SUPPLEMENT

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CITY AND GUILDS OF LONDON INSTITUTE EXAMINATIONS 1974–75

Contents

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QUESTIONS AND ANSWERS

Answers are occasionally omitted or reference is made to earlier Supplements in which questions of substantially the same form, together with the answers, have been published. Some answers contain more detail than would be expected from candidates under examination conditions.

For economic reasons, alternate issues of the Supplement are published in 32-page and 16-page sizes.

LINE PLANT PRACTICE C, 1974 (*continued*)

Q 8 Describe 5 important factors which influence the size of an area served by a telephone exchange.

A 8 For completeness, 6 factors influencing the size of an area served by a telephone exchange are described.

(*a*) The technical requirements of the exchange and transmission equipment include the need to be able to operate the exchange equipment by switch-hook control and dialling. This places a limitation on the d.c. resistance of the cable pairs linking the telephones to the exchange; in modern British practice, this limit is 1 kΩ. Similarly, the need to be able to speak over the completed circuit imposes a limit on the length of circuits between the telephones and the exchange. In modern British practice, the limit is a loss of not more than 10 dB at 1·6 kHz

(*b*) The density of subscribers per unit area of territory influences the siting of the telephone exchange, when taken into account in relation to the items discussed below.

(*c*) Natural boundaries such as rivers, estuaries and railways, affect the siting of telephone exchanges and the division of exchange areas into cabinet, pillar and distribution-point areas.

(*d*) Line-plant costs considerably influence the choice of sites for exchanges; a balance has to be struck between the costs of the provision of subscribers' and junction cables. Within exchange areas, line-plant costs and the rate of growth largely determine the siting of cabinets and distribution points.

(*e*) Exchange-equipment, building and accommodation costs have a major influence on determining the size of an exchange area. Concentration of equipment into one building tends to reduce these costs, although the high costs of building land and maintaining a large building in the centre of a town could favour smaller exchanges on the periphery of the conurbation.

(*f*) Within an exchange area, the method of distribution is influenced by the number of subscribers per unit area and by the forecast rate of growth of the number of subscribers. The use of cross-connexion cabinets has been shown to be economic in most circumstances. In a densely-populated area, a large cabinet serving a compact area could be economic. In a less-dense area, the cabinet could be smaller, although it would serve a larger area. The use of cabinets saves main-cable pairs and, hence, the size of a cabinet area is influenced by its distance from the exchange.

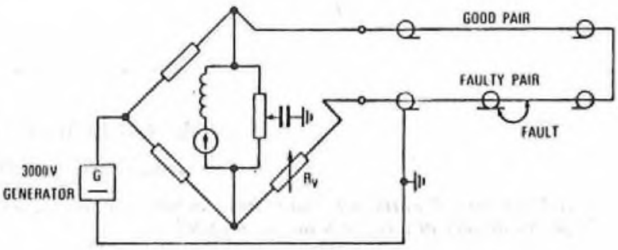
The governing factor influencing the planning of the area served by a telephone exchange is the economic consideration, judged over the costing period (usually 20 years), taking into account existing internal and external plant and future requirements based on a forecast of growth, both of subscribers' lines and traffic.

Q 9 (*a*) *Describe how the high-voltage Varley test can be used to locate a fault which has lowered the dielectric strength of a coaxial pair in a cable.*

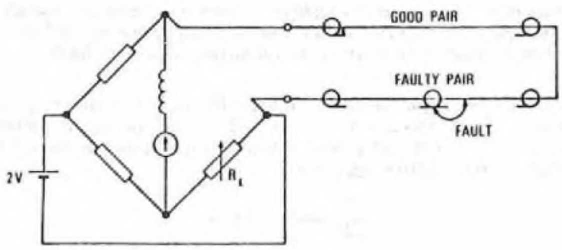
(*b*) *What safety precautions must be taken before carrying out this test?*

A 9 (*a*) The fault can be found using the high-voltage Varley test in conjunction with a loop-resistance test. The circuit shown in sketch (*a*) is connected for the Varley measurement, and sketch (*b*) shows how the bridge is used to make the loop measurement.

The circuit of sketch (*a*) is, essentially, a conventional Varley earth-fault test circuit, but a high-voltage d.c. generator is used in place of a battery, and protection is provided for the galvanometer. Equal ratio arms are used, the resistance of each being approximately that of the



(*a*)



(*b*)

inner conductor of 10 km of 2·6/9·5 mm coaxial cable. The variable resistance arm is a 4-dial decade box, adjustable in steps of 0·01 Ω. A ballistic balance circuit is connected across the galvanometer to eliminate fluctuations of the needle that occur due to dielectric breakdown. Two balance circuits are provided: for 5 km and 10 km sections of cable. The series inductor is used to limit current surges which could otherwise damage the galvanometer when breakdown occurs.

The loop resistance is measured first, using the circuit shown in sketch (*b*). The Varley measurement is then made using the circuit in sketch (*a*). At first, the galvanometer sensitivity is reduced to a minimum, and the output of the high-voltage generator increased until approximately 2 breakdowns occur per second; this is observed on an output voltmeter of the generator. The sensitivity of the galvanometer is then increased, while the bridge is balanced on variable resistor R_V and the ballistic balance control in turn, since these 2 adjustments affect each other. The resistance to the fault, x ohms, is then given by

$$x = \frac{R_L - R_V}{2},$$

where R_L is the loop resistance (Ω), and R_V is the Varley resistance (Ω).

(*b*) Since a high voltage is used for the test, safety precautions are essential. Before the test is made, it should be ensured that no person can touch the cable conductors at the ends, or at any intermediate stations, and that the power-feeding circuits and terminal apparatus are disconnected. Warning notices should be displayed. At the end of the test, the equipment should be left for a few minutes after switching off and, before the cable is disconnected, it should be confirmed that it has discharged, by using a neon tester.

Q 10 (a) Describe a method of providing an earth connexion to a telephone exchange using deep-driven electrodes.

(b) The resistance of a single earth rod is 20 Ω. By reference to Fig. 1, determine how many rods will be required to make the earth-system resistance 7 Ω. Assume a spacing-to-length-of-rod ratio of 2.

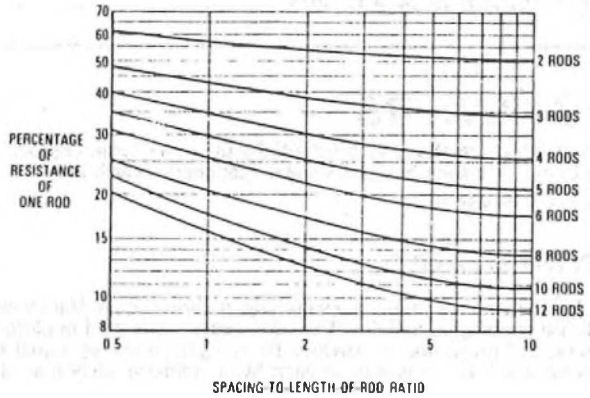


Fig. 1

A 10 (*a*) A deep-driven earth-electrode system uses mild-steel earth rods, 1·5 m long and 16 mm in diameter, threaded at both ends to permit them to be screwed together to form a long continuous rod. The site for the earth electrode is chosen to be as far as possible from any lead-covered cables entering the building and adjacent electricity cables and gas and water pipes.

A hole, about 600 mm deep and 450 mm in diameter, is excavated. A tip and driving-head are screwed to the ends of a rod, and the rod is driven vertically into the ground at the bottom of the hole. When the first rod has been driven in, the driving head is removed, a second rod is secured to the first, the driving head is replaced, and driving is continued. More rods are added until a sufficiently low resistance value has been obtained.

If driving becomes difficult before a sufficiently low resistance value is obtained, further rods are driven into the ground. The spacing between the rods should be approximately equal to the depth to which the spikes are driven, but spacing need not exceed 12 m. It is usually found that additional spikes can be driven to about the same depth as the first spike, and the resistances of all the spikes can be assumed to be approximately the same value.

Connexion between the spikes of the electrode system, and between the complete system and the main bus-bar, is by means of a suitable size of green PVC-insulated cable and soldering sockets. The cable connecting the spikes is laid in a trench not less than 300 mm deep. The cable between the electrode system and the bus-bar is laid at the same depth but protected by ducting. The top connexions of the spikes, which are liable to corrosion, are wrapped with sealing tape.

(*b*) For a 7 Ω earth system, the percentage ratio of the resistance of the system to that of one rod is

$$\frac{7}{20} \times 100\% = 35\%.$$

From the graph in Fig. 1, for a spacing-to-length-of-rod ratio of 2, it can be seen that 4 rods are required.

BASIC MICROWAVE COMMUNICATION C, 1974

Students were expected to answer any 6 questions

Q 1 (a) If a power, P watts, is radiated from an isotropic source, show that the power density at a range R metres is equal to

$$\frac{P}{4\pi R^2} \text{ watts/metre}^2.$$

(b) Calculate the power reaching a correctly aligned paraboloid, having an aperture of 3 m², at an unobstructed range of 20 km, if the transmitter radiates 1 W from an aerial having a gain of 2000.

A 1 (*a*) An isotropic source radiates its power uniformly in all directions. The surface area of a sphere, of radius *R* metres, is $4\pi R^2$ metres², so that the power density at a distance *R* metres from an isotropic source radiating *P* watts is

$$\frac{P}{4\pi R^2} \text{ watts/metre}^2.$$

(*b*) The power density at a range of 20 km from a 1 W isotropic source is $1/\{4\pi(2 \times 10^4)^2\}$ W/m². Therefore, by definition, the power density at this range from an aerial having a gain of 2000 is

$$\frac{2000}{4\pi(2 \times 10^4)^2} = 4 \times 10^{-7} \text{ W/m}^2.$$

Therefore, the power received by an aerial of aperture 3 m² is

$$3 \times 4 \times 10^{-7} \text{ W} = 1 \cdot 2 \ \mu\text{W}.$$

Q 2 (a) Explain why the characteristic impedance of an open-wire transmission line is considerably greater than that of a coaxial cable.

(b) State, with reasons, whether or not each of the following factors influences the power loss in a low-loss coaxial cable at radio frequencies:

(i) the diameter of the inner conductor,
(ii) the inner diameter of the outer conductor,
(iii) the permittivity of the insulating material, and
(iv) the standing-wave ratio.

A 2 (*a*) The characteristic impedance, Z_0 ohms, of a transmission line, where the series resistance and shunt leakage are negligible, is given by

$$Z_0 = \sqrt{\left(\frac{L}{C}\right)} \text{ ohms},$$

where *L* is the loop inductance (H), and *C* is the loop capacitance (F).

The conductors of an open-wire transmission line are well-spaced and have both a high self-inductance and a low capacitance between them. The characteristic impedance of an open-wire line is given by $276 \log_{10} d/r$ ohms, where *d* and *r* are the spacing and radius of the wires respectively (m). For a typical line, $Z_0 \simeq 600\ \Omega$.

In a coaxial cable, the conductors are close together, reducing the self-inductance and increasing the capacitance. Both factors reduce the value of Z_0. For an air-spaced coaxial cable, the characteristic impedance is given by $138 \log_{10} b/a$ ohms, where *b* and *a* are the inner diameter of the outer conductor and the outer diameter of the inner conductor respectively (m). For a typical air-spaced cable, $Z_0 = 75\ \Omega$ when $b/a = 3 \cdot 6$.

The value of Z_0 falls to about 50 Ω for a cable with a solid dielectric.

(*b*) The power loss in a coaxial cable can be expressed in terms of the attenuation, α, where

$$\alpha = \left(\frac{\omega\rho\epsilon_r}{2}\right)^{1/2} \frac{\left(\frac{1}{2b} + \frac{1}{2a}\right)}{\log_e \frac{b}{a}} \text{ nepers/metre},$$

or,

$$\alpha = \frac{1}{2b}\left(\frac{\omega\rho\epsilon_r}{2}\right)^{1/2} \frac{\left(1 + \frac{b}{a}\right)}{\log_e \frac{b}{a}} \text{ nepers/metre},$$

where ω is the angular velocity (rad/s), ρ is the resistivity (Ω m), and ϵ_r is the relative permittivity.

(*i*) If *b* is held constant while *a* is varied, the attenuation, and hence the power loss, reaches a minimum when $b/a = 3 \cdot 6$. This represents an optimum condition between the high resistive loss of a small inner conductor and the low $b : a$ ratio which reduces Z_0 but requires a heavier current for a given transmitted power.

(*ii*) If *a* is held constant while *b* is varied, the power loss is reduced as *b* is increased. This is because the reduced resistance of the outer conductor reduces the resistive loss, and because the increased characteristic impedance requires a lower current for a given power and the resistive losses are further reduced.

(*iii*) It can be seen from the formula that, if the permittivity is increased, the loss, which is primarily due to the resistivity, is also increased. The increase in permittivity reduces Z_0 and the current is, therefore, increased for a given transmitted power. This increases the resistive loss.

(*iv*) If the standing-wave ratio is other than unity, power is reflected by the load which, therefore, receives less power. The reflected wave itself causes additional losses in the cable. The conductor losses from the forward wave nevertheless remain the same.

