SUPPLEMENT

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Contents

QUESTIONS AND ANSWERS

Answers are occasionally omitted or reference is made to earlier Supplements in which questions of substantially the same form, together with the answers, have been published. Some answers contain more detail than would be expected from candidates under examination conditions.

For economic reasons, alternate issues of the Supplement are published in 32-page and 16-page sizes.

ENGINEERING SCIENCE, 1976

Students were expected to answer 2 questions from Q1–4 and 4 questions from Q5–10

Q 1 (*a*) *What is the meaning of the following terms used with reference to machines:*

(*i*) *velocity ratio,*
(*ii*) *mechanical advantage, and*
(*iii*) *efficiency?*

(*b*) *A screw jack raises a mass of 600 kg through a vertical distance of 50 mm in 15 s. If the screw jack has an efficiency of 25%, find*

(*i*) *the work done on the load, and*
(*ii*) *the required input power.*

A 1 (*a*) (*i*) The velocity ratio (VR) of a machine is the ratio of the distance moved by the effort, which must be applied to the machine to move the load, to the distance moved by that load.

Thus, $$\text{VR} = \frac{\text{distance moved by effort}}{\text{distance moved by load}}.$$

(*ii*) The mechanical advantage (MA) of a machine is the ratio of the load to be moved by the machine to the effort that must be applied to the machine to move that load.

Thus, $$\text{MA} = \frac{\text{load on the machine}}{\text{effort applied to the machine}}.$$

(*iii*) For an ideal machine,

effort × distance moved by effort = load × distance moved by load.

Thus, $$\text{VR} = \text{MA}, \text{ or } \text{MA}/\text{VR} = 1.$$

In practice, because of losses in the machine, more work must be done by the effort than is required by the load, and the VR is greater than the MA.

$$\text{The efficiency of a machine} = \frac{\text{MA}}{\text{VR}} \times 100\%.$$

(*b*) (*i*) The force, F newtons, required to raise a mass of 600 kg against gravity is given by the mass multiplied by the acceleration due to gravity. Thus,

$$F = 600 \times 9{\cdot}81 \text{ N} = 5{\cdot}886 \text{ kN}.$$

The work done, W joules, in raising the load is the product of the force applied and the distance through which the load moves. Thus,

$$W = 5{\cdot}886 \times 10^3 \times 50 \times 10^{-3} = \underline{294{\cdot}3 \text{ J}}.$$

(*ii*) Power is defined as the rate of doing work. Therefore, the power, P_L watts, required to raise the load is given by the work done divided by the time taken. Thus,

$$P_L = \frac{294{\cdot}3}{15} = 19{\cdot}6 \text{ W}.$$

But the screw jack is only 25% efficient and, therefore, 4 times as much power must be applied to the jack to raise the load through 50 mm in 15 s. The input power, P_{in} watts, is therefore given by

$$P_{in} = 19{\cdot}6 \times 4 = \underline{78{\cdot}4 \text{ W}}.$$

Q 2 *A lift descends from rest at the top of a building with a constant acceleration of 2·0 m/s² until its speed is 4·0 m/s, and then moves with uniform speed for a distance of 32 m. The lift then decelerates uniformly and finally completes the descent in a further distance of 8·0 m.*

(*a*) *Find the time during which the lift is accelerating.*
(*b*) *Calculate the total distance travelled by the lift.*
(*c*) *Determine the deceleration of the lift.*
(*d*) *Sketch a velocity/time graph for the motion of the lift.*

A 2 (*a*) The time, t seconds, during which the lift is accelerating is obtained from the formula

$$v = u + at \text{ metres/second},$$

where v is the final velocity (metres/second), u is the initial velocity (metres/second), and a is the acceleration (metres/second²).

$$\therefore\ 4 = 0 + 2t \text{ m/s}.$$
$$\therefore\ \underline{t = 2 \text{ s}}.$$

(*b*) The distance travelled, s metres, is given by

$$s = ut + \frac{at^2}{2} \text{ metres}.$$

During the initial period in which the lift is accelerating,

$$s = 0 \times 2 + \frac{2 \times 2^2}{2} = 4 \text{ m}.$$

The lift then travels at a constant velocity for a distance of 32 m and, finally, completes the descent by decelerating over a further distance of 8 m.

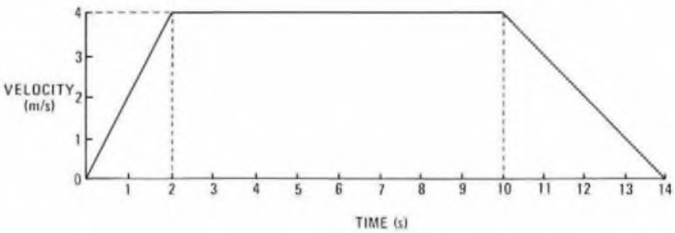
Therefore, the total distance travelled by the lift

$$= 4 + 32 + 8 = \underline{44 \text{ m}}.$$

(*c*) The lift decelerates uniformly from a speed of 4 m/s to rest over a distance of 8 m. Thus, the average velocity over this distance is (4 + 0)/2 = 2 m/s. Therefore, the time taken is 8/2 = 4 s.

Now, $$\text{deceleration} = \frac{\text{change in velocity}}{\text{time taken}},$$

$$= \frac{4 - 0}{4} = \underline{1 \text{ m/s}^2}.$$



(*d*) A velocity/time graph for the motion of the lift is shown in the sketch. For the period of constant velocity, since the lift travels 32 m at 4 m/s, the time taken is 32/4 = 8 s.

Q 3 (*a*) *What is meant by*

(*i*) *tensile stress,*
(*ii*) *tensile strain, and*
(*iii*) *safety factor?*

(*b*) *Sketch a typical load/extension graph for a mild-steel wire, and label the various stress regions.*

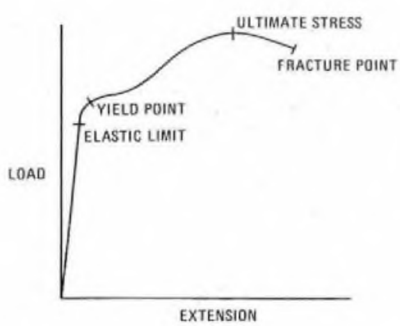
(*c*) *A steel structure contains a tie-bar under tensile stress. Young's modulus of elasticity for steel is $2 \cdot 0 \times 10^{11}$ N/m², and the ultimate tensile strength is $6 \cdot 0 \times 10^{8}$ N/m². Find the tensile strain in the bar under maximum-load conditions, assuming a safety factor of 3·0.*

A 3 (*a*) (*i*) Tensile stress is the force per unit cross-sectional area set up in a body as a reaction to an external force applied to the body that tends to pull it apart. The tensile stress is given by the applied force divided by the cross-sectional area.

(*ii*) The tensile strain of a body subjected to tensile stress is the ratio of the increase in length to the original length.

(*iii*) The safety factor is the ratio of the ultimate tensile strength (or ultimate tensile *stress*) of a material to the maximum working stress to which it is actually subjected.

(*b*) The sketch shows a typical load/extension graph for a mild-steel wire.



(*c*) From part (*a*) (*iii*), the maximum stress in the tie-bar

$$= \frac{\text{ultimate tensile stress}}{\text{safety factor}},$$

$$= \frac{6 \times 10^8}{3} = 2 \times 10^8 \text{ N/m}^2.$$

Now, Young's modulus of elasticity is equal to the stress divided by the strain, so that the strain in the tie-bar

$$= \frac{2 \times 10^8}{2 \times 10^{11}} = \underline{1 \times 10^{-3}}.$$

Q 4 *A truck, of mass 400 kg, is held by a brake at the top of a ramp 2·0 m high and 20 m long.*

(*a*) *Find the potential energy of the truck at the top of the ramp.*

(*b*) *Determine the force acting on the truck in a downward direction parallel to the surface of the ramp.*

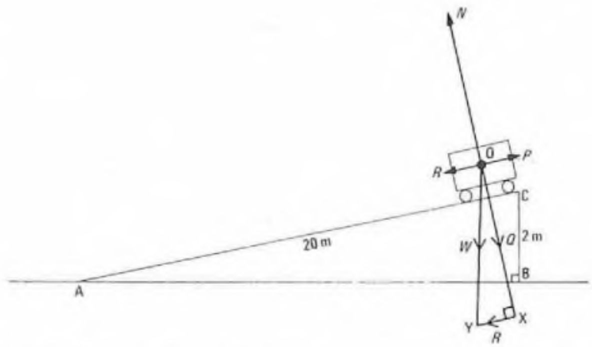
(*c*) *The brake is released and the truck moves down the incline. It is subject to an average retarding frictional force of 200 N. Find*

(*i*) *the work done against the retarding force when the truck reaches the bottom of the ramp, and*

(*ii*) *the speed of the truck at the bottom of the ramp.*

A 4 (*a*) The potential energy of the truck is the amount of work that would be done if the truck were allowed to fall through a vertical distance of 2 m, and is therefore given by the force (which is the weight of the truck) times the distance. Thus, the potential energy

$$= 400 \times 9 \cdot 81 \times 2 \text{ J} = \underline{7 \cdot 848 \text{ kJ}}.$$



(*b*) The forces acting on the truck are shown in the sketch. *W* is the gravitational force on the truck acting vertically downwards; that is, the weight of the truck. *N* is the normal reaction between the truck and the surface of the ramp, and *Q* is a force, equal and opposite to *N*, which just keeps the truck in contact with the ramp. *P* is the force preventing the truck rolling down the ramp, and comprises the force applied by the brake and any frictional forces that are present. *P* is equal and opposite to *R*, which is the component of *W* tending to pull the truck down the ramp; that is, *R* is the force acting on the truck in a downward direction parallel to the surface of the ramp.

Triangles ABC and OXY are similar.

$$\therefore \frac{BC}{AC} = \frac{XY}{OY} = \frac{R}{W}.$$

$$\therefore \frac{2}{20} = \frac{R}{400 \times 9 \cdot 81}.$$

$$\therefore \underline{R = 392 \cdot 4 \text{ N}}.$$

(*c*) (*i*) The work done is equal to the force times the distance. In this case, the force is that force needed to overcome the retarding frictional force. Thus, the work done when the truck reaches the bottom of the ramp

$$= 200 \times 20 \text{ J} = \underline{4 \text{ kJ}}.$$

(*ii*) Force *R* acts against a retarding force of 200 N, so that a force of 392·4 − 200 = 192·4 N is available to accelerate the truck down the ramp. The acceleration, *a* metres/second², is given by the force divided by the mass. Thus,

$$a = \frac{192 \cdot 4}{400} = 0 \cdot 481 \text{ m/s}^2.$$

Now, the time taken to travel down the ramp, *t* seconds, is obtained from the formula

$$s = ut + \frac{at^2}{2} \text{ metres},$$

where *s* is the distance (metres), and *u* is the initial velocity (metres/second).

$$\therefore 20 = 0 \times t + \frac{0 \cdot 481 t^2}{2} \text{ metres}.$$

$$\therefore t^2 = \frac{2 \times 20}{0 \cdot 481} = 83 \cdot 16 \text{ s}^2.$$

$$\therefore t = 9 \cdot 12 \text{ s}.$$

The final velocity at the bottom of the ramp, *v* metres/second, is given by the acceleration multiplied by the time taken.

$$\therefore v = 0 \cdot 481 \times 9 \cdot 12 = \underline{4 \cdot 39 \text{ m/s}}.$$

Q 5 (*a*) *Assuming that the resistance of the ammeter shown in the circuit in Fig. 1 is negligible, find*

(*i*) *the total resistance of the circuit,*
(*ii*) *the current supplied by the battery,*
(*iii*) *the power dissipated by the 20 Ω resistor, and*
(*iv*) *the potential difference across the 5 Ω resistor.*

(*b*) *Suggest a fault that would cause the ammeter to read 0·2 A instead of the expected value.*

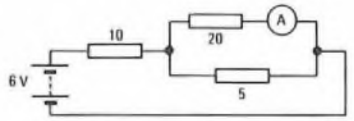


Fig. 1

A 5 (*a*) (*i*) The total resistance of the circuit

$$= 10 + \frac{20 \times 5}{20 + 5} = \underline{14\ \Omega.}$$

(*ii*) The current supplied by the battery

$$= \frac{6}{14}\ \text{A} \approx \underline{430\ \text{mA.}}$$

(*iii*) The current in the 20 Ω resistor

$$= \frac{5}{20 + 5} \times 430 \times 10^{-3}\ \text{A} = 86\ \text{mA.}$$

Therefore, the power dissipated in the 20 Ω resistor

$$= 86^2 \times 10^{-6} \times 20\ \text{W} \approx \underline{150\ \text{mW.}}$$

(*iv*) The potential difference across the 5 Ω resistor is equal to that across the 20 Ω resistor

$$= 86 \times 10^{-3} \times 20 = \underline{1 \cdot 72\ \text{V.}}$$

(*b*) If the ammeter reads 0·2 A, the potential difference across the 20 Ω resistor is 0·2 × 20 = 4 V.

As the resistance of the ammeter is negligible, the potential difference across the 10 Ω resistor is 6 − 4 = 2 V.

Therefore, the current in the 10 Ω resistor is 2/10 = 0·2 A, which is the same as that in the 20 Ω resistor.

Hence, no current is flowing in the 5 Ω resistor and, so, a fault that would cause the ammeter to read 0·2 A is a disconnexion of the 5 Ω resistor.

Q 6 (*a*) *Sketch the construction and give 3 properties of a lead-acid cell.*

(*b*) *The terminal voltage of a battery rises from 8·4 V to 10 V while charging from a 24 V source through a 4 Ω resistor. Find the charging current*

(*i*) *at the beginning, and*
(*ii*) *at the end*
of the process.

(*c*) *Describe one way of assessing the state of charge of a lead-acid cell.*

A 6 (*a*) and (*c*) See A7, Engineering Science, 1975, and A3, Elementary Telecommunication Practice, 1975, *Supplement*, Vol. 68, pp. 95 and 87, Jan. 1976.

(*b*) The charging current, I amperes, is given by

$$I = \frac{V_S - V_B}{R}\ \text{amperes,}$$

where V_S is the source voltage, V_B is the terminal voltage of the battery, and R is the resistance in the circuit.

(*i*) At the beginning of the process,

$$I = \frac{24 - 8 \cdot 4}{4} = \underline{3 \cdot 9\ \text{A.}}$$

(*ii*) At the end of the process,

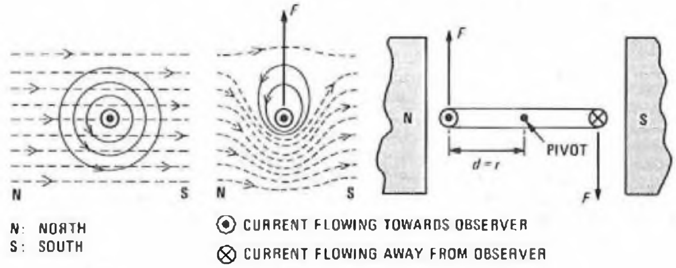
$$I = \frac{24 - 10}{4} = \underline{3 \cdot 5\ \text{A.}}$$

Q 7 (*a*) (*i*) *Explain how a torque is produced on a coil that is carrying current in a magnetic field.*

(*ii*) *Show how this torque depends on the position of the coil relative to the field.*

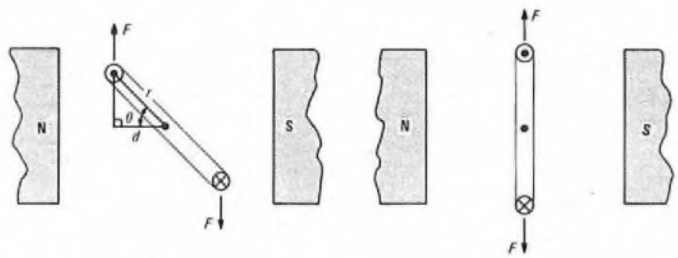
(*b*) *With the aid of sketches, describe the construction and action of a moving-coil ammeter.*

A 7 (*a*) (*i*) Sketch (*a*) shows the lines of force (solid lines) for a current-carrying conductor in a magnetic field (dashed lines). The main field is reinforced on one side of the conductor and weakened on the other, producing the combined field shown in sketch (*b*). Thus, the conductor is subject to a force at right angles to the main field, the direction of the force being given by Fleming's left-hand rule. The magnitude of the force, F, is given by $F = BIl$ newtons, where B is the flux density of the main field (teslas), I is the current in the conductor (amperes), and l is the length of the conductor in the magnetic field (metres).



(*a*) (*b*) (*c*)

For a coil positioned as shown in sketch (*c*), parallel to the main field, forces act on each side of the coil. Torque is the product of a force and the perpendicular distance to the line of action of the force. Therefore, the coil is subjected to a torque of $2BIld$ newton metres, where d is the perpendicular distance to the line of action of the force, equal in this case to the radius of the coil, r metres. (For a coil with more than one turn, this expression must be multiplied by the number of turns.)



(*d*) (*e*)

(*ii*) If the coil is allowed to move to the position shown in sketch (*d*), d is reduced to $r \cos \theta$ metres, where θ is the angle through which the coil has rotated from the horizontal position (degrees). The torque is thus reduced accordingly.

When the coil is perpendicular to the main field, as shown in sketch (*e*), θ is 90° and d is therefore zero. Thus, the torque is zero.

Hence, the torque depends on the orientation of the coil in the field, and its magnitude is given by $2BIlr \cos \theta$ newton metres.

(*b*) See A9, Engineering Science, 1975, *Supplement*, Vol. 68, p. 95, Jan. 1976.

Q 8 (*a*) *Describe 2 ways by which a power transistor can be cooled when operating in a piece of electronic equipment.*

(*b*) *A photoconductive device consists of a 30 × 10⁻³ mm thick semiconductor film evaporated onto a transparent conducting electrode, as shown in Fig. 2. Electrical contact is made to the top surface of the film by completely covering it with a gold electrode. A constant-voltage d.c. supply of 2·0 V is applied across the electrodes.*

(*i*) *Find the resistance between the electrodes if the resistivity of the film is 10^7 Ω m with the device in the dark.*

(*ii*) *Calculate the charge passing through the film when a 100 ms pulse of light is directed onto the device and the film resistivity decreases by 50% relative to that in the dark.*

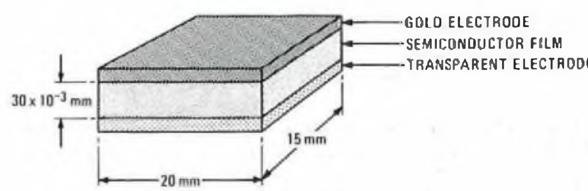


Fig. 2

A 8 (*a*) Heat can be transferred from a body by conduction, convection or radiation. Any device designed to cool a power transistor must make use of one or more of these methods.

The transistor can be bolted to a heat sink, which is a metal plate of good thermal conductivity, so that the heat generated by the transistor is transferred by conduction through the case of the transistor to the metal plate, and thence to its surroundings by conduction, convection and radiation. Also, a power transistor can be fitted with a heat sink having fins to increase the surface area open to the atmosphere. Heat is transferred from the transistor to the heat sink by conduction, and from the heat sink to the surroundings by convection and radiation. The material of the heat sink must have a high thermal conductivity, and is usually finished in matt black to obtain maximum radiation.

(*b*) (*i*) The resistance, R ohms, of a body is given by

$$R = \frac{\rho l}{a} \text{ ohms,}$$

where ρ is the resistivity of the material (ohm metres), l is the length (depth) of the material (metres), and a is the cross-sectional area (metres^2). Hence, the resistance between the electrodes

$$= \frac{10^7 \times 30 \times 10^{-3} \times 10^{-3}}{20 \times 10^{-3} \times 15 \times 10^{-3}} \, \Omega = \underline{1 \text{ M}\Omega.}$$

(*ii*) If the resistivity reduces by 50%, R becomes 500 kΩ, since it is proportional to ρ.

By Ohm's law, the current flowing through the film is

$$2/(500 \times 10^3) \text{ A} = 4 \; \mu\text{A}.$$

Now, the charge is given by the current flowing multiplied by the time for which it flows. Therefore, the charge passing through the film when it is activated by a 100 ms light pulse

$$= 4 \times 10^{-6} \times 100 \times 10^{-3} \text{ C} = \underline{400 \text{ nC.}}$$

Q 9 (*a*) *State Faraday's law of electromagnetic induction.*

(*b*) *A rectangular coil of 200 turns has sides of length 100 mm and 400 mm, and is rotated at a constant rate in a uniform magnetic field of flux density 50 mT. Find the rate of rotation if the maximum instantaneous e.m.f. induced in the coil is 125 V.*

(*c*) (*i*) *With the aid of a labelled diagram, describe the construction of a simple a.c. generator using a coil rotating in a magnetic field.*

(*ii*) *Explain how the magnitude of the induced e.m.f. in the coil depends on its orientation in the magnetic field.*

A 9 (*a*) Faraday's law of electromagnetic induction states that, when a magnetic flux linking a conductor changes, an e.m.f. is induced in the conductor with a magnitude proportional to the rate of change of flux.

(*b*) It is assumed that the 400 mm sides of the coil are parallel to the axis of rotation, and are therefore cutting the magnetic flux when the coil rotates.

The e.m.f., e volts, generated by a coil rotating in a uniform magnetic field is given by

$$e = 2NBlv \sin \theta \text{ volts,}$$

where N is the number of turns, B is the flux density (teslas), l is the length of one side of the coil parallel to the axis of rotation (metres), v is the tangential velocity of the coil (metres/second), and θ is the angle of the coil relative to the direction of the field (degrees).

The maximum instantaneous e.m.f. occurs at $\theta = 90°$; that is, at $\sin \theta = 1$. Transposing the above equation for this condition gives

$$v = \frac{e}{2NBl} \text{ metres/second,}$$

$$= \frac{125}{2 \times 200 \times 50 \times 10^{-3} \times 400 \times 10^{-3}} = 15 \cdot 6 \text{ m/s.}$$

The angular velocity, ω radians/second, of the coil is given by the tangential velocity divided by the radius.

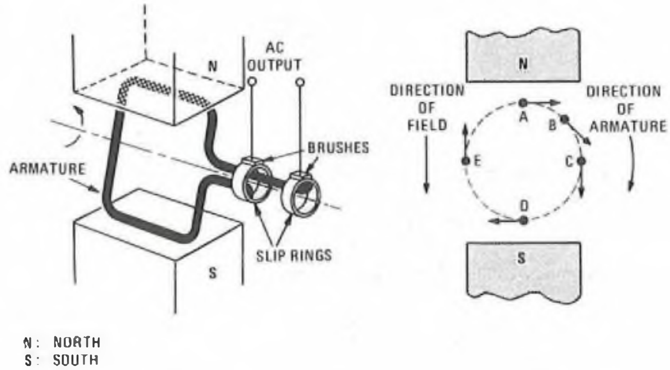
$$\therefore \; \omega = \frac{15 \cdot 6}{50 \times 10^{-3}} = 312 \text{ rad/s.}$$

Therefore, the rate of rotation

$$= \frac{\omega}{2\pi} = \frac{312}{2\pi} \approx \underline{50 \text{ revolutions/s.}}$$

(*c*) (*i*) Sketch (*a*) shows the construction of a simple a.c. generator. A coil of wire, called an *armature*, is made to rotate in a strong magnetic field. The ends of the coil are connected to copper slip rings. Carbon brushes maintain sliding contact with the slip rings to allow the generated e.m.f. to be applied to a load.

(*ii*) Sketch (*b*) shows one side of the coil at various points in a revolution. At point A, the velocity of the coil at right angles to the field is a maximum, equal to the tangential velocity of the coil. The e.m.f. generated is therefore a maximum. At point B, the conductor is moving at about 45° to the field, so that its component of velocity at right angles to the field is less than the tangential velocity, and the



(*a*) (*b*)

induced e.m.f. is less than that produced at point A. At point C, the coil has no component of velocity at right angles to the field, and no e.m.f. is generated. At point D, the e.m.f. is again a maximum, but in the opposite direction to that generated at point A. At point E, the e.m.f. is again zero.

In one revolution, therefore, the generated e.m.f. undergoes one cycle of alternation as the coil changes its orientation in the field. The e.m.f.s generated in each side of the coil are in series-aiding.

Q 10 (*a*) *A moving-coil meter has a resistance of 40 Ω and gives a full-scale deflexion when a current of 1·0 mA passes through it. Show how the the meter can be used to measure*

(*i*) *currents up to 3·0 A, and*

(*ii*) *voltages up to 25 V.*

(*b*) *In the circuit shown in Fig. 3, the voltmeter reads 4·0 V on the 5 V range. What is the voltage sensitivity of the meter in ohms/volt?*

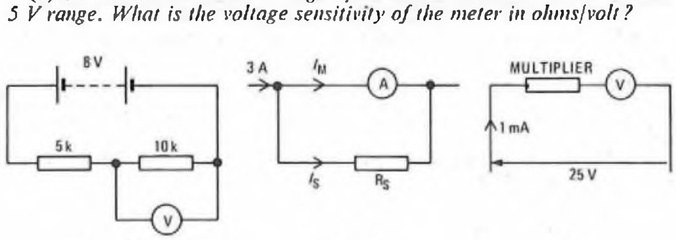


Fig. 3 (*a*) (*b*)

A 10 (*a*) (*i*) If the meter is to be used to measure currents up to 3 A, a shunt must be provided to take 2·999 A while 1 mA flows through the meter to give full-scale deflexion. The arrangement is shown in sketch (*a*).

If R_S is the shunt resistance (ohms), R_M is the meter resistance (ohms), I_S is the shunt current (amperes), and I_M is the meter current (amperes), then, since the voltage across each arm of the parallel circuit is the same,

$$I_S R_S = I_M R_M.$$

$$\therefore \; R_S = \frac{1 \times 10^{-3} \times 40}{2 \cdot 999} \, \Omega = \underline{13 \cdot 3 \text{ m}\Omega.}$$

(*ii*) If the meter is to be used to measure voltages up to 25 V, a series resistor (multiplier) must be provided, as shown in sketch (*b*). For full-scale deflexion, the current in the circuit is 1 mA. By Ohm's law, the total resistance in the circuit

$$= \frac{25}{1 \times 10^{-3}} \, \Omega = 25 \text{ k}\Omega.$$

But the resistance of the meter is 40 Ω. Thus, the resistance of the multiplier

$$= 25 \times 10^3 - 40 \, \Omega = \underline{24 \cdot 96 \text{ k}\Omega.}$$

(*b*) As the voltmeter reading is 4 V and the supply is 8 V, the voltages across the 5 kΩ resistor, and the 10 kΩ resistor in parallel with the meter, are the same. Hence, the combined resistance of the 10 kΩ resistor and the voltmeter must be 5 kΩ.

