnetizing force is removed. Permanent magnets have a high degree of retentivity, and temporary magnets are low in retentivity. The magnetism that remains in a material after the magnetizing force is removed is called the residual magnetism. This property is very uscful in the operation of electric generators.

Classification. It has been common practice in the past to classify materials as being either magnetic or nonmagnetic. Iron, steel, nickel, and such other materials that may easily be magnetized were classed as magnetic; air, copper, brass, and such materials which seemed impossible to magnetize were classed as nonmagnetic. The present practice is to classify materials into one of three groups, namely, ferromagnetic, paramagnetic, and diamagnetic.

Ferromagnetic materials are those which become strongly magnetized in the direction of the magnetizing field. They have high values of permeability. Included in this classification are iron, steel, nickel, cobalt, magnetite, and alloys such as Heusler's alloys, Permalloy, and Alnico.

Paramagnetic materials are those which will become only very weakly magnetized but in the direction of the magnetizing field. The permeability of these materials is greater than one but the values are low. Included in this classification are aluminum, platinum, oxygen, air, manganese, and chromium.

Diamagnetic materials are those which will become very weakly magnetized but in a direction opposite to that of the magnetizing field. The permeability of these materials is less than one. Included in this classification are bismuth, antimony, copper, zinc, mercury, gold, and silver.

In general only iron, steel, Permalloy, Alnico, and a few other alloys are considered as magnetic materials, and practically all other materials are considered nonmagnetic. The permeability of iron and steel varies from 25 to 2500 . Permalloy, which is an alloy containing about 78 per cent nickel and 22 per cent iron, is highly magnetic and has been known to have a value as high as 85,000 .

Alnico is a very powerful magnetic alloy recently developed. Sintered Alnico is an alloy of aluminum, nickel, iron, and cobalt made by pressing the powdered metals together and heating them to almost melting point. Its magnetic qualities are so great that it can lift 500 times its own weight. By assembling a magnet in a special manner, it has been found capable of lifting as much as four thousand times its own weight.


5-11. Magnetic Shapes. Bar Magnets. Magnets are used for many purposes and are therefore to be found in a large variety of shapes. The bar magnets shown in Fig. 5-18a are commonly used in the school laboratories, as they provide a very satisfactory means of demonstrating the laws


Fig. 5-19.-Forms of magnets: (a) horseshoe, (b) U-shape.
of magnetism and the path of magnetic lines. When the bar magnets are not being used, it is desirable to place them so that adjacent ends will be of opposite polarity (see Fig. 5-18b) in order that they may retain their magnetic strength.

Horseshoe Magnets. Magnets for commercial use are generally in the
form of a horsestoe or some variation of this form because they provide a much stronger magnet than the bar magnet of an equal weight of material. This is true because the two poles are closer to one another and also because both poles can probably be utilized. When the horseshoe magnet is not


Fig. 5-20.-Applications of magnets.
in use, a soft iron keeper should be placed across its poles to keep the magnet strong. Forms of the horseshoe magnet are shown in Fig. 5-19. The magnet used in the cone-type radio speaker and the permanent magnet of a popular-type electrical measuring instrument are shown in Fig. 5-20. The compound horseshoe magnet of a mag-


Fig. 5-21.-Compound horseshoe magnet. neto used for some types of ignition systems is illustrated in Fig. 5-21.

Laminated Magnets. The strongth of magnets may be further increased by constructing a magnet of several thin strips instead of one thick strip. This is referred to as a laminated magnet and is illustrated in Fig. 5-18c.

Ring Magnets. Magnets are sometimes made in a circular form as shown in Fig. $5-22 a$ and are called ring magnets. This magnet has no poles, and its magnetic lines form closed loops around the ring. This form may be used to demonstrate the operating principle of the transformer. The magnetic core of the transformer shown in Fig. $5-22 b$ is but a variation of the ring magnet.
If a small piece is cut out of a ring magnet as in Fig. 5-23, it will then have two strong magnetic poles. This type of magnet is sometimes used in electrical instruments.

Magnetic Screens. Another use of a circular (usually a temporary)

(a)

Ring magnet


Transformer cores

Fig. 5-22.-Applications of magnets.
magnet is the magnetic screen. Its purpose is to shield any object from magnetic lines as illustrated by the meter and its screen in Fig. 5-24. As there is no known insulator of magnetic lines, shielding is the practical substitute and is accomplished by placing the object in the center of a ring or another closed circuit. As the screen is usually made of soft iron, it provides the easiest path for the mag-


Fig. 5-23.-Ring magnet used in a meter. netic lines and thereby keeps the object in the center free from these magnetic lines.

5-12. The Earth's Magnetism and the Compass. Magnetic Characteristics of the Earth. The earth possesses magnetic characteristics that make it act like a large bar magnet with its poles located near the geographic


Fig. 5-24.-Magnetic screen.
poles (Fig. 5-25). Because the earth's magnetic north pole lies near its geographic south, the magnetic lines leave the earth near the south geographic pole and flow toward the north near the surface of the earth. They enter near the north geographic pole and travel through the earth to the south to form their closed loop. Thus the earth's surface has a nearly
uniform magnetic field, flowing from the south to the north, commonly called the earth's magnetic field. These magnetic characteristics have greatly aided navigators for many years in guiding their ships. However,


Fig. 5-25.-The earth as a magnet. (From "Unified Physics" by G. L. Fletcher, I. Mosbacher, and S. Lehman, McGraw-Hill Book Company, Inc., New York.)
for greatest accuracy in navigation, corrections must be made to allow for the difference between the location of the magnetic and geographic poles and for the distortions of the earth's magnetic field.

The Compass. It has previously been stated that when an elongated lodestone or a bar magnet is suspended so that it is free to rotate it will


Fig. 5-26.-The simple compass. come to rest in approximately a northsouth position. This is due to the effect of the earth's magnetic field upon the lodestone (or bar magnet), causing it to take a position parallel with the earth's magnetic lines. This is the principle upon which the compass operates. The simple compass is shown in Fig. 5-26 and consists of a small bar magnet, usually called the needle, mounted on a pivot so that it is free to rotate above a printed scale of the compass points $\mathrm{N}, \mathrm{E}$, S , and W . The compass was already in
general use in Columbus's time and was used by him at the time of his historic voyage to America. The mariner's compass has its printed scale attached to the needle, and its indication is noted against a mark on the rim of the compass. 'These compasses must have special mountings to keep them level at all times. Aviation and ship compasses are set in liquid to keep them level.

Declination. The navigators in the days of Columbus knew that the compass did not point to the true geographic north pole, but it was not until 1831 that the location of the northern magnetic pole was discovered. In that year, Sir James Ross located the pole at latitude $70^{\circ} 30^{\prime}$ longitude


Fig. 5-27.-A map of the world showing lines of equal magnetic declination. Such lines are called isogonic lines. (From "Unified Physics" by G. L. Fletcher, I. Mosbacher, and S. Lehman, McGraw-Hill Book Company, Inc., New York.)
$95^{\circ} \mathrm{W}$. It was located again in 1905 by Roald Amundsen a short distance from the previous point. Its approximate location is given as $70^{\circ} 5^{\prime} \mathrm{N}$ and $96^{\circ} 46^{\prime} \mathrm{W}$. This location is a spot in Boothia, northern Canada, approximately $20^{\circ}$ from the north geographic pole; this is about 1000 miles from the geographic pole. The angle by which the compass needle at any given place on the earth points away from the geographical north is called the declination of the compass at that place. As the declination varies considerably with many localities, magnetic maps (see Fig. 5-27) have been prepared for determining the correct values. For example, the declination at New York is approximately $10^{\circ} \mathrm{W}$; at Atlanta, Ga., it is nearly zero; and in California it is about $15^{\circ} \mathrm{E}$. This means that the
compass points in a direction approximately $10^{\circ}$ west of the geographic north at New York. The declination is also often referred to as the magnetic deviation of the compass. In addition to the earth's nonuniform magnetic condition, the deriation of the compass is also affected by the presence of magnetic materials near the compass. The magnetic effect of the hull of a steel ship is corrected to a large extent by placing large balls of magnetic materials near the compass.

The location of the magnetic pole determined by Ross in 1831 and by Amundsen in 1905 indicates that the earth's magnetic condition is continually changing. It is stated that the magnetic pole is continually shifting in a large circle which it is estimated will take about 25,000 years to complete. Thus the declination will be con-


Fig. 5-28.-The dipping needle. tinually changing. This is called the magnetic variaion, and for accurate work, tables are prepared for each day of the year to account for this shift in the declination. Celestial and terrestrial disturbances such as sunspots, earthquakes, and so-called magnetic storms cause errors also called magnetic variations.

Dip. Another effect of the earth's magnetism is the magnetic dip. This effect is clearly demonstrated when a freely suspended needle under the influence of the earth's magnetism alone is permitted to come to rest. It will be found that its position will not be parallel to the earth's surface but will be at an angle to the horizontal that varies with the geographical location. This can be determined by the dipping-needle compass shown in Fig. 5-28. At New York, the dip is about $70^{\circ} \mathrm{N}$. At each magnetic pole, the dip is 90 degrees, while near the equator it is zero.

A complete study of terrestrial and celestial magnetic conditions is essential in navigation and radio location work.

5-13. Magnetic Field about a Wire Carrying a Current. Magnetism has been described as the twin of electricity, but up to this point magnetism has been considered alone. The important part that electricity plays in magnetic work will now be studied.

In 1819, Oersted made the famous discovery that a magnetic field always exists about a wire that is carrying a current. He found that certain relations existed between the magnetic and electric conditions. The principle of Oersted's discovery may be demonstrated by placing a wire vertically through a piece of cardboard and determining its magnetic characteristics when a current is flowing through the wire. This is shown
in Fig. $5-29$ where the wire $A B$ is placed through the cardboard $D$ and connected to either a d-c generator $G$ or a storage battery $S$. (Note that an adjustable resistor $R$ is connected in series to control the amount of current.) If a current of approximately 50 amperes is permitted to flow from $A$ to $B$ and then some iron filings are sprinkled over the cardboard, it will be found that the filings will take an orderly circular form about the wire as shown in Fig. 5-29. It will also be observed that if compasses are set at the positions $C$ the compass needles will come to rest at right angles to the wire and will have their north poles each pointing in the same direction of rotation. If the current is removed by opening the


Fig. 5-29.--Magnetic field about a wire carrying a current.
switch, the compass needles will no longer stay perpendicular to the wire but will take positions in accordance with the earth's or any other nearby magnetic field. If the cardboard is gently tapped, the iron filings will take a haphazard form. Thus it must be concluded that the magnetic field exists about the wire only when a current is flowing through it.

5-14. Relation of Magnetic Field and Electron Flow. Effect of Amount of Electron Flow. If the current flowing through the wire of Fig. $5-29$ is reduced from 50 to 30 amperes, it will be found that while the irgn filings still take the same form a smaller quantity of filings will be affected. If the compasses are moved, first closer to the wire and then farther from the wire, it will be found that the magnetic effect will extend farther from the wire when the current is increascd. From this, it is seen that the strength of the magnetic field about a current-carrying wire increases as the current increases and decreases as the current decreases.

Effect of Direction of Electron Flow. If the leads connected to the wire $A B$ are interchanged and the circuit is adjusted so that 50 amperes
flow from $B$ to $A$, the sprinkling of iron filings on the cardboard will produce the same results as before. Close observation of the compass needles will, however, indicate that a change has taken place, for they will now have their north poles pointing in the direction opposite to that when the current was flowing from $A$ to $B$. This indicates that the direction of the magnetic field about a current-carrying wire depends upon the direction of current flow through that wire.


Fig. 5-30.-Left-hand rule for a wire carrying a current: (a) electrons flowing downward, (b) electrons flowing upward.

Left-hand Rule for a Wire Carrying a Current. The relation between the direction of electron flow and the magnetic field is easily remembered by the left-hand rule for a wire, which states: Grasp the wire with the left hand so that the thumb points in the direction of the electron flow, and the fingers will poiat in the direction of the magnetic field. Thus, if either the direction of electron flow or the direction of the magnetic field is known, the other may be obtained. Figure 5-30 illustrates this rule. It should
be observed that $\oplus$ indicates electron current entering the wire and $\odot$ indicates electron current leaving the wire.

Note: The above left-hand rule is a variation of the old right-hand rule used with the older conventional direction of current flow. This rule stated: Grasp the wire with the right hand so that the thumb points in the direction of current flow, and the fingers will point in the direction of the magnetic field. This rule was used to determine the relationship between the direction of the magnetic field and current flow before the discovery of the electron and the subsequent discovery of electron flow. At that time current was assumed to flow from positive to negative. The discovery of electron flow proved this assumption to be incorrect as current actually flows from negative to positive.


Fig. 5-31.-Magnetic field about a straight wire carrying a current.
5-15. Magnetic Field of a Coil, Left-hand Rule. Magnetic Field of a Coil. The magnetic ficld around a straight wire carrying a current exists at all points along its length and consists of concentric circles in a plane perpendicular to the wire (see Fig. 5-31). If such a long piece of wire is wound on a core as shown in Fig. 5-32, it is called a coil; it is also commonly called a solenoid or helix. When a current is made to flow through this coil, additional magnetic characteristics result.

Considering first the two turns of wire, at $A$ and $B$ of Fig. 5-33, it will be seen that the current is leaving at these points. If the wires are not too close to each other, their magnetic fields will take the form as illustrated in Fig. 5-34a. Notice that the magnetic lines at the right of $A$ are going downward while the adjacent lines (at the left of $B$ ) are going upward. When the two wires $A$ and $B$ are adjacent to each other, the effect of the one's downward and the other's upward lines will neutralize one another at this point, but the field about the two wires will be strengthened as indicated in Fig. 5-34b.

When the entire coil, $A$ to $F$ and $A^{\prime}$ to $G^{\prime}$, is considered, the resulting
magnetic field is shown in Fig. 5-32. The strength of the magnetic field will increase with an increase in the number of turns and also with an increase in the amount of current flowing. Note that the field is the strongest at the ends of the coil and that this corresponds with the definition for a pole; hence the coil is said to have two magnetic poles. The direction of the magnetic lines indicated in Fig. 5-32 shows them leaving


Frg. 5-32.-Magnetic field about a coil carrying a current.
the coil at the left side and entering again at the right. According to the magnetic laws, this indicates that the left pole is a north and the right is a south.

Left-hand Rule for a Coil. As the direction of the magnetic lines was determined by the direction of current flow, it is apparent that reversing the direction of current flow will reverse the polarity of the coil. The relation between the direction of


Fig. 5-33.-View showing the direction of the current in the conductors of the coil of Fig. 5-32. the current flow and the poles of the coil is known as the left-hand rule for a coil and is stated: Grasp the coil with the left hand so that the fingers point in the direction of the electron flow, and the thumb will point in the direction of the coil's north pole. This is illustrated in Figs. 5-35a and 5-35b.

The magnetic strength of the coil is also affected by the type of core upon which the coil is wound. If a ferromagnetic material is used for the core, it will produce a much stronger coil than a paramagnetic core.

5-16. Magnetic Circuits and Calculations. Many electrical devices depend upon magnetism for their operation, and, to have these devices


Frg. 5-34.-Magnetic fields about adjacent conductors of a coil carrying a current: (a) conductors separated, (b) conductors adjoining.


Fig. 5-35.--Left-hand rule for a coil: (a) electrons entering the coil at the right terminal, (b) electrons entering the coil at the left terminal.
function efficiently, engineers work out intricate designs for the required magnetic conditions. The magnets must supply the required strength and must be provided with paths or circuits of suitable shapes and materials. The magnetic circuit is defined as the path (or paths) taken by the magnetic lines of induction leaving a north pole, passing through the entire circuit, and returning to the starting point. A magnetic circuit may be a simple series or parallel circuit or may be a complicated combination circuit depending upon how intricate the device may be.

Many of the magnetic-circuit calculations are similar to the electriccircuit calculations as is apparent in the following table.

Table V-I

| Unit | Electric circuit | Magnetic circuit |
| :---: | :---: | :---: |
| Pressure. | Volt (E) | Gilbert ( $F$ ) |
| Quantity. | Ampere ( ) $^{\text {a }}$ | Maxwell ( $\phi$ ) |
| Resistance. | Ohm (R) | $\mathfrak{R}$ |
| Ohm's law. | $\begin{aligned} E & =I R \\ I & =\frac{E}{R} \\ R & =\frac{E}{I} \end{aligned}$ | $\begin{aligned} F & =\phi \mathcal{R} \\ \phi & =\frac{F}{\mathcal{R}} \\ \mathcal{R} & =\frac{F}{\phi} \end{aligned}$ |
| Specific resistance. | Resistivity (K) $R=\frac{K l}{A}$ | $\begin{aligned} & \text { Reluctivity }(\nu) \\ & \mathcal{R}=\frac{\nu l}{A} \end{aligned}$ |

Examination of Table V-I will show that the magnetic-circuit calculations and the electric-circuit calculations are very similar. There is, however, one point in which the two systems differ, and this occurs in the calculation of the reluctance (resistance) of the magnetic circuit. In the resistance equation of the electric circuit, $K$ represents the specific resistance of the material used to conduct the electric current. The value of $K$ is obtained from Appendix IV; for example, $K$ for copper is 10.4, and this value is the same whether the conductor carries one or five amperes. In the reluctance equation of the magnetic circuit, $\nu$ represents the specific reluctance of the material used to conduct the magnetic lines. This value of $\nu$ is not a constant even for a given material but varies with the number of magnetic lines per unit of area and must thercfore be found separately for each condition considered. This may be done by use of the permeability curve of the kind of material being used. Curves for various materials may be found in engineering handbooks or may be obtained from the manufacturer of the material. A sample permeability curve for cast steel is given in Fig. 5-36. The specific reluctance or re-
luctivity is the reciprocal of the permeability, or

$$
\begin{equation*}
\nu=\frac{1}{\mu} \tag{5-5}
\end{equation*}
$$

where $\nu=$ specific reluctance (or reluctivity)
$\mu=$ permeability


Fig. 5-36.-Permeability curve for cast steel.
Example 5-7. What is the reluctivity of a piece of cast steel four square centimeters in cross section and carrying 16,000 maxwells?

Given:

$$
\begin{aligned}
\phi & =16,000 \\
A & =4 \mathrm{sq} \mathrm{~cm}
\end{aligned}
$$

Find:
$\nu=$ ?

Solution:
To find $\nu$, we must have $\mu$; and to find $\mu$ from Fig. 5-36, we must have $B$ [see Eq. (5-3)].

$$
\begin{aligned}
B & =\frac{\phi}{\Lambda}=\frac{16,000}{4}=4,000 \text { oersteds } \\
\mu & =1200 \text { (from curve, Fig. } 5-36 \text { ) } \\
\nu & =\frac{1}{\mu}=\frac{1}{1200}=0.000833
\end{aligned}
$$

Example 5-8. What is the reluctivity of the piece of cast steel of Example 5-7 if it must carry a flux of 26,000 maxwells?

Given:

$$
\begin{aligned}
& \phi=26,000 \\
& A=4 \mathrm{sq} \mathrm{~cm}
\end{aligned}
$$

Find:
$\nu=$ ?

## Solution:

$$
\begin{aligned}
B & =\frac{\phi}{A}=\frac{26,000}{4}=6500 \text { oersteds } \\
\mu & =1400 \text { (from curve, Fig. } 5-36 \text { ) } \\
\nu & =\frac{1}{\mu}=\frac{1}{1400}=0.000714
\end{aligned}
$$

It is now possible to find the reluctance of a magnetic circuit by the equation

$$
\begin{equation*}
\mathfrak{R}=\frac{\nu l}{\bar{A}} \tag{5-6}
\end{equation*}
$$

where $Q=$ reluctance of the circuit
$\nu=$ reluctivity of the magnetic path
$l=$ length of the magnetic path, centimeters
$A=$ area of the magnetic path, square centimeters
Example 5-9. If the length of the magnetic path of Example 5-7 is 16 centimeters, what is its reluctance?

```
Given:
    \nu=0.000833 (from Example 5-7)
    l=16 cm
    A=4 sq cm
```

Solution:

$$
\begin{aligned}
R & =\frac{\nu l}{A}=\frac{0.000833 \times 16}{4} \\
& =0.003332
\end{aligned}
$$

Find:
$R=$ ?

When the reluctance (resistance) and the flux in maxwells (quantity) are known, it is then possible to find the pressure (magnetomotive force $F$ ) required to push the flux through the magnetic circuit by use of Ohm's law for magnetism.

$$
\begin{equation*}
F^{\prime}=\phi \Omega \tag{5-7}
\end{equation*}
$$

where $F=$ magnetomotive force, gilberts
$\phi=$ flux, maxwells
$Q=$ reluctance
Example 5-10. What pressure is required to push the flux through the circuit of Example 5-9?

> Given: $\begin{aligned} \phi & =16,000 \text { maxwells } \\ \mathfrak{R} & =0.003332 \text { (from Example } 5-9)\end{aligned}$

Find:
$F=$ ?

Solution:

$$
\begin{aligned}
\mathbf{F} & =\phi \mathbb{R} \\
& =16,000 \times 0.003332 \\
& =53.3 \text { gilberts }
\end{aligned}
$$

As electromagnets are used in many cases, the magnetizing force is supplied by passing an electric current through a coil wound around a portion of the magnetic circuit. The strength of this magnetizing force
will depend upon the number of turns and the amount of current flowing and is expressed by the equation

$$
\begin{equation*}
F=1.26 \mathrm{~N} I \tag{5-8}
\end{equation*}
$$

where $F=$ magnetomotive force, gilberts
$N=$ number of turns on the coil
$I=$ amperes flowing through the coil
Example 5-11. How many ampere-turns are required for a coil to supply the magnetic circuit of Example 5-10?

$$
\begin{array}{ll}
\text { Given: } & \text { Find: } \\
F=53.3 & N I=?
\end{array}
$$

Solution:

$$
\begin{gathered}
F=1.26 \mathrm{NI} \\
\text { Therefore, } N I=\frac{F}{1.26}=\begin{array}{l}
53.3 \\
1.26
\end{array}=42.3 \text { ampere-turns }
\end{gathered}
$$

Example 5-12. If the coil of Example 5-11 has 100 turns, how many amperes would be required?

$$
\begin{array}{rrr}
\text { Given: } & \text { Find: } \\
N I & =42.3 & I=? \\
N=100 &
\end{array}
$$

Solution:

$$
\begin{gathered}
N I=42.3 \\
\text { Therefore, } I=\frac{42.3}{N}=\frac{42.3}{100}=0.423 \mathrm{amp}
\end{gathered}
$$

Example 5-13. If the coil of Example 5-11 is connected in an electric circuit in which 0.50 ampere is flowing, how many turns would the coil need?

$$
\begin{array}{rlrl}
\text { Given : } & \text { Find: } \\
N I & =42.3 & N & =? \\
I & =0.50 &
\end{array}
$$

Solution:

$$
\begin{gathered}
N I=42.3 \\
\text { Therefore, } N=\frac{42.3}{I}=\frac{42.3}{0.50}=84.6 \text { turns }
\end{gathered}
$$

It has been stated that magnetic circuits may be simple series or parallel circuits or complicated combination circuits. In any event, they follow the same method of procedure as solving the electric circuits. In the series magnetic circuit, for example, the total reluctance is

$$
\begin{equation*}
\mathfrak{R}_{T}=\mathfrak{R}_{1}+\mathfrak{R}_{2}+\Omega_{3}, \text { etc. } \tag{5-9}
\end{equation*}
$$

Example 5-14. What is the reluctance of a series circuit made of reluctances of $0.0025,0.0005$, and 0.0015 ?


Find:

$$
R_{1}=0.0025
$$

$$
\mathfrak{R}_{T}=?
$$

$$
\mathfrak{R}_{2}=0.0005
$$

$$
\mathfrak{R}_{3}=0.0015
$$

$$
\begin{aligned}
\mathfrak{R T} & =\mathscr{R}_{1}+\mathfrak{R}_{2}+\mathscr{R}_{3} \\
& =0.0025+0.0005+0.0015 \\
& =0.0045
\end{aligned}
$$

For a parallel circuit, the total reluctance is

$$
\begin{equation*}
\mathfrak{R}_{T}=\frac{1}{\frac{1}{\Omega_{1}}+\frac{1}{\mathfrak{R}_{2}}+\frac{1}{R_{3}}, \text { etc. }} \tag{5-10}
\end{equation*}
$$

Example 5-15. What is the reluctance of a parallel magnetic circuit made of reluctances of $0.0025,0.0005$, and 0.0025 ?

$$
\begin{array}{lr}
\text { Given: } & \text { Find: } \\
\Re_{1}=0.0025 & \mathscr{R}_{T}=? \\
\Re_{2}=0.0005 & \\
\Re_{3}=0.0025 &
\end{array}
$$

Solution:

$$
\begin{aligned}
\mathfrak{Q}_{T} & =\frac{1}{\frac{1}{\mathfrak{R}_{1}}+\frac{1}{\mathfrak{R}_{2}}+\frac{1}{\mathfrak{R}_{3}}} \\
& =\frac{1}{\frac{1}{0.0025}+\frac{1}{0.0005}+\frac{1}{0.0025}} \\
& =\frac{1}{\frac{1+5+1}{0.0025}}=\frac{1}{\frac{7}{0.0025}} \\
& =\frac{0.0025}{7}=0.000357
\end{aligned}
$$

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## QUESTIONS

1. What is the relative importance of magnetism and electricity to the study of radio, television, and electronics?
2. Define (a) magnet, (b) magnetic materials.
3. What is meant by (a) a natural magnet? (b) An artificial magnet?
4. (a) Distinguish between permanent and temporary magnets. (b) What materials are used for each?
5. (a) Describe the poles of a magnet. (b) By what name is each pole known?
6. Explain Weber's theory of magnetism.
7. State the law of magnetic polarity.
8. How does the distance between two magnets affect the force of attraction or repulsion between them?
9. State four rules to which the actions of magnetic lines conform.
10. (a) What is meant by the magnetic field? (b) How may the field of a magnet be illustrated?
11. What are magnetic lines?
12. What are lines of force?
13. Distinguish between lines of induction and lines of force.
14. Define flux density.
15. Explain why a magnet draws a piece of magnetic material to it.
16. Describe several methods of inducing magnetism in a steel bar.
17. Define the following propertics of magnets: (a) reluctance, (b) permeability, (c) retentivity.
18. (a) Define reluctivity. (b) With which electrical term does it compare?
19. (a) Define permeance. (b) What is its relation to reluctance?
20. Give the meaning of ferromagnetic, paramagnetic, and diamagnetic as they apply to classification of materials.
21. (a) What is Permalloy? (b) What is its use?
22. (a) What is Alnico? (b) What is its use?
23. Name three shapes of magnets, and give some uses of each.
24. State some precautions to be observed in the use of magnets.
25. Describe the earth's magnetism.
26. (a) Where is the earth's magnetic south pole located? (b) Is the location permanent? Explain.
27. What is meant by the declination of a compass?
28. What is meant by the magnetic variation?
29. What is done to compensate for the effects of declination and variation?
30. (a) What was Oersted's discovery? (b) To what important uses is it applied?
31. State the left-hand rule for a wire carrying a current.
32. What effect does a change in the amount of current flowing in a conductor have upon the magnetic field?
33. What effect does a change in the direction of current flow in a conductor have upon the magnetic field?
34. What is a solenoid?
35. What factors determine the magnetic strength of a solenoid?
36. State the left-hand rule for a coil.
37. Compare magnetic circuits with electric circuits.
38. What is Ohm's law for the magnetic circuit?
39. In what manner does the reluctance of a magnetic circuit differ from the resistance of an electric circuit?
40. Why are magnetic circuit calculations necessary?
41. How is the reluctance of a magnetic circuit affected (a) by the length of the circuit? (b) By the cross-sectional area of the magnetic path?
42. How do the number of turns and the current affect the magnetomotive force of an electromagnet?
43. How are reluctances of a series magnetic circuit added?
44. How are reluctances of a parallel magnetic circuit added?

## PROBLEMS

1. Draw a diagram showing how a bar magnet can be used to magnetize a stec] rod by stroking the rod with the bar magnet. Indicate the area of the steel rod that must be stroked by one pole of the magnet and the area that must be stroked by the other pole. Indicate the polarity of the magnet and also the polarity that the steel rod will have after the stroking operation is completed.
2. If two steel bars were found on a workbench and their magnetic characteristics were not known, describe a simple method of testing whether (a) neither bar was a magnet, ( $b$ ) both bars were magnets, (c) only one bar was a magnet.



Fig 5-37.


Fig. 5-38.
3. If it was found that only one of the two bars of Prob. 2 was a magnet, describe a method to determine (a) which bar is the magnet, (b) which of its poles is the north pole, (c) which of its poles is the south pole.
4. Draw a diagram of two bar magnets in the relative positions shown in Fig. 5-37, and sketch the magnetic field that will result.
5. Reverse the lower magnet of Prob. 4 so that it will be in the position as shown in Fig. 5-38, and sketch the magnetic field that will result.
6. Sketch the magnetic fields of the two bar magnets in the position as shown in Fig. 5-38.
7. Sketch the magnetic field of the permanent magnet shown in Fig. 5-20b.
8. Sketch the magnetic field of the magnet and the soft steel bar shown in Fig. $5-39$. What is the polarity of the bar at $A$ and at $B$ ?
9. A magnetic north pole of 250 -unit pole strength and a south pole of 100 -unit
pole strength are placed 10 cm apart in air. (a) What force in dynes is acting on the poles? (b) Is the force attraction or repulsion?
10. Two magnetic north poles of 500 - and 250 -unit poles, respectively, are phaced 15 cm apart in air. (a) What force in dynes is acting on the poles? (b) Is the foree attraction or repulsion?
11. What is the distance between two magnetic poles of like polarity having strengths of 100 - and 200 -unit poles, respectively, if the force acting on the poles is 2000 dynes?
12. How far apart must two magnetic poles of unlike polarity of 250 -unit pole strength each be spaced in air if the force of attraction is to be 3000 dynes?
13. If two poles of similar strength are to have a force of attraction of 1500 dynes through a distance of 6 cm , what is the strength of each pole?
14. What strength is required of a magnet if it is to exert a repulsion force of 6250 dynes upon a magnet of 250 -unit poles that is 2 cm away?
15. If each pole of the horseshoe magnet of Fig. $5-39$ has a strength of 250 -unit poles, what is the total force acting on the bar $A B$ when the distance between the bar and the magnet is (a) 2 cm ? (b) 1 cm ? (c) $\frac{1}{2} \mathrm{~cm}$ ?
16. What is the force in dynes on a magnet of 75unit pole strength when placed in a field whose inten-


Fic. $5-3 \%$ sity is 100 lines per square centimeter?
17. What is the force in ounces on a magnet of 125 -unit poles when placed in a field whose intensity is 300 oersteds?
18. If a magnet of 40 -unit pole strength is to be acted upon with a force of 3 oz when placed in a magnetic field, what must be the intensity of the field?
19. What is the strength of a magnet that is acted upon with a force of $\frac{1}{4} \mathrm{lb}$ when placed in a field whose intensity is 6450 lines per square inch?
20. What is the flux density in oersteds of a magnet that has a cross-sectional area of 4 sq cm and a flux of 5000 maxwells?
21. What is the flux density in lines per square inch of a magnet that has a crosssectional area of $1 \frac{1}{2}$ by $1_{2}^{\frac{1}{2}}$ sq. in. and that has a flux of 50,000 maxwells?
22. What cross-sectional area must a magnet have if it must carry a flux of 250,000 maxwells and the flux density is to be 50,000 lines per square inch?
23. If a magnet is to be made of an iron rod (round) and it is to carry a flux of 980 maxwells, what diameter rod must be used if the flux density is to be 3000 lines per square inch?
24. Hor many lines of force extend outward from a pole of 50 -unit pole strength?
25. How many lines of force extend outward from a pole of 75 -unit pole strength? What is the flux density 1 cm away? 2 cm away? 4 cm away? What force will it exert on a unit pole 1 cm away? 2 cm away? $t \mathrm{~cm}$ away?
26. Sketch the magnetic field of the loudspeaker magnet shown in Fig. 5-20a. Describe the action when a current flows in the winding on the pivoted bar (a) when a current is flowing inward at the left-hand lead, and (b) when a current is flowing outward at the left-hand lead.
27. Sketch the magnetic field of the electromagnet of a loudspeaker as shown in Fig. 5-40.
28. A horseshoe magnet made of cast steel has a cross-sectional area of 2 by 4 sq cm and carries a flux of 32,000 maxwells. (a) What is its flux density? (b) What is the permeability? (See curve, Fig. j-36). (c) What is the reluctivity?
29. If the magnet of Prob. 28 has a length of 16 cm , what is the reluctance of the magnet? What is its permeance?
30. What is the reluctivity of a piece of cast steel 2 cm in diameter and carrying 15,000 maxwells?


Fig. 5-10.
31. A cast steel bar $\frac{1}{2}$ in. sq. carries a flux of 3000 maxwells. (a) What is its flux density? (b) What is the permeability? (c) What is the reluctivity?
32. What is the reluctance of the bar magnet of Prob. 31 if its length is 4 in.? What is its permeance?
33. The electromagnet shown in Fig. $5-41$ is made of cast steel and carries a flux of 12,000 maxwells. (a) What is the reluctance of the U-shaped bar? (b) What is the reluctance of the straight bar? (c) What is the reluctance of the complete mag-


Fig. 5-41.
netic circuit, assuming that the straight bar fits tightly against the U-shaped bar? (d) What magnetomotive force is required by the magnet?
34. If the coil of the electromagnet of Prob. 33 is connected in a circuit that will have 0.25 amp flowing, how many turns are required?
35. If the coil of the electromagnet of Prob. 34 has a resistance of 24 ohms, what voltage is required to produce the required amount of current?
36. When the straight bar is separated from the U-shaped magnet of Prob. 33, additional reluctances in the form of air gaps are introduced at each pole. If the air gap at each pole is $\frac{1}{8} \mathrm{em}$, find (a) reluctance of each air gap (permeability of air $=1$ ); (b) reluctance of the complete circuit; (c) the magnetomotive force required by the magnetic circuit.
37. If the coil of the electromagnet of Prob. 36 is connected in a circuit carrying 0.25 amp , how many turns are required?
38. A series magnetic path consisting of three reluctance of $0.0065,0.0032$, and 0.0018 is to carry a flux of 5000 maxwells. (a) What is the reluctance of the circuit? (b) How many ampere-turns are required to produce the flux?
39. A series magnetic path consisting of five reluctances of $0.00025,0.00125$ $0.0025,0.0075$, and 0.0050 is to carry a flux of 8000 maxwells. (a) What is the reluctance of the circuit? (b) How many ampere-turns are required to produce the flux?
40. A parallel magnetic circuit contains reluctances of $0.005,0.0025$, and 0.0075 . (a) What is the reluctance of the circuit? (b) How many ampere-turns are required if the total flux is to be 15,000 maxwells?
41. A parallel magnetic circuit contains reluctances of $0.0005,0.0015,0.00025$, 0.0003 , and 0.0005 . What is the reluctance of the circuit?


Fig. 5-42.
42. The electromagnet shown in Fig. 5-42 is made of cast steel and carries a flux of 12,000 maxwells in sections $A A^{\prime}$ and $C C^{\prime}$ and 24,000 maxwells in section $B B^{\prime}$. The length of paths indicated as $l_{1}$ to $l_{8}$ is 5 cm each. Assume that the bar fits tightly against the poles of the magnet. (a) What is the reluctance of each path? (b) How many ampere-turns would be required for a coil to be wound around the center pole?
43. Repeat Prob. 42 for a condition when the bar is $\frac{1}{8} \mathrm{~cm}$ away from the poles of the electromagnet.

## CHAPTER VI

## METERS

Electrical instruments are essential in order that the amount of voltage or current flowing in a circuit can be measured, adjusted, or controlled. There are so many different types of electric meters that a complete description of all of them would not be practical in this text; therefore only those necessary for general electrical and radio measurements will be considered.

6-1. Electrical Instruments. Voltage and current are the two quantitics most generally measured, as it is possible to calculate other quantities such as resistance and power by using these two values. A galvanometer is a mechanism used to measure current or voltage. The following outline classification and the brief descriptions of each type of galvanometer will cover the various kinds of ammeters and voltmeters generally used in taking radio measurements.

1. Principle of operation
a. Electrostatic
b. Flectrothermal
c. Flectromagnetic
2. Mechanism
3. Scale
a. Uniform
b. Irregular
c. Multiple
d. Complex
4. Construction
a. Portable
b. Switchboard
5. Application
a. Galvanometer
b. Ammeter
c. Voltmeter
d. Ohmmeter

6-2. Electrostatic Meters. Electrostatic meters are used for measuring high voltages and are based on the theory that charged bodics of like polarity repel one another and those of unlike polarity attract one another. The essential parts of such a meter are shown in Fig. 6-1. It consists of two hollow cylinders connected by a pivoted bar that also supports the pointer. When a difference of potential is applied at $A B$, the two plates $P_{1}$ and $P_{2}$ become charged and a force of attraction is set up causing the bar to revolve. The amount of motion of the pointer is indicated on a scale calibrated in volts. The scale of such an instrument cannot be uniform because the force causing the bar to revolve is proportional to the voltage squared. For very high voltages the capacitors $C_{1}$ and $C_{2}$ are left in the circuit, but for
lower voltages they are short-circuited, thereby increasing the range of the meter.

The electrostatic mechanism is the only one used for electrical indications that functions because of the presence of a voltage. All the other mechan-


Fig. 6-1.-Missential parts of the electrostatic voltmeter.
isms are current-sensitive, producing pointer movements due to current flowing through the instrument circuit Figure 6-2 shows the internal construction of a commercial voltmeter. using the electrostatic principle of operation. Two sets of plates, one movable and the other stationary,


Fig. 6-2.-Internal construction of a commercial electrostatic voltmeter. (Weston Electrical Instrument Corporation.)
are substituted for the hollow cylinders and bar of the simple electrostatic meter. A protective resistance is normally connected in series in the circuit of these instruments to limit the current flow in case of a short circuit between the moving and fixed plates. The electrostatic voltmeter is particularly useful on a-c circuits where the current taken by other mechan-
isms would result in distorted values because of the $I R$ drop of the instrument.

6-3. Electrothermal Meters. Hot-wire Type. Hot-wire meters depend upon the expansion and contraction of a wire carrying an electric current.


Fig. 6-3.-Principle of the hot-wire meter. (Weston Electrical Instrument Corporation.)
A typical meter of this type is shown in Fig. 6-3. Current is caused to flow through a platinum alloy wire $A B$, or any other material that will function properly. A phosphor-bronze wire is arranged with one end attached to the wire $A B$, one turn of the wire wound around the pulley supporting the needle, and the other end attached to a tension spring. As the wire becomes hot, it sags, causing the pointer to move across the scale. The purpose of the spring is to maintain a steady pull so that the movement of the pointer is proportional to the lengthening of the wire. The expansion of the wire is proportional to the heat, and as the heat is proportional to the current, the scale can be calibrated to indicate amperes. As the voltage is directly proportional to the current, the scale can also be calibrated to indicate volts. Instability due to wire stretch and lack of ambient temperature compensation have made this mechanism impracticable.


Fig. 6-4.-Principle of the thermocouple meter. (Weston Electrical Instrument Corporation.)
Thermocouple Type. A practical application of the thermal effect of electron flow is obtained by using the principle of the thermocouple. Figure 6-4 is a simple diagram of a typical meter of this type. Current is forced to flow through $A B$ and in so doing flows through the junction of the
two dissimilar metals $E C$ and $E D$. The current in $A B$ produces heat which causes a d-c voltage to be produced across $C D$. This voltage forces a current through the galvanometer which is a very sensitive moving-coil instrument. Movement of the pointer is calibrated to indicate the amount of current flowing in $A B$.

Since the voltage developed at the junction of a thermocouple is a function of the temperature difference of its hot and cold ends, this temperature difference must be caused only by the current being measured. For accu-


Fig. 6-5.-A compensated heating element of a thermocouple meter. (Weston Electrical Instrument Corporation.)
rate measurements, points $C$ and $D$ must be at the mean temperature of points $A$ and $B$. This is accomplished by attaching the ends of the couple $C$ and $D$ to the center of separate copper strips whose ends are thermally in contact with $A$ and $B$ but which are electrically insulated from them.

Uses of Thermal-type Meters. Thermo instruments measure the effective value of the current flowing and therefore can be used on d-c, a-c, a-f and r-f currents. They are used extensively where wave forms or frequencies are such as to cause errors in other types of meters. As no standard ampere is available at radio frequencies, thermo instruments are generally calibrated at 60 cycles. Instruments giving accurate readings at frequencies up to 100 megacycles can be obtained.

The amount of heat that is developed in a circuit will vary as the square of the current flow. Increasing the current will cause a greater movement of the pointer as the current goes through a uniform rate of increase. On the meter scale, the divisions for


Fig. 6-6.-Commercial multirange thermocouple meter. (Weston Electrical Instrument Corporation.) equal changes of current will be further apart at the high readings than at the low readings.

## 6-4. Permanent-magnet Moving

 Coil. Basic Theory. Most of the meters used commercially are based on the electromagnetic principle. These meters may be further classified as follows: (1) permanent magnet, (2) iron vane, (3) dynamometer. As the name implies, the operation of the permanent-magnet movablecoil meter depends upon the presence of a magnetic field which acts upon a movable element. In this type, illustrated in Fig. 6-7, a small coil of wire $C$ wound on a light cylindrical frame of aluminum carries the current to be measured. The frame is so pivoted that it will revolve freely between the soft iron core $D$ and the pole picces $N$ and $S$ of the permanent magnet $P$. Attached to the aluminum frame is a pointer which moves across the scale, thus indicating the deflection made. This type of meter is applicable only to direct current, as no resultant forces can be set up when alternating current flows in coil $C$. Instruments of this type may be calibrated to indicate current or, if connected in series with a proper resistance, to indicate voltage.Types of Scales Produced with Pole Pieces Having Concentric Faces. Because the coil moves between the


Fig. 6-7.-Essential parts of the permanentmagnet moving-coil meter. pole picces of the permanent magnet, this type of instrument is also known as a moving-coil mechanism. In 1881, d'Arsonval patented an instrument of this type, and permanent-magnet moving-coil mechanisms are often referred to as d'Arsonval instruments. The conventional permanent-magnet moving-coil mechanism is supplied
with core and pole pieces having concentric faces. Such mechanisms produce the scales shown in Fig. 6-10. Scale $A$ is that of the mechanism itself showing uniform divisions for linear increases in current in the moving coil.


Fig. 6-8.-Mechanism of a commercial permanent-magnet moving-coil meter. (Weston Electrical Instrument Corporation.)


Fig. 6-9.-A commercial meter using the mechanism shown in Fig. 6-8. (Weston Electricat Instrument Corporation.)

Scale $B$ shows the square-law distribution resulting from the use of a heating element with the standard mechanism. Scale $C$ shows logarithmic distribution resulting when the movement is used to measure sound levels (decibel


Fig. 6-10.-Meter scales produced by d'Arsonval instruments whose core and pole pieces have concentric faces: $A$, uniform scale; $B$, square-law distribution; $C$, logarithmic distribution.


Fig. 6-11.-Meter scales produced by d'Arsonval instruments whose core and pole pieces have eccentric faces: $A$, effect of eccentricity on uniform scale; $B$, effect of eccentricity on square-law scale; $C$, effect of eccentricity on logarithmic scale.
measurements). Because of cramping at the left, the upper two scales are difficult to read at the lower values.

Types of Scales Produced with Pole Pieces Having Eccentric Faces. The permanent-magnet moving-coil mechanism may be obtained with specially
shaped or eccentric pole pieces resulting in uneven flux distribution in the air gap and a nonlinear relation between current in the moving coil and pointer movement as shown in Fig. 6-11. Scale $A$ shows the effect of the eccentricity illustrated. When used with square-law heating elements, the resulting scale $B$ becomes more linear. When used with sound-level instruments, the logarithmic scale $C$ becomes more uniform. These scales not only are more uniform but are readable over a larger portion of their


Fig. 6-12.-Early iron-vane instrument of the suction type. (Weston Electrical Instrument Corporation.)
arcs. Meter scales produced by using eccentric pole pieces are called expanded scales and are very useful in adapting permanent-magnet movingcoil mechanisms to r-f and sound-lcvel instruments.

6-5. Iron Vane. Suction or Plunger Type. The principle of the iron vane was first utilized to construct a mechanism for measuring current in the suction or plunger type of instrument illustrated in Fig. 6-12. The current to be measured is made to flow through a low-resistance solenoid, eausing it to suck the movable iron core into the coil. How far the core is drawn into the coil will depend upon the amount of current flow. The pointer being attached to the movable core will move over a scale that has been calibrated to indicate the current flow in amperes. An instrument of this type has large errors due to hysteresis, or lag, and to the excessive mass of its movable iron parts. It is practically obsolete and has been super-
seded by the Thomson inclined-coil instrument and the radial and concentric vane instruments.

Thomson Inclined-coil Instruments. The Thomson inclined-coil instrument illustrated in Fig. 6-13 consists of an energizing coil $A$ which is located about 45 degrees from the horizontal and a rotating element which is merely a rectangular piece of thin soft iron mounted on a shaft. Whenever the coil $A$ is encrgized, the soft-iron vane $V$ seeks to place itself parallel with the magnetic flux of the coil; the shaft is then forced to rotate, causing the pointer $P$ to indicate the deflection on a calibrated scale. Such an instrument may be used as an ammeter or voltmeter for either direct or alternating currents. However, it is gen-


Fig. 6-13.-Essential parts of the 'Thomson inclined-coil instrument. erally used for measuring alternating currents. The deflection of the pointer will vary as the square of the power, thus making the scale irregular.

Comparison of Radial and Concentric-type Movable Iron Vanes. If two similar adjacent iron bars are similarly magnetized, a repelling force is developed between them which tends to move them apart. In the moving iron-vane mechanism, this principle is used by fixing one bar in space and pivoting the second so that it will tend to rotate when the magnetizing current flows through a coil. A spring attached to the moving vane opposes its motion and permits the scale to be calibrated in terms of the current flowing. The movable iron vane may be of the radial type illustrated in Fig. 6-14 or of the concentric type shown in Fig. 6-15. Comparing these two types, the radial vane is the more sensitive and has almost a linear scale. The concentric vane causes the pointer to move by the square law, but its vanes can be shaped to secure special scale characteristics; its shorter magnetic vanes result in smaller d-e reversal and residual errors. Both types can be used as ammeters or voltmeters to measure either direct or alternating currents. However, they are generally used for measuring alternating currents.

6-6. Dynamometer. The dynamometer-type meter is composed of two stationary coils and a movable coil which also supports the pointer. A diagram illustrating the basic principle of this type of meter is shown in Fig. 6-16. When current flows in all the meter windings, the movable coil $C$ tends to place itself parallel to the two stationary coils $S_{1}$ and $S_{2}$ but is opposed by the action of a spiral spring. This type of meter can be used for a-c as well as d-c measurements. When it is used as an ammeter or voltmeter, all coils are connected in series; when it is used as a wattmeter,
the stationary coils form the current element and the movalle coil the voltage element.

Electrodynamometer mechanisms are the most universal of all the indicating devices now used. Distortion of alternating currents does not occur because of the complete absence of magnetic materials such as iron, and the indications are truc effective values. As this mechanism is current-sensi-


Fig. 6-14.-The radial-type moving iron-vane mechanism. (Weston Electrical Instrument Corporation.)
tive, the pointer moves because of current flowing through turns of wire. It is the most versatile of all the basic mechanisms since it can be used to indicate current, voltage, or power, either alternating or direct current. Using a crossed-coil movement, the power factor, phase angle, frequency, or capacitance measurements can be obtained.

6-7. Rectifier-type Meters. Rectifier-type meters are used for measuring alternating currents and voltages of small magnitude, such as milliamperes, millivolts, microamperes, and microvolts, which cannot readily be measured otherwise. They are used in telephone, telegraph, and signal-
corps communications, in monitoring in broadcast studios, and in measuring loss or gain on transmission lines.

A rectifier is a device that changes an alternating current toa direct current. It operates on the principle that the resistance to current flow is high in one direction and low in the opposite direction. Rectifiers used for instrument work are of the copper oxide type. A disk of copper is oxidized on one of its sides by a special heat-treatment, the temperature used being approxi-


Fig. 6-15.-The concentric-type moving iron-vane mechanism. (Weston Electrical Instrument Corporation.)
mately $1800^{\circ}$ Fahrenheit. Resistance to current flow from oxide to copper is very low, while the resistance to current flow in the opposite direction is very high. If only one disk is used, an intermittent current will flow through the meter as no current will flow during that half cycle when the current is supposed to flow from copper to the oxide. In order to obtain full-wave rectification, that is, to utilize both halves of the a-c cycle, a bridge circuit is used. Such a circuit requires four disks connected as shown in Figs. 6-18 and 6-19. The rectified current is now used to activate a permanent-magnet moving-coil instrument. The assembly of disks, illus-
trated in Fig. 6-19, is hermetically sealed in a Bakelite housing (see Fig. $6-18)$ to ensure constancy of performance with time. To use this type of instrument as a voltmeter, it is necessary to connect a resistance in series with the line to limit the current to the rating of the rectifier unit. The efficiency of a rectifier-type instrument is generally about 90 per cent, the losses being due to the changes in temperature, the current blocked out by the rectifier action, and the resistance of the moving coil. It can be used to measure alternating currents of frequencies up to 20,000 cycles per second and has an error of less than 1 per cent per 1000 cycles when calibrated at a definite frequency.

6-8. Ammeters. How to Connect an Ammeter in a Circuit. An ammeter is an instrument used for measuring electric currents. It is always connected in series with that part of a circuit whose current is to be measured. In Fig. 6-20, ammeter 1 is connected in series with the line to indicate the line current, and ammeters 2 and 3 are connected in series with the lamp and fixed resistance, respectively, to obtain their individual currents. Being connected in series, an ammeter must carry the current passing through that part of the circuit in which it is connected. In Fig. 6-20, if ammeter 1 has a resistance of one ohm and the line current is 10 amperes it follows that there would be a 10 -volt drop across the ammeter. This would cause the appliance and


Fig. 6-16.-Essential parts of the dynamometer-type instrument. lamp to operate on a voltage of 10 volts less than intended, thus decreasing their final output and efficiency. The voltage drop across the meter is decreased by making the resistance of the meter as low as possible. The value of this drop varies (usually about 50 millivolts) with the type of meter and the manufacturer. Most ammeters are designed for 50 -millivolt drop at full rated current.

Precautions in the Use of Ammeters. If the current flowing through an ammeter is 10 amperes and its voltage drop 50 millivolts, then its resistance must be $0.050 / 10$, or 0.005 ohm . If the line voltage is 110 volts and the ammeter is connected directly across it, the current would be 110/0.005, or 22,000 amperes. As the meter is designed to carry only 10 amperes, it will be damaged. Therefore, never connect an ammeter across the line.

The current due to an unforeseen short circuit, an overload, or even the starting of a motor is large enough to injure an ammeter if left connected in the circuit. To prevent damage due to factors that cannot be forescen, the ammeter should always be protected by connecting a short-circuiting switch across it, as illustrated by $S_{1}, S_{2}$, and $S_{3}$ in Fig. 6-20. The switch is always
kept closed except when a reading is to be taken. If, upon opening the switch, the needle swings backward or completely across the meter scale, it should be closed instantly to prevent damaging the meter. The necessary


Fig. 6-17.-Cutaway view illustrating the construction of a commercial type of dynamometer mechanism. (Weston Electrical Instrument Corporation.)
changes to the circuit should then be made before the switch is again opened. If the meter needle moves backward, it indicates a reversal of polarity and


Fig. 6-18.-A copper oxide rectifier and the bridge circuit used for a milliammeter. (Weston Electrical Instrument Corporation.)
the lead-in wires must be interchanged to make it move in the proper direction.

6-9. Voltmeters. How to Connect a Voltmeter in a Circuit. A voltmeter is an instrument used for measuring voltage. It is always connected across
that part of the circuit whose voltage drop is to be measured. Figure 6-20 shows the correct way to connect voltmeters in a parallel circuit. Voltmeter 1 indicates the line voltage; voltmeter 2 shows the voltage across the lamp; and voltmeter 3 shows the voltage across the fixed resistance.


Fig. 6-19.-A copper oxide rectifier and the bridge circuit used for a millivoltmeter. (Weston Electrical Instrument Corporation.)

The construction of a voltmeter does not differ materially from that of an ammeter in so far as the movements and magnets are concerned. The moving coil of a voltmeter is usually wound with a greater number of turns and of finer wire than that of the ammeter. As a voltmeter is connected directly across the line, it is desirable that it take as little current as is practicable. The current in the moving coil of a voltmeter is generally the same as that in an ammeter; this is approximately 0.01 ampere. Because


Fig. 6-20.-Correct method of connecting ammeters and voltmeters.
of its comparatively low resistance, approximately 20 ohms, the moving clement cannot be connected directly across the line. It is therefore necessary to connect a high resistance in series with it. The value of this resistance depends on the resistance of the coil, or moving element, the current rating of the instrument, and the full-scale voltage desired.

Example 6-1. If the resistance of the moving coil of a voltmeter is 20 ohms and the current rating of the instrument is 0.01 ampere, what amount of resistance must be added to give a full-scalc reading of 150 volts?

Given:
$\begin{array}{ll}E=150 \text { volts } & R_{S}=? \\ R_{e}=20 \mathrm{ohms} & \\ I_{M}=0.01 \mathrm{amp} & \end{array}$

$$
I_{M}=0.01 \mathrm{amp}
$$

Find:

Solution:

$$
R_{T}=\frac{E}{I_{M}}=\frac{150}{0.01}=15,000 \mathrm{ohms}
$$

As the instrument has a resistance of 20 ohms, this means that

$$
\begin{aligned}
R_{S} & =R_{T}-R_{c} \\
& =15,000-20 \\
& =14,980 \mathrm{ohms}
\end{aligned}
$$

Methods of Obtaining Additional Voltage Ranges. Voltmeters are generally made with more than one voltage range. To accomplish this, the high resistance is tapped to give the voltage range desired (see Fig. 6-21).


Fig. 6-21.-Internal circuit of a voltmeter having a 30 - and 150 -volt range.
Example 6-2. If the voltmeter of Example 6-1 is to have a 30 -volt scale in addition to the 150 -volt scale, where must the 14,980 -ohm resistance be tapped?

Given:

$$
\begin{aligned}
& E=30 \mathrm{volts} \\
& R_{e}=20 \mathrm{ohms} \\
& I_{M}=0.01 \mathrm{amp}
\end{aligned}
$$

Solution:

$$
\begin{aligned}
R_{T}=\frac{E}{I_{M}}= & \frac{30}{0.01}=3000 \mathrm{ohms} \\
R_{S} & =R_{T}-R_{c} \\
& =3000-20 \\
& =2980 \text { ohms }
\end{aligned}
$$

Multirange voltmeters, such as those used in radio test equipment, use the above principle. The resistance must be large enough to withstand the highest voltage and must have a tap for each additional voltage range desired.

Sensitivity of Voltmeters. When a voltmeter is used to measure the voltage across a resistor (or any other type of circuit element), a parallel circuit is formed because the voltmeter acts as a resistor connected in parallel with the circuit element whose voltage is being measured. From Art. 4-9 it can be seen that the resistance of a parallel circuit depends upon the ratio of the resistances of its individual branches. For example, if the resistance of the
voltmeter is equal to that of the circuit element being measured, the total resistance of the parallel combination is one-half (or 50 per cent) of the value of the circuit clement's resistance; if the ratio is increased so that the resistance of the voltmeter is five times that of the circuit element, the total resistance of the parallel combination is 83.3 per cent of the value of the circuit element's resistance; and if the ratio is further increased so that the resistance of the voltmeter is ten times that of the circuit element, the total resistance of the parallel combination is 90.9 per cent of the value of the circuit element's resistance. It can be seen from the above example that unless the resistance of a voltmeter is many times greater than the resistance of the circuit element whose voltage is being measured the total resistance of the newly formed parallel circuit will be appreciably lower than that of the circuit element alone and an incorrect voltage reading may be obtained. This is particularly true in radio, television, and electronic circuits where the circuit element being investigated is often found to be part of a series circuit and considerable error will result from the use of a voltmeter of relatively low resistance. Use of a low-resistance voltmeter also produces error in the voltmeter readings in these types of circuits because the voltage output of the power source may be lowered when the voltmeter is connected owing to the relatively high current taken by the meter.

The sensitivity of a voltmeter is indicated by its ohms per volt rating. This rating is obtained by dividing the resistance of the voltmeter for a particular range by its full-scale voltage value of that range. For example, the sensitivity of a voltmeter having a resistance of 150,000 ohms for a 150 -volt range is 1000 ohms per volt. The sensitivity of a meter increases with an increase in its ohms per volt rating.

When measuring voltages in power circuits, accurate values can be obtained with a voltmeter having a low sensitivity, such as 1000 ohms per volt, as the resistance of these circuits is usually less than 1000 ohms and often less than 100 ohms. The resistance of many electronic circuits is very high, being of the order of more than 100,000 ohms. In measuring voltages in these circuits, accurate values can be obtained only with a voltmeter having a sensitivity of 20,000 ohms per volt or higher. The sensitivity required of a voltmeter depends largely upon the resistance and the type of circuit whose voltage is to be measured.

Irecautions in the Use of Voltmeters. Voltmeters do not form a definite part of a circuit as do ammeters. It is therefore not necessary to connect voltmeters permanently in the circuit. Because a voltmeter usually is connected only when a reading is to be taken, fewer meters are needed as one meter may be used to take readings of several circuits and circuit elements. A voltmeter may be damaged by excessive voltage; but since it is connected only when a reading is to be taken, it may be disconnected instantly if any overvoltage condition is apparent.

6-10. Meter Scales. Uniform and Irregular Scales. In selection of meters, it is well to choose a meter of such a range that the readings will be in the middle two-thirds of the scale. The readings taken from this part of the scale are more accurate than those at the extreme ends, and there is also less danger of damaging the needle. Meter-scale divisions may be uniform or irregular. $A$ uniform scale is one that has the entire scale divided into equal divisions. If full-scale deflection is 150 volts, the middle of the scale will be 75 volts, one-sixth of the scale will be 25 volts, and twothirds 100 volts. To find the middle two-thirds of a scale, subtract onesixth of the full-scale reading from each end. For example, a voltmeter having a 150 -volt range will give best operating results when used for measuring voltages between 25 and 125 volts.


Fig. 6-22.-Meter scales: top, uniform seale; bottom, irregular scale.
On an irrcgular scale, the divisions are not equal. Referring to Fig. 6-22, it can be seen that the scale divisions are very close at the low readings and that the space between divisions increases at the higher readings. Close examination of Fig. 6-22 shows that the middle two-thirds of the irregular ( 150 -volt range) scale extends from approximately 63 to 135 volts.

Multiple and Complex Scales. A common practice is to have one scale indicate more than one value. On such a scale, each division will represent two or more values, each one being a multiple of the others. Scales of this type are therefore referred to as multiple scales. Referring to Fig. 6-22, when the needle is over the 100 mark on the center set of numbers, it will indicate 100 volts if the 150 -volt range is being used, 50 volts for the 75 volt range, and 200 volts for the 300 -volt range.

Test instruments used for communication work usually consist of one meter that is capable of indicating a number of different values, such as d-c current, d-c voltage, a-c voltage, and resistance. Instruments of this type usually consist of two or more multiple seales and are therefore known as complex-scale meters. Care must be taken in using a complex-scale meter to read the correct scale, that is, the one corresponding to the setting for which the meter is adjusted.


Fig. 6-23.-A combination a-c/d-e volt milliammeter illustrating an application of the complex scale. (Weston Electrical Instrument Corporation.)

6-11. Parallax. To read a meter correctly, the eyes must be parallel to the seale of the instrument. If the eyes are not parallel with the meter scale, a false reading will be obtained because of the space between the scale and the needle. Referring to Fig. 6-24, a correct reading can be taken only when the eyes are parallel to the scale; readings taken at any other position will be incorrect. This crror in reading meters is due to parallax.

To avoid error in reading meters due to parallax, a mirror is placed below the scale to be read, as shown in Fig. 6-24. When the eyes are in a direct line with the needle and the seale, the reflection of the needle cannot be seen in the mirror and the reading taken at this point is correct.

6-12. Shunts. Calculation of the Resistance of the Shunt. The current which may safely be led into an ammeter movement is limited by the
current-carrying capacity of the moving element, which must necessarily be small. To increase the range of such instruments, shunts and current transformers are used, the former with direct currents, the latter with alternating currents.


Fig. 6-24.-Frrors possible in reading meters because of parallax.
It has previously been stated that the movement's current must be very small, generally about 0.01 ampere. This is accomplished by connecting a low resistance in parallel with the meter. The ammeter is in


Fig. 6-25.-Proper connections for use of a shunt with a millivoltmeter.
reality now a voltmeter (see Fig. 6-25) indicating the voltage drop across a resistance. This resistance is called a shunt and forms a definite part of all ammeters.

Referring to Fig. 6-25,

$$
\begin{aligned}
I_{S H} & =\text { shunt current } \\
I_{M} & =\text { meter current } \\
R_{S H} & =\text { shunt resistance }
\end{aligned}
$$

$R_{M}=$ meter resistance
$E=$ voltage drop across meter and shunt as indicated by voltmeter $V$
Then

$$
\begin{align*}
E_{S H} & =I_{S H} \times R_{S H}  \tag{6-1}\\
E_{M} & =I_{M} \times R_{M} \tag{6-2}
\end{align*}
$$

As the voltage drop across the shunt and the moving coil are equal, then

$$
\begin{equation*}
I_{S H} \times R_{S H}=I_{M} \times R_{M} \tag{6-3}
\end{equation*}
$$

and

$$
\begin{equation*}
R_{S H}=\frac{I_{M} \times R_{M}}{I_{S H}} \tag{6-4}
\end{equation*}
$$

As the meter current is generally less than 0.01 ampere, the current in the meter shunt will differ from the line current by only a very small amount when the line current is one ampere or more. For ammeters rated at one ampere or more, sufflciently accurate values of shunt resistance can be obtained by using Eq. (6-4) and assuming that the shunt current is equal to the line current.

For ammeters rated at less than one ampere an error of more than one per cent, which is the maximum amount usually permitted, may result if it is assumed that the current flowing in the shunt is equal to the line current. For these meters, more accurate values of the shunt resistance may be obtained by using the value of current actually flowing in the shunt as provided by the equation

$$
\begin{equation*}
I_{S I}=I_{\mathrm{linc}}-I_{M} \tag{6-5}
\end{equation*}
$$

Example 6-3. If an instrument that is to be used as an ammeter has a resistance of five ohms and the meter current required to obtain full-scale deflection is 0.01 ampere, what resistance shunt must be used to give full-scale deflection with 10 amperes flowing?

| Given: | Find: |  |
| ---: | :--- | ---: |
| $R_{M H}$ | $=5 \mathrm{ohms}$ | $R_{S H}=?$ |
| $I_{M}$ | $=0.01 \mathrm{amp}$ |  |

Solution:

$$
R_{S H}=\frac{R_{M} \times I_{M}}{I_{S H}}=\frac{5 \times 0.01}{10}=0.005 \mathrm{ohm}
$$

Example 6-4. A milliammeter movement has a resistance of two ohms and requires one milliampere to obtain full-scale deflection. What per cent error is obtained in shunt resistance if in calculating the value of shunt resistance it is assumed that the full line current of 10 milliamperes flows through the shunt?

Given:

$$
\begin{aligned}
R_{M} & =2 \mathrm{ohms} \\
I_{M} & =1 \mathrm{ma} \\
I_{\text {line }} & =10 \mathrm{ma}
\end{aligned}
$$

Find:
Per cent error $=$ ?

Solution:
(Assuming shunt current equal to line current)

$$
R_{S H}=\frac{R_{M} \times I_{M}}{I_{\mathrm{line}}}=\frac{2 \times 0.001}{0.01}=0.2 \mathrm{ohm}
$$

(Using correct value of shunt current)

$$
\begin{gathered}
I_{S H}=I_{\text {line }}-I_{M}=0.01-0.001=0.009 \mathrm{amp} \\
R_{S H}=\frac{R_{M} \times I_{M}}{I_{S H}}=\frac{2 \times 0.001}{0.009}=0.222 \mathrm{ohm} \\
\text { Per cent error }=\frac{0.222-0.2}{0.222} \times 100 \cong 10 \text { per cent }
\end{gathered}
$$

Types of Shunts. When a shunt is connected permanently inside an ammeter, it is called an internal shunt. When it is desired to increase the range of an ammeter, an external shunt is used. External shunts consist


Fig. 6-26.-Arnmeter shunts: left, multiple-range rotary-switch type; right, single-range external type. (Weston E'lectrical Instrument Corporation.)
of a low resistance, usually manganin brazed to two comparatively large copper blocks. Two sets of binding posts are fastened to these blocks, one for carrying the line current through the shunt and the other for the armmeter leads. These ammeter leads are calibrated to be used with a particular instrument, and therefore no other leads should be used.

For greater convenience, a multiple-range rotary-switch type of shunt can be used. It is so constructed that any one of a number of ranges may be selected by merely turning the switch handle until the index points to the desired range. Multirange ammeters use a similar rotary-type adjustable shunt that is permanently connected to the galvanometer.

6-13. Multipliers. Calculation of the Resistance of a Multiplier. The range of a voltmeter having its resistance incorporated within the instrument may be increased by the use of an external resistance connected in series with the instrument. External resistances used in this way are
called multipliers and are usually placed within a perforated box, the terminals being brought outside. The multiplying power of each terminal is indicated by a marker. The equation showing the relation of the resistance


Fig. 6-27.- Con nections for using a multiplier with a voltmeter.
of the multiplier ( $R_{E X}$ ), the resistance of the meter ( $R_{M}$ ), and the multiplying power ( $M$ ) may be expressed as

$$
\begin{equation*}
M=\frac{R_{E X}+R_{M}}{R_{M}} \tag{6-6}
\end{equation*}
$$

Example 6-5. If a voltmeter has a resistance of 15,000 ohms and an external resistance of $45,000 \mathrm{ohms}$ is connected in series with it, what is the multiplying power of the external resistance?

| Given: | Find: |
| ---: | :--- |
| $R_{M}=15,000$ ohms | $M=?$ |
| $R_{E X}=45,000$ ohms |  |

Solution:

$$
\begin{aligned}
M & =\frac{R_{E X}+R_{M}}{R_{M}} \\
& =\frac{45,000+15,000}{15,000} \\
& =\frac{60,000}{15,000} \\
& =4
\end{aligned}
$$

Commercial Types of Multipliers. Multipliers may be used for alternating currents up to 750 volts as well as for direct currents. For all a-c voltages higher than this, it is advisable to use potential transformers. Series resistors for use as multipliers take various physical forms, depending on the application. Self-contained ranges use small compact spools for the more sensitive instruments and card-mounted types for those requiring more current. These are shown in Fig. 6-28. If the series resistors generate more heat than the instrument case can radiate, the resistors must be mounted externally. The special tubular resistor shown at the top
of Fig. 6-28 is wax filled, hermetically sealed, and electrostatically shielded to ensure long life at high voltages in humid or salty atmosphere.

6-14. Low-range Meters. It is sometimes necessary to measure current strength of only a few thousandths or a few millionths of an ampere; voltage drops may also be only a few thousandths of a volt. Measurements of these kinds are made frequently in radio, telephone, television, and vacuum-tube work. These small quantities are expressed in milliamperes, microamperes, and millivolts.

Instruments used for measuring very small currents are called microammeters and are calibrated to read to millionths of an ampere.

Instruments used for measuring small currents are called milliammeters and are calibrated to read to thousandths of an ampere.


Fig. 6-28.-Resistors for use with voltmeters: top, wax-filled hermetically sealed type; lower left, spool type; lower right, card-mounted type. (Weston Electrical Instrument Corporation.)

Instruments used for measuring small voltages are called millivoltmeters and are calibrated to read to thousandths of a volt.

6-15. Ohmmeters. Ammeter-voltmeter Method of Measuring Resistance. Using an ammeter to measure the current flowing through a resistor and a voltmeter to measure the voltage drop across it (Fig. 6-30), the resistance may be calculated by using Ohm's law, $R=E / I$. If this method is used to measure high resistances and the voltmeter is connected as shown in Fig. 6-30a, the ammeter will measure the sum of the currents taken by the voltmeter and the resistance being measured. The current flowing through the resistor is very small, and the voltmeter current may be as great or greater, depending on the resistence of the voltmeter used. The resistance calculated by using this current value will be in error. Connecting the voltmeter as shown in Fig. 6-30b will eliminate such errors.


Fig. 6-29.-Low-range meters: (a) milliammeter, (b) microammeter, (c) galvanometer. (Weston Electrical Instrument Corporation.)


Fig. 6-30.-Resistance measurement by the voltmeter-ammeter method. (a) circuit for measuring low resistances, (b) circuit for measuring high resistances.

Voltmeter Method of Measuring IIigh Resistances. High resistances may be measured by using a voltmeter whose resistance $R_{M}$ is known and connecting it as shown in Fig. 6-31. A reading is taken with the key closed, thus short-circuiting the resistor; the voltmeter will then indicate the voltage of the power source, $E_{r}$. Another reading is taken with the key
opened, thus adding the unknown resistance $R_{X}$ in series with the voltmeter; this is the same as increasing the range of the voltmeter by adding a series resistor. The voltage reading $E_{M}^{\prime}$ would therefore be the drop across the voltmeter itself. The voltage drop across the unknown resistance will be

$$
\begin{equation*}
E_{X}=E_{T}-E_{M} \tag{6-7}
\end{equation*}
$$

These two voltage drops may be expressed in terms of current and resistance as

$$
\begin{align*}
& E_{X}=I \times R_{X}  \tag{6-8}\\
& E_{M}=I \times R_{M} \tag{6-9}
\end{align*}
$$

Since the current $I$ is the same in each resistance, dividing Eq. (6-8) by (6-9)

$$
\begin{equation*}
\frac{E_{X}}{E_{M}^{\prime}}=\frac{R_{X}}{\bar{R}_{M}} \tag{6-10}
\end{equation*}
$$

Solving for the unknown resistance by multiplying both sides of the equation by $R_{M}$,

$$
\begin{equation*}
R_{X}=R_{M} \times \frac{E_{X}}{E_{M}} \tag{6-11}
\end{equation*}
$$

Substituting for $E_{X}$ its equivalent $E_{T}-E_{M}$,

$$
\begin{equation*}
R_{X}=R_{M} \times \frac{E_{T}-E_{M}}{E_{M}} \tag{6-12}
\end{equation*}
$$

Using this formula, it is very easy to determine the value of high resistances. The resistance of a voltmeter is sometimes marked on the meter; if not, it can be obtained from the manufacturer.

Basic Principle of the Ohmmeter.


Fig. 6-31.- Voltmeter method of measuring high resistances. An ohmmeter is an instrument that indicates the resistance of a circuit or part of a circuit directly in ohms without any need for calculations. Its principle of operation is based on the voltmeter method of measuring resistance. Figure 6-32 is a schematic diagram of the internal circuit of a simple ohmmeter. In this instrument, unit cells are used as the power source, and standard calibrated resistances $R_{1}$ and $R_{A}$ are substituted in place of the voltmeter. The milliammeter seale is calibrated to indicate the resistance in ohms
directly. When the terminals $T_{1}$ and $T_{2}$ are short-circuited and the battery cells are new, so that $E_{B}=4.5$ volts, then $R_{1}+R_{A}$ must be equal to 4500 ohms in order that the milliammeter read its full-scale deflection of one milliampere.

$$
R_{1}+R_{A}=\frac{E_{B}}{I_{M}}=\frac{4.5}{0.001}=4500 \mathrm{ohms}
$$

Variable Series-resistor Method of Compensating for Variations in Cell Voltage. As the voltage of the unit cells will diminish with their age, the resistance $R_{A}$ is made adjustable to compensate for the variations in voltage of the power supply. If the voltage $E_{B}$ decreases to 4.2 volts, then $R_{1}+R_{A}$ must be equal to 4200 ohms in order to obtain one milliampere of current. When the cells become too weak, it will no longer be possible to adjust the resistance $R_{A}$ to obtain a flow of one milliampere and new


Fig. 6-32.-Circuit of a simple ohmmeter.
cells must be substituted in place of the old ones. In the ohmmeter being considered, this will be necessary when the total voltage of the cells drops below four volts. When the terminals $T_{1}$ and $T_{2}$ are short-circuited, the resistance of the external circuit is zero and the milliammeter scale is marked to indicate zero ohms instead of one milliampere. If an unknown resistance $R_{x}$ is placed across the terminals $T_{1}$ and $T_{2}$, the total resistance of the circuit is increased and the current flowing in the circuit, as indicated by the milliammeter, will decrease. Suppose the milliammeter indicates that 0.5 milliampere is flowing in the circuit; then
as
then

$$
\begin{aligned}
R_{1}+R_{A}+R_{X} & =\frac{4.5}{0.0005}=9000 \mathrm{ohms} \\
R_{1}+R_{A} & =4500 \mathrm{ohms} \\
R_{X} & =9000-4500=4500 \mathrm{ohms}
\end{aligned}
$$

Therefore the milliammeter is marked to indicate 4500 ohms instead of 0.5 milliampere. In a similar manner, the entire scale can be calibrated
to indicate the resistance of the external circuit in ohms. The highest reading that can be made with any degree of accuracy is approximately 10 times the center reading, and the lowest reading is approximately onetenth of the center reading. The ohmmeter being discussed will therefore have an approximate range of 450 to 45,000 ohms.


Fig. 6-33.-A commercial ohmmeter. (Weston Electrical Instrument Corporation.)
As the battery voltage decreases, the resistance of the circuit is lowered. In this case $R_{1}+R_{A}$ is decreased to 4200 ohms when the battery voltage drops to 4.2 volts. If the milliammeter indicates that 0.5 milliampere is flowing in the circuit, then
as

$$
R_{1}+R_{\Delta}+R_{X}=\frac{4.2}{0.0005}=8400 \mathrm{ohms}
$$

then

$$
\begin{aligned}
R_{1}+R_{A} & =4200 \mathrm{ohms} \\
R_{X} & =8400-4200=4200 \mathrm{ohms}
\end{aligned}
$$

As the ohmmeter has been calibrated to indicate 4500 ohms at this point, the reading will be in error, as it now should read only 4200 ohms. The readings at all other points on the scale will be decreased accordingly. Resistance values obtained with instruments using this method of adjusting for decreases in battery voltage will not be accurate when the battery voltage drops because of the aging of the cell.

Variable Parallel-resistor Method of Compensating for Variations in Cell Voltage. In order to overcome the difficulty of the series-resistor method of compensation, an adjustable low resistor $R_{P}$ is connected in parallel with the milliammeter as indicated by the dotted lines in Fig. 6-32. This is used in place of the adjustable high resistor $R_{A}$ that was connected in series with the meter and the fixed resistor $R_{1}$. The meter can be adjusted to zero by varying the position of the movable contact on $R_{P}$.

If the resistance of $R_{1}+R_{A}$ is increased to 9000 ohms and the battery voltage $E_{B}$ is increased to nine volts, then the range of the ohmmeter will increase to approximately 900 to 90,000 ohms. Therefore, by using different values of resistance and battery voltage, an ohmmeter can be made to indicate any value of resistance. In Fig. 6-34, $R_{1}, R_{2}, R_{3}$, and $R_{4}$ are multiples of one another; therefore it is only necessary to have a single scale to indicate the values of the lowest range and a multiplying factor


Fig. 6-34.-Diagram showing the use of resistors and switch to select the range of a multirange ohimmeter.
for each of the other resistors. A rotating switch $S R$ is used to connect or to disconnect these resistors into the circuit. The points $1,2,3$, and 4 are marked to indicate the multiplying factor to be used for the resistor that is connected into the circuit.

6-16. Combination Meters. In test and service work on radio, television, and electronic equipment, combination meters are very useful since one instrument is made to take the place of several meters. The value of the multiplier, shunt, and ohmmeter resistors can be calculated by using the formulas explained under Multipliers, Shunts, and Ohmmeters. In order to use a d-c milliammeter for measuring alternating currents, a copper oxide rectifier is used.

Figure 6-36 shows the circuit diagram of a combination meter; its sensitivity for the d-c voltage ranges is 20,000 ohms per volt. This instrument has five ranges of a-c and d-c voltages, namely, $2.5,10,50,250$, and 1000 volts; four ranges of resistance, 3000 ohms, $30,000 \mathrm{ohms}, 3 \mathrm{meg}-$ ohms, and 30 megohms; and eight d-c current ranges, $0.05,0.1,1,10,50$,
and 250 milliamperes, 1 ampere, and 10 amperes. It can also be used to measure capacitance or to indicate the output of an amplifier stage in volts or decibels. This one instrument is very useful in the testing and servicing of a radio receiver, transmitter, public-address system, television receiver, vacuum-tube equipment, or cathode-ray equipment.


Fig. 6-35.-A combination ohmmeter, a-c/d-c voltmeter, and d-c ammeter. (Weston Electrical Instrument Corporation.)

In using combination meters, care should be taken to see that all switches and dial settings are in their correct positions before the instrument is connected into the circuit. If this is not done, the instrument may be damaged, as it might be set for use on direct current and be used on alternating current, or set to be used as an ammeter and connected for use as a voltmeter, or set in some other incorrect fashion.
$\mathbf{6 - 1 7}$. Wheatstone Bridge. The ohmmeter is a simple and convenient means for measuring resistance but the values obtained are not extremely
accurate although for most practical purposes the error involved can be ignored. When precise measurement of resistance is required, the instrument most generally used is the Wheatstone bridge. Essentially, this instrument consists of four resistors connected in series to form a diamond. A source of voltage, usually a battery of one or two cells, is connected in


Fig. 6-36.-Schematic diagram of a combination meter. (Weston Electrical Instrument Corporation.)
series with a switch and across two terminals of opposite junctions as points $A$ and $B$ in Fig. 6-37. An indicating instrument, usually a sensitive galvanometer, is connected in series with another switch and across the other two terminals of opposite junctions shown as points $C$ and $I$ in Fig. 6-37. The unknown resistor $R_{X}$ is used as one of the four resistors
forming the diamond; the other three resistors are of the adjustable type whose value is indicated as it is varied. Two forms of adjustable resistors commonly used in Wheatstone bridges are (1) the plug type, which is gen-


Fig. 6-37.-Wheatstone bridge method of measuring resistance. erally employed where high precision measurements are required, and (2) the dial type, which is utilized when greater speed in obtaining readings is desired. The settings of the three adjustable resistors are varied until, with switches $S_{1}$ and $S_{2}$ closed, no current flow is indicated on the galvanometer. Under this condition the bridge is said to be balanced. The value of the unknown resistance is then determined from the values to which the adjustable resistors were set in order to produce a balance.

The principle of the Wheatstone bridge can be explained in the following manner. When the switch $S_{1}$ is closed, the current from the battery will divide into two paths, namely, $I_{1}$ through the path $A C B$ and $I_{2}$ through path $A D B$. The voltage drops across the individual resistors are

$$
\begin{align*}
E_{A C} & =I_{1} R_{1}  \tag{6-13}\\
E_{C B} & =I_{1} R_{2}  \tag{6-14}\\
E_{A D} & =I_{2} R_{S}  \tag{6-15}\\
E_{D B} & =I_{2} R_{X} \tag{6-16}
\end{align*}
$$

When the bridge is balanced, the junction points $C$ and $D$ are at the same potential. This is evident from the fact that the galvanometer reading is zero, since current will flow between two points only when there is a difference of potential between the two points. Under this condition,
and

$$
\begin{align*}
& I_{1} R_{1}=I_{2} R_{s}  \tag{6-17}\\
& I_{1} R_{2}=I_{2} R_{X} \tag{6-18}
\end{align*}
$$

Dividing Eq. (6-17) by (6-18),

$$
\begin{equation*}
\frac{I_{1} R_{1}}{I_{1} R_{2}}=\frac{I_{2} R_{s}}{I_{2} R_{X}} \tag{6-19}
\end{equation*}
$$

or

$$
\begin{equation*}
\frac{R_{1}}{R_{2}}=\frac{R_{s}}{R_{X}} \tag{6-20}
\end{equation*}
$$

Solving for $R_{X}$ as the unknown resistance

$$
\begin{equation*}
R_{X}=R_{s} \times \frac{R_{2}}{R_{\mathrm{t}}} \tag{6-21}
\end{equation*}
$$

This is the fundamental equation for the Wheatstone bridge. The ratio $R_{2} / R_{1}$ is generally made to equal one or a multiple or a fraction of 10 . For example, $R_{2} / R_{1}$ may be made cqual to $1, \frac{1}{10}, \frac{1}{100}, 10,100$, etc. The value of the unknown resistance is then found by multiplying the value of $R_{s}$ required to produce a balance by the ratio $R_{2} / R_{1}$.

