



*SECTION 2*

ADVANCED  
PRACTICAL  
**RADIO ENGINEERING**

TECHNICAL ASSIGNMENT

INDUCTIVE REACTANCE, IMPEDANCE,  
Q OF COILS

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## INDUCTIVE REACTANCE, IMPEDANCE, Q OF COILS

*INTRODUCTION.*—The reasons for the inductive effects of a conductor have been discussed. It has been shown that *every* conductor, regardless of shape, length, or surrounding medium, will show inductive effects whenever the current through the conductor is caused to vary. This is due to the fact that the density of the magnetic field is a direct function of the current amplitude and varies when the current varies. As the field changes in density (and thus expands or contracts) its magnetic lines of force cut the conductor and other near by conductors.

*ANGLE OF LAG.*—One of the principal effects introduced into the circuit by means of inductance is the LAG of the current behind the voltage. The reason for this lag has been explained. The *amount* of the lag, however, requires a further explanation of the inductive effect than has thus far been attempted.

It has been shown in the explanation of the induction coil, that the voltage induced across a coil by the variation of current through the coil depends upon how rapidly the variations take place; and that in the case of the induction coil where the circuit is suddenly opened, the voltage built up may be much higher than the applied voltage. (A similar effect is to be expected in certain units of television equipment, such as deflection circuits, where a transformer may be required to handle voltages having almost vertical wavefront.)

In the case of an inductance connected across a source of alter-

nating current which varies in the form of a sine curve, the condition is somewhat different than if the inductance is connected across a source of d.c. and the switch closed and opened. In the latter case the variations of the current, and therefore of the magnetic field, are intermittent and of unequal duration. In the case of sinusoidal a.c. the rise and fall of current are of equal duration and take place in the same form. Also, the field is continuously varying *except* at the two maximum points of the cycle,  $90^\circ$  and  $270^\circ$ .

So long as the current is varying, a voltage will be built up that will oppose the variation, and in the case of a pure inductance, (actually an impossible condition), the induced voltage will be EQUAL TO and OPPOSITE TO the voltage that is causing the varying current, that is, the generator voltage.

A perfect inductance, that is, a circuit or piece of apparatus containing nothing but inductance, is impossible to attain because in order to have inductance it is necessary to have a conductor through which the current flows and varies. ALL conductors possess a certain amount of resistance so that theoretically it is not possible to have a purely inductive circuit. However, if the effects of inductance in a circuit are many times, (several hundreds or thousands of times), greater than the effective resistance, it is often possible to neglect the resistance and consider the coil as having only inductance. In studying the theory of inductance a condition of pure inductance will

be temporarily assumed.

In the case of a perfect inductance there will be nothing to oppose the flow of current through the inductance but the opposition offered by the inductive effect itself.

In a circuit containing only resistance the voltage drop across the total resistance of the circuit is at every instant *exactly* equal to the applied voltage. (Kirchoff's Law.) A study of the circuit composed of a generator and a resistance will also show that the polarity of the resistance voltage drop is exactly opposite to that of the applied voltage. If either voltage were greater than the other, there would be voltage unaccounted for somewhere in the circuit, a condition which cannot exist.

A similar situation exists in a pure inductance connected directly across a source of a.c. Since there is no opposition to current flow except that offered by the inductance itself, ALL of the alternating voltage must be expended in forcing current through the inductance. And the counter electromotive force *developed across the inductance* must, AT EVERY INSTANT, be *exactly* equal and opposite to the applied voltage.

This c.e.m.f. is directly due to the variations of current through the inductance; also, the c.e.m.f. is greatest at the instant the current VARIATIONS are the greatest. It must therefore be thoroughly understood and remembered that the induced e.m.f. is greatest, NOT when the magnetic field is the strongest, but when the magnetic field is varying the most rapidly. The variations of the field depend directly upon the variations of current. An inspection of a sine curve of cur-

rent, (See Fig. 1), will show the following: (Current curve marked I.)

1. That the current, and therefore the density of the magnetic field, is at a maximum value at

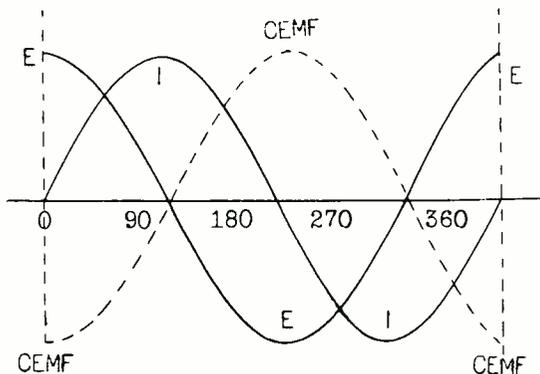


Fig. 1.—Sine wave representation of I, E, and c.e.m.f. in a pure inductance.

rent, (See Fig. 1), will show the following: (Current curve marked I.)

2. That at these maximum points, for an instant, the VARIATIONS of current, and therefore the variations of the magnetic field, are ZERO.

3. That the greatest VARIATION of current takes place as the current is passing through its ZERO value and is just about to reverse.