Thus, the resistance of the voltmeter

$$= \frac{1}{\dfrac{1}{5 \times 10^3} - \dfrac{1}{10 \times 10^3}} \, \Omega = 10 \text{ k}\Omega.$$

Therefore, the sensitivity of the voltmeter on the 5 V range

$$= \frac{10 \times 10^3}{5} \, \Omega/\text{V} = \underline{2 \text{ k}\Omega/\text{V.}}$$

ELEMENTARY TELECOMMUNICATION PRACTICE, 1976

Students were expected to answer any 6 questions

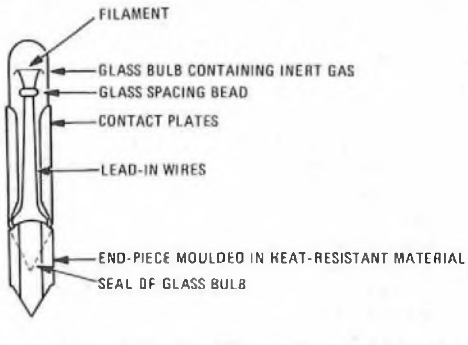
Q 1 (*a*) (*i*) *Sketch the construction of a small lamp of the type commonly used for switchboard supervisory signals. Label the sketch to show the essential parts.*

(*ii*) *Why is tungsten used for the filament, and what action is taken during manufacture of the lamp to ensure that the filament has a reasonable life under normal operating conditions?*

(*iii*) *State what is meant by the rating of a lamp.*

(*b*) *State 3 ways in which a hot body loses heat to its surroundings. In the case of a lamp, describe briefly how each of these contributes to the transfer of heat from the filament to the outside of the lamp.*

A 1 (*a*) (*i*) The sketch shows a small supervisory lamp.



(*ii*) Tungsten is used for the filament because it can be operated at white heat without melting; that is, it has a high melting temperature.

At normal operating temperatures, a tungsten filament would oxidize very rapidly in air. It is therefore necessary during manufacture to replace the air in the glass envelope by an inert gas, such as nitrogen or argon, before the envelope is sealed. The gas filling also helps to cool the filament and reduces the rate at which the filament is evaporated, thus extending the life of the lamp.

(*iii*) The rating of a lamp is the power dissipated by the filament, and is given by the square of the supply voltage divided by the resistance of the filament. The life of the filament and the light output of the lamp depend on the operating temperature which, in turn, depends on the power dissipated. Thus, since the filament resistance is reasonably constant, it is usual to specify the designed working voltage of the lamp as well as its rating.

(*b*) A body can lose heat to its surroundings by conduction, convection and radiation.

Conduction applies mainly to solid materials. Heat applied to a body at one point is conducted along the body to raise progressively the temperature of adjacent cool areas; that is, heat is conducted from the hottest point on the body to the coolest.

If a gas or liquid is heated, it expands and becomes less dense. Gas immediately adjacent to a hot body therefore tends to rise as it is heated, and is replaced by cooler, denser gas. The rising hot gas then cools and falls, so setting up a continuous circulation, or convection current, past the hot body to dissipate its heat.

Heat energy is also radiated through free space as a form of electromagnetic wave, and heats any body in its path independently of the above methods of heat transference.

In a lamp, heat is lost from the filament by all 3 methods. Heat is conducted along the supporting lead-in wires to the metal contacts on the outside of the lamp. Convection of the inert gas around the filament transfers heat to the inside surface of the glass, whence it is conducted through the glass to the outside surface. Radiation also directly heats the glass, the inert gas and the air surrounding the lamp. The outside surface of the lamp is in turn cooled by conduction through the electrical contacts and their supply wires, by convection of the surrounding air, and by radiation or reradiation from the body of the lamp and its holder.

Q 2 (*a*) (*i*) *Using standard symbols, draw a circuit diagram showing how a carbon microphone, a battery, an induction coil and a receiver are connected to form a simple telephone. Label the components and show the connexions to line.*

(*ii*) *Give 2 reasons for including an induction coil in the circuit.*

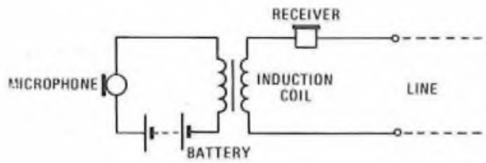
(*b*) *Describe, in general terms only, how*

(*i*) *the carbon microphone converts sound waves into electrical waves, and*

(*ii*) *the receiver converts electrical waves into sound waves.*

(*Detailed sketches or descriptions of the construction of these components are not required.*)

A 2 (*a*) (*i*) The sketch shows the circuit of a simple local-battery telephone. A similar instrument would be connected to the other end of the line.



(*ii*) The audio-frequency signal voltages produced by the carbon microphone are relatively small and their amplitude depends upon the value of the direct current flowing in the microphone. Thus, if the line formed part of the microphone circuit, the performance of the microphone would vary with the resistance, and hence length, of the line. The induction coil isolates the microphone circuit from the line.

The induction coil also isolates the d.c. component in the microphone circuit from the receiver. A direct current flowing in the appropriate direction can affect the performance of the receiver by tending to demagnetize the permanent magnet.

The induction coil also allows the relatively low impedance of the microphone to be matched to that of the line, effectively stepping-up the signal voltage applied to the line.

(*b*) (*i*) The carbon microphone depends for its operation upon the fact that the electrical resistance of carbon granules varies inversely with the mechanical pressure applied to them. Such a microphone is therefore constructed so that the variations in air pressure due to sound waves cause a diaphragm to vibrate in sympathy, the diaphragm transmitting the pressure variations to carbon granules in a closed container having electrical connexions (electrodes) on each side. If a direct voltage is applied across the electrodes, the value of the current flowing depends on the resistance of the carbon at any instant. Thus, the current varies in sympathy with the sound waves, giving an equivalent electrical signal.

(*ii*) The receiver depends for its operation upon the fact that a current flowing in a coil of wire produces a magnetic field, the direction and strength of which is proportional to the direction and value of the current. In the receiver, coils are arranged so that the magnetic field produced by a signal current aids or opposes the field of a permanent magnet. The combined magnetic field acts on a diaphragm of magnetic material (or an armature of magnetic material connected to a non-magnetic diaphragm) which is free to move under the influence of the field. Thus, a signal voltage, such as that produced by the microphone described above, causes the diaphragm to move in sympathy with the signal; this movement causes pressure variations in the air adjacent to the diaphragm, and these reproduce the original sound waves.

Q 3 (*a*) *Describe the signalling code used for teleprinter operation. Explain, with the aid of a sketch, the features of the code that permit the sending and receiving machines to maintain synchronism.*

(*b*) *For a telegraph circuit, draw simple circuit diagrams, and sketch typical line-signal voltages, to show what is meant by*

(*i*) *single-current working, and*

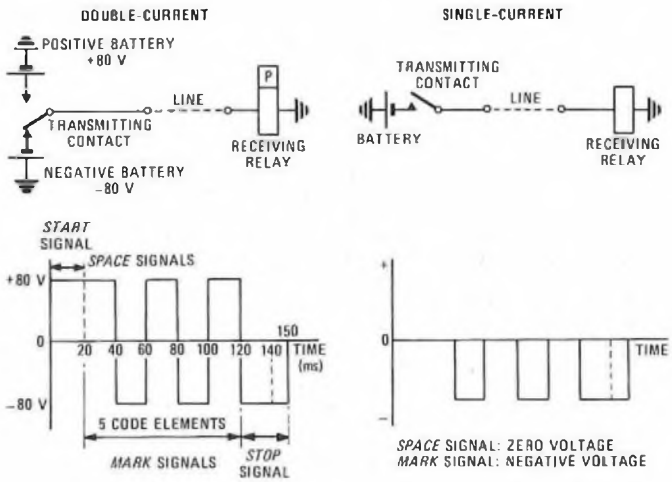
(*ii*) *double-current working.*

A 3 (*a*) The signalling code used for teleprinter operation is the Murray, or 5-unit, code. In this code, each character is represented by a combination of 5 signal elements. Each element can take one of 2 line conditions, usually referred to as the *mark* and *space* signals. The code is therefore capable of $2^5 = 32$ different combinations, and so can transmit 32 individual characters. To transmit all the alphabetical, numerical and punctuation characters, it is necessary to use 2 of the combinations to give *figure-shift* and *letter-shift* signals, so that any one combination can be used either to represent a letter or a figure/punctuation character, depending on the last *shift* signal sent.

The *mark* and *space* signal elements are all of the same duration (20 ms) and can be represented respectively by voltage and no-voltage conditions, negative and positive battery, tone and no-tone, or any similar 2-state scheme, depending on the transmission path.

Because the time taken to transmit each character is the same (that is, $5 \times 20 = 100$ ms), the 5-unit code is more suitable for machine operation than, say, the Morse code, where the transmission time is different for each character. In addition, the 5 code elements of each character are preceded by a *start* signal (which is a *space* signal 1 element in length) and followed by a *stop* signal (which is a *mark* signal equal in duration to $1\frac{1}{2}$ elements). This gives a total transmission time for each character of $7\frac{1}{2}$ elements, or units. The use of *start* and *stop* signals with each character gives character separation and, at the same time, provides signals that can be used to synchronize the transmitting and receiving machines for each character.

The *start* and *stop* signals of a 5-unit code are illustrated in sketch (*a*).



(*a*) (*b*)

(*b*) Sketches (*a*) and (*b*) show circuit diagrams and line-signal voltages for double-current and single-current systems respectively.

Q 4 (*a*) (*i*) *Describe, with the aid of a sketch, the construction of a carbon-film type of resistor.*

(*ii*) *Give 4 advantages of this type compared with wire-wound resistors.*

(*iii*) *In a particular circuit, a 470 Ω resistor is required to carry 100 mA. Could a carbon-film type of resistor be used? Give brief reasons for your answer.*

(*b*) (*i*) *A resistor carries 4 evenly spaced narrow colour bands near to one end. Why is it marked in this way?*

(*ii*) *Draw a simple sketch of this arrangement. Label each band's position and state its significance. Also, sketch one alternative method of marking the resistor with the same colours.*

A 4 See A3, Elementary Telecommunication Practice, 1970, *Supplement*, Vol. 63, p. 75, Jan. 1971, and A7, Elementary Telecommunication Practice, 1975, *Supplement*, Vol. 68, p. 88, Jan. 1976.

Q 5 (*a*) *Explain briefly what is meant by any 3 of the following terms, applied to a telephone system:*

(*i*) *calling rate,*
(*ii*) *traffic,*
(*iii*) *busy hour,*
(*iv*) *a junction circuit, and*
(*v*) *a trunk circuit.*

(*b*) *What factors affect the scale of provision of circuits on a route between 2 distant automatic exchanges?*

A 5 (*a*) For completeness, all 5 terms are explained below.

(*i*) The calling rate is the average number of telephone calls originated per line in a certain period. The period is usually either 1 d, or 1 h at the busiest part of the day.

(*ii*) The traffic is the total number of telephone calls carried by a circuit, a group of circuits or item of equipment. Both the number and duration of the calls are taken into account. The unit of traffic is the erlang and, if 1 circuit is fully occupied for 1 h, it is said to be carrying 1 erlang of traffic.

(*iii*) The traffic carried by an exchange varies widely during a day. The busy hour is that period of 1 h during which the exchange is carrying a maximum volume of traffic. The time when the busy hour occurs can vary from exchange to exchange, depending on the type of community served.

(*iv*) A circuit interconnecting 2 telephone exchanges, one of which is a local exchange, is called a *junction circuit*. The term is applied, for example, to circuits between local exchanges, or between a local exchange and a group switching centre (GSC). Thus, junction circuits are generally short.

(*v*) Circuits interconnecting telephone centres forming part of the main switching network are called *trunk circuits* (or *main-network circuits*). These circuits are often long and include, for example, routes between GSCs.

(*b*) The first consideration, when deciding the number of circuits to be provided between 2 exchanges, is the volume of telephone traffic between them. This is measured at the busiest time of an average day; that is, the busy-hour traffic on the route is assessed. However, the cost of installing and maintaining circuits is high, and it is uneconomic to provide sufficient of them to meet the peak demand. To do so would mean that many of the circuits would be used for only a few minutes each day and, hence, earn little revenue. In practice, it is accepted that some calls during the busy period will be lost due to insufficient lines. The number of calls lost in this way can be expressed as a proportion of the calls made, and is referred to as the *grade of service* offered.

The time needed to plan and install new lines may also be taken into account, particularly if the volume of traffic on a route is expected to grow. Thus, it may be desirable, for organizational reasons, to provide more circuits than are necessary initially, and allow the number of lost calls to rise progressively to the designed level, when the next cable or group of equipment is installed.

Q 6 (*a*) *Describe, with the aid of block diagrams, how a 50 V d.c. power supply using secondary cells is arranged to work on*

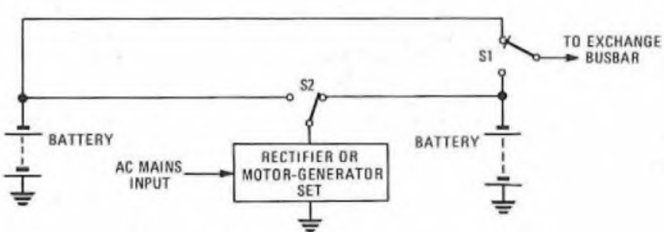
(*i*) *a charge-discharge system, and*
(*ii*) *a float system.*

(*b*) *State 2 reasons why the float system is preferred.*

(*c*) *One secondary cell in a 50 V battery shows a persistently low voltage when compared with the other cells in the battery. There is no indication of undue gassing.*

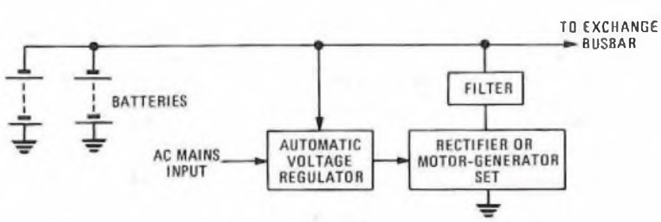
(*i*) *State the probable fault.*
(*ii*) *Suggest 2 causes of the fault.*

A 6 (*a*) (*i*) In the charge-discharge system, illustrated in sketch (*a*), although 2 batteries of secondary cells are provided, only one is connected to the load (that is, the exchange busbar) at any time. The second battery is charged from a suitable power supply, such as a mains-driven motor-generator set or a mains rectifier. When the working battery is discharged, switch S1 is changed over so that the newly charged battery supplies the load, and the discharged battery can then be recharged by operating switch S2. To avoid interrupting the supply to the load, switch S1 must be a make-before-break type. The batteries should each have sufficient capacity to supply the load for a period of 24 h in case of failure of the mains supply. The mains supply is used only while charging the idle battery.



(*a*)

(*ii*) The float system is illustrated in sketch (*b*). Both batteries and the mains-driven power supply are connected to the load. The mains-driven power supply, which is again either a rectifier or a motor-generator set, is sufficiently large to provide most of the maximum current required by the load. Thus, the batteries supply only peak-load currents in excess of the capacity of the rectifier or motor-generator set, and recharge when the load current falls below that capacity. To restrict variation of the busbar voltage, an automatic voltage regulator is provided. This adjusts the mains input voltage as necessary. A filter



(*b*)

is provided to give additional smoothing and minimize noise injected into any telephone circuits supplied by the system. Although a single battery can be used, 2 smaller batteries are frequently provided so that one can be taken out of service for maintenance, when necessary, without interrupting the power supply.

(*b*) The float system is preferred to the charge-discharge system because

(*i*) in the event of a mains failure, the batteries take up the load without interruption, and they are always ready to do so in the fully-charged condition,
(*ii*) as they are fully charged at all times, smaller batteries can be used,
(*iii*) since the batteries are not constantly (daily) being discharged and charged, they tend to have a longer life and need less maintenance.

(*c*) (*i*) The conditions in a secondary cell leading to persistent low voltage are sulphation, loss of active material from the plates, or internal shunting of the cell. With the first 2 faults, however, the cell gives excessive gassing when the battery is charged. An internal shunt is therefore the most probable cause of the fault.
(*ii*) The possible causes of an internal shunt are buckling of the plates (that is, distortion of the plates due to unequal expansion of the active material), or *treeing*, which is the formation of a spongy growth of lead on the negative plate.

Q 7 (a) State, in general terms only, what a telecommunication relay is and what function it performs.
(b) In the case of a non-polarized relay,

(i) state the purpose of the armature air-gap and why it is kept as small as possible,
(ii) explain why the core is provided with an enlarged face at the armature end, and
(iii) name the particular part of the magnetic circuit that ensures prompt and reliable release of the armature when the current in the coil is switched off. Explain the effect on the magnetic flux, and hence on the release of the armature, if this feature is omitted.

(c) Describe 3 ways in which the design of relay contacts helps to ensure reliable electrical contact in use.

A 7 See A9, Elementary Telecommunication Practice, 1973, *Supplement*, Vol. 67, p. 9, Apr. 1974, and A2, Elementary Telecommunication Practice, 1975, *Supplement*, Vol. 68, p. 86, Jan. 1976.

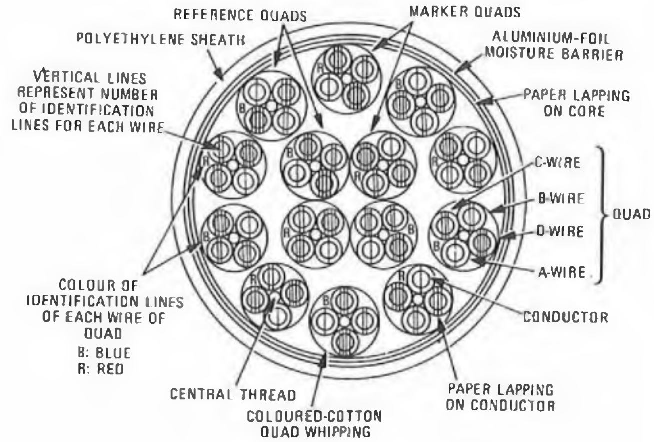
Q 8 (a) Briefly compare local-network types of underground cable with internal cables used in an exchange in respect of

(i) sheath material,
(ii) conductor material,
(iii) conductor insulation material, and
(iv) size range in terms of maximum number of pairs (or wires).

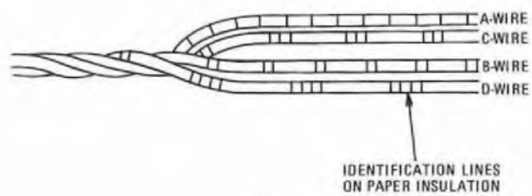
(b) Describe, with the aid of a sketch, the construction of an audio-type external cable suitable for a junction route requiring a small number of circuits.

A 8 (*a*) (*i*) Polyethylene is generally used to sheath external cables because it is tough and light, although lead, or lead with polyethylene protection, is used for some cables. PVC is used to sheath internal cables because it is not as inflammable as polyethylene.
(*ii*) Annealed high-conductivity copper is used as the conductor for both internal and external cables. Usually, the conductors of internal cables are tinned for ease of termination; that is, to assist in any soldering operations. Aluminium alloys are used in some external cables in place of copper.
(*iii*) External cables use polyethylene (either solid or cellular) or paper for insulating the conductors, to give high insulation resistance and low mutual capacitance. Internal cables use PVC for the same reason as given in part (*a*) (*i*).
(*iv*) External cables are available in a range of types and sizes up to about 4800 pairs, while internal cables are generally used in sizes of up to about 200 wires.

(*b*) Sketch (*a*) shows the cross-section of an audio cable of the star-quad type commonly used on small junction routes, and sketch (*b*) shows a single quad in detail.
Each conductor is insulated by a lapped paper tape, and 4 conductors are twisted together around a central thread to form a quad. Each quad has a whipping of cotton thread to help retain the quad formation. The quads are assembled in concentric layers, the outer layer being lapped with paper tapes. The whole core is contained within a polyethylene sheath which has an aluminium-foil moisture barrier bonded to its inside surface.
The individual wires within each quad are identified by ink lines on the paper insulation, the lines being arranged in groups of 1, 2,



(*a*)



(*b*)

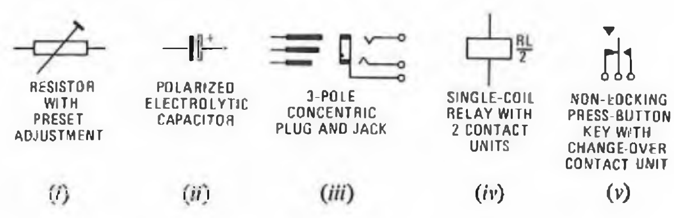
3 or 4, corresponding to the A-, B-, C- and D-wires of the quad. The first, or *marker* quad, in each layer has red ink markings, the next quad has blue ink markings, and this sequence is continued around the layer. The quad-whipping threads are also coloured, being white and black on alternate layers. An additional orange thread is added to the first and last quad in each layer to identify them. (The last quad in each layer is called the *reference* quad.)

Q 9 (a) Draw the standard symbols for the following circuit elements:

(i) a preset variable resistor,
(ii) an electrolytic capacitor,
(iii) a 3-pole concentric plug and jack,
(iv) a single-coil relay having 2 contact units, and
(v) change-over contacts for a non-locking press-button key.

(b) Diagrams representing the physical layout of the components and wiring of a circuit are sometimes used in telecommunication work. Comment on the limitations of such diagrams, and indicate how these are overcome.

A 9 (*a*) The symbols for the circuit elements are shown in the sketch, drawn in accordance with British Standard 3939: *Graphical Symbols for Electrical Power, Telecommunications and Electronics Diagrams.*



(*b*) Diagrams representing the physical layout of components and wiring are generally used only for wiring work; for example, for equipment-production purposes, or for illustrating the connexions needed when a subscriber's instrument is being installed. Except in the simplest circuits, with a limited number of components, it is difficult to deduce correctly the circuit operation or recognize the function of the circuit.
These limitations can be avoided by adopting a symbolic representation known as a *detached-contact* diagram. Components are represented by simple standard symbols, and their positions in the diagram are not restricted by their spatial relationships. Thus, for

example, relay coils and their contacts can be shown separately and placed conveniently in the appropriate functional groups of circuit elements, giving a more explicit diagram. The same principle can be applied to all other components, so that their functional relationships with other components in the circuit can be readily recognized.

Connecting points and component tags can be identified by numbers or letters. Relay coils and contacts can be related by suitable designations; for example, a relay coil may be marked $\frac{A}{2}$. The letter is a component reference related to the function of the relay (the component reference A normally denotes a pulsing relay), and the number indicates that the relay has 2 contact units, labelled A1 and A2, elsewhere in the diagram.

Wiring routes, indicating the order in which component tags are connected to a common wiring run, can be shown adjacent to that run. This compensates for the loss of a plan of the physical layout, a loss that is inevitable with detached-contact diagrams.

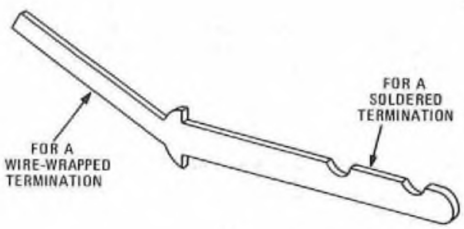
The use of certain conventions, such as drawing contact units in the unoperated condition, enables signal paths, circuit conditions and operational sequences to be readily determined.

Q 10 (*a*) (*i*) *Sketch the forms of tag used for the wire-wrapped and soldered types of wire termination. Explain the reasons for any differences in their shapes.*

(*ii*) *State 3 advantages and 3 disadvantages of the wire-wrapped termination.*

(*b*) *What method is used to fit a spade-type termination to the conductor of a telephone-handset cord? State reasons for adopting this method in this particular case.*

A 10 (*a*) (*i*) Sketch (*a*) shows the typical form of a tag capable of taking a wire-wrapped termination at one end and a soldered termination at the other. This type of tag is used on termination blocks on intermediate distribution frames.



(*a*)

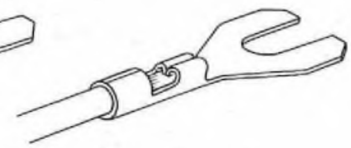
Tags for soldered terminations are rather wider than those for wrapped terminations because they require a reasonable surface area to make contact with the solder. Tags for wrapped terminations, however, need good square corners to bite into the wire as it is wound on under tension. The corners cut into the wire through any oxide film, and thereby lock some tension into the wire to maintain a good intimate contact at each corner. The wire is wrapped closely around the tag, using 7–10 turns, and no notches are required to hold it in position. Two notches are provided on tags for soldered terminations so that the wire can be wound firmly $1\frac{1}{2}$–2 times around the tag. This gives a good support for the wire, so that the solder, when applied, is under no strain. There is the danger of the solder cracking if it is subjected to strain, and this would result in a faulty connexion.

(*ii*) Compared with a soldered termination, a wrapped termination gives less risk of a high-resistance connexion, and the contact resistance is more stable with time. Wrapped terminations are mechanized, so that the procedure is simple and the result is less dependent on skill. No heat is used, so that the risk of damage to adjacent wires and equipment is reduced, and there is no risk of faults occurring through solder splashes.

However, a wrapped wire cannot be removed and reterminated, so that wrapped terminations are suitable only for permanent connexions. Whereas soldered terminations can be made on various shapes of tag, a wrapped termination requires a specially shaped tag, and a wire-wrapping tool is also needed. Wrapped terminations are suitable only for solid conductors, and cannot be used with stranded conductors.

(*b*) Sketch (*b*) illustrates a spade-type terminal in its unused state, and sketch (*c*) shows one fitted to a wire.

(*b*)



(*c*)

A spade-type terminal is fitted to the wire of a telephone-handset cord by crimping it around the wire, using a special crimping tool. The insulation is stripped from the wire, and the bare section is laid in the U-shaped channel of the spade terminal. The crimping tool, as it is squeezed, bends the sides of the channel over the wire, forming a tube that is then further compressed on to the wire to give a good electrical contact. The wire is placed in the tag so that part of its insulation is also trapped, to give extra mechanical strength.

This method is adopted because the conductor of a telephone cord uses either very fine stranded wires or a stranded-tinsel type of construction. Such conductors are difficult to solder or trap under a screw terminal without the risk of breaking many of the strands or, at least, weakening them so that early breakage occurs with movements of the handset cord.

CORRECTIONS

ELEMENTARY TELECOMMUNICATION PRACTICE, 1975

(*Supplement*, Vol. 68, Jan. 1976)

A 1 The 2 short springs that engage with L-shaped projections on the main springs were incorrectly described as stout. In fact, it is the break springs (the 2 centre springs) that are stout.

A 6 The signal coils are wound in series-aiding, not series-opposition as stated.

TELECOMMUNICATION PRINCIPLES A, 1976

Students were expected to answer 6 questions, including at least one from Q9–10

Q 1 (*a*) *An iron-cored coil carries a current. State*

(*i*) *the factors on which the strength of the magnetic field depends, and*

(*ii*) *the reason why doubling the current does not necessarily double the strength of the magnetic field.*

(*b*) *An iron ring has a cross-sectional area of 0·005 m² and a mean length of 1·2 m. It is uniformly wound with a coil of 900 turns. If a current of 2 A in the coil produces a magnetic flux density in the ring of 1·1 T, calculate*

(*i*) *the total magnetic flux in the iron,*
(*ii*) *the magnetizing force, and*
(*iii*) *the relative permeability of the iron under these conditions.*

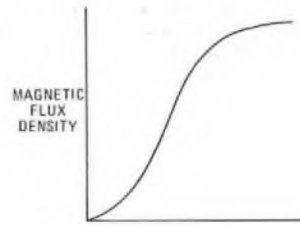
A 1 *Note:* In part (*a*) of this question, the term "strength of the magnetic field" was used to allow relative freedom to candidates of differing academic backgrounds in their approaches to answering the question. However, part (*a*) (*ii*) of the question implies that the non-linearity of the magnetic-flux-density/magnetic-field-strength (*B*/*H*) curve is required to be discussed. Therefore, for the purposes of this model answer, it is assumed that the term "strength of the magnetic field" means the *magnetic flux*. (A strict interpretation of the term as meaning the magnetic field strength, which is given by the current multiplied by the number of turns divided by the length of the magnetic circuit, leads to the conclusion that doubling the current necessarily doubles the magnetic field strength.)