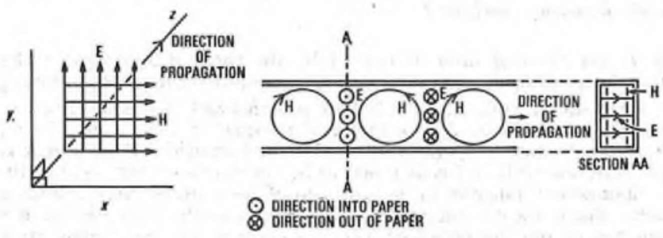
Q 3 (*a*) *With the aid of diagrams, explain*

(*i*) *why a TEM wave cannot be propagated in a hollow waveguide, and*

(*ii*) *how a TE wave can be represented as the resultant of a pair of TEM waves.*

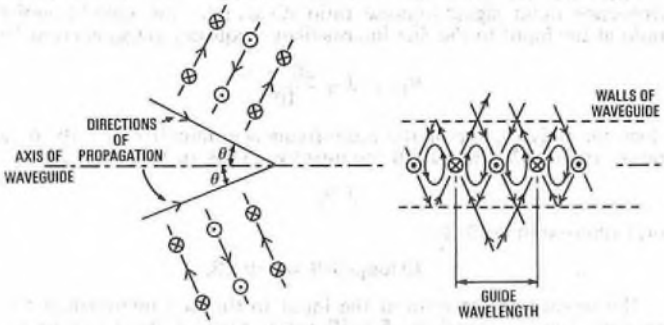
(*b*) *A wave, propagated in the dominant mode in a rectangular waveguide having internal dimensions 28 × 13 mm, is found to have a guide wavelength equal to the cut-off wavelength. Calculate the frequency of the source energizing the waveguide.*

A 3 (*a*) (*i*) A TEM wave has an electric field, E, and a magnetic field, H, at right angles to each other and to the direction of propagation, as shown in sketch (*a*) for a plane TEM wave in free space. This wave cannot be propagated in a hollow waveguide because the magnetic field would be at right angles to the sides of the waveguide containing the *y*-axis. This does not satisfy the boundary condition that a magnetic field perpendicular to a perfectly-conducting boundary must vanish at the boundary.



(*a*) (*b*)

(*ii*) A TE_{10} wave, which has a longitudinal component of magnetic field, is shown in sketch (*b*), and can be propagated along a hollow waveguide (a rectangular waveguide is shown in sketch (*b*)). The boundary condition referred to in part (*a*) (*i*) is met in this case, and the longitudinal component of the magnetic field, parallel with the sides of the guide, is balanced by currents in the walls at right angles to the magnetic field. This wave can be represented by 2 TEM waves travelling at an angle of $\pm\theta$ to the longitudinal axis of the waveguide, as shown in sketch (*c*). The walls of the waveguide can then be inserted as indicated by the dashed lines, and the boundary condition is met.



(*c*)

(*b*) For the dominant mode in a rectangular waveguide,

$$\frac{1}{\lambda_g^2} = \frac{1}{\lambda^2} - \frac{1}{(2a)^2},$$

where λ_g is the guide wavelength (m), λ is the free-space wavelength (m), and a is the broad dimension of the waveguide (m).

The waveguide cuts off at a free-space wavelength of λ_c metres when $\lambda_g \to \infty$. Therefore,

$$\frac{1}{\lambda_g^2} = \frac{1}{\lambda_c^2} - \frac{1}{(2a)^2} = 0,$$

whence $\lambda_c = 2a$.

Now, $\lambda_g = \lambda_c = 2a$.

$$\therefore \frac{1}{(2a)^2} = \frac{1}{\lambda^2} - \frac{1}{(2a)^2},$$

whence $\lambda = a\sqrt{2}$ metres.

But, $$f = \frac{c}{\lambda} = \frac{c}{a\sqrt{2}} \text{ hertz},$$

where c is the velocity of light, equal to 3×10^8 m/s, and f is the frequency (Hz).

$$\therefore f = \frac{3 \times 10^8}{28 \times 10^{-3} \times \sqrt{2}} \text{ Hz},$$

$$= \underline{7{\cdot}58 \text{ GHz.}}$$

Q 4 (*a*) *Explain briefly why the full advantages of frequency modulation cannot be achieved unless*

(*i*) *a very high carrier frequency is used, and*

(*ii*) *the signal at the output of the intermediate-frequency amplifier in the receiver is large.*

(*b*) *A frequency-modulation system, which provides a channel for speech communication (300–3400 Hz), has a carrier frequency of 100 MHz and a deviation ratio of 15. A 12 V, 600 Hz test input signal is found to produce the maximum frequency deviation. Calculate*

(*i*) *the maximum frequency deviation of the system,*

(*ii*) *the frequency deviation that would result from a 10 V, 3 kHz input signal,*

(*iii*) *the modulation index for an 8 V, 1·7 kHz input signal, and*

(*iv*) *the approximate practical bandwidth required for the system.*

A 4 (*a*) (*i*) To pass the most important part of the frequency spectrum of the modulated signal, a frequency-modulation system requires a bandwidth many times wider than that required for an amplitude-modulation system. The wide spectrum reduces the effect of interference occurring in a limited part of the band. The effect of phase-modulated noise on the carrier is proportionately less for wider phase deviations caused by the modulation. The wide spectrum enables a good signal-to-noise ratio to be maintained in the communication channel in the presence of a much poorer carrier-to-noise ratio, provided the latter does not fall below the threshold level.

The large bandwidth required for each channel requires a very high carrier frequency, especially for multi-channel systems where space is required in the frequency spectrum for a number of separate channels. Each transmission must form a low enough percentage of the carrier frequency to be practicably realizable.

(*ii*) A large signal is required at the output of the intermediate-frequency amplifier to enable the amplitude limiter to work efficiently. If it does not work efficiently, impulsive noise and variations in the signal level impair the quality of the communication channel.

(*b*) (*i*) The deviation ratio is given by

$$\frac{\text{maximum frequency deviation}}{\text{maximum modulating frequency}},$$

so that the maximum frequency deviation of the system

$$= 15 \times 3{\cdot}4 = \underline{51 \text{ kHz.}}$$

(*ii*) The frequency deviation is proportional to the modulating voltage, so that the frequency deviation produced by a 10 V modulating signal, for any frequency in the speech-communication band, is

$$51 \times \frac{10}{12} = \underline{42{\cdot}5 \text{ kHz.}}$$

(*iii*) The frequency deviation produced by an 8 V, 1·7 kHz input signal is 51 × 8/12 = 34 kHz.

Now, the modulation index

$$= \frac{\text{frequency deviation}}{\text{modulating frequency}},$$

$$= \frac{34 \times 10^3}{1{\cdot}7 \times 10^3} = \underline{20.}$$

(*iv*) The approximate practical bandwidth required for a frequency-modulation system is $2(D + A)$ hertz, where D is the frequency deviation (Hz), and A is the maximum modulating frequency (Hz).

Therefore, the bandwidth required

$$= 2(51 + 3{\cdot}4) \simeq \underline{110 \text{ kHz.}}$$

Q 5 (*a*) *Fig. 1 represents a section of rectangular waveguide closed at both ends to form a cavity, which is to resonate at its lowest frequency. Make 2 copies of this diagram and,*

(*i*) *on one copy, sketch the H-field pattern,* (*ii*) *on the other, indicate with a line the location of the maximum E-field, and,*

(*iii*) *on either, show the position of a probe to ensure maximum coupling with the cavity.*

(*b*) *Calculate the free-space wavelength of the lowest resonant frequency.*

(*c*) *State briefly why resonance could be obtained at other frequencies.*

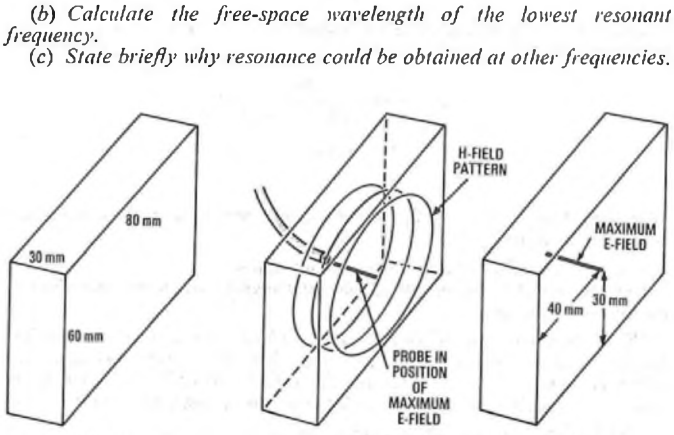


Fig. 1 (*a*) (*b*)

A 5 (*a*) The resonant field-pattern in the cavity can be represented as the resultant of 2 waves moving in opposite directions over half a guide-wavelength of waveguide. The resultant magnetic-field pattern, and the position of the maximum E-field, are shown in sketches (*a*) and (*b*) respectively. The pattern for the lowest resonant frequency is that of the H_{101} resonance pattern. Sketch (*a*) also shows the position of a probe for maximum coupling.

(*b*) For a rectangular waveguide,

$$\frac{1}{\lambda_g^2} = \frac{1}{\lambda^2} - \frac{1}{(2a)^2},$$

where a is the broad dimension of the waveguide (m), λ_g is the guide wavelength, equal to $2l$ where l is the length of the cavity (m), and λ is the free-space wavelength (m).

$$\text{Hence } \lambda = \frac{1}{\sqrt{\left\{\frac{1}{\lambda_g^2} + \frac{1}{(2a)^2}\right\}}} \text{ metres,}$$

$$= \frac{1}{\sqrt{\left\{\frac{1}{(2l)^2} + \frac{1}{(2a)^2}\right\}}} \text{ metres,}$$

$$= \frac{1}{\sqrt{\left\{\frac{1}{(160 \times 10^{-3})^2} + \frac{1}{(120 \times 10^{-3})^2}\right\}}} \text{ m} = \underline{96 \text{ mm.}}$$

(*c*) Higher resonant frequencies can be obtained by exciting the cavity with a probe or loop in the optimum position for the higher mode required. In general, different integers for m, n, and l can be chosen for the TE_{mnl} and TM_{mnl} modes, where m, n and l represent the number of E-field or M-field half-period patterns repeated parallel to the narrow and broad dimensions, and longitudinally in the length of the cavity, respectively. These patterns can occur in 3 different orientations in a given cavity.

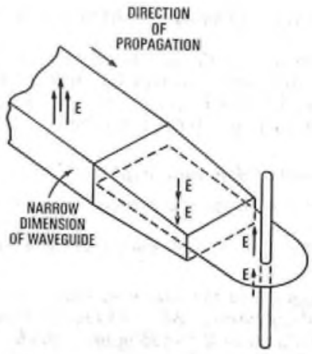
Q 6 (*a*) *Give 2 reasons why the open end of a rectangular waveguide does not provide a suitable feed for a paraboloidal reflector.*

(*b*) *With the aid of carefully-drawn diagrams, explain how a 10 GHz half-wave dipole could be energized by a rectangular waveguide supporting the dominant mode. Show the mounting of the dipole, clearly distinguishing between the broad and narrow dimensions of the waveguide.*

A 6 (*a*) (*i*) The open end of a rectangular waveguide is smaller than one wavelength and, therefore, radiates little energy, most being reflected back along the waveguide by the impedance mismatch.

(*ii*) The energy that is radiated is distributed over a wide angle instead of uniformly over the reflector with a suitable tailing-off, or tapering, towards its edges, the tapering of the illumination being required to reduce the amplitude of the sidelobes.

(*b*) The sketch shows a method of energizing a half-wave dipole radiator from a rectangular waveguide supporting the dominant mode. The overall length of the dipole is half a wavelength; that is, 15 mm at 10 GHz. The dipole is mounted on a conducting plate which bisects the narrow walls of the guide. Being perpendicular to the electric field,



the plate has little effect on radiation. The electric field, E, is split into 2 parts, which energize their respective halves of the dipole in opposite phases. The waveguide is slightly tapered to improve matching.

Q 7 (*a*) *Explain the difference between thermal noise and shot noise.*

(*b*) *The signal-to-noise ratio at the input of a microwave receiver is 25 dB. The radio-frequency amplifier stage has a power gain of 10 and a noise factor of 3 dB. This is followed by a mixer having a noise factor of 13 dB. Calculate the signal-to-noise ratio at the input to the first intermediate-frequency stage.*

(*c*) *Why is the noise factor of the mixer much greater than that of the radio-frequency amplifier?*

A 7 (*a*) Thermal noise is caused by the thermal agitation of the free electrons inside a resistor. It is equivalent to a generator producing a mean-square voltage, $\overline{e^2} = 4kTBR$, in series with a noiseless resistor, where k is Boltzmann's constant and is equal to $1{\cdot}38 \times 10^{-23}$ J/K, T is the absolute temperature (K), B is the bandwidth (Hz), and R is the resistance (Ω) and is assumed to be constant over the bandwidth.

Shot noise is inherent in the emission of electrons in, for example, a valve, due to the current not being perfectly smooth but consisting of a number of moving electrons, each carrying a discrete charge. In a temperature-saturated diode, for example, the mean of the square of the fluctuating noise component of the current is given by $i_n^2 = 2eIB$, where e is the electronic charge and is equal to $1{\cdot}6 \times 10^{-19}$ C, and I is the current (A).

(*b*) The overall noise factor N_{12}, of 2 stages of a receiver is given by

$$N_{12} = N_1 + \frac{N_2 - 1}{G_1},$$

where N_1 is the noise factor of first stage, N_2 is the noise factor of second stage, and G_1 is the gain of first stage, these being expressed as power ratios.

