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Foundations of Wireless and Electronics

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M. G. SCROGGIE<br>B.Sc., C.Eng., F.I.E.E.

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## Preface

Wireless-or radio-has branched out and developed so tremendously that very many books would be needed to describe it all in detail. In ordinary conversation, 'radio' has come to mean sound broadcasting, as distinct from 'TV' which gives pictures as well as sound, but of course the pictures come by radio just as much as the sound. Besides broadcasting sound and vision, radio is used for communication with and between ships, cars, aircraft, satellites and spacecraft; for direction-finding and radar (radiolocation), photograph and "facsimile" transmission, telegraph and telephone links, meteorological probing of the upper atmosphere, astronomy, and other things. Very similar techniques are applied on an increasing scale to industrial control ('automation') and scientific and financial computation. All of these are based on the same foundational principles now generally comprised in the word electronics. The purpose of this book is to start at the beginning and lay these foundations, on which more detailed knowledge can then be built.

At the beginning. . . . If you had to tell some body about a cricket match you had seen, your description would depend very much on whether or not your hearer was familiar with the jargon of the game. If he wasn't and you assumed he was, he would be puzzled. If it was the other way about, he would be irritated. There is the same dilemma with electronics. It takes much less time to explain it if the reader is familiar with methods of expression, such as symbols, that are taken for granted in technical discussion but not in ordinary conversation. This book assumes hardly any special knowledge. But if the use of graphs and symbols had to be completely excluded, or else accompanied everywhere by digressions explaining them, it would be very boring for the initiated. So the methods of technical expression are explained separately in a preliminary Initiation. Those already initiated can of course skip it ; but it might be as well just to make sure, because this is a most essential foundation.

Then there are the technical terms. They are explained one by one as they occur and their first occurrence is distinguished by printing in italics; but in case any are forgotten they can be looked up at the end of the book; and so can the symbols and abbreviations. These references are there to be used whenever the meaning of anything is not understood.

Most readers find purely abstract principles very dull; it is more stimulating to have in mind some application of those principles. As it would be confusing to have all the applications of electronics
or even of radio in mind at once, broadcasting of sound is mentioned most often because it directly affects nearly everyone. But the same principles apply more or less to all the other things. The reader who is interested in the communicating of something other than sound has only to substitute the appropriate word.

1975
M. G. Scroggie

## Initiation into the Shorthand of Electronics

Glancing through this book, you can see numerous strange signs and symbols. Most of the diagrams consist of little else while the occasional appearance of what looks like algebra may create a suspicion that this is a Mathematical Work and therefore quite beyond a beginner.

Yet these devices are not, as might be supposed, for the purpose of making the book look more learned or difficult. Quite the contrary. Experience has shown them to be the simplest, clearest and most compact ways of conveying the sort of information needed.

The three devices used here are Graphs, Circuit Diagrams, and Algebraic Symbols. The following explanations are only for readers who are not quite used to them.

### 0.1 Algebraic Symbols

If a car had travelled 90 miles in 3 hours, we would know that its average speed was 30 miles per hour. How? The mental arithmetic could be written down like this:

$$
90 \div 3=30
$$

That is all right if we are concerned only with that one particular journey. If the same car, or another one, did 7 miles in a quarter of an hour the arithmetic would have to be:

$$
7 \div \frac{1}{4}=7 \times 4=28
$$

To let anyone know the speed of any car on any journey, it would be more than tedious to have to write out the figures for every possible case. All we need say is, ${ }^{\cdot}$ To find the average speed in miles per hour, divide the number of miles travelled by the number of hours taken'.

What we actually would say would probably be briefer still: 'To get the average speed, divide the distance by the time'. Literally that is nonsense, because the only things that can be arithmetically divided or multiplied are numbers. But of course the words 'the number of' are (or ought to be!) understood. The other words'miles per hour', 'miles', 'hours' - the units of measurement, as they are called, may also perhaps be taken for granted in such an easy

## 18

case. In others, missing them out might lead one badly astray. Suppose the second journey had been specified in the alternative form of 7 miles in 15 minutes. Dividing one by the other would not give the right answer in miles per hour. It would, however, give the right number of miles per minute.

In electronics, as in other branches of physics and engineering, this matter of units is often less obvious, and always has to be kept in mind.

Our instruction, even in its shorter form, could be abbreviated by using mathematical symbols as we did with the numbers:

Average speed $=$ Distance travelled $\div$ Time taken.
Here we have a concise statement of general usefulness; note that it applies not only to cars but to aircraft, snails, bullets, space capsules, and everything else that moves.

Yet even this form of expression becomes tedious when many and complicated statements have to be presented. So for convenience we might write:
or alternatively

$$
\begin{aligned}
& S=D \div T \\
& S=\frac{D}{T} \\
& S=D / T
\end{aligned}
$$

### 0.1.1 WHAT LETTER SYMBOLS REALLY MEAN

This is the stage at which some people take fright, or become impatient. They say, 'How can you divide $D$ by $T$ ? Dividing one letter by another doesn't mean anything. You have just said yourself that the only things that can be divided are numbers!' Quite so. It would be absurd to try to divide $D$ by $T$. Those letters are there just to show what to do with the numbers when you know them. $D$, for example, has been used to stand for the number of miles travelled.

The only reason why the $S, D$ and $T$ were picked for this duty is that they help to remind one of the things they stand for. Except for that there is no reason why the same information should not have been written as:
or even

$$
\begin{aligned}
& x=\frac{y}{z} \\
& \alpha=\frac{\beta}{\gamma}
\end{aligned}
$$

so long as we know what these symbols were intended 10 mean. As there are only 26 letters in our alphabet, and fewer still in the Greek, we cannot allocate any one of them permanently to mean "average speed in miles per hour". $x$. in particular. is notoriously capable of meaning absolutely anything. Yet to write out the exact meaning every time would defeat the whole purpose of using the letters. How, then, does one know the meaning?

Well, some meanings have been fixed by international agreement. There is one symbol that means the same thing every time, not only in electronics but in all the sciences-the Greek letter $\pi$ fread as "pie'). It is a particularly good exanuple of abbreviation because it stands for a number that would take eternity to write out in fullthe ratio of the circumference of a circle to its diameter, which begins: $3 \cdot 14 \mid 5926535 \ldots$. (The first three or four decimal places give enough accuracy for most purposes.)

Then there is a much larger group of symbols which have been given meanings that hold good throughout a limited field such as electrical engineering. or one of its subdivisions such as electronics, but are liable to mean something different in. say, astronomy or hydraulics. These have to be learnt by anyone going in for the subject seriously. A list of those that concern us appears on pages 5046 of this book: but do not try to learn them there. It is much easier to wat till they turn up one by one in the body of the book.

Lastly, there are symbols that one uses for all the things that are not in a standard list. Here we are free to choose our own: but there are some rules it is wise to observe. It is common sense to steer clear as far as possible from symbols that already have established meanings. The important thing is to state the meaning when first using the symbol. It can be assumed to bear this meaning to the end of the particular occasion for which it was attached: after that. the label is taken off and the symbol thrown back into the common stock, ready for use on another occasion, perhaps with a different label-provided it is not likely to be confused with the first.

Sometimes a single symbol might have any of several different meanings. and one his to decide which it bears in that particular context. The Greek letter $\mu$ (pronounced "mew') is an example that occurs in this book. When the subject is a radio valve. it can be assumed to mean "amplification factor'. But if iron cores for transformers are being discussed, $\mu$ should be read as "permeability". And if it comes before an upright letter it is an abbreviation for "micro-'. meaning one mitlionth of".

### 0.1.2 SOME OTHER USES OF SYMBOLS

The last of these three meanings of $\mu$ is a different kind of meaning altogether from those we have been discussing. Until then we had
been considering symbols as abbreviations for quantities of certain specified things, such as speed in miles per hour. But there are several other uses to which they are put.

Instead of looking on ' $S=D / T$ ' as an instruction for calculating the speed, we can regard it as a statement showing the relationship to one another of the three quantities, speed, time and distance. Such a statement, always employing the 'equals' sign, is called by mathematicians an equation. From this point of view $S$ is no more important than $T$ or $D$, and it is merely incidental that the equation was written in such a form as to give instructions for finding $S$ rather than for finding either of the other two. We are entitled to apply the usual rules of arithmetic in order to put the statement into whatever form may be most convenient when we come to substitute the numbers for which the letters stand.

For example, we might want to be able to calculate the time taken on a journey, knowing the distance and average speed. We can divide or multiply both 'sides' of an equation by any number (known, or temporarily represented by a letter) without upsetting their equality. If we multiply both sides of $S=D / T$ by $T$ we get $S T=D(S T$ being the recognized abbreviation for $S \times T)$. Dividing both sides of this new form of the equation by $S$ we get $T=D / S$. Our equation is now in the form of an instruction to divide the number of miles by the speed in miles per hour (e.g., 120 miles at 24 m.p.h. takes 5 hours).

When you see books on electronics (or any other technical subject) with pages covered almost entirely with mathematical symbols, you can take it that instead of explaining in words how their conclusions are reached, the authors are doing it more compactly in symbols. It is because such pages are concentrated essence, rather than that the meanings of the symbols themselves are hard to learn, that makes them difficult. The procedure is to express the known or assumed facts in the form of equations, and then to combine or manipulate these equations according to the established rules in order to draw some useful or interesting conclusions from those facts.

This book, being an elementary one, explains things in words, and only uses symbols for expressing the important facts or conclusions in concise form, or for rearranging them.

### 0.1.3 ABBREVIATIONS

Another use for symbols is for abbreviation pure and simple. We have already used one without explanation-m.p.h.-because it is well known that this means 'miles per hour'. The mile-per-hour is a unit of speed. ' $£$ ' is a familiar abbreviation for the pound sterling. The unit of electrical pressure is the volt, denoted by the abbreviation V. Sometimes it is necessary to specify very small voltages, such as

5 millionths of a volt. That could be written $0 \cdot 000005 \mathrm{~V}$. But a more convenient abbreviation is $5 \mu \mathrm{~V}$, read as ' 5 microvolts'. The list on pages 5046 gives the abbreviations commonly used in electronics.

Still another use for letters is to point out details on a diagram. $R$ stands for electrical resistance, but one has to guess whether it is intended to mean resistance in general, as a property of conductors, or the numerical value of resistance in an equation, or the particular resistance marked $R$ in a diagram. Often it may combine these meanings, being understood to mean "the numerical value of the resistance marked $R$ in Fig. So-and-so'.

Attaching the right meanings to symbols probably sounds dreadfully difficult and confusing. So do the rules of a new game. The only way to defeat the difficulties is to start playing the game.

But before starting to read the book, here are a few more hints about symbols.

### 0.1.4 HOW NUMBERS ARE USED

One way of making the limited stock of letters go farther is to use different kinds of type. It has become a standard practice to distinguish symbols for physical quantities by italic (sloping) letters, leaving the roman (upright) ones for abbreviations; for example, ' $V$ ' denotes an unspecified amount of electrical potential difference, reckoned in volts, for which the standard abbreviation is ' $V$ '. ' $R$ ' is the resistance of a resistor $R$. And so on.

Another way of making letters go farther is to number them. To prevent the numbers from being treated as separate things they are written small near the foot of the letter ('subscript'). For example, if we want to refer to several different resistances we can mark them $R_{1}, R_{2}, R_{3}$, etc. Sometimes a modification of a thing denoted by one symbol is distinguished by a tick or dash; $A$ might stand for the amplification of a receiver when used normally, and $A^{\prime}$ when modified in some way.

But on no account must numbers be used "superscript" for this purpose, because that already has a standard meaning. $5^{2}$ (read as ' 5 squared') means $5 \times 5: 5^{3}$ (‘5 cubed') means $5 \times 5 \times 5: 5^{4}$ (' 5 to the 4 th power') means $5 \times 5 \times 5 \times 5 ; x^{2}$ means the number represented by $x$ multiplied by the same number.
$x^{-2}$ seems nonsensical according to the rule just illustrated. It has been agreed, however, to make it mean $1 / x^{2}$ (called the reciprocal of $\lambda$ squared). The point of this appears most clearly when these superscripts, called indices (singular: index), are applied to the number $10.10^{4}=10000,10^{3}=1000,10^{2}=100,10^{1}=10,10^{0}=1$. $10^{-1}=0 \cdot 1,10^{-2}=0 \cdot 01$, and so on. The rule is that the power of 10 indicates the number of places the decimal point has to be away from 1: with a positive index it is on the right: negative, on the left. Advantage is taken of this to abbreviate very large or very small
numbers. It is easier to see that 0.0000000026 is 2.6 thousandmillionths if it is written $2.6 \times 10^{-9}$. Likewise $2.6 \times 10^{12}$ is briefer and clearer than 2600000000000.
$X^{\frac{1}{2}}$ also needs explanation. It is read as 'the square root of $x$ ', and is often denoted by $\sqrt{ } x$. It signifies the number which, when multiplied by the same number, gives $x$. In symbols, $\left(v^{\prime} x\right)^{2}=x$.

Note that $10^{6} \times 10^{6}=10^{12}$, and that $10^{12} \times 10^{-3}=10^{12} / 10^{3}$ $=10^{9}$. In short, multiply by adding indices, and divide by subtracting them. This idea is very important in connection with decibels (Sec. 19.2).

### 0.2 Graphs

Most of us when we were in the growing stage used to stand bolt upright against the edge of a door to have our height marked up. The succession of marks did not convey very much when reviewed afterwards unless the dates were marked too. Even then one had to look closely to read the dates, and the progress of growth was difficult to visualize. Nor would it have been a great help to have presented the information in the form of a table with two columnsHeight and Date.

But, disregarding certain technical difficulties, imagine that the growing boy had been attached to a conveyor belt which moved him horizontally along a wall at a steady rate of, say, one foot per year, and that a pencil fixed to the top of his head had been tracing


Fig. 0.1-Simple (but inconvenient) system for automatic graph plotting of human growth. The equipment consists of a conveyor belt moving at the rate of one foot per year, a pencil, and a white wall
a line on the wall (Fig. 0.1). If he had not been growing at all, this line would, of course, be straight and horizontal. If he had been growing at a uniform rate, it would be straight but sloping upwards.

A variable rate of growth would be shown by a line of varying slope.

In this way the progress over a period of years could be visualized by a glance at the wall.

### 0.2.1 scales

To make the information more definite, a mark could have been made along a horizontal line each New Year"s Day and the number of the year written against it. In more technical terms this would be a time scale. The advantage of making the belt move at uniform speed is that times intermediate between those actually marked can be identified by measuring off a proportionate distunce. If one foot represented one year, the height at, say, the end of May in any year could be found by noting the height of the pencil line 5 inches beyond the mark indicating the start of that year.

Similarly a scale of height could be marked anywhere in a vertical direction. It happens in this case that height would be represented by an equal height. But if the graph were to be reproduced in a book, although height would still represent height it would have to do so on a reduced scale, say half an inch to a foot.

To guide the eye from any point on the information line to the two scales, it is usual to plot graphs on paper printed with horizontal and vertical lines so close together that a pair of them is sure to come near enough to the selected point for any little less or more to be judged. The position of the scales is not a vital matter, but unless there is a reason for doing otherwise they are marked along the lines where the other quantity is zero. For instance, the time scale would be placed where height is nil; i.e., along the foot of the wall. If, however, the height scale were erected at the start of the year 1 A.D. there would be nearly half a mile of blank wall between it and the pencil line. To avoid such inconvenience we can use a 'false zero', making the scale start at or slightly below the first tigure in which we are interested. A sensible way of dong it in this example would be to reckon from the time the boy was born, as in Fig. 0.1. The point that is zero on both scales is called the origin.

False zeros are sometimes used in a slightly shady manner to give a wrong impression. Fig. $0.2 a$ shows the sort of graph that might appear in a company-promoting prospectus. The word 'Profit' would, of course be in a big type, and the figures indistinctly, so that the curve would seem to indicate a sensational growth in profits. Plotted without a false zero, as in Fig. $0.2 b$, it looks much less impressive. Provided that it is clearly admitted, however, a false zero is useful for enabling the significant part of the scale to be expanded and so read more precisely.
Fig. 0.2 , by the way, is not a true graph of the sort a mathematician would have anything to do with. Profits are declared
annually in lumps, and the lines joining the dots that mark each year's result have no meaning whatever but are there merely to guide the eye from one to another. In a true graph a continuous variation


Fig. 0.2-(a) Typical financial graph arranged to make the maximum impression consistent with strict truthfulness. (b) The same information presented in a slightly different manner
is shown, and the dots are so close together as to form a line. This line is technically a curve, whether it is what is commonly understood as a curve or is as straight as the proverbial bee-line.

### 0.2.2 what a 'Curve’ signifies

Every point on the curve in Fig. 0.1 represents the height of the boy at a certain definite time (or, put in another way, the time at which the boy reached a certain height). Since it is possible to distinguish a great number of points along even a small graph, and each point is equivalent to saying 'When the time was $T$ years, the height was $H$ feet', a graph is not only a very clear way of presenting information, but a very economical one.

Here we have our old friend $T$ again, meaning time, but now in years instead of hours. It is being used to stand for the number of years, as measured along the time scale, represented by any point on the curve. Since no particular point is specified, we cannot tell how many feet the corresponding height may be, so have to denote it by $H$. But directly $T$ is specified by a definite number. the number or value of $H$ can be found from the curve. And vice versa.

In its earlier role (Sec. 0.1) $T$ took part in an equation with two other quantities, denoted by $S$ and $D$. The equation expressed the relationship between these three quantities. We now see that a graph is a method of showing the relationship between two quantities. It is particularly useful for quantities whose relationship
is too complicated or irregular to express as an equation-the height and age of a boy, for instance.

### 0.2.3 THREE-DIMENSIONAL GRAPHS

Can a graph deal with three quantities? There are three ways in which it can, none of them entirely satisfactory.

One method is to make a three-dimensional graph by drawing a third scale at right angles to the other two. Looking at the corner of a room, one can imagine one scale along the foot of one wall, another at the foot of the other wall, and the third upwards along the intersection between the two walls. The "curve" would take the form of a surface in the space inside-and generally also outsidethe room. There are obvious difficulties in this.

Another method is to draw a number of cross-sections of the three-dimensional graph on a two-dimensional graph. In other words, the three variable quantities are reduced to two by assuming some numerical value for the third. We can then draw a curve showing the relationship between the two. A different numerical value is then assumed for the third, giving (in general) a different graph. And so on.

Let us take again as an example $D=S T$. If any numerical value is assigned to the speed. $S$, we can easily plot a graph connecting $D$ and $T$, as in Fig. 0.3a. When the speed is zero, the distance


Fig. 0.3a-One way of graphing the relationship between speed, time and distance of a journey. Graphs of speed against time for any given distance can be derived from it (b)
remains zero for all time, so the ' $S=0$ ' curve is a straight line coinciding with the time scale. When $S$ is fixed at 1 mile per hour, $D$ and $T$ are always numerically equal, giving a line ( ${ }^{\circ} S=l^{\prime}$ ) sloping up with a 1 -in-l gradient (measured by the $T$ and $D$ scales). The
' $S=2$ ' line slopes twice as steeply; the ' $S=3$ ' line three times as steeply, etc.

If a fair number of $S$ curves are available, it is possible to replot the graph to show, say, $S$ against $T$ for fixed values of $D$. A horizontal line drawn from any selected value on the $D$ scale in Fig. $0.3 a$ (such as $D=10$ ) cuts each $S$ curve at one point, from which the $T$ corresponding to that $S$ can be read off and plotted, as in Fig. 0.3b. Each point plotted in $a$ corresponds to an $S$ curve in $b$, and when joined up they give a new curve (really curved this time!). This process has been demonstrated for only one value of $D$, but of course it could be repeated to show the time/speed relationship for other distances. Fig. $3 b$ gives numerical expression to the wellknown fact that the greater the speed the shorter is the time to travel a specified distance.

Graphs of this kind are important in many branches of electronics; for example, with transistors, whose relationships cannot be accurately expressed as equations.

Lastly, two (or more) variables can be grouped so as to reduce the total to two. In our $D=S T$. $D$ can be plotted against $S T$, which in mathematical language is called the product of $S$ and $T$. The resulting very simple graph is shown as Fig. 0.4. A point on the $S T$ scale.


Fig. 0.4-A third method of graphing three variables is to combine two of them: in this example the product of $S$ and $T$ is used as one of the two variables in the graph
say 60 , shows the distance travelled in 2 hours at 30 miles per hour. 3 hours at 20 miles per hour, etc. No one would bother to draw a graph for such a simple equation. but this is quite a useful technique for more complicated ones. Fig. 20.7 is an example.

### 0.2.4 SIGNIFICANCE OF SLOPE

Even from simple two-dimensional graphs like Fig. 0.1 it may be possible to extract information that is worth replotting separately.

It is clear that a third quantity is involved-rate of growth. This is not an independent variable; it is determined by the other two. For it is nothing else than rate-of-change-of-height. As we saw at the start. when the curve slopes upward steeply, growth is rapid; when the curve flattens out. it means that growth has ceased; and if the curve started to slope downwards-a negative gradient-it would indicate negative growth. So it would be possiole to plot growth in, say, inches per year, against age in years. This idea of rate-of-change, indicated on a graph by slope, is the essence of no less a subject than the differential calculus, which is much easier than it is often made out to be. If you are anxious to get a clearer view of any electrical subject it would be well worth while to read at least the first few chapters of an elementary book on the differential calculus, such as S. P. Thompson's classic 'Calculus Made Easy’ (Macmillan).

### 0.2.5 NON-LNIFORM SCALES

Finally. although uniform scales for graphs are much the easiest to read, there may quite often be reasons which justify some other sort of scale. The most important. and the only one we need consider here, is the logarithmic scale. In this, the numbers are so spaced out that a given distance measured anywhere along the scale represents. not a certain addition, but a certain ratio or multiplication. Musicians use such a scale (whether they know it or not); their intervals correspond to ratios. For example, raising a musical note by an octave means doubling the frequency of vibration, no matter whereabouts on the scale it occurs. Users of slide-rules are familiar with the same sort of scale. Readers who are neither musicians nor slide-rule pushers can see the difference by looking at Fig. 0.5, where $a$ is an ordinary uniform (or linear) scale and $b$ is logarithmic. If we start with $a$ and note any two scale readings $1 \frac{1}{2}$ inches apart we find that they always differ by 10 . Applying the same test to $b$ we find that the difference may be large or small, depending on whether the inch and a half is near the top or bottom. But the larger reading is always 10 times the smaller.

One advantage of this is that it enables very large and very small readings to be shown clearly on the same graph. Another is that with some quantities (such as musical pitch) the ratio is more significant than the numerical interval. Fig. 20.11 is an example of logarithmic scales.

### 0.3 Circuit Diagrams

It is easier to identify the politicians depicted in our daily papers in the cartoons than in the news photos. There is a sense in which


Fig. 0.5-Comparison between uniform or 'linear' scale (a), and logarithmic scale (b)

Fig. 0.6-Alternative methods of distinguishing crossing from connected wires in circuit diagrams
the distorted versions of those individuals presented by the caricaturist are more like them than they are themselves. The distinguishing features are picked out and reduced to conventional forms that can be recognized at a glance.

A photograph of a complicated machine shows just a maze of wheels and levers, from which even an experienced engineer might derive little information. But a set of blue-prints, having little relation to the original in appearance, would enable him, if necessary, to reproduce such a machine.

There should be no further need to argue why circuit diagrams are more useful than pictures of radio sets or other electronic equipinent.

In a circuit diagram each component or item that has a significant electrical effect is represented by a conventional symbol, and the wires connecting them up are shown as lines. Fortunately. except for a few minor differences or variations, these symbols are recognizably the same all over the world. Most of them indicate their functions so clearly that one could guess what they mean. However, they are introduced as required in this book.

### 0.3.1 ALTERNATIVE METHODS

One convention about which there are differences of opinion is the way in which crossing wires should be distinguished from connected wires. Fig. 0.6a shows the commonest method. Crossing wires are assumed to be unconnected unless marked by a blob. as at $b$. There is obviously a risk that either the blob may be omitted where needed or may form unintentionally (especially when the lines are drawn in ink) where not wanted. Even when correctly drawn the difference is not very conspicious. Method $h$ ought never to be used. especially akong with $a$. Connecting crossings should always be staggered, as at $c$. To distinguish unconnected crossings more clearly they are sometimes drawn as at $d$. To make quite sure, the author uses methods $e$ and $c$ respectively to denote unconnected and connected crossings.

Another variation is that some people omit to draw a ring round the symbols that represent the 'innards' of a valve or transistor. In this book they are drawn, because they make the transistors or valves, which are usually key components. stand out distinctly.

### 0.3.2 IMPORTANCE OF LAYOUT

There is a good deal more in the drawing of circuit diagrams than just showing all the right symbols and connections. If the letters composing a message were written at random all over the paper, nobody would bother to read it even if the correct sequence were
shown by a maze of connecting lines-unless it was a clue to valuable buried treasure. And a circuit diagram, though perfectly accurate, is difficult to 'read' if it is laid out in an unfamiliar manner. The layout has by now become largely standardized, though again there are some variations. It is too soon in the book to deal with layout in detail, but here are some general principles for later reference.

The first is that the diagram should be arranged for reading from left to right. In a radio receiver, for example, the aerial should be on the left and the final item, the loudspeaker, on the right.

Next, there is generally an 'earthed' or 'low-potential' connection. This should be drawn as a thicker line, usually straight across the diagram. The most positive potential connections should be at the top and the most negative at the bottom. This helps one to visualise the potential distribution and the direction of current flow (normally downward).

Long runs of closely parallel wires are difficult to follow and should be avoided.

As with written symbols, facility in drawing and reading circuit diagrams soon comes with practice.

### 0.3.3 WHERE CIRCUIT DIAGRAMS CAN MISLEAD

One warning is needed in connection with circuit diagrams, especially when going on to more advanced work. They are an indispensably convenient aid to thought, but it is possible to allow one's thoughts to be moulded too rigidly by them. The diagram takes for granted that all electrical properties are available in separate lumps, like chemicals in bottles, and that one can make up a circuit like a prescription. Instead of which they are more like the smells from the said chemicals when the stoppers have been left open-each one strongest near its own bottle. but pervading the surrounding space and mixing inextricably with the others. This is particularly true at very high frequencies, so that the diagram must not always be taken as showing the whole of the picture. Experience tells one how far a cirċuit diagram can be trusted.

## CHAPTER 1

## General View of a System

Most of the chapters of this book are about subjects the usefulness of which, taken by themselves. might not be at all obvious. Proverbially, one might not be able to see the wood for the trees. So before examining these things in detail we may find it helpful to take a general view of a complete electronic system. The choice is a broadcasting and receiving system, because it is so familiar in everyday life and it happens to embody most of the principles of electronics.

### 1.1 What Wireless Does

People who remark on the wonders of wireless seldom seem to consider the fact that all normal persons can broadcast speech and song merely by using their voices. We can instantly communicate our thoughts to others, without wires or any other visible lines of communication and without even any sending or receiving apparatus outside of ourselves. If anything is wonderful, that is. Wireless, or radio, is merely a device for increasing the range.

There is a fable about a dispute among the birds as to which could fly the highest. The claims of the wren were derided until, when the great competition took place and the eagle was proudly outflying the rest, the wren took off from his back and established a new altitude record.

It has been known for centuries that communication over vast distances of space is possible-it happens every time we look at the stars and detect light coming from them. We know, then, that a long-range medium of communication exists. Radio simply uses this medium for the carrying of sound and vision.

## Simply?

Well, there actually is quite a lot to learn about it. But the same basic principles can be applied to many other electronic systems.

### 1.2 Nature of Sound Waves

Radio, as we have just observed, is essentially a means of extending natural communication beyond its limits of range. It plays the part of the swift eagle in carrying the wren of human speech to remote distances. So first of all let us see what is required for 'natural' communication.

There are three essential parts: the speaker, who generates sound; the hearer who receives it; and the medium, which connects one with the other.

The last of these is the key to the other two. By suspending a source of sound in a container and extracting the air (Fig. 1.1) one can


Fig. 1.1-Experiment to demonstrate thett somend waves cammot cross empty space. As the air is removed from the glass vessel. the sound of the heli fades aural
demonstrate that sound cannot travel without air (or some other physical substance) all the way between sender and receiver.

Anyone who has watched a cricket match will recall that the smack of bat against ball is heard a moment after they are seen to meet; the sound of the impact has taken an appreciable time to travel from the pitch to the viewpoint. If the pitch were 1100 feet away, the time delay would be one second. The same speed of travel is found to hold good over longer or shorter distances. K nowing the damage that air does when it travels at even one-tenth of that speed (hurricane force) we conclude that sound does not consist in the air itself leaping out in all directions.

When a stone is thrown into a pond it produces ripples, but the ripples do not consist of the water which was directly struck by the stone travelling outwards until it reaches the banks. If the ripples pass a cork floating on the surface they make it bob up and down; they do not carry it along with them.
What does travel, visibly across the surface of the pond, and invisibly through the air, is a wolve, or more often a succession of waves. While ripples or waves on the surface of water do have much in common with waves in general, they are not of quite the same type as sound waves through air or any other medium (including water). So let us forget about the ripples now they have served their purpose, and consider a number of boys standing in a queue. If the boy at the rear is given a sharp push he will bump into the boy in front of him, and he into the next boy; and so the push may be passed on right to the front. The push has travelled along
the line although the boys-representing the medium of communica-tion-have all been standing still except for a small temporary displacement.

In a similar manner the cricket ball gives the bat a sudden push: the bat pushes the air next to it; that air gives the air around it a push: and so a wave of compression travels outwards in all directions like an expanding bubble. Yet all that any particular bit of air does is to move a small fraction of an inch away from the cause of the disturbance and back again. As the original impulse is spread over an ever-expanding wave-front the extent of this tiny air movement becomes less and less and the sound fainter and fainter.

### 1.3 Characteristics of Sound Waves

The generation of sound is only incidental to the functioning of cricket bats, but the human vocal organs make up a generator capable of emitting a great variety of sounds at will. What surprises most people, when the nature of sound waves is explained to them, is how such a variety of sound can be communicated by anything so simple as to-and-fro tremors in the air. Is it really possible that the subtle inflexions of the voice by which an individual can be identified, and all the endless variety of music, can be represented by nothing more than that?

Examining the matter more closely we find that this infinite variety of sounds is due to four basic characteristics, which can all be illustrated on a piano. They are:
(a) AMPLITUDE. A single key can be struck either gently or hard, giving a soft or loud sound. The harder it is struck, the more violently the piano string vibrates and the greater the tremor in the air, or, to use technical terms, the greater the amplitude of the sound wave. Difference in amplitude is shown graphically in Fig. 1.2a. (Note that to-and-fro displacement of the air is represented here by up-and-down displacement of the curve, because horizontal displacement is usually used for time.)
(b) frequency. Now strike a key nearer the right-hand end of the piano. The resulting sound is distinguishable from the previous one by its higher pitch. The piano string is tighter and lighter, so vibrates more rapidly and generates a greater number of sound waves per second. In other words, the waves have a higher frequency. which is represented in Fig. 1.2h.
(c) waveform. If now several keys are struck simultaneously to give a chord, they blend into one sound which is richer than any of the single notes. The first three waves in Fig. 1.2c represent three such notes singly: when the displacements due to these separate notes are added together they give the more complicated waveform shown on the last line. Actually any one piano note itself has a complex waveform, which enables it to be distinguished
from notes of the same pitch played by other instruments.
(d) envelope. A fourth difference in sound character can be illustrated by first 'pecking' at a key, getting brief (staccato) notes, and then pressing it firmly down and keeping it there, producing a sustained (legato) note. This can be shown as in Fig. 1.2d. The


Fig. 1.2-Graphical representation of the four basic ways in which sound (or any other) waves can differ. (a) amplitude; (b) frequency; (c) waveform; and (d) envelope
dotted lines, called envelopes, drawn around the wave peaks, have different shapes. Another example of this type of difference is the contrast between the flicker of conversational sound and the steadiness of a long organ note.

Every voice or musical instrument, and in fact everything that can be heard, is something that moves; its movements cause the air to vibrate and radiate waves; and all the possible differences in the sounds are due to combinations of the four basic differences.

Mathematically, Envelope is covered by Waveform, so may not always be listed separately.

The details of human and other sound generators are outside the scope of this book, but the sound waves themselves are the things that have to be carried by long-range communication systems such as radio. Even if something other than sound is to be carried (pictures, for instance) the study of sound waves is not wasted, because the basic characteristics illustrated in Fig. 1.2 apply to all waves-including radio waves.

The first (amplitude) is simple and easy to understand. The third and fourth (waveform and envelope) lead us into complications that are too involved for this preliminary survey, and will have to be gone into later. The second (frequency) is very important indeed and not too involved to take at this stage.

### 1.4 Frequency

Let us consider the matter as exemplified by the vibrating string or wire shown in Fig. 1.3. The upper and lower limits of its vibration are indicated (rather exaggeratedly, perhaps) by the dotted lines.


Fig. 1.3-A simple sound generator, consisting of a string held raut by a weight. When plucked it vibrates as indicated by the dotted lines

If a point on the string, such as A , were arranged to record its movements on a strip of paper moving rapidly from left to right, it would trace out a wavy line which would be its displacement/ time graph, like those in Fig. 1.2. Each complete up-2nd-down-

Fig. 1.4-The extem of one excle in a scries of waves is shown in heary line. The time occupied by its generation, or by its passing a given point, is marked $T$

and-back-again movement is called a cycle. In Fig. 1.4, where several cycles are shown, one has been picked out in heavy line. The time it takes. marked $T$, is known as its period.

We have already noted that the rate at which the vibrations take place is called the frequency, and determines the pitch of the sound
produced. Its symbol is $f$, and it used to be reckoned in cycles per second (abbreviated $\mathrm{c} / \mathrm{s}$ ) but the cycle per second is now called the hertz (abbr. Hz) in honour of Heinrich Hertz for a reason to be given very soon. Middle C in music has a frequency of 261 Hz . The full range* of a piano is 27 to 3516 Hz . The frequencies of audible sounds-about $20-20000 \mathrm{~Hz}$ for young people with normal hearing-are distinguished as audio frequencies.

You may have noticed in passing that $T$ and $f$ are closely related. If the period of each cycle is one-hundredth of a second, then the frequency is obviously one hundred cycles per second. Putting it in general terms, $f=1 / T$. So if the time scale of a waveform graph is given, it is quite easy to work out the frequency. In Fig. 1.4 the time scale shows that $T$ is 0.0005 sec , so the frequency must be 2000 Hz .

### 1.5 Wavelength

Next, consider the air waves set up by the vibrating string. Fig. 1.5 shows an end view, with point A vibrating up and down. It pushes the air alternately up and down, and these displacements travel outwards from A. Places where the air has been temporarily moved a little farther from A than usual are indicated by full lines, and places where it is nearer by dotted lines; these lines should be imagined as expanding outwards at a speed of about 1100 feet per


Fig. 1.5-End viell of the vibrating string at A in Fig. 1.3. As it moves up and down over the distance AA it sends out air waves which carry some of its energy of vibration to the listener's ear


4 .

Fig. 1.6 - Twenty successive waves from the string shown in Figs. 1.3 and 1.5
second. So if, for example, the string is vibrating at a frequency of 20 Hz , at one second from the start the first air wave will be 1100 feet away, and there will be 20 complete waves spread over that

[^0]distance (Fig. 1.6). The length of each wave is therefore onetwentieth of 1100 feet, which is 55 feet.

Now it is a fact that sound waves of practically all frequencies and amplitudes travel through the air at the same speed, so the higher the frequency the shorter the wavelength. This relationship can be expressed by the equation

$$
\lambda=\frac{v}{f}
$$

where the Greek letter $\lambda$ (pronounced 'lambda') stands for wavelength, $v$ for the velocity of the waves, and $f$ for the frequency. If $v$ is given in feet per second and $f$ in hertz, then $\lambda$ will be in feet. The letter $v$ has been put here instead of 1100 , because the exact velocity of sound waves in air depends on its temperature; moreover the equation applies to waves in water, wood, rock, or any other substance, if the $v$ appropriate to the substance is filled in.

It should be noted that it is the frequency of the wave which affects the pitch, and that the wavelength is a secondary matter depending on the speed of the wave. That this is so can be shown by sending a sound of the same frequency through water, in which the velocity is 4700 feet per second; the wavelength is therefore more than four times as great as in air, but the pitch, as judged by the ear, remains the same as that of the shorter air wave.

### 1.6 The Sender

The device shown in Fig. 1.3 is a very simple sound transmitter or sender. Its function is to generate sound waves by vibrating and stirring up the surrounding air. In some instruments, such as violins, the stirring-up part of the business is made more effective by attaching the vibrating parts to surfaces which increase the amount of air disturbed.

The pitch of the note can be controlled in two ways. One is to vary the weight of the string, which is conveniently done by varying the length that is free to vibrate, as a violinist does with his fingers. The other is to vary its tightness, as a violinist does when tuning, or can be done in Fig. 1.3 by altering the weight W. To lower the pitch, the string is made heavier or slacker. As this is not a book on sound there is no need to go into this further, but there will be a lot to say (especially in Chapter 8) on the electrical equivalents of these two things, which control the frequency of radio waves.

### 1.7 The Receiver

What happens at the receiving end? Reviving our memory of the stone in the pond, we recall that the ripples made corks and other small flotsam bob up and down. Similarly, air waves when they
strike an object try to make it vibrate with the same characteristics as their own. That is how we can hear sounds through a door or partition; the door is made to vibrate, and its vibrations set up a new lot of air waves on our side of it. When they reach our ears they strike the ear-drums; these vibrate and stimulate a very remarkable piece of receiving mechanism, which sorts out the sounds and conveys its findings via a multiple nerve to the brain.

### 1.8 Electrical Communication by Wire

To extend the very limited range of sound-wave communication it was necessary to find a carrier. Nothing served this purpose very well until the discovery of electricity. At first electricity could only be controlled rather crudely, by switching the current on and off; so spoken messages had to be translated into a code of signals before they could be sent, and then translated back into words at the receiving end. A simple electric telegraph consisted of a battery and a switch or 'key' at the sending end, some device for detecting the current at the receiving end, and a wire between to carry the current.

Most of the current detectors made use of the discovery that when some of the electric wire was coiled round a piece of iron the iron became a magnet so long as the current flowed, and would attract other pieces of iron placed near it. If the neighbouring iron was in the form of a flexible diaphragm held close to the ear, its movements towards and away from the iron-cored coil when current was switched on and off caused audible clicks even when the current was very weak.

To transmit speech and music, however, it was necessary to make the diaphragm vibrate in the same complicated way as the sound waves at the sending end. Since the amount of attraction varied with the strength of current, this could be done if the current could be made to vary in the same way as the distant sound waves. In the end this problem was solved by quite a simple device-the carbon microphone, which, with only details improved, is still the type used in telephones.

In its simplest form, then, a telephone consists of the equipment shown in Fig. 1.7. Although this invention extended the range oif speech from yards to miles, every mile of line weakened the electric currents and set a limit to clear communication. So it was not until the invention of the electronic valve, making it possible to amplify the complicated current variations, that telephoning could be done over hundreds and even thousands of miles.

### 1.9 Electric Waves

Wire or line telephones and telegraphs were and still are a tremendous aid to communication. But something else was needed
for speaking to ships and aeroplanes. And even where the intervening wire is practicable it is sometimes very inconvenient.

At this stage we can profitably think again of the process of unaided voice communication. When you talk to another person you do not have to transfer the vibrations of your vocal organs directly


Fig. 1.7-An earty form of one-nay electric selephone, showing how the characteristics of the original air wares are duplicuted in electric currents, which travel farther and quicker, and which are then transformed back into air waves to make them perceptible by the listener
to the ear-drum of your hearer by means of a rod or other "line". What you do is to stir up waves in the air, and these spread out in all directions, shaking anything they strike, including ear-drums.
'Is it possible', experimenters might have asked, 'to stir up electric waves by any means, and if so would they travel over greater distances than sound waves? Gradually scientists supplied the answer. Clerk Maxwell showed mathematically that electric waves were theoretically possible, and indeed that in all probability light, which could easily be detected after travelling vast distances, was an example of such waves. But they were waves of unimaginably high frequency, about $5 \times 10^{14} \mathrm{~Hz}$. A few years later the German experimenter Hertz actually produced and detected electric waves of much lower (but still very high) frequency, and found that they shared with sound the useful ability to pass through things that are opaque to light.

Unlike sound, however, they are not carried by the air. They travel equally well (if anything, better) where there is no air or any other material substance present. So the broadcasting expression 'on the air' is misleading. If the experiment of Fig. 1.1 is repeated with a source of electric waves instead of sound waves, extracting the air makes no appreciable difference. We know, of course, that light and heat waves travel to us from the sun across 93000000 miles of empty space. What does carry them is a debatable question. It was named ether (or aether, to avoid confusion with the anaesthetic liquid ether), but experiments which ought to have given results if there had been any such thing just didn't. So scientists deny its existence. But it is very hard for lesser minds to visualize how electric waves, whose behaviour corresponds in so many respects with waves
through a material medium, can be propagated by nothing.
There is no doubt about their speed, however. Light waves travel through space at about 186282 miles, or nearly 300000 (more accurately, 299 792) kilometres, per second. Other sorts of electric waves, such as X-rays, ultra-violet and radio waves, travel at the same speed, and differ only in frequency. Their length is generally measured in metres; so filling in the appropriate value of $v$ in the formula in Sec. 1.5 we have, very nearly,

$$
\lambda=\frac{300000000}{f}
$$

If. for example, the frequency is 1000000 Hz , the wavelength is 300 metres.

### 1.10 Why High Frequencies are Necessary

Gradually it was discovered that electric waves of these frequencies, called radio waves, are capable of travelling almost any distance, even round the curvature of the earth to the antipodes.

Their enormous speed was another qualification for the duty of carrying messages; the longest journey in the world takes less than a tenth of a second. But attempts to stir up radio waves of the same frequencies as sound waves were not very successful. To see why this is so we may find it helpful to consider again how sound waves are stirred up.

If you try to radiate air waves by waving your hand you will fail, because the highest frequency you can manage is only a few hertz, and at that slow rate the air has time to rush round from side to side of your hand instead of piling up and giving the surrounding air a push. If you could wave your hand at the speed of an insect's (or even a humming bird's) wing, then the air would have insufficient time to equalize the pressure and would be alternately compressed and rarefied on each side of your hand and so would generate sound waves. Alternatively, if your hand were as big as the side of a house. it could stir up air waves even if waved only a few times per second. because the air would have too far to go from one side to the other every time. The frequency of these air waves would be too low to be heard, it is true; but they could be detected by the rattling of windows and doors all over the neighbourhood.

The same principle applies to radio waves. But because of the vastly greater speed with which they can rush round, it is necessary for the aerial-which corresponds to the waving hand-to be correspondingly vaster. To radiate radio waves at the lower audible frequencies the aerial would have to be miles high, so one might just as weli (and more conveniently) use it on the ground level as a telephone line.

Even if there had not been this difficulty, another would have arisen directly large numbers of senders had started to radiate waves in the audible frequency band. There would have been no way of picking out the one wanted; it would have been like a babel of giants.

That might have looked like the end of any prospects of radio telephony, but human ingenuity was not to be beaten. The solution arose by way of the simpler problem of radio telegraphy, so let us follow that way.

### 1.11 Radio Telegraphy

Hertz discovered how to stir up radio waves; we will not bother about exactly how, because his method is obsolete. Their frequency was what we would now call ultra-high; in the region of hundreds of millions of hertz, so their length was only a few centimetres. Such frequencies are used nowadays for radar. He also found a method of detecting them over distances of a few yards. With the more powerful senders and more sensitive receivers developed later. ranges rapidly increased. until in 1901 Marconi actually signalled across the Atlantic. The various sorts of detectors that were invented from time to time worked by causing an electric current to flow in a local receiving circuit when radio waves impinged on the receiving aerial. Human senses are unable to respond to these "wireless" waves directly-their frequency is far too high for the ear and far too low for the eye - but the electric currents resulting from their detection can be used to produce audible or visible effects.

At this stage we have the radio counterpart of the simple telegraph. Its diagram, Fig. 1.8, more or less explains itself. The


Fig. 1.8-Elementary radio tekgraph. Currents flow in the receiving headphones and cause sound whenever the sender is radiating waves
sending key turns on the sender, which generates a rapid succession of radio waves, radiated by an aerial. When these reach the receiver they operate the detector and cause an electric current to flow.
which can make a noise in a telephone earpiece. By working the sending key according to the Morse code, messages can be transmitted. That is roughly the basis of radio telegraphs to this day, though some installations have been elaborated almost out of recognition, and print the messages on paper as fast as the most expert typist.

### 1.12 Tuning

One important point to note is that the current coming from the detector does not depend on the frequency of the waves, within wide limits. (The frequency of the current is actually the frequency with which the waves are turned on and off at the sender.) So there is a wide choice of wave frequency. In Sec. 17.12 we shall consider how the choice is made; but in the meantime it will be sufficient to remember that if the frequency is low, approaching audibility, the aerials have to be immense; while if the frequency is much higher than those used by Hertz it is difficult to generate them powerfully. and they are easily obstructed, like light waves. The useful limits of these radio frequencies are about 15000 to 50000000000 Hz .

The importance of having many frequencies to choose from was soon apparent, when it was found to be possible-and highly advantageous-to make the receiver respond to the frequency of the sender and reject all others. This invention of nuning was essential to effective radio communication, and is the subject of a large part of this book. So it may be enough just now to point out that it has its analogy in sound. We can adjust the frequency of a sound wave by means of the length and tightness of the sender. A piano contains nearly a hundred strings of various fixed lengths and tightnesses, which we can select by means of the kevs. If there is a second piano in the room, with its sustaining pedal held down so that all the strings are free to vibrate, and we strike a loud note on the first piano, the string tuned to the same note in the second can be heard to vibrate. If we had a single adjustable string. we could tune it to respond in this way to any one note of the piano and ignore those of substantially different frequency. In a corresponding manner the electrical equivalents of length (or weight) and tightness are adjusted by the tuning controls of a radio receiver to select the desired sending station.

### 1.13 Radio Telephony

The second important point is that (as one would expect) the strength of the current from the detector increases with the strength of the waves transmitted. So, with Fig. 1.7 in mind, we need no further hint to help us to convert the radio telegraph of Fig. 1.8 into a
radio telephone or a broadcasting system, by substituting for the Morse key a device for varying the strength of the waves. just as the microphone varied the strength of the line current in Fig. 1.7. Such devices are called modulators, and will be referred to in more detail in Chapter 15. The microphone is still needed, because the output of the radio sender is not controlled directly by the impinging sound waves but by the wavily-varying currents from the microphone.

A practical system needs amplifiers in various places, firstly to amplify the feeble sound-controlled currents from the microphone until they are strong enough to modulate a powerful wave sender; then to amplify the feeble radio-frequency currents stirred up in a distant receiving aerial by the far flung waves: then another to


Fig. I.9-The radio telegraph of Fig. 1.8 converted into a radio telephone by controlling the output of the sender by sound waves instend of just turning it on and oft. In practice is is also gencrally necessury to use amplifiers there shoutn
amplify the audio-frequency currents from the detector so that they are strong enough to work a loudspeaker instead of the less convenient headphones. This somewhat elaborated system is indicated -but still only in broadest outline-by the block diagram, Fig. 1.9.

### 1.14 Recapitulation

At this stage it may be as well to review the results of our survey. We realize that in all systems of communication the signals must,
in the end, be detectable by the human senses. Of these, hearing and seeing are the only ones that matter. Both are wave-operated. Our ears respond to air waves between frequency limits of about 20 and 20000 Hz ; our eyes respond to waves within a narrow band of frequency centred on $5 \times 10^{14} \mathrm{~Hz}$-waves not carried by air but by space, and which have been proved to be electrical in nature.

Although our ears are remarkably sensitive, air-wave (i.e., sound) communication is very restricted in range. Eyes, it is true, can see over vast distances, but communication is cut off by the slightest wisp of cloud. If, however, we can devise means for 'translating' audible or visible waves into and out of other kinds of waves, we have a far wider choice, and can select those that travel best. It has been found that these are electric waves of lower frequencies than the visible ones, but higher than audible frequencies. The lowest of the radio frequencies require excessively large aerials to radiate them, and the highest are impeded by the atmosphere. All travel through space at the same speed of nearly 300000000 metres per second, so the wavelength in metres can be calculated by dividing that figure by the frequency in Hz .

The frequency of the sender is arranged by a suitable choice and adjustment of the electrical equivalents of weight and tightness (or, more strictly, slackness) in the tuning of sound generators. In the same way the receiver can be made to respond to one frequency and reject others. There are so many applicants for frequenciesbroadcasting authorities, telegraph and telephone authorities, naval, army, and air forces, space organisations, weather stations, police, the merchant navies, airlines, research scientists, amateurs and others-that even with such a wide band to choose from it is difficult to find enough for all, especially as only a small part of the whole band is generally suitable for any particular kind of service.

The change-over from the original sound waves to radio waves of higher frequency is done in two stages: first by a microphone into electric currents having the same frequencies as the sound waves; these currents are then used in a modulator to control the radio-frequency waves in such a way that the original sound characteristics can be extracted at the receiver. Here the changeback is done in the same two stages reversed: first by a detector, which yields electric currents having the original sound frequencies; then by ear-phones or loud speakers which use these currents to generate sound waves.

Since the whole thing is an application of electricity, our detailed study must obviously begin with the general principles of electricity. And it will soon be necessary to pay special attention to electricity varying in a wavelike manner at both audio and radio frequencies. This will involve the electrical characteristics that form the basis of tuning. To generate the radio-frequency currents in the sender, as well as to amplify and perform many other services, use is made of various types of electronic devices, such as valves and transistors.
so it is necessary to study these in some detail. The process of causing the r.f. currents to stir up waves, and the reverse process at the receiver, demands some knowledge of aerials and radiation. Other essential matters that need elucidation are modulation and detection. Armed with this knowledge we can then see how they are applied in typical senders and receivers. By then, most of the basic principles of electronics in general will have been covered, but a few more chapters will be needed to fill in the special requirements in such things as television, radar, electronic instruments and computers.

## CHAPTER 2

## Electricity and Circuits

### 2.1 Electrons

The exact nature of electricity is a mystery that may never be fully cleared up, but from what is known it is possible to form a sort of working model or picture which helps us to understand how it produces the results it does, and even to think out how to produce new results. The reason why the very existence of electricity went unnoticed until comparatively recent history was that there are two opposite kinds which exist in equal quantities and, unless separated in some way, cancel one another out. This behaviour reminded the investigating scientists of the use of positive ( + ) and negative ( - ) signs in arithmetic: the introduction of either +1 or -1 has a definite effect, but $+1-1$ equals just nothing. So, although at that time hardly anything was known about the two kinds of electricity, they were called positive and negative. Both positive and negative electricity produce very remarkable effects when they are separate, but a combination of equal quantities shows no signs of electrification.

Further research led to the startling conclusion that ail matter consists largely of electricity. All the thousands of different kinds of matter, whether solid, liquid, or gaseous, consist of atoms which contain only a very few different basic components. Of these we need take note of only one kind-particles of negative electricity, called electrons. For a simple study it is convenient to imagine each atom as a sort of ultra-microscopic solar system in which a number of electrons are distributed around a central nucleus, somewhat as our earth and the other planets around the sun (Fig. 2.1). (The subject is treated in much more detail in the author's The Electron in Electronics.) The nucleus is generally a more or less composite structure; it makes up nearly all the weight of the atom, and the number and type of particles it contains determine which element it is-oxygen, carbon, iron, etc. It is possible to change some elements into others by breaking up the nucleus under very intense laboratory treatment. For our present purpose, however, the only things we need remember about the nucleus are that it is far heavier than the electrons, and that it has a surplus of positive electricity.

In the normal or unelectrified state of the atom this positive surplus is exactly neutralised by the planetary electrons. But it is a comparatively easy matter to dislodge one or more of these electrons from each atom. One method is by rubbing; for example, if a glass rod is rubbed vigorously with silk some of the electrons belonging
to the glass are transferred to the silk. Doing this produces no change in the material itself-the glass is still glass and the silk remains silk. As separate articles they show no obvious evidence of the transfer. But if they are brought close together it is found that they attract one another. And the glass rod can pick up small

Fig. 2.1-Diagrammatic picture of an atom, showing the nucleus surrounded by a number of electrons

scraps of paper. It is even possible, in a dry atmosphere, to produce sparks. These curious phenomena gradually fade away, and the materials become normal again.

### 2.2 Electric Charges and Currents

The surplus of electrons on the silk is called a negative electric charge. The glass has a corresponding deficiency of electrons, so its positive nuclear electricity is incompletely neutralised, and the result is a positive charge. It is a pity that the people who decided which to call positive did not know anything about electrons, because if they had they would certainly have called electrons positive and so spared us a great deal of confusion. As it is. however, one just has to remember that a surplus of electrons is a negative charge.

Any unequal distribution of electrons is a condition of stress. Forcible treatment of some kind is needed in order to create a surplus or deficiency anywhere, and if the electrons get a chance they move back to restore the balance; i.e., from negatively to positively charged bodies. This tendency shows itself as an attraction between the bodies themselves. That is what is meant by saying that opposite charges attract.

The space between opposite charges, across which this attraction is exerted, is said to be subject to an electric field. The greater the charges, the more intense is the field and the greater the attractive force.

If a number of people are forcibly transferred from town $A$ to town B , the pressure of overcrowding and the intensity of desire to leave $B$ depend, not only on the number of displaced persons, but
also on the size of B and possibly on whether there is a town C close by with plenty of accommodation. The electrical pressure also depends on other things than the charge, reckoned in displaced electrons; so it is more convenient to refer to it by another term. namely difference of potential, often abbreviated to p.d. We shall consider the exact relationship between charge and p.c'. in the next chapter. The thing to remember now is that wherever there is a difference of potential between two points there is a tendency for electrons to move from the point of lower (or negative) potential to that of higher (or positive) potential in order to equalise the distribution.* If they are free to move they will do so; and their movement is what we call an electric current.

This is where we find it so unfortunate that the names 'positive' and 'negative' were allocated before anybody knew about electrons. For it amounted to a guess that the direction of current flow was the opposite to that in which we now know electrons flow. By the time the truth was known this bad guess had become so firmly established that reversing it would have caused worse confusion. Moreover, when the positively charged atoms (or positive ions, as they are called) resulting from the removal of electrons are free to do so they also move, in the direction + to - , though much more slowly owing to their greater mass. So this book follows the usual custom of talking about current flowing from + to - or high to low potential, but it must be remembered that most often this means electrons moving from - to + .

### 2.3 Conductors and Insulators

Electrons are not always free to move. Except in special circumstances (such as high temperature, which we shall consider in Chapter 9) atoms do not allow their electrons to fly off completely on their own, even when they are surplus. The atoms of some substances go so far as to allow frequent exchanges, however, like dancers in a Paul Jones, and in fact such exchanges go on all the time, even in the normal unelectrified state. The directions in which the electrons flit from one atom to another are then completely random, because there is nothing to influence them one way or another.

But suppose the whole substance is pervaded by an electric field. The electrons feel an attraction towards the positive end; so between partnerships they tend to drift that way. The drift is what is known as an electric current, and substances that allow this sort of thing to go on are called conductors of electricity. All the metals are more

[^1]or less good conductors; hence the extensive use of metal wire. Carbon and some liquids are fairly good conductors.

Note that although electrons start to go in at the negative end and electrons start to arrive at the positive end the moment the p.d. is set up, this does not mean that these are the same electrons. which have instantaneously travelled the whole length of the conductor. An electron drift starts almost instantaneously throughout miles of wire, but the speed at which they drift is seldom much more than an inch a minute.

Other substances keep their electrons, as it were. on an elastic leash which allows them a little freedom of movement, but never "out of sight" of the atom. If such a substance occupies the space between two places of different potentiat, the electrons strain at the leash in response to the positive attraction, but a continuous steady drift is impossible. Materials of this kind. called insulators, can be used to prevent charges from leaking away. or to form boundaries restricting currents to the desired routes. Dry air, glass, polythene, rubber, and paraffin wax are among the best insulating materials. None is absolutely perfect. however; electrified glass rods, for example, graduatly lose their charges.

We shall see in Chapter 9 that there is a third class of substances called semiconductors, of immense importance in electronics.

### 2.4 Electromotive Force

If electrons invariably moved from - to + , as described, they would in time neutralize all the positive charges in the world, and that, for all practical purposes. would be the end of electricity. But fortunately there are certain appliances, such as batteries and dynamos (or generators) which can force electrons to go from + to - . contrary to their natural inclination. In this way they can continuously replenish a surplus of electrons. Suppose A and B in Fig. 2.2 are two insulated metal terminals, and a number of electrons have been transferred from $A$ to $B$. so that $A$ is at a higher potential than B. If now they are joined by a wire, electrons will drift along it. and. if that were all. the surplus would soon be used up and the potential difference would disappear. But A and B happen to be the terminals of a battery, and as soon as electrons leave B the battery provides more, white at the same time it withdraws electrons arriving at $\mathbf{A}$. So while electrons are moving from $B$ to $A$ through the wire, the battery keeps them moving from A to B through itself. The battery is, of course, a conductor; but if it were only that it would be an additional path for electrons to go from B to A and dissipate the charge all the quicker. It is remarkable, then, for being able to make electrons move against a p.d. This ability is called electromotive force (usually abbreviated to e.m.f.).

Fig. 2.2 shows what is called a closed circuit, there being a continuous endless path for the current. This is invariably necessary for a continuous current, because if the circuit were opened any-


Fig. 2.2-A very sinuple electrical circait, round which the electromotive force of a battery makes a current of clectrons flow comtimuously
where, say by disconnecting the wire from one of the terminals, continuation of the current would cause electrons to pile up at the break, and the potential would increase without limit; which is impossible.

### 2.5 Electrical Units

The amount or strength of an electric current might reasonably be reckoned as the number of electrons passing any point in the circuit each second, but the electron is so extremely small that such a unit would lead to inconveniently large numbers. For practical purposes it has been agreed to base the unit on the metric system. It is called the ampere (or more colloquially the amp), and happens to be about 6240000000000000000 (or $6.24 \times 10^{18}$ ) electrons per second.

As one would expect. the number of amps caused to flow in a given circuit depends on the strength of the electromotive force operating in it. The practical unit of e.m.f. is the volt.

Obviously the strength of current depends also on the circuitwhether it is made up of good or bad conductors. This fact is expressed by saying that circuits differ in their electrical resistance. The resistance of a circuit or of any part of it can be reckoned in terms of the e.m.f. required to drive a given current through it. For convenience the unit of resistance is made numerically equal to the number of volts required to cause one amp to flow, and to avoid the cumbersome expression 'volts per amp' this unit of resistance has been named the ohm.

By international agreement the following letter symbols have been allocated to denote these electrical quantities and their units, and will be used from now on:

| Quantity | Symbol for <br> Quantity | Unit of <br> Quantity | Symbol for <br> Unit |
| :--- | :---: | :---: | :---: |
| E.M.F. | $E$ | Volt | V |
| Current | $I$ | Ampere | A |
| Resistance | $R$ | Ohm | $\Omega$ |

( $\Omega$ is a Greek capital letter omega)
These three important quantities are not all independeni. for we have just defined resistance in terms of e.m.f. and current. Expressing it in symbols:

$$
R=\frac{E}{I}
$$

### 2.6 Ohm's Law

The question immediately arises: does $R$ depend only on the circuit, or does it depend also on the current or voltage? This was one of the first and most important investigations into electric currents. We can investigate it for ourselves if we have an instrument for measuring current, called an ammeter, a battery of identical cells, and a length of thin wire or other resistive conductor. A closed circuit is formed of the wire, the ammeter, and a varying number of the cells; and the ammeter reading corresponding to each number

Fig. 2.3-Graph showing the results of an experiment on the relationship between currem in amps and e.m.f. in volts, for two different circuits

of cells is noted (Fig. 2.5). The results can be plotted in the form of a graph (Fig. 2.3). If the voltage of a cell is known, the current can be plotted against voltage instead of merely against number of cells.

When all the points are joined up by a line it will probably be
found that the line is straight and inclined at an angle and passes through the origin (0). (If the line is not quite straight, that may be for the reason given at the end of Sec. 2.11.) If a different resistance is tried, its line will slope at a different angle. Fig. 2.3 shows two possible samples resulting from such an experiment. In this case each cell gave 2 volts, so points were plotted at $2,4,6,8$, etc., volts. The points at $-2,-4$, etc., volts were obtained by reversing the battery. A current of -2 A means a current of the same strength as +2 A , but flowing in the opposite direction.
Looking at the steep line, we see that an e.m.f. of 2 V caused a current of $2 \mathrm{~A}, 4 \mathrm{~V}$ caused 4 A , and so on. The result of dividing the number of volts by the number of amps is always 1 . And this, according to our definition, is the resistance in ohms. So there is no need to specify the current at which the resistance must be measured, or to make a graph; it is only necessary to measure the voltage required to cause any one known current (or the current caused by any one voltage). The amount (or, as one says in technical language, the value) of resistance so obtained can be used to find the current at any other voltage, or voltage at any other current. For these purposes it is more convenient to write the relationship $R=E / I$ in the form

$$
I=\frac{E}{R} \quad \text { or } \quad E=I R
$$

Thus if any two of these quantities are known, the third can be found.

Conductors whose resistance does not vary with the amount of current flowing through them are described as linear, because the line representing them on a graph of the Fig. 2.3 type (called their current/voltage characteristic) is straight. Although the relationship $I=E / R$ is true for any resistance, its greatest usefulness lies in the fact, discovered by Dr. Ohm, and known as Ohm's law, that ordinary conductors are linear. From Chapter 9 onwards we shall come across some exceptions. And often the relationship $I=E \mid R$ is itself rather loosely called Ohm's law.

As an example of the use of Ohm's law, we might find. in investigating the value of an unknown resistance, that when it was connected to the terminals of a 9 V battery a current of 0.01 A was driven through it. Using Ohm's law in the form $R=E / I$, we get for the value of the resistance $9 / 0 \cdot 01=900 \Omega$. Alternatively, we might know the value of the resistance and find that an old battery, nominally of 9 V , could only drive a current of 0.007 A through it. We could deduce, since $E=I \times R$, that the voltage of the battery had fallen to $900 \times 0.007=6.3 \mathrm{~V}$. Note that in driving current through a resistance an e.m.f. causes a p.d. of the same voltage between the ends of the resistance. This point is elaborated in Sec. 2.14.

### 2.7 Larger and Smaller Units

It is unusual to describe a current as 0.007 ampere, as was done just now: one speaks of ' 7 milliamperes', or, more familiarly, ' 7 milliamps'. A milliampere is one-thousandth part of an ampere. Several other prefixes are used; they are tabulated in Appendix 2. the commonest being

| Prefix | Meaning | Symbol |
| :--- | :--- | :---: |
| milli- <br> micro- <br> nano- | one thousandth of <br> one millionth of <br> one thousand- <br> millionth of | m |
| pico- | n <br> one billionth of <br> (million millionth) | p |
| kilo- <br> me thousand <br> mega- | k |  |

( $\mu$ is the Greek letter mu)
The prefixes can be put in front of any unit ; one speaks commonly of milliamps, microamps, megohms, and so on. 'Half a megohm' is easier to say than 'Five hundred thousand ohms', and ' $\frac{1}{2} \mathrm{M} \Omega$ ' is quicker to write than ' $500000 \Omega$ '.

It must be remembered, however, that Ohm's law in the forms given on p. 22 assumes volts, ohms, and amps. If a current of 5 mA is flowing through $15000 \Omega$, the voltage across that resistance will not be 75000 V . But since most of the currents we shall be concerned with are of the milliamp order it is worth noting that there is no need to convert them to amps if $R$ is expressed in thousands of ohms $(\mathrm{k} \Omega)$. So in the example just given one can get the correct answer, 75 V , by multiplying 5 by 15 .

### 2.8 Circuit Diagrams

At this stage it will be as well to start getting used to circuit diagrams. In a book like this, concerned with general principles rather than with constructional details, what matters about (say) a battery is not its shape nor the design of its label, but its voltage and how it is connected in the circuit. So it is a waste of time drawing a picture as in Fig. 2.2. All one needs is the symbol shown as Fig. 2.4a. which represents one cell. An accumulator cell gives 2 V , and a dry cell about 1.4 V . The longer stroke represents the + terminal. To get higher voltages, several cells are connected one after the other, as in Fig. $2.4 b$, and if so many are needed that it is tedious to draw them all one can represent all but the end ones by a dotted line, as in Fig. 2.4c, which also shows how to indicate the voltage. Other
symbols in Fig. 2.4 are ( $d$ ) a circuit element called a resistor, because resistance is its significant feature, (e) a switch shown 'open' or 'off', and $(f)$ a measuring instrument, the type of which can be shown by ' mA ' for milliammeter, ' $V$ ' for voltmeter, etc., and the range of measurement marked as here. A simple line represents an electrical

(d)

(b)

(e)

(c)

(f)

Fig. 2.4-A first instalment of circuit-diagram symbols, representing (a) a cell, (b) a battcry of three cells. (c) a battery of many cells, (d) a resistor. (e) a switch, and (f) a milliammeter
connection of negligible resistance, often called a lead (rhyming with 'feed').

The circuit diagram for the Ohm's law experiment can therefore be shown as in Fig. 2.5. The arrow denotes a movable connection, used here for including anything from one to eight cells in circuit.

Fig. 2.5 is a simple example of a series circuit. Two elements are said to be connected in series when, in tracing out the path of the current, we encounter them one after the other. With steadily


Fig. 2.5-Circuit diagram of the apparatus used for obtaining Fig. 2.3
flowing currents the electrons do not start piling up locally, so it is clear that the same current flows through both elements. In Fig. 2.5, for instance, the current flowing through the ammeter is the same as that through the resistor and the part of the battery 'in circuit'.

Two elements are said to be in parallel if they are connected so as to form alternative paths for the current; for example, the resistors in Fig. 2.8a. Since a point cannot be at two different potentials at once, it is obvious that the same potential difference exists across both of them (i.e., between their ends or terminals). One element in parallel with another is often described as shunting the other.

The method of connection can sometimes be regarded as either
series or parallel, according to circumstances. In Fig. 2.6, $\mathrm{R}_{1}$ and $R_{2}$ are in series as regards battery $B_{1}$, and in parallel as regards $B_{2}$.

Looking at the circuit diagrams of radio sets-especially television receivers-one often sees quite complicated networks of resistors. Fortunately Ohm's law can be applied to every part of a

Fig. $2.6-R_{1}$ is in series with $R_{2}$ as regards $B_{1}$. but in pacrallel with it as regards $B_{2}$

complicated system of e.m.fs and resistances as well as to the whole. Beginners often seem reluctant to make use of this fact, being scared by the apparent difficulty of the problem. So let us see how it works out with more elaborate circuits.

### 2.9 Resistances in Series and in Parallel

Complicated circuit networks can be tackled by successive stages of finding a single element that is equivalent (as regards the quantity to be found) to two or more. Consider first two resistances $R_{1}$ and $R_{2}$, in series with one another and with a source of e.m.f. E (Fig. 2.7a). To bring this circuit as a whole within the scope of calculation


Fig. 2.7-Resistances in series. The circuit b is equivalent to the circuit $u_{0}$ in the sense that both take the same current from the battert: if $R=R_{1}+R_{2}$
by Ohm's law we need to find the single resistance $R$ (Fig. 2.7b) equivalent to $R_{1}$ and $R_{2}$ together.

We know that the current in the circuit is everywhere the same: call it $I$. Then, by Ohm's law, the voltage across $R_{1}$ is $I R_{1}$, and that across $R_{2}$ is $I R_{2}$. The total voltage across both must therefore be $I R_{1}+I R_{2}$, or $I^{2}\left(R_{1}+R_{2}\right)$, which must be equal to the voltage $E$.

In the equivalent circuit, $E$ is equal to $I R$, and since, to make the circuits truly equivalent, the currents must be the same in both for the same battery voltage, we see that

$$
R=R_{1}+R_{2} .
$$

Generalizing from this result, we conclude that: The total resistance of several resistances in series is equal to the sum of their individual resistances.

Turning to the parallel-connected resistances of Fig. 2.8a, we see that they have the same voltage across them; in this case the e.m.f.


Fig. 2.5-Resistances in parallel. The circuit b is equivalent to the circuit a in the sense that both take the current from the hatsery, if $\|R=\| R_{1}+\| R_{2}$
of the battery. Each of these resistances will take a current depending on its own resistance and on the e.m.f.-the simplest case of Ohm's law. Calling the currents respectively $I_{1}$ and $I_{2}$, we therefore know that $I_{1}=E / R_{1}$ and $I_{2}=E / R_{2}$. The total current drawn is the sum of the two: it is

$$
I=\frac{E}{R_{1}}+\frac{E}{R_{2}}=E\left(\frac{1}{R_{1}}+\frac{1}{R_{2}}\right)
$$

In the equivalent circuit of Fig. $2.8 b$ the current is $E / R$, which may also be written $E(l / R)$. Since, for equivalence between the circuits, the current must be the same for the same battery voltage, we see that

$$
\frac{1}{R}=\frac{1}{R_{1}}+\frac{1}{R_{2}}
$$

Generalizing from this result, we conclude that: If several resistances are connected in parallel, the sum of the reciprocals of their individual resistances is equal to the reciprocal of their total resistance.
If the resistances of Fig. $2.8 a$ were $100 \Omega$ and $200 \Omega$. the single resistance $R$ which, connected in their place, would draw the same current is given by $1 / R=1 / 100+1 / 200=0.01+0.005=0.015$. Hence, $R=1 / 0.015=66 \cdot 67 \Omega$. This could be checked by adding together the individual currents through $100 \Omega$ and $200 \Omega$, and comparing the total with the current taken from the same voltagesource by $66.67 \Omega$. In both cases the result is 15 mA per volt of battery.

We can summarize the two rules in symbols:

1. Series Connection: $\quad R=R_{1}+R_{2}+R_{3}+R_{4}+$
2. Parallel Connection: $1 / R=1 / R_{1}+1 / R_{2}+1 / R_{3}+$

For only two resistances in parallel, a more convenient form of
the same rule can be obtained by multiplying above and below by $R_{1} R_{2}$ :

$$
R=\frac{1}{1 / R_{1}+1 / R_{2}}=\frac{R_{1} R_{2}}{R_{1}+R_{2}}
$$

### 2.10 Series-Parallel Combinations

How the foregoing rules, derived directly from Ohm's law, can be applied to the calculation of more complex circuits can perhaps best be illustrated by a thorough working-out of one fairly elaborate network, Fig. 2.9. We will find the total current flowing, the equivalent

Fig. 2.9-The current through and voltage across cach resistor in this complicated network cun be calculated by applying the noo simple rules derived from Figs. 2.7 and 2.8

resistance of the whole circuit, and the voltage and current of every resistor individually.

The policy is to look for any resistances that are in simple series or parallel. The only two in this example are $R_{2}$ and $R_{3}$. Writing $R_{23}$ to symbolise the combined resistance of $R_{2}$ and $R_{3}$ taken together, we


Fig. 2.10-Successive stages in simplifying the circuit of Fig. 2.9. $R_{33}$ stands for the single resistunces equivalent to $R_{2}$ and $R_{3}$ : and so on. $R$ represents the whole system
know that $R_{23}=R_{2} R_{3} /\left(R_{2}+R_{3}\right)=200 \times 500 / 700=142.8 \Omega$. This gives us the simplified circuit of Fig. 2.10a. If $R_{23}$ and $R_{4}$, were one resistance, they and $R_{5}$ in parallel would make another simple
case, so we proceed to combine $R_{23}$ and $R_{4}$ to make $R_{234}=R_{23}+R_{4}$ $=142 \cdot 8+150=292 \cdot 8 \Omega$. Now we have the circuit of Fig. 2.10b. Combining $R_{234}$ and $R_{5}, R_{2345}=292.8 \times 1000 / 1292.8=226.5 \Omega$. This brings us within sight of the end; Fig. 2.10 c shows us that the total resistance of the network now is simply the sum of the two remaining resistances; that is, $R$ is Fig. $2.10 d$ is $R_{2345}+R_{1}=$ $226 \cdot 5+100=326 \cdot 5 \Omega$.

From the point of view of current drawn from the $40-\mathrm{V}$ source the whole system of Fig. 2.9 is equivalent to a single resistor of this value. The current taken from the battery will therefore be $40 / 326 \cdot 5$ $=0.1225 \mathrm{~A}=122.5 \mathrm{~mA}$.

To find the current through each resistor individually now merely means applying Ohm's law to some of our previous results. Since $R_{1}$ carries the whole current of 122.5 mA , the potential difference across it will be $100 \times 0.1225=12.25 \mathrm{~V} . R_{2345}$ also carries the whole current ( $2.10 c$ ); the p.d. across it will again be the product of resistance and current, in this case $226.5 \times 0.1225=27.75 \mathrm{~V}$. This same voltage also exists, as comparison of the various diagrams will show, across the whole complex system $R_{2} R_{3} R_{4} R_{5}$ in Fig. 2.9. Across $R_{5}$ there comes the whole of this voltage; the current through this resistor will therefore be $27 \cdot 75 / 1000 \mathrm{~A}=27.75 \mathrm{~mA}$.

The same p.d. across $R_{234}$ of Fig. 2.10b, or across the system $R_{2} R_{3} R_{4}$ of Fig. 2.9, will drive a current of $27 \cdot 75 / 292 \cdot 8=94.75 \mathrm{~mA}$ through this branch. The whole of this flows through $R_{4}(2.10 a)$, the voltage across which will accordingly be $150 \times 0.09475=14.21 \mathrm{~V}$. Similarly, the p.d. across $R_{23}$ in Fig. $2.10 a$, or across both $R_{2}$ and $R_{3}$ in Fig. 2.9, will be $0.09475 \times 142.8=13.54 \mathrm{~V}$, from which we find that the currents through $R_{2}$ and $R_{3}$ will be respectively $13 \cdot 54 / 200$ and $13.54 / 500 \mathrm{~A}$, or 67.68 and 27.07 mA , making up the required total of 94.75 mA for this branch.

This completes an analysis of the entire circuit; we can now collect our scattered results in the form of the following table.

| Resistance <br> $(\Omega)$ |  | Current through <br> it $(\mathrm{mA})$ | Potential Difference <br> across it $(\mathrm{V})$ |
| :---: | :---: | :---: | :---: |
| $R_{1}$ | 100 | 122.5 | 12.25 |
| $R_{2}$ | 200 | 67.68 | 13.54 |
| $R_{3}$ | 500 | 27.07 | 13.54 |
| $R_{4}$ | 150 | 94.75 | 14.21 |
| $R_{5}$ | 1000 | 27.75 | 27.75 |
| $R$ | 326.5 | 122.5 | 40 |

Note that by using suitable resistors any potential intermediate between those given by the terminals of the battery can be obtained. For instance, if the lower and upper ends of the battery in Fig. 2.9 are regarded as 0 and +40 (they could equally be reckoned as -40 and 0 , or -10 and +30 , with respect to any selected level of
voltage), the potential of the junction between $R_{3}$ and $R_{4}$ is 14.21 V . The arrangement is therefore called a potential divider, and is often employed for obtaining a desired potential not given directly by the terminals of the source. If a sliding connection is provided on a resistor, to give a continuously variable potential, it is generally known-through not always quite justifiabiy-as a potentiometer.

### 2.11 Resistance Analysed

So far we have assumed the possibility of almost any value of resistance without considering exactly what determines the resistance of any particular resistor or part of a circuit. We understand that different materials vary widely in the resistance they offer to the flow of electricity, and can guess that with any given material a long piece will offer more resistance than a short piece, and a thin piece than a thick. We have indeed actually proved as much and more; by the rule for resistances in series the resistance of a uniform wire is exactly proportional to its length-doubling the length is equivalent to adding another equal resistance in series, and so on (Fig. $2.11 a$ ). Similarly, putting two equal pieces in parallel, which halves

Fig. 2.11-The rule for resistances in series proves that doubling the length of a uniform conductor doubles its resistance (a). By the rule for resistances in parallel, doubling its cross-section area halves its resistance (b): and doubling its diameter quarters its resistance (c)

the resistance, is equivalent to doubling the thickness; so resistance is proportional to the reciprocal of thickness. Or, in more precise language, to the cross-section area (Fig. 2.11 b ). Altering the shape of the cross-section has no effect on the resistance-with steady currents, at least. Fig. 2.11c shows how doubling the diameter of a piece of wire divides the resistance by four.

To compare the resistances of different materials it is necessary to bring them all to the same standard length and cross-section area, which, in the system of units known as SI (Système Internationale d'Unités), is one metre long by one square metre. This comparative resistance for any material is called its resistivity' (symbol: $\rho$, pronounced 'roe'). Knowing the resistivity (and it can be found in
almost any electrical reference book*) one can easily calculate the resistance of any wire or piece having uniform section by multiplying by the length in metres ( $)$ ) and dividing by the cross-section area in square metres ( $A$ ). In symbols:

$$
R=\frac{\rho l}{A}
$$

Such calculations are seldom necessary, because there are tables showing the resistance per metre or per yard of all the different gauges of wire, both for copper (usually used for parts of the circuit where resistance should be as low as possible) and for the special alloys intended to have a high resistance.

Resistance varies to some extent with temperature, so the tables show the temperature at which they apply, and also the proportion by which the resistance increases with rise of temperature.

### 2.12 Conductance

It is often more convenient to work in terms of the ease with which a current can be made to flow, rather than the difficulty; in other words, in conductance rather than resistance. The symbol for conductance is $G$, and its unit (called the siemens, $\mathbf{S}$; formerly, the mho) is equal to the number of amps caused to flow by one volt. So $I=E G$ is an alternative form of $I=E / R$; and $G=1 / R$.

It is not difficult to see that corresponding to the equation above for calculating resistance there will be one for conductance:

$$
G=\frac{\sigma A}{l}
$$

where $\sigma$ (Greek ‘sigma') stands for conductivity (siemens per metre).
The advantage of conductance appears chiefly in parallel circuits, for which it is easy to see that the formula is $G=G_{1}+G_{2}+\cdots \cdots$

### 2.13 Kirchhoff's Laws

We have already taken it as obvious that no one point can be at more than one potential at the same time. (If it is not obvious, try to imagine a point with a difference of potential between itself; then electrons will flow from itself to itself, i.e., a journey of no distance and therefore non-existent.) So if you start at any point on a closed circuit and go right round it, adding up all the voltages

[^2]on the way, with due regard for + and - , the total is bound to be zero. If it isn't, a mistake has been made somewhere: just as a surveyor must have made a mistake if he started from a certain spot, measuring his ascents as positive and his descents as negative, and his figures told him that when he returned to his starting.point he had made a net ascent or descent.

This simple principle is the modern form of one of what are known as Kirchhoff's laws, and is a great help in tackling systems of resistances where the methods already described fail because there are no two that can be simply combined. One writes down an equation for each closed loop or mesh, by adding together all the voltages and equating the total to zero, and one then solves the resulting simultaneous equations. That can be left for more advanced study, but the same law is a valuable check on any circuit calculations. Try applying it to the several closed loops in Fig. 2.9, such as that formed by $E, R_{1}$, and $R_{5}$. If we take the route clockwise we go 'uphill' through the battery, becoming more positive by 40 V . Coming down through $R_{1}$ we move from positive to negative so add -12.25 V , and, through $\mathrm{R}_{5},-27.75 \mathrm{~V}$, reaching the starting point again. Check: $40-12.25-27.75=0$.

Taking another clockwise route via $R_{3}, R_{5}$, and $R_{4}$. we get $13 \cdot 54-27 \cdot 75+14 \cdot 21=0$. And so on for any closed loop.

The other Kirchhoff law is equally obvious; it states that if you count all currents arriving at any point in a circuit as positive, and those leaving as negative, the total is zero. This can be used as an additional check, or as the basis of an alternative method of solving circuit problems.

### 2.14 P.D. and E.M.F.

The fact that both e.m.f. and p.d. are measured in volts can be confusing. So it will be time well spent if, before going any farther, we look into the matter rather carefully.

We have just been likening difference of potential to difference in height. This analogy is implied whenever we talk about low and high potential. But although we measure height in feet or metres what we have in mind is not just a distance; we are really concerned with the force of gravity close to a massive body, usually the earth. ('Height' has no meaning in outer space.) This force tends to make things move downwards, or fall. Corresponding to this gravitational field is the electric field between opposite electric charges, which tends to make mobile charges move in the direction of that field, towards the low-potential or negative end if the mobile charges are positive ones, but "upwards' if they are electrons. Just as an effort. which we might call a weight-motive force, is required to lift a weight against the force of a gravitational field, so an electromotive force is needed to lift positive electric charges to a point of higher
potential; that is to say, against the force of an electric field. And just as the amount of weight-motive force could be reckoned as the height it could lift a weight-any weight-so an e.m.f. is specified as the p.d. it can overcome. If water has to be supplied to the top floor of a 200 ft block of flats, then a water-motive force which could be specified as not less than 200 ft must be provided by a pump or other active device. The difference in height that the pressure of the water source can overcome is the most directly informative way of stating it. Similarly the p.d. in volts that a source of electricity can overcome is the most helpful way of stating its e.m.f., which indeed is sometimes referred to as its pressure.

Another reason for reckoning e.m.f. in volts is that it cannot be measured as such, but only in terms of the p.d. it can overcome or create.

Note that the p.d. between two points is something quite definite, but the potential of a point (like the height of a hill) is relative, depending on what one takes as the reference level. Heights are usually assumed to be relative to sea level, unless it is obvious that some other 'zero' is used (such as street level for the height of a building). The potential of the earth is the corresponding zero for electrical potentials, where no other is implied. To ensure definiteness of potential, apparatus is of en connected to earth.

The p.d. across a resistor (or, as it is sometimes called, the voltage drop, or just 'drop') is, as we know, equal to its resistance multiplied by the current through it; and the positive end is that at which the current enters (or electrons leave). The positive end of a source of e.m.f., on the other hand, is that at which current would leave if its terminals were joined by a passive conductor. That is because an e.m.f. has the unique ability of being able to drive a current against a p.d. Note however that the positive terminal of a source of e.m.f. is not always the one at which current actually does leave; there may be a greater e.m.f. opposing it in the circuit.

Fig. $2.12 a$ shows a circuit invented to illustrate questions that can arise in the early stages of studying these matters. Let us assume that each cell in it has an e.m.f. of 2 V .

All five cells in the battery $B_{1}$ are connected in series and all tend to drive current clockwise, so the total clockwise e.m.f. of $B_{1}$ is $5 \times 2=10 \mathrm{~V}$, and we can show this as in Fig. 2.12b.
$B_{2}$ seems to be a flat contradiction of Kirchhoff's voltage law. Here we have an e.m.f. of 2 V between two points which must be at the same potential because they are joined by a wire of negligible resistance! The cell is, as we say, short-circuited. or 'dead-shorted'. Part of the answer to this apparent impossibility is that no real source of e.m.f. can be entirely devoid of resistance. Often it is not enough to matter, so is not shown in diagrams. But where the source is short-circuited it is vital, because it is the only thing to limit the current. To represent a source of e.m.f. completely, then, we must include between its terminals not only a symbol for e.m.f.
but also one for resistance. When a scurce of e.m.f. is open-circuited, so that no current flows, there is no voltage drop in its internal resistance, so the terminal voltage is equal to the e.m.f. When current is drawn the terminal voltage falls, and if the source is deadshorted the internal drop must be equal to the e.m.f., as shown in


Fig. 2.12-(a) is am imaginary circtuit for illustrating the meanings of e.mr.f. and p.d. The voltages and currems in different parts of it are marked in (h). Arrows alongside roltages show the direction of rise of potentia!

Fig. 2.12b. The other part of the answer is that when the source has a very low resistance (e.g., a large accumulator or generator) such an enormous current flows that even a wire of normally negligible resistance does not reduce the terminal voltage to zero, and the wire is burnt out, and perhaps the source too.

In our example we shall assume that all the resistance is internal, so the external effect of $B_{2}$ is nil.

If the loop containing $B_{3}, B_{4}$, and $R_{2}$ is considered on its own as a series circuit, we see two e.m.fs each of 4 V in opposition, so the net e.m.f. is zero, and no current due to them flows through $\mathrm{R}_{2}$. As regards the main circuit, $\mathrm{B}_{3}$ and $\mathrm{B}_{4}$ are in parallel, and contribute an e.m.f. of -4 V (because in opposition to $\mathrm{B}_{1}$, which we considered positive). This can be indicated either by ' -4 V ' and a clockwise arrow, or by ' 4 V ' and an anticlockwise arrow as shown.

It would be wise to stop for a moment and re-read that last sentence. So often there are disputes about which is the "right' direction for a voltage or current direction arrow, when all the time either direction could be right. The important thing is to say what the arrow is intended to mean-direction of positive voltage rise or current flow, or direction of actual voltage rise or current flow (Sec. 5.9).
We see that putting a second battery in parallel does not increase the main circuit e.m.f., but it does reduce the internal resistance by putting two in parallel, so is occasionally done for that reason. In our example the $20 \Omega$ in series with $\mathrm{B}_{4}$ is probably far greater than the internal resistance of $\mathrm{B}_{3}$, so practically all the current will flow
through $B_{3}$, and the effect of $B_{4}$ will be negligible.
We have, then (ignoring $B_{2}$ ), $10-4=6 \mathrm{~V}$ net clockwise e.m.f. in the circuit, and (neglecting the internal resistances of $\mathrm{B}_{1}$ and $\mathrm{B}_{3}$ ) a total resistance of $7+5=12 \Omega$. The current round the circuit is therefore $6 / 12=0.5 \mathrm{~A}$, as marked, and the voltage drops across $\mathrm{R}_{1}$ and $\mathrm{R}_{3}$ are $0.5 \times 7=3.5 \mathrm{~V}$ and $0.5 \times 5=2.5 \mathrm{~V}$ respectively.

Finally, the symbol at the bottom left-hand corner means that the negative end of $\mathrm{B}_{1}$ is connected to earth (or 'earthed'). So its potential is reckoned as zero, and the potential of every other point in the circuit is thereby defined.

To distinguish between e.m.f. and p.d. the symbol $E$ is sometimes reserved for e.m.f., p.ds being denoted by $V$. But often there is no need to make any distinction, and we shall see in Sec. 5.11 that attempting to do so can lead to controversy and confusion, so there is much to be said for reckoning in voltage rises (falls being negative rises) as for example those indicated in Fig. 2.12b.

### 2.15 Electrical Effects

It is time now to see what electricity can do. One of the most familiar effects of an electric current is heat. It is as if the jostling of the electrons in the conductor caused a certain amount of friction. But whereas it makes the wire in an electric fire red hot, and. in an electric light bulb white hot, it seems to have no appreciable effect on the flex and other parts of the circuit-in a reputable installation, anyway. Reasoning of the type we used in connection with Fig. 2.11 shows that the rate at which heat is generated in a conductor containing no e.m.fs is proportional to the product of the current flowing through it and the voltage across it (i.e., the p.d. between its ends); in symbols, $E I$. Since both $E$ and $I$ are related to the resistance of the conductor, we can substitute $I R$ for $E$ or $E / R$ for $I$, getting $I^{2} R$ or $E^{2} / R$ as alternative measures of the heating effect. Thus for a given current the heating is proportional to the resistance. Electric lamps and heaters are designed so that their resistance is far higher than that of the wires connecting them to the generator. $I^{2} R$ shows that the heating increases very rapidly as the current rises. Advantage is taken of this in fuses, which are short pieces of wire that melt (or 'blow') and cut off the current if, owing to a shortcircuit somewhere, it rises dangerously above normal.

Another effect of an electric current is to magnetize the space around it. This effect is particularly strong when the wire is coiled round a piece of iron. As we shall see, it is of far more significance than merely being a handy way of making magnets.

An effect that need not concern us much is the production of chemical changes, especially when the current is passed through watery liquids. A practical example is the charging of batteries. The use of the word 'charging' for this process is unfortunate, for
no surplus or deficiency of electrons is accumulated by it.
All these three effects are reversible; that is to say, they can be turned into causes, the effect in each case being an e.m.f. The production of an e.m.f. by heating the junction of two different metals (a thermocouple) is of only minor importance, but magnetism is the basis of all electrical generating machinery, and chemical changes are very useful on a smaller scale in batteries.

### 2.16 Instruments for Measuring Electricity

All three effects can be used for indicating the strength of currents; but although thermocouple ammeters (Sec. 5.5) are used in radio senders, the vast majority of current meters are based on the magnetic effect. There are two main kinds: the moving-iron instrument, in which the current coil is fixed, and a small piece of iron with a pointer attached moves to an extent depending on the amperage; and the moving-coil (generally preferred) in which the coil is deflected by a fixed permanent magnet.

Besides these currents effects there is the potential effect-the attraction between two bodies at different potential. The force of attraction is seldom enough to be noticeable, but if the p.d. is not too small it can be measured by a delicate instrument. This is the principle of what is called the electrostatic voltmeter, in which a rotatable set of small metal vanes is held apart from an interleaving fixed set by a hairspring. When a voltage is applied between the two sets the resulting attraction turns the moving set through an angle depending on the voltage. This type of instrument, which is a true voltmeter because it depends directly on potential and draws no current, is practical and convenient for full-scale readings from about 1000 V to 10000 V , but not for low voltages. ('Full-scale reading' means the reading when the pointer is deflected to the far end of the scale.)

Since $V$ or $E=I R$, voltage can be measured indirectly by measuring the current passing through a known resistance. If this additional conducting path were to draw a substantial current from a source of voltage to be measured, it might give misleading results by lowering that voltage appreciably; so $R$ is made relatively large. The current meter (usually of the moving-coil type) is therefore a milliammeter, or better still a microammeter. For example, a voltmeter for measuring up to 1 V could be made from a milliammeter reading up to 1 mA by adding sufficient resistance to that of the moving coil to bring the total up to $1000 \Omega$. The same instrument could be adapted to read up to 10 V (all the scale readings being multiplied by 10 ) by adding another resistance, called a multiplier, of $9000 \Omega$, bringing the total to $10000 \Omega$. Such an arrangement, shown in Fig. 2.13a, can obviously be extended.

The same milliammeter can of course be used for measuring
currents up to 1 mA by connecting it in series in the current path; but to avoid increasing the resistance of the circuit more than can be helped the voltage resistances would have to be short-circuited or otherwise cut out. Care must be taken never to connect a current meter directly across a voltage source, for its low resistance might


Fig. 2.13-Showing how a single moving-coil indicating instrument cun be used for measuring (a) voltage, (b) current, (c) resistance. The moving coil and pointer attached are shown diagrammaticall!
pass sufficient current to destroy it. Higher currents can be measured by diverting all but a known fraction of the current through a bypass resistance called a shunt. If, for example, the resistance of the $0-1$ milliammeter is $75 \Omega$, shunting it with $75 / 9=8.33 \Omega$ would cause nine times as much current to flow through the shunt as passes through the meter, thereby multiplying the range by 10 (Fig. 2.13b).
Thus a single moving-coil instrument can be made to cover many ranges of current and voltage measurement, simply by connecting known resistances as shunts and multipliers. The operation can be reversed by incorporating a battery sufficient to give full-scale deflection; then, when an unknown resistance is connected, either as a shunt or multiplier, according to whether it is small or large, the extent to which the reading is reduced depends on the value of that resistance. It is therefore possible to graduate the scale in ohms and we have an ohmmeter (Fig. 2.13c).

### 2.17 Electrical Power

When everyday words have been adapted for scientific purposes by giving them precise and restricted meanings, they are more likely to be misunderstood than words that were specially invented for use as scientific terms-'electron', for example. Power is a word of the first kind. It is commonly used to mean force, or ability, or authority. In technical language it has only one meaning: rate of doing work. Work here is also a technical term, confined to purely physical activity such as lifting weights or forcing things to move against
pressure or friction-or potential difference. If by exerting muscular force you lift a 10 lb weight 3 ft against the opposing force of gravity you do 30 ftlb of work. And if you take a second to do it, your rate of doing work-your power output-is 30 ft lb per second, which is rather more than $1 / 20$ th of a horse-power. Correspondingly, if the electromotive force of a battery raises the potential of 10 electrons by 3 V (i.e., causes them to flow against an opposing p.d. of 3 V ) it does 30 electron-volts (eV) of work; and if it takes one second to do it the output of power is 30 volt-electrons per second. The electron per second is, as we saw, an extremely small unit of electric current, and it is customary to use one more than $6 \times 10^{18}$ times larger-the ampere. So the natural choice for the electrical unit of power might be called the volt-ampere. Actually, for brevity (and for another reason which appears in Chapter 6), it has been given the name watt.

We now have the following to add to the table which was given in Sec. 2.5:

| Quantity | Symbol for <br> Quantity | Unit of <br> Quantity | Symbol for <br> Unit |
| :---: | :---: | :---: | :---: |
| Electrical power | $P$ | Watt | W |

We also have the equation $P=E I$. So when your 3 V torch battery is lighting a 0.2 A bulb it is working at the rate of $0.2 \times 3=0.6 \mathrm{~W}$. In this case the battery e.m.f. is occupied in forcing the current through the resistance of the filament in the bulb, and the resulting heat is visible. We have already noted that the rate at which heat is produced in a resistance is proportional to $E I$-the product of the e.m.f. applied and the current flowing-and now we know this to be a measure of the power expended.
Alternative forms of the power relationship can be found as in Sec. 2.15:

$$
P=I R=\frac{E^{2}}{R}
$$

Thus if any two of the four quantities $E, I, R$, and $P$ are given, all are known. For example, if a $1000 \Omega$ resistor is to be used in a circuit where 50 V will be maintained across it, the power dissipated as heat will be $50^{2} / 1000=2.5 \mathrm{~W}$. In choosing such a resistor one must take care not only that the resistance is correct but also that it is large enough to get rid of heat at the rate equivalent to 2.5 W without reaching such a high temperature as to damage itself or anything near it.

To familiarize oneself with power calculations it would be a good exercise to add another column to the table in Sec. 2.10, headed 'Power (watts)', working out the figures for each resistor and
checking that they are the same whether derived from $E I, I^{2} R$, or $E^{2} / R$. A further check is to add up all the wattages for $R_{1}$ to $R_{5}$ and see that the total is the same as that for $R$.

It is often necessary to be able to tell whether a certain part of a circuit is acting as a giver (or generator, or source) of power, or as a receiver (or load). The test is whether the current is going through it in the direction of rising potential, in which case power is being generated there, or of falling potential, in which case power is being absorbed. In Fig. 2.12 the current is going through $\mathrm{B}_{1}$ from - to + , so $B_{1}$ is a generator, as one would expect of a battery. But it is going through $\mathrm{B}_{3}$ from + to - , so although in a different circuit $\mathrm{B}_{3}$ could be a generator, in this one it is a load, into which power is being delivered, perhaps for charging it. Through any resistance alone, current can only flow from + to - , because a resistance is essentially a user-up of power.

### 2.18 A Broader View of Resistance

It will be found later on in this book that there are other ways in which electrical power can be 'lost' than by heating up the circuit through which the current flows. This leads to a rather broader view of resistance; instead of defining it as $E / I$, resulting from an experiment such as that connected with Fig. 2.3, one defines it as $P / I^{2}$ (derived from $P=I^{2} R$ ), $P$ being the expenditure of power in the part of the circuit concerned, and $I$ the current through it. The snag is that $P$ is often difficult to measure. But as a definition to cover the various sorts of 'resistance' encountered in radio it is very useful.

A feature of electrical power expended in resistance-however defined-is that it leaves the circuit for good. Except by some indirect method it is not possible to recover any of it in the form of electrical power. But there are ways in which electrical power can be employed to create a store of energy that can be released again as electrical power. Energy here is yet another technical term, meaning the amount of work that such a store can do. It can be reckoned in the same units as work, and so power can be defined alternatively as rate of release of energy. To return to our mechanical analogy, 30 ft lb of energy expended by exerting a force of 101 b to push a box 3 ft across the floor is all dissipated as heat caused by friction.* But 30 ft lb used in raising a 101 b weight 3 ft is stored as what is called potential energy. If the weight is allowed to descend, it delivers up the 30 ftlb of energy, which could be used to drive a

[^3]grandfather clock for a week. Another way of storing the energy would be to push a heavy truck mounted on ball bearings along rails. In this case very little of the pressure would be needed to overcome the small amount of friction; most of it would have the effect of giving it momentum which would keep it going long after the 3 ft push had finished. A heavy truck in motion is capable of doing work because the force setting it in motion has been used to store energy in it; this kind is called kinetic energy.

In the next chapter we shall begin the story of how electrical energy can be stored and released in two ways, corresponding to potential and kinetic energy, and how this makes it possible to tune in to different stations on the radio.

## CHAPTER 3

## Capacitance

### 3.1 Charging Currents

Although most of the last chapter was about steady e.m.fs driving currents through resistances, you may remember that the whole thing began with the formation of an electric charge. We likened it to the deportation of unwilling citizens from town A to town B . Transferring a quantity of electrons from one place to another and leaving them there sets up a stress between the two places, which is only relieved when the same quantity of electrons has been allowed to flow back. The total amount of the stress is the p.d., measured in volts. And the place with the surplus of electrons is said to be negatively charged. The original method of charging-by frictionis inconvenient for most purposes and has been generally superseded by other methods. If an e.m.f. of, say, 100 V is used, electrons will flow until the p.d. builds up to 100 V ; then the flow will cease because the charging and discharging forces are exactly equal.

It was at this stage that we provided a conductive path between the negatively and positively charged bodies, allowing the electrons to flow back at a rate proportional to the p.d.; and, as this p.d. is being maintained by a constant e.m.f., a continuous steady current is set up. This state of affairs could be represented as in Fig. 3.1. Our attention then became attached to this continuous current and the circuit through which it flowed. The electron surplus at the negative end-and the deficit at the positive end-seemed to have no bearing on this, and dropped out of the picture. So long as e.m.f. and current are quite steady, the charges can be ignored.

But not during the process of charging. While that is going on there must be more electrons entering B than are leaving it; and vice versa during discharging. Assuming that there are parts of a circuit requiring a substantial number of electrons to charge them to a potential equal to the applied e.m.f., then it is clear that the ordinary circuit principles we have been studying become more complicated whenever the e.m.f. varies. For one thing, it seems as if the current is no longer the same in all parts of a series circuit. The moment the e.m.f. is applied, a charging current starts to flow, over and above any current through conducting paths. If the e.m.f. is increased at any time, a further charging current flows to raise the p.d. correspondingly. Reducing the e.m.f. causes a discharging current. Since telecommunications and most other applications of electronics involve e.m.fs that are continually varying, it is evidently important to know how far the charging and discharging currents affect matters.

### 3.2 Capacitance: What It Is

The first thing is to find what decides the number of electrons needed to charge any part of a circuit. The number of people that could be transferred to another town would probably depend on (a) the pressure brought to bear on them. (b) the size of the town, and (c) its remoteness. The number of electrons that are transferred as a charge certainly depends on the electrical pressure appliedthe e.m.f. As one would expect. it is proportional to it. If the battery in Fig. 3.1 gave 200 V instead of 100 , the surplus of electrons

Fig. 3.1-Before a p.d. can be established hetween amy two parts of a circuit (such as A and B) they must be charged, and when the p.d. is rapidlly varied the charging current mal be importunt

forced into B would be just twice as great. The quantity of electrons required to charge any part of a circuit to l V is called its capacitance.

We have already noted in Sec. 2.5 that as a practical unit of electric current one electron-per-second would be ridiculously small. and the number of electrons per second that fits the metric system (SI) is rather more than $6 \times 10^{18}(=1 \mathrm{~A})$. So it is natural to take the same number of electrons as the unit of electric charge or quamity. of electricity (symbol: Q). It is therefore equal to the number that passes a given point every second when a current of 1 A is flowing, and the name for it is the coulomb. So the unit of capacitance is that which requires 1 coulomb to charge it to 1 volt. It is named the farad (abbreviation: F) in honour of Michael Faraday, who contributed so much to electrical science. In symbols, the relationship between electric charge, voltage, and capacitance is

$$
Q=V C
$$

which should be compared with the relationship between electric current, voltage, and conductance (Sec. 2.12). $V$ is used instead of $E$ because fundamentally it is the p.d that is involved rather than the charging e.m.f. If the conducting path and e.m.f. in Fig. 3.1 were both removed, leaving A and B well insulated, their charge and the p.d. would remain. In practice one need not bother to distinguish between e.m.f. and p.d. every time (Sec. 2.14). All that
matters is the voltage, and $E$ and $V$ do equally well.
As capacitance is equal to the charge required to set up a given potential difference between two parts of a circuit or other objects, one has to speak of the capacitance between those objects, or of one to another.

### 3.3 Capacitance Analysed

We saw that when any object is being charged the current going into it is greater than that coming out. So if Kirchhoff's law about currents is true in all circumstances, the object must be more than just a point; it must have some size. We would expect the capacitance of objects to one another to depend in some way on their size : the question is, what way?

When analysing resistance (Sec. 2.11) we found that it depended on the dimensions of the conductor and what it was made of. So, of course, does conductance, though in inverse ratio (Sec. 2.12). Capacitance, however, has to do with the space between the conductors; it results from the force of attraction that can be set up in that space, or from what is called the electric field. Suppose A and B in Fig. 3.2 are our oppositely-charged terminals. The directions


Fig. 3.2-The dotted lines indicate the direction and relative intensit! of electric field between (wo oppositely-charged terminals. $A$ and $B$
along which the force acts are indicated by dotted lines, called lines of electric force. These show the paths electrons would take if they were free to move slowly under the force of attraction to + . (But to agree with the convention for direction of electric currents the arrows point in the opposite direction.) To represent a more intense field, lines are drawn more closely together. Each can be imagined to start from a unit of positive charge and end on a unit of negative charge. It must be understood that it is a matter of convenience what size of unit is taken for this purpose; and that the lines are not meant to convey the idea that the field itself is in the form of lines, with no field in the spaces between.

The fact that the space between two small widely-separate objects
such as these is not confined to a uniform path makes the capacitance between them a more difficult thing to calculate than the resistance of a wire. But consider two large flat parallel plates, shown edge-on in Fig. 3.3a. Except for a slight leakage around the edges, the field


Fig. 3.3-Consideration of the electric field bencen chosely-spaced parallel plates leads to the relationship between dimensions and capacitance
is confined to the space directly between the plates, and this space, where the field is uniformly distributed, is a rectangular slab.

Suppose, for the sake of example, the plates are charged to 100 V . Since the field is uniform, the potential changes at a uniform rate between one plate and the other. Reckoning the potential of the lower plate as zero, the potential half way between the plates is therefore +50 V . A third plate could be inserted here, and we would then have two capacitances in series, each with a p.d. of 50 V . Provided that the thickness of the third plate was negligible, its presence would make no difference to the amount of the charge required to maintain 100 V across the outer plates. Each of the two capacitances in series would therefore have the same charge but only half the voltage. To raise their p.d. to 100 V each, the amount of charge would have to be doubled (Fig. $3.3 b$ ). In other words (remembering that $C=Q / V$ ) halving the spacing doubles the capacitance. In general, $C$ is inversely proportional to the distance between the plates (which we can denote by $t$, for thickness of the space).

Next, imagine a second pair of plates, exactly the same as those in Fig. 3.3a. Obviously they would require the same quantity of electricity to charge them to 100 V . If the two pairs were joined together as in Fig. 3.3c, they would form one unit having twice the cross-section area and requiring twice the charge to set up a given p.d.; in other words, twice the capacitance. In general, $C$ is directly proportional to the cross-section area of the space between the plates (which we can denote by $A$ ).

So the capacitance between two plates separated by a slab of space in which the electric field is uniform depends on the dimensions of the slab in the same way as does the conductance of a piece of material (Sec. 2.12):

$$
C=\frac{\varepsilon A}{t}
$$

Here $\varepsilon$ ('epsilon') is the 'constant', corresponding to conductivity, needed to link the chosen system of units with the experimentally observed facts. Formerly the symbol $\kappa$ was used. One would expect it to be called 'capacitivity', but actually the name is absolute permittivity. When, as in SI units (Sec. 2.11), C is reckoned in farads, $t$ in metres, and $A$ in square metres, and the space is completely empty, $\varepsilon$ turns out to be $8.854 \times 10^{12}$. This particular value is sometimes called the 'electric space constant' and is given the distinctive symbol $\varepsilon_{0}$.

Filling the space with air makes very little difference: but if you use solid or liquid materials such as glass, paper, plastics or oil, the capacitance is considerably increased. The amount by which the capacitance is multiplied in this way by filling the whole of the space with such a material is called the relative permittivity of the material, or oflen just permittivity (symbol: $\varepsilon_{\mathrm{r}}$ ).

Insulating materials in this role are called dielectrics, and another name for relative permittivity is dielectric constant. For most solids and liquids it lies between 2 and 10, but for special materials it may be as much as several thousands.

Just as the embodiment of resistance is a resistor, a circuit element designed for its capacitance is called a capacitor; the older name condenser is also used. Usually it consists of two or more parallel plates spaced so closely that most of the field is directly between them. as in Fig. 3.3.

The capacitance of a capacitor thus depends on three factors: area and thickness of the space between the plates, and permittivity of any material there. For convenience, the permittivity specified for materials is the relative value, $\varepsilon_{r}$; and since $\varepsilon$ is $\varepsilon_{r}$ times $\varepsilon_{0}$ the formula is usually adapted for $\varepsilon_{\mathrm{r}}$ by filling in the value of $\varepsilon_{0}$ :

$$
C=\frac{8.854 \times 10^{-12} \varepsilon_{\mathrm{r}} A}{t}
$$

This practice is so common that the little $r$ is often omitted, ' $\varepsilon$ ' being understood to be the relative value.

Supposing, for example, the dielectric had a permittivity of 5 and was 0.1 cm thick, the area needed to provide a capacitance of 1 F would be about 9 square miles. In practice the farad is far too large a unit, so is divided by a million into microfarads $(\mu \mathrm{F})$, or even by $10^{12}$ into picofarads ( pF ). Adapting the above formula to the most convenient units for practical purposes, we have:

$$
C=\frac{\varepsilon A}{11 \cdot 3 t} \mathrm{pF}
$$

### 3.4 Capacitors

The general symbol for a capacitor in circuit diagrams is Fig. 3.4a. The capacitors themselves appear in great variety in circuits, according to the required capacitance and other circumstances. Some actually do consist of a single sheet of insulating material sandwiched between a pair of metal plates, but sometimes there are


Fig. 3.4-The standard symbol for a fixed capacitor is at a; it often has many plates interleaved as at b. Electrolytic capacitors are distinguished as at $c$. in which the white plate is positive. d denotes a variable capacitor
a number of plates on each side as indicated in Fig. 3.4b. The effective area of one dielectric is of course multiplied by their number-in this case four.

A very common form of capacitor consists of two long strips of aluminium foil separated by polyester film and rolled up into a compact block. Above a few microfarads it is often more economical to separate the aluminium electrodes by chemically impregnated paper or other material which causes an extremely thin insulating film to form on one of them. Apart from this film-forming, the solution also acts as a conductor between the film and the other plate. These electrolytic capacitors must always be connected and used so that the terminal marked + never becomes negative relative to the other. Even when correctly used there is always a small leakage current. To help the wireman, the symbol Fig. 3.4c, in which the + plate is distinguished by outlining, is used for electrolytic types.

Air is seldom used as the dielectric for fixed capacitors, but is quite common in variable capacitors. These, indicated in diagrams as $3.4 d$, consist of two sets of metal vanes which can be progressively interleaved with one another, so increasing the effective area, to obtain any desired capacitance up to the maximum available, seldom more than 500 pF . Plastic film dielectric is used in variable capacitors where compactness is essential.

Besides the capacitance, an important thing to know about a capacitor is its maximum working voltage. In Sec. 2.3 we visualized insulators as substances in which the electrons are unable to drift through the material in large numbers under the influence of an electric field but instead they 'strain at the leash', being slightly
displaced from their normal positions in the atoms. If the intensity of electric field is increased beyond a certain limit the elastic leashes tethering the electrons snap under the strain, and the freed electrons rush uncontrollably to the positive plate, just as if the dielectric were a good conductor. It has, in fact, been broken down or punctured by the excessive voltage. For a given thickness, mica stands an exceptionally high voltage, or, as one says, has a relatively high dielectric strength. Air is less good, but has the advantage that the sparking across resulting from breakdown does it no permanent harm.

In our concentration on intentional capacitance between plates, we should not forget that, whether we like it or not, every part of a circuit has capacitance to surrounding parts. When the circuit e.m.fs are varying rapidly, these stray capacitances may be very important.

### 3.5 Charge and Discharge of a Capacitor

It is instructive to consider the charging process in greater detail. In Fig. 3.5 the capacitor can be charged by moving the switch to A, connecting it across a battery; and discharged by switching to $\mathbf{B}$. $R$ might represent the resistance of the wires and capacitor plates, but as that is generally very small it will be easier to discuss the


Fig. 3.5-Circuit used for obtaining the charge and discharge curves shown in Fig. 3.6
matter if we assume we have added some resistance, enough to bring the total up to, say, $200 \Omega$. Suppose the capacitance $C$ is $5 \mu \mathrm{~F}$ and the battery e.m.f. 100 V .

At the exact moment of switching to A the capacitor is as yet uncharged, so there can be no voltage across it (Sec. 3.2); the whole of the 100 V of the battery must therefore be occupied in driving current through $R$, and by Ohm's law that current is found to be $0 \cdot 5$ A (extreme left of Fig. 3.6a). Reckoning the potential of the negative end of the battery as zero, we have at this stage the positive end of the battery, the switch, the upper plate of the capacitor, and (because the capacitor has no p.d. across it) the lower plate also, all at +100 V . The lower end of the resistor is zero, so we have

100 V drop across the resistor, as already stated. Note that immediately the switch is closed the potential of the lower plate of the capacitor as well as that of the upper plate jumps from zero to +100 . The use of a capacitor to transfer a sudden change of potential to another point, without a conducting path, is very common.

We have already seen that the number of coulombs required to charge a capacitance of $C$ farads to $V$ volts is $V C$. In this case, it
(a)


Fig. 3.6-These curves shonv the current and voltuge in the circuit of Fig. 3.5 during churge and discharge
is $100 \times 5 \times 10^{-6}$, which is 0.0005 . As a coulomb is an amperesecond, the present charging rate of 0.5 A , if maintained, would fully charge the capacitor in 0.001 sec . The capacitor voltage would rise steadily as shown by the sloping dotted line in Fig. 3.6b. Directly it starts to do so, however, there are fewer volts available for driving current through $R$, and so the current becomes less and the capacitor charges more slowly. When it is charged to 50 V , 50 V are left for the resistor, and the charging rate is 0.25 A . When $C$ is charged to $80 \mathrm{~V}, 20$ are left for $R$, and the current is 0.1 A . And so on, as shown by the curves in Fig. 3.6.

Curves of this type are known as exponential, and are characteristic of many growth and decay phenomena. Note that the current curve's downward slope, showing the rate at which the current is decreasing, is at every point proportional to the current itself. It can be shown mathematically that in the time a capacitor would take to charge at the starting rate it actually reaches $63.2 \%$ of
its full charge. By using the principles we already know we have calculated that this time in the present example is 0.001 sec . Try working it out with letter symbols instead of numbers, so that it covers all charging circuits, and you should find that it is $C R$ secs, regardless of the voltage $E . C R$-equal to the time in seconds taken to reach $63.2 \%$ of final charge-is of course a characteristic of the circuit, and is called its time constant. $C$ in this example being $5 \times 10^{-6} \mathrm{~F}$ and $R 200 \Omega, C R$ is 0.001 sec , confirming Fig. 3.6.

Theoretically, the capacitor is never completely charged, because the charging depends on there being a difference between the applied e.m.f. and the capacitor voltage, to drive current through the resistance. If the current scale is multiplied by 200 (the value of $R$ ), the upper curve is a graph of this voltage across $R$. Note that the voltages across $C$ and $R$ at all times up to 0.004 sec add up to 100 . just as Kirchhoff's voltage law says they ought (Sec. 2.13).

For practical purposes the capacitor may be as good as fully charged in a very short time. Having allowed 0.004 sec for this to happen in the present case, we move the switch to B. Here the applied voltage is zero, so the voltages across $C$ and $R$ must now add up to zero. We know that the capacitor voltage is practically 100. so the voltage across $R$ must be -100 , and the current -0.5 A ; that is to say, it is flowing in the opposite direction, discharging the capacitor. And so we get the curves in Fig. 3.6 from 0.004 sec onwards.

A point to remember is that the voltages set up across $R$ at the instants of switching to $A$ and $B$ are short-lived; some idea of their duration in any particular case can be found by multiplying $C$ by $R$ in farads and ohms respectively, giving the answer in seconds. These temporary changes are called transients.

### 3.6 Where the Power Goes

The height of the charging curve above the zero line in Fig. 3.6a represents the current supplied by the battery. Multiplied by its e.m.f. ( 100 V ) it represents the power. When first switched on, the battery is working at the rate of $0.5 \times 100=50 \mathrm{~W}$, but at the end of 0.004 sec this has fallen off almost to nothing. At first the whole 50 W goes into the resistance, so all its energy is lost as heat. But the effort of the e.m.f. immediately becomes divided, part being required to drive the current through $R$ and the rest to charge $C$. It is clear that energy is going into the capacitance (Sec. 2.18); what happens to it? The answer is that it is stored as an electric field, in a similar way to the storage of mechanical energy in a spring when it is compressed by an applied force. During the charging process as a whole, then, part of the release of energy from the battery is lost as heat in the resistor, and part is stored in the capacitor.

When we come to the discharge we find that the voltage across $R$
and the current through $R$ follow exactly the same curves as during the charge, except that they are both reversed. But multiplying two negatives together gives a positive. so the energy expended in $R$ is the same amount as during charge. The capacitor voltage and current during the discharge are the same as for the resistor, so the amount of energy is equal; in fact, it is the same energy, for what goes into the resistor has to come out of the capacitor. Applying the test described at the end of Sec. 2.17. we can check that the capacitor is a source of energy; the current that is going into the resistor at its positive terminal is coming out from the capacitor's positive terminal.

At the end of the discharge, all the energy supplied by the battery during the charging process has been dissipated in the resistance as heat. And as the amounts dissipated therein during charge and discharge are equal, it follows that during charge the energy from the battery was shared equally by $R$ and $C$.

Summing up: during the charging period half the energy supplied by the battery is stored as an electric field in the capacitor and half is dissipated in the resistor; during discharge the half that was stored is given back at the expense of the collapsing field and is dissipated in the resistor.

It may be as well to repeat the warning that the 'charging' and 'discharging' of accumulator batteries is an entirely different process, accompanied by only minor variations in voltage. The electricity put into the battery is not stored as an electric charge; it causes a reversible chemical change in the plates.

### 3.7 Recapitulation

By this time you may be getting a little confused about the meanings and relationships of the various quantities we have met. So let us recapitulate. First, those that do not necessarily depend on time. There is electric charge ( $Q$ ), reckoned in coulombs (C), which can be thought of as surpluses or deficiencies of electrons. Transferring electrons from A to B charges A positively and B negatively (as we say), and sets up a p.d. ( $V$ ) between them, which is a kind of stress and is reckoned in volts ( V ). $Q / V$ is called the capacitance $(C)$ between A and B , and its unit is the farad ( F ). To make the electrons move from A to B against the p.d. created by the separation of the charges we need an e.m.f. $(E)$ which is reckoned in terms of the p.d. it is able to counteract, so its unit also is the volt. The p.d. is always tending to restore the charge to the normal uncharged state, and if it is allowed to do so the electrical energy stored in $C$ is released. That energy is equal to the effort (called work) that the source of e.m.f. had to do in order to charge $C$. Because energy and work, like income and expenditure, are the same thing looked at from different points of view, they are both denoted by $W$ and
have the same unit-the joule ( J )-new information, this.
We know (Sec. 3.2) that if a charge $Q$ is imparted to a capacitance $C$ the resulting p.d., $V$, is equal to $Q / C$. But as the starting p.d. was zero and increased in proportion to the charge, the average p.d.and therefore the average e.m.f. needed to transfer the chargemust be half as much, $Q / 2 C$. The work done by the e.m.f., and therefore the energy stored in $C$, is $Q$ multiplied by the average e.m.f., that is to say

$$
W=Q \times Q / 2 C=Q^{2} / 2 C=V^{2} C^{2} / 2 C=\frac{1}{2} V^{2} C
$$

The important thing to remember about energy is that it never ceases to exist. Even when the switch in Fig. 3.5 is moved to $B$ the electrical energy stored in $C$ is converted into an equal amount of heat energy in $R$.

Time comes into things as rates of happening. The rate at which charge is moved is called current ( $I$ ), reckoned in amperes (A). So if a current of $I$ amps continues steadily for $t$ secs, the charge moved past a fixed point en route is $I t$ coulombs and $I=Q / t$. If the charge is moved against a constant p.d. $V$, energy equal to $V Q$ is transferred every second. and an equal amount of work has to be done by a source of e.m.f. The rate at which energy is transferred and work is done is called power $(P)$ and is reckoned in watts (W). So $P=$ $V Q / t=V I$ (Sec. 2.17).

### 3.8 Displacement Currents

If any readers are worrying about reconciling capacitance with Kirchhoff's current law, which implies that current in a series circuit is the same throughout, they deserve a note on the subject.

There should be no difficulty so far as the extra capacitance due to dielectrics is concerned. A charging or discharging current in the circuit connected to the capacitor plates can be regarded as continued through the dielectric by the displacements of electrons as they 'strain at the leash'. The fact that this movement is very restricted is no problem, for it is consistent with the very limited quantity of electricity that can be allowed to flow in one direction as a charging current. To take some practical figures, $\operatorname{InF}\left(=10^{-9} \mathrm{~F}\right)$ is typical of capacitors in radio circuits, and the voltages across them are usually quite small. But let us go up to 1000 V as a typical breakdown voltage. Then using the basic equation $Q=V C$ we find the charge needed to bring it to that point is only $10^{-6} \mathrm{C}$, which would be delivered in 1 second at the rate of only $1 \mu \mathrm{~A}$. The enormous number of electrons in the dielectric would not have to move very far to match such a small drift of their colleagues in the charging circuit.

The real problem concerns the capacitance left when the dielectric
is removed and even the air between the plates is pumped out, so that no electrons or any other charges are there. If looks as if Kirchhoff's current law then breaks down completely.

Clerk Maxwell, who has already come into this story (Sec. 1.9), was well aware of this difficulty, and got himself out of it by imagining what he called a displacement current in the empty space, equal to the charging or discharging current in the rest of the circuit. (The name connected it with a quantity already well known in the theory of electricity as displacement.) This seems rather a desperate expedient for a respectable scientist to fall back on merely in order to save Kirchhoff's law from failing in a particular case, and so perhaps it would be if that were all. But as we saw earlier (Sec. 1.9) Maxwell was concerned with a much more important matter than that: the possibility of electric waves in space. These could not exist, and neither could we, if something equivalent to electric currents did not function in empty space. After all, we do not have to understand everything fully in order to use it.

