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HOW POWER STATION FREQUENCY IS CONTROLLED

By A. T. Dover, M.I.E E.

This subject is becoming of increasing importance to electrical engineers, first on account of the grid transmission system which is now being developed all over the country, and secondly on account of the possibilities it offers for the use of mains driven clocks which depend for accuracy on the correct periodicity of the supply being maintained.

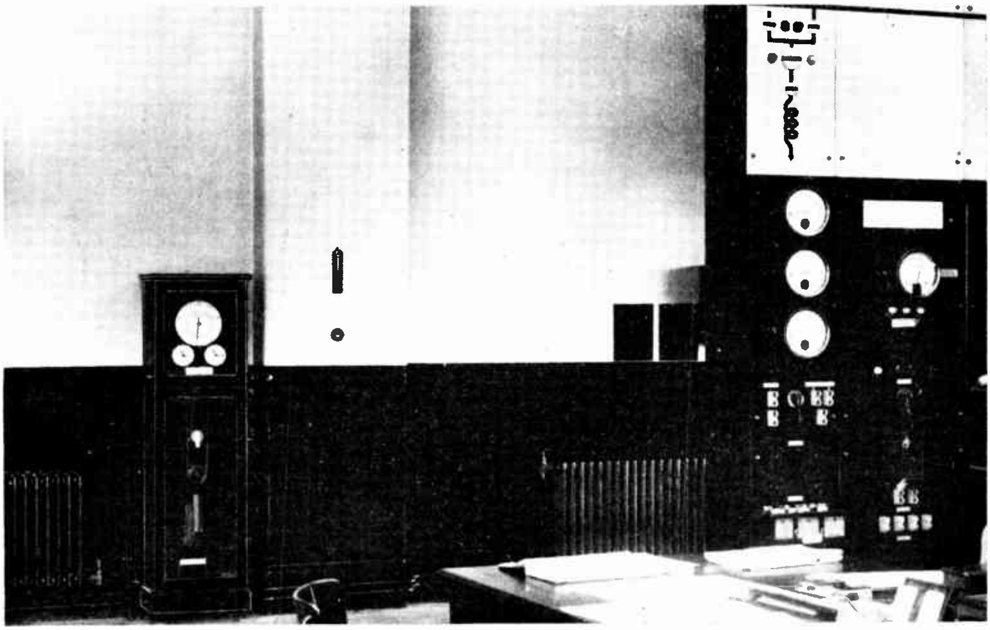


Fig. 1.—The Control Room at Croydon Corporation's Power Station Showing Early Type of Warren Master Frequency Meter. (*Everett Edgcumbe.*)

The clock is driven by weights and is wound automatically by a Warren synchronous motor. The time hand of the frequency meter (upper dial) is driven directly by the clock, and the frequency hand is driven by a Warren motor.

The Advantages of a Controlled Frequency.

THE maintenance of the frequency of an A.C. supply system at a standard mean value is of considerable advantage in practice. Thus (1) large generating stations in which the frequency is rigidly controlled can operate satisfactorily in parallel, and can share or interchange load so as to obtain the most economical operation ; (2) timing devices —such as clocks, graphic instruments, time switches, traffic control signals, etc.—

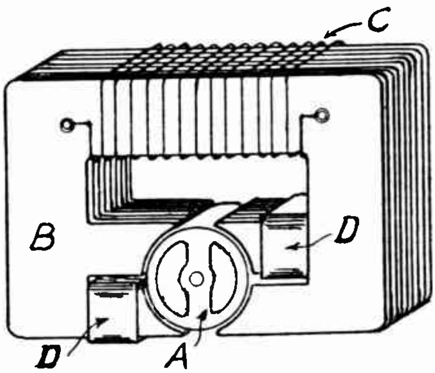


Fig. 2.—ARRANGEMENT OF MAGNETIC CIRCUIT OF WARREN SELF-STARTING SYNCHRONOUS MOTOR.

This type is made by Everett Edgcumbe and is used in their "Synclocks" and master frequency meters. A, armature of hardened steel; B, laminated field magnet with divided pole faces; C, exciting winding; D, short-circuited copper bands ("shading coils") encircling one half of each pole face.

may be operated from the supply mains, and *accurate* functioning of these devices is assured with the minimum of attention and maintenance; (3) industries operating continuously (e.g., textile, paper making, flour milling) are assured of a constant rate of output when their machines are driven by synchronous or induction motors.

How the Idea of a Controlled Standard Frequency Originated.

The idea of controlling the frequency with the object of supplying time service from the supply mains originated in Germany in about 1909, but could not be successfully carried out in practice because, at that time, no suitable self-starting synchronous motor was available. The idea remained undeveloped until about 1916, when the Warren self-starting synchronous motor was produced in the United States of America. Warren's classic paper on "Utilising the time characteristics of alternating current" was presented to the American Institute of Electrical Engineers on May 7th, 1919. This paper gave practical details of a successful miniature synchronous motor (which developed about one-millionth of a horse-power) and its application to frequency control and time service. Warren fully recognised the importance and the necessity of an accurately controlled frequency for time service, and for the interconnection (i.e., parallel operation) of large power stations.

A more recent paper dealing with "Synchronous Electric Time Service" was presented to the A.I.E.E. on January 25th, 1932.

The Warren Synchronous Motor.

This motor is built as a single-phase, bipolar motor with shaded poles, and the arrangement of its parts is shown in Fig. 2. The magnetic circuit consists of laminations B and is excited by the single coil C. Each pole piece is divided into two portions, one of which is encircled by a short-circuited copper band D, called a "shading coil." The eddy currents induced in these coils by the alternations of the main flux produce a magnetic reaction which causes a time phase difference between the fluxes in the two portions of the pole face, the flux in the "shaded" portion lagging by about 50° relatively to that in the unshaded portion. Thus, an irregular rotating field is produced in the air gap.

The armature or rotor A is constructed of hardened steel, either as a single disc or a pair of discs, according to the output required. Each disc is in the form of an annulus with a diametrical crosspiece as shown in Fig. 2. When two or more discs are used the crosspieces are set in line. The spindle is directly connected to a built-on speed reducing gear which is immersed in oil.

How the Synchronous Motor Operates.

The motor is self-starting on load and runs up to synchronous speed in a fraction of a second. The starting torque is due to magnetic attraction between the pole faces of the field magnets and the induced poles in the armature. On account of magnetic hysteresis in the hard steel armature disc a brief time lag exists between the induced poles in the core and the inducing flux at the pole faces. Hence, the magnetic pull between these poles is tangential to the armature core and produces torque. At low speeds there is a progressive shifting of the induced poles through the mass of the disc, but at synchronous speed the induced poles remain in step with the rotating field.

The starting torque is more than 50 per cent. of the maximum running torque and is maintained up to synchronous speed, thus giving rapid acceleration.

The synchronous operation of the motor is unaffected by considerable changes in the supply voltage, e.g., a 50 per cent. variation of voltage will not interfere with the running of the motor.

The Master Frequency Meter.

To ensure that the frequency at all the power stations is maintained rigidly at a constant average value, a master frequency meter is used at each station to indicate the average frequency. This meter measures the frequency by a direct reference to a time standard, e.g., a pendulum-type standard clock, the time-keeping of which can be accurately controlled to within a fraction of a second a day.

The Warren Master Frequency Meter.

In the master frequency meter originally developed by Warren, two hands or pointers are arranged concentrically over a common dial, which is the upper dial in Fig. 1. One hand is driven by the pendulum of the standard clock and the other is driven through gearing from a Warren synchronous motor connected to the supply mains.

The hand driven by the standard clock is enamelled black and that driven by the synchronous motor is enamelled red or some other distinctive colour.

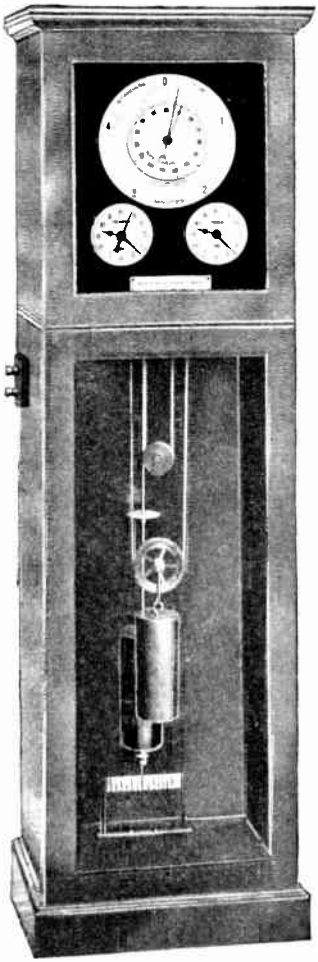


Fig. 3.—LATEST TYPE OF EVERETT EDGCUMBE ALL-ELECTRIC "SYNCLOCK" MASTER FREQUENCY METER.

With impulse driven standard clock.

The lower left-hand clock dial is driven by a synchronous motor and the right-hand clock dial is an electromagnetic impulse type clock movement.

The gearing is so arranged that at the normal supply frequency both hands complete one revolution in five minutes. Thus, if the frequency is maintained rigidly at its correct value, both hands will rotate at exactly the same speed, and will, therefore, show no *relative* change of position.

Small momentary variations in the frequency will not perceptibly affect the relative position of the hands, and as these fluctuations are equally likely to be positive and negative, only the cumulative error will be shown by a change in the relative positions of the hands. But any steady or gradual change of frequency from the mean value will cause one hand to gain upon the other, so that the relative positions of the hands will show a gradual change.

Obviously, this is the only error in the frequency which should be corrected by the engineer in charge, because the governors of the turbines will take care of the momentary fluctuations.

The two lower dials shown in Fig. 1 are ordinary clock dials, one of which (on the right) is driven by the pendulum and the other (on the left) is driven by an independent Warren synchronous motor. The left-hand dial is intended as a check on the frequency meter and acts as a stand-by in the unlikely event of failure of the supply to the motor driving the frequency meter.

Latest Type of Master Frequency Meter.

The latest type of Everett Edgcumbe master frequency meter is illustrated in

Fig. 4.—MASTER FREQUENCY METER AND CLOCK DIALS FOR OPERATION FROM STANDARD TIME SYSTEM OPERATED BY IMPULSE-TYPE MASTER CLOCK. (*Everett Edgcumbe.*)

This type of master frequency meter is used when a standard time (impulse) system is already available or when duplication of the indications of an impulse-driven clock type master frequency meter (Fig. 3) is required.

Fig. 3. Although in principle it is essentially the same as the earlier type there are important differences. Thus, the "time" hand of the frequency meter (upper dial) rotates with an even, steady movement, instead of jumping forward in half-seconds beats; the time of one revolution of the "time" hand is three minutes instead of five minutes; the "frequency" hand is shorter in length and is painted on a concentric graduated "frequency" disc which rotates synchronously with the "time" hand; the standard clock is of the electric impulse type instead of the weight-driven escapement type, i.e., the pendulum derives its energy from impulses supplied to it by an electromagnet.

The Advantages of the Impulse-driven Standard Clock.

This type of clock (which is now used in the Everett Edgcumbe master frequency meter) possesses several advantages over the ordinary escapement type. Thus it is much cheaper to manufacture owing to its simpler construction; its time-keeping is better because of the reduced friction losses owing to the elimination of the escapement mechanism; the frame can be built lighter owing to the elimination of the heavy driving weights required for the escapement type of clock; no winding is necessary; any number of clock dials giving "standard time" may be driven without affecting the time-keeping. These dials are simple electromagnetic impulse movements.

The Frequency Disc.

The frequency disc is shown in detail in Figs. 4 and 5. It has a red hand or pointer painted in a definite position on the dial, and the latter has 180 graduations representing seconds. The concentric "time" dial is similarly graduated.

How the Frequency Disc and Time Hand are Driven.

The frequency disc and the time hand are *both* driven by a Warren synchronous

Fig. 5.—MASTER FREQUENCY METER WITHOUT CLOCK DIALS. (*Everett Edgcumbe.*)

This type of master frequency meter is operated from a standard time (impulse) system, and is used for duplicating the indications of a clock type master frequency meter (Fig. 3) at the switchboard or at the turbine controls.

motor. The former is rigidly connected to the driving sleeve (which is geared to the motor), and the spindle of the time hand is driven from this sleeve through a friction clutch as shown diagrammatically in the upper part of Fig. 6. The spindle carries a star wheel having six V-shaped depressions and a roller is forced into one of these depressions every 30 seconds by an electromagnetically operated striker. The gear ratio and the number of depressions in the star wheel are such that if the frequency is correct the roller will exactly fit into the appropriate depression and no slipping of the clutch will occur. But if the frequency is high or low the roller will engage with the star wheel and force the latter into its correct position thereby causing the clutch to slip.

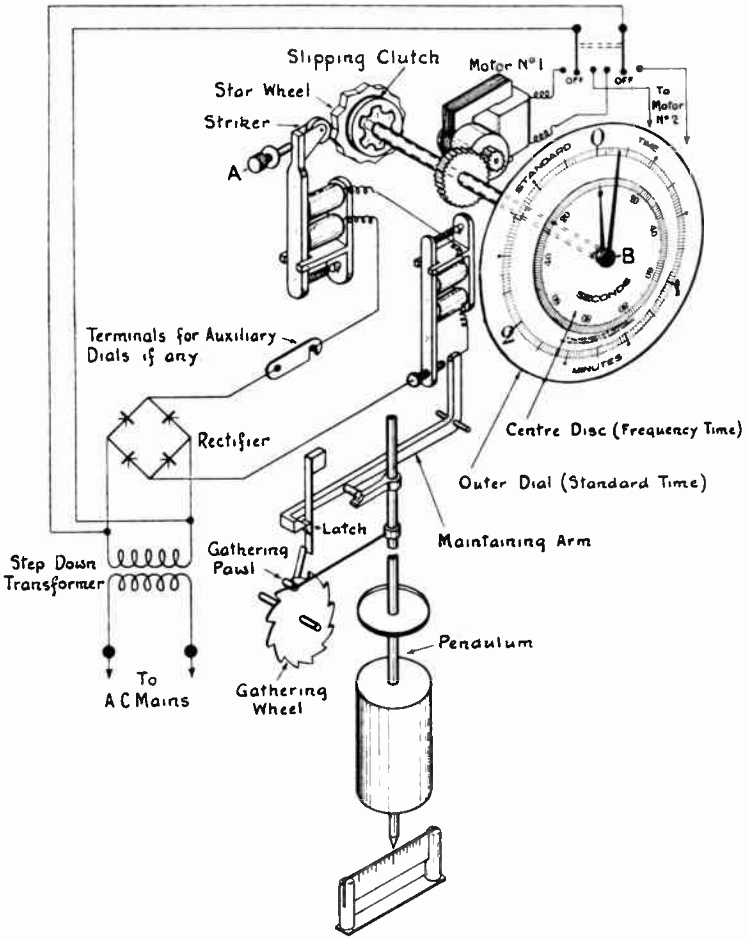


Fig. 6.—GENERAL ARRANGEMENT OF EVERETT EDGCUMBE MASTER FREQUENCY METER WITH IMPULSE-DRIVEN CLOCK (FIG. 3).

Duplicate motors with a change-over switch are provided for driving the frequency disc and time hand.

The impulse electromagnets are supplied with direct current (at about 12 volts), which is obtained from a rectifier and a step-down transformer.

Thus the clock is used only to regulate, and not to drive, the time hand. Moreover, this regulation is effected without adding any friction or other load to the pendulum.

The Mechanism of the Impulse-driven Standard Clock.

This mechanism is very simple and is of the type commonly used in electric impulse-driven clocks. The seconds invar pendulum (which is accurately compensated for temperature) is supported on knife edges so as to swing freely. The lower end of the pendulum is fitted with a pointer which moves over a graduated scale, so that the arc of swing can be observed, and just above the bob is a platform upon which weights can be placed for adjusting the time-keeping. At the upper part of the pendulum a light "gathering pawl" is fitted, which engages with the teeth of a gathering wheel in the manner shown in Fig. 6. Hence at each swing of the pendulum from left to right the pawl moves the gathering wheel through a definite arc.

How an Impulse is Imparted to the Pendulum.

At a predetermined point in the revolution of the gathering wheel a latch is tripped which releases the "maintaining arm" (or striking lever), and causes a roller attached to the latter to impart an impulse to the pendulum.

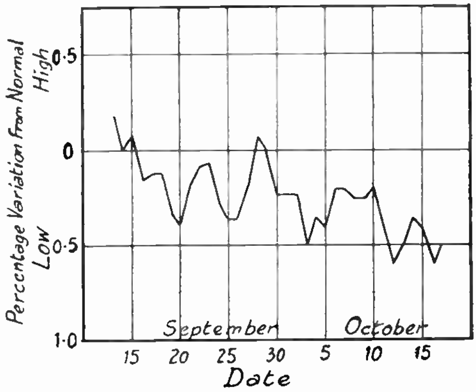


Fig. 7.—Daily Variation of Mean Frequency on Large Power System Employing Indicating Type of Frequency Meter.

When this system was changed over to frequency control on the Warren system the variation of mean frequency was zero.

Automatic Replacement of Maintaining Arm.

When the maintaining arm reaches its lowest position, a contact is closed and the circuit of an electromagnet is completed. The operation of this electromagnet automatically replaces the maintaining arm on the latch.

How the Time-keeping of the Standard Clock is Controlled.

The time-keeping is controlled by two methods. Initial adjustments are made by slight alterations to the effective length of the pendulum (e.g., by changing slightly the position of the bob on the pendulum). Final or service adjustments are made by slight alterations to the position of the effective centre of gravity of the bob, calibrated weights being added or subtracted on a platform above the bob while the pendulum is in motion. Each weight is marked with the number of seconds by which it will alter the daily rate.

How the Time-keeping of the Standard Clock is Checked.

The checking of the time-keeping of the standard clock is a very simple matter nowadays owing to the wireless transmission of time signals at definite times throughout the day from Greenwich and other Observatories. Thus all that is necessary is to install a wireless receiving set in the control room, and to check the clock against the wireless time signals. The set can be operated automatically by a time switch, which switches the set on a few seconds before the signal time and switches it off a few seconds later The time switch is driven by a synchronous motor instead of the ordinary clock spring and gearing. Such an arrangement is in use at a number of power stations.

Why the Ordinary Type of Indicating Frequency Meter cannot be used as a Master Frequency Meter.

The ordinary indicating frequency meter shows only the instantaneous value of the frequency, which may be, and usually is, fluctuating momentarily above and below the average value. When the control engineer observes the reading of the meter from time to time, the indication at a particular instant only tells him the frequency at that instant, and he is entirely unaware of whether the average frequency during the interval from the previous reading is above or below the frequency indicated by the meter. If, therefore, he makes an adjustment of the frequency according to the reading of the meter, the average frequency may be affected either adversely, or favourably, relatively to the standard frequency. The more frequently these adjustments are made the more necessary they apparently become. Hence, the average frequency over a period of 24 hours may differ appreciably from the standard value, and with such control the supply system would be useless for time service.

Moreover, the indications of the meter may not be accurate due to (1) slight inaccuracies in calibration; (2) change of calibration with use; (3) variation of temperature and voltage.

Actual Variations of Average Frequency in Practice.

That relatively large variations in average frequency actually occur in practice when an indicating frequency meter is used is shown by the reproduction in Fig. 7 of a record of the average daily frequency for a large generating station in which first-class indicating frequency meters were used and in which the operating staff endeavoured to maintain the frequency at the standard value, as indicated by the meter. The actual record of the average frequency was obtained by dividing the total number of cycles during each day (as measured by a counter and synchronous motor) by the number of seconds. Incidentally, it is of interest to note that the variation of the average frequency from the standard value is due almost entirely to variations of temperature in the frequency meters.

On the other hand, when the frequency is controlled by the master clock method, the variation of the average frequency from the standard value depends only upon the accuracy of the clock, and if the latter varies to the extent of a second a day (24 hours), the mean error in the average frequency will be less than $\frac{1}{1000}$ of one per cent.

How the Frequency is Maintained at the Required Value.

In a large power station the master frequency is installed in the control room in view of the control engineers. The meter is observed at frequent intervals, and if the "frequency" (red) hand is gaining or losing relatively to the "time" (black) hand, the speeds of the turbines are adjusted slightly so as to restore normal conditions. The adjustments are effected electrically by operating small switches in the control desk or switchboard, each of which controls a small reversible motor coupled to the governor mechanism of the appropriate turbine. The operation of this motor alters the setting of the governor. By these means the speed is raised or lowered slightly from the control room, and the governor maintains the speed constant (within the usual momentary limits) at the new value.

