SUPPLEMENT

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TEC, SCOTEC & CGLI: GUIDANCE FOR STUDENTS

Contents

CITY AND GUILDS OF LONDON INSTITUTE

Questions and Answers

Answers are occasionally omitted or reference is made to earlier Supplements in which questions of substantially the same form, together with the answers, have been published. Some answers contain more detail than would be expected from candidates under examination conditions.

PRACTICAL MATHEMATICS 1978

Students were expected to answer any 6 questions

Q1 (*a*) *If* $y = \dfrac{4{\cdot}491 \times 0{\cdot}837}{59{\cdot}62}$,

(*i*) *use logarithms only to calculate an accurate value of y, showing all working,*
(*ii*) *rewrite the above equation, expressing each number to 2 significant figures, and*
(*iii*) *hence estimate the value of y.*

(*b*) *Using mathematical tables, evaluate*

(*i*) $\dfrac{1}{0{\cdot}12} + \dfrac{1}{0{\cdot}096}$, *and*

(*ii*) $(1{\cdot}67)^2 - (0{\cdot}91)^2$.

(*c*) *Find x if*

(*i*) $10^x = \dfrac{1}{100}$, *or*

(*ii*) $\log_{10} x = 4$.

A1 (*a*) (*i*) $\log y = \log 4{\cdot}491 + \log 0{\cdot}837 - \log 59{\cdot}62,$

$= 0{\cdot}6523 + \bar{1}{\cdot}9227 - 1{\cdot}7753,$

$= \bar{2}{\cdot}7997.$

$\therefore \quad y = \text{antilog } \bar{2}{\cdot}7997 = \underline{0{\cdot}06305}.$

(*ii*) $\underline{y = \dfrac{4{\cdot}5 \times 0{\cdot}84}{60}}.$

(*iii*) $y = \dfrac{0{\cdot}3 \times 0{\cdot}84}{4} = \underline{0{\cdot}063}$ to 2 significant figures.

(*b*) (*i*) $\dfrac{1}{0{\cdot}12} + \dfrac{1}{0{\cdot}096} = 8{\cdot}333 + 10{\cdot}42$ (from reciprocal tables),

$= \underline{18{\cdot}753}.$

(*ii*) $(1{\cdot}67)^2 - (0{\cdot}91)^2 = 2{\cdot}790 - 0{\cdot}8281$ (from table of squares),

$= \underline{1{\cdot}9619}.$

(*c*) (*i*) $10^x = \dfrac{1}{100} = \dfrac{1}{10^2} = 10^{-2}.$

$\therefore \quad \underline{x = -2}.$

(*ii*) $\log_{10} x = 4.$

$\therefore \quad x = 10^4 = \underline{10\,000}.$

Q2 (*a*) *Evaluate*

(*i*) $\left(\dfrac{3x}{2y}\right)^3 \div \left(\dfrac{x}{4y}\right)^2$, *and*

(*ii*) $\sqrt{\left(\dfrac{4}{9}\right)^3}$.

(*b*) *Find x if* $\dfrac{1}{2x} + \dfrac{1}{3x} = \dfrac{1}{60}$.

(*c*) *The constituents of a mixture are: A, 38%; B, 42%; and C the remainder. To a sample of the mixture of mass 20 kg, a further 4 kg of C is added. Find the percentage of constituent B in the new mixture.*

A2 (*a*) (*i*) $\left(\dfrac{3x}{2y}\right)^3 \div \left(\dfrac{x}{4y}\right)^2 = \left(\dfrac{3x}{2y}\right)^3 \times \left(\dfrac{4y}{x}\right)^2,$

$= \dfrac{27x^3 \times 16y^2}{8y^3 \times x^2} = \underline{\dfrac{54x}{y}}.$

(*ii*) $\sqrt{\left(\dfrac{4}{9}\right)^3} = \left(\dfrac{2}{3}\right)^3 = \underline{\dfrac{8}{27}}.$

(*b*) $\dfrac{1}{2x} + \dfrac{1}{3x} = \dfrac{1}{60}.$

$\therefore \quad \dfrac{3+2}{6x} = \dfrac{1}{60}.$

$\therefore \quad 5 \times 60 = 6x.$

$\therefore \quad \underline{x = 50}.$

(*c*) The weight of constituent B in the original mixture

$= \dfrac{42}{100} \times 20 \text{ kg} = 8{\cdot}4 \text{ kg}.$

The weight of the new mixture = 20 + 4 = 24 kg.
Therefore, the percentage of constituent B in the new mixture

$= \dfrac{8{\cdot}4}{24} \times 100 = \underline{35\%}.$

Q3 (*a*) *Simplify*

(*i*) $3a(a+b) - 2b(a-2b)$, *and* (*ii*) $2d\,[4d - 3(d+4)] + 6d$.

(*b*) *If* $C = \dfrac{M_1 V}{M_1 + M_2}$,

(*i*) *make* M_2 *the subject of the above formula, and*
(*ii*) *calculate* V *if* $M_1 = 540$ *kg*, $M_2 = 362$ *kg, and* $C = 37\cdot 2$ *m/s.*

(*c*) *If f is inversely proportional to the square root of the product of L and C*
(*i*) *write down a formula for f using a constant, k, and*
(*ii*) *if L is increased twelve times and C is increased three times, by how many times is f decreased?*

A3 (*a*) (*i*)

$$3a(a+b) - 2b(a-2b),$$
$$= 3a^2 + 3ab - 2ab + 4b^2,$$
$$= \underline{3a^2 + ab + 4b^2}.$$

(*ii*)

$$2d[4d - 3(d+4)] + 6d,$$
$$= 2d[4d - 3d - 12] + 6d,$$
$$= 2d(d-12) + 6d,$$
$$= 2d^2 - 24d + 6d,$$
$$= 2d^2 - 18d = \underline{2d(d-9)}.$$

(*b*)

$$C = \frac{M_1 V}{M_1 + M_2}.$$

(*i*) $C(M_1 + M_2) = M_1 V.$

$$\therefore \quad CM_2 = M_1 V - M_1 C = M_1(V - C).$$

$$\therefore \quad \underline{M_2 = \frac{M_1(V-C)}{C}}.$$

(*ii*)

$$V = \frac{C(M_1 + M_2)}{M_1},$$
$$= \frac{37\cdot 2(540 + 362)}{540},$$
$$= \frac{37\cdot 2 \times 902}{540} = \underline{62\cdot 13\dot{7}\ \text{m/s}}.$$

Note: Since $\dfrac{M_1 + M_2}{M_1}$ is a ratio of masses, the units of V must be the same as those of C; that is, a velocity in metres/second.

(*c*) (*i*)

$$f \propto \frac{1}{\sqrt{(LC)}}.$$
$$\therefore \quad \underline{f = \frac{k}{\sqrt{(LC)}}}.$$

(*ii*) If L_1 and C_1 are the new values of L and C, then

$$f_1 = \frac{k}{\sqrt{(L_1 C_1)}} = \frac{k}{\sqrt{(12L \times 3C)}},$$
$$= \frac{k}{\sqrt{(36LC)}} = \frac{k}{6\sqrt{(LC)}}.$$

Hence, $\dfrac{f_1}{f} = \dfrac{1}{6}$; that is, <u>$f$ is decreased 6 times.</u>

Q4 (*a*) *If* $g = -5$ *and* $h = 2$, *find the value of*

(*i*) $g^2 - h^2$, *and* (*ii*) $5h - 2g$.

(*b*) *Express as single fractions, simplifying as far as possible,*

(*i*) $\dfrac{3x}{2y} + \dfrac{2x^2}{3y^2} + \dfrac{x}{4y}$, *and* (*ii*) $\dfrac{3}{(x+6)} - \dfrac{2}{(x+4)}$.

(*c*) *Solve for m and n the equation* $3m - n = m + 2n = 7$.

Q5 (*a*) *Simplify* $(2gh^2)^{-3} \times (4g^2h^2)^2$.

(*b*) (*i*) *If* $\dfrac{1}{R} = \dfrac{1}{R_1} + \dfrac{1}{R_2} + \dfrac{1}{R_3}$, *calculate R when* $R_1 = 15$, $R_2 = 20$, *and* $R_3 = 30$.

(*ii*) *If* $W = \frac{1}{2}LI^2$, *calculate W when* $L = 1\cdot 2 \times 10^{-5}$, *and* $I = 1\cdot 3 \times 10^{-2}$, *giving your answer in the same form as L and I.*

(*c*) *On a slide rule, the actual distance between the figures 1 and 2 is 3 cm. Find the actual distance between the figures*

(*i*) *2 and 10, and* (*ii*) *5 and 70.*

(*d*) *The areas of 2 similar triangles are in the ratio 1:9. The perimeter of the smaller one is 12 cm. Find the perimeter of the larger one.*

A5 (*a*)

$$(2gh^2)^{-3} \times (4g^2h^2)^2 = \frac{16g^4h^4}{(2gh^2)^3},$$
$$= \frac{16g^4h^4}{8g^3h^6} = \underline{\frac{2g}{h^2}}.$$

(*b*) (*i*)

$$\frac{1}{R} = \frac{1}{15} + \frac{1}{20} + \frac{1}{30},$$
$$= \frac{4+3+2}{60} = \frac{9}{60} = \frac{3}{20}.$$
$$\therefore \quad R = \frac{20}{3} = \underline{6\tfrac{2}{3}}.$$

(*ii*)

$$W = \tfrac{1}{2} \times 1\cdot 2 \times 10^{-5} \times (1\cdot 3 \times 10^{-2})^2$$
$$= 0\cdot 6 \times 10^{-5} \times 1\cdot 69 \times 10^{-4}$$
$$= \underline{1\cdot 014 \times 10^{-9}}.$$

(*c*) The physical distances along a slide-rule scale are proportional to the logarithms of the numbers on the scale.

Now, $\log_{10} 1 = 0$ and $\log_{10} 2 = 0\cdot 3010$.

Thus, the difference between these logarithms, $0\cdot 3010$, is represented by 3 cm.

(*i*) $\log_{10} 2 = 0\cdot 3010$ and $\log_{10} 10 = 1\cdot 000$.
The logarithmic difference is $1\cdot 000 - 0\cdot 3010 = 0\cdot 6990$.

A difference of $0\cdot 301$ is represented by 3 cm. Therefore, a logarithmic difference of $0\cdot 699$ is represented by

$$\frac{3}{0\cdot 301} \times 0\cdot 699 = \underline{6\cdot 967\ \text{cm}}.$$

(*ii*) $\log_{10} 5 = 0\cdot 6990$ and $\log_{10} 70 = 1\cdot 8451$.
Similarly to part (*i*), the logarithmic difference of $1\cdot 8451 - 0\cdot 699 = 1\cdot 1461$ and is represented by,

$$\frac{3}{0\cdot 301} \times 1\cdot 1461 = \underline{11\cdot 423\ \text{cm}}.$$

Note: If, as seems likely, the question was intended to relate to the usual slide rule with 2 scales on the stock (upper and lower), the numbers 5 and 70 would appear together only on the upper scale. As this scale is half the physical or logarithmic length of the lower scale, the distance between 5 and 70 would be half of the above answer; that is $5\cdot 711$ cm.

(*d*) If k is the constant of proportionality between the linear dimensions of one triangle and the corresponding linear dimensions of a similar triangle, then

$$\frac{\text{area of first triangle}}{\text{area of similar triangle}} = k^2 = 9.$$
$$\therefore \quad k = \sqrt{9} = 3.$$

Hence, the perimeter of the larger triangle

$$= 3 \times 12 = \underline{36\ \text{cm}}.$$

Q6 A current, I amperes, passing through a meter causes a deflexion, θ degrees, as shown in the table.

I	0	0·5	1·0	1·5	2·0	2·5	3·0
θ	0	20	36	47	55	60·5	65

(a) Draw a graph plotting I against tan θ.

(b) Show that the law $I = k \tan\theta$ is approximately true and find the value of the constant k.

(c) From the graph, find the value of

(i) I, when θ = 50, and (ii) θ, when I = 1·2.

Q7 (a) Use your tables to find the value of

(i) cos 48° 34′, (ii) tan 67° 42′, and (iii) sin 52° 39′.

(b) Triangle ABC is right-angled at B. AB = 5 cm and BC = 12 cm. N is the foot of the perpendicular from B to AC.

(i) Calculate the length of AC,
(ii) calculate the area of the triangle, and
(iii) use the result in part (b)(ii) to calculate the length of BN.

(c) PQR is an equilateral triangle whose sides are of length 2a. QN is the perpendicular from Q to PR. Use the right-angled triangle PQN to work out, leaving your answer in terms of $\sqrt{3}$ where appropriate,

(i) sin 30°, and (ii) cos 30°.

From the triangle, prove that $\cos 60° = (\cos 30°)^2 - (\sin 30°)^2$.

Q8 (a) Write down the equations of the following lines, using positive whole number coefficients:
(i) with the gradient of $+\frac{1}{2}$ and intercept on the y-axis of +3, and
(ii) with a gradient of −3 and intercept on the y-axis of −1.

(b) Make a sketch graph of each line in part (a).

(c) If $3x + y = 18$ and $5x - 2y = 8$, calculate
(i) the co-ordinates of the point of intersection of these straight-line graphs, and
(ii) the area contained between the 2 lines and the x-axis.

(d) Two points, A and B, on a straight-line graph have coordinates (2, 5) and (−2, 3); a third point, C, has co-ordinates (0, k). Obtain the value of k.

A8 (*a*) The equations of the lines are of the form

$$y = mx + c,$$

where *m* is the gradient and *c* is the intercept on the *y*-axis.

(*i*) The equation of the line is

$$y = \tfrac{1}{2}x + 3,$$

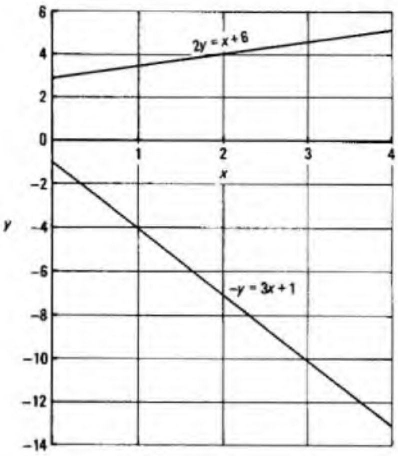
or $$\underline{2y = x + 6.}$$

(*ii*) The equation of the line is

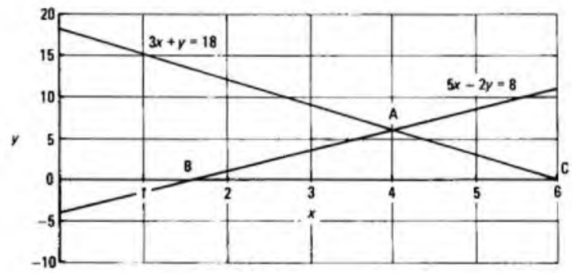
$$y = -3x - 1,$$

or $$\underline{-y = 3x + 1.}$$

(*b*) The graphs are shown in sketch (*a*).



(*a*)



(*b*)

(*c*) (*i*) The lines intersect at the point where the same value of *x* and the same value of *y* satisfy both equations. Therefore, solving the 2 equations as a pair of simultaneous equations

$$3x + y = 18 \qquad \ldots\ldots (1)$$

$$5x - 2y = 8 \qquad \ldots\ldots (2)$$

Multiplying equation (1) by 2 gives

$$6x + 2y = 36 \qquad \ldots\ldots (3)$$

Adding equations (2) and (3) gives

$$11x = 44.$$

$$\therefore \quad x = 4$$

Substituting for *x* in equation (1) gives

$$y = 18 - 12 = 6.$$

Thus, the co-ordinates of the intersection point of the 2 lines are $\underline{(4, 6)}$.

(*ii*) $$3x + y = 18.$$

When $x = 0$, $y = 18$ and when $y = 0$, $x = 6$. The graph of the straight line may then be drawn, as shown in sketch (*b*), the intersection point acting as a check.

In a similar way, the graph of $5x - 2y = 8$ passes through the points (0, −4) and ($1\frac{3}{5}$, 0).

The area contained between the 2 lines and the *x*-axis is the triangle ABC shown in sketch (*b*).

$$\text{The area contained} = \tfrac{1}{2}(6 - 1\tfrac{3}{5}) \times 6,$$

$$= 3 \times 4\tfrac{2}{5} = \underline{13\tfrac{1}{5} \text{ square units.}}$$

(*d*) The gradient of the line is

$$\frac{5-3}{2-(-2)} = \frac{1}{2}.$$

The equation of the line may therefore be written in the form $y = mx + c$.

$$\therefore \quad y = \frac{x}{2} + c.$$

Substituting the co-ordinates (2, 5) into this equation gives

$$10 = 2 + 2c.$$

$$\therefore \quad c = 4.$$

Hence the equation of the line is

$$2y = x + 8.$$

Substitution of the co-ordinates of point C into the above equation gives

$$2k = 8 \text{ or } \underline{k = 4.}$$

Q9 The maximum current, I, passing through a circuit is determined by the formula

$$I = \frac{24}{(V + 2)}.$$

(a) Give a table of values for I as V increases from 0 to 10 at 2-unit intervals.

(b) Using the table in part (a), plot a graph of I against V.

(c) From the graph, estimate,
(i) I when $V = 4{\cdot}5$, and (ii) V when $I = 7$, showing the points clearly on your graph.

(d) By drawing a straight line on your graph from the points where $V = 3$ to $V = 8$, determine the average change in I per unit of V.

Q10 A semicircular duct is cut into a block of rectangular cross-section, as shown in Fig. 1, where
$$AX = DY = EZ = 1 \text{ cm, and } BC = 15 \text{ cm.}$$

(a) Calculate

(i) the radius of the semicircle,
(ii) the length of AB, and
(iii) the cross-sectional area of the block remaining.

(b) If, when the duct is full, water flows at 3 cm/s along it, what volume of water will pass along the duct in 5 min. Express your answer in litres.

A10 (*a*) (*i*) The diameter, XY, of the semicircle is
$BC - AX - YD = 15 - 1 - 1 = 13$ cm.

Therefore the radius of the semicircle is $\underline{6{\cdot}5 \text{ cm}}$.

(*ii*) AB = the radius of the semicircle + EZ

$$= 6{\cdot}5 + 1 = \underline{7{\cdot}5 \text{ cm}}.$$

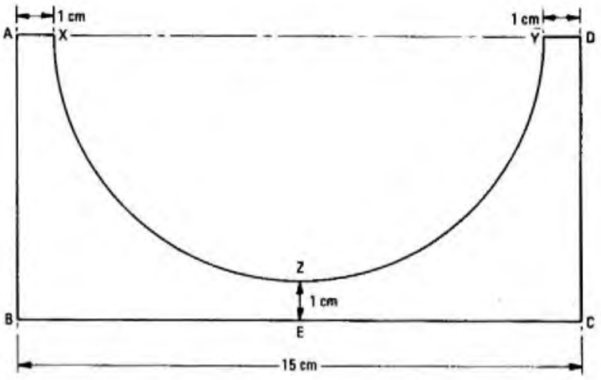


Fig. 1.

(*iii*) The remaining cross-sectional area of the block is the area of the rectangle ADCB minus the area of semicircle XYZ,

$$= AB \times BC - \frac{\pi r^2}{2} \text{ where } r = 6{\cdot}5 \text{ cm},$$

$$= 7{\cdot}5 \times 15 - \frac{\pi \times 6{\cdot}5^2}{2} = 112{\cdot}5 - 66.366,$$

$$= \underline{46{\cdot}134 \text{ cm}^2}.$$

(*b*) The volume of water passing along the duct in 1 s is the cross-sectional area of the duct times 3 cm³.

From part (*a*)(*iii*), the cross-sectional area of the duct is $66{\cdot}366 \text{ cm}^2$.

Therefore, the volume of water flowing in 5 min

$$= 66{\cdot}366 \times 3 \times 5 \times 60 \text{ cm}^3,$$

$$= \frac{66{\cdot}366 \times 15 \times 60}{1000} = \underline{59{\cdot}73 \text{ litres}}.$$

ENGINEERING SCIENCE 1978

Students were expected to answer 2 questions from Q1–4 and 4 questions from Q5–10. The use of pocket calculators was permitted

Q1 (a) State the principle of moments and give the SI unit for the moment of a force about a point.

(b) A uniform steel bridge of mass 3×10^3 kg and length 6 m is used to allow vehicles to be driven from a landing stage onto a car ferry, as shown in Fig. 1.

(i) Find the reaction at the pivot, A, when the bridge is horizontal.
(ii) Determine the compressive force in the bridge when it is raised by the cable to make an angle of 30° with the vertical.

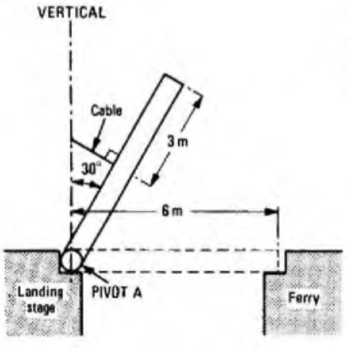
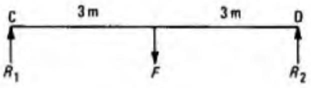
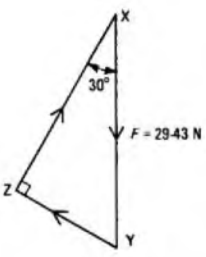


Fig. 1



(*a*)



(*b*)

A1 (*a*) See A1, Engineering Science 1975, *Supplement*, Vol. 68, p. 93, Jan. 1976.

(*b*) (*i*) It is assumed that, when the bridge is horizontal, there is no tension in the cable, so that the mass of the bridge is supported by the landing stage and the ferry. The forces acting on the bridge are shown in sketch (*a*). R_1 and R_2 are the reactions at the pivot and ferry respectively, and F is the force due to gravity acting on the mass of the bridge vertically downwards through the centre of the bridge. F is equal to $3 \times 10^3 \times 9{\cdot}81 = 29{\cdot}43 \times 10^3$ N.

Taking moments about D, $3F = 6R_1$.

$$\therefore\ R_1 = \frac{3 \times 29{\cdot}43 \times 10^3}{6} = \underline{14{\cdot}72 \times 10^3 \text{ N}}.$$

(*ii*) The compressive force in the bridge when it is raised to make an angle of 30° with the vertical can be found by applying the principle of the triangle of forces to the point at which the cable is connected to the bridge.
The triangle is shown in sketch (*b*). XY represents the force due to gravity acting vertically downwards. YZ represents the tension in the cable, and XZ represents the compressive stress in the bridge. XZY is a right angle. Therefore, cos 30° = XZ/XY, so that the compressive stress in the bridge

$$= XY \cos 30^\circ = 29{\cdot}43 \times 0{\cdot}866 = \underline{25{\cdot}5 \text{ N}}.$$

Q2 (a) Explain the following terms, used with reference to machines:
(i) velocity ratio, and
(ii) efficiency.

(*b*) *A machine raises a mass of 120 kg through a vertical distance of 8 m in 30 s. The machine has an efficiency of 40% and a velocity ratio of 60. Find*

(*i*) *the increase in potential energy of the load,*
(*ii*) *the mechanical advantage of the machine, and*
(*iii*) *the required power input.*

A2 (*a*) See A1, Engineering Science 1976, *Supplement*, Vol. 70, p. 1, Apr. 1977.

(*b*) (*i*) The increase in the potential energy of the load is the work that would be done if it were allowed to fall to the position it occupied before it was raised. This work is due to the force of gravity acting on the mass of the load over the vertical distance concerned. Therefore, the increase in potential energy

$$= 120 \times 9{\cdot}81 \times 8 = \underline{9418\ \text{J}}.$$

(*ii*) The efficiency of a machine can be expressed as the ratio of the mechanical advantage to the velocity ratio, times 100%. Thus, the mechanical advantage

$$= \frac{40 \times 60}{100} = \underline{24.}$$

(*iii*) Power is the rate of doing work. Therefore the power, *P* watts, required to raise the load is given by the work done divided by the time taken. The work done in raising the load is the same as the resulting increase in potential energy.

$$\therefore\ P = \frac{9418}{30} = 313{\cdot}9\ \text{W}.$$

However, the machine has an efficiency of 40%, and efficiency can also be expressed as the ratio of the output power to the input power, times 100%. Thus, the required input power is

$$\frac{313{\cdot}9 \times 100}{40} = \underline{784{\cdot}75\ \text{W}.}$$

Q3 A car travels along a straight length of motorway and passes 3 lamp posts, A, B and C, which are equally spaced along the centre of the motorway, as shown in Fig. 2. The car passes lamp A at 72 km/h, accelerates uniformly between A and B, and moves at a uniform speed between B and C. The car takes 2·0 s to pass from lamp A to lamp B.

(*a*) *Find the acceleration of the car.*
(*b*) *Find the speed of the car as it passes lamp B.*
(*c*) *Sketch a velocity/time graph of the motion of the car between lamps A and C.*
(*d*) *Using the velocity/time graph, or otherwise, determine the time taken by the car to travel 60 m from lamp A.*

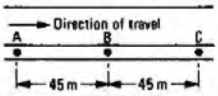
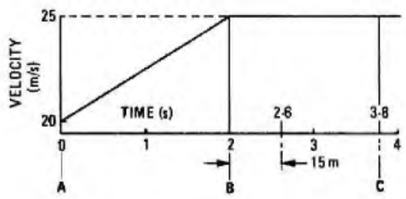


Fig. 2



(*a*)

A3 (*a*) Acceleration = $\underline{2{\cdot}5\ \text{m/s}^2}$ (from $s = ut + \frac{1}{2}at^2$).

(*b*) Speed = $\underline{25\ \text{m/s}}$ (from $v = u + at$).

(*c*) Graph shown in sketch (*a*). Note that 72 km/h = 20 m/s. Car passes C 1·8 s after passing B (from $v = s/t$).

(*d*) 60 m from A is 15 m past B. Car passes this point 0·6 s after passing B (from $v = s/t$).

a: acceleration
s: distance
t: time
u: initial velocity
v: final velocity

Q4 (*a*) *Describe 3 ways in which a hot body can transfer heat to its surroundings.*

(*b*) *Sketch a typical heat sink, and explain briefly the various design features.*

Q5 For the circuit shown in Fig. 3, calculate

(*a*) *the total resistance of the circuit,*
(*b*) *the current in the 6 Ω resistor,*
(*c*) *the potential difference across the 5 Ω resistor, and*
(*d*) *the charge drawn from the battery in 3 min.*

Assume that the internal resistance of the cells is negligible.