## CHAPTER 4

## Inductance

### 4.1 Magnets and Electromagnets

If a piece of paper is laid on a straight bar magnet, and iron filings are sprinkled on the paper, they are seen to arrange themselves in a pattern something like Fig. 4.1a. The lines show the paths along which the attraction of the magnet exerts itself. (Compare the lines of electric force in Fig. 3.2.) As a whole, they map out the magnetic field, which is the sphere of influence, as it were, of the magnet. The field is most concentrated around two regions, called poles,* at the ends of the bar. The lines may be supposed to continue right through the magnet, emerging at the end marked N and returning at S . This direction, indicated by the arrows, is (like the direction of an electric current) purely conventional, and the lines themselves are an imaginary representation of a condition occupying the whole space around the magnet.

The same result can be obtained with a previously quite ordinary and unmagnetized piece of iron by passing an electric current through a wire coil round the iron-an arrangement known as an electromagnet. It is not even necessary to have the iron core (as it is called); the coil alone, so long as it is carrying current, is interlinked with a magnetic field having the same general pattern as that due to a bar magnet, as can be seen by comparing Fig. 4.la and $b$. Without the iron core, however, it is considerably weaker. Finally, if the wire is unwrapped, every inch of it is still found to be surrounded by a magnetic field. Though very much less concentrated than in the coil, if the current is strong enough it can be demonstrated with filings as in Fig. 4.2.

The results of these and other experiments in electromagnetism are expressed by saying that wherever an electric current flows it surrounds itself with a field which sets up magnetic flux isymbol: $\Phi$, the Greek capital phi). This flux and the circuit link one another like two adjoining links in a chain. By winding the circuit into a compact coil, the field due to each turn of wire is made to operate in the same space, producing a concentrated magnetic flux (Fig. 4.1b). Finally, certain materials, notably iron and its alloys, described as ferromagnetic, are found to have a very large multiplying effect on the flux, this property being called the relative permeability of the material, denoted by the symbol $\mu_{\mathrm{r}}$, or often just $\mu$. The permeabilities

[^4]of most things, such as air, water, wood, brass, etc., have practically no effect on the flux and are therefore generally reckoned as 1 . Certain alloys, not all containing iron, have permeabilities running into many thousands. Unlike resistivity and permittivity, however, the permeability of ferromagnetic materials is not even approximately constant but varies greatly according to the flux density (i.e., the flux passing through unit area at right angles to its direction).


Fig. 4.1-The dotted lines indicate the direction and distrihution of magnetic field around (a) a permanent har magnet. and (b) a coil of wire carrying current


Fig. 4.2-Conventional direction of magnetic field around a straight wire carrying current


Fig. 4.3-Deflection of a campass needle bry a magnet, proving that like poles repel and unlike poles attract

It is now known that all magnets are really electromagnets. Those that seem not to be (called permunent magnets) are magnetized by the movements of the electrons in their own atomic structure. The term 'electromagnet' is however understood to refer to one in which the magnetizing currents flow through an external circuit.

### 4.2 Interacting Magnetic Fields

The ' N ' and ' S ' in Fig. 4.1 stand for North-seeking and Southseeking respectively. Everybody knows that a compass needle points to the north. The needle is a magnet, and turns because its own field interacts with the field of the earth, which is another magnet. Put differently, the north magnetic pole of the earth attracts the northseeking pole of the needle, while the earth's south pole attracts its south-seeking pole. The poles of a magnet are often called just north and south, but strictly, except in referring to the earth itself, this is incorrect. By bringing the two poles of a bar magnet-previously identified by suspending it as a compass--in turn towards a compass needle it is very easy to demonstrate, as in Fig. 4.3, that unlike poles attract one another. This reminds one of the way electric charges behave.

Now suppose we hang a coil in such a way that it is free to turn, as suggested in Fig. 4.4. So long as no current is passing through


Fig. 4.4-A coil carring corrent, and free to rotate. sets itself with its axis pointing $N$ and $S$. a is a side vien' and h an end vion'
the coil it will show no tendency to set itself in any particular direction, but if a battery is connected to it the flow of current will transform the coil into a magnet. Like the compass needle, it will then indicate the north, turning itself so that the plane in which the turns of the coil lie is east and west, the axis of the coil pointing north. If now the current is reversed the coil will turn through 180 degrees, showing that what was the N pole of the coil is now, with the current flowing the opposite way, the S .

The earth's field is weak, so the force operating to turn the coil is small. When it is desired to make mechanical use of the magnetic effect of the current in a coil it is better to provide an artificial field of the greatest possible intensity by placing a powerful magnet as close to the coil as possible. This is the basis of all electric motors.

In electronics, however, we are more interested in other applications of the same principle, such as the moving-coil measuring instruments described very briefly in Sec. 2.16. The coil is shaped somewhat as in Fig. 4.4, and is suspended in the intense field of a
permanent magnet. Hairsprings or taut metal ribbons at top and bottom of the coil serve the double purpose of conducting the current to the coil and keeping the coil normally in the position where the pointer attached to it indicates zero. When current flows through it the coil tends to rotate against the restraint of the springs, and the angle through which it moves is proportional to the magnetic interaction, which in turn is proportional to the value of the current.

### 4.3 Induction

In Sec. 4.1 we noted that the magnetic circuit formed by the imaginary flux lines is linked with the electric circuit that causes them, like two adjoining links in a chain (Fig. 4.5). Now suppose the electric circuit includes no source of e.m.f. and consequently no current,

Fig. 4.5-An electric current and the magnetic flux set up thereby are linked with one another as indicated in this diagram, which show's the relative directions established by convention

but that some magnetic flux is made to link with it. There are various ways in which this can be done: a permanent magnet or an electro-magnet-or any current-carrying circuit-could be moved towards it, or current could be switched on in a stationary circuit close to it.
 is generated around the path

Whatever the method, the result will be the same: so long as the flux linked with the circuit is increasing, an e.m.f. is generated in the circuit. The magnitude and direction of this induced e.m.f. can be found by means of a suitable voltmeter, as in Fig. 4.6. The amount
of e.m.f. is proportional to the rate at which the amount of flux linked with the circuit is changing. So a constant flux linkage yields no e.m.f. When the flux is removed or decreased, a reverse e.m.f. is induced. In Fig. 4.6. the increasingly linked flux has the same direction as in Fig. 4.5; the e.m.f. induced thereby would tend to drive current in the direction shown; i.e., in the opposite direction to the current in Fig. 4.5.

These results. first discovered by Michael Faraday in 1831, are the basis of all electrical engineering, including of course radio. The electrical generators in power stations are devices for continuously varying the magnetic flux linked with a circuit, usually by making electromagnets rotate past fixed coils. The alternative methed of varying the flux linkages-by varying the current in adjacent fixed coils-is adopted in transformers (Sec. 4.11) and. as we shall see, in many kinds of radio equipment.

The Sl unit of flux. the weher $(\mathrm{Wb})$, is such that the induced e.m.f. 'in volts' is equal to the rate at which the linked flux is changing, in webers per second.

### 4.4 Self-Inductance

We have just been considering an e.m.f. being induced in a circuit by the varying of magnetic flux due to current in some other circuit. But in Fig. 4.5 we see a dotted loop representing the flux due to current flowing in that circuit itself. Before the current flowed there was no flux. So, directly the current was switched on, the flux linked with the circuit must have been increasing from zero to its present value, and in the process it must have been inducing an e.m.f. in its own circuit. This effect is-very naturally-known as self-induction. Since Fig. 4.5 can be taken to represent any circuit carrying current. and because a flow of current inevitably results in some linked magnetic flux, it follows that an e.m.f. is induced in any circuit in which the current is varying.

The amount of e.m.f. induced in a circuit (or part of a circuit) when the current is varied at some standard rate is called its selfinductance, or often just inductance (symbol: L). Its unit, the henry (abbreviation: H), was so defined that the inductance of a circuit in henries is equal to the number of volts self-induced in it when the current is changing at the rate of 1 ampere per second.

Actually it is the change of fux that induces the e.m.f., but since information about the flux is less likely to be available than that about the current, inductance is defined in terms of current. What it depends on, then, is the amount of flux produced by a given current, say I A. The more flux, the greater the e.m.f. induced when that current takes one second to grow or die, and so the greater the inductance. The flux produced by a given current depends, as we have seen, on the dimensions and arrangement of the circuit or part of circuit, and on the permeability.

### 4.5 Lenz's Law

In Sec. 4.3 we noted the direction of the induced e.m.f. in relation to the direction of the flux inducing it. By comparing Fig. 4.6 with Fig. 4.5 we see that a self-induced e.m.f. would tend to send current round the circuit in the opposite direction to the current producing the tlux that is inducing it. In more general terms, any induced e.m.f. tends to oppose its cause. This statement is known as Lenz's law, and is really inevitable, because if induced e.m.fs aided their cause there would be no limit to their growth. So it is easy to remember.

When you try to stop the clockwise current in Fig. 4.5 by switching off, the flux in the direction shown begins to decrease. and in doing so it induces an e.m.f. in the same direction as that of the battery, tending to oppose the switch-off and keep the current going.

This is a good point at which to learn another easily remembered rule - the corkscrew rule. Imagine that the current in Fig. 4.2 is a corkscrew being driven downwards. The direction the corkscrew has to be turned is the direction of the resulting flux. This can be seen in Fig. 4.5 too. And it holds good if the roles are interchanged; turning the corkscrew in the direction of the current around the circuit, the direction in which it moves, in or out, is the direction of the resulting flux through the circuit. With this rule and Lenz's law we are fully equipped to deal with induction polarity questions. But we must realize that the directions are conventions, not physical facts.

### 4.6 Inductance Analysed

We had no great difficulty in finding a formula expressing resistance in terms of resistivity and the dimensions of the circuit element concerned. because currents are usually confined to wires or other conductors of uniform cross-section. Capacitance was not so easy, because it depends on the dimensions of space between conductors. and the distribution of the electric field therein is likely to be anything but uniform, but for the particular and practically-important case of a thin space between parallel plates we found a formula for capacitance in terms of permittivity and the dimensions of the space. Inductance is even more difficult, because the magnetic fields of many of the circuit forms used for providing it. known in general as inductors, are not even approximately uniform or easily calculated. And some of the flux may link with only part of the circuit. In coils, for example, the flux generated by one turn usually links with some of the others but not all.

However, Fig. 4.7 shows a type of inductor in which the coil is linked with an iron core of nearly uniform cross-section area. The permeability of the iron is usually so high that the core carries very
nearly all the magnetic flux; the remainder-called leakage fluxthat strays from it through the surrounding air or the material of the coil itself, can be neglected. So it is nearly true to assume that all the flux links all the turns, and that it is uniformly distributed through the whole core. On that assumption, the inductance of a


Fig. 4.7-Showing how a closed iron core links with the turns of the cail and carries nearly all the flux set up by the coil
coil is related to the dimensions in the same way as for conductance and capacitance:

$$
L=\frac{\mu A}{l}
$$

where $A$ is the cross-section area of the core, $l$ is its length, and $\mu$ is its absolute permeability. As with permittivity, the 'absolute' value is the one that gives the answer in the desired units without the assistance of other numbers ('constants'). In SI units (Sec. 2.11), in which $L$ is in henries, $A$ in square metres and $/$ in metres, its value for empty space (sometimes called the 'magnetic space constant'), denoted by $\mu_{0}$, is $1.257 \times 10^{-6}$. The same value is practically correct for air and most other substances except the ferromagnetic materials. Although strictly the permeability $(\mu)$ is this $\mu_{0}$ multiplied by the relative permeability $\mu_{\mathrm{r}}$ (Sec. 4.1), the values quoted for materials are invariably relative, so the formula is usually adapted for them by incorporating $\mu_{0}$ :

$$
L=\frac{1.257 \times 10^{6} \mu_{\mathrm{r}} A}{l}
$$

And, as with permittivity, $\mu_{\mathrm{r}}$ is then usually written ' $\mu$ ' and called just 'permeability'. For air, etc., it can be omitted altogether, being practically 1. If the dimensions are in centimetres, $10^{-8}$ must be substituted for $10^{-6}$.

The above equation is for only one turn. If there are $N$ turns the same current flows $N$ times around the area $A$, so the flux (assuming $\mu$ to be constant) is $N$ times as much. Moreover, this $N$-fold flux links the circuit $N$ times, inducing $N$ times as much
e.m.f. when it varies as in one turn. So the inductance is $N^{2}$ times as great:

$$
L=\frac{1.257 \times 10^{-6} \mu_{\mathrm{r}} A N^{2}}{l}
$$

### 4.7 Practical Considerations

In practice this formula is not of much use for calculating the inductance of even iron-cored coils. For one thing, in order to get the core into position around the coil it is necessary to have it in more than one piece, and the joint is never so magnetically perfect as continuous metal. Even a microscopic air gap makes an appreciable difference, because it is equivalent to an iron path perhaps tens of thousands of times as long. Then the permeability of iron depends very much on the flux density and therefore on the current in the coil. So one cannot look it up in a list of materials as one can resistivity or permittivity. It is usually plotted as a graph against the strength of the magnetizing field. which has the symbol $H$ and is reckoned as the total current encircling the iron (i.e., the number of ampere-turns) per unit length of iron carrying the flux. For many purposes it is even more helpful to have graphs of the flux


Fig. 4.8-The fux density in many magnetic materials such as iron varies with the magnetizing field in this sort of way
density per square metre, which has the symbol $B$ and is reckoned in teslas (i.e., webers of flux per square metre) against $H$, as in Fig. 4.8. The permeability, equal to $B / H$, can be found from this for any particular value of $H$ or $B$. It is represented by the steep-
ness of the slope of the graph. A characteristic of ferromagnetic materials is that increasing the ampere-turns yields progressively smaller increases in flux density. In other words, the permeability gets less and less. This effect is called magnetic saturation. Another effect in ferromagnetic materials will come to our notice in Sec. 25.7.

If you are finding this section rather heavy going, with so many new quantities cropping up, comfort yourself with the assurance that they will not be found essential within the scope of this book. But if you are keen enough to pursue the magnetic relationships farther you may care to note, starting from the fact that $\mu=B / H$, that one can derive the first formula for inductance in Sec. 4.6:

So

$$
\begin{aligned}
\mu & =\frac{B}{H}=\frac{\Phi l}{A l} \\
\frac{\mu A}{l} & =\frac{\Phi}{l}=L
\end{aligned}
$$

With 'air-core' coils, even when all the turns are close together it is not correct to assume that all the flux due to each turn links with the whole lot; and the error is of course much greater if the turns are widely distributed as in Fig. 4.1b. An even greater difficulty is that the flux does not all follow a path of constant area $A$ and equal length $l$.

A general formula for inductance has nevertheless been given, to bring out the basic similarity in form to those for conductance and capacitance, but for practical purposes it is necessary either to measure inductance or calculate it from the various formulae and graphs that have been worked out for coils and straight wires of various shapes and published in radio reference books.

Between a short length of wire and an iron-cored coil with thousands of turns there is an enormously large range of inductance. The iron-cored coil might have an inductance of many henries; the short wire a small fraction of a microhenry. For most purposes the inductance of connecting wires is small enough to ignore. But not always. The maximum current may be much less than 1 A, but if it comes and goes a thousand million times a second its rate of change may well be millions of amps per second and the induced voltage quite large.

It is, of course, this induced voltage that makes inductance significant. Just how significant will appear in later chapters, but we can make a start now by comparing and contrasting it with the voltage that builds up across a capacitance while it is being charged.

### 4.8 Growth of Current in an Inductive Circuit

What happens when current is switched on in an inductive circuit can be studied with the arrangement shown in Fig. 4.9, which
should be compared with Fig. 3.5. To make comparison of the

results easy, let us assume that the inductance, for which the curliness marked $L$ is the standard symbol, is 0.2 H , and that $R$ (which includes the resistance of the coil) is $200 \Omega$ and $E$ is 100 V as before.

With the switch as shown, no current is flowing through $L$ and there is no magnetic field. At the instant of switching to A, the full 100 V is applied across $L$ and $R$. The current cannot instantly leap to 0.5 A , the amount predicted by Ohm's law, for that would mean increasing the current at an infinite rate, which would induce an infinite voltage opposing it. So the current must rise gradually, and at the exact moment of closing the circuit it is zero. There is therefore no voltage drop across $R$, and the battery e.m.f. is $\vartheta$ pposed solely by the inductive e.m.f., which must be 100 V (Fig. 4.10a). That
(a)


Fig. 4.10-Curves showing the current and voltage in the circuit of Fig. 4.9 when the switch is moved first to $A$ and then to $B$
enables us to work out the rate at which the current will grow. If $L$ were 1 H , it would have to grow at 100 A per sec to induce 100 V . As it is only 0.2 H it must grow at $500 \mathrm{~A} / \mathrm{s}$.

In the graph (Fig. 4.10b) corresponding to Fig. 3.6b for the capacitive circuit, the dotted line represents a steady current growth of $500 \mathrm{~A} / \mathrm{s}$. If it kept this up, it would reach the full 0.5 A in 0.001 sec . But directly the current starts to flow it causes a voltage drop across $R$. And as the applied voltage remains 100 , the induced voltage must diminish. The only way this can happen is for the current to grow less rapidly. By the time it has reached 0.25 A there are 50 V across $R$, therefore only 50 across $L$, so the current must be growing at half the rate. The full line shows how it grows; here is another exponential curve and, as in the capacitive circuit, it theoretically never quite finishes. In the time that the current would take to reach its full Ohm's law value if it kept up its starting pace, it actually reaches $63.2 \%$ of it. This time is, as before. called the time constant of the circuit. A little elementary algebra based on the foregoing shows it to be $L / R$ seconds.

The voltage across $L$ is also shown. Compared with Fig. 3.6. current and voltage have changed places. The voltage across $R$ is of course proportional to the current and therefore its curve is the same shape as Fig. 4.10b. Added to the upper, it must equal 100 V as long as that is the voltage applied.

When the current has reached practically its full value, we flick the switch instantaneously across to B. The low resistance across the contacts is merely to prevent the circuit from being completely interrupted in this process. At the moment the switch is operated, the magnetic field is still in existence. It can only cease when the current stops, and the moment it begins to do so an e.m.f. is induced which tends to keep it going. At first the full 0.5 A is going, which requires 100 V to drive it through $R$; and as the battery is no longer in circuit this voltage must come from $L$, by the current falling at the required rate: $500 \mathrm{~A} / \mathrm{s}$. As the current wanes, so does the voltage across $R$, and so must the induced voltage, and therefore the current dies away more slowly, as shown in the continuation of Fig. 4.10. In $L / R(=0.001)$ secs it has been reduced by the inevitable $63.2 \%$.

### 4.9 Power During Growth

In reckoning how the power in the circuit varies during these operations, we note firstly that the output from the battery (calculated by multiplying the current at each instant by its 100 V ) instead of starting off at 50 W and then tailing off almost to nothing, as in the capacitive example, starts at nothing and works up towards 50 W . If there had been no $C$ in Fig. 3.5 there would have been no flow of power at any time ; the effect of $C$ was to make possible a
temporary flow, during which half the energy was dissipated in $R$ and half stored in $C$. In Fig. 4.9, on the other hand, absence of $L$ would have meant power going into $R$ at the full $50-\mathrm{W}$ rate all the time; the effect of $L$ was to withhold part of that expenditure from $R$ for a short time. Not all this energy withheld was a sheer saving to the battery, represented by the temporarily sub-normal current; part of it-actually a half-went into $L$, for the current had to be forced into motion throt:gh it against the voltage induced by the growing magnetic field. The energy was in fact stored in the inagnetic field, in a similar way to the storage of mechanical energy when force is applied to set a heavy vehicle in motion.
Just as the vehicle gives up its stored energy when it is brought to a standstill, the result being the heating of the brakes, so the magnetic field in $L$ gave up its energy when the current was brought to a standstill, the result being the heating of $R$. Self-inductance is therefore analogous to mechanical inertia.

The amount of energy stored when a current $I$ is flowing is again equal to the charge $Q$ transferred during the magnetizing process, multiplied by the p.d. it has been transferred against. Supposing the current grew at the starting rate in Fig. 4.10 it would reach its full value $I$ in time $L / R$, so $Q=I L / R$. The average p.d. across the coil is clearly half the maximum value $I R$. So

$$
W=\frac{I L}{R} \times \frac{1}{2} I R=\frac{1}{2} P^{2} L
$$

### 4.10 More Comparison and Contrast

In the previous chapter we saw that besides the conducted currents. reckoned by dividing voltage by resistance, there is an extra current whenever the voltage is varying. This current is made up of movements of charges caused by the electric field adjusting itself to the new voltage. It is proportional not only to the rate at which the voltage is changing but also the capacitance (intentional or otherwise) of the part of the circuit considered. In this chapter we have seen that in addition to the resistive voltage, reckoned by multiplying current by resistance, there is an extra voltage whenever the current is varying. This voltage is caused by the magnetic field adjusting itself to the new current. It is proportional not only to the rate at which the current is changing but also to the inductance (intentional or otherwise) of the part of circuit considered.

### 4.11 Mutual Inductance

If a coil with the same number of turns as that in Fig. 4.9 were wound so closely around it that practically all the magnetic flux
due to the original primary winding embraced also this secondary

Fig. 4.11-How a transformer is represented in circuit diagrams

winding, then an equal voltage would at all times be induced in the secondary. If an appreciable part of the flux is not linked with the secondary winding, the voltage induced in it is correspondingly less. But by giving the secondary more turns than the primary, it is possible to obtain a greater voltage in the secondary than in the primary. Remember that this voltage depends on the varying of the primary current. A device of this kind, for stepping voltages up or down, or for inducing voltages into circuits without any direct connection, is named (rather inappropriately) a transformer. It is represented in circuit diagrams by two or more coils drawn closely together. An iron core is shown by one or more straight lines drawn between them (Fig. 4.11).

The effect that one coil can exert, through its magnetic field, on another, is called mutual inductance (symbol: $M$ ), and like selfinductance is measured in henries. The definition is similar too: the mutual inductance in henries between two coils is equal to the number of volts induced in one of them when the current in the other is changing at the rate of 1 A per second.

Transformers are an immensely useful application of the inductive effect, and in fact are the main reason why alternating current (a.c.) has almost superseded direct current (d.c.) for electricity supply. So we now proceed to study a.c.

## CHAPTER 5

## Alternating Currents

### 5.1 Frequencies of Alternating Current

In our first chapter we saw that speaking and music are conveyed from one person to another by sound waves, which consist of rapid vibrations or alterations of air pressure. And that to transmit them over longer distances by telephone it is necessary for electric currents to copy these alternations. And further, that for transmitting them across space, by radio, it is necessary to use electric currents alternating still more rapidly. We have now just noted the fact that the great advantage of being able to step voltages up and down as required is obtainable only when the electricity supply is continually varying, which is most conveniently done by arranging for it to be alternating. Public electricity supplies which light and heat our houses and provide the power that works our television receivers and many other facilities are therefore mostly of the alternating-current kind.

The only essential difference between all these alternating currents is the number of double-reversals they make per second; in a word, the frequency. We have already gone fairly fully into the matter of frequency (Sec. 1.4), so there is no need to repeat it ; but it may be worth recalling the main divisions of frequency.

There is no hard-and-fast dividing line between one lot of frequencies and another; but those below 100 Hz are used for power (the standard in Britain is 50 Hz ); those between about 20 and 20000 Hz are audible, and therefore are classed as audio frequencies (a.f.); while those above about 15000 are more or less suitable for carrying signals across space, and are known as radio frequencies (r.f.). Certain of these, such as 525000 to 1605000 , are allocated for broadcasting. All those above about 1000 MHz (wavelengths shorter than about 1 ft ) are often lumped together as microwave frequencies.

What has been said about currents applies to voltages, too; it requires an alternating voltage to drive an alternating current.

The last two chapters have shown that when currents and voltages are varying there are two circuit effects-capacitance and inductance -that have to be taken into account as well as resistance. We shall therefore be quite right in expecting a.c. circuits to be a good deal more complicated and interesting than d.c. Before going on to consider these circuit effects there are one or two things to clear up about a.c. itself.

### 5.2 The Sine Wave

An alternating current might reverse abruptly or it might do so gradually and smoothly. In Fig. $3.6 a$ we have an example of abrupt reversal. There is, in fact, an endless variety of ways in which current or voltage can vary with time, and when graphs are drawn showing how they do so we get a corresponding variety of wave shapes. The steepness of the wave at any point along the time scale shows the rapidity with which the current is changing at that time.

Fortunately for the study of alternating currents, all wave shapes, however complicated, can be regarded as built up of waves of one fundamental shape, called the sine wave (adjective: sinusoidal). Fig. 1.2c shows examples of this. Note the smooth, regular alternations of the component sine waves, like the swinging of a pendulum. Waves of even such a spiky appearance as those in Fig. 3.6 can be analysed into a number of sine waves of different frequencies.

Most circuits used in radio and other systems of telecommunication consist basically of sources of alternating e.m.f. (often called generators) connected to combinations of resistance, inductance and/or capacitance (usually called loads). With practice one can reduce even very complicated-looking circuits to standard generator-and-load combinations.

We have just seen that although there is no end to the variety of waveforms that generators can produce, they are all combinations of simple sine waves. So it is enough to consider the sine-wave generator. Sources of more complicated waveforms can be imitated by combinations of sine-wave generators. Theoretically this dodge is always available. but in practice (as, for example, with square waves) there are sometimes easier alternatives, as we shall see in Chapter 24. Unless the contrary is clear, it is assumed from now on that a generator means a sine-wave generator.

### 5.3 Circuit with Resistance only

The simplest kind of a.c. circuit is one that can be represented as a generator supplying current to a purely resistive load (Fig. 5.1). We have already found (Sec. 2.6) how to calculate the current any


Fig. 5.1-C'ircuit consisting of an a.c. generator (G) feeding a purely resistive load $(R)$
given e.m.f. will drive through a resistance. Applying this method to an alternating e.m.f. involves nothing new. Suppose. for example, that the voltage at the peak of each wave is 200 , and that its fre-
quency is 50 Hz . Fig. 5.2 shows a graph of this e.m.f. during rather more than one cycle. And suppose that $R$ is $200 \Omega$. Then we can calculate the current at any point in the cycle by dividing the voltage at that point by 200, in accordance with Ohm's law. At the peak it is, of course, $200 / 200=1$ A; half-way up the wave it is 0.5 A. Plotting a number of such points we get the current wave, shown dotted. It is identical in shape, though different in vertical scale.

### 5.4 R.M.S. Values

How is an electricity supply that behaves in this fashion to be rated? Can one fairly describe it as a $200-\mathrm{V}$ supply, seeing that the actual voltage is changing all the time and is sometimes zero? Or should one take the average voltage?

The answer that has been agreed upon is based on comparison with d.c. supplies. It is obviously very convenient for a lamp or


Fig. 5.2-Time graphs of e.m.f., current and power in the circuit Fig. 5.1. Note that the frequency' of the power waves is twice that of the e.m.f. and current
heater intended for a $200-\mathrm{V}$ d.c. system to be equally suited to a.c. mains of the same nominal voltage. This condition will be fulfilled if the average power taken by the lamp or heater is the same with both types of supply, for then the element will reach the same temperature and the light or heat will be the same in both cases.

We know (Sec. 2.17) that the power in watts is equal to the voltage multiplied by the amperage. Performing this calculation for a
sufficient number of points in the alternating cycle in Fig. 5.2, we get a curve showing how the power varies. To avoid confusion it has been drawn below the $E$ and $I$ curves. A significant feature of this power curve is that although during half of every cycle the voltage and current are negative, their negative half-cycles always exactly coincide, so that even during these half-cycles the power is (by the ordinary rules of arithmetic) positive. This mathematical convention agrees with and represents the physical fact, checked by the test given at the end of Sec. 2.17, that while both current and voltage reverse their direction the flow of power is always in the same direction, namely, out of the generator.

Another thing one can see by looking at the power curve is that its average height is half the height of the peaks. The peak height is $200 \times 1=200 \mathrm{~W}$; so the average power must be 100 W . This can be checked by cutting out the areas of wave above 100 W and finding that they exactly fill the empty troughs below 100 W , giving a steady 100 W of d.c. The next step is to find what direct voltage would be needed to dissipate 100 W in $200 \Omega$; the answer is again in Sec. 2.17: $P=E^{2} / R$, from which $E^{2}=P R$, which in this case is $100 \times 200=$ 20000 . So $E=\sqrt{ } 20000=141$ approximately.

Judged on the basis of equal power, then, an a.c. supply with a maximum or peak voltage of 200 is equivalent to a d.c. supply at 141 V.

Generalizing this calculation, we find that when an alternating voltage is adjusted to deliver the same power to a given resistance as a direct voltage, its peak value is $\checkmark^{2}$ times the direct voltage, or about 1.414 times as great. Put the other way round, its nominal or equivalent or effective voltage-called its $r$.m.s. (root-mean-square) value-is $1 / \checkmark 2$ or about 0.707 times its peak value. Since the resistance is the same in both cases, the same ratio exists between r.m.s. and peak values of the current. What is called a $240-\mathrm{V}$ a.c. supply therefore alternates between + and $-\sqrt{ } 2 \times 240=339 \mathrm{~V}$ peak.

The r.m.s. value is not the same as the average voltage or current (which is actually 0.637 times the peak value, if averaged over a half cycle, or zero over a whole cycle). If you have followed the argument carefully you will see that this is because we have been comparing the a.c. and d.c. on a power basis, and power is proportional to voltage or current squared.

Remember that the figures given above apply only to the sine waveform. With a square wave (Fig. 6.2h) for example, it is obvious that peak, r.m.s., and average (or mean) values are all the same.

There is one other recognized 'value' - the instantaneous value, changing all the time in an a.c. system. It is the quantity graphed in Fig. 5.2. It can be found for any stage of a cycle by dividing the cycle into 360 degrees of angle as in Sec. 5.7 and then looking up the sine of that angle in a trigonometrical table and multiplying the peak value by it.

As regards symbols, plain capital letters such as $E, V$ and $I$ are
generally understood to mean r.m.s. values unless the contrary is obvious. Instantaneous values, when it is necessary to distinguish them, are denoted by small letters, such as $i$; peak values by $I_{\text {max }}$; and average (or mean) values by $l_{\text {ave }}$.

Using r.m.s. values for voltage and current we can forget the rapid variations in instantaneous voltage, and, so long as our circuits are purely resistive, carry out all a.c. calculations according to the rules discussed in Chapter 2.

### 5.5 A.C. Meters

If alternating current is passed through a moving-coil meter (Sec. 4.2) the coil is pushed first one way and then the other, because the current is reversing in a steady magnetic field. The most one is likely to see is a slight vibration about the zero mark. Certainly it will not indicate anything like the r.m.s. value of the current.

If, however, the direction of magnetic field is reversed at the same times as the current, the double reversal makes the force act in the same direction as before, and the series of pushes will cause the pointer to take up a position that will indicate the value of current. The obvious way to obtain this reversing magnetic field is to replace the permanent magnet by a coil and pass through it the current being measured. When the current is small the field also is very weak and the deflection too small to be read, so this principle is seldom used, except in wattmeters, in which the main current is passed through one coil, and the other-the volt coilis connected across the supply.

A more usual type is that in which there is only one coil, which is fixed. Inside are two pieces of iron, one fixed and the other free to move against a hairspring. When either d.c. or a.c. is passed through the coil, both irons are magnetized with the same polarity, and so repel one another, to a distance depending on the strength of current. These moving-iron meters are useful when there is plenty of power to spare in the circuit for working them, but they tend to use up too much in low-power circuits.

Another method is to make use of the heating effect of the current. When a junction of two different metals is heated, a small unidirectional e.m.f. is generated, which can be measured by a moving coil meter. Instruments of this kind, called thermojunction or thermocouple types, if calibrated on d.c. will obviously read r.m.s. values of a.c. regardless of waveform. They are particularly useful for much higher frequencies than can be measured with instruments in which the current to be measured has to pass through a coil.

The electrostatic instrument (Sec. 2.16) can be used for alternating voltages and responds to r.m.s. values.

But perhaps the most popular method of all is to convert the
alternating current into direct by means of a rectifier-a device that allows current to pass through it in one direction only-so that it can be measured with an ordinary moving-coil meter. The great advantage of this is that by adding a rectifier a multi-range d.c. instrument can be used on a.c. too. Most of the 'multimeters' used in radio and electronics are of this kind. Because the movingcoil instrument measures the average current, which in general is not the same as the r.m.s. current, the instrument is arranged to take account of the factor necessary to convert one to the other. This, as we have seen, is approximately $0.707 / 0.638(=1 \cdot 11)$ for sine waves. It is different for most other waveforms, so the rectifier type of meter reads them incorrectly.

### 5.6 Phase

Looking again at the current graph in Fig. 5.2 we see that it not only has the same shape as the e.m.f. that causes it but it is also exactly in step with it. Consideration of Ohm's law proves that this must be so in any purely resistive circuit. The technical word


Fig. 5.3-The tinte lag hetween the start of $A$ and the start of $B$ is the same in a and b. but in a the tho voltages are quarter of a cycle out of phase and in th are in phase. Time is therefore not the basic measure of phase
for being in step is being in phase. The idea of phase is very important, so we had better make sure we understand it.

Suppose we have two alternating generators, A and B. The exact voltages they give will not matter, but to make it easier to distinguish their graphs we shall suppose A gives double the voltage of B. If we start drawing the voltage graph of A just as it begins a cycle the result will be something like waveform A in Fig. 5.3a.

Next, consider generator $B$, which has the same frequency, but is out of step with $\mathbf{A}$. It starts each of its cycles, say, quarter of a cycle later than $A$, as shown by waveform $B$. This fact is stated by saying that voltage $B$ lags voltage $A$ by a phase difference of quarter of a cycle. Seeing that it is a time graph, it might seem more natural to say that B lagged A by a phase difference of 0.005 sec . The incorrectness of doing so is shown in Fig. $5.3 b$, where the frequency of $A$ and B is four times as great. The time lag is the same as before, but the two voltages are now in phase. So although phase is usually very closely related to time, it is not advisable to think of it as time. The proper basis of reckoning phase is in fractions of a cycle.

### 5.7 Phasor Diagrams

The main advantage of a time graph of alternating voltage, current, etc., is that it shows the waveform. But on those very many occasions when the form of the wave is not in question (because we have agreed to stick to sine waves) it is a great waste of effort to draw a number of beautifully exact waves merely in order to show the relative phases. Having examined Figs. 1.2, 5.2 and 5.3, we ought by now to be able to take the sine waveform for granted, and be prepared to accept a simpler method of indicating phases.

In Fig. 5.4, imagine the line OP to be pivoted at O and to be capable of rotating steadily in the direction shown (anticlockwise),

Fig. 5.4-Starting position of an alternative method of representing sinusoidal variation

which by mathematical convention is the positive direction of rotation. Then the height of the end $P$ above a horizontal line through $O$ varies in exactly the same way as a sine wave. Assuming OP starts, as shown in Fig. 5.4, at ' 3 o'clock', $P$ is actually on the horizontal line, so its height above it is, of course, nil. That represents the starting point of a sine-wave cycle. As OP rotates, its height above the line increases at first rapidly and then more slowly, until, after a quarter of a revolution, it is at right angles to its original position. That represents the first quarter of a sine-wave cycle. During the third and fourth quarter of the revolution, the height of $P$ is negative, corresponding to the same quarters of a cycle.

Readers with any knowledge of trigonometry will know that the ratio of the height of P to its distance from O (i.e., $\mathrm{PQ} / \mathrm{PO}$ in Fig. 5.5) is called the sine of the angle that OP has turned through; hence the name given to the waveform we have been considering and which is none other than a time graph of the sine of the angle $\theta$ (abbreviated 'sin $\theta$ ') when $\theta$ (pronounced 'theta') is increasing at a steady rate.

We now have a much more easily drawn diagram for representing sine waves. The length of a line such as OP represents the peak value of the voltage, etc., and the angle it makes with the ' 3 o'clock' position represents its phase. The line itself is called a phasor. The angle it turns through in one whole revolution (which, of course, brings it to the same position as at the start) is $360^{\circ}$, and that


Fig. 5.5-The Fig. 5.4 phasor after having turned through an angle, $\theta$


Fig. 5.6-Phasor diagram corresponding to Fig. S.3a
corresponds to one whole cycle of the voltage. So a common way of specifying a phase is in angular degrees. Quarter of a cycle is $90^{\circ}$, and so on. Usually even a waveform diagram such as Fig. 5.3 is marked in degrees.

Besides the common angular measure, in which one whole revolution is divided into $360^{\circ}$, there is the mathematical angular measure in which it is divided into $2 \pi$ radians. Seeing that $\pi$ is a very odd number (Sec. 0.1.1) this may seem an extremely odd way of doing things. It arises because during one revolution P travels $2 \pi$ times its distance from O . Even this may not seem a good enough reason, but the significance of it will appear very soon in the next chapter.

As an example of a simple phasor diagram. Fig. 5.6 is the equivalent of Fig. 5.3a. In the position shown it is equivalent to it at the beginning or end of each cycle of $A$, but since both phasors rotate at the same rate the $90^{\circ}$ phase difference shown by it between A and B is the same at any stage of any cycle. If the voltages represented had different frequencies, their phasors would rotate at different speeds, so the phase difference between them would be continually changing.

It should be stated here that the subject of phasor diagrams is currently treated in a number of different ways, which are examined in the author's book Phasor Diagrams. Phasors are still often called vectors, but vectors are the subject of a branch of mathematics dealing with vector quantities, which current and voltage are not. The likelihood of misunderstanding through using the same name for things that are essentially different is increased by the common practice of attaching an arrow head to the end of each phasor, making it look exactly like a vector. The arrow head is quite unnecessary, and can be confusing until it is realized that it does not indicate a direction of motion.

Worthy of more respect is the doctrine that phasors should not be supposed to rotate. Some of the reasons are rather too subtle for the present level of study, and in any case the difference between
the two schools of thought is theoretical rather than practical, because the diagrams that even rotating-phasor exponents draw on paper are inevitably fixed there. The effort of imagining them to be rotating at $3000 \mathrm{rev} / \mathrm{min}$ to represent a frequency of 50 Hz (let alone the speeds corresponding to radio frequencies!) might well induce a feeling of dizziness. So long as all the phasors in a diagram represent quantities having the same frequency, the shape of the diagram does not change. Nevertheless, in the early stages of study we may find it helpful to turn the diagram around in order to see "in slow motion' how each voltage and current in an a.c. circuit changes relative to the others as the cycle progresses.

It is an essential part of the rotating phasor concept that the length of the phasor represents the peak value. But without forgetting this there is really no reason why we should not make use of the fact that the ratio between peak and r.m.s. values is constant $\left(v^{\prime} 2\right)$ to regard the phasors alternatively as representing r.m.s. values. It is merely a matter of choice of scale. This is convenient and is often done, expecially when one has become used to interpreting stationary diagrams.

### 5.8 Adding Alternating Voltages

A great advantage of the phasor diagram is the way it simplifies adding and subtracting alternating voltages or currents that are not in phase. To add voltages A and B in Fig. 5.3a, for example, one would have to add the heights of the $A$ and $B$ waves at close intervals of time and plot them as a third wave. This would be a very laborious way of finding how the peak value and phase of the combined voltage (or resultant, as it is called) compared with those of its component voltages. In a phasor diagram it is quickly done by 'adding' phasor $\mathbf{B}$ to the rotating end of phasor $\mathbf{A}$, as in Fig. 5.7,

and drawing a straight line from 0 to the 'loose' end of B . This line is the phasor of the combined voltage, $\mathbf{A}+\mathbf{B}$. Its length represents its peak value (to the same scale as $A$ and $B$ ) and its angle represents its relative phase. Alternatively of course $\mathbf{A}$ can be added to $\mathbf{B}$. Note that the result of adding two voltages that are not in phase is less than the result of adding them by ordinary arithmetic, and its phase is somewhere between those of the components. Since it is representable by a phasor, it is sinusoidal, like its components.

### 5.9 Direction Signs

Have you ever been directed to a destination by someone who supposed that it was being approached from the opposite direction? It is a very perplexing and frustrating experience. Yet quite a common one for students of a.c., because much of the literature on the subject is ambiguous in its direction signs.

All our talk about voltages and currents will do us no practical good unless we can relate them to circuits. Already we have been drawing graphs and phasor diagrams of voltages A and B without any reference to where they could be found. So let us suppose that voltage A (call it $V_{A}$ ) occurs between terminals c and d in a circuit, and $V_{B}$ between terminals e and $f$ in the same or another circuit. As these circuits might be very complicated, in Fig. 5.8 only the


Fig. 5.8-If $V_{B}$ is known to lag quarter of $a$ cycle behind $V_{1}$, which of its terminals. first becomes positive after d?
terminals are shown. A very common way of indicating the voltages between them is as shown there, with two-way arrows. Now let us try to relate this to the waveform graph, Fig. $5.3 a$, which tells us that the positive peak of $V_{B}$ occurs quarter of a cycle later than the positive peak of $V_{A}$. The phasor diagram, Fig. 5.6, tells us the same thing in a simpler way. Suppose that at a certain instant $d$ is at positive peak relative to c . Then we know that a quarter of a cycle later either e or $f$ will be at peak positive relative to the other. Which one? The answer we have is like that of the man who was asked 'Will Oxford or Cambridge win the boat race?' and replied 'Yes'. Quite true, but quite useless. So are two-way arrows as voltage signposts.

One solution to the difficulty is to insist on arrows pointing one way only, like any sensible signpost. The same applies of course to currents. When we were dealing with d.c. circuits, such as Fig. 2.12, one point that had to be made clear was that there needed to be agreement on whether the arrow heads marked what we knew were (say) the positive ends, or merely the ends that were to be taken as positive, so that if any voltage, at first unknown, turned out to be negative we would know that the arrow head would mark what was actually the negative end. With a.c. the voltages and currents are continually reversing, so no terminal or direction is any more positive than negative; but one-way arrows or even + and - signs can be used for a.c. to show the polarity that is arbitrarily taken as positive at a given instant. Half a cycle later everything will be reversed, but things that were opposite in sign will still be so and like
signs will still be like. Provided that all phasors are correctly marked to correspond (hence the arrow-head custom) this method can be made to work. Phasor Diagrams mentions eight disadvantages it has, however. and advises a system that is free from all of them. A brief outline follows.

### 5.10 Subscript Notation

If the terminals are labelled. as in Fig. 5.8, there is no need to mark the voltages or their directions on the circuit diagram at all; $V_{A}$ is automatically $V_{\mathrm{cd}}$ or $V_{\mathrm{dc}}$, and so on. In practice there is no need even to write the $V$; it is much easier to write (and especially to type) just $c d$ or $d c$. Similarly there is no need to mark the phasor with an $A$, or $V_{A}$ as in Fig. 5.7; nor with an arrow. Marking its ends c and d makes it correspond completely with the circuit. We do not then have to make an arbitrary choice of polarity: both sets of halfcycles are equally represented. If necessary, we can distinguish between a voltage and its phasor by following the usual convention of italic type for a voltage (e.g., c(f) and roman letters for its phasor (cd).

Only one thing remains to be settled. When $c d$ is positive, which terminal is the positive one? Some people who use a subscript voltage notation write the letters in the order that makes a voltage rise negative. It seems more natural, and fits in with other conventions better, to regard a rise as positive, and that is uhat will be done here. So whenever terminal d is positive relative to c voltage $c d$ will be taken as positive, signifying in fact the change in potential on moving from c to d . It follows that $d c$ would be negative.

Fig. 5.9 shows how Fig. 5.7 wouid be redrawn according to the foregoing conventions. It tells us that if terminals $d$ and $e$ were

Fig. 5.9-Fig. 5.7 redrawn, using double-subscript convention reluted to Fig. 5.8

connected together, the voltage between c and f would be correctly represented by the phasor between c and f . The angle by which it lags $c d$ is marked $\varphi$ (Greek small phi), the usual symbol for a phase difference. It answers all questions unambiguously. For example, when d is at peak positive with respect to c , what is the potential of $f$ relative to c , and when will it reach its positive peak? If we turn Fig. 5.9 quarter of a turn anticlockwise. so that cd is bolt upright and therefore appropriately representing $c d$ at positive peak, $d$ being at its farthest above $c$, corresponding to Fig. 5.3a
0.005 sec from the origin, cf shows that $c f$ is nearing its positive peak and will reach it in a fraction $\varphi / 360^{\circ}$ of a cycle.

If instead of connecting e to d we had connected it to c , Fig. 5.10 would be the appropriate diagram. It shows the potential at f a full quarter-cycle behind that at d (both relative to $c$ ). This agrees with


Fig. 5.10-If one of the circuit connections was reversed, this would apply instead of Fig. 5.9

Fig. 5.3a, but until we had a proper notation we were unable to tell which was the right connection, and in fact went astray in Fig. 5.7.

One of the advantages of not using arrow heads in phasor diagrams is that we are not tied to any one point as the centre about which the thing is supposed to rotate. Whichever point in the circuit we choose to regard as the reference point, or zero potential, with respect to which all other voltages are reckoned, is the appropriate centre, and we can mark it with a circle or stick a pin through it to serve as an axis.

### 5.11 Current Phasors

Attempts to extend subscript notation to currents usually run into trouble, because people write $I_{\mathrm{cd}}$ to mean the current flowing from c to d . But whereas there can be only one voltage at a time between two points, there can be as many currents as there are parallel paths. This difficulty can be avoided by labelling the meshes or loops of the circuit. There are excellent theoretical reasons for doing this, but

it also works in practice. To distinguish currents from voltages, capital letters can be used for the meshes. All we need is to agree on which order of letters to use in order to denote the positive direction of current. Either would do, but to agree with Phasor Diagrams let us say that $I_{A B}$ (which we can convenientiy write as $A B$ ) in Fig.
$5.11 a$ means positive current moving up through the generator and down through the resistor. Note that although this simplest of circuits appears to have only one mesh we must mark the external one (A) too. Note also that $I_{B}$, or $B$. means the current circulating clockwise around mesh B , and $A B$ means the net current crossed in moving fron: A to B .

For such a simple circuit the phasor diagram is very simple. There are only two terminals, so only one voltage, $a b$ or $b a$, between them and therefore one voltage phasor (Fig. 5.11b). When $a b$ is peak positive, as shown, we know (Sec. 5.3) that the current $A B$, which according to our convention is the one flowing upward through the generator, is also peak positive, so is represented by the vertically upward phasor AB.

It can be said fand many of those who use other conventions probably will say) that there are two voltages in this circuit. And in a sense there are, but they are really the same voltage looked at from two opposite points of view. The only way of finding two voltages with a voltmeter is to measure it once and then measure it again with reversed leads. If we think of Kirchhoff's voltage law and start at a, going clockwise around the circuit with the current, at the instant represented by the phasors, we find that when we have reached $b$ the potential has risen. In other words, $a h$ is positive. Going now through the resistor R to b we find the potential falls. In other words, ba is negative. This may seem too obvious to have to explain. but it is surprising how confused the people who use arrows can get about it. And if the confusion is possible with this simplest of circuits the need to be quite clear before going on to more complicated ones needs no emphasis.

One objection that is sometimes raised against the foregoing notation is that it introduces a minus sign into Ohm's law, since the voltage across $R$ in the direction of $A B$ is ba. But this is not so in the form of the law which says that the current is proportional to the voltage applied. (This is usually implied by using $E$ for the voltage, in $I=E / R$.) In Fig. $5.11 a$ the voltage applied is the generator voltage, $a b$.

## CHAPTER 6

## Capacitance in A.C. Circuits

### 6.1 Current Flow in a Capacitive Circuit

The next type of basic circuit to consider is the one shown in Fig. 6.1. If the generator gave an unvarying e.m.f. no current could flow, because there is a complete break in the circuit. The most that could happen would be a momentary current when switching on, as shown in Fig. 3.6.

Some idea of what is likely to happen when an alternating e.m.f. is applied can be obtained by an experiment similar to Fig. 3.5 but with an inverted battery in the B path, as shown in Fig. 6.2a. Moving the switch alternately to $A$ and $B$ at equal time intervals will then provide an alternating voltage of square waveform (Fig. 6.2b).

Whenever the switch is moved to A there is a momentary charging current in one direction, and moving it to B causes a similar current in the reverse direction.

We know that for a given voltage the quantity of electricity transferred at each movement of the switch is proportional to the capacitance (Sec. 3.2). And it is obvious that the more rapidly the switch is moved to and fro (that is to say, the higher the frequency of the alternating e.m.f.) the more often this quantity of electricity will surge to and fro in the circuit (that is to say, the greater the quantity of electricity that will move in the circuit per second, or, in other words, the greater the current).

So although the circuit has no conductance an alternating e.m.f. causes an alternating current. It does not flow conductively through the capacitor, whose dielectric is normally a very good insulator, but can be visualized as flowing in and out of it. Alternatively, Sec. 3.7 suggested how the current can be regarded as existing even between the capacitor plates.

The fact that an alternating e.m.f. can make a current flow in a circuit blocked to d.c. by a capacitance in series is easily demonstrated by bridging the open contacts of an electric light switch by a capacitor (of suitable a.c. rating!); say $2 \mu \mathrm{~F}$ for a 40 W lamp (Fig. 6.3). The lamp will light and stay alight as long as the capacitor is connected. But it will not be as bright as usual. The less the capacitance, the dimmer the light. The amount of current due to a given voltage thus depends on two things: the capacitance and the frequency.

Fig. 6.1-Circuit consisting of an a.c. gentrator with a purely capacitive load, for comparison with Fig. S.i


(a)


Fig. 6.2-If the generator in Fig. 6.1 consisted of the periodically switched batteries as shown at a, the waveform would be as at $h$, and the current (assuming a certain amount of resistance) could be calculated as discussed in connection with Fig. 3.6

Fig. 6.3-Demonstration of current without conduction



Fig. 6.4-E.mf. and current diagram for Fig. 0.1, draun to correspond with Fig. 5.2

### 6.2 Capacitive Current Waveform

Let us now consider the action of Fig. 6.1 in greater detail, by drawing the graph of instantaneous e.m.f. (Fig. 6.4) exactly as for the resistive circuit, in which we assumed a sine waveform. But unlike the resistive case there is no Ohm's law to guide us in plotting the current waveform. We have, however, a rather similar relationship (Sec. 3.2):

$$
V=Q / C
$$

where $V$ is the p.d. across $C$ and $Q$ is the charge in $C$. This is true at every instant, so we can rewrite it $v=q / C$, to show that we mean instantaneous values. In Fig. 6.1, $v$ is always equal in magnitude to $e$, the instantaneous e.m.f. So we can say with confidence that at the moments when $e$ is zero the capacitor is completely uncharged, and at all other moments the charge is exactly proportional to $e$. If we knew the right scale, the voltage wave in Fig. 6.4 would do also as a charge curve. But we are not so much interested in the charge as in the current; that is to say, the rate at which charging takes place. At points marked $a$, although the voltage and charge are zero they are growing faster than at any other stage in the cycle. So we may expect the current to be greater than at any other times. At points marked $b$, the charge is decreasing as fast as it was growing at $a$, so the current is the same in magnitude but opposite in direction and sign. At points marked $c$, the charge reaches its maximum, but just for an instant it is neither growing nor waning; its rate of increase or decrease is zero, so at that instant the current must be zero. At intermediate points the relative strength of current can be estimated from the steepness of the voltage (and charge) curve. Joining up all the points gives a current curve shaped like the dotted line.

If this job is done carefully the shape of the current curve is also sinusoidal, as in the resistive circuit, but out of phase, being quarter of a cycle ( or $90^{\circ}$ ) ahead of the voltage.

On seeing this, students are sometimes puzzled and ask how the current can be ahead of the voltage. 'How does the current know what the voltage is going to be, quarter of a cycle kater? This difficulty arises only when it supposed, quite wrongly, that the current maximum at $a$ is caused by the voltage maximum quarter of a cycle later, at $c$; whereas it is actually caused by the voltage at $a$ increasing at its maximum rate.

### 6.3 The 'Ohm's Law' for Capacitance

What we want to know is how to predict the actual magnitude of the current, given the necessary circuit data. Since, as we know (Sec. 3.2), when there is 1 V between the terminals of a 1-F capacitor
its charge must be 1 C , we can see that if the voltage were increasing at the rate of $1 \mathrm{~V} / \mathrm{s}$ the charge would be increasing at $1 \mathrm{C} / \mathrm{s}$. But we also know that $1 \mathrm{C} / \mathrm{s}$ is a current of 1 A ; so from our basic relationship, $Q=V C$. we can derive

$$
\text { Amps }=\text { Volts-per-second } \times \text { Farads }
$$

The problem is how to calculate the volts-per-second in an alternating e.m.f., especially as it is varying all the time. All we need do, however, is find it at the point in the cycle where it is greatest-at $a$ in Fig. 6.4. That will give the peak value of current, from which all its other values follow (Sec. 5.4).

Going back to Fig. 5.5, you may remember that one revolution of P about O represents one cycle of alternating voltage, the fixed length OP represents the peak value of the voltage, and the length of PQ (which, of course, is varying all the time) represents the instantaneous voltage. When P is on the starting line, as in Fig. 5.4. the length of PQ is zero, but at that moment it is increasing at the rate at which $P$ is revolving around $O$. Now the distance travelled by P during one cycle is $2 \pi$ times the length of OP. And if the frequency is 50 Hz , it does this distance 50 times per second. Its rate is therefore $2 \pi \times 50$ times OP. More generally, if $f$ stands for the frequency in hertz, the rate at which $\mathbf{P}$ moves round O is $2 \pi f$ times OP. And as OP represents $E_{\max }$, we can say that P's motion represents $2 \pi f E_{\text {max }}$ volts per second (Fig. 6.5). But we have just seen that it

Fig. 6.5-From this diagram it can be inferred that the maximum rate at which $E$ increases is $2 \pi f E_{\text {max }}$ volts per second

also represents the maximum rate at which PQ , the instantaneous voltage, is growing. So, fitting this information into our equation, we have

$$
I_{\max }=2 \pi f E_{\text {max }} C
$$

where $I$ is in amps. $E$ in volts, and $C$ in farads. Since $I_{\text {max }}$ and $E_{\text {max }}$ are both $\nu^{\prime} 2$ times $I$ and $E$ (the r.m.s. values) respectively, it is equally true to say

$$
I=2 \pi f E C
$$

And if we rearrange it like this:

$$
\frac{E}{I}=\frac{1}{2 \pi f C}
$$

and then take another look at one of the forms of expressing Ohm's law:

$$
\frac{E}{I}=R
$$

we see they are the same except that $1 / 2 \pi f C$ takes the place of $R$ in the role of limiting the current in the circuit. So, although $1 / 2 \pi f C$ is quite a different thing physically from resistance, for purposes of calculation it is of the same kind, and it is convenient to reckon it in ohms. To distinguish it from resistance it is named reactance. To avoid having to repeat the rather cumbersome expression $1 / 2 \pi f C$, the special symbol $X$ has been allotted. The 'Ohm's law' for the Fig. 6.1 circuit can therefore be written simply as

$$
\frac{E}{I}=X \quad \text { or } I=\frac{E}{X} \quad \text { or } E=I X
$$

For example, the reactance of a $2 \mu \mathrm{~F}$ capacitor to a 50 Hz supply is $1 /\left(2 \pi \times 50 \times 2 \times 10^{-6}\right)=1592 \Omega$, and if the voltage across it were 240 V the current would be $240 / 1592=0 \cdot 151 \mathrm{~A}$.

The factor group $2 \pi f$ occurs so often in connection with a.c. circuits that it is commonly denoted by the Greek small omega, $\omega$. So an alternative way of writing the reactance is $1 / \omega C$. But in this elementary book the formulae will be used mainly for numerical examples, so it may be clearer if the separate factors are shown.

### 6.4 Capacitances in Parallel and in Series

The argument that led us to conclude that the capacitance between parallel plates is proportional to the area across which they face one another (Sec. 3.3) leads also to the conclusion that capacitances in parallel add up just like resistances in series.

We can arrive at the same conclusion by simple algebraic reasoning based on the behaviour of capacitors to alternating voltage. Fig. 6.6 represents an a.c. generator connected to two capacitors in parallel. The separate currents in them are respectively $E \times 2 \pi f C_{1}$ and $E \times 2 \pi f C_{2}$. As these currents are in phase with one another (because both lead $E$ by the same angle), the total current is $E \times 2 \pi f$ ( $C_{1}+C_{2}$ ), which is equal to the current that would be taken by a single capacitance equal to the sum of the separate capacitances of $C_{1}$ and $C_{2}$.

We also saw that if a single capacitor is divided by a plate placed midway between its two plates, the capacitance between the middle plate and either of the others is twice that of the original capacitor. In other words, the capacitance of two equal capacitors in series is half that of each of them. Let us now consider the more general case of any two capacitances in series.

If $X_{1}$ and $X_{2}$ are respectively the reactances of $C_{1}$ and $C_{2}$ in Fig. 6.7, their combined reactance $X^{\prime}$ is $\left(X_{1}+X_{2}\right)$, as in the case of

Fig. 6.6-In this circuit the capacitance of $C_{1}$ and $C_{2}$ together is equal to $C_{1}+C_{2}$

resistances in series. By first writing down the equation $X=X_{1}+$ $X_{2}$, and then replacing each $X$ by its known value, of form $1 / 2 \pi f C$, we deduce that

$$
\frac{1}{C}=\frac{1}{C_{1}}+\frac{1}{C_{2}}
$$

That is, the sum of the reciprocals of the separate capacitances is equal to the reciprocal of the total capacitance.

So the rule for capacitances in series is identical with that for resistances or reactances in parallel. From the way it was derived


Fig. 6.7-If the one capacitor $C$ in $b$ is to take the same current as the two in series at a, its capacitance must be equal to $I /\left(1 / C_{1}+1 / C_{2}\right)$
it is evidently not limited to two capacitances only, but can be applied to any number. It implies that if capacitors are connected in series, the capacitance of the combination is always less than that of the smallest.

Applied to two capacitances only, the rule can, as with resistances (end of Sec. 2.9), be put in the more easily workable form

$$
C=\frac{C_{1} C_{2}}{C_{1}+C_{2}}
$$

### 6.5 Power in a Capacitive Circuit

Fig. 6.4 was drawn to match Fig. 5.2 as regards peak voltage and current. The only difference is the current's $90^{\circ}$ phase lead. Let us now complete the picture by calculating the wattage at a sufficient number of points to draw the power curve (Fig. 6.8). Owing to the phase difference between $E$ and $I$, there are periods in the cycle when


Fig. 6.8-The current and voltage graphs for a capacitance load (Fig. 6.4) are here repeated, with a power graph derived from it plotted below'
they are of opposite sign, giving a negative power. In fact, the power is alternately positive and negative, just like $E$ and $I$ except for having twice the frequency. The net power, taken over a whole cycle, is therefore zero. Since we are applying the sign ' + ', to positive current going through the generator towards positive potential, positive power means power going out of the generator, and negative power is power returned to the generator by the load (end of Sec. 2.17).

The power curve in Fig. 6.8 represents the fact that although a pure capacitance draws current from an alternating generator it does not permanently draw any power-it only borrows some
twice per cycle, repaying it in full with a promptitude that might be commended to human borrowers.

A demonstration of this fact is given by the capacitor in series with the lamp (Fig. 6.3). The lamp soon warms up, showing that electrical power is being dissipated in it, but although the capacitor is carrying the same current it remains quite cold.

### 6.6 Phasor Diagram for Capacitive Circuit

In dealing with such a simple circuit as Fig. 6.1 we are not likely to become uncertain about the directions of e.m.f. and current represented in graphs such as Fig. 6.8, especially as we have just stated them fully in words in order to check the directions of power flow. But in order to get into practice for more complicated circuits let us draw the phasors for Fig. 6.1, as we did for the simple resistive circuit in Fig. 5.11.

Fig. $6.9 a$ shows the circuit suitably labelled. We can always choose the angle at which to draw the first phasor, so to make it easy to


Fig. 6.9-Simplest capacirive circuit and corresponding phasor diagrams
compare with Fig. 5.11 let us draw the voltage phasor in the same way, indicating that terminal $\mathbf{b}$ is at peak positive potential with respect to a. At that moment, therefore, the voltage across $C$ has just stopped increasing and is about to decrease; so $C$ is charged to the maximum extent, positive on the upper plate. The current, which has been flowing clock wise to deliver that charge, is now at zero and about to discharge anticlockwise. The current phasor must therefore be drawn horizontal, to signify zero current. And as anticlockwise current, which our convention denotes by $B A$, is the one about to become positive, it must be drawn left to right as shown (Fig. $6.9 b$ ). As $a b$ is the generator voltage applied to $C$, one would usually refer to the clock wise current $A B$, which the diagram shows to be a quarter of a cycle ahead of $a b$. This confirms the rule for sinusoidal a.c., well known to electrical engineers, that the current into a pure capacitance leads the applied voltage by $90^{\circ}$.

### 6.7 Capacitance and Resistance in Series

Fig. 6.10 shows the next type of circuit to be considered: $C$ and $R$ in series. We know how to calculate the relationship (magnitude and phase) between voltage and current for each of these separately. When they are in series the same current will flow through both, and this current cannot at one and the same time be in phase with the generator voltage and $90^{\circ}$ ahead of it. But it is quite possible-and in fact essential-for it to be in phase with the voltage across the resistance and $90^{\circ}$ ahead of the voltage across the capacitance. These two voltages, which must therefore be $90^{\circ}$ out of phase with one another, must add up to equal the generator voltage. We have already added out-of-phase voltages by two different methods (Fig. 5.3 and Fig. 5.7), so that part of it should not be difficult. Because the current, and not the voltage, is common to both circuit elements, it will be easier to reverse the procedure we have adopted until now, and, starting with a current, find the e.m.f. required to drive it. When we have in that way discovered the key to this kind of circuit, we can easily use it to calculate the current resulting from a given e.m.f.

We could, of course, represent the two voltages by drawing waveform graphs, and, by adding them, plot the graph of the total generator voltage and compare it for phase and magnitude with the current graph. But seeing we have taken the trouble to learn the much quicker phasor method of arriving at the same result, we


Fig. 6.10-C and $R$ in series, and corresponding phasor diagrans
might as well use it. (If you prefer not to, there is nothing to stop you drawing the waveforms.)

The first thing, then, is to draw a current phasor AB at any angle we please. In Fig. 6.10 it is vertical. $A B$ being a clockwise current, we know that it is in phase with the voltage applied clockwise, via G and C , to the resistor R ; that is to say ac. So we draw ac parallel -
to AB to show that they are in phase. We know that $A B$ leads by $90^{\circ}$ the voltage applied clockwise, via $R$ and $G$, to $C$; that is to say $c b$. So cb should be drawn to $\operatorname{lag} \mathrm{AB}$ by $90^{\circ}$. The generator e.m.f., $a b$, is then indicated in magnitude and phase. The current is leading it by the angle $\varphi$, less than $90^{\circ}$.

You may ask how we know how long to draw these phasors. The current one has to be drawn first, and at that stage we do not know how much current there is, even if we know $a b, f, C$ and $R$. Whatever $A B$ may turn out to be, however, any length of phasor will be right if a suitable scale is chosen. As for $a c$ and $c b$, those veltages are proportional to $R$ and $X(=1 / 2 \pi f C)$, so we can draw their phasor lengths in the ratio $R: X$. Then if we know the generator e.m.f., $a b$ represents $i t$, so the voltage scale is found and hence the values of $a c$ and $c b$. Lastly. $A B$ can be calculated by Ohm's law as $a c / R$.

Most of the time, however, phasor diagrams are drawn just to show the principle without bothering about the exact values they represent. At low frequencies at least, it is likely that C would impede current more than $R$ and so take the major share of the available e.m.f., so cb has been drawn longer than ac.

### 6.8 Impedance

A new name is needed to refer to the current-limiting properties of this circuit as a whole. Resistance is appropriate to $R$ and reactance to $C$; a combination of them is called impedance (symbol: $Z$ ). Resistance and reactance themselves are special cases of impedance. So we have still another relationship in the same form as Ohm's law; one that covers the other two, namely:

$$
I=\frac{E}{Z}
$$

Although $Z$ combines $R$ and $X$, it is not true to say $Z=R+X$. Suppose that in Fig. 6.10 C is the $2 \mu \mathrm{~F}$ capacitor which we have already calculated has a reactance of $1592 \Omega$ at 50 Hz , and that $R$ is a round $1000 \Omega$. $A B$, shall we say, is $0 \cdot 1 \mathrm{~A}$. Then ac must be 100 V and $c b 159 \mathrm{~V}$. If the phasor diagram was drawn accurately to scale it would show $a b$ to be 188 V , so $Z$ must be $1880 \Omega$, which is much less than $1592+1000$.

As the three voltage phasors refer to the same current, to a suitable scale they also represent the corresponding three impedances: resistive, reactive and total. Also, as they make a rightangled triangle, the celebrated Theorem of Pythagoras tells us that

$$
\begin{aligned}
Z^{2} & =R^{2}+X^{2} \\
\text { or } Z & =v^{\prime}\left(R^{2}+X^{2}\right)
\end{aligned}
$$

In trigonometry the ratio $\mathrm{cb} / \mathrm{ac}$, which we have just seen is equal to $X / R$, is known as the tangent of the angle $\varphi$, abbreviated to $\tan \varphi$. As there are tables from which the angle corresponding to any tangent can be seen, phase angles can be found from

$$
\tan \varphi=\frac{X}{R}
$$

So we have the alternative of computing $Z$ by arithmetic instead of drawing a phasor diagram or a waveform diagram accurately to scale.

The phasor triangle also tells us, of course, that

$$
a b=v^{\prime}\left(a c^{2}+c b^{2}\right)
$$

(Note that here ' $a c^{\prime}$ ' is really one symbol, all squared; not $a \times c^{2}$.) We will no doubt have already realized that 100 V and 159 V , the voltages across R and C , do not add up in the ordinary way to equal 188 V , the voltage across both. This would be very puzzling-in fact, quite impossible, and a serious breach of Kirchhoff's voltage law-if these figures were instantaneous values. But of course they are r.m.s. values, and the explanation of the apparent discrepancy is that the maximum instantaneous values do not occur all at the same time. This is most clearly shown by a waveform graph, tedious though it is to draw as a means of calculating the total voltage, etc., compared with a phasor diagram or algebra.

But this type of phasor diagram does very neatly illustrate Kirchhoff's voltage law, because the voltage phasors for any whole circuit always necessarily form a closed geometrical figure, such as our triangle.

### 6.9 Power in Mixed Circuit

By now we should hardly need to draw waveform diagrams such as Fig. 6.8 in order to find how much power the generator delivers to the Fig. 6.10 circuit. Fig. 6.8, together with Fig. 6.9, has shown that when phasors are at right angles, as they are when they refer to a perfect capacitor, energy flows to and fro between generator and load, but none is permanently delivered. That is because a capacitor can store electrical energy and returns it in the same form, unlike a resistor, which dissipates it all as heat just as fast as it is received. In Sec. 5.4 we found that the power $P$ delivered to and dissipated by resistance $R$ is $E^{2} / R$, where $E$ is the r.m.s. voltage across it. As $R=E / I, I$ being the r.m.s. current through $R$, by substituting for $R$ in the first equation we get $P=E I$, just as with d.c. (Sec. 2.17). The quantities corresponding in Fig. 6.10 to $E$ and $I$ are AB and ac, if those are being used to represent r.m.s. values. If however they
are used more strictly to represent peak values, and since peak values are $v^{\prime} 2 \times$ r.m.s. values, $P$ would be equal to $A B$. ac $/ 2$.

In a circuit such as this, including both $R$ and $C$ in series, the part of the total voltage that should be multiplied by the current to find the power dissipated is represented by the phasor that is parallel to the current phasor: ac in Fig. 6.10b. In terms of the total voltage, this voltage is $a b . a c / a b$, or $E R / Z$. So

$$
P=\frac{E I R}{Z}
$$

$E$ and $I$ being the total voltage and the current.
In trigonometry the ratio ac/ab is called the cosine of the angle $\varphi$, abbreviated to $\cos \varphi$. So another form of the same equation is

$$
P=E I \cos \varphi
$$

$\varphi$ being the phase angle between $E$ and $I . \operatorname{Cos} \varphi$ is known to electrical engineers as the power factor.

### 6.10 Capacitance and Resistance in Parallel

In solving this type of circuit, Fig. 6.11, we revert to the practice of starting with the voltage, because that is common to both. What we do, in fact, is exactly the same as for the series circuit


Fig. 6.11-A parallel circuit such as this is similar to a series circuit such as Fig. 6.10 with voltages and currents interchanged
except that currents and voltages change places. We now have two currents, $C B$ and $A C$, one in phase with the e.m.f. $a b$ and the other $90^{\circ}$ ahead of it. By drawing the phasors accordingly, Fig. 6.11b, we find the total current, $A B$, and the phase angle $\varphi$ by which it leads $a b$.

By the same methods as we used in Sec. 6.8 we find that

$$
A B=\mathcal{V}^{\prime}\left(A C^{2}+C B^{2}\right)
$$

This is in fact the same equation, apart from the capital letters for currents instead of small ones for voltages. As with them, $A C$ is regarded as one symbol, all squared.

Calculating the impedance is slightly more complicated, however, because the currents are inversely proportional to the impedances through which they flow. So $1 / Z=\sqrt{ }\left(1 / R^{2}+1 / X^{2}\right)$, or

$$
Z=\frac{1}{\sqrt{ }\left(\frac{1}{R^{2}}+\frac{1}{X^{2}}\right)}
$$

which can be simplified to

$$
Z=\frac{R X}{v^{\prime}\left(R^{2}+X^{2}\right)}
$$

Compare the formula for two resistances in parallel at the end of Sec. 2.9.

In Sec. 2.12 we found that calculations in parallel resistance circuits can be made as simple as those for series circuits by substituting conductance, $G$, for $1 / R$. This idea has been extended to


Fig. 6.12-Example of a complex circuit that can be reduced to Fig. 6.11 by applying the rultes for resistances and capacitances in series and in parallel
a.c. circuits by introducing susceptance, $B$, for $1 / X$, and admittance, $Y$, for $1 / Z$. The equations for parallel circuits then correspond to those for series circuits (Sec. 6.8):

$$
\begin{aligned}
Y & =\sqrt{\prime}^{\prime}\left(G^{2}+B^{2}\right) \\
\text { and } \quad I & =E Y \quad \text { (compare } I=E G \text { in Sec. 2.12.) }
\end{aligned}
$$

Tan $\varphi$ in Fig. $6.11 b$ is equal to $C A / B C$, and as these currents are
proportional to susceptance and conductance respectively we have

$$
\tan \varphi=\frac{B}{G}
$$

Note that the method by addition of squares can be applied only to circuits in which the two currents or voltages are exactly $90^{\circ}$ out of phase, as when one element is a pure resistance and the other a pure reactance. Combining series impedances which are themselves combinations of resistance and reactance in parallel, or vice versa, is a rather more advanced problem than will be considered here. Such a circuit as Fig. 6.12, however, can be tackled by rules already given. $R_{2}$ and $R_{3}$ are reduced to a single equivalent resistance (Sec. 2.10) which is then added to $R_{1} . C_{1}, C_{2}$ and $C_{3}$ are just added together, and so are $C_{4}$ and $C_{5}$; the two resulting capacitances in series are reduced to one (end of Sec. 6.4). The circuit has now boiled down to Fig. 6.11.

## Inductance in A.C. Circuits

### 7.1 Current Flow in an Inductive Circuit

Although the phenomenon of magnetism, which gives rise to inductance, differs in many ways from electrostatics, which gives rise to capacitance, it is helpful to draw a very close parallel or analogy between them. Inductance and capacitance are, in fact, like opposite partners; and this chapter will in many ways be a repetition of the last one, but with a few basic things reversed.

We have already noticed some striking similarities as well as differences between capacitance and inductance (Sec. 4.10). One thing that could be seen by comparing them in Figs. 3.6 and 4.10 was an exchange of roles between voltage and current.

The simple experiment of Fig. 6.2 showed that, in a circuit consisting of a square-wave alternating-voltage generator in series with a capacitor, the current increased with frequency, beginning with zero current at zero frequency. This was confirmed by examining in more detail a circuit (Fig. 6.1) with a sine-wave generator, in which the current was found to be exactly proportional to frequency.

Comparing Fig. 4.10 with Fig. 3.6, we can expect the opposite to apply to inductance. At zero frequency it has no restrictive influence on the current, which is limited only by the resistance of the circuit. But when a high-frequency alternating e.m.f. is applied the current has very little time to grow (at a rate depending on the inductance) before the second half-cycle is giving it the 'about turn'. The gradualness with which the current rises, you will remember, is due to the magnetic field created by the current, which generates an e.m.f. opposing the e.m.f. that is driving the current. It is rather like the gradualness with which a heavy truck gains speed when you push it. If you shake it rapidly to and fro it will hardly move at all.

For finding exactly how much current a given alternating voltage will drive through a given inductance (Fig. 7.1) we have the basic relationship (Sec. 4.4):

## Volts $=$ Amps-per-second $\times$ Henries (Compare Sec. 6.3.)

Adapting the same basic method we begin with a sinusoidal current and find what voltage is required. The dotted curve in Fig. 7.2 represents such a current during rather more than one cycle. At the start of a cycle ( $a$ ), the current is zero, but is increasing faster than at any other phase. So the amps-per-second is at its maximum, and so therefore must be the voltage. After half a cycle (b) the cur-
rent is again zero and changing at the same rate, but this time it is decreasing, so the voltage is a maximum negative. Halfway between (c) the current is at its maximum, but for an instant it is neither increasing nor decreasing, so the voltage must be zero. And so on. Completing the voltage curve in the same way as we did the current

Fig. 7.1-Circuit consisting of an a.c. generator with a purely inductive load



Fig. 7.2-E.m.f. and current graphs for Fig. 7.1. drann to correspond with Figs. 5.2 and 6.4
curve in Fig. 6.4, we find that it also is sinusoidal. This is very fortunate, for it allows us to say that if the sinusoidal voltage represented by this curve were applied to an inductance, the current curve would represent the resulting current, which would be sinusoidal.

### 7.2 The 'Ohm's Law' for Inductance

We have already found (Sec. 6.3) that when an alternating e.m.f. $E$, of frequency $f$, is sinusoidal, its r.m.s. volts-per-second is $2 \pi f E$. The same method applies equally to current, so the r.m.s. amps-persceond is $2 \pi f I$. Fitting this fact into our basic principle we get

$$
E=2 \pi f I L
$$

where $L$ is the inductance in henries.
What we set out to find was the current due to a given e.m.f., so the appropriate form of the above equation is

$$
I=\frac{E}{2 \pi f L}
$$

Again, this is in the same form as Ohm's law, $2 \pi f L$ taking the

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place of $R$. This $2 \pi f L$ can therefore also be reckoned in ohms, and, like $1 / 2 \pi f C$, it is called reactance and denoted by $X$. Whenever it is necessary to distinguish between inductive reactance and capacitive reactance they are denoted by $X_{L}$ and $X_{C}$ respectively. $X$ is the general symbol for reactance.

A question that may come to mind at this point is: can $X_{L}$ and $X_{C}$ in the same circuit be added in the same simple way as resistances? The answer is so important that it is reserved for the next chapter.

Note in the meantime that $X_{L}$ is proportional to $f$, whereas $X_{C}$ is proportional to $l / f$; this expresses in exact terms what our early experiments had led us to expect about the opposite ways in which frequency affected the current due to a given voltage.

An example may help to clinch the matter. What is the reactance of a 2 H inductor at $50 \mathrm{~Hz} ? X=2 \pi f L=2 \pi \times 50 \times 2=628 \Omega$. So if the voltage is 240 , the current will be $240 / 628=0.382 \mathrm{~A}$.

### 7.3 Inductances in Series and in Parallel

The previous section has shown that the reactive impedance of an inductance is directly proportional to the inductance-oppositely to that of capacitance (which is proportional to $1 / C$ ), but exactly like resistance. So inductances can be combined in the same way as resistances. The total effect of two in series, $L_{1}$ and $L_{2}$, is equal to that of a single inductance equal to $L_{1}+L_{2}$. And inductances in parallel follow the reciprocal law: $1 / L=1 / L_{1}+1 / L_{2}$. or

$$
L=\frac{L_{1} L_{2}}{L_{1}+L_{2}}
$$

These principles can easily be verified by adding the reactances when they are in series, and the currents when the inductances are in parallel.

The above rules are subject to one important condition, however: that the inductors are so placed that their mutual inductance (Sec. 4.11) or magnetic coupling is negligible. If mutual inductance, $M$, does exist between two coils having separate self-inductances $L_{1}$ and $L_{2}$, the total self-inductance of the two in series is $L_{1}+L_{2}+2 M$ or $L_{1}+L_{2}-2 M$, according to whether the coils are placed so that their separate magnetic fields add to or subtract from one another. The eeason $M$ is doubled is that each coil is affected by the other, so in the combination the same effect occurs twice.

The corresponding elaboration of the formula for two inductances in parallel gives the result

$$
L=\frac{\left(L_{1} \pm M\right)\left(L_{2} \pm M\right)}{L_{1}+L_{2} \pm 2 M}
$$

where ' $\pm$ ' signifies ' + or - '.

Complications arise also when the fields of two capacitors interact, but with the usual forms of construction such interaction is seldom enough to take into account.

### 7.4 Power in an Inductive Circuit

Comparing Figs. 6.4 and 7.2 we see one difference. Whereas the capacitive current leads its terminal voltage by quarter of a cycle $\left(90^{\circ}\right)$, the inductive current lags by that amount. If you want something to do you might care to calculate the instantaneous power at intervals throughout the cycle in Fig. 7.2 and draw a power curve, as in Fig. 6.8. But the same result can be achieved with less trouble simply by reversing the time scale in Fig. 6.8, making it read from right to left. So except for relative phases the conclusions regarding power in a pure inductance are exactly the same as for a capacitance: the net power taken over a whole cyele is zero, because during half the time the generator is expending power in building up a magnetic field. and during the other half the energy thus built up is returning power to the generator. So in the example we took, with 240 V passing 0.382 A through $2 \mathrm{H}, 240 \times 0.382$ does not represent watts dissipated in the circuit (as it would with resistance) but half that number of volt-amps tossed to and fro between generator and inductor at the rate of 100 times a second.

If you try an experiment similar to Fig. 6.3. but using a coil of several henries in place of the capacitor, you may find that it does become perceptibly warm. This is because the coil is not purely inductive. Whereas the resistance of the plates and connections of a capacitor is usually negligible compared with its reactance, the resistance of the wire in a coil is generally very appreciable. Although the resistance and inductance of a coil are physically inseparable, it is allowable to consider them theoretically as separate items in series with one another.

### 7.5 Phasor Diagram for Inductive Circuit

For comparison with Fig. 6.9 we draw the simple circuit diagram, Fig. 7.3a, of a generator feeding a load consisting of pure (i.e.. resistanceless) inductance. We start with the current phasor and by applying the basic principles we know, as in Sec. 6.6, we find that the correct relative angle of the voltage phasor is as shown (Fig. $7.3 b$ ). With the same conventions as before, it expresses the wellknown rule that the current through a pure inductance lags the applied sinusoidal voltage by $90^{\circ}$.

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### 7.6 Inductance and Resistance in Series

$R$ in Fig. 7.4a represents the resistance of the inductor whose inductance is represented by $L$, plus any other resistance there may be in series. This circuit can be tackled in exactly the same manner as Fig. 6.10. As the current is common to both $L$ and $R$ we again begin with its phasor. The voltage phasor ac is again parallel to $A B$,


Fig. 7.3-The circuit of Fig. 7.1 repeated with its phasor diagrams


Fig. 7.4-L and $R$ in series, and corresponding phasor diagrams
and in fact the only real difference is that the phasor representing the voltage $c b$ applied to $L$ must be drawn so that AB is $90^{\circ}$ behind in phase, which is the same thing as saying cb must lead $A B$ by that angle.

As ac is parallel to $\mathrm{AB}, \varphi$ is the angle by which $A B$ lags the total applied voltage, $a b$. Again, as in Secs 6.8 and 6.9, and by similar reasoning,

$$
\begin{aligned}
Z & =\sqrt{\prime}\left(R^{2}+X^{2}\right) \\
\tan \varphi & =\frac{X}{R} \\
\text { and } \quad P & =E I \cos \varphi
\end{aligned}
$$

### 7.7 Inductance and Resistance in Parallel

Fig. 7.5 corresponds to Fig. 6.11 and the phasor diagram is derived in the same way. As the current $A C$ through $L$ lags the applied voltage $a b$ by $90^{\circ}$ instead of leading it as it would with a capacitor, AC must be drawn in the opposite direction, and $\varphi$ is the angle by which the total current supplied by the generator lags its voltage.

If $R$ is independent of $L$, then this circuit is somewhat unrealistic, as the resistance of the wire with which L is wound is not allowed


Fig. 7.5-Parallel LR circtuit and phasor diagrams.
for. If its resistance were included (in series with $L$ ) the problem could be solved by combining the methods for series and parallel impedances, as in Sec. 8.9. We shall see however. in Sec. 8.16, that Fig. $7.5 a$ is an alternative way of taking account of the resistance of a reactive component.

The formulae for $Z, Y$, etc., found in Sec. 6.10, apply equally here. $X$ standing for inductive instead of capacitive reactance.

The more advanced books extend the methods and scope of circuit calculation enormously; but before going on to them one should have thoroughly grasped the contents of these last two chapters. Some of the results can be summarized like this:

Applicd voltage:


With $R$ and $X$ in series, $Z=\imath^{\prime}\left(R^{2}+X^{2}\right)$ and $\tan \varphi=X / R$.


With $G$ and $B$ in parallel, $Y=\downarrow^{\prime}\left(G^{2}+B^{2}\right)$ and $\tan \varphi=B / G$.

### 7.8 Transformers

In Sec. 4.11 we had a brief introduction to the transformer, which is an appliance very widely used for electrical purposes, including radio and electronics. We are now equipped to study it in a little more detail, beginning with Fig. 7.6a. This represents a transformer


Fig. 7.6-Unloaded I : I transformer with $100 \%$ coupling
connected to a generator giving the usual sinusoidal waveform. Let us assume that the primary and secondary coils have the same number of turns, and at this stage such complications as their resistance are conveniently assumed to be absent. An iron core is used to ensure that we can also assume, without serious error, that all the magnetic flux generated by the primary coil also links with the secondary, which at present is not connected to anything. The dots marking the top ends of the coils are a standard convention to exclude uncertainty about the relative direction of the windings. Without it one could not be sure about the polarity of the secondary voltage. If both coils are wound around the core in the same direction, then the dots mark both starting ends or both finishing ends.

With the secondary coil open-circuited, as shown, the circuit is essentially the same as Fig. 7.3, so the phasor diagram is the same. We saw in Sec. 7.2 that the current driven by any applied voltage $E$
through an inductance $L$ is inversely proportional to $L$. And in Sec. 4.6 we noted that the inductance of an ideal inductor of the type we are considering is directly proportional to the permeability, $\mu$ of the core, and to the square of $N$, the number of turns. The iron core of a transformer normally has a $\mu$ running into thousands, and $N$ is usually considerable, so the inductance is likely to be very large and the current therefore quite small. That is why AB is drawn so short in Fig. 7.6b. $A B$ is called the magnetizing current because its purpose is to generate sufficient alternating magnetic flux in the core to induce an e.m.f. in the primary coil exactly equal to that applied. As the flux is due directly to the current (not to its rate of change), it is in phase with the current, so the same phasor will do for both. This is shown by marking it with the flux symbol, $\Phi$ (Greek capital phi) in Fig. $7.6 b$; and this also picks out $A B$ as the magnetizing current. Actually, as mentioned in Sec. 4.7, $\mu$ depends on the flux density in the core, varying throughout each cycle; so to induce a sinusoidal e.m.f.-necessary to match the applied one at every instant-the magnetizing current has to be distorted, or nonsinusoidal, and therefore cannot strictly be represented by a phasor. This awkward complication is usually ignored, especially as the magnetizing current is such a little one. With the ideal conditions assumed, it is clear that the average power required from the generator is nil, but that does not mean that there is no point in keeping the magnetizing current as small as possible; the resistances of coil, generator and leads, which we are neglecting, give rise to a power loss proportional to the square of the current.

Because the secondary coil in the assumed transformer has the same number of turns as the primary and is linked by the same alternating flux, the secondary voltage $c d$ must be the same as the primary, $a b$, so is represented by a phasor of the same length and angle. In fact, a single phasor would do, and this corresponds to the fact that a could be connected to $c$ and $b$ to $d$ without affecting the electrical conditions. This is where the dots are helpful; if the secondary dot was at the cend it would indicate a reversal of winding and consequent polarity of secondary voltage, so paralleling the two coils as described would be disastrous!

### 7.9 Load Currents

The next step is to connect a load (for full explanation of this term, see Sec. 11.2) to the secondary terminals, and continuing our ideal simplicity we make it a resistor, R in Fig. 7.7a. This creates a new current mesh, C , and also raises a question about how this should be related to mesh $B$. Are we to suppose that the external mesh $A$ penetrates between the windings, or should we think of them as too close together for that? According to our current notation $B C$ is the current crossed on passing from $B$ to $C$, and this is so whether
we chose to see an intermediate mesh A , so that $B C=B A+A C$, or $A$ is ignored and $B C$ is seen as the same as the sum of the currents upwards in the two coils. Both ways come to the same thing. The important point is that the magnetic flux is the result of current in both coils. As the generator voltage $a b$ is the same as before, the net


Fig. 7.7-The transformer of Fig. 7.6 with a resistive load
magnetizing current, which now is $B C$, must be the same as $B A$ in Fig. 7.6. So in Fig. $7.7 b$ we must draw BC accordingly.

Having settled that point we turn to the secondary circuit, which is identical with Fig. 5.11 except for the different letters. Following the same principle we draw AC parallel to cd, which now has been made to coincide with ab as we saw it could have been in Fig. 7.6b. The total current that the generator has to supply, $A B$, follows from this construction, and we see that it lags the generator voltage by the small phase angle $\varphi$.

The procedures for other types of load than $R$ have been explained so need not be repeated.

We have already noted that in this example the coils could have been joined together at both ends, as we have represented by the coinciding voltage phasors. The coils themselves, indeed, could be made to coincide without affecting the working principles. Having done so, we could regard the secondary load current $A C$ as flowing direct from the generator by the route bdca, and the combined primary and secondary coil as carrying only the magnetizing current, $C B$. This is equivalent to going straight from C to B in the phasor current diagram, instead of via $A$, which takes account of the separate coil currents. These largely oppose one another in order to maintain the magnetizing current $C B$ unchanged. The very close correspondence between this type of phasor diagram and what it represents gives a clear insight into the working of the circuit and makes it almost impossible to fall into such a common error as to suppose that the secondary voltage is in opposite phase to the primary.

### 7.10 Transformer Losses

In real transformers the primary coil has some resistance as well as inductance. Moreover, the alternating flux in the core generates a certain amount of heat in it (Sec. 8.14), which has to come from somewhere, and in accordance with our wider definition of resistance (Sec. 2.18) can be considered as equivalent to some extra resistance in the primary. The primary resistance, augmented in this way, causes the magnetizing current to lag the primary voltage by less than $90^{\circ}$.

Nor is the power from the supply transferred to the load without loss. The primary and secondary load currents have to pass through the resistances of the windings. These resistances are represented in Fig. 7.8 by $R_{1}$ and $R_{2}$. Moreover in real transformers some of the


Fig. 7.8 -Fig. 7.7 a is here elaborated to include the resistances of the coils and their leartage inductances
flux linking the primary coil fails to link with the secondary; similarly when secondary current is flowing it creates some flux not linked with the primary. These fluxes can be represented by small inductances $L_{1}$ and $L_{2}$, called leakage inductances. Note that the transformer terminals are still $\mathrm{a}-\mathrm{b}$ and $\mathrm{c}-\mathrm{d}$; the internal junctions are of course not accessible as they are entirely imaginary. Fig. 7.8 is an example of an equivalent circuit - equivalent in this case, approximately, to a real transformer, so far as any actual electrical measurements can show. It happens to be an example of a very common type of equivalent circuit, with two input and two output terminals; and because one may have to guess, from external measurements, what circuit best represents its actual behaviour, it is often called a black box.

If you feel confident about applying the phasor diagram rules, Fig. 7.8 would be a good exercise. Although more complicated than anything we have tackled so far, it is quite straightforward, if one labels the four extra junctions and starts with the $c d$ phasor. However, as it may be too big a jump for some, let us deal with a simplified version.

We have already seen that if the transformer turns ratio is $1: 1$ the two coils can be reduced to one. And as the magnetizing current is
normally quite small compared with the load current its contribution to the voltage drop across $R_{1}$ and $L_{1}$ can reasonably be neglected, by transferring them to the secondary circuit, so that suitably increased values of $R_{2}$ and $L_{2}$ include them. Fig. $7.9 a$ shows the simplified diagram. For easy comparison with the preceding ones, c is still shown, though it is now electrically the same as a.
AC and cd are first drawn parallel to one another. If the load had not been purely resistive they would of course have had to be drawn


Fig. 7.9-Simpler phasor diagrams (b) than those applicable to Fig. 7.8 are obtained by transferring the primary resistance and leakage inductunce to the secondary circuit and combining the fully coupled parts of the windings (a)
with the appropriate phase angle between them. de is simply an extension of cd, but eb must lead it by $90^{\circ}$ because eb is across an inductance. We now see $a b$, the input voltage, and can therefore draw CB at right angles to it to represent an ideal magnetizing current, or at a somewhat smaller angle to be more realistic. As the diagram clearly shows, one effect of coil resistance and leakage inductance is that the output voltage falls as the current taken is increased. This effect is confirmed by practical experience. And if we calculate the output power, equal to $\mathrm{cd} . A C$, and compare it with the input power, which in Sec. 6.9 we found to be $a b \cdot A B \cos \varphi$, we can find how much power is lost in the transformer. The ratio of output to input power, which is always less than I, is called the efficiency of the transformer.

These considerations are most important in mains transformers; in audio-frequency transformers (which are less used than they once were) the emphasis is rather different, and in radio-frequency transformers it is almost completely different, as will be seen when they come up for attention.

### 7.11 Impedance Transformation

So far our transformer diagrams have been related to those having the comparatively little-used I : I ratio. There is no reason why
other ratios should not be illustrated by means of the relative lengths of the phasors. except that if the ratio were large there would be difficulty in drawing all the phasors clearly to the same scale. Of course the primary and secondary phasors of our type would need to be shown separately. On the other hand there is no strong reason why phasor diagrams sloould be used to show ratios other than $1: 1$. Transformer designers usually work in ampere-turns and volts per turn. which are the same for both primary and secondary and for any other windings embracing the whole core. So the phasors can very well be regarded as referring to ampere-turns and volts per turn, the actual amps and volts being derived by dividing or multiplying by the numbers of turns.

For we already know that when the same alternating flux links all the turns it induces the same voltage in every turn. And you have


Fig. 7.10-So fur as the generutor is concerned, the perfect transformer and load at a call be replaced by the impedonce as an b. An iusperfect masformer can be simuluted os at c
probably realised that if (say) the secondary coil has twice as many turns as the primary, so that its no-load voltage is twice that applied to the primary, the primary current dete to any load current in the secondary must be twice as great. because the primary has only haff the number of turns with which to offset the magnetizing effect of the secondary current. It has to be so, too, to make the input watts the same (neglecting transformer loss) as the output.

Now if the turns ratio of the transformer is $n: 1$ and the load impedance is $Z$ (Fig. 7.10a) the primary voltage $V_{\mathrm{p}}$ is $n$ times the secondary voltage $V_{\mathrm{s}}$, and the primary load current $I_{\mathrm{p}}$ is $I_{\mathrm{s}} / n$. So, while $Z=V_{s} / I_{s}$, the impedance "looking into" the transformer from the primary side is thus $n^{2} V_{\mathrm{V}} / I_{\mathrm{s}}$, or $n^{2} Z$. The transformer and its load $Z$. then, can be replaced by a load $n^{2} Z$. as in Fig. 7.10h, without making any difference from the generator's point of view, except for magnetizing current and losses. If we want to be more precise and simulate these too we can do it by adding suitable parallel inductance and resistance, as in Fig. 7.10c. Here is another example of an equivalent circuit. Obviously the equivalence is valid only on the primary side.

One of the uses of a lransformer is for making a load of a certain impedance. say $Z_{\mathrm{s}}$, equivalent to some other impedance, say $Z_{\mathrm{F}}$. So

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it is often necessary to find the required turns ratio. Since $Z_{\mathrm{p}}=n^{2} Z_{\mathrm{s}}$, it follows that $n$ must be $\left(Z_{\mathrm{n}} Z_{\mathrm{s}}\right)$.

Another use is for insulating the load from the generator. That is why 1 : 1 transformers are occasionally seen. But if it is not necessary to insulate the secondary winding from the primary, there is no need for two separate coils, even when the ratio is not $1: 1$.


Fig. 7.11-For some purposes it is practicuhle to obtain a voltage stef-tip or down withour a separate secondary winding

The winding having the smaller number of turns can be abolished, and the connections tapped across the same number of turns forming part of the other winding as in Fig. 7.11. This device is called an allto-transformer.

Transformers are not limited to two windings. In practice it is quite usual to have several secondary windings delivering different voltages for various purposes, all energized by one primary coil.

## CHAPTER 8

## The Tuned Circuit

### 8.1 Inductance and Capacitance in Series

In the previous chapters we have seen what happens in a circuit containing capacitance or inductance (with or without resistance), and the question naturally arises: what about circuits containing both? We know that two or more reactances of either the inductive or capacitive kind in series can be combined just like resistances, by adding; and either sort of reactance can be combined with resistance by the more complicated square-root process. The outcome of combining reactances of opposite kinds is so fundamental to radio and many other electronic applications that it needs a chapter to itself.

We shall begin by considering the simplest possible series circuit, Fig. 8.1 with capacitance and inductance in series. The method is exactly the same as for reactance and resistance in series; that is to say, since the current is common to both it is easiest to start from it


Fig. 8.1-(a) circuil consisting of an a.c. gericrator in series with capwcitance and inductance. and $(t)$ corresponding phasor diagedms.
and work backwards to find the e.m.f. needed to drive it. As in Fig. 7.4, we first draw a current phasor $A B$ at any angle we like. The voltage applied to $L(c h)$ must lead $A B$ by $90^{\circ}$ : ac on the other hand must lag $A B$ by $90^{\circ}$. And so we get the complete phasor diagram. As a further check we may care to take another look at phasor and waveform diagrams for $C$ alone (Figs. 6.9 and 6.4) and for $L$ alone (7.3 and 7.2). The conclusion must be the same: that in Fig. 8.1 the voltages across $C$ and $L$ are in opposite phase, or $180^{\circ}$ out of phase. So one of them must be subtracted from the other to give the necessary driving e.m.f. And since reactance is equal to applied voltage divided by current (the same in both cases) it follows that the total
reactance of the circuit is also equal to one of the separate reactances less the other.

Which voltage or reactance must be subtracted from which to give the total? There is no particular reason for favouring either, but the agreed convention is to call $X_{L}$ positive and $X_{C}$ - negative.

For example, suppose $L$ in Fig. 8.1 is $2 \mathrm{H}, C$ is $2 \mu \mathrm{~F}$, and $a b$ is 240 V at 50 Hz . We have already calculated the reactances of 2 H and $2 \mu \mathrm{~F}$ at 50 Hz (Secs. 7.2 and end of 6.3) and found them to be $628 \Omega$ and $1592 \Omega$. So the total reactance must be $628-1592=$ $-964 \Omega$, which at 50 Hz is the reactance of a $3 \cdot 3 \mu \mathrm{~F}$ capacitor. The generator, then, would not notice any difference (at that particular frequency) if a $3 \cdot 3 \mu \mathrm{~F}$ capacitor were substituted for the circuit consisting of $2 \mu \mathrm{~F}$ in series with 2 H . The magnitude and phase of the current would be the same in both cases; namely, $240 / 964=0.249 \mathrm{~A}$, leading by $90^{\circ}$.

To make sure, let us check it by calculating the voltage needed to cause this current. $c b$ is equal to $X_{C}$ times $A B=0.249 \times 1592=$ 396 V . ac is $X_{L}$ times $A B=0.249 \times 628=156 \mathrm{~V}$. The resultant of $a c$ and $c b$, namely $a b$, must be (as shown in Fig. 8.1b) $396-156=$ 240 V .

The fact that the voltage across the capacitor is greater than the total supplied may be surprising, but it is nothing to what we shall see soon!

## 8.2 $L, C$ and $R$ all in Series

Bringing $R$ into the circuit introduces no new problem, because we have just found that $L$ and $C$ can always be replaced (for purposes of the calculation) by either $L$ or $C$ of suitable value, and the method of combining this with $R$ was covered in the previous two chapters. Elaborating the equation given therein (Sec. 6.8) to cover the new information, we have

$$
Z=v^{\prime}\left[\left(X_{L}-X_{c}\right)^{2}+R^{2}\right]
$$

If the three circuit elements are connected in the order shown in Fig. $8.2 a$, the phasor diagram, drawn according to the now familiar rules, will be as at $b$. Note that ad, representing the voltage across $C$, is longer than $a b$ representing the generator voltage. If $L$ and $C$ were replaced by a larger $C$ to form the equivalent circuit mentioned above, $b c$ would disappear and ad be shortened by that amount, but $a b$ would still have the same length and phase angle relative to $A B$.

### 8.3 The Series Tuned Circuit

We have already seen that the reactance of a capacitor falls and that of an inductor rises as the frequency of the current supplied to
them is increased (Secs. 6.1 and 7.1). It is therefore going to be interesting to study the behaviour of a circuit such as Fig. 8.3 over a range of frequencies. For the values given on the diagram, which are typical of practical broadcast receiver circuits, the reactances of coil and capacitor for all frequencies up to 1800 kHz are plotted as curves in Fig. 8.4. The significant feature of this diagram is that


Fig. 8.2-Inductance, resistance and copacilance all in scrics, with (h) corresponding phasor diagrams


Fig. 8.3-The way the reactances in this circuit vary with the frequensy of E, graphed in Fig. 8.4, leads to interesting conchusions regarding this type of circuit in general


FREQUENCY, $f$, IN KILOHERTZ
Fig. 8.4-Reactances of the coil and capacitor in Fig. 8.3 plonted against frequency
at one particular frequency, about 800 kHz (more precisely, 796), $L$ and $C$ have equal reactances, each amounting to $1000 \Omega$. So the total reactance, being the difference between the two separate reactances, is zero. Put another way, the voltage developed across the one is equal to the voltage across the other; and since they are, as always, in opposition, the two voltages cancel out exactly. The circuit would therefore be unaltered, so far as concerns its behaviour
as a whole to a voltage of this particular frequency, by completely removing from it both $L$ and $C$. This, leaving only $R$, would result in the flow of a current equal to $E / R$.

Let us assume a voltage not unlikely in broadcast reception, and see what happens when $E=5 \mathrm{mV}$. The current at 796 kHz is then $5 / 10=0.5 \mathrm{~mA}$, and this current flows, not through $R$ only, but through $L$ and $C$ as well. Each of these has a reactance of $1000 \Omega$ at this frequency; the potential difference across each of them is therefore $0.5 \times 1000=500 \mathrm{mV}$, which is just one hundred times the voltage $E$ of the generator to which the flow of current is due.

That a simple circuit like this, without any amplifier, can enable so small a voltage to yield two such relatively large voltages, may seem incredible, but practical radio communication would not be possible without it.

### 8.4 Magnification

In the particular case we have discussed, the voltage across the coil (or across the capacitor) is one hundred times that of the generator. This ratio is called the magnification of the circuit.
We have just worked out the magnification for a particular circuit; now let us try to obtain a formula for any circuit. The magnification is equal to the voltage across the coil divided by that from the generator, which is the same as the voltage across $R$. If $I$ is the current flowing through both, then the voltage across $L$ is $I X_{L}$, and that across $R$ is $I R$. So the magnification is equal to $I X_{L} \mid I R$, which is $X_{L} / R$ or (because in the circumstances considered $X_{C}$ is numerically equal to $\left.X_{L}\right) X_{C} / R$. At any given frequency, it depends solely on $L / R$, the ratio of the inductance of the coil to the resistance of the circuit, or on $C R$. (Both of these, you may have noticed, are time constants.)

To obtain high magnification of a received signal (for which the generator of Fig. 8.3 stands), it is thus desirable to keep the resistance of the circuit as low as possible.

The symbol generally used to denote this voltage magnification is $Q$ (not to be confused with $Q$ denoting quantity of electricity).

In any real tuned circuit the coil is not pure inductance as shown in Fig. 8.3, but contains the whole or part of $R$ and also has some stray capacitance in parallel with it. Consequently the magnification obtained in practice may not be exactly equal to $Q$ (as calculated by $X_{L} / R$ ); but the difference is usually unimportant.

### 8.5 Resonance Curves

At other frequencies the impedance of the circuit is greater, because in addition to $R$ there is some net reactance. At 1250 kHz , for example, the individual reactances are 1570 and $637 \Omega$ (see Fig. 8.4),
leaving a total reactance of $933 \Omega$. Compared with this, the resistance is negligible, so that the current, for the same driving voltage of 5 mV , will be $5 / 933 \mathrm{~mA}$, or roughly $5 \mu \mathrm{~A}$. This is approximately one hundredth of the current at 796 kHz . Passing through the reactance of $L(1570 \Omega)$ it gives rise to a voltage across it of $1570 \times 5=$ $7850 \mu \mathrm{~V}=7.85 \mathrm{mV}$, which is only about one sixty-fourth as much as at 796 kHz .

By extending this calculation to a number of different frequencies we could plot the current in the cireuit, or the voltage developed across the coil, against frequency. This has been done for two circuits in Fig. 8.5. The only difference between the circuits is that


Fig. 8.5-Showing how the woltuge developed across the coil in a tumed circuit varies with frequency. Curves are plotsed for $L=180 \mu \mathrm{H}, \mathrm{C}=14 \mathrm{p} F, E$ (injected inltage) $=$ 0.5 mV . and $R=15 \Omega$ for $\mathrm{tr} \mathrm{m} Q=75$ circuit and $5.63 \Omega$ for the $Q=200$ circuit
in one the resistance is $15 \Omega$, giving $Q=75$, and in the other it is $5.63 \Omega$, giving $Q=200$. In both, the values of $L$ and $C$ are such that their reactances are equal at 1000 kHz . These curves illustrate one thing we already know-that the response (voltage developed across the coil) when the reactances are equal is proportional to $Q$. They also show how it falls off at frequencies on each side, due to unbalanced reactance. At frequencies well off 1000 kHz the reactance is so much larger than the resistance that the difference between the two circuits is insignificant. The shapes of these curves show that the response of a circuit of the Fig. 8.3 type is far greater to voltages of one particular frequency-in this case 1000 kHz than to voltages of substantially different frequencies. The circuit
is said to be tumed to, or to resonate $10,1000 \mathrm{kHz}$; and the curves are called resonance curves. This electrical resonance is very closely analogous to acoustical resonance: the way in which hollow spaces or pipes magnify sound of a particular pitch.

The principle on which a receiver is tuned is now beginning to appear: by adjusting the values of $L$ or $C$ in a circuit such as that under discussion one can make it resonate to any desired frequency. Signal voltages received from the aerial at that frequency will be anmplified so much more than those of substantially different frequencies that these others will be more or less tuned out.

### 8.6 Selectivity

This ability to pick out signals of one frequency from all others is called selectivity. It is an even more valuable feature of tuned circuits than magnification. There are alternative methods (which we shall consider later) of magnifying incoming signal voltages, but


Fig. 8.6-In this diagram the curve ' $Q=200^{\prime}$ is the same' as in Fig. 8.5. but, to enable the sele ctivity of the $Q=75$ circuit to be more easily compared, the voltage injected into it has heen ruised from 0.5 ml ' 101.33 mV , so as to make the output voltage at resonamce equal to that across the $Q=200$ circuit
by themselves they would be useless, because they fail to distinguish between the desired programme and others.

The way the curves in Fig. 8.5 have been plotted focuses attention on how the value of $Q$ affects the response at resonance. The
comparative selectivity is more easily seen, however. if the curves are plotted to give the same voltage at resonance, as has been done in Fig. 8.6. This necessitates raising the input to the low- $Q$ circuit from 0.5 V to 1.33 V . Since the maximum voltage across both the coils is now 100 mV , the scale figures can also be read as percentage of maximum. Fig. 8.6 brings out more clearly than Fig. 8.5 the better selectivity of the high- $Q$ circuit. For a given respense to a desired station working on 1000 kHz , the $200-Q$ response to 990 k Hz voltages is less than half that of the $75-Q$ circuit. In general. the rejection of frequencies well off resonance is nearly proportional to $Q$.

### 8.7 Frequency of Resonance

It is obviously important to be able to calculate the frequency at which a circuit containing known $L$ and $C$ resonates, or to calculate the $L$ and $C$ required to tune to a given frequency. The required equation follows easily from the fact that resonance takes place at the frequency which makes the reactance of the coil equal that of the capacitor:

$$
2 \pi f_{\mathrm{r}} L=\frac{1}{2 \pi f_{\mathrm{r}} C}
$$

where the symbol $f_{\mathrm{r}}$ is used to denote the frequency of resonance. Rearranging this, we get

$$
f_{\mathrm{r}}^{2}=\frac{1}{(2 \pi)^{2} L C}, \quad \text { so } f_{\mathrm{r}}=\frac{1}{2 \pi_{\mathrm{J}}(L C)}
$$

If $L$ and $C$ are respectively in henries and farads, $f_{r}$ will be in hertz: if $L$ and $C$ are in henries and microfarads, $f_{\mathrm{r}}$ will be in kHz . But perhaps the most convenient units for radio purposes are $\mu \mathrm{H}$ and pF . and when the value of $\pi$ has been filled in the result is

$$
f_{\mathrm{r}}(\text { in } \mathrm{kHz})=\frac{159.155}{1(L C)} \text { or } f_{\mathrm{r}}(\text { in } \mathrm{MHz})=\frac{159 \cdot 15}{1^{\prime}(L C)}
$$

If the answer is preferred in terms of wavelength, we make use of the relationship $f=3 \times 10^{8} / \lambda$ (Sec. 1.9) to give

$$
\lambda_{\mathrm{r}}=1.885,(L C) \quad\left(\lambda_{\mathrm{r}} \text { in } \mathrm{m}: L \text { in } \mu \mathrm{H}: C \text { in } \mathrm{pF}\right)
$$

Note that $f_{\mathrm{r}}$ and $\lambda_{\mathrm{r}}$ depend on $L$ multiplied by $C$; so in theory a coil of any inductance can be tuned to any frequency by using the appropriate capacitance. In practice the capacitance cannot be reduced indefinitely, and there are disadvantages in making it very large. A typical tuning capacitor for the medium-frequency band
has a range of about 16 to 270 pF . The capacitance of the wiring and circuit components necessarily connected in parallel may add another 14 pF . This gives a ratio of maximum to minimum of $284 / 30=9 \cdot 47$. But the square root sign in the above equations means that if the inductance is kept fixed the ratio of maximum to minimum $f_{\mathrm{r}}$ (or $\lambda_{\mathrm{r}}$ ) is $1^{\prime} 9 \cdot 47$, or $3 \cdot 075$. Any band of frequencies with this range of maximum to minimum can be covered with such a capacitor, the actual frequencies in the band depending on the inductance chosen for the coil.

Suppose we wished to tune from 1620 MHz to $1620 / 3.075$ or 527 kHz , corresponding to the range of wavelengths 185 to 569 metres. For the highest frequency or lowest wavelength the capacitance will have its minimum value of 30 pF ; by filling in this value for $C$ and 1.620 MHz for $f_{\mathrm{r}}$, we have $1 \cdot 620=159 \cdot 15 / \mathrm{s}^{\prime}(30 L)$, from which $L=321.5 \mu \mathrm{H}^{*}$.

If we calculate the value of $L$ necessary to give 0.527 MHz with a capacitance of 284 pF , it will be the same.

In the same way the inductance needed to cover the short-wave band 9.8 to 30 M Hz ( 30.6 to 10 metres) can be calculated, the result being $0.938 \mu \mathrm{H}$.

Notice how large and clumsy numbers are avoided by a suitable choice of units, but of course it is essential to use the equation having the appropriate numerical 'constant'. If in any doubt it is best to go back to first principles ( $2 \pi f_{\mathrm{r}} L=1 / 2 \pi f_{\mathrm{r}} C$ ) and use henries, farads, and Hz .

## 8.8 $L$ and $C$ in Parallel

Having studied circuits with $L$ and $C$ in series we come naturally to the simple parallel case, Fig. 8.7a. It should hardly be necessary by now to point out that the voltage $a b$ is the quantity common to both $L$ and $C$, and so we start with its phasor. Drawing the current phasors in the appropriate directions and assuming that in this particular example more current goes through $L$ than through $C$ we get the complete diagram, Fig. $8.7 b$.

If we go back to Fig. 8.1 and compare the two phasor diagrams we see that they are identical except that current and voltage have changed places. We seem to be coming across quite a lot of this sort of thing: see Secs. $4.8,4.10$ and 6.10 for example. It has a special name, but going into that just now would divert attention from our study of tuned circuits so will have to wait until Sec. 12.2.

[^5]Meanwhile we note the fact of current-voltage interchange sufficiently to be able to grasp, without too much repetition along the lines of Sec. 8.1, that in the parallel circuit the branch currents are in opposite phase, so that the total current to be supplied is the difference between them (or their sum if one of them is taken as


Fig. 8.7-L and C in parallel. and cerresponding phasor diagrams
negative). Similarly the two susceptances, $B_{L}$ and $B_{C}$, are opposite in sign, and this time it is the inductive one that is taken as negative. So the total susceptance in our example is negative and smaller numerically then either $B_{L}$ or $B_{C}$.

Suppose now that the frequency is such as to make the susceptances (and therefore the reactances) equal. The currents will therefore be equal, so that the difference between them will be zero. We then have a circuit with two parallel branches, both with currents flowing in them, and yet the current supplied by the generator is nil!

It must be admitted that such a situation is impossible, the reason being that no practical circuits are entirely devoid of resistance. But if the resistance is small, when the two currents are equal the system does behave approximately as just described. It is a significant fact, which can easily be checked by calculation of the type given in Sec. 8.7, that the frequency which makes them equal is the same as that which in a series circuit would make $L$ and $C$ resonate.

### 8.9 The Effect of Resistance

When the resistance of a parallel LC circuit has been reduced to a minimum, the resistance in the C branch is likely to be negligible compared with that in the L branch. Assuming this to be so, let us consider the effect of resistance in the L branch (Fig. 8.8), representing it by the symbol $r$ as a reminder that it is small compared with the reactance. We have already studied this branch on its own (Fig. 7.4) and noted that one effect of the resistance is to make the phase angle between current and voltage less than $90^{\circ}$; the greater the resistance the smaller the angle. It would be quite easy
to modify Fig. $8.7 h$ by reducing the phase angle slightly, but to do the job thoroughly, so that if necessary we could draw the diagram to scale. let us start from scratch without reference to other diagrams and just follow the basic rules of phase relationships.

We start with the series branch made up of $L$ and $r$, and as current is common to both we draw its phasor, AC, horizontally. Now we


Fig. 8.8 (a) Ideal capacitor in parallel with inductor having tosses represented by resistance $r$ in series. and (b) corresponding phasor diagrams
can draw ac in phase, small because $r$ is small: and $\mathrm{cb} 90^{\circ}$ ahead. That completes the voltage part. Having now got $a b$, not perfectly vertical this time, we can draw CB $90^{\circ}$ ahead of it: let us assume it is equal to AC . That shows us the supplied current. $A B$. Because $\mathrm{AC}=\mathrm{CB}$, the angle ABC is less than a right angle, but as we have drawn CB at right angles to $a b$ we deduce that although the supplied current $A B$ looks as if it might be in phase with its e.m.f. $a b$ it is not exactly so. It could be made so by a very slight reduction in frequency, which would increase $A C$ and reduce $C B$. (This sort of information would be almost impossible to obtain from wavelorm diagrams, even at cost of much greater time and trouble.) In practice $r$ is likely to be less than one-hundredth as much as $X_{L}$, so even when $X_{L}$ is made exactly equal to $X_{C}$ the current supplied by the generator would be small and practically in phase with its e.m.f. In other words, it is identical with the current that would flow if the two reactive branches were replaced by one consisting of a high resistance.

### 8.10 Dynamic Resistance

It is of considerable practical interest to know how high this resistance is (let us call it $R$ ), and how it is related to the values of the real circuit: $L, C$ and $r$. We have already seen that if $r=0, A B$ is zero, so in that case $R$ is infinitely large. Fig. $8.8 h$ shows that as $r$
is increased, making the voltage drop across it, ac, greater, $a b$ is swung round clockwise, and $B C$ with it, increasing $A B$. This increase in current from the generator signifies a fall in $R$. So at least we can say that the smaller we make $r$ in Fig. 8.8, the greater is the resistance that the circuit as a whole presents to the generator.

It can be shown mathematically that, so long as $r$ is much less than $X_{L}$, the resistance $R$, to which the parallel circuit as a whole is equivalent at a certain frequency, is practically equal to $X_{L}{ }^{2} / r$. The proof by algebra is given in textbooks, but geometry-minded readers should not find it difficult to prove from Fig. 8.8b.

To take an example, suppose $C$ and $L$ are the same as in Fig. 8.3. and $r$ is equal to $10 \Omega$ (Fig. $8.9 a$ ). At 796 kHz the reactances are

(a)

(b)

Fig. 8.9- The circuil composed of C. $L$ and $r$ having the values marked (a) can at 796 kHz he reploced by a high resistonce ( $h$ ), so far as the phase and magnitude of the current taken from the getrerator is concerned
each $1000 \Omega$ (Fig. 8.4). So $R$ is $1000^{2} / 10=100000 \Omega$ (Fig. 8.9b). If $L . C$ and $r$ were hidden in one box and $R$ in another, each with a pair of terminals for connecting the generator, it would be impossible to tell. by measuring the amount and phase of the current taken. which box was which. Both would appear to be resistances of $100 \mathrm{k} \Omega$. at that particular frequency. We shall very soon consider what happens at other frequencies, but in the meantime note that the apparent or equivalent resistance, which we have been denoting by $R$, has the special name dynamic resistance.

### 8.11 Parallel Resonance

Suppose $E$ in Fig. 8.9 is 1 V. Then under the conditions shown the current $l$ must be $1 / 100000 \mathrm{~A}$, or $10 \mu \mathrm{~A}$. Since the reactances of $C$ and $L$ are each $1000 \Omega$, and the impedance of the $L$ branch is not appreciably increased by the presence of $r$, the branch currents are each $1000 \mu \mathrm{~A}$. But let us now change the frequency to 1000 kHz . Fig. 8.4 shows that this makes $X_{C}$ and $X_{L}$ respectively $800 \Omega$ and $1250 \Omega$. The current taken by $C$ is then $1 / 800 \mathrm{~A}=1250 \mu \mathrm{~A}$ and the $L$ current is $1 / 1250 \mathrm{~A}=800 \mu \mathrm{~A}$. Since $r$ is small, these two currents are so nearly in opposite phase that the total current is practically
equal to the difference between them, $1250-800=450 \mu \mathrm{~A}$. This balance is on the capacitive side, so at 1000 kHz the two branches can be replaced by a capacitance having a reactance $=E / I=$ $1 / 0.00045=2222 \Omega$, or 71.6 pF . The higher the frequency the greater is the current. and the larger this apparent capacitance; at very high frequencies it is practically 200 pF -as one would expect, because the reactance of $L$ becomes so high that hardly any current can flow in the $L$ branch.

Using the corresponding line of argument to explore the frequencies below 1000 kHz , we again find that the total current becomes larger, and that its phase is very nearly the same as if $C$ were


Fig. 8.10-Plotting the current and impedance of the Fig. 8.9a circuit as a whole at various frequencies gives the shapes shown here: (a) current, (b) impedance
removed, leaving an inductance greater than $L$ but not much greater at very low frequencies.

Plotting the current over a range of frequency, we get the result shown in Fig. 8.10a, which looks somewhat like a series-circuit resonance curve turned upside down.

The graph of impedance against frequency (Fig. 8.10b) is still more clearly a resonance curve.

### 8.12 Frequency of Parallel Resonance

Before we complicated matters by bringing in $r$ we noticed that the frequency which would make $L$ and $C$ resonate in a series circuit had the effect in a parallel circuit of reducing $I$ to zero and making the impedance infinitely great. In these circumstances the frequency of parallel resonance is obviously the same as that of series resonance.

But actual circuits always contain resistance, which raises some awkward questions about the frequency of resonance. In a series circuit it is the frequency that makes the two reactances equal. But in a parallel circuit this does not make the branch impedances exactly equal unless both contain the same amount of resistance, which is rarely the case. The two branch currents are not likely to be exactly equal, therefore. So are we to go on defining $f_{r}$ as the frequency that makes $X_{L}=X_{C}$, or as the frequency which brings $I$ into phase with $E$ so that the circuit behaves as a resistance $R$, or as the frequency at which the current $I$ is least?

Fortunately it happens that in normal radio practice this question is mere hair-splitting: the distinctions only become appreciable when $r$ is a large-sized fraction of $X_{L}$; in other words, when $Q$ ( $=X_{\mathrm{L}} / \cdot$ ) is abnormally small-say less than 5 . On the understanding that such unusual conditions are excluded, therefore, the following relationships are so nearly true that the inaccuracy is of no practical importance. The frequency of parallel resonance ( $f_{r}$ ) is the same as that for series resonance, and it makes the two reactances $X_{L}$ and $X_{C}$ numerically equal. The impedance at resonance $(R)$ is resistive, and equal to $X_{L}^{2} / r$ and also to $X_{C}{ }^{2} / r$ (Sec. 8.10). And, because $X_{L}=X_{C}$.
from which

$$
\begin{gathered}
R=\frac{X_{L} X_{C}}{r}=\frac{2 \pi f_{\mathrm{r}} L}{2 \pi f_{\mathrm{r}} C r}=\frac{L}{C r} \\
r=\frac{L}{C R}
\end{gathered}
$$

In all these, $r$ can be regarded as the whole resistance in series with $L$ and $C$, irrespective of how it is distributed between them. Since $Q=X / r$.

$$
R=Q X, \quad r=\frac{X}{Q}, \quad \frac{R}{X}=\frac{X}{r} \text { and } \frac{R}{Q}=Q r
$$

All these relationships assume resonance.

### 8.13 Series and Parallel Resonance Compared

The resonance curve in Fig. 8.10 b showing $R=100 \mathrm{k} \Omega$ applies to the circuit in Fig. 8.9a, in which $Q=X_{L} / r=2 \pi f L / r=2 \pi \times$ $796000 \times 0.0002 / 10=100$. If $r$ were reduced to $5 \Omega Q$ would be 200 , and $R$ would be $200 \mathrm{k} \Omega$. But reducing $r$ would have hardly any effect on the circuit at frequencies well off resonance; so the main result would be to sharpen the resonance peak. Increasing $r$ would flatten the peak. This behaviour is the same as with series resonance, except that a peak of impedance takes the place of a
peak of voltage across the coil or of current from the generator. Current from the generator to the parallel circuit reaches a minimum at resonance (Fig. 8.10a), as does impedance of the series circuit.

At resonance a series circuit takes the maximum current from the generator, so is called an acceptor circuit. A parallel circuit, on the contrary, takes the minimum current so is called a rejector circuit. One should not be misled by these names into supposing that there is some inherent difference between the circuits; the only difference is in the way they are used. Comparing Fig. $8.9 a$ with Fig. 8.3 we see the circuits are identical except for where the generator is connected. To a generator connected in one of the branches, Fig. 8.9a would be an acceptor circuit at the same time as it would be a rejector circuit to the generator shown.

