SUPPLEMENT

TO THE POST OFFICE ELECTRICAL ENGINEERS' JOURNAL

Vol. 68 Part 3 October 1975

CITY AND GUILDS OF LONDON INSTITUTE EXAMINATIONS 1974

Contents

Page

QUESTIONS AND ANSWERS

Answers are occasionally omitted or reference is made to earlier Supplements in which questions of substantially the same form, together with the answers, have been published. Some answers contain more detail than would be expected from candidates under examination conditions.

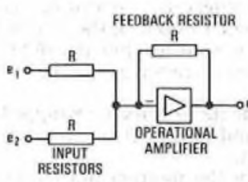
For economic reasons, alternate issues of the Supplement are now published in 32-page and 16-page sizes.

COMPUTERS B, 1974 (*continued*)

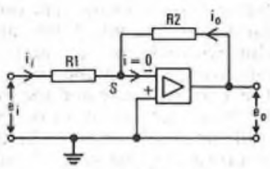
Q 10 (*a*) *Draw a diagram of an operational amplifier which is used to sum 2 analogue values, showing clearly the position of input and feedback components.*

(*b*) *Discuss the virtual-earth concept as applied to an ideal operational amplifier. Using this concept, derive, from basic principles, a formula which gives the output voltage in terms of the input voltage and feedback resistances.*

A 10 (*a*) A diagram of an operational amplifier which is used to sum 2 analogue voltage values, e_1 and e_2, is shown in sketch (*a*). The 2 input resistances and the feedback resistance are equal. The output voltage, e_o, is equal to the sum of e_1 and e_2.



(*a*)



(*b*)

(*b*) An operational amplifier, together with input and feedback resistors R1 and R2 respectively, is shown in sketch (*b*). An ideal operational amplifier has infinite gain and takes no current; that is, $i = 0$ A. Therefore, as long as the output voltage, e_o, of the amplifier is of a reasonably finite value, point S is virtually at earth potential. This type of amplifier is often referred to as a virtual-earth amplifier.

The virtual-earth concept is applied to the circuit in the following derivation, in which e_i is the input voltage, i_i the input current, and i_o the output current.

Now, $$e_i = i_iR_1, \qquad \ldots\ldots (1)$$

and $$i_i = -i_o.$$

Also, $$e_o = i_oR_2,$$
$$= -i_iR_2. \qquad \ldots\ldots (2)$$

Dividing equation (2) by equation (1) gives

$$\frac{e_o}{e_i} = -\frac{i_iR_2}{i_iR_1},$$

$$= -\frac{R_2}{R_1}.$$

$$\therefore\ e_o = -\frac{R_2}{R_1}e_i.$$

RADIO AND LINE TRANSMISSION B, 1974

Students were expected to answer any 6 questions

Q 1 (*a*) (*i*) *Describe, with the aid of a simplified circuit diagram, the operation of a Q-meter.*

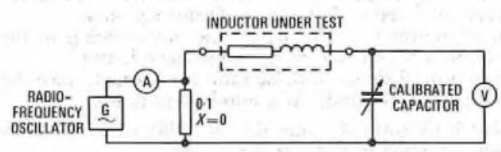
(*ii*) *Show how the Q-factor of an inductor is related to the voltage and current in the circuit.*

(*b*) *At a frequency of 1·2 MHz, the Q-factor of an inductor was measured as 180 when tuned with a capacitance of 320 pF. Calculate*

(*i*) *the value of the inductor,*
(*ii*) *the effective series resistance of the inductor, and*
(*iii*) *the dynamic impedance at resonance of a parallel circuit containing the inductor and a capacitor having a value of 320 pF.*

A 1 (*a*) (*i*) A simplified diagram of a Q-meter is shown in the sketch. The Q-meter consists of a radio-frequency oscillator, that can be tuned over the frequency range of interest, connected to a low-value non-inductive resistor of known resistance via a radio-frequency ammeter. This allows the current delivered to the resistor to be set in conjunction with a control on the oscillator. Connected in series with the non-inductive resistor are the inductor under test and a high-grade calibrated variable capacitor. A high-impedance radio-frequency voltmeter is connected across the capacitor.

In operation, the oscillator is set to the desired frequency for the measurement and the current is set to a suitable value; on most instruments, there is a calibration mark on the ammeter for this purpose. The variable capacitor is then tuned for series resonance, this being



when the voltage across the capacitor has a maximum value as indicated by the voltmeter. The Q-factor, or magnification factor, is the ratio of this voltage to that applied in series with the circuit across the non-inductive resistor. The scale of the voltmeter is usually calibrated directly in Q-factor values, and these may be used if the current has been set to the calibration mark on the ammeter. Alternatively, the Q-factor can be calculated directly, as shown in part (*ii*).

(*ii*) At resonance, the Q-factor is given by

$$Q = \frac{V_C}{V},$$

where V_C is the voltage across the capacitor (V), and V is the voltage across the circuit resistance (V); that is, the voltage applied to the circuit.

Now, the current, I amperes, supplied to the circuit is given by

$$I = \frac{V}{\dfrac{rR}{r+R}},$$

where r is the resistance of the low-value non-inductive resistor (Ω), and R is the effective series resistance of the inductor (Ω).

$$\therefore\ Q = \frac{V_C}{I \times \dfrac{rR}{r+R}}.$$

But, $r \ll R$.

$$\therefore\ Q = \frac{V_C}{I \times \dfrac{rR}{R}} = \frac{V_C}{Ir}.$$

Since r is known, Q can be calculated. Alternatively, if I is always fixed at a particular known value, and r is known, the voltmeter may be directly calibrated in terms of Q-factor values.

(*b*) (*i*) At resonance, the reactance of the inductor equals that of the capacitor.

$$\therefore\ \omega L = \frac{1}{\omega C},$$ where L is the inductance (H), C is the capacitance (F), and $\omega = 2\pi f$, where f is the frequency (Hz).

$$\therefore\ L = \frac{1}{\omega^2 C}\ \text{henrys},$$

$$= \frac{1}{(2\pi \times 1\cdot 2 \times 10^6)^2 \times 320 \times 10^{-12}}\ \text{H} = \underline{55\ \mu\text{H}.}$$

(*ii*) Now, $$Q = \frac{\omega L}{R},$$

or $$R = \frac{\omega L}{Q}\ \text{ohms},$$

$$= \frac{2\pi \times 1\cdot 2 \times 10^6 \times 55 \times 10^{-6}}{180} = \underline{2\cdot 3\ \Omega.}$$

(*iii*) The dynamic impedance, Z_D ohms, of a parallel circuit at resonance is given by

$$Z_D = \frac{L}{CR}\ \text{ohms},$$

$$= \frac{55 \times 10^{-6}}{320 \times 10^{-12} \times 2\cdot 3}\ \Omega = \underline{74\cdot 7\ \text{k}\Omega.}$$

Q 2 (a) State 3 advantages of using single-sideband operation compared with double-sideband operation.

(b) Draw a block diagram showing the assembly of a 12-channel group of telephone channels.

(c) Indicate the type of modulator used and explain why single-sideband operation is normally employed.

(d) How is the required sideband selected in the transmitting assembly?

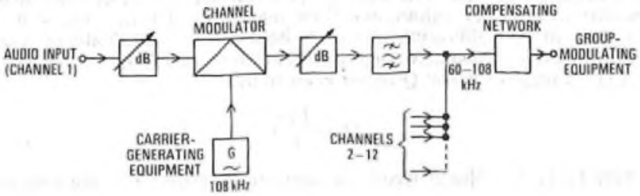
A 2 (*a*) Three advantages of single-sideband operation over double-sideband operation are

(*i*) a reduced frequency spectrum is obtained, so that more channels can be accommodated within a given frequency band,

(*ii*) greater power is available, all the power being in the wanted signal and not in the carrier or second sideband, and

(*iii*) an improved signal-to-noise ratio is obtained, since the receiver bandwidth is narrower and, thus, admits less noise.

(*b*) A block diagram showing the assembly of a 12-channel group of telephone channels is given in the sketch.



(*c*) A balanced modulator is used for channel modulation.