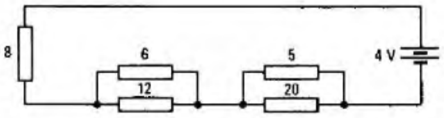


Fig. 3

A5 (*a*) The total resistance of 2 resistors, R1 and R2, connected in parallel is given by $R_1R_2/(R_1 + R_2)$, so that the total resistance of the circuit is

$$8 + \frac{6 \times 12}{6 + 12} + \frac{5 \times 20}{5 + 20} = 8 + 4 + 4 = \underline{16\ \Omega.}$$

(*b*) By Ohm's law, the total current in the circuit is 4/16 = 0·25 A. For the 6 Ω and 12 Ω resistors in parallel, the current divides in inverse ratio to the values of the resistors; that is, twelve-eighteenths of the current flows in the 6 Ω resistor. This current is therefore

$$\frac{12}{18} \times 0{\cdot}25\ \text{A} = \underline{167\ \text{mA}.}$$

(*c*) By Ohm's law, the voltage across the parallel combination of the 5 Ω and 20 Ω resistors (and therefore the voltage across the 5 Ω resistor) is given by the current in the circuit multiplied by the resistance of this combination, and is therefore

$$0{\cdot}25 \times 4 = \underline{1\ \text{V}.}$$

(*d*) Charge (in coulombs) is given by current (in amperes) multiplied by time (in seconds). Therefore the charge drawn from the battery in 3 min

$$= 0{\cdot}25 \times 3 \times 60 = \underline{45\ \text{C}.}$$

Q6 (*a*) *Compare the resistivities of conductors, semiconductors and insulators, and give one example of each material.*

(*b*) *A nichrome strip is used as a heating element, and has a resistance of 15 Ω at 0°C and a temperature coefficient of resistance* $1{\cdot}0 \times 10^{-4}$*/°C.*

(*i*) *What is the resistance of the heating element at 400°C?*
(*ii*) *If the resistivity of the nichrome strip is* $1{\cdot}0 \times 10^{-6}$ *Ω m at 0°C and it has a rectangular cross-section 5 mm × 0·005 mm, find the length of the strip.*

A6 (*a*) The resistivity of common conductors is of the order of 10^{-8} Ω m, and one of the most common conductors is copper. The resistivity of common insulators is generally greater than 10^{7} Ω m, a common insulator being paper. The resistivity of semiconductors lies between the values for conductors and insulators, and is of the order of 10^{-3} Ω m. Germanium is an example of a semiconducting material.

(*b*) (*i*) The resistance, R_T ohms, of the element at a given temperature, *T* degrees Celsius, is obtained from the formula $R_T = R_0(1 + \alpha t)$, where R_0 is the resistance at 0°C (ohms), α is the temperature coefficient of resistance (per degree Celsius), and *t* is the increase in temperature from 0°C.

$$\therefore\ R_{400} = 15(1 + 10^{-4} \times 400) = \underline{15{\cdot}6\ \Omega.}$$

(*ii*) The resistance, *R* ohms, of a uniform conductor at a constant temperature is given by the formula $R = \rho l/a$, where ρ is the resistivity (ohm metres), *l* is the length of the conductor (metres), and *a* is the cross-sectional area (metres²).

$$\therefore\ l = \frac{Ra}{\rho} = \frac{15 \times 5 \times 0{\cdot}005 \times 10^{-6}}{1 \times 10^{-6}}\ \text{m} = \underline{375\ \text{mm}.}$$

Q7 (*a*) *Explain how a torque is produced on a coil carrying current in a magnetic field.*

(*b*) *A 30-turn square coil of side 80 mm is mounted in, and at right angles to, a uniform field of flux density* $6{\cdot}0 \times 10^{-3}$ *T, and carries a current of 5·0 mA. Determine the magnitude, and show (with the aid of a sketch) the direction, of the resultant maximum torque.*

(*c*) *Why is a radial magnetic field used in a moving-coil ammeter?*

A7 (*a*) and (*b*) See A7, Engineering Science 1976, *Supplement*, Vol. 70, p. 3, Apr. 1977.

The maximum torque occurs when the coil is in the position shown in sketch (*c*) of the above reference. Its magnitude is $2BlldT$ newton metres $= 2 \times 6 \times 10^{-3} \times 5 \times 10^{-3} \times 80 \times 10^{-3} \times 40 \times 10^{-3} \times 30$ N m $= \underline{5{\cdot}76\ \mu\text{N m}}$.

(*c*) See A9, Engineering Science 1975, *Supplement*, Vol. 68, p. 96, Jan. 1976.

Q8 *(a) The EMF of each cell in Fig. 4 below is 2·0 V and the internal resistance of each cell is 0·20 Ω. Calculate the voltage across the 8 Ω resistor when the switch is connected to*

(i) terminal A, and
(ii) terminal B.

(b) Explain why overcharging of batteries should be avoided, and list the steps which should be taken to prevent it.

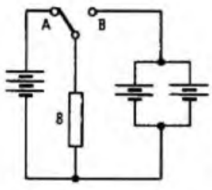
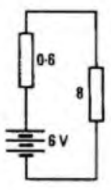
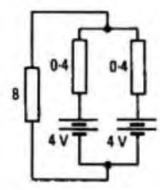


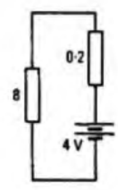
Fig. 4



(*a*)



(*b*)



(*c*)

A8 (*a*) The internal resistance of a cell causes a voltage drop when a current is drawn from the cell. This voltage drop can be calculated using Ohm's law. In a theoretical circuit, the internal resistance can be shown as a resistance external to the cell and in series with it.

(*i*) With the switch connected to terminal A, the circuit can be drawn as shown in sketch (*a*), which shows the internal resistances in theoretical form. By Ohm's law, the current is 6/8·6 A, so that the voltage across the 8 Ω resistor

$$= \frac{6}{8{\cdot}6} \times 8 = \underline{5{\cdot}58\ \text{V}}.$$

(*ii*) With the switch connected to terminal B, the circuit can be represented as shown in sketch (*b*). The effective EMF of two 4 V batteries in parallel is still 4 V. The effective resistance of the two 0·4 Ω resistors in parallel is 0·2 Ω. The circuit thus simplifies to that shown in sketch (*c*). The current is 4/8·2 A, so that the voltage across the 8 Ω resistor

$$= \frac{4}{8{\cdot}2} \times 8 = \underline{3{\cdot}9\ \text{V}}.$$

(*b*) See A5, Elementary Telecommunication Practice 1974, *Supplement*, Vol. 68, p. 13, Apr. 1975.

Q9 *(a) A coil of 200 turns is threaded by a flux of 60 × 10⁻⁶ Wb. If the flux is reduced to 40 × 10⁻⁶ Wb in 0·08 s, what is the induced EMF?*

(b) Describe and explain, with the aid of diagrams, one method of generating an alternating EMF.

(c) What is the function of a commutator?

A9 (*a*) The induced EMF is equal to the rate of change of flux linkages. Thus, the induced EMF

$$= \frac{200(60 \times 10^{-6} - 40 \times 10^{-6})}{0{\cdot}08}\ \text{V} = \underline{50\ \text{mV}}.$$

(*b*) See A9, Engineering Science 1976, *Supplement*, Vol. 70, p. 4, Apr. 1977.

(*c*) A commutator is a device (fitted to a generator) which systematically reverses the connexions to the coil as it rotates in such a way as to produce a DC output.

Q10 *(a) Show how a meter, having a full-scale deflexion of 1 mA and a resistance of 40 Ω, may be adapted for use as an ammeter with a full-scale deflexion of 100 mA.*

(b) The 1 mA meter is used in the circuit shown in Fig. 5 for the measurement of resistance. When the terminals A and B are short-circuited, a full-scale deflexion is obtained.

(i) Determine the value of resistor R.
(ii) Find the resistance which, on connexion between A and B, gives a meter reading of 0·25 mA.
(iii) Calculate the current reading of the meter when a resistor of 6·0 kΩ is connected between A and B.

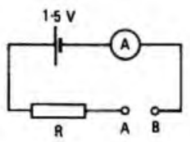
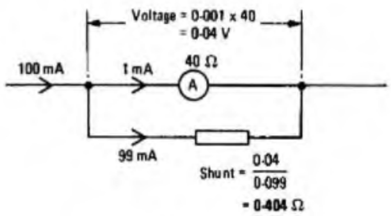


Fig. 5



(*a*)

A10 (*a*) The meter must be shunted so that, when 100 mA are supplied, 1 mA flows through the meter and 99 mA flows through the shunt, as shown in sketch (*a*).

(*b*) (*i*) Voltage across meter $= 1 \times 10^{-3} \times 40$ V $= 40$ mV. Thus, voltage across R is $1{\cdot}5 - 0{\cdot}04 = 1{\cdot}46$ V, so that $R = 1{\cdot}46/0{\cdot}001 = \underline{1{\cdot}46\ \text{k}\Omega}$.

(*ii*) Total resistance in circuit $= 1{\cdot}5/0{\cdot}25 \times 10^{-3} = 6$ kΩ. Thus, resistance between A and B is $6000 - (1460 + 40) = \underline{4{\cdot}5\ \text{k}\Omega}$.

(*iii*) Total resistance $= 1460 + 40 + 6000 = 7{\cdot}5$ kΩ, so that the current in the circuit $= 1{\cdot}5/7500 = \underline{0{\cdot}2\ \text{mA}}$.

TELECOMMUNICATION PRINCIPLES A 1978

Students were expected to answer not more than 5 questions from Q1–8, and at least one question from Q9–10.
The use of electronic pocket calculators was permitted

Q1 *(a) Three resistors are connected as shown in Fig. 1. Calculate the resistance between points (i) M and N, and (ii) M and P.*

(b) Three capacitors are connected as shown in Fig. 2. Calculate the capacitance between (i) A and B, and (ii) A and D.

(c) Two inductors of 2 H and 3 H, with a mutual inductance of 0·5 H, are connected in series. Calculate the two possible values of the resultant inductance.

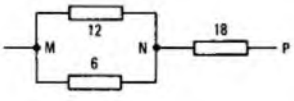


Fig. 1

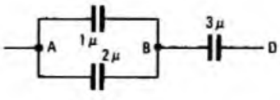


Fig. 2

A1 (*a*) (*i*) The points M and N are connected by resistances of 12 Ω and 6 Ω in parallel. The resultant resistance is therefore

$$1/\{(1/12) + (1/6)\} = 1/(3/12) = 12/3 = \underline{4\ \Omega}.$$

(*ii*) The resistance between points M and P is the total resistance between points M and N, in series with the 18 Ω resistor, and is therefore $4 + 18 = \underline{22\ \Omega}$.

(*b*) (*i*) The capacitance between points A and B is the resultant of capacitances of 1 μF and 2 μF in parallel, and is therefore $1 + 2 = \underline{3\ \mu\text{F}}$.

(*ii*) The capacitance between points A and D is the total capacitance between points A and B, in series with the 3 μF capacitor, and is therefore

$$1/\{(1/3) + (1/3)\} = 1/(2/3) = 3/2 = \underline{1{\cdot}5\ \mu\text{F}}.$$

(*c*) When inductors L1 and L2, with mutual inductance M, are connected in series, the resultant inductance, L, is given by $L = L_1 + L_2 \pm 2M$ (all values in henrys). The last term ($2M$) in the expression is positive for a series-aiding connexion and negative for a series-opposing connexion. The two possible answers to the given problem are therefore

$$L = 2 + 3 \pm 2 \times 0{\cdot}5 = 5 \pm 1 = \underline{6\ \text{H}} \text{ or } \underline{4\ \text{H}}.$$

Q2 (*a*) *The waveform of current in a 10 Ω resistor is given by the expression $i = 6 \sin 200\pi t$ amperes.*

(*i*) *Find the frequency.*

(*ii*) *Find the RMS value of the current.*

(*iii*) *Write down an expression for the waveform of the voltage across the resistor.*

(*b*) *A current, given by the expression $i = 6 \sin 200\pi t$, is passed through an inductor of 0·04 H and negligible resistance. Find an expression for the waveform of the voltage across the inductor.*

(*c*) *Calculate the cost per week of supplying power to the resistor in part (a) at a tariff of 2·3 pence/kW h.*

A2 (*a*) (*i*) The instantaneous value at time t seconds of a sinusoidal current of frequency f hertz and peak value I amperes is $i = I \sin 2\pi f t$ amperes. Comparing this with the given expression shows that the frequency of the latter is 100 Hz.

(*ii*) The RMS value of a sine wave is the peak value divided by $\sqrt{2}$. For the given waveform, the RMS value is $6/\sqrt{2} = \underline{4{\cdot}24\ \text{A}}$.

(*iii*) For a resistive AC circuit, the voltage and current are in phase and are related by Ohm's law, so that the voltage across the 10 Ω resistor is

$$10 \times 6 \sin 200\pi t = \underline{60 \sin 200\pi t \text{ volts.}}$$

(*b*) When alternating current flows in a pure inductance, the voltage is given by the current multiplied by the reactance of the inductor, and leads the current by 90° ($\pi/2$ rad). The reactance is $2\pi f L$ ohms, where f is the frequency (hertz) and L the inductance (henrys), and is therefore $2\pi \times 100 \times 0{\cdot}04 = 25{\cdot}13\ \Omega$. The peak voltage is thus $25{\cdot}13 \times 6 = 150{\cdot}8$ V, so that the expression for the instantaneous voltage, v, is

$$\underline{v = 150{\cdot}8 \sin(200\pi t + \pi/2) \text{ volts.}}$$

(*c*) The RMS power supplied to a resistance of 10 Ω carrying an RMS current of 4·24 A is $4{\cdot}24^2 \times 10 = 179{\cdot}8\ \text{W} = 0{\cdot}1798$ kW. The energy supplied in a week (in kilowatt-hours) is the power multiplied by the number of hours in a week, and is therefore $0{\cdot}1798 \times 7 \times 24 = 30{\cdot}21$ kW h. At 2·3 pence/kW h, this represents a total cost of

$$2{\cdot}3 \times 30{\cdot}21 = \underline{69{\cdot}5 \text{ pence.}}$$

Q3 (*a*) (*i*) *Outline briefly the principle of operation of a junction diode.*

(*ii*) *Explain the effect on the rectifying action if the diode temperature increases.*

(*b*) *An experiment is to be set up to demonstrate that a junction diode will operate as a rectifier. Sketch a suitable circuit, specify briefly the essential equipment, and describe the procedure to be followed.*

Q4 *Two resistors, one (R1) of 1000 Ω and the other (R2) of 2000 Ω, are connected in series across a 6 V DC supply. Two voltmeters are available, each of range 0–10 V, voltmeter A having a sensitivity of 100 Ω/V and voltmeter B 1000 Ω/V.*

(*a*) *Calculate*

(*i*) *the true voltage across each resistor without either voltmeter connected,*

(*ii*) *the readings shown on voltmeter A when it is connected across each resistor in turn, and*

(*iii*) *the readings shown on voltmeter B when it is connected across each resistor in turn.*

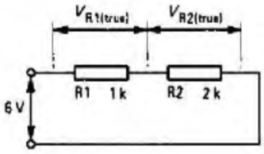
(*b*) *Explain the reason for the differences between the readings in (a)(i), (a)(ii) and (a)(iii).*

A4 The resistance of voltmeter A is $10 \times 100 = 1000\ \Omega$, and that of voltmeter B is $10 \times 1000\ \Omega = 10\ \text{k}\Omega$. (These are fixed values, and do not vary with the current supplied to the meters.)

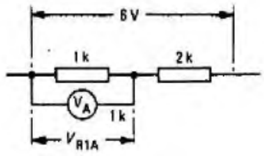
(*a*) (*i*) The circuit is illustrated in sketch (*a*). The current in the circuit is $6/(1000 + 2000)$ A $= 2$ mA, so that $V_{R1(true)}$ is $2 \times 10^{-3} \times 1000 = \underline{2\ \text{V}}$, and $V_{R2(true)}$ is $2 \times 10^{-3} \times 2000 = \underline{4\ \text{V}}$.

(*ii*) Sketch (*b*) shows the circuit with voltmeter A connected across resistor R1. The resistance of voltmeter A (1 kΩ) in parallel with that of R1 (also 1 kΩ) gives a resultant resistance of 500 Ω, and this is in series with R2 across the 6 V supply.

$$\therefore V_{R1A} = 6 \times 500/(500 + 2000) = \underline{1{\cdot}2\ \text{V}}.$$



(*a*)



(*b*)

Similarly, the parallel combination of voltmeter A and R2 gives a resultant resistance of 0·67 kΩ, and this is connected in series with R1 across the 6 V supply.

$$\therefore V_{R2A} = 6 \times 0{\cdot}67/1{\cdot}67 = \underline{2{\cdot}4\ \text{V}}.$$

(*iii*) Voltmeter B in parallel with R1 gives a resistance of 0·909 kΩ, so that $V_{R1B} = 6 \times 0{\cdot}909/2{\cdot}909 = \underline{1{\cdot}88\ \text{V}}$.

Voltmeter B in parallel with R2 gives a resistance of 1·67 kΩ, so that $V_{R2B} = 6 \times 1{\cdot}67/2{\cdot}67 = \underline{3{\cdot}75\ \text{V}}$.

(*b*) The voltmeter readings are different because of alterations to the circuit resistance caused by the shunting effect of the voltmeters. The higher the resistance of the meter, the less is the shunting effect, and the error caused by the presence of the meter is smaller.

Q5 (*a*) (*i*) *Explain the effect of reactance in an AC circuit.*

(*ii*) *Show, by a phasor diagram, how inductive and capacitive reactances are represented.*

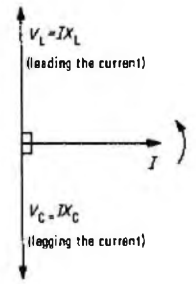
(*b*) *A current of 0·5 A at a frequency of 500 Hz flows in a circuit consisting of a capacitor of 1·5 μF in series with a resistor of 183 Ω.*

(*i*) *Sketch a phasor diagram to represent the voltages and current in the circuit.*

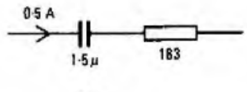
(*ii*) *By using the diagram in (b)(i), or otherwise, find the voltage applied to the series circuit, and the phase angle between the applied voltage and the current.*

A5 (*a*) Reactance is the ability of an inductance or capacitance to resist the flow of alternating current. It is analogous to resistance in a DC circuit (except that no energy is dissipated). The value of reactance depends on the frequency of the alternating current and, for pure reactances, there is a phase difference of 90° ($\pi/2$ rad) between the applied voltage and the current flowing in the reactive component.

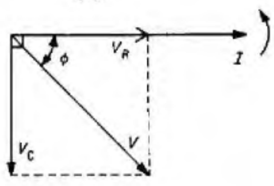
The phasor diagram in sketch (*a*) shows how pure inductive and capacitive reactances in series are represented.



(*a*)



(*b*)



(*c*)

I: Current
V_R: Voltage across resistance
V_C: Voltage across capacitance
V_L: Voltage across inductance
X_C, X_L: Capacitive, inductive reactances
R: Resistance

(*b*) (*i*) Sketch (*b*) shows the series circuit, and sketch (*c*) is the relevant phasor diagram. The current is the reference phasor, and V_R is in phase with I and has a magnitude of IR. V_C lags I by 90° and has a magnitude of IX_C. V is the resultant voltage vector for the circuit, and has the phase angle ϕ.

(*ii*) Now, $V_R = 0{\cdot}5 \times 183 = 91{\cdot}5$ V, and

$$V_C = 0{\cdot}5/(2\pi \times 500 \times 1{\cdot}5 \times 10^{-6}) = 106{\cdot}1\ \text{V}.$$

$$\therefore V = \sqrt{(91{\cdot}5^2 + 106{\cdot}1^2)} = \underline{140\ \text{V}}.$$

Also, $\phi = \tan^{-1} 106{\cdot}1/91{\cdot}5 = \underline{49{\cdot}2°}$.

Q6 (*a*) *Sketch a typical B/H curve for soft iron, and use it to explain the meaning of* (*i*) *relative permeability, and* (*ii*) *magnetic saturation.*

(*b*) *The mean length of the magnetic path of an iron core in the shape of a ring is 0·5 m, and the cross-sectional area is 0·004 m*^{2}. *A coil of 1200 turns is wound uniformly round the ring. When the coil carries a current of 0·5 A, the total flux in the iron is 4·4 mWb. Calculate the relative permeability of the iron under this condition. Ignore magnetic leakage.*

A6 (*a*) See A1, Telecommunication Principles A 1976, *Supplement*, Vol. 70, p. 8, Apr. 1977, and A1, Telecommunication Principles A 1975, *Supplement*, Vol. 69, p. 1, Apr. 1976.

(*b*) The flux density, B, in the core is given by Φ/A teslas, where Φ is the magnetic flux (webers) and A is the cross-sectional area of the core (metres^2). Thus, $B = 4{\cdot}4 \times 10^{-3}/0{\cdot}004 = 1{\cdot}1$ T.

The magnetizing force, H, is given by NI/l amperes/metre, where N is the number of turns on the coil, I is the current (amperes), and l is the length of the magnetic circuit (metres). Thus, $H = 1200 \times 0{\cdot}5/0{\cdot}5 = 1200$ A/m.

Now, $B/H = \mu_0\mu_r$, where μ_0 is the permeability of free space (equal to $4\pi \times 10^{-7}$ H/m) and μ_r is the relative permeability. Thus,

$$\mu_r = 1{\cdot}1/(1200 \times 4\pi \times 10^{-7}) = \underline{730.}$$

Q7 (*a*) *Describe briefly, with the aid of a sketch, the essential features in the construction and operation of a moving-coil milliammeter.*

(*b*) *A milliammeter has a coil resistance of 50 Ω and gives full-scale deflexion at 2·0 mA. How can this milliammeter be converted to operate as a voltmeter with a range of 0–1·0 V? Calculate any necessary resistance.*

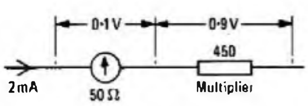
A7 (*a*) See A5, Telecommunication Principles A 1975, *Supplement*, Vol. 69, p. 3, Apr. 1976, and A9, Engineering Science 1975, *Supplement*, Vol. 68, p. 96, Jan. 1976.

(*b*) If the milliammeter has an internal resistance of 50 Ω and gives full-scale deflexion when a current of 2 mA flows through it, then a voltage of 100 mV is needed across the meter to give full-scale deflexion. Thus, to give a range of up to 1 V, a resistor must be connected in series with the meter such that, when full-scale-deflexion current is flowing (2 mA), the voltage across the resistor is 900 mV.

By Ohm's law, the value of the series resistor

$$= 900 \times 10^{-3}/2 \times 10^{-3} = \underline{450\ \Omega.}$$

The arrangement is shown in the sketch. The series resistor is sometimes known as a *multiplier*.



Q8 (*a*) (*i*) *Explain why a coil has self-inductance.*
(*ii*) *State 2 factors which determine the value of inductance.*

(*b*) *When a coil of 2500 turns carries a direct current of 0·4 A, a flux of 20 μWb is produced. Neglecting magnetic leakage, calculate*
(*i*) *the inductance of the coil, and*
(*ii*) *the energy stored in the coil.*

(*c*) *When the circuit in* (*b*) *is opened, the current of 0·4 A falls to zero in 1·0 ms. Calculate the average EMF induced in the coil.*

A8 (*a*) See A2, Telecommunication Principles A 1976, *Supplement*, Vol. 70, p. 9, Apr. 1977, and A8, Telecommunication Principles A 1974, *Supplement*, Vol. 68, p. 17, Apr. 1975.

(*b*) (*i*) For a coil having a magnetic circuit of constant reluctance, the inductance, L, is given by $L = N\Phi/I$ henrys, where N is the number of turns on the coil, Φ is the flux (webers) and I is the current (amperes).

$$\therefore\ L = 2500 \times 20 \times 10^{-6}/0{\cdot}4\ \text{H} = \underline{125\ \text{mH.}}$$

(*ii*) The energy stored is given by

$$\tfrac{1}{2}LI^2 \text{ joules} = 125 \times 10^{-3} \times 0{\cdot}4^2/2\ \text{J} = \underline{10\ \text{mJ.}}$$

(*c*) The induced EMF is given by $-L\mathrm{d}i/\mathrm{d}t$ volts, where $\mathrm{d}i/\mathrm{d}t$ is the rate of change of current (amperes/second). The negative sign indicates that the EMF opposes the circuit change producing it. Thus, the EMF

$$= -125 \times 10^{-3} \times 0{\cdot}4/1 \times 10^{-3} = \underline{-50\ \text{V.}}$$

Q9 (*a*) *Give a brief account of an experiment to determine the static output characteristics of a transistor operating in the common-emitter configuration. Draw a circuit and comment on the measuring equipment and experimental procedure.*

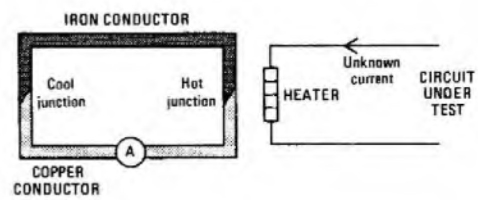
(*b*) *Sketch a typical family of common-emitter output characteristics, and show how these can be used to obtain*
(*i*) *the output resistance for a given base current, and*
(*ii*) *the current gain for a given collector voltage.*

A9 See A5, Telecommunication Principles B 1975, *Supplement*, Vol. 69, p. 25, Apr. 1976.

Q10 (*a*) *State the principle of operation of a thermocouple ammeter.*

(*b*) *Why can this type of instrument function on either direct or alternating current?*

(*c*) *What factors limit the operating frequency at which it can be used?*



A10 (*a*) The thermocouple ammeter is an application of the Seebeck effect: namely, that a current is generated in a closed circuit consisting of 2 different metals when each junction is at a different temperature. As shown in the sketch, one junction is kept cool at a constant temperature while the other is heated by a small resistor through which the current to be measured is passed. A microammeter in the thermocouple circuit measures the thermoelectric current generated, which is proportional to the temperature of the hot junction, which in turn is proportional to the heating current.

(*b*) Because it operates on the heating effect of an electric current (and not the magnetic effect), the thermocouple meter can be used for AC or DC circuits. (It thus measures the RMS value of alternating currents.) The meter can be accurately calibrated using direct currents before test measurements are made.