Since the amplitude of the c.e.m.f. depends directly upon the variation of current, the maximum amplitude of c.e.m.f. must occur at the instant the current *variation* is most rapid. This is shown on curve CEMF, (dotted line), in Fig. 1. It will be seen that at the instant the current is passing through its zero value the c.e.m.f. will be at maximum, and at the instant the current is at maximum AND IS NOT VARYING the c.e.m.f. is zero.

A reference to trigonometry, or a study of the variations of the

sines of the various angles in a table of trigonometric functions, will demonstrate the truth of the preceding paragraph. The sine of the angle *varies* rapidly as the angle approaches zero, and as the angle approaches ninety degrees the sine value, while large, varies by very small amounts. Since the current varies in the form of a sine curve and the sine curve varies directly as the sine of the angle, the preceding paragraph must be correct.

It has been shown that the c.e.m.f. is *exactly equal and opposite* at every instant to the applied voltage. Thus the curve of the applied voltage,  $E$ , may be drawn in one hundred eighty degrees out of phase, (opposite), with the c.e.m.f. curve. These curves will then express the relation between the current and voltage in a purely inductive circuit, as shown in Fig. 1.

A study of this set of curves will show that at the instant the current,  $I$ , is beginning its cycle, the voltage,  $E$ , has completed  $90^\circ$  of its cycle and is at maximum.

By the time the voltage rises to maximum and falls back to zero the current has reached only its  $90^\circ$  position, (Maximum).

From this it may be stated that in a purely inductive circuit, *regardless of the amount of the inductance*, the current lags  $90^\circ$  behind the voltage at every instant along the cycle.

In plotting the curves it must be assumed that several cycles have already been completed, because the perfect sine curves as shown in Fig. 1 will not be the form of the current the instant the circuit is closed. The form of the applied voltage is controlled mechanically by the construction of the alter-

nator and its driving equipment. When the switch is closed and a difference of potential is applied across the circuit, the current at first will try to rise instantly in a surge, but after several cycles have elapsed the current will have assumed its correct relation with respect to the voltage and the original surge of current will have died out.

The effect of inductance on the phase relation between the current and voltage is to cause the current to lag  $90^\circ$  behind the voltage. Since it is not possible to have a circuit without some resistance, and since the effect of resistance is such as to cause the current and voltage to be in phase, the angle of lag of the current with respect to the voltage is always something less than  $90^\circ$  and will be between  $0^\circ$  and  $90^\circ$ , the exact angle depending upon the relative effects of the inductance and the resistance of the circuit.

*INDUCTIVE REACTANCE.*—It has been stated that in a purely inductive circuit the only opposition to current flow is that due to the inductance itself. In a resistance circuit there is opposition to current flow due directly to the friction and other opposition the electron meets with as it travels through the conductor. In a purely inductive circuit there would be none of that kind of opposition, and on first thought it would seem that the current should be infinite. Such, however, is not the case. The induced voltage across the inductance must be *exactly equal and opposite to the applied voltage*. The induced voltage is a generated voltage produced by an alternator, and depends directly upon the three factors: the number of conductors,

that is, the number of turns of the coil that are cut by the expanding and collapsing magnetic field; the velocity of cutting of the turns by the field, which velocity depends directly upon how rapidly the current varies, this in turn being governed mechanically by the construction and r.p.m. of the alternator; the average density of the magnetic field, this depending entirely, for a given design of coil, upon the average current flow through the coil.

In a given coil the number of turns, shape of turns, and spacing of turns is fixed, as is the core material. The frequency of the alternator is also fixed mechanically. Therefore, the ONLY VARIABLE factor is the density of the field which depends directly upon the current flow.

Since the amplitude of the induced voltage is fixed by the amplitude of the applied voltage, it will be seen that the average current through the inductance must be EXACTLY sufficient to produce an average magnetic field which, cutting the given number of turns at the given frequency, will produce an average c.e.m.f. exactly equal to the average applied voltage. The current can be neither more nor less than this required amount if the circuit consists of a pure inductance.

If any one of the other factors is changed, the current necessary to produce the correct value of magnetic field will change accordingly. If the r.p.m. of the alternator is increased, increasing the frequency and therefore the velocity of cutting of the conductors by the field, a weaker magnetic field moving with increased velocity will produce the same c.e.m.f. that the stronger

field produced at the lower frequency; so with a given inductance, if the frequency is increased the current is decreased in exactly the same proportion.

In a similar manner, if the inductance of the coil is increased by adding more turns, a weaker field cutting the increased number of conductors will produce the same c.e.m.f. and the current will reduce accordingly in order that the magnetic field may be reduced to the proper flux density.

If the number of turns is kept the same, but the inductance increased by increasing the permeability of the surrounding medium, for example, by placing an iron core within the windings, the same density of field will be required to produce the required c.e.m.f. This field, however, due to the increased permeability of the medium within the field, will be created by a smaller current flow and the current flow will be correspondingly reduced.

Thus for a given applied voltage, the current will vary inversely as both the frequency and the inductance, and for a given frequency a given inductance will offer a certain constant amount of opposition to current flow. The opposition to current flow offered by the inductance of the circuit is called the INDUCTIVE REACTANCE and it varies DIRECTLY AS THE FREQUENCY AND AS THE INDUCTANCE. If the inductance is doubled the inductive reactance is doubled. If the frequency is doubled the inductive reactance is doubled.