If the generator in a circuit such as Fig. $8.9 a$ were to deliver a constant current instead of a constant e.m.f., the voltage developed across it would be proportional to its impedance, and so would vary with frequency in the manner shown in Fig. 8.5. So the result is the same, whether a tuned circuit is used in series with a source of voltage or in parallel with a source of current. In later chapters we shall come across examples of both.

### 8.14 The Resistance of the Coil

In the sense that it cannot be measured by ordinary direct-current methods-by finding what current passes through on connecting it across a 2 V cell, for example-it is fair to describe $R$ as a fictitious resistance. Yet it can quite readily be measured by any method suitable for measuring resistances at the frequency to which the circuit is tuned; in fact, those methods by themselves would not disclose whether the thing being measured was the dynamic resistance of a tuned circuit or the resistance of a resistor.

If it comes to that, even $r$ is not the resistance that would be indicated by any d.c. method. That is to say, it is not merely the resistance of the wire used to make the coil. Although no other cause of resistance may appear, the value of $r$ measured at high frequency is always greater, and may be many times greater, than the d.c. value.

One possible reason for this has been mentioned in connection with transformers (Sec. 7.10). If a solid iron core were used, it would have currents generated in it just like any secondary winding: the laws of electromagnetism make no distinction. These currents passing through the resistance of the iron represent so much loss of energy, which as we have seen, brings the primary current more nearly into phase with the e.m.f., just as if the coil's impedance included a larger proportion of resistance. To stop these eddy currents, as they are called, iron cores are usually made up of thin sheets
arranged so as to break up circuits along the lines of the induced e.m.f. At radio frequencies even this is not good enough, and if a magnetic core is used at all it is either iron in the form of fine dust bound together by an insulator, or a non-conducting magnetic material called a ferrite.

Another source of loss in magnetic cores, called hysteresis, is a lag in magnetic response, which shifts the phase of the primary current still more towards that of the applied e.m.f., and therefore represents resistance, or energy lost.

The warming-up of cores in which a substantial number of watts are being lost in these ways is very ioticeable.

Although an iron core introduces the equivalent of resistance into the circuit, its use is worth while if it enables a larger amount of resistance to be removed as a result of fewer turns of wire being needed to give the required inductance. Another reason for using iron cores, especially in r.f. coils, is to enable the inductance to be varied, by moving the core in and out.

Even the metal composing the wire itself has e.m.fs induced in it in such a way as to be equivalent to an increase in resistance. Their distribution is such as to confine the current increasingly to the surface of the wire as the frequency is raised. This skin effect occurs even in a straight wire; but when wound into a coil each turn lies in the magnetic field of other turns, and the resistance is further increased; so much so that using a thicker gauge of wire sometimes actually increases the r.f. resistance of a coil.

### 8.15 Dielectric Losses

In the circuits we have considered until now, $r$ has been shown exclusively in the inductive branch. While it is true that the resistance of the capacitor plates and connections is usually negligible (except perhaps at very high frequencies), insulating material coming within the alternating electric field introduces resistance rather as an iron core does in the coil. It is as if the elasticity of the 'leashes' (Sec. 3.4) were accompanied by a certain amount of friction, for the extra circuit current resulting from the electron movements is not purely capacitive in phase. The result is the same as if resistance were added to the capacitance.

Such materials are described as poor dielectrics, and obviously would not be chosen for interleaving the capacitor plates. Air is an almost perfect dielectric, but even in an air capacitor a part of the field traverses solid material. especially when it is set to minimum capacitance. Apart from that. wiring and other parts of the circuitincluding the tuning coil-have 'stray' capacitances: and if inefficient dielectrics are used for insulation they will be equivalent to extra resistance.

### 8.16 H.F. Resistance

All these causes of loss in high-frequency circuits come within the general definition of resistance (Sec. 2.18) as (electrical power dissipated) $\div$ (current-squared). In circuit diagrams and calculations it is convenient to bring it all together in a single symbol such as $r$ in Fig. 8.9a, in conjunction with a perfectly pure inductance and capacitance, $L$ and $C$. But we have seen that at resonance the whole tuned circuit can also be represented as a resistance $R=L / C r$ (Fig. 8.9b). We also know that a perfect $L$ and $C$ at resonance have an infinite impedance, so could be shown connected in parallel with $R$ as in Fig. 8.11a. That they are 'invisible' to the generator is


Fig. 8.11-L.C and $R$ all in parallel, and corresponding phasor diagrams
shown in the phasor diagram $(b)$ by the points A and C coinciding, meaning that the current $A C$ is zero. At the same time the capacitor and coil currents, $C D$ and $D A$, are relatively large, but equal and opposite. This current diagram is unusual in being a four-sided figure with two diagonally opposite points coinciding (to put it paradoxically!). If $R$ had been placed between $C$ and $L$ the current diagram would have opened out into a slim rectangle. Fig. 8.11a represents the actual circuit not only at resonance but (except in so far as $R$ varies somewhat with frequency) at frequencies on each side of resonance.

More generally, for every resistance and reactance in series one can calculate the values of another resistance and reactance which, connected in parallel, at the same frequency, are equivalent. And if (as we are assuming) the reactance is much greater than the series resistance, it has very nearly the same value in both cases. The values of series and parallel resistance, $r$ and $R$ respectively, are connected by the same approximate formula $R=X^{2} / r$, which we have hitherto regarded as applying only at the frequency of resonance (Sec. 8.9). This ability to reckon resistance as either in parallel or in series is very useful. It may be, for example, that in an actual circuit there is resistance connected both ways. Then for simplifying
calculations it can be represented by a single resistance, either series or parallel.

An example, worked out with the help of relationships explained in this chapter, may make this clearer. A certain coil has, say, an inductance of 1.2 mH , and at 300 kHz its resistance, reckoned as if it were in series with a perfect inductance, is $25 \Omega$ (Fig. 8.12a). At that frequency the reactance, $X_{L}$, is $2 \pi f L=2 \pi \times 300 \times 1 \cdot 2=$ $2260 \Omega$. Now although $Q$ has been reckoned in Sec. 8.12 as the ratio of reactance to series resistance ( $X / r$ ) of a coil, it has so far been considered as a property of a complete resonant circuit, in which $X$ could be the reactance of either coil or capacitor, since at resonance they are equal. And $r$ is the resistance of the whole tuned circuit. (Likewise in the alternative form for $Q: R / X$.) But this meaning of $Q$ is often extended to cover separate reactive parts of circuits, as a measure of their 'goodness'. The $Q$ of the coil in our example, which we could distinguish as $Q_{L}$, is therefore $2260 / 25=90 \cdot 4$. If tuned by a completely loss-free capacitor, that would also be the $Q$ of the combination.

The same coil can be represented alternatively as a perfect inductance of very nearly 1.2 mH in parallel with a resistance $R_{L}=$ $X_{L}{ }^{2} / r_{L}=2260^{2} / 25=205 \mathrm{k} \Omega$ (Fig. 8.2b). If $r_{L}$ remained the same at


Fig. 8.12-Example to illustrate series and parallel equivalents. $b$ is equivalent to $a$; and $d$ and e are alternative ways of expressing the result of combining $c$ with $b$

200 kHz , the value of $\mathrm{R}_{L}$ would be $(2 \pi \times 200 \times 1 \cdot 2)^{2} / 25=91 \mathrm{k} \Omega$; so when the equivalent parallel value has been found for one frequency it must not be taken as holding good at other frequencies. As a matter of fact, neither $r_{L}$ nor $R_{L}$ is likely to remain constant, but in most practical coils $Q$ is fairly constant over the useful range
of frequency; which is a good reason for using it as a figure of merit. So it is likely that at $200 \mathrm{kHz} R_{L}$ would be somewhere around $135 \mathrm{k} \Omega$, and $r_{L}$ would be about $17 \Omega$.

The capacitance required to tune the coil to 300 kHz , or 0.3 MHz , is (Sec. 8.7, footnote) $C=25333 / /_{\mathrm{r}}^{2} L=25333 /\left(0 \cdot 3^{2} \times 1200\right)=$ 235 pF . This, of course, includes the tuning capacitor itself plus the stray capacitance of the wiring and of the coil. The whole of this is equivalent to a perfect 235 pF in parallel with a resistance of, shall we say, $0.75 \mathrm{M} \Omega$ (Fig. $8.12 c$ ). The reactance, $X_{C}$, is bound to be the same as $X_{L}$ at resonance; but to check it we can work it out from $1 / 2 \pi f C=1 /(2 \pi \times 0.3 \times 0.000235)=10^{6} /(2 \pi \times 0.3 \times 235)=$ $2260 \Omega$. $Q_{C}$ is therefore $R_{C} / X_{C}=750000 / 2260=332$.

If the two components are united as a tuned circuit, the total loss can be expressed as a parallel resistance, $R$, by reckoning $R_{L}$ and $R_{C}$ in parallel: $R_{L} R_{C} /\left(R_{L}+R_{\mathrm{C}}\right)=0.205 \times 0.75 / 0.955=0.161 \mathrm{M} \Omega$ (Fig. 8.12d). Although this is less than $R_{L}$, it indicates a greater loss. The $Q$ of the tuned circuit as a whole is $R / X=161000 / 2260=71$. Incidentally, it may be seen that this is $Q_{L} Q_{c} /\left(Q_{L}+Q_{c}\right)$. The total loss of the circuit can also be expressed as a series resistance, $r=X^{2} / R=2260^{2} / 161000=31 \cdot 8 \Omega$ (Fig. 8.12e). Calculating $Q$ from this, $X / r=2260 / 31 \cdot 8$, gives 71 as before.

Before going on, the reader would do well to continue calculations of this kind with various assumed values, taking note of any general conclusions that arise. For instance, a circuit to tune to the same frequency could be made with less inductance; assuming the same $Q$, would the dynamic resistance be lower or higher? Would an added resistance in series with the lower- $L$ coil have less or more effect on its $Q$ ? And what about a resistance in parallel?

## CHAPTER 9

## Diodes

### 9.1 Electronic Devices

We now have all the essential basic principles of circuits, and it is time to strike out along a new line-electronics. Although all the electric currents we have been discussing in the last six chapters consisted of electrons in motion there has been no real need to consider them as such. It would have been just about as easy to deal with the subject using one of the theories of the electric current in vogue before electrons were discovered. But now we are going to study electronic devices, which could not be satisfactorily explained without reference to electrons. It is these devices that are responsible for the rapid and multifarious developments included under the broad name of electronics.

There are two main divisions of electronic devices: those in which the currents pass through vacuum or gas, and those in which they pass through semiconducting solids. The first includes those called valves in Britain and tubes in America, and the second includes transistors.

Although some simple semiconductor devices were used, without knowing how they worked, for several decades before 1948, it was not until that year, when the transistor was invented as a result of research into the electronic nature of semiconductors, that they began to overtake thermionic valves (as they are named more fully), which are reckoned to have started in 1904.

Electronic devices can be, and often are classified according to the number of their electrodes. An electrode is not very easy to define precisely, but roughly it is any part of the device where a current leaves or enters or by means of which it is electrically controlled. Often each electrode has a terminal or wire by which connection is made to a circuit, but sometimes electrodes are connected together inside the device.

### 9.2 Diodes

The least number of electrodes is two, and devices so constituted are called diodes. Although they are relatively limited in what they can do-their chief capability is allowing currents to flow only one way--they embody most of the basic principles needed for understanding all electronic devices. So as not to be too theoretically minded, however, we may at once note two most important practical
uses of diodes. Working at low (or power) frequencies such as 50 Hz , they enable a.c. to be converted to d.c. such as is required for running nearly all electronic equipment. This function is called rectification. And by rectifying radio-frequency currents, diodes enable radio signals to be made evident, as mentioned in Sec. I.11. This function, although basically rectification, is often distinguished as detection and is the subject of Chapter 18.
Although thermionic diodes have been superseded almost entirely by semiconductor (or solid-state) types, their principles are easier to understand and are a necessary introduction to other valves and to cathode-ray tubes. So we begin with them.

### 9.3 Thermionic Emission of Electrons

So far we have considered an electric current as a stream of electrons along a conductor. The conductor is needed to provide a supply of 'loose" or mobile electrons, ready to be set in motion by an e.m.f. An e.m.f. applied to an insulator causes no appreciable current because the number of mobile electrons in it is negligible.

To control the current it is necessary to move the conductor, as is done in a switch or rheostat (a variable resistor). This is all right occasionally, but quite out of the question when (as in radio) we want to vary currents millions of times a second. The difficulty is the massiveness of the conductor; it is mechanically impracticable to move it at such a rate.

This difficulty can be overcome by releasing electrons, which are inconceivably light, from the relatively heavy metal conductor Although in constant random agitation in the metal, they have not enough energy at ordinary temperature to escape beyond its surface. But if the metal is heated sufficiently they 'boil off' from it. A source of free electrons such as this is called a cathode. To prevent the electronic current from being hindered by the surrounding air, the space in which it is to flow is enclosed in a glass bulb and as much as possible of the air pumped out, giving a vacuum. We are now well on the way to manufacturing a thermionic valve.

Cathodes are of two main types: directly heated and indirectly heated. The first, known as a filament and now seldom used, consists of a fine wire heated by passing a current through it. To minimize the current needed, the wire is usually coated with a special material that emits electrons freely at the lowest possible temperature. The indirectly heated cathode is a very narrow tube, usually of nickel, coated with the emitting material and heated by a separate filament, called the heater, threaded through it. Since the cathode is insulated from the heater, three connections are necessary compared with the two that suffice when the filament serves also as the source of electrons, unless two are joined together internally. In either case only one connection counts as an electrode-the cathode; that it is
heated electrically is only for convenience. So except in Fig. 9.1 the heating arrangement will be omitted.

One advantage of indirectly heated cathodes is that they can be heated by a.c., as explained in Sec. 26.11; another is that the whole cathode is at the same potential.

The electrons released by the hot cathode do not tend to move in any particular direction unless urged by an electric field. Without such a field they accumulate in the space around the cathode. Because this accumulation consists of electrons, it is a negative charge, known as the space charge, which repels new arrivals back again to the cathode, so preventing any further build-up of electrons in the vacuous space.

### 9.4 The Vacuum Diode Valve

To overcome the stoppage caused by the negative space charge it is necessary to apply a positive potential. This is introduced into the valve by means of the second electrode, a metal plate or cylinder called the anode.

When the anode is made positive relative to the cathode, it attracts electrons from the space charge, causing its repulsion to diminish so that more electrons come forward from the cathode. In this way current can flow through the valve and round a circuit such as in Fig. 9.1. But if the battery is reversed, so that the anode is more


Fig. 9.1-(a) Electron flow from cathode to anode in a directly-heated diode. The alternative indirect method of heating is shown at $b$
negative than the cathode, the electrons are repelled towards their source, and no current flows. The valve therefore permits current to flow through it in one direction only, like the air valve attached to a tyre, and this is why it was so named. The anode battery or other supply is often called the h.t. (signifying "high tension') and the filament or heater source the l.t. ('low tension').

If the anode of a diode is slowly made more and more positive with respect to the cathode, as for example by moving the slider of
the potentiometer in Fig. 9.2 upwards, the attraction of the anode for the electrons is gradually intensified and the current increases. To each value of anode voltage $V_{a}$ there corresponds some value of anode current $I_{\mathrm{a}}$, and if each pair of readings is recorded on a graph a curve like that of Fig. 9.3 (called a valve $I_{a} / V_{a}$ characteristic curve) is obtained.

The shape of the curve shows that at low voltages the anode collects few electrons, being unable because of its greater distance


Fig. 9.2-Circuin for finding the relationship hetween the anote vollage ( $V_{\mathrm{a}}$ ) and the anode cimrent ( $I_{3}$ ) in a diode


Fig. 9.3-Characteristic carve of a thermionic diode
from the cathode to overcome much of the repelling effect of the space-charge. The greater the positive anode voltage the greater the negative space charge it is able to neutralize; that is to say. the greater the number of electrons that can be on their way between cathode and anode : in other words, the greater the anode current. By the time the point $C$ is reached the voltage is so high that electrons are reaching the anode practically as last as the cathode can emit them; a further rise in voltage collects only a few more strays, the current remaining almost constant from $D$ onwards. This condition is called saturation. Because the saturation current is limited by the emission of the cathode, and that depends on the temperature of the cathode, it is often called a temperature-limited current. to distinguish it from the space-charge limited current lower down the curve. In modern valves the saturation current is far above the working range.

At B an anode voltage of 50 V drives through the valve a current of 40 mA ; the valve could therefore be replaced by a resistance of $50 / 40=1.25 \mathrm{k} / 2$ without altering the current flowing at this voltage. This value of resistance is therefore the equivalent d.c. resistance of the valve at this point. The curve shows that although the valve is in this sense equivalent to a resistor, it does not conform to Ohm's law (compare Fig. 2.3); its resistance depends on the voltage applied. To drive 10 mA . for example, needs $25 \mathrm{~V}: V / I=25 / 10=$ $2.5 \mathrm{k} \Omega$.

This is because the valve. considered as a conductor, is quite different from the material conductors with which Ohm experimented. Its so-called resistance-really the action of the space charge-is like true resistance in that it restricts the current that flows when a given voltage is applied, but it does not cause the current to be exactly proportional to the voltage as it was in Fig. 2.3 In other words, it is non-linear.

The kind of non-linearity just examined is particularly important in an undesirable way in valves having more than two electrodes, as we shall see later. The most significant non-linearity of diodes is that because the anode enits no electrons there is no possibility of current when the anode voltage is negative compare Fig. 9.3 again with Fig. 2.3. It is this that makes diodes so useful. Fig. 9.3 makes clear that if a source of alternating voltage of, say, 50 V peak were applied instead of the battery in Fig. 9.1 to the anode the positive half-cycles would cause a peak current of 40 mA , but the negative half-cycles would cause none. So the anode current would consist of d.c. pulses.

For this process of rectification the thermionic diode has two disadvantages, both of them wasteful of power: it requires an auxiliary source of current to heat the cathode. and the space charge necessitates quite a number of volts to neutralize it. In high-power applications the latter disadvantage can be mitigated by admitting a controlled amount of gas or vapour into the tube. Directly electrons start moving across to the anode with sufficient velocity-which can be imparted by a fairly low anode voltage - they collide with the gas atoms (or small groups of atoms called molecules) violently enough to knock out some of their electrons. These electrons join the stream and augment the anode current, but the more important result is that the gas molecules with electrons missing are positively charged ions (Sec. 2.2). (The process just described in ionization, which is important in many connections.) These ions are therefore attracted to the cathode. but being far heavier than the electrons they move comparatively slowly. So each one lingers in the space charge long enough to neutralize the negative charge not only of the electron torn from it but also of many of those emitted from the cathode. In fact, the negative space charge is practically eliminated, and with it nearly all the resistance of the diode.

Even this interesting improvement succeeded only in delaying
for a time the replacement of thermionic diodes by semiconductor ones, both for power and signals (rectifiers and detectors). So we go on now to study these.

### 9.5 Semiconductors

In Sec. 2.3 solid materials were grouped into conductors and insulators. The difference was explained by supposing that conductors are substances in which the electrons belonging to their atoms are free to move, under the influence of an electric field, in one direction (towards the positive pole) and so to create an electric current, whereas the electrons of insulators are, as it were, held on elastic


Fig. 9.4-The four valence electrons in a tetravalentatom can be visualized as satellites around the rest of the atom. These parts are electrically charged as shown
leashes so that an electric field (unless extremely strong) is unable to move them far from the home atoms. This is a very much oversimplified view of the matter, but it will do as a first approach. For a more advanced but still not highly mathematical treatment, see the author's The Electron in Electronics.

Not all the electrons in atoms are involved, but only the outermost ones, known as valence electrons. This term first arose in chemistry, because it happens that the chemical behaviour of substances depends mainly on the number and arrangement of these outer electrons. There are never more than seven of them per atom, though the normal total number of electrons per atom, called the atomic number, can be anything up to about 100 .

Substances in which all the atoms have the same atomic number are called elements. The semiconductor materials most important in electronics are the elements silicon and germanium, which are tetravalent (meaning that they have four valence electrons per atom). Their atomic numbers are 14 and 32 respectively, but for our purpose all except the four valence electrons may be lumped together with the remainder of the atom; that is to say, the positively charged nucleus. Around this relatively heavy main body of the atom the valence electrons can be visualized as satellites (Fig. 9.4). The positive charge on any atom is exactly neutralized by the whole normal complement of electrons, so if four electrons are subtracted there must be a positive charge of 4 electron units, as shown.

Now it happens that the full capacity of the outermost or valence orbit is eight electrons. Atoms show a general tendency to fill up vacancies in such orbits in some way or other, and one of these ways is the creation of what are called covalent bonds. If each atom shares each of its four valence electrons with another atom, so that both atoms have two electrons in common, every atom has eight electrons in its valence orbit and is attached to four other atoms. These are arranged symmetrically in three dimensions, so a model would be needed to represent the system properly; but Fig. 9.5 shows the result of squashing a bit of such a model flat so as to get it on a sheet of paper. Because every atom is identical with every other, the structure so formed is made up of atoms in perfectly regular array, like a case of tennis balls packed for export. But we must remember that the atoms are inconceivably small and numerous. Broken-off pieces of such an atomic structure naturally tend to


Fig. 9.5-Diagrammatic representation of the four-fold tinkages of tetravalent atoms such as silicon
have flat faces at certain definite angles to one another, and this description fits what we call crystals.

### 9.6 Holes

A crystal of this kind, if perfect, would have no electrons free to roam around and would therefore be a perfect insulator. And in fact one of the tetravalent elements, carbon, crystallizes in this way (though not very readily!) as diamond, which is a very good but rather expensive insulator. But the atoms of silicon-and even more those of germanium-hold on to their electrons less firmly,
and at ordinary room temperatures there is enough heat energy to 'shake out' a valence electron here and there, making it available for electrical conduction. The atom from which it came is then an electron short.

Now 'an electron short' is, in effect, a positive charge. We have already noted that the inner part of the atom, taken by itself, is positively charged, as shown in Fig. 9.4, because its four valence electrons are being considered separately. Taken as a whole, with all its electrons, the atom is neutral. But when one of the valence electrons moves off, the atom as a whole becomes a positive ion (Sec. 2.2). The electron vacancy thus created is called a hole. The curious and interesting thing is that this hole can wander about and cause conduction, almost like the loose electron. Although the exact way in which this happens is difficult to understand, the following analogy or working model should give sufficient insight for practical purposes into the significance of electronic holes.

Fig. 9.6 shows part of a floor laid with small square tiles. One tile has been removed and laid aside. It represents an electron, and the


Fig. 9.6-Tile analogy of the creation of a 'hole' by the displacement of an electron from an atom
lump it makes on the floor represents its negative charge. The hole it has left behind, being the reverse of a lump, represents the positive charge created by an electronic hole. Now introduce a tile-motive force towards yourself. That draws the displaced tile towards you and represents a tiny electric current away from you. Most of the other tiles (representing electrons fixed in the crystal structure) are unable to move in response to the t.m.f., but the one marked $\mathbf{A}$ can do so because there is an adjacent hole into which it can slide. That leaves tile B free to move into the hole left by $A$, and so on. The net effect is that the hole moves away from you, and as it represents a positive charge it too means an electric current away from you.

An electronic hole can for most purposes be regarded as a mobile electron, except for its being electrically positive instead of negative. When a meter reads 1 mA it could be due to $6.24 \times 10^{15}$ electrons per second moving through it one way, or an equal number of holes the other way-or a mixture of both to the same total. Conduction caused by heat disturbance is therefore twofold: each electron released creates a hole, and when there is an electric field (due, very likely, to an e.m.f.) the resulting current is made up of electrons
moving towards the positive end and holes towards the negative end. Owing to the more complicated way in which holes 'move', their mobility is less than that of electrons. This means that they move slower.

Just as the only things that really moved in Fig. 9.6 were tiles, so the only things that move in conduction by holes are electrons. Teachers are sometimes so anxious to emphasize that holes are not real positive charges that there may be difficulty later on when certain experiments show that holes really do behave like positive charges. Although not strictly scientifically, Fig. 9.6 covers this point too. We can all agree that both kinds of tile current are brought about solely by movements of tiles. That corresponds to the statement that a hole movement one way is really an electron movement the opposite way. But looking at the two 'current carriers' we must also admit that one is a hump and the other a hollow. As these irregularities stand for negative and positive polarities respectively, one kind of current is a negative charge moving one way, and the other kind is a positive charge noving the other way. So although the causes are the same the results are different. That corresponds to the statement that a hole behaves physically as a positive charge.

### 9.7 Intrinsic Conduction

Having grasped the significance of holes. we may perhaps gain further insight into this kind of conduction by considering a vast array of small balls on a flat tray. They are kept in their places by resting in shallow depressions ('holes') in the tray. A difference of potential between the ends of the tray is represented by raising one end slightly, the raised end being 'negative'. Such a tilt would cause no movement of balls ('electrons'). except perhaps a slight lurch to the side of each hole nearest the 'positive' end, representing a displacement or capacitive current (Sec. 3.8). This is like a crystal at a very low temperature when it is an insulator.

The effect of raising the temperature could be shown by making the tray vibrate with increasing vigour. At a certain intensity a ball here and there would be shaken loose, and on a perfectly level tray (no e.m.f.) would wander around at random. In doing so it would sooner or later fall into a hole - not necessarily the one from which it came. There would therefore be some random 'movement' of holes. At any given degree of vibration (temperature) a balance would eventually be reached between the rate of shaking loose and the rate of re-occupation (called in electronics recombination), but the greater the vibration the greater the number of free electrons and holes at any one time. and the greater the drift of balls towards the 'positive' end of the tray when tilted. One result of this movement would he a tendency for more holes to be reoccupied at the
lower end than the upper; in effect, there would be a movement of holes up the tray towards the negative end.

To represent a sustained e.m.f. it would be necessary for the balls that reached the bottom of the tray to be conveyed by the source of the e.m.f. to the top end, where their filling of the holes would be equivalent to a withdrawal of the holes there.

If you have been able to visualize this model sufficiently you should by now have gathered that at very low temperatures a semiconductor crystal is an insulator, but that as the temperature rises it increasingly conducts. In other words its resistance falls, in contrast to that of metals, which rises with temperature. This sort of conduction is called intrinsic, because it is a property of the material itself.

A similar effect is caused by light; a fact that is turned to advantage in the use of semiconductors as light detectors and measurers.

Unless a substance crystallizes under ideal conditions, it tends to be a jumble of small crystals, at the boundaries of which there are numerous breaks in the regularity of the lattice, releasing spare electrons: this is yet another cause of conduction.

Even although conduction due to these causes is contributed to by both electrons and holes, the combined number falls far short of the free electrons in metals, so the result is only 'semiconduction'. In fact, as we shall see later, to be suitable material for most electronic devices the intrinsic conduction at working temperatures should be as nearly as possible negligible. It is because silicon fulfils this condition better than germanium that it has tended to supersede germanium, especially for use at high temperatures.

### 9.8 Effects of Impurities

So far we have assumed that our crystal, however it may be disturbed by heat, light, or its original formation, is at least all of one material. In practice however nothing is ever perfectly pure. Now if the impurities include any element having five valence electrons (such as phosphorus, arsenic or antimony) its atoms will be misfits. When they take their places in the crystal lattice-as they readily do-one electron in each atom will be at a loose end, more or less free to wander and therefore to conduct. Such an incident is represented in Fig. 9.7. Intruding atoms that yield spare electrons are called donors. It might be supposed that because each surplus electron leaves behind it an equal positive charge it thereby creates a hole, but it should be understood that the technical term 'hole' applies only to a mobile positive charge. In this case all four valence electrons needed for the lattice are in fact present, so there is no vacancy into which a strolling electron can drop. Therefore the positive charges due to donor impurities are fixed, and conduction is by electrons only. This is a most important thing to remember.


Fig. 9.7-Showing the release of an electron wherever a pantaralent (domor impuritu) atom finds its way into a tetravelent crystal latice structure


Fig. 9.8-Where a trivalent (acceptor impuritr) atom takes a place in the wetravalent lative a hole is created b! its capturing an elecrron

Because electrons are negative charges, a semiconductor with donor impurity is classified as $n-t y p e$.

An incredibly small amount of impurity is enough to raise the conductivity appreciably; far less than can be detected by the most sensitive chemical analysis. As little as one part in a hundred thousand million ( 1 in $10^{11}$ ) can be significant. This is the ratio of about a thirtieth of a person in the entire population of the world! So the manufacture of semiconductor devices is a tricky business.

Trivalent elements, such as aluminium, gallium and indium, have only three valence electrons per atom; and when any of these substances is present the situation is the opposite of that just described. The vacancy in such an atom's valence orbit is a hole, and because trivalent atoms attract electrons more strongly that the surrounding tetravalent atoms hold on to theirs, the hole is quickly filled (Fig. 9.8). The impurity atom in this case is an acceptor. Note that when the hole has migrated from the acceptor-in other words, an electron from elsewhere has filled up or cancelled the hole in the acceptor atom-that atom constitutes a negative charge, because its main body has only three net positive charges to offset what are now four valence electrons; but such negative charges are fixed and take no direct part in current flow. Because conduction is by holes only, which are positive charges, a semiconductor with acceptor impurity is called p-type.

So far as impurity conduction is concerned, we see that the semiconductor itself plays only a passive part, as a framework for the impurity atoms. In fact, it corresponds to the vacuum in a thermionic valve, and ideally would be non-conducting. Intrinsic conduction is just a nuisance. In the diagrams that follow, therefore, all except the relatively few impurity atoms will be omitted in order to focus attention on them. Fig. 9.9 shows the symbols used for this purpose.


Fig. 9.9-Simplified representation of pieces of impure semiconductor, shoving only the electric charges of broken impurity atoms. In p-type material the mobile charges are predominamly positive (holes), and in n-type material negative (electrons). The uniform distrihution of mobile charges is represented below

The mobile charges-electrons and holes-are represented by simple minus and plus signs respectively; and the fixed chargesdonor and acceptor atoms-are distinguished by encircled signs. Although these atoms are all members of the crystalline lattice, they are such a small minority that when the tetravelent atoms are
omitted, as here, the pattern can no longer be discerned. In each type of material the mobile charges are distributed as uniformly as the fixed, and clearly they must be equa! in number, so the material is electrically neutral thronghout.

One might ask what happens when both donor and acceptor impurities are present at the same time. The answer is that they tend to cancel one another out by electrons filling holes, and when present in equal quantities the semiconductor is much as if it had no impurity at all. But although this compensation as it is called is used for converting n-type into p-type (or vice versa) by adding more than enough opposite impurity to neutralize the impurity already present, the balance would be too fine to enable heavily contaminated material to be made apparently pure by the same method.

We must however take care not to concentrate so exclusively on the important impurity current carriers that we forget all about the background intrinsic conduction, for it is always present in semiconductor devices and in fact sets an upper limit to the temperatures they can be allowed to reach. Do the two sorts of conduction go on independently, so that the total numbers of electrons and holes are the sum of those due to impurity atoms and those due to heat and other disturbances? The answer is no, because there is a law that the product of the densities of electrons and holes (i.e.. number per unit volume of material) is constant. Suppose that in a certain very small volume of pure material there are 14 free electrons and 14 holes. The product, $14^{2}$. is 196 . Now add n-type impurity to the extent of 96 electrons. The total numbers are now 110 and 14 , and the product is $110 \times 14=1540$. which is much too large. What happens is that 12 of the impurity electrons fill 12 of the intrinsic holes, reducing the numbers to 98 and 2. to restore the product to 196. The electrons in this example are called majority carriers and the holes are minority carriers. In p-type material the roles are reversed, of course.

More hole-and-electron pairs are continually being generated by heat, but these are balanced by recombinations, so at a fixed temperature the numbers remain practically constant. The numbers taken in our example were extremely small, to make the calculation clear: in such a small sample they would fluctuate considerably about the averages quoted. But we should realize that in as little as one cubic millimetre (about the size of a pin head) of typical material at ordinary room temperature the number of carriers is of the order of $10^{7}$ for silicon and $10^{10}$ for germanium. Even so, they are very rare compared with the atoms.

### 9.9 P-N Junctions

We can have, then, p-type material, in which conduction is by holes (positive carriers); n-type, in which it is by electrons (negative
carriers); and what is sometimes called i-type (for 'intrinsic'), in which it is by both in equal numbers. None of these by itself is particularly useful; it is when more than one kind are in contact that interesting things begin to happen. Just bringing two pieces together is not good enough, however; the combination has to be in one crystalline piece, divided into two or more regions having different impurities. Various manufacturing methods have been devised for achieving this. The process of introducing a desired proportion of p or n impurity is called doping.

Fig. 9.10 shows with standard symbols a small section of such a combination at its boundary between $p$ and $n$ materials, both


Fig. 9.10-Where $p$ and $n$ regions adjoin in the same crystal, diffusion across the junction and reciprocal cancellation of mobile charges creates a depletion layer. Here the fixed charges set up a potential difference that stops further diffusion
equally doped let us assume. The boundary is called a p-n junction. Compare it with Fig. 9.9 where the two materials are shown separately. Let us suppose first that there is no electric field to influence matters. But at room temperature, which is nearly $300^{\circ} \mathrm{C}$ above absolute zero ( $-273^{\circ} \mathrm{C}$ ), the spare electrons on the n side and holes on the $p$ side are in continuous random motion, like the intrinsic ones illustrated by the tray experiment. This motion tends to make any concentration of them gradually spread out into a uniform distribution-a process known as diffusion. It is by this process that when a bottle of ether is unstoppered in a room the smell pervades the whole room within seconds. Unlike intrinsic mobile charges, which for the moment we are ignoring, and impurity mobile charges within their own separate zones in Fig. 9.9, all of which from the start are distributed uniformly throughout the material because that is how they are generated, the holes have the entirely holeless $n$ region in front of them. So they begin to diffuse across the junction. Directly they do so they encounter electrons diffusing in the opposite direction and combine with them, eliminating both so far as our diagram (restricted to electric charges) is concerned. The same story applies in reverse to the electrons. Because of this recombination process the mobile charges are prevented from diffusing far into the opposite territory. It is like a frontier between two equally matched warring nations. As a result of the casualties there is a thin layer on each side of the junction in which there are hardly any mobile charges; a sort of no man's land. It is called a depletion layer (Fig.
9.10). And it really is thin; typically one thousandth as thick as a sheet of paper.

The fixed charges are still there, of course; positive on the $n$ side and negative on the $p$. The cancelling out of the mobile charges therefore builds up a difference of potential between the two regions, of such a polarity as to repel mobile charges moving towards the junction, until it is strong enough -a few tenths of a volt-to bring further traffic across the junction to a halt. The array of unneutralized negative fixed charged on the $p$ side behaves rather like the space charge in a thermionic diode towards electrons seeking to follow up from the $n$ side, but this time there is a corresponding set-up of opposite polarity on the other side, holding back the holes.

If you connect a sensitive voltmeter to the two sides of a p-n junction to measure this p.d. you will not get a reading. This does not prove that the p.d. does not exist. In the complete circuit there are inevitably other inter-material p.ds that bring the total to zero, provided all is at the same temperature.

Next, consider what happens when an external e.m.f. is applied, as in Fig. 9.11. Electrons in the $n$ region are attracted by its positive pole and tend to move to the right, while holes are attracted to the left. There are no current carriers of the correct polarity to be drawn across the junction. Instead, more fixed charges on each side of it are 'uncovered' by retreating carriers, and the barrier p.d. is


Fig. 9.11-If an external volnage is applied, positive to n. the p.d. in Fig. 9.10 is angmemed and the slepletion laver widened


Fig. 9.12-If sufficient extermal voltage is applied, positive to p . the internal p.d. is overcome and current flows
thereby increased. When this process has gone on enough for the p.d. between the regions to be equal and opposite to the applied e.m.f. another no-current balance is established. In other words, to an e.m.f. of this polarity (called a reverse bias) the junction behaves as a non-conductor. Because it develops a back voltage equal to that applied, it is like a small capacitor.

Suppose however that the e.m.f. is applied the other way round, as in Fig. 9.12, and is increased gradually from zero. The polarity of this forward bias, as it is called, is now such as to repel electrons and holes towards the junction, and the first effect is to neutralize
some of the fixed charges there and reduce the potential barrier. This movement constututes only a small temporary 'capacitive' current. But when the applied e.m.f. is sufficient to exceed the initial barrier p.d. the carriers are able to flow across the junction. Because of recombinations when the electrons and holes meet, the flow of holes to the right begins to die out even before the junction is reached, and has done so completely a short distance beyond it. So current through the semiconductor, except close to the junction. is by holes on the p side and by electrons on the n side, as indicated at the foot of Fig. 9.12. The fact that near the junction there are some carriers of the 'wrong' polarity will be found later to be very important. Their number can be greatly increased if one region is doped more heavily than the other, instead of equally as we have been assuming.

If we remember that there are no positive current carriers in metals, we may wonder where the supply of holes comes from to replenish the $p$ zone as they are drawn to the right and are neutralized by electrons. They are in fact created by withdrawal of electrons at the positive terminal of the semiconductor.

### 9.10 The Semiconductor Diode

Summing up, a p-n junction allows current to flow one way (provided that the e.m.f. applied is not too small) but not the other.

In short, it is a rectifier. The foregoing explanation of why it rectifies may seem complicated in comparison with the action of the vacuum diode (Sec. 9.4), so let us look at them side by side, as in Fig. 9.13. Here diodes of both vacuum and p-n junction types are


Fig. 9.13-Showing how in diodes the presence or absence of current depends on the presence or absence of the right kind of current carriers. fis the general symbol for a diode
connected both ways across a battery. At $a$ we are reminded that current can consist either of negative charges flowing to + or positive charges to -. Or, of course, both at ence. When a vacuum diode is connected as at $b$, current can flow, because the heated cathode is an emitter of electrons, which are negative charges. But when reversed, as at $c$, there is no current. because the anode
does not emit electrons. And neither electrode emits positive charges. In a $p$-n diode connected $p$ to + as at $d$ current flows. because the $n$ region emits electrons. and because the $p$ region emits holes, which are positive charges. When reversed (e) both regions emit the wrong charges, so there is no current. The general symbol for a diode is shown at $f$. It could replace both $h$ and $d$ if there was no need to specify which kind.

The semiconductor diode rectifier has some important advantages over the vacuum type: it needs no cathode heating: it is less fragile: and the emitted electrons do not form a space charge tending to oppose the anode voltage. because their charge is neutralized in the crystal by the fixed positive charges. so less voltage is iost in the diode when current flows and as a rectifier it is more efficient. And beealuse the working region of a semiconductor diode is contained in a microscopically thin layer the whole thing can be made exceedingly small.

On the other hand there are one or two complications. So far we have ignored the intrinsic conduction which, as we lave seen, results from the releasing of electrons and holes in equai numbers throughout the material, so does not depend on the direction of the e.m.f. As a small extra current in the forward direction it is unimportant, but there should be no current in the reverse direction. So to the extent that intrinsic conduction results in a reverse or leakage current it reduces the effectiveness of the diode as a rectifier. One cause of intrinsic or minority current-light - can easily be excluded by an opaque covering. But the amount of heat at ordinary temperatures is sufficient for the current of a germanium diode to be appreciable, and it approximately doubles with every $9^{\circ} \mathrm{C}$ rise in temperature. This is one of the most serious limitations of germanium. In this respect silicon is an attractive alternative because its intrinsic conductivity is about a thousand times less. On the other hand silicon has nearly three times the barrier potential to be overcome, so the forward current does not 'get off' the ground" so easily.

This imperfection of a semiconductor diode cannot be fully represented in an equivalent circuit diagram by a resistor in parallel with a perfect diode, because the current through an ordinary ohmic resistor is proportional to the applied voltage, whereas intrinsic semiconductor current is limited by the number of carriers released at the temperature of the material and cannot be increased by applying more volis-short of a limit to be mentioned later. With no bias, there is an exact balance between intrinsic electrons released on the $p$ side (where they are minority carriers) moving to the $n$ side because of the aftraction of the positive space charge there, and electrons released on the n side with suflicient force to propel them across the barrier. Hole flow is balanced in the same way. So there is no net current. But once enough reverse bias has been applied to raise the barrier too high for any majority carriers to get
over it, the only current flow consists of minority carriers only, and is in the direction of the bias.

The behaviour of this leakage current, in reaching a saturation value unaffected by further increase in voltage but steeply increasing with rise in temperature, may remind us of the temperature-limited vacuum diode (Sec. 9.4) but the mechanism of the effect is quite different.

### 9.11 Diode Characteristics

We can now account for the features of the current/voltage characteristics of a typical germanium diode, Fig. 9.14. To positive voltages below about $\frac{1}{4} \mathrm{~V}$ the response is very small, as they are occupied mainly in dismantling the potential barrier at the junction. Such current as does flow is mostly intrinsic. Above that level of applied


Fig. 9.14-Current/voltage characteristics of a typical germanium diode at two different temperatures
voltage the barrier is down and the forward current increases steeply; soon it is limited chiefly by the resistance of the semiconductor material and external circuit. Compared with such a large impurity current the intrinsic current at ordinary temperatures is small, so the effect of a moderate change in temperature on the total current is also small. To the left the curves show the reverse current (needing a ten-thousand-fold change in scale to make it clear) reaching saturation value at a low reverse voltage and
determined mainly by temperature. A silicon diode would have much less reverse current, and the forward current would not rise steeply until the voltage reached about 0.6 V . Even so, it gets away at at much lower anode voltage than does a thermionic diode (compare Fig. 9.3).

If an attempt were made to extend the curves in the forward direction, the power dissipated in the diode would sooner or later overheat it so much that the intrinsic current would no longer be a minor effect but the major one. And because it can flow equally in both directions the diode would cease to be an effective rectifier. It could hardly even be called a semiconductor, since the material would be more like a metal. The junction would be destroyed and one would say that the diode was burnt out.

If the voltage is increased in the reverse direction, the current remains nearly constant until a point is reached at which the intrinsic carriers are driven so fast that they knock out more electrons from the crystal atoms and start a chain reaction. called avalanche effect, that increases the current until either it is limited by external means such as resistance or the diode is destroyed, rather as a capacitor is destroyed by excessive voltage (Sec. 3.4).

And indeed a semiconductor diode is also a capacitor, as we have already noted. Because its dielectric-the depletion layer-is so extremely thin it has quite a large capacitance per square millimetre. It provides a by-pass for a.c.. which may be a serious disadvantage. High-power rectifiers need large junction areas to pass heavy current without getting too hot, but fortunately the frequencies for which such diodes are needed are usually low -50 Hz , for example and the capacitance is then not too serious. But at the very high frequencies used for television it is difficult to make junctions small enough for the capacitance to be tolerable, even although there is no power problem. This is why signal diodes are usually of the poini-contact type, consisting of a small piece of n-type crystal with

Fig. 9.15-Construction of a point-comact germaniam rectifier

a pointed springy wire impinging on it, enclosed in a glass bulb (Fig. 9.15). During manufacture a pulse of current is passed, which has the effect of giving p-type characteristics to the germanium immediately surrounding the point. Because of the small area of the boundary so formed, the capacitance is less than in the junction type, so these diodes are effective at much higher frequencies. 2 pF is a typical value.

A wide variety of characteristics are obtainable by varying the impurity in the material, and other details. Owing to its relatively
high starting voltage for forward current, silicon is not normally used for signal diodes, but has other applications (Sec. 26.4). One of them is as a voltage-varied capacitor, or varactor. The capacitance of a diode, being dependent on the thickness of the depletion layer, and that thickness varying with the reverse voltage. as shown by comparison of Fig. 9.11 with 9.10 , the capacitance must vary with reverse voltage. Since increasing the voltage increases the thickness of the depletion layer. it reduces the capacitance (Sec. 3.3). This property has some uses. such as tuning receivers (Sec. 21.6).

One quite essential thing we have been taking for granted is the possibility of connecting the p and n regions of a diode to an external circuit. This routine necessity is actually too difficult a subject to discuss in this book, and the science of it is still not fully understood, so manufacturers have to rely mainly on practical experience. The problem is to make a joint between the semiconductor crystal and a metal wire, that does not rectify on its own. The ideal is a connection that has no resistance at all; that being impossible, the aim is a resistance that is low and linear, or ohmic.

### 9.12 Recapitulation

Even an elementary introduction to semiconductors is not easy to grasp at once, and as it is important to have at least the main points clear before going any farther. let us recapitulate. To avoid a lot of 'ifs' and 'buts' let it be assumed that the basic semiconductor material is silicon ('Si') or germanium (' Ge ') and that exceptional conditions are excluded.

At low temperatures, perfectly pure material would be an insulator, because the active or valence electrons would all be occupied in holding the atoms together in regular crystalline array and there would be none to spare as current carriers. At slightly below room temperatures ( Si ) or well below ( Ge ) the heat energy is sufficient to ionize enough atoms (i.e.. release electrons from them) to cause perceptible conduction. Each displaced electron leaves behind a hole, which behaves like a positive charge. So negative and positive current carriers are created in equal numbers throughout the material. At any given temperature this population increases until balanced by the rate at which pairs recombine. Their density (number per unit volume) then remains steady at that temperature. But it increases so fast with temperature that the conductivity doubles with about every $9^{\circ} \mathrm{C}$ rise. The ultimate conductivity can be very high-comparable with that of metals-because all the atoms are involved. But at room temperatures only perhaps 1 in $10^{10}$ happens to be ionized at any moment, so the material behaves like a poor insulator. So much for intrinsic conduction.

When trivalent impurities are present they provide one hole per impurity atom for conduction, so there is a surplus of holes. Some
of them mop up some of the intrinsic electrons, so the net effect is to reduce electrons and increase holes. Electrons are then minority carriers and holes are majority carriers, and the material is p-type. At anything above an extremely low temperature all the impurity atoms are ionized, so even if they are outnumbered perhaps $1: 10^{11}$ by the basic material they are still likely to contribute more current carriers at room temperature. But as their contribution is already at its maximum it is rapidly rendered insignificant at high temperatures by the huge intrinsic reserves. At the same time, of course, the disparity in numbers between electrons and holes practically vanishes too. As semiconductor devices rely on the disparity they must be worked below the temperature at which intrinsic conduction begins to take over.

Pentavalent impurity has the same results except for introducing electrons instead of holes. so material with it is n-type. If both kinds of impurity are present, the carriers created by the smaller quantity of impurity are neutralised by those from the larger quantity, so only the surplus impurity of one kind counts.

All the impurity atoms in p-type material are fixed negative charges, and all those in n-type are fixed positive charges. Unless the mobile current carriers are moved by an electric field they mill about at random, and because they are equal and opposite to the fixed charges any volume of the material as a whole is neutral.

This random motion tends to make the carriers fill any available volume uniformly, so at a p-n junction the electrons on the $n$ side begin to diffuse into the $p$ region, and when they meet the holes doing the same thing in the opposite direction they fill them, so creating a depletion layer, in which there is a shortage of both kinds. So the fixed charges of opposite polarity on each side of the junction establish a potential barrier that stops further diffusion. Applying an external voltage, negative to $p$ and positive to $n$, heightens the barrier and no majority-carrier current can flow. Intrinsic (minority) carriers can flow, because they are of the right polarity to do so, but the resulting current is limited by their number, which is increased by a rise in temperature but not by a rise in voltage-short of breakdown voltage. Applying an external voltage of the opposite (forward) polarity overcomes the barrier with roughly its first 0.2 V $(\mathrm{Ge})$ or $0.6 \mathrm{~V}(\mathrm{Si})$, and all above that causes a current limited mainly by incidental resistance.

A reverse-biased diode has a capacitance that decreases with increase in bias voltage.

## CHAPTER 10

## Triodes

### 10.1 The Vacuum Triode Valve

Valuable though diodes are, radio and electronics would not have got very far on only two electrodes. The important date is 1906, when De Forest tried adding a third electrode as a sort of tap to control the flow of current between cathode and anode. It is called the grid, and usually takes the form of an open spiral of thin wire wound closely around the cathode so that in order to reach the anode the electrons from the cathode have to pass between the turns of wire.

If the potential of the grid is made positive with respect to the cathode it will assist the anode to neutralize the space charge of electrons around the cathode, so increasing the anode current; and, being nearer to the cathode, one grid volt is more effective than one anode volt. If, on the other hand, it is made negative it will assist the space charge in repelling electrons back towards the cathode.

Fig. 10.1 shows the apparatus needed to take characteristic curves of this three-electrode valve, or triode. And in Fig. 10.2 are some such curves, showing the effects of both a node and grid voltages. (These curves are somewhat idealized, being straighter and more parallel than in actual practice). Each of them was taken with the fixed grid voltage indicated alongside. Notice that this voltage, like all others relating to a valve, is reckoned from the cathode as zero. If, therefore, the cathode of a valve is made two volts positive with respect to earth, while the grid is connected to earth, it is correct to describe the grid as 'two volts negative', the words 'with respect to the cathode' being understood. In a valve directly heated by d.c., voltages are reckoned from the negative end of the filament.

### 10.2 Amplification Factor

One would expect that if the grid were made neither positive nor negative the triode would be much the same as if the grid were not there; in other words, as if it were a diode. This guess is confirmed by the curve in Fig. 10.2 marked ' $V_{g}=0$ ', for it might easily be a diode curve. Except for a progressive shift towards the right as the grid is made more negative, the others are almost identical. This means that while a negative grid voltage reduces the anode current in the way described, this reduction can be counterbalanced by a suitable increase in anode voltage. In the valve for which curves are
shown, an anode current of 40 mA can be produced by an anode voltage of 120 if the grid is held at zero potential. This is indicated by the point $A$. If the grid is now made 6 V negative the current drops to 16 mA (point $B$ ), but can be brought up again to its original value by increasing the anode voltage to 180 V (point C).

Looked at another way, the distance A to C represents 60 V on the $V_{\mathrm{a}}$ scale and -6 V on the $V_{\mathrm{g}}$ scale, with no change in $I_{\mathrm{a}}$. So we can say that a change of 6 V at the grid can be compensated for by a change of 60 V . or ten times as much, at the a node. For reasons

Fig. 10.1-Circuit for tuking characteristic curves of a thermionic triode



Fig. 10.2-Characteristic curves of triode valve, at different grid voltages
that will soon appear, this ratio of 10 to 1 is called the amplification factor of the valve. It is yet another thing to be denoted by $\mu$.

### 10.3 Mutual Conductance

Another measure of the control that the grid has over the electron stream in a valve is the change of anode current that results from a given change in grid voltage, the anode voltage meanwhile remaining constant. Comparing points A and B in Fig. 10.2 we note that increasing the grid voltage by 6 V increases the anode current by 24 mA . In Sec. 2.12 the number of amps made to flow, per volt. was the definition of the siemens, the basic unit of conductance. Here now we have a current change brought about in a circuit (the anode circuit) by a voltage somewhere else (between cathode and grid). To distinguish this conductance from the ordinary sort it is called mutual conductance and given the special symbol $g_{\mathrm{m}}$. Although its value in this example would correctly be stated as 4 mS ( 4 millisiemens) one is more likely to come across it in milliamps per volt.

To find the value of $g_{\mathrm{m}}$ from an $I_{\mathrm{a}} / V_{\mathrm{a}} \mathrm{graph}$, as we have just done, it must have curves for at least two different grid voltages. A more suitable form of characteristic curve for this purpose is obtained by plotting $I_{\mathrm{a}}$ against $V_{\mathrm{w}}$, taking care to keep $V_{\mathrm{a}}$ constant, as in Fig. 10.3. A single curve of this kind is sufficient to show $g_{\mathrm{m}}$. For instance BC represents the increase in $I_{\mathrm{a}}$ caused by an increase in $V_{8}$ represented by AB ; so $g_{\mathrm{m}}$ is given by $\mathrm{BC} / \mathrm{AB}$-in this case $4 \mathrm{~mA} / \mathrm{V}$, as before. Although they can be drawn by direct measurement, using the Figs. 10.1 set-up, the curves in Fig. 10.3 were actually derived from those in Fig. 10.2, by noting (for example) the anode currents corresponding to the four grid voltages where they cut the vertical $\mathrm{V}_{\mathrm{a}}=100$ line; and so on for the other three values of $V_{\mathrm{a}}$.

The usefulness of identifying $g_{\mathrm{m}}$ with the slope of such curves is that one can see at once that it is not everywhere the same. Near the foot of each curve the slope, and therefore $g_{\mathrm{m}}$, is less. And if we examine genuine valve curves we find that they are not so straight and parallel as these, so that $g_{\mathrm{m}}$ varies to some extent even along the upper stretches. Thus, although $g_{\mathrm{m}}$ used sometimes to be called a valve constant, in actual fact it is not at all constant, so is better referred to as a parameter of the valve. The other parameter we have encountered, $\mu$, also varies to some extent with $V_{\mathrm{a}}$ and $V_{\mathrm{g}}$, but much less so than $g_{\mathrm{m}}$ does.

If a characteristic curve really is appreciably curved (technically, non-linear) then not only does its slope vary from point to point but even at a given point it depends on the amplitude of the signal. In fact, slope hardly has a meaning unless the signal amplitude is small enough for the part of the curve used by it to be practically straight (linear), so a.c. parameters are often called small-signal parameters.


Fig. 10.3-Four samples of anode-current/grid-voltage characteristic curve, each taken at a different anode voltage

### 10.4 Anode Resistance

By this time you may be wondering if the slope of the curves in Fig. 10.2 has any special significance and name. Clearly it is a ratio of change in $I_{\mathrm{a}}$ caused by a change in $V_{\mathrm{a}}, V_{\mathrm{g}}$ being kept constant. So it too is a conductance, and this time a straightforward one, because both current and voltage refer to the same thing: the cathode-to-anode path through the valve. This slope is therefore called anode conductance, and its symbol is $g_{\mathrm{a}}$. More often, however, it is turned upside down, $1 / g_{a}$, and this reciprocal of $g_{a}$ is, as one would expect, called anode resistance and denoted by $r_{\mathrm{a}}$.

When considering the diode curve, Fig. 9.3, we calculated a kind of resistance in the same way as we did with Ohm's law, as $V_{\mathrm{a}} / I_{\mathrm{a}}$, but there was not very much point in doing so, because, as we saw then, a valve is not a proper resistance and does not conform to Ohm's law. We shall see later that in most uses of valves one is more interested in changes in $I_{a}, V_{g}$, etc., than in their values relative to zero, so that is why in regard to triodes we have been working in terms of changes or differences. It represents what a valve does with an alternating voltage or current superimposed on the d.v. or d.c. corresponding to a selected point on the valve curve. So $r_{\mathrm{a}}$ is more precisely termed anode a.c. resistance. In Fig. 10.2 it would be found as $\mathrm{AC} / \mathrm{BA}=60 / 24=2.5 \mathrm{k} \Omega$ ( $\mathrm{k} \Omega$ instead of $\Omega$ because BA is in mA ). And of course $g_{a}$, being $1 / r_{2}$, is $1 / 1.25=0.4 \mathrm{~mA} / \mathrm{V}$ or mS .

If we have been keeping the figures for this example in mind we will remember that $g_{\mathrm{m}}$ is $4 \mathrm{~mA} / \mathrm{V}$, which is now seen to be 10 times $g_{\mathrm{a}}$. And $\mu$ is 10 . This is not just a coincidence. Because $V_{\mathrm{g}}$ has $\mu$ times the effect on $I_{\mathrm{a}}$ that $V_{\mathrm{a}}$ has, and $\mu$ is the ratio of their effectivenesses in this respect, it must always be true (provided all three parameters refer to the same valve under the same conditions) that

$$
g_{\mathrm{m}}=\mu g_{\mathrm{a}}
$$

In terms of $r_{\mathrm{a}}$, therefore

$$
g_{\mathrm{m}}=\frac{\mu}{r_{\mathrm{a}}}, \quad \text { or } \quad \mu=g_{\mathrm{m}} r_{\mathrm{a}}, \quad \text { or } \quad r_{\mathrm{a}}=\frac{\mu}{g_{\mathrm{m}}}
$$

We see, then, that if any two of these three parameters are known they are all known.

### 10.5 Alternating Voltage at the Grid

In Fig. 10.4 we have an $I_{\mathrm{a}} / V_{\mathrm{g}}$ curve for a typical triode. Suppose that, as suggested in the circuit diagram, we apply a small alternating voltage, $v_{\mathrm{g}}$, to the grid of the valve, what will the anode current do? If the batteries supplying anode and grid give 200 and $-2 \frac{1}{2} \mathrm{~V}$ respectively, the a node current will set itself at about $5 \frac{3}{4} \mathrm{~mA}$; point A on the curve.

If $v_{\mathrm{g}}$ has a peak value of 0.5 V , the total voltage on the grid will swing between -2 and -3 V , alternate half-cycles adding to or subtracting from the steady voltage $E_{g}$. The anode current will swing correspondingly with the changes in grid voltage, the points $B$ and $C$ marking the limits of the swing of both. The current, swing-


Fig. 10.4-The result of applying an alternating e.m.f. ( $v_{k}$ ) to the grid of a triode can be worked out from the $I_{a} / V_{z}$ curve, as here. The fixed starting voltage, $E_{v}$, in this case is -2.5 V
ing between $7 \frac{1}{2}$ and 4 mA , is reduced by $1 \frac{3}{4} \mathrm{~mA}$ on the negative halfcycle and increased by the same amount on the positive one. The whole is therefore equivalent to the original steady current with an alternating current of $1 \frac{3}{3} \mathrm{~mA}$ peak superposed on it. Incidentally this. divided by the peak grid voltage, 0.5 , gives 3.5 mA V. the $g_{m}$.

There are two ways in which this a.c. in the anode circuit can be usefully employed. It can be used for producing an alternating voltage, by passing it through an impedance. If the voltage so obtained is larger than the alternating grid voltage that caused it, we have a voltage amplifier. Alternatively, if the alternating current is strong enough it can be used to operate a loudspeaker or other device, in which case the valve is described as a power amplifier. It is this ability to amplify that has made modern radio and the many kindred developments comprised in electronics possible. The subject is so important that several chapters will be devoted to it, beginning with the next one.

### 10.6 Grid Bias

You may have been wondering why the grid of the valve has never been shown as positive. The reason is that a positive grid attracts electrons to itself. It is, in effect, an anode. The objection to this is not so much that it robs the official anode of electrons-for up to a point it continues to increase the flow there as well as to itselfbut because current flowing in the grid circuit calls for expenditure of power there, thus depriving the valve of one of its most attractive features-its ability to release or control power without demanding any.

To prevent the flow of grid current during use, a fixed negative voltage, known as grid bias, is applied. The amount of bias required is equal to-or preferably a volt or so more than-the positive peak of the signal input voltage which the valve is expected to accept. The reason for making the bias greater than the signal peak is that some electrons are emitted from the cathode with sufficient energy to force a landing on the grid against a negative bias of anything up to about one volt.

### 10.7 The Transistor

Considering how enormously the usefulness of the thermionic valve was increased by adding a third electrode to the original diode, one is hardly surprised that attempts were made to do the same with the semiconductor diode. Owing to the greater difficulty of the basic theory, success was not achieved until 1948. But the result, the transistor, in spite of its late start has largely supplanted thermionic valves.

Fig. 10.5 reproduces parts of Fig. 9.13, a being a thermionic diode connected in the forward direction. Current flows because the


Fig. 10.5-To remind us of Fig. 9.13
cathode emits electrons, which are attracted across the vacuum by the positive anode. This was converted into a triode by interposing an electrode (the grid) to control the current-normally by reducing it. A junction diode, also connected in the forward direction, is shown at $b$. Besides electron flow to the anode there is hole flow to the cathode. So the valve control method would not work here; its effect on the electrons would be counteracted by its opposite effect on the holes.

Coming now to $c$, a reverse-biased diode, we recall that the potential barrier at its junction prevents both electrons and holes from crossing it. However, the junction p.d. that is a barrier to those current carriers has the opposite effect on carriers of the opposite polarities, such as those responsible for intrinsic conduction (Sec. 9.10). In rectifiers these are a ṇuisance, to be kept to a minimum, because they are not under control. What we want is a supply of (say) electrons that can be introduced into the $p$ region in numbers that can be controlled. But this is what we already have in diode $b$. The number of electrons introduced into the p region there depends on the positive bias applied to it. So all we have to do to diode $c$ is to add on an n region below as well as above it and apply a positive bias to p , relative to the new n region, as in Fig. 10.6. The upper p -n junction is the reverse-biased one of Fig. 10.5 c and the lower junction is the forward-biased one of Fig. $10.5 b$. This device, which is called

Fig. 10.6-Applying bias of the right polarity to the middle layer causes increased current between the outers

an n-p-n transistor, does indeed work, but if it is to do so to a useful degree at least two important structural features must be included.

In considering p-n junctions with the aid of Figs 9 10-12, we assumed that the $p$ and $n$ regions were doped with opposite impurities equally. As Fig. 9.12 indicates, half the electrons have combined with oncoming holes before they have even crossed the frontier, and the survivors are nearly all mopped up before they get very far. So the chances of reaching the second junction and crossing it would seem poor. In practice, therefore, the lower $n$ region is doped much more heavily, say a hundred times more; thus the electrons, by so greatly outumbering the p-region holes, nearly all


Fig. 10.7-If the n region of a diode is 'doped' much more heavily than the p region, so that electrons greaty oummber holes, ther are able to penetrate an appreciable distance into thep region, as show here and as is required in the middle layer of a transistor
manage to cross the first junction. And when they have arrived in the $p$ layer the rate of recombination is slower, so more of them get farther. This result is represented in Fig. 10.7.

The other feature must now be obvious: make the player as thin as possible, so that the electrons have hardly got into it before they come under the influence of the relatively high positive potential of the upper n region and are pulled into it. For this reason the p layer is usually made less than one thousandth of an inch $(0.004 \mathrm{~cm})$ thick.

Making the journey to the positive n region so short and easy has the effect of making the alternative path to the source of $p$ bias relatively less easy, so that the proportion of electrons finding that route is small. This is very much to the good because it reduces the power needed for control. There would not be much point in being able to control a current if the power needed to do so was not lessand preferably very much less-than the power controlled.

The purpose of the lower n region is to provide a large source of free electrons for emission into the p layer, which without it is virtually an electron vacuum. So it corresponds to the cathode of a vacuum valve, and is called the eminer (a name that could equally well be applied to the valve cathode). The upper n region is there to collect the electrons so emitted, so is called the collector (a name that would equally fit the anode of a valve). The amount of collector current depends on the amount of bias given to the $p$ layer, which is therefore analogous to the valve grid, and is called the base. Unlike
the other two names, this one seems strangely inappropriate. Its origin was historical rather than functional. And the emitter is usually said to inject rather than emit carriers.

Obviously there is an alternative way of making a transistor. by sandwiching an $n$ layer between two $p$ regions. giving a $p-n-p$ type. To make it work, the base and collector polarities have to be negative, instead of positive as in Fig. 10.6.

Although in some ways a transistor is remarkably like a vacuum triode valve, there are important differences. The valve is a rectifier connected to a supply voltage in the right direction to make current How through it. This current can be either increased or decreased by applying grid bias, according to whether it is positive or negative: but to avoid the flow of grid current it is usual to make it negative, so that in practice the anode current is controlled downwards. The transistor, on the contrary, faces the supply with a rectifier in reverse, so except for a small leakage the normal state is no current. Biasing the base negative would merely add another reversed rectifier to the score, so to obtain any useful result it is necessary for the bias to be positive: and although under these conditions base current is inevitable it enables a much larger collector current to be created and controlled. In electronic jargon a valve is said to be usually operated in the depletion mode and a transistor in the enhancement mode.

As with the junction diode, the transistor does not contain an unneutralized space charge to impede current flow, so only a few volts are needed at the collector-certainly not enough to be dignified by the description 'h.t.' The base bias is small, too; usually only a fraction of a volt. And the emitter provides vast numbers of electrons without any heater. So transistors clearly save much power and some circuit complication compared with valves.

The transistor symbols most often seen, Fig. $10.8 a$ and $b$, very aptly suggest the point-contact type of transistor, which has so long been obsolete that it needs no further mention in this book. The symbols at $c$ and $d$ are quicker to draw and fit better into circuit diagrams, as well as suggesting more clearly the function of the base. In all symbols the emitter is distinguished from the collector by carrying an arrow head, which also distinguishes p-n-p from n-p-n types by pointing in the direction of (positive) current.

(a)

(b)

(c)

(d)

Fig. 10.8 - ( $a$ and b) Usual symbol for a transistor, $n-p-n$ and $p-1-p$ respectively. They are very suitable for the obsolete point-contact type. Junction types ( $n-p-n$ and $p-n-p$ respectively) are more appropriately represemted hy cand d

### 10.8 Transistor Characteristic Curves

To plot transistor curves we can use much the same set-up as for the triode valve (Fig. 10.1) except that there is no need for cathode heating. the supply voltages should be lower and both of the same polarity relative to the emitter, and a low-reading milliammeter is needed for base current; see Fig. 10.9. We must remember that


Fig. 10.9-Arrangement, similar to Fig. IO.I, for ohtaining characteristic curves of a transistor
what in Fig. 9.14 was the 'working' half of the graph-the right-hand one-has no place in connection with the collector-to-emitter circuit of a transistor, which always includes one reverse-biased junction no matter what the polarity of the voltage applied. The only place where the forward curve has any relevance is in the control (base) circuit by itself. First, though, let us deal with the collector circuit.

With the base disconnected, what we have is a reverse-biased p-n junction in series with a forward one. The only current that can flow through the reversed junction is the intrinsic or leakage current, which should be small enough to neglect for most practical purposes. The bias needed to make this small current flow through the forward emitter-to-base junction is also negligible. So the first curve, whether we label it ' $I_{B}=0$ ' or ' $V_{B}=0$ ', will show a small and nearly constant (over a wide range of $V_{c}$ ) collector current. $I_{c}$.

As our circuit provides for measuring both $I_{B}$ and $V_{B}$, we have a choice of two methods. We could follow the same procedure as with valves and plot $I_{C} / V_{c}$ curves at regular intervals of positive $V_{B}$ (instead of negative $V_{8}^{\prime}$ ). If we did we would find that instead of the regular spacing of the valve curves as in Fig. 10.2 the transistor curves would be crowded together at first, with progressive thinning out. This would show that the increases in $I_{C}$ are not in constant ratio with the increases in $V_{B}$ causing them. Although such curves are sometimes drawn, the more usual sets are in terms of base current, $I_{B}$. Fig. 10.10 is an example. For most low-power germanium


Fig. $10.10-I_{c} / V_{c}$ characteristic curves for typical small germanium transistor
transistors such as this the curves are spaced more regularly than comparable valve ones. This can be seen more clearly by plotting $I_{C}$ against $I_{B}$ at a constant $V_{C}$, as in Fig. 10.11.

The base voltage is nevertheless important, so besides the $I_{c} / V_{c}$ curves for stepped values of $I_{B}$ (called collector or output characteristics) we really need what is called the base or input characteristic: $I_{B}$ against $V_{B}$. This (Fig. 10.12) is of course essentially a diode forward characteristic, like the right-hand half of Fig. 9.14, which we already know is very non-linear. The Fig. 10.9 circuit is suitable for measuring it, if allowance is made for the voltage drop in the $I_{B}$ milliammeter, which must be deducted from the reading of the $V_{B}$ voltmeter-necessarily a low-range one to suit the low working bias voltages of transistors. If on the other hand the voltmeter is connected direct to the base, the current it takes must be deducted from the reading of the $I_{B}$ meter.

By combining the information shown by curves such as those in Figs. 10.10 and 10.12, or by direct measurement, we can obtain a curve of $I_{C}$ against $V_{B}$ at a fixed $V_{C}$ (Fig. 10.13). This corresponds to Fig. 10.3 for a valve, but it is scarcely necessary to draw more than one such curve because it is so little affected by changing the collector voltage.


Fig. $10.11-I_{C} / I_{B}$ characteristic for a fixed $V_{C}(3 \mathrm{~V})$ derived from Fig. 10.10, showing how directly propertional $I_{C}$ is to $I_{B}$


Fig. 10.12-Input characteristic $\left(I_{B} / V_{B}\right)$ for the same iransistor, showing marked nontinearity. The input resistance at one particular point is represented by the dotted line, a tangent to the curve at that point


Fig. 10.13-From Figs. 10.11 and 10.12 this transfer characteristic ( $I_{C} / V_{B}$ ) is easily derived

All the curves shown are typical of a low-power germanium transistor. With silicon types at room temperature the collector current at zero $I_{B}$ or $V_{B}$ is far too small to show, and in general the curves are less 'ideal'. The base bias voltage required to cause appreciable collector current is around 0.6 V instead of 0.2 V . With both kinds, if the temperature is raised sufficiently all the $I_{C}$ curves are moved upwards.

Curves for p-n-p transistors are essentially the same except for all voltage and current polarities being reversed. Strictly, their curves should be drawn in the lower left-hand quarter where the scales are negative, but more often they are drawn in the upper right-hand quarter like valve and n-p-n transistor curves, sometimes without even minus signs to the scales to show that they refer to a p-n-p type.

We did not bother to measure emitter current in our Fig. 10.9 experiment, because Kirchhoff's current law (Sec. 2.13) assures us that it is equal to $I_{C}+I_{B}$. As both of these flow into the $\mathrm{n}-\mathrm{p}-\mathrm{n}$ transistor, $I_{E}$ must flow out of it. And as $I_{B}$ is normally very much less than $I_{C}, I_{E}$ is approximately equal to $I_{C}$ but a little larger.

### 10.9 Transistor Parameters

The most obvious differences between the $I_{C} / V_{c}$ curves in Fig. 10.10 and the corresponding ones for a valve (Fig. 10.2) are the very rapid rise of $I_{c}$ followed by a nearly flat top. In a triode valve there is a plentiful supply of electrons emitted by the cathode, and the a mount of a node current depends on how much anode voltage there is to counteract the combined throttling-back effect of the negative space charge and (usually) negative grid voltage. An n-p-n transistor, on the other hand, relies on a positive base voltage to make available a supply of electrons, and a fraction of a volt at the collector is enough to get nearly all of them across the junction. Little can be done by further increasing $V_{C}$.

When considering valve curves we saw that the steeper the slope of a current/voltage characteristic the higher the conductance and the lower the resistance to current changes at that part of the curve. So Fig. 10.10 shows a very low resistance for the first fraction of a volt, and a very high. resistance thereafter. Consider the resistance between emitter and collector terminals with $2 \mathrm{~V} V_{C}$ and $30 \mu \mathrm{~A} I_{B}$, so $4.25 \mathrm{~mA} I_{C}$. To a.c. it can be found by noting that an increase of 6 V in $V_{C}$ increases $I_{C}$ by 0.15 mA . So the a.c. resistance is $6 / 0 \cdot 15=$ $40 \mathrm{k} \Omega$. To d.c., however, the resistance at the $2 \mathrm{~V} / 4.25 \mathrm{~mA}$ point is only $0.47 \mathrm{k} \Omega$, or $470 \Omega$. The a.c. resistance, which could be denoted by $r_{c}$, is in practice seldom used, for reasons that will be explained in Chapter 12.

The mutual conductance of a transistor, as we would expect from Sec. 10.3, is given by the slope of the $I_{C} / V_{B}$ curve, Fig. 10.13. We see that its value depends very much on the part of the curve chosen, so this parameter too is less used than with valves.

Because the voltage amplification factor of a transistor would be equal to the above two parameters multiplied together (Sec. 10.4) it likewise would depend very much on working conditions, in contrast to a valve's $\mu$ which is fairly constant. On the other hand the even spacing of the curves in Fig. 10.10 shows that the ratio of collector current to base current is remarkably constant, and this is brought out clearly by plotting one against the other, as in Fig. 10.11. The slope, being practically the same throughout, is easily calculated as the ratio of any corresponding pair of currents, say $10 / 0.068$, so in this case it is 147 . If we consider how a transistor works we shall realise that this figure means that out of 10.068 mA crossing from emitter to base, 10.000 mA is bagged by the collector and only 0.068 mA is lost by flowing into the base circuit. This current ratio parameter is called the curremt amplification factor. Several symbols have been used for it, such as $\alpha^{\prime}$ and $\beta$. The official one is $h_{F}$, again for reasons that will appear in Chapter 12.

It should be admitted that not all types of transistor show such a
constant $h_{F}$ as our example, out the tendency is there.
If $h_{F}$ is truly constant the $I_{B} / V_{B}$ curve, Fig. 10.12, is just as nonlinear as Fig. 10.13. In some types, however, the $I_{C} / I_{B}$ characteristic is appreciably curved, and in such a way that $I_{C} / V_{B}$ is less curved. In any case the input a.c. resistance, which can be found from the $I_{B} / V_{B}$ curve, is important, even if we have to realize that it varies considerably from point to point. Let us try the one marked in Fig. 10.12. Drawing the dotted line to show the slope there, we calculate the resistance to a.c. by dividing the voltage step ( $0.35-0.27$, $=$ 0.08 ) by the corresponding current step ( $59 \mu \mathrm{~A},=0.059 \mathrm{~mA}$ ) and get $0.08 / 0 \cdot 059$, or $1356 \Omega$. So, in this example at least, it is much less than the collector resistance. This is very different from a valve, which when suitably biased has almost infinite resistance between grid and cathode.

For broadly comparing one triode valve's characteristics with another's it is sufficient to know any two of the three parameters we have studied: $r_{\mathrm{a}}, \mu$ and $g_{\mathrm{m}}$. The third can always be very simply calculated from the other two (Sec. 10.4). For a transistor, because there is base voltage and current and another complication that will emerge in Chapter 12, we must know at least four, selected from a bewilderingly large list of possible parameters. Unfortunately at the present time there is no general agreement as to which four. The choice depends largely on why they are needed, or perhaps on what the manufacturer publishes on his data sheets. In the next chapter we shall begin to look at the significance of triode characteristics under working conditions.

### 10.10 Field-Effect Transistors

The general class of triode transistors considered so far is sometimes called bipolar, because it employs current carriers of both polarities: electrons and holes. There is another class, distinguished as unipolar, working on such a different principle that its right to be called a transistor at all is open to doubt, but it is nevertheless known as the field-effect transistor, usually abbreviated to f.e.t. To some extent it combines the most useful features of valves and bipolar transistors; in particular, the power economy and compactness of the transistor and the almost infinite input resistance of the valve. On the other hand it is much more easily destroyed by misuse than valves.

An f.e.t. consists essentially of a channel of semiconductor material, either por $n$ type, on the side of which is a control electrode of the opposite type, as shown in Fig. 10.14. Unlike the bipolar transistor, the p-n junction so formed is biased in the reverse polarity; hence its high resistance. Again, we have to learn a new set of names for the electrodes. In place of the valve cathode or transistor emitter we have the source. In place of the anode or collector there is the drain. There is however a tendency to use the terms emitter and collector for f.e.ts too. But the third electrode in a f.e.t. is always the
gate. Increasing its reverse bias increases the depletion layer, as explained in Sec. 9.9, and thus narrows the conducting channel between source and drain, reducing the amount of current the applied voltage can pass, just like a water or gas tap. If sufficient bias is applied the channel is closed and the drain current cut off.

With an n-type channel, as in Fig. 10.14, the reverse gate bias is

Fig. 10.14-Showing how a field-effect transistor is constructed in principle

normally negative relative to the source, and the drain voltage is positive. In both these respects, and in starting with maximum current at zero bias voltage, this f.e.t. resembles a valve. But drain-current/drain-voltage ( $I_{D} / V_{D}$ ) curves, shown in Fig. 10.15, are more like transistor curves: for quite a different reason however. Increasing the drain voltage accelerates the electrons and so tends to increase the current, as with ordinary resistance, but at the same time it makes all parts of the channel progressively more positive towards the drain end, thereby increasing the reverse bias across the junction and narrowing the channel. Beyond a certain $V_{D}$, called the pinchoff voltage, the two effects almost balance, so the curve flattens out as shown. The a.c. resistance is therefore fairly high. Note however that the curves level out more gradually than in bipolar types, and in general the working drain voltages tend to be higher than collector voltages. F.e.t. mutual conductances are of the same order as in valves. Typical $I_{D} / V_{G}$ curves are so similar to valve $I_{a} / V_{\mathrm{g}}$ curves such as those in Fig. 10.3 that there is really no need to show any. In fact, f.e.ts can often be substituted for valves with little or no circuit modification.

The f.e.t. illustrated so far is an $n$-channel type, corresponding to an $n-p-n$ bipolar transistor. Just as there is the alternative $p-n-p$ type, so there is the $p$-channel f.e.t., requiring negative $V_{D}$ and positive $V_{G}$ bias. Both of them work in what is called the depletion mode, the effect of applying bias being to reduce the drain current.

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GATE VOLTAGE, VG


Fig. 10.15-Typical $I_{D} / V_{D}$ characteristic curves for junction field-effect transistor. Compare Figs. 10.2 and 10.10

(c)

Fig. 10.16-Symbols for n-channel (a) junction f.e.t., (b) depletion insulated-gate f.e.t., (c) enhancement insulated-gate f.e.t. For p-channel types the arrow head is reversed

### 10.11 Insulated-Gate F.E.Ts

There is another group of f.e.ts which offer not only a choice of polarities ( n -channel or p -channel) but also a choice between the depletion mode and the enhancement mode, in which the effect of applying bias is to increase the drain current from zero or thereabouts. Instead of the resistance between gate and drain being in

Fig. 10.17-Basic construction of n-channel depletion igfet

megohms as in the f.e.ts we have been considering, it is in thousands of megohms, or gigohms ( $\mathrm{G} \Omega$ ), at least.

The reason for this remarkably high control resistance, greater even than that of valves, is that the gate electrode is very highly insulated. So in contrast to the junction-gate f.e.ts (jugfets) already described they are known as insulated-gate f.e.ts (igfets) or alternatively, because of a particular construction favoured, metal-oxidesilicon or metal-oxide-semiconductor f.e.ts (mosfets or mosts). This basic difference is indicated in the symbols for igfets (Fig. 10.16b and c) compared with that for a jugfet ( $a$ ). All these symbols refer to nchannel types, with positive drain voltages; for p -channel the arrow heads are reversed. Both depletion and enhancement n-channel igfets are laid down on a block of p-type material (the substrate), making a fourth terminal, which may be connected straight to the source, or not, as required. Below the source and drain terminals are n-type regions, heavily doped to keep their resistance low. Between them is the gate electrode, insulated from the body of the device by a very thin layer of silicon dioxide.

Except for the insulation, a depletion igfet resembles in principle the jugfet, for there is an n-type channel between drain and source, as shown in Fig. 10.17, so that the device normally conducts. Applying a negative bias to the gate repels electrons present in the channel, reducing its conductivity, and ultimately cutting off drain current entirely. The characteristic curves are therefore similar to those for jugfets (Fig. 10.15) except that the zero- $V_{G}$ curve is usually much lower, but because the gate is insulated it can be used to increase as well as reduce the drain current. Doing this with a jugfet would drastically reduce the input resistance.

In an enhancement igfet the channel is missing, so with no gate bias there is no conduction between source and drain. This feature
is indicated in its symbol (Fig. 10.16c) by the gaps between source and drain. To turn the device on, positive gate bias is needed. In this respect it resembles bipolar transistors, but in all other respects it is similar to the depletion igfet. Because the bias has to make the ptype material that lies under the gate intrinsic before it can begin to convert it into an n-type channel, several volts of bias are needed to bring the device to a suitable working point.

The gate insulation in an igfet being so extremely thin, of the order of 0.0001 mm , it can be permanently broken down by quite a low voltage - 25 , say - and because the capacitance is no more than perhaps 4 pF the quantity of electric charge needed to destroy the device instantly is extremely small. Substituting the above figures in $Q=V C($ Sec. 3.2 $), 25 \times 4 \times 10^{-12}=10^{-10} \mathrm{C}$. This can easily be generated by taking a step or two on a dry floor in rubber soled shoes. So precautions have to be taken, such as wrapping the cevice in foil until installed, and shunting it by protective diodes.

### 10.12 Light-Sensitive Diodes and Triodes

We have already seen (e.g., on the left-hand side of Fig. 9.14) that the reverse (leakage) current of a p -n diode depends very much on the temperature. because heat energy splits up atoms into free electrons and holes, both of which are current carriers. Light has the same effect, and for this reason ordinary semiconductor diodes and transistors have opaque containers. But for many purposes small light-sensitive devices are useful. So what are called photodiodes are provided with little windows through which light can fall on the junction in such a way as to reach as many as possible of the semiconductor atoms. The resulting free electrons and holes are driven in opposite directions by the reverse bias voltage, creating a current through the diode. The brighter the light, the more the current. The very small leakage that flows when there is no light is called the dark current.

Often the changes in light that are to be detected or measured cause insufficient change in current to do what is required, so the current changes have to be amplified. A separate amplifier can be attached of course, but it may be more convenient to use instead of the photodiode a phototransistor, in which the light is applied to the base-to-collector junction (which of course is reverse-biased). The current changes here cause changes about $h_{\mathrm{F}}$ times greater in the emitter circuit (Secs. 10.9,13.3). Changes due to temperature, which are comparatively slow, or due to slow changes in light, can be filtered out by a blocking capacitor (Sec. 14.6). Phototransistor characteristic curves are like Fig. 10.10 but with brightness of illumination in place of base current values.

In both of the above types of device, light controls a current driven by a voltage source. There are some applications (e.g.,
photographic exposure meters) where providing this source is inconvenient. Fortunately there are such things as photovoltaic diodes, in which the light itself provides it. We saw in Sec. 9.9 that the free electrons on the $n$ side of a p-n junction and the holes on the $p$ side diffuse across the junction and set up a difference of potential between the two sides. As mentioned there, a voltmeter connected to the two sides of the junction would give no reading, because if both sides are at the same temperature and receive the same light intensity there is a balance. However, by suitable design of the junction and applying the light to one side there develops between the terminals a net voltage, which can drive a current around an external circuit. The strength of the current is proportional to the illumination. and can be used as a measure ol it.

On a larger scale, photovoltaic diodes are used to generate power from sunlight-particularly useful in space vehicles.

When a semiconductor diode is passing forward current, electrons and holes recombine in large numbers, as described at the end of Sec. 9.9. Consequently energy is released, usually as heat. But there are certain semiconductors other than germanium and silicon (such as gallium arsenide) which emit some of the energy as light. The colour of the light is determined by the materials used to make these light-emituing diodes (l.e.ds), and the intensity of the light is controlled by the current, so they have many uses.

Thermionic diodes are now practically obsolete, but if we recall from Sec. 9.3 that certain substances readily emit electrons when heated, and that we have just noted that light has an effect similar to heat as a stimulator of electron movement, it is hardly surprising that light-sensitive devices analogous to thermionic diodes are possible. In fact, they were used extensively until superseded by the semiconductor devices just described. They are still used as 'targets" in television cameras of the photoemissive type (Sec. 23.6).

There are other substances which are sensitive to light in that their resistance is reduced by it. Among other applications these too are used in television cameras of the photoconductive type.

Photoelectric devices, which are in general those that emit or are stimulated by light, are distinguished in circuit diagrams by a pair of diagonal arrows, representing the light, placed above the basic symbol (Fig. 10.18).


Fig. 10.18-Symbols for (a) photodiade, (b) photovoltuic diode, (c) light-emitting diode, and (d) photoconductive cell

## The Triode at Work

### 11.1 Input and Output

Although in the last chapter attention has been focused on triode valves and transistors themselves, various hints along the line (especially Sec. 10.5) must have conveyed some ideas about how these devices are employed. Now is the time to turn our whole attention to this aspect of the matter.

We have seen that basically triodes are devices by which very small amounts of electrical power-sometimes negligibly small, because requiring voltage without appreciable current-are able to control relatively large amounts, somewhat as the power of a car engine can be controlled by slight variations in pressure on a pedal. Two circuits are thus involved: the controlling or input circuit and the controlled or output circuit. Each of these must be connected to the amplifying device (such as a triode) at not less than two points. As a triode has by definition only three terminals, one of them must be common to both input and output circuits. The choice of common electrode is a very important question which we shall have to look into in the next chapter. So as not to complicate things too much right at the start, however, one particular choice has been made for each device considered: cathode, emitter and source, which in principle are all the same. This is the choice we have been assuming, because it is by far the most usual in practice, so will continue to be assumed unless one of the others is indicated.

### 11.2 Source and Load

Running a car engine in neutral gives it no scope for making use of the power it can develop, and is likely to be harmful if the accelerator is pressed hard down. To fulfil its purpose it needs a load. This term is the one used also in electronics and other branches of electrical engineering, but we must beware because it is also used to mean several things that are not quite the same-as indeed it is in general conversation (e.g., 'he's loaded'). When we say something is overloaded we usually mean that too much power is going into it and it is in danger of burning out. But here the word 'load' will be used primarily to mean whatever receives output power; for example, a loudspeaker.

Sometimes a load is fed with power direct from a generator. In this respect a television receiver as a whole is a very small part of the
load connected to a generator in a power station. Simpler still, the lamp in a torch is the load fed by the battery. This book is more concerned with situations in which there is an important link between the source of power and the load; for example, a transformer, which does not itself amplify but has to be suitably adapted to its load. Or the output part of a triode, which is a link between the source of anode or collector voltage and the load. But more often the triode and any power supplies it needs are regarded as one unit (an amplifier, for example) which is a link between the controller and the load.

This controller, typified by the driver of a car, is usually called the source of the voltage or current needed to control the device,


Fig. II.I-Triodes, like many other devices and complete units for such purposes as amplification, have an input and an output, as shown here in general
and is the essential item in its input circuit, just as the load is in the output circuit.

The substance of these first two sections can be shown quite simply as Fig. 11.1. Later on, when we see the importance of the impedances between the four pairs of terminals, we shall have to elaborate this diagram somewhat.

### 11.3 Feeds and Signals

In the meantime there are a few practical details and terms that need to be made clear before we go on.

We have gathered, no doubt, that any triode (and the same is true of electronic devices with more than three electrodes) can only work properly if it is supplied with certain suitably chosen currents and voltages. What is needed can usually be ascertained with the aid of characteristic curves- together with some understanding of how the device works. which by now we are supposed to have. For instance, our study of triodes would have made clear that trying to work a valve with its anode-to-cathode supply voltage reversed would be futile, but that this might not necessarily be true of the drain-to-source voltage of an f.e.t.

We may have gathered that certain combinations of these feeds
would be harmful to the device. For example, too high a collector voltage would puncture the junction. A lower voltage might be all right within a certain range of base voltage, but above that the collector current multiplied by the collector voltage would be a larger power than the transistor could stand without overheating and finally failing. The manufacturers supply information on these maximum ratings, which put certain areas of the curve sheets out of bounds'.
With all this in mind we find a suitable working point for the purpose in view, and provide power supplies to maintain the device in the conditions represented by that point. It is then in a fit state to receive controlling currents or voltages from the source. These are referred to as input signals, whether they are in fact what most people would understand by that word, or are electrical alternations corresponding to sound waves (Sec. 1.3) or to pictures, or even a current we want to measure and which is too small for any meter we have. By signal currents and voltages, then, current and voltage changes are to be understood.

All this was illustrated in Sec. 10.5 by a simple example, but it is worth refreshing the memory about it because the apparatus we have been using so far (for characteristic curves) has been concerned mainly with feed currents and voltages. While these parts of a circuit are absolutely essential, and arranging for them is a large part of practical circuit design, for discussing signal handling in circuits it is clearer, and saves a lot of time, if we take the feed arrangements for granted and do not trouble to show them. It is quite usual for a designer to deal with the two matters separately, each with a circuit diagram showing only one, before combining them in a diagram of the complete circuit. It is also customary, as in Fig. 10.4, to use capital letters for feeds and small ones for signals.

In most (but not all) applications of electronic devices the signals are alternating at a sufficiently high frequency (which actually can be quite low) to be easily separated from the constant feeds in the circuits themselves as well as in the diagrams. Much use is made of the fact that a capacitor is a complete circuit break (open circuit) to d.c., but allows a.c. to pass. And inductance, on its own, offers no impedance to d.c., so is a short circuit, yet may offer such a high impedance to a.c. as to be almost an open circuit. In Fig. 10.4, however, the input feed source of $R_{\mathrm{g}}$ and the input signal source of $V_{\mathrm{g}}$ were connected simply in series, and no load was shown at all. We must now consider how an amplifier is affected by its load.

### 11.4 Load Lines

At this early stage in the study of electronic circuits we must take care not to get so involved in various incidental complications that the basic principles are not clearly seen. For instance, amplification
of high (radio) frequencies is beset with considerable complications, which will be kept back until Chapter 22. So for a start we shall assume signals of low (audio) frequency. They are not only simpler to deal with but are of practical interest in radio, television, tape recorders, record players and much else.
Triode valves are now almost never used for a.f. amplification, but they are freer from sidetracking issues than transistors, so the basic principles of amplification can be demonstrated more clearly at the start using a triode valve as the active device; once grasped, these principles provide a good foundation for studying other types.

Fig. 11.2, then, shows a common-cathode triode valve in the simplest possible amplifier. It is in fact Fig. 10.4 with the addition of


Fig. 11.2-Simple vollage-amplifier (ircuil
a $20 \mathrm{k} \Omega$ load resistance $(\mathrm{R})$ in the anode (output) circuit. This immediately faces us with a difficulty. It is easy enough to calculate the output signal current (a.c.) when $R$ is not there; it is equal to the milliamps of anode signal current per volt of grid signal (i.e., $g_{\mathrm{m}}$ ) multiplied by the grid signal voltage ( $v_{\mathrm{g}}$ ). In Fig. 10.4 we had $g_{\mathrm{m}}=3 \frac{1}{2}$ and the peak $v_{g}$ was $\frac{1}{2} \mathrm{~V}$, so the peak $I_{\mathrm{a}}$ should be $\frac{1}{2} \times 3 \frac{1}{2}=$ $1 \frac{13}{4} \mathrm{~mA}$, and this agrees with what we found from the $I_{\mathrm{a}} / V_{\mathrm{g}}$ graph. But whenever the anode current varies in Fig. 11.2 the voltage drop across R must vary correspondingly, and as the total voltage provided in this anode circuit is fixed at 240 V , the voltage across the valve ( $V_{\mathrm{a}}$ ) must vary oppositely to that across R . But the definition of $g_{\mathrm{m}}$ says $V_{\mathrm{a}}$ must be constant. So that method of calculation fails. And the alternative method, by Ohm's law, using $r_{a}$ as the resistance of the valve (between anode and cathode) also breaks down, because the definition of $r_{\mathrm{a}}$ specifies constant grid voltage. What are we to do?

The answer is to combine the anode characteristic curves of the valve with a "characteristic curve' of $R$ in such a way as to make the total voltage across both of them equal to that provided. in this example 240 V . Suppose the curves in Fig. 11.3 are the ones relating to our valve. They tell us, of course, all the possible combinations of $I_{\mathrm{a}}$ at any $V_{\mathrm{B}}$ for which a curve is provided. When $V_{\mathrm{g}}=-0.5 \mathrm{~V}$,
for example, the combinations include 4 mA and 100 V , and 10 mA and 160 V , and any number of others. But in our loaded circuit the $V_{\mathrm{a}}$ corresponding to any particular $I_{\mathrm{a}}$ is fixed by the fact that $V_{\mathrm{a}}$ must be 240 V less the amount dropped in R , namely $I_{\mathrm{a}} R$. When $I_{\mathrm{a}}=0$, then $V_{\mathrm{a}}=240-0,=240 \mathrm{~V}$; when $I_{\mathrm{a}}=1 \mathrm{~mA}, I_{\mathrm{a}} R=20 \mathrm{~V}$,


Fig. 11.3-The problem of finding the amplification of the valve in Fig. 11.2 is here worked out on an $I_{\mathrm{a}} / V_{\mathrm{a}}$ curve sheet
so $V_{\mathrm{a}}=220 \mathrm{~V}$; when $I_{\mathrm{a}}=2 \mathrm{~mA}, V_{\mathrm{a}}=200 \mathrm{~V}$; and so on. Plotting all these points on Fig. 11.3 we get the straight line ' $R=20 \mathrm{k} \Omega$ '. Since $R$ is a load resistance, this line is called a load line. It would be equally easy to draw a load line for any other value of $R$.'

The only possible values of $I_{\mathrm{a}}$ are those that are on the appropriate load line. So now for any given load line each value of $V_{\mathrm{g}}$ has only one possible corresponding value of $I_{\mathrm{a}}$. To check this, think of a number for $I_{\mathrm{a}}$, say $3 \frac{1}{2} \mathrm{~mA}$. The only point corresponding to this on the $20 \mathrm{k} \Omega$ load line is marked A. It tells us that $V_{\mathrm{a}}$ is 170 V and $V_{\mathrm{g}}$ is $-2.5 \mathrm{~V} .3 \frac{1}{2} \mathrm{~mA}$ flowing through $20 \mathrm{k} \Omega$ drops $3 \frac{1}{2} \times 20=70 \mathrm{~V}$. Deducting this from the anode battery voltage, 240 , leaves 170 V , which checks.

### 11.5 Voltage Amplification

Suppose now we alter the grid voltage to -1.5 V . The working point must move to B , because that is the only point on both the ' $V_{\mathrm{g}}=$ -1.5 V curve and the load line. The anode current rises from 3.5 mA to 4.6 mA : an increase of 1.1 mA . The voltage drop due to $R$ therefore increases by $1 \cdot 1 \times 20=22 \mathrm{~V}$. So the voltage at the anode $\left(V_{\mathrm{a}}\right)$ falls by that amount.

Note that a grid voltage change equal to 1 V has caused an anode
voltage change of 22 V , so we have achieved a voltage multiplication -called amplification or gain-of 22. If the anode had been connected direct to a 170 V battery there could of course have been no change in $V_{a}$ at all, and the working point would have moved to C. representing an $I_{\mathrm{a}}$ increase of $3 \cdot 3 \mathrm{~mA}$. The reason why the increase with $R$ in circuit was only 1.1 mA was the drop of 22 V in $V_{a}$, which partly offset the rise in $V_{g}$.

A nother thing to note is that making the grid less negative caused the anode to become less positive. So an amplifier of this kind reverses the sign of the signal being amplified.

### 11.6 An Equivalent Generator

It would be very convenient to be able to calculate the voltage amplification when a set of curves was not available and only the valve parameters were known. To understand how this can be done, let us go back to Fig. 10.1 with its controls for varying anode and grid voltages. First, leaving the $V_{\mathrm{g}}$ control untouched, let us work the $V_{a}$ control. The result, indicated on the milliammeter, is a variation in anode current. The same current variation for a given variation in $V_{\text {a }}$ would be obtained if an ordinary resistance equal to $r_{\mathrm{a}}$ were substituted for the valve ( Sec .10 .4 ). Considering only the variations (signals), and ignoring the initial $V_{\mathrm{a}}$ and $I_{\mathrm{a}}$ needed to make the valve work over an approximately linear part of its characteristics (feeds), we can say that from the viewpoint of the anode voltage supply the valve looks like a resistance $r_{\mathrm{a}}$.

Now keep $V_{a}$ steady and vary $V_{g}$. To the surprise of the $V_{a}$ supply (which does not understand valves!), $I_{\mathrm{a}}$ again starts varying. If the $V_{\text {a }}^{\prime}$ supply could think, it would deduce that one (or both) of two things was happening: either the resistance $r_{\mathrm{a}}$ was varying, or the valve contained a source of varying e.m.f. To help it to decide between these, we could vary $V_{\text {a }}$ slightly-enough to check the value of $r_{\mathrm{a}}$ by noting the resulting change in $I_{\mathrm{a}}$-at various settings of the $V_{g}$ control. Provided that we took care to keep within the most linear working condition, the value of $r_{\mathrm{a}}$ measured in this way would be at least approximately the same at all settings of $V_{\mathrm{g}}$. So that leaves only the internal e.m.f. theory in the running. We (who do understand valves) know that varying $V_{\mathrm{g}}$ has $\mu$ times as much effect on $I_{a}$ as varying a voltage directly in the a node circuit (namely, $V_{a}$ ).

So we can now draw a diagram. Fig. 11.4 (which should be compared with Fig. 11.2) to show what the valve looks like from the point of view of the anode circuit. Its behaviour can be accounted for by supposing that it contains a source of e.m.f., $\mu v_{\mathrm{g}}$. in series with a resistance. $r_{\mathrm{a}}$. We have already come across examples of substituting (on paper or in the imagination) something which, within limits, behaves in the same way as the real circuit, but is
easier for calculation. One of them was a dynamic resistance in place of a parallel resonant circuit (Fig. 8.9). And now this trick of the equivalent generator, which is one of the most important and useful of all equivalent circuits.

But 'within limits' must be remembered. In this case, for a valve

Fig. 11.4-The 'equivalent generator' circuit af a valve, which can be substituted for Fig. 11.2 for purposes of calculating the performance of the valve

to behave like an ohmic resistance it must be working at feed voltages corresponding to linear parts of its characteristic curves, and they are never perfectly linear, so at best $r_{\mathrm{a}}$ is no more than a fair approximation to an ohmic resistance. And we must remember Sec. 11.3 and realise that the equipment generator takes account only of signals and ignores feeds, which are merely incidental conditions necessary for achieving reasonable linearity.

It is failure to separate in one's mind these two things. feeds and signals, that leads to confusion about the next idea-the minus sign in front of $\mu \nu_{g}$ in Fig. 11.4. Some people argue that because a positive change in $v_{\mathrm{g}}$ (say from -2.5 V to -1.5 V ) increases $I_{\mathrm{a}}$ the imaginary signal voltage $\mu v_{\mathrm{g}}$ must be positive. And then they are stuck to account for $v_{\mathrm{a}}$, the amplified signal voltage, being negative as we noticed at the end of the last section. But if they realised that the feed voltage and current do not come into Fig. 11.4 they would avoid this dilemma. For it just happens that the feed current flows anticlockwise around the circuit. but as we have replaced the real valve and its feeds by an imaginary signal generator there is no reason for departing from the usual convention of reckoning voltages with reference to the common electrode, here the cathode. If the generator voltage were $+\mu v_{\mathrm{g}}$, a positive signal on the grid would make the anode go positive, contrary to fact. To put this right by turning the generator upside down and reckoning the cathode voltage with respect to the anode is contrary to established convention and common sense. The logical course is to reckon the generator voltage as $-\mu v_{g}$.

A further point about equivalent generators is that such equivalence as they have applies only to the external circuit. In general we would be wrong to draw any conclusion from them about conditions within the valve or other device simulated.

Fig. 11.5 is an alternative version of the diagram, using the notation explained in the latter part of Chapter 5. As Fig. 11.4 is the kind of diagram commonly used, and Fig. 11.5 has a number of advantages over it which grow with increasing circuit complexity, it will be as well to be able to interpret both. One of the advantages


Fig. II.5-(a) Alternative version of Fig. 1/.4, adapted for phasor tiagrams (b)
of Fig. 11.5 is that while we can refer to the outflowing $i_{\mathrm{a}}$ as $A B$ the people who insist on reckoning the anode signal current as inwards can call it $B A$, and they can have their positive generator voltage $b k,=\mu k g$. Just now when we are going to discuss valves in general, $i_{\mathrm{a}}$ etc. will be a little more convenient than $A B$ etc., because one does not have to refer to a particular diagram to know what they mean, and in such a simple case we are not likely to get the signs wrong. And without any reactances there is little justification for a phasor diagram. But in preparation for less simple circuits, and as a further check on our understanding of this basic circuit. Fig. $11.5 b$ is its phasor diagram according to the conventions explained in Chapter 5. The absence of reactances is indicated by all the phasors being parallet to one another-a condition which the usual arrowed type of diagram tends to confuse rather than elucidate. In accordance with the equation $k b=-\mu k g, \mathrm{~kb}$ is drawn opposite to kg and $\mu$ times as long. It represents the generator voltage, which is divided between $r_{\mathrm{a}}$ and $R$. Point b is of course inaccessible in a real valve: $a b$ is drawn dotted to emphasise this.

### 11.7 Calculating Amplification

Now let us apply this equivalent generator technique to the problem we set out to solve-finding the amplification of a valve. The signal

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current $i_{\mathrm{a}}$ in Fig. 11.4 is (by Ohm's law) the signal e.m.f. divided by the total circuit resistance: $-\mu v_{\mathrm{g}} /\left(R+r_{\mathrm{a}}\right)$. The signal output voltage, $v_{\mathrm{a}}$, is caused by this current flowing through $R$, so is $=\mu \mathrm{v}_{\mathrm{a}} R /\left(R+r_{\mathrm{a}}\right)$. The voltage amplification (which is often denoted by $A$ ) is $v_{\mathrm{a}} / v_{\mathrm{g}}$, so

$$
A=\frac{-\mu R}{R+r_{\mathrm{a}}}
$$

This minus sign agrees with our finding that the signal output voltage is opposite in polarity to the input. Since it is generally understood that $v_{\mathrm{a}}$ is in opposite phase to $v_{\mathrm{g}}$, the minus is often omitted. Let us apply this equation to the case we considered with the help of Fig. 11.3. Using the methods already described in Secs 10.2 and 10.4 we find that in the region of point A the valve in question has an $r_{\mathrm{a}}$ of about $14.5 \mathrm{k} \Omega$ and $\mu$ about 39 . Substituting these, and $R=20 \mathrm{k} \Omega$, in the formula: $A=-39 \times 20 /(20+14 \cdot 5)=$ $-22 \cdot 6$, which agrees pretty well with the figure obtained graphically. (It must be remembered that with electronic devices we are in the realm of the approximate. Performance depends so much on con-


Fig. 11.6-The full line shows hor: the voltage amplification depends on the value of loud resistance $R$. It is calculated for a value with $\mu=39$ and $r_{\mathrm{a}}=14.5 \mathrm{k} \Omega$. The dotted line shows the peak milliwatts delivered to $R$ for a grid input of $1 V$ peak
ditions, and even when conditions are the same different samples of the same type of device give different results. So there is no point in specifying them with high precision.)

The amplification formula that we have just used shows at once that making $R=0$ would make $A=0$. It also shows that making $R$
so large that $r_{\mathrm{a}}$ was negligible in comparison with it would make $A$ almost equal to $\mu$, which is the reason for calling this parameter the amplification factor. What happens at intermediate values of $R$ can best be seen by using the formula to plot a graph of $A$ against $R$ the full-line curve in Fig. 11.6, in which, incidentally, logarithmic scales (Sec. 0.2.5) have been used to cover wide ranges of values without squeezing most of them into a corner. According to this it looks as if the higher the more, so far as $R$ and $A$ are concerned. But although feeds do not come directly into the equivalent generator (from which this curve was derived) the assumption that they are adequate to maintain the parameters at the values assumed must never be forgotten. Looking at Fig. 11.3 we see that if $R$ were increased to a much larger value the load line would have to swing round until it was nearly horizontal, cutting the valve curves at points where their decreased slopes would mean much larger values of $r_{\mathrm{a}}$ too. So although very high load resistances are sometimes used, it is more likely to be for some other reason, such as keeping the feed current small, than for extracting the maximum possible voltage gain. The alternative solution, increasing the supply voltage to compensate for the loss in $R$, is usually uneconomic with valves, and exposes transistors to serious risk of breakdown when current is cut off. In practice there is often little or no choice of load resistance, and things may have to be designed to fit it, rather than vice versa.

### 11.8 The Maximum-Power Law

For some purposes (such as working loudspeakers) we are not interested in the voltage output so much as the power output. This, of course, is equal to $i_{\mathrm{a}} v_{\mathrm{a}}$ or $i_{\mathrm{a}}{ }^{2} R(\mathrm{Sec} .2 .17)$, and can therefore be calculated by filling in the value of $i_{\mathrm{a}}$ we found namely $-\mu v_{\mathrm{a}} /\left(R+r_{\mathrm{a}}\right)$. Denoting the power by $P$, we therefore have

$$
P=\left(\frac{\mu v_{g}}{R+r_{\mathrm{s}}}\right)^{2} R .
$$

A graph of this expression, for $v_{g}-1 \mathrm{~V}$, and $\mu-39$ and $r_{\mathrm{a}}-14.5 \mathrm{k} \Omega$ as before, is shown doted in Fig. 11.5. The interesting thing about it is that it has a maximum value when $R$ is about $15 \mathrm{k} \Omega$. The only thing in the circuit that seems to give any clue to this is $r_{a}, 14.5 \mathrm{k} \Omega$. Could the maximum power result when $R$ is made equal to $r_{\mathrm{a}}$ ?

It could, and does; as can be proved mathematically. This fact is not confined to this particular valve, or even to valves in general, but (since, you remember. it was based on Fig. 11.4) it applies to all circuits which consist basically of a generator having internal resistance and working into a load resistance. So the fact that it has been demonstrated for a device not now commonly used does not detract from its importance. Making the load resistance equal to
the generator resistance (which in this case simulates an amplifying device's output resistance) is called load matching.

It does not follow that it is always desirable to make the load resistance equal to the generator resistance. Attempting to do so with a power-station generator would cause so much current to flow that it would be disastrous! But this law, that the maximum power from a generator (whether 'equivalent' or real) giving a fixed voltage, a.c. or d.c., is obtained when the load resistance is equal to the generator resistance, is a very important one. In the form just stated, both generator and load are assumed to be purely resistive; if there is reactance too it should be cancelled out by reactance of the opposite kind (Sec. 8.3).

### 11.9 Transistor Load Lines

We have seen that the output current/voltage curves of transistors, both bipolar and field-effect, are shaped very differently from those of vacuum triodes. It happens that valves of the types more often used, such as pentodes (Sec. 15.7) also have the same sort of shape,


Fig. 11.7 -Varions load lines applied to a set of $I_{C} / V_{C}$ curves for a small silicon transistor
rising very steeply at first and then flattening out. So we ought to see how the foregoing principles apply to curves of this general shape.

Fig. 11.7 is a set of output curves for a typical low-power silicon transistor. Note that this time (unlike Fig. 10.10) they have been drawn for equal intervals of base voltage. As we would expect from

Fig. 10.12, the corresponding steps in collector current are not at all equal. If, mindful of the maximum power law, we were to try drawing a load line having the same slope (but the other way) as the transistor lines we would see that whatever our choice the load line would have to be extended a very long way to the right before it hit the zero-current line, so that the point where it did would almost certainly represent a collector voltage above the rated maximum for the transistor. That being so, if the collector current were cut off for any reason (a large negative input signal, or a lack of bias) the transistor's life would probably come to an abrupt end. In practice one is seldom vitally interested in getting the maximum power output per input volt or milliamp regardless of load resistance. A commoner requirement is maximum power output for any reasonable input, and this will come up for discussion in Chapter 19.

Meanwhile, an interesting thing to notice about devices with curves of this shape is that owing to the very steep initial rise of current the load line can be extended nearly to zero $V_{c}$ regardless of the slope at which it is drawn. For the same reason the maximum output voltage obtainable is practically the same for all load resistances. The three sample load lines in Fig. 11.7, based on 9 V as the collectorsupply voltage ( $V_{c c}$ ) bring these factsout clearly. Comparing this diagram with Fig. 11.3 we see that something like twice as much of the supply voltage can be utilised.

It remains true, as with valve triodes, that the higher the load resistance the higher the voltage gain, again with the practical restriction that if we are too greedy we get into trouble with the closing up of the curves at what is called the bottom hend-more clearly seen as such when the relationship between $I_{C}$ and $V_{B}$ is expressed directly as in Fig. 10.13.

While the voltage output is limited at the lower end of the load line by bottom bend, it is stopped abruptly at the top end by coming up against the steep rise of the curves, an action that is commonly called bottoming. This term may appear rather an odd one for something that happens at the top end of the line. farthest from the bottom bend. The explanation is not an unseemly anatomical obsession but a reference to the output voltage, which is here at the bottom of its range-only a fraction of a volt above zero $V_{c}$. An alternative name for this state is saturation.

More appropriate to bipolar transistors than voltage gain is current gain. By looking at Fig. 10.10 and mentally drawing load lines across it we should be able to see that maximum current amplification is obtained with $R=0$. Just as maximum voltage gain with no current is seldom useful, so is maximum current gain with no voltage. Both conditions give zero power output. However, by turning the vertical line representing zero $R$ through a moderate anticlockwise angle we shall find that a useful voltage output is obtained without losing very much current. Try something like $500 \Omega$.

## Transistor Equivalent Circuits

Don't be perturbed if you find this chapter tougher than any of the others, especially the second half, from Sec. 12.7 onward. It is tougher! The first half should be persevered with, and if you then feel the need for some relief you may be glad to know that a complete grasp of the second half is not essential to understanding the rest of the book. At a first approach it should be enough to study Figs. 12.14, 16, 18 and 19.

### 12.1 The Equivalent Current Generator

When we attempted, in Sec. 10.9, to get to grips with the leading transistor parameters on the lines that worked so well and simply with valves, we were put off by statements that what looked like the obvious equivalents for transistors were actually not very popular, and by promises that the reasons would emerge more clearly in due course. That moment has now arrived.

Two of the three main valve parameters relate output to input and are therefore called transfer parameters: $g_{\mathrm{m}}$ is the ratio of output signal current ( $i_{\mathrm{a}}$ ) to input signal voltage ( $v_{\mathrm{g}}$ ), output signal voltage being zero; $\mu$ is the ratio of the voltage of the imaginary output generator to the input voltage. Knowing them both, then for a given input voltage we know the maximum values of both output voltage and current. As $g_{\mathrm{m}}$ is defined for zero load resistance, the whole of that voltage is engaged in driving that current through the internal resistance of the valve, which can therefore be found as the ratio of voltage to current, or $\mu / g_{m}$, and is denoted by $r_{a}$. (It can also be found directly and independently, from $I_{a} / V_{a}$ characteristic curves.)

Both of the transfer parameters for valves refer to the input voltage. That is natural enough, for any input current is incidental. It is the grid voltage that does the controlling. Because the input resistance of a transistor is comparatively low, both voltage and current are needed, but our study of how a transistor worked showed that it was the base current that decides how much collector current is released. So it is the ratio of these two, the current amplification factor, $h_{f}$, that best expresses the control capability of a transistor. Like $\mu$ for a valve, $h_{\mathrm{f}}$ has the merit that its value depends less on working conditions than do most other parameters. It seems, then, that in bipolar transistors (which are the kind to be assumed unless indicated otherwise) current plays a role corresponding to voltage
in valves. So, as our valve equipment was a voltage generator, should a transistor equivalent be a current generator? Is there such a thing?

The answer to the second question is definitely yes; but to the first it is maybe, for in truth either kind of generator can be used to represent either a transistor or a valve, though it is also true that for transistors the current form is usually chosen. For reasons that appear later (Sec. 15.7) it is often used also for valves. So first let us see how it can be applied to them.

In Sec. 11.7 we used the equivalent voltage generator circuit to calculate the usable output voltage from the valve. The generator voltage, $\mu v_{\mathbf{g}}$, was in series with the valve's own internal resistance and the load resistance, $r_{\mathrm{a}}$ and $R$, as shown again in Fig. 12.1a. So the fraction of it that comes across $R$ is $R /\left(R+r_{\mathrm{a}}\right)$, and therefore

$$
v_{\mathrm{a}}=\mu \mathrm{v}_{\mathrm{s}} \frac{R}{R+r_{\mathrm{a}}}
$$

If we remember that $\mu=g_{\mathrm{m}} r_{\mathrm{a}}$ we can rewrite this equation as

$$
v_{\mathrm{a}}=g_{\mathrm{m}} r_{\mathrm{a}} v_{\mathrm{g}} \frac{R}{R+r_{\mathrm{a}}}=g_{\mathrm{m}} v_{\mathrm{B}} \frac{R r_{\mathrm{a}}}{R+r_{\mathrm{a}}}
$$

At the end of the Sec. 2.9 we saw that the product of two resistances, divided by their sum, was the combined resistance of the two in parallel. So the second form of the equation says that the output


Fig. 12.1-(a) Valve equivakent woltage generator as in Fig. 11.5, except that as equivalent generators in general are now being led up to there is no need to retain the minus sign appropriate to valve equivalents in particular, so the current direction is shown reversed. (b) alternative equivalent current generator
voltage of a valve is equal to $g_{\mathrm{m}} v_{\mathrm{g}}$ multiplied by the combined resistance of valve and load in parallel. Now $g_{\mathrm{m}} V_{\mathrm{g}}$ is a currentthe output signal current from a valve with zero load resistance-so if we connect a generator of that amount of current in parallel with $r_{\mathrm{a}}$ and $R$, as in Fig. 12.1b, this diagram also conforms to the equation
and is interchangeable with $a$ so far as the external circuit is concerned. For example, if $R=0$ then all the current from the generator will take that path of least resistance - and that checks with the way $g_{\mathrm{m}}$ is defined. At the opposite extreme, if $R$ is infinitely large (an open circuit) all the current has to go through $r_{\mathrm{a}}$, setting up a voltage $g_{\mathrm{m}} v_{\mathrm{g}} r_{\mathrm{a}}$ between the terminals. This is the same thing as $\mu v_{\mathrm{g}}$, which is the voltage we would get between the open terminals in Fig. 12.1a. If you are still not convinced, try working out some more results for each equivalent circuit in turn, using any numerical values you like for $r_{\mathrm{a}}, g_{\mathrm{m}}$ and $R$.

The same principles hold good for ideal voltage and current generators in general: a voltage generator in series with a resistance (Fig. $12.2 a$ ) can be replaced by a current generator in parallel


Fig. 12.2-(a) voltage and (b) alternative current generators shown in completely general form
with the same resistance, the amount of current being whatever is needed to set up the same voltage by flowing through that resistance (b). Similarly, if you have a current generator giving a certain voltage across its parallel resistance, with no load connected, that is the voltage of the equivalent voltage generator, which should be in series with the same resistance.

This ideal voltage generator, which we have been using in valve equivalent circuits, is often called a constant voltage generator, because (since it has no resistance of its own) it maintains its voltage regardless of the amount of current drawn from it. If it was 'deadshorted' it would deliver an infinitely large current. Although no real generator can do this (for all have some internal resistance) the idea is easy to accept because close approximations to it do exist, such as large accumulator batteries for d.c. and the main electricity supply for a.c. Real generators can be simulated by an ideal voltage generator in series with the appropriate resistance, as we have seen. Note that the current supplied by a constant voltage generator is whatever is necessary to develop a back voltage equal to its own, across whatever resistance is connected to it.

An ideal or constant current generator is one that delivers a certain current regardless of what is connected to it. The voltage between its terminals is whatever is needed to make that current llow. So if it is open-circuited the voltage becomes infinitely large. This kind of generator is even more difficult to imagine. because nothing very like it exists. The nearest is a very high voltage in series with a very large resistance, so that inserting any reasonable load resistance makes very little difference to the amount of current. But we shall see in Chapter 26 that a constant current almost regardless of resistance (within certain limits) can be provided by special electronic circuits. Any real generator can be simulated by an ideal current generator in parallel with the appropriate resistance. or conductance.

Because an ideal current generator has no conductance of its own it is shown in Fig. $12.2 b$ connected by dotted lines. The symbol

Fig. 12.3-This symbol for a curren generator is deprecated

shown in Fig. 12.3 is often used, but is undesirable for this purpose as it is an international standard symbol for a transformer.

### 12.2 Duality

Although it will interrupt our quest for a transistor equivalent circuit, this may be as good a moment as any to take note of an important general circuit concept-duality. As far back as Sec. 2.12 we saw that an alternative to Ohm's law in the form $E=I R$ is $I=E G$. This is because $G$. the conductance, is defined as $1 / R$, and substituting it in the first form of Ohm's law, after having divided both sides by $R$, we get the second. Comparing the two forms, we see that current and voltage have changed places, and so have resistance and conductance. This is not just a coincidence. We can extend the list :

| Voltage, $V$ or $E$ | $\leftrightarrow$ | Current. $I$ |
| ---: | :---: | :--- |
| Resistance, $R$ | $\leftrightarrow$ | Conductance, $G$ |
| Capacitance. $C$ | $\leftrightarrow$ | Inductance, $L$ |
| Reactance, $X$ | $\leftrightarrow$ | Susceptance, $B$ |
| Impedance, $Z$ | $\leftrightarrow$ | Admittance, $Y$ |

From any equation in which any of these quantities appear, another equation can be derived by changing each of them for the other in the pair. In Sec. 7.2 we found that the 'Ohm's law' for inductance was

$$
I=\frac{E}{2 \pi f L}
$$

If we do the exchanging trick we get

$$
E=\frac{I}{2 \pi f C}
$$

which we went to some trouble to prove in Sec. 6.3.
These are simple examples, but the principle holds good for the most complicated ones. Having established one relationship, we can prove another in this easy way; two formulae for the price of one, as it were. They are called duals of one another.

More interesting still, if an equation applies to a certain circuit its dual equation applies to its dual circuit, which is constructed by exchanging series for parallel and short circuits (zero resistance, infinite conductance) for open circuits (zero conductance, infinite resistance), capacitance for inductance, etc. We do not have to look far for an example of this, for Fig. 12.1 shows two circuits that externally are equivalent to one another: the first comprises a voltage generator in series with two resistances; the second, a current generator in parallel with what could be expressed as two conductances - more conveniently, in fact, for they can be simply added to give the total conductance, just as the resistances in the first circuit are added to give the total resistance.

The equation derived from Fig. 12.1 $a$ for finding $v_{a}$ (the part of the generated voltage received by the load) was

$$
v_{\mathrm{a}}=\frac{\mu v_{\mathrm{g}} R}{R+r_{\mathrm{a}}}
$$

We can find the part of the total generated current in Fig. 12.1b reaching the load by applying duality to the above equation, substituting the generator current $g_{m} v_{g}$ for the generator voltage $\mu \mathrm{r}_{\mathrm{g}}$ :

$$
i_{\mathrm{a}}=\frac{g_{\mathrm{m}} v_{\mathrm{g}} G}{G+g_{\mathrm{a}}}
$$

where of course $G=1 / R$ and $\mathrm{g}_{\mathrm{a}}=1 / r_{\mathrm{a}}$. If you still do not trust duality by itself you can work it out from the equation for $v_{a}$, for $i_{\mathrm{a}}=v_{\mathrm{a}} G$. And if you still prefer resistances to conductances you should easily be able to find that

$$
i_{\mathrm{a}}=\frac{g_{\mathrm{m}} \nu_{\mathrm{g}} r_{\mathrm{a}}}{R+r_{\mathrm{a}}}
$$

Lastly, dual circuits have dual phasor diagrams. We have already noted examples of this, such as Figs. 8.1 and 8.7 . Because the circuits concerned are duals, the phasor diagrams are identical except for interchange of current and voltage.

### 12.3 Voltage or Current Generator?

How do we decide whether to use a voltage or current generator in an equivalent circuit? Remember that although they are exactly equivalent externally they are not equivalent inside. Our domestic electricity supply could be regarded as a voltage generator of 240 V r.m.s. in series with perhaps $0.1 \Omega$. With nothing switched on, there would be no power consumption in the equivalent generator either. The equivalent current generator would have to be 2400 A in parallel with $0 \cdot 1 \Omega$. which would be consuming $2400^{2} \times 0.1=$ 570000 W all the time!