(*c*) The thermocouple meter is valuable because it responds accurately to radio frequencies, provided the reactance of the heater is negligible. In practice, the very small, unavoidable series inductance and shunt capacitance of the heater ultimately limit the frequency at which the meter is reliably accurate.

RADIO AND LINE TRANSMISSION A 1978

Students were expected to answer any 6 questions

Q1 (*a*) *With reference to amplitude modulation, explain the following terms*

(*i*) *modulation envelope,*
(*ii*) *side frequencies, and*
(*iii*) *sidebands.*

(*b*) *A 300 kHz carrier wave is amplitude modulated sinusoidally at a frequency of 5 kHz and the amplitude of the modulated waveform varies between ±0·4 V and ±0·8 V. Determine the depth of modulation.*

(*c*) *What frequencies are present in the modulated wave?*

A1 (*a*) (*i*) If the amplitude of an unmodulated carrier wave is V_c, and it is amplitude modulated by a lower frequency of amplitude V_m, then the amplitude of the modulated wave varies, at the frequency of the modulating wave, between the limits of $V_c - V_m$ and $V_c + V_m$. The envelope of the modulated wave is referred to as the modulation envelope.

(*ii*) If a carrier wave of frequency f_c is amplitude modulated by a low-frequency signal of frequency f_1 then the resulting amplitude-modulated wave comprises three frequency components: ($f_c - f_1$), f_c, and ($f_c + f_1$). The components ($f_c - f_1$) and ($f_c + f_1$) are known as the *side frequencies*.

(*iii*) If the carrier wave is amplitude modulated by more than one low-frequency signal, then each modulating signal produces its own pair of side frequencies. The range of side frequencies below the carrier is called the *lower sideband*, and the range above is called the *upper sideband*.

(*b*) Let the amplitude of the unmodulated carrier be V_c volts, and that of the modulating signal be V_m volts. Then,

$$V_c + V_m = 0{\cdot}8 \text{ V}, \qquad \ldots\ldots (1)$$

and

$$V_c - V_m = 0{\cdot}4 \text{ V}. \qquad \ldots\ldots (2)$$

Adding equations (1) and (2) gives,

$$2V_c = 1{\cdot}2 \text{ V}.$$

$$\therefore \quad V_c = 0{\cdot}6 \text{ V}.$$

Substituting for V_c in equation (1) gives,

$$V_m = 0{\cdot}2 \text{ V}.$$

The depth of modulation $= \frac{V_m}{V_c} = \frac{0{\cdot}2}{0{\cdot}6} = \underline{0{\cdot}33 \text{ or } 33\%}.$

(*c*) The modulated wave contains three frequency components: $(f_c + f_1)$, f_c, and $(f_c - f_1)$, where f_c is the frequency of the carrier and f_1 the frequency of the modulating signal. Thus the components are:

$$f_c - f_1 = 300 \text{ kHz} - 5 \text{ kHz} = \underline{295 \text{ kHz}},$$

$$f_c = \underline{300 \text{ kHz}}, \text{ and}$$

$$f_c + f_1 = 300 \text{ kHz} + 5 \text{ kHz} = \underline{305 \text{ kHz}}.$$

Q2 (a) (i) State the formula for expressing a power ratio in decibels.

(ii) Under what conditions may voltage ratios be expressed in decibels?

(b) Give TWO reasons why the use of logarithmic units (decibels) may simplify calculations.

(c) An amplifier has a power gain of 60 dB. Its input and output impedances are 10 kΩ and 100 kΩ respectively. If the input voltage is 100 mV, calculate

(i) the output voltage in volts and in decibels relative to 1 V, and

(ii) the output power in watts and in decibels relative to 1 W.

A2 (*a*) (*i*) If the ratio of two powers, P_1 and P_2, is to be expressed in decibels, the number of decibels, N, is given by,

$$N = 10 \log_{10} \frac{P_1}{P_2} \text{ decibels} \qquad \ldots\ldots (1)$$

(*b*) The use of logarithmic units can simplify calculations because an enormous range of values can be expressed in simple numbers. Also, the operations of multiplication and division are reduced to the simpler operations of addition and subtraction respectively.

(*c*) Let the input power, voltage and impedance be P_{in}, V_{in} and R_{in} respectively.
Let the corresponding output values be P_{out}, V_{out} and R_{out}.

(*i*) From equation (1),

$$60 = 10 \log_{10} \frac{P_{out}}{P_{in}}.$$

The power, P watts, dissipated in a resistor, R ohms, is given by

$$P = \frac{V^2}{R} \text{ watts}.$$

Hence,

$$60 = 10 \log_{10} \frac{V_{out}^2/R_{out}}{V_{in}^2/R_{in}},$$

$$= 20 \log_{10} \frac{V_{out}}{V_{in}} + 10 \log_{10} \frac{R_{in}}{R_{out}}.$$

Substituting the given values gives

$$60 = 20 \log_{10} \frac{V_{out}}{0{\cdot}1} + 10 \log_{10} \frac{10\,000}{100}.$$

$$\therefore \quad \frac{V_{out}}{0{\cdot}1} = \text{antilog } 2 = 100.$$

$$\therefore \quad V_{out} = \underline{10 \text{ V}}.$$

Therefore, the output voltage in decibels relative to 1 V

$$= 20 \log_{10} \frac{10}{1} = \underline{20 \text{ dBV}}.$$

(*ii*)

$$P_{out} = \frac{V_{out}^2}{R_{out}} = \frac{100}{100} = \underline{1 \text{ W}}.$$

Therefore, the output power in decibels relative to 1 W

$$= 10 \log_{10} \frac{1}{1} = \underline{0 \text{ dBW}}.$$

Q3 (a) With the aid of a labelled diagram, explain the principle of operation of a moving-coil microphone.

(b) Explain briefly the constructional difference between a moving-coil microphone and a moving-coil loudspeaker.

(c) Give a typical range for the frequency response of a moving-coil microphone and give ONE example of an application which would require this response.

A3 (*a*) See A6 (*a*), Radio and Line Transmission A 1972, *Supplement*, Vol. 65, p. 93, Jan. 1973.

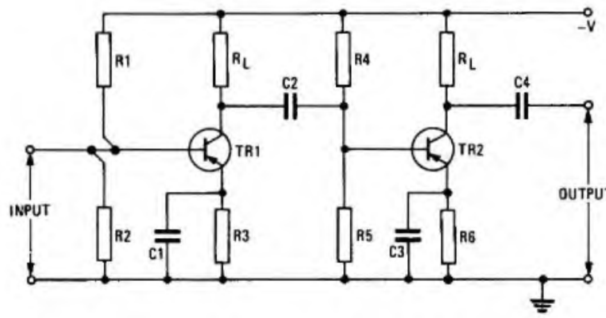
(*b*) The moving-coil loudspeaker is usually larger than the moving-coil microphone. The loudspeaker has a large rigid cone whereas the microphone normally has a more flexible and less conical shaped diaphragm. The magnet of the loudspeaker is generally much larger and more powerful than that of the microphone, and the moving-coil of the loudspeaker is wound with fewer turns of heavier gauge wire.

(*c*) A moving-coil microphone would typically have a frequency range from 50 Hz to 15 kHz. Such a response would make the microphone suitable for use in broadcasting studios.

Q4 (a) Draw the circuit diagram of a 2-stage R–C-coupled audio-frequency transistor amplifier using the common-emitter configuration and indicate typical component values.

(b) Explain what limits the upper frequency response of the amplifier in part (a).

(c) A 2-stage amplifier develops a fault, and a transistor failure is suspected. Describe a simple test or tests using a signal generator and an oscilloscope which could be used to determine which transistor has failed.



A4 (*a*) The circuit diagram required is shown in the sketch. Suitable component values are:

R1 = R4 = 18 kΩ	R2 = R5 = 12 kΩ
RL = 3·3 kΩ	R3 = R6 = 1 kΩ
C2 = C4 = 2 μF	C1 = C3 = 8 μF.

(*b*) At high frequencies, the stray capacitances in the circuit (for example, across the input of the second stage) shunt the signal, and the output of the amplifier is reduced at these frequencies.

(*c*) To identify which transistor is faulty, the following procedure is adopted. Firstly, the signal generator is connected to the input of the first stage; that is, across the base and earth. The output of the signal generator is set to a suitable voltage and frequency for the amplifier concerned. The oscilloscope is connected across the collector of the first transistor and earth. If the first transistor is functioning correctly, an amplified version of the input signal will be seen on the oscilloscope. The second transistor may be checked by repeating the test, but with the signal generator applied to the base of the second transistor and the oscilloscope connected to the collector of this transistor.

Q5 (*a*) *Draw the circuit diagram of a transistor amplifier operating in the common-base configuration and explain the principles of operation of the amplifier.*

(*b*) *If the transistor has a common-base current gain of 0·98 and the amplifier has an input impedance of 100 Ω and a load impedance of 5 kΩ, calculate for the amplifier*

(*i*) *the voltage gain, and*
(*ii*) *the power gain.*

(*c*) *What transistor configuration would be used in an amplifier to provide high current gain?*

A5 (*a*) See A8(*b*), Radio and Line Transmission A 1975, *Supplement*, Vol. 69. p. 11, Apr. 1976.

(*b*) The voltage gain of the common-base amplifier is

$$\frac{\alpha R_L}{R_{in}},$$

where α is the current gain and R_L and R_{in} are the load resistance and input resistance respectively.

Thus, the voltage gain is

$$0·98 \times \frac{5 \times 10^3}{100} = \underline{49.}$$

The power gain is

$$\alpha^2 \frac{R_L}{R_{in}} = 0·98^2 \times \frac{5 \times 10^3}{100} = \underline{48.}$$

(*c*) The common-emitter configuration would normally be used to provide high current gain.

Q6 (*a*) *Draw a labelled block diagram of a 2-wire repeatered audio junction circuit between 2 telephone exchanges.*

(*b*) *Discuss briefly the advantages and disadvantages of 2-wire working compared with 4-wire working.*

(*c*) *Draw the circuit diagram of a 2-wire-to-4-wire terminating set and state its function.*

Q7 (*a*) *Draw the circuit diagram of a tuned-collector transistor amplifier.*

(*b*) *Explain how such an amplifier provides selectivity.*

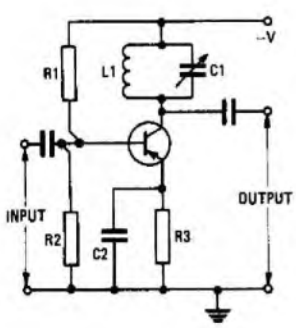
(*c*) *Explain briefly how the amplifier in part (a) may be converted into an oscillator.*

(*d*) *State the formula for determining the frequency of oscillation and give the units for each symbol.*

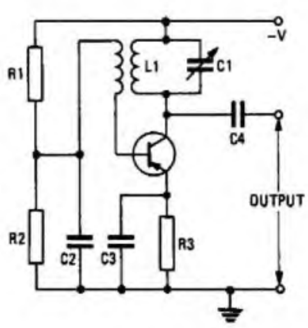
A7 (*a*) The circuit diagram of a tuned-collector transistor amplifier is shown in sketch (*a*).

(*b*) See A5(*c*), Radio and Line Transmission A 1974, *Supplement*, Vol. 68, p. 3, Apr. 1975.

(*c*) To convert the amplifier into an oscillator, it is necessary to provide feedback from the output to the input and to ensure that the feedback is both in the correct phase and is of sufficient amplitude to maintain oscillations. One way of doing this is to couple a second coil to the inductor of the tuned circuit and to connect this coil into the base circuit. The resulting circuit diagram is shown in sketch (*b*).



(*a*)



(*b*)

(*c*) The frequency of resonance, f_0, of a tuned circuit having a capacitance *C* farads and inductance *L* henrys is given approximately by

$$f_0 = \frac{1}{2\pi\sqrt{(LC)}} \text{ hertz.}$$

Q8 (*a*) *Draw the diagram of a circuit for obtaining the static characteristics of a transistor in the common-emitter configuration.*

(*b*) *The data given in the table below refer to a transistor in the common-emitter configuration.*

Collector voltage (V)	*Collector current (mA) for base current (μA)* 40	60	80
2	2·9	4·4	5·8
6	3·7	5·4	7·0
10	4·5	6·4	8·2

Draw the collector-voltage/collector-current characteristic for EACH *base current shown.*

(*c*) *Draw the load line for a collector load resistance of 1 kΩ and a supply voltage of 10 V, and determine the power dissipated at the collector when the base current is 60 μA.*

Q9 (*a*) *List* FIVE *electrical characteristics of a resistor and for* EACH *characteristic compare the relative merits of wire-wound and carbon resistors.*

(*b*) *Describe the constructional details of a small mains transformer and give* TWO *causes of power loss in such a transformer.*

A9 (*a*) Electrical characteristics of a resistor include self-inductance, self-capacitance, stability, tolerance and power rating. A table showing the relative performances of carbon and wire-wound resistors is given below.

Characteristic	Wire-Wound Resistors	Carbon Resistors
Self-inductance and self-capacitance	Quite noticeable, particularly at high frequencies; effect can be reduced if resistor is specially wound.	Negligible
Stability and tolerance	Good	Fair to poor unless specially made
Power rating	High	Low

(*b*) A mains transformer consists of a single primary winding and one or more secondary windings wound on a laminated soft-iron core. The wire used for windings is covered by a layer of varnish which acts as insulation between the turns, and the coils are usually separated from each other, and from the core, by layers of insulating material. In use, the alternating voltage of the mains is applied to the primary and an output voltage appears at the secondary winding(s). The ratio of output to input voltage is equal to the ratio of the number of turns on the secondary to that on the primary.

There are several causes of power loss in transformers. The most important are eddy-current and hysteresis losses in the core, and losses in the windings due to heat being produced in the finite resistance of the wire.

Q10 (*a*) *With reference to a sinusoidal wave, explain, with the aid of sketches where appropriate, the terms*

(*i*) *frequency,*
(*ii*) *wavelength,*
(*iii*) *periodic time, and*
(*iv*) *amplitude.*

(*b*) *A radio wave is propagated through a block of polyethelene in which its velocity of propagation is 2 × 10⁸ m/s. If its wavelength is 1 cm, what is the frequency of the wave?*

(*c*) *A signal is transmitted from the earth to a satellite 36 000 km distant. Calculate the time taken for the signal to reach the satellite.*

A10 (*a*) (*i*) The frequency of the wave is the number of cycles which occur in one second.

(*ii*) The wavelength is the distance covered by one complete cycle of the wave.

(*iii*) The periodic time is the time required to complete one cycle of the wave.

(*iv*) The amplitude of the wave is the maximum or peak value the wave reaches.

(*b*) For a wave of wavelength λ metres, propagated at a velocity of v metres per second, the frequency is

$$v/\lambda \text{ hertz},$$

$$= \frac{2 \times 10^8}{1 \times 10^{-2}} = 2 \times 10^{10}\ \text{Hz},$$

$$= \underline{20\ \text{GHz}}.$$

(*c*) For a distance of d metres and velocity v metres/second, the time taken is d/v seconds.

For propagation in free space, $v = 3 \times 10^8$ m/s.

Therefore, the time taken $= \frac{36\,000 \times 10^3}{3 \times 10^8}$ s $= \underline{120 \text{ ms.}}$

LINE PLANT PRACTICE A 1978

Students were expected to answer any 6 questions

Q1 (*a*) *Describe* TWO *possible methods of repairing a small sheath failure in a polyethylene cable.*

(*b*) *Describe the "wrap-around" sleeve and sheath repair using the heat-shrinking technique.*

A1 (*a*) A damaged sheath can be repaired by heat-welding a polyethylene-backed foil to the sheath and binding it with thin wire and self-amalgamating tape, similar to the method employed during epoxy-resin plumbing. It is also possible in certain circumstances to heat-weld small splits.

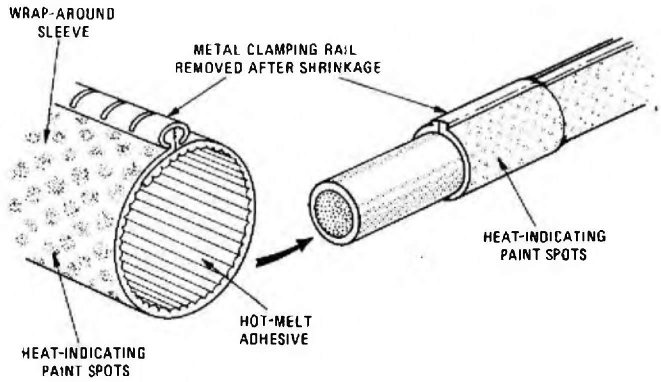
(*b*) The "wrap-around" sleeve is manufactured from specially processed polyethylene which is heat-shrinkable. The material behaves as a normal plastic below its original melting-point temperature, but above this temperature it becomes rubbery instead of melting. The material may be expanded or stretched while in the rubbery state and, if it is held in this position until the temperature is lowered below the melting point, it remains at the expanded size when the expanding device is removed. The material is supplied in this expanded state and, if reheated above the melting-point temperature, it will contract towards its original size.

Heat-shrinkable wrap-around repair sleeving consists of split heat-shrinkable sleeving and a metal clamping rail which grips the longitudinal edges of the split to hold it in the form of a tube during the shrinking operation. The sleeving is coated on the inside face with a hot-melt adhesive to ensure a seal, as the material is not rubbery at normal temperatures.

The damaged sheath is inspected to determine the extent of the damage. If the damage to the sheath is superficial (for example, a small split) the sleeving is applied over it. With more severe cases, a portion of the sleeve is removed entirely, including the aluminium-foil moisture barrier if incorporated. After ensuring that the insulation on the conductors is not damaged, a continuity wire is riveted to both sides of the moisture barrier and overwrapped with paper tape. The sleeving is placed round the cable sheath and the edges joined by fitting the metal clamping rail.

Heat from a propane torch is applied to shrink the sleeve on to the cable, working from the centre outwards so that the shrinking sleeve forces out any trapped air bubbles towards the ends. The correct temperature is indicated by spots of temperature sensitive paint which change from blue to tan. Another indication of satisfactory shrinkage is when the adhesive exudes from the ends of the sleeve. When the sleeve has cooled, the clamping rail is cut off.

The sleeving, together with a repair made with the shrunk sleeving, is shown in the sketch.



Q2 (*a*) *List the basic requirements for a good joint and closure in a polyethylene insulated and sheathed audio cable.*

(*b*) *What* TWO *basic methods are used for jointing conductors in underground cables?*

(*c*) *What* TWO *mechanical methods are available for jointing conductors in underground audio cables?*

(*d*) *In what circumstances is it necessary to solder joints in cable conductors?*

(*e*) *What electrical tests should be made as the jointing of a multi-pair cable proceeds?*

A2 (*a*), (*b*), (*c*) and (*d*) See A2, Line Plant Practice A 1975, *Supplement*, Vol. 69, p. 14, Apr. 1976.

(*e*) (*i*) Prove the continuity of the wires.
(*ii*) Prove the absence of wire crosses between pairs.
(*iii*) Prove freedom from wire and earth contacts.
(*iv*) Measure insulation resistance.

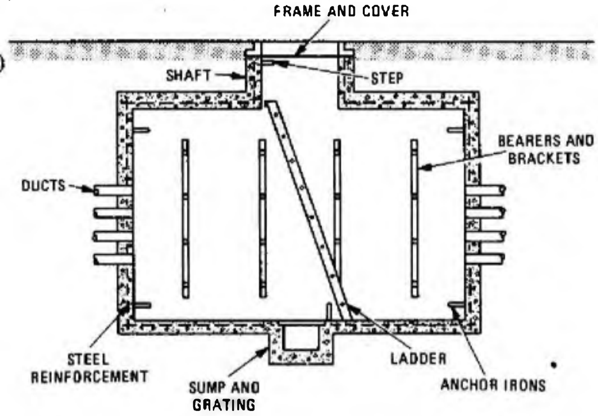
Q3 (*a*) *Draw a simple labelled sketch (preferably cross-section) of a manhole.*

(*b*) *Draw a simple labelled sketch of a brick-built joint box.*

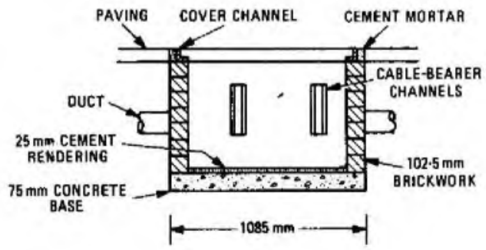
(*c*) *What precautions must be taken when concreting in cold weather?*

(*d*) *What precautions must be taken when concreting in hot or windy weather?*

A3 (*a*)



(*b*)



(*c*) (*i*) Do not concrete when the temperature is below 3°C.

(*ii*) Make sure the temperature of the aggregate and sand is above 3°C. This may be achieved by using lukewarm water.

(*iii*) The concrete when placed must be protected against frost for at least 4 h.

(*d*) The concrete must be prevented from drying too rapidly by shielding it from the sun and wind. After placing, the concrete should be kept moist by frequent sprinkling with a hose or watering can, or by covering the concrete with saturated sackcloth or wet sand throughout the curing period of about 7 d.

Q4 (a) What is a pilot hole in relation to trenching for a duct track?

(b) If another authority's pipe or main is exposed during an excavation for a trench, explain how the pipe or main should be supported.

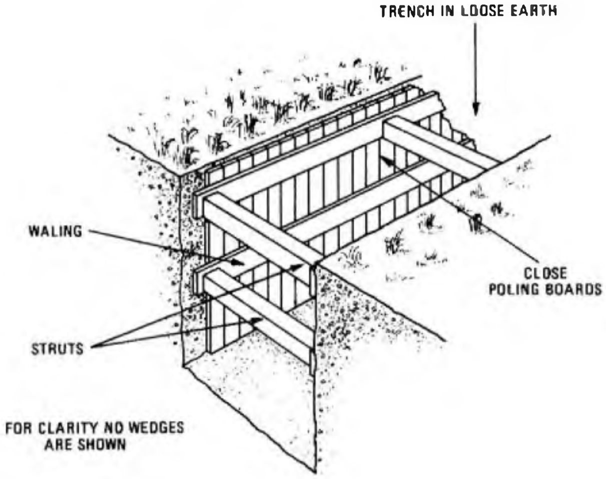
(c) Explain, with the aid of a sketch, how a deep trench in loose earth should be protected against collapse during construction.

(d) State FOUR *situations when it would be necessary to resort to tunnelling methods to lay a duct track.*

A4 (*a*) Before excavating a trench for a duct track it is necessary to dig pilot holes at intervals along the route of the trench to ascertain whether conditions under the surface are suitable. A pilot hole should normally be 150 mm wider than, and deeper than, the proposed trench to disclose the presence of existing plant such as supply mains. Pilot holes are always opened at points selected for jointing chambers.

(*b*) It should be supported by ropes or chains slung from baulks of timber or steel scaffold poles placed across the top of the trench. If there is any possibility of subsidence, brick or concrete piers should be built to support the pipes behind their sockets.

(*c*) The trench is close-timbered and consists of vertical members called *poling boards*, horizontal members known as *walings*, and timber across the trench known as *struts*. Wedges may be driven in between the walings and the struts to tighten the structure.



(*d*) (*i*) When it is undesirable to obstruct the roadway with open trenches; for example, on account of traffic congestion on busy thoroughfares.

(*ii*) When rivers, canals, bridges, etc. lie in the track of the route.

(*iii*) Where the amount of existing underground plant makes it essential that the ducts be laid at abnormal depths.

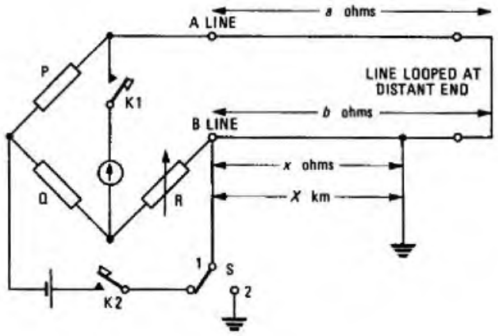
(*iv*) Where a large obstacle, such as an underground chamber or sewer, is present.

(*v*) Where there is a rock subsoil and the cost of breaking down from the surface and removing a large volume of rock exceeds the high cost of tunnelling.

Q5 A long telephone cable pair has a specific resistance of 15 Ω per single-wire kilometre, and the loop resistance of the pair is 105 Ω. If the distance to an earth fault is 2 km, and the bridge ratio is set 1: 10, what readings would be obtained on the bridge to measure the loop resistance and the resistance to the fault? Derive the general formula used and explain, with a diagram, how the test would be performed.

A5 (*i*) The faulty wire and a good wire are looped at the distant end.

(*ii*) The bridge is connected to the pair as shown in the sketch.



(*iii*) With switch S in position 1, the value R_1 for the balanced bridge is obtained using a suitable ratio of $P : Q$ for maximum sensitivity. From this value the loop resistance can be calculated.

(*iv*) With switch S in position 2, the value of R_2 is obtained for the balanced bridge, once again with appropriate ratio of $P : Q$.

(*v*) With these values and the known specific resistance of the wires, the distance to the fault can be determined.

For the first reading, R_1,

$$\frac{P}{Q} = \frac{(a+b)}{R_1}.$$

$$\therefore \quad R_1 = \frac{Q(a+b)}{P} = \frac{(a+b)}{\dfrac{P}{Q}}.$$

For the second reading, R_2,

$$\frac{P}{Q} = \frac{(a+b)-x}{R_2+x}.$$

$$\therefore \quad P(R_2+x) = Q(a+b-x).$$

$$\therefore \quad PR_2 + Px = Qa + Qb - Qx.$$

$$\therefore \quad PR_2 = Qa + Qb - Qx - Px.$$

$$\therefore \quad R_2 = \frac{Qa + Qb - Qx - Px}{P},$$

$$= \frac{Q(a+b-x) - Px}{P} = \frac{(a+b-x) - \dfrac{P}{Q}x}{\dfrac{P}{Q}}.$$

Thus, $R_1 = \dfrac{105}{1/10} = \underline{1050\ \Omega}.$

x = specific resistance times the distance to the fault,
$= 2 \times 15 = 30\ \Omega.$

Thus, $R_2 = \dfrac{(105-30) - 30/10}{1/10} = 10(75-3) = \underline{720\ \Omega}.$

Q6 (a) Describe, with the aid of sketches, the anti-creepage devices which resist

(i) creep in, and (ii) creep out.

Explain how they are fitted.