The symbol for inductive reactance is  $X_L$ : X for reactance and L for inductance. The unit of reactance is the ohm, and since this is the same unit as that of resist-

ance it must express the same amount of opposition. A reactive circuit has a reactance of one ohm when it allows one ampere to flow due to the pressure of one volt. It may be stated:

$$\text{Ohms} = X = E/I = \text{Volts/Amperes}$$

E and I are usually considered from their r.m.s. values but the same X will be obtained by using average or maximum values. However, the same denomination of both E and I MUST be used; that is, both maximum, both r.m.s. or both average.

Since  $X_L$  is a function of both the inductance of the circuit and the frequency, in order to compute the reactance offered by any inductance at any frequency it is necessary to have an equation which takes into consideration both L and F. Expressing  $X_L$  in terms of both L and F is the following equation:

$$X_L = 2\pi FL$$

$\pi$ , Greek letter Pi, is equal to 3.1416;  $2\pi$  is usually taken as 6.28.

DERIVATION OF  $X_L = 2\pi FL$ .—The derivation of this equation is interesting as it shows how exactly the principle of the inductance compares with that of the alternator. This fact is also shown by the definition of the unit of inductance, the henry:

A circuit has one henry of inductance when a change of current of one ampere per second induces an e.m.f. of one volt. This is expressed by the equation:

$$E_{ave} = L \frac{di}{dt}$$

where  $E_{ave}$  = Average INDUCED voltage

L = Inductance of the circuit in henries.

di = CHANGE OF CURRENT in amperes

dt = Time of change in seconds

The condition may also be expressed as follows: The instantaneous voltage induced in any circuit by a change in current is equal to the product of the inductance in henries time the RATE OF CHANGE IN AMPERES PER SECOND:  $E = L \frac{di}{dt}$ , where di expresses the current change and dt expresses the corresponding time variation, di/dt expressing the instantaneous rate of change.

An alternating current makes four complete changes in amplitude each cycle (but only two alternations as shown previously), as follows:

1. From zero to maximum.
2. From maximum to zero.
3. From zero to maximum in the opposite direction.
4. From the second maximum back to zero.

These changes are shown on the sine curve of current in Fig. 2. In each change the current makes a complete variation of from zero to maximum or vice-versa. The amount of change is then equal to the maximum or peak amplitude of the current.

The TIME of one complete change is equal to the time of one-quarter of a cycle. The time of one complete cycle is equal, in seconds, to  $1/F$  second. The time for each change is equal to one-fourth of the time of one cycle, in seconds, and is therefore equal to  $1/4F$  second.

The rate of current change in amperes per second is equal to the total amount of change, in amperes, divided by the time of one change in seconds. Since the amount of change is equal to the maximum amplitude of

and amperes.

The average voltage may be expressed in terms of the maximum voltage. From an earlier equation,

$$E_{ave} = .636E_m$$

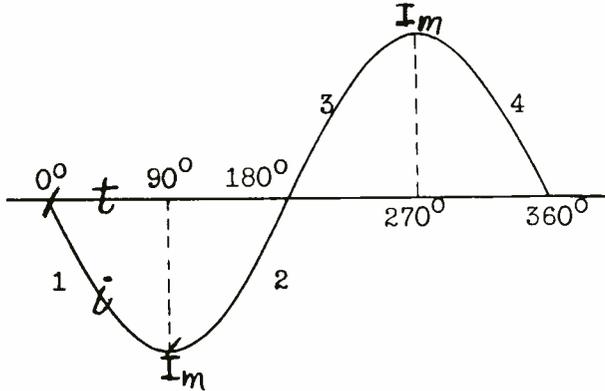


Fig. 2.—An alternating current makes four complete changes in amplitude during each cycle.

current,  $I_m$ , the expression  $di/dt$  must equal

$$\frac{I_m}{(1/4F)}$$

Dividing by the fraction, invert the fraction and multiply, making the expression equal to  $(4FI_m)$  amperes per second. This is the rate of change of an alternating current in amperes per second.

Replacing, in the original equation, the value  $di/dt$  by its equivalent,  $4FI_m$ , the original equation,

$$E_{ave} = L \frac{di}{dt} \text{ becomes } E_{ave} = 4FLI_m$$

Therefore the average voltage induced across a perfect inductance by a sinusoidal current is equal to four times the frequency times the inductance times the maximum current. Note: All values in this equation must be in units; cycles, henries

Substituting,

$$.636E_m = 4FLI_m$$

Dividing through by .636, the equation becomes,

$$E_m = \frac{4FLI_m}{.636}$$

4 divided by .636 equals 6.28 which in turn is equal to  $2\pi$ . Therefore,

$$E_m = 2\pi FLI_m$$

Since the values, E and I, in both sides of the equation are maximum values, it is now possible to substitute average or effective values for both and the equation will still be correct. The values may be stated simply as E and I, remembering that both must be of the same denomination, either both maximum, effective, or average. The

equation becomes.

$$E = 2\pi FLI$$

From an earlier equation,  $E = IX_L$ . Substituting this equivalent value for  $E$ ,

$$IX_L = 2\pi FLI$$

Dividing through by  $I$ ,

$$X_L = 2\pi FL.$$

The value of the inductive reactance in ohms is equal to  $2\pi$  (a constant), times the frequency in cycles per second times the inductance in henries.