Therefore, the noise factor of the receiver, which is the radio-frequency input signal-to-noise ratio divided by the signal-to-noise ratio at the input to the first intermediate-frequency stage, is given by

$$N_{12} = 2 + \frac{20 - 1}{10},$$

since the noise factor of the radio-frequency amplifier is 3 dB, or a power ratio of 2, and that of the mixer is 13 dB, or a ratio of 20,

$$= 3{\cdot}9,$$

or, expressed in decibels,

$$10 \log_{10} 3{\cdot}9 = 5{\cdot}9 \text{ dB.}$$

The signal-to-noise ratio at the input to the first intermediate-frequency stage is, therefore, 5·9 dB worse than at the input to the receiver and, hence, is given by

$$25 - 5{\cdot}9 = \underline{19{\cdot}1 \text{ dB.}}$$

(*c*) A mixer can be considered to consist of 2 equivalent circuits, one as seen by the input or the local-oscillator terminals, and the other representing the circuit as seen looking back from the input to the intermediate-frequency amplifier. The first circuit has a negative gain; that is, the conversion loss of the difference-frequency output power compared with the radio-frequency input power. In the case of a diode mixer, this must be 6 dB or more. The second stage includes the noise power due to the average of the square of the noise current over the local-oscillator cycle, together with noise added by the local oscillator. The noise factor of the 2 circuits in tandem is given by $N_1 + (N_2 - 1)/G_1$. The gain of the first circuit, G_1, is less than 0·25 (i.e. −6 dB) and, so, $(N_2 - 1)$ is multiplied at least 4 times, and degrades the overall noise factor. Furthermore, the noise current depends upon the diode current, which has to be large to switch the diode efficiently. The corresponding currents in the amplifier are kept to a minimum to keep the internal noise power low.

Q 8 (a) How does the formation of a p i n diode differ from that of a simple p n diode? What properties of the p i n diode are of special interest in microwave applications?

(b) Explain, stating one advantage in each case, how a p i n diode may be used as

(i) a microwave modulator, and

(ii) a microwave switch.

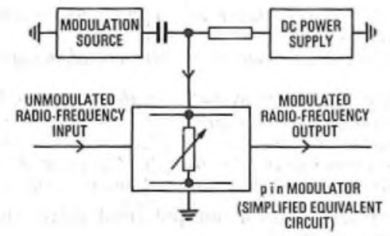
A 8 (*a*) A p n diode is a silicon semiconductor device containing a junction between electron-deficient (p-type) and electron-rich (n-type) materials.

The p i n diode has a layer of intrinsic (high-resistivity) material, of finite thickness, between highly-doped p-type and n-type material, thus giving rise to the abbreviation p i n.

At low frequencies, the p i n diode rectifies as would an ordinary p n junction diode. However, at frequencies above about 100 MHz, rectification ceases, and the current is controlled by the availability of stored charges in the intrinsic layer that do not have time to recombine. The diode then acts as a resistance with a value inversely proportional to the charge, the size of which depends upon the biasing current. With forward-biasing, the charge is high and the resistance to microwave currents low; when reverse-biased, the charge is low and the resistance to microwave currents is high.

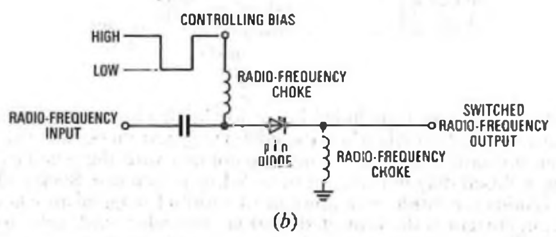
The device is small, and the impedance is substantially resistive, so that it can be built into microwave circuits and used for broadband applications.

(*b*) (*i*) For use as a microwave modulator, the p i n diode is connected across a strip transmission line or a waveguide. It is connected to a biasing circuit as shown in sketch (*a*). The bias is adjusted to give a value of resistance that absorbs and reflects some energy, but allows a proportion to pass to the radio-frequency output. The bias is varied in accordance with the output from the modulator, so varying the amount of attenuation caused by the diode, and modulating the amplitude of the output. This arrangement can, therefore, be used to amplitude modulate the output from a klystron or other microwave source without altering the frequency, provided an isolator is placed between the source and the modulator. Normally, such a modulation system is difficult or impossible to achieve by modulating the power supply to a microwave source.



(*a*)

(*ii*) The arrangement for a microwave switch is similar, and is shown in sketch (*b*). The controlling bias is switched rapidly between high and low values, thus changing the resistance of the diode from a few ohms to over 1 kΩ, so that the radio-frequency power is switched on and off. A greater isolation can be achieved with several diodes spaced at quarter-wavelength intervals along the line, to reflect the incident radio-frequency power.



(*b*)

One advantage of the p i n diode switch is that it can be designed to work over a broad frequency band, because the impedance of the device is free from parasitic elements.

Q 9 (a) State the highest modulation frequency that could be transmitted by a pulse-amplitude-modulation communication system using rectangular pulses of duration 80 μs and unity mark-to-space ratio.

(b) A 60 mm microwave communication system uses unity mark-to-space ratio pulses of duration 5 μs. Give, stating reasons,

(i) the value of the intermediate frequency (IF) suitable for the receiver, and

(ii) the bandwidth required in the IF amplifier.

(c) What are the requirements of the filter placed between the detector and video amplifier in such a system as described in part (b)?

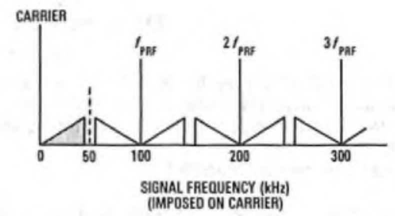
A 9 (*a*) The pulse-repetition frequency (PRF) for sampling a modulation frequency f hertz must be at least $2f$ hertz. One 80 μs pulse occurs, followed by a space of 80 μs. The PRF is, therefore, $1/(160 \times 10^{-6})$ Hz, and the maximum modulation frequency is

$$\frac{1}{2 \times 160 \times 10^{-6}}\ \text{Hz} = \underline{3 \cdot 125\ \text{kHz}}.$$

(*b*) (*i*) The frequency of a 60 mm wavelength signal is $(3 \times 10^8)/(60 \times 10^{-3})$ Hz = 5 GHz. If I is the intermediate frequency (Hz), and the system is one of a number of adjacent systems using the available frequency spectrum, the input filter is required to accept the required carrier, say $5 \times 10^9 + I$ hertz, and reject the image frequency of $5 \times 10^9 - I$ hertz. The design of the filter is simplified when the proportional difference between the 2 frequencies, $2I/(5 \times 10^9)$, is not too small. Again, if the difference is very small, very good frequency-stability is required of the carrier and the local oscillator, so that the IF falls acceptably close to the centre of the IF amplifier pass-band. If, on the other hand, the IF is too high, difficulties may arise in the practical construction of the IF amplifier, although strip-line techniques now make high IFs practical and convenient.

The IF bandwidth required also dictates a minimum value of IF, because the bandwidth must not exceed more than about 10–15% of the IF centre frequency for practical amplifier designs. A suitable frequency, by well-established standards, would be <u>30–70 MHz</u>.

(*ii*) The system uses a 5 μs wide pulse followed by a space of 5 μs. The PRF, f_{PRF} hertz, is, therefore, $1/(10 \times 10^{-6})$ Hz = 100 kHz. The frequency spectrum of a pulse-amplitude-modulated signal is shown in the sketch, where the harmonics of f_{PRF} are each surrounded by sidebands of up to $\pm f_{PRF}/2$ hertz. The bandwidth of the IF amplifier must cover 0–f_{PRF} hertz; that is, it must be <u>100 kHz</u> wide.



(*c*) It can be seen from the sketch that the upper sideband of zero signal frequency is, in fact, the required video band of 50 kHz, since the maximum modulation frequency is half the PRF. The filter between the detector and the video amplifier should, therefore, be of the low-pass type, cutting-off all frequencies above $f_{PRF}/2$ = 50 kHz.

Q 10 (a) Diode D1 in the circuit shown in Fig. 2 may be regarded as equivalent to a resistance of several megohms when cut off, and of 1 kΩ when conducting. Calculate the time constant of the circuit for each condition.

(b) A pulse generator, of negligible output impedance, supplies to the input of this circuit a pair of 100 V pulses of duration 100 μs, as shown. By means of time-related diagrams, show, for the interval 0–300 μs, the variation in potential difference

(i) across capacitor C1,

(ii) across resistor R1, and

(iii) between the output terminals.

(Voltage scales are to be shown on each diagram.)

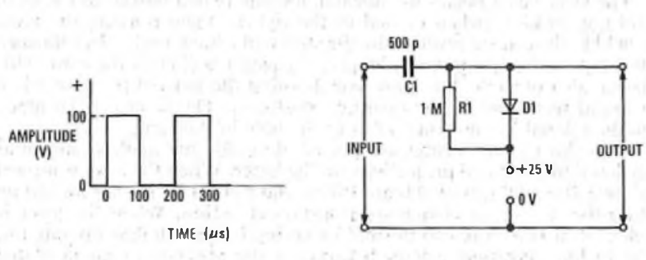
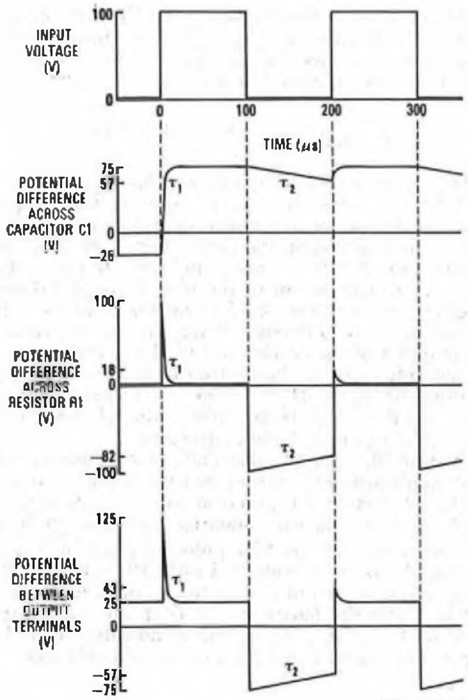


Fig. 2

A 10 (*a*) The circuit, when diode D1 is conducting, consists of capacitor C1 in series with the 1 kΩ forward resistance of diode D1, the 1 MΩ shunt resistance of resistor R1 being negligible. The 25 V source in the circuit does not affect the time constant. The time constant, τ_1 seconds, is therefore

$$\tau_1 = 1000C_1 = 1000 \times 500 \times 10^{-12}\ \text{s} = \underline{0 \cdot 5\ \mu\text{s}}.$$



When the diode is cut off, capacitor C1 is in series with resistor R1, the reverse resistance of diode D1 being assumed to be negligible in parallel with the resistance of resistor R1. The time constant, τ_2 seconds, is, therefore,

$$\tau_2 = C_1R_1 = 500 \times 10^{-12} \times 1 \times 10^6\,\text{s} = \underline{500\ \mu\text{s}}.$$

(*b*) The sketch shows the variation with time of the input voltage and the potential differences across capacitor C1, resistor R1, and between the output terminals.

Capacitor C1 is initially charged to −25 V. When the input voltage first increases from 0–100 V, the potential difference across capacitor C1 remains at −25 V for an instant, and the potential difference across resistor R1 increases from 0–100 V. Capacitor C1 charges with time constant $\tau_1 = 0{\cdot}5\ \mu$s, and can be considered fully charged to 100 − 25 = 75 V after about 2·5 μs, while the potential difference across resistor R1 falls to zero.

When the input voltage returns to zero, the instantaneous voltage across resistor R1 is −100 V, and capacitor C1 discharges with time constant $\tau_2 = 500\ \mu$s. After an interval of 100 μs (that is, when the input voltage is next due to change from 0–100 V), the potential difference across capacitor C1 has fallen by only $100\,(1 - e^{-100/500}) \simeq 18$ V.

Therefore, when the input voltage rises from 0–100 V for the second time, the total voltage applied is 18 V. This instantaneously appears as the potential difference across resistor R1, which falls to zero as capacitor C1 recharges to 75 V with time constant $\tau_1 = 0{\cdot}5\ \mu$s. Thereafter, the cycle is repeated.

The output potential difference is the same as that across resistor R1 plus 25 V due to the source in the circuit.

ELEMENTARY TELECOMMUNICATION PRACTICE, 1975

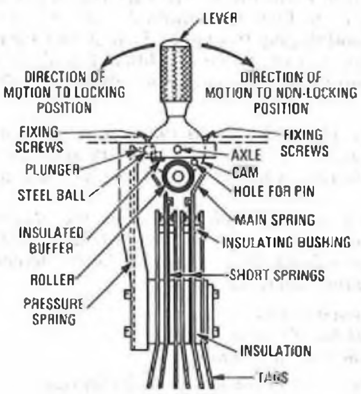
Students were expected to answer any 6 questions

Q 1 (*a*) *With the aid of a sketch, describe a 3-position locking key suitable for switching low-power audio-frequency circuits.*

(*b*) *Describe the type of contact used in the key and explain the reasons for its selection.*

(*c*) *Name a suitable contact material.*

A 1 (*a*) The sketch shows a 3-position locking key suitable for switching low-power audio-frequency circuits.