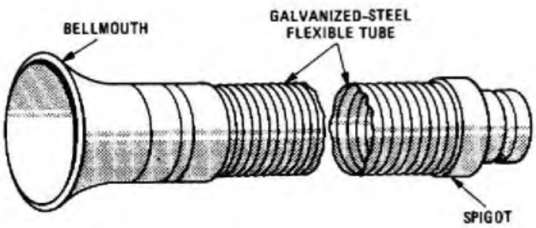
(b) Sketch and describe

(i) a flexible cable guide, and (ii) a split curved guide.

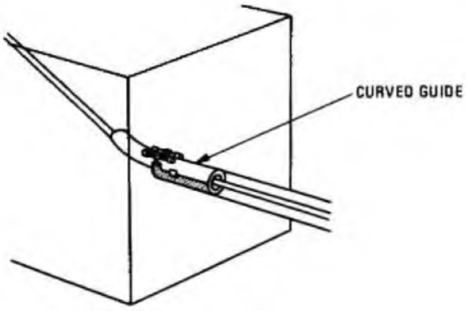
A6 (*a*) See A10(*b*), Line Plant Practice B 1976, *Supplement*, Vol. 70, p. 48, July 1977.

(*b*) (*i*) A flexible cable guide is shown in sketch (*a*). It is a flexible galvanized-steel hose with a spigot at one end, and a separate bellmouth at the other. Guides are available in 1·5 m and 3·0 m lengths and have a bore of 90 mm. They can be joined to make longer lengths by inserting a spigot in the socket which accommodates the bellmouth.

(*ii*) The rigid guide, shown in sketch (*b*), consists of a straight piece and a curved piece held together with clamps. They can be assembled around a cable and are adjustable so that they will be a tight fit in different size duct mouths.



(*a*)



(*b*)

Q7 (*a*) *Explain the difference between a frontage "tee type" of underground service feed to a subscriber's premises and underground "radial" distribution.*

(*b*) *On the street map shown in Fig. 1, draw a typical example of both types of distribution.*

(*c*) *Sketch and describe a typical joint and its housing for use with the "radial" distribution method.*

(*d*) *For which of the methods in* (*a*) *could a jointing post be used? Sketch and describe a typical jointing post.*

(*e*) *Why should either method of underground distribution be used in preference to overhead distribution from a pole?*

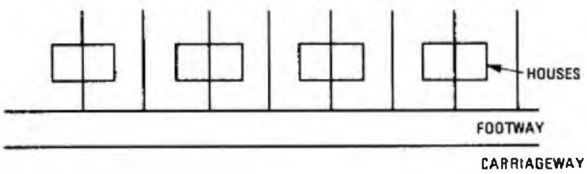


Fig. 1

Q8 (*a*) *Describe the "direct placing" method of erecting an aerial cable using a vehicle-mounted elevated platform.*

(*b*) *Describe the "threading" method of erecting an aerial cable.*

(*c*) *Under what circumstances should the threading method be used instead of the direct placing method?*

(*d*) *Name a third method of erecting an aerial cable.*

Q9 (*a*) *State* FOUR *situations that may make it necessary to move a duct track.*

(*b*) *Describe the operation of slewing and lowering a four-way earthenware duct track containing cables, clearly stating the important precautions that must be taken during the operation.*

A9 (*a*) (*i*) A change in road level may expose a duct track.

(*ii*) The provision of a new kerb line may be over an existing route.

(*iii*) A road change may necessitate the moving of a jointing chamber; for example, it may be in a position that would cause traffic difficulties during working operations.

(*iv*) If a joint box is to be replaced by a manhole, the duct track at this point will need to be lowered.

(*b*) (*i*) Before slewing or lowering operations are commenced, it must be ascertained that adequate slack exists in the cables in the adjacent jointing chambers.

(*ii*) The exact position of the duct line must be established and any obstructions requiring removal must be located.

(*iii*) An adequate labour force must be available to ensure that the whole of the duct is moved gently in one operation.

(*iv*) The trench should be fully excavated, widened and deepened to incorporate the new position of the duct line.

(*v*) The sides of the trench should be supported by poling boards.

(*vi*) Tubular scaffold poles or lengths of timber should be placed along the top of the duct line to act as a *strongback*.

(*vii*) The soil should be excavated from beneath the ducts at intervals to enable each duct length to be securely attached to the strongback.

(*viii*) The ducts and strongback should be firmly lashed together and wedges driven between them to tighten the ties, thereby making the arrangement rigid.

(*ix*) A *sling pole* should be supported by an arrangement of scaffolding or wooden poles straddling the trench and supported on edging boards on either side of the trench.

(*x*) The strongback should then be hung from the sling pole.

(*xi*) The earth beneath the ducts should be removed until the duct line is freely supported from the sling poles.

(*xii*) The duct should be slewed gradually throughout its length by sliding the sling poles along the cross members.

(*xiii*) The rope ties between the sling pole and strongback should be gradually eased off at the sling poles to lower the duct.

(*xiv*) When the duct is firmly bedded in its new position, the ties and supports should be removed.

(*xv*) The duct track must be tested for damage and repaired if necessary.

(*xvi*) Back-filling can then be undertaken.

Q10 (*a*) *Describe the construction of a telephone cable, having aluminium conductors, suitable for use in a subscribers' distribution network.*

(*b*) *Describe the pair identification colour scheme used.*

(*c*) *What advantages and disadvantages have such cables compared with the equivalent cables using copper conductors?*

COMPUTERS A 1978

Students were expected to answer 6 questions

Q1 *Perform the following arithmetic operations in binary showing all working and convert the results of each operation to denary*

(*a*) *add 10 111 010 to 11 011,*

(*b*) *subtract 1 110 010 from 11 010 111,*

(*c*) *divide 101 101 110 by 1 100, expressing the answer in quotient and remainder form, and*

(*d*) *complete the division in* (*c*) *and express the new answer in denary form.*

A1 (*a*)

	1 0 1 1 1 0 1 0
	0 0 0 1 1 0 1 1 +
	1 1 0 1 0 1 0 1
Carry	0 1 1 1 0 1 0
Answer =	11010101_2

Converting to denary (radix 10) is performed by summing the product of each bit with its 2-to-the-power equivalent:

```
  1    1    0    1    0    1    0    1
  ×    ×    ×    ×    ×    ×    ×    ×
 2^7  2^6  2^5  2^4  2^3  2^2  2^1  2^0
128 + 64 + 0 + 16 + 0 + 4 + 0 + 1 = 213₁₀
```

$128 + 64 + 0 + 16 + 0 + 4 + 0 + 1 = \underline{213_{10}}$

(*b*)

```
  1 1 0 1 0 1 1 1
  0 1 1 1 0 0 1 0 −
  0 1 1 0 0 1 0 1
Borrow 1 1
```

Converting to denary:

```
  0    1    1    0    0    1    0    1
  ×    ×    ×    ×    ×    ×    ×    ×
 2^7  2^6  2^5  2^4  2^3  2^2  2^1  2^0
```

$0 + 64 + 32 + 0 + 0 + 4 + 0 + 1 = \underline{101_{10}}$

(*c*) The numbers can be divided by a shift and subtract technique:

```
                1 1 1 1 0
1 1 0 0) 1 0 1  1 0 1  1 1 0
         1 1 0  0
         1 0 1  0 1
           1 1  0 0
           1 0  0 1 1
              1 1 0 0
              1 1 1 1
              1 1 0 0
                  1 1 0
                  0 0 0
                  1 1 0
```

Quotient = $1\ 1\ 1\ 1\ 0_2$, remainder = $1\ 1\ 0_2$

Converting the quotient to denary:

```
  1    1    1    1    0
  ×    ×    ×    ×    ×
 2^4  2^3  2^2  2^1  2^0
```

$16 + 8 + 4 + 2 + 0 = \underline{30_{10}}$

Converting the remainder to denary:

```
  1    1    0
  ×    ×    ×
 2^2  2^1  2^0
```

$4 + 2 + 0 = \underline{6_{10}}$

(*d*) The division performed in part (*c*) is completed as shown below

```
                1 1 1 1 0 · 1
1 1 0 0) 1 0 1  1 0 1  1 1 0 · 0
         1 1 0  0
         1 0 1  0 1
           1 1  0 0
           1 0  0 1 1
              1 1 0 0
              1 1 1 1
              1 1 0 0
                  1 1 0
                  0 0 0
                  1 1 0 0
                  1 1 0 0
                  . . . .
```

Quotient = $1\ 1\ 1\ 1\ 0 \cdot 1_2$

Converting to denary

```
  1    1    1    1    0    ·1
  ×    ×    ×    ×    ×    ×
 2^4  2^3  2^2  2^1  2^0  2^-1
```

$16 + 8 + 4 + 2 + 0 + \cdot 5 = \underline{30 \cdot 5_{10}}$

Q2 (*a*) *Explain the meaning of the term "byte".*

(*b*) *An instruction in a digital computer is 4 bytes long, and comprises a function field (1 byte), an operand field (1½ bytes) and a miscellaneous field (1½ bytes) in which bits 0 and 1 are coded as follows:*

01 means that the operand field contains the address of an item of data,

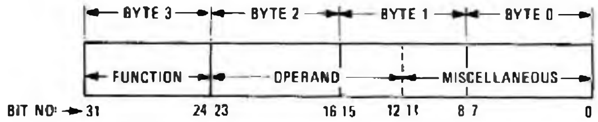
11 means that the operand field contains a literal value.

(*i*) *How many different functions could be provided?*

(*ii*) *What maximum value of literal could be contained?*

(*iii*) *How many different items could be located?*

(*iv*) *If in (ii) the literal value is signed, what are its maximum positive and negative values?*



A2 (*a*) A byte is a binary character string operated upon as a unit. It is usually a subdivision of a computer word and normally is 8 bit long.

(*b*) The instruction can be represented as shown in the sketch. It is assumed that the byte has 8 bit.

(*i*) The function field is one byte and therefore the range of functions is 00 000 000 to 11 111 111 or $2^8 = \underline{256 \text{ functions}}$.

(*ii*) The operand field contains a literal when bits 0 and 1 of the miscellaneous field contain 11.

The operand field consists of 1½ bytes which in this case is 12 bits. The maximum value of the literal, assuming it is unsigned, is therefore

$$1111\ 1111\ 1111_2$$
$$= 2^{12} - 1 = \underline{4095_{10}}$$

(*iii*) The operand field contains the address of an item of data when bits 0 and 1 of the miscellaneous field contain 10. The field contains 12 bit and therefore the number of different items which can be addressed is $2^{12} = \underline{4096.}$

(*iv*) If the literal is signed, then the most significant bit is used as a sign bit. The field is 12 bit consisting of

```
  1        111 1111 1111
sign          number
 bit
```

By convention, if the sign bit is 0, the number is positive and if the sign bit is 1, the number is negative and in 2's complement form.

The maximum positive value is

$$0\,111\ 1111\ 1111_2$$
$$= 2^{11} - 1 = \underline{2047_{10}}$$

The maximum negative number using the 2's complement method is

```
1000 0000 0000
0111 1111 1111   1's complement
             1   +
1000 0000 0000   2's complement
```

The 2's complement of the number is the same as the original number and in this case the most significant bit is used as part of the literal. The maximum negative number is therefore $-2^{11} = -2048_{10}$.

Q3 (*a*) (*i*) *Explain the doubling-and-halving method of multiplication using the binary values 1 001 001 and 101 as multiplicand and multiplier respectively.*

(*ii*) *What advantage has this method over that of repeated addition?*

(*b*) *Write a program, using a machine code of your own choice (give a key), that will multiply two integers and output the result; it is to be assumed that a multiply function is not available.*

A3 (*a*) (*i*) To perform multiplication by the doubling-and-halving method, the multiplicand and the multiplier are first placed in two separate registers. The multiplier is then checked to determine whether it is odd or even (that is, the least significant bit contains 1 or 0). If the content is 1, the multiplicand is added to an *accumulator* register, which was initially set to zero. If the content is 0, no addition occurs.

The multiplier is then divided by the modulo 2 method. This is simply achieved by shifting the contents of the multiplier register one place to the right and discarding the least significant bit. Simultaneously, the multiplicand is doubled by shifting the contents of the multiplier register one place to the left.

The above process is repeated until the contents of the multiplier register is zero, when the result of the multiplication is found within the accumulator.

As is normal when performing multiplication, the capacity of the accumulator must be sufficient to store the number which is the result of multiplying the largest multiplier and multiplicand. Also, the multiplicand register must be capable of storing a similar length number.

Using the value $1\,001\,001_2$ (73_{10}) as the multiplicand and the value 101_2 (5_{10}) as the multiplier, the method of doubling-and-halving multiplication is demonstrated in the table.

Multiplicand Register	Multiplier Register	Operation	Accumulator
1 001 001	101	Add 1 001 001	1 001 001
10 010 010	010	Nothing	1 001 001
100 100 100	001	Add 100 100 100	101 101 101
1 001 001 000	000	Answer in Accumulator	101 101 101

The answer obtained is 101 101 101_2 (365_{10}).

(*ii*) The main advantage of the doubling and halving multiplication technique compared to the repeated addition method is that the minimum of necessary additions are performed in obtaining the result. Using repeated addition, the number of times it is necessary to add the multiplicand to the accumulator equals the value of the multiplier.

Also, all the other operations (shifting one place to the left or right, checking the least significant bit, checking for zero) are very quick and simple and are basic to a computer. Thus the method is very simple to implement and very quick in its operation.

(*b*) A machine-code program which performs multiplication by the doubling-and-halving method is shown below together with a key to the operations used.

PROGRAM			COMMENTS
	LOAD	ZERO	0 → Accumulator, Acc
	STORE	MULAC	(Acc) → Multiplication Accumulator
	READ	MCD	Read in Multiplicand
	READ	MPL	Read in Multiplier
	LOAD	MPL	(MPL) → Acc
REPEAT:	AND	ONE	(Acc) AND (ONE) → Acc
	JEQ	L1	Jump to L1 if (Acc) = 0
	LOAD	MULAC	(MULAC) → Acc
	ADD	MCD	(Acc) + (MCD) → Acc
	STORE	MULAC	(Acc) → MULAC
L1:	SHL	MCD	Shift (MCD) one place to left
	SHR	MPL	Shift (MPL) one place to right
	LOAD	MPL	(MPL) → Acc
	JNZ	REPEAT	Jump to REPEAT if (Acc) ≠ 0
	OUT	MULAC	Output (MULAC)
	STOP		

CONSTANTS ZERO: 0, ( ) indicates *contents of*
ONE : 1.

Key

OPERATION	DESCRIPTION
LOAD X	Set Accumulator to contents of store location X.
STORE X	Store contents of Accumulator in store location X.
READ X	Read in next value from input device and store in store location X.
AND X	Perform logical AND function on each bit of the Accumulator and contents of store location X.
JEQ	If contents of the Accumulator are zero then jump to store location L.
ADD X	Add contents of store location X to Accumulator.
SHL X	Shift contents of store location X one place to left. Lose most significant bit of store location.
SHR X	Shift contents of store location X one place to right. Lose least significant bit.
JNZ L	If contents of Accumulator not equal to zero then jump to location L.
OUT X	Output contents of store location X to output device.
STOP	Halt program.

Q4 (*a*) *Using the Venn diagram in Fig. 1 below, write down Boolean expressions for areas 1, 2 and 3.*

(*b*) *Give* ONE *expression to define the combination of these areas and sketch the appropriate logic circuit to provide this function.*

(*c*) *Draw a Karnaugh or Veitch map, marking those squares that satisfy the function in part* (*b*) *above.*

(*d*) *Write down an expression for area 4 and mark the map to correspond to this.*

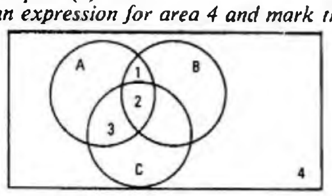


Fig. 1

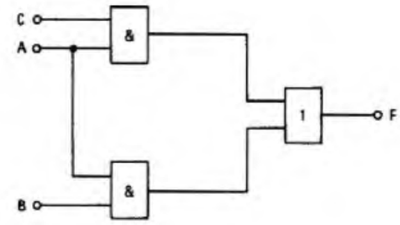
A4 (*a*) The expressions F1, F2, and F3 are Boolean expressions for the areas 1, 2 and 3 respectively in the Venn diagram:

$$F1 = A.B.\bar{C}$$
$$F2 = A.B.C$$
$$F3 = A.\bar{B}.C$$

(*b*) The combination of the areas 1, 2 and 3 is defined by the Boolean expression:

$$F = A.C + A.B$$

A logic circuit to provide this function is shown in sketch (*a*).



(*a*)

(*c*) A Karnaugh map is shown in sketch (*b*). The squares marked 1 satisfy the function

$$F = A.C + A.B$$

(*d*) Area 4 is defined by the Boolean expression:

$$F4 = \bar{A}.\bar{B}.\bar{C}$$

The square marked 4 on the Karnaugh map in sketch (*b*) satisfies this function.

	A.B	A.$\bar{B}$	$\bar{A}$.$\bar{B}$	$\bar{A}$.B
C	1	1		
$\bar{C}$	1		4	

(*b*)

Q5 (*a*) *With the aid of a circuit diagram, voltage and logic truth tables, explain how diodes may be used to provide a 3-input* AND *logic element.*

(*b*) *By adding a transistor stage, show how this element may provide a* NAND *function.*

(*c*) *By reversing the "logic" of the input signals in* (*a*) *and* (*b*), *what functions are obtained? Use truth tables to support your answer.*

(*d*) *State* TWO *advantages gained by using transistors rather than simple diode elements.*

A5 (*a*) See A7, Computers A 1973, *Supplement*, Vol. 67, p. 38, July 1974.

(*b*) See A10, Computers A 1977, *Supplement*, Vol. 71, p. 19, Apr. 1978.

(*c*) If the logic values of the input signals is reversed (that is, 0 volts = 1 and $+V_{cc}$ volts = 0) the following truth tables are obtained for the circuits without (circuit (*a*)) and with (circuit (*b*)) the transistor stage respectively.

Inputs A	B	C	Output O/P1
0	0	0	0
0	0	1	1
0	1	0	1
0	1	1	1
1	0	0	1
1	0	1	1
1	1	0	1
1	1	1	1

Circuit (*a*)

Inputs A	B	C	Output O/P2
0	0	0	1
0	0	1	0
0	1	0	0
0	1	1	0
1	0	0	0
1	0	1	0
1	1	0	0
1	1	1	0

Circuit (*b*)

The function obtained from circuit (*a*) is OR and from circuit (*b*) NOR.

(*d*) Two reasons why transistor gates are preferable to simple diode gates are:

(*i*) The transistor stage provides a degree of amplification which overcomes the effects of loading by successive gates. This enables the output to be fed to a larger number of gates; that is, it increases the *fan-out*.

(*ii*) The transistor introduces inversion of the logic states; that is, NAND and NOR gates are realized. These can be used in combination to produce any logic function.

Q6 (*a*) *Describe briefly* TWO *types of immediate access store.*

(*b*) *State the relative advantages of* EACH *type.*

(*c*) *Explain the meaning of the terms "cycle time" and "access time".*

Q7 (*a*) *Write down the general form of radix notation and explain it with reference to the octal scale.*

(*b*) *Convert the denary number 4087 to its binary equivalent.*

(*c*) *Write down the octal equivalent of* 4087_{10}.

(*d*) *If the number* 3476_8 *was shifted one place to the left in a binary register, what octal number would be produced? (Assume the register is large enough to contain the result.)*

A7 (*a*) The radix is the basis of a number system. Each digit of a number represents a multiple of a power of the radix. For example, each digit in an octal number represents a multiple of a power of 8, and thus the octal system has the radix 8. The powers of the radix are determined by the position of the digits within the number. The expression of numbers by this positional representation is called *radix notation.* Any number, N, can be represented by radix r and digits D_n, D_{n-1}, D_{n-2} etc., thus:

$$N = D_n r^n + D_{n-1} r^{n-1} \ldots + D_0 r^0 + D_{-1} r^{-1} \ldots D_{-m} r^{-m}$$

where terms up to and including $D_0 r^0$ represent the integral part, terms after $D_0 r^0$ represent the fractional part, and D can take any integral value from 0 to $r - 1$.

(*b*) The number 4087_{10} can be converted into radix-2 notation by repeatedly dividing by 2 and writing down the remainder in reverse order, as shown.

	Remainder
2)4087	
2)2043	1
2)1021	1
2) 510	1
2) 255	0
2) 127	1
2) 63	1
2) 31	1
2) 15	1
2) 7	1
2) 3	1
2) 1	1
0	1

$\therefore\ 4087_{10} = \underline{111\ 111\ 110\ 111_2}$

(*c*) Similarly, 4087_{10} can be converted to radix-8 notation by repeated division by 8 as shown

	Remainder
8)4087	
8) 510	7
8) 63	6
8) 7	7
0	7

$\therefore\ 4087_{10} = \underline{7767_8}$

(*d*) Shifting a number one place to the left in a binary register is equivalent to multiplying that number by 2. Hence, shifting 3476 one place to the left is equivalent to multiplying the number by 2 using octal arithmetic.

```
       3476
         2×
       7174
Carry   111
```

Thus, the octal result is $\underline{7174_8}$

Q8 (*a*) *Describe briefly* ONE *typical input device and* ONE *typical output device of a different type, used with digital computer systems.*

(*b*) *State the main features of* EACH *device which distinguish it from other parts of the computer and explain how these features are catered for in the system.*

Q9 (*a*) *Variables in digital and analogue computers are represented in differing ways. Describe these ways and include in your answer any factors such as component tolerance which affect this representation.*

(*b*) *What does "scale factor" mean?*

(*c*) *Show how scale factor affects a passive summing network.*

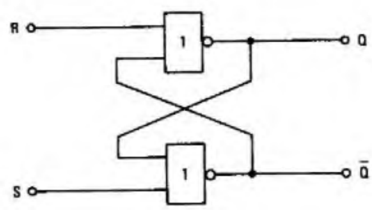
Q10 (*a*) *Draw a block diagram of a bistable device and explain its operation.*

(*b*) (*i*) *State* TWO *possible applications for a circuit containing three or more of these devices.*

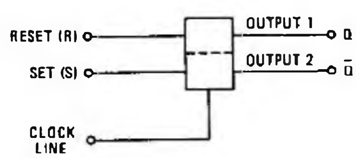
(*ii*) *Show how the bistables are interconnected to perform the required function in these applications.*

(*iii*) *Describe, with the aid of a waveform diagram, the operation of* ONE *of the networks in part (ii) above.*

A10 (*a*) A bistable can be represented by the block diagram shown in sketch (*a*). The usual symbol for this element is shown in sketch (*b*).



(*a*)



(*b*)

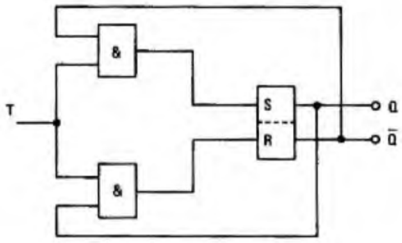
The common *flip-flop* circuit is used as the basis of the gated-bistable device. The device can be in either of two stable states representing binary 1 or 0. A bistable element is an asynchronous device, which means that as soon as the required logic levels are present on inputs R and S, the outputs Q and $\bar{Q}$ will immediately take up the logic states dictated by the inputs. Asynchronous operation of bistables leads to transient problems in some applications and therefore a clocking system is used to synchronize the operation of the bistable with an external CLOCK signal.

When Q = 1 and $\bar{Q}$ = 0, the bistable is said to be in the SET condition and when Q = 0 and $\bar{Q}$ = 1, it is said to be in the RESET condition. The bistable is set when the inputs are set to S = 1, R = 0; to reset the bistable the inputs are set to S = 0, R = 1. When both inputs are zero, the circuit outputs remain stable in the condition induced by the previous states of the inputs. The input condition of both R = 1 and S = 1 is normally prohibited and the outputs of the bistable are said to be *indeterminate*. The truth table defining the operation of the R-S bistable is shown below.

R	S	Q_n	Q_{n+1}
0	0	0	0
0	0	1	1
0	1	0	1
0	1	1	1
1	0	0	0
1	0	1	0
1	1	0	} Indeterminate
1	1	1	

In this table, the Q_n column gives the state of the Q output before the inputs R and S are applied. Q_{n+1} gives the state of the Q output after the inputs have been applied and the circuit has settled. Obviously, the $\bar{Q}$ output is always opposite to the Q output.

A simple extension of the R-S bistable is the triggered-T-type bistable. This type of bistable consists of a basic R-S bistable with feedback connexions from Q and $\bar{Q}$ to AND gates and the third T input. A block diagram of the conversion of an R-S bistable to a T-type bistable is shown in sketch (*c*).



(*c*)

Each time a positive pulse is applied to the T input, the R-S bistable changes state but a negative input has no effect.

The truth table for the T-type bistable is

T	Q_n	Q_{n+1}
0	0	0
1	0	1
0	1	1
1	1	0

This bistable is the fundamental unit of the binary counter and its function is to divide the frequency of the T input signal by two; that is, Q and Q change state at half the frequency of T.

(*b*) (*i*) Two possible applications of circuits containing three or more bistables are a three-bit counter and a three-bit shift register.

For the descriptions of operation of both networks see A5, Computers A 1975, *Supplement*, Vol. 69, p. 18, Apr. 1976.

TELECOMMUNICATION PRINCIPLES B 1978

Students were expected to answer any 6 questions

Q1 (*a*) *Sketch typical B/H curves of materials suitable for*
(*i*) *transformer cores, and*
(*ii*) *steel for permanent magnets.*

(*b*) *Label the curves in* (*a*) *to illustrate the terms*
(*i*) *hysteresis,*
(*ii*) *coercivity,*
(*iii*) *remanence, and*
(*iv*) *relative permeability.*

(*c*) *A transformer core of iron laminations of relative permeability 500 is assembled in the shape of a square with mean magnetic path length 0·40 m. The cross-section is uniformly square, 2 cm × 2 cm.*

(*i*) *Calculate the MMF needed to produce a flux density of 0·01 T in the core.*

(*ii*) *An air-gap of 1 mm length is now cut across the core. Find the MMF needed to produce a flux density of 0·01 T in the air-gap, assuming that 10% of the total MMF is ineffective due to leakage.*

A1 (*c*) The iron core is square with a mean path for magnetic flux of 0·40 m and the flux density (B) is 0·01 T.