This equation,  $X_L = 2\pi FL$ , may, of course, be rearranged to find any other factor if  $X_L$  and one other factor are known.

$$X_L = 2\pi FL$$

$$L = X_L / 2\pi F$$

$$F = X_L / 2\pi L$$

Thus if the frequency and reactance are known it is a simple matter to determine the inductance. If the inductance and reactance are known, the frequency may be easily determined.

**IMPEDANCE.**—The effects of self-inductance in a circuit are such as to cause opposition to the flow of current through the circuit and to cause the current to lag behind the voltage. It has been shown that the effect of the inductance itself is to tend to cause a current lag of exactly ninety degrees. Since all circuits must contain resistance, and since Ohm's law holds true at every instant so far as the resist-

ance effects on the current in the circuit are concerned, it is evident that the current lag in any circuit can never be quite as great as ninety degrees. If the inductive reactance is very large compared to the resistance (if  $X_L/R$  is large), the angle of lag will be large. Unless the ratio  $X_L/R$  is equal to infinity (zero resistance) the angle must be less than  $90^\circ$ .

This is easily shown by vector analysis.  $R$  tends to keep the current in phase with the voltage and  $X_L$  tends to cause the current to lag  $90^\circ$  behind the voltage. These effects are  $90^\circ$  apart and can be shown in a vector diagram as in Fig. 3.

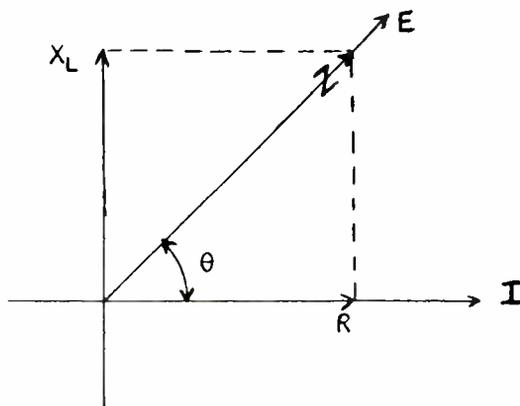


Fig. 3.—Vector illustrating current lag when  $R = X_L$ .

In a series circuit the reference vector is taken as the current, which is the same throughout the circuit. If  $X_L$  is drawn perpendicular to the reference vector and  $R$  along the reference vector, their effects are  $90^\circ$  apart as in Fig. 3. If  $R$  is equal to  $X_L$ , the angle  $\theta$  between  $Z$  and  $I$  will be  $45^\circ$ .  $Z$  represents the impedance of the series circuit and shows graphically its

magnitude if the figure is drawn to scale. The angle  $\theta$  can be calculated as will be shown or can be measured if the figure is drawn accurately.

$Z$  is the total opposition to the flow of current in the series circuit and may be found by vector (geometric) addition, as discussed in a previous assignment.  $Z$  is expressed in ohms the same as  $X_L$  and  $R$ .

In a series circuit containing only inductance and resistance, such as a coil of wire, (the resistance distributed through the coil is always considered as being in series with the inductance), the impedance becomes

$$Z = \sqrt{R^2 + X_L^2}$$

$$\text{IMPEDANCE} = \sqrt{\text{RESISTANCE}^2 + (\text{INDUCTIVE REACTANCE})^2}$$

i.e.,  $Z$  is the hypotenuse of a right triangle of which  $X_L$  and  $R$  are arms, as is clear from Fig. 3.

Regardless of the relative values of  $R$  and  $X_L$  their effects are always  $90^\circ$  apart, the one which is the greater having the greater effect on the angle of lag of the current. This is shown in Fig. 4 in

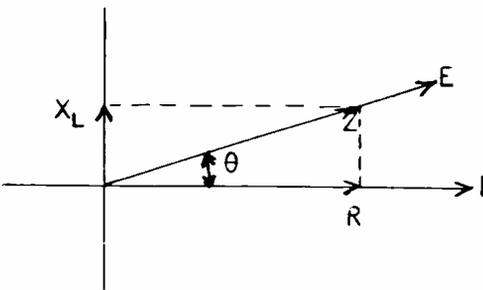


Fig. 4.—Vector illustrating current lag when  $R > X_L$ .

which  $R$  is large compared to  $X_L$ .

In Fig. 3,  $X_L = R$  and both have an equal effect on the current, the lag being  $45^\circ$ .

In Fig. 4,  $R$  is considerably larger than  $X_L$ , so that the effect of  $R$  tending to bring the current in phase with  $E$  is therefore greater than the effect of  $X_L$  tending to cause a current lag of  $90^\circ$ . The current lags the voltage by the angle  $\theta$  which is considerably less than  $45^\circ$ .

In Fig. 5,  $X_L$  is much greater than  $R$ ; the angle of current lag therefore is greater than  $45^\circ$ .

In series circuit calculations, the angle of lag of the current with

respect to the voltage is equal to the angle between the impedance vec-

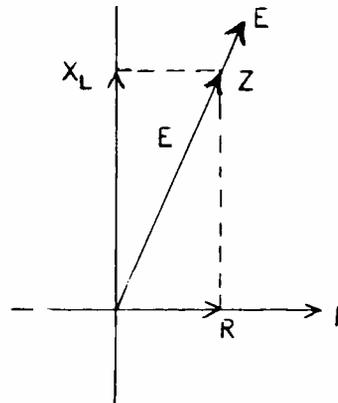


Fig. 5.—Vector illustrating current lag when  $R < X_L$ .

tor,  $Z$ , and the resistance vector  $R$ .