(a)

(b)

Fig. 6-38.-Basic a-c bridge circuits: (a) for capacitance measurements, (b) for inductanct. measurements.

Example 6-6. A balance is obtained in a Wheatstone bridge whose circuit is similar to Fig. 6-37 when $R_{1}=10$ ohms, $R_{2}=1000$ ohms, and $R_{S}=75$ ohms. What is the value of the unknown resistance $R_{X}$ ?

| Given: | Find: |
| :--- | :--- |
| $R_{1}=10$ ohms | $R_{X}=?$ |
| $R_{2}=1000 \mathrm{ohms}$ |  |
| $R_{S}=75$ ohms |  |

Solution:

$$
R_{X}=R_{S} \times \frac{R_{2}}{R_{1}}=75 \times \frac{1000}{10}=7500 \mathrm{ohms}
$$

6-18. A-C Bridge. The unknown value of the inductance of an inductor or the capacitance of a capacitor can also be measured by means of the
bridge circuit. The basic circuits for measuring inductance or capacitance are shown in Fig. 6-38. The voltage source is a steady a-c voltage, usually 1000 cycles. The indicating instrument may be a set of earphones or a sensitive a-c milliammeter. Resistors $R_{1}$ and $R_{2}$ are also used in this circuit as ratio arms just as in the Wheatstone bridge. A known standard value of inductance, $L_{s}$, or of capacitance $C_{s}$ is used in place of the standard resistor, $R_{s}$, of Fig. 6-37. The unknown inductor or capacitor is connected in a manner similar to $R_{X}$ of Fig. 6-37. A balance is obtained by adjusting the ratio resistors $R_{1}$ and $R_{2}$ until the sound in the earphones or the indication on the a-c milliammeter is at its minimum. The unknown value of inductance or capacitance can then be found by using the following cquations:

$$
\begin{align*}
& L_{X}=L_{S} \times \frac{R_{2}}{R_{1}}  \tag{6-22}\\
& C_{X}=C_{s} \times \frac{R_{2}}{R_{1}} \tag{6-23}
\end{align*}
$$

If a standard value of resistance $R_{s}$ and an unknown value of resistance $R_{X}$ are substituted for $L_{S}$ and $L_{X}$ in the circuit shown in Fig. 6-38, the circuit can also be used for measuring the unknown resistance value of a resistor.

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## QUESTIONS

1. Why are electrical instruments essential to the study of electricity, radio, and television?
2. How may electrical instruments be classified?
3. What is the difference between a galvanometer, an ammeter, and a voltmeter?
4. (a) Explain the principle of operation of an electrostatic meter. (b) What type of instruments operates on this principle?
5. (a) Explain the principle of operation of an electrothermal meter. (b) Name the different types of currents it can measure, and explain why this is possible.
6. How may the various types of electromagnetic instruments be classified?
7. Explain the principle of operation of each classification mentioned in Question 6.
8. What type of electromagnetic meter is (a) most versatile, (b) limited to d-c operation, (c) most widely used?
9. (a) What is the principle of operation of the copper oxide rectifier? (b) How is this principle used in electrical measuring instruments?
10. What is the efficiency of rectifier-type instruments?
11. (a) How is an ammeter connectod in a circuit? (b) How is it protected when readings are not being taken?
12. Is the resistance of an ammeter high or low? Explain.
13. Can an ammeter be used as a millivoltmeter? Explain.
14. (a) How is a voltmeter connected in a circuit? (b) How is it protected when readings are not being taken?
15. Is the resistance of a voltmeter high or low? Explain.
16. (a) What is meant by the sensitivity of a voltmeter? (b) How is the sensitivity rating of a voltmeter usually indicated?
17. Why is it desirable to use a voltmeter having a high sensitivity rating when measuring voltages in electronic circuits?
18. What is the essential difference between the construction of a voltmeter and that of an ammeter?
19. What is meant by a multirange voltmeter?
20. What is meant by (a) A linear scale? (b) A square law scale? (c) A logarithmic scale?
21. Is it possible to expand the scale of certain types of meters in order to make an irregular scale practically uniform? Explain.
22. On what part of the meter scale is it desirable to take readings? Explain.
23. What is the difference between a multiple scale and a complex scale?
24. What errors can easily be made when reading multiple or complex scales?
25. (a) What is meant by parallax? (b) How are errors due to parallax counteracted?
26. (a) What is the purpose of a shunt? (b) What is the difference between an internal and an external shunt?
27. In calculating the resistance of a shunt is it always practical to assume that the line current and shunt current are equal? Explain your answer.
28. (a) What is the purpose of a multiplier? (b) Can multipliers be used with a-c meters? (c) Can multipliers be used with d-c meters?
29. (a) Can the voltmeter-ammeter method be used to measure low resistances? (b) High resistances? (c) Explain.
30. How can high resistances be measured with just a voltmeter?
31. (a) What is an ohmmeter? (b) Explain its basic principle of operation.
32. (a) How does a decrease in the battery voltage affect the reading of an ohmmeter? (b) What provision is made to compensate for this?
33. What is meant by a combination meter?
34. Why are combination meters useful in test and service work on radio, television, and electronic equipment?
35. What precautions are necessary in using combination meters?
36. For what purpose is the Wheatstone bridge generally used?
37. Explain the principle of operation of the Wheatstone bridge.
38. Name three types of circuit elements whose value may be determined by means of a bridge circuit.

## PROBLEMS

1. A galvanometer has a resistance of 125 ohms. What resistance shunt must be used with this galvanometer if it is desired that (a) one-tenth of the line current should pass through the galvanometer? (b) One twenty-fifth of the line current should pass through the galvanometer?
2. An instrument has a rating of 50 mv , and its moving coil has a resistance of

20 ohms. If it is to measure a current of 1 amp , how much current flows through (a) the meter? (b) The shunt?
3. A $150 / 15$ scale voltmeter has a total resistance of 150,000 ohms. (a) What is the resistance between its 15 -volt terminals? (b) What resistance multiplier is necessary to give this instrument a range of 600 volts?
4. If the resistance of an instrument is 40 ohms and its current rating is 10 ma , what amount of resistance must be added to give a full-scale reading of (a) 50 volts? (b) 150 volts? (c) 250 volts?
5. An instrument has a resistance of 25 ohms, and full-scale deflection is obtained when 25 ma flows through the instrument. What value of shunt resistance must be used to produce full-scale deflection when the current flowing in the line is (a) 250 ma ? (b) 500 ma ? (c) 1 amp ?

Note: Assume that the shunt current is equal to the line current.
6. What is the per cent error in the resistance obtained for each current range in Prob. 5 by assuming that the shunt current is equal to the line current?
7. It is desired to extend the range of a microammeter having a full-scale deflection of $50 \mu \mathrm{a}$ and whose resistance is 1200 ohms. (a) What resistance shunt is required to obtain full-scale deflection with a $100-\mu$ a line current? (b) What resistance shunt is required to obtain full-scale deflection with a $500-\mu$ a line current?
8. A voltmeter having a sensitivity of 1000 ohms per volt has ranges of 10,50 , 100,250 , and 1000 volts. What is the resistance of the voltmeter for each of these ranges?
9. A voltmeter having a sensitivity of 1000 ohms per volt is used to measure a voltage drop of approximately 40 volts that exists across a 50,000 -ohm resistor. What is the per cent decrease in this circuit's resistance when the voltmeter is connected across the resistor and the full-scale range of the meter is (a) 50 volts? (b) 100 volts?
10. What is the per cent decrease of a circuit's resistance when a voltmeter having a sensitivity of 20,000 ohms per volt is used to measure the voltage drop across a 200,000 -ohm resistor (a) using its 100 -volt range? (b) Using its 250 -volt range?
11. Two 50,000 -ohm resistors are connected in series across a 100 -volt source of power. A voltmeter having a sensitivity of 1000 ohms per volt is used to measure the voltage drop across each resistor. (a) What voltage will the voltmeter indicate when using its 100 -volt range? (b) What is the per cent error?
12. A voltmeter having a sensitivity of 20,000 ohms per volt is substituted for the meter used in Prob. 11. (a) What voltage will the voltmeter indicate when using its 100 -volt range? (b) What is the per cent error?
13. A voltmeter having a sensitivity of 10,000 ohms per volt and a full-scale reading of 150 volts is connected as shown in Fig. 6-31 to measure the resistance values of three unknown resistors. The voltage of the power source is 100 volts, and the meter reads (a) 10 volts, (b) 50 volts, (c) 60 volts, respectively, when each of the resistors is connected in the circuit. What is the value of each resistance?
14. If the voltmeter in Prob. 13 had a sensitivity of 1000 ohms per volt, could it be used to measure these resistances? If so, what voltage would be indicated for each of the resistors measured?
15. The voltmeter-ammeter method is used to determine the resistance of a 10 ohm resistor. The voltmeter has a sensitivity of 100 ohms per volt and a full-scale deflection of 150 volts. The ammeter has a $50-\mathrm{mv}$ drop for its full-scale deflection of 10 amp . The voltage of the power supply is 50 volts. Two sets of readings are taken under the following conditions: (1) the voltmeter connected so that it reads the voltage drop across the resistor and the ammeter; (2) the voltmeter connected so that the ammeter reads the current taken by the voltmeter and the resistor.
(a) What are the voltmeter and ammeter readings when connected as in (1) and in (2), respectively? (b) Which connection gives a more accurate reading?
16. Repeat Prob. 15 for the following conditions: 10,000 -ohm resistor to be measured instead of the $10-\mathrm{ohm}$ resistor; use same voltmeter; substitute a $50-\mathrm{ma}$ ammeter (also requiring a $50-\mathrm{mv}$ drop) in place of the $10-\mathrm{amp}$ ammeter; the voltage of the power supply is increased to 100 volts.
17. In the circuit shown in Fig. 6-32, what should the resistance of $R_{1}+R_{\mathrm{A}}$ be in order that the milliammeter indicate 9000 ohms in the center of its scale when the applied voltage is 9 volts?
18. A voltmeter has a full-scale reading of 15 volts and a sensitivity of 1000 ohms per volt. This meter is connected in series with a 30 -volt battery and unknown resistances. What are the values of these unknown resistances if the meter indicates (a) 5 volts? (b) 10 volts? (c) 15 volts?
19. A d-c milliammeter having a resistance of 30 ohms and a full-scale deflection of 1 ma is to be connected to various shunts and multiplier resistors so that it will indicate the following d-c voltages and currents when a rotating switch connects the proper unit into the circuit: $10,50,100,250$, and 500 volts; $1,10,50,100$, and 1000 ma. As a voltmeter, the instrument should have a sensitivity of 1000 ohms per volt.
a. Draw a circuit diagram of the meter, shunts, multipliers, and rotary switch.
$b$. Indicate the resistance value required for each shunt and multiplier.
20. A balance is obtained in a Wheatstone bridge whose circuit is similar to Fig. $6-37$ when $R_{1}=100 \mathrm{ohms}, R_{2}=1 \mathrm{ohm}$, and $R_{S}=30 \mathrm{ohms}$. What is the value of the unknown resistor $R_{X}$ ?
21. A balance is obtained in a Wheatstone bridge whose circuit is similar to Fig. $6-37$ when $R_{1}=10,000 \mathrm{ohms}, R_{2}=10$ ohms, and $R_{S}=78.5 \mathrm{ohms}$. What is the value of the unknown resistor $R_{X}$ ?
22. A bridge circuit similar to Fig. 6-38a is used to measure the unknown value of a capacitor. When a balance is obtained, it is found that $R_{1}=250 \mathrm{ohms}, R_{2}=100$ ohms, and $C_{B}=10 \mu \mathrm{f}$. What is the value of the unknown capacitor?
23. A bridge circuit similar to Fig. 6-38a is used to measure the unknown value of a capacitor. When a balance is obtained, it is found that $R_{1}=3750 \mathrm{ohms}, R_{2}=100$ ohms, and $C_{S}=10 \mu \mathrm{f}$. What is the value of the unknown capacitor?
24. A bridge circuit similar to Fig. $6-38 b$ is used to measure the unknown value of an inductor. When a balance is obtained, it is found that $R_{1}=80 \mathrm{ohms}, R_{2}=1200$ ohms, and $L_{s}=100 \mathrm{mh}$. What is the value of the unknown inductor?
25. A bridge circuit similar to Fig. $6-38 b$ is used to measure the unknown value of an inductor. When a balance is obtained, $i$ is found that $R_{1}=4 \mathrm{ohms}, R_{2}=1680$ ohms, and $L_{S}=100 \mathrm{mh}$. What is the value of the unknown inductor?

## CHAPTER VII

## ELECTRICAL POWER APPARATUS

Most users of radio are content with the knowledge that the electrical energy required to operate a receiver is obtained by merely inserting its plug into a convenient outlet in the wall at home or by inserting batteries in the case of portable receivers. Instructions attached to new radio or television receivers, such as "Operate at 115 volts, 50 or 60 cycles, a-c only"; "For 120 -volt a-c or d-c operation"; "For 220 -volt, $50 / 60$ cycles, alternating current"; "Operate on 32 -volt, direct current only," have little or no meaning to many users. In order to understand the principles of radio and television operation, it is necessary to have a knowledge of the various types of power systems, the methods used to produce the power, and the characteristics of each.

7-1. Types of Power Supply and Equipment. Of the six kinds of electric currents described in Art. 2-10, only the continuous, direct, and alternating are used as sources of power supply.

Continuous Current. The continuous current, which is obtained from battery cells, is used largely for operating portable receivers and transmitters. The method of producing this type of current has been discussed in Chap. III, and the circuit characteristics studied in Chap. IV will apply to this type of current.

Direct Current. The direct current, usually referred to as $d-c$, is that type obtained from a rotating machine called the $d$-c generator. The theory of operation of this machine is based upon the principle of electromagnetic induction and is described in the following articles. Direct-current power systems are found mainly in rural and isolated sections or in some instances where an individual building may have its own power plant. Such power systems are generally 110 volts or in a few instances 220 volts. Another common d-c system is the farm-lighting system used where no other source of electrical power is available. To have power available 24 hours per day without running a gasoline-engine-driven generator continuously, it becomes necessary to use storage batteries. The power is supplied by the batteries, and the generator is operated periodically to keep the batteries charged. To keep the number of cells at a reasonable amount, it has been common practice to make these systems 32 volts. The external-circuit
characteristics of the d-e gencrator systems are similar to those of battery systems and have been studied in Chap. IV.

Alternating Current. The alternating current, usually referred to as $a-c$ is that type obtained from a rotating machine called an alternator or $a-c$ generator. The theory of this machine is also based on the principle of electromagnetic induction and is described in the following articles. It is estimated that over 90 per cent of the electrical power is generated as alternating current, and hence most power systems are of this type. The circuit characteristics of a-e systems differ in some respects from the d-c systems, and these new characteristics will be explained in the following chapters.

Voltage of A-C Systems; the Transformer. 'The generated voltage of the alternator in the large modern power plant is much higher than the voltage supplied to the homes. It is not unusual for this to be as high as 16,600 volts, which is indeed high when compared with the 110 volts used in the homes. This higher voltage increases the efficiency of operation for the power companies but makes it necessary to reduce the voltage before connecting it to the home. The extensive use of a-c systems is due largely to the ability to raise or lower the voltage easily by means of a device called the transformer. The operation of this device is also based on the principle of electromagnetic induction and is explained later in this chapter.

7-2. Faraday's Discovery, Electromagnetic Induction. Faraday's Discovery. The operation of electrical power apparatus such as the d-c generator, the alternator, and the transformer is based on the principle of electromagnetic induction. The discovery of this principle is credited to Michael Faraday, who, in 1831, found that if a conductor which was part of a closed circuit was moved through a magnetic field a current would flow in the conductor. He proved this to be true by connecting a sensitive instrument, called a galvanometer, in the closed circuit; when the conductor was moved in the magnetic field, it caused a deflection of the galvanometer needle.

Demonstration of Faraday's Discovery. F'araday's discovery may be demonstrated by moving a conductor through the field of a strong electromagnet as shown in Fig. 7-1. If the conductor $C$ is moved upward through the magnetic field, it will cause a voltage to be induced in the conductor and the galvanometer $G$ will indicate the presence of this induced voltage. It is common practice to call this induced voltage the induced emf. If the conductor is held stationary in the magnetic field, the galvanometer will show zero, indicating that no voltage is being induced. Moving the conductor downward through the magnetic field will cause the galvanometer to indicate a voltage but in the opposite direction to that when the conductor was moved upward. If the conductor is moved in the field in a


Fig. 7-1.-Demonstration of Faraday's discovery.
sidewise direction, that is, parallel to the field from $N$ to $S$, the galvanometer will show zero, indicating that no voltage is being induced. Reversing the polarity of the magnet, that is, interchanging the location of the $N$ and $S$ poles, would also cause a reversal of the galvanometer indication.


Fig. 7-2.-Fleming's right-hand rule for determining the direction of the induced emf.
Fleming's Right-hand Rule. The foregoing leads to the conclusion that the conductor must cut through the magnetic field in order to have a voltage induced and also that the direction of the induced voltage depends
upon the direction of motion of the conductor and the direction of the magnetic field. This relation is expressed by Fleming's right-hand rule shown in Fig. 7-2, which may be stated as follows:

1. Hold the thumb, forefinger, and middle finger of the right hand at right angles to one another.
2. Point the thumb in the direction of motion of the conductor.
3. Point the forefinger in the direction of the magnetic field.
4. The middle finger will then point in the direction of the induced voltage, namely, toward the positive terminal.

Methodsof Producing an Induced


Fig. 7-3.-Electromotive force induced by magnetic lines cutting a conductor. EMF. So far it has been shown that a voltage is induced when a conductor is moved through a magnetic field. An induced emf can also be set up in a stationary conductor by having a magnetic field move so that its lines cut the conductor. Referring


Fig. 7-4.-A simple generator.
to Fig. 7-3, it will be seen that if the bar magnet $A$ is moved into the center of a coil $C$ the field of the bar magnet will cut the conductors of the coil, and, if the coil circuit is closed through a galvanometer $G$, a current flow
will be indicated by the galvanometer. By using the left hand in place of the right, Fleming's rule may be applied to finding the direction of the induced emf when the moving magnetic field cuts the conductor.

The principle of electromagnetic induction may now be restated as follows: whenever there is motion between a conductor and a magnetic field, an emf will be induced in the conductor; the motion may be produced by moving the conductor through a stationary magnetic field or by having the magnetic field cut a stationary conductor.

7-3. The Simple Generator; Value of Induced EMF. The Simple Generator. The principle of electromagnetic induction as it applies to generators is illustrated in Fig. 7-4. This fundamental generator has two poles, $N$ and $S$, which sct up the magnetic field. The coil $C$ is mounted on a shaft $S$ so that it can be rotated in the magnetic field; this part is called the armature. When the armature is rotated, its conductors, which are really the coil sides $a a^{\prime}$ and $b b^{\prime}$, cut through the lines of the magnetic field and a voltage is induced in the conductors.

Value of the Induced EMF. The value of the induced emf in any conductor is proportional to the rate of cutting lines. A fundamental electrical law states that whenever a conductor cuts lines (or is cut by lines) at the rate of 100 million lines per second, an emf of one volt is induced in that conductor. Since this is a rate of cutting lines, the speed of rotation as well as the number of lines cut will affect the induced emf.

In Fig. 7-4, the armature coil $C$ consists of only one turn of wire, but it has two conductors which are connected in series. In many gencrators, the coils have more than one turn and hence have a greater number of conductors. This results in a higher induced emf, because when these conductors are connected in series their voltages will add. Each conductor will cut the flux twice in one revolution. Combining these facts, we may express them mathematically by the equation

$$
\begin{equation*}
E=\frac{2 \phi C S}{60 \times 10^{8}} \tag{7-1}
\end{equation*}
$$

where $E=$ induced emf of the generator
$\phi=$ total flux going from the $N$ to $S$ pole
$C=$ number of conductors connected in series
$S=$ speed, rpm
Example 7-1. A generator operating at 1800 rpm has 500 conductors on its armature. If the flux from the $N$ pole to the $S$ pole is 500,000 lines, what will the induced emf of the generator be?

Given:

$$
\begin{aligned}
& \phi=500,000 \\
& C=500 \\
& S=1800
\end{aligned}
$$

Find:
$E=$ ?

Solution:

$$
\begin{aligned}
E & =\frac{2 \phi C S}{60 \times 10^{8}} \\
& =\frac{2 \times 500,000 \times 500 \times 1800}{60 \times 10^{8}} \\
& =150 \text { volts }
\end{aligned}
$$

Example 7-2. If the armature of the generator of Example 7-1 had only one single-turn coil, what would the induced emf of the generator be?

Given:

$$
\begin{aligned}
& \phi=500,000 \\
& C=2 \\
& S=1800
\end{aligned}
$$

Solution:

$$
\begin{aligned}
& =\frac{2 \times 500,000 \times 2 \times 1800}{60 \times 10^{8}} \\
& =0.6 \text { volt }
\end{aligned}
$$

The above examples illustrate that the practical generator requires a large number of series-connected conductors in order to obiain a standard commercial voltage.

Collector Rings and Brushes. If the coil $C$ of Fig. $7-4$ is to be rotated and its induced emf applied to an external circuit, it will be necessary to connect the conductors $a a^{\prime}$ and $b b^{\prime}$ to the collector rings $R_{a}$ and $R_{b}$, respectively, and these rings must be insulated from the shaft. lieces of carbon, called brushes, make sliding contact with the collector or slip rings, and the external circuit is connected to these brushes, indicated by $B_{1}$ and $B_{2}$.

7-4. The A-C Generator. The simple generator of Fig. 7-4 is really an a-c generator as will be shown in the following discussion. The coil is assumed to be rotating at a uniform speed in a counterclockwise direction, and the magnetic field is considered to be uniform, that is, each square inch of pole surface emits the same number of magnetic lines. Figure $7-5 a$ shows one conductor in 12 positions 30 degrees apart, and Fig. 7-5b shows the voltage corresponding to each of these positions. At position 1 , the voltage is indicated as zero, which is explained by the fact that when the conductor moves a very small amount, say zero to 1 degree, or ${ }_{360}$ of a revolution, its motion is practically parallel with the magnetic lines. Under this condition, it cuts none of the lines and the induced emf is zero. At position 4 , the conductor has moved 90 degrees, and the voltage as indicated on the curve is 100 volts. In this zone, any small amount of motion, say one additional degree, will be in practically a vertical direction, and the conductor's motion will be perpendicular to the magnetic lines. Under
this condition, it will cut the lines at the greatest rate possible, and the voltage will be at its maximum value, assumed for convenience as 100 volts. By applying Fleming's right-hand rule (see Fig. 7-2), it will be found that the direction of the induced emf is outward as indicated on Fig. 7-5a by $\odot$. For any position between 1 and 4 , the direction of the conductor's motion will be neither parallel nor perpendicular to the magnetic lines but will be at an angle to the lines. Therefore the induced emf will have


Fig. 7-5.-Induced emf of a conductor in the simple a-e generator.
values greater than zero but less than 100 volts. Figure $7-5 b$ shows that at 30 degrees the induced emf is 50 volts and at 60 degrees it is 86.6 volts. When the conductor has passed position 4, the induced emf decreases until it reaches position 7, when it is again zero. After passing position 7, the conductor cuts lines again, but its motion is now upward. Applying Fleming's rule, it will be seen that the direction of the induced emf has been reversed. The values of the emf for positions between 7 and 13 (same as 1) are all negative as indicated on Fig. 7-5b. Any continued motion will
merely result in additional similar voltage cycles. As the flow of electric current is dependent upon the voltage impressed, its flow will vary in a manner similar to the voltage.

From the foregoing, it is now possible to set up a definition for an alternating current. An alternating current is one that is continually changing in magnitude and alternates in direction or polarity at regular intervals.

7-5. A-C Characteristics: Alternation, Cycle, Frequency. The a-c voltage of Fig. $7-5 b$ is shown starting at zero volts, increasing in a positive direction to its maximum value at 90 degrees, then decreasing to zero at 180 degrees when it reverses in polarity, or alternates; next it increases to its maximum negative value at 270 degrees and again decreases to zero at 360 degrees. From this point on indefinitely, the voltage repeats this procedure. The degrees mentioned here are called electrical degrees. In this case, they also correspond to the mechanical degrees of coil motion, a condition that occurs only with two-pole alternators.

Alternation, Cycle, Frequency. The term alternation is used to define a period of 180 electrical degrees. The span of one complete set of values, that is, all the positive and all the negative values, is called a cycle. The cycle corresponds to 360 electrical degrees and also corresponds to two alternations. The number of times these complete sets of values occur in a second is called the frequency and is expressed in cycles per second.

The frequency of an alternator depends on its speed of rotation and the number of poles. Expressed mathematically, this is

$$
\begin{equation*}
f=\frac{P \times S}{120} \tag{7-2}
\end{equation*}
$$

where $f=$ frequency, cycles per second
$P=$ number of poles of the alternator
$S=$ speed of the alternator, rpm
Example 7-3. What is the frequency of a four-pole alternator which is being driven at 1800 rpm ?

| Given: | Find: |
| :--- | :---: |
| $P=4$ | $f=?$ |
| $S=1800$ |  |

Solution:

$$
\begin{aligned}
f & =\frac{P \times S}{120}=\frac{4 \times 1800}{120} \\
& =60 \text { cycles per second }
\end{aligned}
$$

Period. The time required for a voltage (or current) to complete one cycle is called the period and is expressed mathematically as

$$
\begin{equation*}
t=\frac{1}{f} \tag{7-3}
\end{equation*}
$$

where $t=$ time in seconds required to complete one cycle
$f=$ number of cycles per second.
For a 60 -cycle circuit $t$ equals $\frac{1}{60}$ second, while for a 25 -cycle circuit $t$ would be $\frac{1}{25}$ second.

Frequencies Used in Power-supply Systems. The frequency of power systems is low, the most common being 60 cycles, although $25,30,40$, and 50 are also used. The 60 -cycle is the most popular because it gives the best results when used for operating both lights and machinery. Alternators can be built to produce frequencies of 500 cycles and in a few special cases have been built for a frequency of 20,000 cycles.

Frequencies Used in Radio Systems. In radio work, higher frequencies such as thousands or hundred thousands of cycles (kilocycles) and millions of cycles (megacycles) per second are used. For these higher frequencies, it is common practice to use vacuum tubes which make it possible to attain frequencies up into the hundreds of million cycles per second.

7-6. A-C Voltage and Current Characteristics. Instantaneous Values. It has been shown that the induced emf of the alternator is continually changing in magnitude and is also periodically alternating in polarity. When the emf is produced by rotating a coil at constant speed in a uniform magnetic field, the value of the voltage at any instant of time may be found by the equation

$$
\begin{equation*}
e_{\theta}=E_{\text {max }} \times \sin \theta \tag{7-4}
\end{equation*}
$$

where $e_{\theta}=$ instantancous value of the emf when the coil has gone through $\theta$ electrical degrees
$E_{\text {max }}=$ maximum value of the emf
$\sin \theta=$ value obtained from the table in Appendix XI.
Example 7-4. An alternator produces a sine-wave voltage whose maximum value is 500 volts. What is the instantaneous value at (a) 7 degrees, (b) 73 degrees, (c) 162.5 degrees, (d) 195 degrees, (e) 322.5 degrees?

| Given: | Find: |  |
| ---: | :--- | ---: |
| $E_{\text {max }}$ | $=500$ | $e=?$ |
| $\theta$ | $=7^{\circ}$ |  |
|  | $=73^{\circ}$ |  |
|  | $=162.5^{\circ}$ |  |
|  | $=195^{\circ}$ |  |
|  | $=322.5^{\circ}$ |  |

Solution:
(a)

$$
\begin{align*}
e_{\theta} & =E_{\max } \times \sin \theta \\
e_{i^{\circ}} & =500 \sin 7^{\circ}=500 \times 0.122=61 \text { volts } \\
e_{73^{\circ}} & =500 \sin 73^{\circ}=500 \times 0.956=478 \text { volts }  \tag{b}\\
e_{169.5^{\circ}} & =500 \sin 162.5^{\circ}=500 \times 0.301=150.5 \text { volts } \\
e_{195^{\circ}} & =500 \sin 195^{\circ}=500 \times-0.259=-129.5 \text { volts } \\
e_{322.5^{\circ}} & =500 \sin 322.5^{\circ}=500 \times-0.609=-304.5 \text { volts }
\end{align*}
$$

(c)
(d)
(e)

Note: For values of sine, see Appendives XII and XII.
Figure $7-5 b$ shows an a-c voltage whose maximum value is 100 volts and indicates that the instantancous value at 30 degrees is 50 volts, at 60 degrees is 86.6 volts, etc. These values may be verified by use of Eq. ( $7-4$ ) as follows:

$$
\begin{aligned}
& e_{30^{\circ}}=E_{\max } \times \sin 30^{\circ}=100 \times 0.500=50 \text { volts } \\
& e_{60^{\circ}}=E_{\max } \times \sin 60^{\circ}=100 \times 0.866=86.6 \text { volts }
\end{aligned}
$$

The Sine Wave. If a number of instantaneous values obtained by use of Eq. (7-4) are plotted and a curve is drawn, the curve is called a sine wave. A voltage corresponding to this shape is called a sine-wave voltage.

A simple method of drawing a sine-wave voltage is by means of the wheel diagram illustrated in Fig. 7-6. This may be done by first drawing a circle whose radius is made equal to the value of $E_{\max }$. Next, set off a number of equally spaced spokes. The number used must correspond to the number of points desired to draw the sine wave, and the greater the number of points, the more accurate the sine-wave drawing. Figure 7-6b is drawn with as many equally spaced vertical lines as there are spokes. By projecting horizontal lines from the ends of the spokes in Fig. 7-6a to the corresponding vertical lines on Fig. $\overline{7}$ - $6 b$, points of the sine-wave curve are obtained and may then be connected by a smooth line.

Most alternators produce a voltage that is a sine-wave voltage or so close to the sine wave that the entire study of alternating current from this point on will be based on sine-wave voltages and currents. Figure 7-7 shows several nonsinusoidal voltage waves, the study of which requires a college level of electrical engineering and hence is beyond the scope of this text.

The Maximum Talue. This is simply the highest value reached in a cycle as is shown in Fig. 7-8. This value is important in some parts of the study of alternating voltages and current, but it is not used as the rated value. It is designated as $E_{\text {max }}$ and $I_{\text {max }}$.

The Average Value. Examination of the sine wave of Fig. $\overline{\mathrm{i}}-8$ shows that it is not a straight line from zero to maximum value but a smooth curved line. The average value of a complete cycle is zero, because the
positive and negative areas under the curve are equal to each other. The average value generally referred to is, however, for only one-half of a cycle.


Fig. 7-6.-Wheel-diagram method of drawing a sine-wave voltage.
This value may be determined by finding a large number of equally spaced instantaneous values from 0 to 180 degrees (or from 0 to 90 degrees),


Fig. 7-7.-Several nonsinusoidal voltage waves recorded with the use of an oscillograph.
getting their sum, and then dividing it by the number of cases used. This may be expressed mathematically as

$$
\begin{equation*}
E_{\mathrm{ave}}=\frac{e_{1}+e_{2}+e_{3}+e_{n}}{n} \tag{7-5}
\end{equation*}
$$

Figure 7-9 illustrates this method of finding the average value. The accuracy of the result will increase as the number of instantaneous values
used is increased. A more accurate result may be obtained by higher mathematics (calculus), which shows that the average value is equal to $2 / \pi$ times the maximum. This is the commonly acecpted valuc and is expressed as

$$
\begin{equation*}
E_{\mathrm{ave}}=0.637 E_{\max } \quad\left(\text { Note }: \frac{2}{\pi}=0.637\right) \tag{7-6}
\end{equation*}
$$

The average value of the sinc-wave voltage of Fig. 7-8 is 63.7 volts and is indicated by the line drawn through this point. While the average value is used in some engineering calculations; it is not the value used to represent the sine wave.


Fig. 7-8.-Relative values of an a-c sine-wave voltage.
The A-c Ampere. The current flowing in a circuit is proportional to the voltage, and therefore the maximum, instantaneous, and average values as described for sine-wave voltages will also apply to sine-wave currents. Thus Fqs. (7-4) and (7-6) may be stated as

$$
\begin{gather*}
i_{\theta}=I_{\max } \times \sin \theta  \tag{7-7}\\
I_{\mathrm{ave}}=0.637 I_{\max } \tag{7-8}
\end{gather*}
$$

The maximum, instantaneous, and average values of current and voltage are used in some engineering calculations, but they are not used in practical work because they do not provide means of comparison with direct current.

The a-c ampere, which is the practical unit, is based upon the heating effect of the current. As the heating effect is the same regardless of the direction of the current flow, it provides a suitable means of comparing alternating and direct current effects. The a-c ampere may then be defined as that amount of alternating current which will produce the same effect as one ampere of continuous or direct current.

The Effective Value. The instantaneous value of the sine wave chosen to represent the a-c ampere is called the effective value, since it must be such
an amount that it will produce the same heating effect as one ampere of direct current.

The heating effect of any current, whether alternating or direct current, is equal to the current squared, times the resistance ( $P=I^{2} R ; p=i^{2} R$ ).

(a)

| $\theta^{\circ}$ | $\sin \theta$ | $e$ |
| ---: | ---: | ---: |
| 0 | 0.000 | 0.0 |
| 10 | 0.173 | 17.3 |
| 20 | 0.342 | 34.2 |
| 30 | 0.500 | 50.0 |
| 40 | 0.643 | 64.3 |
| 50 | 0.766 | 76.6 |
| 60 | 0.866 | 86.6 |
| 70 | 0.940 | 94.0 |
| 80 | 0.985 | 98.5 |
| 90 | 1.000 | 100.0 |

Total $=621.5$
Average $=\frac{621.5}{10}=62.15$

| $\theta^{\circ}$ | $\sin \theta$ | $e$ |
| :---: | :---: | :---: |
| 0 | 0.000 | 0.0 |
| 5 | 0.087 | 8.7 |
| 10 | 0.173 | 17.3 |
| 15 | 0.259 | 25.9 |
| 20 | 0.342 | 34.2 |
| 25 | 0.422 | 42.2 |
| 30 | 0.500 | 50.0 |
| 35 | 0.574 | 57.4 |
| 40 | 0.643 | 64.3 |
| 45 | 0.707 | 70.7 |
| 50 | 0.766 | 76.6 |
| 55 | 0.819 | 81.9 |
| 60 | 0.866 | 86.6 |
| 65 | 0.906 | 90.6 |
| 70 | 0.940 | 94.0 |
| 75 | 0.966 | 96.6 |
| 80 | 0.985 | 98.5 |
| 85 | 0.996 | 99.6 |
| 90 | 1.000 | 100.0 |
| $\text { Total }=1195.1$ |  |  |
|  |  |  |

(c)
(b)

Fig. 7-9.-Method of determining the average value of a sine-wave voltage: (a) portion of a sine-wave voltage curve, (b) average value obtained by taking 10 -degree intervals, (c) average value obtained by taking 5 -degree intervals.

In alternating current, the heating effect will vary continually because the current is varying continually. The comparison of a-c and d-c heating effect is thercfore based on the average rate at which the heat is produced. This may be expressed mathematically as

$$
\begin{equation*}
I_{\mathrm{dc}}{ }^{2} R=\frac{i_{1}{ }^{2} R+i_{2}{ }^{2} R+i_{3}{ }^{2} R+i_{4}{ }^{2} R+i_{n}{ }^{2} R}{n} \tag{7-9}
\end{equation*}
$$

In order to make a comparison, $R$ must have the same value for both sides of the above equation and mathematically the equation may be simplified as

$$
\begin{equation*}
I_{\mathrm{dc}}{ }^{2}=\frac{i_{1}{ }^{2}+i_{2}{ }^{2}+i_{3}{ }^{2}+i_{4}{ }^{2}+i_{n}{ }^{2}}{n} \tag{7-10}
\end{equation*}
$$

lexamining the right-hand member of this equation, it will be seen that by adding the instantaneous $i^{2}$ values and dividing this sum by the number of cases, $n$, the average of the instantaneous $i^{2}$ values is obtained. Thus the equation may be restated as
or

$$
\begin{align*}
& I_{\mathrm{dc}}{ }^{2}=\text { average of the instantaneous squares }  \tag{7-11}\\
& I_{\mathrm{dc}}{ }^{2}=\left(\text { ave } i^{2}\right) \tag{7-11a}
\end{align*}
$$

As the a-c and d-c amperes are to have the same heating effect, the equation may also be stated as

$$
\begin{equation*}
I_{\mathrm{dc}}{ }^{2}=I_{\mathrm{ac}}{ }^{2}=\left(\mathrm{ave} i^{2}\right) \tag{7-12}
\end{equation*}
$$

Taking the square root of each member of this equation, it becomes

$$
\begin{equation*}
I_{\mathrm{dc}}=I_{\mathrm{ac}}=\sqrt{\left(\mathrm{ave} i^{2}\right)} \tag{7-13}
\end{equation*}
$$

The effective value of $a$ sine wave is therefore equal to the square root of the average of the instantaneous squares. Figure 7-10 illustrates a simple method of finding the effective value. The effective value may also be obtained by higher mathematics (calculus), which shows that the effective value of a sine wave is always equal to maximum value divided by $\sqrt{2}$ This is the commonly accepted value and is expressed as

$$
\begin{equation*}
I=\frac{I_{\max }}{\sqrt{2}} \tag{7-14}
\end{equation*}
$$

This may be simplified as

$$
\begin{align*}
& I=\frac{1}{\sqrt{2}} \times I_{\max }=\frac{1}{1.414} \times I_{\max }=0.707 I_{\max } \\
& I=0.707 I_{\max } \tag{7-15}
\end{align*}
$$

Since alternating voltages and currents both follow sine-wave forms, the same relation applies to voltages; thus

$$
\begin{align*}
& E=0.707 E_{\max }  \tag{7-16}\\
& E=\frac{E_{\max }}{\sqrt{2}}=\frac{E_{\max }}{1.414} \tag{7-17}
\end{align*}
$$

The effective value is sometimes called the rms (root-mean-square) value because it is found by taking the square root of the mean (or average) of
the squares of a large number of instantaneous values. The effective value of the voltage wave of Fig. $7-8$ is 70.7 volts and is indicated by the line drawn through this point. The effective value is represented by the capital

(a)

(b)

| $\theta^{\circ}$ | $\sin \theta$ | $i$ | $i^{2}$ |
| :---: | :---: | :---: | :---: |
| 0 | 0.000 | 0.000 | 0.000 |
| 10 | 0.173 | 0.173 | 0.030 |
| 20 | 0.342 | 0.342 | 0.117 |
| 30 | 0.500 | 0.500 | 0.250 |
| 40 | 0.643 | 0.643 | 0.413 |
| 50 | 0.766 | 0.766 | $\mathbf{0 . 5 8 7}$ |
| 60 | 0.866 | 0.866 | $\mathbf{0 . 7 5 0}$ |
| 70 | 0.940 | 0.940 | $\mathbf{0 . 8 8 4}$ |
| 80 | 0.985 | 0.985 | $\mathbf{0 . 9 7 0}$ |
| 90 | $\mathbf{1 . 0 0 0}$ | $\mathbf{1 . 0 0 0}$ | $\mathbf{1 . 0 0 0}$ |

Total $=5.001$

$$
\begin{aligned}
& \text { ave } i^{2}=\frac{5.001}{10}=0.5001 \\
& \sqrt{\text { ave } i^{2}}=\sqrt{0.5001}=0.707
\end{aligned}
$$

(c)

Fig. 7-10.-Method of determining the effective value of a sine-wave current: (a) portion of a sine-wave current showing the variation of $i$ and $i^{2}$, (b) curves of $i$ and $i^{2}$ for one complete cycle, (c) effective value obtained by taking 10-degree intervals.
letters $E$ and $I$; note that no subscript is used as was the case in $E_{\text {ave }}$ and $I_{\text {max }}$.

Alternating-current measuring instruments such as voltmeters or ammeters are calibrated to indicate the effective values; thus if the voltage measurement on an a-c circuit is 110 volts, it will produce the same effects
on an incandescent lamp as a 110 -volt d-c circuit even though the a-c circuit at some instants reaches the maximum of $110 \times 1.414$, or 155 volts.

Example 7-5. (a) What is the effective value of the voltage of the alternator in Example 7-4? (b) What is its average value?

## Given:

$$
E_{\max }=500
$$

Find:

$$
\begin{aligned}
E & =? \\
E_{\text {ave }} & =?
\end{aligned}
$$

Solution:
(a)

$$
\begin{align*}
E & =0.707 E_{\max } \\
& =0.707 \times 500=353.5 \mathrm{volts} \\
E_{\mathrm{av}} & =0.637 E_{\max }  \tag{b}\\
& =0.637 \times 500=318.5 \mathrm{volts}
\end{align*}
$$

Example 7-6. The rms value of a current in an a-c circuit is 10 amperes. (a) What is the maximum value? (b) average value?

$$
\begin{array}{lr}
\text { Given: } & \text { Find: } \\
I=10 \mathrm{amp} & I_{\max }=? \\
I_{\mathrm{ave}}=?
\end{array}
$$

Solution:
(a)
(b)

$$
\begin{aligned}
I_{\max } & =1.414 \times I \\
& =1.414 \times 10=14.14 \mathrm{amp} \\
I_{\mathrm{ave}} & =0.637 I_{\max } \\
& =0.637 \times 14.14 \\
& =9.00 \mathrm{amp}
\end{aligned}
$$

7-7. The D-C Generator. The Simple Generator. The current produced by a d-c generator is unidirectional; that is, one terminal of the generator is always positive and the other always negative. On the a-c generator described in Art. 7-4, one brush, $B_{1}$, for example, is positive during the first half cycle and negative during the second half cycle, while the other brush $B_{2}$ is negative during the first half cycle and positive during the second. The simple a-c generator can be made a d-c generator by installing a rotary reversing switch in place of the two slip rings. Such a device, called a commutator, is used to reverse the connections automatically between the coil leads and the brushes.

Figure 7-11 shows a simple d-c generator with a single armature coil, a commutator, and two brushes. The commutator consists of two segments $K_{1}$ and $K_{2}$ that are assembled onto the shaft but insulated from it and also insulated from each other. The coil side $a a^{\prime}$ is connected to the commutator bar $K_{1}$, and coil side $b b^{\prime}$ is connected to bar $K_{2}$. The carbon brushes $B_{1}$ and $B_{2}$ are mounted in a stationary position and make a sliding or wiping contact with the commutator.

By Fleming's right-hand rule, it will be seen that the conductor moving downward under the $N$ pole will be positive because its induced emf is
outward in direction. This is true whether it is the conductor $a a^{\prime}$ or the conductor $b b^{\prime}$. As the brush $B_{1}$ is always making contact with that commutator bar which is connected to the conductor moving downward under


Fig. 7-11.—A simple d-c generator the $N$ pole, $B_{1}$ will always be the positive terminal of the generator. By similar, reasoning, it can be shown that $B_{2}$ will be negative.

The voltage of the $\mathrm{d}-\mathrm{c}$ generator coil will be exactly the same as for the a-c generator if the coil, poles, magnetic field, and speed are considered to be the same for each. However, since the commutator has been added in place of the slip rings, the voltage output of the generator will be unidirectional as shown in Fig. 7-12. This figure is similar to Fig. $7-5 b$ with its negative loop reversed.

Voltage of Single-coil and Multicoil Generators. The voltage of this single-coil generator is a pulsating voltage, and it does not approach the continuous current as is expected of a $d-c$ generator. If the generator is built with two coils as shown in Fig. 7-13a, it will result in an improved output voltage. The voltages of the two coils individually are shown as $C_{1}$ and $C_{2}$ in Fig. $7-13 b$. The voltage at the brushes is obtained by adding these two. For example, at point 1 the resultant volt-


Fig. 7-12.-Voltage of a single-coil d-c generator.
age is $0+100=100$ volts; at point 2 the resultant is $50+86.6=136.6$ volts; at point 3 the resultant is $70.7+70.7=141.4$ volts; at point 4 the resultant is $86.6+50=136.6$ volts; at point 5 the resultant is $100+0=$ 100 volts.

Figure 7-14 shows the voltages of four separate coils and also their resultant voltage. It becomes apparent from these illustrations that as
more coils are added the resultant voltage approaches a straight line and becomes similar to the continuous current.

Use of Direct Current. When this current is used for operating motors, lights, heaters, and other power devices, it is usually considered equivalent to the continuous current; but when used to operate sound devices such as radio equipment and public-address systems, it will not produce satisfac-


Fig. 7-13.-A two-coil d-c generator: (a) the generator, (b) voltage of the generator.
tory results. In these cases, the wave shape of the output voltage, referred to as commutator ripple, causes disturbing noises in sound equipment, and special filter circuits (see Chap. XII) must be used.

7-8. Commercial Generators. The simple generators discussed in Arts. 7-3, 7-4, and 7-7 are the fundamental machines; that is, the poles are only shown as bar magnets, the armatures have only single coils, and in general they contain only the bare necessities. This is done intentionally to make it easier to understand the theory of the gencrators.

In commercial machines, the simple poles are replaced by a welldesigned magnetic circuit as shown in Fig. 7-15. It consists of two or
more laminated pole pieces bolted to a frame. The frame in addition to supporting the bearings and armature also is a part of the magnetic circuit. In order to get a strong magnetic field, the bar magnets are replaced by


Fig. 7-14.- Voltage of a four-coil d-c generator.
electromagnets. The coils of the electromagnets are called the field coils and usually consist of a large number of turns of fairly small wire. The


Fig. 7-15.-The magnetic circuit of a commercial two-pole generator.
armature is constructed of a number of coils placed in the slots of a laminated iron core. The iron core provides a low reluctance path for the magnetic lines. The air gap between the armature and the pole pieces is kept
as small as possible to reduce the reluctance of the magnetic circuit. An air gap of approximately 0.015 inch is often found in small machines, while air gaps up to 0.125 inch may be found in the medium sizes. The method of constructing and mounting the slip rings or the commutator and the brush holders is also varied to provide adequate current-collecting methods.

Alternating-current generators, with the exception of very small sizes, are usually made with the field poles on the rotating unit and the armature


Fig. 7-16.-Disassembled view of an a-e generator. (General Electric Company.)
winding, now called the stator, as the stationary part. This requires fewer slip rings, provides a better magnetic circuit for the armature, permits higher speeds, and makes higher voltages possible. A disassembled view of an a-c generator of the rotating-field type showing its stator, rotating field unit, bearing brackets, and brush holders is given in Fig. 7-16. An a-c generator, whether of the rotating-field or rotating-armature type, requires direct current for its field circuit. In many installations, no d-c power line is available, and under these conditions it is common practice to mount a small d-c generator on the end of the alternator so that it may be operated from the same driving force. This d -c generator need only supply current to the alternator's field coils and is called the exciter. Its rating is usually less than five per cent of the alternator rating. Figure

7-17 shows an alternator with its own direct-connected exciter. Alternators are usually made three-phase to obtain maximum efficiency of the alternator and power system. Even though an alternator is a three-phase machine, it can efficiently supply power to single-phase circuits.


Fig. 7-17.-A $1250-\mathrm{kva}, 720 \mathrm{rpm}, 2400$-volt, 60-cycle, a-e generator with a direct-connected exciter. (Westinghouse Electric Corporation.)

Direct-current gencrators are made only of the stationary-field type. An internal view of a d-e generator is given in Fig. 7-18. The armature and commutator are constructed as shown in Fig. 7-19 and a complete generator in Fig. 7-20.

7-9. Transformers. Use of Transformers in Power Systems. Onc of the most important reasons for the greater use of alternating current over direct current is the ease with which the voltage may be raised or lowered by use of transformers. This makes it possible to generate power in large quantities at the source of energy such as a hydroelectric station. The
voltage may then be raised to transmission-line values as high as 300,000 volts and thereby efficiently transmit power to citie's several humdred miles


Fig. 7-18.-Internal view of a d-c generator. (Westinghouse Electric Corporation.)


Fig. 7-19.-The armature of a d-c generator. (Westinghouse Electric Corporation.)
from the generating station. At the outskirts of each city, a transformer substation is installed to reduce the voltage to reasonable amounts for distribution throughout the city, and it is then further stepped down by additional transformers for supplying consumers with power.

Principle of Operation. Electromagnetic induction is also the basis of operation of the transformer. Figure 7-21 illustrates the fundamental transformer consisting of a core and two windings called the primary and


Fig. 7-20.-A d-c generator. (Electro Dynamic Works.)


Fig. 7-21.- A fundamental transformer.
secondary windings. The core provides a path for the magnetic field and is generally built up of a large number of thin, high-grade sheet-steel laminations. The primary is the winding that receives the energy from
the line, and the secondary is the winding that delivers the energy to the load.

The theory of operation of the transformer is as follows:

1. When the primary is connected to a source of alternating emf, an alternating current is caused to flow in that winding.
2. Whenever a current flows in a conductor, a magnetic field is set up about the conductor. When the current is continually changing in magnitude and alternating in polarity, the magnetic field that it sets up in the iron core will do likewise.
3. The alternating magnetic field is therefore continually expanding and contracting. As the magnetic circuit is a closed path, the variation of the magnetic field is the same anywhere on the core.
4. The expanding and contracting magnetic lines will therefore cut conductors placed anywhere on the core, and according to Faraday's experiment, an induced emf will be set up in the conductors.
5. As the same flux cuts each conductor on the core, the induced emf per turn will be the same. Therefore the voltage of each winding will be proportional to the number of turns; expressed mathematically, this is

$$
\begin{equation*}
\frac{E_{P}}{E_{S}^{\prime}}=\frac{N_{P}}{\overline{N_{s}}} \tag{7-18}
\end{equation*}
$$

6. From this equation, it may be seen that the secondary voltage may be raised or lowered by using the proper ratio of turns.

Example 7-7. It is desired to have a radio power transformer step up the voltage from 110 volts to 750 volts. How many turns will be required on the secondary winding if the primary has 120 turns?

| Given: | Find: |
| :---: | :---: |
| $E_{P}=110$ | $N_{S}=?$ |
| $E_{S}=750$ |  |
| $N_{P}=120$ |  |

Solution:

$$
\begin{aligned}
& \frac{E_{P}}{E_{S}}=\frac{N_{P}}{N_{S}} \\
& \text { Therefore } N_{S}=N_{P} \frac{E_{S}}{E_{P}} \\
& =120 \times \frac{750}{110} \\
& =818 \text { turns }
\end{aligned}
$$

Operation of Transformer When Loaded. If the secondary is connected to a load, a current will flow through the load and also through the secondary winding. The power consumed by the load must come from the line; hence the primary load must vary in the same manner as the secondary.

Examination of Fig. 7-21 shows that there is no electrical connection between the primary and secondary windings. The power consumed by the load is transferred from the primary winding to the secondary winding by means of the magnetic flux. The efficiency of the transformer is very high, often above 95 per cent; hence the watts on the secondary side are


Fig. 7-22.-Commercial types of power transformers: (a) 0.5-kva lighting transformer, (b) 100-kva distribution transformer, (c) 2000-kva power transformer. (General Electric Company.)
nearly equal to the watts on the primary side. Under this condition, the currents vary inversely with the voltages. Mathematically, this is

$$
\begin{equation*}
\frac{E_{P}}{\bar{E}_{S}}=\frac{I_{S}}{\bar{I}_{P}} \quad \text { or } \quad E_{P} I_{P}=E_{S} I_{S} \tag{7-19}
\end{equation*}
$$

It can be seen from this equation that as the voltage is stepped up by a transformer the current is stepped down. This is a decided advantage in power-transmission systems.

Example 7-8. A certain city requires 5000 kilovolt-amperes for its lighting load. If the transmission line bringing the power to the city is operated at 132,000 volts, how much current will flow in the transmission lines?

| Given: | Find: |  |
| ---: | :--- | ---: |
| kva | $=5000$ | $I_{P}=?$ |
| $E_{P}$ | $=132,000$ |  |

Solution:

$$
\begin{aligned}
I_{P} & =\frac{\mathrm{kva} \times 1000}{E_{P}} \\
& =\frac{5000 \times 1000}{132,000} \\
& =37.8 \mathrm{amp}
\end{aligned}
$$

Example 7-9. A transformer substation is used to reduce the voltage to 4.400 volts for the power system of Example 7-8. What is the secondary current?

| Given: | Find: |
| ---: | ---: |
| $E_{P}=132,000$ | $I_{S}=?$ |
| $E_{S}=4400$ |  |
| $I_{P}=37.8$ |  |

Solution:

$$
\begin{aligned}
\frac{E_{P}}{E_{S}} & =\frac{I_{S}}{I_{P}} \\
I_{S} & =I_{P} \frac{E_{P}}{E_{S}} \\
& =37.8 \times \frac{132.000}{4400} 0 \\
& =1134 \mathrm{amp}
\end{aligned}
$$

Example 7-10. The power of Example 7-9 is passed through additional transformers to reduce the voltage from 4400 to 110 volts. What is the total current at 110 volts?

$$
\begin{aligned}
& \text { Given: } \\
& E_{P}=4400 \\
& E_{S}=110 \\
& I_{P}=1134
\end{aligned}
$$

Solution:

$$
\begin{aligned}
I_{S} & =I_{P} \frac{E_{P}}{E_{S}} \\
& =1134 \times \frac{4400}{110} \\
& =45,360 \mathrm{amp}
\end{aligned}
$$

It has been shown that the transformer operates because of the changing magnetic field. The transformer is therefore an a-c device and will not operate on direct current. It should be evident also that the frequency of the secondary circuit will be the same as that of the primary.

Commercial transformers will differ in construction from that shown in Fig. 7-21, which is merely a convenient form of illustration. Several commercial transformers are shown in Figs. 7-22 and 7-23.


Fig. 7-23.-Cutaway views of two types of radio power transformers. (Slandard Transformer Corporalion.)

7-10. Efficiency. Every generator, motor, transformer or other device that transforms energy from one form to another loses some of the energy in the process. If a gasoline engine is used to rotate a generator, the engine supplies mechanical energy to the shaft of the generator, and this energy is converted to the electrical energy supplied to the load. Only part of the mechanical energy will be transformed to electrical energy because of the friction, iron, and copper losses of the generator. Efficiency is the expression used to indicate what portion of the energy received by a device can be given out by it. Efficiency may be defined as the ratio of the output to input of any device; mathematically it is expressed as

$$
\begin{equation*}
\text { Efficiency }=\frac{\text { output }}{\text { input }} \tag{7-20}
\end{equation*}
$$

It is more commonly expressed in per cent, as

$$
\begin{equation*}
\text { Per cent efficiency }=\frac{\text { output }}{\text { input }} \times 100 \tag{7-20a}
\end{equation*}
$$

Example 7-11. A generator that is delivering 10 kilowatts to a load requires the gasoline engine driving it to supply 15 horsepower to the generator shaft. What is the per cent efficiency of the generator?

```
Given:
Output \(=10 \mathrm{kw}\)
        Input \(=15 \mathrm{hp}\)
Find:
    Per cent eff \(=\) ?
```

Solution:

$$
\begin{aligned}
\text { Per cent eff } & =\frac{\text { output }}{\text { input }} \times 100 \\
& =\frac{10 \times 1000}{15 \times 746} \times 100 \\
& =\frac{10,000}{11,190} \times 100 \\
& =89.3
\end{aligned}
$$

Note: To use the efficiency equation, both the output and input must be in the same kind of units. To satisfy this requirement, both the kilowatts and horsepower were converted to watts.

The efficiency of a transformer is the ratio of power output (secondary watts) to the power input (primary watts). As the transformer has no moving parts, there will be no friction loss, but there will be losses in the iron core and in the copper of the windings.

Example 7-12. $\Lambda$ transformer is used to supply the filament current to six 2.5 volt tubes. The tubes are connected in parallel, and each requires 2.5 amperes.

The voltage at the secondary of the transformer is 2.65 volts. What is the efficiency of the transformer if its primary draws 50 watts from the line?

```
Given:
\(E_{S}=2.65 \quad\) Per cent eff \(=\) ?
\(I_{S}=2.5 \times 6\)
\(W_{P}=50\)
```

Solution:

$$
\begin{aligned}
\text { Per cent eff } & =\frac{\text { watts output }}{\text { watts input }} \times 100 \\
& =\frac{2.65 \times 2.5 \times 6}{50} \times 100 \\
& =\frac{39.75}{50} \times 100 \\
& =79.5
\end{aligned}
$$

Wiring systems carrying current are actually being used to transport electrical energy from a source to a load. In doing this, some energy is lost by the wire. 'This loss appears in the form of heat which the wire gives off to the surrounding air, and it is equal to $I^{3} R$.

Example 7-13. The wires from the secondary of the transformer to the first tube of Example 7-12 have a resistance of 0.01 ohm . (a) What is the power lost in the wires? (b) What power is available at the tubes? (c) How efficiently is the wiring doing the job?

$$
\begin{aligned}
& \text { Given: } \\
& I=2.5 \times 6 \\
& R_{W}=0.01 \\
& P_{s}=39.75
\end{aligned}
$$

Find:
$P$ wire $=$ ?
$p$ tubes $=$ ?
Per cent eff $=$ ?

Solution:
(a)

$$
\begin{aligned}
P_{W} & =I^{2} R=(2.5 \times 6)^{2} \times 0.01 \\
& =2.25 \text { watts } \\
P_{T} & =P_{S}-P_{W} \\
& =39.75-2.25 \\
& =37.5 \text { wat ts } \\
\text { Per cent eff } & =\frac{P \text { at tubes }}{P \text { at secondary }} \times 100 \\
& =\frac{37.5}{39.75} \times 100 \\
& =94.3
\end{aligned}
$$

(b)
(c)

Efficiency of most electrical apparatus is high, and generally the larger the device the higher its efficiency. Large generators and motors have efficiencies around 90 per cent, and large transformers as used by power companies may have efficiencies of 98 per cent.

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## QUESTIONS

1. What kinds of current are used to operate radio receivers?
2. Name several applications where battery power is used to operate radio equipment.
3. Where is the d-c power system most likely to be found?
4. (a) Why are a-c systems used so extensively? (b) What percentage of power is generated as alternating current?
5. (a) With what fundamental principle of electricity is Faraday's name associated? (b) Describe a simple experiment that illustrates this principle.
6. State Fleming's right-hand rule.
7. What conditions are necessary in order to have a voltage induced in a conductor?
8. Describe the fundamental generator.
9. Under what conditions will the induced emf have a value of one volt?
10. To what factors is the induced emf proportional?
11. Describe the voltage induced in a conductor that is being rotated at a constant rate of speed through a uniform magnetic field.

Note: Consider this as a simple two-pole a-c generator.
12. Give a definition of an alternating current.
13. Define (a) alternation, (b) cycle, (c) frequency, (d) period.
14. Compare the values of frequencies used in power systems with those used in radio and television.
15. Define (a) maximum value, (b) instantaneous value, (c) average value, (d) effective value.
16. Describe a simple method of drawing a sine wave.
17. Describe the a-c ampere.
18. How does the construction of a simple d-c generator differ from that of a simple a-c generator?
19. (a) What is a commutator? (b) How is it constructed? (c) What is its purpose?
20. (a) What is the disadvantage of a single-coil generator? (b) How is this disadvantage overcome?
21. (a) What is meant by commutator ripple? (b) What effect does it have on the operation of sound-producing equipment? (c) What must be done to correct it?
22. Describe the field and frame construction of a commercial generator.
23. Describe the armature construction of a commercial generator.
24. (a) What is the location of the armature and field poles of the commercial a-c generator? (b) What are the advantages of this type of construction?
25. Why is an exciter required with an alternator?
26. How does the rating of the exciter compare with that of the alternator?
27. What is the importance of transformers to a-c systems?
28. What are the fundamental parts of the transformer?
29. (a) What is the purpose of the core? (b) How is it constructed?
30. Define (a) primary, (b) secondary.
31. Explain the principle of operation of the transformer.
32. Can a transformer be operated on direct current? Explain.
33. What is the relation of the primary voltage and turns to the secondary voltage and turns?
34. How does the voltage per turn on the secondary side compare with that on the primary side? Why?
35. How do the primary and secondary currents vary with the voltages?
36. (a) What is meant by efficiency? (b) How is it usually expressed?
37. Is the efficiency of most electrical apparatus high or low? Explain.

## PROBLEMS

1. A gencrator having 20 coils, each consisting of 24 turns, operates at a speed of 1200 rpm , and the flux per pole is 650,000 lines. What is the value of the induced emf?

Note: Each turn has two conductors; therefore $C=2 \times 24 \times 20$, or 960 .
2. $\Lambda$ generator having 15 coils of eight turns each is operated at a speed of 3600 rpm. What is the induced emf if the flux per pole is 765,000 maxwells?
3. How many turns per coil are required on each of the coils of a generator that is to have an emf of 125 volts if it has 15 coils? The flux per pole is $1,150,000$ maxwells, and the speed is 1800 rpm .
4. What would the voltage of the generator of Prob. 3 be if the speed is increased to 3600 rpm ?
5. What is the flux per pole of a generator that has 33 coils of eight turns each and that produces an emf of 115 volts when rotated at a speed of 1500 rpm ?
6. What is the frequency of a four-pole alternator operating at a speed of (a) 1800 rpm ? (b) 1500 rpm ? (c) 750 rpm ?
7. What is the frequency of a six-pole alternator operating at a speed of (a) 1200 rpm ? (b) 800 rpm ? (c) 500 rpm ?
8. At what speed must a two-pole alternator be driven in order to have a frequency of (a) 25 cycles? (b) 60 cycles? (c) 120 cycles?
9. How many poles must an alternator have if $i t$ is to produce 500 cycles when driven at (a) 1500 rpm ? (b) 3000 rpm ?
10. What is the period of an alternating voltage whose frequency is (a) 30 cycles? (b) 50 cycles? (c) 120 cycles?
11. What is the frequency of an alternating current whose period is (a) 0.04 sec ? (b) 0.0167 sec ? (c) 0.002 sec ?
12. An a-c sine-wave voltage has a maximum value of 250 volts. What is the instantaneous value at (a) $10^{\circ}$ ? (b) $41.5^{\circ}$ ? (c) $107.5^{\circ}$ ? (d) $311^{\circ}$ ? (e) $342.5^{\circ}$ ?
13. An a-c sine-wave current has a maximum value of 15 amp . What is the instantancous value of current at (a) $63^{\circ}$ ? (b) $218.5^{\circ}$ ? (c) $270^{\circ}$ ? (d) $290.5^{\circ}$ ? (e) $345^{\circ}$ ?
14. What is the maximum value of a sine-wave voltage whose value is 75 volts at $14.5^{\circ}$ ?
15. A sine-wave voltage has an instantaneous value of 106 volts at $32^{\circ}$. What is its value at $10.5^{\circ}$ ?
16. Draw one cycle of a sine-wave voltage whose maximum value is 300 volts. Use at least 36 points to draw the curve.
17. What is the average value of a voltage whose maximum is 250 volts?
18. Using the method illustrated in Fig. 7-9, find the average value of a sinewave voltage whose maximum is 300 volts. Use values $2^{\circ}$ apart. What is the ratio of the average to the maximum value?
19. What is the average value of a voltage whose instantaneous value is 97 volts at $29^{\circ}$ ?
20. What is the maximum value of a current whose average value is 7.5 amp ?
21. Using the method illustrated in Fig. 7-10, find the effective value of the sinewave of Prob. 18. What is the ratio of effective to maximum value?
22. What is the effective value of a current whose maximum value is 3.54 amp ?
23. What is the effective value of a voltage that has an instantancous value of 152 volts at $49.5^{\circ}$ ?
24. What is the effective value of a current that has an instantaneous value of -3.00 amp at $270^{\circ}$ ?
25. What is the effective value of a voltage that has an instantaneous value of -150 volts at $345^{\circ}$ ?
26. What is the effective value of a voltage whose a verage value is 200 volts?
27. A sine-wave voltage has an instantancous value of 145 volts at $133.5^{\circ}$. Find (a) the maximum value, (b) the average value, (c) the effective value.
28. Draw a diagram showing the voltage of a two-coil d-c generator.


Fig. 7-24. Each coil delivers a sine-wave voltage whose maximum value is 85 volts, and the individual coil voltages are 90 electrical degrees apart. What is the maximum and minimum value of the resultant voltage?
29. Repeat Prob. 28 for a threc-coil d-c generator. Each coil voltage has a maximum value of 60 volts and is $60^{\circ}$ from its adjacent voltage.
30. Repeat Prob. 28 for a four-coil d-c generator. Each coil voltage has a maximum value of 100 volts and is $45^{\circ}$ from its adjacent voltage.
31. A transformer is required to step up the voltage from 110 volts to 480 volts. How many turns are required on the secondary winding if the primary has 150 turns?
32. A transformer is required to step up the voltage from 120 volts to 1500 volts. How many turns are required on the secondary winding if the primary has 140 turns?
33. How many turns are required on the primary winding of a $120 / 1500$ volt step-up transformer if the secondary has 1200 turns?
34. The transformer shown in Fig. 7-24 has 192 turns on its primary winding. (a) How many turns are there on winding cd? (b) How many turns are there on winding ef? (c) How many turns are there on winding gi? (d) How many turns are there at $g h$ and $h i$ ?

35 A 110/700 volt step-up transformer has a current of 150 ma on the secondary side. What is the primary current, assuming the losses to be negligible?
36. A $120 / 1500$ volt step-up transformer has a current of 350 ma on the secondary side. What is the primary current, assuming the losses to be negligible?
37. A 4400/110 volt step-down transformer is used to supply a lighting load.
(a) If the primary current is 5 amp , what current will flow in the secondary assuming the losses to be negligible? (b) What is the kilovolt-ampere load on the transformer? (c) How many 100 -watt lamps can it supply with electrical energy?
38. What is the current in the primary winding of the transformer in Prob. 34 if the current in Sec. cd is 6 amp , in ef is 3.5 amp , and in $g i$ is $\mathbf{1 8 0 \mathrm { ma } \text { ? }}$
39. What is the efficiency of a transformer that draws 28 watts from the line when it delivers 9 amp at 2.5 volts?
40. What is the efficiency of a transformer similar to that of Prob. 34 if it takes 90 watts from the line when it supplies 5 amp at 2.5 volts, 3.3 amp at 6.3 volts, and 120 ma at 375 volts (center-tapped winding)?
41. A power transformer of a radio set has an efficiency of 78 per cent when it delivers a load of 40 watts. How much power does it draw from the line?
42. What is the power input of a transformer that has an efficiency of 83 per cent when its load is 65 watts?
43. How many watts will a transformer deliver at its secondary if it takes 75 watts from the line and its efficiency is 87.5 per cent?
44. What horsepower engine is required to drive a $3-\mathrm{kw}$ generator if the efficiency of the generator is 80 per cent?
45. A 400 -watt generator requires ${ }_{4}^{3} \mathrm{hp}$ from its gasoline-driven engine. What is the efficiency of the generator?

## CHAPTER VIII

## INDUCTANCE

The discoveries of Oersted and Faraday are among the most important in the entire fields of radio and clectronics. Understanding the principle of the magnetic field about a current-carrying conductor and that of electromagnetic induction led to the development of electrical machines. The development of radio circuits also was made possible by the same


Fig. 8-1.-Alternating current flowing in a coil.


Fig. 8-2.-Magnetic field set up when electrons are flowing outward at conductor $A$.
principles. In radio and television, these principles are employed in the functioning of inductance coils, or choke coils, as they are generally called, and of a-f, i-f, and r-f power transformers.

8-1. Inductance, Lenz's Law. Inductance. In Arts. 5-13 to 5-15 the effects of the magnetic field about a conductor and about a group of conductors in the form of a coil were studied. At that point, the study was based entirely on the effects produced by a current that was constant in amount. When the current is changing in amount, a new effect called inductance must be considered. Inductance is the property of a circuit that opposes any change in the amount of current.

When an alternating voltage is applied to the coil shown in Fig. 8-1, it will cause an alternating, and therefore continually changing, current to flow in the coil. If the conditions concerning conductor $A$ of Fig. 8-2
are considered, it will be seen that during the positive half cycle of the alternating voltage, electrons will be flowing outward at this conductor. During this half cycle, the current starts at zero, gradually increases to its maximum value and then gradually decreases to zero.

Circuit Reactions with Increasing Field. Considering first the increasing values occurring from 0 to 90 degrees, it follows that the magnetic field too will be increasing in strength (Art. 5-14). Reviewing the four rules concerning magnetic lines (Art. 5-8), it is evident that as the current

(a)

(b)

Fig. 8-3.--Voltage induced in conductor $B$ when the current in conductor $A$ is increasing in strength: (a) direction of flow of electrons in conductor $A$, (b) left hand used to determine the direction of the induced emf.
is increasing the magnetic lines about conductor $A$ will be expanding and in doing so they will cut conductor $B$, which is adjacent to it. Faraday's experiment has shown that whenever there is motion between a conductor and magnetic lines an emf will be induced in that conductor. Conductor $B$ will therefore have an emf induced in it. The direction of this induced emf may be determined by the left-hand rule as illustrated in Fig. $8-3$; it is found to be coming out at conductor $B$. This induced cmf is in the opposite direction of the impressed voltage. It reduces the effect of the impressed voltage to push current through the coil. The more rapid the change in the amount of current, the greater this emf will be and the greater is the opposition to the change in current. In general, then, the induced emf opposes any increase in the amount of current. The value of the emf is equal to the product of the number of turns of the coil and the magnetic flux divided by the time in seconds necessary for the flux to change from its maximum value to zero. Expressed mathematically

$$
\begin{equation*}
e=\frac{N \phi}{t \times 10^{8}} \tag{8-1}
\end{equation*}
$$

where $e=$ induced voltage
$N=$ number of turns
$\phi=$ number of lines linking the coil
$t=$ time, seconds
Example 8-1. A flux of $1,800,000$ lines links a coil having 300 turns. The flux in the coil decreases from maximum value to zero in 0.18 second. What is the value of the induced voltage?

| Given: | Find: |  |
| ---: | ---: | ---: |
| $N$ | $=300$ | $e=?$ |
| $\phi$ | $=1,800,000$ |  |
| $t$ | $=0.18$ |  |

Solution:

$$
\begin{aligned}
e & =\frac{N \phi}{t \times 10^{8}} \\
& =\frac{300 \times 1,800,000}{0.18 \times 10^{8}} \\
& =\frac{3 \times 1.8}{0.18} \\
& =30 \mathrm{volts}
\end{aligned}
$$

From the above example, it can be seen that the voltage induced in a coil is proportional to the number of turns in that coil and the rate of change of flux.

Circuit Reactions with a Decreasing Field. In considering the valucs from 90 to 180 degrees, it will be scen that the impressed voltage is decreasing and that the current in conductor $A$ will then also decrease. According to the rules concerning magnetic lines, these lines will now be collapsing, and in doing so they will again cut conductor $B$. The direction of motion of the magnetic lines has been reversed, and by applying the left-hand rule the direction of the induced emf will be inward at conductor $B$ as is illustrated in Fig. 8-4. This induced emf is now in the same direction as the impressed voltage, and it will aid the line voltage in pushing the current through the coil, thereby opposing the decrease in current. The more rapid the decrease in current, the greater the induced emf, and the greater will be its effort to oppose the change in current. In general, then, the induced emf opposes any decrease in the amount of current.

Combining these two explanations, it can be seen that when the current increases the induced emf is in such a direction that it opposes the
increase, and that when the current decreases the induced emf is in such a direction that it opposes the decrease. This effect is called inductance and conforms to the definition that inductance is the property of a circuit which opposes any change in the amount of current. It should be noticed that change is a very important word in this definition.

Lenz's Law. It will be noticed that the induced emf in any case tends to oppose any change in the amount of current. This is commonly referred to as Lenz's law. This law has been stated in various forms by


Fig. 8-4.-Voltage induced in conductor $B$ when the current in conductor $A$ is decreasing in strength: (a) direction of flow of electrons in conductor $A$, (b) left hand used to determine the direction of the induced cmf .
different texts; the one best serving our purpose may be stated as follows: When the current in a circuit is increasing, the induced emf opposes the applied voltage and tends to keep the current from increasing; and when the current is decreasing, the induced emf aids the line voltage and tends to keep the current from decreasing. Another way of stating Lenz's law is: When a current flowing through a circuit is varying in magnitude, it produces a varying magnetic field which sets up an induced emf that opposes the current change producing it.

Presence of Inductance. The above discussion was based upon what took place at conductor $A$ during the positive half cycle. Applying the same reasoning to conductor $A$ for the negative half cycle, it can be shown that a similar effect takes place. Applying this reasoning to any other conductor, it will be seen that the same conditions exist at all conductors.

It should now be evident that inductance is effective only when there is a changing current present in a circuit. With d-c circuits, this condition usually exists only at the instant of time when a circuit is closed or opened by means of a switch; therefore, the inductance in such circuits is
generally disregarded. In a-e circuits, the current is continually changing and inductance is always present, and its effect must be considered in all $a-c$ circuits.

8-2. Self-inductance. Self-inductance is defined as the property of a single circuit that opposes any change in the amount of current in that circuit. The preceding article actually presented an explanation of selfinductance. The discussion in the following paragraphs will present the unit of inductance and the factors affecting the value of inductance.

Unit of Inductance. The unit of inductance is the henry; it was named in honor of an early American scientist, Joseph Henry. A circuit has a self-inductance of one henry when a current changing at the rate of one ampere per second induces an average of one volt. The symbol used to represent inductance is the capital letter $L$.

Factors Affecting the Inductance of a Coil. While practically all circuits are likely to have some inductance, it is common to think of inductance only in terms of a coil. When a coil is used expressly for its property of inductance, it is called an inductor. The self-inductance of a coil depends upon its physical characteristics, that is, its dimensions, number of turns, and the magnetic qualities of its core. Because it is sometimes difficult accurately to predict the magnetic conditions in a circuit, several equations are used to express the relations of the factors affecting the inductance of a coil.

When the length of a coil is several times its diameter, it is called a solenoid. For a solenoid whose length is at least 10 times its diameter, the inductance may be calculated by the equation

$$
\begin{equation*}
L=\frac{1.26 N^{2} \mu A}{10^{8} l} \tag{8-2}
\end{equation*}
$$

where $L=$ inductance of the coil, henries
$N=$ number of turns
$\mu=$ permeability of the core
$A=$ area of the core, square centimeters
$l=$ length of the core, centimeters
Example 8-2. What is the inductance of a tuning coil that has 300 turns wound on a cardboard tubing 4 centimeters in diameter and 40 centimeters long?

Note: As cardboard is nonmagnetic, $\mu=1$.

| Given: | Find: |
| ---: | :--- |
| $N$ | $=300$ |
| $\mu$ | $=1$ |
| $d$ | $=4 \mathrm{~cm}$ |
| $l$ | $=40 \mathrm{~cm}$ |

Solution:

$$
\begin{aligned}
A & =\frac{\pi d^{2}}{4}=\frac{3.14 \times 4 \times 4}{4}=12.56 \mathrm{sq} \mathrm{~cm} \\
L & =\frac{1.26 N^{2} \mu A}{10^{8} l} \\
& =\frac{1.26 \times 300 \times 300 \times 1 \times 12.56}{10^{8} \times 40} \\
& =0.000356 \mathrm{heary} \\
& =356 \mu \mathrm{~h}
\end{aligned}
$$

Example 8-3. What inductance would the coil of Example 8-2 have if it was wound on an iron core whose permeability was 4000 ?

$$
\begin{array}{rlrl}
\text { Given }: & \text { Find: } \\
N & =300 & L & =? \\
\mu & =4000 & \\
A & =12.56 & \\
l & =40 &
\end{array}
$$

Solution:

$$
\begin{aligned}
L & =\frac{1.26 N^{2} \mu A}{10^{8} l} \\
& =\frac{1.26 \times 300 \times 300 \times 4000 \times 12.56}{10^{8} \times 40} \\
& =1.424 \text { henries }
\end{aligned}
$$

Example 8-4. What inductance would the coil of Example $8-3$ have if there were 900 turns of wire on the coil?

$$
\begin{aligned}
& \text { Given: } \\
& N=900 \\
& \mu=4000 \\
& A=12.56 \\
& l=40
\end{aligned}
$$

Find:

Solution:

$$
\begin{aligned}
L & =\frac{1.26 N^{2} \mu A}{10^{8} l} \\
& =\frac{1.26 \times 900 \times 900 \times 4000 \times 12.56}{10^{8} \times 40} \\
& =12.81 \text { henries }
\end{aligned}
$$

The above examples indicate both that the inductance can be increased considerably by winding the coil on a ferromagnetic core and that it also increases as the square of the number of turns. Changing the turns from 300 to 900 or making their number three times as great increases the inductance by $3 \times 3$, or nine times.

Multilayer, Pancake, Solenoid Coils. Equation (8-2) was used because it illustrates by the simplest mathematics how the various factors affect the inductance of a coil. The dimensions of coils used in radio circuits are such that they seldom conform to the conditions of this equation. In


Fig. 8-5.-Types of inductance coils: (a) multilayer, (b) flat or pancake, (c) solenoid.
most cases it becomes necessary to resort to one of the three following equations. Figure 8-5 shows three classifications of coil shapes for which the three equations are used. Figure $8-5 a$ is called a multilayer coil, and its inductance may be found by use of the equation

$$
\begin{equation*}
L=\frac{0.8 a^{2} N^{2}}{6 a+9 b+10 c} \tag{8-3}
\end{equation*}
$$

where $L=$ inductance of coil, microhenries
$N=$ number of turns
$a, b, c=$ dimensions, inches (Fig. 8-5a)
Figure 8-5b shows a flat or pancake coil whose inductance may be calculated by the equation

$$
\begin{equation*}
L=\frac{a^{2} N^{2}}{8 a+11 c} \tag{8-4}
\end{equation*}
$$

where $L=$ inductance of coil, microhenries
$N=$ number of turns
$a, c=$ dimensions, inches (Fig. 8-5b)
Figure 8-5c shows a form of solenoid in which the length does not exceed its diameter by any great amount. The inductance of such a coil may be found by the equation

$$
\begin{equation*}
L=\frac{a^{2} N^{2}}{9 a}+10 b \tag{8-5}
\end{equation*}
$$

where $L_{L}=$ inductance of coil, microhenries
$N=$ number of turns
$a, b=$ dimensions, inches (Fig. 8-5c)
Example 8-5. What is the inductance of a multilayer coil that has 1200 turns and whose dimensions are $a=1 \frac{1}{2}, b=\frac{3}{4}$, and $c=1 \frac{1}{2}$ inches?

$$
\begin{array}{rlr}
\text { Given: } & \text { Find: } \\
N & =1200 & L=? \\
a & =1.5 & \\
b & =0.75 & \\
c & =1.5 &
\end{array}
$$

Solution:

$$
\begin{aligned}
L & =\frac{0.8 a^{2} N^{2}}{6 a+9 b+10 c} \\
& =\frac{0.8 \times 1.5 \times 1.5 \times 1200 \times 1200}{6 \times 1.5+9 \times 0.75+10 \times 1.5} \\
& =\frac{2.592,000}{30.75} \\
& =84,292 \mu h \\
& =84.292 \mathrm{mh}
\end{aligned}
$$

The above equations will give reasonably accurate results for coils without iron cores such as may be found in r-f circuits. When iron cores are used, more accurate results can be obtained by securing readings of volts, amperes, and either watts or resistance, as will be explained later. Numerous tables and charts have been prepared to facilitate calculations of inductance or number of turns by short-cut methods. Some of these tables or charts may be found in the references listed in the bibliography at the end of this chapter.

8-3. Inductive Reactance, Angle of Lag. It has been shown that inductance is the property of a circuit which opposes any change in the amount of current flowing in a circuit. The effects of inductance in an a-c circuit are twofold; namely, it sets up an opposition to the flow of current, and it causes a delay or lag in the current.