More important for us are the terms in which the two components are expressed. For a valve the generator part can be in terms of $\mu \mathrm{r}_{\mathrm{g}}$ or $g_{\mathrm{m}} \mathrm{g}_{\mathrm{g}}$ - voltage or current. The other part is the same in both, though in one it is normally expressed as $r_{\mathrm{a}}$ and in the other as $g_{\mathrm{a}}$. In a triode valve particularly, $r_{\mathrm{a}}$ is usually moderate, and we could (if we used such a valve at all) make $R$ relatively large. So for a rough approximation we could neglect $r_{a}$ in comparison with $R$ in $R+r_{\mathrm{a}}$ in the equation we found in Sec. 11.7 for a valve's voltage gain:

$$
A=\frac{\mu R}{R+r_{\mathrm{a}}}
$$

It would then reduce to the simple approximation $A \simeq \mu$ ( $\simeq$ means "is approximately equal to'.) See also Fig. 11.6. The parameter thus gives a better idea of the valve's capability than $\boldsymbol{g}_{\mathrm{m}}$. for the current generator circuit shows that when $R$ is much greater than $r_{\mathrm{a}}$ only a small part of the generated current gets into it. When $R>r_{\mathrm{a}}$, then. the voltage generator circuit is usually preferred. We have seen that both bipolar and field-effect transistors have such large a.c. output resistances that $R$ is usually small by comparison. The same is even more true of many valves other than triodes. If then in the equation for $A$ we neglect $R$ in the denominator (lower part) we get

$$
A \simeq \frac{\mu R}{r_{\mathrm{a}}}
$$

and as $\mu / r_{\mathrm{a}}=g_{\mathrm{m}}$ this further simplifies to

$$
A \simeq g_{\mathrm{m}} R
$$

which is a very useful approximation. So where the load resistance is less than the device's output resistance the current generator,
with $g_{\mathrm{m}}$, is usually preferred. This preference is strengthened by the fact that in practical circuits there are a number of resistances and reactances in parallel at the output, and if they are expressed as conductances and susceptances they can simply be added to find the total admittance.

### 12.4 Transistor Equivalent Output Circuit

Our conjecture in Sec. 12.1 that a current generator might be the one to use for representing a transistor is thus justified. The output current with $R=0$ is, as we know, $h_{\mathrm{f}} i_{\mathrm{b}}$, so that is the correct output of the imaginary generator. In parallel with it is the internal resistance of the transistor, preferably expressed as a conductance. We might make its symbol $g_{c}$, but for a reason to appear later $h_{o}$ has been adopted.

So now we have the two components needed for constructing the output part of a transistor equivalent circuit. But a question arises again about the sign of the generator's output. In Sec. 11.6 the reason was given for preferring a minus sign for the voltage, $\mu \mathrm{l}$ 'g. It seems logical that when a current generator is substituted the sign of the current should be the same. However there is a very widely used convention that instead of a minus sign for $h_{\mathrm{f}} i_{\mathrm{b}}$ the direction of the current should be shown reversed, so that the positive direction of current through the load is into the collector, as in Fig. 12.4. The effect of a positive $i_{c}$ is to make the emitter end

of $R$ positive, so for $v_{\mathrm{c}}$ to be positive it would have to point downwards. As the convention is to make it point upwards, for positive inputs $+v_{c}$ will always have negative values. This seems confusing, but fortunately our arrowless notation does not tie one down to either convention but covers both. So Fig. 12.5a shows the same thing as Fig. 12.4, the convention therein being expressed by $D A=h_{\mathrm{f}} i_{\mathrm{b}}$. The other, $A D=-h_{\mathrm{f}} i_{\mathrm{b}}$, means exactly the same thing, but because ec is in any case equal to $R$ multiplied by the current
flowing down through it ( $C A$ ) and up through the generator $(A D)$ a negative value of it comes naturally as a result of the minus sign in front of $h_{\mathrm{i}} i_{\mathrm{b}}$, corresponding to positive values of $i_{\mathrm{b}}$. Fig. $12.5 h$ is the correspording phasor diagram, and by comparing it with Fig. 11.5 h we see that it is essentially the same except that voltage and

Fig. 12.5-Al a, Fig. 12.4 is adapted for phasor diagrams (b)

current have changed places. These are therefore dual phasor diagrams. Again, a dotted line has been used for what is inaccessible in the reat circuit. If you choose to regard the collector current as inwards, it is represented by $C A$ (instead of $A\left({ }^{\prime}\right)$. and ec shows that the voltage $e c$ is negative relative to it.

So far an n-p-n transistor has been assumed, but no polarity changes in equivalent circuit diagrams are needed for $p-n-p$ types. because when they are all reversed they are still relatively the same. That is why it is possible to use one-way arrows for a.c. signals, which are reversing all the time. But in circuits with reactances. where phase relationships are not just simple positive or negative. arrows can be confusing, whereas the notation explained in the latter part of Chapter 5 is always unambiguous.

We next have to consider the input part of the transistor, which is more complicated than that of a valve, because both voltage and current are involved. The ratio of the two is the input resistance. which we found from the slope of the $I_{B} / V_{B}$ characteristic as in Sec. 10.9. It has been given the symbol $h_{\mathrm{i}}$, and its value depends very much or: $V_{B}$. as we can tell from the variableness of the slope in Fig. 10.12. And that is not the end of the matter, for a further complication arises which involves a principle of such importance that we shall have to devote some time to it.

### 12.5 Some Box Tricks

Suppose we are given a "black box’ with two terminals and are told to measure the resistance between them. We do this in a simple

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straightforward way by applying a known voltage and measuring the current going through the box. Suppose we try 6 V and the current meter reads 50 mA . We conclude that the box contains a resistance of $6 / 0.05=120 \Omega$. However, just to make sure we try 12 V , and expecting the current to be 100 mA we are disconcerted to find it is 200 mA . After having just been thinking of a non-linear


Fig. 12.6-Graph showing results of an experiment with a sealed box


Fig. 12.7-Contents of the box revealed


Fig. 12.8 - In this box the internal voltage is directly proportional to the voltage applied
characteristic curve, we may put forward the theory that the box contains a non-linear resistor; a transistor, perhaps. Determined to find out, we plot more points in order to trace the curve, and find it is a perfectly straight line, Fig. 12.6. Odder still, when we apply 4 V we get no current at all, and when we join the terminals by the milliammeter alone we get -100 mA !

The straight line tells us that the box contains a linear resistance. Its slope, which being constant can of course be measured anywhere to give the same result, tells us that the resistance is $40 \Omega$. But obviously there must be a source of e.m.f. too. If we assume it is a voltage source, its voltage is best measured when there is no current to cause a voltage drop in the resistance. The point where the line crosses the zero-current line tells us that it is 4 V . So we deduce that the contents of the box could be as in Fig. 12.7. (Alternatively they could be-in theory at least-a current source of 0.1 A in parallel with $40 \Omega$.) Actually the box could contain a whole lot of batteries
and resistors connected in an elaborate network, in which case Fig. 12.7 would be the simplest possible equivalent circuit.

Next, imagine that instead of the e.m.f. in the box being constant it is directly proportional to the voltage applied between the terminals; say, always half as much, and in the reverse direction, Fig. 12.8. The effect of applying any voltage would now be the same as applying half that voltage to a plain $40 \Omega$ resistor. There would be only half the expected current. so the box would behave exactly like an $80 \Omega$ resistor. There would be no means, without opening the box or X-raying it, of telling whether it contained just an $80 \Omega$ resistor or some other resistance and a source of e.m.f. proportional to that applied. And we can easily see that if the internal source in Fig. 12.8 was reversed, so that it increased the flow of current, it would make the resistance appear less than it really was.

Or imagine again that instead of being in series with a proportional voltage source our resistor was in parallel with a proportional current source, as in Fig. 12.9. For every milliamp we put into the resistor

Fig. 12.9-Box thut inchudes a current source equal to that applied

from outside, the unseen current generator puts another through it in the same direction. So the voltage drop across the resistor, which we measure between the terminals. is twice what it should be with $40 \Omega$. It would be reasonable to deduce, therefore, that the box contained an $80 \Omega$ resistor. If the internal source sent current through the resistor in the reverse direction it would make the resistance seem less than it actually was.

Here we have another example of duality: voltage in opposition simulates higher resistance; current in opposition simulates higher conductance. And conversely for assisting direction.

Let us call the apparent resistance, as inferred from the externally applied voltage $V$ and current $I . R_{\text {app }}$, and the actual resistance in the box. $R_{\text {scl }}$. Let $v$ be the internal voltage aiding $V$ (negative value if it is opposing it). Then

$$
\text { (1) } R_{\mathrm{app}}=\frac{V}{I} \quad \text { and } \quad \text { (2) } \quad R_{\mathrm{act}}=\frac{V}{I}
$$

From (2), $\quad I=\frac{V+v}{R_{\text {act }}}$, and substituting this in (1) we get

$$
R_{\mathrm{app}}=R_{\text {act }} \frac{V}{V+}, \text { or } \frac{R_{\text {act }}}{1+1 / V}
$$

Check this with our example. Then use duality and get

$$
G_{\mathrm{app}}=\frac{G_{\mathrm{act}}}{1+i / \Gamma} \quad \text { or } \quad R_{\mathrm{app}}=R_{\mathrm{act}}(1+i / I)
$$

where $i$ is an unseen current augmenting $I$, in Fig. 12.9 for example.

### 12.6 Input Resistance

The previous section with its imaginary voltage and current sources adjusting themselves as if by magic to fixed proportions of whatever we cared to try at the input terminals, deceiving us into believing that the box contained only an ordinary resistor of a different value, was not just a vain flight of fancy. It demonstrates one of the most important basic principles of practical electronic circuits, known as feedback. We need it now in order to continue our study of transistors.

So far we have taken for granted that we could connect our test circuit, such as Fig. 10.9, direct to the p-n junction. Inevitably however there is some resistance between them and each of the connecting wires. These resistances are represented in Fig. 12.10 by resistors.


The important one just now is $r_{e}$ because it is in the common lead so has both input and output currents flowing through it, and the output current is usually much the larger, so is likely to have a drastic effect on the value of $r_{e}$ as seen by the input signai.

Feeds are again omitted and $v_{b}$ is a small d.v. signal applied between emitter and base (input). It causes an input current $i_{b}$, and as we are assuming small signals we take $v_{\mathrm{b}} / i_{\mathrm{b}}$ as the input resistance, for which we have noted the symbol is $h_{i}$. The main result of this input signal is to cause an output current $h_{\mathrm{f}}$ times as large, if there is zero load resistance, as is normally so when making measurements on transistors or valves. The positive direction of this current, $i_{\mathrm{c}}$, is as shown; check with Fig. 12.4. We now have exactly the sort of situation imagined in Fig. 12.9. For every microamp of $i_{\mathrm{b}}$ flowing through $r_{\mathrm{e}}$ via $r_{\mathrm{b}}$ there are $h_{\mathrm{f}}$ extra microamps flowing through it via $r_{\mathrm{c}}$. So to the input circuit $r_{\mathrm{e}}$ behaves as if it were $r_{\mathrm{e}}\left(1+i_{\mathrm{c}} / i_{\mathrm{b}}\right),=$ $r_{\mathrm{e}}\left(1+h_{\mathrm{f}}\right)$; and $h_{\mathrm{i}}=r_{\mathrm{b}}+\left(1+h_{\mathrm{f}}\right) r_{\mathrm{e}} . r_{\mathrm{c}}$ includes a reversed junction,
so its resistance is high enough for its shunting effect to be neglected.
Typically for a small transistor, $r_{r}$, might be $1000 \Omega . r_{\mathrm{e}} 10 \Omega$ and $h_{\mathrm{f}} 100$. So $h_{\mathrm{i}}$ would in this case be practically $2000 \Omega$.

For the purpose of deriving characteristic curves under the simple conditions provided by the apparatus used (Fig. 10.8) this internal complication can be ignored. It makes no difference whether we attribute the measured results to the signal voltage applied or to the somewhat smaller voltage actually reaching the base-emitter junction plus the fed-back voltage. But under working conditions there is normally a non-zero load resistance, so $i_{c}$ is less than $h_{\mathrm{f}} i_{\mathrm{b}}$, the difference (in a current-generator equivalent circuit) being diverted into the $h_{0}$ by-pass. So the feedback is that much less, and the apparent extra resistance due to it is less, and the input resistance is less than $h_{\mathrm{i}}$ (which assumes $R=0$ ). In short, the input resistance of a transistor depends to some extent on the load resistance.

Without the feedback effect the input resistance would be $r_{\mathrm{b}}+r_{\mathrm{e}}$. The extra resistance introduced by full feedback is $h_{r} r_{e}$. This could be represented by a voltage generator opposing the input signal. However, because $R$. if not exactly zero, is normally much less than $1 / h_{0}$, it is much more convenient to start from $R=0$ as the reference condition, allowing in $h_{\mathrm{i}}$ for full feedback, and to correct for the reduction in feedback due to non-zero $R$ by imagining a voltage generator assisting the signal and thereby reducing the apparent input resistance, as in Fig. 12.11.

The problem now is to find what this voltage is, in terms of $R$ or something related to it. It must be equal to $r_{e}$ multiplied by the reduction in current through it due to $R$ not being zero. That reduction is equal to the current by-passed through $h_{0}$, which must be equal

Fig. 12.11-Consideration of Fig. 12.10 shows that the input circuit of a transistor can be represented by a fixed resistance ( $h_{\mathrm{i}}$ ) in series with a voltage generator to simulate the feedback effect

to $v_{\mathrm{c}} h_{\mathrm{o}}$. So the output of the voltage generator is $v_{\mathrm{c}} h_{\mathrm{o}} r_{\mathrm{e}}$, in the direction shown. Now $h_{0} r_{e}$ is really a ratio between two resistances, $r_{\mathrm{e}} / r_{\mathrm{c}}$, and typically is a few ten-thousandths. It has been allocated the symbol $h_{\mathrm{r}}$ and named the reverse voltage feedback ratio.

As shown in Fig. 12.11 it is not reverse, since it seems to be assisting the input signal to drive $i_{\mathrm{b}}$. That is because we have departed from our practice of reckoning all voltages with respect to the common terminal, the emitter in this case. If we reverse the arrow the voltage of the generator must be negative with respect to

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the input, just as with the output voltage generator in a valve equivalent. The conventions shown in Fig. 12.4 bring this right automatically, because $v_{c}$ is always negative for positive input, so $h_{\mathrm{r}} v_{\mathrm{c}}$ must be negative, since $h_{\mathrm{r}}$ like all the other three $h$ parameters is taken as positive. Again, either direction we care to give the output


Fig. 12.12-The complete h-paramerer transistor equivalent circuit is shown at a in conventional form and at h adapted for phasor diagrams (c)
current is catered for in the arrowless notation. So in Fig. 12.12 both systems are shown, as they were for valves (Figs. 11.4, 11.5).

As $D A=h_{\mathrm{f}} A B$, in the phasor diagram AB is in the opposite direction to AD (compare kg and kb in Fig. 11.5b), and, as $e a=$ $h_{\mathrm{r}} e c$, ea is in the same direction as ec; and ab, representing the voltage applied to $h_{\mathrm{i}}$ to cause $A B$, is in phase with it.

### 12.7 Complete Transistor Equivalent Circuit

We now have a complete equivalent circuit; one of many, but worthy of a special place because its four $h$ parameters are measurable practically. By the way, the $h$ stands for hybrid, because they are a mixed bag: $h_{i}$ is a resistance, $h_{0}$ a conductance, $h_{\mathfrak{f}}$ a current ratio and $h_{\mathrm{r}}$ a voltage ratio.

We have seen that when measuring $h_{i}$ the load resistance must be kept at zero, or as near it as makes little difference. This obviously applies also to measuring $h_{f}$ so that all the current passes through the external measuring instrument. When measuring $h_{\mathrm{r}}$, by applying a signal voltage to the output terminals and observing the resulting voltage (or more likely microvoltage) at the input terminals, no signal current must be allowed to flow in that circuit, or some of the voltage would be lost in $r_{\mathrm{b}}$. Lastly, $h_{\mathrm{o}}$ can also be measured by applying a voltage to the output terminals and noting the current that goes in. We would again have to make sure that no feedback signal current flows in the input circuit, because an amplified current due to it would pass through $h_{\mathrm{o}}$ and confuse the measurement. (For the same reason, when plotting $I_{C} / V_{C}$ curves for fixed values of $I_{B}$ it may be necessary to adjust $V_{B}$ slightly to keep $I_{B}$ constant.)

Just as the input resistance is exactly equal to $h_{\mathrm{i}}$ only when there is zero output voltage and therefore nothing from the internal voltage generator, so the output conductance is exactly equal to $h_{0}$ only when there is zero input current and therefore nothing from the internal current generator. (Note duality again.) And just as the effective input resistance is in general not $h_{\mathrm{i}}$ but depends on the load resistance (for that is what gives rise to the $v_{c}$ or ea factor in the reverse feedback voltage), so the effective output conductance is in general not $h_{0}$ but depends on the signal source conductance (for that is what gives rise to the $i_{\mathrm{b}}$ or $A D$ factor in the forward transfer current).

Suppose we try to measure $h_{0}$ and the condition of zero input source conductance is not fulfilled. Then the measuring current applied to the output, say by making ec positive so that it flows into c, develops a positive $h_{\mathrm{r}} e c$ which drives a positive current $B A$ round the input circuit. This is a negative $A B$, so $A D$ is negative twice and therefore positive. Some of this current goes through $h_{0}$, in the same direction as that supplied. So (see end of Sec. 12.5) it makes the output conductance appear less than $h_{0}$. So the greater the load resistance the less the input resistance, and the greater the input source conductance the less the output conductance.

### 12.8 A Simpler Equivalent

It is because bipolar transistor input and output circuits react on one another in this way that calculating their circuit performance is more complicated than for valves and f.e.ts. But if the load resistance, which we should now distinguish as $R_{L}$, is considerably less than $1 / h_{0}$, as it very often is, and the input signai source or


Fig. 12.13-Simplified transistor equivalent circuit, good enough in many cases. The three diagrams correspond with those in Fig. 12.12
generator resistance ( $R_{G}$ ) is considerably greater than $h_{\mathrm{i}}$, as it often is or can be made, then the effect of $h_{\mathrm{r}}$ is small and its voltage generator can be omitted, as in Fig. 12.13, and calculations are quite easy. Even if this simplification gave rise to errors of the order of $30 \%$ these could be acceptable if we kept in mind that the para-

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meters of individual transistors of the same type are likely to differ even more widely. We shall see later (Sec. 13.1, for example) that one of the objects of good circuit design is to minimize the influence of transistor parameters on the results.

To get the feel of the thing let us try a few figures. Suppose $h_{\mathrm{i}}$ is $3 \mathrm{k} \Omega, h_{\mathrm{r}}$ is 80 and $R_{L}$ is $4 \mathrm{k} \Omega$. The approximate current gain, $A_{\mathrm{i}}$, is simply $h_{\mathrm{f}}$, so is 80 . The voltage gain is

$$
A_{\mathrm{v}}=\frac{v_{\mathrm{c}}}{v_{\mathrm{b}}} \simeq-\frac{i_{\mathrm{c}} R_{\mathrm{L}}}{i_{\mathrm{b}} h_{\mathrm{i}}}=-\frac{h_{\mathrm{r}} R_{L}}{h_{\mathrm{i}}}=-\frac{80 \times 4}{3}=-107
$$

The input resistance, $r_{\mathrm{in}} \simeq h_{\mathrm{i}}$, and the output conductance, $g_{\text {out }}$, would on our assumption be zero, but if $h_{0}$ is known it might as well be used as an approximate $g_{\text {out }}$. And we get better approximations for $A_{i}$ and $A_{\mathrm{v}}$, without adding much to the work, by taking account of it. Suppose it is $25 \mu \mathrm{~S}\left(=25 \times 10^{-6} \mathrm{~S}\right)$. The load conductance, $G_{L}=1 / R_{L}$, is $1 / 4000$, or $250 \mu \mathrm{~S}$. So the total conductance into which the output current goes is $275 \mu \mathrm{~S}$, and the load's share being 250 our figure for $A_{i}$ is reduced to $80 \times 250 / 275$, or $73 . A_{v}$ is reduced in proportion, to 97.

Compare these approximations with the results using the full equivalent circuit. At this stage the derivation of the full formulae would be rather too much, so here they are just stated:

$$
\begin{array}{ll}
A_{\mathrm{i}}=\frac{h_{\mathrm{r}}}{h_{\mathrm{o}} R_{L}+1} & A_{\mathrm{v}}=\frac{-h_{\mathrm{r}}}{h_{\mathrm{i}}\left(h_{\mathrm{o}}+G_{L}\right)-h_{\mathrm{r}} h_{\mathrm{r}}} \\
r_{\text {in }}=h_{\mathrm{i}}-\frac{h_{\mathrm{r}} h_{\mathrm{f}}}{h_{\mathrm{o}}+G_{L}} & g_{\text {out }}=h_{\mathrm{o}}-\frac{h_{h_{\mathrm{r}}} h_{\mathrm{r}}}{h_{\mathrm{i}}+\overline{R_{G}}}
\end{array}
$$

Suppose $h_{\mathrm{r}}$ is $5 \times 10^{4}$ and $R_{\mathrm{G}}$ is $5 \mathrm{k} \Omega$. The results of filling in the values in the above, with the previous approximate results for comparison, are

|  | Withour $h_{\mathrm{r}}$ or $h_{\mathrm{o}}$ | Without $h_{\mathrm{r}}$ | Fult working |
| :---: | :---: | :---: | :---: |
| $A_{\mathrm{i}}$ | 80 | 73 | 73 |
| $A_{\mathrm{v}}$ | -107 | -97 | -102 |
| $r_{\mathrm{in}}$ | $3 \mathrm{k} \Omega$ | $3 \mathrm{k} \Omega$ | $2.855 \mathrm{k} \Omega$ |
| $g_{\text {out }}$ | 0 | $25 \mu \mathrm{~S}$ | $20 \mu \mathrm{~S}$ |

So the simplest diagram gives very good approximation (for this kind of work) to $A_{\mathrm{v}}$, and with $h_{\mathrm{o}}$ gives correct $A_{\mathrm{i}} . r_{\mathrm{in}}$ is not much less than $h_{i}$; but $g_{\text {out }}$ is rather more affected by the finite value of $R_{G}$, though still not enough to worry about. Even without allowing for $h_{\text {o }}$ the answers, obtainable in a few seconds. are good enough for a preliminary estimate.

Sometimes the values of $r_{\mathrm{b}}$ and $r_{\mathrm{e}}$ (Fig. 12.10) are given instead of $h_{\mathrm{i}}$ and $h_{\mathrm{r}}$. The working $r_{\text {in }}$ is then $r_{\mathrm{b}}+\left(h_{\mathrm{f}}+1\right) r_{\mathrm{e}}$ as explained in Sec. I2.6, and as I is usually negligible compared with $h_{\mathrm{f}}$ the formula can be simplified to $r_{\mathrm{b}}+\bar{h}_{\mathrm{f}} r_{\mathrm{e}}$. Given $h_{\mathrm{f}}\left(\right.$ and perhaps $\left.h_{\mathrm{o}}\right)$ the current and voltage gains are calculable as before.

### 12.9 Other Circuit Configurations: Common Collector

So far we have considered only the circuit configuration or mode in which the emitter (or cathode, or source) is the electrode common to both input and output. As a triode has two other electrodes there are two other possibilities-unless we make three more by interchanging output and input, but as these do not amplify they are not usually counted.

Fig. 12.14 shows the three useful ones as applied to a bipolar $n-p-n$ transistor. With the feed batteries reversed the same diagrams would apply to a p-n-p transistor. And there are corresponding


Fig. 12.14-The three useful transistor configurations, or modes of use: (a) commoncmitfer, (h) common-collector. whd (c) common-hase
varieties for f.e.ts and valves. Although in $b$ the junction between input and output circuits is not actually at the collector (since the collector feed source comes in between) so far as signals are concerned an ideal source is non-existent and is omitted from our theoretical diagrams.

One of the advantages of the $/ 2$ parameters, already noted, is that they are suited to practical measurement. A nother advantage is that, being related to input and output terminals and not to parts of the transistor itself, they are not aflected in principle by the way the transistor is connected between those terminals. For the same reason, neither are the formulae for $A_{\mathrm{i}}$, etc., given in the previous section. But because the $h$ parameters are measured between different points of the transistor in the three different modes their values are different and so therefore are $A_{\mathrm{i}}$ and the other derived properties. To prevent confusion all the $h$ parameters are given an additional subscript letter to denote the common electrode. So what we have been calling $h_{\mathrm{f}}$ is more explicitly $h_{\mathrm{fe}}$, and similarly for the others.

The common-emitter mode (Fig. 12.14a) having been assumed in all we have done on transistors so far. it should already be fairly
familiar. It is also the most used. So perhaps the best line to take with the other two is to find how the $h$ values. and consequently the circuit properties, compare.

Beginning with the common-collector mode we redraw our common-emitter equivalent circuit (Fig. 12.12) to fit the commoncollector input-output terminal scheme, Fig. 12.15a. Our object is

(a)

(b)

Fig. 12.15-(a) Common-emitter transistor equivalemt circuit (Fig. 12.12a) redrawn to fit the common-sollector terminal arrangement. The standard h-parameter circuit (b) can he made equivalent to this by changing the values of the parameters where necessary
to find the values of $h_{\mathrm{c}}$ parameters that make the standard $h$ form of circuit ( $b$ ) equivalent to this.

The simplest way of regarding the change from common-emitter to common-collector is as merely a transfer of the load from collector to emitter. So whenever the output terminals are shortcircuited (as they are for measuring $h_{\mathrm{r}}$ and $h_{\mathrm{i}}$ ) the two circuits are exactly the same. As ${ }^{5}$ e is then zero, $h_{\mathrm{re}}{ }^{\prime}{ }^{\prime}$ is likewise, and to the input Fig. 12.15 consists only of $h_{\mathrm{ie}}$. So

$$
h_{\text {ic }}=h_{\text {ie }}
$$

It would be falling into a trap to assume without further thought that $h_{\mathrm{fc}}=h_{\mathrm{fe}}$, for if we look at the diagrams again we see that when the output terminals are shorted not only does the whole of $h_{\mathrm{fe}_{\mathrm{e}} i_{\mathrm{b}}}$ flow that way but $i_{\mathrm{b}}$ as well. Moreover they flow towards the common terminal (c) instead of away from it as in Fig. 12.12a. So

$$
h_{\mathrm{fc}}=-\left(h_{\mathrm{fc}}+1\right)
$$

Remembering that the positive $h_{\text {re }}$ went along with the fact that the output was reversed relative to the input, would we be correct in interpreting the negative $h_{\mathrm{rc}}$ as giving an output of the same sign as the input? We would indeed. as we can check by putting a positive input signal on Fig. 12.14h; it increases the emitter current and so makes the emitter more positive.

For measuring $h_{0}, R_{L}$ is removed and the input terminals are open-
circuited (to signals) so that there is no $i_{\mathrm{b}}$ and the $\nu_{\mathrm{e}}$ and $i_{\mathrm{e}}$ used in the test see only $h_{\text {oe }}$. So

$$
h_{\mathrm{oc}}=h_{\mathrm{oc}}
$$

Where the most drastic difference comes is in regard to $h_{\mathrm{rc}}$, the reverse voltage transfer ratio. Multiplied by the output voltage across the load, it tells us the voltage fed back to the input. If we look again at Figs. $12.14 b$ and 12.15 we see that the whole of the output voltage is fed back to the input. So instead of being usually a few parts in 10000 like $h_{\text {re }}$,

$$
h_{\mathrm{rc}}=1
$$

We must note again however that because the output voltage for a positive input is positive instead of negative the fed-back voltage, $h_{\mathrm{rc}} v_{\mathrm{e}}$, is positive, so opposes $v_{\mathrm{b}}$ around the input circuit, and instead of slightly reducing the zero- $R_{L}$ input resistance it greatly increases it. We can quite easily find how much it does so. If only $i_{\mathrm{b}}$ were flowing through $1 / h_{\mathrm{oe}}$ and $R_{L}$ in parallel, their combined resistance would seem to the input to be $1 /\left(h_{\mathrm{oe}}+G_{L}\right)$, where $R_{L}$ is transformed into a conductance for easy addition. But we know that $h_{\mathrm{fe}}+1$ (or $-h_{\mathrm{fc}}$ ) times as much current is passing through in the same direction. so its resistance seems like $\left(h_{\mathrm{fe}}+1\right) /\left(h_{\mathrm{oc}}+G_{L}\right)$, or - $h_{\mathrm{fc}} /\left(h_{\mathrm{oc}}+G_{L}\right)$, beside which $h_{\mathrm{ie}}$ can usually be neglected. With $h_{\mathrm{rc}}=1$, this agrees with the formula for $r_{\mathrm{in}}$ given in the previous section.

To compare the configurations let us use the formulae on the same data as before, namely

| $h_{\text {ie }}=3 \mathrm{k} \Omega$ | so $h_{\text {ic }}=3 \mathrm{k} \Omega$ |
| :--- | ---: |
| $h_{\text {he }}=25 \mu \mathrm{~S}$ | $h_{\text {oc }}=25 \mu \mathrm{~S}$ |
| $h_{\text {fe }}=80$ | $h_{\text {fc }}=-81$ |
| $h_{\mathrm{re}}=5 \times 10^{-4}$ | $h_{\mathrm{rc}}=1$ |
| $R_{L}=4 \mathrm{k} \Omega$ and | $R_{G}=5 \mathrm{k} \Omega$ |

The results are

$$
\begin{aligned}
& A_{\mathrm{i}}=\frac{-81}{0 \cdot 1+1}=-73.6 \\
& A_{\mathrm{v}}=\frac{81}{0.82}+81 \\
& 0.9 .99 \\
& r_{\text {in }}=3000+\frac{81}{(25+250) \times 10^{-6}}=298 \mathrm{k} \Omega \\
& g_{\text {out }}=25 \mu \mathrm{~S}+\frac{81 \times 10^{6}}{8000} \mu \mathrm{~S}=25+10125 \mu \mathrm{~S}=10.15 \mathrm{mS}
\end{aligned}
$$

$$
\text { or } r_{\text {out }}=98.6 \Omega
$$

Some intermediate steps are shown here, to give an idea of how much the various data contribute to the results. Provided that $R_{L}$ is not abnormally small, $h_{\text {ic }}$ is a negligible part of $r_{\text {in }}$ compared with the amount due to the $100 \%$ voltage feedback. So where a far higher input resistance is needed than a common-emitter connection can provide, the common-collector is indicated. This goes along with a far lower output resistance than $1 / h_{\mathrm{oe}}$ alone. The current gain is practically the same, but the voltage 'gain' is less than 1, which means that there is a loss. Admittedly the loss is very small in any practical case, but one would not choose common-collector for voltage gain! Nor is it any better than common-emitter for current gain, but it is used quite a lot for matching high to low resistance.

The reason for $\mathrm{A}_{v}$ being always less than 1 is very clear in the phasor diagram. We could draw phasor diagrams corresponding to Figs 12.12 and $12.15 b$ but they will be clearer if they are referred to the actual transistor circuits, as in Fig. 12.16. (Helpful though


Fig. 12.16-Basic transistor amplifier circuits and the ir phasor diagrams: (a) commonemitter: (b) common-collector. Note that the phasor diagrams themselves ave identical
equivalent circuits are as a logical basis for calculations, we must never become so at-home in their world of make-believe that we forget that they are not real circuits, any more than the algebraical symbols.) The phasor diagrams for common-emitter and commoncollector turn out to be identical. The only distinction we can make is to show which is the common terminal by encircling its point. We must remember that so far as the transistor is concerned the working input terminals are always e and b. In Fig. 12.16a these also happen to be the external input terminals, but in Fig. 12.16 $b$ the input terminals embrace the output voltage too, so the gross.input must always be as much more than the output as is needed to provide the working input. This is a result of $100 \%$ voltage feedback. The voltage phasors in Fig. 12.16 show that whereas in a the input and output are in opposite directions and very different in magnitude, in $b$ the emitter output potential is nearly the same in both magnitude and direction as the input, so a transistor in this mode is often called an emitter follower. The emitter (output) potential follows that of the base (input). Similarly the common-anode valve is called a cathode follower.

Substituting various values of $R_{L}$ and $R_{G}$ in the formulae shows
that the input and output resistances depend very much on them. When we have dealt with the common-base mode we shall look at some comparative curves for all three modes.

### 12.10 Common Base

Although now perhaps the least used of the three modes. historically this was the first. Whereas the common collector has $100 \%$ voltage feedback, common base has $100 \%$ current feedback. The consequences can be seen if we follow the same plar as in the last section.

Fig. 12.17 shows the common-emitter equivalent circuit redrawn to fit the common-base terminal scheme. We begin again by shortcircuiting $R_{L}$. Then, the basic physical behaviour of a transistor is that any input current $i_{\mathrm{e}}$ going towards the emitter divides inside the transistor, a small part going direct to the base, and $h_{f_{e}}$ times as much going via the collector. So for every $\left(h_{\mathrm{fe}}+1\right)$ microamps of input, the output is $h_{\mathrm{fe}}$ micioamps, slightly less. This mode therefore gives a small current loss. This current 'gain' is also negative in


Fig. 12.17-Fig. 12.12a redrann again to fit common-hase terminals
phase because the output current flows in the opposite direction to that from the $h_{\mathrm{fe}}$ generator in Fig. 12.17. So

$$
h_{\mathrm{rb}}=-\frac{h_{\mathrm{fe}}}{h_{\mathrm{fe}}+1}
$$

The historical symbol for $h_{\mathrm{fb}}$, still sometimes used, is $\alpha$. corresponding to $\beta$ for $h_{\mathrm{fe}}$.

As the shorted $R_{L}$ makes $v_{c}$ zero, we can ignore the internal voltage generator, but the effect of the current generator, as we have just seen, is to cause $h_{\text {re }}+1$ times as much input current to flow as

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could be accounted for by the resistance $h_{\mathrm{ic}}$ alone. So

$$
h_{\mathrm{ib}}=\frac{h_{\mathrm{ie}}}{h_{\mathrm{fe}}+1}
$$

Next, we leave the input open, remove $R_{L}$ and substitute a source to make current flow into the collector. As $i_{e}=0$, the same current must flow out from the base, via $i_{\mathrm{ie}}$. Positive $i_{\mathrm{b}}$ is reck oned into the base. so $i_{\mathrm{b}}=-i_{\mathrm{c}}$. So for every microamp of $i_{\mathrm{c}}$ we send through $1 / h_{\mathrm{oe}}$ from outside, the internal current generator sends $h_{\mathrm{fe}}$ microamps through it in the same direction. So it sets up $\left(h_{\mathrm{fe}}+1\right)$ times as much $v_{c}$ as can be accounted for by $i_{c}$ alone, and therefore its resistance a ppears to be $\left(h_{\mathrm{fe}}+1\right) / h_{\mathrm{oe}}$. Compared with this, the effects of the other two items in circuit are negligible, and

$$
h_{\mathrm{ob}} \simeq \frac{h_{\mathrm{oe}}}{h_{\mathrm{fe}}+1}
$$

While we are applying $p_{c}$ we should notice that the equivalent circuit forms a potential divider. Using double-subscript notation.

$$
v_{\mathrm{c}}=v_{\mathrm{bc}}=v_{\mathrm{be}}+v_{\mathrm{ec}}
$$

The purpose of the parameter $h_{\mathrm{rb}}$ is to indicate what proportion of $v_{b c}$ appears as ${ }^{\prime}$ be:

$$
h_{\mathrm{rb}}=\frac{v_{\mathrm{be}}}{v_{\mathrm{bc}}}
$$

We have just found that the resistance between c and e is $\left(h_{\mathrm{fe}}+1\right) /$ $h_{\mathrm{oe}}$, and the resistance between e and b is $h_{\mathrm{ie}}$. So the part of $h_{\mathrm{rb}}$ due to these resistances is

$$
\frac{h_{\mathrm{ie}}}{\left(h_{\mathrm{fe}}+1\right) / h_{\mathrm{oe}}}=\frac{h_{\mathrm{ie}} h_{\mathrm{oe}}}{h_{\mathrm{fc}}+1}
$$

Fig. 12.17 shows that there is another voltage contributing to $v^{\prime}$ : it is $-h_{\mathrm{re}} v_{\mathrm{c}}$. Divided by $v_{\mathrm{c}}$ it is $-h_{\mathrm{re}}$. So the total $h_{\mathrm{rb}}$ is

$$
h_{\mathrm{rb}}=\frac{h_{\mathrm{i}} h_{\mathrm{oe}}}{h_{\mathrm{fe}}+1}-h_{\mathrm{re}}
$$

With the same data as before for an example, the results of using the above formulae are

$$
\begin{array}{ll}
h_{\mathrm{ib}}=37 \Omega & h_{\mathrm{ob}} \simeq 0.31 \mu \mathrm{~S} \\
h_{\mathrm{fb}}=-0.9875 & h_{\mathrm{rb}}=4.26 \times 10^{4}
\end{array}
$$

$$
\begin{aligned}
A_{\mathrm{i}} & =\frac{-0.9875}{0.00124+1}=-0.986 \\
A_{\mathrm{v}} & =\frac{0.9875}{\left.19.26 \times 10^{-3}\right)+\left(0.42 \times 10^{-3}\right)}=102 \\
r_{\text {in }} & =37+1.68=38.7 \Omega \\
g_{\text {out }} & =0.31 \mu \mathrm{~S}+0.084 \mu \mathrm{~S}=0.394 \mu \mathrm{~S} \\
& \quad \text { or } r_{\text {out }}=2.54 \mathrm{M} \Omega
\end{aligned}
$$

We see that unless $R_{L}$ is exceptionally large $A_{\mathrm{i}}$ is practically the same as $h_{\mathrm{fb}}$, and with the same stipulation we can neglect $h_{\mathrm{ob}}$ compared with $G_{L}$, and can also neglect $h_{\mathrm{rb}} h_{\mathrm{rb}}$, so that

$$
A_{\mathrm{v}} \simeq \frac{R_{L}}{h_{\mathrm{ib}}}
$$

that is to say, the ratio of load resistance to zero- $R_{L}$ input resistance. And if $R_{G}$ is large, $g_{\text {out }}$ is not much more than $h_{\text {ob }}$, but in practice there could be an appreciable difference. Note the much lower $r_{\text {in }}$ and $g_{\text {out }}$ than with common emitter. The contrast with common-collector is even greater. So choice of mode (and to some


Fig. 12.18-Conunon-base amplifier circuit and phasor diagrams for comparison with those for the other modes (Fig. 12.16)
extent of $R_{G}$ and $R_{L}$ ) gives one a wide range of control over input and output resistances of a transistor.

We do not really need another diagram to compare with Figs. $12.12 a$ and $12.15 b$, for the only differences would be the terminal labels and the second subscripts to the $h$ parameters. But Fig. 12.18 is for comparison with Fig. 12.16, and again we see the phasor diagram is the same except for the encircled point marking the common
electrode. The voltage diagram shows at once that only common emitter gives a signal reversal. Study of the three phasor diagrams helps one to see how the common-emitter gives both voltage and current gain, whereas common-collector gives only current gain and common-base only voltage gain. The obtainable power gain of the common-emitter mode is therefore obviously much greater than that of the other two, which accounts for its popularity. But it must be re-emphasized that we are assuming low-frequency signals.

Note that $A_{i}$ and $A_{\mathrm{y}}$ are in terms of the input current and voltage actually reaching the input terminals, not those supplied by the input current or voltage signal generator. So although in our numerical example the common-base mode had the same $A_{v}$ as the commonemitter, the higher input current with common-base would mean far greater loss in the generator resistance, $R_{G}$.

### 12.11 A Summary of Results

Perhaps the best way of taking in the results of the last three sections is to look at Fig. 12.19. This shows how current, voltage and power gain, and input resistance, of our typical transistor vary with load resistance over the range $100 \Omega$ to $1 \mathrm{M} \Omega$, and also how output resistance depends on the resistance of the input signal source over the same range. To cover this wide range, and the ranges of gain, etc., the scales are logarithmic.

The curves tell the story without further comment, but three qualifications should be underlined: these curves are derived from one particular set of transistor parameters; they are small-signal parameters: and they are restricted to frequencies low enough for reactances to be neglected. But the general shapes of the curves are affected hardly at all by substituting other values of the parameters typical of transistors intended for small-signal amplification. We shall see later (in Chapter 24) that transistors serve other purposes, for which $h$ parameters are not the most helpful approach.

To complete this summary, here follow the formulae and values used for plotting the curves in Fig. 12.19. You may notice that for values of $R_{L}$ within the range $100 \Omega$ to $10 \mathrm{k} \Omega$ the curves are either almost flat or sloping at $45^{\circ}$, for which simpler approximate formulae given at the end are good enough.
$h_{\mathrm{i}}=$ input resistance with output shorted (i.e., $R_{L}=0$ )
$h_{\mathrm{o}}=$ output conductance with input open (i.e., $G_{G}=0$ )
$h_{f}=$ forward current transfer ratio
$h_{\mathrm{r}}=$ reverse voltage transfer ratio
$R_{L}=$ load resistance. $\quad G_{L}=$ load conductance, $=1 / R_{L}$
$R_{G}=$ external resistance between input terminals. $\quad G_{G}=1 / R_{G}$


A

(b)


Fig. 12.19-(a) current gairr given her a specified typical ramsistor in the there modes. related to loud resiskurce: (t) veltage gain related to load resistance: (c) power gain related to load resistance: (d) input resistance related to loud resistance and output resistance rclated to signal generator resistance

Fig. 12.20-This circuit diagram illustrutes the symbols lissed opposite


TYPICAL VALUES

| common emitter |  | common collector |  | common base |  |
| :--- | :--- | :--- | :--- | :--- | :---: |
| $h_{\mathrm{he}}$ | $3 \mathrm{k} \Omega$ | $h_{\mathrm{ic}}=h_{\mathrm{ie}}$ | $3 \mathrm{k} \Omega$ | $h_{\mathrm{ib}}=\frac{h_{\mathrm{ie}}}{h_{\mathrm{fe}}+1}$ | $37 \Omega$ |
| $h_{\mathrm{oe}}$ | $25 \mu \mathrm{~S}$ | $h_{\mathrm{oc}}=h_{\mathrm{oe}}$ | $25 \mu \mathrm{~S}$ | $h_{\mathrm{ob}} \simeq \frac{h_{\mathrm{oc}}}{h_{\mathrm{fe}}+1}$ | $0.31 \mu \mathrm{~S}$ |
| $h_{\mathrm{fe}}$ | 80 | $h_{\mathrm{fc}}=-\left(h_{\mathrm{fe}}+1\right)$ | -81 | $h_{\mathrm{rb}}=-\frac{h_{\mathrm{fe}}}{h_{\mathrm{fe}}+1}$ | 0.9875 |
| $h_{\mathrm{re}}$ | $5 \times 10^{-4}$ | $h_{\mathrm{rc}}=1$ | 1 | $h_{\mathrm{rb}}=\frac{h_{\mathrm{ic}} h_{\mathrm{oc}}}{h_{\mathrm{fe}}+1}-h_{\mathrm{re}}$ | $4.26 \times 10^{-4}$ |
| $h_{\mathrm{re}} h_{\mathrm{fe}}$ | 0.04 | $h_{\mathrm{r}} h_{\mathrm{fc}}$ | -81 | $h_{\mathrm{rb}} h_{\mathrm{fb}}$ | $-4.21 \times 10^{-4}$ |

Current gain, $A_{\mathrm{i}}=\frac{h_{\mathrm{f}}}{h_{\mathrm{o}} R_{\mathrm{L}}+1}$
Voltage gain, $A_{v}=\frac{-h_{\mathrm{f}}}{h_{\mathrm{i}}\left(h_{\mathrm{o}}+G_{\mathrm{L}}\right)-h_{\mathrm{r}} h_{\mathrm{f}}}$
Power gain, $A_{\mathrm{p}}=A_{\mathrm{i}} A_{\mathrm{v}}$
Input resistance, $r_{\text {in }}=h_{\mathrm{i}}-\frac{h_{\mathrm{r}} h_{\mathrm{r}}}{h_{\mathrm{o}}+} \overline{G_{L}}$
Output conductance, $g_{\text {out }}=h_{\mathrm{o}}-\frac{h_{\mathrm{r}} h_{\mathrm{f}}}{h_{\mathrm{i}}+R_{\mathrm{G}}}$
Output resistance, $r_{\text {out }}=\frac{1}{g_{\text {our }}}$
approximations ( $R_{L}$ or $R_{G}=10^{2}-10^{4}$ unless stated)

|  | common emitter | common collector | common base |
| :---: | :---: | :---: | :---: |
| $A_{\text {i }}$ | $h_{\text {re }}$ | $h_{\text {fc }}$ | 1 ( $R_{L}$ up to $10^{5}$ ) |
| $A_{v}$ | $h_{\mathrm{fe}} \frac{R_{L}}{h_{\text {ie }}}$ | 1 ( $R_{L}$ above 10 ) | $h_{\text {fe }} \frac{R_{\mathrm{L}}}{h_{\text {ie }}}$ |
| $A_{\mathrm{p}}$ | $h_{\mathrm{fe}^{2}} \frac{R_{f}}{h_{\mathrm{ie}}}$ | $A_{i}\left(R_{L} 10^{3}-10^{4}\right)$ | $A_{v}\left(R_{L}\right.$ up to $\left.10^{5}\right)$ |
| $r_{\text {in }}$ | $h_{\text {ie }}$ | $-h_{\pi} h_{\text {rc }} R_{L}$ | $h_{\text {ib }}$ |
| gout | $h_{\text {o }} \quad\left(R_{G}\right.$ above $3 \times 10^{3}$ ) | $-\frac{h_{\mathrm{rc}} h_{\mathrm{rc}}}{h_{\mathrm{rc}}+R \mathrm{G}}$ | $h_{\text {ob }} \quad\left(R_{L}\right.$ above $\left.3 \times 10^{3}\right)$ |

### 12.12 F.E.Ts and Valves

With their near-infinite low-frequency input resistance, field-effect transistors resemble valves much more than they resemble the bipolar transistors we have been considering, so that the same simple equivalent circuits and gain calculations studied for valves in Chapter 11 will do for f.e.ts. If the valves are of the pentode type (Sec. 15.7) the output-current/output-voltage curves are very similar 100 (compare Figs. 10.15 and 15.1) and in this respect they both differ little from bipolar transistors except for the greater voltage before the nearly flat (high-resistance) parts are reached. Unless the contrary is mentioned, operation is assumed to be confined to these parts. Because of the normally high output resistance, the equivalent current generator is usually preferred to the voltage generator.

Under the simple low-frequency small-signal conditions assumed. then, the bipolar parameters $h_{\mathrm{r}}$ and $h_{\mathrm{i}}$ are not applicable, nor (except in common-gate or common-grid) are $A_{\mathrm{i}}$ and $A_{n}$, nor $r_{\text {in }}$. And $g_{\text {out }}$ is simply $g_{\mathrm{a}}$ for valves $\left(=1 / r_{\mathrm{a}}\right)$ and $g_{\mathrm{d}}$ for f.e.ts. These correspond to $h_{0}$ for bipolar transistors. The most important parameter is $g_{\mathrm{m}}$. the mutual conductance or ratio of short-circuit output signal current to input voltage (Sec. 10.3). To bring it more into line with bipolar transistor nomenclature, it is alternately called $g_{\mathrm{fs}}$, the subscripts indicating "forward' and '(common) source".

This is a reminder that so far we have considered valves and f.e.ts only in their common-cathode or common-source mode. Although it is the most used. the other two also have their uses. Their circuit diagrams are of course the same as those in Figs. 12.14. 12.16 and 12.18 except for device symbols and electrode initials. So too are the phasor diagrams if the current $B A$ is reduced to zero.

Fig 12.21 compares the equivalent circuits for the three configurations; all three are actually the same circuit internally. only the positions of the terminals and the ways the source and load are connected being different. In $b$ and $c$ the current phasors are reversed. but this is not a discrepancy: it arises because the mesh labels $\mathrm{A}, \mathrm{B}$ and $C$ carnot be arranged so that the current directions they indicate are the same in both actual and equivalent sets of diagrams. In Fig. 12.21h and $c$. AB is opposite to AB in $a$. so to agree with physical behaviour the minus sign is omitted from $b$ and $c$.

Comparing now the common-drain and common-anode modes. $b$ (also known as source follower and cathode follower for the reason given in Sec. 12.9) there are two main effects. both similar to those already studied in bipolar transistors: the voltage gain is reduced to a little less than 1. and the effective output resistance is also greatly reduced. The reason for the first effect is exactly the same as before: the whole of the output voltage is fed back in opposition to the input, which must therefore be augmented equally to overcome it. The reason for the low output resistance can be seen by using the same method as before : substituting for $R_{L-}$ a source of signal voltage,
which in this case would be called $d s$, and noting the resulting current. To ensure again that no current comes from the equivalent generator the input sg must be zero, and as this time it is a voltage instead of a current the no-signal condition is obtained by short-circuiting the input terminals. In diagram a under this condition we would en-


Fig. 12.21-Field-effect iransistor equivalent current-generator circuits and corresponding phasor diagrams (compare Figs. 11.5 and 12.5) : (a) common-source, (b) commondrain, ard (c) common-gate
counter only the resistance $1 / g_{d}$. But in $h$ the test voltage $d s$ comes reversed between $s$ and $d$, so $s g=s d=-d s$, and the generator consequently gives a current $A B=s d$. $g_{\mathrm{m}}$. This means the same as $B A=s d . g_{\mathrm{m}}$, and this current flows in the same direction as the current $d s . g_{d}$ directly due to $d s$. So the total current is $d s . g_{d}+d s . g_{m}$,
and the total conductance is therefore

$$
g_{\text {out }}=g_{\mathrm{m}}+g_{\mathrm{d}}
$$

Usually $g_{d}$ is negligible compared with $g_{m}$. so it is near enough to say that the output conductance is $g_{\mathrm{m}}$. A typical value is 3 mS , which is a resistance of $333 \Omega$.

The voltage 'gain' can quite easily be worked out : it can be put in various forms, of which perhaps the simplest and clearest is

$$
A_{\mathrm{v}}=\frac{g_{\mathrm{m}}}{g_{\mathrm{m}}+g_{\mathrm{d}}+G_{\mathrm{L}}}
$$

in which $g_{d}$ is usually small enough to neglect, and $G_{L}$ can be made fairly small, so that $A_{v}$ is only slightly less than 1 .

Both the above results apply to valves if $g_{\mathrm{a}}$ is substituted for $g_{\mathrm{d}}$.
Lastly. Fig. 12.21c shows very clearly that input and output are in series, so the same current passes through both and $A_{i}=1$. Input and output both affect one another, $R_{G}$ comes into the account and the situation is nearly as complicated as with a bipolar transistor. We can see from the generator equation that current is injected into the circuit in the same direction as that due directly to the signal and $g_{m}$ times as much. A little of it is lost in $g_{d}$, but most of it flows right round the circuit. So the apparent input resistance is much less than $r_{\mathrm{d}}+R_{L}$. It is in fact

$$
r_{\text {in }}=\frac{r_{\mathrm{d}}+R_{L}}{g_{\mathrm{m}} r_{\mathrm{d}}+1}=\frac{r_{\mathrm{d}}+R_{L}}{\mu+1}
$$

The output resistance is measured with no input voltage $g a$, but this does not mean that the internal current generator is doing nothing, for the circuit current flowing through $R_{G}$ sets up an input voltage. The internal generator current due to this sets up a voltage across $r_{\mathrm{d}}$ opposing the applied voltage $g d$, so the effect of $R_{G}$ is to increase $r_{\text {out }}$, which comes to

$$
r_{\text {out }}=r_{\mathrm{d}}+R_{\mathrm{G}}\left(g_{\mathrm{m}} r_{\mathrm{d}}+1\right)=r_{\mathrm{d}}+R_{G}(\mu+1)
$$

The safest way of arriving at these results is to apply Kirchholl's laws to the circuit and solve the equations so obtained. If we continue this work to find the voltage gain, as before in terms of the input voltage reaching the input terminals, we get

$$
A_{v}=\frac{R_{L}\left(g_{\mathrm{m}} r_{\mathrm{d}}+1\right)}{R_{L}+r_{\mathrm{d}}}=\frac{R_{L}(\mu+1)}{R_{L}+r_{\mathrm{d}}}
$$

If we allow for the loss in $R_{G}$ and reckon the input as $g a$, we get a smaller gain; call it $A_{v}^{\prime}$ :

$$
A_{\mathrm{v}}^{\prime}=\frac{R_{L}}{R_{G}+\frac{R_{L}+r_{\mathrm{d}}}{g_{\mathrm{m}} r_{\mathrm{d}}+1}}=\frac{R_{L}}{R_{G}+\frac{R_{L}+r_{\mathrm{d}}}{\mu+1}}
$$

All the above results are given alternatively in terms of $\mu$, which is often known, for valves at least, in which cases of course $r_{\mathrm{a}}$ would take the place of $r_{d}$.

As an example let us assume $g_{\mathrm{m}}=3 \mathrm{~mA} / \mathrm{V}, r_{\mathrm{d}}=30 \mathrm{k} \Omega, R_{L}=10 \mathrm{k} \Omega$ and $R_{G}=0.5 \mathrm{k} \Omega$. Then

$$
r_{\text {in }}=0.44 \mathrm{k} \Omega(440 \Omega) ; r_{\text {out }}=75.5 \mathrm{k} \Omega ; A_{\mathrm{v}}=22.75 ; A_{\mathrm{v}}^{\prime}=10.65 .
$$

We can review this section by looking again at the phasor diagrams in Fig. 12.21. The chief advantage of f.e.ts and valves over bipolar transistors is the absence of input current in $a$ and $b$, but in $c$ this advantage is sacrificed, so common-gate and common-grid are less attractive than the other modes.

Having now achieved complete transistor and valve equivalent circuits, at least for frequencies at which reactances can be neglected, we may find this a good moment for pausing to realize that within the limits of the assumptions made we can represent all actual circuits by equivalent circuits made up of only five different elements: resistance, capacitance, inductance, voltage generators and current generators. Even this short list could be reduced, because voltage and current generators are alternatives, and at any one frequency inductances and capacitances can be expressed as positive and negative reactances.

So much attention having been given to the rather artificial concept of small-signal equivalent circuits, before turning to applications of transistors and valves it will be as well to consider their working requirements; in a word, feeds.

## The Working Point

### 13.1 Feed Requirements

We have been studying at some length the parameters and performance of transistors and valves, but of course these qualities cannot be obtained unless the devices are fed with d.c. power adjusted to achieve a suitable working point. What is suitable depends on the kind of work they will be required to do, and for a start we shall continue to assume low-frequency low-power amplification. Modifications for other k!nds of work will appear when we come to them.

First of all let us deal with n-p-n transistors. The same information can be applied to $\mathrm{p}-\mathrm{n}-\mathrm{p}$ types if all polarities are reversed. F.e.ts and valves will follow.

As we have seen, the emitter-to-base junction is a diode which under working conditions is forward biased. The easily remembered rule for this is that positive bias supply goes to the $p$ side of the junction. The collector-to-base junction is always reverse biased. (A transistor as a whole is more than just two diodes connected together, of course.) With reference to the emitter, the collector and base biases therefore have the same polarity. In this respect they differ from valves and most f.e.ts, in which the control diode is reverse biased (negative to gate or grid) and the other diode, which has source or cathode as one of its electrodes, is forward biased, with positive to drain or anode. (With valves at least, the term 'bias' is usually restricted to the grid supply.)

Fig. 13.1 summarizes these requirements. It also shows the symbols used for feed voltages (and currents, where applicable): a capital $V$ (or $l$ ) with double capital subscripts to indicate the electrode for which intended, followed if necessary by a third letter to indicate the reference electrode. (Note, this is opposite to the convention used for phasor diagrams.) K rather than C is used for cathode, as C is reserved for collector or capacitance. As anything to do with signals is omitted, there is no need to show commoncollector, etc., arrangements separately; so far as feeds are concerned they are the same as common-emitter, though the actual voltages required may differ. For the same reason the feed voltages look the same as those actually applied to the electrodes, but where loads. etc., intervene, the electrode voltages are distinguished by single subscripts as used in previous chapters.

Although two batteries are shown for each device, arrangements are usually made for both electrodes to be fed from one source. The procedure for obtaining device data, plotting characteristic curves
and drawing load lines, and thereby deciding on a suitable working point. by which the feed voltages and currents are specified, has already been described in Secs 10.1 and 10.8. What we now have to realize is that this procedure takes far too much time to put into practice for every individual transistor. Yet individual devices, even


BIPOLAR
TRANSISTORS
$I_{E}=I_{C}+I_{\theta}$


JUGFET
$n$ chonnel
$I_{D}=I_{S}$
(normally $I_{G}=0$ )


Fig. 13.1-Comparison of polarities and comparative magnitudes of feed voltages needed by various amplifying devices in their three configurations
when they bear the same type number, differ so widely that their feeds seem to need individual tailoring. Even if they got it, satisfaction could still not be guaranteed, for temperature changes can upset the most carefully adjusted working points.

The answer to this problem is so to design the circuit that any
changes tending to displace the working point are automatically counteracted to a sufficient extent to maintain performance reasonably close to that intended. To do this we must know about the causes of working point displacement.

### 13.2 Effect of Amplification Factor Variations

Fig. 13.2 shows a typical $I_{c} / V_{c}$ curve sheet with load line. P has been selected as the working point because it is in the middle of the usable part of the load line, so that positive and negative input


Fig. 13.2-Typical $l_{c} \mid V_{C}$ curves and load line for low-power transistor. The designed warking point ( $\mathbf{P}$ ) may be drastically allered ( 10 ( $\mathrm{P}^{\prime}$ or $\mathbf{P}^{\prime \prime}$, for example) by changes in $h_{\mathrm{fe}}$.
half-cycles, each of $20 \mu \mathrm{~A}$ peak value, could be accepted and would yield apparently equal output half-cycles, each of about $5 \frac{1}{2} \mathrm{~V}$ peak.

The curves tell us that for each increase of $10 \mu \mathrm{~A}$ in $I_{B}$ the collector current increases by about $500 \mu \mathrm{~A}$, so $h_{\mathrm{fe}}$ is about 50 . If this was classed as 'typical', for others of the same type it could range from 25 to 100 . With a $25 h_{\mathrm{fe}}$ the $20 \mu \mathrm{~A} I_{B}$ line would be about where the $10 \mu \mathrm{~A}$ line is in Fig. 13.2, so the working point would be displaced to $\mathrm{P}^{\prime}$, halving the maximum signal that could be handled without
cutting off its peaks. With a $100 h_{\text {fe }}$ sample the $20 \mu \mathrm{~A}$ line would be about where the $40 \mu \mathrm{~A}$ line is in Fig. 13.2, so the working point would be $\mathrm{P}^{\prime \prime}$ and while negative half-cycles could be handled splendidly the positive halves would be cut off almost completely, since $V_{c}$ is only a fraction of a volt. It looks as if our aim should be to maintain the designed working collector current rather than base current.

### 13.3 Influence of Leakage Current

Although leakage current came into Chapter 10 it has been ignored since, because it has no place in equivalent circuits that are concerned only with signals. Even now it may not appear to be important, for it is represented in graphs like Fig. 13.2 by the height of the zero- $I_{B}$ line above the zero- $I_{C}$ line, and unless it is exaggerated it is often too small to show clearly, even for a germanium transistor, let alone silicon. All it seems to do is shorten to an insignificant extent the useful part of the load line at the high- $V_{C}$ end. But that is leaving temperature out of account. And we must remember that the temperature concerned is that of the junction itself, which is affected not only by the external ('ambient') conditions but by the electrical power dissipated in the transistor. Temperature is what limits the power a transistor can handle, and the reason is the rapid increase in leakage current that it causes.

There are actually three leakage currents, depending on which of the three electrodes is open-circuited, or at least reduced to zero current. Of these, $I_{\text {Ebo }}$, the current from emitter to base with collector unconnected, is unimportant, because this diode is normally forward biased. $I_{C B O}$ is the reverse current of the collector-


Fig. 13.3-For the elucidation of leakage currents in transistors
base diode, as shown in Fig. 13.3a, and is the one most often given on data sheets. The leakage current represented by the $I_{B}=0$ line in Fig. 13.2 is $I_{\text {CEO }}$, which passes through both diodes (Fig. 13.3b) and is the most important in practice. Note that the reverse voltage used for measuring leakage current is not specified. Provided it is not
less than say I V and is not high enough to threaten breakdown, its value hardly matters, as Fig. 13.2 shows.

Thoughtful readers looking at Fig. 13.3 h may be puzzled. It doesn't seem to conform to the rule that $I_{C}=h_{F E} I_{B}$, for here $I_{B}$ is zero but $I_{C}$ is not. Observant readers will have noticed that although the voltage between collector and base in Fig. 13.3a has the normal working polarity for an n-p-n transistor the base current is flowing the wrong way for normal operation. If an adjustable bias were added between base and emitter, as at $c$, there would be an adjustment at which the currents into and out from the base were equal, leaving zerc net current and so reproducing the conditions of diagram $b$. The current going into the base, which we have nade equal and opposite to $I_{\text {CBO }}$, by transistor action causes $h_{F E}$ times as much current to flow to the collector. The total collector current, which in these circumstances is by definition $I_{\text {CEO }}$, is thus equal to $I_{C B O}\left(h_{\text {FE }}+1\right)$. It is therefore many times larger than $I_{C B O}$. Note the capital letter subscripts in $h_{F E}$, signifying that it is the d.c. no-signal ratio $I_{C} / I_{B}$, not the signal ratio $i_{c} i_{\mathrm{b}}$ as in the last chapter. $h_{\text {FE }}$ and $h_{\mathrm{fe}}$ are usually of the same order, at least; but both depend somewhat on the working point and temperature.

A typical value of $I_{C B O}$ for a low-power germanium transistor at $25^{\circ} \mathrm{C}$ is $3 \mu \mathrm{~A}$. Iceo could therefore be something like $200 \mu \mathrm{~A}$. or 0.2 mA . As the leakage doubles for about every $9^{\circ} \mathrm{C}$, at $70^{\circ} \mathrm{C}$ this would have risen to almost 6 mA . In a case such as that shown in Fig. 13.2, lifting the whole array of $I_{B}$ lines so that the zero one was at about $6 \mathrm{~mA} I_{C}$ would result in the working point being buried in the nearly vertical saturation part of the curves, putting the transistor virtually out of action. Negative base bias would be needed.

Again, it seems clear that stabilizing collector current, rather than base current, should be the aim.

### 13.4 Methods of Base Biasing

The simplest method of providing base bias current is from the collector supply through a resistor, $\mathrm{R}_{B}$ in Fig. 13.4. Suppose for example that the required bias is $30 \mu \mathrm{~A}$ and $V_{C C E}$ is 9 V ; then $R_{B}$ should be $9 / 30=0.3 \mathrm{M} \Omega$. In this calculation the difference between the potential of the base and that of the emitter has been neglected. As we noted in Sec. 9.12, the forward voltage bias for a germanium diode is about 0.2 V and for silicon 0.6 V , and because the current rises steeply (see Fig. 10.13, for example), which means that the $g_{\mathrm{m}}$ of bipolar transistors is high, the working range of bias voltage is quite small and never far from the above values. For d.v. calculations the base-to-emitter voltage can therefore be assumed to be approximately these values. Anyone who wanted to calculate $R_{B}$ more accurately by deducting the appropriate p.d. from $V_{\text {CCE }}$ could do
so, but he would be wasting his time because nothing about transistors is as accurate as that. In any case in practice the choice of resistance is confined to "preferred values'. l"urthermore, this method of biasing is unsuitable for most practical circuits, because the required $R_{B}$ is generally much larger than any variations of the


Fig. 13.4-A simple but not entirely satisfactory method of providing base bias for an amplifying transistor
input resistance of the transistor, so in conjunction with $V_{C C E}$ is virtually a constant-current generator (Sec. 12.1). The last two sections have shown that constant bias current is unlikely to maintain a satisfactory working point against changes in temperature and transistor parameters.

Fig. 13.4 does however show two capacitors, which are usually necessary to prevent the base and collector potentials from upsetting or being upset by whatever is connected to the input and output terminals. At the same time, if of sufficient capacitance in relation to the resistances, they allow signals to pass freely (Sec. 14.6).

Because the full working range of collector current results from varying the base voltage within a range which typically is only a fraction of a volt (Fig. 10.13), establishing the working point by using a constant-voltage source might seem to be even more unlikely to succeed. And so it would be if such bias were introduced directly between emitter and base. Adjustment of the bias voltage would be extremely critical, and when carried out for one transistor might well be quite unsuitable for others of the same type. But the very steep control of collector current by base voltage that is the reason for dismissing this method is actually the basis for a modification which is the most popular of all base biasing methods.

Fig. 13.5 shows an experimental circuit in which a resistor $R_{E}$ has been introduced in the emitter lead. Suppose $R_{E}$ is of the order of $1 \mathrm{k} \Omega$. With the base voltage control slider at the bottom, $V_{B}=0$ and the transistor is "cut off". Now raise the slider gradually. At about 0.2 V (for a germanium transistor) a base current begins to flow, causing a much larger emitter current. Flowing through $R_{E}$, this $I_{E}$ causes the emitter to become more positive, reducing the working bias of base relative to emitter. So $I_{E}$ is not as much as it would have been without $\mathrm{R}_{E}$. As $V_{B}$ is increased, $V_{E}$ follows close behind all the way, for this is a form of emitter follower (Sec. 12.9). Even for quite a large $I_{E}$ the increase of $V_{B E}$ above the initial 0.2 V
or so is quite small, so we are not far out in saying that $V_{E}$ is $V_{B}-0.2$ over the whole range of adjustment, at least until saturation is reached ( $V_{B}$ and $V_{C}$ nearly equal). Note that the value of $R_{E}$ does not enter into this. So long as it is not too small relative to $R_{C}$ it is not very important. At a fixed setting of $V_{B}, V_{E}$ remains almost constant regardless of $R_{E}$ or the amount of current drawn. The arrangement is a form of nearly constant voltage generator. It is also a basic circuit that has other applications. But just now we are interested in it because the amount of emitter current is fixed within fairly close limits by the setting of $V_{B}$ and the value of $R_{E}$. From the foregoing

$$
I_{E}=\frac{V_{E}}{R_{E}} \simeq \frac{V_{B}-0 \cdot 2}{R_{E}}
$$

for germanium; for silicon, substitute 0.6 for 0.2 . $I_{C}$ is of course nearly the same as $I_{E}$.

Substituting a transistor having a different value of $h_{F E}$, and therefore demanding a different value of $I_{B}$ to maintain the desired $I_{C}$, merely causes $I_{E}$ to change to the small extent needed to adjust


Fig. 13.5-Experimental circuit for studying base biasing


Fig. 13.6-This base-biasing scheme is more stable than the one shown in Fig. 13.4 and is usually preferred
$V_{B E}^{\prime}$ sufficiently to change to the new $I_{B}$. The larger $V_{E}$ is, the smaller the necessary changes in $I_{E}$ and the less any differences in actual $V_{B E}$ required by different transisters matter.
Fig. 13.6 shows how this biasing scheme is usually arranged in practice. Success depends on $V_{B}$ being reasonably constant, so the values of the potential dividing resistors $\mathrm{R}_{\mathrm{B}_{1}}$ and $\mathrm{R}_{\mathrm{B}_{2}}$ should not be so high that the potential of the base is affected to a substantial extent by changes in base current. In other words, the current flowing through them when the base is disconnected should be at least several times $I_{B}$. But we do not want the resistances to be so low that they waste a lot of power from the collector supply, or reduce the input resistance enough to cause serious loss of signal.

The extra capacitor $C_{E}$ (which is not invariably used) is to shortcircuit $R_{E}$ to signal currents.

There are other methods of biasing, such as keeping the base at zero and biasing the emitter negatively, but this calls for an extra current-supplying source and is not always convenient. The only other one that perhaps deserves showing now is outlined in Fig. 13.7. The object of connecting $R_{B}$ to the collector instead of as in Fig. 13.4


Fig. 13.7-This base-biasing circuit is much simpler than Fig. 13.6 and does have stabilizing action. but owing to certain disadvantages is less used
is to compensate in a simpler way for changes in $I_{c}$. If $I_{c}$ is too large, it increases the voltage drop in $R_{L}$ and so reduces the voltage applied to $\mathrm{R}_{\mathrm{B}}$, reducing $I_{B}$ and therefore $I_{C}$; and oppositely if $I_{C}$ is too small. There are several disadvantages, however, one of them being that signal as well as feed current is fed back from collector to base.

Common-collector transistors call for no separate attention as they automatically provide compensated bias of the Fig. 13.6 type to a maximum degree.

There is no difference in principle in biasing transistors in the common-base mode; for example, Fig. 13.6 can be converted by short-circuiting the input terminals (so that $C_{B}$ earths the base to signals), the input signal source being connected between emitter and $R_{E}$.

### 13.5 Biasing the F.E.T.

The shape of the f.e.t. output characteristic curves (Fig. 10.15, for example) is similar to those for bipolar transistors, and so is the drawing of load lines and finding a working point, allowance being made for the higher saturation or bottoming voltage; about 5 V in the example referred to. (Drain voltages are usually higher than collector voltages, too.) But when biasing a depletion-mode device we must remember that instead of beginning at the bottom with base current rising from zero and driven by a voltage of the same polarity as the collector we begin at the top with gate voltage falling from zero and opposite in polarity to the drain.

The oppositeness of polarity does not mean that a special gate
voltage source must be provided. We have seen in Fig. 13.6 how an emitter can be held at a positive potential by making use of the voltage drop in a resistor connected in the emitter lead. The base, because it must be positive to the emitter, has to be provided with a slightly larger positive voltage. Things are simpler with an f.e.t. because the required gate bias is negative, so the gate can be left at zero potential and the resistance $R_{s}$ in Fig, 13.8 adjusted to bias the source positively to the required extent. This is the same thing as negative gate bias, except that it is obtained at the expense of $V_{D S}$, the effective drain voltage. And it has the advantage, as in Fig. 13.6, of automatically adjusting the amount of bias to counteract any tendency for the source current (= drain current) to depart from the designed value-but less effectively, because f.e.t. mutual conductances are lower.

If the device is permanently connected to a signal source which provides a d.c. path but no d.v.. then $\mathrm{C}_{\mathrm{G}}$ and $\mathrm{R}_{\mathrm{G}}$ can be omitted. The purpose of $\mathrm{R}_{\mathrm{G}}$ is to ensure that the gate is held at the commonterminal potential (reckoned as zero). Because of the nearly zero gate current with negative bias, $R_{G}$ can be high; typically $10 \mathrm{M} \Omega$.

Because f.e.t. characteristics are especially subject to temperature and manufacturing variations, the degree of compensation provided by simple source bias as in Fig. 13.8 may not always be enough.

Fig. 13.8-This method of providing gate bias for f.e.ts is applicable also to valves


More compensation is obtained by increasing $R_{s}$, but if this alone were done it would develop excessive bias, so it must be accompanied by positive gate bias, which can conveniently be provided by connecting an appropriate high resistance from $V_{D D}$ to gate, making a potential divider similar in form to that in Fig. 13.6 but easier to design because of the absence of gate current. This new resistor would normally have to be several times higher in resistance than $R_{G}$, and such values are not usually very satisfactory in situations such as this. To enable conveniently lower values to be used without losing the potentially high input resistance of the f.e.t., a single high resistance can be connected between the junction of the potential divider and the gate.

### 13.6 Valve Biasing

Hardly anything need be said under this heading at the present stage, because the methods for f.e.ts can be used for valves, the simpler form (Fig. 13.8) usually giving adequate compensation. Other methods for both valves and f.e.ts are commonly used however, but they will be easier to understand in connection with the applications considered later (Sec. 26.8).

### 13.7 Biasing by Diodes

In many circuits there is a need for a fairly small constant bias voltage. Provided one can make do with about 0.2 V or 0.65 V or a combination of these, a convenient, reliable and cheap method is to pass a current through a germanium or silicon junction diode or through two or more in series, in the forward or low-resistance direction, and use the voltage drop across it or them. As Fig. 13.9 shows, the precise current used does not matter, nor does it have to be anywhere near constant. Fig. 19.15 is an example of this technique, but see Sec. 26.9 for the production of higher constant voltages by special reverse-connected diodes.


Fig. 13.9-Showing how the voltage across a typical silicon junction diode changes little with the current through it, over a wide range

## Oscillation

### 14.1 Generators

Having now a foundational knowledge of the few basic electrical and electronic circuit elements, we can begin to look at their applications. Elementary amplifiers have already appeared, and frequent references have been made to signals. Generators of signals have so far been represented by conventional symbols. The only sort of a.c. generator actually slown (Fig. 6.2) is incapable of working at the very high frequencies needed in radio and other systems, and produces a square waveform which for many purposes is less desirable than the sine shape (Fig. 5.2).

The alternating currents of power frequency-usually 50 Hz in Great Britain and many other countries-are generated by rotating machinery which moves conductors through magnetic fieids produced by electromagnets (Sec. 4.3). Such a method is quite impracticable for frequencies of millions per second. This is where the curious properties of inductance and capacitance come to the rescue.

### 14.2 The Oscillatory Circuit

Fig. 14.1 shows a simple circuit comprising an inductor and a capacitor in parallel, connected as the load of a f.e.t. The f.e.t. is not essential in the first stage of the experiment, which could be performed with a battery and switch, but it will be needed fater on. A valve would do just as well, or (with minor modifications) a bipolar transistor. With the switch as shown. there is no negative bias,

Fig. 14.1-Apparatus for setting up oscillations in the circuit LC


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and we can assume that the current flowing through the coil L is fairly large (Fig. 10.15). It is a steady current, and therefore any voltage drop across L is due to the resistance of the coil only. As none is shown, we assume that it is too small to take into account, and therefore there is no appreciable voltage across L and the capacitor is completely uncharged. The current through $L$ has set up a magnetic flux which by now has reached a steady state, and represents a certain amount of energy stored up, depending on the strength of current and on the inductance L. (Sec. 4.9). This condition is represented in Fig. 14.2 by $a$, so far as the circuit LC is concerned.

Now suppose the switch is moved from A to B, putting such a large negative bias on the gate that drain current is completely cut off. If the current in L were cut off instantaneously, an infinitely high voltage would be self-induced across it (Sec. 4.4), but this is impossible owing to C . What actually happens is that directly the current through L starts to decrease, the collapse of magnetic flux induces an e.m.f. in such a direction as to oppose the decrease, that is to say, it tends to make the current continue to flow. It can no longer flow through the f.e.t., but there is nothing to stop if from flowing into the capacitor C , charging it up. As it becomes charged, the voltage across it rises (Sec. 3.2), demanding an increased charging voltage from L. The only way in which the voltage can rise is for the current through it to fall at an increasing rate, as shown in Fig. 14.2 between $a$ and $b$.

After a while the current is bound to drop to zero, by which time C has become charged to a certain voltage. This state is shown by the curves of $I$ and $V$ and by the circuit sketch at the point $b$. Because the current is zero there can no longer be any magnetic field; its energy has been transferred to C as an electric field.

The voltage across C must be balanced by an equal voltage across $L$, which can result only from the current through $L$ continuing to fall; that means it must become negative, reversing its direction. The capacitor must therefore be discharging through L . Directly it begins to do so it inevitably loses volts, just as a punctured tyre loses pressure. So the rate of change of current (which causes the voltage across L ) gradually slackens until, when C is completely discharged and there is zero voltage, the current is momentarily steady at a large negative value (point $c$ ). The whole of the energy stored in the electric field between the plates of C has been returned to L, but in the opposite polarity. Assuming no loss in the to-and-fro movement of current (such as would be caused by resistance in the circuit), it has now reached a negative value equal to the positive value with which it started.

This current now starts to charge C, and the whole process just described is repeated (but in the opposite polarity) through $d$ to $e$. This second reversal of polarity has brought us to exactly the same condition as at the starting point $a$. And so the whole thing begins
all over again and continues to do so indefinitely, the original store of energy drawn from the battery being forever exchanged between coil and capacitor; and the current will never cease oscillating. Note that from the moment the switch was moved the f.e.t. played no part whatever. The combination of L and C is called an oscillatory circuit.

It can be shown mathematically that voltage and current vary sinusoidally.

### 14.3 Frequency of Oscillation

From the moment the f.e.t. current was switched off in Fig. 14.1 the current in L was bound to be always equal to the current in C because, of course, it was the same current flowing through both. Obviously, too, the voltage across them must always be equal. It


Fig. 14.2-Sequence of events in Fig. 14.1 after the switch has been-moved to B. Stages a to e cover one complete oscillation
follows that the reactances of $L$ and $C$ must be equal, that is to say $2 \pi f L=1 / 2 \pi f C$. Rearranging this we get

$$
f_{\mathrm{o}}=1 / 2 \pi v^{\prime}(L C)
$$

where $f_{0}$ denotes frequency of oscillation. This is the same as the formula we have already had (Sec. 8.7) for the frequency of resonance. It means that if energy is once imparted to an oscillatory circuit (which is now seen to be the same thing as a tuned circuit) it goes on ascillating at a frequency called the natural frequency of
oscillation, which depends entirely on the values of inductance and capacitance. By making these very small-using only a few small turns of wire and small widely-spaced plates-very high frequencies can be generated, running if necessary into hundreds of millions per second, or even more with combined structures (Sec. 16.10).

### 14.4 Damping

The non-stop oscillator just described is as imaginary as perpetual motion. It is impossible to construct an oscillatory circuit entirely


Fig. 14.3-The first few cycles of a train of damped oscillations
without resistance of any kind. Even if it were possible it would be merely a curiosity, without much practical value as a generator, because it would be unable to yield more than the limited amount of energy with which it was originally furnished. After that was exhausted it would stop.

The inevitable resistance of all circuits-using the term resistance in its widest sense (Sec. 2.18)-dissipates a proportion of the original energy during each cycle, so each is less than its predecessor, as in Fig. 14.3. This effect of resistance is called dumping; a highly damped oscillatory circuit is one in which the oscillations die out quickly.

In Sec. 8.4 the symbol $Q$ was given to the ratio of reactance (of either kind) to resistance in a tuned circuit. So a highly damped circuit is a low- $Q$ one. If the $Q$ is as low as $\frac{1}{2}$ the circuit is said to be critically damped; below that figure it is non-oscillatory and the voltage and current do not reverse after the initial kick due to an impulse but tend towards the shape shown in Figs. 3.6 and 4.10. A high- $Q$ circuit continues oscillating for many cycles. Oscillation is created in any tuned circuit by any sudden electrical impulse, and the circuit is said to be shock-excited. The same kind of thing happens to a car when it encounters a hole or bump in the road, unless sufficient mechanical resistance is imported to the springs in the form of shock absorbers (or dampers).

### 14.5 Maintaining Oscillation

What we want is a method of keeping an oscillator going, by supplying it periodically with energy to make good what has been usefully drawn from it as well as what has unavoidably been lost in its own resistance. We have plenty of energy available from batteries. etc.. in a continuous d.c. form. The problem is to apply it at the right moments.

The corresponding mechanical problem arises in a clock. The pendulum is an oscillatory system which on its own comes to a standstill owing to frictional resistance. The driving energy of the mainspring is 'd.c.' and would be of no use if applied direct to the pendulum. The problem is solved by the escapement, which is a device for switching on the pressure from the spring once in each oscillation, the 'switch' being controlled by the pendulum itself.

In Fig. 14.1 oscillation was started by moving the switch from $A$ to B at the stage marked $a$ in Fig. 14.2. By the time marked $c$ a certain amount of energy would have been lost in resistance, so that the negative current would be a little less than the positive current with which it started, and by $e$ the loss would be greater. But by switching positive current on again at $c$ until $e$ the upsweep of the current $I$ would be augmented. The amount of current required would not be as much as the original (before a) but only enough to make good the loss. In a high- $Q$ circuit this would be comparatively little.

Operating an actual switch by hand to reduce the bias during half of every cycle is not practicable, but it can easily be arranged to be done antomatically by the oscillation itself. From $a$ to $c$ in Fig. $14.2, V$ is positive. This is the period during which we want the downward swing of $I$ to be speeded. Reducing the f.e.t. drain current by applying or increasing negative bias does just this. During the

Fig. 14.4-Showing how the cincuit in Fig. 14.1 ccm be modified to enable oscillations to be self-maintained continuaus!

negative half-cycle of $V$ we want to increase the current again by reducing bias. If we inductively couple a small coil to L as in Fig. 14.4 we generate in it a voltage that alternately augments and diminishes the fixed bias provided by the bias battery. This voltage is exactly synchronized with $V$; its amount can be adjusted by the

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number of turns in the gate-circuit coil and by how closely it is coupled to L , and the relative polarity of the voltage applied to the gate is decided by the way the coil is turned. One of these ways is right for maintaining oscillation. This system is often called positive feedback, and the coil is a feedback or reaction (or, more precisely, retroaction) coil.

When the coupling is at least sufficiently close and in the right direction to maintain oscillation there is no need to do anything to start it; the ceaseless random movement of electrons in conductors (Sec. 2.3) provides impulses to start a tiny oscillation which soon builds up.

There is another way of looking at a self-oscillator circuit. Suppose the f.e.t. or other active device is arranged in a way with which we are quite familiar: Fig. 14.5. $v_{\mathrm{g}}$ is a small alternating input

(b)

Fig. 14.5-Diagram $b$ shows the relative amplitude and phase of grid and anode voltages when their frequency is the same as the natural frequency of the circuit LC
signal, graphed at $b$. We have seen in Sec. 8.10 that at the frequency of oscillation a tuned circuit is equivalent to a high resistance. If L and C are tuned to the frequency of $v_{\mathrm{g}}$ they are such a resistance, so voltage amplification takes place and an enlarged and inverted copy of $v_{\mathrm{g}}$ is obtained across LC. This is $v_{\mathrm{d}}$, also shown at $b$.

If then we take a fraction $v_{\mathbf{g}} / v_{d}$ of the output voltage and reverse it, as in Fig. 14.4 for example, we can substitute it for the generator in Fig. 14.5a. The amplifier supplies its own input by positive feedback and has become an oscillator.

In one respect this amplifier approach can be misleading. An amplifier has input and output pairs of points, and its behaviour depends on which electrode of the amplifying device is made common to both. Chapter 12 was largely about this. It is perfectly allowable to consider oscillator circuits in this way, and it is often done, but doing so tends to obscure the basic unity of oscillator circuits. Fig. 14.6 shows an oscillatory circuit LC with a potential divider across it. If a triode of any kind that can be used for amplification is connected as shown (where, in order to focus attention on signals, feeds have been omitted) and its gain is
sufficient to cancel losses, oscillation is maintained and the question of which electrode is common to input and output does not arise. If we choose to regard $c$ as the common point (as in Fig. 14.4 for example) then the input $c a$ and the output $c b$ need to be opposite in polarity, as they in fact are with common source (or emitter or cathode).

Fig. 14.6-A basic oscillator, with feeds omitted for clarity. Note that the question of which electrode of the amplifying device is the common one does not arise


But anyone is entitled to say that $a$ is the common point and that this is a common-gate circuit, in which the amplified output $a b$ has the same sign as the input ( $a c$ ). A third party could equally maintain that this was a common-drain circuit, in which the output voltage $b c$ is always not only of the same sign as the input ( $b a$ ) but slightly smaller, and oscillation is equally well maintained on this basis, for different points of view of observers make no difference to the circuit!

### 14.6 Practical Oscillator Circuits

Basically, then, an oscillator consists of an oscillatory circuit (L and C in Fig. 14.6) and a maintaining amplifier (the rest of Fig. 14.6). Owing largely to the necessity for feeding the amplifier with d.c., practical oscillator circuits usually look very different from Fig. 14.6, and it is often quite difficult to see that in principle they are the same.

A resistive potential divider is rarely used, for it adds to the losses. And it is quite unnecessary; the oscillatory circuit can be made to act as its own potential divider. One method is to 'tap' the coil, as in Fig. 14.7. (To show no bias in favour of any particular amplifying device, here a bipolar transistor is used.) This is the basic form of what is called the Hartley circuit. Alternatively we can divide the oscillatory voltage on the C side. This could be done by inserting a third plate between the two, but usually it is more convenient to use two separate capacitors as in Fig. 14.8, of such capacitances that the two in series are equal to $C$. This is the Colpitts oscillator circuit.

The coil in Fig. 14.7 can be regarded as an auto-transformer (Fig. 7.11). In Sec. 7.11 we saw that a resistance connected across one winding of a transformer affects the other similarly to connecting across it a resistance of a different value, depending on the turns ratio.

The same principle applies to an auto-transformer, and if a capacitor of a suitably larger capacitance than $C$ is connected across part of the coil it may still be tuned to the same frequency. Fig. 14.9 is therefore essentially the same as Fig. 14.7. but we can more easily recognize it as the retroaction coil circuit with which we began, Fig. 14.4. In general, the Hartley and Colpitts types enable the amplifier to be connected more effectively to the tuned circuit, so (especially the Colpitts) are particularly useful in 'difficult' cases, such as very high frequencies, at which oscillation is less easily maintained. A practical advantage of the separate retroaction coil type is that no special devices are needed to prevent feeds from being short-circuited, but in the circumstances just mentioned there might be difficulty in getting close enough coupling.

Fig. 14.10 is a practical Hartley circuit, in which the biasing components have the same labels as in Fig. 13.6 for easy identification. $\mathrm{C}_{\mathrm{B}}$, which completes the oscillation-frequency connection between one end of $L$ and the base without upsetting the bias by shortcircuiting $\mathrm{R}_{\mathrm{B}_{1}}$ to d.c., is called a blocking capacitor. The resistance $R_{B 1}$ must be high enough not to load the left-hand part of $L$ excessively. The tapping connection for the emitter goes via the battery and $\mathrm{C}_{\mathrm{E}}$ (for which reason this is called a series-fed Hartley circuit), and to provide an easier path for the a.c. a capacitor is often connected from tap to emitter. The same object is achieved in a different manner in what is called the parallel-fed Hartley circuit, Fig. 14.11, where for another change a valve is the active device. Ch is a coil of relatively high inductance, called a choke coil, which keeps a.c. out of the anode current supply while allowing the d.c. feed to flow freely. $\mathrm{C}_{\mathrm{a}}$ serves the same purpose as $\mathrm{C}_{\mathrm{B}}$ in Fig. 14.10. In this particular circuit the grid bias is fed in series with part of $L$, but often the oscillatory circuit is tapped straight to the common or earth terminal and bias is parallel-fed too. Fig. 14.12 shows a parallelfed Colpitts transistor circuit in which $R_{c}$ and $C_{C}$ take the place of Ch and $\mathrm{C}_{\mathrm{a}}$ as the parallel-feed components.

Practice in 'reading' circuit diagrams is needed before one can tell quickly and certainly which components are for blocking d.c. or a.c. and which are parts of the oscillatory circuit. One must bear in mind, too, that commercial designers like to make one component do the jobs of two or more. Then the intentional tuning capacitance is always augmented by stray capacitances due to other components and wiring. Diagrams of very-high-frequency oscillators are likely to be especially confusing because the tuning and coupling capacitances required are often so small that inter-electrode capacitances of the active device, and other 'strays', are sufficient.

An example of this is what is known as the tuned-anode tunedgrid oscillator, Fig. 14.13. It relies on the fact, shown in Fig. 8.1, that when a current flows through inductance and capacitance in series the voltages across them are in opposite phase. So if the reactance of $C_{2}$ is greater than that of $L_{1}$ the voltage across $L_{1}$ is in opposition

Fig. 14.7-This variety of Fig. 14.6 is the basis of the important Hartiey' oscillator circitit


Fig. 14.8-If the oscillatory circuit is tapped on the cupacitive side instead of on the inductive side as in Fig. 14.7 it is a Colpitts circuit

Fig. 14.9-The capacitor in the oscillatory circuit is often connected to the tapping point: When provided with feeds this circuit is seen to be the same as in Fig. 14.4


Fig. 14.11-Parallel-fed Hartley circuit, with series grid bias

to that across the output tuned circuit LC, and therefore in the correct phase for maintaining oscillation. In practice the capacitance between the anode and grid is usually enough to serve as $\mathrm{C}_{2}$, which therefore does not appear in the diagram. The value of the grid-tocathode inductance is usually adjusted by $\mathrm{C}_{1}$, which looks like a


Fig. 14.12-Parallel-fed Colpitts circuit


Fig. 14.13-Tuned-anode tuned-grid (T.A.T.G.) oscillator circuit. The phase-reversing part of the circuit is $C_{2}$ (normally the valve itself) and $L_{1}$
tuning capacitor. The name of the circuit suggests the same thing, but must not be taken to mean that grid and anode circuits are both tuned to the frequency of oscillation. If $\mathrm{L}_{1} \mathrm{C}_{1}$ were so tuned it would be equivalent to a resistance (Sec. 8.10) and not an inductive reactance as required. Without $C_{1}$ this reactance could not be adjusted so conveniently, and $\mathrm{L}_{1}$ would have to be much larger.

Although transistors also have internal capacitance between collector and base, the input impedance between base and emitter is usually too low to allow this type of oscillation.

### 14.7 Resistance-Capacitance Oscillators

We have approached the subject of oscillators by considering an $L C$ circuit with its natural frequency of oscillation, and then connecting an amplifier to feed it with regular periodical doses of energy to prevent those natural oscillations from dying out owing to circuit resistance or loading. We can alternatively regard the $L C$ circuit simply as a positive-feedback path from output to input of the amplifier, so designed that only a.c. of one particular frequency is able to reach the input at the strength and phase relationship capable of maintaining itself via the amplifier. A circuit that selects or rejects particular frequencies or bands of frequencies is known as a filter. Filters are not necessarily oscillatory circuits, or even combinations of $L$ and C. Fig. 6.10 shows how, when a voltage such as $a b$ is applied to a capacitance and resistance in series, the voltages across C and R separately lead or lag $a b$ by a phase angle determined by $C$ and $R$ and the frequency of the voltage. Now, as Fig. 14.5 showed, the input to a common-source (or cathode, or emitter) amplifier required to maintain oscillation must be $180^{\circ}$ different in phase from the output, and at least $1 / A$ times the output in strength, $A$ being the gain of the amplifier. Study of the situation illustrated by Fig. 6.10 shows that by the time the phase difference is $90^{\circ}$ the ooutput' voltage is nil. But by using three stages of this elementary form of filter, as in Fig. 14.14, a total phase difference of $180^{\circ}$ can

Fig. 14.14-Three-stage CR filter circuit for obtsining the $180^{\circ}$ pliase shift required by a


Fig. 14.15-The filter in Fig. 14.14 can be developed into an oscillator by connecting a simple amplifier between OUT and IN
be achieved with an overall loss which can be matched by the gain of a transistor or valve. The output of this filter is connected to the input of the amplifier, whose output is connected to the input of the filter, as in Fig. 14.15.

This type of oscillator has a rather limited use, because the component values and the active device have to be well chosen to ensure oscillation; varying the frequency of oscillation is not very convenient because several component values have to be varied in order to do so; because of stray capacitances it is not really suitable for high frequencies; and the waveform is prone to depart from the sine shape. But it does illustrate the possibility of a sinusoidal oscillator without the inductance that is essential in a natural oscillatory circuit, and $R C$ oscillators in more elaborate forms are very useful as audio-frequency signal generators.

### 14.8 Negative Resistance

Going back to the $L C$ oscillatory circuit again, we recall that the reason for the dying out of its oscillation was resistance. In Chapter 8 we studied the total effective resistance of such circuits and came to the conclusion (Sec. 8.16) that it could be represented as a (normally relatively low) resistance in series with $L$ and $C$, or as a (normally relatively high) resistance in parallel with them. Fig. 14.16

(a)

(b)

Fig. 14.16-The losses that cause damping in an oscillatory circuit can be represented by either series (a) or parallel (b) resistance
may bring it to mind. Since then we have come across duality (Sec. 12.2) and can now see $a$ and $b$ as duals of one another and be inclined to refer to the high resistance as a low conductance. Now if we had such a thing as negative resistance we could insert $-r$ ohms in series in Fig. 14.16a and thereby reduce the total resistance to zero, so that oscillations once started would continue unabated. Or if we had negative conductance we would connect $-1 / R$ siemens of it in parallel with Fig. $14.16 b$ and get the same result.

Although there is no such thing as negative resistance or conductance as a passive component, an active device can be arranged to simulate it, and this is how we can regard the maintaining amplifier in each of our oscillator circuits.

We have seen that in a graph of current against voltage applied to a conductance the value of the conductance is represented by the slope of the graph. The slope of a graph of voltage against current represents the resistance concerned. Our first graphs of this kind, such as Fig. 2.3, referred to ordinary resistors and were straight lines passing through the origin. When we came to active devices we found that graphs of their voltages and currents were not by any
means straight; their varying slopes implied resistances and conductances that varied with the values of current and voltage. But whatever the slopes, an increase in voltage always went with an increase in current, so that the slopes, which are the ratio of the increases, were always positive. Load lines are not an exception, because their voltage scales (not shown) run from right to left. However, there are a few active devices which have current/voltage characteristics which over parts of their ranges slope the 'wrong' way. An increase in applied voltage, say, causes a fall in current.

One such device is a type of semiconductor diode in which the impurity content is vastly greater than in the usual types. It is called a tunnel or Esaki diode, and for reasons which would take too long to go into here its $I / V$ characteristic takes the form shown in Fig. 14.17. We can say that over the range $A$ to $B$ its conductance is


Fig. 14.17-Current/voltage characteristic curve of a tunnel diode, showing negativeresistance range. A to B
negative. Connecting it in parallel with an oscillatory circuit having a positive conductance not greater than the negative conductance at the selected working point results in sustained oscillations. If however the positive conductance is the greater, as shown by the greater slope of the load line cutting the curve at C , the net conductance is positive, the working point remains stable at C , and any oscillations are damped-though not so heavily as without the diode.

Devices are known in which there is a negative slope in part of the $V / I$ characteristic, representing negative resistance, which is capable of cancelling positive resistance in series.

Negative conductances and resistances can be represented in an extension of the equivalent generator method explained in Sec . 12.5.

Suppose, as in Fig. 14.18, an internal voltage generator provides a reverse voltage exceeding that applied. The applied voltage $V$ then apparently makes current flow out from instead of into the resistor. In the example shown, the total circuit voltage corresponding to $+V$ applied would be $V-1 \frac{1}{2} V=-\frac{1}{2} V$, and the current would be $\frac{1}{2} V / R$, so the apparent resistance would be $V \div-\frac{1}{2} V / R,=-2 R$.


Fig. 14.18-When, as here, the imernal feedhack voltage in Fig. 12.8 exceeds the applied voltage, the box appears to contain a megative resistance

This could of course be expressed as a conductance $-1 / 2 R$, and could alternatively be simulated by the equivalent current generator in parallel with $R$ (Sec. 12.1).

An advantage of tunnel diodes and other devices that have inherent negative conductance or resistance characteristics is that they need only be connected to the oscillatory circuit at two points, instead of three (at least) required with the amplifier maintaining circuits discussed previously. This is seldom a vital point ; where it is, amplifier circuits can be designed for two-point connection, as we shall see in Sec. 14.11.

### 14.9 Amplitude of Oscillation

So far it has been assumed that the amplifier or other negativeresistance device used for maintaining oscillation was applied only just sufficiently to make good the losses. This is quite an unpractical condition, because the adjustment would be impossibly delicate in the first place, and even if it could be achieved it would be upset by the slightest change in temperature, component or device parameters, etc., so that oscillation would either start dying out or building up. In the latter event, what stops it growing so that it levels out? The answer is that the range of voltage or current over which the negative-resistance or amplifying characteristics exist is limited. This is very easily seen in Fig. 14.17; and we have already noted that the amplifying parameters we studied in Chapter 12 are valid only so far as the slopes of the characteristics at the working
point continue linearly in both directions. That is why they were called small-signal parameters. Sooner or later those slopes come to an end, and with them the properties that made the amplitude of oscillation increase.

This can be illustrated by an example, Fig. 14.19. Although not identical with any of the previous oscillator circuits, all its features have already been explained. The $I_{\boldsymbol{c}} / V_{C}$ characteristics and load line are the same as in Fig. 13.2, and so is the working point P , but there is an interesting difference in the conditions represented thereby. In Fig. 13.2 the load was a real resistance, so of the supply voltage $V_{c c}$, which was 12 V , half was lost in the resistance and only the remaining half reached the transistor as $V_{c}$ at point $P$. In Fig. 14.19 however the resistance of the coil L to d.c. is likely to be negligible, so only a 6 V battery is needed to maintain the working $V_{c}$ at 6 V .
The load line represents the dynamic resistance of the $L C$ circuit, at the frequency of oscillation only (Sec. 8.10) and indicates by its


Fig. 14.19-A simple oscillator circuit investigated by the load-line technique
slope that this is $6 \mathrm{k} \Omega$. It is the parallel equivalent of the oscillationfrequency losses in L and C and also the resistance across the feedback coil multiplied by the square of the effective transformer step-up ratio from it to $L$ (Sec. 7.11). Part of the resistance across the feedback coil is the input resistance of the transistor, which we
know depends very much on the working point - see Fig. 10.12, for instance-but let us suppose that its variableness is reduced to a sufficiently small proportion by the constant resistance of the biasing potential divider for it not to have much effect on the total load resistance of the transistor. Suppose also that the amplitude of oscillation across L is 3 V peak. which means that $V_{c}$ varies between 3 V and 9 V . Tracing this along the load line shows that in order to maintain this amplitude $L$ needs to induce in the feedback coil sufficient voltage to make the base current oscillate between $30 \mu \mathrm{~A}$ and $10 \mu \mathrm{~A}$; i.e., $10 \mu \mathrm{~A}$ peak reckoned from the working point level, $20 \mu \mathrm{~A}$. If in fact it induces a shade more, the amplitude of oscillation grows, but by the time it has doubled, swinging hetween 0 and $40 \mu \mathrm{~A}$, there is practically no scope for further increase.

In this case there is a sudden limiting at both ends, but sometimes the working point and load line are so placed that cut-off oceurs first at one end, or more gradually - especially in the $I_{B}^{\prime} V_{B}^{\prime}$ characteristic, the effect of which we have been neglecting. The amplitude at one end may then continue to increase, until the reduction of $h_{\text {fe }}$ at the other end, perhaps to zero, over an increasingly large proportion of each cycle, automatically limits the feedback power to a point where a balance is reached.

Sometimes a diode is used to provide a sharp amplitude-limiting effect that is more easily controlled in this respect than the amplifying device itself.
F.e.ts and valves are normally biased to a point that allows a range of oscillation between cut-off at one end and the start of grid or gate current at the other. Within this range practically no input current flows and the oscillatory circuit is not loaded thereby. But beyond it the amplitude of oscillation tends to be limited by output current being cut off, or the $L C$ circuit being loaded by input current, or both.

### 14.10 Distortion of Oscillation

The last section can be summarized by saying that the amplitude of oscillation grows until it is limited by non-linearity of the charac-


Fig. 14.20- If the amplitude of oscillution soes pasis the ends of the loud line in Fig. 14.19 the peuks of collector current will he flattened as shown dotted
teristics of the maintaining device. As we shall be seeing more fully in Chapter 19, non-linearity causes distortion of the oscillation waveform, which in its natural state is sinusoidal. One cycle appears in full line in Fig. 14.20. Under the conditions shown in Fig. 14.19 both peaks of collector current would be cut off fairly sharply, as
shown by the dotted line, representing severe distortion of the current waveform. But it would be wrong to suppose that this represents distortion of the oscillation waveform. To repeat: this is the waveform of the collector current. Its job is merely to make good the loss of energy in the oscillatory circuit through which it flows. Our study of this basic circuit in Chapter 8 showed that the oscillatory circulating current is $Q$ times as much as needs to be supplied from outside. So no matter how badly this supplied current is distorted, it cannot affect the waveform of the oscillatory current much if $Q$ is large. To go back to our clock analogy, the pendulum maintains its steady oscillation 'undistorted' although the periodical pushes administered by the escapement are anything but simusoidal in 'wavetorm'. And it is quite usual for electronic oscillators to be kept going by square waves. pulses or what have you.

But to cbtain a very pure waveform-and this is often required of an oscillator-it is sensible to make sure that the oscillatory circuit itself has as high a $Q$ as practicable. and that this is not spoilt by resistance transferred into it by connecting the maintainer. In other words, the maintainer should not be coupled more closely to the $L C$ circuit than is necessary to ensure adequate oscillation in all conditions of use. The higher the $Q$ the less maintenance is needed. The loose-coupling policy also tends to prevent even the small maintaining part of the current from being badly distorted.

For many purposes non-sinusoidal waveforms are required: methods of generating them will be deferred to Chapter 24.

### 14.11 Constancy of Frequency

The frequency of an oscillator, as we have seen, is adjusted by varying the inductance and/or capacitance forming the oscillatory circuit. For most purposes it is desirable that when the adjustment has been made the frequency shall remain perfectly constant. This means that the inductance and capacitance (and to a less extent the resistance) shall remain constant. For this purpose inductance includes not only that which is intentionally provided by the tuning coil, but also the inductance of its leads and the effects of any coils or pieces of metal (equivalent to single short-circuited turns) inductively coupled to it. Similarly capacitance includes that of the coil, the wiring and the maintaining device. Resistance depends on many things, such as loading and feed voltage variations. So some of the conditious for constant frequency are the same as those for undistorted waveform: high $Q$ and minimum coupling with the maintaining device.

An example of an oscillator circuit (due to C. S Franklin) designed for constancy of frequency is shown in Fig. 14.21. Here two stages of amplification are used, one result of which is to give a much greater amplification, so that only a correspondingly very small fraction of the output need be fed back in order to maintain
oscillation. $\mathrm{C}_{1}$ is in fact the feedback connection and typically is only 1-2 pF. $\mathrm{C}_{2}$, which connects the oscillatory circuit LC to the input, has a similarly very small capacitance. The total effective capacitance of the amplifier being only a small fraction of the total frequency-determining capacitance (most of it being $C$ ), any small variations in it affect the frequency to a very small extent indeed. Another result of using two stages is that the phase reversal needed in single-stage amplifiers is provided by the second stage, so no phase-reversing tapping is needed. Sufficient information has already been given to enable the purposes of all the components


Fig. 14.21-By' using a lot of amplification and the minimum of feedback, good constancy of frequency can be obrained
shown in Fig. 14.21 to be understood, but if there are any doubts they should be cleared up when we consider amplifiers in greater detail, in Chapters 19 and 20.

Inductance and capacitance depend mainly on dimensions, which expand with rising temperature. If the tuning components are shut up in a box along with the maintaining device, the temperature may rise considerably, and the frequency will drift for some time after switching on. Frequency stability therefore is aided by placing any components which develop heat well away from the tuning components; by arranging effective ventilation; and by designing tuning components so that expansion in one dimension offsets the effect of that in another. Fixed capacitors are obtainable employing a special ceramic dielectric material whose variations with temperature oppose those of other capacitors in the circuit; this reminds one of temperature compensation in watches and clocks.

An alternative method of achieving constant frequency is explained in Sec. 15.8.

## Radio Senders

### 15.1 Essentials of a Sender

We now have the material for tackling in greater detail what was indicated in barest outline in Fig. 1.9. Considering the sending or transmitting system first, we have as the essentials for, say, broadcasting sound:
(a) A radio-frequency generator.
(b) A microphone, which may be regarded as an audio-frequency generator.
(c) A modulator, combining the products of (a) and (b).
(d) An aerial, radiating this combination.

In a television sender the camera takes the place of the microphone, and it generates a considerably wider band of frequencies, called video frequencies.

### 15.2 The R.F. Generator

The need for constancy of frequency (Sec. 14.11) is particularly important in radio senders. Their ever-increasing number often necessitates several working on the same frequency, and to avoid excessive interference their frequencies must be kept correct within one part in millions. And when working frequencies are hundreds or even thousands of millions of hertz, constancy of a high order is needed to prevent interference between senders on different but neighbouring frequencies and to avoid need for readjusting receiver tuning. As we have seen, policies for achieving this constancy include making the $Q$ factor of the oscillatory circuit as high as possible and minimizing the influence on it of the load, by loose coupling. On the other hand, especially with high-power senders, a main object is to ensure that as much as possible of the d.c. feed power expended is delivered at radio frequency to the load-in this case the aerial.

As these two basic requirements are quite incompatible, the functions of oscillating at the required frequency and of generating r.f. power for delivery to the aerial are kept separate. The frequency is generated by an oscillator in which all necessary precautions are taken to ensure constancy of frequency and amplitude. This can best be done at low power, and by actually using only a small part even of that (in order to limit the influence of the load, as explained
in Sec. 14.11). The r.f. power is generated by an amplifier (called the power amplifier) designed for maximum power efficiency consistent with other requirements such as waveform. Even in quite a lowpower sender the output available from the oscillator would be inadequate as the input for the power amplifier, and at least one and usually several driver stages of amplification are needed in between.

### 15.3 The Power Amplifier

Until the final (power amplifier) stage is designed one cannot tell what will be required of the others. So that is where we begin. Transistors can be used for relatively low powers, but valves are available for all powers up to 100 kW or more, and stand up better to moderate or temporary misuse. The higher the power the sender is required to produce, the more important it is to get maximum efficiency, which is defined as the ratio of useful output power to input feed power.

In considering valves (and f.e.ts) as small-signal amplifiers we noted that normal operation was restricted to the negative range of


Fig. 15.i-Example of $I_{\mathrm{a}} / V_{\mathrm{a}}$ characteristic curves and loal line for a high-power triode value
grid voltage, the reason being that when the grid is positive it forms, with the cathode, a forward-biased diode, so current flows and one of the main advantages of a valve is lost. This restriction if applied to sending (or transmitting) valves would limit the power output to a fraction of their capability, so the grid is driven positive as well as negative, and the need for the driver to supply power rather than
merely voltage is accepted. So the term 'power amplifier' for the final stage is fully appropriate.

Fig. 15.1 shows $I_{a} / V_{a}$ characteristic curves for a Mullard TY12-15A triode valve, and while the negative-grid curves have the same general shape as the triode curves we have already seen (Fig. Il.3 for example) if allowance is made for the compressing effect of the $I_{\mathrm{a}}$ scale, in the positive-grid area they are more like f.e.t. or even bipolar transistor curves. They enable one to work much farther to the left and thereby obtain a greater voltage swing. Let us suppose that the maximum allowable peak anode voltage is 10 kV , and that the grid can be allowed to swing between -200 V and +1000 V . Then, working on the same lines as hitherto. we would give the valve +400 V bias and arrive at a load line something


Fig.15.2-Dynamic $I_{\mathrm{s}} / V_{\mathrm{g}}$ characteristic curve derived from Fig. 15.1, showing how to plot the output current wavefarm corresponding to a given imput voltage'
like that shown. with an anode supply voltage of 5.2 kV . The anode current at this working point would be 22 A . so the input to the anode is $22 \times 5.2=114.4 \mathrm{~kW}$.

Fig. 15.2 is another way of presenting the situation, enabling the grid and anode voltage swings to be seen instead of just being imagined. It is derived from Fig. 15.1 by plotting $I_{\mathrm{a}}$ against $V_{\mathrm{g}}$, but because it takes account of the working variations in $V_{a}$ along the load line it is called a dynamic characteristic curve and ought not to be confused with the type of $I_{\mathrm{a}} / V_{\mathrm{g}}$ curve shown in Fig. 10.4. in which the anode voltage is kept constant. One cycle of input from the driver stage is shown lying on its side to correspond with the
grid voltage scale. The resulting output current is plotted point by point. An example is shown: at one eighth of a cycle along the input voltage scale ( W ) the voltage is given by point X ; projecting this upwards to the characteristic curve gives point $Y$, and projecting this along to one eighth of a cycle on the output current time scale locates $Z$, a point on the output graph. And so on. The curve being roughly linear, the output has nearly the same waveform as the input. Corresponding to the input swing between -200 V and +1000 V is an output swing between 2 A and 38 A . Fig. 5.1 shows that the corresponding output voltage swing is 10 kV to 1.4 kV . The voltage peak-to-peak would therefore be $10-1.4=8.6 \mathrm{kV}$, so the peak voltage is 4.3 kV , or $4 \cdot 3 / \mathcal{V}^{\prime} 2=3 \cdot 04 \mathrm{kV}$ r.m.s. Similarly, the output current would be $(38-2) / 2 \vee 2=12.73$ A r.m.s. Provided they were exactly in phase (because going into a pure resistance load) the output power would be $3.04 \times 12.73=38.7 \mathrm{~kW}$. The efficiency would then be $38 \cdot 7 / 114.4=33.8 \%$.

Ideally, the input to the valve under conditions like this would be the same ( 114.4 kW ) whether it was working or not, because when working the voltage and current swings are equally above and below the point $P$ so they average out. This being so, it may be difficult to see how an output of 38.7 kW can be obtained when the input is just the same as if there were no output at all. The explanation is that when the anode voltage is highest the current is nearly cut off, and vice versa, so the amount of power wasted in the valve is less than when both current and voltage are constant at $P$. It works out that the amount less is exactly equal to the output. So under working conditions $114.4-38.7=75.7 \mathrm{~kW}$ is lost as heat at the anode. The maximum rating of this particular valve is 15 kW , so its life under these conditions would be very brief! Even at 15 kW the heat has to be carried away by forced-air cooling, and in some types by water cooling.

The mode of working just described, in which the a.c. swings equally above and below the working point, is termed Class $A$. While it is excellent for non-oscillatory amplifiers in which absence of distortion is important and power consumption unimportant, in high-power applications it is clearly very wasteful. Even with an imaginary perfect d.c.-to-a.c. converter device the theoretical maximum efficiency is $50 \%$, and in practice $35 \%$ is quite good. The overall efficiency is much lower still when the swing is not at maximum all the time; for example, when amplifying a sound or vision programme. The full feed power has to be maintained even during intervals.

Incidentally, because sinusoidal a.c. power into a resistive load is $I V, I$ and $V^{\prime}$ being the r.m.s. values, and r.m.s. values are peak-topeak values ( $I_{p-p}$ and $V_{p p p}$ ) divided by $2 \sqrt{ } 2$, we can calculate the power quickly as

$$
P=\frac{I_{p-p} V_{p-p}}{2 v^{\prime} 2 \times 2 \sqrt{2}}=\frac{I_{p-p} V_{p-p}}{8}
$$

### 15.4 High-Efficiency Amplifiers

If we bias the valve almost to cut-off and double the grid voltage swing, as in Fig. 15.3. the non-working anode feed is 2 A at 10 kV , or 20 kW . It could of course be reduced to zero by increasing the bias a little more. When working at full stretch the anode current is cut off entirely during alternate half cycles, and during the other half


Fig. 15.3 -For comparison with Fig. 15.2 (Class A) this diagram shows Class B operation


Fig. 15.4-The push-pull configuration has advanages for nearly all power amplifiers. but especially for Class B and Class $C$
cycles, when it is at its peaks, the anode voltage is at its minimum. The output waveform is of course nothing like the input waveform, but that is not an essential requirement with an ossillatory output circuit (Sec. 14.10). However. this mode of operation, called Cluss $B$, can be made to give an almost perfectly undistorted output, acceptable even for audio amplification, if two valves are connected in what is called push-pull, shown in outline in Fig. I5.4. $\mathrm{L}_{1} \mathrm{C}_{1}$ is the input oscillatory circuil receiving the output of the driver stage and across which is built up the grid voltage swing for the twin valve
stage shown. $\mathrm{L}_{\mathrm{o}} \mathrm{C}_{\mathrm{o}}$ is the output oscillatory circuit, to which the aerial is coupled in such a way as to be equivalent to a resistance load that absorbs the output of the amplifier most efficiently. In this context, $L_{1} C_{1}$ and $L_{o} C_{o}$ are often called tank circuits, because they store energy. While one valve is cut off the other is contributing its half cycle, and as the two contributions are in opposite phase they add together to form complete cycles. We shall be considering the pushpull circuit in its a.f. applications in Chapter 19.

The ideal efficiency of a Class B amplifier (push-pull or singleended) is $\pi / 4,=78.5 \%$, and over $60 \%$ is obtainable in practice.

The Class B policy can be pressed farther, by increasing the grid bias to well beyond cut-off and increasing the grid swing correspondingly. The effect, as we can find by drawing another diagram of the Fig. 15.3 type, is to reduce the period during which current flows to less than half of each cycle, concentrating it still more into the period when the anode voltage is at its lowest. The output power is reduced by this narrowing of the current pulses, but the feed power is reduced in greater proportion, so that ideally the efficiency approaches $100 \%$. But to obtain a reasonable output a compromise is necessary, and current flow is typically adjusted to one-third of each cycle, or $120^{\circ}$ in angular measure (Sec. 5.7). Up to $80 \%$ efficiency can be obtained in practice. But of course a very large input to the grid must be provided, including a significant amount of power not included in the foregoing efficiency figures.

This mode of operation is distinguished as Class $C$.

### 15.5 Constant-Current Curves

It must be admitted that the last section, in so far as it applied to amplifiers with oscillatory (tank) output circuits. cheated a little. In Figs. 15.1-3 we used the ordinary load line technique, on the assumption that the output load, being a resonant tuned circuit, at the working frequency was equivalent to a resistance. When the valve output current and voltage are sinusoidal and therefore are respectively proportional to the current circulating in the tank circuit and equal to the voltage across it, as is the case in Class $A$, this is perfectly sound. But in Classes B and C it is not so; the output tank circuit, if reasonably high in $Q$, forces the circulating current and consequently the voltage across it, to be very nearly sinusoidal, whereas the valve current and voltage waveforms are far different. The effect of this is that the load is not equivalent to a resistance that is constant throughout each cycle, so the load line is not straight and in fact is quite difficult to determine. Fig. 15.3 is therefore invalid. As it happens, it is near enough to demonstrate the principle of Class B on a basis of what has become familiar, but would not do for numerical design purposes. So the type of characteristic curves used
in practice and usually provided by the makers of high-power valves should now be looked at, Fig. 15.5.

Here grid and anode voltages are plotted against one another, anode current being kept constant at a different value for each curve. These could be called $\mu$ curves, because the definition of the voltage amplification factor, $\mu$ (Sec. 10.2) shows that it is equal to the reciprocal slope of these curves. Incidentally, they support the statement in Sec. 10.3 that $\mu$ is very nearly constant over a wide range of

Fig. 15.5-Example of constantcurrent characteristic curves, needed for designing high-power high-efficiency amplifiers

operation. To enable the input circuit also to be designed, the dotted constant-grid-current curves are usually included.

On the assumption that grid and anode voltages are both across reasonably high $-Q$ circuits tuned to the same frequency and therefore in exactly opposite phase, we can draw sine waves parallel to grid and anode voltage scales and from these plot the line linking them up, in the same way as we could have arrived at the sloping line connecting input voltage and output current in Fig. 15.2 if we had been given them and not the line. And when, as we are now assuming, input and output waveforms are identical. the line must be a straight one. Alternatively, and more practically, we can draw the straight line on the curve sheet to agree with the valve rated limits and the desired mode (Class B, etc.) and from it derive the corresponding input and output voltage swings. Fig. 15.6 shows a positive sinusoidal half-cycle of grid voltage, working from a Class C bias of -1000 V . Using the $V_{g}$ scales in this diagram and in Fig. 15.5 to transfer points on this half-cycle to the load line in Fig. 15.5, we can read off the corresponding values of $I_{\mathrm{a}}$ and $I_{\mathrm{g}}$ and transfer them back to Fig. 15.6 to yield the half-cycles shown there. Although the
negative half-cycle of $V_{g}$ exists, there is no point in showing it, because $I_{\mathrm{a}}$ and $I_{\mathrm{g}}$ are of course zero during that phase. Incidentally, a load line of the Fig. 15.1 type can be derived from Fig. 15.5 and its non-linearity confirmed.

### 15.6 Neutralizing

Comparing Fig. 15.4 with Fig. 14.13 we can see that it is potentially a t.a.t.g. self-oscillator, which is not at all what it is meant to bean amplifier. Remembering that an essential requirement for oscillation in this circuit is that the input circuit is inductive, while $\mathrm{C}_{1} \mathrm{~L}_{1}$ is tuned to the working frequency, we might think that self-


Fig. 15.6-Anode and grid current waveforms corresponding to a given inpur. obtained by the use of Fig. 15.5
oscillation would be impossible. But at a frequency slightly lower than the working frequency this requirement is fulfilled. In any case, the feedback via anode-to-grid capacitances makes practical tuning of the circuits virtually impossible. While it is true that selfoscillation is prevented by negative bias when there is no drive, it would occur directly drive was applied.

To prevent this, equal and opposite feedback is applied via neutralizing capacitors connected from each valve's anode to the opposite valve's grid. In single-valve r.f. amplifying circuits, such as are generally used at least in the earlier stages, either grid or anode tuning circuit is centre-tapped as shown in Fig. 15.4, providing a point of opposite potential for the neutralizing capacitor, NC in Fig. 15.7.

### 15.7 Tetrodes and Pentodes

An alternative method of preventing self-oscillation is to get rid of the capacitance that causes it. The capacitance between any two objects can be eliminated by placing between them as a screen an earthed metal sheet of sufficient size. The operation of such a screen can be understood by considering Fig. $15.8 a$, in which a stray

Fig. 15.7-Showing how a neutralizing capacitor (NC) can be introduced into an r.f. amplifier circuit to prevent self-oscillation


Fig. 15.8-Illustrating the theory of screening
capacitance is represented by two plates A and B separated from one another by an air-space. The a.c. e.m.f. $E$ will therefore drive a current round the circuit Earth- $E-\mathrm{A}-\mathrm{B}-Z_{1}-$ Earth. Across $Z_{1}$, which is an impedance of some kind between B and earth, the current will develop a potential difference, and this p.d. will be the voltage appearing on B as a result of current passing through the capacitance AB.

At $b$ a third plate S , larger than either of the two original plates, is inserted between them in such a way that no electric field passes directly from A to B. We now have no direct capacitance between A and B, but we have instead two capacitances, AS and SB, in series. If an impedance $Z_{2}$ is connected between $S$ and earth the current round the circuit Earth- $E-\mathrm{A}-\mathrm{S}-Z_{2}$ - Earth will develop a p.d. across $Z_{2}$. Since $Z_{2}$ is also included in the right-hand circuit the voltage across it will drive a current round the circuit Earth - $Z_{2}$ -$\mathrm{S}-\mathrm{B}-Z_{1}$-Earth, and this will give rise to a potential on B. So far, $S$ has not screened $A$ from $B$; there remains an effective capacitance between them which, if $Z_{2}$ is infinitely large, amounts

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to the capacitance equivalent to that of AS and SB in series.
Now imagine $Z_{2}$ to be short-circuited. Current will flow round the first circuit, but since there is now no impedance common to both there will be no driving voltage to produce a current in the latter. No matter what alternating voltages are applied to $A$, none will appear on $B$, even though large currents may flow via $S$ to earth. The effective capacitance between $\mathbf{A}$ and B has therefore been reduced to zero, and $B$ is completely screened from $A$.

It is very important to note that $\mathbf{S}$ is only effective as a screen if it is connected to earth (or to a constant-potential point in the circuit) by a negligible impedance.

This is the principle used in reducing the anode-to-grid capacitance of a valve. A screen is put between anode and grid, while capacitance between the wires running to grid and anode is avoided by extending the screening down to and below the base, or in some valves by taking one or other of them out through the top of the bulb.

Clearly, a solid metal sheet, while providing perfect screening. would cut off the electron flow from cathode to anode. The screen actually used therefore consists either of metal gauze or a wire grid similar to the first or control grid. The spaces between the wires, while allowing electrons to pass freely, affect the screening remarkably little. Its effectiveness can be judged from the fact that it reduces $c_{\text {ag }}$ to about one-hundredth of that capacitance in a triode of similar rating.

