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Commentary

ASTRONOMY in Great Britain was probably at its zenith towards the end of the last century and its position was due in no small part to the early pioneering work done by the Earl of Rosse and Sir William Herschel in the construction of high power telescopes.

However, as the light gathering properties of the telescopes increased it became apparent that climatic conditions in this country were setting a limit to further progress and, as a result, the American continent, where the climate is more favourable, has been the principal centre for astronomy during the twentieth century.

Now, due to the enormous progress made during the Second World War in radar and radio techniques, an enormously powerful new tool known as radio astronomy is being placed in the hands of astronomers for the exploration of the space surrounding our planet, and one which, at the same time, bids fair to place this country in the position where it can pursue its studies of the universe without the handicap of fog and cloud.

Radio astronomy is of comparatively recent origin, for it was only in 1931 that an American engineer, Jansky, made the surprising discovery that radio waves were being generated in outer space and were capable of being received on earth. This discovery was published at the time but it created little interest among astronomers, although Grote Reber working in his garden at Illinois with apparatus of his own design was able to confirm Jansky's work.

Reber constructed what was, in fact, the first radio telescope and, pointing it to different parts in the sky, he was able to show that the radio signals were strongest from directions near the centre of the Milky Way, the strength of the signals being roughly proportional to the concentration of stars in the directions to which the telescope was pointing.

The war years brought work to a halt and no further progress was made until 1948 when the centre of interest had shifted to Sydney and Cambridge. By using many of the radar and radio techniques developed during the war it was discovered that at least some of the radio waves arriving from outer space were coming from discrete or localized sources which were called radio stars.

From 1948 onwards progress has been rapid, due principally to the great work carried out by A. C. B. Lovell, Professor of Radio Astronomy at the University of Manchester, on the radio telescope at Jodrell Bank in Cheshire. Much outstanding research was done with this telescope which, with its aperture of 220ft is still the largest in existence.

It had, however, certain limitations. It was fixed to the earth and only a small part of space could be explored. Experience with the instrument soon demonstrated that a completely steerable radio telescope of this order of size was essential for further exploration by this radio method.

At a recent lecture to the Royal Society of Arts Professor Lovell outlined some of the engineering difficulties associated with the construction of this giant telescope. With financial backing to the extent of approximately £500,000 provided by the Department of Scientific and Industrial Research and the Nuffield Foundation, the world's largest steerable radio telescope is now taking shape at the Jodrell Bank Experimental Station of the University of Manchester. It will have a parabaloidal reflector of 250ft aperture weighing some 500 tons and will be supported on towers rising 180ft from the ground.

The beam width of the telescope will be about 1° when used on a wavelength of one metre and its power gain will be over sixteen thousand, while on a wavelength of 21cm its beamwidth will be reduced to a few minutes of arc only.

This great discrimination and power gain will be used to elucidate the nature of the radio sources and to plot detailed maps of the sky, particularly in those regions which are obscured by interstellar dust.

During the last few years a great deal of attention has been given to a particular type of radio emission from space which is generated in the neutral hydrogen gas in the Milky Way on a wavelength of 21cm and great attention, therefore, will be attached to 21cm emissions when the telescope is in operation.

At the moment it seems likely that the importance of the problems associated with radio emissions from space will occupy nearly the whole time of the telescope for the first few years, but it is intended that it should be used to cover all aspects of radio astronomy, including those in which radio waves are first transmitted from the earth.

It is hoped that preliminary tests on the telescope in a fixed position will be made in the autumn of this year and that it will be in full operation by 1956.

At the present moment Britain has a lead of some four years in this new and fascinating subject of radio astronomy and, when the telescope is completed, studies of the universe and nearby space can once more be pursued in this country.

A Transient
Pulse Width and Pulse Amplitude Meter

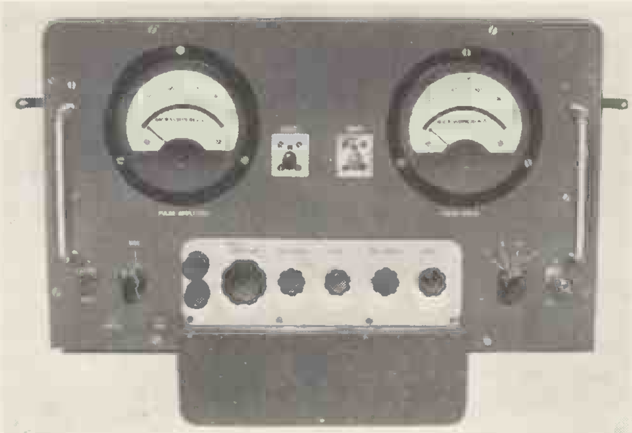
By F. Hart*, A.M.I.E.E.

The basic principle of measuring the amplitude of the transient is to charge a low leakage capacitor to the peak amplitude of the pulse and measure the voltage across the capacitor, with an electrometer type voltmeter, before the charge leaks away.

To measure the pulse width (at the base) a similar capacitor is allowed to charge linearly to an amplitude which is proportional to pulse width. The amplitude of the sawtooth thus produced is measured by a second electrometer voltmeter which gives a linear scale reading of pulse width.

STRAIN gauges, crystal pick-ups and other forms of transducer are used as electro-mechanical elements to produce pulses which correspond to the displacement, velocity or acceleration of a component under shock test.

The measurement of transient phenomena is normally accomplished by observing or photographing the waveform displayed on a c.r.t. There are many applications such as shock testing of structures, packages, electronic components, etc., in which the amplitude or duration of the impulse are the only quantities required. Where the work is purely repetitive, photography becomes laborious and uneconomical.



The front panel of the pulse width-amplitude meter

The instrument to be described was developed to read the peak amplitude of transient pulses originating from a valve under vibration. In the particular application for which the equipment was developed the combination of pulse amplitude and pulse width was of importance. The instrument therefore gives a meter indication of the peak amplitude and width of non-recurrent pulses.

Noise pulses originating in other components or assemblies, can also be measured.

Action

Pulse Amplitude

The input pulse should be positive and should have an amplitude greater than 0·5V and less than 10V.

If the transient to be measured has insufficient amplitude, any conventional amplifier, having a bandwidth suitable to pass the pulse without distortion, can be used to amplify the transient before feeding to the instrument.

Referring to Fig. 1, V_1 and V_4 constitute anode-followers having stable gains of approximately ten and unity respectively. The anode-follower type amplifier was used because of its ability to amplify a wide range of signal amplitudes without amplitude distortion. Fig. 2 shows the combined amplitude response of V_1 and V_4. The response is linear to inputs as great as 30V peak-to-peak.

Care should be taken with the layout and the quality of the components determining the gain of the amplifier. The resistors which determine the gain should have low self inductance and low capacitance. Capacitors should have no resistive or inductive components. The anode-follower is particularly susceptible to uncontrolled feedback circuits with subsequent modification of amplifier characteristics. With this in mind, the components should be spaced away from each other and the chassis.

Without the trimmer capacitor C_2 the high frequency response of the amplifier would be poor because of the h.f. filtering action of the resistor R_2 and the effective input capacitance of V_1.

Fig. 3(b) shows the response of V_1 to the 10μsec input square wave shown in Fig. 3(a).

Fig. 3(c) shows the square wave response of the two amplifiers.

Examination of Fig. 3(c) shows that the amplifier rise time-constant is approximately 1μsec.

The input positive-going transient, after linear amplification, by V_1 and V_4, is fed through the cathode-follower V_6 to the charging capacitor C_{18}. As the output pulse from the cathode-follower builds up positively, the diode V_{8b} conducts, keeping the right-hand plate of the charging capacitor slightly above earth potential. When the pulse has reached peak amplitude, the diode stops conducting and leaves a charge in the capacitor proportional to the pulse amplitude. Fig. 3(d) shows the response of the charging capacitor to a single impulse input.

To obtain a true representation of the capacitor charge, it is not sufficient to observe the waveform at the cathode of the cathode-follower since the resistance of the conducting diode must be considered as part of the charging source. We require the time to charge the capacitor from a source impedance consisting of the cathode-follower in series with

* *Ministry of Supply.*

the conducting diode. To achieve this, the capacitor and the diode were interchanged and the oscilloscope was connected across the capacitor with one terminal at earth potential. To simulate single impulse conditions, while keeping a reasonable c.r.t. brightness for photographic purposes, the " reset " switch was shunted with a 47kΩ resistor to allow the capacitor to discharge completely between impulses of low p.r.f.

From the photographs it will be seen that the value of the charging capacitor determines the minimum width of pulse that can be measured without amplitude distortion.

right-hand plate of the 0·01μF capacitor charged to a negative potential proportional to the amplitude of the input pulse. This negative potential is measured by the cathode-follower electrometer valve V_{10}. To maintain a high input resistance, the heater voltage is limited to 4·3V and the anode to 90V. A typical graph showing the relationship between the negative potential applied to the grid of electrometer valve and the output meter reading is shown in Fig. 5. It will be seen that the circuit is quite linear over the specified range of input voltages.

Fig. 6 shows the amplitude response of the cathode-

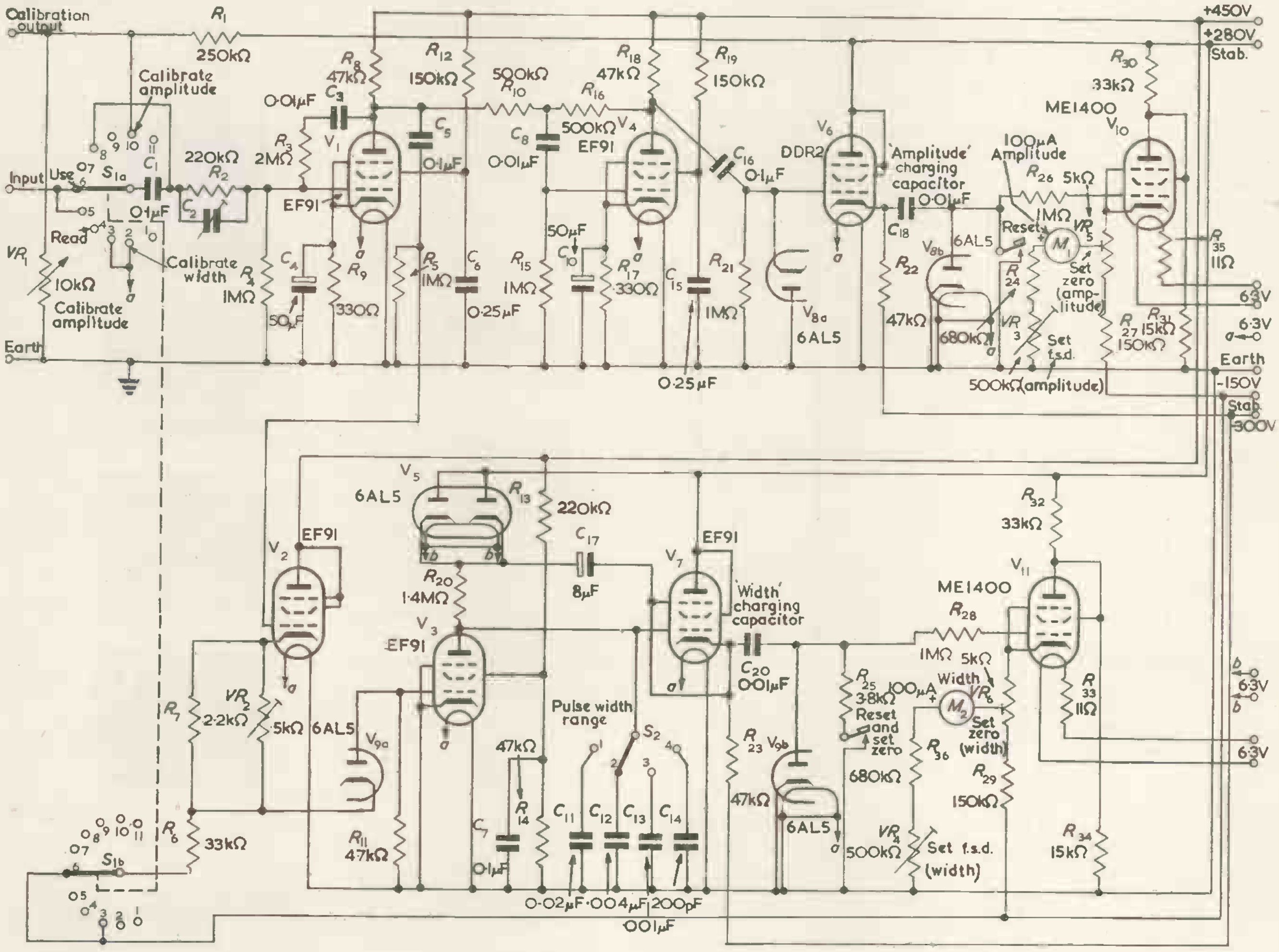


Fig. 1. The pulse width-amplitude meter

It will be seen from Fig. 3(d) that the charge time-constant is approximately 3μsec. Thus a pulse of 12μsec would rise to 98 per cent of the maximum value and a pulse of 15μsec to 99·3 per cent. The measured relationship between the pulse amplitude meter reading and a constant amplitude single impulse of varying width is shown in Fig. 4. It will be seen that amplitude limiting occurs on pulses of less than 15μsec width.

During the pulse the left-hand plate of the charging capacitor C_{18} stays at a positive potential proportional to the incoming pulse amplitude. At the end of the pulse the left-hand plate of the capacitor starts to fall and the right-hand plate of the capacitor falls by an amount equal to the positive potential which existed on the left-hand plate. Thus at the cessation of the pulse we are left with the

follower V_6, to a positive potential. Here again the amplification is linear to inputs up to +200V. With sufficient input to give full scale deflexion on the output meter, there is still an effective bias of 5V on the cathode-follower which precludes the passing of grid current.

It will have been observed that the amplifier has a better frequency response than the cathode-follower and grid current will be taken by the cathode-follower V_6 during a fast rising input pulse. However, the effective output impedance of V_4 is approximately 500Ω, so that amplitude limiting can be neglected.

At the end of the pulse the 0·01μF capacitor C_{18} starts to discharge through any available leakage paths. The main leakage paths available are:

(1) The insulation resistance of the capacitor C_{18}—greater than 20 × 10^6MΩ.

(2) The ME1400 effective input resistance—greater than 2 × 10^6MΩ.

(3) The CV140 (6AL5) p.t.f.e. valve holder—greater than 2 × 10^6MΩ.

(4) The CV140 (6AL5) valve total leakage between anode and earth—greater than 0·5 × 10^6MΩ.

It will be seen that the limiting component tending to reduce the leakage time-constant is the CV140 valve so that there is nothing to be gained by excluding the diode holder and holding the diode in the wiring.

In like applications the insulation resistance of capacitors is invariably the limiting factor determining circuit operability. The capacitor used in this application is a T.C.C. Plastapack polystyrene film dielectric type which is admirably suited, having such a high insulation resistance. The measured effective discharge time-constant of 100 minutes is such that when an input transient of amplitude 10V has registered full scale, the meter reading then falls by 1 per cent in 50 seconds, giving ample time to take a reading. Pulses narrower than 15μsec can be measured, at the expense of a faster decay time, by reducing the value of the charging capacitor C_{18}.

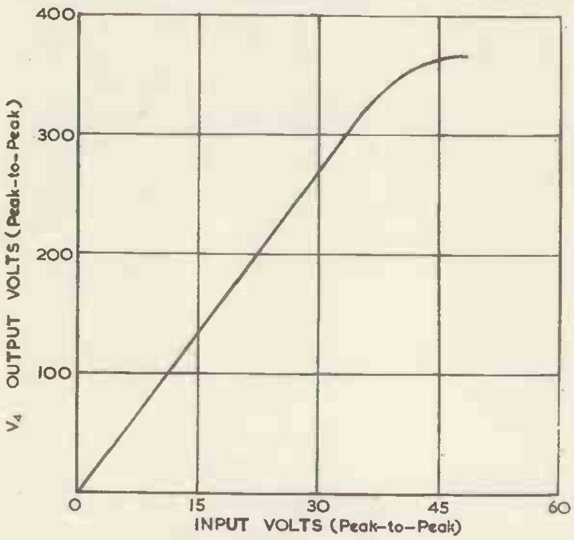


Fig. 2. Amplifier amplitude distortion
Input frequency—500c/s; h.t.—450V; gain—×9.

To reset the meter after taking a reading a non-locking discharge switch (S_3) is operated to discharge the capacitor. To reduce the leakage paths the switch consists of a phosphor bronze spring strip which is remotely controlled by a Bowden cable. The charging capacitor carries no switch contact or associated leakage paths, the phosphor bronze strip simply wiping across the capacitor terminal to effect a discharge.

Pulse Width

To measure the transient pulse width, the input pulse, having been phase inverted in V_1, is fed through the cathode-follower V_2 to the pentode V_3. This valve, normally conducting, has a fixed anode current of 200μA determined by the high stability anode load of 1·4MΩ in series with the conducting diode V_5 and the stabilized supply voltage of +280V. When the control grid of V_3 is cut off the capacitor C_{12} starts to charge through the 1·4MΩ resistor. This rising potential is fed via the cathode-follower V_7 to the high potential end of the anode load resistor. This raises the potential on the cathode of the diode V_5 above +280V and the diode ceases to conduct. Assuming that the cathode-follower was perfect, the high potential end of the charging resistor would now follow exactly the rise in potential across the capacitor C_{12}. This means that the steady current of 200μA which was flowing through the valve would now be supplied from the cathode-follower, through the 8μF capacitor C_{17} and into the capacitor C_{12} since the diode and pentode are both cut off and virtually out of circuit.

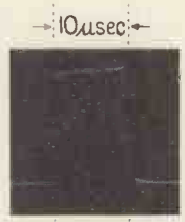


Fig. 3(a). Input pulse

Fig. 3(b). Response of V_1 to input pulse

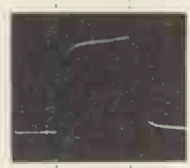


Fig. 3(c). Response of V_4 to input pulse

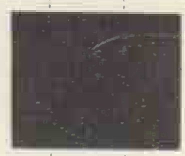


Fig. 3(d). Response of charging capacitor C_{18}

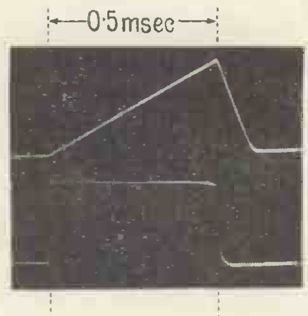


Fig. 3(e). Linear charge of C_{12} due to 0·5msec input pulse

Assuming the cathode-follower gain were unity the current (i) flowing into the capacitor C_{12} would be constant and hence the voltage rise (V_c) would be linear governed by the expression:

$$V_c = it/C_{12} \qquad (1)$$

The cathode-follower gain is slightly less than unity and the expression (1) for the voltage across the capacitor is given by:

$$V_c = \frac{V_{ht}}{1 - G}[1 - \exp(-t(1 - G)/C_{12}R] \quad \ldots\ldots (2)$$

where G is the gain of the cathode-follower.

$$G = \frac{\mu R_k}{r_a + (\mu + 1)R_k} \quad \ldots\ldots\ldots\ldots (3)$$

So, making R_k large so that $(\mu + 1) R_k > r_a$

$$G \simeq \frac{\mu}{\mu + 1}$$

i.e. $1 - G \simeq 1/\mu$.

Referring to equation (2) the capacitor appears to be

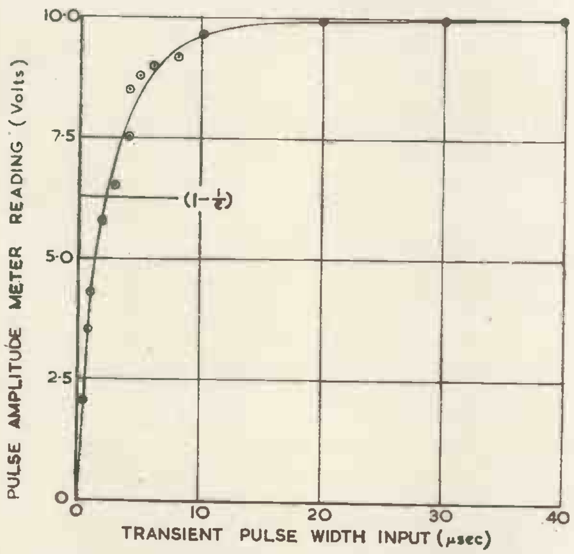


Fig. 4. Pulse response of transient amplitude meter
Constant amplitude input pulse of 10V; effective CR approximately 2·4μsec.

charging on a time-constant of $\mu C_{12}R$ from a potential of μ times the h.t. voltage.

There are four ranges for pulse width as shown in Table 1.

TABLE 1

RANGE	SCALE MULTIPLIER	CHARGING CAPACITOR
10—100 μsec	0.1	200 pF*
50—500 μsec	0.5	·001 μF ± 1% h.s.
0·2—2 msec	2	·004 μF ± 1% h.s.
1—10 msec	10	0·02 μF ± 1% h.s.

** This capacitor comprises a 170pF±5% h.s. in parallel with a trimmer which is adjusted so that the total capacitance, including strays, is 200pF.*

Considering the 10msec range and assuming perfect linearity of charge, substituting in equation (1) it will be seen that full scale deflexion is reached when $V_c = 100$V.

Substituting in equation (2) $R = 1{\cdot}4\text{M}\Omega$

$$C_{12} = 0{\cdot}02\mu\text{F}$$
$$\mu = 35$$
$$V_{ht} = 280\text{V}$$

we find that $V_c = 99{\cdot}5$V, giving a theoretical loss of linearity of $-0{\cdot}5$ per cent.

Fig. 3(e) shows the linear charge of the capacitor C_{12} to an input pulse of 0·5msec width.

When the input pulse falls to zero, the pentode V_3 again conducts and the capacitor C_{12} is discharged nearly to earth potential. The potential of the cathode of V_7 and hence of the 8μF capacitor also fall; the diode V_5 conducts and the 8μF capacitor is supplied with the small amount of charge lost due to charging the capacitor C_{12}. Thus the circuit returns to its original state ready for the next transient.

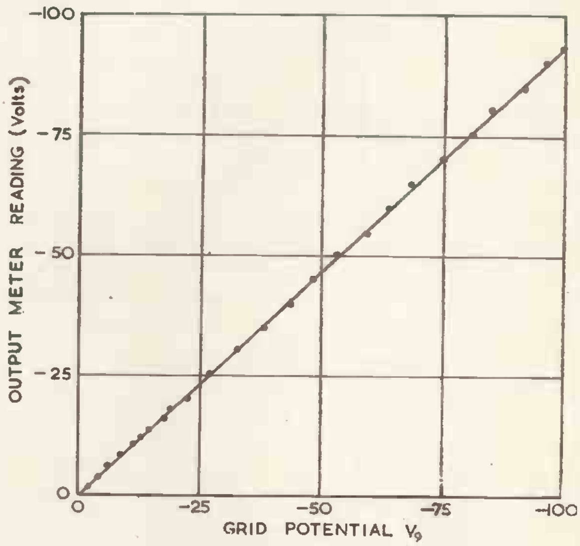


Fig. 5. Electrometer cathode-follower linearity

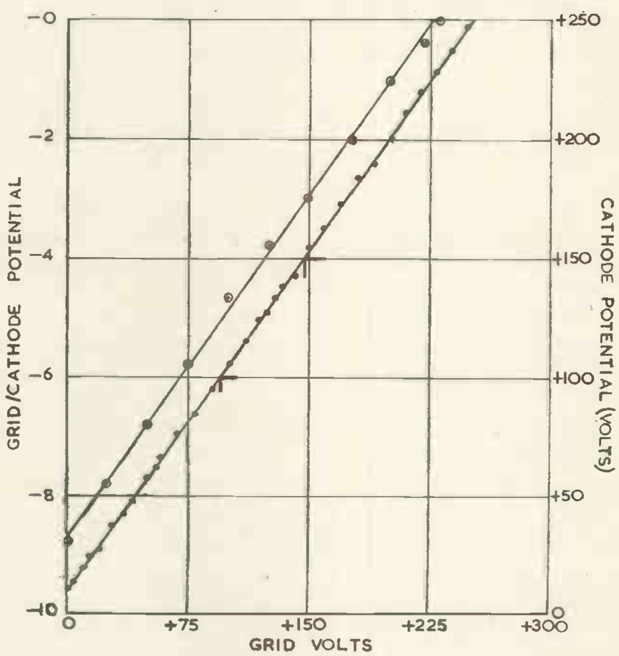


Fig. 6. V_5 d.c. characteristics

It will be seen that capacitor C_{17} must be large compared with C_{12} to keep the change of potential across R_{20} to a minimum and preserve the linearity of the charge.

The operation of the remainder of the circuit is similar to the pulse amplitude circuit except that the "Width" charging capacitor C_{20} is negatively charged to the peak value of a sawtooth instead of an exponential waveform. It is obvious that the linear increase of potential across C_{12} is directly proportional to the applied pulse width giving a linear scale meter reading.

Calibration

SET ZERO—PULSE AMPLITUDE AND PULSE WIDTH

To set the zero meter reading in both the pulse amplitude and width meters, the "set zero" non-locking switches are closed and the "set zero" potentiometers adjusted until the meters read zero. When the switches are released it will be noticed that the meters read approximately one per cent of f.s.d. This effect is mainly due to diode contact potential. This false zero is the more correct setting since a positive input pulse to the diode having an amplitude equal to the contact potential, say 1 volt, would cause no current to flow into the charging capacitor, but the meter would still indicate correctly at 1 volt. Similarly a pulse of, say, 70V would cause charging current to flow due to 69V of the pulse. The false zero setting thus compensates for contact potential error. In any case the diode input pulses are high compared with contact potential, so that the resultant percentage error is very small and can be neglected.

SET FULL SCALE DEFLEXION—PULSE AMPLITUDE

Any d.c. voltmeter of known calibration accuracy is connected, and remains connected during the pulse amplitude calibration, to the terminals labelled "calibration output".

The "calibrate amplitude" potentiometer VR_1 is adjusted until the voltage, as read on the external d.c. voltmeter, is 10V. To set f.s.d. the calibration switch is simply turned to "calibrate amplitude". A d.c. potential of 10V is thus passed to the input of the amplifier producing an amplified transient to the cathode-follower V_6, causing the meter to read. The set f.s.d. potentiometer is then adjusted to give full scale reading on the meter. To repeat, the calibration switch is rotated anti-clockwise to the "use" position and again turned to the "calibrate amplitude" position to re-check the calibration. The input capacitor C_1 is automatically discharged as the switch is returned to the "use" position. This is necessary otherwise the capacitor would only discharge slowly through its own leakage resistance, giving an effective calibration signal of much less than 10V due to the charge remaining in the input capacitor C_1.

To check other points on the meter scale the "calibrate amplitude" potentiometer is adjusted to read the required input voltage and the calibration switch is rotated from "use" to "calibrate amplitude" and the output meter indication is checked against the steady voltage reading on the external voltmeter.

SET FULL SCALE DEFLEXION—PULSE WIDTH

To set the f.s.d. calibration of the pulse width meter, first turn the pulse width range switch to the 10msec range (position 1, $C_{11} = 0{\cdot}02\mu F$). Next turn the calibration switch from "use" to "calibrate width". During the time that the switches are in position 3 a 50c/s sine wave of approximately 180V peak-to-peak appears on the cathode of V_2. The diode V_{9a} conducts on the negative half cycles so that the pentode is cut off for 10msec, allowing the capacitor C_{11} to charge to 100V. The pulse width meter reading can now be adjusted to read full scale. The calibration on the other ranges will now be correct assuming that the high stability capacitors C_{12} to C_{14} have been chosen to suit the scale ranges required.

Further detail is appropriate at this stage. Referring to the switch contacts S_{1b}, it will be seen that when on the "calibrate width" position the $-150V$ supply is not being supplied to V_2. Had the switch been left in position 3 the capacitor C_{20} would carry the 50c/s sawtooth component found at the anode of V_3 in addition to the peak d.c. component and the meter would read the mean value which would have a form factor of approximately $\frac{3}{4}$ for an equal mark-space sawtooth. Thus, as the switch is rotated from the "use" to "calibrate width" position, so long as the switch remains on position 3 for more than 0·02sec, at least one half-cycle of the 50c/s calibration signal will pass to V_3. The capacitor C_{20} will then be fully charged to the peak value of the sawtooth and the a.c. component will be automatically removed when the switch reaches the "calibrate width" position (position 2). To remove the a.c. component, it was found better to disconnect the negative supply to V_2 rather than switch off the input calibration signal. Switching off at the input produces sudden discontinuities giving an erroneous calibration. If the supply to V_2 is switched off, the calibration signal simply falls to zero without any discontinuity. With this in mind the switch S_{1a} has make before break contacts to ensure that the input signal is not momentarily switched off when moving from one contact to another.

During the quiescent state in the "use" condition it is essential that the diode V_{9a} be non-conducting. If the cathode potential of this diode were negative, V_3 would be permanently cut off, C_{12} would be fully charged and the meter would not indicate on subsequent pulses. To achieve this, the current through the cathode-follower V_2 is stabilized by the 33kΩ h.s. resistor and the stabilized $-150V$ supply so that the diode cathode potential can be set at, say, 2V positive ensuring that the diode is not conducting. This means that V_3 will not be cut off until the negative half-cycle input to V_2 has moved through about 3V. The 90V negative half-cycle calibration signal would give an angle of "no flow" of 176°. Thus there is an inherent error of $+2{\cdot}2$ per cent when measuring the width of pulses having very fast rise and fall times. It has already been shown that there already exists a $-0{\cdot}5$ per cent error in the amplitude of the sawtooth waveform. The resultant error is small enough to be neglected. Furthermore, there is an advantage in using such a conservative bias of $+2V$ on the cathode of the diode V_{9a}. Without this amplitude discrimination unwanted hum signals which are fed to the input will be amplified by V_1 and tend to cut off V_3 and give full scale readings, particularly on the narrower width ranges. With this in mind the potentiometer VR_2 has been included in the cathode circuit of V_2. The diode bias can now be adjusted to eliminate the effect of extraneous input hum signals. Similarly unwanted noise signals can be rejected.

Stable supplies and components are not essential in most parts of the circuit since the built-in calibration facilities permit easy setting up. With the circuit as shown continuous re-calibration due to mains and component variation is reduced giving a calibration accuracy of approximately ± 3 per cent.

Subsidiary Uses

FREQUENCY METER

It will be understood from the action of the 50c/s calibration signal producing full scale deflexion on the 10msec range that the same principle applies on the other ranges at different frequencies. Thus, sine waves at frequencies other than 50c/s and having amplitude of 6·3V r.m.s. would also cut off V_3 for the same fraction (176/360) of a cycle. Therefore, if the meter is set to read f.s.d. on 50c/s there will be no errors on the other ranges at higher frequencies.