In stations where the speed is controlled by hand, the frequency disc and time dial shown in Fig. 5 are installed on the turbine instrument panel. The mechanism of this instrument is similar to that of the clock type master frequency meter (Fig. 3), the time hand being synchronised at half-minute intervals by impulses controlled by the clock mechanism.

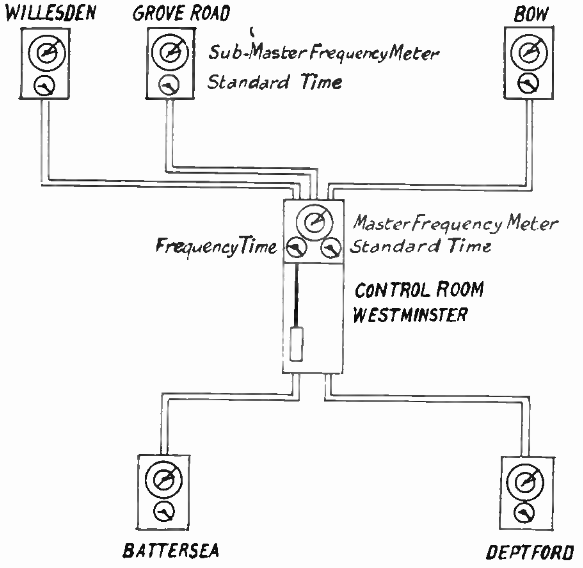


Fig. 8.—LAYOUT OF LONDON POWER CO.'S FREQUENCY CONTROL SYSTEM.

The clock type master frequency meter is installed in the central control room at Westminster, and electrically operated repeaters are installed in each of the generating stations.

In practice with large interconnected stations the average frequency over considerable time intervals, such as 30 minutes, remains sensibly constant, due, to a large extent, to the flywheel action of the generating plant and load. Hence relatively few adjustments of the frequency are necessary compared with those which would be necessary if an indicating frequency meter were used.

Applications to the Super-power Stations of the National Grid.

All the large power stations of the "National Grid" will have their frequency controlled on the Warren system. In some cases the master frequency meter is of the type illustrated in Fig. 1, but the more recent stations have the latest type of meter which is illustrated in Fig. 3. In some of the older stations which were already provided with a standard time system, the master frequency meter is of the type illustrated in Fig. 4, which consists of the three dials of the clock-type master frequency meter (Fig. 3) without the pendulum.

How the London Power Company Controls the Frequency of its Interconnected Generating Stations.

The London Power Company has a central control room at Westminster, from which the frequency of the present four generating stations, viz., Grove Road, Bow, Deptford and Willesden, is controlled. When the new station at Battersea is put into service, its frequency will also be controlled from this room. Fig. 8 is a diagram of the layout of the frequency control system.

The master frequency meter is installed in the control room, and is of the clock type. A submaster frequency meter is installed in each of the generating stations, and the clock of each of these meters is operated electrically from the standard clock in the control room; the latter sending out, through special pilot wires, a current impulse every second to each submaster clock. The dials of the submaster frequency meters, therefore, give at all times an exact replica of the frequency dial and the standard-time dial of the master frequency meter in the control room, so that each generating station has full information of the mean frequency of the system as well as of "standard time."

How the Manchester Corporation Controls the Frequency of its Interconnected Generating Stations.

At Manchester a standard time system (operated by an electric impulse standard clock) was in use prior to frequency control, and this time system is utilised to provide the "standard time" for the master frequency meter. As the standard clock sends out current impulses at half-minute intervals, these impulses cannot be directly utilised for driving the "time" hand of the master frequency meter owing to the jerky movement which would result.

When frequency control was first adopted the half-minute impulses were converted into one-second impulses, and these impulses were used for driving the "time" hand of the master frequency meter, the "frequency" hand of which was driven by a Warren synchronous motor.

In the Barton station the Everett Edgcumbe frequency disc type of master frequency meter illustrated in Figs. 4 and 5 is used, the time hand being synchronised at half-minute intervals by impulses received from the standard time system.

Acknowledgment.

Acknowledgment is due to Messrs. Everett Edgcumbe for their kindness in supplying blocks and details of their new type of master frequency meter illustrated in Figs. 3, 4, 5, 6.

HOW TO DESIGN SMALL POWER TRANSFORMERS

By H. E. J. Butler

THE design of small power transformers of outputs ranging from 10 to 1,000 volt-amperes is simplified here so that the essential data of any particular size of transformer is quickly found by reference to the tables given.

The First Consideration.

The first consideration in the design of any transformer is the output. The output is stated in so many volt-amperes, because in alternating current apparatus it is not usual to speak of an output in watts. This is because the volts multiplied by the amperes, in an alternating circuit, do not always give the true watts. For most purposes, however, the watts output of a small transformer will be the product of the volts and amperes.

Estimating the Output of a Transformer.

To estimate the output of a transformer consider a practical example. A battery charger is to have a rectifying valve requiring 45 volts at 3 amperes, and a filament winding of 1.8 volts at 5.5 amperes. The transformer is to be connected to a 230-volt, 50-cycle supply. The output of this transformer is found as follows:—

$$\begin{array}{rcr} 45 \times 3 & = & 135 \text{ volt-amperes} \\ 1.8 \times 5.5 & = & \underline{10} \text{ volt-amperes} \\ & & 145 \text{ volt-amperes.} \end{array}$$

Thus the output is determined by multiplying the volts by the amperes, and, where there is more than one output winding, the total output is found by adding together the separate outputs. It will be seen from the example given that the nature of the supply does not enter into the calculation of the output.

Core Section.

When the total output of the transformer has been determined, the next consideration is the sectional area of the core. The core of a transformer is that part of the iron on which the coils are wound. The yoke of the transformer, or that part of the iron outside the coils, must have a total cross-sectional area at least equal to the area of the core. A simple rectangular core has a uniform section, the coils being wound on one side of the rectangle. In the more usual arrangement of T—U stampings

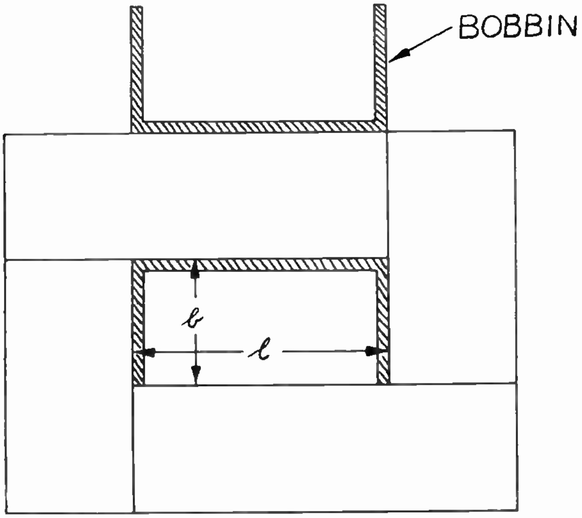


Fig. 1.—A Transformer Core Built up from Strips of Stalloy.

Each succeeding layer of strips is arranged to bridge the previous layer. The product of *l* and *b* gives the gross winding area of the transformer.

the core section is twice the area of the yokes since there are two paths for the magnetic flux from the ends of the core. Figs. 1 and 2 show the difference between the two types of cores.

A Trouble Saving Table.

The mathematical determination of the optimum area of the core is extremely complicated and the best average values are given for outputs of 10-1,000 volt-amperes in Table I. It will be seen from Table I that the sectional area depends on two factors, the output and the frequency of the supply mains. To continue with the design of the practical example, where an output of 145 volt-amperes is required, refer to Table I and choose the next largest output, which is 150 volt-amperes. If the frequency of the supply is 50 cycles, the core section will be 1.75 square inches of stalloy transformer steel. The area of the core, for a given output, is inversely proportional to the frequency of the supply. Thus the area of the core is doubled at 25 cycles, that is, 3.5 square inches for an output of 150 volt-amperes.

TABLE I.

SHOWING BEST AVERAGE VALUES OF CORE AREA FOR OUTPUTS OF 10–1,000 VOLT-AMPERES.

Output (volt-amperes).	Core Section (square inches) Stalloy.				Approx. Winding Area (square inches).
	25 Cycles.	40 Cycles.	50 Cycles.	60 Cycles.	
10	.6	.4	.3	.25	5/8
15	.8	.5	.4	.3	3/4
20	1.0	.6	.5	.4	7/8
25	1.25	.75	.6	.5	15/16
50	2.0	1.25	1.0	.8	1
75	2.5	1.5	1.25	1.0	1 1/4
100	3.0	1.75	1.5	1.25	1 3/8
125	3.25	2.0	1.75	1.5	1 1/2
150	3.5	2.25	1.75	1.5	1 5/8
175	4.0	2.5	2.0	1.75	1 3/4
200	4.0	2.5	2.0	1.75	1 7/8
250	5.0	3.0	2.5	2.0	2
300	5.0	3.25	2.5	2.5	2 1/4
350	6.0	3.5	3.0	2.5	2 3/8
400	6.0	3.75	3.0	2.5	3
450	6.5	4.0	3.25	2.75	3 1/8
500	7.0	4.5	3.5	2.75	4
750	8.0	5.0	4.0	3.5	5
1,000	9.0	6.0	4.5	3.75	5 3/4

How Many Turns ?

When the area of the core has been found from Table I, the number of turns on the various windings is estimated from Table II. To continue with the example design, look down column 1 for a core section of 1.75 and then across on this line to column 4. This gives a figure of 4 turns per volt. That is to say, that for every volt across a winding on this particular transformer there must be four turns. Take the primary winding first. This is to be connected to a 230-volt main, therefore the number of turns will be 4 × 230 = 920.

When estimating the number of secondary turns an allowance of 5 per cent. extra turns is made to compensate for the drop in volts due to the current in the windings. The 45-volt secondary winding will have, therefore, 45 × 4 × 1.05 = 189 turns. Similarly, the 1.8-volt winding has 1.8 × 4 × 1.05 = 7½ turns.

Where extreme accuracy in the voltage output is required, the resistance of the windings must be predetermined and the number of turns necessary to compensate for the drop in volts on load, or the regulation as it is called, must be calculated. An easier and more certain method is to adjust the number of secondary turns until an A.C. voltmeter gives the desired reading when connected across the winding loaded at the normal current.

The Gauge of Wire.

Table III gives the wire sizes and their current-carrying capacity at 1,500 amperes per square inch, which is a satisfactory value for continuous working. Where the current shown in column 1, Table III, does not exactly correspond with the output current, the next highest value is taken. Thus, in the practical example, the 45-volt, 3-ampere winding is wound with 17 S.W.G. D.S.C. wire, and the 1.8-volt, 5.5-ampere coil is wound with 15 S.W.G. D.S.C. wire.

Wire Covering.

The wire covering specified in Table III gives the smallest, and consequently the most efficient, coil, consistent with good insulation. It is not generally safe to wind enamel covered wire thicker than 19 S.W.G., so that double silk covering is used for the heavier sizes. Double cotton covered wire is also suitable and may be used when the necessary extra winding space is available.

TABLE II.

Giving Sizes and Quantities of Stampings for Various Core Sections.

Sectional Area of Core (square inch).	Turns per Volt.				Actual Size of Core (inches × inches).	Stampings.	
	25 Cycles.	40 Cycles.	50 Cycles.	60 Cycles.		Width of Tongue (inches).	Quantity .014 inch thick.
.25	—	—	—	23.3	$\frac{1}{2} \times \frac{9}{16}$	$\frac{1}{2}$	36
.3	—	—	23.3	19.5	$\frac{5}{8} \times \frac{9}{16}$	$\frac{5}{8}$	36
.4	—	22.0	17.5	14.6	$\frac{5}{8} \times \frac{3}{4}$	$\frac{5}{8}$	48
.5	—	17.5	14.0	11.7	$\frac{3}{4} \times \frac{3}{4}$	$\frac{3}{4}$	48
.6	23.4	14.6	11.7	9.75	$\frac{3}{4} \times \frac{7}{8}$	$\frac{3}{4}$	56
.7	20.0	12.5	10.0	8.35	$\frac{7}{8} \times \frac{7}{8}$	$\frac{7}{8}$	56
.8	17.5	11.0	8.75	7.3	$\frac{15}{16} \times \frac{15}{16}$	$\frac{15}{16}$	60
.9	15.6	9.75	7.8	6.5	1 × 1	1	64
1.0	14.0	8.75	7.0	5.85	$1 \times 1\frac{1}{8}$	1	71
1.25	11.2	7.0	5.6	4.7	$1\frac{1}{4} \times 1\frac{1}{8}$	$1\frac{1}{4}$	71
1.5	9.3	5.85	4.65	3.9	$1\frac{1}{4} \times 1\frac{3}{8}$	$1\frac{1}{4}$	88
1.75	8.0	5.0	4.0	3.34	$1\frac{1}{4} \times 1\frac{9}{16}$	$1\frac{1}{4}$	100
2.0	7.0	4.37	3.5	2.92	$1\frac{1}{2} \times 1\frac{1}{2}$	$1\frac{1}{2}$	96
2.25	6.2	3.9	3.1	2.6	$1\frac{5}{8} \times 1\frac{5}{8}$	$1\frac{5}{8}$	104
2.5	5.6	3.5	2.8	2.34	$1\frac{5}{8} \times 1\frac{3}{4}$	$1\frac{5}{8}$	112
2.75	5.1	3.2	2.5	2.13	$1\frac{3}{4} \times 1\frac{3}{4}$	$1\frac{3}{4}$	112
3.0	4.7	2.9	2.33	1.95	$1\frac{7}{8} \times 1\frac{3}{4}$	$1\frac{7}{8}$	120
3.25	4.3	2.7	2.16	1.8	$2 \times 1\frac{7}{8}$	2	120
3.5	4.0	2.5	2.0	1.67	2 × 2	2	128
3.75	3.74	2.34	1.86	1.56	$2 \times 2\frac{1}{8}$	2	136
4.0	3.5	2.2	1.75	1.46	$2 \times 2\frac{1}{4}$	2	144
4.5	3.1	1.95	1.55	—	$2\frac{1}{4} \times 2\frac{1}{4}$	$2\frac{1}{4}$	144
5.0	2.8	1.75	—	—	$2\frac{1}{4} \times 2\frac{1}{2}$	$2\frac{1}{4}$	160
5.5	2.54	1.59	—	—	$2\frac{1}{2} \times 2\frac{1}{2}$	$2\frac{1}{2}$	160
6.0	2.34	1.46	—	—	$2\frac{1}{2} \times 2\frac{3}{4}$	$2\frac{1}{2}$	176
6.5	2.15	—	—	—	$2\frac{3}{4} \times 2\frac{3}{4}$	$2\frac{3}{4}$	176
7.0	2.0	—	—	—	$2\frac{3}{4} \times 2\frac{7}{8}$	$2\frac{3}{4}$	184
7.5	1.97	—	—	—	$2\frac{7}{8} \times 3$	$2\frac{7}{8}$	192
8.0	1.75	—	—	—	3 × 3	3	192
9.0	1.56	—	—	—	$3\frac{1}{8} \times 3\frac{1}{4}$	$3\frac{1}{8}$	213

It must be remembered that when enamel insulated wire is used for coils consisting of more than two or three layers, paper insulation between the layers is necessary. This is because enamel insulation provides no resilience which is necessary to allow the winding to expand as it warms up on load.

Primary Current.

Before the gauge of wire for the primary or mains winding can be settled, it is necessary to calculate the primary current for full load. This is found by dividing the mains voltage into the output watts or volt-amperes. In the example under consideration the primary current is $\frac{145}{230} = .63$ ampere, at 100 per cent. efficiency. A transformer of this size would have an efficiency of about 92 per cent., so that the primary current would actually be .63 × 1.09 = .69 amperes. It would be permissible to use 23 S.W.G. wire for this current, since the extra, .01 ampere, is too small to matter. The efficiency of the transformers dealt with here would range from 80 per cent. in the smallest sizes to 96 per cent. in the largest.

Winding Area.

When the number of turns and the gauges of the wires to be used have been found, the next step is to calculate the area of the opening, or window, in the stampings which the winding will occupy. The values of winding area given in Table I may be taken as a rough guide. Since the winding area varies considerably with the gauges of wires used in the coils it is not possible to fix a definite value

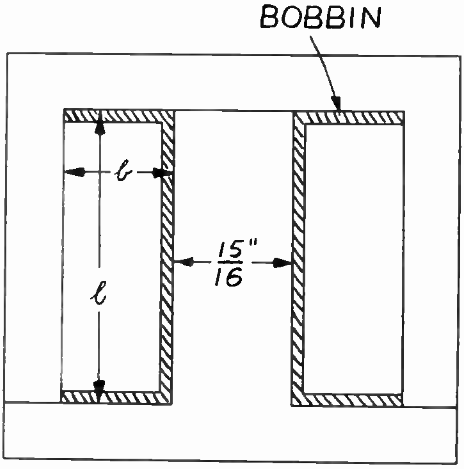


Fig 2.—A TRANSFORMER CORE WITH BOBBIN IN POSITION BUILT FROM STANDARD T-U STAMPINGS.

In the example discussed in the article, $l=2\frac{3}{16}''$ and $b=\frac{7}{8}''$. The product of l and b gives the winding area for this type of core.

for each size of transformer. It is advisable, therefore, to check the value given in Table I to ensure that it is not too small.

How to Check the Winding Area.

For the transformer which is being considered as an example, this is done as follows:—

The primary winding has 920 turns of 23 S.W.G. enamel covered wire. From Table III it is seen that this gauge of wire winds 1,510 turns per square inch. The area occupied by the primary is therefore :

$$\frac{920}{1510} = .61 \text{ square inch.}$$

Similarly, the 45-volt winding, which has 189 turns of 17 S.W.G. double silk covered wire, has a winding area of :—

$$\frac{189}{272} = .7 \text{ square inch.}$$

The 1.8 volt winding, which will occupy less than one layer will take up about .09 square inch if the length of the winding space is 1 inch. The total winding area is now :—

$$.61 + .7 + .09 = 1.4 \text{ square inches.}$$

It is now necessary to add about 20 per cent. allowance for the insulation between the layers, the space occupied by the bobbin and insulation between the windings. The area now becomes :—

$$1.4 \times 1.2 = 1.68 \text{ square inches.}$$

The Size of Stampings.

When the area of the core has been found from Table I and the winding area has been calculated as in the previous paragraph, it is now possible to determine the size of stampings.

The best shape for the core would be cylindrical, but as this is not practicable with flat stampings, the next best shape, square, is used whenever possible. Table II gives the sizes and quantities of stampings for various core sections within the range of transformers under consideration.

It is not always possible to find standard stampings which will give a square core for any particular area, so that one of two alternatives are adopted.

The first is to make the core from strips of stalloy of the width shown in column 7, Table II. The other way is to make the core section rectangular by using stampings with a narrower tongue than is specified.

Suppose for the example transformer it is decided to use the stampings shown in Fig. 2 as these are a standard product

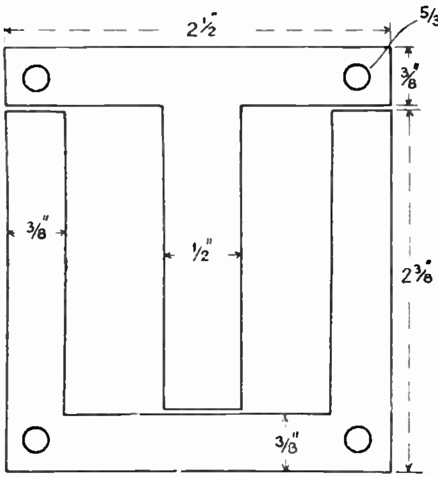
TABLE III.

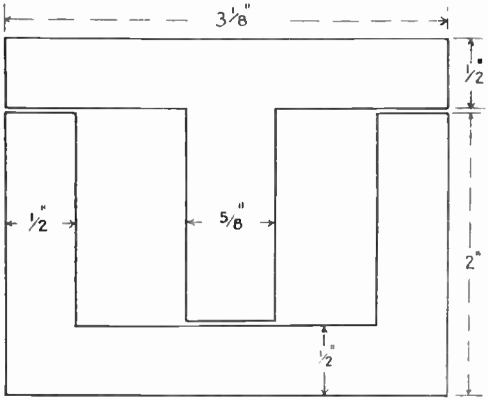
SHOWING WIRE SIZES AND THEIR CURRENT-CARRYING CAPACITY AT 1,500 AMPERES PER SQ. IN.

