SUPPLEMENT

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Contents

QUESTIONS AND ANSWERS

Answers are occasionally omitted or reference is made to earlier Supplements in which questions of substantially the same form, together with the answers, have been published. Some answers contain more detail than would be expected from candidates under examination conditions.

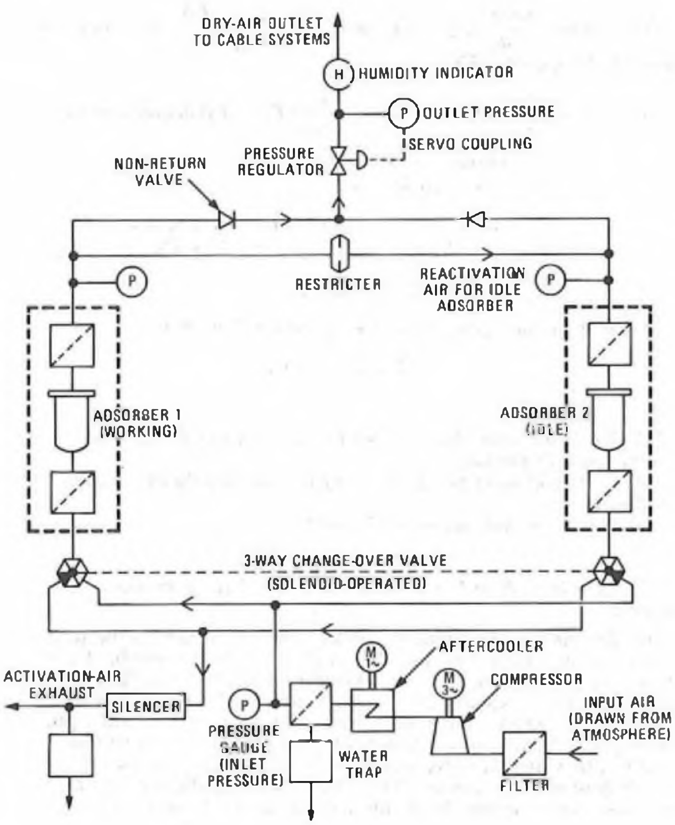
For economic reasons, alternate issues of the Supplement are published in 32-page and 16-page sizes.

LINE PLANT PRACTICE C, 1975

Students were expected to answer any 6 questions

Q 1 With the aid of a labelled block diagram, describe the operation of an adsorption type of compressor-desiccator used in a continuous-flow gas-pressurization system.

A 1 The compressor-desiccator is in 2 distinct units: the compressor and the desiccator. The compressor is an oil-sealed sliding-vane rotary air-compressor designed to run continuously without an associated air receiver. It is directly coupled to a 1·5 kW 3-phase a.c. motor, the whole unit being bolted to a tripod stand. The compressed air is passed through an aftercooler, which cools it to the ambient temperature. The aftercooler consists of a fan driven by a single-phase motor, and is bolted to the rear of the tripod with a permanently connected automatic air-filter and water-trap.



The desiccator is of the self-regenerating type, requiring no heating. It consists of 2 beds of activated alumina (called *adsorbers*), each of which is in service for 1 min at a time while the other is being regenerated by a current of air from the bed in service. The change-over is effected by a solenoid-operated valve controlled by an electronic process timer. The desiccator delivers a maximum of 3·7 kg/h at 62 kPa, and there is a large difference between the air delivery of the compressor and that of the desiccator to allow a large flow of air to reactivate the idle adsorber bed. A block diagram of the compressor-desiccator is shown in the sketch, where adsorber 1 is shown working and adsorber 2 is idle.

The air is drawn into the compressor via a filter and passed into the aftercooler. The cooled compressed air then passes to the desiccator via a filter and a moisture trap, the latter collecting any moisture condensed in the air line between the compressor and the change-over valve. The dried air from the adsorber bed passes through a non-return valve to a pressure-regulating valve that reduces the pressure of the dried air fed to the cables to the required value of 62 kPa.

To reactivate the idle adsorber bed, some of the dried air is rapidly expanded through a restrictor (an orifice) to zero pressure, to produce super-dried air. This air is then passed over the idle adsorber bed. During adsorption, the surface temperature of the bed is raised slightly and any moisture tends to evaporate. The evaporation, coupled with the passage of cool super-dried air over the alumina, cools the desiccant and returns it to a suitably dry state.

The damp exhaust air is fed through a silencer, together with the surplus water from the inlet filter. The water is collected in a small container and the air is exhausted to the atmosphere.

Q 2 (a) Explain the relationship between stress and strain in a steel wire.

(b) A steel control wire working a remote-control switch is 1·2 km long and 5 mm in diameter. The switch is operated by a pull of 2 kN. If the switch requires a movement of 250 mm to operate it, find the movement required at the controlling end of the wire. (Young's modulus for steel, E_S, is 200 GPa.)

A 2 (*a*) Within the elastic limits of a material, strain is proportional to the stress producing it. The ratio of stress to strain is constant for a material and is known as Young's modulus.

(*b*) The cross-sectional area of the wire, A, is given by

$$A = \pi \times 2{\cdot}5^2 = 19{\cdot}63 \text{ mm}^2.$$

Now, the strain, e, of the wire is given by

$$e = \frac{f}{E_S},$$

where f is the stress (pascals), given by the tension in the wire divided by its cross-sectional area.

$$\therefore\ e = \frac{2 \times 10^3 \times 10^6}{19{\cdot}63 \times 200 \times 10^9}.$$

Now, the strain is equal to the extension, x metres, divided by the original length, l metres, so that

$$x = \frac{2 \times 10^3 \times 10^6 \times 1{\cdot}2 \times 10^3}{19{\cdot}63 \times 200 \times 10^9} \text{ m} = 611 \text{ mm}.$$

Thus, the movement necessary at the controlling end

$$= 611 + 250 = \underline{861 \text{ mm}}.$$

Q 3 (a) What is the practical centre of a telephone-exchange area?

(b) Describe briefly 3 processes that assist in locating the practical centre.

(c) Fig. 1 shows the forecasts of routes leading to 3 possible exchange sites, A, B and C. Which is the practical centre and why?

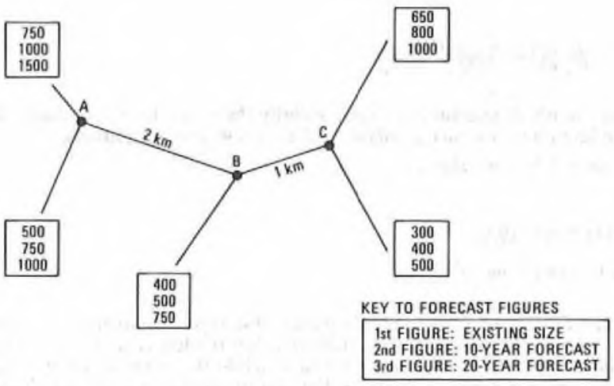


Fig. 1

A 3 (*a*) The planning of a local-line network is done on the assumption that the telephone exchange is situated at the practical centre of the area. Considering the external plant required to provide local and junction services to the existing and forecast subscribers, the practical centre is the point in the exchange area at which the cost, expressed as the present value of the annual charges, is a minimum, taking into account the use that can be made of existing line plant. No account is taken of whether a suitable site for the exchange is available at the practical centre.

(*b*) The 3 processes that assist in locating the practical centre are described below.

(*i*) On a *summary map* showing the subscriber and junction forecast distribution summarized in small sections, with the junction forecasts marked at the points at which the existing or proposed routes enter the exchange area, a north–south line is drawn in such a position as to divide the forecast into 2 equal parts; that is, so that there are as many local and junction lines east of the line as there are west of it. A similar east–west line is then drawn and the intersection of the 2 lines indicates where the practical centre can be expected to lie.

(*ii*) If a straight-line diagram showing the main line-plant requirements has 3 or more principal routes radiating from one point, and the number of lines forecast (including junctions) on any one route does not exceed the sum of those on the remaining routes, then that point is likely to be the practical centre.

(*iii*) A density map is prepared, and the location of the practical centre within the densest part of the area may be apparent. If the summary map shows only one point of intersection of principal routes within the densest area, then that point is likely to be the practical centre.

(*c*) The above methods may indicate several possible positions, as in the example in Fig. 1. Progress is then made by finding the pair-kilometre centre; this is the point from which the pair-kilometre value to meet the 20-year forecast is a minimum. The table compares the pair-kilometre values for the 3 sites.

Site of Exchange	Pair-Kilometre Value
A	2 × 750 + 3 × (1000 + 500) = 6000
B	2 × (1500 + 1000) + 1000 + 500 = 6500
C	3 × (1500 + 1000) + 750 = 8250

Thus, the practical centre is at $\underline{A}$.

Q 4 An aerial-cable suspension wire is erected over a span of 50 m at a tension of 4·2 kN and a temperature of 12°C. A temperature change reduces the tension to 3·9 kN. Calculate

(a) the dip of the wire when the tension is 4·2 kN,

(b) the dip of the wire when the tension is 3·9 kN, and

(c) the temperature when the tension is 3·9 kN.

Young's modulus for steel, E_S, is 200 GPa, the diameter of the wire, d, is 4 mm, the weight of the wire, W, is 1·1 N/m, and the coefficient of linear expansion, α, is 20×10^{-6}/°C.

A 4 (*a*) The dip, D metres, of a wire is given by

$$D = \frac{WL^2}{8T} \text{ metres,}$$

where L is the length of the span (metres), and T is the tension (newtons).

Therefore, at a tension of 4·2 kN,

$$D_1 = \frac{1{\cdot}1 \times 50^2}{8 \times 4{\cdot}2 \times 10^3} \text{ m} = \underline{81{\cdot}85 \text{ mm}}.$$

(*b*) At a tension of 3·9 kN,

$$D_2 = \frac{1{\cdot}1 \times 50^2}{8 \times 3{\cdot}9 \times 10^3} \text{ m} = \underline{88{\cdot}14 \text{ mm}}.$$

(*c*) The alteration in the length of the wire from l_1 to l_2 metres when the temperature rises from t_1 to t_2 degrees celsius is given by

$$l_2 - l_1 = \alpha l_1(t_2 - t_1) - \frac{l_1(T_1 - T_2)}{AE_S} \text{ metres,} \quad \ldots\ldots (1)$$

where A is the cross-sectional area of the wire (metres²). (The first term of the right-hand side of equation (1) is the expansion due to the rise in temperature, and the second term is the elastic contraction due to the reduction in tension from T_1 to T_2 newtons.)

Now, $$l_1 = L + \frac{8D_1^2}{3L} \text{ metres and } l_2 = L + \frac{8D_2^2}{3L} \text{ metres.}$$

Substituting for l_1 and l_2 in equation (1) gives

$$\frac{8}{3L}(D_2^2 - D_1^2) = \alpha L(t_2 - t_1) + \frac{8\alpha D_1^2}{3L}(t_2 - t_1)$$

$$- \frac{L(T_1 - T_2)}{AE_S} - \frac{8D_1^2(T_1 - T_2)}{3AE_SL} \text{ metres.}$$

The terms $\frac{8\alpha D_1^2}{3L}(t_2 - t_1)$ and $\frac{8D_1^2(T_1 - T_2)}{3AE_SL}$ are very small and can be ignored. Thus,

$$t_2 - t_1 = \frac{8}{3\alpha L^2}(D_2^2 - D_1^2) + \frac{1}{\alpha AE_S}(T_1 - T_2) \text{ degrees celsius,}$$

$$= \frac{8 \times (0{\cdot}088^2 - 0{\cdot}082^2)}{3 \times 20 \times 10^{-6} \times 50^2}$$

$$+ \frac{4{\cdot}2 \times 10^3 - 3{\cdot}9 \times 10^3}{20 \times 10^{-6} \times \pi \times 2^2 \times 10^{-6} \times 200 \times 10^9} \text{ °C,}$$

$$= 0{\cdot}054 + 5{\cdot}968 = 6{\cdot}022\text{°C}.$$

Thus, the temperature when the tension is 3·9 kN is

$$12 + 6 = \underline{18\text{°C}}.$$

Q 5 (a) State and describe briefly the 3 main groups into which natural rock is divided.

(b) State and describe the 5 main fractions into which a mineral soil is divided.

(c) What is a moisture-unstable soil?

A 5 (*a*) Natural rock is divided into the 3 main groups described below.

(*i*) *Igneous Rocks* Igneous rocks can be regarded as the ultimate origin of all rocks forming the solid crust of the earth. They are formed by the cooling and crystallization of molten rock that has been injected into the earth's crust from below, and have a very complex chemical structure. Igneous rocks are estimated to form approximately 95% by volume of the outer 15 km of the earth's crust but, at the surface, they are extensively covered by sedimentary rocks.

(*ii*) *Sedimentary Rocks* The effects of weathering of exposed igneous rocks cause both mechanical and chemical breakdown, often called *primary* and *secondary weathering*. Primary-weathering processes include the disruption of rock due to differential expansion and contraction following temperature changes, and abrasion by wind and water-borne particles. Secondary-weathering processes include the leaching action of water containing dissolved carbon dioxide. The larger pieces of rock (mainly products of mechanical disintegration) form gravels, and the smaller pieces (products of chemical breakdown) are carried away by rivers and streams to form new rock beds. These deposited beds are the sedimentary rocks.

(*iii*) *Metamorphic Rocks* Periodically, the rocks of the earth's surface are subjected to great pressures and heat due to shrinkage and buckling of the earth's crust. These great pressures and the accompanying heat can occur over relatively large areas for long periods and cause considerable modification of the mineral structure of the rocks. Mineral changes also occur when, due to surface movements, the solid rock comes into contact with molten rock erupting from below.