Inductive Reactance. The study of inductance has shown that the changing magnetic field induces a voltage in such a direction that it opposes any change in the amount of current. This results in the current being lower than if inductance were not present, and inductances must therefore introduce an opposition to the flow of current. This opposition is called inductive reactance and is expressed in ohms; its symbol is $X_{L}$. The value of the inductive reactance is affected by two factors, one being
the inductance of the circuit and the other the rate or speed at which the current is changing. The inductance of a circuit depends on the physical characteristics of the circuit and has been explained in Art. 8-2. The rate of speed at which the current is changing is directly proportional to the frequency of the power supply to which it is connected. The effect of these two factors results in the equation

$$
\begin{equation*}
X_{L}=2 \pi f L \tag{8-6}
\end{equation*}
$$

where $X_{L}=$ inductive reactance, ohms
$f=$ frequency, cycles per second
$L=$ inductance, henries
If a circuit is assumed to consist of inductance only, the amount of current flowing in such a circuit would be equal to its voltage divided by the inductive reactance, or

$$
\begin{equation*}
I_{L}=\frac{E_{L}}{X_{L}} \tag{8-7}
\end{equation*}
$$

Example 8-6. The choke coil of a filter circuit has an inductance of 30 henries. (a) What is its inductive reactance on a 60 -cycle circuit? (b) What current will flow when the voltage across the coil is 250 volts?

| Given: | Find: |
| :--- | ---: |
| $L=30$ | $X_{L}=?$ |
| $f=60$ | $I_{L}=?$ |
| $E=250$ |  |

Solution:
(a)
(b)

$$
\begin{aligned}
X_{L} & =2 \pi f L \\
& =2 \times 3.14 \times 60 \times 30 \\
& =11,304 \mathrm{ohms} \\
I_{L} & =E_{Z} \\
& =\frac{250}{11,304} \\
& =22.1 \mathrm{ma}
\end{aligned}
$$

Example 8-7. The primary of an r-f transformer has an inductance of $350 \mu \mathrm{~h}$. (a) What is its inductive reactance at 1200 kc ? (b) What current will flow when the voltage across the primary is 10 volts?

$$
\begin{array}{rrr}
\text { Given : } & \text { Find: } \\
L=350 \times 10^{-6} & X_{L} & =? \\
f & =1200 \times 10^{3} & I_{L}
\end{array}=?
$$

$$
E=10
$$

Solution:
(a)
(b)

$$
\begin{aligned}
X_{L} & =2 \pi f L \\
& =2 \times 3.14 \times 1200 \times 10^{3} \times 350 \times 10^{-1} \\
& =2637.6 \mathrm{ohms}
\end{aligned}
$$

$$
I_{L}=\frac{E_{L}}{X_{L}}
$$

$$
=\frac{10}{2637.6}
$$

$$
=3.79 \mathrm{ma}
$$

Effect of the Resistance of an Inductive Circuit. In actual practice it is impossible to have a circuit containing only inductance because the wire of which the inductor is wound has some resistance. The resistance is usually so small compared with the inductive reactance that it is ignored and the circuit is assumed to contain inductance only.

If the resistance is to be considered, its ohmic effect must be combined with the ohmic effect of the inductive reactance. The combined ohmic effect is called the impedance and is represented by the symbol $Z$. Mathematically it is equal to

$$
\begin{equation*}
Z=\sqrt{R^{2}+X_{L}{ }^{2}} \tag{8-8}
\end{equation*}
$$

where $Z=$ impedance of the circuit, ohms
$R=$ resistance of the circuit, ohms
$X_{L}=$ inductive reactance of the circuit, ohms
When both the resistance and the inductive reactance of a circuit are taken into consideration, the current flowing in the circuit will be equal to the voltage of the circuit divided by its impedance, or

$$
\begin{equation*}
I=\frac{E}{Z} \tag{8-9}
\end{equation*}
$$

where $I=$ current flowing in the circuit, amperes
$E=$ voltage of the circuit, volts
$Z=$ impedance of the circuit, ohms
Example 8-8. If the coil of Example 8-6 has a resistance of 400 ohms , (a) what is the impedance of the coil? (b) What current will flow when the voltage across the coil is 250 volts?

$$
\begin{aligned}
\text { Given } & = \\
X_{L} & =11,304 \\
R & =400 \\
E & =250
\end{aligned}
$$

```
Find:
    Z= ?
    I= ?
```

Solution:
(a)
(b)

$$
\begin{aligned}
Z & =\sqrt{R^{2}+X_{L^{2}}} \\
& =\sqrt{400^{2}+11,304^{2}} \\
& =\sqrt{160,000+127,780,416} \\
& =\sqrt{127,940,416} \\
& =11,311 \text { ohms }
\end{aligned}
$$

$$
\begin{aligned}
I & =\frac{E}{Z} \\
& =\frac{250}{11,311}=22.1 \mathrm{ma}
\end{aligned}
$$

Comparison of the results of Examples 8-6 and 8-8 shows that the impedance and inductive reactance are practically equal and that the current


Fig. 8-6.-Time required for the current to build up in a circuit containing only resistance: (a) the circuit diagram, (b) a graph showing the relation between current and time.
too is practically the same whether the resistance is considered or neglected. This is always the case when the inductive reactance is ten (or more) times greater than the resistance.

Time Constant. That inductance causes a delay or lag in the current is shown in the following manner. If a length of wire is arranged so that it has no inductance (for example, kept straight so that at no point will two sections of the wire be near each other), its only effect will be that of resistance. If the length of the wire is such that it has a resistance of 12 ohms and it is connected through a switch and ammeter to a six-volt battery (Fig. $8-6 a$ ), a current of $\frac{6}{12}$, or 0.5 , ampere will flow when the switch
is closed. Furthermore, because the circuit contains resistance only, the current will reach this value practically instantly as indicated by the graph of Fig. 8-6b.

If the piece of wire, or an identical piece, is wound around a cylindrical form, it will have inductance as well as its resistance of 12 ohms . If the coil is connected to a circuit as in Fig. 8-7a, the current upon closing the switch will become $\frac{6}{12}$, or 0.5 , ampere. The current however will not



Fig. 8-7.-Time required for the current to build up in a circuit containing resistance and inductance: ( $a$ ) the circuit diagram, ( $b$ ) a graph showing the relation between current and time.
attain this value instantly because of the inductance now present in the circuit. The amount of time required for the current to reach its final value depends upon the relative amount of inductance and resistance in the circuit. If the inductance of this circuit is 0.36 henry, the time required for the current to build up is that which is shown in Fig. 8-7b. The ratio of inductance to resistance is called the time constant and represents the time in seconds required for the current to build up to 63.2 per cent of its final value. Mathematically it is expressed

$$
\begin{equation*}
t=\frac{L}{\bar{R}} \tag{8-10}
\end{equation*}
$$

where $t=$ time in seconds for current to reach 63.2 per cent of its final value
$L=$ inductance of the circuit, henries
$R=$ resistance of the circuit, ohms

Example 8-9. What time is required for the current in the circuit of Fig. 8-7a to build up to 63.2 per cent of its final value?
Given:

$$
\begin{aligned}
L & =0.36 \\
R & =12
\end{aligned}
$$

Find:
$t=$ ?

Solution:

$$
\begin{aligned}
t & =\frac{L}{R} \\
& =\frac{0.36}{12} \\
& =0.03 \mathrm{sec}
\end{aligned}
$$

Angle of Lag. If the battery in Fig. 8-7a is replaced with a source of alternating current, the effect of the inductance in the circuit will cause the current to lag continually behind the voltage. The amount of lag is dependent upon the relative amount of inductance and resistance in the circuit and is generally expressed in electrical degrees instead of time in seconds. It is determined mathematically by equation

$$
\begin{equation*}
\cos \theta=\frac{R}{Z}=\frac{R}{\sqrt{R^{2}+X_{L}^{2}}} \tag{8-11}
\end{equation*}
$$

Example 8-10. If the circuit of Fig. 8-7a is connected to a 60-cycle six-volt a-c power supply, find (a) the inductive reactance, (b) the impedance, (c) the angle of current lag, (d) the current.

## Given:

$$
\begin{aligned}
R & =12 \\
L & =0.36 \\
E^{\prime} & =6 \\
f & =60
\end{aligned}
$$

Solution:
(a)
(b)

$$
\begin{aligned}
X_{L} & =2 \pi f L \\
& =2 \times 3.14 \times 60 \times 0.36 \\
& =135.6 \mathrm{ohms} \\
Z & =\sqrt{\overline{R^{2}}+X_{L^{2}}} \\
& =\sqrt{12^{2}+135.6^{2}} \\
& =\sqrt{144+18,387} \\
& =136.1 \mathrm{ohms}
\end{aligned}
$$

(c)

$$
\begin{aligned}
\cos \theta & =\frac{R}{Z} \\
& =\frac{12}{136.1} \\
& =0.0881 \\
\theta & =85^{\circ} \text { (from Appendix XI) }
\end{aligned}
$$

Find:
$\begin{aligned} X_{L} & =\text { ? } \\ Z & =? \\ 0 & =? \\ I & =?\end{aligned}$
(d)

$$
\begin{aligned}
I & =\frac{F}{Z} \\
& =\frac{6}{136.1} \\
& =0.0440 \mathrm{amp}
\end{aligned}
$$

By examining Eq. (8-11), it can be seen that if it were possible to build an inductance without any resistance the value of $\cos \theta$ would be zero and the angle by which the current would lag the voltage would be 90 degrees. It may also be seen that if no inductance is present the value of $\cos \theta$ would be 1 and the angle would be 0 degrees. In actual practice, inductance coils cannot be built without resistance, therefore, a 90 -degrec angle of lag cannot be obtained. Practical inductances often achieve angles up to 80 or 85 degrees.

The angle between the current and voltage may be illustrated by sine waves as in Fig. 10-2 or by vector diagrams as in Fig. 10-11a. The vector method of representation is the one most commonly used.

8-4. Mutual Inductance. When two windings are placed so that a change of current in one will cause its changing magnetic field to cut the turns of the other, an induced emf will be set up in the second coil. The two circuits are then said to possess mutual inductance.

Principle of Mutual Inductance. This may be demonstrated by the circuits shown in Fig. 8-8. A coil $A A^{\prime}$ is formed by winding a number of turns on a core of cardboard tubing. A second piece of cardboard tubing, one whose inside diameter is slightly larger than the outside diameter of the completed coil, is then placed over the coil to act as an insulator and as a form for the second winding $B B^{\prime}$. The coil $B B^{\prime}$ is then wound over this tubing. If the winding $A A^{\prime}$ is connected to a source of alternating current and if a voltmeter is connected to the winding $B B^{\prime}$, the voltmeter will indicate that a voltage is being induced in the winding $B B^{\prime}$ by the expanding and contracting magnetic field caused by the current flowing in the winding $A A^{\prime}$.

Calculation of Mutual Inductance. If the circuit $B B^{\prime}$ is connected to a load, it will cause a current to flow in the winding $B B^{\prime}$ and set up a magnetic field of its own. Each circuit can, and under such condition will, have an inductance of its own. The inductance of each may be calculated separately by a suitable equation from the group (8-2) to (8-5). The winding that receives the energy from the power line is called the primary winding, and its self-inductance is usually designated as $L_{1}$. The other winding is called the secondary, and its self-inductance is designated as $L_{2}$.

Unit of Mutual Inductance. When the current changes in one circuit
as $A A^{\prime}$, at the rate of one ampere per second and it induces an average emf of one volt in the second circuit, as $B B^{\prime}$, the two circuits have a mutual inductance of one henry. If the self-inductances of the two coils are known and it is assumed that all the magnetic lines set up by the first coil cut all


Fig. 8-8.-Principle of mutual inductance.
the turns of the sccond coil, the mutual inductance may be found by the equation

$$
\begin{equation*}
M=\sqrt{\overline{L_{1}} \times L_{2}} \tag{8-12}
\end{equation*}
$$

where $M=$ mutual inductance of the coils, henries
$L_{1}=$ self-inductance of first coil, henries
$L_{2}=$ self-inductance of second coil, henries
Example 8-11. What is the mutual inductance of two coils wound adjacent to one another? (Assume that all the magnetic lines set up in the first coil cut all the turns of the second coil.) The primary coil consists of 2000 turns wound on a cardboard core two inches in diameter and four inches long. The secondary coil consists of 4000 turns wound on a cardboard core $2 \frac{1}{4}$ inches in diameter and $3 \frac{1}{2}$ inches long.

| Given: | Find: |  |
| ---: | :--- | ---: |
| $N_{1}$ | $=2000$ | $M=?$ |
| $a_{1}$ | $=1$ |  |
| $b_{1}$ | $=4$ |  |
| $N_{2}$ | $=4000$ |  |
| $a_{2}$ | $=1 \frac{1}{8}$ |  |
| $b_{2}$ | $=3 \frac{1}{2}$ |  |

Solution:
Note: The coils meet conditions of Fig. 8-5c; we therefore use Eq. (8-5).

$$
\begin{aligned}
L_{1} & =\frac{a_{1}{ }^{2} N_{1}{ }^{2}}{9 a_{1}+10 b_{1}} \\
& =\frac{1 \times 1 \times 2000 \times 2000}{9 \times 1+10 \times 4} \\
& =\frac{4,000,000}{49} \\
& =81,632 \mu \mathrm{~h} \\
& =81.6 \mathrm{mh} \\
L_{2} & =\frac{a_{2}{ }^{2} N_{2}{ }^{2}}{9 a_{2}+10 b_{2}} \\
& =\frac{1.125 \times 1.125 \times 4000 \times 4000}{9 \times 1.125+10 \times 3.5} \\
& =\frac{20,250,000}{45.125} \\
& =448,753 \mu \mathrm{~h} \\
& =448.75 \mathrm{mh} \\
M & =\sqrt{L_{1} \times L_{2}} \\
& =\sqrt{81.6 \times 448.75} \\
& =\sqrt{36,618} \\
& =191.3 \mathrm{mh}
\end{aligned}
$$

8-5. Coefficient of Coupling. Calculation of Coefficient of Coupling. When two circuits are arranged so that energy from one circuit may be transferred to the other, the circuits are said to be coupled. Mutual inductance is an example of coupled circuits.

In the case of mutual inductance, if all the magnetic lines set up by the current in the first circuit cut all the turns of the second circuit, the circuits are coupled perfectly. If only half the lines set up in the first circuit cut the turns of the second circuit, the coupling is only 50 per cent. The percentage of coupling is usually referred to as the coefficient of coupling and is designated by the letter $K$. It is expressed mathematically by the equation

$$
\begin{equation*}
K=\frac{M}{\sqrt{L_{1} \times L_{2}}} \tag{8-13}
\end{equation*}
$$

where $K=$ the coefficient of coupling (expressed as a decimal)
$M=$ the mutual inductance of the two circuits
$L_{1}=$ the self-inductance of the first coil
$L_{2}=$ the self-inductance of the second coil

Example 8-12. What is the coefficient of coupling of two coils whose mutual inductance is 1.0 henry and whose self-inductances are 1.2 and 2.0 henries?

$$
\begin{array}{rr}
\text { Given: } & \text { Find: } \\
M=1.0 & K=? \\
L_{1}=1.2 & \\
L_{2}=2.0 &
\end{array}
$$

Solution:

$$
\begin{aligned}
K & =\frac{M}{\sqrt{L_{1} L_{2}}} \\
& =\frac{1.0}{\sqrt{1.2 \times 2.0}} \\
& =\frac{1.0}{\sqrt{2.4}} \\
& =\frac{1.0}{1.55} \\
& =0.645
\end{aligned}
$$

The coefficient of coupling depends upon the construction of the coils and also largely upon whether the coils are wound on an iron core or on an air core. The highest possible value is one and the lowest is zero. The power transformers described in Art. 7-9 often achieve the high value of 0.98 , which is considered excellent. The coefficient of coupling for aircore transformers used in radio circuits is very low and will vary considerably depending upon the design of the coils and the frequency of the circuits in which they are used. In radio circuits, a low value of coefficient of coupling is often desired as it aids sharpness of tuning.

Calculation of Voltage Induced in the Secondary. The voltage induced in the second circuit by a change of current in the first may be expressed by the following equation:

$$
\begin{equation*}
e_{2}=\frac{N_{2} K \phi}{t \times 10^{8}} \tag{8-14}
\end{equation*}
$$

where $e_{2}=$ voltage induced in second circuit
$N_{2}=$ turns on coil in the second circuit
$K=$ coefficient of coupling
$\phi=$ flux set up by current in first circuit
$t=$ time in seconds for current in first circuit to change from maximum value to zero, or vice versa

Example 8-13. Two coils, the first having 200 turns and the second 350 turns, are placed so that only 40 per cent of the lines set up by coil 1 link coil 2 . If 600,000
lines are set up when three amperes flow through coil 1 , what voltage will be induced in coil 2 if the current decreases from three amperes to zero in 0.10 second?

```
Given:
\(N_{2}=350\)
\(K=0.40\)
\[
\phi=600,000
\]
\[
t=0.10
\]
```

Solution:

$$
\begin{aligned}
e_{2} & =\frac{N_{2} K \phi}{t \times 10^{8}} \\
& =\frac{350 \times 0.40 \times 600,000}{0.10 \times 10^{8}} \\
& =8.40 \mathrm{volts}
\end{aligned}
$$

During one stage of the development of radio, it was common practice to tune a radio receiver by means of a variable inductor. One such device, shown in Fig. 8-9, had its inductance varied by changing the number of


Fig. 8-9.-A variable inductor.
turns used by means of the sliding contactor. Another device, called the variocoupler or variometer, shown in Fig. 8-10c, varied its inductance by changing the position of one coil with respect to another, which, in effect, was really varying the coefficient of coupling.

8-6. Series and Parallel Inductances. Basic Series and Parallel Connections. Circuits often contain several inductances connected in series or in parallel. When the separate inductors are located far enough apart so that there is no coupling between them, the inductances may be added in the same manner in which resistances are added. The inductance of a circuit containing several inductors connected in serics and far enough apart so that no coupling exists between them will be

$$
\begin{equation*}
L_{T}=L_{1}+L_{2}+L_{3}, \text { etc. } \tag{8-15}
\end{equation*}
$$

The inductance of a circuit containing several inductors connected in parallel but far enough apart so that no coupling exists between them will be

$$
\begin{equation*}
L_{T}=\frac{1}{\frac{1}{\bar{L}_{1}}+\frac{1}{\bar{L}_{2}}+\frac{1}{\bar{L}_{3}}, \text { etc. }} \tag{8-16}
\end{equation*}
$$

Effect of Inductance on Series-connected Inductors. When two coils are located close enough to each other so that there will be coupling be-


Fig. 8-10.-Two coupled inductors connected in series.
tween them, the total inductance of the two coils when connected in series must be found in a different manner. The value of the total inductance will depend on how the coils are connected, that is, whether they are aiding or opposing and also upon their mutual inductance and coupling. Figure $8-10$ shows three conditions that may exist. In Fig. 8-10a, two coils are
connected in series so that their magnetic fields aid each other; the inductance of the circuit will then be

$$
\begin{equation*}
L=L_{1}+L_{2}+2 K \sqrt{\overline{L_{1} L_{2}}} \tag{8-17}
\end{equation*}
$$

where $L=$ inductance of the circuit
$L_{1}=$ inductance of the first coil
$L_{2}=$ inductance of the second coil
$K=$ coefficient of coupling
In Fig. 8-10b, the two coils are connected in series so that their magnetic fields oppose each other; the inductance of the circuit will then be

$$
\begin{equation*}
L=L_{1}+L_{2}-2 K \sqrt{\overline{L_{1} L_{2}}} \tag{8-18}
\end{equation*}
$$

In Fig. 8-10c, the second coil is considered as being mounted on a shaft so that it may be rotated. If it can be rotated through 180 degrees, it may be changed from aiding to opposing or it may also be stopped at any point in between. The inductance of the circuit would be

$$
\begin{equation*}
L=L_{1}+L_{2} \pm 2 K \sqrt{\overline{L_{1} L_{2}}} \tag{8-19}
\end{equation*}
$$

If two similar coils are used for Fig. 8-10c and perfect coupling could be achieved, the inductance could be varied between zero and $4 L_{1}$. This is true because when the coefficient of coupling is unity the value of $\sqrt{L_{1} L_{2}}$ is equal to $L_{1}$ (or $L_{2}$ since they are equal). When the two coils are aiding,

$$
\begin{equation*}
L=L_{1}+L_{2}+2 K \sqrt{\overline{L_{1} L_{2}}}=4 L_{1} \tag{8-20}
\end{equation*}
$$

and when the two coils are opposing,

$$
\begin{equation*}
L=L_{1}+L_{2}-2 K \sqrt{\overline{L_{1} L_{2}}}=0 \tag{8-21}
\end{equation*}
$$

When the coils are at a 90 -degree angle, the value of $\sqrt{L_{1} L_{2}}$ is zero and $L=L_{1}+L_{2} \pm 0=2 L_{1}$. This is the basis of the variometer and variocoupler shown in Fig. 8-10c.

Example 8-14. Two coils, each with an inductance of four henries, are arranged so that they may be connected in series in the various ways shown in Fig. 8-10. What is the inductance of the circuit when the two coils are connected in series so that they are (a) aiding and the coupling is 100 per cent? (b) Opposing and the coupling is 100 per cent? (c) In a position that produces zero coupling? (d) Aiding and the coupling is 50 per cent?

Given:

$$
\begin{aligned}
& L_{1}=4 \\
& L_{2}=4
\end{aligned}
$$

Find:
(a) $L=$ ?
(b) $L=$ ?
(c) $L=$ ?
(d) $L=$ ?

Solution:
(a)
(b)
(c)
(d)

$$
\begin{aligned}
L & =L_{1}+L_{2}+2 K \sqrt{L_{1} L_{2}} \\
& =4+4+(2 \times 1 \times \sqrt{4 \times 4}) \\
& =4+4+8 \\
& =16 \text { henries } \\
L & =L_{1}+L_{2}-2 K \sqrt{L_{1} L_{2}} \\
& =4+4-8 \\
& =0 \\
L & =L_{1}+L_{2} \pm 2 K \sqrt{L_{1} L_{2}} \\
& =4+4 \pm 2 \times 0 \times 4 \\
& =8 \text { henries } \\
L & =L_{1}+L_{2}+2 K \sqrt{L_{1} L_{2}} \\
& =4+4+2 \times 0.5 \times 4 \\
& =12 \text { henries }
\end{aligned}
$$

8-7. Low-frequency Inductance Coils. Power-supply Applications. Inductance coils used in filter circuits of power supplies and in a-f circuits are generally classed as low-frequency coils. These coils must have a high inductance in order to get a high value of impedance at the frequencies for which they are used. As the impedance is practically equal to the inductive reactance (see Examples 8-6 and 8-8), it may be seen by Eq. (8-6), $X_{L}=2 \pi f L$, that at low frequencies the inductance $L$ must be high in order to attain the desired high values of impedance. In order to get high values of inductance, the coils have a large number of turns and are wound on iron cores.

Coils in power-supply filter circuits have a pulsating current flowing through them, and the pulsations are usually in the order of 60 or 120 pulsations per second. 'I'hese coils are generally referred to as chokes. It is not uncommon to have 20 - or 30 -henry chokes, and values as high as 100 henries are known to be used. It is common practice to use the field coil of the radio set's loudspeaker as a filter choke in many radio receivers. Low-frequency filter chokes are shown in Fig. 8-11.

A-f Circuit Applications. In a-f circuits, inductance coils are used as a coupling device in order to transfer energy from one stage to another. Applications of coils in such circuits are as follows: (1) transformers used to transfer energy from one circuit to another, (2) parallel feed supply of the $B$ voltage to the plate of a tube, (3) impedance coupling, (4) a-f filter circuits.

Transformers are used in a-f circuits as coupling devices and operate at frequencies of about 100 to 5000 cycles. These transformers have a primary and secondary wound on a laminated iron or steel core. The secondary usually has a greater number of turns than the primary, and ratios of 2 to 1 or 4 to 1 are commonly used. This ratio aids in stepping up the voltage of the a-f signal whose energy it is to transfer. The increase in voltage obtained from the a-f transformer is not wholly dependent upon
the ratio of the turns, but rather it is affected to a great extent by the ratio of the impedance of the transformer's primary winding to the impedance of the tube to which it is connected. In general, however, the primary of the transformer should have a high impedance to match the high impedance of the tube. To attain the high impedance, the primary wind-


Fig. 8-11.-Low-frequency filter chokes. (Thordarson Electric Manufacturing Dvvision, Maguire Industries, Inc.)
ing has many thousand turns of small wire, which results in a high value of resistance and inductance. The resistance is of the order of 2000 to 5000 ohms, and its inductance is about 100 henries. The inductive reactance will vary directly with the frequency as seen from the equation $X_{L}=$ $2 \pi f L$; at 50 cycles the inductive reactance of the 100 -henry winding will be $6.28 \times 50 \times 100$, or $31,400 \mathrm{ohms}$; and at 500 cycles it will be $6.28 \times$ $500 \times 100$, or $314,000 \mathrm{ohms}$. The core of the transformer is usually made of a higher grade steel than is used in filter chokes because it operates on much higher frequencies. These higher grades of steel such as silicon steel and several kinds of alloys have very low losses, thereby making it possible to operate at the higher frequencies.

Other Applications. Another example of coupling is the output transformer that is used to couple the output circuit of a receiver to the voice coil of a dynamic speaker. This transformer must have a high-impedance primary to match the impedance of the tube and a low-impedance secondary to match the impedance of the voice coil of the dynamic speaker. Primary impedances range from 2000 to 20,000 ohms, while the secondary impedances range from 2 to 30 ohms.

Low-frequency coils are also used in parallel-feed 13 supply circuits to keep the direct current out of the devices in the output circuit of the tube. These devices may be either the primary winding of a transformer or the audible device.

When an a-f coil is used in conjunction with a high resistance and a capacitor to couple one a-f stage with another, the circuit is called an impedance-coupled circuit. Such circuits are used with a-f amplifiers.

Audio-frequency coils are also used in some types of amplifier circuits to keep out or to filter certain parts of the current from the amplificr


Fig. 8-12.-Low-frequency inductance coils: (a) a-f transformers, (b) output transformers. (Thordarson Electric Manufacturing Division, Maguire Industries, Inc.)
to improve its operation. These coils have high inductance, generally in the order of 100 henries. They offer a high impedance to the a-f currents and a low resistance to the direct current, thereby choking off the a-f currents and causing them to take another path but permitting the direct current to flow freely.

8-8. High-frequency Inductance Coils. R-f Choke Coils. Choke coils used in r-f circuits and transformers used in r-f and i-f circuits operate at frequencies above 100 kilocycles and are classed as high-frequency coils. Because the magnetic effects of high-frequency currents are difficult to predict and to control, various designs of coils have been developed.

Choke coils are used in r-f circuits to provide a high impedance to the r-f currents and a low impedance to direct current. To accomplish this, the r-f chokes should have a high inductive reactance and a low resistance. Because they operate on high frequencies, a high inductive reactance can be obtained with a relatively low inductance as compared with low-frequency coils. For example, an 80-millihenry choke operated at 500 kilocycles has an inductive reactance of $\left(X_{L}=2 \pi f L\right) 2 \times 3.14 \times 500 \times 10^{3} \times$ $80 \times 10^{-3}$, or 251,200 ohms. Standard r-f chokes are available at ratings from 2.5 to 125 millihenries and have low values of resistance compared with their impedance, the resistance varying from about 20 to 250 ohms depending upon their inductance rating and design.

Many of the r-f coils used in f-m, television, and high-frequency electronic circuits operate at frequencies of 5 to 500 megacycles. Special single-layer coils wound on plastic cores or on steatite tubes are used in these circuits. These chokes have inductances ranging from 0.2 to 80 microhenries.

Magnetic Core. Many of the r-f coils are wound on nonmagnetic cores and are referred to as air-core coils. Some of the newer r-f coils are wound on a specially prepared magnetic core consisting of finely powdered iron particles held together with a magnetic insulating binding substance. This special type of iron core reduces the losses in the iron, making it possible to work on higher frequencies. Because of the iron core, a high inductance can be obtained with a smaller number of turns, resulting in a lower resistance, smaller coil, and a higher value of $Q$. This value of $Q$, which is the ratio of the inductive reactance to the resistance of an inductor, or $X_{L} / R$, is explained in Chap. XI. At this time, it is sufficient to state that high values of $Q$ are desirable and that the values of $Q$ may be 100 or more.

Adjustable Magnetic Core. A powdered iron core arranged so that its position may be varied is used in the manufacture of adjustable inductance coils. The inductance of the coil is adjusted by varying the amount of iron inserted in the coil. This is accomplished by means of a screw-driver adjustment. When the adjusting screw is turned so that the core is all the way in the coil, maximum inductance is obtained; when the core is withdrawn from the coil, minimum inductance is obtained. This type of core is used in universal oscillator coils in order to adjust them to track at the proper frequency. It is also used in coils requiring a high $Q$ factor, such as the i-f transformers used in frequency modulation and television and the series and shunt peaking coils used in video circuits.

Methods of Winding. Another feature of the r-f coils is the special way in which they are wound. Instead of just winding the turns alongside of each other and the layers on top of one another, special winding
methods have been developed to reduce the distributed capacitance (see Chap. IX) of the coil. These methods employ (among others) the universal, honeycomb, lattice, spider-web, bank, figure-of-eight, and binocular types of windings. Several types of windings are shown in Fig. 8-13; of these the universal and the honeycomb are the most commonly used.

R-f Transformers. Radio-frequency transformers are used to couple one stage of a tuning circuit to the next stage. These transformers have


Fig. 8-13.-Various methods of winding high-frequency coils.
two windings, referred to as the primary and secondary winding. They are usually of the universal type and may be wound on either an air core or the special powdered-iron type core. Often they are wound with a special wire called litz wire, and the windings are given special treatment to help them withstand varying temperature and humidity. These transformers have a low number of turns, and the primary is of a lower number than the secondary. The r-f transformers operate at the frequency range of the receiver in which they are connected; thus, for a broadeast-band receiver, the minimum operating frequency range should be 530 to 1550 kilocycles. The r-f transformers in an $\mathrm{f}-\mathrm{m}$ receiver must cover a minimum range of 88 to 108 megacycles, and the r-f coils in a television receiver must cover a range of 54 to 216 megacycles. Receivers used by amateur radio operators generally cover two or more of the numerous bands allocated
to the amateurs. When a receiver, such as a television or multiband radio recciver, is to operate on several frequency ranges, separate $r$-f coils may be used for each channel or band. In some instances, a group of coils may be mounted in a single container or several separate coils of the plug-in type may be used.

I-f Transformers. Intermediate-frequency transformers are a form of r-f transformers that are designed for operation at a definite frequency. Standard values of intermediate frequency have been set up for each of the various types of service for which i-f transformers are used. For a-m broadcast receivers several values of intermediate frequency are in use, and standard transformers can be obtained for frequencies of $175,262,370$, and 456 kilocycles. Of these, the 175 - and 456 -kilocycle transformers are the most commonly used. In f-m receivers the i-f value most generally used is 10.7 megacycles, although higher and lower values have been used. In many television receivers two or more i-f stages of slightly different frequency values are used in order to obtain the necessary wideband characteristics by the method referred to as stagger tuning. The i-f values most generally used in television receivers is in the order of 21 to 27 megacycles, although higher and lower values also have been used. The i-f transformers used in television receivers are made adjustable in order to obtain the proper wide-band amplifier characteristics. For a-m shortwave and amateur receivers, i-f values of 1500 and 1600 kilocycles are commonly used.

Radio-frequency and i-f transformers are usually mounted in shielding containers, and their over-all dimensions are approximately $1 \frac{1}{2}$ to 2 inches round (or square) and 2 to 4 inches high. The leads are generally each of a different color to conform with the RMA (Radio Manufacturers' Association) color code (see Appendix IX). This makes it possible to connect the transformer easily and properly. Transformers must be selected according to their frequency range, and their primary and secondary impedances are usually chosen to match the parts of the radio circuit in which they are to be connected. Several types of r-f and i-f coils are shown in Fig. 8-14.

Single winding r-f coils are used as chokes in the output of the detector circuit to separate the r-f currents from the a-f currents. These coils are small in size and are often mounted in metal shiclds. Even when mounted in shields, they seldom exceed $1 \frac{1}{2}$ inches in diameter and $1 \frac{3}{4}$ inches in height. Several types of r-f choke coils are shown in Fig. 8-15.

8-9. Shielding. High-frequency circuits, such as the i-f and r-f circuits using the transformers just described, often have undesired coupling between adjacent circuits due to their magnetic fields. To prevent this undesired magnetic coupling, a metal shield, usually of aluminum or brass,
is placed about the coil. When a magnetic field passes through such at shield, an cmf is induced in it, and as the shield acts as a closed circuit a current will flow in the shield. This current will set up its own magnetic field which, according to Lenz's law, will oppose the original magnetic field and tend to keep it from spreading beyond the shield.

Shields should be carefully designed as to size because the currents set up in them act as a loss that must be subtracted from the power in the coil circuit. This result has a tendency to increase the resistance and also to


Ftg. 8-14.-Commercial r-f and i-f coils. (Meissner Manufacturing Division, Maguire Industries, Inc.)
reduce the inductance. Both of these effects result in a lower $Q$ and hence reduce the effectiveness of the coil. The shield should be of a heavy material that is a good conductor, such as copper, brass, or aluminum, and should be large enough so that it is not too close to the coil. Copper shields are seldom used because of the corroding effect that air has on the copper. Aluminum is used extensively because of its greater mechanical strength, lower cost, and good conductivity.

8-10. Resistance of Coils. D-c Resistance. The resistance of coils that operate on d-c low-frequency power circuits and audio-frequencies is generally taken to be that amount which may be calculated by Ohm's law from voltage and current readings obtained when direct current is made to flow through the coil. This resistance is often referred to as the $d$-c resistance of the coil.

When coils such as r-f chokes and transformers are used in high-frequency circuits, the resistance to the high-frequency currents is much greater than the d-c resistance of the coils. There are several reasons for this increased resistance to the higher frequency currents.


Fig 8-15.-Commercial r-f chokes. (IIammarlund Manufacturing Company, Inc.)
Skin Effect. Skin effect, which is onc cause for this increased resistance, is explained in the following manner. Any flow of current through a conductor is considered as a flow of electrons. When the nature of the electron flow is a direct current, the clec-


Fig. 8-16.-. Distribution of electrons over the area of a conductor carrying a current: (a) electrons distributed over the entire area with a flow of direct current or low-frequency alternating current, (b) electrons concentrated near the surface with high-frequency currents. trons will be evenly distributed throughout the entire cross-sectional area of the conductor, as shown in Fig. 8-16a. However, when the nature of the electron flow is a high-frequency current, the electrons will be concentrated near the surface of the wire and the center of the cross-sectional area will carry practically no electrons, as shown in Fig. 8-16b. This reduces the effective area of the conductor, and in accordance with Eq. (4-2), $R=K L / A$, any reduction in the area will increase the ressistance. The electrons of high-frequency currents move near the surface of the conductor because at high frequencies the magnetic lines (see Fig. 5-31) expand and contract so rapidly that they cause sufficient induced voltage in the conductor itself to repel and (according to Lenz's law) to push the electrons from the center of the conductor to the outer area.

Eddy Currents. The varying concentric magnetic fields shown in Fig. 5-31 also induce voltages at many points along the conductor itself, and these voltages, even though they are small, set up additional currents in the conductor. These currents, known as eddy currents, flow back and forth in small areas of the conductor and therefore do no useful work but instead are actually a loss in the conductor. The eddy currents cause additional heating of the conductor, and, as they are a loss, they have the same effect as an increased resistance.

Combined High-frequency Resistance. At high-frequency, it is common practice to combine all the resistance effects, that is, the d-c or ohmic resistance, the skin effect, and the eddy-current effect into a single value called the a-c resistance of the coil. This a-c resistance of a coil increases as the frequency at which the coil is operated is increased. The skin effect and hence the a-c resistance increase at a greater rate for large conductors than for small ones. To reduce the value of the a-c resistance, a special type of conductor called litz wire is made by weaving a large number of small insulated wires to form the conductor. As the individual wires are insulated, they act as separate conductors and will result in a fairly uniform distribution of current at broadcast and even at short-wave frequencies, but they are not highly effective for ultrahigh frequencies. Another method used to reduce the value of the a-c resistance is to use hollow conductors or


Fig. 8-17.-Conductors used for highfrequency currents: (a) hollow conductor, (b) flat-strip conductor. thin, flat strip conductors as shown in Fig. 8-17. The litz wire is used mostly for receivers, while the hollow conductor is used mostly for transmitters.

8-11. Measuring Inductance. Methods of Measuring Inductance. The method of calculating the inductance from the physical characteristics of a coil was described in Art. 8-2. It is often desired to determine the inductance of a coil, but some factor required for calculating the inductance may not be readily obtained. In such cases it becomes necessary to measure the inductance of the coil. There are two methods available for measuring the inductance of a coil, the comparison and the impedance methods.

Comparison Method. The comparison method involves the use of a standard inductor and some form of a bridge circuit. The circuits involved and the theory of such bridge circuits were described in Art. 6-18. A more detailed discussion may be obtained from the references at the end of this chapter. 'There are a number of commercial bridges available
for measuring inductance. Use of such a bridge is not difficult, and quite accurate results may be obtained.

Impedance Method. The impedance method involves the taking of voltmeter, ammeter, frequency, and wattmeter readings of the coil when connected to an a-c power source. The inductance may then be calculated by use of Eqs. (8-9), (2-14), (8-8), (8-6), as illustrated in the following example.

Example 8-15. It is desired to determine the inductance of a coil by the impedance method. The coil is connected to the circuit as shown in Fig. 8-18, and the


Fig. 8-18.
following meter readings are obtained: voltmeter, 250 volts; ammeter, 0.100 ampere; wattmeter, 0.50 watt; frequency, 60 cycles. (a) What is the inductance of the coil? (b) What is the $Q$ of the coil?

| Given: | Find: |  |
| ---: | :--- | ---: |
| $E$ | $=250$ | $L=?$ |
| $I$ | $=0.100$ | $Q=?$ |
| $W$ | $=0.500$ |  |
| $f$ | $=60$ |  |

Solution:
(a)

$$
\begin{aligned}
Z & =\frac{E}{I}=\frac{250}{0.100}=2500 \mathrm{ohms} \\
R & =\frac{P}{I^{2}}=\frac{0.500}{0.100 \times 0.100}=50 \mathrm{ohms} \\
X_{L} & =\sqrt{Z^{2}-R^{2}}=\sqrt{2500^{2}-50^{2}}=\sqrt{6,250,000-2500} \\
& =\sqrt{6,247,500}=2499 \mathrm{ohms} \\
L & =\frac{X_{L}}{2 \pi f}=\frac{2499}{2 \times 3.14 \times 60}=6.63 \text { henries }
\end{aligned}
$$

$$
\begin{equation*}
Q=\frac{X_{L}}{R}=\frac{2499}{50}=50(\text { approx }) \tag{b}
\end{equation*}
$$

If a wattmeter is not available, the resistance may be determined by connecting the coil to a d-c power supply and taking voltmeter and ammeter readings. The resistance may then be calculated by Ohm's law, $R=E / I$. If the coil is then connected to an a-c power supply of known frequency and voltmeter and ammeter readings are taken, the inductance may be found in the manner indicated in Example 8-15. Examining the
results of Example 8-15, it may be seen that, when $R$ is small compared with $Z$, then $X_{L}$ is practically equal to $Z$. Under this condition, l may be neglected, and yet it will not produce any appreciable error in the value of $X_{L}$ and $L$. Further examination of the results of Example 8-15 shows that the $Q$ of the coil is approximately 50 . As the value of $Q$ for coils ranges from 30 to several hundred and since with such values of $Q$ the resistance is low compared with the inductive reactance, it is permissible to neglect the resistance entirely in the equation $X_{L}=\sqrt{\overline{Z^{2}}-R^{2}}$ and $X_{L}$ is then considered equal to $Z$. On this basis, the inductance may be found by taking a voltmeter and ammeter reading with the coil connected to an a-c power supply of known frequency.

Example 8-16. What is the inductance of a high- $Q$ coil that draws five milliamperes when connceted to a 25 -volt, 1000 -cycle power supply?

| Given: | Find: |  |
| ---: | :--- | ---: |
| $E=25$ | $L=?$ |  |
| $I$ | $=0.005$ |  |
| $f$ | $=1000$ |  |

Solution:

$$
\begin{aligned}
Z & =\frac{E}{I}=\frac{25}{0.005}=5000 \text { ohms } \\
X_{L} & =Z=5000 \text { ohms (approximately) } \\
L & =\frac{X_{L}}{2 \pi f}=\frac{5000}{2 \times 3.14 \times 1000} \\
& =0.796 \text { henry }
\end{aligned}
$$

The three steps used in the solution of Example 8-16 can be combined into a single equation

$$
\begin{equation*}
L=\frac{E}{2 \pi f I} \tag{8-22}
\end{equation*}
$$

Example 8-17. What is the inductance of a high- $Q$ coil that draws five milliamperes when connected to a 25 -volt, 1000 -cycle power supply?

| Given: | Find: |  |
| ---: | :--- | ---: |
| $E$ | $=25$ | $L=?$ |
| $I$ | $=0.005$ |  |
| $f$ | $=1000$ |  |

Solution:

$$
\begin{aligned}
L & =\frac{E}{2 \pi f I}=\frac{25}{2 \times 3.14 \times 1000 \times 0.005} \\
& =0.796 \text { henry }
\end{aligned}
$$

In the measurement of inductance of low-frequency coils wound on iron cores, it is recommended that the amount of current used for this meas-
urement be about the same as the rated current of the coil. If the coil has a direct current flowing in addition to the a-f currents under its normal operating conditions, then a similar current should be applied when taking measurements to be used for determining its inductance. Measuring the inductance without its proper d-c component may result in errors as great as several hundred per cent.

The inductance of high-frequency coils, between 1 and 100 millihenries, may be measured


Fig. 8-19.-Noninductive winding: (a)' a loop of wire, (b) a noninductively wound coil. at ordinary power frequencies, but a special low-current-drain voltmeter such as a vacuum-tube voltmeter should be used. Inductances below one millihenry should be measured with highfrequency currents applied, and care should be exercised that suitable high-frequency ther-mocouple-type meters are used.

8-12. Noninductive Windings. The effect of inductance is useful in radio, television, and clectronic circuits, as is indicated by the preceding description of the various choke coils and transformers; this will be further explained in the chapter on Resonance. There are cases, however, in which as little inductance effect as possible is desired.

When it is desired that a coil of wire have the lowest possible amount


Fig. 8-20.-Noninductive wiring: (a) wires twisted to reduce the inductance, (b) wires placed parallel to one another to reduce the inductance.
of inductance, the coil is wound so that each turn of wire with a current flowing in one direction has its inductive effect neutralized by a turn with its current flowing in the opposite direction. The coil is then said to be noninductively wound.

If a coil requires only a few turns, it may be noninductively wound by first looping the wire as shown in Fig. 8-19a and then winding the loop
around the core. The completed coil will be as shown in Fig. 8-196. When the coil requires a large number of turns, it is difficult to loop the wire first. In this case, the coil is usually wound in two sections, cach having two leads. When completed, the two winding sections are connected in series in such a mamer that the currents in each section set up magnetic fields of opposite directions which will neutralize one another.


Fig. 8-21.--A typical superheterodyne circuit. The various inductors are shown enclosed in the heavy broken-line boxes.

The wires used to connect various parts of a receiver sometimes cause unwanted inductance. This is usually true of filament or heater circuits in a-c operated sets, especially if the wires are spaced a small distance apart and are parallel to one another. The inductive effect may be reduced to a negligible amount by twisting the wires, as shown in Fig. 8-20a, or even by running the wires parallel to each other but right alongside of one another, as shown in Fig. 8-20b.

8-13. Use of Inductors in Radio, Television, and Electronic Circuits. Radio Circuits. In the preceding articles, the theory, construction, and uses of the various types of inductors have been discussed in detail. Figure $8-21$ is a circuit diagram of a typical superheterodyne receiver. The various inductors used in this circuit are shown in the heavy broken-line
boxes. The following table indicates the coil numbers and their application in the circuit:

| Coil Number | Description |
| :---: | :---: |
| 1. | R-f transformer |
| 2 | R-f transformer |
| 3 | I-f transformer |
| 4 | I-f transformer |
| 5 | Power transformer |
| 6 | Filter-circuit choke |
| 7. | Filter-circuit choke |
| 8. | Oscillator-circuit coil |
| 9. | A-f transformer |
| 10. | Output transformer |
| 11. | Voice coil of loudspeaker |

The inductors used in the circuit of Fig. 8-21 are represented by their symbolic representations as is common practice in preparing such diagrams. To associate the symbols with the actual parts, refer to Fig. 9-34, which is a photograph of a receiver showing some of the coils in the form in which they actually appear. Figure $9-34$ is not for the same receiver as Fig. 8-21 and hence has a different number of coils. Two high-frequency transformers are shown, but as they are shielded the actual coils cannot be seen. An output transformer is mounted behind the speaker and between the two i-f transformers.

Television Circuits. A television receiver, in addition to reproducing a video signal, is also required to reproduce an audio signal. Therefore, practically all the various types of inductors usually found in a radio receiver are also present in television receivers. In general, the construction of these components are the same as those found in broadcast radio receivers except that they are designed to operate at the frequencies used for television receivers. The power transformer is much larger since it must provide the plate and filament currents for 20 or more tubes. These currents may be as high as 500 milliamperes for the plate supply and 20 amperes for the filament supply. The relatively high-filament current introduces several new problems among which are the following: (1) a larger size of wire must be used in the filament circuits in order to prevent overheating and excessive voltage drops; (2) care must be exercised in the arrangement of the wiring in order to minimize the hum picked up in the amplifier circuits because of the strong magnetic field about the filament wiring. Low-filament circuit currents are obtained in many television receivers by using a transformer with two or more secondaries for supplying the filament current.

The video circuit of a television receiver also uses a number of inductors among which are: (1) a magnetic deflection coil for deflecting the electron
beam (either vertically or horizontally) in the cathode-ray tube; (2) a focusing coil for focusing the electron beam on the fluorescent screen of the cathode-ray tube; (3) a variable reactor with a powdered-iron core for adjusting the width of the picture; (4) filament chokes for eliminating undesirable r-f currents from the filament circuits; (5) a video shunt-peaking coil; (6) a video series peaking coil; (7) a vertical output transformer for coupling the vertical output to the deflection coil; (8) a vertical blockingoscillator transformer (which also produces a 60 -cycle pulse); (9) a horizontal synchronizing-discriminator transformer for supplying the synchronizing voltage for the horizontal oscillator; (10) a horizontal output transformer for driving the deflection coil (which is also sometimes used for supplying the high voltage for the cathode-ray tube).

Electronic Circuits. In industrial electronic equipment a number of different types of inductors are used. Many of these inductors are similar to those used in radio and television except for their current and voltage ratings and the frequency at which they are designed to operate. In addition to the inductors that are common to radio and television circuits, various types of control and timing circuits use special inductors and relays that are designed for a particular purpose.

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## QUESTIONS

1. Define inductance.
2. Why is a knowledge of inductance important in the study of (a) radio? (b) Television?
3. (a) What is meant by induced emf? (b) How is it produced?
4. Explain the relation between the induced emf and the impressed voltage during the time that the magnetic field is expanding.
5. Explain the relation between the induced emf and the impressed voltage during the time that the magnetic field is collapsing.
6. State Lenz's law.
7. Why is inductance always present in a-c circuits?
8. Define the basic unit of inductance.
9. What is a millihenry? Microhenry?
10. Define the following terms: (a) self-inductance, (b) inductor, (c) solenoid.
11. What are the factors that determine the inductance of a coil?
12. Describe (a) multilayer coil, (b) pancake coil.
13. (a) What is meant by inductive reactance? (b) In what unit is it expressed?
(c) What is its symbol?
14. What two factors affect the amount of inductive reactance of a circuit?
15. (a) What is meant by the impedance of a coil? (b) What is the unit of impedance? (c) What is its symbol?
16. (a) Under what condition may the resistance of an inductor be ignored?
(b) Why?
17. Explain how inductance causes a delay or lag in current flow.
18. How is the amount of delay or lag usually expressed?
19. What are the factors affecting the amount of lag in current flow?
20. Explain the following terms: (a) mutual inductance, (b) primary winding, (c) secondary winding.
21. What is meant by the coefficient of coupling?
22. Explain what is meant by coils connected in series: (a) aiding, (b) opposing.
23. Describe the principle of the variometer and the variocoupler.
24. (a) What is meant by low-frequency inductance coils? (b) Where are they generally used in radio circuits?
25. Is the inductance of a low-frequency coil high or low? Why?
26. (a) Describe the construction of a-f transformers. (b) Why must the impedance of the primary be high in value?
27. What relative impedance values are required for the primary and secondary of an output transformer? Why?
28. Describe how a-f coils are used in filter circuits.
29. (a) What is meant by high-frequency inductance coils? (b) Where are they generally used in radio circuits?
30. (a) What is the purpose of $r$-f choke coils? (b) How do they differ in construction from a-f choke coils?
31. What are the advantages of using powdered-iron-core r-f coils in place of air-core r-f coils?
32. What is meant by an i-f transformer?
33. What frequencies are used for the i-f transformers in (a) a-m broadcast radio receivers? (b) A-m short-wave receivers? (c) F-m receivers? (d) Television receivers?
34. What are the various types of construction used to reduce the effects of distributed capacitance in r-f coils?
35. What is the purpose of r-f transformers?
36. How are the values of frequency and impedance used to determine the r-f transformer to be used?
37. Why must $r$-f and i-f coils be shielded?
38. Explain the principle of shielding.
39. What is meant by (a) skin effect? (b) Eddy-current effect?
40. What methods of construction are used to reduce skin effect and eddy-current effects?
41. What methorls are used to determine the inductance of a coil?
42. (a) Describe two impedance methods of determining the inductance of a coil. (b) What are the advantages of each method?
43. What precautions should be observed in measuring the inductance of lowfrequency coils?
44. What precautions should be observed in measuring the inductance of highfrequency coils?
45. How are coils wound when it is desired to reduce their inductance to practically zero?
46. How are undesirable inductive effects reduced in the general wiring of a radio receiver?
47. What types of inductors are used in television receivers that are also used in radio receivers?
48. How does the construction of a power transformer designed for use in a radio receiver differ from one designed for use in a television receiver?
49. Why is it desirable for a power transformer used in a television receiver to have more than one secondary winding for the filament supply?
50. What is the purpose of (a) the deflection coil? (b) The focusing coil?
51. What is the purpose of (a) the vertical output transformer? (b) The vertical blocking-oscillator transformer?
52. What is the purpose of (a) the horizontal synchronizing-discriminator transformer? (b) The horizontal output transformer?

## PROBLEMS

1. A flux of $2,000,000$ lines links a coil having 250 turns. The flux in the coil decreases from maximum value to zero in 0.125 sec . What is the value of the induced voltage?
2. How many turns must a coil have if its induced voltage is to be 125 volts when a flux of 500,000 lines changes from maximum value to zero in 0.004 sec .?
3. A flux of 10 lines links a coil having 50 turns. The flux in the coil varies in accordance with an alternating current having a frequency of 465 kc . What is the value of the induced voltage? Note: The flux varies from zero to maximum and vice versa, in one-quarter of a cycle.)
4. What is the maximum value of the flux in a coil having 75 turns if it induces a voltage of 12 volts? The flux varies in accordance with an alternating current having a frequency of 175 kc .
5. What is the inductance of a tuning coil that has 250 turns wound on cardboard tubing 4 cm in diameter and 20 cm long? [Use Eq. (8-2).]
6. How many turns are necessary to obtain an inductance of $250+\mathrm{h}$ if a coil is to be wound on fiber tubing 3 cm in diameter and its length is to be 24 cm ?
7. What would the inductance of the coil in Prob. 6 be if it were wound on an iron core whose permeability was 3600 ?
8. What would the inductance of the coil in Prob. 5 be if the number of turns were increased to 750 ?
9. What would the inductance of the coil in Prob. 7 be if the number of turns were doubled?
10. A transformer has 150 turns on its primary winding and 875 turns on its secondary. The length of the core is 30 cm , its cross-sectional area is $8 \mathrm{sq} . \mathrm{cm}$, and the permeability is 3500 . (a) What is the inductance of the primary winding? (b) What is the inductance of the secondary winding?
11. What is the inductance of a multilayer coil of 750 turns if its dimensions (Fig. 8-5a) are $a=1 \frac{1}{2}$ in., $b=\frac{3}{4}$ in., $c=1 \mathrm{in}$.?
12. What is the inductance of a multilayer coil of 1500 turns if its dimensions (Fig. 8-5a) are $a=1 \frac{1}{2}$ in., $b=1 \frac{1}{2}$ in., $c=1$ in.?
13. What is the inductance of a flat coil (Fig. 8-5b) having 200 turns if its dimensions are $a=2$ in., $c=1 \frac{1}{2}$ in.?
14. What is the inductance of a flat coil (Fig. 8-5b) having 100 turns if its dimensions are $a=1 \frac{3}{4}$ in., $c=1$ in.?
15. What is the inductance of a solenoid (Fig. 8-5c) having 600 turns if its dimensions are $a=1 \frac{1}{2}$ in., $b=5$ in.?
16. What is the inductance of a solenoid (Fig. 8-5c) having 1200 turns if its dimensions are $a=2$ in., $b=8 \mathrm{in}$.?
17. What is the inductive reactance of a 30 -henry choke coil at (a) 60 cycles? (b) 120 cycles?
18. What is the inductive reactance of a 5 -henry choke coil at (a) 50 cycles? (b) 500 cycles? (c) 1000 cycles? (d) 5000 cycles?
19. What is the inductive reactance of a $250-\mu \mathrm{h}$ choke coil at (a) 550 ke ? (b) 1000 kc ? (c) 1500 kc ? (d) 7.5 mc ?
20. What is the inductance of a coil that has an inductive reactance of 4500 ohms at 60 cycles?
21. If the transformer of Prob. 10 is operated at 60 cycles, what is the inductive reactance of (a) the primary winding? (b) The secondary winding?
22. The secondary of an r-f transformer has an inductance of $185 \mu \mathrm{~h}$. What is its inductive reactance at (a) 500 kc ? (b) 1000 kc ? (c) 1500 kc ?
23. A 10 -henry choke coil that has a resistance of 475 ohms is connected to a 110 volt 60 -cycle power supply. (a) What is its inductive reactance? (b) What is its impedance? (c) What current does it draw from the line?
24. Repeat Prob. 23 for a 6 -henry choke coil whose resistance is 200 ohms.
25. What current will a 4 -henry choke coil whose resistance is 160 ohms take when connected to a 120 -volt 60 -cycle power supply line?
26. A coil having an inductance of $300 \mu \mathrm{~h}$ and a resistance of 6 ohms is connected across a 3 -volt battery. What time is required for the current to build up to 63.2 per cent of its final value?
27. A coil having an inductance of 30 henries and a resistance of 250 ohms is connected across a 3-volt battery. What time is required for the current to build up to 63.2 per cent of its final value?
28. If the coil of Prob. 26 is connected to a 1000 -kc 3 -volt a-c power supply, find (a) the inductive reactance, (b) the impedance, (c) the angle of lag, (d) the current.
29. If the coil of Prob. 27 is connected to a 60 -cycle 300 -volt a-c power supply, find (a) the inductive reactance, (b) the impedance, (c) the angle of lag, (d) the current.
30. What is the mutual inductance of two coils wound adjacent to one another? Assume that all the magnetic lines set up in the first coil cut all the turns of the second coil. The primary coil consists of 800 turns wound on a cardboard core 1 in . in diameter and 4 in. long. The secondary coil consists of 1600 turns wound on a cardboard core $1 \frac{1}{4} \mathrm{in}$. in diameter and 4 in . long.
31. What is the mutual inductance of the transformer of Prob. 10 if the coefficient of coupling is 0.95 ?
32. What is the coefficient of coupling of two coils whose self-inductances are 0.04 and 0.06 mh and whose mutual inductance is 0.01 mh ?
33. What is the coefficient of coupling of two coils whose self-inductances are 50 and $75 \mu \mathrm{~h}$ and whose mutual inductance is $5 \mu \mathrm{~h}$ ?
34. What is the coefficient of coupling of two coils whose self-inductances are 5 and 8 mh and whose mutual inductance is $75 \mu \mathrm{~h}$ ?
35. Two coils, the first having 300 turns and the second 450 turns, are placed so that only 5 per cent of the lines set up by coil 1 link coil 2 . If 1200 lines are set up when 3 ma flow through coil 1 , what voltage will be induced across coil 2 if the current decreases from its maximum value to zero in 0.00003 sec ?
36. Two coils, one having an inductance of $100 \mu \mathrm{~h}$ and the second $400 \mu \mathrm{~h}$, are arranged so that they may be connected in scrics in the various ways shown in Fig. $8-10$. What is the inductance of the circuit when the two coils are connected in series so that they are (a) aiding and the coupling is 100 per cent? (b) Opposing and the coupling is 100 per cent? (c) In a position that produces zero coupling? (d) Aiding and the coupling is 50 per cent?
37. Two identical coils are connected in series aiding with a coupling coefficient of 0.20 . What must be the inductance of each coil to produce a circuit inductance of $185 \mu$ h?
38. What is the inductance of each coil in Prob. 37 if the coefficient of coupling is reduced to 0.02 ?
39. It is desired to determine the inductance of a coil by the impedance method. The coil is connected to a circuit as shown in Fig. 8-18, and the following meter readings are obtained: voltmeter 115 volts, ammeter 1.0 amp , wattmeter 5 watts, frequency 60 cycles. (a) What is the inductance of the coil? (b) What is the $Q$ of the coil?
40. The coil in Prob. 39 is connected to a signal source of 12 volts and 500 cycles. (a) What is the inductive reactance? (b) What is the resistance of the coil if the coil $Q$ is assumed to be the same as in Prob. 39? (c) What is the impedance of the coil? (d) What current will the coil draw?
41. What is the inductance of a high- $Q$ coil that draws 3 ma when connected to a 10 -volt 5000 -cycle power supply?
42. A coil having a $Q$ of 150 and an inductive reactance of 1200 ohms at 60 cycles is connected to a 15 -volt 1000 -cycle source of power. Find (a) the inductance of the coil, (b) the resistance of the coil, (c) the current.
43. A coil is connected as shown in Fig. 8-18 in order to determine its inductance by the impedance method. The following meter readings were obtained: voltmeter 110 volts, ammeter 10 ma , wattmeter 0.012 watt, frequency 60 cycles. (a) What is the inductance of the coil? (b) What is the $Q$ of the coil?
44. If the inductance of the coil of Prob. 43 was determined by assuming that the inductive reactance is approximately equal to the impedance, what is the percentage of error in the answer obtained by this method?

## CHAPTER IX

## CAPACITANCE

Radio and television receivers and transmitters (and industrial electronic equipment) require resistance, inductance, and capacitance in one or more of their circuits to make them operate satisfactorily. The effects of resistance and inductance have already been presented. A knowledge of capacitance and of its effects in a circuit is therefore very necessary.

9-1. Capacitance, Capacitor Action. Principle of Capacitance. Capacitance is defined as the property of a circuit that opposes any change in the amount of voltage. Notice how much this definition conforms with the one for inductance, which states that inductance is the property of a circuit that opposes any change in the amount of current. Whereas inductance is caused by the magnetic field and it opposes any change in the amount of current, capacitance is caused by the electrostatic field and it opposes any change in the amount of voltage. The capacitance is also referred to as the capacity, but the preferred designation is capacitance.

Capacitance is present in practically all a-c circuits; in some instances it is desired, while in others it is not. In many radio circuits it is desired, and in such cases a special device called a capacitor is used to obtain the required amount of capacitance. The capacitor is also known as a condenser, and even though the name condenser was used a great deal, capacitor is the better name and has rapidly become the more popular.

A capacitor is formed whenever two conductors are placed close to one another but are separated by an insulator which is called the dielectric. The conductors are usually made of thin plates in order to obtain sufficient area, and the insulator is usually a very thin piece of material. The details of construction are fully described in the following articles. The theory of the action that takes place in a capacitor will now be described.

Electron Theory of Capacitor Action. According to the electron theory (see Chap. II), all matter is made up of electrons and the flow or movement of electrons constitutes an electric current. In Chap. IV, conductors were described as being such materials in which electrons are free to move and which offer very little resistance to the flow of electrons. Insulators were described as such materials in which the electrons have a very strong attachment to their nucleus and hence are not free to move far away from the nuclei; they also offer considerable resistance to the flow of electrons.

Figure $9-1 a$ shows a capacitor consisting of two metal conductors $A$ and $B$ separated by the dielectric $C$. The capacitor is connected to a
switch, a meter, and a battery. With the switch open, as shown in the figure, the battery emf can have no effect on the capacitor and the meter indicates zero current flow. The capacitor has no charge, and in terms of the electron theory the materials of the plate and the dielectric are in a neutral state. The nucleus and some orbital electrons of two atoms of the dielectric are shown. It should be remembered that the dielectric has too large a number of atoms to attempt to show all of them; therefore for convenience only two are shown. Furthermore, since the actual material of the dielectric has not been named, no specific electron structure of the


Fig. 9-1,-Principles of capacitor action: (a) capacitor in neutral electron state, (b) a charged capacitor.
atom can be shown and the atom is represented by a nucleus and six orbital electrons.

If the switch shown in Fig. 9-1 $a$ is closed at $M$, the following actions take place.

1. The emf of the battery causes electrons from plate $A$ to flow through the circuit to plate $B$ as indicated by the arrows; this electron flow will also be indicated by a movement of the necdle of the meter.
2. The transfer of electrons from plate $A$ to plate $B$ causes plate $A$ to have less than its normal amount of electrons, and it is said to have a positive charge. As plate $B$ has taken on some electrons, it now has an excess of electrons and is said to possess a negative charge. Tnder these conditions, an electrostatic field is said to exist between plates $A$ and $B$ of the capacitor.
3. The dielectric $C$, being in the electrostatic field, will be affected by it. Because of the nature of the dielectric material, its electrons do not become separated from their nuclei but the electrostatic field does cause their orbital motion to become distorted as indicated in Fig. 9-1b.
4. This distortion causes an additional electrostatic field within the dielectric that is opposite in direction to the field between the plates, which in turn causes more electrons to flow from plate $A$ to plate $B$ and restores
the electrostatic field between the plates to its original strength. The dielectric has thereby increased the charge on the capacitor.

The action just described is referred to as charging a capacitor, because through this action plate $A$ has become positively charged and plate $B$ negatively charged. This entire action takes place in a very short time, usually a small fraction of a second. The flow of electrons from plate $A$ to plate $B$ is completed in this short interval of time, and hence the current exists only for that time. The movement of the meter needle referred to above will thercfore be only a momentary movement, that is, the needle will indicate a current flow for just a short period of time and then return to zero indication.

The discussion so far concerning the action of the capacitor should make clear two important characteristics: (1) A capacitor is a device that actually stores electrical energy. (2) The current does not flow through the capacitor, but the current flow indicated by the meter represents the transfer of electrons from one plate to another.

Effect of Capacitor Losses. If the switch, Fig. 9-1b, is now opened so that it does not make contact with either $M$ or $N$, the capacitor will be disconnected from the line. At the instant of time when the switch is being opened, the capacitor is charged; that is, one plate has an excess of electrons and the other has a deficiency of electrons. Also, electrostatic fields exist between the plates and in the dielectric. If the capacitor is assumed to be a perfect one, that is, having no losses, it will remain in a charged condition until connected into a closed circuit and then it will discharge through that circuit. Unfortunately, all capacitors have some losses, as will be described later. However, the losses are small, and some types of high-grade capacitors may hold their charges for several hours or more.

Electron Flow in a Capacitor Circuit. If a capacitor is connected in a circuit similar to Fig. 9-1b and the switch is closed through point $M$, the capacitor will become charged. If the switch is opened and kept in the open position, the capacitor will remain charged. If the switch is now closed through point $N$, it will form a closed circuit from plate $B$ through the connecting wires, the resistor $R$, the switch, and the ammeter to plate $A$. As plate $B$ of the charged capacitor has an excess of electrons, some electrons will now flow through the circuit back to plate $A$. This flow of electrons constitutes a current and will cause an indication on the ammeter. The current of a capacitor discharging through a resistor will decrease quite rapidly. The time required to discharge the capacitor through a resistor will be dependent on the product of $R$ and $C$. In most cases, however, the current will decrease so rapidly that the ammeter will merely show an indication for an instant and then return to zero.

If the battery terminals in Fig. 9-1b are interchanged and the switch is closed through point $M$, the capacitor will become charged, but now plate $A$ will have a negative charge and plate $B$ will be positive.

If the capacitor is connected in a circuit similar to Fig. 9-2a and the switch is closed toward $F$, a momentary current flow will be indicated on the meter and plate $A$ will take on a positive charge, while plate $B$ will become negatively charged. If the switch is changed so that it is closed toward $G$, a momentary current flow will again be indicated on the meter, but this time the needle will move in the opposite direction to that when the switch was closed toward $F$. Plate $A$ of the capacitor will now have a negative charge and plate $B$ a positive. Each time the switch position is changed, a momentary current flow will be indicated by the meter.


Fig. 9-2.-Hffect of reversing the polarity at the terminals of a capacitor: (a) reversals produced by means of a switch, (b) reversals produced by connecting the capacitor to an a-c power supply.

If a capacitor is connected to an a-c power supply as in Fig. 9-2b, the polarity as applied to the plates is continually being reversed. The number of reversals will be double the number of cycles; hence a capacitor connected to a 60 -cycle power line will have 120 reversals per second. With alternating current applied to a capacitor, the electrons are flowing continually from one plate through the external circuit to the other plate and then back through the external circuit to the first plate, etc. This is called capacitor action.

9-2. Factors Affecting the Capacitance. Capacitance is defined as the property of a circuit that opposes any change in the amount of voltage. The action of the capacitor has just been explained, and we shall now study the unit of capacitance and the factors that affect its value.

Unit of Capacitance. The unit of capacitance is the farad; it was named in honor of an early scientist, Michael Faraday. A circuit has a capacitance of one farad when a voltage changing at the rate of one volt per second causes an average current of one ampere to flow. The symbol used to repre-
sent capacitance is the letter $C$. The farad is too large a unit for practical purposes, and the microfarad (one-millionth of a farad) is commonly used. Several forms of abbreviations such as $\mu \mathrm{f}, \mathrm{mf}$, and mfd are used to represent the word microfarad; the symbol $\mu \mathrm{f}$ will be used in this text. In radio work the microfarad is often still too large a unit, and it is common practice to use the micromicrofarad (one-millionth of a millionth of a farad), abbreviated as $\mu \mu \mathrm{f}$.

Factors Affecting the Capacitance. Since a capacitor (or a circuit) has a capacitance of one farad when a change of one volt per second causes a current of one ampere to flow, it can be said that the capacitance depends upon the number of electrons set in motion by the specified rate of voltage change. From the definition that a capacitor consists of two conductors separated by an insulator (the dielectric) and from the discussion in the preceding article, it is evident that the number of electrons flowing from one conductor to the other will be directly proportional to the active area of one conductor. This is true whether the conductor has only one plate or several plates connected in parallel (see Art. 9-3). A second factor that affects the capacitance is the thickness of the dielectric, because if its thickness is reduced, fewer distorted electron orbits will oppose the electrostatic field. This results in a greater number of electrons being stored in the conductor, which increases the capacitance. As a decrease in the thickness causes an increase in capacitance, it may be stated that capacitance varies inverscly with the thickness of the diclectric. The third factor affecting the capacitance is the kind of material used as the dielectric. Because the atoms of different materials have different orbital electron arrangements, the effect upon the charge on the plates will vary. This effect is accounted for by a value called the dielectric constant $K$ of the material. The capacitance is directly proportional to the value of $K$. A list of $K$ values for materials used in the construction of capacitors is given in Appendix VI. These three factors affecting the capacitance are represented in each of the following equations for capacitance.

The capacitance of a simple capacitor, that is, one having two flat plates separated by a dielectric, may be found by the equation

$$
\begin{equation*}
C=\frac{22.45 K A}{10^{8} t} \tag{9-1}
\end{equation*}
$$

where $C=$ capacitance, microfarads
$A=$ area of one plate, square inches
$K=$ dielectric constant (Appendix VI)
$t=$ thickness of dielectric, inches
Example 9-1. What is the capacitance of a capacitor made of two plates of tin foil, each $6 \times 8$ inches, separated by a sheet of mica 0.01 inch thick?

Given:

$$
\begin{aligned}
A & =6 \times 8 \\
K & =5.5 \\
t & =0.01
\end{aligned}
$$

Solution:

$$
\begin{aligned}
C & =\frac{22.45 K A}{10^{8} t} \\
& =\frac{22.45 \times 5.5 \times 6 \times 8}{10^{8} \times 0.01} \\
& =\frac{5926}{10^{6}} \\
& =0.005926 \mu \mathrm{f}
\end{aligned}
$$

9-3. Fixed Capacitors, Commercial Types. Classification of Capacitors. Capacitors are manufactured in various forms and may be divided into two


Fig. 9-3.-A parallel-plate capacitor.
fundamental classes, namely, fixed capacitors and variable capacitors The fixed capacitors may be further classified as to the type of material used for the dielectric, such as mica, paper, ceramic, oil, and electrolytes. The variable capacitors are described in Art. 9-4. The electrolytic capacitors and their actions are described separately in Arts. 9-10 to 9-12.

A capacitor constructed so that it can have only one value of capacitance is called a fixed capacitor (or fixed condenser). Practically all radio circuits contain one or more fixed capacitors. Of the four types to be described in this article, the paper and mica capacitors are the most common bccause they are used in a-m radio receivers; the ceramic and mica capacitors are employed in high-frequency circuits such as $f-m$ and television; the oil dielectric type is used mostly in high-voltage applications.

Mica Dielectric Capacitors. Mica capacitors are made by stacking plates of tin foil (or aluminum foil) and thin shects of mica in alternate layers. Figure 9-3 shows that alternate plates are connected to form one terminal; for example, plates $1,3,5,7,9$ are connected as one terminal, and plates $2,4,6,8$ form the second terminal. Examination of the figure will show that both sides of the plates, except the outside plates, are now active and that the active area will be equal to the area of one plate times a number equal to one less than the total number of plates. Applying this consideration of area to Eq. (9-1) will result in the following equation for the capacitance of a multiple-parallel-plate capacitor:

$$
\begin{equation*}
C=\frac{22.45 K A(N-1)}{10^{8} t} \tag{9-2}
\end{equation*}
$$

Example 9-2. What is the capacitance of a capacitor made up of 103 plates of lead foil, each two inches square and separated by layers of mica 0.01 inch thick?

| Given: | Find: |
| :--- | ---: |
| $K=5.5$ | $C=?$ |
| $A=2 \times 2$ |  |
| $N$ | $=103$ |
| $\ell=0.01$ |  |

Solution:

$$
\begin{aligned}
C & =\frac{22.45 K A(N-1)}{10^{8} t} \\
& =\frac{22.45 \times 5.5 \times 2 \times 2 \times 102}{10^{8} \times 0.01} \\
& =0.050 \mu \mathrm{f}
\end{aligned}
$$

For convenience in manufacture, the plates are usually made of tin foil because it can be stamped into thin flexible strips and easily soldered. Mica can be split into very thin sheets and makes an excellent material for use as the dielectric. The complete capacitor, consisting of the plates and dielectric, is usually enclosed in a molded Bakelite container to keep out the moisture. Mica capacitors have low dielectric loss and a low power factor, and will withstand much higher voltages than paper capacitors. Because of the stacked plate construction, the inductance of the capacitors is very low; therefore the losses due to inductance at the high and very high frequencies are practically negligible. Its low dielectric loss, low power factor, low value of inductance, and high voltage rating make the mica capacitor almost indispensable in high-frequency circuits such as are found in f-m and television receivers. Mica capacitors are made in various sizes (see Fig. 9-4) and may arbitrarily be grouped as small, medium, and large. The small-size capacitor usually has a capacitance of 1 to 10,000 micromicrofarads with a d-c working voltage of 300 to 1000 volts. Medium-size
capacitors have a capacitance range of 30 to 00,000 micromicrofarads with d-c working voltages of 600 to 2500 volts. Large capacitors have a capacitance range of 50 to $1,000,000$ micromicrofarads with d-e working voltages to 10,000 volts.


Fig. 9-4.-Commercial types of mica dielectric capacitors. (Solar Manufacturing Corporation.)

Paper Dielectric Capacitors. Capacitors of this type are usually made with tin foil for the conductors and with some form of specially treated paper for the dielectric. The tin foil and paper are made in long, narrow


Fig. 9-5.-Typical construction of paper dielectric capacitors.
strips and then rolled together to form a more compact unit, as shown in Figs. 9-5 and 9-7. The strips of tin foil are placed as shown in Fig. 9-5; this arrangement permits a lead to be soldered at each end in such a manner that each turn of a conductor may be connected to the terminal for that
conductor. This reduces the inductive effect of the various turns; such a capacitor is called a noninductive type. The paper used for the dielectric is usually arranged so that two or more layers of paper separate the plates (see Fig. 9-5). The use of two or more thicknesses of paper in place of a single sheet reduces the possibility


Fig. 9-6-End view of a rolled paper dielectric capacitor. of faulty capacitors owing to the tiny holes that may appear in the paper; even high-quality paper is likely to have some minute holes. When several layers are used, it is unlikely that the holes will be present in each layer at exactly the same locations and hence the possibility of capacitor failures is reduced. When a capacitor having two plates is rolled into a cylindrical or flat form, one of the plates will have two active surfaces, as may be seen from examination of Fig. 9-6. The active area of such a capacitor would be two times the area of one plate.

Example 9-3. What is the capacitance of a capacitor consisting of two plates, each four inches wide and 26 feet long, separated by a paraffined paper dielectric that is 0.005 inch thick?

Note: The capacitor is rolled into a compact cylindrical form.

| Given: | Find: |  |
| ---: | :--- | ---: | :--- |
| $l$ | $=26 \mathrm{ft}$ | $C=?$ |
| $w$ | $=4 \mathrm{in}$. |  |
| $K$ | $=2.2$ |  |
| $t$ | $=0.005$ |  |

Solution:

$$
\begin{aligned}
A & =l \times w \times 2=26 \times 12 \times 4 \times 2 \\
& =2496 \mathrm{sq} \text { in. } \\
C & =\frac{22.45 K A}{10^{8} t}=\frac{22.45 \times 2.2 \times 2496}{10^{8} \times 0.005} \\
& =0.247 \mu \mathrm{f}
\end{aligned}
$$

Paper capacitors are used extensively in radio work because they have the advantage of being smaller in size and lower in cost than mica capacitors. They have higher losses, higher power factor, and a shorter life than mica capacitors. The inductance of the so-called noninductive-type paper capacitor is relatively low, and there is no appreciable loss in capacitance due to inductance at the low radio frequencies encountered in a-m broadcast receiver circuits. However, at the higher radio frequencies used in $\mathrm{f}-\mathrm{m}$ and television receivers, the loss in capacitance due to inductance is appreciable; therefore this kind of capacitor is not generally used for these
types of service. The completed capacitor is generally placed in a metal or cardboard container and then sealed with wax or pitch to keep out moisture. The sizes of paper capacitors range from 0.0002 to several microfarads. Many of the larger sizes are now replaced by electrolytic capacitors.

Ceramic Dielectric Capacitors. Ceramic diclectric capacitors are generally made by applying a silver coating on baked ceramic. A ceramic is a substance that is made from earth by the use of heat. The composition of the ceramic can be controlled so that it will produce a dielectric that makes possible the manufacture of capacitors having positive, negative, or practically zero temperature coefficients. The kinds of ceramics most generally used as a dielectric for capacitors are (1) titanium dioxide, (2) a hydrous magnesium silicate called steatite, (3) a complex magnesium aluminum silicate called cordierite, (4) a mixture of ground mica and leadborate glass called Mycalex. Ceramic capacitors are made in two forms (1) the tubular type in which a silver coating is applied to both sides of a ceramic tube so that two silver coaxial cylinders are formed, which are separated by a ceramic dielectric; (2) a feed-through type in which a silver coating is applied to both sides of a ceramic disk.

Ceramic capacitors have a relatively high dielectric constant and therefore have a very low dielectric loss; one commercial form has a dielectric constant of 6000 . These capacitors also have a very low power factor, practically no inductance, excellent capacitance stability, and high voltage rating, and they are quite compact.

Ceramic capacitors are used in radio and television circuits wherever efficient high-frequency by-passing or coupling is required. This type of capacitor is also used in oscillator and other types of circuits in which the capacitance must be maintained constant, regardless of temperature changes, in order to prevent frequency drift.

Oil Dielectric Capacitors. In radio transmitter circuits, it is not unusual to have voltages above 600 volts, and under such conditions paper capacitors and, in some instances, mica capacitors will not have a very long life. In such cases, special capacitors make use of oil or even a combination of glass and oil as the dielectric. These capacitors are more expensive and are used mostly in transmitters.

9-4. Variable Capacitors. Construction. In certain parts of a radio circuit, it is necessary to use a capacitor whose capacitance can be varied or adjusted to obtain a definite condition in that part of the radio circuit in which it is used. $\Lambda$ variable capacitor consists of two sets of plates, a rotating set called the rotor and a stationary set called the stator (see Fig. $9-8$ ). The stator is generally made with one more plate than the rotor. The capacitor is so constructed that the rotor plates will move freely in between the stator plates, thus causing the capacitance to be varied.

It has been mentioned that the material used in making the capacitor plates must be a good conductor. The plates of a variable capacitor must also be strong and rigid in order to maintain a uniform capacitance and to prevent a possible short circuit between plates. As the plates are often exposed to the air, the material used should be noncorrosive, for corrosion will increase the resistance of the plates. For this reason, brass and aluminum are used rather than pure copper, even though copper


Fig. 9-7.-Commercial types of paper dielectric capacitors. (Solar Manufacturing Corporation.)
is a better conductor. There is very little difference between the resistance of brass and aluminum, although brass will corrode and aluminum is practically free from corrosion. Brass plates were formerly used and had to be lacquered to prevent corrosion. Capacitors are now made with aluminum plates or with cadmium-plated brass plates.

Two different metals joined together will corrode in time unless the joint is well soldered, welded, or brazed. This corrosion will increase the resistance between the plates. To guard against having this high resistance, the spacers and shaft of the rotor and the bar that acts as the separator for the stator plates are all made of the same material. In addition to this, the plates are usually soldered or brazed to their shaft or other support.

The thickness of capacitor plates has no effect on the capacitance. To reduce the losses due to skin effect and eddy currents, these plates are made as thin as practically possible. Most of the plates used are approximatcly 0.025 inch thick. As the capacitance will vary inversely
with the distance between the rotor plates and their adjacent stator plates, it is desirable to have this distance as small as practicable. This distance is usually 0.025 inch, which is the same as the thickness of the plates. When the rotor plates are in mesh with the stator plates, there will be two air gaps and one stator plate between two adjacent rotor plates and


Fig. 9-8.-Commercial types of variable capacitors: (a) standard-size single-broadcast capacitor, (b) midget split-stator capacitor, (c) midget single capacitor, (d) micro capacitor. (Hammarlund Manufacturing Company, Inc.)
two air gaps and one rotor plate between two adjacent stator plates. This means that the distance between two adjacent rotor or stator plates would have to be approximately 0.075 inch.

Ranges of Capacitors. For ordinary broadcast reception, the capacitors used will range from 250 to 500 micromicrofarads when all the rotor plates are completely in mesh with the stator. For short-wave reception, the capacitance is generally less than 150 micromicrofarads. For high frequencies and ultrahigh frequencies, the capacitance required is very small and midget variable capacitors are used (Fig. 9-8). These capacitors are constructed in the same manner as regular capacitors, the only difference
being in the surface area of the plates. A midget capacitor may have a maximum capacitance ranging from 25 to 150 micromicrofarads. When a smaller capacitance is required, micro capacitors are used (see Fig. 9-8). Such capacitors have a maximum capacitance as low as five micromicrofarads.


Fig. 9-9.-Commercial types of variable capacitors: (a) capacitor with opposed rotor and stator for perfect counterbalancing, (b) double-spaced transmitting capacitor. (Hammarlund Manufacturing Company, Inc.)

Transmitting Capacitors. In transmitters, the voltage between plates is very high and double spacing must be used. This means that the spacing between two adjacent rotor and stator plates would be 0.05 inch and the spacing between two adjacent rotor or two adjacent stator plates would be 0.125 inch (see Fig. 9-9). In all other respects, they are constructed in the same manner as any other variable capacitor.

Split-stator Capacitors. In certain radio circuits, it is necessary that the capacitance of the circuits be perfectly balanced. In order to obtain this condition, a split-stator capacitor is used. The stator of this type of capacitor is separated into two equal parts, each half being electrically
insulated from the other. In all other respects, it is constructed in the same manner as any regular capacitor. When the rotor is made with a large number of plates, the semicircular shape of the plates will place all the weight of the rotor on one side. This shift of weight will tend to move the rotor out of the position to which it has been adjusted. To overcome this difficulty, the rotor is balaneod mechanically. This is accomplished by having half of the stator and rotor plates mounted 180 degrecs mechanically from each other (see Fig. 9-9).

Multiple or Gang Capacitors. Radio receivers generally contain more than one stage of tuning, and a variable capacitor is required for each stage. When separate capacitors are used, each circuit must be tuned separately. This method formerly meant that a large number of dials had to be adjusted to tune in a station. An easier method is to tune all the circuits simultaneously. This is accomplished by placing all sets of rotating plates on a common shaft. These capacitors may consist of two or more units and are called multiple or gang capacitors (see Fig. 9-10).