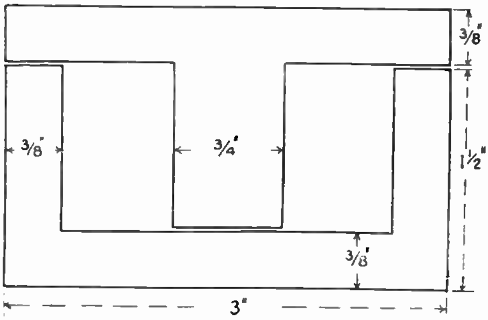
Current Amperes.	Wire Size S.W.G.	Wire Covering.	Turns per Square Inch.
19.3	10	Double Silk	57
15.8	11	,,	69
12.7	12	,,	85
9.9	13	,,	108
7.5	14	,,	139
6.1	15	,,	172
4.8	16	,,	213
3.7	17	,,	272
2.7	18	,,	376
1.9	19	Enamel	560
1.5	20	,,	680
1.2	21	,,	865
.92	22	,,	1,110
.68	23	,,	1,510
.57	24	,,	1,775
.47	25	,,	2,120
.38	26	,,	2,560
.32	27	,,	3.120
.26	28	,,	3.760
.22	29	,,	4.390
.18	30	,,	5.380
.158	31	,,	6,060
.137	32	,,	6 890
.118	33	,,	7.900
.100	34	,,	9,610
.083	35	,,	11,250
.068	36	,,	13.450
.054	37	,,	16.400
.042	38	,,	20.400
.032	39	,,	28,250
.027	40	,,	32,450

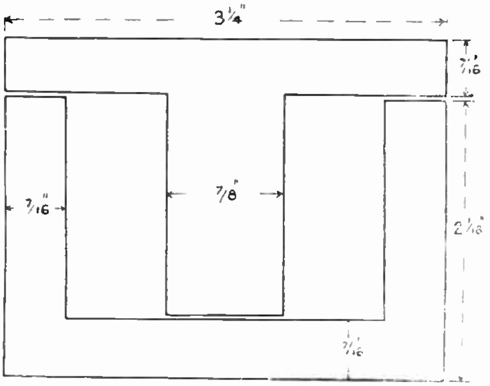
DIMENSIONS OF TYPICAL "STALLOY" LAMINATIONS FOR TRANSFORMERS AND CHOKES.

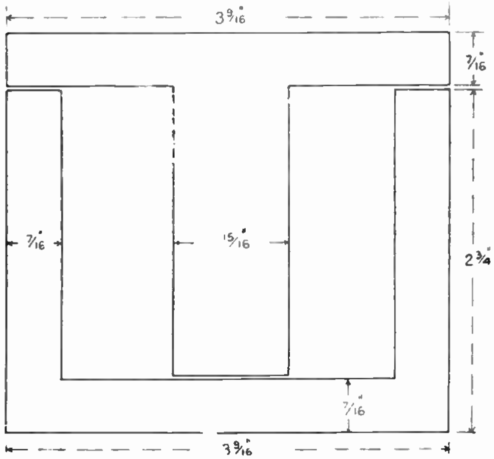
(The Standard Thickness of "Stalloy" is .014".)

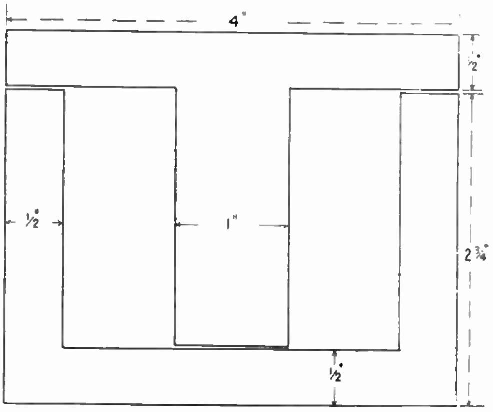




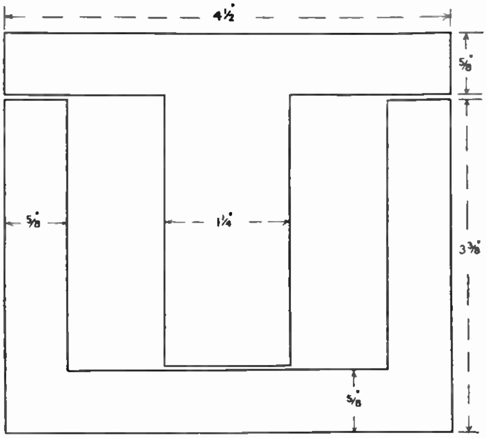


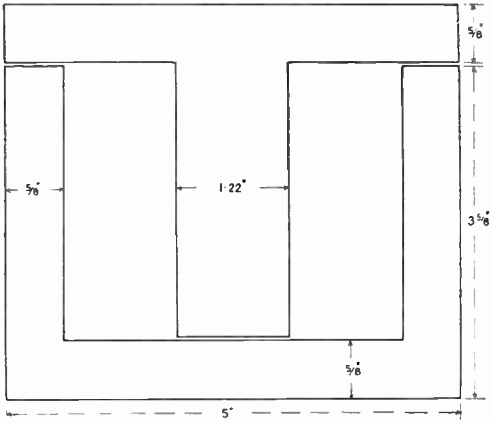


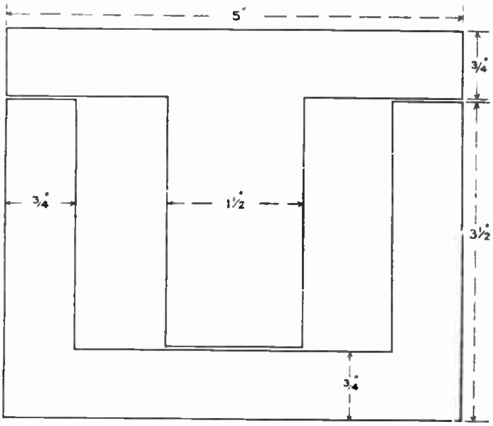




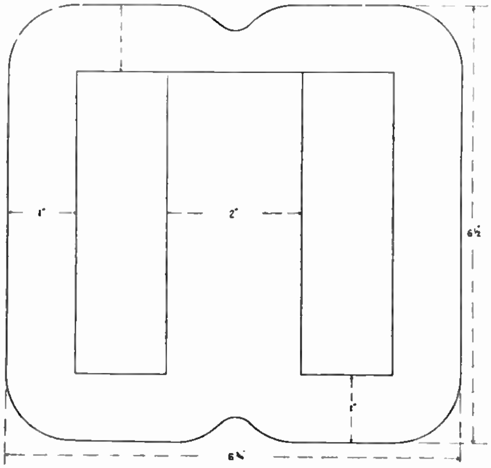
DIMENSIONS OF "STALLOY" LAMINATIONS—(*continued*).

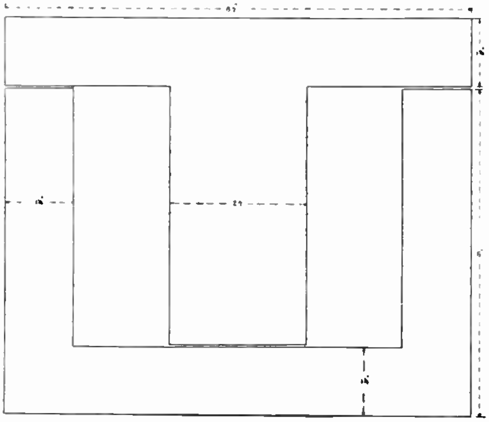






The laminations shown on these two pages are selected from the large range of sizes stocked by Joseph Sankey & Sons, Ltd., Bilston.





easily obtainable. The width of the core is $\frac{13}{16}$ inch and the thickness of the stampings is .014 inch, so that the quantity of the stampings is given by :—

$$\frac{A}{w} \times 71.4$$

where A is the area of the core and w the width of the tongue of the stampings. This works out at 133 stampings in the present example.

The thickness occupied by the stampings is found from :—

$$\frac{A}{w} \times 1.11$$

This gives a thickness of 2.07 inches for the example under consideration. The actual size of the core is therefore $\frac{13}{16}$ inch × 2.07 inches. A bobbin with a hole 1 inch × $2\frac{1}{8}$ inches would be quite suitable to accommodate this.

Centre-tapped Windings.

A full wave valve rectifier requires a centre-tapped winding of approximately twice the total voltage of the rectifier output voltage. A 500-volt D.C. rectified output from a valve rectifier would require a 1,000-volt centre-tapped H.T. winding on the transformer. This winding, however, does not carry the full current. Each half of the winding carries half the value of the current output of the rectifier. Suppose a rectifier is to have an output of 500 volts, 100 nilliamperes. The H.T. winding on the transformer is then wound for 1,000 volts, 50 milliamperes, with a centre-tapping.

Useful Formulæ.

The following formulæ are useful in connection with the design of power transformers :—

If v = Voltage of winding
f = Frequency of supply mains
T = Total turns of winding
A = Area of core transformer in square inches
B = Maximum flux in core (lines per square inch),

then :—

$$v = \frac{4.44 f\ T\ A\ B}{10^8}$$

A satisfactory value for B, when stalloy is used, is 65,000 lines per square inch.

If t = turns per volt, the above formula may be simplified to :—

$$t = \frac{14}{A} \text{ for 25 cycles.}$$

$$t = \frac{8.75}{A} \text{ for 40 cycles.}$$

$$t = \frac{7}{A} \text{ for 50 cycles.}$$

$$t = \frac{5.85}{A} \text{ for 60 cycles.}$$

If t = turns per volt
$v\ I$ = output in volt-amperes,

then the approximate winding area is given by :—

Winding area = $t v\ I \times .00254$ sq. in., or :—

$$v\ I \text{ (output)} = \frac{\text{Winding area} \times 394}{t}$$

From the last formula it is possible to estimate the approximate output obtainable from a particular assembly of stampings.

Calculating Resistance of a Transformer Winding.

The resistance of a transformer winding is calculated from the following formula :—

If l = Length of mean turn inches.
T = Total turns of winding.
r = Resistance of wire per yard in ohms.
R = Resistance of coil in ohms, then

$$R = \frac{l \times T \times r}{36}$$

Finding the Length of the Mean Turn.

The length of the mean turn may be found in two ways. The first is to set out the section of the winding to scale and measure it. A quicker way is to measure the length of a turn in the first layer before commencing the winding, then wind on the calculated number of turns. Now measure the length of a turn in the final layer. Half the sum of the two values obtained gives the length of the mean turn.

A.C. CIRCUIT CALCULATIONS

By A. T. DOVER, M.I E.E.

How Alternating Currents and E.M.F.'s differ from Direct Currents and E.M.F.'s.

ALTERNATING e.m.f.'s and currents are *periodic quantities*, i.e., their values change, both in *magnitude* and *direction* (or sign), from instant to instant according to a definite law, and any particular value is repeated at regular intervals. In practice one complete set of changes of e.m.f. or current occurs during $\frac{1}{50}$ second. Thus, the e.m.f. or current flows in one direction for $\frac{1}{100}$ second, during which time its value changes from zero to a maximum and again to zero; the e.m.f. or current then flows in the opposite direction for the next $\frac{1}{100}$ second and the previous set of changes are repeated in the reverse direction.

Cycle, Period and Frequency.

The complete set of changes between two corresponding successive values in the *same* direction (e.g., two successive positive maximum values) is called a *cycle*, and the time during which these changes occur is called a *period*. The number of cycles per second is called the *frequency*. Hence in the above case the frequency is 50 cycles per second.

How Alternating E.M.F.'s and Currents are Represented Graphically.

In ordinary calculations of alternating-current circuits the e.m.f. and current are considered to vary according to a *sine* law, and are represented graphically by sine curves as shown in Fig. 1. The period—which in Fig. 1 is $\frac{1}{50}$ second—is considered as equivalent to 360° [or 2π (i.e., 6.28) radians] and the number of degrees, or radians, corresponding to a given instant in the period is called the *time angle*.

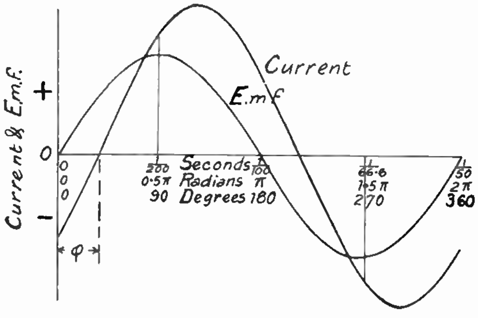


Fig. 1.—GRAPHIC REPRESENTATION OF ALTERNATING CURRENT AND E.M.F. VARYING ACCORDING TO A SINE LAW, AND HAVING A PHASE DIFFERENCE ϕ.

The horizontal axis represents the time-angle, and is proportional to time.

How Sine Quantities are Calculated.

With quantities varying according to a sine law the value of the quantity at any instant is proportional to the sine of the corresponding time-angle; the sine of an angle being the trigonometrical ratio (height/hypotenuse) of a right-angled triangle constructed with this angle at the base. Numerical values of the sines of certain angles are given in Table I.

Thus the e.m.f. represented by the sine curve in Fig. 1 is zero when the time-angle is 0°, 180°, 360°, etc., the corresponding times in seconds (measured from zero time) being 0, $\frac{1}{100}$, $\frac{1}{50}$, etc. The positive maximum value of the e.m.f. occurs when the time-angle is 90° (the corresponding time being $\frac{1}{200}$ second) and the negative maximum value occurs when the time angle is 270° (the corresponding time being $\frac{1}{66.6}$ second). Subsequent positive maximum values occur when the time-angle is 450°, 810°, etc., and subsequent negative maximum values occur when the time-angle is 630°, 990°, etc.

How the Instantaneous Value of the E.M.F. is Calculated.

If we wish to know the value of the e.m.f. at any particular instant, we calculate the corresponding time-angle and obtain the value of the sine

TABLE I.—VALUES OF SINES OF ANGLES.

Angle.		Sine.	Angle.		Sine.	Angle.		Sine.	Angle.		Sine.
Degrees.	Radians.		Degrees.	Radians.		Degrees.	Radians.		Degrees.	Radians.	
0	0	0	90	1.57	+1.0	180	3.14	0	270	4.71	−1.0
15	0.262	+0.259	105	1.83	+0.966	195	3.4	−0.259	285	4.97	−0.966
30	0.524	+0.5	120	2.09	+0.866	210	3.66	−0.5	300	5.23	−0.866
45	0.785	+0.707	135	2.35	+0.707	225	3.92	−0.707	315	5.49	−0.707
60	1.047	+0.866	150	2.62	+0.5	240	4.19	−0.866	330	5.76	−0.5
75	1.309	+0.966	165	2.88	+0.259	255	4.45	−0.966	345	6.02	−0.259
90	1.57	+1.0	180	3.14	0	270	4.71	−1.0	360	6.28	0

from Table I or a complete table of sines. This quantity will then represent the value of the e.m.f. in terms of its maximum value.

For example, if the particular instant is $\frac{1}{1000}$ second, the time angle is $\frac{1}{1000} \times (360 \div \frac{1}{50}) = 18^\circ$, and the sine of this angle (obtained from a table of sines) is $+0.309$. Hence at $\frac{1}{1000}$ second the e.m.f. is 0.309 of its positive maximum value. Again, if the value of the e.m.f. is required at the instant 0.012 second, the time angle is $0.012 \times (360 \div \frac{1}{50}) = 216^\circ$, and the sine of this angle (obtained from a table of sines) is -0.573. Hence at this instant the e.m.f. is 0.573 of its *negative* maximum value.

Root-Mean-Square (R.M.S.) Value.

The root-mean-square value of an alternating e.m.f. or current is the square root of the mean value of the *squared* ordinates over half a period. For example, in Fig. 2, curve A is a sine curve and curve B has ordinates which are proportional to the squares of the corresponding ordinates of curve A, i.e., at the instant t, when the ordinate of curve A is represented by e, the ordinate of curve B is represented by e^2. If now a rectangle is drawn with the base equal to the half-period and its area is equal to the area enclosed by a half-period of curve B, then the height of the rectangle will represent the mean height or the mean value of curve B. Hence the square root of the height of the rectangle will represent the R.M.S. value of the sine curve A.

Why R.M.S. Values are Used in Practice.

R.M.S. values of alternating e.m.f.'s and currents are used in all calculations because the quantitative effect (electric stress, magnetic, heating) produced by an e.m.f. or current of given R.M.S. value is the same as that produced by a direct or steady e.m.f. or current of this value. Thus an alternating current of 10 amperes R.M.S. value produces the same heating effect in a given resistance as a direct current of 10 amperes. Hence hot-wire ammeters and voltmeters calibrated on a direct-current circuit will read R.M.S. values of alternating currents and e.m.f.'s. Similar results are obtained with modern electromagnetic (moving-iron) ammeters and voltmeters, and also with electrostatic voltmeters.

R.M.S. Value of Sine Curve.

The R.M.S. value of a sine curve is equal to

$$\text{maximum value} \div \sqrt{2}$$
$$= 0.707 \times \text{maximum value} \quad \ldots\ldots\ldots(1)$$

This result can be deduced from Fig. 2, for the height of the rectangle is one-half of the maximum height of curve B, since the four shaded areas C are equal to one another.

Phase Difference.

When two sine quantities of the same frequency are not "in step" with each other (i.e., their zero or maximum values do not occur at the same instant), the time angle between their zero values is called phase difference. This quantity is usually denoted by the Greek letter φ (*phi*). Phase difference is a relative term, and to avoid ambiguity, one of the alternating quantities must be regarded as a reference quantity. The phase difference between this quantity (e.g., an e.m.f.) and a second quantity (e.g., a current) is then leading or lagging according to whether the zero value of the second quantity (e.g., current) occurs before or after the corresponding zero value of the reference quantity (e.g., e.m.f.).

With sine quantities of the same frequency, the phase difference is constant.

How Vectors are Used to Represent Alternating Quantities.

In the calculation of A.C. circuits we are chiefly concerned with the R.M.S. values and phase differences of the e.m.f.'s and currents.

The R.M.S. values of sine quantities of the same frequency may be represented graphically by straight lines (called *vectors*) drawn, in the same plane, from a given point or origin. The *length* of each vector represents the *magnitude* (R.M.S. value) of the e.m.f. or current to a particular scale.

The phase differences are represented by the *angles* between each vector and a reference vector. An angle measured in the *counter-clockwise* direction denotes a *leading* phase difference, and an angle measured in the *clockwise* direction denotes a *lagging* phase difference. Fig. 3 shows how lagging and leading currents are represented by vectors.

Addition of Vectors.

Two vectors not in the same straight line are added *geometrically* by constructing a parallelogram on the vectors and drawing the diagonal from the origin. Then the *length of this diagonal represents the sum of the vectors.*

For example, in Fig. 4 the two vectors are represented by O A and O B, lagging by the angles φ_1, φ_2, respectively. To obtain their sum, complete the parallelogram O A C B by drawing A C parallel to O B, and B C parallel to O A. Join O and C. Then the length of O C measured to the same scale as O A and O B represents the sum of the vectors O A and O B.

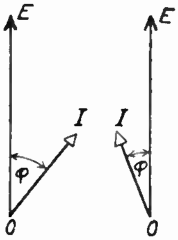


Fig. 3.—Showing How a Lagging (Left) and a Leading (Right) Current are Represented by Vectors.

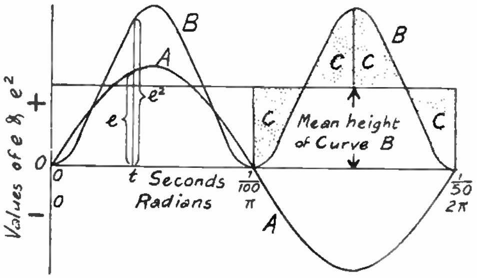


Fig. 2.—Showing How the R.M.S. Value of an Alternating E.M.F. (A) is Obtained. The curve B is drawn with ordinates proportional to the squares of the corresponding ordinates of A. The rectangle encloses an area equal to that enclosed by B. The four shaded areas C are equal to one another.

When three or more vectors are to be dealt with we construct a polygon, the several sides of which are equal and parallel to the several vectors taken in order. The closing line of the polygonthen represents the sum of the vectors.