Rocks altered by heat alone, without any considerable pressure, are called *thermal metamorphic rocks*, and this process usually results in a rock that is much harder and tougher than the original. Rocks of simple composition merely undergo recrystallization without chemical change. Thus, sandstones are converted to quartzite and limestones are converted to marble. Rocks of a more complex composition, such as igneous rocks and impure sedimentary rocks, can be so changed chemically that the nature of the original rocks is completely altered. The resulting rock is called *hornfels*. Thermal metamorphism almost always improves rocks from the road-making point of view, and some hornfels are among the best road-making aggregates.

Rocks altered by pressure and heat occur over larger areas than thermal metamorphic rocks and are called *regional metamorphic rocks*. The main characteristic of these rocks is a banded, or laminated, structure. Examples of regional metamorphic rocks are shale, slate and granulite. Few of these rocks are suitable for road-making aggregates.

(*b*) The 5 main fractions into which a mineral soil is divided are described below.

(*i*) *Boulders* Boulders are stones with an equivalent particle diameter of over 60 mm. (The equivalent particle diameter of a soil is based on the diameter of a hypothetical sphere of the same material which gives the same soil characteristics as the original particle.)

(*ii*) *Gravel Fraction* The gravel fraction consists of particles between 60 mm and 2 mm equivalent particle diameter. Gravel consists of particles of coarse material resulting from the disintegration of rocks. These particles have often been transported by water and, as a result, they are worn and have rounded shapes. The particles are generally found in sand or sand mixed with clay.

(*iii*) *Sand Fraction* The sand fraction consists of particles between 2·0 mm and 0·06 mm equivalent particle diameter. Sands are usually composed of particles of silica and quartz. Dry or damp sand is a very good foundation material provided that it cannot spread laterally or be washed out.

(*iv*) *Silt Fraction* The silt fraction consists of particles between 0·06 mm and 0·002 mm equivalent particle diameter. Silt particles are similar, physically and chemically, to particles in the sand fraction; the differences in characteristics are due mainly to the smaller particle size. Silt is an unsuitable load-bearing material. Sub-divisions exist for both sand and silt, embracing coarse, medium and fine fractions.

(*v*) *Clay Fraction* The clay fraction consists of all particles smaller than 0·002 mm equivalent particle diameter. The particles in the clay fraction differ from those in the other fractions in their chemical constitution and physical properties. Chemically, they consist mainly of hydrated alumino-silicates. Physically, the clay particles differ from the coarser fractions in that they are flat and elongated. Clay soils vary widely in character, ranging from soft wet clays, which squeeze out laterally under a light vertical pressure, to hard clays capable of bearing heavy loads without yielding.

(*c*) When water is added to a group of closely-packed particles, there is mutual attraction between the water and the particle surfaces due to adsorption and surface tension. This mutual attraction causes the particle surfaces to be coated with water. The distance between adjacent particles is therefore increased and the total volume of the material is increased. The water film is extremely thin, but, for soils having a small equivalent particle diameter, its effect is very pronounced. The volume of these soils thus varies considerably with changing moisture content. This effect occurs in clay and silt, and both soils are therefore considered to be *moisture unstable*. Some clays show a change in volume of up to 30% with a change in moisture content.

Q 6 A cable bracket is supported in a channel bearer having an 8 mm diameter steel pin, as shown in Fig. 2. If the total weight, W, of the cables and the bracket is taken to be 600 N acting as shown, calculate

(*a*) *the resultant force on the securing pin, and*
(*b*) *the shear stress, in pascals, on the securing pin.*

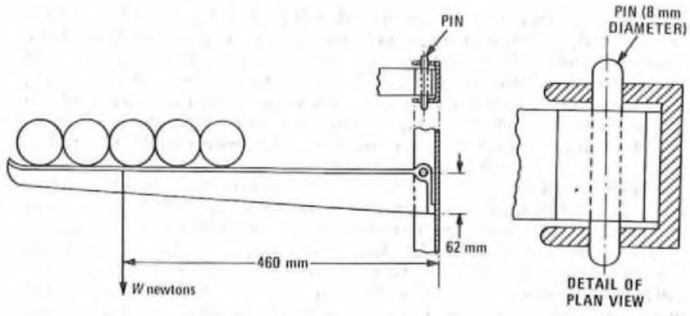


Fig. 2

A 6 Taking moments about the pin,

$$600 \times 460 = 62P \text{ newton millimetres,}$$

where P is the reaction at the rear lower edge of the bracket (newtons).

$$\therefore\ P = \frac{600 \times 460}{62}\ \text{N} = 4{\cdot}452\ \text{kN}.$$

The resultant force, F newtons, on the pin is the resultant of W and P, and is given by

$$F = \sqrt{(W^2 + P^2)} = \sqrt{(600^2 + 4452^2)}\ \text{N} = \underline{4{\cdot}492\ \text{kN}}.$$

Since the pin is in double shear, half this force acts at each bearing point, so that the shear stress in the pin, s pascals, is given by

$$s = \frac{F}{2\pi r^2}\ \text{pascals,}$$

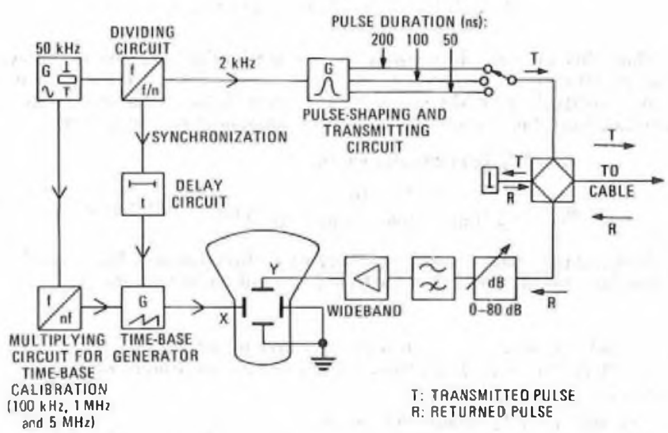
where r is the radius of the pin (metres).

$$\therefore\ s = \frac{4492}{2\pi \times 4^2 \times 10^{-6}}\ \text{Pa} = \underline{44{\cdot}68\ \text{MPa}}.$$

Q 7 (*a*) *Draw a labelled block diagram of a pulse tester used to locate faults on cable pairs.*

(*b*) *Describe briefly the principle involved in using this equipment to locate a cable fault.*

A 7 (*a*) The sketch shows a block diagram of a pulse tester.



(*b*) The pulse-testing method of fault location, used on coaxial cables, is based on the principle that, if an electric pulse is applied to a line on which an impedance irregularity exists (possibly due to a fault), a portion of the pulse (the *echo*) is reflected back to the sending end. The interval between the transmitted and reflected pulse is measured by displaying the reflected pulse on a cathode-ray tube having a specially calibrated time base. As the velocity of propagation of the pulse is constant in a cable of uniform construction, the delay is a measure of the distance between the fault and the testing end.

The pulse-echo method of testing enables the type of fault to be established. A fault that increases the impedance at the fault point (a series fault) produces a pulse in the same direction as the incident pulse. A fault that reduces the impedance at the fault point (a shunt fault) produces a pulse in the opposite direction to the incident pulse.

A undirectional pulse is used and, while the shape of the pulse is not critical, a pulse from which the lower frequencies have been removed avoids distortion of the shape of the reflected pulse. The removal of the lower frequencies from the pulse before amplification allows the output of the higher frequencies to be increased without overloading the amplifying equipment associated with the test set.

The duration of the pulse is indicated by the width at the half pulse-height. The pulse width must be narrow enough to allow complete transmission of the pulse before any reflection is likely to be returned from the nearest fault. A narrow pulse also enables a greater sensitivity to be obtained because smaller impedance irregularities can be detected. The use of a narrow pulse has disadvantages when testing on long lengths of cable. The high frequencies making up the narrow pulse suffer considerable attenuation in a long cable, and the reflected pulse may therefore be distorted beyond recognition. To accommodate these conflicting factors, 3 pulse widths are provided. The smallest pulse width is used for short lengths of cable (for example, factory lengths), while the largest width is used for overall tests on repeater sections.

The pulse-repetition rate must be such that the interval between each pulse is sufficient to allow any reflections from the most distant fault point to be returned before the next pulse is transmitted. For coaxial cables, where the distance between repeater points is approximately 10 km (a return path of 20 km), a pulse-repetition rate of 2000 pulses/s is used.

Q 8 (a) What is meant by double reinforcement in connexion with reinforced concrete?

(b) When is reinforcement of this type used?

(c) Calculate the dimensions of a doubly-reinforced concrete beam that will support a load of 10 kN applied at the centre of a 3 m span. The stress in the steel must not exceed 130 MPa, and the depth of the beam must not exceed 400 mm.

A 8 (*a*) Double reinforcement describes the situation where reinforcement is provided in both the compressive and tensile sides of a reinforced member, such as a beam. The concrete then accepts no force at all, merely serving to locate the reinforcing rods. The neutral axis is therefore equidistant from the top and bottom.

(*b*) Double reinforcement is useful where space is limited and the cross-sectional area of concrete required using single reinforcement is greater than can be allowed. The use of compressive reinforcement reduces the amount of concrete required and, hence, reduces the cross-section needed.

(*c*) If a depth of 50 mm of concrete is allowed to cover the reinforcing bars, the effective depth of the beam is 400 − 100 = 300 mm.

The maximum bending moment, M newton metres, is at the centre of the beam, and is given by

$$M = \frac{10 \times 10^3}{2} \times \frac{3}{2}\,\text{N m} = 7{\cdot}5\ \text{kN m}.$$

Half this moment is resisted by the steel reinforcement in each flange. Thus, if A_S is the cross-sectional area of the steel in each flange (metres²), f_S is the stress in the steel (pascals), and h is the distance from the neutral axis to the reinforcement (metres), then

$$M = 2f_SA_Sh \text{ newton metres.}$$

$$\therefore\ A_S = \frac{7{\cdot}5 \times 10^3}{2 \times 130 \times 10^6 \times 150 \times 10^{-3}}\,\text{m}^2 = 192{\cdot}3\ \text{mm}^2.$$

Now, 12 mm diameter bars have a cross-sectional area of 113·1 mm². Therefore, two 12 mm diameter bars are required in each flange.

Q 9 (a) For what purpose is a psophometer used?

(b) With the aid of sketches, describe how measurements of the following are made:

(i) longitudinal psophometric voltage,

(ii) transverse psophometric voltage across a telephone pair, and

(iii) transverse psophometric voltage across a circuit when a call has been set up.

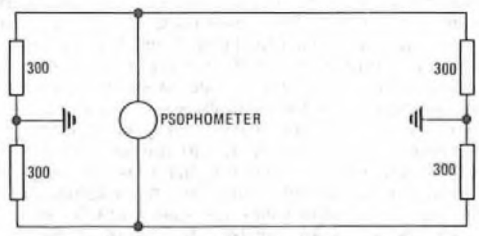
A 9 (*a*) A psophometer is an instrument that gives a visual indication of the aural effect of circuit noise. The instrument consists essentially of a network having a response equivalent to that of the combination of a telephone receiver and human ear. The network is followed by an electronic voltmeter calibrated in millivolts.

(*b*) (*i*) Sketch (*a*) illustrates the method of measuring longitudinal psophometric voltage. The psophometer is connected in series with one wire of the telephone pair, the wire being earthed at both ends. Thus, the psophometer measures the voltage induced into the wire.

(*ii*) Sketch (*b*) illustrates the method of measuring the transverse psophometric voltage across a telephone pair. The test is made with

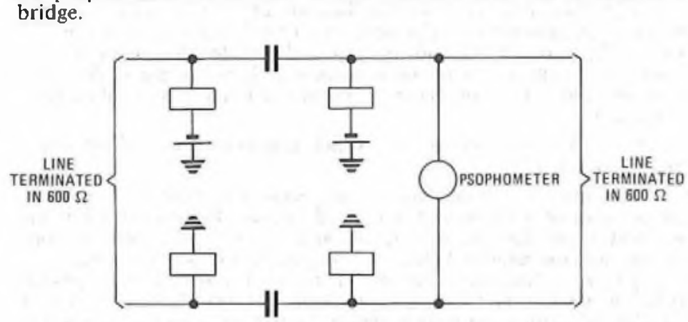
(*a*)

a balanced 600 Ω termination at each end of the pair, and the psophometer is connected across the pair at one of the terminations. In this case, the psophometer measures the transverse voltage due to the unbalance of the line.



(*b*)

(*iii*) Sketch (*c*) illustrates the method of making a transverse measurement across a circuit when a call has been set up. The telephones at each end of the line are replaced by 600 Ω terminations, and the psophometer is connected across the line side of the transmission bridge.



(*c*)

Q 10 (a) For each of the following ground conditions, describe, with sketches, a type of lining that could be used when building a 2·1 m diameter tunnel:

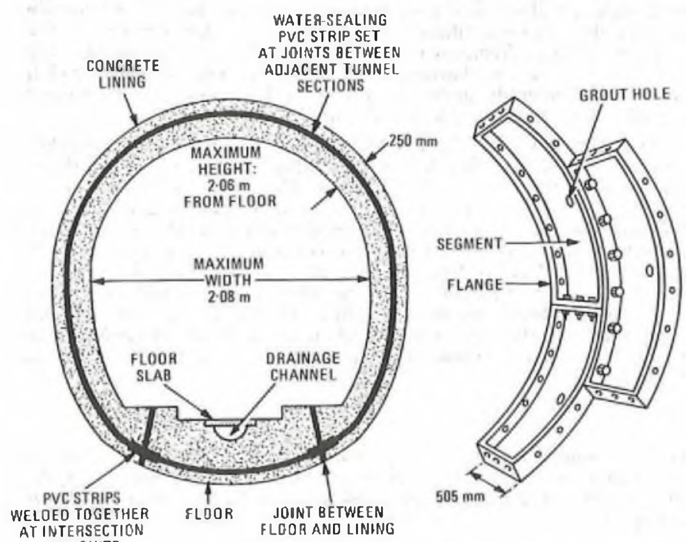
(i) stable rock,

(ii) hard clay, and

(iii) soft and loose soil.