Calculation of Capacitance. The capacitance of variable capacitors can be calculated by using Eq. (9-2). The area of the rotor plates in mesh with the stator is the area to be used. When the rotor plates are completely in mesh with the stator, maximum capacitance is obtained. Rotor plates do not have uniform shapes, and therefore their areas cannot be measured or calculated in the usual manner. If an accurate value of area is desired, a planimeter is used. For the purpose of illustrating the use of this equation we can assume an approximate area for the plates. It can be found by approximation that the area of the average rotor plate used in broadcast receivers will generally average about 2.75 square inches. In variable capacitors, the dielectric is air; therefore $K$ is always equal to 1 .

Example 9-4. What is the maximum capacitance of a 15-plate variable capacitor if each rotor plate has an area of 2.75 square inches and each air gap is 0.025 inch?

| Given: | Find: |  |
| ---: | :--- | ---: | :--- |
| $A$ | $=2.75 \mathrm{sq}$ in. | $C=?$ |
| $N$ | $=15$ |  |
| $K$ | $=1$ |  |
| $t$ | $=0.025$ |  |

Solution:

$$
\begin{aligned}
C & =\frac{22.45 \times K \times A \times(N-1)}{10^{8} t} \\
& =\frac{22.45 \times 1 \times 2.75 \times 14}{10^{8} \times 25 \times 10^{-3}} \\
& =0.000346 \\
& =346 \mu \mu \mathrm{f}
\end{aligned}
$$

Variable capacitors having 15 plates usually have a capacitance of about 350 mieromicrofarads; therefore it can be seen that this equation can be used for variable as well as fixed capacitors.

Adjustable Capacitors. Adjustable capacitors, sometimes referred to as trimmers or padders, are used to adjust the capacitance in a circuit, balance the capacitance between two or more tuning circuits, or neutralize a circuit. This type of capacitor consists of two plates insulated from each other by means of a mica sheet, constructed so that the distance separating the two


Fig. 9-10.-Commercial types of multiple, or gang, capacitors: (a) two-gang capacitor, (b) three-gang capacitor. (Meissner Manufacturing Division, Maguire Industries, Inc.)
plates can be varied by means of adjusting a small setscrev (see Fig. 9-11). Trimmer and padder capacitors are generally rated as to their minimum and maximum capacitance. Trimmers having minimum capacitance as low as 0.5 micromicrofarad and maximum capacitance up to 500 micromicrofarads can be obtained. All variable or adjustable capacitors have a certain amount of minimum capacitance. This is ordinarily very small in relation to the maximum value and generally can be ignored. Capacitors used for trimmers and padders have a very small maximum capacitance; therefore, in order to obtain adjustments over a large range, it is essential that the minimum capacitance be as small as possible.

9-5. Capacitive Reactance, Angle of Lead. It has been shown that when a voltage is applied to the plates of a capacitor a momentary current will flow and cause the plates to become charged, one negative and the other positive. The negative plate will have an excess of electrons, and if a higher voltage of the same polarity is applied to the capacitor, the effect will be to tend to increase the number of electrons on that plate. Since the plate already has an excess of electrons, its action will tend to oppose
the increase. This corresponds with the definition that capacitance opposes any change in the amount of voltage. However, if the increased voltage is maintained, it will in a short time cause additional electrons to be transferred from one plate to the other. From this it can be seen that capacitance offers an opposition to the flow of current just as inductance causes an opposition to the flow of current. It can also be seen that capacitance delays a change in voltage just as inductance delays a change in current.

Capacitive Reactance. The opposition to the flow of current offered by a capacitance is called the capacitive reactance. It is expressed in ohms, and its symbol is $X_{c}$. The value of the capacitive reactance is affected by two factors: the capacitance of the circuit and the rate of speed at which the voltage is changing. The capacitance of a circuit depends on the physical characteristics of the capacitor (or circuit) and has been explained in Art. 9-2. The rate of speed at which the voltage is changing is proportional


Fig. 9-11.-Commercial types of adjustable capacitors. (Hammarlund Manufacturing Company, Inc.) to the frequency of the power supply to which it is connected. The effect of these two factors is indicated in the following equation

$$
\begin{equation*}
X_{C}=\frac{1}{2 \pi f C} \tag{9-3}
\end{equation*}
$$

where $X_{C}=$ capacitive reactance, ohms
$f=$ frequency, cycles per second
$C=$ capacitance, farads
Since the capacitance is usually expressed in microfarads the following equation will find greater use.

$$
\begin{equation*}
X_{c}=\frac{10^{6}}{2 \pi f C}=\frac{159,000}{f C} \tag{9-4}
\end{equation*}
$$

where $X_{c}=$ capacitive reactance, ohms
$f=$ frequency, cycles per second
$C=$ capacitance, microfarads
Example 9-5. What is the capacitive reactance of a 10 -microfarad capacitor when connected to a 60 -cycle circuit?

$$
\begin{array}{lr}
\text { Given: } & \text { Find: } \\
f=60 & X_{C}=? \\
C=10 &
\end{array}
$$

Solution:

$$
\begin{aligned}
X_{C} & =\frac{10^{6}}{2 \pi f C} \\
& =\frac{10^{6}}{6.28 \times 60 \times 10} \\
& =265 \mathrm{ohms}
\end{aligned}
$$

If a capacitor could be built without any losses, the voltage change would be delayed by such an amount that the current flow would lead the voltage by 90 electrical degrees. Furthermore, the only opposition to the flow of current would be the capacitive reactance. Therefore, the current taken by a perfect capacitor would be

$$
\begin{equation*}
I_{C}=\frac{E_{C}}{X_{C}} \tag{9-5}
\end{equation*}
$$

Substituting $10^{6} / 2 \pi f C$ for $X_{c}$ in Eq. (9-5), it may be restated as

$$
\begin{equation*}
I_{C}=\frac{2 \pi f C E_{C}}{10^{6}} \quad(\text { when } C \text { is in microfarads }) \tag{9-6}
\end{equation*}
$$

Example 9-6. What current will the capacitor of Example 9-5 draw from a $110-$ volt 60 -cycle line?

| Given: | Find: |  |
| ---: | :--- | ---: |
| $C$ | $=10$ | $I=?$ |
| $X_{C}$ | $=265$ |  |
| $E$ | $=110$ |  |
| $f$ | $=60$ |  |

Solution:

$$
\begin{aligned}
I & =\frac{E}{X_{C}} \\
& =\frac{110}{265} \\
& =0.415 \mathrm{amp}
\end{aligned}
$$

Example 9-7. How much current will a 0.05 -microfarad capacitor take when connected to an a-f circuit of 25 volts and 5000 cycles?

$$
\begin{array}{lr}
\text { Given: } & \text { Find: } \\
C=0.5 & I=? \\
f=5000 & \\
E & =25
\end{array}
$$

Solution:

$$
\begin{aligned}
I & =\frac{2 \pi f C E}{10^{6}} \\
& =\frac{6.28 \times 5000 \times 0.05 \times 25}{10^{6}} \\
& =0.0392 \mathrm{amp} \\
& =39.2 \mathrm{ma}
\end{aligned}
$$

Effect of the Resistance of a Capacitive Circuit. The discussion on capacitors up to this point has assumed a perfect capacitor, that is, one that has no resistance. In actual practice, it is impossible to have a circuit containing capacitance only, because the plates and the connectors of the capacitor have some resistance. This resistance is usually so small compared with the capacitive reactance that it is ignored and the circuit is assumed to contain capacitance only.

If, however, the resistance is to be considered, its ohmic effect must be combined with the ohmic effect of the capacitive reactance. The combined ohmic effect is called the impedance and is represented by the symbol Z. Mathematically it is equal to

$$
\begin{equation*}
Z=\sqrt{R^{2}+X_{c}^{2}} \tag{9-7}
\end{equation*}
$$

where $Z=$ impedance of the circuit, ohms
$R=$ resistance of the circuit, ohms
$X_{c}=$ capacitive reactance of the circuit, ohms.
When both the resistance and the capacitive reactance of a circuit are taken into consideration, the current flowing in the circuit will be equal to the voltage of the circuit divided by its impedance, or

$$
\begin{equation*}
I=\frac{E}{Z} \tag{8-9}
\end{equation*}
$$

where $I=$ current flowing in the circuit
$E=$ voltage of the circuit
$Z=$ impedance of the circuit.
When both the resistance and the capacitive reactance of a circuit are 1 i ken into consideration, the angle by which the current leads the voltage will depend upon the relative amount of resistance and capacitive reactance
in the circuit. The angle by which the current leads the voltage is expressed in electrical degrees and is sometimes referred to as the phase angle. Its value may be determined mathematically by the equation

$$
\begin{equation*}
\cos \theta=\frac{R}{\bar{Z}}=\frac{R}{\sqrt{R^{2}+\overline{X_{c^{2}}}}} \tag{9-8}
\end{equation*}
$$

Example 9-8. What are the impedance, current, and phase angle of a 10 -microfarad capacitor connected to a 110 -volt 60 -cycle line? Note that the capacitor is not a perfect one and that its resistance has the same effect as a 10 -ohm resistor connected inseries with the capacitor.

$$
\begin{aligned}
& \text { Given: } \\
& \begin{aligned}
C & =10 \\
E & =110 \\
f & =60 \\
R & =10
\end{aligned}
\end{aligned}
$$

$$
\begin{aligned}
& \text { Find: } \\
& \begin{aligned}
Z & =? \\
I & =? \\
\theta & =?
\end{aligned}
\end{aligned}
$$

Solution:

$$
\begin{aligned}
X_{C} & =\frac{10^{9}}{2 \pi f C} \\
& =\frac{10^{6}}{6.28 \times 60 \times 10} \\
& =265 \mathrm{ohms} \\
Z & =\sqrt{ } R^{2}+X_{c^{2}} \\
& =\sqrt{10^{2}+265^{2}} \\
& =265.2 \mathrm{ohms} \\
I & =\frac{E}{Z} \\
& =\frac{110}{265.2} \\
& =0.414 \mathrm{amp} \\
\cos \theta & =\frac{R}{Z} \\
& =\frac{10}{265.2} \\
& =0.0377 \\
\theta & =88^{\circ} \text { (from Appendix XI) }
\end{aligned}
$$

Examining the results of this cxample, it may be seen that the value of the capacitive reactance and the impedance are practically the same, and hence the current of the circuit too will be practically the same whether the
resistance is considered or ignored. This is always the case when the capacitive reactance is 10 or more times greater than the resistance.

9-6. Losses in a Capacitor. It has just been stated that capacitors cannot be built without some amount of losses. The losses of a capacitor may be divided into three groups: (1) leakage losses, (2) dielectric losses, (3) resistance losses.

Leakage Losses. If a capacitor is connected to a source of d -e voltage, it will become charged, and when it is disconnected from the sourco it will still be in a charged condition. If the capacitor were a perfect one, the charge would remain indefinitely. Actually, this condition is not achieved because there will be a flow of electrons from the negative plate, through the diclectric, to the positive plate. This flow of electrons through the dielectric causes heating of the dielectric, though only slight, and is classed as a loss commonly called the leakage loss. This loss cannot be eliminated because there is no known perfect insulator. It can be kept at a low value by using a high-resistance material for the dielectric. The quality of the dielectric may be determined from its value of insulation resistance. Mica and ceramics have excellent qualities and when used in capacitors produce low leakage losses for the capacitor. Paper dielectric capacitors have higher leakage losses than mica and ceramie capacitors, the amount of losses depending largely upon the quality of paper used.

Dielectric Losses. If the terminals of a charged capacitor are connected to one another momentarily, the capacitor will be discharged, as may be seen by connecting a voltmeter to its terminals. If the capacitor is left disconnected for a short time and again tested with a voltmeter, it will be found that it has some amount of charge, generally called the absorbed charge. This is believed to be due to the fact that the distorted electron orbits do not return to their normal state immediately. When the capacitor is connected to an a-c voltage, this effect becomes greater as the frequency of the power source is increased. At the higher frequencies, this absorbed charge is lost and is called the dielectric absorption loss. Also, when a-c power is applied to the capacitor, the electron orbits of the dielectric are continually being distorted in different directions due to the reversals of the a-c voltage. Energy is required to change the electron orbits, and this energy is considered as a loss, commonly called the dielectric hysteresis loss. These two losses, the diclectric absorption loss and the dielectric hysteresis loss, are generally combined and referred to merely as the dielectric losses.

Resistance Loss. The plates of the capacitor, the connections to the plates, and the leads (terminals) all carry the capacitor current, and all have some amount of resistance. The current flowing through this resistance produces a loss which is called the resistance loss.

Combined Loss. The effect of these three losses, leakage, dielectric, and resistance, is usually combined and called the capacitor losses. At low frequencies the effect is so small that it is often disregarded, but at higher frequencies it must be considered. When the losses are considered, the capacitor may be treated either as (1) a perfect capacitor connected in series with a pure resistor or as (2) a perfect capacitor connected in parallel with a pure resistor. The value of the capacitor should be the same as that of the actual capacitor, and the value of the resistor should be such that it will produce a phase angle equal to that of the actual capacitor.

9-7. Voltage Ratings of Capacitors. Effect of Voltage. In the study of capacitors, it was shown that the capacitance varies inversely with the thickness of the dielectric [Eq. (9-1)]. For this reason, thin dielectrics are used in order to get the desired capacitance without making the capacitor too large. On the other hand, the dielectric must have sufficient strength to prevent the electrons from flowing through it. The choice of the dielectric material and its thickness therefore has an important bearing on the construction of the capacitor.

When a voltage is applied to the plates of a capacitor, it causes a strain on the electron orbits of the dielectric separating the plates. As the voltage is increased, the strain will be increased, and it seems natural to conclude that at some value of voltage the strain will be sufficient to cause a breakdown. When this occurs, the electrons find a path from one plate to the other. This causes a discharge through the dielectric, usually in the form of a spark, which burns a small hole through the dielectric. This results in a permanent change in the physical characteristics of the dielectric, and the capacitor should then be discarded because it is no longer useful.

Breakdown Voltage. The ability to withstand the strain caused by a voltage varies with different materials. The voltage at which a material breaks down is called the breakdown voltage. Tests are made for the breakdown voltage of various materials, and the results are arranged in tabular form as in Appendix VI. The breakdown voltage is expressed for a certain thickness of material such as one-thousandth of an inch or one centimeter. This value is also known as the dielectric strength.

Voltage Rating of Capacitors. For reasons now obvious, capacitors are rated for the maximum voltage at which they may be safely operated as well as for their capacitance. The voltage rating is usually specified for two conditions: (1) the d-e working voltage, (2) the a-c working voltage. The d-c working voltage will have a higher value than the a-c because the a-c line voltages referred to are effective values. An a-c power supply has a maximum voltage value that is 1.414 times as great as the effective value (see Art. $7-6$ ). For example, a capacitor that is rated at 450 volts $\mathrm{d}-\mathrm{c}$ will
have an a-c rating of 450 divided by 1.414 , or 318 volts. The a-c workingvoltage ratings are further reduced to account for: (1) non-sine-wave voltages, which may cause the maximum voltage to be more than $1 .+14$ times the effective value; (2) heating of the capacitor, which reduces the dielectric strength; (3) operation at higher frequencies, which increases the losses and heating of a capacitor.

The working voltage of variable-type air capacitors must also be taken into consideration. This is done by the spacing of the plates. In radio receivers, this is not of great importance because the voltages are of moderatc values. Transmitting capacitors of ten have very high voltages applied across their plates, and greater spacing is used.

9-8. Capacitors in Series and Parallel. Voltage Distribution of Seriesconnected Capacitors. When the voltage rating of a capacitor is less than the voltage of the circuit to which it is to be connected, it is possible to use several capacitors connected in series for this condition. According to the principles of the series circuit, the voltage will divide among the series members of the circuit. When several capacitors are connected in series on an a-c circuit, the voltage will divide among them in proportion to their capacitive


Fig. 9-12.-Effect of two capacitors connected in series. reactances. For example, three $10-$ microfarad capacitors connected in series across a 600 -volt a-c power supply will have one-third of 600 volts, or 200 volts, across each capacitor.

When several series-connected capacitors are used on a d-c power line, the voltage will divide in proportion to their d-e resistances. As the $\mathrm{d}-\mathrm{c}$ resistance of capacitors varies considerably, the d-c voltage distribution will also vary considerably. For example, if two 10 -microfarad capacitors with d-c resistances of 30 and 10 megohms, respectively, are connected to a 600 -volt d-c power supply, the voltage distribution will then be $\frac{30}{40}$ of 600 and $\frac{10}{40}$ of 600 , or 450 volts and 150 volts, respectively.

Capacitance of a Series-connected Group. When two capacitors are connected in series, the combined capacitance will be lower than either individual capacitance. This is true because connecting them in series has the same effect as increasing the thickness of the dielectric. If two capacitors are connected in series as in Fig. 9-12, the electrons from plate $A$ are transferred through the external circuit to plate $D$ and the orbital electrons of both dielectrics, $F$ and $G$, are under strains. The effect is therefore the same as one dielectric of a thickness equal to the sum of $F$ and $G$.

The combined capacitance of capacitors connected in scries may be found by the equation

$$
\begin{equation*}
C_{T}=\frac{1}{\frac{1}{C_{1}}+\frac{1}{C_{2}}+\frac{1}{C_{3}}, \text { etc. }} \tag{9-9}
\end{equation*}
$$

Example 9-9. What is the total capacitance of a circuit containing two 10 -microfarad and two 5 -microfarad capacitors connected in series?


Fig. 9-13.-Effect of two capacitors connected in parallel.

Given: Find:
$C_{1}=10 \quad C_{T}=$ ?
$C_{2}=10$
$C_{3}=5$
$C_{4}=5$
Solution:

$$
\begin{aligned}
& C_{T}=\frac{1}{\frac{1}{C_{1}}+\frac{1}{C_{2}}+\frac{1}{C_{3}}+\frac{1}{C_{4}}} \\
&=\frac{1}{\frac{1}{10}+\frac{1}{10}+\frac{1}{5}+\frac{1}{5}} \\
&=\frac{1}{\frac{1+1+2+2}{10}=\frac{1}{\frac{6}{10}}=\frac{10}{6}} \\
&=1.67 \mu \mathrm{f}
\end{aligned}
$$

Capacitance of a Parallel-connected Group. When several capacitors are connected in parallel, the combined capacitance will be greater than the highest individual capacitance. This is true because connecting them in parallel has the same effect as increasing the area of the plates. If two capacitors are connected in parallel as in Fig. 9-13, the electrons from plates $A$ and $C$ are transferred through the external circuit to plates $B$ and $D$. The effect is therefore the same as one capacitor whose plate area is equal to the total area of the plates of both capacitors.

The combined capacitance of capacitors connected in parallel may be found by the equation

$$
\begin{equation*}
C_{T}=C_{1}+C_{2}+C_{3}, \text { etc. } \tag{9-10}
\end{equation*}
$$

Example 9-10. What is the total capacitance of a circuit containing two $10-$ microfarad and two 5 -microfarad capacitors connected in parallel?

| Given | Find: |
| ---: | :--- |
| $C_{1}$ | $=10$ |
| $C_{2}$ | $=10$ |
| $C_{3}$ | $=5$ |
| $C_{4}$ | $=5$ |

Soultion:

$$
\begin{aligned}
C_{T} & =C_{1}+C_{2}+C_{3}+C_{4} \\
& =10+10+5+5 \\
& =30 \mu \mathrm{f}
\end{aligned}
$$

When capacitors are connected in parallel, full voltage exists across each capacitor; therefore each must have a voltage rating sufficiently high to meet full line-voltage conditions.


Fig. 9-14.-Distributed capacitances of a coil.
9-9. Distributed Capacitance. Coils used in radio and television work are designed to have a definite amount of inductance. Because of the manner in which they have to be constructed, all coils have some resistance and some capacitance. Both these factors increase the encrgy loss of a coil, and it is therefore desirable to keep the resistance and capacitance of the coil at a minimum. The factors affecting the effective resistance of a coil have been taken up in the previous chapter. How the distributed capacitance of a coil increases this effective resistance and how it can be reduced will now be taken up.

In the definition of a capacitor, it was stated that two conductors separated by an insulator will form a capacitor and that if an a-e voltage is impressed across these two conductors, capacitor action will result. Because of this, any inductance coil will have capacitances between adjacent turns, capacitances between turns that are not adjacent, capacitances between terminal leads, and capacitances between the ground and each
turn (see Fig. 9-14). The amount of energy stored in any one of these capacitors is equal to the product of its capacitance and the voltage existing between the coil turns involved. If an a-c voltage is impressed across a coil, the greatest voltage will exist between the two ends of the coil, but there also will be a voltage between each adjacent turn or any two turns. For example, if 100 volts is impressed across a coil having 50 turns, the voltage between any two adjacent turns will be two volts and between every second turn four volts, etc. The amount of voltage between each turn and ground will depend on how the coil is connected in the circuit in respect to the ground. Fach turn is separated from its adjacent turn by a small space which may be filled by the wire's insulation or air. As the capacitance between two conductors will vary inversely with the distance between them, the greatest capacitance of a coil will be between two adjacent turns. The total effect produced by each of the capacitors can usually be represented to a high degree of accuracy by assuming that these


Fig. 9-15.--The equivalent distributed capacitance of a coil considered as a capacitor $C_{D}$ connected in parallel with the coil. capacitors can be replaced by a single capacitor of an equivalent value, connected across the terminals of the coil (Fig. 9-15). This equivalent capacitance $C_{D}$ is called the distributed capacitance of the coil.

Effects of Distributed Capacitance. One effect of distributed capacitance is to bypass a certain amount of the r-f currents. The amount of current that is by-passed will increase directly with the frequency. This can be seen by substituting the inductance of the coil in Eq. (8-6), $X_{L}=2 \pi f I$, and the distributed capacitance in Eq. (9-3), $X_{C}=$ $1 / 2 \pi f C$. The inductive reactance of the circuit will increase as the frequency increases, thus allowing less current to flow through the coil. The capacitive reactance will decrease as the frequency increases and will allow more current to flow through the distributed capacitance. At low frequencies, the effects of distributed capacitance are negligible and can be ignored. At very high frequencies, the distributed capacitance of a coil may be of greater importance than its inductance. This leakage of current through the distributed capacitance causes a loss of energy that will increase directly with frequency increase. This loss in energy is equivalent to a resistance loss, and the effect produced is the same as if the effective resistance of the coil was increased.

Another effect of distributed capacitance is the manner in which it affects the tuning circuit. It can be seen from Fig. 9-15 that the distributed capacitance is connected in parallel with the variable tuning capacitor, thus increasing the effective capacitance of the circuit. The distributed capacitance is a poor capacitor having a large amount of resistance. Increasing the amount of resistance in a circuit will decrease the amount of current
flow; therefore the resistance of the distributed capacitance further increases the energy loss.

Distributed capacitance may sometimes proluce another effect. The distributed capacitance and coil form a parallel resonant circuit. 'This circuit will be resonant at some rather high frequency. At this frequency, oscillating currents will circulate in the winding and distributed capacitors. This effect will be taken up in greater detail in the chapter on Resonance.

Methods of Reducing Distributed Capacitance. The distributed capacitance of a coil may be reduced by using one or more of the following methods: (1) Increasing the length of the coil. This is true because it, increases the distance between the coil ends or points of greatest voltage. (2) Decreasing the diameter of a coil. This increases the number of turns required and decreases the voltage between turns, thus decreasing the distributed capacitance of the coil. (3) Using wire whose insulation has a low dielectric constant. The capacitance between two conductors will vary directly with the value of its dielectric constant. Air, whose value is one, has the lowest constant. A coil wound with bare wire will therefore have the least distributed capacitance. Cotton is next, then silk; enamel is the highest. (4) Decreasing the size of wire used. This will decrease the surface area of each turn, thus decreasing the distributed capacitance. However, decreasing the size of the wire will also increase its resistance. (5) Using a core with a low dielectric constant. The lower the dielectric constant of the form on which a coil is wound, the less will be its distributed eapacitance. Dry paper or cardboard is best from this standpoint. Bakelite has a higher dielectric constant and increases the distributed capacitance.

9-10. Electrolytic Capacitors. Irinciple of Electrolytic Capacitors. An electrolytic capacitor consists of two metallic plates separated by an electrolyte. From the description of the action and construction of capacitors, it would seem that the purpose of the electrolyte is to act as the dielectric or insulator. This supposition is inaccurate because the electrolyte is not the actual dielectric material but is the negative electrode. The dielectric material consists of an extremely thin oxide film which is formed on the surface of the positive capacitor plate. A peculiar characteristic of aluminum and a few other metals is that, when they are immersed in certain electrolytic solutions and a current is passed through the metal and the electrolyte, a nonconducting film will be formed on the metal. This action can be illustrated by the following experiment.

Action of Electrolytic Capacitor. Two aluminum plates are immersed in a suitable electrolyte, as shown in Fig. 9-16. 'This arrangement is now connected in a d-c circuit so that current will pass from one plate to the other. When the circuit is closed, the current will be very high and will taper off until there is very little or no current flowing in the circuit. This
action is due to a process called forming, which means that an insulating film is being formed on the surface of one of the plates. If the electrodes are aluminum, this film will always form on the plate connected to the positive terminal. The formation of this film on the plate retards the flow of current. If the polarity of the voltage applied to the terminals is reversed, the film will form on the opposite plate and current will flow in the circuit until this forming process is sufficient to build up a large enough resistance to block all current flow. From this experiment, we can see that the film acts as an insulator only as long as the same polarity used in forming is maintained on its terminals. This principle of one-way current conduction was used for years in the electrolytic rectifier, previous to its application to the construction of capacitors.


Fig. 9-16.-Construction of a simple electrolytic capacitor.


Fig. 9-17.-Construction of a simple electrolytic capacitor for direct current.

Factors Affecting the Capacitance. The capacitance of the electrolytic capacitor depends on (1) the area of the plates, (2) the thickness of the diclectric, (3) the dielectric constant. By means of an electrochemical etching process, the effective area of the surface of the plates can be increased without increasing its physical dimensions. Electrolytic capacitors using etched plates can be made considerably smaller than a capacitor of similar capacitance using smooth plates. Recently, a method has been developed further to reduce the size of electrolytic capacitors. An aluminum plate, known as a fabricated plate, is formed by spraying molten aluminum on a special carrier. The effective surface area of a plate is considerably increased by using this method. Reducing the physical size of a capacitor by reducing the physical dimensions of the plates by either of the two methods explained does not generally affect the useful life of the capacitor.

Dielectric. The dielectric used in electrolytic capacitors is the oxide film formed on the surface of the positive clectrode. The thickness of this film will depend on the material used for the plates and the amount of
voltage used in its forming process. The higher the applied voltage, the thicker the film becomes and the lower will be the resultant capacitance. The maximum working voltage of an electrolytic capacitor must be less than the voltage used in its forming process. If this voltage is exceeded, the thickness of the oxide film will be increased and the capacitance of the capacitor will be reduced. As long as a capacitor is operated on voltages less than that used in forming, the thickness of the oxide film and the capacitance will remain constant.

Forming Voltage. The maximum voltage that may be used in forming will depend on the electrolyte used. Concentrations of borates, phosphates, citrates, silicates, etc., of sodium or ammonia dissolved in water are used in making electrolytic solutions. Acid electrolytes are not used because of the corrosive properties of acids. Solutions of borax are used extensively because of the high voltages that they can withstand and because of their noncorrosive properties. Oxide films formed by using aluminum plates and a borax (sodium tetraborate) solution will withstand as much as 480 volts. Inasmuch as the thickness of the oxide film is determined by the voltage used in forming, the capacitance of a capacitor will depend on this voltage, which in turn is dependent on the material of the plates, the electrolyte used, and the surface area of each plate.

Self-healing Characteristic. The oxide film formed on the surface of a plate is very thin, usually less than 0.00001 inch. Since this thickness is only a fraction of the thickness of any other dielectric of similar voltage rating, large capacitance may be obtained in a small space. When the voltage applied exceeds the maximum critical voltage that the clectrolyte can withstand, the thin oxide film will be punctured. This causes a short circuit between its terminals. When the applied voltage is removed or reduced to a value less than its maximum critical voltage, the break will be monded by the formation of a new insulating film. This self-healing characteristic of electrolytic capacitors is another advantage they have in comparison with capacitors using paper dielectrics. Momentary overvoltages applied to capacitors using paper as a dielectric will generally cause a permanent breakdown. If a momentary overvoltage is applied to an electrolytic capacitor, its film gives way while the surge lasts but will be restored as soon as the excessive strain is removed. Thercfore temporary excessive voltages will not permanently destroy the dielectric insulation.

The capacitor shown in Fig. 9-16 is suitable for use in a-c circuits as it will not allow the flow of electricity through it in either direction. In radio work, electrolytic capacitors are generally used with direct or pulsating currents which do not reverse polarity. In such circuits, a capacitor whose terminals have a fixed polarity (Fig. 9-17) is used. This capacitor consists of an aluminum plate to form the positive electrode, an electrolyte to form the negative electrode, and an oxide film as the dielectric. In order to
make a connection to the electrolyte, an inactive plate is used. This plate serves merely to make an electrical connection from the solution to the negative side of the circuit and has no capacitor action whatever. It is impossible for the electric current to pass from the aluminum to the electrolyte; therefore this type of capacitor provides the necessary insulating effect as long as the positive side of the circuit is connected to the positive terminal (aluminum plate).


Fig. 9-18.—Cross-sectional view showing the construction of a commercial wet-electrolytic capacitor.

9-11. Wet and Dry Electrolytic Capacitors. Construction of Wet Electrolytic Capacitors. Electrolytic capacitors may be either wet or dry and consist of one or more metal electrodes immersed in an electrolyte enclosed by some suitable container. The electrolyte used in wet electrolytic capacitors is always in liquid form, and the container must be made leakproof. In practically all commercial capacitors, the container also serves as the inactive electrode, since it makes direct contact with the electrolyte and is usually made of copper or aluminum. The positive electrode is generally made of aluminum, and a number of different methods are used to increase its surface area. Some of these methods are: (1) concentrically winding a thin sheet of material; (2) using a combination of a hollow and corrugated material; (3) a combination of crimping and corrugating of the electrode. These types of construction are known commercially as the
coil type, accordion-pleated type, corrugated type, helical type, and radial type (see Fig. 9-20). The solution fills up all the empty space so that a large active surface arra is obtained. The stem of the positive electrode is brought through a tight-fitting hole in the cover and acts as the positive terminal. It is usually threaded to receive terminal nuts for attaching the connecting wire. The container forms the negative terminal, and its connection is usually made by mounting it directly on the chassis, which is generally the common negative terminal of a radio set. A shield of insulating material is placed between the positive plate and the container to prevent an accidental contact causing a short circuit. This shield is generally made of a sheet of celluloid that has been perforated to permit circulation of the electrolyte. In order to prevent the electrolyte from leaking out,


Fig. 9-19.-Commercial wet-electrolytic capacitors. (P. R. Mallory \& Co., Inc.)
wet electrolytic capacitors should always be mounted in a vertical position. The construction of this type of capacitor is shown in Fig. 9-18.

Construction of Dry Electrolytic Capacitors. By use of a jellylike electrolyte, electrolytic capacitors can be constructed in a dry form. They are considered dry in the same sense that dry cells are considered dry, that is, because the electrolyte cannot be spilled or poured from its container. Dry electrolytic capacitors provide high values of capacitance in relatively small dimensions and are the most economical type for many applications. Most of the electrolytic capacitors used in radio are of the dry type. In general, a dry electrolytic capacitor consists of a positive foil, a negative foil, and a separator containing an electrolyte, which are all wound into a roll and provided with means for electrical connection, housing, and mounting (see Fig. 9-21).

The positive foil, usually made of aluminum, is subjected to a special electrochemical forming process which completely covers it with an extremely thin oxide film. The nature and thickness of this film will govern


Fig. 9-20.-Types of construction used for the positive electrode.
its voltage and capacitance per unit area. The separator is made of some absorbent material, usually gauze, paper, nonfibrous cellulose, or various combinations of these; it serves to hold the electrolyte in position and keep the positive and negative foils from making physical contact. The electrolyte consists of a chemical solution essentially similar to a dry paste and serves as the negative electrode. In addition, it tends to maintain the film on the positive electrode. The negative foil, generally aluminum, is usually
unformed and acts merely as a means of making contact with the electrolyte, which is the negative electrode of the capacitor.

The earliest capacitors used plain aluminum foil for the positive plate.


Fig. 9-21.-Construction of a dry-electrolytic capacitor. (P. R. Mallory \& Co., Inc.)


Fre. 9-22.-Construction of a fabricated-type dry-electrolytic capacitor: (a) Internal construction, (b) eomplete capacitor. ( $P$. R. Mallory \& Co., Inc.)

Later on, an etching process was developed for roughening the surface of this foil, thereby increasing its effective area. The increased arca gave increased capacitance ranging from 2.75 to 1 at high voltages to roughly 7
to 1 at low voltages. With a special fabricated plate (Fig. 9-22), the area was further increased; capacitors using such a plate have a normal capacitance ratio, as compared with plain aluminum foil, of 10 to 1 . Ratios of 20 to 1 or higher are possible under certain conditions but are not used at present. Figure 9-23 illustrates the comparative size of a capacitor using the three different types of plates discussed.

Capacitor Blocks. A great many


FIg. 9-23.-Comparative size of plain-, etched-, and fabricated-type capacitors. (P. R. Mallory \& Co., Inc.) capacitor applications require or permit the negative terminals of the various sections to be ganged together and connected to one common point in the circuit. Since the electrolyte itself is the negative electrode in all electrolytic capacitors, it is obvious that several positive electrodes could be included in one common electrolyte. In actual production, this type of unit consists of one long negative foil and the required number of positive foils laid end to end and arranged parallel to it, which, together with the proper separators, are rolled into one complete unit (see Fig. 9-24). Each positive foil is provided with a positive terminal. Care is taken to see that sufficient space separates the ends of each positive foil to prevent a short circuit at these points. Capacitors so constructed are entirely satisfactory electrically and mechanically and represent an appreciable saving from a production-cost standpoint over a similar combination of individual capacitors.


Fig. 9-24.-A capacitor block with a common negative and three positive plates.
Types of Containers. Dry electrolytic capacitors may be housed in cardboard tubes, cardboard cartons, and round or rectangular metal cans. Various types of mounting features are available, and either soldering lugs,
screw terminals, or flexible leads are provided for external connections (see Fig. 9-25). In order to centrol the quality of a capacitor and at the same time reduce its cost of manufacture, the shape and material of which the


Fig. 9-25-Various forms of dry-electrolytic capacitors. (P' R. Mallory \& Co., Inc.)
container is made have been standardized. Practically all dry electrolytic capacitors are now constructed in round containers. In the better quality capacitors, the container is always made of metal. As there is no danger


Fig. 9-26.-Types of small dry-electrolytic tubular capacitors. (P. R. Mallory \& Co., Inc.)
of the electrolyte leaking out, dry clectrolytic capacitors may be mounted in any position.

9-12. Electrical Characteristics of Electrolytic Capacitors. Comparison of Wet and Dry Types. The electrical characteristics of dry and wet elec-
trolytic capacitors are similar in the manner in which they are rated. Actually, the power factor or filtering efficiency of wet electrolytic capacitors is not so high as that of the modern dry electrolytic capacitors, and therefore wet electrolytic capacitors can be replaced with dry electrolytics of less total capacitance than originally specified. For this reason, practically all electrolytic capacitors are now of the dry type. Because the wet electrolytic capacitor is becom-


Fig. 9-27.-Types of large dry-electrolytic capacitors. (P. R. Mallory \& Co., Inc.) ing obsolete, only the electrical characteristics of dry electrolytic capacitors will be discussed.

Electrolytic capacitors are of large capacitances, 0.25 to 200 microfarads or more, and are generally used in power-supply circuits or to by-pass large quantities of a-f currents. Most dry electrolytic capacitors are designed for continuous use with direct current or intermittent direct current (rectified alternating current). Capacitors designed for a-c applications are usually restricted to intermittent operation, as in starting service on capacitor-induction motors.

Polarized Types. In radio applications, electrolytic capacitors are limited to circuits that are polarized with or without limited a-c components. For this reason, most electrolytic capacitors are usually polarized. In other words, one terminal is marked positive and the other negative. Consequently, polarized capacitors should not be subjected to reversed polarity as the heavy current passing through the capacitor under this condition will increase the internal temperature and may cause serious damage to the unit. The negative electrode will tend to form an oxide coating when the polarity is reversed. Repeated applications with a reversed polarity will cause a drop in capacitance. This is obvious, as under such a circumstance two capacitors are now in series.

Nonpolarized Type. Certain types of dry electrolytic capacitors are so constructed that they function equally well in either direction on d-c lines
from a polarizing standpoint. They are not designed for alternating currents and therefore should not be used on a-c circuits. In this construetion, both plates are formed to exactly the same voltage. In such a case, the plates lose their identity and simply become electrodes. As both have the same surface area, the unit is considerably larger than a similar polarized capacitor rated at the same capacitance. Nonpolarized capacitors are used wherever the voltage supply may become reversed and remain so indefinitely.

Voltage Rating. The voltage rating of a dry electrolytic capacitor is determined by the character of the oxide film, the forming voltage, and the electrolyte used in the finished capacitor. This type of capacitor is rated at its continuous d -c working voltage. Its maximum over-all peak voltage, maximum superimposed a-c component or ripple voltage, and its surge voltage are also important characteristics.

D-C Working Voltage. This is the maximum d-c voltage the capacitor will withstand satisfactorily under continuous operating conditions within its normal temperature range.

Peak Ripple Voltage or A-C Component. This is the maximum instantaneous value of alternating voltage across the capacitor due to the a-e component in the capacitor.

Peak Voltage. This represents the direct voltage plus the peak alternating ripple voltage and refers to continuous operation.

Surge Volitage. This term is used in reference to tests made for comparative purposes. It is the maximum voltage the capacitor will withstand without injury for a period of five minutes when applied to a series combination of the capacitor and a resistor having a value in ohms equal to 20,000 divided by the rated capacitance in microfarads of the capacitor being tested. Momentary surges are sometimes encountered in service and will not damage the capacitor if they do not excced this rating.

Current Characteristics. The leakage-current characteristic of a dry electrolytic capacitor represents the amount of direct current flowing through the capacitor other than its momentary charging current. It is an indication of the quality of the oxide film and is a direct expression of the insulation resistance of the capacitor.

Normal Leakage Current. This represents the d-e leakage current in actual service and should become lower with continued use. A wellmade capacitor will have an exccedingly small leakage current when in continuous use. On intermittent operation, the normal value of leakage current may vary between 50 and 100 milliamperes per microfarad, depending on the capacitance and voltage rating.

Initial Leakage Current. This represents the amount of current drawn by the capacitor when first applied to the voltage source. The
initial current is relatively high as compared with the normal leakage current, but it should drop quickly at first and then more gradually until it reaches the normal leakage value.

Power Factor and Equivalent Series Resistance. The power factor of a capacitor for all practical purposes is the ratio between the equivalent series resistance and the capacitive reactance at a given frequency. It is expressed in per cent and indicates mainly the amount of energy consumed by the capacitor.

The equivalent series resistance may be used as a comparative char-


Fig. 9-28.-Factors affecting the equivalent series resistance of a dry-electrolytic capacitor. (P. R. Mallory \& Co., Inc.)
acteristic similar to power factor. It is more generally used for calculating purposes and has been found preferable to power factor. The equivalent series resistance represents the total losses in a capacitor divided by the square of the current flow. It is due to (1) dielectric loss of the oxide film, (2) contact resistance, (3) electrolyte resistance, (4) insulation resistance. These factors are illustrated in Fig. 9-28. Because of the nature of an electrolytic capacitor, it would be difficult to ascertain accurately the value of all these losses separately. The combined effect of these losses is expressed as the equivalent resistance required to produce an $I^{2} R$ loss of the same magnitude.

Impedance and Temperature Characteristics. The value of a low impedance characteristic at high frequencies is becoming more important with the development of efficient all-wave portable and automobile radio re-
ceivers. It is possible to obtain dry electrolytic capacitors having a r-f impedance low enough, at 10 or 20 megacycles, to assist in the suppression of vibrator noise or other high-frequency disturbanees in rectifier circuits.

Temperature is an important consideration from an application standpoint as it is closely related to all the characteristics of a capacitor. In planning the location of the capacitor with respect to other component


Fig. 9-29.-Capacitor analyzer (Solar Manufacturing Corporation.)
parts, serious consideration should be applied to the capacitor's proximity to transformers, tubes, and high-current resistors because of the usual temperature rise involved in these components.

9-13. Measurement of Capacitance. Methods Used. The method of calculating capacitance from the physical characteristics of a capacitor has been discussed in Art. 9-2. It is often desired to determine the capacitance of a capacitor, but some factor required for calculating the capacitance may not be readily obtained. In such cases, it becomes necessary to measure the capacitance of the capacitor. There are two methods available for measuring the capacitance of a capacitor, namely, the comparison method and the impedance method.

Comparison Method. The comparison method involves the use of a standard capacitor and some form of bridge circuit. The circuits involved and the theory of such bridge circuits may be obtained from Arts. 6-17 and $6-18$ and also from the references at the end of this chapter. There are a number of commercial bridges available for measuring capacitance, one of which is shown in Fig. 9-29. The use of such a bridge is not very difficult, and quite accurate results may be obtained.

Impedance Method. The impedance method involves the taking of voltmeter, ammeter, frequency, and wattmeter readings of the capacitor when connected to an a-c power source. The capacitance may then be calculated by use of Eqs. (2-14), (8-9), (9-4), (9-7), (9-8) as illustrated in the following example.

Example 9-11. It is desired to determine the capacitance of a capacitor by the impedance method. The capacitor is connected to a circuit as shown in Fig. 9-30, and the following meter readings are obtained: voltmeter 110 volts, ammeter 0.415 ampere, wattmeter 1.5 watts, frequency 60 cycles. (a) What is the capacitance of the capacitor? (b) What is the power factor of the capacitor? (c) What is its angle of lead?

| Given: | Find: |  |  |
| ---: | :--- | ---: | :--- |
| $E$ | $=110$ | $C$ | $=?$ |
| $I$ | $=0.415$ | $\mathrm{P}-\mathrm{F}$ | $=?$ |
| $P$ | $=1.5$ | $\theta$ | $=?$ |



Fig. 9-30.
Solution:
(a)

$$
\begin{aligned}
Z & =\frac{E^{\prime}}{I}=\frac{110}{0.415}=265 \mathrm{ohms} \\
R & =\frac{P}{I^{2}}=\frac{1.5}{0.415 \times 0.415}=8.72 \mathrm{ohms} \\
X_{C} & =\sqrt{Z^{2}-R^{2}}=\sqrt{265^{2}-8.72^{2}} \\
& =\sqrt{70,225-76}=\sqrt{70,149} \\
& =264.86 \mathrm{ohms} \\
C & =\frac{10^{\mathrm{B}}}{2 \pi f X_{C}} \\
& =\frac{10^{6}}{6.28 \times 60 \times 264.86}=\frac{10^{6}}{100,000}=10 \mu \mathrm{f}
\end{aligned}
$$

(b)

$$
\begin{aligned}
\mathrm{P}-\mathrm{F} & =\frac{R}{Z} \\
& =\frac{8.72}{265}=0.0329
\end{aligned}
$$

(c)
$\theta=88$ degrees
Voltmeter-ammeter Method. Measuring the capacitance of a capacitor by the impedance method involves the use of two meters that are not ordinarily found in most shops or laboratories, namely, a frequency meter and a low-reading low-power-factor wattmeter. A quick, yet practical, method involves the use of just an ammeter and a voltmeter, two instru-


Fig. 9-31.-Impedance method of determining capacitance: (a) voltmeter-ammeter method, (b) voltmeter-ammeter method including a neon lamp to test for a short-circuited capacitor.
ments usually found in most laboratories. In its simplest form, the circuit in Fig. 9-31a is employed. This method does not take into consideration any series equivalent resistance that the capacitor may have, and it also assumes that the impedance of the ammeter is zero and that the frequency of the line is known. For these conditions

$$
\begin{equation*}
C=\frac{10^{6} I}{2 \pi f E} \quad[\text { from Eq. (9-6)] } \tag{9-11}
\end{equation*}
$$

The error involved in using this equation (due to ignoring the capacitor's resistance) is generally very small, and therefore this method can be used for a quick check on the capacitance of a capacitor.

When measuring capacitors, care should be taken to determine whether the capacitor is short-circuited before connecting it in series with the ammeter and the line. If a short-circuited capacitor were connected to a circuit similar to Fig. 9-31a, it would probably cause the ammeter to be damaged. A test for a short-circuited capacitor can be included by connecting a neon lamp and a single-pole switch in the circuit, as shown in Fig. 9-31b. The switch $A$ is left open, and if the neon lamp lights, the capacitor is shorted and no further test should be made. If the lamp does not light, then the capacitor is not shorted and the switch $A$ may be closed and the capacitor may then be tested for its capacitance.

Example 9-12. It is desired to determine the capacitance of a capacitor using the voltmeter-ammeter method and the circuit shown in Fig. 9-31. The voltmeter reads three volts and the ammeter nine milliampercs. What is the capacitance if the frequency is 60 cycles?
Given:
$E=3 \mathrm{volts}$
$I=9 \mathrm{ma}$
$f=60 \mathrm{cycles}$

Find:
$C=$ ?

Solution:

$$
\begin{aligned}
C & =\frac{I \times 10^{5}}{2 \pi f E} \\
& =\frac{0.009 \times 10^{\circ}}{6.28 \times 60 \times 3} \\
& =7.96 \mu \mathrm{f}
\end{aligned}
$$

Electrolytic Capacitors. Although an approximate value of the capacitance of an electrolytic capacitor can be obtained by using the impedance methods just described, it is preferable to test this type of capacitor by using an a-c voltage of either 60


Fig. 9-32.-Impedance method of determining capacitance using a wattmeter, a voltmeter, and an ammeter. or 120 cycles, not to be in excess of the maximum rated ripple voltage, plus a d-c polarizing voltage equal to the rated operating voltage. Such a circuit is shown in Fig. 9-32. The capacitor $C_{1}$ is used to by-pass the power pack and should be as large as possible. The capacitor $C_{2}$ should be of oil or wax paper and of such a value as to provide a reactance which is small compared with the resistance of the voltmeter. Readings should be taken on all three instruments simultaneously. The desired quantities may then be found by the equations

Capacitance

$$
\begin{equation*}
C_{x}=\frac{10^{\mathrm{b}}}{2 \pi f \sqrt{\binom{E}{I}^{2}-\left(\frac{P}{\overline{I^{2}}}\right)^{2}}} \tag{9-12}
\end{equation*}
$$

Equivalent series resistance

$$
\begin{equation*}
R_{x}=\frac{P}{I^{\overline{2}}} \quad \text { [from Eq. (2-14)] } \tag{9-13}
\end{equation*}
$$

Power factor

$$
\begin{equation*}
P-F=\frac{P}{E I} \tag{9-14}
\end{equation*}
$$

The simplified formula for the capacitance, $C=10^{6} I / 2 \pi f \mathrm{E}[\mathrm{Lq} .(9-11)]$, can be used if the equivalent series resistance is neglected.

9-14. Uses of Capacitors. Properties of Capacitors. From a practical standpoint, a capacitor has the property to store electrical energy in the form of an electrostatic charge. The capacitor can be connected in a circuit so that this stored energy can be made to flow in a desired circuit. Three important characteristics of capacitors are as follows:

1. A capacitor does not provide a path for direct current but does provide a path for alternating current.


Fig. 9-33.-Circuit diagram of a typical superheterodyne receiver with its capacitors indicated by numbers.
2. When a capacitor is used in a-c circuits, its reactance is expressed in ohms and varies inversely with the frequency as well as with the capacitance.
3. When a capacitor is used in a-c circuits, there is a time-phase difference between the current and the voltage. For a perfect capacitor, the current would lead the voltage by 90 electrical degrees.

The various applications of capacitors can now be classified according to which of the three characteristics are to be utilized.

1. In the first group of applications are those which make use of only the first characteristic. These include (a) coupling capacitors, (b) by-pass capacitors connected across bias resistors or voltage dividers, (c) capacitors used in the plate, screen, and grid circuits.
2. In the second group of applications are those which make use of characteristics 1 and 2. These applications include all the capacitors used


where frequency discrimination is desired. The common tone-control and the simple resistor-capacitor filter circuits are examples of this classification.
3. In the third group of applications are those which make use of all three characteristics, for example, capacitors used with an inductance and in tuned circuits.

Radio Applications. In order to understand the purpose of each capacitor used in a radio receiver, it is necessary to have a thorough knowledge of radio as well as an electrical knowledge of radio circuits. However, the purpose of the various capacitors in a radio receiver can be considered in terms of the classifications just described.

Figure 9 - 33 is the circuit diagram of a typical superheterodyne receiver with each of its capacitors indicated by a number. Capacitors 4 to 26 and 31 and 32 are applications listed under group 1. There are no capacitors in this circuit that would be listed under group 2. The remaining capacitors, $1,2,3,27,28,29,30,33,34$, and 35 , are all classified under group 3. The following table gives a further classification of the capacitors and their application to the circuit.

| Capacitors | Application |
| :---: | :---: |
| 1,2,3 | Tuning capacitors |
| 4, 5, 6, 7 . | Decoupling capacitors |
| 8, 9, 10, 11, 12 | Cathode-resistor by-pass capacitors |
| 13, 14, 15 . | Filter capacitors (power supply) |
| 16, 17, 18 | Screen-grid by-pass capacitors |
| 19, 20, 21, 22. | Plate-circuit by-pass capacitors |
| 23, 24, 25, 26. | D-c blocking capacitors |
| 27, 28, 29, 30 | Adjustable capacitors in i-f amplifiers |
| 31, 32. | Automatic-volume-control filter |
| 33. | Series capacitor in oscillator circuit |
| 34. | Trimmer |
|  | Padder |

The capacitors used in the circuit of Fig. 9-33 are represented by their symbolic representations, as is common practice in preparing such diagrams. In order to associate the symbols with the actual parts, refer to Fig. 9-34, which is a photograph of a receiver showing some of the capacitors in the form in which they actually appear. Figure $9-34$ is not for the same receiver as Fig. 9-33 and therefore has a different number of capacitors.

Television Circuits. Since a television receiver contains many more tubes than a radio receiver, it also contains many more capacitors. The variable capacitor is seldom used in television receivers as it is impractical to employ this means of tuning for the high-frequency range used in television broadcasting. Adjustable capacitors are used in both the sound and picture i-f circuits. Fixed capacitors are used for by-pass, coupling, de-
coupling, and tuning in high- and low-voltage and high- and low-frequency circuits.

Mica and ceramic capacitors are ordinarily used in high-frequency circuits requiring a capacitor with a low dielectric loss, practically no inductance, and a very low temperature coefficient in order to prevent frequency drift. Mica capacitors are generally used in the high voltage circuits, although paper capacitors with d-c ratings of 3000 and 6000 volts are also available.

The low-voltage power supply must be designed so that the hum level in the output voltage is kept at as low a level as is practicable, since a hum that might be inaudible in a radio receiver might cause an appreciable amount of distortion in the visible picture. The power supply must also be designed so that it will have good voltage regulation characteristics in order to prevent variations in voltage in the numerous circuits from interfering with each other and with the picture. Filtering is therefore quite extensive in the low-voltage power supply, and a large number of highvalue capacitors are generally used. The large amount of filtering also serves to prevent any undesirable coupling between the many different circuits supplied by the one power supply. Electrolytic capacitors of the single and multiple types are used for filtering in these circuits; commonly used ratings are of 500 to 2500 microfarads, with a working voltage of 3 to 6 volts d-c, and 5 to 200 microfarads, with a working voltage of 25 to 450 volts d-c.

Electronic Circuits. Industrial electronic circuits are quite similar in principle to those used in radio and television. Capacitors are used in these circuits to serve the same purposes as in radio and telcvision circuits. The value of capacitance and the type of capacitor used will be determined by the particular circuit requirements. Although many industrial electronic circuits use capacitors whose capacitance, voltage, and current ratings are similar to those used in radio and television, there are also many circuits that require capacitors having much higher current ratings.

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## QUESTIONS

1. Define capacitance, capacitor.
2. Explain the cause of capacitance and its effect.
3. Describe the action of a simple capacitor, in terms of the electron theory, during its period of charge.
4. What are the two important characteristics of capacitor action?
5. Describe the action of a simple capacitor, in terms of the electron theory, during its period of discharge.
6. Describe capacitor action when alternating current is applied.
7. (a) Define the basic unit of capacitance. (b) Define microfarad, micromicrofarad. (c) Which units are generally used?
8. Name and explain how the various factors affect the capacitance of a capacitor.
9. Where are oil dielectric-type capacitors used?
10. What are the advantages and disadvantages of mica dielectric capacitors?
11. Define the classifications of small, medium, and large mica capacitors.
12. Why are two thicknesses of paper used in place of a single thickness in the manufacture of paper capacitors?
13. What are the advantages and disadvantages of paper dielectric capacitors?
14. What is the objection to the use of paper capacitors in high-frequency circuits?
15. What are the advantages of ceramic capacitors?
16. What materials are used in making ceramic capacitors?
17. What are the essential parts of a variable capacitor?
18. What factors affect the resistance of a variable capacitor?
19. How is the resistance of a variable capacitor reduced to a minimum?
20. How are the losses due to skin effect and eddy currents minimized in the construction of variable capacitors?
21. What are the approximate ranges of variable capacitors to be used for (a) broadcast? (b) Short wave? (c) High frequencies? (d) Ultrahigh frequencies?
22. What is the difference between a variable capacitor that is used in a transmitter and one that is used in a receiver?
23. (a) What is meant by a split-stator capacitor? (b) Why is such a capacitor used?
24. (a) What is meant by a multiple capacitor? (b) What is its advantage?
25. (a) Describe the construction of adjustable capacitors. (b) Where are they used?
26. (a) What is meant by capacitive reactance? (b) In what unit is it expressed? (c) What is its symbol?
27. (a) What is meant by the impedance of a capacitor? (b) In what unit is it expressed? (c) What is its symbol?
28. (a) What two factors affect the capacitive reactance? (b) What two factors affect the impedance?
29. Explain the following: (a) leakage loss, (b) dielectric loss, (c) resistance loss, (d) capacitor losses.
30. (a) What is meant by breakdown voltage? (b) How is this voltage usually expressed?
31. How are capacitors generally rated in regard to voltage?
32. What is the reason for connecting capacitors in (a) series? (b) Parallel?
33. What is meant by distributed capacitance?
34. What effect does the distributed capacitance of a coil have on the circuit in which it is connected?
35. How may the distributed capacitance of a coil be reduced?
36. Explain the principle of electrolytic capacitors.
37. What factors affect the capacitance of an electrolytic capacitor?
38. What factors determine the thickness of the diclectric in an electrolytic capacitor?
39. Explain what is meant by sclf-healing.
40. Describe some of the methods used to increase the surface area of the positive electrode used in wet electrolytic capacitors.
41. How does the construction of the dry electrolytic capacitor differ from the construction of the wet electrolytic capacitor?
42. What are the advantages of dry electrolytic capacitors over the wet type?
43. Describe some of the methods used to increase the surface area of the positive electrode used in dry electrolytic capacitors.
44. Describe the construction of a capacitor block.
45. What is meant by a polarized capacitor?
46. What is meant by a nonpolarized capacitor?
47. Explain the following voltage ratings: (a) d-c working voltage, (b) peak ripple voltage, (c) peak voltage, (d) surge voltage.
48. Fxplain the following current ratings: (a) leakage current, (b) normal leakage current, (c) initial leakage current.
49. What does the equivalent series resistance of an clectrolytic capacitor represent?
50. What is the relation between the power factor of a capacitor and its equivalentseries resistance?
51. Describe the impedance method of determining capacitance.
52. What are the assumptions on which the voltmeter-ammeter method of determining capacitance is based?
53. Why is it necessary to use a d-c polarizing voltage in addition to the a-c voltage in determining the capacitance of electrolytic capacitors?
54. Describe the three important characteristics of a capacitor?
55. In what type of circuits of a television receiver are fixed capacitors used?
56. In what type of circuits is it desirable to use (a) mica capacitors? (b) Ceramic capacitors?
57. Why is it necessary to use a considerable amount of filtering in the lowvoltage power-supply circuits of a television receiver?
58. How do the voltage and capacitance rating of electrolytic capacitors used in radio receivers compare with those used in television receivers?

## PROBLEMS

1. What is the capacitance of a capacitor made of two plates of tin foil, each 1 in. square, separated by a sheet of mica 0.01 in . thick?
2. If paraffined paper is substituted for the mica in the capacitor of Prob. 1, what will its capacitance be?
3. If beeswaxed paper is substituted for the mica in the capacitor of Prob. 1, what will its capacitance be?
4. If it is desired that the capacitor of Prob. 3 have the same capacitance as that of Prob. 1, what length must the plates be if their width is to be kept at 1 in .?
5. If it is desired that the capacitor of Prob. 3 have the same capacitance as that of Prob. 1, what thickness of dielectric is required if the size of the plates is to remain 1 in . square?
6. What is the capacitance of a capacitor made of 25 plates of lead foil, each $\frac{1}{2}$ by 1 in ., separated by layers of Bakelite 0.01 in . thick?
7. What is the capacitance of a capacitor made of 720 plates of aluminum foil, each 2 by 3 in ., separated by beeswaxed paper dielectric 0.006 in. thick?
8. How many plates must be used to make a capacitor of $\dot{0} .0125 \mu \mathrm{f}$ if they are to be 2 in. square and the dielectric is to be mica sheets 0.012 in. thick?
9. How many plates must be used to make a capacitor of $0.0005 \mu \mathrm{f}$ if the plates are $\frac{3}{4}$ in. square and the dielectric mica sheets 0.007 in. thick?
10. What is the capacitance of a capacitor consisting of two plates, each 1 in . wide and 81 in . long and separated by two sheets of paraffined paper each 0.005 in . thick?
11. If the capacitor of Prob. 10 is to have a capacitance of $0.01 \mu \mathrm{f}$, what length is required for each plate?
12. If the width of the plate of the capacitor of Prob. 10 is increased to 2 in., what length will be required for a capacitor of $0.10 \mu \mathrm{f}$ ?
13. It is desired to construct a capacitor from two tin-foil sheets 1 in . wide and separated by a beeswaxed paper 0.001 in . thick. The capacitor is to be rolled into a compact cylindrical form. What is the length of each tin-foil plate if the capacitance is to be $0.5 \mu \mathrm{f}$ ?
14. What is the maximum capacitance of a 23 -plate variable capacitor if each plate has an area of 2.75 sq in.? The air gap is 0.025 in .
15. A midget capacitor has 43 plates, and its maximum capacitance is $320 \mu \mathrm{ff}$. The air gap between adjacent plates is 0.025 in . What is the approximate area of each rotor plate?
16. What is the capacitance of an 11-plate midget variable capacitor if the area of each rotor plate is 0.89 sq in . and the air gap is 0.025 in .?
17. What is the capacitance of a 19 -plate double-spaced midget variable capacitor if the area of each plate is 0.89 sq in . and the air gap is 0.0715 in .?
18. What is the capacitance of a 27 -plate micro variable capacitor if the area of each plate is 0.35 sq in . and the air gap is 0.0205 in .?
19. What is the capacitance of a 14 -plate micro variable capacitor if the area of each plate is 0.35 sq in. and the air gap is 0.0205 in .?
20. (a) What is the capacitive reactance of an $8-\mu \mathrm{f}$ fixed capacitor at a frequency of 60 cycles? (b) What current will flow if the capacitor is connected to a 110 -volt 60 -cycle power supply?
21. (a) If the capacitor of Prob. 20 has a resistance of 10 ohms , what is its impedance at 60 cycles? (b) What current will flow if the capacitor is connected to a 110 -volt 60-cycle power supply? (c) What is its power factor? (d) What is its phase angle? (e) What power will it take from the line?
22. What are the impedance, current, power factor, and phase angle of a $10-\mu \mathrm{f}$ capacitor when connected to a 250 -volt 60 -cycle line? The equivalent series resistance of the capacitor is 10 ohms.
23. What are the impedance, current, and phase angle of a $20-\mu \mathrm{f}$ capacitor when connected to a 300 -volt 60 -cycle line? The equivalent series resistance of the capacitor is 8 ohms.
24. What is the capacitive reactance of a $0.001-\mu \mathrm{f}$ capacitor when connected in a circuit of the following high-frequeney currents: (a) 750 kc ? (b) 1250 kc ? (c) 1450 kc ? (d) 60 mc ? (e) 100 mc ? ( $f$ ) 216 mc ?
25. What current will flow if the capacitor of Prob. 24 is connected in a circuit of (a) 500 mv and 750 kc ? (b) 1000 mv and 1250 kc ? (c) 2.5 volts and 1450 kc ? (d) 1 volt and 60 mc ? (e) 1 volt and 216 mc ?
26. How much current will a $0.5-\mu \mathrm{f}$ capacitor take when connected to an a-f circuit of 30 volts and 3000 cycles?
27. What is the capacitance of a circuit containing two $8-\mu \mathrm{f}$ and one $4-\mu \mathrm{f}$ capacitors connected in serics?
28. If the series circuit of Prob. 27 is connected across a 250 -volt 60 -cycle power line, find ( $a$ ) the current, (b) the voltage across cach capacitor.
29. What is the capacitance of a circuit containing an 8-, a 4-, a $2-$, and a $1-\mu \mathrm{f}$ capacitor connected in series?
30. If the series circuit of Prob. 29 is connected across a 250 -volt 60 -cycle power line, find ( $a$ ) the current, (b) the voltage across each capacitor.
31. What is the capacitance of a circuit containing an 8 -, a 4 -, and a $2-\mu \mathrm{f}$ capacitor connected in parallel?
32. If the parallel circuit of Prob. 31 is connected to a 110 -volt 60 -cycle power line, find (a) the voltage across each capacitor, (b) the current in each parallel branch, (c) the line current.
33. What is the capacitance of a circuit containing an 8 -, a $4-$, a $2-$, and a $1-\mu \mathrm{f}$ capacitor connected in parallel?
34. If the parallel circuit of Prob. 33 is connected to a 110 -volt 60 -cycle power line, find (a) the voltage across each capacitor, (b) the current in each branch circuit, (c) the line current.
35. What value of capacitance must be connected in series with a $10-\mu \mathrm{f}$ capacitor in order to obtain a capacitance of $6.66 \mu$ ?
36. What value of capacitance must be connected in series with a $10-\mu \mathrm{f}$ capacitor in order to obtain a capacitance of $3.33 \mu$ f?
37. What value of capacitance must be connected in parallel with a $10-\mu \mathrm{f}$ capacitor in order to obtain a capacitance of $15 \mu \mathrm{f}$ ?
38. What is the capacitance of the circuit shown in Fig. 9-35?


Fig. 9-35.


Fig. 9-36.
39. What is the capacitance of the circuit shown in Fig. 9-36?
40. What is the capacitance of the circuit shown in Fig. 9-37?
41. What is the capacitance of the circuit shown in Fig. 9-38?


Fig. 9-37.
Fig. 9-38.
42. It is desired to obtain a capacitance of $5 \mu \mathrm{f}$. A number of standard sizes, including $2-, 4-$, and $8-\mu \mathrm{f}$ capacitors, are available. What circuit arrangement will provide a capacitance of $5 \mu \mathrm{f}$ with a minimum number of capacitors? What is the minimum number of capacitors, and what are their sizes?
43. It is desired to determine the capacitance of a capacitor by the impedance method. The capacitor is connected to the circuit as shown in Fig. 9-30, and the following readings are obtained: voltmeter 220 volts, ammeter 0.55 amp , wattmeter 10 watts, frequency 60 cycles. (a) What is the capacitance of the capacitor? (b) What is the power factor of the capacitor? (c) What is its angle of lead?
44. It is desired to determine the capacitance of a capacitor using the volt-meter-ammeter method and the circuit shown in Fig. 9-31. The voltmeter indicates 3 volts and the ammeter 23 ma . What is the capacitance if the frequency is 60 cycles?
45. A $0.1-\mu \mathrm{f}$ capacitor is tested by the method and circuit used in Prob. 44. The line voltage is 100 volts and the frequency 60 cycles. What is the ammeter reading?

## CHAP'TER X

## ALTERNATING CURRENT CIRCUITS

The relation of voltage, current, and power in electric circuits as presented in Chap. IV was on the basis of a continuous or direct current flowing in the circuit, as the circuit characteristics are more easily understood when direct currents are considered. The study of a-c generators and transformers has shown that there are advantages of alternating current over direct current; because over 90 per cent of the electrical power is generated as alternating current, the need for knowing the circuit characteristics with alternating current can be recognized.

10-1. Circuit Characteristics. $A-C$ vs. $D-C$. Because of the difference in the voltage and current characteristics of alternating current and direct current, it is necessary to introduce some new terms in the study of a-c circuits. The difference in the characteristics referred to above is shown in Fig. 10-1. The voltage and current as indicated by voltmeters and ammeters are the same for each circuit, namely, 110 volts and five amperes.

The voltage for the d-c circuit is 110 volts at all instants of time, and its current is five amperes at all instants as indicated in Fig. 10-1a. All the opposition to the flow of current is accounted for in the resistance of the circuit, and Ohm's law, $I=E / R, E=I R$, and $R=E / I$, applies to all d-c circuits. The power consumed by the d-e circuit may be calculated by $P=E \times I, P=I^{2} R$, or $P=E^{2} / R$.

Phase Relation of Current and Voltage. The a-c circuit conditions shown in Fig. 10-1b indicate that the effective voltage and effective current are the same as for the d-c circuit shown in Fig. 10-1a. While this is so, it is also evident that the a-c voltage and current are continually changing in magnitude. It should be noticed, too, that the current goes through its cycle at the same time that the voltage goes through its cycle. They are said to be of the same time phase, or, more simply stated, to be in phase. This condition of being in phase may be more fully described by the following statement: The two waves start from zero and rise together, each reaching its maximum at the same instant; they descend together, go through zero, and alternate in polarity at the same time; they reach their maximum negative values at the same time; and then they decrease to zero to complete their cycles together. With the current and voltage in phase, the circuit will follow the same laws that applied to the d-c circuit, namely, that $I=E / R, E=I R, R=E / I, P=E I, P=I^{2} R$, and $P=$ $E^{2} / R$.

The study of inductance and capacitance has shown that it is possible for conditions to exist whereby the current wave may not be in phase with the voltage wave. Such conditions exist in a-c circuits more often than do inphase currents. To study a-c circuits, it is therefore necessary to account for these new conditions. When the current is not in phase with the voltage, only one of the laws mentioned above will apply, that is, $P=I^{2} R$; the others cannot be used.


Fig. 10-1.-Direct current vs. alternating current characteristics: (a) current and voltage when the circuit is connected to a d-c power supply, (b) current and voltage when the circuit is connected to an a-c power supply, (c) the circuit diagram.

Reactance. In d-c circuits, the only opposition to the flow of current considered is resistance. In a-c circuits, two additional factors must be considered, namely, inductive reactance and capacitive reactance. Inductive reactance is the opposition to the flow of current offered by an inductor; its unit is the ohm; its symbol is $X_{L}$. Capacitive reactance is the opposition to the flow of current offered by a capacitor; its unit is the ohm; its symbol is $X_{c}$.

When an a-c circuit contains resistance only, the current and voltage will be in phase. If an a-c circuit contains either capacitive reactance or inductive reactance, the current and voltage will be out of phase except in the case of resonance. (Resonance is described in Chap. XI.)

10-2. Effects of Inductive and Capacitive Reactances. Inductive Reactance. Inductive reactance is generally caused by an alternating current flowing through a coil of wire. The alternating current sets up a magnetic field about the turns of wire which is continually varying in strength and direction. These magnetic lines cut the turns of wire and thereby set up an induced emf which opposes a change in current strength. The effect of this is twofold: it results in the inductive reactance opposing



Fig. 10-2.-Voltage and current characteristics if a perfect inductor were connected to an a-c power supply.
the flow of current and also delays the current, causing it to lag behind the voltage. A perfect inductor, that is, one having no resistance, has its current lagging the voltage by 90 electrical degrees, and under this condition it takes no power because the coil stores up the energy in the magnetic field during one half cycle and returns this cnergy to the line in the following half cycle. The current taken by a perfect inductor would be equal to its voltage divided by its inductive reactance, or

$$
\begin{equation*}
I_{L}=\frac{E_{L}}{\overline{X_{L}}} \tag{8-7}
\end{equation*}
$$

Figure $10-2$ shows the current and voltage waves for an inductive reactance of 22 ohms connected across a 110 -volt circuit.

Capacitive Reactance. Capacitive reactance is caused when an a-c voltage is applied to a capacitor. The alternating current sets up a charge on the capacitor which opposes any change in the voltage. The effect of this is twofold: it results in the capacitive reactance, which opposes the flow of current, and also causes the current to lead the voltage. A perfect capacitor, that is, one having no resistance, has its current leading the voltage



Fig. 10-3.-Voltage and current characteristics if a perfect capacitor were connected to an a-c power supply. by 90 electrical degrees, and under this condition, it takes no power because the capacitor stores up energy in the electrostatic field during one half cycle and returns this energy to the line in the following half cycle. The current taken by a perfect capacitor would be equal to its voltage divided by its capacitive reactance,

$$
\begin{equation*}
I_{C}=\frac{E_{C}}{\overline{X_{C}}} \tag{9-5}
\end{equation*}
$$

Figure $10-3$ shows the current and voltage waves for a capacitive reactance of 22 ohms connected across a 110 -volt circuit.

The conditions described above -are for perfect inductors and capacitors which require that they have no resistance. While it is impossible to manufacture such an inductor or capacitor, these conditions are used to explain the effect of each when considered alone.

## 10-3. Series A-C Circuits Con.

 taining Resistance, Capacitance, and Inductance. Impedance. It has been shown that a-c circuits may consist of any combination of resistance, inductive reactance, and capacitive reactance. As in the d-e circuit, we can consider the total opposition to the flow of current in the entire circuit, or we can consider the opposition to the flow of current in any of its parts. The combined effect of resistance and reactance is called impedance. It, too, is expressed in ohms, and its symbol is $Z$. When a circuit contains resistance and reactance, it is necessary to combine their ohmic effects by a different method than that used for resistance only.Calculation of Impedance. The reactance of any circuit will always be 90 degrees out of phase with its resistance. Whether it leads or lags, the
resistance will depend on whether the reactance is capacitive or inductive (see Fig. 10-4). Observation of the diagrams in this figure will show that the resistance, reactance, and impedance form a right-angle triangle. Their

(a)

(b)

Fig. 10-4.--Relation of resistance, reactance, and impedance: (a) resistance, inductive reactance, and impedance; (b) resistance, capacitive reactance, and impedance.
relations to one another when connected in series can therefore be expressed by the following equations. The impedance of a circuit containing resistance and inductive reactance is

$$
\begin{equation*}
Z=\sqrt{R^{2}+X_{L}^{2}} \tag{8-8}
\end{equation*}
$$

The impedance of a circuit containing resistance and capacitive reactance is

$$
\begin{equation*}
Z=\sqrt{R^{2}+\overline{X_{c}}} \tag{9-7}
\end{equation*}
$$

Example 10-1. What is the impedance of a circuit having a resistance of 30 ohms and a capacitive reactance of 40 ohms?

$$
\begin{aligned}
& \text { Given: } \\
& R=30 \\
& X_{C}=40
\end{aligned}
$$

Solution:

$$
\begin{aligned}
Z & =\sqrt{R^{2}+X c^{2}} \\
& =\sqrt{30^{2}+40^{2}} \\
& =\sqrt{2500} \\
& =50 \mathrm{ohms}
\end{aligned}
$$

Example 10-2. An inductance coil has a resistance of five ohms and a reactance of 10 ohms . What is the impedance of the coil?
Given:
Find:
$R=5$
$Z=$ ?

$$
X_{L}=10
$$

Solution:

$$
\begin{aligned}
Z & =\sqrt{R^{2}+X_{L^{2}}} \\
& =\sqrt{5^{2}+10^{2}} \\
& =\sqrt{1 \overline{25}} \\
& =11.18 \text { ohms }
\end{aligned}
$$

The impedance of a circuit containing resistance, inductive reactance, and capacitive reactance is

$$
\begin{equation*}
Z=\sqrt{R^{2}+\left(X_{L}-X_{c}\right)^{2}} \tag{10-1}
\end{equation*}
$$

Example 10-3. What is the impedance of a series circuit containing 40 ohms resistance, 40 ohms inductive reactance, and 70 ohms capacitive reactance?

| Given: | Find: |  |
| ---: | :--- | ---: |
| $R$ | $=40$ | $Z=?$ |
| $X_{L}$ | $=40$ |  |
| $X_{C}$ | $=70$ |  |

Solution:

$$
\begin{aligned}
Z & =\sqrt{\bar{R}^{2}+\left(\overline{\left.X_{L}-X_{C}\right)^{2}}\right.} \\
& =\sqrt{40^{2}+(40-70)^{2}} \\
& =\sqrt{40^{2}+(-30)^{\frac{2}{2}}} \\
& =\sqrt{1} \overline{600}+900
\end{aligned}=50 \mathrm{ohms} .
$$

Current. The current flowing in an a-c circuit will be equal to the voltage applied to the circuit divided by the impedance of the circuit, or

$$
\begin{equation*}
I=\frac{t}{Z} \tag{8-9}
\end{equation*}
$$

Example 10-4. What is the total current flowing through the coil in Example 10-2 when 10 volts is impressed across its terminals?

| Given: | Find: |
| :--- | ---: |
| $E=10$ | $I_{T}=?$ |
| $Z=11.18$ |  |

Solution:

$$
\begin{aligned}
I_{T} & =\frac{E}{Z} \\
& =\frac{10}{11.18} \\
& =0.895 \mathrm{amp}
\end{aligned}
$$

10-4. Complex Series Circuits. $\Lambda$ series a-c circuit is formed whenever one or more resistances are connected in series with one or more reactances. Since all coils or capacitors contain some resistance in addition to their reactance, a series circuit is formed whenever a coil or a capacitor is connected across an a-c power supply (see Fig. 10-5). The resistance and reactance are actually part of the coil and capacitor and not two separate devices as the circuit seems to indicate. In calculating a-c circuits, the reactance and resistance are always treated as separate units.

If the coil and capacitor in Fig. 10-5 are connected in serjes, the circuit diagram will be as shown in Fig. 10-6. As all resistances produce currents that are in phase with one another, the total resistance will be equal to the sum of the two resistances. The two reactances are 180 degrees out of
 Fig. 10-5.-Circuit diagrams for a coil and a capacitor.
phase with one another, and therefore the resultant reactance will be equal to the difference between the two. The inductive ractance in this case is larger; therefore the resultant reactance must be inductive.

If two or more inductive reactances are connected in series, the total inductive reactance will be equal to the sum of the individual inductive reactances, as they are all in phase with one another. In a like manner, the total capacitive reactance of a scries circuit will be equal to the sum of the individual capacitive reactances.


Fig. 10-6.-Circuit diagram representing a coil and a capacitor connected in series.

From the foregoing statements, the following equation can be used to solve for the impedance of any series circuit:

$$
Z=\sqrt{\left(R_{1}+R_{2}+R_{3}, \text { etc. }\right)^{2}+\left(X_{L 1}+X_{L 2}+X_{L 3},\right. \text { etc. }} \frac{\left.-X_{C 1}-X_{C 2}-X_{c 3}, \text { etc. }\right)^{2}}{-X_{0}}
$$

This one equation may be used for all series circuits by placing all the values of $R$ in the first parentheses and all the reactances in the second. When adding the reactances in the second parentheses, all inductive reactances are given plus signs and all capacitive reactances minus signs.

Example 10-5. What is the impedance of the circuit shown in Fig. 10-6?

| Given |  |
| ---: | :--- |
| $R_{L}$ | $=10$ |
| $R_{C}$ | $=15$ |
| $X_{L}$ | $=100$ |
| $X_{C}$ | $=25$ |$\quad$| Find: |
| :--- |

Solution:

$$
\begin{aligned}
Z & =\sqrt{\left(R_{L}+R c\right)^{2}+\left(\overline{\left.X_{L}-X_{C}\right)^{2}}\right.} \\
& =\sqrt{\left(\overline{10+15)^{2}}+(100-25)^{2}\right.} \\
& =\sqrt{25^{2}+75^{2}} \\
& =\sqrt{625+5625} \\
& =\sqrt{6250} \\
& =79.05 \mathrm{ohms}
\end{aligned}
$$

Example 10-6. The series circuit shown in Fig. 10-7 is connected to a 150 -volt 60 -cycle power line. (a) What is the impedance of the circuit? (b) What current will the circuit draw from the line?

| Given: | Find: |  |
| ---: | :--- | ---: |
| $E$ | $=150$ | $Z=?$ |
| $R_{1}$ | $=28$ | $I=?$ |
| $X_{L 1}$ | $=10$ |  |
| $X_{L 2}$ | $=20$ |  |
| $X_{C 1}$ | $=100$ |  |
| $R_{2}$ | $=2$ |  |
| $X_{L 3}$ | $=60$ |  |
| $X_{C 2}$ | $=50$ |  |
| $R_{3}$ | $=10$ |  |



Fig. 10-7.
Solution:
(a)

$$
\begin{aligned}
Z & =\sqrt{\left(R_{1}+R_{2}+R_{3}\right)^{2}+\left(X_{L 1}+X_{L 2}+X_{L 3}-X c_{1}-X c_{2}\right)^{2}} \\
& =\sqrt{(28+2+10)^{2}+(10+20+60-100-50)^{2}} \\
& =\sqrt{(40)^{2}+(-60)^{2}} \\
& =\sqrt{1600+3600} \\
& =\sqrt{5200} \\
& =72.1 \text { ohms }
\end{aligned}
$$

(b)

$$
\begin{aligned}
I & =\frac{E}{Z} \\
& =\frac{150}{72.1} \\
& =2.08 \mathrm{amp}
\end{aligned}
$$

10-5. Vectors-Voltage and Current. Vector Representation of Voltage. The sine-wave diagrams provide excellent illustrations of a-c voltages and currents, but they require a great deal of time to prepare, especially if accurate drawings are necessary. A simpler method, used extensively because of its timesaving feature, is the vector representation. By definition, a vector is a line that has both magnitude and direction and is used to represent an alternating voltage or current.

By the vector method, the sine-wave


Scale: 1 inch $=100$ volts
Fic. 10-8.-Vector representation of a sine-wave voltage. voltage illustrated in Fig. 7-5b is represented by the single line shown in Fig. 10-8. Vectors are usually drawn to a suitable scale as is indicated in the figure where one inch represents 100 volts.

The wheel diagram of Figs. 7-6 and 10-9a might be analyzed in terms of vectors since each "spoke" is really a vector used to obtain an instantaneous value as is shown in Fig. 10-9. Four vector diagrams are included in this figure to illustrate that a vector has both magnitude and direction. The first vector diagram (Fig. 10-9b) corresponds to position 1 of the wheel diagram, which is for zero degrees. This is the starting point, and it is generally laid off in a horizontal position and to the right of the point of origin $O$. The second vector (Fig. 10-9c) corresponds to position 2 , or 30 degrees. As the standard direction of vector rotation about its origin is counterclockwise, the vector $E_{\text {max }}$ has been advanced 30 degrees from the starting position and in that direction. A vertical line projected from the end of the vector to the horizontal plane will be equal to the instantancous value of the voltage, $e$. The line $e$, when measured, is found to be five units, and its voltage therefore will be $5 \times 10$, or 50 volts. The third vector diagram (Fig. 10-9d) corresponds to position 3, and the vector is drawn 60 degrees from the horizontal. The vertical projection $e$ represents the instantaneous voltage which is found to be approximately 87.5 volts (actual value is 86.6 volts). The fourth vector diagram (Fig. 10-9e) is for position 9 , or 240 degrees. The vector is now below the horizontal line, and its instantaneous value will be negative. The value of $e$ found by the vertical projection is approximately -87.5 volts. For any position, the instantaneous value may be found in a similar manner. This method of solving for $e$ by constructing a diagram to scale, measuring the length of the line $e$, and then converting its length from inches to volts is called the graphical method of solution. The values of $e$ in Figs. 10-9d and 10-9e solved by this method have been found to be approximately 87.5 volts and -87.5 volts, while their accurate values are 86.6 volts and -86.6 volts. The accuracy of the graphical method of solution depends upon the size of the diagram and the accuracy of drawing.