The operation would be to apply the signal, having a peak amplitude of less than 10V, to the input terminal, then switch from "use" to "read" and the meter reading would then indicate the input frequency, e.g. on the 2msec to 0·2msec range a 500c/s sine wave applied to the input would read half full scale when switched to the "read" position.

The ranges of frequency measurements corresponding to the pulse width ranges are shown in Table 2.

TABLE 2

PULSE WIDTH RANGE	FREQUENCY COVERAGE
10—1msec	50—500c/s.
2—0·2msec	250—2500c/s.
500—50μsec	1—10kc/s.
100—10μsec	5—50kc/s.

For small input amplitude the angle of no flow decreases, giving a negative error in the indication. However, measurements on the instrument give negligible errors so long as the input does not drop below 1·5V r.m.s.

Peak-to-peak Voltmeter

Repetitive signals applied to the input become negative with respect to earth by action of the diode V_{8b} On disconnecting the input by switching to "read", the a.c. component across C_{18} disappears, leaving the peak-to-peak voltage of the input signal across C_{18}, i.e. 1 to 3 mark-space positive square wave (mean/peak $= \frac{1}{4}$) of amplitude 10V, applied to the input which would be restored negatively by the diode V_{8b} to give a mean/peak ratio of $\frac{3}{4}$ and a meter reading of 7·5V. If the input is switched off, the meter would then read the true peak value of 10V.

The diode V_{8a}, although not essential when measuring transient pulses, is included to restore recurrent signals positively with respect to earth. This ensures that the pulses into the cathode-follower V_6 always start their positive excursion from the same fixed potential. Without the diode the meter reading would increase as the p.r.f. is reduced.

With recurrent pulses the charging capacitor C_{18} is fully charged during the first pulse so that the high frequency response to subsequent pulses is improved. With this particular amplifier the minimum width of a recurrent pulse which can be measured is 3μsec as compared with 15μsec for a transient pulse.

For a sine wave input the negative restoration by the diode V_{8b} produces a mean/peak-to-peak ratio of 2/1 and the meter reads half the peak-to-peak value of any particular input. On switching off the input the meter reading is doubled to give the true peak-to-peak value.

Interval Timer

If the interval to be measured is in the form of a positive square wave, intervals between 10μsec and 10msec can obviously be measured. If the interval is not available in this form it is generally a simple matter to produce a positive square wave from the "start" and "stop" pulses or the "make" and "break" contacts of the particular item under test.

Acknowledgments

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Acknowledgment is made to the Chief Scientist, Ministry of Supply, for permission to publish this article. Reproduced with the permission of the Controller of H.M. Stationery Office.

REFERENCE

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Electronic Machine for Sorting Parcels

An automatic parcel-sorting machine, the first of its kind in Australia, installed at the Melbourne General Post Office, has as its basis a moving belt which drops parcels through a series of gates into their correct sorting bins. Electronic controls enable a parcel to be sent to the required gate, and this closes again after the parcel falls sideways off the conveyor belt into the receiving bin.

The conveyor belt is 80ft long, and one side of its enclosing frame is composed of 12 gates, representing the classified destination of the postal articles. Thus, there is one gate above the appropriate conveyor bin for City parcels, another for suburban, and others for country and interstate destinations.

There are two operating positions at the launching end of the belt (to which the parcels in bulk are conveyed by another belt, and to a hopper-like table). On each side of the table are two identical control panels, operated by two sorting officers.

These panels function by compressed air under the control of the keyboard pre-selector mechanism.

The sorting officers feed the articles into the belt from the pile on the table, but before each presses his button for the gate to which the article is to be directed, he has to get a green signal light on the panel. This is because the preceding parcel has to pass a photocell in the early stages of its journey. Past this point the selector panel automatically receives its green light signal, and the way is clear for another parcel to be placed on the belt.

The arrangement provides for appropriate spacing of the postal articles, and the speed of the general sorting is thus governed by the speed of the moving belt, which is capable of dispatching about 36 a minute.

An additional guide for each of the two operators of the panels is a red light which automatically shows if a button is pressed before a moving parcel has passed the magic eye. Addresses on the parcels are, of course, studied by the operators before pressing the button.

Technically the important units comprise a revolving drum, which is mechanically linked with the belt-driving apparatus, and 12 annular magnetic steel rings mounted on the drum. The latter completes a full revolution during the period in which the sloping belt has travelled from a point near the sorting position to the end of the belt run. Associated with each annular ring, one for each drop panel, is a recording head, magnetic relay and obliterator.

The two control panels; the conveyor belt can be seen passing between the two operators

The Matrix Approach to Filters and Transmission Lines

By M. E. Fisher*

(Part 1)

The use of matrices in the treatment and solution of problems in electronic engineering has become increasingly popular. The matrix approach is flexible and gives a broad and unified outlook on the problems discussed. In recent years it has been applied to various passive network problems[1,2], to the design of phase shifting networks for oscillators[3] and to the theory of quadripoles including thermionic valves as elements[4]. This article is intended to show how the theory of lumped-parameter and continuous transmission lines and filters finds a natural development in the language of matrices. Important concepts in this theory such as the propagation and attenuation constants, the surge or iterative impedances and the nature of the pass and stop bands turn out to be closely related to basic properties of matrices. Only the normal algebra used in a.c. theory is required for matrix methods which thus obviate the need for the calculus and differential equations in the theory of lines.

A simple graphical method for finding the nature of the pass bands of any filter is developed. The method also gives the attenuation and the speed of the waves on a filter, and their variation with frequency. The reason for the large number of circle diagrams used in transmission line theory[6,5] is seen to arise from the properties of a matrix transformation.

THE fundamentals of matrix algebra—addition and multiplication—and the matrix equations for quadripoles have been discussed before in this journal[2-4]. It will be assumed that the reader is familiar with these fundamentals and they will be restated merely to introduce the notation used in this article.

Fig. 1, General quadripole or four-terminal net

Fig. 1 shows a general quadripole and the conventions for the input and output voltages and currents. In terms of these we have the equations:

$$\left.\begin{aligned} V_0 &= a_{11}V_1 + a_{12}I_1 \\ I_0 &= a_{21}V_1 + a_{22}I_1 \end{aligned}\right\} \quad \text{(1)}$$

The coefficients a_{ij} are determined by the structure of the particular quadripole under discussion[3]. If this contains no active elements, such as amplifiers, then the following relation between the coefficients holds as a result of the reciprocity theorem[4].

$$|a_{ij}| = a_{11}a_{22} - a_{12}a_{21} = 1 \quad \text{(2)}$$

We will only consider such passive quadripoles.

It is useful to notice that a_{11} and a_{22} are pure numbers while a_{12} has the dimensions of impedance and a_{21} those of admittance.

Equations (1) may be written in matrix notation as:

$$\begin{bmatrix} V_0 \\ I_0 \end{bmatrix} = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \begin{bmatrix} V_1 \\ I_1 \end{bmatrix} \quad \text{(3)}$$

It is convenient to introduce single symbols to represent the matrices in equation (3). These are distinguished from normal algebraical symbols by the use of heavy type.

$$\mathbf{x}_0 = \begin{bmatrix} V_0 \\ I_0 \end{bmatrix} \quad \mathbf{x}_1 = \begin{bmatrix} V_1 \\ I_1 \end{bmatrix} \quad \mathbf{A} = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \quad \text{(4)}$$

The fundamental equation then becomes:

$$\mathbf{x}_0 = \mathbf{A}\mathbf{x}_1 \quad \text{(5)}$$

This can be inverted to give the output conditions in terms of the input conditions, i.e.:

$$\mathbf{x}_1 = \mathbf{A}^{-1}\mathbf{x}_0 \quad \text{(6)}$$

$\mathbf{A}^{-1}$ is the inverse or reciprocal matrix given in this case by:

$$\mathbf{A}^{-1} = \begin{bmatrix} a_{22} & -a_{12} \\ -a_{21} & a_{11} \end{bmatrix} \quad \text{(7)}$$

Reverse Quadripole

If a matrix **A** characterizes a given quadripole the matrix corresponding to the reverse quadripole, formed by turning the quadripole around and using the original output terminals as input terminals, can be obtained as follows:

we write $\quad \mathbf{y}_0 = \mathbf{R}\mathbf{y}_1 \quad \text{(8)}$

by analogy with equation (5). Here **R** is the matrix of the reverse quadripole and $\mathbf{y}_0$ and $\mathbf{y}_1$ are the input and output column matrices for the reverse quadripole. From equation (7) and noting that the sense of the currents must be altered when using the quadripole in reverse we obtain:

$$\mathbf{R} = \begin{bmatrix} a_{22} & a_{12} \\ a_{21} & a_{11} \end{bmatrix} \quad \text{(9)}$$

A symmetric or reversible quadripole is one which has the same effect whichever pair of terminals is used for the input. In this case the matrices **A** and **R** must be the same which gives as a condition for a quadripole to be reversible that:

$$a_{11} = a_{22} \quad \text{(10)}$$

(compare equations (4) and (9)).

This condition is illustrated by the matrices of some

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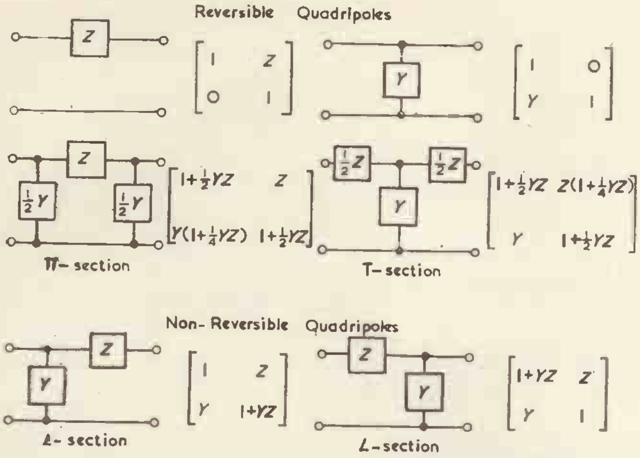


Fig. 2. Some common quadripoles and their "A" matrices.
Z—impedance, Y—admittance

common reversible and non-reversible quadripole networks shown in Fig. 2.

Quadripoles in Cascade

Many quadripole networks encountered in practice are made up of various combinations of simpler quadripoles. In these cases it is very useful to have a direct method of calculating the matrix of the more involved quadripole in terms of those of its constituent quadripoles. Combinations of quadripoles with the terminals in series, parallel or series-parallel are easily dealt with[1]. In particular the matrix

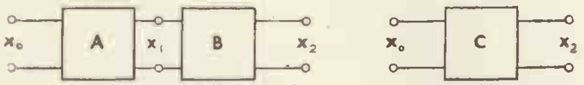


Fig. 3. Two quadripoles in cascade equivalent to a single quadripole

for the overall quadripole formed by connecting two quadripoles in cascade (Fig. 3) can be found by the method of matrix multiplication[3]. In the notation of Fig. 3 we have:

$$\mathbf{x}_0 = \mathbf{A}\mathbf{x}_1 \quad \mathbf{x}_1 = \mathbf{B}\mathbf{x}_2 \quad \ldots\ldots\ldots\ldots (11)$$

Thus:

$$\mathbf{x}_0 = \mathbf{A}\mathbf{B}\mathbf{x}_2 = \mathbf{C}\mathbf{x}_2 \quad \ldots\ldots\ldots\ldots (12)$$

where the matrix **C**, representing the overall quadripole, is the matrix product **AB** of the two matrices **A** and **B**.

For ladder networks, that is for longer chains of quadripoles, we have further factors in the product. For example, we would have:

$$\mathbf{x}_0 = \mathbf{A}\mathbf{B}\mathbf{C}\mathbf{D}\mathbf{x}_4$$

for a row of four different quadripoles in cascade. Practical application of this principle to *RC* phase shifting networks are illustrated in Ref. 3.

A filter or artificial line is a row of similar quadripoles each with the same matrix. The overall matrix for a chain of *n* similar quadripoles with matrix **A** is $\mathbf{A}^n$ (repeated application of equation (12) with **A** = **B**).

We have:

$$\mathbf{x}_0 = \begin{bmatrix} V_0 \\ I_0 \end{bmatrix} = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}^n \begin{bmatrix} V_n \\ I_n \end{bmatrix} = \mathbf{A}^n\mathbf{x}_n \quad \ldots (13)$$

and inverting this:

$$\mathbf{x}_n = \begin{bmatrix} V_n \\ I_n \end{bmatrix} = (\mathbf{A}^n)^{-1} \begin{bmatrix} V_0 \\ I_0 \end{bmatrix} = \mathbf{A}^{-n}\mathbf{x}_0 \quad \ldots\ldots (14)$$

These two matrix equations relating the input and output voltages and currents are the basic equations of the theory of lines and filters. Later they will appear in more familiar form. ($\mathbf{A}^{-n}$ is the matrix inverse of $\mathbf{A}^n$).

Vector Interpretation

Equation (13) relates the two column matrices $\mathbf{x}_0$ and $\mathbf{x}_n$. Each of these depends on two numbers, one representing a voltage and the other a current. These two numbers can be regarded as the two components of a vector in a plane. Thus $\mathbf{x}_0$ can be interpreted as a vector with components (I_0, V_0) and $\mathbf{x}_n$ as a vector with components (I_n, V_n). These vectors can be represented on a diagram (Fig. 4) using two perpendicular axes to represent current and voltage. As, however, we are in general concerned with alternating currents the components of the vector **x** (*I*, *V*) will be complex numbers which cannot be represented directly as distances along a single axis. In spite of this it is useful to regard *V* and *I* as single magnitudes and interpret them, symbolically if not directly, by the distances along two axes in a plane.

With this interpretation of $\mathbf{x}_0$ and $\mathbf{x}_n$ the matrix $\mathbf{A}^n$ appears as an " operator " which when applied to the vector $\mathbf{x}_n$ transforms it, by rotation and extension into the new vector $\mathbf{x}_0$. $\mathbf{A}^{-n}$ is the inverse operator which generates $\mathbf{x}_n$

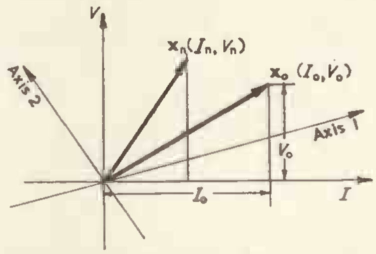


Fig. 4. Showing the interpretation of x_0 and x_n as vectors and the two axes of a typical A matrix

when applied to $\mathbf{x}_0$, i.e., cancels the effect of $\mathbf{A}^n$. The matrix **A** alone is an operator which when it operates *n* times in succession on $\mathbf{x}_n$ transforms it successively into $\mathbf{x}_{n-1}, \mathbf{x}_{n-2}$ . . . and finally into $\mathbf{x}_0$. The interpretation of a square matrix as a vector operator is a useful aid in considering the effect of a line or filter on a given applied voltage and current.

Propagation Along an Artificial Line

Consider an infinite line of similar quadripoles each with matrix **A** connected in cascade. A single wave propagating along this line is characterized by the fact that the voltage and current suffer the same change in phase and the same attenuation (or amplification) as the wave passes across any quadripole. In other words, voltage and current at the n^{th} pair of terminals are equal to the voltage and current at the $(n+1)^{th}$ pair multiplied by a constant factor (possibly complex). The value of this factor will determine the nature and the speed of the propagation. We can write:

$$\mathbf{x}_n = \xi\mathbf{x}_{n+1} \quad \ldots\ldots\ldots\ldots (15)$$

where ξ is a constant. Furthermore, since the matrix of the quadripoles is **A**

$$\mathbf{x}_n = \mathbf{A}\mathbf{x}_{n+1} \quad \ldots\ldots\ldots\ldots (16)$$

Thus for a particular type of quadripole the constant ξ must be determined from the equation.

$$\mathbf{A}\mathbf{x} = \xi\mathbf{x} \quad \ldots\ldots\ldots\ldots (17)$$

The subscript $(n+1)$ may be omitted since it is irrelevant here. This equation may be interpreted directly in terms of our vector representation of x. It implies that x must be a vector such that the effect of operating on it with **A** is to leave it unaltered except for a change in magnitude by a factor ξ. The direction of x is not affected. Any vector, say w, fulfilling this condition is called an *eigen-vector* of the operator matrix **A** (from the German *eigen* meaning self, since, except for the magnitude, w is transformed into itself). The value of ξ corresponding to a particular eigen-vector **w** is called the *eigen-value* of the matrix for the vector **w**. The terms *characteristic* or *latent roots* of the matrix or *proper value* are also used for ξ. The directions determined by the eigen vectors of a particular matrix are sometimes called the *axes* of the matrix[7]. (see Fig. 4.)

We can determine the eigen-values of the matrix **A** as follows. Equation (17) is equivalent to:

$$\begin{aligned}(a_{11}-\xi)V + \quad a_{12}I &= 0\\ a_{21}V + (a_{22}-\xi)I &= 0\end{aligned} \qquad (18)$$

These two equations can only be consistent if:

$$\begin{vmatrix} a_{11}-\xi & a_{12} \\ a_{21} & a_{22}-\xi \end{vmatrix} = 0 \qquad (19)$$

This equation is the *characteristic equation* of the matrix and is well known in many applications of matrix theory. For example it is used to determine the frequencies of vibration of an aircraft wing and the behaviour of an electron in a crystal.

Using the fact that $|a_{ij}| = 1$ it can be written:

$$\xi^2 - (a_{11} + a_{22})\xi + 1 = 0 \qquad (20)$$

Solving this quadratic equation gives two values for ξ, the eigen-values

$$\xi_1 = \tfrac{1}{2}a_0 + \sqrt{[(\tfrac{1}{2}a_0)^2 - 1]}$$

and

$$\xi_2 = \tfrac{1}{2}a_0 - \sqrt{[(\tfrac{1}{2}a_0)^2 - 1]} \qquad (21)$$

where:

$$a_0 = a_{11} + a_{22} \qquad (22)$$

is the sum of the diagonal elements of the matrix and is called the *trace* or *spur* of the matrix **A**. This plays an important part in what follows.

From equation (21):

$$\xi_1\xi_2 = 1 \qquad (23)$$

If we write: $\xi_1 = e^{\gamma}$

then: $\xi_2 = e^{-\gamma} \qquad (24)$

where γ is a complex number

$$\gamma = \alpha + j\beta \qquad (25)$$

(α and β real) called the *propagation constant*.

The significance of the propagation constant is well known and can be seen if we suppose that w_1 and w_2 are two eigen-vectors of **A** corresponding to the eigen-values ξ_1 and ξ_2. w_1 and w_2 represent the voltages and currents at some point of the chain. Then by equation (19):

$$\mathbf{A}\mathbf{w}_1 = \xi_1\mathbf{w}_1 \quad \mathbf{A}\mathbf{w}_2 = \xi_2\mathbf{w}_2 \qquad (26)$$

If the voltages and currents are varying harmonically in time with frequency $\nu = \omega/2\pi$ we may write:

$$\mathbf{w}_1 = \mathbf{w}^{I} e^{j\omega t} = \begin{bmatrix} V^{I} e^{j\omega t} \\ I^{I} e^{j\omega t} \end{bmatrix} \qquad (27)$$

and:

$$\mathbf{w}_2 = \mathbf{w}^{II} e^{j\omega t} = \begin{bmatrix} V^{II} e^{j\omega t} \\ I^{II} e^{j\omega t} \end{bmatrix}$$

Using $\mathbf{w}_1$ and $\mathbf{w}_2$ as $\mathbf{x}_0$ in equation (14) we obtain:

$$\mathbf{x}_n = \mathbf{A}^{-n}\mathbf{x}_0 = \mathbf{A}^{-n}\mathbf{w}_1 = \xi_1^{-n}\mathbf{w}^{I} e^{j\omega t} \qquad (28)$$

or:

$$\mathbf{x}_n = \mathbf{A}^{-n}\mathbf{x}_0 = \mathbf{A}^{-n}\mathbf{w}_2 = \xi_2^{-n}\mathbf{w}^{II} e^{j\omega t}$$

Then from equations (24) and (25):

$$\mathbf{x}_n = \mathbf{w}^{I} e^{j\omega t - \gamma n} = \mathbf{w}^{I} e^{-\alpha n} e^{j(\omega t - \beta n)} \qquad (29)$$

or:

$$\mathbf{x}_n = \mathbf{w}^{II} e^{j\omega t + \gamma n} = \mathbf{w}^{II} e^{\alpha n} e^{j(\omega t + \beta n)} \qquad (30)$$

Equations (29) and (30) represent waves of frequency:

$$\nu = \omega/2\pi = 1/\tau \qquad (31)$$

(where τ is the period and ω the pulsatance or angular frequency) and wave number:

$$k = \beta/2\pi = 1/\lambda \qquad (32)$$

(λ is the wavelength of the wave measured in units of the spacing of the quadripoles or filter sections). β may also be interpreted as the phase change per quadripole. The wave represented by equation (29) travels to the right with decaying amplitude controlled by the decrement or attenuation α. The power attenuation of the wave due to one quadripole or filter section is given by:

$$A = 8{\cdot}68\alpha \text{ decibels} \qquad (33)$$

Equation (30) represents a similar wave travelling to the left. If $\alpha = 0$ both these waves propagate freely without attenuation. The speed of the waves is given by:

$$c = \lambda/\tau = \omega/\beta \qquad (34)$$

in units of quadripole spacing per second.

Thus the two eigen-values ξ_1 and ξ_2 of the matrix **A** correspond to two independent waves of voltage and current travelling in opposite directions with speed and attenuation determined by the propagation constant γ. This in turn is fully determined by the coefficients of the **A** matrix.

Note that the reverse quadripole with matrix **R** given by equation (10), has the same eigen-values and eigen-vectors as the direct quadripole, but in reverse order.

Attenuation and Pass-Bands

If the waves are to be transmitted along the line of quadripoles without attenuation we must have $\alpha = 0$; in other words γ must be purely imaginary, so that:

$$\xi = e^{\pm\gamma} = e^{\pm j\beta} \qquad (35)$$

For a given quadripole this will usually be possible only for certain frequencies. Our aim is to determine what frequencies can be freely transmitted. Equation (20) gives ξ in terms of the trace $a_0 = a_{11} + a_{22}$, but it can be rewritten to give a_0 in terms of ξ.

$$a_0 = \xi + 1/\xi \qquad (36)$$

If there is to be no attenuation, ξ is restricted by equation (35) so a_0 is also restricted,

$$a_0 = e^{\mp j\beta} + e^{\pm j\beta} = (\cos\beta \pm j\sin\beta) + (\cos\beta \mp j\sin\beta)$$

i.e.:

$$a_0 = 2\cos\beta = 2\cos 2\pi k \qquad (37)$$

using equation (32).

Now $\cos\beta$ can take any values between $+1$ and -1 inclusive and is always real since β is real by definition. The condition for unattenuated propagation is thus:

(i) The trace of the matrix, a_0, must be real, $a_0 = a_0^*$; and (ii):

$$-2 \leqslant a_0 \leqslant 2 \qquad (38)$$

a_0 depends on the coefficients of the **A** matrix which in turn are determined by the impedances and admittances of the quadripole. These impedances and admittances vary with the frequency, i.e., with ω, so that equation (37) determines those frequencies for which the line acts as a pass-band filter.

Frequencies for which a_0 does not satisfy equation (38) will be attenuated. In a filter with negligible resistance a_0 is always real since it depends only on products or ratios of pure reactances. Then if a_0 is outside the range -2 to $+2$ the eigen-value ξ will be purely real by equation (21). In this case β is zero and $\gamma = \alpha$ so that equation (30) becomes:

$$|a_0| = \xi + 1/\xi = e^{\pm\alpha} + e^{\mp\alpha}$$

i.e.:

$$|a_0| = 2\cosh\alpha \quad \text{.................} \quad (39)$$

($|a_0|$ is the modulus or absolute value of a_0 irrespective of sign.)

This shows how the attenuation depends on the value of the trace of the matrix outside a pass band. If a_0 is large equation (39) can be approximately written $\alpha = \ln|a_0|$ which gives the power attenuation per section as:

$$A = 20{\cdot}0 \log_{10}|a_0| \text{ decibels} \quad \text{..........} \quad (40)$$

Application to Practical Filters

We will illustrate the use of the criterion equation (38)

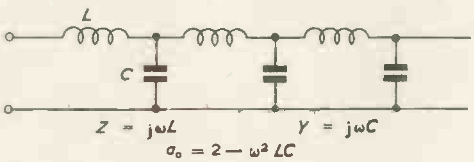


Fig. 5(a). Low-pass filter

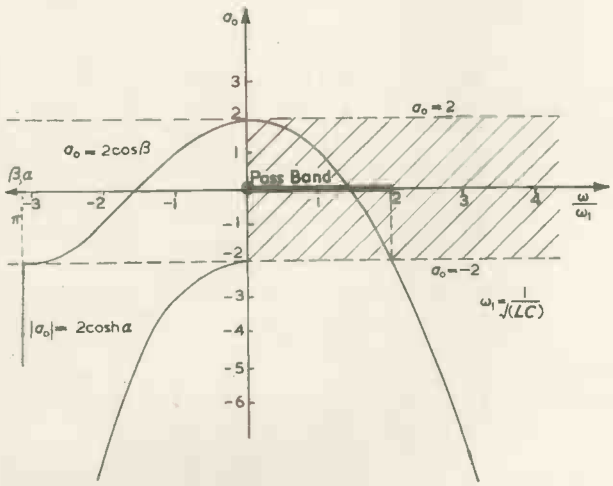
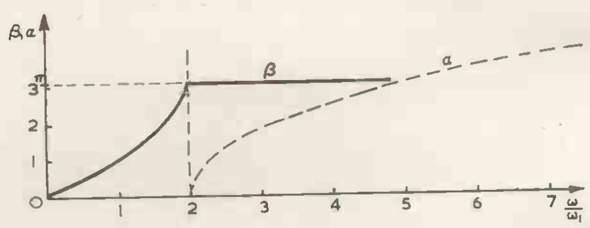


Fig. 5(b). Graph of the trace a_0 against frequency ω

Fig. 5(c). Graph of attenuation α, and phase charge β, against ω



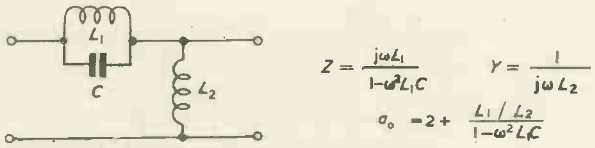


Fig. 6(a). High-pass filter section

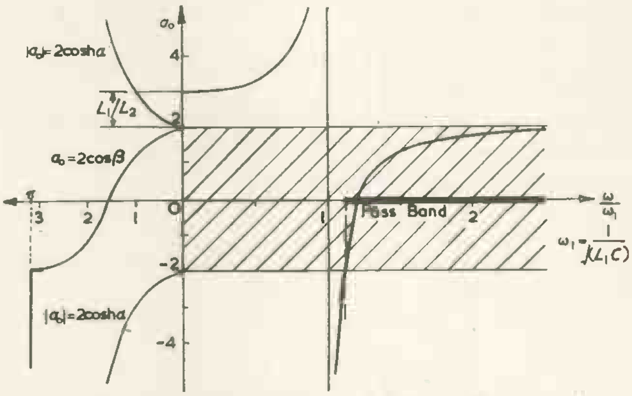


Fig. 6(b). Graph of a_0 against ω for high-pass filter ($L_1 = L_2$)

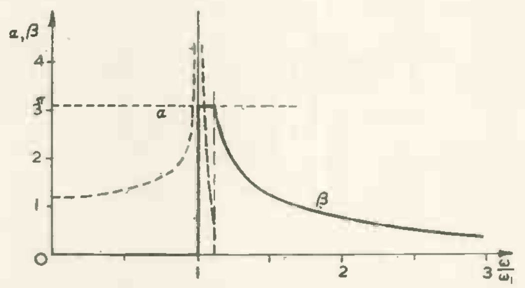


Fig. 6(c). Graph of attenuation α and phase charge β against ω

by discussing various filters formed of L-type sections shown in Fig. 2. The argument would equally well apply to the reversible π or T networks. For all these types of quadripole the trace of the matrix is:

$$a_0 = 2 + YZ \quad \text{................} \quad (41)$$

where Z is the series impedance and Y the parallel admittance of one section.

Consider the simple low-pass filter of Fig. 5(a). Here the trace is:

$$a_0 = 2 - \omega^2 LC \quad \text{..............} \quad (42)$$

which will always be real, hence fulfilling the first condition of equation (38). In Fig. 5(b) a graph of a_0 against ω the pulsatance or angular frequency is plotted for positive ω. In this case the graph is merely an inverted parabola. Now by equation (38) when a_0 is between -2 and $+2$ the attenuation is zero. To find what band of frequencies the filter will pass we examine the curve of a_0 against ω. Where this curve falls between the two horizontal lines $a_0 = 2$ and $a_0 = -2$ shown in Fig. 5(b) the filter will pass the waves. This shows that this filter will pass low frequencies from $\omega = 0$ to $\omega = 2/\sqrt{LC}$, while frequencies greater than this will be attenuated.

Fig. 5(c) shows graphs of the attenuation α, and β the phase change per section, against ω. These can be obtained directly from the graph of a_0 by use of the two graphs:

$$a_0 = 2\cos\beta \text{ and } |a_0| = 2\cosh\alpha$$

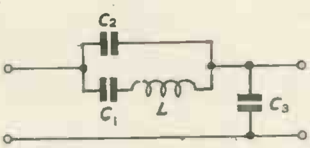
(equations (37) and (39)).

which are plotted on the left-hand side of Fig. 5(b) using the same a_0 axis. For example, to find the attenuation for a given pulsatance ω the first step is to find the a_0 corresponding to this from the right-hand portion of Fig. 5(b). The second step consists in transferring this value of a_0 to the left-hand portion where a_0 is plotted against α, and finding the value of α corresponding to this value of a_0. This value of α is then the attenuation appropriate to the original ω. The same method is used to find β for a given ω.