(b) Describe, with a sketch, a method of limiting water seepage through both the longitudinal and circumferential joints of a cast-iron lining.

A 10 (*a*) (*i*) For tunnels built in stable rock, a concrete lining is usually made *in situ*. The tunnel is normally completely excavated before the lining is placed in position, arch supports being used to support the tunnel roof if there is a possibility of collapse. The tunnel



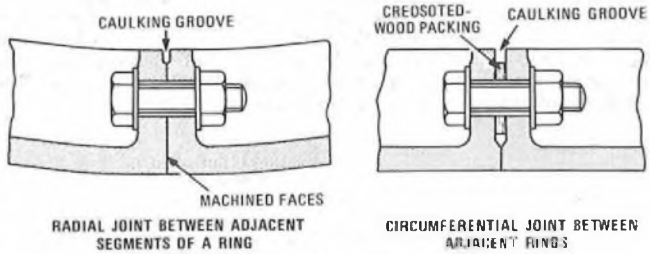
(*a*) (*b*)

is first cut roughly to shape, and then trimmed to size using a template. The concrete lining is formed using a specially designed shutter fitted on a carriage running on rails. Sketch (*a*) illustrates the lining used in a rock tunnel.

(*ii*) For tunnels in hard clay, precast concrete or cast-iron lining segments made up into rings are used. The rings are placed in position as the excavation proceeds, and provide support for the clay. The space between the rings and the surrounding clay is grouted with cement. Sketch (*b*) shows a type of cast-iron lining that could be used for tunnelling in clay.

(*iii*) For tunnels in soft or loose soil, the lining is again normally cast-iron or concrete segments, but fitting the lining presents more difficulty than working in clay; the ground is not self-supporting long enough for the rings to be fitted in the normal way. To overcome this, a device known as a *shield* is used. The shield supports the excavation until the lining segments are assembled in position.

(*b*) The surfaces between adjacent segments in a ring are machined to give intimate contact between the faces. The flanges of adjacent rings have a strip of creosoted wood inserted between them. These precautions effectively prevent water passing through the joints. However, if water does tend to pass through joints, they can be caulked with lead or a waterproof asbestos compound in the caulking grooves provided, the wood packing first being removed. It is normal to caulk the joints underneath the floor before the invert is cast.



(*c*)

The 2 types of joint are illustrated in sketch (*c*).

BASIC MICROWAVE COMMUNICATION C, 1975

Students were expected to answer any 6 questions

Q 1 (*a*) *Give 3 reasons why frequencies about 4 GHz are well suited to multichannel point-to-point microwave links.*

(*b*) *Fig. 1 represents an electromagnetic wave propagated in the direction A–B and having its E-component in the direction H–J. Reflection from a perfectly conducting surface, XY, occurs at point B. With the aid of diagrams, explain what happens at reflection to the E-component in the directions*

(*i*) *parallel to XY, and*

(*ii*) *perpendicular to XY.*

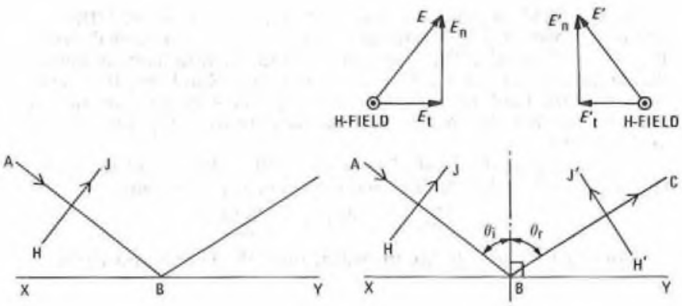


Fig. 1 (*a*)

A 1 (*a*) Frequencies about 4 GHz are well suited to multichannel point-to-point microwave links because

(*i*) a frequency of 4 GHz is sufficiently high to permit the use of a bandwidth capable of accommodating wideband multichannel systems,

(*ii*) high gains are obtainable from aerials of a manageable size, permitting the use of transmitter powers of only a few watts, and

(*iii*) the resulting narrow beams minimize mutual interference with other systems and enable free-space propagation to be achieved over line-of-sight radio paths.

(*b*) The tangential and normal components of the incident E-field are represented in sketch (*a*) as E_t (parallel to XY) and E_n (perpendicular to XY) respectively.

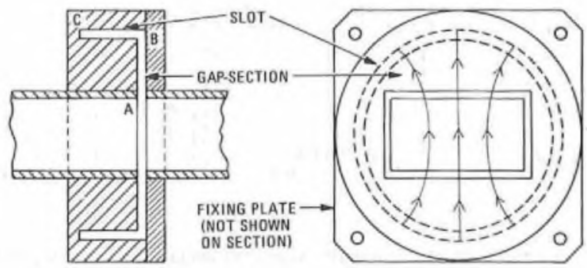
A tangential E-field must be zero adjacent to a perfectly conducting surface because it is impossible to generate a potential gradient in a perfect conductor. This condition is satisfied by making the reflected field, E_t', equal to $-E_t$, as shown. A perpendicular E-field can exist at the conducting surface, so that $E_n' = E_n$. The reflected components, E_t' and E_n', add vectorially to give E', the field of a wave reflected at an angle θ_r equal to the angle of incidence, θ_i. The field is in the direction H′–J′, as shown, and is propagated in the direction B–C.

(The H-field is perpendicular to the paper and is of the same sense and magnitude for both incident and reflected waves. The H-field adjacent to the conducting surface induces a sheet of current at right-angles to the H-field.)

Q 2 (*a*) *Draw, and briefly describe, a choke flange suitable for joining 2 lengths of rectangular waveguide.*

(*b*) *For dominant-mode propagation in a waveguide having dimensions 50 × 24 mm, the attenuation is found to be least when the guide wavelength is 60 mm. Calculate the frequency for minimum attenuation.*

A 2 (*a*) A choke flange is shown in the sketch. It incorporates a series-connected branching line, ABC, formed by the gap-section AB and the slot BC, where AB = BC. ABC is half a wavelength long so that the short circuit at C presents zero impedance at A. The mechanical contact of the joint is at B, the high-impedance point half-way along the line. The wall currents cross the joint on only the wide sides of the guide. On the narrow sides, the wall currents are parallel to the gap, so that the segments of the slot adjacent to the narrow sides are not strictly necessary.



(*b*) For a rectangular waveguide, the guide wavelength, λ_g metres, the free-space wavelength, λ_0 metres, and the broad dimension of the guide, *a* metres, are related by the expression

$$\frac{1}{\lambda_g{}^2} = \frac{1}{\lambda_0{}^2} - \frac{1}{4a^2}.$$

$$\therefore\ \lambda_0 = \left(\frac{1}{\dfrac{1}{\lambda_g{}^2} + \dfrac{1}{4a^2}}\right)^{1/2} \text{ metres.}$$

Since $c = \lambda_0 f$ metres/second, where *c* is the speed of light (equal to 3×10^8 m/s), and *f* is the frequency of operation (hertz), then

$$f = c \times \sqrt{\left(\frac{1}{\lambda_g{}^2} + \frac{1}{4a^2}\right)} \text{ hertz,}$$

$$= 3 \times 10^8 \times \sqrt{\left(\frac{1}{60^2 \times 10^{-6}} + \frac{1}{4 \times 50^2 \times 10^{-6}}\right)} \text{ Hz,}$$

$$= 3 \times 10^8 \times 10^3 \times \sqrt{\left(\frac{1}{60^2} + \frac{1}{4 \times 50^2}\right)} \text{ Hz,}$$

$$= \underline{5 \cdot 83 \text{ GHz.}}$$

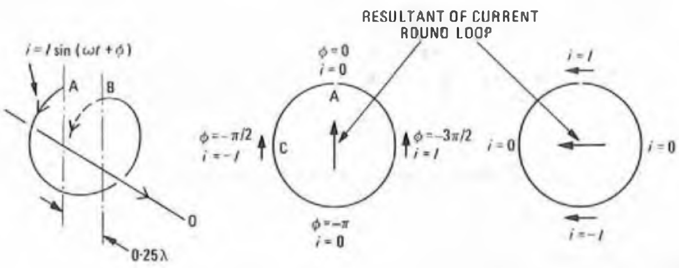
Q 3 (*a*) *An aerial has the form of a helix, the pitch of which is 0·25λ and the length of each turn 1·25λ, where λ is the wavelength. By considering a single turn, explain why the aerial*

(*i*) *forms a beam, and*

(*ii*) *radiates a circularly-polarized wave.*

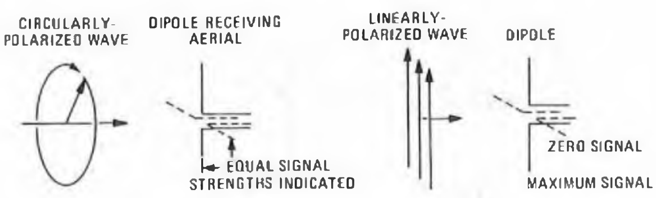
(*b*) *With the aid of a diagram, explain a practical method of verifying that the aerial radiates a circularly-polarized wave.*

A 3 (*a*) (*i*) Consider a sinusoidal current, $i = I \sin \omega t$, fed into the helical aerial at position A, as shown in sketch (*a*). The phase of the wave is delayed by $5\pi/2$ rad in travelling $1{\cdot}25\lambda$ along the helix to position B. The direct wave from A in the direction of the helix to O is delayed by $\pi/2$ rad by the time it reaches position B since the direct distance from A to B is $0{\cdot}25\lambda$. The phase difference between the waves is therefore 2π rad, and they appear to be in phase to an observer in the far-field at O. Similarly, the waves from each turn of the helix all add in phase, but this is not so off the axis, where the combined signal is weaker. This interference effect produces the familiar radiation pattern of a main beam with weaker side-lobes.



(*a*) (*b*) (*c*)

(*ii*) To an observer at O, the helix looks like a single loop of circumference λ, as shown in sketch (*b*). At time $t = 0$, the current at point A is $I \sin 0 = 0$. At point C, the phase angle ϕ, is delayed by $\pi/2$ rad, so that the value of the current is $I \sin(-\pi/2) = -I$, and so on round the loop. The resultant current is as shown. At the instant in time when $\omega t = \pi/2$ rad, the resultant current has rotated by $\pi/2$ rad, as shown in sketch (*c*). The resultant current phasor thus rotates once for every cycle of the feed current, and the helix therefore radiates a circularly-polarized wave.



(*d*) (*e*)

(*b*) A dipole or other linearly-polarized aerial, used in conjunction with a detector, indicates a constant signal strength from a circularly-polarized wave regardless of the angle to which the dipole is rotated about the axis of the direction of propagation, as illustrated in sketch (*d*). If the dipole is rotated in a linearly-polarized wave, the received signal varies from a maximum when the planes of polarization of the wave and the aerial coincide, to zero when the planes are at right-angles, as illustrated in sketch (*e*). If a mixture of plane and circular polarization is present in the wave, the resultant polarization is elliptical, and the detector shows maxima and minima as the dipole is rotated. A steady signal therefore indicates circular polarization.

Q 4 A $\lambda/8$ length of loss-free coaxial cable (where λ is the wavelength) is terminated in a resistive load. The voltage and current conditions at the input to the cable are represented by the phasor diagrams in Fig. 2, where V_i represents the incident voltage, V_r the reflected voltage, I_i the current towards the load, and I_r the current away from the load.

(*a*) (*i*) *What is the characteristic impedance of the cable?*

(*ii*) *State, with reasons, whether the input impedance of the cable is resistive, purely inductive or purely capacitive.*

(*b*) *Beneath copies of Fig. 2, draw phasor diagrams representing the voltage and current conditions at the load at the same instant. Hence, determine the load resistance.*

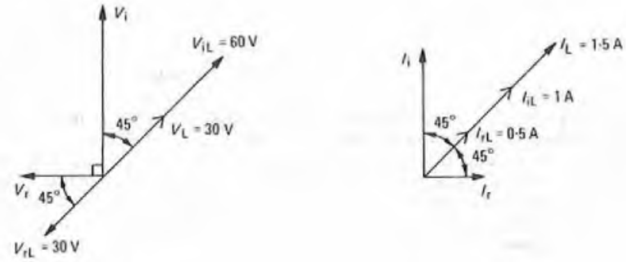
Fig. 2

A 4 (*a*) (*i*) The characteristic impedance, Z_0 ohms, of the cable is given by

$$Z_0 = \frac{V_i}{I_i} = \frac{60}{1} = \underline{60\ \Omega.}$$

(*ii*) The phase-angle of the impedance at the input to the cable is the angle between the current and voltage at that point. The voltage phasor is the sum of the incident and reflected voltage phasors, which, from Fig. 2, is $\sqrt{(30^2 + 60^2)} = 67{\cdot}1$ V at an angle of $+\tan^{-1}(30/60) = +26{\cdot}57°$ relative to V_i (assuming the normal convention of counter-clockwise rotation of phasors). Similarly, the current phasor is $\sqrt{(1^2 + 0{\cdot}5^2)} = 1{\cdot}12$ A at an angle of $-\tan^{-1}(0{\cdot}5/1) = -26{\cdot}57°$ relative to I_i. (From Fig. 2, phasors V_i and I_i are in phase; hence, the characteristic impedance is resistive.)