(*a*) (*i*) The magnetic flux produced, Φ, is given by

$$\Phi = \frac{\mu_0 \mu_r A N I}{l} \text{ webers,}$$

where μ_0 is the absolute permeability of free space ($4\pi \times 10^{-7}$ H/m), μ_r is the relative permeability, A is the cross-sectional area of the core (metres²), N is the number of turns on the coil, I is the current flowing (amperes), and l is the length of the magnetic circuit (metres).

(*ii*) For the magnetic circuit given, the parameters A, N and l can be assumed to be constant, and μ_0 is an absolute constant. Thus, $\Phi \propto I$ if μ_r is a constant. However, for a given material, μ_r is not constant for all values of Φ.



The sketch shows a typical B/H curve for iron, where $B = \Phi/A$ (so that B is proportional to the magnetic flux), and $H = NI/l$ (so that H is proportional to the current). The ratio $B/\mu_0 H$ is the relative permeability; that is, μ_r is directly related to the gradient of the curve. The shape of the curve shows that μ_r is not constant as the magnetic field strength varies. Hence, B is not proportional to H and, therefore, the magnetic flux is not proportional to the current. Thus, doubling the current does not necessarily double the magnetic flux.

(*b*) (*i*) The total magnetic flux in the iron

$$= BA = 1 \cdot 1 \times 0 \cdot 005 \text{ Wb} = \underline{5 \cdot 5 \text{ mWb}}.$$

(*ii*) Magnetizing force is a deprecated term synonymous with magnetic field strength. The magnetic field strength

$$= \frac{NI}{l} = \frac{900 \times 2}{1 \cdot 2} = \underline{1 \cdot 5 \times 10^3 \text{ A/m}}.$$

(*iii*) The relative permeability

$$= \frac{B}{\mu_0 H} = \frac{1 \cdot 1}{4\pi \times 10^{-7} \times 1 \cdot 5 \times 10^3} = \underline{583 \cdot 6}.$$

Q 2 (a) Explain what happens when an inductive coil is first connected across a d.c. supply, and hence explain self-inductance.

(b) What is meant by mutual inductance? Give one practical application where this effect is used to advantage and one where it is a disadvantage.

(c) Two coils, A and B, are mutually coupled, the mutual inductance being 200 mH. Calculate the average value of the induced e.m.f. in coil B if the current in coil A increases from zero to 10 A in 500 ms.

A 2 (*a*) When an inductive coil is connected across a d.c. supply, current begins to flow, and the change of current causes the flux linking with the coil to change. By Faraday's law of electromagnetic induction, this induces in the coil a back-e.m.f., the polarity of which, by Lenz's law, is such as to oppose the change in the current. Because of this opposition, the rise of current in the coil is delayed, and the current builds up progressively, eventually reaching a steady value given by the ratio of the applied voltage to the resistance of the coil (from Ohm's law).

The magnitude of the back-e.m.f., e, is the product of the instantaneous rate of change of current, $\mathrm{d}i/\mathrm{d}t$ amperes/second, and a constant, L, that depends on the number of turns on the coil and its physical dimensions. This constant is called the *self-inductance* of the coil (sometimes abbreviated to *inductance*); the unit of inductance is the henry. Thus,

$$e = -L\frac{\mathrm{d}i}{\mathrm{d}t} \text{ volts,}$$

the minus sign indicating that the induced e.m.f. is in opposition to the applied voltage. If the applied voltage is V volts, the voltage available to drive a current, i amperes, is $V + e$ volts, where $V + e = iR$ volts, and R is the resistance of the coil (ohms).

$$\therefore\ i = \frac{V + e}{R} = \frac{1}{R}\left(V - L\frac{\mathrm{d}i}{\mathrm{d}t}\right) \text{ amperes,}$$

$$= \frac{V}{R} - \frac{L\mathrm{d}i}{R\mathrm{d}t} \text{ amperes.}$$

The first term of this equation is the steady-state (Ohm's law) value of current, and the second is a decreasing amount of opposing current due to the back-e.m.f.

(*b*) Two circuits have mutual inductance if a change of current in one circuit causes an e.m.f. to be induced in the other; that is, there is a common flux linking the 2 circuits. The mutually-induced e.m.f. is given by

$$e = -M\frac{\mathrm{d}i}{\mathrm{d}t} \text{ volts,}$$

where M is the mutual inductance (henrys).

Mutual inductance is used to advantage in a transformer. It is a disadvantage when it produces unwanted induced e.m.f.s (crosstalk) between 2 telephone circuits.

(*c*) The e.m.f. induced in coil B

$$= -200 \times 10^{-3} \times \frac{10}{500 \times 10^{-3}} \text{ V,}$$

$$= \underline{-4 \text{ V}}.$$

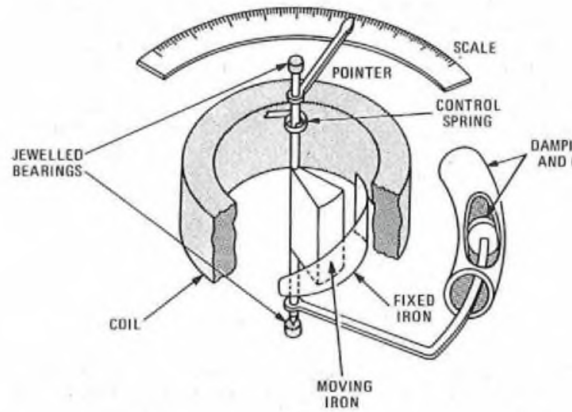
This is the average value of induced e.m.f. because the total current change is spread over a period of 500 ms.

Q 3 (a) Why is some form of damping required in electrical indicating instruments?

(b) Draw a labelled diagram showing the construction of a moving-iron instrument. Explain briefly the principle of operation of an instrument of this type.

(c) A moving-iron ammeter is wound with 60 turns and indicates a full-scale reading with 2 A flowing through it. If the coil has a tapping point at 48 turns, what current supplied at this point will produce full-scale deflexion?

A 3 (*a*) The movement of an electrical indicating instrument is produced by electromagnetic forces acting on a rotating system against a restoring force applied by control springs. In operation, the control springs are wound up until they produce a restoring torque that just equals the deflecting torque supplied by the electromagnetic forces. However, the rotating parts have inertia, and the kinetic energy stored during the movement must be given up before the pointer can stop at the point of equilibrium. This excess energy tends to make the pointer overshoot until the energy is absorbed by the control springs, which then return the energy to the rotating system, causing the pointer to swing back past the point of equilibrium. An oscillation thus occurs about the point of equilibrium. The oscillation can be damped by adding a means of absorbing and dissipating the excess kinetic energy. A piston-and-cylinder arrangement, such as that shown in the sketch, is often used. The resistance offered by the air to the movement of the piston is sufficient to prevent oscillations. Damping does not affect the deflecting or control torques, and so does not affect the reading of the instrument. It ensures that the pointer moves smoothly and unidirectionally to the true scale reading.



(*b*) The sketch shows the construction of a moving-iron instrument. The current to be measured is passed through the coil, setting up a magnetic field that magnetizes the fixed and moving irons. Because of the repulsive action of similar magnets, the moving iron tends to move away from the larger end of the fixed iron towards the smaller end, where the repulsion is less. The repulsion is eventually neutralized by the control spring. The movement of the moving iron rotates the pointer, and the deflexion given is proportional to the square of the current.

(*c*) If a current of 2 A gives a full-scale deflexion in a coil of 60 turns, the magnetomotive force required to give full-scale deflexion on the instrument is 120 A.

Thus, if 48 turns are used, the current needed to give full-scale deflexion

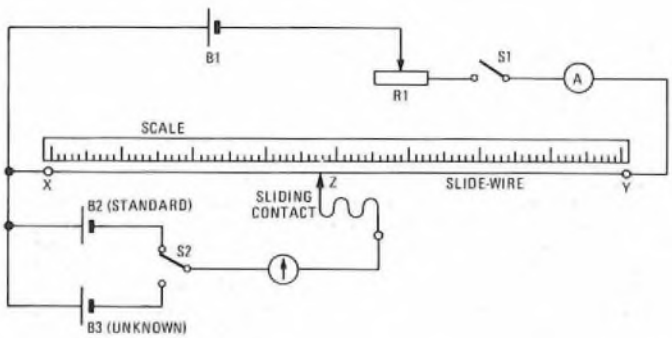
$$= \frac{120}{48} \text{ A,}$$

$$= \underline{2 \cdot 5 \text{ A}}.$$

Q 4 (a) Explain the advantage of using a potentiometer rather than a voltmeter to measure an e.m.f.

(b) Sketch the circuit arrangement of a simple d.c. potentiometer, and explain carefully how it can be used to measure the e.m.f. of a cell. Give 2 possible sources of error.

A 4 (*a*) The potentiometer, when used as a voltage-measuring device, operates by comparing the unknown voltage with a known standard potential difference, so that each reading is reliably calibrated. When the comparison is exact, no current is taken from the circuit under test. The potentiometer thus does not disturb the circuit conditions; that is, it has an infinite input impedance. Moreover, the potentiometer, being a slide-wire instrument, is sensitive to small movements of its sliding contact, and readings can thus be taken with considerable accuracy. Also, greater scale accuracy is possible with a linear change of resistance than with the angular change of a voltmeter's magnetic movement.



(*b*) The circuit of a slide-wire potentiometer, arranged to measure an unknown voltage, is shown in the sketch. XY is a resistive wire, accurate in section throughout its length, stretched alongside a linear scale. Current is supplied to the slide-wire by battery B1, controlled by switch S1. Resistor R1 and the ammeter enable the current in the slide-wire, and hence the voltage across XY, to be accurately set and maintained. Switch S2 connects either a standard cell (battery B2) or an unknown voltage (represented by battery B3), via a sensitive centre-zero galvanometer, to the sliding contact, Z.

The slide-wire is calibrated by connecting the standard cell to the sliding contact and adjusting resistor R1 to give a balance point, indicated by a null reading on the galvanometer, about half-way along the slide-wire. The scale reading, d_1, is noted for the standard potential (usually 1·0186 V), and the ammeter reading is noted and maintained throughout the measurements to preserve the accuracy of the calibration.

The unknown voltage is then connected to the sliding contact, and a new balance reading, d_2, taken. The potential difference between the sliding contact and the left-hand end of the slide-wire is proportional to the linear distance XZ. Therefore,

$$\frac{d_1}{d_2} = \frac{1{\cdot}0186}{E},$$

where E is the unknown e.m.f. (volts). Hence,

$$E = \frac{1{\cdot}0186 d_2}{d_1} \text{ volts.}$$

A possible source of error is a change of resistance of the slide-wire due to a rise in temperature. The effect can be minimized by making measurements rapidly so that the wire is not heated significantly by the current, and by using wire that has a virtually zero temperature coefficient. Other possible sources of error are

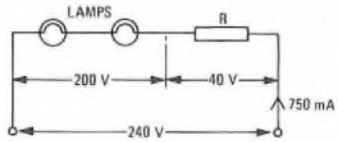
(*i*) a lack of uniformity of cross-section along the slide-wire, so that the potential difference per unit length is not constant,
(*ii*) variation in the slide-wire current,
(*iii*) variation in the contact resistance of the sliding contact, and
(*iv*) an inaccurately centred galvanometer.

Q 5 (a) Two 100 V, 75 W lamps are to operate in series. The supply available is 240 V. Calculate

(i) the value of a resistor to be connected in the circuit so that the lamps function at their correct rating,
(ii) the power loss in the resistor, and
(iii) the cost of supplying the lamps for 70 h per week for 10 weeks, with the price of energy at 2p per kilowatt hour.

(b) If the resistor in part (a) were to decrease in value, explain the effect on the circuit.

A 5 (*a*) If the lamps are to operate at their correct rating, each must have a voltage of 100 V across it, and an internal resistance, r ohms, such that $100^2/r = 75$ W. Therefore, $r = 133{\cdot}3\ \Omega$, and the current, I, through each lamp is $100/133{\cdot}3$ A $= 750$ mA.



(*i*) The sketch shows the circuit arrangements for the 2 lamps in series with a fixed resistor, R. The current in the circuit is 750 mA, and the total voltage across the 2 lamps is 200 V, leaving 40 V as the voltage across the resistor.

By Ohm's law, $R = 40/(750 \times 10^{-3}) = \underline{53{\cdot}3\ \Omega}.$

(*ii*) The power loss in resistor R

$$= I^2R = 750^2 \times 10^{-6} \times 53{\cdot}3 = \underline{30\ \text{W}}.$$

(*iii*) The total power of the circuit

$$= 75 + 75 + 30 = 180\ \text{W}.$$

Therefore, the energy used in 700 h

$$= 700 \times 180\ \text{W h} = 126\ \text{kW h}.$$

At 2p per kilowatt hour, the total cost

$$= 2 \times 126 = 252\text{p} = \underline{£2{\cdot}52}.$$

(*b*) Assuming the supply voltage to remain constant, if resistor R were to decrease in value, the current in the circuit would increase. The lamps would consequently be overloaded, raising the temperature of their filaments and increasing their brightness. The life of the lamps would be significantly reduced if the increase in current were large.

Q 6 (a) State the relationship between charge, capacitance and voltage for a charged capacitor. Give the unit of each quantity.

(b) What factors affect the capacitance of a capacitor?

(c) Why is it necessary to take precautions when working with circuits containing capacitors?

(d) Calculate the value of a capacitor which, when combined with a capacitor of 2·0 μF gives an equivalent capacitance of 1·5 μF. State how the 2 capacitors are connected to give the required value.

A 6 (*a*) The charge, Q coulombs, of a charged capacitor is related to the capacitance, C farads, and the voltage, V volts, by the equation

$$\underline{Q = CV \text{ coulombs.}}$$

(*b*) A capacitor can be considered as consisting of 2 parallel plates, each of effective area A metres², separated by a uniform sheet of dielectric of relative permittivity ϵ_r and thickness d metres. The capacitance is given by

$$C = \frac{\epsilon_0 \epsilon_r A}{d} \text{ farads,}$$

where ϵ_0 is the permittivity of free space ($10^{-9}/36\pi$ F/m).

(*c*) Ignoring leakage, capacitors can store charge indefinitely, unless a discharge path is provided. If the terminals of a charged capacitor are touched, an electric shock can be received, the severity depending on the charge stored and the voltage. To avoid shocks when working with circuits containing large capacitors, the capacitors should first be discharged by initially shunting, and then short-circuiting, their terminals. The terminals of large capacitors held in store should also be short-circuited.

(*d*) To give a resultant capacitance less than the value of either component, 2 capacitors must be connected in series.

For 2 capacitors, C1 and C2, connected in series, the total capacitance, C farads, is given by the equation

$$\frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2} \text{ farads}^{-1}.$$

Thus, if C1 is the known capacitor,

$$\frac{1}{C_2} = \frac{1}{1{\cdot}5 \times 10^{-6}} - \frac{1}{2 \times 10^{-6}} \text{ F}^{-1},$$

$$= (0\cdot667 - 0\cdot5) \times 10^6\,\mathrm{F}^{-1}.$$

$$\therefore\ C_2 = \frac{1}{0\cdot167 \times 10^6}\,\mathrm{F} = \underline{6\ \mu\mathrm{F}}.$$

Q 7 *(a) Explain briefly the meaning of the following terms relating to an alternating voltage represented by $v = V \sin 2\pi ft$:*

(i) frequency,
(ii) cycle,
(iii) peak amplitude, and
(iv) r.m.s. value.

(b) Two generators, with e.m.f.s given by $v_1 = 100 \sin 100\pi t$ and $v_2 = 80 \sin (100\pi t + 45°)$, are operating in series.

(i) Sketch the 2 voltage waveforms on the same axes and, from these, sketch the resultant voltage waveform.

(ii) Draw to scale a phasor diagram of v_1 and v_2, and hence obtain the magnitude of the resultant and its phase relative to v_1.

A 7 (*a*) In the expression for a sinusoidal voltage, $v = V \sin 2\pi ft$, v is the instantaneous voltage at time t seconds after a point at which the instantaneous voltage is zero, V is the peak value of the voltage, and f is the frequency (hertz).

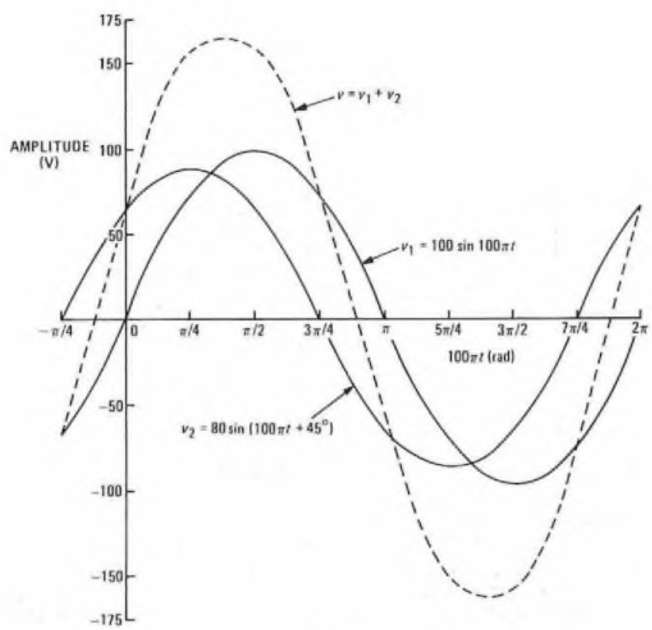
(*i*) The frequency is the number of cycles of alternation occurring per second, and is represented by the symbol f in the given equation.

(*ii*) A cycle is one complete excursion of the voltage, during which the angle represented by the term $2\pi ft$ makes one complete revolution.

(*iii*) The peak amplitude is the maximum value of voltage above or below zero, and is represented by the symbol V in the given equation.

(*iv*) The r.m.s. value is the value of a direct voltage that would have the same heating effect as the alternating voltage when driving current through a given resistor. For a sinusoidal wave, $V_{\mathrm{r.m.s.}} = 0\cdot707V$.

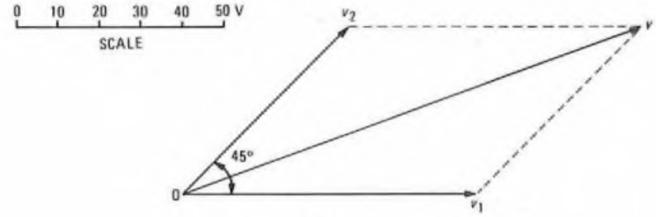
(*b*) (*i*) The waveforms are shown in sketch (*a*). The resultant, v, is obtained by adding ordinates for v_1 and v_2 at various points along the horizontal axis.



(*a*)

(*ii*) A sinusoidal wave can be represented by a phasor. The length of the phasor represents the magnitude of the wave (usually, the r.m.s. value), and the direction of the phasor relative to some datum line represents the phase angle of the wave. It is usual to draw the datum line horizontally from left to right, with positive phase angles being measured in the counter-clockwise direction.

Sketch (*b*) shows to scale phasors representing v_1 and v_2. Phasor v_1 has a magnitude of $0\cdot707 \times 100 = 70\cdot7$ V; it has a phase angle of zero, and is thus drawn along the datum line. Phasor v_2 has a magnitude of $0\cdot707 \times 80 = 56\cdot56$ V, and its phase angle is $+45°$. The resultant, v, is found by completing the parallelogram and drawing the diagonal from the common origin, O.



(*b*)

From the phasor diagram, by measurement, the r.m.s. value of the resultant is $\underline{117\cdot7\ \mathrm{V}}$, and its phase angle is $\underline{+20°}$. Hence, the trigonometrical expression for the resultant is

$$v = 1\cdot414 \times 117\cdot7 \sin (100\pi t + 20°),$$

$$= \underline{166\cdot4 \sin (100\pi t + 20°)}.$$

Q 8 *Describe one of the following experiments:*

(a) the determination of the static characteristics of a vacuum diode, or

(b) the measurement of a low resistance and a high resistance by the ammeter-voltmeter method.

Set out the answer in the form of a laboratory report, which should include a circuit diagram, details of the equipment used, the procedure, typical results and conclusions.

A 8 See A1, Telecommunication Principles A, 1974, *Supplement*, Vol. 68, p. 15, Apr. 1975.

Q 9 *The data in the table below refer to a transistor in the common-emitter configuration.*

Collector Voltage (V)	*Collector Current (mA) for Base Current =* 50 μA	80 μA	110 μA
2	*3·3*	*4·8*	*6·2*
4	*3·7*	*5·3*	*6·8*
6	*4·1*	*5·8*	*7·4*
8	*4·5*	*6·3*	*8·0*
10	*4·9*	*6·8*	*8·6*

(a) Plot the collector-current/collector-voltage characteristics for the base currents shown and, from the characteristics, find

(i) the current gain for a constant collector voltage of 6 V, and

(ii) the output resistance for a base current of 80 μA.

(b) On the same characteristics, draw the load line for a supply of 12 V and a load resistance of 1·5 kΩ. If the transistor operates with a base current of 80 μA and a sinusoidal input signal of 60 μA peak-to-peak, determine the r.m.s. value of the alternating current in the load resistor.

A 9 (*a*) The collector-current/collector-voltage characteristics are shown in the sketch.

(*i*) As the transistor is in the common-emitter configuration, an input signal is represented by a change in the base current, and an output signal by a change in the collector current. The gain is the ratio of the latter change to the former for a given value of collector voltage.

From the characteristics, for a collector voltage of 6 V, the gain

$$= \frac{\mathrm{AB}}{110\ \mu\mathrm{A} - 50\ \mu\mathrm{A}} = \frac{(7\cdot4 - 4\cdot1) \times 10^{-3}}{(110 - 50) \times 10^{-6}},$$

$$= \frac{3\cdot3 \times 10^{-3}}{60 \times 10^{-6}},$$

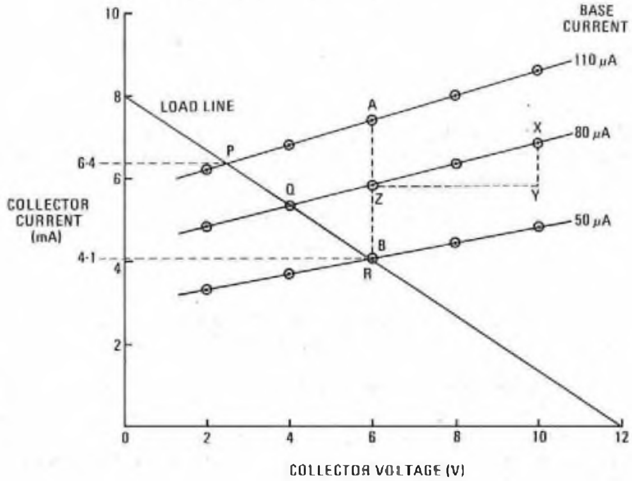
$$= \underline{55.}$$

Thus, under static conditions, a change in base current of 60 μA produces a change in collector current of 3·3 mA.

(*ii*) The output resistance is given by the ratio of a change in collector voltage to the corresponding change in collector current for a constant value of base current. From the characteristics, for a base current of 80 μA, the output resistance

$$= \frac{YZ}{XY} = \frac{10 - 6}{(6 \cdot 8 - 5 \cdot 8) \times 10^{-3}}\,\Omega,$$

$$= \frac{4}{1 \times 10^{-3}}\,\Omega = \underline{4\ \text{k}\Omega.}$$



Thus, the output resistance is given by the reciprocal of the gradient of the characteristic. The value obtained is accurate since the characteristic is virtually linear over the portion considered.

(*b*) The load line is used to indicate the performance of the circuit under a.c. conditions. For a resistive load, the load line is a straight line with a gradient that is the inverse of that of the load resistance. For zero collector current, the supply voltage appears as the collector voltage, and the load line thus intersects the collector-voltage axis at the supply voltage; that is, 12 V in this case. For zero collector voltage, the supply voltage appears across the load resistance, and the load line thus intersects the collector-current axis at a value given by the supply voltage divided by the load resistance; that is, $12/(1 \cdot 5 \times 10^3)$ A = 8 mA in this case. The load line is shown on the characteristics.

For a quiescent (that is, no-input-signal) base current of 80 μA, a peak-to-peak input signal of 60 μA causes the base current to vary between 80 μA − 30 μA and 80 μA + 30 μA; that is, between 50 μA and 110 μA. The signal thus uses the portion PQR of the load line, where Q is the quiescent point. Projecting points P and R to the collector-current axis, the peak-to-peak variation in current is $6 \cdot 4 - 4 \cdot 1 = 2 \cdot 3$ mA. This is the peak-to-peak alternating current in the load resistor, and has a peak value of $2 \cdot 3/2 = 1 \cdot 15$ mA and an r.m.s. value of $1 \cdot 15 \times 0 \cdot 707$ mA $\approx$ $\underline{800\ \mu\text{A}}$.

Q 10 *(a) Describe the principle of operation of a microphone for use in a telephone handset.*

(b) Sketch the construction of a modern type of microphone and label each part.

A 10 See A9, Elementary Telecommunication Practice, 1972, *Supplement*, Vol. 66, p. 10, Apr. 1973.

MATHEMATICS A, 1976

Students were expected to answer any 6 questions

Q 1 *(a) Draw the graphs of $y_1 = 2 \sin \theta$ and $y_2 = 3 \cos 2\theta$ between $\theta = 20°$ and $\theta = 60°$, plotting points at intervals of 10°.*

(b) Use the graphs to solve the equation $2 \sin \theta - 3 \cos 2\theta = 0$.

(c) Add the ordinates at each interval and hence draw the graph of $y_3 = 2 \sin \theta + 3 \cos 2\theta$. Use this graph to estimate a solution of the equation $2 \sin \theta + 3 \cos 2\theta = 2$.

A 1 (*a*) The graphs of y_1 and y_2 are shown in the sketch, plotted from the values given by the table below.