Thus in Fig. 5 three vectors are represented by O A, O B, O C, lagging by the angles φ_1, φ_2, φ_3, respectively. The polygon O A D F is constructed on the vector O A, the sides A D, D F being drawn equal to and parallel to the vectors O B, O C, respectively. The closing side O F represents the sum of the vectors O A, O B, O C; and the angle φ between O E and the reference axis is the phase difference.

Application of Vectors to the Calculation of Parallel Circuits.

Problems relating to currents in parallel circuits are easily solved by parallelogram and polygon of vectors. Thus if the currents in the branch circuits are given, the supply or line current is obtained by determining the vector sum of the branch-circuit currents. For example, in Fig. 4 the vectors O A, O B may be considered to represent the currents in two branches of a simple parallel circuit, the reference axis containing the vector (O E) representing the line voltage. Hence the vector O C represents the line current, and the angle φ represents the phase difference between this current and the supply voltage.

To obtain an accurate result the diagram must be drawn to a large scale and the vectors must be set out carefully.

Numerical Examples on Currents in Parallel Circuits.

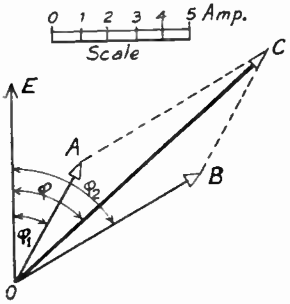


Fig. 4.—Showing How the Sum of Two Currents is Obtained.

A parallelogram is constructed and the diagonal is drawn through the origin.

(1) *A parallel circuit consists of two branches. The current in one branch (A) is 5 amperes and its phase difference is 30° (lagging) relative to the supply voltage. The current in the other branch (B) is 8 amperes and its phase difference is 60° (lagging) relative to the supply voltage. What is the supply current and its phase difference?*

Solution.—This problem is easily solved graphically by vectors. A sheet of paper (about 10-in. × 8-in. minimum), a sharp-pointed pencil, a scale (divided into inches and tenths, or centimetres and millimetres), and a protractor are required.

Proceed as follows: Near one edge of the paper draw a line to represent the reference axis, O E, Fig. 4. Select a convenient scale for the current vectors, e.g., 0.5 inch (or 1 centimetre, if a metric scale is used) to represent 1 ampere. Draw O A and O B to represent the currents in the branches A and B. O A is $\frac{1}{2}\times 5=2\frac{1}{2}$ in. long, and is drawn at an angle of 30° (in the clockwise direction) to the reference axis. O B is $\frac{1}{2}\times 8=4$ in. long, and is drawn at an angle of 60° (in the clockwise direction) to the reference axis. From A draw A C parallel to O B, and from B draw B C parallel to O A. These lines intersect at C. Join O C and measure carefully its length (which should be 6.3 in.) and inclination to the reference axis (which should be 48°). Hence, supply current

$$= \frac{\text{length of O C}}{\text{scale (i.e. inches per ampere)}}$$

$$= \frac{6.3}{0.5} = 12.6 \text{ amp.}$$

Phase difference = 48° (lagging).

(2) *A parallel circuit consists of two branches A, B. The current in A is 10 amperes and its phase difference relative to the supply voltage is 30° (lagging). The supply current is 25 amperes and its phase difference is 48° (lagging). What is the current in branch B?*

Solution.—This problem is solved graphically as follows:—

Draw the reference vector O E, and the vectors O A and O C to represent the currents in the branch A and the supply respectively. A suitable scale would be $\frac{1}{4}$ in. to represent 1 ampere. Join A and C. From O draw O B parallel to A C, and from C draw C B parallel to O A, these lines intersecting at B. Then O B represents the current in the branch B. Fig. 4 shows the completed vector diagram.

By measurement, O B = 4 in., and its inclination to the reference axis is 60°. Hence, current in branch B

$$= \frac{4}{0.25} = 16 \text{ amp.}$$

Phase difference = 60° (lagging).

Resistance of Conductors Carrying Alternating Currents.

When the current is uniformly distributed over the cross section of the conductors the resistance of an A.C. circuit is calculated in the same manner as that of a D.C. circuit. But with large conductors carrying heavy currents, or with magnetic conductors, the magnetic effect of the current causes a concentration of the current to the outer portions of the conductors. This is called the *skin effect*. In these cases the conductors offer a greater resistance to alternating currents than to direct currents. The increased resistance due to the non-uniform distribution of the current is difficult to calculate, as it depends upon the frequency, the cross section and the permeability of the conductor. Tables and formulæ are given in the larger electrical engineering pocket books. For the conductors used in ordinary electric lighting and power circuits the skin effect is negligible, and the resistance of these circuits is calculated in the usual manner.

Reactance of an A.C. Circuit.

When a circuit contains coils or other apparatus in which the current produces an alternating flux which links with the conductors, the self-induced e.m.f. due to this linkage has the effect of causing the current in the circuit to be smaller than that calculated by simply dividing the supply voltage by the resistance of the circuit. This additional opposition to the current (which is not due to resistance) is called *reactance*, and is calculated by the formula :—

Reactance = 2 π × frequency × inductance (henries).

or $$X = 6.28 \times f \times L \quad \ldots\ldots\ldots\ldots (2)$$

where f is the frequency and L is the inductance. For 50 cycles

$$X = 314\ L \quad \ldots\ldots\ldots\ldots\ldots\ldots (3)$$

The inductance (L) is a physical property of the circuit, and its value depends upon the dimensions of the coils, the *square* of the number of turns, and the permeability. It is, therefore, difficult to calculate.

What Occurs when a Condenser is Connected to an Alternating E.M.F.

When a condenser is connected to an A.C. supply it is continually being charged and discharged. A charge and discharge take place at each half-cycle of the e.m.f. Hence with a 50-cycle supply the condenser is charged 100 times a second and is discharged the same number of times. This rapid charging and discharging produce an alternating current in the supply system. This current is called the *charging current* of the condenser.

The charging current does not pass through the dielectric, as does the leakage current (due to an imperfect dielectric). Moreover, the charging current has a phase difference of 90° (leading) with respect to the supply voltage, whereas the leakage current is in phase with the supply voltage and is, of course, calculated by Ohm's Law.

The charging current is calculated from the formula :—

Charging current
= 2 π × frequency × capacity (farads) × supply voltage,

or $$I_c = 6.28 \times f \times C \times E \ldots (4)$$

Condensive or Capacitive Reactance.

From formula (4) we obtain

$$\frac{E}{I_c} = \frac{1}{6.28 \times f \times C} \quad \ldots\ldots\ldots (5)$$

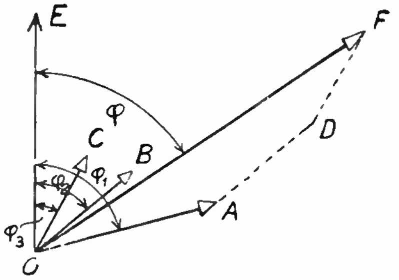


Fig. 5.—SHOWING HOW THE SUM OF THREE CURRENTS IS OBTAINED.
In this case a polygon is constructed

This quantity—1/(6.28 f C)—is called the *reactance* of the condenser.

Usually the capacity is expressed in microfarads (1 microfarad = one-millionth of a farad). Hence if C is expressed in microfarads, the formula for condensive reactance is

$$X_c = \frac{1{,}000{,}000}{6.28\,f\,C} = \frac{10^6}{6.28\,f\,C} \quad \ldots (6)$$

or for 50-cycle supply

$$X_c = \frac{10^6}{314\ C} \quad \ldots\ldots\ldots\ldots\ldots\ldots (7)$$

Impedance of an A.C. Circuit.

The ratio (applied volts/amperes) in an A.C. circuit is called *impedance*, and is denoted by Z.

In general impedance is a compound quantity, comprising resistance and reactance (which may be inductive or condensive, or a combination of each). The resistance and reactance of a simple series circuit must be added *geometrically* to obtain the impedance, because in a simple series circuit containing resistance, inductance and capacity the line voltage is distributed across the several parts of the circuit in the following manner :—

Voltage across resistance :—
$R\ I$, in phase with the current ;
Voltage across inductance :—
6.28 f L I, phase difference 90° (*leading*) with respect to the current ;
Voltage across condenser :—
$I \times 10^6 / (6.28\ f\ C)$, phase difference 90° (*lagging*) with respect to the current.

These voltages, together with the line

voltage and the current, are shown in the vector diagram of Fig. 6, the current vector being the vector of reference.

Hence the impedance may be represented graphically by the hypotenuse of a right-angled triangle, as shown in Fig. 7, of which the two sides containing the right angle represent the resistance and resultant reactance.

Formulæ for Impedance.

The following formulæ are used in calculating the impedance of series circuits :—

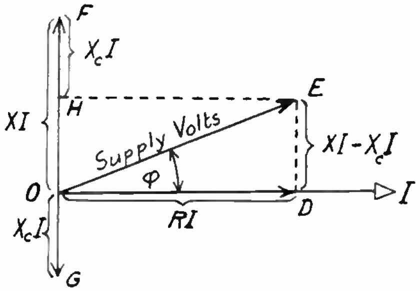


Fig. 6.—Vector Diagram for a Series Circuit Containing Resistance, Inductance and Capacitance.

(1) Resistance and inductance :—

$$Z = \sqrt{(R^2 + X^2)} \quad \ldots\ldots\ldots\ldots\ldots (8)$$

$$\text{or } Z = \sqrt{[R^2 + (6.28\, f\, L)^2]} \quad \ldots\ldots\ldots\ldots (9)$$

(2) Resistance and capacitance :—

$$Z = \sqrt{(R^2 + X_c^2)} \quad \ldots\ldots\ldots\ldots\ldots (10)$$

$$\text{or } Z = \sqrt{[R^2 + (10^6/6.28\, f\, C)^2]} \quad \ldots\ldots (11)$$

(3) Resistance, inductance and capacity:—

$$Z = \sqrt{[R^2 + (X - X_c)^2]} \quad \ldots\ldots\ldots\ldots (12)$$

$$\text{or } Z = \sqrt{\left[R^2 + \left(6.28 f L - \frac{10^6}{6.28\, f\, C}\right)^2\right]} \quad \ldots\ldots\ldots\ldots (13)$$

Calculation of Current in a Simple Series Circuit.

The current in a simple series circuit or one of the branches of a parallel circuit is calculated by first calculating the impedance, using the appropriate formula (8), (10), or (12), and then dividing this impedance into the supply voltage, i.e. :—

Current = supply voltage/impedance,

$$\text{or } I = E/Z \quad \ldots\ldots\ldots\ldots\ldots\ldots (14)$$

in all cases.

The phase difference (φ) between the supply voltage and current is obtained by first calculating cos φ from formula (15), and then determining φ from a table of cosines.

Thus :—

$$\cos\varphi = \text{resistance/impedance}$$
$$= R/Z \quad \ldots\ldots\ldots\ldots\ldots\ldots (15)$$

Numerical Examples on Simple Series Circuits.

(1) *A series circuit has a resistance of* 30 *ohms and an inductance of* 0.15 *henry. What will be the current and its phase difference when the circuit is connected to* 230-*volt* 50-*frequency supply mains?*

Solution.—The reactance is calculated by using formula (3). Thus :—

$$X = 314 \times 0.15 = 47.1 \text{ ohms.}$$

The impedance is calculated by using formula (8). Thus :—

$$Z = \sqrt{(30^2 + 47.1^2)} = 55.8 \text{ ohms.}$$

Whence, from formula (14), the current is :—

$$I = 230/55.8 = 4.12 \text{ amp.,}$$

and from formula (15) :—

$$\cos\varphi = 30/55.8 = 0.538.$$

Referring to a table of cosines, we obtain φ = 70.2°.

Hence the phase difference is 70.2° (lagging).

(2) *Calculate the charging current of a* 5-*microfarad condenser when it is connected to a* 230-*volt* 50-*cycle supply.*

Solution.—From formula (4) the charging current is :—

$$I_c = \frac{314 \times 5 \times 230}{1{,}000{,}000} = 0.362 \text{ amp.}$$

(3) *If the condenser in example* (2) *has a dielectric of low insulation resistance, say* 5,000 *ohms, what is the leakage current, and what is the current taken from the supply?*

Solution.—The leakage current is calculated by Ohm's Law. Thus :—

$$I_d = \frac{230}{5{,}000} = 0.046 \text{ amp.}$$

This current is in phase with the supply voltage, and the charging current leads the supply voltage by 90°, i.e., the two currents have a phase difference of 90°. Hence the current taken from the supply is :—

$$I = \sqrt{(I_c^2 + I_d^2)} = \sqrt{(0.362^2 + 0.046^2)}$$
$$= 0.365 \text{ amp.}$$

The phase difference of this current is obtained by calculating cos φ from the formula :—

$$\cos\varphi = \frac{\text{leakage current}}{\text{supply current}} = \frac{I_d}{I}$$

$$= \frac{0.046}{0.365} = 0.0126.$$

Whence, from a table of cosines, $\varphi = 89.3^\circ$ (leading).

(4) *A series circuit consists of a non-inductive resistance of* 100 *ohms and a condenser of* 15 *microfarads. The supply voltage is* 230, *and the frequency is* 50 *cycles per second. What is the supply current, its phase difference, the voltages across the resistance and condenser?*

Solution.—The reactance of the condenser is calculated from formula (7). Thus :—

$$X_c = \frac{10^6}{314\times15} = 212 \text{ ohms.}$$

Therefore the impedance of the circuit, calculated from formula (10), is :—

$$Z = \sqrt{(100^2 + 212^2)} = 234.5 \text{ ohms.}$$

Whence, from formula (14), the current is :—

$$I = 230/234.5 = 0.981 \text{ amp.,}$$

and from formula (15)

$$\cos\varphi = 100/234.5 = 0.426.$$

Referring to a table of cosines, we obtain $\varphi = 64.8^\circ$.

Hence the phase difference is 64.8° (leading).

The voltage across the resistance is equal to :—

$$IR = 0.981 \times 100 = 98.1 \text{ volts,}$$

and is in phase with the current.

The voltage across the condenser is equal to :—

$$IX_c = 0.981 \times 212 = 208 \text{ volts,}$$

and lags 90° relatively to the current.

As a check on the calculations, the square root of the sum of the squares of these voltages should equal the supply voltage. Thus :—

$$\sqrt{(98.1^2 + 208^2)} = 230.$$

(5) *A series circuit consists of an inductive resistance* (R = 25 *ohms*, $L = 0.2$ *henry*) *and a condenser of* 30 *microfarads. The supply voltage is* 100 *and the frequency is* 50. *Calculate the current, its phase difference and the voltage across the condenser.*

Solution.—The equivalent reactance of the circuit is equal to: (inductive reactance — condensive reactance), i.e.,

$$314 \times 0.2 - \frac{10^6}{314\times30} = 62.8 - 105.4$$

$$= -42.6 \text{ ohms.}$$

Hence the impedance of the circuit :—

$$= \sqrt{(25^2 + 42.6^2)} = 49.4 \text{ ohms.}$$

Therefore, the current

$$= 100/49.4 = 2.02 \text{ amp.,}$$

and $\cos\varphi = \dfrac{25}{49.4} = 0.506.$

Whence $\varphi = 59.6^\circ$.

The voltage across the condenser

$$= IX_c = 2.02 \times 105.4 = 213 \text{ volts.}$$

Observe that this voltage is over twice the supply voltage. The reason for this high voltage is that the supply voltage is equal to the vector sum of the voltages across the resistance, inductance and capacity; the two latter voltages have a phase difference of 180° relatively to each other and phase differences of 90° (leading and lagging) relatively to the voltage across the resistance, as is shown in Fig. 6.

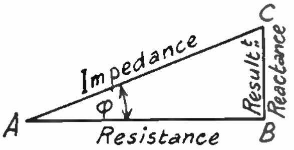


Fig. 7.—IMPEDANCE TRIANGLE FOR A SERIES CIRCUIT.

This is obtained from the triangle O D E, Fig. 6, by a change of scale (e.g., by dividing throughout by the current).

In general, the voltage across the condenser will be less than, equal to, or greater than the supply voltage according to whether the condensive reactance is less than, equal to, or greater than the impedance of the circuit. In fact, the voltage across the condenser is given simply by the expression :—

$$\text{Supply voltage} \times \frac{\text{condensive reactance}}{\text{impedance}}$$

Hence, with a circuit of high condensive reactance and low impedance, the voltage across the condenser may be several times the supply voltage.

Calculation of Currents in Parallel Circuits.

The current in each branch is calculated separately by considering each branch as a simple circuit. The branch currents are then added *vectorially* to obtain the supply current. This process may be carried out graphically as already explained, but in some cases a graphical solution is difficult or impracticable. In these cases the supply current is calculated by the following method:—

Resolve each branch-current vector into components along perpendicular axes; one axis containing the vector of the supply e.m.f., as shown in Fig. 8. This axis is called the *in-phase* axis; the other (*perpendicular*) axis is called the *quadrature* axis. Then the supply current is equal to

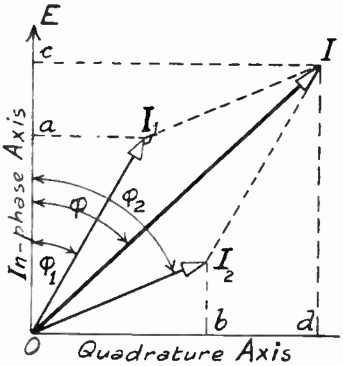


Fig. 8.—Showing How Currents are Resolved into Quadrature and In-phase Components for the Purpose of Determining the Vector Sum.

$$\sqrt{\left[\begin{array}{l}(\text{sum of in-phase components})^2+\\(\text{sum of quadrature components})^2\end{array}\right]}$$

and

$$\cos\varphi=\frac{\text{sum of in-phase components}}{\text{supply current}}$$

Thus if I_1, I_2, . . . denote the branch-circuit currents, φ_1, φ_2, . . . their phase differences, the in-phase components are $I_1 \cos\varphi_1$, $I_2 \cos\varphi_2$, etc., and the quadrature components are $I_1 \sin\varphi_1$, $I_2 \sin\varphi_2$, etc. Hence the line or supply current is

$$I=\sqrt{\left[\begin{array}{l}(I_1\cos\varphi_1+I_2\cos\varphi_2+\ldots)^2\\+(I_1\sin\varphi_1+I_2\sin\varphi_2+\ldots)^2\end{array}\right]} \qquad (16)$$

$$\text{and } \cos\varphi=\frac{I_1\cos\varphi_1+I_2\cos\varphi_2+\ldots}{I} \qquad (17)$$

Why the A.C. Parallel Circuit Cannot be Calculated by the Method Used for a D.C. Circuit.

In calculating a D.C. parallel circuit we calculate directly the equivalent resistance and obtain the line current as explained on p. 1385. But with an A.C. circuit we cannot proceed in this simple manner because impedances must be treated *vectorially* instead of arithmetically. It is, therefore, more straightforward and usually simpler to proceed by calculating the branch-circuit currents separately and adding these currents vectorially as already explained. If the equivalent impedance of the circuit is required this is obtained by dividing the line current into the line voltage.

Numerical Example on Parallel Circuits.

One branch (A) of a parallel circuit has a resistance of 2 ohms and an inductive reactance of 3 ohms. The other branch (B) has a resistance of 2 ohms and an inductive reactance of 1.5 ohms. Calculate (1) the current taken from a 100-volt, 50-cycle supply, (2) the joint impedance of the circuit.

Solution.—The impedances of the branches A, B, are calculated from formula (8). Thus:—

$$Z_A = \sqrt{(2^2+3^2)} = 3.6 \text{ ohms.}$$
$$Z_B = \sqrt{(2^2+1.5^2)} = 2.5 \text{ ohms.}$$

Hence the branch-circuit currents are:—

$$I_A = 100/3.6 = 27.8 \text{ amp.}$$
$$I_B = 100/2.5 = 40 \text{ amp.}$$

To calculate the in-phase and quadrature components of these currents, we require $\cos\varphi_A$, $\sin\varphi_A$, $\cos\varphi_B$, $\sin\varphi_B$. These quantities are easily obtained if we remember that generally:—$\cos\varphi$ = resistance/impedance, $\sin\varphi$ = reactance/impedance.