If H is the magnetizing force, for the iron circuit

$$H = B/\mu_0\mu_r \text{ where } \mu_0 = 4\pi \times 10^{-7} \text{ and } \mu_r = 500.$$

$$\text{Hence, } H = \frac{0{\cdot}01}{4\pi \times 10^{-7} \times 500} = 15{\cdot}9 \text{ A/m}.$$

$$\text{Magneto-motive-force needed} = H \times \text{path length}$$
$$= 15{\cdot}9 \times 0{\cdot}40 = \underline{6{\cdot}36 \text{ A.}}$$

(*ii*) When the air-gap is introduced, an increase in magnetizing force will be needed to overcome the increased reluctance of the magnetic path. The total MMF will be the sum of that needed for the iron plus that for the air-gap. Assume that the path length in iron is unchanged by the cut for the air-gap. The MMF is then given in (*c*)(*i*) for the iron.

For the air, $\mu_r = 1$ and $\mu_0 = 4\pi \times 10^{-7}$.

$$\text{Then, } H_{\text{air}} = \frac{0{\cdot}01}{4\pi \times 10^{-7}} = 7{\cdot}95 \times 10^3 \text{ A/m}.$$

$$\text{MMF(air)} = H \times 10^{-3} = 7{\cdot}95 \text{ A}.$$

$$\text{Total MMF} = (6{\cdot}36 + 7{\cdot}95) = 14{\cdot}31 \text{ A}.$$

Leakage accounts for a further 10% so that the actual MMF required = $\underline{15{\cdot}9 \text{ A.}}$

Q2 (*a*) *Describe the principle of operation of* ONE *type of electronic voltmeter.*

(*b*) *What type of input circuit to the voltmeter is usual when an extension probe is to be used?*

(*c*) *Explain how the range of measurement can be altered.*

Q3 (*a*) *Explain briefly the principle of operation of the cathode ray tube* (*CRO*), *including focusing and beam deflexion.*

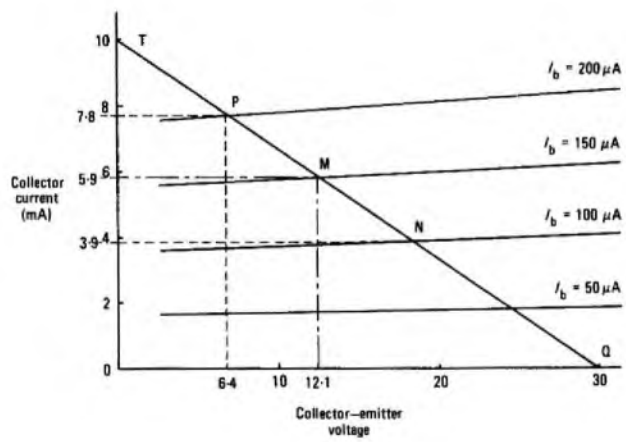
(*b*) *Explain how a CRO can be used to measure the frequency and amplitude of an alternating voltage.*

Q4 The table below gives collector–emitter voltage and collector-current characteristics for a transistor amplifier in common-emitter configuration.

(*a*) *Plot the curves and draw the load line for a collector load resistance of 3000 Ω and supply voltage of 30 V.*

(*b*) *Find*
(*i*) *the maximum power available in the load resistor with a peak base current of 200 μA,*
(*ii*) *the output resistance when* $I_b = 150\ \mu A$, *and*
(*iii*) *the collector-current/base-current gain for* $I_c = 6$ *mA.*

Collector–emitter voltage V_c	Collector Current (I_c mA) at Base Current (I_b μA)			
	50	100	150	200
2·5	1·65	3·60	5·70	7·70
10·0	1·72	3·80	5·90	8·00
20·0	1·82	3·94	6·15	8·30
30·0	1·93	4·10	6·40	8·60



A4 (*a*) The collector-current/collector-voltage curves are plotted in the sketch. The characteristic for each value of base current is nearly a straight line over the useful range.

The load-line is shown as QT. It intersects the V_c axis at V = 30 which, as the collector current at that point is zero, gives the maximum of 30 V across the load resistance. At the point T, where the load line cuts the current axis, the collector voltage will be zero, all the voltage being dropped across the 3000 Ω resistor.

The collector current will therefore be 10 mA.

(*b*) (*i*) The uppermost curve of the family corresponds to $I_b = 200\ \mu A$ and so the point P, where the load line cuts this curve, is the peak operating point. At this point, from the graph, $I_c = 7{\cdot}8$ mA and $V_c = 6{\cdot}4$ V. Since the maximum power is required in the load resistor, the other peak must be $I_c = 0$ (point Q), where the load line cuts the voltage axis, and $V_c = 30$ V.

The maximum voltage swing in the load = 30 − 6·4 = 23·6 V p-p.

The corresponding current swing = 7·8 − 0 = 7·8 mA p-p.

∴ RMS voltage = $23{\cdot}6/2\sqrt{2}$ = 8·34 V, and

RMS current = $7{\cdot}8/2\sqrt{2}$ = 2·76 mA.

∴ Maximum power in load = 8·34 × 2·76 = <u>23·02 mW.</u>

(*ii*) The curve for $I_b = 150\ \mu A$ cuts the load line at point M, where

$V_c = 12{\cdot}1$ V and $I_c = 5{\cdot}9$ mA.

∴ output resistance = $V_c/I_c = 12{\cdot}1/5{\cdot}9 \times 10^{-3}$ = <u>2050 Ω.</u>

(*iii*) When $I_c = 6$ mA, the load line cuts the 150 μA curve.

Taking this as the centre point of a signal swing of ±50 μA (that is from point P to point N) the corresponding peaks of I_c are ±2 mA.

$$\therefore \text{ the current gain } = \frac{2 \times 10^{-3}}{50 \times 10^{-6}} = \underline{40}.$$

Q5 (a) When the expression a − jb is used in connexion with a circuit what do a and b represent?

How is this interpreted in a phasor diagram?

(b) For the circuit shown in Fig. 1, calculate

(i) the equivalent impedance at the input,
(ii) the phase angle between input voltage and current, and
(iii) the input current to give 10 V output across AB.

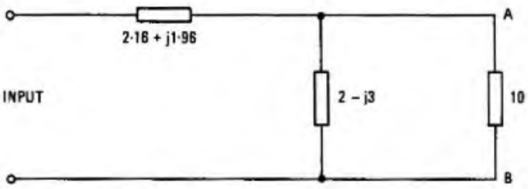


Fig. 1

A5 (*a*) The expression *a* − j*b* represents the impedance of the circuit. The term *a* is the resistance and *b* is the magnitude of the reactance. The negative sign denotes negative reactance due to a capacitance. A positive reactance would be due to an inductance.

In a phasor diagram, the operator j indicates that the corresponding reactance phasor is at right angles to the resistance phasor. This is illustrated in sketch (*a*).

The correct way to consider a phasor diagram is as if it were rotating round an axis at O; convention has established a counter-clockwise rotation as positive. The phasor which is then rotationally "in front" is said to be leading on those "behind".

Thus, in sketch (*b*), phasor OB is lagging phasor OA.

OA represents the resistance *a* to a suitable scale,

OB represents the reactance −j*b* to the same scale, and

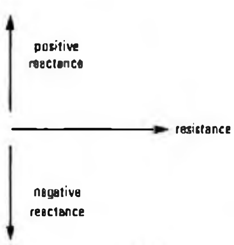
the resultant OC represents the impedance *a* − j*b*.

(*b*) Since the 10 Ω and 2 − j3 Ω are in parallel, they can be replaced by one equivalent impedance *Z'* where

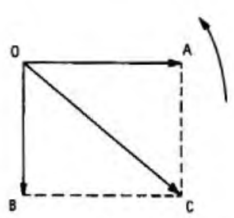
$$\frac{1}{Z'} = \frac{1}{2 - j3} + \frac{1}{10} = \frac{10 + 2 - j3}{10(2 - j3)} = \frac{3(4 - j)}{10(2 - j3)}.$$

$$\text{Hence, } Z' = \frac{10}{3}\frac{(2 - j3)(4 + j)}{(4 - j)(4 + j)} = \frac{10}{3 \times 17}(8 + 3 - 12j + 2j),$$

$$= \frac{10}{51}(11 - j10) \qquad = 2{\cdot}16 - j1{\cdot}96\ \Omega.$$



(*a*)



(*b*)

The equivalent input impedance is now *Z'* in series with 2·16 + j1·96. The j terms are equal and of opposite sign and so cancel out.

(*i*) Input impedance = 2·16 + 2·16,

= <u>4·32 Ω with zero reactance.</u>

(*ii*) Since the input impedance is a pure resistance the input current and voltage are in phase.

(*iii*) The phasor diagram representing the impedance *Z'* across AB is shown in sketch (*b*). OA represents the resistance 2·16 Ω and OB the reactance −j1·96 Ω. OC is the impedance.

By Pythagoras's theorem, OC = $\sqrt{(2{\cdot}16^2 + 1{\cdot}96^2)}$,

= $\sqrt{8{\cdot}508}$ = 2·917 Ω.

Therefore, by Ohm's law, an input current of

10/2·917 A = <u>3·43 A</u> is required.

Q6 (a) Explain briefly, with the aid of sketches, the principle of the direct current generator. What is the basic difference between the AC and the DC generators?

(b) A DC generator with a shunt-connected 240 Ω field winding is supplying a 60 Ω load at 200 V. The effective armature resistance is 5 Ω. The mechanical losses in the generator can be considered as an additional 120 Ω load shunted across the generator.

(i) Calculate the power actually generated in the armature.

(ii) What percentage of this power is taken by the 60 Ω load?

A6 (*b*) Sketch (*a*) shows the equivalent circuit of the given generator and load. The terminals A and B are the output connexions, so that the 240 Ω field coils, the 60 Ω load and the 120 Ω resistance equivalent to the mechanical losses are all effectively connected across AB. The resistance of the armature winding, 5 Ω, is in series with this total load of three resistances in parallel. The EMF generated in the armature must be large enough to produce 200 V across AB, plus the voltage drop in the 5 Ω internal resistance.

From sketch (*a*), the equivalent load across AB is

$$\frac{1}{\frac{1}{240} + \frac{1}{60} + \frac{1}{120}} = \frac{240}{7} = 34{\cdot}3\ \Omega.$$

The effective equivalent circuit, shown in sketch (*b*), consists of 5 Ω and 34·3 Ω in series, with a potential difference of 200 V across AB.

The current flowing in 34·3 Ω to produce 200 V = $\frac{200}{34{\cdot}3}$ = 5·83 A.

∴ the armature drop = 5 × 5·83 = 29·2 V.

Therefore the EMF generated in the armature is

200 + 29·2 = 229·2 V.

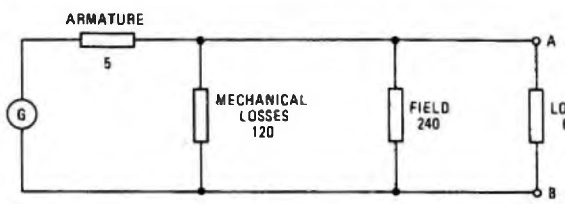
(*i*) The power actually generated in the armature is

229·2 × 5·83 = <u>1336·2 W.</u>

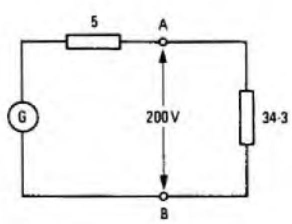
(*ii*) The power absorbed by the 60 Ω load is

$$\frac{200^2}{60} = 667\ \text{W}.$$

This represents $\frac{667}{1336} \times 100\%$ = <u>50%</u> of the total power generated.



(*a*)



(*b*)

Q7 A 10 μF capacitor is charged through 20 kΩ from a DC source that can give either a constant current or a constant voltage as required.

(a) When the capacitor is charged by a constant current, indicate graphically from the instant of switching on

(i) the charge/time relation for the capacitor,
(ii) the voltage/time relation for the capacitor, and
(iii) the voltage/time relation across the resistance-capacitance circuit.

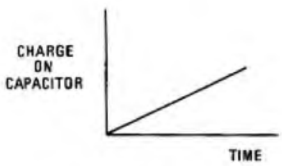
(b) Give an expression for the instantaneous voltage across the capacitor t seconds after connexion to a constant voltage source.

(c) When the capacitor is charged through the resistor from a 100 V supply, find

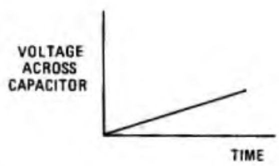
(i) the time constant,
(ii) the initial current, and
(iii) the maximum energy stored.

A7 (*a*) A charge Q coulombs produces a voltage V across a capacitance C farads according to the relationship $Q = CV$.

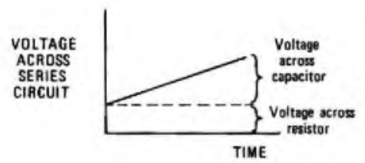
A small increase in charge, dQ, will give a small increase in voltage, dV, where $dV = dQ/C$. But the rate of increase of charge per second is the charging current in amperes. So the charge/time relation is a straight line having as gradient the value of the current.



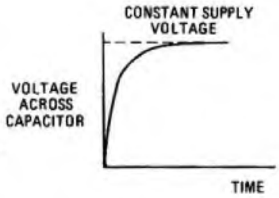
(*a*)



(*b*)



(*c*)



(*d*)

(*i*) The charge/time relation for the capacitor is shown in sketch (*a*) for the constant charging current condition.

(*ii*) The voltage/time relation for the capacitor is shown in sketch (*b*). This is also a straight line because voltage is proportional to charge in the fixed capacitor.

(*iii*) When a resistance is in series with the capacitor and the current is constant, by Ohm's law there must be a constant voltage across the resistor. So the voltage across the series circuit will be that across the capacitor increased by the constant voltage across the resistor, as shown in sketch (*c*).

(*b*) When the series circuit is energized from a constant-voltage source the voltage across the capacitor will rise exponentially, and will ultimately reach the voltage of the source. It cannot rise further than this. The capacitor voltage/time relation is shown in sketch (*d*).

If v is the voltage across the capacitor C at time t seconds from switching-on and the fixed source voltage is V, then

$$v = V(1 - e^{-t/CR}) \text{ where } R \text{ is the series resistance.}$$

$$v = 100(1 - e^{-t/0\cdot0001 \times 20\,000}) = \underline{100(1 - e^{-5t})}.$$

(*c*) (*i*) The time constant $= CR = 10 \times 10^{-6} \times 20\,000 = \underline{0\cdot2 \text{ s}}$

(*ii*) At switch-on, all the source voltage will appear across the resistor: therefore the initial current is $100/20\,000 = \underline{5 \text{ mA}}$.

(*iii*) The energy stored in the capacitor $= \frac{1}{2}CV^2 = \frac{10^{-5}}{2} \times 100^2$,

$$= \underline{0\cdot05 \text{ J}}.$$

Q8 (a) (i) What features are essential in a circuit for resonance to occur? Use reactance/frequency curves as illustration.

(ii) Explain the different effects produced by series and parallel resonance.

(b) Derive a formula for the frequency of resonance in a series circuit.

(c) A resonant circuit is concealed in a box with only the two terminals accessible. Describe briefly an experiment to decide whether it is a series or parallel resonance.

Give the answer in the form of a short experimental report.

A8 (*a*) (*i*) Resonance will occur in an AC circuit that contains inductance and capacitance (that is, positive and negative reactances) when the frequency applied to the circuit makes these two reactances equal in magnitude and opposite in sign. The circuit is then non-reactive (that is, a pure resistance), at the frequency of resonance. Sketch (*a*) shows the reactance/frequency relationships for inductance and capacitance. The capacitive reactance, $-1/2\pi fC$, being negative, is drawn below the frequency axis; the inductive reactance, $2\pi fL$, is positive and is above the frequency axis. The frequency at which the two reactances are equal is the resonant frequency. Sketch (*a*) shows the algebraic sum of the two reactances in series, obtained by adding vertical ordinates algebraically. The frequency of resonance is that giving zero reactance (at point Z).

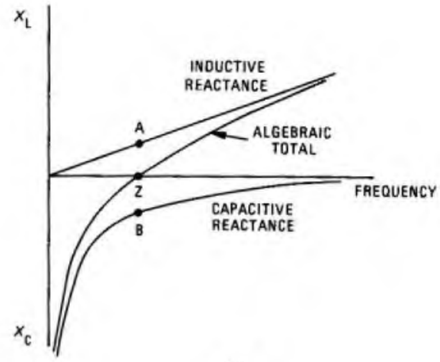
(*ii*) When inductance and capacitance are in series, the impedance at resonance is resistive; it is usually a low value being the resistance of the resonant circuit which is effectively the DC resistance of the inductance. The alternating current flowing is maximum because the impedance is minimum.

When the inductance and capacitance are in parallel, the impedance of the combination is a maximum at resonance. The external current into the circuit is therefore a minimum, but there is a high circulating current within the enclosed parallel reactive loop. This gives a high voltage across the external terminals if the external circuit conditions permit, as in the tuned circuit of a radio receiver.

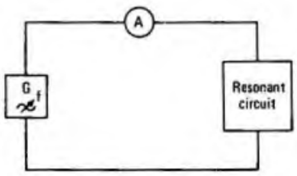
(*b*) As shown in part (*a*), resonance occurs when the inductive reactance equals the capacitative reactance.

$$\therefore \quad 2\pi fL = \frac{1}{2\pi fC}.$$

$$\therefore \quad \underline{f = \frac{1}{2\pi\sqrt{(LC)}}}.$$



(*a*)



(*b*)

(*c*) A circuit to determine whether the two-terminal network is a series or parallel resonant circuit is shown in sketch (*b*). The equipment needed is a wide-range variable-frequency oscillator as source, and either an AC ammeter to be connected in series with, or a high impedance AC voltmeter to be connected across, the unknown circuit.

The oscillator frequency is slowly increased and the behaviour of the meter noted. If the series-connected ammeter shows a sharp rise in the current followed by an equally sharp fall as the frequency is further increased, there is series resonant circuit in the box. If it shows a sharp fall followed by a rise, there is a parallel resonant circuit.

If an AC voltmeter is used instead of an ammeter, the voltage across the terminals will become high at parallel resonance and very low at series resonance.

Q9 (*a*) (*i*) *Explain the principle of the transformer.*

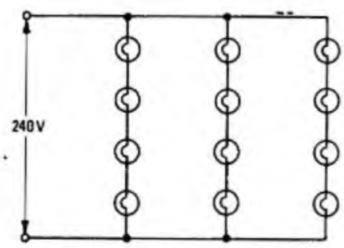
(*ii*) *Deduce the relation between the secondary resistive load and the resistance that can be measured across the primary. Assume that there are no losses in the transformer.*

(*b*) *Twelve 60 V lamps of rating 60 W each are to be operated from a 240 V supply. A transformer with a winding ratio of 4 : 1 is available if required.*

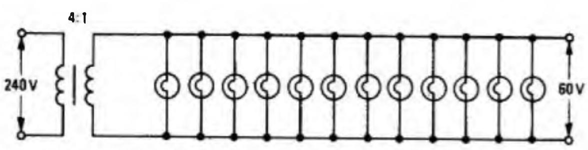
(*i*) *Give* TWO *practical methods of operating the lamps.*
(*ii*) *State the relative advantages of the* TWO *methods.*

A9 (*b*) (*i*) The circuits of 2 methods of feeding the twelve lamps are shown in sketches (*a*) and (*b*). Since four 60 V lamps in series need 240 V, three such groups of four across the 240 V supply will operate correctly, without any inefficiency (sketch (*a*)). In sketch (*b*), the transformer provides a 60 V supply and all 12 lamps operate in parallel.

(*ii*) The method shown in sketch (*a*) is more economical in plant cost but less reliable because failure of one lamp extinguishes four. The method shown in sketch (*b*) gives maximum reliability, as each lamp is independent: but an extra transformer is needed, involving capital cost and wasted power in losses. The safety aspect of low-voltage (60 V) distribution could be important.



(*a*)



(*b*)

Q10 (*a*) *Define the decibel.*

(*b*) (*i*) *Explain the difference between "dB" and "dBm".*
(*ii*) *What is the meaning of −3 dB?*
(*iii*) *What is the meaning of 6 dBW?*

(*c*) *The network in Fig. 2 is supplied from a 20 V source across AB. Express the power in load CD in decibels relative to that in load EF in Fig. 3.*

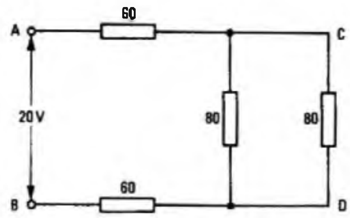


Fig. 2

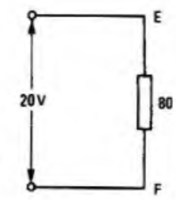
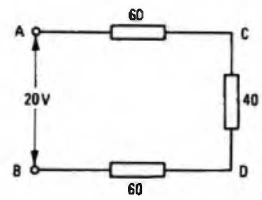


Fig. 3

A10 (*c*) The equivalent circuit of Fig. 2 is shown in the sketch.



The total series resistance = 60 + 60 + 40 Ω,
= 160 ohms.

Since the same current flows in each of the components, the voltage across CD $= \frac{40}{160} \times 20 = 5{\cdot}0$ V.

Then the power in the 80 Ω resistor across CD $= \frac{5^2}{80} = 0{\cdot}3125$ W.

In Fig. 3 the power in 80 Ω resistor across EF $= \frac{20^2}{80} = 5{\cdot}0$ W.

The ratio of the two powers $= 10 \log_{10} \frac{0{\cdot}3125}{5{\cdot}0} = 10 \log_{10} 0{\cdot}0625$,
$= -12$ dB approx.

MATHEMATICS B 1978

Students were expected to answer any 6 questions

Q1 (*a*) *By completing the square, find the maximum or minimum value of the quadratic expression* $5 + 7x - 3x^2$, *and state the value of x which corresponds to this.*

(*b*) *Solve to 2 significant figures the simultaneous equations*

$5x - 2z = 7{\cdot}8$,
$3y + 5z = 11{\cdot}6$, and
$x + y + 3z = 9{\cdot}5$.

A1 (*a*) Let $y = 5 + 7x - 3x^2$,

$$= -3\left(x^2 - \frac{7x}{3} - \frac{5}{3}\right),$$

$$= -3\left[x^2 - \frac{7x}{3} + \left(\frac{7}{6}\right)^2 - \frac{5}{3} - \left(\frac{7}{6}\right)^2\right],$$

$$= -3\left[\left(x - \frac{7}{6}\right)^2 - \frac{60 + 49}{36}\right],$$

$$= \frac{109}{12} - 3\left(x - \frac{7}{6}\right)^2.$$

The term $3\left(x - \frac{7}{6}\right)^2$ is positive irrespective of the sign of x, and hence, y must decrease for any increase in the value of x. Hence, there will be a maximum value of y when $3\left(x - \frac{7}{6}\right)^2$ is zero, which occurs when $x = \frac{7}{6}$.

$\therefore\ y_{max} = \frac{109}{12} = 9{\cdot}08\dot{3}$ at $x = 1{\cdot}1\dot{6}$.

(*b*) Numbering the 3 equations (1), (2) and (3) respectively.
Multiplying equation (1) by 3 gives

$15x - 6z = 23{\cdot}4.$ .... (4)

Multiplying equation (2) by 5 gives

$15y + 25z = 58.$ .... (5)

Adding equations (4) and (5) gives

$15x + 15y + 19z = 81{\cdot}4$ or,

$x + y + \frac{19}{15}z = \frac{81{\cdot}4}{15} = 5{\cdot}42\dot{6}.$ .... (6)

Subtracting equation (6) from equation (3) gives

$$z(3 - 1\cdot 2\dot{6}) = 9\cdot 5 - 5\cdot 42\dot{6}.$$

$$\therefore \quad z = \frac{4\cdot 07\dot{3}}{1\cdot 73} = 2\cdot 350.$$

Substituting for z in equation (1) gives

$$5x = 7\cdot 8 + 4\cdot 7$$

$$\therefore \quad x = \frac{12\cdot 5}{5} = 2\cdot 5$$

Substituting for z in equation (2) gives

$$3y = 11\cdot 6 - 11\cdot 75.$$

$$\therefore \quad y = \frac{-0\cdot 15}{3} = -0\cdot 05.$$

Hence, to 2 significant figures, $x = 2\cdot 5$, $y = -0\cdot 05$ and $z = 2\cdot 4$.

Q2 (a) Sketch the graph of $i = 8 \cos\left(\omega t + \frac{\pi}{6}\right)$, *where i is measured in milliamperes and* $\omega = 5000$ *rad/s, from* $t = 0$ *to* $t = \frac{2\pi}{\omega}$ *seconds.*

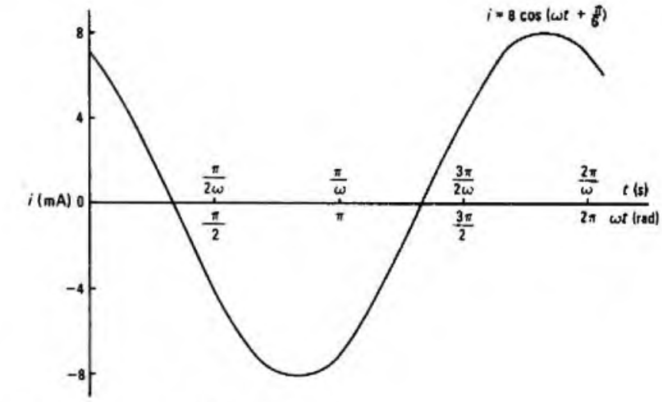
(b) Calculate i when $t = 1\cdot 6 \times 10^{-4}$ *s.*

(c) Find the time ($t > 0$) *when (i) the current is first zero and (ii) the current first attains its positive peak value.*

A2 (*a*) When $t = \frac{2\pi}{\omega}$, the angle is $\left(2\pi + \frac{\pi}{6}\right)$ or 2π rad (360°) greater than the value of the angle $\left(\frac{\pi}{6}\right)$ when $t = 0$. Thus, there is one complete period of the cosine wave from $t = 0$ to $t = \frac{\omega}{2\pi}$. Since $\cos\theta = \cos(-\theta)$, the curve may be sketched from salient values of i from $t = 0$ to $t = \frac{\omega}{\pi}$; these are tabulated below.

t (s)	0	$\frac{\pi}{6\omega}$	$\frac{\pi}{3\omega}$	$\frac{\pi}{2\omega}$	$\frac{2\pi}{3\omega}$	$\frac{5\pi}{6\omega}$	$\frac{\pi}{\omega}$
ωt (rad)	0	$\frac{\pi}{6}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	$\frac{2\pi}{3}$	$\frac{5\pi}{6}$	π
ωt °	0	30	60	90	120	150	180
$\left(\omega t + \frac{\pi}{6}\right)^\circ$	30	60	90	120	150	180	210
$\cos\left(\omega t + \frac{\pi}{6}\right)$	0·866	0·5	0	−0·5	−0·866	−1	−0·866
i	6·928	4	0	−4	−6·928	−8	−6·928

The curve is shown in the sketch.