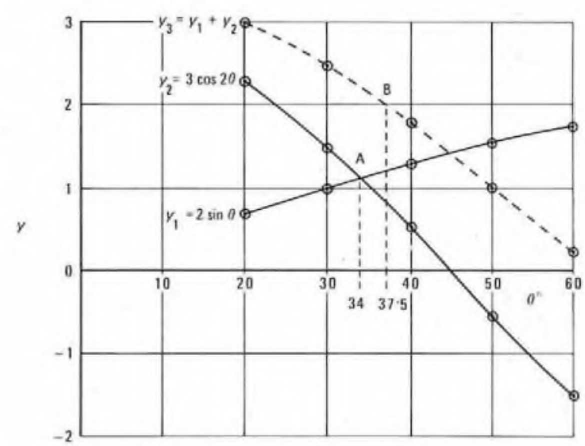
$\theta°$	20	30	40	50	60
$2\theta°$	40	60	80	100	120
$\sin \theta$	0·342	0·500	0·6428	0·766	0·866
$\cos 2\theta$	0·766	0·500	0·1736	−0·1736	−0·500
$y_1 = 2 \sin \theta$	0·684	1·000	1·286	1·532	1·732
$y_2 = 3 \cos 2\theta$	2·298	1·500	0·521	−0·521	−1·500
$y_3 = y_1 + y_2$	2·982	2·500	1·807	1·011	0·232

(*b*) If $2 \sin \theta - 3 \cos 2\theta = 0$, then $2 \sin \theta = 3 \cos 2\theta$; that is, $y_1 = y_2$. Thus, the point of intersection, A, of the 2 graphs gives the solution of the equation. From the sketch, at point A, $\underline{\theta \approx 34°}$.

(*c*) The ordinates for y_1 and y_2 are added in the table above, and the graph of y_3 is shown in the sketch. The graph intersects the line $y = 2$ at point B, where

$$y_3 = 2 \sin \theta + 3 \cos 2\theta = 2.$$

The abscissa of point B therefore gives an estimate of the solution of the equation. From the sketch, at point B, $\underline{\theta \approx 37 \cdot 5°}$.



Q 2 *(a) Evaluate, with the aid of mathematical tables:*

(i) sin 220° 32′,

(ii) tan 163° 19′, and

(iii) cos (−0·6), where the angle is given in radians.

(b) Fig. 1 shows the phasor diagram for 2 alternating potentials, $v_1 = 6 \sin \{2\pi t + (\pi/3)\}$ and $v_2 = 8 \sin \{2\pi t - (\pi/6)\}$. By the method of resolution, or otherwise, calculate their resultant and the angle it makes with the reference axis, OX.

(c) Check your answers for part (b) by drawing an accurate diagram.

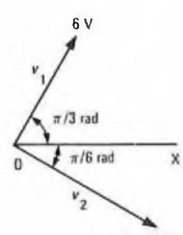


Fig. 1

A 2 (*a*) (*i*) $\sin 220° 32' = \sin(180° + 40° 32')$,

$= -\sin 40° 32' = \underline{-0{\cdot}6498}.$

(*ii*) $\tan 163° 19' = \tan(180° - 16° 41')$,

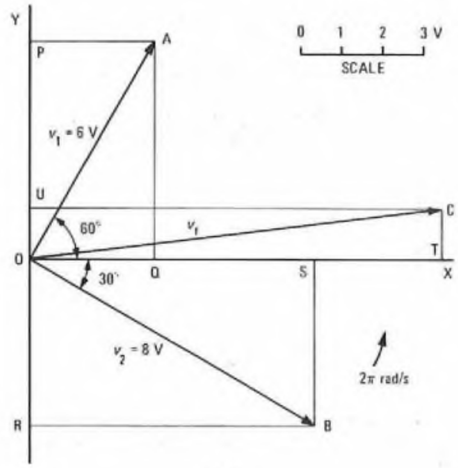
$= -\tan 16° 41' = \underline{-0{\cdot}2997}.$

(*iii*) Now, $\cos(-0{\cdot}6 \text{ rad}) = \cos(0{\cdot}6 \text{ rad})$.

From a conversion table,

$$\cos(0{\cdot}6 \text{ rad}) = \cos 34° 23',$$
$$= \underline{0{\cdot}8253}.$$

(*b*) *Note:* The accurate diagram of sketch (*a*), drawn for part (*c*) of this question, serves also to illustrate the method of resolution used below.



(*a*)

In sketch (*a*), OA represents v_1 in magnitude and phase, angle AOX being $\pi/3$ rad or 60°. Similarly, OB represents v_2, lagging OX by $\pi/6$ rad or 30°.

OA is resolved into 2 components, OP and OQ along axes OY and OX respectively. Similarly, OB is resolved into components OR and OS.

Now, $OP = 6 \sin 60° = 6 \times 0{\cdot}866 = 5{\cdot}196$,

and $OQ = 6 \cos 60° = 6 \times 0{\cdot}5 = 3$.

Also, $OR = 8 \sin 30° = 8 \times 0{\cdot}5 = 4$,

and $OS = 8 \cos 30° = 8 \times 0{\cdot}866 = 6{\cdot}928$.

Hence, the total component along axis OX

$$= OQ + OS = 3 + 6{\cdot}928 = 9{\cdot}928,$$

and the total component along axis OY

$$= OP - OR = 5{\cdot}196 - 4 = 1{\cdot}196.$$

These components are shown as OT and OU along axes OX and OY respectively, so that the resultant of phasors v_1 and v_2 is given by OC, the diagonal of rectangle OUCT.

Now, $OC^2 = OT^2 + OU^2 = 9{\cdot}928^2 + 1{\cdot}196^2$,

$= 98{\cdot}57 + 1{\cdot}43 = 100.$

$\therefore$ OC = 10 V.

Also, $\tan \angle COT = \dfrac{CT}{OT} = \dfrac{1{\cdot}196}{9{\cdot}928} = 0{\cdot}1205.$

$\therefore \angle COT = 6° 52' = 0{\cdot}1199$ rad.

Therefore, the resultant, v_r, is given by

$$v_r = 10 \sin(2\pi t + 0{\cdot}12),$$

and thus has a magnitude of $\underline{10 \text{ V}}$ and makes an angle of $\underline{+0{\cdot}12 \text{ rad}}$ with the reference axis, OX.

(*c*) An accurately constructed phasor diagram is shown in sketch (*a*), from which v_r has a measured magnitude of $\underline{10 \text{ V}}$ and makes an angle of approximately 7°, or $\underline{0{\cdot}122 \text{ rad}}$, with axis $\underline{\text{OX}}$.

Q 3 (*a*) (*i*) *Convert 2·3 rad to degrees and minutes.*

(*ii*) *Express 130° 45′ in radians, correct to 2 decimal places.*

(*b*) *In Fig. 2, triangle PQR is right-angled at Q, and QS is the perpendicular from Q to PR.*

(*i*) *Name the 3 similar triangles.*

(*ii*) *If PQ = 120 mm, PR = 200 mm and QR = 160 mm, use the properties of similar triangles to calculate the lengths of QS and RS.*

(*c*) *Establish the truth of the following identities:*

(*i*) $2\cos^2 A - 1 = (\cos A + \sin A)(\cos A - \sin A)$, *and*

(*ii*) $(1 - \sin^2 B)\tan^2 B = \sin^2 B$.

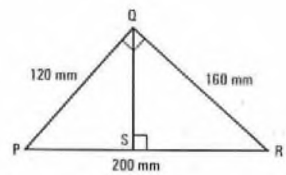


Fig. 2

A 3 (*a*) (*i*) Since π rad = 180°

then $2{\cdot}3 \text{ rad} = \dfrac{2{\cdot}3}{\pi} \times 180°$,

$= 131{\cdot}8°$,

$= \underline{131° 48'}.$

Number	Logarithm
2·3	0·3617
180	2·2553+
	2·6170
π	0·4971−
131·8	2·1199

(*ii*) $130° 45' = \dfrac{130{\cdot}75°}{180°} \times \pi$ rad,

$= \underline{2{\cdot}28 \text{ rad}}$,

correct to 2 decimal places.

Number	Logarithm
130·75	2·1164
π	0·4971+
	2·6135
180	2·2553−
2·281	0·3582

(*b*) (*i*) The 3 similar triangles are $\underline{\text{PQS, QRS and PQR}}$.

(*ii*) Since triangles QRS and PQR are similar,

$$\frac{QS}{160} = \frac{PQ}{PR} = \frac{120}{200}.$$

$$\therefore QS = \frac{160 \times 120}{200} = \underline{96 \text{ mm}}.$$

Also,
$$\frac{RS}{160} = \frac{QR}{PR} = \frac{160}{200}.$$

$$\therefore RS = \frac{160 \times 160}{200} = \underline{128 \text{ mm}}.$$

(*c*) (*i*) The left-hand side of the identity $= \cos^2 A + \cos^2 A - 1$.

But $\sin^2 A + \cos^2 A = 1$, or $\cos^2 A - 1 = -\sin^2 A$, so that the left-hand side

$= \cos^2 A - \sin^2 A$,

$= \underline{(\cos A + \sin A)(\cos A - \sin A)}.$ QED.

(*ii*) Since $\sin^2 B + \cos^2 B = 1$, the left-hand side of the identity

$= \cos^2 B \tan^2 B$,

$= \cos^2 B \times \dfrac{\sin^2 B}{\cos^2 B}$,

$= \underline{\sin^2 B}.$ QED.

Q 4 *In Fig. 3, A, B and C are 3 points at the same ground level. AF is a vertical mine shaft of depth 1 km. Tunnels are to be constructed from B and C to meet at F.*

If AB = 5 km, AC = 3 km and ∠BAC = 100°,

(*a*) *show that BF = 5·099 km, CF = 3·162 km and BC = 6·262 km, and*

(*b*) *calculate angle BFC.*

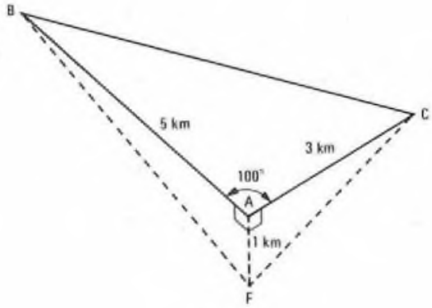


Fig. 3

A 4 (*a*) Applying the theorem of Pythagoras to right-angled triangle BAF, and using a table of square roots, gives

$$\mathrm{BF} = \sqrt{(5^2 + 1^2)} = \sqrt{26} = \underline{5 \cdot 099 \text{ km}}. \qquad \text{QED.}$$

Similarly, from right-angled triangle CAF,

$$\mathrm{CF} = \sqrt{(3^2 + 1^2)} = \sqrt{10} = \underline{3 \cdot 162 \text{ km}}. \qquad \text{QED.}$$

Applying the cosine rule to triangle BAC gives

$$\mathrm{BC}^2 = \mathrm{BA}^2 + \mathrm{AC}^2 - 2 \times \mathrm{BA} \times \mathrm{AC} \times \cos \angle \mathrm{BAC},$$
$$= 25 + 9 - 2 \times 5 \times 3 \times \cos 100°,$$
$$= 34 - 30 \times (-\cos 80°) = 34 + 30 \times 0 \cdot 1736.$$
$$\therefore\ \mathrm{BC} = \sqrt{(34 + 5 \cdot 208)} = \sqrt{39 \cdot 208} = \underline{6 \cdot 262 \text{ km}}. \quad \text{QED.}$$

(*b*) Applying the cosine rule to triangle BFC gives

$$\cos \angle \mathrm{BFC} = \frac{\mathrm{BF}^2 + \mathrm{CF}^2 - \mathrm{BC}^2}{2 \times \mathrm{BF} \times \mathrm{CF}},$$
$$= \frac{26 + 10 - 39 \cdot 21}{2 \times 5 \cdot 099 \times 3 \cdot 162},$$
$$= -\frac{3 \cdot 21}{10 \cdot 198 \times 3 \cdot 162},$$
$$= -0 \cdot 09954.$$

Number	Logarithm
10·198 3·162	1·0085 0·5000 +
	1·5085
3·21	0·5065 1·5085 −
0·09954	$\bar{2}$·9980

$$\therefore\ \angle \mathrm{BFC} = 180° - 84°\,17' = \underline{95°\,43'}.$$

Q 5 The surface area of a sphere of radius r is equal to the total surface area of a closed cylinder with diameter d and length l.
(a) Show that $8r^2 = d^2 + 2dl$.
(b) If the radius of the sphere is half the diameter of the cylinder, show that the ratio of the length of the cylinder to its diameter is 1 : 2.
(c) Using the results of part (b), calculate the ratio of the volume of the sphere to that of the cylinder.

A 5 (*a*) The surface area of the sphere is $4\pi r^2$, and that of the closed cylinder is $2 \times (\pi d^2/2^2) + \pi dl$. Hence,

$$4\pi r^2 = \frac{\pi d^2}{2} + \pi dl.$$

Dividing throughout by π and multiplying by 2 gives

$$\underline{8r^2 = d^2 + 2dl.} \qquad \text{QED}$$

(*b*) Substituting for $r = d/2$ in the above equation gives

$$\frac{8d^2}{4} = d^2 + 2dl.$$
$$\therefore\ d^2 = 2dl.$$
$$\therefore\ \underline{\frac{l}{d} = \frac{1}{2}}. \qquad \text{QED.}$$

(*c*) The ratio of the volume of the sphere, V_S, to that of the cylinder, V_C, is given by

$$\frac{V_S}{V_C} = \frac{\dfrac{4\pi r^3}{3}}{\dfrac{\pi d^2 l}{2^2}}.$$

But $r = d/2$ and $l = d/2$.

$$\therefore\ \frac{V_S}{V_C} = \frac{\dfrac{4}{3} \times \left(\dfrac{d}{2}\right)^3}{\left(\dfrac{d}{2}\right)^2 \times \dfrac{d}{2}} = \underline{\frac{4}{3}}.$$

Q 6 (a) A current, i, has the periodic waveform shown in Fig. 4. Calculate the average value of current over 1 cycle.
(b) The width, x, of a metal cover-plate, measured at stated distances, d, along one straight edge, is given in the table below, where x and d are in millimetres.

d	*0*	*10*	*20*	*40*	*60*	*80*	*100*	*110*	*120*
x	*40*	*48*	*52*	*55*	*51*	*41*	*28*	*23*	*20*

(i) Plot accurately the values given and so obtain the shape of the plate.
(ii) Divide the figure into 6 equal-width strips and apply the mid-ordinate rule to calculate the area of the plate.

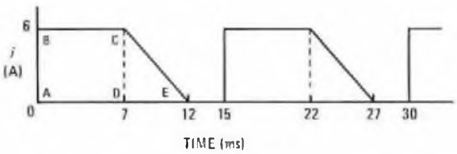
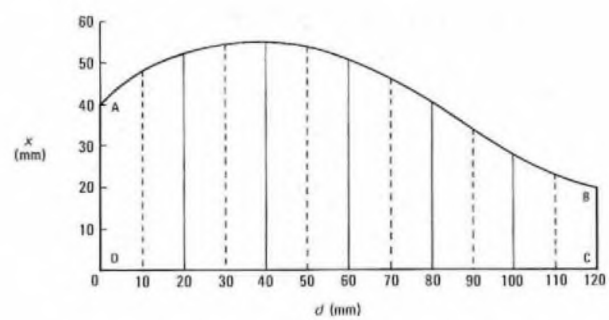


Fig. 4

A 6 (*a*) Considering the first complete cycle (of period 15 ms) and labelling the salient points of the waveform A, B, C, D and E as shown, the average value of i

$$= \frac{\text{total area under curve between 0–15 ms}}{15 \text{ ms}},$$
$$= \frac{\text{area of rectangle ABCD + area of triangle CDE}}{15},$$
$$= \frac{6 \times 7 + \frac{1}{2} \times (12 - 7) \times 6}{15} = \underline{3 \cdot 8 \text{ A.}}$$

(*b*) (*i*) The graph of x/d is shown in sketch (*a*). The shape of the plate is thus given by the figure ABCD.



(*a*)

(*ii*) The figure is divided into 6 equal-width strips by erecting ordinates at $d = 20$, 40, 60, 80 and 100 mm. The middle of each strip (the *mid-ordinate*) is shown by a dashed line.

From the mid-ordinate rule, the area under the graph AB is given by the width of a strip multiplied by the sum of the mid-ordinates. The mid-ordinates for $d = 10$ and 110 mm can be taken from the table; the rest are read from the graph. Thus, the area under AB (that is, the area of the plate)

$$= 20(48 + 54 + 54 + 46 + 34 + 23) \text{ mm}^2,$$
$$= 20 \times 259 \text{ mm}^2,$$
$$= \underline{5180 \text{ mm}^2}.$$

Q 7 (*a*) *Rearrange the formulae:*

(*i*) $n = \dfrac{CR}{E - Cr}$, *to obtain an expression for C, and*

(*ii*) $Z = \sqrt{\{R^2 + (X_1 - X_2)^2\}}$, *to obtain an expression for* X_1.

(*b*) *The impedance, Z ohms, of a series circuit containing resistance R ohms, inductance L henrys and capacitance C farads, is given by*

$$Z = \sqrt{[R^2 + \{\omega L - (1/\omega C)\}^2]}.$$

(*i*) *Calculate* Z *when* $R = 15$, $L = 0{\cdot}02$, $\omega = 2100$ *and* $C = 8 \times 10^{-6}$.

(*ii*) *if* $\omega = 2\pi f$, *where f is the frequency (hertz), obtain an expression for the frequency at which* $Z = R$.

A 7 (*a*) (*i*) $n = \dfrac{CR}{E - Cr}$.

$$\therefore\ CR = nE - nCr.$$

$$\therefore\ C(R + nr) = nE.$$

$$\therefore\ \underline{C = \frac{nE}{R + nr}}.$$

(*ii*) $Z = \sqrt{\{R^2 + (X_1 - X_2)^2\}}$.

$$\therefore\ Z^2 = R^2 + (X_1 - X_2)^2.$$

$$\therefore\ (X_1 - X_2)^2 = Z^2 - R^2.$$

$$\therefore\ X_1 - X_2 = \sqrt{(Z^2 - R^2)}.$$

$$\therefore\ X_1 = \underline{X_2 + \sqrt{(Z^2 - R^2)}}.$$

(*b*) (*i*) Substituting the values gives

$$Z = \sqrt{\left\{15^2 + \left(2100 \times 0{\cdot}02 - \frac{1}{2100 \times 8 \times 10^{-6}}\right)^2\right\}}\ \Omega,$$

$$= \sqrt{\{225 + (42 - 59{\cdot}52)^2\}}\ \Omega,$$

$$= \sqrt{(225 + 307)} = \underline{23{\cdot}07\ \Omega}.$$

(*ii*) When $Z = R$ (that is, when $Z = \sqrt{(R^2)}$),

$$\left(\omega L - \frac{1}{\omega C}\right)^2 = 0.$$

$$\therefore\ \omega L = \frac{1}{\omega C}.$$

$$\therefore\ \omega^2 = \frac{1}{LC} = (2\pi f)^2.$$

$$\therefore\ 4\pi^2 f^2 = \frac{1}{LC}.$$

$$\therefore\ f^2 = \frac{1}{4\pi^2 LC}.$$

$$\therefore\ \underline{f = \frac{1}{2\pi\sqrt{(LC)}}\ \text{hertz}.}$$

Q 8 (*a*) *Form the equation whose roots are* $x = -2$ *and* $x = 3$, *expressing the result in the form* $x^2 + px + q = 0$.

(*b*) *Solve the following equations:*

(*i*) $5w^2 + 3w = 0$,
(*ii*) $(2x - 7)^2 = 25$, *and*
(*iii*) $5t^2 - 14t - 3 = 0$.

(*c*) *A standard cell has an e.m.f. of 1·0183 V at 20°C. The e.m.f. changes with temperature in such a way that, when the temperature rises above 20°C by x°C, the e.m.f. falls by* $(x^2 + 41x) \times 10^{-6}$ *V. Calculate the temperature at which the e.m.f. is 1·0178 V.*

A 8 (*a*) The equation is given by

$$\{x - (-2)\}\{x - 3\} = 0.$$

$$\therefore\ x^2 + 2x - 3x - 6 = 0.$$

$$\therefore\ \underline{x^2 - x - 6 = 0}.$$

(*b*) (*i*) $5w^2 + 3w = 0$.

$$\therefore\ w(5w + 3) = 0.$$

$$\therefore\ w = 0 \text{ or } 5w + 3 = 0.$$

$$\therefore\ \underline{w = 0 \text{ or } -3/5}.$$

(*ii*) $(2x - 7)^2 = 25$.

$$\therefore\ 2x - 7 = \pm 5.$$

$$\therefore\ 2x = 7 + 5 \text{ or } 2x = 7 - 5.$$

$$\therefore\ \underline{x = 6 \text{ or } 1}.$$

(*iii*) Factorizing the equation gives

$$(5t + 1)(t - 3) = 0.$$

$$\therefore\ \underline{t = -1/5 \text{ or } 3}.$$

(*c*) Let E volts be the e.m.f. at 20°C. Since the e.m.f. falls by $(x^2 + 41x) \times 10^{-6}$ volts when the temperature increases by x°C, the new value of e.m.f., E_x volts, is given by

$$E_x = E - (x^2 + 41x) \times 10^{-6}.$$

$$\therefore\ 1{\cdot}0178 = 1{\cdot}0183 - (x^2 + 41x) \times 10^{-6}.$$

$$\therefore\ 0{\cdot}0005 = (x^2 + 41x) \times 10^{-6}.$$

$$\therefore\ x^2 + 41x = 0{\cdot}0005 \times 10^6 = 500.$$

$$\therefore\ x^2 + 41x - 500 = 0.$$

By the method of completing the squares,

$$x^2 + 41x + \left(\frac{41}{2}\right)^2 = 500 + \left(\frac{41}{2}\right)^2.$$

$$\therefore\ x + \frac{41}{2} = \pm\sqrt{\frac{2000 + 1681}{4}} = \pm\frac{60{\cdot}67}{2}.$$

$$\therefore\ x = \frac{60{\cdot}67}{2} - \frac{41}{2} \text{ or } -\frac{60{\cdot}67}{2} - \frac{41}{2},$$

$$= 9{\cdot}835 \text{ or } -50{\cdot}835.$$

The negative value of x is inadmissible because the temperature must exceed 20°C. Therefore, the temperature at which the e.m.f. is 1·0178 V is 20 + 9·8°C = <u>29·8°C</u>.

Q 9 (*a*) *The anode current,* I_a, *in a thermionic tube can be expressed as* $I_a = p + qV_g$, *where* V_g *is the grid potential, and p and q are constants. Given that* $I_a = 0{\cdot}5$ *when* $V_g = -8$, *and* $I_a = 1{\cdot}7$ *when* $V_g = -4$, *calculate*

(*i*) *the values of p and q, and*
(*ii*) *the value of* V_g *when* $I_a = 1{\cdot}25$.

(*b*) *Supplies of 12 V d.c. and 6 V d.c. are connected to the network shown in Fig. 5. Currents* I_1, I_2 *and* $(I_1 + I_2)$ *amperes flow in the parts of the circuit indicated.*

(*i*) *Form 2 simultaneous equations relating* I_1 *and* I_2.
(*ii*) *Solve the equations to find* I_1 *and* I_2.

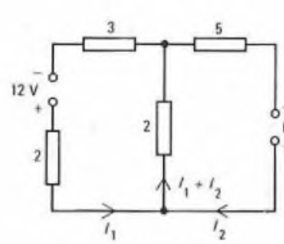
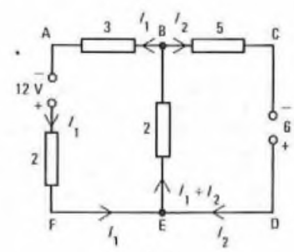


Fig. 5



(*a*)

A 9 (*a*) (*i*) Substituting values in the equation gives

$$0{\cdot}5 = p - 8q, \qquad \ldots\ldots (1)$$

$$\text{and } 1{\cdot}7 = p - 4q. \qquad \ldots\ldots (2)$$

Subtracting equation (1) from equation (2) gives

$$1{\cdot}2 = -4q - (-8q) = 4q.$$

$$\therefore\ \underline{q = 0{\cdot}3}.$$

Substituting for q in equation (1) gives

$$0\cdot5 = p - 8 \times 0\cdot3.$$

$$\therefore \underline{p = 2\cdot9.}$$

(*ii*) Using the above values of p and q, when $I_a = 1\cdot25$,

$$1\cdot25 = 2\cdot9 + 0\cdot3V_g.$$

$$\therefore V_g = \frac{1\cdot25 - 2\cdot9}{0\cdot3} = \underline{-5\cdot5.}$$

(*b*) (*i*) Fig. 5 is amplified in sketch (*a*) to show the current entering each resistor. Applying Kirchhoff's law to the network ABEF gives

$$12 = 2I_1 + 2(I_1 + I_2) + 3I_1.$$

$$\therefore \underline{12 = 7I_1 + 2I_2.} \qquad \ldots\ldots (1)$$

Applying Kirchhoff's law to the network BCDE gives

$$6 = 2(I_1 + I_2) + 5I_2.$$

$$\therefore \underline{6 = 2I_1 + 7I_2.} \qquad \ldots\ldots (2)$$

(*ii*) Multiplying equation (1) by 7, and equation (2) by 2, gives

$$49I_1 + 14I_2 = 84, \qquad \ldots\ldots (3)$$

$$\text{and } 4I_1 + 14I_2 = 12. \qquad \ldots\ldots (4)$$

Subtracting equation (4) from equation (3) gives

$$45I_1 = 72.$$

$$\therefore \underline{I_1 = 1\cdot6 \text{ A.}}$$

Substituting for I_1 in equation (4) gives

$$4 \times 1\cdot6 + 14I_2 = 12.$$

$$\therefore 14I_2 = 12 - 6\cdot4.$$

$$\therefore \underline{I_2 = 0\cdot4 \text{ A.}}$$

Q 10 (*a*) *Using tables other than logarithmic tables, determine the values of*

(*i*) $0\cdot4871^2$, (*ii*) $\sqrt{0\cdot3954}$, (*iii*) $\sqrt{542\cdot6}$, *and* (*iv*) $1/21\cdot63$.

(*b*) *The calculation* $\dfrac{48\cdot67 \times \sqrt{63\cdot12} \times \tan 43^\circ 37'}{3\cdot82^2 \times 0\cdot546}$ *gives a result having the significant figures 4624. By suitable approximation and cancellation, insert the decimal point in its correct position in the result.*

(*c*) *If* $\log(2x + 5) + \log 4 - 2\log 6 = 1$, *and all logarithms are to the base 2, find the value of* x.

(*d*) *Express the denary number 175 in binary form.*

(*e*) *Working in binary notation throughout, determine*

(*i*) *11 001 + 10 011,*
(*ii*) *11 001 − 10 011, and*
(*iii*) *11 001 × 10 011.*

A 10 (*a*) (*i*) $0\cdot4871^2 = \underline{0\cdot2373.}$

(*ii*) $\sqrt{0\cdot3954} = \underline{0\cdot6288.}$

(*iii*) $\sqrt{542\cdot6} = \underline{23\cdot29.}$

(*iv*) $1/21\cdot63 = \underline{0\cdot04623.}$

(*b*) Since tan 45° is unity,

$$\frac{48\cdot67 \times \sqrt{63\cdot12} \times \tan 43^\circ 37'}{3\cdot82^2 \times 0\cdot546} \approx \frac{50 \times \sqrt{64} \times \tan 45^\circ}{4^2 \times 0\cdot5},$$

$$= \frac{50 \times 8 \times 1}{16 \times 0\cdot5} = 50.$$

Thus, the true result of the calculation is $\underline{46\cdot24.}$

(*c*) $\log_2(2x + 5) + \log_2 4 - 2\log_2 6 = 1.$

$$\therefore \log_2\{(2x + 5) \times 4\} - \log_2 6^2 = 1.$$

$$\therefore \log_2 \frac{4(2x + 5)}{36} = 1.$$

$$\therefore \frac{8x + 20}{36} = 2^1.$$

$$\therefore 8x + 20 = 72.$$

$$\therefore \underline{x = 6\cdot5.}$$

(*d*) See A9, Mathematics A, 1975, *Supplement*, Vol. 69, p. 8, Apr. 1976.