Combining the equation for  $Z$  with the equation for  $X_L$ , the relation between current and voltage in a coil of known inductance and re-

sistance may be calculated at any frequency.

For example, assume a coil of 3 henries inductance and a resistance of 120 ohms, connected across a 60 cycle line, the effective voltage of which is 200 volts. To find the current and the phase relation between I and E.

To calculate the impedance, Z, first determine R and  $X_L$ .

$$R = 120 \text{ ohms}$$

$$X_L = 2\pi FL = 6.28 \times 60 \times 3 = 1130 \text{ ohms}$$

$$Z = \sqrt{R^2 + X_L^2} = \sqrt{120^2 + 1130^2} =$$

$$1136 \text{ ohms}$$

Since  $X_L$  is considerably larger than R the condition is somewhat as shown in Fig. 5. It will be seen that the three values, Z, R, and  $X_L$ , form a right triangle, the hypotenuse of which is Z, with R for the adjacent side and  $X_L$  for the opposite side. By trigonometry, the tangent of a given angle in any right triangle is equal to the opposite side divided by the adjacent side. In this case,

$$\text{Tangent } \theta = \frac{\text{Opposite Side}}{\text{Adjacent Side}} = \frac{X_L}{R} = \frac{1130}{120} = 9.41$$

$$\theta = 84^\circ \text{ lag. (See Trig. Table of Tangents.)}$$

The current through the coil will be equal to the voltage divided by the impedance, that is,

$$I = E/Z = \frac{200}{1136} = .176 \text{ ampere.}$$

Therefore if a coil having a resistance of 120 ohms and an inductance of 3 henries is connected across a 200 volt 60 cycle line,

there will be a current flow through the coil of .176 ampere and the current will lag  $84^\circ$  behind the voltage.

Calculations such as the above are commonly encountered in both audio-frequency and radio-frequency work, the only difference being in the values of L, R, and F.

An inspection of the values of  $X_L$  and Z in this problem demonstrates that when  $X_L$  is many times larger than R, the R practically may be neglected in computing the total impedance. For example, assume this is to be the impedance in the plate circuit of an impedance coupled amplifier and L has been increased to 30 henries, a size of wire being used to keep R down to a few hundred ohms.

At 60 cycles, the inductive reactance will be 11,300 ohms. At 600 cycles  $X_L$  will equal 113,000 ohms. At 6,000 cycles,  $X_L$  will equal 1,130,000 ohms. This is practically the width of the broadcast band of audio frequencies. Over this entire range the Z of the coil is practically equal to  $X_L$  and R may be neglected. (It will be noted that  $X_L$  varies directly as F. This

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is shown in the equation  $2\pi FL$ .)

If it were desired to compute the necessary inductance to obtain a certain high impedance at any given frequency, assuming that  $X_L$  is very large compared to R and that R may be neglected, considering  $Z = X_L$ .

$$\text{Then, } L = \frac{X_L}{2\pi F}$$

This equation is particularly useful in audio-frequency amplifier calculations for transformers and impedance.

For example, a 6C5 vacuum tube is to be used as an audio amplifier. The tube plate-filament impedance  $R_p$  is 10,000 ohms and it is desired that the load impedance  $Z_L$  in the plate circuit shall be not less than  $R_p$  at 40 cycles. The arrangement is shown schematically in Fig. 6. To

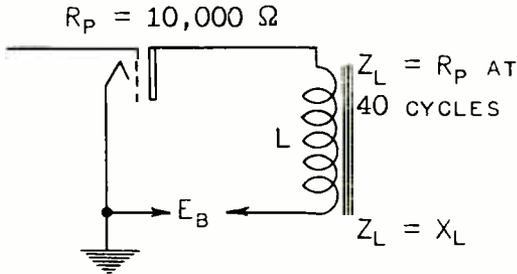


Fig. 6.—Schematic of an impedance-coupled audio-amplifier stage.

find the proper value of  $L$ . (Assume  $R$  of winding to be negligible.)

$$L = \frac{X_L}{2\pi F} = \frac{Z_L}{2\pi F} = \frac{R_p}{2\pi F}$$

$$L = \frac{10,000}{6.28 \times 40} = 40 \text{ henries}$$

At any frequency higher than 40 cycles the reactance of  $L$  will be greater than  $R_p$ , a desirable condition for vacuum tube amplifier operation. In any practical winding having an inductance of 40 henries,  $R$  will be negligible as compared with  $X_L$  so far as  $Z$  is concerned,

and for a practical calculation as shown in Fig. 6 may be neglected.