The angle between the current and voltage at the cable input is therefore $2 \times 26{\cdot}57° = 53{\cdot}14°$ (and the magnitude of the impedance is $67{\cdot}1/1{\cdot}12 = 60\ \Omega$). This impedance is resistive with an inductive component because the voltage phasor is leading the current phasor. The input impedance is equivalent to a resistance of $60 \cos 53{\cdot}14° = 36\ \Omega$ in series with an inductive reactance of $60 \sin 53{\cdot}14° = 48\ \Omega$.



(*a*) (*b*)

(*b*) The incident voltage at the load, V_{iL}, lags V_i by 45° (the phase difference over a $\lambda/8$ length of cable), while the reflected voltage, V_{rL}, leads V_r by 45°. The 2 phasors are thus in antiphase, as shown in sketch (*a*). There are no losses in the cable. Similarly, the incident current at the load, I_{iL}, lags I_i by 45°, and the reflected current at the load, I_{rL}, leads I_r by 45°. These 2 phasors are thus in phase, as shown in sketch (*b*).

The voltage at the load, V_L, is $60 - 30 = 30$ V, and the current, I_L, is $1 + 0{\cdot}5 = 1{\cdot}5$ A. The load resistance is therefore

$$V_L/I_L = 30/1{\cdot}5 = \underline{20\ \Omega.}$$

(Note that V_L and I_L are in phase; thus, the load is resistive.)

Q 5 A metal diaphragm, having a small central hole, forms a cavity at the end of a length of rectangular waveguide of cross-section 30 × 16 mm, as shown in Fig. 3.

(*a*) *Determine the length, l metres, of the cavity if it is to resonate at 6·25 GHz.*

(*b*) *On a copy of Fig. 3, show where a small probe should be located in the cavity to detect resonance.*

(*c*) *Explain whether, at the resonant frequency, there is a large or small standing-wave ratio in the portion of the waveguide feeding the cavity.*

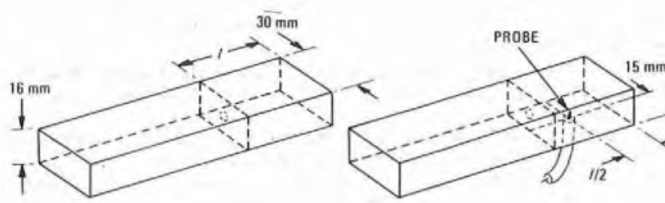


Fig. 3 (*a*)

A 5 (*a*) The small central hole couples a small proportion of the energy in the waveguide into the cavity. The dominant H_{101} mode resonates in the cavity when the frequency is such that the guide wavelength, λ_g metres, is twice the length of the cavity; that is, when $\lambda_g = 2l$ metres.

The equation for the guide wavelength of an H_{10} wave is

$$\frac{1}{\lambda_g^2} = \frac{1}{\lambda_0^2} - \frac{1}{4a^2}, \qquad \ldots\ldots (1)$$

where λ_0 is the free-space wavelength (metres), and a is the broad dimension of the waveguide (metres).

Now, $\lambda_0 = c/f$ metres, where f is the frequency (hertz), and c is the velocity of light, equal to 3×10^8 m/s. Therefore, at $f = 6{\cdot}25$ GHz,

$$\lambda_0 = \frac{3 \times 10^8}{6{\cdot}25 \times 10^9}\ \text{m} = 48\ \text{mm}.$$

Substituting $\lambda_g = 2l$ in equation (1) and rearranging gives

$$l = \frac{1}{2} \times \frac{1}{\sqrt{\left(\frac{1}{\lambda_0^2} - \frac{1}{4a^2}\right)}}\ \text{metres,}$$

$$= \frac{1}{2} \times \frac{1 \times 10^{-3}}{\sqrt{\left(\frac{1}{48^2} - \frac{1}{4 \times 30^2}\right)}}\ \text{m} = \underline{40\ \text{mm}}.$$

(*b*) To detect resonance, a small detector probe should be placed in line with the electric field and at the point of maximum intensity, as shown in sketch (*a*); that is, vertically at the centre of the cavity.

(*c*) The small coupling hole extracts only a small proportion of energy from the waveguide. The remainder of the energy is reflected back along the guide from the near short-circuit of the metal diaphragm. The reflected wave is thus nearly as strong as the incident wave. The 2 waves add at certain points to give almost twice the incident voltage, and subtract at other points to give a value approaching zero. Hence, the standing-wave ratio is large.

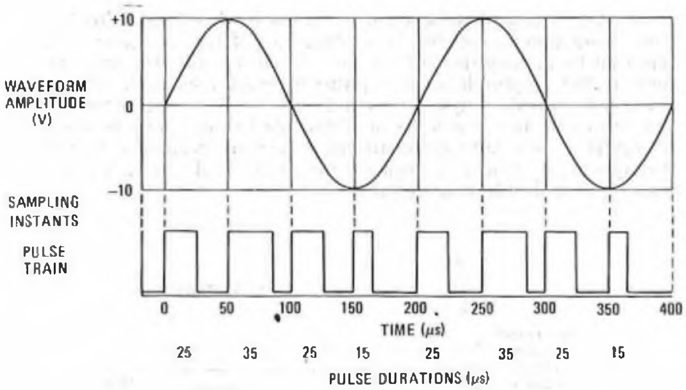
Q 6 (*a*) *A communication system using pulse-duration modulation is capable of handling sine waves of maximum amplitude 25 V and frequencies up to 10 kHz. State*

(*i*) *the minimum sampling rate that can be permitted, and*
(*ii*) *the maximum pulse duration that can be permitted.*

(*b*) *The system is used to sample a 5 kHz sine wave having an amplitude of 10 V. Sketch the sine wave to be sampled. Beneath this, in the appropriate time relationship, plot the pulse train that will result. The duration of each pulse must be indicated.*

A 6 (*a*) (*i*) The minimum sampling rate is twice the modulation frequency, and is therefore $\underline{20 \times 10^3\ \text{samples/s}}$.

(*ii*) The pulse duration must not exceed the reciprocal of the sampling rate; that is, $1/(20 \times 10^3)\ \text{s} = \underline{50\ \mu\text{s}}$. Otherwise, the pulse stream forms a continuous signal. In practice, a guarding period also has to be inserted to maintain separation between the pulses.



(*b*) The sketch shows the sine wave to be sampled and the resulting pulse train. It is assumed that a value of +25 V produces a pulse duration of 50 μs, and that a value of −25 V produces zero pulse duration. This gives proportional values of a duration of 15 μs for a value of −10 V, 25 μs for 0 V, and 35 μs for +10 V.

Q 7 (*a*) *What is meant by the term white noise?*

(*b*) *Write down an expression for the noise power available from a resistor and explain each term in the formula.*

(*c*) *A microwave receiver, having a bandwidth of 8 MHz, is required to give an output signal-to-noise ratio of better than 55 dB for a noise-free input signal of 0·1 μW. Taking kT as 4×10^{-21} J, determine whether this requirement will be achieved if the receiver has a noise factor of 12 dB.*

A 7 (*a*) Noise is said to be white when the noise power per unit bandwidth is spread uniformly over the frequency range concerned.

(*b*) The noise power available from a resistor is $\underline{kTB\ \text{watts}}$, where k is Boltzmann's constant ($1{\cdot}38 \times 10^{-23}$ J/K), T is the absolute temperature of the resistor (kelvins) and can be taken to be 290 K at normal ambient temperature, and B is the bandwidth (hertz) over which the noise power is measured.

Boltzmann's constant is the power due to thermal agitation available at the terminals of a resistor per kelvin per hertz.

(*c*) The noise power available at the input to the microwave receiver

$$= kTB\ \text{watts},$$

$$= 4 \times 10^{-21} \times 8 \times 10^6\ \text{W} = 32 \times 10^{-9}\ \mu\text{W}.$$

The noise-free input-signal level is 0·1 μW, and the input signal-to-noise ratio

$$= 10 \log_{10} \frac{0{\cdot}1}{32 \times 10^{-9}} = 64{\cdot}95\ \text{dB}.$$

The output signal-to-noise ratio is degraded by the noise factor (12 dB) due to additional noise generated in the receiver. The output signal-to-noise ratio is therefore

$$64{\cdot}95 - 12 = 52{\cdot}95\ \text{dB},$$

which does <u>not</u> achieve the required value of 55 dB.

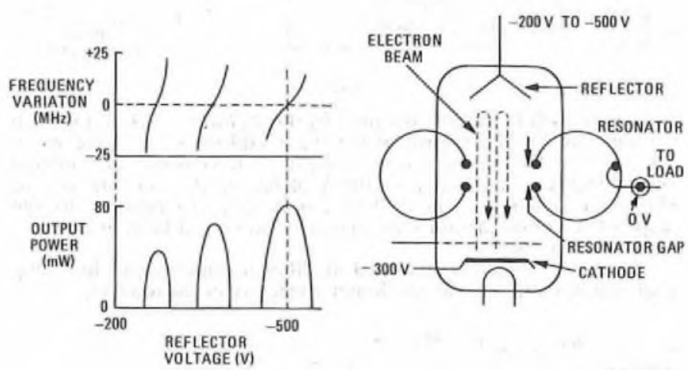
Q 8 (*a*) *With the reflector voltage held at − 500 V, the cavity of a reflex klystron is tuned to give a maximum output of 80 mW at a requency of 9·5 GHz. Explain, with the aid of diagrams, the variation in*

(*i*) *the frequency, and*
(*ii*) *the ouput power*

as the reflector voltage is adjusted continuously to about −200 V.

(*b*) *Explain briefly one practical application of the properties of this type of klystron as revealed by these variations.*

A 8 (*a*) (*i*) and (*ii*) The variations in frequency and output power as the reflector voltage of a reflex klystron is progressively changed from −500 V to −200 V are shown in sketch (*a*).



(*a*) (*b*)

An explanatory diagram of a reflex klystron is shown in sketch (*b*). At −500 V, the electron beam from the cathode to the reflector is velocity-modulated as the electrons pass through the oscillating electric field in the resonator gap. This causes bunching of the electrons, and this effect becomes very pronounced when the electrons reach the resonator gap on their return from the reflector. The resonator is mechanically tuned to give maximum output power when the reflector voltage is −500 V. This requires the bunches to be phased so that the resonator extracts maximum energy as each bunch is retarded by the opposing voltage across the resonator gap. Under these conditions, the number of space-charge wavelengths in the drift distance of the electrons from the resonator to the reflector and back must be $N + \frac{3}{4}$, where N is a low integer. As the reflector voltage is reduced from −500 V, reflection occurs later and the drift distance increases, causing a phase shift in the feedback. The component of feedback at 90° is equivalent to a reactance coupled to the resonator circuit, and the resonant frequency is reduced as shown.

At the same time, the in-phase component is gradually reduced and the power falls until oscillations suddenly cease when the feedback is insufficient to replace the energy lost in the resonator and load. As the reflector voltage is further reduced, the correct phase relationship is restored when N increases by 1. The klystron then operates in the new mode, and the power output passes through another maximum as the reflector voltage is reduced. The frequency again starts high and falls through about 50 MHz during this process.

The klystron may pass through 3 or 4 modes of operation over the reflector-voltage range quoted, but the peak output power of each mode is less as the voltage reduces, with the frequency varying as shown.

(*b*) A practical application is suggested by the variation of frequency with reflector voltage. This characteristic can be used to frequency-modulate a klystron in the transmitter circuit of a microwave radio-relay system. A linear portion of the characteristic is chosen, and the modulating signal could, for example, be the baseband signal of a frequency-division-multiplex multichannel telephony system.

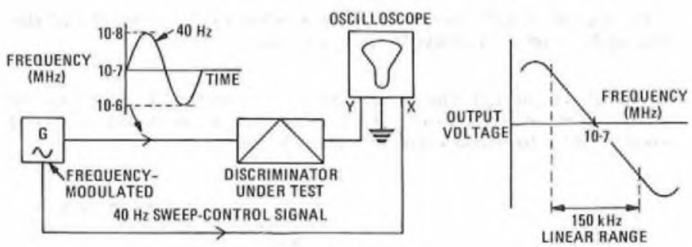
Q 9 (a) A discriminator, which is to operate with a maximum deviation of 75 kHz, has been designed for a frequency-modulation receiver having an intermediate frequency of 10·7 MHz. Describe in detail, with the aid of block diagrams, how the discriminator can be checked for

(i) linearity, and

(ii) adequate limiting.

(b) Draw and explain labelled graphs to illustrate the results expected from the tests.

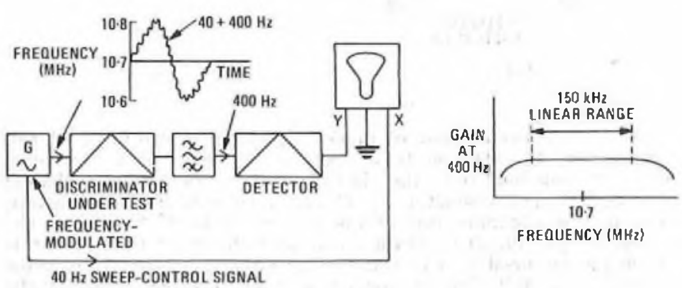
A 9 The linearity of the discriminator can be checked using the arrangement shown in sketch (*a*). A low-distortion signal generator is swept over the range, say, 10·7 MHz ± 100 kHz at a low frequency, say 40 Hz. The sweep voltage controlling the frequency is also applied to the X-plates of the oscilloscope. The output voltage from the discriminator is applied to the Y-plates. The oscilloscope thus displays the discriminator transfer characteristic, as shown. The departure from a straight line over the working range of ±75 kHz can be checked visually with the aid of a straight edge.