(*e*) (*i*)
$$\begin{array}{r} 11\,001 \\ 10\,011+ \\ \hline \underline{101\,100} \end{array}$$

(*ii*)
$$\begin{array}{r} 11\,001 \\ 10\,011- \\ \hline \underline{110} \end{array}$$

(*iii*)
$$\begin{array}{r} 11\,001 \\ 10\,011\times \\ \hline 11\,001 \\ 110\,01 \\ 110\,01 \\ \hline \underline{111\,011\,011} \end{array}$$

LINE PLANT PRACTICE A, 1976

Students were expected to answer any 6 questions

Q 1 (*a*) *State the reasons for guarding road works, and the general principles to be observed.*

(*b*) *List 7 items of equipment suitable for use in guarding road works, and briefly describe 2 of them.*

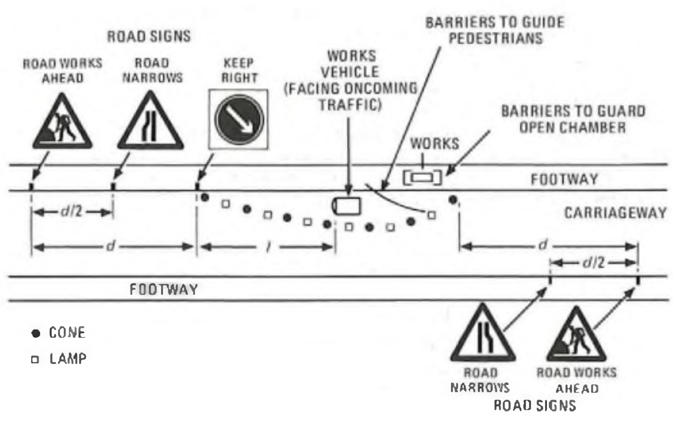
(*c*) *Sketch a layout of guarding equipment suitable for use when a narrow footway has to be completely blocked by work in a jointing chamber, thus causing pedestrians to use the carriageway.*

A 1 (*a*) Road works are guarded to protect

(*i*) workmen, when work is to be undertaken in the carriageway,
(*ii*) pedestrians, where the work blocks the footway, and
(*iii*) drivers, where the work (or the pedestrians' right of way) encroaches on the carriageway.

The general principles of guarding are to

(*i*) provide a temporary kerb line to guide traffic away from the hazard,
(*ii*) give early warning of the hazard,
(*iii*) protect the pedestrians' right of way, and
(*iv*) provide adequate illumination of the hazard at night.



Note: Distances *d* and *l*, and the number of cones, are dependent on the speed of the traffic and the width of the hazard (*b*) Items of equipment used in guarding road works are

(*i*) traffic lights or STOP-GO boards,
(*ii*) ROAD WORKS AHEAD signs,
(*iii*) other signs, such as KEEP RIGHT and ROAD NARROWS signs,
(*iv*) reflecting cones,
(*v*) lamps,
(*vi*) guards (for example, barriers) to protect and guide pedestrians, and
(*vii*) free-standing and vehicle-mounted beacons.

Some of these items are described briefly in A8, Line Plant Practice A, 1973, *Supplement*, Vol. 67, p. 35, July 1974.

(*c*) The sketch shows a layout of guarding equipment suitable for use when a footway is blocked by work in a jointing chamber.

Q 2 (*a*) *What types of explosive gas can be encountered in underground structures?*
(*b*) *What non-explosive-gas problem can be encountered in underground structures?*
(*c*) *Describe the principle of operation of an instrument suitable for detecting the presence of explosive gas.*
(*d*) *Describe 4 tests and 2 operations that must be made on such an explosive-gas-detecting instrument before it is used.*
(*e*) *What non-explosive-gas test must be made before an underground structure is entered? On what principle does it operate?*

A 2 (*a*) Explosive gases that can be encountered in underground structures are

(*i*) piped gas from the national distribution network, such as North Sea gas or manufactured town gas,
(*ii*) liquid-petroleum gas, such as propane or butane, normally used for stoves, blow torches and generator sets,
(*iii*) naturally occurring methane, and
(*iv*) petrol vapour.

(*b*) A non-explosive, but still dangerous, type of gas that can be encountered is an asphyxiating gas; that is, an atmosphere deficient in oxygen.
(*c*) An instrument suitable for detecting the presence of explosive gas is the battery-operated combustion-type detector. Using an aspirator bulb, a sample of air is drawn into the detector and passed over a filament that is electrically heated to approximately 600°C. If the sample is an explosive gas-air mixture, the gas burns, thereby further raising the temperature of the filament. The filament is arranged as one of the resistance arms of a Wheatstone bridge, and any change in resistance due to a rise in temperature can be measured. The change in resistance is indicated on a meter calibrated in terms of the concentration of the explosive gas.
(*d*) The following tests are made on an explosive-gas detector before it is used.

(*i*) The aspirator bulb is squeezed. If it does not restore within about 2 s, a blockage is indicated.
(*ii*) With a finger over the detector's inlet, the bulb is squeezed again. If it restores in less than 5 s, a leak is indicated.
(*iii*) The detector is switched on. Correct operation of the electrical circuit is indicated by the meter needle moving across its scale and returning.
(*iv*) The state of the batteries is checked by attempting to adjust the needle to give a concentration reading of 20%. If this is achieved, the batteries are satisfactory.

The following operations are carried out before an explosive-gas detector is used.

(*i*) The detector is purged by squeezing the bulb 10 times while the inlet is in fresh air. (This operation is carried out before tests (*iii*) and (*iv*) above are made.)
(*ii*) The needle is adjusted to zero.

(*e*) Before an underground structure is entered, a test is made for the presence of an asphyxiating gas. This test is made using a safety lamp, which operates on the principle that its flame will not be sustained by an atmosphere deficient in oxygen.

Q 3 (*a*) *Name the 3 main methods of strengthening a pole.*
(*b*) *State the reasons for strengthening poles.*
(*c*) *Which method is the most efficient?*
(*d*) *Describe 3 situations where pole strengthening may be needed.*
(*e*) *Describe the circumstances under which a gallows stay may be needed.*
(*f*) *Describe the procedure for erecting a gallows stay.*

A 3 (*a*) The 3 main methods of strengthening a pole are *staying*, *strutting* and *blocking*. The process of strengthening a pole is often referred to as *stabilization*.

(*b*) Poles are stabilized

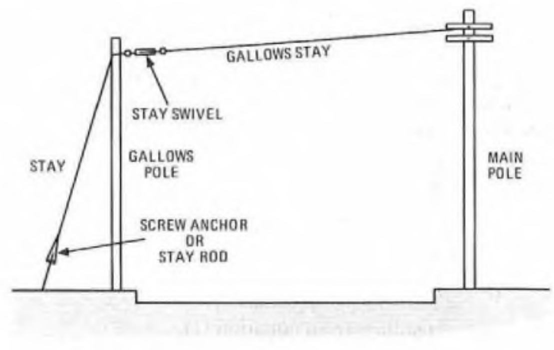
(*i*) to counteract the effect of high static head loads, such as those that occur at terminal poles and at points where the line of a route deviates,
(*ii*) to counteract high dynamic loads caused by high winds, and
(*iii*) in ground that does not itself provide good stability.

(*c*) Staying is the most efficient method of stabilization.

(*d*) Five situations where stabilization may be needed are at

(*i*) terminal poles,
(*ii*) angle poles,
(*iii*) road or rail crossings,
(*iv*) points where the longitudinal load changes, and
(*v*) exposed sites on high ground.

(*e*) A gallows stay may be needed where an ordinary stay cannot be fitted because of site restrictions. For example, when a pole is situated in a grass verge on a bend in a country lane, an ordinary stay is precluded because it would necessarily be anchored in the carriageway, but a gallows stay may be feasible.



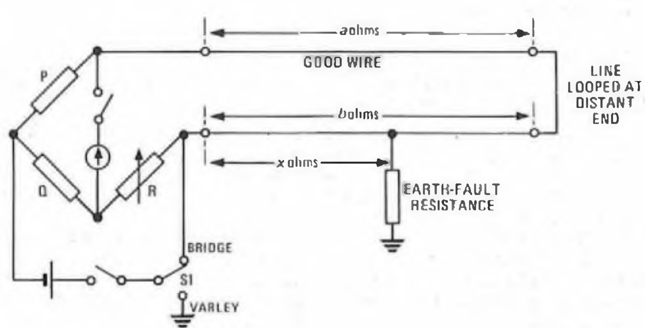
(*f*) The sketch illustrates the arrangement of a gallows stay. The procedure for its erection is described below.

(*i*) A gallows pole, similar to the pole being stayed, is erected.
(*ii*) The gallows pole is stayed conventionally, and the stay is tensioned so that the pole remains approximately vertical during subsequent operations.
(*iii*) A gallows stay is erected between the 2 poles, with a tensioning screw (stay swivel) at the gallows-pole end.
(*iv*) The gallows-stay tension is adjusted to give the correct stabilization, while the gallows-pole stay is adjusted to ensure that the gallows pole is vertical.

Q 4 (*a*) *With the aid of a diagram, explain how a Varley earth test is performed using a Wheatstone bridge.*
(*b*) *Derive the formula for the resistance to the earth fault.*
(*c*) *With the ratio arms set to 1 : 100 in each case, the figures in the decade resistance window read 6251 on* BRIDGE *and 3827 on* VARLEY. *Calculate the distance to the earth fault if the single-wire resistance is 15 Ω/km.*

A 4 (*a*) The sketch shows the arrangement of a Wheatstone bridge for performing a Varley earth test to find the distance to an earth fault. The procedure for the test is as follows.

(*i*) A good wire in the cable is selected and connected to the faulty wire at the distant end.



(*ii*) The near ends are connected to the bridge as shown.

(*iii*) With switch S1 in the BRIDGE position, the loop resistance, $a + b$ ohms, is given at balance by resistor R (with due allowance being made for the ratio in force between resistors Q and P).

(*iv*) With switch S1 in the VARLEY position, a new balance, R ohms, is obtained.

(*v*) Using the formula given in part (*b*) below, the resistance to the fault, x ohms, is obtained and, if the resistance per unit length of conductor is known, the distance to the fault can be calculated.

(*b*) With switch S1 in the VARLEY position, the earth-fault resistance does not form part of the bridge circuit and, therefore, does not affect the balance conditions, which are given by

$$\frac{P}{Q} = \frac{a + b - x}{R + x}.$$

$$\therefore\ PR + Px = Q(a + b) - Qx.$$

$$\therefore\ x(P + Q) = Q(a + b) - PR.$$

$$\therefore\ x = \frac{Q(a + b) - PR}{P + Q}\ \text{ohms.} \qquad \ldots\ldots (1)$$

For equal ratio arms, $P = Q$.

$$\therefore\ x = \frac{(a + b) - R}{2}\ \text{ohms.}$$

(*c*) For ratio arms set to 1 : 100, $P/Q = 1/100$ for both the BRIDGE and VARLEY readings.

For the BRIDGE reading,

$$\frac{P}{Q} = \frac{(a + b)}{R}.$$

$$\therefore\ \frac{1}{100} = \frac{(a + b)}{6251}.$$

$$\therefore\ (a + b) = 62{\cdot}51\ \Omega.$$

For the VARLEY reading, from equation (1),

$$x = \frac{100 \times 62{\cdot}51 - 1 \times 3827}{1 + 100} = 24\ \Omega.$$

Since the resistance per unit length of conductor is 15 Ω/km, the distance to the fault is

$$\frac{24}{15} = 1{\cdot}6\ \text{km}.$$

Q 5 (*a*) *What is the main purpose of a preliminary survey carried out prior to planning an underground duct scheme?*

(*b*) *Certain ground conditions can affect the installation, life or performance of a duct route. List 3 such conditions that must be looked for during the preliminary survey.*

(*c*) *List 6 factors that must be observed in the preparation of a detailed survey to determine the best positions of a duct track and its jointing chambers in the highway.*

(*d*) *What minimum clearance is required between underground telecommunications plant and high-voltage single-core power cables?*

A 5 (*a*) The main purpose of a preliminary survey is to determine tentatively the most suitable and economic route for an underground duct.

(*b*) Six ground conditions that must be looked for during a preliminary survey are

(*i*) conditions that can cause cable creepage (for example, heavy traffic on hills),
(*ii*) flooding,
(*iii*) subsidence,
(*iv*) running sand,
(*v*) rock, and
(*vi*) obstructions.

(*c*) Eleven factors that must be observed in the preparation of a detailed survey are given below.

(*i*) The Public Utilities Street Works Act, regarding the breaking-up of newly resurfaced roads, must be observed.

(*ii*) Where possible, plant should lie under grass verges.

(*iii*) Preference should be given to placing plant under a footway rather than under a carriageway.

(*iv*) If constructed under a footway, the duct track should be kept at least 450 mm from the kerb line; if constructed under a grass verge, it should be at least 600 mm from the edge of the verge.

(*v*) A trench should not be cut at the foot of an embankment, because of the danger of the bank collapsing.

(*vi*) A trench in the carriageway should not be closer than 600 mm to the kerb or 750 mm to the edge of a grass verge.

(*vii*) Transitions of the route between grass verges, footways and carriageways should be kept as short as possible.

(*viii*) Foundations of buildings must not be impaired.

(*ix*) Track conditions favourable to cable creepage should be avoided where possible.

(*x*) Entry shafts to manholes should be sited where they will not hinder the flow of traffic when work is in progress in the manholes.

(*xi*) Manholes and tracks should be sited so as to avoid difficulties in drawing-in cables, and permit cabling to be carried out in long lengths.

(*d*) The minimum clearance between underground telecommunications plant and high-voltage single-core power cables is 460 mm.

Q 6 (*a*) *How should a cable drum be rolled? Give reasons for using the method you describe.*

(*b*) *With the aid of a sketch, show the arrangement at the cable-drum end when a large cable is pulled into a duct at a manhole.*

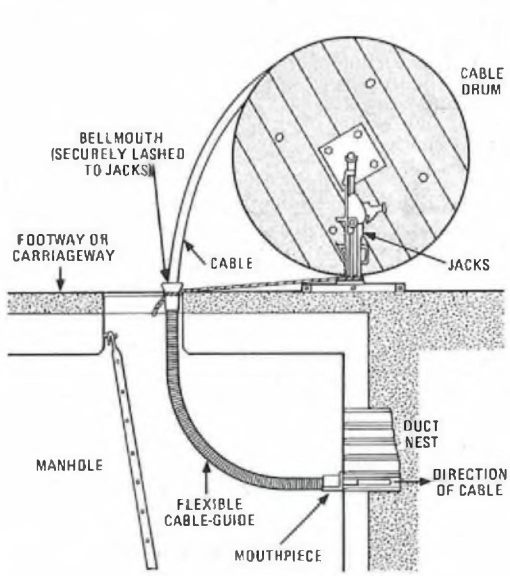
(*c*) *Explain, with the aid of a sketch, how a capstan is used for pulling-in a cable. What important safety precaution must be observed?*

(*d*) *It is sometimes necessary to cable from an intermediate cabling point, pulling-in the cable from opposite directions. Explain how this is done.*

(*e*) *What speed of pulling-in is generally considered to be satisfactory for cabling? What must be avoided when applying tension to a cable?*

A 6 (*a*) A cable drum should be rolled in such a direction as to tend to wind the cable more tightly onto the drum. Otherwise, a sudden movement of slack turns of cable could make the drum unmanageable and cause damage.

(*b*) Sketch (*a*) shows the arrangement of a cable drum for pulling-in a large cable.



(*a*)

(*c*) Sketch (*b*) illustrates how a capstan winch is used to pull in a cable. The winch operator holds the free end of the rope. By applying tension to the rope, the friction between the rope and the capstan is increased, causing the capstan to apply a pulling tension to the cable. To stop the cable, the tension on the free end is relieved, causing the rope to slip on the capstan.

To avoid the danger of clothing or hands being caught between the rope and the capstan, it is essential for the operator to stand well clear of the capstan.

(*d*) The operations necessary when cabling from an intermediate point are as follows.

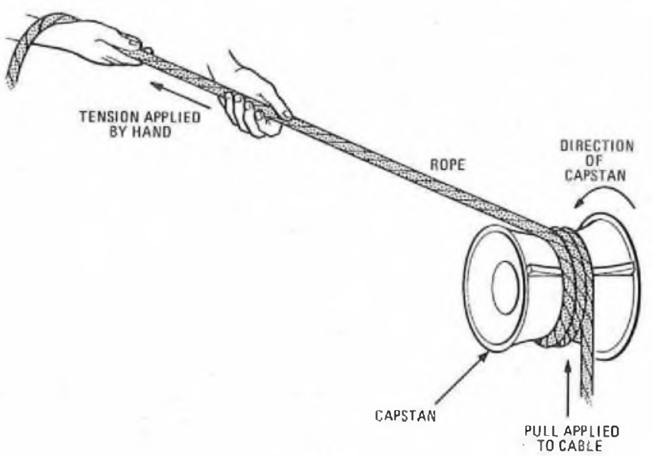
(*i*) The cable drum is set up at the intermediate chamber.

(*ii*) If the 2 lengths to be cabled are different, the longer is cabled first.

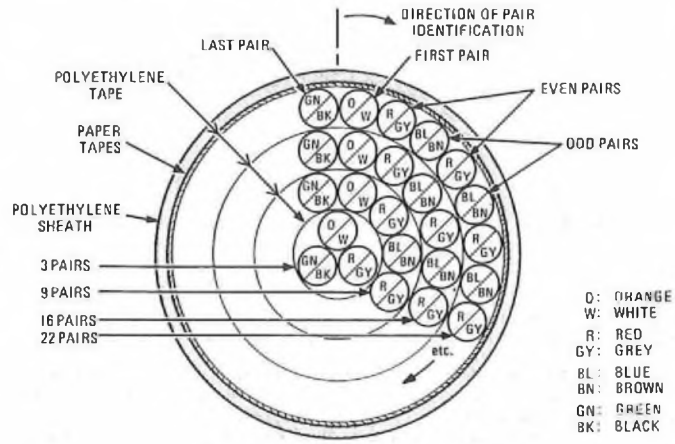
(*iii*) Without cutting the cable, the length of cable remaining on the drum is unwound and laid in figure-of-eight layers on a tarpaulin alongside the jointing chamber.

(*iv*) The cable is then drawn-in in the opposite direction, care being taken not to twist it.

(*e*) A satisfactory speed for cabling is about 500 mm/s. When applying tension to the cable, it is necessary to avoid surging due to



(*b*)



the momentum of the cable, and the consequent snatching that would occur as the winch took up the load again after a surge. The above cabling speed generally helps to avoid surging and snatching.

Q 7 (*a*) *In what 2 situations is overhead construction normally used?*

(*b*) *What is the difference between open wires and covered wires? Where are the latter used?*

(*c*) *What material is used for open wires? What 2 conductor sizes are used? State 2 situations where each size of conductor is used.*

(*d*) (*i*) *What is meant by factor of safety in relation to the wire tension?*

(*ii*) *What value of factor of safety is used for cadmium-copper wires?*

(*iii*) *What value is used for covered wires?*

(*e*) *What is meant by transposition? Why is it necessary?*

A 7 (*a*) Overhead construction is normally used for subscribers' distribution and for lightly-loaded rural routes.

(*b*) Open wires are bare (uninsulated) conductors. They are used in pairs and separated from each other by the physical arrangement at the point of suspension.

Covered wires are insulated for added protection. They are used as dropwires for subscribers' terminations, at power crossings to protect against electrical contact, for crossing electrified railways, on routes used jointly with power authorities, for protection against corrosive atmospheres, and to reduce fault liability due to contact with trees.

(*c*) Cadmium-copper alloy is used for open wires. The diameters of conductor in general use are 1·7 mm and 2·5 mm. Conductors of 1·7 mm diameter are used for

(*i*) subscribers' lines in exposed conditions,

(*ii*) coastguards' circuits in sheltered positions, and

(*iii*) junction circuits.

Conductors of 2·5 mm diameter are available for use in exceptional circumstances, such as

(*i*) long spans,

(*ii*) important circuits in exposed situations, and

(*iii*) long junction routes in remote areas, consisting of only a few circuits.

(*d*) The factor of safety of a conductor is the ratio of its minimum breaking tension to the maximum permitted working tension at −7°C. For open cadmium-copper wire, the factor of safety is 3 : 1. For covered wires, the value specified is 5 : 1.

(*e*) Transposition is the changing of the relative positions of conductors in a long route. The changes are obtained by cross-connecting the conductors at selected points. The purpose of transposition is to minimize inductive interference in a circuit from adjacent and parallel telephone, telegraph, or power circuits. The selective cross-connexions cause the 2 conductors of a circuit to be subjected equally to the interfering signals and, therefore, the induced voltages cancel out around the circuit loop.

Q 8 (*a*) *Describe the make-up of a 50-pair aluminium-conductor cable used for telephone distribution purposes.*

(*b*) *What advantages and disadvantages has this type of cable over an equivalent cable using copper conductors?*

A 8 (*a*) The cross-section and pair-identification colour scheme of a 50-pair aluminium-conductor cable are shown in the sketch.

The conductors are 0·5 mm in diameter, and are insulated with extruded cellular polyethylene and uniformly twisted to form pairs. The pairs are stranded in layers around a 3-pair centre, with successive layers being stranded in opposite directions. An open helical wrapping of polyethylene terephthalate tape is applied to each layer, and the completed core is wrapped in 2 overlapping paper tapes. The core is sheathed by a 1·2 mm thick polyethylene extrusion containing butyl rubber and carbon black; the butyl rubber improves the stress-cracking resistance, and the carbon black reduces deterioration of the polyethylene when exposed to ultra-violet light in sunlight. The cable is filled with petroleum jelly for protection against the ingress of moisture.

(*b*) The advantages of aluminium-conductor cable are that

(*i*) it is cheaper than the equivalent copper-conductor cable,

(*ii*) it is lighter than the equivalent copper-conductor cable, and therefore less cabling tension is required for a given length, and

(*iii*) the price of aluminium is more stable than that of copper; this is helpful when estimates of costs are being made.

The disadvantages are that

(*i*) because aluminium has reduced electrical and mechanical properties compared with copper, larger conductors must be used, and hence cables are larger,

(*ii*) aluminium cables have a lower breaking strength and are less ductile than copper cables, and

(*iii*) protection against corrosion is necessary.

Q 9 (*a*) *State 3 circumstances in which a thrust-boring machine may be used to advantage.*

(*b*) *Describe the survey for a thrust-boring operation.*

(*c*) *Explain how the thrust-boring machine is set up and used for the provision of a duct.*

A 9 See A6, Line Practice A, 1975, *Supplement*, Vol. 69, p. 15, Apr. 1976.

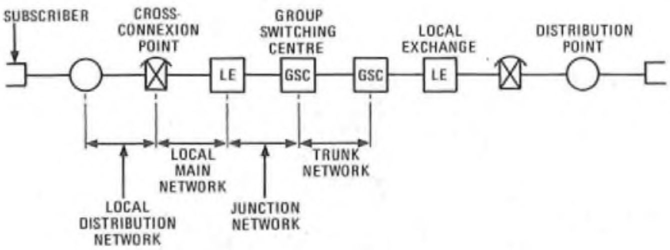
When surveying for a thrust-boring operation, besides ascertaining whether the soil is suitable, it is necessary to locate and plot other services, such as gas, water and electricity services. The route of the bore is then planned to give the necessary minimum clearance from these services, and give the required minimum depth beneath any road surfaces.

Q 10 (*a*) *Describe, with a simple block diagram, how 2 subscribers in different parts of the country may be connected together through a typical telephone network. Name the items in the diagram.*

(*b*) *Briefly describe the types of cable that could be used in the various parts of the network.*

A 10 (*a*) The sketch shows how 2 subscribers in different parts of the country may be interconnected through the telephone network.

Each subscriber is connected to a local telephone exchange, and between the local exchange and the subscriber are flexibility points. These are *cross-connexion points*, which are normally cabinets, and *distribution points*, which are often constructed in overhead practice. The local exchange is connected to the group switching centre for the area, and group switching centres are interconnected by the trunk network, which includes the transit network. The actual routing and the number of exchanges used in the trunk network usually depend on the destination of the call.



Note: The trunk network is to become known as the *main network*, and group switching centres are to become known as *main-network switching centres*. These terms are already in use to some extent.

(*b*) Between an overhead distribution point and a subscriber's house, dropwire is normally used. This consists of a pair of copper-clad steel conductors in a single moulding of PVC insulation.

The local distribution network consists of petroleum-jelly-filled polyethylene-insulated-and-sheathed copper or aluminium pair-type cables. Such cables may be directly buried, in which case they are armoured.

In the local main network, pressurized polyethylene-sheathed unit-twin cables are used. The conductors can be either copper or aluminium alloy, insulated by paper or polyethylene.

For junction circuits between the local exchange and group switching centre, pressurized polyethylene-sheathed cables are used. The conductors are of copper with paper insulation, and are made up in quad formation.

In the trunk network, pressurized polyethylene-protected lead-sheathed coaxial cables are normally used.

CORRECTION

LINE PLANT PRACTICE A, 1975 (*Supplement*, Vol. 69, Apr. 1976)

A 2 Crimped joints are preferred for aluminium conductors because they overcome the insulating effect of an oxide film that forms on aluminium. Also, crimped joints are generally more reliable than dry twist joints. Aluminium is not too brittle to be twist jointed, as stated.

COMPUTERS A, 1976

Students were expected to answer any 6 questions

Q 1 Write down a brief history of mechanical calculating machines leading to the development of modern digital computers.

A 1 A digital computer is a device that performs arithmetic, and a simple example is the abacus, widely used in Asian countries since earliest times. Strictly, the abacus is not in itself a computer, but a means of recording intermediate results; it is the user who does the computing.

It is generally recognized that true mechanical digital computing began with Pascal's calculator of 1642. Pascal, a Frenchman, made calculators to reduce the tedium of his work in a tax office. His machines could perform addition and subtraction, and were capable of working to the non-denary bases demanded by the units of money used. Numbers were entered by means of dials and stored on number wheels, to which the dials were geared. The positions of the number wheels could be inspected through windows. Carry digits were conveyed to higher-order number wheels by ratchets.

In 1672, Leibniz, a German, improved on Pascal's calculators by building a device capable of multiplication and division.