In Fig. 6 a similar graphical analysis is performed for a high-pass filter having a frequency at which there is infinite attenuation just below the cut-off frequency. The expression for the trace of the matrix in this case is:

$$a_0 = 2 + \frac{L_1/L_2}{1 - \omega^2 L_1 C}$$

from which the cut-off frequency and frequency for infinite attenuation can be obtained directly.



$$Z = \frac{1 - \omega^2 L C_1}{j\omega(C_2 + C_1 - \omega^2 L C_1 C_2)} \qquad Y = j\omega C_3$$

$$a_0 = 2 + \frac{C_3}{C_2} \cdot \frac{1 - \omega^2 L C_1}{1 + \frac{C_1}{C_2} - \omega^2 L C_1}$$

Fig. 7(a). Band-pass filter section

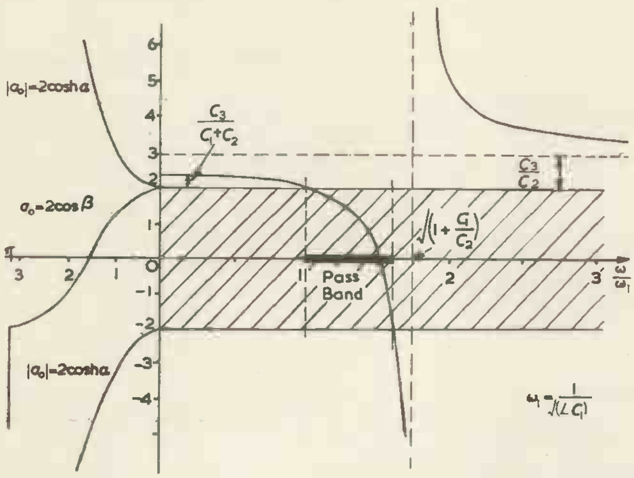
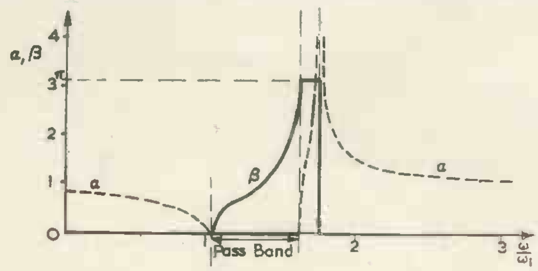
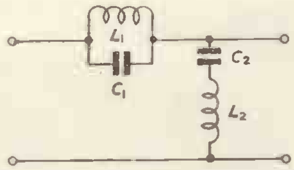


Fig. 7(b). Graph of a_0 against ω. for band-pass filter ($C_1 = 2C_2 = 2C_3$)

Fig. 7(c). Graph of attenuation α, and phase charge β against ω





$$Z = \frac{j\omega L_1}{1 - \omega^2 L_1 C_1} \qquad Y = \frac{j\omega C_2}{1 - \omega^2 L_2 C_2} \qquad a_0 = 2 - \frac{\omega^2 L_1 C_2}{(1 - \omega^2 L_1 C_1)(1 - \omega^2 L_2 C_2)}$$

Fig. 8(a). Band-stop filter section

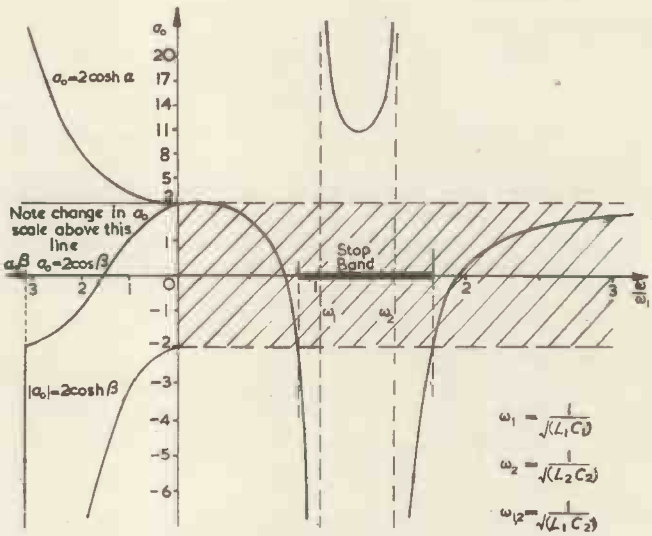


Fig. 8(b). Graph of a_0 against ω for band-stop filter. ($\omega_1 = \omega_{12} = \frac{2}{3}\omega_2$)

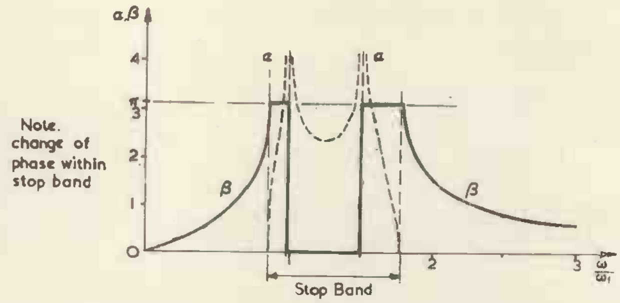


Fig. 8(c). Graph of attenuation α and phase charge β

A band-pass filter utilizing three capacitors and one inductance is analysed in Fig. 7. The expression for the trace is slightly more involved, although the graph is readily drawn. In this case:

$$a_0 = 2 + C_3/C_1 \cdot \frac{1 - \omega^2 L C_1}{1 + C_2/C_1 - \omega^2 L C_2}$$

Finally Fig. 8 shows the attenuation and phase change obtained from a band-stop filter with two frequencies of infinite attenuation in the band.

Here:

$$a_0 = 2 - \frac{\omega^2 L_1 C_2}{(1 - \omega^2 L_1 C_1)(1 - \omega^2 L_2 C_2)}$$

Using this graphical analysis it is easy to devise filters with any particular pass characteristics or to predict the behaviour of a more involved filter section with one or more passing bands.

Impedances Corresponding to the Axes of the Matrix

The matrix equation (17) $\mathbf{Ax} = \xi\mathbf{x}$ enabled us to determine ξ_1 and ξ_2, the two eigen-values of the matrix. On the basis of the vector interpretation of matrices it was shown how only certain of the vectors $\mathbf{x}$ could possibly satisfy this equation. These are the eigen-vectors of $\mathbf{A}$, $\mathbf{w}_1$ and $\mathbf{w}_2$, corresponding to eigen-values ξ_1 and ξ_2 respectively. One can write:

$$\mathbf{w}_1 = \begin{bmatrix} V' \\ I' \end{bmatrix} \qquad \mathbf{w}_2 = \begin{bmatrix} V'' \\ I'' \end{bmatrix} \quad \ldots\ldots\ldots (43)$$

These two eigen-vectors determine the *axes of the matrix* (see Fig. 9). The slopes of the two axes are:

$$m_1 = V'/I' \text{ and } m_2 = V''/I'' \quad \ldots\ldots\ldots (44)$$

It should be noted that if $\mathbf{w}_1$ is an eigen-vector of a matrix $\mathbf{A}$ for eigen-value ξ_1 then any constant multiple of $\mathbf{w}_1$ such as $c\mathbf{w}_1$ is also an eigen-vector for the same eigen-value. So that whatever particular eigen-vector is chosen the slope of the axis is completely determined by equation (44). All possible eigen-vectors lie along one or other of the two axes.

Now the slopes m_1 and m_2 have the dimensions of

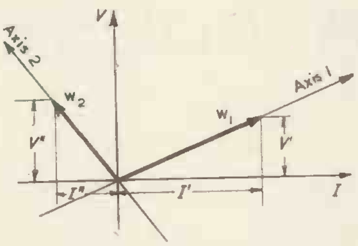


Fig. 9. Showing two eigen-vectors of a matrix

impedance (voltage divided by current). We use them to define two *characteristic impedances* for the particular quadripole.

$$z_1 = m_1 = V'/I' \qquad z_2 = m_2 = V''/I'' \quad \ldots\ldots (45)$$

It will be seen that these impedances are closely related to the two *surge* or *iterative impedances* of a quadripole.

To evaluate z_1 and z_2 in terms of the matrix elements of the quadripole we write out the matrix equation $\mathbf{Ax}=\xi\mathbf{x}$ in full using the two known values of ξ_1 and ξ_2.

For the first eigen-value this gives

$$\begin{aligned} a_{11}V' + a_{12}I' &= \xi_1 V' \\ a_{21}V' + a_{22}I' &= \xi_1 I' \end{aligned} \quad \ldots\ldots\ldots\ldots (46)$$

From this we have

$$z_1 = V'/I' = \frac{a_{12}}{\xi_1 - a_{11}} \text{ or } z_1 = \frac{\xi_1 - a_{22}}{a_{21}} \quad \ldots\ldots (47)$$

and a similar expression for z_2 involving ξ_2 in place of ξ_1. Substituting for ξ in terms of the coefficients (equation 21) one obtains

$$z_1 = \frac{(a_{11} - a_{22})}{2a_{21}} + \frac{\sqrt{\{(\frac{1}{2}a_0)^2 - 1\}}}{a_{21}}$$

$$z_2 = \frac{(a_{11} - a_{22})}{2a_{21}} - \frac{\sqrt{\{(\frac{1}{2}a_0)^2 - 1\}}}{a_{21}} \quad \ldots\ldots\ldots (48)$$

where $a_0 = a_{11} + a_{22}$, is the trace of the matrix as before.

For a symmetric or reversible quadripole such as the usual π or T sections the matrix elements a_{11} and a_{22} are equal (equation 10)). In this case the expression for the characteristic impedances greatly simplifies.

$$z_1 = \sqrt{(a_{12}/a_{21})} \qquad z_2 = \sqrt{(a_{12}/a_{21})} \quad \ldots\ldots (49)$$

so that $z_1 = -z_2$ for a reversible quadripole.

Surge or Iterative Impedances

We will now examine the relation between these characteristic impedances of a quadripole determined from the slopes of the axes of the matrix, and the two iterative impedances of a quadripole met with in normal filter theory. The forwards iterative impedance of a quadripole is defined as the impedance looking into an infinite line of quadripoles connected in cascade. The reverse iterative impedance is similarly defined to be the impedance looking into an infinite chain of reversed quadripoles. We denote these impedances by ζ_1 and ζ_2 respectively*. The iterative impedances can be evaluated in terms of the matrix coefficients by considering the effect of removing one quadripole from the beginning of the line. Obviously since the line is infinite the situation is really unaltered and the impedance looking into the chain is still ζ_1 (Fig 10).

As always if the matrix of the quadripole is $\mathbf{A}$ we have that

$$\mathbf{x}_0 = \mathbf{A}\mathbf{x}_1 \quad \ldots\ldots\ldots\ldots\ldots\ldots (50)$$

where in this case

$$\mathbf{x}_0 = \begin{bmatrix} V_0 \\ I_0 \end{bmatrix} = \begin{bmatrix} \zeta_1 I_0 \\ I_0 \end{bmatrix} \qquad \mathbf{x}_1 = \begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = \begin{bmatrix} \zeta_1 I_1 \\ I_1 \end{bmatrix} \quad \ldots (51)$$

The second relation in each of these equations follows since the impedance is the same whether measured at the 0th pair of terminals or the 1st pair, i.e., $V_0 = \zeta_1 I_0$ and

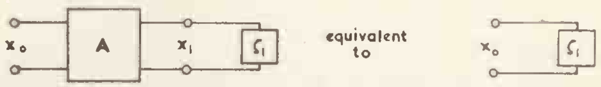


Fig. 10. Illustrating the fact that the input impedance of an infinite chain is unaltered by removal of one quadripole

$V_1 = \zeta_1 I_1$. Substituting these values for $\mathbf{x}_0$ and $\mathbf{x}_1$ and writing out the matrix equation in full we obtain

$$\begin{aligned} \zeta_1 I_0 &= (a_{11}\zeta_1 + a_{12})I_1 \quad \ldots\ldots\ldots\ldots\ldots (52) \\ I_0 &= (a_{21} + a_{22}\zeta_1)I_1 \end{aligned}$$

Eliminating the currents from these equations gives

$$a_{21}\zeta_1^2 - (a_{11} - a_{22})\zeta_1 - a_{12} = 0 \quad \ldots\ldots (53)$$

Solving this gives the forwards iterative impedance ζ_1 in terms of the matrix coefficients:

$$\zeta_1 = \frac{(a_{11} - a_{22})}{2a_{21}} + \frac{\sqrt{\{(\frac{1}{2}a_0)^2 - 1\}}}{a_{21}} \quad \ldots\ldots\ldots (54)$$

The positive sign for the root must be taken since ζ_1 must have a positive real part; resistance is always positive. The reverse iterative impedance can be determined by an exactly similar argument using the matrix of the reverse quadripole (equation 9) which merely has a_{11} and a_{22} interchanged. Thus:

$$\zeta_2 = -\left[\frac{(a_{11} - a_{22})}{2a_{21}} - \frac{\sqrt{\{(\frac{1}{2}a_0)^2 - 1\}}}{a_{21}}\right] \quad \ldots\ldots (55)$$

* *The Greek letter* ζ, zeta, *is used here to avoid writing further subscripts on the over-worked letter Z.*

If these expressions for the iterative impedances are compared with those for the characteristic impedances (equation 48) it will be seen that:

$$\zeta_1 = z_1 \text{ and } \zeta_2 = -z_2 \quad \ldots\ldots\ldots\ldots \quad (56)$$

In other words, except for a change of sign the iterative impedances are exactly the characteristic impedances determined by the slopes of the axes or eigen-vectors!

On reflexion this is not so surprising since, as we have shown, the eigen-vectors of the two eigen-values correspond to two independent waves propagating in opposite directions with a relation between voltage and current determined by the slope of the eigen-vectors. As is well known, such waves are not reflected if the line is matched, that is terminated with the appropriate iterative impedance. With such an arrangement the correct relationship between voltage and current is maintained at the end of the line for propagation in one direction only.

For a reversible quadripole $a_{11} = a_{22}$ and the two iterative impedances become identical.

$$\zeta_1 = \zeta_2 = \zeta = \sqrt{(a_{12}/a_{21})} \quad \ldots\ldots\ldots\ldots \quad (57)$$

as would be expected on physical grounds.

(To be continued)

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A REVERSIBLE BINARY COUNTER

By R. W. Fenemore*

A binary counter is described, which will both "add" and "subtract" from the total registered. Control and input signals are required. A control switching unit is described, and reference is made to a practical version of the complete reversible binary counter. One example is the digital analogue convertor, and the other use of a counter in conjunction with an interpolator.

COUNTERS are used extensively in industrial process and control systems, as well as in general scientific investigations. Usually these counters are purely summation devices in that they record the sum of the total number of input signals received.

For some applications it may be necessary to both add and subtract from the total registered by the counter. In this case, the counter must be supplied with both control and input signals. The control signals indicate to the counter whether particular input pulses should be added or subtracted from the total registered by the counter. Such counters are generally termed "reversible," and this article will describe a reversible binary counter.

For an example of the use of a reversible counter, it may be required to indicate the total number of items already in store, to and from which items are being added or removed. Another example is the application of corrections to counts already made and registered in the counter. These corrections may be either negative or positive. A final illustration is the use of a reversible counter to indicate the actual position of a member when it has been subjected to a series of forward and reverse movements. The counter is used to keep a running total of instructions to the member and so indicate its true position. This system is employed as part of the control of a milling machine operating from input information, which is digital in form.

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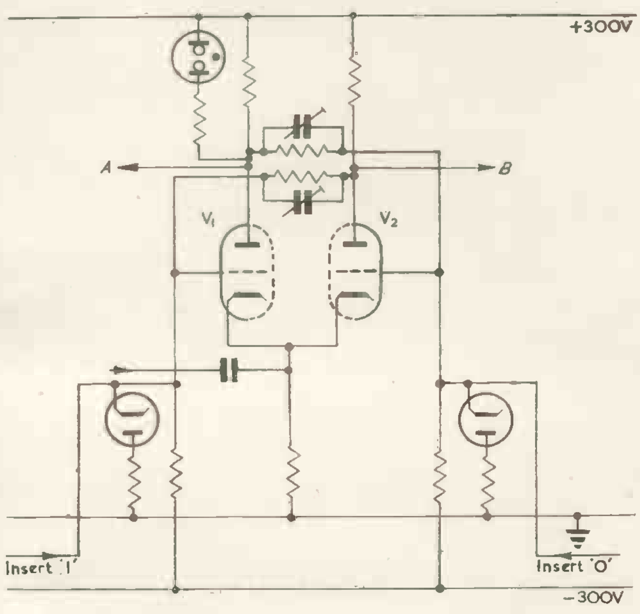
Bistable Multivibrators as Counters

Multivibrators have been used as binary counters for a long time. An example of such a counter stage is shown

Fig. 1. Basic counter stage



in Fig. 1. Let this counter stage represent "0" when valve V_1 is conducting and "1" when it is cut off, i.e. when valve V_2 is conducting.

For the process of addition, a carry-forward signal is required to operate the next stage when the counter changes from "1" to "0." This signal may be obtained in several ways, two of which are:

(a) from the anode of V_1 when it changes from being cut off to conducting,

(b) from the anode of V_2 when it changes from being conducting to cut off.

A sharper edge pulse is obtained at the anode of a valve when it is made to conduct than when it is cut off. This is because drawing current through the valve charges the stray capacitances at the anode of the valve more quickly than the charge leaks away when the valve is cut off. Hence the most suitable signal for triggering the

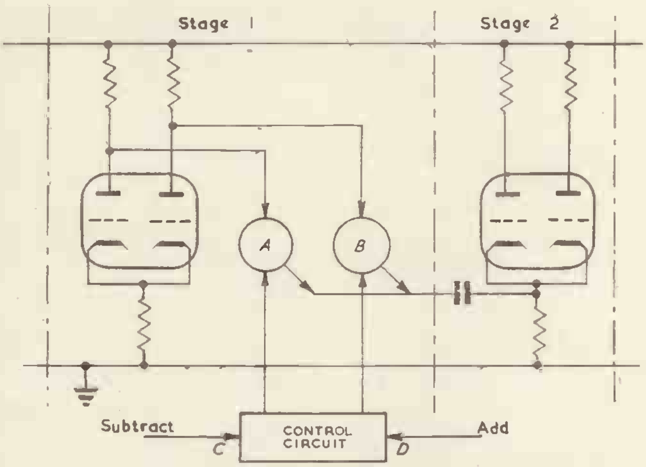


Fig. 2. Basic add-subtract counter stage

succeeding stage of the counter is obtained from condition (a) above—point A Fig. 1. For subtraction, a similar argument leads to the selection of a triggering pulse for the succeeding stage from the anode of V_2 when it is made to conduct—point S Fig. 4.

Modification for Add-Subtract Facilities

For a counter that will both add and subtract, the foregoing leads to the circuit arrangement of Fig. 2. In this arrangement, gates A and B are placed in the triggering circuits between consecutive stages of the counter. Trigger pulses for the second stage of the counter are selected from either one of the anodes of the valves in the first stage by the gates A and B. The gates are operated differentially and only one is open at any time. The control circuit which determines the gate to be opened operates on the receipt of the add or subtract control signals associated with the counting pulses which are fed to the input of the counter.

Practical Circuit Arrangement

A practical circuit arrangement for a reversible binary counter is shown in Fig. 3. Two counter stages only are shown, one represented by valves V_1 and V_2, the other by valves V_5 and V_6. The interstage gating circuits are provided by diodes V_3 and V_4. Differential potentials applied to lines X and Y determine whether the counter shall add or subtract. In this particular arrangement these biasing potentials are about either −150V and 0V.

The biasing potentials for controlling the interstage gating circuits may be obtained from an electronic switch, e.g. a circuit similar to that employed in the counter stage itself. The biasing potentials are taken from the two

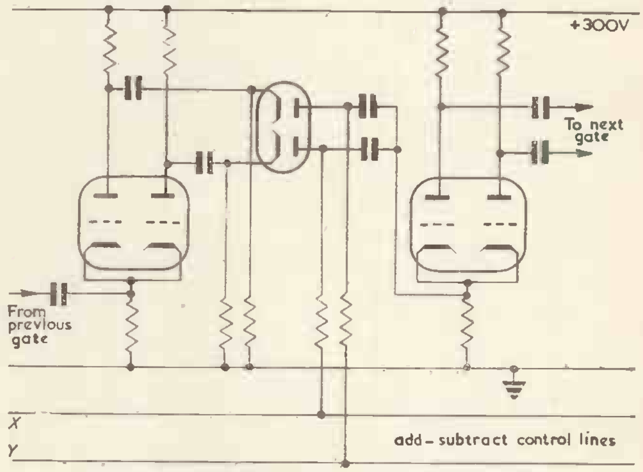


Fig. 3. Circuit of single stage add-subtract counter

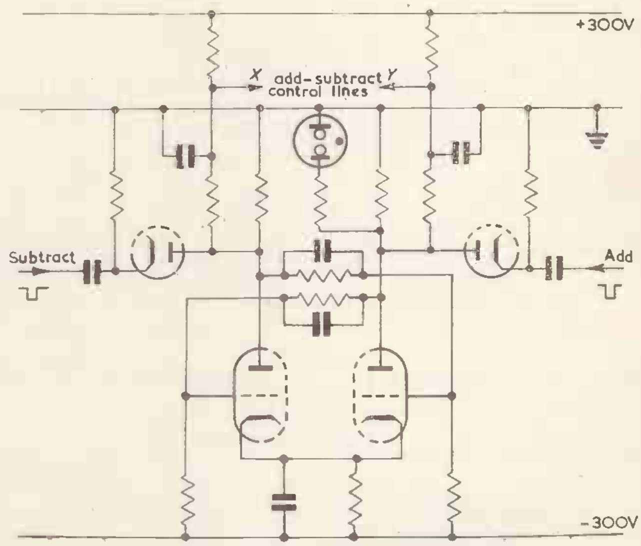


Fig. 4. Add-subtract switch

anodes of the valves forming the bistable pair. An example of such a switch is shown in Fig. 4.

The switch is tripped into one of its two stable states by negative control pulses on leads A and B. Biasing potentials are supplied to the main counter stages from the anode of the valves V_3 and V_4 at points X and Y. When valve V_3 is conducting, the potential of point Y is about −150V and that of point X about earth. With the switch in this position, the counter will be in the subtract condition. When separate control pulses are not available, they must be generated from the counting pulses. It is necessary to introduce a small delay in the signal path to the counter input. This permits the electronic switch to operate and set the counter in the appropriate state before counting pulses reach the input of the counter.

The design of the add-subtract switch requires to be related to that of the counter stage. If this is not done, the switching of the bias potentials at the interstage gating circuits may cause spurious trigger pulses to appear at succeeding counter stages. To overcome this, the counter stages are made to operate only on trigger pulses which have a rise time smaller than the switching time of the biasing potentials.

In the particular arrangement of Figs. 3 and 4, the switching time of the control circuits is about 10μsec and trigger pulses for operation of the counter stages need to have a rise time of about 0·01μsec and amplitude of about 20V negative.

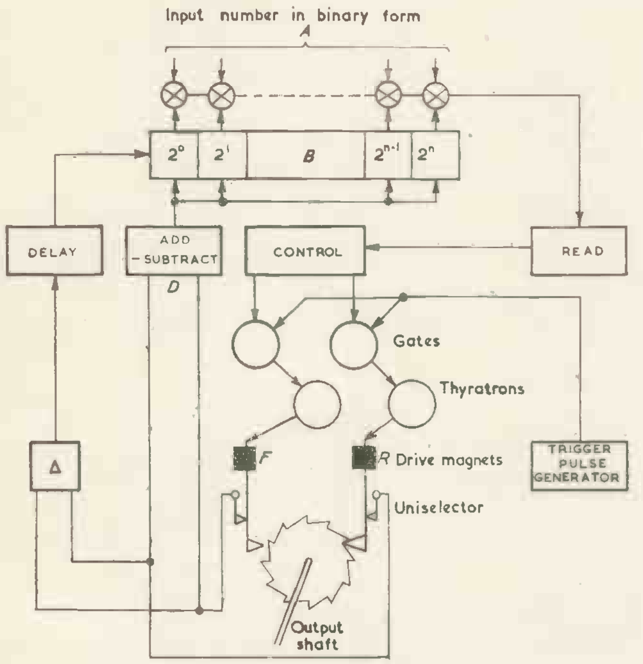


Fig. 5. Example of digital-analogue convertor

Applications of the Counter

Uses of a reversible binary counter are manifold, but only two will be described by way of example.

The first illustration is the use of the counter as a component in a digital analogue convertor. The general arrangement of the system is shown in Fig. 5. It is required to convert to a shaft rotation information which is presented to the convertor in the form of a binary number. The output shaft is driven through suitable reduction gearing by a bothway uniselector. The position of this shaft is registered as a binary number in the counter *B*. Auxiliary contacts on the uniselector generate pulses each time the shaft is stepped in either the forward or reverse direction. Pulses representing forward motion are added and those representing reverse motion are subtracted from the total in the counter. The required position of the output shaft in binary form is compared with the position of the output shaft as registered by the binary counter. The difference is used to generate an error signal which controls the motion of the uniselector. The driving magnets of the uniselector may be energized through thyratrons which are controlled by two gate circuits; the gates pass triggering pulses to the appropriate driving circuit thyratron when there is an error signal.

The system is a closed loop system and hence conditions for stability of such systems must be satisfied. However, it is a sufficient, though not a necessary, condition for stability if the repetition rate of the trigger pulses is such that, after operation of the uniselector, the new error signal is fed to the control gates before the next trigger pulse appears. This limits the maximum rate of change of input information which the system will follow without large transient errors.

The second example concerns the use of the counter as part of a digital data interpolator. A block schematic arrangement is shown in Fig. 6 (a), and the mode of operation will be clear by reference to Fig. 6 (b). The value of Y_N is inserted in parallel into the counter at the beginning of an interpolation interval, e.g. at point X_N for the interval X_N to $X_N + 1$. Data for the calculation of the value ΔY is inserted in the interpolator at the same time. The output of the interpolator consists of a series of pulses. The repetition rate of the pulse is proportional to the slope of the curve. Pulses are fed to the reversible counter, and the total indicated by the counter is the value $Y_N + \Delta Y$, i.e. the value of Y corresponding to the value of X. At the end of the interpolation interval, the new value Y, i.e. $Y_N + 1$, is inserted into the counter and interpolation proceeds as before.

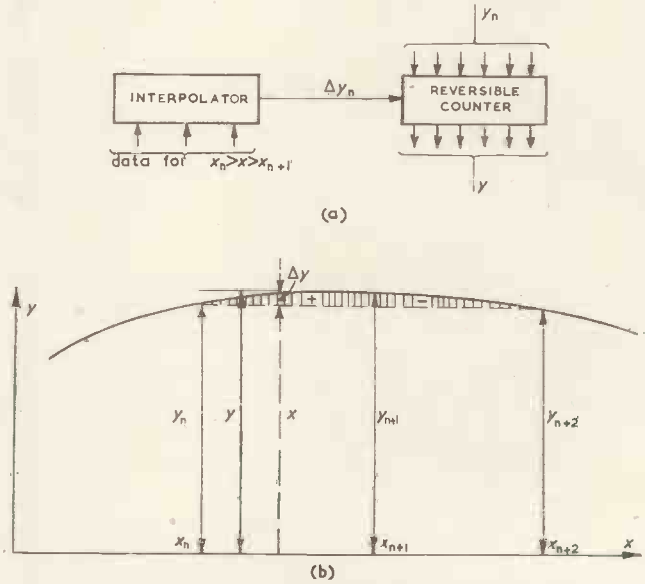


Fig. 6. Reversible counter used in interpolator

Acknowledgments

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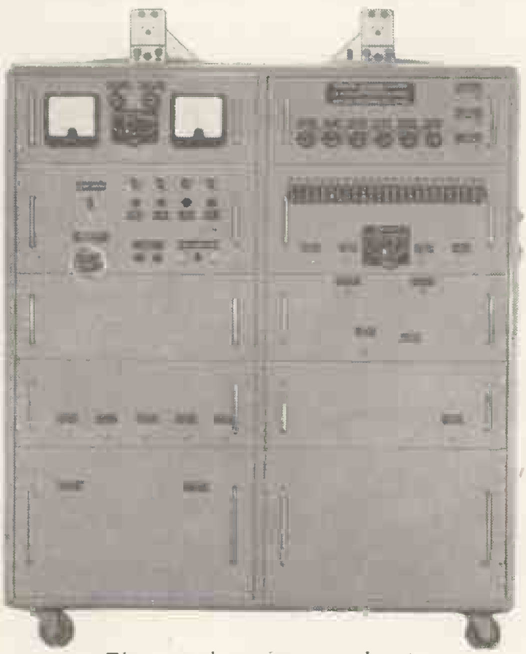
A HARMONIC RESPONSE PLOTTER

By Z. Czajkowski*, B.Sc.

An instrument is described which simultaneously plots a harmonic response diagram of a servo system and its first derived plot. The range of the instrument is from 0·2c/s to 104c/s in 100 steps which are marked on both diagrams. The advantage of the instrument described is that it is almost completely insensitive to waveform distortion which may be introduced in the servo system under test, so that only the fundamental of the test frequency is taken into account in the plot computation. The instrument contains its own generator and provision for automatic running.

THE instrument has been built to satisfy the urgent need of a servo system design laboratory for an instrument giving a quick indication of the frequency response of a servo system, and also predicting its response under closed loop conditions. The instrument can also be used for determining the low frequency characteristics of electrical networks, transductors, or, in fact, of any device which is capable of accepting and producing a signal in the form of an electric voltage.

Though some Nyquist diagram plotters have been built[1,2], the instrument described here is the first to provide simultaneously both simple harmonic response and derived plots. An electro-mechanical analogue computor system is used, incorporating six servo mechanisms.



The complete response plotter

Test Frequency Generation

The test frequency voltage which is applied to the system under test is generated by a magslip driven with variable speed by a velodyne type motor-generator (see magslip 3 in "velocity servo" (Fig. 1). This magslip has a single-phase stator which is supplied with constant voltage from a carefully stabilized generator of carrier frequency which is in this case 2·4kc/s. The output from the rotor, which is also single-phase will therefore be:

$$E_1 \sin \omega_m t \sin \omega_c t \quad \ldots\ldots\ldots\ldots \quad (1)$$

where ω_m is the radial velocity of the rotor corresponding to the test frequency.

$\omega_c = 2\pi\ 2400$ is the radial velocity of the carrier.

This voltage is then fed to a phase sensitive rectifier (P.S.R.5 on Fig. 1) and after rectification and filtering goes to an output d.c. amplifier and to the system under test. The output attenuator is used only for the initial output level adjustment.

The output signal:

$$E_1 \sin \omega_m t \quad \ldots\ldots\ldots\ldots \quad (2)$$

is not dependent on the test frequency and remains constant throughout the test. This output voltage from the instrument, which is also the input voltage to the system under test is marked A_0 on the vector diagram in Fig. 2.

Input Circuits

The output signal from the system under test which has now the amplitude E_0 and is displaced in phase by angle α is accepted as an input to the instrument (vector A in Fig. 2). After passing through a d.c. cathode-follower, preset attenuator and a d.c. amplifier, this signal is used to modulate a 2·4kc/s carrier. The modulator circuit presented a considerable difficulty because of the high degree of accuracy required for the d.c./a.c. amplitude conversion, and by the need of keeping the phase-angle constant throughout the test frequency range. Obviously, any phase difference introduced at this stage would be detected by the sensitive resolver system which follows. Finally, a circuit utilizing the curvature of anode characteristics of two triodes in push-pull has been found satisfactory. Even so, the phase lag throughout this stage of the instrument, due to various causes, amounted to about 4° at 100c/s and a suitable phase-advance network has been incorporated to neutralize this effect.