(*a*)

A more useful display is obtained by the *derivative method*, in which a small (say 400 Hz) sinusoidal voltage is added to the 40 Hz sweep voltage. The 400 Hz component of the discriminator output is filtered out, detected and applied to the Y-plates of the oscilloscope, as shown in sketch (*b*). The Y-deflexion is then proportional to the slope of the discriminator characteristic and should be constant over the deviation range.

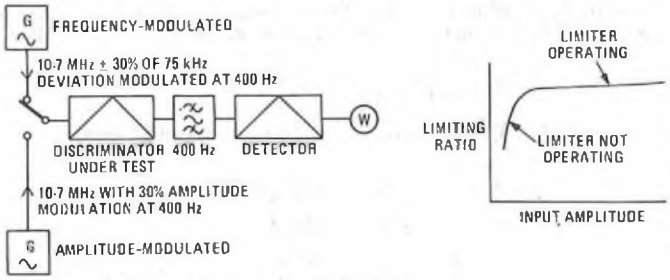
These tests should be repeated at different input levels, including that corresponding to the maximum sensitivity of the receiver.



(*b*)

Note: Under examination conditions, students would not be expected to describe both of the above methods.

Sketch (*c*) illustrates a method of checking for adequate limiting. The frequency-modulated signal generator is adjusted to give 30% of the maximum deviation (that is, ±30% of 75 kHz) with a 400 Hz modulation frequency. The level is adjusted to the lowest at which the discriminator is expected to operate. The 400 Hz output power, P_{FM}, from the discriminator is noted. An amplitude-modulated signal generator is then substituted, giving a signal at the same level and with 30% modulation at 400 Hz. The output power due to unwanted response to amplitude modulation, P_{AM}, is noted. The limiting ratio P_{FM}/P_{AM} is an indication of the adequacy of the limiting, and is plotted, as the level of the modulated input signal is varied, to give the graph shown. The ratio of the amplitude modulation to the frequency modulation is kept constant.



(*c*)

Q 10 (a) Explain, by choosing a typical example, why d.c. restoration may be necessary in the processing of a pulse train.

(b) A sinusoidal signal of peak voltage 25 V and frequency 20 kHz is applied at the input of the circuit shown in Fig. 4. Sketch to scale and explain fully the voltage waveforms to be expected between points X and E, and between points Y and E, when

(i) resistor R2 is 50 kΩ and capacitor C is 100 pF, and

(ii) resistor R2 is 200 kΩ and capacitor C is 2·5 nF.

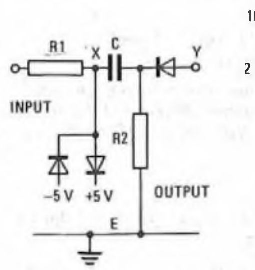
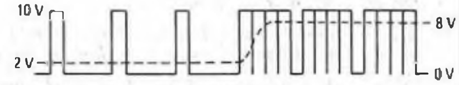
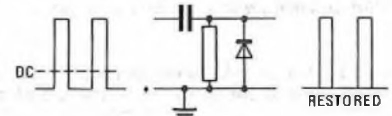


Fig. 4

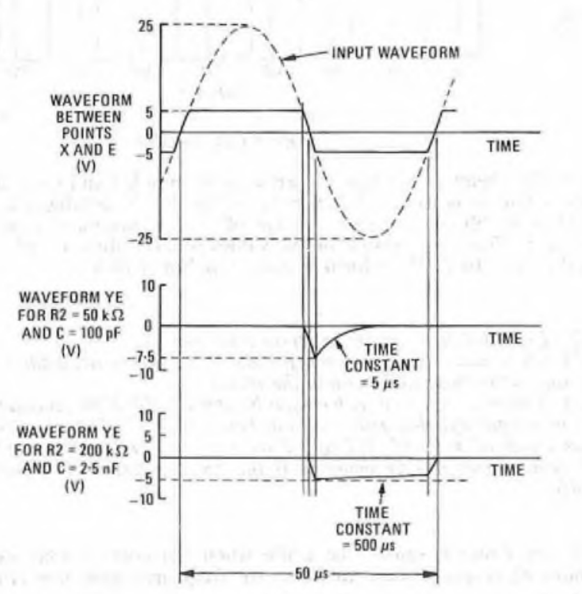


(*a*)



(*b*)

A 10 (*a*) A typical pulse train is shown in sketch (*a*), with its d.c. component represented by the dashed line. If the d.c. component is removed by passing the pulse train through a capacitor or a transformer, the amplitude of the positive peaks relative to the mean value is 8 V for the single pulses and only 2 V for the grouped pulses. This may be unacceptable to the following circuit, and it is necessary to restore the d.c. component to make the peak positive voltage 10 V throughout. A typical d.c. restoration circuit is shown in sketch (*b*), with input and output waveforms.



(*c*)

(*b*) (*i*) When the input voltage exceeds ± 5 V, one of the diodes connected to the $+5$ V and -5 V supplies conducts and, provided that the diode's resistance is much less than R1, voltage XE is clamped to the appropriate voltage. Thus, the waveform expected between points X and E is as shown in sketch (*c*). The transition from $+5$ V to -5 V follows the waveform of the input voltage. The instantaneous value of the input waveform is $25 \sin \omega t$ volts, where $\omega = 2\pi f$ radians/second and f is the frequency (hertz), so that the gradient at zero voltage is $25\omega \cos \omega t$, where $\cos \omega t = 1$. Since $\omega = 2\pi \times 20 \times 10^3$ rad/s, the gradient is $50\pi \times 20 \times 10^3 = 3{\cdot}14 \times 10^6$ V/s, so that the transition from $+5$ V to -5 V takes approximately $10/(3{\cdot}14 \times 10^6)$ s $= 3{\cdot}2$ μs (assuming the curve to be linear over the small range about zero voltage).

To assess the waveform between points Y and E, it is necessary to know the time constant of capacitor C and resistor R2, which is given by $100 \times 10^{-12} \times 50 \times 10^3$ s $= 5$ μs. This time constant, compared with the half-period of the input waveform of $1/(2 \times 20 \times 10^3)$ s $= 25$ μs, causes circuit CR2 to behave as a differentiator, but the $3{\cdot}2$ μs transition time is comparable with the time constant, and the action is not perfect. For the negative-going transition, the voltage across resistor R2 reaches some value less negative than -10 V; say $-7{\cdot}5$ V. The diode adjacent to point Y thus conducts and, assuming the output load to be of a sufficiently high impedance not to affect the circuit voltages, voltage YE also reaches $-7{\cdot}5$ V. The positive-going transition produces a similar positive pulse but, assuming the diode then has a much higher (backward) impedance than the load, the output voltage is zero. During the remainder of the cycle, voltage XE is clamped to ± 5 V, the rate of change is zero, and thus voltage YE is also zero. The waveform is shown in sketch (*c*).

(*ii*) For R2 $= 200$ kΩ and C $= 2{\cdot}5$ nF, waveform XE is the same as for part (*i*).

However, the time constant of capacitor C and resistor R2 is now $2{\cdot}5 \times 10^{-9} \times 0{\cdot}2 \times 10^6$ s $= 500$ μs. This is long compared with both the transition period and the half-period of the waveform. Thus, the waveform across resistor R2 is practically the same as voltage XE, while voltage YE again follows the negative pulses but not the positive ones. The waveform is shown in sketch (*c*).

PRACTICAL MATHEMATICS, 1976

Students were expected to answer any 6 questions

Q 1 (*a*) *Simplify as far as possible the following expressions:*

(*i*) $a(a - 2b) - b(a + 3b) + 3ab$, *and*

(*ii*) $\dfrac{3x^2y}{yz} \times \dfrac{yz^2}{6x}$.

(*b*) *Solve each of the following equations:*

(*i*) $\dfrac{1}{6p} = \dfrac{1}{12}$,

(*ii*) $(3q)^2 = 3^4$,

(*iii*) $(2r)^{-2} = 4$, *and*

(*iv*) $10^s = 1{\cdot}42$.

A 1 (*a*) (*i*) $a(a - 2b) - b(a + 3b) + 3ab$

$$= a^2 - 2ab - ab - 3b^2 + 3ab,$$
$$= \underline{a^2 - 3b^2}.$$

(*ii*)
$$\frac{3x^2y}{yz} \times \frac{yz^2}{6x} = \underline{\frac{xyz}{2}}.$$

(*b*) (*i*)
$$\frac{1}{6p} = \frac{1}{12}.$$
$$\therefore\ 6p = 12.$$
$$\therefore\ \underline{p = 2}.$$

(*ii*)
$$(3q)^2 = 3^4.$$
$$\therefore\ 3q = \pm 3^2$$
$$\therefore\ \underline{q = \pm 3}.$$

(*iii*)
$$(2r)^{-2} = 4.$$
$$\therefore\ \frac{1}{(2r)^2} = 4.$$
$$\therefore\ \frac{1}{2r} = \pm 2.$$
$$\therefore\ \underline{r = \pm 1/4}.$$

(*iv*)
$$10^s = 1{\cdot}42.$$

Taking logarithms gives

$$s \log_{10} 10 = \log_{10} 1{\cdot}42.$$
$$\therefore\ \underline{s = 0{\cdot}1523}.$$

Q 2 (*a*) *The resistance, R, of a wire is proportional to its length, l, and inversely proportional to the square of the diameter, d. Express R in term of l, d and a constant, k. If l is increased by 50% and d increased by 25%, find the percentage change in R.*

(*b*) *Calculate the percentage error in assuming that $1{\cdot}1^3$ is equal to $1{\cdot}3$.*

(*c*) *Express the following in the form $a \times 10^b$, where a is a number between 1 and 10, and b is a positive or negative whole number:*

(*i*) *65 200,*

(*ii*) *0·003 65, and*

(*iii*) *26 400 × 10^{-7}.*

A 2 (*a*) Since R is proportional to l, and inversely proportional to d^2,

$$R \propto \frac{l}{d^2}.$$
$$\therefore\ \underline{R = \frac{kl}{d^2}},$$

where k is the constant of proportionality.

If l' and d' are the new values of l and d respectively, then

$$l' = l + (50/100)l = 1{\cdot}5l, \text{ and } d' = d + (25/100)d = 1{\cdot}25d.$$

Hence, the new value of R

$$= \frac{kl'}{(d')^2} = \frac{1{\cdot}5kl}{(1{\cdot}25d)^2},$$
$$= 0{\cdot}96 \times \frac{kl}{d^2} = 0{\cdot}96R.$$

Thus, the percentage change in R

$$= \frac{R - 0{\cdot}96R}{R} \times 100\% = \underline{4\%}.$$

(*b*) From a table of cubes, $1{\cdot}1^3 = 1{\cdot}331$. Hence, the percentage error in assuming $1{\cdot}1^3 = 1{\cdot}3$

$$= \frac{1{\cdot}331 - 1{\cdot}3}{1{\cdot}331} \times 100\%,$$
$$= \frac{0{\cdot}031 \times 100}{1{\cdot}331}\ \%,$$
$$= \underline{2{\cdot}329\%}.$$

Number	Logarithm
3·1	0·4914
1·331	0·1242−
2·329	0·3672

(*c*) (*i*) $65\,200 = \underline{6{\cdot}52 \times 10^4}$.

(*ii*) $0{\cdot}003\,65 = \underline{3{\cdot}65 \times 10^{-3}}$.

(*iii*) $26\,400 \times 10^{-7} = 2{\cdot}64 \times 10^4 \times 10^{-7} = \underline{2{\cdot}64 \times 10^{-3}}$.

Q 3 (*a*) *Remove the brackets and simplify as far as possible:*

(*i*) $3a - [\{a - 3(2a + b)\} + 2b]$, *and*

(*ii*) $3l[2m - 3\{l - (3m - 2l)\}]$.

(*b*) *Factorize completely each of the following expressions:*

(*i*) $x^2 - x$, *and*

(*ii*) $2x^3 - 2x^2$,

and hence, or otherwise, simplify $\dfrac{2x^3 - 2x^2}{x^2 - x}$.

(*c*) *If $r = -2$, $s = 3$ and $t = -1$, evaluate*

(*i*) $r^2 - s(r + t) + t^2$, *and*

(*ii*) $3rs^2t^3$.

A 3 (*a*) (*i*)
$$\begin{aligned} &3a - [\{a - 3(2a + b)\} + 2b] \\ &= 3a - [\{a - 6a - 3b\} + 2b], \\ &= 3a - [-5a - b], \\ &= 3a + 5a + b, \\ &= 8a + b. \end{aligned}$$

(*ii*)
$$\begin{aligned} &3l[2m - 3\{l - (3m - 2l)\}] \\ &= 3l[2m - 3\{l - 3m + 2l\}], \\ &= 3l[2m - 9l + 9m], \\ &= 3l(11m - 9l). \end{aligned}$$

(*b*) (*i*) $x^2 - x = x(x - 1)$.

(*ii*) $2x^3 - 2x^2 = 2x^2(x - 1)$.

Hence $\dfrac{2x^3 - 2x^2}{x^2 - x} = \dfrac{2x^2(x - 1)}{x(x - 1)} = 2x.$

(*c*) (*i*)
$$\begin{aligned} &r^2 - s(r + t) + t^2 \\ &= (-2)^2 - 3\{-2 + (-1)\} + (-1)^2, \\ &= 4 - 3(-3) + 1 = 14. \end{aligned}$$

(*ii*)
$$\begin{aligned} 3rs^2t^3 &= 3 \times (-2) \times 3^2 \times (-1)^3, \\ &= 27 \times (-2) \times (-1) = 54. \end{aligned}$$

Q 4 Evaluate the following using the appropriate mathematical tables:

(*a*) $34 \cdot 8^2 - 9 \cdot 81^2$,

(*b*) $\sqrt{0 \cdot 007\,25}$,

(*c*) $\dfrac{1}{6 \cdot 51} + \dfrac{1}{8 \cdot 26}$, *and*

(*d*) *i, where* $i = \dfrac{e}{\sqrt{(R^2 + x^2)}}$, $e = 46 \cdot 8$, $R = 24 \cdot 1$, *and* $x = 8 \cdot 3$.