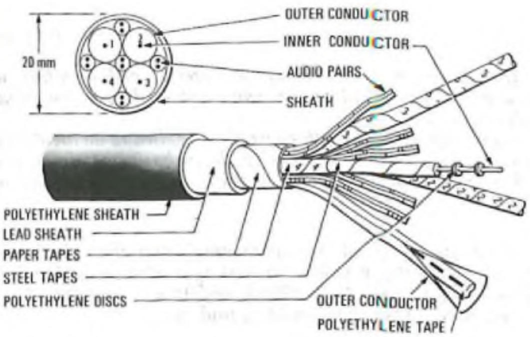
By transmitting only one sideband in each channel, twelve 4 kHz channels can be accommodated in the 60-108 kHz frequency band, making the system more economical.

(*d*) The required sideband is selected by the band-pass filter shown in the sketch. This is usually a crystal filter, so that sufficient rejection is provided for out-of-band frequencies. This filter also removes any unwanted carrier signal still present after the modulating stage.

Q 3 (a) Describe, with the aid of a sketch, the construction of a coaxial cable suitable for use in inland multi-channel telephony systems.

(b) A multi-channel telephony signal of 4 MHz bandwidth is transmitted through a coaxial cable. Calculate the signal-to-noise ratio in a 4 kHz channel at 4 MHz, after transmission through 9 km of cable, given that the transmitted signal power is 10 mW, the attenuation of the cable is equal to $2\cdot 3 \times \sqrt{f}$ decibels/kilometre, and the noise-power level in the cable is equal to kTB watts, where T is the absolute temperature and has the value 290 K, B is the bandwidth (Hz), f is the frequency (MHz), and k is a constant equal to $1\cdot 38 \times 10^{-23}$.

A 3 (*a*) The construction of a coaxial cable suitable for use in inland multi-channel telephony systems is shown in the sketch. The cable is made up of 4 coaxial pairs and additional audio pairs within a single sheath.



The coaxial pairs comprise a copper inner conductor with polyethylene spacing discs placed at regular intervals, supporting a coaxial copper outer conductor. The outer diameter of one of the coaxial pairs illustrated is 4·4 mm. Steel tapes are wound around the outer conductor, with intermediate and outer paper wrappings, the latter being numbered to identify the pair.

The 4 coaxial pairs and the interstitial audio pairs are wrapped with paper tapes, and have an outer lead and polyethylene covering. The overall sheath diameter is about 20 mm.

In operation, the coaxial pairs carry the multi-channel telephony traffic, and the audio pairs are used for supervisory and engineering speech circuits.

(*b*) At $f = 4$ MHz, the attenuation of the cable is $2\cdot 3 \times \sqrt{4} = 4\cdot 6$ dB/km. Therefore, the total loss in 9 km of cable is $41\cdot 4$ dB.

If the ratio of the transmitted signal power to the received signal power is $P_T : P_R$, then

$$41\cdot 4 = 10 \log_{10}\frac{P_T}{P_R} = 10 \log_{10}\frac{10 \times 10^{-3}}{P_R}.$$

$$\therefore\ \frac{10 \times 10^{-3}}{P_R} = \text{antilog}_{10}\, 4\cdot 14 = 1\cdot 38 \times 10^4.$$

$$\therefore\ P_R = 7\cdot 24 \times 10^{-7}\ \text{W}.$$

The noise power in the receiver passband

$$= 1\cdot 38 \times 10^{-23} \times 290 \times 4 \times 10^3 = 1\cdot 6 \times 10^{-17}\ \text{W}.$$

Therefore, the signal-to-noise ratio

$$= 10 \log_{10}\frac{7\cdot 24 \times 10^{-7}}{1\cdot 6 \times 10^{-17}}\ \text{dB},$$

$$= \underline{106\cdot 6\ \text{dB}.}$$

Q 4 Explain, with the aid of diagrams, 5 of the following terms used in relation to aerials.

(a) Isotropic radiator.
(b) Half-wave dipole.
(c) Unipole.
(d) Radiation pattern.
(e) Aerial gain.
(f) Half-power beamwidth.

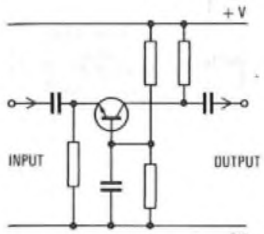
Q 5 (a) Draw circuit diagrams of single-stage resistance-loaded amplifiers, showing typical biasing arrangements of transistors in the following configurations:

(i) common-base, and
(ii) common-emitter.

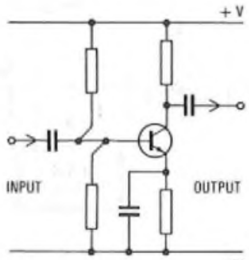
(b) State typical current gains for the transistors in configurations (i) and (ii), and give a reason for not using the common-base configuration in both stages of a 2-stage resistance-capacitance-coupled amplifier.

(c) What form of coupling would you recommend for the common-base configuration?

A 5 (*a*) Circuit diagrams for single-stage common-base and common-emitter resistance-loaded amplifiers are shown in sketches (*a*) and (*b*) respectively, together with typical biasing arrangements.



(*a*)



(*b*)

(*b*) For a transistor in the common-base configuration, the current gain is less than unity, and is typically 0·98. For a transistor in the common-emitter configuration, the current gain is typically in the range 50–200.

A reason for not using the common-base configuration in both stages of a 2-stage resistance-capacitance-coupled amplifier is that the matching between the stages would be poor. The common-base configuration has a high output impedance and a low input impedance, so that the potential-dividing action of these impedances would tend to attenuate the signal between the stages.

(*c*) Transformer coupling would be used to couple the stages of a common-base amplifier, with the turns ratio chosen to match the 2 impedances.

Q 6 Fig. 1 shows a variable tuned circuit used in the high-frequency amplifying stage of a radio-communication receiver.

(a) Calculate the range of frequencies covered by the circuit.

(b) Calculate, for the extremes of the range of frequencies in part (a),

(i) the Q-factor values,
(ii) the bandwidths at the −3 dB points, and
(iii) the dynamic impedances at resonance.

Assume that the resistance value is independent of frequency.

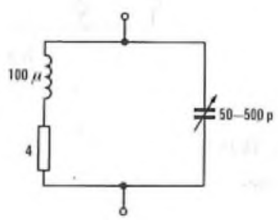


Fig. 1

A 6 (*a*) At resonance, $\omega L = 1/\omega C$, where L is the value of the inductance (H), C is the value of the capacitance (F), and $\omega = 2\pi f$, where f is the frequency (Hz).

$$\therefore f = \frac{1}{2\pi\sqrt{(LC)}} \text{ hertz.}$$

Substituting values for the extremes of the capacitance range gives

$$f_1 = \frac{1}{2\pi\sqrt{(100 \times 10^{-6} \times 50 \times 10^{-12})}} \text{ Hz} = 2{\cdot}25 \text{ MHz},$$

and $$f_2 = \frac{1}{2\pi\sqrt{(100 \times 10^{-6} \times 500 \times 10^{-12})}} \text{ Hz} = 712 \text{ kHz}.$$

Therefore, the range of frequencies covered by the circuit is 712 kHz–2·25 MHz.

(*b*) (*i*) The Q-factor is given by

$$Q = \frac{\omega L}{R}, \text{ where } R \text{ is the value of the resistance } (\Omega).$$

$$\therefore Q_1 = \frac{2\pi \times 2{\cdot}25 \times 10^6 \times 100 \times 10^{-6}}{4} = \underline{353},$$

and $$Q_2 = \frac{2\pi \times 712 \times 10^3 \times 100 \times 10^{-6}}{4} = \underline{112}.$$

(*ii*) The bandwidth, B hertz, is given by

$$B = \frac{f}{Q} \text{ hertz.}$$

$$\therefore B_1 = \frac{2{\cdot}25 \times 10^6}{353} \text{ Hz} = \underline{6{\cdot}37 \text{ kHz}},$$

and $$B_2 = \frac{712 \times 10^3}{112} \text{ Hz} = \underline{6{\cdot}36 \text{ kHz}}.$$

Hence, the bandwidth of the circuit is independent of frequency.