If the screen were earthed by connecting it straight to cathode, it would cut off the attraction of the positive anode; electrons near the grid would then not be drawn onwards, and the anode current would fall practically to zero. But since, as Fig. 15.8 shows, the requirements of screening can be met by making $Z_{2}$ negligibly small, we can connect a large capacitance between the screen grid and cathode of the valve, and then supply a suitable positive potential to the screen grid.

The portion of the valve comprising cathode, control grid, and screen grid is a triode, and could be used as one, the screen grid acting as an anode. The current through the valve is practically unaffected by whatever voltage may be connected to the real anode, because its electric field is screened off. But if, as is usual, the screen grid is given a constant positive potential, the electrons from the cathode are accelerated towards it so rapidly that unless they chance to hit one of its wires they go right through. What happens to them then depends on the potential of the anode. If it is zero or less, practically all the electrons are attracted back to the screen grid, just as a ball thrown up into the air reverses and comes back. But if the anode is substantially positive it attracts and retains those electrons that come near it. Since the number that do so depends mainly on the potential of the screen grid and very little on the potential of the anode, increasing the positive anode voltage beyond a certain amount has very little effect on the anode
current. The slope of the $I_{\mathrm{a}} / V_{\mathrm{a}}$ characteristic curve is therefore very gradual, like the slope in the corresponding curves for transistors, and the $r_{\mathrm{a}}$ is consequently large; much larger than in a triode. As such valves have four electrodes they are called tetrodes, and Fig. 15.9 shows the symbols for it and its electrodes.

A complication arises, however. When electrons bombard electrodes they often dislodge electrons already there and enable them

Fig. 15.9-Symbol for a tetrode (screened) valve

to be emitted. This effect is called secondary emission. Such emission takes place at the anodes of valves, but it is not noticeable in triodes because the anode is more positive than any other electrode and its attraction prevents the emitted electrons from getting away. This happens also in tetrodes when the anode voltage is higher than the screen-grid voltage, but when it is lower the screen-grid outbids the anode in attracting the secondary electrons. The higher the anode voltage (within the range below $V_{g 2}$, the screen grid voltage) the greater the secondary emission and the greater the current awo from the anode, to be deducted from the electronic current reaching it from the cathode. Increasing the anode voltage therefore reduces the net anode current, causing $r_{\mathrm{a}}$ to be negative. It is only when $V_{\mathrm{a}}$ is greater than $V_{\mathrm{g} 2}$ that the $I_{\mathrm{a}} / V_{\mathrm{a}}$ curve resumes its normal shape.

The secondary emission effect makes a big hole in the useful range of anode voltage. One way of eliminating it is to shape the two grids and design the whole structure of the valve so as to make the electrons travel in beam formation through both grids to the anode. Valves of this type, called beam tetrodes, not only are free from secondary emission distortion of characteristic curves but the screen-grid current (which is a dead loss) is reduced. A nd $V_{82}$ can usually be supplied direct from the same source as $V_{\mathrm{a}}$ instead of a lower voltage having to be provided.

An alvernative solution is to have a third grid, called the suppressor grid, between $g_{2}$ and the anode and kept at the same potential as the cathode, to which it is of ten connected internally. This fiveelectrode valve is called a pentode. The zero potential of $g_{3}$ turns back secondary electrons to the anode so that they are not lost. The faster-moving primary electrons have sufficient momentum to carry them past $g_{3}$ into the zone of anode attraction. The addition of $g_{3}$, kept at constant potential, reduces still further the influence of anode voltage on anode current; in other words, pentodes tend to have even higher $r_{a}$ than do comparable tetrodes. And because $\mu$

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is a measure of the effect of $V_{\mathrm{g} 1}$ on $I_{\mathrm{a}}$ as compared with that of $V_{\mathrm{a}}$, both tetrodes and pentodes have values of $\mu$ far higher than triodesof the order of 50 times greater, in fact. Fig. 15.10 shows how typical
$I_{a}, A$


Fig. 15.10-Typical $1_{a} / V_{a}$ curves for a power pentode valve
$I_{\mathrm{a}} / V_{\mathrm{a}}$ curves for a power tetrode are similar in general shape to those of transistors, especially f.e.ts.

Although anode-to-grid capacitances are so small that neutralizing is hardly needed, in practice a correspondingly small amount is sometimes provided in sender circuits.

### 15.8 The Master Oscillator

We now go right to the other end of the sender, because the number and design of the intermediate stages depend not only on the amount of amplification needed but also on matters not yet determined. The function of what is usually called the master oscillator is to define the frequency of the station at least as accurately and constantly as the now very narrow international regulations require. We looked at this problem in the last chapter, especially Sec. 14.11, but one important aid to constant frequency was not included there. It exploits the remarkable properties of certain crystals, notably quartz. Such crystals are capable of vibrating mechanically up to high radio frequencies and in doing so they develop an alternating voltage between two opposite faces. Conversely, an a.v. applied
between the faces causes vibration. This is known as the piezoelectric effect.

The subject is a large and specialized one, but for our purpose the crystal can be regarded as a tank circuit of remarkably high inductance and $Q$. Unlike the coil-and-capacitor tank circuit, a crystal once it has been cut to the size and shape needed to vibrate at the required frequency has hardly anything about it that can vary. There is a small change in frequency with temperature, so where requirements are stringent the crystal is kept in a temperaturecontrolled box.

The crystal is mounted between two metal plates, which are sometimes coated on to it, and for best results is enclosed in a vacuum. Fig. 15.11 shows an oscillator circuit in which the crystal itself is the

Fig. 15.11-One of many types of crystal oscillator circuit

only tuned circuit. The nature of this circuit is not clear unless the capacitances of the valve's anode and grid (shown dotted) are taken into account; it then turns out to be a Colpits oscillator (Fig. 14.8). A disadvantage of this circuit is that most crystals are capable of oscillating at what are called spurious frequencies. To prevent this the anode coupling resistor is often replaced by an $L C$ tuning circuit, which has insufficient impedance to sustain oscillations at any frequency other than the one to which it is tuned. Many other crystal-controlled oscillator circuits, both valve and transistor, have been devised.

Although the frequency of a crystal-controlled oscillator can be adjusted within very narrow limits (a few parts in 10000 ) by means of a variable capacitor in parallel, it is virtually a fixed-frequency oscillator. Crystals being relatively inexpensive, they can be used in a sender working on a number of spot frequencies. But if it is necessary to be able to vary the oscillator over a band of frequencies an $L C$-tuned variable frequency oscillator (v.f.o.) must be used, with appropriate precautions.

### 15.9 Frequency Multiplication

As far back as Fig. 1.2c we saw that when sine waves having frequencies in the ratios $1: 2: 3$ etc. are added together the result is a non-sinusoidal waveform which repeats at its lowest frequency the one represented by 1 . In general the waveforms generated by musical instruments, buzzers, sirens, etc-and, because of distortion, electronic oscillators-are more or less non-sinusoidal. It has been proved that repetitive waveforms can be analysed into sine waves having exactly multiple frequencies as above. These component waves are called harmonics, but the one of lowest frequency (which is also the frequency of the composite waveform) is usually called the findamental rather than the first harmonic. The composite waveform depends on the number, amplitudes and relative phases of the harmonics. Fig. 15.12 shows a simple example, $a$ being


Fig. 15.12- A fundamental sine wave (a) plas a third harmonic (b) has a waveform (c) similar to that in the ourpur of a push-pull Class C amplifier.
the fundamental and $b$ a third harmonic phased so that two of its peaks coincide with those of $a$. The result of adding them together is shown at $c$, and from a study of Fig. 15.3 it should be recognizable as not unlike the output of a Class $C$ push-pull amplifier, driven by a grid voltage such as $a$. One can then quite correctly infer that the output of such an amplifer includes not only a component having the same frequency as the drive but at least one other with 3 times the frequency-the third harmonic, in fact.

The principle of the frequency multiplier is thus established. The beauty of the thing is that the frequency of the harmonics is as constant as that of the fundamental, at least on a percentage basis.

It is difficult to cut quartz crystals thin enough to vibrate at more than $10-20 \mathrm{MHz}$. Even $L C$-tuned oscillators work best at the lower frequencies. So for higher frequency senders frequency multiplication is needed. Up to about 100 MHz , crystals can be used as their own frequency multipliers. by getting them to vibrate at an over-
tone (a kind of harmonic) of the fundamental mode. This is done by suitable mounting. or by tuning the oscillator circuit to the overtone, or both. They are called overtones, rather than harmonics, because they are not exact multiples of the fundamental frequency.

A Class C push-pull amplifier is actually not a commonly-used form of frequency multiplier, but circuits in great variety have been devised. some in order to emphasize one particular harmonic, others to provide a whole range of harmonics.

Because the amplitudes of the harmonics tend to be progressively less as their order (i.e., multiple of frequency) is higher, a large frequency multiplication is usually done in a number of stages. The input to the first stage is tuned to the fundamental, and the output is tuned to the required harmonic, say the third. Because the impedance of this output circuit is low at all frequencies except that of the third harmonic, the third harmonic voltage is amplified and all others are more or less suppressed. It is thus quite usual to combine the functions of frequency multiplier and r.f. amplifier in the stages between master oscillator and final driver.

### 15.10 Modulation

We have now reached the point-in theory at any rate-at which we can generate a continuous stream of waves of constant frequency and (if desired) high power. This. however, is completely useless for communication of messages, let alone speech, music or pictures. For communication some kind of organized change is needed. The technical word for it is modulation. The rau material that we have provided ready for modulation is called the carrier wave, although really it is a continuous succession of waves.

To take a simple case, suppose the 'programme' to be sent is a 1 kHz tuning note, which is heard as a high-pitched whistle. If it is to be a 'pure' sound its waveform must be sinusoidal, as in Fig. 15.13a. It can be generated by a suitable a.f. oscillator but, for the reasons given in Chapter 1, waves of such a low frequency cannot be radiated effectively through space. We therefore choose an r.f. carrier wave, say 1000 kHz . As 1000 of its cycles occur in the time of every one of the a.f. cycles, they cannot all be drawn to the same scale, so the cycles in Fig. 15.13 b are representative of the much greater number that have to be imagined. What we want is something which, like $b$, is entirely radio-frequency (so that it can be radiated), and yet carries the exact imprirt of $a$ (so that the a.f. waveform can be extracted at the receiving end). One way of obtaining it is to vary the amplitude of $b$ in exact proportion to $a$, increasing the amplitude when $a$ is positive and decreasing it when $a$ is negative, the result being such as $c$. As we can see, this modulated wave is still exclusively r.f. and so can be radiated in its entirety. But although it contains no a.f. current, the tips of the waves trace
out the shape of the a.f. waveform, as indicated by the dotted line, called the envelope of the r.f. wave train.

The same principle applies to the far more complicated waveforms of music and speech, or of light and shade in a televised scene-their complexity can be faithfully represented by the envelope of a modulated carrier-wave.

Since receivers are designed so that an unmodulated carrier wave (between items in the programme) causes no sound in the loudspeaker, it is evident that a graph such as Fig. 15.13c represents
(a)

(b)

(c)


Fig. 15.13-Curve a represents two cycles of audio-frequency current, such as might be flow'ing in a microphone circuit: b represents the much larger number of radio-frequency cycles generated by the sender. When these have been amplitude-modulated by: $a$, the result is as shown at $c$
a note of some definite loudness, the loudness depending on the amount by which the r.f. peaks rise and fall above and below their mean value. The amount of this rise and fall, relative to the normal amplitude of the carrier wave, is the depth of modulation.

For distortionless transmission the increase and decrease in carrier amplitude, corresponding to positive and negative half-cycles of the modulating voltage, must be equal. The limit to this occurs when the negative half-cycles reduce the carrier-wave amplitude exactly to zero, as shown in Fig. 15.14. At its maximum it will then rise to double its steady value. Any attempt to make the maximum higher than this will result in the carrier actually ceasing for an appreciable period at each minimum; over this interval the envelope of the carrier-amplitude can no longer represent the modulating voltage and there will be distortion.

When the carrier has its maximum swing. from zero to double its mean value, it is said to be modulated to a depth of $100 \%$. In general, half the variation in amplitude, maximum to minimum, expressed as a percentage of the mean amplitude. is taken as the measure of modulation depth. Thus, when a $100-\mathrm{V}$ carrier wave is


Fig. 15.14-Carrier-wave modulated to a depth of $100^{\circ}{ }_{0}$. At its minima (points A) the r.f. currem just drops to zero
made to vary between 40 V and 160 V , the depth of modulation is $60 \%$.

In transmitting a musical programme, variations in loudness of the received music are produced by variations in modulation depth, these producing corresponding changes in the amount of a.f. output from the receiver.

### 15.11 Methods of Modulation

In the early days, modulation was usually effected by varying the voltage of one of the electrodes of the oscillator valve by means of an amplified version of the sound, etc., to be communicated. This inevitably varied the frequency of oscillation to a small but no longer acceptable extent, so the practice now is to modulate the carrier wave in one of the r.f. amplifier stages. This is preferably not done in the first stage, in case it reacts on the oscillator, but the earlier in the chain it is done the less is the a.f. power needed for modulation. This advantage of what is called low-level modulation is offset by the need to avoid any distortion of the envelope from there on. That rules out Class C, for a start. Class B r.f. amplification of a modulated carrier within tolerable limits of distortion is possible but difficult. So in spite of the very large amount of a.f. power needed, modulation is often applied at the final power amplifier stage.

Fig. 15.15 shows in outline the classical method of modulation, often called the Heising method, which varies the anode voltage. As before, $L_{1} C_{1}$ is the input tank circuit and $L_{0} C_{Q}$ the output tank circuit. C has sufficient capacitance to provide a direct path for r.f. current to cathode but not enough to shunt the a.f. transformer T appreciably. This transformer couples the output of an a.f. amplifier to the r.f. power amplifier. The amount of a.f. power needed is anything up to half as much as that supplied by the d.c. anode feed
source, and of the same order as the r.f. output, so for a high-power sender the transformer has to be a very substantial one, and so has the a.f. amplifier feeding it. At $100 \%$ modulation its peak output voltage is almost equal to the feed d.v., and it therefore swings the total anode supply voltage from nearly zero to almost double the


Fig. 15.15-Ouline of Heising method of amplitude modulation
normal, thereby making the tank circuit current vary as shown in Fig. 15.14.

It used to be common practice to use a single inductor (a "choke coil') in place of the transformer, and to connect the final a.f. amplifying valve across $C$, so that it was fed from the same source as the r.f. valve. This form of anode modulation was known as choke control.

Because of the large amount of power and heavy equipment needed for anode modulation, the a.f. voltage is sometimes applied to the grid. This is not so simple or satisfactory as might be expected; it tends to increase distortion and certainly reduces the r.f. efficiency, so is not much favoured. Other methods of modulation have been devised, some of them very complicated, in attempts to reduce power consumption without increasing distortion.

### 15.12 Frequency Modulation

Instead of keeping the frequency constant and varying the amplitude, one can do vice versa. Fig. 15.16 shows the difference; $a$ represents a small sample-two cycles-of the a.f. programme, as in Fig. 15.13a, and $b$ is the corresponding sample of r.f. carrier wave after it has been amplitude-modulated by $a$. If, instead, it were frequency-modulated it would be as at $c$, in which the amplitude remains constant but the frequency of the waves increases and decreases as the a.f. voltage rises and falls. In fact, with a suitable vertical scale of hertz, $a$ would represent the frequency of the carrier wave.

The chief advantage of frequency modulation ('f.m.') is that under certain conditions reception is less likely to be disturbed by interference or noise. Since with f.m. the sender can radiate at its full power all the time, it is more likely to outdo interference than with amplitude modulation ('a.m.'), which has to use a carrier wave of half maximum voltage and current and therefore only one quarter maximum power. At the receiving end, variations in amplitude caused by noise, etc., coming in along with the wanted signals can be cut off without any risk of distorting the modulation.

Another contribution to noise reduction results from the fact that modulation can be made to swing the carrier-wave frequency up and down over a relatively wide range or 'band' as it is called. Exactly how this advantage follows is too involved to explain here, but since the change of frequency imparted by the modulator is proportional to the a.f. voltage it is obvious that the greater the change the greater the audible result at the receiving end.

In a.m., $100 \%$ modulation varies the amplitude between zero and double. If an f.m. station varied its frequency between zero and double it would (apart from other impracticabilities!) leave no frequencies clear for any other stations, so some arbitrary limit must be imposed, to be regarded as $100 \%$ modulation. This limit is called the deviation, and for sound broadcasting is commonly 75 kHz above and below the frequency of the unmodulated carrier wave. Stations working on the 'medium' broadcast frequencies


Fig. 15.16-Cwve a represents 2 cycles of audio frequency used to modilate an r.f. carrier wave. If amplitude modulafion is used, the result is represented by $b$ : if frequency modulution, by $c$

( 525 to 1605 kHz ) are spaced only 9 or 10 kHz apart, and if they used f.m. they would spread right across one another. For this and other reasons it is necessary for f.m. stations to use much higher frequencies, usually in the v.h.f. band, 30 to 300 MHz . Although the principle of f.m. is simple, the actual methods of modulation adopted in practical senders are too complex to describe here.

A silent background to reception is especially desirable in broadcasting, and for this reason $\mathrm{f} . \mathrm{m}$. is much used for sound programmes, with or without accompanying vision. It is also widely used in radio telegraphy and still-picture transmission.

### 15.13 Telegraphy and Keying

To produce telegraph signals such as morse, a simple but extreme form of modulation is used: its depth is always either 0 or $100 \%$. It used to be done by a morse key-a form of switch that makes a contact when pressed and breaks it when let go-in such a way that it turned the output of the sender on and off. Nowadays most of the remaining radiotelegraph communication is done at high speed by punched tape controlling contactors, which are electrically operated switches. There are a great many alternative positions in the sender circuits where a key or contactor would start and stop the output, but in practice quite a number of problems arise. So a m . has been largely displaced by f.m., or frequency-shift keying, in which the radiation is going all the time but alternates between two closely spaced frequencies.

### 15.14 Microphones

So far we have learnt something about two of the boxes at the sending end of Fig. 1.9: the r.f. generator and the modulator.

A device by which sound, consisting of air waves, is made to set up electric currents or voltages of identical waveform, which in turn are used to control the modulator, is one that most people handle more or less frequently, in telephones. It is the microphone; and the type used in telephones, but not for radio broadcasting, is essentially the same as the original invention by Hughes in 1878. The sounds are directed on to a circular diaphragm, to the centre of which is attached a carbon button. Beyond it is a fixed carbon button, and the space between is filled with carbon granules. So between the terminals, which are connected to the diaphragm and


Fig. 15.17-How a carhon microphone (M) is connected to give an a.f: voltage
fixed button, there are many not very firm contacts between pieces of carbon. Their resistance depends on the pressure, which is varied by the air waves striking the diaphragm.

A carbon microphone is, in fact, a resistance that varies to correspond with the sound waves, and so the current delivered by the battery in Fig. 15.17 varies in the same way. Since this current passes through the primary of a transformer, a varying voltage is
produced at the secondary terminals. Sound waves of normal intensity are very feeble, and even with a step-up transformer the result is generally only a few volts at most, so for modulating a powerful sender several stages of amplification are necessary.

While such a microphone can be designed to give a reasonably large output over the band of frequencies corresponding to the essentials of intelligible speech, many of the frequencies needed to give full naturalness to speech and music are not fairly represented. The carbon microphone is also, on account of its loose contacts, noisy. So various other systems have been tried; but generally speaking it is true that the higher the fidelity-i.e., the more faithfully the electrical output represents the sound-the lower the output.

The electrostatic microphone consists of a diaphragm, usually metal, with a metal plate very close behind. When vibrated by sound, the capacitance varies, and the resulting charging and discharging currents from a battery, passed through a high resistance, set up a.f. voltages. lts disadvantages are very low output and high impedance.

The crystal or piezo-electric microphone depends on the same properties as are applied in crystal control; the varying pressures directly give rise to a.f. voltages. Crystals of Rochelle salt are particularly effective.

The ty pe most commonly used for broadcasting and the better class of 'public address' is the electromagnetic. There are many varieties: in some a diaphragm bears a moving coil in which the voltages are generated by flux from a stationary magnet; while in the most favoured types a very light metal ribbon is used instead of the coil. Electromagnetic and crystal microphones generate e.m.fs directly, so need no battery; but electromagnetic types need a transformer to step them up.

The counterpart of the microphone in a vision system is the camera, which is described in Sec. 23.6.

### 15.15 Loading the Power Amplifier

The last main division of a sender is the aerial, which is the subject of Chapter 17, and the connecting link is considered in Chapter 16, but one aspect that should be looked at just now is the manner in which the aerial loads the power amplifier, because the power efficiency is greatly affected thereby.

The output tank circuit, $\mathrm{L}_{0} \mathrm{C}_{\mathrm{O}}$ in Fig. 15.18 , is equivalent (at the frequency of oscillation) to a high resistance, represented by $R_{O}$, which is equal to $Q_{o} X_{o}, Q_{o}$ being the $Q$ of the circuit without $R_{L}$, and $X_{o}$ the reactance of $L_{O}$ or $C_{O}$ (Sec. 8.12). The load resistance $R_{L}$ should be so much lower than $R_{O}$ that as large a proportion as possible of the output power goes into it rather than into $R_{\boldsymbol{O}}$ where

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it would just be wasted. At the same time it must not be so low that $Q_{L}$, the $Q$ of the circuit when loaded by $R_{L}$, is inadequate for keeping the oseillation waveform reasonably sinusoidal in spite of the very non-sinusoidal Class B or C driving current. About 12 is a typical compromise figure for $Q_{L}, Q_{o}$ for a good tank circuit usually being


Fig. $15.18 C_{o}$ and $L_{o}$ together are an outpat oscillatory civcmit, its losses being represented in parallol by $k_{o} R_{L}$ is the effective load resistance
in the 100300 range. Suppose for example it was 156 . Then $R_{o}$ would be $156 X$, and if $R_{L}$ was $13 X$ the two in parallel would be $X .156 \times 13 /(156+13)=12 X$ and $Q_{1}$ therefore 12 as suggested. The power going into $R_{L}$ would be $V^{2} / 13 X,=0.077 V^{2} / X$, compared with $V^{2} / 156 X,=0.0064 V^{2} / X$ lost in the tank circuit, and the efficiency of transfer is $0.077 /(0.077+0.0064)=92 \cdot 3^{\circ}$. The total load on the valve is, as we have seen, $12 X$ ohms.

We saw in Sec. 11.6 that a valve can be regarded as a generator having internal resistance $\left(f_{a}\right)$ and in Sec. 11.8 that the maximum output nower for a given alternating voltage at the grid is obtained by making the load resistance equal to the internal resistance, the efficiency then being $50 \%$. Is this the principle to use when connecting the aerial load to the output circuit of the power amplifier?

There are several reasons why it is not. Firstly, we are not really interested in getting the maximum power for a given grid drive. The main limiting factor is not the grid a.v. - which can readily be varied-but the amount of heat that the valve can dissipate. So it is more important to increase efficiency. As we have seen, efficiencies greater than the theoretical maximum of $50^{\circ}$ (when load resistance is equal to generator resistance) are obtainable in practice by departing from this load-matching condition. In any case it is valid only for Class A working, which is not normally used in thes position. The whole concept of load resistance then breaks down, and the only principle is to make the load resistance seen by the valve the value that gives the best results within the various rating limits of the valve. This is a very complicated process, but fortunately the valve makers do it for us.

If this optimum load resistance happens to be the same as the resistance of the aerial to be fed, then neglecting the loss in the tank circuit ( $R_{o}$ ) we can simply connect the aerial as $R_{L}$ in Fig. 15.18. This would be a happy but very unlikely coincidence, so one must usually connect $R_{L}$ in such a way as to int roduce a step up or down. Doing this by using $L_{o}$ as an auto-transformer (Sec. 7.11) has the
disadvantage that if the cathode of the valve was earthed the aerial would receive the full d.v. of the anode, which might be thousands of volts. To avoid such a hazard it is poss:ble, but somewhat inconvenient, to earth the positive of the d.v. feed supply. Using $L_{o}$ as the primary of a double-wound transformer leaves us free to earth what we choose and enable the coupling to be adjusted more conveniently. Other methods sometimes used involve capacitors; see also Sec. 22.12.

## Transmission Lines

### 16.1 Feeders

In dealing with circuit make-up we have for the most part assumed that resistance, inductance and capacitance are concentrated separately in particular components, such as resistors. inductors and capacitors. True, any actual inductor (for example) has some of all three of these features, but it can be imitated fairly accurately by an imaginary circuit consisting of separate L. $C$ and $R$ (Sec. 8.16). In the next chapter, however, we shall turn attention to aerials, in which $L ; C$ and $R$ are all mixed up and distributed over a considerable length. The calculation of aerials is therefore generally much more difficult than anything we have attempted, especially as they can take so many different forms.

But one example of distributed-circuit impedance does lend itself to rather simpler treatment, and makes a good introduction to the subject, as well as being quite important from a practical point of view. It is the transmission line. The most familiar example is the cable connecting the television aerial on the roof to the set indoors. For reasons to be explained later (Sec. 23.6) television uses signals of very high frequency, which are best received on aerials of a definite length (Sec. 17.7). It is desirable that the wires connecting aerial to receiver should not themselves radiate or respond to radiation, nor should they weaken the v.h.f. signals. The corresponding problem at the sending end is even more important, because a large amount of power has to be conveyed from the sender on the ground to an aerial hundreds of feet up in the air. A transmission line for such purposes is usually called a feeder.

To reduce radiation and pick-up to a minimum, its two conductors must be run very close together, so that effects on or from one cancel out those of the other. One form is the parallel-wire feeder (Fig. 16.1a); still better is the coaxial feeder (b), in which one of the conductors totally encloses the other.

### 16.2 Circuit Equivalent of a Line

With either type, and especially the coaxial, the closeness of the two leads causes a large capacitance between them; and at very high frequencies that means a low impedance, which, it might be supposed, would more or less short-circuit a long feeder, so that very little of the power put in at one end would reach the other.

But it must be remembered that every inch of the feeder not only has a certain amount of capacitance in parallel, but also, by virtue of the magnetic flux set up by any current flowing through it (Fig. 4.2), a certain amount of inductance in series. Electrically, therefore, a piece of transmission line can be represented with standard circuit

Fig. 16.1-The two main t!pes of ref: transmission line or feeder cable: (a) parallel-wire, and (b) coaxial


Fig. 16.2-Approximate electrical analysis of a short length of transmission line


Fig. 16.3-Synhesis of trunsmission line from basic circuit elements
symbols as in Fig. 16.2, in which $L$ and $C$ are respectively the inductance and capacitance of an extremely short length, and so are extremely small quantities. Each inductance is, of course. contributed to by both wires, but it makes no difference to our argument if for simplicity the symbol for the total is shown in one wire; it is in series either way. As we shall only be considering lines that are uniform throughout their length. every $L$ is accompanied by the same amount of $C$; in other words, the ratio $L / C$ is constant.

Having analysed the line in this way, let us consider the load at the receiving end; assuming first that its impedance is a simple resistance. $R$ (Fig. 16.3a). If we were to measure the voltage across its terminals and the amperage going into it, the ratio of the two readings would indicate the value $R$ in ohms.

Next. suppose this load to be elaborated by adding a small inductance in series and a small capacitance in parallel. as at $h$. Provided that the capacitance really is smatl, its reactance. $\lambda_{C}$. will be much greater than $R$. By an adaptation of the method used in Sec. 8.16 a relatively large reactance in parallel with a resistance can. with negligible error, be replaced by an equivalent circuit consisting of the same resistance in series with a relatively small reactance. Calling this equivalent series capacitive reactance $X_{C^{\prime}}$, we have $X_{c^{\prime}}{ }^{\prime} / R=R X_{C}$, or $X_{c^{\prime}}=R^{2} / X_{C}$. And if we make this $X_{C^{\prime}}$ equal to $X_{L}$, it will cancel out the effect of $L$ (Sec. 8.3), so that the whole of Fig. 16.3 b will give the same readings on our measuring instruments as $R$ alone. $X_{L}$, is. of course. $2 \pi / L$. and $X_{C}$ is $1 / 2 \pi f C$; so the condition for $X_{C}{ }^{\prime}$ and $X_{L}$ cancelling one another out is

$$
\begin{aligned}
& 2 \pi / L=R^{2} \times 2 \pi f C \\
& \text { or. } \quad L=R^{2} C \\
& \text { or. } \quad R=, ~\left(\frac{L}{C}\right)
\end{aligned}
$$

Notice that frequency does not come into this at all. except that if it is very high then $L$ and $C$ may have to be very small indeed in order to fulfil the condition that $X_{C}$ is much larger than $R$.

Assuming, then, that in our case , $(L / C)$ does happen to be equal to $R$. so that $R$ in Fig. $16.3 a$ can be replaced by the combination $L C R$ in $b$, it will still make no difference to the instrument readings if $R$ in $b$ is in turn replaced by an identical $L C R$ combination, as at $c . R$ in this load can be replaced by another $L C R$ unit, and so on indefinitely ( $d$ ). Every time. meters connected at the terminals will show the same readings. just as if the load were $R$ only.

The smaller $L$ and $C$ are, the more exactly the above argument is true at all practical frequencies. This is very interesting, because by making each $L$ and $C$ smaller and smaller and increasing their number we can approximate as closely as we like to the electrical equivalent of a parallel or coaxial transmission line in which inductance and capacitance are uniformly distributed along its length. In such a line the ratio of inductance to capacitance is the same for a metre or a mile as for each one of the infinitesimal 'units', so we reach the conclusion that provided the far end of the line is terminated by a resistance $R$ equal to , $(L / C)$ ohms the length of the line makes no difference; to the signal-source or generator it is just the same as if the load $R$ were connected directly to its terminals.

That, of course is exactly what we want as a feeder for a load which has to be located at a distance from the generator. But there is admittedly one difference between a real feeder and the synthetic one shown in Fig. 16.3d; its conductors are bound to have a certain
amount of resistance. Assuming that this resistance is uniformly distributed, every 10 metres of line will absorb a certain percentage of the power put into it. Obviously, a line made of a very thin wire, of comparatively high resistance, will cause greater loss than one made of low-resistance wire. So, too. will a pair separated by poor insulating material (Sec. 8.15). Although the loss due to resistance is an important property of a line, it is generally allowable to neglect the resistance itself in calculations such as we have just been making.

### 16.3 Characteristic Resistance

The next thing to consider is how to make the feeder fit the load. so as to fulfil the necessary condition,,$(L / C)=R .,(L / C)$ is obviously a characteristic of the line itself, and to instruments connected to one end the line appears to be a resistance: so for any particular line it is called its characteristic resistance. denoted by $R_{00}$. (If loss resistance is taken into account the expression is slightly more complicated than $v(L / C)$, and includes reactance; so the more comprehensive and strictly accurate term is characteristic (or surge) impedance. $Z_{0}$. With ceasonably low-loss lines there is not much difference.)

If you feel that the resistance in question really belongs to the terminating load and not the line, then you should remember that the load resistance can always be replaced by another length of line. so that ultimately $R_{a}$ can be defined as the input resistance of an infinitely long line. Alternatively (and more practically) it is a resistance equal to whatever load resistance can be fed through any length of the line without making any difference to the generator.

In any case. $L$ and $C$ depend entirely on the spacing and diameters of the wires or tubes of which the line consists. and on the permittivity and permeability of the spacing materials. So far as possible, to avoid losses. feeders are air-spaced. The closer the spacing, the higher is $C$ (as one would expect) and the smatler is $L$ because the magnetic field set up by the current in one wire is nearly cancelled by the field of the returning current in the other). So a closely-spaced line has a small $R_{0}$, suitable for feeding lowresistance loads. Other things being equal, one would expect a coaxial line to have a greater $C$. and therefore lower $R_{0}$. than a parallel-wire line. Formulae have been worked out for calculating $R_{0}$; for a paralle-wire line (Fig. 16.4a) it is practically 276 $\log _{10}(2 D / d)$ so long as $D$ is at least 4 or 5 times $d$. and for a coaxial line (Fig. 16.4b) $138 \log _{10}\left(D^{\prime}(d)\right.$. Graphs such as these can be used to find the correct dimensions for a feeder to fit a load of any resistance, within the limits of practical construction. It has been found that the feeder dimensions causing least loss make $R_{0}$ equal to about $600 \Omega$ for parallel-wire and $80 \Omega$ for coaxial types: but a reasonably efficient feeder of practical dimensions can be made to


Fig. 16.4-Graphs showing characteristic resistance ( $R_{0}$ ) of $(a)$ parallel-wire and (b) couxial transmission lines in terms of dimensions

(b)

Fig. 16.5-(a) Fig. 16.3d labelled for phasor diagrams (b)
have an $R_{0}$ of anything from. say 200 to 650 and 40 to $140 \Omega$ respectively. We shall see later what to do if the load resistance cannot be matched by any available feeder.

In the meantime, what if the load is not a pure resistance". Whatever it is, it can be reckoned as a resistance in parallel with a reactance. And this reactance can always be neutralized or tuned out by connecting in parallel with it an equal reactance of the opposite hind (Sec. 8.8). So that little problem is soon solved.

### 16.4 Waves along a Line

Although interposing a loss-free line (having the correct $R_{0}$ ) between a generator and its load does not affiect the voltige and current at the load terminals-they are still the same as at the generator-it does affect their phase. The mere fact that the voltage across R in Fig. 16.3 b is equal to the voltage across the terminals makes that inevitable. as one can soon see by drawing phasor diagrams. Fig. 16.5a shows a few sections as in Fig. 16.3. and the phasor diagrams (b) are drawn by beginning at the load R and working backwards. They take the form of contimously opening fans. We can easily judge from this that for an actual line in which the sections are infinitesimal. the diagrams would be circles. If we


Fig. 16.6--(a) Electrical represconation of the generotor end of a transmission line. and (b) instantancous sollage (and current) diagran $\frac{3}{4}$ of a cycle from thre start of generating a sine ware
consider what happens in a line when the generator starts generating (Fig. 16.6) we can see that what the gradual phase lag along it really means is that the current takes time to travel from generator to load.

During the first half-cycle (positive, say) current statis flowing into $C_{1}$, charging it up. The inductance $L_{11}$ prevents the voltage across $C_{1}$ rising to its maximum until a litte later than the generator voltage maximum. The inertia of $L_{2}$ to the growth of current

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through it allows the charge to build up in $C_{1}$ a little before $C_{2}$, and so on. Gradually the charge builds up on each bit of the line in turn. Meanwhile, the generator has gone on to the negative half-cycle, and this follows its predecessor down the line. At its first negative maximum, the voltage distribution will be as shown at $h$. With a little imagination one can picture the voltage wave flashing down the line, rather like the wave motion of a long rope waggled up and down at one end.

Our theory showed that wherever the line is connected to measuring instruments it appears as a resistance, which means that the current is everywhere in phase with the voltage, so the wave diagram implies power flowing along the line.

The speed of the wave clearly depends on its capacitance and inductance: the larger they are the longer the time each bit of line takes to charge and the longer the time for current to build up in it. We have already noted that $C$ is increased by spacing the two conductors more closely, but that the $L$ is reduced thereby. As it happens, the two effects exactly cancel one another out, so the speed of the wave is unaffected : the only result is that the ratio $L / C$-and hence $R_{0}$-is decreased. The only way of increasing $C$ without reducing $L$ is to increase the permittivity ( $\varepsilon$ ) in the space between the conductors (Sec. 3.3); similarly $L$ is affected by the permeability $(\mu)$ there (Sec. 4.6). It has been discovered that the speed of the waves is equal to $l / \wedge \varepsilon \mu$. The lowest possible permittivity and permeability are in empty space (though air is practically the same); in SI units they are $8.854 \times 10^{-12}$ and $1.257 \times 10^{-6}$ respectively, so the speed of the waves works out at $1 / 1\left(8.854 \times 1.257 \times 10^{-18}\right)=$ very nearly 300 million metres per second. Significantly, this is the same as the speed of light and electric waves in space (Sec. 1.9).

There have to be solid supports for the conductors, and sometimes the space between is completely filled. In practice the permeability of such material would not differ appreciably from that of empty space, but the permittivity would be higher. Consequently the speed of the waves would be lower: for example, if the relative permittivity were 4 the speed would be reduced to a half. But it would still be pretty high! So the time needed to reach the far end of any actual line is bound to be very short-a small fraction of a second. But however short, there must be some interval between the power first going into the line from the generator and its coming out of the line into the load. During this interval the generator is not in touch with the load at all: the current pushed into the line by a given generator voltage is determined by $R_{0}$ alone, no matter what may be connected at the far end: which is further evidence that $R_{0}$ is a characteristic of the line and not of the load.

During this brief interval, when the generator does not "know' the resistance of the load it will soon be required to feed, the $R_{0}$ of the line controls the rate of power flow tentatively. In accordance with the usual law (Sec. 11.8), the power delivered to the line by a
constant-voltage generator during this transient state is a maximum if $R_{0}$ is equal to $r$, the generator resistance.

Neglecting line loss. the voltage and current reaching the load end will be the same as at the generator. So if the load turns out to be a resistance equal to $R_{0}$, it will satisfy Ohm's law, and the whole of the power will be absorbed just as fast as it arrives. For example, if a piece of line having an $R_{0}$ of $500 \Omega$ is connected to a 1000 )-V generator having an $r$ or $500 \Omega$, the terminal voltage is bound to be 500 V and the current 1 A until the wave reaches the far end, no matter what may be there. If there is a load resistance of $500 \Omega$, the r.m.s. current will, of course, continue at 1 A everywhere.

### 16.5 Wave Reflection

But suppose that the load resistance is, say, $2000 \Omega$. According to Ohm's law it is impossible for 500 V applied across $2000 \Omega$ to cause a current of 1 A to flow. Yet 1 A is arriving. What does it do? Part of it, having nowhere to go, starts back for home. More scientifically, it is reflected by the mismatch. The reflected current, travelling in the opposite direction, can be regarded as opposite in phase to that arriving, giving a total which is less than 1 A . The comparatively high resistance causes the voltage across it to build up above 500 ; this increase can be regarded as an in-phase reflected voltage driving the reflected current. If half the current were reflected, leaving 0.5 A to go into the load resistance, the voltage would be increased by a half, making it 750 . A voltage of 750 and current 0.5 A would fit a $1500 \Omega$ load, bui not $2000 \Omega$, so the reflected proportion has to be greater-actually $60 \%$ giving 800 V and 0.4 A at the load. In general terms, if the load resistance is $R$ the fraction of current and voltage reflected, called the reflection coefficient or return loss and denoted by $\rho$, is $\left(R-R_{0}\right) /\left(R+R_{0}\right)$.

We now have 1 A , driven by 500 V , travelling from generator to load, and 0.6 A . driven by 300 V . returning to the generator. (The ratio of the reflected voltage to the reflected current must, of course, equal $R_{0}$.) The combination of these two at the terminals of the load gives, as we have seen. $1-0.6=0.4 \mathrm{~A}$, at $500+300=$ 800 V . But at other points on the line we have to take account of the phase lag. At a distance from the load end equal to quarter of a wavelength $(\lambda / 4)$ the arriving and returning waves differ in phase by half a wavelength ( $\lambda / 2$ ) or $180^{\circ}$ as compared with their relative phases at the load (because a return journey has to be made over the $\lambda / 4$ distance). So at this point the current is $1+0.6=1.6 \mathrm{~A}$ at $500-300=200 \mathrm{~V}$. At a point $\lambda / 8$ from the load end the phase separation is $\lambda / 4$ or $90^{\circ}$, giving $1 \cdot 16 \mathrm{~A}$ at 582 V . At intervals of $\lambda / 2$. the two waves come into step again.

### 16.6 Standing Waves

Calculating the current and voltage point by point in this way and plotting them, we get the curves in Fig. 16.7. It is important to realize that these are not, as it were, flashlight photographs of the waves travelling along the line; these are r.m.s. values set up continuously at the points shown, and would be indicated by meters


Fig. 16.7-Diagrams of voltage, current, and (as a result) impedance at the load end of an unmatched transmission line, showing standing waves
connected in or across the lines at those points (assuming the meters did not appreciably affect the $R_{0}$ of the line where connected). Because this wavelike distribution of current and voltage, resulting from the addition of the arriving and reflected waves travelling in opposite directions, is stationary, it is called a standing wave. For comparison, the uniform distribution of current and voltage when the load resistance is equal to $R_{0}$ is shown dotted.

The ratio of maximum to minimum current or voltage is called the standing wave ratio (s.w.r.); in our example it is $800 / 200=4$. (Because the ratio is the same for voltage and current the common practice of referring to voltage standing wave ratio (v.s.w.r.) is unnecessary and liable to mislead one into thinking that the ratio must be different for current.) Applying simple algebra to the way we have found the s.w.r. in this example we can easily find that in general it is equal to $R / R_{0}$. Its usual symbol is $S$.

In due course the reflected wave reaches the generator. It is in this indirect way that the load makes itself felt by the generator. If the $2000-\Omega$ load had been directly connected to the generator terminals, the current would have been $1000 /(500+2000)=0 \cdot 4 \mathrm{~A}$, and the terminal voltage $0.4 \times 2000=800 \mathrm{~V}$. and the power $800 \times 0.4=320 \mathrm{~W}$. This, as we have seen, is exactly what the load at the end of the line is actually getting. But the power that originally went out from the generator, being determined by $R_{0}$, was $500 \times 1=500 \mathrm{~W}$. The reflected power is $300 \times 0.6=180 \mathrm{~W}$. so the net outgoing power is $500-180=320 \mathrm{~W}$. just as it would have been with the load directly connected.

### 16.7 Line Impedance Variations

So the power adjustment is (in this case) quite simple. But the current and voltage situation at the generator is complicated by the time lag, and is not necessarily the same as at the load. Unless the length of the line is an exact multiple of $\lambda / 2$, the phase relationships are different. Suppose, for example, it is an odd multiple of $\lambda / 4$; say. $5 \lambda / 4$. as in Fig. 16.7. Then the current and voltage at the generator end will be 1.6 A and 200 V respectively. That makes 320 W all right-but compare the power loss in the generator. 1.6 A flowing through $r=500 \Omega$ is $1.6^{2} \times 500=1280 \mathrm{~W}$, whereas at an 800 - $V$ point the current would be only 0.4 A and the loss in the generator only $0.4^{2} \times 500=80 \mathrm{~W}$ ! So when there are standing waves, the exact length of the line (in wavelengths) is obviously very important. If there are no standing waves, it does not matter if the line is a little longer or shorter or the wavelength is altered slightly; and that is one very good reason for matching the load to the line. The usual way of making sure that the load is right is by running a voltmeter or other indicator along the line and seeing that the reading is the same everywhere. except perhaps for a slight gradual change due to line loss.

Connecting the generator to a point on the line where the voltage and current are 200 V and 1.6 A respectively is equivalent to connecting it to a load of $200 / 1 \cdot 6=125 \Omega$. The impedance of the line at all points can easily be derived from the voltage and current curves, as at the foot of Fig. 16.7. This curve indicates the impedance measured at any point when all the line to the left of that point is removed. The impedance depends, of course, not only on the distance along the line to the load but also on $R_{0}$ and the load impedance. From a consideration of the travelling waves it can be seen that at $\lambda / 4$ intervals the phases of the currents and voltages are exactly the same as at the load, or exactly opposite; so that if the load is resistive the input resistance of the line is also resistive. At all other points the phases are such as to be equivalent to introducing reactance.

In our example we made the generator resistance, $r$, equal to $R_{0}$. If it were not, the situation would be more complicated still, because the reflected wave would not be completely absorbed by the generator, so part would be reflected back to the load, and so on, the reflected power being smaller on each successive journey, until finally becoming negligible. The standing waves would be the resultant of all these travelling waves. Even quite a long line settles down to a steady state in a fraction of a second, but it is a state that is generally undesirable because it means that much of the power is being dissipated in the line instead of being delivered to the load.

It should be noted that mismatching at the generator end does not affect the standing-wave ratio, but does affect the values of current and voltage attained.

Another result of making $r=R_{0}$ is that any reflection from the load or elsewhere inevitably impairs the generator-to-line matching. That is so even if the generator is connected to a point where the impedance curve coincides with the $500-\Omega$ line in Fig. 16.7. because then the impedance is reactive. But if $r$ were, say, $2000 \Omega$. so that it would be mismatched to the $500-\Omega$ line during the brief moment following the start, there would be a chance that when the reflected wave arrived it would actually improve the matching. even to making it perfect (e.g., if connected at A or B in Fig. 16.7): but on the other hand it might make it still worse (if connected at C. D, or E).

It should be remembered that 'perfect' matching is that which enables maximum power to be transferred: but in practice, as we have seen in Sec. 15.15, there may be good reasons for deliberately mismatching at the generator. To take figures we have already had, it may be considered better to deliver 320 W with a loss of 80 W than the maximum ( 500 W ) with a loss of 500 W .

### 16.8 The Quarter-Wave Transformer

An interesting result of the principles exemplified in Fig. 16.7 is that a generator of one impedance can be perfectly matched to a load of another by suitably choosing the points of connection. For instance, a generator with an internal resistance of $125 \Omega$ would be matched to the $2000 \Omega$ load if connected at $C, D$ or $E$. The line then behaves as a $1: 4$ transformer. Points can be selected giving (in this case) any ratio between $1: 4$ and $4: 1$, but except at the lettered points there is reactance to be tuned out.

It is not necessary to use the whole of a long line as a matching transformer: in fact. owing to the standing waves by which it operates it is generally undesirable to do so. We can see from Fig. 16.7 that the maximum ratio of transformation, combined with
non-reactive impedance at both ends, is given by a section of line only quarter of a wavelength long. We also see that the mismatch ratio to the line is the same at both ends; in this particular case it is $1: 4(125 \Omega$ to $500 \Omega$ at the generator end and $500 \Omega$ to $2000 \Omega$ at the load end), which, incidentally, is equal to the voltage ratio of the whole transformer. In general terms, if $R_{I}$ denotes the input resistance of the line (with load connected), $R_{t}$ is to $R_{0}$ as $R_{0}$ is to $R$; which means $R_{t} / R_{0}=R_{0} / R$, or $R_{0}=\downarrow^{\prime} R_{t} R$.

This formula enables us to find the characteristic resistance of the quarter-wave line needed to match two unequal impedances, $R_{t}$ and $R$. Suppose we wished to connect a $70-\Omega$ load to a $280-\Omega$ parallel-wire feeder without reflection at the junction. They could be matched by interposing a section of line $\lambda / 4$ in length, spaced to


Fig. 16.8 Example of a quarter-wavelength of line being wed as a matching transformer
give a $R_{0}$ of $\jmath^{\prime}(70 \times 280)=140 \Omega$. Parallel wires would have to be excessively close (Fig. $164 a$ ), but the problem could be solved by using a length of $70-\Omega$ coaxial cable for each limb and joining the metal sheaths together as in Fig. 16.8, putting the $70-\Omega$ impedances in series across the ends of the $280-\Omega$ line.

### 16.9 Fully Resonant Lines

Going now to extremes of mismatch. we inquire what happens when the 'load" resistance is either infinite or zero; in other words. when the end of the line is open-circuited or short-circuited. Take the open circuit first. If this were done 10 our original 50()$-\Omega$ example, the current at the end would obviously be nil, and the voltage would rise to 1000 double its amount across the matched load. This condition would be duplicated by the standing waves at points $A$ and $B$ in Fig. 16.7: while at $C, D$, and $E$ the voltage would be nil and the current 2 A . The impedance curve would consequently fluctuate between zero and infinity.

With a short-circuited line. there could be no volts across the end, but the current would be 2 A : in fact, exactly as at E with the open line. A shorted line, then, is the same as an open line shifted quarter of a wavelength along. Reflection in both cases is complete. because there is no load resistance to absorb any of the power.

If the generator resistance is very large or very small, nearly all
the reflected wave will itself be reflected back, and so on, so that if the line is of such a length that the voltage and current maximum points coincide with every reflection, the voltages and currents will build up to high values at those maximum points-dangerously high with a powerful sender. This reminds us of the behaviour of a high- $Q$ resonant circuit (Sec. 8.3). The maximum current or voltage points are called antinodes, and the points where there is no current or voltage are nodes.

When the length of a short-circuited or open-circuited line is a whole number of quarter-wavelengths, the input impedance is


Fig. 16.9-Showing how the reactances of short-circuited and open-circuited lines vary. with their length


(b)

Fig. 16.10-(a) Conventional aerial input circuit using 'lumped' components, and (b) distributed equivalent using a quarter-wave coaxial line
approximately zero or infinity. An odd number of quarter-wavelengths gives opposites at the ends-infinite resistance if the other end is shorted, and vice versa. An even number of quarter-wavelengths gives the same at each end.

In between, as there is now no load resistance, the impedance is a pure reactance. At each side of a node or antinode there are opposite
reactances-inductive and capacitive. If it is a current node, the reactance at a short distance each side is very large; if a voltage node, very low. Fig. 16.9 shows how it varies. It is clear from this that a short length of line-less than quarter of a wavelength - can be used to provide any value of inductance or capacitance. For very short wavelengths, this form is generally more convenient than the usual inductor or capacitor, and, under the name of a stub if often used for balancing out undesired reactance.

### 16.10 Lines as Tuned Circuits

A quarter-wave line shorted at one end is, as we have just seen, a high-almost infinite-resistance. But only at the frequency that makes it quarter of a wavelength (or an odd multiple of $\lambda / 4$ ). This resistance is closely analogous to the dynamic resistance of a parallel resonant circuit; and in fact a line can be used as a tuned circuit. At wavelengths less than about 1 metre it is generally more efficient and easily constructed than the conventional sort.

The lower the loss resistance of the wire, the higher the dynamic resistance across the open ends; and to match lower impedances all that is necessary is to tap it down, just as if it were any other sort of tuned circuit.

Fig. $16.10 a$ is an example of a conventional tuning circuit, such as might be used in a receiver, and $b$ is the coaxial equivalent. A coaxial feeder is also used to connect the aerial, and being normally about $80 \Omega$ it is tapped low down, near the earthed end. The impedance at the top end is normally thousands of ohms, and is too high for the input of a transistor.

At still higher frequencies, with waves only a few centimetres long, the innes conductor becomes unnecessary, and power can be transmitted along hollow pipes (called waveguides) and tuned by cavities. But that is a subject in itself.

## Radiation and Aerials

### 17.1 Bridging Space

In the first chapter of this book the processes of radio communication were traced very briefly from start to finish; since then we have considered the sender in some detail. Before going on to the receiver we should know something about how the space in between is bridged.

We have just seen how electrical power can be transmitted from one place to another at the not inconsiderable speed of nearly 300000 km ( 186282 miles) per second, in the form of waves along a line consisting of two closely spaced conductors. A southerner who was asked to explain how it was possible to signal by telegraph from New York to New Orleans replied by pointing out that when you tread on the tail of a dog the bark comes out at its head. A telegraph was like a very long dog, with the tail in New York and the head in New Orleans. 'But what about wireless?' he was asked. 'Same thing, 'cept the dawg am imagin'ry'. This reply, though ingenious, is not perhaps altogether satisfying. How is it possible to transmit the waves without any conductors to carry the currents to and fro?

The significant thing about the section on how waves are transmitted along a line (Sec. 16.4) is that conduction was hardly mentioned; the important properties of a line are its distributed capacitance and inductance. In other words, transmission of waves along a line is primarily a matter of oscillating electric and magnetic fields, located in the space between and around the two conductors. These conductors are not required to carry current from one end to the other, any more than waves on a pond carry the water from one side to the other; all that the electrons in the conductors do is oscillate a very short distance locally.

### 17.2 The Quarter-Wave Resonator Again

Near the end of the same chapter we found that a line only quarter of a wavelength long and open at the end, as in Fig. 17.1a, is equivalent from the generator's point of view to a series resonant circuit ( $b$ ); that is to say, the only impedance the generator faces is the incidental resistance of the circuit. We saw in Sec. 8.4 that if this resistance is small a very little generator voltage can cause a very large current to flow and build up large voltages across C
and L . This remarkable behaviour of C and L is due to the electric field concentrated mainly between the plates of C and the magnetic field concentrated mainly inside and close around the turns of L . The main difference between $a$ and $b$ is that in $a$ the inductance and capacitance-and therefore the fields-are mixed up together and uniformly distributed along the length. The distance of the dotted lines from the conductors in Fig. 17.1a indicates the distribution of


Fig. 17.1-(a) A quarter-wavelength line, showing the distribution of current and voltage, and ( $b$ ) its circuit equivalent
current and voltage along them; compare Fig. 16.7. The dotted lines can be regarded as marking the limits of the alternations of current and voltage, just as the dotted lines in Fig. 1.3 marked the limits of vibration of a stretched string.

An essential feature of a transmission line, even such a short one as this, is that the distance between the conductors is small compared with a wavelength -or even quarter of a wavelength. The reason is that the explanation of its action is based on neglecting the time taken for the fields to establish themselves when a current flows or a p.d. is created. Provided nearly all of each field is close to the conductors, this assumption is fair enough. But when a current is switched on, the resulting magnetic field cannot come into existence instantaneously everywhere; just as a current takes time to travel along the line, so its effect in the form of a magnetic field takes time to spread out. And when the current is cut off, the effect of the remoter part of the magnetic field collapsing takes time to get back to the conductor. At points in the field one eighth of a wavelength from the conductor of the current, the combined delay is quarter of a wavelength, or quarter of a cycle, which upsets the whole idea of self-inductance. In fact, this phase delay between a change of current and the self-induced e.m.f. calls for a driving e.m.f. in phase with the current. just as if there were a resistance in the circuit-so far as that piece of field is concerned.

Suppose that in order to study this we open out the two conductors of our quarter-wave parallel line to the fullest possible extent, as in Fig. 17.2. Corresponding to our lumped $L$ and $C$ approximation to the original parallel line we could draw Fig. 17.3. The opening

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out greatly increases the area through which the magnetic field can pass, so increasing $L$. The lines of electric field are on the average much longer, so $C$ is decreased. Again, these two effects almost exactly cancel out, so the line is still nearly resonant to the same generator frequency. It can be brought exactly into tune by shortening it a very few per cent, which will be assumed to be done even though (as in Fig. 17.2) the total length is for simplicity marked


Fig. 17.2-The quarter-wave line of Fig. 17.1 opened fully oul


Fig. 17.3-Approximate representation of the distribured inductance and caparitance in the half-wavelength sistem of Fig. 17.2
$\lambda / 2$. So the distribution of current and voltage along the conductors is much the same as before. Looking at Fig. 17.3 we would say that the current was greatest at the middle because it has to charge all the capacitance in parallel, whereas near the ends there is hardly any to charge.

Fig. 17.4 shows some typical electric and magnetic lines of force. At any one radius from the generator they would trace out a sort of terrestrial globe, on which the magnetic lines were the parallels of latitude and the electric lines the meridians of longitude. This is helpful as a picture, but we must remember once again that field lines are as imaginary as lines of latitude and longitude.

### 17.3 A Rope Trick

It is clear that quite an appreciable portion of each field is now far enough away to be subject to a phase delay in its reaction on
the conductors. To the generator the effect is as if some resistance had been inserted in the circuit. If so, that means the generator is now supplying energy. Where does it go?

When we were young we must all have taken hold of one end of a rope and waved it up and down. If this is done slowly, nothing


Fig. 17.4-The distributed inductance and capacitance in Fig. 17.2 are hetter indicated by drawing a few lines of force The fill lines are electrix and the broken lines magneric
much happens except that the part of the rope nearest us moves up and down in time with our hand, as in Fig. 17.5a. But if we shake it rapidly, the rope forms itself into waves which run along it to the far end (b). This is quite an instructive analogy. The up-and-down movement of the hand represents the alternation of
(a)

(b)


Fig. 17.5—Well-known results of waving a rope up and down, (a) slowly, (b) yuickly charge in the $\lambda / 2$ rod caused by the generator. When the current is upwards it charges the upper half positively and the lower half negatively, resulting in an electric field downwards between the two. Half a cycle later the pattern is reversed. The up-and-down displacement of the rope from its middle position represents this
alternating electric field. The displacement travels outward from the hand as each bit of rope passes it on to the next. Ir the time of each cycle of hand movement is long compared with the time for the movement to pass along the rope, all the rope for at least quite a distance appears to move as a whole, "in phase". Because the electric field has to spread out in all directions, and not just in one direction like the rope, its amplitude falls off much more rapidly with distance from the source than does that of the rope.

When the hand is moved so fast that the rope displacement has not gone far before the hand reverses, the displacements become progressively out of phase with one another, and this is why travelling waves are formed, as in Fig. 17.5b. The same applies to the alternations of electric field; they form a wavelike pattern which travels outwards, just like the electric field between the conductors of a long line in that direction, except for a falling off in amplitude because it is not going out only along one radius but all around the 'equator".

### 17.4 Electromagnetic Waves

A good question at this stage would be: what keeps it going? The rope wave would not keep going unless the rope had not only displacement but inertia. The fact that we have already noted a resemblance between inertia and inductance (Sec. 4.9) may remind us that we seem to have forgotten about the magnetic field. But of course that too is spreading out, and in fact is essential to the wave motion. When we did the experiments with iron filings around a wire carrying current (Sec. 4.1) we said that the cause of the magnetic field was the electric current in the wire. But we know that an electric current consists of electric charges in motion, and charges are surrounded by an electric field. So the electric current, which generates the magnetic field, is really a moving electric field. We know, too, that a moving magnetic field generates an e.m.f. We tend to think, perhaps, of an e.m.f. only in a circuit, but it is really an electric field, whether there happen to be any circuits or conductors about or not. Putting all this concisely: Moving electric field causes magnetic field; moving magnetic field causes electric field.

Without going into the rather advanced mathematics of the thing, one might guess the possibility of the two fields in motion keeping one another going. Such a guess would be a good one. for they can and do. The speed they have to keep up in order to be mutually sustaining is the same as when they are guided by the transmission line. The combination of fast-moving fields is known as an electromagnetic wave. The union of the two fields is so complete that each depends entirely on the other and would disappear if it were destroyed. The speed of the wave is the same as that of lighta significant fact, which led to the discovery that light waves are
the same as those used for radio communication, except for their much higher frequency.

### 17.5 Radiation

An important thing to note about the rope trick is that the rope itself does not travel away from us-only a wavy pattern. But there is another thing: some of the energy we put into it travels to the far end, for it is capable of setting a support there into vibration. Electromagnetic waves are similar in this respect too; they convey energy away from the source. The outward movement of the waves from the source is called radiation. All variations in current cause radiation. but when the field energy is mainly localized, say between the conductors of a transmission line or the plates of a capacitor, the amount is much less than when it is spread out as in Fig. 17.4. Radiation also increases with the rate of variation: the 50 Hz variations of ordinary a.c. power systems cause negligible radiation. but 50 MHz is quite a different matter. So although we have spoken of the waveforms of low-frequency a.c., this should not be taken as suggesting that they are waves in the sense used in this chapter-electromagnetic waves.

We saw in connection with the lumped tuned circuit of Fig. 14.2 that the electric and magnetic fields, responsible for the capacitance and inductance, are quarter of a cycle out of phase with each other. Even in the most spread-out circuit, such as our half-wavelength rod, the majority of the energy of the surrounding fields is able to return to the circuit twice each cycle, to be manifested there as its distributed capacitance and inductance. This majority portion of each field, mainly near the source, is similarly out of phase with the other. It is distinguished by the term indiction field, and it falls off inversely as the square of the distance, which means that most of it is quite close to the source.

In order to convey energy right away from the source, however, there have to be components of fields in phase with one another. and it is this portion, called the radiation field, in which we are interested now. It falls off inversely as the distance, which means that although it starts as a minority it spreads much more, and far away from the source it is very much in the majority.

### 17.6 Polarization

Fig. 17.4 shows that the electric and magnetic fields are at right angles to one another and to the direction in which they radiate outwards as electromagnetic waves. Imagine we are looking towards the vertical source and the onconing waves. Then the electric field alternates up and down and the magnetic field side to
side, as shown in Fig. 17.6. This pattern can be imagined as due to a transmission line consisting of parallel strips top and bottom. At the instant depicted, the potential is positive at the top, so the electric field is downwards from it. This p.d. would be driving current towards us through the top conductor and away from us


Fig. 17.6-Relative direction of electric (full line) and magnetic (broken line) fields in an electromagnetic wave advancing towards the viewer
through the bottom one. The "corkscrew rule’ (Fig. 4.2) indicates a clockwise magnetic field round the bottom current and anticlockwise round the top one; both combine to give a magnetic field from left to right, as shown.

If for any reason we prefer the directions of the two fields to be interchanged-electric horizontal and magnetic vertical-that can easily be arranged, by turning the radiator horizontal. Similarly for any other angle. The direction of the field in an electromagnetic wave is known as its polarization. Since there are two fields, mutually at right angles, there would be ambiguity if it had not been agreed to specify the polarization of a wave as that of its electric field. This custom has the advantage that the angle of polarization is the same as that of the radiator.

Because radiation starts off with a certain polarization it does not follow that it will arrive with the same. If it has travelled far, and especially if on its way it has been reflected from earth or sky, it is almost certain to have become disarranged. Another thing is that polarization is not necessarily confined to a single angle. After reflection, radiation often has both vertical and horizontal components, and the maximum may be at some odd angle. If it is equal at all angles, the wave is said to be circularly polarized. because at any point it consists of fields of constant strength rotating continuously at the rate of one revolution per cycle. Sometimes for special purposes the radiator is designed to make the waves circularly polarized from the start.

### 17.7 Aerials

Having considered the process of radiation, we are due to turn attention to the radiator. If intended for that purpose it is called an aerial or antenna. The form we developed from a quarterwavelength transmission line, known as a half-wave dipole, is perhaps the most important both from the theoretical and (since the
arrival of television) practical point of view.
We have already noted that this straight metal wire or rod is a tuned circuit, and that its sound-wave analogue is a stretched cord as in Fig. 1.3. The fundamental frequency at which it resonates is the one that makes half a wavelength equal to (or, to be precise, a few per cent greater than) the length of the rod.

There are certain higher frequencies at which the same length of aerial can be set into vibration at harmonic frequencies; but because of complications due to currents flowing in opposite directions, tending to cancel out radiation, this condition is seldom used. It is not essential for an aerial to be in resonance at all, but as the radiation is proportional to the current, which is a maximum at resonance, aerials are practically always worked in that condition.

Just as with the tuned circuits in Chapter 8, the originator of the current has been shown as a generator in series; actually at the centre of the dipole. But in practice a tuned circuit is often 'excited' by an external field, as for example a magnetic field in inductive coupling. The same applies to our metal rod. If it is exposed to electromagnetic waves, the magnetic field pattern sweeping across it induces an e.m.f. in it. This statement must not be taken to mean that if there were no metal there would be no e.m.f. It is the movement of the magnetic field that induces the electric field, as we have seen; and that electric field constitutes an alternating e.m.f. in space.

It is unimportant whether the e.m.f. acting on the aerial is credited to the electric field or the magnetic field-they are simply two different descriptions of a single action-but it is usually convenient to work in terms of the electric component of the waves. Near a radiator it is reckoned in volts per metre, or microvolts per metre if far off. If a vertical wire two metres long is exposed to vertically polarized radiation of $100 \mu \mathrm{~V}$ per metre, then an e.m.f. of $200 \mu \mathrm{~V}$ will be induced in it. But if either the wire or the radiation is horizontal, no e.m.f. is induced. In general, e.m.f. is proportional to the cosine of the angle between the wire and the direction of polarization.

It is in this way that an aerial is used at the receiving end. The only difference in principle between a sending and a receiving aerial is that one is excited by a r.f. generator connected to it, and the other by the electromagnetic waves created by the former. The factors that make for most effective radiation in any particular direction are identical with those for the strongest reception from that direction, so whatever is said about aerials as radiators can readily be adapted from the reception standpoint.

### 17.8 Radiation Resistance

We have noted that the dipole aerial, like the quarter-wave line from which we developed it, has inductance and capacitance.

What about resistance? There is of course the resistance of the metal of which it is made, together with the various sources of loss resistance described in Secs. 8.14 and 15. But we have seen that the waves radiated from a sending aerial carry away energy from it, otherwise they would not be able to make currents flow in receiving aerials. It is convenient to express any departure of energy from a circuit in terms of resistance (Sec. 2.18). But whereas the other kinds of aerial resistance are wasteful, this kind, called radiation resistance, measures the ability of an aerial to do its job.

The radiation resistance of an ordinary tuning circuit is generally very small compared with its other resistance (radiation, after all, is not its job!): at the other extreme the loss resistance of a dipole, made perhaps of copper tubing. is small compared with its radiation resistance, which at resonance is about $70 \Omega$ if it is measured at the centre where the current is greatest. If therefore a current of say, 2 A can be generated there, the power radiated, calculated in the usual way as $I^{2} R$, is $2^{2} \times 70=280 \mathrm{~W}$. If the radiation resistance were measured at some other point along the dipole it would be greater, but this would not mean greater radiation, because the current there would be less.

A receiving aerial too has radiation resistance, and some of the energy imparted to it by the waves is re-radiated - a fact of great importance in the design of receiving aerials, as we shall see.

### 17.9 Directional Characteristics

We based our study of radiation from a dipole on the production of waves along a rope. But of course there is no reason for electromagnetic radiation to be confined along a single line like this; it goes out equally in all directions around the 'equator'. So the wave pattern in this plane can be likened to the ripples created on the surface of a pond by dropping in a stone. A skeleton of it could be produced by having ropes radiating from one"s hand like the spokes of a horizontal wheel. But what about other angles, upwards and downwards?

Is it too much to imagine oneself in outer space, where ropes have no weight, at the centre of great numbers of them stretching in every direction? Those immediately above and below receive no wavy pattern; only lengthwise stretching and relaxing. At intermediate angles there would be a combination of the two effects, as illustrated by the few samples in Fig. 17.7. By filling in as many others as possible, we can get an idea of the whole pattern of radiation: most strongly around the equator, and none at all along the axis. The pattern would be still more completely represented by the effect on a vast light elastic jelly of making a point at its centre vibrate up and down. The relative strength of radiation in different directions is however usually represented by what is
called a polar diagram. This is a curve drawn around a centre, such that the distance from the centre to the curve in any direction represents the radiation in that direction. Ideally, a polar diagram would be a three-dimensional surface, but on paper one usually has to be content with the curves, which are sections of the surface at particular planes.

Fig. $17.8 a$ is the horizontal polar diagram of a vertical dipole,


Fig. 17.7-If the rope trick of Fig. 17.5 (b) were performed with ropes radiating in all directions, the results would look something like this


Fig. 17.8-Polar diagrams of radiation from a simple halfwavelength dipole. seen (a) end-on, and (b) side view
showing that radiation (or reception) is equal in all horizontal directions. At $b$ is the vertical polar diagram, showing how the radiation varies from maximum at right angles to the dipole to nil end-on. So if a dipole is placed horizontally it can be used to cut out undesired reception from two opposite directions. As regards reception, these diagrams assume, of course, that the radiation arriving has the correct polarization. A horizontal dipole does not respond to vertically polarized waves from any direction.

### 17.10 Reflectors and Dïrectors

Any metal rod is a tuned circuit which will have maximum currents set going in it by waves twice its own length. These currents radiate
waves. just as if they were caused by a directly connected generator. If such a rod is placed close to a radiating dipole, the re-radiation from it is strong enough to modify the primary radiation very considerably, subtracting from it in some directions and adding to it in others. When the parasitic aerial (as it is called) is about the same length as the driven one, and quarter of a wavelength away from it, the phase of the re-radiation is such as to reverse much of the radiation beyond the rod, which is therefore called a reflectorsee Fig. 17.9. The same principle applies to a receiving aerial, and reflectors are frequently used: not so much to strengthen the wanted signal (though that is usually helpful) but to cut out interference from the opposite direction.

The effect just described depends on the phase relationship


Fig. 17.9—Showing hou the equal-allround radiation of Fig. 17.8 (1) (repecated here dotfed) is motified by (an unconnected dipole placed quarter of a wavelengih awar from the first, and of such a lengith as to have a $45^{\circ}$ phase angle
between the primary and secondary radiation. When considering tuned circuits we may have noticed (from Fig. 8.8, for example) that a small departure from exact tuning causes a large phase change between voltage and current. It is the same with dipoles; a small increase or decrease in length alters the phase and therefore the polar diagrams. The phase can also be varied by the spacing between the rods. If the parasitic rod is shorter than the driven one, and only about $0 \cdot 1 \lambda$ away from it, the effect is opposite to that shown in Fig. 17.9, so such a rod is called a director. The use of reflectors or directors affects the resistance of the driven dipole as a load, usually lowering it.

The one-way tendency can be increased by using both a reflector and a director. one on each side; or even several directors, as in the array shown in Fig. 17.10 b. which is called a Yagi aerial. Its polar diagram (a) shows a very considerable advantage over that of the single dipole.

But this is only the beginning of what can be done. The number of driven (or receiver-connected) dipoles can be increased. by placing them side by side, or in stacks one above the other, or both, and they can also be provided with parasitic elements. The underlying principle of all is the same: to arrange them so that the separate
radiations add up in phase in the desired directions and cancel out in the undesired directions. It is necessary to consider not only the distribution horizontally but also the angle of elevation above the horizon. By using a sufficient number of dipoles, suitably arranged,

Fig. 17.10-(a) Polar diagram of an aerial system consisting of two parallel units each made up of a dipole, one reflector and four directors (b). The diagram for a single dipole is again shown dotted for comparison

the radiation can be concentrated into a narrow beam in any desired direction.

### 17.11 Aerial Gain

Concentrating the radiation in the direction or directions where it will be most useful increases the apparent power of the sender. The number of times the intensity of radiated power in any particular direction is greater than it would be if the same amount of power were radiated equally in all directions is called the power gain (or just 'gain') of the aerial in that direction. For example, if the radiated power were concentrated uniformly into one tenth of the whole surrounding sphere the gain would be 10 . Within that beam, the radiation would be the same as if ten times the power were being radiated equally in all directions. The effective power of a sender is therefore equal to the actual radiated power multiplied by the aerial gain; this figure is known as the effective radiated power (e.r.p.).

In broadcasting, the aim is to increase the e.r.p. by directing as much of the radiation as possible horizontally, rather than upwards where it would be wasted. The apparent power of a sender, so far as any particular receiver is concerned, can of course, be further

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increased by using a receiving aerial with gain in the direction of the sender.

A simple dipole, by favouring the equatorial as compared with polar directions, has a gain of 1.64 , and the gains of other aerials are sometimes quoted relative to this rather than to the theoretical spherical radiator.

### 17.12 Choice of Frequency

A feature of the dipole aerial and its derivations which we have considered so far is that their dimensions are strictly related to the wavelength and therefore to the frequency (Sec. 1.9). What decides


Fig. 17.11-'Geostationary' satellite for long-distance microwave communication
the choice of frequency: a convenient aerial length, or something else?

The shorter the wavelength and higher the frequency, the smaller and cheaper the aerial and the more practical it is to direct its radiation and increase its gain. For example, as long ago as 1931 a radio link was established across the English Channel on a wavelength of 17 cm , a half-wave aerial for which is only about $3 \frac{1}{2}$ in long! Why, then, erect at vast expense aerials hundreds of feet high?

The main reason is that such waves behave very much like the still shorter waves of light, in not being able to reach anywhere beyond the direct line of 'view' from the sending aerial. So if the cheapness is to be maintained the range is usually restricted to a few miles. It can be increased to one or even two hundred miles by setting up the aerials on high towers or mountains, and to thousands of miles by using man-made satellites, for re-transmission, as in Fig. 17.11. The most useful type of satellite for this purpose is in orbit at such a distance ( 22283 miles) that its period of revolution around the earth is 24 hours, so that if established in the same direction as the earth's daily rotation the satellite maintains a constant station relative to places on earth.

Wavelengths shorter than 10 m are used for television, not bebecause the shortness of wavelength in itself has any particular merit for that purpose, but because it corresponds to a very high frequency, which, as we shall see in Chapter 23, is necessary for a carrier wave that has to carry modulation frequencies of up to several MHz . They are aiso used for short-distance communication such as police cars, radiotelephone links between islands and mainland, and other specialized short-range purposes.

### 17.13 Influence of the Atmosphere

At various heights between 60 and 300 miles above the earth, where the atmosphere is very rarified, there exist conditions which cause radio waves to bend, rather as light waves bend when they pass from air into water. The very short (high-frequency) waves we have just been considering normally pass right through these layers and off into space, as shown in Fig. 17.12a. They are therefore suitable for communication with and control of spacecraft. Where they are close to the ground, they are rapidly absorbed, as indicated by the shortness of the ground-level arrow. So their effective range there is limited.

As the frequency is reduced below about 30 MHz -ine exact dividing line depends on time of day and year, sun-spot activity, and other conditions--the sky wave is bent so much that it returns to earth at a very great distance, generally several thousands of miles. Meanwhile the range of the wave-front traveiling along the surface of the earth (and therefore termed the ground wave) increases slowly (Fig. 17.12b). Between the maximum range of the ground wave and the minimum range of the reflected wave there is an extensive gap, called the skip distance, which no appreciable radiation reaches.

As the frequency is further reduced this gap narrows, and earth reflections may cause the journey to be done in several hops (c). Since the distances at which the sky waves return to earth vary according to time and other conditions as mentioned, it is rather a complicated business deciding which frequency to adopt at any
given moment to reach a certain distance. But a vast amount of data has been accumulated on this and a judicious choice enables reliable communication to be maintained at nearly all times. As waves usually arrive at the receiver by more than one path simultaneously and tend to interfere with one another, fading and distortion are usual unless elaborate methods are adopted for sorting them out. At a certain frequency, of the order of 2 MHz , the ground wave and reflected wave begin to overlap at night (e), while during daylight the reflected wave is more or less absent. Over


Fig. 17.12-Showing (nol to scale) the relative ranges of ground wave and reflected wave from very high frequencies (a) lo medium (e)
the ranges at which there is overlap the two waves tend to interfere and cause fading and distortion, as they do with more than one reflected wave.

Finally, the range of the ground wave increases and becomes less affected by daylight or darkness, so that frequencies of $50 \mathrm{kHz}-20 \mathrm{kHz}$
have a range of thousands of miles and are not at the mercy of various effects that make long-distance short-wave communication unreliable. For this reason they were originally selected as the only wavelengths suitable for long ranges, but now only a small fraction of long distance communication is borne by very long waves. The disadvantages of the latter are (1) the enormous size of aerial needed to radiate them; (2) the low efficiency of radiation even with a large and costly aerial system; (3) the higher power needed to cover long ranges, due largely to (4) the great intensity of atmo-spherics-interference caused by thunderstorms and other atmospheric electrical phenomena; and (5) the very limited number of stations that can work without interfering with one another, because the waveband is so narrow in terms of frequency-which is what matters; see Chapter 20.

### 17.14 Earthed Aerials

We see, then, that although for convenience of dipole aerial size one would choose frequencies of a few hundred MHz , these are not suitable for all purposes. For distances of more than about 50 miles it is usually necessary to use longer waves (lower frequencies). Until now we have assumed that the aerial is suspended well clear

Fig. 17.13-The earth can be used as one half of a radiator, leaving a quarter-wavelength wire above earth

of the earth, but this is not convenient with a dipole for wavelengths of many metres, and the length of the aerial itself may be unwieldy. Marconi used the ground itself as the lower half of a vertical dipole; the remaining half sticking up out of it is therefore often called a quarter-wave Marconi aerial. The current and voltage distribution are shown in Fig. 17.13, which is the same as the top half of Fig. 17.2.

For the earth to be a perfect substitute for the lower half of a dipole it must be a perfect conductor, which of course it never is. Sea water is a better approximation. To overcome the loss due to imperfectly conducting earth, it is a common practice to connect the lower end of the aerial to a radial system of copper wires. An insulated set of wires stretched just above the ground is known as a counterpoise. Alternatively, to avoid tripping up people or animals, the wires are often buried just below the surface, as shown dotted in Fig. 17.13.

### 17.15 Feeding the Aerial

One advantage of the Marconi aerial is that it places the middle of the dipole at ground level, which is helpful for coupling it to the sender or receiver. Even so, it is not always convenient to have the output circuit of a sender close enough to the foot of the aerial to couple it directly, and it is seldom convenient to have either sender or receiver at the centre of a suspended dipole, so there is a need for some means of linking the aerial with the ground equipment. Chapter 16 showed how the problem can be conveniently and efficiently solved by means of a transmission line designed to match the impedance of the aerial. The object of matching, we recall, is to avoid reflections, which prevent the maximum transfer of power to or from the aerial and cause excessive line currents and voltages at a sender, or signal echo effects at a receiver.

We have noted that an ordinary centre-fed dipole has a radiation resistance of about $70 \Omega$. At resonance, which is the normal working condition, inductance and capacitance cancel one another out. Other kinds of resistance are usually negligible, so the dipole can be treated as a $70 \Omega$ load. The simplest scheme is to use a line having a characteristic resistance, $R_{0}$, of about $70 \Omega$, as in Fig. 17.14.


Fig. 17.14 (left)—A simple dipole connected to a sender (or receiver) by means of a matchedimpedance transmission line

Here the dipole can be regarded as a radiating extension of the line. At the other end, the line should be matched to the sender output circuit (Sec. 15.15).

Fig. 16.4 shows that $70 \Omega$ is an awkward $R_{0}$ for a parallel-wire line, for the wires have to be almost touching: it is also a long way from the lowest-loss value. On the other hand it is about optimum for coaxial cable. Moreover the coaxial type radiates less and picks up less interference. But it has the disadvantage of being unsymmetrical: that is to say, unbalanced with respect to earth. The outer conductor is in fact itself normally earthed, so although
the line can easily be coupled to the ground equipment (which is usually earthed) it is not strictly correct to connect it to a dipole, which is symmetrical. Where maximum efficiency and optimum radiation pattern are important, as in a sender, a balance-tounbalance transformer (commonly called a balun) is used; for receiving. the unbalance is often tolerated. Alternatively, parallelwire line of relatively high $R_{0}$ can be matched to the dipole by means of a transformer such as the one shown in Fig. 16.8.

In some cases it may be more convenient to alter the input resistance of the aerial itself. For example, in a large array where many dipoles have to be fed in parallel the total resistance would be very low if they were fed at their centres, which is analogous to feeding tuned circuits in series. We found that a tuned circuit in parallel was a relatively high resistance, and the equivalent of this is feeding a dipole at its end. With one dipole this resistance


Fig. 17.15-Development of folded dipole
(about $3600 \Omega$ ) would be too high to match any kind of feeder, but a large number in parallel can be so matched.

It is often convenient to feed a dipole from a parallel-wire line having an $R_{0}$ of the order of $300 \Omega$. There is a method of adapting the dipole to match this: its theory can be developed as in Fig. 17.15, where $a$ shows the ordinary dipole with its input resistance, at its centre, of about $70 \Omega$. Imagine now that it is split down the middle, as at $b$. Both halves are still connected to the generator, which is maintaining the same voltage, $V$, so half the original current ( $I$ ) goes into one half dipole and the other half into the other. Next, suppose the energizing of the two half aerials is entrusted to separate generators (c) each supplying $I / 2$. If they are in phase with one
another, the ends of the two dipoles will be at the same potential so can be joined together without making any difference. Finally ( $d$ ) the two voltages $V$ are supplied by one generator, which must therefore provide twice the voltage of the original one (a) to make half the original current flow into the modified aerial, called a folded dipole. Its resistance is therefore four times that of an ordinary dipole, so is matched by a $280 \Omega$ feeder.

A folded dipole is shown as the main element in the Yagi aerial in Fig. 17.10, because with many directors and reflectors the impedance of a dipole is much reduced and a folded type may be a better match than an ordinary one for a coaxial line. Moreover, the centre point of the non-fed half is at zero potential so needs no insulation if the aerial is supported there.

### 17.16 Tuning

A dipole aerial is self-tuned, by virtue of its chosen length, to a particularly frequency. But an aerial is often required to work over a very wide range of frequency, especially for receiving. Instead of varying the length of the aerial wire, it is usual to augment its distributed inductance and capacitance by means of variable capacitors or


Fig. 17.16-A common method of tuning an aerial to a desired wavelength
inductors, as for example in Fig. 17.16. These tuning components reduce the resonant length of the aerial for a given frequency, which may be very desirable; for example, even a quarter-wave self-tuned aerial for 30 kHz would have to be 8000 feet high! It is quite usual for a considerable proportion of the reactance of an aerial system, especially for medium and long waves, to be in concentrated form; but it must be understood that this is at the expense of radiation or reception.

Where height is limited, by restrictions imposed by aviation or by the resources of the owner, it is possible to increase radiation moderately by adding a horizontal portion, as in the familiar T or
inverted L aerials, Fig. 17.17. The effect of the horizontal extension is to localize the bulk of the capacitance in itself so that the current in the vertical part, instead of tailing off to zero, remains at nearly the maximum and therefore radiates more. The top portion radiates too, but with a different polarization, so the addition due to it may not be very noticeable in a vertical receiving aerial.

### 17.17 Effective Height

In general, the greater the radiation resistance of an aerial the more effective it will be for receiving as well as for sending. But a more useful piece of information about an aerial when it is used for receiving is what is called its effective height. It is the height which, when multiplied by the electric field strength, gives the e.m.f.

Fig. 17.17-To increase the radiation from a verical aerial without increaseing its height, a horizontal top is commonly adsed, forming (a) the $T$ or (b) the inverted L type. Note the more uniform current distribution in the vertical part (b)

(a)

(b)
generated in it. For example, if the wave from a sender has a strength of 2 mV per metre at a receiving station, and the effective height of the aerial is 5 metres, the signal voltage generated in the aerial will be 10 mV . The effective height is less than the physical height. It can be calculated for a few simple shapes, such as a vertical wire rising from flat open ground, but usually has to be measured, or roughly estimated by experience.

### 17.18 Microwave Aerials

What happens if, instead of making the wavelength longer than is most handy for dipoles, it is made much shorter-say a few
centimetres? The radiation from a single dipole is of course limited by its size, and to bring the e.r.p. (Sec. 17.11) up to a useful figure a relatively large reflector is needed. Instead of an enormous number of small dipoles, it is made of continuous metal, or, to reduce weight and wind resistance, wire netting. The currents induced in this by the driven dipole cause reflection, but in a more controllable pattern, for the metal can be formed into a parabolic reflector, like a large headlight or bowl heater. In this way the radiation or reception can be confined very closely in the required direction.

Just as at these frequencies the two-conductor transmission line gives place to a waveguide (Sec. 16.10), the dipole gives place to a horn, which is an expanding end to the waveguide, arranged to direct the radio power on to the reflector.

Such frequencies and aerials are commonly used for radar, where narrow beaming is essential, and for point-to-point transmission of television signals.


Fig. 17.18--The effectiveness of a receiving aerial consisting of a loop or coil of wire depends on the direction from which the waves are arriving

### 17.19 Inductor Aerials

In all the aerials considered, the circuit between the ferminals is completed by the capacitance from one arm to the other or to earth. Although they have inductance, they can be regarded roughly as opened-out capacitors. It is possible instead to use an opened-out inductor, in which of course there is a wire path all the way. This type is known as a frame or loop aerial, and because its loss resistance is usually larger than the radiation resistance it is seldom used as a radiator, but mainly for receiving, especially for direction finding.

We noted that reception by an open aerial is usually calculated from the strength of the electric field, although the same answer would be given by the magnetic field. The opposite applies to loop aerials. Taking as example a single square turn of wire as in Fig. 17.18, one can see that waves arriving broadside on, whatever their polarization, induce no net e.m.f. in the aerial. A vertical electric field would cause e.m.fs in the portions $A B$ and $C D$, but these would be exactly in opposition around the loop. It is easier,
however, to see that the magnetic field, being in the same plane as the loop, would not link with it at all, and that is sufficient to guarantee no signal pick-up.

From any direction in the plane of the paper-say that indicated by the arrow-the magnetic field is at right angles to the loop, so links it to the maximum extent, provided the width is not more than half a wavelength. Although the e.m.fs from A to B and C to D are equal (and most of the time opposite, if the loop is small compared with a wavelength) they are out of phase, so there is a net e.m.f. around the loop.

At intermediate angles the results are intermediate, so the polar diagram is very like that of a dipole at right angles to the loop.

The size of a loop aerial in a portable broadcast receiver is, at most, very small compared with the wavelengths to be received, so the signal e.m.f. obtained by it is relatively weak. If, in order to embrace as much field flux as possible, aerial wire is wound around the set just inside its outer covering, it also embraces a good deal of assorted material, much of which causes considerable losses of the kind referred to in Sec. 8.15. If, however, the aerial winding is provided with a high-permeability core, the magnetic flux over a wide area tends to pass through the core, as shown in Fig. 17.19, just as electric current favours a low-resistance path.


Fig. 17.19-The concentrating effect of a suitable magnetic core on the magnetic field of the waves greatly increases the effectiveness of a small coil as an aerial

In this way a coil less than one inch in diameter is equivalent to a very much larger one without the core. It is essential, of course, for the core to be of low-loss material; the non-conducting magnetic materials known as ferrites are used. The tuning inductance so created is too much for very high frequencies, so receivers with facilities for v.h.f. usually include a telescopic vertical aerial.

## Detection

### 18.1 The Need for Detection

Turning now to the reception end in Fig. 1.9, we see a number of boxes representing sections of a receiver. Although all of these sections are included in most receivers, they are not all absolutely essential. So we begin with the detector, because it is the one thing indispensable, for reasons now to be explained.

At the receiving aerial, the modulated carrier wave generates e.m.fs whose waveforms exactly match those of the currents in the sending aerial. The frequency of those currents, as we know, is so high as to be inaudible. It has been chosen with a view to covering the desired range most economically (as discussed in the previous chapter), and bears no relationship to the frequencies of the sounds being transmitted. In order to reproduce those sounds, it is necessary to produce currents having the frequencies and waveforms of the sounds. For example, given the amplitude-modulated carrier wave shown in Fig. 15.13c (or 18.1a), the problem is to obtain from it Fig. 15.13a. The device for doing so is the detector, or, as it is often called, the demodulator.

For receiving frequency-modulated signals (Sec. 15.12) it is necessary also to include means for converting the frequency variations into the amplitude variations on which the detector works. This device-the discriminator-is considered at the end of this chapter.

### 18.2 The Detector

The simplest method of detection is to eliminate half of every cycle, giving the result shown in Fig. 18.1b. At first glance, this might not seem to differ fundamentally from $a$, for the original frequency appears to be still there, though at half-strength. It is quite true that the original frequency is there, and means have to be provided for getting rid of it. The fundamental difference is that whereas the average value of each cycle in $a$ is zero (because the negative half cancels the positive half), in $b$ it is proportional to the amplitude (at that moment) of the modulated carrier wave, which in turn is proportional to the instantaneous value of the a.f. modulating signal. The dotted line in Fig. $18.1 b$ indicates the average level of the current, and it can be seen to vary at the same frequency and with the same waveform as Fig. 15.13a. There is
admittedly some loss in amplitude, but we shall see that the types of detector in common use manage to avoid most of this loss and give an output almost equal to the peak values.

The process of suppressing half of each cycle, converting alternations of current into a series of pulses of unidirectional current, is known by the general term rectification. The detector usually consists of a rectifier adapted to the particular purpose now in view, and may be taken to include means for extracting the desired


Fig. 18.1-(a) Sample of an amplitude-modulated carrier wave, similar to Fig. 15.13c. The average value of each cycle is zero: but if one half of each is eliminated (b) the average value of the other halves, shoun dotted, fluctuates in the same manner as the modulating waveform (e.g., Fig. 15.13a)
a.f. from a mixture like Fig. 18.1b. It should be noted that this rectified signal contains not only r.f. and a.f. components, but also a unidirectional component (d.c.). That is, the desired a.f. current does not alternate about zero, as in Fig. 15.13a, but about some steady current-positive or negative, depending on which set of half-cycles was eliminated by the rectifier. Although this d.c. is unnecessary and in fact undesirable for reproducing the original sounds, it has its uses as a by-product (Sec. 22.11).

### 18.3 Rectifiers

A perfect rectifier would be one that had no resistance to current flowing in one direction, and an infinite resistance to current in the opposite direction. It would, in fact, be equivalent to a switch, completing the circuit during all the positive half-cycles in Fig. 18.1a, thus enabling them to be passed completely, as in $b$; and interrupting it during all the negative half-cycles, suppressing them completely, as also shown in $b$. For very low-frequency alternating currents such as 50 Hz it is possible to construct such a switch, known as a vibrator rectifier, but any device involving mechanical switching is impracticable at radio frequencies.

The two main types of electronic diode rectifier have already been considered in Chapter 9. In the early days of radio, point-contact semiconductor rectifiers using various minerals were much used under the name of crystal detectors. Between the world wars these were almost entirely superseded by thermionic valves, but were revived and improved during the second world war as the pointcontact silicon diodes used for radar. Now, radio detectors are nearly always semiconductor diodes.

In considering resistance we began with the current/voltage graphs of ordinary conductors (Fig. 2.3), and noted that because such graphs were straight lines the conductors described by them were called linear, and that since linear resistances were covered by the very simple relationship known as Ohm's law there was really no need to spend any more time drawing their graphs. Then we came to valves, and again drew current/voltage graphs, such as Fig. 9.3, and found that they were curved, indicating non-linear characteristics, not conforming to Ohm's law (except in the modified and limited sense of Fig. 11.4, and then only approximately). All non-linear conductors are capable of some degree of rectification, and all rectifiers are necessarily non-linear in the above sense; that is, their resistance depends on the amount of voltage or current, instead of being constant as in Ohm's law.

Here now in Fig. 18.2 are some more current/voltage characteristics, the lines COD and AOB representing (as can be calculated


Fig. 18.2-Characteristic curves of various linear resistances. The line $C O D$ is seen to represent $500 \Omega$. If the resistance to negative voltages is different from that to positive, as shown for example by the line COB, the result is a partial rectifier
from the current and voltage scales) $500 \Omega$ and $2000 \Omega$ respectively. As we noticed before, the steeper the slope the lower the resistance. To go to extremes, the line FOG represents zero resistance, and HOJ infinite resistance; so FOJ is the graph of a perfect rectifier. COB is an example of a partial rectifier-a resistance of $500 \Omega$ to positive currents and $2000 \Omega$ to negative currents. Now apply an alternating voltage to a circuit consisting of any of the foregoing imaginary resistances in series with a fixed resistance of $1000 \Omega$ (Fig. 18.3a). The voltage across the $1000 \Omega$-call it the output voltage-is equal to the applied voltage only if $R$ is zero. Both voltages are then indicated by the full line in Fig. 18.3b. If $R$ were $500 \Omega$ (COD), the output would be represented by the dotted line $C D$, and if $2000 \Omega$ by the dotted line AB.

If now a rectifier is substituted for R, as shown in Fig. 18.3c, the positive and negative output voltages are unequal. For example, the perfect rectifier (FOJ) gives the output FJ in $b$; and the partial rectifier gives CB. The average voltage, taken over each whole cycle, is, of course, nil when $R$ is not a rectifier. Using the perfect

Fig. 18.3-The results of applying a sinusoidal voltage to a circuit (a) consisting of any of the resistances represented by Fig. 18.2 (denoted by $R$ ) in series with $1000 \Omega$ are indicated by b, which is lettered to correspond with the various characteristics in Fig. 18.2. When $R$ is a rectifier it is represented as in $c$

rectifier, the average of the positive half is $64 \%$ of the maximum (Sec. 5.4), while the negative is, of course, nil; so the average rectified current taken over a whole cycle is $32 \%$ of the peak value. Using the rectifier with the graph COB, the negative half-cycle ( B in Fig. 16.3b) cancels out half of the positive half (C), which is twothirds of the input; so the rectified result is less than $11 \%$ of the peak input.

### 18.4 Linearity of Rectification

Whether complete or partial, however, the degree of rectification from rectifiers having characteristics like those in Fig. 18.2 would be independent of the amount of voltage, provided, of course, that the rectifier was arranged to work exactly at the sharp corner in its characteristic, whether that happened to correspond with zero voltage, as in Fig. 18.2, or not. So if we were to draw a graph showing the net or average or rectified voltage output against peak
alternating voltage input to a rectifier of this kind we would get a straight line, such as the two examples in Fig. 18.4, corresponding to characteristics FOJ and COB in Fig. 18.2. These are therefore called linear rectifiers, it being understood that the term is in reference to a graph of this kind and not to one of the Fig. 18.2 kind.

It should be clear from Fig. $18.1 b$ that it is necessary for the rectifier to be linear if the waveform of the modulation is not to be


Fig. 18.4-The rectified outputs obtained by applying alternating voltages to various rectifiers are shown in these characteristic curves, lettered to correspond with Figs. 18.2 and 18.3
distorted. It is therefore unfortunate that in reality there is no such thing as a perfectly linear rectifier. The best one can do is to approach very close to perfection by intelligent use of the rectifiers available. The weak feature of all of them is the more or less gradual transition from high to low resistance, exemplified by the typical 'bottom bend' in Fig. 9.3, which causes weak signals to be rectified less completely than strong ones. Such a characteristic is represented in Fig. 18.4 by the curved line, and is explained in greater detail later (Sec. 18.11). Semiconductor diode characteristics are considerably better, if we allow for the offset or displacement from zero, especially with silicon (Sec. 9.11).

### 18.5 Rectifier Resistance

There were two reasons for the poor results of rectifier COB in Fig. 18.2 compared with FOJ. One was its forward resistance ( $500 \Omega$ ), which absorbed one-third of the voltage, leaving only two-thirds for the $1000 \Omega$ load. The other was its finite backward resistance ( $2000 \Omega$ ) which allowed current to pass during the negative half-cycles, so neutralizing another one-third of the voltage. With such a rectifier it would clearly be important to choose the load resistance carefully in order to lose as little as possible in these ways. As it happens, $1000 \Omega$ does give the largest output voltage in this case, the general formula being $\downarrow^{\prime}\left(R_{\mathrm{F}} R_{\mathrm{B}}\right)$, where $R_{\mathrm{F}}$ and $R_{\mathrm{B}}$ are respectively the forward and backward resistances of the rectifier.

As compared with crystal diodes, the vacuum diode does have one
point in its favour. Although it is not perfect in the forward direction, for it always has some resistance, it is practically perfect in the reverse direction; that is to say, it passes negligible backward current, even when quite large voltages are applied. So even if, owing to a high forward resistance, the forward current were very small, the output voltage could, theoretically at least, be made to approach that given by a perfect rectifier, merely by choosing a sufficiently high load resistance. Silicon diodes are now available with very high backward resistance, and lower forward resistance

Fig. 18.5-Diode rectifier with the load resistor $(R)$ shuntea by a reservoir capacitor, $C$

than thermionic types. They have a sharper rectification bend, and of course reed no cathode heater. But they need a little forward bias.

Although there are limits to the load resistance that can be used in practice the loss of rectification can usually be made insignificantly small. In fact, by a very simple elaboration of Fig. 18.3c-a capacitor in parallel with the load resistance-it is possible to approach a rectification efficiency of $100 \%$ instead of the $32 \%$ that is the maximum without it, and so do nearly three times better than our 'perfect' rectifier. This point deserves closer consideration.

### 18.6 Action of Reservoir Capacitor

The modified rectifier circuit is now basically as Fig. 18.5. Suppose. in order to get the utmost rectified voltage, we made the load resistance $R$ infinitely great, and that the backward resistance of the diode was also infinite. The capacitor $C$ would then be in series with the diode, with no resistor across it. To simplify consideration we shall apply a square wave instead of a sine wave; amplitude 10 V (Fig. 18.6a). We shall also assume that its frequency is 500 kHz , so that each half cycle occupies exactly one millionth of a second (l $\mu \mathrm{s}$ ). At the start C is uncharged, and therefore has no voltage across it. It can acquire a charge only by current flowing through the rectifier, which offers a certain amount of resistance, and so when the first positive half-cycle arrives its 10 V at first appears wholly across that resistance, as shown by the full line $V_{D}$ in Fig. 18.6 $b$. The current driven by this voltage starts charging the capacitor, whose voltage thereby starts to rise towards 10 V , as shown by the dotted
line. It is clear that the voltages across rectifier and capacitor, being in series, must add up to give 10 V so long as that is the applied voltage; and the full-line curve in Fig. 18.66 has been drawn accordingly. We have in fact exactly the same situation as the one we discussed in connection with Fig. 3.6, so no wonder the curves have the same shapes as there. The greater the resistance


Fig. 18.6-Analysis of application of a square alternating voltage (a) to the circuit Fig. 16.5. The dotted line in $b$ represents the voltage ucross C; the full line, that across the diode
of the rectifier and the capacitance of the capacitor, the slower the rate of charge, just as with a very large balloon inflated through a very narrow tube. As we noted with Fig. 3.6, the capacitance in microfarads multiplied by the resistance in megohms is the time constant-the number of seconds required for the capacitor voltage $V_{C}$ to reach $63 \%$ of the applied voltage. Suppose that the rectifier resistance (assumed constant) is $0.01 \mathrm{M} \Omega$ and the capacitance $0.0001 \mu \mathrm{~F}$. Then the time constant is 0.000001 second or $1 \mu \mathrm{~s}$. In this case that very conveniently happens to be the time occupied by one half-cycle of the 500 kHz applied voltage. So at the end of the first positive half-cycle $V_{\mathrm{C}}$ is $6 \cdot 3$, while $V_{\mathrm{D}}$ has dropped to $10-6 \cdot 3=$ $3 \cdot 7$. Then comes the negative half-cycle. The diode ceases to conduct, and while the capacitor therefore cannot charge up any more it likewise has no conducting path through which to discharge, and so remains at 6.3 V until the second positive half-cycle. Meanwhile. $V_{C}(6.3 \mathrm{~V})$ plus $V_{D}$ must now be equal to the new applied voltage, - 10. The voltage across the rectifier must therefore be -163 . Another way of looking at this is to say that the input changes from +10 V to -10 V , so $V_{\mathrm{D}}$ becomes 20 V more negative than +3.7 V ;i.e., -16.3 V .

The net voltage applied to the capacitor when the second positive half-cycle arrives is $10-6.3=3.7 \mathrm{~V}$, so at the end of this half-cycle the voltage across the capacitor will increase by $63 \%$ of 3.7 , or $2 \cdot 3 \mathrm{~V}$, which, added to the $6 \cdot 3$ it already possessed, makes $8 \cdot 6$. The charge thus gradually approaches the peak signal voltage in successive cycles, while the average voltage across the rectifier falls by the same amount, as shown in Fig. 18.6 $b$. The rectified output voltage therefore approaches $100 \%$ of the peak applied r.f. voltage.

A similar result is obtained (but more slowly) with a sine wave signal.
Because C maintains the voltage during the half-cycles when the supply is cut off, it is called a reservoir capacitor.

### 18.7 Choice of Component Values

This, of course, is excellent so far as an unmodulated carrier is concerned. When it is modulated, however, the amplitude alternately increases and decreases. The increases build up the capacitor voltage still further; but the decreases are powerless to reduce it, for the capacitor has nothing to discharge through. To enable $V_{C}$ to follow the modulation it is necessary to provide such a path, which may be in parallel with either the capacitor or the diode, so long as in the latter case there is a conducting path through the input to complete the discharge route. The resistance of the path should be low enough to discharge the capacitor $C$ at least as fast as the modulation is causing the carrier amplitude to decline. The speed of decline increases with both depth and frequency of modulation, and calculating the required resistance properly is rather involved, but a rough idea can be obtained as follows.

Suppose, for example, the highest modulation frequency is 10000 Hz . Then the time elapsing between a peak of modulation and a trough ( $p$ to $t$ in Fig. $18.1 b$ ) is half a cycle, or $1 / 20000$ th of a second $(50 \mu \mathrm{~s})$. If C had to discharge at least $63 \%$ of the difference in carrier amplitude in that time, the time constant would have to be not more than $50 \mu \mathrm{~s}$ and to avoid appreciable loss and distortion of top frequencies had better be less. Of the two factors making up the time constant $C R, R$ is decided on the grounds just about to be considered, leaving us with the choice of $C$ to make the time constant suitable.

We have no doubt realized by now that the discharge resistance $R$ needed to enable the rectified output to follow the modukation is the same thing as what previously we regarded as the load resistance. If the load is the final outlet of the receiver, such as a pair of headphones, then its impedance is limited in practice to a few kilohms. More often however the detector is followed by an amplifier to increase the power to a level suitable for working a loudspeaker or television tube. With a valve or f.e.t. amplifier the input impedance is many megohms - too high and indefinite to serve as the discharge path. So something of the order of $0.5 \mathrm{M} \Omega$ is chosen for $R$. This suits the thermionic diode used with valves (and which often shares the cathode of the first amplifying valve) but is on the high side for semiconductor diodes. The value of $C$ that makes $C R$ equal to $50 \times 10^{-6}$ is 100 pF . For good reproduction of the highest audio frequencies either $C$ or $R$ or both should be reduced. The limit begins to be reached when $C$ discharges appreciably between one r.f. peak and the next. causing a loss of detection efficiency at all
modulation frequencies. The choice of $C R$ is most difficult when the carrier-wave frequency is not many times greater than the top modulation frequency, and for this reason (among others) the carrier frequencies chosen for 'hi-f' broadcasts are always very high (v.h.f.).

If a bipolar transistor is used as the first modulation amplifier, its input resistance is normally of the order of one or two kilohms, or rather more if it is increased by means of an unbypassed emitter biasing resistance (Sec. 12.9). To maintain a sufficiently high reservoir action at the lowest r.f. (which we shall see in Chapter 21 is usually 470 kHz or thereabouts) $C$ must be much greater, usually about 10 nF , which with $4 \mathrm{k} \Omega$ gives a time constant of $40 \mu \mathrm{~s}$.

The diode is chosen for a forward resistance that is small and a backward resistance large compared with $R$; for $R$ equal to a few kilohms this requirement is readily met by point-contact germanium types.

The much higher modulation frequencies used for vision necessitate a much shorter time constant, of the order of $20 \mathrm{~ns}(=1 / 50 \mu \mathrm{~s}$ ), but the carrier frequencies too are very high, or 'ultra' high (u.h.f.).

### 18.8 The Diode Detector in Action

As the action of the deceptively simple-looking circuit in Fig. 18.5 is not very easy to grasp, and is also most important, it will be worth spending even more time on it. Fig. 18.7 is a diagram which, compared with Fig. 18.6, is speeded up so as to show some modulation, and uses proper sine waves for the carrier. Also, the voltage $V_{\mathrm{C}}$ across the capacitor-which, of course, is the output voltageis drawn together with the r.f. input voltage $V_{1}$ at $a$. Notice that whenever $V_{1}$ is more positive than $V_{c}$ the diode conducts and $C$ charges rapidly, because the time constant is $C R_{F}, R_{F}$ (the forward resistance of the diode) being much less than $R$. Directly $V_{1}$ drops below $V_{C}, C$ discharges through the only path available ( R ) with the much longer time constant $C R$. It is too long in this case, because $V_{\mathrm{c}}$ loses contact with $V_{\mathrm{I}}$ altogether during the troughs, and its waveform is obviously a distorted version of the modulation envelope. But that is because for the sake of clearness the carrier frequency shown is excessively low for the modulation frequency.

Note too that in addition to the desired frequency, $V_{c}$ has also a saw-tooth ripple at the carrier frequency, but much less of it than in Fig. $18.1 b$ where there was no reservoir capacitor. Not only has C improved the rectification efficiency but it has also got rid of most of the unwanted r.f.

The third component making up $V_{C}$ is a steady positive voltage (zero frequency), represented by the average height of the dotted line above the zero line. The voltage across the diode must, of course, be equal to $V_{\mathrm{I}}$ minus $V_{\mathrm{C}}$ (Fig. 18.7b). So it also contains the z.f. and a.f. voltages, and therefore could be used alternatively

Fig. 18.7-In diagram $a, V_{1}$ represents a modulated r.f. input voltage and $V_{c}$ the resulting oufput voltage. In diagram $b$, the full line shows $V_{1}-V_{c}$ : i.es the voltage across the diode


Fig. 18.8-The principle varieties of diode detector circuit
as the output voltage, but with the disadvantage of having nearly the full r.f. voltage along with it.

### 18.9 The Detector as a Load

In Chapter 20 we shall see that the $Q$ of receiver tuning circuits is very important. The detector receives its input from such a circuit, across which it is connected. Doing so is bound to reduce the $Q$ of that circuit, and how much it does so can be calculated (Sec. 8.16) if we know the resistance of the detector. But do we?

With a perfect diode in series with a resistance $R$, as was assumed in Sec. 18.2 and shown in Fig. 18.8a, the resistance during one set of half-cycles is $R$ and during the other set is infinite, or, with a practical diode, so much larger than $R$ as to have negligible effect. We know (Sec. 14.5) that the energy-storing capability of a circuit having a high enough $Q$ to be useful for tuning enables its waveform and amplitude to continue almost unchanged even though the circuit receives its maintaining energy in pulse form. The same applies to the removal of energy by resistance. So a resistance load that exists only during every alternate half-cycle is virtually the same as half the load all the time. The power, or rate of energy, delivered to a resistance $R$ is $V^{2} R, V$ being the voltage developed across $R$ (Sec. 2.17). So half the load due to $R$ all the time is the same as the load due to $2 R$ all the time.

What about the reservoir or peak detector, Fig. $18.8 b$ ? This looks difficult to assess because, as Fig. 18.7 shows, the tuned circuit is unloaded during most of each cycle, but for an unspecified period at each positive peak it is connected by the diode to $R$ and $C$ in parallel. But if we assume that the nearly steady voltage across $C$ is equal to the peak input voltage (in practice, with suitable component values, it nearly is), we can get at it this way: Denote the resistance which, if connected across the tuned circuit in place of the detector, would have the same loading effect, by $R^{\prime}$. If $V$ is the r.m.s. voltage supplied by the tuned circuit, the output voltage according to the above assumption is $\sqrt[2]{ } / 2 \mathrm{~V}$. As $R$ is the only component dissipating power, that power will be ( $12 V)^{2} / R_{1}=2 V^{2} / R$. But from the way in which we have defined $R^{\prime}$ this power is also equal to $V^{2} / R^{\prime}$ :

$$
\frac{V^{2}}{R^{\prime}}=\frac{2 V^{2}}{R} \quad \text { So } R^{\prime}=\frac{R}{2}
$$

Provided that the input circuit is a low-resistance d.c. path, one end of R can be transferred to the other end of it as shown in Fig. $18.8 c$, but now (assuming $C$ has negligible impedance to the carrier) $\mathbf{R}$ is across the r.f. input all the time, in addition to the
detector loading as already calculated. So $R^{\prime}$ is now $R$ and $R / 2$ in parallel, which is equal to $R / 3$, the loading effect being $50 \%$ more than in circuit $b$. This arrangement is therefore seldom used. but there are situations (especially in valve circuits) in which the input circuit is at a d.v. potential that would upset the action of the diode if it reached it. So the output is taken from across the diode, as in Fig. 18.8d. Not only is there the greater loading of the tuned circuit but since $R$ is not bypassed by $C$ the output contains the full r.f. as well as the modulation frequency (full line in Fig. 18.7b).

It should hardly be necessary to say that when an output of the opposite polarity is required the diode is simply reversed.

The above calculations of loading effects were all based on the assumption of a tuned input circuit. For some purposes (in measuring r.f. voltages, for example) a peak type of diode detector may be applied to a non-resonant circuit. This has no stored energy to supply the extremely non-uniform load, which requires strong pulses of charging current at each alternate peak of the input voltage. If the input source has much resistance these pulses cause a more or less considerable drop in voltage and therefore a loss of output voltage.

There is a drop in voltage even with a tuned circuit if its dynamic resistance is not small compared with the equivalent resistance of the detector. The few-kilohm load where the detector is followed by a transistor loads a tuned circuit excessively, so a suitable step-down transformer ratio from the tuned circuit is chosen.

### 18.10 Filters

The outputs from detector circuits $b$ and $c$ in Fig. 18.8 are something like the dotted line in Fig. 18.7b, and that from circuit $d$ is like the continuous line. Both types of output contain the desired modulation signal, and both contain also a unidirectional voltage-the average level of the output waveform in the diagram. We shall see in Chapter 22 that good use of this by-product can often be made, but it is usually undesirable in the amplifier that follows the detector. It can easily be removed by a blocking capacitor (Sec. 14.6). In the outputs of the $b$ and $c$ circuits, there is also a trace of the original r.f., which appears as the notches in the dotted lines in Fig. 18.7, and practically the full r.f. in the output from d. If r.f. were allowed to go forward into the amplifier it might upset its working conditions, or even find its way back at amplified sirength and cause selfoscillation. So it is desirable with $b$ and $c$ and essential with $d$ to remove or at least reduce it.

Devices for separating currents of different frequencies are known as filters, and are themselves a vast subject. Those used in detectors are usually of the very simplest types, however. All filters depend on the fact that inductive and capacitive impedances
vary with frequency. The simple circuits of Figs. 6.10, 7.4 and 8.1 can all be used as filters.

Take Fig. 6.10: C and R in series. At very low frequencies C offers a high impedance compared with R, and nearly all the voltage applied across them both appears across C. As the frequency rises the impedance of C falls, until, at a very high frequency, most of the voltage appears across R . The dividing frequency, at which the voltages across C and R are equal to one another (and to the input voltage divided by $\sqrt{ } 2$ ), is given by $X_{c}=1 / 2 \pi / C=R$, so $f^{\prime}=1 / 2 \pi C R$. (Notice that here again we have $C R$ cropping up.)

Suppose the lowest carrier frequency to be handled by a detector is 150 kHz , and the highest modulation frequency is 10 kHz . One might make the dividing frequency 15 kHz . Then $C R=1 /(2 \pi \times$ $15000)=10.6 \times 10^{6}$. The separate values of $C$ and $R$ depend on the general impedance of the detector circuit. Irrespective of this, and assuming that the filter output is not loaded, Fig. 18.9 shows


Fig. 18.9-The filtering action of $C R$ is shown here as a graph of $V_{0} / V_{1}$ against frequency
how much of the filter input from the detector would get through at frequencies from 1 to 1000 kHz . There is very little loss at even the highest modulation frequencies, but the residual r.f. is substantially reduced, especially when, as is usual in broadcast receivers, the lowest r.f. is about 470 kHz .

Logically the detector circuit would now be as in Fig. 18.10, in which $\mathrm{R}_{\mathrm{F}}$ and $\mathrm{C}_{\mathrm{F}}$ are the filter components, $\mathrm{C}_{\mathrm{B}}$ the blocking capacitor, and $\mathrm{R}_{\mathrm{v}}$ the volume control which is usually placed here, between the detector and the a.f. amplifier, which is often a transistor with an input resistance of only a few kilohms. A usual value for $R_{V}$ is $5 \mathrm{k} \Omega$, so the load resistance at maximum volume may be only about $2 \mathrm{k} \Omega . R_{F}$ should therefore be only a fraction of this if it is not to reduce the signal appreciably. R still further adds to the load, but quite often it is removed and $R_{F}+R_{V}$ made to do its duty, in which
case $C_{B}$ is transferred to the output side of $R_{V}$. Fig. 18.11 shows a typical detector for a transistor broadcast receiver. Note the step down from the tuned circuit to match the comparatively low impedance of the whole thing. With an f.e.t. or valve amplifier the loading problem is much relieved by its virtually infinite input


Fig. 18.10-Complete detector circuit, comprising Fig. 18.8b, Fig. 18.9 (inset) and a.f. volume control with blocking capacitor $\mathrm{C}_{\mathrm{B}}$.


Fig. 18.11-Another detector circuit for transistor receiver, with typical component values
resistance. Resistances are usually about 100 times larger and capacitances about 100 times smaller. For television picture modulation the shunt capacitances are very small-only a few picofarads.

### 18.11 Detector Distortion

In discussing rectifiers we noted that they had to be non-linear conductors in order to rectify, but that if the non-linearity consisted of a sharp corner between two straight portions (e.g., COB in Fig. 18.2), the rectifier itself could be described as linear (e.g., COB in Fig. 18.4). And that no real rectifiers have perfectly sharp bends. so all are more or less non-linear. Now that we have seen the details of complete detector circuits we are returning to this topic, to consider how to ensure that the non-linearity is less rather than more.

The simple Fig. 18.8a rectifier circuit would be the easiest to analyse, but because the reservoir type is the one used almost invariably let us assume it, even though we shall have to do some
'guesstimating'. Fig. 18.12 shows the characteristic curve of a germanium diode of a type used for this purpose, and the load resistance ( $R$ ) is say $10 \mathrm{k} \Omega$. Let us suppose that an unmodulated r.f. signal of 1 V peak is applied. And let us guess that with this input the detector has an efficiency of $70 \%$. In other words the d.v. out-


Fig. 18.12-Analysis of germanium diode detector performance by means of characteristic curve of the diode
put across C and R which back-biases the diode is 0.7 V . So we draw a cycle of the input voltage from -0.7 V as its base line, between $A$ and B. With this voltage across $R$, the continuous current through it must be 0.07 mA , as shown dotted to the right of the characteristic curve, from $\mathrm{A}^{\prime}$ to $\mathrm{B}^{\prime}$. Current through the diode is limited to the part of the input cycle ( C to D ) which makes the voltage positive, and by referring the input cycle via the characteristic curve to the current scale we get the diode current waveform. The area under it, being current multiplied by time, is a measure of the
charge per cycle going into $C$. It must be equal to the discharge per cycle, represented by the area under the dotted line $\mathrm{A}^{\prime} \mathrm{B}^{\prime}$. It looks roughly right, or perhaps hardly enough; if insufficient, a very slight decrease in negative bias, and therefore in detector efficiency, would correct it.

Now imagine the input cycles slowly changing in amplitude above and below 1 V peak due to modulation. Because the slope of the curve at P is steep, the increases in bias will be very nearly proportional to the increases in peak value. And so will moderate decreases. But if the modulation depth approaches $100 \%$. making the minimum input amplitude approach zero, clearly the decreases in bias will not be in anything like exact proportion. Since the modulation signal we are extracting consists of the ups and downs in bias, when the modulation is deep its waveform will be distorted.

Repeating this kind of diagram for different unmodulated amplitudes leads to the conclusion that the greater the amplitude the higher the efficiency and the less the distortion. Further thought should make clear that the loss of efficiency and the distortion are due mainly to the low load resistance necessitating a strong peak of charging current each cycle to keep the bias going for the rest of the cycle, and that better performance all round (including less damping of the input circuit and less loss of signal voltage due to the need for a step down) would be obtained if the load resistance were raised to the hundred-kilohm order as it can be when a f.e.t. or valve amplifier follows.

Both low and high resistance detectors are open to another cause of distortion of deeply modulated signals if the type of circuit in Fig. 18.10 is used. Here the resistance to d.c. is $R$ alone, for the other circuit paths are all blocked by capacitors. But a.c. at the modulation frequency has another path in parallel via the lowimpedance $C_{B}$ to $R_{V}$, so the re - istance to it is lower than the resistance to d.c. This makes the current through the diode swing more widely above and below the mid position during modulation. With $100 \%$ modulation this cannot happen at the negative peaks, because the diode current cannot be less than zero. So the negative peaks are clipped. The circuit in Fig. 18.11 is free from this objection.

Looking at Fig. 18.12 again we see that if a very weak station is tuned in, its amplitude being 0.1 V or less. detection will be very inefficient because the current passed by the diode at up to 0.1 V is almost negligible. For this reason it is usual to arrange for the diode to have a positive bias of up to about 0.2 V to bring it to the most sharply bent part of the characteristic curve.

In all the foregoing we have assumed that there is no capacitance across the diode. If there were it would form with C a potential divider across the signal source, reducing the efficiency of detection. For this reason junction diodes. which have enough self-capacitance to be appreciable, especially at v.h.f. and u.h.f., are generally unsuitable, and point-contact types with their very small contact area
are used. External stray capacitance must be avoided by attention to the circuit layout.

### 18.12 Triode Detectors

A diode detector is sometimes combined with a modulation amplifier, for the base-emitter junction is a diode, and the rectified base current causes a collector current $h_{\mathrm{fe}}$ times as large. Fig. 18.13 shows a typical circuit, which should be compared with Fig. 18.11. If the transistor were being used as an amplifier it would be given sufficient forward bias to enable it to work over a reasonably linear part of its characteristic curve, but if the input diode is to work as a rectifier the transistor must be adjusted to its 'bottom bend", at which the collector current is nearly zero. A very small forward bias is


Fig. 18.13-A transistor detector circuit
usually helpful, and $R_{B}$ in Fig. 18.13 is for this purpose. Inevitably the full r.f. goes through the transistor, so a filter is needed in the collector circuit to remove it.

The main advantage of this kind of detector is that the r.f. input is in tenths of a volt rather than volts, making it possible perhaps to save a preceding stage of amplification. Because the input of the transistor is working at its bottom bend its loading effect on the tuned circuit is comparatively small, and this helps in the same direction. But both efficiency and linearity of detection tend to be poor. And if the input voltage is allowed to go high the collector circuit may run into saturation, causing severe distortion.

A triode valve used in the same mode is called an anode-bend detector, and because of its negative bias its input loads the r.f. circuit negligibly. But it too is non-linear, and its maximum rating is limited. An f.e.t. can likewise be used as a detector in this mode.

Valves can alternatively be used in the grid detector mode, shown in Fig. 18.14, in which the grid and cathode are used as a diode in a reservoir detector of the Fig. 18.8d type, and the output across this diode is amplified in the valve. The only bias is the
negative one produced by rectification of the input signal, so as a triode the talve begins well up on its characteristic curve where its mutual conductance is high, and works down from that. It is therefore more sensitive and more linear than the anode-bend type, but if the input is allowed to become high enough to bias the valve to


Fig. 18.14-Grid delector circuil
its bottom bend there will be two modes of detection working in opposite directions, with resulting distortion and loss of efficiency.

### 18.13 F.M. Detectors

The performance of all the detectors considered so far depends hardly at all on the frequency of the carrier wave between very wide limits. Consequencly they treat a frequency-modulated carrier wave (See. 15.12) as if it were unmodulated. In order to receive f.m. signals it is necessary either to convert the variations of frequency into corresponding variations of amplitude so that an ordinary detector can be used. or else use a different kind of detector altogether.

The first method can be employed in a very simple manner with an ordinary a.m. receiver by adjusting the tuning so that instead of the incoming carrier-wave frequency coinciding with the peak of the resonance curve (Fig. 8.5) it comes in on one of the slopes. As its frequency rises and falls by modulation, the amplitude of response rises and falls. But because the slope of the resonance curve is not quite linear, the amplitude modulation is not a perfect copy of the frequency modulation, so there is some distortion. More serious still. the potential benefits of f.m. are not achieved, because the receiver is open to amplitude modulation caused by interference. And both selectivity and efficiency are poor.

Receivers designed for f.m. reception therefore include means for rejecting amplitude modulation and responding only to frequency modulation. The first of these functions is performed by a limiter
and the second by a discriminator. In one of the most popular types of f.m. detector-the ratio detector-these functions are combined. However, for a first approach it will be easier to consider a system in which they are separate.

In connection with oscillators we saw in Sec. 14.9 that if the amplitude of oscillation exceeds a certain amount the transistor or valve cuts off the positive or negative peaks or both. A limiter is usually one of these devices so arranged that it does this at anything above quite a small signal amplitude, so that however much greater the incoming signal may be it is cut down or limited to the same size. Variations in amplitude are therefore almost completely removed, and with them much of the effects of interference.