A 4 (*a*) Using a table of squares,

$$34 \cdot 8^2 - 9 \cdot 81^2 = 1211 - 96 \cdot 24 = 1114 \cdot 76.$$

(*b*) Using a table of square roots,

$$\sqrt{0 \cdot 007\,25} = 0 \cdot 085\,15.$$

(*c*) Using a table of reciprocals,

$$\frac{1}{6 \cdot 51} + \frac{1}{8 \cdot 26} = 0 \cdot 1536 + 0 \cdot 1211 = 0 \cdot 2747.$$

(*d*)
$$i = \frac{e}{\sqrt{(R^2 + x^2)}} = \frac{46 \cdot 8}{\sqrt{(24 \cdot 1^2 + 8 \cdot 3^2)}}.$$

Using a table of squares,

$$i = \frac{46 \cdot 8}{\sqrt{(581 + 68 \cdot 89)}} = \frac{46 \cdot 8}{\sqrt{649 \cdot 89}}.$$

Using a table of square roots,

$$i = \frac{46 \cdot 8}{25 \cdot 5}, \quad = 1 \cdot 835.$$

Number	Logarithm
46·8	1·6702
25·5	1·4065—
1·835	0·2637

Q 5 (a) If the measured distance between the figures 1 and 10 on a slide rule is 200 mm, what are the distances between the figures

(i) 1 and 2, and

(ii) 3 and 7?

(b) Using a slide rule, or otherwise, evaluate the following:

(*i*) *V, where* $V = RI$, $R = 2600$, *and* $I = 1 \cdot 28$,

(*ii*) *C, where* $C = \dfrac{C_1 + C_2}{C_1C_2}$, $C_1 = 15 \cdot 6$, *and* $C_2 = 8 \cdot 41$, *and*

(*iii*) *E, where* $E = LI^2/2$, $L = 4 \cdot 82$, *and* $I = 3 \cdot 27$.

(*c*) *Make I the subject of the formula* $E = LI^2/2$.

A 5 (*a*) The measured distance between any 2 numbers on a slide-rule scale is proportional to the difference of the logarithms of the numbers.

Thus, since $\log_{10} 1 = 0$ and $\log_{10} 10 = 1$, the measured distance of 200 mm is proportional to $\log_{10} 10 - \log_{10} 1 = 1$.

(*i*) Since $\log_{10} 1 = 0$ and $\log_{10} 2 = 0 \cdot 301$, the measured distance between the figures 1 and 2 is $0 \cdot 301 \times 200 = 60 \cdot 2$ mm.

(*ii*) Since $\log_{10} 3 = 0 \cdot 4771$ and $\log_{10} 7 = 0 \cdot 8451$, the measured distance between the figures 3 and 7 is

$$(0 \cdot 8451 - 0 \cdot 4771) \times 200 = 73 \cdot 6 \text{ mm}.$$

(*b*) The following have been evaluated using a slide rule. The accurate answers are given in parenthesis to 4 significant figures.

(*i*)
$$\begin{aligned} V &= RI = 2600 \times 1 \cdot 28, \\ &= 3330 \text{ (accurate answer: 3328).} \end{aligned}$$

(*ii*)
$$\begin{aligned} C &= \frac{C_1 + C_2}{C_1C_2} = \frac{15 \cdot 6 + 8 \cdot 41}{15 \cdot 6 \times 8 \cdot 41}, \\ &= \frac{24 \cdot 01}{15 \cdot 6 \times 8 \cdot 41} = 0 \cdot 183 \text{ (accurate answer: } 0 \cdot 1830). \end{aligned}$$

(*iii*)
$$\begin{aligned} E &= \frac{LI^2}{2} = \frac{4 \cdot 82 \times 3 \cdot 27^2}{2}, \\ &= 2 \cdot 41 \times 3 \cdot 27^2 = 25 \cdot 8 \text{ (accurate answer: } 25 \cdot 77). \end{aligned}$$

(*c*)
$$\begin{aligned} E &= \frac{LI^2}{2}. \\ \therefore LI^2 &= 2E. \\ \therefore I^2 &= \frac{2E}{L}. \\ \therefore I &= \sqrt{\frac{2E}{L}} = \pm 1 \cdot 414 \times \sqrt{\frac{E}{L}}. \end{aligned}$$

Q 6 (a) Without drawing graphs, find the slope and 2 intercepts of the straight lines for each of the following:

(*i*) $y = 3x + 4$,

(*ii*) $4y = 3x - 12$, *and*

(*iii*) $3x + 5y = 30$.

(b) (i) When a car moves on a straight road with constant acceleration, the velocity, v, and time, t, are related by the linear law $v = u + at$, *where u and a are constants. Calculate the values of u and a if* $v = 15$ *m/s when* $t = 5$ *s, and* $v = 25$ *m/s when* $t = 10$ *s.*

(ii) If this problem were represented graphically, what values would be expected for the slope and intercepts of the straight-line graph?

A 6 (*a*) To determine the slope in each case, the equation is compared with the general equation for a straight line: $y = mx + c$, where m is the slope (and c is the point of intersection with the y-axis). The intercepts are found by putting $y = 0$ and $x = 0$ in turn into the equation.

(*i*) The slope of the equation $y = 3x + 4$ is 3.

The x-axis intercept occurs at $y = 0$; that is, at $x = -4/3$.
The y-axis intercept occurs at $x = 0$; that is, at $y = 4$.

(*ii*)
$$\begin{aligned} 4y &= 3x - 12. \\ \therefore y &= (3/4)x - 3. \end{aligned}$$

Hence, the slope is 3/4, the x-axis intercept occurs at $x = 4$, and the y-axis intercept occurs at $y = -3$.

(*iii*)
$$\begin{aligned} 3x + 5y &= 30. \\ \therefore 5y &= -3x + 30. \\ \therefore y &= -(3/5)x + 6. \end{aligned}$$

Hence, the slope is $-3/5$, the x-axis intercept is at $x = 10$, and the y-axis intercept is at $y = 6$.

(*b*) (*i*) Substituting the 2 sets of data into the equation $v = u + at$ gives the following 2 equations:

$$15 = u + 5a, \qquad \ldots\ldots (1)$$

and

$$25 = u + 10a. \qquad \ldots\ldots (2)$$

Subtracting equation (1) from equation (2) gives

$$10 = 5a.$$

$$\therefore\ \underline{a = 2\ \text{m/s}^2.}$$

Substituting for a in equation (1) gives

$$15 = u + 10.$$

$$\therefore\ \underline{u = 5\ \text{m/s}.}$$

(*ii*) The linear equation is $v = 5 + 2t$, so that the slope of the graph is $\underline{2\ \text{m/s}^2}$; this represents the constant acceleration.

The v-axis intercept occurs at $t = 0$; that is at $\underline{v = 5\ \text{m/s}}$. (Thus, $v = u$, the initial velocity.)

The t-axis intercept occurs at $v = 0$; that is, at $\underline{t = -2{\cdot}5\ \text{s}}$. (This implies that, under the given conditions, the velocity would have been zero 2·5 s before the point in time at which the initial velocity is defined.)

Q 7 (a) Copy and complete the tables of values for the equations given below.

(i) $2y + 3x = 1$:

x	-1	1	3
y			

(ii) $y = 4x - 8$:

x	1	2	3
y			

(b) Draw the straight lines $2y + 3x = 1$ and $y = 4x - 8$, and hence estimate the values of x and y that satisfy both equations simultaneously.

(c) State the values of the intercept with the x-axis of each line drawn for part (b), and hence find the area enclosed between the 2 lines and the x-axis.

A 7 (*a*) (*i*)

$$2y + 3x = 1.$$

$$\therefore\ y = -\frac{3x}{2} + \frac{1}{2}.$$

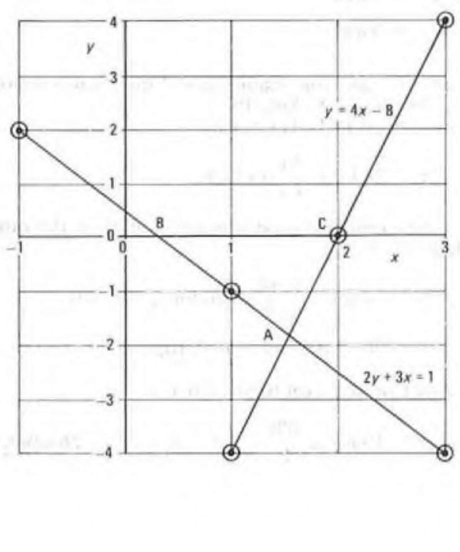
Hence, the values appropriate to complete the table are given below.

x	-1	1	3
y	2	-1	-4

(*ii*) For $y = 4x - 8$, the values appropriate to complete the table are given below.

x	1	2	3
y	-4	0	4

(*b*) The graphs are shown in the sketch, drawn from the values derived in part (*a*).



The graphs intersect at point A, where $x \approx 1{\cdot}55$ and $y \approx -1{\cdot}82$. Thus, the values of x and y that simultaneously satisfy both equations are $\underline{1{\cdot}55}$ and $\underline{-1{\cdot}82}$ respectively.

(*c*) The line $2y + 3x = 1$ intercepts the x-axis at $\underline{x = 0{\cdot}\dot{3}}$, and the line $y = 4x - 8$ intercepts the x-axis at $\underline{x = 2}$.

The area enclosed by the 2 lines and the x-axis is triangle ABC, and the area of a triangle is given by one half of the base multiplied by the perpendicular height.

The base, BC, is $2 - 0{\cdot}\dot{3} = 1{\cdot}\dot{6}$. The perpendicular height is given by the ordinate of point A; that is, 1·82.

Thus, the area

$$= \frac{1}{2} \times \frac{5}{3} \times 1{\cdot}82 \approx \underline{1{\cdot}52\ \text{square units.}}$$

Q 8 (a) Solve the equations:

(i) $2a - 6 = 3a + 1$,

(ii) $0 = 3(b - 1) - 2(3b + 2)$, and

(iii) $5 - \frac{1}{4}(2c - 1) = \frac{3}{5}$.

(b) Evaluate t, where $t = \dfrac{R(1 - p)}{p(R + 1)}$, $R = 67{\cdot}58$, and $p = 0{\cdot}572$.

A 8 (*a*) (*i*)

$$2a - 6 = 3a + 1.$$

$$\therefore\ \underline{a = -7.}$$

(*ii*)

$$0 = 3(b - 1) - 2(3b + 2),$$

$$= 3b - 3 - 6b - 4.$$

$$\therefore\ 3b = -7.$$

$$\therefore\ \underline{b = -7/3.}$$

(*iii*)

$$5 - \frac{1}{4}(2c - 1) = \frac{3}{5}.$$

Multiplying the equation by 20, the lowest common multiple of 4 and 5, gives

$$100 - 5(2c - 1) = 12.$$

$$\therefore\ 100 - 10c + 5 = 12.$$

$$\therefore\ -10c = -93.$$

$$\therefore\ c = 9{\cdot}3.$$

(*b*)

$$t = \frac{R(1 - p)}{p(R + 1)},$$

$$= \frac{67{\cdot}58(1 - 0{\cdot}572)}{0{\cdot}572(67{\cdot}58 + 1)},$$

$$= \frac{67{\cdot}58 \times 0{\cdot}428}{0{\cdot}572 \times 68{\cdot}58},$$

$$= \underline{0{\cdot}7372.}$$

Number	Logarithm
67·58 0·428	1·8298 $\bar{1}$·6314+
	1·4612
0·572 68·58	$\bar{1}$·7574 1·8362+
	1·5936
Numerator Denominator	1·4612 1·5936−
0·7372	$\bar{1}$·8676

Q 9 Fig. 1 shows 2 coastguard stations, A and B, 55 km apart at sea level. A ship, P, is observed simultaneously by stations A and B in directions E 36° N and W 82° N respectively, where E stands for east, N for north, and W for west.

(a) Find the distance of the ship from each station by drawing an accurate scale diagram.

(b) Calculate the distance of the ship from each station correct to 3 significant figures.

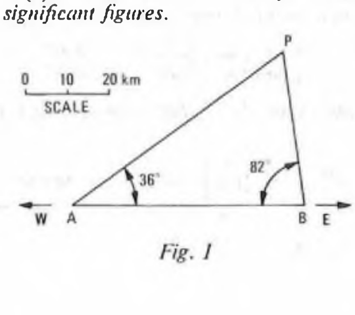
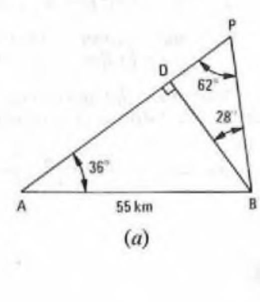


Fig. 1



(*a*)

Note: On the examination paper, Fig. 1 was not drawn to scale, but was intended to give only a rough plan of the situation. To avoid reproducing 2 very similar diagrams, Fig. 1 has here been drawn accurately to scale so that it can be used to answer part (*a*).

A 9 (*a*) The scale drawing (Fig. 1) is constructed by marking distance AB in a west–east direction according to the scale shown. A line originating at A is drawn in the direction A–P at an angle of 36° north of east, and a line originating at B is drawn in the direction B–P at an angle of 82° north of west. The intersection gives point P, the position of the ship.

By measurement, $\underline{AP = 61 \cdot 8 \text{ km}}$ and $\underline{BP = 36 \cdot 7 \text{ km}}$.