(*iii*) The dynamic impedance, Z_D ohms, at resonance, is given by

$$Z_D = \frac{L}{CR} \text{ ohms.}$$

$$\therefore Z_{D1} = \frac{100 \times 10^{-6}}{50 \times 10^{-12} \times 4}\ \Omega = \underline{500\ \text{k}\Omega},$$

and $$Z_{D2} = \frac{100 \times 10^{-6}}{500 \times 10^{-12} \times 4}\ \Omega = \underline{50\ \text{k}\Omega}.$$

Q 7 (a) State, with reasons, 3 advantages of applying overall negative feedback to an amplifier.

(b) Explain in particular why negative feedback is advantageous in transmission-line amplifiers carrying multi-channel telephony signals.

(c) The gain of a coaxial-line amplifier is 60 dB at 4 MHz. What gain would be required at 60 kHz to obtain uniform output if the attenuation of the line in decibels is proportional to the square root of the frequency?

A 7 (*a*) One advantage of applying negative feedback to an amplifier is the reduction obtained in distortion for a given output level. If the level of distortion, expressed as a fraction of the fundamental output, is D, then the resulting level of distortion with feedback applied, D_o, is given by $D_o = D/(1 + A\beta)$, where A is the amplifier gain without feedback, and β is the feedback fraction.

A second advantage of applying negative feedback is better gain stability. The gain of an amplifier with feedback, A', is given by $A' = A/(1 + A\beta)$. Hence, the overall gain of the amplifier with feedback is less dependent on changes in the stage gain, such as may occur as a valve ages.

A third advantage is that a nearly constant gain/frequency response is obtained. Since the overall gain is less dependent on variations in stage gain, then variations in overall gain, due to changes in device gain with frequency, are reduced. Hence, the frequency response is constant, provided that β is independent of frequency, as will be the case if resistors are used in the feedback network. The gain-bandwidth product of the amplifier is unaffected by negative feedback, so that, at the lower gain obtained with feedback applied, a wider frequency response is achieved.

(*b*) The principal advantage of applying negative feedback to transmission-line amplifiers carrying multi-channel telephony signals is the reduction obtained in unwanted harmonic components, sum-and-difference products and intermodulation distortion. These would otherwise tend to cause crosstalk between subscribers, which is clearly undesirable. The use of negative feedback also allows a higher output-signal level to be used before the distortion products become obtrusive and, in coaxial-line amplifiers, allows the gain/frequency response to be matched to that of the cable, by making the feedback network frequency sensitive.

(*c*) If the attenuation, a decibels, of the line is proportional to the square root of the frequency, f hertz, then $a = k\sqrt{f}$, where k is a constant.

At $f = 4$ MHz, a coaxial-line amplifier of 60 dB gain is required. Hence the cable attenuation may be assumed to be 60 dB.

$$\therefore k = \frac{60}{\sqrt{(4 \times 10^6)}} = 0{\cdot}03.$$

Therefore, at $f = 60$ kHz,

$$a = 0{\cdot}03 \times \sqrt{(60 \times 10^3)} = 7{\cdot}35 \text{ dB}.$$

Hence, a gain of 7·35 dB is required at 60 kHz.

Q 8 (*a*) *Describe, with the aid of a circuit diagram, a crystal-controlled oscillator suitable for use as a stable frequency source.*
(*b*) *Draw and describe the equivalent electrical circuit of the crystal, and sketch a typical reactance/frequency characteristic for the circuit.*
(*c*) *Indicate on the sketch the approximate operating frequency of the oscillator.*

A 8 See A8, Radio and Line Transmission B, 1973, *Supplement*, Vol. 68, p. 11, Apr. 1975.

Q 9 (*a*) *Describe briefly the effects of the ionosphere on radio communication over long distances.*
(*b*) *State suitable frequencies below 300 MHz for transmissions providing the following types of service:*

(*i*) *broadcasts over an area of approximate radius 40 km,*
(*ii*) *point-to-point communication over a distance of 200–300 km, and*
(*iii*) *global communication over distances in excess of 5000 km.*

(*c*) *Explain the mode of propagation in each of the above cases.*

A 9 (*a*) Ultra-violet radiation from the sun has the effect of separating electrons from their atoms in the outer regions of the Earth's atmosphere. A spherical shell of free electrons and ions is thus created in the rarified outer atmosphere. The process is known as *ionization*, and the region in which it occurs is called the *ionosphere*. The electron density, as a result of this ionization, increases with height above the Earth's surface, and reaches a maximum at between 300–500 km, and is dependent upon the time of day and the season of the year. The propagation of radio waves begins to become influenced at a height of about 100 km, where there is a fairly distinct layer of electrons known as the *E-layer*. Fairly constant conditions prevail at this height with little diurnal or seasonal variation. Above this layer, the electron density increases, but is considerably influenced by the sun. During the day, there are 2 main layers: the *F1*-layer and the *F2*-layer, at heights of approximately 200 km and 300 km respectively. At night, the *F2*-layer descends to merge with the *F1*-layer, providing a maximum electron density at a height of about 250 km.

A radio wave, passing through the ionosphere, transfers energy to the electrons which, in consequence, oscillate in sympathy with, but not in phase with, the radio field. The oscillating electrons reradiate energy which, in combination with the original field, has the effect of changing the direction of travel of the radio wave. The magnitude of the electron oscillations is inversely proportional to the frequency, so that the bending effects of the ionosphere, and the attenuation through it, are greater at lower frequencies. The effect of the ionosphere on radio communication becomes negligible at frequencies above about 30 MHz. Below this frequency, the ionosphere is used as a reflecting medium to provide propagation over distances of several thousands of kilometres.

(*b*) (*i*) Broadcasting over an area of an approximate radius of 40 km can be achieved either by using medium frequencies in the range up to 3 MHz, or very-high frequencies in the range 30–300 MHz. Typically, a frequency of 100 MHz is used for frequency-modulated radio communication.

(*ii*) Point-to-point communication over a distance of 200–300 km can be achieved using low frequencies of the order of about 200 kHz.

(*iii*) Global communication over distances in excess of 5000 km can be achieved using either very-low frequencies, typically 15 kHz, or high frequencies, typically in the range 3–30 MHz.

(*c*) (*i*) The mode of propagation would be by ground-waves for medium frequencies, and space-waves for very-high frequencies.

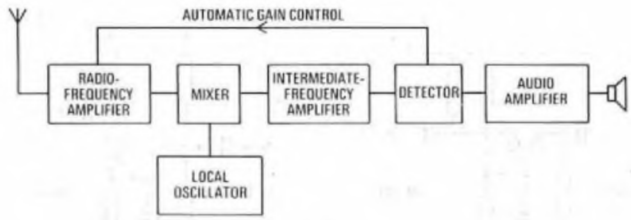
(*ii*) Low-frequency point-to-point propagation would be by ground-waves.

(*iii*) Global communication at very-low frequencies would use ground-wave propagation and, at high frequencies, sky-wave propagation.

Q 10 (*a*) *Draw a block diagram of a superheterodyne receiver.*
(*b*) *For a superheterodyne receiver,*

(*i*) *state 2 reasons for using a radio-frequency amplifying stage, and*
(*ii*) *explain how continuously-variable tuning is obtained when the local oscillator and the radio-frequency stage resonate at different frequencies.*

A 10 (*a*) The block diagram of a superheterodyne receiver is shown in sketch (*a*).



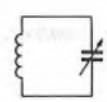
(*a*)

(*b*) (*i*) A radio-frequency amplifying stage is used to improve image-channel rejection and reduce local-oscillator radiation.

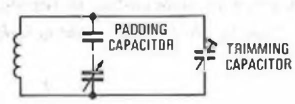
The image channel is separated from the wanted channel by twice the intermediate frequency. It is, therefore, important to have a band-pass-filter characteristic, before the mixer, that admits the wanted channel whilst greatly attenuating the image channel. The inclusion of a radio-frequency amplifier with its own tuned circuit assists this.

Similarly, the tuned circuits of the radio-frequency amplifying stage help to isolate the local oscillator from the aerial connexion and, thus, prevent the radiation of unwanted signals.