The lever has 3 positions: normal, locking (when moved to the left), and non-locking (when moved to the right). When normal, the lever is held in the central position by the steel ball which, under the influence of the pressure spring and plunger, engages a socket in the cam. This device also prevents the lever overshooting the normal position when restored from one of the operated positions. The lever can be made unidirectional by inserting a pin in the hole in the cam.

Two short stout springs are placed alongside the main springs and enclosed in L-shaped projections on the latter. When the lever is moved to the left or right, both a main spring and a short spring are moved by the roller to give a change-over spring-set action. When the lever is released, it is restored to normal by spring tension, unless operated to the locking position where, because of the absence of an insulated buffer, the spring tension holds the roller in the operated position. It is then necessary to move the lever slightly towards the normal position before the spring tension can become effective in restoring the roller.

(*b*) The contacts used are of the point-and-disc type, so that the contact force is concentrated in a small area, thereby ensuring low electrical resistance at the point of contact. The springs have a degree of flexibility and there is a slight wiping action which helps to break through any surface contamination. Because of the large surface area of the discs, perfect alignment of the springs is unnecessary.

This type of contact is used only for low-power applications since the small area of contact is not capable of carrying a heavy current.

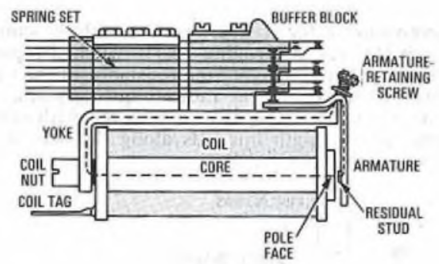
(*c*) A gold-silver alloy is used as the contact material.

Q 2 (*a*) *Draw a labelled sketch of a non-polarized relay, and clearly mark its magnetic circuit.*

(*b*) *Explain why it is necessary to include a residual gap in the magnetic circuit.*

(*c*) *Two similar relays are wound with the same number of turns on their coils. The coil on the first relay has wire of twice the cross-sectional area of that used on the second. The first relay is connected to a 6 V battery and the second to a 12 V battery. Compare the forces exerted on the lifting pins of the relays, giving reasons for your answer.*

A 2 (*a*) The sketch shows a non-polarized relay; the dashed line indicates its magnetic circuit.



(*b*) A residual gap is included in the magnetic circuit to ensure that the relay releases correctly when the energizing current ceases. The residual gap prevents the armature making contact with the pole face and forming a closed magnetic circuit of very low reluctance. Such a closed circuit retains a considerable amount of residual magnetism when the energizing current is disconnected, making the relay extremely slow to release. With a residual gap, when the current is disconnected, the magnetic flux immediately begins to fall rapidly, so that the relay releases quickly.

(*c*) The force exerted on the armature of a relay is proportional to the square of the magnetic flux which, in turn, is proportional to the product of the current flowing in the coil and the number of turns on the coil. Since the 2 relays are similar, the constants of proportionality can be assumed to be the same and, because both relays have the same number of turns on their coils, the forces on their armatures depend only on the respective currents flowing.

The coil of the first relay is wound with wire of twice the cross-sectional area of that of the second and, therefore, its resistance is half that of the second relay's coil. However, because the supply voltage to the first relay is half that to the second, the current is the same in both coils.

Hence, the forces exerted on the lifting pins of each relay are the same.

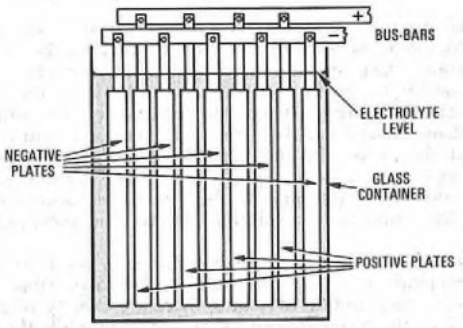
Q 3 (*a*) *State the principal difference between primary and secondary cells.*

(*b*) *Describe, with the aid of a sketch, a simple lead-acid secondary cell.*

(*c*) *Explain, in some detail, why the specific gravity of the electrolyte is an indicator of the state of charge of a cell, and say how the specific gravity is measured.*

A 3 (*a*) In a primary cell, the chemical reactions which release electrical energy are not reversible. In a secondary cell, electrical energy can be supplied to the cell in its discharged condition to restore the chemical make-up prevailing when the cell is charged.

(*b*) The sketch shows the components of a simple lead-acid secondary cell. Two interleaved sets of plates, each connected to a bus-bar, are immersed in an electrolyte within a glass container. Essentially, in the charged state, the positive plates consist of lead peroxide, and the negative plates of lead. The electrolyte is dilute sulphuric acid.



(*c*) The electrolyte is a mixture of sulphuric acid and water. Because sulphuric acid has a specific gravity higher than that of water, the specific gravity of the mixture is greater than unity, the actual value depending on the proportions of the 2 constituents. In a typical case, the specific gravity of the electrolyte of a fully-charged cell is 1·215. The electrochemical processes, which take place during discharge, result in the conversion of sulphuric acid to water, and this change in the relative amounts of the 2 constituents produces a fall in the specific gravity of the mixture. Because the amount of acid converted is directly related to the energy released by the cell, the specific gravity is a direct measure of the state of charge of the cell.

Specific gravity is measured using a hydrometer, which consists of a graduated pencil of glass attached to a weighted bulb so that it floats in a vertical position. The level to which the hydrometer sinks in a liquid is a measure of the specific gravity of that liquid. In pure water, the level to which the hydrometer sinks is calibrated as 1·000.

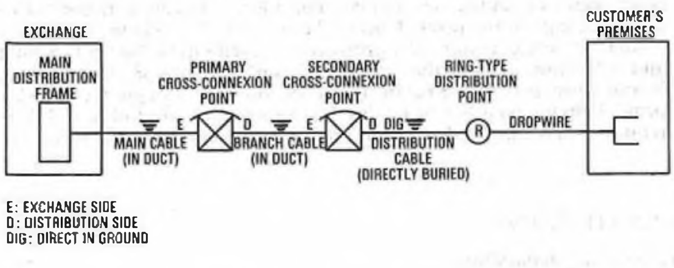
Q 4 (*a*) *With the aid of a diagram, describe the normal stages of a telephone line connecting an exchange and a customer's premises.*

(*b*) *For part* (*a*) *above, give a description of either*

(*i*) *the equipment at a branching point, or*

(*ii*) *a typical cable used at some specified point.*

A 4 (*a*) The sketch shows typical elements in a telephone-exchange–customer's-premises distribution scheme. The main cable is unit-twin polyethylene cable, and the branch and distribution cables are twin polyethylene cable. Cable conductors can be either copper or aluminium alloy, except at the main distribution frame and cross-connexion-point exchange-side terminations, where copper conductors must be used. Main cables are pressurized; others are petroleum-jelly-filled.



The layout shown is typical, and variations can occur; for example, houses on many of the newer estates, and many office buildings, are fed by underground leads-in.

(*b*) Because of the wide choice of options offered, no model answer to this part is given.

Q 5 *Telephone circuits between 2 large centres may be provided by audio cable, coaxial cable or microwave-radio link. By reference to their individual characteristics, state the factors that influence the choice in particular cases.*

A 5 Audio cables are normally restricted to one-circuit-per-pair or one-circuit-per-quad working, and are therefore relatively expensive. If the route is long, it is necessary to provide amplification at intermediate points. The cable must be accessible at all points for installation and repair operations. An audio-cable route need not initially be fully equipped with terminal equipment if the traffic requirement is less than the ultimate capacity. The terminal equipment is simple and inexpensive. Because of these characteristics, audio cables are generally used over fairly short distances, and where growth is expected.

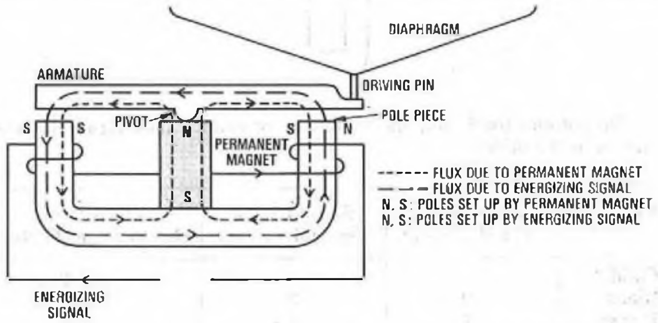
Coaxial cables also require full accessibility. Many circuits can be routed over each pair, but complex and expensive terminal equipment is needed, with a potential risk of large-scale failure. Amplifiers are needed along the route at fairly close spacings, adding to the cost of the system. Coaxial-cable systems, therefore, are generally used where large numbers of circuits are required over long distances.

Microwave-radio links need a line-of-sight propagation path, over which they provide a large number of circuits. They require expensive terminal equipment, but intermediate amplifying stations are widely separated. There is a risk of extensive failure unless special precautions are taken. Microwave-radio systems are, therefore, used when a large number of circuits is required over long distances, especially where it is impracticable or uneconomic to lay cable.

Q 6 (*a*) *With the help of a labelled sketch, describe the action of a rocking-armature* (*balanced-armature*) *type of telephone receiver.*

(*b*) *Show the path of the magnetic flux in the receiver, and describe the action of the permanent magnet.*

A 6 (*a*) The sketch shows the elements of a rocking-armature receiver.



The permanent magnet sets up a polarizing magnetic field having 2 symmetrical parallel paths and, thus, in the absence of a signal current in the coil windings, the armature is balanced. The coils are wound in series opposition so that, when current flows, the magnetic field is strengthened at one pole piece and weakened at the other, causing the armature to move towards the pole piece with the stronger field. When the current is reversed, the armature moves towards the other pole piece. Hence, the armature is driven in phase with an energizing signal current.

(*b*) The magnetic flux paths due to the permanent magnet and an energizing signal are shown in the sketch. The direction of the latter corresponds to the direction of current flow indicated. The magnetic poles set up by the permanent magnetic field are shown in bold type, and those set up by the energizing signal are shown in normal type.

If the magnet flux density at each pole piece, due to the permanent magnet, is B teslas, and that due to the energizing signal is b teslas, the force of attraction at one pole piece is proportional to $(B + b)^2$ and, at the other, to $(B - b)^2$. Therefore, the effective force on the armature

$$\begin{aligned} &\propto (B + b)^2 - (B - b)^2, \\ &= (B^2 + 2Bb + b^2) - (B^2 - 2Bb + b^2), \\ &= 4Bb. \end{aligned}$$

Since B can be made much larger than b, $4Bb$ can be made much larger than b^2. Thus, the sensitivity of the receiver is improved by the presence of the permanent magnet. Also, without the permanent magnet, the forces of attraction at each pole piece, due to the energizing signal, would be equal, and the armature would not respond. (Note that there is no possibility of frequency doubling occurring in this type of receiver under such conditions.)

Q 7 (*a*) *Name the circuit symbols shown in Fig. 1.*

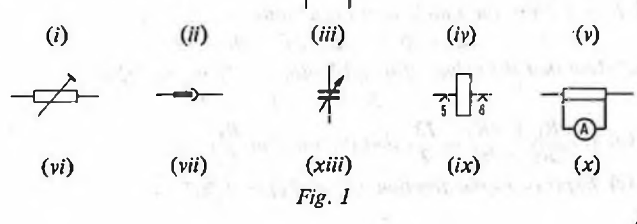


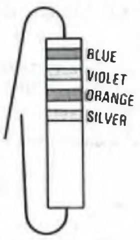
Fig. 1

(b) A small fixed resistor has a value of 67 kΩ. Describe, with the aid of a sketch, how it can be marked so that its value can be determined from all sides. Explain how a resistance tolerance of 10% is indicated.

A 7 (*a*) The circuit symbols are listed below.

(*i*) Fuse.
(*ii*) Signal lamp.
(*iii*) Transformer with ferromagnetic core.
(*iv*) Microphone.
(*v*) Change-over (make-before-break) contact unit.
(*vi*) Resistor with preset adjustment.
(*vii*) Plug and socket.
(*viii*) Variable capacitor.
(*ix*) Relay between plug-in points (U-points) 5 and 6.
(*x*) Ammeter with shunt.

(*b*) The sketch shows a method of indicating the value of a 67 kΩ resistor by means of coloured bands.



The colours used, and the values of resistance they represent, are shown in the table.

Colour	First Band (First Figure)	Second Band (Second Figure)	Third Band (Multiplying Factor)
Gold	–	–	0·1
Black	0	0	1
Brown	1	1	10
Red	2	2	100
Orange	3	3	**1 000**
Yellow	4	4	10 000
Green	5	5	100 000
Blue	6	6	1 000 000
Violet	7	7	–
Grey	8	8	–
White	9	9	–

The tolerance value is indicated by the colour of the fourth band as follows.

Silver: 10% tolerance.
Gold: 5% tolerance.
Red: 2% tolerance.
Brown: 1% tolerance.

The absence of a fourth band implies a tolerance of 20%.

For a 67 kΩ resistor of 10% tolerance, bands 1–4 are therefore coloured blue, violet, orange and silver respectively.

Q 8 (a) List the steps involved in terminating a single-strand plastics-insulated wire by soldering to a tag. Indicate at each step what precautions are necessary to obtain good results.
(b) Briefly describe one other method of terminating a similar wire.