The fundamental principles of digital computing, as they stand today, were first propounded in 1833 by Babbage, an English mathematician. He proposed a machine having a memory, a control unit, an arithmetic unit, and input and output devices—all the major elements, in fact, of a modern computer. Sequences of punched cards were to be the method of control, but many of Babbage's ideas were beyond the technology of the day. Babbage also conceived that important feature of modern computing known as *conditional branching*, whereby the subsequent course of a calculation can be modified by an intermediate result. In 1854, Scheutz, a Swede, exhibited a machine based on Babbage's design and capable of printing its output.

In 1889, Hollerith, an American, patented a tabulating machine that used a punched-card input system to produce population statistics.

Another American, Shannon, produced in 1938 his important work on the analysis of switching circuits by Boolean algebra.

The close of the mechanical computing era came in 1939 with Bell Telephone Laboratory's relay-based computer. In the same year, work on another electromechanical computer was started by IBM, using the advanced ideas put forward by Babbage.

Electronic computers, using tubes, began to appear in 1943, pioneered by the University of Pennsylvania in the USA, and by the British Post Office. Computers based on transistors became available in the late-1950s.

Q 2 (a) What is meant by the term radix, or base?
(b) Show how the denary number 174 can be represented by numbers having the radices 2, 4 and 8.
(c) Why are the radices 2 and 8 used extensively in digital computers?

A 2 (*a*) The radix, or base, is the basis of a number system. Each digit of a number represents a multiple of a power of the radix. For example, each digit in a denary (decimal) number represents a multiple of a power of 10, and thus the denary system has the radix 10. The powers of the radix are determined by the position of the digits within the number. The expression of numbers by this positional representation is called *radix notation*.

Any number, N, can be represented by radix r and digits D_n, D_{n-1}, D_{n-2} etc., thus:

$$N = D_n r^n + D_{n-1} r^{n-1} \ldots + D_0 r^0 + D_{-1} r^{-1} \ldots + D_{-m} r^{-m},$$

where terms up to and including $D_0 r^0$ represent the integral part, terms after $D_0 r^0$ represent the fractional part, and D can take any integral value from 0 to $r - 1$.

For example, for the denary number 486·27,

$$486{\cdot}27_{10} = 4 \times 10^2 + 8 \times 10^1 + 6 \times 10^0 + 2 \times 10^{-1} + 7 \times 10^{-2}.$$

(*b*) The number 174_{10} can be converted into radix 2 notation by repeatedly dividing by 2 and writing down the remainder in reverse order, as shown in the following table.

Quotient	Remainder
2) 174	
87	0
43	1
21	1
10	1
5	0
2	1
1	0
0	1

$$\therefore\ 174_{10} = 10\,101\,110_2.$$

Check: $10\,101\,110_2 = 1 \times 2^7 + 0 \times 2^6 + 1 \times 2^5 + 0 \times 2^4 + 1 \times 2^3 + 1 \times 2^2 + 1 \times 2^1 + 0 \times 2^0,$

$$= 128 + 32 + 8 + 4 + 2 = 174_{10}.$$

Similarly, 174_{10} can be converted into radix 4 and radix 8 notation by repeated division by 4 and 8 respectively, as shown in the following tables.

Quotient	Remainder
4) 174	
43	2
10	3
2	2
0	2

Quotient	Remainder
8) 174	
21	6
2	5
0	2

$$\therefore\ 174_{10} = 2232_4, \text{ and } 174_{10} = 256_8.$$

Check: $2232_4 = 2 \times 4^3 + 2 \times 4^2 + 3 \times 4^1 + 2 \times 4^0,$

$$= 128 + 32 + 12 + 2 = 174_{10},$$

and $256_8 = 2 \times 8^2 + 5 \times 8^1 + 6 \times 8^0,$

$= 128 + 40 + 6 = \underline{174_{10}}.$

(*c*) Most digital computers are constructed from 2-state devices that are either open or closed, such as electronic switches. These 2 states are represented by 0 and 1, which are the only permissible digits in a radix 2 number system. Thus, the radix 2 number system is extensively used in digital computers.

The radix 8 number system, known as the *octal* system, is used in digital computers because it allows large binary numbers to be conveniently expressed in a shorter form; conversion between the radix 2 and radix 8 systems can readily be performed. Basically, the octal system is a shorthand method for replacing groups of three radix 2 digits by a single radix 8 digit, which can have a value between 0–7. Thus, because $2^3 = 8$, each 3 bit group of a binary number is replaced by its equivalent denary value to give the octal form of the number. Two examples are shown below. To illustrate the convenience of the method, the figures in the first example can be compared with the values obtained from the calculations in part (*b*).

Binary Number	010 101 110	111 010 011
Denary Value of 3 bit Groups	2 5 6	7 2 3
Octal Number	256	723

Q 3 Calculate the following binary arithmetic functions, showing all working:

(*a*) *101 101 + 10 111,*
(*b*) *1 110 111 − 1 001,*
(*c*) *110 101 × 1 101, and*
(*d*) *100 011 ÷ 111.*

A 3 (*a*) The given binary numbers can be stored in a computer as 00 101 101 and 00 010 111. This assumes a word length of 8 bit, where the most significant bit is used to indicate the sign of the number, and is called the *sign bit*. If the sign bit is 0, the number is positive, and if the sign bit is 1, the number is negative.

```
 00 101 101
 00 010 111 +
 ----------
 01 000 100
```

Thus, the binary sum is 1 000 100.

(*b*) As described above, the given binary numbers can be stored as 01 110 111 and 00 001 001. Let the former number be represented by the symbol A, and the latter by B.

The subtraction of B from A can be achieved by creating the negative value of B and adding it to A. A negative number can be stored in a form known as the *2's complement*. Each 1 is replaced by a 0, and each 0 is replaced by a 1, to create the *one's complement*; the 2's complement is created by adding 1 to the one's complement, as shown below.

B:	00 001 001
One's complement of B:	11 110 110
Add 1:	1 +
$-B$ (i.e., the 2's complement of B):	11 110 111

Note that the process of creating the 2's complement automatically gives the correct sign bit.

To obtain the required result, $-B$ is added to A, as shown below.

```
 01 110 111
 11 110 111 +
 ----------
101 101 110
```

The most significant bit is lost in the computer since it is outside the word length used. Thus, the binary difference is 01 101 110. (The sign bit indicates that the result is positive.)

Note: Executing a subtraction in this way is analogous to subtracting 4 from 7 in the denary system by creating the number (10 − 4), which is analogous to the 2's complement, and adding it to 7. This gives 7 + (10 − 4) = 13. Ignoring the left-most digit as being outside the scope of the operation gives the correct answer of 3, and is equivalent to subtracting the extraneous value of 10 introduced by the process of creating the number (10 − 4).

(*c*) Binary multiplication can be carried out using a successive shifting-and-adding technique, as shown below.

Multiplicand:	110 101
Multiplier:	1 101 ×
Copy multiplicand:	110 101
Copy multiplicand, shifted 2 places:	11 010 100 +
Copy multiplicand, shifted 3 places:	110 101 000 +
Sum:	1 010 110 001

The binary product is therefore 1 010 110 001.

The shifting operation for each 1 in the multiplier is equivalent to multiplying by the power of 2 represented by the position of that digit.

(*d*) Binary division can be carried out by a successive shifting-and-subtracting technique, as shown below. This technique is, effectively, a long-division technique.

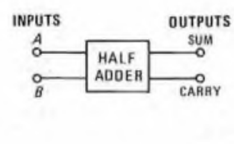
Quotient:	101
Divisor and dividend:	111) 100 011
Shift divisor:	11 1
Subtract:	111
Shift divisor 2 places:	111
Subtract:	000

Thus, the binary quotient is 101.

Q 4 (a) Write down the truth table for a binary half adder, and explain why it is so called.
(b) With the aid of a truth table, explain the operation of a full adder.
(c) Draw a block diagram of 2 binary half adders, showing how they are connected together to form a binary full adder.

A 4 (*a*) The truth table for a half adder is shown below; the block diagram of a half adder is shown in sketch (*a*).

Inputs		Outputs	
A	B	SUM	CARRY
0	0	0	0
0	1	1	0
1	0	1	0
1	1	0	1



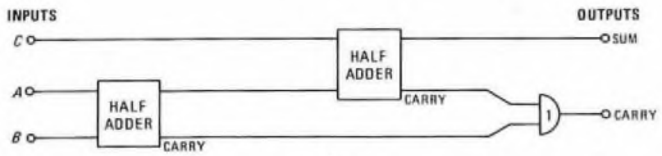
(*a*)

This type of circuit is called a half adder because it performs part of the function of binary addition. It can obtain the sum of 2 inputs, A and B, and produce a carry, but it cannot take account of the carry from other such circuits. Full binary addition can be accomplished with 2 half adders.

(*b*) The full adder is a device that adds together 2 binary numbers, one digit at a time, together with the carry digit, C, from the previous stage of addition. It provides 2 outputs: the SUM and CARRY. The truth table for a full adder is shown below.

Inputs			Outputs	
A	B	C	SUM	CARRY
0	0	0	0	0
0	0	1	1	0
0	1	0	1	0
0	1	1	0	1
1	0	0	1	0
1	0	1	0	1
1	1	0	0	1
1	1	1	1	1

The SUM output is the output for one stage of the addition, and the CARRY output is the carry to be added to the next stage of addition.



(*b*)

(*c*) A full adder, formed from 2 half adders, is shown in the block diagram in sketch (*b*).

Q 5 Explain what is meant by the following terms, used in connexion with digital-computer storage, and give a list of the major features in each case:
(*a*) *working store,*
(*b*) *random access,*
(*c*) *backing store, and*
(*d*) *volatile store.*

A 5 (*a*) *Working Store* A working store is an area of storage used to hold temporary values of variables or partial results during the calculation of complex arithmetic or logic functions. It is sometimes known as a *work-area* or an *intermediate store.*

As an example, if it is required to compute the function $(A + B) \times (C + D)$, it is first necessary to calculate $A + B$ and temporarily store the result in a working store until $C + D$ is calculated. The 2 results are then multiplied together to obtain the required result.

The main features of a working store are that

(*i*) it is a fast read/write type of storage, and typically consists of special electronic registers within the computer's central processing unit,

(*ii*) the number of bits that each location can hold is greater than the main-storage word length to allow the storage of partial results to a higher precision or within a wider range, and

(*iii*) it is an expensive form of storage.

(*b*) *Random Access* Any form of store that has the same access time to all locations is known as *random-access storage*. A magnetic-core store is the most common form of random-access storage.

The main features of random-access storage are that it

(*i*) is used as main storage in a computer to hold the programs that are currently being executed, and

(*ii*) usually has a short access time.

(*c*) *Backing Store* Backing storage is used to hold information that is not required to be permanently resident in the main memory, until this information is required by the program being executed. Such storage is also known as *bulk storage*, *auxiliary storage* or *secondary storage*. Examples of backing stores are magnetic discs and drums, and slow-access magnetic-core stores.

The main features of backing storage are that

(*i*) it is cheaper than main storage,

(*ii*) the access time is longer than for main storage, and

(*iii*) it is capable of storing large quantities of information in an acceptable volume.

(*d*) *Volatile Store* Volatile storage is dependent on a constantly available power supply for the retention of its contents. Registers using electronic bistable circuits and circulating delay lines are examples of volatile stores.

The main features of volatile stores are that

(*i*) they use storage devices that do not possess naturally occurring stable states,

(*ii*) information is lost when the power supply is removed, and must be rewritten into the store when the power is reconnected,

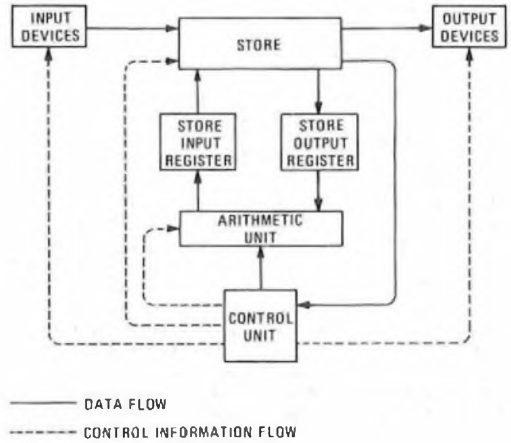
(*iii*) they are more susceptible to transients in the power supply than are non-volatile stores, and

(*iv*) they have a higher power consumption than non-volatile stores.

Q 6 (*a*) *Draw a block diagram of a digital computer, showing clearly the interconnexions and the directions of control information and data flow. Explain the function of each block.*

(*b*) *With reference to the block diagram in part* (*a*), *explain the sequence of operations for a digital computer performing the addition and multiplication functions.*

A 6 (*a*) The main functional units of a digital computer are shown in the sketch, and are described below.



Store The store contains the program instructions and necessary data. An instruction is held within a store location, and each instruction is obeyed in sequence under the control of the control unit.

Arithmetic Unit The arithmetic unit performs arithmetical and logic operations on information transferred from the store. The results of these operations can be placed in the store, if required. The arithmetic unit contains an *accumulator*, the purpose of which is to store partial results.

Control Unit The control unit sequentially decodes the instructions and provides control signals to the other units.

Input and Output Devices Input devices are used to enter information into the store from the outside world. Output devices are used to transfer processed information from the store to the outside world.

(*b*) The basic sequence of operations for most instructions in a digital machine of the single-address type is contained within a period called the *instruction cycle*, followed by a period called the *execution cycle*. At the beginning of the instruction cycle, the contents of a program register within the control unit are transferred into the store's input register. The memory then uses the address in the input register to obtain the next instruction, which is subsequently placed in the store's output register. The operation-code part of the instruction is transferred to an operation-code register within the control unit, and the address portion is transferred to the store's input register.

Following the above sequence of operations, for an ADD instruction, the contents of the store's output register are added to the accumulator within the arithmetic unit. Then, the program register is incremented, and the next instruction is selected from storage and transferred to the control unit.

Multiplication is carried out using the shift-and-add method. Whenever the digit 1 appears in the multiplier, the multiplicand is shifted by the number of places indicated by the position of the multiplier digit and added to the sum of the partial products. The sequence of operations is the same as for an ADD instruction, except that the execution of the operation code takes a number of cycles. The next instruction is then selected. Each control pulse from the control unit instructs the accumulator and multiplier registers to shift one place to the right. The process is repeated until all the multiplier digits have been examined.

Q 7 Give a Veitch or Karnaugh map, a binary expression and a simple logic element for each of the Venn diagrams shown in Fig. 1.

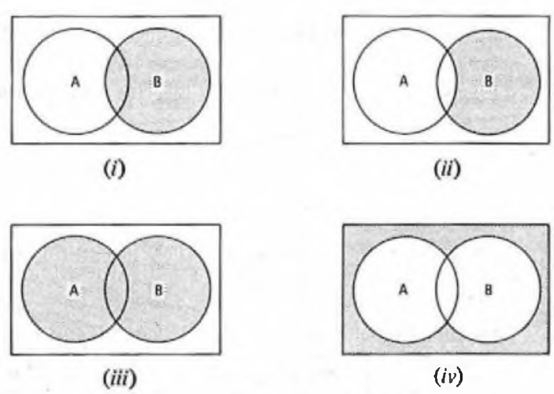
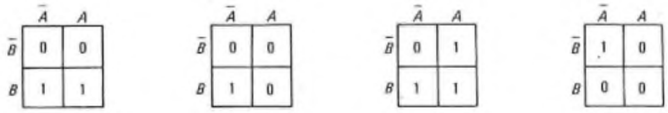


Fig. 1

A 7 Karnaugh maps for each of the logic functions represented by the Venn diagrams are shown in sketch (*a*).



(*i*) (*ii*) (*iii*) (*iv*)

(*a*)

Binary expressions for each of the logic functions, F, are given below.

(*i*) $F = B$, (*ii*) $F = B \cdot \bar{A}$,

(*iii*) $F = A + B$, (*iv*) $F = \overline{A + B}$.

Simple logic elements for each of the logic functions are shown in sketch (*b*).

(*i*) (*ii*) (*iii*) (*iv*)

(*b*)

Q 8 With the aid of sketches, compare the operation of the following types of backing store, with particular reference to average access times, storage capacity and application:

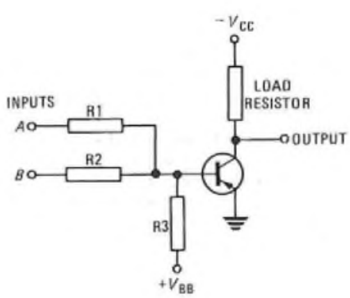
(a) magnetic tape,
(b) magnetic disc, and
(c) magnetic drum.

A 8 See A6, Computers A, 1970, *Supplement*, Vol. 63, p. 92, Jan. 1971.

Q 9 (a) Draw the circuit diagram of a positive-logic resistor-transistor NAND *logic element and, with the aid of voltage and truth tables, explain its operation.*

(b) What advantages have diode-transistor logic elements over resistor-transistor types?

A 9 (*a*) The circuit diagram of a positive-logic resistor-transistor NAND logic element is shown in the sketch.



The input resistors, R1 and R2, have the same value. The value of the base-bias resistor, R3, is such that, when both inputs A and B are at 0 V (or a very low negative voltage), the base of the transistor is at a positive potential sufficient to hold the transistor in the OFF state. The output voltage is then approximately equal to the supply voltage, $-V_{CC}$ volts. If either (or both) of inputs A and B is at a large negative voltage (say $-V_{CC}$ volts), the base of the transistor is at a negative potential, and the transistor is in the ON state. The output voltage is then approximately zero. These conditions are shown in the following voltage table.

Inputs (V)		Output (V)
A	B	
$-V_{CC}$	$-V_{CC}$	0
$-V_{CC}$	0	0
0	$-V_{CC}$	0
0	0	$-V_{CC}$

In positive logic, the logic value 1 is assigned to the more positive voltage (in this case, 0 V), and logic value 0 to the more negative voltage ($-V_{CC}$ volts). The truth table for the circuit is therefore as shown below.

Inputs		Output
A	B	
0	0	1
0	1	1
1	0	1
1	1	0

This is the truth table for a NAND function.

(*b*) Compared with resistor-transistor logic, diode-transistor logic

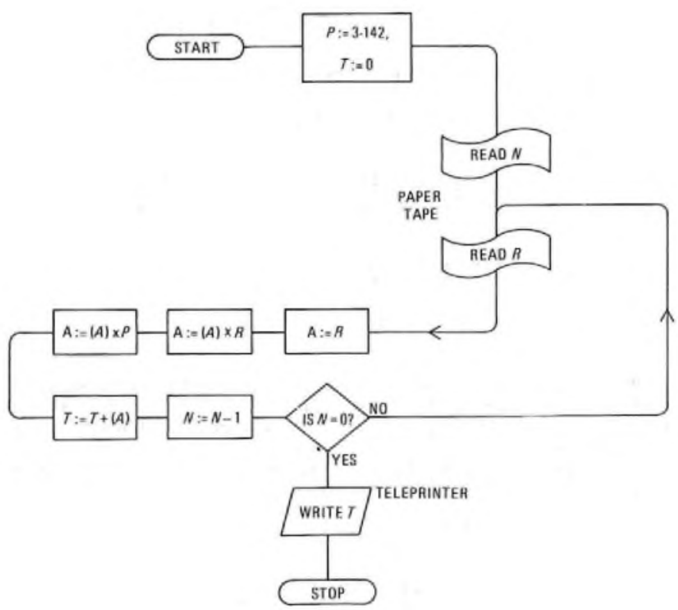
(*i*) gives faster switching speeds,
(*ii*) has better noise immunity,
(*iii*) gives less current drain on preceding stages, and therefore has a greater fan-out capacity, and
(*iv*) has a simpler manufacturing process in integrated-circuit form.

Q 10 Draw a detailed flow chart for a program that will accept from paper tape a set of values representing radii of circles, and process them to

(a) calculate the area of each circle (the square of the radius multiplied by π),
(b) maintain a running total of the areas calculated to date, and
(c) print the final total on a teleprinter when all values have been read.

The first number on the tape is the number of values in the set.

A 10 A flow chart for a program that performs the required functions is shown in the sketch. The table defines the symbols used.



Symbol	Interpretation
A	Accumulator
(A)	Content of accumulator
N	Number of values in set
P	π
R	Radius of a circle
T	Total of areas calculated to date
:=	. . . . takes the value of . . . .

MATHEMATICS B, 1976

Students were expected to answer any 6 questions

Q 1 (*a*) *Solve for q the equations:*

$$15p - 5q = -4,$$
$$25p + 10(q + r) = -2,$$
$$5q - 20r = 8.$$

(*b*) (*i*) *By rearranging the function $y = 3x^2 - 14x + 20$ in the form $y = a(x + b)^2 + c$, show that y cannot be less than $3\frac{2}{3}$.*
(*ii*) *Calculate the 2 roots of $3x^2 - 14x + 20 = 0$, to 3 significant figures.*

A 1 (*a*)

$$15p - 5q = -4, \qquad \ldots\ldots (1)$$
$$25p + 10(q + r) = -2, \qquad \ldots\ldots (2)$$
$$5q - 20r = 8. \qquad \ldots\ldots (3)$$

From equation (1),

$$15p = 5q - 4.$$

Multiplying by 5/3 gives

$$25p = \frac{5}{3}(5q - 4). \qquad \ldots\ldots (4)$$

From equation (3),

$$20r = 5q - 8.$$

Dividing by 2 gives

$$10r = \frac{5q}{2} - 4. \qquad \ldots\ldots (5)$$

Substituting for $25p$ and $10r$ (from equations (4) and (5) respectively) in equation (2) gives

$$\frac{5}{3}(5q - 4) + 10q + \frac{5q}{2} - 4 = -2.$$

Multiplying throughout by 6 gives

$$50q - 40 + 60q + 15q - 24 = -12.$$
$$\therefore\ 125q = 64 - 12 = 52.$$
$$\therefore\ q = \frac{52}{125} = \underline{0{\cdot}416.}$$

(*b*) (*i*)

$$y = 3x^2 - 14x + 20,$$
$$= 3\left(x^2 - \frac{14x}{3} + \frac{20}{3}\right),$$
$$= 3\left\{\left(x^2 - \frac{14x}{3} + \left(\frac{7}{3}\right)^2 + \frac{20}{3} - \left(\frac{7}{3}\right)^2\right\},\right.$$
$$= 3\left\{\left(x - \frac{7}{3}\right)^2 + \frac{60 - 49}{9}\right\},$$
$$= \underline{3\left(x - \frac{7}{3}\right)^2 + \frac{11}{3}}.$$

In the above expression, the term, $3\{x - (7/3)\}^2$ must always be positive. Hence, the smallest value of y occurs when $x = 7/3$; that is, when $3\{x - (7/3)\}^2$ is zero. The value of y is then $11/3$. Thus, y cannot be less than $3\frac{2}{3}$.

(*ii*) From part (*b*) (*i*), the equation $3x^2 - 14x + 20 = 0$ can be expressed as

$$3\left(x - \frac{7}{3}\right)^2 + \frac{11}{3} = 0.$$
$$\therefore\ 3\left(x - \frac{7}{3}\right)^2 = -\frac{11}{3}.$$
$$\therefore\ \left(x - \frac{7}{3}\right)^2 = -\frac{11}{9}.$$
$$\therefore\ x - \frac{7}{3} = \pm\frac{\sqrt{(-11)}}{3}.$$
$$\therefore\ x = \frac{7}{3} \pm \frac{j3{\cdot}317}{3},$$

where $j = \sqrt{(-1)}$.

$$\therefore\ x = 2{\cdot}\dot{3} \pm j1{\cdot}105\dot{6}\,.$$

The 2 roots are therefore imaginary and, to 3 significant figures, are expressed as $\underline{2{\cdot}33 \pm j1{\cdot}11.}$

Q 2 *The charge, q coulombs, on a capacitor of C farads, t seconds after discharge commences through a resistor of R ohms, is given by $q = VCe^{-t/CR}$.*
(*a*) *Express t in terms of q, V, C, and R.*
(*b*) *If $C = 4 \times 10^{-9}$ F and $R = 5 \times 10^6\,\Omega$, calculate, to the nearest millisecond, the time taken for the charge to fall to 10% of its initial value.*
(*c*) *Sketch the graph of q/t for the values of C and R given in part (b), if V = 80.*

A 2 (*a*)

$$q = VCe^{-t/CR}.$$
$$\therefore\ e^{-t/CR} = \frac{q}{VC}.$$

Taking logarithms to base e gives

$$-\frac{t}{CR} = \log_e \frac{q}{VC}.$$
$$\therefore\ t = \underline{-CR \log_e \frac{q}{CV} \text{ seconds.}}$$

(*b*) When $t = 0$ s, the initial value of charge, q_0, is given by

$$q_0 = VCe^0 = VC \text{ coulombs.}$$

Hence, from part (*a*),

$$t = -CR \log_e \frac{q}{q_0} \text{ seconds.}$$

When q is 10% of its initial value, $q/q_0 = 0{\cdot}1$.

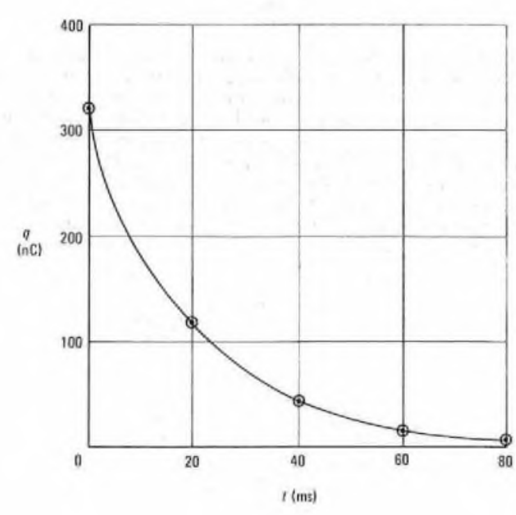
$$\therefore\ t = -4 \times 10^{-9} \times 5 \times 10^6 \times \log_e 0{\cdot}1 \text{ s},$$
$$= -20 \times 10^{-3} \times (-2{\cdot}3026) \text{ s},$$
$$= \underline{46 \text{ ms}} \text{ (to the nearest millisecond).}$$

(*c*) When $V = 80$,

$$q = 80 \times 4 \times 10^{-9} \times e^{-t/0{\cdot}02} \text{ coulombs},$$
$$= 320 \times 10^{-9} \times e^{-50t} \text{ coulombs.}$$

Since, from part (*b*), the charge falls to 10% of its initial value after 46 ms, it is reasonable to sketch the graph of q/t over a period of, say, 80 ms. Suitable values are derived in the table, and the graph is shown in the sketch.

t (ms)	0	20	40	60	80
$-50t$ (s)	0	-1	-2	-3	-4
e^{-50t}	1·0000	0·3679	0·1353	0·0498	0·0183
$q = 320e^{-50t}$ (nC)	320	118	43·3	15·9	5·9



Q 3 The flow of water, q litres/hour, over a weir is measured for different values of the depth of water, x centimetres, coming over the weir, with the following results.

x (cm)	6	9	15	24
q (l/h)	$5 \cdot 9 \times 10^3$	$12 \cdot 0 \times 10^3$	$23 \cdot 5 \times 10^3$	$48 \cdot 0 \times 10^3$

(a) Show, by the use of a straight-line graph, that the formula $q = kx^n$ reasonably describes the variation in flow as a function of depth.
(b) Derive from the graph estimates of the constants k and n.
(c) Estimate the depth of water when 18×10^3 l/h are flowing over the weir. Give your answer to the nearest centimetre.