(*ii*) The local oscillator and radio-frequency stage of a superheterodyne receiver are tuned to different frequencies, and this difference, which is equal to the intermediate frequency, is required to remain constant as the receiver is tuned across its range. This is achieved by using a different (usually smaller) value of inductance for the oscillator circuit, and providing a shunt capacitance across this inductance, and a series capacitor between the inductor and the tuning capacitor. These are known as *trimming* and *padding* capacitors respectively, and can be adjusted to provide a nearly constant difference between the radio-frequency-stage and local-oscillator tuned-circuit frequencies. This process is known as *tracking*. Sketch (*b*) shows a conventional oscillator tank circuit, and sketch (*c*) shows the modified oscillator tank circuit using trimming and padding capacitors to provide tracking.



(*b*)



(*c*)

TELECOMMUNICATION PRINCIPLES C, 1974

Students were expected to answer any 6 questions

Q 1 (*a*) *Two alternative equivalent circuits for a capacitor are shown in Fig. 1 and Fig. 2. Sketch a phasor diagram for the voltage, V, and current, I, and show also the quantities I_{R1}, I_C, V_{R2} and V_C.*
(*b*) *A capacitor may be represented, at a frequency of 2 MHz, by a capacitance of 200 pF in series with a resistance of 0·8 Ω. Determine, for the capacitor,*

(*i*) *the reactance,*
(*ii*) *the power factor, and*
(*iii*) *the equivalent parallel resistance.*

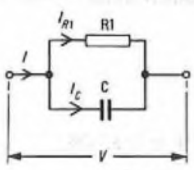


Fig. 1

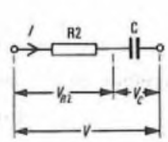


Fig. 2

A 1 (*a*) A phasor diagram for V, I, I_{R1}, I_C, V_{R2} and V_C is shown in the sketch. The direction of phasor I is taken as the reference direction. Phasor V_{R2} is in phase with I, and phasor V_C lags I by 90°; phasor V is their resultant. Phasor I_{R1} is in phase with V, and phasor I_C leads V by 90°; phasor I is their resultant, in phase with V_{R2}.

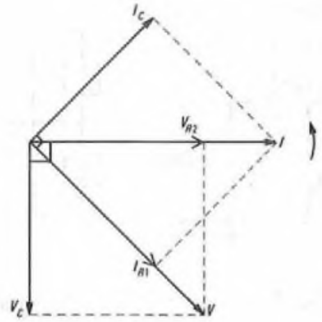
(*b*) (*i*) The reactance of the capacitor, X_C ohms, is given by

$$X_C = \frac{1}{2\pi f C} \text{ ohms, where } f \text{ is the frequency (Hz), and } C \text{ is the capacitance (F),}$$

$$= \frac{1}{2\pi \times 2 \times 10^6 \times 200 \times 10^{-12}} = \underline{398\ \Omega.}$$

(*ii*) Power factor $= \dfrac{\text{resistance}}{\text{impedance}}$,

$$= \frac{0{\cdot}8}{\sqrt{(0{\cdot}8^2 + 398^2)}} = \underline{0{\cdot}002.}$$



(*iii*) The impedance of the equivalent parallel circuit must equal the impedance of the series circuit.

$$\therefore \quad \frac{1}{\dfrac{1}{R_p}+\mathrm{j}\dfrac{1}{X_p}} = R_s - \mathrm{j}X_s,$$

where R_p and X_p are the resistance and reactance, respectively, of the parallel circuit, and R_s and X_s are the resistance and reactance, respectively, of the series circuit.

$$\therefore \quad \frac{1}{R_p}+\mathrm{j}\frac{1}{X_p} = \frac{1}{R_s - \mathrm{j}X_s},$$

$$= \frac{R_s + \mathrm{j}X_s}{R_s^2 + X_s^2}.$$

$$\therefore \quad R_s + \mathrm{j}X_s = (R_s^2 + X_s^2) \times \frac{1}{R_p} + (R_s^2 + X_s^2) \times \mathrm{j}\frac{1}{X_p}.$$

Equating real parts gives

$$R_s = \frac{R_s^2 + X_s^2}{R_p}.$$

$$\therefore \quad R_p = \frac{R_s^2 + X_s^2}{R_s},$$

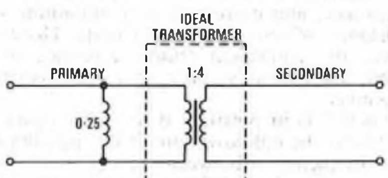
$$= \frac{0{\cdot}8^2 + 398^2}{0{\cdot}8}\ \Omega = \underline{198\ \mathrm{k}\Omega}.$$

Q 2 *(a) An audio-frequency transformer, which has a primary inductance of 0·25 H, may otherwise be considered ideal. It has 4 times as many secondary turns as primary turns. A potential difference of 5 V, at a frequency of 159 Hz, is maintained across the primary winding. Calculate the secondary voltage, and the primary and secondary currents, when the secondary winding is*

(i) open-circuited, and
(ii) connected to a 160 Ω resistor.

(b) Explain briefly why the secondary voltage of a practical transformer ultimately falls at high frequencies.

A 2 (*a*) The sketch shows the circuit of an ideal transformer with a primary inductance of 0·25 H and a primary-to-secondary turns ratio of 1:4.



(*i*) The secondary voltage, E_S volts, is given by

$$E_S = \frac{E_P}{N} \text{ volts, where } N \text{ is the turns ratio, and } E_P \text{ is the primary voltage (V),}$$

$$= 5 \times \frac{4}{1} = \underline{20\ \mathrm{V}}.$$

As the secondary winding is open-circuited, the secondary current, I_S, is <u>zero</u>.

The primary current, I_P amperes, is equal to the magnetizing current, I_M amperes, and is given by

$$I_P = I_M = \frac{E_P}{\mathrm{j}2\pi fL} \text{ amperes, where } f \text{ is the frequency (Hz), and } L \text{ is the primary inductance (H).}$$

$$= -\mathrm{j}\frac{5}{2\pi \times 159 \times 0{\cdot}25}\ \mathrm{A} = \underline{-\mathrm{j}20\ \mathrm{mA}}.$$

(*ii*) When a resistance of 160 Ω is connected to the secondary winding, the secondary voltage remains unchanged.

$$\therefore \quad \underline{E_S = 20\ \mathrm{V}}.$$

Therefore, the secondary current is given by

$$I_S = \frac{20}{160}\ \mathrm{A} = \underline{125\ \mathrm{mA}}.$$

The equivalent primary circuit becomes an inductance of 0·25 H in parallel with a resistance of $160 \times (\frac{1}{4})^2 = 10\ \Omega$. Hence, the primary current is the sum of the current taken by this resistance and the magnetizing current.

$$\therefore \quad I_P = \frac{5}{10} - \mathrm{j}0{\cdot}02\ \mathrm{A} = \underline{500 - \mathrm{j}20\ \mathrm{mA}}.$$

(*b*) At high frequencies, the secondary voltage ultimately falls due to the increasing effects of the leakage inductance, which appears in series with the output, and the self-capacitance of the windings, which appears in parallel with the output.

Q 3 *(a) Describe a laboratory method for determining the Thévènin equivalent circuit of a resistive source.*

(b) Determine the Thévènin equivalent circuit for the terminals A and B of the circuit shown in Fig. 3.

(c) Hence, or otherwise, determine the impedance which takes maximum current when connected to terminals A and B, and the value of that current.

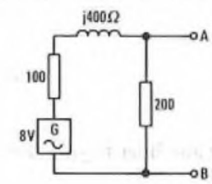
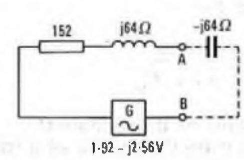


Fig. 3



(*a*)

A 3 (*a*) To determine, experimentally, the Thévènin equivalent circuit of a resistive source, 2 measurments are required. Firstly, the open-circuit output voltage, V_{oc}, is measured across the output terminals, using a high-impedance voltmeter; this gives the value of the equivalent Thévènin generator. Secondly, a calibrated, variable, non-reactive resistance is connected across the terminals, and adjusted until the voltmeter indicates a voltage of $V_{oc}/2$ volts. Since the source is resistive, the value of the variable resistance is equal to that of the Thévènin equivalent resistance of the source.