Hence
$$\cos\varphi_A = 2/3.6 = 0.555;$$
$$\sin\varphi_A = 3/3.6 = 0.833;$$
$$\cos\varphi_B = 2/2.5 = 0.8;$$
$$\sin\varphi_B = 1.5/2.5 = 0.6.$$

The in-phase components are, therefore,

$$I_A\cos\varphi_A + I_B\cos\varphi_B = 27.8 \times 0.555 + 40 \times 0.8 = 50.45 \text{ amp.,}$$

and the quadrature components are

$$I_A\sin\varphi_A + I_B\sin\varphi_B = 27.8 \times 0.833 + 40 \times 0.6 = 47.2 \text{ amp.}$$

Therefore, the line current is

$$I = \sqrt{(50.45^2 + 47.2^2)} = 69 \text{ amp.}$$

The phase difference (φ) of this current is calculated from formula (17). Thus:—

$$\cos\varphi = \frac{\text{in-phase components}}{\text{line current}} = \frac{50.45}{69} = 0.732,$$

and $\varphi = 42.9$.

The joint impedance of the circuit is

$100/I = 100/69 = 1.45$ ohms.

If the equivalent resistance and reactance of this impedance were required, they would be calculated by the aid of formulæ (15), (8). Thus :—

Resistance = Impedance × cos φ

$= 1.45 \times 0.732 = 1.06$ ohms.

Reactance

$= \sqrt{[(\text{impedance})^2 - (\text{resistance})^2]}$

$= \sqrt{(1.45^2 - 1.06^2)} = 0.985$ ohm.

Calculation of Current in a Series-parallel Circuit.

The first step is to calculate the joint impedance of the parallel portion of the circuit. The easiest way of doing this is to calculate the branch currents, the joint impedance, and the equivalent resistance and reactance exactly as for a simple parallel circuit, following the method given in the preceding example. The calculations can be made without a knowledge of the voltage across the parallel portion of the circuit (which, of course, is unknown at the present stage). The method of procedure is best shown by a numerical example (2) which follows.

(1) *If an inductive resistance* (R = 0.8 *ohm*, X = 1.0 *ohm*) *is connected in series with the parallel circuit of the preceding example, what will be the line current if the supply voltage is 230 volts ?*

Solution.—Using the results of the preceding example, the parallel portion of the circuit is equivalent to a single inductive resistance having R = 1.06 ohms, X = 0.985 ohm.

Having obtained the joint impedance of the parallel portion of the circuit, this is added vectorially to the series impedance to obtain the joint impedance of the whole circuit. Whence the current is readily obtained in the usual manner.

To obtain the joint impedance of the complete circuit we add the joint impedance of the parallel portion to that of he series portion of the circuit. Thus the resistance term of the joint impedance is equal to the sum of the resistance terms of the separate impedances, i.e., 1.06 + 0.8 = 1.86 ohms. Similarly, the reactance term is equal to the sum of the reactance terms of the separate impedances, i.e., 0.985 + 1.0 = 1.985 ohms. Hence the joint impedance of the series-parallel circuit is

$\sqrt{(1.86^2 + 1.985^2)} = 2.72$ ohms.

Whence the line current = 230/2.72 = 0.845 amp.

(2) *A series-parallel circuit consists of an inductive resistance* (R = 10 *ohms*, L = 0.05 *henry*) *in series with two parallel branches A, B. Branch A consists of an inductive resistance* (R = 50 *ohms*, L = 0.15 *henry*). *Branch B consists of a non-inductive resistance* (R = 100 *ohms*) *in series with a 40-microfarad condenser. Calculate the line current and the joint impedance of the circuit when it is connected to a 230-volt, 50-cycle supply.*

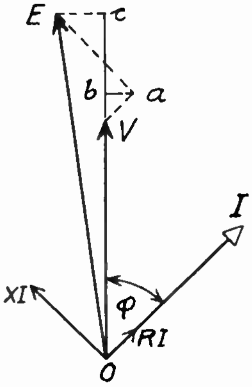


Fig. 9.—VECTOR DIAGRAM FOR A SIMPLE TRANSMISSION LINE, SHOWING HOW THE VOLTAGE AT THE GENERATOR END IS DETERMINED.

The actual voltage drop is given approximately by Vc, which is smaller than the voltage drop due to impedance (represented by the line joining V and E).

Solution.—First calculate the separate impedances of the branch circuits. Thus

$Z_A = \sqrt{[50^2 + (314 \times 0.15)^2]}$

$= \sqrt{(50^2 + 47.1^2)} = 68.8$ ohms.

$\cos\varphi_A = 50/68.8 = 0.727.$

$\sin\varphi_A = 47.1/68.8 = 0.687.$

$$Z_B = \sqrt{\left[100^2 + \left(\frac{10^6}{314 \times 40}\right)^2\right]}$$

$= \sqrt{(100^2 + 79.6^2)} = 127.7$ ohms.

$\cos\varphi_B = 0.783.$

$\sin\varphi_B = -0.622$ (the minus sign is added to denote that φ_B is leading).

Next calculate the branch circuit currents and their in-phase and quadrature components. These currents and their components are expressed in terms of the voltage across the branch circuits, which, although unknown in magnitude, is denoted by V. Thus

$$I_A = V/68.8.$$

$$I_A \cos \varphi_A = \frac{V}{68.8} \times 0.727 = \frac{V}{100} \times 1.05.$$

$$I_A \sin \varphi_A = \frac{V}{68.8} \times 0.687 = \frac{V}{100} \times 1.0.$$

$$I_B = V/127.7.$$

$$I_B \cos \varphi_B = \frac{V}{127.7} \times 0.783 = \frac{V}{100} \times 0.612.$$

$$I_B \sin \varphi_B = \frac{V}{127.7} \times (-0.622)$$

$$= -\frac{V}{100} \times 0.487.$$

The joint impedance of the parallel circuit is next calculated by first calculating the line current (in terms of V) and dividing this into the voltage V. Thus from formula (16)

$$I = \frac{V}{100} \sqrt{[(1.05 + 0.612)^2 + (0.612 - 0.487)^2]}$$

$$= \frac{V}{100} \times 1.74.$$

Whence the joint impedance is

$$Z_1 = \frac{V}{I} = \frac{V}{V} \times \frac{100}{1.74} = 57.5 \text{ ohms.}$$

This must be split up into resistance and reactance. Thus if φ_1 is the phase difference,

$$\cos \varphi_1 = \frac{1.05 + 0.612}{1.74} = 0.955,$$

$$\sin \varphi_1 = \frac{0.612 - 0.487}{1.74} = 0.295,$$

whence

$$R_1 = Z_1 \cos \varphi_1 = 57.5 \times 0.955 = 54.9 \text{ ohms.}$$

$$X_1 = Z_1 \sin \varphi_1 = 57.5 \times 0.295 = 16.95 \text{ ohms.}$$

We now add this resistance and reactance to the resistance and reactance of the series portion of the circuit, and calculate the joint impedance of the whole circuit.

The resistance of the series portion of the circuit is 10 ohms, and the reactance is $(314 \times 0.05 =)$ 15.7 ohms.

Whence

joint impedance of whole circuit =

$$\sqrt{[(54.9 + 10)^2 + (16.95 + 15.7)^2]}$$

$$= 72.7 \text{ ohms.}$$

Hence, the line current

$$= 230/72.7 = 3.16 \text{ amps.}$$

Calculation of Three-phase Circuits.

Three-phase circuits are calculated by considering each phase separately, and calculating the phase currents from the phase voltages and impedances in the same manner as for single-phase circuits. In practice, three-phase systems are symmetrical and the loads are usually balanced, in which cases the calculations are simple and straightforward. For the methods of calculating when the loads are unbalanced or the system is unsymmetrical reference should be made to the larger text-books (such as *Theory and Practice of Alternating Currents*, by A. T. Dover).

Having calculated the phase currents, the line currents are obtained from the following simple rules:—

With a *star connected system*—

line current = phase current ; ......(18)

line voltage = 1.73 × phase voltage..(19)

With a *delta-connected system*—

line current = 1.73 × phase current ;..(20)

line voltage = phase voltage ....(21)

Power and Power Factor in Single-phase Circuits.

In a single-phase circuit the power at any instant is equal to the product of the instantaneous values of the e.m.f. and current at that instant. The power, therefore, pulsates or alternates throughout each cycle of the current or e.m.f. as shown by the curves of Fig. 32, p. 1310. The mean power (i.e., the average rate at which energy is supplied during a period) is calculated from the formula:—

$$P = E\,I \cos \varphi \quad \text{................(22)}$$

where E and I are the R.M.S. values of the supply e.m.f. and current, respectively, and φ is the phase difference between E and I when these are sine quantities.

The power factor is the ratio:—

(power/volt-amperes) or $P/E\,I$, ...(23)

and, in the case of sine quantities, is equal to $\cos \varphi$.

Hence generally

Power = volts × amperes
× power factor........(24)

Power and Power Factor in Three-phase Circuits.

In a symmetrical three-phase circuit with balanced loads the power is constant from instant to instant, and is calculated from the formula :—

$$P = \sqrt{3}\ V I \cos\varphi \qquad \ldots\ldots\ldots\ldots (25)$$
$$= 1.73\ V I \cos\varphi.$$

where V and I denote *line* voltages and currents respectively and cos φ is the power factor. With sine quantities φ is the phase difference between the *phase* voltage and the *phase* current.

Calculation of Voltage Drop in Single-phase Circuits and Transmission Lines.

The voltage drop in the system is the arithmetical difference between the voltages at the generator end and the load end of the line. This voltage drop is not generally equal to the actual voltage lost in the line conductors due to their impedance owing to the phase difference between the former and the latter. The conditions are represented in the vector diagram of Fig. 9, in which the reference vector O V represents the voltage at the load ; O I the current lagging φ° with reference to O V (cos φ being equal to the power factor of the load) ; O R, the voltage drop due to the resistance of the line conductors ; O X, the voltage drop due to the reactance of the line conductors (O X leads O I by 90°) ; and O E the voltage at the generator end of the line. O E is the vector sum of O V, O R, and O X.

The arithmetical difference between O E and O V may be determined graphically from the vector diagram, but it is usually more convenient to calculate it from the approximate formula :—

$$I\ (R \cos\varphi + X \sin\varphi) \ldots\ldots\ldots\ldots (26)$$

where R is the resistance and X the reactance.

[Note.—If the current is leading the plus sign must be replaced by a minus sign.]

This formula is easily derived from the vector diagram by projecting the points *a* and E on to O V produced. Then the voltage drop is approximately equal to V*c* which is equal to V*b* (or V*a* cos φ) = R I cos φ) *plus b c* (or *a* E sin φ = X I sin φ).

Calculation of Voltage Drop in Three-phase Circuits and Transmission Lines.

The voltage drop in the system (i.e., the arithmetical difference between the *line* voltages at the generator end and load end of the line) is calculated by the approximate formula :—

$$\sqrt{3}\ I\ (R \cos\varphi + X \sin\varphi)$$
$$= 1.73\ I\ (R \cos\varphi + X \sin\varphi) \ldots\ldots\ldots (27)$$

where I is the *line* current ; R, the resistance *per line* ; and X the reactance *per line*.

[Note.—If the current is leading the plus sign must be replaced by a minus sign.]

Numerical Example on Voltage Drop in Three-phase Circuit.

A three-phase motor is supplied from a transformer—some distance away—by overhead lines, each line having a resistance of 0.1 *ohm and a reactance of* 0.15 *ohm. If the motor is taking a current of* 80 *amp. at* 0.9 *power factor, and the terminal voltage at the transformer is* 415 *volts, what is the voltage at the motor ?*

Solution.—In this case

$$\cos\varphi = 0.9;\ \sin\varphi = \sqrt{(1 - 0.9^2)} = 0.436.$$

Hence, from formula (27), the voltage drop is

$$1.73 \times 80\ (0.1 \times 0.9 + 0.15 \times 0.436)$$
$$= 21.5 \text{ volts}.$$

whence the voltage at motor

$$= 415 - 21.5 = 393.5 \text{ volts}.$$

ARCHITECTURAL LIGHTING

By E. H. Freeman, M.I.E.E.

Fig. 1.—A Special Example of Architectural Lighting.
This shows the lighting in toilet saloon at Messrs. Austin Reed's premises in Regent Street, designed by Mr. P. J. Westwood. (*Photo by Bedford Lemere & Co.*)

The methods of supplying artificial illumination to buildings generally described as "architectural lighting" provide an entirely new phase in illumination design that scarcely allows of exact calculation. The description is somewhat indefinite but may be taken as including methods of illumination in which the sources of light form a part of the structure of the building. Pendants hanging from ceilings and brackets projecting from walls are in general not included, although such light sources may be used to supplement or contrast with other built-in light sources. Cornice lighting; concealed laylight fittings; illuminated wall panels and various kinds of indirect lighting are all forms of what is usually understood by architectural lighting, whilst there are occasional other special examples which may be reasonably included, such as the toilet saloon at Messrs. Austin Reed's premises in Regent Street—see Fig. 1.

Efficiency of Architectural Lighting.

As has already been mentioned in the article on Hotel Lighting (see page 1468), schemes of this character (cornice lighting in particular being referred to in that article) must be "inefficient" in the sense of not providing the maximum number of foot candles for the lowest consumption in watts. The need for efficiency in all commercial buildings has been the inevitable result of high charges for current, but the modern tendency to adopt a "two part tariff" system of charging with a fixed annual payment and a low charge for units used, of the order of 1d. per unit, is already leading to a different conception of lighting values.

Comparative Costs for Current.

In the example given in Hotel Lighting the extra annual cost of the scheme adopted, taking 14,400 watts actually in use, as compared with a strictly "efficient" scheme requiring 2,400 watts, would be £600 a year, with current at 6d. a unit and allowing about 2,000 hours' use a year—a not unreasonable time for a hotel reception room. Such a charge would be prohibitive, but if current is reduced to 1d., plus a fixed sum irrespective of the units used, the extra cost is only £100 a year, which is not sufficient to justify spoiling the decorative scheme for the building.

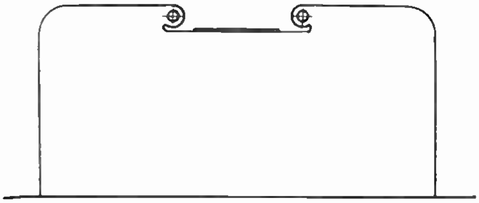


Fig. 2.—DIAGRAM SHOWING PRINCIPLE OF CENTRAL CORNICE LIGHTING.

The commercial possibility of using lighting schemes of this kind thus follows directly as a result of low charges for current, as although there have been many such installations in the past they have been the exception rather than the rule, due to the excessive cost for current.

Cornice Lighting.

The most common form of architectural lighting in the past has been that known as cornice lighting in which the lamps are concealed, the light being reflected from the ceiling.

This scheme of lighting, as has been seen from the example at Claridge's Hotel, can be designed to give most attractive effects, but its limitations must be kept fully in mind when considering its application.

Unsuitable for Small Rooms.

In the first place cornice lighting is quite unsuitable for small rooms. Even allowing for a low efficiency it will be found that it is not possible to distribute the "wattage" round the walls evenly unless very small lamps are used. Thus for a room 15 ft. by 12 ft. the total watts required might be about 500 watts, whilst the length of wall is 54 ft. A lamp of only 10 watts for every foot of cornice would be required and such lamps are not made for ordinary voltages.

Larger lamps at wider spacing would give patches of alternate light and shade on the ceiling and spoil the whole effect, an essential feature of cornice lighting being even distribution of the light on the ceiling.

Also Unsuitable for Low Rooms.

For rooms that are low in proportion to their size, cornice lighting is also unsuitable. The use of two or three cornices as at Claridge's Hotel would not be practicable in such a case.

In small rooms the size of cornice required to conceal the lamps would be out of proportion to the other dimensions of the room and would look unsightly, whilst the necessary breaks in the cornice (or in the lighting) at windows would cause unpleasant, dark patches on the ceiling.

Difficulties in Larger Rooms.

Difficulties would also arise in larger rooms of normal "commercial" height, i.e. 10 to 12 ft. from floor to ceiling. The space between ceiling and cornice should be not less than one-twelfth of the width of the room for even lighting, so that for a room 30 ft. across—a reasonable width for a shop area—the top of the cornice must be 2 ft. 6 in. down and the bottom about 3 ft. 6 in. down, which is quite impracticable if the walls are occupied, as they usually are in business premises, with stock cabinets or display cases. If the cornice is kept nearer the ceiling, the centre portion of the ceiling will be left badly lit with an effect of dullness.

The general appearance with cornice lighting is also apt to be dull and uninteresting and there is not the brightness of effect that is an essential to good shop lighting, unless very heavy consumption of current is permissible.

Applications to Larger Rooms.

In large reception rooms and in such places as theatres, cinemas, and so on, the objections mentioned do not apply to the same extent. For example, a reception room 60 ft. by 30 ft. would require about 5,000 watts corresponding to 25 to 30 watts per foot run of wall. The cornice should be 2 ft. 6 in. down from the ceiling and this will not be out of keeping in such a room which might be 18 to 20 ft. high. The mass of the cornice might also be disguised as at Claridge's Hotel by using a double or triple cornice, but this will have the effect of increasing the length of cornice and reducing the lamp size.

Fig. 3.—A Special Application of Cornice Lighting.

This shows the method employed in Messrs. Austin Reed's Tropical Room, designed by Mr. P. J. Westwood. The lighting is from chain pendants and a concealed cornice is run round the room about 3 ft. from the wall to give special lighting for the decorated frieze. (*Photo by Bedford Lemere & Co.*)

Effect of Decorations and Colour.

In the above examples it has been assumed that white walls and ceilings—particularly ceilings—are used and if this is not the case the wattage required may be greatly increased. Any definite colouring in the ceiling will have a marked effect, as will be seen from the relative reflective values for different colours given below.

White paint, matt finish ..	77%
White paint, glossy finish ..	78%
Cream paint, matt finish ..	62%
Rose pink	50%
Light green, matt finish ..	40%
French grey	38%
Dark grey	22%
Crimson..	10%

These figures are all very approximate as one shade fades into another, but they will

Fig. 4.—INTERIOR OF UNDERGROUND RAILWAY STATION WITH INDIRECT LIGHTING FROM STANDARDS
The lamps are concealed in bowls at the top of the standards and the light is partly reflected from the inverted cones fixed in the bowls above the lamps and partly from the ceilings. (*Fox Photo.*)

illustrate the immense differences in the results obtained with coloured reflective surfaces instead of white.

A further point to be noted in connection with coloured ceilings is the effect of the yellow light of the lamps on the colour. This is particularly troublesome with blue shades which, though clear and distinct by daylight, change to unpleasant greys and greens when the light from the concealed lamps is thrown directly on to them.

Shadows on Ceiling.

Another frequent cause of failure of cornice lighting is the existence of projecting features in the ceiling. These will cast heavy shadows, if near the cornice, with probably most unpleasing effects.

Central Cornice Lighting.

A variation from ordinary cornice lighting round the walls has been proposed by Mr. Waldo Maitland of the Lighting Service Bureau. This consists of a central feature with a " cornice " somewhat as illustrated in Fig. 2. This would provide the advantages of this form of lighting with a much reduced length of cornice in proportion to the size of the room, thus allowing the use of larger and more efficient lamp units.

With such a scheme the bottom of the central panel might be illuminated or might contain a heating element for warming the room.

A special application of this proposal is provided in Messrs. Austin Reed's Tropical Room at Regent Street, the concealed lighting in that case (see Fig. 3) being used primarily to illuminate the decorative frieze that runs round the upper part of the walls.

Panel Lighting.

This form of lighting can also be used in suitable conditions to give most effective results, but it also must not be used unless the conditions are suitable. The principle involved is to illuminate the room from large areas of glass that are lit from behind. The glass areas may be in the wall or the ceilings, but the principle is the same in each case.

Even Lighting Essential.

The results will not be satisfactory unless the area of glass is evenly lit, and this involves one or both of two conditions, viz., the lamps must be a sufficient distance back from the glass panel or the glass must be very opaque and correspondingly inefficient.

Much experimental work has been carried out in recent years with a view to finding forms of glass that will help to overcome the difficulties imposed by these conditions and great advances have been made, but even now glass with a high diffusing value (i.e., which will enable the lamps to be placed reasonably close without patchy effects) has a correspondingly low " transmission " efficiency.

Spacing Required for Even Lighting.

The following table gives the results of experiments carried out by the Lighting Service Bureau and the differences in the spacing required with different glasses are very marked, ranging as will be seen with lamps 10 in. back, from 3¾ in. with frosted Stippolyte to 21½ in. with acid-etched flashed opal glass.