(*b*) When $t = 1\cdot 6 \times 10^{-4}$ s,

$$i = 8\cos\left(5000 \times 1\cdot 6 \times 10^{-4} + \frac{\pi}{6}\right),$$
$$= 8\cos(0\cdot 8 + 0\cdot 5236),$$
$$= 8\cos 1\cdot 3236 \text{ rad} = 8\cos 75^\circ\, 50',$$
$$= 8 \times 0\cdot 2447 = 1\cdot 9576 \text{ mA}.$$

(*c*) (*i*) When $i = 0$, $\cos\left(\omega t + \frac{\pi}{6}\right) = 0$.

$$\therefore \quad \omega t + \frac{\pi}{6} = \frac{\pi}{2} \text{ or } \frac{3\pi}{2} \text{ in the range 0 to } 2\pi.$$

$$\therefore \quad \omega t = \frac{\pi}{2} - \frac{\pi}{6} = \frac{\pi}{3} \text{ when } i \text{ is first zero.}$$

$$\therefore \quad \omega t = \frac{\pi \times 10^3}{3 \times 5000} = 0\cdot 2094 \text{ ms.}$$

(*ii*) When $i = 8$ mA, $\cos\left(\omega t + \frac{\pi}{6}\right) = 1$.

$$\therefore \quad \omega t + \frac{\pi}{6} = 0 \text{ or } 2\pi \text{ in the range 0 to } 2\pi.$$

For $t > 0$, the first solution is inadmissible because it would give a negative value of t.

Hence, $\quad \omega t = \frac{11\pi}{6}$.

$$\therefore \quad t = \frac{11\pi \times 10^3}{6 \times 5000} = 1\cdot 152 \text{ ms.}$$

Q3 The number (y) of employees at a factory on 1 January is shown in the table, between the years 1962 and 1969.

Year	*1962*	*1964*	*1966*	*1969*
y	*435*	*507*	*601*	*754*

Assume the formula $y = ak^x$ *connects these figures x years from 1 January 1962.*

(a) Plot suitable variables to test this assumption.

(b) From the graph, obtain estimates of the constants a and k.

(c) Estimate the number employed on 1 January 1972.

(d) What is the annual percentage growth in the number employed?

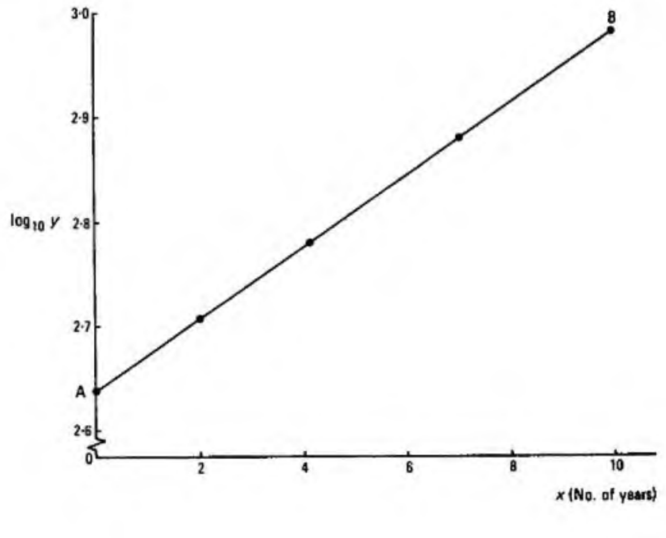
A3 (*a*) If $y = ak^x$, then, taking logarithms,

$$\log_{10} y = \log_{10} a + x \log_{10} k.$$

Since $\log_{10} a$ and $\log_{10} k$ are constants, this law is of the linear form $y = mx + c$, and plotting $\log_{10} y$ against x should yield a straight line. The appropriate variables are shown in the table.

Year	1962	1964	1966	1969
x	0	2	4	7
y	435	507	601	754
$\log_{10} y$	2·6384	2·7050	2·7788	2·8773

The graph of $\log_{10} y$ against x is shown in the sketch, from which it



is evident that it is possible to draw a straight line to pass very close to all 4 points. Hence, the assumption of the law $y = ak^x$ is justified.

(*b*) When $x = 0$, $\log_{10} y = \log_{10} a$, and hence $y = a$.

Reading from the graph, at $x = 0$, $\log_{10} y = 2{\cdot}636$.

$\therefore \quad a = \text{antilog}_{10}\, 2{\cdot}636 = \underline{432{\cdot}5.}$

The gradient of the graph is equal to $\log_{10} k$. From the widely separated points A and B, which actually lie on the straight line,

$$\text{gradient} = \frac{2{\cdot}983 - 2{\cdot}636}{10 - 0} = 0{\cdot}0347.$$

$\therefore \quad k = \text{antilog}_{10}\, 0{\cdot}0347 = \underline{1{\cdot}083.}$

(*c*) At 1 January 1972, $x = 10$.

From the graph, at $x = 10$, $\log_{10} y = 2{\cdot}983$.

$\therefore \quad y = \text{antilog}_{10}\, 2{\cdot}983 = 961{\cdot}6.$

Therefore the number employed at 1 January 1972 = $\underline{962.}$

(*d*) Since x and y are not related by a linear law, the annual growth rate is not constant, but will depend on the number of years considered. Taking $x = 7$, to correspond with the available data,

$\log_{10} y = 2{\cdot}88$ (from the graph), and

$y = 758{\cdot}6 = 759$ to the nearest integer.

At $x = 0$, $y = 433$ to the nearest integer (from part (*b*)).

Therefore the annual percentage growth rate from 1962 to 1969

$$= \frac{759 - 433}{7 \times 433} \times 100\% = \underline{10{\cdot}76\%.}$$

Q4 (a) From the definition of a logarithm, state and prove a rule for converting the logarithm of a number to base e to its logarithm to base 10.

(b) Show that $\log_b a = \dfrac{1}{\log_a b}$.

(c) Calculate $\log_e (0{\cdot}72)$ *using logarithms to base 10 only.*

(d) Solve for x in the equation, $\log_e (x + 2) = -1{\cdot}2$.

($e = 2{\cdot}7183$.)

A4 (*a*) If N is the given number, then

$$\underline{\log_{10} N = \log_e N \times \log_{10} e = 0{\cdot}4343 \log_e N.}$$

Proof: Let $\log_e N = p$, whence, by definition

$N = e^p$.

Taking logarithms to base 10, $\log_{10} N = \log_{10} e^p$,

$= p \times \log_{10} e$.

But $\quad p = \log_e N$.

$\therefore \quad \log_{10} N = \log_e N \times \log_{10} e. \qquad$ QED

(*b*) Let $\log_b a = q$.

$\therefore \quad a = b^q$

$\therefore \quad a^{1/q} = b$ and

$\log_a (a^{1/q}) = \log_a b$.

$\therefore \quad 1/q \times 1 = \log_a b$.

$\therefore \quad \dfrac{1}{\log_b a} = \log_a b. \qquad$ QED

(*c*) From part (*a*), $\log_e 0{\cdot}72 = \dfrac{\log_{10} 0{\cdot}72}{\log_{10} e}$,

$$= \frac{\bar{1}{\cdot}8573}{0{\cdot}4343} = \frac{-0{\cdot}1427}{0{\cdot}4343},$$

$$= -0{\cdot}3285 = \underline{\bar{1}{\cdot}6715.}$$

(*d*) $\quad \log_e (x + 2) = -1{\cdot}2.$

$\therefore \quad \log_{10} (x + 2) = -1{\cdot}2 \times 0{\cdot}4343 = -0{\cdot}5212 = \bar{1}{\cdot}4788.$

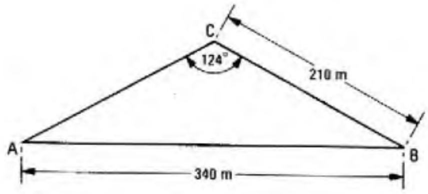
$\therefore \quad x + 2 = \text{antilog}_{10}\, \bar{1}{\cdot}4788 = 0{\cdot}3012.$

$\therefore \quad x = 0{\cdot}3012 - 2 = \underline{-1{\cdot}6988.}$

Q5 On a triangular building site ABC, AB = 340 m, BC = 210 m and angle BCA = 124°.

(a) Calculate the remaining side and angles of the triangle.

(b) Calculate the area of the site in hectares, given 1 hectare = $10^4\ m^2$.



A5 (*a*) The triangle is shown in the sketch.

By the sine rule, $\dfrac{\sin \angle CAB}{BC} = \dfrac{\sin \angle ACB}{AB}$.

$$\therefore \quad \sin \angle CAB = \frac{210 \times \sin 124°}{340} = 0{\cdot}5120.$$

$\therefore \quad \angle CAB = \underline{30°\ 48'.}$

Hence, $\angle ABC = 180° - (124° + 30°\ 48')$,

$= \underline{25°\ 12'.}$

Again, using the sine rule, $\dfrac{CA}{\sin \angle ABC} = \dfrac{AB}{\sin \angle ACB}$.

$$\therefore \quad CA = \frac{340 \times \sin 25°\ 12'}{\sin 124} = \underline{174{\cdot}6\ m.}$$

(*b*) The area of the triangle $= \frac{1}{2} \times AB \times BC \times \sin \angle ABC$,

$= \frac{1}{2} \times 340 \times 210 \times \sin 25°\ 12'$,

$= 15\,200\ m^2$,

$= \underline{1{\cdot}52 \text{ hectares.}}$

Q6 (a) Using the expansions for $\sin (A + B)$ *and/or* $\cos (A + B)$ *and without the use of trigonometrical tables*

(i) calculate sin 75° to 3 significant figures, and

(ii) deduce a formula for $\tan 2\theta$ *in terms of* $\tan \theta$.

(b) Calculate in vulgar-fraction form

(i) $\cos 2\alpha$, *and (ii)* $\sin (\alpha + \beta)$

when $\sin \alpha = 3/5$, $\tan \beta = 5/12$ *and angle* α *is obtuse, while angle* β *is acute.*

A6 (*a*) $\quad \sin (A + B) = \sin A \cos B + \cos A \sin B$, and

$\cos (A + B) = \cos A \cos B - \sin A \sin B$.

(*i*) $\quad \sin 75° = \sin (45° + 30°)$,

$= \sin 45° \cos 30° + \cos 45° \sin 30°$,

$$= \frac{1}{\sqrt{2}} \times \frac{\sqrt{3}}{2} + \frac{1}{\sqrt{2}} \times \frac{1}{2},$$

$$= \frac{\sqrt{2}(1{\cdot}732 + 1)}{4} = \underline{0{\cdot}966} \text{ to 3 significant figures.}$$

(*ii*) $$\tan 2\theta = \frac{\sin 2\theta}{\cos 2\theta} = \frac{2 \sin \theta \cos \theta}{\cos^2 \theta - \sin^2 \theta}.$$

Dividing the numerator and the denominator by $\cos^2 \theta$ gives

$$\tan 2\theta = \frac{2\dfrac{\sin \theta}{\cos \theta}}{1 - \dfrac{\sin^2 \theta}{\cos^2 \theta}} = \underline{\frac{2 \tan \theta}{1 - \tan^2 \theta}}.$$

(*b*) (*i*) $\cos 2\alpha = \cos^2 \alpha - \sin^2 \alpha$,

$= 1 - 2 \sin^2 \alpha$ (since $\cos^2 \alpha + \sin^2 \alpha = 1$),

$$= 1 - 2 \times \left(\frac{3}{5}\right)^2 = \underline{\frac{7}{25}}.$$

(*ii*) $\sin (\alpha + \beta) = \sin \alpha \cos \beta + \cos \alpha \sin \beta$.

Since α is an obtuse angle, its cosine will be negative. Also, since $\sin \alpha = 3/5$, the supplement of α is an angle in a 3, 4, 5 triangle and $\cos \alpha = -4/5$. Also, since β is an acute angle and is an angle in a 5, 12, 13 triangle, $\cos \beta = 12/13$ and $\sin \beta = 5/13$.

$$\text{Hence, } \sin (\alpha + \beta) = \frac{3}{5} \times \frac{12}{13} + \left(\frac{-4}{5}\right) \times \frac{5}{13} = \underline{\frac{16}{65}}.$$

Q7 (a) If $s = ut + \frac{1}{2}gt^2$, *where u and g are constants, derive from first principles an expression for* $\dfrac{ds}{dt}$.

(b) For the function $y = x^3 - 12x + 4$, find the critical values of x at which $\frac{dy}{dx} = 0$ and discuss their significance.

(c) Sketch the graph of $y = x^3 - 12x + 4$, showing both positive and negative values of x.

A7 (*a*) Let t increase by a small amount δt and let δs be the corresponding increase in s.

Then, $s + \delta s = u(t + \delta t) + \frac{1}{2}g(t + \delta t)^2$.

$\therefore\ (ut + \frac{1}{2}gt^2) + \delta s = u(t + \delta t) + \frac{1}{2}g(t^2 + 2t\delta t + \delta t^2)$.

$$\delta s = ut + u\delta t + \tfrac{1}{2}gt^2 + gt\delta t + \tfrac{1}{2}g\,\delta t^2 - ut - \tfrac{1}{2}gt^2,$$
$$= u\delta t + gt\delta t + \tfrac{1}{2}g\,\delta t^2.$$
$$\therefore \quad \frac{\delta s}{\delta t} = u + gt + \tfrac{1}{2}g\delta t.$$

In the limit, as $t \to 0$, $\frac{\delta s}{\delta t} \to \frac{ds}{dt}$ and $\frac{1}{2}g\delta t$ becomes zero.

Hence, $\frac{ds}{dt} = \underline{u + gt}$.

(*b*)
$$y = x^3 - 12x + 4.$$
$$\therefore \quad \frac{dy}{dx} = 3x^2 - 12.$$

When $\frac{dy}{dx} = 0$, $3x^2 - 12 = 0$.

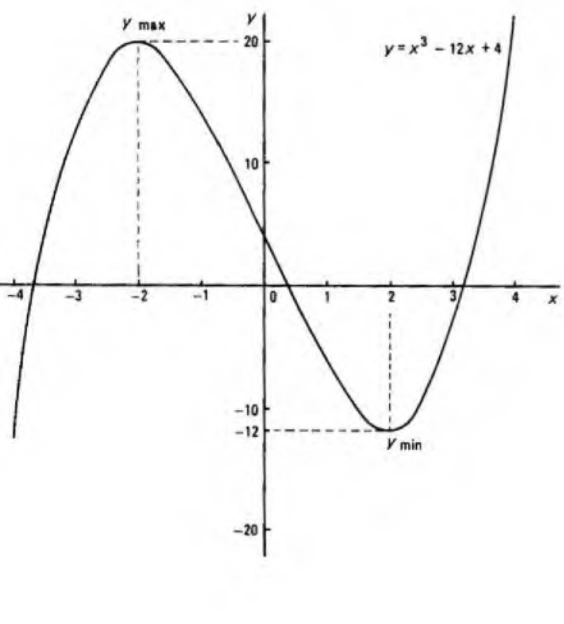
$$\therefore \quad \underline{x = \pm 2}.$$

The first derivative of a function, $\frac{dy}{dx}$, is a measure of the gradient of the curve at any point and when $\frac{dy}{dx} = 0$, this indicates that the gradient is zero or, in other words, that the tangent to the curve at that point is parallel to the x-axis. Such points are usually called *turning points* because, in passing through zero, the sign of $\frac{dy}{dx}$ changes from positive to negative or vice versa and the curve reaches a maximum value if, as x increases, the change in sign is from positive to negative; if the change is from negative to positive, a minimum value occurs at the point where $\frac{dy}{dx}$ is zero.

To determine whether a maximum or minimum value of y occurs at, say $x = +2$, it is necessary to examine the values of the function in the vicinity of $x = +2$, where $y = -12$. At $x = 1\frac{1}{2}$, $y = -10\frac{5}{8}$, which is greater than $y = -12$. Hence the curve is ascending towards $x = 0$ and a minimum value must therefore occur at $x = +2$. Similarly, it may be shown that, at $x = -2$, a maximum value of y occurs.

(*c*) The graph of $y = x^3 - 12x + 4$ is shown in the sketch with the maximum and minimum values of y clearly marked.

Notes: (*i*) The maximum or minimum values of a function are not necessarily the greatest or least values for any value of x, but only in relation to values in their immediate vicinity. As can be seen from the sketch, y becomes greater than 20 when $x > 4$ and less than -12 when $x < -4$.



(*ii*) The condition $\frac{dy}{dx} = 0$ does not inevitably indicate a maximum or minimum value. Exceptionally, it may indicate a different feature known as a *stationary point*, where the second derivative, $\frac{d}{dx}\left(\frac{dy}{dx}\right)$ or $\frac{d^2y}{dx^2}$, is also zero; this however is beyond the scope of the syllabus.

Q8 A constant voltage, V, is applied across a capacitor (C farads) in series with a resistor (R ohms) at time $t = 0$. The charge, q coulombs on the capacitor t seconds later is given by the approximate formula

$$q = \frac{Vt}{R}\left(1 - \frac{t}{2CR} + \frac{t^2}{6C^2R^2}\right)$$

provided t is substantially less than CR seconds.

(a) Assuming these conditions, write a formula for the charging current, i amperes, given by $i = \frac{dq}{dt}$.

(b) Calculate this current in milliamperes, when $t = 4 \times 10^{-3}$, $C = 2 \times 10^{-6}$, $R = 3 \times 10^4$ and $V = 60$.

A8 (*a*)
$$q = \frac{Vt}{R} - \frac{Vt^2}{2CR^2} + \frac{Vt^3}{6C^2R^3}.$$
$$\therefore \quad i = \frac{dq}{dt} = \frac{V}{R} - \frac{2Vt}{2CR^2} + \frac{3Vt^2}{6C^2R^3},$$
$$= \underline{\frac{V}{R}\left(1 - \frac{t}{CR} + \frac{t^2}{2C^2R^2}\right)}.$$

(*b*) When $t = 4 \times 10^{-3}$ s, $C = 2 \times 10^{-6}$ F, $R = 3 \times 10^4\ \Omega$ and $V = 60$ V.

$$CR = 2 \times 10^{-6} \times 3 \times 10^4 = 0{\cdot}06 \text{ s},$$

which is substantially greater than $0{\cdot}004$ s.

$$\therefore \quad i = \frac{60}{3 \times 10^4}\left[1 - \frac{0{\cdot}004}{0{\cdot}06} + \frac{1}{2}\left(\frac{0{\cdot}004}{0{\cdot}06}\right)^2\right] \text{A},$$
$$= 20 \times 10^{-4}(1 - 0{\cdot}0\dot{6} + \tfrac{1}{2} \times 0{\cdot}00\dot{4}) \text{ A},$$
$$= 2 \times 0{\cdot}93\dot{5} = \underline{1{\cdot}87\dot{1} \text{ mA}}.$$

Q9 (a) Evaluate $\int_1^4 (2 + \sqrt{x})^2 dx$.

(b) With the aid of a sketched graph, calculate by integration the area enclosed between the straight line $y = 5 - 2x$ and the parabola $y = 5 + 3x - 2x^2$.

A9 (*a*)
$$\int_1^4 (2 + \sqrt{x})^2\,dx = \int_1^4 (4 + 4x^{\frac{1}{2}} + x)\,dx,$$
$$= \left[4x + \frac{4x^{3/2}}{3/2} + \frac{x^2}{2}\right]_1^4,$$
$$= 16 + \frac{8}{3} \times 4^{3/2} + \frac{16}{2} - \left(4 + \frac{8}{3} \times 1^{3/2} + \frac{1}{2}\right),$$
$$= 24 - \tfrac{64}{3} - (4\tfrac{1}{2} + \tfrac{8}{3}) = \underline{38\tfrac{1}{6}}.$$

(*b*) For the straight line, when $x = 0$, $y = 5$ and when $y = 0$, $x = 2\frac{1}{2}$. The line may therefore be drawn through these points as shown in the sketch.

For the parabola, when $x = 0$, $y = 5$ and when $y = 0$,

$$5 + 3x - 2x^2 = 0 \text{ or } 2x^2 - 3x - 5 = 0.$$

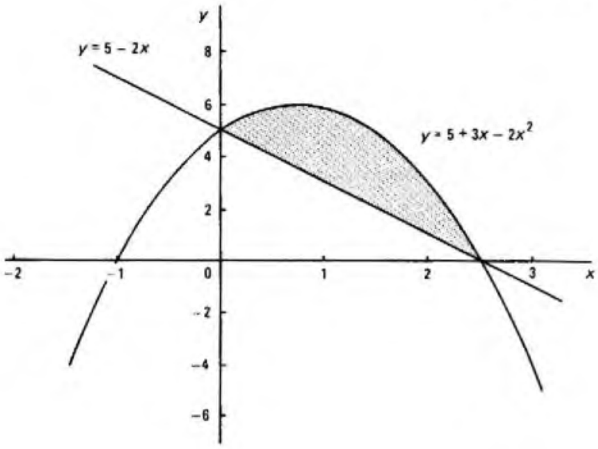
Solving the quadratic equation by means of the general formula gives values of x of $2\frac{1}{2}$ or -1. Hence, the parabola passes through the points $(-1, 0)$ and $(2\frac{1}{2}, 0)$.

Also, $\frac{dy}{dx} = 3 - 4x$ which is equal to zero for a maximum or minimum value of y.

Hence, at $x = \frac{3}{4}$, $y = 6\frac{1}{8}$, and this must be a maximum as it is greater than the value of $y = 5$ at $x = 0$.

The parabola may therefore be sketched from the values established above.

The required area, shown shaded in the sketch, is the difference between the area under the parabola and the area under the straight line, both between the limits of $x = 0$ and $x = 2\frac{1}{2}$ (where the 2 graphs intersect).



$$\therefore \text{ enclosed area} = \int_0^{2\frac{1}{2}} (5 + 3x - 2x^2)\,dx - \int_0^{2\frac{1}{2}} (5 - 2x)\,dx,$$

$$= \int_0^{2\frac{1}{2}} (5 + 3x - 2x^2 - 5 + 2x)\,dx,$$

$$= \int_0^{2\frac{1}{2}} (5x - 2x^2)\,dx,$$

$$= \left[\frac{5x^2}{2} - \frac{2x^3}{3}\right]_0^{2\frac{1}{2}},$$

$$= \frac{5}{2} \times \frac{25}{4} - \frac{2}{3} \times \frac{125}{8},$$

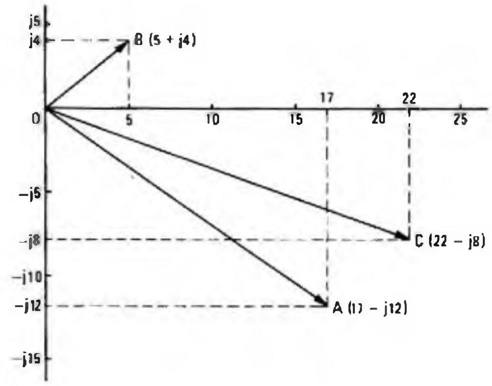
$$= \underline{5 \cdot 208 \text{ units.}}$$

Q10 *(a) Given $Z_1 = 17 - j12$ and $Z_2 = 5 + j4$*

(i) display Z_1, Z_2 and $(Z_1 + Z_2)$ as phasors in the complex plane, and

(ii) express in the form $a + jb$, the complex number $\dfrac{Z_2}{Z_1 - Z_2}$.

(b) Express the admittance $Y = \dfrac{1}{R + j\omega L} + j\omega C$ in the form $a + jb$ (to 2 significant figures) given $R = 30$, $L = 20 \times 10^{-3}$, $C = 12 \times 10^{-6}$ and $\omega = 1000$.



A10 (*a*) (*i*) The phasors are shown in the sketch. OA represents Z_1, OB represents Z_2 and OC represents the vector sum of $Z_1 + Z_2$ where $Z_1 + Z_2 = 17 - j12 + 5 + j4 = 22 - j8$.

(*ii*)
$$\frac{Z_2}{Z_1 - Z_2} = \frac{5 + j4}{17 - j12 - (5 + j4)} = \frac{5 + j4}{12 - j16},$$

$$= \frac{(5 + j4)(3 + j4)}{4(3 - j4)(3 + j4)},$$

$$= \frac{15 + j12 + j20 - 16}{4(9 - j12 + j12 + 16)} = \frac{-1 + j32}{100},$$

$$= \underline{-0 \cdot 01 + j0 \cdot 32.}$$

(*b*)
$$Y = \frac{1}{R + j\omega L} + j\omega C = \frac{R - j\omega L}{R^2 + \omega^2 L^2} + j\omega C.$$

$$\omega L = 1000 \times 20 \times 10^{-3} = 20, \text{ and}$$

$$\omega C = 1000 \times 12 \times 10^{-6} = 0 \cdot 012.$$

$$\therefore \quad Y = \frac{30 - j20}{900 + 400} + j0 \cdot 012,$$

$$= \frac{30}{1300} - j\frac{20}{1300} + j0 \cdot 012,$$

$$= 0 \cdot 0231 - j0 \cdot 01538 + j0 \cdot 012,$$

$$= \underline{0 \cdot 023 - j0 \cdot 0034} \text{ correct to 2 significant figures.}$$

TELEPHONY C 1978

Students were expected to answer any 6 questions

Q1 *(a) A translator undergoing test is offered 900 random calls from an artificial-traffic machine over a period of 15 min. Calls offered when the translator is free have a holding time of 0·2 s, and calls offered when it is busy are lost. It is observed that the occupancy of the translator is only 16·5%; show by calculation whether this is the expected percentage. Why is there a difference between the observed and the calculated occupancy?*

(b) In an actual register-translator installation, how and for what reason does the offering of calls to a common translator differ from that described in (a)?