(*b*) For Fig. 3, the open-circuit voltage appearing across terminals A and B is given by

$$V_{oc} = \frac{200}{100 + \mathrm{j}400 + 200} \times 8\ \mathrm{V},$$

$$= \frac{16}{3 + \mathrm{j}4} = \frac{16(3 - \mathrm{j}4)}{3^2 + 4^2}\ \mathrm{V},$$

$$= \frac{48 - \mathrm{j}64}{25} = 1{\cdot}92 - \mathrm{j}2{\cdot}56\ \mathrm{V}.$$

The impedance, Z, appearing across terminals A and B when the source is short-circuited is given by

$$Z = \frac{(100 + \mathrm{j}400) \times 200}{100 + \mathrm{j}400 + 200}\ \Omega,$$

$$= \frac{200 + \mathrm{j}800}{3 + \mathrm{j}4} = \frac{(200 + \mathrm{j}800)(3 - \mathrm{j}4)}{3^2 + 4^2}\ \Omega,$$

$$= \frac{3800 + \mathrm{j}1600}{25} = 152 + \mathrm{j}64\ \Omega.$$

Hence, the Thévènin equivalent circuit consists of a voltage generator of <u>1·92 − j2·56 V</u> in series with an impedance of <u>152 + j64 Ω</u>. The circuit is shown in sketch (*a*).

(*c*) Maximum current is drawn from the generator when the value of the external component is such that the circuit is in series resonance;

that is, when the circuit is purely resistive. For this condition, the external component must have an impedance of $-j64\ \Omega$, and is, therefore, a capacitance (shown dashed in sketch (*a*)).

Hence, the maximum current, I_M amperes, is given by

$$I_M = \frac{1\cdot92 - j2\cdot56}{152}\ \text{A} = 12\cdot6 - j16\cdot8\ \text{mA}.$$

$$\therefore\ |I_M| = \sqrt{(12\cdot6^2 + 16\cdot8^2)} = \underline{21\ \text{mA}.}$$

Q 4 (a) Explain the Miller effect.

(b) A triode valve has an anode a.c. resistance of 10 kΩ, a mutual conductance of 4 mS, a grid–anode capacitance of 1·5 pF, and a grid–cathode capacitance of 8·5 pF. The valve is connected as a voltage amplifier with a resistive anode load.

Calculate the voltage gain and input capacitance of the amplifier for load resistances of

(i) zero, and
(ii) 10 kΩ.

A 4 (*a*) The inherent grid–anode capacitance of a triode valve allows a leakage path to exist whereby feedback can occur from the anode circuit to the grid circuit. The input impedance of the valve is modified in operation by this feedback, and the resulting change in input impedance is known as the Miller effect.

(*b*) The amplification factor, μ, of a triode valve is given by

$\mu = r_a g_m$, where r_a is the anode a.c. resistance (Ω), and g_m is the mutual conductance (S),

$= 10 \times 10^3 \times 4 \times 10^{-3} = 40.$

The voltage gain, A, of a triode amplifier connected to a resistive load is given by

$$A = \frac{\mu R_L}{r_a + R_L}, \qquad \ldots\ldots (1)$$

where R_L is the load resistance (Ω).

The input capacitance, C_{in}, of a triode amplifier is given by

$$C_{in} = C_{gk} + C_{ag}(1 + A)\ \text{farads}, \qquad \ldots\ldots (2)$$

where C_{gk} is the grid–cathode capacitance (F), and C_{ag} is the grid–anode capacitance (F).

(*i*) From equations (1) and (2), when the load resistance is zero,

$$\underline{A = 0,}$$

and $C_{in} = 8\cdot5 + 1\cdot5(1 + 0) = \underline{10\ \text{pF}.}$

(*ii*) Similarly, when the load resistance is 10 kΩ,

$$A = \frac{40 \times 10 \times 10^3}{10 \times 10^3 + 10 \times 10^3} = \underline{20,}$$

and $C_{in} = 8\cdot5 + 1\cdot5(1 + 20) = \underline{40\ \text{pF}.}$

Q 5 (a) Fig. 4 shows a simple 3-winding transformer bridge, or admittance bridge. Explain its principle of operation. What is the purpose of switch S?

(b) A circuit is connected across the UNKNOWN *terminals of the bridge, and balance is obtained at source frequencies of 1 kHz and 2 kHz as indicated in the table.*

Source Frequency (kHz)	*1*	*2*
R (kΩ)	*10*	*10*
C (pF), with switch S in position A	—	*1550*
C (pF), with switch S in position B	*4420*	—

Determine an equivalent circuit for the unknown circuit at each frequency. Suggest a probable form for the unknown circuit.

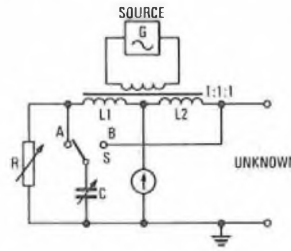
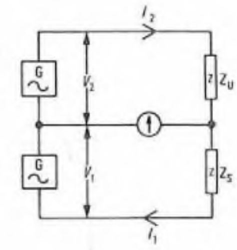


Fig. 4



(*a*)

A 5 (*a*) The 3-winding transformer bridge is designed to eliminate inaccuracies due to the effects of stray capacitances when measuring unknown impedances.

Windings L1 and L2 constitute a pair of equal e.m.f. sources, and are wound such that, at balance, the resultant current in the detector limb is zero. Resistor R and capacitor C form a calibrated variable standard impedance, Z_S ohms, switch S being assumed to be set to position A. The unknown impedance to be measured, Z_U ohms, is connected across the UNKNOWN terminals, and is assumed to be capacitive.

At balance, indicated by a null reading on the detector and obtained by varying resistor R and capacitor C, the voltage, V_1, produced by winding L1, drives a current, I_1, through the standard impedance. Similarly, the voltage, V_2, produced by winding L2, drives a current, I_2, through the unknown impedance. Currents I_1 and I_2 are equal. The circuit can be redrawn as shown in sketch (*a*).

From sketch (*a*), at balance,

$$V_1 = I_1 Z_S, \qquad \ldots\ldots (1)$$

$$\text{and } V_2 = I_2 Z_U. \qquad \ldots\ldots (2)$$

Dividing equation (1) by equation (2) gives

$$\frac{V_1}{V_2} = \frac{I_1 Z_S}{I_2 Z_U},$$

and, since $V_1 = V_2$ and $I_1 = I_2$, then

$$Z_U = Z_S. \qquad \ldots\ldots (3)$$

The bridge is also called an admittance bridge, since the standard components are in parallel, making it more convenient to work in terms of admittances. From equation (3),

$$Y_U = Y_S,$$

where Y_U and Y_S are the admittances of the unknown and standard impedances respectively (S).

Hence,

$$G_U + jB_U = G_S + jB_S,$$

where G_U, G_S, B_U and B_S are the conductance and susceptance components of Y_U and Y_S (S).

Therefore, by equating the real and imaginary terms, the value of resistor R gives the equivalent shunt resistance of the unknown impedance, and the value of capacitor C gives its capacitance.

For an inductive unknown impedance, switch S is set to position B, so that capacitor C is connected in parallel with the unknown impedance. At balance, capacitor C is in parallel resonance with the unknown inductance, and current I_2 is a minimum, dependent only upon the resistance of the unknown circuit. Hence, the value of resistor R gives the equivalent shunt resistance of the unknown impedance, and the reactance of capacitor C equals that of the unknown inductance.

(*b*) Since switch S is in position B at a frequency of 1 kHz and position A at 2 kHz, the unknown circuit is a parallel resonant circuit with a resonant frequency of between 1–2 kHz.