Fig. 18.15a shows the essentials of the Foster-Seeley or phase discriminator. It consists of two Fig. $18.8 b$ type a.m. detectors

(a)

(b)

Fig. 18.15-(a) Circuit of phase discriminator for f.m. detection, arranged to bring out the principle: $L_{p}$ and $L_{\mathrm{p}}$ are the primary and secondary windings of the input r.f. transformer; (b) phasors of voltages marked, for carrier wave frequency. Dotted lines indicate the effect of f.m. deviation
back to back, both receiving the incoming r.f. carrier-wave voltage $d a$ from a tuned circuit $\mathrm{L}_{\mathrm{p}} \mathrm{C}_{\mathrm{p}}$, and each receiving in addition half the voltage from the tuned circuit $L_{s} C_{s} . L_{p}$ and $L_{s}$ are the primary and secondary windings of an r.f. transformer, and because both windings are exactly tuned to the carrier wave and are coupled only very loosely the voltages across them are $90^{\circ}$ out of phase, as indicated by the phasors da, ab and ac in Fig. 18.15b. The total r.f. input to diode $\mathrm{D}_{1}$ is therefore $d b$, and to diode $\mathrm{D}_{2}$ it is $d c\left(\mathrm{C}_{1}\right.$ and $\mathrm{C}_{2}$ being r.f. short-circuits). These inputs are equal, so the rectified voltages set up across the load resistors $R_{1}$ and $R_{2}$ will also be equal. But as they are connected in opposite polarity, the net output is nil.

When the carrier wave is frequency-modulated it swings off the resonance peak and, as explained in Sec. 8.11, the impedance of a tuned circuit changes from resistance to inductance on one side and capacitance on the other. The phase swings correspondingly, as indicated by the broken lines in Fig. 18.15b, changing the one-to-
one ratio of $d b$ to $d c$ to something alternately greater and less. The value of $d b-d c$ therefore becomes alternately positive and negative at the frequency of modulation, and the modulating signal is reproduced at the output. Practical phase-discriminator circuits differ in detail from Fig. 18.15 but the main principle is the same.

The ratio detector circuit combines the function of limiter with that of discriminator, so is usually preferred, in spite of slightly


Fig. 18.16-Circuit of ratio discriminator, much used in f.m. receivers


Fig. 18.17-In the pulse-counter type of f.m. discriminator the carrier-wave cycles are reduced to pulses of uniform size. When rectified, the ir mean value is directly proportional to their frequency
higher distortion. It looks very much like the phase discriminator, but one of the diodes is reversed, so when the carrier is unmodulated the total output from the diodes is not zero but is equal to the sum of the two. When modulation takes place this sum tends to remain constant but the ratio between the two rectifier outputs varies.

The a.f. potential of the junction between them therefore varies with respect to the tuned circuits, so the a.f. signal is developed between these two points.

Fig. 18.16 shows this basic circuit, but in practice there are quite a number of variations in the connections. Sometimes the a.f. output is taken from between the junctions of $\mathrm{R}_{1} \mathrm{R}_{2}$ and $\mathrm{C}_{1} \mathrm{C}_{2}$, and in the unbalanced form one end of $R_{1} R_{2}$ rather than the centre is chosen.

Amplitude is kept constant by C , which has a capacitance so large that the voltage across it is practically unaffected at even the lowest modulation frequency.

There is an entirely different type of frequency discriminator which uses diodes to reduce the incoming signal cycles to unidirectional pulses whose shape and duration are unaffected by the signal frequency. The number of them occurring per second, and therefore their average voltage, is of course directly proportional to the signal frequency, as shown in Fig. 18.17. This linear relationship can be very accurately maintained, ensuring freedom from distortion, but the practical limit of carrier-wave frequency is a few hundred kilohertz.

## Low-Frequency Amplification

### 19.1 Recapitulation

Amplification has already been considered in a general way and we have studied the aids used for designing amplifiers: characteristic curves and equivalent circuits. We have also looked at r.f. amplifiers as used in senders. These take a weak signal generated by a master oscillator and amplify it in stages-sometimes with one or more changes of frequency on the way-until a stage is reached at which the output power is sufficient, when radiated, to give the required range of communication.

At the receiving end the r.f. signal picked up by the aerial usually needs amplification before being presented to the detector, which the last chapter has shown needs at least a volt or two if it is to do its job well. Besides amplifying, the receiver r.f. amplifier has to select the desired signals and exclude all others. And although it can be allowed to maltreat the waveform of the r.f. cycles, the waveform of the modulation must be carefully preserved. With f.m. this is no problem. but a.m. demands some care.

Our detailed study of transistor and valve paranseters and equivalent circuits was for the most part based on neglecting reactances, so that phase relationships were just plus or minus. To what is called a first order of approximation, audio-frequency amplification can be treated on this basis, so we shall now proceed to do so, leaving receiver r.f. amplification (which certainly cannot be) until Chapter 22. Besides radio receivers, a.f. amplifiers are used in hearing aids, tape and disk recording and reproduction, and sound reinforcement. For applications in instruments and certain types of computers. amplification is often needed below the a.f. range. right down to zero frequency, in fact: so the whole range included in this chapter is called low frequency.

The main problem is not in providing any desired amount of amplification: it is in doing so without perceptible distortion or extraneous noise. Perceptible, note. It is the human ear that judges the sound resulting from a.f. distortion. so it is important that the amouns of distortion as well as the amount of amplification is reckoned as nearly as possible as the listener judges it. There are two main kinds of distortion: one is called amplitude frequency distortion (often just 'frequency distortion'), and occurs when the amplification or gain is not the same at all frequencies, so that sounds of different pitch are not reproduced at the correct relative strengths; the other is non-linearity distortion (or waveform
distortion). In general it is more unpleasant than frequency distortion. But first we should consider how amplitude and frequency are judged by hearers.

### 19.2 Decibels

Suppose a 0.5 V signal is put into an amplifier which raises it to 12 V . This 12 V is then put into another amplifier which raises it to 180 V . Which amplifier amplifies most: the one that adds 11.5 V to the signal, or the one that adds 168 V ? Put this way, the figures tend to suggest the second. But if we work out $A$, the voltage amplification ratio (Sec. 11.5) we find it is 24 for the first and only 15 for the second. This method of reckoning is the right one because we do not judge by the amount by which a sound is increased in strength, but by the ratio in which it is increased. Doubling the power into a loudspeaker produces much the same impression of amplification whether it be from 50 to 100 mW or from 500 to 1000 .

This is like the way the ear responds to the pitch of sounds. While it is pleased by combinations of sounds whose frequencies have certain fixed ratios to one another, it can find no significance at all in any particular additive frequency intervals, say an increase of 250 Hz . Musical intervals are all ratios, and the significant ratios have special names, such as the octave, which is a doubling of frequency wherever on the frequency scale it may come. The octave is subdivided into tones and semitones.

The same principle is appropriate for the strength of sound or electrical signals, and amplification-or indeed any change in power-is reckoned in ratios. Instead of the 2 to 1 ratio, or octave, a power ratio of 10 to 1 was chosen as the unit. The advantage of this choice is that the number in such units is simply equal to the common logarithm of the power ratio. It was named the bel (abbreviation: B) after A. Graham Bell, the telephone inventor. So an amplifier that multiplies the power by 10 can be said to have a gain of 1 bel. A further power multiplication by 10 , giving a total of 100 , adds another bel, making 2 bels.

The bel, being rather a large ratio for practical purposes, is divided into 10 decibels ( dB ). One tenth of a 10 to 1 ratio is not at all the same as one tenth of 10 : it is 1.259 to 1 . If you have any doubts, try multiplying 1 by 1.259 ten times.

So a gain of 1 dB is a $25.9 \%$ increase in power. The voltage or current increase giving a $25.9 \%$ increase in power is $12.2 \%$ provided that the resistance is the same for both voltages (or currents) in the ratio, because power is proportional to the square of voltage or current (Sec. 2.17). The fact that a voltage or current decibel is a smaller ratio than a power decibel does not mean that there are two different sizes of decibels, like British and American gallons; it arises because the ratio of volts or amps is smaller than the ratio of
power in the same situation. Decibels are fundamentally power ratios.

If 1 is multiplied by 1.259 three times, the result is almost 2 , so a doubling of power is a gain of 3 dB . Doubling voltage or current quadruples the power, so is a gain of 6 dB . A table connecting decibels with power and voltage (or current) ratios is printed on p. 507. Since the number of bels is $\log _{10} P_{2} / P_{1}$, or $2 \log _{10} V_{2} / V$, (where $P_{2} / P_{1}$ is the power ratio and $V_{2} / V_{1}$ the voltage ratio) the number of decibels is $10 \log _{10} P_{2} / P_{1}$ or $20 \log _{10} V_{2} / V_{1}$. If Fig. $0 \cdot 5 u$ is a scale of dB , then $b$ is the same scale in power ratios. The dB scale, being linear, is obviously much easier to interpolate (i.e., to subdivide between the marked divisions). Fig. 11.6 would be improved if the logarithmic voltage-ratio scale were replaced or supplemented by a uniformly divided scale with its 0 at 1 and its 20 at 10 , which would indicate dB .

An advantage of working in dB is that instead of having 10 multiply successive changes together to give the net result, as with the $A$ figures, they are simply added. A voltage amplification $\left(A_{V}\right)$ of 24 is +27.6 dB , and $A_{V}=15$ is +23.5 dB , so the total $\left(A_{V}=\right.$ $360)$ is $+51 \cdot 1 \mathrm{~dB}$. The + indicates amplification or gain; attenuation or loss shows as a - . For instance, if the output voltage of some circuit were half the input, so that the power was one quarteri.e., a power 'gain' of 0.25 -the 1 to 4 power ratio would be denoted by the same number of dB as 4 to 1 , but with the opposite sign: -6 dB . Zero dB, equivalent to $A=1$, means 'no change'. The power losses in lines (Chap. 16) are reckoned in dB ; if the line is uniform, the loss in dB is proportional to its length.

An important thing to realize is that decibels are not units of power, still less of voltage. So it is nonsense to say that the output from an amplifier is 20 dB , unless one has also specified a reference level such as $0 \mathrm{~dB}=50 \mathrm{~mW}$. Then 20 dB , being 100 times the power, would mean 5 W .

To the general public, decibels are units of loudness, but they can be used as such only if a standard reference level is understood. If they only knew, dB can just as well be used for betting odds!

### 19.3 Frequency Distortion

The amount of frequency distortion in any piece of equipment is shown by means of a graph in which gain (or loss) is plotted against frequency. For the reasons we have just been considering, both scales are usually logarithmic, the gain scale being linear in dB and the frequency scale usually running from 20 Hz to 20 kHz in three decades (i.e., $10: 1$ ranges).

Fig. 19.1 is a typical example. A curve having the same shape at a higher or lower level would mean the same amount of distortion. If a linear $A$ scale were used the shape would vary with the level and
so be misleading. An ideal characteristic curve would of course be flat over the whole range of frequencies to be reproduced.

As a guide to interpreting frequency characteristics, it is worth noting that 1 dB is about the smallest change in general level of sound that is perceptible. If the loss or gain is confined to a small


Fig. 19.1-A typical a.f. gain|frequency curve, with logarithmic scales in both directions
part of the audible frequency range, several dB may go unnoticed. So far as audible distortion is concerned, therefore, there is no sense in trying to 'iron out' irregularities of a fraction of a decibel. A peak of one or two decibels, though unnoticeable as frequency distortion, may however cause non-linearity distortion by overloading the amplifier. An average fall of 10 dB over all frequencies above, say, 1 kHz would be easily noticeable as muffled and indistinct reproduction. A rise of the same amount, or a falling off below 1 kHz , would be heard as thin shrill sound.
A cheering thought is that frequency distortion in one part of the system can usually be compensated in another by deliberately introducing distortion of the opposite kind, emphasizing the weakened frequencies. But such methods, called equalization, should not be relied on more than necessary, because they may cause difficulty in using one piece of equipment with others.

Means for the listener to adjust the frequency curve to suit his taste or to counteract distortion are called tone controls.

### 19.4 Non-Linearity Distortion

Non-linearity can be expressed as a graph of output against input. A diagonal straight line passing through the origin, as in Fig. 2.3,
represents perfect linearity, and means that any given percentage increase or decrease of input changes the output in exactly the same proportion. Transistors are examples of non-linear devices, as shown by the curvature in their characteristics (Fig. 10.13, etc.).

Judging from an output-input graph, the most obvious effect of non-linearity might seem to be that the volume of sound produced would not be in exact proportion to the original. It is true that reducing the signal strength at the detector to one-tenth is likely to reduce the a.f. output to far less than one-tenth, because of the bottom bend (Sec. 18.11). But that is about the least objectionable of the effects of non-linearity. What matters much more is the distortion of waveform. Because the waveforms of a.f. signals correspond to those of the original sound waves, we must preserve their form with the utmost care. Otherwise the sound will be harsh and we shall not be able to distinguish one musical instrument from another (Sec. 1.3).

### 19.5 Generation of Harmonics

Fig. 15.12 showed that the addition of a three-times frequency (third harmonic) distorts a sinusoidal waveform in a somewhat similar way to a push-pull Class $C$ amplifier, and we concluded that such an amplifier would introduce third harmonics into whatever signals it was amplifying. Let us now follow up this line of thought.

Fig. 19.2 is a typical collector-current/base-voltage curve. If the load resistance were small compared with the output resistance of the transistor, the output voltage would be proportional to $I_{C}$. Suppose P is the working point, from which an input signal swings the base between 0.4 V and 0.7 V . Plotting the output current waveform, we see that whereas the positive peak is 48 mA the negative peak is only 32 mA . The average is 40 mA . In Fig. 19.3 a sine wave of that size has been plotted at $a$, a second harmonic having an amplitude equal to half the difference between average and actual peak output ( 4 mA ) is shown at $b$, and $c$ is the result of adding $a$ and $b$ together. It is almost identical with the output in Fig. 19.2. We conclude that characteristic curves of the Fig. 19.2 type, in which the slope increases all the way, generate second harmonics. Unless they are exactly the shape called parabolic they also generate other even-number harmonics. which account for any slight differences between the Fig. 19.2 output and Fig. 19.3c. In this example the amplitude of the harmonic (Fig. 19.3b) is $0 \cdot 1$ of that of the fundamental (a) so the second-harmonic distortion is said to be $10 \%$. Without drawing any waveforms we can find the percentage from the dynamic characteristic curve by joining the extreme points of the swing, $A$ and $B$, by a straight line and expressing the height of its centre above P as a percentage of the vertical height of B above A .

In the same way one can show that a characteristic curve having a double or S bend, seen in Fig. 19.4, generates third and other odd harmonics. (Note that because the centre of the curve lies on the diagonal between its ends the percentage of even harmonic, found by the method just described, is nil.) This type of curve often


Fig. 19.2-Showing the waveform distortion introduced by a non-lineur characteristic curve
results from attempting to get as much power as possible out of a transistor or valve. Try drawing a load line across Fig. 10.15, for example, and deriving from it a curve of output voltage against input voltage, $V_{G}$. Most output/input characteristics show a combination of both types of curvature, but usually one or other predominates. Unless there are sharp bends (which should always be avoided) the percentages of harmonics decrease as their order (i.e. number of times their frequency is greater than that of the fundamental) increases.

The ear tolerates a fairly large percentage of second-harmonic distortion, less of third harmonic, and so on. A fraction of $1 \%$ of the higher harmonics such as the eleventh introduces a noticeable harshness; the reason being that a second harmonic differs by an exact octave from the original tone, whereas the high harmonics form discords.

### 19.6 Intermodulation

Although for simplicity the distortion of a pure sine wave has been shown, the more complicated waves corresponding to musical instruments and voices are similarly distorted. A most important principle. associated with the name of Fourier, is that all repetitive



Fig. 19.4-In contrast to the type of characteristic curve in Fig. 19.2, which generates evennumber harmonics, this double bend generates odd-number harmonics

Fig. 19.3-Method of estimating the harmonic distortion in the Fig. 19.2 output
waveforms, no matter how complicated, can be analysed into pure sine waves, made up of a fundamental and its harmonics, which are all exact multiples of the fundamental frequency. So when one is dealing with waveforms other than sinusoidal it is usual to perform this analysis, and then tackle them separately on the basis of simple
sine-wave theory; like the man in the fable who found it easier to break a bundle of faggots by undoing the string and attacking them one by one. Each harmonic current and voltage can be calculated as if it were the only one in the circuit-provided that the circuit is linear. But just now we are studying non-linear conditions, in which it is not true to say that two currents flowing at the same time have no effect on one another. We shall see in Sec. 21.3 that when two voltages of different frequency are applied together to a non-linear device the resulting current contains not only the original frequencies but also two other frequencies, equal to the sum and difference of the original two. For some purposes that is a very useful result, but extremely undesirable in an a.f. amplifier. It means that all the frequencies present in the original sound-and there may be a great many of them when a full orchestra is playinginteract or intermodulate to produce new frequencies; and, unlike the lower harmonics, these frequencies are generally discordant. Even such a harmonious sound as the common chord (C, E, G) is marred by the introduction of discordant tones approximating to $D$ and $F$. When non-linearity distortion is severe, these intermodulation or combination tones make the reproduction blurred, tinny, harsh, rattling, and generally unpleasant. And onice they have been introduced it is practically impossible to remove them.

### 19.7 Allowable Limits of Non-Linearity

Whenever one attempts to get the greatest possible power from a transistor or a valve, one comes up against non-linearity. So the problem tends to be most acute in the output stage. If the signal amplitude is reduced, so that it operates over the most nearly linear parts of the characteristics, the distortion can be made very small, but then the output will be uneconomically small in relation to the power being put in. It is not a question of adjusting matters until distortion is altogether banished, because that could be done only by reducing the output to nil. It is necessary to decide how much distortion is tolerable, and then adjust for maximum output without exceeding that limit.

Fixing such a limit is no simple matter, because it should (in reproduction of music, etc.) be based on the resulting unpleasantness judged by the listener. Different listeners have different ideas about this, and their ideas vary according to the natur- of the programme. And, as has just been pointed out, the amount of distortion that can be heard depends on the order of the harmonic (second, third, etc.) or of the intermodulation tones, as well as on the percentage present.

Various schemes for specifying the degree of distortion in terms that can be measured have been proposed, but the nearer they approach to a fair judgment the more complicated they are; so the admittedly unsatisfactory basis of percentage harmonic content
is still commonly used. Preferably the percentage of each harmonic is specified, but more of ten they are all lumped together as "total harmonic distortion'.

### 19.8 Phase Distortion

The main result of passing a signal through any sort of filter, even if it is only a resistance and a reactance in series, is to alter the amplitudes of different frequencies to different extents (Fig. 18.9). If it discriminates appreciably between frequencies within the a.f. band it introduces frequency distortion, which, of course, may be intentional or otherwise. It also shifts the phases to a differing extent; see Eig. 6.10 for example. If the signal is a pure sine wave, the phase shift has no effect on the waveform; but few people spend their time listening to continuous pure sine waves.

To take a very slightly more complex example, a pure wave is combined with a third harmonic, shown separately in Fig. 19.5a. The waveform of this combination, obtained by adding these two together, tends towards squareness, as shown at $b$. Suppose now that after passing through various circuits the relative phases of the component waves have been altered, as at $c$. Adding these together, we see the resulting waveform, $d$, which is quite different from $b$. Yet oddly enough the ear, which is so sensitive to some changes of waveform (such as those caused by the introduction of new frequencies), seems quite incapable of detecting the difference between $d$ and $b$, or indeed any differences due to phase shifts in continuous waves.

One has to be much more careful about phases when dealing with television signals, however, because the result is judged by the eye, to which form is of paramount importance. Phase distortion such as that shown in Fig. 19.5 would be very objectionable.

Generally, it can be said that if care is taken to avoid phase distortion, frequency distortion will take care of itself.

### 19.9 Loudspeakers

Having considered distortion we begin now to apply this knowledge to 1.f. amplifiers, and at first will particularly have in mind the requirements of a.f. amplifiers. Their end product is usually sound from a loudspeaker, which forms the load for the final or output stage of the amplifier. Until we know the requirements here we cannot tell how many and what stages will be needed between the a.f. signal source-detector, tape head, microphone, etc.) and the output. So let us begin at the end with the loudspeaker. Its object, of course, is to convert the electrical power delivered to it from the output stage into acoustical power having as nearly as possible the same waveforms.

A great many types have been devised, but the only one in general use to-day is the moving-coil. As the name suggests, it works on essentially the same principle as the moving-coil meter (Sec. 4.2). The mechanical force needed to stir up air waves is obtained by the interaction between two magnetic fields. One of the fields is unvarying and very intense, and the other is developed by passing the signal currents through a small movable coil of wire.

Fig. 19.6 shows a cross-section of a typical speaker, in which the


Fig. 19.5-Showing that the waveform resulting from adding together two component waves is greatly altered by shifting the relative phase of the components
steady magnetization is provided by a ring-shaped permanent magnet A. Its magnetic circuit is completed by iron pole-pieces $B B$, except for a narrow circular gap $G$. The high permeability of the iron enables the magnet to set up a very intense magnetic flux across
the gap. In this gap is suspended the moving coil C, through which the alternating signal currents from the amplifier are passed. The mechanical force is proportional to both the flux density in the gap (which is large and constant) and the current in the moving coil. As the latter is alternating, the force is alternating, acting along the directions of the double-headed arrow in Fig. 19.6 and so tending

Fig. 19.6-Cross-section of a moving-coil loudspeaker

to move the coil to and fro in the gap. The coil is attached to a conical diaphragm D , which is flexibly supported at its centre by a 'spider' S and at its circumference by a ring R , so the vibrations of the coil are communicated to the diaphragm and hence to the air.

It is more difficult to avoid distortion due to the loudspeaker than any other in the whole system. Just as inductance and capacitance in a circuit without much resistance have a frequency of resonance, at which current to and fro builds up, so inertia and springiness in a mechanical system without much friction tend to make it resonate at a particular frequency. With the moving system of a loudspeaker this frequency is often in the region of 80 Hz ; its bad effects can be alleviated by a low output-stage resistance. To reproduce the lowest frequencies fully, it is necessary to move a large volume of air. If this is done by permitting a large amplitude of movement, either a very large magnet system is needed, or there is non-linearity due to the coil moving beyond the uniform part of the magnetic field in the gap. If it is obtained by using a very large cone, this makes it impossible for the cone to vibrate as a whole at the high audio frequencies and they are badly reproduced. The design is therefore a compromise. For the highest quality of reproduction it is usual to have two or more loudspeakers of different sizes, each reproducing a part of the whole a.f. band.

When the diaphragm in Fig. 19.6 is moving, the air in front is compressed and the air behind is rarefied. If the period of one cycle of movement is long enough to give the resulting air wave time to travel round its edge from front to back, these pressures tend to
equalize and not much sound will be sent out. To prevent this loss, evidently worst at the lowest frequencies, the loudspeaker is mounted so that it 'speaks' through a hole in a baffle. This consists of a piece of wood, flat or in the form of a cabinet, designed to lengthen the air-path from front to back of the diaphragm and so to ensure that the bass is adequately radiated. Alternatively a horn can be used.

In a large building use is often made of the same principles as with aerials for directing the radiation in useful directions, instead of towards the roof, from which confusing echoes would be reflected. If a number of loudspeakers are mounted above one another in a column-shaped enclosure their combined output gives reinforced sound horizontally and some degree of mutual cancellation at other angles, owing to phase differences.

As a load, a moving-coil loudspeaker can be regarded very roughly as a resistance, usually in the range 2-15 $\Omega$. For a better approximation a certain amount of inductance in series should be assumed. Of the resistance, part is the resistance of the wire in the moving coil, and the remainder (the useful part) is analogous to the radiation resistance of an aerial (Sec. 17.8).

The maximum amount of a.f. power needed to drive a loudspeaker depends on two things: the maximum volume of sound required, and the efficiency of the speaker. Well-designed horn types have efficiencies ranging up to as much as $50 \%$ but usually a lot less, and with domestic cabinet types figures more like $5 \%$ are usual. The ear is far more sensitive to middle frequencies, say $300-3000 \mathrm{~Hz}$, than the lower ones, so small portable speakers give an apparent loudness far greater than their limited power input might suggest. The maximum 'resonably distorted' power varies from about $\frac{1}{3}$ to I W for portable sets, up to about 5 W for ordinary domestic receivers, and from 10 W upwards for 'hi-fi'.

### 19.10 The Output Stage

In some respects, and especially for obtaining high efficiency and finding the best load resistance, the principles of audio output stages are similar to those for senders. The main difference of course is that the couplings between one stage and the next, and between output stage and load, in a sender are tuned to one working frequency, but in an a.f. amplifier they have to be 'flat' over the whole audio range of frequencies, and there is no tank circuit effect to smooth out non-linearity.

In considering how much power can be got out of a transistor or valve, regard has to be paid to the maker's maximum ratings. Thesecan be quite complicated, but the most important is the amount of power that can safely be dissipated continuously by the device. Fig. 19.7 shows a set of $I_{c} \mid V_{c}$ curves for a small output transistor
rated at 0.7 W . This power could be 0.7 A at $1 \mathrm{~V}, 0.35 \mathrm{~A}$ at 2 V , and so on for any number of combinations of output current and voltage. If we plot a sufficient selection of them we can draw the dotted curve through them to represent all such points. (A curve of this shape, by the way, is known as a hyperbola.) Every point above it


Fig. 19.7-I $I_{c} / V_{c}$ curves of a small a.f. output transistor, showing power-limit curve (dotted) and Class A load line
represents more than 0.7 W , so is out of bounds except transiently during a signal swing. But not all the area below it is allowable; there are maximum voltage and current ratings too, which depend on other conditions but which for simplicity we shall assume are 25 V and l A .

Consider Class $\mathbf{A}$ amplification, in which the current is made to swing equally above and below the working point. The average
current therefore remains the same whether there is any signal or not. The power dissipated in the transistor is actually less when it is working (Sec. 15.4), but as it could be 'silent' for indefinite periods we cannot take advantage of this to put the working point above the dotted curve. The limit we can go to is putting it on the curve. The problem is to fit a load line around this point in such a way as to represent the maximum power output, while at the same time avoiding distortion by seeing that the $I_{B}$ curves are cut at equal intervals all the way. To get any reasonable output we shall have to amend these instructions to allow a reasonable amount of distortion. If the load line is nearly vertical we shall get a big swing of current but very little voltage, and vice versa with a nearly horizontal line.

It looks as if we cannot do very much better than the load line shown, in which the current and voltage swings, peak to peak, are 0.38 A and 5 V , giving a power output (Sec. 15.3) of $0.38 \times 5 / 8=$ 0.24 W . The feed is $0.225 \times 3=0.675 \mathrm{~W}$, so the efficiency is 0.24 / 0.675 , just over $35 \%$, which is well below the $50 \%$ for ideal Class A, even though it looks as if we are accepting a fair amount of distortion. The resistance represented by the load line is $13 \Omega$, so loudspeakers of $10-15 \Omega$ could be worked directly from this transistor. This sort of investigation cannot be much more than a rough approximation, particularly if we remember that at top and bottom frequencies a loudspeaker load line is an ellipse instead of a straight line, which is valid only for a pure resistance.

One can generally do a little better than this with valves, but their optimum loads are thousands of ohms, so an output transformer is needed to match a loudspeaker to the valve, and any extra output could quite easily be lost in it, to say nothing of the cathode-heating power.

The positive half-cycles of $i_{B}$ in Fig. 19.7 obviously yield smaller half-cycles of output than the negative ones. If however two transistors (or valves) are connected in push-pull (Sec. 15.4) the one smaller and one larger half-cycle are added together both for positive and negative, so this particular form of distortion (second-harmonic) is largely cancelled out and the nearly-undistorted output is more than twice the equivalent output from one transistor only. The end-to-end load resistance should be made double that suitable for one.

While the heavy power consumption of Class A amplifiers may perhaps be tolerable in amplifiers worked from the mains, and at least has the merit of being constant, it is quite intolerable when batteries are used, their cost per unit being enormously greater. The particularly undesirable feature is the continuance of full power consumption during quiet periods, so that the average efficiency is very low indeed.

### 19.11 Class B Amplification

The inadequate power output indicated when the transistor curves in Fig. 19.7 are examined on a Class A basis prompts us to look into
the methods so successful in sender output stages (Sec. 15.4). Class C however is out, because the half-cycles are so distorted that even when the positive and negative halves are put together the result serves us only as an illustration of third-harmonic distortion (Fig. 15.12). In Class B the amplifying valves or transistors are biased close to cut-off, so that the quiescent power consumption is nearly zero, yet yield complete half-cycles for putting together to make whole ones. At all volumes - not only at maximum-most of the power expended is returned usefully. Translating this ideal into practice has its problems of course, as we shall see.

The load-line procedure we used with Class A does not work with Class B, because the working point is near cut-off, where the power input is very small. In fact, it is usually referred to as the quiescent point. And even when a load line has been drawn it does not directly indicate the power which the transistor will be called upon to dissipate. So we must try a different approach.
Fig. 19.8 shows the basic circuit diagram and ideal characteristics with zero saturation voltage and no bottom bend. So the


Fig. 19.8-Basic circuit diagram and ideal transistor characteristics for Class B output stage
quiescent point ( P ) can be placed on the zero- $I_{c}$ level. The circuit diagram shows that when one transistor is fully conducting, so that the signal voltage ( $=V_{c c}$ ) exists across its half of the output transformer, an equal and opposite voltage exists across the other half, raising the collector voltage of the cut-off transistor to $2 V_{c c}$. Therefore (subject to minor adjustments for factors we have neglected) supply voltage $V_{c c}$ up to half the maximum $V_{c}$ rating for the transistor can be allowed. Available battery voltages must of course be considered. On these lines we decide the position of P .

The position of the other end of the load line ( $I_{C M}$ ) must not be
higher than the maximum peak current rating for the transistor. If we go to this limit, the load line can then be drawn and its resistance ascertained ( $=V_{C C} / I_{C M}$ ). The peak signal voltage is $V_{C C}$ and peak current $I_{C M}$, so (if we remember that r.m.s. values are peak values divided by $\ell^{\prime 2}$ ) the maximum signal power is $V_{C C} I_{C M} / 2$.

One transistor delivers this for only half of each cycle, so this is the power output per pair.

The next thing is to find the power input to each transistor under these ideal conditions. It is equal to the supply voltage ( $V_{C C}$ ) multiplied by the average current. The average value of half a sine wave is $2 / \pi$ times the peak value, $I_{C M}$ in this case. So the total input is $2 V_{C C} I_{C M} / \pi$ over the active half-cycle and $V_{C C} I_{C M} / \pi$ over a whole cycle. The efficiency is the signal power per transistor divided by the feed input: $V_{C C} I_{C M} / 4 \div V_{C C} I_{C M} / \pi,=\pi / 4$, which is 0.785 , or $78.5 \%$. This compares with $50 \%$ for ideal Class A. The power left to be dissipated by the transistor is the input power less the output power, which works out at $0.068 V_{C C} I_{C M}$.

This power could possibly be found to exceed the rating for the transistor. But it assumes a continuous sinusoidal signal at maximum volume. The programmes that people listen to are not usually of this nature, and even loud sustained music has an average power appreciably less. The specifications of amplifiers of this kind should therefore state the type of programme to which the claimed output refers.

It is often helpful to have the output power, $P_{0}$, in terms of load resistance, $R_{L}$, rather than $I_{C M}$. Substituting $V_{C C} / R_{L}$ for $I_{C M}$ in $V_{C C} I_{C M} / 4$ we get

$$
\mathrm{P}_{0}=\frac{V_{C C^{2}}}{4 R_{L}}
$$

Notice that in all the foregoing analysis the transistor characteristics do not come into it at all, only the maximum ratings. Notice too that if you short-circuit the output, so that $R_{L}=0$, the output power appears to go up to infinity, provided that there is sufficient input drive. Of course it doesn't in practice; for one thing, the drive would not be sufficient. But it is near enough to the truth to make it unwise to risk an output short-circuit, unless a fast-acting fuse is fitted in that circuit (or there is an adequate supply of spare transistors). If $V_{C C}$ is given, then the smaller the transistor (in terms of maximum power rating) the higher the load resistance must be.

The load-line resistance $R_{L}$ refers to the resistance the transistor sees across its half of the output transformer. If the turns ratio between that half and the secondary winding is $1: 1$, this resistance (with an ideal transformer) is equal to that of the loudspeaker. But usually the transformer ratio is stated in terms of the whole primary; and because there is a $2: 1$ turns ratio between that and each half, the equivalent loading across the whole primary is $4 R_{L}$ (Sec. 7.11). In Class A push-pull, both transistors are working dur-
ing both half-cycles, instead of one being cut off, so the wholeprimary resistance is $2 R_{L}$.

In Fig. 19.9 a more practical load line is shown, though still with zero quiescent current, on the same curve sheet as Fig. 19.7. The resistance it represents is $15 \Omega$, and the signal peak is 11 V 0.72 A ,


Fig. 19.9-Same characteristic curves as in Fig. 19.7, but Class B load line giving an output power per transistor of practically 2 W . The input is $12 \times 0.72 / \pi,=2.75 \mathrm{~W}$, so with this type of signal the dissipation is just over the rated 0.7 W .

### 19.12 Distortion with Class B

In small-signal transistors the $I_{C} / I_{B}$ relationship ( $h_{\mathrm{fe}}$ ) is remarkably linear, as we saw in Fig. 10.11, whereas $I_{C} / V_{B}$ is very non-linear be-
cause the working range is near the bottom bend. In power transistors on the other hand they are worked high up the input curve, where $I_{C} / V_{B}$ tends to be linear and $I_{C} / I_{B}$ shows a marked falling off as $I_{c}$ increases. This is noticeable in Fig. 19.9, where the spaces between the $I_{B}$ lines get progressively smaller as one moves up the load line. Incidentally, these characteristics depend hardly at all on $V_{c}$, so the published no-load curves are reasonably valid for loaded conditions. If Fig. 19.9 had been plotted with $V_{B}$ curves instead of $I_{B}$, the tendency would be for them to become farther apart rather than closer together. Unlike Class A push-pull, Class B does not compensate for distortion of these kinds, which blunt or sharpen both half-cycles equally if the two transistors are alike. The driver


Fig. 19.10-Showing the production of crossover distortion in a Class B output stage
actually supplies both voltage and current, and with a bit of luck the two tendencies might at least partly cancel out. If luck fails, a more scientific remedy awaits in Sec. 19.15.

But in any case the bottom bend remains. If therefore the two transistors were biased exactly to cut-off, this bend would be reproduced in both half-cycles, as shown in Fig. 19.10, where the
two $I_{C} / V_{B}$ curves are drawn in opposite directions to represent the push-pull connection. The resulting crossover distortion is particularly objectionable because it generates high-order harmonics (Sec, 19.5).

The remedy is to bias the transistors to about the middle of the bottom bends, as in Fig. 19.11. If the biasing has been well judged


Fig. 19.11-Showing how, by suitable biasing, crossover distortion can be eliminated
the two output waveforms will add up to a very acceptable enlarged copy of the input. The crossover region is shown inset on a larger scale. Obviously the success of this method depends on how accurately the biasing has been set in the first place and kept there in spite of changes in transistor characteristics, due mainly to temperature changes. It is not much good having a beautifully undistorted output immediately after switching on if severe distortion sets in when the transistors have warmed up. This is where we must look at actual circuits.

### 19.13 Class B Circuits

What might be called the conventional push-pull circuit is as shown in Fig. 19.8; whether it is Class A, B or C (or one of their hybrids) depends on the amount of bias used. Unlike the tuned r.f. input and output transformers in Fig. 15.4, the a.f. transformers are designed to give uniform results over the whole a.f. band (or as much of it as the designer can manage for the money). The input transformer is needed to apply positive signal to one transistor or valve while the other is receiving negative signal. In Class A both are working all the time, each delivering full cycles; in Class B each works half time, alternately delivering half cycles. An output transformer is needed to combine the outputs in the correct phase to give to the load. With valves (and some transistors) it is also needed to match their high output resistance to the low loudspeaker impedance. As we have seen, the most suitable load resistances for transistors are often in the same range as those of loudspeakers. A.f. transformers-and especially output transformers-are heavy and expensive if they are to be good, and if they are not good they cause both distortion and loss, so it seems a pity to use one merely for combining the outputs from the two transistors. And it is in fact unnecessary to do so.

If Class B transistors are fed in series, as in Fig. 19.12, instead of in parallel as in Fig. 19.8, the two outputs flow through the load alternately in opposite directions, so reconstituting the whole waveform.


Fig. 19.12-One form of Class B oulpul circuil. Its disadvantage is the centre-lap on the power supply

Here two p-n-p transistors are shown; for simplicity the bias arrangements are again omitted. To avoid the need for a centre-tap on the d.c. power supply it is usual to put a high-C blocking capacitor in series with the loudspeaker, which can then be connected to either end of the supply (Fig. 19.14).

As regards the input transformer, it too can be dispensed with by making use of what is called a phase splitter as the drive stage. Fig. 19.13 shows one type, sometimes called the concertina circuit, in which outputs of opposite phase are obtained from resistors across each side of the transistor or valve. When transistors are to be driven, the outputs have to supply current as well as voltage, so R is needed to equalize the output resistances of the driver, since
the output resistance at the emitter is relatively low (Sec. 12.9). Phase splitters are essential for transformerless push-pull using valves, but when transistors are used for the output stage advantage can be taken of the fact that they are available in both $\mathrm{p}-\mathrm{n}-\mathrm{p}$ and $\mathrm{n}-\mathrm{p}-\mathrm{n}$ forms. If one of each is used their inputs can be driven with the


Fig. 19.13-Phase-splitter circuit for driving a push-pull stage


Fig. 19.15-Transformerless combined driver and output slage using complemertary transistors


Fig. 19.14-Complementary pair methad of doing without a twophase input


Fig. 19.16-Compound pair of transistors for multiplied current gain
same polarity, the reversal of phase being brought about by the opposite polarities of the output transistors themselves. They are termed a complementary pair, and Fig. 19.14 shows the basic circuit.

In this particular formation, with the input returned to either $b$ or c, both transistors are working in common-collector mode, so the
input signal voltage would have to be actually greater than the output. To avoid this disadvantage by using the common-emitter mode it would seem that the input should be returned to the emitters, point a. But as this point is alternating at the full output signal voltage the driver would have to be coupled to the output stage by a transformer, which is what we are trying to avoid. A way out of this difficulty is shown in Fig. 19.15, where the input voltage is developed across $R_{1}$, the top end of which is returned to output emitters signalwise via C, while the required d.c. feed is obtained via the loudspeaker. The two output transistors receive the usual current-adjusting emitter bias by means of $\mathrm{R}_{2}$ and $\mathrm{R}_{3}$, which because of the large currents flowing through them are typically only an ohm or two. Base bias is supplied by the voltage drop across D (Sec. 13.7). This voltage changes comparatively little over quite a wide range of current. One silicon diode usually provides enough voltage (about 0.6 V ) for a pair of germanium transistors; if they are silicon, two diodes in series will be needed.

### 19.14 Compound Transistors

A convenient method of obtaining a higher current gain than is provided by a single transistor is by combining two as shown in Fig. 19.16, forming what is known as a compound pair, or Darlington pair. The essential feature is that the emitter current of the first transistor is the base current of the second, so the current gain of the pair is equal to the product of the current amplication factors of the two transistors; i.e., $h_{\mathrm{fe} 1} \times h_{\mathrm{fe} 2}$. This result is achieved with a minimum of components, so the idea is used for many purposes besides power amplifiers.

Biasing arrangements are similar to those for one transistor, but of course the actual bias voltage required-the difference between $V_{B_{1}}$ and $V_{E_{2}}$-is equal to the sum of the two separate bias voltages.

In Fig. 19.16 the pair is shown in common-emitter mode. When used in common-collector mode the feedback effect (Sec. 12.9) also is multiplied, so that the input resistance is exceptionally high.

### 19.15 Negative Feedback

The effect on the apparent resistance of a circuit of injecting into it a voltage or current proportional to the voltage or current applied was explored in Chapter 12, where in most cases the injected signal was in opposition to the applied signal. In amplifiers this is called negative feedback. It is often used intentionally to increase or reduce the input or output resistance of an amplifier. It is also much used for reducing distortion. Transistor and valve characteristic curves are never quite linear, and with Class B there is the risk of
crossover distortion. The variation of loudspeaker impedance with frequency introduces frequency distortion too, and sometimes the gain of the transistors falls off at the highest audio frequencies. Loudspeaker resonances cause sounds of certain frequercies to be unduly prolonged.

Fig. 19.17 shows an amplifier in which a fraction (B) of the output


Fig. 19.17-Showing the basic principle of voltage negative feedback
is fed back in opposition to the input. (Note that with this method of application the amplifier must be phase-reversing.) The voltage gain of the amplifier itself is denoted by $A$. This means that for every signal millivolt (say) applied to the input terminals, $A$ millivolts appear between the output terminals, and $A B$ millivolts are fed back to the input. The signal source is therefore required to supply, not 1 mV but $1+A B \mathrm{mV}$.* If we denote the overall gain with feedback by $A^{\prime}$, then

$$
A^{\prime}=\frac{\text { output }}{\text { input }}=\frac{A}{1+A B}
$$

If $A B$ is so large that 1 is negligible by comparison with it, then

$$
A^{\prime} \simeq \frac{1}{B}
$$

The great significance of this is that $B$ depends on a potential divider or something of the kind that can be perfectly linear, so the non-linearities involved in $A$ are more or less removed. Also the effects of temperature on the characteristics of transistors, and the very large variations among samples of the same type, are largely overcome.

These major advantages have to be paid for by the increased input to be provided. As a general rule the distortion and overall gain are both reduced by the same divisor, $1+A B$. The overall gain can usually be restored by, say, an extra stage of amplification. But the rule is misleading if it makes us suppose that we need not bother

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much about avoiding distortion in the basic design because it can always be linearized by negative feedback. The fallacy of this can be seen by imagining a Class B amplifier in which, owing to incorrect bias, both transistors were momentarily cut off at the crossover. The value of $A$ could be normal over the rest of each cycle, but at the crossover $A^{\prime}$ would be $0 /(1+0)$, so the gap would be just as bad as without feedback; in fact, even worse because of the sharper transitions between linear and non-amplifying regimes (Sec. 19.5). Negative feedback can make a good amplifier better, not a bad amplifier good.

To illustrate this, Fig. 19.18 shows at $a$ an output/input curve that is perceptibly but not grossly non-linear between A and B -it would


Fig. 19.18-(a) Typical outputiinput characteristic with slight curvature. (b) The same after applying negative feedback
in fact cause about $5 \frac{1}{2} \%$ second-harmonic distortion. P , the centre of the input swing, is the working point. Curve $b$ is the result of applying sufficient negative feedback to reduce the gain (reckoned as the average between A and B) to one tenth, but for ease of comparison the input scale has been reduced to one tenth, as if a distortionless 20 dB amplifier had been added before the input. Also for clarity the curve has been shifted to the right, and the corre-
sponding input swing around $\mathrm{P}^{\prime}$ as working point is from $\mathrm{A}^{\prime}$ to $\mathrm{B}^{\prime}$. The positive and negative output half-cycles are now much more nearly equal than in $a$. The second-harmonic distortion is too small to read off the curve, but if calculated would be found to be one tenth as much as in $a$.

Suppose now that the input is increased by $50 \%$ so as to include the dotted portions. In contrast to the practically linear $\mathrm{D}^{\prime} \mathrm{A}^{\prime}$, the bit from $\mathrm{A}^{\prime}$ to $\mathrm{C}^{\prime}$ would cause a more drastic increase in distortion than AC. So one has to be careful when using an amplifier with negative feedback, not to push the volume control too far.

What about frequency distortion? Suppose that in the example just considered the gain falls off at the highest a.f. to the extent of 6 dB , or $50 \%$. At the lower frequencies, where the full gain operates, the reducing factor of the feedback is $1 /(1+9)=0 \cdot 1$ as we assumed. When A is halved this is $0.5 /(1+4.5)=0.091$. So the relative gain at the top frequencies, with feedback, is $0.091 / 0 \cdot 1,=0.91$, or -0.8 dB , which is hardly noticeable. Negative feedback therefore tends to flatten out irregularities in the gain/frequency characteristic.

### 19.16 Phase Shift with Feedback

The main cause of frequency distortion in amplifiers is reactance. In Sec. 18.10 we considered the use of a resistance-capacitance potential divider as an elementary sort of filter for removing r.f. signals from the output of a detector. Either kind of reactance causes an increasing loss at either higher or lower frequencies, depending on where it comes in relation to resistance; both kinds of reactance together can cause either a loss or gain around one particular frequency, since they form a tuned circuit. In Sec. 6.7 and elsewhere this frequency discriminating effect was shown to be accompanied by a phase shift. So frequency distortion is accompanied by phase distortion. As negative feed back reduces frequency distortion it seems reasonable to infer that it reduces phase distortion too. And so it does, but with important reservations as we shall see.

For manufacturing reasons, inductance tends to be avoided in modern a.f. circuits. Where a.f. transformers are still used they are analysed into their equivalent circuits (Sec. 7.10) for calculating frequency effects, which can be done by drawing phasor diagrams for a number of typical frequencies. There is inevitably some inductance in loudspeakers. some of it purely electrical and some simulated by the motion of the coil in the field of the magnet. But most phase shifts in circuits are due to capacitances. intentional or otherwise.

Fig. 19.19 shows at $a$ and $b$ the two basic combinations of capacitance and resistance with voltage input and output, and at $c$ and $d$ with current input and output. Diagram $a$ might be a resistive loud-
speaker and its blocking capacitor as in Fig. 19.14; $b$ we have come across in Sec. 18.10; $C$ in $d$ might be stray capacitance across a load resistance, but is not likely to be significant at a.f. other than in high-resistance valve circuits.

All these oft-recurring combinations can be treated at one go by relating all frequencies to what we could call the turning frequency $\left(f_{1}\right)$ because it marks a transition in the frequency characteristic from resistance to reactance regimes. As in Sec. 18.10, $1 / 2 \pi f_{1} \mathrm{C}=R$, so

$$
f_{\mathrm{t}}=\frac{1}{2 \pi C R}
$$

In Fig. 19.19 $C$ can be regarded at high frequencies as a shortcircuit, so can be more or less ignored in $a$ and $c$; similarly in $b$ and $d$ at low frequencies, which make it almost an open-circuit.

Fig. 19.20 shows the results of phasor diagrams as curves of relative output against frequency relative to $f_{t}$, with and without

feedback in which $A B=10$. The 11 -fold loss resulting from this feedback is compensated, for easy comparison. Curves are shown for one Fig. 19.19b or $d$ combination and also for two having the same $C R$ value. The curves for Fig. 19.19a and $c$ are a left-to-right mirror image of Fig. 19.20. So all combinations of $C$ and $R$ and values of $C R$ are covered. Fig. 19.21 shows the corresponding phase shifts. Fig. 6.10 shows that in Fig. 19.19a the phase of the output voltage is ahead of the input, so it is regarded as positive. In $b$ the output lags, so the phase shift is negative. A dual phasor diagram would show the shift to be positive in $c$ and negative in $d$.

Figs. 19.20 and 21 show clearly the effect of feedback in extending the flat part of the frequency characteristic curves; with one $C R$ the


Fig. 19.20-Relative output plotted against frequency (relative to the turning frequency $f_{1}$ ) for one and two $C R$ circuits with and without $10: 1(=20.8 \mathrm{~dB})$ negative feedback


Fig. 19.21-Phase shift graphs corresponding to Fig. 19.20
extension is 11 -fold-the same as the reduction in gain, $1+A B$. Note that at $f_{l}$, where the loss without feedback is 3 dB , the phase shift is $45^{\circ}$. The same applies to $(1+A B) f_{1}$ with feedback. But with two $C R$ units the picture changes somewhat. The curves for no feedback are obtained simply by doubling the dB and phase-angle figures, but this time negative feedback does not increase the frequency coverage 11 -fold.

Frequency and phase distortions are certainly much reduced, but beyond about $2 f_{\text {t }}$ the phase shift takes a steep plunge; meanwhile the output actually goes higher than at the 'flat' frequencies. We see that at the peak the corresponding phase shift is $90^{\circ}$. Feedback here can hardly be described as either negative or positive. Most of the suppressed gain of the amplifier is therefore released. At still higher frequencies the phase shift approaches $180^{\circ}$, so feedback is largely positive, but meanwhile the loss due to the $C R$ units is so great that the overall output curve continues to drop away steeply.

At about $1.7 f_{1}$ the phase shift per $C R$ unit is $60^{\circ}$ and the gain is still quite high, so if three units were used the total phase shift would be $180^{\circ}$ and the conditions for self-oscillation (Sec. 14.5) would be fulfilled. The amplifier would in fact have turned into the oscillator circuit of Fig. 14.15. Such an amplifier would be described as unstable.

A single $L C$ combination can cause a phase shift of nearly $180^{\circ}$, which is another reason for avoiding $L$ in amplifiers But even where there is none, the existence of $C$ between stages, and in the transistors and valves themselves, makes it difficult to prevent instability if one attempts to reduce distortion throughout by making the feedback loop embrace the whole amplifier. It is not uncommon for oscillation to take place at some frequency too high to be audible itself but only as distortion or loss of output. With amplifiers of very high gain care has to be taken to avoid instability due to unwanted feedback via stray capacitances or impedance common to high and low level signal circuits. And one is only too familiar with the howl set up when a microphone picks up too much of the output from its loudspeaker; in this case the feedback is acoustical.

So far it has been assumed that $B$ is the same at all frequencies. But sometimes it is designed to vary with frequency, usually by including capacitance as well as resistance in the feedback path. Obviously if negative feedback is reduced at some frequencies the gain at those frequencies is increased. That may be the object, or a modification of the value and phase angle of $B$ may be needed to improve stability.

### 19.17 Input and Output Resistance

The theory of the effects of feedback on input and output resistance is given in Sec. 12.5; now we can glance at it again with particular
reference to amplifiers. Usually we would be glad to increase the input resistance of a bipolar transistor so as to make it as nearly as possible voltage-operated, more like an f.e.t. or valve. We saw in Sec. 12.9 how a great increase is obtainable by working it in commoncollector (or emitter-follower) mode, and in Sec. 19.14 a further increase by using a pair of transistors in this way. The effect is produced by what amounts to total negative feedback; all the output voltage is fed back to the input. So the input voltage to cause a given input current must be increased $(1+A)$ times and the apparent input resistance is therefore $(1+A)$ times greater.

An important practical point is that even when a load or coupling resistance is on the collector or anode side there is usually a resistor on the emitter or cathode side for biasing purposes. This feeds back a fraction of the total output voltage to the input, reducing the gain and increasing the input resistance. This local negative feedback might seem to be a good thing, but in general a feedback loop around more stages is preferable. So the biasing resistor is often bypassed to signals by a capacitor having a reactance (at the lowest signal frequency) much less than the resistance. But when the resistance is so low that this calls for an uneconomically large capacitance it may be omitted.

In Fig. 19.17 some of the output voltage was fed back in series with the input. An alternative method in which current is fed back in parallel is shown in Fig. 19.22. It is sometimes called shunt feed-


Fig. 19.22-Showing the basic principle of current negative feethack
back. Treating the system as a current amplifier we again call the input 1 , this time say $1 \mu \mathrm{~A}$. With a current amplification $A$, there is an output of $A$ microamps in the direction shown. Of this, a fraction $B$, controlled by $R$, is fed back to the input. (The fact that the arrows seem to show it being fed forward is because it is negative feedback.) So for every microamp going into the amplifier the signal current source has to supply ( $1+A B$ ) microamps. The current gain (neglecting the fraction of output bypassed via $R$ ) is therefore reduced to $A /(1+A B)$ as in the voltage case, and because the current taken from the source is $(1+A B)$ times as much as is actually going into the amplifier the input conductance seems to be $(1+A B)$ times as great, and the resistance is that much less.

Output resistance, as we saw in Sec. 12.7, is measured by substituting a signal generator for the load and short-circuiting the input voltage generator, the only input then being what is fed back. This is not difficult to do theoretically by calculation, and the result is that voltage feedback reduces the apparent output resistance of an amplifier and current feedback increases it. But the extent of these effects is greater than the factor $(1+A B)$, in which $A$ is the gain with load connected. The effect of voltage feedback is to divide the output resistance by $\left(1+A_{0} B\right)$, where $A_{0}$ is the voltage gain with output terminals open-circuited; and current feedback multiplies it by $\left(1+A_{\mathrm{s}} B\right)$, where $A_{\mathrm{s}}$ is the current gain with output terminals short-circuited. $A_{0}$ and $A_{\mathrm{s}}$ can be considerably greater than $A$.

Fig. 19.23 might look like voltage feedback, but it is actually current feedback, because the voltage fed back is proportional to


Fig. 19 23-Although this looks ralher like Fig. 19.17 (voltage feedback) it is actuallv current feedback
the current flowing through the load $R_{L}$, whereas in Fig. 19.17 it is proportional to the voltage across $R_{L}$.
When a short-lived signal (called a transient) having a frequency at which the loudspeaker it is fed into resonates, the loudspeaker continues to vibrate and emit sound after the signal has ceased. This effect is called transient distortion. The oscillation of the moving coil in the strong magnetic field (Sec. 19.9) generates an e.m.f., and as it is in parallel with the output of the amplifier it will make a current flow therein. How much current depends on the output impedance. If it is very small-say a fraction of the loudspeaker's own impedance-the current will be correspondingly large and will quickly damp out the undesirable oscillation. This is obviously a good thing, which can be achieved by quite a moderate amount of negative voltage feedback.

The $100 \%$ negative feedback in emitter followers and cathode followers not only greatly increases the input resistance but reduces the output resistance. So although the voltage gain is less than 1 the current gain can be very large. These devices are therefore largely used as a sort of current transformer to feed low-impedance loads from high-impedance sources. They have two great advantages over inductive transformers: the voltage loss is very small; and the effective frequency band can be very large, including zero frequency. Fig. 19.22 can be adapted to illustrate $100 \%$ negative feedback by transferring $R_{L}$ to the $R$ position so that all the output current is fed back.

### 19.18 Linearizing the Input

The advantages of negative feedback in a.f. amplifiers are so valuable that it is practically always used. The number of ways in which it can be applied are too many to show here, but armed with the foregoing principles we ought to be able to follow most of the circuit diagrams we see.

Many of the low-power transistors used in the initial stages of amplifiers have very linear $I_{C} / I_{B}$ characteristics, as in Fig. 10.11. and very non-linear $I_{C}$ (or $I_{B}$ ) $V_{B}$, as in Figs. 10.12 and 13. Obviously they should be worked from signal current sources rather than voltage. Where the impedance of the actual source was not sufficiently high to qualify it as a current source, a common practice was to

Fig. 19.24-One way of linearizing a transistor is by means of a relatively high resistance $R$ in series with the input

bring it up to that standard by adding resistance. Fig. 19.24 shows an input-stage circuit (feeds omitted) in which the transistor is assumed to have a perfectly linear $I_{C} / I_{B}$ curve (i.e., constant $h_{\mathrm{fe}}$ ). With the voltage signal source $S$ connected direct there would be serious distortion because of the very non-linear input resistance ( $I_{B} V_{B}$ curve). Inserting a relatively large linear $R$ makes the total resistance nearly linear and therefore the input current, and therefore also the output current, nearly proportional to the input signal voltage. But of course most of the signal is lost in $R$.

If instead negative voltage feedback were used the result would be just the same; the voltage drop due to $I_{B}$ flowing through $R$ would be replaced by a voltage drop due to $h_{\mathrm{fe}_{\mathrm{e}}} I_{B}$ flowing through part of $R_{L}$. But whereas the series-resistance method linearizes only the input circuit, and depends on linear $I_{C} / I_{B}$ (which is now less often found). feedback can be applied to as much of the amplifier as one wants and corrects all non-linearities, so has virtually superseded series resistance.

### 19.19 Some Circuit Details

To gather logether the fragmentary studies of this chapter and to illustrate some further features. a complete a.f. amplifier circuit is shown in Fig. 19.25. Typical component values are given only to indicate the orders of magnitude to be expected. In such diagrams the main unit abbreviations are often omitted, as the component symbol implies it : for example, ' 1 m ' for the loudspeaker blocking capacitor means $1 \mathrm{mF},=1000 \mu \mathrm{~F}$.

The output stage follows the lines of Fig. 19.15-a complementary pair-each half being made up of two transistors as suggested in Sec. 19.14. The two biasing diodes are supplemented by a resistor $\mathrm{R}_{1}$, the special symbol for which indicates that it can be pre-set to any value up to $50 \Omega$ in order to minimize crossover distortion. A


Fig. 19.25-A upical complete andio amplifier circuit
slightly different arrangement is used for feeding the collector of $T_{R_{2}}$ : instead of coming from the positive supply rail (as it is called) via the loudspeaker, it comes via $\mathrm{R}_{2}$, and the output signal voltage is added via $C_{1}$, so that the signal potential at the top end of $R_{3}$ is about the same as $\mathrm{T}_{\mathrm{R}_{2}}$ applies at the bottom end. $\mathrm{R}_{3}$ is thus made to look to $T_{R_{2}}$ like a very high resistance, and a good stage gain is obtained. This particular method of applying the principles of Sec. 12.5 is called bootstrapping.

The emitter resistor of $\mathrm{T}_{\mathrm{R}_{2}}$ is bypassed to prevent negative feedback here, but the emitter resistor for $T_{R_{1}}$ is left unbypassed because it forms part of the overall feedback loop right from the loudspeaker via $R_{4} C_{2}$ provides subsidiary feedback to ensure stability at very high frequencies. Biasing follows normal practice (Sec. 13.4). $R_{5}$ and $C_{3}$ together are what is known as a decoupler the purpose and working of which are explained in Sec. 26.7.

### 19.20 Noise

While the gain of an amplifier such as that just described is sufficient to give full output from a radio detector or some types of gramophone pick-up, it is usually not enough for the better types of pickups and microphones or for tape playback heads. Amplifiers for
many other purposes also have to deal with even smaller signals, perhaps only a few microvolts. Designing for the required gain is easy; the real problem is noise. This word is used to refer to electrical disturbances which, in sound reproducers, cause audible noise; the same disturbances are also objectionable when the end product is not sound-they mar the picture in television, for example.

Some noise is picked up from external electrical equipment. There is hum from the public a.c. supply. And anything that causes abrupt changes of current generates noise over a wide frequency band; such things as switching, thermostat contacts, appliances using commutator motors, and faulty lighting. Such disturbances can be excluded from a.f. amplifiers by properly arranged earthing, electric and magnetic screening, filters and careful layout of wiring. Printed wiring and integrated circuits are less likely to pick up induced noise.

But when all such noise has been suppressed or excluded there is a never entirely avoidable residue generated in the amplifier circuits themselves. An electric current is not a smoothly continuous flow; it is made up of individual electrons, and even when there is no net current in any one direction the electrons in, say, a resistor are in a continuous state of agitation. Their random movements, like the aimless jostling of a crowd which as a whole remains stationary, are equivalent to tiny random currents, which give rise to small voltages across the resistor. Quite a moderate gain, such as can be obtained with three or four stages, is enough to enable them to be heard as a hissing sound, or seen on the television screen as animated graininess. Only the part of an amplifier at or near its input is a noise problem, because all the other parts are followed by less amplification.

This electron agitation increases with temperature and can only be quelled altogether by reducing the temperature to absolute zero $\left(-273^{\circ} \mathrm{C}\right)$ which is hardly practical for ordinary purposes, though for special purposes such as reception from space vehicles the important parts are immersed in liquid helium ( $-269^{\circ} \mathrm{C}$ ). It is known as thermal agitation or Johnson noise. Although the voltages occur in a completely random fashion, so that the peak values are continually fluctuating, their r.m.s. value over a period of time is practically constant; somewhat as the occurrence of individual deaths in a country fluctuates widely but the death rate taken over a year changes little. Since the frequency of the jostlings is also completely random, the power represented by it is distributed uniformly over the whole frequency spectrum. By analogy with light, it is often called white noise. So if the amplifier is selective, accepting only frequencies within a certain band, $B$ hertz wide, the noise is reduced. The formula is

$$
V=2 \imath^{\prime}(k T B R)
$$

where $V$ is the r.m.s. noise voltage, $k$ is what is called Boltzmann's
constant and is $1.37 \times 10^{-23}, T$ is the temperature in kelvins $\left({ }^{\circ} \mathrm{C}+273\right)$ and $R$ is the resistance in ohms. For example, suppose the resistance of a signal source is $0.1 \mathrm{M} \Omega$ and the full a.f. band ( 20 kHz ) is accepted. Room temperature can be taken as 290 K . The noise voltage is then $2, ~\left(1.37 \times 10^{-23} \times 290 \times 2 \times 10^{4} \times 10^{5}\right)$, $=5.6 \mu \mathrm{~V}$. If the total voltage gain were one million ( 120 dB ) this noise ( 5.6 V ) would be very disturbing.

A somewhat similar cause of noise, usually called shot effect. is due to the current in diodes, transistors and valves being made up of individual electrons. It is not so easily calculated as Johnson noise because it depends on the structure of the device, space charges and other things, but like Johnson noise it is proportional to $\imath^{\prime} B$. The r.m.s. noise current tends to be proportional to $\downarrow^{\prime} I$, where $I$ is the current flowing through the device, so in the first stage this current should be made small, so long as it is not reduced past the point where the gain of the device begins to be reduced faster. The aim is the highest possible signal/noise ratio. Multielectrode valves such as pentodes are avoided in early stages because the extra electrodes introduce extra noise called partition noise. Special silicon bipolar transistors are made for low noise, and in general f.e.ts are low-noise devices. Transistors have a lownoise band from moderately low a.f. up to a certain frequency depending on the design. Above that frequency the signal/noise ratio falls off, and at very low frequencies there is what is called ficker noise, or $1 / f$ noise because it is inversely proportional to frequency.

The noisiness of a device or stage, or even of a whole amplifier or receiver, is specified as its noise factor or noise figure, which is defined in various ways, of which a simple one is

$$
\frac{\text { input signal/noise power ratio }}{\text { output signal/noise power ratio }}
$$

It is usually expressed in $\mathrm{dB} ; 0 \mathrm{~dB}$ is perfection, and the higher the figure the worse.

### 19.21 Operational Amplifiers

The shunt feedback circuit (Fig. 19.22) was treated in Sec. 19.17 as a current amplifier. In practice a signal current source takes the form of a voltage source in series with such a high impedance that the current is determined almost entirely by it rather than by whatever circuit it works into. If this high impedance takes the form of a resistance that is a permanent part of the amplifier, the basis is laid for an interesting and important class of feedback voltage amplifiers.

Fig. 19.26 is essentially Fig. 19.22 plus this resistance, $R_{1}$. If the
current gain is sufficiently large the part of the input current that goes into the amplifier at $Y$ is negligible compared with what goes through $\mathrm{R}_{2}$ to the 'live' output terminal. Z . For the feedback to be negative, the potential of Z has to be opposite to that of Y


Fig. 19.26-The simplest form of operational amplifier circuit
(relative to E). And because of the high gain of the amplifier it must be very much larger. The only way that these conditions can be fulfilled is for the signal potential to vary as shown in Fig. 19.27. If $V_{\mathrm{x}}$ alternates, the line see-saws about point P as the fulcrum, so the author named Fig. 19.26 the see-saw circuit. If the voltage gain


Fig. 19.27-Showing the distribution of potential along $\mathrm{R}_{\mathbf{1}} \mathrm{R}_{2}$ in Fig. 19.26
of the amplifier is large enough, the position of Y on this diagram is so near $P$ as to be hardly distinguishable from it and we can say that

$$
V_{\mathrm{z}}^{\prime} \simeq-\frac{R_{2}}{R_{1}} V_{\mathrm{x}}
$$

So the first use of such an arrangement is as a voltage inverter in
which the output is almost as nearly equal and opposite to the input as $R_{2}$ can be made equal to $R_{1}$.

Next, a voltage can not only be inverted but reduced or enlarged by choosing $R_{1}$ and $R_{2}$ in the desired ratio.

Because these results depend negligibly on the characteristics or changes thereof in the amplifier (always assuming its gain is large), this sort of system is used in computers of the analogue class; that is to say, those in which voltages or currents represent the data in the calculations. They come up for attention in Sec. 25.2. Because amplifiers of this type can be used for performing mathematical operations they are usually known as operational amplifiers.

Another technical term arises from the fact, which Fig. 19.27 shows so clearly, that point $Y$ is always very close to zero or earth potential without actually being connected to it. It is therefore called a virtual earth, and has found a number of useful applications in circuitry.

Operational amplifiers usually have to work down to zero frequency, so must be direct-coupled throughout. This immediately raises a difficulty. The gain of the amplifier having to be large, every little variation in d.c. conditions in the first stage, due to temperature changes especially, is amplified and will confuse the results. Two matched transistors are likely to vary to nearly the same extent, so the difference between them will be relatively small. Z.f. amplifiers therefore almost always have at least a first stage that responds to the difference between two signals (one of which of course can be zero) and this adds considerably in other ways to its usefulness

as an operational amplifier. Fig. 19.28 shows the most used differential (or balanced) amplifier stage, usually called the long-tailed pair, the 'long tail' being the high resistance $R$. It should have as high a resistance as possible, and to save negative supply voltage an electronic substitute for a high resistance (a constant-current source') is sometimes used. The idea is to keep the total current to both transistors virtually constant, so that increasing it by a positive
signal at one of the inputs communicates via $R$ an equal and opposite signal to the other input. Equal signals of the same polarity at both inputs (called common-mode signals) therefore have very little effect. False signals due to such things as temperature changes are usually of this kind. Ability to distinguish between difference signals and common-mode signals is called common-mode rejection.

Amplifiers are often represented in diagrams by triangles, as in Fig. 19.29, in which the direction in which the signals get larger is

Fig. 19.29-Convertional representation of (a) inverting and (b) noninverting balanced operational amplifiers

represented by the direction in which the width of the triangle gets smaller. Balanced amplifiers have two inputs along the base line as shown, and the common output is at the point. (Normally only one of the two available outputs in Fig. 19.28 is used.)

Except for its balanced input, Fig. 19.29a is the same as Fig. 19.26, and Fig. $19.29 b$ is a non-inverting version of it. $R_{1}$ is still in the input circuit, but it is also in the output circuit, so instead of the overall gain being $-R_{2} / R_{1}$, it is $\left(R_{2}+R_{1}\right) / R_{1}$. If $R_{2}$ is made zero, we get a 1:1 non-inverting condition. Except that the gain is not large enough for operational purposes because it has only one stage, emitter followers and cathode followers are included in this form. Among other things, it can be used to make one point in a circuit maintain virtually the same potential as another to which it is not connected. (The virtual earth is a particular example.) In this way the effect of an unavoidable capacitance can sometimes be circumvented by preventing appreciable differences of potential from occurring across it.

### 19.22 Integrated Circuits

Because operational amplifiers consist of resistors and silicon transistors and diodes, and involve only very low power, the integrated circuit form of manufacture is ideal for them. A whole batch of identical amplifiers is laid down on a slice of silicon, the various $p$
and n regions being produced by successive exposures to appropriate vapours which diffuse into the material. The conductive paths for resistors and 'wiring' are formed by metal deposits. The patterns of these actions are controlled by microscopic masks obtained by photographic reduction. Silicon dioxide prevents diffusion, so is laid down on the unmasked areas and can later be removed in preparation for the next process. The fact that pairs of transistors can be laid down very close together in identical circumstances minimizes differences in performance that could result in commonmode effects.

A complete amplifier may occupy only one or two square millimetres. The capital cost is large but the unit cost is small, so if large numbers are required they can be made cheaply. Inductors and large capacitors cannot be included, but transistors and diodes are cheap. Reactances can be simulated by transistors with feedback phase-shifted by $C$ and $R$, and we have seen how negative feedback can be used to obtain precise performance from imprecise active devices. Design philosophy for integrated circuits is therefore quite different from that appropriate to circuits made up of separate components connected together with wire. But the principles on which they work are the same, so are covered by this book and there is no need to spend any more time on integrated circuits (i.cs) as such.

In practical applications i.cs usually have to take their places as components in systems where components of other kinds must also be included. If the minute size of the i.cs is not to be wasted, these other components should be as small as possible. An example of this is the ceramic filter to be described in Sec. 22.7. For compactness, stability and ease of production the wiring usually takes the form of a copper sheet laid down on insulating material (substrate) and eaten away by acid to the desired pattern (printed circuit) rather like the i.c. itself but on a larger scale. On this substrate the components, including the i.cs, are mounted. Inductors of low value can be 'printed' on the sheet as spirals.

## Selectivity and Tuning

### 20.1 Need for R.F. Amplification

In Sec. 18.11 we saw how the effectiveness of detectors deteriorates quite rapidly as the r.f. input is reduced much below about I V. Signals picked up by a receiving aerial are seldom more than a few millivolts and are often only microvolts, so (except from a powerful sender at short range) they are too weak to operate a detector satisfactorily. Amplification of the r.f. signals is therefore a practical necessity in order to give a receiver adequate sensitivity, which is usually expressed as the input microvoltage needed to yield a specified a.f. output (often 50 mW ) at a specified modulation depth (usually $30 \%$ ).

Another practical necessity is selectivity, which as we saw in Sec. 8.6 is the ability of a receiver to discriminate in favour of the wanted signal against all others, some of which may be much stronger and only slightly different in frequency. As will have been gathered, the chief means for obtaining selectivity is the tuned circuit. Although tuned circuits are not necessarily associated with amplification they almost invariably are, for reasons that will appear. Meanwhile we shall do well to make a closer study of selectivity.

### 20.2 Selectivity and Q

We saw in Chapter 8 that both selectivity and sensitivity are improved by increasing the $Q$ of a tuned circuit. Fig. 8.5 , for example. shows how the response at the frequency to which a circuit is tuned increases in direct proportion to $Q$, whereas the response at other frequencies is less affected: in fact, at frequencies far off tune it hardly changes at all. The problem of r.f. amplification might therefore seem to be simply one of getting a high enough $Q$. We found (Sec. 8.4) that the $Q$ of a circuit is the ratio of its reactance (either inductive or capacitive) to its series resistance, which is the same thing as the ratio of its dynamic resistance to its reactance:

$$
Q=\frac{X}{r}=\frac{R}{X}
$$

Now the amount of reactance is decided by the requirements of tuning, so attention must be confined to reducing the series resistance, $r$. This will have the effect of increasing the dynamic
resistance, $R$. We noted (Sec. 8.14) that $r$ is a sort of hold-all figure including everything that causes energy loss from the tuned circuit; not only the resistance of the coil of wire, but loss in the solid core (if any), in adjacent metal such as screens, and in dielectrics associated with the circuit. These can all be reduced by careful choice of materials and careful design, but size and cost limit what can be done, so the result is a $Q$ seldom very much better than about 200.

In Chapter 14 we saw how the loss that damps out oscillations in tuned circuits can be completely neutralized by energy fed to it at the right moment in each cycle (positive feedback), so that oscillations once started continue indefinitely. This would not do in a receiver, because we want the strength of oscillation in the tuning circuit to depend all the time on what is coming from the distant sender. But if the positive feedback is reduced, the amount of energy fed back will partly neutralize the losses. Since we are already using the symbol $r$ to include not only resistance in its strictest sense but all other causes of loss, there is no reason why we should not also make it take account of a source of energy gain. In this sense, then, positive feedback reduces $r$ and therefore increases $Q$.

We can put this to the test, using an oscillator of the type shown in Fig. 14.4, by increasing the distance between the two coils just sufficiently to make the continuous oscillation stop. The tuned output circuit will then behave exactly as if its resistance were abnormally low; any small voltage injected in series with it at its resonant frequency is magnified much more than if the transistor were not working. The same result is obtained if the tuned circuit and the reaction coil are interchanged; in this way an input tuning circuit likewise can be given a sharper resonance.

In Fig. 20.1 are plotted some of the resonance curves that might be obtained, using different amounts of positive feedback on a circuit tuned to 1 MHz . The ' $Q=40^{\prime}$ curve represents a rather poor circuit with no feedback, and the others with progressively increased amounts. To raise the $Q$ from 40 to 8000 necessitates very careful adjustment of the coupling, as it is then very near the selfoscillation point, and a slight change in supply voltage or in transistor temperature would either carry it past this point or in the opposite direction where $Q$ would be substantially less. In other words, the adjustment of coupling is extremely critical. In practice it would be difficult to maintain the $Q$ close to such a high value for long, and one would be wise to be content with rather less. Even so, the benefit that can be obtained by feedback is so great that it could not be shown clearly on a graph like those that were used in Chapter 8; Fig. 20.1 has therefore been drawn to a logarithmic voltage scale, so compared with Fig. 8.5 the increase is much greater than it looks,

The required selectivity and sensitivity seem now to be readily obtainable. If only a simple carrier wave were being received, that might be true. But it is the modulation of the carrier that conveys
the desired information (using that word in its widest sense). If the carrier is frequency-modulated (Sec. 15.12) it is obvious that a definite band of frequencies, with the carrier frequency at its centre,

Fig. 20.1-Showing how r.f. voltage across a tuned circuit varies with frequency at different Q values, obtained by adjusting positive feedback

must be received to an equal degree. The need for covering a band of frequencies when the carrier is amplitude-modulated is less obvious, but it none the less exists.

### 20.3 Theory of Sidebands

Strictly speaking, it is only an exactly recurring signal that can be said to have one definite frequency. The continuous change in amplitude of a carrier uave during amplitude modulation makes the r.f. cycle of the modulated wave non-recurring, so that in acquiring its amplitude variations it has lost its constancy of frequency.

A mathematical analysis shows that if a carrier of $f_{1} \mathrm{~Hz}$ is modulated at a frequency $f_{2} \mathrm{~Hz}$ the modulated wave is exactly the same as the result of adding together three separate waves of frequencies $f_{1},\left(f_{1}+f_{2}\right)$, and $\left(f_{1}-f_{2}\right)$. It is not easy to perform the analysis
of the modulated wave into its three components by a graphical process, but the corresponding synthesis, adding together three separate waves, demands little more than patience.

Fig. 20.2 shows at $a, b$, and $c$ three separate sine-wave trains, there being 35,30 , and 25 complete cycles, respectively, within the width
(a)

##  UPPER SIDEBAND $\left(f_{1}+f_{2}\right)$, HERTZ

(b)

(c)
 LOWER SIDEBAND $\left(f_{1}-f_{2}\right)$, HERTZ


MODULATED CARRIER
CARRIER FREQUENCY $f_{1}$ MODULATED $50 \%$ AT $f_{2}$, HERTZ
Fig. 20.2-Showing that a modulated carrier wave (d) is identical with three unmodulated waves ( $a, b$ and $c$ ) added together
of the diagram. By adding the heights of these curves above and below their own zero lines point by point, the composite waveform at $d$ is obtained. It has 30 peaks of varying amplitude, and the amplitude rises and falls five times in the period of time represented. If this is a thousandth part of a second, curve $d$ represents what we have come to know as a 30 kHz carrier amplitude-modulated at 5 kHz . Fig. 20.2 shows it to be identical with three constant-
amplitude wave-trains having frequencies of $30,30 \div 5$, and $30-5 \mathrm{kHz}$.

Thus a carrier modulated at a single frequency is equivalent to three simultaneous signals: the unmodulated carrier itself and two other steady frequencies spaced from the carrier on each side by an amount equal to the frequency of modulation.

The same facts can be illustrated by a phasor diagram. Ir such a diagram, as we know (Sec. 5.7), a single unmodulated carrier-wave of frequency $f$ is represented by a phasor such as OP in Fig. 5.5, pivoted at one end $(\mathrm{O})$ and rotating $f$ times per second. The length of the phasor represents (to some convenient scale) the peak voltage or current, and the vertical 'projection' PQ represents the instantaneous value, which of course becomes alternately positive and negative once in each cycle.

If this were related to a waveform diagram such as Fig. 20.2, it should be clear that amplitude modulation would be represented by a gradual alternate lengthening and shortening of the phasor as it rotated. Thus, modulation of a 600 kHz carrier to a depth of $50 \%$ at a frequency of 1 kHz would be represented by a phasor revolving 500000 times per second, and at the same time increasing


Fig. 20.3-Progressive series of phasor diagrams, showing how adding a pair of oppositely-rotating phasors (PA and AB) alternately lengthens and shortens the original plasor (OP! without affecting its phase
and decreasing in length by $50 \%$ once during each 600 revolutions.
The attempt to visualize this rapid rotation makes one quite dizzy, so once we have granted the correctness of the phasor representation we may forget about the rotation and focus our attention on how the alternate growing and dwindling can be brought about by phasorial means.

OP in Fig. 20.3a, then, is the original unmodulated-carrier phasor, now stationary. Add to it two other phasors, PA and AB, each one quarter the length of OP. If they are both in the same direction as OP, they increase its length by $50 \%$; if in the opposite direction they decrease it by $50 \%$. The intermediate stages. corresponding to sinusoidal modulation, are represented by the two small phasors
rotating in opposite directions at modulation frequency. Various stages in one cycle of modulation are shown at Fig. $20.3 b j$, the total length of atl three vectors added together being $O B$ in every case. Notice that in spite of the small phasors going through complete $360^{\circ}$ cycles of phase relative to OP , the triple combination OB is always exactly in phase with OP. It therefore gives the true lengthening and shortening effect we wanted.

Having agreed about this, we can now set OP rotating att 600000 rev/s again. In order to maintain the cycle of changes shown in Fig. 20.3 the little phasor $A B$ will therefore have to rotate at 600000 rev/s plus once in every 600 ; that is 601000 : while PA must rotate at 600000 less once in every $600: 599000$.

Translated back into electrical terms, this confirms the statement that amplitude-modukating a carrier-wave of $f: \mathrm{Hz}$ at a frequency of $f_{2} \mathrm{~Hz}$ creates side-frequencies, $f_{1}+f_{2}$ and $f_{1}-f_{2}$. And Fig. 20.3 also shows that the amplitude of each of these side-waves relative to that of the carrier is half the modulation depth (Sec. 15.10).

In a musical programme, in which a number of modulation frequencies are simultaneously present, the carrier is surrounded by a whole array of extra frequencies. Those representing the lowest musical notes are close to the carrier on each side, those bringing the middle notes are farther out, and the highest notes are the farthest removed from the carrier frequency. This spectrum of associated frequencies on each side of the carrier is called a sidehand, and as a result of their presence a musical programme, nominally transmitted on a frequency of, say, $1(0)(0) \mathrm{kHz}$, spreads over a band of frequencies extending from about 990 to 1010 kHz . Expressed generally, the band of frequencies occupied by an amplitude-modulated r.f. signal is double the highest modulation frequency.

With frequency modulation we might expect the band of frequencies to be double the deviation (Sec. 15.12), $f_{D}$. In fact however the matter is very involved mathematically, and the pattern of sidebands is affected by the modulation frequency $\left(f_{\mathrm{m}}\right)$ as well as by its depth. Only at low modulation frequencies is the bandwidth not appreciably higher than $2 f_{D}$. In Fig. 20.4 the bandwidth is plotted against $f_{\mathrm{m}}$ for $f_{D}=75 \mathrm{kHz}$, which is the usual figure for broadcasting.

Many people have had the bright idea that by making $f_{D}$ smallsay 1 kHz or even less-the bandwidth required could also be made small. These attempts to defeat the course of nature all come to grief on the fact that the sidebands always extend across at least $2 / \mathrm{m}$, not less with f.m. than with a.m. but rather more.

A most important point to remember is that with either system of modulation the frequency band occupied depends entirely on the modulation and not at all on the frequency of the carrier wave.

### 20.4 Over-Sharp Tuning

It is now clear that in order to receive a modulated signal completely the receiver must accept not one frequency but a whole band, pre-
ferably with equal effectiveness throughout. If we look back at Fig. 20.1 we see that the curve for $Q=8000$ fails conspicuously to meet this requirement for a.f. modulation. The sidebands corresponding to a modulation frequency of 5 kHz are shown to be received only to the extent of $1.3 \%$ as much as the carrier wave and the lowest a.f. sidebands. The high notes are quite literally tuned out by overselectivity. Higher pitched sounds still would be even more drastically attenuated. The result would be indistinct speech and boomy


Fig. 20.4-Showing how the frequency bandwidth of a frequency-modulated carrier wave depends on the maximum modulation frequency, $f_{\mathrm{m}}$
music. Even with a $Q$ as low as $100,30 \%$, or 3 dB , of 5 kHz modulation would be lost, but this would not be too much to restore by an equalizer (Sec. 19.3).

Because the effective frequency bandwidth for a given $Q$ is proportional to the carrier frequency, whereas the bandwidth required is unaffected by it, the sideband cutting effect is less at higher carrier frequencies and greater at lower carrier frequencies.

With frequency modulation there is the same need to preserve sidebands, but the results of failing to do so are different. Fig. 20.4 might certainly suggest that if the bandwidth was limited to 150 kHz all except perhaps the lowest audio frequencies would be lost. They would however all be represented at least to some extent by the sidebands up to 150 kHz . But this graph represents $100 \%$ modulation, whereas most of the time the percentage is much less. The main effect of sideband cutting is therefore to limit the depth of modula-tion-and hence the resulting a.f. amplitude-that can be correctly
reproduced. In other words, non-linearity distortion, in contrast to frequency distortion of a.m. signals.

This undesirable effect is at least partly counteracted by the limiter in an f.m. receiver, which tends to bring all r.f. signal amplitudes down to one level, but it could not cope with really severe sideband cutting.

Any hopes we may have entertained of achieving adequate sensitivity and selectivity quite simply by pushing up $Q$ are therefore bound to fade.

### 20.5 Channel Separation

Having looked at the anti-selectivity demands of modulation, let us now turn to the selectivity demands of the frequency spacing of broadcasting stations. For sound, the modulation frequencies are audio frequencies, which go up to at least 15 kHz . The corresponding sidebands would occupy at least 30 kHz . Ideally, then, the carrier waves of sound broadcasting stations should be spaced at intervals of at not less than 30 kHz , plus a margin for selectivity, as suggested in Fig. 20.5a, where the dotted line is an ideal overall receiver


Fig. 20.5-At (a) is shown how broadcasting stations' carrier waves ought to be spaced on the frequency scale in order to permit full reception of each without interference from others. How they actually are spaced (b), shows that interference is almost inevitable if full reception of sidebands is attempted
response curve, embracing the whole of transmission A and completely rejecting transmission $B$.

Unfortunately the pressure of national demands for broadcasting 'channels' has squeezed this ideal severely; the standard interval in Europe on the low and medium frequencies is only 9 kHz . Obviously this faces receivers with an impossible task, for it brings 9 kHz modulation of one station on to the same frequency as the carrier wave of the next and the 9 kHz modulation of the next but one. Therefore few, if any, broadcasting stations on these frequencies actually radiate sidebands up to $15 \mathrm{kHz} ; 8 \mathrm{kHz}$ is a more realistic maximum. There is therefore some loss at the start. And even what is transmitted cannot be fully reproduced without interference, unless the wanted transmission is so much stronger than
those on adjacent frequencies as to drown them. So receivers for 'other than local-station reception must be designed to cut off at about 4.5 kHz , sacrificing quality of reproduction in the interests of selectivity (Fig. 20.5b), and even then liable to some interference from adjacent-channel sidebands if they are strong enough. This is heard as what is called monkey chatter.

Because the spacing of carrier waves is related to modulation it is on a frequency basis, not on a wavelength basis. If a station transmits on 200 kHz , the two nearest in frequency will be on 191 and 209 kHz . The corresponding wavelengths (Sec. 1.9) are 1571.1500 and 1435 m , so the average wavelength spacing is 68 m . Three consecutive carrier-wave frequencies higher in the scale, with the same 9 kHz spacing, are 1502,1511 and 1520 kHz . The corresponding wavelengths are $199.7,198.6$ and 197.3 metres, so the wavelength spacing here is only 1.2 metres.

The modulation for the picture in television is a complicated subject (Sec. 23.7) and at this point it is sufficient to say that the modulation frequencies, called video frequencies, extend to several megahertz. One of the several standard channel widths is 8 MHz nearly 1000 times as much as a sound broadcasting channel! In fact, it exceeds the combined width of all the bands available for broadcasting on low, medium and high frequencies. Television is therefore accommodated in the much wider "very high frequency" (v.h.f.) and 'ultra high frequency' (u.h.f.) bands, 30 to 300 MHz and 300 to 3000 MHz respectively. Their disadvantage is their comparatively short range (Sec. 17.12).

Since the carrier itself conveys no information, and the second sideband duplicates the first. by omitting them one can get twice as many channels of communication into a given frequency band, and there is a large saving in power. So in point-to-point radio, in which the necessary complications in sender and receiver equipment can be accepted, this is usually done. But to equip millions of broadcast radio recelvers to work on this system seems impracticable. However, in television most of one sideband is omitted (Sec. 23.7) without creating difficulties at the receiving end.

### 20.6 A General Resonance Curve

Whether we are considering channel separation or loss of sidebands, then, the basis of reckoning is 'frequency off-tune'. To distinguish this particular frequency from others we shall denote it by $f^{\prime \prime}$.

The ideal shape for a tuning curve would seem to be one which was level over a frequency band wide enough to include all the wanted sidebands and then cut off completely to exclude everything else. None of the resonance curves we have seen so far looks anything like this-see Fig. 20.1, for example. And except for Fig. 8.6 they are arranged for comparing sensitivity rather than selectivity.

We need to take the response at resonance as the starting point and see how much it falls at off-tune frequencies. This falling-off can be reckoned in dB. An alternative scale for any who may still have difficulty in thinking in dB is in what we shall call $S$ numbers, which are the numbers of times by which response off-tune is less than that at resonance. We now want to know what determines $S$ at any given $f^{\prime}$-in general; not like Fig. 8.6, which applies to only two values of $Q$ and one carrier frequency.

To discover this we go back to tuned-circuit fundamentals. Fig. $20.6 a$ is a tuned circuit containing a source of e.m.f. $E$. It could be a signal picked up inductively by $L$, but it is shown separately to keep things clear. The phasor diagram $b$ is drawn according to the rules in Chapter 5 , beginning with the current $A B$. It shows the condition of resonance, in which $X_{L}$ and $X_{C}$, and therefore the voltages across $L$ and $C$, are equal and opposite. So the whole of $E(=a b)$ is available for driving current through $r . A B=a b / r$, in fact. If $f_{r}$ is the frequency at resonance, $X_{L}\left(=-X_{C}\right)=2 \pi f_{r} L$.

Next suppose that the frequency of $E$ increases from $f_{r}$ to $f_{r}+f^{\prime}$. $X_{L}$ increases in direct proportion, to $2 \pi\left(f_{r}+f^{\prime}\right) L$; so the increase is $2 \pi f^{\prime} L$. Provided that $f^{\prime}$ is small compared with $f_{r}, X_{C}$ decreases by practically the same amount. So the total reactance, $X_{L}+X_{C}$, has


Fig. 20.6-(a) A tuned circuit, with its phasor diagrams (b) at resonance and (c) slightly off resonance
increased from zero to $4 \pi f^{\prime} L$. Fig. $20.6 c$ shows the corresponding phasor diagram. Because we have to start with the current before we know what it is going to be, we draw AB the same as at $b$. As however it is $E$ that we are assuming to be constant, the current must actually decrease in the same ratio as the total circuit impedance is increased by altering the frequency. At $f_{\mathrm{r}}$ this impedance is $r$, and at $f_{\mathrm{r}}+f^{\prime}$ it is $v^{\prime}\left[r^{2}+\left(4 \pi f^{\prime} L\right)^{2}\right]$ (Sec. 6.8).

The output, or response, of the tuned circuit is the voltage
developed across C by the current. (The whole loss resistance $r$ is assumed to be in the inductor.) Now $X_{C}$ is not quite the same at $f_{r} \pm f^{\prime}$ as at $f_{\mathrm{r}}$, as the phasor diagrams show; but within our assumption that $f^{\prime} \ll f_{\mathrm{r}}$ the difference can be neglected Our selectivity number, $S$, is the ratio of response at $f_{\mathrm{r}}$ to response at $f_{\mathrm{r}} \pm f^{\prime}$, and we now see that this is equal to the current ratio, which is the reciprocal of the impedance ratio. That is to say,

$$
S=\sqrt{\left[r^{2}+\left(4 \pi f^{\prime} L\right)^{2}\right] / r=\sqrt{ }\left[1+\left(\frac{4 \pi f^{\prime} L}{r}\right)^{2}\right]}
$$

The only variable part of this is $f^{\prime} L / r$, so if we know or assume the $L / r$ ratio for a tuned circuit we can find $S$ for any values of $f$ we like and plot a selectivity graph. Whether we will care to undertake this work after looking at the formula above and realising that the result of the effort holds good for only one value of $L / r$ is more doubtful. If however $S$ is plotted against $f^{\prime} L / r$ a curve is obtained which can be adapted for any $L / r$ ratio merely by dividing the horizontal scale numbers ty it. Fig. 20.7 is that curve. It shows how much the response of any one tuned circuit falls off as the frequency is altered from resonance. Using a logarithmic scale for $f^{\prime} L / r$ as well as for $S$ has the advantage that the greater part of the curve is a straight line with a $1:$ I slope, or 6 dB per octave as it is usually specified. This curve is in fact the same as those in Figs. 18.9 and 19.20 and like them has a 'turning frequency', $f_{1}$, at which the straight slope continued would meet the zero-dB line. $f_{t}$ is the $f$ that makes the reactance ( $4 \pi f^{\prime} L$ ) equal to the resistance $(r)$, so occurs where $f^{\prime} L / r=1 / 4 \pi=$ 0.08 . (It is 80 in Fig. 20.7 because of the units used, listed in the caption.) As before, the loss at $f_{\mathrm{t}}$ is 3 dB . What is called the bandwidth $\left(f_{B}\right)$ is the frequency band between the -3 dB points; that is to say $2 f_{1}$. From the foregoing, $f_{B}$ is $r / 2 \pi L$. And because $Q=$ $2 \pi f_{r} L / r, f_{B}$ is also equal to $f_{r} / Q$, which is a very useful result. We also find that $Q f^{\prime} / 2 \pi f_{\mathrm{r}}$ is the same as $f^{\prime} L / r$ in Fig. 20.7.

There is no need to show the other half of the selectivity curve, because it is practically the same so long as $f^{\prime} \ll f_{\mathrm{r}}$. If it is not, then a lack of symmetry does appear at frequencies far from resonance, but within our assumption we can regard $f^{\prime}$ as either + or - .

As an example of using Fig. 20.7, suppose $L$ is $150 \mu \mathrm{H}$ and $r$ is $24 \Omega$, so that $L / r=6 \cdot 25$. 200 on the $f^{\prime} L / r$ scale then means 32 kHz , at which $S$ is $2 \cdot 7$, or $-8 \frac{1}{2} \mathrm{~dB}$. Or suppose we wanted to find the resistance of a coil to give a certain selectivity, say -20 dB at 40 kHz off-tune. The curve shows that for this $f^{\prime} L / r$ is about 800 , so, at $f^{\prime}=$ $40, L / r$ is 20 . If $L$ is $5000 \mu \mathrm{H}$ then $r$ should be $250 \Omega$. The loss of 7.5 kHz side frequency due is such a coil is found by looking up $S$ for $f L / r=7.5 \times 20=150$, namely about 7 dB .

So far from a tuned circuit offering any choice of shape to combine good selectivity with retention of sidebands, what we have found is that there is only one shape, and the most we can do is
slide it horizontally along the $f^{\prime}$ scale until the best compromise is obtained; $L / r$ is then indicated. Once we have decided on the response at any one off-tune frequency, the response at all others is thereby decided too, whether we like it or not.

### 20.7 More Than One Tuned Circuit

Let us now consider what happens when additional tuned circuits are used. If we can assume that these circuits do not react on one another, then it is simple. If one circuit reduces signals at a certain off-tune frequency 3 times (relative to the response at resonance), and another circuit is somewhat more lightly damped and reduces them 4 times, then the total relative reduction is 12 times. If a third circuit gives a further reduction of 3 times, the total is 36 . In other words, an r.f. amplifier incorporating these three tuned circuits amplifies this certain frequency 36 times less than it amplifies the frequency to which all three circuits are tuned. In decibels, the three circuits cause reductions of $9 \cdot 5,12$ and 9.5 dB ; the total is found by simple addition: 31 dB .

If the problem is simplified by assuming all the circuits have the same $L / r$, then the curve in Fig. 20.7 is made to apply to a pair of circuits by squaring $S$; to three circuits by cubing it; and so on. Or by multiplying the dB scale by 2,3 , etc.

Comparison between one and several tuned circuits can be made in different ways. First of all we shall compare them on the basis of equal gain, in order to dispose finally of the question of positive feedback versus r.f. amplification. Taking some typical figures, suppose that the r.f. amplifier is a single stage giving a gain of 50 times, with input and output tuned circuits for which $L / r=10$. Squaring the $S$ values in Fig. 20.7, we get curve $a$ in Fig. 20.8. To obtain equal gain by means of feedback, it would be necessary to apply enough of it to multiply $Q$ by 50 . This would multiply $L / r$ by the same factor. So curve $b$ is drawn for a single tuned circuit with $L / r=500$. We see that the modulation frequency corresponding to a 5 kHz note is reduced 3 dB by the r.f. amplifier; that is to say, it retains about $70 \%$ of its relative strength. But the receiver with feedback reduces it by 30 dB , leaving only about $3 \%$. True, it gives excellent selectivity; but it would take all the life out of music and make speech difficult to follow. At 9 kHz off tune the r.f. amplifier is very inselective. But, significantly, at the more remote frequencies its selectivity is beginning to overtake its rival.

The excessive sharpness of $b$ in Fig. 20.8 was a result of having to make the tuned circuit provide as much gain as a stage of amplification. But now let us disregard gain and compare one circuit with several on a basis of equal selectivity. Suppose the intention is to reduce signal voltages at two channels off-tune ( 18 kHz ) to onehundredth ( -40 dB ) relative to the on-tune voltage. Fig. 20.7 shows


Fig. 20.7-Generalized resonance curve for any single tuned circuit. To make it refer to any particular circuit, divide the numbers on the horizontal scale by $L / r$ (or by $Q / 2 \pi f_{r}$ ) to convert them to a scale of $f^{\prime}$. The units are: $L$ in $\mu H ; r$ in $\Omega ; f^{\prime}$ in $k H z ; f_{\mathrm{r}}$ in MHz
that the required $L / r$ for a single circuit would be $8000 / 18=445$; nearly as sharp as $b$ in Fig. 20.8. With two circuits, $S$ for each would be -20 dB , so $L / r=800 / 18=44 \cdot 5$. With four circuits, $S$ would be -10 dB , and $L / r=13$. With six, $S$ would be $-6 \frac{2}{3} \mathrm{~dB}$, and $L / r=8.4$. The overall response at other frequencies can be derived from Fig. 20.7 by taking $S^{2}$ or twice the number of dB for two circuits, and so on. Plotting the results for one and six circuits on one graph, we get Fig. 20.9, which shows very clearly that a single tuned circuit, whether its extreme sharpness is necessitated by the requirements of gain or of selectivity, can fulfil these requirements only at the cost of drastic cutting of the wanted sidebands, and that a large number of relatively flat circuits is much to be preferred. Extensions of the curves show, too, that the selectivity at frequencies beyond the chosen reference point of 18 kHz goes on increasing much more rapidly with many circuits than with few.

Lastly, we can compare one circuit with six on a basis of equal retention of sidebands. Fig. 20.9 shows that with six circuits the loss of 5 kHz is just over 6 dB . The effect of flattening the single tuned circuit until its loss at this point is the same is represented by sliding curve $a$ along until it coincides with $b$ there (position $c$ ). This shows how poor the single-circuit selectivity would then be, in spite of the appreciable sideband cut.

The way things are going, it looks as if six or even more tuned circuits would hardly give a fair approximation to the ideal with which we started Sec. 20.6. And here it is necessary to remember that the way in which we have been deriving the overall results of several circuits depends on their not reacting on one another, for that would modify their response curves. The usual way of using such circuits is as couplings between one stage and the next in an r.f. amplifier. Here it is difficult to avoid enough coupling via the amplifying devices to cause appreciable interaction. Anyway, the use of anything like six r.f. stages, even if technically feasible, would provide far too much gain, so most of it would have to be thrown away. But is inter-circuit coupling necessarily bad?

Introducing coupling between tuned circuits makes it more difficult to predict their combined resonance curve. But by a rather more mathematical investigation than is appropriate in a book of this kind it can be shown that beneficial results are obtainable in this way, as we shall now see.

### 20.8 Coupled Tuned Circuits

There are many ways of coupling two tuned circuits: Fig. 20.10 shows some of them. The feature common to all is a certain amount of shared (or mutual) reactance. In type $a$ it is capacitance, $C_{m}$; in $b$, self-inductance, $L_{\mathrm{m}}$; and in $c$, mutual inductance, $M$. Fortunately for calculation, the result does not depend (at any one


Fig. 20.8-Comparison on an equal-gain basis of a stage of amplification with two tuned circuits (curve a) with a single tuned circuit given a high magnification by positive feedback (b)


Fig. 20.9-Response curves of one (a) and six (b) tuned circuits on a basis of equal selectivity at 18 kHz , showing how the six preserve sideband frequencies much better. Curve c refers to a single tuned circuit with its resonance flattened to give equal response at 5 kHz off-sune
frequency) on the kind of mutual reactance but only on its amount; so we can denote it simply by $x$. As a matter of fact, method $c$ is nearly always used, because no extra component is needed and the coupling can be adjusted simply by varying the spacing of the two coils. Just as $L / r$ is the factor determining the selectivity of a single tuned circuit, $x / r$ determines the modifying effect of the coupling on the combined resonance curves of two similar circuits.

This result can be shown most clearly in the form of the overall resonance curve for two circuits, (a) with no coupling, except through a one-way electronic device, and $(b)$ with reactance coupling. The question at once arises: with how much? The shape of the curve is different for every value of $x$. Since the primary purpose of the coupling is to transfer the signal from one circuit to the other, a


Fig. 20.10-Three methods of coupling two tuned circuits: (a) by capacitance common to both, $C_{M}$; (b) by common inductance, $L_{M}$; and (c) by mutual inductance, $M$
natural coupling to try for a start is the amount that transfers it most effectively. This is not, as might at first be supposed, the greatest possible coupling. It is actually (with low-loss circuits) quite small, being simply $x=r$.

For this value, called the critical coupling, the curve is Fig. 20.11b. The curve applies to a pair in which the two circuits have the same $L / r$; if there is any difference, an approximation can be made by taking the mean $L / r$. Curve $a$, for uncoupled circuits, is the same as Fig. 20.7 but with all the $S$ figures squared. Although the difference in shape is not enormous, it is quite clear that the curve for the coupled circuits is considerably flatter-topped. But it is also less selective. The advantage gained by the coupling is more obvious if the resistance of the coupled circuits is reduced to half that of the uncoupled circuits, as in Fig. 20.11c. If, for example, the $L / r$ of each of the uncoupled circuits represented by $a$ were 10 , the $L / r$ for each of the coupled circuits represented by $c$ would be 20 , and the off-tune or sideband frequency treated equally by both combinations (represented by the intersection of $a$ and $c$ ) would be 6 kHz . Although the coupled circuits would thus retain sideband frequencies up to 6 kHz better than the uncoupled circuits they would also give substantially more selectivity.

### 20.9 Effects of Varying Coupling

Even so, an adjacent-channel selectivity of less than 8 is hardly an adequate projection against interference equal in strength to the wanted signal, still less against relatively strong interference. We may reasonably ask whether any other degree of coupling than critical would give a still better shape; for, if so, any loss in signal transference caused thereby would be well worth while, as it could easily be made up by a little extra amplification. Fig. 20.12 shows what happens when the coupling between two tuned circuits is


Fig. 20.11-Comparison betwcen the overall resonance curve for a pair of similar tumed circuits. (a) mincoupted, and (b) "critically" coupled. Curve $c$ is the' same as $b$ but shifted to the left to represem ahalving of resistance
varied. The curves have been ploted to a linear frequency scale both sides of resonance to show the kind of shapes seen on an oscilloscope screen (Sec. 23.4).

Reducing the coupling below critical, by making $x$ less than $r$, not only reduces the signal voltage across the second circuit but also
sharpens the peak, so is wholly undesirable. Increasing the coupling splits the peak into two, each giving the same voltage as with the critical coupling at resonance; and the greater the coupling the farther they are apart in frequency. Here at last we have a means by which the selectivity can be increased by reducing the damping of the circuits, while maintaining sideband response at a


Fig. 20.12-Changes in resonance curve of a pair of coupled circuits. caused by varying the coupling: curve $a$ is for critical coupling; $b$ is for less than critical; and c for more
maximum. Coupled pairs of circuits are one type of what are known, for obvious reasons, as band pass filters. There is a limit, however, because if one goes too far with the selectivity one gets a result something like Fig. 20.13, in which a particular sideband frequency is accentuated excessively. Symmetrical twin peaks are in any case difficult to achieve in practice and still more difficult to retain over a long period, because the coupling adjustments are so extremely critical. Moreover, they make the set difficult to tune by ear because one tends to tune it to the frequency that gives the loudest sound, which in this case is not the carrier frequency.

Perhaps the best compromise is to have one pair of circuits critically coupled, and sufficiently selective to cause appreciable
sideband cutting; and the other pair slightly over-coupled, to give sufficient peak-splitting to compensate for the loss due to the first pair. The ultimate cut-off will then be very sharp and give excellent selectivity.

Mention should also be made of yet another technique for combining a flat top with sharp cut-off. It is called stagger tuning and has certain practical advantages, particularly for v.h.f. A number of single or double circuits are peak-tuned to slightly different frequencies in the pass band, as shown in Fig. 20.14.

### 20.10 Practical Tuning Difficulties

It is easy enough to talk about choosing $L / r$ ratios and adjusting couplings to give the best possible response curves. It is not usually very hard to achieve something like the desired results at one frequency. But the demand for receivers permanently tuned to one

Fig. 20.13-The result of trying to maintain sidebands when using excessively selective coils in a bandpass filter-the 'rabbit's ears' produced by over-coupling


Fig. 20.14-'Stagger-tuning', in which the separate tuned circuits are set to 'peak' at slightly different frequencies las shown at a), so that the combined result b has a fairly fat top and sharp cut-off

channel is not large. It is when one attempts to vary the tuning over wide bands of frequency that the troubles really begin. Means must be provided for varying the tuning capacitance or inductance of every circuit continuously and exactly together by one control.

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Because of the limited frequency band that can be covered in one sweep (Sec. 8.7) switching of inductors is nearly always needed to cover several bands. It would be extremely difficult to maintain all the couplings at exactly the right adjustments and practically impossible to keep $L / r$ always right. When, as is usual, a coil is tuned by a variable capacitor, $L$ of course remains the same over the whole band, but $r$ is much less at the lowest frequency (maximum capacitance) than at the highest. So the selectivity varies accordingly. Even if some way out of this varying selectivity trouble were found, it would only be to run into another. For if $L / r$ is kept constant over a band of frequency, the dynamic resistance $R$ which as we know is equal to $L / C r$ (Sec. 8.12)-must be inversely proportional to the tuning capacitance, and therefore directly proportional to the square of the frequency. This means very much less amplification at the low frequency end of the band than at the high.

Tuning by varying $L$ instead of $C$ could theoretically avoid both these difficulties at once, if $r$ were made to vary in proportion, for then $L / r$ and $L / C r$ would both be constant. But this is easier said than done. And we have not begun to think about keeping all this lot stable at all frequencies. So it is not surprising that attempts to produce a really practical high-performance receiver on the lines of variable-frequency r.f. amplification have failed. The generallyaccepted solution is the subject of the next chapter.

## CHAPTER 21

## The Superheterodyne Receiver

### 21.1 A Difficult Problem Solved

Fortunately it is possible to combine the advantages of fixedtuned selectivity, using a sufficient number of tuning circuits carefully adjusted once and for all in the factory, with the ability to receive over wide ranges of frequency. It is done in the great majority of present-day receivers, under the name of supersonic heterodyneusually abbreviated to superheterodyne or just 'superhet'. A device called a frequency-changer shifts the carrier wave of any desired station, together with its sidebands, to whatever fixed frequency has been chosen for the selective circuits. This frequency is called the intermediate frequency (i.f.), and the following choices are typical:

| For receiving- | I.F. |
| :--- | :---: |
| Sound broadcasts (a.m.) on low and medium frequencies | 470 kHz |
| Sound broadcasts (f.m.) on very high frequencies | $10-7 \mathrm{M} \mathrm{Hz}$ |
| Television broadcasts $\left\{\begin{array}{l}\text { vision } \\ \text { sound }\end{array}\right\}$ on v.h.f. and u.h.f. | $\left\{\begin{array}{l}33.5 \mathrm{M} \mathrm{Hz} \\ 39.5 \mathrm{M} \mathrm{Hz}\end{array}\right.$ |

The frequencies of all incoming signals are shifted by the frequency changer to the same extent. So if, for example, the original carrier-wave frequency of the wanted programme was 865 kHz and it had been shifted to 470 kHz , adjacent-channel carrier waves at 856 and 874 kHz would be passed to the i.f. amplifier as 461 and 479 kHz .

After the desired signal has been amplified at its new frequency and at the same time separated from those on neighbouring frequencies. it is passed into the detector in the usual way, and the remainder of the receiver is in no way different from a 'straight' set.

### 21.2 The Frequency-Changer

The most important thing to grasp in this chapter is the principle on which the frequency-changer works. All else is mere detail.

Suppose a sample of the carrier wave coming from the desired station during one hundred-thousandth of a second to be represented by Fig. 21.1a. As it contains 10 cycles, the frequency must be 1000 kHz . Similarly $b$ represents a sample of a continuous wave-
train having a frequency of 1450 kHz , which is being generated by a small oscillator in the receiver. Now add the two together. Since the local oscillation alternately falls into and out of step with the incoming signal, it alternately reinforces and weakens it, with the result shown at $c$ : waves varying in amplitude at a frequency of


Fig. 21.1-Showing the action of a frequency-changer

450 kHz . This part of the process is often referred to as $b$ beating or heterodyning with $a$, and 450 kHz in this case would be called the beat frequency. Being above audibility, it is described as supersonic. The same kind of thing takes place audibly when two musical notes of nearly the same pitch are sounded together.

Notice particularly that what is happening at 450 Hz is only amplitude variation (or modulation) of waves of higher frequency. No signal of 450 kHz is present, for each rise above the centre line is neutralized by an equal and opposite fall below the line. (Fig. $20.2 d$ is another example of the same thing, except that it is made up of waves of three frequencies, all higher than the frequency at which its amplitude is varying.) The average result, when applied to a circuit tuned to 450 kHz , is practically nil. We have been up against this kind of thing already (see Fig. 18.1) and the solution is the same now as then-rectify it. After eliminating the negative halfcycles (d) the smoothed-out or averaged result of all the positive half-cycles (shown dotted) is a signal at 450 kHz ; that is to say, the difference between the frequencies of $a$ and $b$. The mixture of 450 kHz and the original frequencies is then passed to the i.f.
circuits, which reject all except the 450 kHz , shown now alone at $e$.
So far, the result is only a 450 kHz carrier wave of constant amplitude. corresponding to the 1000 kHz carrier wave received from the aerial. But if this 1000 kHz wave has sidebands, due to modulation at the sender, these sidebands when added to the 1450 kHz oscillation and rectified give rise to frequencies which are sidebands to 450 kHz . So far as the i.f. amplifier and the rest of the receiver are concerned, therefore, the position is the same as if they were a "straight' set receiving a programme on 450 kHz .

Any readers who, in spite of Fig. 20.3, have difficulty in seeing amplitude modulation as sidebands, may find Fig. 21.2 helpful. This shows a rather longer sample of the carrier wave $(a)$, the individual cycles being so numerous that they are hardly distinguishable. And this time it is fully modulated; the sample is sufficient to show two whole cycles of modulation frequency. The local oscillation. which is certain to be much stronger. and of course is unmodulated, is shown at $b$. When $a$ is added to it (c). nothing happens to it at the troughs of modulation. where $a$ is zero. but elsewhere the cycles of $a$ alternately strengthen and weaken those of $b$, causing its amplitude to vary at the beat frequency, in the manner shown. The amplitude of beating is proportional to the amplitude of $a$, and the frequency of beating is equal to the difference between the frequencies of $a$ and $b$. The next picture, $d$. shows the result of rectifying $c$. We now have, in addition to other frequencies, an actual signal at the beat frequency (or i.f.), shown at $e$ after it has been separated from the rest by the i.f. tuned circuits. The result of rectifying $e$ in the detector is $f$; and after the remains of the i.f. have been removed by the detector filter the final result $(g)$ has the same frequency and waveform as that used to modulate the original carrier wave.

Fig. 21.3 shows an elementary way in which the process just described can be carried out. The incoming signal and the local oscillation are added together by connecting their sources in serles with one another, and the two together are fed into a rectifier-some kind of diode. $C_{1}$ is used for tuning $L_{2}$ to the signal, so as to develop the greatest possible voltage at that frequency. There is, of course, no difficulty in getting an ample voltage from the local oscillator. $\mathrm{C}_{2}$ serves the double purpose of tuning the transformer primary ( $\mathrm{L}_{5}$ ) to the intermediate frequency, and of providing a short-circuit for the two input frequencies, which are not wanted there.

### 21.3 Frequency-Changers as Modulators

If the rectifier in Fig. 21.3 had been omitted, or replaced by an ordinary linear resistor, no new signal frequency would have been created. In linear circuits, currents of different frequencies can flow sinultaneously without affecting or being affected by the

## INCOMING MOOULATED CARRIER WAVE

(a)


LOCAL OSCILLATION
(b)

(c)

(c) RECTIFIED
(d)


## BEAT FREQUENCY (IF) EXTRACTED FROM (d)

(e)
(e) RECTIFIED IN DETECTOR
(f)

MODUL ATION FREQUENCY EXTRACTED FROM (f)
(g)


Fig. 21.2-In this diagram the sequence of Fig. 21.1 is elaborated to show what happens when the incoming carrier wave is modulated. The additional lines, $(f)$ and $(g)$, show the extraction of the modulation frequency


Fig. 21.3-Simple type of frequency-changer, in which the incoming signal and the local oscillation are added together by connecting their sources in series, and then rectified
others. In non-linear circuits that is not so. A rectifier is an extreme kind of non-linear resistance. Fig. 21.4 shows the current/voltage characteristic curve of a milder kind. In diagram a a single small alternating voltage - frequency, say $f_{1}$-is applied, and the resulting current waveform is derived and shown to the right of the graph. Such a small portion of the characteristic curve is not very different


Fig. 21.4-Showing the effect of adding a strong signal to a weak signat through a non-linear device stach as a transistor. The amplitude of the weak signal is varied (or modulated by the strong
from a straight line, so the distortion is only slight. In any case each of the output cycles is the same as all the others. Diagram $b$ shows the result of applying the same voltage plus another, which has been chosen to have a much larger amplitude and lower frequency $\left(f_{2}\right)$ to make it easily distinguishable. Its positive half-cycles carry the $f_{1}$ signal up the curve to where the resistance is less, so the $f_{1}$ current cycles ate larger than in $a$. The $f_{2}$ negative half-cycles carry the $f_{1}$ signal to where the resistance is greater, so the $f_{1}$ current cycles are smaller than in $a$. The $f_{2}$ signal has left its mark on the $f_{1}$ signal, by amplitude-modulating it. The depth of modulation obviously depends on the degree of non-linearity of the circuit.

We already know that the process of amplitude modulation creates signals having frequencies different from both the modulating and modulated frequencies; to be precise, these frequencies are equal to the sum and difference of $f_{1}$ and $f_{2}$-see Fig. 20.2 again. These are not necessarily the only new frequencies created-that depends on the kind of non-linearity but we shall confine our attention to them.

In Fig. 21.1 the difference frequency is clearly visible, especially at $e$, after it has been extracted from the mixture. The fact that in $d$ there is a frequency equal to that of $b$ plus that of $a$ is not at all obvious, but its existence can be shown mathematically, or by act ual
experiment—picking it out from the rest by tuning a sharply resonant circuit to it.

The purpose of the local oscillator can now be seen as a means of varying the conductance of a part of a circuit carrying the incoming signal, amplitude-modulating it and thereby creating signals of at least two new frequencies.

Either of these could be chosen as the i.f. in a superhet, but the lower or difference frequency is the one usually favoured, because it is easier to get the desired amplification and selectivity at a relatively low frequency.

### 21.4 Types of Frequency-Changer

A frequency-changer therefore consists of two parts: an oscillator, and a device whereby the oscillation modulates or varies the amplitude of the incoming signal. This device is commonly called the mixer. If the word 'mix' is understood in the sense a chemist (or even an audio engineer) would give it-just adding the two things together-then it is a very misleading word to use. Simply adding voltages or currents of different frequencies together creates no new frequencies, just as merely mixing oxygen and hydrogen together creates no new chemical substance. So from this point of view 'combiner' would be a better name for the second part of a frequencychanger. It can also be fairly termed the modulator.

The oscillator presents no great difficulty; almost any of the circuits shown in Chapter 14 could be used. It is, of course, necessary to provide for varying the frequency over a suitable range, in order to make the difference between it and the frequency of the wanted station always equal to the fixed i.f.

The simplest type of modulator or mixer is the plain non-linear element-a diode-already seen in Fig. 21.3. Frequency-changers essentially of this type are used in radar and other receivers working on frequencies higher than about 800 MHz , for reasons which will appear shortly. But they have certain disadvantages which have resulted in their having been superseded for most other purposes. For one thing, adjustment of $\mathrm{C}_{1}$ is liable to affect the frequency of the oscillator. The local oscillations on their part are liable to find their way via $L_{2}$ and $L_{1}$ to the aerial, causing interference with other receivers. In short, the signal and oscillator circuits interact, elsewhere than in the proper direction-through the rectifier. And the rectifier does not amplify; on the contrary, it and its output circuit tend to damp the signal presented to it by the tuned circuit $\mathrm{L}_{2} \mathrm{C}_{1}$. The principle of a commonly used type of frequency changer is shown in Fig. 21.5. A transistor triode is used both as oscillator and modulator. $L_{1}$ and $C_{2}$ tune the aerial and input circuit. If a wire aerial is used, $\mathrm{C}_{1}$ limits the amount of extra capacitance the aerial can add to $C_{2}$. More often $L_{1}$ is itself the aerial, wound on a ferrite rod (Sec. 17.19). To suit the relatively low input impedance of the
transistor, $\mathrm{L}_{1}$ and $\mathrm{L}_{2}$ act as a step-down transformer. $\mathrm{L}_{5} \mathrm{C}_{3}$ is the oscillator tuning circuit, coupled by $L_{3}$ and $L_{4}$ to emitter and collector in such phase as to maintain oscillation. This has sufficient amplitude to swing the transistor alternately between cut-off and the steep part of its output/input curve, fully modulating the


Fig. 21.5-Typical transistor frequency changer circuit for medium and low r.f.
relatively weak signal across $L_{2}$ and thereby producing sum and difference frequencies in the collector circuit. A fraction of a volt is enough for a transistor, but the more gradual control of the grid in a valve (and to some extent the gate in a f.e.t.) calls for several volts. When $C_{3}$ is adjusted to make the difference frequency equal to the chosen i.f., the i.f. part of the mixture of signals is selected by $\mathrm{L}_{6} \mathrm{C}_{4}$ and passed on from there to the i.f. amplifier.

This is a simple and economical type of frequency changer, but combining the two functions in one device makes it difficult to ensure the best operation for both at all times, and especially to avoid the circuit interactions already mentioned. So where the best performance is desired separate devices are preferred. This is especially so with valves, even though the two valves concerned are enclosed in the same 'bottle', but electrically screened from one another. One example is the triode-pentode, the triode being used as the oscillator and the pentode as modulator.

So far the modulation has been effected by adding the signal and oscillation together and passing both through the non-linear device. This is therefore called the additive principle. If however the device that amplifies the signal has another electrode which directly controls its gain the signal is thereby modulated without being swung between extremes of the transfer characteristic. The amplitude of the signal is, in effect, being multiplied by the alternating voltage of the oscillation, so this is called a multiplicative type of frequency changer. Fig. 21.6 shows one of several types of multiplicative

frequency changer valve, which in this case (though not in all) comprises two valves. The triode part again is used as the oscillator, but its grid is connected internally to an electrode between the signal grid and anode-and carefully screened from both. The alternating grid voltages modulate the electron stream passing through the hexode and thereby the signal output from the anode is modulated and an i.f. component introduced.

### 21.5 Conversion Conductance

The eflectiveness of an ordinary amplifying or oscillating type of valve or transistor is, or can be. expressed as the mutual conductance, in milliamps of signal current in the output circuit per voli of signal applied at the input. In a frequency-changer, however, the current at signal frequency is a waste product, to be rejected as soon as possible: what we are interested in is the milliamps of i.f. current per volt of signal-frequency. This is known as the conversion conductance. denoted by $g_{\mathrm{c}}$. Obviously it does not depend on the device alone but also on the oscillation voltage. If this is too small. it does not vary the mutual conductance over anything like its full range; consequently the signal current is modulated to only a fraction of its full amplitude, and the i.f. current is less than it need be. The conversion conductance reaches its maximum when the local oscillation is enough to bring the mutual conductance to zero during its negative half-cycles and to maximum during as much as possible of its positive half-cycles. The positive part of Fig. $21.2 c$ is therefore reproduced with the full $g_{m}$, while the negative part is completely suppressed : the net result is the same as with a perfect rectifier. Fig. 21.2d. The incoming signal, however weak, is swept right round the bend by the oscillation. That explains why a superhet is sensitive to the feeblest signals, which, if applied straight to the detector, would make no perceptible impression.

In Fig. 21.7a, a small sample of oscillation plus signal (such as Fig. $21.2 c$ or $21.1 c$ ) is shown in relationship to the $V_{g}$ scale of an $I_{\mathrm{a}} / V_{g}$ valve curve. (The fact that the oscillation and signal may actually be applied to different grids does not affect the essential
principle.) Since the i.f. fluctuations are due to the signal alternately strengthening and weakening the local oscillation, their amplitude is, as we have seen, equal to that of the signal. Neglecting the effect of the load in the anode circuit, as we generally can with these types of valve, corresponding fluctuations in anodecurrent half-cycles are practically equal to those that would be


Fig. 21.7-Diagram showing how conversion conductance compares with mutual conductance
caused by an i.f. voltage of the same amplitude as the signal voltage amplified straightforwardly, as indicated at $b$. If the i.f. current amplitude in $a$ were the same thing as the amplitude of the i.f. envelope, this would mean that the conversion conductance was practically equal to the mutual inductance. But, as is shown more clearly in Fig. 21.1d, the fluctuation in mean value of the half-cycles is $1 / \pi$ or only about $32 \%$ of the peaks (Sec. 18.3). So $g_{c}$ is only about one-third of $g_{\mathrm{m}}$ under ideal conditions, and in practice is between one-third and one-quarter.