(*b*) Sketch (*a*) again shows the plan (not drawn to scale) with line BD drawn perpendicular to AP.

Since the sum of the angles of triangle ABP is 180°,

$$\angle APB = 180° - (36° + 82°) = 62°.$$

Also, in triangle DPB, since $\angle PDB = 90°$,

$$\angle DBP = 90° - 62° = 28°.$$

Now, $$\frac{AD}{AB} = \cos 36° = 0 \cdot 8090.$$

$$\therefore AD = 55 \times 0 \cdot 809 = 44 \cdot 496 \text{ km}.$$

By the theorem of Pythagoras,

$$DB^2 = AB^2 - AD^2 = 55^2 - 44 \cdot 496^2.$$

$$\therefore DB = \sqrt{(3025 - 1980)} = \sqrt{1045} = 32 \cdot 33 \text{ km}.$$

Also, $$\frac{DP}{DB} = \tan 28° = 0 \cdot 5317.$$

$$\therefore DP = 32 \cdot 33 \times 0 \cdot 5317 = 17 \cdot 19 \text{ km}.$$

Hence, $AP = AD + DP = 44 \cdot 496 + 17 \cdot 19 = 61 \cdot 69$ km.

Also, $BP^2 = DP^2 + DB^2 = 17 \cdot 19^2 + 1045.$

$$\therefore BP = \sqrt{(295 \cdot 5 + 1045)} = \sqrt{1340 \cdot 5} = 36 \cdot 61 \text{ km}.$$

Thus, by calculation, $\underline{AP = 61 \cdot 7 \text{ km}}$ and $\underline{BP = 36 \cdot 6 \text{ km}}$, correct to 3 significant figures.

Q 10 Fig. 2 shows an isosceles triangle, ABC, with sides AB = 60 mm, BC = 60 mm, and CA = 72 mm, circumscribed by a circle with centre O and radius r millimetres.

(a) Calculate

(i) the height, BN, of triangle ABC, and

(ii) the area of triangle ABC.

(b) Using triangle ONA, verify that r = 37·5 mm.

(c) Find, in degrees and minutes, the angles ABN and AON, and comment on the results.

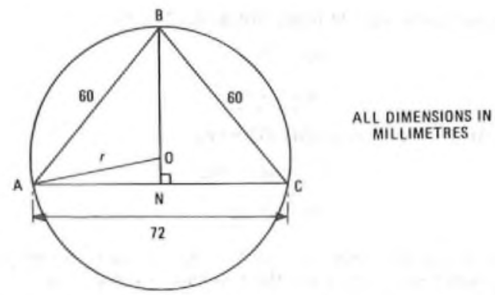


Fig. 2

A 10 (*a*) Since triangle ABC is isosceles, AB = BC. Also, $\angle BNA = \angle BNC = 90°$. Therefore, as side BN is common, triangles ABN and CBN are congruent. Therefore, AN = NC = 36 mm.

(*i*) By the theorem of Pythagoras,

$$BN^2 = AB^2 - AN^2 = 60^2 - 36^2.$$

$$\therefore BN = \sqrt{(3600 - 1296)} = \sqrt{2304} = \underline{48 \text{ mm}}.$$

(*ii*) The area of triangle ABC

$$= \tfrac{1}{2} \times \text{base} \times \text{perpendicular height},$$

$$= 36 \times 48 = \underline{1728 \text{ mm}^2}.$$

(*b*) In triangle ONA,

$$AO^2 = AN^2 + ON^2 = AN^2 + (BN - OB)^2.$$

$$\therefore r^2 = 36^2 + (48 - r)^2,$$

$$= 1296 + 2304 - 2 \times 48r + r^2.$$

$$\therefore 96r = 3600.$$

$$\therefore \underline{r = 37 \cdot 5 \text{ mm}}. \qquad \text{QED.}$$

(*c*) Now, $$\sin \angle ABN = \frac{AN}{AB} = \frac{36}{60} = 0 \cdot 6.$$

$$\therefore \underline{\angle ABN = 36° \, 52'}.$$

Also, $$\sin \angle AON = \frac{AN}{r} = \frac{36}{37 \cdot 5} = 0 \cdot 96.$$

$$\therefore \underline{\angle AON = 73° \, 44'}.$$

Thus, angle AON is exactly double angle ABN, and this is confirmed by the geometry of the figure. Angle AON, which is an angle external to triangle ABO, is equal to the sum of the opposite internal angles ABO (or ABN) and OAB. However, since AO = BO = *r*, triangle ABO is isosceles, so that angle ABO is equal to angle OAB. Thus, angle AON must be twice angle ABO, or twice angle ABN.

RADIO AND LINE TRANSMISSION A, 1976

Students were expected to answer any 6 questions

Q 1 (a) Define the decibel.

(b) Briefly explain why it is a convenient unit for use in radio and line communication work.

(c) A fault develops in an amplifier and, for a constant input voltage, the output voltage falls from 3·54 V to 1·6 V. Calculate the loss in decibels introduced by the fault.

(d) The following data refer to the network shown in Fig. 1:

input power = 400 mW,	*gain of amplifier 1 = 20 dB,*
loss in link = 23 dB,	*gain of amplifier 2 = 25 dB.*

Determine the input and output power levels for each amplifier in decibels relative to 1 mW (dBm).

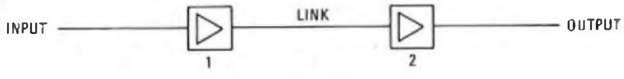


Fig. 1

A 1 (*a*) and (*b*) See A6, Radio and Line Transmission A, 1974, *Supplement*, Vol. 68, p. 3, Apr. 1975.

(*c*) The loss, *N* decibels, is given by

$$N = 20 \log_{10} \frac{V_1}{V_2} \text{ decibels},$$

where V_1 is the original output voltage, and V_2 is the output voltage when the fault is present.

$$\therefore N = 20 \log_{10} \frac{3 \cdot 54}{1 \cdot 6} = 20 \log_{10} 2 \cdot 21 \text{ dB},$$

$$= 20 \times 0 \cdot 3449 = \underline{6 \cdot 9 \text{ dB}}.$$

(*d*) The input power level to amplifier 1

$$= 10 \log_{10} \frac{400}{1} = 10 \times 2 \cdot 6021 = \underline{26 \text{ dBm}}.$$

The output power level from amplifier 1

= the input power level + the gain,

= 26 + 20 = 46 dBm.

The input power level to amplifier 2

= the output from amplifier 1 − loss in link,

= 46 − 23 = 23 dBm.

The output power level from amplifier 2

= the input power level + the gain,

= 23 + 25 = 48 dBm.

Q 2 *(a) Briefly explain why most long-distance public telephone systems involve the use of*

(i) a 2-wire–4-wire terminating set, and
(ii) a carrier.

(b) Draw labelled block diagrams of 2 of the following connexions:

(i) a telephone subscriber to the local telephone exchange (that is, a local-line network),
(ii) an outside-broadcast event to the national broadcasting station, and
(iii) a long-distance overseas radio-telephone call.

(c) State suitable values for any carrier frequencies used in parts (b) (ii) or (b) (iii).

A 2 (*a*) (*i*) When telephone signals are transmitted over a long-distance circuit, they are attenuated. To maintain a good signal-to-noise ratio at the output of the circuit, amplifiers are needed. An amplifier is a unidirectional device, whereas telephone circuits are bidirectional. Thus, to amplify a telephone circuit, its GO and RETURN paths must be separated, and this is achieved using a 2-wire–4-wire terminating set.

(*ii*) In the case of long-distance line transmission, it is more economical to transmit more than one signal over a pair of wires or a coaxial cable. To avoid mutual interference between the various signals, each is arranged to amplitude-modulate one of a series of different carrier frequencies.

In the case of long-distance circuits involving radio links, it is necessary to use a carrier frequency to obtain the optimum propagation conditions for the path between the transmitter and the receiver.

(*b*) See A10, Radio and Line Transmission A, 1975, *Supplement*, Vol. 69, p. 12, Apr. 1976.

(*c*) For a radio link between an outside-broadcast point and the point of connexion to the Post Office network, a carrier frequency in the range 2–12 GHz may be used.

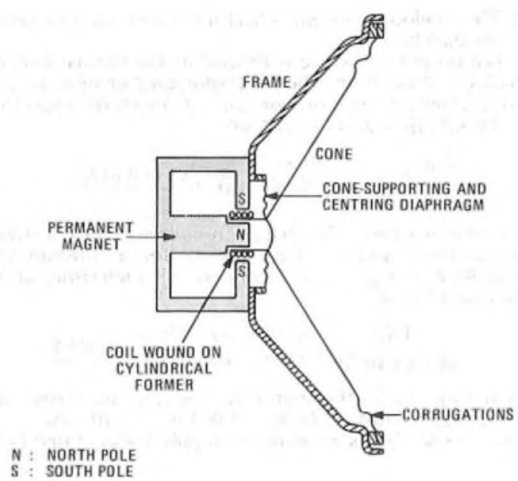
For a long-distance overseas radio-telephone call, a carrier frequency in the high-frequency band (3–30 MHz) would be used over the radio path.

Q 3 *(a) Explain, with the aid of a sketch, the principle of operation of a moving-coil loudspeaker.*

(b) State the approximate frequency range over which the response of a moving-coil loudspeaker is reasonably uniform.

(c) Why are baffle boards normally used with moving-coil loudspeakers?

A 3 (*a*) The moving-coil loudspeaker operates on the principle that a force acts on a conductor carrying a current in a magnetic field. The sketch shows a cross-section of a typical loudspeaker.



A coil is wound on a light cylindrical former, constrained by a diaphragm to move axially within the annular air-gap of a permanent-magnet system. Connexions to the coil are made by flexible leads. Signal currents, flowing in the coil, set up a varying magnetic field around the coil that interacts with the permanent magnetic field, causing a varying force to be exerted on the coil. This force causes the coil to move axially in sympathy with the amplitude and frequency of the signal currents and, since the coil is attached to a cone, this movement is transmitted to it. The movement of the cone creates sympathetic pressure variations in the air adjacent to it, thus giving rise to sound waves that reproduce the intelligence contained in the signal currents.

The cone is light and firmly constructed, so that it acts as a piston without distorting during its action. It is attached to a frame by flexible corrugations.

(*b*) Typically, a cheap loudspeaker may have a fairly uniform frequency response over the range 100–8000 Hz. An expensive high-quality loudspeaker may have a uniform frequency response over the range 50–15 000 Hz.

(*c*) Considering the cone of a loudspeaker to be moving forward, the air in front of the cone is compressed and the air behind it is rarefied. Thus, the movement of the cone produces a sound wave from the rear in antiphase with that produced from the front. A baffle board prevents the 2 sound waves from cancelling each other and reducing the volume of sound produced.

Q 4 *(a) For a resistor, briefly explain the meaning of each of the following terms:*

(i) self-inductance,
(ii) self-capacitance,
(iii) stability,
(iv) tolerance, and
(v) power rating.

(b) Prepare a table to show the relative performances of carbon and wire-wound resistors for the 5 characteristics above.

A 4 (*a*) (*i*) Because a wire-wound resistor is essentially a coil consisting of a number of turns of wire, a magnetic field is set up around the coil when current flows. If the current changes, an e.m.f. is induced in the winding, and this effect is known as *self-inductance*. A similar but less-pronounced effect can occur between the leads of a carbon resistor.

(*ii*) Adjacent turns in the winding of a wire-wound resistor, together with the insulation separating them, constitute small capacitances. These collectively give the resistor a capacitive effect, known as the *self-capacitance*.

(*iii*) The stability of a resistor is the amount by which its resistance can be expected to change in operation during its working life. It is usually expressed as a percentage.

(*iv*) The tolerance of a resistor is the accuracy to which it is made compared with its nominal value. It is usually expressed as a percentage.

(*v*) The power rating of a resistor is the maximum power it can dissipate without its temperature exceeding a critical value above which the specification is no longer met or damage occurs.

(*b*) A table showing the relative performances of carbon and wire-wound resistors is given below.

Characteristic	Wire-Wound Resistors	Carbon Resistors
Self-Inductance and Self-Capacitance	Quite noticeable, particularly at high-frequencies; effect can be reduced if resistor is specially wound	Negligible
Stability and Tolerance	Good	Fair to poor unless specially made
Power Rating	High	Low

Q 5 *(a) A carrier frequency, f_c, is amplitude-modulated by 5 audio frequencies, f_1–f_5. Sketch a frequency-spectrum diagram to show the frequencies produced.*

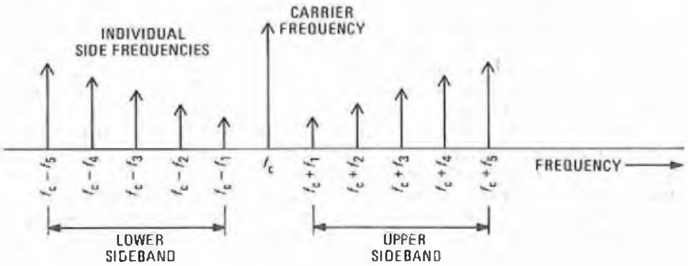
(b) With reference to the diagram drawn for part (a), explain the difference between side frequencies and sidebands.

(c) A carrier frequency of 100 kHz is amplitude-modulated by an audio frequency of 120 Hz together with its third and fifth harmonics. Determine

(i) the side frequencies produced,
(ii) the limits of the sidebands, and
(iii) the overall bandwidth of the transmission.

(d) If the unmodulated carrier of 100 kHz is propagated through a cable with a velocity of 2×10^8 m/s, determine the wavelength.

A 5 (*a*) The frequency spectrum is shown in the sketch.