Angle Resolving System

The basic principle underlying the action of resolving the angle and magnitude of a vector is explained in Fig. 1.

The signal received from the network under test, after modulation, is of the form:

$$E_0 \sin (\omega_m t + \alpha) \sin \omega_c t \quad \ldots\ldots\ldots\ldots \quad (3)$$

this voltage is then applied to the rotor of a magslip which is being driven by the same motor as the generator magslip 3 and is therefore rotating with velocity ω_m. The stator of this magslip is joined by a three-phase connexion to that of another stationary magslip (in "α servo", Fig. 1). The winding X of this magslip forms an angle ϕ with a datum line. Hence the voltage X will be:

$$X = E_0 \sin (\omega_m t + \alpha) \sin \omega_c t \cos (\omega_m t + \phi) \quad \ldots \quad (4)$$

$$\therefore X = \tfrac{1}{2} E_0 [\sin (2\omega_m t + \alpha - \phi) + \sin (\alpha - \phi] \sin \omega_c t \quad \ldots \quad (5)$$

Hence it can be seen that, neglecting the carrier, the voltage will consist of a component of twice the test frequency

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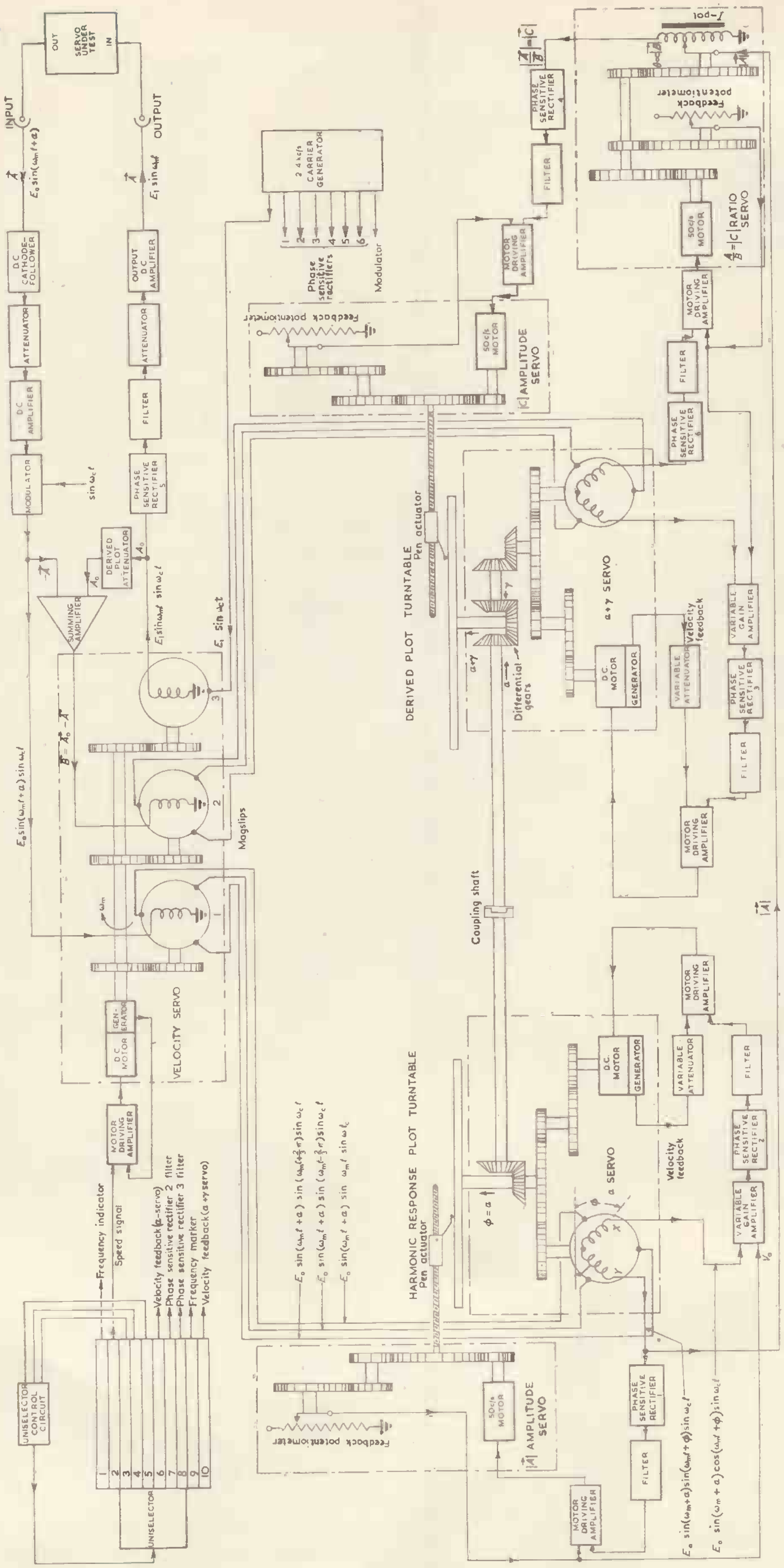


Fig. 1. Block diagram of the instrument

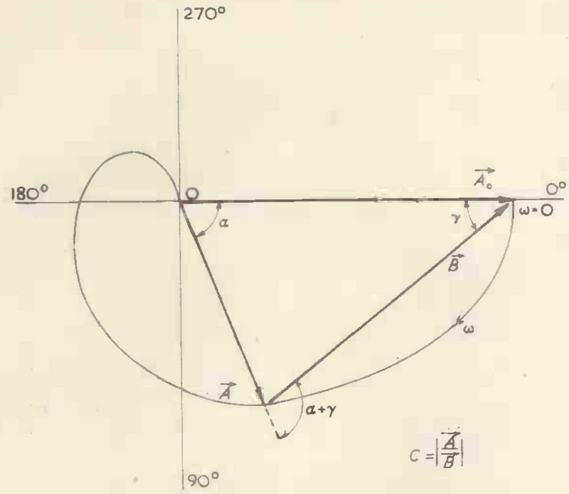


Fig. 2. Locus of the output vector

A_o = *input signal vector*, A = *output vector* $\vec{B} = \vec{A_o} - \vec{A}$

plus a constant, which will, however, disappear if:

$$\sin(\alpha - \phi) = 0 \quad (6)$$

i.e.:

$$\alpha = \phi \quad (7)$$

A servo which seeks a null position of this winding will therefore set the magslip shaft exactly at an angle corresponding to phase shift α. A turntable with the plot is coupled to the magslip shaft.

Assume now that the n^{th} harmonic of the test frequency of amplitude E_n is present in the output of the system under test due to distortion. Further, assume that the phase relation of this harmonic to the fundamental is represented by angle β. The expression (3) will now become:

$$[E_o \sin(\omega_m t + \alpha) + E_n \sin(n\omega_m t + \beta)] \sin \omega_c t \quad (8)$$

After this signal has been resolved in the magslip system as before, the voltage from winding X will be:

$$X = [E_o \sin(\omega_m t + \alpha) \cos(\omega_m t + \phi) + E_n \sin(n\omega_m t + \beta) \cos(\omega_m t + \phi)] \,.\, \sin \omega_c t \quad (9)$$

$$\therefore X = \tfrac{1}{2}\left\{E_o \sin(2\omega_m t + \alpha + \phi) + E_o \sin(\alpha - \phi) + E_d \sin[(n+1)\omega_m t + \beta + \phi] + E_n \sin[(n-1)\omega_m t + \beta - \phi]\right\} \sin \omega_c t \quad (10)$$

By inspection, it can be seen that the only d.c. component (after removal of the carrier $\sin \omega_c t$) is the term $(E_o/2)\sin(\alpha - \phi)$. Therefore, the presence of harmonics in the waveform will not affect the computed angle ϕ.

Nulling Servo

The servo-mechanism used in the magslip resolver which was mentioned before represents rather a special problem. The nulling signal after rectification is of the form:

$$V = \tfrac{1}{2}E_o \sin(\alpha - \phi) \quad (11)$$

First, it may be noticed that the amplitude of the signal to be resolved is variable over a range which in this instrument is 1 : 100. If a servo loop of constant gain were used, it would result in both low angular resolution for small values of K, and servo instability because of too high gain for large values of K. To prevent this, a variable gain amplifier (see Fig. 1) was inserted into the loop. The gain of this amplifier was made inversely proportional to a voltage V_a derived from the feedback potentiometer of servo A. This voltage is negative and proportional to the magnitude of vector A, i.e. to E_o.

The variable gain amplifier itself consists of two stages (see Fig. 3), the first of them a variable-mu pentode has the grid bias automatically adjusted to give the desired gain. The second valve is a conventional voltage amplifier.

The input resistor network is so arranged that the gain is the desired function of V_a, i.e.:

$$G = G_o/K \quad (12)$$

Another difficulty encountered in this servo system was due to the signal ripple of frequency $2\omega_m$. For the lowest test frequency of 0·2c/s the servo would show a tendency to follow this ripple. To prevent this, an unusually large velocity damping has been applied. To avoid, however, a very slow servo response for higher frequencies, the velocity feedback is applied through an attenuator operated by the automatic switch coupled to the test frequency selector, which reduces the velocity feedback at the rate inversely proportional to the test frequency.

Amplitude Servo

The function of this servo is to plot the magnitude of the vector A (see Fig. 1). The required voltage is derived from the winding Y on the magslip resolver in the "α servo". This winding is at right-angles to the winding X and the output voltage, corresponding to equation (4) will therefore be:

$$Y = E_o \sin(\omega_m t + \alpha) \sin \omega_c t \sin(\omega_m t + \phi) \quad (13)$$

$$\therefore Y = \tfrac{1}{2} E_o [\cos(\alpha - \phi) - \cos(2\omega_m t + \alpha + \phi)] \sin \omega_c t \quad (14)$$

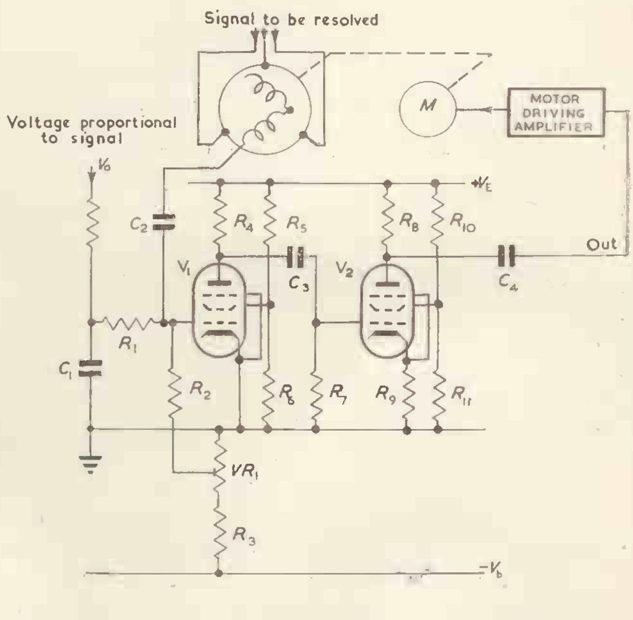
but from (7): $\alpha = \phi$

hence:

$$Y = \tfrac{1}{2} E_o [1 - \cos(2\omega_m t + 2\alpha)] \,.\, \sin \omega_c t \quad (15)$$

After removal of the carrier component $\sin \omega_c t$ in the phase sensitive rectifier 1 (Fig. 1) a d.c. voltage is obtained with superimposed ripple of twice the test frequency. A filter which is used to smooth this ripple has the components automatically selected by the main test frequency

Fig. 3. Variable gain amplifier in a servo loop



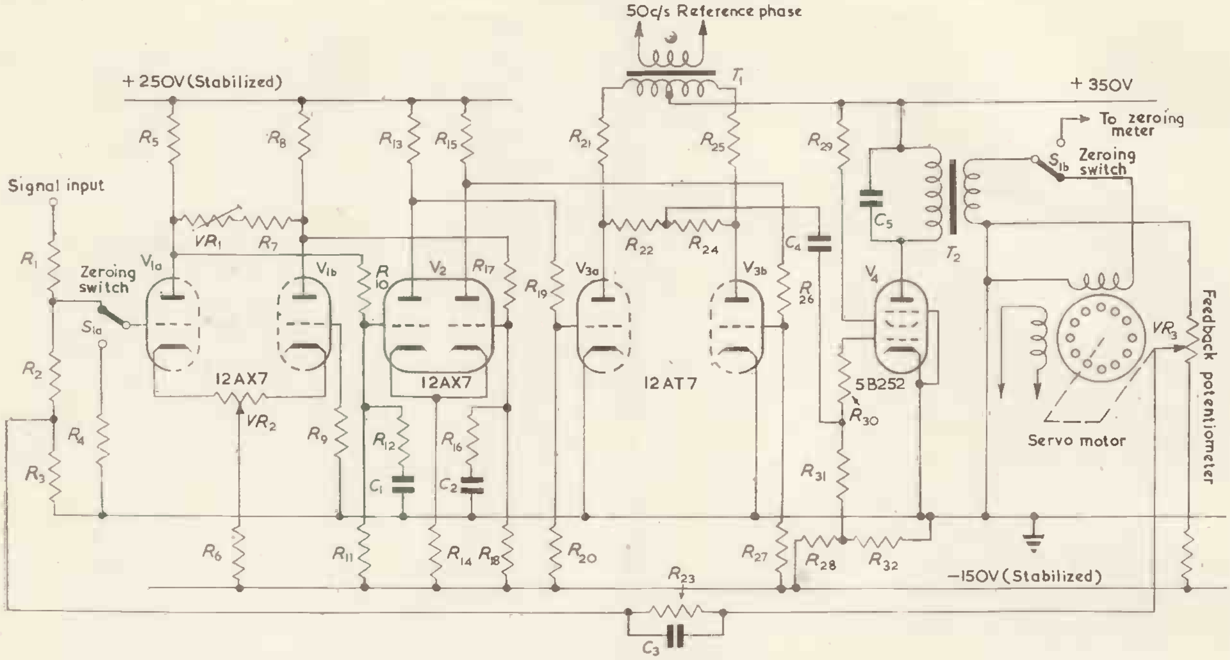


Fig. 4. Motor driving amplifier

selector. This voltage is fed into the "A amplitude servo" which is coupled by means of a lead screw to the writing pen above the graph paper. The motor driving amplifier is shown in Fig. 4.

The first two stages form a conventional d.c. amplifier with pre-set control VR_1 used for gain adjustment. V_3 is a very simple 50c/s modulator and V_4 an output stage which drives through a matching transformer an Evershed & Vignoles servo motor type FZ2/C. No velocity feedback is used because in view of a low inertia of moving parts the "self-stabilizing" characteristics of such a motor have been found sufficient to stabilize the system.

Derived Plot

Referring to Fig. 2, a graph of vector C and angle $\alpha + \gamma$ is required, where:

A_0 — is zero frequency or input voltage.

A — the output voltage at the test frequency.

B — is the difference between A_0 and A.

γ — the angle contained between A and B.

$$C = A'/B' = A/B \underline{|\alpha + \gamma} \quad \ldots\ldots\ldots (16)$$

(see Ref. 3, 4). To obtain the difference vector B, signals A_0 and A are fed to the summing amplifier (see Fig. 1). Both voltages are of modulated carrier form and can be applied to magslip 2, which is driven from the same shaft as magslip 1 and 3.

The resultant vector $A_0 - A$ is then resolved in exactly the same way as vector A. The derived plot turntable is driven through a differential gear from the harmonic response plot turntable so that the angle displayed is equal to the sum of α and γ.

To obtain the ratio of A/B, an I-pot driven by a servo ("A/B ratio servo" in Fig. 1) is used. The slider of the I-pot is moved to a position proportional to B and a signal of amplitude A is applied to it. The output, proportional to A/B is then used to position the pen on the derived plot turntable.

Control Circuit

The automatic action of the instrument is governed by the control circuit. It consists of a twenty-five position uniselector operated by a thyratron circuit and a four-position range-selecting switch. The functions of this unit are:—

(1) To provide a voltage proportional to test frequency which is fed into the velodyne driving the magslip generators. As the speed range of 500:1 is required in the instrument a simple two-speed gearbox is used, in which the gear change is achieved by the change of direction of rotation of the driving motor.

(2) To provide the variable time intervals, corresponding to the time required to stabilize the instrument at different frequencies.

Fig. 5. The two plotting tables

(3) To change filter constants.

(4) To adjust velocity feedback in servo 2 and 3, as mentioned before.

(5) To provide frequency markings in the form of short transients introduced to the pen drivers at each fifth step.

(6) To operate a visual frequency indicator.

It is possible to stop both plots at any time, for instance, on pressing of which the input to the amplifier becomes connected to earth and the output to the large centre-zero meter on the front panel.

Stabilized d.c. and heater supplies are available at the front panel of the instrument for connexion of any auxiliary apparatus.

Performance

The angular accuracy of both plots is 2°; the magnitude

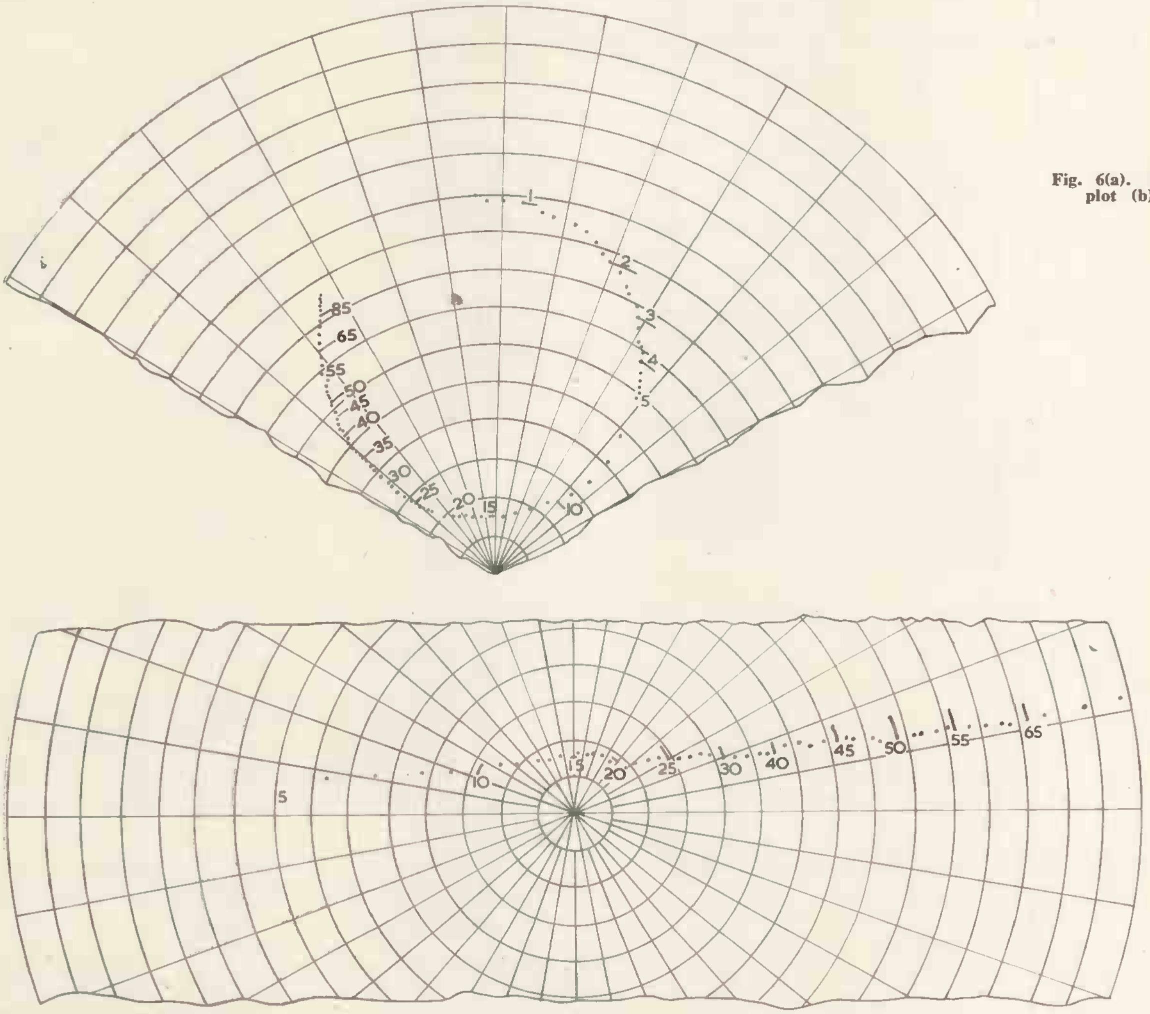


Fig. 6(a). Harmonic response plot (b) Derived plot

to see what effect the change of some constants in the servo under test has on the plots.

Layout and Construction

The instrument is housed in a double 19in rack 4ft 4in high. Both plotting tables, to take 30cm polar diagram paper, are mounted on top (Fig. 5). A pen is moved across the table by a lead screw. The pens are normally not in contact with the paper and are actuated only when the instrument changes to a new frequency so that the plot is obtained in the form of a series of dots and dashes marking each fifth frequency step. At the back access is available to a number of tags situated on the chassis and interconnecting cableforms, thus facilitating fault finding.

All amplifiers are provided with spring-loaded switches

accuracy is ± 2 per cent for the harmonic response plot and ± 3 per cent for the derived plot.

Two specimen diagrams are shown in Fig. 6. These were obtained with a bridged-*T* filter, used as a test network.

Acknowledgments

The author wishes to thank the Management of De Havilland Propellers Limited and Winston Electronics Limited for permission to publish this article.

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An Outline of

An Electronic Arithmetic Unit

By W. Woods-Hill*,

Since the first acceptance of electronic calculators into the business world, the call has been for greater reliability and a means of automatically checking calculations. The need is greater in electronic machines than in their mechanical counterpart because of the very much greater output per unit-time. In this article an electronic arithmetic unit is described and a method of checking calculations is detailed.

An arithmetic unit was required which could perform at least the following:

$A \times B \pm D = C$ and check.

The "and check" indicates that the calculation must be verified and proved correct before the answer leaves the machine.

The factors were to be supplied from punched holes in a Hollerith card in decimal form and the answer punched back into the same card also in decimal.

The cards were to feed at 100 per minute (on a Type 237 Gang Punch), and the time available for the calculation was the gap between cards, i.e. 90msec.

The system of multiplication used is interesting, and for obvious reasons called "Doubling and Halving." To multiply 7 × 3, set up the 7 and halve it consecutively, discarding anything less than one,

i.e. 7

$\frac{7}{2} = 3(\cdot 5)$ 3

$\frac{3}{2} = 1(\cdot 5)$ 1

$\frac{1}{2} = 0(\cdot 5)$ 0

then set up the 3 beside it and double it consecutively,

i.e.

7	3	
3	6	(= 3 × 2)
1	12	(= 6 × 2)
0	24	(= 12 × 2)

Now inspect the digits in the left-hand column, starting at the top, and every time the digit is an odd number, take the digit in the right-hand column and put it down. Add up this new column,

i.e.

7	odd	3	3
3	odd	6	6
1	odd	12	12
0	even	24	
			21

i.e. 7 × 3 = 21

"0" is looked upon as an even number. This will work with any notation, including £ s. d., and is why it is used here.

* The British Tabulating Machine Co. Ltd.

Those familiar with binary code will realize that this is simply a way of extracting the binary terms in a decimal number and doing binary multiplication by column shift in the scale of 2. For example:

7 × 3 = 21

```
7 =     111
3 =     011
        ---
        111
       111
      000
      -----
      10101  =  21
```

i.e. 16 + 0 + 4 + 0 + 1 = 21

The great point to be got out of all this is that to double or halve a number expressed in binary requires but a shift of one binary place.

We have here the five components needed to make a multiplier (Fig. 1).

(1) Register *A* holds the multiplier numbers to be halved.

(2) Register *B* holds the multiplicand numbers to be doubled.

(3) Register *C* receives the product numbers from *B* before they are doubled, and by means of box ADD accumulates them.

The fifth component is the O.E.D. box (odd-even detector), which only allows numbers to go from *B* to *C* when *A* is odd.

Shifting Registers

Consider register *B*. Each square in Fig. 2(a) is an Eccles-Jordan trigger. The values allocated to these triggers are 8, 4, 2, 1 across the top (*X* axis) and units—hundreds—thousands—etc. up the side (*Y* axis).

That means that the number 325 would be written:

		8421
3	=	0011
2	=	0010
5	=	0101

In the register it would cause the triggers marked *X* (Fig. 2(a)) to be ON and all the rest OFF. (Note that the pattern is the same as the binary equivalent above.)

Now register *B* (like *A* and *C*) is a "shifting register." That is to say, a single pulse applied to all the triggers together will cause the "pattern" representing 325 to shift down one place along the *Y* axis, so that the "5" drops off the end, and the "3" and the "2" move down one place.

The number now reads 32 (Fig. 2(b)). If four connexions had been made from the bottom to the top of the register (one to each binary column), then the "5" would not have been lost but would have "fed back" and appeared at the top. (Fig. 2(c).)

Thus, if the right number of pulses were applied to the register (in this case 10), the pattern representing 325 would have moved right round the register, one step per pulse, to come back to its starting position. The system of presenting numbers just described is known as "sequential parallel."

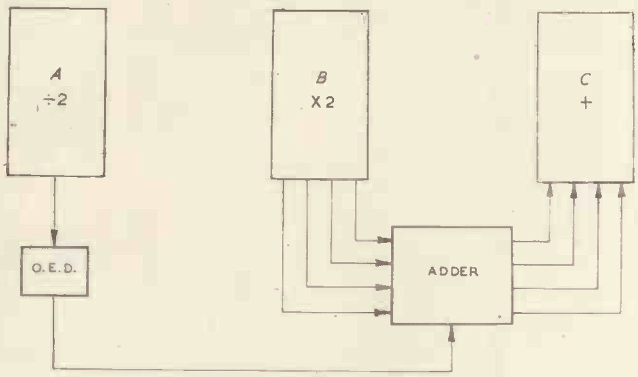


Fig. 1. Basis of multiplier

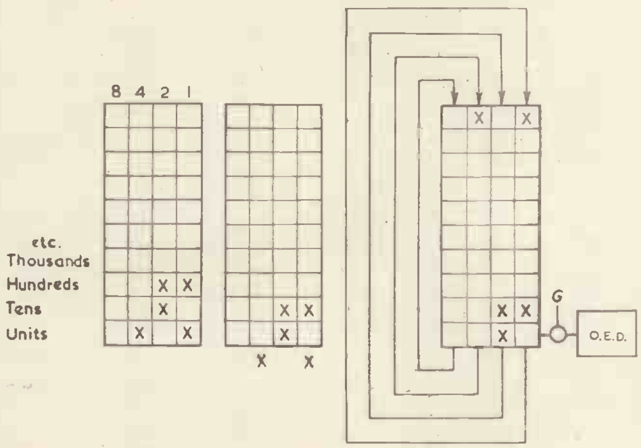


Fig. 2. Shifting registers

TRANSFER

To transfer a number from register *B* to register *C*, one simply needs to take four connexions across and couple them to *C*'s feedback loop.

Pulsing both registers through one cycle (10 pulses) will cause the number 325 to appear in register *C* while still retaining it unchanged in *B*.

This is, of course, due to *C* being unable to distinguish between a stimulation from another register and its own feedback loop.

If the registers are cycled again, nothing further will occur and 325 will simply circulate round both registers.

If it is required to repeatedly add 325 to *C* from *B* so that after each cycle it reads:

325
650 (+325)
975 (+325)
etc.

then an ADDER must be inserted into the transfer connexions. This is the fourth component in Fig. 1, and will be described later.

O.E.D.

Component five, the O.E.D. (odd-even detector), is a very simple device doing just what its name implies, and consists of nothing more than a trigger connected by a gate (*G*) (Fig. 2(c)) to the lowest binary-decimal position of the register *A* in such a way that if the "1" trigger of the "units" denomination is "on" the O.E.D. will also come on. This will only occur, of course, if gate *G* is momentarily opened.

It can be shown that if ever this "1" trigger is ON when a number is resting in its "at home" position, then that whole number is odd. Example:

8 = binary 1000 = even

5 = binary 0101 = odd.

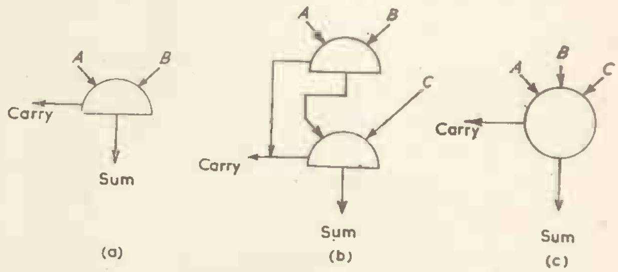


Fig. 3. A half adder, two half adders and an adder

Therefore opening gate *G* at the correct time will cause the O.E.D. to "remember" if the number was odd or even. The voltage from this O.E.D. trigger is used to prevent, or allow, transfer (from *B* to *C*, Fig. 1).

COMPONENT 4: THE FULL ADDER

The full adder is built up of a series of adders which in turn consist of two ½ adders. This adder (Fig. 3(a)) must be capable of doing the following binary addition:

	A		*B*		SUM	CARRY
Binary	0	+	0	=	0	0
,,	1	+	0	=	1	→ 0
,,	0	+	1	=	1	→ 0
,,	1	+	1	=	0	→ 1

and can handle two inputs *A* and *B*. If a third input is to be handled (incident carry) then two ½ adders in cascade can be used connected as shown (Fig. 3(b)). This will handle:

A		*B*		*C*		SUM	CARRY
0	+	0	+	0	=	0	→ 0
0	+	0	+	1	=	1	→ 0
1	+	0	+	1	=	0	→ 1
0	+	1	+	1	=	0	→ 1
1	+	1	+	1	=	1	→ 1

or any permutation of *A*, *B* and *C* to give these answers. An adder (2 × ½ adder) is abbreviated to Fig. 3(c) for simplicity.

If it is wished to add two numbers from register *B* and *C* and put the resultant back into *C*, then it will be necessary to handle four pairs of binary inputs, four outputs (sums) and a grand carry.

How this is done is shown in Fig. 4. C.M. is the Carry Memory Trigger needed to store a grand carry. A grand carry, of course, is a carry between vertical-denominations requiring a digit time delay, as against a horizontal binary carry requiring no delay.

The four adders receive the four possible pairs of inputs simultaneously, sums them, and sets a carry if the sum exceeds 16.