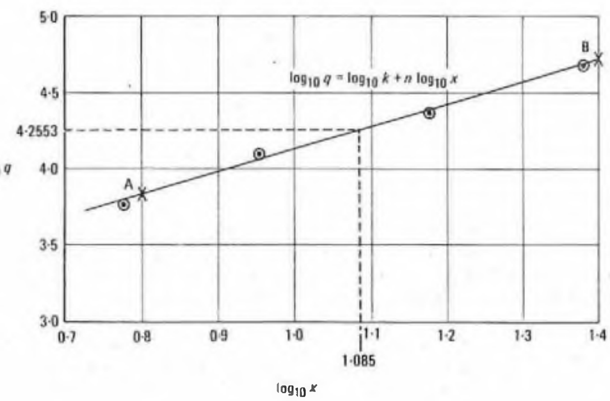
A 3 (*a*) Taking logarithms for the given formula gives

$$\log_{10} q = \log_{10} k + n \log_{10} x. \qquad \ldots\ldots (1)$$

Since k and n are constants, the above logarithmic form is a linear law, with n being the slope.

The graph of $(\log_{10} q)/(\log_{10} x)$ is shown in the sketch, plotted from the values derived in the table for each measurement given in the question.

$\log_{10} x$	0·778	0·954	1·176	1·380
$\log_{10} q$	3·771	4·079	4·371	4·681



It can be seen from the graph that a straight line can be drawn passing reasonably closely to the plotted points. Hence, the assumed formula for q is reasonably true for the given data.

(*b*) The slope of the graph is derived from the co-ordinates of 2 widely separated points, A and B, which lie on the graph. Thus,

$$n = \frac{4 \cdot 72 - 3 \cdot 82}{1 \cdot 4 - 0 \cdot 8} = \underline{1 \cdot 5.}$$

Substitution of the co-ordinates of any point on the graph (say point A) and the derived value of n into equation (1) gives

$$3 \cdot 82 = \log_{10} k + 1 \cdot 5 \times 0 \cdot 8.$$

$$\therefore \log_{10} k = 3 \cdot 82 - 1 \cdot 2 = 2 \cdot 62.$$

$$\therefore k = 416 \cdot 9 \approx \underline{417.}$$

(*c*) When $q = 18 \times 10^3$ l/h, $\log_{10} q = 4 \cdot 2553$. From the graph, at this value, $\log_{10} x = 1 \cdot 085$, from which $x = 12 \cdot 16$ cm. Thus, to the nearest centimetre, $\underline{x = 12 \text{ cm}}$.

Q 4 (a) Evaluate $\log_2 e$.
(b) If $\log_2 N = -1 \cdot 2$, calculate N.
(c) Solve the equation $\log_e (10 + 9x) - \log_e (11 - x) = 2$.
Give all your answers correct to 3 significant figures.

A 4 (*a*) Using tables of natural logarithms and reciprocals,

$$\log_2 e = \frac{1}{\log_e 2} = \frac{1}{0 \cdot 6931},$$

$$= \underline{1 \cdot 44,} \text{ to 3 significant figures.}$$

Note: This can alternatively be evaluated from

$$\log_2 e = (\log_{10} e)/(\log_{10} 2).$$

(*b*) Now, $\log_2 N = (\log_{10} N)/(\log_{10} 2)$.

$$\therefore \log_{10} N = \log_2 N \times \log_{10} 2,$$

$$= -1 \cdot 2 \times 0 \cdot 301,$$

$$= -0 \cdot 3612 = \bar{1} \cdot 6388.$$

$$\therefore \underline{N = 0 \cdot 435,} \text{ to 3 significant figures.}$$

(*c*) $\log_e (10 + 9x) - \log_e (11 - x) = 2$.

$$\therefore \log_e \frac{10 + 9x}{11 - x} = 2.$$

$$\therefore \frac{10 + 9x}{11 - x} = e^2 = 7 \cdot 3891.$$

$$\therefore 10 + 9x = 7 \cdot 3891(11 - x) = 81 \cdot 280 - 7 \cdot 3891x.$$

$$\therefore 16 \cdot 389x = 71 \cdot 280.$$

$$\therefore \underline{x = 4 \cdot 35,} \text{ to 3 significant figures.}$$

Q 5 (a) Sketch the sinusoidal waveform

$$y = 400 \sin \{100\pi t - (\pi/6)\},$$

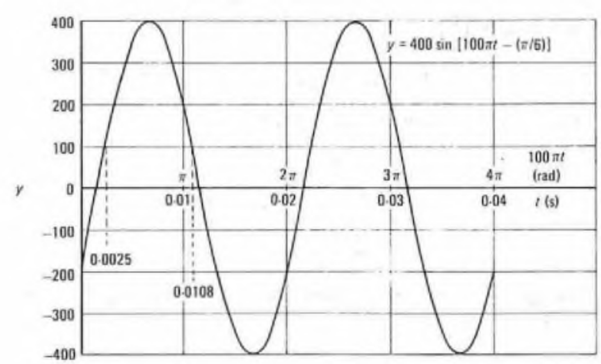
showing 2 complete cycles from $t = 0$.
(b) If t is measured in seconds,
(i) state the waveform frequency in hertz,
(ii) give its period in milliseconds,
(iii) from the graph, estimate (in milliseconds) the 2 instants in the first cycle at which $y = 100$, and
(iv) confirm the estimates in part (b) (iii) by calculation.

A 5 (*a*) When $t = 0$, $y = 400 \sin(-\pi/6)$,

$$= 400 \times (-0 \cdot 5) = -200.$$

As t increases from zero, y increases, and becomes zero when $\sin \{100\pi t - (\pi/6)\} = 0$; that is, when $100\pi t = \pi/6$, or $t = 1/600 = 0 \cdot 001\dot{6}$ s.

The value of y then increases to a maximum of 400 when $\sin \{100\pi t - (\pi/6)\} = 1$; that is, when $100\pi t - (\pi/6) = \pi/2$, or $t = 0 \cdot 00\dot{6}$ s. The graph follows the sine law, decreasing to a minimum of -400 after a further period of 0·01 s, and completes the first cycle in a total elapsed time of 0·02 s. The second cycle is a repetition of the first, taking a further period of 0·02 s. The graph is shown in the sketch.



(*b*) (*i*) The frequency is $\underline{50 \text{ Hz}}$.

(*ii*) The period is $\underline{20 \text{ ms}}$.

(*iii*) From the graph, when $y = 100$, $t \approx \underline{2 \cdot 5 \text{ ms and } 10 \cdot 8 \text{ ms}}$.

(*iv*) When $y = 100$,

$$100 = 400 \sin \{100\pi t - (\pi/6)\}.$$

$$\therefore \sin \{100\pi t - (\pi/6)\} = 0 \cdot 25.$$

$$\therefore 100\pi t - (\pi/6) = 14° 29' \text{ or } 180° - 14° 29',$$

over the range 0–180° (0–π rad).

For ease of calculation, this is best expressed as
$100\pi t - (\pi/6) = \pi \times 14\frac{1}{2}°/180°$ or $\pi \times 165\frac{1}{2}°/180°$.

Note: The error in taking 14° 29′ as $14\frac{1}{2}°$ is only 1 in 869, and is much less for the alternative answer.

$$\therefore\ 100t = \frac{1}{6} + \frac{29}{360} \text{ or } \frac{1}{6} + \frac{331}{360} \text{ s},$$

$$= 0{\cdot}1\dot{6} + 0{\cdot}0806 \text{ or } 0{\cdot}1\dot{6} + 0{\cdot}9194 \text{ s},$$

$$= 0{\cdot}2472 \text{ or } 1{\cdot}0861 \text{ s}.$$

$$\therefore\ t = 2{\cdot}47 \text{ ms or } 10{\cdot}86 \text{ ms}.$$

The values obtained by calculation confirm the estimates obtained graphically.

Q 6 (a) If $r\cos(\theta + \alpha) = 5\sin\theta + 3\cos\theta$, use the expansion of $\cos(A + B)$ to calculate r (assuming this to be positive) and the angle α, between $-180°$ and $+180°$.

(b) In the triangle ABC, $a = 24$ mm, $b = 36$ mm, and $c = 18$ mm. Calculate

(i) the largest angle, and
(ii) the area of the triangle.

A 6 (*a*) Now, $\cos(A + B) = \cos A\cos B - \sin A\sin B$.

Hence, $r\cos(\theta + \alpha) = r(\cos\theta\cos\alpha - \sin\theta\sin\alpha)$,

$$= -r\sin\alpha\sin\theta + r\cos\alpha\cos\theta.$$

Comparing this expansion with the given equation, it can be seen that

$$-r\sin\alpha = 5, \qquad \ldots\ldots (1)$$

$$\text{and } r\cos\alpha = 3. \qquad \ldots\ldots (2)$$

Dividing equation (1) by equation (2) gives

$$-\tan\alpha = 5/3 = 1{\cdot}6667.$$

$$\therefore\ \tan\alpha = -1{\cdot}6667.$$

Since angles in only the second and fourth quadrants have negative tangents,

$$\alpha = 180° - 59° 2' \text{ or } 360° - 59° 2',$$

$$= 120° 58' \text{ or } 300° 58'.$$

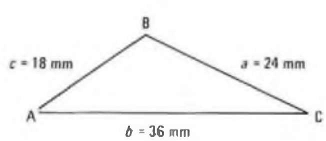
However, from equation (1), $\sin\alpha$ must be negative since r is positive. Hence, α cannot lie in the second quadrant.

$$\therefore\ \alpha = 300° 58' \text{ or } -59° 2'.$$

From equation (2),

$$r = \frac{3}{\cos(-59° 2')} = \frac{3}{0{\cdot}5145} = 5{\cdot}83.$$

(*b*) The triangle is shown in the sketch.



(*i*) The largest angle is opposite the largest side of the triangle, and is therefore angle B. From the cosine rule,

$$\cos B = \frac{c^2 + a^2 - b^2}{2ca},$$

$$= \frac{18^2 + 24^2 - 36^2}{2 \times 18 \times 24} = -0{\cdot}4583.$$

$$\therefore\ B = 180° - 62° 43' = 117° 17'.$$

(*ii*) The area of the triangle

$$= \frac{ac\sin B}{2},$$

$$= \frac{24 \times 18 \times \sin 117° 17'}{2} \text{ mm}^2,$$

$$= 216\sin 62° 43' \text{ mm}^2,$$

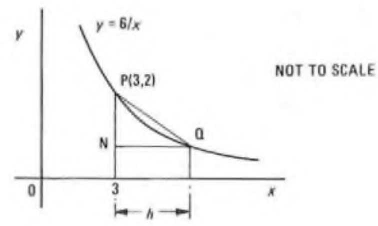
$$= 216 \times 0{\cdot}8887 = 192 \text{ mm}^2.$$

Q 7 (a) (i) Point P is the point (3,2) on the curve $y = 6/x$. If Q is the neighbouring point on the curve where $x = 3 + h$, find an expression in terms of h for the gradient (slope) of the straight line PQ.

(ii) Hence, deduce the slope of the tangent at point P.

(b) (i) Plot the graph of $y = \cos 100\pi t$ from $t = 0$ to $t = 20$ ms.
(ii) Obtain from this graph an estimate of dy/dt at a time 2·5 ms after the positive peak value.
(iii) Verify, for these values of angular frequency and time, the formula for differentiating $\cos\omega t$ (where $\omega = 100\pi$).

A 7 (*a*) (*i*) The points P and Q, on the curve $y = 6/x$, are shown in sketch (*a*).



(*a*)

If QN is perpendicular to the ordinate through P, then NQ = h. When $x = 3 + h$ (that is, at point Q), $y = 6/(3 + h)$.
Hence, the gradient of the straight line PQ

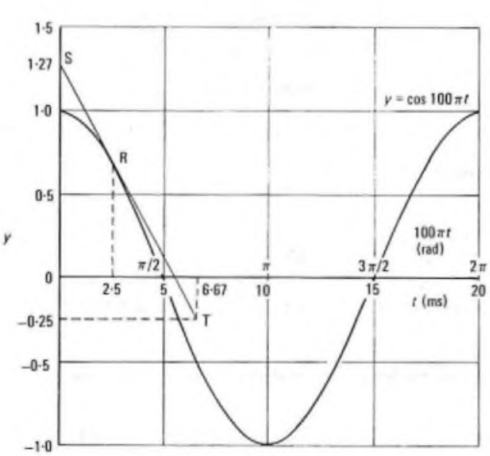
$$= \frac{NP}{NQ} = \frac{\{6/(3 + h)\} - 2}{h},$$

$$= \frac{6 - 6 - 2h}{h(3 + h)} = -\frac{2}{3 + h}.$$

(*ii*) As point Q moves closer to point P, the gradient of the line PQ becomes more nearly equal to that of the tangent to the curve at point P. Eventually, when $h = 0$, the gradients are coincident. Hence, in the limit, as h approaches 0, the gradient of the tangent at point P is $-2/3$.

(*b*) (*i*) For the given curve, when $t = 0$, $y = \cos 0 = 1$, and when $t = 20 \times 10^{-3}$ s, $y = \cos 2\pi = 1$. The graph is therefore one complete cycle of a cosine curve, and is plotted from the convenient values derived in the table.

t (ms)	0	5/3	10/3	5
$100\pi t$ (rad)	0	$\pi/6$	$\pi/3$	$\pi/2$
$100\pi t$ (degrees)	0	30°	60°	90°
$y = \cos 100\pi t$	1	0·866	0·5	0



(*b*)

The graph is shown in sketch (*b*). The numerical values from $100\pi t = 120°$ to $100\pi t = 360°$ are those shown in the table, but with the appropriate signs assigned.

(*ii*) The tangent, ST, to the cosine curve is drawn at point R, where $t = 2\cdot5$ ms. The gradient of the tangent gives the value of dy/dt at point R. Taking the co-ordinates of points S and T, the estimated value of dy/dt

$$= \frac{-0\cdot25 - 1\cdot27}{6\cdot67 \times 10^{-3}} = -\frac{1\cdot52}{6\cdot67 \times 10^{-3}} = \underline{-228.}$$

(*iii*)
$$y = \cos 100\pi t.$$

$$\therefore \frac{dy}{dt} = -100\pi \sin 100\pi t.$$

When $t = 2\cdot5$ ms,

$$\frac{dy}{dt} = -100\pi \sin(\pi/4),$$

$$= -100\pi \times 0\cdot7071 = \underline{-222\cdot1.}$$

The estimated value for dy/dt is reasonably close to the calculated value, verifying the formula for differentiation.

Q 8 (*a*) *Differentiate from first principles the function*

$$y = ax^2 + \frac{b}{x}.$$

(*b*) (*i*) *Sketch graphs of* $y = ax^2$ *and* $y = b/x$, *where the constants a and b are both positive.*

(*ii*) *By graphical addition, obtain the graph of* $y = ax^2 + \dfrac{b}{x}$.

(*iii*) *Show that this function has only one maximum or minimum value. Find this value when* $a = 4$ *and* $b = 12$.

A 8 (*a*) $y = ax^2 + \dfrac{b}{x}$.

Let δx be a small increment of x, and δy the corresponding change in y. Then,

$$y + \delta y = a(x + \delta x)^2 + \frac{b}{x + \delta x}.$$

$$\therefore \delta y = a(x + \delta x)^2 + \frac{b}{x + \delta x} - \left(ax^2 + \frac{b}{x}\right),$$

$$= a\{x^2 + 2x\delta x + (\delta x)^2\} - ax^2 + \frac{b}{x + \delta x} - \frac{b}{x},$$

$$= 2ax\delta x + a(\delta x)^2 + \frac{bx - b(x + \delta x)}{x(x + \delta x)}.$$

$$\therefore \frac{\delta y}{\delta x} = 2ax + a\delta x - \frac{b}{x(x + \delta x)}.$$

$$\therefore \frac{dy}{dx} = \lim_{\delta x \to 0} \frac{\delta y}{\delta x} = \underline{2ax - \frac{b}{x^2}}.$$

(*b*) (*i*) Assuming suitable arbitrary values for the positive constants a and b, the graphs of $y = ax^2$ and $y = b/x$ are plotted from the readily deduced values for y in each case. The graphs are shown in the sketch.

The curve $y = ax^2$ is parabolic, with a minimum value at $x = 0$. As x increases, either positively or negatively, the value of y increases indefinitely in the positive direction. The curve $y = b/x$ is a rectangular hyperbola, and is asymptotic to the x-axis for large positive or negative values of x, and asymptotic to the y-axis as $\pm x$ approaches zero.

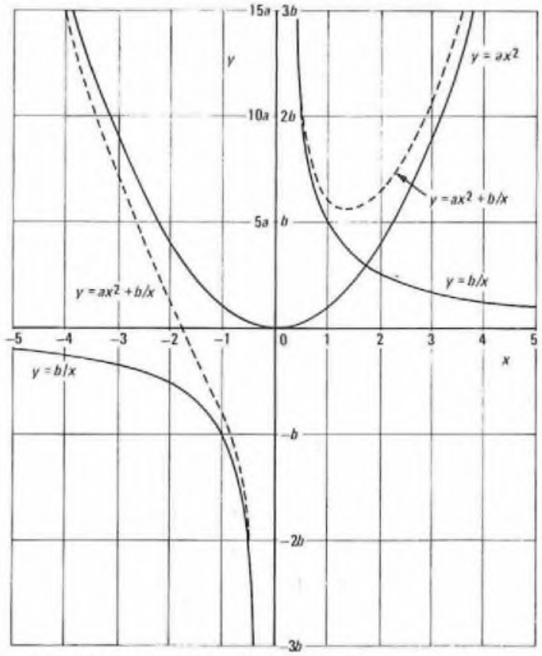
(*ii*) The graph of $y = ax^2 + b/x$ is shown by the dashed lines, drawn to an arbitrary y-axis scale by graphical addition.

(*iii*) For large positive or negative values of x, $|ax^2| \gg |b/x|$, and the function $y = ax^2 + b/x$ is asymptotic to the curve $y = ax^2$. As $\pm x$ approaches zero, $|b/x| \gg |ax^2|$, and the function is asymptotic to the curve $y = b/x$. Thus, for negative values of x, there is no maximum or minimum value, but, as x increases positively from zero, the function has a single minimum value.

When y is a minimum,

$$\frac{dy}{dx} = 2ax - \frac{b}{x^2} = 0.$$

$$\therefore x = \left(\frac{b}{2a}\right)^{1/3}.$$



Hence, the minimum value of y is given by

$$y_{\text{min.}} = a\left(\frac{b}{2a}\right)^{2/3} + \frac{b}{\left(\dfrac{b}{2a}\right)^{1/3}},$$

$$= 2^{-2/3}a^{1/3}b^{2/3} + 2^{1/3}a^{1/3}b^{2/3},$$

$$= a^{1/3}b^{2/3}\left(\frac{1}{4^{1/3}} + 2^{1/3}\right).$$

When $a = 4$ and $b = 12$,

$$y_{\text{min.}} = 4^{1/3} \times 12^{2/3} \times \left(\frac{1}{1\cdot587} + 1\cdot26\right),$$

$$= 4 \times 3^{2/3} \times (0\cdot63 + 1\cdot26),$$

$$= 4 \times 1\cdot442^2 \times 1\cdot89 = \underline{15\cdot72.}$$

Q 9 (*a*) *Find the function y such that* $\dfrac{dy}{dx} = 4x + \dfrac{16}{x^2}$, *given that* $y = 20$ *when* $x = 4$.

(*b*) *Use integration to calculate the area enclosed by the curve* $y = 5x^2 - 2x^3$ *and the x-axis.*

A 9 (*a*)

$$y = \int \left(4x + \frac{16}{x^2}\right) dx,$$

$$= 2x^2 - \frac{16}{x} + c,$$

where c is a constant.

Substituting the given values gives

$$20 = 2 \times 16 - \frac{16}{4} + c.$$

$$\therefore c = 20 - 32 + 4 = -8.$$

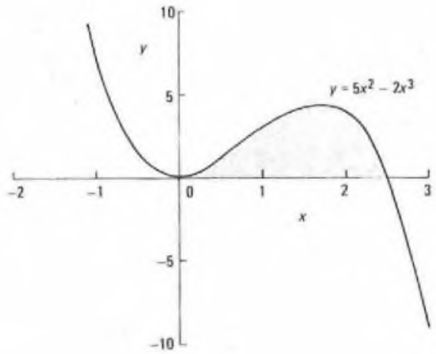
Hence,
$$\underline{y = 2x^2 - \frac{16}{x} - 8.}$$

(*b*) Putting $y = 0$ in the equation $y = 5x^2 - 2x^3$ gives

$$5x^2 = 2x^3.$$

$$\therefore x = 0 \text{ or } 2\cdot5.$$

When x is negative, the terms in the expression for y are positive, and the value of y increases rapidly as $|x|$ increases. When $x = 1$, $y = 3$, and when $x = 2$, $y = 4$. As x increases above $2\cdot5$, the term $2x^3$ exceeds $5x^2$, and the value of y is therefore negative. Thus, the graph of the function is as shown in the sketch. The area enclosed by the curve and the x-axis is shown shaded.



The shaded area

$$= \int_0^{2\cdot5} (5x^2 - 2x^3)\,\mathrm{d}x,$$

$$= \left[\frac{5x^3}{3} - \frac{2x^4}{4}\right]_0^{5/2},$$

$$= \frac{5}{3} \times \frac{125}{8} - \frac{625}{2 \times 16},$$

$$= 26\cdot04 - 19\cdot53,$$

$$= \underline{6\cdot5 \text{ square units.}}$$

Q 10 *(a) Evaluate*

(i) $(2 - j3)(5 + j2)$, *and*

(ii) $\dfrac{3 + j1}{5 - j3}$,

expressing each in the form $a + jb$.

(b) (i) Express, in the form $A + jB$, *the impedance*

$$Z = R + j\omega L + \frac{1}{j\omega C},$$

when $R = 1\cdot2\ \mathrm{k}\Omega$, $L = 8\ \mathrm{mH}$, $C = 2\ \mathrm{nF}$, *and* $\omega = 10^5\ \mathrm{rad/s}$.

(ii) Find, to 3 significant figures, the magnitude of this impedance.

A 10 *(a) (i)* $(2 - \mathrm{j}3)(5 + \mathrm{j}2) = 10 + \mathrm{j}4 - \mathrm{j}15 + 6,$

$$= \underline{16 - \mathrm{j}11.}$$

(ii) $$\frac{3 + \mathrm{j}1}{5 - \mathrm{j}3} = \frac{(3 + \mathrm{j}1)(5 + \mathrm{j}3)}{(5 - \mathrm{j}3)(5 + \mathrm{j}3)},$$

$$= \frac{15 + \mathrm{j}9 + \mathrm{j}5 - 3}{25 + 9},$$

$$= \frac{12 + \mathrm{j}14}{34} = \underline{0\cdot353 + \mathrm{j}0\cdot412.}$$

(b) (i) $$Z = R + \mathrm{j}\omega L + \frac{1}{\mathrm{j}\omega C},$$

$$= R + \mathrm{j}\omega L - \frac{\mathrm{j}}{\omega C},$$

$$= R + \mathrm{j}\left(\omega L - \frac{1}{\omega C}\right),$$

$$= 1\cdot2 \times 10^3 + \mathrm{j}\left(10^5 \times 8 \times 10^{-3} - \frac{1}{10^5 \times 2 \times 10^{-9}}\right)\Omega,$$

$$= 1\cdot2 \times 10^3 + \mathrm{j}(800 - 5 \times 10^3)\ \Omega,$$

$$= \underline{1\cdot2 - \mathrm{j}4\cdot2\ \mathrm{k}\Omega.}$$

(ii) $$|Z| = \sqrt{(1\cdot2^2 \times 10^6 + 4\cdot2^2 \times 10^6)}\ \Omega,$$

$$= \sqrt{(1\cdot44 + 17\cdot64)} \times 10^3\ \Omega,$$

$$= \sqrt{19\cdot08}\ \mathrm{k}\Omega,$$

$$= \underline{4\cdot37\ \mathrm{k}\Omega}, \text{ to 3 significant figures.}$$

TELECOMMUNICATION PRINCIPLES B, 1976

Students were expected to answer any 6 questions

Q 1 *An uncharged 2 μF capacitor, connected in series with a 10 kΩ resistor, is to be charged from a 40 V battery. The rise of voltage across the capacitor with time is to be displayed on an oscilloscope.*

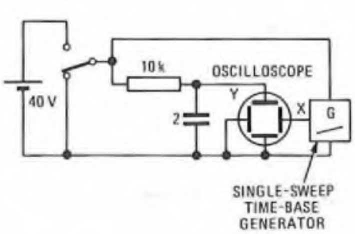
(a) Briefly describe an experiment in which this can be done. Comment on the circuit arrangements.

(b) (i) Write down the expression for the voltage across the capacitor in terms of the time from the start of charging.

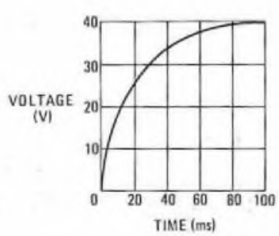
(ii) Find the time constant of the circuit.

(iii) Calculate the initial current in the circuit.

A 1 *(a)* Sketch *(a)* shows a circuit suitable for displaying the rise of voltage across a 2 μF capacitor charged from a 40 V source through a 10 kΩ resistor.



(a)



(b)

Operating the switch starts both the charging sequence and the single-sweep time-base generator. The generator applies a linearly-rising potential difference across the X-plates, drawing the trace horizontally across the oscilloscope screen linearly with time over a certain period. The capacitor is connected across the Y-plates, so that the rising voltage across the capacitor simultaneously deflects the trace vertically, the deflexion being proportional to the voltage at any instant. This results in a trace of the form shown in sketch *(b)*. For a charging capacitor, the trace is exponential. The oscilloscope does not affect the operation of the circuit since it presents a very high-impedance input. The cathode-ray tube used must give a long afterglow to ensure that the trace persists sufficiently for satisfactory observation. A more elaborate timing circuit can be used that causes the cycle to be repeated automatically, with the capacitor being discharged before recharging begins.

The charging sequence is as follows. When the switch is operated, the supply voltage appears across the resistor, since the uncharged capacitor offers no opposing voltage. The initial current is therefore a maximum, but the voltage across the capacitor is initially zero. As the current charges the capacitor, the voltage across the capacitor rises. The voltage effective across the resistor thus decreases, reducing the charging current, so that the rate of rise of voltage across the capacitor is reduced. Ultimately, the voltage across the capacitor can be considered to reach 40 V, and the charging current ceases. This condition is effectively reached after a period equal to about 5 times the time constant of the circuit.