A1 (*a*) Offered traffic = No. of calls offered per hour × average holding time in hours,

$$= (900 \times 4) \times \frac{0 \cdot 2}{3600} = 0 \cdot 2 \text{ erlangs.}$$

If B is the grade of service and A is the offered traffic, for one trunk,

$$B = \frac{A}{1 + A} = \frac{0 \cdot 2}{1 \cdot 2} = 0 \cdot 167.$$

$$\text{Translator occupancy} = (\text{offered traffic} - \text{traffic lost}) \times 100\%,$$

$$= [0 \cdot 2 - (0 \cdot 2 \times 0 \cdot 167)] \times 100\%,$$

$$= 16 \cdot 7\%.$$

The 0·2% difference between the observed and calculated occupancy would have been due to factors such as:

(*i*) the artificial traffic generated may not have been of a truly random nature,

(*ii*) the sampling period was only 15 min and not an hour,

(*iii*) the number of traffic sources was probably low (<100), and

(*iv*) there may have been statistical variations within the sampling period which would have been smoothed out over a longer sampling period.

(*b*) In an actual register-translator installation, calls offered to a common translator when it is busy, queue and wait until it is free, so that no offered calls are lost. This is necessary to ensure that every call handled by the registers has the possibility of maturing to a successful *register release* state without being affected by the fact that the translator may be busy when applying for a translation. The register equipment is dimensioned to take into account the small amount of extra holding time due to the probability of queuing delays.

Q2 *(a) Describe briefly* THREE *circuit features of a 600-line Strowger rural automatic exchange (UAX) which lead to economy of equipment or junction line plant, and which distinguish it from a similar size non-director Strowger local exchange.*

(b) With the aid of sketches of the circuit elements concerned, explain the circuit operation of ONE *of the features given in answer to (a).*

Q3 (*a*) *Sketch hysteresis loops of magnetic materials suitable for use in a TXE2 exchange for*

(*i*) *the calling-number generator, and*

(*ii*) *the calling-number-generator store.*

Explain the need for any differences.

(*b*) *What determines the number of wires and windings associated with each core or ring?*

(*c*) *Why are the functions of calling-number generation and storage separated in a TXE2 exchange?*

Q4 (*a*) *Sketch the circuit elements of an operator's cord circuit to show how the application of call-timing pulses to a chargeable-time clock (CTC) is controlled. Explain how the control is effected.*

(*b*) *Explain, with the aid of a sketch, how the CTC returns a periodic tone signal to the subscriber.*

A4 See A5, Telephony C 1974, *Supplement*, Vol. 68, p. 65, Oct. 1975.

Q5 (*a*) *Sketch the circuit arrangements used in a crossbar local exchange to select and mark a free line when a call is made to a private branch exchange (PBX) served by a small group of exchange lines.*

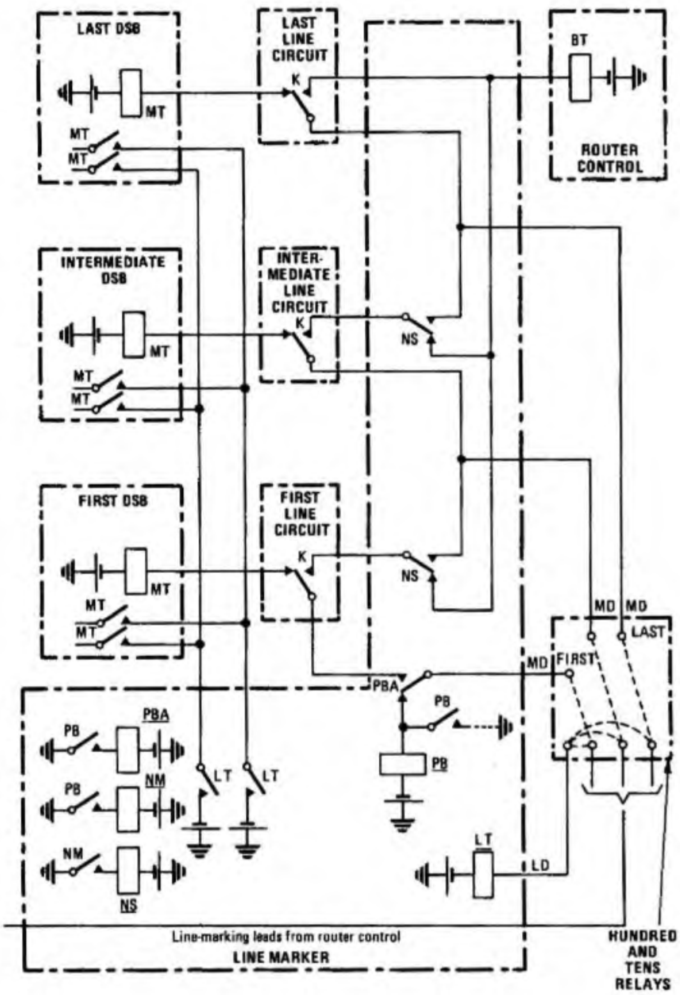
(*b*) *Describe how the above arrangement works when*

(*i*) *the first number of the group is dialled, and*

(*ii*) *the number of an intermediate line is dialled.*

(*c*) *What sets a limit to the maximum number of exchange lines in a PBX group served by the arrangements described?*

A5 (*a*) The circuit arrangements are shown in the sketch.



(*b*) (*i*) When the first line in the group is dialled, the marking earth is extended from the *router control* over the line-marking leads and, after further decoding by the hundreds and tens relays, marks the MD and LD leads of the first line. Relays PB and LT operate; relay PB holds via its own contact and also operates relays PBA and NM. Relay NM operates relays NS (there is more than one NS relay). A PBA contact transfers the marking earth to the K-relay contact of the first line circuit. If the line is free, relay K is normal and relay MT in the first DSB operates and prepares the crosspoints for operation via the MT and LT contacts (the auxiliary and SELECT magnets are not shown). If the first line is busy, relay K is operated and the marking earth is extended via the NS contact to the K-relay contact of the next line circuit. If the next line circuit is free, the relay MT in that DSB operates and prepares those crosspoints as just explained. If the next line circuit is busy, the marking earth is extended to a further line circuit until the last line circuit is reached. If the last line circuit is busy, the marking earth operates relay BT in the router control to cause the return of busy tone to the calling subscriber from the transmission relay group.

(*ii*) If an intermediate line is dialled, relays PB and PBA, NM and NS will not be operated. The marking earth from the hundreds and tens relays will be connected directly to the K-relay contact of the intermediate line via the MD lead. If the line circuit is free, relay MT operates as previously explained. If the line circuit is busy, the marking earth operates relay BT in the router control to cause the return of busy tone, as relay K will be operated but relay NS will be normal; therefore, no search for a free line takes place.

(*c*) Each line in such a PBX has to be located on a different DSB switch from the others within the same distributor. As there is a maximum of 20 DSB switches in a distributor (each distributor caters for 500 lines), the maximum number of lines in such an arrangement is 20.

Q6 (*a*) *Draw a sketch to show how in-band voice frequency (VF) signalling and pulsing equipment is associated with a speech channel provided by a carrier transmission system.*

(*b*) *What factors influence the point of connexion of the signalling equipment?*

(*c*) *Sketch and explain the circuit element of the VF signalling receiver which enables it to respond satisfactorily to a range of received signal levels.*

Q7 (*a*) *Give a sketch showing the method of connecting the following items of exchange equipment which are designed to monitor the quality of service given to subscribers*

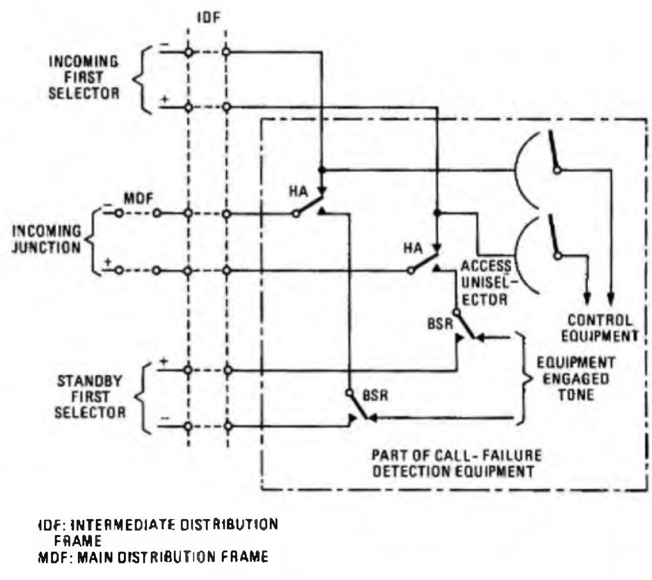
(*i*) *equipment which automatically monitors a proportion of all subscribers' dialled calls, and*

(*ii*) *equipment which generates artificial traffic and records the results.*

(*b*) *State the main facilities of* ONE *of the above items of equipment.*

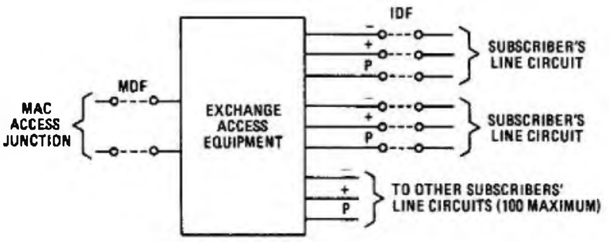
(*c*) *Compare the value of the information obtained from the above equipments.*

A7 (*a*) (*i*) Sketch (*a*) shows how a call-failure detection equipment (CFDE) is connected to a first selector in, for example, a tandem exchange. Incoming calls are monitored by the control equipment and via the access uniselector. If a call failure is detected, relay HA is operated to return equipment engaged tone (EET) to the subscriber. The failed call may be held for tracing if desired and, under these



IDF: INTERMEDIATE DISTRIBUTION FRAME
MDF: MAIN DISTRIBUTION FRAME

(*a*)



(b)

conditions, relay BSR is subsequently operated, so that follow-on calls are not lost but able to use the standby first selector.

(ii) Sketch *(b)* shows measurement and analysis centre (MAC) access equipment at a local exchange. The MAC control equipment gains access to subscribers' line circuits for the purposes of injecting artificial test traffic via the exchange access equipment, and selects one of up to 100 subscribers' line circuits by sending a 2-digit steering code to the exchange access equipment.

(b) The main facilities of a CFDE are

(i) by automatic monitoring of live calls, assessing the quality of service given to subscribers,
(ii) the storage of observed-call particulars and details,
(iii) the printing out of failed-call particulars and details,
(iv) a punched tape record of all monitored calls,
(v) line-splitting, to permit the holding of faulty calls, the return of EET to the subscriber on such calls, and switching to a standby selector to maintain service on the incoming junction when a faulty call is held, and
(vi) access to up to 192 traffic circuits.

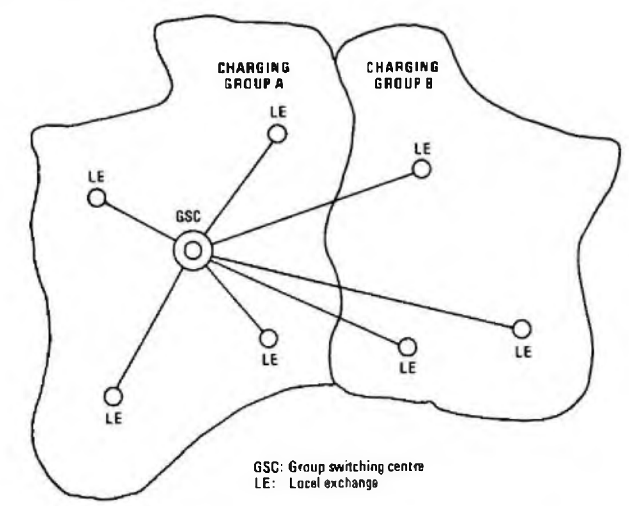
(c) Although both CFDEs and MACs provide quality-of-service statistics, there are some differences as follows:

(i) CFDEs monitor live calls, whereas MACs only monitor artificial calls.
(ii) CFDEs monitor calls to various subscribers on an exchange, whereas MACs send calls to specific test numbers on an exchange.
(iii) MACs check the transmission level, which the CFDE does not..
(iv) MACs can concentrate test calls onto troublesome routes, whereas CFDEs rely on normal traffic.

Q8 (a) A number of local exchanges are dependent on one controlling register-translator installation (group routing and charging equipment). Some of them may be charged at different rates for subscriber trunk dialled (STD) calls.

(i) Explain how this situation arises.
(ii) Describe a method by which charging rates can be made to differ.

(b) With the aid of a sketch of the circuit elements, describe how periodic meter pulses can be transmitted to the exchange of origin via the two-wire junction used for speech.



(a)

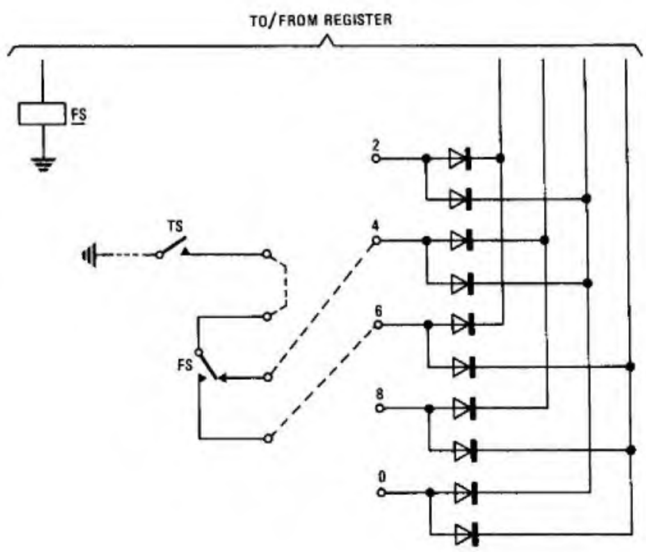
A8 *(a) (i)* In many places, a group switching centre (GSC) (containing the register-translator installation) serves not only its own charging group but one or more adjacent charging groups, the latter being known as dependent charging groups. Sketch *(a)* shows such a situation with charging group A containing the GSC and various local exchanges (LEs), and the LEs in the dependent charging group B relying on the GSC in charging group A for incoming and outgoing STD calls.

As the charging rate for STD calls is dependent on the distance between the calling and called charging groups, then, for STD calls, the distance between charging group A and the called charging group, and charging group B and the called charging group will be different. In some cases, this difference in distance will mean that different charge rates will need to be applied to particular calls, depending on whether they originate from LEs in charging group A, or LEs in charging group B. Hence the register-translator installation at the GSC may need to apply different charge rates to calls which originate in charging group B from those charge rates applied to calls which originate in charging group A, even though the calls may be to the same destination.

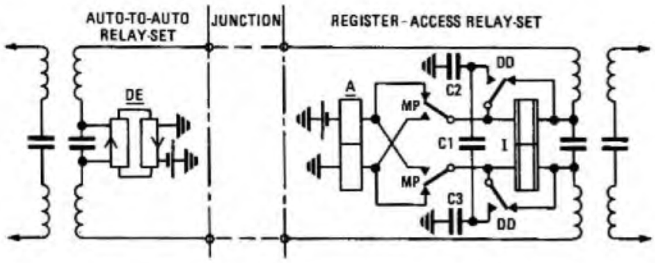
(ii) Sketch *(b)* shows how charging rates may be made to differ depending in which charging group the call originates. An STD call originating in the home charging group, would, for a specific destination, cause the operation of a TS relay in the translator when the register applied for a translation. In the sketch it is assumed that the fee digit 4 is returned to the register for subsequent sending to the register-access relay-set.

STD calls originating in the dependent charging group would use a different group of register-access relay-sets from those used for home-charging-group calls. In this case, it is arranged that the register-access relay-set passes a discrimination signal (such as −50 V battery) to the register, which further extends it to the translator to operate the FS relay. This allows a different fee digit to be returned (in the example: 6) to the register (and ultimately, to the register-access relay-set), even though the same TS relay is operated for supplying the routing information for the same destination.

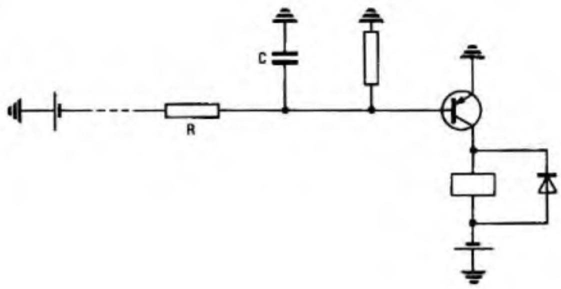
(b) Sketch *(c)* shows the circuit elements involved in sending metering-over-junction signals from a register-access relay-set in a GSC to an auto-to-auto relay-set in a local exchange. Upon receipt of the *called subscriber answer* signal, relay DD (not shown) operates, the contacts of which introduce a filter circuit comprising capacitors C1, C2, C3 and impedance coil I into the line-reversal circuit-element. The filter ensures that any electrical disturbance produced by the line reversing contacts, MP, is reduced to an inaudible level. Another contact of relay DD operates relay MP for 250 ms. The MP contacts reverse the line potential for 250 ms and, in consequence, relay DE in the local exchange also operates for 250 ms. A contact of relay DE activates the meter-guard circuit and a meter pulse is returned to the subscriber's line circuit to operate the meter. Further periodic metering pulses are sent back in the same way.



(b)



(*c*)



(*b*)

Q9 Using discrete electronic components, show with the aid of circuit diagrams how the performance of a standard relay may be modified to give the following effects

(*a*) *operation by a very small current,*

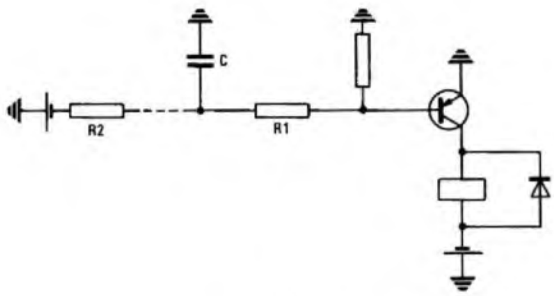
(*b*) *normal operate lag, very slow to release, and*

(*c*) *very slow to operate, normal release.*

In each case, briefly explain the function of the principal components.

A9 (*a*) See A1, Telephony C 1975, *Supplement*, Vol. 69, p. 72, Oct. 1976.

(*b*) Referring to sketch (*a*), when resistor R2 is connected, the transistor will conduct and the relay will operate. As resistor R2 has a low value, capacitor C will charge very quickly to −50 V (approximately) with no noticeable effect on the operate time of the relay. However when resistor R2 is disconnected, capacitor C will discharge through resistor R1 and the base–emitter junction to keep the transistor conducting until the current has fallen to a very low value and the transistor ceases to conduct and the relay releases. Resistor R1 (which has a high value in comparison to resistor R2) and capacitor C determine the release lag by their time-constant.



(*a*)

(*c*) Referring to sketch (*b*), when the battery is connected to resistor R, the transistor will not conduct and the relay will not operate until the voltage across capacitor C has reached a suitable value. This operate time lag is determined by the values of resistor R and capacitor C. When the battery is disconnected, capacitor C will discharge rapidly via the base–emitter junction and the transistor will cease to conduct without any noticeable release lag of the relay.

Q10 The circuit diagram shown in Fig. 1 below contains circuit elements typical of a long-distance direct-current signalling system.

(*a*) *Explain how relay L responds to the pulsing of contact A irrespective of the position of contacts W and X.*

(*b*) *Assuming that A is operated and X is normal, explain what happens when contact W operates.*

(*c*) *In what respect will the circuit fail to function correctly if resistor R3 is disconnected?*

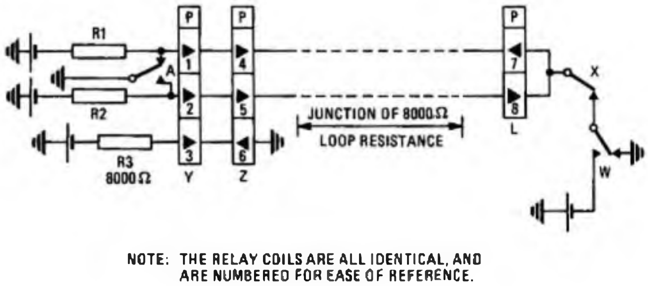


Fig. 1

A10 See A4, Telephony C 1976, *Supplement*, Vol. 70, p. 62, Oct. 1977.

MATHEMATICS A 1978

Students were expected to answer any 6 questions

Q1 (*a*) *A length of wire has resistance R and diameter d. R is directly proportional to the length l and inversely proportional to* d^2.

(*i*) *Write a formula for R in terms of l and d with a constant k.*
(*ii*) *If l is doubled and d is halved, what will be the change in R?*
(*iii*) *If R is kept constant and l increased four times, what will be the change in d?*

(*b*) *Evaluate, using the identity* $\cos^2 A + \sin^2 A = 1$, *the following*

(*i*) $\dfrac{\sin^2 A - \cos^2 A}{1 - 2\sin^2 A}$, *and*

(*ii*) $\dfrac{\sqrt{\{(1-\sin^2 A)(1-\cos^2 A)\}}}{2\sin A\cos A}$.

(*c*) (*i*) *Express 2·62 radians in degrees and minutes.*
(*ii*) *Express 152° 35′ in radians correct to two decimal places.*

A1 (*a*) (*i*) $R = \dfrac{kl}{d^2}$.

(*ii*) If R_1, l_1 and d_1 are the new values, then $l_1 = 2l$ and $d_1 = \dfrac{d}{2}$.

$$\therefore \quad R_1 = \frac{kl_1}{d_1{}^2} = \frac{2kl}{\left(\frac{d}{2}\right)^2},$$

$$= \frac{8kl}{d^2} = 8R.$$

Thus, R is increased in the ratio of 8 : 1.

(*iii*) $d^2 = \dfrac{kl}{R}$ from part (*i*).

When l is quadrupled, d', the new value of d, is given by

$$(d')^2 = \frac{4kl}{R} = 4d^2.$$

$$\therefore \quad \frac{d'}{d} = \sqrt{4} = 2.$$

$\therefore$ d is doubled in value.

(*b*) (*i*) $\dfrac{\sin^2 A - \cos^2 A}{1 - 2\sin^2 A} = \dfrac{\sin^2 A - (1 - \sin^2 A)}{1 - 2\sin^2 A}$,

$$= \frac{2\sin^2 A - 1}{1 - 2\sin^2 A} = \underline{-1.}$$

(*ii*) $\dfrac{\sqrt{\{(1 - \sin^2 A)(1 - \cos^2 A)\}}}{2\sin A\cos A} = \dfrac{\sqrt{(\cos^2 A \sin^2 A)}}{2\sin A \cos A} = \underline{\tfrac{1}{2}}.$

(*c*) (*i*) 2·62 rad = 1·5 + 1·12 rad,
= 85° 57′ + 64° 10′ from tables = 150° 7′.

(*ii*) 152° 35′ = 80° + 72° 35′,
= 1·3963 + 1·2669 rad from tables,
= 2·6632 rad,
= 2·66 rad correct to 2 decimal places.

Q2 (*a*) *Evaluate*

(*i*) *tan 422° 36′*,
(*ii*) *cos (−282° 12′), and*
(*iii*) *sin 231° 24′.*

(*b*) *Two vectors 5∠140°, 8∠−70° act at a point 0. A third vector V∠θ° is such that all three vectors have resultant of zero.*

(*i*) *Calculate the values of V and θ in degrees and minutes.*
(*ii*) *Make a sketch to illustrate the results obtained in part* (*b*) (*i*).

A2 (*a*) (*i*) tan 422° 36′ = tan (360° + 62° 36′),
= tan 62° 36′ = 1·9292.

(*ii*) cos (−282° 12′) = cos (−360° + 77° 48′),
= cos 77° 48′ = 0·2113.

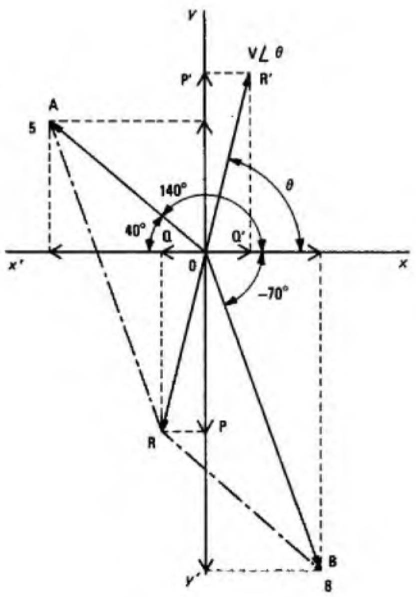
(*iii*) sin 231° 24′ = sin (180° + 51° 24′),
= −sin 51° 24′ = −0·7815.

(*b*) (*i*) Resolving the first two vectors into components acting along the y-axis gives:

$$5\sin 140° + 8\sin(-70°) = 5 \times \sin 40° - 8 \times \sin 70°,$$
$$= 5 \times 0{\cdot}6428 - 8 \times 0{\cdot}9397,$$
$$= 3{\cdot}2140 - 7{\cdot}5176 = -4{\cdot}3036.$$

Resolution along the x-axis gives:

$$5\cos 140° + 8\cos(-70°) = -5\cos 40° + 8\cos 70°,$$
$$= -5 \times 0{\cdot}7660 + 8 \times 0{\cdot}342,$$
$$= -3{\cdot}830 + 2{\cdot}736 = -1{\cdot}094.$$



In order to give a resultant of zero, the vector $V\angle\theta°$ must have components of opposite phase to those of the resultant of the first two vectors; that is, its vertical and horizontal components must be 4·3036 and 1·094 respectively.

Hence, $V^2 = 4{\cdot}3036^2 + 1{\cdot}094^2$,
$= 18{\cdot}5210 + 1{\cdot}1968$,
$= 19{\cdot}7178$.

$\therefore$ $V = \sqrt{19{\cdot}7178} = 4{\cdot}4405$,

and $\tan\theta = \dfrac{4{\cdot}3036}{1{\cdot}094} = 3{\cdot}9338$.

$\therefore$ $\theta = 75° 44'$.

Thus, $V\angle\theta° = \underline{4{\cdot}4405\angle 75° 44'}$.

(*ii*) The sketch shows the vectors OA and OB representing 5∠140° and 8∠−70° respectively, together with their resolved components along the x and y-axes.

OP and OQ are the net resolved components of the two vectors OA and OB, giving the resultant OR. OP′ and OQ′ are the components of the equilibrant OR′ representing $V\angle\theta°$.