Q 9 (a) Explain what is meant by a signalling code.
(b) Describe the Morse code, the 5-unit code and the 2-out-of-5 code, and give examples of the circumstances in which each is used.

A 9 (*a*) A signalling code is a means of expressing information in a form different from the original, and suitable for transmission over a communication system. It implies a means of encoding the information at the transmitting end, and decoding it at the receiving end.

(*b*) *The Morse Code* The Morse code consists of combinations of 2 signals of similar nature but of different lengths, in the ratio 1 : 3. The signals are called the *dot* and *dash* respectively. Each alphabetical and numerical character is represented by a unique combination of dots and dashes with between 1–5 elements, resulting in a considerable variation in character lengths. Signal elements are separated by an interval equal to the dot, letters by an interval equal to 3 dots, and words by an interval equal to 5–7 dots. The Morse code is used for hand-sending; that is, where the coded message is generated manually.

The 5-Unit Code Each character in the 5-unit code consists of a discrete combination of 5 signal elements, each of the same duration, derived from 2 basic signal conditions. As all characters are therefore of the same length, no interval is required between the elements, but it is necessary to distinguish between characters. The introduction of a single-element *start* signal before each character, and a 1·5 element *stop* signal after, gives this distinction and, at the same time, provides signals that can be used to synchronize transmitting and receiving machines. Because of this, and the fact that all elements are of the same duration, the 5-unit code is suitable for machine telegraphy.

The 2-out-of-5 Code The 2-out-of-5 code is used to transmit (or store) numerical or control information. For 5 possible signals, each having an ON state and an OFF state, there are exactly 10 arrangements by which 2 of the 5 signals can be in one state while the remaining 3 are in the other. Thus, for numerical information, each of the 10 digits 1–0 is uniquely coded by one of these 10 arrangements. The signals may consist, for example, of a combination of 2 voice-frequency tones transmitted simultaneously out of 5 possible tones available, or of d.c. conditions applied to 2 wires of a group of 5 wires. As each of the 5 possible signals has 2 states, a signal combination can be stored on any group of five 2-state devices. One application of the 2-out-of-5 code is in the multi-frequency signalling system used in the trunk transit network. Here, control information in the backward direction is coded in 2-frequencies-out-of-5 form, filters being used to separate the frequencies at the receiving end for decoding purposes.

Q 10 (a) A panel-mounted amplifier is redesigned to have similar electrical characteristics, but be mounted on a printed-circuit board. With the aid of a sketch, describe the general construction of the new design.
(b) Write brief notes on the difference between the 2 designs in respect of the
(i) size,
(ii) replacement of faulty components, and
(iii) ease of construction.

A 10 (*a*) See A3, Elementary Telecommunication Practice, 1972, *Supplement*, Vol. 66, p. 7, Apr. 1973.
(*b*) (*i*) An amplifier designed for manufacture on a printed-circuit board uses modern lightweight components, and is small in size and weighs little.
(*ii*) Identification of components is more readily achieved on a printed-circuit board because the components are individually mounted rather than being suspended in the wiring, as is the case in the more traditional designs. In replacing them, however, it is necessary to use components of similar size and configuration, and care is necessary to avoid damage to the printed-circuit board and its tracking.
(*iii*) The construction of a prototype printed-circuit board is a long and laborious process that requires detailed physical design work before construction can begin. However, once the design is complete, printed-circuit boards can readily be mass-produced, and less skill is required than was needed in traditional methods of manufacture.

PRACTICAL MATHEMATICS, 1975

Students were expected to answer any 6 questions

Q 1 (a) Solve the simultaneous equations

$$4a + 3b = 5 \text{ and } 5a - 2b = 12,$$

and show that the values of a and b also satisfy the equation

$$2a - b = 5.$$

(b) If $\dfrac{3R_1 + 2R_2}{5R_1 - R_2} = \dfrac{12}{7}$, *find the value of* $\dfrac{R_1}{R_2}$.

(c) Express, as one fraction, $(3\tfrac{1}{2} - 2\tfrac{3}{8}) \times 4\tfrac{4}{15}$.

A 1 (*a*)

$$4a + 3b = 5, \qquad \ldots\ldots (1)$$
$$5a - 2b = 12. \qquad \ldots\ldots (2)$$

Multiplying equation (1) by 2 gives

$$8a + 6b = 10. \qquad \ldots\ldots (3)$$

Multiplying equation (2) by 3 gives

$$15a - 6b = 36. \qquad \ldots\ldots (4)$$

Adding equations (3) and (4) gives

$$23a = 46.$$

$$\therefore\ \underline{a = 2.}$$

Substituting for a in equation (1) gives

$$8 + 3b = 5.$$

$$\therefore\ 3b = -3.$$

$$\therefore\ \underline{b = -1.}$$

Substituting for a and b in the equation $2a - b = 5$ gives, for the left-hand side,

$$2a - b = 4 - (-1) = 5,$$

$$= \text{the right-hand side.}$$

Hence the values obtained for a and b also satisfy the equation $2a - b = 5$.

(*b*) $$\frac{3R_1 + 2R_2}{5R_1 - R_2} = \frac{12}{7}.$$

Cross-multiplying the equation gives

$$21R_1 + 14R_2 = 60R_1 - 12R_2.$$

$$\therefore\ 26R_2 = 39R_1.$$

$$\therefore\ \frac{R_1}{R_2} = \frac{26}{39} = \underline{\frac{2}{3}}.$$

(*c*) $$(3\tfrac{1}{2} - 2\tfrac{3}{8}) \times 4\tfrac{4}{15} = \left(\frac{7}{2} - \frac{19}{8}\right) \times \frac{64}{15},$$

$$= \frac{28 - 19}{8} \times \frac{64}{15},$$

$$= \frac{9}{8} \times \frac{64}{15},$$

$$= 3 \times \frac{8}{5} = \underline{\frac{24}{5}}.$$

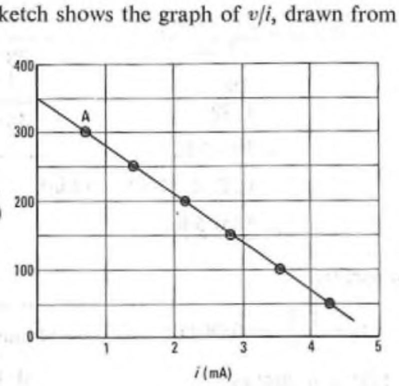
Q 2 When a capacitor discharges through a resistor, the current, i amperes, is related to the voltage, v volts, across the capacitor by the expression $v = V - iR$, where V and R are constants.

(a) Draw a suitable straight-line graph of v against i using the table of values given below.

$v\,(V)$	50	100	150	200	250	300
$i\,(mA)$	4·28	3·54	2·82	2·14	1·40	0·70

(b) Obtain from your graph estimates for the values of V and R.

A 2 (*a*) The sketch shows the graph of v/i, drawn from the table of values given.



(*b*) It can be seen from the graph that the plotted points lie almost exactly on a straight line and, hence, the values given in the table conform very closely to the law $v = V - iR$.

When $i = 0$, $v = V$. Hence, the value of V may be estimated by extending the curve to the vertical axis and reading off the value of v at $i = 0$.

From the graph, when $i = 0$, $v \simeq 350$ V.

$$\therefore\ \underline{V = 350 \text{ V.}}$$

To obtain the value of R, the co-ordinates of any point on the graph, together with the value of V obtained above, are substituted in the equation $v = V - iR$. Taking the co-ordinates at point A (0·7, 300) gives

$$300 = 350 - 0{\cdot}7R.$$

$$\therefore\ 0{\cdot}7R = 50.$$

$$\therefore\ R = \frac{50}{0{\cdot}7} = \underline{71{\cdot}43}.$$

Q 3 (a) The measured distance between the readings 5 and 2 on a slide-rule scale calibrated from 1–10 is 79·6 mm.

(i) Calculate the measured length of the full scale.

(ii) Illustrate your answer with a sketch.

(b) Using a slide-rule, or otherwise, calculate the values for

(i) R, where $R = \dfrac{R_1R_2}{R_1 + R_2}$ *and* $R_1 = 13$ *and* $R_2 = 16$, *and*

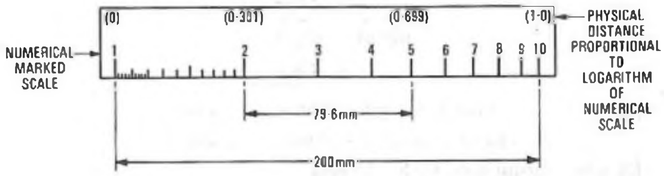
(ii) E, where $E = \dfrac{Li^2}{2}$, *and* $L = 3{\cdot}64$ *and* $i = 5{\cdot}36$.

A 3 (*a*) (*i*) Distances along a slide-rule scale are proportional to the logarithms of the numbers shown on the scale. Since $\log_{10} 2 = 0{\cdot}301$ and $\log_{10} 5 = 0{\cdot}699$, the measured distance of 79·6 mm is proportional to the difference between 0·699 and 0·301; that is, 0·398. The total length of the scale is proportional to the difference between $\log_{10} 10$ and $\log_{10} 1$; that is, unity.

Hence, $$0{\cdot}398 \propto 79{\cdot}6 \text{ mm.}$$

$$\therefore\ 1 \propto \frac{79{\cdot}6}{0{\cdot}398} = 200 \text{ mm.}$$

Therefore, the measured length of the full scale is 200 mm.



(*ii*) The sketch illustrates the lower (1–10) scale of the slide of a slide-rule. For clarity, most of the intermediate markings have been omitted. It can be seen that the numeral 5 occurs not half-way along the scale, but at approximately seven-tenths of the actual distance between markings 1 and 10. Because of its logarithmic nature, the scale is more open at the left-hand end, towards unity, and the numbers increasingly bunch together towards the right-hand end.

(*b*) (*i*) $$R = \frac{R_1R_2}{R_1 + R_2}.$$

Substituting the values given,

$$R = \frac{13 \times 16}{13 + 16},$$

$$= \frac{13 \times 16}{29},$$

$$= \underline{7{\cdot}17}, \text{ by slide-rule.}$$

Note: The accurate value for R is 7·172, to 3 decimal places.

(*ii*) $$E = \frac{Li^2}{2}.$$

Substituting the values given,

$$E = \frac{3{\cdot}64}{2} \times 5{\cdot}36^2,$$

$$= 1{\cdot}82 \times 5{\cdot}36^2,$$

$$= \underline{52{\cdot}3}, \text{ by slide-rule.}$$

Note: The accurate value for E is 52·288, to 3 decimal places.

Q 4 (a) Evaluate, using mathematical tables,

(i) $$\frac{98{\cdot}54 \times 8{\cdot}161}{4287 \times 0{\cdot}0002025}, \text{ and}$$

(ii) $\sqrt{0{\cdot}462} - \sqrt{0{\cdot}0952}$.

(b) (i) Express in terms of log 3, without using logarithmic tables, log 9 and log 81.

(ii) Hence, or otherwise, find the value of x that satisfies the equation

$$3 \log 3 + \log 9 - \log 81 = x \log 3.$$

A 4 (*a*) (*i*) $\dfrac{98\cdot54 \times 8\cdot161}{4287 \times 0\cdot0002025}$.

Evaluating the numerator and denominator by logarithms:

Numerator		Denominator	
Number	Logarithm	Number	Logarithm
98·54	1·9936	4287	3·6322
8·161	0·9117+	0·0002025	$\bar{4}$·3064+
	2·9053		$\bar{1}$·9386

Evaluating the division of the numerator by the denominator by logarithms and taking the anti-logarithm of the result:

Number	Logarithm
	2·9053
	$\bar{1}$·9386−
926·2	2·9667

$$\therefore \quad \frac{98\cdot54 \times 8\cdot161}{4287 \times 0\cdot0002025} = \underline{926\cdot2.}$$

(*ii*) $\sqrt{0\cdot462} - \sqrt{0\cdot0952} = 0\cdot6797 - 0\cdot3085$ from a table of square roots,

$$= \underline{0\cdot3712.}$$

(*b*) (*i*)

$$\log 9 = \log 3^2,$$
$$= \underline{2 \log 3.}$$
$$\log 81 = \log 3^4,$$
$$= \underline{4 \log 3.}$$

(*ii*)

$$3 \log 3 + \log 9 - \log 81 = x \log 3.$$
$$\therefore\ 3 \log 3 + 2 \log 3 - 4 \log 3 = x \log 3.$$

Dividing throughout by log 3 gives

$$3 + 2 - 4 = x.$$
$$\therefore\ \underline{x = 1.}$$

Q 5 (a) When a battery of internal resistance 2 Ω is connected in series with an external resistance of 10 Ω, the current flowing is 2 A. Calculate

(i) the e.m.f. of the battery, and
(ii) the current flowing when the external resistance is 6 Ω.

(b) Solve the following equation for t:

$$\frac{1}{4}(3t + 1) - \frac{1}{3}(1 - 2t) = \frac{1}{2}(t + 1).$$

(c) If N cells, each with internal resistance r ohms, are connected in series with an external resistance, R ohms, then N is given by the formula

$$N = \frac{iR}{E - ir}.$$

Rearrange this formula to make i the subject.