In an r.f. tetrode or pentode the valve resistance is so much higher than any normal load resistance $(R)$ that the voltage gain is approximately equal to $g_{\mathrm{m}} R$. The corresponding quantity in a frequencychanger, the conversion gain, is equal to $g_{c} R$ and is the ratio of output i.f. voltage to input r.f. voltage. Allowing for the usually somewhat lower output impedance of transistors, the same is broadly true of them. Even though a frequency changer gives only about a quarter of the gain of a comparable r.f. amplifier, it makes a very welcome contribution to the gain of a receiver, in addition to its main function.

Gain is not everything, however. At the input to any amplifier it is noise that decides how much gain can usefully be sought (Sec. 19.20). A frequency changer is inherently more noisy than a comparable straight amplifier. In a multi-electrode valve the extra electrodes introduce partition noise. In transistors, noise depends
mainly on the construction and they range from very bad to very good. If a high sensitivity is required, combined with a good signal/noise ratio, there should be at least one low-noise stage of amplification at the original signal frequencies. The part of the receiver that deals with those frequencies is known as the preselector.

### 21.6 Ganging the Oscillator

We have seen that the intermediate frequency is equal to the difference between the signal frequency and the oscillator frequency. With an i.f. of 470 kHz , for example, the oscillator must be tuned to a frequency either 470 kHz greater or 470 kHz less than the signal.

Suppose the set is to tune from 525 to 1605 kHz . Then, if higher in frequency, the oscillator must run from $(525+470)$ to $(1605+$ 470 ), i.e., from 995 to 2075 kHz . If, on the other hand, the oscillator is lower in frequency than the signal, it must run from (525-470) to ( $1605-470$ ) or 55 to 1135 kHz . The former range gives 2.09 as the ratio between highest and lowest frequency: the latter 20.6 . Since even the signal-circuit range of 3.06 is often quite difficult to achieve, owing to the irreducible capacitance likely to be present in a finished set, the oscillator range from 995 to 2075 kHz is the only practical choice.

The next problem is to control both signal-frequency circuit tuning and oscillator frequency with one knob. Because there must be a constant frequency difference between the two, it hardly seems that what is called a ganged variable capacitor-consisting, in effect, of two identical capacitors on the same spindle-would do. As we have just seen, the ratio of maximum to minimum capacitance is quite different. This difference, however, can easily be adjusted by putting a fixed capacitance either in parallel with the oscillator capacitor to increase the minimum capacitance, or in series to reduce the maximum. The ratio of maximum to minimum having been corrected in either of these ways, correct choice of inductance for the oscillator coil will ensure that it tunes to the right frequency at the two ends of the tuning-scale.

In the middle, however, it will be widely out, but in opposite directions in the two cases. It is found that a judicious combination of the two methods, using a small parallel capacitor to increase the minimum a little, and a large series capacitor to decrease the maximum a little, will produce almost perfect 'tracking' over the whole waveband.

Fig. 21.8 shows these tuning arrangements applied to one frequency band. $\mathrm{L}_{1} \mathrm{C}_{1} \mathrm{C}_{3}$ is a preselector tuning circuit : there is often at least one other, either at the input and output of a stage of amplification or loosely coupled to one another by reactance. Sometimes $L_{1}$ can be preset to the designed value by the screw adjustment of the magnetic core, which is what the symbol indicates. $\mathrm{C}_{1}$ is one
section of the gang tuning capacitor, and $\mathrm{C}_{2}$ is a preset trimmer to bring the total effective capacitance to the designed value when $C_{1}$ is at its minimum, since in this condition the frequency is critically dependent on stray circuit capacitance. $C_{2}$ is another section of the tuning capacitor, identical with $\mathrm{C}_{1}$, for varying the oscillator frequency; $\mathrm{C}_{4}$ is the oscillator trimmer, and $\mathrm{C}_{5}$ the series capacitance


Fig. 21.8 - Preselector and oscillator tuning circuils arranged to maintain a constant frequency difference
or padder. Sometimes (especially if $L_{2}$ is not adjustable) it can be preset by a small variable capacitor in parallel with a fixed one.

For each further frequency band additional inductors are fitted, the unused ones being short-circuited by the band switch to prevent them from acting as resonators.

Although the frequency band used for v.h.f. sound broadcasting is much wider than the medium and low frequency bands in terms of hertz, when reckoned as the ratio of maximum to minimumwhich is what counts in designing the tuning system-it is narrower. Television channels are not selected by continuous tuning as assumed so far, but by switching in the appropriate sets of inductors. The most that the user can control, apart from the channel switch, is a fine tuning adjustment which enables the oscillator frequency to be varied within narrow limits if the receiver drifts off the preset adjustments. Channel switching may be used instead of or in addition to continuous tuning on the sound broadcast bands, and the preset tuning done by either trimmers or inductor cores.

At the higher frequencies the tuning is sometimes controlled by variable-capacitance diodes, or varactors. As we saw in Sec. 9.11, the depletion layer between the two conducting regions in a junction diode is like the dielectric in a capacitor, but it has the useful property that its thickness-and hence the capacitance-depends on the voltage applied. This obviously provides scope for convenient and compact ganged tuning.

### 21.7 Whistles

A superheterodyne is liable to certain types of interference from which an ordinary set is free. The most noticeable result of these is a whistle, which changes in-pitch as the tuning control is rotated, as when using a 'straight' receiver in an oscillating condition, but
generally less severe. There are many possible causes, some of which are quite hard to trace.

The best-known is second-channel or image interference. In the preceding section we have seen that it is usual for the oscillator to be adjusted to a frequency higher than that of the incoming signal. But if, while reception is being obtained in this way, another signal comes in on a frequency higher than that of the oscillator by an equal amount, an intermediate frequency is produced by it as well. If the frequency difference is not exactly the same, but differs by perhaps 1 kHz , then two i.f. signals are produced, differing by 1 kHz , and the detector combines them to give a continuous note of 1 kHz .
An example will make this clear: Suppose the station desired works on a frequency of 200 kHz , and the i.f. is 470 kHz . When the receiver is tuned to 200 kHz the oscillator is 470 kHz higher. 670 kHz , and yields a difference signal of 470 kHz , to which the i.f. amplifier responds. So far all is well. But suppose now a signal of 1137 kHz is also able to reach the frequency changer. It will combine with the oscillation to produce a difference signal of 1137 $670,=467 \mathrm{kHz}$. This is too near 470 kHz for the i.f. amplifier to reject, so both are amplified together by it and are presented to the detector, which produces a difference frequency, 3 kHz , heard as a high-pitched whistle. Slightly altering the tuning control alters the pitch of the note, as shown in the table:

| Set <br> tuned <br> to: | Oscillator <br> at: | I.F. signal <br> due to <br> 200 kHz signal <br> (wanted) | I.F. signal <br> due to <br> 1,137 kHz signal <br> (interfering) | Difference <br> (pitch of <br> whistle) |
| :---: | :---: | :---: | :---: | :---: |
| kHz | kHz | kHz | kHz | kHz |
| 195 | 665 | 465 | 472 | 7 |
| 196 | 666 | 466 | 471 | 5 |
| 197 | 667 | 467 | 470 | 3 |
| 198 | 668 | 468 | 469 | 1 |
| $198 \frac{1}{2}$ | $668 \frac{1}{2}$ | $468 \frac{1}{2}$ | $468 \frac{1}{2}$ | 0 |
| 199 | 669 | 469 | 468 | 1 |
| 200 | 670 | 470 | 467 | 3 |
| 201 | 671 | 471 | 466 | 5 |
| 202 | 672 | 472 | 465 | 7 |
| 203 | 673 | 473 | 464 | 9 |

An example of how second-channel interference is produced
Since both stations give rise to carriers falling within the band to which the i.f. amplifier must be tuned to receive one of them, this part of the set can give no protection against interference of this sort. So the superhet principle does not allow one to do away entirely with selective circuits tuned to the original signal frequency.

Another form of interference, much more serious if present, but
fortunately easy to guard against, is that due to a station operating within the i.f. band itself. Clearly, if it is able to penetrate as far as the i.f. tuning circuits it is amplified by them and causes a whistle on every station received. Again, a good preselector looks after this; but the 525 kHz end of the medium frequency band may be dangerously close to a 470 kHz i.f. If so, a simple rejector circuit tuned to the i.f. and placed in series with the aerial will do the trick. It is known as an i.f. trap.

The foregoing interferences are due to unwanted stations. But it is possible for the wanted station to interfere with itself! When its carrier arrives at the detector it is, of course, always at intermediate frequency. The detector, being a distorter, inevitably gives rise to harmonics of this frequency (Sec. 19.5). If, therefore, these harmonics are picked up at the aerial end of the receiver, and the frequency of one of them happens to be nearly the same as that of the station being received, the two combine to produce a whistle. It is easy to locate such a defect by tuning the set to two or three times the i.f. The cure is to by-pass all supersonic frequencies that appear at the detector output, preventing them from straying into other parts of the wiring and thence to the preselector circuits or aerial.

Oscillator harmonics are bound to be present, too; and are a possible cause of whistles. Suppose again the receiver is tuned to 200 kHz and the i.f. is 470 . Then the oscillator frequency is 670 , and that of it: second harmonic is 1340 kHz . If now a powerful local station were to be operating within a few kHz of 870 , its carrier would combine with the harmonic to give a product beating at an audio frequency with the 470 kHz signal derived from the 200 kHz carrier. Such interference is likely to be perceptible only when the receiver combines poor preselector selectivity and excessive oscillator harmonics.

Excluding most of these varieties of interference is fairly easy so long as there are no overwhelmingly strong signals. But if one lives under the shadow of a sender it is liable to cause whistles in a number of ways. Besides those already mentioned, an unwanted carrier strong enough to force its way as far as the frequencychanger may usurp the function of the local oscillator and introduce signals of intermediate frequency by combining with other unwanted carriers. Any two stations transmitting on frequencies whose sum or difference is nearly equal to the i.f. may cause interference of this kind. Here again, preselector selectivity is the cure, but at sites exceptionally close to a source of interference a special trap circuit may have to be added to reduce the effects.

The foregoing list of possible causes of interference is by no means exhaustive, but it is only in exceptional situations that whistles are conspicuous in a receiver of good design.

## CHAPTER 22

## High-Frequency Amplification

### 22.1 Amplification So Far

We have examined methods of low-frequency amplification at some length in Chapter 19, mainly with reference to a.f. amplifiers. Broadly, they consist of amplifying devices-transistors or valvesconnected together by couplings which are sometimes called aperiodic because they are designed to have no natural period or resonance but to treat all frequencies alike, at least those within the audio band. Since then we have assumed the practicability of amplifiers that respond only to the very limited band of frequencies included in one sound channel or the much wider (but, in proportion to the carrier frequency, still small) band in a television channel. In these the inter-stage couplings consist of tuned circuits. For an i.f. amplifier these couplings are preset, whereas for amplification at the original signal frequency they have to be varied by the user if he wants to receive on more than one channel. Although 'r.f.' includes i.f., where both are mentioned in the same context it can be taken that it means the original incoming signal frequency.

Then there are the video frequencies (v.f.) which include the whole of the low frequencies referred to in Chapter 19 and a large range of r.f. as well, up to several megahertz, for which an aperiodic amplifier is needed, since they correspond in a television picture to a.f. in sound reproduction. As we have already studied aperiodic amplification for a.f. there might seem to be no need to take up any more time over v.f. But this chapter is necessitated not only by the nature of inter-stage couplings for high frequencies but by departures from low-frequency characteristics of the active devices themselves.

### 22.2 Valve Interelectrode Capacitances

The electrodes of valves and cathode-ray tubes, together with their connecting leads, act as tiny capacitors, with effects on the behaviour of the circuit as a whole. Taking a simple case, a triode, Fig. 22.1a shows the three interelectrode capacitances as if they were external components. (Note that the fact that they are actually internal is indicated by a small $c$, in conformity with the usual convention for properties of valves and transistors.) At $b$ they have been incorporated in the equivalent current generator circuit. In receiving valves and in c.r. tubes they are usually a few picofarads. To get some idea of what they mean in practice, let us calculate the reactance of 4 pF at
various frequencies (Sec. 6.3):

| Frequency | Reactance |
| :---: | :---: |
| 10 kHz | $4 \mathrm{M} \Omega$ |
| 100 kHz | $400 \mathrm{k} \Omega$ |
| 1 MHz | $40 \mathrm{k} \Omega$ |
| 10 MHz | $4 \mathrm{k} \Omega$ |
| 100 MHz | $400 \Omega$ |
| 1 GHz | 40 S 2 |

If a triode is used in the usual common-cathode mode. then $c_{\mathrm{gk}}$ comes in parallel with the input and $c_{a k}$ in parallel with the output. So it would seem that the effects of these capacitances would be negligible at a.f., and not very significant at low r.f., but at high r.fs they would increasingly short-circuit the valves and prevent them from working properly. This is certainly true of untuned (or aperiodic) circuits, but in tuned circuits $c_{\mathrm{gk}}$ and $c_{\mathrm{ak}}$ form parts of the

(0)

(b)

Fig. 22.1-(a) Triode valve showing capacitances benween its electrodes. (b) Corresponding eyiaivalent current generator circuit
input and output tuning circuits and are cancelled out by the tuning inductances. Their disadvantage is that they raise the minimum turing capacitance and therefore limit the frequency to which one can tune with a given inductance.
$c_{a g}$ is different, however. This is because neither of its 'plates' is earthed. In Chapter 12 (and especially Sec. 12.5) we saw how the value of a resistance as calculated from the voltage and current applied at one end can be quite different from its own real value if the other end is fed with a current or voltage directly proportional to that applied. We were then considering only resistances (or conductances) and only plus or minus, so that the only effects were increases or decreases in value. In this chapter, however, we have to
take into account reactances as well as resistances, and consequently all possible phase relationships as well as just $0^{\circ}$ and $180^{\circ}$.

### 22.3 Miller Effect

To take a simple case first, consider Fig. $22.2 a$, in which C is a capacitor hidden in our now well-worn black box, in series with a special sort of meter that measures the charge passing into the capacitor when the switch is closed. As capacitance is equal to the charge required to raise the p.d. across it by 1 V (Sec. 3.2) we have


Fig. 22.2-Showing how the introduction of an extra voltage, such as that fed back (b) from anode to grid in a valve, can be equivalent to a multiplied capacitance

here a means of measuring $C$. But suppose ( $b$ ) that when we connect our I $V$ to one end of C a battery giving $A$ volts is applied at the other end with polarity as shown. The charging voltage is $A+1$, so the charge is $A+1$ times as much as that due to 1 V only. Knowing nothing of the hidden battery, we would suppose the box contained a capacitance equal to $(A+1) C$, as at $c$. So far as external electrical tests can tell, the boxes $b$ and $c$ are identical.

This is exactly what happens in a common-cathode amplifier having a voltage amplification $A$, if the anode load is purely resistive at the frequency concerned-it could be the dynamic resistance of a tuned circuit. As $A$ might be 50 or more, a $c_{\text {as }}$ actually only 2 pF would have the same effect as one of 100 pF or more. This would be serious enough, but at a slightly different frequency or anode tuning the phase angle would shift nearly $90^{\circ}$ one way or the
other. The effect can be considered by analysing the feedback into two parts. one exactly negative ( $180^{\circ}$ ) and the other $90^{\circ}$ different. The $180^{\circ}$ component magnifies $c_{a s}$ as explained. In so far as the anode load was capacitive, however, the current fed back to the grid would be in such phase as to appear to input voltages as conductance, which would have a loading or damping effect. An inductive load, on the other hand, would feed back current in the opposite phase, giving the effect of negative conductance. If this was enough to offset the loss conductance of the input circuit, oscillation would result. We would in fact have a t.a.t.g. oscillator (Fig. 14.13).

We have also come across this phenomenon in Sec. 15.6 in connection with senders, and noted that the cure was to apply neutralization or to substitute a valve in which $c_{\text {ag }}$ had been almost eliminated by internal screening, supplemented in both cases by external screening where necessary. Now that we are considering receiver r.f. amplifiers, the same applies.

These feedback effects via $c_{\text {ag }}$ are named after their discoverer, J. M. Miller. With tuned amplifier couplings, Miller effect causes at least tuning difficulties and usually oscillation; now we see that if resistance coupling was used at high frequencies the inevitable stray capacitance across it would lead to a capacitive component of feedback current tending to increase the input conductance-which one tends to assume is practically zero with a valve. So Miller effects are, in general, bad And except with specially screened valves (Sec. 15.7) they are appreciable at quite low radio frequencies; sometimes even at high a.f.

There are two other causes of apparent input conductance which with normal design do not seriously impair the usefulness of valves as amplifiers at frequencies below about 50 MHz , but need to be considered at very high frequencies. One is inductance of the cathode lead (Fig. 22.3). This is common to both input and output

Fig. 22.3-Feedhack due 10 cathode-lead inductance and cathodegrid capaciance of a valve

circuits and produces the effect of input conductance by feedback. The other is the time taken by electrons to travel across the vacuum between the electrodes-transit time, as it is called. Besides the ordinary capacitive input current due to the input voltage across $c_{g k}$, an additional current is induced by the movements of electrons towards and away from the grid. Even though they never land on it they form a moving space charge which must be matched by a

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change of charge on the grid, constituting a capacitive current. But at very high frequencies the transit time is appreciable compared with the time of one cycle, so a phase lag develops in the response, which changes a capacitive current into a conductive one. This effect is of course lessened by making the interelectrode distances very small-say a fraction of a millimetre-and this has the further advantage of tending to increase the mutual conductance, but unfortunately it also increases interelectrode capacitances.

### 22.4 High-Frequency Effects in Transistors

The problem of transit time in transistors is worse in one respect: movement of current carriers between emitter and collector is by diffusion (Sec. 10.7) whereas in valves there is a substantial electric field, due to the anode voltage, to urge them on. On the other hand solid material can be made thinner than a vacuous space, especially when that space has a heated cathode on one side of it. The thickness of the base in transistors for h.f. amplification is of the order of 0.001 mm . This would lead to unacceptably high capacitance were it not that the low power involved in receiver r.f. amplifiers enables the area to be correspondingly minute. By feats of technology, types of transistors are manufactured that give useful gain not only at v.h.f. but u.h.f. Types designed for high-power a.f. output stages necessarily have relatively very large junction areas causing high capacitances, and thicker bases causing greater delay effects, so some falling-off in performance may be detectable at the top end even of the a.f. band.

The combined influence of internal capacitances and delayed response is very difficult to represent in an equivalent circuit that is both simple and accurate enough for practical purposes, even though 'accurate' is not a very demanding word where transistors are concerned. There is also the complication that depletion-layer capacitance depends on the applied voltage (Sec. 9.11). The $h$ parameter diagram (Chapter 12) can be used, but the values of the parameters are no longer simple numbers; they are 'complex' numbers, having both magnitude and phase angle. The algebra is the same in form, but the in-phase (positive or negative) and at-rightangles or 'quadrature' or reactive components have to be kept in separate accounts, as explained in mathematical books on a.c. And the complex $h$ parameters vary with frequency.

For this introduction, at least, a clearer picture of the behaviour of a transistor at high frequencies is presented by what is called the hybrid $\pi$ equivalent circuit, Fig. 22.4. A great advantage is that its parameters are roughly constant with frequency. Typical values are shown for an r.f. transistor under working conditions. Another advantage is that these parameters are fairly simply related to the complex $h$ values. Incidentally, with $r_{b b}$, and $r_{b, c}$ removed and some
of the values altered, the diagram would do for a valve. Over a small band of frequency such as the 470 kHz i.f. band the hybrid $\pi$ circuit of Fig. 22.4 can be replaced by a slightly simpler equivalent in which $r_{\text {bb }}$ is eliminated by allowing for it in the values of the other components. However, to see how the more comprehensive circuit works in practice let us draw phasor diagrams for it at two widely different frequencies.

### 22.5 Transistor Phasor Diagrams

Fig. 22.5 shows the Fig. 22.4 equivalent circuit fitted with a resistive load of $20 \mathrm{k} \Omega$ and labelled for phasors. Let us first draw diagrams for a frequency low enough for the capacitances to be neglected. As usual, the best starting place is the output, and to get realistic proportions let us calculate actual values, assuming an output ec of 1 V .


Fig. 22.4-Hybrid $\pi$ transistor equivalent circuit


Fig. 22.5-Hybrid $\pi$ circuit marked with typical values and phasor-diagram labels
Because the input voltage will be relatively small, we choose a scale that brings only $\frac{1}{400}$ th of ec into the diagram, Fig. 22.6. We calculate and plot $A F, F E$ and $D A$, which are 50,15 and $0.5 \mu \mathrm{~A}$ respectively. For this purpose $r_{b / e}$ is negligible compared with $r_{b \prime c}$. So the current generator has to supply $65 \cdot 5 \mu \mathrm{~A}$, and making use of
the generator equation given we find $e b^{\prime}$ to be $65 \cdot 5 / 40=1.64 \mathrm{mV}$. $D C$ is zero in this case, so $D B$ is $1 \cdot 64 / 1=1.64 \mu \mathrm{~A}$. That completes the current diagram and reveals $A B$ as $0.5+1 \cdot 64=2 \cdot 14 \mu \mathrm{~A}$, from which $b b^{\prime}$ follows as 0.214 mV , to complete the voltage diagram. The input voltage, $e b$, is $1.64+0.214=1.854 \mathrm{mV}$, so the voltage gain


Fig. 22.6-Low-frequency phasor diagram for Fig. 22.5
(ec/eb) is $1000 / 1 \cdot 854=540$, the current gain is $23 \cdot 4$, and the power gain 12600 , or 41 dB . Note that $r_{\mathrm{b}, \mathrm{c}}$ causes some reduction in gain by negative feedback ( $B D<B A$ ). The input resistance, $e b \mid A B$, is $866 \Omega$.

Let us now see what happens at 470 kHz . At this frequency the reactance of $c_{\mathrm{b}, \mathrm{e}}$ is $1.7 \mathrm{k} \Omega$. And $c_{\mathrm{b}, \mathrm{c}}$ is $68 \mathrm{k} \Omega$, which in parallel with $r_{b, c}$ is very nearly equivalent to the same capacitance in series with $2.3 \mathrm{k} \Omega$ of resistance. The total impedance (Sec. 6.8) is hardly more than $68 \mathrm{k} \Omega$. We begin as before with 1 V of ec, and assuming the same load we get the same $A F$ and $F E$ (Fig. 22.7). But $D A$ is very


Fig. $22.7-470 \mathrm{kHz}$ phasor diagram for Fig. 22.5
different both in magnitude and phase angle. The feedback impedance is still large enough for $r_{b}$ to be neglected in comparison with it, but is much less than before, so $D A$ is $1000 / 68=14.7 \mu \mathrm{~A}$; and by drawing the impedance triangle (or by calculation) we find that it leads ec by $88^{\circ}$. Plotting it fixes $D$ and gives us $E D$ as $67.5 \mu \mathrm{~A}$, the total generator current. Using the equation again we find $e b^{\prime}$ to be 1.7 mV , so $D C$ is $1 \mu \mathrm{~A}$ leading it by $90^{\circ} . C B$ is $1.7 \mu \mathrm{~A}$, in phase, making $B A 16 \mu \mathrm{~A}$ and $b b^{\prime} 1.6 \mathrm{mV}$. The completed diagram tells us that $e b$ is 2.7 mV , so the voltage gain is down to 370 and the current gain more drastically down to $3 \cdot 1$. The power gain is not simply $A_{v} A_{\mathrm{i}}$ this time, because there is a phase difference of $37^{\circ}$ between input voltage and current. The input power is therefore not eb. $A B$ but eb. $A B \cos 37^{\circ}$ (Sec. 6.9), so the power gain is $A_{v} I_{v} / \cos 37^{\circ},=$ 1450 , or $31 \cdot 6 \mathrm{~dB}$. The ratio $e b / A B$ indicates an input impedance of
$170 \Omega$, which cannot be accounted for by the values of $r_{\mathrm{bb}}, r_{\mathrm{b}, \mathrm{e}}$ and $c_{b, e}$ alone, but clearly points to Miller effect. The apparent value of $c_{\text {bec }}$ is in fact $3 \cdot 2 \mathrm{nF} ; 16$ times its real value.

Nevertheless this transistor is capable of useful i.f. gain, and in a suitable circuit could go considerably higher. The data we have gathered by drawing the phasor diagrams will help us soon in designing an i.f. amplifier.

### 22.6 Limiting Frequencies

The very important transistor parameter that we have so often been using in our studies at low frequencies is the short-circuited-output current gain, $h_{\mathrm{f}}$, or $h_{\mathrm{fc}}$ when applied to the common-emitter mode in particular. It should be interesting to see what happens to it when account is taken of high-frequency effects. This is very easy, after what we have just been cioing, because the short-circuited-output condition enables Fig. 22.4 to be drastically simplified. Because $r_{\text {ce }}$ is short-circuited it can be removed for a start. And as there can be no output voltage across a short-circuit there is no feedback via $c_{b / c}$ and $r_{b r c}$. Instead, they are simply in parallel with $c_{b / c}$ and $r_{b, c}$, and their values are normally such that they too can be removed. And


Fig. 22.8-This greatly simplified equivalent circuit often gives a useful approximation to the high-frequency performance of a transistor
since $h_{\mathrm{fe}}$ is a current ratio, $r_{\mathrm{bb}}$, does not come into the problem. So the circuit boils down to Fig. 22.8, in which $c_{b, e}$ is the only highfrequency influence.

In Fig. 22.5 the output current $D E$ is given in terms of $e b^{\prime}$, equal to the impedance of $r_{b, e}$ and $c_{\mathrm{b}, \mathrm{c}}$ in parallel (which we can call $z_{b / e}$ ) multiplied by the input current $A B$. This part of the circuit is the same as Fig. 6.11, and the calculation of $z \mathrm{~b}, \mathrm{c}$ is as shown in connection therewith (Sec. 6.10). As with the other combinations of a resistance and a reactance we have encountered, what happens at different frequencies can be seen most clearly if we consider two
ranges of frequency and the point at which they meet. But first let us get $h_{f e}$ in terms of $z_{b, e}$ :

$$
\begin{aligned}
h_{\mathrm{fe}} & =\frac{D E}{A B} \\
& =\frac{g_{\mathrm{m}} b^{\prime} \mathrm{e}}{A B} \\
& =\frac{g_{\mathrm{m}} A B z_{\mathrm{b} r \mathrm{e}}}{A B} \\
& =g_{\mathrm{m}} z_{\mathrm{b} / \mathrm{e}}
\end{aligned}
$$

This means simply that, if $g_{\mathrm{m}}$ is constant, $h_{\mathrm{fe}}$ is directly proportional to $z_{\mathrm{b} \text { ee }}$, so that a frequency graph of $h_{\mathrm{re}}$ has the same shape as one of $z_{b, e}$. Over the range of frequency low enough for the effect of $c_{b r e}$ to be negligible, $h_{\mathrm{fe}}$ is constant and is represented by a horizontal line. That is the low-frequency $h_{\mathrm{fe}}$ we have been using hitherto. At frequencies high enough for the effect of $r_{b}$ ee to be negligible (because practically all the current is going via $c_{b, e}$ ) $z_{\mathrm{b}, \mathrm{e}}$, and therefore $h_{\mathrm{fe}}$, is inversely proportional to frequency so is represented by a line sloping at 6 dB per octave (Sec. 20.6). The two lines join at the turning frequency $\left(f_{\mathrm{t}}\right)$ at which $r_{\mathrm{b}, \mathrm{e}}$ and $x_{\mathrm{b}, \mathrm{e}}(=1 / 2 \pi$


Fig. 22.9-Gain|frequency graph of a transistor derived from Fig. 22.8, by which its frequency parameters can be defined
$c_{\mathrm{b}, \mathrm{e}}$ ) are equal. Because the numerical value of $z_{\mathrm{b}, \mathrm{e}}$ at this frequency is equal to $r_{b e} / \mathrm{y}^{\prime} 2$ there is a 3 dB loss there. We should by now be quite familiar with this kind of curve, which is the same as in Figs. 18.9, 19.20 and 20.7.

Fig. 22.9 shows it again, calculated for the values given in Fig.
22.5. As $g_{\mathrm{m}}$ is there $40 \mathrm{~mA} / \mathrm{V}$ and $r_{\mathrm{b} \text { 栚 }}$ is $1 \mathrm{k} \Omega$, the low-frequency $h_{\mathrm{fe}}$ is 40 , or 32 dB . The frequency at which $c_{\mathrm{b} e}(200 \mathrm{pF})$ has a reactance of $1 \mathrm{k} \Omega$ is almost 0.8 MHz . That is what we have been calling the turning frequency, but in this particular context it is called the cut-off frequency and (in the common-emitter mode) it has the symbol $f_{\mathrm{hfe}}$ or $f_{\beta}$. As we see, it is the frequency at which $h_{\mathrm{fe}}$ has dropped 3 dB below its low-frequency value. The phasor diagram for this point is shown in Fig. 22.10, for comparison with Fig. 22.7.


Fig. 22.10-Simplified high-frequency phasor diagram for a transistor, derived from Fig. 22.9

Because $h_{\mathrm{fe}}=D E / A B$ it implies a phase angle as well as a numerical ratio.

If the $1-i n-1$ ( 6 dB per octave) gradient is drawn from this point until it meets the 0 dB line it does so at $40 \times 0.8=32 \mathrm{MHz}$. This frequency also has a special symbol, $f_{1}$, because at it the value of $h_{\mathrm{fe}}$ is $1(\operatorname{or} 0 \mathrm{~dB}$ ) so there is no current gain. A more commonly used symbol for it is $f_{T}$, but this one is sometimes taken to mean the product of low-frequency $h_{\mathrm{fe}}$ and bandwidth. As the bandwidth is regarded as the frequency band up to the -3 dB point, this interpretation of $f_{T}$ gives it the same value: $40 \times 0.8 \mathrm{MHz}$ in this example. But this is only so in this rather over-simplified theory of the relationship between $h_{\mathrm{fe}}$ and frequency, and in practice the different definitions of $f_{1}$ and $f_{r}$ may result in slightly different values.

The same zero- dB frequency is also normally nearly the same as that at which $h_{\mathrm{rb}}$ is 3 dB down. It therefore is a measure of commonbase bandwidth. And as the low-frequency $h_{\mathrm{fb}}$ is almost equal to 1 the gain $x$ bandwidth definition of $f_{T}$ leads to about the same $f_{T}$ for common-base as for common-emitter. Although therefore a transistor in common-base gives no current gain and so less voltage and power gain than in common-emitter, what it has it maintains up to a much higher frequency: about $h_{\mathrm{fe}}$ times higher, in fact.

### 22.7 An I.F. Amplifier

For this first attempt at designing an i.f. amplifier we take single tuned circuit couplings, such as are used in most portable transistor receivers. We start from the bandwidth, $f_{B}$, which we found in Sec. 20.6 is equal to the resonant frequency $(470 \mathrm{kHz}$ in this case) divided by the $Q$ of the tuned circuit. A practical bandwidth is 4 kHz each side of resonance, or 8 kHz altogether. So the required $Q$ is $470 / 8$ : say 60 . This, it should be understood, is the $Q$ under
working conditions, loaded by the output resistance of the preceding transistor and the input resistance of the following one. At the moment we do not have either of these data for the transistor represented by Fig. 22.5. If we can assume that the load is $20 \mathrm{k} \Omega$ resistance we have the input impedance at $470 \mathrm{kHz}, 170 \Omega$, but Fig. 22.7 shows that the input current leads the input voltage by $37^{\circ}$. A little calculation on the lines of Chapter 6 shows that this is equivalent to $216 \Omega$ in parallel with 1.23 nF . The output resistance is more difficult. Another phasor diagram is needed to derive it from Fig. 22.5, and we would have to know the signal source impedance as well. If we assume it is made equal to the input resistance, the output resistance comes out at about $25 \mathrm{k} \Omega$, in parallel with capacitance.

Although the tuning product $L C$ is fixed for us by the resonant frequency (Sec. 8.7) we are free to choose the ratio $L / C$. If $C$ is small, so that most of it is made up of transistor and wiring capacitances, it cannot be relied upon to be constant. About 250 pF total is a typical choice. The corresponding $L$ is $460 \mu \mathrm{H}$, and its reactance at 470 kHz is $1.36 \mathrm{k} \Omega$. The $Q$ of the tuned circuit itself is ideally infinite; in practice it might be 140. The corresponding dynamic resistance (Sec. 8.10) is $190 \mathrm{k} \Omega$. When loaded ( $Q=60$ ) it will be $82 \mathrm{k} \Omega$, so the combined input and output loading is $144 \mathrm{k} \Omega$ (for 144 and 190 in parallel are 82). If we suppose that input and output loadings are equal, then each must be $288 \mathrm{k} \Omega$. For $25 \mathrm{k} \Omega$ to load the tuned circuit to that extent it must be connected across only part of the tuning coil, the fraction of turns being $\downarrow^{\prime}(25 / 288), \simeq 0.3$ (Sec. 7.11). Similarly the following transistor input should be connected across $\checkmark^{\prime}(0.216 / 288),=0.027$. The usual way in which this is done is shown in Fig. 22.11. The symbol for the inductor shows that the value of $L$ is preset by what is called permeability tuning, which usually takes the form of a ferrite core that can be screwed in and out of the coil to vary the proportion of it in the magnetic field.

Although this design would work after a fashion, the transistors (if correctly represented by Fig. 22.5) really are not quite good enough for it at this frequency. There is effectively a nearly $11: 1$ voltage step-down from one transistor to the next, necessitated by the low input impedance due to Miller effect. Because even a smaller amount of Miller effect has objectionable results (when setting up the tuning, for example) it is sometimes worth while to neutralize the tuning circuits even at such a low frequency as 470 kHz . This can be done in the same way as was described for senders in Sec. 15.6, but the alternative shown dotted in Fig. 22.11 may be more convenient.

Because the internal feedback is not purely capacitive, to neutralize it completely a certain amount of resistance needs to be put in series with $C_{\mathrm{n}}$. If exactly the right values are chosen, the transistor should work entirely as a one-way device. This superior form of neutralization has been given the even grander name of unilateralization.

For a better combination of selectivity and sideband retention, and especially for getting the flat top needed for vision i.f. amplifiers,


Fig. 22.11-Circuit diagram of Iypical i.f. amplifier for 470 kHz

Fig. 22.12-Alternative i.f. interstage coupling, double-tuned

double tuned circuits with suitably adjusted couplings (Sec. 20.8) are used, as in Fig. 22.12. In receivers working at one i.f. for a.m. and another for f.m., the i.f. transformers are usually connected with primaries in series and secondaries in series, no switching being needed. So far from tapping down being needed at the high i.f. and wide bancuidth necessary for vision, not only is the whole winding usually in series with the collector but sometimes there is even a damping resistance in parallel.

In both a.m. and f.m. i.f. stages the LC tuning circuits are sometimes supplemented or even substituted by ceramic filters. These are similar in principle to the quartz crystals described in Sec. 15.8 and
consist of a small ceramic disk with electrodes deposited on each side. As the frequency and bandwidth are determined by the dimensions, these are fixed, normally at about 470 kHz or 10.7 MHz . Ceramic filters combine good i.f. tuning with small size.

### 22.8 R.F. Amplification

Amplification at the original signal frequency, if that is lower than about 30 MHz , is now seldom used except in high-performance 'communications' receivers; but when it is used the techniques are similar to those for i.f., with the addition of variable tuning. This is done by one or more extra sections on the ganged tuning capacitor. Because the i.f. amplifier is relied upon for most of the selectivity, the preselector tuning circuits do not have to be very sharp, so slight errors in ganging are not serious. So long as the tuning capacitor sections are reasonably alike, all that is needed is to make the inductances and minimum capacitances equal. The only difficulty arises in the first tuned circuit, to which the aerial is coupled; this problem is considered in Sec. 22.12.

Preselector (r.f.) amplifiers are most used at frequencies higher than about 30 MHz , because noise generated in the frequency changer tends to be greater the higher the frequency, and signals tend to be weaker. So the all-important signal/noise ratio is usually not good enough without some amplification at the original frequency. This is rather awkward, because the difficulties in achieving such amplification increase with frequency. And of course only lownoise devices can be considered.

Before the development of transistors that could perform adequately at v.h.f. and u.h.f., valves had to be used. At the lower radio frequencies, pentodes were chosen in order to avoid Miller effects, but at the higher frequencies where such effects are proportionately greater the use of pentodes (or any other multi-electrode valves) was unsatisfactory because of partition noise (Sec. 19.20). To enable the quieter triodes to be used in spite of their relatively large anode-to-grid capacitance, $c_{\mathrm{ag}}$, there have been several lines of approach.

One of them is to use the control grid as a screen, by connecting in the common-grid mode, Fig. 22.13. As with transistors, this results in a relatively low input impedance and absence of current gain. But because the dynamic resistances of tuning circuits at these frequencies are also low they are less affected by the low input impedance than in lower r.f. practice, and even a moderate gain is worth having, in terms of signal/noise ratio.

The next idea is to connect two triodes together in such a way that they function very much as a pentode but with a lower overall noise factor. This combination is called a cascode. It is also valid for transistors and will be described when we come to them.

Neutralization is an obvious solution, but at v.h.f. the large $c_{\mathrm{ag}}$
of the ordinary triode has to be very precisely neutralized if its effect is to be negligible. So it is not a very popular remedy. But more recently triodes have been produced with a sort of screen kept at earth potential (and therefore hardly an electrode, as no electrons alight on it) which is nevertheless compatible with the high $g_{m}$ that


Fig. 22.14-Common-hase preselector stage suitable for v.h.f.
is needed if a valve is to give appreciable gain at v.h.f. $c_{\mathrm{ag}}$ in these types is of the order of $\frac{1}{3} \mathrm{pF}$, and little or no neutralization is needed.

More or less the same story applies to transistors. Fig. 22.14 shows a preselector stage suitable for the 87 to $95 \mathrm{MHz} \mathrm{f.m}$. broadcasting band. Because this band is all within $4 \frac{1}{2} \%$ of its centre frequency a fixed-tuned input circuit, damped as it is by the low input impedance of a common-base transistor, covers it adequately. The output circuit is inductance tuned, and tapped down at the output so as to have a high enough $Q$ to give a useful contribution to selectivity.

For low noise, f.e.ts are better than either valves or bipolar transistors. In most other respects they are similar to valves. So if used in common-source mode they need neutralizing, and can be used unneutralized in common-gate; for example, an f.e.t. can take the place of the valve in Fig. 22.13.

Two triode valves or f.e.ts are often combined in cascode connection to give a low-noise high-overall-gain stage: Fig. 22.15.


Fig. 22.15-Cascode v.h.f. preselector slage using f.e.ts
Here the output f.e.t. $\left(\mathrm{T}_{\mathrm{R}_{2}}\right)$ is working as a voltage amplifier in common-gate mode and therefore presenting the common-source $\mathrm{T}_{\mathrm{R}_{1}}$ with such a low load impedance that it gives little or no voltage gain. Miller effect in $\mathrm{T}_{\mathrm{R}_{1}}$ is thus too small to be troublesome. But its own input impedance is high, so the input tuned circuit gives good voltage magnification, and $\mathrm{T}_{\mathrm{R}_{1}}$ good current gain. Some inductance ( $L$ ) is sometimes included to separate the stray capacitances of the two f.e.ts and prevent them from reducing the gain at very high frequencies; the way it does this will be explained in connection with Fig. 22.26.

Bipolar transistors can be used in cascode, but their noise contribution is likely to be rather higher.

The need for amplification at v.h.f. and u.h.f. usually goes along with wide bandwidth; for example, television and radar. Because of the difficulties of unilateralization at such frequencies a common policy is to use transistors with the lowest possible feedback capacitances in conjunction with stagger tuning (Sec. 20.9) or overcoupling. Both of these types of tuning help to give the desired wide flat top to the frequency response, and at the same time reduce feedback, either because the gain is less or because the input and output circuits are not tuned to exactly the same frequency.

Another feature of u.h.f. amplifiers is that the inductances required for tuning are too small to be provided in the form of coils,
so tuned circuits in the form of quarter-wave 'lines' as described in Sec. 16.10 are used.

Fig. 22.16 shows a typical arrangement, in which $L_{1}$ is the input tuned circuit consisting of a quarter-wave rod which is the inner conductor and the surrounding metal screen (shown dotted) is the outer conductor. It is inductively coupled to the u.h.f. aerial feeder on one side and to the emitter of the common-base transistor on the other. The collector is connected to a tapping on the output tuned circuit $\mathrm{L}_{2}$. The d.c. is fed through a choke coil that prevents the signal voltage from being short-circuited. Note that all the feeds pass through the screen via I nF 'feed-through' capacitors which


Fig. 22.16-Complese circuil diagram of u.h.f. 'from end (preselector and frequencychanger slages) using 'line' iuning
effectively earth all signals to the screen. $L_{2}$ is loosely coupled to $L_{3}$ through a hole in the screen, so that the pair are a band-pass filter represented in conventional symbols by Fig. 20.10c. 1ts output is inductively coupled to the common-base frequency changer. $\mathrm{L}_{4}$ is the oscillator tuning circuit, back-coupled to the emitter via $\mathrm{C}_{2}$. The u.h.f. and i.f. signals in the collector circuit are separated by a choke coil, and the i.f. signals are developed across $L_{5}$. All four line circuits are tuned by a four-section ganged capacitor $\mathrm{C}_{1 \mathrm{a}}$ d.

### 22.9 Screening

The basic principle of screening was explained in Sec. 15.7 with particular reference to valve electrodes; the same principle applies
to the external screening of circuits. The object is to prevent capacitive coupling between two circuit elements by interposing a conducting sheet which is either directly earthed or has such a large capacitance to earth that it is effectively at zero signal potential. Metal sheet or foil is the usual screening material, but its effectiveness is not greatly reduced by perforations, and for some largescale purposes wire netting is good enough. Because even very small stray capacitances have sufficiently low impedance at very high frequencies to form undesired circuit paths, the need for screening generally increases with frequency. The complete screening of the u.h.f. circuits in Fig. 22.16 is an example of this.

One cannot always assume that because two points are connected to such a screen they must both be at the same signal potential. If a substantial signal current flows between points some distance apart there may well be enough impedance between them to give rise to an appreciable signal p.d. and therefore a risk of undesired coupling. So care has to be taken in laying out u.h.f. circuits.

Signal leakage via d.c. or relatively low-frequency leads is another possibility to watch. Fig. 22.16 includes two examples of u.h.f. chokes for this purpose. Sometimes the lead itself can be given sufficient inductance by threading it through one or more ferrite beads designed for this application.

So far we have considered screening as a means of confining electric fields. The fact that they are often effective in confining magnetic fields too should not prevent us from distinguishing clearly between the two kinds of screening. The essential feature of electric screening is negligible impedance to a point of zero potential. A thin sheet of copper or other low-resistance metal connected to earth or its equivalent is usually effective. The effect


Fig. 22.17-Showing the principle of one method of magnetic screening
of stray magnetic fields is to induce e.m.fs in any conductors encountered. Magnetic screens work on two different principles. One kind surrounds the source of the field with such a low magnetic im-pedance-reluctance is the term-that practically all the magnetic flux passes through it in preference to the high-reluctance space outside, as in Fig. 22.17. To ensure the low reluctance the screening material must have a high permeability. Special alloys such as mumetal are used for this purpose.

The other type of screening works by forming short-circuited paths magnetically coupled to the source of the field, which induces currents in them, and these currents set up magnetic fields in opposition to the inducing fields. It is in fact an example of transformer action. The important property of the screening material is not permeability (which would impede its action) but, as with electric screening, conductivity. There is no need for a magnetic screen to be earthed, but there must be continuous low resistance around the paths of induced currents (which is not essential for electric screening). If the 'can' in Fig. 22.17 was made of copper instead of magnetic alloy it would have currents induced in it in planes parallel to the turns of wire in the coil, and the net external field would be greatly reduced. If earthed, it would also be an effective electric screen. If it had a saw cut made across it, dividing it into two paths and they were both earthed, it would still be an electric screen, but the magnetic screening would be almost completely destroyed by the interruption of induced currents.

So it is possible to screen both kinds of field, or either of them alone.

In general the permeability screen is the more effective magnetic type at low frequencies, including z.f., and the conductive screen at high frequencies. The permeability type increases the inductance of the enclosed inductor, especially if it is shaped so as not to be a good conductive screen. Provided that this is not objectionable, the screen need not be widely spaced from the inductor. A closed magnetic core acts as a magnetic screen by short-circuiting the magnetic flux. A conductive screen reduces the inductance and tends also to increase its resistance, the closer it is to the inductor. So the inductor itself should be designed to have as little external field to screen as possible.

### 22.10 Cross-Modulation

The emphasis on distortion in the chapter on I.f. amplification has been notably lacking in the present chapter. The reason is that so long as the shape of the modulation waveform is preserved, the shapes of the individual r.f. cycles that carry the modulation are unimportant. They could all be changed from sine waves to square waves or any other shape, and indeed this is actually done in f.m. receivers by the limiter, which chops all the cycles down to a constant size, regardless of distortion. That would not be allowable in an a.m. receiver, of course (the purpose of the limiter being in fact to remove any a.m.) but so long as the relative sizes of the cycles were preserved they would still define the original modulation envelope. A perfect detector cuts off half of every cycle, yet this drastic r.f. distortion introduces no distortion of the modulation. It is the imperfect or non-linear detector (Sec. 18.4) that distorts

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the modulation by altering the relative proportions of the r.f. cycles. But unless the modulation is fairly deep it is surprising how little it is distorted by even a considerable curvature in the r.f. amplifier's output/input characteristic. It must be remembered that except for the output of the stage feeding the detector the amplitude of the r.f. signals is so small that it is unlikely to experience appreciable nonlinearity.

There is an important exception to this. If signals very much stronger than those to which the receiver is tuned are not very widely different in frequency they may get some way into the receiver before being tuned out by its selectivity. In a superhet most of the selectivity is concentrated in the i.f. amplifier, for the reasons given in Sec. 20.10. So such signals may well be present in the first stage at considerable amplitude. What happens there is very similar to the frequency-changer action illustrated in Fig. 21.4b, the interfering strong signal now playing the part of the local oscillation. If a transistor or valve handling these signals simultaneously is appreciably non-linear within the amplitude of the strong signal, other signals at sum and difference frequencies will be created and may cause whistles as already mentioned at the end of Sec. 21.7. But unlike the local oscillations generated in the frequency changer, the unwanted signals will usually be modulated.

Fig. 22.18 shows a characteristic curve, with O as the working point. The amount of non-linearity between say $A$ and $B$ is so


Fig. 22.18-Showing how the average slope of an amplifier curve varies during each modulation cycle of a strong signal
slight that even if as large a section of the curve as that was being covered by the wanted signal in the first stage the distortion of its modulation would be negligible. But now consider what would happen if there was also a strong modulated signal as depicted below, sweeping between C and D at the peaks of its modulation and between A and B at the troughs. The small wanted signal would be swept with it. Owing to the markedly different slopes at $C$ and $D$,
the output amplitude of the wanted signal would be forced to vary according to the modulation of the unwanted signal. In other words the wanted signal is to some extent modulated ty the unwanted modulation. So even if the unwanted signal is removed entirely by subsequent selectivity it has left its mark indelibly on the wanted carrier which, with its two-programme modulation. passes through the receiver and both are heard together.

If the wanted transmission were switched off the programme of the interfering station would cease with it, proving that the interference was due to cross-modulation and not to insufficient overall selectivity

Clearly the remedies for cross-modulation are two-fold: avoid excessive non-linearity in the early stages, and provide enough selectivity to proted them against overwhelmingly strong interference. Such interference is unlikely on the v.h.f. and u.h.f. bands, so the problem seldom intrudes there. which accounts for the fact that sometimes there is no variable tuning at all before the first stage (Figs. 22.14 and 15 , for example), but broadcast receivers for the medium frequency band respecially if used with large aerials) and communications and other specialist receivers must be designed with cross-modulation in mind.

It is fortunate that when a carrier wave is tuned in it affects the detector in such a way that if some interfering modulated carrier wave is present its modulation is rendered more or less ineffective. In this way the selectivity of the receiver is apparently increased. With sharp tuning, the percentage modulation of the wanted station is also reduced, reducing detector distortion. So it may pay to tune sharply in the interests of selectivity and detector linearity, and make up for the resulting weakening of outer sidebands by upper frequency compensation in the a.f. department.

### 22.11 Automatic Gain Control

Receivers for sound are usually designed for a certain signal amplitude at the detector, such that at a moderate depth of modulation ( say $30 \%$ ) there is enough a.f. gain to give maximum output, plus a bit in reserve. The volume control, to enable the output to be adjusted between more-than-maximum and zero, is usually placed between the detector and the first a.t. stage. Ideally, all carrier waves to which the receiver is tuned would arrive at the detector at the same amplitude. This implies that the rif. gain is always adjusted inversely to the strength of the signal as it comes from the aerial. This adjustment is no: one that the user of the receiver wants to be bothered to make. so most receivers are designed to do it automatically, by means of what is called attomatic gain control (a.g.c.). To get close to the ideal some fairly complicated circuitry is needed. Here we shall look at only the simplest methods.

The general principle is to use the rectified carrier wave voltage part of the detector output (the d.v.) to alter the bias of one or more of the preceding stages in such a way as to reduce their gain. The choice of stages calls for careful consideration. Control of preselector stages, if any, is most desirable, but the process usually increases non-linearity so the risk of cross-modulation must be watched. The type of frequency changer that combines oscillator and 'mixer' is not available for a.g.c., because oscillation might be stopped thereby, or at least mistuned. The last i.f. stage is handling signals at their maximum strength, so must not be allowed to become non-linear enough to distort the modulation. The non-linearity problem was largely solved in valve receivers by using valves (often called variable-mu valves, but they had several other names) in which the grid was wound with varying pitch so as to maintain reasonably linear amplification while gain was controlled by grid bias over a wide range. A lower standard of a.g.c. seems to be accepted for most transistor receivers, in which control of the first i.f. stage only is customary.

In all valve receivers and most transistor receivers the rectified carrier-wave voltage is applied to the amplifier in the polarity that reduces output current, to which the mutual conductance and consequently gain are roughly proportional. To prevent the control from being upset by modulation of the carrier, the detector output is first put through a filter with a time constant long enough to remove even the lowest modulation frequencies but not so long as to


Fig. 22.19—Circuit of a typical i.f. amplifier and detector showing a.g.c. system
cause a noticeable time lag on tuning from one carrier wave to another.

Fig. 22.19 shows the parts involved, with typical values. Two i.f. stages are shown, the second followed by a diode detector, with filter for removing the residual i.f. The rectification of the carrier wave makes the point X more positive than the positive side of the power supply. The modulation is removed by the filter made up of
$R_{1}$ and $C$. and the positive potential reaches the base of $T_{R_{1}}$ via the secondary coil of the tuned i.f. transformer at the frequency-changer output. Note that as these are p-n-p transistors they normally work with a small negative bias; for $\mathrm{T}_{\mathrm{R}_{1}}$ it comes from the negative side of the power supply via $R_{2}$. $R_{1}, R_{2}$ and $R_{3}$ have been chosen so that with no output from the detector the bias for $T_{R_{1}}$ is about the best for good i.f. gain. The a.g.c. voltage, by making the bias less negative, reduces the collector current and gain. Because only a fraction of a volt is needed to reduce the collector current from its initial value to nearly zero, control is very 'steep', not requiring a very large signal voltage at the detector, and one-stage control is tolerably effective. With valyes, several volts of a.g.c. bias are needed, and usually at least two stages are controlled.
A.g.c. systems are often designed so that only when an adequate working voltage at the detector is exceeded does it reduce the gain. This feature is called-not very aptly, for time is not involveddelaved a.g.c.

Note that the a.g.c. potential divider system in Fig. 22.19 applies a small forward bias to the detector diode, sufficient to bring the working point to the sharpest point on its characteristic curve (see Fig. 9.14) and thereby increase its response to weak signals.

Oddly enough, a.g.c. is possible with transistors by applying control bias of the opposite polarity to that just indicated. For this forward a.g.c. (as it is called) to work, there must be a suitable amount of resistance in the collector circuit. So when a.g.c. bias is applied and increases the collector current, the voltage reaching the collector falls and as the transistor begins to run into saturation its gain falls fairly steeply. At the same time its output resistance falls and damps the tuned circuit into which it works, increasing the bandwidth and sideband acceptance. The associated loss of selectivity can be accepted because it takes place only when a comparatively strong carrier wave is tuned in and the gain is low, so interference is unlikely.
A.g.c. is hardly necessary for f.m., because the limiter irons out the differences in strength between incoming signals. In the exceptional cases where signals of such widely different strengths are encountered that the limiter would have difficulty in coping with the full range, the nearly constant d.c. output of the f.m. detector would obviously be ineffective for a.g.c. purposes, so a separate diode is used at some i.f. point just ahead of the limiter.
In the vision section of a television receiver the provision of a.g.c. is complicated by the fact that the vision signal has no constant mean carrier amplitude; it varies with the average light and shade in the picture. So if a.g.c. is applied in the same way as for sound, it tends to brighten scenes that are supposed to be dark and darken scenes that are supposed to be bright. In designs where trouble is taken to exclude this defect, certain parts of the composite vision signal that are not reproduced on the screen, and which represent the strength
of the incoming signal, are selected by what is called a gating circuit for use as a.g.c. bias.

### 22.12 The Aerial Coupling

The basic problem in coupling a sender to its aerial (Sec. 15.15) is to ensure the maximum transfer of power. The amount of power developed in the output stage can be varied by controlling the drive voltage, so the maximum-power law (Sec. 11.8), which assumes a fixed available power, does not apply. The limiting factor is the amount of power that can be allowed to be lost as heat in the output stage. The receiving aerial can be regarded as a signal generator and the receiver as the load, and this time the signal voltage is fixed by the incoming field strength and the nature of the aerial, and there is no heating problem, so the power law does apply. But the transfer of power has to be compatible with selectivity and easy tuning.

Three main classes of aerial have been considered in Chapter 17: (1) the dipole, in single or multiple form, which works at or near its resonant frequency, decided by its dimensions: (2) the loop or coil, which usually takes the place of the first tuning inductor in the receiver and is tuned by variable capacitor; (3) the wire-and-earth aerial, which can be almost any size and is usually worked below its resonant frequency, so has capacitive reactance. Aerials in class 1 , which normally are connected to the receiver via a cable can be regarded as resistances equal to the characteristic resistance of the cable, such as $70 \Omega$. Class 2 aerials are of course essentially' inductances, but when tuned can be regarded as high (dynamic) resistances. Aerials of class 3 may be anything from a few feet of wire indoors to a large outdoor structure. with an impedance of about $20-1000 \mathrm{pF}$ in series with perhaps $50-5000 \Omega$ resistance. And in conjunction with multi-band receivers they may be required to work at


Fig. 22.20- The three sections involved in coupling the aerial to a receiver: the aerial, the unning circuit and the amplifier input
almost any radio frequency. So the most suitable coupling arrangements are rather difficult to determine! We shall now consider this most difficult of the three cases.
The nature of the problem is illustrated by Fig. 22.20, in which $R_{\mathrm{ac}}$ and $C_{\mathrm{ac}}$ represent the series resistance and capacitance of the aerial (which vary somewhat with frequency) and $E$ the e.m.f. it picks up from space. On the right there is the first active device (a common-emitter transistor in this example), and L and C are available for tuning purposes. The problem is how to interconnect these three systems.

Ideally, in order to put into effect the maximum-power law any reactance in the aerial's impedance should be tuned out by reactance of the opposite kind, leaving only resistance. This resistance should then be connected to the transistor in such a way that it matches (i.e.. is equal to) its input resistance. As it is very unlikely that the two resistances would happen to be equal. some impedancetransforming (or matching) device such as a transformer is indicated.

Basically this is the problem we have already met in connecting one amplifier stage to another. But in this case it is complicated by a number of things: whoever designs a receiver to work from an open aerial usually has no control over or knowledge of the electrical characteristics of the aerials that may be used; the arrangement often


Fig. 22.21-Use of r.f. transformer to match aerial impedance to receiver inmat impedance
has to work well enough over a very wide range of frequency; and there are the selectivity and tuning considerations already mentioned but not yet gone into in detail.

First let us assume ideal conditions: a purely resistive aerial and transistor input, and a perfect transformer to couple them (Fig. 22.21 (a). If the transformer ratio is denoted by $1: n$. from the load's point of view Fig. 22.21b is the same as $a$. The maximum power is delivered to $R_{L}$ (and therefore the greatest voltage, $V$, set up across it) when $n^{2} R_{\mathrm{ac}}=R_{\mathrm{L}}$. So $n=\sqrt{\prime}^{\prime}\left(R_{L} / R_{\mathrm{ac}}\right)$, and

$$
V_{\max }=\frac{E}{2},\left(\frac{R_{L}}{R_{\mathrm{ae}}}\right)
$$

If you assume some values for $R_{L}$ and $R_{\mathrm{x}}$ and calculate $n$ required to give $V_{\text {max }}$, and then calculate $V$ for values of $n$ each side of the optimum, you will find that $V$ falls very little below $V_{\text {max }}$ even when $n$ is as much as $30 \%$ above or below its best value. In other words, $n$ is not critical. Which is just as well, for practical cases are far from ideal.

Where does the tuned circuit come into it? If it was omitted, so that there was no selectivity in front of the first stage of amplification, serious cross-modulation would be likely, especially if the aerial was a large one. The tuned circuit should interfere as little as possible with the ideal conditions shown in Fig. 22.21 so far as the wanted signals are concerned, and short-circuit all others as much as possible. It would do this if it was connected in parallel with either primary or secondary side of the transformer in Fig. 22.21. But if $\mathrm{R}_{\mathrm{ac}}$ and $R_{L}$ were not unusually high they would probably reduce the $Q$ of the tuned circuit so much that its selectivity would be poor, unless the $L / C$ ratio was so low that $C$ would be right out of line with the capacitances used to tune the other circuits in the receiver. If the receiver is to be used by unskilled persons we can assume that only one tuning control is allowable. So $C$ has to be provided by a section on the ganged tuning capacitor (Sec. 21.6).

The solution is the same as we used for our i.f. amplifier: tap the input or output circuits or both across part of the tuning circuit, or use separate windings. In other words, combine tuning circuit and matching device. Fig. 22.22 shows what we could now have, where both varieties of method are included. Except that the tuning circuit


Fig. 22.22-Coupling aerial to receiver by means of $a$ combined tuning circuit and impedance-matching transformer
is fed from an aerial instead of a transistor, the circuit is identical in principle to an i.f. amplifier (Fig. 22.19). But in practice it is very different, because of the wide range of frequencies (instead of one fixed one), the wide range needed for $C_{1}$, dictated by the requirements of the ganged tuning capacitor, and the great range of possible aerial impedances. So far we have conveniently been assuming that they are all at least resistive. While that is roughly true of aerials in class 1 , open-wire aerials usually have substantial capacitance, and
because the amount is unknown (it could be hundreds of pF ) and varies with frequency and with the size of the aerial connected, and even a few pF can mistune the circuit seriously, a major problem exists.

Both over-coupling and under-coupling between aerial and tuning circuit reduce signal transfer, but over-coupling increases the


Fig. 22.23-Two other mothods of cterial coupling: (a) a relatively high-inductance primary coil $\left(\mathrm{L}_{1}\right)$ angmented if mecessary by rop capacitance $\left(C_{3}\right)$; 'h, small series capacitance
disturbance to tuning and reduces the selectivity (both for that reason and by reducing $Q$ ), whereas under-coupling improves both tuning and selectivity. So the usual policy is to under-couple considerably, say by tapping well below the optimum even for the largest aerial. Signal strength is then likely to be poor with a small aerial. One solution, not much used now, is to provide alternative tappings. In another (Fig. 22.23a) the primary ( $\mathrm{L}_{1}$ ) of a transformer is designed so that with the aerial capacitance it resonates at a frequency below the lowest in the range of the receiver. The larger the aerial, the more the resonant frequency of this coil departs from that of the secondary and so the less the capacitance that is transferred into it. The greater signal loss is compensated by the stronger signal the larger aerial brings in. Signal transfer tends to be poor at the highest frequencies, at which $\mathrm{L}_{2} \mathrm{C}_{2}$ is most out of tune with the aerial, so a small capacitor $\mathrm{C}_{3}$ is sometimes included to help things along. Another method (Fig. $22.23 b$ ) has the merit of simplicity and of ensuring that whatever aerial is used the capacitance it adds to the tuned circuit must be less than $C_{4}$. And the range of signal strength brought in bv aerials of different sizes is to some extent levelled out, for the larger the aerial the more its coupling is reduced by the capacitance $C_{4}$ (which is usually only a few pF) being so much less than its own. At the same time the effects of diverse aerial resistances are reduced.

Providing for v.h.f. dipole acrials is comparatively easy because their resistance is specified, their reactance is farly small or even zero, and the frequency ratio is usually quite limited.

Portable receivers with internal aerials that are in effect large
tuning coils are easier still, for the designer has everything under his control.

### 22.13 V.F. Amplification

As we shall see in the next chapter, the signals needed to reproduce fine detail on a television screen may include all frequencies up to about 5 MHz . And this is true of the sharp-cornered pulses needed to keep the picture steady. A wide frequency band is needed also for the pulse signals used in radar, and for the signal amplifiers in oscilloscopes. There is no difficulty in choosing transistors or valves capable of uniform service up to 5 MHz ; the main problem is the coupling impedance needed to develop the output voltage. We shall see in a moment that the greatest practical impedance is generally small compared with the output impedance of the transistor or valve to which it is connected, so the gain can be regarded as proportional to the coupling impedance. It should therefore be constant over the whole working frequency band, and large enough for satisfactory gain.

Although transformers can be designed to cover a wide band of high frequencies or from say 30 Hz to 30 kHz , it is not practicable to cover both at once, and even if it were its size, weight and cost would rule it out. Simple resistance coupling is the ideal solutionor would be but for stray capacitance. Counting the cathode-ray tube for which the output of the v.f. amplifier is required, the circuit wiring, and the v.f. transistor or valve itself, this capacitance is usually in the range $15-25 \mathrm{pF}$. It is shown as $C$ in Fig. 22.24. At


Fig. 22.24-The frequency bandwidth of a v.f. amplifier is normally limited by stray capacitance, $C$
$4 \mathrm{MHz}, 25 \mathrm{pF}$ has a reactance of only $1.6 \mathrm{k} \Omega$, effectively in parallel with the coupling resistance $R$. If $R$ also was $1.6 \mathrm{k} \Omega$, there would be a loss of 3 dB at 4 MHz compared with low frequencies (Sec. 19.16). Increasing $R$ would increase the low-frequency gain, but the comparative fall-off at 4 MHz would be all the greater. Decreasing $R$ increases the frequency at which the reactance of $C$ is equal to it (the 3 dB loss frequency) but decreases the gain of the stage and increases the signal current needed to set up across $R$ the required voltage output, which may be as much as 100 V for a television tube. To get 100 V output across $1.6 \mathrm{k} \Omega$ requires $100 / 1 \cdot 6=63 \mathrm{~mA}$, which is quite substantial.

The fall-off point can be shifted to a higher frequency, or the allowable $R$ increased, by counterbalancing $C$ by means of inductance. The inductance $(L)$ cannot be connected directly in parallel with $C$, because at low frequencies its impedance would be so low that there would be very little gain. This objection can be avoided by putting L in series with R , but calculations are more difficult because the equivalent parallel inductance and resistance vary in rather a complicated way with frequency. A useful rule is that by making $L=$ $C R^{2} / 2$ the fall-off at the -3 dB point is reduced to zero. You might perhaps guess that this is the frequency at which $C$ is exactly tuned out by $L$, but actually there is no such frequency. A substantial part of the benefit is due to the fact that at the frequencies where $C$ appreciably shunts $R$ the presence of $L$ makes the equivalent parallel resistance higher than $R$. The equivalent parallel inductance does not completely neutralize $C$ at any frequency.

Suppose $C$ is 20 pF and that in the hope that the scheme just described will work we make $f_{t}$ rather lower than the maximum required frequency- 3 MHz , say. $R=1 / 2 \pi f_{1} C$, $=2.65 \mathrm{k} \Omega$. So $L=20 \times 2.65^{2} / 2=70 \mu \mathrm{H}$. Fig. $22.25 a$ shows the calculated


Fig. 22.25-(a) Calculated high-frequency loss (due to C in Fig. 22.24) in an example of a v.f. amplifier. (b) The result of inserting a suitable amount of inductance in series with $R$
high-frequency loss without $L$, and we note that it is 3 dB at 3 MHz , as planned. Curve $b$ shows the result of inserting the calculated value of $L$, and we see that the -3 dB point has been moved up to about 5.5 MHz . At 3 MHz the expected zero loss is achieved. At slightly lower frequencies there is a flat peak; it is not really enough to worry about, but if you insist on what is called a maximally flat response the choice of $L$ is $0.414 C R^{2}$.

The curves make no allowance for self-capacitance of $L$, and in practice some extra capacitance or a damping resistance may be connected across it. In some wide-band amplifiers inductance is


Fig. 22.26-An alternative position for the h.f. lossreducing inductor in a $v . f$. amplifier. It has the advantage of separating the stray capacitances $C_{1}$ and $C_{2}$

Fig. 22.27-As an alternative to the frequency curves in Fig. 22.25, the performance of a v.f. amplifier can be expressed on a time basis. The use of inductance is seen here to result in slight overshoot
used between the output and input capacitances as shown in Fig. 22.26, instead of or in addition to the position just considered. and with or without parallel resistance.

For oscilloscopes and some other purposes a top frequency of 30 MHz or even higher is needed. Use of inductance in both the positions mentioned helps; apart from that the general policy is to reduce the effiect of stray capacitances by reducing the coupling resistance. To retain a useful amount of gain, the mutual conductance of the amplifying device must be made high, and in general this means a high output current at the working point. So although the power output required from a very wide-band amplifier may be quite small it is likely to need power types of device and means for getting rid of quite a lot of heat. This is not easy to combine with the necessary very high $f_{T}$.

It may seem a roundabout way to deal with this sort of problem in terms of frequency, when in television, radar, pulse telegraphy, etc., one is not really concerned with frequency as such but with timethe time taken to jump from white to black on the television screen, for example. This takes into account phase, which so far we have ignored. Time and frequency are, as we know, very closely related, one being the reciprocal of the other: and given response and phase curves (or their equation) on a frequency or steady-state basis, the equivalent on a time or transient basis can be derived, and vice versa (Sec. 24.2). You may however remember that we have dealt with two very simple cases on a transient basis: the response of capacitance and resistance in series to a step signal ( Sec .3 .5 ) and the same for inductance instead of capacitance (Sec. 4.8). The time responses corresponding to the frequency responses in Fig. 22.25 are shown in Fig. 22.27. With inductance, there is a much faster rise than without, but also almost $10 \%$ 'overshoot'. This would not seriously spoil a television picture, but would disqualify the amplifier for an cscilloscope (Sec. 23.4) intended to reveal just such shortcomings in transient signals such as pulses.

## CHAPTER 23

## Cathode-Ray Tubes: Television and Radar

### 23.1 Description of Cathode-Ray Tube

The preceding chapters have covered the general principles underlying all radio systems (and incidentally quite a number of other things such as deaf aids and industrial r.f. heating), together with sufficient examples of their application to provide at least the outlines of a complete picture of the broadcasting and reception of sound programmes. There remains one device which is so important for such purposes as television and radar that this book would be incomplete without some account of it: the cathode-ray tube.

In many respects it resembles a valve; the differences arise from the different use made of the stream of electrons attracted by the anode. Both devices consist of a vacuum tube containing a cathode for emitting electrons, an anode for attracting them, and (leaving diodes out of account) a grid for controlling their flow. But whereas valves are designed for using the electron stream (or anode current) externally, cathode-ray tubes use it inside by directing it against one end of the tube, where its effects can be seen. This controlled stream of electrons was originally known as a cathode ray; hence the name of the tube.

Fig. 23.1 shows a section of a c.r. tube. The collection of electrodes at the narrow end is termed the gun, because its purpose is to produce the stream of high-speed electrons. There is the cathode with its heater, and close to it the grid. Although the names of these electrodes follow valve practice, their shapes are modified; the emitting part of the cathode is confined to a spot at its tip, and the grid is actually more like a small cup with a hole in the bottom. Provided that the grid is not so negative with respect to the cathode as to turn back all the emitted electrons, a narrow stream emerges from this hole, attracted by the positive anode voltage, which for television is usually between 10 and 25 kV . The anode current is comparatively small; seldom much more than 1 mA and often much less. Many different shapes of anode have been favoured, almost the only feature common to all being a hole in the middle for allowing the electrons, greatly accelerated by such a high voltage, to pass on to the enlarged end of the tube. Usually the anode is extended in the form of a coating of carbon inside the tube, as shown in Fig. 23.1. One might perhaps expect the electrons to be attracted straight to the anode itself instead of going on to the screen. One reason why they do not is that during most of the journey they are surrounded by the anode so there is no difference of potential or electric field.

As with valves, the grid is normally kept negative and takes negligible current.

Although the electron target is generally called the screen, the name has nothing to do with screening as we considered it in Sec.


Fig. 23.1-Section of cathode-ray rube for television. $H=$ heater; $C=$ cathode; $G=$ grid; $A=$ anode $; S=$ fuorescent screen
22.9. The glass is coated with a thin layer of a substance called a phosphor, which glows when it is bombarded by electrons. This effect is known as fluorescence, and is essential in a c.r. tube; but there is also an effect. called phosphorescence, which is a continuance of the glow after bombardment. Phosphorescence depends on the choice of phosphor, and with some dies away in a few thousandths of a second, and with others (for 'long after-glow' tubes) lasts as much as a minute. The colour of glow also is determined by choice of phosphor. The brightness of the glow is controlled by the grid bias, which varies the anode current and hence the rate at which the electrons bombard the screen.

When the electrons have done their bombarding they have to be withdrawn from the screen, or they would soon charge it so negatively that the following electrons would be repelled from it. The bombardment causes secondary emission (Sec. 15.7), and enough secondary electrons are drawn away to the anode to balance those arriving from the gun. In modern television tubes the phosphor is covered with an extremely thin film of aluminium, which provides a conducting path to the anode. The film is too thin to stop the electrons from reaching the phosphor through it, and it is sufficiently mirror-like to reflect the glow and thereby increase its brightness as seen from outside the tube.

The only thing that can be done with the tube as described so far is to vary the brightness of the patch of light on the screen by vary-
ing the grid bias. To make it useful, two more things are needed: a means of focusing the beam of electrons so that the patch of light can be concentrated into a small spot; and a means of deflecting the beam so that the spot can be made to trace out any desired path or pattern. Both of these results are obtained through the influence of electric or magnetic fields. The fact that electrons are so infinitisimally light that they respond practically instantaneously, so that the trace of the spot on the screen faithfully portrays field variations corresponding to frequencies up to millions per second, is the reason for the great value of the cathode-ray tube, not only in television but in almost every branch of scientific and technical work.

In television tubes both focusing and deflection can be done by magnetic fields, but as the electric methods are rather easier to understand we shall consider them first.

### 23.2 Electric Focusing and Deflection

When considering electric fields we visualized them by imagining lines of force mapping out the paths along which small charges such as electrons would begin to move under the influence of the field forces (Sec. 3.3). For example, if two parallel circular plates, A and B in Fig. 23.2a, were maintained at a difference of potential, the lines would be somewhat as marked-parallel and uniformly distributed except near the edges. As an electron moves from, say, A to B, its potential at first is that of $A$ and at the end is that of $B$; on the journey it passes through every intermediate potential, and the voltages could be marked, like milestones, along the way. If this


Fig. 23.2-Diagrams to illustrate electrostatic focusing
were done for every line of force we could join up all the points marked with the same voltage, and the result would be what is called an equipotential line. If potential is analogous to height above sealevel (Sec. 2.14) equipotential lines are analogous to contour lines.

In Fig. 23.2a the equipotential lines (dotted) are labelled with their voltages, on the assumption that the total p.d. is 100 V . An important relationship between the lines of force and the equipotential lines is that they always cross at right angles to one another.

Now introduce a third electrode, C in Fig. 23.2b, consisting of a cylindrical ring maintained at say -20 V . As C is a conductor, the


Fig. 23.3-C.r. tube gun with electric focusing, including potential divider for feeding and control
whole of it must be at this voltage; so the equipotential lines between C and B must be very crowded. Along the axis between A and B , where C's influence is more remote, they open out almost as they were in $a$. When the lines of force are drawn it is seen that in order to be at right angles to the equipotential lines they must bend inwards towards the centre of B.

This is the principle employed in electric focusing. At least one extra anode (corresponding to C ) is adjusted to such a voltage as to make the convergence bring all the electrons to the same spot on the screen. The process is analogous to the focusing of rays of light by a lens, and in fact the subject in general is called electron optics. Just as a lens has to be made up of more than one piece of glass if it is to give a really fine focus, so a good electron lens generally contains at least three anodes at different voltages. The focus is usually adjusted by varying one of the voltages. There are too many different varieties of electron lenses in c.r. tubes to illustrate even a selection, but Fig. 23.3 shows one of the simpler arrangements, with a potential divider for maintaining the electrodes at suitable voltages.

Electric deflection is quite simple. If two parallel plates are placed, one above and the other below the beam as it emerges from the gun
( $Y_{1} Y_{2}$ in Fig. 23.4), and there is a difference of potential between the plates, the electric field between them will tend to make the electrons move from the negative to the positive plate. This tendency, combined with the original axial motion, will deflect the beam upwards or downwards, depending on which plate is positive.


Fig. 23.4-Arrangement of deflector plates in a cathode-ray tuhe
A second pair of plates, $X_{1} X_{2}$, is placed on each side of the beam for deflecting it sideways. By applying suitable voltages between $Y_{1}$ and $Y_{2}$ and between $X_{1}$ and $X_{2}$ the spot of light can be deflected to any point on the screen.

### 23.3 Magnetic Deflection and Focusing

Magnetic deflection depends on the force which acts on an electric current flowing through a magnetic field. In this case the current consists of the electron beam, and the field is produced by coils close to the neck of the tube. The force acts at right angles to the directions of both the current and the field (Sec. 4.2), so if the coils are placed at the sides of the beam as in Fig. 23.5, so as to set up a magnetic field in the same direction as the electric field between the X plates in Fig. 23.4, the beam is deflected, not sideways, but up or


Fig. 23.5-Arrangement of deflector coils in a cathode-ray tube
down. A second pair of coils, above and below, provides sideways deflection.

With electric deflection, the angle through which the beam is deflected is inversely proportional to the final anode voltage. So if, say, 100 V between the plates is sufficient to deflect the spot to the edge of the screen when the voltage on the final anode is 1000 , raising the anode voltage to 2000 makes it necessary to raise the deflecting voltage to 200 . With magnetic deflection, the angle is inversely proportional to the square root of the anode voltage; so if, say, 20 mA in the deflecting coils was sufficient in the first case, it would only have to be raised to 28.3 mA in the second.

Raising the anode voltage thus necessitates more deflecting current or voltage, but it brightens the glow on the screen and generally improves the focus.

For focusing magnetically, it is necessary for the direction of the field to be along the tube's axis. Electrons travelling exactly along the axis are therefore parallel to the field and experience no force. Those that stray from this narrow path find themselves cutting across the field and being deflected by it. That much is fairly obvious, but it is not at all easy to predict from first principles the final result of such deflection. Actually, by suitably adjusting the field strength, the electrons can be made to converge to a focus, but instead of doing so in the same plane as the divergence (as they would in electric focusing) the path is a spiral one. As with electric focusing, the beam is subjected to the focusing field before being deflected.

The magnetic field can be produced by a coil wound around the neck of the tube, in which case the focus is adjusted by varying the current through the coil. Alternatively one can use a ring-shaped permanent magnet, with a moveable yoke for focus control.

### 23.4 Oscilloscopes

To the electronics engineer a cathode-ray oscilloscope is at least as essential as X -ray equipment to the surgeon. In fact there is scarcely any branch of science or technology in which oscilloscopes are not immensely helpful. Some are quite simple; others extremely elaborate.

In order to use a c.r. tube at all there must of course be a power unit to provide the high anode voltage, a variable grid bias (usually labelled 'Brightness', Fig. 23.3), a variable focusing anode voltage, and of course cathode heating.

It is obvious that an electrically-deflected c.r. tube with these auxiliaries can be used as a voltmeter, by applying the voltage to be measured between a pair of deflection plates and noting how far the spot of light moves across the screen. It has the advantage of being a true voltmeter, since current is not drawn; but of course the
anode voltage must be maintained at a constant value or the reading will be inaccurate.

If the deflector voltage alternates at any frequency above a few cycles per second, the movement of the spot is too rapid to be followed by eye; what one sees is a straight line of light, the length of which is proportional to the peak-to-peak voltage. But the possibilities of the c.r. tube are more fully realized when voltages are simultaneously applied to both pairs of plates. For example, if a source of test input voltage to an amplifier is applied to one pair and the output is applied to the other, the appearance of the line or trace on the screen is very informative. If the amplifier is linear and free from phase shift, it is a diagonal straight line. Nonlinearity shows up as curvature of the line, and phase shift as an opening out into an ellipse.

A particularly useful range of tests can be performed if to the horizontally-deflecting (or X) pair of plates is applied a voltage that increases at a steady rate. The c.r. tube then draws a time graph of any voltage applied to the other pair (Y plates), and in this way its waveform can be seen. The usual procedure is to arrange that when the 'time' voltage has swept the spot right across the screen it returns it very rapidly to the starting point and begins all over again. If the cycles of the waveform to be examined are all identical and the start of every sweep is made to occur at the same phase of the waveform, the separate graphs coincide and appear to the eye as a steady 'picture'. The equipment for generating this repetitive time voltage is called a time-base or sweep unit.

Another practically essential facility in an oscilloscope is a system of adjustable deflection-plate bias voltages for enabling the starting point of the spot of light to be placed anywhere on the screen. Usually there is an X -shift control and a Y -shift control.

Magnetic deflection also can be applied to oscilloscopes by means of external coils as in Fig. 23.5, but seldom is, because of the current required and the variableness of coil impedance with frequency. In television however deflection is practically always magnetic.

The usefulness of an oscilloscope would be very severely limited without amplifiers to enable adequate deflection to be obtained from weak signals. Ideally, such amplifiers would give a known constant linear gain at all frequencies from zero to the highest in any signals likely to be applied, and would introduce no phase shift. As we saw in Sec. 22.13, the greater the frequency band to be covered the less the gain that can be expected-or else the greater the amplitude/ frequency distortion. The cost of an oscilloscope depends in no small degree on the specifications of its amplifiers, and especially on their maximum frequency. On some of the cheaper models there is no $\mathbf{X}$ amplifier, the assumption being that either the time base will be used, or some other source of comparable voltage.

Because of the difficulties of providing high gain without appreciable distortion of any kind, high deflection sensitivity of the c.r.
tube is desirable; but this conflicts with the essential requirements of good focus and bright spot trace, which call for high anode voltage and consequently low deflection sensitivity. Tubes for oscilloscopes therefore often include a feature called post-deflection acceleration (p.d.a.), a name that really explains itself. In such a tube the voltage on the anode that accelerates the electron beam ready to pass between the deflection plates may be as low as 1 kV , enabling good deflection sensitivity to be obtained: after deflection the beam is further accelerated by an anode held at 10 kV or even more. Sometimes the acceleration is imparted progressively by a very highresistance spiral of carbon deposited on the inside of the flared portion of the tube and acting as a continuous potential divider between the final anode and the pre-deflection anode.

### 23.5 Time Bases

The ideal waveform for the time-base (X-plate) deflecting voltage (or current, if magnetic deflection is used) as described in the preceding section is shown in Fig. 23.6, which explains why it is called


Fig. 23.6-Sawtooth waveform used for c.r. Whe fime-base deflection
a saw tooth waveform. The most important feature is the constant slope of the sweep, necessary if the time scale of the graph is to be linear.

Time base generation is so important in nearly all applications of c.r. tubes that it deserves some attention now, at least in general principle, although the details come within the scope of the next chapter. The subject is a very large one, and hundreds of time-base circuits have been devised. The basis of many of the methods is to charge a capacitor at a controllable rate through a resistor, and then discharge it quickly by short-circuiting it. With an ordinary resistor the charging is not linear; it follows the exponential curve shown in Fig. $3.6 b$, the reason being that as the voltage across the capacitor rises the charging voltage falls off and so (with an ordinary resistor) does the current.

There are two main ways of overcoming this defect: one is to distort the non-linear sawtooth into a linear one, and the other is to use some sort of resistor that passes a constant current regardless of the decline in voltage across it. Often a combination of these methods is used

In some designs the fly-back is brought into action by the timebase voltage itself which, when it reaches a predetermined value, causes a low resistance to occur suddenly across the charged capacitor. In others the fly-back is initiated independently. A truly instantaneous fly-back would necessitate discharge at an infinite rate, implying an infinitely large current, which is impossible. In practice an appreciable time is required. So in oscilloscopes no attempt is made to use every cycle of a signal waveform to build up the stationary trace on the screen. Fig. 23.7a shows for example four successive cycles of a waveform to be examined. Below at $b$ we see a linearly rising deflection voltage which continues slightly beyond the duration of $a$, to make sure that the whole cycle is included. The fly-back then occurs. During it a negative bias is automatically applied to the grid of the c.r. tube to suppress the beam. At the start of the third cycle of $a$ the time base comes into action again and the waveform is reproduced on the screen; and so on.

To make the time base begin always at exactly the same phase in the signal cycle, so that a steady trace appears on the screen, some kind of synchronizing device is needed. Here again, many techniques have been worked out. The simplest method is to arrange for a controllable amount of the signal voltage to be fed to the timebase generator at such a point that it prods it into action.

If magnetic deflection is used, as in television, a sawtooth current is required. The voltage waveform needed to cause such a current would itself have a sawtooth form only if the impedance of the deflection coils were pure resistance, but they are bound to have some inductance as well, and at high frequencies the inductance may predominate. As we saw in Fig. 4.10, the rate at which current grows


Fig. 23.7-(a) Four cycles of a waveform that is to be exhibited on an oscilloscope screen. To make sure of including the whole cycle, only alternate cycles are projected, the others heing suppressed to allow time for flyback (b)
in an inductance is proportional to the voltage across it. So if we want the current to grow at a constant rate we have to apply a constant voltage.

In general the deflection coils have both resistance and inductance, so if $a$ in Fig. 23.8 is the required deflection current the

Fig. 23.8-ia) Required waveform of magnetic deflection coil current. The voltage waveform to overcome the coil resistance is shown at $b$, the roltage for the coil inductance at c, and the total roltage at $d$

part of the applied voltage needed for the coil resistance would have the form shown at $b$, the part needed for the inductance at $c$, and the total applied voltage at $d$. Quite a large amount of ingenuity has been expended on means for economically generating acceptable approximations to the ideal waveforms (Sec. 23.10).

### 23.6 Application to Television

In cinematography the appearance of motion is produced by showing successive still pictures at a sufficiently high rate to deceive the eye-say 25 per second. Each picture can be regarded as made up of a great number of small areas of light and shade; the greater the number of these picture elements, the better the definition. For entertainment, hardly less than 200000 is acceptable. Now it is obviously impracticable to communicate as many as 200000 radio signals, indicating degrees of light and shade, simultaneously; the only way is to send them in succession. If the whole lot has to be transmitted 25 times per second, the total picture element frequency is $25 \times 200000=5000000$ per second. If light and dark picture elements alternate, each pair necessitates one cycle of signal; so the signal frequency is 2.5 MHz . In practice this calculation has to be slightly modified to take account of other things, but it does give some idea of the problem.

These picture-element or video-frequency (v.f.) signals cannot be directly radiated, because they are liable to have any frequency from at least 2.5 MHz down to almost zero; so they must be used to
modulate a carrier wave just like a.f. signals. But for reasons discussed in Chapter 20 it is necessary for the carrier-wave frequency to be a good many times higher than the highest modulation frequency. That is why television cannot be satisfactorily broadcast on frequencies less than about 40 MHz , and the bandwidth is not a mere 20 kHz or so as in sound broadcasting but extends to several megahertz.

To deal with picture elements successively, it is necessary to scan the scene in such a way as to include them all. The usual method is like that of the eye in reading a page of printed matter; it moves in lines from left to right, with rapid flyback between lines; and this horizontal movement is combined with a much slower vertical movement down the page. It should be clear that scanning of this kind can be achieved in a cathode-ray tube by means of a highfrequency time base connected to the horizontally-deflecting plates or coils, and a low-frequency vertical time base. The spot then traces out a succession of lines, and if its brightness is meanwhile being controlled by the v.f. signals in proportion to the brightness of the corresponding point in the scene being broadcast, a picture will be produced on the screen of the c.r. tube.

To cover all the picture elements, the scene has to be scanned in some hundreds of lines; the greater the number of lines the better the possible definition of the picture. Clearly the scanning at the receiving end must be the same as that in the camera, so a standard number of lines must be agreed upon. Several different line standards are or have been used; for the sake of example the 625 -line standard used in Britain will be assumed. The corresponding number of times the complete picture is scanned is 25 per second, so the line frequency is $625 \times 25=15625$.

25 pictures per second is a low enough frequency to cause noticeable flicker. To avoid this the scene is actually scanned 50 times per second, first the odd lines only, then the even lines. So although the complete picture frequency is still 25 , the frequency of the vertical time base is 50 and fly-back occurs after every $312 \frac{1}{2}$ lines. This subdivision of the scanning is called interlacing, and each scan making up half the complete picture is called a field (rather confusingly, because it has nothing to do with electric or magnetic fields).

A scanning diagram showing all 625 lines would be difficult to follow, so Fig. 23.9 shows a raster (as it is called) of only 9 lines. The principle is the same, however. Beginning at $A$, the line time base scans line No. 1, at the end of which the flyback (shown dotted) brings the spot of light back to the left-hand edge. But not to A, for the field time base has meanwhile moved it downward two spaces to $B$. In the same way all the odd lines are scanned, until No. 9, half-way along which, at $C$, the field time base flies back to the top of the picture. Although for clearness it is shown as doing so vertically, this is not practicable as it would mean infinitely high speed. In a.625-line system a time of up to 31 lines is allowed,
so fewer than 600 lines are actually available for the picture. During field and line fly-back the cathode ray is cut off, so nothing appears. The line time base can now fill in all the even lines, shown dashed.


Fig. 23.9-Nine-line interlaced 'raster'. The dotred lines are fly-backs, which normally are invisible

At the end of the last (D) the second field fly-back returns the spot to the starting point A, though again not by the idealized dotted line.

At the sending end an image of the scene is focused as in a photographic camera; but in place of the film the television camera contains a plate (called the target) inside a special form of cathoderay tube. This plate is sensitive to light, not chemically as in photography, but electrically ( Sec 10.12 ). There are different types in use, but the general result is a charge pattern over the surface, corresponding to the light pattern of the image projected on to the other side of the plate. The electron beam is made to scan the plate as already described and picks off the charges; their voltages are amplified, and used to amplitude-modulate the carrier wave of the sender.
An essential addition to the system is some means for ensuring that all the receiving scanners work in exact synchronism with the scanner in the camera; otherwise the picture would be all muddled up. Synchronizing signals are transmitted during the fly-back periods of the camera scanner, and the receivers separate these from the picture signals and apply them to the time base generators to control their exact moments of fly-back.

### 23.7 Characteristics of Television Signals

Two sample lines of combined signals, as used to modulate the carrier wave, are shown in Fig. 23.10. This is the so-called negative
modulation used with the 625 -line standard, in which the minimum carrier-wave strength ( $10 \%$ of maximum) represents the brightest parts of the picture and $75 \%$ represents zero brightness, or black. The $75 \%$ to $100 \%$ strength is reserved for synchronizing signals. (405and 819 -line systems use positive modulation in which these levels are reversed.) Each line in a 625 -line signal lasts for $1 / 15625 \mathrm{~s}$, or $64 \mu \mathrm{~s}$. Of this, about one sixth of the time is used for fly-back and for emitting a single-pulse line synchronizing signal, which itself lasts for only about $6 \mu \mathrm{~s}$.

In Fig. 23.11 eight lines are shown, but to concentrate attention on the line sync. signals the picture modulation between them is represented merely by the letter P . The rising front of each sync. pulse. marked by a dot, is used to make the receiver line time base transfer the spot of light on the screen to the left-hand edge of the picture ready to start the next line.

Looking again at Fig. 23.9 we see that the last line of the first (and all odd) fields must be interrupted exactly half way in order to ensure accurate interlacing. So at this point, marked X in Fig. $23.11 h$, line pulses begin to occur twice as often. The line time base ignores these extra pulses and continues to respond only to those marked in the diagram with a dot. The six half-line pulses after the last half-line of picture are used to mark time (for less than 0.0002 sec!) in preparation for the field sync. pulses, which consist of six much wider pulses as shown. The rising fronts of those marked with a dot act also as line sync. signals, for the line time base must be kept running steadily even when nothing is appearing on the screen. The field pulses make the spot return to nearly the top of the screen ready to begin an even-numbered field, and after a few more halfline pulses normal service is resumed (Fig. 23.11 a).

Following this field the fly-back takes place at the end of a full


Fig. 23.10-Complete television modulation signal waveform of 2 lives duration
line, so the field sync. signals are as shown in Fig. 23.11c, which needs no further explanation.

An important point is that modulation of a television carrier wave does not take place equally above and below an unmodulated
(a)

(b)

(c)


Fig. 23.11-Television synchronizing pulses. The rises used for line synchronizing are marked with a dot: P denotes a Ime of picture signals. Line pulses only are shown at a; the modifications required at the end of each field for field synchronization are as at $b$ and $c$
level as in the transmission of sound. During intervals between scenes the sync. signals must be continued steadily, and their peaks indicate maximum ( $100 \%$ ) carrier level. If the receiver is to reproduce the black level correctly it must be able to distinguish a carrier wave at $75 \%$ of maximum strength from lower strengths, even when it is unmodulated by a picture (as when a blank screen is shown). In other words, it must be able to respond to the zero-irequency (d.c.) part of the received signal.

As about 600 lines appear on the screen in a 625 -line system, there are a possible 600 picture elements in a vertical direction. The standard picture width is one third greater than the height, so if the horizontal definition were the same as the vertical there would be 800 picture elements horizontally; that is to say, along every line. The duration of the visible part of a line is about $52 \mu \mathrm{~s}$, so the number per second is $800 \times 10^{6} / 52=15.4 \times 10^{6}$. On the assumption that the maximum v.f. is half the frequency of the picture elements, it amounts to $15.4 / 2=7.7 \mathrm{MHz}$. The most allowed for in the $625-$ line specification is 5.5 MHz , so the horizontal definition is necessarily less than the vertical.

If amplitude modulation were used in the same way as for sound broadcasts, even this reduced v.f. would occupy a radio bandwidth of $5.5 \times 2=11 \mathrm{MHz}$. And room must be made for the accompanying sound, and also some separation from signals of other stations.

So the total width of a television channel would be so great that there would be insufficient channels even in the considerable v.h.f. and u.h.f. bands available. Moreover the gain of each amplifier stage in the receiver is inversely proportional to the bandwidth (Sec. 22.6), so many stages would be needed. We have seen how amplitude modulation at any one frequency results when a carrier wave and two other waves, one at a lower frequency and the other at a higher, are added together, as in the phasor diagram Fig. 20.3. If we omit one of the two sideband phasors there is still amplitude modulation, and when rectified in the receiver the modulation frequency is still extracted. We can easily see that there would be some distortion of the modulation waveform, but at low modulation depths it would be very slight.

Now it happens that in television the depth of modulation is normally small at the upper video frequencies. It has been found that little is lost if the receiver is made sufficiently selective to tune out most of one sideband. The tuning is adjusted by the makers so that the carrier-wave frequency is half-way down the lower-frequency slope of the overall selectivity curve, as shown by the broken line in Fig. 23.12, where the theoretical receiver acceptance is shaded. So the receiver bandwidth problem is almost halved. Note that at very low video side frequencies (those closest to the carrier frequency) reception is practically ordinary double sideband. Up to 1.25 MHz one side becomes progressively weaker but the other progressively stronger, so that the total continues to be the same, as it does also above 1.25 MHz where reception is entirely singlesideband.

Radiating the whole of the lower sideband would merely broaden the channel unnecessarily, so all of it beyond 1.25 MHz is more or


Fig. 23.12--The ideal television receiver tuning acceptance is shown shaded; the broken line indicates a more practical performance
less filtered off at the sending end. The frequencies broadcast are therefore as shown in Fig. 23.13. (The received part is again shaded.) The width of an entire channel is thus reduced to 8 MHz . Note that these diagrams do not show the actual amplitudes at the various frequencies in the broadcast signal; they naturally depend on the
scene being televised at any moment, and usually fall off with increasing video frequency.

Either a.m. or f.m. can be used for television sound ; in the $625-$ line standard it is f.m. The v.f. modulation is always a.m.

The specification shown faces a receiver with quite a difficult job, for the vision section ought to respond as in Fig. 23.12 over a


Fig. 23.13-Diagram of the frequency allocations in one complete 8 MHz relevision channel. The shaded portion is again the ideal receiver acceptance
6.75 MHz band and yet reject the associated sound only 0.5 MHz higher-and the adjacent-channel sound 0.75 MHz lower. Not infrequently receivers fail to do this, and the reproduced picture is visibly disturbed by the sound signal.

The foregoing information refers to signals for plain television (i.e., without colour). The main modifications needed for colour come in Sec. 23.11. By the way, 'black and white' is a clumsy and inaccurate description for pictures without colour, and 'monochrome' is wholly wrong because it means 'one colour', whereas white is composed of every colour! So, recalling the old advertising slogan, 'penny plain, twopence coloured', why not 'plain'?
lgnoring the quite considerable problems at the sending end, let us now take a broad look at what a television receiver must include.

### 23.8 Television Receivers

Fig. 23.14 is a block diagram of a typical receiver for 625 lines. Usually there is a single preselector stage to ensure that the signal is strong enough when it comes to the relatively noisy frequency changer. We have already seen an example of this part of a television receiver; Fig. 22.16. It should be designed to amplify the full channel frequency band of 8 MHz equally; for example, from 470 to 478 MHz for the lowest-frequency u.h.f. channel, No. 21. If this is done, the adjacent-channel selectivity is likely to be hardly noticeable, but fortunately the range of reception on such frequencies is
limited to the fairly local area and any other senders in that area would be allotted widely different frequencies. Change of channel at v.h.f. is usually obtained by switching different pre-tuned inductors into circuit, but with the rod-and-cylinder tuners used for u.h.f. this is not practicable and tuning is by ganged capacitor.

The frequency changer shifts the whole frequency pattern of the channel to the i.f., usually 33.5 MHz for vision and 39.5 MHz for sound. Because the amplifier has to cover such a wide band in relation to this i.f., the gain per stage is limited and several stages are needed. In our example, vision and sound continue together through the detector and even the v.f. amplifier. By that stage the frequencies present are as shown in Fig. 23.13, if the vision carrier is taken as zero frequency. The sound carrier wave, beating with the vision carrier, yields a 6 MHz signal, with the sound sidebands around it. The vision detector, in fact, acts as a second frequency changer for the sound signal, having an output i.f. of 6 MHz , which is amplified in the v.f. stage along with the video frequencies. By means of various tuned rejector circuits (often called traps) the


Fig. 23.14-Block diagram of typical television receiver (plain)
vision and sound signals are separated from one another. For the principles of the v.f. amplifier see Sec. 22.13.
The sound signals may be further amplified at what is called the intercarrier i.f. of 6 MHz before being passed through the f.m. detector (limiter and discriminator) to the a.f. amplifier and loudspeaker. All this follows the principles described in earlier chapters.

Alternatively the sound and vision may be separated earlierperhaps somewhere along the main i.f. amplifier-or there may be a separate intercarrier detector for sound.

If the v.f. signals are applied to the cathode of the c.r. tube, less signal voltage is needed than if the grid were used, because making
(a)



Fig. 23.15-Showing television signal voltage levels with (a) and without (b) d.c. restoration
the cathode more positive increases the negative bias and reduces the first anode voltage (relative to cathode), both of which reduce brightness. Moreover the polarity of the picture signals that is correct for the cathode normally happens also to be right for dealing with the synchronizing signals. So cathode modulation of the beam is usual, the grid being reserved for control of general picture brightness by the viewer; there is no scarcity of voltage for that.

Because of the polarity of cathode modulation just referred to, the vision detector and v.f. amplifier must be arranged so that an increase in radio signal amplitude makes the cathode more positive, the $75 \%$ (black-level) amplitude being just enough to extinguish the spot altogether. The sync. pulses then make the cathode more positive still, so are unable to produce any visible effect, being 'blacker than black'. The same applies to a positive modulation system if the detector is connected the opposite way.

If a series capacitor has had to be used somewhere in the v.f. system, the d.c. part of the signal is blocked off. This means that in a waveform diagram there will be equal areas above and below the zero-signal line, marked 0 in Fig. 23.15a, which shows two sample lines of video signal, the first corresponding to a bright scene and the second to a dark one. The brightness control in the receiver should have the effect of biasing the c.r. tube so that the signal level marked 'black' corresponds to extinction of the screen illumination by the beam. The diagram shows that if correctly adjusted for the bright scene it would be quite incorrect for the dark scene. This fault can be overcome by using what is called a d.c. restorer. It is simply a diode rectifier which 'pushes the signal down' so that the tips of the sync. pulses are at a constant level as shown in Fig. 23.15b and explained fully in Sec. 18.8. See also Sec. 24.10. Designers do not
always bother to include a d.c. restorer and the result is that scenes supposed to be in near-darkness appear on the screen fairly bright.

### 23.9 Synchronizing Circuits

A third outlet from the v.f. amplifier goes to the sync. separator, which removes the picture part of the signal, leaving only the sync. signals. It consists of what looks like a perfectly ordinary stage of resistance coupled amplification, Fig. 23.16 for example. As shown, the input is the composite v.f. signal with the sync. pulses positivegoing. (It would have to be reversed for a p-n-p transistor.) Here


Fig. 23.16-Example of a sync. separator circuit
again the d.c. component is removed by C , with the result that the sync. pulses move up and down with scene brightness as shown in Fig. 23.15a, and a bias setting suitable for removing all but the sync. signals during a bright scene would remove the sync. signals too during a dark one. But it is possible to put this right by d.c. restorer action. $R_{1}$ in Fig. 23.16 is given an unusually high value, so that the base-to-emitter diode is on its bottom bend. It therefore rectifies the input signal, pushing it down until during the whole of the picture part the transistor is cut off. Only the sync. pulses appear at the output, where, owing to inversion by the amplifier, they are negative-going. The d.c. restorer action also automatically ensures that any excess pulse amplitude is cut off by the excess bias it develops.

If a valve or f.e.t. is used, no bias at all is needed, as the desired range of operation as a pulse amplifier is obtained between zero and cut-off bias.

Next, the field sync. signals have to be sorted out from the line signals which function at regular $64 \mu$ s intervals all the time. The principle of this sorting out can be understood by referring back to our capacitor-charging experiment, Fig. 3.5. Line signals (Fig. $23.11 a$ ) which consist of brief pulses separated by relatively long
intervals. can be simulated by flicking the switch alternately between A and B, the B periods being (say) 10 times as long as the A periods. The charging voltage thus varies as in Fig. 23.17a. The voltage across the capacitor has hardly begun to rise by the time the charging voltage drops back to zero. So it never gets a chance to build up (b). But if the A and B periods are reversed to simulate a train of field pulses ( $c$ ) the capacitor charge builds up as shown at $d$. The corresponding voltage in a television receiver is fed into the field time base generator to bring the fly-back into action.

The difference between the odd and even fields, necessary for interlacing, has been explained in connection with Fig. 23.11.
(a)


Fig. 23.17-The normal narrom line symc. signals (a) do not build up a field sync. signal (b), but the wide signals (c) do (d). Both a and $c$ yield line sync. pulses it the same phase (e and f)

As the dots in Fig. 23.11 indicate, line synchronizing is effected by the rising fronts of the pulses. Apart from the half-line pulses, which the line time-base generator ignores (see Sec. 24.6 with reference to Fig. 24.16) these fronts are equally spaced whether the pulses themselves are narrow or broad. To prevent the different widths of the field signals from disturbing the steady rhythm of the line time base, the return strokes of all the pulses are eliminated before the line signals are fed to the line time base generator. This is done by applying the combined sync. signals to another capacitance and resistance in series, but this capacitance is smaller and it is the voltage across the resistance that is used to initiate the fly-back. Fig. 23.17e shows this voltage, which has the same shape (speeded up) as in Fig. $3.6 a$ showing the current in the circuit and therefore also the voltage across the resistance. Substituting field sync. signals alters the pattern to Fig. 23.17f, but there is no difference in the rising pulses, and the falling ones are eliminated by biasing off.

If the fly-back for every line were initiated directly by its own individual sync. pulse, and spasmodic interference was coming in along with the sync. signals, the lines affected would be displaced and the picture torn apart horizontally. To avoid this, a system called flywheel synchronization is commonly used. Without going into details one can say that the line time base is allowed to run continuously, producing pulses of its own at every fly-back. These are compared with the incoming sync. signals in what is called a phase detector. If both occur at the same frequency, nothing happens. But directly the line generator gets out of step, the phase generator produces a voltage, the polarity of which depends on whether the generator is running fast or slow. This voltage is applied to the generator in such a way as to correct its speed. The time constant of the system is chosen so that correction is gradual; occasional mutilation of the line sync. pulses by interference then has little or no effect, as there is an averaging out of the control.

### 23.10 Scanning Circuits

The line time base is the greater problem because of the greater amplitude required and its much higher frequency and the correspondingly short time - only a few microseconds-for the current through the deflection coils to be changed from maximum one way to maximum the other way during fly-back. This is not remarkably fast by oscilloscope standards, but television is faced with the demand for maximum screen size combined with minimum distance from back to front. That means a very much larger angle through which the beam has to be deflected, and therefore a correspondingly larger effort from the time base generator (or scanner). Electric deflection is not practicable, so substantial current has to be reversed very rapidly. And as this is only one of many functions
in a complete receiver, only a limited amount of power and space can be allowed for it.

The subject is extremely complex, and circuit diagrams of the many varieties of line scanner that have been used usually give little clue to the exact working conditions. In spite of the wide variations in detail, especially between valve and transistor circuits, the basic principle is the same for all. Instead of providing a new lot of energy for each stroke- 15625 every second-energy is made to circulate to and fro, only the losses needing to be made up.

The basic method is so like the generation of continuous oscillation as described in Sec. 14.2 that it might be wise at this point to re-read that section. Although the line time base output stage proved to be the most difficult in a television receiver to transistorize, the transistor circuits are on the whole simpler than the valve forms. The example now to be described is the simplest of all, but it should be understood that its working has been idealized and that in practice there are many complications.

Fig. 23.18 shows the circuit. At the time of writing the transistor is more likely to be a p-n-p type, but an n-p-n type is more convenient for explanation and corresponds more closely in appearance with valve circuits; apart from reversed polarities the p-n-p circuit and functioning are the same. The type of transistor is one that can pass a heavy current, say 10 A , and withstand a relatively high collector voltage, say 150 V -but not both at the same time, as we shall


Fig. 23.18-Ideally simple circuit for line deflection current shaping
see. Because it can pass current only in one direction, whereas the current in the deflection coils is required to alternate equally in both directions, a diode D is connected in parallel to provide a path for the opposite current. Although the line deflection coils are normally connected to a transformer winding and include some resistance, they are represented ideally by an inductance $L . \mathrm{C}$ is an energystoring capacitor, which is effectually in parallel with $L$ because the $V_{C C}$ supply is shunted by a capacitor of negligible impedance.

The base input consists of negative-going pulses as shown in Fig. 23.18 and also in Fig. 23.19a, corresponding approximately to line
sync. pulses. All the time between them the base is positive. Let us begin half-way through this inter-pulse phase. The transistor is bottomed, passing current with hardly any emitter-to-collector voltage ( $v_{c}$ ). Practically all of $V_{c c}$ is therefore across $L$, so the current through it grows at a steady rate. Fig. $23.19 b$ shows $v_{c}$ almost zero, and at $c$ the current $i_{T}$ growing steadily. Because $v_{C}$, such as it is, is positive, no current $i_{D}$ can flow through $D$, and only a very small charging current $i_{c}$ flows into $C$, so the voltage across it $\left(v_{c}\right)$ rises almost imperceptibly-see Fig. 23.19d, $e$ and $b$. The current
(a)


Fig. 23.19—Waveform diagram for Fig. 23.18
through the deflection coils is therefore practically $i_{T}$ so the spot on the screen moves steadily from the vertical centre towards the right-hand edge.

By the time $i_{L}$ has become enough to deflect the spot to the edge, the input pulse arrives and cuts of $i_{T}$. This moment corresponds to the point $a$ in Fig. 14.2 in a practically identical circuit. What happens is that $i_{L}$ (which cannot change suddenly because of the inductance $L$ ) is diverted to C and starts charging it-note the
sudden jump in $i_{c}$, equal to the drop in $i_{T}$. The voltage $v_{\mathcal{C}}$ builds up to a positive peak, where momentarily it is neither rising nor falling, so $i_{L}$ and $i_{C}$ must be zero. This corresponds to $b$ in Fig. 14.2. C then starts discharging, so $i_{C}$ (and therefore $i_{L}$ ) reverses. And so $v_{C}$ and $i_{c}$ continue to follow the same half-sine-wave shape as in Fig. 14.2, $a$ to $c$. This makes the spot on the screen (extinguished by a negative pulse on the grid of the c.r. tube) fly back to the left-hand edge). From there things are different, at first because of D. Directly the negative half-cycle of Pc begins, D conducts and being practically a short-circuit it holds $v_{c}$ nearly constant at a very small negative value. The voltage across L is therefore again virtually equal to $V c c$. So $i_{L}$ again rises steadily, carrying the spot from left to right. Because $v_{c}$ hardly varies, C hardly charges at all, and $i_{L}$ and $i_{D}$ are almost the same current.

This state of affairs continues until about half-way between the input pulses, by which time the transistor is once more ready to conduct. Directly $v_{c}$ becomes positive it does so, taking over from D. This is where we came in, so the whole programme repeats, continuously.

The current supplied by $V_{C c}$ is $i_{L}$, and according to Fig. 23.19f it is entirely a.c., so can be supplied by a large capacitance across the $V_{c c}$ source. No net d.c. power is needed. This ideal cannot of course be achieved in practice; we have already noted the resistance of the coils as one cause of loss. But the power to be supplied is only a fraction of what would be needed if it were not for the energystoring ability of C .

That is not the only economy effected by this type of circuit. We have noted the rapid change in $i_{L}$ during fly-back, giving rise to a peak of $v_{c}$. The inductance $L$ must be quite small if this voltage peak is to be within the limit of what the transistor can stand. That is to say, it must have only a few turns of wire and the current $i_{L}$ must be large in order to deflect the beam sufficiently. (A valve in this role can stand far more voltage but less current, so the number of turns is much greater.) As was mentioned earlier, the actual deflection coils are connected across a low-voltage transformer winding, through which the rapid change in current occurs at fly-back. Another winding having a very large number of turns of fine wire steps up the voltage peak to something like 16 kV , and a highvoltage low-current rectifier suppresses the reverse voltage so that the d.c. output is available for the final anode of the c.r. tube. Because of the small current load, sufficient smoothing is obtained by the capacitance (of the order of 1.5 nF ) between the inner conducting coating of the tube and an external one laid on for that purpose. Economy again!

In transistor sets there is also a need for other voltages greater than $V_{c c}$; these too can be obtained in similar manner from this line output transformer. With these omitted, the line output stage is likely to look something like Fig. 23.20. $\mathrm{D}_{2}$ is the e.h.t. (extra
high tension) rectifier. If a thermionic type is used its heater is fed from a few highly-insulated turns of wire on the transformer.

Owing to the comparatively slow rate at which current in the field deflection coils changes, their inductance is less significant than their


Fig. 23.20-Showing how the type of line scanning circuit shown in Fig. 23.18 is adapted to produce also e.h.t. supply for the c.r. mube
resistance. The voltage needed has therefore a mainly sawtooth form and the stage supplying the coils is basically an amplifier of such a waveform provided by the field scanning generator. Neither field nor line deflecting currents would be sufficiently linear without means for linearizing, details of which can be found in books on television receiver design.

### 23.11 Colour Television

Reproduction of scenes in colour, by photography, printing and television, relies on the fact that any colour can be acceptably made up from three primary colours in the right proportions. The most apparently straightforward way of applying this fact to television is to transmit three complete pictures-one in each primary colourin place of each one in the system we have just been studying. Assuming for the moment that pictures in the three separate primary colours (to each of which the term 'monochrome' correctly' applies) can be reproduced on a single screen within $1 / 25$ th sec, 'persistence of vision' makes them appear collectively as a picture in full colour.

Although this scheme has been successfully demonstrated, the practical objection is that because everything has to be done three times as fast the bandwidth required is three times the already awkwardly wide requirement for plain television. Alternatively, the three primary colours could be broadcast concurrently on three channels,
but the total frequency band would of course still be three times that for one ordinary channel. This factor of 3 could no doubt be cut down to something slightly less by taking advantage of the fact that with colour a rather lower standard of definition is acceptable. Even so, the economic disadvantages of this approach are so considerable that very much more complicated systems have been devised in order to avoid it and keep colour television within the standard channels. These are the systems in actual use. Although there are several, the differences between them are only in details which are too involved to include in a book of this kind. An essential requirement of any practical colour' system is that it be compatible; that is to say, colour programmes must be receivable on non-colour receivers just as if they were uncoloured, and uncoloured programmes receivable as such on colour receivers.

The basic problem-to restate it in a different way-is to convey extra picture information (colour) without widening the frequency band. A solution might appear impossible, for a fundamental law of communication (Hartley's law) is that the product of bandwidth and time needed to convey a given amount of information is constant. So if you try to convey more information in a given time you have to use more bandwidth. This law assumes that the information is coded in the most economical manner. The compatibility requirement is the apparently insuperable difficulty, for it rules out any more economical coding of the basic picture (without colour), even if that were possible. So no room seems to be left for colour information.

The solution is to superimpose the colour information on the plain picture information (luminance) within the same frequency band, coding the colour information so unobtrusively that it does not ruin the picture. Provided that the normal standard of luminance definition is preserved, quite a low definition of colour is acceptable, as infants demonstrate with their painting books. The colour v.f. is


Fig. 23.21-For comparison with Fig. 23.13 here are the frequency allocasions for a colour television signal
therefore limited to about 1 MHz . And it is put high up in the luminance v.f. band, centred on a sub-carrier-wave frequency of approximately 4.43 MHz , because the interference with the luminance information is least noticeable in this region of frequency. The subcarrier itself certainly would be very noticeable if it was broadcast, but it is suppressed at the sender and restored in the receiver. Fig. 23.21 shows, for comparison with Fig. 23.13, the resulting channel structure. By the way, the technical term for the colour information is chrominance, and it comprises two parts: hue and saturation. Hue is the colour itself-red, blue, etc.-and saturation is the intensity of the colour. In an uncoloured picture all colours have zero saturation, and 'pastel shades' are low in saturation compared with 'geranium red'. Saturation is conveyed by the amplitude of the chrominance signal, and hue by its phase displacement. The whole subject, theory and practice, is a vast one, and even an outline of it is enough to fill a book, so cannot be included here.

### 23.12 Application to Radar

If we make a sound and hear an echo $t$ seconds later, we know that the object reflecting the wave is $550 t$ feet distant, for sound travels through air at about 1100 feet per second and in this case it does a double journey. Radar is based on the fact that radio waves also are reflected by such objects as aircraft, ships, buildings and coastlines. If the time taken for the echo to return to the point of origin is measured, the distance of the reflecting object is known to be $150000 t \mathrm{~km}$ ( $93141 t$ miles) away, the speed of radio waves through space being almost 300000 km per second. Since distances which it is useful to measure may be less than a kilometre, it is obvious that one has to have means for measuring very small fractions of a second. In radar, as in television, time is measured in microseconds ( $\mu \mathrm{s}$ ).

This is where the cathode-ray tube again comes in useful. With a time base of even such a moderate frequency as 10 kHz one has a scale that can be read to less than $1 \mu \mathrm{~s}$, and a much faster time base can easily be used if necessary. The time between sending out a wave and receiving its echo can be measured by making both events produce a deflection at right angles to the time base line. This can best be understood by considering a typical sequence of operations.

We begin with the spot at A in Fig. 23.22, just starting off on a stroke of the time base. Simultaneously a powerful sender is caused to radiate a burst or pulse of waves. This is picked up by a receiver on the same site, connected to the $Y$ plates so that the spot is deflected, say to B. The pulse must be very short, so that echoes from the nearest objects to be detected do not arrive while it is still going on. In practice it may have to be a fraction of a microsecond, and as it has to consist of a reasonable number of r.f. cycles, their
frequency obviously has to be many MHz . A more pressing reason for extremely high frequency is that it enables one 10 radiate the pulses precisely in any desired direction (Sec. 17.18). Typical frequencies are 3 GHz and 10 GHz ( 10 cm and 3 cm wavelengths).

The time base voltage continues to move the spot across the screen, and if the receiver picks up one or more echoes they are

Fig. 23.22-Appearance of trace on cathoderay tube in one type of radar receiver

made visible by deflections such as $C$. When the spot reaches the end of its track, at $D$, it flies back to the start; then, either at once or after a short interval, begins another stroke. The sender, being synchronized, registers another deflection at $A$, and the echo is again received at $C$. The repetition frequency is normally high enough for the trace of the spot to be seen steadily as a whole.

The time for an echo to return from a distance of one kilometre being $1 / 149896 \mathrm{~s}$, is equal to the time occupied by one cycle of a 149896 Hz oscillation. By switching the Y plates over to a 149896 Hz oscillator, so that the cycles are seen as a wavy line, the time base can be marked off in distances corresponding to kiłometres.

This is only a mere outline of one of many ways in which the c.r. tube is used in radar. Fig. 23.22 shows what is known as an A-type display. The only direct information it yields is distance. For most applications one wants to know the direction of the reflecting object in the horizontal plane, the term for which is azimuth. A ship's navigator, for example $e_{\text {in }}$ in fog wants to be able to see the positions of coasts and other ships all around within a certain range. It is possible to use the A-type display for this by having an extremely directive aerial system and turning it around until the echo is at a maximum the angle of the aerial then reveals the direction of the echoing object, or target, as it is called.

This early method has been superseded by the plan position indicator (p.p.i.) in which the time base starts from the centre of the screen and moves radially outward like a spoke of a wheel. The aerial rotates steadily and the time base deflection coils rotate in synchronism with it so that the 'spoke' moves around and covers
the whole screen in each revolution. Normally the glow is almost completely suppressed by negative bias at the grid of the c.r. tube, so the rotating line can hardly be seen; echo signals are made to reduce the bias, bringing up the glow at the appropriate radial distance and at an angle which indicates the direction of the target. The phosphor of the screen has a sufficiently long after-glow to keep echoes visible until they are renewed the next time round. If


Fig. 23.23-P.P.I. type of radar display
there is relative movement between the target and the radar, the echo moves gradually across the screen and the relative speed and direction of motion of the target can be found. Fig. 23.23 shows a typical p.p.i. trace. The range scale is marked by a series of fixed circles. This is the normal form of display for shipborne radar and for airport surface movement control.

For military and police purposes the p.p.i. method of detecting and measuring movement is not nearly fast enough, so an entirely different technique is used. Everyone is familiar with the change in pitch of a police siren or other source of pitched sound as it passes along the street. While it is getting nearer the pitch is higher than if the source were kept at a constant distance, because each successive cycle starts from a nearer position and so has a shorter journey and arrives after a shorter interval. When the source is receding, conditions are reversed and the pitch is lowered. This is called the Doppler effect, and applies also to radio waves. A Doppler radar is designed to measure small differences in radio frequency between the signals radiated and the echoes received. A meter indicating this difference is scaled in relative speed towards or away from the observer.
For purposes such as air traffic control, the height of the aircraft observed is important. One way of obtaining this information is to use an aerial system that is highly directional in the vertical phane and to move it up and down ('nodding') instead of or in addition to rotating it continuously or from side to side. Fig. 23.24 shows that the height of the target aircraft depends not only on the measured
angle of elevation of the radar beam ( $\varphi$ ) but also on the distance along the beam (given by the echo delay) and at long ranges is further complicated by the curvature of the earth. So determining the height is not altogether simple. The display of information presents problems too. One method is to show the echoes as spots on a graph on the c.r. tube screen, angle of elevation being plotted against range. Direction has to be shown separately by the azimuthal angle of the aerial. Another scheme is to have several beams at different fixed angles of elevation, together wide enough to cover the whole range, with some overlapping of adjacent beams. The whole system rotates continuously in azimuth, and there is a p.p.i. for each beam as well as one showing the combined echoes of them all. Each of the range zones on any of the separate p.p.is, between a consecutive pair of the range circles, is marked with the upper and lower limits of height of any echo in that zone, depending on the beam angle for that p.p.i.

Nearly all types of radar use radio pulses, and we have already noted that for short ranges the pulse has to be brief enough to be over by the time the nearest echo could arrive. For an object 50 m away the return journey is 100 m and is done in $\frac{1}{3} \mu \mathrm{~s}$, so the pulse would have to be shorter than that. Short pulses also enable the shape of the target to be traced in greater detail than long ones.

Another basic specification of a radar system is its pulse recurrence frequency (p.r.f.). The p.r.f. must not be so great that more than one pulse is radiated in the time represented by the longest range. If that range were, for example, 150 km , the time between


Fig. 23.24-Showing how the height of an aircraft detected by a radar beam depends on angle of elevation $(\varphi)$ and distance
pulses would have to be at least $1000 \mu \mathrm{~s}$, or 1 ms , so that the p.r.f. could not be more than 1 kHz .

So a moderately long-range radar might have a pulse length of $1 \mu \mathrm{~s}$ and a p.r.f. of 1 kHz . The sender would thus be radiating during only one thousandth of the time it was on-a proportion called the duty cycle. The pulse power in this example would be 1000 times the average or mean power, which is the figure that counts in reckoning heating of the sender equipment. An installation of quite moderate size can radiate a pulse of the order of 1 MW -enormously more than it could do continuously. Pulse radar can thus achieve ranges of hundreds of miles even though the amount of power reflected from a small distant target is only a minute fraction of that radiated, and though only a minute fraction of the total reflected power is received back at the radar.

As in television, the shorter the pulse the higher the maximum modulation frequency and the greater the bandwidth. Even if the pulse has rounded corners instead of being square-cut the bandwidth must be reckoned as not less than $2 / T$, where $T$ is the pulse length; for example, 4 MHz for a $0.5 \mu \mathrm{~s}$ pulse. The design of radar receivers is thus something like that of television. But the incoming signal is far weaker, and usually at a much higher radio frequency. Amplification at that frequency is usually impracticable by ordinary methods, so the signal may be fed straight into a crystal diode, which in conjunction with an oscillator does the job of frequency changer. There will be two or three times as many i.f. stages as in a domestic television receiver, with an i.f. of the order of 50 MHz , followed by detector and 'video' amplifier to bring the signal level up to that needed to deflect or modulate the c.r. tube beam.

The same sharply directive aerial is used for both sending and receiving, with special devices to direct the power along the appropriate feeders and to protect the sensitive receiver from the high sender power.