*Q OF COILS.*—While in many audio-frequency and power supply circuits the resistance practically may be neglected in calculations involving the impedance of a reactor or transformer winding, this is far from true in most radio-frequency circuits. Most radio-frequency circuits are worked at or near resonance. This means that there is sufficient capacitive reactance to cancel out the effect of the inductive reactance so that the current in the circuit is limited almost entirely by the effective resistance. In most capacitors as used in radio-frequency circuits the losses are exceedingly small so that, except at the *very high* radio frequencies, the circuit losses are due almost entirely to the effective resistance of the coil. In most r-f circuit calculations this is assumed; where the effective resistance of the circuit is actually measured, for purpose of calculation the total effective resistance—including that reflected from other circuits—is lumped and considered as coil resistance. A thorough study will be made in later assignments of capacity reactance, resonant circuits, and reflected resistance. These factors are merely mentioned at this point to emphasize the difference between the conditions under which coils operate in audio- and radio-frequency circuits.

*R-F AND D-C RESISTANCE.*—The resistance of a coil winding at power and audio frequencies can be accurately measured with a wheatstone bridge, or calculated from the coil dimensions and a set of wire tables. The effective resistance of a coil at radio frequencies varies

widely with frequency variation, with the shape of the coil, etc. The effective resistance of a given length of wire at high radio frequency is greatly different from the same length of the same wire in a d-c circuit. Also the effective r-f resistance of this same piece of wire when wound into a coil is greatly different when the wire is straight. In addition, the ratio of the r-f resistance to the d-c resistance of a coil varies with the shape of the coil. These differences in the effective resistance are due to skin effect as previously mentioned. Skin effect is due to the reaction between the magnetic fields set up in the different parts of the conductor. In a straight wire the field of the current traveling near the surface causes induced currents in the opposite direction to flow in the center of the conductor. The higher the frequency, the greater is the tendency toward induced current within the conductor and hence the greater the tendency to crowd the primary current toward the surface of the conductor. This in effect reduces the cross-section of the conductor in which current can flow and of course increases the resistance. To this condition is also added the heat loss due to the reverse current in the center of the conductor.

In practice it works out that the current penetration into a conductor is a function of frequency, the higher the frequency the less the penetration and the greater the skin effect. This brings up a peculiar condition. Since the current tends to travel on the surface, it would seem that the effective resistance of a conductor should vary inversely with diameter since the

circumference is equal to  $\pi d$  or  $2\pi r$ , where  $d$  = diameter and  $r$  = radius. However, the cross-section area of a round conductor is equal to  $\pi r^2$  so that the amount of copper in a given length of wire is a function of  $r^2$  while the surface area of the conductor is a function of  $r$ . Thus as the diameter of the conductor is increased, the total volume of copper increases at a much more rapid rate than does the surface area, so that the ratio of r-f resistance to d-c resistance is greater for a large wire than for a small wire. This does *not* mean that the effective resistance of a large wire is not less than that of a small wire—it means that it is not nearly so much less as a wire table would seem to indicate. An example of this will be given a little later.

When the wire is formed into a coil with adjacent turns close together, the skin effect is further complicated by the reaction of the magnetic fields of adjacent turns which tends to cause the current to travel on the surface of only one side of the wire. This further decreases the effective current carrying copper in the conductor and adds to the effective resistance. The mathematical relation between the r-f and d-c resistance of a coil is so complex that the only case for which an exact formula is available is that of a single layer winding of wire having rectangular cross-section with an insulation of negligible thickness between turns, the length of the winding being assumed to be very great compared with the mean radius, and the latter being assumed to be very great compared with the thickness of the wire. It will be seen that this is a somewhat impractical type of coil for use in a

radio circuit so that the formula itself will not be discussed at this point.

However, the formula does give some means of arriving at an approximation of the r-f resistance of a coil. Using the formula in a later assignment in calculations involving a comparison of two coils, the only difference between them being in the size of wire used: coil 1 was wound with 40 feet of No. 20 wire the d-c resistance being .406 ohm. Coil 2 was wound with 40 feet of No. 28 wire the d-c resistance of which is 2.6 ohms. Using the formula and tables from Bureau of Standards Circular 74 the following resistance ratios and r-f resistance values were obtained:

Frequency	Coil 1 No. 20		Coil 2 No. 28	
	R.F./D.C.	R.F. Res.	R.F./D.C.	R.F. Res.
600 kc/s	6.025	2.445	2.63	6.83
1,000 kc/s	7.66	3.106	3.275	8.52
1,500 kc/s	9.21	3.77	3.92	10.18

One fact should particularly be noted. The d-c resistance of Coil 2 is 6.4 times greater than that of Coil 1, ( $2.6/.406$ ), but the r-f resistance is only 2.5 times as great. Also the r-f resistance at 1,500 kc/s is approximately 1.5 times as great as at 600 kc/s. It should also be noted that with a coil of this shape  $R$  increases approximately as  $\sqrt{F_2/F_1}$ .

A good idea of the effect of coil shape on the r-f resistance may be had by a comparison of the figures above with figures by Terman based on measurements of two coils wound respectively with No. 20 and No. 28 wire, the diameter and length of winding both being three inches. In that case the difference between the resistance of the two coils is only

about 35 per cent, that is, the r-f resistance of the coil of No. 28 wire is only about 35 per cent greater than when wound with No. 20 wire. In the long coil as shown in the table below, one is 2.5 times greater than the other. In both cases, however, lower resistance is obtained by the use of larger wire.