A more accurate method of solution is the mathematical method. In this method, the value of $e$ may be found by Eq. (7-4). Using this method to check the graphical solution,

$$
\begin{aligned}
e_{0^{\circ}} & =E_{\max } \times \sin 0^{\circ}=100 \times 0.000=0 \text { volts } \\
e_{30^{\circ}} & =E_{\max } \times \sin 30^{\circ}=100 \times 0.500=50 \text { volts } \\
e_{60^{\circ}} & =E_{\max } \times \sin 60^{\circ}=100 \times 0.866=86.6 \text { volts } \\
e_{240^{\circ}} & =E_{\max } \times \sin 240^{\circ}=100 \times(-0.866)=-86.6 \mathrm{volts}
\end{aligned}
$$



Fig. 10-9.-Vector representation of the wheel diagram: (a) the wheel diagram showing 12 equally spaced points, (b) vector representin? ooint 1 , (c) vector representing point 2 . (d) vector representing point 3 , (e) vector represenung point 9 .

Vector Representation of Voltage and Current. When several sine waves are plotted together as in Fig. 10-1b, the vector representation requires two lines, one for the voltage and one for the current. The vector diagram for this condition is given in Fig. 10-10. As the current and voltage are in phase with each other, the two vectors must be drawn on the same line. Because of the great difference in the numerical value of the current and voltage, it is permissible and also common practice to use separate scales as in Figs. 10-10 and 10-11. By careful observation, it may also be noted that these vectors are for effective values, whereas in Fig. 10-9 maximum values were used; either values may be used as long as they are properly
marked. Figure 10-11a shows the vector diagram corresponding to Fig. $10-2$ in which the current lags the voltage by 90 degrees. As vector rotation is counterelockwise, the current is downward, or lags the voltage by 90 degrees. Figure $10-116$ is the vector diagram corresponding to Fig. $10-3$ in which the current leads the voltage by 90 degrees. The arrows on the current vectors are closed, while on the voltage they are left open in order to distinguish currents from voltages.

10-6. Vectors-Resistance, Reactance, and Impedance. Vectors


Scale: $/$ inch $=100$ volts; 1 inch $=4$ amperes
Fig. 10-10.-Vector representation of current and voltage when the current is in phase with the voltage. are also used to show the relation among resistance, inductive reactance, capacitive reactance, and impedance. This was taken up in Art. 10-3, but the values were not considered as vectors at that time. If the reactance and resistance vectors are drawn to scale, the impedance of the circuit can be determined graphically.


Fig. 10-11.-Vector representation of current and voltage: (a) current lagging the voltage by 90 degrees, (b) current leading the voltage by 90 degrees.

Example 10-7. By means of vectors, determine the impedance of a series circuit containing a resistance of 40 ohms and an inductive reactance of 30 ohms. Check the answer by solving for impedance mathematically.

Given:

$$
\begin{aligned}
R & =40 \\
X_{r_{t}} & =30
\end{aligned}
$$

Find:
$Z=$ ?

Solution:
Since the resistance causes an inphase current and the inductive reactance causes a 90 -degree lagging current, the inductive reactance is drawn 90 degrees from the resistance. The combination of the two as determined graphically is shown in Fig. 10-12 and is equal to 50 ohms.


Fig. 10-12.
Checking mathematically:

$$
\begin{aligned}
Z & =\sqrt{R^{2}+X_{L}^{2}} \\
& =\sqrt{40^{2}+30^{2}} \\
& =\sqrt{1600}+900 \\
& =\sqrt{2500} \\
& =50 \text { ohms }
\end{aligned}
$$

Example 10-8. A circuit has a resistance of 30 ohms connected in series with a capacitive reactance of 40 ohms. Determine its impedance graphically and mathematically.

| Given: | Find: |
| ---: | ---: | ---: |
| $R=30$ | $Z=?$ |
| $X_{C}=40$ |  |

Solution:
The capacitive reactance causes a 90 -degree leading current and is therefore drawn 90 degrees from the resistance. The combination of the two as determined graphically is shown in Fig. 10-13 and is equal to 50 ohms.


Fig. 10-13.
Checking mathematically:

$$
\begin{aligned}
Z & =\sqrt{R^{2}+X c^{2}} \\
& =\sqrt{30^{2}+40^{2}} \\
& =\sqrt{2500} \\
& =50 \mathrm{ohms}
\end{aligned}
$$

Series circuits may contain resistance, inductance, and capacitance. In such circuits, the total impedance is determined graphically by combining the resultant resistance vector with the resultant reactance vector.

Example 10-9. A series circuit contains a resistance of 40 ohms, an inductive reactance of 30 ohms , and a capacitive reactance of 60 ohms . Determine the impedance of this circuit, using vectors. Check the answer mathematically.

$$
\begin{aligned}
& \text { Given: } \\
& R=40 \\
& X_{L}=30 \\
& X_{C}=60
\end{aligned}
$$

Find:
$Z=$ ?

Solution:
Referring to Fig. 10-14, th inductive reactance and the capacitive reactance are first combined into the single value ( $X_{L}-X_{C}$ ) which is then added to the resistance to form the triangle for finding the impedance. Using this method, $Z=50$ ohms.

Checking the answer mathematically:

$$
\begin{aligned}
Z & =\sqrt{R^{2}+\left(X_{L}-X c\right)^{2}} \\
& =\sqrt{40^{2}+(30-60)^{2}} \\
& =\sqrt{40^{2}+(-30)^{2}} \\
& =\sqrt{2500} \\
& =50 \text { ohms }
\end{aligned}
$$

10-7. Power in A-C Circuits. Apparent Power. The power consumed by a d-c circuit was given in Art. 2-13 as $P=E \times I[E q$. (2-12)]. In a-c circuits this will be true only when the current is in phase with the voltage as is the case for circuits containing resistance alone and for resonant circuits to be studied later. In cases where the current is not in phase with the voltage, the product of the volts and amperes will not be equal to the power actually consumed by the circuit. Instead, the product of the volts and amperes is called the apparent power and is expressed in volt-amperes. To avoid confusion, it is preferable always to call the product of the volts and amperes the apparent power.


Fig. 10-14.

Actual Power. The actual power consumed by an a-c circuit is best determined by the use of a wattmeter, which always indicates the true power, regardless of whether the current is in phase with the voltage or not. The power may, how ever, be determined without the use of a wattmeter if the angle between
the current and voltage is known. Power may then be calculated by the equation

$$
\begin{equation*}
P=E \times I \times \cos \theta \tag{10-3}
\end{equation*}
$$

where $P=$ power, watts
$E=$ voltage of the circuit
$I=$ current of the circuit
$\theta=$ angle between current and voltage
$\cos \theta=$ cosine of the angle $\theta$ (from tables in Appendix XI)
The power may also be found by Eq. (2-14), $P=I^{2} R$, but care must be exercised that only the resistance is used and that it is not confused with either inductive reactance, capacitive reactance, or impedance.

Power Factor. The value of $\cos \theta$ is often referred to as the power factor. When the current is in phase with the voltage, the angle $\theta$ is zero and the power factor, cos 0 degrees, is 1.000 . When the current is out of phase by 90 degrees, cither leading or lagging, the power factor, cos 90 degrees, is zero. For lagging or leading currents with angles between zero and 90 degrees, the cosine value, and hence the power factor, varies from 1.000 to zero as indicated in the cosine table. Substituting the apparent power $V A$ for $E \times I$ and P-F for $\cos \theta$ in Eq. (10-3), it becomes

$$
\begin{equation*}
P=V A \times \mathrm{P}-\mathrm{F} \tag{10-4}
\end{equation*}
$$

or

$$
\text { Actual power }=\text { apparent power } \times \text { power factor }
$$

From this equation a definition may be derived for power factor, namely, power factor is a factor or number whose value varies from zero to 1.000 , by which the apparent power must be multiplied in order to determine the actual power. From the power equations, it may be seen that when the power factor is unity (1.0) all the volt-amperes become actual power, while if the power factor is less than unity, only part of the volt-amperes becomes actual power. Since only the actual power does useful work, it is evident that it is desirable to have the power factor as near to unity as possible. Combining the two facts that (1) only resistance takes power and (2) impedance is the combined effect of resistance and the reactances, it follows that the power factor can also be expressed in terms of the resistance and impedance of a circuit. Hence

$$
\begin{equation*}
\mathrm{P}-\mathrm{F}=\frac{R}{Z} \tag{10-5}
\end{equation*}
$$

In practical work, the power factor of a circuit is generally determined from meter readings. When the volts, amperes, and watts of a circuit are known, the power factor may be calculated by the equation

$$
\begin{equation*}
\text { Power factor }=\frac{\text { watts }}{\text { volts } \times \text { amperes }} \quad \text { or } \quad \mathrm{P}-\mathrm{F}=\frac{P}{E \times I} \tag{9-14}
\end{equation*}
$$

It may be seen from this equation that the power factor may also be defined as the ratio of the actual power to the apparent power. A powerfactor meter that gives a direct indication of the power factor of a circuit is also available, but this method of determining the power factor is not used so much as the voltmeter, ammeter, wattmeter method. Its greatest use is for circuits in which the power factor may be varied as by a rheostat in a synchronous-motor field circuit.

Phase Angle. The phase angle, represented by the Greek letter $\theta$ (pronounced theta), is the angle between two a-c quantities (most commonly between the current and voltage, although it is often used with two voltages, two currents, resistance and impedance, etc.). When it is used with current and voltage, especially as the power-factor angle, it is standard practice also to state what the current does with respect to the voltage, that is, whether the current lags or leads the voltage by the angle $\theta$.

10-8. Series Circuit Problems. As most of the principles presented in Arts. $10-1$ to $10-7$ are so closely related, the following examples will illustrate several of them in each example.

Example 10-10. A 10 -henry choke coil is connected to a 110 -volt 60 -cycle power line. It is assumed that the resistance of the coil is zero. (a) How much current does the coil draw from the line? (b) What are the power, power factor, and phase angle? (c) Draw a vector diagram showing the current and voltage relations.

## Given:

$$
\begin{aligned}
E & =110 \\
f & =60 \\
L & =10
\end{aligned}
$$

Find:
$I=$ ?
$P=$ ?
P-F $=$ ?
$\theta=$ ?
Vector diagram $=$ ?

Solution:
(a)

$$
\begin{aligned}
I_{L} & =\frac{E_{L}}{X_{L}}=\frac{E_{L}}{2 \pi f L} \\
& =\frac{110}{6.28 \times 60 \times 10}=\frac{110}{3768} \\
& =0.291 \mathrm{amp}=29.1 \mathrm{ma} \\
P & =I^{2} R=0.0291^{2} \times 0=\text { zero }
\end{aligned}
$$

(b)

$$
\begin{aligned}
\text { P-F } & =\frac{R}{Z}=\frac{0}{3768}=\text { zero } \\
\theta & =90 \text { degrees (lagging current) }
\end{aligned}
$$

(c) Vector diagram, Fig. 10-15b


Fig. 10-15.
Example $10-11$. A 10 -microfarad capacitor is connected to a 250 -volt 60 -cycle circuit. It is assumed that the resistance of the capacitor is zero. Find (a) the current, (b) the power, (c) the volt-amperes, (d) the power factor, (c) the phase angle. (f) Draw a vector diagram of the current and voltage relations.

> Given: $\begin{aligned} E & =250 \\ f & =60 \\ C & =10\end{aligned}$

Find:
$I=$ ?
$P=$ ?
$V A=$ ?
$\mathrm{P}-\mathrm{F}=$ ?
$\theta=$ ?
Vector diagram $=$ ?

Solution:
(a)

$$
\begin{aligned}
I & =2 \pi f C E 10^{-6}=6.28 \times 60 \times 10 \times 250 \times 10^{-6} \\
& =0.943 \mathrm{amp}
\end{aligned}
$$

(b)

$$
P=I^{2} R=0.943^{2} \times 0=\text { zero }
$$

(c)

$$
V A=E I=250 \times 0.943=235.7
$$

(d)

$$
\text { P-F }=\frac{P}{V A}=\frac{0}{235.7}=\text { zero }
$$

(e) $\theta=90^{\circ}$ leading current
(f) Vector diagram, Fig. 10-16b


Fig. 10-16.
Example 10-12. A scries circuit containing a resistor of 40 ohms, an inductor of 80 millihenries, and a capacitor of 40 microfarads is connected across a 250 -volt 60 -cycle line as shown in Fig. 10-17a. The resistances of the inductor and the capacitor are assumed to be zero. Find (a) the inductive reactance, (b) the capacitive
reactance, (c) the impedance, (d) the current, (e) the power, ( $f$ ) the apparent power, ( $g$ ) the power factor, ( $h$ ) the phase angle, ( $i$ ) the voltage drops across the resistor, inductor, and capacitor. (j) Draw a vector diagram.

Given:
$E=250$ volts
$f=60$ cycles
$R=40 \mathrm{ohms}$
$L=80 \mathrm{mh}$
$C=40 \mu \mathrm{f}$


Find:
$X_{L}=$ ?
$X_{C}=$ ?
$Z=$ ?
$I=$ ? $P=$ ?
$V A=$ ?
$\mathrm{P}-\mathrm{F}=$ ?
$\theta=$ ?
$E_{R}=$ ?
$E_{L}=$ ?
$E_{C}=$ ?
Vector diagram $=$ ?

Fig. 10-17a.
Solution:
(a)

$$
\begin{aligned}
X_{L} & =2 \pi f L=6.28 \times 60 \times 80 \times 10^{-3} \\
& =30.1 \text { ohms }
\end{aligned}
$$

(b)

$$
\begin{aligned}
X_{C} & =\frac{10^{6}}{2 \pi f C}=\frac{10^{6}}{6.28 \times 60 \times 40} \\
& =66.2 \mathrm{ohms}
\end{aligned}
$$

(c)

$$
\begin{aligned}
Z & =\sqrt{R^{2}+\left(X_{L}-X_{C}\right)^{2}}=\sqrt{40^{2}+(30.1-66.2)^{2}} \\
& =\sqrt{40^{2}+(-36.1)^{2}}=\sqrt{ } 1600+1303=\sqrt{2903} \\
& =53.8 \mathrm{ohms}
\end{aligned}
$$

(d)

$$
I=\frac{E}{Z}=\frac{250}{53.8}=4.65 \mathrm{amp}
$$

(e)

$$
P=I^{2} R=4.65^{2} \times 40=865 \text { watts }
$$

(f)

$$
V A=E I=250 \times 4.65=1162
$$

(g)

$$
\mathrm{P}-\mathrm{F}=\frac{P}{V A}=\frac{865}{1162}=0.744
$$

(h)

$$
\theta=42^{\circ}
$$

(i)

$$
\begin{aligned}
& E_{R}=I R=4.65 \times 40=186 \mathrm{volts} \\
& E_{L}=I X_{L}=4.65 \times 30.1=140 \mathrm{volts} \\
& E_{C}=I X_{C}=4.65 \times 66.2=309 \mathrm{volts}
\end{aligned}
$$

(j) Vector diagram, Fig. 10-17b


Fig. 10-17b.
Note: The voltage drop across the capacitor is greater than the line voltage. This is not in error, for this can occur in a series circuit that contains resistance, inductance, and capacitance. Whether or not it occurs depends upon the relative values of resistance, inductive reactance, and capacitive reactance.

Example 10-13. An inductor (inductance coil) is connected in a circuit as shown in Fig. 10-18a. The readings taken show 110 volts, 60 cycles, 3 amperes, and 66 watts. What are the (a) impedance, (b) resistance, (c) apparent power, (d) power factor, (e) phase angle, $(f)$ inductive reactance, and ( $g$ ) inductance of the coil? (h) Draw a vector diagram.


Fig. 10-18a.

Given:

$$
\begin{aligned}
E & =110 \\
I & =3 \\
P & =66 \\
f & =60
\end{aligned}
$$

Find:
$Z=$ ?
$R=$ ?
$\mathrm{A}-\mathrm{P}=$ ?
$\mathrm{P} \cdot \mathrm{F}=$ ?
$\theta=$ ?
$X_{L}=$ ?
$L=$ ?
Vector diagram $=$ ?

Solution:
(a)

$$
Z=\frac{E}{I}=\frac{110}{3}=36.7 \mathrm{ohms}
$$

$$
\begin{equation*}
R=\frac{P}{I^{2}}=\frac{66}{3^{2}}=7.33 \mathrm{ohms} \tag{b}
\end{equation*}
$$

(c)

$$
\Lambda-\mathrm{P}=E I=110 \times 3=330 \mathrm{v}-\mathrm{a}
$$

$$
\begin{equation*}
\mathrm{P}-\mathrm{F}=\frac{P}{V A}=\frac{66}{330}=0.200 \tag{d}
\end{equation*}
$$

(e)

$$
\begin{aligned}
\theta & =\text { angle whose cosine is } 0.200 \\
& =78.5^{\circ}
\end{aligned}
$$

(f)

$$
\begin{aligned}
X_{L} & =\sqrt{Z^{2}-R^{2}}=\sqrt{36.7^{2}-7.33^{2}}=\sqrt{1345.9-53.7} \\
& =\sqrt{1292.2}=35.9 \mathrm{ohms}
\end{aligned}
$$

(g)
$L=\frac{X_{L}}{2 \pi f}=\frac{35.9}{377}=0.0952$ henry
(h) Vector diagram, Fig. 10-18b

10-9. Parallel Circuits. Parallel circuits are used more frequently in electrical systems than are series circuits. In electronic equipment, scries, parallel, and combination circuits are used. Because all inductors and capacitors have some resistance, it is not pos-
 sible to set up a circuit containing pure reactances connected in parallel. However, in some inductors and capacitors, especially capacitors, the resistance is so low in comparison to the reactance that the resistance is ignored, that is, assumed to be zero. Under these conditions, a circuit may be treated as if it would contain any combination of pure resistances and pure reactances connected in parallel. The characteristics of such a parallel circuit can best be illustrated by solving for the impedance, currents, power, etc., of a typical parallel circuit.

The various factors of a parallel circuit can be determined with the least difficulty by observing the following order:

1. Find the impedance of each branch.
2. Find the current of each branch.
3. Draw a vector diagram of the currents.
4. Find the total inphase or resistance current.

5 . Find the total reactance current.
6. Find the line current.
7. Find the impedance of the circuit.
8. Find the line power (sum of the separate powers).
9. Find the line volt-amperes ( $E_{\text {line }} \times I_{\text {line }}$ ).
10. Find the line power factor (line nower divided by the line voltamperes).

This order is recommended but is not essential; in fact it may have to be altered to meet the needs of the problem being considered. If the impedance of a parallel circuit is desired but no line voltage is specified, any convenient value may be assumed for calculating purposes.

The equation for the line current in a parallel circuit containing only pure resistances and pure reactances (see Fig. 10-19a) is obtained by combining steps 1 to 6 listed above and is

$$
\begin{equation*}
I_{\mathrm{line}}=\sqrt{\left(I_{R 1}+I_{R 2}+I_{R 3}\right)^{2}+\left(I_{X C 1}+I_{X C 2}-I_{X L 1}-I_{X L 2}\right)^{2}} \tag{10-6}
\end{equation*}
$$

In the second parentheses, all leading currents (caused by capacitive reactance) are plus, and all lagging currents (caused by inductive reactance) are minus. When the sum of the values in the second parentheses is a plus, it indicates that the line current is leading the voltage, and if negative it indicates that the line current is lagging.

Example 10-14. The parallel circuit shown in Fig. 10-19a consists of only pure resistances and pure reactances. Find (a) the current of each branch, (b) the line current, (c) the impedance of the circuit, (d) the power consumed by each branch, (e) the power taken by the complete circuit, $(f)$ the volt-amperes of the complete circuit, ( $g$ ) the line power factor, ( $h$ ) the phase angle between line current and line voltage. (i) Draw a vector diagram of the voltage and all the currents.

Given:


Fig. 10-19a.
Solution:
(a) Use steps 1 and 2.

$$
\begin{aligned}
& I_{1}=\frac{E_{1}}{Z_{1}}=\frac{100}{20}=5 \mathrm{amp} \\
& I_{2}=\frac{E_{2}}{Z_{2}}=\frac{100}{20}=5 \mathrm{amp} \\
& I_{3}=\frac{E_{3}}{Z_{3}}=\frac{100}{40}=2.5 \mathrm{amp} \\
& I_{4}=\frac{E_{4}}{Z_{4}}=\frac{100}{40}=2.5 \mathrm{amp} \\
& I_{5}=\frac{E_{5}}{Z_{5}}=\frac{100}{40}=2.5 \mathrm{amp} \\
& I_{6}=\frac{E_{6}}{Z_{6}}=\frac{100}{80}=1.25 \mathrm{amp}
\end{aligned}
$$

(b) Use steps $3,4,5,6$.

Step 3, vector diagram (Fig. 10-19b).


Fig. 10-19b.
Steps 4, 5, 6 are combined in Eq. (10-6).

$$
\begin{aligned}
I_{\text {live }} & =\sqrt{\left(I_{R_{1}}+I_{R 2}\right)^{2}+\left(I X C+I X C_{2}-I_{X L_{1}}-I_{\left.X L_{2}\right)^{2}}\right.} \\
& =\sqrt{(5+2.5)^{2}+(2.5+1.25-5-2.5)^{2}} \\
& =\sqrt{(7.5)^{2}+(-3.75)^{2}}=\sqrt{56.25+14.06} \\
& =\sqrt{70.31} \\
& =8.38 \mathrm{amp}
\end{aligned}
$$

(c) Use step 7.

$$
Z_{\mathrm{cct}}=\frac{E_{\text {line }}}{I_{1 \mathrm{ine}}}=\frac{100}{8.38}=11.93 \mathrm{ohms}
$$

(d) Power may be calculated by Eq. (2-14); note that circuits containing no resistance take no power.

$$
\begin{aligned}
& P_{1}=I_{1}{ }^{2} R_{1}=5^{2} \times 20=500 \text { watts } \\
& P_{2}=\text { no resistance in circuit }=\text { zero } \\
& P_{3}=I_{3}{ }^{2} R_{2}=2.5^{2} \times 40=250 \text { watts } \\
& P_{4}=\text { no resistance in circuit }=\text { zero } \\
& P_{5}=\text { no resistance in circuit }=\text { zero } \\
& P_{6}=\text { no resistance in circuit }=\text { zero }
\end{aligned}
$$

(e) Use step 8.

$$
\begin{aligned}
P_{\text {line }} & =P_{1}+P_{2}+P_{3}+P_{4}+P_{5}+P_{6} \\
& =500+0+250+0+0+0 \\
& =750 \mathrm{watts}
\end{aligned}
$$

(f) Use step 9.

$$
\begin{aligned}
V A_{\text {line }} & =E_{\text {line }} \times I_{\text {line }} \\
& =100 \times 8.38 \\
& =838
\end{aligned}
$$

(g) Use step 10.

$$
\begin{aligned}
P-\mathrm{F}_{\text {line }} & =\frac{P_{\text {line }}}{V A_{\text {line }}}=\frac{750}{838} \\
& =0.895
\end{aligned}
$$

(h) $\theta_{\text {line }}=26.5^{\circ}$ lagging (from Appendix XI)
(i) Vector diagram, Fig. 10-19c


Scale: linch=4amperes
linch $=50 \mathrm{volts}$
Fig. 10-19c.
10-10. Parallel-series Circuits. When one or more series groups become part of a parallel system, the combined circuit is called a parallelseries circuit. The following procedure is suggested for solving such circuits:

1. Find the impedance, current, power, $\cos \theta$, and $\sin \theta$ for each branch.
2. Resolve cach current into its inphase and quadrature ( 90 -degree angle) components. This is done in the following manner. For example, a current of five amperes lags its voltage by 30 degrees, as shown in Fig. 10-20. As a similar current could be caused by a resistance current $I_{R}$


Fig. 10-20.-The current $I$ lagging the voltage by 30 degrees resolved into its inphase or resistance component and its quadrature or reactance component. This procedure may also be applied to any leading current.
and a reactance current $I_{X}$, we may resolve the current $I$ into two such components. By trigonometry (see Appendix X)

$$
\begin{aligned}
& I_{R}=I \times \cos \theta=5 \times \cos 30^{\circ}=5 \times 0.866=4.33 \\
& I_{X}=I \times \sin \theta=5 \times \sin 30^{\circ}=5 \times 0.500=2.5
\end{aligned}
$$

3. Calculate the line current by combining the inphase components and the quadrature components of all the branch currents. This may be done in the form of the following equation
$I_{\text {line }}=\sqrt{\left(I_{1} \cos \theta_{1}+I_{2} \cos \theta_{2}, \text { etc. }\right)^{2}+\left( \pm I_{1} \sin \theta_{1} \pm I_{2} \sin \theta_{2}, \text { etc. }\right)^{2}}$
Note: $\pm I_{1} \sin \theta_{1}$, etc.: Use + for leading currents and - for lagging currents. When the sum of the second parentheses is + , it indicates that $I_{1 \text { ine }}$ is a leading current, and if the sum is - , it indicates that $I_{\text {line }}$ is lagging.
4. Find the impedance of the circuit.
5. Find the power taken by the circuit.
6. Find the volt-amperes of the rircuit.
7. Find the line power factor.

This order is recommended but is not essential. It may even have to be altered to meet the needs of the problem being considered.

Example 10-15. A parallel-series circuit is shown in Fig. 10-21a. (a) Find the current, power, power factor, and phase angle for each branch. (b) Find the current, power, apparent power, power factor, and phase angle for the complete circuit. (c) Draw a vector diagram.

Given:


Fig. 10-21a.

Solution:
(a)

$$
\begin{aligned}
Z_{1} & =\sqrt{R^{2}+X_{L}{ }^{2}}=\sqrt{20^{2}+40^{2}}=\sqrt{2000}=44.7 \mathrm{ohms} \\
I_{1} & =\frac{E_{1}}{Z_{1}}=\frac{100}{44.7}=2.23 \mathrm{amp} \\
P_{1} & =I_{1}{ }^{2} R_{1}=2.23^{2} \times 20=99.5 \mathrm{watts} \\
P-\mathrm{F}_{1} & =\cos \theta_{1}=\frac{R_{1}}{Z_{1}}=\frac{20}{44.7}=0.447 \\
\theta_{1} & =63.5^{\circ} \text { lagging } \\
I_{2} & =\frac{E_{2}}{Z_{2}}=\frac{100}{20}=5 \mathrm{amp} \\
P_{2} & =I_{2}{ }^{2} R_{2}=5^{2} \times 20=500 \mathrm{watts}
\end{aligned}
$$

$$
\begin{aligned}
& \mathrm{P} \cdot \mathrm{~F}_{2}=\frac{R_{2}}{Z_{2}}=\frac{20}{20}=1.00 \\
& \theta_{2}=0^{\circ} \\
& Z_{3}=\sqrt{R_{3}{ }^{2}+X c^{2}}=\sqrt{40^{2}+80^{2}}=\sqrt{8000}=89.4 \mathrm{ohms} \\
& I_{3}=\frac{F_{3}}{Z_{3}}=\frac{100}{89.4}=1.12 \mathrm{amp} \\
& P_{3}=I_{3}{ }^{2} R_{3}=1.12^{2} \times 40=50 \text { watts } \\
& \mathrm{P}_{-\mathrm{F}_{3}}=\frac{R_{3}}{Z_{3}}=\frac{40}{89.4}=0.447 \\
& \theta_{3}=63.5^{\circ} \text { leading } \\
& Z_{4}=\sqrt{R^{2}+\left(X_{L}-X_{C}\right)^{2}}=\sqrt{20^{2}+(75-150)^{2}}=\sqrt{6025}=77.6 \mathrm{ohms} \\
& I_{4}=\frac{E_{4}}{Z_{4}}=\frac{100}{77.6}=1.29 \mathrm{amp} \\
& P_{4}=I_{4}{ }^{2} R_{4}=1.29^{2} \times 20=33.3 \text { watts } \\
& \mathrm{P}_{-\mathrm{F}_{4}}=\frac{R_{4}}{Z_{4}}=\frac{20}{77.6}=0.257 \\
& \theta_{4}=75^{\circ} \text { leading } \\
& I_{5}=\frac{E_{5}}{Z_{5}}=\frac{100}{50}=2 \mathrm{amp} \\
& P_{5}=I_{5}{ }^{2} R_{5}=2^{2} \times 0=0 \text { watts } \\
& \text { P-F } \mathrm{F}_{5}=\frac{R_{5}}{Z_{5}}=\frac{0}{50}=0 \\
& \theta_{5}=90^{\circ} \text { lagging }
\end{aligned}
$$

(b) $\quad I_{1 \mathrm{ine}}=\sqrt{\left(I_{1} \cos \theta_{1}+I_{2} \cos \theta_{2}+I_{3} \cos \theta_{3}+I_{4} \cos \theta_{4}+I_{5} \cos \theta_{5}\right)^{2}}$

$$
\begin{aligned}
& +\left( \pm I_{1} \sin \theta_{1} \pm I_{2} \sin \theta_{2} \pm I_{3} \sin \theta_{3} \pm I_{4} \sin \theta_{4} \pm I_{5} \sin \theta_{5}\right)^{2} \\
= & \sqrt{(2.23 \times 0.447+5 \times 1.000+1.12 \times 0.447+1.29 \times 0.257} \\
& \frac{(2 \times 0.000)^{2}+(-2.23 \times 0.895+5 \times 0.000+1.12}{\times 0.895+1.29 \times 0.966-2 \times 1.000)^{2}} \\
= & \sqrt{(1.00+5+0.50+0.331+0)^{2}+(-2.0+0+1.0+1.25-2.0)^{2}}
\end{aligned}
$$

$$
\begin{aligned}
& =\sqrt{6.831^{2}+(-1.75)^{2}}=\sqrt{40.72}=7.05 \mathrm{amp} \\
P_{\mathrm{ling}} & =P_{1}+I_{2}+P_{\mathrm{a}}+P_{4}+P_{5}=99.5+500+50+33.3+0 \\
& =682.8 \mathrm{watts} \\
\mathrm{~A}-\mathrm{P} & =E_{\mathrm{lino}} \times I_{\mathrm{line}}=100 \times 7.05=705 \mathrm{v}-\mathrm{a} \\
\mathrm{P}-\mathrm{F} & =\frac{P_{\mathrm{line}}}{V A_{\mathrm{lide}}}=\frac{682.8}{705}=0.968 \\
\theta & =14.5^{\circ} \mathrm{lagging}
\end{aligned}
$$

(c) Vector diagram, Fig. $10-21 b$


Fig. 10-21b.
10-11. Series-parallel Circuits. When one or more parallel groups become part of a series circuit, the combined circuit is called a seriesparallel circuit. The following procedure is suggested for solving such circuits:

1. Reduce each parallel group to a single value of resistance and reactance (either inductive or capacitive) so that when connected in series they will produce the same effect as the original parallel group. These values are called the equivalent resistance and the equivalent reactance. This may be done by (a) assigning a convenient assumed voltage to the group which is to be used only for finding the impedance and phase angle of the group; (b) finding the equivalent resistance by $R_{\mathrm{eq}}=Z \times \cos \theta$; (c) finding the equivalent reactance by $X_{\text {eq }}=Z \times \sin \theta$.
2. Draw a new circuit diagram, substituting the equivalent resistances and reactances in place of the parallel groups. This circuit will be a series circuit similar to Example 10-6.
3. Solve for the impedance, current, power, apparent power, power factor, and phase angle of the circuit as in Example 10-6.
4. If it is required to get the division of current in the parallel parts of the circuit, this may be done by first finding the voltage drop (IZ) of that part and then getting the separate currents.

Example 10-16. A series-parallel circuit is shown in Fig. 10-22a. (a) Find the current, power, apparent power, power factor, and phase angle of the line. (b) Find the voltage and current of each part of the circuit. (c) Draw a vector diagram.

## Given:



Fig. 10-22a.
Solution:
Step 1a. Equivalent series circuit of group I. (Assume 120 volts.)

$$
\begin{aligned}
I_{R} & =\frac{120}{30}=4 \mathrm{amp} \\
I_{X L} & =\frac{120}{40}=3 \mathrm{amp} \\
I_{\text {line }} & =\sqrt{4^{2}+3^{2}}=\sqrt{25}=5 \mathrm{amp} \\
Z_{\mathrm{eq}} & =\frac{120}{5}=24 \mathrm{ohms} \\
\cos \theta & =\frac{I_{R}}{I_{\text {line }}}=\frac{4}{5}=0.800 \\
\sin \theta & =\frac{I_{X}}{I_{\text {live }}}=\frac{3}{5}=0.600 \\
\theta & =37^{\circ} \operatorname{lagging} \\
R_{\mathrm{eq}} & =Z \times \cos \theta=24 \times 0.800=19.2 \mathrm{ohms} \\
X_{\text {Leq }} & =Z \times \sin \theta=24 \times 0.600=14.4 \mathrm{ohms}
\end{aligned}
$$

Step 1b. Equivalent series circuit of group II. (Assume 240 volts.)

$$
\begin{aligned}
I_{n} & =\frac{240}{60}=4 \mathrm{amp} \\
I_{X C} & =\frac{240}{80}=3 \mathrm{amp} \\
I_{\text {line }} & =\sqrt{4^{2}+3^{2}}=\sqrt{25}=5 \mathrm{amp} \\
Z_{\text {eq }} & =\frac{240}{5}=48 \mathrm{ohms}
\end{aligned}
$$

$$
\begin{aligned}
\cos \theta & =\frac{I_{R}}{I_{\text {line }}}=\frac{4}{5}=0.800 \\
\sin \theta & =\frac{l_{x c}}{I_{\text {line }}}=\frac{3}{5}=0.600 \quad \text { (see Appendix X) } \\
\theta & =37^{\circ} \text { leading } \\
R_{\text {eq }} & =Z \times \cos \theta=48 \times 0.800=38.4 \text { ohms } \\
X_{\text {Ceq }} & =Z \times \sin \theta=48 \times 0.600=28.8 \text { ohms }
\end{aligned}
$$

Step 2. Equivalent-series-crieuit diagram (Fig. 10-22b)


Fig. 10-22b.
Step 3.

$$
\begin{aligned}
Z_{\text {cet }} & =\sqrt{(19.2+10+38.4)^{2}+(14.4-28.8)^{1}} \\
& =\sqrt{67.6^{2}+(-14.4)^{2}}=\sqrt{4777}=69.1 \mathrm{ohms}
\end{aligned}
$$

(a)

$$
\begin{aligned}
I_{\text {line }} & =\frac{E_{\text {line }}}{Z_{\text {cet }}}=\frac{120}{69.1}=1.73 \mathrm{amp} \\
P_{\text {cet }} & =I^{2} R_{\text {cet }}=1.73^{2} \times 67.6=203 \mathrm{watts} \\
\mathrm{~A}-\mathrm{P}_{\mathrm{cet}} & =E_{\text {line }} \times I_{\text {line }}=120 \times 1.73=208 \mathrm{v}-\mathrm{a} \\
\mathrm{P}-\mathrm{F}_{\text {cet }} & =\frac{\mathrm{P}_{\text {cct }}}{\mathrm{A}-\mathrm{P}_{\text {cet }}}=\frac{203}{208}=0.976 \\
\theta_{\text {cet }} & =12.5^{\circ} \text { leading }
\end{aligned}
$$

(b)

Voltage at group $I=I_{\text {line }} \times Z_{\text {eq }}=1.73 \times 24=41.5$ volts Voltage at $R_{2}=I_{2} R_{2}=1.73 \times 10=17.3$ volts
Voltage at group II $=I_{G R \cdot I I} \times Z_{\text {eq }}=1.73 \times 48=83$ volts

$$
\begin{gathered}
I_{R 1}=\frac{E_{G R \cdot I}}{R_{1}}=\frac{41.5}{30}=1.38 \mathrm{amp} \\
I_{X L 1}=\frac{E_{G R \cdot I}}{X_{L 1}}=\frac{41.5}{40}=1.04 \mathrm{amp}
\end{gathered}
$$

Check:

$$
\begin{gathered}
\sqrt{1.38^{2}+1.04^{2}} \text { should equal } 1.73=\sqrt{1.91+1.08}=\sqrt{2.99}=1.73 \\
I_{R 2}=I_{\text {lipe }}=1.73 \mathrm{amp} \\
I_{R 3}=\frac{E_{G R \cdot I I}}{R_{3}}=\frac{83}{60}=1.38 \mathrm{amp} \\
I_{X C}=\frac{E_{G R \cdot I I}}{X_{C}}=\frac{83}{80}=1.04 \mathrm{amp}
\end{gathered}
$$

The voltage and current distribution for this circuit is shown in Fig. 10-22c.


Fig. 10-22c.
(c) Vector diagram, Fig. 10-22d


Fig. 10-22d.

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## QUESTIONS

1. In what manner do the voltage and current characteristics of alternating current differ from those of direct current?
2. What is meant by the expression that the current and voltage are in phase?
3. What opposition is offered to the flow of current in a d-c circuit?
4. What opposition is offered to the flow of current in an a-c circuit?
5. Define inductive reactance.
6. Explain the cause of inductive reactance.
7. Describe two effects of inductive reactance upon the current in an a-c circuit.
8. Define capacitive reactance.
9. Explain the cause of capacitive reactanch.
10. Describe two effects of capacitive reactance upon the current in an a-c circuit.
11. What symbol is used to represent (a) inductive reactance, (b) capacitive reactance?
12. What unit is used to cxpress the amount of (a) inductive reactance, (b) capacitive reactance?
13. What is meant by a perfect inductor? Capacitor?
14. Is it possible to construct a perfect inductor? Capacitor? Fxplain your answer.
15. What power would be consumed by a perfect inductor? Capacitor? Explain your answer.
16. Define impedance.
17. (a) What symbol is used to represent impedance? (b) What unit is used to express the amount of impedance?
18. (a) What is an impedance triangle? (b) How is an impedance triangle drawn for a circuit containing resistance and inductive reactance?
19. How are the resistance and inductive reactance of a practical inductor considered in calculating a-c circuits?
20. How are the resistance and capacitive reactance of a practical capacitor considered in calculating a-c circuits?
21. What is meant by a complex circuit?
22. In solving complex series circuits, how are the following individual factors considered: (a) resistance, (b) inductive reactance, (c) capacitive reactance?
23. (a) What is a vector? (b) What quantities may be represented by vectors? (c) What is the standard direction of vector rotation?
24. (a) Describe the graphical method of solving vector values. (b) What factors determine the accuracy of this method of solution?
25. Explain the use of vectors when both voltage and current are to be represented.
26. Explain what is meant by the expression: the current and voltage are out of phase.
27. What is meant by the expression: the current lags the voltage?
28. What is meant by the expression: the current leads the voltage?
29. Explain the construction of an impedance triangle for a circuit containing (a) resistance and inductance; (b) resistance and capacitance; (c) resistance, inductance, and capacitance.
30. Describe the vector method of solving a-c circuits.
31. What is meant by the apparent power? How is its value obtained?
32. What is meant by the volt-amperes of a circuit? How is its value obtained?
33. What is meant by the actual power of a circuit? How may its value be obtained?
34. What is meant by the power factor of a circuit? How may its value be obtained?
35. Under what conditions will the power factor of a circuit be unity (one)? Zero?
36. What is the most desired value for the power factor of a circuit? Why?
37. Describe two methods of obtaining the power factor of a circuit by meter readings.
38. What is meant by phase angle?
39. How is a leading or lagging angle expressed in terms of the voltage and current?
40. Compare the line current in a series a-c circuit with the current in its component parts.
41. Compare the impedance of a series circuit with the impedance of its component parts when the circuit contains: (a) resistance and inductive reactance; (b) resistance and capacitive reactance; (c) resistance, inductive reactance, and capacitive reactance.
42. Compare the line voltage of a series circuit with the voltage of its component parts when the circuit contains: (a) resistance and inductive reactance; (b) resistance and capacitive reactance; (c) resistance, inductive reactance, and capacitive reactance.
43. Compare the line voltage of a parallel a-c circuit with the voltage of its component parts.
44. Compare the impedance of a parallel circuit with the impedance of its component parts when the circuit contains: (a) resistance and inductive reactance; (b) resistance and capacitive reactance; (c) resistance, inductive reactance and capacitive reactance.

Note: Assume the inductor and capacitor to be perfect.
45. Compare the line current of a parallel circuit with the current in its component parts when the circuit contains: (a) resistance and inductive reactance; (b) resistance and capacitive reactance; (c) resistance, inductive reactance, and capacitive reactance.

Note: Assume the inductor and capacitor to be perfect.
46. What procedure is recommended in solving parallel a-c circuits?
47. What is meant by a parallel-series a-c circuit?
48. What procedure is recommended in solving parallel-series a-c circuits?
49. What is meant by a series-parallel a-c circuit?
50. What procedure is recommended in solving series-parallel circuits?

## PROBLEMS

1. An audio choke coil used as a parallel feed in the plate circuit of a radio receiver has a d-c resistance of 405 ohms . The inductive reactance of this coil will vary with the frequency of the audible signal. For a certain low note, its inductive reactance is 2763 ohms and for a certain high note $690,800 \mathrm{ohms}$. What is its impedance ( $a$ ) for the low note? (b) For the high note?
2. A 10 -henry filter choke has a d-c resistance of 475 ohms. (a) What is its inductive reactance at 60 cycles? (b) What is its impedance?
3. An inductance coil has an impedance of 72 ohms and an inductive reactance of 66 ohms. What is its d-e resistance?
4. An $8-\mu \mathrm{f}$ filter capacitor used in the power supply of a radio receiver has an effective series resistance of 8 ohms. (a) What is its capacitive reactance at 60 cycles? (b) What is its impedance?
5. What current will flow through the coil in Prob. 2 when 150 volts is impressed across its terminals?
6. What is the value of signal current flowing through the coil in Prob. 1 when the signal voltage is 10 volts (a) for the low note? (b) For the high note?
7. A filter choke has an inductive reactance of 5655 ohms and a d-c resistance of 375 ohms. (a) What is the impedance of the coil to alternating current? (b) What is the d-c voltage drop when 85 ma (direct current) flows through the coil?
8. A coil having a resistance of 5 ohms and an inductive reactance of 6280 ohms is connected in series with a capacitor having a reactance of 3200 ohms . (a) What is the impedance of the circuit? (b) What current will flow in this circuit when the impressed voltage is 20 volts?
9. What current will flow in an a-c series circuit having a resistance of 5 ohms, an inductive reactance of 80 ohms, and a capacitive reactance of 140 ohms when the impressed voltage is 120 volts?
10. Two inductors are connected in scries with one another across a 300 -volt power supply. One inductor has a resistance of 160 ohms and an inductive reactance of 750 ohms; the second inductor has a resistance of 230 ohms and an inductive reactance of 2250 ohms. Find (a) the impedance of the circuit, (b) current flowing in the line, (c) voltage across each inductor.
11. (a) What is the impedance of a series circuit consisting of a capacitive reactance of 34 ohms , an inductive reactance of 276 ohms , and a resistance of 10 ohms? (b) How much current flows through the circuit when 80 volts is impressed across it? (c) What is the voltage across the resistor, capacitor, and inductor?
12. The antenna circuit of a radio receiver may be represented by two inductors connected in series with a capacitor as shown in Fig. 10-23. (a) What impedance will an antenna circuit offer to a $1200-\mathrm{kc}$ signal if $L_{A}=120 \mu \mathrm{~h}, L_{\mu}=50 \mu \mathrm{~h}$, and $C=100 \mu \mu \mathrm{f}$ ? (Assume the resistance to be so low that it may be ignored.) (b) How much current will flow in the circuit when a signal of 1200 kc and $120 \mu \mathrm{v}$ is impressed across the circuit? (c) What is the voltage across the inductor $L_{P}$ ?


Fig. 10-23.
13. What is the impedance of the circuit shown in Fig. 10-24 for a frequency at which $X_{C 1}=1590 \mathrm{ohms}, R_{C 1}=10 \mathrm{ohms}, X_{C 2}=7840 \mathrm{hms}, R_{C 2}=5 \mathrm{ohms}, X_{L 1}=90,000$ ohms, $R_{L 1}=8$ ohms, $X_{L 2}=10,000 \mathrm{ohms}, R_{L 2}=2 \mathrm{ohms}$ ? The impressed voltage $F^{\prime}$ is equal to 180 volts. Find the line current and the voltages $V_{1}, V_{2}, V_{3}$, and $V_{4}$.


Fig. 10-24.
14. By means of vectors, determine the impedance of a series circuit containing a resistance of 12 ohms and an inductive reactance of 32 ohms . Check the answer by solving for the impedance mathematically.
15. A series circuit has a resistance of 50 ohms and a capacitive reactance of 100 ohms. Determine its impedance graphically and mathematically.
16. A series circuit contains a resistance of 250 ohms, an inductive reactance of 3000 ohms, and a capacitive reactance of 1500 ohms. Determine the impedance of this circuit, using vectors. Check the answer mathematically.
17. By means of vectors, determine the impedance of the circuit used in Prob. 7. How does this answer check with the value obtained mathematically?
18. A series circuit contains a resistance of 150 ohms, a capacitive reactance of 850 ohms , and an inductive reactance of 600 ohms . Determine the impedance of this circuit by means of vectors. Check the answer mathematically.
19. An 8-henry choke coil as used in the power supply of a radio receiver is to be tested by connecting it to a 120 -volt 60 -cycle circuit. (a) Assuming the resistance of the coil to be zero, determine the current, power, volt-amperes, power factor, and phase angle. (b) Draw a vector diagram showing the voltage and current relation.
20. The choke coil in Prob. 19 has a d-c resistance of 290 ohms. Assume that the same voltage is impressed across the coil. (a) Find the current, power, voltamperes, power factor, and phase angle. (b) Draw a vector diagram showing the voltage and current relations.
21. A $30-\mu \mathrm{f}$ capacitor has a capacitive reactance of 88.4 ohms when connected to a 300 -volt 60 -cycle circuit. The resistance of the capacitor is assumed to be zero. (a) Find the current, power, volt-amperes, power factor, and phase angle of the circuit. (b) Draw a vector diagram showing the relation between the current and voltage.
22. If the capacitor in Prob. 21 has a series resistance of 8 ohms and the same voltage is impressed across the capacitor, (a) find the current, power, volt-amperes, power factor, and phase angle of the circuit. (b) Draw a vector diagram showing the relation between the current and voltage.
23. The maximum voltage that the capacitor of Probs. 21 and 22 can withstand is 400 volts. Is the capacitor rating satisfactory for the power line used?
24. A series circuit containing a resistance of 30 ohms , an inductive reactance of 60 ohms , and a capacitive reactance of 40 ohms is connected across a 220 -volt 60 cycle line. (a) Find the impedance, current, power, apparent power, power factor, and phase angle. (b) What is the voltage drop across the resistance, inductive reactance, and capacitive reactance? (c) Draw a vector diagram showing the relation of the voltages and current for this circuit.
25. A series circuit containing a 5000 -ohm resistor, a $0.5-\mu \mathrm{f}$ capacitor, and a $10-$ henry inductor is connected across a 250 -volt 100 -cycle power supply. (a) Find the capacitive reactance, inductive reactance, impedance, current, power, apparent power, power factor, and phase angle. (b) What is the voltage drop across the resistor, capacitor, and inductor, respectively? (c) Draw a vector diagram showing the voltages and current for this circuit.
26. An inductor is connected in a circuit as shown in Fig. 10-18a. The readings taken are 150 volts, 60 cycles, 40 ma , and 0.75 watt. (a) What is the impedance, resistance, apparent power, power factor, phase angle, inductive reactance, and inductance of the coil? (b) Draw a vector diagram showing current and voltage relations.
27. A capacitor is connected in the circuit in place of the inductor (Fig. 10-18a). The readings taken are 150 volts, 60 cycles, 150 ma , and 0.5 watt. (a) What are the impedance, resistance, apparent power, power factor, phase angle, capacitive reactance, and capacitance of the capacitor? (b) Draw a vector diagram showing current and voltage relations.
28. The parallel circuit shown in Fig. 10-25 consists of only pure resistances and pure reactances. Find (a) the current of each branch, (b) the line current, (c)


Fig. 10-25.
the impedance of the circuit, (d) power consumed by each branch, (e) power taken by the complete circuit, $(f)$ the volt-amperes of the complete circuit, (g) the line power factor, ( $h$ ) the phase angle between line current and line voltage. (i) Draw a vector diagram of voltage and all currents.
29. Repeat Prob. 28 for the circuit shown in Fig. 10-26.


Fig. 10-26.
30. Repeat Prob. 28 for the circuit shown in Fig. 10-27.


Fig. 10-27.
31. Repeat Prob. 28 for the circuit shown in Fig. 10-28.

32. Repeat Prob. 28 for the circuit shown in Fig. 10-29.


Fig. 10-29.
33. A parallel-series circuit is shown in Fig. 10-30. (a) Find the current, power, power factor and phase angle for each branch. (b) Find the current, power, apparent power, power factor, and phase angle for the complete circuit. (c) Draw a vector diagram.


Fig. 10-30.
34. Repeat Prob. 33 for the circuit shown in Fig. 10-31.


Fig. 10-31.
35. A series-parallel circuit is shown in Fig. 10-32. (a) Find the current, power, apparent power, power factor, and phase angle of the line. (b) Find the voltage and current of each part of the circuit. (c) Draw a vector diagram.


Fig. 10-32.
36. Repeat Prob. 35 for the circuit shown in Fig. 10-33.


Fig. 10-33.
37. The circuit shown in Fig. 10-34 is sometimes called a low-pass filter. (a) What is the impedance of the capacitor to a 3000 -cycle a-f current? (b) What is the impedance of the capacitor to a 750 -ke r-f current? (c) What percentage (approximate) of the a-f current flows through the capacitor? The output resistor $R_{0}$ ? (d) What percentage (approximate) of the r-f current flows through the capacitor? The output resistor $R_{0}$ ? (e) From the percentage of a-f and r-f currents that flow through the output resistor, justify the designation of the circuit as a low-pass filter.


Fig. 10-34.
38. The circuit shown in Fig. 10-35 is sometimes called a high-pass filter, (a) What is the impedance of the inductor to a 3000 -cycle a-f current? (b) What is the impedance of the inductor to a 750 -ke r-f current? (c) What percentage (approximate) of the a-f current flows through the inductor? The output resistor $R_{0}$ ? (d) What percentage (approximate) of the r-f current flows through the inductor? The output resistor $R_{0}$ ? (e) From the percentage of a-f and $\mathbf{r}$-f currents that flow through the output resistor, justify the designation of the circuit as a high-pass filter.


Fig. 10-35.
39. Figure $10-36$ shows a combination of capacitors and inductors. (a) What is the impedance of the capacitors to an a-f current of 2000 cycles? An r-f current of 1500 ke ? (b) What is the impedance of the inductors to an a-f current of 2000 cycles? An r-f current of 1500 kc ? (c) Can this circuit be classed as cither a highpass or a low-pass filter? Explain.


Fig. 10-36.
40. Figure 10-37 shows a combination circuit of capacitors and inductors. (a) What is the impedance of the inductors to an a-f current of 500 cycles? An r-f current of 550 ke ? (b) What is the impedance of the capacitors to an a-f current of 500 cycles? An r-f current of 550 kc ? (c) Can this circuit be classed as either a high-pass or a low-pass filter? Explain.


Fig. 10-37.

## CHAPTER XI

## RESONANCE

Resonant circuits are one of the most important types of circuits used in radio and television receivers or transmitters. The principles of resonance as applied to these circuits are used to increase the signal voltage and current at a desired frequency and to decrease the signal voltages and currents for all other frequencies. In operating a radio or television receiver, the desired program is obtained by tuning the receiver. Actually, the tuning circuit of the receiver is adjusted so that it is in resonance with the carrier frequency of the station transmitting the desired program. This is just one of the many applications of resonant circuits as applied to radio and electronics.

11-1. Resonance. Resonance is a condition that exists when the inductive reactance and the capacitive reactance of a circuit are equal. Under this condition, the effective or total reactance of a series resonant circuit will be zero and its impedance will be equal to the resistance of the circuit [see Eq. (10-1)]. The impedance will therefore be at its minimum value, and the current will be at its maximum value. As the current is at its maximum value, the power of the circuit will also be at its maximum value.

Resistance is the only opposition that a direct current has when it flows through an electric circuit. When an alternating current flows through a circuit, its flow is opposed by the combined effects of resistance and reactance which is called impedance. The opposition to alternating current offered by a resistance is the same as that offered to a direct current. In Chap. VIII, it was shown that the current due to inductive reactance lags the voltage by 90 degrees. In Chap. IX it was shown that the current due to capacitive reactance leads the voltage by 90 degrees. Therefore the effects of inductive reactance and capacitive reactance are 180 degrees out of phase with one another. The resultant reactance of the circuit is therefore equal to the algebraic sum of its inductive and capacitive reactances.

If either the capacitor or inductor in a series circuit can be adjusted so that their individual reactances are equal, the total reactance of the circuit will be zero and the impedance of the circuit will be equal to its resistance. Such a circuit is called a series resonant circuit. The action of a resonant circuit is best explained by reference to graphs illustrating the variation in the circuit conditions at and near resonance.

11-2. Graphs. Use of Graphs. A graph is a pictorial representation illustrating the manner in which one factor varies with changes in any other factor on which it is dependent. Graphs are used in all branches of radio to illustrate the operating conditions of the various circuits and parts used. Radio manufacturers use graphs to illustrate the operating features of their products. Many of the answers to radio and television problems can be explained better if the results of the problem are plotted in the form of a graph. The characteristics of a circuit for various operating conditions can be compared with one another more easily and understood much better when illustrated on a graph than from using the comparison of a number of figures representing the results for the same operating conditions. It is therefore desirable to know how to plot and interpret graphs.

Simple Graphs. The simplest type of graph is one illustrating the variation of two quantities with one another. This type of variation is generally plotted on cross-section paper called graph paper. This paper consists of a series of vertical and horizontal lines drawn so that the squares formed by these lines are all of equal area, as shown in Fig. 11-1. One of the horizontal lines is used as a reference line to represent one of the quantities and is marked with a suitable scale of values as $0,1,2,3$, shown on Fig. 11-1; all distances in a horizontal direction are called abscissas (or abscissae). One of the vertical lines is used as a reference line to represent the other quantity and is also marked with a seale (though not necessarily the same scale), such as $0,5,10,15$, as shown on Fig. 11-1; all distances in a vertical direction are called ordinates. The point where these two lines meet is called the point of origin and represents zero value for both quantities. For ease in plotting and reading graphs, each square is made to represent either one, two, or five units or some multiple thereof. The square farthest from the point of origin should represent a value equal to or slightly greater than the maximum valuc to be plotted, and each square between these points represents a definite fraction of this value.

Independent and Dependent Variables. When plotting graphs, the independent variable is plotted as the abscissa and the dependent variable as the ordinate. For example, in Fig. 11-1 the line $A$ was plotied to show the values of voltage required to push currents of $2,4,6,8$, and 10 amperes through a five-ohm resistance. Because current is the unit to which values have been assigned, it is the independent variable and hence is plotted as the abscissa. The value of the voltage is dependent on the assigned value of the current, $E=I R$, and is called the dependent variable; hence it is plotted as the ordinate. Values of the ordinates must be obtained for definite values of the abscissas. These values are then plotted on the graph paper by making points along the ordinate scale at values corresponding to the amount that was obtained for the definite values of abscissa. A line is
drawn through this series of points to form a straight or curved line. This line represents the variation of the two quantities with respect to one another and is very uscful for finding the value of the variable quantity for any value of the quantity being varied. These operations and principles can best be explained by plotting a few typical curves.

11-3. Plotting, Use, and Interpretation of Curves. By substituting a series of values in any mathematical equation, the results obtained will show how one quantity is affected by any change in another. If these results are plotted in graph form for each of the values used, a curve can be obtained that will illustrate how a factor of this equation varies with the values substituted. Therefore we can plot a curve for any equation in electricity to illustrate how the variable quantity changes with the value being varied.

Example 11-1. Plot a curve showing how the voltage required to force a current through a five-ohm resistor varies with currents of 1 to 10 amperes.

Using Fq. (2-6), $E=I \times R$, we can obtain five points for the curve by substituting $2,4,6,8$ and 10 amperes for the current, the resistance remaining constant at five ohms. These values should be listed in the following tabular form:

Table XI-I

| $R$, ohms | $I$, amperes | $E$, volts |
| :---: | :---: | :---: |
| 5 | 2 | 10 |
| 5 | 4 | 20 |
| 5 | 6 | 30 |
| 5 | 8 | 40 |
| 5 | 10 | 50 |

For this graph, the current is the quantity being varied and is, therefore, the abscissa. The voltage is the dependent variable and is therefore the ordinate. The reference abscissa is divided in equal divisions representing current in amperes, and the reference ordinate is divided into cqual divisions representing the voltage required in volts. Referring to Fig. 11-1, a line is drawn parallel to the ordinates at the two-ampere mark on the reference abseissa; another line is drawn parallel to the abscissas at the 10 -volt mark on the reference ordinate. The point where these two lines meet gives one point for the curve to be drawn. Repeating this process for the results obtained for $4,6,8$, and 10 amperes, four more points are obtained for the curve. $A$ line is drawn through points $1,2,3,4$, and 5 . This line illustrates the manner in which the voltage in a circuit varies with the amount of current flowing.

Graphs Containing a Series of Curves. If necessary, more than one curve can be plotted on the same graph. If the resistance in Example 11-1 were increased to eight ohms, we should substitute eight ohms for the resistance
value and obtain five points for the curve as before. This curve is shown in Fig. 11-1 as curre $B$. Graphs having a series of eurves are quite common in radio work as they are very useful in illustrating more than one variable condition. For example, by referring to Fig. 11-1 we can sce how much voltage is required for increases in current and how this voltage varies with an increase in resistance.


Fig. 11-1.-Curves showing the variation of voltage with current in a circuit. Curve $A$ for a circuit whose resistance is 5 ohms and curve $B$ for a circuit whose resistance is 8 ohms.

Nonlinear Curves. The curves in Fig. 11-1 are straight lines. An illustration of how a curved line graph is obtained is shown by the following example.

Example 11-2. By means of a graph, illustrate how the power in a circuit having a resistance of 10 ohms changes as the current is varied from 0 to 10 amperes.

Using Eq. (2-14),$P^{\prime}=I^{2} R$, we can obtain points for the curve by substituting 1 ,
$2,3,4,5,6,7,8,9$, and 10 for the current $I$. Listing the results in tabular form we have:

Table XI-II

| $R$, ohms | $I$, amperes | $P$, watts |
| :---: | :---: | :---: |
| 10 | 1 | 10 |
| 10 | 2 | 40 |
| 10 | 3 | 90 |
| 10 | 4 | 160 |
| 10 | 5 | 250 |
| 10 | 6 | 360 |
| 10 | 7 | 490 |
| 10 | 8 | 640 |
| 10 | 9 | 810 |
| 10 | 10 | 1000 |

For this graph, the current is the quantity being varied and is therefore the abscissa. The power is the dependent variable and is therefore the ordinate. The abscissas and ordinates are divided in equal divisions and marked to represent current and power, respectively. The 10 points for the curve are now located in the same manner as in the previous example. This graph is shown in Fig. 11-2. The line joining these points is a curved line and should be drawn with the aid of a French curve. In order to obtain an accurate representation of the variation of two quantities when this variation produces a curved line, it is necessary to plot as many points as practical.


Fig. 11-2.-Curve showing the variation of power with current in a circuit whose resistance is 10 ohms .

Point of Origin. The point of origin is not always drawn at the extreme ends of the abscissa and ordinate. When negative values are used, the point of origin may be at the center of either or both of the reference lines. Figure 11-3 illustrates the variation in plate current of a tube with various values of grid voltages. As the potential on the grid varies from negative values to positive values, the point of origin is drawn at the center of the abscissa.


Fig. 11-3.-Curve showing the variation in the plate current of a tube with changes in the grid voltage. Commonly called an $F_{G} I_{P}$ curve.

Interpretation of Curves. Curves are very useful in determining the value of any quantity for any condition shown on the graph. An ordinate may be drawn from any point on the abscissa, and the corresponding value of the variable quantity is read from the point where this line crosses the curve. Referring to Fig. 11-1, 25 volts is required to force five amperes through a resistance of five ohms, and 40 volts will be required if the resistance of the circuit is increased to eight ohms. Referring to Fig. 11-2, the power in the circuitwith 3.5 amperes flowing is 122 watts. In Fig. 11-3 the plate current output for the tube is 3.35 milliamperes when its grid voltage is adjusted to negative 0.5 volt.

A graph is a visual illustration of the relation between two factors. When one factor is increased, the other will either increase or decrease.

If joining the series of points plotted on the graph paper forms a straight line, the two factors are proportional to one another. If one increases when the other is increased, the two factors are directly proportional to one another. If one increases when the other is decreased, the two factors are inversely proportional to one another. When the series of points forms a curved line, the two factors may be varying with each other according to the square law, as in Fig. 11-2, or according to some complex equation.

Curves are also useful in indicating whether or not the figures secured in an experiment are correct. F'or example, the curve shown in Fig. 11-3 was plotted from results obtained from a laboratory experiment; points $A$ and $B$ do not fall on the curve that goes through the other points. Therefore, it is assumed that these points were incorrectly taken, and the adjustments that produced these values should be repeated in order to check the results. There are a number of other interpretations of curves, and these will be taken up as the need arises.

11-4. Series Resonance. Condition of Resonance. A fixed inductance coil and a variable capacitor are connected in series with a signal input to form a resonant circuit as shown in Fig. 11-4a. The resistance $R$ is not, a separate resistance but represents the total resistance of the coil, capacitor, and the conductors in the circuit. Referring to Eq. (8-6), $X_{L}=2 \pi f L$, it can be seen that the inductive reactance of a circuit will increase with an increase in frequency. Referring to Eq. (9-3), $X_{C}=1 / 2 \pi f C$, it can be seen that the capacitive reactance of a circuit will increase as the frequency decreases. A resonant circuit will have its inductive reactance equal to its capacitive reactance at one value of frequency.

If this frequency is kept constant and the capacitance of the circuit is varied, the capacitive reactance will vary. When the capacitance is increased, the capacitive reactance will decrease and the resultant reactance ( $X_{L}-X_{c}$ ) will increase. When the capacitance is decreased, the capacitive reactance will increase and the resultant reactance ( $X_{L}-X_{c}$ ) will also increase. The current flowing in this circuit will be equal to the input signal voltage divided by the impedance, $i=e / z$. Therefore, when the impedance is large, the signal current will be small. By adjusting the variable capacitor, the capacitance of the circuit can be changed. The capacitance of the circuit can be adjusted so that for a given input frequency the capacitive reactance can be made equal to the inductive reactance. The series circuit is then at resonance for the frequency of the input signal, and the current flowing in the circuit is at its maximum value since its only opposition is the resistance of the circuit.

Relation between $f, L$, and $C$ at Resonance. $\Lambda$ t resonance, the inductive reactance is equal to the capacitive reactance; therefore

$$
\begin{equation*}
2 \pi f_{r} L=\frac{1}{2 \pi f_{r} C} \tag{11-1}
\end{equation*}
$$

where $f_{r}=$ resonant frequency, cycles per second
$L=$ inductance, henries
$C=$ capacitance, farads
We can now solve for the resonant frequency by multiplying both sides of the equation by $f_{r}$ and dividing both sides by $2 \pi L$.

$$
\begin{equation*}
f_{r}^{2}=\frac{1}{4 \pi^{2} L C} \tag{11-2}
\end{equation*}
$$

Taking the square root of both sides of the equation, we have

$$
\begin{equation*}
f_{r}=\frac{1}{2 \pi \sqrt{\overline{L C}}} \tag{11-3}
\end{equation*}
$$

This is one of the most important equations in radio work. Using this equation as a basis, the equations used for calculating all tuned circuits, filters, oscillators, etc., are derived. In some radio circuits, the inductance may be fixed in amount, and it is then necessary to determine the amount of capacitance needed to meet the specified conditions of the circuit, while in others the capacitance may be fixed and the amount of inductance needed must be calculated. The value of inductance or capacitance requircd can be easily found by transposing Eq. (11-2) to solve for the inductance in terms of capacitance and frequency or to solve for the capacitance in terms of inductance and frequency. Multiplying both sides of Eq. (11-2) by $L^{\prime}$ and dividing both sides by $f_{\mathrm{r}}^{2}$, the equation for the inductance of a resonant circuit becomes

$$
\begin{equation*}
L=\frac{1}{4 \pi^{2} f_{r}^{2} C} \tag{11-4}
\end{equation*}
$$

Multiplying both sides of Eq. (11-2) by $C$ and dividing both sides by $f_{r}{ }^{2}$, the equation for the capacitance of a resonant circuit becomes

$$
\begin{equation*}
C=\frac{1}{4 \pi^{2} f_{r}^{2} h} \tag{11-5}
\end{equation*}
$$

At radio frequencies, frequency is generally expressed in kilocycles, the inductance in microhenries, and the capacitance in microfarads. A more practical form of Eq. (11-3) is one in which the factors are expressed in the above units. If $L$ is expressed in microhenries, $C$ in microfarads, $f_{r}$ in kilocycles, and 1 is divided by $6.28(\angle \pi)$, Eq. (11-3) may be written as

$$
\begin{equation*}
f_{r}=\frac{159}{\sqrt{L C}} \tag{11-6}
\end{equation*}
$$

In a similar manner, Eqs. (11-4) and (11-5) can be expressed in terms of microhenries, microfarads, and kilocycles; thus

$$
\begin{equation*}
L=\frac{25,300}{f_{r}^{2} \bar{C}} \tag{11-7}
\end{equation*}
$$

and

$$
\begin{equation*}
C=\frac{25,300}{f_{r}^{2} L} \tag{11-8}
\end{equation*}
$$

The following examples will illustrate the practical application of these equations.

Example 11-3. What is the resonant frequency of a series circuit having an inductance of 250 microhenries if the capacitor is adjusted to 350 micromicrofarads?

$$
\begin{array}{lc}
\text { Given: } & \text { Find: } \\
C=0.00035 \mu \mathrm{f} & f_{r}=? \\
L=250 \mu \mathrm{~h} &
\end{array}
$$

Solution:

$$
\begin{aligned}
f_{r} & =\frac{159}{\sqrt{I C}} \\
& =\frac{159}{\sqrt{250 \times 350 \times 10^{-6}}} \\
& =\frac{159}{0.296}=538 \mathrm{kc}
\end{aligned}
$$

Example 11-4. What valuc of inductance is required in a series circuit having a capacitance of 250 micromicrofarads to produce resonance with a 500 -kilocycle signal voltage input?

$$
\begin{array}{lr}
\text { Given: } & \text { Find: } \\
C=0.00025 \mu \mathrm{f} & L=? \\
f=500 \mathrm{kc} &
\end{array}
$$

Solution:

$$
\begin{aligned}
L & =\frac{25,300}{f_{r}^{2} C} \\
& =\frac{25,300}{500 \times 500 \times 250 \times 10^{-6}} \\
& =405 \mu \mathrm{~h}
\end{aligned}
$$

Example 11-5. To what value of capacitance must the variable capacitor of a series circuit be adjusted to produce resonance at 600 kilocycles if the inductance of the circuit is 300 microhenries?

Given:

$$
\begin{aligned}
L & =300 \mu \mathrm{~h} \\
& =600 \mathrm{kc}
\end{aligned}
$$

Find:
$C=$ ?

Solution:

$$
\begin{aligned}
C & =\frac{25,300}{f_{r} L_{\mu}} \\
& =\frac{25,300}{600 \times 600 \times 300} \\
& =0.000234 \mu \mathrm{f} \\
& =234 \mu \mu \mathrm{f}
\end{aligned}
$$

11-5. Resonance Curves. If a voltage of constant frequency is applied to a series circuit containing a fixed inductance and a variable capacitance, the circuit may be adjusted so that it will be resonant at this frequency. If the frequency of the applied voltage is then varied from a value starting below the frequency of resonance and gradually increased to a value beyond the frequency of resonance, the current flowing through the circuit will vary


Fig. 11-4.-Series resonance: (a) a series resonant circuit, (b) resonance curves of the circuit shown in (a).
from a very low value at a frequency below resonance, increasing until the frequency of resonance is reached. At this point the current will be at its maximum value and will decrease to a very low value again when the frequency is increased beyond the frequency of resonance. This variation in signal current with frequency change is best shown by a resonance curve.

Method of Obtaining Resonance Curves. Figure 11-4b shows two resonance curves for the series circuit of Fig. 11-4a with two values of $R$. The values used to plot these resonance curves are given in Table XI-III. They
were obtained by substituting the values of $R, X_{L}$, and $X_{c}$ in Eq. (10-1) for a number of values of frequency above and below the frequency of resonance. As the circuit is resonant at 1500 kilocycles values were obtained for frequencies between 1470 and 1530 kilocycles. Lxamination of the curves shows that no appreciable current flows until the frequency is

Table XI-III

| Frequency, <br> kc. | $X_{L}$ | $X_{C}$ | $R$ | $Z$ | ma. when <br> $E=0.5$ |
| :---: | :---: | :--- | :--- | :--- | :--- |
| 1470 | 980 | 1020 | 5 | 40.31 | 12.4 |
| 1475 | 983.5 | 1016.5 | 5 | 33.37 | 14.98 |
| 1480 | 987 | 1013 | 5 | 26.47 | 18.8 |
| 1485 | 990 | 1010 | 5 | 20.61 | 24.2 |
| 1490 | 993 | 1007 | 5 | 14.86 | 33.6 |
| 1495 | 997 | 1003 | 5 | 7.81 | 64.02 |
| 1500 | 1000 | 1000 | 5 | 5.0 | 100 |
| 1505 | 1003 | 997 | 5 | 7.81 | 64.02 |
| 1510 | 1007 | 993 | 5 | 14.86 | 33.6 |
| 1515 | 1010 | 990 | 5 | 20.61 | 24.2 |
| 1520 | 1013 | 987 | 5 | 26.47 | 18.8 |
| 1525 | 1016.5 | 983.5 | 5 | 33.37 | 14.98 |
| 1530 | 1020 | 980 | 5 | 40.31 | 12.4 |
|  |  |  |  |  |  |
| 1470 | 980 | 1020 | 10 | 41.23 | 12.1 |
| 1475 | 983.5 | 1016.5 | 10 | 34.48 | 14.5 |
| 1480 | 987 | 1013 | 10 | 27.85 | 17.9 |
| 1485 | 990 | 1010 | 10 | 22.36 | 22.3 |
| 1490 | 993 | 1007 | 10 | 17.20 | 29.1 |
| 1495 | 997 | 1003 | 10 | 11.68 | 42.8 |
| 1500 | 1000 | 1000 | 10 | 10.0 | 50.0 |
| 1505 | 1003 | 997 | 10 | 11.68 | 42.8 |
| 1510 | 1007 | 993 | 10 | 17.20 | 29.1 |
| 1515 | 1010 | 990 | 10 | 22.36 | 22.3 |
| 1520 | 1013 | 987 | 10 | 27.85 | 17.9 |
| 1525 | 1016.5 | 983.5 | 10 | 34.48 | 14.5 |
| 1530 | 1020 | 980 | 10 | 41.23 | 12.1 |

approximately 1470 kilocycles. The current increases slowly until the frequency is very close to the resonant frequency. Then the current increases very rapidly until the maximum current is reached at the frequency of resonance. As the frequency is increased beyond resonance, the current decreases very rapidly at first and then decreases more slowly until no appreciable current flows at approximately 1530 kilocycles. In radio, a circuit that is or can be adjusted so that it is resonant for a definite frequency is referred to as a tuned circuit.

Effect of Resistance in Series Resonant Circuits. At resonance, the strength of the current flowing in a series tuned circuit is dependent entirely
upon the resistance of the circuit as is shown in the following steps:

$$
\begin{equation*}
I=\frac{E}{Z} \tag{8-9}
\end{equation*}
$$

and

$$
\begin{equation*}
Z=\sqrt{R^{2}+\left(X_{L}-X_{c}\right)^{2}} \tag{10-1}
\end{equation*}
$$

but with resonance $X_{L}=X_{C}$
then

$$
\begin{equation*}
Z=\sqrt{R^{2}+(0)^{2}}=\sqrt{R^{2}}=R \tag{11-9}
\end{equation*}
$$

Figure 11-4b shows the resonance curves for two circuits tuncd to the same frequency, one having a resistance of five ohms and the other a resistance of 10 ohms . At resonance, the current flowing through the circuit having five ohms resistance is twice the amount flowing through the circuit having 10 ohms resistance. Therefore, to produce maximum current, it is best that the resistance of a scries tuned circuit be as small as possible.

11-6. Circuit Q. Definition of Circuit $Q$. The ratio of the inductive reactance of a tuned circuit to the resistance of the circuit is referred to as the $Q$ of the circuit. Expressed mathematically

$$
\begin{equation*}
Q=\frac{X_{L}}{R} \tag{11-11}
\end{equation*}
$$

As $R$ is the resistance of the entire series circuit, its value will be higher, though only slightly higher, than the resistance of the coil. The circuit $Q$ will therefore be less than the coil $Q$. The difference between the values of the circuit $Q$ and the coil $Q$ is normally small because the resistance of the capacitor and connecting wires of the circuit is small compared with the resistance of the coil.

Relation between Circuit $Q$ and the Slope of the Resonance Curves. The slope of the resonance curve is determined by the $Q$ of the resonant circuit. This can be seen by the relative slopes of the two curves in Fig. 11-4b and can be explained in the following manner.

At resonance, the impedance of the series circuit becomes equal to $R$ and therefore the current is inversely proportional to the resistance as shown by
also,

$$
\begin{align*}
I_{+} & =\frac{E}{\bar{R}}  \tag{11-12}\\
R & =\frac{X_{L}}{Q}  \tag{11-13}\\
I_{r} & =\frac{E}{\frac{X_{L}}{Q}}=\frac{E Q}{X_{L}} \tag{11-14}
\end{align*}
$$

The current at resonance is therefore also directly proportional to the circuit $Q$. It may now be stated that the higher the resistance, the lower the current and circuit $Q$ will be, while the lower the resistance, the higher the current and circuit $Q$ will be.

The impedance of the circuit for frequencies differing appreciably from the resonant frequency is practically equal to the reactance of the circuit, because at these frequencies the reactance is much greater than the resistance and the impedance becomes practically equal to the reactance of the circuit. The current at these frequencies is therefore practically independent of the circuit resistance. Increasing the circuit resistance, which also lowers the circuit $Q$, will decrease the current at resonance without affecting the current at frequencies differing appreciably from resonance. This action can be seen by observation of the two curves in Fig. 11-4b. The slope of the curve having a high circuit $Q$ is steep. When the resistance of the circuit is increased, the current at resonance will decrease, thus decreasing the slope of the curve. This slope is very important in tuning and filter circuits and will be discussed in more detail in subsequent articles.

Relation between the Width of the Resonance Curve and the Circuit $Q$. The width of the resonance curve can be used as a measure of the ability of a circuit to select the signal at a desired frequency and to eliminate the signals of all other frequencies. This characteristic is very important in tuning and filter circuits. It is general practice to measure the width of resonance curves at the point where the current is 0.707 times the current at resonant frequency.

This condition of $I=0.707 I$, will occur when the effective reactance $X_{L}-X_{c}$ is equal to the resistance $R$. This is shown mathematically in the following steps:

$$
\begin{equation*}
I=\frac{E}{\sqrt{R^{2}+\left(X_{L}-X_{C}\right)^{2}}} \tag{11-15}
\end{equation*}
$$

Substituting $R$ for $X_{L}-X_{c}$, then

$$
\begin{equation*}
I=\frac{E}{\sqrt{2 R^{2}}}=0.707 \frac{E}{\bar{R}} \tag{11-16}
\end{equation*}
$$

Substituting $I_{r}$ for $\frac{E}{R}$,

$$
\begin{equation*}
I=0.707 I_{r} \tag{11-17}
\end{equation*}
$$

If the frequency of the impressed voltage is varied both above and below the frequency of resonance, two values of frequency $f_{1}$ and $f_{2}$ can be obtained that will produce currents $I_{1}$ and $I_{2}$ each equal to $0.707 I_{r}$, as shown in Fig. 11-5. The width of the resonance curve at this point is $f_{2}-f_{1}$ and is generally referred to as the width of the band. The circuit $Q$
can be expressed in terms of these frequencies and is equal to the resonant frequency divided by the difference between the two frequencies required to obtain a current of $0.707 I_{r}$. 'This is shown mathematically in the following steps.

$$
\begin{equation*}
Q=\frac{X_{L}}{R}=\frac{2 \pi f_{r} L}{R} \tag{11-18}
\end{equation*}
$$

Substituting $X_{L}-X_{c}$ for $R$, then

$$
\begin{equation*}
Q=\frac{2 \pi f_{r} L}{X_{L}-X_{C}} \tag{11-19}
\end{equation*}
$$



Fig. 11-5.-Resonance curves with the width of the band ( $f_{2}-f_{1}$ ) indicated.
However, as the condition of $X_{L}-X_{c}=R$ occurs at $f_{1}$ and $f_{2}$, then $X_{L}-$ $X_{c}$ will also be equal to $2 \pi f_{2} L-2 \pi f_{1} L$, and substituting this in the above equation, it becomes

$$
\begin{equation*}
Q=\frac{2 \pi f_{r} L}{2 \pi f_{2} L-2 \pi f_{1} L}=\frac{f_{r}}{f_{2}-f_{1}} \tag{11-20}
\end{equation*}
$$

The quantity $f_{2}-f_{1}$ is referred to as the width of the frequency band and may be expressed mathematically as

$$
\begin{equation*}
f_{2}-f_{1}=\frac{f_{r}}{Q} \tag{11-21}
\end{equation*}
$$

Substituting $X_{L} / R$ or $2 \pi f_{\tau} L / R$ for $Q$ in the above equation, it becomes

$$
\begin{equation*}
f_{2}-f_{1}=\frac{f_{r} R}{2 \pi f_{r} L}=\frac{R}{2 \pi L} \tag{11-22}
\end{equation*}
$$

From this equation, it can be seen that the width of the resonance curve is directly proportional to its circuit resistance and inversely proportional to its inductance. In order to obtain high selectivity in tuning circuits of a-m radio reccivers, it is essential that the width of the resonance curve be as narrow as practicalıy possible. This means that the resistance of the circuit must be small and the inductanen of the coil high; in other words, the circuit $Q$ must be high.

Example 11-6. A series tuned circuit has a resistance of five ohms and an inductance of 225 microhenries. (a) What is the width in cycles of its resonance curve at a point where the current in the circuit is equal to 0.707 times the current at resonance? (b) If the resistance of the circuit is increased to 10 ohms, how does this change affect the width of the resonance curve?

Given:

$$
\begin{aligned}
& R=5 \mathrm{ohms} \\
& L=225 \mu \mathrm{~h} \\
& R=10 \mathrm{ohms} \\
& L=225 \mu \mathrm{~h}
\end{aligned}
$$

## Find:

$f_{2}-f_{1}=$ ?
$f_{2}-f_{1}=?$

Solution:
(a)

$$
\begin{aligned}
f_{2}-f_{1} & =\frac{R}{2 \pi L} \\
& =\frac{5}{6.28 \times 225 \times 10^{-6}}=3538 \text { cycles }
\end{aligned}
$$

$$
\begin{equation*}
f_{2}-f_{1}=\frac{10}{6.28 \times 225 \times 10^{-6}}=7076 \text { cycles } \tag{b}
\end{equation*}
$$

11-7. LC Product. Relation of $f_{r}, L$, and $C$. Obscrvation of the equation for finding the frequency of resonance of a resonant circuit [ Fq. . (11-3)] will indicate that there is a definite relation between the frequency of resonance and the inductance and capacitance of the circuit. If the values of any two of these quantities are known, the third can be found by substituting the known values in this equation and solving for the unknown quantity. Solving by use of this equation is not always practicable; therefore other simple means have been devised.

LC Product. Close observation of the above equation will show that the frequency of resonance is dependent on the product of the inductance and capacitance of the circuit. Therefore, for each value of $L$ times $C$ there can be only one frequency at which resonance occurs and, conversely, there can be only one $L C$ product for each resonant frequency. If the $L C$ product for a desired frequency is known, the capacitance required can be found by dividing this product by the value of inductance used, or the inductance required can be found by dividing the $L C$ product by the capacitance used. Values of $L$ times $C$ for commonly used frequencies are
often listed in tabular or graphical form. These charts or tables can be found in many radio reference books. A tabular listing of the $L C$ product is found in the authors' Essentials of Radio.

LC Checker. Another practical means of finding the value of the unknown variable quantity of a resonant circuit is by the use of an instrument


Fig. 11-6.-An $L C$ checker and two of its applications. Left, the $L C$ checker; upper right, checking the inductance of a coil; lower right, checking the capacitance of a capacitor in a radio receiver. (Aerovox Corporation.)
such as the $L C$ checker shown in Fig. 11-6. This instrument tests combinations of inductance and capacitance for their resonant frequencies. It can therefore be used to adjust a circuit or system to its proper operating point. The capacitance of any capacitor can also be checked at any radio frequency without removing the capacitor from the circuit.