At a frequency, $f_1 = 1$ kHz, from part (*a*), the equivalent shunt resistance is $\underline{10\ \text{k}\Omega.}$

Also, $$2\pi f_1 L_1 = \frac{1}{2\pi f_1 C_1},$$

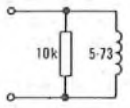
where L_1 is the inductance of the unknown impedance at frequency f_1 (H), and C_1 is the value of capacitor C at that frequency (F).

$$\therefore\ L_1 = \frac{1}{4\pi^2 \times 10^6 \times 4420 \times 10^{-12}} = \underline{5\cdot73\ \text{H}.}$$

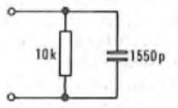
Hence, the equivalent circuit at a frequency of 1 kHz is as shown in sketch (*b*).

At a frequency, $f_2 = 2$ kHz, from part (*a*), the equivalent shunt resistance is 10 kΩ, and the capacitance, C_2 farads, of the unknown impedance at frequency f_2 is 1550 pF.

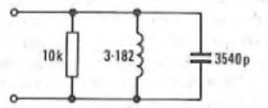
Hence, the equivalent circuit at a frequency of 2 kHz is as shown in sketch (*c*).



(*b*)



(*c*)



(*d*)

Now, at frequency f_1, the unknown circuit is inductive and has a susceptance, B_1 siemens, given by

$$B_1 = B_{LU} - B_{CU},$$

where B_{LU} and B_{CU} are the susceptances of the unknown inductance, L_U, and capacitance, C_U, respectively (S).

$$\therefore \frac{1}{2\pi f_1 L_1} = \frac{1}{2\pi f_1 L_U} - 2\pi f_1 C_U. \qquad \ldots\ldots (4)$$

Similarly, at frequency f_2, the unknown circuit is capacitive and has a susceptance, B_2 siemens, given by

$$B_2 = B_{CU} - B_{LU}.$$

$$\therefore 2\pi f_2 C_2 = 2\pi f_2 C_U - \frac{1}{2\pi f_2 L_U}.$$

But,

$$f_2 = 2f_1.$$

$$\therefore 4\pi f_1 C_2 = 4\pi f_1 C_U - \frac{1}{4\pi f_1 L_U}. \qquad \ldots\ldots (5)$$

Multiplying equation (5) by 2 gives

$$8\pi f_1 C_2 = 8\pi f_1 C_U - \frac{1}{2\pi f_1 L_U}. \qquad \ldots\ldots (6)$$

Adding equations (4) and (6) gives

$$\frac{1}{2\pi f_1 L_1} + 8\pi f_1 C_2 = 6\pi f_1 C_U.$$

$$\therefore \frac{1}{2\pi \times 10^3 \times 5\cdot73} + 8\pi \times 10^3 \times 1550 \times 10^{-12}$$
$$= 6\pi \times 10^3 \times C_U.$$

$$\therefore C_U = 3540 \text{ pF}.$$

Substituting for C_U in equation (4) gives

$$\frac{1}{2\pi \times 10^3 \times 5\cdot73} = \frac{1}{2\pi \times 10^3 \times L_U}$$
$$-2\pi \times 10^3 \times 3540 \times 10^{-12}.$$

$$\therefore L_U = 3\cdot182 \text{ H}.$$

Hence, a probable form for the unknown circuit is as shown in sketch (*d*).

Note: It is unlikely that students would have been required to calculate the values of L_U and C_U. The question asks only for a suggestion of the probable form of the unknown circuit, and sketch (*d*), without the values marked in, would be sufficient.

Q 6 A 1 mH coil, having a Q-factor of 200, is connected in parallel with a 4000 pF capacitor of negligible loss. A 4 V source, of constant voltage, negligible impedance and variable frequency, is connected across the parallel combination.

(*a*) *Determine*

(*i*) *the frequency at which the current taken from the source is a minimum,*

(*ii*) *the value of this minimum current, and*

(*iii*) *the value of the circulating current in the tuned circuit.*

(*b*) *To what values would the answers to parts* (*a*) (*ii*) *and* (*iii*) *change if the source had a resistive impedance of 25 kΩ?*

A 6 (*a*) The circuit is shown in sketch (*a*).

(*i*) The frequency at which the current taken from the source is a minimum is the resonant frequency, f_0 hertz, and is given by

$$f_0 = \frac{1}{2\pi\sqrt{(LC)}} \text{ hertz, where } L \text{ is the inductance of the coil (H), and } C \text{ is the capacitance of the capacitor (F),}$$

$$= \frac{1}{2\pi\sqrt{(1 \times 10^{-3} \times 4000 \times 10^{-12})}} \text{ Hz} = 79\cdot6 \text{ kHz}.$$

(*ii*) Now, for a coil with parallel resistance, the Q-factor is given by

$$Q = \frac{R}{2\pi f_0 L}, \text{ where } R \text{ is the equivalent shunt resistance } (\Omega).$$

$$\therefore R = 2\pi f_0 LQ \text{ ohms},$$

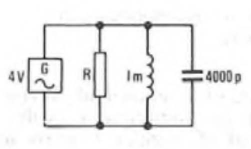
$$= 2\pi \times 79\cdot6 \times 10^3 \times 1 \times 10^{-3} \times 200\ \Omega = 100 \text{ k}\Omega.$$

Hence, the current taken at resonance, I_S amperes, is given by

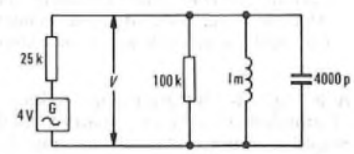
$$I_S = \frac{4}{100 \times 10^3} \text{ A} = 40\ \mu\text{A}.$$

(*iii*) The circulating current at resonance, I_C amperes, is equal to the current flowing in the inductance or the capacitance. The current flowing in the inductance and, hence, I_C is given by

$$I_C = \frac{4}{2\pi \times 79\cdot6 \times 10^3 \times 1 \times 10^{-3}} \text{ A} = 8 \text{ mA}.$$



(*a*)



(*b*)

(*b*) The modified circuit is shown in sketch (*b*).

The current taken from the source at resonance is now given by

$$I_S = \frac{4}{25 \times 10^3 + 100 \times 10^3} \text{ A} = 32\ \mu\text{A}.$$

Hence, the voltage, V volts, appearing across the coil in parallel with the capacitance is given by

$$V = 32 \times 10^{-6} \times 100 \times 10^3 = 3\cdot2 \text{ V}.$$

Therefore, the circulating current is now given by

$$I_C = \frac{3\cdot2}{2\pi \times 79\cdot6 \times 10^3 \times 1 \times 10^{-3}} \text{ A} = 6\cdot4 \text{ mA}.$$

Q 7 (*a*) *Explain what is meant by frequency modulation of a sinusoidal carrier.*

(*b*) *A 2 V peak, 100 kHz sinusoidal carrier is frequency modulated by a 2·5 V peak, 400 Hz sinusoidal signal. The modulator has a sensitivity of 5 kHz/V.*

Write down expressions for

(*i*) *the instantaneous frequency of the modulated carrier, and*

(*ii*) *the instantaneous voltage of the modulated carrier.*

(*c*) *Estimate the minimum transmission bandwidth required for the modulated carrier.*

A 7 (*a*) Frequency modulation of a sinusoidal carrier is that modulation in which the instantaneous frequency of the modulated carrier differs from the carrier frequency by an amount proportional to the instantaneous amplitude of the modulating signal. The number of times per second the instantaneous frequency of the modulated carrier varies about the carrier frequency is equal to the modulating frequency. The amplitude of the modulated carrier is maintained at a constant value equal to the amplitude of the unmodulated carrier.

(*b*) (*i*) The instantaneous frequency, f hertz, of a frequency-modulated carrier is given by

$$f = f_c + f_d \sin 2\pi f_m t \text{ hertz},$$

where f_c is the carrier frequency (Hz), f_d is the frequency deviation (Hz), f_m is the modulating frequency (Hz), and t is the time (s).

Now, the sensitivity of modulation

$$= \frac{f_d}{V_m} \text{ hertz/volt, where } V_m \text{ is the modulating voltage (V).}$$

$$\therefore f_d = 5 \times 10^3 \times 2\cdot5 \text{ Hz} = 12\cdot5 \text{ kHz}.$$

Hence, $f = 100 \times 10^3 + 12\cdot5 \times 10^3 \times \sin(800\pi \times t)$ Hz.