For a given amount of copper, the r-f resistance can be decreased by the use of Litz wire, that is, a cable made up of a large number of strands of very fine wire, each strand insulated from the others. It is found that the ratio of R.F./D.C. resistance of 20-36 Litz at 465 kc/s is 3.33 and for an equivalent amount of copper in a

solid wire, 5.7. Thus the r-f resistance of the Litz wound coil at this frequency is slightly less than 60 per cent of that of an equivalent coil of solid wire. This is of course for a single layer coil and the relation will not necessarily be correct for other types of windings, but a considerable decrease of  $R$  is obtained with any type of winding by the use of Litz. (Litz wire is only effective at frequencies lower than three or four thousand kilocycles as explained elsewhere). In modern broadcast receivers Litz wire is most commonly used in the intermediate-frequency transformers where compact coils of comparatively large inductance are required.

It is seen that a coil has two principal characteristics, inductive

reactance and resistance, both of which vary with frequency. In a radio-frequency circuit, for example the tuned-grid circuit of a receiver r-f amplifier, it is usually required that a given power input to the circuit build up the highest possible voltage across the inductance. This is shown in Fig. 7.

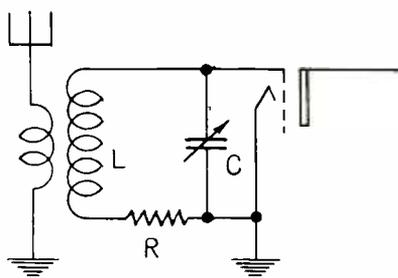


Fig. 7.—Typical tuned-grid circuit of an r-f amplifier.

$L$  is the inductance of the grid coil and  $R$  represents all the tuned circuit resistance which here consists almost entirely of the r-f resistance of the coil. It is seen that the voltage impressed between tube grid and filament is equal to  $IX_c$ . Since LCR is operated at resonance,  $X_L$  is cancelled by  $X_c$  and the circulating current in the circuit is limited entirely by  $R$ .

However, with a given  $R$ , and hence a given circulating current in the tuned circuit,  $E_g$  (voltage between grid and filament) is a direct function of  $L$  because  $E_g = IX_L$  and  $X_L = 2\pi fL$ . Therefore a direct figure of merit for the coil at any given frequency will be an expression of the ratio of  $X_L$  to  $R$ . This ex-

pression is called  $Q$ , and

$$Q = \frac{X_L}{R}$$

In view of the fact that to cover a certain frequency band with a given variable capacitor requires a certain fixed amount of inductance, the  $Q$ 's of a number of coils designed for a certain purpose will be an inverse function of resistance, because all having the same  $L$ , all will likewise offer the same  $X_L$  at a given frequency. It is thus necessary to design the coil for minimum resistance at a given inductance. This means, with other factors equal, a coil in which the required inductance is obtained with the minimum length of wire.

It is not possible to make a statement as to the "best" shape of coil, because that depends so much upon the conditions under which the coil is to be used. An examination of the equation previously given for the inductance of a single layer coil will bring out some of the factors which must be taken into consideration.

$$L = \frac{.03948 r^2 N^2}{l} K$$

Where

$r$  = Radius of Coil (cm)

$l$  = Length of Winding (cm)

$K$  = Constant which is a function of  $2r/l$

For a given number of turns and radius, if the turns are spread out so that  $l$  is greater, the in-

ductance decreases directly as  $l$  is increased. This is compensated for somewhat by the increase of  $K$  as  $l$  is increased. However, with a given  $r$  and  $N$ , if the length of winding is increased from  $l = 2r$  to  $l = 4r$ , (in other words, if the ratio of diameter to length is changed from 1 to .5—See Fig. 8),  $K$  is only increased

wire size as does the d-c resistance.

With these ideas in mind, it is seen that coil design is more than using a simple formula and getting the proper amount of inductance. Efficient circuit operation requires coils having the highest possible value of  $Q$ . In obtaining an efficient coil design for a particular

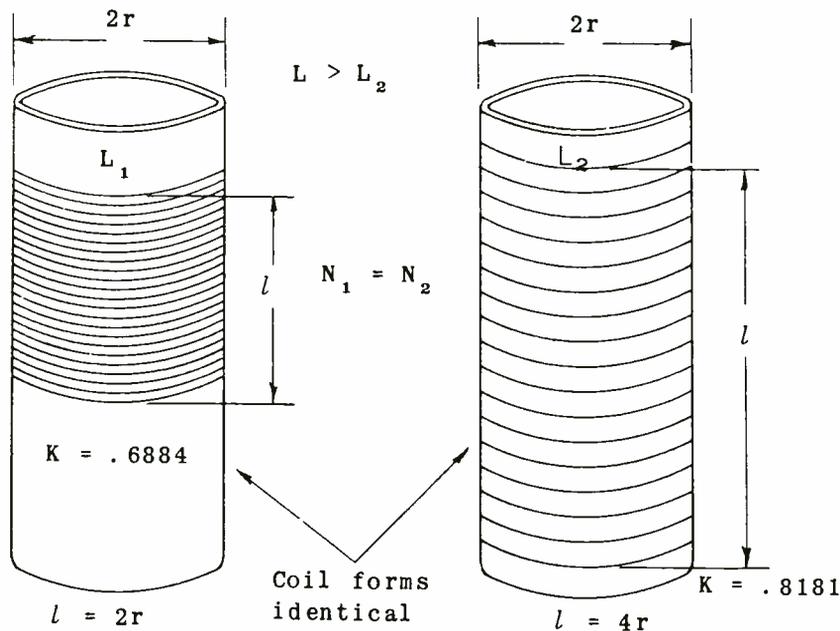


Fig. 8.—For a given number of turns, if the spacing between turns is increased  $L$  will be decreased.