The device, as it is, would be excellent to handle numbers in the scale of 16 but quite useless for the scale of 10, the main trouble being that the carry occurs at 16 instead of 10. This can be solved by building a "10 or

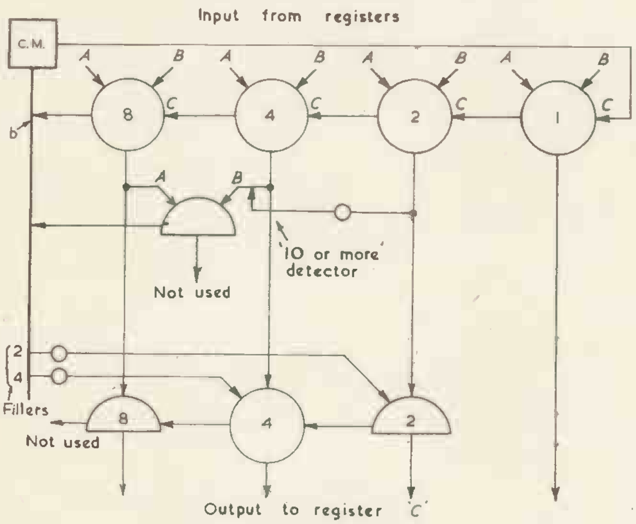


Fig. 4. A full adder

more" detector which would stimulate a false carry as soon as a sum greater than 10 was produced by the incident digits. This is quite simple, and consists of a ½ adder connected to the sum line value 8, 4 and 2 (Fig. 4), so that if the sum contains an 8 and a 2, or an 8 and a 4, or an 8 and a 4 and a 2 (these are the only combinations which can exceed 10), then the carry bussbar (Fig. 4) is stimulated.

This solves the carry problem and will cause a "1" to be put in the next denomination higher (later), but this denomination now has a value of 10 by stipulation and the carry was stimulated by any number between 10 and 16.

If it can be arranged to "pay back 6" into the present denomination at the same time as storing a carry for the next, then the conditions will have been fulfilled and a carry will indeed be worth 10.

This is done by inserting a further bank of adders in the SUM lines 2s, 4s, 8s, (Fig. 4) and causing the carry bussbar to stimulate a 2 and a 4 filler input to these every time it is energized. This causes 6 to be added to whatever the sum lines were showing.

For example, 5 + 8 = 13, "Greater than 10 detector" stimulates carry bussbar, carry bussbar sets carry memory and causes 4 and 2 (6) to be added to the sum lines via a second bank of adders, i.e. 13+6=19. 19 in the scale of 16 is 3 and a carry: this second carry is lost (not

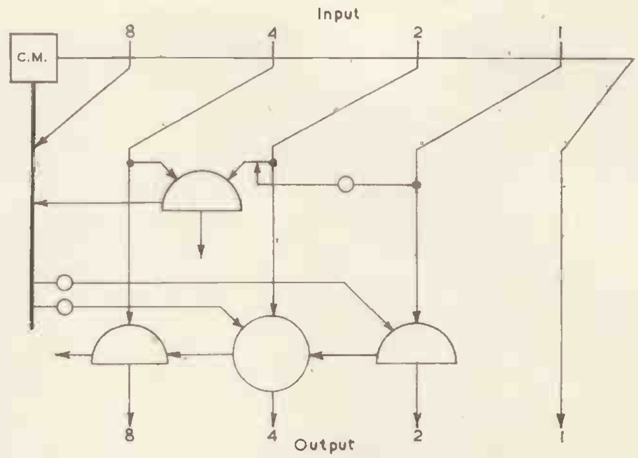


Fig. 5. A doubler

connected). Net result: 5 + 8 = 3 → 1 carried.

All this may sound rather laborious, but due to the d.c. nature of the ½ adders all is over in less than ½ a pulse time. It should be noted that some of the adders in the second bank are only receiving two inputs and are therefore only ½ adders. The largest incident digits to this full adder would be 9 + 9 = 18.

Doubler

As previously mentioned, to double a number expressed in binary one needs but to shift one place to the left; i.e. binary 0011 = 3, shift one left: binary 0110 = 6. Therefore, if in register *B* the feedback loops were misconnected so that the 1 fed back to the 2, the 2 to the 4, the 4 to the 8, the 8 to a carry memory, then the number would be doubled simply by applying 10 pulses to cycle it. Unfortunately, the carry is in the scale of 16 and all the remarks about numbers exceeding 10 applicable to the full adder apply here and the same circuits must be used, but this time in the feedback loop. Fortunately, the bulk of the circuits (the first bank of adders) can be left out because there is only one set of incident digits, not two.

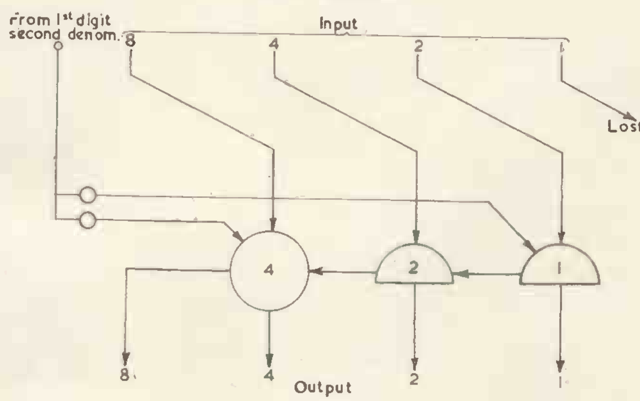
Fig. 5 shows the same sort of circuits as the full adder with the first bank missing and the input wires crossed to the left to produce doubling.

The largest incident digit to be doubled would be 9 × 2 = 18.

Halver

To halve, simply cross the connexions to the right

Fig. 6. A halver



(Fig. 6). There is no need for rationalizing circuits because no number halved can exceed 9.

One bank of adders is required, however, because halving an odd number (this means any individual denomination, not the whole number) necessitates the adding of 5 into the denomination below. Example:

$$\frac{50}{2} = 2(5) \text{ in binary/ten.}$$

Second Denom.	0101 =	5	÷ 2 =	0010 =	2
First Denom.	000 =	0		0101 =	5

This infers a carry of 5 downwards every time a binary digit is detected in the 1s column of the second denomination. As shown in Fig. 6, this is achieved by tying a detector to this digit which, when energized, causes 4 and 1 to be added via the adders to the denomination being fed back, after it was halved, i.e., beyond the point where the connexions cross.

Multiplication $A \times B$

At this point we can elaborate on Fig. 1 and follow the sequence of events when multiplying 9 × 4 = 36. In Fig. 7, the 9 is standing in the multiplier register A, 4 in the multiplicand B, and the product register C is empty. By momentarily opening G_1, the O.E.D. can be set or unset before each cycle. All registers are pulsed together 10 times each cycle. Before the start of each cycle the registers will look like this:

REGISTER		A	B	C	
Cycle	1	9	4	0	odd transfer
„	2	4	8	4	even
„	3	2	16	4	even
„	4	1	32	4	odd transfer
„	5	0	64	36	even
„	6	0	128	36	even
	etc.	etc.	etc.	etc.	etc.

The following is the state of the binary patterns at the end of each cycle, showing in more detail what happens during each cycle.

	A		B		C	
1	(9)	1001	(4)	0100	(0)	0000

Test A for odd-even by gate G_1. Odd, so O.E.D. sets and opens gate G_2 (B to C): cycle once.

	A		B		C	
2	(4)	0100	(8)	1000	(4)	0100

Test A. Even, so O.E.D. unsets. Gate G_2 closed: cycle once.

	A		B			C	
3			(16)	(1)	0001		
	(2)	0010		(6)	0110	(4)	0100

Test A: even. O.E.D. stays unset. G_2 closed: cycle once.

	A		B			C		
4			(32)	(3)	0011			
	(1)	001		(2)	0010		(4)	0100

Test A: odd. O.E.D. set. G_2 opens: cycle once.

	A		B			C		
5			(64)	(6)	0110	(36)	(3)	0011
	(0)	0000		(4)	0100		(6)	0110

Test A: even. O.E.D. unset. G_2 closed.

(The numbers in parentheses are the decimal equivalent of the binary numbers.)

Any further cycling of the machine after A has reached zero cannot affect C as G_2 never opens again.

It is obvious from the above that the length of time

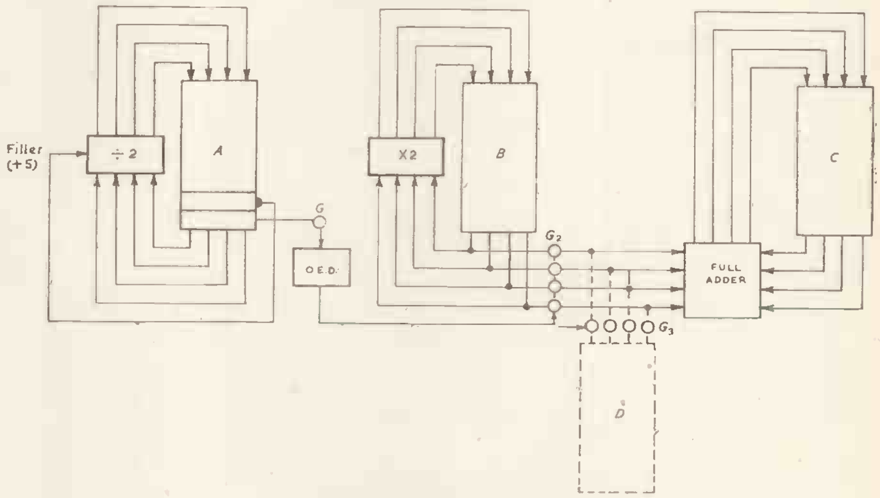


Fig. 7. An arithmetic unit

taken to multiply a number will be governed by the size of the multiplier (A) and quite independent of the multiplicand (B). As a matter of interest, the number of cycles required to ensure that a 10-digit number reading all 9's is "halved" below "1" is 36. The length of time it will take, therefore, to multiply 10 digits by 10 digits to give a 20 decimal digit answer is 18msec with a pulse repetition rate of 20kc/s. These 20 digits exceed the capacity of the counter C described, and is of academic interest only.

Circuits

The three basic circuits used are:

(1) The Shifting Register[1],

(2) The Gate, and

(3) The Half Adder.

(1) Shifting Register

Fig. 8 shows two stages of a shifting register V_1 and V_2. MR_1 and MR_2 are two diode gates connected to the left-hand and right-hand anodes of trigger V_1. If V_1 is 'on,' then gate MR_1 is primed; if V_1 is 'off,' gate MR_2 is primed. MR_1 is connected to the input of V_2 trigger so that a pulse through it will set V_2, and MR_2 will unset V_2. Pulsing the common pulse line P will therefore cause V_2 to assume whatever state V_1 might be. V_2 itself will influence V_3, and V_3 will influence V_4, and so on.

Thus any trigger will be made to assume the state of

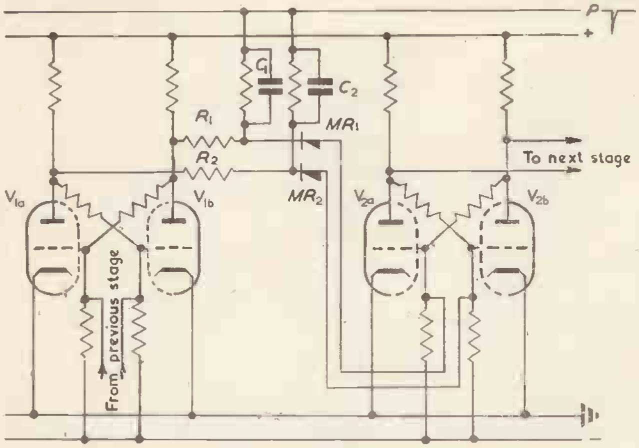


Fig. 8. A shifting register

its predecessor. The fact that any stage may be changing while influencing the next is taken care of by the relatively long time-constant made by R_1C_1, R_2C_2, "remembering" what state the trigger was in for the duration of the action pulse (5μsec).

(2) Gate

Fig. 9 shows a pentode gate connected to a trigger. The grid voltage of a trigger varies between + 2 – 30V, depending upon whether it is 'on' or 'off' respectively.

The anode voltage of the pentode will not vary, whatever state the grid is in, as long as —30V is maintained on the suppressor. Bringing the suppressor up to earth will allow electrons to reach the anode and an amplified inverted version of the grid condition will appear thus, i.e.

Grid Voltage	Anode (supp. +2) Voltage	Anode (supp. –30) Voltage
+ 2	+ 50	+160
–30	+160	+160

One trigger connected to the suppressors of four pentodes can therefore control the four output lines of a register.

(3) Half Adder

A digit in this machine is a fall in voltage from +160 to +50V (negative going 110V swing). Because these ½ adders are to be connected in chains, their outputs must be of the same polarity and swing as their input requirements.

How this is done is shown in Fig. 10.

With no incident digit (i.e. *A* and *B* at +160V), both halves of V_1 are conducting heavily because both grids are trying to go positive. Both anode voltages are therefore low, and when this condition is transmitted to V_3, an invertor, via R_1R_2 and R_3R_4, results in V_3's grids being negative. Both invertor anodes are therefore cut off and at a high voltage (= 0), i.e.

A		*B*		SUM	CARRY
0	+	0	=	0	→ 0

With one incident digit *A* = +50V, *B* = +160V; the grid of V_{1a} is cut off, because it is near the negative end of the voltage divider, but V_{1a} is still positive and its anode therefore still low. This one anode low and one high condition of V_1 is interpreted at the invertor V_3 anodes as a low "sum" +50 and a high "carry" +160, i.e. 1 + 0 = 1 → 0.

With two incident digits (*A* and *B* = +50 V), both grids of V_1 are cut off and both its anodes should be high, but V_2 anode is also coupled to V_{1b} anode, and because V_2 under these conditions is conducting, this latter anode will remain low and clamp V_{1b}. This condition is interpreted by the invertor as "sum" high +160 and "carry" low +50, i.e. 1 + 1 = 0 → 1.

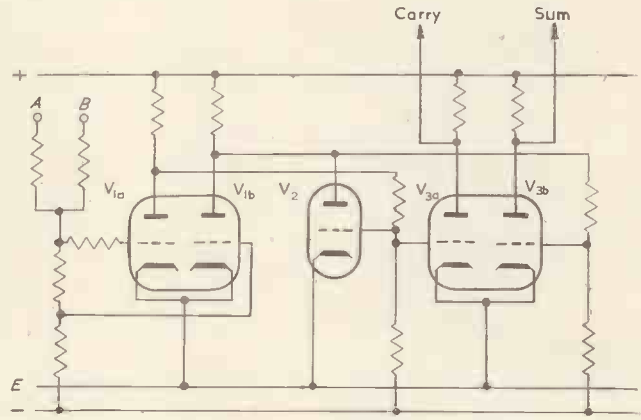


Fig. 10. A half adder

Why V_2 should choose to clamp V_{1b} anode under these conditions is because its grid (V_2) has been connected by a voltage divider chain R_5R_6 to V_{1a} anode, which until this moment has always been low, maintaining the grid of V_2 below cut-off.

The interesting thing about these adders is that they are entirely d.c. connected, so that statically inserting dummy digits at the input of the full adder (causing the voltage to fall from +160 to +50V) one can inspect the output by means of neons connected for the purpose; this enables a check to be done with the machine at rest and see that the adder is doing its sums correctly.

(4) ± *D* Register

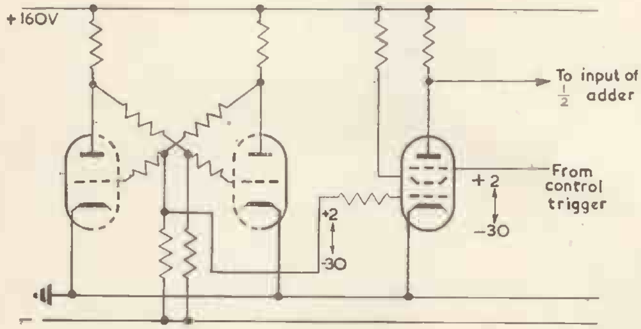
If a further register *D* (shown dotted Fig. 7) is added and connected by gates (G_3) under separate control to the input of the full adder, then when multiplication is finished the contents of *D* can be "rolled" on top of the product standing in *C* by simply opening G_2 and cycling once.

D can be subtracted from *C* by setting *D* up in complement and doing as above.

"And Check"

The system of checking to be described is applicable to any arithmetic unit capable of multiplication, and will

Fig. 9. A gate



be described in general terms without reference to this sequential parallel machine.

Basically it involves doing the multiplication twice and comparing the two results to see if they agree. This in itself is of no great value because a fault (such as an open-circuit heater) will produce the same wrong answer both times and agree.

On the other hand, if the factors to be multiplied are altered in some way the second time, some way which will produce the same product and yet cause the machine to go through a different routine to get there, then a fault such as mentioned will produce a wrong answer both times, but the wrong answer the second time will not be the same as the first and will therefore not agree.

One system proposed used "factor reversal," i.e. exchange the number in the multiplier for that of the multiplicand. This falls down if the two factors happened to be the same, say 4444 × 4444, and anyway cannot be used if the scales of notation are different, 33·7 × £9 19 11. Some systems do not deal with the arithmetic of the machine, but concentrate on checking the digits in the counters: even or odd summing of the digits within counters; biquinary (always two elements must be in use); automatic periodic zero test; pilot multiplication. This last is really a periodic arithmetic check.

All these give some measure of safeguard, but very few, if any, are capable of checking to see if the answer is right *after it has been punched out.*

Stress is laid on this because one of the major sources of non-repetitive faults can be traced to the coupling of the electronic portion to the output organ.

The mysterious quality of some of these faults can be readily understood when the relative speeds of the two machines are compared — electronic equipment and mechanical card feed.

The electronics might repeat a duty cycle 1 000 times in one second, so that a marginal fault will reappear quite quickly.

A mechanical device will take many minutes, if not hours, to do 1 000 passages; and if, as is quite often the case, a combination set of circumstances is required between electronic and mechanical to produce the fault, it may come once and never return. Further, a one-thousandth of an inch unexpected movement on a cam may equal 5 or more complete pulse times of the electronics, and thereby make the same fault produce different evidence each time.

To try to take care of all these factors, the following system was developed.

The factors are read from the card twice: once as the card enters the machine and once as it leaves. Two completely separate sensing mechanisms (brushes) and paths are provided to the electronic portion. The first time, the arithmetic operation is done quite normally, and the answer punched out in the card, and not retained. As the card leaves the machine, it is sensed once again and the factors are read back again to the electronic portion *along with the answer just punched.*

The arithmetic operation is checked by doing the multiplication again, but this time the multiplier factor is increased by a factor of 10 (col. shift up once) and the multiplicand decreased by a factor of 10 (col. shift down) to compensate, and in that way give the same correct answer as the first time. This infers a means of dividing by 10 if the multiplicand factor is expressed in a non-decimal scale, i.e. £ s. d., T. Cwt. Qrts.

The first answer just picked up off the card is now compared with this new answer (by rolling in complement and adding "one"), and if they agree the card is allowed to drop into the hopper.

The following example will help to illustrate how the machine handles this check.

Take the case previously mentioned of 4444 × 4444. This now looks like this: 44440 × 0444·4, and gives the same correct answer.

Suppose it had a fault in counter three and read in like this:

First Multiplication:	*A* 04444	×	4344·0	=	19304736
Second Multiplication:	*B* 44440	×	0344·4	=	15205136

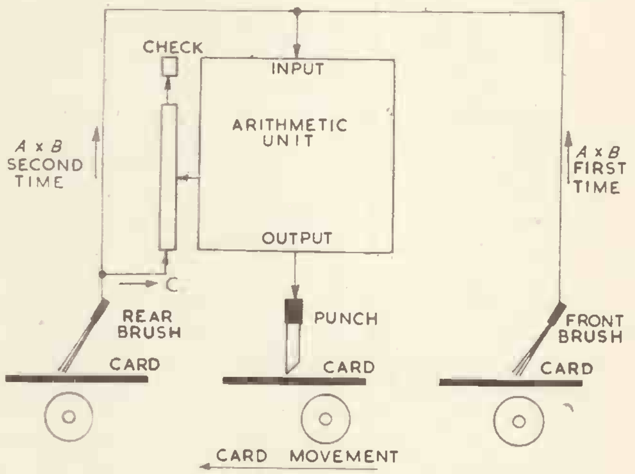


Fig. 11. A print unit ($A \times B = C$ *and check*)

Line *A* does not give the same answer as line *B* and would not check even though the fault is consistent.

Supposing the fault was more subtle, and consisted of a faulty transfer during multiplication on one cycle only, or where there was a special combination of numbers in the multiplicand, or both.

The multiplier is repeatedly halved, and would halve (on the second run) quite differently than it did the first. It would require more cycles (it is now a 5-digit number) and the transfers will occur on different cycles; further, the multiplicand doubling up is doubling from different counters, which in all leaves very little chance for compensating errors.

Finally, when £ s. d. are used, because a tenth of, say, £1 11s. 1d. is 3s. 1·3d., the number in the multiplicand counter bears no resemblance to the one used in the first multiplication.

The same principles could be applied to a print unit by signalling back from the type bars.

The block schematic of such a machine is shown in Fig. 11.

Acknowledgments

The author wishes to offer his thanks to Mr. L. Knight, Mr. A. Trussell, and the members of the Electronic Laboratory, for their contributions to this project, and The British Tabulating Machine Company Limited for permission to publish this article.

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The Display of Transformer Magnetizing Ampere-Turns on a Cathode-Ray Oscilloscope

By M. E. Bond*

If in a transformer the current is not sinusoidal, or if the working conditions are altered by disconnecting the associated circuits, then the normal method of finding the magnetizing ampere-turns is not satisfactory. In this method, small resistors proportional to the number of turns are placed in series with each winding. The resultant voltages are then correctly mixed and displayed on a cathode-ray oscillograph. The application of this method to the line output transformer of a television receiver is described.

IT is often desired to measure the operating conditions of a transformer core. To achieve this it is necessary to measure the magnetizing ampere-turns, various physical constants, and have a knowledge of the *BH* curve of the material concerned. When the magnetization is sinusoidal, the magnetizing ampere-turns are normally found by direct measurement of the no-load current through the transformer. If, due to overloading or a non-sinusoidal applied voltage, the current is not sinusoidal, or if the current under working conditions has a waveform different from that of the no-load condition, then it is necessary to measure the magnetizing ampere-turns by taking the difference of the instantaneous ampere-turns in each winding at many points during the cycle and reconstituting graphically the magnetizing ampere-turns cycle. This involves the measurement of the separate current waveforms in each winding and accurately determining their relative time relationship. The resultant magnetizing ampere-turns can then be calculated and plotted. An easier method is to arrange for the direct display of the magnetizing ampere-turns on a cathode-ray oscillograph.

Consider a transformer with several secondary windings. The instantaneous primary current is i_1 and the instantaneous currents in the other windings are i_2, i_3, etc., respectively. The number of turns on each winding is N_1, N_2, N_3, and so on.

Then the magnetizing ampere-turns =

$$N_1 i_1 - (N_2 i_2 + N_3 i_3 + \ldots) \quad (1)$$

If a small series resistor is placed in each circuit, then the voltage across these resistors will depend on the value of the resistance and the current flowing. The resistors in each circuit are chosen so that the resistance is proportional to the number of turns, thus (see Fig. 1):

$$N_1 = KR_1 \qquad N_2 = KR_2 \qquad N_3 = KR_3 \qquad \text{etc.}$$

The voltage developed across each resistor will then depend on the current flowing and the constant K,

$$v = iR = \frac{iN}{K}$$

or $iN = vK$ where K is expressed in turns per ohm. So that the equation (1) can now be rewritten:

Magnetizing ampere-turns $= (v_1 - v_2 - v_3 - \ldots)K$

Thus if these four voltages can be mixed, the resultant, when multiplied by K, will equal the magnetizing ampere-turns. It is essential for this that one side of each winding should be earthy and that the right phasing of each voltage is obtained, the voltage from the primary being in opposite phase to those from the secondary. This can often be arranged by inserting the resistors where either the winding or its load is connected to earth. The voltages are mixed by applying them to a network of carefully matched resistors (Fig. 2).

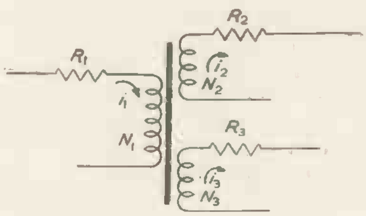

Fig. 1. Transformer with series resistors

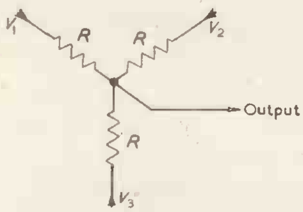

Fig. 2. Mixing circuit

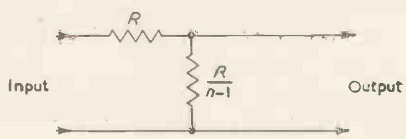

Fig. 3. Circuit seen by each input

If these resistors R are very much larger than R_1, R_2, R_3, etc., then for each input the circuit approximates to Fig. 3.

Thus the voltage fed to the oscilloscope $= V/n$ where n is the number of windings and $V = (V_1 - V_2 - V_3 - \ldots)$.

Hence the magnetizing ampere-turns $= Vnk$ where n and K are known and V can be measured on the oscilloscope tube face.

Where direct instead of a.c. coupling to the oscilloscope is possible, the input to the oscilloscope can be shorted intermittently to show zero level as a horizontal line on the tube face.

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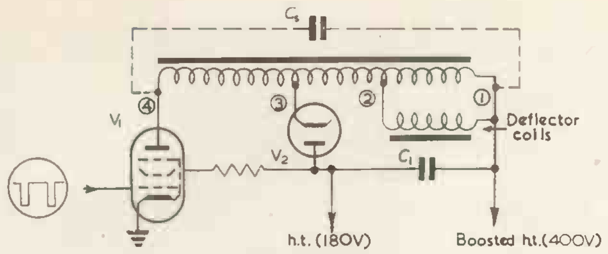


Fig. 4. Basic circuit of television receiver line output stage

This method has been used to find the operating conditions of the ferroxcube core of the line output transformer of a television receiver.

Basically the circuit is as shown in Fig. 4.

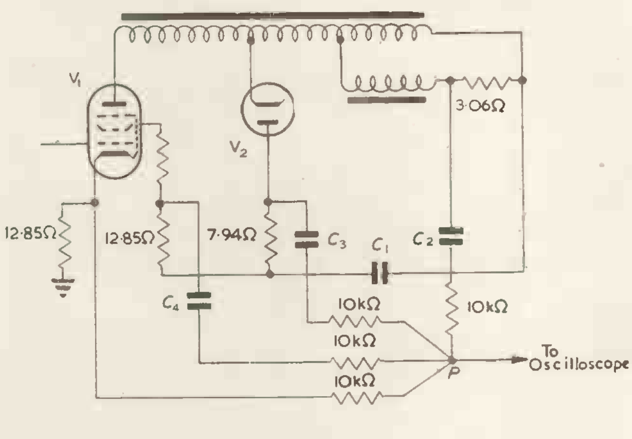
The waveform applied to the control grid of valve V_1 switches this valve on or off. When the valve is switched on, a current flows through it and the transformer so that a potential difference exists across the transformer. The tap (3) does not fall below the h.t. line potential, and a reasonably steady voltage exists across the transformer during this period. This causes an exponentially increasing current to flow through the deflector coils.

When V_1 is made non-conducting, the current then flowing in the circuit charges C_s the self-capacitance, to a very high voltage opposite in direction to the original voltage across the transformer, raising point (4) to a very high positive potential. The current through the deflector coils changes direction very rapidly.

C_s now starts to discharge in an oscillatory manner through the transformer. The potential at (4) drops rapidly until (3) is negative relative to the h.t. line. Valve V_2 then conducts and clamps (3) at the h.t. voltage, causing point (1) to rise to a voltage higher than that of the h.t. line. C_1 is thereby charged. When the charge from C_s has been transferred to C_1, V_1 is again switched on and maintains the voltage gradient across the transformer. The current through the deflector coils increases exponentially from its negative peak at the end of the flyback to provide the receiver line scan. The whole process is repeated at a frequency of the order of 10kc/s.

This has only described the basic principles of the circuit. In practice the circuit would be modified, for instance, to provide a linear and not an exponential scanning current. It has shown that no part of the circuit can be disconnected without materially affecting its working. Measurement of the core working conditions cannot therefore be made in the normal way.

Fig. 5. Circuit adapted for measuring magnetizing ampere-turns



From the winding data it is found that section (1) to (2), (N_1), of the transformer has 180 turns, section (1) to (3), (N_2), has 470 turns, and section (1) to (4), (N_3), has 720 turns. It has also been found by experiment that the maximum resistance that can be added to the deflector coil circuit without appreciably altering its working is of the order of 3Ω. Another difficulty is that V_1 anode current waveform is not readily available. This can, however, be obtained by subtracting the screen waveform from the cathode waveform in the same manner as the transformer magnetizing ampere-turn waveforms would be subtracted.

If N_1 = 180t and a resistor of 3·06Ω is inserted in this circuit, then K = 59·2 turns per ohm

$$\text{so that } R_2 = N_2/K = 7{\cdot}94\Omega$$
$$\text{and } R_3 = N_3/K = 12{\cdot}85\Omega.$$

These are connected in the circuit as shown (Fig. 5). The mixing resistors are 10kΩ because this value is of the order of 1 000 times R_1, R_2 and R_3, but is not high enough with the stray capacitances to limit the upper frequency response of the oscilloscope input circuit. Capacitors C_2, C_3 and C_4 are included to isolate the oscilloscope from the direct h.t. and boosted h.t. voltages.

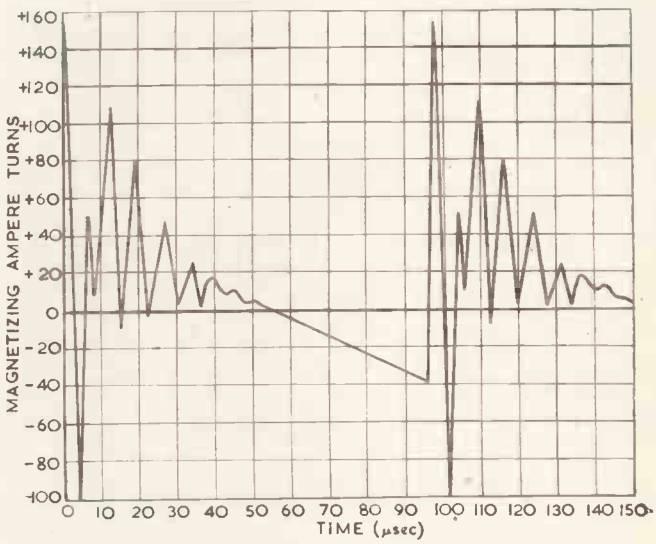


Fig. 6. Magnetizing ampere-turns of line output transformer

The sensitivity of the oscilloscope reckoned in r.m.s. sine wave values is 0·1V/cm. Thus the scale for magnetizing ampere-turns is:

$$0{\cdot}1 \times 2\sqrt{2} \times 4 \times 59{\cdot}2 = 67 \text{ ampere-turns/cm.}$$

The oscilloscope time-base is synchronized with the television line scan waveform. A drawing of the results is shown (Fig. 6), and from this the operating conditions of the ferroxcube core can easily be determined. In this case much tedious work is saved by using this method, as the accurate relative time location of the several peaks in the waveforms and their subsequent subtraction would be very difficult.

Acknowledgments

The author wishes to thank the Directors of Philips Electrical Ltd. for permission to publish this article.

The Tapped Bridged-T Rejector

By D. J. S. Westwood*, B.Sc.

Two forms of tapped bridged-T rejectors are developed. The balance equations are determined in each case and approximations applied to reduce the solutions for practical use. By differentiation of certain equations the maximum values of the balancing resistors are established.

IN television receiver practice the tapped tuned circuit rejector shown in Fig. 1(a) is often used in series with coupling elements in i.f. or r.f. amplifiers to obtain rejection of the sound or adjacent sound channel. The symmetrical bridged-*T* rejector shown in Fig. 1(b) is also occasionally used. The inherent resistances of the coils in Fig. 1 are omitted for simplicity in the diagrams. It is sometimes desirable to combine the features of these circuits to obtain a rejector which gives a very large attenuation at the rejection frequency, but does not appreciably affect the passband of the coupling into which it is inserted. The form of this

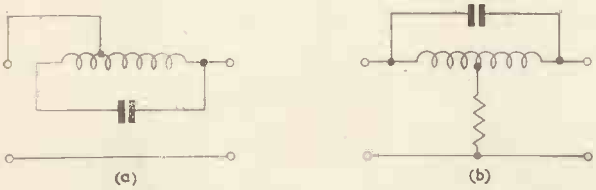


Fig. 1(a). Tapped rejector (b). Symmetrical bridged-T

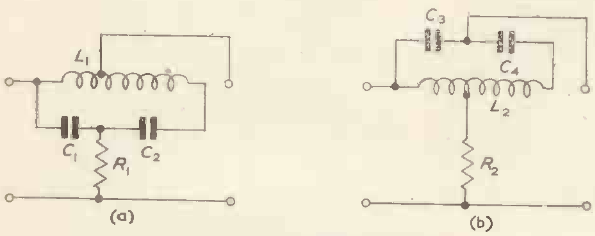


Fig. 2. Tapped bridged-T rejectors

circuit is shown in Fig. 2(a) and an alternative in Fig. 2(b) with the bridging resistor tapped into the inductive arm. The conditions for infinite attenuation will be established in both cases and approximations given which yield simple formulæ for practical use.

The method employed is to change the network into the form of Fig. 3 and, ignoring the impedances Z_1, Z_2, apply the stated conditions to the parallel arms. Fig. 4 is the circuit of Fig. 2(a) with the coil given an equivalent circuit and the inherent resistances added. Then the impedance of the upper arm is:

$$R_A + jX_A = R_3 + j\omega(L_3 + M) \quad \ldots\ldots\ (1)$$

The T to π transformation is applied to the lower arm and yields:

$$R_B + jX_B = R_4 - \frac{1}{\omega^2 C_1 C_2 R_1} + \frac{L_4 + M}{C_1 R_1} + j\,[\omega\,(L_4 + M) - 1/\omega C_2 - 1/\omega C_1] \quad \ldots\ldots\ (2)$$

Applying the conditions stated in Fig. 3, equating real and imaginary parts:

$$R_1 = \frac{1}{R_3 + R_4}\left(\frac{1}{\omega^2 C_1 C_2} - \frac{L_4 + M}{C_1}\right) \quad \ldots\ (3)$$

$$\omega^2(L_3 + L_4 + 2M) = 1/C_1 + 1/C_2 + R_4/C_1 R_1 \quad \ldots\ (4)$$

$R_3 + R_4$ may be replaced by $\omega L_1/Q$, where Q is the Q-factor of the whole coil and, since $L_3 + L_4 + 2M = L_1$:

$$R_1 = Q/\omega L_1\left(\frac{1}{\omega^2 C_1 C_2} - \frac{L_4 + M}{C_1}\right) \quad \ldots\ldots\ (5)$$

$$\omega^2 L_1 = 1/C_1 + 1/C_2 + R_4/C_1 R_1 \quad \ldots\ldots\ (6)$$

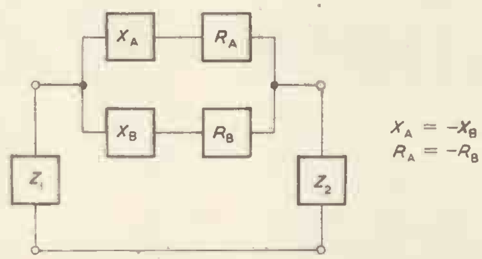


Fig. 3. Equivalent circuit and infinite attenuation conditions

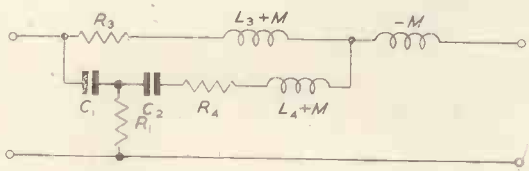


Fig. 4. Equivalent circuit of Fig. 2(a)

These equations give the required conditions and may be simplified by certain approximations. In practice R_4 can be small compared to R_1, so that the last term in equation (6) may be neglected. If close coupling is assumed between the two coil sections and if n_1 is the turns ratio of L_4 to L_3:

$$L_4 + M = \frac{n_1 L_1}{1 + n_1} \quad \ldots\ldots\ (7)$$

Let the total series tuning capacitance be C_s where

$$1/C_s = 1/C_1 + 1/C_2 \quad \ldots\ldots\ (8)$$

* *Ferguson Radio Corporation Limited.*

Equation (6) reduces to:

$$\omega^2 L_1 C_s = 1 \quad \dots \quad (9)$$

Then equation (5) may be written:

$$R_1 = Q/\omega C_1 = \left(\frac{1}{1+n_1} - C_s/C_1\right) \dots (10)$$

Thus for given values of Q, L_1, n_1, C_1, equations (8), (9) and (10) enable R_1, C_s and C_2 to be calculated. The maximum permissible value of R_1 may be found by differentiating equation (10).

$$dR_1/dC_1 = Q/\omega C_1^2\left(2C_s/C_1 - \frac{1}{1+n_1}\right) = 0$$

giving:

$$C_1 = 2C_s(1+n_1) \quad \dots \quad (11)$$

$$R_1 = \frac{Q}{4\omega C_s(1+n_1)^2} \quad \dots \quad (12)$$

Putting $n_1 = 0$ the symmetrical case is obtained

$$R_1 = Q/4\omega C_s \quad \dots \quad (13)$$

$$C_1 = 2C_s \quad \dots \quad (14)$$

The alternative circuit of Fig. 2(b) is developed in Fig. 5

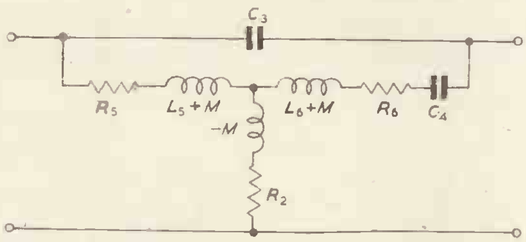


Fig. 5. Equivalent circuit of Fig. 2(b)

and may be treated in a similar manner. The top arm has the impedance:

$$R_A + jX_A = -j/\omega C_3 \quad \dots \quad (15)$$

Applying the T to π transformation to the lower arm:

$$R_B + jX_B = R_5 + j\omega(L_5+M) + R_6 + j\omega(L_6+M) - j/\omega C_4 + \frac{[R_5 + j\omega(L_5+M)]\,[R_6 + j\omega(L_6+M) - j/\omega C_4]}{R_2 - j\omega M} \dots (16)$$

Using the conditions in Fig. (3), equating real and imaginary parts:

$$R_2(R_5+R_6) = \omega M[1/\omega C_3 + 1/\omega C_4 - \omega(L_5+L_6+2M)] + \omega^2(L_5+M)(L_6+M) - 1/C_4\,(L_5+M) - R_5R_6 \quad \dots \quad (17)$$

$$R_2[1/\omega C_3 + 1/\omega C_4 - \omega(L_5+L_6+2M)] + \omega M(R_5+R_6) = R_5[\omega(L_6+M) - 1/\omega C_4] + \omega R_6(L_5+M) \quad \dots \quad (18)$$

It is appropriate to introduce approximations at this stage to reduce the complexity of the equations. Assuming, as before, that the coil has closely coupled sections with a turns ratio n_2 between L_6 and L_5, that Q is the Q-factor of the whole coil and effective resistance is proportional to turns:

$$M = \frac{n_2 L_2}{(1+n_2)^2} \quad \dots \quad (19)$$

$$L_5 + M = \frac{L_2}{1+n_2} \quad \dots \quad (20)$$

$$L_6 + M = \frac{n_2 L_2}{1+n_2} \quad \dots \quad (21)$$

$$L_5 + L_6 + 2M = L_2 \quad \dots \quad (22)$$

$$R_5 + R_6 = \omega L_2/Q \quad \dots \quad (23)$$

$$R_5 = \frac{\omega L_2}{Q(1+n_2)} \quad \dots \quad (24)$$

$$R_6 = \frac{\omega L_2 n_2}{Q(1+n_2)} \quad \dots \quad (25)$$

Equation (17) becomes:

$$R_2 = \frac{1}{(1+n_2)^2}\left[Qn_2/\omega C_3 + Qn_2/\omega C_4 - \frac{\omega L_2 n_2}{Q} - \frac{Q(1+n_2)}{\omega C_4}\right] \quad (26)$$

Equation (18) becomes:

$$1/\omega C_3 + 1/\omega C_4 - \omega L_2 = \frac{\omega L_2}{QR_2(1+n_2)^2}\left(\omega L_2 n_2 - \frac{1+n_2}{\omega C_3}\right) \quad (27)$$

Substitution from (27) in (26) for R_2 gives:

$$Qn_2/C_3^2 + Qn_2/C_3C_4 - Q/C_3C_4 - Q/C_4^2 = \omega^2(Qn_2L_2/C_3 + L_2n_2/QC_3 - QL_2/C_4 - L_2/QC_4) \dots (28)$$

Neglecting L_2n_2/QC_3 compared to Qn_2L_2/C_3 and L_2/QC_4 compared to QL_2/C_4

$$\omega^2 L_2 = \frac{C_3 + C_4}{C_3C_4} \quad \dots \quad (29)$$

Let the total tuning capacitance be C_T where:

$$1/C_T = 1/C_3 + 1/C_4 \quad \dots \quad (30)$$

Then:

$$\omega^2 L_2 C_T = 1 \quad \dots \quad (31)$$

From (26) putting $\omega L_2 = 1/\omega C_T$:

$$R_2 = \frac{Q}{(1+n_2)^2}(n_2/\omega C_3 - 1/\omega C_4 - n_2/\omega C_T Q^2) \dots (32)$$

Neglecting $n_2/\omega C_T Q^2$ compared to $1/\omega C_4$ and using equation (30):

$$R_2 = \frac{Q}{\omega C_3(1+n_2)^2}(1 + n_2 - C_3/C_T) \quad \dots \quad (33)$$

Equations (30), (31) and (33) enable C_4, L_2 and R_2 to be found for given values of Q, C_T, C_3 and n_2. The maximum permissible value of R_2 is found by differentiation of equation (33):

$$dR_2/dn_2 = \frac{Q}{\omega C_3(1+n_2)^2} - \frac{2Q}{\omega C_3(1+n_2)^3}(1 + n_2 - C_3/C_T) = 0 \quad (33)$$

Then:

$$n_2 = 2\,C_3/C_T - 1 \quad \dots \quad (34)$$

$$R_2 = \frac{Q}{\omega C_T(1+n_2)^2} \quad \dots \quad (35)$$

The symmetrical case is given when $n_2 = 1$.

Then:

$$R_2 = Q/4\omega C_T \quad \dots \quad (36)$$

$$C_4 = \infty \text{ (short-circuit)} \quad \dots \quad (37)$$

Adjustment of these circuits may be made by varying one of the capacitors or by means of an iron dust core in the coil. In the latter case the coil tap may be moved slightly or the bridge resistor adjusted to allow for the unbalance introduced by the presence of the core.

A Simple Quartz Crystal Oscillator driven by a Junction Transistor

By H. G. Bassett*, B.Sc.(Eng.), A.M.I.E.E.

The oscillator uses a junction transistor with grounded-base, together with a quartz crystal operating in its series mode. Using present-day transistors the circuit can operate at frequencies up to about 300kc/s and power outputs up to 10mW or so. An experimental oscillator operates from a 6V supply and delivers 7mW at 245·9kc/s.

SIMPLE quartz crystal oscillators employing thermionic valves make use of the parallel resonance of the crystal because of the high input impedance of the thermionic pentode when used with grounded cathode. In the design of simple transistor oscillators, however, the low impedance of the emitter circuit of the transistor makes the use of the series resonance of the quartz crystal much more attractive. The oscillator described accordingly uses a series resonant quartz crystal and a grounded-base junction transistor; the simplicity, low power consumption and good frequency stability of the oscillator make it attractive where the power output is not required to be more than a few milliwatts and where the frequency is not higher than about 300kc/s (for currently available transistors). High-frequency junction transistors such as the thin-base unit described by Mueller & Pankove[1] and the pnip unit[2] will enable higher operating frequencies to be attained.

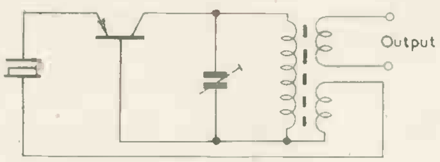


Fig. 1. Basic circuit of oscillator

Description of Circuit

Fig. 1 shows the basic circuit arrangement; d.c. supply circuits are omitted for simplicity. The transistor is used with grounded base because it can then operate usefully at frequencies in the carrier telephony range; the positive feedback is provided from the collector to the emitter via a tuned transformer and the quartz crystal which is connected in series with the emitter circuit. At the series resonance the impedance of the quartz crystal is low and the tuned transformer is so designed that at this frequency there is an overall current gain round the feedback loop when the third (output) winding of the transformer is closed with its designed load impedance. The d.c. supply circuit is arranged so that the transistor is biased to the centre of its linear operating regime; it stabilizes the emitter current of the transistor in the manner described by Shea[3]. Fig. 2 shows the complete circuit diagram. Amplitude limiting occurs under operating conditions by collector-current cut-off at one end of the linear regime and by collector voltage "bottoming" at the other end; in the simple oscillator described these limiting processes control the amplitude of oscillation and make it almost proportional to the supply voltage. Inevitably harmonic components are introduced into the output signal, but by suitable design of the output transformer the harmonics are kept to an acceptably low level. A feature of the circuit is that the attenuation of harmonic frequencies around the feedback loop is large (more than 50dB in the experimental oscillator). The discrimination against harmonic frequencies assists frequency stability.

Choice of Operating Conditions

The choice of operating conditions for the transistor is not as straightforward as the choice of operating conditions

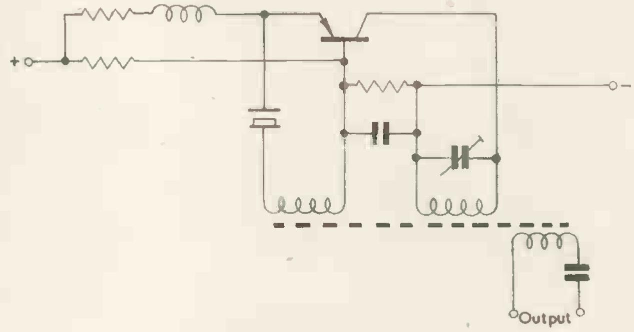


Fig. 2. The complete oscillator circuit

in a thermionic valve oscillator. Thus, if the transistor is operated with an amplitude of voltage V and an amplitude of current I in its collector circuit, the power in the collector circuit, P_c, is $VI/2$. If the current gain is α and the input resistance (crystal + emitter circuit) is R_e, the power, P_e, fed back to the input circuit must be $(I/\alpha)^2 R_e/2$.

The external output power, P_o, of the oscillator is then $VI/2 - (1/\alpha)^2 R_e/2$ if we neglect transformer losses. If we assume that the transformer ratio is suitably changed, the change of output power with change of V and I is given by:

$$dP_o = \partial P_o/\partial V \, dV + \partial P_o/\partial I \, dI$$
$$= I/2 \, dV + (V/2 - I/\alpha^2 R_e) \, dI$$

and maximum external power output is obtained from the oscillator when:

(a) V is as large as possible;

(b) $V/2 = I/\alpha^2 R_e$.

It follows from the second condition that $I = V\alpha^2/2R_e$, and hence $P_{o(max)} = V^2\alpha^2/8R_e$. There is therefore an upper limit to the output power obtainable from the oscillator, independent of any limitation upon output imposed by maximum collector dissipation.

Using present-day transistors, $P_{o(max)}$ is often of the same

* *Post Office Engineering Department.*

order as the maximum collector dissipation. It follows from condition (a) that V should be as large as possible; even if maximum power output is not desired, the efficiency of the oscillator increases as V is raised.

Performance of an Experimental Oscillator

An experimental oscillator has been constructed which delivers 7mW at 245·9kc/s when powered with a 6V dry battery. The current consumption is 6·2mA and the transistor operates with $V = 5$V, $I = 6$mA. The quartz crystal is a CT-cut unit, mounted in an evacuated envelope and having a series resonant impedance of about 200Ω; the value of R_e is then about 250Ω. The oscillator is designed to operate at maximum power output in accordance with the foregoing analysis; it has at the time of writing completed over 2 000 hours of operation. The circuit is very tolerant of variations in transistor parameters; changes in collector impedance and collector cut-off current are unimportant and the oscillator operates satisfactorily with a wide range of transistors from various manufacturers. The oscillator functions satisfactorily up to at least 50°C; the change of output level with temperature is 0·02dB/°C and the change of frequency with temperature is almost entirely attributable to the quartz crystal. The frequency stability against 10 per cent change of supply voltage is 3×10^{-7}.

Acknowledgments

The author wishes to acknowledge advice given by Mr. F. F. Roberts and Mr. F. G. Clifford, and is indebted to the Engineer-in-Chief of the G.P.O. for permission to make use of the information contained in this article.

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Colour Television Equipment

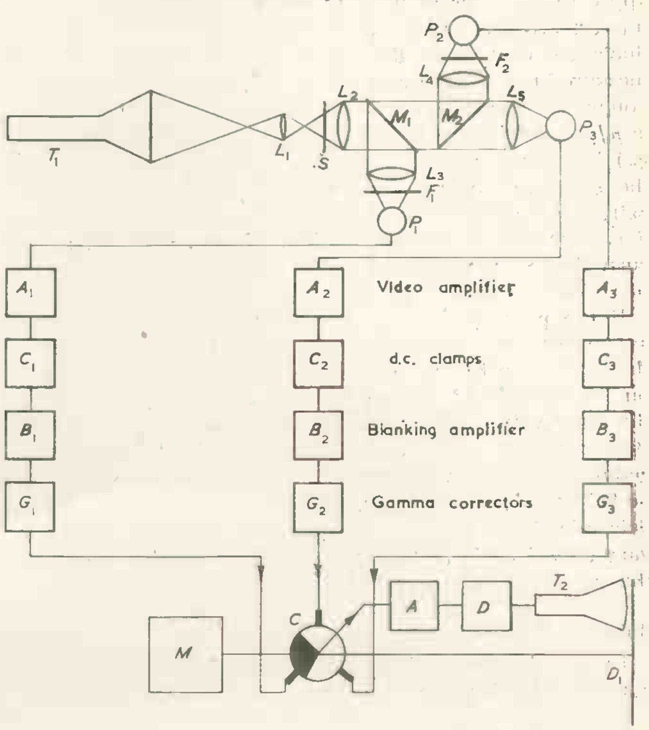
COLOUR television equipment has been constructed by the staff of the Electrical Engineering Department of the Bradford Technical College under the direction of the Head of the Department, Dr. G. N. Patchett in order to show the principles of colour transmission and reception to students. The equipment consists of a flying spot colour slide camera and a receiver using a rotating colour disc.

The flying spot colour slide camera uses a special flying spot scanning tube T_1 (see Fig. 1) which operates with a final anode voltage of 25kV. The tube has a green fluorescent screen with an extremely short afterglow, an essential property of any flying spot scanning tube. The raster on this tube is focused on to the colour slide S by a 5cm, f1·5 lens L_1. The light leaving the colour slide is made into an approximately parallel beam by lens L_2. The light is now split into the three primary colours, red, green and blue. This is achieved by dichroic mirrors M_1 and M_2. Mirror M_1 is made so as to pass red and green light but reflect blue light, this being focused, by lens L_3, on to the cathode of the photo-multiplier cell P_1, which produces a signal corresponding to the blue image of the colour slide. A correcting filter F_1 is placed in front of the cell to correct the colour response. The red light passing through M_1 is now reflected from the mirror M_2 through lens L_4 and filter F_2 on to the cathode of the photo-multiplier P_2 which produces a signal corresponding to the red image. The green light passes through mirrors M_1 and M_2 and is focused by the lens L_5 on to the photo-multiplier cell P_3 which produces a signal corresponding to the green image.

The three signals are now passed through video amplifiers A_1, A_2 and A_3 with adjustable gains so as to adjust the contrast of each colour image. It is now necessary to set the black level of the signals which is done by the three d.c. clamping circuits C_1, C_2 and C_3. During both line and frame flyback periods the beam on the flying spot tube is suppressed and clamping is applied during the line flyback periods. Adjustment of the clamping voltage controls the brightness (or d.c. level) of each colour on the receiving tube. A blanking signal is now applied to cut off any signal during the line and frame flyback by the blanking amplifiers B_1, B_2 and B_3. After this gamma correction is applied (in G_1, G_2 and G_3) to compensate for the non-linearity of the receiving cathode-ray tube. The three signals are now fed to a commutator C driven at 1 000 rev/min by a synchronous motor M which is constructed so as to feed the three signals, in order, to the receiving tube T_2. through the video amplifier (A) and d.c. restorer (D). Attached to the same shaft is a disk D_1 which rotates in front of the receiving tube. The disk is divided into three sectors of 120°. Over the sectors are placed red, green and blue filters so arranged that when the green filter is in front of the cathode-ray tube the green photo-multiplier is connected through the commutator to the grid of the tube, and so on. In this way successive red, green and blue images are seen by the observer, which, due to the persistence of vision, results in a colour picture. The system runs at 405 lines with 50 frames/second. Since the time between similar colour frames is 3/50 second flicker is rather bad, but, as the equipment was designed to show the principle only it is of little importance. It could, of course, be improved by raising the frame frequency but this has been kept at 50 frames/second so that a common pulse generator could be used for both the black and white camera and the colour equipment.

The colour rendering of the equipment is very satisfactory when carefully adjusted but, like all colour systems, it is very critical to the correct relationship between the three primary colours. The definition corresponds to about 2Mc/s. Interesting demonstrations can be given of the effect of reducing the bandwidth of the various channels. Considerable reduction can be made in the bandwidth of the blue channel without altering the picture detail. A small reduction in the bandwidth of the red, and particularly of the green channel soon shows up in the definition of the resulting picture.

Fig. 1. Arrangement of the apparatus



A New Circuit for Balancing the Characteristics of Pairs of Valves

By R. E. Aitchison*

Various methods for adjusting the balance of a pair of triodes are considered. A new method is suggested which is based upon the adjustment of the cathode temperature so that the static anode currents are made equal. Measurements indicate that under these conditions the mutual conductances are then nearly equal, and the behaviour with respect to overall heater voltage changes is also improved.

IN many applications, and particularly in the case of d.c. amplifiers, it is desirable to have a pair of identical valves. For example, in push-pull power output circuits this is necessary to obtain an exactly balanced push-pull output. In the case of d.c. amplifiers the pair of valves are arranged so that the effect of changes in supply voltages on the characteristic of one valve are compensated by changes in a similar, preferably identical, valve. Similarly, compensation is obtained for any change in the valve characteristics with time. Unfortunately, it is very difficult to obtain identical valves, particularly in the case of valves with an oxide cathode; and some means must be adopted in the associated circuits of adjusting for differences between pairs of valves.

Triode Parameters

The small signal parameters of a valve (for the sake of simplicity a triode is chosen, but similar arguments apply to pentodes at fixed screen voltage) which are of interest are the amplification factor (μ), the mutual conductance (g_m), and the static anode current (I_a), all measured under fixed conditions, i.e., heater voltage, anode voltage, and grid bias. (Note the dynamic anode resistance is μ/g_m and is hence implied from a knowledge of μ and g_m). The static anode current is important, as it determines the static d.c. balance of balanced circuits, obviously of extreme importance in the case of d.c. amplifiers, although it is desirable from many points of view that the anode currents be equal in push-pull a.c. circuits—i.e., reduction of ripple, and of d.c. saturation of the output transformer.

If the parameters of a series of valves of the one type are measured it is found that μ shows small deviations of the order of a few per cent from the mean. The g_m and I_a, on the other hand, deviate by well over ten per cent from the mean value. This is to be expected, as the amplification factor is, under ideal conditions, determined by the valve geometry, which with normal construction methods can be held to close tolerances, whereas the g_m and I_a are also determined by the cathode emission, which is known to vary considerably in the case of oxide cathodes, even with the most carefully controlled production.

Balance Circuits

In any double triode circuit it is possible to adjust for equality of either the static anode currents, the stage gain, or the effect of heater changes, but not for all three simultaneously. Normally, adjustments are made for equality of the static anode currents, or zero adjustment. Methods of doing this are shown in Fig. 1. For example, the anode supply may be taken through a potentiometer as in 1(a), and this adjusted for static balance. Alternatively, the bias can be changed as in 1(b) so as to give static balance. Finally, the valve may be shunted with a resistor as in 1(c). Unfortunately, if other adjustments are

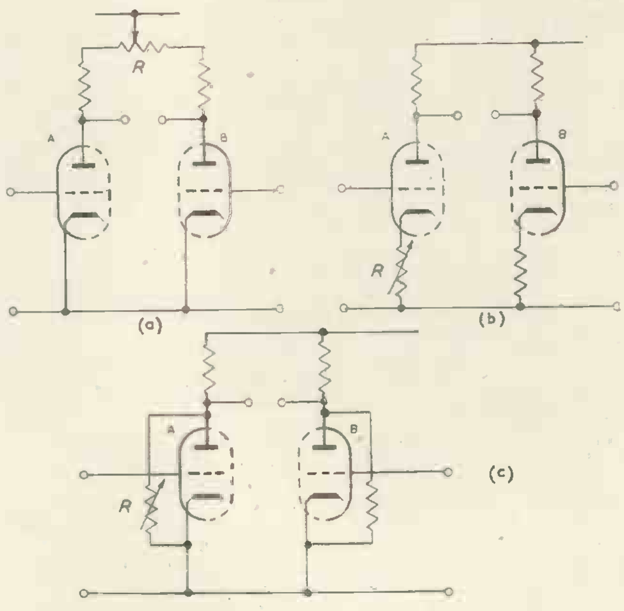


Fig. 1. Three basic methods used to balance the static properties of a pair of triodes

introduced so as to equalize also the stage gain and the effect of heater changes, the various adjustments are not independent, and in a multi-stage amplifier the use of three interacting controls on each stage would be impracticable.

More elaborate circuits have been devised to balance the anode current and the gain simultaneously[1]. However, these methods are based on the use of a large cathode resistance common to both valves, this serving to nearly balance the dynamic properties, and one of the methods of Fig. 1 is then used to balance the static currents. An

* Electrical Engineering Department, University of Sydney, Australia.

TABLE 1

Differences Between Twin Triodes Before and After Balancing by Heater Compensation

VALVE NUMBER	HEATER VOLTAGE DIFFERENCE (V)	GRID BIAS (V)	I_{a1} (mA)	I_{a2} (mA)	g_{m1} (μA/V)	g_{m2} (μA/V)	ΔI_a (mA)	Δg_m (μA/V)
1	0·0	2·2 2·0 1·8	1·00 1·37 1·82	0·815 1·18 1·545	2 160	1 825	0·19	335
	0·67	2·2 2·0 1·8	0·905 1·275 1·68	0·91 1·275 1·68	1 945	1 925	0·0	20
2	0·0	2·2 2·0 1·8	1·04 1·41 1·875	0·97 1·31 1·745	2 155	1 940	0·10	215
	0·23	2·2 2·0 1·8	0·98 1·39 1·83	0·99 1·39 1·82	2 110	2 050	0·0	60
3	0·0	2·2 2·0 1·8	1·11 1·49 1·965	0·925 1·27 1·68	2 160	1 890	0·22	270
	0·57	2·2 2·0 1·8	1·015 1·39 1·86	1·015 1·39 1·87	2 110	2 140	0·0	30
4	0·0	2·2 2·0 1·8	1·03 1·41 1·84	0·99 1·365 1·785	2 075	1 990	0·045	85
	0·13	2·2 2·0 1·8	1·03 1·41 1·825	1·04 1·41 1·83	1 990	1 975	0·0	15
5	0·0	2·2 2·0 1·8	0·89 1·285 1·68	1·04 1·44 1·84	1 975	2 115	0·155	140
	0·38	2·2 2·0 1·8	0·96 1·35 1·78	0·97 1·35 1·775	2 045	2 010	0·0	35
6	0·0	2·2 2·0 1·8	2·975 1·36 1·79	0·98 1·36 1·78	2 040	2 000	0·0	40
7	0·0	2·2 2·0 1·8	0·86 1·20 1·60	0·83 1·15 1·52	1 860	1 725	0·05	135
	0·18	2·2 2·0 1·8	0·835 1·18 1·6	0·83 1·18 1·52	1 815	1 850	0·0	35
8	0·0	2·2 2·0 1·8	1·10 1·49 1·93	1·11 1·47 1·89	2 075	1 950	0·02	125
	0·06	2·2 2·0 1·8	1·11 1·50 1·95	1·12 1·50 1·93	2 050	2 025	0·0	25
9	0·0	2·2 2·0 1·8	1·33 1·76 2·30	0·925 1·28 1·72	1 550	1 985	0·48	565
	0·73	2·2 2·0 1·8	1·11 1·55 2·01	1·12 1·55 2·03	2 310	2 300	0·0	10

example of this type of balance circuit is given in Fig. 2. However, this method is applicable only when using parallel balanced triodes with a large value of the common cathode resistor R, and hence is not applicable in many cases.

New Balance Circuit

As the differences between triodes of the same type are mainly caused by differences in the emission from the cathode, it should be possible to balance valves by making the cathode emission the same. A new circuit[4] based upon this fact is shown in Fig. 3. A potentiometer of a few ohms resistance is connected as shown so as to introduce a small difference ΔV_h in the heater voltages of the valves 1 and 2. The simplest procedure is to adjust R so that the static anode currents of the valves are equal. If necessary, the overall heater supply can be increased slightly, but in many applications, for example in the case of d.c. amplifiers, it is desirable to run the heaters at a reduced voltage and the overall drop in R is not important.

It was found experimentally that the maximum difference in voltage between the two triodes amounted to 10 per cent of the normal for a batch of 50 twin triodes type 12AX7,

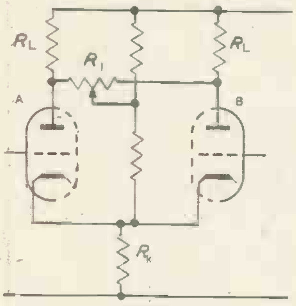


Fig. 2. A method for balancing both the static and dynamic properties of a pair of triodes

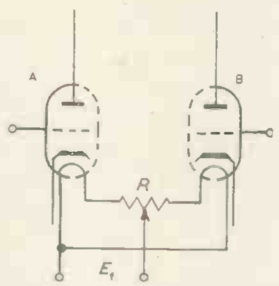


Fig. 3. New circuit for the overall balance of a pair of triodes

and that of these approximately half could be balanced with heater voltage differences of only 5 per cent.

As the potentiometer R is set so as to equalize the static anode currents, the valves will be perfectly balanced if the mutual conductances are equal and they respond equally to the effect of heater supply changes.

These two points were checked by measurements on a number of valves type 12AX7. Measurements on the mutual conductance are given in Table 1. The twin triode type 12AX7 was first tested at fixed heater voltage, then the heater voltages were adjusted for static balance at the recommended test conditions (anode voltage 250V, grid bias $-2{\cdot}0$V). The dynamic balance was next measured by measuring the mutual conductance, and also the anode current at a grid bias of $-2{\cdot}2$V and $-1{\cdot}8$V. It is obvious that for all the valves tested the mutual conductances after balancing for I_a are almost identical. This would probably hold only for triodes of near-identical geometry, where the unbalance is caused solely by the differences in work function of the cathode. Once the emissions are made equal by adjustment of the cathode temperatures, the valves should then be identical apart from a slight difference in the escape voltage of the electrons from the cathodes. It is possible that the differences in temperature will introduce other side effects such as differences in life of the two triodes, different rates of poisoning, etc. However, balance adjustments are normally a routine adjustment in the case of d.c. amplifiers. The twin triode No. 1 of Table 1 was tested after 100 hours and the balance adjustment was found to have changed by 0·07V, the new value of ΔV_h being 0·60V. It should be noted that out of the nine valves tested, one valve (No. 6) showed anode currents and mutual conductances to be nearly balanced without any further compensation.

Of major importance in d.c. amplifier design is the effect of heater supply variations, and several circuits are used to balance a pair of triodes for equality in changes due to heater supply changes[2, 3]. Although the new circuit makes the cathode temperatures different, all other properties are nearly balanced, and to a first order it would be expected that the stability to heater supply variations would be also balanced. Measurements in Table 2 give the effect of a change of + and −5 per cent on the overall heater supply. As is clear from these measurements, the

TABLE 2

VALVE NUMBER	HEATER SUPPLY	HEATERS NOT COMPENSATED		HEATERS COMPENSATED	
		I_{a1} (mA)	I_{a2} (mA)	I_{a1} (mA)	I_{a2} (mA)
4	Normal	1·41	1·365	1·40	1·40
	+5%	1·50	1·46	1·47	1·47
	−5%	1·37	1·32	1·33	1·33
5	Normal	1·285	1·44	1·325	1·325
	+5%	1·35	1·49	1·39	1·395
	−5%	1·21	1·37	1·27	1·275
6	Normal	1·36	1·36	—	—
	+5%	1·425	1·42	—	—
	−5%	1·275	1·28	—	—
7	Normal	1·20	1·15	1·18	1·18
	+5%	1·28	1·22	1·6	1·575
	−5%	1·16	1·11	0·835	0·83
8	Normal	1·48	1·47	1·50	1·50
	+5%	1·58	1·56	1·55	1·55
	−5%	1·44	1·42	1·43	1·425
9	Normal	1·76	1·28	1·55	1·55
	+5%	1·85	1·35	1·66	1·65
	−5%	1·67	1·22	1·50	1·49

compensation is good but not perfect. However, this result is typical of all heater compensation circuits, which rarely give an improvement of better than 20 : 1 (Verhagen[1]).

Conclusions

It is concluded that the new balance circuit of Fig. 3 is capable of giving, simultaneously, an accurate balance of the static anode currents and mutual conductances of pairs of triodes. As the amplification factors are normally also balanced, this gives a pair of triodes, without selection, identical to within a few per cent. At the same time the balance with respect to overall heater supply changes is improved sufficiently to render heater compensation circuits unnecessary.

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Notes from North America

Highlights from I.R.E. Convention

New medical, dental and industrial uses of ultrasonics were announced at the national convention of the Institute of Radio Engineers held recently in New York.

John M. Reid and John J. Wild, M.D., reported that ultrasonic energy has been successfully used at St. Barnabas Hospital, Minneapolis, to diagnose lumps in the human breast before operation. When the narrow sound beam encounters human tissue, a pattern of echoes is returned and displayed on a television picture tube.

Irregularities such as cancer, non-malignant solid tumors and liquid-filled cysts can be recognized from their characteristic pictures. Several echo photographs of typical lesions were shown for the first time at a session of ultrasonic specialists at the Belmont Plaza Hotel.

In another paper at the same session, Douglas H. Howry, M.D., University of Colorado Medical Center, disclosed a technique which permits use of ultrasonic echo equipment for studying vital organs deep within the human body. Ultrasonics may supplement X-rays as a diagnostic aid. Although X-rays are invaluable in studying the body's bony structure, they cannot always give an adequate picture of soft tissue.

The use of ultrasonics in clinical dentistry was described by Alvin E. Strock, Peter Bent Brigham Hospital in Boston. Much of the discomfort associated with visits to the dentist may be alleviated by an ultrasonic dental drill developed by the Cavitron Corp. of Long Island City and described in a companion paper by Lewis Balamuth. An ultrasonic drill is used to abrade or wear away the decayed parts of the tooth. It does its work rapidly and without force being exerted by the dentist. The patient is spared the uncomfortable sensations of heat and pressure caused by a mechanical drill.

Industrial applications of ultrasonics were also described including studying river currents to determine the best place to build a hydro-electric station, removing the chaff from textile fabrics and determining the mechanical strength of manufactured products non-destructively.

A single colour television camera tube which does the work of the three tubes used in present day cameras was reported under development by RCA Laboratories. The new tube is capable of responding to all three primary colours at once instead of just to a single colour. The development was announced in a paper presented by P. K. Weimer, S. Gray, H. Borkan, S. A. Ochs, and H. C. Thompson, of RCA.

A system by which radar information may be transmitted by ordinary telephone lines from a distant radar station to a radar viewing screen was disclosed by C. W. Doerr and J. L. McLucas of Haller, Raymond and Brown, Inc. At the present time coaxial cables or expensive microwave relay systems are required for this purpose at airports and for radar networks.

Thirty tons of printed matter on paper 8in wide in an 8-hour day or a printing speed of 5 000 characters a second; this is the possible output of a new and basic recording technique developed by the Burroughs Corporation.

H. Epstein and F. Innes in a paper titled "Electrographic Recording", revealed the development by which a mark may be put on a paper in a duration as small as one microsecond and that printed characters, formed by combination of such marks, could be recorded at rates exceeding 5 000 characters per second. This system will be particularly useful in electronic computer systems, teletypewriter and telemetering services, facsimile, etc., where the output of the machine is to be displayed as a printed message.

The development of a new type of high-gain transmitting antenna for television broadcasting in the u.h.f. band was described by O. M. Woodward of RCA, and James Gibson of Sweden. In another session, a new type of television transmitting antenna which eventually should work to the advantage of the set-owner (especially in colour television) as well as to the broadcaster and manufacturer, was outlined by R. W. Masters and C. J. Rauch of Ohio State University.

The technical aspects of television in Europe, with its many different engineering standards creating problems in channel allocation, set manufacture, and international programme exchange, were summarized by Hubert A. S. Gibas of Switzerland.

The possibility of using man-made space satellites as a means of relaying television across the ocean was disclosed by Dr. John R. Pierce, director of electronics research of Bell Laboratories.

Good reception would be electronically feasible by using a 100ft satellite in an orbit 22 000 miles above the earth to reflect television signals from one continent to another. This would require an antenna 250ft in diameter at the sending and receiving stations on earth and a transmitter power of only 50kW. At such time as it becomes possible to construct such a satellite, the chief problem would be keeping the satellite's reflecting surface steadily aimed in the proper direction.

Among other speakers at the symposium was Prof. S. F. Singer, physics department of the University of Maryland, who reported that the technical problems connected with launching, control and instrumentation of a very small man-made satellite called the MOUSE (Minimum Orbital Unmanned Satellite of the Earth) are well within the range of present techniques. Among the many scientific instruments which would be installed in the MOUSE would be one which measures the sunlight reflected by the earth. This would give a measure of the total world cloud coverage which, in turn, could be used to forecast long-range climatic changes.

Edward F. Feldman, speaking at a session on instrumentation, described an electronic test instrument which automatically analyses sounds and noises and displays their frequency components on a cathode-ray tube. The instrument is useful in such projects as silencing business machines, measuring high fidelity equipment, and finding faults in precision ball bearings.

The Physical Society's 39th Exhibition

A description, compiled from information supplied by the manufacturers, of selected exhibits at the Physical Society's Exhibition held at the New Hall of the Royal Horticultural Society, London, from 25th to 28th April.

Boulton Paul

Load Cells

(Illustrated below)

A NEW range of load cells is being developed, using differential transducers as the means of measuring the displacement under load. This feature almost completely eliminates errors due to non-axial loading and avoids difficulties of creep associated with strain gauges. These cells will be of interest to manufacturers and users of rolling mills and allied machinery. The accurate measurement of loading can be used as the safety device, ensuring that the machine is never overloaded.

Displacement Transducer

THE EP 303 is an entirely new displacement transducer, capable of measuring displacements of the order of 100 micro-inches. It is robustly constructed and is sufficiently small to enable it to be fitted to apparatus where other methods have proved impossible. It can be fitted to a capsule or diaphragm, so making a very sensitive pressure measuring device. It has been designed with a view to long life and should give many years of service without maintenance.

Boulton Paul Aircraft Ltd.,
Pendeford Lane,
Wolverhampton.

B.P.L.

Pulse-height Valve-voltmeter

THIS instrument, Model PV 812 will read up to 100V with an accuracy of ±2 per cent and is designed to measure the height of periodically recurring pulses, being independent of pulse width and repetition rate.

Pulse Generator

(Illustrated above right)

THE pulse and square wave generator, Model PG712, has been developed as a piece of ancillary equipment to the audio-frequency signal generator which has now an extended frequency range to cover from 10c/s up to 100kc/s. It provides a means of converting sinusoidal voltages into steep fronted pulses of short duration or alternatively into square waves. The output for pulse operation is

variable up to 15V positive or negative to earth, while the maximum output for square wave generation is 10V. Both the above items are being manufactured under licence from the N.R.D.C.

British Physical Laboratories,
Houseboat Works,
Radlett,
Hertfordshire.

B.T.H.

Microwave T.R. Devices

A RANGE of X-band t.r. and t.b. broadband cells was shown. The special developments to which attention is drawn are the form of compression window seal, the copper brazed steel method of vacuum envelope construction, and the shielded gap construction which hinders deposits of material sputtered off the primer electrode from detuning the r.f. circuit. These features, together with special processing techniques, enable an average life well in excess of 1 000 hours to be achieved.

The British Thomson-Houston Co. Ltd.,
Rugby,
Warwickshire.

Cinema-Television

Flying Spot Microscope

THIS is the first production model of the prototype shown previously and incorporates a number of new features. Basically the equipment permits any normal microscope slide to be viewed on a television type screen with the advantages of greater magnification, variable contrast control and large audience viewing facilities.

A special feature of the exhibit was a demonstration of automatic particle counting.

Miniature Multi-channel Recording Equipment

THIS new equipment has been designed to meet the demand for a miniature multi-channel recording apparatus and permits from one to twelve channels to be used simultaneously. Photographic recording of the tube traces can be carried out with either a continuous feed or drum type of camera.

Each amplifier and tube unit is only $9\frac{1}{4}$in high × 2in wide × 15in deep thus permitting the units to be closely stacked together. Associated units comprise the main amplifier and power supplies.

Cinema-Television Ltd.,
Worsley Bridge Road,
Lower Sydenham,
London, S.E.26.

Dawe Instruments

Vibration Meter

(Illustrated below)

THE new type 1403 vibration meter is a mains-operated instrument which uses a moving-coil pick-up in place of

the crystal pick-up of the portable type 1402 battery-operated equipment. This new pick-up provides an output voltage which is directly proportional to the vibration velocity instead of to the vibration acceleration as with the crystal pick-up. Electrical integrating and differentiating circuits are, however, incorporated in the new meter to provide a direct calibration in terms of vibration displacement and acceleration, as well as velocity.

Dawe Instruments Ltd.,
99 Uxbridge Road,
Ealing,
London, W.5.

Edison Swan

Process Controller

THIS equipment is designed to be the controlling element in automatic production plant. It is suitable for any type of machine which has controls that can be electrically or pneumatically operated.

The controller is basically a universal timer controlling up to six separate and individual functions so that they are automatically performed in the desired sequence and for the requisite time, in relation to one another. Auxiliary units are also provided so that if one or more of the functions required are to turn on or off, for example, gas/air or gas/oxygen burners, the burner supplies are turned on and off in the right sequence and with a suitable time delay to prevent explosions.

Industrial and Special Valves

THE ES1001 is a radiation cooled triode with thoriated tungsten filament and with a maximum anode dissipation of 1kW at 40Mc/s.

The ESA1002 is a forced air cooled triode with a thoriated tungsten filament and with a maximum anode dissipation of 12kW at 40Mc/s.

The 13E1 is a beam tetrode d.c. control valve for use as either a series or shunt control valve in stabilized power supplies. It is also eminently suitable for servo control motor systems. Mutual conductance approximately 40mA/V. Maximum anode dissipation 90W.

The Edison Swan Electric Co. Ltd.,
155 Charing Cross Road,
London, W.C.2.

Edwards

Quartz Crystal Vibrator Coating Plant

EDWARDS model 12QE coating plant is capable of coating batches of 16 quartz crystal vibrators in one pumping cycle and provision is made for individual frequency calibration of each crystal during the process.

A new crystal coating unit has been developed for individual crystals making use of valveless pumping in order to provide the simplest possible construction both for experimental and small batch quantities of crystals. The work-chamber is of about ½ litre capacity and can be exhausted to 10^{-4}mm of mercury by a "Speedivac" model A203 air-cooled vapour pump backed by a "Speedivac" model 1S50 rotary pump in a total time of 2½ minutes. It is sealed by one of two covers so mounted that they can be rotated horizontally as a joined pair through 180°. Each cover is fitted with a crystal holder and an evaporation source. In operation a crystal may be coated and frequency calibrated whilst the other cover is being loaded. At the finish of the coating cycle air is admitted, the covers are rotated bringing a new one into service, and the operation is repeated.

W. Edwards & Co. (London) Ltd.,
Manor Royal,
Crawley,
Sussex.

E.I.C.

A.C. Comparator

(*Illustrated below*)

THIS is a non-electronic alternating current bridge suitable for the measurement and grading of capacitors, chokes and resistors from mains supply.

An external power supply and an external standard are required. For capacitance measurements between 0·25 and 25μF mains frequency (50c/s) is suitable, but for lower values a higher frequency is required. Two versions of the comparator are made; an industrial model intended for factory use and a laboratory model, the latter being five times more sensitive than the former.

A capacitance decade box covering from 50pF to 1μF and suitable for use with the comparator was also shown.

The Electrical Instrument Co. (Hillingdon) Ltd.,
Boswell Square Industrial Estate,
Hillingdon,
Glasgow, S.W.2.

Elliott

Microwave Equipment

THE range of microwave equipment shown included the following:

Absolute Method of Attenuation Calibration: Equipment for 10 000 *Mc/s Band.*

Equipment is being developed which operates as a microwave bridge system, enabling attenuators to be calibrated by the provision of signals of accurately related amplitude.

Standing-Wave Indicator for 10 000*Mc/s Band.*

This is a prototype instrument of advanced mechanical and electrical design. It employs an electroformed waveguide of great accuracy, a high-precision but long-wearing travelling surface, and an improved electrical system. The whole instrument is totally enclosed, and is fitted with a dial gauge for direct reading of the probe position.

The instrument forms part of

A Complete High Precision Low-Power Test Bench for 10 000*Mc/s Band.*

This prototype test equipment enables the transmission and reflexion characteristics of most microwave components to be measured. Demonstration was made of the direct methods of measuring voltage standing wave ratio by the use of the Elliott rotary attenuator and a standing wave indicator.

A Broad-Band, High Directivity Directional Coupler for 35 000*Mc/s Band.*

This instrument, at present under development, employs a multi-slot coupling hole system and a first-grade glass vane matched load.

D.C. Magnetic Amplifier

THIS instrument is a high stability d.c. amplifier of rugged construction. By the use of second harmonic modulator input stages, stability is appreciably better than that obtained with conventional types of amplifiers, and errors are reduced to the equivalent of 10^{-12}W signal input powers. For inputs from 0 to 4mV across impedances of 20Ω to 20kΩ, the amplifier delivers 0 to 5mA into 500Ω to drive an industrial pattern recorder or indicator. Response time is one second approximately. An accuracy of ±1 per cent full scale deflexion is maintained with mains variations of ±10 per cent voltage and ±5 per cent frequency. The amplifier was demonstrated in conjunction with a magnetic amplifier relay. By means of a calibrated dial the operating point of the relay is adjustable to any point on the output characteristic of the magnetic amplifier.

Elliott Brothers (London) Ltd.,
Century Works,
Lewisham,
London, S.E.13.

Ferranti

C.R.T. Afterglow Measurement

THIS equipment is designed to determine the decay curve of luminescence of a cathode-ray tube phosphor as a function of time. It consists of two units: one unit is for short-afterglow measurement in the range from a fraction of a microsecond up to about 200msec, and the other unit for medium- and long-afterglow measurements. In the short-afterglow unit an electron beam sweeps across the phosphor under investigation in a linear trace; the light pulse passing through a narrow slit in a mask placed in front of the cathode-ray tube screen is converted by a photo-multiplier to a corresponding voltage pulse for display on a cathode-ray tube. In the second unit, when used for medium-afterglow measurements, the electron beam sweeps across the phosphor under investigation in a circular trace: the light intensity of the afterglow is measured with a photo-multiplier placed in front of a slit in the mask which rotates in front of the cathode-ray tube screen at the same speed as that of the spot, the adjustable phase difference between luminous spot and viewing slit representing the time interval of decay. If the unit is used for long-afterglow measurements, the photo-multiplier remains stationary in front of the screen and the decay of light output is timed with a watch.

Ferranti Ltd.,
Hollinwood,
Lancashire.

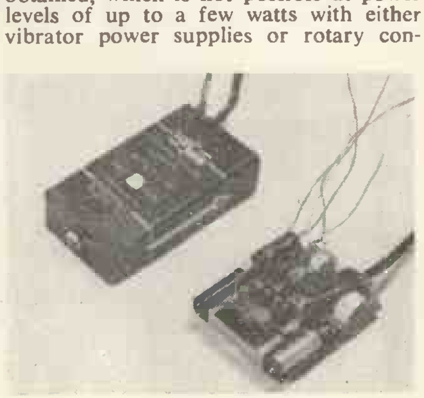
Fortiphone

Transistor Components and Applications

(*Illustrated below*)

A RANGE of subminiature transformers designed for use in conjunction with transistors was shown.

Transistors have proved to be a very efficient means of converting low voltages to higher voltages by using an oscillator running at 10 to 20kc/s and simple smoothing, which is all that is necessary at that frequency. Conversion efficiencies of as high as eighty per cent have been obtained, which is not possible at power levels of up to a few watts with either vibrator power supplies or rotary con-



verters. Circuits of this type and also audio frequency transistor circuits are being made up as small potted special assemblies in rectangular form as illustrated.

The unit shown contains two transformers, two transistors, two amplifiers and a number of other components and measures only ½in × 1in × 1⅞in.

Fortiphone Ltd.,
Fortiphone House,
247 Regent Street,
London, W.1.

Johnson Matthey
Wax-Protected Precision Silvered Mica Capacitors

A RANGE of wax-protected capacitors embodying silvered mica plates, covering capacitances from 5pF to 0·25μF and adjusted to a minimum capacitance tolerance of ±0·5 per cent or ±1pF (whichever is the greater). High stability, low power factor and absence of scintillation are among their characteristics. The capacitors are available with three voltage ratings—500, 350 and 200V peak—in both eyeleted and fired types.

Moulded Precision Silvered Mica Capacitors

ROBUST moulded components embodying silvered mica plates, designed to give efficient service over long periods in the temperature range −60°C to +120°C. The relationship between temperature and capacitance change is linear and cyclic. The capacitors are rated for 500V peak working, and are available in two sizes covering the capacitance range 5pF to 0·01μF.

Waveguide Tubes

SEAMLESS waveguide tubes of high accuracy and excellent internal finish in copper, brass, 7½ per cent and 10 per cent copper-silver, and also in silver-lined copper. Rectangular tubes, with sharp interior angles, are available with external dimensions up to 1·25 × 0·625in. Round tubes are available with outside diameters up to 1·5in.

Johnson Matthey & Co. Ltd.,
78 Hatton Garden,
London, E.C.1.

Labgear
Time Period Counter
(Illustrated below)

THIS instrument is designed for the accurate measurement of the time interval between two events. The design

incorporates the requirements of a counter chronograph operating within the range of 10 to 10^{-4}sec.

The 'time source' oscillator employs a 10kc/s quarts crystal having a frequency stability better than 0·02 per cent. The counter section consists of a single GC10D stage and four stages of GC10B Dekatron glow-transfer tubes with hard-valve couplings.

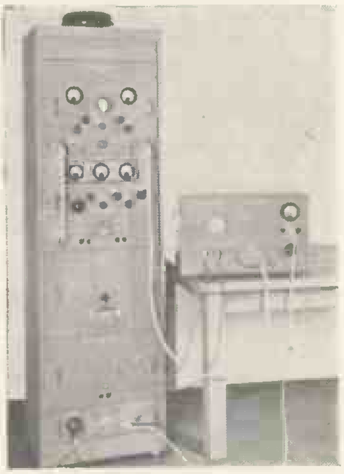
Provision is made for extension of the normal 10sec count range to 10 000sec by the addition of an external Dekatron register.

A flexible triggering arrangement, employing a pentode gate of negligible resolving time in conjunction with two gas-trigger holding tubes, permits the use of pulses derived from either a common origin or from independent sources. Alternatively, a mechanical contact sequence may be accommodated.

Labgear (Cambridge) Ltd.,
Willow Place,
Cambridge.

Marconi Instruments
E.H.F. Signal Generator (0A 1000)
(Illustrated below)

ELECTRICALLY this equipment divides into three main parts: (1) a klystron microwave oscillator which can



be preset for operation at frequencies between 33 300 and 37 500Mc/s, i.e., 8 to 9mm; (2) a separate and versatile klystron power supply and modulation system; and (3) an independent system of frequency stabilization based on a design due to R.V. Pound. The whole equipment is assembled in two main sections. One is the signal generator unit comprising the klystron, an output meter, a precision calibrated attenuator, and a waveguide hybrid-T circuit in which the frequency of the klystron output is compared with that of a built-in standard reference cavity. The other is a cabinet rack containing both the electronically regulated power supply units feeding the klystron and the Pound stabilizer units which constitute an automatic klystron frequency control system actuated by the magnitude and phase of the correction signal fed back from the hybrid-T circuit. All the millimetre-band components are confined to the separate signal generator unit and all connexions between this unit and the cabinet rack are made by means of plug-and-socket connectors. Because of this, and because the main outputs from the power supply sections are monitored and each independently variable over a wide range, the OA 1000 is not only an e.h.f. signal generator but also a versatile and comprehensive equipment suitable for supplying most types of klystron at present available.

Marconi Instruments Ltd.,
St. Albans,
Hertfordshire.

Metropolitan-Vickers
Null Indicator

THIS 50c/s null indicator is intended particularly for use with high voltage bridges, e.g., Schering bridges, and has a very high rejection factor, of the order of several thousand times, for high frequency corona discharge currents. Its performance at 50c/s is superior to that of single tuned circuit or paralelled-T feedback selective amplifiers since it has a higher bandwidth and greater rejection of third harmonic. The response is substantially flat from 25 to 75c/s and the rejection of 150c/s is 130 times. The instrument, which is an *RC* filter with valve anode impedances replacing some of the resistance elements, is simple to use and free from zero or gain controls; also a variation of ±20 per cent in the value of any of the components is tolerable.

Metropolitan-Vickers Electrical Co. Ltd.,
Trafford Park,
Manchester, 17.

Mullard
Millimicrosecond Photography

THE use of image converter tubes as the shutters of ultra high-speed cameras is now well established. The image of the object to be photographed is focused on the sensitive photo-cathode of the tube. When operating voltages are applied, a duplicate image appears at the opposite end of the tube. To photograph fast transient phenomena, the tube is switched on by voltage pulses synchronized with the phenomena, the actual photographic record being made by an ordinary camera at the viewing end of the tube.

In the past, circuit limitations have fixed the lower limit of exposure at about 30 × 10^{-9}sec. In the present demonstration, the use of a coaxial pulsing system enables the exposure time to be reduced ten times, i.e., to 3 × 10^{-9}sec.

In the demonstration, a voltage pulse produced by a very fast spark discharge travels down a coaxial line to trigger on the tube. At the same time, the light generated by the spark is guided by mirrors to the photo-cathode of the image tube. When the light path is lengthened by moving the mirrors, a point is reached where the image of the spark on the viewing screen disappears, because the light arrives at the photo-cathode after the voltage pulse which triggers the tube has died away. By adjusting the length of the light path, the various stages in the formation of the spark can be seen, and the effective exposure may be reduced to about 3 × 10^{-10}sec, the picture on the tube being visible to the eye and recordable by film. (Light travels 9cm in 3×10^{-10} sec.)

Mullard Ltd.,
Century House,
Shaftesbury Avenue,
London, W.C.2.

Plessey
Computing Components
(Illustrated below)

SHOWN for the first time was a magnetic store drum (*illustrated below*) and input-output buffer units, which are being developed for use with the Plessey payroll computer. The store has a capacity of 100 000 bits in a surface medium of cobalt-rich cobalt-nickel alloy. A track packing of 16 to the inch and a digital packing of approximately fifty to the inch is used. The mumetal heads have an impedance of 25Ω and a playback amplitude of about 10mV. The peripheral velocity of the drum is approximately 900in/sec.

The input and output buffer storage units each store 16 decimal digits. Five such units may be used to store, or punch all eighty columns of a standard Hollerith card using standard equipment. The use of input-output buffer storage allows computing to take place as cards are being read and punched. The individual storage elements in the buffers are dekatrons, and these tubes are used in the arithmetic unit of the computer proper. The power consumption is approximately 250mW per decimal digit for the input buffer and approximately 1W per digit for the output buffer.

The Plessey Co. Ltd.,
Ilford,
Essex.

Pye
"Scalamp" Megohm Voltmeter
(Illustrated below)

THE usefulness of the pivoted voltmeter is limited in many applications by its comparatively low resistance which can seldom be higher than 20kΩ/V. By use of a suspended galvanometer it is possible to obtain much higher resistances and exhibited was the prototype of a voltmeter having a resistance of 1MΩ/V. A further advantage is the possibility of measuring low voltages and this instrument has ranges as low as 10mV for full-scale deflexion. The scale is approximately $5\frac{1}{2}$in long and the light spot has a hair line which allows readings to be taken with great accuracy. The specified accuracy is to ±1 per cent of full-scale deflexion and there are 10 ranges from 10mV to 300V. The whole instrument is robust and simple to use. There is provision for shorting the coil automatically when the instrument is turned off and when it is lifted from the bench.

W. G. Pye & Co. Ltd,
"Granta" Works,
Newmarket Road,
Cambridge.

S.T.C.
Radar P.P.I. Simulator
(Illustrated below)

THE "Standard" radar p.p.i. simulator is a simple non-electronic device affording a realistic p.p.i. display for practical radar instructional purposes.

A feature of the apparatus is that all information displayed on the artificial radar screen is prepared and pre-set as a programme of events by the instructor and remains under his full control throughout the period of instruction.

The principle on which the radar trainer operates ensures a high degree of realism in the simulated display. For example, the time-base is formed by a rotating beam or shaft of ultra-violet light activating a coating of television powder on the "tube" face and causing it to fluoresce. Target and other information is superimposed on this time-base by the interposition between the light source and the tube face of a substance giving a more pronounced fluorescence with correspondingly longer persistence. In the version suggested for the Army, nylon thread with globules of thin substance in colourless lacquer provide the moving target traces in association with an ingenious system of pulleys and motors which afford an infinitely variable programme of targets and courses with the choice of two or more speeds. Emulating conventional radar sets at present in service, the simulator is equipped with a cursor for reading off bearings on the tube face and appropriate range markers.

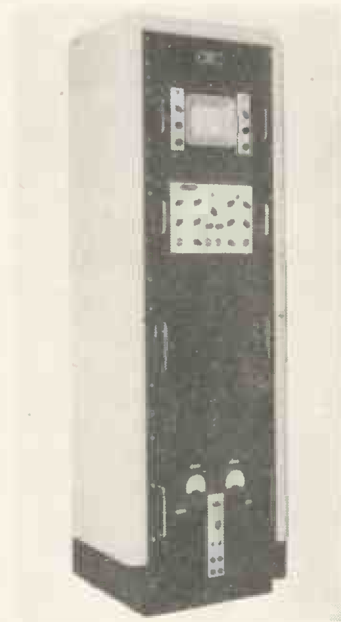
Standard Telephones & Cables Ltd,
Connaught House,
Aldwych,
London, W.C.2.

Sunvic
Pulse Height Analyser
(Illustrated below)

THE pulse height analyser is an instrument for the analysis of a complex pulse amplitude spectrum into groups of known height according to their voltage amplitude.

It displays up to a maximum of 120 channels and can store 1 200 binary digits. The design is based on that of Hutchinson and Scarrot (*Phil. Mag.* 42, 792, 1951) and the instrument is made under licence from the N.R.D.C.

An input pulse which is to be sorted passes in turn through a pre-amplifier with a gain of four, a biased amplifier which removes a controllable amount of the base of the pulse, an inverting amplifier of variable gain and a gate circuit and is then fed into a pulse lengthener. A linear sweep circuit generates a voltage which is linear with time and the analyser measures the time interval which elapses between the



beginning of the linear sweep and the instance at which the linear sweep voltage equals the output voltage from the pulse lengthener. This interval is then used to add unity to the appropriate binary number in the display.

The display unit, incorporates the sorting, memory and display circuits, the display being presented on the screen of a cathode-ray tube in the form of a raster of vertical lines, one line for each channel. The count in each channel is represented on a binary scale by an array of brightened dots. Each dot represents a "1" while a space represents an "0". The least significant figure in each channel appears at the bottom of the line and hence the pulse spectrum is approximately plotted on a logarithmic scale by the dots representing the most significant figure in each channel.

A calibration pulse generator is included in the apparatus.

Sunvic Controls Ltd,
10 Essex Street,
London, W.C.2.

BOOK REVIEWS

Electroacoustics (The analysis of transduction, and its historical background.)

By Frederick V. Hunt. 260 pp. 60 figs. Demy 8vo. Harvard Monographs in applied science, No. 5. John Wiley & Sons, Inc., New York. Chapman & Hall Ltd., London. 1954. Price 48s.

PROFESSOR Hunt's first book contains 235 pages of text, of which the first 91 are a historical survey of the developments in sound transmission and reception. This is fascinating but rather insular, as is common with American texts. True, Henry Hunning's invention (British Patent 3647/1878) of the carbon granule transmitter is mentioned, but not with much enthusiasm; yet this, the basis of all current designs of telephone transmitter, was a far-reaching and revolutionary innovation. No mention is made of Dr. N. W. MacLachlan's moving-coil free cone loudspeaker, which certainly pre-dated the Rice and Kellog patents. This reviewer remembers hearing it in 1924. Nor is P.G.A.H. Voigt's work on capacitive transmitters described. The reader may feel that about 15s. is rather a lot to pay for this collection of memoirs and details of patent litigation.

The second chapter examines electromechanical coupling in general terms, motional impedance and the effect of frequency on mechanical impedance being introduced as a series of vectors relating phase angle to magnitude in a general sense, so that the data is applicable to any coupled vibrating system, e.g., string oscillographs.

In the next chapter the difficulties of relating the antireciprocal nature of electromagnetic systems to the principle of reciprocity for coupled electromechanical systems leads to a discussion of the shortcomings of existing sign conventions in such analyses, and allows the author to introduce a new form of space operator to bring analytical symmetry into antireciprocal cases involving magnetic fields. This concept permits a unified treatment for all types of electro-mechanical coupling and is certainly ingenious if difficult to manipulate; it is observed that the author does not seek to apply this method very much in the book.

Chapter IV is a long and detailed analysis of transducer performance on an admittance basis as contrasted with an impedance basis. One is left with the feeling that the author recognizes that experimental requirements really dictate which approach is of most value.

Chapter V is an excellent discussion on electrodynamic loudspeakers. Unfortunately the only valid information relates to the low-frequency response range. Only a conjectural approach is made, covered in but two pages, to the vexed problems of the behaviour above the frequency at which the cone ceases to act as a solid piston. The author says, "Eliminating these irregularities in response throughout the middle and upper range of frequency is one of the major and not yet fully solved problems of modern loudspeaker design." It is clear that this most important part of the range does not yield to any of the theories developed in this book; rather disappointing, this is something we all want to know a lot more about.

In Chapter VI we come to an excellent and most refreshing analysis of electrostatic loudspeakers. This promising alternative to the conventional moving coil system could profitably be re-examined since the advent of new dielectric materials, sputtering techniques, etc.

The last chapter purports to describe electromagnetic transducers of the gramophone pick-up type, but is really devoted to an analysis of the ring diaphragm telephone receiver. It seems unfortunate that other patterns of transducer were not included, since the title of the monograph is very broad and the inference is that the inclusion of all types might be anticipated.

The book is up-to-date, stimulating in parts, and contains much valuable food for thought. It is written in a scholarly style and a second edition should give Professor Hunt the opportunity to include further examples of well-known methods of transduction which would greatly increase the present limited appeal.

ALAN DOUGLAS.

Zirkonium Seine Herstellung Eigenschaften und Anwendungen in der Vakuumtechnik (Zirconium — Its production, properties and applications in vacuum technique)

By Werner Espe. 174 pp. 14 figs. 20 tables. Demy 8vo. C. F. Winter'sche Verlagshandlung, Fuessen/Bayern. 1953.

THE author of this little monograph is in charge of development of the now nationalized electronic valve manufacture in Czechoslovakia. He is the co-author (with Knoll) of that well known standard work *Stoffkunde der Hochvakuumtechnik* (Materials for High Vacuum Technique) which appeared in 1936. In the slender book under review he gives a comprehensive, competent and very valuable survey on the production and the properties of zirconium and its various applications in the manufacture of vacuum tubes. After some brief introductory remarks about how and where the raw material is found, the production of zirconium powder, of zirconium-hydride powder and of compact, ductile zirconium are described in detail and illustrated. The physical and chemical properties of the metal are discussed and particularly those which make it suitable for gettering purposes. A brief section deals with the manufacturing of intermediate products of zirconium, like sponge, brickets, ingots, disks, sheets, bars and wires. Special precautionary methods are described which must be applied in order to reduce fire risks due to the high inflammability of zirconium power. The second part of the book deals in detail with the application of the metal in vacuum technique and in particular with the methods applied for producing gettering layers by painting, dipping or spraying or by cataphoresis.

A special feature of the book is the large number of tables containing the most important physical, chemical and mechanical properties of the material and some recipes in concise form for producing the suspensions of zirconium powder or zirconium hydride as used in the various gettering processes.

A bibliography and a subject matter index conclude the book which will be welcomed by all those interested in gettering technique.

R. NEUMANN.

Active Networks

By V. C. Rideout. 485 pp. 75 figs. Demy 8vo. Constable & Co. Ltd. 1954. Price 42s.

THIS is a useful and well balanced textbook on valves and valve circuits for students who possess a fair mathematical background. Although the author devotes space to explain the principles of four terminal network analysis, Fourrier analysis and the use of Laplace transformations, as far as this is possible in a book of this size, he does assume a fair knowledge of passive networks, Bessel functions and the use of determinants.

Each chapter concludes with a most comprehensive collection of references, mostly to American books and journals unfortunately, and a set of problems of a practical type. There are about 150 problems in the book and a collection of valve and transistor characteristics are provided which are required for some of the solutions. No answers are available but a number of numerical problems are worked out in the text.

The jacket states that transistors are dealt with in considerable detail although there are in fact only fourteen pages on this subject distributed throughout the book. References to magnetic amplifiers and position controlled servo-mechanisms are brief.

In the chapter on power amplifiers the usual discussion arises on the optimum value of load for a power output triode. A value of load resistance equal to twice that of the anode slope resistance of an ideal valve is determined once the h.t. is fixed and is not a compromise due to non-linear characteristics as the author suggests.

Again, when dealing with phase shift oscillators, the author chooses to analyse the case where the valve load resistance is small compared with the input impedance of the phase shift network. This treatment leads to values of frequency and attenuation that may be materially at variance with practically observed values.

Relationships between f.m., p.m. and a.m. are well illustrated. Modern techniques of producing wide band f.m. and p.m. using the serrasoid modulator and klystron contrast clearly with earlier systems such as Armstrongs. One must, however, move on to the chapter on noise and information theory to discover the improvement in signal-to-noise ratio which is a virtue of wide band f.m. and the further improvement pre-emphasis affords.

Errors in the text are very few and detract in no way from a work which manages to cover a wide field in a stimulating manner.

H. HENDERSON.

Studio Engineering for Sound Broadcasting

Edited by J. W. Godfrey. 208 pp., 108 figs. Demy 8vo. Iliffe & Sons Ltd. 1955. Price 25s.

THIS book has been compiled for the primary purpose of training BBC technical staff in the general principles underlying operational procedures at the Corporation's studio centres. It is now made available outside the Corporation in the belief that broadcasting staff throughout the world, on both the engineering and non-engineering sides, will find a great deal of interest and practical value in its pages.

All six authors are members of the BBC Engineering Division, and each is a specialist in his field. The technical level is practical rather than academic and should present no difficulty to readers.

Mathematics of Engineering Systems (Linear and Non-Linear).

By D. F. Lawden 388 pp., 50 figs. Demy 8vo. Methuen & Co. Ltd. 1954. Price 30s.

THE techniques described in this book are very carefully explained in detail and copiously illustrated by a useful selection of examples from electrical engineering. After an introductory chapter on mathematical accessories—readers are assumed to know the first part of the mathematics for an engineering degree—the author discusses the solution of linear differential equations with constant coefficients, using the D-operator, Laplace transform and then complex variable for response curves, but not dealing with inverse transforms. There is a chapter on Fourier analysis leading up to Fourier transforms. The final chapter describes very well the now standard methods of dealing with the non-linear equations of Van der Pol and Duffing. My real criticism of the book is that the title is completely misleading. There are vast numbers of engineering systems whose mathematics are not described: for example, Rayleigh's method and partial differentiation are not mentioned. A further criticism concerns the presentation of methods for non-linear equations. The methods developed so far, excellent though they are, are severely limited in their application, unless numerical, and this should be made quite clear. Probably the only reason for spending so much time on the mathematics is that this is one field where the beginner can be brought into contact with fresh developments in the mathematics.

G. J. Kynch.

The Gyroscope Applied

By K. I. T. Richardson. 384 pp., 120 figs. Royal 8vo. Hutchinson's Scientific and Technical Publications. 1954. Price 30s.

THIS is an excellently produced book based on "The Gyroscope and its Applications" which was published in 1946 when secrecy restrictions prevented reference to many interesting achievements and possibilities. This new book has been almost entirely rewritten, describing much that is new but at the same time incorporating most of the information given in the first version, although this is presented in a different manner and in some cases from a different view point.

Unlike the first book which was written by several authors. this is the work of one man who is closely associated with the development of many forms of gyroscopic application in many different spheres. It is believed to be one of the very few books that embraces in one cover as many of the different types of application as possible ranging from small intricate aircraft instruments to large engineering projects, such as the roll stabilizers for 40 000 ton liners.

The Physics of Particle Size Analysis

217 pp. Royal 8vo. The British Journal of Applied Physics Supplement No. 3. The Institute of Physics. 1954. Price 35s.

THIS supplement presents the papers of the eight sessions of a conference held in the University of Nottingham from 6-9 April, 1954. Comments are included at the end of each paper, together with the authors' replies.

LETTERS TO THE EDITOR

(We do not hold ourselves responsible for the opinions of our correspondents)

Feedback Amplifiers

Dear Sir,—Despite the opening remarks in his letter in the March issue. Mr. Moorby makes a rather confusing analysis of the simple feedback amplifier.

First, the symbol Z_1 is used to denote two entirely different impedances (Figs. 2 and 3) and then in deriving the expression for g, e_{in} and E_{in} are interchanged although they are totally different voltages. Z_2 is simply left by the wayside! Also, the terminals A-B in Fig. 3 cannot be regarded as the input terminals of the circuit.

However, it so happens that the Z_2 of Fig. 2 can often be omitted without modifying the overal gain appreciably (though not in an exact analysis—and Mr. Moorby makes no assumptions regarding the value of Z_2). This is easily shown as follows with reference to Fig. 4.

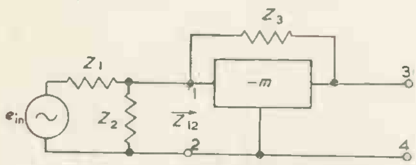


Fig. 4. Rearrangement of feedback amplifier

From Fig. 4 $Z_{12} = Z_3\ (1+m)$. This follows since the voltage across Z_3 is $(1+m)e_{12}$ and the input current due to e_{12} is therefore $\dfrac{(1+m)e_{12}}{Z_3}$

Now Z_{12} is normally a very low impedance which allows Z_2 (being in shunt with Z_{12}) to be neglected in many cases. The overall gain (g) then becomes:

$$g \simeq -m\left(\frac{Z_{12}}{Z_1 + Z_{12}}\right) \quad \ldots\ldots\ldots\ldots (1)$$

$$\doteq \frac{-m}{Z_1(1+m)/Z_3+1} \quad \ldots\ldots (2)$$

If Z_2 is not much larger than Z_{12} it is a simple matter to modify the value of Z_{12} in equation (1) to take account of the shunting effect of Z_2. This gives an exact expression for g:—

$$g = \frac{-m}{(1/Z_2+(1+m)/Z_3)} \quad \ldots\ldots\ldots (3)$$

In many cases Z_2 may be chosen to provide the correct d.c. conditions without appreciably affecting the overall gain; Z_1 and Z_3 being chosen to fix the overall gain and with regard to loading of the source e_{in}, loading of the output or by frequency response effects. In some cases Z_1 may include or consist entirely of the source impedance of the generator e_{in}, in which case only Z_3 need be determined to fix the gain.

No assumptions regarding the value of Z_3 need be made since the value of m can be modified to take Z_3 into account; Z_3 behaving as a load of $m/1+m\ Z_3$ on the amplifier. Usually $m \gg 1$ and this approximates to Z_3.

Turning to output impedance, Mr. Moorby's remarks concerning maximum power output may be very confusing to the inexperienced. He implies that the maximum power transfer theorem does not hold for feedback amplifiers. If e_{in} is fixed at a sufficiently low value, so that the amplifier is never overloaded, and the load impedance (Z_L) varied, then maximum power output is obtained when $Z_L = Z_O$ regardless of the application of feedback. It is true, of course, that maximum available power is obtained with Z_L equal to the appropriate optimum (not necessarily R_a) and with e_{in} increased to fully load the stage, but a linear analysis made without regard to purely practical limitations such as available current swing, d.c. operating points, etc.. cannot take this into account and one could define Z_O as being equal to the load required for maximum power output—"linear operation" understood.

Such operation is of course taken for granted in Mr. Moorby's own analysis, where he considers G_O as "the overall mutual conductance of the amplifier at the short-circuited output terminals." This measurement would require a very low value of e_{in}, and must be done in a manner which will not disturb the d.c. operating conditions.

Mr. Moorby points out a useful approach in quoting $Z_O = g_\infty/G_O$ (I assume gD was a misprint for g_∞) but he does not make clear that G_O is the ratio of the short-circuit output current to e_{in} and that it depends on Z_1, Z_2 and Z_3. In fact he falls into his own trap in his last example when he quotes $G_O = G_m$ which is obviously not true. I am also puzzled in this example as to the significance of n_∞ (again I assume $m = 22.3$ is a misprint for $n = 22{\cdot}3$). This figure is used instead of g in calculating Z_O; G_O being incorrectly stated as $= G_m$.

G_O is easily derived with reference to Fig. 5.

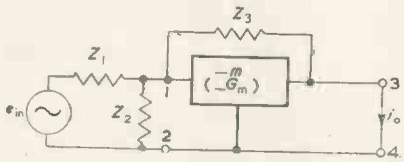


Fig. 5. Derivation of G_O

$$G_O = i_o/e_{in} = \frac{-e_{12}(G_m - 1/Z_3)}{e_{in}}$$

$$= -\frac{(G_m - 1/Z_3)}{1 + Z_1/Z_3 + Z_1/Z_2}$$

$$\approx -\frac{G_m}{1 + Z_1/Z_3 + Z_1/Z_2}$$

(since in general $G_m \gg 1/Z_3$)— (4)

It is interesting to note that $Z_O\ (= g/G_O)$ is a function of Z_2 unless $Z_2 \gg Z_1$ whereas the overall gain may be substantially independent of Z_2 even when $Z_2 < Z_1$ providing $Z_2 \gg Z_3/(1+m)$, which is often true.

Finally, referring back to Mr. McDonnell's letter in the December issue, it will be seen that for the simple "see-saw" phase-splitter (anode-follower) where $Z_1 = Z_3$ and $Z_2 \gg Z_1$ then providing $m \gg 1$ so that $g \simeq 1$ then $Z_O = g/G_O = 1/G_m/2 = 2/G_m$, assuming the output is taken direct from the anode. It will be seen that the presence of the two R's in Mr. McDonnell's less usual circuit modifies G_O so that this expression does not hold.

Yours faithfully,
C. W. Ward,
Plymouth,
Devon.

Amplifier Output Impedance

Dear Sir,—I regret that in reading the proofs of my letter in the March issue, I missed some small errors, mainly in suffixes. May I therefore briefly correct the basic formulae.

(1) Suffixes, o refers to zero load, ∞ refers to infinite load, f to a finite (non-zero) load.

(2) m is the gain of the amplifier measured from the grid to the "hot" end of the load, i.e. for a single stage, from grid to anode. g is the gain from the input terminal to the load. Obviously, if the impedance between the grid and input terminal is negligible, $m = g$. Please note, that to agree with Fig. 3, in the derivation of equations for m, n, ∞, g, the first equation in the last paragraph of column one should read

$$\frac{1 - E_{in}}{Z_1} = \frac{m-1}{Z_2}$$

We then get

(a) $g = \dfrac{mn}{1+n-m}$ (b) $n = \dfrac{g(m-1)}{g-m}$

(c) $m = \dfrac{g(n+1)}{g+n}$

I should like to modify my definition of the distinction between voltage and current feedback, as I find some small ambiguities in my original statement. I think that to get a true definition, we must proceed as follows.

Consider Fig. 4, which is a development of Fig. 3 of my original letter. Here, e A.B. is the input, and e 3·4 the output, and 1 is the grid of the first stage of the amplifier since $g = e_{3.4}/e_{AB}$ and $m = e_{3.4}/e_{1.2}$ both of which may be measured. We can determine $n = \dfrac{g(m-1)}{g-m}$ (ante (b)).

In the case of current feedback, n varies with load impedance, while in the case of voltage feedback, n is independent of load impedance. This I now consider is the only concise and sufficiently rigorous definition available.

In my earlier letter I put $G_o = G$. This is incorrect as $G = G\dfrac{n}{1+n}$ as will be obvious if we consider the anode earthy. The grid potential is then $E_{in} = \frac{n}{1+n}$ whence $G_o = G\frac{n}{1+n}$. This gives $Z_o = \frac{-m_\infty(n+1)}{G(-m_\infty+n+1)}$

Perhaps, having reduced the problem by the equations, to which we add $Z_o = g\infty/G_o$, we may assume that the distinction is now dedundant, and that "feedback" alone is a sufficient statement, since if we attempt a definition in general terms, it is not always useful, and all we

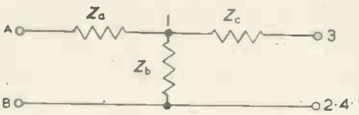


Fig. 4. Development of Fig. 3 (April)

should really be concerned with is overall gain (g) and output impedance (Z_o).

Yours faithfully,

H. MOORBY,

Farnborough, Hants.

The Effect of Instrument Impedance in the Measurement of "Quality Factor" by Parallel Resonance

DEAR SIR,—When making electrical measurements, instrument impedances usually affect circuit behaviour, however slightly. Here is one case where they have no overall effect whatever. In the circuit shown the aim is to find Q the Quality Factor for the imperfect physical inductance L. An a.c. voltage-generator maintains a constant voltage output E of any chosen angular frequency, ω. C represents the setting of a decade capacitance box and R that of a non-inductive decade resistance box. The losses in the capacitance C are assumed to be negligible compared with those of inductance L.

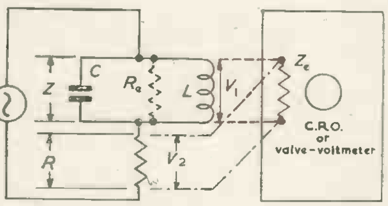


Fig. 1. Measurement of Q

With the measuring instrument out of circuit and at the resonant frequency $\omega_n = 1/\sqrt{(LC)}$, the impedance Z is purely resistive and equal to R_e the loss resistance of L at frequency ω_n.

Quality Factor is defined at $Q = L\omega_n/R_e$. If V_e and V are the true voltages across R_e and R respectively

$$R_e = R.\ V_e/V \quad \text{(1)}$$

The Quality Factor Q is, therefore,

$$Q = (L\omega_n/R)(V/V_e) \quad \text{(2)}$$

As $L\ \omega_n$ and R are known, Q can be computed from equation (2) if the voltage ratio V/V_e is known.

To measure the ratio of voltages across R_e and R, an instrument such as a cathode-ray oscillograph or a valve-voltmeter might be used. Whatever the instrument, it will in general have an input impedance Z_c having both reactive and non-reactive components. The connexion of the instrument across R_e and R in turn as shown by dotted and chain-dotted lines will result in the recording of voltages V_1 and V_2 which will differ from V_e and V respectively. It is shown below that provided the circuit is resonated by some means without the measuring instrument in circuit

$$V_2/V_1 = V/V_e \quad \text{(3)}$$

If the resonance at frequency ω_n is detected by the measuring instrument by noting when the adjustment of C makes V_1 a maximum, then the above result still holds providing Z_c is non-reactive.

It can be seen from equations (2) and (3) that

$$Q = (L\omega_n/R)(V_2/V_1) \text{ for all values of } Z_c \quad \text{(4)}$$

This is proved as follows. In the dotted position for measuring V_1, the line-current i_1 is

$$i_1 = \frac{E}{\frac{1}{(1/R_e + 1/Z_c)} + R}$$

$$\therefore V_1 = \frac{\frac{E}{(1/R_e + 1/Z_c)}}{\frac{1}{(1/R_e + 1/Z_c)} + R} = \frac{E}{1 + R(1/R_e + 1/Z_c) + 1}$$

In the chain-dotted position for measuring V_2 the line-current i_2 is

$$i_2 = \frac{E}{R_e + \frac{1}{(1/R + 1/Z_c)}}$$

$$\therefore V_2 = \frac{\frac{E}{(1/R + 1/Z_c)}}{R_e + \frac{1}{(1/R + 1/Z_c)}} = \frac{E}{R_e(1/R + 1/Z_c) + 1}$$

Hence

$$V_2/V_1 = \frac{1 + R(1/R_e + 1/Z_c)}{R_e(1/R + 1/Z_c) + 1} = R/R_e\left[\frac{R_eZ_c + R(Z_c + R_e)}{R_e(Z_c + R) + RZ_c}\right] = R/R_e \quad \text{(5)}$$

By comparing equations (1) and (5) equation (3) is proved. Hence equation (4) follows from equations (2) and (3).

Yours faithfully,

J. P. DUNCAN,

Engineering Department,
University of Manchester.

Computers and Computors

DEAR SIR,—I am writing with regard to recent letters on the spelling of "computer/or". I don't think the matter worth prolonging in print, as my point of view is not altogether one of great importance to the practising computer/or engineer.

The suffix -er is the normal English suffix for the agent, thus:

Computer, one who or that which computes.

Commenter, one who comments.

The suffix -or is the normal Latin suffix for the agent, thus:

Computator, from "computare" = computer,

Commentator, from "commentari" = commenter,

Commentor, from "comminisceri" an inventor,

usually in the sense of a faker, but never adopted into English. I cannot agree that "commentor" is a correct form of "commentator". It has been used in this sense, as a variant spelling of "commenter", see the OED, but, to quote A. P. Herbert, the dictionary "is a pusillanimous work, preferring feebly to record what has been done rather than stating firmly what should be done". Notwithstanding this, the OED will have nothing of "computor"—"bad spelling of 'computer'", and from the purely classical standpoint one has to agree.

The case for "computor", I therefore take it, is not one of good philology but necessity for the distinction to be invented where none previously existed. I submit that there is still no necessity. If a context is such as to leave one in any doubt, it should be rewritten, rather than having to rely on an inaudible distinction to make the matter clear.

(I should, parenthetically, remark that "computor" is not, nor is it ever made out to be, part of the systematic terminology that has given us such philological horrors as "impedance", which if it had to be a Latin word with a Latin tail would have been "impedience"—compare "expedient".)

As regards a distinction which some claim by which -er is a person and -or a thing, I know of no other example, except that I suppose that the wartime underground movement wasn't made up of resistors! Indeed, I am not aware of any significant use of the suffix -or in English (as opposed to in words of Latin origin taken over intact) except in the electronics use of "a component embodying -ance", and it is manifest that "computor" is indefensible on this ground.

Perhaps there is a case for the defence of "computor"—if so I should like to hear it. I have a difficult enough time in the Cambridge computer world persuading people that "program(me)" doesn't mean anything like what the Greeks meant by it—a public notice—but what the French adapted it to mean, and so that there is a case for continuing to spell it as the French do and most of the English, if not the Americans.

Yours faithfully,

JOHN LEECH.

King's College,
Cambridge.

Several other letters on this subject have been received but we feel that the one printed above summarizes the matter fairly and this correspondence is now closed. In view of the arguments put forth (and with which Mr. Puckle, our original contributor on this subject, is now in agreement) the spelling in this journal will be changed to computer.—EDITOR.

Short News Items

The Governors of The College of Aeronautics announce the creation of a Department of Aircraft Electrical Engineering and the appointment of Mr. G. A. Whitfield as Professor and Head of Department. Mr. Whitfield will take up his appointment on 1 June. He is at present head of the Controlled Weapons Division in the Armament Department at the Royal Aircraft Establishment, Farnborough. This appointment is in fulfilment of the Governors' policy of expanding the Electrical Section of the Department of Aircraft Design into a full teaching department.

Metropolitan-Vickers Electrical Co Ltd has endowed four Post-Graduate Bursaries in Electrical and Mechanical Engineering, each of the value of £400 per annum, in the University of Manchester. Two of these bursaries will normally be tenable in the Faculty of Technology of the University at the Manchester College of Technology. The Company has also endowed three similar bursaries tenable in the Mechanical and Electrical Engineering Departments at the Imperial College of Science and Technology, London. A particular purpose of these bursaries is to encourage graduates in physics and mathematics to undertake post-graduate study or research in engineering in preparation for entry into the electrical industry.

The Royal Society recently elected 25 persons into the Fellowship, among whom were the following. D. R. Bates, Professor of Applied Mathematics, The Queen's University, Belfast. S. Devons, Professor of Physics, Imperial College of Science and Technology, London. A. C. B. Lovell, Professor of Radio Astronomy, The University of Manchester.

Export Packing Service Ltd have opened a new Packing Design and Development Department at the Chipping Warden centre. There are depots at Banbury, Sittingbourne, Merthyr Tydfil and Cardiff.

Aero Research Ltd of Duxford, makers of "Araldite" epoxy resins, state that greatly increased output of these products has made it possible to reduce manufacturing costs and, with them, the prices at which "Araldite" can be purchased. These reductions apply to nearly all forms of the resin.

St. Erik's Fair, Stockholm. will be held this year from 27 August-11 September. Last year twenty-nine countries participated, the average attendance being 400 000, including 80 000 trade buyers.

The British Plastics Convention will be held at the National Hall, Olympia, from 1-11 June. Details may be obtained from the Press Officer, British Plastics Federation, 11 Garrick Street, London, W.C.2.

The Board of Extra-Mural Studies, University of Cambridge, announce that a Summer School in programme design for automatic digital computing machines will be held in the University Mathematical Laboratory at Cambridge during the period 12-23 September. It will be along the same lines as those held previously. A detailed syllabus and form of application for admission may be obtained from Mr. G. F. Hickson, Secretary of the Board of Extra-Mural Studies, Stuart House, Cambridge, to whom the completed application form should be returned by 15 June.

The University of Cambridge is also arranging a third course for managers in industry. It will be held in Madingley Hall from 27 June to 22 July, and will be open to managers who have some years' experience in industry and commerce and are under the age of 40. Firms can obtain details from Mr. G. F. Hickson at the address mentioned above.

The Radio Industry Council announce that exports of British radio equipment reached new high levels in 1954. The provisional value is over £29 100 000. This is £3 300 000. more than 12 per cent, higher than the value for 1953, the previous highest figure.

W. Bryan Savage Ltd., have appointed John Ould Ltd., of 389 Fifth Avenue, New York 16, N.Y., to act as their sole concessionaires for the United States of America. This new British enterprise has been formed to operate as a sales organization for manufacturers of electronic and allied equipment in this country. American inquiries for W. Bryan Savage Ltd. products should be sent to Mr. Howard M. Layton, Sales Director, at the address mentioned.

The BBC announces that the first of its new v.h.f. transmitting stations, using Frequency Modulation, will be brought into service at Wrotham, Kent, on 2 May. The new station will transmit the three BBC programmes, the Light Programme on a frequency of 89·1Mc/s, the Third Programme on 91·3Mc/s, and the Home Service on 93·5Mc/s. The effective radiated power will be 120kW in each case, and the transmissions will be horizontally polarized. The station will serve the London area and South-East England within a range of about fifty miles.

Mr. Isaac Shoenberg has been appointed a Director of Electric and Musical Industries Ltd. Mr. Shoenberg has been eminent in the research and development of radio for fifty years. He has been with E.M.I. since its formation in 1931. Last year he was awarded the Faraday Medal for his outstanding contribution to the development of High Definition Television.

The Ministry of Supply announces that Mr. C. J. Carter has been promoted to Deputy Chief Scientific Officer and has been appointed Director of Instrument Research and Development.

Air Marshal Sir Victor Goddard, K.C.B., C.B.E., has joined Elliott Brothers (London) Ltd as an Assistant (Air) to the Managing Director.

Mr. R. L. S. Saunders has been appointed Assistant Engineer-in-Chief of Cable and Wireless Ltd.

Mr. J. P. A. Meldrum and Mr. R. H. S. Turner have been appointed Directors of Metropolitan-Vickers Electrical Co Ltd.

The Independent Television Authority have made the following appointments. Mr. A. M. Beresford-Cooke, Senior Planning Engineer. Mr. R. C. Harman, Superintendent Engineer (Operations and Maintenance). Mr. W. N. Anderson, Senior Lines Engineer.

Mr. H. T. Greatorex is now Assistant Head of the Engineering Information Department of the BBC. Mr. T. P. Douglas has been appointed Engineer-in-Charge of the Sutton Coldfield Television Station.

Mullard Ltd exhibited electronic equipment and electron tubes at the recent exhibition of the Institute of Radio Engineers in New York. Some of the exhibits represented the latest developments of British electronic research.

The Council of the Radio and Electronic Component Manufacturers' Federation recently elected Mr. C. M. Benham as Chairman in succession to Mr. W. F. Randall. Mr. Benham is Managing Director of Painton & Co Ltd, Kingsthorpe, Northampton. Mr. S. H. Brewell, Chairman and Managing Director of A. H. Hunt Capacitors Ltd, was elected Vice-Chairman. Mr. Hector V. Slade remains Honorary Treasurer.

Erratum. In the description of "A Very High Speed Camera" on page 153 of the April issue the frame rates given in the table should read 2×10^6, 1×10^6, etc., and not 2 × 106, 1 × 106, etc.