(*ii*) The instantaneous voltage, v volts, of a frequency-modulated carrier is given by

$$v = V_c \sin\{2\pi f_c t - (f_d/f_m)\cos 2\pi f_m t\} \text{ volts},$$

where V_c is the carrier voltage (V).

$$\therefore\ v = 2 \times \sin\left\{2\pi \times 100 \times 10^3 \times t - \frac{12 \cdot 5 \times 10^3}{400} \times \cos(2\pi \times 400 \times t)\right\} \text{V},$$
$$= \underline{2 \times \sin\{2\pi \times 10^5 \times t - 31 \cdot 25 \times \cos(800\pi \times t)\}\text{V}}.$$

(*c*) An empirical formula for obtaining the bandwidth, B hertz, of a frequency-modulated carrier is given below.

$$B \simeq 2(f_d + f_m) \text{ hertz},$$
$$= 2 \times (12 \cdot 5 \times 10^3 + 400) \text{ Hz} = \underline{25 \cdot 8 \text{ kHz}}.$$

Q 8 (a) A carrier, $e_c = E_c \sin \omega t$, is amplitude modulated by a signal, $e_s = E_s \sin pt$, to give a modulation factor m.

Give the corresponding expression for the modulated carrier, and expand this expression to show that the modulated carrier contains 3 separate frequency components. Show how the modulated carrier may be depicted on a phasor diagram.

(b) If the values of the quantities E_c and m in part (a) are 10 V and 0·7 respectively, determine

(i) the maximum instantaneous voltage,
(ii) the r.m.s. value of each separate frequency component, and
(iii) the r.m.s. value of the modulated carrier.

A 8 (*a*) The instantaneous voltage, e volts, of a sinusoidal carrier of angular velocity ω radians/second and peak amplitude E_c volts, amplitude modulated by a sinusoidal signal of angular velocity p radians/second to give a modulation factor m, is given by

$$\underline{e = E_c(1 + m \sin pt) \sin \omega t \text{ volts}}, \qquad \ldots\ldots (1)$$

where t is the time (s).

Expanding the expression gives

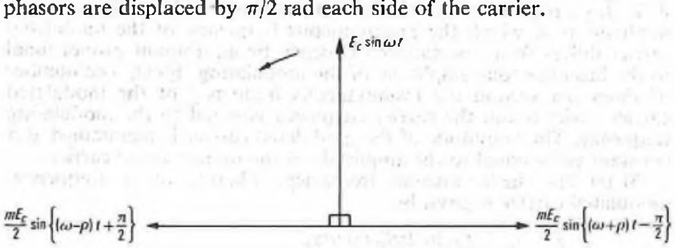
$$e = E_c \sin \omega t + mE_c \sin \omega t \sin pt \text{ volts},$$
$$= E_c \sin \omega t + \frac{mE_c}{2}\{\cos(\omega t - pt) - \cos(\omega t + pt)\} \text{ volts},$$
$$= E_c \sin \omega t - \frac{mE_c}{2}\cos(\omega + p)t + \frac{mE_c}{2}\cos(\omega - p)t \text{ volts}. \qquad \ldots\ldots (2)$$

Thus, the modulated carrier contains 3 separate frequency components: the carrier frequency, $\underline{\omega/2\pi}$ hertz, the upper side-frequency, $\underline{(\omega + p)/2\pi \text{ hertz}}$, and the lower side-frequency, $\underline{(\omega - p)/2\pi \text{ hertz}}$.

From equation (2),

$$e = E_c \sin \omega t + \frac{mE_c}{2}\sin\left\{(\omega + p)t - \frac{\pi}{2}\right\} + \frac{mE_c}{2}\sin\left\{(\omega - p)t + \frac{\pi}{2}\right\} \text{ volts}.$$

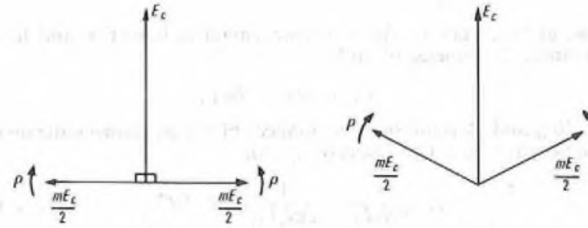
At any instant, the modulated carrier can be represented on a phasor diagram having 3 phasors, representing the carrier and the upper and lower side-frequency components. The amplitude of the modulated carrier at any instant is given by the sum of the 3 phasors. Sketch (*a*) shows the phasor diagram at an instant when the side-frequency phasors are displaced by $\pi/2$ rad each side of the carrier.



(*a*)

Since the 3 phasors are rotating with different angular velocities, the relative positions shown in sketch (*a*) are not maintained. The upper side-frequency-component phasor gains on the carrier phasor at the same rate as the latter gains on the lower side-frequency-component phasor. Thus, a phasor diagram which shows the relative angular velocities can be plotted by subtracting the angular velocity of the carrier, ω radians/second, from those of the side-frequency components, $(\omega + p)$ and $(\omega - p)$ radians/second for the upper and lower side-frequency components respectively.

Therefore, the side-frequency-component phasors rotate in opposite directions, with angular velocity p radians/second, about the carrier phasor, which is considered to be stationary.



(*b*) (*c*)

Sketch (*b*) shows the resulting phasor diagram for the instant illustrated in sketch (*a*). Sketch (*c*) shows the phasor diagram for an instant a short interval after that shown in sketch (*b*). (The phasors in sketches (*a*), (*b*) and (*c*) are not drawn to scale.)

(*b*) (*i*) From equation (1),

$$e = 10 \times (1 + 0 \cdot 7 \sin pt) \sin \omega t \text{ volts},$$

and the maximum value occurs when $\sin pt$ and $\sin \omega t$ are both unity. Hence,

$$e = 10 \times (1 + 0 \cdot 7) = \underline{17 \text{ V}}.$$

(*ii*) The r.m.s. value of the carrier is given by

$$\frac{E_c}{\sqrt{2}} = \frac{10}{\sqrt{2}} = \underline{7 \cdot 07 \text{ V}}.$$

The r.m.s. value of each side-frequency component is given by

$$\frac{mE_c}{2 \times \sqrt{2}} = \frac{0 \cdot 7 \times 10}{2 \times \sqrt{2}} = \underline{2 \cdot 48 \text{ V}}.$$

(*iii*) The r.m.s. value of the modulated carrier is given by the root of the sum of the squares of the r.m.s. values of its components.

$$\therefore\ E = \sqrt{\left\{\left(\frac{E_c}{\sqrt{2}}\right)^2 + \left(\frac{mE_c}{2 \times \sqrt{2}}\right)^2 + \left(\frac{mE_c}{2 \times \sqrt{2}}\right)^2\right\}} \text{ volts},$$
$$= \sqrt{(7 \cdot 07^2 + 2 \cdot 48^2 + 2 \cdot 48^2)} = \underline{7 \cdot 89 \text{ V}}.$$

Q 9 (a) Sketch a circuit diagram for a transistor oscillator which produces sinusoidal oscillations.

(b) Explain the purpose of each component in your circuit, and describe the circuit operation, paying particular attention to those factors which determine

(i) the waveform,
(ii) the frequency, and
(iii) the maintenance of the oscillations.

Q 10 (a) Sketch a typical set of collector characteristics for a common-emitter n p n transistor.

(b) Show, on your characteristics, the load lines you would expect when a load resistor is

(i) connected in series with the collector, and
(ii) coupled by a transformer in the collector circuit.

The load lines should be clearly marked and carefully explained.

(c) Briefly discuss the relative merits of the 2 methods of load connexion for a power amplifier.

COMMUNICATION RADIO C, 1974

Students were expected to answer any 6 questions

Q 1 (a) Which ionized regions are present in the atmosphere during a winter night? Give the approximate height of each of these regions.

(b) What do you understand by the following terms in connexion with ionospheric propagation:

(i) skip distance, and
(ii) gyro frequency?