LC Ratio. As the frequency of resonance of a circuit is only dependent on its $L C$ product, it can be seen that any number of different combinations of $L$ and $C$ can be used to obtain the same resonant frequency. The value of inductance can be made greater than, less than, or equal to that of
capacitance. The manner in which the resonant circuit is to be used will determine the $L C$ ratio that should be used.

When resonant circuits are used in the tuning circuit of a radio receiver, it is desirable to have high values of inductance. This is due to the fact that increasing the inductance increases the circuit $Q$, thereby increasing the slope of the response curve. The steeper this slope, the greater will be the degree of selectivity that can be obtained. However, increasing the selectivity beyond a certain point should be avoided as it will decrease the fidelity of the receiver.


Fig. II-7.-Voltages across the parts of a series resonant circuit at resonant frequency.

This is but onc example of the many applications of resonance to radio circuits. Its application to filter circuits is taken up in the following chapter. This subject is treated in more detail in Essentials of Radio, which deals with the applications of the principles of electricity to radio circuits.

11-8. Voltage Ratios in Series
Resonant Circuits. In a series resonant circuit, the impedance of the circuit at resonance is very small, thus allowing a comparatively large current to flow. This large current flowing through the capacitor and inductor causes a voltage to be developed across these reactances which is greater than the voltage applied to the circuit. The amount of voltage developed across either reactance is equal to the product of the current flowing through the circuit and the value of the reactance. Expressed mathematically,

$$
\begin{align*}
& E_{X_{L}}=I_{r} X_{L}  \tag{11-23}\\
& E_{X_{C}}=I_{r} X_{C} \tag{11-24}
\end{align*}
$$

where $E_{X_{L}}=$ voltage developed across the inductor at resonance
$E_{x_{c}}=$ voltage developed across the capacitor at resonance
$I_{r}=$ current in the circuit at resonance
$X_{L}=$ inductive reactance at resonance
$X_{c}=$ capacitive reactance at resonance
The current flowing through the circuit at resonance is equal to the voltage applied, divided by the resistance of the circuit, or

$$
\begin{equation*}
I_{r}=\frac{E}{\bar{R}} \tag{11-12}
\end{equation*}
$$

Substituting $E / R$ for $I_{r}$ in Eq. (11-23)

$$
\begin{equation*}
E_{X_{L}}=\frac{E X_{L}}{R} \tag{11-25}
\end{equation*}
$$

Substituting $Q$ for $X_{L} / R$,

$$
\begin{equation*}
E_{X_{L}}=E Q \tag{11-26}
\end{equation*}
$$

Relation between the Circuit $Q$ and the Reactance Vollages. The voltages across the reactances have the same value because at resonance the reactances are equal. From this fact and from Eq. (11-26) it can be seen that the voltage across either the inductor or capacitor at resonance is dependent on the circuit $Q$. The value of $Q$ for many of the series resonant circuits used in radio is greater than 100 . Therefore, a series resonant circuit will develop high reactive voltages with a low applied voltage. This is possiblo because the voltages across the inductor and capacitor are equal and opposite. This leaves the resultant voltage across the circuit equal to tha voltage across the resistor, which is equal to the voltage applied to the circuit. This ratio of voltages is illustrated in Fig. 11-7 and is explained in the following example.

Example 11-7. The resistance of a series resonant circuit is 10 ohms, and its inductive reactance is equal to 500 ohms at resonant frequency. What voltage is developed across the inductor, capacitor, and resistor when the applied voltage is five volts?

$$
\begin{array}{rr}
\text { Given: } & \text { Find: } \\
E=5 & E_{X_{L}}=? \\
R=10 & E_{X_{C}}=? \\
X_{L}=500 & E_{R}=?
\end{array}
$$

Solution:

$$
\begin{aligned}
I_{\mathrm{r}} & =\frac{E}{R}=\frac{5}{10}=0.5 \mathrm{amp} \\
E_{X_{L}} & =I_{\tau} X_{L}=0.5 \times 500=250 \mathrm{volts}
\end{aligned}
$$

At resonance, $X_{L}=X_{C}$.

$$
\text { Therefore } \begin{aligned}
E_{X_{C}} & =H_{X_{X}}=250 \text { volts } \\
E_{R} & =I_{r} \times R=0.5 \times 10=5 \text { volts }
\end{aligned}
$$

11-9. Parallel Resonance. If a coil, a capacitor, and a voltage source are connected in parallel, the combination is generally called a parallel resonant circuit. The resistance of the circuit cannot be treated as a single value of $R$ as was the case in series resonant circuits. It is generally true that most of the resistance of the circuit is in the coil and that the resistance of the capacitor is so small that it may be neglected. On this basis, the circuit is considered as an inductor with a small amount of resistance in series in one branch and a perfect capacitor (resistance neglected) in the other branch; such a circuit is shown in Fig. 11-8a. This circuit is actually a parallel-series circuit, and, to understand its action, it should be treated as
such. This may best be done by assigning valucs to $R, L$, and $C$ and solving for the reactances and impedances at a number of frequency values.

Circuit Calculations. The frequency of resonance for the circuit of Fig. $11-8 a$ is

$$
\begin{aligned}
f_{r} & =\frac{159}{\sqrt{L C}} \\
& =\frac{159}{\sqrt{106 \times 106 \times 10^{-6}}}=1500 \mathrm{kc}
\end{aligned}
$$



Fig. 11-8.-Parallel resonance: (a) a parallel resonant circuit, (b) current resonance curves (c) impedance resonance curves.

The inductive reactance of the circuit at resonance is

$$
\begin{aligned}
X_{L} & =2 \pi f L \\
& =6.28 \times 1500 \times 10^{3} \times 106 \times 10^{-6}=1000 \mathrm{ohms}
\end{aligned}
$$

The capacitive reactance of the circuit at resonance is

$$
\begin{aligned}
X_{c} & =\frac{1}{2 \pi f C} \\
& =\frac{1}{6.28 \times 1500 \times 10^{3} \times 106 \times 10^{-12}}=1000 \mathrm{ohms}
\end{aligned}
$$

The impedance of the circuit can be found in the manner described in Art. 10-10. This requires assuming a voltage for the circuit, and for ease in mathematics a value of 1000 volts will be used. Then, at resonance

$$
I_{1}=\frac{E_{1}}{Z_{1}}=\frac{E_{1}}{\sqrt{R_{1}^{2}+X_{L_{1}}{ }^{2}}}=\frac{1000}{\sqrt{10^{2}+10000^{2}}}=\frac{1000}{1000.04}=1.0 \mathrm{amp}
$$

Note. $R_{1}$ is so small compared with $X_{L_{1}}$ that its effect on $Z_{1}$ may readily be neglected.

$$
\begin{gathered}
\cos \theta_{1}=\frac{R_{1}}{Z_{1}}=\frac{10}{1000}=0.01 \quad \sin \theta_{1}=\frac{X_{L_{1}}}{Z_{1}}=\frac{1000}{1000}=1.0 \\
I_{2}=\frac{K_{2}}{Z_{2}}=\frac{K_{2}}{X_{C}}=\frac{1000}{1000}=1.0 \text { ampere } \\
\cos \theta_{2}=\frac{R_{2}}{Z_{2}}=\frac{0}{1000}=0 \quad \sin \theta_{2}=\frac{X_{c}}{Z_{2}}=\frac{1000}{1000}=1.0
\end{gathered}
$$

Using Eq. (10-7)

$$
\begin{aligned}
I_{\text {line }} & =\sqrt{\left(I_{1} \cos \theta_{1}+I_{2} \cos \theta_{2}\right)^{2}+\left( \pm I_{1} \sin \theta_{1} \pm I_{2} \sin \theta_{2}\right)^{2}} \\
& =\sqrt{(1 \times 0.01+1 \times 0)^{2}+(-1 \times 1.0+1 \times 1.0)^{2}} \\
& =\sqrt{(1 \times 0.01)^{2}+0}=0.01 \text { ampere }
\end{aligned}
$$

Then

$$
\begin{equation*}
Z=\frac{E_{\text {line }}}{I_{\text {line }}}=\frac{1000}{0.01}=100,0000 \mathrm{ohms} \tag{7}
\end{equation*}
$$

Parallel Resonance Curves. By repeating the above steps for a number of different frequencies, a set of values may be obtained for the plotting of a resonance curve. A parallel resonance curve is gencrally plotted with frequency as the abscissa and impedance as the ordinate, although occasionally curves may be plotted with current as the ordinate. In Table XI-IV a set of values for the circuit of Fig. 11-8a is shown, and resonant curves plotted from these values are shown in Figs. 11-8b and $c$.

Simplified Method of Calculating the Impedance of a Parallel Circuit. The method used above for calculating the impedance of a parallel circuit can be used for any circuit with two or more parallel branches. The method is lengthy but gives accurate results. When a circuit consists of only two parallel branches, the impedance may also be found by the equation

$$
\begin{equation*}
Z_{p}=\frac{Z_{1} Z_{2}}{Z_{1}+Z_{2}} \tag{11-28}
\end{equation*}
$$

where $Z_{p}=$ impedance of the parallel circuit
$Z_{1}=$ impedance of branch 1
$Z_{2}=$ impedance of branch 2
$Z_{1}+Z_{2}=$ series impedance of the circuit

This equation provides an easier means of calculating the impedance and will produce accurate results if the resistance is small compared with the reactances, as is the case in radio circuits with high values of $Q$. With high values of $Q$, the resistance may be ignored in the numerator of the equation but must be included in the denominator, for $Z_{1}+Z_{2}$ is equal to

Table XI-IV

| Frequency, <br> kc. | $X_{L}$ | $X_{C}$ | $R$ | $Z$ <br> (approx.) | $I$ ma. at <br> $E=100$ |
| :---: | :--- | :--- | :--- | ---: | :---: |
| 1470 | 980 | 1020 | 5 | 25,000 | 4.00 |
| 1475 | 983.5 | 1016.5 | 5 | 30,000 | 3.33 |
| 1480 | 987 | 1013 | 5 | 38,000 | 2.63 |
| 1485 | 990 | 1010 | 5 | 48,000 | 2.08 |
| 1490 | 993 | 1007 | 5 | 67,000 | 1.49 |
| 1495 | 997 | 1003 | 5 | 128,000 | 0.78 |
| 1500 | 1000 | 1000 | 5 | 200,000 | 0.50 |
| 1505 | 1003 | 997 | 5 | 128,000 | 0.78 |
| 1510 | 1007 | 993 | 5 | 67,000 | 1.49 |
| 1515 | 1010 | 990 | 5 | 48,000 | 2.08 |
| 1520 | 1013 | 987 | 5 | 38,000 | 2.63 |
| 1525 | 1016.5 | 983.5 | 5 | 30,000 | 3.33 |
| 1530 | 1020 | 980 | 5 | 25,000 | 4.00 |
|  |  |  |  |  |  |
| 1470 | 980 | 1020 | 10 | 24,250 | 4.12 |
| 1475 | 983.5 | 1016.5 | 10 | 29,000 | 3.44 |
| 1480 | 987 | 1013 | 10 | 36,000 | 2.78 |
| 1485 | 990 | 1010 | 10 | 44,500 | 2.25 |
| 1490 | 993 | 1007 | 10 | 58,000 | 1.72 |
| 1495 | 997 | 1003 | 10 | 85,500 | 1.17 |
| 1500 | 1000 | 1000 | 10 | 100,000 | 1.00 |
| 1505 | 1003 | 997 | 10 | 85,500 | 1.17 |
| 1510 | 1007 | 993 | 10 | 58,000 | 1.72 |
| 1515 | 1010 | 990 | 10 | 44,500 | 2.25 |
| 1520 | 1013 | 987 | 10 | 36,000 | 2.78 |
| 1525 | 1016.5 | 983.5 | 10 | 29,000 | 3.44 |
| 1530 | 1020 | 980 | 10 | 24,250 | 4.12 |

the impedance that the circuit would have if it were connected as a series circuit.

Example 11-8. Calculate the impedance of the parallel resonant circuit of Fig. $11-8 a$ for a frequency of (a) 1485 kilocycles, (b) 1500 kilocycles, (c) 1530 kilocycles.

Note 1: As the circuit $Q$ is high, $Z_{1} Z_{2}$ may be taken as $X_{I} X_{C}$. These values appear in Table XI-IV and will be taken from the table.

Note 2: $Z_{1}+Z_{2}$ is equal to the series impedance of the circuit, and at points near resonance it must include the resistance whether it is large or small. Therefore, $Z_{1}+Z_{2}=\sqrt{R^{2}+\left(X_{L}-X_{C}\right)^{2}}$.

Solution:

$$
\begin{aligned}
R & =10 & \text { (a) } Z_{p} & =\frac{Z_{1} Z_{2}}{Z_{1}+Z_{2}} \\
X_{L} & =990 & & =\frac{990 \times 1010}{\left.\sqrt{10^{2}+(990}-1010\right)^{2}} \\
X_{C} & =1010 & & 44,718 \mathrm{ohms} \\
R & =10 & \text { (b) } Z_{p} & =\frac{Z_{1} Z_{2}}{Z_{1}+Z_{2}} \\
X_{L} & =1000 & & =\frac{1000 \times 1000}{\sqrt{10^{2}+(1000-1000)^{2}}} \\
X_{C} & =1000 & & =100,000 \mathrm{ohms} \\
R & =10 & \text { (c) } Z_{p} & =\frac{Z_{1} Z_{2}}{Z_{1}+Z_{2}} \\
X_{L} & =1020 & & =\frac{1020 \times 980}{\sqrt{10^{2}+(1020-980)^{2}}} \\
X_{C} & =980 & & =24,244 \mathrm{ohms}
\end{aligned}
$$

Relation among $Z, X$, and $Q$. The impedance at resonance can be found by a simpler method as is shown in the following explanation. In the foregoing solution, the impedance was found by

$$
\begin{equation*}
Z_{p r}=\frac{E_{\text {line }}}{I_{\text {line }}} \tag{11-29}
\end{equation*}
$$

Examining the step where Eq. (10-7) was used to find $I_{\text {line }}$, it will be observed that the value of the second parentheses, which represents the total reactance current, is zero and also that the value of $I_{2} \cos \theta_{2}$, which represents the inphase or resistance component of the capacitor current, is zero. This is always true when there is resonance and the capacitor's resistance is disregarded; hence at resonance $I_{\text {line }}$ becomes equal to $I_{1} \cos \theta_{1}$. However, $I_{1}=E / X_{L}$, and $\cos \theta_{1}=R / X_{L}$; therefore

$$
\begin{equation*}
I_{\text {line }}=I_{1} \cos \theta_{1}=\frac{E}{X_{L}} \times \frac{R}{X_{L}}=\frac{E R}{X_{L}^{2}} \tag{11-30}
\end{equation*}
$$

Substituting these values in Eq. (11-29),

$$
\begin{equation*}
Z_{p r}=\frac{E_{\text {line }}}{I_{\text {line }}}=\frac{E_{\text {line }}}{\frac{E R}{X_{L}^{2}}}=\frac{X_{L}^{2}}{R} \tag{11-31}
\end{equation*}
$$

At resonance $X_{L}=X_{c}$; therefore this equation may be expressed in any of the following forms:

$$
\begin{equation*}
Z_{p r}=\frac{X_{L}{ }^{2}}{R} \text { or } \frac{X_{C}{ }^{2}}{R} \text { or } \frac{X_{C} X_{L}}{R} \text { or } \frac{X_{L} X_{L}}{R} \tag{11-32}
\end{equation*}
$$

Using the last form and substituting $Q$ for $X_{L} / R$, the equation can be stated as

$$
\begin{equation*}
Z_{p r}=X_{L} Q \tag{11-33}
\end{equation*}
$$

Example 11-9. What is the circuit $Q$ and the impedance at resonance for the circuit shown in Fig. 11-8a? The frequency of resonance and the value of $X_{L}$ at resonance may be taken from Table XI-IV and are 1500 kilocycles and 1000 ohms, respectively.

$$
\begin{array}{rlrl}
\text { Given: } & \text { Find: } \\
R & =10 & Q & =? \\
X_{L} & =1000 & Z_{p r} & =?
\end{array}
$$

Solution:

$$
\begin{aligned}
Q & =\frac{X_{L}}{R}=\frac{1000}{10}=100 \\
Z_{p r} & =X_{L} Q \\
& =1000 \times 100=100,000 \mathrm{ohms}
\end{aligned}
$$

The error involved in Eqs. (11-31) and (11-33) by disregarding the resistance of the capacitor will depend on the amount of the capacitor resistance and the circuit $Q$. This error is negligible for ordinary calculations and can be ignored if the circuit $Q$ is not extremely low. Examination of Figs. $11-8 c$ and $11-4 b$ will show that the resonance curve for the impedance of a parallel circuit varies in the same manner as the resonance curve for current in a series circuit. The rules for the interpretation of the series resonance curve in regard to the slope and width of the curve will apply to the parallel circuit as well, and the frequency of resonance can be determined by the same equations used for the series resonant circuit.

At resonance, the impedance of the parallel circuit is at its maximum value. Fquation (11-33) shows that the impedance at resonance is equal to the product of $Q$ and the reactance of either branch and is therefore much greater than the impedance of either branch. As the circuit $Q$ is generally larger than 100 , the impedance of the circuit will be more than one hundred times the impedance of either branch.

If the resistance of the circuit of Fig. 11-8a were reduced to five ohms, it would result in a higher value of the circuit $Q$ as indicated by Eq. (11-11) and also a higher circuit impedance as indicated by Eq. (11-31). The effect of a resistance change upon the impedance and current can also be seen by examination of the resonance curves of Figs. 11-8b and 11-8c. This property of a parallel resonant circuit has many applications in radio circuits, some of which will be discussed in another part of this chapter.

11-10. Currents in Parallel Resonant Circuits. Line Current. The line current in a parallel resonant circuit is equal to the applied voltage divided by the impedance of the circuit. At resonance, the impedance of the circuit is maximum; therefore the line current is at its minimum. Be-
cause the impedance at resonance is very high (more than one hundred times either reactance), the amount of current flowing in the line is very small. As the frequency changes from resonance, the circuit impedance decreases very rapidly and the line current will therefore increase in the same manner.

Branch Currents. The current in each branch of a parallel resonant circuit is equal to the voltage applied to the circuit divided by the impedance of that branch. The impedance of each branch varies slowly with frequency change, and therefore the currents in the branch circuits do not go through resonant action. At resonance, the impedance of each branch is relatively small compared with the line impedance; therefore the currents flowing through the capacitor and inductor are much greater than the line current. This resonant action is due to the fact that the currents in each branch of a parallel resonant circuit are largely reactive and approximately 180 degrees out of phase with one another; therefore the resultant current will be practically zero. At resonance, a large current will keep circulating between the inductance and the capacitance. The resistance of resonant circuits is generally very small, and hence the energy losses will be small. The current flowing in the line is only the amount needed to supply the circuit losses, and as the circuit losses are small the line current is also relatively small.

Currents at Frequencies off Resonance. For frequencies other than resonance, the reactive currents are not equal. The resultant reactive current must be supplied by the line. As the difference in the reactive currents in each circuit will increase with the amount of deviation from resonant frequency, the line current will increase in the same manner. The relation between the line and branch currents is shown in the graph of Fig. 11-9a, while the graph in Fig. 11-9b shows the relation between the branch impedances and the line impedance.

11-11. Comparison of Series and Parallel Resonant Circuits. Resonance Curves and Frequency. In series resonant circuits, the resistance, inductance, capacitance, and the supply voltage are connected in series with one another. In parallel resonant circuits, an inductance containing a small amount of resistance is connected in parallel with a capacitance of negligible resistance. In both circuits, resonance occurs when the capacitive reactance equals the inductive reactance. The resonant frequency of both circuits can be calculated by Eqs. (11-3) and (11-6); both have similar resonance curves. Increasing the circuit $Q$ will increase the slope of the resonance curves in both circuits, thus decreasing the width of their resonance curves.

Characteristics of Series Resonant Circuits. At resonance, the impedance of a series resonant circuit is at its minimum, and its value is equal to the resistance of the circuit. The circuit acts the same as a resistor. The
current through all parts of the circuit is the same and is equal to the line current; the current is at its maximum value and is in phase with the applied voltage. The power factor of the circuit will therefore be unity. The


Fig. 11-9.-Characteristic curves of a parallel resonant circuit: (a) current curves, (b) impedance curves.
voltages across the reactances are approximately equal and nearly 180 degrees out of phase with each other, and the voltage across the resistance is equal to the applied voltage. Increasing the value of the resistance will decrease both the current in the line and the voltage across each reactance.

For frequencies below resonance, the capacitive reactance is greatest and the current is leading. For frequencies above resonance, the inductive reactance is greatest and the current is lagging.

Characteristics of Parallel Resonant Circuits. At resonance, the impedance of parallel resonant circuits is at its maximum, and its value is equal to the product of either reactance and the circuit $Q$. The circuit acts the same as a resistor, and the current and voltage are in phase; the power factor of the circuit will therefore be unity. The line current is at a minimum and is equal to the applied voltage divided by the impedance of the circuit. The voltages across the inductance and the capacitance are the same and are equal to the applied voltage. At resonance, the currents in the inductor and capacitor are approximately equal and nearly 180 degrees out of phase with each other. Increasing the resistance of the circuit decreases the circuit impedance, thereby increasing the line current. For frequencies below resonance, the current in the inductor increases and the line current is lagging. For frequencies above resonance, the current in the capacitor increases and the line current is leading.

11-12. Uses of Resonant Circuits. Fundamental Uses. In radio, television, and electronic circuits, the principles of resonance are used to increase the strength of a desired signal and to decrease to a minimum the strength of undesired signals. Series resonant circuits are used wherever maximum current is desired for a definite frequency or band of frequencies. Parallel resonant circuits are used wherever the signal strength of any one frequency or band of frequencies is to be reduced to a minimum.

Classification of Circuits as Series or Parallel. Resonant circuits form only a part of the complete radio circuit. Because of this, it is sometimes very difficult to ascertain whether the resonant circuit is of the series or parallel type. In radio work, some circuits that have all the appearances of a parallel resonant circuit are actually classed as series resonant circuits. This is usually the case when the inductor is the secondary of a transformer and the voltage of the circuit is an induced voltage rather than a voltage applied from an entirely separate source. For example, Fig. 11-10c shows two resonant circuits (drawn in heavy lines), each of which has the general appearance of a parallel circuit, but in analyzing the complete circuit the one at the left is considered a parallel resonant circuit and the one at the right is considered a series resonant circuit. The circuit at the left is considered a parallel resonant circuit because it receives its electrical energy from the plate circuit of tube 1 . The circuit at the right is considered a series resonant circuit because no separate voltage is applied to the inductor and capacitor, but instead a voltage is induced in the inductor (secondary of an r-f transformer), which is considered as a voltage connected in series with the inductor and the capacitor. The following procedure will help to determine whether a circuit should be classed as parallel or series: (1) locate
the inductive and capacitive components forming the resonant circuit; (2) locate the source of a-c voltage for these components; (3) determine whether the components are in series or parallel with the source of a-c voltage. In radio circuits, the source of a-c voltage may be derived from any one of the following circuits: antenna, output of a vacuum tube, or the induced


Fig. 11-10.-Applications of parallel resonant circuits in radio receivers.
voltages from other circuits. Some of the applications of resonant circuits as applied to radio are illustrated in Figs. 11-10 and 11-11. The resonant circuit in each illustration is shown in heavy lines.

Application of Resonant Circuits in the Antenna Circuit of a Receiver. A simple illustration of the use of the principle of resonance is given in the circuit shown in Fig. 11-10a. Here a parallel resonant circuit is connected in series with the antenna circuit of a receiver. This circuit will offer a very high impedance to any signal current at the frequency for which it is resonant and will therefore reduce such a signal current to a minimum. It
thereby hinders currents of this frequency from entering the receiver but does not prevent currents of any other frequency from entering the receiver. Such a circuit is sometimes used when one particular station causes troublesome interference, and it is referred to as a wave trap.

Another illustration is given in the circuit of Fig. 11-11a. Here a series resonant circuit is connected to the antenna circuit of a recciver. Being a series resonant circuit, it will offer a very low impedance to any signal current at the frequency for which it is resonant and a high impedance to


Fig. 11-11.-Applications of series resonant circuits in radio receivers.
currents of all other frequencies. It provides a low impedance path to the ground for any signal current of the resonant frequency to which it is adjusted. It thereby keeps signal currents of the undesired frequency from entering the receiver and acts as a wave trap.

Use of Series Resonant Circuits in the Tuning Circuit of a Receiver. In order to listen to any audible program being transmitted, the tuning circuit of the receiver must be adjusted so that its capacitive and inductive components produce resonance at the frequency of the carrier wave of the station transmitting the program desired. The scries resonant circuit shown in Fig. 11-11b is typical of all tuning circuits. The input signal voltage $e$ may come from the antenna circuit or from the output of a previous tuning stage. The amount of signal voltage across the secondary of the transformer will be much greater because of the high reactive voltages


Fig. 11-12.-Theoretical parallel resonant circuit of an inductance coil. developed in a series resonant circuit at its resonant frequency. The value of either reactive voltage is more than that which could have been obtained by direct transformer ratio. This increase in voltage represents a gain in signal strength obtained by means of coupling. This voltage is now applied to the input of the following tube.

Use of Parallel Resonant Circuits to Obtain Selectivity and High Impedance. Parallel resonant circuits are used as the output load of a tube where it is necessary that this circuit, in addition to having a high selectivity, must also have a high impedance. Such a circuit is shown in Fig. 11-10b.

The advantages obtained by proper use of series and parallel resonant circuits is further illustrated by the manner in which they are used in the intermediate stages of superheterodyne receivers. Such a circuit is shown in Fig. 11-10c. The resonant circuit connected to the output of tube 1 must have a high selectivity and high impedance; a parallel resonant circuit is therefore used. The resonant circuit connected to the input of tube 2 must have a high selectivity but a low impedance in order to obtain a high


Fig. 11-13.-Applications of resonant circuits: (a) tuning unit with an adjustable capacitor, (b) tuning unit with an adjustable capacitor and adjustable inductor. (Hammarlund Manufacturing Company, Inc.)
reactive voltage; a series resonant circuit is therefore used. This circuit arrangement has excellent selective characteristics and is used in practically all modern receivers.

Resonant Frequency of Choke Coils. In a previous chapter, it has been shown that the distributed capacitance existing between the turns of a coil may be considered theoretically as a lumped capacitance placed across the terminals of the coil. If a coil is considered in this manner (see Fig. 11-12), it can be seen that it is equivalent to a parallel resonant circuit and therefore has a resonant frequency of its own. Choke coils are designed to have a comparatively high inductive reactance for a wide band of frequencies below its resonant frequency when considered as a parallel resonant circuit. At resonance its impedance is very high, and for frequencies above resonance its reactance becomes capacitive and the coil acts as a capacitor.

If there is no disadvantage in having the reactance capacitive instead of inductive, a coil with high distributed capacitance is superior for frequencies lower than its resonant frequency. Above this frequency, the actual impedance of the coil is less than it would be for a coil having a negligible amount of distributed capacitance.

Figure 11-14 represents a schematic diagram of a typical superheterodyne receiver. This is the same circuit diagram that was used in the


Frg. 11-14-Schematic diagram of a typical superheterodyne receiver with its resonant circuits indicated by heavy-line boxes.
chapters on inductance and capacitance. In this diagram, the resonant circuits are enclosed in dashed lines and used as follows: 1 and 2 are series resonant circuits used to sclect the signals from a desired station; 3 and 5 are parallel resonant circuits used in the primary side of the i-f transformers; 4 and 6 are series resonant circuits used in the secondary side of the i-f transformers; 7 is a parallel resonant circuit used in the local oscillator.

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## QUESTIONS

1. Why are resonant circuits considered one of the most important types of circuits used in radio apparatus?
2. What is meant by the term resonance as applied to electric circuits?
3. (a) What is a graph? (b) Where are graphs used in radio? (c) Why is the study of the plotting and interpretation of graphs essential to understanding radio, television, and electronic circuits?
4. What is meant by (a) the abscissa? (b) The ordinate? (c) The point of origin?
5. (a) In plotting a graph, which variable is plotted as the ordinate? (b) What is the other variable called?
6. What type of curve will a graph of the following equations illustrate: (a) Plotting $P$ vs. $E$ for the cquation $P=E^{2} / R$ ? (b) Plotting $X_{L}$ vs. $f$ for the equation $X_{L}=2 \pi f L$ ? (c) Plotting $f_{r}$ vs. $L C$ for the equation $f_{r}=1 / 2 \pi \sqrt{ } L C$ ?
7. Why is it necessary to obtain more points when plotting a graph having a curved line than for one having a straight line?
8. If a graph is plotted from results obtained from a laboratory experiment, is the curve drawn through all the points obtained? Explain.
9. In a scries circuit containing inductance and capacitance, how do the inductive and capacitive reactances vary when the frequency of the input voltage is (a) increased? (b) Decreased?
10. Why should the resistance of series resonant circuits be kept at a minimum?
11. What quantities are generally used to plot (a) a series resonance curve? (b) A parallel resonance curve?
12. What is meant by a tuned circuit?
13. How does the shape of the resonance curve vary when its resistance is (a) increased? (b) Decreased?
14. How does the circuit $Q$ of a tuned circuit compare with the coil $Q$ ?
15. How does the slope of the resonance curve for a tuned circuit vary when its circuit $Q$ is (a) increased? (b) Decreased?
16. When the following factors are increased, how is the circuit $Q$ of a tuned circuit affected: (a) resistance, (b) inductance, (c) frequency?
17. In order to obtain high selectivity in tuning circuits, should the circuit $Q$ be large or small? Explain your answer.
18. Fxplain how the voltage across either reactance in a series tuned circuit can be greater than the applied voltage.
19. Will the reactive voltages in a series resonant circuit increase or decrease when the circuit $Q$ is increased? Explain.
20. (a) When solving parallel resonant circuits, why must they be treated as parallel-series circuits? (b) Why is the resistance of the capacitor generally ignored?
21. Under what conditions may the impedance of a parallel resonant circuit be taken as the product of the inductive reactance and the circuit $Q$ ?
22. How do parallel resonant circuits differ from series resonant circuits?
23. How do the rules for the interpretation of parallel resonance curves compare with those for a series circuit? Why?
24. How does a decrease in the circuit resistance affect the impedance of a parallel resonant circuit?
25. How does the impedance of each branch of a parallel resonant circuit compare with the impedance of the circuit (a) at the resonant frequency? (b) For froquencies below resonance? (c) For frequencies above resonance?
26. How does the current flowing in each branch of a parallel resonant circuit compare with the line current (a) at the resonant frequency? (b) For frequencies below resonance? (c) For frequencies above resonance?
27. How does the power factor of the inductance and capacitance of parallel and series resonant circuits compare with the power factor of the line (a) at the resonant frequency? (b) For frequencies below resonance? (c) For frequencies above resonance?
28. In what respects are parallel resonant circuits similar to series resonant circuits?
29. In what respects do parallel resonant circuits differ from series resonant circuits?
30. What factor determines whether a resonant circuit should be classed as a series or parallel type?
31. What procedure should be followed in determining whether a resonant circuit is of the series or parallel type?
32. Name four applications of series resonant circuits to radio.
33. Name four applications of parallel resonant circuits to radio.
34. What is meant by the distributed capacitance of a coil?
35. Why must the distributed capacitance of a choke coil be taken into consideration?

## PROBLEMS

1. Plot a curve showing how the power varies with current changes from zero to 10 amp if the voltage is kept constant at 125 volts.
2. Plot a curve showing how the power varies with voltage changes from zero to 125 volts for a circuit whose resistance is 0.25 megohm.
3. The following data were taken for plotting a curve for a certain tube characteristic. The grid voltage of a 6F7 tube is kept constant at -5 volts. The following readings of plate current (milliamperes) were obtained for the values of plate voltage listed below. Plot an $E_{p} I_{p}$ curve.

| $E_{p}$ | $I_{p}$ |
| ---: | :--- |
| 50 | 0.35 |
| $\mathbf{7 5}$ | $\mathbf{1 . 2}$ |
| 100 | 2.5 |
| 125 | 4.0 |
| 150 | $\mathbf{5 . 5}$ |
| 175 | 7.2 |
| 200 | 9.0 |

4. A resonant circuit has an inductance of $316 \mu \mathrm{~h}$ and a capacitance of $80 \mu \mu \mathrm{f}$. Plot a curve showing how the inductive reactance changes when the frequency is varied from 975 kc to 1025 kc . On the same paper, using the same reference abscissa,
plot another curve showing how the capacitive reactance changes for the same frequency range. In plotting these curves, obtain values for every 5 kc .
5. Using the curves obtained in Prob. 4, determine the resultant circuit reactance at the following frequencies: (a) 980 , (b) 990 , (c) 1000 , (d) 1010 , (e) 1020 kc .
6. Determine the resonant frequency of the circuit used in Prob. 4 (a) by referring to the curves obtained in Prob. 4, (b) by substituting the values of inductance and capacitance in the equation for finding the frequency of resonance.
7. $\Lambda$ serics tuned circuit has an inductance of $316 \mu \mathrm{~h}$. 'To what value of capacitance must its variable capacitor be adjusted in order to obtain resonance for the following frequencies: (a) 500 kc ? (b) 1500 kc ?
8. A variable capacitor having a maximum capacitance of $350 \mathrm{~m} \mathrm{\mu f}$ is used for tuning a broadcast receiver. (a) What inductance is required to make the circuit resonant at the lowest frequency 500 ke ? (b) If the minimum capacitance of the capacitor is $15 \mu \mu \mathrm{f}$, what is the highest frequency that can be obtained with the inductance determined in part (a)?
9. The coil used in Prob. 8 plus the circuit wiring has a distributed capacitance of $15 \mu \mu \mathrm{f}$ thus increasing the circuit capacitance by this amount. What is the frequency range of the circuit?
10. It is desired to cover a short-wave band whose lowest frequency is to be 1700 ke by connecting a different coil to the capacitor in Prob. 8. (a) Find the inductance of the coil. (b) What is the highest frequency that can be tuned if the distributed circuit capacitance ( $15 \mu \mu f$ ) is to be taken into consideration and the inductance coil is used as determined in part (a)? (c) What is the highest frequency to which the circuit may be tuned if the distributed circuit capacitance is ignored? (d) What is the highest frequency to which the circuit may be tuned if the distributed circuit capacitance is ignored and the minimum value of the variable capacitor is $10 \mu \mu \mathrm{f}$ ?
11. A variable capacitor having a maximum capacitance of $140 \mu \mu \mathrm{f}$ and a minimum capacitance of $10 \mu \mu \mathrm{f}$ is used to tune a short-wave band whose lowest frequency is 1700 kc . (a) Find the inductance of the coil. (b) What is the highest frequency to which the circuit can be tuned if the distributed circuit capacitance ( $10 \mu \mu \mathrm{f}$ ) is to be taken into consideration? (c) What is the highest frequency to which the circuit may be tuned if the distributed circuit capacitance is ignored? (d) What is the highest frequency to which the circuit may be tuned if the distributed capacitance is ignored and the minimum value of the variable capacitor is $5 \mu \mu \mathrm{f}$ ?
12. The capacitor in Prob. 11 is to be used to tune a short-wave band whose lowest frequency is 6.5 mc . In this problem the distributed circuit capacitance is to be ignored. (a) Find the inductance of the coil. (b) What is the highest frequency that can be tuned?
13. It is desired to cover a short-wave band whose lowest frequency is to be 1700 kc by the use of a fixed capacitor $C_{s}$ connected in series with the tuning capacitor $C_{T}$, as shown in Fig. 11-15. The maximum and minimum capacitance of the tuning capacitor is $350 \mu \mu \mathrm{f}$ and $15 \mu \mu \mathrm{f}$, respectively, the inductance of the secondary is 290 $\mu \mathrm{h}$, and the distributed circuit capacitance is $15 \mu \mu \mathrm{f}$. (a) Find the capacitance of the series capacitor $C_{S}$. (b) What is the highest frequency to which the circuit may be tuned if the distributed circuit capacitance is to be taken into consideration and the series capacitor is used as determined in part (a)? (c) What is the highest frequency to which the circuit may be tuned if the distributed circuit capacitance is ignored? (d) What is the highest frequency to which the circuit may be tuned if the distributed circuit capacitance is ignored and the minimum value of the variable capacitor is $10 \mu \mu \mathrm{f}$ ?
14. A variable capacitor having a maximum capacitance of $140 \mu \mu \mathrm{f}$ and a minimum capacitance of $10 \mu \mu \mathrm{f}$ is used to tune a short-wave band whose lowest frequency
is 4.63 mc . The distributed circuit capacitance is $5 \mu \mu \mathrm{f}$, and the inductance of the secondary winding is $58.5 \mu \mathrm{~h}$. (a) What value of eapracitance must be contuected in series with the tuning capacitor in order to obtain this minimum frequency when all its plates are in mesh? (b) What is the highest frequency to which the cireuit may be tuned if the distributed circuit capacitance is to be taken into consideration and the series capacitor is used as determined in part (a)? (c) What is the highest frequency to which the circuit may be tuned if the distributed cireuit capacitance is ignored? ( $d$ ) What is the highest frequency to which the circuit may be tuned if the distributed circuit capacitance is ignored and the minimum value of the variable capacitor is $2.5 \mu \mu f$ ?
15. A series resonant circuit is to be used as a wave trap to eliminate the cffect of a $1200-\mathrm{kc}$ signal. What value of capacitance must be used if the coil has an inductance of $80 \mu \mathrm{~h}$ and a distributed capacitance of $10 \mu \mu \mathrm{f}$ ?
16. A coil having a distributed capacitance of $10 \mu \mu \mathrm{f}$ is connected in series with a $850-\mu \mu \mathrm{f}$ capacitor in order to pass a $750-\mathrm{kc}$ signal. Find the inductance of the coil.
17. A $120-\mu \mu \mathrm{f}$ adjustable capacitor and an induetance coil are connected in parallel to form the primary side of an i-f transformer whose resonant frequency is 460 kc (Fig. 11-16). (a) What is the inductance of the primary winding? (b) What is the $Q$ of the primary winding if its resistance is 9.85 ohms?


Fig. 11-15.


Fig. 11-16.
18. A $2.5-\mathrm{mh}$ coil and an adjustable capacitor are connected to form the secondary side of an i-f transformer whose resonant frequency is 460 ke (Fig. 11-16). (a) What is the capacitance of the adjustable capacitor? (b) What is the $Q$ of the secondary winding if its resistance is 13.75 ohms ?
19. A $10-\mathrm{mv}$ signal is applied to a series resonant circuit having an inductance of $316 \mu \mathrm{~h}$, a capacitance of $80 \mu \mu \mathrm{f}$ (same as Prob. 4), and a resistance of 10 ohms. Plot the resonance curve for this circuit.
20. Determine the width of the frequency band of the circuit used in Prob. 19 (a) from the resonance curve as plotted, (b) substituting the values of inductance and resistance in the equation for finding the width of the frequency band.
21. The resistance of a series resonant circuit is 12 ohms , and its inductive reactance is equal to 300 ohms at the resonant frequency. (a) What is the value of the circuit $Q$ ? (b) What voltage is developed across the inductor, capacitor, and resistor when the applied voltage is 10 volts?
22. (a) What voltage is developed across the resistor, inductor, and capacitor of the series resonant circuit used in Prob. 19? (b) What is the value of the circuit $Q$ ?
23. The primary side of an i-f transformer forms a parallel resonant circuit consisting of a $3-\mathrm{mh}$ coil and a $39.03-\mu \mu \mathrm{f}$ capacitor. The resistance of the coil is 20 ohms. (a) What is its frequency of resonance? (b) What is the impedance of the circuit at resonance?
24. What is the impedance of the parallel resonant circuit used in Prob. 17?
25. A parallel resonant circuit is to be used as a wave trap to eliminate the effects of a $1300-\mathrm{ke}$ signal. The circuit has a resistance of 1.5 ohms and a capacitance (distributed and wiring) of $10 \mu \mu \mathrm{f}$. (a) What value of inductance must be used with a capacitor whose value is $65 \mu \mu \mathrm{f}$ ? (b) What is the circuit $Q$ ? (c) What is the width of the band being eliminated?
26. A coil having an inductance of $320 \mu \mathrm{~h}$ is connected in parallel with an adjustable capacitor in order to by-pass a band of frequencies between 999.25 and 1000.75 kc. The distributed capacitance of the coil and circuit is equal to $9 \mu \mu \mathrm{f}$. (a) What is the value of the adjustable capacitor? (b) What is the resistance of the circuit?
27. Using the method outlined in the article on parallel resonance, find the impedance for the following points for the curves in Fig. 11-8c: (a) $R=5$ ohms, $f=1492.5 \mathrm{kc}, f=1502.5 \mathrm{kc}$. (b) $R=10 \mathrm{ohms}, f=1497.5 \mathrm{kc}, f=1507.5 \mathrm{kc}$.
28. Plot a parallel resonance curve for a circuit having a capacitance of $80 \mu \mu \mathrm{f}$, an inductance of $316 \mu \mathrm{~h}$, and a resistance of 10 ohms [use Eq. (11-28)].

Note: These are the same values used in Prob. 4.

## CHAPTER XII

## BASIC ELECTRONIC CIRCUITS

A circuit diagram of a radio receiver, a television recciver, or a piece of industrial electronic equipment is generally quite complex and when examined as a whole unit may be very confusing. Every such electronic circuit is made up of a number of individual circuits, and, to understand the complete circuit, each individual circuit must be analyzed separately.

The purpose of this chapter is to study the electrical effects of each circuit and its relation to other circuits. The applications of these circuits in producing specific actions such as amplification, detection, or oscillation are taken up in the authors' text, Essentials of Radio.

12-1. The Electric Circuit. Circuit Elements. Every electric circuit must contain at least one conducting element. This element may be a resistor, inductor, or capacitor. The purpose of each type of element and its effects on the electric circuit have been discussed in previous chapters and should be reviewed. In general, a resistor in a circuit will limit the amount of current flowing in that circuit. A reduction in current flow will also reduce the voltage drop across the remainder of the circuit elements. Inductance in a circuit will oppose any change in the amount of current flow. Capacitance in a circuit will oppose any change in the amount of voltage impressed across the circuit.

Stray Resistance, Capacitance, and Inductance. Every wire-wound resistor will have, in addition to its resistance, an inductance effect due to the turns of wire and a capacitance effect due to the space (insulation) between each turn. Every coil will have, in addition to its inductance, a resistance whose value will depend on the size, length, and kind of wire used and a capacitance effect due to the insulation between each turn. The current flowing in and out of a capacitor must flow through a path consisting of its leads and plates. There will therefore be some resistance in this path. In order to obtain the maximum capacitance in the minimum amount of space, the plates of fixed capacitors are wound in the form of a coil. This construction causes an inductance effect to exist in all capacitors so constructed.

Stray resistance, inductance, and capacitance effects will therefore exist in all resistors, inductors, and capacitors. Many of these units are designed in such a way as to minimize these effects. At audio frequencies and the lower radio frequencies, stray effects can usually be ignored, but, at the higher frequencies, these effects are troublesome and have to be taken into
consideration when selecting the units to be used. It is therefore appropriate at this time to sum up the action of these stray effects on each of the circuit elements.

Equivalent Circuits. The inductance of a resistor can be considered as an inductor connected in series with the resistance. The capacitance of the resistor can be considered as a capacitor connected in parallel with the resistance. The equivalent circuit for the resistor is then as shown in Fig. 12-1a.

The resistance of a coil can be considered as a resistor connected in series with the inductance. The distributed capacitance between adjacent turns,


Fig. 12-1.-Wquivalent circuits for the three kinds of circuit elements: (a) a resistor, (b) an inductor, (c) a capacitor used on direct current, (d) a capacitor used on alternating current.
nonadjacent turns, and between each turn and the ground can be considered as a single lumped capacitance connected in parallel with the coil. The equivalent circuit for the inductance coil is then as shown in Fig. 12-1b.

Capacitors are used in both d-c and a-c circuits. When a capacitor is used in a d-e circuit, there is no inductive effect and the resistance of the plates and leads is considered as a resistor connected in parallel with the capacitor, Fig. 12-1c. When a capacitor is used in a-c circuits, the equivalent circuit may be considered as either series or parallel. The series equivalent circuit is usually more useful and is shown in Fig. 12-1d. Often the inductance is negligible, and for capacitors with low losses the equivalent series resistance approaches zero.

Example 12-1. The inductance of a 5000 -ohm 100 -watt resistor is equal to 11 microhenries. What is the equivalent impedance of this resistor, if the distributed capacitance is ignored at (a) 500 kilocycles? (b) 1500 kilocycles?

Given:

$$
\begin{aligned}
L & =11.0 \mu \mathrm{~h} \\
R & =5000 \mathrm{ohms} \\
f & =500 \mathrm{kc} \\
f & =1500 \mathrm{kc}
\end{aligned}
$$

Find:
$Z=$ ?

## Solution:

(a) At 500 kc

$$
\begin{aligned}
X_{L} & =2 \pi f L \\
& =6.28 \times 500 \times 10^{3} \times 11 \times 10^{-6} \\
& =34.6 \text { ohms } \\
Z_{500} & =\sqrt{R^{2}} \frac{+X_{L^{2}}}{} \\
& =\sqrt{(5000)^{2}+(34.6)^{2}} \\
& =5000.11 \text { ohms }
\end{aligned}
$$

(b) At 1500 kc

$$
\begin{aligned}
X_{L} & =2 \pi f L \\
& =6.28 \times 1500 \times 10^{3} \times 11 \times 10^{-6} \\
& =103.6 \mathrm{ohms} \\
Z_{1500} & =\sqrt{R^{2}+X_{L^{2}}} \\
& =\sqrt{(5000)^{2}+(103.6)^{2}} \\
& =5001.07 \text { ohms }
\end{aligned}
$$

Example 12-2. An r-f plate choke is used in a receiver whose r-f range is $\mathbf{1 . 8 7 5}$ to 15.0 megacycles. The inductance of this coil is equal to 200 microhenries and its resistance nine ohms. Find the impedance of this coil at the two frequency limits.

| Given: | Find: |
| :--- | ---: |
| $R=9 \mathrm{ohms}$ | $Z=?$ |
| $L=200 \mu \mathrm{~h}$ |  |
| $f=1.875 \mathrm{mc}$ |  |
| $f$ | $=15.0 \mathrm{mc}$ |

## Solution:

(a) At 1.875 mc

$$
\begin{aligned}
X_{L} & =2 \pi f L \\
& =6.28 \times 1.875 \times 10^{6} \times 200 \times 10^{-5} \\
& =2355 \mathrm{ohms} \\
Z_{1,875} & =\sqrt{R^{2}+X_{L^{2}}} \\
& =\sqrt{(9)^{2}+(2355)^{2}} \\
& =2355.01 \text { ohms }
\end{aligned}
$$

(b) At 15 mc

$$
\begin{aligned}
X_{L} & =2 \pi f L \\
& =6.28 \times 15 \times 10^{6} \times 200 \times 10^{-6} \\
& =18,840 \text { ohms } \\
Z_{15} & =\sqrt{(9)^{2}}+\left(18,8 \overline{40)^{2}}\right. \\
& =18,840 \text { ohms }
\end{aligned}
$$

Example 12-3. A five-microfarad capacitor has a series resistance of 47 ohms . What is its impedance at (a) 60 cycles? (b) 120 cycles?

| Given: | Find: |
| :--- | ---: |
| $R=4 \overline{7}$ ohms | $Z=?$ |
| $C=5 \mu \mathrm{f}$ |  |
| $f=60$ cycles |  |
| $f=120$ cycles |  |

Solution:
(a)
(b)

$$
\begin{aligned}
X_{C} & =\frac{10^{6}}{2 \pi f C}=\frac{159,000}{f C}=\frac{159,000}{60 \times 5}=530 \mathrm{ohms} \\
Z & =\sqrt{R^{2}+X c^{2}}=\sqrt{(47)^{2}+(530)^{2}}=532 \mathrm{ohms} \\
X_{C} & =\frac{159,000}{f C}=\frac{159,000}{120 \times 5}=265 \mathrm{ohms} \\
Z & =\sqrt{R^{2}+X c^{2}}=\sqrt{(47)^{2}+(265)^{2}}=269.1 \text { ohms }
\end{aligned}
$$

From these three examples it can be seen that the stray effects can usually be ignored.

Individual Circuits. Resistors, inductors, or capacitors can be connected individually, in multiple, or in combination with one another. The electric circuit resulting from the proper connection of the circuit elements may be a simple, series, parallel, or complex circuit. The analysis and use of these circuits for both direct current and alternating current have been discussed in the previous chapters.

Combined Circuit. It has been stated that electronic circuits consist of a number of individual circuits, each designed to accomplish a definite purpose. It is the combined effect of each of its individual circuits which produces the desired performance of a radio receiver or transmitter. In combining two or more individual circuits, the folloring two factors must be taken into consideration: (1) the means used to transfer energy from one circuit to another, (2) the separation of the different types of current so that they will flow through the proper circuit.

Coupling. In order to transfer electrical energy from one circuit element to another, a conducting material, usually a wire, is used. In order to transfer electrical energy from one circuit to another, a common impedance must exist between the two circuits. This common impedance is called the coupling element. When two circuits are connected with one another by means of a coupling element, the circuits are said to be coupled. There are various ways in which two circuits may be coupled with one another, and each method has a definite use and produces different effects. The various types of coupling circuits and their effects are discussed later in this chapter.

Filters. When two or more individual electronic circuits are combined, they form a complex circuit through which the following kinds of curreuts may constantly flow: (1) direct, (2) low frequency ( 60 cycle), (3) audio frequency, (4) radio frequency. One side or part of any number of these


Fig. 12-2.-A capacitor and an inductor used to separate alternating current from direct current and low-frequency currents from high-frequency currents.
circuits may be completed through a common wire or through the chassis. It is the purpose of a filter to separate these currents at any desired point and to direct each of them into the conductor or circuit through which it is desired to have them flow.


Fig. 12-3.-Effects of inductance and frequency on current flow: (a) variable inductance, constant frequency; (b) variable frequency, constant inductance.

12-2. Filter Action. A filter circuit consists of a combination of capacitors, inductors, and resistors connected so that it will separate alternating currents from direct currents or alternating currents within a band of frequencies from those alternating currents outside of this band. Filter circuits will range from a very simple circuii to a very intricate circuit, depending upon their application. However, no matter how simple or involved a filter circuit may be, its action must depend upon the following principles of a-c circuits:

1. The opposition offered to the flow of alternating currents by a circuit containing only inductance will increase with frequency increase. Such a circuit will offer comparatively little opposition to the flow of direct, pulsating, or low-frequency alternating currents. The opposition offered to r-f currents by such a circuit will be comparatively high (see Figs. 12-2 and 12-3).



High-frequency current Low reactance


Fig. 12-4.-Effects of capacitance and frequency on current flow: (a) variable capacitance, constant frequency; (b) variable frequency, constant capacitance.
2. The opposition offered to the flow of alternating currents by a circuit containing only capacitance will decrease with frequency increase. Such a circuit will offer a comparatively high opposition to low-frequency currents, little opposition to r-f currents, and will block the flow of direct currents (see Figs. 12-2 and 12-4).


Fig. 12-5.-Effect of resonant circuits on the amount of current flow at or near the resonant frequency: (a) series resonant circuit, (b) parallel resonant circuit.
3. A series resonant circuit has a low impedance at resonance and will offer little opposition to the flow of all currents whose frequencies lie within a narrow band above and below the resonant frequency. Such a circuit will offer a comparatively high opposition to the flow of currents of all other frequencies (see Fig. 12-5a).
4. A parallel resonant circuit has a high impedance at resonance and will offer a comparatively high opposition to the flow of all currents whose frequencies lie within a narrow band above and helow the resonant fre-
quency. Such a circuit will offer little opposition to the flow of currents of all other frequencies (see Fig. 12-5b).
5. Resistors do not provide any filtering action when used alone, as they oppose the flow of all currents regardless of their frequency. When connected in series with a capacitor or inductor, or both, it increases the impedance of the circuit, thus decreasing the sharpness of the filter circuit. This can be seen by observing the resonance curves in Fig. 11-4b. Increasing the resistance of the series resonant circuit decreases the current at the resonant frequency, thus decreasing the slope of the curve on either side of the resonant frequency.

Example 12-4. A 10-henry filter choke has a resistance of 475 ohms. (a) What is the amount of opposition it offers to direct current? (b) What is the amount of opposition it offers to 60 -cycle alternating current?

| Given: | Find: |
| :--- | :--- |
| $R=475$ ohms | (a) $R=?$ |
| $L=10$ henries | (b) $Z=?$ |

Solution:
(a)

$$
\begin{aligned}
& R=475 \text { ohms } \\
& X_{L}=2 \pi f L=6.28 \times 60 \times 10=3768 \text { ohms } \\
& Z=\sqrt{R^{2}+X_{L^{2}}}=\sqrt{475^{2}+3768^{2}}=3797 \mathrm{ohms}
\end{aligned}
$$

From the above example it can be seen that the opposition offered to the flow of 60 -cycle alternating current is approximately eight times that offered to the d-c flow. This type of coil can therefore be used to pass direct current and block the flow of low-frequency alternating currents.

Example 12-5. (a) To which type of current will a 0.2 -microfarad capacitor offer the greater opposition, a 5000 -cycle audio signal or a 5000 -kilocycle r-f signal?
(b) How many times greater is the larger impedance than the smaller impedance?
(c) What type of current is blocked by a capacitor of this size?

| Given: | Find: |
| :--- | ---: |
| $C=0.2 \mu \mathrm{f}$ | $X_{C}=?$ |
| $f=5 \mathrm{kc}$ |  |
| $f=5000 \mathrm{kc}$ |  |

Solution:
(a) At 5000 cycles

$$
X_{C}=\frac{159,000}{f C}=\frac{159,000}{5 \times 10^{3} \times 2 \times 10^{-1}}=159 \mathrm{ohms}
$$

At 5000 kc

$$
X_{C}=\frac{159,000}{f C}=\frac{159,000}{5 \times 10^{6} \times 2 \times 10^{-1}}=0.159 \mathrm{ohm}
$$

(b)

$$
\text { Ratio of opposition }=\frac{159}{0.159}=1000
$$

(c) The a-f currents are blocked.

12-3. Types of Filter Circuits. Filter circuits are composed of inductors and capacitors having losses as low as commercially obtainable. In the elementary consideration of filter circuits, it is assumed that the inductors and capacitors have no internal effective resistance. There are four


Fig. 12-6.--Characteristic curve for a simple lowpass filter circuit. gencral types of filter circuits, namely, low-pass filter, high-pass filter, band-pass filter, band-stop filter.

Low-pass Filter. A low-pass filter circuit is used to allow all currents having a frequency below a certain value to pass into a desired circuit while opposing or diverting the flow of all currents having a frequency above this value (see Fig. 12-6). An inductance coil inserted in the line (Fig. 12-7a) will offer little opposition to the flow of low-frequency currents and a large amount of opposition to the flow of high-frequency currents. In order to divert the undesired highfrequency currents back to the source, a capacitor is used as a by-pass, as shown in Fig. 12-7b. The capacitance of the by-pass should be of such a value that it will offer little opposition to the flow of current for all frequencies above a definite value and greatly oppose the flow of current for all frequencies below this value. By connecting the coil and capacitor, as shown in Fig. 12-7c, the simplest type of a low-pass filter circuit is obtained.


Fig. 12-7.-Basic units of a low-pass filter circuit.

Example 12-6. A five-millihenry choke coil and a 0.001 -microfarad capacitor are connected as shown in Fig. 12-7c to form a low-pass filter circuit. What opposition is offered by the capacitor ( $a$ ) to the highest frequency audio signal usually obtained in a radio recciver ( 5000 cycles)? (b) To the lowest frequency carrier wave usually obtained in a radio receiver ( 500 kilocycles)? What opposition is offered by the inductor to (c) a 5000 -cycle signal? (d) A 500 -kilocycle signal?

| Given: | Find: |
| :--- | ---: |
| $C=0.001 \mu \mathrm{f}$ | $X_{C}=?$ |
| $L=5 \mathrm{mh}$ | $X_{L}=?$ |
| $f=5000$ cycles |  |
| $f=500 \mathrm{kc}$ |  |

Solution:
(a)

$$
\begin{equation*}
X_{C}=\frac{159,000}{f C}=\frac{159,000}{5 \times 10^{3} \times 1 \times 10^{-3}}=31,800 \mathrm{ohms} \tag{b}
\end{equation*}
$$

$X_{C}=\frac{159,000}{500 \times 10^{3} \times 1 \times 10^{-3}}=318 \mathrm{ohms}$
(c)
(d)
$X_{L}=2 \pi f L=6.28 \times 5 \times 10^{3} \times 5 \times 10^{-3}=157 \mathrm{ohms}$
$X_{L}=6.28 \times 5 \times 10^{5} \times 5 \times 10^{-3}=15,700$ ohms
Analyzing the values obtained in this example, it can be seen that the high-frequency currents will find the path of least opposition through the capacitor and hence will take that path. The low-frequency currents will find the path of least opposition through the inductor and hence will take that path. A measure of the ability of a filter circuit to by-pass undesired currents may be expressed in terms of the ratio of the impedances at the frequency limits.

High-pass Filter. A high-pass filter circuit is used to allow all currents having a frequency above a cer-


Fig. 12-8.-Characteristic curve for a simple high-pass filter circuit. tain value to pass into a desired circuit while opposing or diverting the flow of all currents having a frequency below this value (see Fig. 12-8). A capacitor inserted in the line (Fig. 12-9a) will offer little opposition to the flow of high-frequency currents, a large amount


Fig. 12-9.-Basic units of a high-pass filter circuit.
of opposition to the flow of low-frequency currents, and will block the flow of direct currents. The capacitor used should be of such a value that it will allow the passage of all currents whose frequencies are above a definite value and greatly oppose the flow of current for all frequencies below this value.

In order to divert the undesired low-frequency currents back to the source, an inductance coil is used as a by-pass as shown in Fig. 12-9b. The inductance of this coil is of such a value that it carrics off the currents whose frequencies are below the cutoff point and rejects the currents whose frequencies are above this value, thus forcing them to pass on through the circuit. Connecting the coil and capacitor as shown in Fig. 12-9c, the simplest type of high-pass filter circuit is obtained.

Example 12-7. A two-henry choke coil and a 0.5 -microfarad capacitor are connected as shown in Fig. 12-9c to form a high-pass filter circuit. What lopposition is offered by the capacitor (a) to 60 cycles (power disturbances)? (b) To a 1200 -cycle a-f signal? What opposition is offered by the inductor (c) to 60 cycles? (d) To 1200 cycles?

| Given : | Find: |
| :--- | ---: |
| $C=0.5 \mu \mathrm{f}$ | $X_{C}=?$ |
| $L=2$ henries | $X_{L}=?$ |
| $f=60$ cycles |  |
| $f=1200$ cycles |  |

Solution:

$$
\begin{align*}
X_{C} & =\frac{159,000}{f C}=\frac{159,000}{60 \times 0.5}=5300 \mathrm{ohms}  \tag{a}\\
X_{C} & =\frac{159,000}{1200 \times 0.5}=265 \mathrm{ohms} \\
X_{L} & =2 \pi f L=6.28 \times 60 \times 2 \\
& =753.6 \mathrm{ohms} \\
X_{L} & =6.28 \times 1200 \times 2 \\
& =15,072 \mathrm{ohms}
\end{align*}
$$

Analyzing, the values obtained in this example, it can be seen that the high-frequency currents will pass on through the capacitor and the lowfrequency currents will be by-passed through the inductor.

Band-pass Filter. The purpose of a band-pass filter is to allow the current of a narrow band of frequencies to pass through a circuit and to exclude all currents whose frequencies are either greater or less than the extreme limits of the band (Fig. 12-10).

Resonant circuits can be made to serve as filters in a manner similar to the action of individual capacitors and inductors. The series resonant circuit offers a very low impedance to those currents whose frequencies are at or near the resonant frequency of the circuit and a very high impedance to the currents of all other frequencies. The series resonant circuit (Fig. 12-11a) replacing the inductor of Fig. 12-7a would act as a band-pass filter, passing currents whose frequencies are at or near its resonant frequency and blocking the passage of all currents whose frequencies are outside this
narrow band. A parallel resonant circuit offers a very high impedance to those currents whose frequencies are at or near its resonant frequency and at relatively low impedance to currents of all other frequencies. The parallel resonant circuit (Fig. 12-11b) replacing the capacitor of Fig. 12-7b, if tuned


Fig. 12-10.-Characteristic curve for a simple band-pass filter circuit.
to the same frequency as the series resonant circuit, will provide a path for all currents whose frequencies are outside the limits of the frequency band passed by the series resonant circuit. Connecting the two resonant circuits


Fig. 12-11.-Basic units of a band-pass filter circuit.
as shown in Fig. 12-11c, the simplest type of band-pass filter circuit is obtained.

Band-stop Fitter. The purpose of a band-stop filter is to oppose the flow of current for a narrow band of frequencies while allowing the current to flow for all frequencies above or below this band (Fig. 12-12). Bandstop filters are also known as band-suppression and band-exclusion filters.

Its purpose is opposite to that of a band-pass filter, and the relative position of the resonant circuits in the filter circuit will have to be interchanged.

The parallel resonant circuit (Fig. 12-13a), replacing the capacitor of Fig. 12-9a, would act as a band-stop filter, blocking the passage of all currents whose frequencies are at or near its resonant frequency and passing


Fig. 12-12.-Characteristic curve for a simple band-stop filter circuit.
all currents whose frequencies are outside this band. The series resonant circuit (Fig. 12-13b), replacing the inductor of Fig. 12-9b, if tuned to the same frequency as the parallel resonant circuit, would act as a by-pass and


Fig. 12-13.-Basic units of a band-stop filter circuit.
provide a path for the undesired band of frequencies. Connecting the two resonant circuits as shown in Fig. 12-13c, the simplest type of band-stop filter circuit is obtained.

12-4. Multisection Filter Circuits. Need for Multisection Filter Circuits. All the filter circuits explained in the previous article have only one section. Using these circuits as is, it is impossible to obtain a sharp reduction of current at the cutoff frequency. Adding a capacitor, inductor, or resonant circuit in series or parallel (depending on the circuit) with the filter circuit will improve its filtering action, thus sharpening the reduction of current at
the desired frequency. When the above units are added to a filter circuit, the form of the resulting circuit will resemble the letter ' I ' or the symbol $\pi$. They are therefore called T- or $\pi$-type filters, depending on which symbol they resemble. 'Two or more sections of either the 'I'- or' $\pi$-type filters may be joined to produce a sharper cutoff.

T-type Filter Circuits. Connecting an inductor in series with the simple low-pass filter (Fig. 12-7c), a T-type low-pass filter circuit is formed (Fig. 12-14a). Connecting two of these filter circuits, as shown in Fig. 12-14b, it can be seen that the two connecting inductors $L_{a}$ and $L_{b}$ can be replaced

(a)

Single-section T-type low-pass filter circuit

(b)

A two-section T-type low-pass filter circuit

(c)

Practical two-section low-pass filter circuit

Fig. 12-14.-T-type low-pass filter circuits.
by a single inductor $L_{e}$ (Fig. 12-14c) whose value is equal to $L_{a}+L_{b}$. As all the inductors originally are of the same size, the center inductor $L_{\varepsilon}$ should have a value of twice the inductance of either of the end inductors.


Fig. 12-15.-T-type high-pass filter circuits.
Connecting a capacitor in series with the simple high-pass filter (Fig. 12-9c), a T-type high-pass filter circuit is formed (Fig. 12-15a). Connecting two of these filter circuits, as shown in Fig. 12-15b, it can be seen that the two connecting capacitors $C_{a}$ and $C_{b}$ can be replaced by a single capacitor $C_{e}$ (Fig. 12-15c) whose value is equal to $1 /\left(\frac{1}{C_{a}}+\frac{1}{C_{b}}\right)$. As all the capacitors originally are of the same size, the center capacitor $C_{6}$ should have a value of one-half the capacitance of either of the end capacitors.

Connecting a series resonant circuit in series with the simple band-pass
filter circuit (Fig. 12-11c), a T-type band-pass filter is formed (Fig. 12-16a). Connecting a parallel resonant circuit in series with the simple band-stop filter circuit (Fig. 12-13c), a T-type band-stop filter is formed (Fig. 12-16b).

Pi-type Filter Circuits. Connecting a capacitor to the simple low-pass filter (Fig. 12-7c), so that the line is shunted at both ends of the inductor


Fig. 12-16.-T-type band-pass and band-stop filter circuits.
by a capacitor, a $\pi$-type low-pass filter is formed (Fig. 12-17a). Connecting two of these filter circuits as in Fig. 12-17b, it can be seen that the two connecting capacitors $C_{a}$ and $C_{b}$ can be replaced by a single capacitor $C_{0}$


Frg. 12-17.-A $\pi$-type low-pass filter circuits.
(Fig. $12-17 c$ ) whose value is equal to $C_{a}+C_{b}$. As all the capacitors originally are of the same size, the center capacitor should have a value of twice the capacitance of either of the end capacitors.


Single-section T-type high-pass filter circuit


A two-section $\pi$-type high-pass filter circuit


Practical two-section high-pass filter circuit

Fig. 12-18.-A $\pi$-type high-pass filter circuits.
Connecting an inductor to the simple high-pass filter (Fig. 12-9c) so that the line is shunted at both ends of the capacitor by an inductor, a $\pi$-type high-pass filter is formed (Fig. 12-18a). Connecting two of these filter circuits as in Fig. 12-18b, it can be seen that the two connecting induc-
tors $L_{a}$ and $L_{b}$ can be replaced by a single inductor $L_{b}$ (Fig. 12-18c) whose value is equal to $1 /\left(\frac{1}{L_{a}}+\frac{1}{L_{b}}\right)$. As all the inductors originally are of same size, the center inductor should have a value of one-half the inductance of either of the end inductors.


Fig. 12-19.-A $\pi$-type band-pass and band-stop filter circuits.
Connecting a parallel resonant circuit to the simple band-pass filter circuit (Fig. 12-11c) so that the line is shunted at both ends of the scries resonant circuit by a parallel resonant circuit, a $\pi$-type band-pass filter circuit is formed (Fig. 12-19a). Connecting a series resonant circuit to the simple band-stop filter circuit (Fig. 12-13c) so that the line is shunted at both ends of the parallel resonant circuit by a series resonant circuit, a $\pi$-type band-stop filter circuit is formed (Fig. 12-19b).

12-5. Filter Circuits as a Whole. The design of filter circuits is a subject for advanced communication study; therefore the calculation of the component parts is left to advanced texts. The choice of the type of circuit to be used is therefore a matter for the designing engineer to decide. However, the following terms, generally used in connection with filter circuits, should be understood.

Source Impedance. Impedance of the circuit leading into the filter circuit is called the source impedance. This may be the plate circuit of a tube, a high resistance, etc.

Load Impedance. The impedance of the circuit into which the filter circuit feeds is called the load impedance. This may be a voltage divider, the plate resistor of a resistance coupled amplifier unit, ctc.

Image Impedances. These are the impedances at each end of the filter. In order that there will be no reflection loss, the image impedance at the load end should equal the load impedance, and the image impedance at the source end should equal the source impedance.

Characteristic Impedance. This is sometimes called the iterative impedance and is equal to the impedance that the filter circuit offers the source. Filter circuits are generally designed so that the load impedance equals the input impedance. The image impedances for this condition are equal and also equal to the characteristic impedance. The image impedance and
characteristic impedance generally vary with frequency; therefore the two measurements must be taken at the same frequency.

12-6. Other Filter Circuits. m-Derived Filter. These circuits are derived from the basic filter types explained in Art. 12-3, and their behavior depends on a factor that is a function of a constant, $m$. This type of filter circuit produces a sharper cutoff than the basic types. Additional impedances are inserted into the basic circuit to form either a shunt-derived or series-derived type filter. If the additional impedances are added to the shunt arm of the section, the filter circuit is series derived (Fig. 12-20). If the additional impedances are added to the series arm of the section, the filter circuit is shunt derived (Fig. 12-21).


Fig. 12-20.-Series-derived $m$-type filter circuits. The dotted impedances are those added to the shunt unit to form the $m$-derived filter circuit.

Resistor-capacitor Circuits. Many circuits in radio receivers and amplifiers carry both alternating and direct current. A circuit may carry direct current for plate supply and an a-c signal at the same time. It is necessary to provide a path for the signal voltages so that they may be applied only to certain portions of the circuit. In other words, it is necessary to separate the direct current and alternating signal current.

A convenient means of obtaining this separation is to use a capacitor to provide a path for the alternating current and a resistor to provide a path for the direct current. This action is illustrated in Fig. 12-22. The circuit of Fig. 12-22a uses a capacitor to allow the passage of the alternating signal current from the screen-grid circuit of a tube to ground. The resistor keeps the alternating signal current from getting into the B supply where it might cause trouble. The resistor is also used to provide the correct voltage for the screen grid by acting as a dropping resistor.

The resistor shown in Fig. 12-22b connected between cathode and ground is used to supply a negative voltage for the grid of the tube. This resistor is usually of several thousand ohms resistance and offers an impedance of this value to the flow of the signal current. This large imped-


Fig. 12-21.-Shunt-derived $m$-type filter circuits. The dotted impedances are those added to the series unit to form the $m$-derived filter circuit.
ance to the flow of the signal current will reduce it to a critical value. At this point in a circuit this large reduction in signal current introduces degeneration; this action should be avoided. If a capacitor is connected across the resistor as shown in Fig. 12-22b, it will provide a path for the alternating signal current. This diverting of flow of the signal current will not affect the obtaining of the voltage drop across the resistor necessary to produce the correct negative grid voltage.

There are a number of other applications of resistor-capacitor filter circuits; these circuits will be best understood in their application to a definite need in a circuit. Therefore, their circuits and actions are

(a)
(b)

Fig. 12-22.-Resistor-capacitor filter circuits: (a) filter action in the plate or sereen-grid circuit, (b) cathode grid-bias circuit. discussed in detail as to their applications to radio-amplifier circuits, audio-amplifier circuits, detector circuits, etc., in the authors' Essentials of Radio.

12-7. Coupling of Circuits. Principles of Coupling. Two circuits are said to be coupled when they have a common impedance that permits the
transfer of electrical energy from one circuit to another. This common impedance, called the coupling element, may be a resistor, an inductor, a capacitor, a transformer, or a combination of two or more of these elements.

Coupling elements are usually required to perform some filter action in addition to transferring energy from one circuit to another. Conversely, every filter circuit contains a section that acts as a coupling device. Coupling circuits and filter circuits are so much alike that it is difficult to state whether they should be called filters or coupling units. The choice of name, which is really unimportant, may be governed by that function which is considered of major importance. The type of impedance used will be


Fig. 12-23.-Types of simple-coupled circuits.
determined by the kinds of current flowing in the input circuit and the kind desired in the output circuit. The characteristics of each type of impedance have already been presented in the study of filters.

Simple-coupled Circuits. A simple-coupled circuit is one in which the common impedance consists of only a single element. A group of simplecoupled circuits is shown in Fig. 12-23.

The resistance-, inductive-, and capacitive-coupled circuits are also called direct-coupled circuits. In these circuits the coupling is accomplished by the current of the input circuit flowing through the common impedance, where it produces a voltage drop. This voltage is applied to
the output circuit, thus resulting in a transfer of electrical energy from the input to the output circuit. The output voltage is equal to the product of the current in the coupling element and its impedance.

The transformer-coupled circuit shown in Fig. 12-23d is also referred to as indirect coupling, magnetic coupling, or mutual-inductive coupling. In this type of coupling the transfer of energy is accomplished by the alternating current of the input circuit flowing through the primary winding and setting up an alternating magnetic field. The magnetic lines of this field link the turns of the secondary winding and induce the voltage that supplies the energy for the output circuit.

In some applications of coupling devices the input circuit may have both alternating and direct current flowing, and it is desired that the coupling unit transfer only the alternating current to the output circuit. The transformer-coupled unit will serve this purpose satisfactorily as it will pass only the alternating current. The other simple-coupled circuits (Fig. 12-23a, b, and c) can be modified by inserting a capacitor in series with the output side so that no direct current can reach the load. This is illustrated by the circuit of Fig. 12-23f.

Complex-coupled Circuits. A complex-coupled circuit is one in which the common impedance consists of two or more circuit elements. A fow of the numerous types of complex coupling are shown in Fig. 12-24.

The proportion of energy transferred in a simple inductive-coupled circuit increases as the frequency increases, while, with simple capacitive coupling, the proportion of energy transferred decreases as the frequency increases. Uising combinations of two or more elements in the coupling unit makes it possible to obtain various proportions of energy transfer for inputs of varying frequency. For example, the coupling element of Fig. $12-24 a$ is really a series tuned circuit and hence will have a minimum impedance at its resonant frequency. The proportion of energy transfer, too, will be at a minimum value when the frequency of the input circuit is equal to the resonant frequency of the coupling unit. At frequencies above resonance the proportion of energy transfer will increase and will be inductive. At frequencies below resonance the proportion of energy transfer will also increase but will be capacitive. The fact that the energy transfer is minimum at the resonant frequency may be more clearly understood when it is stated that the input side of the filter is generally a part of a series circuit, for example, the plate circuit of a tube, as shown in Fig. 12-24e. It can now be seen that at resonance, when the impedance of the coupling unit is minimum, its voltage drop will be at its minimum and the proportion of energy transfer must also be at its minimum.

In general, the amount of energy transferred will be proportional to the current flowing through the coupling unit and to the impedance of the
unit. For purpose of analysis, complex-coupled circuits may be represented by a simple equivalent circuit, as shown in Fig. 12-24f.

Coefficient of Coupling. The ratio of the energy of the output circuit to the energy of the input circuit is called the coefficient of coupling. Criti-


Fig. 12-24.-Typical complex coupled circuits.
cal, tight, and loose coupling are terms used to express the relative value of the coefficient of coupling for mutual-inductive (transformer)-coupled circuits.

Figure 12-25 shows the response curves for tight, critical, and loose coupling. When the maximum amount of energy is transferred from one circuit to another, the circuits are said to possess critical coupling; this is also referred to occasionally as optimum coupling. If the coefficient of coupling is higher than that necessary to produce critical coupling, it is referred to as being tight; if it is lower than that required for critical coupling, it is referred to as being loose.

The effect of varying the coupling between two circuits may be seen from the response curves of ligs. 12-25 and 12-30. When two circuits are very tightly coupled, resonance will be obtained at two new frequencies, one below and the other above the normal frequency of resonance for the capacitor and inductor used. As the coupling is decreased, the two peaks
come closer together until critical coupling is reached and a single peak of maximum height obtained. If the coupling is decreased below the critical value, a single peak of reduced height is obtained.

Air-core transformers, commonly used in radio circuits, illustrate the importance of the amount of coupling between the primary and the second-


Fig. 12-25.- Response curves showing the effect of various amounts of coupling.
ary windings. As it is difficult to design an air-core transformer in which a large portion of the magnetic lines set up by the primary winding will link the turns of the secondary winding, the coefficient of coupling is generally low. A low value for the cocfficient of coupling is not objectionable in some circuits, as it provides certain desirable characteristics which will be presented in the following article.

12-8. Characteristics of Mutual-inductive-coupled Circuits. Inductive coupling, particularly mutual inductance as provided by the transformer, is the means most commonly used to transfer energy from one circuit to another. The characteristics of these circuits depend upon the type of circuit, that is, whether a capacitor is connceted to the primary, to the secondary, or to both. The characteristics are also dependent upon the amount of coupling between the two circuits.

Coupled Impedance. The primary and the secondary circuits of a transformer are separate electrical circuits that are magnetically coupled. Each circuit has an impedance of its own generally designated as $Z_{p}$ and $Z_{s}$. The impedance of the primary winding, when no load is applied to the secondary, consists of the resistance and inductance of the primary winding. The impedance of the secondary circuit consists of the resistance and inductance of the secondary winding plus the impedance of any load connected to the circuit. When the secondary circuit is left open, that is, when no load is applied to its terminals, the impedance of the secondary will be infinity or so large that it is immeasurable. Under this condition the presence of the sccondary will have no effect upon the primary circuit.

When a load is applied to the secondary, the impedance will have a significant value and a current will flow in the secondary circuit. The amount of energy in this circuit will depend upon the secondary voltage and impedance. The secondary voltage, however, is dependent upon the number of magnetic lines linking the two circuits. The number of linkages is proportional to the coefficient of coupling; therefore the amount of energy transferred is also dependent upon the coefficient of coupling. As the energy in the secondary circuit must come from the primary, it is evident that the primary impedance will be affected by the impedance of the secondary circuit. The effect of the secondary circuit upon the primary is equivalent to adding an impedance in series with the primary. This added impedance is generally referred to as the coupled impedance.

The numerical value of the coupled impedance of a mutual-inductivecoupled circuit may be found by the equation

$$
\begin{equation*}
Z_{p-s}=\frac{(2 \pi f M)^{2}}{Z_{s}} \tag{12-1}
\end{equation*}
$$

where $Z_{p-s}=$ impedance coupled into the primary by the secondary, ohms
$f=$ frequency of the power source, cycles per second
$M=$ mutual inductance, henries
$Z_{s}=$ secondary impedance, ohms
The derivation of this equation is explained in the following steps:

1. From the definition, two circuits have a mutual inductance of one henry when a current in one circuit, changing at the rate of one ampere per second, induces an average emf of one volt in the second circuit; the induced voltage in the second circuit may be expressed as

$$
\begin{equation*}
\epsilon_{s}=M \frac{I_{p 2}-I_{p 1}}{t_{2}-t_{1}} \tag{12-2}
\end{equation*}
$$

This equation indicates that, when the mutual inductance $M$ is one henry and the rate of current change $\frac{I_{p 2}-I_{p 1}}{t_{2}-t_{1}}$ is one ampere per second, the average value of the induced voltage $e_{s}$ will be one volt. In other words, this equation is derived from the definition of the unit of mutual inductance.
2. When an alternating current $I_{p}$ is flowing, the current is continually changing from a maximum value to zero in a positive and negative direction and at a rate proportional to the frequency. As the alternating current $I_{p}$ is an effective value, the maximum current will be $I_{p}$ divided by 0.707 . Also, a change in current from the maximum value to zero occurs in a period of time corresponding to one-quarter of a cycle. Therefore

$$
\begin{equation*}
\frac{I_{p 2}-I_{p 1}}{t_{2}-t_{\mathrm{l}}}=\frac{I_{\mathrm{max}}-I_{o}}{\frac{1}{4 f}}=\frac{I_{\mathrm{max}}}{\frac{1}{4 f}}=\frac{\frac{I_{p}}{0.70 \overline{7}}}{\frac{1}{4 f}}=\frac{4 f I_{p}}{0.707} \tag{12-3}
\end{equation*}
$$

Applying Eq. (12-3) to Eq. (12-2), then

$$
\begin{equation*}
e_{\mathrm{s}}=M \frac{4 f I_{p}}{0.707} \tag{12-4}
\end{equation*}
$$

3. The induced secondary voltage $e_{s}$ is expressed as an average value, and in practical work it is desired to have it expressed as the effective value $E$. As the average value is equal to $2 / \pi$ (or 0.637 ) times the maximum value and the effective value is equal to 0.707 times the maximum value, then the effective value may be expressed as

$$
\begin{equation*}
E_{s}=\frac{e_{s}}{\frac{2}{\pi}} \times 0.707=\frac{0.707 \pi e_{s}}{2} \tag{12-5}
\end{equation*}
$$

or

$$
\begin{equation*}
e_{s}=\frac{2 E_{s}}{0.707 \pi} \tag{12-6}
\end{equation*}
$$

Substituting Eq. (12-6) in Eq. (12-4),
or

$$
\begin{gather*}
\frac{2 E_{s}}{0.707 \pi}=M \frac{4 f I_{p}}{0.707}  \tag{12-7}\\
E_{s}=2 \pi f M I_{p} \tag{12-8}
\end{gather*}
$$

4. The secondary current $I_{s}$ will, therefore, be

$$
\begin{equation*}
I_{s}=\frac{E_{s}}{Z_{s}}=\frac{2 \pi f M I_{p}}{Z_{s}} \tag{12-9}
\end{equation*}
$$

5. This secondary current upon flowing through the secondary winding sets up a magnetic field of its own that induces a voltage in the primary. This induced voltage will be 180 degrees out of phase with the primary impressed voltage and is referred to as a counter, or back, voltage. By the same reasoning as was used to derive the secondary induced voltage, it may be shown that this counter voltage induced in the primary will be

$$
\begin{equation*}
E_{\text {counter }}=2 \pi f M I_{s} \tag{12-10}
\end{equation*}
$$

Substituting Eq. (12-9) for $I_{s}$ in Eq. (12-10),

$$
\begin{align*}
E_{\text {counter }} & =(2 \pi f M) \frac{(2 \pi f M) I_{p}}{Z_{s}}  \tag{12-11}\\
& =\frac{(2 \pi f M)^{2}}{Z_{s}} I_{p} \tag{12-12}
\end{align*}
$$

6. As this voltage represents the effect that the secondary has upon the primary and as an a-c voltage is equal to the product of impedance and current, it may be stated [from Eq. (12-12)] that the effect of the secondary impedance upon the primary is

$$
\begin{equation*}
Z_{p-s}=\frac{(2 \pi f M)^{2}}{Z_{s}} \tag{12-1}
\end{equation*}
$$

The coupled impedance expressed by Eq. (12-1) may be represented by an equivalent resistance and an equivalent reactance connected in series with the primary circuit. The numerical values of the equivalent resistance and equivalent reactance are expressed by the following equations:

$$
\begin{align*}
& R_{p-s}=\frac{(2 \pi f M M)^{2} R_{s}}{Z_{s}{ }^{2}}  \tag{12-13}\\
& X_{p-s}=-\frac{(2 \pi f M)^{2} X_{s}}{Z_{s}{ }^{2}} \tag{12-14}
\end{align*}
$$

where $R_{p-s}=$ resistance coupled into the primary by the secondary, ohms
$X_{p-s}=$ reactance coupled into the primary by the secondary, ohms
$f=$ frequency of the power source, cycles per second
$M=$ mutual inductance, henries
$R_{s}=$ resistance of the secondary circuit, ohms
$Z_{s}=$ impedance of the secondary circuit, ohms
$X_{s}=$ reactance of the secondary circuit, ohms
Note: When $X_{s}$ is inductive, then $X_{p-s}$ has a negative sign, and when $X_{s}$ is capacitive, $X_{p-s}$ has a positive sign.