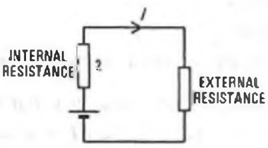
A 5 (*a*) The sketch illustrates the circuit arrangements. Let E be the e.m.f. of the battery (V), R be the value of the external resistance (Ω), r be the value of the internal resistance (Ω), and I be the current in the circuit (A). By Ohm's law,

$$E = I(r + R) \text{ volts.}$$

(*i*) When $R = 10\ \Omega$, $\quad E = 2 \times (2 + 10) = \underline{24\ \text{V}.}$

(*ii*) When $R = 6\ \Omega$, $\quad 24 = I(2 + 6)$ volts.

$$\therefore\ I = \frac{24}{8} = \underline{3\ \text{A}.}$$



(*b*) $\frac{1}{4}(3t + 1) - \frac{1}{3}(1 - 2t) = \frac{1}{2}(t + 1).$

Multiplying the equation by 12 gives

$$3(3t + 1) - 4(1 - 2t) = 6(t + 1).$$
$$\therefore\ 9t + 3 - 4 + 8t = 6t + 6.$$
$$\therefore\ 9t + 8t - 6t = 6 - 3 + 4.$$
$$\therefore\ 11t = 7.$$
$$\therefore\ \underline{t = \frac{7}{11}.}$$

(*c*)

$$N = \frac{iR}{E - ir}.$$
$$\therefore\ NE - Nir = iR.$$
$$\therefore\ iR + Nir = NE.$$
$$\therefore\ i(R + Nr) = NE.$$
$$\therefore\ \underline{i = \frac{NE}{R + Nr}.}$$

Q 6 (a) Use the approximate conversion 5 ft = 1·52 m to evaluate

(i) 11 ft in metres, to 2 decimal places,
(ii) 5·6 m in feet and inches, to the nearest inch, and
(iii) 8 yd² in square metres, to 2 decimal places.

(b) Simplify, giving your results with positive indices only,

(i) $\dfrac{2x^{-2}}{x^{-3}}$,

(ii) $\sqrt{(25a^4b^2)}$, *and*

(iii) $\dfrac{1 - 4x^2}{2x + 1}$.

(c) Evaluate $2(x + y)^2 - (y - z)^2 - z^2$ *when* $x = -3$, $y = 1$ *and* $z = -2$.

A 6 (*a*) (*i*) If $\quad 5 \text{ ft} \simeq 1\cdot52 \text{ m},$

then

$$1 \text{ ft} \simeq \frac{1\cdot52}{5} \text{ m}.$$
$$\therefore\ 11 \text{ ft} \simeq \frac{1\cdot52 \times 11}{5} = \underline{3\cdot34 \text{ m}.}$$

(*ii*) $\quad 1\cdot52 \text{ m} \simeq 5 \text{ ft}.$

$$\therefore\ 1 \text{ m} \simeq \frac{5}{1\cdot52} \text{ ft}.$$
$$\therefore\ 5\cdot6 \text{ m} \simeq \frac{5 \times 5\cdot6}{1\cdot52} \text{ ft},$$
$$= \frac{28}{1\cdot52} \text{ ft},$$
$$= 18\cdot42 \text{ ft},$$
$$= 18 \text{ ft} + (0\cdot42 \times 12 \text{ in}),$$
$$= \underline{18 \text{ ft } 5 \text{ in}.}$$

Number	Logarithm
28	1·4472
1·52	0·1818−
18·42	1·2654

(*iii*) From part (*i*),

$$1 \text{ ft} \simeq \frac{1\cdot52}{5} = 0\cdot304 \text{ m}.$$
$$\therefore\ 1 \text{ ft}^2 \simeq 0\cdot304^2 \text{ m}^2.$$

Now, $\quad 1 \text{ yd}^2 = 9 \text{ ft}^2.$

$$\therefore\ 8 \text{ yd}^2 = 72 \text{ ft}^2,$$
$$= 72 \times 0\cdot304^2 \text{ m}^2,$$
$$= 6\cdot654 \text{ m}^2,$$
$$= \underline{6\cdot65 \text{ m}^2 \text{ to 2 decimal places.}}$$

Number	Logarithm
0·304	$\bar{1}$·4829
0·304	$\bar{1}$·4829+
	$\bar{2}$·9658
72	1·8573+
6·654	0·8231

(*b*) (*i*) $\dfrac{2x^{-2}}{x^{-3}} = 2x^{-2+3} = \underline{2x}.$

(*ii*) $\sqrt{(25a^4b^2)} = \underline{5a^2b}.$

(*iii*) $\dfrac{1-4x^2}{2x+1} = \dfrac{(1-2x)(1+2x)}{2x+1} = \underline{1-2x}.$

(*c*) $2(x+y)^2 - (y-z)^2 - z^2 = 2\times(-3+1)^2 - (1+2)^2 - (-2)^2,$

$= 2\times 4 - 9 - 4 = \underline{-5}.$

Q 7 (*a*) *Fig. 1 shows a coastguard station at the top of a cliff, 100 m above sea level. A ship is observed from the station due east at an angle of depression of 10°. After 1 min, the angle of depression is 6°. If the ship is travelling due north,*

(*i*) *show that AB = 567 m and AC = 951 m, and*
(*ii*) *calculate the speed of the ship in kilometres/hour.*

(*b*) *Evaluate, using mathematical tables,*

(*i*) *sin 63° 21′,*
(*ii*) *tan 42° 35′, and*
(*iii*) *cos 76° 49′.*

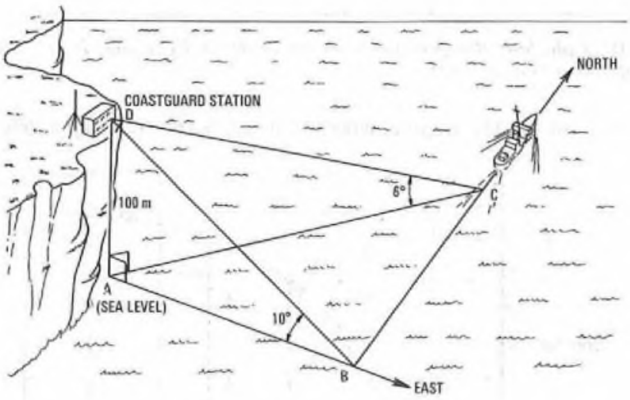


Fig. 1

A 7 (*a*) (*i*) When the ship is observed to be due east of the coastguard station, the angle of depression is as illustrated in sketch (*a*). In triangle DAB,

$\dfrac{DA}{AB} = \tan 10°.$

$\therefore AB = \dfrac{100}{0\cdot 1763},$

$= 567\cdot 1$ m,

$= \underline{567 \text{ m}}$ (to 3 significant figures). QED.

Number	Logarithm
100	2·0000
0·1763	$\bar{1}$·2463−
567·1	2·7537

The angle of depression 1 min later is illustrated in sketch (*b*). In triangle DAC,

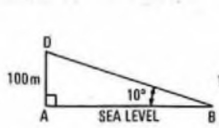
$\dfrac{DA}{AC} = \tan 6°.$

$\therefore AC = \dfrac{100}{0\cdot 1051},$

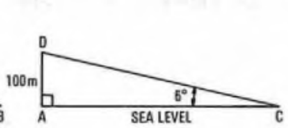
$= 951\cdot 4$ m,

$= \underline{951 \text{ m}}$ (to 3 significant figures). QED.

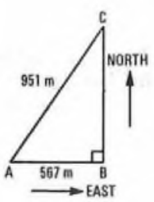
Number	Logarithm
100	2·0000
0·1051	$\bar{1}$·0216−
951·4	2·9784



(*a*)



(*b*)



(*c*)

(*ii*) The path of the ship at sea level is shown in sketch (*c*). Since the ship is travelling due north, and AB is in an easterly direction, ∠ABC is a right angle. By the theorem of Pythagoras,

$BC^2 = AC^2 - AB^2,$

$= 951^2 - 567^2,$

$= (951 - 567)(951 + 567),$

$= 384 \times 1518,$

$= 582\,900.$

Number	Logarithm
384	2·5843
1518	3·1813+
582 900	5·7656

∴ BC = 763·5 m from a table of square roots.

Thus, in 1 min, the ship travels 763·5 m. Therefore, the speed of the ship

$= \dfrac{763\cdot 5 \times 60}{1000} = \underline{45\cdot 81 \text{ km/h}}.$

(*b*) (*i*) sin 63° 21′ = $\underline{0\cdot 8938}$.

(*ii*) tan 42° 35′ = $\underline{0\cdot 9190}$.

(*iii*) cos 76° 49′ = $\underline{0\cdot 2281}$.

Q 8 (*a*) *The angles of a triangle, in degrees, are given by the expressions*

x, 2x − 24 and 3x − 30.

Calculate the value of x and the values of the other 2 angles.

(*b*) *Solve the following equations:*

(*i*) *2 sin A = 1·462,*
(*ii*) *tan B = 0·8,*
(*iii*) *antilog x = 13·58, and*
(*iv*) *10^t = 9·426.*

A 8 (*a*) The sum of the angles of a triangle is 180°.

$\therefore x + 2x - 24 + 3x - 30 = 180°.$

$\therefore 6x = 180° + 54° = 234°.$

$\therefore \underline{x = 39°}.$

Evaluating the other 2 angles gives

$2x - 24 = 2 \times 39° - 24° = \underline{54°},$

and $3x - 30 = 3 \times 39° - 30° = \underline{87°}.$

(*b*) (*i*) $2 \sin A = 1\cdot 462.$

$\therefore \sin A = 0\cdot 731.$

$\therefore \underline{A = 46° 58'}.$

(*ii*) $\tan B = 0\cdot 8.$

$\therefore \underline{B = 38° 40'}.$

(*iii*) antilog $x = 13\cdot 58.$

$\therefore \underline{x = 1\cdot 1329}.$

(*iv*) $10^t = 9\cdot 426.$

$\therefore t \log_{10} 10 = \log_{10} 9\cdot 426.$

$\therefore \underline{t = 0\cdot 9743}.$

Q 9 (*a*) *A template is made from a circular sheet of metal of radius 80 mm, as shown in Fig. 2. The shaded parts are cut away. Calculate*

(*i*) *the area of the equilateral triangle ABC,*
(*ii*) *the area of the segment BCD, and*
(*iii*) *the total area of the template.*

(*b*) *A metal tank consists of a right circular cylinder, of length 4·2 m and diameter 2·4 m, with hemispherical ends domed inwards. Neglecting the thickness of the material, calculate the internal capacity of the tank.*

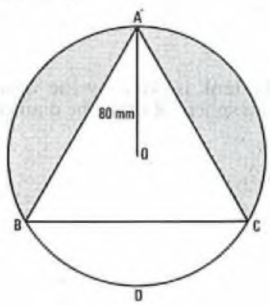
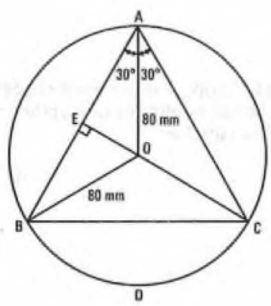


Fig. 2



(*a*)

A 9 (*a*) The circular sheet of metal is shown in sketch (*a*). Since triangle ABC is equilateral, it is also equiangular and, hence, $\angle A = \angle B = \angle C = 60°$. Also, the radius OA bisects $\angle A$ and, therefore, $\angle BAO = \angle CAO = 30°$.

If points B and O are joined, and the line COE is drawn perpendicular to AB, then OE divides triangle ABO into 2 equal triangles AEO and BEO.

(*i*) In triangle AEO,

$$\frac{EO}{AO} = \sin 30°.$$

$$\therefore\ EO = 80 \times 0{\cdot}5 = 40 \text{ mm}.$$

Also $$\frac{EA}{AO} = \cos 30°.$$

$$\therefore\ EA = 80 \times \frac{\sqrt{3}}{2} = 40\sqrt{3} \text{ mm}.$$

Now, the area of triangle AEO

$$= \tfrac{1}{2} \times EO \times EA,$$

$$= \frac{40 \times 40\sqrt{3}}{2} \text{ mm}^2,$$

$$= 800\sqrt{3} \text{ mm}^2.$$

Therefore, the area of triangle ABO

$$= 1600\sqrt{3} \text{ mm}^2.$$

From the symmetry of the figure,

$$\Delta ABO = \Delta AOC = \Delta BOC.$$

Hence, the area of triangle ABC

$$= 3 \times 1600\sqrt{3} \text{ mm}^2,$$

$$= \underline{8314 \text{ mm}^2}.$$

(*ii*) Now, the area of circle ABC

$$= \pi \times 80^2 \text{ mm}^2.$$

Therefore, the total area of the 3 segments formed by triangle ABC

$$= 6400\pi - 8314 \text{ mm}^2,$$

$$= 11\,792 \text{ mm}^2.$$

Since the 3 segments are of equal area, the area of segment BCD

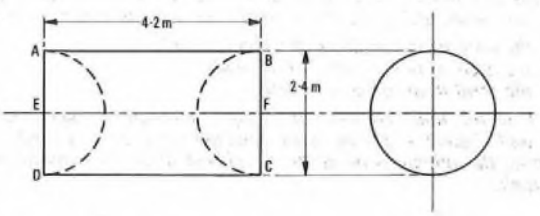
$$= \frac{11\,792}{3} = \underline{3931 \text{ mm}^2}.$$

(*iii*) The total area of the template

$$= \text{area of } \Delta ABC + \text{area of segment BCD},$$

$$= 8314 + 3931 = \underline{12\,245 \text{ mm}^2}.$$

(*b*) The tank is shown in sketch (*b*). The 2 hemispherical ends together form a sphere with the same diameter as that of the cylinder.