from .6884 to .8181, not enough to compensate for the effect of doubling  $l$ , so that to maintain the same inductance  $N$  must be increased, which will require a greater length of wire. On the other hand,  $L$  varies as  $N^2$  so, although a few more turns are required, a larger size of wire may be used. This in turn tends to be compensated for by the fact that the r-f resistance does not decrease nearly so rapidly with increased

purpose, it is best to design a number of coils, taking into careful consideration the space limitations. Several sizes of wire should be used and several shapes of coils wound, all having the same measured inductance. Then measure the r-f resistance at several points over the frequency range at which the coil is to operate, calculate  $Q$  at each frequency, and plot curves of  $Q$  against  $F$  for each coil. From these

curves it will be easy to select the coil that best meets the particular requirements, or to determine the trend of design and thus wind a coil that is a compromise between several previously wound.

An instrument called a Q-meter is now commercially available, by means of which the Q of coils or circuits may be measured directly. Such an instrument will save many hours of calculation and allow much greater accuracy than can ordinarily be obtained by indirect measurement and calculation.

The Q of the coils used in high grade receivers in the broadcast band may be in the order of 75-150 depending upon the type of construction; in i-f transformers wound

with Litz wire the Q of the secondary winding may be in the order of 150. In the high frequency bands Q need not differ so greatly from that in the broadcast band if care is used in the selection of coil materials.

Transmitter coils may be designed with Q as high as eight or nine hundred. This is possible because instead of small wire, large diameter thin wall copper tubing or high grade Litz may be used with well spaced turns.

(It must be understood that the figures given in the two paragraphs above are simply typical examples. Actually the Q of a coil may exceed the limits given in either direction).

INDUCTIVE REACTANCE, IMPEDANCE, Q OF COILS

EXAMINATION

1. (A) What is meant by the "Counter E.M.F."? To what is it equal?

(B) At what instant is the Counter E.M.F. the greatest in an inductive circuit?

2. (A) When a current in a circuit is in the form of a sine curve, at what instant or instants during a cycle is the rate of change in current most rapid? *Why?* At what times is the rate of change of current equal to zero? *Why?*

INDUCTIVE REACTANCE, IMPEDANCE, Q OF COILS

EXAMINATION, Page 2.

2. (Continued)

(B) Explain in detail the phase relation between the current and voltage in a purely inductive circuit. What is the relation and why?

INDUCTIVE REACTANCE, IMPEDANCE, Q OF COILS

EXAMINATION, Page 3.

2. (Continued)

3. (A) In a purely inductive circuit would the amount of inductance have any bearing on the phase relation between the current and voltage? Why?

(B) What effect has the amount of inductance on the amplitude of the current? Does it increase or decrease?

INDUCTIVE REACTANCE, IMPEDANCE, Q OF COILS

EXAMINATION, Page 4.

3. *(Continued)*

4. What is INDUCTIVE REACTANCE? *Explain.*

5. Has a variation of frequency any effect on the inductive reactance of a circuit? *Why? Explain.* What is the effect of increasing L?

INDUCTIVE REACTANCE, IMPEDANCE, Q OF COILS

EXAMINATION, Page 5.

5. (Continued)

6. (A)  $F = 560 \text{ KC/s}$ ;  $X_L = 470 \text{ ohms}$ . Find  $L$ . Express  $L$  in microhenries.

(B)  $L = 21.8 \text{ microhenries}$ ;  $X_L = 260 \text{ ohms}$ . Find  $F$ .

INDUCTIVE REACTANCE, IMPEDANCE, Q OF COILS

EXAMINATION, Page 6.

6. (Continued)

7. (A)  $L = 24.0$  microhenries;  $F = 500$  KC/s. Find  $X_L$ .

(B)  $L = 6.5$  henries;  $F = 2,600$  cycles. Find  $X_L$ .

INDUCTIVE REACTANCE, IMPEDANCE, Q OF COILS

EXAMINATION, Page 7.

7. *(Continued)*
  
8. A series circuit consisting of 2 henries inductance and 1,000 ohms of resistance is connected across the terminals of a 50-cycle alternator whose effective voltage is 300 volts. Find the impedance, current, and phase relation between current and voltage. Draw a vector showing this relation.

INDUCTIVE REACTANCE, IMPEDANCE, Q OF COILS

EXAMINATION, Page 8.

8. (Continued)

9. (A) How does the r-f resistance of a video peaking coil compare with the d-c resistance?

(B) Why has Litz wire a lower r-f resistance at 1,000 KC/s than equivalent solid wire?

INDUCTIVE REACTANCE, IMPEDANCE, Q OF COILS

EXAMINATION, Page 9.

9. (Continued)

10. A coil having an inductance of  $180 \mu\text{H}$  is operated in a circuit at 1,200 KC/s. The r-f resistance of the coil at that frequency is 10.2 ohms. What is the Q of the coil at 1,200 KC/s?