(*b*) For a modulated carrier, each modulating frequency, f_m, produces 2 *side frequencies,* one above the carrier ($f_c + f_m$) and one below the carrier ($f_c - f_m$). Thus, each of the 10 components $f_c \pm f_1$ to $f_c \pm f_5$ is a side frequency. The range of side frequencies below the carrier ($f_c - f_5$ to $f_c - f_1$) is called the *lower sideband,* and the range above ($f_c + f_1$ to $f_c + f_5$) is called the *upper sideband.*

(*c*) (*i*) The modulating frequencies are 120 Hz, 360 Hz and 600 Hz. Hence, the side frequencies are

100 kHz ± 120 Hz = $\underline{100{\cdot}12\text{ kHz and }99{\cdot}88\text{ kHz}}$,

100 kHz ± 360 Hz = $\underline{100{\cdot}36\text{ kHz and }99{\cdot}64\text{ kHz}}$,

and 100 kHz ± 600 Hz = $\underline{100{\cdot}6\text{ kHz and }99{\cdot}4\text{ kHz}}$.

(*ii*) The lower sideband is $\underline{99{\cdot}4\text{–}99{\cdot}88\text{ kHz}}$, and the upper sideband is $\underline{100{\cdot}12\text{–}100{\cdot}6\text{ kHz}}$.

(*iii*) The bandwidth of the transmission

$$= 100{\cdot}6 - 99{\cdot}4 = \underline{1{\cdot}2\text{ kHz}}.$$

(*d*) The wavelength, λ metres, is given by the velocity of propagation divided by the frequency.

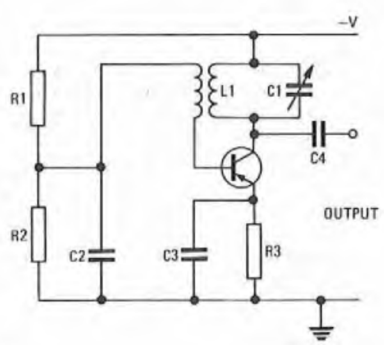
$$\therefore\ \lambda = \frac{2 \times 10^8}{100 \times 10^3}\text{ m} = \underline{2\text{ km}}.$$

Q 6 (a) Sketch the circuit and outline the operation of a simple mutual-inductance-coupled transistor oscillator, explaining how the oscillations are self-starting and of constant amplitude.

(b) The envelope of an amplitude-modulated carrier wave varies sinusoidally between maximum values of ±9 V and minimum values of ±5 V. Determine

(*i*) *the amplitude of the unmodulated carrier,*
(*ii*) *the amplitude of the modulating signal, and*
(*iii*) *the percentage modulation.*

A 6 (*a*) The circuit of a mutual-inductance-coupled transistor oscillator is shown in the sketch.



The circuit is basically that of a common-emitter amplifier with resistors R1, R2 and R3 providing the conventional bias and stabilization arrangements. The tuned circuit, consisting of capacitor C1 and inductor L1, determines the frequency of oscillation, which is approximately equal to $1/2\pi\sqrt{(L_1C_1)}$ Hz. Oscillations are maintained by feeding back energy from the collector circuit, via the transformer, to the base circuit. The transistor, acting as an amplifier, is arranged just to overcome the circuit losses; that is, the loop gain of the circuit is unity.

For the oscillator to be self-starting when first switched on, the transistor is initially biased into the class-A mode by resistors R1, R2 and R3, thus providing high gain. Transient voltages due to the action of switching on cause the tuned circuit to oscillate at its resonant frequency. Oscillations are fed back in series with the base-bias voltage so that the base-emitter voltage is increased in one half-cycle and reduced in the other. As oscillations build up, the base-emitter junction becomes cut off for part of the positive half-cycle, so that the amplitude of the waveform in one half-cycle is greater than that in the other. Hence, capacitor C3 is charged to a greater extent in one direction than in the other, and a negative potential builds up at the emitter. This effectively increases the bias so that the transistor is driven into class-C operation. In this way, the loop gain is reduced to unity to give oscillations of a constant amplitude.

(*b*) (*i*) Let the amplitude of the unmodulated carrier be V_C volts, and that of the modulating signal be V_M volts. Then,

$$V_C + V_M = 9\text{ V}, \qquad \ldots\ldots (1)$$

and

$$V_C - V_M = 5\text{ V}. \qquad \ldots\ldots (2)$$

Adding equations (1) and (2) gives

$$2V_C = 14\text{ V}.$$

$$\therefore\ \underline{V_C = 7\text{ V}.}$$

(*ii*) Subtracting equation (2) from equation (1) gives

$$2V_M = 4\text{ V}.$$

$$\therefore\ \underline{V_M = 2\text{ V}.}$$

(*iii*) The percentage modulation

$$= \frac{V_M}{V_C} \times 100\%,$$

$$= \frac{2}{7} \times 100\% = \underline{28{\cdot}6\%}.$$

Q 7 (a) The data in the table refer to a transistor in the common-emitter configuration.

Collector–Emitter Voltage (V)	*Collector Current (mA) for Base Current =* −*60 μA*	−*80 μA*	−*100 μA*
−*2*	−*3·0*	−*4·5*	−*6·0*
−*4*	−*3·4*	−*5·0*	−*6·5*
−*6*	−*3·8*	−*5·5*	−*7·0*
−*8*	−*4·2*	−*6·0*	−*7·6*
−*10*	−*4·6*	−*6·5*	−*8·2*

Draw the collector-current/collector-voltage characteristics for each of the base currents shown.

(b) Use the characteristics to determine

(*i*) *the output resistance of the transistor for a base current, I_B, of −80 μA, and*

(*ii*) *the current gain for a collector–emitter voltage of −7 V.*

(c) The transistor is to be used in a common-emitter amplifier with a load resistance of 900 Ω and a supply voltage of −9 V. Draw the load line and use it to determine the power dissipated at the collector when I_B = −80 μA.

A 7 (*a*) The collector-current/collector-voltage characteristics are shown in the sketch.

(*b*) (*i*) The output resistance is defined as the change in collector–emitter voltage divided by the corresponding change in collector current for a constant value of base current. From the characteristics, at $I_B = -80\ \mu$A, the output resistance

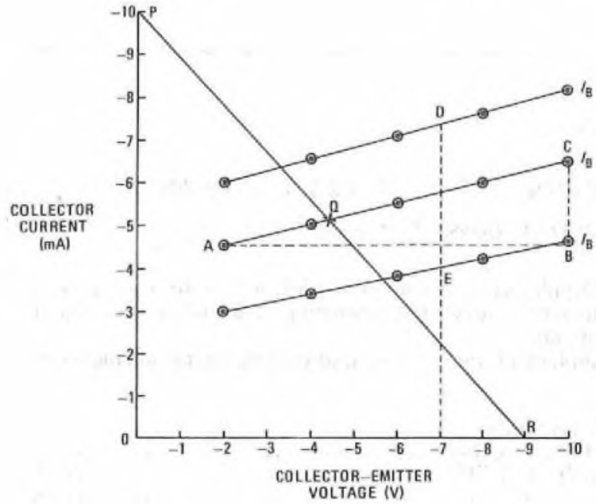
$$= \frac{\text{AB}}{\text{BC}} = \frac{10 - 2}{(6{\cdot}5 - 4{\cdot}5) \times 10^{-3}}\ \Omega = \underline{4\text{ k}\Omega.}$$

(*ii*) The current gain is the change in collector current divided by the corresponding change in base current for a constant value of collector–emitter voltage, V_{CE}. From the characteristic, at V_{CE} = −7 V, the current gain

$$= \frac{\text{DE}}{\text{change in }I_B} = \frac{(7{\cdot}3 - 4) \times 10^{-3}}{(100 - 60) \times 10^{-6}} = \underline{82{\cdot}5.}$$

(*c*) When V_{CE} = 0 V, the supply voltage appears across the load, so that the collector current, I_C, is −9/900 A = −10 mA.

When I_C = 0 A, V_{CE} is equal to the supply voltage; that is, V_{CE} = −9 V.



From these 2 points, the load line, PR, is drawn, as shown on the characteristics.

For $I_B = -80$ μA, the operating point is at point Q. At this point, the power dissipated

$$= V_{CE} \times I_C \text{ watts,}$$
$$= 4 \cdot 4 \times 5 \cdot 1 \times 10^{-3} \text{ W} = \underline{22 \cdot 4 \text{ mW}}.$$

Q 8 (a) Draw circuit diagrams showing how a junction transistor can be used in a single-stage resistance-loaded audio-frequency amplifier in the

(i) common-base configuration, and
(ii) common-emitter configuration.

(b) Compare, or give typical values of, the current gain, the voltage gain, the power gain, the input impedance and the output impedance of the 2 types of amplifier in part (a).

A 8 (*a*) See A8, Radio and Line Transmission A, 1975, *Supplement*, Vol. 69, p. 11, Apr. 1976.

(*b*) The table compares the characteristics of the 2 configurations.

Characteristic	Common-Base Configuration	Common-Emitter Configuration
Current Gain	Low (less than unity)	Medium (30–80)
Voltage Gain	Medium–good	Very good
Power Gain	Good (15–30 dB)	Very good (30–40 dB)
Input Impedance	Low (30–100 Ω)	Medium (200–50 000 Ω)
Output Impedance	High (10^5–10^6 Ω)	High (10^4–10^5 Ω)

Q 9 (a) Sketch the impedance/frequency characteristic of a parallel tuned circuit near resonance.

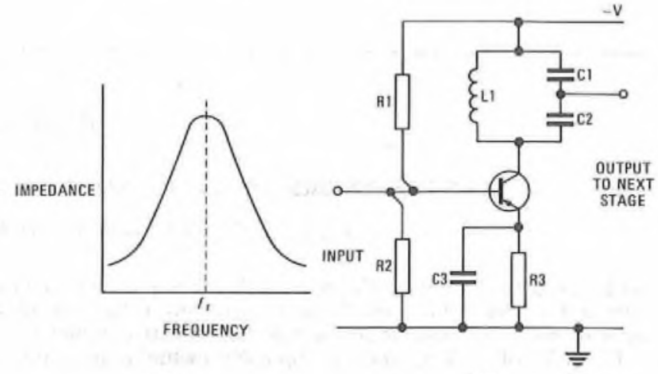
(b) With reference to the characteristic in part (a), explain how a parallel tuned circuit can be used to provide selectivity in a radio-frequency amplifier.

(c) Sketch the circuit of a tuned-collector transistor radio-frequency amplifier. Mark clearly the input, output and biasing arrangements.

(d) A parallel tuned circuit consists of an inductance of 640 mH and a capacitance of 900 pF. Determine the frequency of resonance.

A 9 (*a*) Sketch (*a*) shows a typical impedance/frequency characteristic for a parallel tuned circuit near resonance. The impedance is a maximum at the resonant frequency, f_r, and decreases each side of f_r.

(*b*) A radio-frequency amplifier is required to amplify wanted incoming signals and reject the unwanted signals. It is therefore required to amplify a narrow band of frequencies centred on the wanted-signal frequency. Considering the characteristic curve in



(*a*) (*b*)

sketch (*a*), it can be seen that this requirement is met if a parallel tuned circuit, tuned to the wanted-signal frequency, is included in the frequency-determining circuit of the amplifier. The gain of an amplifier is proportional to the load impedance. If the tuned circuit forms the load of the radio-frequency amplifier, then maximum amplification is obtained for a narrow band of frequencies about the resonant frequency. In addition, the resonant frequency can be varied over a wide range by adjusting the value of the capacitor or inductor in the tuned circuit.

(*c*) Sketch (*b*) shows a tuned-collector transistor radio-frequency amplifier. The biasing arrangements are provided by resistors R1, R2 and R3.

(*d*) The resonant frequency of a tuned circuit is given by $f_r \approx 1/2\pi\sqrt{(LC)}$ hertz, where L is the inductance (henrys), and C is the capacitance (farads).

$$\therefore f_r = \frac{1}{2\pi\sqrt{(640 \times 10^{-3} \times 900 \times 10^{-12})}} \text{ Hz,}$$
$$= \underline{6 \cdot 63 \text{ kHz}}.$$

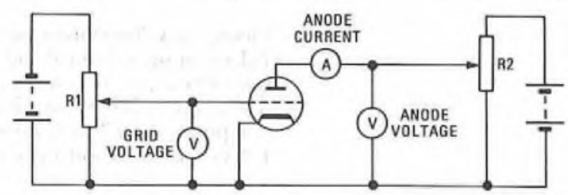
Q 10 (a) Describe, with the aid of a circuit diagram, a method of determining, for a triode tube,

(i) the anode characteristics, and
(ii) the mutual characteristics.

(b) Using the triode mutual characteristics, draw input/output waveform diagrams to illustrate what is meant by each of the following conditions of amplification:

(i) class-A,
(ii) class-B, and
(iii) class-C.

A 10 (*a*) The sketch shows a circuit suitable for use in determining the anode and mutual characteristics of a triode tube.



(*i*) To plot the anode characteristics, the grid voltage is set to a convenient value and maintained at that value by adjusting potentiometer R1 as necessary. Potentiometer R2 is varied to give a range of values of anode voltage, and the anode current is recorded at each step. The measurements are repeated for different values of grid voltage to obtain a family of curves.

(*ii*) To plot the mutual characteristics, the anode voltage is set to a suitable value and maintained at that value by adjusting potentiometer R2 as necessary. Potentiometer R1 is varied to give a range of values of grid voltage, and the anode current is recorded at each step. The measurements are repeated for different values of anode voltage to obtain a family of curves.

(*b*) See A5, Radio and Line Transmission A, 1975, *Supplement*, Vol. 69, p. 10, Apr. 1976.

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