Distance of Lamp behind Glass " D " Inches.	Type of Glass.			
	Frosted Stippolyte.	Acid-etched Sheet.	Flashed Opal.	Acid-etched Flashed Opal.
	Lamp Spacing " S " Inches.			
4	3	3	5½	6
6	3	3¼	8	9½
8	3½	4	12	15
10	3¾	5	17	21½

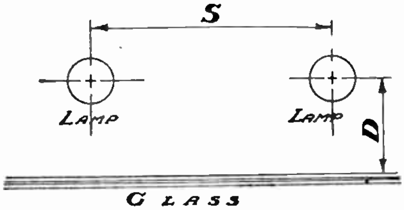


Fig. 5.—Spacing Required for Even Lighting.

Application of Above Results.

The effect of these experiments can be seen if a scheme is worked out for panel lighting in a room, say, 30 ft. × 15 ft.

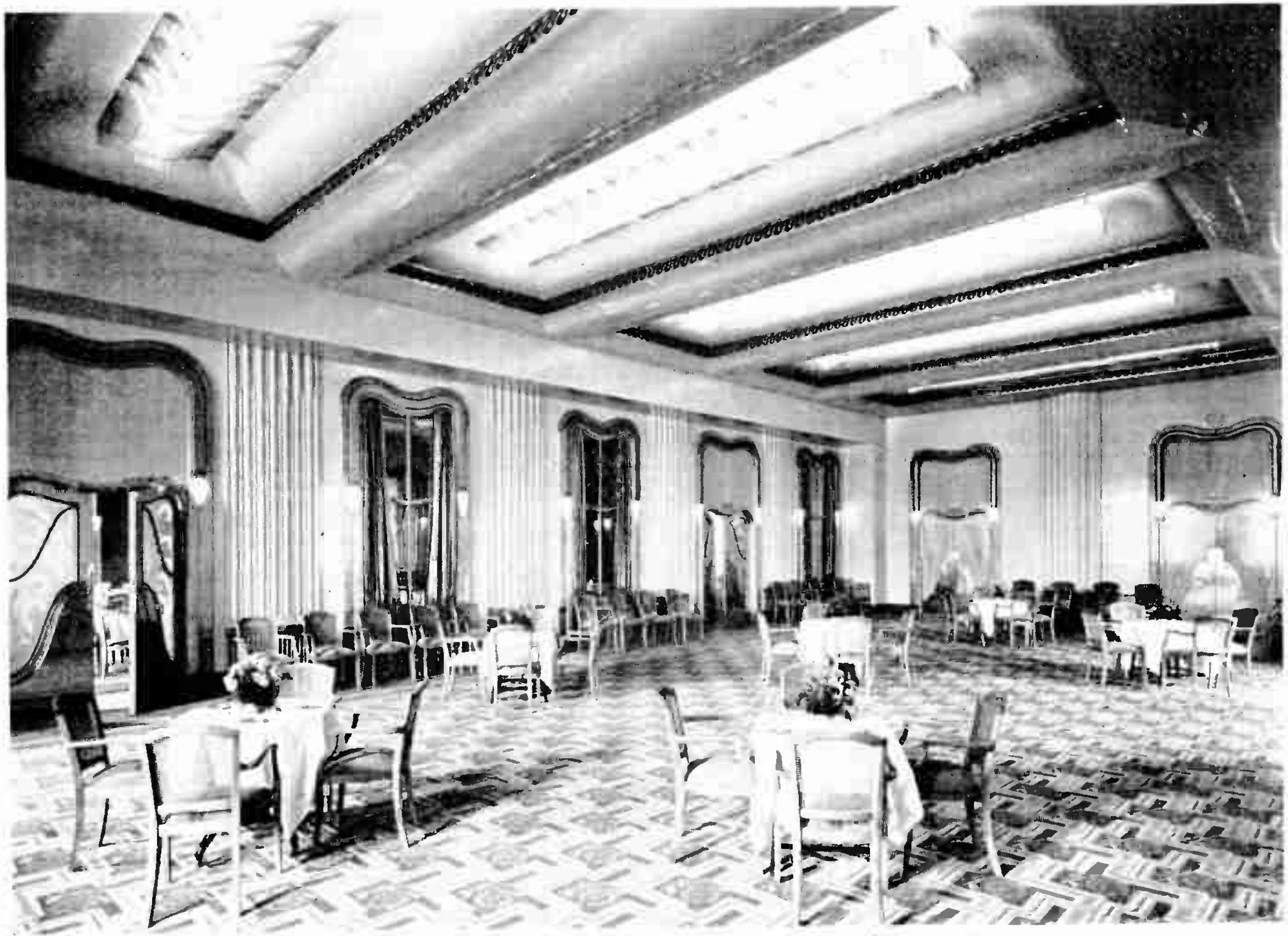


Fig. 6.—INTERIOR OF EMPIRE ROOM AT TROCADERO RESTAURANT.
The lighting in this case is from ceiling trough lights, in which the lamps are placed behind a trough built up of horizontal glass tubes.
(Photo by courtesy of Lighting Service Bureau.)

This might be lit from two ceiling panels and allowing for 30 per cent. efficiency the total wattage required would be about 1,500 watts. With panels 4 ft. × 2 ft. 4 ins., it would thus be necessary to distribute about 750 watts in each panel.

Using flashed opal glass the lamps could be 15 ins. apart if they are 8 ins. back from the glass—corresponding to a total depth behind the panel of 12 ins.

The lamps might be arranged as Fig. 7 giving spacings of 14 ins. and 16 ins. in the two directions, which should be satisfactory. This would allow each panel to be lit with six lamps of either 100 or 150 watts—according to whether rather better or worse lighting is desired than the calculated result.

On the other hand, if Stippolyte glass is used, the lamps can only be 3½ ins. apart and no fewer than 112 lamps must be used, spaced about as in Fig. 8. Each lamp will be only about 15 watts and the efficiency of the scheme appreciably reduced, as such lamps give little more than half the number of lumens per watt that are obtained from 150-watt lamps.

Size of Lamp.

One result of the experiments referred to below was very unexpected, viz., that the spacing required for even illumination was independent of the lamp size—at any rate within the limits of 15-watt and 100-watt lamps.

Transmission Values of Glass.

Another important range of experimental work has been carried out by various authorities to ascertain the extent to which various glasses transmit the light, and the following figures are taken from the booklet issued by Messrs. Chance Bros., dealing with this subject.

Clear plain sheet glass (15 oz.)	95%
Rolled plate (⅛ inch)	83%
Dewdrop	75½%
Flashed opal (plain)	66%
Stippolyte	91%
Plain sheet (acid obscured) ..	79%
Plain sheet (sand blasted) ..	72½%
Dewdrop (sand blasted) ..	55%
Flashed opal (sand blasted) ..	47%

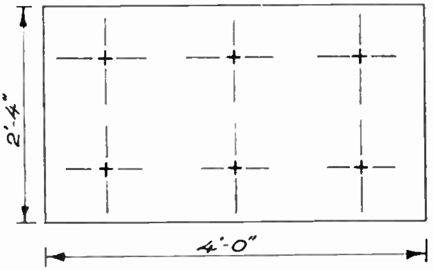


Fig. 7.—DIAGRAM OF SPACING OF LAMPS USING FLASHED OPAL GLASS OF 100-150-WATT LAMPS.

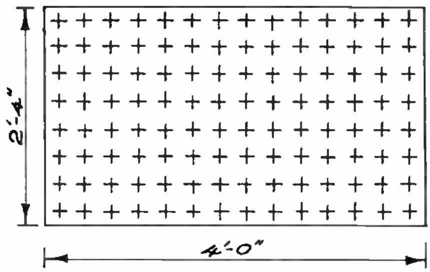


Fig. 8.—DIAGRAM OF SPACING OF LAMPS USING STIPPOLYTE GLASS AND 15-WATT LAMPS.

These figures are only approximate as glass samples from different makers vary widely whilst the effects of aciding and sand blasting show still wider variations.

They are also for clean glass and it would probably be necessary to allow for a further loss of 20 per cent. or more for average conditions with panel lighting to cover the effects of even a very slight layer of dust. It must be remembered that glass panels such as would be used for ceiling panel lighting are usually not easily accessible so that frequent cleaning may be difficult involving the necessity for a substantial allowance for the effect of dust on both the glass panel and on the lamp.

Difficulty of Calculation.

The difficulty of designing this type of installation will be appreciated from the above figures, particularly when it is remembered how difficult it is at the early stages of a building scheme to obtain a final decision on such points as the exact colour of walls and ceiling; the exact kind of glass that will harmonize with the decorative treatment and, what may be worst of all, the possibility that slight tinting of the glass may be found desirable.

The only course is to allow a very ample margin for all such contingencies on the whole of the wiring scheme and to adjust the lamp sizes and the spacing of the lamps behind the panels at a comparatively late stage in the execution of the work.

Other Special Methods of Lighting.

Many variations of the two main schemes described above may be used and illustrations of several installations of this kind are shown in Figs. 3, 4 and 6.

Fig. 4 shows a type of indirect lighting unit at one of the Underground Railway stations. The lamps are concealed in the bowls at the tops of the standards and the light is partly reflected from the inverted cones fixed in the bowls above the lamps and partly from the ceilings.

Fig. 6 shows the Empire Room at the Trocadero Restaurant. The lamps are placed behind a form of trough built up of horizontal glass tubes, one such trough being arranged in each of the large recessed panels of the ceiling stretching the whole width of the room. Supplementary lighting, mainly for contrasting effects, is obtained from wall brackets at the sides of the windows and doors.

In a typical scheme of lighting adopted at Lloyds Bank and designed by Messrs. Sir John Burnet and Partners, the lighting is obtained from standards which carry large glass bowls which enclose the lamps. The lighting is obtained partly by reflection from the ceiling and partly by transmission through the glass bowls. Such a scheme provides a method of lighting that should give very satisfactory results but it is difficult to arrange the fittings so that they are not unduly obtrusive. Probably such a scheme would be seen at its best if it could be arranged with the light sources carried on the tops of low partitions screening off private offices.

General Applications.

At present these forms of lighting are probably mainly suitable to special buildings such as cinemas, theatres, concert and lecture halls, hotel reception rooms and in a modified degree to banking and other large commercial offices. It is but rarely that they can be applied successfully to smaller rooms and to domestic installations but as the materials available are developed and as the desirability of using light as a form of decoration is recognised by architects and their clients, so will such methods of illumination be gradually adopted for every class of building.

QUESTIONS AND ANSWERS

What are the approximate relative reflective values of some typical colours ?

White paint, matt finish ..	77%
White paint, glossy finish ..	78%
Cream paint, matt finish ..	62%
Rose pink	50%
Light green, matt finish ..	40%
French grey	38%
Dark grey	22%
Crimson	10%

What is a frequent cause of failure of cornice lighting ?

The existence of projecting features in the ceiling. These will cast heavy shadows, if near the cornice, with probably most unpleasing effects.

What is the principle involved in panel lighting ?

To illuminate the room from large areas of glass that are lit from behind. The glass areas may be in the wall or the ceilings. The lamps must be a sufficient distance back from the glass panel or the glass must be very opaque and correspondingly inefficient.

What types of glass are most suitable and least suitable for transmitting light ?

Clear plain sheet glass (15 ounce), 95 per cent., and Stippolyte, 91 per cent., are most effective, while Dewdrop (sand blasted), 55 per cent., and Flashed Opal (sand blasted), 47 per cent., are least effective.

LOAD TESTING OF D.C. AND A.C. MACHINES

By H. W. Johnson

ELECTRICAL dynamos and motors are given load tests before they are placed into commission.

These tests are generally carried out on the test beds of the constructors, but may, under certain circumstances, be made where the machines will be installed. Large machines installed at generating stations, for example, are often tested on the site.

The Tests which are Made.

The machines are tested for output, efficiency, losses, temperature rise and insulation resistance. They are also given a high voltage flash test for one minute applied between the windings and frame of the machine. The testing voltage applied is 1,000 + twice the rate voltage of the machine, the minimum value being 2,000 volts.

Induction motors are also subjected to a running-up torque test.

The Testing Plant and Equipment.

In order to facilitate the testing of various types of machines special testing plant and equipment are installed by the constructors. The scope and nature of this plant and equipment will be governed by the size and nature of the machines made by the constructors. Special attention will be given to the equipment of the test beds and control panels which will be used for testing standard type machines, so as to reduce to a minimum the time required for fitting these machines on the beds and completing the tests.

The Test Beds.

The floor space will be divided into sections, and each of the sections will be allotted for testing one particular type of machine. The sections are fitted with cast iron bed plates, which are bolted to the floor. The bed plates are provided with a number of parallel slots which are spaced equidistant from each other. These slots receive and hold the heads of the fixing bolts required to bolt the machine under test down to the bed.

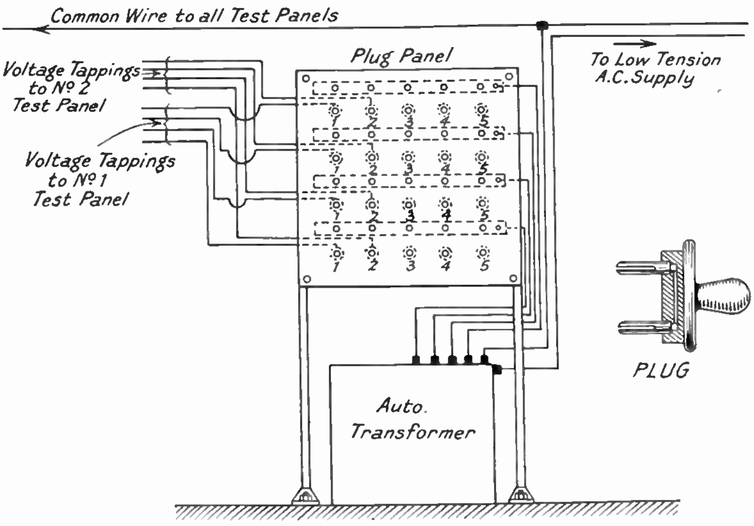


Fig. 1.—The Plug Board to Give a Variable Voltage L.T. A.C. Supply to the Test Panels.

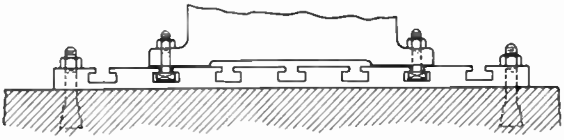


Fig. 2.—THE TEST MACHINE BOLTED TO THE TEST BED. The bolts are slid into position from the edge of the bed.

The stator of the alternator will be specially constructed to allow its windings to be connected in various ways so as to give a variable voltage; the variable frequency would be obtained by the speed control of the driving motor. A synchronising equipment is fitted to allow the supply to be paralleled to alternators, and synchronous motors under test.

The Panels.

Slate panels fitted to iron frames will be provided for each testing bed. The panels will be equipped with suitable measuring instruments, control gear, and switches.

A supply of current at various voltages is given to each panel.

Cable Runs from the Panels to the Test Beds.

These will be taken from the panels through ducts under the floor and emerge at various outlet pits on the test beds.

The Supply of Testing Current.

This is obtained from the public supply company and distributed through a main switchboard to the various testing panels through insulated cables fixed in cleats. These cleats are fitted on floor standards fixed along the routes to the testing panels.

The nature of the supply to a panel will depend upon the type of machines—D.C., A.C. or both—which are to be tested on the bed and controlled by that panel.

Multiple A.C. Voltage L.T. Supply.

This may be obtained from a centrally fixed auto-transformer whose tappings are connected to a plug switchboard. This switchboard may be connected to any selected A.C. panel with plug connectors.

Multiple A.C. Voltage H.T. Supply at Variable Frequencies.

These would be given by a special alternator coupled to a variable speed motor fed from the public supply mains.

Multiple D.C. Voltage Supply.

This supply would be required to test D.C. machines whose normal voltages may differ widely from that of the D.C. supply.

A motor-driven booster set would be able to give any desired change above, or below, that of the supply voltage. A reversible field rheostat for the booster will change the direction of the "booster" voltage as required.

Starting Switches for the Test Machines.

Various types of D.C. and A.C. starting units are fitted on the test beds and may be connected as desired to the machines under test. The panels are thus free from any bulky starting gear.

E.H.T. A.C. Supply for Flash Testing.

E.H.T. current for flash testing is obtained by stepping up the L.T. A.C. supply through suitable transformers.

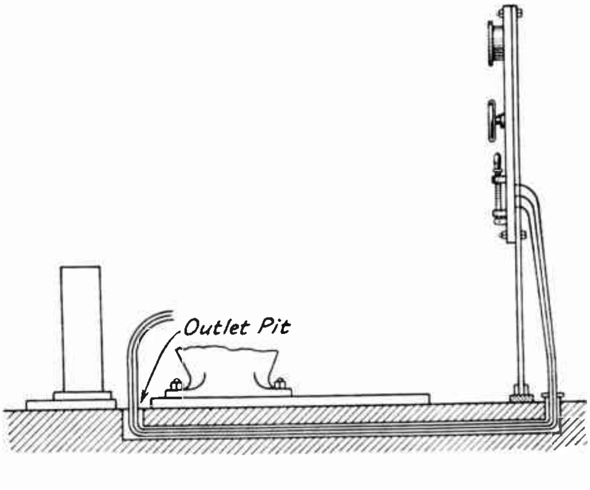


Fig. 3.—CABLE RUN FROM A TEST PANEL TO THE TEST BED.

EFFICIENCY, OUTPUT AND TEMPERATURE TESTS.

A modification of the Hopkinson test is used for all D.C. machines, excepting those of very small output, say, below 3 k.w. or 4 b.h.p.

The original Hopkinson test was first used for testing the efficiencies of similar D.C. shunt machines. The machines were mechanically coupled together and connected electrically in opposition. The machines were driven by a belt. With the field currents adjusted so that equal E.M.F.'s were induced in the two armatures, no current circulated between the two armatures. When the field of one machine was weakened, the balance of E.M.F. was upset and current circulated in the armature circuit. The circulating current may be increased by gradually reducing the value of the field current of one machine. The machines were loaded up to any desired value. The losses due to friction, windage, iron, copper, were measured by a dynamometer fitted to the belt drive.

A series of readings of armature current, voltages and losses recorded by the dynamometer were taken.

$$\text{The combined efficiency} = \frac{\text{Generator output}}{\text{Generator output} + \text{Losses recorded by dynamometer}}$$

$$\text{Efficiency of either machine} = \sqrt{\text{Combined efficiency}}.$$

In practice the modification of the Hopkinson test, by Dr. Kapp, of supplying the losses electrically is generally used.

The Test.

Two similar shunt machines are connected in parallel and run up to speed through a single starter, having first ascertained that they develop a starting torque in the same direction.

The field of each machine is adjusted so that each takes the same armature current when run at the normal voltage and speed.

The field rheostat of one machine is adjusted so as to weaken the field current of this machine. The effect is to cause the machine to accelerate and act as a motor driving the other machine which now becomes a generator and supplies current to the motor. The machine acting as a generator can be loaded up by further weakening the field of the motor.

Using this method of testing the total power required from the supply mains is only about 10 per cent. of the full load output of large machines under test, also the temperature rise test on full load can be taken.

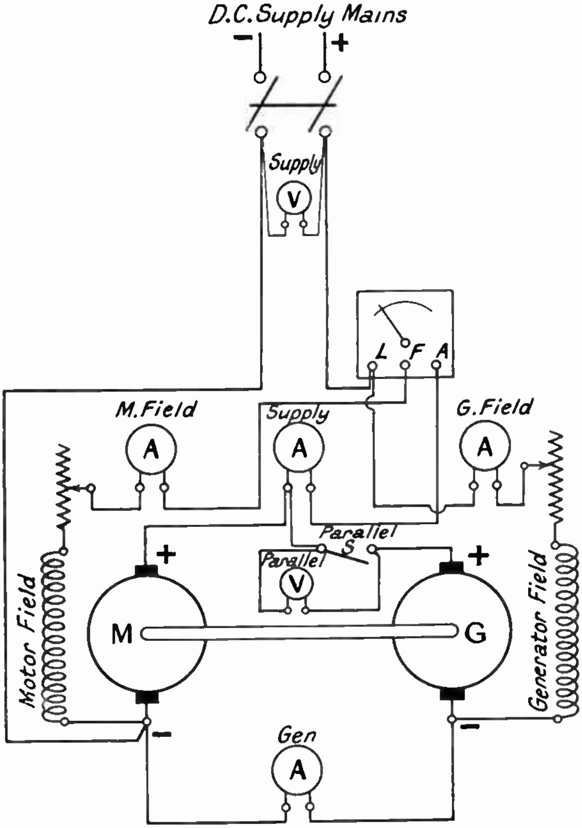


Fig. 4.—Connections for the Hopkinson Test on Two Similar D.C. Shunt Machines.