Example 12-8. A mutual-inductance-coupled circuit is shown in Fig. 12-26 together with the circuit values. (a) What coupled impedance does the secondary present to the primary? (b) What is the value of the equivalent resistance component of the coupled impedance? (c) What is the value of the reactance component of the coupled impedance? (d) Draw the equivalent-circuit diagram. (e) What is the effective impedance of the primary circuit? ( $f$ ) What is the primary circuit current if the voltage of the 175 -kilocycle signal is 10 volts? ( $g$ ) What is the secondary voltage? ( $h$ ) What is the secondary current?

Given:


Fig. 12-26.

Find:
(a) $Z_{p-}=$ ?
(b) $R_{p-s}=$ ?
(c) $X_{p-1}=$ ?
(d) Diagram
(e) $Z_{p T}=$ ?
(f) $\quad I_{p}=$ ?
(g) $\quad E_{s}=$ ?
(h) $\quad I_{z}=$ ?

## Solution:

(a)

$$
\begin{aligned}
Z_{p-s} & =\frac{(2 \pi f M)^{2}}{Z s}=\frac{(2 \pi f M)^{2}}{\sqrt{\left(R_{s}+R_{\text {toat }}\right)^{2}+\left(2 \pi f L_{s}\right)^{2}}} \\
& =\frac{\left(6.28 \times 175 \times 10^{3} \times 0.50 \times 10^{-3}\right)^{2}}{\sqrt{(20+80)^{2}+\left(6.28 \times 175 \times 10^{3} \times 3.5 \times 10^{-3}\right)^{2}}} \\
& =\frac{301,950}{3847} \\
& =78.5 \mathrm{ohms}
\end{aligned}
$$

(b)

$$
\begin{aligned}
R_{p-s} & =\frac{(2 \pi f M)^{2} R_{s}}{Z_{s}^{2}} \\
& =\frac{301,950 \times 100}{(3847)^{2}} \\
& =2.04 \mathrm{ohms}
\end{aligned}
$$

(c)

$$
\begin{aligned}
X_{p-s} & =\frac{(2 \pi f M)^{2} X_{s}}{Z_{s}{ }^{2}} \\
& =\frac{301,950 \times 3846}{(3847)^{2}} \\
& =78.5 \text { ohms }
\end{aligned}
$$


(d)

Fig. 12-27.
(e)

$$
\begin{aligned}
Z_{p T} & =\sqrt{\left(R_{p}+R_{p-s}\right)^{2}+\left(X_{p}-X_{p-s}\right)^{2}} \\
& =\sqrt{(10+2.04)^{2}+(1868-78.5)^{2}} \\
& =1789.6 \mathrm{ohms}
\end{aligned}
$$

(f)

$$
\begin{aligned}
I_{p} & =\frac{E}{Z_{p T}} \\
& =\frac{10}{1789.6} \\
& =0.00558 \mathrm{amp} \\
& =5.58 \mathrm{ma}
\end{aligned}
$$

(g)

$$
\begin{aligned}
E_{s} & =2 \pi f M I_{p} \\
& =549.5 \times 0.00558 \\
& =3.06 \mathrm{volts}
\end{aligned}
$$

(h)

$$
\begin{aligned}
I_{s} & =\frac{E_{s}}{Z_{s}} \\
& =\frac{3.06}{3847} \\
& =0.000795 \mathrm{amp} \\
& =0.795 \mathrm{ma}
\end{aligned}
$$

Examining the results of this example, one can see that the effect of the coupled equivalent resistance is to increase the effective resistance of the primary circuit. The equivalent reactance that is coupled into the primary by a secondary whose reactance is inductive is opposite in phase to the primary reactance and hence reduces the effective reactance of the primary circuit. The net result is a reduction in the effective primary impedance, more current thereby being allowed to flow in the primary circuit, thus making possible the transfer of more energy to the secondary circuit.

Many of the important characteristics of coupled circuits are explained by the effects of coupled impedance. Examination of Eq. (12-1) indicates that the coupled impedance will be low when the coefficient of coupling is low because the value of $M$ decreases when the coefficient of coupling is decreased. Also, the coupled impedance will be low when the secondary impedance is high. Thus, when the coefficient of coupling is low or when very little load is applied to the secondary (high secondary impedance), the coupled impedance will be low and the effect of the secondary upon the primary will be negligible. However, when the cocfficient of coupling is high or when the secondary carries considerable amounts of load (low secondary impedance), the coupled impedance will be high and the secondary will produce considerable effect upon the primary circuit.

Circuit with Untuned I'rimary and Untuned Secondary. The simplest type of transformer coupling would be a circuit having an untuned primary and an untuned secondary with a resistance or inductance load. Such a circuit is shown in Fig. 12-28a. This circuit is often used as an equivalent circuit to represent the effects produced by a shield, metal panel, or other metal object located near a coil. The effect of the shield or panel upon the coil would be the same as that of a secondary winding consisting of inductance and resistance in scrics. The coupled impedance of such a circuit will increase the effective resistance of the primary and reduce its effective reactance. It also indicates that losses of the coil circuit are increased by
an amount proportional to the resistance coupled into the primary by the secondary, which is actually the shield or nearby metal pancl.

Circuit with Untuned Primary and Tuned Secondary. The circuit shown in Fig. 12-28b differs from Fig. 12-28a in that a capacitor is used in place


Fig. 12-28.-Fundamental transformer-coupled circuits.
of the resistor in the secondary circuit. The commonly used tuned-radiofrequency amplifier circuit, or its equivalent circuit, is similar to this fundamental circuit.

The secondary is similar to the series tuned circuit studied in Chap. XI. Its characteristics will be the same as those of the series tuned circuit. At resonant frequency the impedance will be at its minimum and the current at its maximum. The impedance coupled into the primary will be large and will have a critical effect upon the primary current. At frequencies above or below resonance the secondary impedance increases and its current decreases. The impedance coupled into the primary decreases, and the effect on the primary circuit is decreased.

Circuit with Tuned Primary and Tuned Secondary. This type of circuit (Fig. 12-28c) is used extensively in radio receivers. A common example of this circuit is the i-f amplifier of the superheterodyne receiver. This circuit is very useful for amplifiers because it can be designed to provide an approximately uniform secondary current response over the range of frequencies that are normally applied to the primary.

12-9. Band-pass Amplifier Circuits. Ideal Response Curve. The ideal resonance curve for the tuning or i-f circuits of an a-m broadeast receiver would be one having a flat top and very stecp sides. The flat top should be approximately 10 kilocycles wide. This band of 10 kilocycles is not arbitrarily chosen but represents a five-kilocycle side band above and below the carrier frequency of any transmitting station. These side bands are part of every modulated wave, and the amount its frequency varies from the transmitter's carrier frequency will depend on the frequency of the audio signal. The frequency of an audible signal will vary from 40 to more than 10,000 cycles. However, most of the radio receivers in use today generally reproduce sounds only up to 5000 cycles. Although many of the larger broadcasting stations are cquipped to transmit all audio frequencies up to

10,000 cycles, they do not do so because of these frequency limits. The upper limit of most stations is between five and six kilocycles. The sharper the sides of the response curve, the more selective the receiver will be. Circuits possessing such characteristics are known as band-pass filters, band-pass amplifiers, or band-pass circuits.

This ideal can be approached by the proper use of coupled circuits. A band-pass amplifier circuit consists of two resonant circuits tuned to the same frequency and coupled together (Fig. 12-29). The important characteristic of this type of circuit is


Fig. 12-29.-Band-pass amplifier circuit. the manner in which the current in the secondary circuit varies with the frequency when a constant voltage is applied to the primary circuit. The amount of current flow in the secondary is directly proportional to the amount of coupling between the two windings. The shape of the resonance curve will, therefore, be dependent on the coefficient of coupling and may be a narrow peaked curve, a flat-top curve, or a curve having two separate peaks with a hollow between them.

Figure 12-30 illustrates the manner in which the current in the secondary of such a circuit varies with the amount of coupling for frequencies above and below its resonant frequency. When the coefficient of coupling is low, $K=0.01$ for resonance at 500 kilocycles, the secondary current will be small, and the curve will be peaked. As the amount of coupling between the two circuits is increased, the amount of secondary current will increase and there will be a reduction in the sharpness of the peak, $K=0.015$. With critical coupling $K=0.02$, the maximum amount of secondary current is obtained and the resonance curve will be comparatively flat at the top and will have steep sides. With tight coupling, the coupled impedance at resonance is large, thus reducing the primary current which in turn reduces the amount of voltage induced into the secondary, finally therefore, reducing the secondary current. This accounts for the decrease in secondary currents at resonance for coefficients of coupling greater than the critical value as indicated by the curves in Fig. 12-30. The reactance coupled into the primary is inductive for frequencies below resonance and capacitive for frequencies above resonance. This reactance is opposite to that of the primary circuit and will therefore reduce the equivalent impedance offered to the applied voltage. The primary current, and therefore the voltage induced into the secondary, will increase for frequencies off
resonance. When the coupling is tight, this action will introduce new resonant frequencies above and below resonance corresponding to the amount of secondary current. This accounts for the humps in the resonance curves when the coupling is greater than the critical value. The current at these peaks is practically the same as the peak current with critical coupling. The spacing between these peaks is directly proportional to the coefficient of coupling.

Width of Band Pass. With critical coupling, the maximum possible transfer of energy to the secondary at the resonant frequency is obtained


Fig. 12-30.-Curves showing the variation in secondary current with frequency for a bandpass circuit.

With tight coupling, a fairly constant secondary current and voltage can be obtained for a narrow band of frequencies. The width of this band, measured at 0.707 of the maximum response, is directly proportional to the coefficient of coupling and the resonant frequency of the tuned circuits. An approximate value of the width of this band can be obtained by the following equation:

$$
\begin{equation*}
\text { Width of band pass }=K \times f_{r} \tag{12-15}
\end{equation*}
$$

where $K=$ coefficient of coupling
$f_{r}=$ resonant frequency of the tuned circuits
From this equation it can be seen that the larger the value of $K$, the wider the band pass will be. The coupled impedance will also increase with an increase in the amount of coupling, thus causing a decrease in the output current (see Fig. 12-30).

Example 12-9. What is the approximate width of the frequency band of a bandpass filter circuit having a resonant frequency of 456 kilocycles and a coefficient of coupling of 0.02 ?

$$
\begin{aligned}
& \text { Given: } \\
& K=0.02 \\
& f_{r}=456 \mathrm{kc}
\end{aligned}
$$

Find:
Width of band pass $=$ ?

Solution:

$$
\begin{aligned}
\text { Width of band pass } & =K \times f_{r} \\
& =0.02 \times 456 \\
& =9.12 \mathrm{kc} \\
& =9120 \text { cycles }
\end{aligned}
$$

The most important properties of a band-pass circuit are the width of the band of frequencies it allows to pass and the uniformity of response within this band. Referring to Fig. 12-30, it can be seen that when $K=0.02$ the secondary current is fairly constant for a band of frequencies between 495 and 505 kilocycles. The response will, therefore, be uniform for this band of frequencies with a coefficient of coupling of 0.02 at 500 kilocycles. As the coefficient of coupling is increased ( $K=0.04, K=0.08$, $K=0.15$ ), it can be seen that the band becomes wider and less uniform.

The coefficient of coupling of band-pass circuits is usually of such a value that uniform response is obtained for a band of 10 kilocycles. The response should decrease rapidly for frequencies beyond these limits (see Fig. 12-30).

The uniformity of response within a band pass of frequencies is dependent on the circuit $Q$ and the value of the coefficient of coupling. The equation for finding the amount of coupling required to produce maximum transfer of energy is usually expressed in terms of the primary and secondary $Q$ as follows:

$$
\begin{equation*}
K_{C}=\frac{1}{\sqrt{ } Q_{p} Q_{s}} \tag{12-16}
\end{equation*}
$$

where $K_{c}=$ critical coupling
$Q_{p}=Q$ of the primary circuit
$Q_{s}=Q$ of the secondary circuit
$\sqrt{Q_{p} Q_{s}}=Q$ of the complete circuit
Figure 12-31 shows that if the circuit $Q$ is too high, pronounced double humps occur; and if too low, the response curve is round instead of flat. Experiments have shown that the best value of $Q$ is approximately 50 per cent more than that required to produce critical coupling. Substituting this value in Eq. (12-16) and solving for the circuit $Q$, we have

$$
\begin{equation*}
\sqrt{\bar{Q}_{p} Q_{s}}=\frac{1.5}{\overline{K_{C}}} \tag{12-17}
\end{equation*}
$$

or

$$
\begin{equation*}
Q_{p} Q_{s}=\frac{2.25}{K_{C}^{2}} \tag{12-18}
\end{equation*}
$$

From Eq. (12-15) it can be seen that the coefficient of coupling would be equal to the width of the band pass divided by the resonant frequency. As the width of the band pass is usually about 10 kilocycles and the resonant frequency would generally be 175 kilocycles or more, the coefficient of


Fig. 12-31.-Characteristics of a band-pass amplifier circuit showing the effect of circuit $Q$ on uniformity of response within the band being passed.
coupling must therefore be less than 0.057 . Substituting this value of $K$ in Eq. (12-18), we obtain circuit Q's greater than 25.

Example 12-10. A band-pass filter circuit is tuned to a resonant frequency of 456 kilocycles. What values of circuit $Q$ 's are necessary to produce uniform response for an 8-kilocycle band?

$$
\begin{aligned}
& \text { Given: } \\
& \begin{aligned}
\text { Width of band pass } & =8 \mathrm{kc} \\
\qquad f_{r} & =456 \mathrm{kc}
\end{aligned}
\end{aligned}
$$

Find:

$$
Q_{p}=?
$$

$$
Q_{0}=?
$$

Solution:

$$
\begin{aligned}
K & =\frac{\text { width of band pass }}{f_{r}} \\
& =\frac{8}{456}=0.0175 \\
Q_{p} Q_{s} & =\frac{2.25}{K^{2}} \\
& =\frac{2.25}{(0.0175)^{2}}=7346
\end{aligned}
$$

If $Q_{p}=Q_{s}$, then

$$
Q_{p}=\sqrt{7346}=85.7
$$

12-10. Wide-band-pass Amplifier Circuits. Methods Used to Obtain a Wide Band Pass. The width of the band of frequencies to be passed in the r-f and i-f amplificr circuits of f-m radio reccivers and television receivers is much greater than that which can normally be obtained by the use of a tuned circuit consisting only of a ca-


Fig. 12-32.-Variable-coupling i-f transformer used to adjust the width of band pass. (Hammarlund Manufacturing Company, Inc.) pacitor and an inductor. In order to obtain a wide band pass, one or more of the following methods are generally used: (1) decreasing the value of the circuit $Q,(2)$ increasing the value of the coefficient of coupling, (3) stagger tuning.

The value of circuit $Q$ can be decreased by either adding resistance to the external circuit or by increasing the $L / C$ ratio. In the first method a resistor is connected either in series or in parallel with the tuned circuit. As the addition of resistance to a tuned circuit will lower the value of circuit $Q$, the desired value can be obtained by adding the proper amount of resistance. The second method is based on the fact that the $Q$ of a tuned circuit varies directly with the amount of capacitance in the circuit. Thus, by using a larger value of inductance and a smaller value of capacitance, a lower value of $Q$ is obtained. The effect of various values of $Q$ on the width of band pass was discussed in Art. 11-6 and is illustrated in Fig. 11-5.
Obtaining a wide band pass by increasing the value of the coefficient of coupling has been discussed in the preceding article. The relation between the coefficient of coupling and the width of the band pass is illustrated in Fig. 12-30 and can be seen by a study of Eq. (12-15).

A wide band pass can also be obtained by stagger-tuning several stages of tuned amplifier circuits. In this method, each tuned circuit in the various stages of amplification involved is tuned to a different value of frequency, but all are within the desired range of the band to be passed. In order to obtain the over-all characteristics desired, the $Q$ of each circuit is adjusted to produce a certain amount of overlap.

Band Pass in F-m Receivers and in the Sound Section of Television Receivers. The band pass in the i-f section of $\mathrm{f}-\mathrm{m}$ broadcast receivers is 150 kilocycles as compared with the 10 -kilocycle band pass of the i-f section of a-m broadeast receivers. The sound section of television receivers is
frequency modulated and operates with a band pass of 100 kilocycles. The frequency of operation of the i-f section of f-m receivers is also much higher than the i-f section of a-m receivers, being of the order of 10.7 megacycles for f-m receivers as compared with 456 kilocycles for a-m receivers (see Fig. 12-33). The i-f value of the $\mathrm{f}-\mathrm{m}$ sound section of a television receiver is of the order of 25 megacycles.

A wide band pass is obtained in f-m receivers by using either stagger tuning or overcoupled i-f transformers, the latter being the method most


Fig. 12-33.-A double peak i-f response curve for an $f$-m receiver.
generally used. When overcoupling is employed to obtain a wide band pass, three i-f transformers tuned to the same frequency are generally used. The first and third transformers are single-peaked and just under critical coupling. The second transformer is overcoupled to produce a double-peaked response curve (see Fig. 12-34). By applying Eq. (12-15), it can be seen that in order to produce a 150 -kilocycle band pass for an f-m receiver the coefficient of coupling would have to be of the order of 0.014 . In order to produce a 100 -kilocycle band pass as is employed in the $\mathrm{f}-\mathrm{m}$ sound section of a television receiver, the cocfficient of coupling would have to be of the order of 0.004 for an i-f value of 25 megacycles.

Band Pass of the Video Section in Television Receivers. The over-all band pass of the video i-f section of a television receiver cannot be symmetrical because of the vestigial-side-band method of transmission. The ideal response curve for reception of this type of transmission is shown in

Fig. 12-35. The operating frequency of video i-f transformers generally ranges from 21 to 27 megacycles, and the width of its unsymmetrical band pass approximates four megacycles. The ideal response curve is difficult to obtain; however, by using various combinations of stagger tuning, a close approximation can be achicved. The combination used will vary with the designer. In general, three or more stages of tuned amplification are used. Each circuit may be tuned to a single frequency or may be overcoupled to produce peaks at two (or more) frequencies within the desired range of the band pass. It is beyond the scope of this text to analyze each of the variations that may be used. An example of one of the combinations


Fig. 12-34. Wide pass band for the i-f response curve in an $\mathrm{f}-\mathrm{m}$ receiver obtained by use of an over-coupled i-f transformer. that may be used is illustrated in Fig. 12-36. In this combination, the first and third stages are overcoupled and stagger-tuned to different values of frequency and the second stage is a single tuned circuit. From this figure it can be observed that the over-all response curve approximates the ideal response curve of Fig. 12-35.

12-11. Delayed-action Circuits. Inductors or capacitors may be used in electric, radio, television, and electronic circuits to control the time required for the current or voltage to reach a certain value. The operation of these circuits is based on the time constant of the resistance-inductance or the resistance-capacitance combination. These circuits are generally referred to as $R-L$ and $R-C$ circuits.
Time Constant of Resistance-inductance Circuits. Inductance, by definition (Art. 8-1), is the property of a circuit that opposes any change in the amount of current flowing in that circuit. The opposition to a change in the amount of current is caused by the induced voltage due to the selfinductance of the circuit. This induced emf will be in a direction opposite to that of the impressed voltage whenever the current is increasing in amount and in the direction of the impressed voltage when the current is decreasing in amount.

If an inductor, which may be considered as a resistance and inductance
in series, is connected to a d-c power source, a current will flow in the circuit. The amount of current that will flow will be its Ohm's law value, namely, the voltage applied to the circuit divided by the resistance of the circuit. In a circuit having only resistance (Fig. 12-37a), the current will rise to its


Fig. 12-35.-The ideal video i-f response curve for the reception of vestigial-side-band transmission of television signals.


Fig. 12-36.-Wide pass band for the i-f response curve in the video section of a television receiver obtained by use of both stagger tuning and over-coupled i-f transformers.

Ohm's law value practically instantancously, as indicated in Fig. 12-37b. However, as the inductor has the effect of a resistance and inductance connected in scries (Fig. 12-38a) the current will require an appreciable amount of time to reach its Ohm's law value, as is shown in Fig. 12-38b. This is explained by the fact that,


Frg. 12-37.-Characteristics of current vs. time for a circuit containing only resistance: (a) the circuil, (b) current vs. time characteristics. for the current to reach its final value of five amperes, it must progressively pass through its lesser values such as one, two, three, and four amperes. Under these conditions, the current is changing in amount, and the circuit will have an emf induced in it owing to the selfinductance of the circuit. This induced emf will oppose the impressed voltage and thus will prevent the current from reaching its Ohm's law value as long as the induced emf is prosent. The current will, however, eventually reach its Ohm's lavv value, the time required to accomplish this depending upon the relative values of the inductance and resistance. The current increases in a manner indicated by the graph shown in Fig. 12-38b and will rise to 63.2 per cent of its final value in a period of time, expressed in seconds, equal to the inductance of the circuit divided by the resistance of the circuit. This is called the time constant of the circuit and is expressed mathematically as

$$
\begin{equation*}
t=\frac{L}{R} \tag{8-10}
\end{equation*}
$$

where $t=$ time, seconds, for the current to reach 63.2 per cent of its final value
$L=$ inductance of the circuit, henries
$R=$ resistance of the circuit, ohms
Example 12-11. An $R$ - $L$ circuit is used to control the time of closing a relay. The relay closes when the current reaches 63.2 per cent of its final value and the circuit resistance and inductance are 12 ohms and 2.4 henries, respectively. What is the time interval between the closing of the lineswitch and the operation of the relay?

Given:
$R=12 \mathrm{ohms}$
$L=2.4$ henries

Find:
$t=$ ?

Solution:

$$
t=\frac{L}{R}=\frac{2.4}{12}=0.2 \mathrm{sec}
$$

The time required for the current to reach values other than 63.2 per cent of the final value follows a curve known mathematically as an expo-


Fig. 12-38.-Characteristics of current and voltage vs. time for a circuit containing resistance and inductance: (a) the circuit, (b) current vs. time characteristics, (c) voltage vs. time characteristics.
nential curve. The universal time-constant curves of Fig. 12-42 provide a simple means of finding the current at any instant of time.

Further analysis of the $R$-L circuit will show that, when the current is increasing, the voltage drop across the resistance will increase at the same
time rate as the current. This is so because the voltage drop across the resistance at any instant of time is equal to the product of the current and the resistance. Furthermore, as the sum of the voltages around the circuit must be equal to the applied voltage, the induced emf at any instant of time duc to the inductance must be equal to the applied voltage less the


Fig. 12-39.- Characteristics of current and voltage vs. time: (a) the circuit, (b) current vso time characteristics with the switch in position 2, (c) voltage vs. time characteristics with the switch in position 2.
$I R$ drop. Figure 12-38c shows the voltage characteristics of the circuit when the current is building up.

The circuit shown in Fig. 12-39a is arranged so that the $R$ - $L$ circuit may either be connected to the d-c power source or be connected so that the inductance will be short-circuited through the resistance. If the circuit is connected to the d-c power source, a current will flow in the circuit and will rise to its Ohm's law value according to the current-time curve already
described. When a current is flowing in the circuit, energy is transferred to the magnetic field. If the switch (Fig. 12-39a) is changed from position 1 to position 2, so that the inductance is disconnected from the power source and then instantaneously short-circuited across the resistance, the energy in the collapsing magnetic field will induce a voltage in the turns of the coil and cause a current to flow in the circuit. The current will decrease as the energy is dissipated in the resistance. The rate at which the current decreases will depend upon the relative values of the inductance and the resistance. The current-time changes will also follow an exponential curve but will be a descending curve. As the inductance is now actually in parallel with the resistance, the resistance voltage drop and the induced emf will be equal in value and will decrease according to an exponential curve. The current-time characteristics are shown in Fig. 12-39b, and the voltage-time characteristics are shown in Fig. 12-39c. The time in seconds as determined by $L$ divided by $R$ now represents the time in which the current (and voltage) decreases 63.2 per cent; hence the current and voltage will drop to 36.8 per cent of their maximum values in $L / R$ seconds. The time required for the current and voltage to decrease to values other than 36.8 per cent of their maximum values can be found by use of the universal exponential curves presented at the end of this article.

Time Constant of Resistance-capacitance Circuits. Capacitance, by definition (Art. 9-1), is the property of a circuit that opposes any change in the amount of voltage. The opposition to a change in the voltage across a capacitor may be explained by the fact that in order to accomplish a change in voltage the number of electrons at the plates of the capacitor must be changed. This requires a passage of electrons from one plate of the capacitor to the other, and hence a current must flow before there can be a change in voltage. If the voltage across the capacitor is increased, electrons will flow from the positive plate to the negative; if the voltage across the capacitor is decreased, electrons will flow from the negative plate to the positive. In either case a current flow must precede a change in voltage at the plates of the capacitor.

If a perfect capacitor, that is, one having no resistance, is connected to a d-c power source, a high current surge will flow instantly and will charge the capacitor. As the capacitor becomes charged almost instantaneously, the amount of current flow will decrease rapidly. The capacitor will charge to the value of the impressed voltage, and the current flow will diminish to zero practically instantaneously.

If the capacitor or its circuit contains resistance in addition to the capacitance (Fig. 12-40a), the capacitor will become charged to the same value of voltage but will require a longer period of time to reach its final value. The voltage increases in a manner indicated by the graph shown
in Fig. 12-40b and will rise to 63.2 per cent of its final value in a period of time, expressed in seconds, equal to the product of the capacitance and


Fig. 12-40.-Characteristics of current and voltage vs. time for a circuit containing resistance and capacitance: (a) the circuit, (b) capacitor volts vs. time characteristics, (c) current and resistance volts vs. time characteristics.
resistance of the circuit. This is called the time constant of the circuit and is expressed mathematically as

$$
\begin{equation*}
t=C R \tag{12-19}
\end{equation*}
$$

where $t=$ time, seconds, for the voltage across the capacitor to reach 63.2 per cent of its final value
$C=$ capacitance of the circuit, farads
$R=$ resistance of the circuit. ohms

Example 12-12. What is the time constant of an automatic-volume-control filter circuit that uses a 1.25 -megohm resistor and a 0.25 microfarad capacitor?

Given:
$R=1.25 \mathrm{megohms}$ $C=0.25 \mu \mathrm{f}$

Find:
$t=$ ?

## Solution:

$$
t=C R=0.25 \times 10^{-6} \times 1.25 \times 10^{8}=0.3125 \mathrm{sec}
$$

The time required for the voltage to reach values other than 63.2 per cent of the final value follows an exponential curve. The universal timeconstant curves of Fig. 12-42 provide a simple means of finding the voltage at any instant of time.


Fig. 12-41.-Characteristics of current and voltage vs. time: (a) the circuit, (b) current vs. time characteristics with the switch in position 2, (c) voltage vs. time characteristics with the switch in position 2.

If the switch in the circuit of Fig. 12-41 $a$ is closed to position 1, the voltage and current characteristics of the circuit will conform to the voltage-time and current-time curves shown in Fig. 12-40. While a current is flowing in the circuit, energy is being stored in the capacitor. If the


Fig. 12-42.-Universal time-constant curves.
switch (Fig. 12-41a) is changed from position 1 to position 2, the energy stored in the capacitor will cause a current to flow in the resistor and the capacitor will discharge through the resistor. At the instant of closing the switch the current will be at its highest value (Ohm's law value) and will decrease exponentially, as shown in Fig. 12-41b. The voltage across the capacitor and resistor will be equal in amount and will also decrease exponentially with time, as is shown in Fig. 12-41c.

Table XII-I.-Ascending Curve

| k <br> time <br> constants | Per cent of <br> maximum <br> value | k <br> time <br> constants | Per cent of <br> maximum <br> value | k <br> time <br> constants | Per cent of <br> maximum <br> value |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 0.00 | 0.000 | 0.70 | 50.3 | 2.50 | 91.8 |
| 0.05 | 4.9 | 0.80 | 55.1 | 3.00 | 95.0 |
| 0.10 | 9.5 | 0.90 | 59.3 | 3.50 | 97.0 |
| 0.15 | 14.0 | 1.00 | 63.3 | 4.00 | 98.2 |
| 0.20 | 18.1 | 1.20 | 69.9 | 4.50 | 98.9 |
| 0.30 | 25.9 | 1.40 | 75.3 | 5.00 | 99.3 |
| 0.40 | 33.0 | 1.60 | 79.8 | 5.50 | 99.6 |
| 0.50 | 39.3 | 1.80 | 83.5 | 6.00 | 99.8 |
| 0.60 | 45.1 | 2.00 | 86.5 | 7.00 | 99.9 |

Universal Time-constant Curves. The time required for the current of an $R$ - $L$ circuit or the voltage across the capacitor of an $R-C$ circuit to reach values other than 63.2 per cent of their final values may be determined mathematically by use of suitable equations. The mathematics

Table XII-II.-Descending Curve

| k <br> time <br> constants | Per cent of <br> maximum <br> value | k <br> time <br> constants | Per cent of <br> maximum <br> value | k <br> time <br> constants | Per cent of <br> maximum <br> value |
| :---: | :---: | :---: | :---: | :---: | :---: |
|  | 100 | 0.70 | 49.7 | 2.50 | 8.2 |
| 0.00 | 95.1 | 0.80 | 44.9 | 3.00 | 5.0 |
| 0.10 | 90.5 | 0.90 | 40.7 | 3.50 | 3.0 |
| 0.15 | 86.0 | 1.00 | 36.8 | 4.00 | 1.8 |
| 0.20 | 81.9 | 1.20 | 30.1 | 4.50 | 1.1 |
| 0.30 | 74.1 | 1.40 | 24.7 | 5.00 | 0.7 |
| 0.40 | 67.0 | 1.60 | 20.2 | 5.50 | 0.4 |
| 0.50 | 60.7 | 1.80 | 16.5 | 6.00 | 0.2 |
| 0.60 | 54.9 | 2.00 | 13.5 | 7.00 | 0.1 |

involved is beyond the scope planned for this text. A shorter and more convenient method of determining the time required to attain any percentage of the final value is by use of time-constant curves. As all the current-time and voltage-time relations vary exponentially, it is possible to represent these variations by the two general exponential curves shown in Fig. 12-42. These curves are plotted from values obtained mathematically and listed in Tables XII-I and XII-II.

Example 12-13. An $R-L$ circuit used to control the action of a switch has a resistance of 12 ohms and an inductance of 0.5 henry and is connected to a six-volt battery. (a) If the switch operates when the current attains 63.2 per cent of its final value, what time is required to operate the switch? (b) If the switch requires 400 milliamperes to operate, what is the time between the start of current flow and the closing of the switch?

Given:

$$
\begin{aligned}
& R=12 \text { ohms } \\
& L=0.5 \text { henry }
\end{aligned}
$$

Find:
(a) $t=$ ?
(b) $t=$ ?

Solution:
(a) $t=\frac{L}{R}=\frac{0.5}{12}=0.0416 \mathrm{sec}$
(b) Maximum current value $=\frac{E}{R}=\frac{6}{12}=0.5 \mathrm{amp}$

Per cent of maximum current required to operate the switch $=\frac{400}{500} \times 100$
$=80$ per cent

From curve, Fig. 12-42, $k=1.6$
$t=k \frac{L}{R}=\frac{1.6 \times 0.5}{12}=0.0666 \mathrm{sec}$
Example 12-14. A 0.005-microfarad capacitor and a two-megohm resistor are connected to form an $R-C$ circuit. If the $R-C$ combination is connected to a 300 -volt source of d-c power, what time is required for the voltage across the capacitor to reach (a) 100 volts, (b) 200 volts, (c) 270 volts? If the capacitor becomes fully charged ( 300 volts) and is then discharged through the two-megohm resistor, what time is required to discharge the capacitor to (d) 250 volts, (e) 200 volts, ( $f$ ) 110 volts, ( $g$ ) 50 volts?

| Given: | Find: |
| :--- | ---: |
| $R=2$ megohms | $t=?$ |
| $C=0.005 \mu \mathrm{f}$ |  |

Solution:
(a)

Per cent of maximum value $=\frac{100}{300} \times 100=33.3$ per cent
$k$ (from Fig. 12-42) $=0.40$
$t=k C R=0.40 \times 0.005 \times 10^{-6} \times 2 \times 10^{6}=0.004 \mathrm{sec}$
(b)

Per cent of maximum value $=\frac{200}{300} \times 100=66.6$ per cent
$k$ (from Fig. 12-42) $=1.1$
$t=k C R=1.1 \times 0.005 \times 10^{-6} \times 2 \times 10^{6}=0.011 \mathrm{sec}$
(c)

Per cent of maximum value $=\frac{270}{300} \times 100=90$ per cent $k$ (from Fig. 12-42) $=2.27$
$t=k C R=2.27 \times 0.005 \times 10^{-6} \times 2 \times 10^{6}=0.0227 \mathrm{sec}$
(d)

Per cent of maximum value $=\frac{250}{300} \times 100=83.3$ per cent
$k$ (from Fig. 12-42) $=0.19$
$t=k C R=0.19 \times 0.005 \times 10^{-6} \times 2 \times 10^{6}=0.0019 \mathrm{sec}$
(e)

Per cent of maximum value $=\frac{200}{300} \times 100=66.6$ per cent
$k=($ from Fig. 12-42) $=0.40$
$t=k C R=0.40 \times 0.005 \times 10^{-6} \times 2 \times 10^{6}=0.004 \mathrm{sec}$
(j)

Per cent of maximum value $=\frac{110}{300} \times 100=36.6$ per cent
$k=($ from Fig. 12-42) $=1$
$t=k C R=1 \times 0.005 \times 10^{-6} \times 2 \times 10^{6}=0.01 \mathrm{sec}$
(g) Per cent of maximum value $=\frac{50}{300} \times 100=16.6$ per cent
$k=$ (from Fig. 12-42) $=1.8$
$t=k C R=1.8 \times 0.005 \times 10^{-6} \times 2 \times 10^{6}=0.018 \mathrm{sec}$
Uses of Delayed Action Circuits. There are numerous applications of $R-C$ and $R$ - $L$ circuits both in radio and in industrial electronics. A few of the applications are as follows.

A grid-leak detector circuit uses a capacitor and a resistor connected in parallel in the grid circuit of the detector tube. Actually this $R-C$ combination is a time-constant circuit, and the values of $R$ and $C$ are chosen to produce a time constant of sufficient duration so that the charge on the capacitor gained during the positive half cycles does not have time to completely discharge through the resistor during the negative half cycles. Other examples of time-constant circuits in radio apparatus include automatic volume control, relaxation oscillator, and trigger circuits.

Industrial applications of time-constant circuits include controlling the length of time for a specific manufacturing operation, timing of electric welders, timing the exposure of photofinishing processes, timing of instruments, producing repeated action for life tests, and motor control.

The electrical principles presented in this text provide the necessary background for the study of radio, television, and electronic circuits as presented in the authors' Essentials of Radio.

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## QUESTIONS

1. (a) What is meant by the stray effects of inductors, capacitors, and resistors?
(b) How is each of the circuit elements affected by the stray effects?
2. Why is it necessary to have filter circuits in a radio receiver?
3. (a) What are the essential parts of a filter circuit? (b) Explain the action of each part.
4. Explain the purpose of using each of the following units in a filter circuit: (a) capacitor, (b) inductor, (c) resistor, (d) series resonant circuit, (e) parallel resonant circuit.
5. (a) What is meant by a low-pass filter circuit? (b) Draw a simple circuit, and explain its action.
6. (a) What is meant by a high-pass filter circuit? (b). Draw a simple circuit, and explain its action.
7. (a) What is meant by a band-pass filter circuit? (b) Draw a simple circuit, and explain its action.
8. (a) What is meant by a band-stop filter circuit? (b) Draw a simple circuit, and explain its action.
9. Why is it necessary to use multisection filter circuits?
10. What is meant by (a) T-type filter? (b) $\pi$-type filter?
11. Draw a circuit diagram of one section of a T-type filter circuit for (a) low pass, (b) high pass, (c) band pass, (d) band stop.
12. Draw a circuit diagram of a two-section T-type filter circuit for (a) low pass, (b) high pass.
13. When two or more T-type low-pass filter circuits are joined, why should the connecting inductor have a value of twice the inductance of either of the end inductors?
14. When two or more 'T-type high-pass filter circuits are joined, why should the connecting capacitor have a value of one-half the capacitance of either of the end capacitors?
15. Draw a circuit diagram of one section of a $\pi$-type filter circuit for (a) low pass, (b) high pass, (c) band pass, (d) band stop.
16. Draw a circuit diagram of a two section $\pi$-type filter circuit for (a) low pass (b) high pass.
17. When two or more $\pi$-type low-pass filter circuits are joined, why should the connecting capacitor have a value of twice the capacitance of either end capacitor?
18. When two or more $\pi$-type high-pass filter circuits are joined, why should the connecting inductor have a value of one-half the inductance of either end inductor?
19. What is meant by the following terms: (a) Source impedance? (b) Load impedance? (c) Image impedance? (d) Characteristic impedance?
20. What is meant by (a) m-derived filter? (b) Shunt-derived? (c) Seriesderived?
21. Explain how a resistor and a capacitor combine to form a filter circuit.
22. Why is it necessary to couple circuits?
23. What is meant by the coupling element?
24. What is meant by (a) simple coupled circuit? (b) Complex coupled circuit?
25. Draw the circuit diagram and explain the action of four simple coupling circuits using a different coupling element for each.
26. Draw the circuit diagram and explain the action of three complex coupling circuits using a different coupling clement for each.
27. What is the general purpose for using complex circuits?
28. What is the important factor to remember about the coupling element?
29. Explain what is meant by (a) critical coupling, (b) loose coupling, (c) tight coupling.
30. Where is each term of Question 29 used?
31. What type of coupling is generally used to couple circuits in a radio receiver?
32. Explain what is meant by coupled impedance.
33. How does the amount of coupled impedance affect the primary impedance?
34. How does the coupled impedance vary with the amount of coupling?
35. Why is a $10-\mathrm{ke}$ band ordinarily used in band-pass circuits?
36. Where are band-pass amplifier circuits used?
37. Explain the relation between the width of band passed and the amount of coupling.
38. Explain the effect on secondary current of two resonant circuits coupled by means of a transformer with (a) critical coupling, (b) tight coupling, (c) loose coupling.
39. What are the important properties of a band-pass circuit?
40. How does the amount of circuit $Q$ affect the uniformity of response?
41. Name and explain three methods of obtaining a wide band pass.
42. What is the width of the band pass in (a) the i-f section of $\mathrm{f}-\mathrm{m}$ receivers? (b) The i-f sound section of television receivers? (c) The i-f video section of television receivers?
43. What is the frequency of operation of (a) the i-f section of f-m receivers? (b) The i-f sound section of television receivers? (c) The i-f video section of television receivers?
44. Name and describe the method gencrally used for obtaining a wide band pass in the i-f section of an f -m receiver.
45. Describe how the unsymmetrical wide band pass required by the i-f video section of a television receiver is obtained.
46. What is the essential purpose of time-delay circuits?
47. Explain the operation of a delayed-action (a) $R-L$ circuit, (b) $R-C$ circuit.
48. What is meant by the time constant of a delayed-action (a) R-L circuit, (b) $R-C$ circuit?
49. (a) What is the purpose of the universal time-constant curves? (b) How are they used in the solution of time-delay circuits?
50. (a) Name four applications of time-delay circuits. (b) Explain the circuit actions of one of these applications.

## PROBLEMS

1. A 2500 -ohm 50 -watt resistor has an inductance of $3 \mu \mathrm{~h}$. What is the impedance of this resistor at the broadcast frequencies of (a) 500 kc ? (b) 1500 kc ?
2. If the resistor of Prob. 1 is used in a short-wave receiver, what is its impedance at (a) 4 mc ? (b) 15 mc ?
3. If the resistor of Prob. 1 is used in an f -m receiver, what is its impedance at (a) 88 mc ? (b) 108 mc ?
4. If the resistor of Prob. 1 is used in a television receiver, what is its impedance at (a) 60 mc ? (b) 216 mc ?
5. An r-f plate choke used in a broadcast receiver has an inductance of 2.5 mh and a resistance of 70 ohms. What is its impedance at (a) 550 kc ? (b) 1450 kc ?
6. If the inductor of Prob. 5 is used in the i-f circuit of an f-m or television receiver, what is its impedance at (a) 10.7 mc ? (b) 26.4 mc ?
7. An 8 - $\mu \mathrm{f}$ capacitor has a resistance of 60 ohms. What is its impedance at (a) 60 cycles? (b) 120 cycles?
8. It is desired that a filter choke, having a resistance of 45 ohms, oppose the flow of a 60 -cycle current with 10 times the opposition that it offers to direct current. What is the inductance of the coil?
9. (a) To which type of current will a $0.04-\mu \mathrm{f}$ capacitor offer the greater opposition, a 4000 -cycle a-f signal or a 1500 -ke r-f signal? (b) How many times greater is the larger impedance than the smaller impedance? (c) Which type of current is blocked by this capacitor?
10. A $10-\mathrm{mh}$ choke and a $250-\mu \mu \mathrm{f}$ capacitor are connected, as shown in Fig. 12-7c, to form a low-pass filter circuit. What opposition is offered by the capacitor (a) to the highest frequency audio signal usually obtained in a radio receiver (5000
cycles)? (b) To the lowest frequency carrier wave usually obtained in a radio receiver ( 500 kc )? What opposition is offered by the inductor to (c) A 5000 -cycle signal? (d) A 500 -ke signal?
11. A 4 -henry choke coil and a $0.4-\mu \mathrm{f}$ capacitor are connected as shown in Fig. $12-9 c$ to form a high-pass filter circuit. What opposition is offered by the capacitor (a) to 60 cycles (power disturbances)? (b) To a 1200 -cycle a-f signal? What opposition is offered by the inductor (c) to 60 cycles? (d) To 1200 cycles?
12. A resistor and a capacitor are connected as shown in Fig. 12-22. The resistor has a value of 5000 ohms , and the capacitor has a value of $0.5 \mu \mathrm{f}$. (a) What impedance does this capacitor offer to a 5000 -cycle current? (b) Which path will the 5000 -cycle current take?
13. A circuit similar to the one shown in Fig. 12-22a is to be used in the r-f stage of a receiver. It is desired that the capacitor offer an impedance of 100 times that of the resistor whose value is 6500 ohms . What size capacitor is required if the signal is 1500 ke ?
14. A circuit similar to the one shown in Fig. 12-22b has the following values: the resistor has 7200 ohms, and the capacitor has a value of $20 \mu \mathrm{f}$. (a) What impedance does the capacitor offer to a 5000 -cycle current? (b) Will the a-f signal flow through the resistor or the capacitor?
15. What value of inductance is required in a $27.6-\mathrm{mc}$ trap of a television video i-f circuit if the capacitor used has a value of $12 \mu \mu$ ?
16. What value of inductance is required in a $20.6-\mathrm{mc}$ trap of a television video i-f circuit if the capacitor used has a value of $24 \mu \mu \mathrm{f}$ ?
17. What is the approximate width of the frequency band passed by a band-pass amplifier circuit having a resonant frequency of 260 kc and a coefficient of coupling of 0.038 ?
18. What is the approximate width of the frequency band passed by a band-pass amplifier circuit having a resonant frequency of 465 kc and a coefficient of coupling of 0.02 ?
19. What coefficient of coupling is required to produce a $150-\mathrm{kc}$ band pass at an operating frequency of (a) 8.3 mc ? (b) 9 mc ?
20. What coefficient of coupling is required to produce a $100-\mathrm{kc}$ band pass at an operating frequency of (a) 21.5 mc ? (b) 26.25 mc ?
21. It is desired that a band-pass amplifier circuit having a resonant frequency of 460 kc pass a band of frequencies 8 kc wide. What value of coefficient of coupling is required?
22. It is necessary that the critical coupling of an inductance-coupled band-pass circuit be equal to 0.008 . If the circuit $Q$ of the primary and secondary circuit are equal to each other, what is their value?
23. What is the resonant frequency of the circuit in Prob. 22 if the band passed is to be 10 kc wide?
24. A band-pass filter to be used in the i-f amplifier circuit of an a-m broadcast radio receiver must pass a band 10 kc wide centering about a frequency of 465 kc . (a) What is the coefficient of coupling? (b) What is the circuit $Q$ ?
25. What is the capacitance of the tuning capacitors used.in Prob. 24 if the primary and secondary inductances are both 2 mh ?
26. The tuning capacitors used in the band-pass circuit of Prob. 17 have a capacitance of $35 \mu \mu \mathrm{f}$. (a) What is the value of the inductance of the primary and secondary circuits? (b) What is their value of $Q$ ?
27. What would be the value of the induct,ance in Prob. 17 if the tuning capacitors have a capacitance of $70 \mu \mu$ ?
28. It is desired that a band-pass circuit used in the i-f amplifier of an a-m broadcast radio receiver pass a band 10 ke wide. The circuit $Q$ is equal to 68.5 . (a) What value of coefficient of coupling is required? (b) What are the extreme limits of the frequency band passed?
29. The primary and secondary inductances in Prob. 28 are both 4 mh . What is the capacitance of the tuning capacitors?
30. It is desired that a band-pass circuit used in the i-f amplifier of an a-m broadcast radio receiver pass a band 10 kc wide. The $Q$ of the circuit is equal to 40. (a) What value of coefficient of coupling is required? (b) What are the extreme limits of the frequency band passed? (c) What is the capacitance of the tuning capacitors if the inductance of the primary and secondary windings are 6 mh each?
31. A low-current d-c relay that has an inductance of 25 henries is connected in series with a $1000-\mathrm{ohm}$ resistor to form an $R-L$ time-delay control circuit operated on a 110 -volt d-c circuit. (a) What is the time constant of the circuit? (b) If the relay closes when the current is 88 ma , what time elapses between closing the line switch and operation of the relay?
32. A low-current d-c relay having an inductance of 10 henries is to close 0.02 sec after the line switch is closed. What value of resistor should be connected in series with the relay if it closes when the current reaches (a) 63.2 per cent of its final value? (b) 80 per cent of its final value?
33. An a-m broadcast band receiver is to have an $R-C$ circuit with a time constant of 0.2 sec for its ave circuit. (a) What value resistor is required if a $0.1-\mu \mathrm{f}$ capacitor is used? (b) What value resistor is required if a $0.15-\mu f$ capacitor is used? (c) What value capacitor is required if a 1 -megohm resistor is used?
34. A grid-leak detector circuit contains a $250-\mu \mu \mathrm{f}$ capacitor, shunted by a 1-megohm resistor. (a) What is the time constant of this circuit? (b) If the highest a-f signal to be applied to the circuit is 5000 cycles, what is the time required to complete one of these cycles? (c) Under the conditions of (a) and (b) will the capacitor ever become completely discharged? (d) Explain answer to (c)?
35. A $0.05-\mu \mathrm{f}$ capacitor and a 0.5 -megohm resistor are connected to form an $R$ - $C$ circuit. The $R-C$ combination is connected to a 250 -volt d-c source. (a) What time is required for the voltage across the capacitor to reach 50 volts, 100 volts, 200 volts? (b) What current flows when the switch is closed? (c) What current flows when the voltage across the capacitor reaches 200 volts? (d) If the capacitor is fully charged and is then discharged through the 0.5 -megohm resistor, what is the current at the instant it starts to discharge? (e) What is the value of the voltage $R C$ seconds after the capacitor starts to discharge? ( $f$ ) At what time will the capacitor be discharged to half voltage?
36. What is the time constant of an $R$ - $C$ circuit in the sweep section of a television receiver if the value of the resistor is 2.2 megohms and the value of the capacitor is $0.05 \mu$ ?
37. What is the time constant of an $R-C$ circuit in the vertical deflection circuit of a television receiver if the value of the resistor is $470,000 \mathrm{ohms}$ and the value of the capacitor is $0.1 \mu \mathrm{f}$ ?
38. What is the time constant of an $R-C$ circuit in the horizontal deflection circuit of a television receiver if the value of the resistor is 680,000 ohms and the value of the capacitor is $820 \mu \mu \mathrm{f}$ ?

## APPENDIX I

## DRAWING SYMBOLS USED IN ELECTRONICS



Coil or loop antenna.


Doublet antenna.



Counterpoise.
(Body of car forms a counterpoise.)


Battery cell.
(Positive terminal indicated by a long line.)



Milliammeter.


Voltmeter.


Frequency meter or wave meter.


Conductors joined.


Binding post.


Line plug.


Fuse.


Key.


Closed circuit jack.


Open circuit jack.



Single-pole single-throw switch.


Single-pole double-throw switch.


Double-pole single-throw switch.


Double-pole double-throw switch


Double-pole double-throw reversing switch.


Single-deck circuit-selector switch.


Multiple-deck circuit-selector awitch


Fixed resistor.


Adjustable resistor.


Tapped resistor.


Potentiometer.



Push-pull transformer.


Power transformer.


Variable-core transformer.


Relay; circuit A open when deenergized.


Relay; circuits $B_{1}$ and $B_{2}$ closed when deenergized.


Fixed capacitor.
(Paper, mica, or oil.)


Electrolytic capacitor.


Variable capacitor.


Adjustable capacitor. (Trimmer.)


Adjustable capacitor.
(Padder.)


Split-stator variable capacitor.


Twin coaxial cable.


Shielded cable.


Shielded capacitor.



Permanent-magnet dynamic speaker.


Magnetic phonograph-pickup.



Single-button carbon microphone.


Double-button carbon microphone.


Crystal microphone.


Velocity microphone. (Ribbon.)


Dynamic microphone. (Moving coil.)


Crystal.


Contact rectifier, half-wave.



Contact rectifier, full-wave.




Neon lamp.


Photo tube.
 (Gaseous.)


Voltage regulator, cold cathode. (Gaseous.)


Half-wave rectifier, directly heated cathode.


Full-wave rectifier, directly heated cathode.


Half-wave rectifier, indirectly heated cathode.


Full-wave rectifier, indirectly heated cathode.


Triode, directly heated cathode.


Triode, indirectly heated cathode.


Tetrode, directly heated cathode.


Tetrode, indirectly heated cathode.


Pentode, indirectly heated cathode.


Beam-power amplifier.


## APPENDIX II

SYMBOLS AND ABBREVIATIONS USED IN ELECTRONICS

| Term | Symbol | Abbreviation |
| :---: | :---: | :---: |
| Ampere (also see Current). | I | a or amp |
| Milliampere |  | ma |
| Microampere. |  | $\mu \mathrm{a}$ |
| Ampere-turn | $N I$ | A-T |
| American Wire Gauge. |  | AWG |
| Amplitude-modulated |  | a-m |
| Angle........ | $\angle$ |  |
| Phase angle. | $\theta$ (theta) |  |
| Antenna. |  | ant |
| Apparent power (also see Volt-amperes) | $V A$ | A-P |
| Area |  | A |
| Circular mils. |  | cm or cir mils |
| Square centimeters. |  | sq cm |
| Square inches. |  | sq in |
| Capacitance. |  | C |
| Capacitor. |  | C |
| Conductance | G |  |
| Constant, dielectric. | K |  |
| Cosine. |  | $\cos$ |
| Coulomb. | $q$ |  |
| Coupling, coefficient. | $K$ |  |
| Current... | $I$ |  |
| Alternating |  | a-c or A-C |
| Average value. | $I_{\text {ave }}$ |  |
| Direct. |  | d-c or D-C |
| Effective value. | $I$ |  |
| Instantaneous value. | $i_{\theta}$ |  |
| Maximum value | $I_{\text {max }}$ |  |
| Signal. | $i$ |  |
| Cycles.... | $\sim$ |  |
| Per second (also see Frequency). | $\sim$ | c/sec |
| Kilocycle. ....... |  | ke |
| Megacycle. . |  |  |
| Decibel. | db | db |
| Density, flux. |  |  |
| Diameter.. | $d$ | d or diam |
| Distance. | $d$ |  |
| Efficiency |  | eff |


| Term | Symbol | Abbreviation |
| :---: | :---: | :---: |
| Electromotive force | E | emf or $E M F$ |
| Electrostatic unit. |  | esu |
| Energy | W | en |
| Farad. |  | f |
| Microfarad. |  | $\mu \mathrm{f}$ |
| Micromicrofarad. |  | $\mu \mu \mathrm{f}$ |
| Flux, magnetic | $\phi$ |  |
| Density. | $B$ |  |
| Force...... | $F$ |  |
| Magnetomotive |  | mmf |
| Frequency.. | $f$ |  |
| Audio. |  | a-f |
| Intermediate. |  | i-f |
| Modulation. |  | f-m |
| Radio.. |  | r-f |
| Resonance. | $f_{r}$ |  |
| Ultrahigh |  | uhf |
| Gausses (magnetic induction). | $B$ |  |
| Gilbert (unit of magnetomotive force) | $F$ |  |
| Ground... |  | gnd |
| Henry (unit of inductance). | $L$ | h |
| Millihenry.. |  | mh |
| Microhenry. |  | $\mu \mathrm{h}$ |
| Impedance... | $Z$ |  |
| Inductance, self-. | $L$ |  |
| Mutual. | M |  |
| Intensity, magnetic field. | H |  |
| Kilo. |  | k |
| Length... | $l$ |  |
| Maxwell (one magnetic line). | $\phi$ |  |
| Megohm...... | Ms2 |  |
| Meter (measure of length). |  | m |
| Centimeter. |  | cm |
| Millimeter. |  | mm |
| Oersted (unit of magnetic intensity).. | H |  |
| Ohm.... | $\Omega$ or $\omega$ |  |
| Permeability. | $\mu$ |  |
| Permeance. | $\rho$ |  |
| Pi.... | $\pi$ (3.1416) |  |
| Pole, North seeking. | $N$ |  |
| South seeking | $S$ |  |
| Power. |  | p or $P$ |
| Power factor. |  | P-F |
| Primary. | $P$ | p or pri |
| Reactance. | $X$ |  |
| Inductive. | $X_{L}$ |  |
| Capacitive. | $X_{C}$ |  |


| Term | Symbol | Abbreviation |
| :---: | :---: | :---: |
| Reluctance. | a |  |
| Resistance | $R$ or $r$ | resis |
| Root mean square. |  | rms |
| Secondary.. | $S$ | sec |
| Sine. |  | $\sin$ |
| Switch. |  | sw |
| Single-pole single-throw. |  | spst |
| Single-pole double-ihrow. |  | spdt |
| Double-pole single-throw. |  | dpst |
| Double-pole double-throw. |  | dpdt |
| Three deck, four circuit, eigh |  | 3D-4C-8P |
| Temperature, coefficient. | TC |  |
| Degrees centigrade... | ${ }^{\circ} \mathrm{C}$ |  |
| Degrees Fahrenheit. | ${ }^{\circ} \mathrm{F}$ |  |
| Thickness. | t |  |
| Time. | $t$ or $T$ |  |
| Turns, number of. | $N$ |  |
| Volt. | $E$ | v |
| Kilovolt. |  | kv |
| Millivolt. |  | mv |
| Microvolt. |  | $\mu \mathrm{V}$ |
| Voltage. | $E$ |  |
| Average value. | $E_{\text {ave }}$ |  |
| Effective value. | E |  |
| Instantaneous value. | $e_{\theta}$ |  |
| Maximum value. | $E_{\text {max }}$ |  |
| Signal. | $e$ |  |
| Volt-ampere. | VA | $\mathrm{V}-\mathrm{a}$ or V-A |
| Kilovolt-ampere. | KVA | kva |
| Watt.. | $W$ or $P$ | w |
| Kilowatt. |  | kw |
| Kilowatt-hours | ..... | kwhr or kwh |
| Milliwatt. |  | mw |
| Microwatt. |  | $\mu \pi$ |
| Wave, continuous. |  | c-w |
| Wavelength. | $\lambda$ |  |
| Wire, single-cotton-covered. |  | sce |
| Double-cotton-covered. |  | dec |
| Single-silk-covered. |  | ssc |
| Double-silk-covered. |  | dse |
| Enamel, single-cotton-covered |  | esce |
| Enamel, double-silk-covered. |  | edse |

## APPENDIX III

## FORMULAS COMMONLY USED IN ELECTRONICS

Note: The numbers appearing opposite the equations correspond to the numbers of the same equations in the text or to the equations from which they were derived. These numbers are included to facilitate reference to figures, text, and nomenclature when such reference is desirable.

## DIRECT CURRENT

## Ohm's Law

$$
\begin{array}{ll}
\text { Voltage } & =I R=\frac{P}{I}=\sqrt{R P} \\
\text { Current } & =\frac{E}{R}=\frac{P}{E}=\sqrt{\frac{P}{R}} \\
\text { Resistance } & =\frac{E}{I}=\frac{P}{I^{2}}=\frac{E^{2}}{P} \\
\text { Power } & =E I=I^{2} R=\frac{E^{2}}{R} \\
\text { Energy } & =P T ; \quad \text { power }=\frac{W}{T} ; \quad \text { time }=\frac{W}{P} \tag{2-15}
\end{array}
$$

## Series Circuit



$$
\begin{align*}
R_{T} & =r_{1}+r_{2}+r_{3}, \text { etc. }  \tag{4-9}\\
r_{1} & =R_{T}-\left(r_{2}+r_{3}, \text { etc. }\right)  \tag{4-9}\\
E_{T} & =e_{1}+e_{2}+e_{3}, \text { etc. }  \tag{4-8}\\
e_{2} & =E_{T}-\left(e_{1}+e_{3}, \text { etc. }\right)  \tag{4-8}\\
I_{T^{\prime}} & =i_{1}=i_{2}=i_{3}, \text { etc. }  \tag{4-7}\\
P_{T} & =p_{1}+p_{2}+p_{3}, \text { etc. } \tag{4-10}
\end{align*}
$$

## Parallel Circuits

Two resistors in parallel


Fig. A-2.

$$
\begin{align*}
R_{T} & =\frac{1}{\frac{1}{r_{1}}+\frac{1}{r_{2}}}=\frac{r_{1} r_{2}}{r_{1}+r_{2}}  \tag{4-14a}\\
r_{2} & =\frac{R_{T} r_{1}}{r_{1}-R_{T}} \tag{4-14b}
\end{align*}
$$

Any number of resistors in parallel


Fig. A-3.

$$
\begin{align*}
R_{T} & =\frac{1}{\frac{1}{r_{1}}+\frac{1}{r_{2}}+\frac{1}{r_{3}}, \text { etc. }}  \tag{4-14}\\
E_{T} & =e_{1}=e_{2}=e_{3}, \text { etc. }  \tag{4-13}\\
I_{T} & =i_{1}+i_{2}+i_{3}, \text { etc. }  \tag{4-12}\\
i_{3} & =I_{T}-\left(i_{1}+i_{2}, \text { ett. }\right)  \tag{4-12}\\
P_{T} & =p_{1}+p_{2}+p_{3}, \text { etc. } \tag{4-10}
\end{align*}
$$

ALTERNATING CURRENT

## Ohm's Law

$$
\begin{align*}
\text { Voltage } & =I Z=\frac{P}{I \mathrm{P}-\mathrm{F}}  \tag{8-9}\\
\text { Current } & =\frac{E}{Z}=\frac{P}{E \mathrm{P}-\mathrm{F}}  \tag{8-9}\\
\text { Impedance } & =\frac{E}{I}=\frac{R}{\mathrm{P}-\mathrm{F}}=\sqrt{R^{2}+\left(X_{L}-X_{C}\right)^{2}} \\
\text { Power } & =E I \mathrm{P}-\mathrm{F}=I^{2} R \\
\text { Power factor } & =\frac{P}{E I}=\frac{R}{Z}=\cos \theta \\
\text { Energy } & =P T
\end{align*}
$$



Resistance and inductive reactance


Resistance, inductive reactance and capacitive reactance


Any combination of resistances, inductive reactances and capacitive reactances
Fig. A-4.

## Series Circuit

| Series circuit | Resistance and inductance (Fig. A-4a) | Resistance and capacitance <br> (Fig. A-4b) | Resistance, inductance, and capacitance (Fig. A-4c) |
| :---: | :---: | :---: | :---: |
| $Z=$ | $\sqrt{\overline{R^{2}+X_{L}{ }^{2}}}$ | $\sqrt{\overline{R^{2}+X} \overline{c^{2}}}$ | $\sqrt{R^{2}+\left(X^{2}{ }^{2}-X C\right)^{2}}$ |
| $R=$ | $\sqrt{Z^{2}-X L^{2}}$ | $\sqrt{Z^{2}-X c^{2}}$ | $\sqrt{Z^{2}-\left(X_{L}-X_{C}\right)^{2}}$ |
| $X_{L}=$ | $\sqrt{Z^{2}-R^{2}}$ | ........... | $\sqrt{Z^{2}-R^{2}+X_{c}}$ |
| $X c=$ |  | $\sqrt{Z^{2}-R^{2}}$ | $X_{L}-\sqrt{Z^{2}-R^{2}}$ |

For any combination of resistance, inductance, and capacitance (Fig. A-4d)

$$
\begin{align*}
Z & =\sqrt{\left(R_{1}+R_{2}+R_{3}, \text { etc. }\right)^{2}+\left(X_{L_{1}}+X_{L_{2}}+X_{L_{3}},\right. \text { etc. }} \\
& \left.-X_{C_{1}}-X_{C_{2}}-X_{C_{3}}, \text { etc. }\right)^{2}  \tag{10-2}\\
E_{T} & =e_{1}+e_{2}+e_{3}+e_{4}, \text { etc. (to be added vectorially) } \\
I_{T} & =i_{1}=i_{2}=i_{3}=i_{4}, \text { etc. } \\
P_{T} & =p_{1}+p_{2}+p_{3}+p_{4}, \text { etc. }
\end{align*}
$$

## Parallel Circuits

Two impedances in parallel


Fig. A-5.

$$
\begin{align*}
Z_{T} & =\frac{Z_{1} Z_{2}}{Z_{1}+Z_{2}}  \tag{11-28}\\
Z_{1} & =\frac{Z_{T} Z_{2}}{Z_{2}-Z_{T}} \tag{11-28}
\end{align*}
$$

Any number of impedances in parallel
$Z_{T}$ : No single equation is available for this type circuit. For solution see Art. 10-9. $E_{T}=e_{1}=e_{2}=e_{3}$, etc.
$I_{T}=i_{1}+i_{2}+i_{3}$, etc. (to be added vectorially)
$P_{T}=p_{\mathbf{1}}+p_{2}+p_{3}$, etc.
INDUCTORS

## Single-layer Coil

$$
\begin{align*}
L & =\frac{(a N)^{2}}{9 a+10 b}  \tag{8-5}\\
X_{L} & =2 \pi f L \quad L=\frac{X_{L}}{2 \pi f}  \tag{8-6}\\
Z & =\sqrt{R^{2}+X_{L}{ }^{2}}  \tag{8-8}\\
Q & =\frac{X_{L}}{R}=\frac{2 \pi f L}{R} \tag{11-11}
\end{align*}
$$


Fig. A-6

## Inductors in Series

$$
\begin{align*}
L_{T} & =L_{1}+L_{2}+L_{3} \text {, etc. (no flux linkage between coils) }  \tag{8-15}\\
X_{L_{T}} & =X_{L_{1}}+X_{L_{2}}+X_{L_{3}} \text {, etc. (no flux linkage between coils) } \\
L_{T} & =L_{1}+L_{2} \pm 2 K \sqrt{L_{1} L_{2}} \text { (flux linking the coils) } \tag{8-19}
\end{align*}
$$

## Inductors in Parallel

$$
\begin{align*}
L_{T} & =\frac{1}{\frac{1}{L_{1}}+\frac{1}{L_{2}}+\frac{1}{L_{3}}, \text { etc. }}  \tag{8-16}\\
X_{L_{T}} & =\frac{1}{\frac{1}{X_{L_{1}}}+\frac{1}{X_{L_{2}}}+\frac{1}{X_{L_{3}}}}, \text { etc. }=2 \pi f L_{T}
\end{align*}
$$

## Mutual Inductance

$$
\begin{equation*}
M=K \sqrt{L_{1} L_{2}} ; \quad L_{1}=\frac{M^{2}}{K^{2} L_{2}} \tag{8-13}
\end{equation*}
$$

## Coefficient of Coupling

$$
\begin{gather*}
K=\frac{M}{\sqrt{L_{1} L_{2}}}  \tag{8-13}\\
\text { CAPACITORS } \\
C=\frac{22.45 A K(N-1)}{10^{8} t}  \tag{9-2}\\
X_{C}=\frac{10^{6}}{2 \pi f C}=\frac{159,000}{f C} \quad \text { (when } C \text { is in microfarads) }  \tag{9-4}\\
Z=\sqrt{R^{2}+X_{C^{2}}}  \tag{9-7}\\
\text { P-F }=\frac{R}{Z}=\frac{R}{\sqrt{R^{2}+X_{C^{2}}}} \tag{9.8}
\end{gather*}
$$

## Capacitors in Series

Two capacitors

$$
\begin{align*}
C_{T} & =\frac{C_{1} C_{2}}{C_{1}+C_{2}}  \tag{9-9}\\
C_{1} & =\frac{C_{T} C_{2}}{C_{2}-C_{T}} \tag{9-9}
\end{align*}
$$

Any number of capacitors

$$
\begin{align*}
C_{T} & =\frac{1}{\frac{1}{C_{1}}+\frac{1}{C_{2}}+\frac{1}{C_{3}}, \text { etc. }}  \tag{9-9}\\
X_{C_{T}} & =X_{C_{1}}+X_{C_{2}}+X_{C_{8}}, \text { etc. }=\frac{10^{6}}{2 \pi f C_{T}}
\end{align*}
$$

## Capacitors in Parallel

$$
\begin{align*}
& C_{T}= C_{1}+C_{2}+C_{3}, \text { etc. }  \tag{9-10}\\
& X_{C_{T}}= \frac{1}{\frac{1}{X_{C_{1}}}+\frac{1}{X_{C_{2}}}+\frac{1}{X_{C_{3}}}}=\frac{10^{6}}{2 \pi f C_{T}} \\
& \text { RESONANCE }
\end{align*}
$$

## Series and Parallel Circuits

$$
\left.\begin{array}{rl}
f_{r} & =\frac{159}{\sqrt{L C}}  \tag{11-7}\\
L & =\frac{25,300}{f_{r}^{2} C} \\
C & =\frac{25,300}{f_{r}^{2} L}
\end{array}\right\} \begin{aligned}
& f_{r} \text { is expressed in kilocycles } \\
& L \text { is expressed in microhenries } \\
& C \text { is expressed in microfarads }
\end{aligned}
$$

## Series Resonant Circuit



At resonance

$$
\begin{align*}
Z & =R \text { (minimum value possible) }  \tag{11-10}\\
I & =\frac{E}{R} \text { (maximum value possible) }  \tag{11-12}\\
E_{X_{L}} & =E_{X_{C}}=E Q \tag{11-26}
\end{align*}
$$

## Parallel Resonant Circuit



Fig. A-8.

$$
\begin{equation*}
Z=\frac{Z_{1} Z_{2}}{Z_{1}+Z_{2}} \tag{11-28}
\end{equation*}
$$

At resonance

$$
\begin{align*}
Z & =Z_{L} Q \text { (maximum value obtainable) }  \tag{11-33}\\
I & =\frac{E}{Z} \text { (minimum value obtainable) } \tag{11-29}
\end{align*}
$$

Width of Frequency Band for Single Resonant Circuit at 0.707 of the Maximum Re sponse

$$
\begin{equation*}
f_{2}-f_{1}=\frac{f_{r}}{Q}=\frac{R}{2 \pi L} \tag{11-21}
\end{equation*}
$$

## COUPLED RESONANT CIRCUITS

Width of Band Pass

$$
\begin{equation*}
f_{2}-f_{1}=K f_{r} \tag{12-15}
\end{equation*}
$$

## Critical Coupling

$$
\begin{align*}
K_{c} & =\frac{1.5}{\sqrt{Q_{p} Q_{t}}}  \tag{12-17}\\
Q_{p} Q_{\mathrm{e}} & =\frac{2.25}{K_{\mathrm{c}}{ }^{2}} \tag{12-18}
\end{align*}
$$

RELATION BETWEEN WAVELENGTH AND FREQUENCY

$$
\left.\begin{array}{l}
\lambda=\frac{300,000}{f} \\
f=\frac{300,000}{\lambda}
\end{array}\right\} \begin{aligned}
& f \text { is expressed in kilocycles }  \tag{1-4}\\
& \lambda \text { is expressed in meters }
\end{aligned}
$$

DELAYED-ACTION CIRCUITS

## Time Constants

Resistance-inductance circuit

$$
\begin{equation*}
t=\frac{L}{R} \tag{8-10}
\end{equation*}
$$

Resistance-capacitance circuit

$$
\begin{equation*}
t=C R \tag{12-19}
\end{equation*}
$$

## APPENDIX IV

TABLE OF SPECIFIC RESISTANCE AND TEMPERATURE COEFFICIENT OF VARIOUS METALS AT $20^{\circ} \mathrm{C}$

| Material | Description | Specific resistance, ohms per cir-mil-ft | Temperature coefficient, per ${ }^{\circ} \mathrm{C}$ |
| :---: | :---: | :---: | :---: |
| Advance. | Copper, nickel | 295 | 0.000018 |
| Aluminum. | Wire | 17 | 0.00388 |
| Bismuth. |  | 700 | 0.00435 |
| Brass. | Copper, zinc | 40 | 0.002 |
| Cadmium |  | 46 | 0.0038 |
| Carbon. | Graphite | 2600-7500 | -0.0003 |
| Climax. |  | 530 | 0.0007 |
| Constantan. | Same as advance | 295 | 0.000018 |
| Copper. | Ifard-drawn | 10.4 | 0.004 |
| Excello. |  | 560 | 0.00016 |
| German silver. | Nickel, copper, zinc | 200 | 0.00038 |
| Gold. |  | 14.5 | 0.00342 |
| Iala. | Copper, nickel | 295 | 0.000005 |
| Iron. | Cast | 450-570 | 0.006 |
| Lead. |  | 130 | 0.0038 |
| Manganin. | Manganese, copper, nickel | 265 | 0.000006 |
| Mercury . |  | 565 | 0.00072 |
| Monel. | Copper, nickel | 265 | 0.00198 |
| Nichrome. | Nickel, chromium | 600-660 | 0.0004 |
| Nickel. |  | 46 | 0.0062 |
| Phosphor bron |  | 47 | 0.0012 |
| Platinum | Pure | 60 | 0.00367 |
| Silver. |  | 9.8 | 0.00377 |
| Steel. | Soft | 95.4 | 0.005 |
| Steel. | Hard | 275 | 0.0016 |
| Tin. |  | 69 | 0.00425 |
| Tungsten. |  | 34 | 0.0046 |
| Zinc. |  | 35.2 | 0.00372 |

## APPENDIX V

BARE COPPER WIRE TABLES AT 25 DEGREES CENTIGRADE, 77 DEGREES FAHRENHEIT

| A.W.G. and Brown and Sharpe gauge | Diameter, mils | Area, circular mils | Ohms per 1000 ft |
| :---: | :---: | :---: | :---: |
| 1 | 289.3 | 83,690 | 0.1264 |
| 2 | 257.6 | 66,370 | 0.1593 |
| 3 | 229.4 | 52,640 | 0.2009 |
| 4 | 204.3 | 41,740 | 0.2533 |
| 5 | 181.9 | 33,100 | 0.3195 |
| 6 | 162.0 | 26,250 | 0.4028 |
| 7 | 144.3 | 20,820 | 0.5080 |
| 8 | 128.5 | 16,510 | 0.6405 |
| 9 | 114.4 | 13,090 | 0.8077 |
| 10 | 101.9 | 10,380 | 1.018 |
| 11 | 90.74 | 8,234 | 1.284 |
| 12 | 80.81 | 6,530 | 1.619 |
| 13 | 71.96 | 5,178 | 2.042 |
| 14 | 64.08 | 4,107 | 2.575 |
| 15 | 57.07 | 3,257 | 3.247 |
| 16 | 50.82 | 2,583 | 4.094 |
| 17 | 45.26 | -2,048 | 5.163 |
| 18 | 40.30 | 1,624 | 6.510 |
| 19 | 35.89 | 1,288 | 8.210 |
| 20 | 31.96 | 1,022 | 10.35 |
| 21 | 28.46 | 810.1 | 13.05 |
| 22 | 25.35 | 642.4 | 16.46 |
| 23 | 22.57 | 509.5 | 20.76 |
| 24 | 20.10 | 404.0 | 26.17 |
| 25 | 17.90 | 320.4 | 33.00 |
| 26 | 15.94 | 254.1 | 41.62 |
| 27 | 14.20 | 201.5 | 52.48 |
| 28 | 12.64 | 159.8 | 66.17 |
| 29 | 11.26 | 126.7 | 83.44 |
| 30 | 10.03 | 100.5 | 105.2 |
| 31 | 8.93 | 79.70 | 132.7 |
| 32 | 7.95 | 63.21 | 167.3 |
| 33 | 7.08 | 50.13 | 211.0 |
| 34 | 6.31 | 39.75 | 266.0 |
| 35 | 5.62 | 31.52 | 335.5 |
| 36 |  | 25.00 | 423.0 |
| 37 | 4.45 | 19.83 | 533.4 |
| 38 | 3.96 | 15.72 | 672.6 |
| 39 | 3.53 | 12.47 | 848.1 |
| 40 | 3.14 | 9.89 | 1069.0 |
| 41 | 2.80 | 7.84 | 1323 |
| 42 | 2.50 | 6.22 | 1667 |
| 43 | 2.22 | 4.93 | 2105 |
| 44 | 1.98 | 3.91 | 2655 |
| 45 | 1.75 | 3.06 | 3460 |

## APPENTDIX VI

## DIELECTRIC CONSTANT ( $K$ ) AND DIELECTRIC STRENGTH (VOLTS PER 0.001 IN.) OF VARIOUS MATERIALS

| Material | Dielectric constant, $K$ | Dielectric strength, volts per 0.001 in . |
| :---: | :---: | :---: |
| Air. | 1 | 80 |
| Aluminum oxide layer. | 10 |  |
| Bakelite. | 6 | 500 |
| Cambric, varnished. | 4.5 | 1200 |
| Ceramics. | 5-4000 |  |
| Cordierite. | 5-5.5 |  |
| Mycalex. | 6-8 |  |
| Steatite. | 6.1 |  |
| Titanium dioxide. | 90-170 |  |
| Cotton. | .... | 300 |
| Fiber. | 6.5 | 50 |
| Glass, common. | 4.2 | 200 |
| Isolantite.. | 3.5 |  |
| Mica. | 5.5 | 2000 |
| Oil, castor | 4.7 | 380 |
| Pyranol. | 4.2 | 350 |
| Transformer. | 2.4 | 250 |
| Paper, beeswaxed | 3.1 | 1800 |
| Paraffined. | 2.2 | 1200 |
| Shellacked. | 3.4 |  |
| Porcelain. | 5.5 | 750 |
| Quartz | 4.5 |  |
| Resin. | 2.5 |  |
| Styrene (polymerized) | 2.4-2.9 |  |
| Tantalum oxide layer. | 11.5 |  |
| Water, pure. | 81 |  |

Note: The values given in the above table may vary considerably depending upon the quality and manufacture of the material. The values in the table are average values; for greater accuracy, values should be obtained from the manufacturer of the materials used.

## APPENDIX VII

## STANDARD COLOR CODING FOR RESISTORS

There are two methods of placing the color identification on a resistor. In the first method, illustrated by Fig. A-9, the body color $A$ represents the first figure of the resistance value; one end or tip $B$ is colored to represent the second figure; a colored band or dot $(C)$ near the center of the resistor represents the number of zeros following the first two figures. By this system, a 150,000 -ohm resistor would be colored as follows:

Body, brown Tip, green Dot or band, yellow
In the second method, illustrated by Fig. A-10, the colors are indicated by a series of bands or dots generally placed at one end of the resistor. In order to obtain the value of a resistor, with this method, the colors are read starting from the end or tip and going toward the center. With this system, a $750,000-\mathrm{ohm}$ resistor would be colored as follows:

Band $A$, violet Band $B$, green Band $C$, yellow
An auxiliary color code has been established, covering the tolerances of resistors. The tolerances are indicated by the following colors, which appear as a fourth band or dot placed on one end of the resistor:

$$
\text { Gold, } 5 \% \quad \text { Silver, } 10 \% \quad \text { None, } 20 \%
$$

[^0]| 0-Black | 5-Green |
| :--- | :--- |
| 1-Brown | 6-Blue |
| 2-Red | 7-Violet |
| 3-Orange | 8-Gray |
| 4-Yellow | 9-White |

Three colors are used on each resistor to identify its value.


Fig. A-9.
Fig. A-10.
Application of Color Code

| Resistance, ohms | A | $B$ | C | Resistance, ohms | A | B | C |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 50 | Green | Black | Black | 25,000 | Red | Green | Orange |
| 75 | Violet | Green | Black | 30,000 | Orange | Black | Orange |
| 100 | Brown | Black | Brown | 40,000 | Yellow | Black | Orange |
| 150 | Brown | Green | Brown | 50,000 | Green | Black | Orange |
| 200 | Red | Black | Brown | 60,000 | Blue | Black | Orange |
| 250 | Red | Green | Brown | 75,000 | Violet | Green | Orange |
| 300 | Orange | Black | Brown | 100,000 | Brown | Black | Yellow |
| 350 | Orange | Green | Brown | 120,000 | Brown | Red | Yellow |
| 400 | Yellow | Black | Brown | 150,000 | Brown | Green | Yellow |
| 450 | Yellow | Green | Brown | 200,000 | Red | Black | Yellow |
| 500 | Green | Black | Brown | 250,000 | Red | Green | Yellow |
| 600 | Blue | Black | Brown | 300,000 | Orange | Black | Yellow |
| 750 | Violet | Green | Brown | 400,000 | Yellow | Black | Yellow |
| 1,000 | Brown | Black | Red | 500,000 | Green | Black | Yellow |
| 1,200 | Brown | Red | Red | 600,000 | Blue | Black | Yellow |
| 1,500 | Brown | Green | Red | 750,000 | Violet | Green | Yellow |
| 2,000 | Red | Black | Red | $1 \mathrm{M} \Omega$ | Brown | Black | Green |
| 2,500 | Red | Green | Red | $1 \frac{1}{2} \mathrm{M} \Omega$ | Brown | Green | Green |
| 3,000 | Orange | Black | Red | 2M $\Omega$ | Red | Black | Green |
| 3,500 | Orange | Green | Red | $3 \mathrm{M} \Omega$ | Orange | Black | Green |
| 4,000 | Yellow | Black | Red | $4 \mathrm{M} \Omega$ | Yellow | Black | Green |
| 5,000 | Green | Black | Red | $5 \mathrm{M} \Omega$ | Green | Black | Green |
| 7,500 | Violet | Green | Red | $6 \mathrm{M} \Omega$ | Blue | Black | Green |
| 10,000 | Brown | Black | Orange | $7 \mathrm{M} \Omega$ | Violet | Black | Green |
| 12,000 | Brown | Red | Orange | $8 \mathrm{M} \Omega$ | Gray | Black | Green |
| 15,000 | Brown | Green | Orange | $9 \mathrm{M} \Omega$ | White | Black | Green |
| 20,000 | Red | Blåck | Orange | $10 \mathrm{M} \Omega$ | Browi: | Black | Blue |

## APPENIIX VIII

Mica capacitors that are not stamped with their capacitance values usually are marked with three or more colored dots and with an arrow or other symbol indicating the sequence in which the dots are to be read. The capacitance values are in micromicrofarads, and the color code is the same as the one used for resistors. The threedot RMA color code, shown in Fig. A-11a, is used for capacitors whose working voltage is 500 volts and for which one or more of the following conditions apply: (1) the tolerance is greater than 10 per cent, (2) the capacitance rating is less than $10 \mu \mu \mathrm{f}$, (3) the capacitance rating has only one or two significant figures. In this system, the color of the first dot indicates the value of the first significant figure of the capacitance; the second dot indicates the second figure; and the third dot indicates the value of the multiplying factor. For indicating the capacitance of capacitors having three significant figures the five- and six-dot systems, shown in Figs. A-11c, A-11d, and A-11e, are used. The systems using more than three dots provide a dot to indicate the capacitance tolerance. The six-dot system also provides a dot to indicate the d-c working voltage. The use of the capacitor color code can best be understood by reference to the following table, the examples listed in the accompanying table of applications of the color code, and the diagrams of the various systems shown in Fig. A-11.