(b) (i) The expression for the voltage, v, across the capacitor in terms of the time, t seconds, from the commencement of charging is

$$v = V(1 - \mathrm{e}^{-t/CR}) \text{ volts,}$$

where V is the supply voltage, C is the capacitance (farads), and R is the resistance (ohms). Substituting the given values gives

$$v = 40(1 - \mathrm{e}^{-t/0\cdot000\,002 \times 10\,000}) = \underline{40(1 - \mathrm{e}^{-50t}) \text{ volts.}}$$

(ii) The time constant, τ seconds, of the circuit is defined as

$$\tau = CR = 2 \times 10^{-6} \times 10 \times 10^3\ \mathrm{s} = \underline{20\ \mathrm{ms.}}$$

(iii) By Ohm's law, the initial current

$$= \frac{V}{R} = \frac{40}{10 \times 10^3}\ \mathrm{A} = \underline{4\ \mathrm{mA.}}$$

Q 2 Shielding is found to be necessary for

(a) a wideband carrier amplifier situated near a radio transmitting station,

(b) an audio-frequency transformer connecting an oscillator to an a.c. inductance bridge, and

(c) the pick-up head of a tape recorder.

What are the probable causes of the interference for any 2 of the above cases, and what form of shielding would you suggest?

A 2 See A3, Telecommunication Principles B, 1974, *Supplement*, Vol. 69, p. 21, Apr. 1976.

Q 3 (a) Explain the meaning of impedance. Show how it is represented using j notation.

(b) Two impedances, 6 + j11 Ω and 6 − j6 Ω, are connected in series. Calculate the equivalent single impedance. Find the values of the 2 components represented by this impedance at a frequency of 10 kHz.

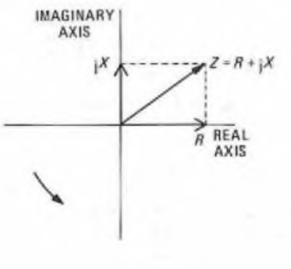
(c) The r.m.s. current supplied to the impedance in part (b) is 5 A at 10 kHz. Find

(i) the magnitude of the supply voltage, and

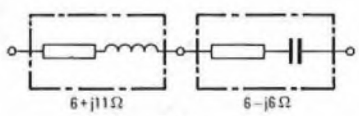
(ii) the phase angle of the voltage relative to the current.

A 3 (*a*) The impedance of a circuit is a measure of its opposition to the flow of alternating current. It is the ratio of the applied voltage to the current taken by the circuit.

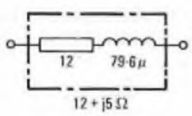
For a series circuit, the general form of the impedance, Z ohms, expressed in j notation, is $Z = R + \mathrm{j}X$ ohms, where R is the resistance of the circuit (ohms), and X is the reactance (ohms). The j notation is an algebraic representation of a phasor diagram in which the reactance is in quadrature to the resistance. The j term, or *imaginary* term, is positive if the reactance is inductive (that is, the reactance phasor leads the resistance phasor), and negative if the reactance is capacitive (that is, the reactance phasor lags the resistance phasor). Sketch (*a*) shows the phasor diagram for a resistance in series with an inductive reactance, assuming the conventional counter-clockwise rotation of phasors. The impedance is given by the resultant phasor.



(*a*)



(*b*)



(*c*)

(*b*) Sketch (*b*) shows the 2 impedances in series. The equivalent single impedance, Z_T, is calculated by adding the real and imaginary parts of each impedance.

$$\therefore\ Z_T = 6 + \mathrm{j}11 + 6 - \mathrm{j}6\ \Omega,$$
$$= (6 + 6) + \mathrm{j}(11 - 6)\ \Omega,$$
$$= \underline{12 + \mathrm{j}5\ \Omega.}$$

The equivalent impedance is therefore a 12 Ω resistance in series with an inductive reactance, X_L, of 5 Ω. The value of the inductance, L henrys, is calculated from the equation

$$X_L = 2\pi f L \text{ ohms},$$

where f is the frequency (hertz).

$$\therefore\ L = \frac{5}{2\pi \times 10 \times 10^3}\,\mathrm{H} = \underline{79 \cdot 6\ \mu\mathrm{H}.}$$

The equivalent impedance is illustrated in sketch (*c*).

(*c*) (*i*) From sketch (*a*), it can be seen that the magnitude of the impedance of the circuit is found using the theorem of Pythagoras. Thus,

$$Z_T = \sqrt{(R^2 + X_L{}^2)} \text{ ohms},$$
$$= \sqrt{(12^2 + 5^2)} = \sqrt{169} = 13\ \Omega.$$

By Ohm's law, the r.m.s. voltage across this impedance when a current of 5 A flows is $5 \times 13 = \underline{65\ \mathrm{V}.}$

(*ii*) The phase angle is the angle between the impedance and resistance phasors. Thus, the phase angle

$$= \tan^{-1}\frac{X_L}{R} = \tan^{-1}\frac{5}{12} = \tan^{-1} 0 \cdot 417 = \underline{22^\circ\ 37'.}$$

Q 4 (a) Describe an experiment using a bridge to measure the capacitance and loss angle of a capacitor of the order of 10 nF at about 100 kHz. Draw a circuit diagram of the bridge, and derive expressions for the balance conditions from which the unknown values can be calculated.

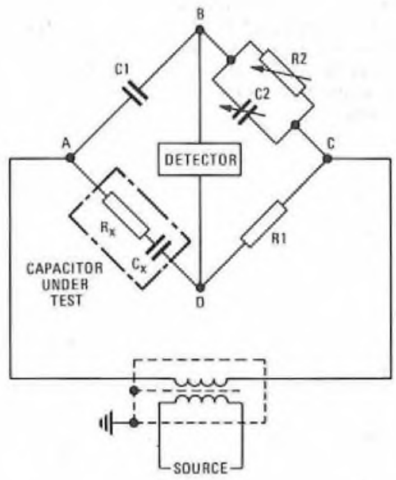
(b) Is the frequency stability of the supply important in the measurements? Give reasons for your answer.

(c) Mention any 2 precautions that must be taken when using the bridge.

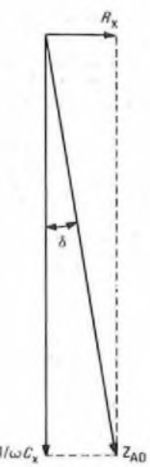
A 4 (*a*) The Schering bridge, shown in sketch (*a*), can be used to measure the capacitance and loss angle of a capacitor. At balance, the signal in the detector must be zero, which, for an a.c. bridge, implies that both amplitude and phase must be identical at points B and D. The impedances of the 4 arms must therefore satisfy the balance equation

$$Z_{AB}Z_{DC} = Z_{AD}Z_{BC}, \qquad \ldots\ldots (1)$$

where Z_{AB} is the impedance of a good quality fixed capacitance (capacitor C1), Z_{DC} is the impedance of a stable known resistance (resistor R1), Z_{AD} is the impedance of the unknown capacitor (capacitor C_x in series with its loss resistance, R_x), and Z_{BC} is the impedance of an adjustable capacitor (capacitor C2) in parallel with a variable resistor (resistor R2).



(*a*)



(*b*)

The 100 kHz source must be a stable oscillator, and is usually amplitude modulated by, say, a 1 kHz signal to enable aural detection via an amplifier-detector unit. The source is connected to the bridge through a screened transformer to minimize the effect of longitudinal capacitance. The capacitor under test is connected by short, carefully positioned leads to minimize induced voltages. Balance is obtained by alternate adjustments of capacitor C2 and resistor R2.

From equation (1),

$$-\frac{\mathrm{j}}{\omega C_1} \times R_1 = \left(R_x - \frac{\mathrm{j}}{\omega C_x}\right) \times \frac{1}{\dfrac{1}{R_2} + \mathrm{j}\omega C_2}.$$

$$\therefore\ R_x - \frac{\mathrm{j}}{\omega C_x} = -\frac{\mathrm{j}R_1}{\omega C_1}\left(\frac{1}{R_2} + \mathrm{j}\omega C_2\right).$$

Equating real terms gives

$$\underline{R_x = \frac{R_1 C_2}{C_1}.} \qquad \ldots\ldots (2)$$

Equating imaginary terms gives

$$\underline{C_x = \frac{R_2 C_1}{R_1}.} \qquad \ldots\ldots (3)$$

The calculation of the loss angle, δ, is illustrated by the impedance diagram of sketch (*b*). Because R_x is very small, δ is small. Thus, the loss angle, expressed in radians, is equal to the tangent of the angle.

Hence,

$$\delta = \frac{R_x}{\frac{1}{\omega C_x}} = \underline{\omega R_x C_x}. \qquad \ldots\ldots (4)$$

(*b*) As the angular frequency, ω, does not appear in equations (2) and (3), the frequency stability of the source does not affect the accuracy of the calculated values for R_x and C_x. However, ω appears in equation (4) for the calculation of the loss angle, and hence the frequency must remain stable while balance is being obtained, and the actual frequency must be known for δ to be calculated.

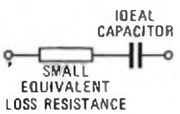
(*c*) The source must be screened to prevent any stray electromagnetic field from affecting the bridge or the detector. Coaxial connexions should be used for the source and the detector, and leads and components should be carefully placed and oriented to minimize coupling.

Q 5 (*a*) *What are the causes of a.c. power loss in a capacitor?*
(*b*) *How can the power loss in a capacitor be represented in an equivalent circuit?*
(*c*) *Explain, with the aid of phasor diagrams, the meaning of loss angle.*
(*d*) *A power loss of 50 μW occurs in a capacitor of 1 μF when a current of 1 A flows at a frequency of 100 kHz. Calculate the loss resistance and the loss angle.*

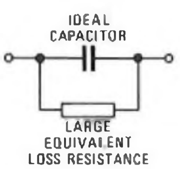
A 5 (*a*) Power loss in a capacitor operating under a.c. conditions occurs in 2 ways: loss in the resistances of the electrodes, and dielectric loss.

Dielectric loss is usually the greater, and is proportional to the square root of the frequency. Resistive loss is independent of frequency, although eddy-current loss in the electrodes can be noticeable at high frequencies.

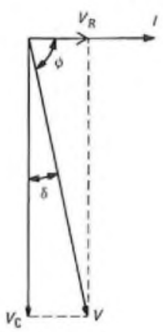
(*b*) Both sources of loss (which, even when taken together, amount to only a small loss) can be represented by a single series resistor that, in the case of a good quality capacitor, has a very small value. The equivalent circuit is shown in sketch (*a*). Alternatively, the loss can be represented by a parallel resistor, as shown in sketch (*b*); in this case, the resistor has a very large value.



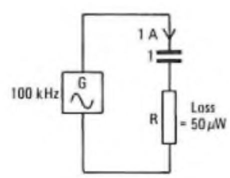
(*a*)



(*b*)



(*c*)



(*d*)

(*c*) Sketch (*c*) shows the phasor diagram for the equivalent series circuit of sketch (*a*). Phasor V_R represents the voltage across the equivalent loss resistance, phasor V_C represents the voltage across the capacitor, phasor I represents the current in the circuit, and phasor V represents the voltage across the circuit. The angle δ, between V_C and V, is called the *loss angle* and, as V_R is small, the loss angle, expressed in radians, is numerically equal to its tangent, V_R/V_C. Since $V_R = IR$ and $V_C = IX_C$, where R is the equivalent loss resistance and X_C is the reactance of the capacitor, then

$$\underline{\delta = \frac{R}{X_C} \text{ radians.}}$$

Note that the power factor of the circuit is cos ϕ.

(*d*) The equivalent series circuit for the given conditions is shown in sketch (*d*). The loss incurred is entirely due to resistor R, and is equal to I^2R watts.

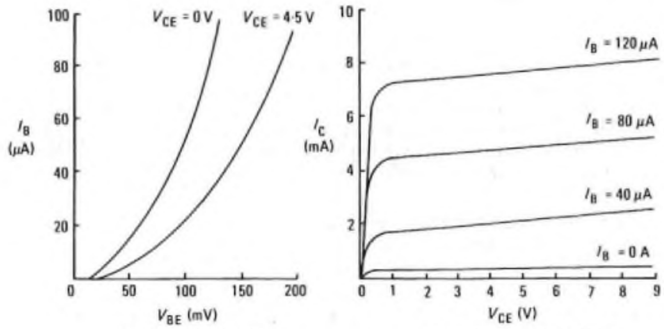
$$\therefore\ R = \frac{50 \times 10^{-6}}{1}\ \Omega = \underline{50\ \mu\Omega.}$$

At a frequency of 100 kHz, the capacitor has a reactance of $1/(2\pi \times 100 \times 10^3 \times 1 \times 10^{-6}) = 1{\cdot}59\ \Omega$.

$$\therefore\ \delta = \frac{50 \times 10^{-6}}{1{\cdot}59} \text{ rad,}$$
$$= \underline{31{\cdot}5\ \mu\text{rad.}}$$

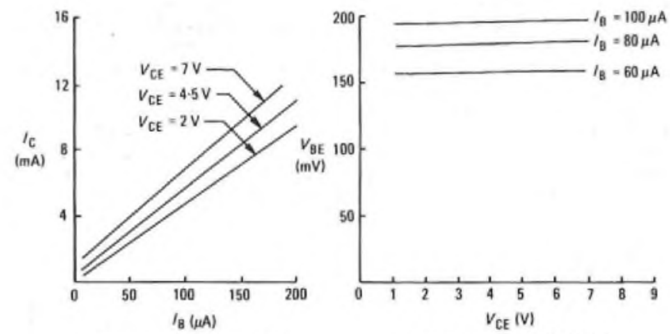
Q 6 (*a*) *Sketch typical characteristics of a transistor, including representative values.*
(*b*) *Define the h-parameters of a transistor.*
(*c*) *Show how these can be obtained from the characteristic curves.*

A 6 (*a*) Typical input, output, transfer and feedback characteristics, for a transistor connected in the common-emitter configuration, are shown in the sketch.



INPUT CHARACTERISTIC

OUTPUT CHARACTERISTIC



TRANSFER CHARACTERISTIC

FEEDBACK CHARACTERISTIC

(*b*) The *h*-parameters (hybrid parameters) for a transistor are related to the gradients of the 4 main static characteristic curves given in part (*a*). They describe the low-frequency small-signal performance of the transistor in terms of the input and output alternating signal voltages (V_{be} and V_{ce} respectively) and currents (I_b and I_c respectively).

The 4 *h*-parameters are the input parameter (h_i), the output parameter (h_o), the forward current gain (h_f), and the reverse voltage ratio (h_r). A second suffix, e, to each parameter denotes that the transistor is connected in the common-emitter configuration.

The *h*-parameters are defined as follows.

(*i*) $h_{ie} = V_{be}/I_b$ with the output short-circuited; i.e., $V_{ce} = 0$.
(*ii*) $h_{oe} = I_c/V_{ce}$ with the input open-circuited; i.e., $I_b = 0$.
(*iii*) $h_{fe} = I_c/I_b$ with the output short-circuited; i.e., $V_{ce} = 0$.
(*iv*) $h_{re} = V_{be}/V_{ce}$ with the input open-circuited; i.e., $I_b = 0$.

(*c*) The static characteristics are plotted from measurements of the direct input and output voltages (V_{BE} and V_{CE} respectively) and currents (I_B and I_C respectively). The relationships between the characteristics and the *h*-parameters are described below.

(*i*) The input characteristic shows the graph of I_B/V_{BE} for various fixed values of V_{CE}. The gradient of the curve gives the input admittance of the transistor, the reciprocal of which is the input parameter, h_{ie}.

(*ii*) The output characteristic shows graphs of I_C/V_{CE} for a series of fixed values of I_B. The gradient of the curves gives the output admittance of the transistor, or the output parameter, h_{oe}.

(*iii*) The transfer characteristic shows the graph of I_C/I_B for various fixed values of V_{CE}. The gradient of the curve gives the forward current gain, h_{fe}.

(*iv*) The feedback characteristic shows graphs of V_{BE}/V_{CE} for a series of fixed values of I_B. The gradient of the curves gives the reverse voltage ratio, h_{re}.

Q 7 (*a*) *Why are at least 2 reactances needed to obtain electrical resonance? Explain why resonance can be obtained with the 2 reactances either in series or in parallel.*

(*b*) *Show, by means of reactance/frequency curves, why there is only one frequency of resonance for a given capacitance and inductance.*

(*c*) *Calculate the capacitance needed to resonate with a series inductance of 150 mH at a frequency of 50 kHz.*

A 7 (*a*) Resonance occurs in an a.c. circuit when the applied frequency is such that the total reactance of the circuit (assessed algebraically) is zero; that is, when the circuit is purely resistive. For this algebraic cancellation to occur, there must be at least 2 reactances, one being of opposite sign to the other. Hence, the simplest resonant circuit consists of a capacitor and an inductor.

For an inductor and capacitor in series, the impedance, Z, of the circuit is given by $Z = R + jX_L - jX_C$, where R is the resistance of the circuit (mainly that of the inductor), X_L is the reactance of the inductor, and X_C is the reactance of the capacitor. For the circuit to be purely resistive, the algebraic sum of the imaginary terms must be zero, so that $X_L = X_C$.

For a parallel circuit, the admittance is given by

$$\frac{1}{Z} = \frac{1}{R + jX_L} + \frac{j}{X_C},$$

$$= \frac{R - jX_L}{R^2 + X_L^2} + \frac{j}{X_C},$$

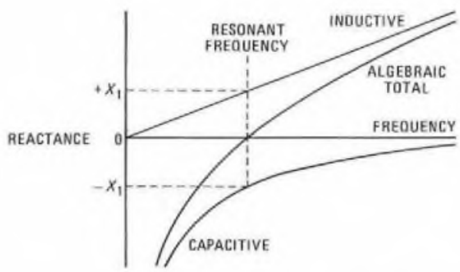
$$= \frac{R}{R^2 + X_L^2} - \frac{jX_L}{R^2 + X_L^2} + \frac{j}{X_C}.$$

Assuming R to be small compared with X_L gives

$$\frac{1}{Z} \approx \frac{R}{X_L^2} - \frac{jX_L}{X_L^2} + \frac{j}{X_C}.$$

For the circuit to be purely resistive, the algebraic sum of the imaginary terms must be zero, so that, again, $X_L = X_C$.

Thus, resonance can be obtained with the reactances connected either in series or in parallel.



(*b*) The sketch shows the graphs of inductive reactance and capacitive reactance plotted against frequency. As the frequency increases from zero, it can be seen that there is a unique point at which the reactances, X_1, are equal but opposite in sign.

(*c*) For resonance, the inductive and capacitive reactances are equal.

$$\therefore\ 2\pi f L = \frac{1}{2\pi f C},$$

where f is the frequency (hertz), L is the inductance (henrys), and C is the capacitance (farads).

$$\therefore\ C = \frac{1}{4\pi^2 f^2 L}\ \text{farads},$$

$$= \frac{1}{4\pi^2 \times 50^2 \times 10^6 \times 150 \times 10^{-3}}\ \text{F} = \underline{67\cdot6\ \text{pF}}.$$

Q 8 (*a*) *Explain the principle of a d.c. motor. What factors in the motor determine the torque that the armature can produce?*

(*b*) *The armature of a d.c. motor is in the form of a cylinder of effective diameter 10 cm and length 12·5 cm. The armature has 10 coils, each of 10 turns, and rotates in a magnetic field of 50 mT. Calculate the torque developed when the average current in each coil throughout one revolution is 5 A.*

A 8 (*a*) See A7, Engineering Science, 1976, in this *Supplement*, p. 3. In that answer, it is demonstrated that the maximum torque, T, is given by

$$\underline{T = 2NBIlr\ \text{newton metres},}$$

where N is the number of turns, B is the flux density (teslas), I is the current (amperes), l is the length of the coil side cutting the magnetic field (metres), and r is the radius of the coil (metres).

(*b*) Assuming the construction of the motor to be such that the magnetic field can be considered to be radial, the above expression for the maximum torque becomes that for the continuous torque produced.

$$\therefore\ T = 2 \times 100 \times 50 \times 10^{-3} \times 5 \times 12\cdot5 \times 10^{-2} \times 5 \times 10^{-2}\ \text{N m},$$

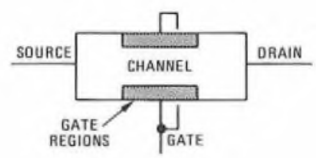
$$= \underline{0\cdot313\ \text{N m}}.$$

Q 9 (*a*) *Explain, with the aid of diagrams, the principle of the field-effect transistor (FET). Indicate the relative potentials of the electrodes.*

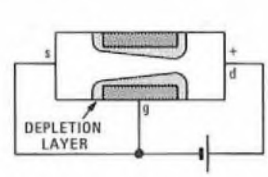
(*b*) *Explain why a high input impedance can be obtained with an FET.*

(*c*) *Sketch typical characteristic curves relating to the FET used as an amplifier.*

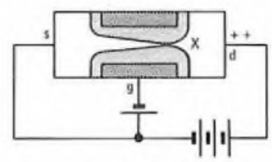
A 9 (*a*) The FET is a recent development in semiconductor triodes, and its characteristics in many ways resemble those of a thermionic tube. The FET, however, is very small, has a very high input impedance, and requires very little power. It basically consists of a semiconductor channel through which the flow of current is controlled by an electric field applied perpendicularly to the direction of current flow.



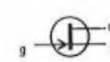
(*a*)



(*b*)



(*c*)

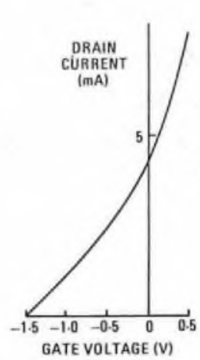


(*d*)

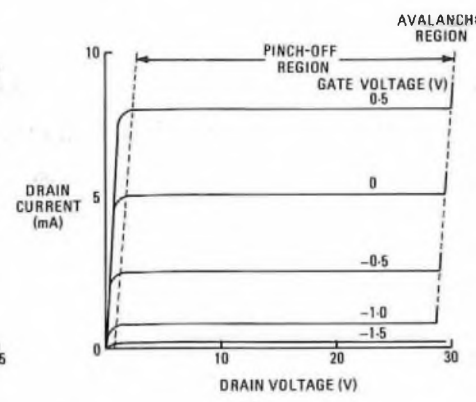
Sketch (*a*) illustrates the physical arrangement of an FET of the type known as the *n-type-channel junction-gate FET*. The channel is a tiny bar of n-type silicon. Connexions are welded to each end of the bar, and are known as the *source* (s) and the *drain* (d). Near the centre of the bar, on opposite sides, small regions of p-type silicon are formed by a diffusion process; these regions are known as the *gate* (g).

When the source, drain and gate are all at earth potential, there is no depletion layer surrounding the gate. If the drain is made positive with respect to the source, as shown in sketch (*b*), there is a flow of electrons along the channel from the source to the drain. This reverse biases the p n junctions between the gate regions and the channel, and depletion layers build up, restricting the cross-section of the channel. If the drain voltage is increased, the depletion layers widen until they touch, as shown at point X in sketch (*c*), and the resistance of the channel increases further. This condition is known as the *pinch-off* point. The current in the channel (the drain current) then remains virtually constant as the drain voltage is further increased, until avalanche breakdown occurs; this constant-current region is called the *pinch-off region*.

Referring to sketch (*c*), in the pinch-off region, electrons reaching point X from the source are injected into the depletion layer and



(*e*)



(*f*)

swept across to the drain under the influence of the reverse-biased gate–drain p n junction, in a manner similar to the action of the base–collector junction of a conventional bipolar transistor. The current flow is thus dependent on the rate at which electrons are injected into the depletion layer, and this depends on the potential difference between point X and the source. If the gate is made negative with respect to the source, as illustrated, the potential across the gate–channel junction increases, thus increasing the resistance of the channel and reducing the drain current. If the gate is made positive, the drain current increases. This action is analogous to that of the grid in a thermionic triode.

Thus, the drain current is controlled by the gate–source voltage, and an amplifier can be constructed. The symbol for an FET is shown in sketch (*d*).

(*b*) Because the gate–channel junction is a reverse-biased p n junction, the input impedance of the FET is very high.

(*c*) Sketches (*e*) and (*f*) respectively show typical drain-current/gate-voltage and drain-current/drain-voltage curves for an FET.

Q 10 (*a*) *Explain briefly how an electronic voltmeter can be used to measure the alternating current in a circuit. Why is an electronic voltmeter particularly suitable for this purpose?*

(*b*) *The circuit ABC, shown in Fig. 1, is used to measure the effective resistance of an inductor of 2 mH. When a 50 kHz supply is connected across AC, a reading of 10·0 V is measured across the 250 Ω resistor (AB), and 30 V across the inductor (BC). Find, using a phasor diagram or otherwise, the effective resistance of the inductor.*

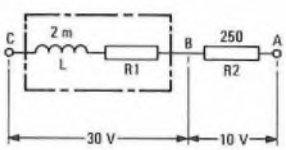
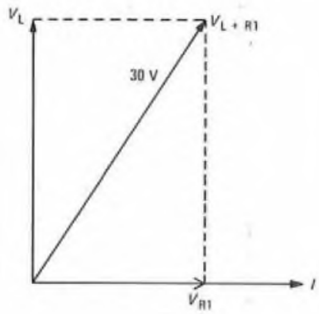


Fig. 1



(*a*)

A 10 (*a*) An electronic voltmeter can be used to measure the alternating current in a circuit by connecting it such that it records the alternating voltage across a resistor of known value in the circuit. To minimize the effect on the circuit, the resistor must have a relatively low value, and must be capable of carrying the circuit current. It must be stable and possess no self-capacitance or self-inductance. The current can then be calculated using Ohm's law.

An electronic voltmeter is particularly suitable because it has a very high input impedance, and thus takes negligible current from the circuit under test, which is then not disturbed by the instrument. Such a meter can be accurately calibrated and read over a wide range of values although, if necessary, the value of the test resistor can be altered to change the range of the measurement.

(*b*) A phasor diagram for the inductor and its resistance is shown in sketch (*a*). Let the resistance of the inductor be designated R1, and that of the 250 Ω resistor R2.

Phasor I represents the current in the circuit, and is the reference phasor. The current is given by the voltage across resistor R1 divided by the value of that resistor.

$$\therefore\ I = \frac{10}{250}\text{ A} = 40\text{ mA}.$$

Phasor V_{R1} represents the voltage across resistor R1, in phase with the current, and phasor V_L represents the voltage across the inductance, leading the current by 90°. The resultant, V_{L+R1}, is the voltage across points BC, equal to 30 V.

Now, the reactance of the inductor, X_L, is given by

$$X_L = 2\pi f L \text{ ohms,}$$

where f is the frequency (hertz), and L is the inductance (henrys).

$$\therefore\ X_L = 2\pi \times 50 \times 10^3 \times 2 \times 10^{-3} = 628{\cdot}3\ \Omega.$$

$$\therefore\ V_L = IX_L = 40 \times 10^{-3} \times 628{\cdot}3 = 25{\cdot}13\text{ V}.$$

Hence, by the theorem of Pythagoras,

$$V_{R1} = \sqrt{\{(V_{L+R1})^2 - V_L^2\}}\text{ volts,}$$

$$= \sqrt{(30^2 - 25{\cdot}13^2)} = 16{\cdot}39\text{ V}.$$

$$\therefore\ R_1 = \frac{V_{R1}}{I} = \frac{16{\cdot}39}{40 \times 10^{-3}} \approx \underline{410\ \Omega}.$$

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