(*b*)

Therefore, the internal capacity of the tank is given by the volume of the right cylinder minus the volume of a sphere of the same diameter, and is equal to

$$\pi r^2 h - \frac{4}{3}\pi r^3,$$

where r is the radius of the sphere or cylinder (m), and h is the length of the tank (m). Thus, the internal capacity

$$= \pi r^2\left(h - \frac{4r}{3}\right),$$

$$= \pi \times 1{\cdot}2^2 \times \left(4{\cdot}2 - \frac{4 \times 1{\cdot}2}{3}\right) \text{ m}^3,$$

$$= 1{\cdot}44\pi \times (4{\cdot}2 - 1{\cdot}6) \text{ m}^3,$$

$$= 1{\cdot}44\pi \times 2{\cdot}6 \text{ m}^3,$$

$$= \underline{11{\cdot}76 \text{ m}^3}.$$

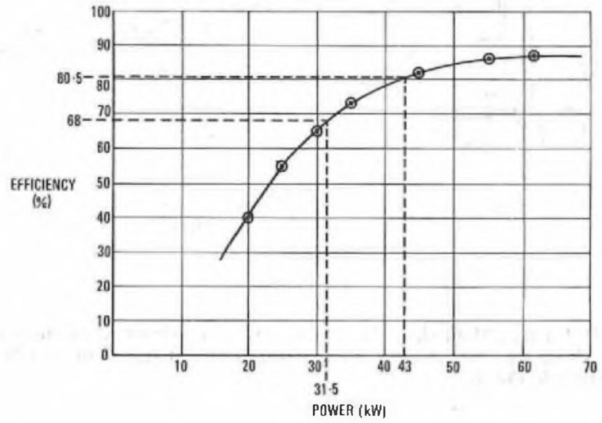
Q 10 (a) (i) The efficiency of an alternator varies according to the power developed. Using the table of values below, draw the graph of efficiency/power, with power as the horizontal axis.

(ii) Determine from the graph, as accurately as possible, the power generated with an efficiency of 68%, and the efficiency obtained with a power of 43 kW.

Efficiency (%)	*40*	*55*	*65*	*73*	*81·8*	*85·9*	*87*
Power (kW)	*20*	*25*	*30*	*35*	*45*	*55*	*61*

(b) Calculate the percentage error incurred by using 1·60 as an approximation to $1{\cdot}15^4$.

A 10 (*a*) (*i*) The graph of efficiency/power is shown in the sketch.



(*ii*) From the graph, when the efficiency is 68%, the power generated is $\underline{31{\cdot}5 \text{ kW}}$.

Also, when the power generated is 43 kW, the efficiency is $\underline{80{\cdot}5\%}$.

(*b*) Now, $1{\cdot}15^4 = 1{\cdot}749$.

Number	Logarithm
1·15	0·0607 4×
1·7490	0·2428

Therefore, the percentage error in using 1·60 as an approximation to $1{\cdot}15^4$

$$= \frac{1{\cdot}60 - 1{\cdot}749}{1{\cdot}749} \times 100\%,$$

$$= -\frac{0{\cdot}149}{1{\cdot}749} \times 100\%,$$

$$= \underline{-\,8{\cdot}52\%}.$$

Number	Logarithm
14·9	1·1732
1·749	0·2428−
8·519	0·9304

Note: The negative sign indicates that the approximate value is below the true figure.

Students were expected to answer 2 questions from Q1–4 and 4 questions from Q5–10

Q 1 (*a*) *State the principle of moments, and give the SI unit for the moment of a force about a point.*

(*b*) *A uniform beam, AB, of mass 5·0 kg and length 1·6 m, is hinged to a wall at A, as shown in Fig. 1. It is supported horizontally by a wire, BC, of circular cross-section.*

(*i*) *Find the tension in the wire.*

(*ii*) *Calculate the tensile stress in the wire if it has a diameter of 1·2 mm.*

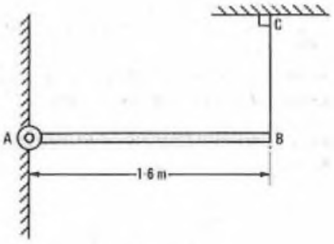


Fig. 1

A 1 (*a*) The principle of moments states that, if a body is at rest under the action of several forces, the total clockwise moment of the forces about any axis is equal to the total counter-clockwise moment of the forces about the same axis.

The moment of a force about a point is the magnitude of the force multiplied by the perpendicular distance from the point to the line of action of the force. The SI unit for the moment of a force about a point is, therefore, the *newton metre*.

(*b*) (*i*) As the beam is held in equilibrium by the wire, the clockwise moment about A must equal the counter-clockwise moment about A.

The mass of the beam is considered to act through its mid-point; that is, at a distance 0·8 m from A.

Therefore, the clockwise moment about A

$$= 5{\cdot}0 \times 9{\cdot}81 \times 0{\cdot}8 \text{ N m},$$

$$= 39{\cdot}24 \text{ N m}.$$

The counter-clockwise moment is due to the tension in the wire and must equal 39·24 N m.

$$\therefore \text{ tension in wire} \times 1{\cdot}6 = 39{\cdot}24.$$

$$\therefore \text{ tension in wire} = \frac{39{\cdot}24}{1{\cdot}6} \text{ N},$$

$$= \underline{24{\cdot}53 \text{ N.}}$$

(*ii*) Now, tensile stress $= \dfrac{\text{tension}}{\text{cross-sectional area}}$.

Therefore, the tensile stress in the wire

$$= \frac{24{\cdot}53}{\pi \times \left(\frac{d}{2}\right)^2} \text{ newtons/metre}^2,$$

where d is the diameter of the wire (m),

$$= \frac{24{\cdot}53}{\pi \times 0{\cdot}6^2 \times 10^{-6}} \text{ N/m}^2,$$

$$= \underline{21{\cdot}69 \times 10^6 \text{ N/m}^2.}$$

Q 2 *A motorist, travelling at 144 km/h in a car of mass 800 kg, observes an obstruction in the road and applies the brakes, which reduce his speed uniformly to 36 km/h in a time of 10 s. Find*

(*a*) *the deceleration when the brakes are applied,*
(*b*) *the braking force applied to the car,*
(*c*) *the distance travelled during the braking time, and*
(*d*) *the energy dissipated during the braking time.*

A 2 (*a*) Deceleration is the change in speed divided by the time for that change to take place.

Now, the change in speed

$$= (144 - 36) \times \frac{1000}{3600} = 30 \text{ m/s}.$$

Therefore, the deceleration when the brakes are applied

$$= \frac{30}{10} = \underline{3 \text{ m/s}^2.}$$

(*b*) Force is equal to mass multiplied by acceleration. In this case, the braking force is equal to the mass of the car multiplied by its deceleration.

Therefore, the braking force applied to the car

$$= 800 \times 3 = \underline{2400 \text{ N.}}$$

(*c*) Distance travelled is given by the average speed over a period of time multiplied by that period of time.

As the change in speed is uniform, the average speed

$$= \frac{144 + 36}{2} = 90 \text{ km/h}.$$

Therefore, the distance travelled during the braking time

$$= 90 \times \frac{1000}{3600} \times 10 = \underline{250 \text{ m.}}$$

(*d*) The energy dissipated during the braking time is the difference between the kinetic energy of the car before braking and the kinetic energy of the car after braking.

Now, kinetic energy $= \frac{1}{2}mv^2$ joules,

where m is the mass (kg), and v is the velocity (m/s).

Therefore, the energy dissipated

$$= \tfrac{1}{2}mv_1^2 - \tfrac{1}{2}mv_2^2 \text{ joules},$$

where v_1 and v_2 are the initial and final velocities respectively,

$$= \tfrac{1}{2}m(v_1^2 - v_2^2) \text{ joules},$$

$$= \tfrac{1}{2} \times 800 \times (144^2 - 36^2) \times \left(\frac{1000}{3600}\right)^2 \text{ J},$$

$$= \underline{600 \text{ kJ.}}$$

Q 3 (*a*) *Describe 3 ways in which a hot body can transfer heat to its surroundings, and give an example in each case.*

(*b*) *A central-heating system has a solar heat-exchanger having an effective area of 1·5 m² and containing 30 kg of water. Find the temperature rise of the water in 10 min, assuming that the sun provides 1·0 kW/m² at the earth's surface. (The specific-heat capacity of water is 4·2 × 10³ J/(kg K), and the efficiency of energy transfer is 60%.)*

A 3 (*a*) The 3 ways in which a hot body can transfer heat to its surroundings are by conduction, convection and radiation.

Conduction When a body is heated, the absorption of energy by the body causes an increase in the thermal vibration of the molecules of that body. The increase in vibration is passed from molecule to molecule until all parts of the body are affected. If one end of a poker is placed in a fire, that end becomes very hot. After a while, the other end also becomes very hot, due to the conduction of heat energy along the poker.

Convection If a body is at a higher temperature than the medium surrounding it, heat energy is transferred from the body to the medium. If that medium is a gas or liquid, it becomes less dense in the region of the hot body, and the less-dense parts of the medium therefore rise, to be replaced by cooler more-dense parts of the medium which, in turn, absorb heat energy, become less dense and also rise. Heat can, therefore, be transferred by convection currents in a gas or liquid. Air heated by an open fire rises up a chimney, and a cold draught can be felt in the region adjacent to the fire, due to the movement of cool air replacing that which has risen up the chimney.

Radiation Any hot body radiates heat energy in the form of electromagnetic waves, which can then be absorbed by another body in their path. This form of heat transfer does not require the presence of a medium between the 2 bodies. The heat of the sun felt on a sunny day is due to electromagnetic radiation.

(*b*) The effective area of the heat exchanger is 1·5 m², and the power provided by the sun is 1·0 kW/m². Therefore, the total power available

$$= 1{\cdot}5 \text{ kW}.$$

The energy available (J) is given by the power (W) multiplied by the time (s). Therefore, the energy available in 10 min

$$= 1{\cdot}5 \times 10^3 \times 10 \times 60 \text{ J},$$

$$= 900 \text{ kJ}.$$

As the efficiency of energy transfer is 60%, the useful energy available to heat the water

$$= 900 \times 10^3 \times \frac{60}{100} \text{ J},$$

$$= 540 \text{ kJ}.$$

Since $4\cdot 2 \times 10^3$ J are required to raise the temperature of 1 kg of water by 1 K, the rise in temperature of 30 kg of water due to 540 kJ of energy

$$= \frac{540 \times 10^3}{4\cdot 2 \times 10^3 \times 30}\ \text{K},$$

$$\simeq \underline{4\cdot 3\ \text{K}}.$$

Q 4 (a) State what is meant by the coefficient of friction between 2 plane surfaces.

(b) A box of mass 15 kg is pulled with uniform speed along a horizontal surface by a rope. The coefficient of friction between the box and the surface is 0·2.

(i) Find the tension in the rope if it acts parallel to the surface.

(ii) Find the normal reaction between the box and the surface if the rope is rearranged to make an angle of 30° with the horizontal.

A 4 (*a*) The coefficient of friction, μ, between 2 plane surfaces is the ratio of the frictional force, F newtons, which is to be overcome in moving one surface over the other to the normal reaction between the surfaces, R newtons. It is constant for a given pair of surfaces.

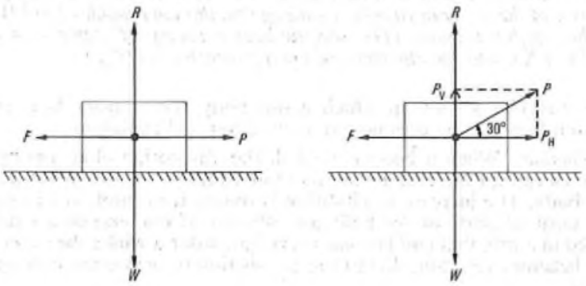
Thus, $$\mu = \frac{F}{R}. \qquad \ldots\ldots (1)$$

(*b*) (*i*) From equation (1), $F = \mu R$, and, for a body on a horizontal surface where the pulling force is parallel to the surface, the normal reaction is equal to the weight, W newtons, of the body. Also, since the box moves at uniform speed, the pulling tension, P newtons, in the rope equals the frictional force, as illustrated in sketch (*a*).

$$\therefore\ P = \mu W \text{ newtons},$$

$$= 0\cdot 2 \times 15 \times 9\cdot 81\ \text{N},$$

$$= \underline{29\cdot 43\ \text{N}}.$$



(*a*) (*b*)

(*ii*) The forces acting on the box are shown in sketch (*b*). The pulling tension can be resolved into a horizontal component, P_H, and a vertical component, P_V.