## Electronic Waveform Generators and Switches

### 24.1 Waveforms

Up to the last chapter, 'signals' were normally sinusoidal. That chapter was an introduction to applications in which sine waves hardly appear, except as radio carrier waves used to carry extremely non-sinusoidal forms. Most of these are of two main kincs. There is the pulse, which is often made to recur at regular intervals. as in Fig. $24.1 a$. If the pulses and infervals are equal, or nearly so, as at $b$, they are commonly called square waves, but in principle are the same. The next most important waveform is the ramp or triangle, which in recurrent form is commonly called sawtooth (c). We have already seen in Fig. 23.8d an example of how other waveforms can be compounded of these two. We have also had some inkling of the ease with which the sharp corners of such ideal forms as in Fig. 24.1 can


Fig. 24.1-Examples of the most important non-sinusoidal waveforms: (a) narrow pulse, (b) square wave and (c) ramp or sawtooth
be rubbed off in real circuits with their stray capacitances and in-ductances-even in circuits where sine waves are well preserved. On the other hand circuits suitable for pulses are quite likely to distort sine waves very seriously. So, having given so much attention to sine waves, we now need to look a little into the very different philosophy of sharp-cornered waveforms, the relative importance of which has for some time been increasing.
In spite of the very different principles and practice ruling sinusoidal and non-sinusoidal signals, there are connecting links in theory, and it is possible in practice to convert one to the other.

As far back as Fig. I. 2 we saw how, by adding together several
pure sine waves, we could get an obviously non-sinusoidal form. If this form is to repeat exactly, at a frequency $f$, it must be made up exclusively of sine waves having frequencies $f, 2 f, 3 f, 4 f$, etc. (but not necessarily all of them) ; that is to say, a fundamental (frequency $f$ ) and its harmonics. Even though Fig. 1.2c does show some signs of squarishness, however, we might perhaps doubt that it is even theoretically possible to make a perfect square wave - still less a narrow pulse-out of pure sine waves. We might have even stronger doubts about the possibility of square waves produced simply by switching, as in Fig. 3.5, being composed of sine waves.

Admittedly a perfect square wave, with vertically rising fronts, requires an infinite number of harmonics, but they are of steadily decreasing amplitude, so a very good approximation to a square wave can be made within practical limits of frequency. The ideal recipe is to add together all the odd harmonics only, in phase at the start, the amplitude of each harmonic being inversely proportional to its number. So if 1 is the relative amplitude of the fundamental, the amplitude of the third harmonic is $\frac{1}{3}$, the fifth $\frac{1}{5}$, and so on. Fig. 24.2 shows the result of adding together a fundamental and its first seven odd harmonics (i.e., up to the 15 th) in this way. The flat parts are still rather wavy; higher harmonics are needed to flatten them out.

It should now be clearer why a wide frequency band is needed to reproduce square waves without distortion. The result of passing a


Fig. 24.2-This approximately stuare wave was obtained hy adding logether a fundamental sine wave and its first seven odd harmonics in the right proportions, in phase at the start
perfect square wave through a filter that cuts off sharply at a frequency just over 15 times its fundamental frequency would be a distorted square wave like Fig. 24.2. In practice the loss of signal with increasing frequency is usually more gradual and the distorted forms not exactly like this. The main defects are: appreciable rise and fall times; 'overshoot', in which the fronts rise too far and then fall back, as in Fig. 22.27b; and 'ringing'-damped oscillation, due to circuit resonance. If the low frequencies (mainly the fundamental) are reduced the flat parts drop. Fig. 24.3 shows all these defects.

Narrow pulses are more difficult still to amplify without distortion because the proportions of the higher harmonics are greater. Triangular waves like Fig. 24.1c but with equal up and down slopes,
on the contrary, are easier because the harmonic amplitudes decline inversely as the square of the harmonic number.

The foregoing are just a few examples of the Fourier principle (Sec. 19.6) which states that any periodic (i.e., exactly repeating)

Fig. 24.3-Typical result of passing a nearly perfeal square wave through circuit having restricted frequency coverage

waveform can be analysed into harmonic sine waves. Put in the reverse way, any periodic waveform can be synthesized by adding together sine waves, as we have been doing in Fig. 24.2 for the particular case of square waves. Reference books on signals often include formulae for the most useful waveforms. As with those we have been considering, they are all unending series, but one can slop at the point where a close enough approximation has been obtained.

### 24.2 Frequency and Time

By now it should have become clear that there are two alternative ways with signals. They can be treated either on a frequency or a time basis. If they are sinusoidal, frequency is much the more convenient. This particular waveform is unique in having only one frequency and in not being altered by any linear circuit composed of $R, C$ and $L$. All the work we have done (notably in Chapters 6-8) on reactance, impedance, phasor diagrams, etc., may seem to have been quite hard -and it is only an introduction to much more advanced study of this kind-but it is dead simple compared with the mathematics of non-sinusoidal signals in circuits. That is one reason why the sine waveform is so much used. Reactance is based on frequency, and by enabling a.c. circuits to be calculated by an extended form of Ohm's law it provides a marvellous short cut. So much so that when dealing with non-sinusoidal signals it may actually be easiest (as we noted in Sec. 19.6) to analyse them into theircomponent sine waves, find out what happens to these separately in the circuit, and then put the results together at the end. In general the output waveform will not be the same as at the input (Fig. 19.5). This procedure is like running round a tennis ball to avoid having to play it backhand. And in any case it relies on the circuit law (Superposition theorem) that the parts of a total current in a circuit can be

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calculated separately, not being affected by the other parts; and this is not true of non-linear circuits. (In frequency changers, for example, currents obviously behave differently according to whether they are together or separate.)

At the opposite extreme are simple on-off square or pulse waveforms in certain simple circuits. We have considered several of these: the charging capacitor (Sec. 3.5); the energizing inductor (Sec. 4.8);


Fig. 24.4-Simple combination of circuit elements used as a starting point for study of two alternative approaches to signals
the oscillator (Sec. 14.2); and quite recently the more complicated linear time base (Figs. 23.18, 19). Significantly, a warning was necessary that the actual behaviour of the last of them is a good deal more complicated than as shown. These were all treated on a time basis. Any reference to frequency was at once converted into time; pulse recurrence frequency, for example. This is very easy because frequency and time are reciprocals of one another: $f=1 / t$.

Take the extremely simple circuit shown in Fig. 24.4. If a sinusoidal voltage were applied to this, calculation of the resulting current and the separate voltages across C and R would obviously start with calculating the reactance of $C, 1 / 2 \pi f C$. That is the frequency approach. If, however, the input were a square wave we would deal with it as in Sec. 3.5, which would involve time. (Remember, the time constant is $C R$.) It is quite possible, but infinitely tedious, to arrive at the same result by breaking down the square wave into its sinusoidal harmonics, calculating the separate currents according
(a)


Fig. 24.5-Three different signal
(b)
 waveforms as they appear when graphed against time (left) and frequency (right)
(c)

to the impedance of the circuit at each of their frequencies, and adding together all the resulting waves on a base of time. On the other hand, it would be quite a puzzle to deal with the single sine wave by the method that was so easy with the square wave, in terms
of the time taken to charge the capacitor, and then convert to frequency.

Because frequency and time are mutually reciprocal, high frequency means short time (per cycle). And we have been learning that the shorter the time in which a pulse occurs the wider the frequency band it occupies; if the pulse is to be perfect, an infinite band. At the opposite extreme is a perfect sine wave, which if it is to have zero frequency bandwidth (because it has only one frequency) must continue in time for ever, because any stoppage or variation in amplitude would be modulation and this introduces sidebands. These extreme comparisons are shown in Fig. 24.5a and b. There is one particular waveform, occupying a central position between these extremes, which has the same shape on both bases $(c)$.

These Irequency-time relationships are fundamental to communications (Hartley's law, Sec. 23.11). For the rest of this chapter let us look at some of the basic circuits used for non-sinusoidal 'signals'. Some of these we have met already, notably in the last chapter, but we can now begin to classify them.

### 24.3 Differentiating and Integrating Circuits

These names may seem rather formidable but we are already familiar with the things themselves. The names refer to mathematical operations. If $y$ is some quantity that varies, differentiating is finding the rate at which it varies, often (but not necessarily) with time. Integrating is the opposite process: given the rate at which $y$ varies, find $y$. In Fig. 24.6 on the top line we have a $y$ that varies in a whole range of different ways for our instruction. Below it is plotted the rate at which $y$ is varying; the mathematical shorthand for this,


Fig. 24.6-The rop line shows rarious ways in which a quantity can varl; below are the corresponding rates of variation
when the variation is with respect to time ( $t$ ) is $\mathrm{d} y / \mathrm{d} t$. From $a$ to $b$, $y$ is increasing at the same steep rate all the time, so $d y / d /$ has a large positive value. From $b$ to $c, y$ is not changing at all, so $d y / \mathrm{d} t$ is zero. From $c 10 d$ it is decreasing steadily, at a slower rate than the increase from $a$ to $b$, so $\mathrm{d} y / \mathrm{d} t$ is less and negative. de and ef are curves of the type we found in charging capacitors and switching on inductors, and the interesting thing is that $\mathrm{d} y / \mathrm{d} t$ has the same shape, though obviously it must be upside down in this case. Lastly $y$ goes through a complete sine wave. We have already dealt with this one in Sec. 6.2, so we can finish the job by completing the lower line showing that $\mathrm{d} y / \mathrm{d} t$ is also a sine wave but shifted quarter of a cycle to the left. This is called a cosine wave. Here we get yet another angle on the unique status of this paricular waveform; it is the only repetitive one that retains its shape (though not its phase) when differentiated. And, since integrating is the reverse of differentiating, integrating $\mathrm{d} \cdot / \mathrm{d} /$ in Fig. 24.6 (lower line) gives the upper line, in which the sine wave still appears but shifted quarter of a cycle to the right. Recalling Fig. 6.4 we can see that capacitance can truly be said to differentiate the applied voltage by the current it accepts. In the same way, looking again at Fig. 7.2 we can say that inductance integrates. On the other hand if we start with current we see that, as regards the terminal voltage it sets up, capacitance integrates and inductance differentiates. They do this to any waveform, repetitive or not, but only the sinusoidal repetitive shape and the 'growth' (exponential) shape (e.g., Figs. 3.6 and 4.10 , and $24.6 /$ to $f$ ) are preserved. Other shapes (e.g., Fig. 24.6u to $d$ ) are drastically changed. We notice especially that our two basic non-sinusoidal shapes, pulse and ramp, are directly related to one another by differentiation and integration.

All this being so, the reason why circuits containing capacitance and inductance are in certain circumstances called differentiating or integrating circuits should now be clearer.

We have already come across some examples here and there. A good one was the line time base generator (Sec. 23.10) in which the requirement was a steadily growing current in a pair of coils. The method was to apply a constant voltage (Figs. 23.18, 19). This is therefore an integrating circuit-getting Fig. 24.6a to $b$ (upper line) from the lower line.

Except in this kind of example, where inductance is an essential component, capacitance is usually more convenient. We have just seen that either $C$ or $L$ can both differentiate and integrate. So commonly $C$ is preferred for both purposes. For differentiation, the input is voltage and the output is current; for integration, vice versa. This is all very well if one can accept the change from voltage to current or from current to voltage. But often one wants to keep it all in voltages. A current can be transformed into a proportional voltage by passing it through a resistace. But unfortunately the circuit to which the input voltage is applied is no longer just $C$ but
is $C$ and $R$ (Fig.24.4) so it is no longer perfect either as a differentiator or as an integrator.

In Fig. 24.7 square voltage waves ( $V$ ) are applied to this $C R$ circuit. If $R$ is relatively small, then the current will be very much the same as it would through $C$ alone; that is to say, $V_{D}$ will be an approximate differential of $V$. If on the contrary $R$ is relatively large

Fig. 24.7-Showing the tendency towards integrution and differentiation of the CR combination (Fig. 24.4)

the current through it (and therefore through $C$, which also is large so that its reactance is small) will be an approximate copy of $V$, and $V_{1}$ therefore approximately the integral of $V$. For differentiating, then, $C R$ (the time constant) should be small, and for integrating it should be large, compared with the time of a cycle of V. In Fig. 24.7 $C R$ is medium, being equal to a cycle, so neither function is done well. Ideally, $V_{D}$ should correspond only to the varying parts of $V$-the steep rises and falls-and $V_{1}$ should have steady slopes corresponding to the flats in $V$. Fig. 24.8 shows how the output waveforms depend on the ratio of $C R$ to the square-wave cycle time, $T$.


Fig. 24.8-How the ratio of tome constant (CR) to signal period (T) affects the waveforms in Fig. 24.7

An important point to remember is that the sum of $V_{I}$ and $V_{D}$ must at every moment be equal to $V$; obvious, one would suppose, but the necessity is surprisingly often overlooked by students drawing the supposed output waveforms corresponding to given inputs in actual circuits.

Note too that a differentiating circuit tends to discriminate in favour of high frequencies, and an integrating circuit in favour of low frequencies, so they are often used singly or in combination as filters; for example, Fig. 18.10.

The inferior performance due to the presence of an $R$ in the circuit can sometimes be tolerated. We came across an example of this in the $C R$ circuit for sorting out the sync. pulses in a television receiver; the typical waveforms in Fig. 23.17 are similar to those in Fig. 24.8 and elsewhere.

A good standard of integration is often wanted, because it is the basis of time base generators. The curves for $V_{I}$ in Fig. 24.8 show the dilemma: the better the linearity the less the amplitude, and a fairly large amplitude is needed for time bases. Some of the ways of dealing with this problem are included in the next section, but meanwhile let us look at a most important general method for obtaining a high standard of integration and differentiation; so good that it is used in computers (Sec. 25.2) for performing these two essential mathematical operations. It makes use of two more varieties of the operational amplifiers to which we were introduced in Sec. 19.21.

There, we saw only the types in which the input and feedback impedances were resistances. In. Fig. 24.9 a capacitance $C$ takes the place of the feedback resistance. As before, the gain of the amplifier (represented by the triangle) is sufficient for the input voltage to be relatively very small; so much so that the input point Y is a "virtual earth', because its potential is hardly distinguishable from earth's. So if a constant input voltage is imposed (indicated by the step input 'waveform') a constant current flows, determined by $R$. Only a minute part of this current is needed by the amplifier, so the constant current flows into C and the potential across it therefore rises

linearly, as indicated at the output. In this way the spoiling effect of resistance in series with C is virtually removed.

Suppose now that the whole thing is reversed; the output becomes the input, to which a linearly rising voltage is fed, and the amplifier


Fig. 24.10-And this one is the corresponding differentiating circuit
is reversed, as in Fig. 24.10. This input, applied to C with its other terminal virtually earthed, ensures a constant input current. Flowing through R, this current sets up a constant output voltage, which of course is the differential of the input; and similarly any other input form is differentiated.

### 24.4 Waveform Shapers

The raw material for most non-sinusoidal waveforms-and sometimes sinusoidal ones - is the pulse or square waveform. It can be produced either by reshaping existing signals or by generating them direct by special oscillators. We consider shapers first.

Available signals are likely to be more or less sinusoidal-the mains supply, for example, or the product of an oscillator-and if the frequency is too low it can be multiplied (Sec. 15.9). They can easily be converted into nearly square waves by passing them through an amplifier that very readily overloads, or limits. Fig.


Fig. 24.11-Simple type of circuit for obtaining a nearly square wave from a sine wave
24.11 shows how simple this is. The positive half-cycles are all removed because the input voltage is then negative and allows only leakage current (negligible with silicon transistors) to flow through R. By using a low value of $V_{C C}$ and plenty of input signal the negative half-cycles are nearly all removed by bottoming. However much the amplifier limits, the fronts and backs of the squares obviously must slope to some extent, and the corners are rounded a little by the less than perfectly sharp cut-off in each direction. These defects can be reduced by another stage of amplification. What can be done in this way is limited by circuit and transistor capacitances; put another way, if a really high sandard of pulse shape is needed, the circuits that handle it must be of the wide-band v.f. type (Sec. 22.13).

With the simple arrangement in Fig. 24.11 $a$ the negative halfcycles of output will be slightly briefer than the positive, because


Fig. 24.12-Circuit for producing narrow pulses from other waveforms
both lie on the same side of the zero line (here, at $V_{c c}$ ); this can be corrected by a suitable amount of bias. Different positive/negative (or mark/space) ratios than $1: 1$ can be obtained by biasing nearer to positive or negative peaks, but one cannot go too near without impairing the waveform.

So if a large ratio is wanted-short pulses-the preferred scheme is to follow the limiter by a differentiating circuit and then bias off the unwanted pulses, exactly as described in connection with Fig. 23.17 (and, incidentally, in the pulse-counter type of f.m. discriminator, Sec. 18.13); see Fig. 24.12. In the output stage, bias and bottoming are used to cut off not only the pulses of unwanted polarity but also for topping and tailing the wanted ones. In situations where clipping off unwanted portions of waveforms is needed without also amplification, suitably biased diodes are often used (Sec. 24.10).

This pulse shaping is often used with a view to producing the sawtooth form, for an oscilloscope time base for instance. In the preceding section we saw that if a constant voltage or current, corresponding to the flat parts of pulses or square waves, is integrated the result is a linear slope. The television line time base system (Fig. $23.18,19$ ) gave us a practical example with a current output.

A capacitor is an integrator, but the problem is to keep the charging current constant without using such a large resistance in series that the required input voltage is unreasonably high. What is needed is something that behaves as a very high resistance but does not need a correspondingly high voltage to pass current through it.

At one time pentode valves were much used for this purpose because, as we saw in Sec. 15.7, they have a considerable range of anode voltage in which the current varies little; yet the amount of current, which together with $C$ determines the rate of voltage rise across C , is easily controlled by grid bias. F.e.ts, with their similar characteristics (Sec. 10.10) also can be used in this role. And when studying the characteristics of bipolar transistors in the commonbase mode (Sec. 12.10) we found that collector current is remarkably independent of collector voltage, as indicated by its very high $v_{\mathrm{c}} / i_{\mathrm{c}}$ resistance, at much lower total voltage than is feasible with a pentode. So if a capacitor C is connected as in Fig. 24.13 and is charged at a comparatively rapid rate by closing the switch, opening it allows C to discharge linearly through the transistor. The sequence is then repeated continuously to yield a sawtooth voltage waveform across C. In practice the switch is another transistor controlled by a pulse waveform.

In one respect transistors are less suitable than valves for voltage outputs: the maximum voltages that can be used are much lower. And step-up transformers for waveforms of this type are not easy to design or make. However, transistors are being developed all the time with higher working voltages.

Operational amplifiers offer a very precise solution, but although their necessarily high internal gain is needed for such applications as computers a single stage may do as much as is needed for normal time base purposes. And it gives the inversion required for the essential negative feedback. And so we get a type of circuit that is


Fig. 24.13-Circuit illustrating principles of ramp waveform production by charging and discharging capacitor
usually called the Miller integrator but which is more appropriately named after its inventor, Blumlein. Fig. 24.14 shows a transistor version, complete with pulse-operated switch $\mathrm{T}_{\mathrm{R}_{1}}$. During the input pulse $T_{R_{1}}$ is 'turned on' and is in the bottomed state, so its collector and the base of $\mathrm{T}_{\mathrm{R}_{2}}$ and the terminal of C connected to it are all held at a potential only slightly above 0 , while $\mathrm{T}_{\mathrm{R}_{2}}$ being cut off allows the other terminal of C to be practically at $V_{c c}$ volts. C is therefore
charged to nearly that voltage. When the pulse ceases, $\mathrm{T}_{\mathrm{R}_{1}}$ is cut off and $T_{R_{2}}$ is turned on, so $C$ discharges through $R_{1}$ and $T_{R_{2}}$. In the ordinary way the rate of discharge would fall off exponentially as the voltage of C fell, but owing to negative feedback Y is a virtual


Fig. 24.14-A form of Blumlein integrator circuit, closely related to Fig. 24.9
earth so the current through $\mathrm{R}_{1}$-and therefore the discharge current, except only for some small variation in the base current of $\mathrm{T}_{\mathbf{R}_{2}}$ - is kept constant. To provide for a quick fly-back, $\boldsymbol{R}_{\mathbf{2}}$ is much less than $R_{1}$.

### 24.5 Waveform Generators: Relaxation Oscillators

If a sinusoidal waveform is not required in addition to others developed from it as described in the last section, there is no point in carefully designing an oscillator for producing shapely sine waves only to distort them drastically. Oscillators can be designed to produce square waves or pulses directly; in fact, that is what an oscillator tends to do anyway if it is given its head. Besides dispensing with at least one stage, this method more readily yields a useful range of mark/space ratios.

When considering sine-wave oscillators we found that the most important conditions for least distortion of this waveform and for most precise frequency maintenance were the same: to use only just enough positive feedback to keep oscillation going. It follows that direct pulse generators, which depend on excessive positive feedback, are in general not so good at keeping themselves within very close frequency limits. That does not matter if, as in television, they can be synchronized from without. There is a whole series of types of pulse generator ranging from the completely independent to the completely dependent shaping types already discussed:
(1) Independent free-running oscillators establishing their own frequency.
(2) Oscillators which without control would operate in group 1,
but which are pulled into step by external synchronizing signals. I and 2 are both classed as astable systems.
(3) Oscillators which cannot run continuously on their own but which on being triggered from an external source will execute one complete cycle and then wait until the next triggering signal. This class is termed monostable.
(4) Oscillators similar to 3 except that when triggered they execute only part of a cycle and need another signal to make them complete it. These are called bistable. Although so dependent on outside sources, they do at least determine the duration and waveform of their part-cycles, and in this important respect differ from waveform shapers.
All these groups of oscillators are distinguished from the sinewave kind, in which the transistor or valve works continuously as an amplifier, as relaxation oscillators, in which it works intermittently as a quick-acting switch. The frequency is determined not by an $L C$ tuned circuit but by delays due to time constants, either $C R$ or $L / C$ (Secs. 3.5 and 4.8) or both. It is the relaxation or gradual discharge of the time-constant elements between the active or switching phases that gave rise to the name for this class of circuits. For an understanding we must once again keep in mind the fundamental principle that the voltage across a capacitor cannot change abruptly but only as a result of current flowing into or out of it for a time, and the dual principle, that the current in an inductor cannot change abruptly but only as a result of voltage across it for a time. Although all relaxation oscillators are basically the same in principle, having a large amount of positive feedback coupling through a time-delay circuit, one particular variety-the blocking escillatoris usually distinguished from the rest.

### 24.6 The Blocking Oscillator

The distinguishing feature is inductive coupling, which makes blocking oscillators look deceptively like the ordinary oscillators described in Chapter i4; see for example Fig. 24.15. Attempts, in the interests of brevity, to explain their working along the same lines as ordinary oscillators are likely to mislead, for it is really quite different.

The cycle of oscillation comprises two phases, one of them being a small fraction of the whole period, in which a lot of things are happening very quickly, so let us begin with the other by supposing that C is charged so as to make point a in Fig. 24.15 negative. As this point is connected through R to $+V_{C C}, \mathrm{C}$ gradually discharges as shown by the exponential curve of $v_{\mathrm{a}}$, which would eventually carry it to the $V_{c c}$ level. The transistor being cut off by this negative bias, the collector potential $v_{c}$ is the same as $V_{c c}$, while $V_{b}$ is the same as $v_{a}$. Directly $v_{a}$ becomes slightly positive, collector current
begins to flow through the primary coil of the transformer. The rate of current increase through its inductance sets up a voltage across the coil. Assuming that the transformer has $100 \%$ coupling and $1: 1$ ratio, an equal voltage must appear across the secondary coil. The dots indicate the terminals having the same polarity; at this moment, negative. So the increasingly positive potential of the base gets a sharp boost from the transformer, which intensely accelerates the action just described. The voltages across the two transformer coils are now equal to $V_{C C}$ less the small collector-to-emitter voltage in the bottomed state. The secondary voltage recharges $C$ through the base-to-emitter resistance, which being much less than $R$ accounts for the relatively steep downward slope of $v_{\mathrm{a}}$ and $v_{\mathrm{b}}$. Directly $v_{\mathrm{b}}$ drops back near to 0 it starts to reduce collector current, and this reduction induces transformer voltages of the opposite polarity. Again there is the intensely accelerated action, this time switching off the transistor. The very fast decay of current makes the induced voltage $v_{\mathrm{c}}$ highly positive; so highly, in fact, that a diode and resistor are sometimes connected as shown dotted to save the transistor from being broken down. (Compare the action of a television line oscillator, Sec. 23.10) The secondary voltage now makes the base more negative even than $v_{\mathrm{a}}$, but as it is already negative and nonconducting this is not very important. The high induced voltage has charged stray capacitances, and when it ceases (as of course it


Fig. 24.15-Blocking oscillator circuit
very quickly does) these discharge as shown by the downward curve of $v_{c}$ and the upward curve of $v_{b}$.

The exact waveforms depend on many details of transformer and transistor, and in general are less sharp than those shown in Fig. 24.15b. Correspondingly the action is far more complicated than the foregoing simplified description. But the main features are present: a pulse of $v_{c}$, which can be made as brief as a small fraction of $1 \mu \mathrm{~s}$
by suitable choice of transformer and transistor and $C$, followed by a relatively long interval determined by $C R$, during which $\nu_{a}$ and $v_{\mathrm{b}}$ are the first half of an exponential curve. The p.r.f. can therefore be conveniently controlled by varying $R$.

Because R tends to bias the transistor on, this form of blocking oscillator is free-running. But if a synchronizing signal is injected


Fig. 24.15-Showing combined synchronizing and frequency-dividing action
at point a , just before the pulse is due, $v_{\mathrm{b}}$ crosses the zero line at once and triggers the transistor into pulse action. To ensure synchronism the free-running p.r.f. is adjusted to be slightly lower than the sync. pulse frequency.

Note that oscillators such as this, whose frequency depends on a time constant, can be synchronized not only to the frequency of the sync. pulses but to sub-multiple frequencies. Fig. 24.16 shows the graph of $v_{\mathrm{b}}$ again, with regular sync. pulses superimposed. It is not until the fifth of these that the voltage is able to rise above zero and start the pulse phase. The oscillator is here acting as a frequency divider.

This circuit or variations of it are commonly used in television receivers to generate the field scanning waveform. It must be followed by an output stage, with provision for linearizing the sawtooth. In an alternative circuit R and C are on the collector side.

The circuit shown can easily be converted inte a monostable one by connecting R across C , or al some other way arranging that the normal base-emitter voltage is zero, so that no action takes place until a triggering pulse is received, of the polarity that turns the transistor on.

### 24.7 The Multivibrator

The blocking oscillator is a remarkably simple device for obtaining very brief fulses. Transformers do not however fit well into modern
circuit construction (Sec. 19.22) so for square waves or longer pulses the choice is usually a circuit in which the phase inversion is obtained by a second transistor. The only other components needed are resistors and capacitors, so a whole pulse generator can be made up as an integrated circuit on a tiny chip. There is a wide choice of circuits, and a wide choice of books about them, so only a very brief introduction is included here.

The most important basic circuit is shown in Fig. 24.17 and is still known by its original name of multivibrator, given to it in 1918 because it was found to be a convenient source of many harmonics for


Fig. 24.17-(a) Basic form of multivibrator circuit and (b) relevant waveforms
frequency multiplying purposes. Hundreds of them can be detected if care is taken to obtain a sharp-cornered waveform by minimizing stray capacitances. It has since been developed mainly for its waveform as a whole rather than for the separate harmonics. One can easily see that it is just a two-stage resistance-coupled amplifier with the whole of its output fed back to the input. If $A$ is the total gain, then the amount of positive feedback is $A$ times as much as is needed just to maintain oscillation, and as $A$ can easily run into thousands there is a very fast switch-over from each half-cycle to the next, and therefore very steep wave fronts. The original valve circuit is the same except that the biasing resistors $\mathrm{R}_{3}$ and $\mathrm{R}_{4}$ are connected to the 0 V rail.

When $V_{c c}$ is applied, the inevitable absence of perfect symmetry will ensure that one transistor is slightly ahead in passing collector current. Suppose it is $T_{R_{1}}$. Then $v_{c_{1}}$ will be slightly more negative than $v_{\mathrm{c} 2}$, and as $v_{\mathrm{c}_{1}}$ via $\mathrm{C}_{1}$ immediately makes $v_{\mathrm{b} 2}$ more negative,
tending to switch $\mathrm{T}_{\mathrm{R}_{2}}$ off and prevent $\nu_{\mathrm{b}_{1}}$ from going negative, $\mathrm{T}_{\mathrm{R}_{1}}$ becomes firmly on and $T_{R_{2}}$ firmly off. This is where we come in at the start of Phase 1 in Fig. 24.17h. Because $\mathrm{T}_{\mathrm{R}_{1}}$ is bottomed, $v_{\mathrm{c}_{1}}$ is only just above zero. And because $\mathrm{C}_{1}$ has had no time to charge, $v_{\mathrm{b} 2}$ is pushed down to the same extent. Although in doing so it cuts off $\mathrm{T}_{\mathrm{R}_{2}}, v_{\mathrm{c}_{2}}$ cannot instantly spring up to $+V_{\mathrm{Cc}}$, because $\mathrm{C}_{2}$ has to charge through $R_{2}$ and $T_{R_{1}}$ (base to emitter). So $v_{c 2}$ rises exponentially as shown. $v_{b t}$ is thereby pushed positive, but only slightly and temporarily because of the charging of $\mathrm{C}_{2}$. When that is complete $v_{b_{1}}$ is held at a small positive bias by current via $\mathrm{R}_{4}$. This fall-off in $v_{b_{1}}$ reduces the $T_{R_{1}}$ collector current slightly, so there is a slight rise in $v_{\mathrm{c} 1}$. Meanwhile $\mathrm{C}_{1}$, held on the left at the near-zero $v_{\mathrm{c}_{1}}$, is charging through $\mathrm{R}_{3}$, so $v_{b_{2}}$ rises exponentially. Assuming, as is usual, $R_{3}$ and $R_{4}$ are considerably greater than $R_{1}$ and $R_{2}$, this growth is comparatively slow.

The end of Phase 1 is brought about by $\nu_{b_{2}}$ (similarly to $\nu_{\mathrm{b}}$ in Fig. 24.15) becoming positive, starting current in $\mathrm{T}_{\mathrm{R}_{2}}$ and thereby a negative-going $v_{c_{2}}$, which is communicated by $\mathrm{C}_{2}$ to the base of $\mathrm{T}_{\mathrm{R}_{1}}$, making $v_{\mathrm{c}_{1}}$ rise. The amplification ensures a rapid reversal of conditions, $\mathrm{T}_{\mathrm{R}_{1}}$ being switched off and $\mathrm{T}_{\mathrm{R}_{2}}$ on. By the end of Phase 2 a complete cycle has been accomplished and the whole process repeats continuously.

Phases 1 and 2 being equal indicates that the component values are the same for each half of the circuit, or at least that $C_{1} R_{3}=C_{2} R_{4}$, these time constants clearly being mainly responsible for the durations of the phases. Any desired mark/space ratio within reason can be obtained by choosing component values to make these time constants suitably unequal.

Like the blocking oscillator, the multivibrator is naturally freerunning but can easily be synchronized by pulses injected almost anywhere in the circuit-usually at one of the bases. Both types of circuit are also self-generators of non-linear ramps. By making $R_{1}$ and $R_{2}$ as low as possible, a nearly square waveform can be obtained from either collector, and if necessary it can easily be improved by shapers (Sec. 24.4).

### 24.8 Bistables

If the two capacitors in the astable multivibrator are replaced by the base-biasing resistors as in Fig. 24.18 the circuit ceases to have a self-triggering action and has to be externally triggered both ways. In other words it is stable in both conditions (either transistor conducting) so is termed bistable. It has ceased altogether to be a selfoscillator, and in its basic form has no time constants; these, we recall, served as delay-action self-triggers.

Like the astable, this particular type of bistable was invented as far back as 1918, in valve form of course. It was named after the in-
ventors as the Eccles-Jordan relay, and is now one of the main 'building blocks' in digital computers. Whereas the multivibrator is a two-stage $100 \%$ positive-feedback a.c. amplifier (because the


Fig. 24.18-Basic form of bistable circuit
stages are coupled to one another by capacitors) the Eccles-Jordan is a d.c. amplifier. Like the multivibrator it cannot maintain a balanced stage with each transistor passing the same amount of current, because the slightest disturbance of the balance, say by circuit noise, starts a movement that is rapidly accelerated by the feedback until one transistor is bottomed and the other cut off. By this time it should be unnecessary to go through the whole sequence in detail. The essential feature is that it remains stably in that state until switched from without.

Suppose $T_{R_{1}}$ is the bottomed one. The $c_{1}$ is near enough to zero potential for insufficient bias current to flow through $\mathrm{R}_{3}$ to make $\mathrm{T}_{\mathrm{R}_{2}}$ conduct. At least, that is the intention, but, to make quite sure, $\mathrm{T}_{\mathrm{R}_{2}}$ is biased off-the usual practice is to connect both bases through a pair of resistors to a negative voltage, as we shall see in a moment.


Fig. 24.19-Practical bistable circuit, with diode pulse-forming and steering system for alternate on-off
$\mathrm{T}_{\mathrm{R}_{2}}$ being cut off, $\mathrm{c}_{2}$ is at $+V_{\mathrm{CC}}$ potential, which applies a large positive base bias to $\mathrm{T}_{\mathrm{R}_{1}}$, ensuring that it is fully on. And there is no internal reason why this state should ever change.

Now apply a negative triggering pulse to $b_{1}$. Provided that it is enough at least to reduce the collector current in $\mathrm{T}_{\mathrm{R}_{1}}$, a positive amplified pulse will be applied to $b_{2}$, making $T_{R_{2}}$ conduct and reinforce the triggering pulse at $b_{1}$, ensuring a speedy and complete switch-over. A negative pulse applied to $\mathrm{b}_{2}$ restores the status quo. (With p-n-p transistors of course all the polarities are reversed.)

Often it is more uselul to be able to reverse state by successive pulses from the same source, like the type of electric light switch in which one push turns the light on and the next turns it off. Fig. 24.19 shous this and other practical elaborations. $R_{7}$, with the emitter current of the 'on' transistor passing through it, makes both emitters several volts positive from $0:+V_{c c}$ is increased by the same amount to make $V_{C E}$ the same as before. $\mathrm{R}_{5}$ extends the $R_{2} R_{4}$ potential divider so that. when $T_{R_{2}}$ is on, the base of $T_{R_{1}}$ is definitely negative. $R_{6}$ similarly continues $R_{1} R_{3} . C_{1}$ and $C_{2}$ are

Fig 24.20-Schmill bistable circunt, with d.c. triggering

usually fitted to speed up the switch-over by providing a lowimpedance path for the high-frequency components of the waveform. $C_{3}$ and $D_{3}$ sharpen the trigger waveform by differentiating action. When $T_{R_{1}}$ is on, $b_{1}$ is positive to the common emitters whereas $b_{2}$ is negative, so $D_{1}$ conducts and $D_{2}$ does not, and the negative-going pulse affects $T_{R_{1}}$ only. switching it off. When the next pulse comes the situation is reversed and $\mathrm{T}_{\mathrm{R}_{2}}$ is switched off.

One other bistable circuit is important enough to be mentioned even in an introduction: the Schmitt trigger, Fig. 24.20. Comparing it with the similarly numbered Fig. 24.19 we see that it is essentially the same except that $T_{R_{1}}$ is controlled directly instead of by $T_{R_{2}}$. The input must therefore be d.c. instead of pulses. If the voltage applied to $b_{1}$ exceeds a certain critical positive value $V_{1}, T_{R_{1}}$ is switched on, and $\mathrm{T}_{\mathrm{R}_{2}}$ switches off in the now familiar way. If now
the input level is reduced to a certain voltage $V_{2}$, less positive than $V_{1}, \mathrm{~T}_{\mathbf{R}_{1}}$ is switched off and $\mathrm{T}_{\mathbf{R}_{2}}$ on. In this circuit $\mathrm{R}_{7}$ does more than simply create a potential more negative than the emitters, as in Fig. 24.19. Because the current through $\mathrm{T}_{\mathrm{R}_{1}}$ is controlled by the input potential, so thereby is the voltage across $R_{7}$, and this in turn controls the base-emitter voltage of $\mathrm{T}_{\mathrm{R}_{2}}$. In other words $\mathrm{TR}_{1}$ and $\mathrm{T}_{\mathbf{R}_{2}}$ are cathode coupled.

### 24.9 Monostables

The monostable multivibrator, or flip-flop, comes half-way between the astable multivibrator, which can trigger itself both ways continuously, and the bistable, which can trigger itself neither way so stays put until it is triggered externally. It needs to be triggered from its stable condition, but triggers itself back to the original state after an interval determined by a built-in time constant. So we are not surprised to find that one half of the basic circuit is the same as in the astable type and the other half is the same as in the bistable type. Fig. 24.21 shows quite a practical form of the circuit, in which the negative bias to ensure that $T_{R_{1}}$ remains completely cut off when in the stable state is supplied from a negative source instead


Fig. 24.21-Practical form of monostable circuit with trigger, and its waveforms
of by the voltage drop due to emitter current through $\mathrm{R}_{7}$ in Figs. 24.19 and 20. A suitable triggering circuit is also shown. For easy comparison with the previous circuits the corresponding components have the same numbers.

Typical waveforms are attached. A sample triggering pulse appears as $v_{\mathrm{a}}$; its shape is not very important so long as the rising
front is reasonably steep. It is differentiated by $\mathrm{C}_{3} \mathrm{R}_{8}$ which has a relatively short time constant to give the short sharp pulses $\nu_{\mathrm{a}}{ }^{\prime}$. Only the positive one ( $\mathrm{p}_{\mathrm{b}_{1}}$ ) reaches the base of $\mathrm{T}_{\mathrm{R}_{1}}$, the negative one being blocked by the diode $\mathrm{D}_{1}$. The switch-over started by such a positive pulse sends $\mathrm{b}_{2}$ sharply negative (see $v_{\mathrm{b}_{2}}$ ) and correspondingly $c_{2}$ sharply positive to form the front of the output waveform. Note that because there is no capacitor $\mathrm{C}_{2}$ this front rises steeply instead of in a charging curve as in Fig. 24.17. The positive voltage thus imparted to the top end of $R_{4}$ maintains $v_{b_{1}}$ positive and $\mathrm{T}_{\mathrm{R}_{1}}$ on after the triggering pulse has died away. As soon as $\mathrm{C}_{1}$ has charged sufficiently to make $\nu_{b_{2}}$ positive, the switch-ever from $T_{R_{1}}$ to $T_{R_{2}}$ takes place in the same manner as in the astable multivibrator.

There are many uses for this kind of circuit, such as generating pulses of controllable length, reshaping pulses that have deteriorated in transmission, and creating adjustable time delays.

### 24.10 Clampers and Clippers

Fig. 24.22 shows a simple multi-purpose device that we have already met more than once. It goes by various names according to the purpose. One of these, known as clamping, is shifting a wave-train as a whole, lining up either positive or negative peaks to any desired potential. As it is shown it lines up or clamps the positive peaks to what is arbitrarily regarded as zero voltage or earth ( 0 V ). To clamp the negative peaks, reverse the diode. To clamp to some other

Fig. 24.22—Diode clamping circuit

potential, bias the lower end of the diode to that voltage. The action is precisely as already described in Sec. 18.6. The upward slope of the negative peaks of the output is due to the negative charge on $C$, accumulated through the diode conducting at the positive peaks, leaking off slightly through R . To minimize distortion of the waveform due to such leakage, the time constant $C R$ should be many times greater than the period of one cycle. While the scheme works more or less with any waveform, one that is very sharply peaked at the polarity of clamping (such as the sync. pulses being treated in this way in Fig. 23.15) is likely to have its peaks clipped somewhat because of the very short time available for topping up C .

If the wave-train is amplitude-modulated, the time constant $C R$ has to be more carefully chosen, according to what is required. If

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it is made sufficiently long the charge will remain in C not just between one cycle and the next, but between one maximumamplitude cycle and the next, with perhaps many weaker cycles in between. Fig. 24.23 shows this, with modulated sine waves clamped positive of zero. $C R$ in this case must be long compared with $T$, which is the length of one cycle of the modulation frequency.

Note that this output contains the original modulated carrier wave plus a z.f. (d.c.) component. So, as with the unmodulated pulses, the circuit can be used as a d.c. restorer. It does not contain the modulation frequencies. To get them, $C R$ must be short enough for the charge on $C$ to leak fast enough to follow the highest-frequency modulation, as explained in Sec. 18.8. In that role the circuit is a diode detector. Its output contains a z.f. component nearly equal to the unmodulated peak value of the carrier. This component is often used for purposes such as a.g.c.
D.C. restoration is often applied, as in television receivers, to the modulation waveform, after the carrier wave has been removed. This was shown in Fig. 23.15, where the peaks that are clamped to a fixed potential are the tips of the sync. pulses. In this case the time constant should be long compared with the duration of one line ( $64 \mu \mathrm{~s}$ in a 625 -line system) but could be less than that of one field. If d.c. restoration is applied by rectifying the v.f. picture signal only, after the sync. pulses have been removed (Fig. 24.24) the z.f. so obtained should be averaged over the whole picture, so as to be a measure of its average brightness, and therefore $C R$ should be longer than $1 / 25 \mathrm{~s}$ but short enough to respond reasonably quickly to changes in brightness of scene; 0.1 s , say.

Note that the diode in Fig. 24.22 can be the input of a transistor or valve which is performing some other function such as amplifying (Sec. 18.12).

In all these applications one object is to preserve the wave-train or signal as far as possible from mutilation. Sometimes however


Fig. 24.23-Showing action of clamping circuit having lony dime constam on modulated waveform
one needs to clip off unwanted bits. We have seen in Sec. 24.4 how to do this by means of transistors. Where their amplification is not needed, diodes can be used instead. The simplest method is to substitute a resistor for C in Fig. 24.22; see Fig. 24.25, in which two diodes are used in order to clip both half-cycles. The batteries represent suitable sources of bias. Until the instantaneous signal voltage
exceeds $+V_{1}$ or $-V_{2}$ neither diode is conducting and the arrangement is a potential divider reducing the signal in the ratio $R_{2} /\left(R_{1}+R_{2}\right) . R_{2}$ is not strictly necessary as a component but represents the resistance of whatever is being fed, combined with the


Fig. 24.24-D.c. restoration of television signals. When applied positively, b yields a much larger hias voltage ithan a
(b)

Fig. 24.26-Series-drode type of clipping circuit

leakage of the diodes if that is appreciable. $R_{1}$ should therefore not be larger than it need be to make the resistance of the diodes when conducting, plus their bias sources, negligible in comparison with it, so that an appreciable fraction of what is supposed to be removed is not developed across them.

That is the shunt type of clipper. The series type (Fig. 24.26) is more suitable for performing the reverse operation: rejecting all of a waveform that does not exceed the bias voltage. Note that this voltage appears at the output all the time. With no bias this clipper
is a simple rectifier giving a mean output equal to half the mean value of a half-cycle of the input (Sec. 26.3).

### 24.11 Gates and Choppers

In computers and other electronic equipment much use is made of electronic switches or gates. They fall into two main classes: linear or transmission gates, in which the signal gets through in its


Fig. 24.27-This type of gating circuit can be used as a chopper for z.f. (d.c.) amplification
original form when the gate is open and not at all when it is closed; and logical gates, in which the output is due to the combination of signals present at the input and is not necessarily of the same form.

Diodes can be used as linear gates, but as many as six may be needed in specially balanced circuits in order to keep the controlling and controlled signals separate without serious loss of signal. This problem is reduced by using transistors, in which the base is available as a control electrode to render the collector-emitter path conducting or non-conducting as required. Fig. 24.27 shows a p-n-p transistor in this role. So long as the gating voltage $v_{G}$ is negative the transistor is non-conducting and can be ignored; the signal voltage $v_{S}$ gets through in the ratio $R_{0} /\left(R_{S}+R_{0}\right)$. When $v_{G}$ is positive the transistor is highly conducting and short-circuits $R_{0}$. The transistor has a lower 'off' resistance and a higher 'on' resistance when connected as shown rather than with $v_{s}$ connected between base and emitter. Only positive signals can be controlled by the form of the circuit shown; for negative signals use an n-p-n transistor, and for both use one of each in parallel, controlled by gate signals of opposite polarity.

Fig. 24.27 shows the gate controlling a d.c. 'signal', in which role it is known as a chopper. As we saw in Sec. 19.21, it is very difficult to obtain high gain in a straightforward d.c. amplifier because the slightest change in transistor characteristics or supply voltages in the first stage is amplified too and cannot be distinguished from the signal. So a common technique is to chop the signal into pulses at a frequency suitable for amplifying. A chopper can be regarded as a frequency changer, changing the signal frequency from zero to an 'i.f.' Conversely the conventional frequency changer in a radio receiver can be regarded as a kind of gating circuit.

Linear gates are used also for control purposes where faster action is needed than mechanical switches or relays can provide.

Logical gates are used to obtain output signals (usually pulses) when certain combinations of input signals occur. An or gate, for example, gives an output whenever any one or more of a number of possible input pulses occur. Suppose there are two inputs, A and B ; then an output is given by a signal at A or B or both. This kind of gate can be devised using one diode per input, as in Fig. 24.28. If the input pulses were negative-going the diodes would of course have to be reversed. $R$ must be large compared with a forwardbiased diode and small compared with a reverse-biased diode.

An and gate gives an output only when inputs A and B , etc., occur, as in Fig. 24.29. So long as either input is at zero volts at


Fig. 24.28-A logical OR gate; its output corresponds to a pulse at A or B

Fig. 24.29-A logical AND gatte: an output is obtained only when there is a pulse at A and B


least one of the diodes is conducting and the output terminal is held almost at zero potential. Only when A and B are both at least $V$ volts positive can the output go positive and register a signal.

Transistors also can be used in OR or AND gates, with the advantage of gain as well if that is needed. Another facility provided by transistors and much used in logic circuits is inversion, which changes "signal' to "no signal' and vice versa. Although any commonemitter transistor amplifier inverts, so hardly qualifies as a gate, in the context of electronic logic it is called a NOT gate. Most often however it combines the NOT function with being an OR or AND gate, the combination being termed a NOR or NAND gate.
A simple nOR gate is shown in Fig. 24.30. With both A and B at zero volts the base is sufficiently negative to cut off the transistor and give +12 V output. If either A or B input is made not less than about +3 V , the base goes slightly positive, making the transistor
conduct and bring the output voltage near zero. To reduce interaction between the input sources and also to reduce loading on them, the input resistors may be replaced by diodes as in Fig. 24.28.

If a p-n-p transistor is substituted in Fig. 24.30 and all the polarities consequently reversed the result is a NAND gate, Fig. 24.31. If any


Fig. 24.30-A transistor NOR gate; all output is obtained whenever there is a pulse at neither A nor B


Fig. 24.31-A transistor nand gute; an output is obtuined whenever there is not a pulse at A and B
input is at zero volts the base is slightly negative, which makes a $\mathrm{p}-\mathrm{n}-\mathrm{p}$ transistor conduct and keeps the output voltage only slightly less positive than 0 V . Only if all inputs are positive can the base go positive and cut off the transistor, making the output voltage fully negative.

### 24.12 Design

You may be disappointed not to find component values given for all the circuits in this chapter, or at least detailed instructions on how to calculate them. Component values are seldom given because they depend so much on the precise circumstances and requirements, so without full discussion would be misleading. And full discussion would swell this book excessively and duplicate the
adequate literature on the subject. The hope is that anyone who has studied this book will be able to design his own pulse-handling circuits, which are very largely an application of Ohm's law, plus $C R$ time constants. The electronics side, at least for the less demanding applications, is easy: the power levels are usually low, so problems of heat dissipation, thermal runaway and leakage currents are absent; and the diodes and transistors are used in only two states: fully conducting or non-conducting. In the conducting state the forward bias can be reckoned as 0.2 V for germanium and 0.6 V for silicon. The conducting ('bottomed') collector-emitter voltage, or knee voltage, is usually a small fraction of a volt; if a closer figure is needed the makers' curves or data should be consulted.
If very high-speed operation is needed then the choice of diodes and transistors is rather important. Here again the makers provide plenty of information. But except as exercises or for specialists the design of pulse circuits is hardly necessary, as most of those likely to be needed are available as integrated circuits. But it does help if one has some idea how they work-to provide which is the purpose of this chapter.

## CHAPTER 25

## Computers

### 25.1 Analogue and Digital

It would of course be absurd to attempt to cover the subject of computers in one short chapter. All that can be done is to indicate broadly what they involve and how they make use of the principles we have been studying.

The first thing is to distinguish between analogue and digital computers. In analogue computers the magnitudes being processed are represented by continuously variable quantities, such as voltage, current or charge, whereas in digital computers they are represented by numbers, which are not continuously variable. One third cannot be expressed by any finite number of decimal digits (being $0.3333 \ldots$ for ever) but one third of a distance is theoretically possible. Paradoxically, however, digital computers are capable in practice of far higher precision than analogue types.

Electrical instruments such as voltmeters illustrate the distinction. The ordinary pointer types are analogue instruments, voltage being indicated in terms of a different quantity: the angle of the pointer. Such voltmeters are simple in principle and have comparatively few parts, but the standard of workmanship goes up so steeply with the degree of precision demanded that they are limited in practice to very moderate precision. Digital voltmeters, in which the reading is exhibited in figures, are very much more complicated and contain very many parts, so for ordinary $1 \%$ accuracy cannot compete in price with pointer instruments. But the number of kinds of parts is quite small and they are individually cheap, and the precision of reading can be increased several decimal places by increasing the number of parts of the same few kinds, little or no improvement in quality being necessary.

The automatic telephone is a digital system. Seven twists of its simple mass-produced dial, generating seven groups of up to ten pulses, could be used to select any one from ten million different numbers. An analogue scale capable of being read unmistakably to one in ten million would be a formidable task for the instrument maker!

Coming to actual computers, we may consider that very familiar analogue type, the sliderule, in which numbers are represented by lengths along scales, proportional to the logarithms of the numbers. A digital computer to perform to the same standard, which is sufficient for most engineering purposes, would be very much more
expensive. But it could work incomparably faster. And digital computers can be made incomparably more precise.

### 25.2 Analogue Computers

Electronic analogue computers are particularly useful for engineering purposes, for several reasons: few engineering data are known, or need be known, very precisely (think, for example of the inevitable tolerances in transistors, and the uncertainty of maximum loading on a structure); quantities such as temperature, pressure and velocity can easily be 'transduced’ into proportional voltages or currents instead of first into numbers; and the effects of varying the data can be observed continuously. The engineering designer used to have to arrive at his specifications by laboriously calculating for several alternative values of each parameter and choosing the best combination. Now it is possible to vary them and see the results at once, notwithstanding that the mathematical relationships may be quite complex. Or he may even hand the whole problem over to the computer and get it to indicate the optimum specification.

The same techniques are involved in the many kinds of simulators, such as those used for the early stages of learning to fly or to drive, without the expense and risk of using actual aircraft in flight or cars on the road with novices in control.

The basic components of analogue computers are operational amplifiers, to which we were introduced in Sec. 19.21. They are available as microscopic integrated circuits (Sec. 19.22) so although large quantities of them are needed they occupy little space. They use voltages as analogues of whatever numerical data are being computed. An inverter amplifier (Fig. 19.29a) changes the mathematical sign; a potential divider (up or down) multiplies or divides. If several input voltages are applied, each through its own resistance equal to $R_{1}$ in Fig. 19.29a, the total input current at $Y$ is equal to the sum of the separate input currents, so the output voltage is equal to minus the sum of the applied voltages. That gives addition-and subtraction, as negative voltages can be applied. Factors can be introduced by using different values of $R_{1}$. Differentiation is accomplished by substituting a capacitor for $R_{1}$ and integration by substituting a capacitor for $R_{2}$. (Sec. 24.3). By suitable combinations of operational amplifiers, differential equations are almost instantly solved. Waveform generators or shapers are sometimes used to introduce mathematical functions corresponding to them.

The basic disadvantage of analogue computers is that any departure from exact proportionality of the voltages to the quantities represented by them introduces error. It is only the use of large amounts of negative feedback which, by nearly eliminating the effects of variations in the active devices in the operational amplifiers, keeps such errors within reasonable limits.

### 25.3 Digital Computers

In order to understand what a digital computer has inside it and why, let us consider what we do when we compute mentally or with the aid of pencil and paper. First we have to receive the data and instructions on what to do with them; for example, 'Add 27 to 189 and multiply the result by 17 '. The first part of a computer, correspondingly, is an input unit, to receive data and instructions in a


Fig. 25.1-A compuler can be divided into these five departments
form the computer can use, such as punched cards, punched or magnetic tape, teleprinter signais, etc.

Next, we perform the arithmetical operations of adding and multiplying. The regular procedure with the above problem would be to add 7 to 9 , get 16 , put down 6 in the units column, carry 1 to the tens column and add it to 2 and 8 , getting 11 , put down 1 and carry 1 to the hundreds column, adding it to 1 to get 2. Partial answer: 216. Then we would multiply this 216 by 7 and add it to 2160 . Or we might have a $\times 17$ multiplication table or ready reckoner giving the answer directly. The part of a computer that manipulates the given digits to provide a partial or final answer is called the arithmetic unit. The part that uses the given instructions to bring this unit into action in the appropriate ways at the right moments is the control unit, and the part that stores partial results until they are needed (taking the place of the paper on which we write them down, or our memory if it is good enough) and such useful information as the multiplication table to shorten frequentlyneeded arithmetical operations, is called the store or memory.

Finally, the part that converts the answers from computer form to a form that is convenient for the user of the computer, is the output unit. Its 'readout' may be displayed or printed numbers or words, or signals to control a machine.

These divisions are shown diagrammatically in Fig. 25,1.
If the computer is a general-purpose one of moderate capability, with plenty in the arithmetic unit but not much in the control unit, the instructions (called the programme) have to be given in con-
siderable detail, so the services of an expert programmer are needed. Very elaborate computers, or special-purpose ones, have programmes built into them. They work rather like a military organisation in which it is not necessary to give a complex multitude of orders in detail to bring about, say, an invasion, but only a simple 'Operate Plan B' which has already been worked out. These programmes are stored in the memory.

A number of different types of store are used, sometimes more than one type in a single computer, depending on whether the stored information is much used and has to be found very quickly, or not. Most stores are magnetic, using tapes, drums, disks, or a matrix of magnetic cores (which will be described later).

### 25.4 The Binary Scale

We normally use a decimal system of numbers, presumably because we normally have 10 fingers (including thumbs) on which to count. A primitive tribe is reputed to use only three numbers: one; two; plenty. When the need arose for numbers greater than a reasonable variety of symbols, the simple and ingenious convention was adopted of placing a symbol one space to the left to mean the number of groups of ten, and a further space to the left to mean the number of ten groups of ten (i.e., hundreds); and so on. Using only ten different symbols, this system extends the range of numbers indefinitely. We may be so used to it that we hardly realise that 47023 (to take an example of a decimal number) is really shorthand for

$$
\left(3 \times 10^{0}\right)+\left(2 \times 10^{1}\right)+\left(0 \times 10^{2}\right)+\left(7 \times 10^{3}\right)+\left(4 \times 10^{4}\right)
$$

And of course we shift to the right for subdivisions, $10^{-1} .10^{-2}$, etc., the units position ( $\times 10^{\circ}$ or 1 ) being identified by a dot on its righthand side.

For the purposes of digital computers this system has the disadvantage of necessitating ten different unmistakable electronic states, such as values of collector current, to represent the ten numerals. Even if extensive negative feedback is used to counteract changes in transistor characteristics and supply voltages, it is very difficult to alter the states at will by the required number of units without risk of error. And even one unit of error is serious when it is in the millions column!

But if the number of different states is reduced from ten to two everything becomes very easy. We need only consider two states: 'off' and 'on'. The last chapter was mainly about circuits that are in an 'off' or an 'on' state, and signals that can be used to change them from one to the other. So long as these signals are not weaker than a certain minimum, their strength is unimportant; and so is the exact amount of current or voltage in the 'on' state.

The symbol corresponding to 'off' is 0 , and 'on' is 1 . That is not very hard to remember. Except for the smaller variety of numerals, the procedure is the same as in decimal notation. That is to say, when the variety of numerals runs out (as it does above 1!) the units position drops back to 0 and 1 is carried. So the binary numbers go up like this:

| Decimal <br> notation | Binary <br> notation |
| :---: | :---: |
| 0 | 0 |
| 1 | 1 |
| 2 | 10 |
| 3 | 11 |
| 4 | 100 |
| 5 | 101 |
| 6 | 110 |
| 7 | 111 |
| 8 | 1000 |
| 9 | 1001 |
| 10 | 1010 |
| etc. | etc. |

The binary number 1010111 therefore is shorthand for $\left(1 \times 2^{0}\right)+$ $\left(1 \times 2^{1}\right)+\left(1 \times 2^{2}\right)+\left(0 \times 2^{3}\right)+\left(1 \times 2^{4}\right)+\left(0 \times 2^{5}\right)+\left(1 \times 2^{9}\right)$ $=1+2+4+16+64=87$ in decimal notation.

An obvious disadvantage of binary notation is the greater number of digits required; about $3 \frac{1}{3}$ times as many on the average. But this is usually more than offset by the simpler circuitry, even allowing for the decimal-to-binary and binary-to-decimal converters at input and output.

The smallest piece of information (using that word in its technical sense) is a 'yes' or 'no' answer to a question, which is given by 1 or 0 and is therefore equal to one binary digit, commonly abbreviated to bit. A specified number of bits, making up a binary number, is a word.
The basic operation in digital computers is addition. Multiplication can be regarded as repeated addition; $4 \times 3$, for example, means $4+4+4$. This is a tedious way for humans to multiply, but not for computers that can do millions of operations per second. Where high-speed multiplication is necessary, provision can however be made for the computer to do it directly. 'Long multiplication' is extremely simple in the binary scale. For example, multiply 11010 by 101 :
11010
00000
$\frac{11010}{10000010}$

In practice one would not write down the second row, or any row corresponding to 0 in the multiplier. As we see, the essential processes are shifting the whole multiplicand successively to the left and adding all the results. Means for doing both these things will appear in the next section.

There are several possible electronic methods of subtracting, corresponding to methods people use. In one. we replace each digit of the number to be subtracted by its nines complement-the number needed to make it up to nine-and add, finally transferring the 1 at the left-hand end to the right-hand end; for example, take 108 from 762:

```
7 6 2
891 (nines complement of 108)
1653 (sum)
& Answer:654
```

A binary computer similarly adds the ones complement, which it obtains simply by means of a NOT gate (Sec. 24.11); for example, take 1101 from 10110:
10110

0010 \begin{tabular}{l}
(ones complement of 1101) <br>
11000 <br>
(sum) <br>
<br>

 

Answer: 1001
\end{tabular}

Division is repeated subtraction. To take a decimal example, $34 \div 9 ; 9$ can be taken from 34 three times, leaving 7 over, so the answer is $3 \frac{7}{7}$. The binary computer has to count the number of times the subtraction can be done before the remainder becomes negative.

### 25.5 Logical Circuits

Obviously we need a set of symbols for the logical gates described in Sec. 24.11, not only for brevity but also for clarity because the


Fig. 25.2-Standard symbols for binary logic components. Inputs are on the left; output on the right



circuit diagrams of gates do not instantly reveal what kind they are. Various symbols are in use; those in Fig. 25.2 are British standard. The number inside the circle indicates how many of the inputs have to be 'on' in order to give an 'on' output.

The basic binary operation is to add two digits. Let us call them $A$ and $B$. There are four possibilities, listed in the following table:

| $A$ | $B$ |  | Sum | Carry |
| :---: | :---: | :---: | :---: | :---: |
| $0+0$ | $=$ | 0 | 0 |  |
| $0+1$ | $=$ | 1 | 0 |  |
| $1+0$ | $=$ | 1 | 0 |  |
| $1+1$ | $=$ | 1 |  |  |

On the right we see the results the computer has to obtain. A whole system of algebra, described by Boole in 1854, has been resurrected in order to arrive at the necessary computer circuitry for all requirements in the quickest and most certain way, but there is not enough space for it here. Instead, a logical circuit for solving our elementary problem appears ready-made in Fig. 25.3. Note that a circle across an input or output means NOT, which operation is performed by an inverter (Sec. 24.11). Let us now check that it does do its job, by making a table of inputs and outputs of the four gates for each of the four possible values of $A$ and $B$. In the circuit, $I$ is represented by a pulse and 0 by no pulse (or a space). It is assumed that $A$ and $B$ are presented simultaneously to the system at the terminals so marked.

| Inputs <br> $A$ | $\boldsymbol{B}$ | $\mathbf{G}_{1}$ <br> inputs | $\mathbf{G}_{1}$ <br> output | $\mathbf{G}_{2}$ <br> inputs | $\mathbf{G}_{2}$ <br> output | $\mathbf{G}_{3}$ <br> output | $\mathbf{G}_{4}$ <br> output | Answer |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 0 | 0 | 0 | 1 | 0 | 1 | 0 | 0 | 0 | 0 |
| 0 | 1 | 0 | 0 | 0 | 1 | 1 | 1 | 1 | 0 |
| 1 | 0 | 1 | 1 | 1 | 0 | 0 | 0 | 1 | 0 |
| 1 | 1 | 1 | 0 | 0 | 0 | 1 | 0 | 0 | 1 |

The $B$ input to $\mathrm{G}_{1}$, being via a NOT, is stown reversed in the table; likewise the $A$ input to $\mathrm{G}_{2} . \mathrm{G}_{1}$ and $\mathrm{G}_{2}$, being and gates, give a 1 output only when both $A$ and $B$ are I. Their outputs are the inputs to $\mathrm{G}_{3}$, which gives 1 when either of them is 1 . Its output is the required sum for that particular digital place. But in the case of $1+1$ only we need to carry 1 , and this result is indicated by the output of the single AND gate, $\mathrm{G}_{4}$.

We see then that this logic circuit does all that is required of it for the first (units) digit, but in all higher places there may be a carry from the one below. So a third input must be provided for and the table extended to include altogether eight possibilities.

In designing an adder for more than one place, two courses are open to us. We can use a single adder to deal with the digit places in succession. This is called the series system, and is economical in

Fig. 25.3-A half-adder circuit-a building block' in binary computers

circuits but wasteful of time. And it must include some means for storing the carried 'bit' produced by the adding operation for one digital place until the next higher one is being dealt with. A bistable is commonly used for this purpose. Suppose TR1 in Fig. 24.18 receives a periodical negative pulse at $b_{1}$. If $T_{R_{1}}$ was originally 'on', the first pulse would turn it off and subsequent pulses would do nothing. But if $T_{R_{2}}$ receives a pulse from some other source (the Carry output of Fig. 25.3, say), turning it off and $\mathrm{T}_{\mathrm{R}_{1}}$ on, the next pulse to $\mathrm{T}_{\mathrm{R}_{1}}$ would turn it off and thereby generate a positive

Fig. 25.4-Showing how half-adders are combinea in a computer

pulse at $c_{1}$ and a negative pulse at $c_{2}$. Whichever of these was of the appropriate polarity would be fed back into the adding system as a carried digit. The system shown in Fig. 25.3 is known as a half adder and is readily available in integrated circuit form. A complete or full adder is one that copes with a carried digit.

The alternative to the series system is the parallel one, in which addition of all the digits takes place at the same time. A full adder is needed for every place except the lowest. Fig. 25.4 shows the circuitry for the first two places (ones and twos). For higher places the
twos circuit is repeated to the left as many times as may be necessary. Each half-adder block comprises the elements in Fig. 25.3. We can see at once that any carried 1 resulting from addition of the $A_{1}$ and $B_{1}$ is added to the sum of $A_{2}$ and $B_{2}$ and comes out as $S_{2}$. If there is


Fig. 25.5-Showing the successive steps in the action of a shift register
a carry from either of the half adders in the full adder, it comes out as $C_{2}$. And so on. To check the performance of the full adder, make a table listing the eight possible combinations of $A_{2}, B_{2}$ and $C_{1}$, and with the aid of our table for the performance of a half adder find the corresponding values of $S_{2}$ and $C_{2}$, comparing these with what you get by doing the sum mentally. They should of course be the same.

Another essential component of any arithmetic unit is the shift register. This is a row of as many bistables as there are bits in a word. Its purpose is to transfer the bits one place to the right every time a regular periodical shift pulse occurs. In this way a whole word can be stored. Fig. 25.5 shows this function for a simple example of only five digit places. The number, 10011, is first shown ready to enter, at the left. One pulse transfers the units digit to the first bistable, where it exists as one of its two possible states. Which one it is determines what happens as a result of the next shift pulse. If it is an 'on' or 1 state, as here, it reverses the next bistable from 0 to 1 . And so in successive stages the whole word is taken in. The last diagram shows the first digit emerging as a 1 pulse. If the last bistable's output is fed back to the first, as in some shift registers, the word is kept circulating and is thus stored until required and made to emerge by some control signal. Or it may be directly applied to
the $A$ input of an adder and added one digit at a time to another numerical word now coming in directly, or shifted one place at a time for multiplication.

For its various functions the basic bistable circuit is subject to slight modifications. The one shown in Fig. 24.19 is arranged so that successive input pulses reverse its state. In a shift register type the $\mathrm{T}_{\mathbf{R}_{1}}$ output of one triggers $\mathrm{T}_{\mathbf{R}_{2}}$ in the next bistable, and the $\mathrm{T}_{\mathbf{R}_{2}}$ output of the first triggers the $T_{R_{1}}$ of the next.

### 25.6 Counters

Electronic counters are used in computers for counting the number of times an adding process has been carried out in multiplying, or subtracting in dividing. But they are also used much more widely; for example, counting articles passing on a conveyor belt, each article generating a pulse by interrupting a beam of light falling on a light-sensitive type of transistor. In high-precision frequency meters a counter is switched on for (say) exactly one second, during which the number of cycles is counted. Or by using a known frequency, an unknown time interval can be measured, also with great precision.

We are familiar with mechanical decimal counters, on car dashboards, for example. A mechanical binary counter would have only two symbols, 0 and 1, on each wheel. The first incoming 'pulse' would move it from 0 to 1 ; the second, back to 0 and at the same time the wheel would pulse the second wheel from 0 to 1 . And so on. Obviously we can do the same thing electronically, using a chain of bistables of the Fig. 24.19 type in place of wheels. Fig. 25.6 shows in some detail how.

At $a$ the circuits of the first two bistables in the series are shown in simplified form, omitting for clarity such practical details as the


Fig. 25.6-(a) A wo-digital place binary counter, using one bistable per place, with (b) waveforms during a count of 3 pulses
diodes in Fig. 24.19 needed to delete positive pulses and steer negative ones to the 'on' transistor. The waveforms at the points marked are shown at $b$. On the top line are the pulses to be counted. They need not be at regular intervals. 'Voltage positive' is taken to mean 1 , so is read from the first collector in each bistable, but equally well it could be 'transistor on' or 'current flowing' and read from the second. The second bistable is triggered at the even-numbered input pulses and indicates the second digit. If each bistable has, say, a lamp to show whether its second half is on or off, and the lamps are placed in a row, right to left, the total of a series of pulses from one or more sources could be read off in binary notation, as shown on the last line in Fig. 25.6b for the first two. For convenience a binary-to-decimal converter could be interposed.

### 25.7 Memories

As we have seen, a shift register can be used as a store, or memory, but its capacity is limited to as many bits as it has bistablesnormally, one word. Large computers have provision for storing many thousands of words, and on this scale magnetic methods are cheaper, more compact and extremely reliable. Where the contents are required to be referred to in sequence, magnetic tape is particularly suitable, but for picking out data here and there, as we do when using arithmetical tables or reference books (random retrieval, as it is called) such types as the magnetic core matrix are better.

The property exploited in magnetic stores is hysteresis which we noted briefly in Sec. 8.14 in the undesired role of a cause of power


Fig. 25.7-Hysteresis curve for magnetic material of the type used in binary store systems (Fig. 25.8)
loss in magnetic cores of inductors. If a strong current is passed through an iron-cored coil and is then switched off, the core is found to have retained some of its magnetism. How much depends
on the core material. Some magnetic materials have a hysteresis curve of the nearly square type shown in Fig. 25.7, where $H$ is the magnetizing field produced by the current through the coil and $B$


Fig. 25.8-A small section of a binary computer store of the magnetic core type
is the resulting magnetic flux density in the core. (The curve for a perfectly non-hysteretic material would be a diagonal straight line.) If the current in the coil alternated between $H_{\mathrm{m}}$ and $-H_{\mathrm{m}}$ the core magnetization would follow the curve anticlock wise. The important point is that once $H_{\mathrm{m}}$ has been applied any half-strength currents, making $H$ equal to $H_{\mathrm{m}} / 2$ or $-H_{\mathrm{m}} / 2$, have very little effect on $B$, which remains in the region of $B_{1}$. A field equal to $-H_{\mathrm{m}}$ is needed to reverse the magnetization, and once that has been done any half measures fail to change it significantly from $B_{0}$.

The subscripts ${ }_{1}$ and ${ }_{0}$ indicate that these are the two 'states' by which such a core can be used to store one bit. Fig. 25.8 is a sketch showing a sample of how a large number of cores-say 1024 ( $32 \times 32$ ) -can be assembled in compact form. Here the usual forms of coil and core have been interchanged, but the fact that the magnetizing coil is a straight wire encircled by the core makes no difference to the basic principle (Fig. 4.2). In practice it means that large pulse currents are needed, at very low voltages, and this fits in well with transistor characteristics.

Suppose that current from left to right through any of the horizontal wires, or downwards through any of the vertical wires, would if strong enough ensure that all the cores around it were in the 'on'

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state, and currents in the reverse direction ensured an "off state; then, if both currents were always half full strength the only core in the matrix that could have its state changed would be the one encircling wires through both of which pulses were sent simultaneously. So any core in the matrix can be changed at will by passing pulses of current through the appropriate horizontal and vertical wires.

A change of state in any core is detected by the diagonal wire threaded through all the cores. The slight changes in magnetization of cores through which only one current passes are cancelled out by the alternate reversals in direction of the diagonal wire. Suppose then that the computer has been instructed to find out what information is stored in a particular core. It passes current pulses in the 0 direction through both wires passing through that core. If it is already in the 0 state, nothing happens, and the answer deduced is ' 0 '. But if it is in the I state the currents reverse the magnetization and this reversal induces a pulse in the diagonal wire, and this pulse the computer interprets as ' 1 '.

In this simple scheme, reading the information also destroys it, but magnetic matrices have been devised in which the information is either retained or restored.

## Power Supplies

### 26.1 The Power Required

In the preceding chapters the power supplies needed to make things work have been assumed. Let us now look at them in detail. Power for valves and cathode-ray tubes can be divided into three classes: cathode-heating, anode (including screen-grid) supplies, and grid bias.

So far as cathode heating is concerned, it is purely a matter of convenience that it is done electrically. In principle there is no reason why it could not be done by a bunsen burner or by focusing the sun on it with a lens. But there are overwhelming practical advantages in electric heating. Nearly always it is done, for convenience, by a.c. Cathode heating power has to be supplied all the time the valves are required for work, whether or not they are actually doing any.

Power supplied to the anode is more directly useful, in that a part of it is converted into the output of the valve. This is generally not true of power taken by auxiliary electrodes, so valves are designed to reduce that to a minimum.

Not all valves need any grid bias; and, of those that do, not all have to be provided with it from a special supply. And only in large sending valves is any appreciable power consumed, the reason being that with this exception care is taken to keep grid current as nearly as possible down to nil (Sec. 10.6). This restriction, desirable for most purposes, would unduly limit output from high-power valves.

Transistors have the great advantages of needing no heater power at all and of making moreeconomical use of the power that is supplied. Often the sole source of power is a small dry battery, or (where available) a car battery. Larger, permanently-installed transistor equipment can more conveniently and economically be supplied (like most valve equipment) from the mains.

### 26.2 Batteries

A battery consists of a number of cells, and although a single cell is commonly called a battery it is no more correct to do so than to call a single gun a battery.

A cell consists of two different sorts of conducting plates or electrodes separated by an 'exciting' fluid or electrolyte. The 485

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e.m.f. depends solely on the materials used, not on their size, and cannot greatly exceed 2 V . The only way of obtaining higher voltage is to connect a number of cells in series to form a battery.

Other things being equal, a small cell has a higher internal resistance than a large one, so the amount of current that can be


Fig. 26.1-Cross-section of a dry cell, showing the essential parts
drawn from it without reducing its terminal voltage seriously is small. To obtain a large current a number of cells of equal voltage must be connected in parallel, or (preferably) larger cells used.

Many types of cell have been devised, but only two are commonly used. They represent the two main classes of batteries--primary and secondary.

Primary cells cease to provide current when the chemical constituents are exhausted; whereas secondary cells, generally called accumulators, can be 'recharged' by passing a current back through them by means of a source of greater e.m.f.

The most commonly used primary batteries are the so-called dry batteries-a badly chosen term, because a really dry battery would not work. The name refers to cells in which the electrolyte is in the form of a paste instead of a free liquid. The essential ingredient is ammonium chloride or 'sal-ammoniac' and the plates are zinc ( - ) and carbon $(+)$. The e.m.f. is about 1.4 V per cell. When current is drawn, each coulomb causes a certain quantity of the zinc and electrolyte to be chemically changed. Towards the end of its life, the e.m.f. falls and internal resistance rises. Ideally, the cell would last indefinitely when not in use, but in practice a certain amount of 'local action' goes on inside, causing it to deteriorate; in time, therefore, a cell becomes exhausted even if never used.

If only the ingredients named above were included in a dry cell, its voltage would drop rapidly when supplying current, owing to the formation of a layer of hydrogen bubbles on the surface of the carbon. To reduce this layer a depolarizer is used, consisting largely of manganese dioxide around the carbon electrode. Fig. 26.1 shows
a section of the cylindrical form of dry cell. Dry batteries of a rectangular shape are built up of interleaved flat plates.

Where the need for compactness and long life justifies the greater cost, the mercury type of cell is preferred. The positive electrode is steel. the negative zinc, the electrolyte potassium hydroxide and the depolarizer mercuric oxide. The voltage remains remarkably constant at about $1 \cdot 2$.

The most commonly used secondary cell is the lead-acid type, in which the plates consist of lead frames or grids filled with different compounds of lead, and the electrolyte is dilute sulphuric acid. Accumulators have an e.m.f. of 2 V per cell and possess the great advantages of very low internal resistance and of being rechargeable.

Against this there are certain drawbacks. Lead is heavy. The acid is very corrosive, and liable to cause much damage if it leaks out or creeps up to the terminals. If the cell is allowed to stand for many weeks, even if unused, it becomes discharged, and if left in that condition it 'sulphates', that is to say, the plates become coated with lead sulphate which cannot readily be removed and which permanently reduces the number of ampere-hours the cell can yield on one charge. If the terminals are short-circuited, the resistance of the accumulator is so low that a very heavy current-possibly hundreds of amps-flows and is liable to cause permanent damage. Other types of secondary battery have been devised to avoid these snags but have had only limited success.

### 26.3 D.C. from A.C.

Power from the mains costs not more than about one fortieth that of power from dry batteries, so there is a strong incentive to use it except where very little power is needed or portability is essential. Almost everywhere mains power is in the form of a.c. Except for cathode heating, a.c. cannot be used as it is, because the requirement is for constant current or voltage in one direction. So we have to convert from a.c. to d.c. and therefore an essential part of a power unit is a rectifier.

If the a.c. is sinusoidal (as it usually nearly is) it can be represented as in Fig. 26.2a. The result of connecting a rectifier in series with it is to abolish half of every cycle, as shown as $b$. The average of this is only about one-third of the peak value (Sec. 18.3) and in any case it would be useless in this form because it is not continuous. By using a resermir capacitor (Sec. 18.6 ) this defect is removed and at the same time the average is brought nearly up to the peak voltage.

The conditions are not quite the same as in a detector because the current drawn is usually much larger and the frequency much lower. Unless the reservoir capacitance is enormous, therefore, it has time to discharge appreciably between one cycle and the next, as in Fig. 26.2c. This is described as d.c. with a ripple, or unsmoothed d.c.,
which would cause a hum if used to feed a receiver. The next requirement, then, is a smoothing circuit, which is a low-pass filter (Sec. 18.10) designed to impede the a.c. ripple while allowing the d.c. to pass freely. If the filter is effective the result is as in Fig. 26.2d, practically indistinguishable from current supplied by a battery.

In some applications, especially those in which varying amounts of current are drawn and passing through the resistance of the rectifier and filter cause undesirable variations in voltage, there is a need for stabilization (Sec. 26.9) to keep the voltage automatically constant.

Although in many domestic receivers the rectifier is connected straight to the mains, this is not suitable if the voltage required is much different from the mains voltage, and even when it is not the use of an intervening transformer is better practice because it improves safety by isolating the circuits from the mains, enabling them to be earthed.

### 26.4 Types of Rectifier

At one time vacuum diodes (Sec. 9.4) were used not only as signal rectifiers (detectors) but also as power rectifiers. To handle the relatively large currents in power units, correspondingly large


Fig. 26.2-(a) Original a.c.; (b) after rectification; (c) the effect of a reservoir capacitor: (d) after passing through a smoothing filter
cathodes (consuming much power to heat) and anodes were needed; and to withstand high voltages the spacing between cathode and anode had to be relatively large, resulting in a high internal resistance
and hence loss of voltage and power. So they have given place to semiconductor diodes, which have no heaters to waste power or delay starting while heating up. The voltage drop across them seldom exceeds 1.5 V , so the loss of power here is far less, and because

Fig. 26.3-Symbol for a thyristor

there is much less heat to dissipate they can be much smaller. And they are reasonably cheap and practically unbreakable.

Copper oxide, selenium and germanium rectifiers have been used, in that historical order, and to some extent still are, but the tendency is for silicon to prevail. Although its voltage drop is about three times that of germanium it is still quite small, being of the order of 1 V , and the reverse voltage it can withstand is higher: up to about 1.5 kV , according to type. For higher voltages, diodes are connected in series, usually integrally. Another advantage of silicon over germanium is the smaller reverse current, or alternatively the higher allowable temperature and therefore output. The rectifiers are made not only in very small sizes for domestic equipment but also to provide many kilowatts for senders and industrial uses, and even these rectifiers are surprisingly small.

Thyristors are silicon rectifiers provided with a third electrode, the gate, which is capable of exercising a limited amount of control. Its symbol is shown in Fig. 26.3. If the gate is kept at or near the same potential as the cathode, the thyristor behaves like a reversed rectifier in both directions; whatever the polarity of the voltage applied between anode and cathode, only leakage current will flow. But if the gate is made a few volts positive so that a small gate current flows, the thyristor behaves as an ordinary silicon rectifier. Although the gate can be used to turn the rectifier on, it is powerless to turn it off; the original state is restored only when forward anode voltage is taken off. This normally happens to a rectifier during half of every cycle. The usefulness of the gate is that it enables forward current to be prevented during any desired proportion of the other half-cycle.

### 26.5 Rectifier Circuits

There are various ways in which rectifiers can be connected in power unit circuits, and one of the simplest is indicated in Fig. 26.4a.

A transformer T is shown for stepping the mains voltage up or down as required, and a single rectifier-the symbol denotes any type-is connected in series with it and a reservoir $C_{1}$. A simple filter circuit consisting of a choke $L$ and a capacitor $C_{2}$ completes the apparatus.

This is called a half-wave rectifier circuit, because only one-half of each a.c. cycle is utilized, the other being suppressed (Fig. 26.2b).


Fig. 26.4-The voltages and rectifier current in a simple halfwave rectifier power-unit circuit (a) are shown in diagram b

If, as is desirable, the capacitance of $\mathrm{C}_{1}$ is large enough for the voltage not to drop much between half-cycles, the proportion of each cycle during which the alternating voltage exceeds it is very small; and, as the recharging of $\mathrm{C}_{1}$ has to take place during these brief moments, the rectifier is obliged to pass a current many times greater than that drawn off steadily at the output. If a large output current is needed, then the reservoir capacitance must be very large, the rectifier must be able to pass very heavy peak currents, and there are difficulties in designing the transformer to work under these conditions. So this circuit is used mainly for low-current highvoltage applications and in a.c./d.c. sets, which are fed direct from the mains without a transformer. When such a unit is used on d.c., the rectifier serves no useful purpose. The set must of course be connected to d.c. mains in the way that enables the rectifier to pass current.

Note that at the moment when the input voltage is at its negative peak it is in series with the rectified voltage across $C_{1}$, which is nearly as great and in the same direction. So the peak inverse voltage (p.i.v.) across the diode is nearly twice the peak input voltage, or getting on for three times the input r.m.s. voltage. This fact has to be remembered when choosing rectifiers.

In Fig. 26.5 the half-wave circuit (a) is shown along with more

(e) VOLTAGE DOUBLER (2)

(f) VOLTAGE QUADRUPLER

Fig. 26.5-Comparison of various rectifier circuits, the voltage supplied from the transformer heing the same in all. Note the comparative outpur voltages
elaborate arrangements; and for comparison the transformer is assumed to supply the same total peak voltage in all of them.
(a) Half-wave. The no-load output voltage (i.e., the voltage when no current is being drawn) is very nearly equal to the peak input voltage; but, for the reasons given above, the voltage tends to drop considerably on load.
(b) Full-wave. By centre-tapping the transformer secondary, two type $a$ circuits can be arranged, in series with the a.c. source, to feed the load in parallel. For the same voltage from the whole secondary, the output voltage is halved; but for a given rectifier rating the current is doubled, and at the same time is steadier, because each rectifier takes it in turn to replenish the reservoir. The resulting ripple, being at twice the a.c. frequency, is easier for the filter to smooth out; but it is more audible.

This is sometimes described as a 2 -phase circuit (the transformer gives two phases $180^{\circ}$ apart); and though less suitable than $a$ for high voltage is better for large current. It is suitable for valve rectifiers because one cathode will do for both halves.
(c) Bridge. Circuit $b$ yields a terminal which is either positive or negative with respect to the centre tap by a voltage slightly less than half the total transformer peak voltage. By connecting a second pair of rectifiers in parallel with the same transformer winding, but in the opposite polarity, a terminal of opposite polarity is obtained. The total voltage between these two terminals is therefore twice that between one of them and the centre tap; i.e., it is nearly equal to the peak voltage across the whole transformer. The outputs of the two pairs of rectifiers are effectively in series. Unless a halfvoltage point is wanted, the centre-tap can be omitted.

This arrangement gives approximately the same no-load output voltage as $a$, but with the advantages of full-wave rectification. Because three separate cathodes are needed the circuit is not commonly used with valve rectifiers; it is, however, quite usual with semiconductor rectifiers, especially for low voltages.
(d) Voltage Doubler: first method. In contrast to $b$, the rectifiers are fed in parallel off the source to give outputs in series. As compared with $a$, the voltage is doubled and the ripple is twice the frequency. The voltage drop on load tends to be large, because each reservoir is replenished only once per cycle.

Like $c$ (with centre tap), this circuit gives two supplies, one positive and one negative, from one transformer winding.
(e) Voltage Doubler: second method. $\mathrm{R}_{1}$ and $\mathrm{C}_{1}$ act, as explained in connection with Fig. 18.6, to bring the negative peaks (say) to earth potential, so that the positive peaks are twice as great a voltage with respect to earth. $\mathrm{R}_{2}$ and $\mathrm{C}_{2}$ employ this as the input of a type $a$ system, giving an output which, on no-load at least, rises to almost twice the transformer secondary peak voltage. Compared with $d_{\text {, }}$ one output terminal is common to the source, which may be convenient for some purposes. As $\mathrm{R}_{2} \mathrm{C}_{2}$ is a half-wave rectifier
this system suffers from the disadvantages of that type, as well as the losses in $\mathrm{R}_{1}$, so is confined to high-voltage low-current power units.
( $f$ ) Voltage Quadrupler. By connecting a second type $e$ circuit in parallel with the transformer, to give an equal voltage output of opposite polarity, the total output voltage tends towards four times the peak of the supply. The conversion of $e$ to $f$ is analogous to the conversion of $a$ to $d$.

### 26.6 Filters

We have already used a simple filter circuit ( $\mathrm{R}_{F}$ and $\mathrm{C}_{F}$ in Fig. 18.10) to smooth out the unwanted r.f. left over from the rectification process in a detector. The same circuit is often used in power units, but the values of $R_{F}$ and $C_{F}$ are greatly different. The frequency of the ripple to be smoothed following a half-wave rectifier is usually 50 Hz , and 100 Hz with full-wave. For effective smoothing, the reactance of $C_{F}$ at this frequency must be low compared with the resistance $R_{F}$. And because of the relatively heavy current in a power unit, $R_{F}$ must itself be quite low if there is not to be an excessive wastage of volts.

For this reason an iron-cored choke coil is often substituted. It has to be carefully designed if the core is not to be saturated by the d.c. (Fig. 4.8) which greatly reduces the inductance and therefore the effectiveness of the coil for smoothing. A typical filter, $\mathrm{LC}_{2}$, is seen in the half-wave rectifier circuit in Fig. 26.4.

Although a reservoir capacitor brings the rectified voltage almost up to the peak input voltage when no current is being taken, the voltage falls off fairly steeply with increase of load current. In technical language, the regulation is not very good. For purposes where it is important that the voltage should remain steady in spite of a fluctuating load current-for example, high-power Class B amplifiers-what is called a choke-input filter is used. In the circuit diagram this looks the same as, say, Fig. 26.4a, except that the reservoir capacitor is missing. This is not the only difference, however, for the choke has to be of a special kind, known as a swinging choke. Instead of being designed so that its inductance varies as little as possible, its inductance increases substantially as current falls, with the aim of maintaining a nearly constant voltage across it. The output voltage falls very rapidly up to, say, one tenth full load current (below which the current is not allowed to fall) and thereafter is relatively steady. An advantage is that the current through the rectifier is relatively constant, instead of having peak values much greater than the mean as in Fig. 26.4b.

Because of the cost and weight of a choke having a high reactance at low frequencies, a resistor is used for smoothing whenever possible, and for this reason electrolytic capacitors of very high
capacitance-1 mF or more-are now customary. Even so, their impedance to the signal-frequency currents flowing through the power source may not be negligible, and a potential difference set up across this impedance by (say) the signal current in the output stage is passed on to all the other stages fed from the same supply and may seriously upset the working, possibly even causing continuous oscillation. We have, in fact, a form of feedback.

### 26.7 Decoupling

To obviate such undesirable conditions, decoupling is used. It is simply an individual filter for those stages liable to be seriously affected. As those stages are generally the preliminary ones, taking only a small proportion of the total current, a resistor having sufficient impedance at the frequencies concerned can be used. The loss in voltage may actually be desirable, as these stages are often required to run at lower voltages than the power output stage; in which case a single cheap component is made to serve the double purpose of a voltage-dropping resistance and (in conjunction with a capacitor) a decoupler. Of such is the essence of good commercial design.


Fig. 26.6-Example of the use of decoupling
In Fig. 26.6, which is an outline circuit diagram of part of an a.f. amplifier, R and C are decoupling components. They serve as a filter, preventing signal voltages set up at point X by signal currents flowing through the power source from reaching the previous stage and then being amplified. In this example the output stage is fed direct from the reservoir capacitor, so there will be significant hum voltage at X . This may not matter to the output stage because it is push-pull which tends to balance out hum, but it could be serious if it came in via the previous stage, so the extra smoothing given by

R and C is needed for this reason too. Because of the comparatively small current flowing through R its resistance can be made very much larger and therefore more eflective than could possibly be used in the main supply line. Signal currents in the stage fed by it find an easy path via C and so are kept out of other parts of the amplifier.

It is the extensive use of decoupling and filtering that makes multi-stage circuit diagrams look so complicated. But once their purpose has been grasped it is easy to sort them out from the main signal circuits. Note the use of a detached "chassis' symbol at the foot of C to avoid drawing a confusing crossing wire to the base line. Considerable clarification of complicated circuits can be obtained in this way.

### 26.8 Bias Supplies

From time to time we have considered various methods of providing bias, and we now review them.

The most obvious way is by means of a battery or other source in series with the circuit connecting the two electrodes concerned, as in Figs. 10.6 and 11.2. A slight modification is the parallel-feed system (Fig. 13.4 and many others) where $R_{B}$ is a resistance so high that it causes negligible loss of the signal applied via C. A choke coil is sometimes used instead of high resistance if the bias voltage must not be subject to drop due to current through it.
Obviously it would be possible to replace a bias battery by a mains power unit. Except for high-power equipment this is not done, because the power required is so small and there are more economical alternatives.

With valves and f.e.ts, which normally are biased opposite in polarity to the anode or drain, one of those alternatives is to insert a resistor as explained in Secs. 13.5 and 6. The required amount of bias is obtained by choice of resistance. To some extent this source of bias is self-adjusting; if an excessive current flows the bias voltage is correspondingly large and the rise in current is restrained. Also it is convenient for providing each stage with its own most suitable bias. Strictly it is cathode or source bias that is supplied, at the expense of anode or drain voltage, and this loss may sometimes need to be taken into account.

If only a simple resistor is used, it carries not only the steady (or z.f.) component of current but also the signal current, which produces a voltage in opposition to the input. We have, in fact, negative feedback (Sec. 19.15). If that is wanted, well and good; but if not, then something has to be done to provide a path of relatively negligible impedance for the signal currents, without disturbing the z.f. resistance. A by-pass capacitor, $\mathrm{C}_{S}$ in Fig. 13.8, serves the purpose ; for r.f. a value of $0.1 \mu \mathrm{~F}$ or even less is enough, but for the

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lowest audio frequencies $50 \mu \mathrm{~F}$ or $100 \mu \mathrm{~F}$ may be needed.
Sometimes it is convenient to have a bias voltage that is equal to the peak value of the incoming signal. This is obtained by using the clamping principle (Sec. 24.10). With a.f. signals it is sometimes used for the stage immediately following a detector. Although a separate diode detector is commonly used, the gate-source or gridcathode diode can be made to serve this purpose.

A very common application of this type of bias is for oscillators; the fact that the bias voltage is nearly equal to the peak oscillation voltage means that it increases and so acts as a governor to prevent the amplitude of oscillation from increasing excessively. If the positive feedback in the oscillator is very large and the time constant equal to the duration of many cycles, the first burst of oscillation may generate such a large bias that the device so biased is cut off, so quenching the oscillation, which cannot resume until some of the charge has had time to leak off the grid. Oscillation then restarts, is again quenched, and so on. This phenomenon of intermittent oscillation is known as squegging, and is sometimes usefully employed. Elsewhere, it is a nuisance, which can be cured by reducing the positive feedback or the time constant of the biasing circuit or both. The blocking oscillator (Sec. 24.6) is sometimes considered as an extreme form of squegging.

### 26.9 Stabilization

Often there is a need to maintain the terminal voltage of a power unit constant despite changes in the amount of current drawn. Sometimes the dual problem arises : to keep the output current constant regardless of the voltage. In the first case the desired result would be obtained if the internal resistance of the power unit was reduced to zero; in the second, if it was increased to infinity. We have seen (Sec. 19.17) that the tendency of negative feedback is to do one or other of these things, according to how it is applied, and in fact many stabilized power units can be regarded as negativefeedback systems. But it is perhaps rather clearer to consider them from the following angle.

The general principle of a voltage stabilizer, illustrated in Fig. 26.7 , is to compare its output voltage (or a certain fraction of it) with a known constant voltage to which it should be equal. If it is not, the difference voltage is amplified and used to vary the output voltage in the direction that brings it toward the desired level. The purpose of the amplification is to ensure that an extremely small 'error' in the actual output voltage is sufficient to exercise effective control. Besides rendering the output voltage almost unaffected by variations (within limits) in the impedance of the load, it does the same with regard to fluctuations in mains voltage.

An essential requirement of such a stabilizer is a source of con-
stant voltage. A much-used device for obtaining it is a special kind of diode, known as a Zener diode, which is distinguished by a very constant (and usually low) breakdown voltage (Sec. 9.11). One takes great care, with other types of diode and transistors, not to apply a reverse voltage sufficiently large to risk breakdown, for


Fig. 26.7-Showing the principles of most voltage-stabilized power units
that would suddenly destroy nearly all resistance and the device would probably be ruined instantly by the uncontrolled current flow. Zener diodes, however, are used in a perpetual state of breakdown. They are no more immune from sudden death than any other kind; survival depends on their being used with sufficient resistance in series to limit the current to a safe amount. This resistance serves also to absorb changes in the voltage of the supply. Fig. 26.8 shows a typical Zener characteristic curve, together with a load line, R. With 12 V applied to the diode and R in series (note the modified


Fig. 26.8-Zener diode characteristic curve and circuit
symbol to indicate that the diode is a Zener type), $R$ being $1 \mathrm{k} \Omega$, the voltage across R is shown to be 6.3 V and the current through $R$ therefore 6.3 mA .

The amount of impurity ('doping') in this particular silicon Zener diode is such that its voltage at 6.3 mA is 5.7 V , which happens to be about the least affected by temperature so is often chosen although Zeners are made for lower and much higher voltages. If the supply voltage fell even as much as 2 V , so that the load line took up the dotted position, the Zener voltage would fall by only 0.02 V . It therefore gives a 100 -fold stabilization. Fed from a higher voltage through a higher resistance, the ratio would be even greater. And any hum ripple in the supple is greatly reduced. So the value of Zener diodes as a source of known constant voltage is obvious.

The simple arrangement shown in Fig. 26.8 can be used by itself as a stabilized power unit. Suppose 2 mA is drawn from across the diode; the current from the source remains practically at 6.3 mA because there is negligible change in voltage across $R$. So the diode current drops to $4 \cdot 3$, but this has hardly any effect on the voltage across it. Obviously this particular design is very limited in the power it can provide, but Zener diodes are available that can pass 100 mA at 75 V and supply many watts. One disadvantage is that the stabilized voltage is fixed by the particular Zener voltage. Another is that the supply voltage must be much greater and a lot of power is wasted in the series resistor as well as in the diode. But in the role of constant-voltage device in Fig. 26.7 it is not required to supply more than a few milliwatts. And because its voltage is compared with a fraction $R_{2} /\left(R_{1}+R_{2}\right)$ of the output voltage, any convenient Zener voltage can be used to stabilize almost any output voltage. Further, by making the $\mathrm{R}_{1} \mathrm{R}_{2}$ potential-divider tapping variable the output voltage can be varied as required.

The design of voltage and current stabilizers has attracted an extraordinary amount of attention, and thousands of books and articles have been written on the subject, in which details of the contents of the boxes in Fig. 26.7 can be found.

This section will now be rounded off with a simple currentstabilizing device, Fig. 26.9. The Zener diode Z, in conjunction with $\mathrm{R}_{\mathrm{B}}$ which is chosen to pass a reasonable current through it, corresponds to the 'constant voltage' box in Fig. 26.7. The current to be kept constant, passing through $R_{E}$, provides the voltage $V_{E}$ to be compared with $V_{\mathrm{z}}$. The difference between them is applied to $T_{\mathrm{R}}$. Because of the high mutual conductance of the transistor chosen for this role, a very small change in $V_{B E}$, caused by any change in $I_{E}$, brings into play a very steep correcting action. Changes in load resistance, $R_{L}$, cause corresponding changes in $V_{E C}$, but we know (from Fig. 11.7, for instance) that this has very little effect on $I_{C}$ (and hence $I_{E}$ ) and such effect as it has is corrected as just explained, But of course there is a limit to the load resistance through which any chosen current can be passed. It occurs when the voltage across
$R_{L}$ nearly equals $V_{c c}$ and the transistor is bottomed. So a fairly high value of $V_{c c}$ (but not exceeding the rated maximum for the transistor) is desirable. The higher $V_{z}$, the stronger the correcting action. The amount of current to be stabilized can be controlled by varying $R_{E}$.

Fig. 26.9-Simple type of current stabilizer circuit


### 26.10 E.H.T.

Cathode-ray tubes in television and radar equipment need a voltage which is measured in kilovolts; 25 kV is a usual figure for colour television receivers. Fortunately the amount of current needed is quite small, of the order of 1 mA . A step-up mains transformer to develop such a voltage in the conventional way by a circuit such as Fig. 26.4 would be impracticably large, heavy and expensive, and also lethal. These objections are only partly removed by using the voltage quadrupler circuit (Fig. 26.5f). So, as mentioned in Sec. 23.10, use is made of the very high rate of change of current in the line-scanning transformer, seen in Fig. 23.20. The voltage developed in an inductor being proportional to the rate of current change, far fewer turns of wire are needed than with the very slow rate obtained with 50 Hz sinusoidal current for equal peak values. And the 15625 Hz frequency, combined with the smallness of the current, enables all smoothing components to be dispensed with, apart from the reservoir capacitor formed by the conducting coatings on the c.r. tube.

It is of course not necessary to have line-scanning equipment in order to make use of the same convenient principle for generating e.h.t. for other purposes. All that is needed is a blocking oscillator (Sec. 24.6) of suitable frequency, with an extra winding on its transformer to step up the voltage, and a rectifier and reservoir capacitor.

It can sometimes work out more economically to use a lower stepup ratio (fewer turns, of thicker wire) and to multiply the voltage by what is known as the Cockroft-Walton circuit. This is simply an extension of the principle of the voltage-doubler circuit, Fig. 26.5e. A voltage sextupler of this type is shown in Fig. 26.10. The


Fig. 26.10-Cockcroft-Walton voltage multiplier circuit
first part of $\mathrm{it}, \mathrm{abc}$, is a straightforward type of rectifier in which a steady voltage very nearly (when the current is small, as it is in this type of equipment) equal to the input peak voltage $E$ is maintained across $C_{1}$. The potential of $c$ (relative to earth, $b$ ) therefore alternates between 0 and $2 E$. This is the input to the second rectifier circuit, bed, but, the peak voltage being twice that in the first, $\mathrm{C}_{2}$ is charged to $2 E$. The third rectifier circuit, cde, similarly charges $C_{3}$ to $2 E$. And so on, finally giving an output voltage of nearly $6 E$ Note that the potentials across the capacitors in both left and right hand columns remain practically constant, but relative to the right-hand (output) column the left-hand one as a whole rises and falls in potential between $-E$ and $+E$ owing to the generator.

### 26.11 Cathode Heating

As stated in Sec. 9•3, the cathode of a valve or c.r. tube may be either directly or indirectly heated. The power for either type can be obtained from a battery, but in practice directly-heated valves are
then used because they require only about one-tenth of the power needed by the indirectly-heated types, and therefore a given battery will run them for much longer, or alternatively a much smaller battery can be used. These points are important, because battery power is so much more expensive than that drawn from the mains.

The vast majority of public electricity supplies are a.c., which can be stepped down to any convenient voltage for valve heating by means of a transformer. If an attempt were made to run directlyheated valves in this way it would be unsuccessful, because the slight variation in temperature between peak and zero of the a.c. cycle, and also the variations of potential between grid and filament from the same cause, would produce variations in anode current, which would be amplified by the following stages and cause a loud lowpitched hum. A possible exception is a carefully arranged output stage, which of course is not followed by any amplification.

The temperature of the relatively bulky indirectly-heated cathode changes too slowly to be affected by 50 Hz alterations, and as it carries none of the a.c. its potential is the same all over. Incidentally this improves the valve characteristics, and so does its larger area compared with a filament. Further, its greater rigidity allows the grid to be mounted closer to it, resulting in still greater mutual conductance.
A usual rating for a.c. valve heaters is 6.3 V . This odd voltage is to enable these valves to be run alternatively from 6 V or 12 V car batteries, which average 6.3 V or 12.6 V under working conditions. They are usually connected in parallel like battery valves, but if they are all designed to take the same current, so that they can be run in series, the rated voltage is anything from 6.3 to 40 according to type. A common practice is to connect all the valve heaters and c.r.t. heater of a set in series, together with sufficient resistance to enable them to be run direct from $200-250 \mathrm{~V}$ mains, either a.c. or d.c.

## Appendix 1

## ALTERNATIVE TECHNICAL TERMS

The reader of books and articles on radio is liable to be confused by the use of different terms to mean the same thing. In the following list the first alternatives to be mentioned are those preferred in this book. The associated terms are not necessarily exact equivalents. Terms distinctively American are printed in italics. The section numbers refer to where the terms are defined or explained.
Section
Accumulator-Secondary battery ..... $26 \cdot 2$
Aerial-Antenna ..... $17 \cdot 7$
Amplification-Gain ..... $11 \cdot 5$
Anode-Plate ..... $9 \cdot 4$
Anode a.c. resistance-Anode incremental resistance-Valve im- pedance-Plate impedance ..... 10.4
Anode battery-H.T.-'B' battery ..... $9 \cdot 4$
Astable-Multivibrator ..... $24 \cdot 7$
Atmospherics-Strays-X's-Static ..... $17 \cdot 13$
Audio frequency (a.f.)-Low frequency (I.f.)-Speech frequency- Voice frequency ..... $1 \cdot 4$
Automatic gain control (a.g.c.)-Automatic volume control (a.v.c.) ..... $22 \cdot 11$
Beam tetrode-Kinkless tetrode ..... $15 \cdot 7$
Bistable-Eccles-Jordan relay ..... 24.8
Blumlein integrator-Miller integrator ..... $24 \cdot 4$
Bottom bend-Cut-off ..... 11.9
Bottoming-Saturation ..... 11.9
Capacitance-Capacity ..... $3 \cdot 2$
Capacitor-Condenser ..... $3 \cdot 3$
Characteristic resistance-Characteristic impedance--Surge im- pedance. ..... $16 \cdot 3$
Coaxial-Concentric ..... $16 \cdot 1$
Common-anode valve - Cathode follower ..... $12 \cdot 9$
Common cathode (or grid, anode, emitter, base, collector, source, gate, drain)-Earthed cathode (etc.)-Grounded cathode (etc.) ..... $12 \cdot 9$
Common-collector transistor-Emitter follower ..... $12 \cdot 9$
Compound pair (of transistors)-Darlington pair-Super-alpha pair-Double emitter follower. ..... $19 \cdot 14$
Detection- Rectification-Demodulation (British usage originally reserved this term for a different phenomenon) ..... 18.1
Dielectric-Insulating material ..... $3 \cdot 3$
Dynamic resistance-Antiresonant impedance ..... $8 \cdot 10$
Earth-Ground . ..... 2. 14
Filament battery-L.T.- 'A' battery ..... 9.4
Frame aerial-Loop antenna ..... 17.19
Frequency-Periodicity ..... 1.4
Frequency-changer-Mixer-First detector ..... $21 \cdot 2$
Grid battery-Bias battery-G.B.-'C' battery ..... $10 \cdot 6$
Harmonic-Overtone-Partial ..... $15 \cdot 9$
Hum-Ripple ..... $26 \cdot 3$
Igfet-Insulated-gate field-effect transistor-M.O.S.T.-MOSFET ..... 10.11
Image interference-Second-channel interference ..... 21.7503
Inductor-Coil ..... 4.6
Integrated circuits-Solid circuits-S.I.Cs ..... 19.22
Interference-Jamming ..... $20 \cdot 5$
Jugfet-Junction-gate field-effect transistor ..... $10 \cdot 11$
Long-tailed pair-Emitter-coupled amplifier-Cathode-coupled am- plifier ..... $19 \cdot 21$
Loss-Attenuation ..... 19.2
Monostable-Flip-flop-One-shot multivibrator ..... 24.9
Moving coil (of loudspeaker)-Speech coil-Voice coil ..... 19.9
Moving-conl loudspeaker-Dyramic loudspeaker ..... 19.9
Mutual conductance-Slope--Transconductance ..... $10 \cdot 3$
Negative feedback-Degeneration. ..... 19.15
Noise-Machine interference-Man-made static ..... $19 \cdot 20$
Parallel-Shunt ..... 2.8
Peak value-Crest value-Maximum value ..... $5 \cdot 4$
Permittivity-Dielectric constant-Specific inductive capacity (s.i.c.) ..... 3.3
Phasor-Vector-Complexor ..... $5 \cdot 7$
Picofarad-Micromicrofarad ..... 3.3
Q-Q facter-Magnification-Storage factor . ..... 8.4
Quality (of reproduced sound)-Fidelity ..... $19 \cdot 4$
Radar-Radiolocation ..... $23 \cdot 12$
Radio-Wireless ..... Preface
Radio frequency (r.f.)-High frequency (h.f.) ..... $1 \cdot 12$
Reaction-Retroaction-Positive feedback-Regeneration ..... $14 \cdot 5$
Reaction coil-Tickler ..... $14 \cdot 5$
Reflection coefficient-Return loss ..... $16 \cdot 5$
Screen-Shield ..... $15 \cdot 7$
See-saw circuit-Floating paraphase ..... $19 \cdot 21$
Sender-Transmitter ..... 1.6
Siemens (unit of conductance)-Mho ..... 2.12
Telephones-Phones-Earphones-Headphones-Headset ..... 1.8
Tetrode-Screen-grid valve (but not all tetrodes are screen-grid valves) ..... 15.7
Thyristor-Silicon controlled rectifier (s.c.r.) . ..... $26 \cdot 4$
Time base-Sawtooth generator-Scanning generator -Sweep ..... $23 \cdot 4$
Tuned circuit-LC circuit-Resonant circuit-Tank curcuit ..... 8.3
Tunnel diode-Esaki diode ..... $14 \cdot 8$
Turning frequency-Break frequency-Roll-off frequency ..... $19 \cdot 16$
Valve-Vacuum tuhe-Tube. ..... 9.1
Variable-mu valve-Supercontrol tube-Remote cut-of tube ..... $22 \cdot 11$

## Appendix 2

## SYMBOLS AND ABBREVIATIONS

General Abbreviations

| a.c. | alternating current | h.t. | high tension |
| :--- | :--- | :--- | :--- |
| a.f. | audio frequency | i.c. | integrated circuit |
| a.g.c. | automatic gain control | i.f. | intermediate frequency |
| a.m. | amplitude modulation | l.f. | low frequency |
| a.v. | alternating voltage | p.d. | potential difference |
| c.r. | cathode ray | p.i.v. | peak inverse voltage |
| d.c. | direct current | r.f. | radio frequency |
| d.v. | direct voltage | r.m.s. | root-mean-square |
| e.h.t. | extra-high tension | s.w.r. | standing wave ratio |
| e.m.f. | electromotive force | u.h.f. | ultra-high frequency |
| f.e.t. | field-effect transistor | v.f. | video frequency |
| f.m. | frequency modulation | v.h.f. | very high frequency |
| h.f. | high frequency | 2.f. | zero frequency |

## Greek Letters

| Letter | Name | Usual Meaning |
| :---: | :---: | :---: |
| $\propto$ | alpha | current amplification factor (of transistor) |
| $\boldsymbol{\beta}$ | beta | current amplification factor, common emitter |
| $\varepsilon$ | epsilon | permittivity |
| $\theta$ | theta | an angle |
| $\lambda$ | lambda | wavelength |
| $\mu$ | mu | (1) permeability |
|  |  | (2) voltage amplification factor (of valve) |
|  |  | (3) one millionth of (as a prefix to a unit symbol) |
| $\pi$ | pi | $\frac{\text { circumference }}{\text { diameter }} \text { of circle }(=3.14159 . \ldots . \text { ) }$ |
| $\rho$ | rho | resistivity |
| $\varphi$ | phi (small) | angle of phase difference |
| © | (capital) | magnetic flux |
| $\omega$ | omega (small) | $2 \pi f$ |
| $\Omega$ | (capital) | ohm |

## Quantities and Units (S.I.)

| Quantity | Symbol | Unit | Abbreviation for unit |
| :---: | :---: | :---: | :---: |
| Time | $t\}$ | second | s or sec |
| Time period of one cycle | $T\}$ |  | , |
| Frequency | $f$ | hertz (cy | $\begin{gathered} \mathrm{Hz} \\ \text { cond) } \end{gathered}$ |
| Wavelength | $\lambda$ | metre | m |
| Electromotive force | E | volt | V |
| Potential difference | $V$ | volt | V |
| Current | I | ampere | A |


| Quantity of electricity, or charge | $Q$ | coulomb | C |
| :---: | :---: | :---: | :---: |
| Power | $P$ | watt | W |
| Capacitance | C | farad | F |
| Self inductance | $L$ | henry | H |
| Mutual inductance | $M$ | henry | H |
| Resistance | $\boldsymbol{R}$ | ohm | $\Omega$ |
| Reactance $\left\{\begin{array}{l}\text { Capacitive } \\ \text { Inductive }\end{array}\right.$ | $X\left\{\begin{array}{l}X_{C} \\ X_{L}\end{array}\right.$ | ohm | $\Omega$ |
| Impedance | $Z$ | ohm | $\Omega$ |
| Conductance | $G$ | siemens | S |
| $\text { Susceptance }\left\{\begin{array}{l} \text { Capacitive } \\ \text { Inductive } \end{array}\right.$ | ${ }^{B}\left\{\begin{array}{l}B_{C} \\ B_{L}\end{array}\right.$ | siemens | S |
| Admittance | $Y{ }^{\text {r }}$ | siemens | S |
| Magnetic field strength | H | ampere/metre |  |
| Magnetic flux | $\Phi$ | weber | Wb |
| Magnetic flux density | B | tesla | T |
| $Q$ factor, $X / R$ | $Q$ |  |  |
| Signal gain or loss | A | decibel | dB |

Note.-Small letters ( $e, v, i$, etc. $)$ are used to indicate instantaneous values.

## Unit Multiple and Submultiple Prefixes

| Symbol <br> G | Read as <br> giga- | Means |
| :---: | :--- | :--- |
| M | one thousand million $\left(\times 10^{9}\right)$ |  |
| k | mega- | one million $\left(\times 10^{6}\right)$ |
| m | kilo- | one thousand $\left(\times 10^{3}\right)$ |
| $\mu$ | milli- | one thousandth $\left(\times 10^{-3}\right)$ |
| n | micro- | one millionth $\left(\times 10^{-6}\right)$ |
| p | nano- | one thousand-millionth $\left(\times 10^{-9}\right)$ |
| Examples: | pico- | one billionth $\left(\times 10^{-12}\right)$ |
|  | $1 \mathrm{MHz}=1,000,000 \mathrm{~Hz}$ |  |
|  | $1 \mathrm{k} \Omega=1.000 \Omega$ |  |
|  | $1 \mathrm{~mA}=0.001 \mathrm{~A}$ |  |
|  | $1 \mu \mathrm{H}=0.000001 \mathrm{H}$ |  |
|  | 1 pF | $=0.00000000001 \mathrm{~F}$ |

Transistor Abbreviations and Symbols

Bipolar
e emitter
b base c collector

Field Effect
s source
g gate
d drain

C-E common-emitter C-C Common-collector C-B common-base

| C-E | $\mathrm{C}-\mathrm{C}$ | $\mathrm{C}-\mathrm{B}$ | Unspecified |
| :--- | :--- | :--- | :---: | :---: |
| $h_{\mathrm{fe}}$ | $h_{\mathrm{fc}}$ | $h_{\mathrm{fb}}$ | $h_{\mathrm{f}} \quad$ forward current transfer ratio with output short- | circuited

$h_{\text {ie }} \quad h_{\text {ic }} \quad h_{\text {ib }} \quad h_{1} \quad$ input impedance with output short-circuited
$h_{o c} \quad h_{\mathrm{oc}} \quad h_{\mathrm{ob}} \quad h_{0} \quad$ output admittance with input open-circuited reverse voltage transfer ratio with input opencircuited
The above are small-signal values. D.C. values are distinguished by capital letter subscripts; e.g., $h_{\mathrm{FE}}$. This also applies to such symbols as $I_{C}$ (collector d.c.). $I_{\mathrm{c}}$ (collector r.m.s. signal current), $i_{*}$ (collector instantaneous signal current). $V_{c}$

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(static voltage at collector), etc. To define the voltage more precisely, $V_{\text {CE }}$ means collector voltage with respect to emitter. A doubled subscript ( $V_{\text {cc }}$, for example) means supply voltage (Fig. 13.1). For $I_{\text {Cbo }}$ and $I_{\text {ceo }}$ see Fig. 13.3. For elements of hy brid- $\pi$ circuit, see Fig. 22.4.
$\begin{array}{ll}f_{\text {nfe }} & \text { frequency at which } h_{\text {fe }} \text { is } 3 \mathrm{~dB} \\ f_{1} & \text { frequency at which } h_{\text {fe }}=1\end{array}$
$f_{1} \quad f_{\text {hre }} \times$ low-frequency $h_{\text {fe }} \simeq f_{1}$
$g_{m} \quad$ mutual conductance

## Valve Abbreviations and Symbols

k cathode
g grid
$\mathrm{g}_{1} \quad$ first grid (nearest cathode)
$\mathrm{g}_{2}$ second grid; and so on
a anode
$\mu \quad$ voltage amplification factor
$r$ anode a.c. resistance
$g_{m} \quad$ mutual conductance
$g_{c} \quad$ conversion conductance (of frequency-changer)
Nole.-Symbols are frequently combined, thus-
$I_{2}=$ anode current
$V_{\mathrm{Rz}}=$ voltage at second grid
$R_{z}=$ resistance connected externally to anode
$c_{\mathrm{Qk}}=$ internal capacitance from grid to cathode
Capital letters are used for associated items outside the valve; small letters for items inside the valve itself.

## Special Abbreviations used in this Book

| $f_{F}$ | frequency of resonance <br> frequency of oscillation |
| :--- | :--- |
| $f_{o}$ | frequency off-tune |
| $f_{E}$ | turning frequency |
| $f_{B}$ | voltage amplification |
| feedback factor |  |
| $R_{L}$ | load resistance |
| $R_{G}$ | generator resistance |

## Appendix 3

## DECIBEL TABLE

The decibel figures are in the centre column: figures to the left represent decibel loss, and those to the right decibel gain. The voltage and current figures are given on the assumption that there is no change in impedance.

| Voltage or current ratio | Power ratio | $\leftarrow \begin{gathered} \overline{d B} \\ + \end{gathered}$ | Voltage or current ratio | Power ratio |
| :---: | :---: | :---: | :---: | :---: |
| 1.00 | 1.000 | 0 | 1.000 | 1.000 |
| 0.989 | 0.977 | 0.1 | 1.012 | 1.022 |
| 0.977 | 0.955 | 0.2 | 1.023 | 1.047 |
| 0.966 | 0.933 | $0 \cdot 3$ | 1.035 | 1.072 |
| 0.955 | 0.912 | $0 \cdot 4$ | 1.047 | 1.096 |
| 0.944 | 0.891 | 0.5 | 1.059 | 1.122 |
| 0.933 | 0.871 | $0 \cdot 6$ | 1.072 | 1.148 |
| 0.912 | 0.832 | 0.8 | 1.096 | 1-202 |
| 0.891 | 0.794 | 1.0 | 1-122 | [-259 |
| 0.841 | 0.708 | 1.5 | 1-189 | [.413 |
| 0.794 | 0.631 | 2.0 | 1.259 | 1.585 |
| 0.750 | 0.562 | 2.5 | 1.334 | 1.778 |
| 0.708 | 0.501 | $3 \cdot 0$ | 1.413 | 1.995 |
| 0.668 | 0.447 | 3.5 | 1.496 | $2 \cdot 239$ |
| 0.631 | 0.398 | 4.0 | 1.585 | 2.512 |
| 0.596 | 0.355 | 4.5 | 1.679 | 2.818 |
| 0.562 | 0.316 | 5.0 | 1.778 | 3.162 |
| 0.501 | 0.251 | 6.0 | 1.995 | 3.981 |
| 0.447 | 0.200 | 7.0 | 2.239 | 5.012 |
| 0.398 | 0.159 | 8.0 | 2.512 | 6.310 |
| 0.355 0.316 | 0.126 0.100 | 9.0 | 2.818 3.162 | 7.943 |
| 0.316 0.282 | 0.100 0.0794 | 10 | 3.162 3.55 | 10.00 12.6 |
| 0.251 | 9.0631 | 12 | 3.98 | 15.9 |
| 0.224 | 0.0501 | 13 | 4.47 | 20.0 |
| $0 \cdot 200$ | 0.0398 | 14 | 5.01 | $25 \cdot 1$ |
| 0.178 | 0.0316 | 15 | 5.62 | 31.6 |
| 0.159 | 0.0251 | 16 | 6.31 | 39.8 |
| 0.126 | 0.0159 | 18 | 7.94 | 63.1 |
| $0 \cdot 100$ | 0.0100 | 20 | 10.00 | $100 \cdot 0$ |
| $\frac{3.16}{} \times 10^{-2}$ | $10^{-3}$ | 30 | $3.16 \times 10$ | $10^{3}$ |
| $10^{-2}$ | $10^{-4}$ | 40 | $10^{2} \times 10$ | $10^{4}$ |
| $\underset{10^{-3}}{3 \cdot 16} \times 10^{-3}$ | 10-5 | 50 | ${ }^{3.16} \times 10^{2}$ | $10^{6}$ |
| $10^{-3}$ $3.16 \times 10^{-4}$ | $10^{-6}$ $10^{-7}$ | 60 70 | ${ }_{\substack{10^{3} \\ 3.16}}^{10} 10^{3}$ | $10^{6}$ |
| $10^{-4} \times 10$ | $10^{-8}$ | 80 | $10^{4}{ }^{4} \times 16$ | $10^{8}$ |
| $3.16 \times 10^{-5}$ | $10^{-9}$ | 90 | $3.16 \times 10^{4}$ | $10^{\circ}$ |
| $10^{-5}$ | $10^{-10}$ | 100 | $10^{5}$ | $10^{10}$ |
| ${ }_{10.16}^{3.6} \times 10^{-6}$ | $10^{-11}$ | 110 | $3.16 \times 10^{5}$ | $10^{11}$ |
| $10^{-6}$ | $10^{-12}$ | 120 | $10^{6}$ | $10^{12}$ |

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[^0]:    * In wireless the word 'range' means distance that can be covered, so to avoid confusion a range of frequencies is more often referred to as a band.

[^1]:    * Note that +1 is positive with respect to -1 or even to 0 , but it is negative with respect to $+2 ;-1$ or 0 are both positive with respect to -2 . There is nothing inconsistent in talking about two oppositely charged bodies one minute, and in the next referring to them both as positive (relative to something else).

[^2]:    * In the older books, resistivity is given in ohms per centimetre cube, or ohm-cm. Resistivity in metre units is a number 100 times smaller.

[^3]:    * Electrical units are defined in terms of mechanical forces, etc., in metric units. Because of the relationships between metric and British units, a power of 746 watts is equal to $550 \mathrm{ft}-\mathrm{lb}$ per second ( 1 horse-power). It is confirmed by experiment that the rate of heating is the same, whether produced by dissipation of 1 mechanical HP or 746 electrical watts.

[^4]:    * The terminals of a battery or other source of e.m.f., between which electric field lines are imagined, are sometimes also called poles. The distinction between + and - in such sources, and N and S in magnets, is called its polarity.

[^5]:    *For this reverse process it saves time to adapt the formula, thus:

    $$
    \begin{aligned}
    & f_{r}=159 \cdot 15 / L^{2}(L C) \quad(\mu \mathrm{H}, \mathrm{pF}, \mathrm{MHz}) . \\
    & f_{\mathrm{r}}^{2}=159 \cdot 15^{2} / L C \\
    & L=25333 / f_{r^{2}} C
    \end{aligned}
    $$

    The corresponding adaptation for calculating $C$ is
    $C=25333 / j_{r}^{2} L$

[^6]:    * On the ground that $A B$ is the measure of negative feedback it would seem logical to regard $B$ as negative: the overall input is then $I-A B$, which comes to the same thing as the above if you remeber that $B$ is negative. Treating $B$ as positive when feedback is negative may be illogical, but when we are talking only about negative feedback it is probably less confusing than having to remember to use a double negative every time.