The Readings to be Taken.

Supply voltage.
Supply current.
Generator output current.
Generator field current.
Motor field current.

Generator armature circuit resistance.
Motor armature circuit resistance.

The machines should run at their normal speed during the test. The run is continued until the windings have reached a steady temperature, and the resistance of the armature circuits of the machines taken when the machines are shut down. The resistance is obtained by passing the full load current through the armatures, and taking the voltage drop across the terminals. The field windings should be disconnected when making this test.

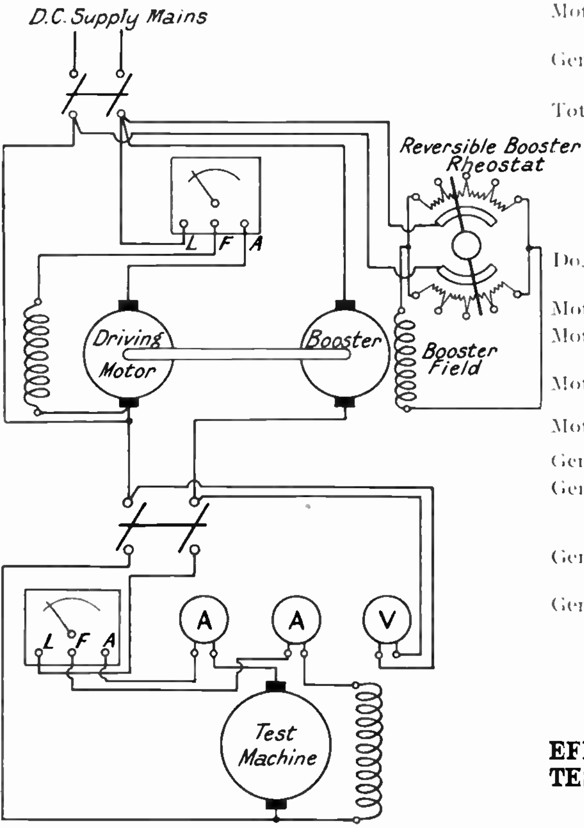


Fig. 5.—CONNECTIONS OF MOTOR BOOSTER TO GIVE A VARIABLE VOLTAGE D.C. SUPPLY REQUIRED FOR TESTING MACHINES.
The boost is varied, by adjusting the excitation of the booster.

Calculation of Efficiency and Losses.

Supply volts = 460.
Supply current = 20 amperes.
Generator armature circuit resistance = .09 ohm.
Generator output current = 100 amperes.
Motor armature circuit resistance = .1 ohm.
Generator field current = 1.0 ampere.
Motor field current = .8 ampere.
Motor armature circuit losses (Copper)
$= (120)^2 \times .1 = 1440$ watts.
Generator armature circuit losses (Copper)
$= (100)^2 \times .09 = 900$ watts.
Total armature circuit losses (Copper)
= 2340 watts.
Supply input power
$= 460 \times 20 = 9200$ watts.
Friction, windgae iron, losses for both machines $= 9200 - 2340 = 6860$ watts.
Do., per machine
$= 6860 \div 2 = 3430$ watts.
Motor input $= (100 + 20) \times 460 = 55200$ watts.
Motor losses (total) $= 3430 + 1440 + (460 \times .8)$
$= 5238$ watts.
Motor output $= 55200 - 5238 = 49962$ watts.
Motor efficiency $= \frac{49962}{55200} \times 100 = 90.5$ per cent.
Generator output $= 460 \times 100 = 46000$ watts.
Generator losses (total)
$= 3430 + 900 + (460 \times 1.0)$
$= 4790$ watts.
Generator input $= 46000 + 4790$
$= 50790$ watts.
Generator efficiency
$= \frac{46000}{50790} \times 100 = 90.5$ per cent.

EFFICIENCY AND TEMPERATURE TEST ON A SINGLE D.C. SHUNT MACHINE.

The machine is run unloaded as a motor at the normal voltage and speed and the armature and field current obtained.

It is now mechanically connected to a suitable load. This load, except in the case of very small machines, will consist of a generator whose output is fed back into the mains. A paralleling switch and voltmeter will be necessary to perform this operation.

The machine is run at its full rated capacity until the temperature of the windings, commutator, core plates, frame, etc., have become constant. The machine is now shut down, taking the precaution to open the paralleling switch of the loading generator before opening the motor circuit.

The temperature of the various parts and windings of the machine is taken immediately, using a thermometer for each part, so that all readings are taken simultaneously. The thermometers should have a bulb with a flat surface so that a good heat contact may be obtained. A wad of cotton wool should be placed over the part of the bulb not in contact with the part whose temperature is to be taken.

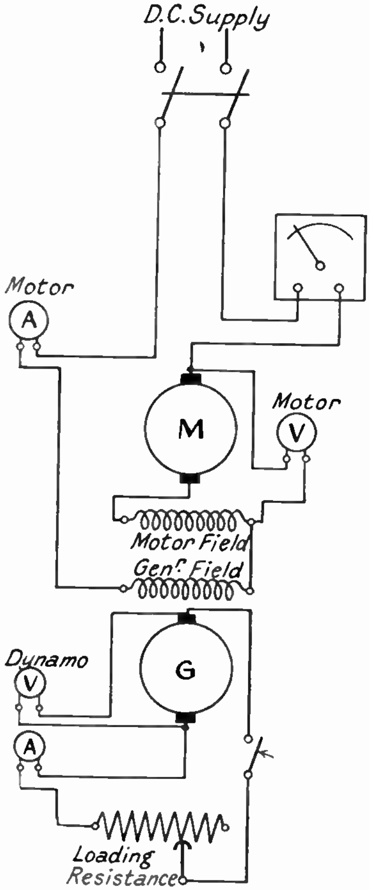


Fig. 6.—Connections for Field's Test of Two Similar D.C. Series Motors.

The stand-still resistance of the armature is taken by obtaining the pressure drop across the recorded full load current passing.

Calculation of Efficiency Readings.

Supply pressure = 600 volts.
Unloaded armature current = 5 amperes.
Field current = 1.0 ampere.
Armature circuit resistance = .08 ohm.
Rated output 50 B.H.P.

Friction, windage and iron losses

$$= (600 \times 5) - (5^2 \times .08) \text{ (negligible)}$$

$$= 3{,}000 \text{ watts approximately.}$$

Approximate armature current at full load

$$= \frac{50 \times 746}{600} + 5 = 67 \text{ amperes}$$

Armature losses (Copper) at full load

$$= 67^2 \times .08 = 359 \text{ watts.}$$

Field winding losses (Copper)

$$= 600 \times 1.0 = 600 \text{ watts.}$$

Total losses

$$= 3000 + 600 + 359 = 3959 \text{ watts.}$$

Machine input

$$= (50 \times 746) + 3959 = 41259 \text{ watts.}$$

Machine output

$$= 50 \times 746 = 37300 \text{ watts.}$$

$$\text{Efficiency} = \frac{37300}{41259} \times 100 = 90.4 \text{ per cent.}$$

VARIABLE D.C. VOLTAGE SUPPLY OF TESTING CURRENT.

When testing motors which require a voltage differing from that of the supply, a motor booster set may be used. A separately excited booster is coupled to a motor which is supplied from the supply mains.

The "boost" may be varied in either direction, with the reversible field rheostat to suit the desired voltage for the machine to be tested.

If the machine is a dynamo it is driven by the motor supplied from the mains, and the booster armature connected in series with the armature of the test machine. The boost is varied in either direction until the generator volts ± the boost is that of the supply, and the output may then be fed back again into the mains supply.

Load Tests on Series Motors.

A special form of test known as Field's test is used when testing series motors.

In this test two similar series motors are mechanically coupled; one of them is run as a motor which drives the other as a dynamo. The output from the dynamo is absorbed by a water or other form of loading resistance. The special point in this test is that the two fields are joined in series, so that the excitation of the

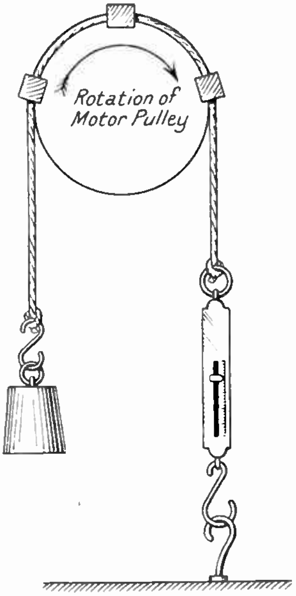


Fig. 7. Arrangement of Rope Brake on the Motor Pulley for B.H.P. Tests on Small Motors.

The torque is obtained by taking the difference between the weight and the spring balance reading and multiplying it by the radius of the pulley in feet.

two machines will be the same during the test.

The Test.

The motor and dynamo switches are closed and the motor gradually run up to speed with the starting resistance. At the same time the value of the loading resistance in the dynamo circuit is reduced in order to prevent the motor speed becoming excessive. Adjust the voltage across the motor terminals to the normal value and the dynamo is then loaded up to any desired rating by adjustment of the loading resistance.

The resistances of the dynamo armature and field, also that of the motor field, are taken with a voltage drop test.

A temperature test may be taken at the end of the run when the temperature of the various windings has reached a constant value.

Readings.

Motor ammeter = 60 amperes.
Motor voltmeter = 500 volts.
Dynamo ammeter = 54 amperes.
Dynamo voltmeter = 480 volts.
Resistance motor armature = .08 ohm.
Resistance motor field = .06 ohm.
Resistance dynamo armature = .08 ohm.
Resistance dynamo field = .06 ohm.

Motor input + Loss in dynamo fields − Dynamo output = Total losses.

Motor input − Dynamo output = Friction, windage and iron losses for both machines + Copper losses in motor arm and field and dynamo armature.

$(60 \times 500) - (54 \times 480) =$ Friction, windage, iron losses for both machines $+ (.14 \times 60^2) + (.08 \times 54^2)$.

Friction, winding, iron losses for both machines $= 3343$.

$$\text{Per machine} = \frac{3343}{2} = 1671 \text{ watts.}$$

The efficiency of each machine may now be calculated as indicated in tests on shunt machines.

EFFICIENCY AND B.H.P. TESTS ON SMALL D.C. MOTORS.

The output of the motor is measured mechanically with a Prony or other form of brake. Readings are taken of the input current and supply voltage to the motor at various loadings of the brake. The speed of the motor and the reading

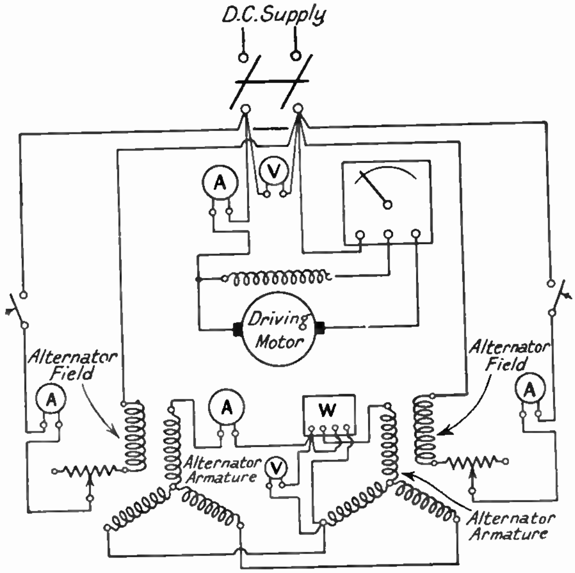


Fig. 8. Connections for Back to Back Test on Two Similar Alternators.

Current is circulated between the alternator stators by altering the field current of one of the machines.

of the spring balance and value of the weight used for each loading is also recorded.

Torque exerted by the motor = (Value of fixed weight − Spring balance reading) × Radius of motor pulley in feet.

B.H.P. of the motor

$$= \frac{\text{Torque} \times 2\pi \times \text{Revs. per min.}}{33000}$$

Input E.H.P.

$$= \frac{\text{Input current} \times \text{Terminal voltage of motor}}{746}$$

$$\text{Efficiency} = \frac{\text{B.H.P.}}{\text{E.H.P.}} \text{ per cent.}$$

Readings.

Motor ammeter = 18 amperes.
Motor voltmeter = 200 volts.
Fixed weight = 56 lbs.
Spring balance = 14 lbs.
Speed = 1000 r.p.m.
Radius of pulley = 6 inches.
Torque = (56 − 14) × .5 = 21 lb. ft.

$$\text{B.H.P.} = \frac{21 \times 6.28 \times 1{,}000}{33{,}000} = 4 \text{ approx.}$$

$$\text{E.H.P.} = \frac{200 \times 18}{746} = 4.8$$

$$\text{Efficiency} = \frac{4}{4.8} \times 100 = 83 \text{ per cent.}$$

BACK-TO-BACK TEST OF TWO SIMILAR ALTERNATORS FOR EFFICIENCY.

The two alternators are mechanically coupled so that an angle of 25 degrees exists between their relative positions of their armature coils and field poles.

The alternators are driven by a D.C. motor of known efficiency and large enough to supply power for the losses in the alternators. The armature windings are connected so as to be electrically opposed to each other. An ammeter and the current coil of a wattmeter are connected in one of the lines which connect the armatures together, the pressure coil of the wattmeter and a voltmeter being connected between one pair of connecting lines.

The input to the driving motor is recorded on an ammeter and voltmeter in the motor circuit.

The Test.

The alternators are run at the normal speed and their excitation currents equalised and adjusted to give the normal voltage.

The excitation of one alternator is now adjusted to give a circulating current between the two machines.

This current is adjusted to any desired value. Readings are obtained from the wattmeter, the motor ammeter and voltmeter and the field ammeters of the alternators.

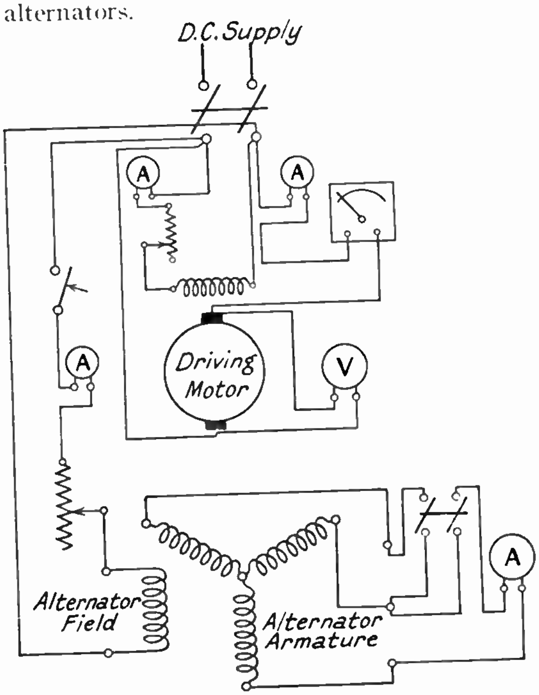


Fig. 9.—EFFICIENCY TEST ON A SINGLE ALTERNATOR.

The full load armature current is obtained by short circuiting the armature at a low exciting current.

Calculating the Efficiency.

The power supplied to make up the alternator losses = Input power to the motor at this input − The motor losses which are known.

The losses for each alternator

$$= \frac{\text{Input to motor} - \text{Losses in motor}}{2} + \text{Excitation copper losses.}$$

Efficiency of an alternator

$$= \frac{\text{Wattmeter reading}}{\text{Wattmeter reading} + \text{Alternator losses}} \times 100 \text{ per cent.}$$

EFFICIENCY TEST ON A SINGLE ALTERNATOR WHEN A SUITABLE PRIME MOVER AND LOAD MAY NOT BE AVAILABLE.

Couple the alternator to a D.C. motor

whose output is from 5 to 10 per cent. of the alternator and whose efficiency is known. Measure the input to the motor with the alternator not excited.

This input power = The losses in both machines for friction, winding and core + Copper losses in the driving motor.

Switch on the exciting current to the alternator and again measure the input to the driving motor.

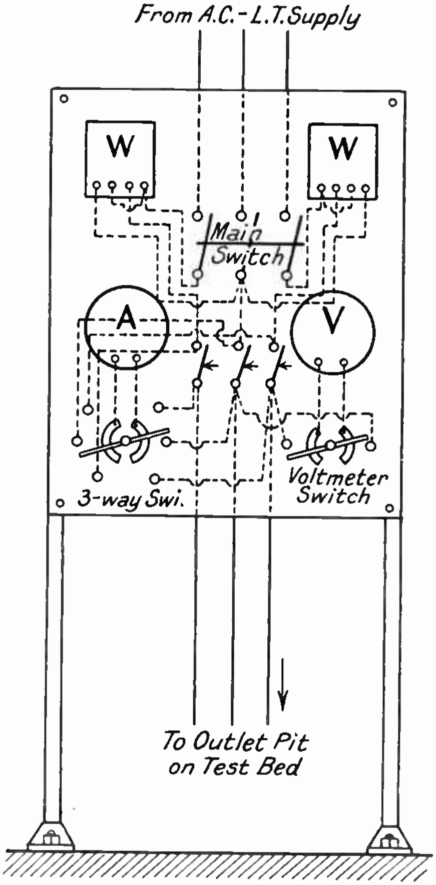


Fig. 10.—AN A.C. TEST PANEL FOR THREE-PHASE INDUCTION MOTORS.

The ammeter may be connected in any one of the three lines with the three-way switch.

This input power = All the above-mentioned losses + Armature core loss due to the excitation current passing.

A series of readings with varying exciting currents is now taken so that the iron loss in the armature core may be determined for any excitation current up to the normal value.

The speed of the motor and the exciting current should be maintained constant during the test.

The exciting current is now switched off and the armature is short circuited. The exciting current is again switched on and the value is adjusted to give any desired current up to the normal value through the armature. The input to the motor is again measured.

The difference between this input and the input which was obtained with the armature circuit open at the same excitation will give the armature copper losses + a small iron loss, which may be neglected.

The excitation copper losses of the alternator are obtained from the value of the exciting current obtained from the open circuit saturation curve of the alternator, which gives the relation between the excitation current and volts generated.

Readings.

(1) Motor ammeter and voltmeter with alternator not excited V and A.

(2) Motor ammeter and voltmeter with alternator excited V_1 and A_1. Field current of alternator and from the open circuit saturation curve of the alternator obtain the armature volts for these values.

(3) Motor ammeter and voltmeter with alternator excited V_2 and A_2, and armature short circuited, taking readings of alternator armature current A_3 for corresponding readings of the motor input current and terminal voltage.

Calculation.

$V_1A_1 - VA$ = Iron loss in alternator armature.
$V_2A_2 - V_1A_1$ = Copper loss in alternator armature.
VA − (Losses in driving motor) = Friction and windage loss in alternator.
(Exciting current)^2 × resistance of alt. field = Copper loss in alternator field.
Alternator output = A_3 × Alternator volts from saturation curve at the correct excitation.

Efficiency may now be obtained from the above data. This test has the advantage that temperature rises may be obtained although the power actually developed by the alternator is only small.

The open circuit saturation curve should be obtained previously by taking a series of readings of armature volts and exciting current with the armature on open circuit and running at normal speed.

INDUCTION MOTOR TESTS—EFFICIENCY, POWER FACTOR, OUTPUT, SLIP, AND TORQUE.

The motor is run at the rated voltage and is loaded with either a mechanical brake, or a D.C. generator which is paralleled back on the D.C. mains supply, the efficiency of the generator being known.

The input power and volt amperes to the motor at various loads is recorded and the output power measured at the brake, or electrically on the output of the generator.

The slip may be obtained in various ways.

One method is to record exactly at the same time the speed of the alternator which is generating the A.C. supply current to the motor, and the speed of the motor, and observe the number of pairs of poles on each of the machines. Another method, which is more convenient where the frequency of the A.C. supply cannot easily be obtained, is to place a large search coil, the shunt coil of a D.C. motor will do, in a favourable position near the induction motor. A sensitive galvanometer is connected across the terminals of the search coil and the number of kicks on the galvanometer in each direction is observed over a certain time. The speed of the motor is also taken.

The running-up torque is obtained by running the motor against various brake loads, these loads being not taken above the capacity of the motor, and observing the voltage at the terminals of the motor necessary to start the motor against the load applied.

The Readings Taken.

Input motor ammeter.

Input motor voltmeter.

The input wattmeters.

The reading of the fixed weight and the spring balance if a mechanical brake is used.

The speed of the motor.

The observations previously indicated for determination of the slip.

The input voltage of motor to start up against varied loads for running-up torque determination.

The output of the D.C. generator if an electrical load is used.

The Method of Calculating the Results Required.

Input volts amperes

$$= \text{Input volts} \times \text{Input amperes} \times \sqrt{3}.$$

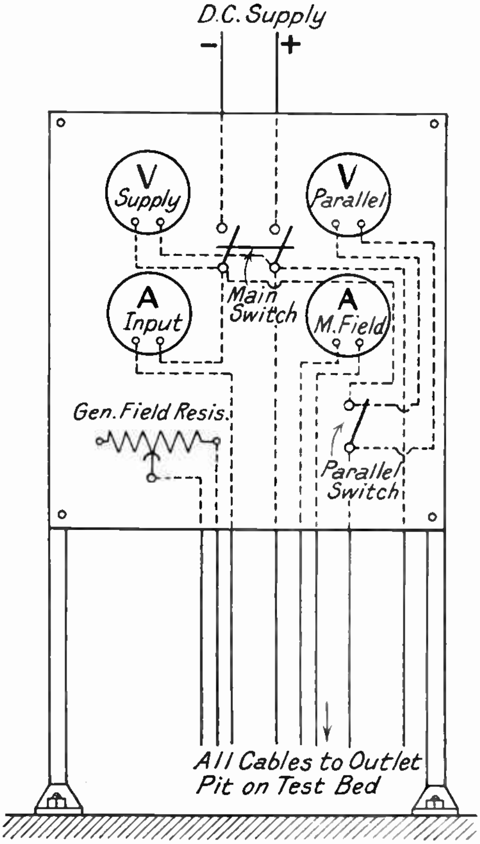


Fig. 11.—A D.C. Test Panel with Arrangement for Paralleling the Loading Generator Back to the Mains Supply.

Input watts

= The sum of the two wattmeter readings.

$$\text{Power factor} = \frac{\text{Input watts}}{\text{Input volt amperes}}$$

The output of the motor

$$= \frac{\text{Torque} \times 2\pi \times \text{Revs. per min.}}{33000} \quad \text{B.H.P.}$$

or = Output of loading generator — The losses.

$$\text{Input E.H.P.} = \frac{\text{Input watts}}{740}$$

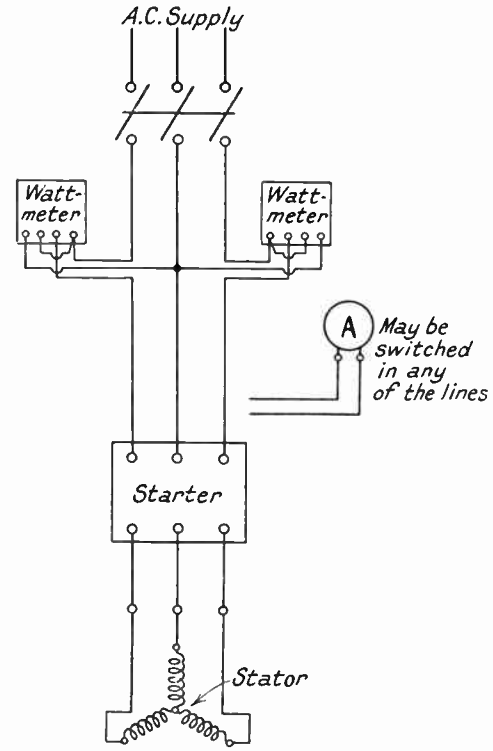


Fig. 12. CONNECTIONS TO THREE-PHASE INDUCTION MOTOR FOR EFFICIENCY AND POWER FACTOR TEST.

The efficiency of the motor

$$= \frac{\text{B.H.P.}}{\text{Input E.H.P.}}$$

$$\text{or} = \frac{\text{Output of loading gen.} - \text{The losses}}{\text{Input watts}}$$

The slip =

$$\frac{\text{Speed of A.C. generator} \times \text{No. of pairs of poles} - \text{Speed of motor} \times \text{No. of pairs of poles}}{\text{Speed of A.C. generator} \times \text{No. of pairs of poles}} \times 100 \text{ per cent.}$$

$$\text{or} = \frac{\text{No. of kicks of galv. needle per min.}}{\text{Speed of motor in revs. per min.}} \times 100 \text{ per cent.}$$

Calculation for a 3-phase Induction Motor.

Motor input voltmeter = 400 volts.
Motor input current = 24 amperes.
Wattmeter readings = 7000 and 7000 watts.
Motor speed = 1000 revs. per min.
Galv. needle kicks per min. = 45.
Generator output voltmeter = 500 volts.
Generator output ammeter = 20 amperes.
Efficiency of generator at this output = 90 per cent.

Input volt amperes

$$400 \times 24 \times \sqrt{3} = 16608 \text{ watts.}$$

$$\text{Input watts} = 7000 + 7000 = 14000 \text{ watts.}$$

$$\text{Power factor} = \frac{14{,}000}{16{,}608} = .84$$

$$\text{Motor output} = \frac{500 \times 20 \times 100}{90} = 11111 \text{ watts.}$$

$$= \frac{11111}{746} = 14.9 \text{ B.H.P.}$$

$$\text{Efficiency of motor} = \frac{11111 \times 100}{14000} = 80 \text{ per cent.}$$

$$\text{Slip} = \frac{45 \times 100}{1000} = 4.5 \text{ per cent.}$$

INSULATION RESISTANCE TESTS.

These are taken with a megger which will give a D.C. voltage of about 500 volts, and the resistance taken between the windings and frame of the machine.

The test should be taken immediately after the high voltage flash test.

The value of the insulation resistance in megohms should not be less than :—

$$\frac{\text{The rated volts}}{1{,}000 + \text{Rated output in K.V.A. or B.H.P.}}$$

The value obtained in the test should be taken when the needle of the megger has become practically steady.

QUESTION AND ANSWER

What process is employed for a back-to-back test of two similar alternators for efficiency ?

The alternators are run at the normal speed and their excitation currents equalised and adjusted to give the normal voltage. The excitation of one alternator is now adjusted to give a circulating current between the two machines. This current is adjusted to any desired value. Readings are obtained from the wattmeter, the motor ammeter and voltmeter and the field ammeters of the alternators.

SPECIAL TYPES & COMBINATIONS OF ROTATING ELECTRICAL MACHINERY

By ROBERT RAWLINSON

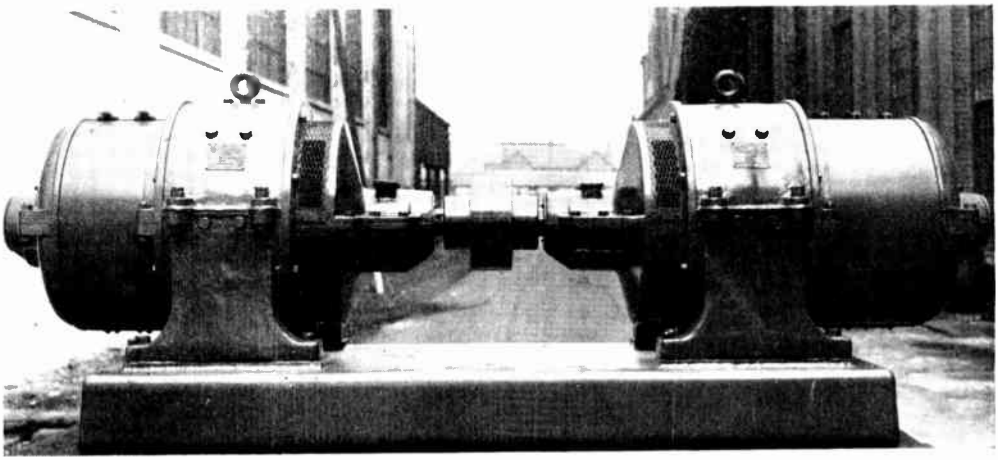


Fig. 1.—A TYPICAL D.C.-D.C. MOTOR GENERATOR SET.
This illustration is typical of most balancer and booster sets. It actually shows a battery booster (non-automatic), and the split yokes, bearings and end brackets should be noted. These assist in maintenance. Note also the fabricated steel bedplate which is stronger and more rigid than the old-fashioned cast iron bedplate construction which it has superseded. (*Metropolitan-Vickers.*)

FROM time to time, as industry develops, the need arises for special forms or combinations of generators and motors. To fill the demand special machines are designed, and these, together with their methods of construction and control, then become available for purposes other than that for which they were specifically developed.

In the following notes, we will deal with such types and combinations of machines as are commonly encountered in modern industrial practice; no attempt will be made to present a full list of special machines, neither will arrangements of purely historic interest be considered, and our notes will be confined to a description of any special points of mechanical or electrical design and construction, together with a brief description of the basic principles of the machines' operation. No detailed operating instructions can be usefully given, since various manufacturers' arrangements are so widely different and for the same reason the circuit diagrams are not in full detail as regards meters, switches, fuses and protective circuits.

Balancers.

When considering the D.C. distribution of electrical power it can be said, subject to insulation and space considerations, that the higher the voltage the lower the capital outlay on cables. This is obvious since the power transmitted is the product of the voltage and current, while the amount of copper in a cable, to maintain a certain current density, is only proportional to the current. This consideration led supply authorities to adopt the highest possible voltage for their D.C. distribution systems and we thus have 460, 480 and 500 volts circuits in common use.

Why D.C. Power is Usually Distributed on the 3-wire System.

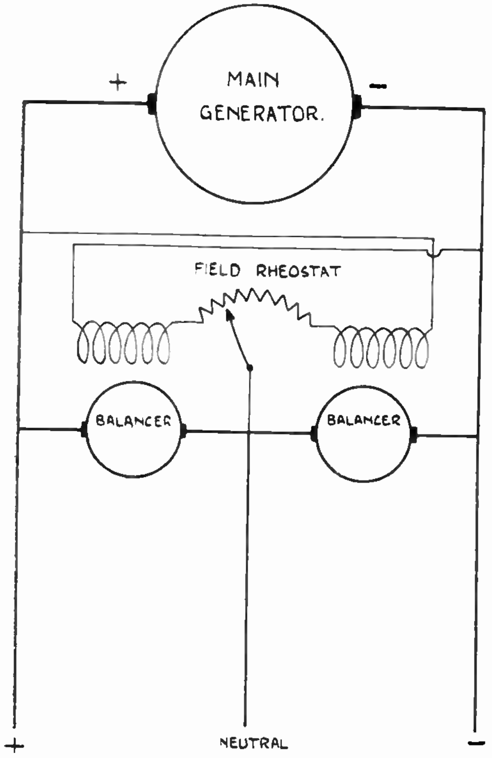


Fig. 2.—DIAGRAM OF CONNECTIONS OF 3-WIRE BALANCER.

The two balancer machines are coupled together and mounted on a bedplate, as shown in Fig. 1. In this diagram the main generator field is not shown. Observe the balancer field cross connections and the connections of the field rheostat.

These higher voltages are very satisfactory so far as power circuits are concerned, but when we consider lighting, we find that about 250 volts is the limit, both as regards satisfactory lamp manufacture and safety in domestic use, and we therefore find that D.C. power is usually distributed on the 3-wire system. This method involves the use of three cables, two "outers" and one "mid-wire"; the mid-wire (or neutral) is kept at a potential half-way between the two outers and it is usually earthed. For instance, on a 500-volt 3-wire system the potentials would be, considering negative as zero :—

Negative outer . .	Zero.
Mid-wire . .	250 volts positive.
Positive outer . .	250+250 volts positive=500 volts.

If now the neutral is earthed, its potential must be considered as zero and the potential distribution becomes

Negative outer . .	250 volts negative.
Earthed mid-wire	Zero.
Positive outer . .	250 volts positive.

Notice that the potential difference between the outers is still 500 volts, and that they may therefore be used for power supply, while lighting load may be taken on a circuit between either outer and the mid-wire.

Balancing the Load.

It is, of course, necessary to balance the load on the two sides of a 3-wire system, and the supply authorities endeavour to do this by connecting equal numbers of houses, for instance, on both positive and negative sides, but it will be clear that under certain conditions the load will not be equal and that a current will flow in the mid-wire, back to the generating station. This will result in a greater voltage drop in the cables on the highly loaded side, the voltage of which will thus tend to drop, while due to the definite difference (of 500 volts in our example) between outers, the voltage on the lightly loaded side will tend to rise. When this happens we lose the advantages of half voltage, which the three-wire system gives us, and it is therefore necessary to ensure that the voltage difference between the two sides shall not exceed a certain definite amount when the loading differences are the greatest that experience has taught us may be expected.

A "3-wire Balancer Set."

This function of voltage balancing on the two sides is done by a "3-wire balancer set," which is designed with regard to the maximum out of balance current which the supply company considers it necessary to legislate for, and which usually consists of two exactly similar shunt or compound wound D.C. machines, with their shafts coupled together, mounted on a bedplate. Each machine is wound for the voltage which is to be maintained between one outer and the neutral, and the armatures are connected in series across the outer cables. The mid-wire comes from the centre point of the two series connected armatures, and the fields are also connected in series across the outers; a field rheostat is usually connected in the shunt circuit, the tapping point being connected to the mid-wire.

The diagram of connections will make this clear, and it will be noted that the fields are shown cross connected, i.e., the field of the positive side machine is connected across negative and mid-wire, while the negative side machine has its field circuit fed between positive outer and neutral. This cross connection gives better balancing. The action of the set is as follows:—

Tendency to Change Speed.

When there is no current in the mid-wire, the two machines (being so connected) will run as unloaded motors, each one dropping half the total voltage and running at the same speed, since both are identical and are wound for half voltage; the mid-wire is thus maintained accurately at half potential between the outers. Imagine now that a load comes on the positive side; as explained before, the voltage on that side will tend to drop and the voltage on the negative side will tend to increase, and it follows that the positive side balancer machine will tend to drop in speed while the machine on the negative side will tend to run at a higher speed.

This tendency to change speed is further increased by the field cross connection previously referred to; the positive machine's field being supplied from the higher voltage side is strengthened, tending to a lower speed, while the negative machine's field is weakened by its connection on the lower voltage side, this field weakening tending towards a higher speed.

How the Unbalanced Power is Divided Between the Two Sides of the System.

Now the two machines are coupled together and cannot run at different speeds, so that the total effect of the varying voltages on the two sides is to cause the negative side machine to run as a motor and drive the positive side machine as a generator, raising the voltage on the more highly loaded positive side. In other words, the unbalanced power, represented by the mid-wire current, is used partly to drive the negative machine as a motor, the remainder of the unbalanced current being passed through the positive machine which is generating and supplying power to the more highly loaded side. By this method the unbalanced power is divided between the two sides of the

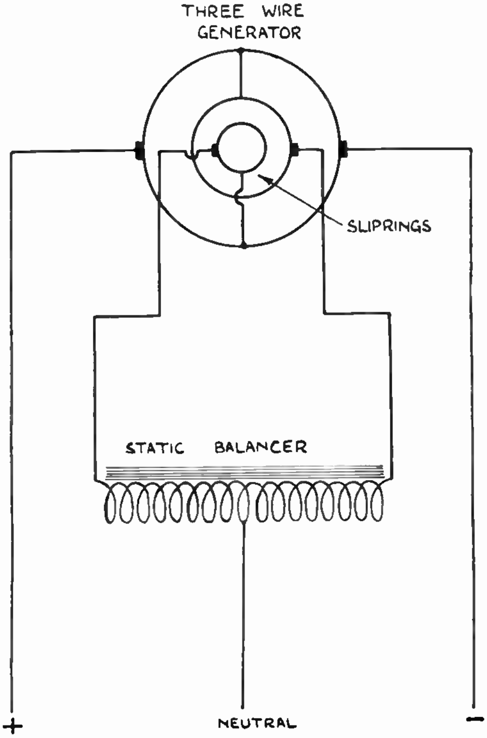


Fig. 3.—CONNECTIONS OF A STATIC BALANCER. The 3-wire generator is similar to a standard D.C. generator with the addition of the sliprings for the static balancer connections. The static balancer itself is simply a choking coil with a centre tap for the neutral connection.

system, and balance, with the mid-wire potential within the correct limits, is attained.

STATIC BALANCER.

Although this is not a rotating machine, its use entails modifications to the normal form of D.C. generator, and we will, therefore, discuss its use briefly.

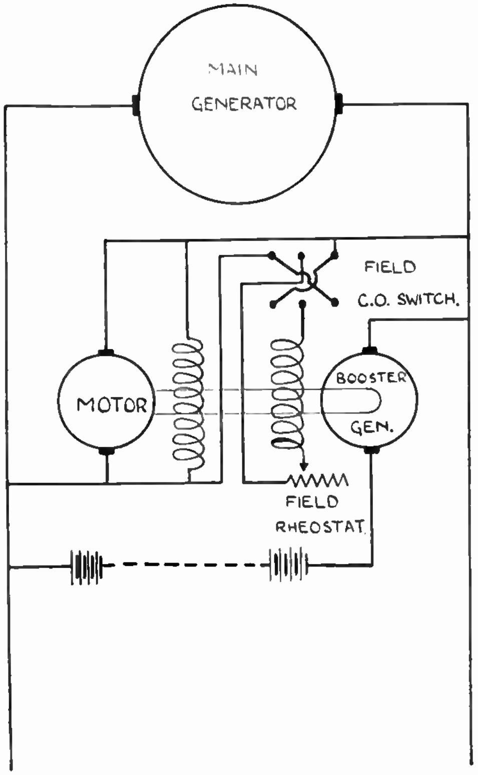


Fig. 4.—The Reversible Battery Booster. This diagram shows the connections of a non-automatic reversible battery booster. The main generator field is not shown and the booster motor is shown of the ordinary shunt wound type. The amount of boost may be varied by the field rheostat and the direction of the booster generator E.M.F. may be altered by means of the field change-over switch, so as to assist the charging or discharging of the battery.

When a three-wire supply is required, and it is not desired to use a balancer machine set such as has previously been described, a static balancer may be used in conjunction with a D.C. generator, which is fitted with slip-rings for connection to the balancer. These slip-rings are connected to points in the armature which are one pole pitch apart, and they thus have the full generated voltage between them.

Wave Wound and Lap Wound Generators.

For the benefit of those readers who are interested in armature winding, we may say that with wave wound generators, each ring is connected to one point only in the armature winding, and the two points are situated as nearly as possible one pole pitch apart in the winding plan. In cases where the generator armature is lap wound, it is usual to connect each ring as though it were an equaliser, i.e., each ring is connected to as many points as there are pairs of poles, and each point must be situated two pole pitches away from the next nearest point connected to the same ring; the second slip-ring is connected to points displaced by one pole pitch from those to which the first slip-ring is connected. Thus, in both cases, the full voltage of the generator exists between the slip-rings, and since the current has not been commutated—having passed from the winding via the slip-ring—it is alternating as in an ordinary A.C. circuit.

How the 3-wire Supply is Taken.

Across these slip-rings a choking coil with a centre tap is connected, and the 3-wire supply is taken as follows:—

Positive outer ..	Generator positive terminal.
Mid-wire ..	Centre tap of choking coil.
Negative outer ..	Generator negative terminal.

The Choking Coil.

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THE BISHOP'S FOLLY

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"Stop!" he exclaimed. "Uncle Theodore, don't open that hamper!"

But it was too late. Already the Bishop was cutting the strings with a hand that trembled with eagerness. Chirruping noises proceeded from him. In his eyes was the wild gleam seen only in the eyes of cat-lovers restored to their loved one.

"Webster!" he called in a shaking voice.

And out of the hamper shot Webster, full of strange oaths. For a moment he raced about the room, apparently searching for the man who had shut him up in the thing, for there was flame in his eye. Becoming calmer, he sat down and began to lick himself, and it was then for the first time that the Bishop was enabled to get a steady look at him.

Two weeks' residence at the vet.'s had done something for Webster, but not enough. Not, Lancelot felt agitatedly, nearly enough.

A mere fortnight's seclusion cannot bring back fur to lacerated skin; it cannot restore to a chewed ear that extra inch which makes all the difference. Webster had gone to Doctor Robinson looking as if he had just been caught in machinery of some kind, and that was how, though in a very slightly modified degree, he looked now. And at the sight of him the Bishop uttered a sharp, anguished cry. Then, turning on Lancelot, he spoke in a voice of thunder.

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