Because the box moves at uniform speed, the frictional force equals the horizontal component of the pulling tension; that is, $F = P_H$ newtons. Also, the vertical component of the pulling tension is tending to lift the box and, therefore, assists the normal reaction in supporting the weight of the box, so that

$$R = W - P_V \text{ newtons}. \qquad \ldots\ldots (2)$$

Therefore, substituting in equation (1) gives

$$\mu = \frac{P_H}{W - P_V},$$

$$= \frac{P_H}{W - P_H \tan 30^\circ}.$$

$$\therefore\ \mu W - \mu P_H \tan 30^\circ = P_H.$$

$$\therefore\ P_H = \frac{\mu W}{1 + \mu \tan 30^\circ} \text{ newtons},$$

$$= \frac{0\cdot 2 \times 15 \times 9\cdot 81}{1 + 0\cdot 2 \times 0\cdot 5774}\ \text{N},$$

$$= 26\cdot 38\ \text{N}.$$

Hence, from equation (2),

$$R = W - P_V \text{ newtons},$$

$$= W - P_H \tan 30^\circ \text{ newtons},$$

$$= 15 \times 9\cdot 81 - 26\cdot 38 \times 0\cdot 5774\ \text{N},$$

$$= \underline{131\cdot 9\ \text{N}}.$$

Q 5 (a) What is meant by a linear resistor?

(b) The e.m.f. of each cell in Fig. 2 is 2·0 V, and the internal resistance of each cell is 0·20 Ω. Find

(i) the current in the 4 Ω resistor when switch S is connected to A,

(ii) the voltage across the 4 Ω resistor when switch S is connected to B, and

(iii) the total energy dissipated by the 4 Ω resistor in 20 s when the connexion to B is used.

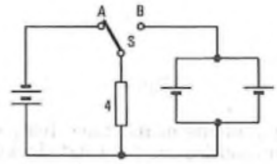


Fig. 2

A 5 (*a*) A linear resistor is one in which the voltage and current are always proportional to each other, assuming that the temperature remains constant.

(*b*) (*i*) With switch S in position A, the 4 Ω resistor is connected in series with 2 cells, also connected in series.

The total e.m.f. in the circuit is, therefore, $2 \times 2\ \text{V} = 4\ \text{V}$, and the total internal resistance of the 2 cells is $2 \times 0\cdot 2\ \Omega = 0\cdot 4\ \Omega$.

Thus, the total resistance in the circuit is $4 + 0\cdot 4 = 4\cdot 4\ \Omega$, and, by Ohm's law, the current in the circuit, and hence in the 4 Ω resistor,

$$= \frac{4}{4\cdot 4}\ \text{A},$$

$$= \underline{0\cdot 91\ \text{A}}.$$

(*ii*) When switch S is in position B, the 4 Ω resistor is connected in series with 2 cells in parallel. The total e.m.f. in the circuit is, therefore, 2 V.

The total internal resistance of the 2 cells in parallel is given by the formula for the total resistance of 2 resistors in parallel,

$$\frac{1}{R_T} = \frac{1}{R_1} + \frac{1}{R_2},$$

where R_T is the total resistance (Ω), and R_1 and R_2 are the values of the resistors (Ω).

$$\therefore\ \frac{1}{R_T} = \frac{1}{0\cdot 2} + \frac{1}{0\cdot 2} = \frac{2}{0\cdot 2}\ \text{S}.$$

$$\therefore\ R_T = \frac{0\cdot 2}{2} = 0\cdot 1\ \Omega.$$

Thus, the total resistance in the circuit is $4 + 0\cdot 1 = 4\cdot 1\ \Omega$, and, by Ohm's law, the current in the circuit

$$= \frac{2}{4\cdot 1}\ \text{A}.$$

Therefore, the voltage across the 4 Ω resistor

$$= \frac{2}{4\cdot 1} \times 4\ \text{V},$$

$$= \underline{1\cdot 95\ \text{V}}.$$

(*iii*) Power is the rate of doing work and, in an electrical circuit, is equal to the square of the current multiplied by the resistance. Hence, the power dissipated by the 4 Ω resistor when connexion B is used

$$= \left(\frac{2}{4\cdot 1}\right)^2 \times 4\ \text{W}.$$

The energy dissipated in a given interval is the power multiplied by the time. Hence, the energy dissipated in the 4 Ω resistor in 20 s

$$= \left(\frac{2}{4\cdot 1}\right)^2 \times 4 \times 20 \text{ J},$$

$$= \underline{19 \text{ J}}.$$

Q 6 (a) Give a detailed explanation of the term temperature coefficient of resistance when referred to a particular base temperature.

(b) The resistance of a copper coil on a transformer is 3·20 Ω at 20°C. After current has passed for some time through the coil, the resistance attains a steady value of 3·80 Ω. If the temperature coefficient of resistance of copper is 3·9 × 10⁻³/°C at 0°C, find the final temperature of the coil.

A 6 (*a*) The temperature coefficient of resistance is defined as the change in resistance of a material due to a temperature rise of 1°C, expressed as a ratio to the resistance at a base temperature of 0°C. For example, if a material has a temperature coefficient, α, of 0·002 /°C, and has a resistance of R_0 ohms at a base temperature of 0°C, its resistance increases by $0\cdot 002R_0$ ohms for every 1°C rise in temperature above 0°C. Hence, the resistance, R_T ohms, of a material at any temperature is given by

$$R_T = R_0\,(1 + \alpha T)\,\text{ohms}, \qquad \ldots\ldots (1)$$

where T is the change in temperature from 0°C.

(*b*) From equation (1), the resistance of the coil at its base temperature of 0°C is given by

$$R_0 = \frac{R_T}{1 + \alpha T}\,\text{ohms},$$

$$= \frac{3\cdot 2}{1 + 3\cdot 9 \times 10^{-3} \times 20} = 2\cdot 97\ \Omega.$$

Also, from equation (1), the change in temperature from 0°C when the resistance of the coil is 3·8 Ω is given by

$$T = \frac{\frac{R_T}{R_0} - 1}{\alpha}\ \text{degrees Celsius},$$

$$= \frac{\frac{3\cdot 8}{2\cdot 97} - 1}{3\cdot 9 \times 10^{-3}} = 71\cdot 7\text{°C}.$$

Hence, the final temperature of the coil is $\underline{71\cdot 7\text{°C}}$.

Q 7 (a) Distinguish between primary and secondary cells.

(b) Describe one test that can be made to assess the state of charge of a lead-acid cell.

(c) A leakage current of 3·0 A flows from a lead-covered cable into moist soil. If the electrochemical equivalent of lead is 1·1 × 10⁻⁶ kg/C, what mass of lead is removed from the cable in 24 h?

A 7 (*a*) A primary cell consists basically of 2 electrodes, of dissimilar metals, immersed in an electrolyte. The resulting chemical action produces an e.m.f. across the terminals of the electrodes. Chemical energy is thus converted into electrical energy but, in a primary cell, this process cannot be reversed. The store of energy in the cell can be renewed only by replacement of the active constituents.

A secondary cell usually consists of 2 lead plates covered with lead compounds and immersed in dilute sulphuric acid. Chemical energy is again transformed into electrical energy but, in this case, the process is reversible, and the secondary cell can be recharged from an external source.

(*b*) The state of charge of a lead-acid cell can be assessed by means of a hydrometer, which is an instrument that measures the specific gravity of the electrolyte in the cell. The specific gravity varies from about 1·175 (for a cell that is discharged) to 1·215 (for a cell that is charged), and is directly related to the state of charge of the cell. This is because the concentration of the electrolyte is strengthened during the charging period and is weakened during the discharging period, due to the chemical action within the cell.

A typical hydrometer consists of a weighted glass bulb with a graduated stem that floats upright in a liquid. The stem rises higher out of the liquid if the liquid becomes more dense, and sinks lower if the liquid becomes less dense. The stem is calibrated to read directly the specific gravity of the liquid in which it is floating. The hydrometer is allowed to float freely in the electrolyte of the cell and, by noting the electrolyte level against the graduations on the stem, the specific gravity of the electrolyte, and hence the state of charge of the cell, can be determined.

A correction may be required, as the specific gravity varies inversely with temperature. Hydrometers are calibrated at 15·5°C, and an allowance must be made for each degree Celsius above or below this temperature.

(*c*) In 24 h, the quantity of electricity that leaks from the cable

$$= 24 \times 3600 \times 3\cdot 0 = 259\cdot 2 \times 10^3\ \text{C}.$$

Since the electrochemical equivalent of lead is $1\cdot 1 \times 10^{-6}$ kg/C, the mass of lead removed in 24 h

$$= 1\cdot 1 \times 10^{-6} \times 259\cdot 2 \times 10^3\ \text{kg},$$

$$= \underline{0\cdot 29\ \text{kg}}.$$

Q 8 (a) State Faraday's law of electromagnetic induction.

(b) What factors determine

(i) the magnitude, and
(ii) the direction

of an induced e.m.f. in a coil?

(c) An alternating voltage, of r.m.s. value 10 V and frequency 200 Hz, is applied across a pure resistor. Sketch a graph to show how the voltage across the resistor varies with time over a period of 0·010 s.

A 8 (*a*) Faraday's law of electromagnetic induction states that, when a magnetic flux linking a conductor changes, an e.m.f. is induced in the conductor whose magnitude is proportional to the rate of change of flux.

(*b*) (*i*) The magnitude, e volts, of an e.m.f. induced in a coil rotating in a magnetic field is given by

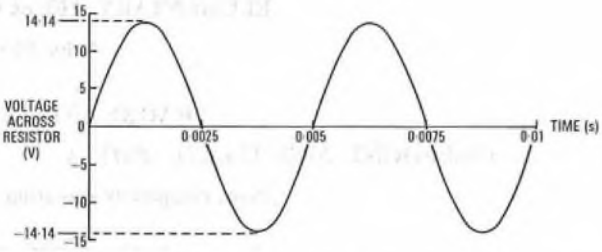
$$e = 2NBlv \sin\theta\ \text{volts},$$

where N is the number of turns on the coil, B is the flux density of the magnetic field (T), l is the length of the side of the coil parallel to the axis of rotation (m), v is the velocity of the coil (m/s), and θ is the angle one side of the coil makes with the direction of the magnetic field at the instant under consideration.

(*ii*) The direction of the induced e.m.f. depends on the directions of the magnetic field and of the motion of the coil. For one side of one turn of the coil, the direction of the induced e.m.f. can be determined by Fleming's right-hand rule, which states that, if the thumb and first 2 fingers of the right hand are held mutually at right angles with the thumb pointed in the direction of motion and the first finger in the direction of the magnetic field, the second finger indicates the direction of the induced e.m.f. It is found that the induced e.m.f.s in the opposite sides of the coil are of opposite polarity and, therefore, act in series-aiding.

(*c*) At a frequency of 200 Hz, the period of 1 cycle is 1/200 = 0·005 s. Therefore, over a period of 0·010 s, 2 cycles occur.

The peak value of the voltage is $\sqrt{2}$ × the r.m.s. value; that is, $\sqrt{2} \times 10 = 14\cdot 14$ V.



The variation of voltage across the resistor with time is shown in the sketch.

Q 9 (a) What 3 properties are desirable in materials used for permanent magnets?

(b) With the aid of sketches, describe the magnetic path and the distribution of magnetic flux for a permanent magnet suitable for use in a moving-coil meter.

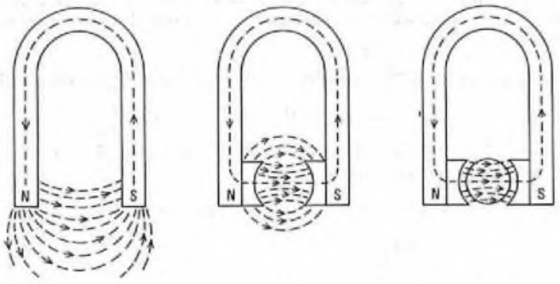
(c) Find the maximum force that can be exerted on a straight wire of length 0·2 m, carrying a current of 0·5 A, if it is placed in a uniform magnetic field of flux density 0·4 T.

A 9 (*a*) The properties desirable in a material used for a permanent magnet are that it should

(*i*) be capable of producing a high magnetic flux density,

(*ii*) retain a high proportion of its magnetic flux after the initial magnetizing force has been removed, and

(*iii*) not be easily demagnetized.



(*a*) (*b*) (*c*)

(*b*) Sketch (*a*) shows a horse-shoe magnet with the north (N) and south (S) poles marked. The direction of the magnetic flux is shown by the dashed lines, and it can be seen that the flux spreads out in passing from the north pole to the south pole. To make the magnet suitable for use in a moving-coil meter, pole pieces are fitted, as shown in sketch (*b*).

The specially-shaped pole pieces concentrate the magnetic flux in the cylindrical gap formed by them, but the flux density is greatest across the narrowest parts of the gap. Sketch (*c*) shows a cylindrical core, around which the moving coil moves, placed in the gap between the pole pieces. The path of the flux is now through the core, and this gives a much more uniform flux density in the air gap. This ensures that a uniform torque is applied to the coil, whatever its position, for a given value of current.

(*c*) The force on the wire

$$= Bli \text{ newtons,}$$

where B is the flux density (T), l is the length of the wire (m), and i is the current (A),

$$= 0{\cdot}4 \times 0{\cdot}2 \times 0{\cdot}5 \text{ N,}$$

$$= \underline{0{\cdot}04 \text{ N.}}$$

(This is the maximum force, and applies when the conductor is a right angles to the magnetic field.)

Q 10 Answer 2 of the following parts.

(*a*) *Describe the construction and action of a bimetal relay.*

(*b*) *Describe the effect of instrument resistance on voltage measurement in a circuit of high resistance. Illustrate your answer with an example.*

(*c*) *Describe an experiment that illustrates mutual inductance.*

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