Note: OR is the diagonal of the parallelogram of vectors OARB.

Q3 (*a*) *Copy and complete the table of values for $i_1 = 5\sin(100t + 0{\cdot}6)$ amperes and $i_2 = 10\cos 200t$ amperes where t is in milliseconds and the angle is in radians, for the values of t between 0 and 6·0 ms at intervals of 1·0 ms.*

t	0	1	2	3	4	5	6
i_1	2·82						4·66
i_2	10·00						3·62

(*b*) *Draw on the same axes the graphs for i_1 and i_2 taking 1 cm = 1 unit for the i-axis and 2 cm = 1 unit for the t-axis.*

(*c*) *Using only the graphs in part* (*b*), *solve for t the equations*

(*i*) $i_1 = i_2$, *and*
(*ii*) $i_1 - i_2 = -4$

Indicate clearly these solutions on the graphs.

A3 (*a*) The completed table is shown below.

t (ms)	0	1	2	3
$(100t + 0{\cdot}6)$ (rad)	0·6	0·7	0·8	0·9
$\sin(100t + 0{\cdot}6)$	0·5646	0·6442	0·7174	0·7833
$i_1 = 5\sin(100t + 0{\cdot}6)$ (A)	2·82	3·22	3·59	3·92
$200t$ (rad)	0	0·2	0·4	0·6
$\cos 200t$	1·0000	0·9801	0·9211	0·8253
$i_2 = 10\cos 200t$ (A)	10·00	9·80	9·21	8·25

t (ms)	4	5	6
$(100t + 0{\cdot}6)$ (rad)	1·0	1·1	1·2
$\sin(100t + 0{\cdot}6)$	0·8415	0·8912	0·9320
$i_1 = 5\sin(100t - 0{\cdot}6)$ (A)	4·21	4·46	4·66
$200t$ (rad)	0·8	1·0	1·2
$\cos 200t$	0·6967	0·5403	0·3624
$i_2 = 10\cos 200t$ (A)	6·97	5·40	3·62

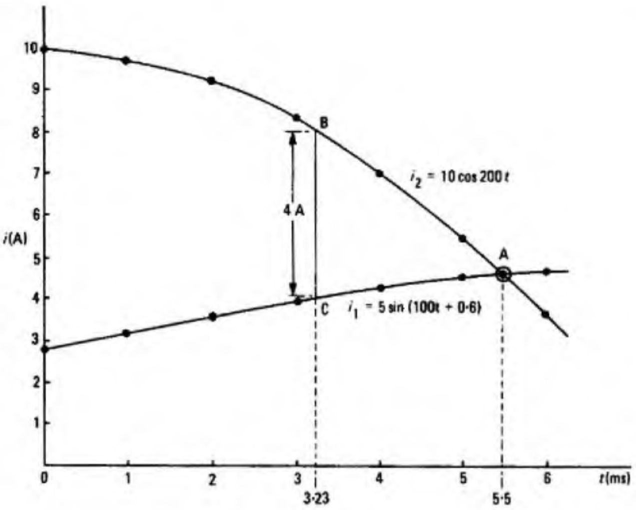
(*b*) The graphs of i_1 and i_2 are shown in the sketch.

(*c*) (*i*) The graphs intersect at point A and, hence, at this point, $i_1 = i_2$. From the sketch, at point A, $t \approx \underline{5{\cdot}5\text{ ms}}$.

(*ii*) $i_1 - i_2 = -4$

or $i_2 - i_1 = 4$

The solution of this equation is obtained from the sketch by finding exactly where the value of i_2 exceeds that of i_1 by 4 A. This was found to be the position shown in the sketch at BC, where $t = \underline{3{\cdot}23\text{ ms}}$.



Q4 (a) Find the positive values of θ in degrees between 0° and 360° which satisfy each of the following equations

(i) $\sin\theta = -0{\cdot}5736$,
(ii) $\cos\theta = 0{\cdot}6428$, and
(iii) $\tan\theta = 1{\cdot}4281$.

(b) If $y = 20\sin(x - 0{\cdot}4)$, where x is in radians, calculate

(i) the value of y for $x = 2\pi$,
(ii) the first positive value of x when $y = 0$, and
(iii) the value of x when $y = 10$.

A4 (a) (i)
$$\sin\theta = -0{\cdot}5736.$$
$$\theta = (180° + 35°) \text{ or } (360° - 35°),$$
$$= \underline{215° \text{ or } 325°}.$$

(ii)
$$\cos\theta = 0{\cdot}6428.$$
$$\theta = 50° \text{ or } (360° - 50°),$$
$$= \underline{50° \text{ or } 310°}.$$

(iii)
$$\tan\theta = 1{\cdot}4281.$$
$$\theta = 55° \text{ or } (180° + 55°),$$
$$= \underline{55° \text{ or } 235°}.$$

(b) (i)
$$y = 20\sin(x - 0{\cdot}4).$$
When $x = 2\pi$,
$$y = 20\sin(2\pi - 0{\cdot}4),$$
$$= -20\sin 0{\cdot}4,$$
$$= -20 \times 0{\cdot}3894 = \underline{-7{\cdot}788}.$$

(ii) When $y = 0$,
$$20\sin(x - 0{\cdot}4) = 0,$$
or $\sin(x - 0{\cdot}4) = 0.$
$$\therefore \quad x - 0{\cdot}4 = 0 \pm n\pi, \text{ where } n = 0, 1, 2 \ldots$$
The first positive value of x occurs when $x - 0{\cdot}4 = 0$,
or $x = \underline{0{\cdot}4 \text{ rad}}.$

(iii) When $y = 10$,
$$\sin(x - 0{\cdot}4) = \frac{10}{20} = \frac{1}{2}.$$
$$\therefore \quad x - 0{\cdot}4 = \frac{\pi}{6} \text{ or } \pi - \frac{\pi}{6}.$$
$$\therefore \quad x = \frac{\pi}{6} + 0{\cdot}4 \text{ or } \frac{5\pi}{6} + 0{\cdot}4 \text{ rad},$$
$$= \underline{0{\cdot}9236 \text{ or } 3{\cdot}0180 \text{ rad}},$$
in the range from $x = 0$ to $x = 2\pi$.

Q5 (a) An open cylindrical tank has height h metres and diameter d metres. State the formulae for the volume V_1 and external surface area S_1 in terms of h and d.

(b) A sphere has radius r metres. State the formulae for the volume V_2 and surface area S_2 in terms of r.

(c) (i) If $S_1 = 3S_2$, show that $d^2 + 4dh = 48r^2$.
(ii) If $d = 4r$, show that $d = 2h$, and calculate the ratio of $V_1 : V_2$.

A5 (a)
$$V_1 = \underline{\frac{\pi d^2 h}{4} \text{ metres}^3}.$$
$$S_1 = \pi dh + \frac{\pi}{4}d^2 \text{ metres}^2.$$
$$= \underline{\pi d\left(h + \frac{d}{4}\right) \text{ metres}^2}.$$

(b)
$$V_2 = \underline{\frac{4}{3}\pi r^3 \text{ metres}^3}.$$
$$S_2 = \underline{4\pi r^2 \text{ metres}^2}.$$

(c) (i)
$$S_1 = 3S_2$$
or
$$\pi dh + \frac{\pi}{4}d^2 = 12\pi r^2.$$
Multiplying by 4 and dividing by π throughout gives
$$4dh + d^2 = 48r^2. \qquad \text{QED}$$

(ii) From the above,
$$4dh + d^2 = 48\left(\frac{d}{4}\right)^2, \quad \text{since } d = 4r,$$
$$= 3d^2.$$
Hence, $4dh = 2d^2$ or $2h = d$. QED

Note: Part (c) of this question should have made it clear that the condition (ii) is dependent on condition (i).
$$\frac{V_1}{V_2} = \frac{\frac{\pi d^2 h}{4}}{\frac{4}{3}\pi r^3} = \frac{3d^2 h}{16r^3}.$$
But $h = \frac{d}{2}$ and $r = \frac{d}{4}$.
$$\therefore \quad \frac{V_1}{V_2} = \frac{3d^2 \times \frac{d}{2}}{16\left(\frac{d^3}{4^3}\right)} = \underline{\frac{6}{1}}.$$

Q6 (a) The depth, h centimetres, of a parabolic dish aerial measured at points x centimetres from the edge along its radius is given in the table below.

x	0	4	9	13	18	22	25	30
h	0	5·00	10·3	13·5	16·7	18·5	19·4	20·0

(i) Plot values of h against x and hence obtain the shape of the semi-cross-section of the dish.

(ii) Divide the figure into six equal intervals of x and compile a table showing the lengths of the mid-ordinates.

(iii) Using the values obtained in part (ii), apply the mid-ordinate rule to determine the area of the figure.

(b) Determine the average value of the periodic waveform shown in Fig. 1.

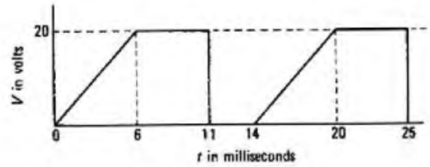


Fig. 1

A6 (a) (i) The graph of h against x, plotted from the given table of values, is shown in sketch (a).

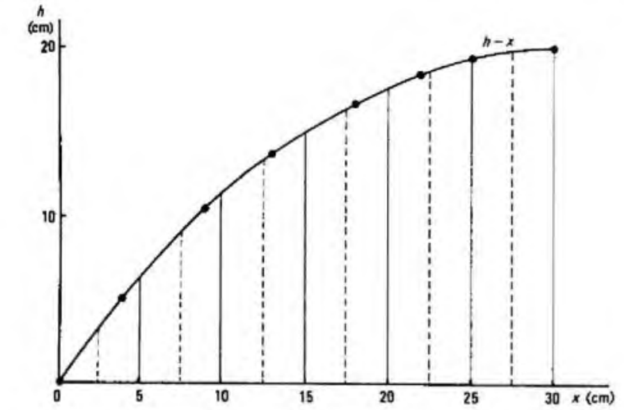
(ii) The figure has been divided into six strips by the ordinates drawn at x = 5, 10, 15, 20, 25 and 30 cm. The mid-ordinates, shown dotted, are erected at x = 2½, 7½, 12½ cm, etc.

The measured lengths of mid-ordinates are shown in the table.

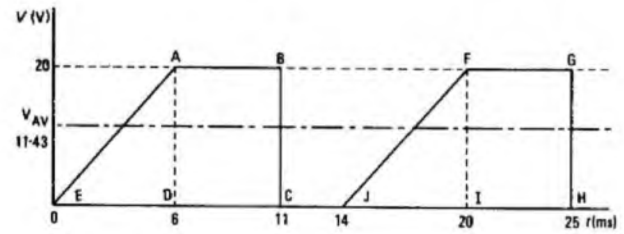
x (cm)	2½	7½	12½	17½	22½	27½
Mid-ordinate (cm)	3·2	8·7	13·2	16·4	18·7	19·8

(iii) Applying the mid-ordinate rule, the area under the graph from $x = 0$ to $x = 30$ cm is given by,

width of each strip × sum of mid-ordinates,
$$= 5 \times (3{\cdot}2 + 8{\cdot}7 + 13{\cdot}2 + 16{\cdot}4 + 18{\cdot}7 + 19{\cdot}8),$$
$$= 5 \times 80{\cdot}0 = \underline{400 \text{ cm}^2}.$$



(*a*)



(*b*)

(*b*) From sketch (*b*), it is seen that the second part of the waveform FGHIJ is exactly the same as the first part ABCDE and that the period of the waveform is 14 ms *and not 11 ms.*

∴ Average value of waveform

$$= \frac{\text{Area ABCDE}}{14} = \frac{\text{Triangle ADE + Rectangle ABCD}}{14},$$

$$= \frac{\frac{1}{2} \times 6 \times 20 + 5 \times 20}{14} = \frac{60 + 100}{14} = \underline{11 \cdot 43 \text{ V}}.$$

Q7 (*a*) *Simplify the following, giving the results with positive indices only*

(*i*) $10^{-3} \times 10^{-2} \div 10^{-6}$, *and*
(*ii*) $[4a^{-3}b^{2/5}]^{-1/2}$.

(*b*) *Rearrange the formulae*

(*i*) $m = \dfrac{\mu L}{L + rCR}$ *to obtain an expression for L, and*

(*ii*) $Z = \dfrac{1}{\sqrt{\left(\frac{1}{R^2} + \omega^2C^2\right)}}$ *to obtain an expression for ω.*

(*c*) *If* $R = \sqrt{\left(\frac{L}{C} - 4\pi^2f^2L^2\right)}$, *calculate the value of R when* $L = 7 \cdot 5 \times 10^{-3}$, $C = 1 \cdot 8 \times 10^{-6}$, $f = 1350$.

A7 (*a*) (*i*) $10^{-3} \times 10^{-2} \div 10^{-6} = 10^{-3-2-(-6)}$
$= \underline{10^1}$.

(*ii*)
$$[4a^{-3}b^{2/5}]^{-1/2} = \frac{1}{[4a^{-3}b^{2/5}]^{1/2}},$$
$$= \frac{1}{2a^{-3/2}b^{1/5}},$$
$$= \underline{\frac{a^{3/2}}{2b^{1/5}}}.$$

(*b*) (*i*)
$$m = \frac{\mu L}{L + rCR}.$$
∴ $m(L + rCR) = \mu L$
or $mL + mrCR = \mu L$.
∴ $\mu L - mL = mrCR$.
$L(\mu - m) = mrCR$.
$$L = \underline{\frac{mrCR}{\mu - m}}.$$

(*ii*)
$$Z = \frac{1}{\sqrt{\left(\frac{1}{R^2} + \omega^2C^2\right)}}.$$
∴ $\sqrt{\left(\frac{1}{R^2} + \omega^2C^2\right)} = \frac{1}{Z}$

or $\frac{1}{R^2} + \omega^2C^2 = \frac{1}{Z^2}$, on squaring both sides.

∴ $\omega^2C^2 = \frac{1}{Z^2} - \frac{1}{R^2}$

or $\omega^2 = \frac{1}{C^2}\left(\frac{1}{Z^2} - \frac{1}{R^2}\right)$.

Whence $\omega = \frac{1}{C}\sqrt{\left(\frac{R^2 - Z^2}{Z^2R^2}\right)}$,

$$= \underline{\frac{\sqrt{(R^2 - Z^2)}}{CZR}}.$$

(*c*)
$$R = \sqrt{\left(\frac{7 \cdot 5 \times 10^{-3}}{1 \cdot 8 \times 10^{-6}} - 4\pi^2 \times 1350^2 \times 7 \cdot 5^2 \times 10^{-6}\right)},$$
$$= \sqrt{(4 \cdot 1\dot{6} \times 10^3 - 4\pi^2 + 1 \cdot 35^2 \times 7 \cdot 5^2)},$$
$$= \sqrt{(4 \cdot 1\dot{6} \times 10^3 - 4 \cdot 047 \times 10^3)},$$
$$= \sqrt{119 \cdot 6\dot{6}} = \underline{10 \cdot 94}.$$

Q8 (*a*) *The resistance, R ohms, of a resistor at temperature t°C is given by* $R = R_0(1 + \alpha t)$ *where* R_0 *is the resistance at 0°C and α is a constant.* $R = 44$ *when* $t = 20$ *and* $R = 48$ *when* $t = 40$. *Calculate*

(*i*) *the value of* R_0 *and α, and*
(*ii*) *the value of t at which* $R = 49$.

(*b*) *In the circuit shown in Fig. 2 below,* $R_1 = 3\ \Omega$, $R_2 = 4\ \Omega$ *and* $R_3 = 2\ \Omega$.
Calculate the values of the currents I_1 *and* I_2.

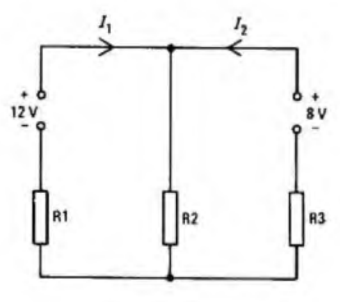


Fig. 2

A8 (*a*) (*i*) $R = R_0(1 + \alpha t)$.
Substituting the given values of t and R:

$44 = R_0(1 + 20\alpha)$, ... (1)
and $48 = R_0(1 + 40\alpha)$. ...... (2)

Dividing equation (2) by equation (1) gives

$$\frac{48}{44} = \frac{1 + 40\alpha}{1 + 20\alpha},$$
or $11(1 + 40\alpha) = 12(1 + 20\alpha)$.
∴ $11 + 440\alpha = 12 + 240\alpha$,
or $200\alpha = 1$.
∴ $\underline{\alpha = 0 \cdot 005}$.

Substituting for α in equation (1):
$44 = R_0(1 + 20 \times 0 \cdot 005)$,
or $R_0 = \frac{44}{1 \cdot 1} = \underline{40}$.

(*ii*) The relationship may now be written as

$$R = 40(1 + 0{\cdot}005t).$$

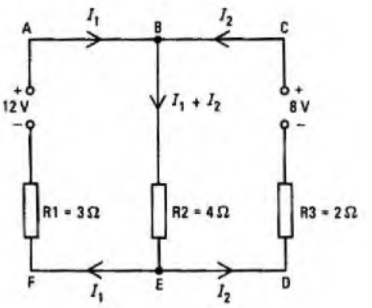
When $R = 49$,

$$1 + 0{\cdot}005t = \frac{49}{40}.$$

$$\therefore \quad 0{\cdot}005t = 1{\cdot}225 - 1$$

or $$t = \frac{0{\cdot}225}{0{\cdot}005},$$

$$= \underline{45°C}.$$



(*b*) The sketch shows that the current through R2 is $(I_1 + I_2)$ amperes and that the return currents through R1 and R3 are I_1 and I_2 respectively.

Applying Kirchhoff's laws to the mesh ABEF,

$$12 = (I_1 + I_2)R_2 + I_1R_1,$$
$$= 4(I_1 + I_2) + 3I_1,$$
$$= 7I_1 + 4I_2. \quad \ldots\ldots (1)$$

Similarly, for the mesh BCDE,

$$8 = 4(I_1 + I_2) + 2I_2,$$
$$= 4I_1 + 6I_2. \quad \ldots\ldots (2)$$

Multiplying equation (1) by 3:

$$36 = 21I_1 + 12I_2. \quad \ldots\ldots (3)$$

Multiplying equation (2) by 2:

$$16 = 8I_1 + 12I_2. \quad \ldots\ldots (4)$$

Subtracting equation (4) from equation (3):

$$20 = 13I_1 \text{ or } I_1 = \frac{20}{13}.$$

Substituting $I_1 = \frac{20}{13}$ in equation (2):

$$8 = \frac{80}{13} + 6I_2,$$

or $$6I_2 = \frac{104 - 80}{13}.$$

$$\therefore \quad I_2 = \frac{4}{13}.$$

Hence $$I_1 = \frac{20}{13} = \underline{1{\cdot}538 \text{ A}},$$

and $$I_2 = \frac{4}{13} = \underline{0{\cdot}308 \text{ A}}.$$

Q9 (a) Form the equation whose roots are $t = -2$ and $t = \frac{1}{3}$. Express the equation in the form $at^2 + bt + c = 0$ where a, b, and c are whole numbers.

(b) Solve the equations

(i) $(5x + 7)^2 = 36$,
(ii) $4y^2 + 7y = 0$, and
(iii) $3w^2 + 2w - 8 = 0$.

(c) A uniform strip x centimetres wide is cut from each of the four sides of a rectangular panel 60 cm by 20 cm. As a result, the area of the original panel is reduced by 15%.

(i) Show that $x^2 - 40x + 45 = 0$.
(ii) Determine the dimensions of the reduced panel.

A9 (*a*) The equation may be written as

$$\{t - (-2)\}\{t - \tfrac{1}{3}\} = 0,$$

or $$(t + 2)(3t - 1) = 0.$$

$$\therefore \quad 3t^2 + 6t - t - 2 = 0,$$

or $$\underline{3t^2 + 5t - 2 = 0}.$$

(*b*) (*i*) $$(5x + 7)^2 = 36.$$

$$\therefore \quad 5x + 7 = \pm 6.$$
$$5x = -1 \text{ or } -13.$$
$$\therefore \quad x = \underline{-\tfrac{1}{5} \text{ or } -2\tfrac{3}{5}}.$$

(*ii*) $$4y^2 + 7y = 0,$$

or $$y(4y + 7) = 0.$$

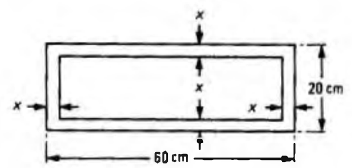
$$\therefore \quad y = 0 \text{ or } 4y + 7 = 0$$

whence, $$y = \underline{0 \text{ or } -1\tfrac{3}{4}}.$$

(*iii*) $$3w^2 + 2w - 8 = 0$$

or $$(3w - 4)(w + 2) = 0.$$

$$\therefore \quad w = \underline{1\tfrac{1}{3} \text{ or } -2}.$$



(*c*) (*i*) The panel is shown in the sketch. As a strip of x centimetres width is cut from each side, the length and breadth of the new rectangle formed will each be $2x$ centimetres less than the original dimensions.

$\therefore$ Area of new panel $= (60 - 2x)(20 - 2x)$ cm^2.

But, new area $= \frac{85}{100} \times 60 \times 20 = 1020$ cm^2.

$$\therefore \quad (60 - 2x)(20 - 2x) = 1020,$$

or $$1200 - 40x - 120x + 4x^2 = 1020.$$

$$\therefore \quad 4x^2 - 160x + 180 = 0,$$

or $$x^2 - 40x + 45 = 0. \qquad \text{QED}$$

(*ii*) Using the general formula for the solution of the above equation,

$$x = \frac{40 \pm \sqrt{(1600 - 180)}}{2},$$
$$= \frac{40 \pm 37{\cdot}68}{2},$$
$$= 38{\cdot}84 \text{ or } 1{\cdot}16 \text{ cm}.$$

The value of $38{\cdot}84$ is inadmissible and hence $x = 1{\cdot}16$ cm.

The dimensions of the reduced panel are therefore $(60 - 2{\cdot}32)$ cm and $(20 - 2{\cdot}32)$ cm; i.e., $\underline{57{\cdot}68 \text{ cm and } 17{\cdot}68 \text{ cm}}$.

Q10 (a) Use tables, other than logarithmic tables, to determine the values of the following

(i) $12{\cdot}36^2$,
(ii) $\sqrt{0{\cdot}00654}$,
(iii) $\sqrt{137{\cdot}8}$, and
(iv) $\frac{1}{23{\cdot}56}$.

(b) Find an approximate value of the expression

$$\frac{\pi \times 8{\cdot}723^2 \times \tan 43° 32'}{0{\cdot}214 \times \sqrt{34{\cdot}94}}.$$

Do not evaluate by logarithms.

(c) If $L = \frac{l}{500}\left[\log_e \frac{4l}{d} - 1\right]$, calculate L when $l = 600$ and $d = 0{\cdot}25$.

(d) (i) Express the denary number 105 in binary form.

(ii) Express the binary number 11 101 in denary form.

(*e*) *Evaluate the following, working in binary notation throughout. Do not convert the binary numbers to denary form.*

(*i*) *11 001 + 1 101.*
(*ii*) *11 001 − 1 101.*
(*iii*) *10 101 × 111.*
(*iv*) *100 011 ÷ 111.*

A10 (*a*) (*i*) Using the table of square roots, 1 to 10,

$$12{\cdot}36^2 = \underline{152{\cdot}8}.$$

(*ii*) Using the table of square roots, 10 to 100,

$$\sqrt{0{\cdot}00654} = \underline{0{\cdot}08087}.$$

(*iii*) Using the table of square roots, 1 to 10,

$$\sqrt{137{\cdot}8} = \underline{11{\cdot}74}.$$

(*iv*) Using the reciprocal table,

$$\frac{1}{23{\cdot}56} = \underline{0{\cdot}04244}.$$

(*b*)

$$\frac{\pi \times 8{\cdot}723^2 \times \tan 43^\circ\, 32'}{0{\cdot}214 \times \sqrt{34{\cdot}94}}.$$

Assuming a fairly broad approximation only is required, π is taken as 3, $8{\cdot}723^2$ as $9^2 \approx 80$, tan 43° 32′ as 1 (since tan 45° = 1), 0·214 as 0·2 and $\sqrt{34{\cdot}94}$ as 6. This then yields

$$\frac{3 \times 80}{0{\cdot}2 \times 6} = \underline{200}.$$

(*c*)

$$L = \frac{1}{500}\left[\log_e \frac{4l}{d} - 1\right],$$
$$= \frac{1}{500}\left[\log_e \frac{2400}{0{\cdot}25} - 1\right],$$
$$= \frac{1}{500}\left[\log_e 9600 - 1\right],$$
$$= \frac{1}{500}(9{\cdot}1696 - 1),$$
$$= \frac{8{\cdot}1696}{500},$$
$$= \underline{0{\cdot}01634}.$$

No.	Nat. log.
9·6	2·2618
10^3	6·9078
9600	9·1696

(*d*) (*i*)

$$105_{10} = 64 + 41,$$
$$= 64 + 32 + 8 + 1,$$
$$= 2^6 + 2^5 + 2^3 + 2^0,$$
$$= \underline{1\,101\,001} \text{ in binary form.}$$

(*ii*)

$$11\,101_2 = 2^4 + 2^3 + 2^2 + 2^0,$$
$$= 16 + 8 + 4 + 1,$$
$$= \underline{29} \text{ in denary form.}$$

(*e*) (*i*)

```
 11 001
  1 101 +
 -------
100 110
```

(*ii*)

```
11 001
 1 101 −
 ------
 1 100
```

(*iii*)

```
     10 101
        111 ×
 ----------
     10 101
    101 010
  1 010 100
 ----------
 10 010 011
```

(*iv*)

```
          101
111 ) 100 011
      11 1
      ----
         111
         111
         ---
         000
```

Answer = $\underline{101}$

CORRECTION

TEC: LINE AND CUSTOMER APPARATUS 1 1977–78

(*Supplement*, Vol. 71, Jan. 1979, p. 91)

A2 The purpose of the permanent magnet was incorrectly stated and referred to a previous type of receiver. In the rocking armature receiver, the permanent magnet induces like magnetic poles at each end of the armature; the ends of the armature are attracted and repelled by the electromagnetic effect of the coils (wound in opposite directions) when an alternating (speech) current is passed through them.

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