Fig. A-11.

Capacitance in Micromicrofarads ( $\mu \mathrm{f}$ )

| Color of dot | Significant figures |  |  | Multiplying factor | Per cent tolerance | I-c working voltage |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | $A$ | B | C | D | $E$ | $F$ |
| Black | 0 | 0 | 0 | 1 |  |  |
| Brown | 1 | 1 | 1 | 10 | 1 | 100 |
| Red | 2 | 2 | 2 | 100 | 2 | 200 |
| Orange | 3 | 3 | 3 | 1000 | 3 | 300 |
| Yellow | 4 | 4 | 4 | 10,000 | 4 | 400 |
| Green | 5 | 5 | 5 | 100,000 | 5 | 500 |
| Blue | 6 | 6 | 6 | 1,000,000 | 6 | 600 |
| Violet | 7 | 7 | 7 | 10,000,000 | 7 | 700 |
| Gray | 8 | 8 | 8 | 100,000,000 | 8 | 800 |
| White | 9 | 9 | 9 | 1,000,000,000 | 9 | 900 |
| Gold | . | . | . | 0.1 | 5 | 1000 |
| Silver | . | $\cdots$ | . | 0.01 | 10 | 2000 |
| No Color | . | . | . |  | 20 | 500 |

Application of Color Code

| System, Fig. A-11 | Capacitance, $\mu \mathrm{f}$ | $\begin{gathered} \text { Capaci- } \\ \text { tance, } \end{gathered}$ $\mu \mu \mathrm{f}$ | Per cent of tolerance | $\begin{gathered} \text { D-c } \\ \text { work- } \\ \text { ing } \\ \text { voltage } \end{gathered}$ | A | B | $C$ | D | $E$ | $F$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| (a) | 0.000005 | 5 |  |  | Black | Green |  | Black |  |  |
| (a) | 0.000012 | 12 |  |  | Brown | Red |  | Black |  |  |
| (a) | 0.00035 | 350 |  |  | Orange | Green |  | Brown |  |  |
| (a) | 0.0004 | 400 |  |  | Yellow | Black |  | Brown |  |  |
| (b) | 0.000025 | 25 | 2 |  | Red | Green |  | Black | Red |  |
| (b) | 0.00075 | 750 | 5 |  | Violet | Green |  | Brown | Green |  |
| (b) | 0.006 | 6000 | 10 |  | Blue | Black |  | Red | Silver |  |
| (c), (d) | 0.0003 | - 300 | 1 |  | Orange | Black | Black | Black | Brown |  |
| (c), (d) | 0.000125 | 125 | 3 |  | Brown | Red | Green | Black | Orange |  |
| (c), (d) | 0.008 | 8000 | 20 |  | Gray | Black | Black | Brown | No color |  |
| (e) | 0.000002 | 2 | 4 | 1000 | Red | Black | Black | Silver | Yellow | Gold |
| (e) | 0.000025 | 25 | 10 | 500 | Red | Green | Black | Gold | Silver | No color |
| (e) | 0.0003 | 300 | 20 | 300 | Orange | Black | Black | Black | No color | Orange |
| (e) | 0.0075 | 7500 | 5 | 600 | Violet | Green | Black | Brown | Gold | Blue |

## APPFNDIX IX

## STANDARD COLOR CODE FOR TRANSFORMER LEADS

In order to identify the various leads of transformers used in radio equipment, the Radio Manufacturers' Association has adopted a set of standards that are used by most manufacturers. The following diagrams i dicate the color of the leads for the three types of transformers most generally used:


Fig. A-12.-(P, R. Mallory \& Co., Inc.).


The upper portion (that code above the dotted line) for single primary and/or secondary fransformers (c)-COLOR CODE AUDIO TRANSFORMERS


## APPENDIX X

## TRIGONOMETRY

The solution of a.c. problems frequently involves adding or subtracting quantities such as voltages, currents, and ohmages by means of vectors. The mathematical solution of these problems requires the use of trigonometry. The method of solution presented in the text makes it possible to solve all such problems by the use of right triangles. The following statements apply to any right triangle and are illustrated in the figure below.

1. A right triangle is one in which one of the angles is a right angle ( 90 degrees).
2. The hypotenuse is the side opposite the right angle.
3. The legs of a right triangle are the two sides that form the right angle.
4. The sine of any angle $\theta$ is equal to the side opposite that angle divided by the hypotenuse.
5. The cosine of any angle $\theta$ is equal to the side adjacent to that angle divided by the hypotenuse.
6. The square of the hypotenuse is equal to the sum of the squares of the two legs of the triangle. (This is also commonly known as the theorem of Pythagoras.)

$$
\begin{array}{rlrl}
\sin A & =\frac{a}{c} & a & =c \sin A \\
\cos A & =\frac{b}{c} & b & =c \cos A \\
\sin B & =\frac{b}{c} & b & =c \operatorname{cin} B \\
\cos B & =\frac{a}{\sin A} & c & =\frac{b}{\cos A} \\
\cos B \\
c^{2} & =a^{2}+b^{2} & a & =c \cos B \\
a^{2} & =c^{2}-b^{2} & c & =\frac{a}{\cos B} \\
b^{2} & =c^{2}-a^{2}
\end{array}
$$

APPENDIX XI
SINE AND COSINE TABLES

| Degrees | $\sin$ | $\cos$ | Degrees | $\sin$ | $\cos$ |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 0.0 | 0.000 | 1.000 | 21.5 | 0.366 | 0.930 |
| 0.5 | 0.009 | 1.000 | 22.0 | 0.374 | 0.927 |
| 1.0 | 0.017 | 0.999 | 22.5 | 0.383 | 0.924 |
| 1.5 | 0.026 | 0.999 | 23.0 | 0.391 | 0.920 |
| 2.0 | 0.035 | 0.999 | 23.5 | 0.399 | 0.917 |
| 2.5 | 0.043 | 0.999 | 24.0 | 0.407 | 0.913 |
| 3.0 | 0.052 | 0.998 | 24.5 | 0.415 | 0.910 |
| 3.5 | 0.061 | 0.998 | 25.0 | 0.422 | 0.906 |
| 4.0 | 0.070 | 0.997 | 25.5 | 0.430 | 0.902 |
| 4.5 | 0.078 | 0.997 | 26.0 | 0.438 | 0.899 |
| 5.0 | 0.087 | 0.996 | 26.5 | 0.446 | 0.895 |
| 5.5 | 0.096 | 0.995 | 27.0 | 0.454 | 0.891 |
| 6.0 | 0.104 | 0.994 | 27.5 | 0.462 | 0.887 |
| 6.5 | 0.113 | 0.993 | 28.0 | 0.469 | 0.883 |
| 7.0 | 0.122 | 0.992 | 28.5 | 0.477 | 0.879 |
| 7.5 | 0.130 | 0.991 | 29.0 | 0.485 | 0.875 |
| 8.0 | 0.139 | 0.990 | 29.5 | 0.492 | 0.870 |
| 8.5 | 0.148 | 0.989 | 30.0 | 0.500 | 0.866 |
| 9.0 | 0.156 | 0.988 | 30.5 | 0.507 | 0.862 |
| 9.5 | 0.165 | 0.986 | 31.0 | 0.515 | 0.857 |
| 10.0 | 0.173 | 0.985 | 31.5 | 0.522 | 0.853 |
| 10.5 | 0.182 | 0.983 | 32.0 | 0.530 | 0.848 |
| 11.0 | 0.191 | 0.981 | 32.5 | 0.537 | 0.843 |
| 11.5 | 0.199 | 0.980 | 33.0 | 0.544 | 0.839 |
| 12.0 | 0.208 | 0.978 | 33.5 | 0.552 | 0.834 |
| 12.5 | 0.216 | 0.976 | 34.0 | 0.559 | 0.829 |
| 13.0 | 0.225 | 0.974 | 34.5 | 0.566 | 0.824 |
| 13.5 | 0.233 | 0.972 | 35.0 | 0.574 | 0.819 |
| 14.0 | 0.242 | 0.970 | 35.5 | 0.581 | 0.814 |
| 14.5 | 0.250 | 0.968 | 36.0 | 0.588 | 0.809 |
| 15.0 | 0.259 | 0.966 | 36.5 | 0.595 | 0.804 |
| 15.5 | 0.267 | 0.963 | 37.0 | 0.602 | 0.798 |
| 16.0 | 0.275 | 0.961 | 37.5 | 0.609 | 0.793 |
| 16.5 | 0.284 | 0.959 | 38.0 | 0.616 | 0.788 |
| 17.0 | 0.292 | 0.956 | 38.5 | 0.622 | 0.783 |
| 17.5 | 0.301 | 0.954 | 39.0 | 0.629 | 0.777 |
| 18.0 | 0.309 | 0.951 | 39.5 | 0.636 | 0.772 |
| 18.5 | 0.317 | 0.948 | 40.0 | 0.643 | 0.766 |
| 19.0 | 0.325 | 0.945 | 40.5 | 0.649 | 0.760 |
| 19.5 | 0.334 | 0.942 | 41.0 | 0.656 | 0.755 |
| 20.0 | 0.342 | 0.940 | 41.5 | 0.663 | 0.749 |
| 20.5 | 0.350 | 0.937 | 42.0 | 0.669 | 0.743 |
| 21.0 | 0.358 | 0.933 | 42.5 | 0.675 | 0.737 |


| Degrees | sin | $\cos$ | Degrees | $\sin$ | $\cos$ |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 43.0 | 0.682 | 0.731 | 67.0 | 0.920 | 0.391 |
| 43.5 | 0.688 | 0.725 | 67.5 | 0.924 | 0.383 |
| 44.0 | 0.695 | 0.719 | 68.0 | 0.927 | 0.375 |
| 44.5 | 0.701 | 0.713 | 68.5 | 0.930 | 0.366 |
| 45.0 | 0.707 | 0.707 | 69.0 | 0.934 | 0.358 |
| 45.5 | 0.713 | 0.701 | 69.5 | 0.937 | 0.350 |
| 46.0 | 0.719 | 0.695 | 70.0 | 0.940 | 0.342 |
| 46.5 | 0.725 | 0.688 | 70.5 | 0.943 | 0.334 |
| 47.0 | 0.731 | 0.682 | 71.0 | 0.945 | 0.326 |
| 47.5 | 0.737 | 0.675 | 71.5 | 0.948 | 0.317 |
| 48.0 | 0.743 | 0.669 | 72.0 | 0.951 | 0.309 |
| 48.5 | 0.749 | 0.663 | 72.5 | 0.954 | 0.301 |
| 49.0 | 0.755 | 0.656 | 73.0 | 0.956 | 0.292 |
| 49.5 | 0.760 | 0.649 | 73.5 | 0.959 | 0.284 |
| 50.0 | 0.766 | 0.643 | 74.0 | 0.961 | 0.276 |
| 50.5 | 0.772 | 0.636 | 74.5 | 0.964 | 0.267 |
| 51.0 | 0.777 | 0.629 | 75.0 | 0.966 | 0.259 |
| 51.5 | 0.783 | 0.622 | 75.5 | 0.968 | 0.250 |
| 52.0 | 0.788 | 0.616 | 76.0 | 0.970 | 0.242 |
| 52.5 | 0.793 | 0.609 | 76.5 | 0.972 | 0.233 |
| 53.0 | 0.798 | 0.602 | 77.0 | 0.974 | 0.225 |
| 53.5 | 0.804 | 0.595 | 77.5 | 0.976 | 0.216 |
| 54.0 | 0.809 | 0.588 | 78.0 | 0.978 | 0.208 |
| 54.5 | 0.814 | 0.581 | 78.5 | 0.980 | 0.199 |
| 55.0 | 0.819 | 0.574 | 79.0 | 0.982 | 0.191 |
| 55.5 | 0.824 | 0.566 | 79.5 | 0.983 | 0.182 |
| 56.0 | 0.829 | 0.559 | 80.0 | 0.985 | 0.174 |
| 56.5 | 0.834 | 0.552 | 80.5 | 0.986 | 0.165 |
| 57.0 | 0.839 | 0.544 | 81.0 | 0.988 | 0.156 |
| 57.5 | 0.843 | 0.537 | 81.5 | 0.989 | 0.148 |
| 58.0 | 0.848 | 0.530 | 82.0 | 0.990 | 0.139 |
| 58.5 | 0.853 | 0.522 | 82.5 | 0.991 | 0.130 |
| 59.0 | 0.857 | 0.515 | 83.0 | 0.992 | 0.122 |
| 59.5 | 0.862 | 0.507 | 83.5 | 0.994 | 0.113 |
| 60.0 | 0.866 | 0.500 | 84.0 | 0.994 | 0.104 |
| 60.5 | 0.870 | 0.492 | 84.5 | 0.995 | 0.096 |
| 61.0 | 0.875 | 0.485 | 85.0 | 0.996 | 0.087 |
| 61.5 | 0.879 | 0.477 | 85.5 | 0.997 | 0.078 |
| 62.0 | 0.883 | 0.469 | 86.0 | 0.997 | 0.070 |
| 62.5 | 0.887 | 0.462 | 86.5 | 0.998 | 0.061 |
| 63.0 | 0.891 | 0.454 | 87.0 | 0.998 | 0.052 |
| 63.5 | 0.895 | 0.446 | 87.5 | 0.999 | 0.043 |
| 64.0 | 0.899 | 0.438 | 88.0 | 0.999 | 0.035 |
| 64.5 | 0.903 | 0.430 | 88.5 | 0.999 | 0.026 |
| 65.0 | 0.906 | 0.423 | 89.0 | 0.999 | 0.017 |
| 65.5 | 0.910 | 0.415 | 89.5 | 1.000 | 0.009 |
| 66.0 | 0.913 | 0.407 | 90.0 | 1.000 | 0.000 |
| 66.5 | 0.917 | 0.399 |  |  |  |

## APPENDIX XII

## SINE AND COSINE VALUES FOR ANGLES GREATER THAN 90 DEGREES

The tables of Appendix XI list the values of sine and cosine for angles between 0 and 90 degrees. In some instances, it is desired to obtain the sine of angles greater than 90 degrees; they may be obtained in the following manner:

When $\theta$ is between 90 and 180 degrees

$$
\sin \theta=\cos (\theta-90)
$$

Example: What is the sine of 137 degrees?

$$
\begin{aligned}
\sin 137^{\circ} & =\cos (137-90) \\
& =\cos 47^{\circ} \\
& =0.682
\end{aligned}
$$

When $\theta$ is between 180 and 270 degrees

$$
\sin \theta=-\sin (\theta-180)
$$

Example: What is the sine of 218 degrees?

$$
\begin{aligned}
\sin 218^{\circ} & =-\sin (218-180) \\
& =-\sin 38^{\circ} \\
& =-0.616
\end{aligned}
$$

When $\theta$ is between 270 and 360 degrees

$$
\sin \theta=-\cos (\theta-270)
$$

Example: What is the sine of 336 degrees?

$$
\begin{aligned}
\sin 336^{\circ} & =-\cos (336-270) \\
& =-\cos 66^{\circ} \\
& =-0.407
\end{aligned}
$$

## APPENDIX XIII

## ANSWERS TO PROBLEMS

Note 1: Answers are provided for approximately 50 per cent of the problems. Except in a few cases, answers are provided for the odd-numbered problems. Instructors using this text can purchase a complete answer book from the publisher.

Note 2: As far as is practicable, all answers are accurate to at least three sig. nificant figures.

## Chapter I

1. 250 meters
2. 30 mc
3. (a) 0.0268 sec
(b) 0.0537 sec
4. (a) 1.345 meters
(b) 4.414 ft
5. 4687.5 cycles
6. 0.0753 ft
7. (a) 4687.5 cycles
(b) 80 cycles
8. (a) 0.0885 sec
(b) 0.00107 sec
9. 151.6 ft
10. (a) 0.113 ft
(b) 1.13 ft
(c) 11.3 ft
11. (a) 350 cycles
(b) 450 cycles
(c) 750 cycles (also 250 cycles)
12. (a) 1000 candle power
(b) 250 candle power
(c) 40 candle power
13. (a) $675,000,000 \mathrm{mc}$
(b) $675,000,000,000,000$ cycles
14. 3.66 meters
15. 1.51 meters or 4.95 ft
1.47 meters or 4.82 ft
16. WOR-422.5 meters

KRLD-288.4 meters
WMBC-211.2 meters
WRUL-15.79 mc
WLAP-250 meters
WPIT-12 mc

## Chapter II

1. 26,667 dyncs (repulsion)
2. 6153 dynes (repulsion)
3. 40,000 dynes (repulsion)
4. 0.15 amp
5. 48 ohms
6. 0.667 ma
7. 0.005 ohm
8. 176 volts
9. The resistor that requires 250 volts for 2.5 ma
10. The 250,000 -ohm resistor
11. 100 watts
12. 33.9 amp
13. The 2-hp motor
14. 335.7 kwh
15. $\$ 0.25$
16. 2.38 kwh
17. 325 volts
18. 400 volts
19. 0.141 amp
20. 9.12 cm
21. (a) 10.5 ohms
(b) 3.78 watts
22. $\$ 0.36$
23. (a) $\$ 1.35$
(b) $\$ 3.60$
24. (a) 97.5 watts
(b) 1.6 watts
(c) 1083 ohms
(d) $250,000,000 \mathrm{ohms}$

## Chapter III

1. 7 cells, series
2. 33 volts
3. 4 cells, parallel
4. $S$ cells, two groups in parallel, each group consisting of 4 cells counected in series
5. 12 cells
6. Approximately 75 per cent greater
7. 15 cells, tapped at 2,3 , and 11 cells
8. 66.66 hr
9. 20 hr
10. (a) 572 sq in .
(b) $190.6 \mathrm{amp}-\mathrm{hr}$

Chapter IV

1. 248,125 ohms
2. 2541 turns
3. 0.214 ohm
4. (a) $10,000 \mathrm{ohms}$
(b) 25 ma
(c) 37.5 volts, 62.5 volts, 25 volts, 125 volts
(d) 0.9375 watt
1.5625 watts
0.625 watt
3.125 watts
(e) 6.25 watts
5. 500 ohms
6. (a) 175 volts
(b) 75 volts
7. 0.52 ohm
8. $14,980 \mathrm{ohms}$
9. (a) 15,000 ohms
(b) 30,000 ohms
10. 9000 ohms, 720 ohms, 1000 ohms, 600 ohms, $R_{T} 240$ ohms
11. 0.0505 ohm
12. (a) 0.01002 ohm
(b) 0.005005 ohm
13. (a) 8.33 ohms
(b) 2.08 ohms
(c) 1.2 amp
14. (a) 6.31 ohms
11.25 ohms
9.47 ohms
(b) 27.03 ohms
(c) 8.88 amp
(d) Group 1-56 volts

Group 2-100 volts
Group 3-84 volts
27. (e) Group 1-5.6 amp

$$
1.87 \mathrm{amp}
$$

1.4 amp

Group 2-6.66amp
2.22 amp

Group 3 1.4amp
1.87 amp
5.6 amp
29. $i_{10.3}=3.88 \mathrm{amp}$
$i_{\text {8. }}=3.95 \mathrm{amp}$
$i_{3.3}=1.81 \mathrm{amp}$
$i_{4.4}=1.36 \mathrm{amp}$
$i_{7.7}=0.78 \mathrm{amp}$
$L_{A B}=6$ volts
$E_{B C}=34$ volts
31. (a) 55 ohms
(b) $I_{1}=0.1818 \mathrm{amp}$
$I_{2}=0.109 \mathrm{amp}$
$I_{3}=0.0727 \mathrm{amp}$
(c) $E_{A B}=4.55$ volts
$E_{B C}=5.45$ volts
$E_{B D}=5.45$ volts
33. $I_{B C}=0.45 \mathrm{ma}$
$I_{A B}=5.45 \mathrm{ma}$
35. $I_{B C}=0.45 \mathrm{ma}$ $I_{A B}=3.78 \mathrm{ma}$
37. (a) Section 1-17,857 ohms

Section 2-10,000 ohms
Section 3-14,705 ohms
Section 4-44.4ohms
Section 5-200 ohms
(b) 195 watts
(c) 2.79 watts
39. $R_{A B}=20$ ohms
$R_{B C}=18.5 \mathrm{ohms}$
$R_{C D}=106.5 \mathrm{ohms}$
$R_{A D}=145$ ohms
40. Section $A_{A B}$
$I_{20}=2.07 \mathrm{mmp}$
Section ${ }_{B C}$
$I_{10}=1.43 \mathrm{amp}$
$I_{25}=0.96 \mathrm{amp}$
$I_{50}=0.48 \mathrm{amp}$
$I_{15}=0.636 \mathrm{amp}$
$I_{45}=0.636 \mathrm{amp}$
Section ${ }_{C D}$ (top)
$I_{100}=0.779 \mathrm{amp}$
$I_{200}=0.518 \mathrm{amp}$
$I_{400}=0.259 \mathrm{amp}$
$I_{50}=0.779 \mathrm{amp}$
40. Section ${ }_{C D}$ (bottom)
$I_{25}=1.102 \mathrm{amp}$
$I_{76}=1.102 \mathrm{amp}$
$I_{100}=1.102 \mathrm{amp}$

$$
\begin{aligned}
& I_{400}=0.184 \mathrm{amp} \\
& I_{800}=0.184 \mathrm{amp} \\
& \text { 41. } E_{A B}=41.4 \text { volts } \\
& E_{B C}=38.2 \text { volts } \\
& E_{C D}=220.4 \text { volts } \\
& \text { 45. } P_{A B}=85.7 \text { watts } \\
& P_{B C}=79 \text { watts } \\
& P_{C D}=456.3 \text { watts } \\
& P_{\text {Total }}=621 \text { watts }
\end{aligned}
$$

## Chapter V

9. (a) 250 dynes
(b) Attraction
10. 3.16 cm
11. 232.2 unit poles
12. (a) 3906 dynes
(b) 15,625 dynes
(c) 62,500 dynes
13. 13.46 oz
14. 17.27 unit poles
15. 22,222 lines per square inch
16. 0.5 in .
17. 942 lines

75 lines per square centimeter
18.75 lines per square centimeter
4.69 lines per square centimeter

75 dynes, 18.75 dynes, 4.69 dynes
28. (a) 4000 lines per square centimeter
(b) 1200
(c) $0.000 \times 33$
29. 0.00166

600
31. (a) 1860 lines per square centimeter
(b) 900
(c) 0.00111
33. (a) 0.00797
(b) 0.00273
(c) 0.0107
(d) 128.4 gilberts
34. 408 turns
35. 6 volts
39. (a) 0.0165
(b) 104.7 ampere-turns
41. 0.0000833
43. (a) $\mathscr{R}_{1}=0.00202$
$\mathfrak{Q}_{2}=0.00202$
$\mathcal{R}_{3}=0.00182$
$\Omega_{4}=0.00171$
$\mathfrak{R}_{5}=0.00182$
$Q_{6}=0.00202$
$\mathcal{R}_{7}=0.00202$
$\mathfrak{R}_{8}=0.00171$
$\mathcal{R}_{A G 1}=0.0708$
$\mathfrak{Q}_{A G 2}=0.0314$
$\mathcal{Q}_{A G 3}=0.0708$
(b) 1345 ampere-turns

## Chapter VI

1. (a) 13.89 ohms
(b) 5.208 ohms
2. (a) $15,000 \mathrm{ohms}$
(b) $450,000 \mathrm{ohms}$
3. (a) 2.5 ohms
(b) 1.25 ohms
(c) 0.625 ohm
4. (a) $R_{\text {sbunt }}=2.78 \mathrm{ohms}$

Error $=10$ per cent
(b) $R_{\text {ghunt }}=1.315 \mathrm{ohms}$ Error $=4.95$ per cent
(c) $R_{\text {shunt }}=0.641 \mathrm{ohm}$ Error $=2.5$ per cent
9. (a) 50 per cent
(b) 33.3 per cent
11. (a) 40 volts
(b) 20 per cent
13. (a) $13,500,000$ ohms
(b) $1,500,000 \mathrm{ohms}$
(c) $1,000,000 \mathrm{ohms}$
15. (a) $E_{1}=50$ volts
$I_{1}=4.9975 \mathrm{amp}$
$E_{2}=49.975$ volts
$I_{2}=5.0008 \mathrm{amp}$
(b) Neither connection, the error is negligible in either case.
17. 9000 ohms
19. Resistors for voltage measurements 10 -volt scale- 9970 ohms
50 -volt scale-49,970 ohms
100 -volt scale- 99,970 ohms
250 -volt scale-249,970 ohms
500 -volt scale-499,970 ohms
Shunts for current measurements
1-ma scale-no shunt needed
$10-\mathrm{ma}$ scale- 3.33 ohms
$50-\mathrm{ma}$ scale- 0.612 ohm
100 -ma scale -0.303 ohm
1000 -ma scale- 0.0300 ohm
21. 0.0785 ohm
23. $0.266 \mu \mathrm{f}$
25. 42 henries

## Chapter VII

1. 249.6 volts
2. 6 turns per coil
3. 435,600 maxwells
4. (a) 60 cycles
(b) 40 cycles
(c) 25 cycles
5. (a) 40 poles
(b) 20 poles
6. (a) 25 cycles
(b) 60 cycles
(c) 500 cycles
7. (a) 13.365 amp
(b) -9.33 amp
(c) -15 amp
(d) -14.055 amp
(e) -3.885 amp
8. 36.4 volts
9. 159.25 volts
10. 127.4 volts
11. 141.4 volts
12. 409.4 volts
13. (a) 200 volts
(b) 127.4 volts
(c) 141.4 volts
14. 120 volts, 103.92 volts
15. 655 turns
16. 96 turns
17. 955 ma
18. (a) 200 amp
(b) 22 kva
(c) 220 lamps
19. 80.3 per cent
20. 51.28 watts
21. 65.6 watts
22. 71.4 per cent

## Chapter VIII

1. 40 volts
2. 9.3 volts
3. 260 turns
4. 0.9 henry
5. 3.6 henries
6. 39.32 mh
7. 4.923 mh
8. 12.75 mh
9. (a) $11,304 \mathrm{ohms}$
(b) $22,608 \mathrm{ohms}$
10. (a) 863.5 ohms
(b) 1570 ohms
(c) 2355 ohms
(d) 11,775 ohms
11. (a) 580.90 hms
(b) 1161.8 ohms
(c) 1742.7 ohms
12. (a) 3768 ohms
(b) 3797.8 ohms
(c) 28.9 ma
13. 79.1 ma
14. 0.12 sec
15. (a) 11,304 ohms
(b) $11,306.7 \mathrm{ohms}$
(c) $88.5^{\circ}$
(d) 26.5 ma
16. 8.87 mh
17. 0.0817
18. 9 volts
19. $77.08 \mu \mathrm{~h}$
20. (a) 0.304 henry
(b) 22.96
21. 106 mh
22. (a) 29.191 henries
(b) 91.66

## Chapter IX

1. $0.000123475 \mu \mathrm{f}$
2. $69.595 \mu \mu \mathrm{f}$
3. 0.00563 in.
4. $0.500 \mu \mathrm{f}$
5. 6 plates
6. $0.0160 \mu \mathrm{f}$
7. 29.93 ft
8. 0.848 sq in.
9. $50.3 \mu \mu \mathrm{f}$
10. $49.82 \mu \mu \mathrm{f}$
11. (a) 331.7 ohms
(b) 0.331 amp
12. (a) 331.8 ohms
(b) 0.331 amp
(c) 0.0301
(d) $88.5^{\circ}$
(e) 1.095 watts
13. (a) 212.3 ohms
(b) 127.3 ohms
(c) 109.80 hms
(d) 2.65 ohms
(e) 1.59 ohms
(f) 0.736 ohm
14. (a) 2.35 ma
(b) 7.85 ma
(c) 22.7 ma
(d) 377 ma
(e) 1.35 amp
15. $0.533 \mu \mathrm{f}$
16. (a) 50.2 ma
(b) 16.67 volts, 33.33 volts, 66.67 volts, 133.33 volts
17. $15 \mu \mathrm{f}$
18. (a) 110 volts (each)
(b) 0.3315 amp 0.1658 amp 0.0829 amp 0.0414 amp
(c) 0.6217 amp
19. $20 \mu \mathrm{f}$
20. $5 \mu \mathrm{f}$
21. $2.5 \mu \mathrm{f}$
22. $6.67 \mu \mathrm{f}$
23. (a) $6.65 \mu \mathrm{f}$
(b) 0.0825
(c) $85^{\circ}$
24. 3.76 ma

## Chapter X

1. (a) 2792 ohms
(b) $690,800.1 \mathrm{ohms}$
2. 28.77 ohms
3. (a) 3.58 ma
(b) $14.4 \mu \mathrm{a}$
4. (a) 5667 ohms
(b) 31.875 volts
5. 1.99 amp
6. (a) 242.2 ohms
(b) 0.330 amp
(c) 3.3 volts, 11.2 volts, 91 volts
7. (a) $97,626 \mathrm{ohms}$
(b) 1.84 ma
(c) $V_{1}=2.92$ volts
$V_{2}=1.44$ volts
$V_{3}=165.6$ volts
$V_{4}=18.4$ volts
8. Check- 111.8 ohms
9. (a) $I=39.8 \mathrm{ma}$
$P=$ zero
$V A=4.776 \mathrm{v}-\mathrm{a}$
P-F $=$ zero
$\theta=90^{\circ}$ (lag)
(b) Vector diagram
10. (a) $I=39.6 \mathrm{ma}$
$P=0.4547 \mathrm{watt}$
$V A=4.752 \mathrm{v}-\mathrm{a}$

P-F $=0.0957$
$\theta=84.5^{\circ}$ (lag)
(b) Vector diagram
21. (a) $I=3.39 \mathrm{amp}$
$P=$ zero
$V A=1017 \mathrm{v}-\mathrm{a}$
$\mathrm{P}-\mathrm{F}=$ zero
$\theta=90^{\circ}$ (leading)
(b) Vector diagram
23. No
25. (a) $X_{C}=3184 \mathrm{ohms}$
$X_{L}=6280$ ohms
$Z=5880 \mathrm{ohms}$
$I=42.5 \mathrm{ma}$
$P=9.03$ watts
$\mathrm{A}-\mathrm{P}=10.625 \mathrm{v}-\mathrm{a}$
$\mathrm{P}-\mathrm{F}=0.850$
$\theta=32^{\circ}$ (lagging)
(b) $E_{R}=212.5$ volts
$E_{C}=135.3$ volts
$F_{L}=266.9$ volts
(c) Vector diagram
27. (a) $Z=1000 \mathrm{ohms}$
$R=22.20 \mathrm{hms}$
$\mathrm{A}-\mathrm{P}=22.5 \mathrm{v}-\mathrm{a}$
$\mathrm{P}-\mathrm{F}=0.0222$
$\theta=88.5^{\circ}$ (leading)
$X_{C}=999.7 \mathrm{ohms}$
$C=2.65 \mu \mathrm{f}$
(b) Vector diagram
29. (a) $I_{1}=250 \mu \mathrm{a}$
$I_{2}=199 \mu \mathrm{a}$
$I_{3}=125 \mu \mathrm{a}$
$I_{4}=398 \mu \mathrm{a}$
(b) $I_{\text {line }}=705 \mu \mathrm{a}$
(c) $Z_{\text {cot }}=354,609 \mathrm{ohms}$
(d) $P_{1}=0.0625$ watt
$P_{2}=$ zero
$P_{3}=0.03125$ watt
$P_{4}=$ zero
(e) $P_{\text {cct }}=0.09375$ watt
(f) $V A_{\text {cet }}=0.17625 \mathrm{v}-\mathrm{a}$
(g) P - $\mathrm{F}_{\text {line }}=0.531$
(h) $\theta_{\text {line }}=58^{\circ}$ (lagging)
(i) Vector diagram
31. (a) $I_{1}=5 \mathrm{ma}$
$I_{2}=104.16 \mathrm{ma}$
$I_{3}=39.06 \mathrm{ma}$
$I_{4}=7.81 \mathrm{ma}$
$I_{5}=52.08 \mathrm{ma}$
$I_{6}=10 \mathrm{ma}$
(b) $I_{\text {line }}=110.3 \mathrm{ma}$
(c) $Z_{\text {cct }}=2266 \mathrm{ohms}$
(d) $P_{1}=1.25$ watts
$P_{2}=$ zero
$P_{z}=$ zero
$P_{4}=$ zero
$P_{5}=$ zero
$P_{6}=2.5$ watts
(e) $P_{\text {oct }}=3.75$ watts
(f) $V A_{\text {ct }}=27.575 \mathrm{v}-\mathrm{a}$
(g) $\mathrm{P}-\mathrm{F}_{\text {line }}=0.136$
(h) $\theta_{\text {line }}=82^{\circ}$ (leading)
(i) Vector diagram
33. (a) $1_{1}=1.31 \mathrm{amp}$
$P_{1}=51.48$ watts
P- $\mathrm{F}_{1}=0.394$
$\theta_{1}=67^{\circ}$ (lagging)
$I_{2}=1 \mathrm{amp}$
$P_{2}=100$ watts
$\mathrm{P}-\mathrm{F}_{2}=1.00$
$\theta_{2}=0^{\circ}$
$I_{3}=1.26 \mathrm{amp}$
$P_{3}=39.7$ watts
P-F ${ }_{3}=0.316$
$\theta_{3}=71.5^{\circ}$ (leading)
(b) $I_{\text {lins }}=1.914 \mathrm{amp}$
$P_{\text {line }}=191.18$ watts
$\mathrm{A}-\mathrm{P}_{\text {line }}=191.4 \mathrm{v}-\mathrm{a}$
$P-F_{\text {line }}=0.9988$
$\theta_{\text {line }}=3^{\circ}$ (lagging)
(c) Vector diagram
35. (a) $I_{\text {line }}=0.676 \mathrm{amp}$
$P_{\text {line }}=67.577$ watts
A- $\mathrm{P}_{\text {line }}=67.6 \mathrm{v}-\mathrm{a}$
$\mathrm{P}-\mathrm{F}_{\text {line }}=0.9996$
$\theta_{\text {line }}=1.5^{\circ}$ (lagging)
(b) $E_{k_{1}}=18.65$ volts
$I_{R_{1}}=0.622 \mathrm{amp}$
$E_{X_{L}}=18.65$ volts
$I_{x_{L}}=0.266 \mathrm{amp}$
$E_{R_{2}}=67.6$ volts
$I_{R_{2}}=0.676 \mathrm{amp}$
$E_{t_{3}}=16$ volts
$I_{R_{3}}=0.64 \mathrm{amp}$
$E_{X_{C}}=16$ volts
$I_{x_{C}}=0.213 \mathrm{amp}$
(c) Vector diagram
37. (a) $53,078 \mathrm{ohms}$
(b) 212 ohms
(c) approximately 10 per cent through $C$
approximately 90 per cent through $R_{0}$
(d) approximately 95 per cent through $C$
approximately 5 per cent
through $R_{0}$
39. (a) 159,235 ohms, 212 ohms
(b) 376.8 ohms

282,600 ohms
(c) high pass

## Chapter XI

5. (a) 86 ohms (capacitive)
(b) 46 ohms (capacitive)
(c) 7 ohms (capacitive)
(d) 33 ohms (inductive)
(e) 73 ohms (inductive)
6. (a) $320 \mu \mu \mathrm{f}$
(b) $35.5 \mu \mu \mathrm{f}$
7. 489 to 1707 kc
8. (a) $62.5 \mu \mathrm{~h}$
(b) 4498 kc
(c) 6360 kc
(d) 9000 kc
9. (a) $15.86 \mu \mu \mathrm{f}$
(b) 1.96 mc
(c) 3.36 mc
(d) 3.77 me
10. $209.6 \mu \mu \mathrm{f}$
11. (a) $996 \mu \mathrm{~h}$
(b) 292
12. (a) 5000 cycles
(b) 5039 cycles
13. (a) 25
(b) $E_{L}=250$ volts
$E_{C}=250$ volts
$E_{R}=10$ volts
14. (a) 465 kc
(b) $3,837,405 \mathrm{ohms}$
15. (a) $199 \mu \mathrm{~h}$
(b) 1083
(c) 1.2 kc
16. (a) 89,443 ohms

171,371 ohms
(b) $95,7850 \mathrm{hms}$

70,719 ohms
Note: Values of $X_{L}$ and $X_{C}$ were obtained from Table XI-IV by interpolation.

## Chapter XII

1. (a) 2500.01 ohms
(b) 2500.150 oms
2. (a) 2998 ohms
(b) 3223 ohms
3. (a) 8635 ohms
(b) 22,765 ohms
4. (a) 337.3 ohms
(b) 176.5 ohms
5. (a) 4000 -cycle audio frequency
(b) 375
(c) 4000-cycle audio frequency
6. (a) 6625 ohms
(b) 331.25 ohms
(c) 1507.2 ohms
(d) 30,144 ohms
7. $0.163 \mu \mu \mathrm{f}$
8. $2.76 \mu \mathrm{~h}$
9. 9880 cycles
10. (a) 0.018
(b) 0.0167
11. 0.0173
12. 1250 kc
13. $58.5 \mu \mu \mathrm{f}$
14. 5.346 mh
15. (a) 0.0375
(b) 261.7 to 271.7 ke
(c) $59.2 \mu \mu \mathrm{f}$
16. (a) 0.025 sec
(b) $0.04 \sec (k-1.60)$
17. (a) 2 megohms
(b) 1.33 megohms
(c) $0.2 \mu \mathrm{f}$
18. (a) $0.005 \mathrm{sec}(k-0.2)$
$0.0125 \mathrm{sec}(k-0.5)$
$0.04 \sec (k-1.6)$
(b) $500 \mu \mathrm{a}$
(c) $100 \mu \mathrm{a}$
(d) $500 \mu \mathrm{a}$
(e) 92 volts
(f) $0.0175 \mathrm{sec}(k-0.7)$
19. 0.047 sec

## INDEX

## A

A battery, 98-99
Abbreviations, 75-76, 487-489
Abscissa, 393-397
Absorption loss, 325
Action, local, 92-93
Actual power, 369-370
Adjustable capacitor, 320
Adjustable magnetic core inductor, 290
Aerial, 9
Air cell A battery, 102-103
Air-core coil, 274, 289-292
Air gap, of capacitor, 317
of generator, 253
Alloys for magnets, 176
Alnico, 176-177
Alternating current, 63, 235, 241
average value of, 243-245
effective value of, $245-249$
instantaneous value of, 242-243, 245
maximum value of, 243,245
power in, 369-371
root-mean-square value of, $247-248$
Alternating-current bridge, 229-230
Alternating-current circuits, character* istics of, 357-358
parallel, 375-378
parallel-series, 378-381
series, 360-375
series-parallel, 381-384
Alternaling-current generator, 239-241, 20̌3-254
Alternating-current meters, 199-202, 205209
(See also Meters)
Alternating-current power systems, 235
Alternating-current resistance, 295
Alternating-current vectors, 365-384
Alternating-current voltage, 242-245
Alternating-current wave, 17, 243
Alternation, 17, 241

Alternator, 235
Alexanderson, 11
construction of, 251-254
frequency of, 241
Goldschmidt, 11
principle of, 238-241
Amalgamation, 93
American Wire Gauge, 121
Ammeter, alternating-current, 200-202, 205-209
connecting of, in circuit, 209-210
D'Arsonval, 202
direct-current, 200-207
dynamometer type, 206-207
hot-wire type, 200
increasing range of, 215-218
iron-vane type, 205-206
permanent-magnetic moving-coil type, 202-205
precautions in using, 209-210
reading of, 215
rectifier-type, 207-209
thermocouple type, 200-201
Ammeter scales, 214-215
Ammeter shunts, 215-218
Ampere, alternating-current, 245-249
definition of, 69
international, 69
microampere, 74
milliampere, 74
Ampere-hour, 109
Ampere-turns, 191
Amplifier, audio, 35
intermediate-frequency, 39
overcoupled, 458-461
picture-signal, 37, 39
radio-frequency, 35
modulated, 37
speech, 35
stagger-tuned, 458-461
video, 37,39
wide-band, 458-460

Antenna, 7, 9, 33, 35, 37, 39
Apparent power, 369
Armature, 238, 251-253
conductors for, 238-239, 249-250
core of, 252
Artificial magnet, 163-164
Atom, 47-49
definition of, 47
structure of, 48-49
Attraction, electrostatic, 50-51
of magnetic poles, 167-168
Audible sound waves, 32,35
Audio-irequency amplifier, 35
Auditory means of communication, 3-8

## B

B battery, 99-101
Back voltage, 449
Band-exelusion filter, 437
Band-pass amplifier circuit, 453-457
Band-pass filter, 436-437
Band-stop filter, 437
Band-suppression filter, 437
Band width, of coupled circuits, 455-457
of resonance curves, 404-406
Bank winding, 291
Bar magnet, 177
Battery, 59, 87-115
A, 9899
Air cell, 102-103
B, 99-101
C, 101-102
chargers for, 113
eharging, methods of, 111-113
principle of, 103-106
containers for, 107-108
deterioration of, 96
discharging, 103-105
dry-cell, 93-96
Edison-Lelande, 87
electrolyte in, 89, 108-109
Layer-Bilt, 100
lead-acid storage, 103-115
Mini-Max, 100-101
nickel-iron storage, 103
separators for, 107, 114
storage, 103-115
testing of, 9.4-95, 109-112
uses of, 59, 94-96
wet-cell, primary, 75, 80-93
secondary, 103-111

Battery terms, 88-89
Binocular coil, 291
Bleeder current, 149
Blocking capacitor, 348
Breakdown voltage, of capacitor, 326-327
of insulator, 123
Bridge, alternating-current, 229-230
for capacitance. 229-230, 344
for inductance, 229-230, 295-296
for resistance, 226-229
Wheatstone, 226-229
Broadcasting, frequency range of, 21,3335,40
Broadcasting operations, 32-39
Broadcasting station, frequency of, 33
Brushes, 239
By-pass capacitor, filter action of, 434435
uses of, 347-349

## C

C battery, 101-102
Camera tube, 33, 37
Candle power, 29
Capacitance, 306-349
calculation of, fixed capacitors, 310-314
impedance method, 344-347
variable capacitors, 319-320
definition of, 306
distributed, of choke coil, 420-421
effect of, 427-430
principle of, 329-331
effect of dielectric on, 310
measurement of, 229-230, 343-347
comparison method of, 344
of electrolytic capacitors, 346-347
impedance method of, 344
voltmeter-ammeter method of, 344345
units of, 309-310
Capacitive reactance, in alternating-current circuits, 360
of capacitor, 321-323
vector analysis of, 367-369
Capacitor, action of, 306-309
adjustable, 320
by-pass, filter action of, 434-435 uses of, 347-351
ceramic, 315
characteristies of, 339-343, 347
charge of, 307-309

Capacitor, charging time of, 465, 46S
dielectric in, 331-334
ceramic, 315
losses of, 325
mica, 312
oil, 315
paper, 313-315
use of, 306
discharge of, 308-309
dry electrolytic, 335-339
electrolytic, 331-343
(See also Electrolytic capacitor)
electrolyte in, 331, 335-337
equivalent circuit of, 428-430
fixed, 311-315
(See also Fixed capacitor)
ganged, 319
leakage of, 325
losses in, dielectric, 325
effect of, 308
leakage, 325
resistance, 325
mica dielectric in, 312-313
micro, 317, 318
midget, 317-318
noninductive, 313-315
oil dielectric in, 315
padder, 320
paper dielectric in, 313-315
in parallel, 328-329
plates of, electrolytic, 331-339
fixed, 311-315
use of, 306
variable, 315-317
properties of, 347-350
ranges of, adjustable, 320
variable, 317-318
reactance of, in alternating-current circuits, 360
capacitor, 321-323
resistance of, effect of, 427-430
electrolytic, 342-343
in series, 327-328
trimmer, 320
uses of, 311, 312, 314, 315, 320, 347351
variable, 315-320
(See also Variable capacitor)
voltage rating, of electrolytic, 341
of transmitting, 318, 327
wet electrolytic, 331-343
(See also Electrolytic capacitor)

Capacitor blocks, 3:38
Capacity, 306
Carrier frequency, 33, 453
Carrier wave, 32, 33
Cell, 59, 87-111
air, 102-103
chemical action of, primary, 80-91, 93-94
secondary, 103-106
combination of, $96-98$
current flow in, 91
dry, 93-96
erosion of negative electrode of, 91
forming plates of, $105-106$
history of, 87
local action of, 92-93
polarization of, 92
primary, chemical action of, $90,93-94$
commercial, 93-96, 98-103
definition of, 88
principles of, 89-93
secondary, care of, 113-115
chemical action of, 103-106
commercial, 106-111
definition of, 89
principles of, 103-106
testing, 111-113
standard dry, 93-96
storage, 103-111
capacity of, 106, 109-111
testing of, dry, 95
secondary, 108-112
unit dry, rating of, 95-96
use of, in B battery, 100
voltaic, 87, 89-92
wet, primary, 87, 89-93
secondary, 103-111
Channel, television, 21, 33, 39
Characteristic impedance, 441-412
Charged bodies, 49-54
Charges, 49-58
electric, 49-51
laws of, 50-51
negative, of battery, 90-91
of capacitor, 307-309
of static electricity, 49-55
positive, of battery, $90-91$
of capacitor, $30-3-309$
of static electricity, 49-55
time of, 53-54
Charging, of capacitor, 307-309
by contact, $51-52$

Charging, of capacitor, by induction, 52-53
of storage batteries, methods of, 111113
principles of, 103-106
Chemical action, in primary cells, 90 , 93-94
in secondary cells, 103-106
Choke coils, audio-frequency, 287-289
distributed capacitance of, 420-421
filter, 287
output, 288-289
power supply, 287
radio-frequency, 289-292
resonant frequency of, 420-421
Circuit $Q$ (see $Q$ )
Circuits, alternating-current, 357-384
(See also Alternating-current circuits)
closed, 118
combination (see Combination circuits)
coupled, 443-460
(See also Coupled circuits)
definition of, 118
delayed aetion, 460-471
direct-current, 129-145, 149-154
(See also Direct-current circuits)
elements of, 427-428
equivalent, 428-430
essential parts of, 118
filter, 431-443
(See also Filter circuits)
magnetic, 186-192
parallel (see Parallel circuits)
parallel-series (see Parallel-series circuits)
resonant, 398-421
(See also Resonant circuits)
series (see Serics circuits)
series-parallel (see Series-parallel circuits)
tuned, 402
voltage divider, 149-154
Circular mil area, 120
Circular mil foot, 120
Closed circuit, 118
Cobalt, 176
Code, Morse, 4, 8

Coefficient, of coupling, calculation of, 282-283
effect of, on response curves, 446-447 on width of band pass, 452-460
temperature, 119
table of, 497
Coils, adjustable magnetic core, 290
aiding, 285-286
air-core, 274, 289-292
bank, 291
binocular, 291
choke (see Choke coils)
distributed capacitance of, effect of, 427-430
principles of, 329-331
equivalent circuit of, 428-430
figure-of-eight, 291
high-frequency inductance, 289-292
honeycomb, 291
impedance of, 287-290
inductance of, calculation of, 271-274
high-frequency, 289-292, 299-301
low-frequency, 287-289
mutual, 280-287
in parallel, 284-285
in series, 284-286
iron-core, 287-289, 290
lattice, 291
low-frequency inductance, 287-289
multilayer, 273
noninductively wound, 298-299
opposing, 285-286
pancake, 273
primary winding of, 256-257
$Q$ (see Q)
resistance of, alternating-current, 293295
direct-current, 293
effect of, 276-277
sccondary winding of, 256-257
shapes and types of, 271-274
shielding of, 292-293
solenoid, 271-274
spider-web, 291
universal-wound, 291
use of, 299-301
variable inductance, 284-286
Collector rings, 239
Color, of light, 29-30
phenomenon of, 30

Color code, audio-frequency transformer, 505
intermediate-frequency transformer, 505
for mica capacitors, 502-503
power transformer, 504
for resistors, 500-501
Combination circuits, alternating-current, 378-384
definitions of, 378,381
parallel-series, 378-381
series-parallel, 381-384
direct-current, 137-145
advanced, 142-145
advantages of, 141
definition of, 137
parallel-series, 139-141
series-parallel, 138-139
solution of, 138-145
uses of, 142
Combination meter, 225-226
Common impedance, 443-444
Communication, auditory, 3-8
radio, history of, 8-11
television, history of, 13-17
visual, 1-2
Commutator, 249
Commutator ripple, 251
Compass, dipping needle, 182
early use of, 162
magnetic, 180-181
Complex currents, 65
Compounds, chemical, 46-48
Condenser, 306
(See also Capacitor)
Conductance, 122-123
Conductor, cross-sectional area of 118
definition of, 70
factors determining choice of material of, 122-123
resistance of, 118-121
Continuous current, 62-63
Copper oxide rectifier battery charger, 113
Copper oxide rectifier meters, 207-209
Copper wire gauge, 122
Cosine, 506
Cosine table, 507-508
Coulomb, 69
Coulomb's law, 57
Counter voltagn, 449

Coupled circuits, amalysis of 447.453 band-pass. 453 -460
characteristies of, 447 460
coefficient of coupling, calculation of. 282-283
effect of, on response curves, 446-477
on width of hand pass, 452-460
complex, 445-446
direct, 444-445
impedance, 289
indirect, 445
inductive, 444-445
magnetic, 445
mutual-inductive, 445-460
principles of, 443-4:44
response curves of, 453-460
simple, 444-445
transformer, 445-460
tuned primary and tuned secondary, 453
untuned primary and tuned secondary, 453
untuned primary and untuned secondary, 452-453
Coupled impedance, 447-452
Coupling, coefficient of, calculation of, 282-283
effect of, on response curves, 446-447 on width of band pass, 452-460
complex, 445-446
critical, 446
direct, 444-445
impedance, 289
indirect, 445
in letive, 444
loose, 446
magnetic, 445
methods of, 443-445
mutual-inductive, 445-460
optimum, 446
principles of, 443-444
simple, 444-445
tight, 446
transformer, 445-460
Coupling element, 444
Current, alternating, 63
average value of, 243-245
effective value of, 245-249
instantaneous value of, 242-243, 244
maximum value of, 243,245
chemical effects of, 62

Current, alternating, complex, 65
continuous, 62-63
direct, 63
effects of, 61-62
electric, method of producing, chemical, 59
magnetic, 60-61
thermal, 59-60
flow of, 49, 58-69
direction of, 58-59, 91-92
hydraulic analogy of, 66, 68
initial leakage, 341-3+2
interrupted, 64-65
kinds of, 62-65
lagging, in alternating-current circuits, 359
of inductor, 275-277
vector representation, 367-369
leading, in alternating-current circuits, 360
of capacitor, 320-322
vector representation, 367-369
magnetic effects of, 62
measurement of, 209-210
methods of producing, 59-61
oscillatory, 63-64
inphase, 357-358
pulsating, 63
thermal effects of, 61-62
units of, 74
electron flow, 69
practical, 70
Curves, analysis of, 397-398
permeability, 189
plotting of, 393-39_
resonance, parallel, 411-415
series, 401-406
response, 453-460
universal time-constant, 463, 469-470
Cycle, 17, 241

## D

D'Arsonval instruments, 202
Declination, angle of, 181-182
Delayed-action circuits, 460-471
$R$-C circuits, 465-468
$R$ - $L$ circuits, $460-465$
universal time-constant curves for, 469-470
uses of, 470-471

Demodulator, 33, 35
Density of field, 172-173
Dependent variable, 393-395
Depolarizer, air-cell, 102
dry-cell, 92-93
purpose of, 92
Detector, 33, 35, 39
Deviation, angle of, 182
Diamagnetic materials, 176
Dielectric absorption loss, 325
Dielectric breakdown voltage, 123
Dielectric constant, 310
table of, 499
Dielectric, effect of, on capacitance, 310
forming voltage of, 333
self-healing characteristic of, 333-334
use of, in capacitor, 306
Dielectric hysteresis loss, 325
Dielectric losses, 325
Dielectric strength, of capacitor dielectrics, 326-327
of insulating materials, 124-125
table of, 499
unit of, 124
Dip, angle of, 182
Dipping-needle compass, 182
Direct current, 63
uses of, 251
Direct-current circuits, advanced, 142145
combination, 137-145
parallel, 133-137
parallel-series, 139-141
series, 130-133
series-parallel, 138-139
simple, 129-130
Direct-current generator, 249-251, 254
Direct-current meters, 200-207
(See also Meters)
Direct-current power systems, 234-235
Direct-current voltage divider, 149-154
Discharge of capacitor, 308,468
Distributed capacitance, of choke coil, 420-421
effect of, 427-430
methods of reducing, 331
principle of, 329-331
Distributed inductance, 427-430
Distributed resistance, 427-430

Dry cell, action of, 93-94
capacity of, 94-95
care of, 93, 96
construction of, 93
rating of, 9:-95
shelf life of, 96
sizes of, $91-96$
Dry electrolytic capacitor, 335-343
(See also Electrolytic capacitor)
Dynamic electricity, 45, 58-59
Dynamometer-type meters, 206-207
Dyne, 56, 167-168

## E

Ear, 22-23
Earth, geographic poles of, 179-180
magnetic characteristics of, 179-180
magnetic field of, 179-180
magnetic poles of, 179-182
Echo, 26
Eddy currents, 295
Edison effect, 11-12
Ffficiency, of generator, 261-262
of transformer, 261-262
Electric current (see Current)
Electrical degrees, 241
Electrical instruments, classification of, 198
Electricity, dynamic, 45, 58-59
fundamental units of, 67-70
nature of, 45
need for knowledge of, 40
static, 45
(See also Static electricity)
Electrode, 88-89
Electrolysis, 62
Electrolyte, chemic ${ }^{-1}$ action of, in battery, 90, 103-106
definition of, 89
use of, in capacitors, 331, 335-337
in storage battery, 108-109
Electrolytic capacitor, 331-343
action of, 331-332
comparison of wet and dry, 339-340
construction of, 334-339
containers for, 339
current characteristics of, 341-342
dielectric used in, 332-333
dry, 335-339
electrical characteristics of, 339-343

Electrolytic capacitor, electrolyte used in, 331, 335-333
equivalent series resistance of, 342
factors affecting capacitance of, 332
forming of, 3:31-3:33
impedance eharacteristics of, 342343
nompolarized type, 340-3.31
polarized type, 30
power factor of, 342
principles of, 331
temperature characteristics of, 3.42-343
uses of, 347-349
voltage rating of, 341
wet, 3:34-335
Electromagnetic field, principles of, 182186
Electromagnetic induction, definition of, 60
principles of, 2:35-238
Flectromagnetic meters, 202-207
Electromagnetism, 182-192
Electromotive force, definition of, 70
induced, 235-238, 283
Electron, free, 49, 70
orbital, 48-49
planetary, 48-49
Electron beam, 37
Electron flow, 49, 58, 69, 91
direction of, 58-50, 91
Electron theory, of capacitor action, 306-309
of matter, 46-49
Electrostatic charge, shielding of, 54-55
unit of, 57
Electrostatic field, principles of, 55-58
Electrostatic lines of force, $55-56$
Electrostatic meters, 198-200
Electrostatic voltmeter, 199-200
Electrostatics, 45-46
Electrothermal meters, 200-202
Elements, chemical, 46
Energy, 73-74
Equations, summary of, 490-496
Equivalent reactance, 381
Equivalent resistance, in alternatingcurrent circuits, 381
of capacitor, 342
Exponents, use of, in calculations, 150151
Eye, 28, 32

## F

Farad, 309-310
Faure plate, 107
Ferromagnetic materials, 176
Field, density of, 169-173
electric, and radio waves, 17,35
electromagnetic, about coil, 185-186
principles of, 182-187
around wire, 182-185
electrostatic, principles of, 55-58
magnetic, 169-173
and radio waves, 17,35
Field coil, 252
Filter choke, 287, 292
Filter circuits, 431-443
Filters, action of, 431-433
band-pass, 436-437
band-stop, 437
high-pass, 435-436
low-pass, 434-435
$m$-derived, 442
multisection, 438-441
$\pi$-type, 440-441
resistor-capacitor, 442-443
T-type, 439-440
types of, 434-443
Fixed capacitor, calculation of, 310-315
ceramic, 315
electrolytic, 331-343
(See also Hlectrolytic capacitor)
mica, 312-313
color code for, 502-503
oil, 315
paper, 313-315
Fixed resistor, 126-127
Fleming valve, 12
Fleming's right-hand rule, 236-237
Flux density, 172-173
Force, of attraction and repulsion, elec-
trostatic, 56-58
magnetic, 167-168
electrostatic, $55-50$
magnetic lines of, 169-171
Formed plate, of battery, 105
of capacitor, 331-333
Free electrons, 49, 70
Frequency, audio-, 24-25
beat, 27
calculation of, 18-20, 241
definition of, 18

Frequency, radio-, 18-21
ranges of, 21
of light waves, 29-30
of power systems, 242
of radio receivers, 25
of radio systems, 242
of sound waves, 24-25
Frequency modulation, 13
Frequency spectrum, 21
Frictional electricity, 49-54

## G

Galvanic cell, 87
Galvanometer, 198, 221
Gang capacitor, 319
Gauss, 175
Generation of voltage, chemical, 59, 8991
magnetic, 60-61, 235-238
thermal, 59-60
Generator, alternating-current, 239-241
construction of, 251-254
power system, 235
principle of, 239-241
direct-current, construction of, 254
power system, 234-235
principle of, 249-251
efficiency of, 261-262
simple, 238-239
Gilbert, 190-191
Graph paper, 393
Graphs, interpretation of, 397-398
plotting of, 393-398
simple, 393
use of, 393, 397-398
Grid, 12

## H

Heating effects, of current, 61-62
Heliograph, 2, 5
Henry, 271
Heusler's alloy, 176
IIigh-frequency choke, 289-292
resonant frequency of, 420-421
High-frequency resistance, 294-295
High-pass filter, 435-436
Honeycomb coils, 291
Horsepower, 72
Horseshoe magnet, 177-178
Hot-wire meters, 200

Hydraulic analogy of current flow, 66, 68
Hydrometer, 110-112
Hypotenuse, 506

## I

Iconoscope, 33
Illumination, intensity of, 20
Image impedance, 441
Impedance, calculation of, 344-347
in alternating-current circuits, $360-$ 362
in capacitor, 323-325
in inductance coil, 276-277
characteristic, 4:41-442
common, 443-444
image, 441
iterative, 441
load, 441
in parallel resonant circuit, 409-414
in series resonant circuit, 402-406
source, 441
in triangle, 361
Impedance-coupled circuit, 289
Independent variable, 393-394
Induced voltage, of generator, 238
of mutual inductance, 283
principles of, 237-239
of self-inductance, 268-269
Inductance, 267-301
calculation of, of air-core coils, 273-274
impedance method, 296-298
mutual, 280-286
self-, 271-274
solenoid, 271-274
measurement of, 229-230, 295-298
alternating-current bridge method, 229-230
nomparison method, 295-296
impedance method, 296-298
mutual, 280-286
calculation of, 280-286
circuil reactions, 280-284
self, 271-274
calculation of, 271-274
circuit reactions with, 268-271
unit, of mutual, 280-281
of self, 271
Inductance coils, 287-292
Induction, magnetic, 174-175
Inductive coupled circuits, 444-445
(See also Coupled circuits)

Inductive reactance, in alternating. current circuits, 359
of coil, 274-275
vector analysis of, 367-360
Inductors (see Coils)
Initial leakage current, 341-3.2
Instantancous value of alternating current, 242-243, 245
Instruments, 198-230 (Sec also Meters)
Insulators, breakdown voltage of, 123
classification of, 125
definition of, 123
dielectric strength of, 124
uses of, 125
Intermediate-frequency amplifier, 39
International standard units, 67-69
Interrupted current, 61-65
Ion, 90
Ionization, 90
Iron-vane meters, 205-206
Isogonic lines, 181
Iterative impedance, 441

## K

Kilo, 18, 74
Kilocycle, 18
Kilovolt, 74
Kilowatt, 74
Kilowatt-hour, 74
Kinescope, 16

## L

Lag, angle of, 279-280
Lagging current, caused by inductance, 276-280
of perfect inductor, 359
vector representation of, 367-368
Layer-Bilt battery, 100
LC checker, 407
LC product, $406-407$
$L C$ ratio, 407-408, 458
Lead, angle of, $322,32 t$
Lead-acid storage battery, 103-111
Leading current, of perfect capacitor, 322, 360
of practical capacitor, 324
vector representation of, 367-368
Leakage loss, in capacitor, 325
Left-hand rule, for a coil, 185-186
for a wire, 181-185, 268

Lenses, applications of, 31-32
concave, 31
convex, 31
definition of, 31
Lenz's law, 270
Letter symbols, 75-76, 487-489
light, 27-32
brightness of, 29
characteristics of, 28-29
color of, 29-30
infrared, 30
intersity of illumination of, 29
propagation of, 30
reflection of, 31
refraction of, 31
speed of, 29
theory of, corpuscular, 28 quantum, 28
transmission of, 28-29
ultraviolet, 30
use of in television, 27
Light waves, frequency of, 29-30
wavelength of, 29-30
Lightning, 54
Lightning rods, 54-55
Lines of force, electrostatic, 55-56
magnetic, 170-171
Litz wire, 295
Local action, 92-93
Lodestone, 163-164, 180
Loudspeaker, 35
Low-pass filter, 434-435

## M

Magnet, actions of, 166-167
artificial, 163-164
attraction of, 167-168
bar, 177
field about, 169-170
horseshoe, 177-178
laminated, 178
law of attraction and repulsion, 167-168
natural, 163
permanent, 164
permeability of, 176
poles of, 165
retentivity of, 176
ring. 178-179
strength of, 171
temporary, 165
uses of, 165

Magnetic calculations, 186-192
Magnetic circuits, 186-192
Magnetic compass, 180-181
dipping needle, 182
early use of, 162
Magnetic declination, 181-182
Magnetic deviation, 181-182
Magnetic dip, 182
Magnetic field, 169-173
about a coil, 185-186
about a wire, 182-185
intensity of, 171
radio waves, 17,35
Magnetic flux density, 172-173
Magnetic force of attraction and repulsion, 167-168
Magnetic induction, 174-175
Magnetic lines, 169-170
of force, 170-171
of induction, 169-170
Magnetic materials, classification of, 175-177
definition of, 163
Magnetic pole strength, 167-168
Magnetic poles, of earth, 179-182
of magnet, 165
Magnetic reluctance, calculation of, 188192
definition of, 175
Magnetic repulsion, 167-168
Magnetic screen, 178-179
Magnetic shielding, 179, 292-293
Magnetic units, 175-176, 188
Magnetic variation, 182
Magnetism, 162-192
electromaguctism, 182-192
molecular theory of, 166
Ohm's law for, 188, 190
residual, 176
theory of, 166-167
Magnetite, 163
Magnetomotive force, 190
Matter, structure of, 46-48
Maximum value, of alternating current, 243,245
Maxwell, 170
Mega, 19
Megacycle, 19
Megohm, 74
Meter scales, complex, 214-215
expanded, 205

Meter scales, irregular, 214
logarithmic distribution, 20t-205
multiple, 214-215
produced with concentric pole faces, 202-204
produced with eccentric pole faces, 204-205
square-law distribution, 201-205
uniform, 203-205, 214
Meters, 198-230
alternating-current, 199-202, 205-209
ammeters (see Ammeters)
combination, 225-226
direct-current, 200-207
dynamometer, 200-207
electromagnetic, 202-207
electrostatic, 198-200
electrothermal, 200-202
hot-wire, 200
how to read, 215-216
iron-vane, 205-206
ohmmeter, 222-225
(See also Ohmmeter)
permanent-magnet type, 202-205
rectifier type, 207-209
voltmeters (see Voltmeter)
Mho, 122
Micro capacitor, 318
Microampere, 74
Microfarad, 310
Micromicrofarad, 310
Microphone, 33, 35, 37
Microvolt, 74
Midget capacitor, 318
Mil, circular, 120
circular mil-foot, 120
square, 120
Milliampere, 74
Millivolt, 74
Mini-Max battery, 100-101
Modulated wave, $33,35,37,39,453$
Modulator, 32, 35
Molecular theory, of magnetism, 166
Molecule, 47
Morse code, 4, 8
Motor-generator set, 113-114
Moving-coil meters, 202-205
Multiplier, voltmeter, 218-220

Mutual inductance, calculation of, 2so282
of scries-connected inductors, $281-286$
unit of, 280-281

## N

Natural magnets, 163
Negative charge, of battery, 90-91
of capacitor, 307-309
of static electricity, 49-55
Negative electrode, 90
Negative ion, 90
Negative plate, 103-107
Nickel-iron storage cell, 103
Noninductice capacitor, 313-315
Noninductive winding, 298-299
Nonmagnetic substance, 176
Nonpolarized capacitor, 340-341
Nonsinusoidal waves, 244
North magnetic pole, 180
Nucleus, 48

## 0

Oersted, 171, 175
Ohm, 70
international, 69
megohm, 74
Ohmmeter, 222-225
parallel-resistor method of compensation, 225
principle of, 222-223
series-resistor method of compensa tion, 223-224
Ohm's law, 66-67, 70
for alternating-current circuits, 361 362
for direct-current circuits, 129
for magnetic circuits, 188,190
Optimum coupling, 464
Orbital electrons, 48-49
Ordinate, 393-397
Origin, point of, 393, 397
Oscillator, 35, 37
Oscillatory current, 63-64
Overcoupled amplifier, 458-461

## P

Padder, 320
Pancake coil, 273
Parallax, 215-216

Parallel circuits, advantages of, 136
alternating-current, solution of, 375378
characteristics of, 135
currents in, 133
definition of, 133
direct-current, resistance of, 134-135
solution of, 136-137
disadvantage of, 136
energy of, 135
magnetic, 192
power of, 135
resonant, 409-421
(See also Resonant circuits)
uses of, 136
voltages of, 134
Parallel-series circuits, alternating-current, 378-381
direct-current, 139-141
Paramagnetic material, 176
Peak ripple voltage, 341
Peak voltage, 341
Period, 241-242
Permalloy, 176
Permanent magnet, 164
Permanent-magnet moving-coil meter, 202-205
Permeability, 176
Permeability curve, 189
Permeance, 175-176
Persistence of vision, 32
Phase angle, in alternating-current circuits, 371
with capacitance and resistance, 324
with inductance and resistance, $279-$ 280
Phase relation, current and voltage, $357-$ 358
Picture-signal amplifier, 37,39
Picture tube, 39
Planetary electrons, 48-49
Planté battery plate, 107
Point of origin, 393,397
Polarization, 92, 93
Poles, of magnet, 165
Positive charge, of battery, $90-91$
of capacitor, 307-309
of static electricity, 49-55
Positive electrode, 90-91
Positive ion, 90

Positive plate, 103-107
Potential, 58
difference of, 58
Potentiometer, 146-149
taper, 147-148
uses of, 147
Poulsen are, 11
Power, actual, 369-370
apparent, 369
calculation of, in alternating-current circuits, 360-371
in direct-current circuits, 72-73
electrical, 71-73
true, 369-370
units of, 72
Power apparatus, 234-262
(See also Generators; Transformers)
Power factor, 370-371
Power supplies, types of, 234-235
Practical units used in electricity, 70
Primary cell, 89-96
Primary impedance, 288
Primary winding, 256-257, 280
Propagation, of light waves, 30
of sound waves, $22-23$
Protons, 48-54
Pulsating current, 63
Q
Q, of coil, 290, 297
definition of, 403
effect of, on impedance of parallel resonant circuit, 413-414
on slope of resonance curves, 403-40.1
on voltages of series resonant circuit, 409
on width of band pass in amplifier circuits, 455-458
on width of resonance curves, 404406
effective, of coupled circuit, 455-458

## R

Radio batteries, 98-103
(See also Battery)
Radio circuit, development of, 12-13
Radio communication, basic electrical principles of, 8-9
early history of, 8-11

Radio-frequency amplifier, 35, 37
Radio-frequency choke, 280-201, 202
Radio-frequency transformer, 291-292
Radio-receiving operations, 32-37
Radio-transmitting operations, 32-37
Radio waves, 17-22
speed of, $\mathbf{1 7}$
Reactance, capacitive, of capacitor, 321323
effect of, in alternating-current circuit, 360
vector analysis of, 367-368
equivalent, 381
inductive, effect of, in alternatingcurrent circuit, 359
of inductor, 274-275
vector analysis of, 367-368
Reciprocal, 122
Rectifier, copper oxide, battery charger, 113
meter, 207-209
tungar, 112-113
Rectifier-type ammeter, 207-209
Rectifier-type voltmeter, 207-209
Reflection, of light, 31
of sound, 26
Refraction, of light, 31
Reluctance, 175, 188-189
Reluctivity, 175
Residual magnetism, 176
Resistance, 66-67
of coil, alternating-current, 295
direct-current, 293
effect of, 276-277
direct-current, 125
equivalent, of alternating-current circuit, 381
of capacitor, 342
high-frequency, 295
measurement of, 220-223, 227
by ohmmeter, 222-223
voltmeter method of, 221-222
voltmeter-ammeter method of, 220
by Wheatstone bridge, 226-229
ohmic, 295
specific, 120-122
table of, 497
temperature coefficient of, 119
table of, 497
units of, 69,70

Resistivity, 119-120
Resisiors, adjustable, 127-128, 145-1.19
automatie resistance-control, 128
carbon, 126-127
classification of, $126-127$
color code for, 129, 500-501
equivalent circuit of, 428-430
fixed, 126-127
power rating of, 129
taper of, 147-148
tapped, 128, 151-154
tolerances of, 128
uses of, 128, 147-149
variable, $127,145-149$
voltage divider, 149-150
wire-wound, 126-128
Resonance, definition of, 392
frequency of, for parallel resonant circuit, 410
for series resonant circuit, 398-400
parallel, 409-414
series, 398-408
Resonance curves, for parallel circuit, 409-414
for series circuit, effect of capacitance on, 406-408
effect of inductance on, 406-408
effect of resistance on, 402-403
effect of $Q$ on, 403-406
methods of obtaining, 401-402
width of, 404-406
Resonant circuits, parallel, characteristics of, 417
circuit calculations of, $410-414$
classification of, 417-418
currents in, 414-415
definition of, 409
impedance of, 410-414
resonance curves of, 411, 415-416
uses of, 417, 419-421
series, characteristics of, 415-417
circuit calculations of, 398-406
classification of, 417-418
definition of, 392
effeet of resistance in, 402-403
impedance of, 402-403
resonance curves of, 401-406
uses of, 417, 419-421
voltage ratios of, 408-409
Response curves, 453-460

Retentivity, 176
Rheostat, advantages of, 149
definition of, 127
taper of, 147-148
uses of, 145, 147
Right-hand rule, Fleming's, 236-237
old, 185
Right triangle, solution of, 506
Rods, lightning, 54-55
Root-mean-square value, of alternating current, 247-248
Rotor, 315
Rotor plates, 315 -317

## S

Scale, meter, 202-205, 214-215 (See also Meter scales)
Scanning, 37-39
electronic, 15-16
mechanical, 14-15
Screen, magnetic, 178-179
Secondary cell, 89, 103-111
Sccondary winding, 256-257, 280
Sclector, 35, 37, 39
Self-inductance, 271-274
Semaphore, 1-2, 4
Sensitivity of voltmeters, 212-213
Separators, battery, 107, 114
Series circuits, alternating-current, complex, 362-364
simple, 360-362
solution of problems for, 371-379
vector analysis of, 365-369
characteristics of, 131
currents in, 130
definition of, 130
direct-current, resistance of, 130
solution of problems for, 132-133
disadvantages of, 132
encrgy of, 131
magnetic, 191-192
power of, 131
resonant, 398-409, 415-421
(See also Resonant circuits)
uses of, 131-132
voltages of, 130
Series-parallel circuits, alternating-current, 381-384
direct-current, 138-139
Shiclding, electrostatic, 54-55
magnetic, 292-203

Shunt, meter, 215-218
Side bands, 453
Signaling, 1-2
Silicon steel, 288
Sine, 506
Sine table, 507-508
Sine-wave voltage, 243
Sine waves, addition of, 250-251
Skin effect, 294
Slope of resonance curve, effect of coupling on, 455-460
effect of resistance on, 403-404
Solenoid, calculating inductance of, 271274
principles of, 185-186
Sound, 22-27
beat frequency of, 27
characteristics of, 22-23
forced vibration of, 27
frequency of, 23-24
frequency ranges of, 24-25
frequency range of radio receivers for, 25
fundamental note in, 26
harmonics in, 26
intensity of, 23
loudness of, 23
musical, 25
noise, 25
overtones, 26
pitch of, 24
quality of, 26
reflection of, 26
speed of, 23
sympathetic vibration caused by, 2627
use of, in radio, 22
wavelength of, 24
Sound waves, 22-27
audible, 32,35
forced vibrations of, 27
frequency ranges of, $24-2 \boldsymbol{0}$
frequency ranges of, in radio receivers, 25
resonance caused by, 27
sympathetic vibrations of, 26-27
Source impedance, 441
South geographic pole, 179-180
Specific reluctance, 189
Specific resistance, $120-122$
Specific resistivity, 119-120

Speech amplifier, 35, 37
Split-stator capacitor, 318-319
Square root, 79-82
Stagger tuning of amplifier circuits, 458-461
Standard units, in electricity, 67-70
Static electricity, definition of, 45
explanation of, 49-57
production of, 45
use of, 46
Stator, alternating-current generator 253
in variable capacitor, 315
Stator plates, 315-317
Storage battery, capacity of, 106, 109-111
care of, 113-115
charging of, 103-106, 111-113
commercial cells for, 106-109
construction of, 106-109
lead-acid type of, 103-111
rating of, 109-111
testing of, 111-112
Strength, dielectric, 124 table of, 499
Subscript method of notation, 76
Substances, for conductors, 118-123
diamagnetic, 176
ferromagnetic, 176
for insulators, 123-125
magnetic, 175-176
nonmagnetic, 176
paramagnetic, 176
Sulphation, 115
Surge voltage, 341
Sweep circuits, 37-39
Symbols, 75-76
drawing, 75, 477-486
electrical and radio, 477-486
letter, 76, 487-489
Synchronizing circuits, 37-39

## T

Tables, of abbreviations, 487-489
color-coded, for resistors, $500-501$
cosine, 507-508
dielectric constant, 499
dielectric strength, 499
of drawing symbols, 477-486
of letter symbols, 487-489
sine, 507-508

Tables, specific resistance, 497
temperature coeflicient, 497
wire, 498
Taper, 147-148
Telegraph, 3-4
Telephone, 4-7
Television, 13-17, 32-35, 37-39
basic principles of, 13-17
iconoscope, 15
kinescope, 16
lines-per-picture, 16
scanning, ${\underset{\tau}{\tau}}^{1} 4-16$
camera tube in, 33,37
iconoscope, 15,33
image dissector, 15
channels in, 33-35, 37
definition of, 27
early history of, 13-17
picture carrier wave in, 39
principles of operation of, 32-35, 3739
sound carrier wave in, 39
use of inductors in, 300-301
use of light in, 27
Television-receiving operations, 32-35, 37-39
Television-transmitting operations, 3235, 37-39
Temperature coefficient of resistance, 119-120
Temporary magnet, 165
Theorem of Pythagoras, 506
Thermal-type meters, 200-202
Thermocouple, 59-60
Thomson inclined-coil meter, 206
Time constant, 277-279, 460-470
for capacitance and resistance, 465-468
for inductance and resistance, 277-279, 460-465
universal curves of, 469-470
'Iime-delay circuits, 460-471
Transformer, audio, 287-289
color code for, 292, 504-505
core of, 288
in coupling, 288-289
efficiency of, 261-262
intermediate-frequency, 292
output, 288
power, 254-260
principles of, 256-259

Transformer, radio-frequency, 291-292
uses of, 254-255, 300
Transverse waves, 28
'Trigonometric functions, 506-509
Trigonometry, 506
Trimmer, 320
Tuned circuit, 402
(See also Resonant circuits)
Tungar rectifier, 112-113
Tuning, 35, 37, 39
Turns ratio for transformers, 257

## U

Unidirectional current, 63
Unit magnet pole, 167
Units, electrical, 67-74
magnetic, 175-176, 188
Universal time-constant curve, 469-470
Cniversal winding, 291

## V

Vacuum tube, 11-13
Variable, dependent, 393-395
independent, 393-395
Variable capacitor, calculation of, 319320
construction of, 315-317
gang, 319
multiple, 319
ranges of, 317-318
split-stator, 318-319
transmitting, 318
Variable inductor, 284-286
Variable resistor, 127, 145-149
Variocoupler, 284-286
Variometer, 284-286
Vector, definition of, 365
direction of rotation of, 365
graphical method of solution by, 365366
mathematical method of solution by, 366
sine function and, 366
solution of circuit impedance by,
with resistance and capacitance in series, 368
with resistance and inductance in series, 367-368
with resistance, inductance, and capacitance in series, 369

Vector method of determining in-phase and quadrature components of current, 378
Vector representation, of sine-wave voltage, 365
of voltage and current, 366-367
with current in phase with voltage, 366
with current lagging voltage, 366-367
with current leading voltage, 366-367
in parallel circuit, 377,378
in parallel-series circuit, 381
in series circuit, 372,374
in series-parallel circuit, 384
of wheel diagram, 365-366
Video amplifier, 37, 39
Vision, persistence of, 32
Visual means of communication, 1-2
Volt, 70
international, 69
microvolt, 74
millivolt, 74
Voltage, 66, 70
average value of alternating-current, 243-245
breakdown, of capacitors, $326-327$
of insulators, 123
counter, 449
drop of, 130
effective value of alternating-current, 245-249
gain of, in series resonant circuit, 408409, 419-420
induced, of generator, 238
of mutual inductance, 283
principle of, 237-239
of self-inductance, 268-269
instantaneous value of alternating current, 242-243, 245
maximum value of alternating current, 243, 245
measurement of, 210-213
methods of producing, 59-61
Voltage divider, 149-154
calculation of, 151-154
power rating of, 153-154
Voltaic cell, 87, 89-91
Voltampere, 369
Voltmeter, alternating-current, 198-202, 205-209
connecting of, 210-211

Voltmeter, D'Arsonval-type, 202
direct-current, 202-207
dynamometer-type, 206-207
electrostatic-type, 198-200
hot-wire type, 200
increasing range of, 212, 218-220
iron-vane-type, 205-206
method of measuring resistance with, 221-222
permanent-magnet moving-coil type of, 202-205
precautions in using, 213
reading, 215, 216
rectifier-type, 207-209
scales on, 214-215
sensitivity of, 212-213
thermocouple-type, 200-202
Voltmeter multipliers, 218-220

## W

Watt, 72
Wave, audio, 32, 35
carrier, 32
longitudinal, 22-23
modulated, $33,35,37,39$
nonsinusoidal, 244
picture carrier, 37-39
radio, 17-22
sine, 243
sound, 22-27
sound carrier, 37-39
transverse, 28

Wave trap, 419
Wavelength, 18-20
calculation of 18,20
definition of, 18
of light waves, 2930
of sound waves, 2:3-24
in spectrum, 21
Weber's theory of magnetism, 166
Wet electrolytic capacitors, 331-335, 339-343
(See also dectrolytic capacitors;
Wheatstone bridge, 226-229
Wheel diagram, 244, 365-366
Wide-band-pass amplifier circuits, 458460
methods of obtaining, 458, 459
in frequency-modulated receivers, 458459
in television receivers, 458-460
Width, of band pass, 455-458
of frequency band, 404-406
Wigwag, 1, 3
Wire, magnetic field around, 182-185
resistance of, $120-122$
table, 498
Wire gauge, 121
Wire table, 498
Wireless, 7-11
Wire-wound resistors, 126-128
Work, 71-72
Z
Zero potential, 58


[^0]:    ${ }^{1}$ For the identification of resistance values of small carbon-type resistors, numbers are represented by the following colors:

