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## Commentary

THE story of the wartime collaboration between American and British scientists in the successful development of the atomic bomb is well known.

It is now a matter of history that at the beginning of the war the possibility of an atomic weapon loomed very prominently in the strategy of the day and after some preliminary studies a committee, disguised under the name of Directorate of Tube Alloys, was set up to further this project.

Parallel work had been going on in America and after a visit of American scientists to this country in 1941 it was later decided that the whole of this work on the development of the atomic bomb should be concentrated in America. The entire British team was sent over, and no further development work was done in this country. So began the joint British, Canadian and American collaboration which continued until the end of the war.

With the cessation of hostilities and the uneasy years which followed America deemed it unwise for her to continue this wartime collaboration, and in 1946 she passed the McMahon Act which virtually brought matters to an end.

While the fundamental principles on which the atomic bombs were based are well known, the actual methods by which the bombs had been produced—the "know how"—continued to remain the secret of the United States. British scientists had, however, contributed much to the original wartime research and working on their own since 1946 have developed atomic weapons of advanced types which are now in production.

All this, too, is well known and the layman can be forgiven if he associates the production of atomic energy only with weapons of mass destruction. The very complexities of this new science make it difficult to appreciate that the enormous energy which can now be released from the nucleus of the atom, far from wiping out civilization can, in a saner world, bring untold comfort and well being to the human race.

It was obvious that a highly industrialized country like Great Britain could not afford to ignore the promising industrial applications of this new form of energy for apart from other considerations, the industrial prosperity of this country is based on coal, adequate supplies of which have already become increasingly difficult and costly to obtain.

Accordingly, towards the end of 1945 the Government decided to set up a research and development establishment at Harwell covering all aspects of atomic energy and this was shortly followed by a Division of Atomic Energy Production "to make available as rapidly as possible material in sufficient quantity to enable us to take advantage rapidly of technical developments as they occur."

Thus was born a vast new industry involving the expenditure of hundreds of millions of pounds and employing thousands of workers in it. It is the industry engaged in the production of fissile material and the story of its birth and early struggles is told for the first time in a fascinating booklet entitled "Britain's Atomic Factories", which has just been published by the Stationery Office.

Much of the work of the Division of Atomic Energy Production still remains secret but "now that we have moved on from the purely research stage into that of regular manufacture", writes Mr. Duncan Sandys, the Minister of Supply, in a foreword to this book, "it is right that the world should be told how fissile material is today being produced in Britain and of the remarkable factories and plant which have been designed and erected to carry out these strange processes."

Strange processes and remarkable factories they are without question, for few of us have the intimate knowledge fully to understand the exact methods by which the fissile materials are produced and "Britain's Atomic Factories" does not set out to enlighten us, but the reader cannot fail to be thrilled as the story of the growth of these enormous laboratories—factories seems an inappropriate word—is told.

It is the story of the task presented to Sir Christopher Hinton—merely a brief instruction to produce annually a certain quantity of plutonium to a given specification in water-cooled piles—and how this task has been successfully accomplished with the bringing into being of the Production Division at Risley and the factories at Springfield, Windscale and Capenhurst.

The production of fissile material may be said to have begun on 4th February, 1946, when the nucleus of the Production Division was formed at Risley and a total staff of twelve moved into their new headquarters. How modest was this beginning may be judged from the fact that of this staff only one member had had previous experience in the development of atomic energy and almost one of the first tasks was to instruct him to explain to the remaining members in the simplest of terms what atomic energy was. Yet from these humble beginnings the development of atomic energy has grown apace—not, of course, without many setbacks which are frankly discussed in the book—but today it is fair to claim that this country is again in the forefront.

# Some Steady-State Characteristics of Short Irregular Lines

By A. Rosen*, A.C.G.I., Ph.D., M.I.E.E.,

*The distribution of irregularities of impedance along a line is represented by a Fourier series, and relations are obtained between the input-impedance deviations and the Fourier coefficients. These are considered in detail at the resonant frequencies, with particular application to the short line; when the input-deviations measured at the two ends are available, their relations to the Fourier coefficients are specially simple, and permit the distribution of impedance irregularities along the line to be synthesized. The measurement of the complex deviations, and the effects of the line irregularities on the open- and short-circuit impedances are considered. Finally the measurement of mean impedance and attenuation coefficient is dealt with, and a brief description is given of a bridge having variable capacitance ratio arms, which is used for testing factory lengths of coaxial cable.*

THE theory of the nominally homogeneous line having small impedance irregularities distributed along its length has been studied extensively[1-6], because of its importance in the transmission of wideband telephony and television through coaxial cables. These cables are usually manufactured in lengths of the order of 0·1 mile, and in this article some of the more important electrical characteristics of such factory lengths are considered, the treatment being confined to the steady-state properties.

So far, one type of coaxial pair has been standardized in this country and abroad for long-distance transmission, and pulse-echo measurements[7,12] have largely superseded the older steady-state tests for the routine measurements of irregularities on such cables. The pulse tests, however, are not yet standardized, and, moreover, they require expensive simulating networks particular to each type of cable. Thus there is still an important field for steady-state irregularity measurements, e.g. in checking and correlating pulse-echo tests on standardized cables and for measuring experimental lengths of a type for which simulating networks are not available.

The disturbing effect of impedance irregularities is due to echoes, produced by double reflexions, which accompany the main signal. A pulse method is available for assessing the doubly-reflected echoes in a short line[10], but no method has so far been devised for measuring these effects in the steady state. However, the irregularities in short lengths are adequately characterized by their single-reflexion effects, and very nearly all factory lengths of coaxial cable are assessed on this basis.

**List of Symbols**

(*The* M.K.S. *System of Units is Employed*)

$j = \sqrt{-1}$
$f$ = frequency
$f_M$ = frequency in megacycles per second
$\omega = 2\pi f$
$C_u$ = capacitance per unit length of uniform line
$L_u$ = inductance per unit length of uniform line
$l$ = length of line
$x$ = distance of point x from sending end
$Z$ = impedance
$R$ = resistive component of $Z$
$X$ = reactive component of $Z$
$Z_m$ = mean characteristic impedance
$\phi_m$ = angle of $Z_m$
$Z_r$ = reference impedance
$\gamma$ = mean propagation coefficient per metre
$\alpha$ = mean attenuation coefficient per metre
$\beta$ = mean phase-change coefficient per metre
$p$ = reflexion coefficient
$\delta Z$ = input-impedance deviation relative to $Z_r$
$\delta r$ = resistive component of $\delta Z$
$\delta x$ = reactive component of $\delta Z$
$\Delta Z$ = input-impedance deviation relative to $Z_m$
$\Delta r$ = resistive component of $\Delta Z$
$\Delta x$ = reactive component of $\Delta Z$
$\sigma$ = deviation of local characteristic impedance from $Z_r$
$S$ = deviation of local characteristic impedance from $Z_m$
$s$ = order of term in Fourier series
$H_s$ = coefficient of sine term in Fourier series
$J_s$ = coefficient of cosine term in Fourier series
$n$ = order of resonant frequency
$k_1, k_2$ = correction terms given by equations (21) and (22)
$M, N$ = factors given by equations (37) and (38)
$\delta f$ = small change of frequency
$\theta$ = small change of phase-change coefficient
$K$ = factor modifying open- and short-circuit impedances
$r$ = difference of "resistance" readings on impedance bridge

SUBSCRIPTS

AB pertaining to line AB
A or B pertaining to end A or B of line AB
x pertaining to point x
r pertaining to reference impedance
m pertaining to mean impedance
op pertaining to line with far end open-circuited
sh pertaining to line with far end short-circuited
n pertaining to order of resonance
u pertaining to unit length of uniform line

ACCENTS

′ pertaining to odd resonant frequency
″ pertaining to even resonant frequency

* *Formerly Siemens Brothers and Company Limited, now with British Insulated Callender's Cables Ltd.*

**Definitions**

The short line with which we are concerned in this article is defined as one for which the overall attenuation coefficient is small, say not greater than 0·2 neper, although the line may be several wavelengths long. Consider such a short line AB, of length $l$, having small impedance irregularities distributed along it. If $Z_X$, $\gamma_X$ are the respective values of the local characteristic impedance and the propagation coefficient per unit length at a point distant $x$ from the sending end, then the mean characteristic impedance $Z_m$ is defined as

$$Z_m = \frac{1}{l}\int_0^l Z_X dx \quad \ldots\ldots\ldots\ldots (1)$$

and the mean propagation coefficient $\gamma$ per unit length is

$$\gamma = \frac{1}{l}\int_0^l \gamma_X dx \quad \ldots\ldots\ldots\ldots (2)$$

The "reference impedance" is an arbitrary impedance close to $Z_m$, and is defined precisely in another paper[13]; here it is the value of the impedance used for terminating the short line. The input-impedance, $Z_{AB}$, is defined as the impedance measured at A when the line is terminated at B by the reference impedance.

The reflexion coefficient, which expresses the relation between the emitted wave and the reflected voltage at the sending end, is also defined precisely elsewhere[13]; for the present purpose, it is sufficient to define the reflexion coefficient $p_{AB}$ by the relation

$$p_{AB} = (Z_{AB} - Z_r)/(Z_{AB} + Z_r) \quad \ldots\ldots\ldots (3)$$

The input-impedance deviation of AB with respect to $Z_r$, $\delta Z_{AB} \equiv \delta r_{AB} + j\delta x_{AB}$, is defined as the difference between $Z_{AB}$ and $Z_r$. Thus

$$p_{AB} = \delta Z_{AB}/(Z_{AB} + Z_r) \quad \ldots\ldots\ldots\ldots (4)$$

and since we are concerned only with small irregularities:

$$p_{AB} = \delta Z_{AB}/2Z_r \quad \ldots\ldots\ldots\ldots (5)$$

The basic equation in the theory of irregular lines was first given by Didlaukis and Kaden[1]. It may be stated as[13]

$$p_{AB} = \frac{\gamma}{Z_r}\int_0^l \sigma_X \exp(-2\gamma x)dx \quad \ldots\ldots\ldots\ldots (6)$$

where

$$\sigma_X = Z_X - Z_r \quad \ldots\ldots\ldots\ldots (7)$$

Thus

$$\delta Z_{AB} = 2\gamma\int_0^l \sigma_X \exp(-2\gamma x)dx \quad \ldots\ldots\ldots\ldots (8)$$

When the mean impedance is selected as reference impedance, we denote the impedance deviation by $\Delta Z_{AB} \equiv \Delta r_{AB} + j\Delta x_{AB}$, and we have

$$\Delta Z_{AB} = Z_{AB} - Z_m$$

$$= 2\gamma\int_0^l S_X \exp(-2\gamma x)dx \quad \ldots\ldots\ldots\ldots (9)$$

where

$$S_X = Z_X - Z_m \quad \ldots\ldots\ldots\ldots (10)$$

Whence

$$\Delta Z_{AB} = \delta Z_{AB} + (Z_r - Z_m)\left\{1 - \exp(-2\gamma l)\right\} \quad . \ (11)$$

[It should be noted that Didlaukis and Kaden[1] and some other writers have used $\Delta Z$ for the *real* part of the input-impedance deviation; in this article, $\Delta Z$, $\delta Z$, are *complex* quantities, as defined above.]

Particular reference is made below to the "104/375" type of coaxial pair, which has been standardized by the British Post Office[11]. This consists of an inner solid copper conductor 0·104in in diameter, insulated with polythene disks at intervals of about 1·3in, with an outer tubular copper conductor having an internal diameter of 0·375in and a thickness of 0·010in. This type of coaxial pair is at present operated within the frequency range 0·06 to 4·5Mc/s, and in this band of frequencies, its attenuation and phase-change per mile are given by the following:

$$\alpha = 0{\cdot}432\ f_M^{\frac{1}{2}} \text{ neper/mile}$$

$$\beta = 34{\cdot}8\ f_M + 0{\cdot}4\ f_M^{\frac{1}{2}} \text{ radian/mile}$$

where $f_M$ is the frequency in Mc/s. The characteristic impedance is given very closely by the expression:

$$Z_m = 74{\cdot}4 + 0{\cdot}9(1 - j)f_M^{-\frac{1}{2}} \text{ ohms.}$$

**Length Distribution of Impedance as a Fourier Series**

The single reflexions from any line can be predetermined over a range of frequencies by means of equations such as (6) providing the length-distribution of impedance is known in detail along the whole line, but such computations are extremely laborious. Cotte[6] has suggested that the impedance-distribution may be represented by a Fourier series, and this generalization enables us to make some useful deductions regarding the properties of lines, particularly when they are short, without imposing any restriction on the distribution of the irregularities; moreover, as shown below we can readily measure the coefficients of the Fourier series for a particular short line and thereby synthesize its impedance-distribution curve.

For any line of length $l$ we may write for $0 < x < l$

$$Z_X = Z_m + \sum_{s=1}^{\infty}\left(H_s \sin\frac{2s\pi x}{l} + J_s \cos\frac{2s\pi x}{l}\right) \ldots\ldots (12)$$

In general the coefficients $H_s$, $J_s$, are complex quantities which vary with frequency. If, however, the line considered is a coaxial pair with small irregularities such as occur in practice, the chief factors causing impedance deviations are local variations of conductor dimensions and of effective permittivity; these produce first order changes in the resistive component of the impedance and only second order changes in the reactive component, which can be neglected. In these circumstances the Fourier coefficients $H_s$, $J_s$, may be regarded as being wholly real quantities; furthermore, provided the frequency is not too low, the coefficients are sufficiently independent of frequency for practical purposes. By combining equations (12) and (8) we obtain

$$\delta Z_{AB} = (Z_m - Z_r)\left\{1 - \exp(-2\gamma l)\right\}$$

$$+ 2\int_0^l \exp(-2\gamma x)\sum_{s=1}^{\infty}\left(H_s \sin\frac{2s\pi x}{l} + J_s \cos\frac{2s\pi x}{l}\right)dx \ \ldots (13)$$

$$\text{Since} \int \exp(ax)\sin bx\, dx = \frac{\exp(ax)(a\sin bx - b\cos bx)}{a^2 + b^2}$$

$$\int \exp(ax)\cos bx\, dx = \frac{\exp(ax)(a\cos bx + b\sin bx)}{a^2 + b^2}$$

we have

$$\delta Z_{AB} = \left\{1 - \exp(-2\gamma l)\right\}\left[Z_m - Z_r + \gamma l\sum_{s=1}^{\infty}\frac{s\pi H_s + \gamma l J_s}{\gamma^2 l^2 + s^2\pi^2}\right] \ldots\ldots (14)$$

The input-impedance deviation is thus composed of two parts: an external component which depends on $Z_m - Z_r$, i.e. on the difference in level between the mean impedance and the reference impedance, and an internal component which depends only on the internal irregularities of the line. When the reference impedance is equal to the mean impedance we have

$$\Delta Z_{AB} = \left\{1 - \exp(-2\gamma l)\right\}\gamma l\sum_{s=1}^{\infty}\frac{s\pi H_s + \gamma l J_s}{\gamma^2 l^2 + s^2\pi^2} \ldots\ldots (15)$$

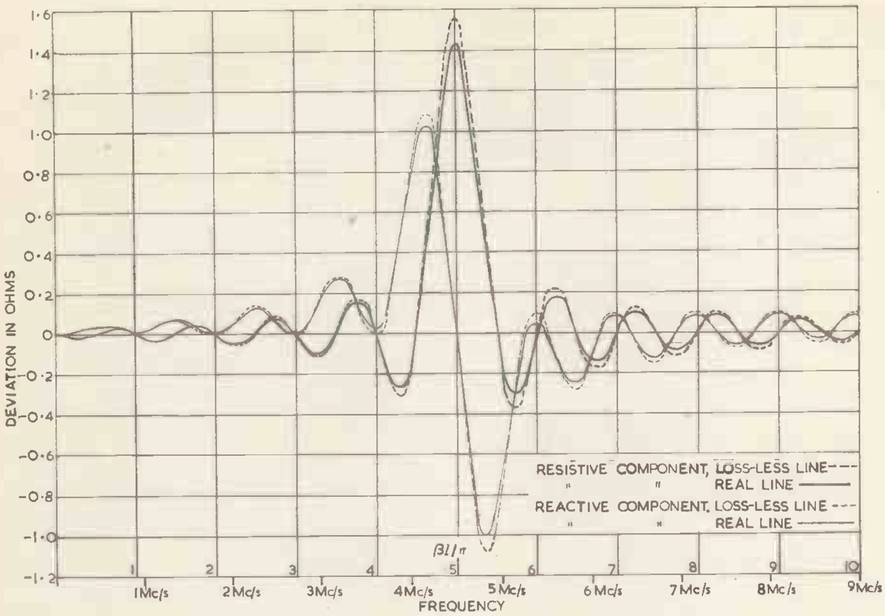


Fig. 1 (a). 0·1 mile, 104/375 coaxial pair

$$S_x = 0{\cdot}1 \sin \frac{10\pi x}{l}$$

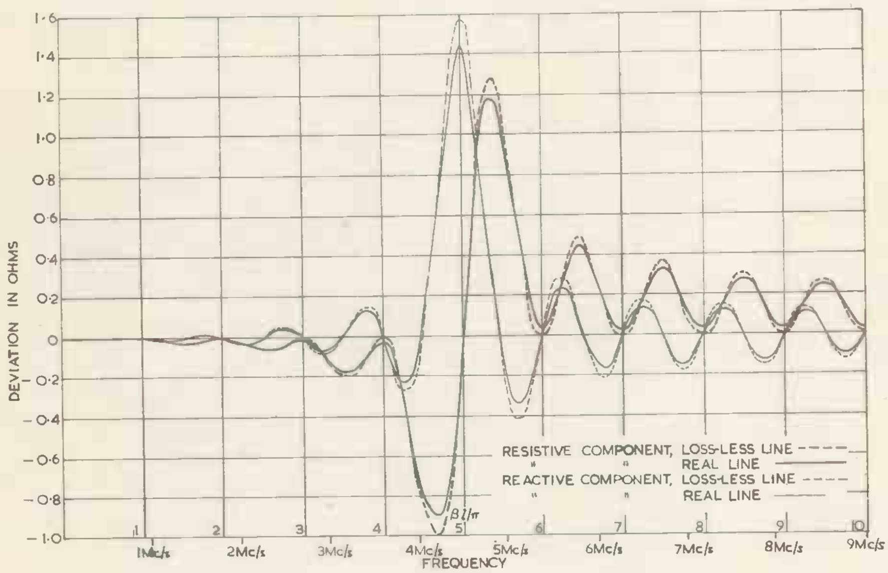


Fig. 1 (b). 0·1 mile, 104/375 coaxial pair

$$S_x = 0{\cdot}1 \cos \frac{10\pi x}{l}$$

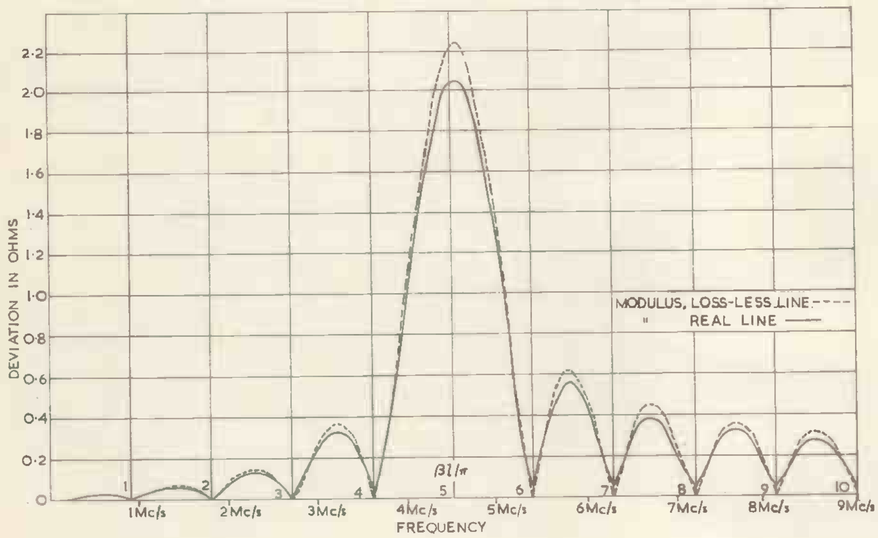


Fig. 1 (c). 0·1 mile, 104/375 coaxial pair

$$S_x = 0{\cdot}1 \sin \frac{10\pi x}{l} + 0{\cdot}1 \cos \frac{10\pi x}{l}$$

To obtain an idea of the contributions of the various terms we may consider a line in which the irregularities consist of pure sine and cosine waves of a single periodicity; thus in Fig. 1, $\Delta Z$ is shown for the case for which $S_x = H_s \sin 10\pi x/l + J_s \cos 10\pi x/l$, the contributions of the sine and cosine terms being drawn separately. The line is 1/10 mile of 104/375 coaxial cable, and $\Delta Z$ is shown for the cases where attenuation is neglected and where it is taken into account. It will be seen that $\Delta Z$ reaches a maximum at the even resonant frequency corresponding to $2\beta l = 10\pi$, and is very small at all the other even resonant frequencies. The resonant frequencies display features of special interest and are considered below; for convenience, quantities related to the odd and even resonant frequencies are distinguished by single and double primes respectively.

INPUT-IMPEDANCE DEVIATIONS AT EVEN RESONANT FREQUENCIES

At even resonance when $2\beta l = n''\pi$, the term in equation (15) for which $s = \frac{1}{2}n''$ predominates over all the others. Thus since since $\alpha \ll \beta$, we have

$$\Delta Z''_{AB} = \left\{1 - \exp(-2\alpha l)\right\}\left[\frac{n''\pi}{4\alpha l}\left((H''_{\frac{1}{2}n} + jJ''_{\frac{1}{2}n}) + \right.\right.$$
$$\left. + \tfrac{3}{4}J''_{\frac{1}{2}n} - \tfrac{1}{4}jH''_{\frac{1}{2}n} + \right.$$
$$\left. + \sum_{\substack{s=1\\(\neq\frac{1}{2}n'')}}^{\infty} \frac{J_s - 2jH_s s/n''}{1-(2s/n'')^2 - 4j\alpha l/n''\pi}\right] \quad \ldots\ldots\ldots\ldots (16)$$

When the line is short, i.e., $\alpha l$ is small,

$$\Delta Z''_{AB} = \tfrac{1}{2}n''\pi(1 - \alpha l)(H''_{\frac{1}{2}n} + jJ''_{\frac{1}{2}n})$$
$$+ 2\alpha l\left[\tfrac{3}{4}J''_{\frac{1}{2}n} - \tfrac{1}{4}jH''_{\frac{1}{2}n} + \sum_{\substack{s=1\\(\neq\frac{1}{2}n'')}}^{\infty} \frac{J_s - 2jH_s s/n''}{1-(2s/n'')^2}\right] \ldots\ldots (17)$$

When $\alpha l$ is very small, the expression simplifies to

$$\Delta Z''_{AB} = \tfrac{1}{2}n''\pi(H''_{\frac{1}{2}n} + jJ''_{\frac{1}{2}n}) \quad \ldots\ldots\ldots\ldots (18)$$

From equation (17), we have for the short line

$$\Delta r''_{AB} = \tfrac{1}{2}n''\pi(1-\alpha l)H''_{\frac{1}{2}n} + k_1 \quad \ldots\ldots\ldots (19)$$
$$\Delta x''_{AB} = \tfrac{1}{2}n''\pi(1-\alpha l)J''_{\frac{1}{2}n} - k_2 \quad \ldots\ldots\ldots (20)$$

where

$$k_1 = 2\alpha l\left[\tfrac{3}{4}J''_{\frac{1}{2}n} + \sum_{\substack{s=1\\(\neq\frac{1}{2}n'')}}^{\infty} \frac{J_s}{1-(2s/n'')^2}\right] \ldots\ldots\ldots (21)$$

$$k_2 = 2\alpha l\left[\tfrac{1}{4}H''_{\frac{1}{2}n} + \sum_{\substack{s=1\\(\neq\frac{1}{2}n'')}}^{\infty} \frac{2H_s s/n''}{1-(2s/n'')^2}\right] \ldots\ldots (22)$$

The terms $k_1$, $k_2$, are generally small, but are not negligible at the higher frequencies. They may be eliminated by combining the deviations measured from the two ends of the line. When considered from the opposite end, the signs of the cosine terms $J_s$ remain unchanged whereas the signs of the sine terms $H_s$ are reversed.

Whence

$$\Delta r''_{BA} = -\tfrac{1}{2}n''\pi(1-\alpha l)H''_{\frac{1}{2}n} + k_1 \quad \ldots\ldots (23)$$
$$\Delta x''_{BA} = \tfrac{1}{2}n''\pi(1-\alpha l)J''_{\frac{1}{2}n} + k_2 \quad \ldots\ldots\ldots (24)$$

Thus

$$H''_{\frac{1}{2}n} = \frac{2\Delta r''_A}{n''\pi(1-\alpha l)} \ldots\ldots\ldots\ldots\ldots (25)$$

$$J''_{\frac{1}{2}n} = \frac{2\Delta x''_A}{n''\pi(1-\alpha l)} \ldots\ldots\ldots\ldots\ldots (26)$$

where

$$\Delta r''_A = \tfrac{1}{2}(\Delta r''_{AB} - \Delta r''_{BA}) \quad \ldots\ldots\ldots\ldots (27)$$
$$\Delta x''_A = \tfrac{1}{2}(\Delta x''_{AB} + \Delta x''_{BA}) \quad \ldots\ldots\ldots\ldots (28)$$

Equations (25) to (28) provide the means for obtaining the Fourier coefficients of a short line from the input-impedance deviations measured at the even resonance frequencies. Note also the relations between the impedance deviations measured from opposite ends of a short line, viz:

$$\Delta r''_{AB} = -\Delta r''_{BA} + 2k_1 \simeq -\Delta r''_{BA} \quad \ldots\ldots (29)$$
$$\Delta x''_{AB} = \Delta x''_{BA} - 2k_2 \simeq \Delta x''_{BA} \quad \ldots\ldots (30)$$

INPUT-IMPEDANCE DEVIATIONS AT ODD RESONANT FREQUENCIES

At odd resonance when $2\beta l = n'\pi$, we have

$$\Delta Z'_{AB} = \left\{1 + \exp(-2\alpha l)\right\}\sum_{s=1}^{\infty}\frac{J_s - 2jH_s s/n'}{1-(2s/n')^2 - 4j\alpha l/n'\pi} \ldots\ldots\ldots\ldots (31)$$

When $\alpha l$ is small

$$\Delta Z'_{AB} = 2(1 - \alpha l)\sum_{s=1}^{\infty}\frac{J_s - 2jH_s s/n'}{1-(2s/n')^2} \ldots\ldots (32)$$

Thus for the short line

$$\Delta r'_{AB} = \Delta r'_{BA} \quad \ldots\ldots\ldots\ldots\ldots (33)$$
$$\Delta x'_{AB} = -\Delta x'_{BA} \quad \ldots\ldots\ldots\ldots (34)$$

It is shown below that the input-impedance deviation can be readily measured at the even resonances, whereas the measurement at the odd resonances necessitates a special network. It may also be noted, however, that for a short line the input-impedance deviations at any odd resonance may be calculated from the values measured at the even resonances. Combining equations (25) and (26) with (32) we have, at the odd resonance $n'$,

$$\Delta r'_{AB} = \sum_{n''=2,4,\ldots} M\Delta x''_{An} \quad \ldots\ldots\ldots\ldots (35)$$

$$\Delta x'_{AB} = \sum_{n''=2,4,\ldots} N\Delta r''_{An} \quad \ldots\ldots\ldots\ldots (36)$$

where

$$M = \frac{4n'^2}{n''\pi(n'^2 - n''^2)} \ldots\ldots\ldots\ldots (37)$$

$$N = \frac{4n'}{\pi(n''^2 - n'^2)} \ldots\ldots\ldots\ldots\ldots (38)$$

and $\Delta r''_{An} + j\Delta x''_{An}$ is the corrected input-impedance deviation at the even resonance $n''$. The factors $M$, $N$, are given in Table 1, and it will be seen that the number of terms required is not large, as the factors diminish rapidly when $n'' > n'$.

## Measurements on Short Lines

MEASUREMENT OF INPUT-IMPEDANCE DEVIATIONS

By definition $\Delta Z$ is the difference between the input impedance and the mean impedance when the line is terminated by the mean impedance. Networks may be constructed which simulate the characteristic impedance of a given type of coaxial cable over a wide range of frequencies[14]; these contain a variable element which permits the network impedance to be adjusted to match the line under test, the mean impedance of the latter having been previously determined. With such networks $\Delta Z$ may be measured directly at any frequency. In the absence of such a network, $\Delta Z$ may be measured at the even resonant frequencies by Kaden's method[1]. In this the line is terminated with a fixed nominally non-reactive impedance $Z_r$ approximately equal to the mean line impedance, which need not be accurately known. Kaden limited his measurements to the resistive component of $\Delta Z$, but it is shown below that the reactive component can also be measured.

If $\delta Z_{AB}$ is the input-impedance deviation with the line terminated by $Z_r$, then at even resonance we have from equation (11)

$$\Delta Z''_{AB} = \delta Z''_{AB} + (Z_r - Z_m)\left\{1 - \exp(-2\alpha l)\right\} \quad \ldots (39)$$

When the line has small attenuation

$$\Delta Z''_{AB} = \delta Z''_{AB} + 2\alpha l(Z_r - Z_m) \quad \ldots\ldots (40)$$

If the mismatch between line and termination is sufficiently small the second term may be neglected, and $\Delta Z$ is equal to the measured input-impedance deviation, i.e. to the difference between the input impedance and the impedance of the termination. If, however, $2\alpha l(Z_r - Z_m)$ cannot be neglected, it may be eliminated by measuring from opposite ends of the line. From equations (29) and (40) we have

$$\Delta r''_{AB} = \tfrac{1}{2}(\delta r''_{AB} - \delta r''_{BA}) + k_1 \quad \ldots\ldots (41)$$

From equations (30) and (40)

$$\Delta x''_{AB} = \tfrac{1}{2}(\delta x''_{AB} + \delta x''_{BA}) + 2\alpha l(X_r - X_m) - k_2 \quad \ldots\ldots (42)$$

where $X_r$, $X_m$, are the reactive components of the termination and mean impedance respectively. Usually $k_1$, $k_2$, can be neglected and we have

$$\Delta r''_{AB} \simeq \Delta r''_A = \tfrac{1}{2}(\delta r''_{AB} - \delta r''_{BA}) \quad \ldots\ldots (43)$$

$$\Delta x''_{AB} \simeq \Delta x''_A = \tfrac{1}{2}(\delta x''_{AB} + \delta x''_{BA}) + 2\alpha l(X_r - X_m) \quad \ldots (44)$$

The input-impedance deviation is generally required as a measure of the internal irregularity of a drum-length, and the difference, if any, between $\Delta Z''_{AB}$ and $\Delta Z''_A$ is of no practical importance. It will be seen that the resistive component of the input-impedance deviation is obtainable directly from the measurements made at opposite ends of the line, but the reactive component has a small correction term $2\alpha l(X_r - X_m)$. The value of $X_r - X_m$ may be determined by measurement from both ends at the odd-resonance frequencies. In this case, for the short line

$$\Delta x'_{AB} = \delta x'_{AB} + 2(1 - \alpha l)(X_r - X_m)$$

and from equation (34) we have

$$X_r - X_m = -\frac{\delta x'_{AB} + \delta x'_{BA}}{4(1 - \alpha l)} \quad \ldots\ldots\ldots (45)$$

The corresponding values of $X_r - X_m$ for the even-resonance frequencies are obtained by interpolation. For a given type of cable, $X_m$ does not change appreciably with the small changes of modulus occurring from length to length in practice, and with a constant termination $X_r - X_m$ depends only on frequency; since the line-length is related to the resonant frequency, the correction factor

**TABLE I**

**Factors Used in Calculating $\Delta Z'$ from $\Delta Z''$**

$M = 4n'^2/n''\pi(n'^2 - n''^2)$

| EVEN RESONANCE $n''$ | ODD RESONANCE $n'$ | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| | 1 | 3 | 5 | 7 | 9 | 11 | 13 | 15 | 17 | 19 | 21 |
| 2 .. | −·21 | +1·15 | +·76 | +·69 | +·67 | +·66 | +·65 | +·65 | +·65 | +·64 | +·64 |
| 4 .. | −·02 | −·41 | +·88 | +·47 | +·40 | +·37 | +·35 | +·34 | +·34 | +·33 | +·33 |
| 6 .. | −·01 | −·07 | −·48 | +·80 | +·38 | +·30 | +·27 | +·25 | +·24 | +·24 | +·23 |
| 8 .. | — | −·03 | −·10 | −·52 | +·76 | +·34 | +·26 | +·22 | +·20 | +·19 | +·19 |
| 10 .. | — | −·01 | −·04 | −·12 | −·54 | +·73 | +·31 | +·23 | +·19 | +·18 | +·16 |
| 12 .. | — | −·01 | −·02 | −·05 | −·14 | −·56 | +·72 | +·29 | +·21 | +·18 | +·16 |
| 14 .. | — | — | −·01 | −·03 | −·06 | −·15 | −·57 | +·70 | +·28 | +·20 | +·16 |
| 16 .. | — | — | −·01 | −·02 | −·04 | −·07 | −·15 | −·58 | +·70 | +·27 | +·19 |
| 18 .. | — | — | −·01 | −·01 | −·02 | −·04 | −·08 | −·16 | −·58 | +·69 | +·27 |
| 20 .. | — | — | — | −·01 | −·02 | −·03 | −·05 | −·08 | −·17 | −·59 | +·69 |
| 22 .. | — | — | — | −·01 | −·01 | −·02 | −·03 | −·05 | −·09 | −·17 | −·59 |
| 24 .. | — | — | — | — | −·01 | −·01 | −·02 | −·03 | −·05 | −·09 | −·17 |
| 26 .. | — | — | — | — | −·01 | −·01 | −·02 | −·02 | −·04 | −·06 | −·09 |
| 28 .. | — | — | — | — | −·01 | −·01 | −·01 | −·02 | −·03 | −·04 | −·06 |
| 30 .. | — | — | — | — | — | −·01 | −·01 | −·01 | −·02 | −·03 | −·04 |
| 32 .. | — | — | — | — | — | −·01 | −·01 | −·01 | −·02 | −·02 | −·03 |
| 34 .. | — | — | — | — | — | — | −·01 | −·01 | −·01 | −·02 | −·02 |
| 36 .. | — | — | — | — | — | — | −·01 | −·01 | −·01 | −·01 | −·02 |
| 38 .. | — | — | — | — | — | — | — | −·01 | −·01 | −·01 | −·01 |
| 40 .. | — | — | — | — | — | — | — | −·01 | −·01 | −·01 | −·01 |

$N = 4n'/\pi(n''^2 - n'^2)$

| EVEN RESONANCE $n''$ | ODD RESONANCE $n'$ | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| | 1 | 3 | 5 | 7 | 9 | 11 | 13 | 15 | 17 | 19 | 21 |
| 2 .. | +·42 | −·76 | −·30 | −·20 | −·15 | −·12 | −·10 | −·09 | −·08 | −·07 | −·06 |
| 4 .. | +·08 | +·55 | −·71 | −·27 | −·18 | −·13 | −·11 | −·09 | −·08 | −·07 | −·06 |
| 6 .. | +·04 | +·14 | +·58 | −·69 | −·25 | −·16 | −·12 | −·10 | −·09 | −·07 | −·07 |
| 8 .. | +·02 | +·07 | +·16 | +·59 | −·67 | −·25 | −·16 | −·12 | −·10 | −·08 | −·07 |
| 10 .. | +·01 | +·04 | +·08 | +·17 | +·60 | −·67 | −·24 | −·15 | −·11 | −·09 | −·08 |
| 12 .. | +·01 | +·03 | +·05 | +·10 | +·18 | +·61 | −·66 | −·24 | −·15 | −·11 | −·09 |
| 14 .. | +·01 | +·02 | +·04 | +·06 | +·10 | +·19 | +·61 | −·66 | −·23 | −·15 | −·11 |
| 16 .. | — | +·02 | +·03 | +·04 | +·07 | +·10 | +·19 | +·62 | −·66 | −·23 | −·14 |
| 18 .. | — | +·01 | +·02 | +·03 | +·05 | +·07 | +·11 | +·19 | +·62 | −·65 | −·23 |
| 20 .. | — | +·01 | +·02 | +·03 | +·04 | +·05 | +·07 | +·11 | +·20 | +·62 | −·65 |
| 22 .. | — | +·01 | +·01 | +·02 | +·03 | +·04 | +·05 | +·07 | +·11 | +·20 | +·62 |
| 24 .. | — | +·01 | +·01 | +·02 | +·02 | +·03 | +·04 | +·05 | +·08 | +·11 | +·20 |
| 26 .. | — | +·01 | +·01 | +·01 | +·02 | +·03 | +·03 | +·04 | +·06 | +·08 | +·11 |
| 28 .. | — | — | +·01 | +·01 | +·02 | +·02 | +·03 | +·03 | +·04 | +·06 | +·08 |
| 30 .. | — | — | +·01 | +·01 | +·01 | +·02 | +·02 | +·03 | +·04 | +·04 | +·06 |
| 32 .. | — | — | +·01 | +·01 | +·01 | +·02 | +·02 | +·02 | +·03 | +·04 | +·05 |
| 34 .. | — | — | +·01 | +·01 | +·01 | +·01 | +·02 | +·02 | +·02 | +·03 | +·04 |
| 36 .. | — | — | +·01 | +·01 | +·01 | +·01 | +·01 | +·02 | +·02 | +·03 | +·03 |
| 38 .. | — | — | — | +·01 | +·01 | +·01 | +·01 | +·02 | +·02 | +·02 | +·03 |
| 40 .. | — | — | — | +·01 | +·01 | +·01 | +·01 | +·01 | +·02 | +·02 | +·02 |

$2\alpha l(X_r - X_m)$ may be simply tabulated in terms of frequency and order of resonance.

### Determination of Resonant Frequencies

The resonant frequencies are those for which $2\beta l = n\pi$, where $n$ is an integer. They are conveniently measured by adjusting the frequency until the input impedance becomes non-reactive, the distant end of the line being open- or short-circuited. Errors arise due to the small reactive component in the characteristic impedance and to the presence of irregularities in the line. Considering the first of these, at resonance $\tanh\gamma l$ and $\coth\gamma l$ are wholly real, and for a uniform line,

$$Z_{op} = Z_m \coth \alpha l \quad (46)$$

$$Z_{sh} = Z_m \tanh \alpha l \quad (47)$$

Thus at resonance the angles of $Z_{op}$ and $Z_{sh}$ are equal to the angle of $Z_m$, i.e. $\phi_m$. Let the frequency be shifted from $f_n$ to $f_n + \delta f$ to make the input impedance non-reactive, and let the corresponding value of $\beta l$ be $\frac{1}{2}n\pi + \theta$. Then, when $n$ is odd,

$$Z'_{op} = Z_m \coth\left\{\alpha l + j(\tfrac{1}{2}n'\pi + \theta)\right\} = Z_m \tanh(\alpha l + j\theta).$$

Expanding by Taylor's theorem, we obtain for values of $\theta$ small compared with $\alpha l$ which is usually the case,

$$\tanh(\alpha l + j\theta) = \tanh \alpha l + \frac{\theta^2 \tanh \alpha l}{\cosh^2 \alpha l} + \frac{j\theta}{\cosh^2 \alpha l} \quad (48)$$

The phase-angle of $\tanh(\alpha l + j\theta)$ is thus $\dfrac{2\theta}{\cosh 2\alpha l}$.

The input impedance $Z'_{op}$ is non-reactive when

$$\frac{2\theta}{\sinh 2\alpha l} = -\phi_m \quad (49)$$

It can be readily shown that for a line having small leakance and for which $\alpha \ll \beta$,

$$\phi_m = -\alpha/\beta \quad (50)$$

whence

$$\frac{2\theta}{\sinh \alpha l} = \alpha/\beta \quad (51)$$

Thus

$$\delta f'_{op} = f'_n\theta/\beta l = f'_n \frac{\sinh 2\alpha l}{2\alpha l}(\alpha/\beta)^2 \quad (52)$$

and when $\alpha l$ is small,

$$\delta f'_{op} = f'_n(\alpha/\beta)^2 \quad (53)$$

Similarly it can be shown that the frequency shift from true odd resonance to make $Z'_{sh}$ non-reactive is $-f'_n(\alpha/\beta)^2$; in the same way we find that the frequency shift from true even resonance to make $Z''_{op}$ non-reactive is $-f''_n(\alpha/\beta)^2$, and for $Z''_{sh}$ it is $+f''_n(\alpha/\beta)^2$. Thus

$$\delta f_{sh} = -\delta f_{op} = (-1)^n f_n(\alpha/\beta)^2 \quad (54)$$

Since for high-frequency cables $\alpha$ is proportional to $f^{\frac{1}{2}}$ and $\beta$ is proportional to $f$, $\delta f$ is the same for all resonant frequencies, i.e. the error becomes relatively less as the frequency rises. To make the input impedance non-reactive the frequency is increased from true resonance when the input impedance is less than $Z_m$, and decreased when it is greater than $Z_m$.

For 104/375 coaxial cable, $\alpha = 0{\cdot}43 f_M^{\frac{1}{2}}$ nepers/mile, $\beta \simeq 34{\cdot}8 f_M$ radians/mile, whence $\delta f = 150$c/s. The small correction in this case may be readily applied, but usually it is negligible.

Considering now the effect of line irregularities, it has been shown elsewhere[13] that when these are small, they modify the open- and short-circuit impedances by a factor $K_{AB}$, given by

$$K_{AB} = 1 + \frac{p_{AB} + p_{BA}}{1 - \exp(-2\gamma l)} + \frac{p_{AB} - p_{BA}}{1 + \exp(-2\gamma l)} \quad (55)$$

When the attenuation is small, we have at even resonance, $1 + \exp(-2\gamma l) \simeq 2$, $1 - \exp(-2\gamma l) \simeq 2\alpha l$. From equations (29) and (30), $p''_{AB} + p''_{BA} \simeq j\Delta x''_{AB}/Z_m$, $p''_{AB} - p''_{BA} \simeq \Delta r''_{AB}/Z_m$. Thus

$$K''_{AB} = 1 + \frac{\Delta r''_{AB}}{2Z_m} + j\frac{\Delta x''_{AB}}{2\alpha l Z_m} \quad (56)$$

At odd resonance, $1 + \exp(-2\gamma l) \simeq 2\alpha l$; $1 - \exp(-2\gamma l) \simeq 2$; from equations (33) and (34), $p'_{AB} + p'_{BA} = \Delta r'_{AB}/Z_m$, $p'_{AB} - p'_{BA} = j\Delta x'_{AB}/Z_m$.

Whence

$$K'_{AB} = 1 + \frac{\Delta r'_{AB}}{2Z_m} + j\frac{\Delta x'_{AB}}{2\alpha l Z_m} \quad (57)$$

Hence the correction factor in terms of input-impedance deviations is the same for all resonant frequencies; we see that the irregularities alter the phase-angle by $\Delta x_{AB}/2\alpha l Z_m$, and necessitate an additional frequency shift

$$\Delta f_{op} = -\Delta f_{sh} = \frac{(-1)^n f_n \Delta x_{AB}}{n\pi Z_m} \quad (58)$$

For example, if $\Delta x_{AB} = 0{\cdot}1\Omega$, $Z_m = 75\Omega$, $f = 1$Mc/s, $n = 2$, then $\Delta f = 210$c/s.

### Determination of Mean Impedance

The mean impedance $Z_m$ plays such an important part in the theory of irregularities that its accurate measurement is a matter of prime interest, particularly for short lines. For this purpose, three methods are available, (i) by measurement of the input impedances with the distant end open and closed, (ii) by measurement of resonant frequencies and capacitance, and (iii) by comparison with a standard resistor at odd resonance.

#### *$Z_m$ from the Open- and Short-circuit Impedances*

The characteristic impedance of a perfectly uniform line is given by

$$Z_m = \sqrt{(Z_{op}Z_{sh})} \quad (59)$$

Inasmuch as impedance measurement at high frequencies is most accurate when the phase-angle is zero, the tests are made at the "non-reactive" frequencies. From equations (56) and (57) it is seen that the line irregularities cause an error in the measurement of the modulus of $Z_m$ equal to $\frac{1}{2}\Delta r_{AB}$. If the measurement is repeated from the opposite end of the line and the average taken, the error will cancel out at the even resonant frequencies, but will be unaffected at the odd resonances.

#### *$Z_m$ from Resonant Frequency and Capacitance*

Since $|Z_m| = \sqrt{(L_u/C_u)}$ and $\beta = 2\pi f\sqrt{(L_u C_u)}$, we have

$$|Z_m| = \frac{\beta}{2\pi f C_u}$$

At the resonance frequencies, $2\beta l = n\pi$, whence

$$|Z_m| = \frac{n}{4fC_u l} \quad (60)$$

The capacitance $C_u l$ is measured at low frequency, e.g. 1 000c/s, since with good quality dielectrics the change with frequency may be neglected. The resonant frequencies are determined from the frequencies at which $Z_{op}$ or $Z_{sh}$ is non-reactive, as discussed previously. From equations (55) and (60) we see that the error caused by the line irregularities is $\Delta x/n\pi$ ohms; as compared with the "open and closed" method, assuming that $\Delta r \simeq \Delta x$, the error is reduced by the factor $2/n\pi$.

#### *$Z_m$ by comparison with a Standard Resistor*

If we measure the input-impedance deviation at odd resonance with the line terminated by a nominally non-reactive resistor $Z_r = R_r + jX_r$, we obtain $\delta r'_{AB}$ and from equation (11), since $R_m = |Z_m|$

$$|Z_m| = R_r + \frac{\delta r'_{AB} - \Delta r'_{AB}}{1 + \exp(-2\alpha l)} \quad (61)$$

We may calculate $\Delta r'_{AB}$ from the reactive components of the input-impedance deviations measured at the even resonant frequencies, using equation (35). However, with sufficiently regular lines and averaging the values obtained at two successive odd resonant frequencies, $\Delta r'_{AB}$ may be ignored.

It is probable that the accuracy of the resonant frequency-capacitance method is better than the accuracy of calibration of the standard resistor at the appropriate frequencies; in fact the best method of calibrating the resistor is to use equation (61) for a line with small irregularities whose mean impedance has been previously determined by the resonant frequency-capacitance method.

### Measurement of Attenuation Coefficient

At the resonant frequencies, the attenuation coefficient is given by

$$\alpha l = \tanh^{-1} \sqrt{(R_{sh}/R_{op})} \text{ nepers} \quad \ldots\ldots\ldots \quad (62)$$

where $R_{sh}$, $R_{op}$ are the real parts of $Z_{sh}$, $Z_{op}$ respectively. When the irregularities are small, the error factor $K_{AB}$ given by equation (55) cancels out, and the measured attenuation coefficient is free from error due to this cause. However, when working in the megacycle frequency range, difficulties arise in measuring both high and low resistances on the same type of bridge. The expedient is adopted with a bridge suitable for high impedances, of measuring $Z_{op}$ at even resonances and $Z_{sh}$ at odd resonances, using the value of $|Z_m|$ obtained by one of the methods described previously, together with the expression

$$\alpha l = \tanh^{-1}(|Z_m|/R) \text{ nepers} \quad \ldots\ldots\ldots \quad (63)$$

Similarly with a bridge suitable for low impedances, $Z_{sh}$

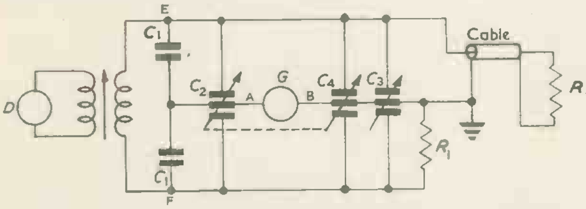


Fig. 2. Bridge for impedance measurements on coaxial cables

is measured at even resonances and $Z_{op}$ at odd resonances, and the appropriate formula is

$$\alpha l = \tanh^{-1}(R/|Z_m|) \text{ nepers} \quad \ldots\ldots\ldots \quad (64)$$

In all these cases a proportional error of the order of $\Delta r/2Z_m$ occurs; however, with normal quality of 104/375 coaxial cable the error is well inside the limits of accuracy expected for attenuation measurements on short lengths.

### Bridges for Impedance Measurement

A bridge for testing coaxial cables has been described by Fuchs and Fenouillet[8]; this contains fixed ratio arms, with variable inductors and resistors, and is used for the measurement of impedance irregularities and of attenuation by the "low-impedance" method over the frequency range of 0·06 to 10Mc/s. A bridge having variable capacitor ratio arms is employed by the British Post Office for tests on repeater sections of coaxial cable[9]; the principle has been adapted for measuring impedance deviations and attenuation by the "high-impedance" method on short lines up to 10Mc/s.

Referring to Fig. 2, the ratio arms consist of two equal fixed mica capacitors $C_1$ in parallel with a small differential air-capacitor $C_2$. As used for measuring irregularities, the cable under test is terminated by a resistor $R_2$ of about 75Ω, and is compared with a standard resistor $R_1$ of about the same value. The differential capacitor $C_2$ is calibrated to read the resistive component of the impedance deviation relative to the standard. The reactive component is measured by the differential air-capacitor $C_3$, calibrated in picofarads, $\delta x$ being $2\pi f \times 75^2 \times \delta C_3$. The supply from a signal-generator $G$ is fed across AB, and a communications receiver $D$, used as detector, is connected through a balanced and screened transformer across EF. A second balance is obtained with $R_2$ replacing the cable on the bridge, and the difference between the two sets of readings gives the input-impedance deviation with respect to $R_2$.

As used for measuring attenuation, the cable (open or closed at the far end according as the resonant frequency is even or odd) is connected to the bridge in parallel with a resistor $R_3$ of about 80Ω, and the frequency is varied until the reactance-capacitor reading is the same with the cable on and off the bridge. The change of ratio occasioned by disconnecting the cable tends to alter the reactance balance, but this effect is compensated by a small differential air-capacitor $C_4$ of appropriate value, ganged to $C_2$. The impedance of the cable is obtained from the difference $r$ in the readings on the "resistance" dial with the cable on and off, and is given by $R_3(R_3/r - 1)$. For lower impedances, smaller resistances are used in place of $R_3$ and $R_1$, an appropriate correction being made to $r$ for the change in the standard resistor, and the cable impedance may be tabulated in terms of $r$ for each pair of resistors. The error introduced by $C_4$ in the measurement of impedance deviations is negligibly small.


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## UNDERWATER TELEVISION

One of the latest types of Marconi-Siebe, Gorman underwater television has recently been flown out to the scene of the Comet crash off Elba.

The camera differs from its predecessors in that it is fitted with a periscope lens which can "see" in any direction within a hemisphere; it being possible to rotate the lens, by remote control from the salvage ship, through an arc of 360°, it can also be elevated through 90°. It overcomes many of the difficulties of television underwater when weather conditions are poor and the apparatus is swinging with the tide, and allows the camera to scan a large area of the sea bed without having to move it either from above, or with the aid of a diver.

The new lens operates quite simply. The tube containing the lens arrangements passes through a brass plate inside the watertight pressure casing, which is fixed between the front of the camera and the viewing dome. The tube can rotate through a complete circle; its "eye"—the front portion—is pivoted to the tube and is able to elevate through 90°.

These movements are operated by servo-motors clamped to the brass plate and camera, and controlled by the operator on board by switches in his control unit. The unit is connected to the camera through the watertight camera cable.

A "fish bowl" viewing dome of inch-thick perspex now replaces the former window for viewing. This has the same refractive index as water so that it does not distort the image, and scratches on its surface are not shown on the monitor screens.

# A Square Root-Law Circuit

By I. G. Baxter, B.Sc., A.R.C.S.

*The output from a negative feedback amplifier is made proportional to the square root of the input, by means of a non-linear feedback network of resistors and miniature selenium rectifiers. The basis of the method is to generate a calibration curve made up of linear segments, which is very nearly parabolic.*

THE circuit is being used as a corrector element in a blood flowmeter[1], in place of a system that depended upon the non-linear characteristics of a selected pentode valve[2]. Basically, it is a variable feedback amplifier; initially the feedback is small and the gain is correspondingly great, but when the output increases, consecutive increments of feedback progressively reduce the gain. The curve relating the input and output voltages (Fig. 1) is accordingly made up of linear segments which each intersect an ideal parabola at two points, resulting in error maxima at the points of inflexion and near the middle of each segment. It can be shown that if the percentage error regardless of sign is made the same at each point where it is greatest, then the points of inflexion or kinks will occur at output voltages which are related geometrically. There are seven kinks in the calibration of the present circuit (Table 1), the first being at 0·04 times peak output voltage. In theory, the greatest departure from a true parabola is 2½ per cent except in the region below the first kink, where the relative error increases rapidly. In practice, despite inaccurate setting up, and the behaviour of components falling short of the ideal, it has been found possible to keep errors well within 5 per cent except near the first kink.

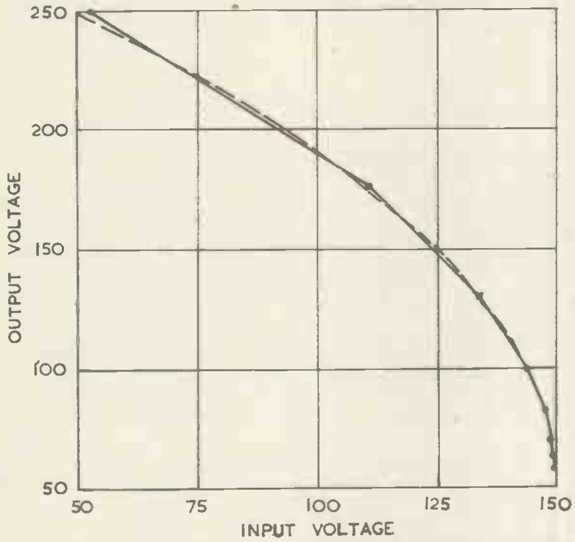


**Fig. 1. Theoretical calibration curve composed of straight segments**
*The broken line represents an ideal parabola.*

In its general principles the system resembles one for squaring described by Marshall[3]. The chief differences in the present case are that the inverse operation is being performed, that miniature selenium rectifiers are used instead of germanium ones, and that the percentage error (not the absolute error) is made minimal.

**TABLE 1**
**Calibration of Circuit**
*The input and output voltages at points of inflexion are tabulated, with corresponding values of gain.*

| INPUT VOLTAGE | OUTPUT VOLTAGE | GAIN |
|---|---|---|
| 52·5 | 250·0 | |
| | | 1·25 |
| 111·5 | 176·0 | |
| | | 2·0 |
| 134·7 | 129·4 | |
| | | 3·2 |
| 143·9 | 100·0 | |
| | | 5·0 |
| 147·6 | 81·5 | |
| | | 8·0 |
| 149·0 | 69·9 | |
| | | 12·7 |
| 149·6 | 62·5 | |
| | | 20·0 |
| 149·9 | 57·9 | |
| | | 52·5 |
| 150·0 | 50·0 | |

## Basic Circuit

A simple arrangement which should, in principle, fulfil the desired requirements is shown in Fig. 2. It comprises an anode-follower in which the feedback network is made up of resistors and rectifiers. The input comes from the anode of a preceding stage. With a high input voltage the output is low and the potential at P is at or below earth, so that most (or all) of the rectifiers are non-conducting; feedback is slight and the gain high. When the input voltage falls, that at P rises with the increased output, and the rectifiers become conducting one after another, so increasing the feedback and diminishing the gain. Unfortunately, if non-thermionic rectifiers are used this scheme has shortcomings, and in any case it entails undesirably high values of feedback resistance.

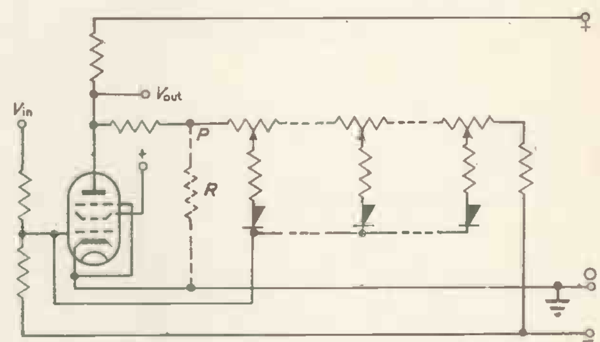


**Fig. 2. Elementary square root law circuit**

It was desired to use type M-1 miniature selenium rectifiers, which are efficient, cheap and very small. They would be working in a moderately high impedance network, where their considerable forward resistance (about $10^4\Omega$) would be unimportant. Their measured backward resistance—at least in the batch tested— was much greater than the 20MΩ specified by the makers, and quite adequate within a certain range of voltage. On the other hand, with reverse potentials greater than about 25 to 30V they passed current significantly.

A large amplitude of output voltage (100V or more) is desirable, for ease of recording and to make the overall calibration less dependent on rectifier and valve characteristics. With the circuit in Fig. 2 this is incompatible with avoiding voltage overload of the rectifiers. Furthermore, at high gains the feedback resistance would be inconveniently high in itself, besides being of the same order as the aggregate shunt leakage path through the rectifiers. There are two objections to resolving these difficulties by providing a resistive connexion ($R$) to earth for reducing the excursion of potential at P. Firstly, variations in the switching voltage of the rectifiers assume greater importance, and secondly, under low gain conditions the feedback resistance becomes of a similar order to the forward resistance of the rectifiers.

**Practical Circuit**

The above considerations led to contriving a feedback network (Fig. 3) that is better matched to the rectifiers, and involves rational values of resistance. The arrangement is potentiometer that gives the required reference potential. This potentiometer, with the valve heaters, is fed from a battery to ensure stability. Regulated H.T. and negative supplies are used.

The effective anode load falls as the rectifiers successively conduct, but this is no drawback because it occurs in harmony with the desired decrease of gain. Maximum gain is conveniently required when the anode current is highest and the $g_m$ large.

**Setting Up**

The resistances were calculated approximately, and the circuit was then built; numerous variable resistors were included so that the calibration could be adjusted experimentally, both as regards the slope of each linear segment and as to positioning of the points of inflexion. Permanently wired in series with each rectifier was a 100Ω resistor, across which a galvanometer could be shunted as a switching indicator when making adjustments.

Calibration was carried out on a straightforward but tedious trial and error basis. The points of inflexion and the gains were set, respectively, by adjusting the tapping

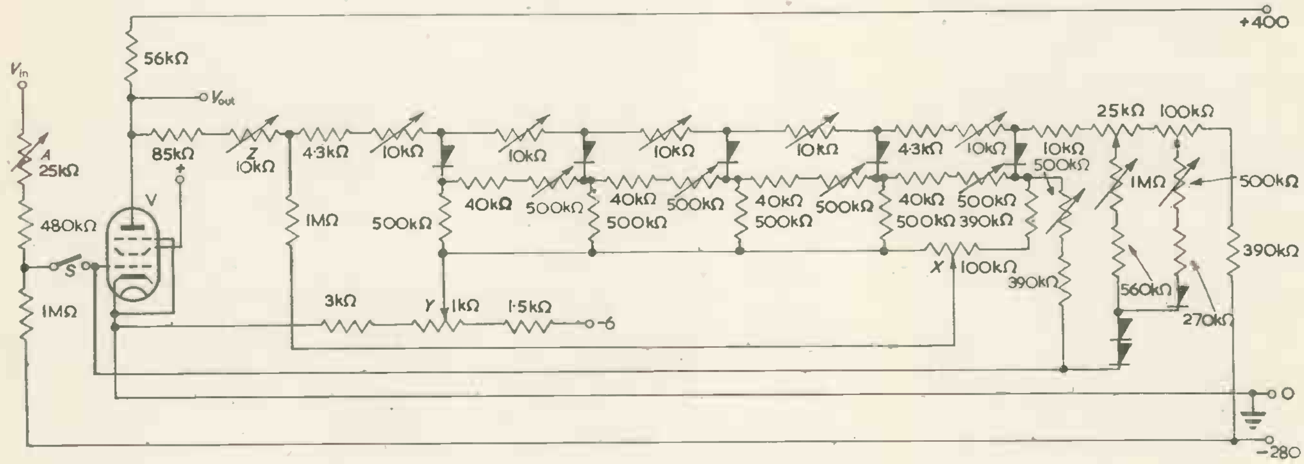


Fig. 3. Practical square root law circuit

essentially a ladder attenuator feeding the control grid of the amplifier valve, with each " rung " connected by a rectifier to the output potentiometer from the anode. When the output is low, only the rectifiers remotest from the grid are conducting, so feedback is slight and the gain high. An increase of output causes additional rectifiers to conduct, one by one, so that feedback is injected closer to the grid and the gain is diminished. Under conditions of high gain the rate of change of voltage across a rectifier approaching conduction is not seriously less than that at the output itself, and this is conducive to accurate switching in the region where it is most needed. As each rectifier cuts in, so does the voltage across the next one change less rapidly in relation to the output, owing to the additional current in the network; the adverse effect of this on switching accuracy matters little when related to the rise in output.

It was found impracticable to use the ladder network for feedback over the whole operating range, because this would have entailed inconvenient values of resistance. The two final increments of feedback, therefore, take place through direct routes to the grid. To avoid voltage overload of the rectifiers concerned, two units in series are used in the penultimate path, and to the ingoing side of this pair is connected the rectifier for the final path.

The initial gain, with all the rectifiers non-conducting, is governed by a connexion from the output potentiometer to a tapping on the first " rung " of the ladder. Grid bias is provided by connecting one side of the ladder to a points on the output potentiometer, and the variable resistors in the ladder network. In was necessary to work upwards from the condition of lowest gain, to minimize the disturbance of one setting by the next. Even so, the sequence of adjustment had to be repeated once or twice before the correct calibration was attained.

The origin was fixed in the following way. Switch $S$ was opened, and potentiometer $X$ set to minimum. Potentiometer $Y$ was then adjusted to make the output meter read correctly, after which potentiometer $Z$ was set so that this reading was not altered by traversing $X$ over its full range. Lastly, with the input voltage at the desired initial value, switch $S$ was closed, and potentiometer $A$ adjusted to restore the correct output reading.

**Conclusion**

The circuit fulfils the requirements it was designed for, at the cost of a large number of fixed and variable resistors. Any appreciable improvement in performance is barred by instability of the components and by the great gain needed near the origin. As it is, origin drift does occur, but this is not objectionable in the present application because there are opportunities for checking, and there is ample input signal most of the time.

If a lower order of accuracy were acceptable, there could be fewer feedback steps, less accurately graded, and it would be possible to use a smaller proportion of variable resistors.

For example, for the computed maximum error to be within 5 per cent from the first kink upwards, five changes of gain would be adequate. For an error of 10 per cent, three kinks would suffice.

The advantages of the system are that it is not dependent upon discovering a single circuit element, such as a pentode valve, that has the desired non-linear characteristics, and it can be adjusted if necessary, to suit curves that are not parabolic.


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# Maintaining Optimum Cycling Intervals in a Two-step Controller


By W. R. Beakley,* M.A., Ph.D.


THE simplest type of automatic temperature control system uses two-step action to regulate the process. With this type of control oscillation about the desired value continuously occurs, but the amplitude of the oscillation may be reduced to a very small value by replacing a large part of the controlled power with a source of constant auxiliary power. Optimum working conditions exist when the on and off parts of the control cycle are approximately equal. Changes in load cause the duration of one or other part of the cycle to increase. In practice an increase in duration of either part of the cycle by a factor of up to 4 compared with the optimum duration still ensures effective control. Control is lost whenever a change in load occurs which is greater than the controlled power, or when large random variations occur in the auxiliary power. When these changes are slow this difficulty may be overcome by controlling the auxiliary power in steps of rather smaller magnitude than that of the primary controlled power. The instrument to be described does this by responding when the duration of the on or off periods exceeds a pre-set value and by remaining inert when the controller is acting effectively.

Fig. 1 is a circuit diagram of the instrument. The rectified output from the switch contacts of the two-step controller actuates the relay $A$. This relay substitutes $C_1$ for $C_2$, discharging the one and connecting the other ($C_1$ say) between anode and grid of the Miller integrator $V_1$. As this capacitor charges, the anode voltage of $V_1$ rises and unless relay $A$ again interchanges the capacitors, it reaches the voltage of the cathode of $V_{2a}$ in a time $t$ given approximately by:

$$t = \frac{CR\ V_c}{V_b}$$

where $C$ is the capacitance of $C_1$ or $C_2$, $R$ is the resistance of the grid leak, $V_c$ the voltage at the cathode of $V_{2a}$ and $V_b$ is the bias voltage. Here $t = 10CR$.

When $V_{2a}$ conducts the Miller action ceases and the grid of $V_1$ is allowed to fall comparatively rapidly, cutting off the screen current and releasing relay $B$. The contact $B_2$ opens and allows $C_4$ to discharge through the selected resistor $R_4$ to $R_9$; the time-constant of these components governs the time for which $V_1$ is cut off. During this part of the cycle $C_1$ is discharged via the contact $B_1$, while the control motor is switched on and runs in the direction set by the state of relay $A$. When the voltage at the grid of $V_1$ rises through cut-off, relay $B$ is again energized, charging $C_4$ and replacing $C_1$ between anode and grid of $V_1$; the initial surge of charging current of $C_1$ prevents $V_1$ being again cut off as the full bias voltage is restored. The circuit is now in its original condition.

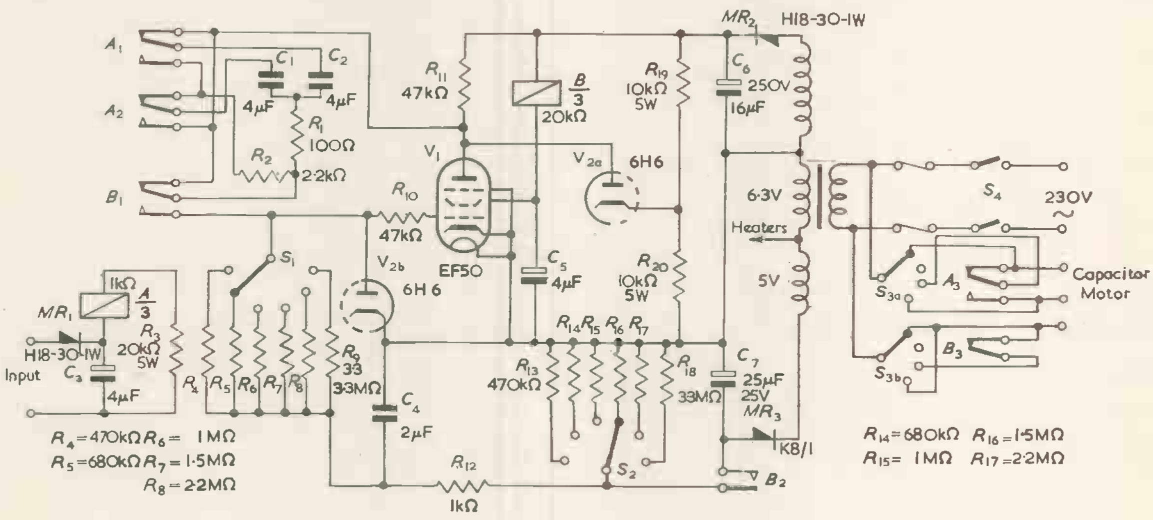



**Fig. 1. Circuit of the controller**

The diode $V_{2b}$ prevents $V_1$ passing excessive grid current at the moment when $C_1$ or $C_2$ is connected between anode and grid.

Should the two-step controller change its state before the anode voltage of $V_1$ reaches the cathode voltage of $V_{2a}$ the circuit is reset by relay $A$ without operation of relay $B$.

The switch $S_3$ is included to allow the auxiliary power to be set manually when required.

This instrument has given trouble-free service controlling conditions in the large-animal psychrometric chamber at this Institute, for a total of 7 000-8 000 hours, during which time it has operated over a large number of periods of a few days each, and also continuously over periods of one to three months.


* *The Hannah Dairy Research Institute, Kirkhill, Ayr.*

The Measurement of

# Very Small Direct Currents

By M. W. Jervis*, B.Sc.Tech., A.M.I.E.E.

*The necessity for measuring direct currents in the range of 1$\mu$A to $10^{-16}$A arises in many branches of physics and engineering. This article is a review of the methods used, particular attention being paid to the thermionic and capacitor modulator electrometers. Reference is made to the factors limiting accuracy and sensitivity obtainable by the methods described, together with some notes on the precautions necessary when measuring very small currents.*

THE necessity for measuring small currents arises in many branches of physics and usually amounts to the determination of the rate of flow of free charged particles. One ampere corresponds to 6 by $10^{18}$ electrons per second and currents as low as 6 electrons per second have been detected.

Fig. 1 indicates the range of currents to be discussed. Typical examples occur in the field of vacuum physics where ion currents of $10^{-7}$A (0·1$\mu$A) are measured in ionization vacuum gauges and $10^{-14}$A in mass spectrometers. In the determination of X-ray and nuclear radiation by ionization chambers, the measurement of currents as low as $10^{-16}$A is required[1].

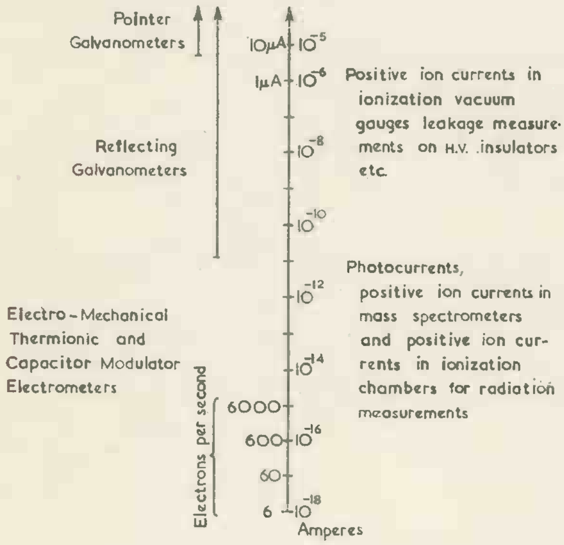



**Fig. 1. The range of currents discussed**

## Common Types of Current Measuring Instruments

### Galvanometers

The moving-coil pointer indicating instrument is the most common current measuring device and micro-ammeters are available to read about 10$\mu$A full-scale. For smaller currents, galvanometers are used in which a beam of light replaces the pointer. These are available with sensitivities of about $10^{-11}$A/mm deflexion[2] on a scale placed 1 metre from the instrument.

A galvanometer of such sensitivity is a delicate laboratory instrument and great precautions must be taken to avoid movement of the spot due to vibration, draughts, etc. Even when these effects have been removed, residual fluctuations remain. These fluctuations arise partly from rotation of the coil caused by its bombardment by air molecules (Brownian motion) and partly from the thermal agitation voltage generated in the resistance of the instrument and circuit. The fluctuations, or " noise " as they are called, cause the mirror of the galvanometer to be in continuous movement, corresponding to about $10^{-12}$A at room temperature, so that they impose this definite limit on the smallest current which can be detected[2].

### Galvanometer Amplifiers

In galvanometer amplifiers, the light spot reflected from the galvanometer is made to fall on a photocell instead of a scale. The photocell is usually split into two and may be of photo-voltaic or photo-emission type. The former will usually operate a secondary less sensitive galvanometer directly[3], while the latter is followed by a thermionic valve amplifier[4]. Such an arrangement gives the effect of a very long optical pointer and the amplifier can be made with a large output for automatic pen recording. In both cases, negative feedback is often applied, as this gives a convenient sensitivity control and improves linearity, response time and permanency of calibration. It can be seen that the Brownian movement limitation remains, however, though the system is a convenient one to use[4].

### Other Low Resistance Devices

The equivalent voltage fluctuations arising in devices such as magnetic and contact modulator amplifiers can be made very small[5].

In the most stable magnetic amplifiers, the input instability is about 2$\mu$A-turns[6]. The number of turns is, however, limited by the winding space and smallest convenient gauge of wire and, in practice, the noise amounts to about $10^{-12}$A.

With the contact modulator amplifier[5], the input noise expressed as a voltage can be made small, but the maximum input resistance which can be used is restricted to a value which does not result in excessive pick-up from the driving coil. A further limitation on the maximum input resistance is set by the requirement that the input time-constant must be small compared with the time during which the contact is in the open or closed position. If this condition is not satisfied, the recorded voltage will depend on the closed and open time of the contactor. These conditions make contact modulators more suitable for measuring voltages than currents, though they have been made[7] with an input noise of $10^{-12}$A. Using a specially screened contact modulator, Kroebel[8] reports noise of 2·5$\mu$V across impedances of $10^{8}\Omega$ for a 1c/s bandwidth.

Low resistance measuring instruments have the advantage of requiring a small voltage drop across them, but, in most cases, a high impedance can be satisfactorily used.

## Measurement of Smaller Currents

### Electrometers

The smallest detectable current with available galvanometers[2] is limited by Brownian movement to about $10^{-12}$A. Much smaller currents can be measured by electrostatic

* *Research Laboratory, Associated Electrical Industries Ltd.*

devices, though these require a relatively high voltage drop.

There are two ways in which such high impedance electrometer type devices can be used:

*Voltage Drop Method*

The current $I$ to be measured is passed through a high resistor $R$ and the electrometer used to measure the voltage drop $V$ across it (Fig. 2). Then:

$$I = V/R \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (1)$$

e.g. $I = 10^{-15}$ A, $R = 10^{12}\Omega$, $V = 1$mV.

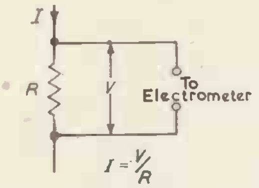



Fig. 2. Voltage drop method

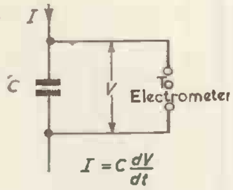



Fig. 3. Capacitor charging method.

*Capacitor Charging Method*

The current $I$ is measured by the rate of rise of voltage across a capacitor $C$ (Fig. 3) then:

$$I = C\,dV/dt \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (2)$$

e.g. $I = 10^{-15}$A, $C = 10$pF, $dV/dt = 100\mu$V/sec.

The leakage resistance must be very high for the charging rate to be constant in the capacitor charging method. This method determines the average current over the time taken for the measurement and so is not suitable for varying currents. The voltage drop method can be used for varying currents only when the rate of change of current is small compared with $1/RC$, where $C$ is now the stray capacitance of the input circuit.

Both voltage drop and capacitor charging methods can be used as a null method, so improving the accuracy and eliminating errors due to changes in electrometer sensitivity.

## Limitations of Accuracy

### Component and Instrumental Limitations

It can be seen that the accuracy of measurement will be no better than that to which the resistor or capacitor is known, but when very small currents are being measured, the major factor limiting accuracy and sensitivity is instability of the electrometer zero in the form of slow drifts or random fluctuations.

The zero fluctuations may be instrumental in origin and therefore characteristic of the type of electrometer. These will be discussed in the section appropriate to the various electrometer types.

There is also one source of noise due to thermal agitation, which is common to all electrometers.

### Thermal Agitation Noise

Whether the voltage drop or capacitor charging method is used there will always be some resistance and capacitance of the electrometer and the effective circuit will be as in Fig. 4.

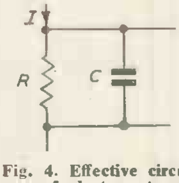



Fig. 4. Effective circuit of electrometer

It can be shown[5] that the thermal agitation voltage $E_n$ across such a circuit is given by $E_n^2 = kT/C$ provided the bandwidth is limited by $R$ and $C$ only; $k$ is Boltzmann's constant and $T$ the temperature in degrees Kelvin.

In the case of the voltage drop method, the signal voltage $E_s = IR$. It can be shown[5] that the peak-to-peak fluctuations of voltage are about eight times the R.M.S. value, so that the smallest current which can be detected with certainty is given by:

$$IR = 8E_n = 8\sqrt{(kT/C)} \therefore I = \frac{8}{R}\sqrt{(kT/C)} \quad \ldots\ldots \quad (3)$$

e.g. for $T = 300°$K (27°C), $C = 9$pF, $R = 10^{12}\Omega$, $I = 8/10^{12} \times \frac{64 \times 10^{-6}}{3} = 2 \times 10^{-16}$A, or about 100 electrons/sec.

For 5 per cent accuracy, the time taken for the measurement will be $3CR$, i.e. 27sec.

In the limiting case when $R$ is made infinite, the current is measured by the capacitance charging method. Assuming that the current $I$ to be measured is constant, the signal voltage $E_s$ after a time $t$ is given by equation (2), i.e.

$$E_s = It/C$$

Therefore, the minimum detectable current for $t = 27$sec (as in above example) is given by:

$$It/C = 8\sqrt{(kT/C)} \therefore I = \frac{8}{t}\sqrt{(kTC)} \quad \ldots\ldots \quad (4)$$

For $C = 9$pF, $T = 300°$K, $I = \frac{8 \times 9 \times 10^{-12}}{27} \times 20 \times 10^{-6} = 6 \times 10^{-17}$A.

### Other Sources of Noise

It should be emphasized that the considerations of the last section concern the limit of the smallest detectable current set by thermal agitation noise. In practice, instrumental difficulties prevent this limit being reached except in cases where great precautions are taken (Appendix 1).

An example of this is the noise generated by the passage of current in the semi-conductor material of which high valued resistors are constructed[9]. This noise can become important in some measurements when using the voltage drop method, but it is being reduced by better manufacturing processes and materials.

### Time Necessary to Measure a Given Current

In the case of the capacitor charging method, the time necessary to detect a given current is given directly from equation (4), i.e.:

$$t = \frac{8}{I}\sqrt{(kTC)} \quad \ldots\ldots\ldots\ldots \quad (5)$$

The time necessary to measure a current to a given accuracy is many times that required to detect it (given by equation (5)) and the time increases as the accuracy required increases.

For the voltage drop method with capacitance present, the voltage across them will follow an exponential law of time-constant $RC$. For the voltage to reach 95 per cent of its final value, a time $3RC$ must elapse after the start of the measurements, e.g. $R = 10^{12}\Omega$, $C = 9$pF, $3RC = 27$sec. From equation (3) it can be seen that $R$ must be large to measure a small current and therefore, for a given capacitance $C$, the measurement time will also be long. In practice, the maximum time available is limited by zero drift of the electrometer.

The effective time-constant can be reduced by the application of negative feedback (Appendix 2).

### Statistical Errors

If the current measured consists of the passage of $n$ randomly spaced discrete charges, the standard deviation in the number measured is $\sqrt{n}$. In practice, the error caused by thermal agitation necessitates the collection of such a large number of charges that statistical errors are small[10,11].

## Practical Instruments

### Electromechanical Electroscopes and Electrometers

In a review of electroscopes and electrometers, Neher[10] defines the difference between them as follows—" electroscopes need only one potential for their operation, while electrometers require auxiliary potentials." An example of an electroscope is the familiar simple gold leaf electroscope while electrometers are generally more refined instruments.

Electroscopes are usually characterized by simplicity, portability and low cost, and since they require no auxiliary potentials for their operation, there is no instability due to supply voltage fluctuations. They are, in general, less sensitive than electrometers, and are not easily used in a null method of reading, but they have considerable application as portable detectors of nuclear particles and radiation[1].

Numerous electrometers are described in the literature[1,2,10]. The most sensitive are the Dolezalek, Compton and Hoffmann types. The latter can be made with a capacitance of 3pF and a charge sensitivity of $1{\cdot}5 \times 10^{-16}$ coulombs/scale division. This enables a current of one electron per second (about $10^{-19}$A) to be registered with a 15-minute measuring time. These instruments require considerable skill in setting up and are not portable. A more convenient and portable instrument is the Lindemann electrometer which has a needle which can be viewed through a microscope and gives a sensitivity about twenty times less than the Hoffmann instrument.

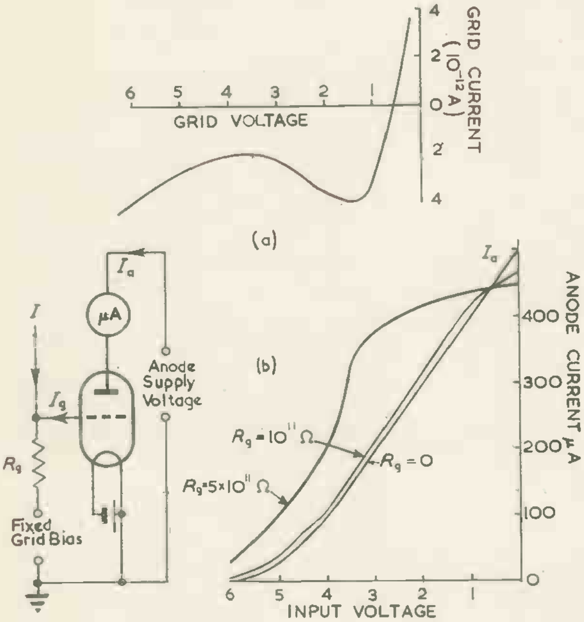



Fig. 5. Simple valve electrometer

Feedback can be applied to some electrometers by a light beam and photocell in a similar manner to galvanometers. This enables air damping to be replaced by electrical damping, the instrument being evacuated[12]. This reduces fluctuations arising from collision of air molecules with the electrometer system and enables smaller currents to be measured. By using this arrangement with a specially designed electrometer, Milatz et al[12] suggests that the R.M.S. error in charge can be reduced to 8 or 9 electrons for a measuring time of 10 seconds.

### Thermionic Electrometers

The desirable properties of an electrometer, i.e. high input resistance and low input capacitance, can be obtained with a thermionic valve.

When a current is to be measured by a valve, fluctuations in the current taken by the measuring electrode, the control grid, must be small compared with the current being measured. Fluctuations in the control grid current occur because of valve electrode potential changes, variations in cathode emission, contact potential changes, shot noise, etc. Also, since the grid current in general varies with control

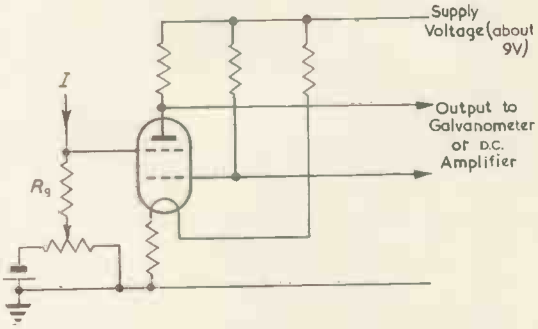



Fig 6. DuBridge and Brown Circuit

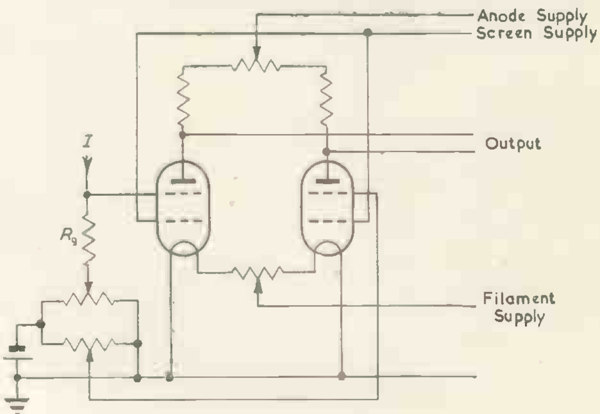
grid potentials (Fig. 5(a)), non-linearity of the relationship between signal current and grid potential may result when large values of grid resistors are used (Fig. 5(b)). For these reasons, "electrometer" valves with small grid currents were developed[13] for measuring very small currents. One popular tetrode electrometer valve has a grid current of only $10^{-16}$ to $10^{-17}$A.

In general, the large grid current of ordinary receiving valves makes them unsuitable for electrometer service. Some valve types, when operated under certain conditions, can be made to operate with grid currents of $10^{-14}$ to $10^{-15}$A. Although such results can be obtained with some specimens of a given valve type, considerable selection is usually necessary[14,15,16].

In the case of tetrode valves, zero drift due to supply voltage variations can be reduced by the DuBridge and Brown[5,9] type of circuit, shown in Fig. 6. In this arrangement, supply voltage variations cause almost equal changes in screen grid and anode current so that their difference remains constant for a constant control grid voltage. Many modifications have been described giving varying degrees of compensation against zero instability[9,17] caused by supply voltage variations.

Two valves can be used in the bridge arrangement[18], illustrated in Fig. 7, so that supply voltage changes have very little effect on the out-of-balance current, since the bridge arms are symmetrical. The two valves in such circuits may be replaced by two valves built into a common envelope with common cathode and space charge grid[19,20] (Fig. 8). If the two sections have characteristics which can be compensated by adjustment of the circuit constants, the effect of supply voltage variations and internal valve changes can be greatly reduced, a zero stability referred to the input of $100\mu$V/hr being possible. The difficulties of precise and stable adjustment of the compensating components make it necessary to use high capacity accumulators or well stabilized mains operated supplies when making very sensitive measurements[20,21].

Fig. 7. Balanced bridge circuit



Although disturbances due to external causes are reduced by balanced circuits, internal fluctuations in the valves, e.g. flicker effect, cause a greater noise level than in single valve circuits. Flicker effect is a source of noise originating from the cathode and becomes important at low frequencies[22]. Tests on noise and grid current in valves[23] show that for non-electrometer valves, the grid current noise greatly exceeds flicker noise. In electrometer valves with grid currents of the order of $10^{-15}$A, flicker noise is often more serious than the shot noise of the grid current.

When used under low grid current conditions, electrometer valves in general have a mutual conductance of about $50\mu$A/V, which is small compared with ordinary valves. The electrometer valve can be operated with a grid resistor of $10^{12}\Omega$, however, and if a current of $10^{-15}$A is passed through this resistor the anode current will be changed by $0{\cdot}05\mu$A. Such a current is easily detected by a reflecting galvanometer. It is interesting to note that a current amplification of 50 million is obtained in one valve.

As an alternative to using the electrometer valve as a current amplifier, it can be used as a voltage amplifying stage, the output being fed into a D.C. amplifier. Since the

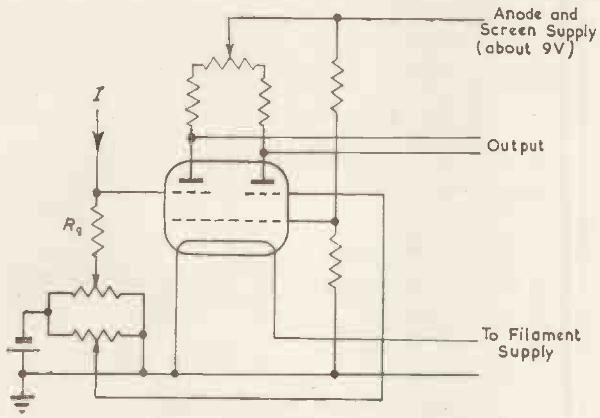



Fig. 8. Balanced tetrode circuit using double valve

voltage amplification of electrometer valves is small, usually about unity, the drift and noise introduced by the subsequent amplifier must be less than that due to the electrometer valve if the latter is to be used to its full capacity. The D.C. amplifier can be made capable of a large power output, so enabling automatic pen recording of small currents. If negative feedback is applied, it can be used to give constant and controllable gain (Appendix 2), this being required in mass spectrometer amplifiers.

Multi-stage D.C. amplifiers are usually used where high gain is required and many circuits have been described. Some of these have highly critical adjustments for best performance and if the adjustments drift for any reason, the performance suffers. The balanced type with a well stabilized power unit, such as that described by Peirson[24], is likely to give the best results over long periods without adjustment. In general, the zero instability of thermionic D.C. amplifiers (referred to the input) will not be less than 50-100$\mu$V (drift + noise) over 30min[22]. An extensive review of various types of D.C. amplifiers is given by Kessler[25].

A convenient arrangement, which has been used by Chance and Legallais[26], is to follow the electrometer valve with a low drift contact modulated amplifier. Alternatively, magnetic or galvanometer amplifiers could be used. Instead of D.C. amplification, electrometer tetrode valves have been employed as D.C. to A.C. convertors by applying suitable modulation to the control grid and the signal to the screen grid[27]. This modulator is followed by an A.C. amplifier and demodulator. The arrangement has some of the advantages of the capacitance modulator system and can be made with a zero stability of 0·5mV in 24 hours and a grid current of $10^{-14}$A[28]. None of these possibilities appear to have been much exploited, however.

As with other types of electrometer, a steady drift is not necessarily serious and for quick measurements, it is the noise which sets a limit to the accuracy obtained. If precautions are taken to remove other sources of instability, the performance of thermionic electrometers will be limited mainly by shot effect of the grid current. Though the algebraic sum of grid current components (positive and negative) is small or zero, each component exhibits its own shot noise. The capacitance charging method is used for very sensitive measurements and currents of $5 \times 10^{-18}$A have been detected by this means[21]. Currents of the order of $10^{-16}$A can be detected with the voltage drop method, though $10^{-15}$ is a value nearer that obtained in routine work.

### Capacitance Modulator Electrometers

In these devices[5], the direct current to be measured is made to charge a capacitor, $C_v$ in Fig. 9, the value of which is periodically varied, so giving rise to an alternating voltage which is then amplified. The voltage generated

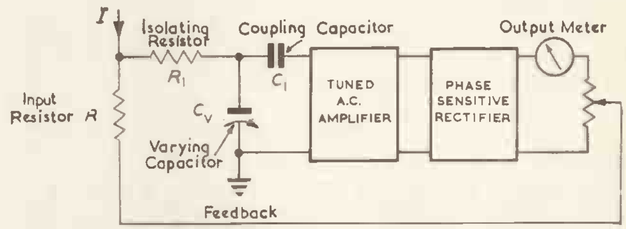



Fig. 9. Capacitance modulator electrometer

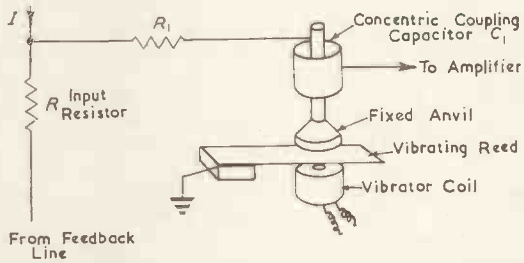



Fig. 10. Principle of vibrating reed electrometer

across capacitor $C_v$ is taken through an isolating capacitor $C_1$ before being passed into the amplifier. In one design, illustrated in Fig. 10, the capacitor $C_v$ is formed by the capacitance between an insulated fixed anvil and an earthed vibrating reed.

The use of A.C. amplification represents a great simplification compared with D.C. throughout. The bandwidth of the A.C. amplifier can be made small and centred on a frequency away from that contained in interference (such as 50c/s, microphony, etc.) and out of the range where flicker effect is troublesome and enables the theoretical limit set by thermal agitation to be more nearly approached.

The high gain A.C. amplifier is followed by a phase sensitive detector, D.C. feedback being applied (Fig. 10) to maintain constant and controllable sensitivity and reduce the input time-constant (Appendix 2).

Although the A.C. amplifier removes the difficulty of drift experienced with D.C. amplifiers, zero drift can originate from contact potential changes between the capacitor modulator electrodes. By paying attention to the treatment of the parts to keep the contact potentials constant, the drift referred to the input can be reduced to 0·1 to 1mV/day[29,30].

This type of electrometer requires considerable mechanical precision and special treatment of the capacitor electrodes, but the instrument is a very convenient and robust device and is not easily damaged by electrical overload.

**Conclusions**

The performance of all three types of electrometer described can be made to approach that fixed by thermal agitation noise, provided precautions are taken to remove instrumental limitations.

The electro-mechanical electrometers have the disadvantage of fragility and difficulty of setting up, though some types, such as the Lindemann, have the virtue of small size and relative simplicity.

The thermionic valve electrometers are more convenient to use. If no subsequent amplifier is used, a very sensitive galvanometer is necessary when measuring very small currents. D.C. amplification after an electrometer valve gives greater power output for automatic recording purposes, but often introduces considerable noise and zero drift, though these can be much reduced by choice of a suitable amplifier.

The capacitor modulator is superior to the other types, being very sensitive and at the same time robust and reliable.

**Acknowledgments**

The author is indebted to his colleagues for helpful discussion and criticism and to Dr. T. E. Allibone, F.R.S., for permission to publish this article.

## Appendix 1.

### General Experimental Techniques

To reduce leakage currents, insulators supporting the input conductor to an electrometer must be of high resistance. It is also desirable to surround such terminals with a guard ring maintained at the potential of the conductor supported. Electrometer valves are usually fitted with external and internal guard rings[19].

As a high insulation material, polished amber has been much used, but nowadays plastic insulating materials such as polished polystyrene, polythene and polytetrafluorethylene (P.T.F.E.) are often employed. These have the advantage of being able to be moulded, and P.T.F.E. that of having a higher working temperature and rigidity, though more expensive. Fused quartz is a good insulator, though care must be taken to ensure that it is free from piezo electric charges caused by strain. Glass is relied upon in electrometer valves for the input terminal insulation, but it has been known to exhibit polarization effects. The effective surface resistivity of glass and ceramic materials under humid conditions can be increased by the use of waxes or silicone varnishes[15]. For measuring the smallest currents, it is preferable to avoid leakage due to moisture by enclosing the electrometer valve in an evacuated enclosure. The enclosing vessel can conveniently be made an electrical screen and serve to screen the valve from light, as this increases the grid current because of the photo-electric effect on the electrodes, and if variable in intensity, leads to spurious signals.

Screening from both static and alternating electric fields is very important. Alternating voltages induced by magnetic fields can be reduced by keeping loops in the wiring as small as possible. Stray alternating voltages are to be avoided since they can be rectified by non-linearity in the electrometer, so giving a spurious indication[11,18]. The disturbing effect of spark coils, switch contacts in inductive circuits, etc., can be reduced by filters in the leads to the electrometer electrodes.

It is recommended that unnecessary insulation should be coated with a conducting material such as colloidal graphite in order to avoid the accumulation of static charges.

Essential insulation should be screened from varying potentials for the same reason. Currents as high as $10^{-12}$A can be caused by very slight movements of concentric flexible cables connected to the input of electrometers. These currents are caused by charges formed on the outer surface of the insulated core of the cable and can be prevented by coating this with graphite. When measuring currents of less than $10^{-13}$A, long term fluctuations caused by recovery of insulation from previous mechanical shock or compression may occur.

In the past, investigators have made their own high valued resistors, but now commercially produced units are available with values up to $10^{13}\Omega$. The semi-conducting nature of these resistors gives rise to rather a high temperature coefficient of resistance and poor stability over long periods, so limiting the accuracy[1]. The best resistors are stable to a few per cent over periods of months and have a temperature coefficient of 0·2 to 0·4 per cent/°C[31].

## Appendix 2.

### Use of Negative Feedback with Electrometers

If an electrometer is followed by an amplifier gain $-A$, negative feedback can be applied through a potential divider ratio $\beta$ to the earthy end of the resistor $R$ (Fig. 11). The output voltage $V_o$ will be given by:

$$V_o = \frac{-AIR}{1 + A\beta}$$

It is well known that the effect on $V_o$ of changes in the amplifier gain and non-linearity in the amplifiers is reduced by a factor $(1 + A\beta)$ so improving the constancy of calibration. Variation of $\beta$ provides a convenient sensitivity control without changing the resistor $R$. It should be noted

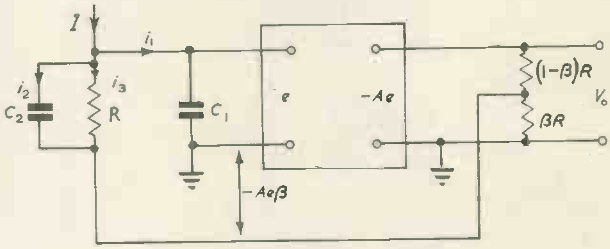



Fig. 11. Use of negative feedback

that the feedback does not reduce the effect of input instability of the electrometer, since this is indistinguishable from the signal.

Without feedback, the input time-constant is given by $R\,(C_1 + C_2)$ where:

$R$ = input resistor

$C_2$ = capacitance across it

$C_1$ = capacitance of input lead to earth.

With feedback, the situation is modified as follows:

$$I = i_1 + i_2 + i_3 \text{ (Fig. 11)}$$
$$= C_1\, de/dt + C_2\,(1 + A\beta)\, de/dt + (1 + A\beta)\, e/R$$
$$\text{where } V_o = eA.$$
$$= [C_1 + C_2\,(1 + A\beta)]\, de/dt + (1 + A\beta)\, e/R$$
$$\therefore e = \frac{IR}{1 + A\beta}\left[1 - \exp - \frac{t}{\dfrac{R[C_1 + C_2\,(1 + A\beta)]}{1 + A\beta}}\right]$$

The effective time-constant is then

$$\frac{R[C_1 + C_2\,(1 + A\beta)]}{1 + A\beta} = R\left[\frac{C_1}{1 + A\beta} + C_2\right]$$

As $C_1$ is usually the larger component, consisting of stray and valve capacitance, a large reduction in time-constant can be made by the feedback.

The capacitance $C_2$ is formed by the distributed and self-capacitance of the resistor $R$ and usually sets a limit on the lowest obtainable time-constant. It has been shown that the time-constant can be reduced further by the use of guard rings connected to the feedback line[32]. These electrodes reduce the charging currents of the capacitance $C_2$

and with a suitable arrangement a bandwidth of 150c/s was obtained for $R = 10^{12}\Omega$ and $C_2 = 10$pF.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## FLIGHT SIMULATORS FOR CANADA

THE Royal Canadian Air Force has recently taken delivery of the first of ten flight simulators being built for them by Messrs. Redifon Ltd, London.

The type F86E simulator consists basically, of a replica of the Sabre cockpit with every instrument and control exactly reproduced, with control panels and recorders for the instructor together with the associated computing equipment.

Practically any navigational problem or emergency condition can be presented to the trainee pilot. These problems and the normal conditions of flight are translated by an analogue computor consisting of a system of electronic and electro-mechanical apparatus into instrument readings and control responses.

*The instructor's console.*

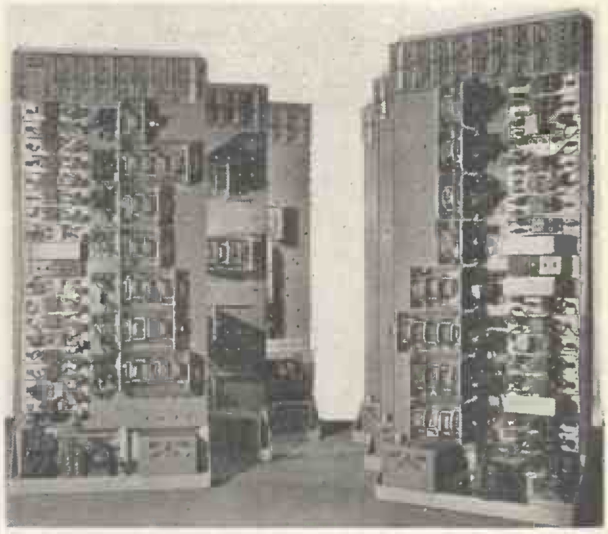
The pilot's handling of the controls and equipment produces the same results in the flight simulator as would be experienced in an actual aircraft—without any of the hazard and at a fraction of the cost of actual flight.

It is thus possible to feed emergency conditions into the simulator—conditions, which, with a student-pilot, might in the air, easily lead to disaster. Such things as engine or instrument failures can be introduced and repeated at will until the instructor is satisfied that the trainee's actions are immediate and correct.

Appropriate aural effects and correct feel of all controls are examples of the faithfulness of simulation. In fact, the only impressions which are not given are the physical effect of accelerations and the external view through the canopy.

When the flight simulator is in operation, signals originating from the controls are fed through valve amplifiers to actuate a number of interconnected electro-mechanical servo units which form the analogue computors.

The servo units are reversible motor-driven integrators or position controlled devices carrying assemblies of potentiometers, with windings contoured to produce various voltage functions. The voltages are combined to interrelate the computing servos and thereby solve the appropriate equations. Every movement of the elevator, ailerons, flaps, rudder and throttle is translated into the reading of the airspeed, rate of climb, rate of turn, pitch and roll instruments in the cockpit. Throttle adjustment results in appropriate variations in the simulated engine noises which also vary with airspeed. Gunfire and brake squeal are examples of other noises generated.



*The analogue computors.*

A large number of failures and effects can be fed in from the instructor's console, including fires in various zones of the aircraft; failure of supplies; flight instrument failures; hydraulic failures; undercarriage faults and errors in all engine and fuel system instruments. The radio aids equipment on the console and the two recorders give full operation—including failures and deficiencies—of the radio facilities. Each aid is independently adjustable for different transmitter locations.

# Voltage Stabilizers for Microwave Oscillators

## Effects on Stability of Variations in Valve Heater Voltage

By F. A. Benson*, M.Eng., Ph.D., A.M.I.E.E., M.I.R.E., and G. V. G. Lusher*, B.Eng., Grad.I.E.E.

*A mathematical analysis is given of the effect of the varying heater voltage on the stability of the types of valve stabilizer commonly used for the operation of low-power microwave oscillators. It is shown that, although analyses assuming constant heater voltage are not perfectly adequate, the effects of heater voltage on the stability of series-parallel arrangements are not as serious as might be expected.*

THE characteristics required of voltage supplies for satisfactory operation of low-power microwave oscillators are now well known. A number of suitable power supply circuits have been given previously by one of the authors[1,2] and have been examined, in detail mathematically. In calculating the performances of these stabilizers it was assumed that the heater voltages of the valves remained constant, but that this is not entirely adequate has been pointed out[3]. Additional calculations

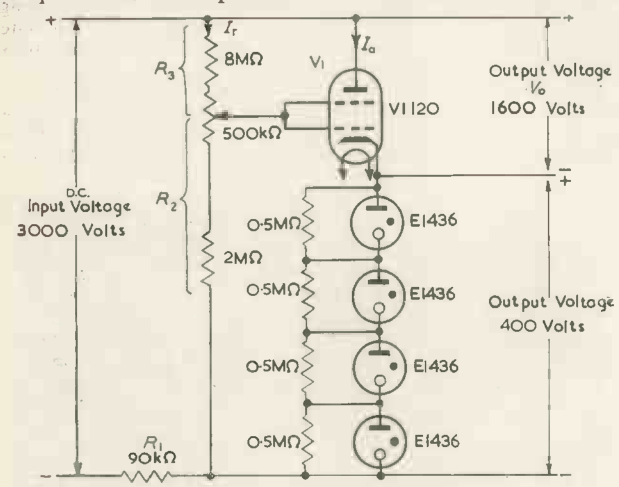



**Fig. 1. Parallel valve stabilizer for microwave oscillator**

have, therefore, now been carried out on each circuit to find the effects of heater-voltage variations on the stability. The results are presented below.

To make the calculations it was found necessary to investigate the variations of triode characteristics with heater voltage and the results of these examinations have already been given[4]. It has been shown that the anode voltage $V_a$, the grid voltage $V_g$ and the anode current $I_a$ of a triode are related by the expression:

$$V_a = r_aI_a - \mu V_g - cr_a \quad \text{(1)}$$

where $\mu$ is the amplification factor of the valve,

$r_a$ is the anode resistance of the valve,

and $c$ is a constant which for many triodes is normally nearly zero.

When the heater voltage of the valve varies it would be expected that $\mu$, $r_a$ and $c$ will all change. Such changes must be determined experimentally. Measurements on a number of triodes which are commonly used in voltage stabilizer circuits have shown, however, that the quantity $c$ is the only one that need be considered because the values of $\mu$ and $r_a$ are reasonably constant even for fairly large heater voltage variations. Thus, in making calculations on a stabilizer where the heater voltages of the valves vary, equation (1) must be used. Then proceeding in the usual way with the calculations an expression is obtained for the output voltage in terms of several known values and certain quantities ($c$'s) which are functions of heater voltages. Consequently the ratio of the percentage change of output voltage to the percentage change of heater voltage can be found providing that it is known how the various $c$'s depend on the heater voltage.

Consider first the parallel-valve stabilizer of Fig. 1. The

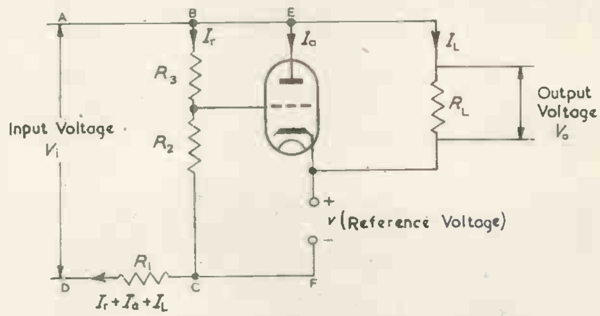



**Fig. 2. Equivalent circuit of Fig. 1**

circuit may be redrawn for the purpose of analysis as in Fig. 2. Let the currents and voltages be as shown.

By Kirchhoff's laws the following equations are obtained:

From A to B to C to D:

$$V_i = I_r(R_2 + R_3) + (I_r + I_a + I_L)R_1 \quad \text{(2)}$$

From A to E to F to D:

$$V_i = (I_r + I_a + I_L)R_1 + v + I_LR_L \quad \text{(3)}$$

Assuming linear valve characteristics:

$$I_LR_L = I_ar_a - \mu V_g - cr_a \quad \text{(4)}$$

In this case:

$$V_g = I_rR_2 - v \quad \text{(5)}$$

From (4) and (5):

$$I_LR_L = I_ar_a - \mu(I_rR_2 - v) - cr_a \quad \text{(6)}$$

From (2), (3) and (6) re-arranging the terms:

$$I_aR_1 + I_r(R_1 + R_2 + R_3) + I_LR_1 = V_i \quad \text{(7)}$$

$$I_aR_1 + I_rR_1 + I_L(R_1 + R_L) = V_i - v \quad \text{(8)}$$

$$I_ar_a - I_r\mu R_2 - I_LR_L = -\mu v + cr_a \quad \text{(9)}$$

Subtracting (8) from (7):

$$I_r = (v + I_LR_L)/(R_2 + R_3) \quad \text{(10)}$$

From (8) and (10):

$$I_a = [V_i - v - R_1(v + I_LR_L)/(R_2 + R_3) - I_L(R_1 + R_L)]/R_1 \quad \text{(11)}$$

* *The University of Sheffield.*

From (9):

$$I_a = [-\mu v + c r_a + I_L R_L + \mu R_2(v + I_L R_L)/(R_2+R_3)]/r_a \quad \ldots \; (12)$$

From (11) and (12) and remembering that $I_L R_L = V_o$:

$$V_o = \frac{V_i r_a(R_2+R_3) + v\{\mu R_1 R_3 - r_a(R_1+R_2+R_3)\} - c r_a(R_2+R_3)R_1}{R_1 r_a(R_2+R_3)/R_L + [R_1(R_2+R_3+\mu R_2) + r_a(R_1+R_2+R_3)]} \quad \ldots\ldots\ldots\ldots \; (13)$$

$$dV_o/dV_h = \frac{-dc/dV_h \; r_a(R_2+R_3)R_1}{R_1 r_a(R_2+R_3)/R_L + [R_1(R_2+R_3+\mu R_2) + r_a(R_1+R_2+R_3)]} \quad \ldots\ldots\ldots\ldots \; (14)$$

Hence the ratio of the percentage change of output voltage to the percentage change of heater voltage $dV_o/V_o \,.\, V_h/dV_h =$

$$\frac{-\,dc/dV_h \; r_a(R_2+R_3)R_1 \,.\, V_h}{V_i r_a(R_2+R_3) + v\{\mu R_1 R_3 - r_a(R_1+R_2+R_3)\} - c r_a(R_2+R_3)} \quad \ldots\ldots\ldots\ldots \; (15)$$

For the particular case under consideration:

$R_1 = 90\text{k}\Omega \quad R_3 = 8{\cdot}4\text{M}\Omega \quad V_i = 3\,000\text{V} \quad \mu = 300$

$R_2 = 2{\cdot}1\text{M}\Omega \quad r_a = 100\text{k}\Omega \quad v = 400\text{V}$

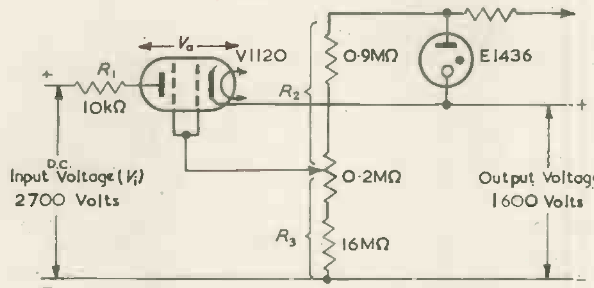



Fig. 3. Series valve stabilizer for microwave oscillator

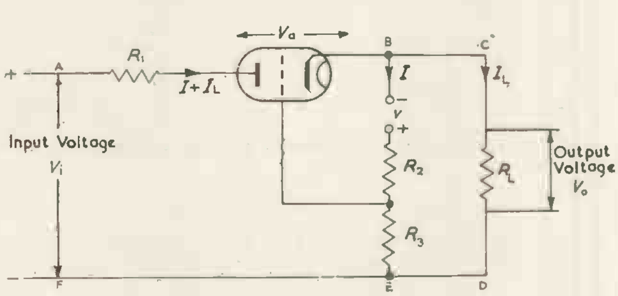



Fig. 4. Equivalent circuit of Fig. 3

Assume $c = 0$. $c$ changed by 0·8mA for a change of $V_h$ of 10 per cent.

Hence $dV_o/V_o \,.\, V_h/dV_h = -0{\cdot}008$

i.e. a change of heater voltage of 10 per cent causes the output voltage to change by 0·08 per cent, i.e. by about 1·3 volts.

It should be noted that previous calculations, made assuming constant valve-heater voltage, showed that a change of input voltage of 10 per cent caused the output voltage to change by about 5 volts. Further, a change in load resistance of 10 per cent causes the output voltage to change by about 1 volt only.

Consider now the series-valve circuit of Fig. 3 which may be redrawn for the purpose of analysis as in Fig. 4. Let the currents and voltages be as shown.

By Kirchhoff's laws the following equations are obtained:

From A to B to C to D to F:

$$V_i = R_1(I_L + I) + V_a + I_L R_L \quad \ldots\ldots\ldots \; (16)$$

From B to E to D to C:

$$v = I(R_2 + R_3) - I_L R_L \quad \ldots\ldots\ldots\ldots \; (17)$$

With linear valve characteristics as in the previous case:

$$V_a = (I_L + I)r_a - \mu V_g - c r_a \quad \ldots\ldots\ldots \; (18)$$

and

$$V_g = v - IR_2 \quad \ldots\ldots\ldots\ldots\ldots \; (19)$$

Therefore, proceeding as in the previous case and remembering that $I_L R_L = V_o$, it is found that:

$$V_o = \frac{R_L[(V_i + c r_a)(R_2+R_3) + v(\mu R_3 - R_1 - r_a)]}{R_L(R_1 + r_a + \mu R_2 + R_2 + R_3) + (R_1 + r_a)(R_2+R_3)} \quad (20)$$

$$dV_o/dV_h = \frac{R_L r_a(R_2+R_3)\, dc/dV_h}{R_L(R_1 + r_a + \mu R_2 + R_2 + R_3) + (R_1 + r_a)(R_2+R_3)} \quad (21)$$

Hence, the ratio of the percentage change of output voltage to the percentage change of heater voltage, i.e.

$$\frac{dV_o}{V_o}\,\frac{V_h}{dV_h} = \frac{dc/dV_h \,.\, r_a(R_2+R_3)\,.\, V_h}{(V_i + c r_a)(R_2 + R_3) + v(\mu R_3 - R_1 - r_a)} \quad (22)$$

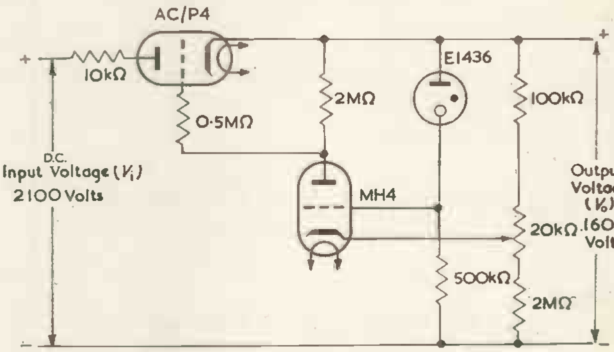



Fig. 5. Series parallel valve stabilizer for microwave oscillator

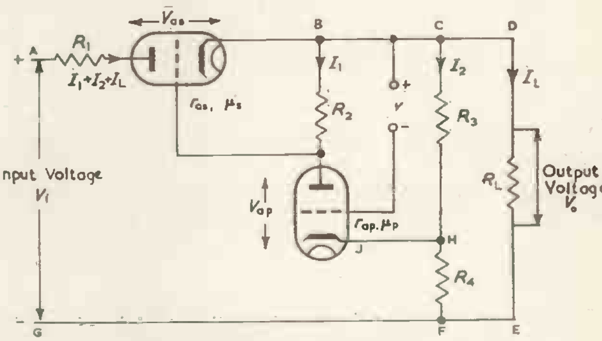



Fig. 6. Equivalent circuit of Fig. 5

For the particular case under consideration:

$R_1 = 10\text{k}\Omega \quad R_3 = 16\text{M}\Omega \quad V_i = 2\,700\text{V}$

$R_2 = 1\text{M}\Omega \quad r_a = 100\text{k}\Omega \quad v = 100\text{V} \quad \mu = 300$

Assume $c = 0$. $c$ changed by 0·8mA for a change of $V_h$ of 10 per cent.

$$\therefore \quad \frac{dV_o}{V_o}\,\frac{V_h}{dV_h} = +0{\cdot}026$$

i.e. a change of heater voltage of 10 per cent causes the output voltage to change by 0·26 per cent, i.e. by about 4·2 volts.

It should be noted that previous calculations, made assuming constant valve heater voltage, showed that a change of input voltage of 10 per cent caused the output voltage to change by about 14 volts. Further, a change in load resistance of 10 per cent causes the output voltage to change by 3·5 volts.

The first arrangement of three series-parallel valve stabilizer circuits will now be considered. It is shown in Fig. 5. The circuit may be redrawn for the purpose of analysis as in Fig. 6. Let the currents and voltages be as shown.

By Kirchhoff's laws the following equations are obtained:

From A to B to D to E to G:

$$V_i = (I_1 + I_2 + I_L)R_1 + V_{as} + I_LR_L \quad \ldots\ldots (23)$$

From B to C to H to J:

$$V_{ap} = I_2R_3 - I_1R_2 \quad \ldots\ldots (24)$$

From C to D to E to F:

$$I_LR_L = I_2R_3 + (I_1 + I_2)R_4 \quad \ldots\ldots (25)$$

With linear valve characteristics:

$$V_{as} = (I_1 + I_2 + I_L)r_{as} - \mu_s V_{gs} - c_s r_{as} \quad \ldots\ldots (26)$$

and

$$V_{gs} = I_1R_2 \quad \ldots\ldots (27)$$

where $r_{as}$, $\mu_s$ and $c_s$ refer to the series valve.

Also

$$V_{ap} = I_1 r_{ap} - \mu_p V_{gp} - c_p r_{ap} \quad \ldots\ldots (28)$$

and

$$V_{gp} = I_2R_3 - v \quad \ldots\ldots (29)$$

where $r_{ap}$, $\mu_p$ and $c_p$ are the constants of the parallel valve.

Therefore, proceeding as before and remembering that $V_o = R_LI_L$ it is found that:

$$V_o = \frac{R_L[aV_i + bv + c_p r_{ap} R_4(R_1 + r_{as}) - (R_1 + r_{as} + \mu_s R_2)(R_3 + R_4) c_p r_{ap} + c_s r_{as} a]}{c} \quad \ldots\ldots (30)$$

where

$$a = (R_3 + R_4)(R_2 + r_{ap}) + R_3R_4(1 + \mu_p) \quad \ldots\ldots (31)$$

$$b = \mu_p[R_3(R_1 + r_{as} + \mu_s R_2) + R_4 \mu_s R_2] \quad \ldots\ldots (32)$$

$$c = a(R_1 + R_L + r_{as}) + R_L(R_1 + r_{as})(R_2 + r_{ap}) + R_3R_L(R_1 + r_{as} + \mu_s R_2)(1 + \mu_p) \quad \ldots\ldots (33)$$

$$\therefore \frac{dV_o}{dV_h} = -\frac{R_L}{c}\left[\frac{dc_p}{dV_h} r_{ap} \left\{(R_4\mu_sR_2 + R_3(R_1 + r_{as} + \mu_sR_2)\right\} - \frac{dc_s}{dV_h} r_{as}a\right] \quad (34)$$

Hence the ratio of the percentage change of output voltage to the percentage change of heater voltage, i.e.

$$\frac{dV_o}{V_o} \cdot \frac{V_h}{dV_h} = -\frac{[dc_p/dV_h\, r_{ap}\left\{R_4\mu_sR_2 + R_3(R_1 + r_{as} + \mu_sR_2)\right\} - dc_s/dV_h\, r_{as}a]V_h}{[aV_i + bv + c_p r_{ap} R_4(R_1 + r_{as}) - (R_1 + r_{as} + \mu_s R_2)(R_3 + R_4) c_p r_{ap} + c_s r_{as} a]} \quad \ldots\ldots (35)$$

For the particular case under consideration:

| | | | |
|---|---|---|---|
| $R_1 = 10\text{k}\Omega$ | $R_3 = 110\text{k}\Omega$ | $V_i = 2\,100\text{V}$ | $r_{as} = 2{\cdot}8\text{k}\Omega$ |
| $R_2 = 2\text{M}\Omega$ | $R_4 = 2\text{M}\Omega$ | $v = 100\text{V}$ | $r_{ap} = 100k\Omega$ |
| | $\mu_s = 18$ | $\mu_p = 25$ | $c_p = c_s = 0$ |

$c_p$ changed by 0·7mA for a 10 per cent change of $V_h$ and $c_s$ changed by 1·1mA for a 10 per cent change of $V_h$.

Therefore $\frac{dV_o}{V_o} \frac{V_h}{dV_h} = -0{\cdot}063$

i.e. changes of heater voltages of 10 per cent cause the output voltage to change by 0·63 per cent, i.e. by about 10 volts.

It should be noted that previous calculations, made assuming constant valve heater voltage, showed that a change in input voltage of 10 per cent caused the output voltage to change by about 16 volts. A change in load resistance of 10 per cent causes the output voltage to change by about 1 volt.

The second series-parallel valve circuit to be considered is shown in Fig. 7 and for the purpose of analysis it may be redrawn as in Fig. 8. Let the currents and voltages be as shown.

By Kirchhoff's laws the following equations are obtained:

From A to B to C to D to E to H:

$$V_i = R_1(I_1 + I_2 + I_L) + V_{as} + I_LR_L \quad \ldots\ldots (36)$$

From A to B to G to H:

$$V_i = R_1(I_1 + I_2 + I_L) + I_1R_2 + V_{ap} \quad \ldots\ldots (37)$$

From C to D to E to F:

$$v = I_2(R_3 + R_4) - I_LR_L \quad \ldots\ldots (38)$$

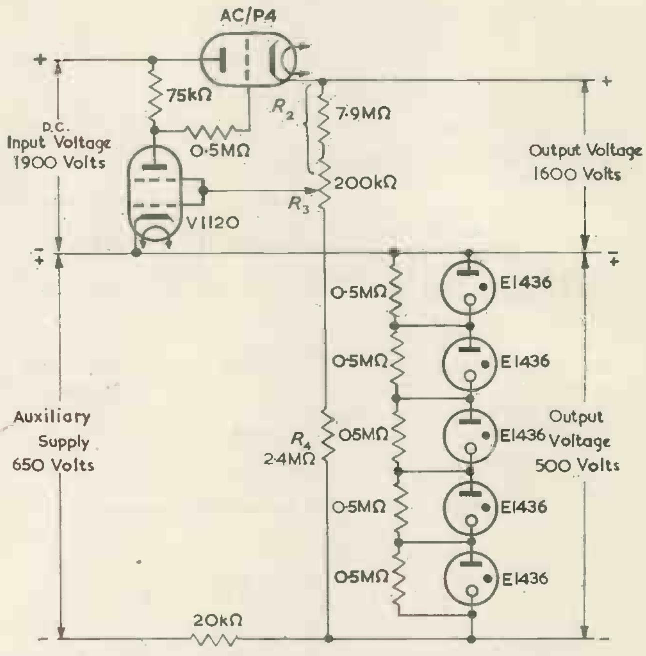



**Fig. 7. Series parallel valve stabilizer for microwave oscillator**

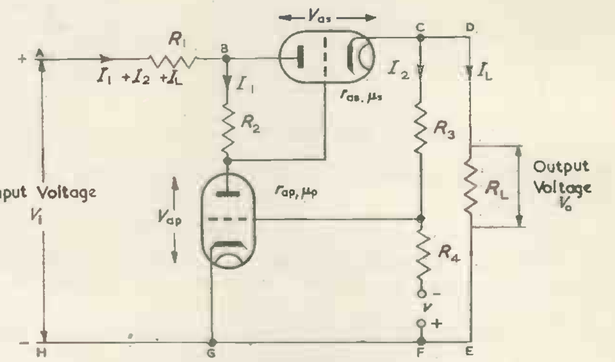



**Fig. 8. Equivalent circuit of Fig. 7**

With linear valve characteristics:

$$V_{as} = (I_2 + I_L)r_{as} - \mu_s V_{gs} - r_{as}c_s \quad \ldots\ldots (39)$$

where

$$V_{gs} = V_{ap} - I_LR_L \quad \ldots\ldots (40)$$

also

$$V_{ap} = I_1 r_{ap} - \mu_p V_p - r_{ap}c_p \quad \ldots\ldots (41)$$

and

$$V_{gp} = I_2R_4 - v \quad \ldots\ldots\ldots\ldots\ldots \quad (42)$$

where $r_{as}$, $r_{ap}$, $\mu_s$, $\mu_p$, $c_s$ and $c_p$ have the same meanings as before.

Therefore, proceeding as before and remembering that $V_o = I_LR_L$ it is found that:

$$V_o = \frac{R_L[\alpha V_i + \beta v + \delta]}{\gamma} \quad \ldots\ldots\ldots\ldots \quad (43)$$

where

$$\alpha = (R_3+R_4)\left\{R_2 + r_{ap}(1+\mu_s)\right\} \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (44)$$

$$\beta = (R_1+R_2)(\mu_s\mu_pR_3 - r_{as}) - R_1\left\{R_2 + r_{ap}(1+\mu_s) - R_3\mu_p\right\} - r_{ap}r_{as} \quad \ldots\ldots\ldots\ldots \quad (45)$$

$$\gamma = (R_3+R_4)[R_1(\mu_sr_{ap}+r_{as}) + (R_1+r_{as})(R_2+r_{ap})] + R_L[R_1(r_{as}+\mu_sr_{ap}) + r_{ap}(R_1+r_{as}) + \mu_pR_4(R_1\mu_s+R_1) + R_2(R_1+r_{as}+\mu_s\mu_pR_4) + (1+\mu_s)(R_3+R_4)(R_1+R_2+r_{ap})] \quad \ldots\ldots\ldots\ldots \quad (46)$$

$$\delta = (R_3+R_4)[r_{as}c_s(R_1+R_2+r_{ap}) - r_{ap}c_p(R_1+\mu_sR_1+\mu_sR_2)] \quad \ldots\ldots\ldots\ldots \quad (47)$$

Therefore:

$$dV_o/dV_h = \frac{R_L[(R_3+R_4)\left\{r_{as}\, dc_s/dV_h\, (R_1+R_2+r_{ap}) - r_{ap}\, dc_p/dV_h (R_1+\mu_sR_1+\mu_sR_2)\right\}]}{\gamma} \quad \ldots\ldots\ldots\ldots \quad (48)$$

Hence, the ratio of the percentage change of output voltage to the percentage change of heater voltage,

$$\frac{dV_o}{V_o}\cdot\frac{V_h}{dV_h} = \frac{[(R_3+R_4)\left\{r_{as}\; dc_s/dV_h\; (R_1+R_2+r_{ap}) - r_{ap}\; dc_p/dV_h\; (R_1+\mu_sR_1+\mu_sR_2)\right\}]\, V_h}{[(R_3+R_4)\left\{R_2+r_{ap}(1+\mu_s)\right\}V_i + [(R_1+R_2)(\mu_s\mu_pR_3 - r_{as}) - R_1\left\{R_2 + r_{ap}(1+\mu_s) - R_3\mu_p\right\} - r_{ap}r_{as}]v + (R_3+R_4)\left\{r_{as}c_s(R_1+R_2+r_{ap}) - r_{ap}c_p(R_1+\mu_sR_1+\mu_sR_2)\right\}]} \quad \ldots\ldots\ldots\ldots \quad (49)$$

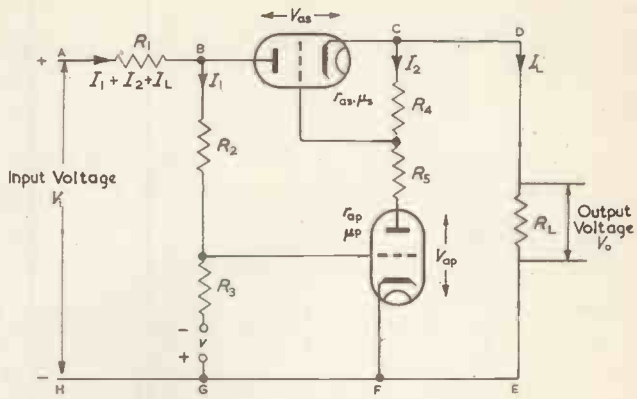



**Fig. 10. Equivalent circuit of Fig. 9**

For this particular case:

| | | | |
|---|---|---|---|
| $R_1 = 0$ | $R_4 = 2{\cdot}5\text{M}\Omega$ | $r_{ap} = 100\text{k}\Omega$ | $V_i = 1\,900\text{V}$ |
| $R_2 = 75\text{k}\Omega$ | $R_L = 260\text{k}\Omega$ | $\mu_s = 18$ | $v = 500\text{V}$ |
| $R_3 = 8\text{M}\Omega$ | $r_{as} = 2{\cdot}8\text{k}\Omega$ | $\mu_p = 300$ | $c_p = c_s = 0$ |

$c_p$ and $c_s$ changed by 0·8mA and 1·1mA respectively for a change in heater voltage of 10 per cent.

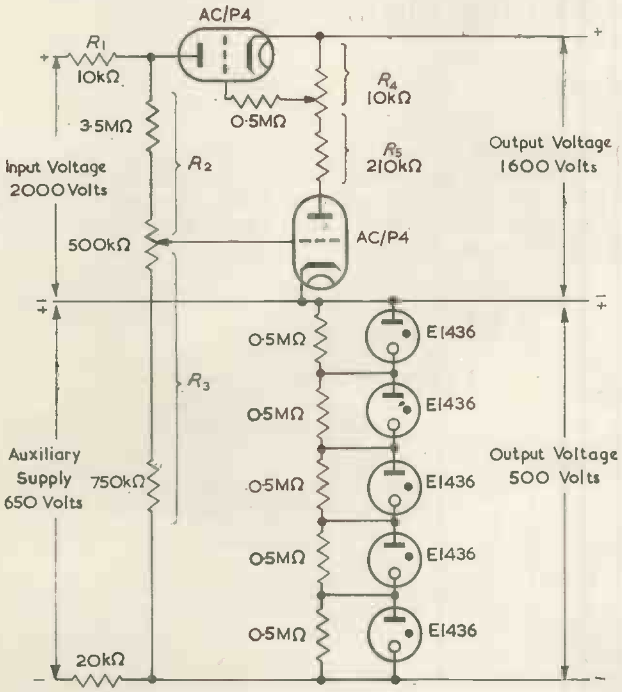
Hence

$$\frac{dV_o}{V_o}\cdot\frac{V_h}{dV_h} = -0{\cdot}007$$

i.e. a change of heater voltage of 10 per cent causes the output voltage to change by 0·07 per cent, i.e. by about 1·1 volts.

It should be noted that previous calculations, made assuming constant valve-heater voltage, showed that a change of input voltage of 10 per cent caused the output voltage to change by about 4 volts. Further, a change in load resistance of 10 per cent causes the output voltage to change by 0·003 volt only.

**Fig. 9. Series parallel valve stabilizer for microwave oscillator**




The third series-parallel circuit is shown in Fig. 9 and may be redrawn as in Fig. 10. Let the currents and voltages be as shown.

By Kirchhoff's laws the following equations are obtained:

From A to B to G to H:

$$V_i = R_1(I_1 + I_2 + I_L) + (R_2 + R_3)I_1 - v \quad \ldots\ldots \quad (50)$$

From A to D to E to H:

$$V_i = R_1(I_1 + I_2 + I_L) + V_{as} + I_LR_L \quad \ldots\ldots \quad (51)$$

From C to D to E to F:

$$V_{ap} = I_LR_L - (R_4 + R_5)I_2 \quad \ldots\ldots\ldots \quad (52)$$

With linear valve characteristics:

$$V_{as} = (I_L + I_2)r_{as} - \mu_s(-R_4I_2) - c_sr_{as} \quad \ldots\ldots \quad (53)$$

and

$$V_{ap} = I_2r_{ap} - \mu_p(I_1R_3 - v) - c_pr_{ap} \quad \ldots\ldots \quad (54)$$

where $r_{as}$, $r_{ap}$, $\mu_s$, $\mu_p$, $c_s$ and $c_p$ have the same meanings as before.

Therefore, proceeding as before and remembering that $V_o = I_LR_L$ it is found that:

$$V_o = R_L/C[Av - BV_i - c_pr_{ap}\left\{\Sigma R(R_1 + r_{as} + \mu_sR_4) - R_1^2\right\} + R_3\mu_pR_1r_{as}c_s + c_sr_{as}\,\Sigma R(R_4 + R_5 + r_{ap})] \quad \ldots\ldots\ldots\ldots \quad (55)$$

where

$$A = \mu_p\left\{\Sigma R(R_1 + r_{as} + \mu_s R_4) - R_1^2\right\} - R_3\mu_p(R_1 + r_{as} + \mu_s R_4) - R_1(R_4 + R_5 + r_{ap}) \quad \ldots\ldots (56)$$

$$B = R_3\mu_p(r_{as} + \mu_s R_4) - (R_2 + R_3)(R_4 + R_5 + r_{ap}) \quad \ldots\ldots (57)$$

$$C = R_3\mu_p R_1(R_L - \mu_s R_4) - (R_4 + R_5 + r_{ap})\left\{R_1^2 - \Sigma R(R_1 + R_L + r_{as})\right\} + R_L\Sigma R(R_1 + r_{as} + \mu_s R_4) - R_L R_1^2 \quad \ldots (58)$$

$$\Sigma R = R_1 + R_2 + R_3 \quad \ldots\ldots (59)$$

therefore:

$$\frac{dV_o}{dV_h} = \frac{R_L}{c}\left[- dc_p/dV_h \,.\, r_{ap}\left\{\Sigma R(R_1 + r_{as} + \mu_s R_4) - R_1^2\right\} + dc_s/dV_h\left\{R_3\mu_p R_1 r_{as} + r_{as}\Sigma R(R_4 + R_5 + r_{ap})\right\}\right] \quad \ldots\ldots (60)$$

$$\therefore \frac{dV_o}{dV_h}\cdot\frac{V_h}{V_o} = \frac{\left[- dc_p/dV_h \,.\, r_{ap}\left\{\Sigma R(R_1 + r_{as} + \mu_s R_4) - R_1^2\right\} + dc_s/dV_h\left\{R_3\mu_p R_1 r_{as} + r_{as}\Sigma R(R_4 + R_5 + r_{ap})\right\}\right]V_h}{\left[Av - BV_1 - c_p r_{ap}\left\{\Sigma R(R_1 + r_{as} + \mu_s R_4) - R_1^2\right\} + R_3\mu_p R_1 r_{as} c_s + c_s r_{as}\Sigma R(R_4 + R_5 + r_{ap})\right]} \quad \ldots\ldots (61)$$

In this case:

$R_1 = 10\text{k}\Omega$ $\quad R_4 = 2{\cdot}5\text{k}\Omega$ $\quad \mu_s = \mu_p = 20$

$R_2 = 3{\cdot}75\text{M}\Omega$ $\quad R_5 = 217{\cdot}5\text{k}\Omega$ $\quad v = 500\text{V}$

$R_3 = 1\text{M}\Omega$ $\quad r_{as} = r_{ap} = 2{\cdot}8\text{k}\Omega$ $\quad c_s = c_p = 0$

$c_s$ and $c_p$ change by 1·1mA for a change of heater voltage of 10 per cent.

$$\text{Therefore: } \frac{dV_o}{V_o}\cdot\frac{V_h}{dV_h} = -0{\cdot}0011$$

i.e. a 10 per cent change of heater voltage produces a change of only 0·011 per cent in the output voltage, i.e. about 0·18 volt.

It should be noted that the output voltage is independent of the input voltage changes, but a 10 per cent change in load resistance causes the output voltage to change by about 1·6 volt.

**Conclusions**

The results of the calculations confirm that analyses of thermionic-valve circuits which assume that the heater voltages of the valves remain constant are not perfectly adequate, because very frequently the heaters are supplied from windings of transformers fed directly by the mains. The effects of heater voltage variations on the stability of series-parallel arrangements are not as serious as might at first sight be expected. This is so because the two valves produce changes in output voltage of opposite sign.

In the case of a single series-valve stabilizer, however, the effects of heater voltage variations are rather serious. It is interesting to note that for some circuits the stability obtained when both input voltage and heater voltages vary simultaneously, is better than when the input voltage alone changes.

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# An Electrical Backlash Circuit

## and its use in a dividing chain


By C. H. Banthorpe*


If two valves are coupled as in Fig. 1, the circuit has the property that only one of the valves will take current and the current may be switched rapidly from one to the other by changing the voltages on $g_1$ of $V_1$. However, there is considerable backlash in the circuit, i.e. the grid voltage required to change from one stable state to the other is quite different to the voltage to change back. This effect was noticed by the author some years ago as an undesirable feature of a circuit, and was later investigated. It has also been investigated, and exploited, by the Radio Corporation of America[1].

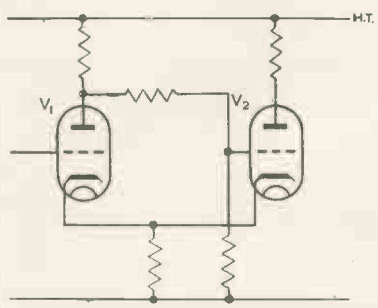


Fig. 1. Basic "switching" circuit having considerable backlash

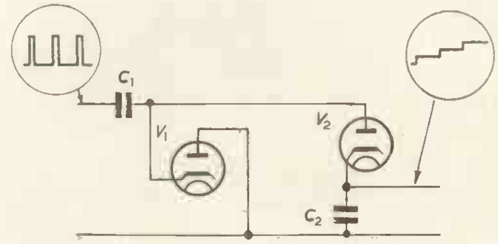


Fig. 2. Divider circuit

One way in which this circuit can be profitably employed is as a discharger in a divider chain. Thyratrons, or blocking oscillators have been used, but this type of circuit is probably more reliable.

The divider circuit is well known, and is shown in Fig. 2.


* *Derwent Radio (Central Equipment Ltd.).*

Briefly, positive pulses are applied to the input, each pulse putting a charge into $C_1$ and $C_2$ as diode $V_2$ conducts. As the capacitance of $C_2$ is much larger than $C_1$ most of the pulse voltage appears across $C_1$. At the end of the pulse $V_2$ becomes open circuit and $V_1$ conducts and discharges $C_1$ or D.C. restores it to H.T. negative. $C_2$ is thus left with a charge until the next pulse, when a second charge is added to the first and so on. The voltage across $C_2$ therefore rises in steps like a staircase. If $C_2$ is discharged when the voltage across it rises to a predetermined value then the waveform across it will be a succession of staircase voltages, the number of steps being pre-set.

If the circuit of Fig. 1 is connected across $C_2$ of Fig. 2 as in Fig. 3 the step voltage will rise until the change-over voltage is reached when current will be transferred from $V_4$ of Fig. 3 to $V_3$. At the same time $V_3$ cathode voltage

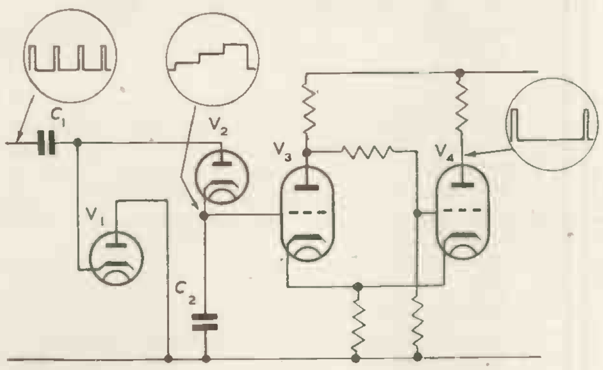


Fig. 3. Use of circuit shown in Fig. 1. as a discharger in a dividing chain

There are several ways in which this undesirable effect can be reduced, but a convenient one is to use a cathode-follower so that the diode $V_1$ does not D.C. restore $C_1$ to H.T. negative, but to a step voltage which follows the voltage across $C_2$. This is done by returning the anode of $V_2$ to the cathode of a cathode-follower whose grid is connected to $C_2$, (Fig. 4). In this way the steps can be maintained extremely uniform in amplitude up to 10 with practical values of pulse amplitude, and step heights sufficiently large so that there is no possibility of the divider stage making a wrong count. The cathode-follower is also a convenient point from which the step voltage waveform may be extracted for other purposes.

For low divide ratios it is quite unnecessary to use a cathode-follower so far as accurate dividing is concerned.

It will be seen that a positive pulse will appear at the anode of $V_4$ during the short time that $C_2$ is being dis-

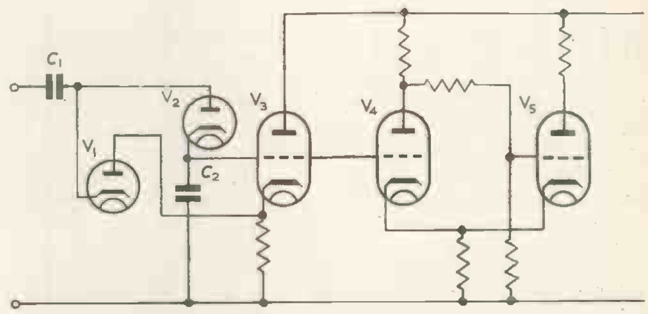


Fig. 4. Method of maintaining voltage steps at a uniform amplitude

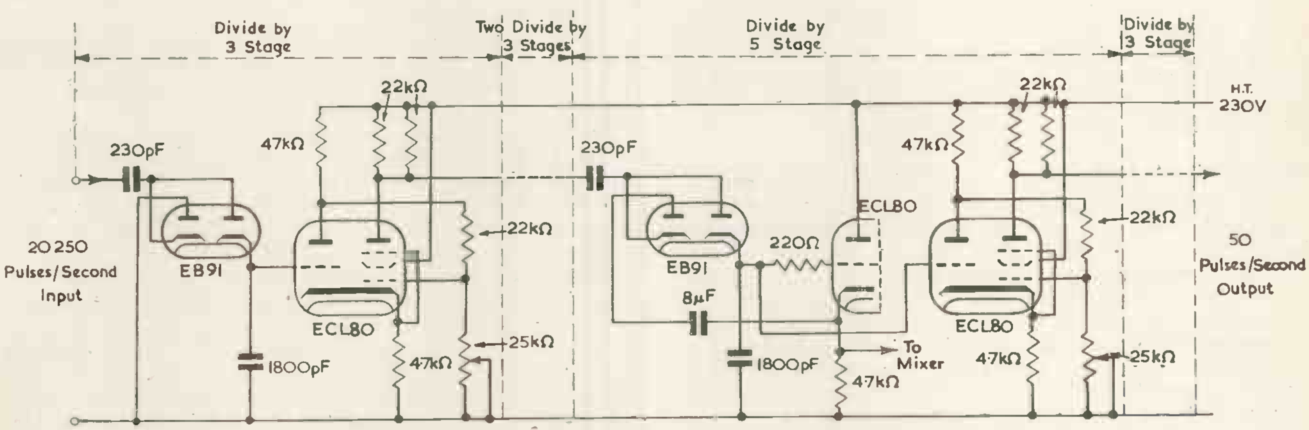


Fig. 5. Circuit for counting down from 20 250c/s to 50c/s for a television pattern generator

*All divide by 3 stages are identical*

will fall and as the grid is still considerably positive, $V_3$ will draw grid current until $C_2$ is discharged to a point at which the anode current of $V_3$ is slightly reduced.

The anode voltage of $V_3$ will then rise slightly, $g_1$ of $V_4$ will rise causing anode current to start in $V_4$, its cathode voltage to rise and with it the cathode of $V_3$. This will bias off $V_3$ still more, its anode current will reduce and its anode voltage rise. The action is cumulative and the condition of no current in $V_3$ and current in $V_4$ will be re-established.

This state will continue until $C_2$ is once more charged up to the point where anode current starts in $V_3$, and the cycle starts again.

As $C_2$ charges, the effective amplitude of the input pulse is reduced, and this means that the amplitude of each step is less than the preceding one, succeeding amplitudes following an exponential law.

charged and this pulse, which occurs once every $N$ input pulses, $N$ being the divide ratio, can be used to operate another divider. The circuit may thus be cascaded for high counts. A circuit used to count down from 20 250c/s to 50c/s ($N = 405$) for a television pattern generator is shown in Fig. 5. As extreme reliability was required the count down was restricted to 3 or 5 per stage, and the order of dividing was chosen so that the ÷ 5 stage waveform could be used as a source of modulation to produce horizontal bars of different intensity from black to white, in the final generated test pattern. This circuit has proved to be extremely stable.

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# A Relay Operated Unit for the Measurement of the Relative Slip Between Two Rollers

By G. J. Parish, B.Sc.*

*When two rollers are pressed together, one driving the other, their peripheral speeds will not in general be the same owing to slip at the nip. This article describes a unit which enables the amount of slip to be calculated. The unit records the number of revolutions of each roller between two instants at which the rollers are in the same relative positions; that is, both rollers make an integral number of revolutions in the recording period. From these numbers of revolutions and the roller circumferences the relative peripheral speeds can be calculated. The unit is designed for use with a wide range of roller sizes and speeds.*

MANY industrial processes include the passage of material between two rollers, one driving the other. In such a system the rollers will not in general have the same peripheral speed, and the difference in speed can have an important effect on the conditions in the nip; its measurement is, therefore, of some importance.

A convenient way of measuring the speed of one roller relative to that of the other is to count the numbers of revolutions made by both rollers in the same time interval; the relative peripheral speeds can then be calculated from the roller circumferences. It is desirable that the numbers of revolutions should be integral; that is, that at the ends of the interval the rollers should be in the same relative positions. With a two-roller system this can generally be achieved to sufficient accuracy without requiring an unduly long time interval.

If a large number and variety of roller combinations are to be investigated it is desirable to have a unit which can be easily connected to the machine under examination and which will perform both the revolution counting and the estimation of the starting and stopping points. This article describes such a unit.

**Design of the Unit**

The unit has been designed for use with machines having rollers of from 18 to 60 inches circumference running at speeds up to 100 yards/minute; it is operated by four sets of contact springs and four cams, two fitted to each roller shaft. The starting and stopping action of the unit is initiated by the simultaneous closing of two of these spring sets (this event will be referred to below as a "coincidence"), the other spring sets operate the revolution counters. By thus separating the estimation of the coincidences and the counting it is possible to make the coincidence cams sufficiently short to give the desired accuracy. To initiate both the starting and stopping actions it is merely necessary to energize a single relay, which holds itself on and then sets in motion the appropriate switching sequence. The closing time of the coincidence spring sets need then be only of the order of the operating time of this relay.

In the normal running condition the unit switches on at coincidences 1, 3, 5 .. .. .. and off at 2, 4, 6 .. .. .. the off periods providing a suitable time for the observer to record the counter readings. If it is desired to count for more than a single coincidence period the unit can be made free-running for any number of periods by closing a manually operated switch; this does not affect the unit's switching on, but prevents its switching off. When the manual switch is reopened the unit switches off at the next coincidence.

As the unit is designed for use with any combination of roller sizes it must not be liable to counting errors due to repeated coincidences on adjacent revolutions, a situation which can occur when the rollers are of closely similar radius and the slip per revolution is less than the coincidence cam width. These repeated coincidences could be eliminated by adjusting the cam size at each condition of roller loading and speed to be investigated, but it is more convenient to set the cams to give a reasonable counting time at the highest speed and thereafter to keep their length unchanged. Since the slip can vary appreciably with speed and roller loading, coincidence groups may then appear at lower speeds. These can be made ineffective by arranging that the unit only operates on the first coincidence of a group and ignores the remainder; it does this by resetting the on-off switching relay only after two revolutions have been made by one roller without an intervening coincidence.

The unit employs eight relays to perform the switching functions, and three counters, two of which record the roller revolutions and the third the coincidences. The counters are of the four-digit type made by the Automatic Telephone Company; the relays, with one exception, are of the 3 000 or 600 types, one of them having two windings. The exception is the start relay, the one which is energized by the closing of the coincidence springs; this is an American type somewhat faster in action than the P.O. types, having an operation time of about 7msec.

With these components the unit has given reliable service within the limits of roller sizes and speeds for which it was designed. The range could be extended to higher speeds or smaller rollers by the use of high-speed relays, providing that sufficient time was allowed in each revolution for the operation of the counters. If the revolution time became too short the counters could then be preceded by suitable scaling circuits[1].

**Circuit Operation**

The operation of the circuit is briefly as follows: none of the relays is energized until the first coincidence of a group switches on the revolution counters. If there is no coincidence on the following revolution the switching relays are reset after the second revolution and counting continues until the unit switches off at the next coincidence. If, however, there is a coincidence during the second revolution the resetting is blocked until two successive revolutions have taken place without an intervening coincidence.

The circuit diagram is shown in Fig. 1. The operation is as follows: initially none of the relays is energized. The first coincidence of a group energizes relay *ST* and counter 3 through $GT_1$, and relay *GB* by its right-hand winding. These three are held by $ST_1$ and $ST_2$ makes the circuit for *GH* and for *CS*, which in its turn makes the circuit for

* *The British Cotton Industry Research Association.*

$CC$, which prepares the revolution counter circuits at $CC_3$, $CC_4$. At the next closing of the counter springs of roller 1 in addition to the operation of counter 1 relay $GT$ is energized through $GH_2$, and $GT_1$ makes the circuit for the left-hand winding of $GB$ before breaking the circuit for $ST$. The operation of $GR$ by $GT_2$ is thus blocked by $GB_2$ and relays $GH$ and $CS$ remain energized through $GH_1$ even though $ST_2$ has opened. When counting springs 1 open $GT$ and $GB$ release in turn. If a second coincidence occurs at the next revolution, $ST$, counter 3 and $GB$ are re-energized, and the subsequent operation of $GT$ again fails to operate $GR$.

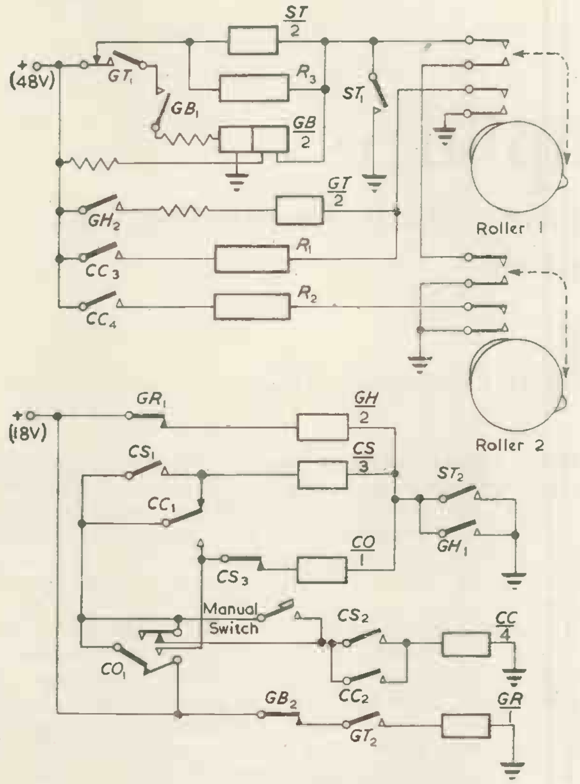


**Fig. 1. Circuit diagram**

Relays :—

| | | | |
|---|---|---|---|
| Start | $ST$ | Group Hold | $GH$ |
| Count Start | $CS$ | Group Test | $GT$ |
| Count Off | $CO$ | Group Block | $GB$ |
| Count Control | $CC$ | Group Release | $GR$ |

This cycle continues, with counter 3 counting the number of coincidences, until the coincidence group has passed. $GT_2$ is then free to operate $GR$, which in turn breaks the circuit for $GH$ and hence for $CS$. $CS_3$ prepares the circuit for $CO$ and the unit has been reset.

At the first coincidence of the next group $ST_2$ makes the circuit for $CO$ through $CO_1$, $CC_1$, $CS_3$. $CO$'s make-before-break holds itself on through $CS_3$ before breaking the circuit for $CC$. The revolution counting is stopped by the opening of $CC_3$, $CC_4$; coincidence counting continues until the end of the group, when $GR$ operates, and the release of $GH$ breaks the circuit for $CO$. The unit is then back in its quiescent state.

The closing of the manual control switch does not affect the operation of the unit when switching on, but prevents switching off. Under this condition the operation of $CO_1$ does not break the circuit for $CC$, and both $CC$ and $CO$ remain energized until the end of the coincidence group, when the release of $CO$ restores the previous condition. The manual switch may thus be used to extend the counting time over any number of coincidence periods.

The relative positions of the coincidence and the counting cams are not critical; it is only necessary that sufficient time should be allowed between the operation of the corresponding spring sets for the sequences of relay operations to be performed. In addition, the counting cams must be sufficiently long to ensure positive counter action at the highest speed.

**Accuracy**

Assuming the counting accuracy to be absolute, the accuracy of the unit is dependent solely upon the size of the coincidence cams and the operating time of the start relay. An estimate of the probable error in a single measurement can be made for the condition of single coincidences in the following manner. Suppose the number of revolutions recorded between two coincidences to be $M$ and $N$ for rollers 1 and 2 respectively. The slip of roller 2 relative to roller 1 is computed from these figures assuming them to be exactly integral. This, however, is not necessarily the case; the limiting conditions for the unit to be operated at each coincidence are as shown in Fig. 2, where $t_1$ and $t_2$ are the closed times of the springs and $\tau$ is the operating time of the start relay. There is thus an uncertainty in the position of roller 2 relative to roller 1 at each coincidence with limits $-(t_2 - \tau)$ and $+ (t_1 - \tau)$, and this uncertainty can be regarded as an error in the determination of $N$, assuming $M$ to be exactly integral.

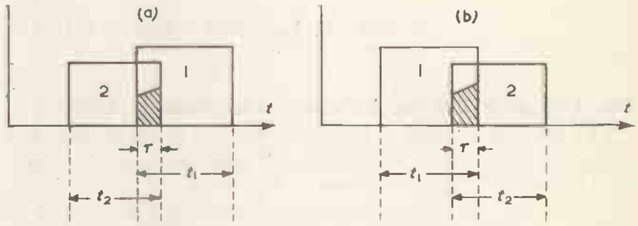


**Fig. 2. Limiting conditions for operation of unit**

For simplicity let $t_1 = t_2 = t_0$. Then, assuming a rectangular distribution of the uncertainty variables in the intervals $\pm (t_0 - \tau)$ at each coincidence, their difference is a triangularly distributed variable[2] with limits $\pm 2 (t_0 - \tau)$, and hence with a probable error ("semi-interquartile range") given by:

$$\Delta t = \pm (1 - 1/\sqrt{2}) \,.\, 2 \,(t_0 - \tau)$$
$$\approx \pm 0{\cdot}6 \,(t_0 - \tau)$$

As stated above, in general, the coincidence cams will be set to give a reasonable counting time at the highest speed. This is usually achieved for $t_0 \approx 3\tau$. At the lower speeds $\tau$ may thus be neglected, giving $\Delta t = \pm 0{\cdot}6 \ t_0$, which can be directly converted into terms of cam length and roller circumference. If $\Delta N$ denotes the error in $N$ as a fraction of a revolution, so that the revolutions of roller 2 can be written as $N \pm \Delta N$, then $\Delta N = \Delta t/T_2$ where $T_2$ is the time for one revolution of this roller. Hence, when $\tau$ is negligible.

$$\Delta N = \pm 0{\cdot}6 \ t_0/T_2$$
$$= \pm 0{\cdot}6 \ \lambda_2/L_2,$$

where $\lambda_2$ is the effective cam length and $L_2$ the roller circumference. The effective cam length is the length of the cam referred to a radius of rotation equal to the roller radius.

The measured slip is then:

$$\frac{M\,L_1 - N\,L_2}{M\,L_1} \pm (0{\cdot}5\,\lambda_2/L_2) \,.\, (L_2/M\,L_1),$$

where $L_1$ is the circumference of roller 1,

$$= \frac{ML_1 - NL_2}{ML_1} \pm 0{\cdot}6\,\lambda_2/NL_2$$

very nearly, since $NL_2 \approx ML_1$.

The error in those cases when $\tau$ is appreciable, or when repeated coincidences occur will be less than this, in the former tending to a limit two-thirds as great (if $t_0 = 3\tau$ at the highest speed), and in the latter being approximately $1/n$ as great, where $n$ is the number of coincidences in a group.

For a cam with an effective length of $1\frac{1}{2}$ inches on a roller of 25 inches circumference $\lambda_2/L_2 = 0{\cdot}06$, and at 100 yards/minute $t_0$ has a value of 25msec. The error is $\alpha = 0{\cdot}036/N$, and hence for $N > 36$ $\alpha < 0{\cdot}1$ per cent. A smaller value than this is probably unnecessary in view of the errors in the measurement of $L_1$ and $L_2$. If required the manual control can be used to obtain a sufficiently high value of $N$.

**Acknowledgments**

The author wishes to thank the Director of the British Cotton Industry Research Association for permission to publish this article.

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# D.C. Amplifiers

## Methods of Amplifying and Measuring Small Direct Currents and Potentials

**(Part 3)**

By J. Yarwood*, M.Sc., F.Inst.P., and D. H. Le Croissette†, M.Sc.

### The Use of Receiving Valves in Electrometer Circuits

The specially designed electrometer valve, which has been discussed previously, operates with the lowest grid current that has so far been achieved. This type of valve should therefore be used when extremely low currents are to be amplified. When the grid current limitations are not quite so stringent, however, it is possible to use valves in the commercial receiving range, operated at reduced potentials. With careful selection of certain valve types it is possible to approach the performance of specially designed electrometer valves.

For some applications receiving valves have advantages over electrometer valves. Hay[6] points out that the design of portable battery operated directly coupled amplifiers for the measurement of ionization current requires valves which are small and light, with a low power consumption. Until recently, when miniature and sub-miniature electrometer valves became available, the existing valves in the electrometer range were of comparatively large physical dimensions.

By comparing values for the amplification factor given in Table 1 to values given previously for electrometer valves, it can be seen that certain selected receiving valves would provide more gain in electrometer circuits. Provided the higher grid current can be tolerated, the increased output of the first stage of an amplifier allows the following stage to operate at a higher level, with a corresponding relaxation of the drift and grid current considerations which would otherwise apply.

* *The Mathematics and Physics Department, The Polytechnic W.1.*
† *The Physics Department, The University of Southampton.*

**TABLE 1.**
**Operating conditions of receiving valves suitable for electrometer use**

| VALVE | $I_g$ (A) | $V_a$ (V) | $I_a$ ($\mu$A) | $V_{g2}$ (V) | $V_f$ (V) | $I_f$ (mA) | $g_m$ ($\mu$A/V) | $\mu$ | $R_a$ (M$\Omega$) | REMARKS | REFERENCE |
|---|---|---|---|---|---|---|---|---|---|---|---|
| RCA 954 | $10^{-13}$ | 30 | 12 | 3·0 | 4·0 | — | — | — | — | Stage gain = 2.2 | Harris[65] |
| RCA 954 | $3\times10^{-16}$ | 4·8 | 6·0 | 4·5 | 3·0 | — | 30 | — | — | $V_{g1} = +0.5$V electrometer connexion | Hay[6] |
| RCA 955 | $5\times10^{-12}$ | 36 | 60 | — | 4·0 | — | 130 | 25 | 0·2 | — | Bishop[63] |
| Brimar 1C5GT | $10^{-13}$ | 16·5 | 20 to 100 | — | 1·0 | 60 | 300 | — | — | Triode connexion. Current amplifier | Hay[6] |
| Brimar 3S4 | $10^{-14}$ | 6·0 | 7·0 | 10·5 | 0·7 | 30 | — | 150 | 1·5 | Half filament in use | Hay[6] |
| Brimar 3S4 | $2\times10^{-14}$ | 6·0 | 80 | 12·0 | 1·4 | 30 | 150 | — | — | Current amplifier | Hay[6] |
| 12BE6 | $5\times10^{-14}$ | 12·0 | — | 7·5 | — | 105 | — | 250 | — | — | Anker[64] |
| RCA 959 | $1{\cdot}7\times10^{-15}$ | 6·0 | 21 | 6·0 | 0·5 | — | 20 | — | — | $V_{g1} = +0{\cdot}5$V Electrometer connexion | Nielsen[7] |

Several workers have investigated the receiving valve range in order to find suitable types of valve for this application[6,7,36,61,65]. Table 1 gives operating conditions of a number of such valves.

Hay[6] and others have noted that two distinct types of valve are often required. One is a voltage amplifier for use as the input stage. It must have a low value of grid current and as high a value of amplification factor as is possible. The other type is a current amplifier which may be operated as an output valve or in a single valve amplifier where high sensitivity is not required. In neither of these latter applications is extremely low grid current essential.

The characteristics necessary in commercial receiving valves for this purpose are similar to those of an electrometer valve.

(1) Low and constant grid current.

(2) Stability of characteristics with time.

(3) Low surface leakage.

(4) Replaceability.

### Valves for the Amplification of Currents Greater Than $10^{-11}$A

Many commercial receiving valves may be used in this range. Both triodes and pentodes are suitable. The use of H.T. potentials below 40 volts considerably reduces the grid current; it is also usually advisable to operate the valve at a reduced heater voltage. When pentodes are in use the anode potential should be less than that of the screen since the grid current is substantially reduced under these conditions[63].

Selection of valves is necessary if the lowest grid current is required since the variation between valves of the same type is considerable.

### Valves for the Amplification of Currents Less Than $10^{-11}$A

Grid currents in certain receiving valves may be reduced almost to values obtained by the best electrometer valves, i.e. to about $10^{-14}$A. The precautions necessary for the use of these valves are generally the same as for electrometer valves. Some additional care must sometimes be taken since the valves are being used for a purpose for which they were not designed. The precautions will be considered in turn.

(1) The use of low anode, screen and heater volts.

(2) Screening from light and stray fields.

(3) Reduction of surface leakage.

(4) " Electrometer connexion ".

#### (1) The Use of Low Anode, Screen and Heater Volts

It is advisable to operate all receiving valves used for low current amplification at a lower heater voltage. This reduces positive ion emission and also reduces photoelectric current produced by light from heater and cathode. Direct current should be used and the potential of the heater should be slightly positive with respect to the cathode, if possible. This is to prevent any electrons which may be emitted by the heater itself from being collected by the control grid.

Low anode and screen volts should be used, as for electrometer valves, to ensure that the electron energy on approaching the anode is less than the amount required for ionization of the residual gas. The valve characteristic must be such that a high enough anode current exists for the purpose required, even when the anode voltage is low. Where a fairly large anode current is desired, anode and screen potentials may be increased only at the expense of increased grid current.

#### (2) Screening from Light and Stray Fields

All valves should be operated in complete darkness when very low grid currents are desired[7]. Stray electric or magnetic fields are to be avoided since any change in the strength or position of the field affects the output of the stage. Johnston[44] discusses electro-magnetic and electrostatic pick-up in balanced stages.

#### (3) Reduction of Surface Leakage

Where the input impedance of a commercial valve is required to be high, of the order of $10^{9}\Omega$ or greater, it is necessary to clean the valve thoroughly. For single ended valves the bakelite valve base, if present, should be removed and the valve washed in alcohol. A coating of paraffin wax may be applied to the glass after cleaning or a high quality moisture resistant varnish used[62,64]. Hay[6] suggests using a silicone coating in the following manner. The valve is washed in alcohol and ether and then allowed to stand in a humid but clean atmosphere for a day. This is necessary, as the formation of a silicone water repellant film is dependent on an initial film of water. The valve is then supported in a large air-tight jar, and an uncorked bottle of B.T.H. " Teddol " placed in the jar for 20 minutes. Vapour from the bottle then reacts with the moisture to form a closely adhering water repelling film. This method gives an insulation between pins of a B7G base greater than $10^{15}\Omega$.

#### (4) " Electrometer Connexion "

It is sometimes possible to use an electrode in a commercial valve in the same way as a space charge grid in an electrometer valve. The valve is then termed " electrometer connected ". This connexion has been used in particular with the acorn valves 954 and 955[7,65,66]. These valves, by virtue of their wide pin spacing, are ideally suited for use as low current electrometer valves. Grid $g_2$ is used as the space charge grid and $g_3$ as the control grid. It is usual to apply a small positive potential of between 0·2 and 1·0 volts between $g_1$ and the cathode. Varying this potential gives some control over the characteristics of the valve which is an advantage if the valve is used in a balanced circuit. As in a specially designed electrometer valve, the use of a space charge grid reduces the control grid current

The use of an electrometer connected acorn pentode has the disadvantage that the input capacitance is higher than when $g_1$ is used as control grid. However, this connexion is advantageous if an extremely low grid current is required[65]. Nielsen[7] notes that all acorn valves tested had a grid current of less than $10^{-14}$A when electrometer connected.

### Interstage Coupling

Harris and Bishop[40] have reviewed the methods of interstage coupling with particular reference to balanced amplifiers. The methods which are mainly used fall in three categories.

(1) Potential divider coupling.

(2) Backing off methods.

(3) Use of a valve in the divider network.

#### (1) Potential divider methods

Direct anode to grid connexion and the use of a divider network, with or without bias, are in this category. When battery supplies are being used it is often possible to supply a low anode potential to the first stage, and couple this anode directly to the following grid. The cathode of the second stage may be returned to a tapping on the battery so that a suitable bias is obtained. Using direct anode to grid coupling in this way, an amplifier can be constructed having a voltage gain of 80db using a 250V H.T. supply[39,40].

The use of a potential divider with negative bias is shown in Fig. 22. This circuit may be arranged so that, with zero input, the grid potential of the second stage is also zero. Although the gain per stage is reduced in this method, it is possible to use any grid in an amplifier constructed on these lines as an input terminal, and servicing is made easier.

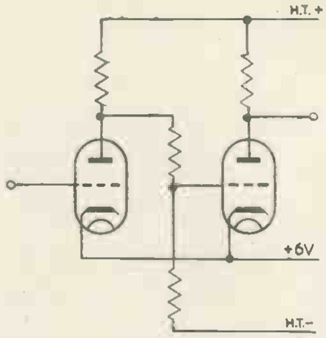


Fig. 22. Use of a potential divider with negative bias for interstage coupling

(2) BACKING OFF METHODS

It is possible to use a battery connected between the anode of the first stage and the grid of the second stage. In this way a low grid potential may be achieved while retaining direct coupling. This method has been used in low gain, battery-operated amplifiers. For the final stages a glow discharge tube may be used in place of the battery[45,39,40]. The operating current, which may be as low as 100$\mu$A, is obtained via the anode load of the previous stage.

(3) THE USE OF A VALVE IN THE DIVIDER NETWORK

The use of a valve has been suggested to replace the grid resistance in the potential divider network. Mezger[48] uses a triode, and Bishop[63], using a pentode, shows that the attenuation produced is only 5 per cent of the total signal. As in the use of valves as cathode loads, a high differential resistance is obtained combined with a low voltage drop across the valve.

Harris and Bishop[40], in assessing the value of the various methods of coupling, consider the use of interstage coupling valves undesirable. They note that glow discharge tubes are more suitable for use in the final stages of an amplifier, whereas in the earlier stages a resistance potential divider is more economical.

**Electrometer Input Circuits**

When it is necessary to amplify small currents obtained from a high impedance source, a low-grid current input valve is essential. As has been previously mentioned, the use of a high resistance in the grid circuit, combined with the inherent input capacitance of the valve and its associated circuit will often result in a high time-constant which will limit the response of the stage[25,68]. In some circumstances this time-constant may not be excessive, and in these cases, the input valve may be an electrometer operating as a voltage amplifier in the conventional manner. This method of connexion is not suitable, however, when a high speed of response is required. It is found, in addition, that when operating valves with high grid resistances, instability may occur as the value of the resistance increases. Sowerby[69] and Crawford[70] have shown that the use of the input valve as a cathode-follower permits a higher grid resistance to be employed before instability sets in. Furthermore, the effect of the feedback in the cathode-follower is to reduce the effective input capacitance of the valve. If the grid to earth capacitance of the input leads is reduced by connecting the screening on the input cable to the cathode of the valve[71], a very high input impedance combined with an extremely low input capacitance may be obtained. The circuit of such a high impedance balanced stage has been published by Bishop[63], using 954 valves.

Another method of using negative feedback to reduce the time-constant of an input stage has been given by Thomas[68]. Here a two-valve amplifier is used, and feedback is obtained by feeding the output voltage back into the input circuit. Since this results in 100 per cent negative feedback, linearity, stability, and speed of response are greatly increased. The output stage is a cathode-follower which serves not only as a link in the feedback chain but also delivers an output voltage across a low impedance. Since the output and input voltages are identical the unit acts as an impedance converter, allowing a robust moving-coil panel meter to be operated.

Several other circuits for this purpose have been given[72,73,65].

**Noise and Drifts**

The sensitivity of a directly-coupled amplifier may be increased until the electrical noise level is easily discernible in the output. Further increase is unprofitable, since a signal may only be detected when its magnitude is sufficient to bring it appreciably above the noise level.

Using balanced circuits and stable supplies, amplifiers having very high sensitivities have been constructed[74,75,43]. The problem of electrical noise is of great importance in the design of such amplifiers.

Noise in directly-coupled amplifiers consists of several components[75,40].

(1) Thermal agitation noise of resistors.

(2) Shot noise of the anode current.

(3) Partition noise.

(4) Shot noise of the grid current.

(5) Flicker noise.

(6) Component noise and supply drift.

(1) The thermal agitation noise of resistors may be given in terms of the mean square voltage fluctuation $\Delta E^2$ which is produced by the resistance.

$$\Delta E^2 = 4RkT.\Delta f$$

where $R$ = value of the resistor over the frequency band $\Delta f$.

$k$ = Boltzmann's constant.

$T$ = Absolute temperature.

(2) Shot noise of the anode current can be represented by:

$$\Delta I^2 = 2eI_aG^2.\Delta f$$

where $e$ = electronic charge.

$I_a$ = anode current.

$G$ = space charge screening factor, a factor varying from 0·1 to 0·01, representing a reduction in anode shot noise due to the presence of space charge. $\Delta I$ is the R.M.S. value of the variation of $I_a$.

(3) Partition noise is the noise in a pentode due to the splitting of cathode current between anode and screen.

(4) Shot noise of the grid current is given by the following expression:

$$\Delta E^2 = 2eI_gR_g^2.\Delta f$$

where $R_g$ = value of the grid resistor over a frequency band $\Delta f$.

$I_g$ = grid current.

and $\Delta E^2$ = mean square voltage fluctuation produced by the grid current.

All the above four sources of noise have a constant frequency spectrum.

(5) At low frequencies, flicker noise is very prominent[75]. The origin and magnitude of this noise has not yet been fully investigated, but the noise is thought to be best

represented by an expression of the type[75]:

$$\Delta I_0^2 = \frac{I_0^2}{\omega^2 + a^2}$$

where $I_0$ = cathode current

$\omega = 2\pi \times$ frequency

$a$ = constant

and $\Delta I_0^2$ = mean square fluctuation of the cathode current due to flicker effect.

It can be seen that flicker noise is of great significance at low frequencies and, as the frequency is reduced, the magnitude of the noise approaches a uniform value. The flicker noise of the input valve assumes great importance in directly coupled amplifiers of high sensitivity.

Flicker effect is thought to be due to variations in contact potential of the cathode surface[40]. Measurements[75] have shown the value of flicker noise to be equivalent to an input of the order of 50 to 100$\mu$V.

(6) Component noise includes noise due to poor contacts, microphony and pick-up[44]. All of these types of noise are avoidable by good design and care in building the amplifier. Semi-conducting resistors also introduce a noise voltage and should not be used.

Drifts which occur in the supply voltages have the same effect as very low frequency noise[43]. It is possible to correct partially for these drifts by using an arbitrary zero as was done by Hafstad[74]. This is only suitable, however, when the drift is of constant magnitude and it is very difficult to achieve over a long period.

**Complete Amplifiers**

From the information which has been gathered here it can be seen that, with careful design, it is possible to build

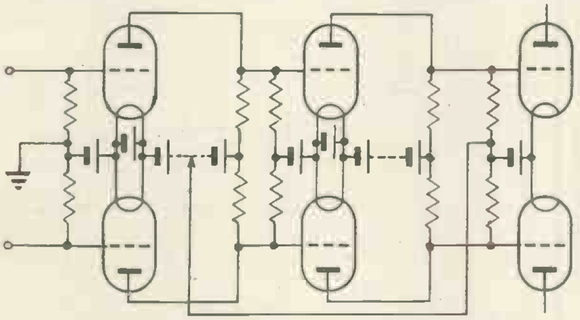

Fig. 23. Mean level feedback in a battery operated amplifier

directly coupled amplifiers having stability and performance approaching the limitations set by random noise. In order to achieve an amplifier of high sensitivity and good performance it is necessary to select valves and components with extreme care. The choice of valves for the input circuit has been treated extensively in view of its importance on the performance of the amplifier. All the valves in the amplifier, however, should be selected with care since their characteristics must remain constant over long periods of time if amplifier stability is to be achieved. Before the final adjustment of any balancing controls it is advisable to age the valves by continuous running.

Semi-conducting types of resistor should be avoided, owing to the high noise which they produce and wire-wound resistors should be used wherever possible. All resistors should be of ample rating so that local heating in the amplifier is avoided.

It has been shown that the use of a balanced stage allows some compensation for variations in supplies to that stage. Thus a balanced amplifier will be inherently less dependent on supply variations than an amplifier containing only single-sided stages. Most of the high gain amplifiers which have been constructed in recent years are of the balanced type.

The use of negative feedback will result in greater linearity and stability. This is true whether single-sided or balanced stages are employed. In the case of balanced amplifiers, however, it is possible to apply negative feedback in two ways.

In the discussion of the balanced stage it was shown that, provided the gain ratio were not zero, any in-phase signal applied to the two valves in the first stage would alter the mean operating potential of succeeding stages. This implies that the in-phase signal is passed on from one stage to the next. Although the amplification of the in-phase signal is usually considerably below that applying to out-of-phase signals, the value of the in-phase input is often quite high. Thus the alteration in the mean operating potential of the output stages of the amplifier due to the in-phase component may be sufficient to disturb the operating conditions of the stages.

Each stage will normally be designed to have as low a gain ratio as possible, so that the combined in-phase gain of the amplifier is low. It is then good practice to introduce 100 per cent mean level negative feedback throughout the amplifier. In this way the operating potentials of all the valves are hardly changed even in the presence of a strong in-phase signal.

Fig. 23 shows the method used by Nielsen[77] to achieve 100 per cent mean level feedback in a battery operated amplifier. The feedback circuit is completed by the lead from the centre-tapped grid resistor of the final stage to the H.T. battery of the first stage. Nielsen shows that the position of the tap on the battery is not very critical. In this way the discrimination factor may be much increased.

Offner[78] has discussed the application of mean level feedback in cathode coupled amplifiers and Johnston[44] and Harris and Bishop[39] have given circuits suitable when mains operated power supplies are in use.

It is advisable to apply the 100 per cent mean level feedback independently of any signal feedback necessary to improve the performance of the amplifier.

In order to eliminate high frequency instability when signal feedback is used, a capacitor may be shunted across the signal path. In this way, marked phase shift at high frequencies, which would cause instability, is avoided.

Further designs of directly coupled amplifiers are considered by Dickinson[79], Goldberg[80], Shepard[81] and Peirson[29].

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The Use of

# Cold Cathode Counting Tubes in Cascade

By D. T. Whelan*

*Circuits for connecting cold-cathode counting tubes (Dekatrons) in cascade, which will allow the passage of a carry pulse to the next higher valve in the train, and which will initiate a resetting, or zeroing, operation are considered. A circuit to eliminate interference between carry and resetting operations is developed.*

AT various times during recent years, articles have been published[1,2] describing the applications of cold-cathode counting tubes (Dekatrons). The connexion of Dekatrons in cascade presents difficulties in that a carry makes necessary a cumbersome circuit for resetting or zeroing. "Carry" is understood as the operation of transferring a pulse from the Dekatron registering " units " to that registering " tens," at the conclusion of one cycle of the " units " tube. Similarly, carry pulses are required to be transferred between each Dekatron and the one following it in the next higher position in the train. " Zeroing " or " resetting " is the operation of forcing the count back to its original starting position before the beginning of a new calculation. It is one of the more attractive features of these tubes that this operation can be carried out without loss of time.

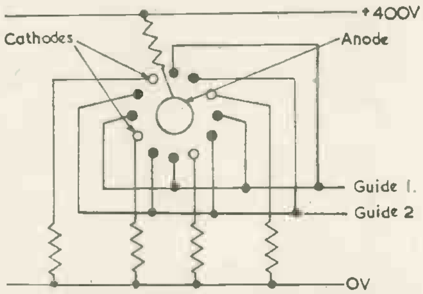


**Fig. 1. Schematic diagram of Dekatron**

It will be as well, perhaps, to consider first of all the circuit used to drive a Dekatron. A schematic diagram of one such tube is shown in Fig. 1. In its quiescent state there will be a glow discharge between one of the cathodes and the anode. The guides, which are in two groups, are held at a point more positive than the cathodes (at about + 60V). If a large negative pulse is applied to the guide 1 line, such that the guides in this group become more negative than the cathode, the discharge will become centred on that guide, connected to this line, which is adjacent to the cathode concerned. At the conclusion of the pulse, the discharge will move back to occupy the cathode, but if, before the pulse has ended, a second similar pulse is applied to the guide 2 line, the discharge will spread to the adjacent second guide, and at the end of guide 1 pulse the discharge will invest the second guide. At the end of the second pulse the discharge will step to the nearest most negative point, which is the adjacent cathode, and will thus have moved one step. In Fig. 1, four cathodes only are shown, but in fact there are available a variety of such tubes, among them being ten- and twelve-cathode types, for use in decimal or duodecimal counting. The type used in the circuits to be described is the GS10B, a cold-cathode selector tube with ten cathodes, each cathode connexion being separately brought out to a tag on the valve base. The principles of operation, however, apply equally to other types of selector tubes in this range.

A published circuit[3] employs a cold-cathode trigger tube (GTE175M), both as an input trigger and as a coupling valve. This circuit is shown in Fig. 2, where the anode resistor of the trigger tube is tapped and two points are connected via capacitors to the guides of the Dekatron to be driven. The total value of the resistance in the anode circuit of the trigger tube is such that insufficient current will

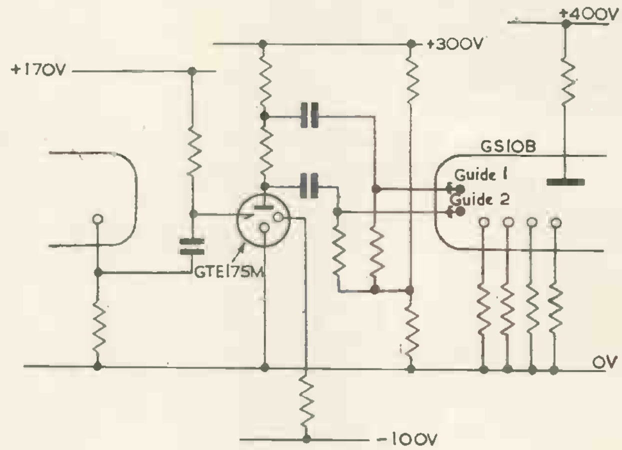


**Fig. 2. Carry circuit**

flow to maintain ionization of the trigger unless supplemented by the discharge current of the two capacitors. When the coupling tube is triggered, the capacitors will discharge and the trigger tube will then extinguish. Due to the differing capacitances of the two capacitors and their positions in the anode circuit of the trigger, a larger negative pulse will be passed to the guide 1 line of the succeeding Dekatron, whose discharge will step to the guide connected to this line. Since the time-constants of the capacitive couplings to the Dekatron guides will differ, the charging of the capacitor in the guide 1 line will be more rapid than that of the capacitor in guide 2 line, with the result that the potential of guide 1 line will rise until it reaches a point when guide 2 line is the more negative, when the discharge of the Dekatron will step to guide 2. When both capacitors are charged, the Dekatron discharge will pass to the adjacent cathode, and will have completed one step.

To fire the coupling tube, pulses of approximately 25V are applied to the trigger electrode via a $0{\cdot}001\mu F$ capacitor. The tube will then fire at the positive-going end of the pulse. Consider the circuit shown in Fig 2. Assume that the cathode shown in the left-hand Dekatron becomes occupied by the discharge, which then steps off to the next cathode. Fig. 3 shows the variation in potential which will occur

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during this period. The trigger electrode will receive two pulses; one positive-going, as the discharge invests the cathode of the Dekatron to which it is coupled; and the second a negative-going pulse as the discharge leaves the cathode. This means that the coupling tube will fire as the discharge arrives at the cathode concerned.

In a machine employing a decimal counter or accumulator, although it is not necessary that the progression of the count should conform to a conventional decimal notation, it is preferable that it should do so, in order to simplify the use and programming of the machine by an operator. In the conventional notation the digits represented on the "units" and "tens" Dekatrons must bear the following relationship to each other during a simple count:

| | |
|---|---|
| Tens : | 0 0 0 0 0 0 0 0 0 0 1 1 1 |
| Units: | 0 1 2 3 4 5 6 7 8 9 0 1 2 etc. |

It will be seen, therefore, that the point at which a carry pulse is required lies between the times when the "9" and "0" cathodes are invested by the discharge. Since the carry pulse will be effective as the discharge arrives at the cathode, then in Fig. 2, the cathode to which the carry or coupling tube is connected must be the "0" cathode of the Dekatron.

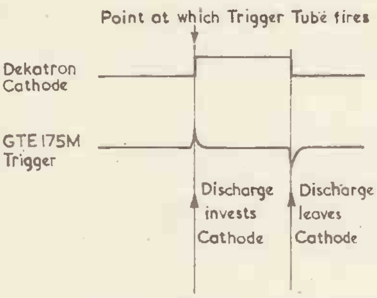



Fig. 3. Variation of potentials in carry circuit

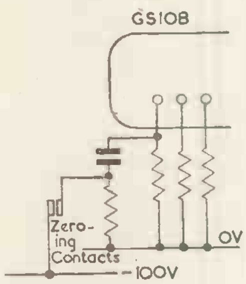



Fig. 4. Zeroing circuit

When a Dekatron circuit is initially switched on, any anode-cathode gap may break down and the glow discharge may occupy any cathode. The same condition obtains at the end of a calculation, when a total of unknown magnitude may be registered in the counter. Before commencing any calculation it is necessary that the count registered by a train of Dekatrons should be brought to a fixed datum. In the case of a multi-stage counter this is conveniently a succession of "0 0 0 0 0—." To bring about this result it is possible to employ a "zeroing relay," whose contacts are arranged to force the discharge to occupy one particular cathode of each Dekatron. Fig. 4 shows a typical resetting or zeroing circuit, in which the operation of the contact connects a large negative pulse to the chosen cathode, and forces the glow discharge to jump to it. Upon removal of the pulse, the discharge will remain at this position.

To obtain a series of 0's the resetting pulses must be connected to the "0" cathodes of the Dekatrons. If the carry or coupling trigger is also connected to this point care must be taken to ensure that the resetting is done cyclically or the operation of resetting will cause a carry pulse to be transmitted to the next higher order in the chain, and a true reset will not result. Resetting in this manner can be accomplished by the zeroing contacts making one after another from the lowest order upwards through the train, but this entails either an involved circuit or a specially adjusted zeroing relay, whose contacts operate in the required order. Neither of these alternatives was regarded with favour, and instead it was decided to use the "9" cathode to initiate a carry transfer, in lieu of the "0" cathode.

In order to maintain the conventional notation in the operation of the counter, the trailing negative pulse shown in Fig. 3. generated as the glow discharge leaves the "9" cathode, is used to cause a carry transfer. This is accomplished by inserting a phase invertor between the carry cathode and the coupling tube, as shown in Fig. 5. The choice of type of valve used for this purpose is dictated in the case of the circuit shown by the interests of standardization, and, in fact, any suitable valve may be used to obtain the conditions shown in Fig. 6. This diagram shows the inversion of the pulse from the "9" cathode, and its application to the trigger tube. It will be seen that a carry pulse is transferred as the discharge leaves the "9" cathode, which satisfies the condition, before described, that this pulse should occur between "9" and "0."

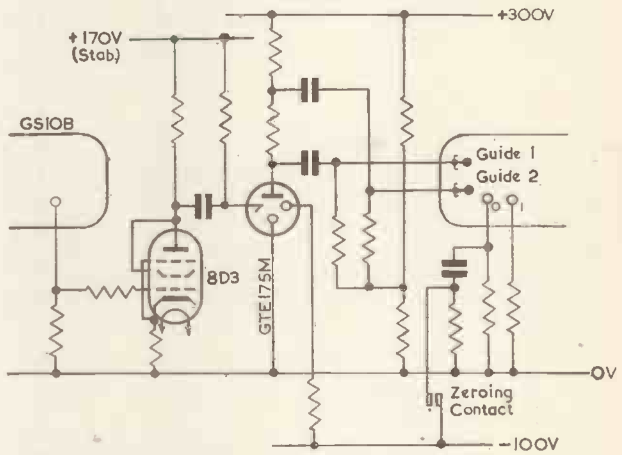
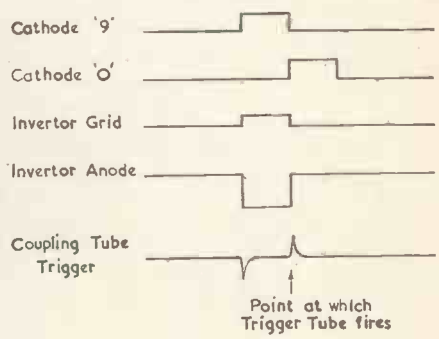



Fig. 5. Carry and zeroing circuit

Fig. 6. Variations in potentials in carry circuit when an invertor is fitted




An alternative method which is applicable to a two-stage accumulator or counter dealing only with tens and units is to arrange to start the count at 1 instead of 0, i.e., the normal reset registration is 01. In this case the carry coupling tube is connected to the "0" cathode of the "units" Dekatron, the invertor stage being omitted. This ensures that a spurious carry pulse is avoided by removing the reset pulse circuit from this cathode and connecting it instead to the "1" cathode. For many purposes it is advantageous to use this 01 condition as a datum, but where there are more than two Dekatrons in cascade, all those above the units position must reset to 0, and invertor stages will be required in all but the "units-tens" carry position.

**Acknowledgments**

The author would like to express his thanks to the British Tabulating Machine Company Limited, by whose permission this article is published.

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# A Low-Pass Filter with a Continuously Variable Cut-off Frequency

By J. W. Thompson*, B.Sc., A.M.I.E.E.

*An electronic feedback system having characteristics similar to those of a conventional low pass filter is described. The cut-off frequency can be continuously changed over a wide range by a simple control. A detailed electrical circuit is given, and calculated and measured frequency response curves are shown.*

THE suppression of electrical disturbances which interfere with submarine cable telegraph signals when electronic receiving apparatus is in use has recently been under investigation. The greater part of the energy of a particular form of disturbance lies in a frequency band above that necessary for adequate signal definition, and the signal-to-noise ratio can therefore be improved by a low-pass filter. The cut-off frequency should be adjusted for each individual case to give the best compromise between signal definition and noise rejection, and for this reason it is desirable to have a filter whose turning point may be continuously varied. Adjustment should be possible over the range 10c/s to 60c/s. This is difficult to achieve with a conventional *LC* network.

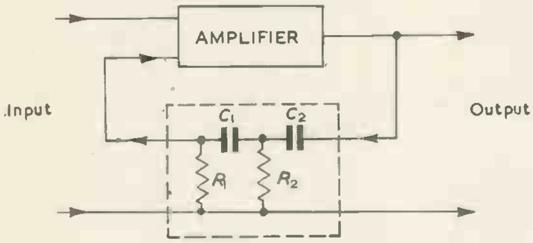


**Fig. 1. Feedback network**

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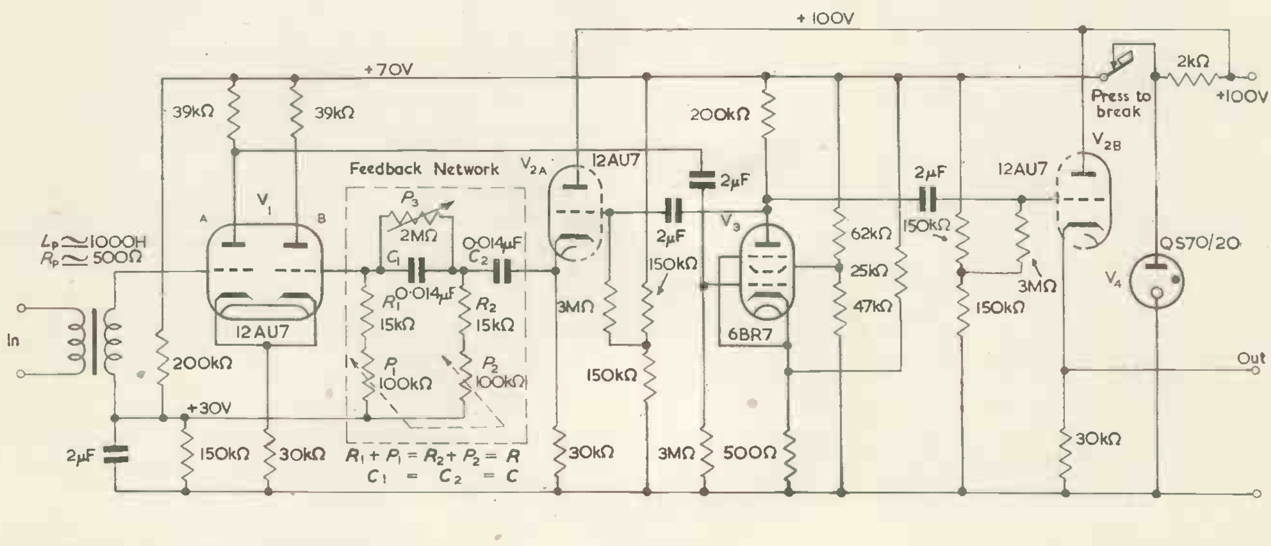
A type of feedback amplifier which can have characteristics similar to those of a low-pass filter, and which can easily be arranged to have a variable cut-off frequency, has been described by Miller[1] and is shown in schematic form in Fig. 1. An inherent feature of this device is a frequency response which rises immediately before cut-off, thus producing a transient characteristic which is unsuitable for some purposes unless an equalizing network is used at a later stage.

An alternative method of levelling the pass-band response and improving the transient characteristic is to shunt capacitor $C_1$ in Fig. 1 by a resistor. It can be seen by inspection that the value of the resistor may be chosen so that it seriously modifies the transfer characteristic of the feedback network only in the region of cut-off, for its effect is masked by the low reactance of $C_1$ at high frequencies and by the high reactance of $C_2$ at low frequencies. The calculation of the response of the feedback amplifier both without and with the additional shunting resistor is shown in the appendix, and the calculated curves are plotted in Fig. 3 (Curves G and H).

The electrical circuit of a low pass *RC* amplifier filter which has been constructed on these lines is shown in Fig. 2. A transformer with a time-constant of about one second is used to couple the input signal to the grid of $V_{1a}$ where it is amplified and fed to the grid of $V_3$. The output voltage from the anode of $V_3$ is fed back to the grid of $V_{1b}$ via the cathode follower $V_{2a}$ and the feedback network. The turning point of the frequency response curve may be

**Fig. 2. RC amplifier filter**



continuously changed by rotation of the ganged potentiometers $P_1$ and $P_2$. The transient response can be modified by adjustment of $P_3$. The output of the unit is taken from the anode of $V_3$ via a second cathode follower, $V_{2b}$. The filter has been designed for a 100 volt supply and it is therefore necessary to operate the "press to break" switch to ensure that $V_4$ is struck.

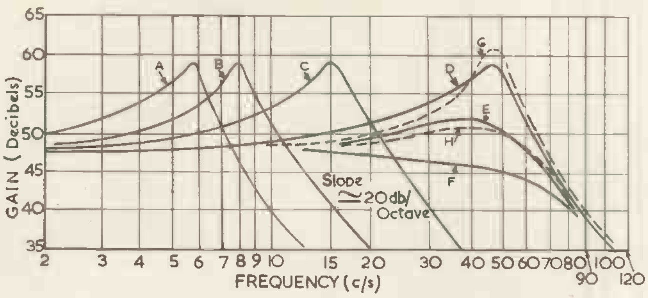



**Fig. 3. Response curves**

*$C=0{\cdot}014\mu F$ $\mu=250$ ; Full line-measured, dotted line-calculated*

| Curve | Setting of $P_1+P_2$ | $P_3$ |
|---|---|---|
| A | 0 ($R=115k\Omega$) | ∞ |
| B | 40 ($R=75k\Omega$) | ∞ |
| C | 75 ($R=40k\Omega$) | ∞ |
| D | 100 ($R=15k\Omega$) | ∞ |
| E | 100 ($R=15k\Omega$) | 400$k\Omega$ |
| F | 100 ($R=15k\Omega$) | 150$k\Omega$ |
| G | *calculated ; parameters as for* D | |
| H | ,, ,, ,, ,, E | |

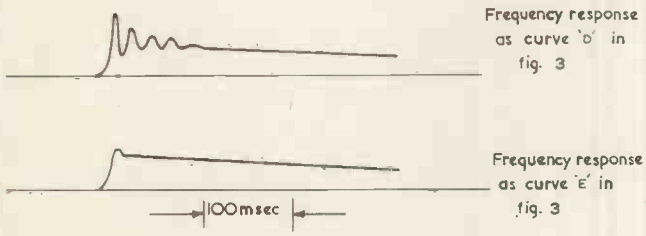



**Fig. 4. Observed transient response**

A family of measured frequency response curves is shown in Fig. 3. Curves A, B, C, and D were obtained with different settings of $P_1$ and $P_2$ when $P_3$ was disconnected. Curves E and F illustrate the effect of $P_3$ for the case where the other parameters correspond to curve D. Tracings from photographs of the response to unit function input, obtained with filter characteristics shown by curves D and E, are given in Fig. 4. Curves A, B and C, together with their associated transient characteristics, are changed in a similar manner by adjustment of $P_3$.


**Acknowledgment**

The author wishes to thank the Engineer-in-Chief, Cable & Wireless Ltd, for permission to publish this article.


## Appendix

### Calculation of Frequency Response (Fig. 5)

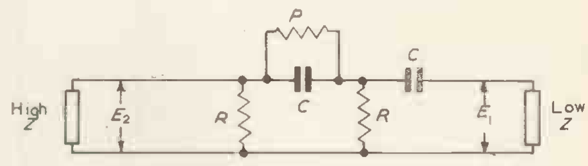



**Fig. 5. Equivalent feedback circuit**

Let $\beta$ = the proportion of the output voltage which is fed back ($= -E_2/E_1$).

$\mu$ = the gain of the amplifier without feedback (measured from grid of $V_{1a}$ to anode of $V_3$).

$G$ = the gain of the amplifier with feedback.

$\theta$ = argument $\beta$.

From the mesh equations for the equivalent feedback circuit it can be shown that:

$$\beta = \frac{(1 + b^2)}{(a^2 - b^2 - ab - 1) + j(3a + 2ab^2 + a^2b)}$$

where $a = 1/\omega CR$ and $b = 1/\omega CP$.
When $P$ becomes infinite this reduces to:

$$\beta' = \frac{1}{(a^2 - 1) + j(3a)}$$

The values of $\beta$ and $\beta'$ were calculated at different frequencies with:

$C = 0{\cdot}014\mu$F
$R = 15$k$\Omega$
$P = 400$k$\Omega$

The determination of the overall gain for a given value of $\beta$ was facilitated by the use of a chart[2]. The theoretically derived response curves for $\mu = 250$ are shown dotted in Fig. 3 (G and H) and the results from which they were plotted are given in Table 1.


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**TABLE 1.**

| | | | | | | | | | $P=\infty$ | | | $P=400$k$\Omega$ | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| $f$ (c/s) | $a$ | $b$ | $a^2$ | $b^2$ | $ab$ | $3a$ | $2ab^2$ | $a^2b$ | $\lvert\beta'\rvert$ (db) | $\theta°\beta'$ | $G'$ (db) | $\lvert\beta\rvert$ (db) | $\theta°\beta$ | $G$ (db) |
| 10 | 75 | 2·8 | 5 630 | 7·8 | 210 | 225 | 1 170 | 15 800 | −75 | − 2° | 48 | −66 | −72·5° | 48 |
| 20 | 38 | 1·4 | 1 440 | 1·96 | 53 | 114 | 149 | 2 020 | −63 | − 4·5° | 50 | −59 | −59° | 49 |
| 30 | 25 | ·94 | 625 | ·88 | 23·5 | 75 | 44 | 590 | −56 | − 7° | 52·5 | −54 | −50° | 50 |
| 40 | 19 | ·71 | 362 | ·50 | 13·5 | 57 | 19 | 257 | −51 | − 9° | 58 | −50 | −44° | 51 |
| 50 | 15 | ·57 | 225 | ·32 | 8·5 | 45 | 9·6 | 128 | −47 | −12° | 60 | −47 | −41° | 50·5 |
| 60 | 12·6 | ·47 | 158 | ·22 | 5·9 | 38 | 5·5 | 74 | −44 | −13·6° | 52 | −44 | −38° | 48 |
| 100 | 7·5 | ·28 | 56·3 | ·08 | 2·1 | 22·5 | 1·2 | 15·8 | −36 | −21·6° | 38·3 | −36 | −37° | 37·8 |

$R = 15$k$\Omega$ $C = 0{\cdot}014\mu$F $\mu = 250$ (48db)

# Step to Frequency Response Transforms for Linear Servo Systems

**(Part 3)**

By L. C. Ludbrook,* B.Eng., A.M.I.E.E.

### Approximate Relations for R.P.C. Servos

OBJECTIVE AND SCOPE OF RULES

A DESIGNER using the step response approach to servo synthesis is concerned with frequency response only as a measure of the undesired amplification given to spurious high frequency signals, and for this restricted purpose a useful indication is given by three salient features of the amplitude response viz: the frequency ($F_{max}$) and per unit amplitude ($Q_{max}$) of maximum transmission and the frequency ($F_{0\cdot5}$) of nominal cut-off. It has already been shown that the amplitude response as a function of frequency is independent of the amount of dead time at the "toe" of the step response; for the restricted purpose any dead time is therefore to be disregarded and time zero taken as the instant at which the step response first attains 2-5 per cent of its crest value.

The risk of excessive resonant amplification is only one of several conflicting factors influencing the choice of step response, and in the early stages of seeking the best compromise it would be useful to have an immediate idea of the way in which the three salient features of the frequency response are affected by changes in the shape of the step response. In the present state of the art, the process of synthesis depends largely on engineering judgement and even the basic data is not precise; for example the servo designer usually has to make his own "educated guess" at the spectrum of spurious input signals and the output that can be tolerated in each particular job. In formulating the rules, the objectives are therefore simplicity and speed of use rather than rigorous accuracy; the complete frequency response should be derived for the step response finally chosen, partly as a check on the rules, but mainly to extend the collection of standard forms originated by Whiteley[12]; these standard forms present the data in the most convenient way for practical use, and leave the designer free to concentrate on methods of attaining the chosen linear mode response, and on the problems encountered in the non-linear region.

$Q_{max}$-$A_{max}$ RELATION

The numerical ratio $Q_{max}/A_{max}$ is very much affected by the way in which the curve of step response recovers to unity after the peak of first overshoot. Consider, for example, two limiting cases, each with $A_{max} = 2$ (i.e. 100 per cent overshoot); if the curve of step response remains at the peak overshoot value then $Q_{max}/A_{max}$ does not exceed unity, but if the first overshoot is followed by others of about the same magnitude in an almost undamped oscillatory response then $Q_{max}/A_{max}$ approaches infinity. Thus if $Q_{max}$ is plotted against $A_{max}$ for typical step responses the points will not lie on a single valued curve; it may, however, be possible to define an area containing the points of technical interest and to indicate the trend of distribution within that area. The boundary cases of practical interest appear to be on the one hand a step response with single mode of damped sinusoidal oscillation and on the other hand a step response with monotonic runback to unity from a single overshoot peak.

An *LRC* filter section formally equivalent to the simplest possible "zero displacement error" servo has already been considered; the values of $Q_{max}$ and $A_{max}$ are plotted as functions of damping resistance on Fig. 15, and show that when $A_{max}$ is made greater than 1·2 the ratio $Q_{max}/A_{max}$ exceeds unity and increases very rapidly. Chestnut and Mayer[13] have reached a similar conclusion after computing the responses of a wide range of linear R.P.C. servos and it seems reasonable to regard the $Q_{max}$-$A_{max}$ relation of Fig. 15 as one boundary line representing systems with single mode damped sinusoidal response to input position step.

The other boundary line for systems with monotonic runback from single overshoot peak may be estimated by

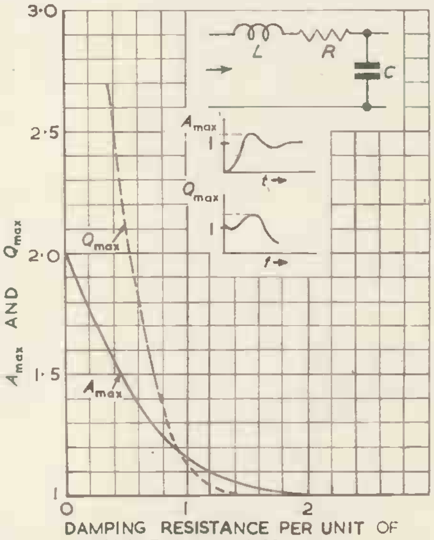


Fig. 15. $Q_{max}$, $A_{max}$ relation

applying equation (3) to a two segment approximation of the step response and examining the trend as the runback time is varied relative to the build-up time, and as the amount of overshoot is varied. The step responses shown in Fig. 16(a) all reach $A_{max} = 1·5$ at one second, but have various runback times $T_R$ between zero and infinity. The corresponding amplitude response curves computed from equation (3) are plotted in Fig. 16(b) as far as the nominal cut-off frequency and show that over the whole range of runback time the ratio $Q_{max}/A_{max}$ lies between 0·73 and 1·09. In practice it is most unlikely that the runback slope will be steeper than the build-up slope and if the lower limit of runback time $T_R$ is taken as 0·5 second, then the spread of the ratio $Q_{max}/A_{max}$ is only 0·9 to 1.09.

In the limiting cases of zero and infinite runback time some unexpected features of the frequency response merit detailed examination; the "build-up vector" $1.5\left(\frac{\sin \pi f}{\pi f}\right)e^{-j\pi f}$ (i.e. originated by the build-up portion of the step response) is common to both cases and its locus is shown as the solid line in the second and third quadrants of Fig. 17(a).

If runback time is zero the "runback vector" $-0{\cdot}5\, e^{-j2\pi f}$ has constant amplitude and rotates in the

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$(X_{(f)} + jY_{(f)})$ plane in the same direction and twice as fast as the build-up vector; the locus of the sum of these vectors is shown as the dotted line in Fig. 17(a) up to 1·25c/s and at higher frequencies it becomes a circle of radius 0·5 centred on the origin. It is physically correct that the per unit response should thus have amplitude 0·5 and phase lag increasing at 360°c/s out to infinite frequency because instantaneous runback in the step response of a linear network could only be achieved by subtracting from the output at the correct instant a portion of the input step signal that had been retarded in a side chain; physically this side chain would be a properly terminated delay cable or equivalent magnetic storage device, and would continue to transmit its portion of input signal out to infinite frequency. This is a particular case of the general postulate that if the graph of per unit response of a linear system to step

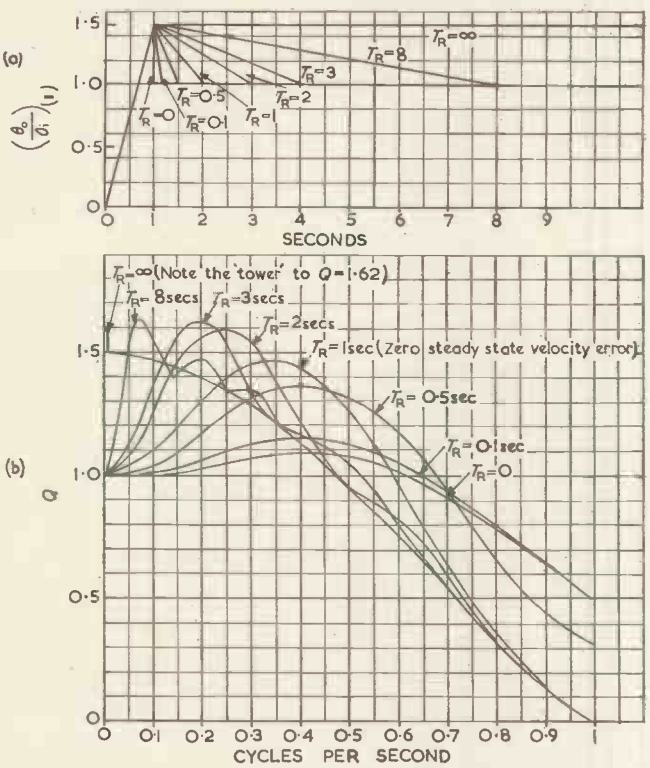


**Fig. 16 (a).** Two segment approximations to step response with single overshoot to $A_{max} = 1.5$ at 1 second and various values of runback time $T_R$.
**(b)** $Q$ curves corresponding to the above single overshoot step responses

function input contains anywhere one vertical rise or fall of amount $A$ then the corresponding per unit amplitude response to sinusoidal input settles to constant amplitude $A$ at high frequencies. If the step response contains two or more vertical segments, transmission is still maintained out to infinite frequency, but the amplitude curve ocillates between two fixed values because the component vector originated by a late vertical segment rotates faster in the $(X_{(f)} + jY_{(f)})$ plane, hence periodically swings in and out of phase with the component vector originated by an earlier vertical segment. Vertical segments in the step response appear to offer no advantage and would not occur in conventional servo systems; they might, however, be inadvertently introduced if finite delay elements were used in side chain feedback circuits as has sometimes been proposed.

Passing to the other extreme of infinite runback time, the locus of the generalized runback vector $-0{\cdot}5\left(\dfrac{\sin \pi T_R f}{\pi T_R f}\right)e^{-j\pi(T_R+2)f}$ acquires the same geometric shape as that of the build-up vector when $T_R$ becomes very large so that $(T_R + 2) \simeq T_R$, but its initial amplitude is only $-0{\cdot}5$ and it is completely traversed for a very small increment of frequency from zero. The locus of the runback vector for the limit case of $T_R = \infty$ is shown as the solid line in the first quadrant of Fig. 17(a) positioned relative to the locus of the build-up vector so that the vector sum can be scaled off directly from the origin. This limiting runback vector is traversed for a vanishingly small increment of frequency from zero and accounts for the "tower" at zero frequency in Fig. 16(b); the effect is reminiscent of Gibbs[14] phenomenon encountered when summing a Fourier series to reconstitute a discontinuous time waveform.

Fig. 16(b) shows that as $T_R$ the duration of runback slope is increased, $Q_{max}$ is increased and $F_{max}$ is reduced. Insertion of a dwell period (i.e. a horizontal segment) at

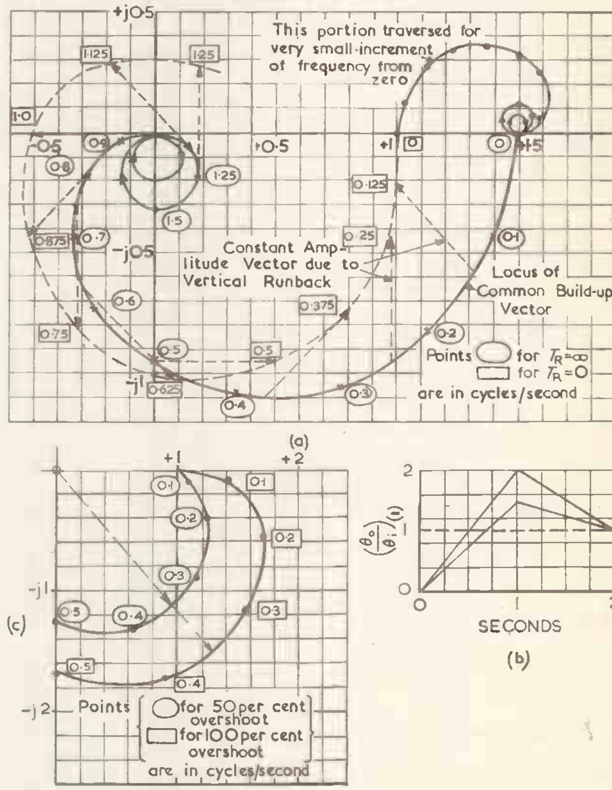


**Fig. 17 (a).** Vector loci for limiting cases of runback times $\begin{cases} T_R = 0 \\ T_R = \infty \end{cases}$
**(b).** Two segment approximations to single overshoot step response with $\begin{cases} 1 \text{ second build-up time} \\ 1 \text{ second runback time} \end{cases}$
**(c).** Portions of Vector Loci $\left(\dfrac{\theta_o}{\theta_i}\right) j\omega$ for the examples of Fig. 17 (b).

peak overshoot before the runback slope gives a similar effect because the faster rotating runback vector then swings round to boost the build-up vector at a lower frequency after less attenuation. In this particular case of $A_{max} = 1{\cdot}5$ the maximum amplitude response approaches $Q_{max} = 2$ as the dwell period is increased; this is intermediate between $Q_{max} = 2{\cdot}32$ given by damped sinusoidal mode and $Q_{max} = 1{\cdot}62$ given by immediate monotonic runback, and the latter case as illustrated in Figs. 16(a) 16(b) appears to define the desired lower boundary for the $Q_{max}$-$A_{max}$ relation.

Having shown that for these elementary single overshoot step responses the ratio $Q_{max}/A_{max}$ lies between 0·9 and 1·09 over the practical range of runback times, it remains to examine the effect of varying the amount of overshoot. The intermediate case of $T_R = 1$ second is used for this purpose and responses with $A_{max} = 1{\cdot}5$ and $A_{max} = 2$ are shown in Fig. 17(b). The corresponding frequency response loci computed from equation (3) are plotted in Fig. 17(c), and show that the ratio $Q_{max}/A_{max} \simeq 1$ and the resonant frequency

0·35c/s are substantially unaffected by the change from 50 per cent to 100 per cent overshoot.

The foregoing crude approximations applied to a few cases suggest the relation $Q_{max} = A_{max}$ as the boundary line for systems having immediate monotonic runback from single overshoot peak. This is plotted on Fig. 18 together with the earlier relation for systems having a single mode of damped sinusoidal oscillation, and it is seen that almost all the points computed by Chestnut and Mayer[13] do lie in the area bounded by these two lines.

A single overshoot in the step response can suffice to give zero steady state velocity error, but if the time scale cannot be shortened far enough to bring the steady state acceleration error down to an acceptably low value then the overshoot area must be increased and an undershoot introduced. Even if sinusoidal terms are avoided and the overshoot and undershoot are obtained as the sum of simple exponential terms, the general effect of these changes is to increase the ratio $Q_{max}/A_{max}$ as compared to that for the single overshoot case. The build-up vector is little affected, but the runback vector has its initial amplitude increased by the amount of undershoot; its increase of attenuation rate due to the longer duration of runback is relatively small and tends to be offset by its increase of

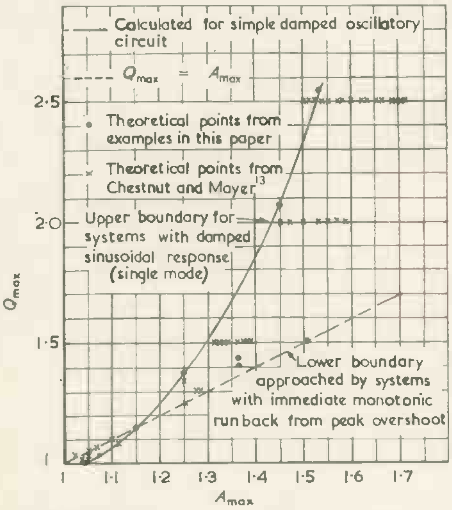


**Fig. 18. Relation between $Q_{max}$ and $A_{max}$ for R.P.C. servos**

rotation rate due to the later average time of occurrence of runback; the runback vector therefore retains a greater amplitude when it has swung round to boost the build-up vector, and the total amplitude response $Q_{max}$ is correspondingly increased. The third vector, originated by the final run-up segment of the step response from trough undershoot to unity has relatively high rates of attenuation and rotation because the run-up segment is of long duration and occurs late in time; nevertheless this run-up vector may contribute significant extra boost in the region of $Q_{max}$. Further experience is required on this problem, but the present picture is that $Q_{max}$-$A_{max}$ points for zero steady state acceleration error systems are likely to lie between the two boundary lines of Fig. 18, and will move nearer the upper boundary as the undershoot amount is increased and duration correspondingly decreased to retain the desired net area.

### $F_{0\cdot5} - T_0$ Relation

An existing rule derived by Guillemin[15] for the idealized low pass filter and confirmed by McColl[16] from experience of practical filters indicates that cut-off frequency is about half the reciprocal of build-up time. As build-up time is constant at one second for the idealized single overshoot step responses of Fig. 16(a) the cut-off frequency should, according to this rule, be constant at 0·5c/s, but the curves of Fig. 16(b) show that, over the practical range of runback time, the spread of $F_{0\cdot5}$ is from 0·72c/s to 0·87c/s. This discrepancy is due partly to the different conventions for "cut-off frequency" and "build-up time," and partly to the over simplified nature of the two segment step responses. The effective build-up time in a practical servo response is certainly less than the time to peak of first overshoot and is more nearly equal to $T_0$ the time to first cross correspondence. Bearing in mind that $T_0$ is an important parameter in the synthesis of step responses and that a simple easily memorized rule is sought for a restricted purpose, the suggested relation is that $F_{0\cdot5} = 0{\cdot}5/T_0$, i.e. that nominal cut-off frequency equals half the reciprocal of the time to first cross correspondence.

### $F_{max} - T_C$ Relation

The curves of Fig. 16(b) show that a given cut-off frequency $F_{0\cdot5}$ can be associated with a wide range of values of resonant frequency $F_{max}$, but the general indication is that for step responses of practical interest $F_{max}$ lies between $0{\cdot}3F_{0\cdot5}$ and $0{\cdot}5F_{0\cdot5}$, the lower values being associated with long duration of runback relative to build-up time in the step response. The relation $F_{max} = 0{\cdot}25/T_0$ is tentatively offered for the restricted purpose but must obviously be applied with discretion and the complete transformation from step response to frequency response carried out in doubtful cases.

## Checks Against Responses of Actual Servos

In this section both the graphical transformation method and the tentative rules are applied to some measured step responses of actual servo systems, and the results compared to the measured frequency responses. In addition to the probable non-linear behaviour of practical servos, two sets of experimental errors are introduced, so that it is not possible to segregate the errors due to the graphical approximation and simplified rules. The overall picture is, however, of interest to the practising engineer who needs to know how closely the predictions of linear theory can be approached in fact.

The test results are taken from existing reports on six servos with power ratings between 0·25 h.p. and 5 h.p. R.M.S. and responses ranging from the fastest possible (consistent with the mechanics of a given gun) to deliberately sluggish (for signal smoothing in an early auto-follow radar). While amplitude response was readily measured by observing the peak-to-peak movement of the servo motor shaft, the special equipment needed for measuring phase lag has only recently become available and the existing test reports contain only amplitude response measurements. The tentative rules can be examined over a wide range, but the graphical transformation cannot be completely checked.

### Step and Frequency Responses of an Auto-follow Radar

The solid lines of Figs. 19(a) 19(b) show respectively the measured responses to $1°_{(1)}$ and $0{\cdot}43° \sin \omega t$. The dashed and dotted lines of Fig. 19(a) show respectively four segment and five segment approximations to the measured step response and it should be noted that due to the use of horizontal segments only three component vectors have to be computed in each case. The corresponding amplitude responses plotted as the dashed and dotted lines of Fig. 19(b) show the increase of $Q_{max}$ and decrease of $F_{max}$ caused by introduction of a dwell at peak overshoot as discussed previously. The slight increase of $F_{0\cdot5}$ is due to the slightly steeper build-up slope used in the five segment case.

Predicted and experimental values are compared in Table 1.

This particular system was unusually linear because the deliberately sluggish response (dictated by the associated predictor) was attained by a local negative feedback loop from aerial acceleration to the input of the electronic amplifier exciting the Amplidyne; there are, however, considerable discrepancies between the results obtained by various methods.

Table 1

| METHOD OF FINDING FREQUENCY RESPONSE | | $Q_{max}$ | $F_{max}$ (c/s) | $F0{\cdot}5$ (c/s) |
|---|---|---|---|---|
| Measured direct .. .. .. | | 1·35 | 0·25 | 0·8 |
| From measured Step response | 4 segment approx. | 1·40 | 0·35 | 1·0 |
| | 5 segment approx. | 1·44 | 0·30 | 1·1 |
| | Tentative rules | 1·35-1·5 | 0·42 | 0·9 |
| Calculated from parameters of idealized system | | 1·20 | 0·28 | 1·2 |

## $Q_{max}$-$A_{max}$ Relations

Fig. 20 records the experimental values of $Q_{max}$ and $A_{max}$ found on the six servo systems 0·25 h.p. to 5 h.p. R.M.S.; most of the points lie outside the zone contained between the two empirical curves, although as shown in the earlier Fig. 18 this empirical zone contained a large number of points calculated for linear servo systems.

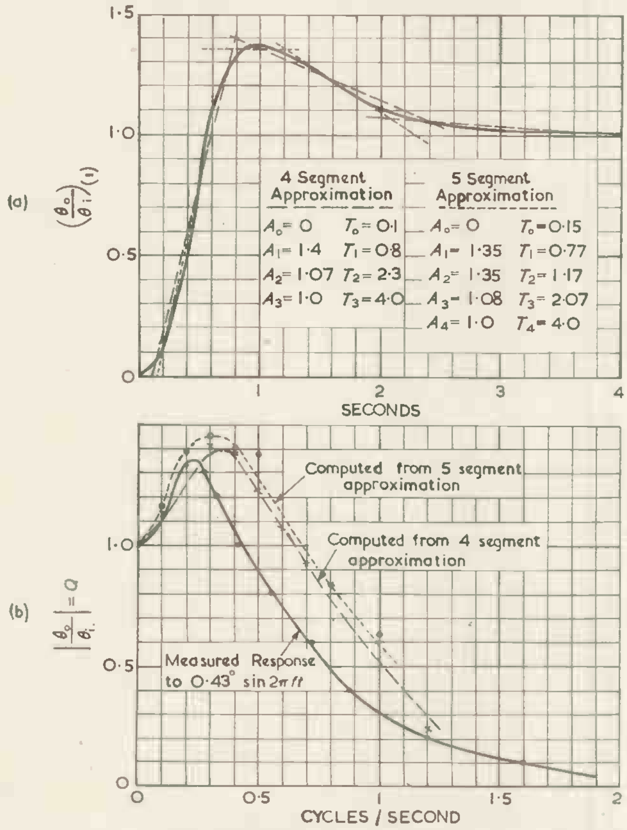


Fig. 19 (a). Measured Response to $1^\circ_{(1)}$. (b) Auto follow radar servo

## $F_{max}$-$T_C$ Relations

Fig. 21(a) records the experimental values of $F_{max}$ and $T_C$ found on the same six servo systems; these points are reasonably near the empirical curve $F_{0\cdot5} = 0{\cdot}25/T_C$.

## $F_{0\cdot5}$-$T_C$ Relations

Fig. 21(b) records the experimental values of $F_{0\cdot5}$ and $T_C$ found on the same six servo systems; these points are reasonably near the empirical curve $F_{0\cdot5} = 0{\cdot}5/T_C$.

## Discussion of Discrepancies

The $F_{max}$-$T_C$ and $F_{0\cdot5}$-$T_C$ relations show their greatest discrepancies at low values of $T_C$—say less than 0·2 second;

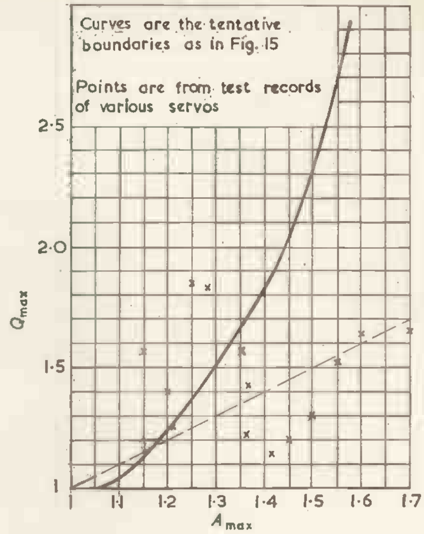


Fig. 20. Experimental $Q_{max}$-$A_{max}$ relations

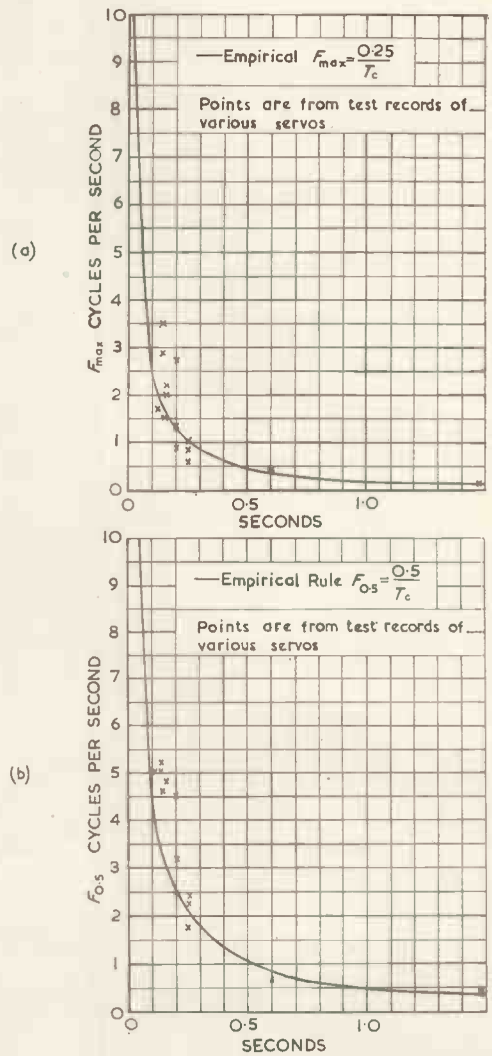


Fig. 21 (a). Experimental frequency at $Q_{max}$. (b). Experimental frequency at cut-off

$T_C$ = *time to first cross correspondence* this would be expected on the following ground:

(a) Systems with $T_C > 0{\cdot}2$ second usually include a stabilizing and/or slugging feedback signal from output acceleration which has the effect of increasing the electro-mechanical time-constant of the servo motor and load by a factor of some hundreds to swamp the prior control delays; such systems do approximate to the simple low pass filters considered in setting up the empirical rules, and may also have improved linearity.

(b) The inherent electro-mechanical time-constant of the servo motor and load is commonly of order 0·2 second, and if $T_C \leqslant 0{\cdot}2$ second has to be achieved, either extreme forcing must be applied to the prior control delays and the linear range correspondingly restricted, or the response contains two oscillatory modes at not very different frequencies; the simple empirical rules relating to $F_{max}$ and $F_{0{\cdot}5}$ can hardly be expected to apply to such two mode systems.

The simple relations $F_{max} = 0{\cdot}25/T_C$ and $F_{0{\cdot}5} = 0{\cdot}5/T_C$ do, however, give a reasonable indication for their intended purpose, and the discrepancies in the $Q_{max}$-$A_{max}$ relations are of more concern. Mechanical non-linear effects such as backlash and friction account for some of the discrepancies, and in at least one case the tooth ripple errors in the synchro data elements were found to be contributing spurious periodic signals of amplitude comparable to that of the intentional sine wave disturbance. Electrical non-linear effects are also very significant; for example the gain and time-constant of a Ward Leonard generator both decrease as mean excitation level is increased and the resulting harmonic distortion is not revealed by the older test method of observing peak-to-peak movement at the motor shaft; also the reversing excitation during sine wave test does not trace the same magnetic hysteresis loop as the unidirectional excitation used in the step response test.

The only criterion for selecting the test results shown in Figs. 20 and 21 was that both step and frequency responses should have been measured on the same equipment; some of these measurements were taken over seven years ago with rather crude test gear. While modern test gear with Lissajous display would reduce the measuring errors and enable the non-linear responses to be disregarded for the purpose of checking linear theory, the existing test results present a truer picture of the behaviour of practical systems. Linear theory gives valuable guidance, but is no substitute for experience based on actual systems.

## Conclusions

Overshoot and undershoot in the response of a servo to input position step are rightly regarded as undesirable because experience has shown that they are associated with resonant amplification of periodic input disturbances within a restricted range of frequencies, and that spurious input signals are likely to occur within this range due to various imperfections of practical equipment and processes. For this reason $A_{max} = 1{\cdot}2$ (i.e. 20 per cent overshoot) or $Q_{max} = 1{\cdot}3$ have been taken as design criteria in past practice.

More recent work on the fundamental "enclosed area" theorems and on the response time scales enforced by practical equipment has shown that some modern specifications of transient crest accuracy during aperiodic input disturbances can be met only if the linear mode response to input position step is shaped to have at least $A_{max} = 1{\cdot}5$ (i.e. 50 per cent overshoot) and probably an appreciable undershoot. Since these step responses are at present synthesized as graphs, the need is felt for a quick method of deriving frequency response direct from a graph so that the final rather tedious breakdown of the graph into Heaviside operational form is carried out only after reaching an acceptable compromise between dynamic error and resonant amplification.

The first stage in filling this need is to combine the best features of existing transformation methods into a new method based on approximating the step response graph by straight line segments. It is shown that for practical servo responses, four segments suffice to give engineering accuracy up to a frequency such that four cycles would occur in the step response settling time; this is well above the conventional "cut-off" frequency $F_{0{\cdot}5}$ at which amplitude response has fallen to half the zero frequency value. The new method shows a useful reduction of computing labour, but it still takes about an hour to transform a typical step response, and the result contains phase-angle information which is superfluous in the particular problem of resonant amplification.

The second stage is therefore to seek for general rules relating $Q_{max}$, $F_{max}$ and $F_{0{\cdot}5}$, the salient features of the amplitude response to features of the step response such as $T_C$ the time to first cross correspondence, $A_{max}$ the peak value, and the shape of the subsequent recovery to unity. The rules are intended only for immediate guidance while shaping a step response curve either on paper or on an analogue computor and are not put forward as substitutes for the complete transformation or measurement.

The following rules indicate the best that can be expected from a step response having immediate monotonic runback from peak overshoot:

(1) $Q_{max}/A_{max} \simeq 1$ (2) $F_{max} \simeq 0{\cdot}25/T_C$ (3) $F_{0{\cdot}5} \simeq 0{\cdot}5/T_C$

If a dwell period is inserted at peak overshoot prior to the runback slope, or if an undershoot is introduced, the cut-off frequency $F_{0{\cdot}5}$ is little affected, but $F_{max}$ is reduced and $Q_{max}/A_{max}$ is increased—both undesirable effects from the point of view of resonant amplification given to spurious periodic input signals. It appears, however, that $Q_{max}/A_{max} < 1{\cdot}2$ can reasonably be expected from these types of step response whereas a single mode damped sinusoidal response can give $Q_{max}/A_{max} > 1{\cdot}5$ in the region $A_{max} = 1{\cdot}5$ of immediate interest.

The third stage is to check the new transformation method and the approximate rules against all the available test records on actual servo systems to find out how far the predictions from linear theory are borne out in practice. There is fair agreement on resonant and cut-off frequencies, but the recorded values of $Q_{max}$ show a much wider scatter than the predicted zone.

The future programme should be partly the collection of more experimental evidence, but mainly the use of the present approximate rules to guide the synthesis of further standard forms extending the collection originated by Whiteley some seven years ago. These standard forms present the data in the most convenient way for practical use because they reduce the linear mode problems to the solution of simple simultaneous equations and leave the designer free to concentrate on the problems encountered in the actual circuits and in the non-linear region.

### Errata

In part 2 the last line of equation (14) should read:

$$= kCR\left(\frac{\omega^2 CR + j\omega}{1 + \omega^2 (CR)^2}\right) \rightarrow Kj\omega \text{ as } CR \rightarrow 0$$

In the index of the exponential in equation (22) $D_D$ should read $T_D$.

### Acknowledgments

The author's thanks are due to his colleague, Mr. J. A. Stokes, for constructive criticisms during the preparation of this article and to the Directors of The British Thomson-Houston Co., Ltd., for permission to publish it.

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# Short News Items

**The Radio Industry Council** announces the award of premiums of 25 guineas each for articles published in the public technical press during 1953 as follows:—

"Spectrum Equalization" by G. G. Gouriet, A.M.I.E.E.
(*Wireless Engineer*, May, 1953.)

"Triode Transformation Groups" by A. W. Keen, M.I.R.E., A.M.I.E.E. (*Wireless Engineer*, October, 1953.)

"A Cylindrical Magnetron Ionization Gauge" by A. H. Beck, B.Sc.(Eng.), A.M.I.E.E., and A. D. Brisbane.
(*Vacuum*, April, 1952—published April, 1953.)

"The Scanning Electron Microscope and the Electron-Optical Examination of Surfaces" by D. McMullan, M.A., Ph.D.
(*Electronic Engineering*, February, 1953.)

"A Linear Sweep Cathode-Ray Polarograph" by H. M. Davis, B.Sc., A.R.I.C., and Miss J. E. Seaborn, B.Sc.
(*Electronic Engineering*, August, 1953.)

"Selective Calling for Radio-Telephone Systems" by J. R. Pollard, M.A., A.M.I.E.E., M.I.R.E.
(*Electronic Engineering*, December, 1953.)

The presentation of the awards will take place at a luncheon to be held by the Public Relations Committee of the Radio Industry Council on Thursday, 11 March.

**The Mond Nickel Fellowships Committee** invites applications for the award of Mond Nickel Fellowships for 1954. The main object of these Fellowships is to enable selected applicants of British nationality and educated to university degree or equivalent standard to obtain additional training and wider experience in industrial establishments at home or abroad, so that they will be better qualified to appreciate the technological significance of research, if they are subsequently employed in executive or administrative positions in the British metallurgical industries. Each Fellowship will occupy one full working year and it is hoped to award five each year of an approximate value of £900 to £1 200 each. Full particulars and forms of application can be obtained from the Secretary, Mond Nickel Fellowships Committee, 4 Grosvenor Gardens, London, S.W.1.

**Mr. E. E. Jones**, who was for some years in charge of the Telecommunications Division of the British Philips Group of Companies and has been Commercial Manager of Messrs. Savage and Parsons Ltd, Watford, since 1949, is relinquishing his position there to take over the direction of the commercial activities of the Solartron Electronic Group of Companies.

**The BBC** announces the appointment of Mr. A. I. Bray as Engineer-in-Charge, Television Outside Broadcasts (London), and Mr. H. O. Sampson as Head of Technical Operations Television Studios.

**The Council of the Telecommunication Engineering and Manufacturing Association** have appointed Mr. H. Faulkner to the office of Director of the Association.

**Precision Rubbers Ltd,** Bagworth, Leicestershire, announce that Dr. C. M. Blow has joined the Board of the Company as Technical Director. He will be responsible for the research and technical development work of the Company.

**Marconi's Wireless Telegraph Co. Ltd,** at the request of the Science Museum, have provided a working demonstration spark transmitter and coherer receiver with morse inker, similar in principle to the original apparatus used by Marconi in his early experiments.

**Pye Ltd** have supplied a camera chain, a telecine camera and vision and sound transmitters for the first television station on the African continent which was set up at Casablanca recently. They have also received an order for three camera chains, together with associated equipment from the Japanese public service broadcasting organization which was established by Government charter in February, 1953.

**Solartron Laboratory Instruments Ltd** have now changed their address to Solartron Works, Queens Road, Thames Ditton, Surrey. A new factory has been acquired and a considerable expansion programme is under way.

**Decca Radar Ltd,** announces that it has been awarded a contract by the Ministry of Supply for a number of Decca 424 Airfield Control Radars. These equipments will be used by the RAF to enable jet aircraft to be landed in rapid succession, in poor visibility.

**Le Salon National de la Pièce Detachée Radio** announce that their annual exhibition of radio parts and components will be held from 12-16 March at the Parc des Expositions, Paris. At the same time as the exhibition there will be a conference on the new techniques and future of the radio and television components industry in France. Further details may be obtained from the offices of Le Salon National de la Pièce Detachée Radio, 23 Rue de Lubeck, Paris.

**The Royal Society** announce that the first appointments to United States (F.O.A.) Research Fellowships have been made by the National Academy of Sciences Washington, on the nomination of the Royal Society, as follows. Mr. G. N. Lance, employed by Hawker Aircraft Ltd, to work on problems in unsteady supersonic aerodynamics at the University of California under Professor John Miles. Mr. S. H. Parker of University College, Leicester, to carry out research in physical-organic chemistry at the Massachusetts Institute of Technology under Professor Gardner Swain. The Fellowships were instituted last year with funds provided by the United States Government, through its Foreign Operations Administration.

**The Regional Advisory Council for Higher Technological Education** has prepared, for the sixth year in succession, a summary of the applied research in electrical engineering in progress in university colleges and technical colleges in London and the Home Counties. In issuing this document the Council hopes to stimulate and further the undertaking of applied research. Copies are available from the Regional Advisory Council, Tavistock House South, Tavistock Square, London, W.C.1.

**The British Council,** in conjunction with the Engineering Department of the British Post Office, is organizing a course in London from 27 June-10 July designed to show how point-to-point radio circuits for the internal and overseas communications of the United Kingdom are planned, equipped and operated. Details may be obtained from the offices of the British Council, 65 Davies Street, London, W.1.

**A. H. Hunt (Capacitors) Ltd,** of London, have announced the formation of Hunt Capacitors (Canada) Ltd. The new company, with plant and production facilities, located at Ajax, Ontario, will manufacture capacitors previously imported from England for the Canadian electrical and industrial markets.

**Multicore Solders Ltd,** of Hemel Hempstead, have recently issued a recommended procedure for the soldering of printed circuits. They have been engaged for some time in the preparation of suitable fluxes and other allied materials and are now able to offer a comprehensive scheme which can be used to give successfully soldered circuits with a good cover of solder over all metal parts.

**The Bronze Medal of the City and Guilds of London Institute** for the best student in Great Britain in the 1953 Intermediate Radio Servicing Examination has been awarded to Mr. J. McCubbin of Glasgow.

**Erratum.** On page 82 of the February issue in the description of the Oscilloscope Type XT.316, "Radio Research Establishment" should read, "Radar Research Establishment".

# BOOK REVIEWS

### Vacuum Tube Oscillators

**By William A. Edson, 476 pp., 250 figs. Demy 8vo. John Wiley & Sons, Inc., New York. Chapman & Hall Ltd., London. 1953. Price 60s.**

THE author states in the preface that this book is intended to serve as a reference work and that the level of the treatment is directed towards the graduate of the usual four year course in electrical engineering.

It would appear that this aim has been reasonably well achieved, although some students may be frustrated by the continual recourse to references, many of which may not be easily available. In many cases important information and relevant discussions are omitted, and the gaps in continuity are intended to be filled from further exploration of a bibliography of 352 references. In this way the author solves the problem of space economy in dealing with a subject which could easily fill many volumes, and it is true to say that although a great deal of very useful material has been provided, the work is by no means a complete treatment of "Vacuum Tube Oscillators."

The book commences with a short introduction to the fundamental principles, properties and types of oscillators. Chapter II reviews the transient behaviour of linear systems and introduces useful ideas (particularly from the mathematical viewpoint) which form the basis of later discussions.

In Chapter III, transitron and dynatron negative resistance oscillators are briefly examined, and methods are given for calculating amplitude and frequency and for calculating the extent to which frequency is modified by the presence of harmonic currents. In Chapter IV, on non-linear oscillations, use is made of isocline diagrams, cyclograms and van der Pol's equation. Many students will find this a very difficult subject, and the presentation is difficult to follow.

Chapter V deals with feedback systems and stability criteria. Nyquist's criterion and plot are rather sketchily explained.

The first five chapters will be of considerable interest to the advanced student and form the theoretical basis for the remaining chapters which are of a more practical nature.

Chapter VI on resonators is extremely useful, and the properties of capacitors and inductors in connexion with oscillators are well explained; quartz crystals are dealt with very briefly.

In Chapters VII, VIII and IX, many linear oscillator circuits are examined, and in Chapter X, several different criteria for the presence or absence of intermittent operation in various types of oscillators are presented. Chapter XI is devoted to the special problems which arise when the power output required is relatively large but the frequency is low enough to be consistent with lumped circuit techniques. The applications discussed are those connected with dielectric and induction heating, diathermy and cyclotrons.

The next and largest chapter (46 pages) provides interesting discussion and analysis of the more familiar and useful relaxation oscillators, including the multivibrator, blocking oscillator and Puckle's saw-tooth generator. Locking and synchronization of oscillators form the subject of Chapter XIII. Several aspects are very helpfully examined, including the synchronization of ordinary harmonic and relaxation oscillators and the use of the locked oscillator as a detector of frequency modulation.

Frequency multiplication and division, tube and thermal noise, amplitude and frequency modulation of oscillators and automatic frequency control are the subjects of four chapters, and finally, Chapter XVIII deals with long line and multiple resonance effects associated with coupled circuits or circuits involving several inductors and capacitors, or lumped elements connected by low-loss electrically long transmission lines. A bibliography, name index and subject index complete the book.

This volume is well written and authoritative, and no important mistakes have been noticed. The author has been able to embody some freshness in his presentation and this is indeed creditable, but in some respects the presentation lacks compactness, and occasionally explanations are not as clear as might be desired, particularly in the first chapters where clarity is particularly important.

At the end of each chapter there are problems designed to stimulate interest and endeavour, and these would be much more valuable if answers to the numerical parts were provided.

It should be understood that this book is designed for advanced students of good mathematical standard, and will be of limited use for beginners and casual readers. Within the framework of the criticisms given, it is a valuable addition to technical literature.

H. V. SIMS.



### Principles of Electronics

**By L. T. Agger. 340 pp., 100 figs. Demy 8vo. Macmillan & Co. Ltd. 1953. Price 18s.**

THIS book lays no claim to prepare the reader for particular examinations, but rather to give an insight into the fundamental principles of electronic devices and their associated circuitry. Knowledge of mathematics and electrical technology to Ordinary National Certificate standard is pre-supposed. An introductory chapter dealing with atomic structure and the electronic theory of conduction is followed by a quantitative treatment of electron dynamics, together with a note on the M.K.S. system. Thermionic emission and conduction through gases lay the foundations for a study of both the vacuum, and the gas-filled diode. Various methods of rectification are then discussed, together with ripple filters; the simple neon stabilizer is also mentioned. The properties of the triode, and its applications as a voltage amplifier are then explained; constructions of dynamic characteristics and of various types of load-line are shown. Tetrodes and pentodes are followed by a description of A.F. amplifiers, frequency distortion, and feedback. Simple tuned amplifiers, Miller effect, and the variable-$\mu$ valve (this rather sketchily) are described in the next chapter then power amplification is dealt with. Attention is then turned to the thyratron and some of its applications. After an introduction showing the analogy between various electrical and mechanical properties, a simple electro-mechanical oscillator is described, using coils encircling a bar-magnet suspended on a spring; the conventional types of oscillator circuits then follow in a readily-assimilable manner. The remaining chapters contain treatments of modulation, detection, the C.R.T., and photo-electric circuits. Each chapter concludes with a set of exercises with answers; appended at the end of the book are selected questions set by various examining bodies. Two mathematical appendices are given on the solution of the differential equation of the free oscillatory circuit, and the phase shift and attenuation of a three-mesh C-R network. On page 219, paragraph (i) should be (1), since paragraphs (2) and (3) follow. On page 220, the fifth word on the fifteenth line should be " damped," instead of " undamped." Due, probably, to a printer's error, the answers to exercise 26 (b) and (c) on page 195 are incorrect. The author has an obvious flair for lucid exposition, using only the minimum of mathematics. The book may be confidently recommended to the venturer into the vast field of electronics, and should be a useful additional text-book for the student.

H. STIBBE.

### The Magnetic Amplifier

**By J. H. Reyner. 119 pp., 80 figs. Demy 8vo. 2nd Edition. Rockliff Publishing Corporation Ltd. 1953. Price 15s.**

THIS is virtually a re-print of the first edition (see *Electronic Engineering*, June, 1951, page 242), with the addition of four recent papers to the group of references which are not specifically mentioned in the text: scrutiny of the contents table and index, and sampling a number of pages of text, failed to reveal the slightest alteration elsewhere. Consequently your reviewer's previous criticism still stands, that series- and parallel-connected transductor arrangements are stated on page 19 to be comparable while it is correctly pointed out on page 46 that the parallel-connected arrangement is likely to have a slower response than the series-connected one. In the light of recent developments, one is now inclined to ask for a clearer distinction between the " bias " and " feedback " functions of " self-excitation," since there is a trend in the literature to use the term " self-excitation " instead of " feedback " when the gain is raised by passing rectified output current through a control winding. None the less, the book is strongly recommended to all who do not possess the first edition, since it is clearly written and gives a reliable introduction to the properties and design of magnetic amplifiers with the minimum of mathematical complication.

D. A. BELL.

### Radio Engineering

**By E. K. Sandeman. 779 pp., 120 figs. Demy 8vo. Volume 1. 2nd Edition. Chapman & Hall Ltd. 1953. Price 60s.**

THIS book is presented as a thoroughly revised edition of the original Volume 1, which appeared in 1947 and has since attained a well-merited measure of popularity as a general text. Minor additions and corrections have been made resulting in a slightly longer volume of 779 pages instead of 775, although space has been saved by the compression of some of the more extravagant illustrations. Due, however, to the use of heavier paper the book is considerably bulkier and perhaps helps to justify the 15s. increase in price.

In its sixteen chapters the book deals thoroughly with the fundamentals of radio engineering (effects and units, sine waves, A.C. theory, resonance and tuning, harmonic analysis, etc.), passes on to valves, amplifiers and oscillators—some 200 pages, and concludes with an exhaustive treatise on transmission and transmitters—300 pages. As formerly, it is a curious blend of an internal instruction and procedure handbook, and a general text. It was written originally as a staff manual for BBC maintenance engineers and this shows all too clearly in some sections where detailed descriptions and operating procedures relating to particular BBC equipments are given; sections which to the general reader have only the doubtful use of illustrating practice instead of theory. The use of terms and descriptions stemming from the author's day-to-day jargon has little to recommend it.

Because of this bias the book is essentially practical in treatment and little touches such as the heavy type reminder " Remember to enter wire radius: not diameter " on Fig. 7/11:13 are typical. Emphasis is predominantly on what might be termed " classical " aspects of radio transmission and the techniques of television and radar are ignored although frequency modulation is treated equally with amplitude modulation in the appropriate sections. This is not to say that the book suffers from this single-mindedness of purpose; it is perhaps in some ways an advantage. Within his chosen terms of reference the author does deal thoroughly with a wealth of topics and does not hesitate to tackle some points conveniently overlooked in other texts—examples being R.F. resistance and the spectra of F.M. signals.

Specific additions are difficult to note but although the book appears unchanged up to page 200 or thereabouts it is clear that minor points have been added. Resetting of the type has also been undertaken with the result that several printer's errors absent in the first edition are present. The use of non-preferred terms such as capacity for capacitance and condenser for capacitor is continued. If the general reader will remember that BBC practice, admirable though it is, is not the only one and that authorities and views—notably American, not represented at all—also exist, the book is a useful revision and will extend the currency already claimed by the first edition.

E. D. HART.

### The Year That Made The Day

**79 pp., 100 figs. Demy 8vo. The British Broadcasting Corporation. 1953. Price 6s.**

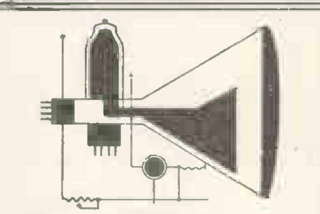
THIS book gives a short account of the planning and preparation by the BBC which enabled millions of people to participate in the ceremonies of Coronation day by television and radio. It is well illustrated and contains photographs from the television screen itself.

### The Practical Electrician's Pocket Book 1954

**Edited by R. C. Norris. 540 pp., 100 figs. Demy 16mo. Odhams Press Ltd. 1953. Price 5s.**

THIS edition includes four new chapters on: Electronics, Maintenance of Cookers, Bell, Alarm and Call Systems, and Fans and Ventilation. There are also revisions of several other sections. In all, some sixty-eight sections contribute towards this popular reference book.





# ELECTRONIC EQUIPMENT

A description, compiled from information supplied by the manufacturers, of new components, accessories and test instruments.

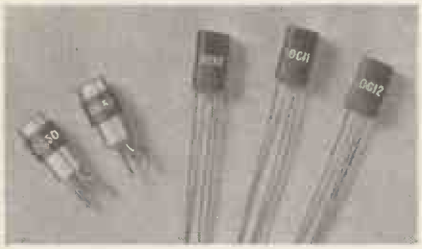
### Point-contact and Junction-type Transistors

*(Illustrated below)*

MULLARD LTD have recently announced the first of a range of point-contact and junction-type transistors for experimental purposes. They are available, in reasonable quantities, to electronic equipment designers wishing to investigate the potentialities of transistors, and to carry out practical experiments in transistor circuits. The range includes two point-contact types, the OC50 and OC51, and three junction types, the OC10, OC11 and OC12.

The OC51 has a better high frequency characteristic and a shorter turn-off time than the OC50. The OC50, however, operates more satisfactorily than the OC51 at the collector "bottomed" condition. In this condition the "fully-on" transistor of extremely low impedance passes a current determined almost entirely by the outside load.

The junction transistors, OC10, OC11



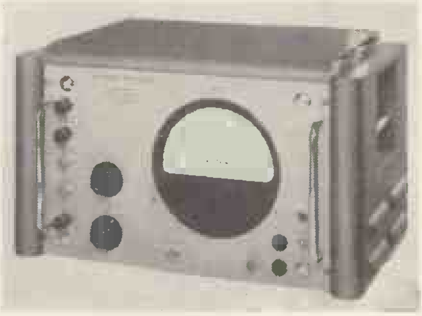

and OC12 are designed for economy in power supplies and, in both amplifier and oscillator circuits, they will operate satisfactorily from H.T. supplies as low as 1·5V, with current consumptions of similar low order. Under suitable conditions they will work with H.T. supplies of only a fraction of a volt.

The OC11 is a general-purpose amplifier which, in a grounded emitter circuit, gives a current amplification factor of 17. Under similar conditions the OC12 gives a current amplification factor of 30, and is intended primarily as an output transistor, although it can, of course, also be used in amplifier circuits. The OC10 is a low-noise version of the OC11 and is intended for use in the early stages of high-gain amplifiers.

**Mullard Ltd,
Century House,
Shaftesbury Avenue,
London, W.C.2.**

### Feedback Valve-Voltmeter

*(Illustrated above right)*



IN the model VF.252 valve-voltmeter the changes in the forward to backward impedance ratio of the meter rectifiers has been overcome, resulting in a higher degree of accuracy than has hitherto been obtainable with rectifier type instruments.

A 6in scale meter with a 1mA movement is employed. Nine sensitivity ranges covering from 15V down to 1·5mV for full-scale indication are provided. On the latter range the scale is calibrated in 20μV steps and useful readings are obtainable down to 30μV. The input impedance is greater than 50MΩ and the accuracy of indication inside ±1 per cent of full scale except on the two highest sensitivity ranges where the impedance is 30MΩ and the accuracy ±2 per cent. An optional 1MΩ 10:1 input attenuator is provided, thereby effectively extending input voltage range up to 150V full scale. In addition, alternative connexions are provided for an isolated input using a wideband 1:1 transformer with a primary inductance of 50H. An output of 3V R.M.S. for full-scale deflexion is brought out to a socket on the front panel and is available for display on an oscilloscope. The signal-to-noise ratio at the output is better than 40db at full-scale.

The instrument covers a frequency range of 10c/s to 100kc/s.

**Solartron Laboratory Instruments Ltd,
Solartron Works,
Queens Road,
Thames Ditton, Surrey.**

### Twin Stabilized Power Supply

*(Illustrated below)*

THE type PSU1 has two independent stabilized outputs which may be connected separately either as lines positive or negative to earth or in series to provide an extended voltage range or a centre-tapped power supply. Where an accurately centre-tapped variable supply is required the unit can be supplied with a self-balancing connexion.

The output of each stabilizer is variable from 180 to 350V at up to 180mA, a negative supply of 170V at 2mA is also available. A total of 72W heater

power at 4 or 6·3V is also provided.

The output is monitored by a 3½in meter which may also be used for measuring external voltages, the sensitivity being 1 000Ω/V.

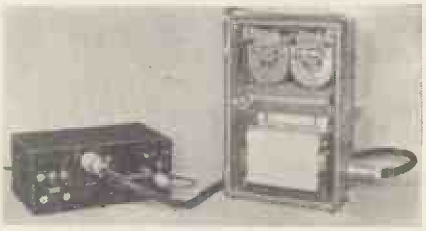
**Omicron (Electronics) Ltd,
Hermitage Buildings,
Longfellow Road,
Walsgrave, Coventry.**

### Duplex Quick Response Recorder

*(Illustrated below)*

THIS instrument, which is of the continuous roll chart type contains two movements which are essentially D.C. recording voltmeters having a rapid response and negligible pen to paper friction. The maximum torque produced by the pen movement is approximately 7oz in.

The recorder is servo operated and works in conjunction with an amplifier unit, the amplifier and recorder being connected by a 5 ft. long 18 core flexible cable with plug and socket ends.



The input to the recorder is opposed by the feedback voltage from a resetting potentiometer which is controlled by the position of the pen. The error, being the difference between these voltages, is amplified and drives the pen movement until the feedback voltage balances the input.

The pen zero may be set in any position and a solenoid operated time marker pen is also included.

The maximum sensitivity may be set to suit requirements between the limits of ±10 volts to ±4 volts for a deflexion of ±1 7/16 inch. It may be continuously reduced down to zero by means of a potentiometer control on the front panel of the amplifier unit. Provision is made to switch this control out of circuit if desired. The input impedance is approximately 50kΩ with the sensitivity control potentiometer in circuit and approximately 2MΩ with it switched out.

The voltage across the resetting potentiometer is stabilized and the sensitivity is thus independent of mains variations.

The instrument will record sine wave voltages up to 15c/s with an amplitude error not exceeding 15 per cent for a peak-to-peak amplitude of 1½ inches. At smaller amplitudes the frequency response is proportionally higher.

The synchronous motor driven chart clock has a four-speed (lever selected) gear box providing for chart speeds of ½, 1, 3, or 6 inches per second in the case of the RD.15 or 2, 4, 12 and 24 inches per minute for the RD.15A.

Recordings can be made either in ink or on teledeltos paper.

**Evershed & Vignoles Ltd,**
**Acton Lane Works,**
**Chiswick,**
**London, W.4.**

**Midget Attenuator**

*(Illustrated above)*

TO meet a demand for a midget stud-type attenuator or fader, Painton & Co. Ltd. have introduced a new midget instrument known as type M.

Primarily designed for audio applications (although in special circumstances it may be satisfactory on carrier frequencies up to 4Mc/s) the unit measures only 1⅛in diameter and 1 5/16in long. Painton ¼-watt high stability carbon resistors are employed and the unit can accommodate either a 10-step twin-arm network (e.g. bridged-T) or a 20-step single-arm network (e.g. unbalanced potentiometer).

A special knob and dial have been designed for use with this attenuator, in which full advantage has been taken of the extreme compactness, so that the dial diameter is no greater than that of the unit.

**Painton & Co. Ltd,**
**Kingsthorpe,**
**Northampton.**

**Capacitor Power Factor Measurement**

*(Illustrated above)*

A RECENT addition to the range of capacitor power factor measuring instruments manufactured by Viduna Instruments is a model covering the range 150 to 150 pF and giving a direct reading of power factor on a 3½in meter.

There are three ranges of indication 0 to 25, 0 to 100 and 0 to $400 \times 10^{-4}$ tangent of loss angle at 1Mc/s. A D.C. polarizing voltage can be incorporated for the application of voltages up to 1 000V in steps of 100V.

Other models cover the range of 4 to 1 000pF.

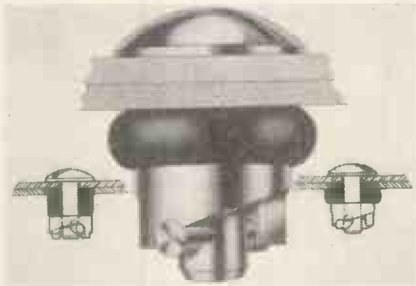
**Viduna Instruments,**
**11 The Ridgeway,**
**Epsom, Surrey.**

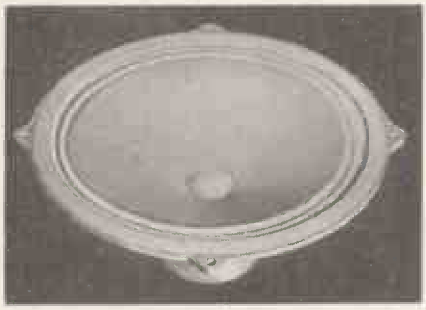
**Panel Fasteners**

*(Illustrated below)*

VIBREX fasteners are designed to lock removable panels in position; a typical application being the locking of the panel and chassis into an instrument case. To fit the Vibrex, which depends on the expansion of rubber for its locking action, only two plain holes are needed, one in the removable panel and the other in the base. The action of the fastener can be seen from the illustration.

**Silentbloc Ltd,**
**Victoria Gardens,**
**London, W.11.**





**Loudspeakers**

*(Illustrated above)*

THE main feature of the latest range of Stentorian loudspeakers is the cone which is made from uncured cambric and bonded pulp, the whole being cured together into a composite cone by a new manufacturing process.

They are available in 6, 8, 9 and 10in sizes; the first two are fitted with 10 000 gauss magnets and the latter two with 12 000 gauss magnets. The 10in unit will handle 10 watts, the frequency range is 30c/s to 14kc/s and the bass resonance 30c/s. All the units are dust proof.

**Whiteley Electrical Radio Co. Ltd,**
**Victoria Street,**
**Mansfield, Notts.**

**Valve Retainers and Extracting Tongs**

*(Retainers illustrated top right)*

THE latest addition to the range of Spearette valve retainers is the type W. These are designed for use with miniature valves and, as shown in the illustration are of extremely simple construction.

Also recently introduced by the same firm is a universal valve and screen can extracting and replacement tool. The tongs are capable of handling all types of miniature valves and their associated cans. The grips in contact with the valve are covered with a Neoprene sleeve.

**Spear Engineering Co. Ltd,**
**Titan Works,**
**Limpsfield Road,**
**Warlingham, Surrey.**

**Shock Absorbent Packing**

*(Illustrated below)*

METPAK shock absorbers are designed to protect, while in transit, heavy and delicate equipment such as electronic control apparatus, radar equipment, television receivers, etc. Each Metpak units consists of a number of springs mounted between a pair of metal plates. The units may be used singly, in pairs or as triple units. This method of packing has been approved and is used by the Ministry of Supply. The illustration below shows an outer container with double Metpaks assembled *in situ* ready for the inner case to be fitted.

**Brayhead Springs Ltd,**
**Full View Works,**
**Kennell Ride,**
**Ascot, Berkshire.**

# LETTERS TO THE EDITOR

*We do not hold ourselves responsible for the opinions of our correspondents)*

### The Wurlitzer Organ

DEAR SIR.—With reference to Alan Douglas's article in the November, 1953, issue, I would like to make two points.

The first one, which is of minor importance, is that if the overtones from a sound producer are not integral multiples of the fundamental, then they cannot be called harmonics.

Secondly, Table 1 (b) in the article is entitled "Typical Overtones for a Free Reed." This table applies only to a cantilever clamped rigidly at one end and having a uniform cross-section throughout its length. While a free reed approximates to a rod clamped rigidly at one end, its cross-section is not uniform throughout the length. Therefore the table given could not apply except in special and unusual cases. As the result of personal work, I have found that the tone produced by a free reed approximates to a harmonic series, typical figures obtained being:—

| | | | |
|---|---|---|---|
| Fundamental | ... | 40 per cent | $f$ |
| Octave ... | ... | 50 per cent | $2f$ |
| Twelfth ... | ... | 75 per cent | $3f$ |
| Fifteenth ... | ... | 25 per cent | $4f$ |
| Seventeenth | ... | 12 per cent | $5f$ |
| Nineteenth | ... | 100 per cent | $6f$ |
| Twentyfirst | ... | 15 per cent | $7f$ |
| Twentysecond | ... | 10 per cent | $8f$ |

The reed thus tested was completely unresonated and fastened to a small block of wood in lieu of a soundboard. The analysis having been done with a Dawe Audio Frequency Analyzer, the harmonics cannot be attributed to faulty analysis technique. It is of interest to notice the effect of the bar's natural overtone at the Nineteenth. In this case it occurred very near to $6f$, and not the $6{\cdot}267f$ of a uniform bar. In the future good voicing technique may consist in so shaping the reed that the fundamental is accentuated, and the first bar overtone is exactly $6f$.

Yours faithfully,
JOHN E. SCOTT.
Birmingham.

**The Author replies :**

DEAR SIR,—Referring to your letter commenting on the reed tone analysis given on page 466 of the November, 1953, issue:

Firstly, the table does not denote the partials as harmonics although I have mentioned them in the text since true harmonics are present in many reed tones.

The harmonic content of your reed is quite representative of that kind of reed voicing. In the case of reeds for electrical pick-up it is not in general important that overtones above the fundamental should be in any particular relationship since the object is to extract as much fundamental as possible by using a pick-up having an area great compared with the vibrating length of the partials. The formula given applies to weighted reeds and many such are used in this class of instrument. The overtones shown numerically apply to a free reed of uniform cross-section, mass and elasticity. Many reeds conform to this condition, particularly those used by the Aeolian Company. The formula for an elemental part of such a reed is:—

$$EI\,\frac{\partial^4 y}{\partial x^4} = -\,m\ddot{y} \quad \ldots\ldots\ldots\ldots\ldots\ (1)$$

where $E$ = Young's Modulus
$I$ = moment of inertia
$y$ = displacement of the section from position of rest
$x$ = distance of section from clamped end
$m$ = mass per unit length

But, in the very common case where the reed contour is variable or the material non-uniform, then formula (1) becomes

$$\frac{\partial^2}{\partial x^2}\left[EI\,\frac{\partial^2 y}{\partial x^2}\right] = -\,m\ddot{y} \quad \ldots\ldots\ (2)$$

This can only be solved by an approximate numerical method[1] if $I$, $m$ and probably $E$ are unspecified functions of $x$. It is a reed of this type which Mr. Scott has analysed, the harmonic content of which has been adjusted empirically. It is clear that whatever harmonic series the reed itself produces, it would be intolerable to simultaneously convert all of these into electrical energy, and the statement on page 469, "careful selection and control of the reeds" is meant to embrace not only the electrical methods adopted to provide the desirable response but also the shape of the vibrating tongue itself.

However, the following additional second order effects greatly influence the final form of the reed vibrations. No reed maintains its physical properties constant throughout its life. Moreover, the ratio of width to length, the aperture between the tongue and the base which determines the rate of shear of the air, introduce complex effects. A thin reed may be set into vibration in another plane due to flexing laterally as the pressure changes. The acceleration at the edges tends to be greater than at the centre due to the rapid increase in wind velocity as the aperture decreases, and this may superimpose further discordant frequencies on the main tone. The building up of a dust film on the edges of the tongue owing to the violent acceleration of air through the slot eventually modifies the generated partials. So does the onset of fatigue in the metal. The damping introduced by mounting in an approximately tuned enclosure affects the overtones audibly. The extent of flexure of the reed tongue with varying wind pressures has a similar effect, and this is partially modified by back pressure in the cavity-pallet system.

The back pressure in the Wurlitzer instrument is controlled by the adjustable mutes as described, and as stated at the end of the article, every single reed is specially contoured and aged in order to reduce inharmonic tones and at the same time have the requisite physical properties for rapid cessation of oscillation. In practice reed tongues vary from approximately 10in × 1·1/4in × 3/32in to 1/4in × 1/40in × ·004in tapering to ·0015in and so far as I am aware, no really comprehensive general analysis has been made for this type of reed dissociated from an air column and resonator; presumably because there has been no demand for such an investigation and the results could not economically be made use of. Mr. H. W. Homer, Mr. D. M. A. Mercer and the Rev. N. Bonavia-Hunt have, of course, examined the properties of the reed organ pipe fairly thoroughly.

The pipe organ reed is quite different since it is unable to pass freely through the slot but strikes it violently since the tongue is larger than the opening. Such a reed is unsuitable for electrical pick-up.

Yours faithfully,
ALAN DOUGLAS.
Radcliffe-on-Trent,
Notts.

REFERENCE

1. TIMOSHENKO, S. Vibration Problems in Engineering. 2nd edition. (Constable & Co. Ltd., 1937.)

### Amplified-torque Clutch

DEAR SIR,—Having had requests for further information relating to the Amplified-torque Clutch, described in your issue of January, 1954, I send you the following characteristics of an experimental version of the clutch.

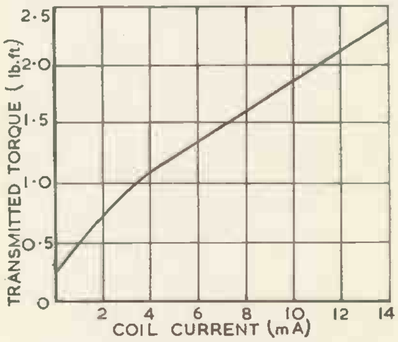



Fig. 1. Relation between output torque and coil current

The magnet has a diameter of 1¼in and its length, including the armature, is 1¼in while a brass shim, 0·007in thick, lies between them. The relation between the attractive force on the armature and the coil current, over the linear region, is 0·5lb/mA. The cylindrical surface of the magnet is covered with a brake material and this, together with three turns of steel tape, forms the amplifier. With a slip speed of 12R.P.M. the relation between the output torque and coil current is as shown in the Fig. 1, the slope of the linear region being 120lb.ft/A; a slight fluctuation of less than 0·1lb.ft was observed when making this measurement of the output torque. It will be seen that some torque is transmitted for zero coil current. When pairs of clutches are used in a servo system, these remanent torques tend to cancel and produce a continuous characteristic.

Yours faithfully,
R. VOLES,
E.M.I. Engineering Development Ltd.

## An Alternating Current Stabilizer

DEAR SIR,—In the article by P. A. V. Thomas in the December, 1953 issue, the circuit of Fig. 2 presents a number of rather bewildering design features. It would be interesting if the author would explain the following seeming anomalies.

(1) If the H.T. line is at approximately 315 volts (the normal voltage across three 105/30 stabilizers) and if the operating current of the control diode $V_1$ is only 15$\mu$A (as indicated in Fig. 3), the anode potential of $V_1$ should be about 165 volts in the absence of grid current in the first section of $V_2$. In fact, however, the absence of such grid current under the conditions indicated, would appear quite impossible. A probable value would be from 12 to 16$\mu$A in which case the control voltage would depend almost as much on the grid current characteristics of $V_2$ as on the emission-temperature characteristic of the control diode $V_1$.

(2) With a 47$\Omega$ cathode resistor in $V_2$ and the second grid taken to H.T. negative, the anode current in the second section of this value is not likely to exceed 100$\mu$A or so. Heavy grid current must therefore flow in the second section of $V_3$ or its grid potential would rise to about 300 volts positive.

(3) Control of $V_1$ heater voltage by means of a variable resistor in the primary of its heater transformer seems a strange method calculated to distort the secondary voltage waveform and probably to increase the frequency dependence of the output.

It would be interesting to know how stable the output of the stabilizer remains over long periods and how sensitive it is to replacement of $V_2$ and $V_3$.

Yours faithfully,
R. C. ROBBINS,
Bedford.

DEAR SIR,—I was interested in the A.C. stabilizer described by Mr. P. A. V. Thomas[1]. I should like to point out that this type of stabilizer gives an excessive output-voltage in the event of diode-failure or certain other types of fault, such as inter-electrode short-circuits in the amplifying valves. Clearly some form of protection is desirable as a maintained overvoltage may well burn out all the valve heaters supplied by the stabilizer.

The requirement for complete protection is simply that the D.C. winding of the saturable-reactor be dis-connected should there be overvoltage at the output. One way of doing this is to use a Mazda 29C1 saturated diode and a relay. The diode filament is run via a 3·5$\Omega$ series resistance from one of the 6·3V windings on the transformer $T_3$ (see Fig. 2, Ref. 1). At the correct A.C. output voltage the diode takes a saturation current of 0·5mA while for a 10 per cent overvoltage the saturation current increases by nearly four times to 1.9mA. This current change is adequate to operate a 20$\Omega$ type 3 000 relay. A suitable H.T. for the diode is about 100V which may be derived from 315V line by two resistors. The relay has two pairs of contacts; one pair disconnects the saturable-reactor control-winding and the other pair earths the diode anode so that the relay remains in the held condition.

Perhaps I should also point out that the curve and working point given in Fig. 3 are in error. Firstly the diode emission is shown going up to 35$\mu$A although there is a 10M$\Omega$ load and a 250V supply (25$\mu$A max) and secondly the working point of 16$\mu$A implies a diode anode voltage of 155V in the stabilizer. An anode voltage of this magnitude, with negligible space charge, would result in the rapid destruction of the oxide-coated filament. However, from the circuit of Fig. 3 it is apparent that the anode potential is only a few volts (less than the grid-base of $V_2$) giving an emission of about 30$\mu$A. A miniature valve run as a saturated diode under these conditions should be quite satisfactory.

Yours faithfully,
V. H. ATTREE,
Fluid Motion Laboratory,
University of Manchester.

REFERENCES

1. THOMAS, P. A. V. An Alternating Current Stabilizer for Supplying Valve Heaters. *Elect. Engng.* 25, 522 (1953).

**The Author replies:**

DEAR SIR,—I would like to thank Mr. R. C. Robbins for his comments and with reference to his point (1), I agree that the total current through the 10M$\Omega$ resistor in Fig. 2 must be of the order of 30$\mu$A and that therefore the operating point (Fig. 3) must be at about $V_h$ = 0·45V and not as indicated (0·35V) though this would be the optimum point; as explained in the text of my article, originally the grid of the second half of $V_2$ was taken to a positive supply but the stabilizer was found to operate better with almost zero voltage on this grid and from Fig. 3 it will be seen that there is not a great difference in sensitivity between 0·35V and 0·45V. Unfortunately I am no longer at the Royal Technical College, Glasgow, and cannot therefore check the actual operating point.

As regards Mr. Robbins' second point, I agree that grid current will flow in $V_3$ due to the high anode potentials of $V_2$; this point had unfortunately been overlooked in the past and is obviously undesirable. As the amplifier must be D.C. coupled, one of two methods might be conveniently used. (a) To tie the anode load of $V_2$ to a lower D.C. potential, easily obtained from the neon chain, or (b) to couple between $V_2$ and $V_3$ using low current neons to drop the D.C. voltage on the latter grids. Of the two methods, (a) would probably be the more satisfactory.

As regards his third point, the method used is simple and the distortion introduced is not likely to be very serious as the secondary load resistance (the valve heater) is small and the distortion introduced can be shown to be approximately proportional to the parallel effect of the source, transformer primary and load impedances which in this case will approximate to the load impedance, though I agree that if the input impedance is reduced then the distortion will also be reduced. As an alternative to the method shown one could use one of the following methods:

(a) A series resistor in the secondary circuit instead of the primary but its value would probably be awkward for a variable resistor.

(b) small continuously variable auto transformer.

(c) No variable component, accepting the output voltage obtained—still of course stabilized.

With regard to the question of stability, I can only state that over a period of several months' operation no change in output was observed and, as stated above, having left the College I unfortunately cannot check the effect of changing $V_2$ and $V_3$ but I should imagine that, as is quite common in D.C. amplifiers (of which this is one), resistor $R$ would have to be altered to reset the output to the original value due to differences in the valve characteristics.

In reply to Mr. Attree, I agree that some form of protection is desirable if the stabilizer is to be used in some permanent equipment, however in an experimental set-up as the particular stabilizer was used this is not so important as one is continually taking readings and any sudden change would undoubtedly give extraordinary results and the stabilizer is quickly checked by observation of the meter $M$, and generally speaking the output voltage would not rise to too high a level. It should be pointed out, however, that the particular faults mentioned are unlikely and using another valve as a safety device, as suggested, does not seem to be the answer but rather some other form of protector such as a maximum voltage voltmeter in which the meter pointer can close a contact at a suitable preset value, say 10 per cent high.

As regards his second point, upon checking my original laboratory notes I find that in Fig. 3 the supply voltage should have been 350V and not 250V as shown and in connexion with the operating point I would ask him to consult my reply to Mr. R. C. Robbins.

Yours faithfully,
P. A. V. THOMAS,
London, S.W.7.

## Characteristics of Mazda Diode 29C1

DEAR SIR,—With reference to the article on the Characteristics of The Temperature—Limited Diode Type 29C1 by Messrs. Benson and Seaman which appeared in the November, 1953, issue, there are a number of points that the writer would like to comment upon and also to add information that may be of interest to users.

These comments are based upon work carried out since 1945 in the Engineering Departments of the Edison Swan Company during the development and production of this valve.

The following are selected points from this work:

(a) The 29C1 valve was designed initially for a specific application and like all valve design was a compromise. It is probably now used for a variety of purposes and requirements for which it is not the most suitable design, or type of valve.

(b) As regards initial characteristics, these are a compromise between economic production and uniformity of performance, and the authors' figure of 15 per cent is reasonable. In order to get the close characteristics required it is necessary to mount and weld accurately to length, a very fine piece of tungsten wire. This tungsten wire itself must be maintained to an accurate diameter and this can only be achieved by careful selection from spools of wire of the same nominal diameter but varying by less than ± 0·0002in.

(c) All valves are aged for 48 hours before being shipped out of the factory.

(d) For our standard life test conditions we use an A.C. stabilized power supply for the filament, which keeps the potential within 2 per cent of the nominal value. The anode is fed with 100V from a generator via a safety lamp. Periodic testing is carried out on separate D.C. test boards.

It should be realized that our object is merely to check the quality of the factory product and hence our life test and testing conditions may not be held so academically accurate as those of the authors', or Mr. Attree. Within the limits shown above we have obtained results that are substantially the same as the authors except that ours have gone on until failure of one filament.

(e) To obtain a better life performance we agree that it would be necessary to run with a lower temperature filament, i.e., lower $I_a$ being taken. Alternatively a higher filament wattage valve which may mean a larger bulb would be required. Such an alternative would slow up the rate of response to change of $V_f$ which was an important point in the initial design requirements. The life if run at 4·5V would be seriously reduced, whereas at 3·5V the life would be greatly increased. Over this range the life factor could be of the order of 20 or so, but the total change of $I_a$ would remain approximately the same.

(f) The rate of response for small filament voltage changes is of the order of 0·5 seconds.

(g) We are aware that the valve has a margin of safety for anode voltage rating but as the valve is a commercial type the ratings must always be conservative and hence we do not recommend exceeding our published figures. As we acquire sufficient indication of reliability then our ratings will be changed.

(h) Referring to short term stability. over a period of 8 hours continuous running, sample valves have shown less than 1 per cent variation of anode current with constant $V_f$. Switching off at night and on again in the morning for a total working period of 20 hours, the stability has been of the order of 2 per cent.

Yours faithfully,
D. L. HALL,
Edison Swan Electric Company, Ltd.
Brimsdown.

# MEETINGS THIS MONTH

## THE BRITISH INSTITUTION OF RADIO ENGINEERS

Date: March 31. Time: 6.30 p.m.
Held at: The London School of Hygiene and Tropical Medicine, Gower Street, London, W.C.1.
Lecture: Radio Astronomy.
By: R. Hanbury Brown.

**North-Western Section**

Date: March 11. Time: 7 p.m.
Held at: The College of Technology, Manchester.
Lecture: Colour Television.
By: G. B. Townsend.

**West-Midlands Section**

Date: March 23. Time: 7.15 p.m.
Held at: The Wolverhampton and Staffordshire Technical College, Wulfruna Street, Wolverhampton.
Lecture: Industrial Application of Electronic Instruments.
By: A. G. Wray.

## THE INSTITUTION OF ELECTRICAL ENGINEERS

All London meetings, unless otherwise stated, will be held at The Institution, commencing at 5.30 p.m.

**Ordinary Meeting**

Date: March 4.
Lectures: Submerged Telephone Repeaters for Shallow Water.
By: R. J. Halsey and F. C. Wright.
The Netherlands-Denmark Submerged Repeater System.
By: A. H. Roche and F. O. Roe.
British Post Office Standard Submerged Repeater Sytem for Shallow-Water Cables, with special mention of the England-Netherlands System.
By: D. C. Walker and J. F. P. Thomas.

**Informal Meeting**

Date: March 8.
Discussion: How Can Electrical Methods Enhance Industrial Productivity?
Opened by: Sir Henry Clay.

**Measurements Section**

Date: March 2.
Lecture: Current and Power Relationships in the Measurement of Iron Losses in a Three-Limbed Transformer Core.
By: F. Brailsford.
and: An Automatic Plotter for Magnetic Hysteresis Loops.
By: H. McG. Ross.
Date: March 16.
Lecture: The Minimum-Moment-of-Error-Squared Criterion: a New Performance Criterion for Servo Mechanisms.
and: The Behaviour of a Remote Position Control Servo Mechanism with Hard-Spring Non-Linear Characteristics.
By: J. C. West and P. N. Nikiforuk.

**Radio Section**

Date. March 10.
Lecture: A Study of some of the Properties of Matter affecting Valve Reliability.
By: E. A. O'Donnell Roberts.
Date: March 22.
Lecture: Colour Television.
By: C. J. Hirsch.

**Supply Section**

Date: March 24.
Lecture: Cathode-Spot Performance Phases and Control of Mass Transfer in Vapour-Discharge Devices.
By: H. von Bertele.

**Cambridge Radio Group**

Date: March 9. Time: 8.15 p.m.
Held at: The Cavendish Laboratory, Cambridge.
Paper on Computing Machines.

**Mersey and North Wales Centre**

Date: March 25. Time: 6.45 p.m.
Held at: The Philharmonic Hall, Liverpool.
Faraday Lecture: Electro-Heat and Prosperity.
By: O. W. Humphreys.

**North-Eastern Radio and Measurements Group**

Date: March 15. Time: 6.15 p.m.
Held at: King's College, Newcastle-upon-Tyne.
Lecture: A Study of some of the Properties of Matter Affecting Valve Reliability.
By: E. A. O'Donnell Roberts.

**North Midland Centre**

Date: March 23. Time: 6.30 p.m.
Held at: Bradford Technical College.
Discussion: High Voltage in Theory and Practice.
Opened by: A. R. Rumfitt.

**North-Western Centre**

Date: March 2. Time: 6.15 p.m.
Held at: The Engineers' Club, Albert Square, Manchester.
Lecture: A Short Modern Review of Fundamental Electromagnetic Theory.
By: P. Hammond.
Date: March 23. Time: 7.30 p.m.
Faraday Lecture: Electro-Heat and Prosperity.
Held at: The Free Trade Hall, Manchester.
By: O. W. Humphreys.

**North-Western Radio Group**

Date: March 17. Time: 6,30 p.m.
Held at: The Engineers' Club, Albert Square, Manchester.
Lecture: Distributed Amplifiers.
By: W. S. Percival.

**North Lancashire Sub-Centre**

Date: March 10. Time: 7.15 p.m.
Held at: The North Western Electricity Board, Demonstration Theatre, North Road, Lancaster.
Lecture: Some Aspects of the Design of V.H.F. Mobile Radio Systems.
By: E. P. Fairbairn.

**South Midland Radio Group**

Date: March 22. Time: 6 p.m.
Held at: The James Watt Memorial Institute, Great Charles Street, Birmingham.
Repeat of North Lancashire Sub-Centre meeting.

**Rugby Sub-Centre**

Date: March 3. Time: 6.30 p.m.
Held at: The Rugby College of Technology and Arts.
Lecture: Transformer-Analogue Network Analysers.
By: M. W. Humphrey Davies and G. R. Slemon.

**Southern Centre**

Date: March 3. Time: 6.30 p.m.
Held at: The SEEB Headquarters, Hove.
Lecture: Technical Arrangements for the Sound and Television Broadcasts of the Coronation Ceremonies on June 2, 1953.
By: W. S. Procter, M. J. L. Pulling and F. Williams.

**Western Centre**

Date: March 8. Time: 6 p.m.
Held at: The South Western Electricity Board Offices, Colston Avenue, Bristol.
Lecture: Telemetering for System Operation.
By: R. H. Dunn and C. H. Chambers.

**South-Western Sub-Centre**

Date: March 10. Time: 3 p.m.
Held at: The Rougemont Hotel, Exeter.
Lecture: Colour Television: Some Subjective and Objective Aspects of Colour Rendering.
By: G. T. Winch.

**Norwich District**

Date: March 15. Time: 7.30 p.m.
Held at: The Royal Hotel, Norwich.
Lecture: Modern Developments in Atomic Energy.
By: T. E. Allibone.

**Oxford District**

Date: March 17. Time: 7 p.m.
Held at: The Southern Electricity Board. 37 George Street, Oxford.
Lecture: Modern Trends in Television.
By: G. G. Gouriet.

## THE TELEVISION SOCIETY

Date: March 12. Time: 7 p.m.
Held at: The Cinematograph Exhibitors' Association, 164 Shaftesbury Avenue, London, W.C.2.
Lecture: Trick Effects in Television Production.
By: D. R. Campbell.

**Leicester Centre**

Date: March 29. Time: 7 p.m.
Held at: The College of Art and Technology, The Newarkes, Leicester.
Lecture: Transistors and other Crystal Valves.
By: D. D. Jones.