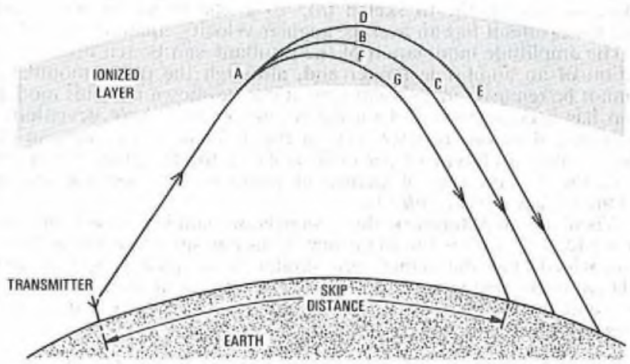
(c) What is the effect on the skip distance of an increase in

(i) transmitted frequency,
(ii) transmitted power, and
(iii) density of ionization of the reflecting layer?

A 1 (*a*) Gases in the atmosphere are ionized when energy from high-frequency radiation, such as ultra-violet radiation, releases electrons from gas molecules. The extent of the ionization depends on the intensity of the radiation and the density of the molecules. The intensity of the radiation falls as it nears the Earth's surface, whereas the density of the gas molecules increases, so that an optimum height for maximum ionization occurs. Furthermore, different optimum heights occur for different radiating frequencies, so that a series of regions of ionized gas are produced. These regions, in order of ascendency, are labelled *D*, *E*, *F1* and *F2*. On a winter night, when the effect of the sun is minimal, the *D*-region disappears, the *E*-region is reduced in density, and the *F1*-region and *F2*-region combine to form the *F*-region.

At such a time, the nominal height for the *E*-region is 100–200 km above the Earth's surface, while that of the *F*-region is 300-400 km.

(*b*) (*i*) The skip distance is the distance between a transmitter and the first point where regular reception is possible for a particular frequency, as shown in the sketch.



(*ii*) If free electrons, set into oscillation by a radio wave, have a component of velocity at right angles to the Earth's magnetic field, a force is exerted on them at right angles to their direction of motion. When the forces resulting from the Earth's field and the radio wave are equal, the path taken by the free electrons becomes a spiral, and extreme absorption of the radio wave results. The frequency at which this occurs, usually about 1·4 MHz, is termed the *gyro frequency*.

(*c*) When a signal, propagated from the Earth's surface, penetrates an ionized layer, the incident radio wave is refracted. As penetration continues, the density of ionization increases, so that the refractive index decreases and the angle of refraction increases. If sufficiently refracted, the wave eventually returns to the Earth along a path such as that illustrated by line ABC in the sketch.

(*i*) If the frequency of the incident signal is increased, the angle of refraction decreases and the wave penetrates deeper into the layer along, say, path ADE, returning to the Earth at a point further away from the transmitter. Thus, the skip distance is increased.

(*ii*) If the transmitter power is increased, the refractive index is unaffected, so that the skip distance remains the same.

(*iii*) If the density of ionization increases, the refractive index is reduced and the wave does not penetrate so far into the layer, following, say, path AFG, and returning to the Earth at a point nearer to the transmitter. The skip distance is, therefore, reduced.

Q 2 (a) Briefly explain why an intermediate frequency is necessary in a superheterodyne radio receiver.

(b) List 5 factors which affect the choice of intermediate frequency in both amplitude-modulated and frequency-modulated receivers. Illustrate their relationship with the intermediate frequency by using numerical examples.

Q 3 (a) A medium-wave superheterodyne receiver is advertised as having 6 active signal components. What is the probable function of each?

(b) Give 3 advantages of using a signal-frequency amplifier in a very-high-frequency radio receiver.

(c) Briefly outline the conflicting factors which determine the choice of time constant in an automatic-gain-control circuit.

(d) If the only components in the automatic-gain-control circuit are an 8 μF capacitor and an 8 kΩ resistor, what is the time constant?

A 3 (*a*) A medium-wave superheterodyne receiver, advertised as having 6 active signal components, would probably have

(*i*) 1 transistor operating as a mixer-oscillator,
(*ii*) 2 transistors operating as intermediate-frequency amplifiers,
(*iii*) 1 diode operating as the detector, and
(*iv*) 2 transistors operating as audio-frequency amplifiers.

(*b*) In a heterodyne receiver, the mixer stage not only mixes wanted signals with an oscillator signal, but also mixes any 2 signals applied to its input. A wide bandwidth prior to the mixer would, therefore, allow many signal components to intermix and produce noise. The use of a signal-frequency amplifier limits the bandwidth preceding the mixer, as well as amplifying the wanted signal, thus improving the signal-to-noise ratio.

The bandwidth of the signal-frequency amplifier is usually too wide to afford any rejection of adjacent signals but, provided that the intermediate frequency is sufficiently high, will significantly suppress the image signal.

The signal-frequency amplifier also reduces the possibility of oscillator frequencies being applied to the aerial and consequently transmitted.

(*c*) The automatic-gain-control circuit is required to maintain a constant receiver output for a given degree of modulation in the event of fluctuation of amplitude of the input carrier. It should, therefore, operate sufficiently fast to counteract a change in the mean level of the carrier, but not so fast that it follows the modulation of the carrier.

(*d*) The time constant, τ seconds, of such a circuit is given by

$$\tau = CR,$$

where C is the capacitance (F), and R is the resistance (Ω).

$$\therefore \tau = 8 \times 10^{-6} \times 8 \times 10^{3} \text{ s},$$
$$= \underline{64 \text{ ms}}.$$

Q 4 A class-B linear amplifier has an HT supply of 1·2 kV. When the tuned-anode load is 20 kΩ, the peak value of the anode-current pulses is 50 mA with the drive unmodulated.

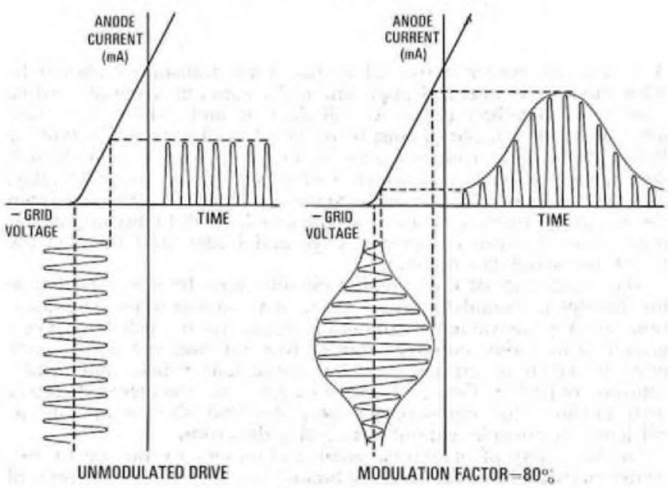
(a) Draw the input and output waveforms for such an amplifier, showing how they are interrelated by the anode-current/grid-voltage characteristic,

(i) for an unmodulated drive, and
(ii) for a drive sinusoidally modulated to a depth of 80%.

(b) Find the efficiency of this amplifier

(i) with the drive unmodulated, and
(ii) with a sinusoidal modulation to a depth of 80%.

A 4 (*a*) The required waveforms are shown in sketches (*a*) and (*b*) respectively.



(*a*) (*b*)

(*b*) (*i*) The anode current, i_a amperes, of a series of sinusoidal half-cycle pulses produced at the output of a class-B tuned amplifier can be represented by a series having the first 2 significant terms

$$i_a = \frac{I_{pk}}{\pi} + \frac{I_{pk}}{2} \sin \omega t + \ldots \text{ amperes},$$

where I_{pk}/π is the mean value of the anode current, and $(I_{pk}/2) \sin \omega t$ is the fundamental-frequency component.

The efficiency, η, of a class-B amplifier is given by

$$\eta = \frac{\text{radio-frequency power developed in the tuned circuit}}{\text{power supplied by HT}}.$$

The radio-frequency power delivered

$$= (I_{r.m.s.})^2 R \text{ watts},$$

where $I_{r.m.s.}$ is the r.m.s. value of the fundamental component and is equal to $I_{pk}/2\sqrt{2}$, and R is the anode load resistance (Ω).

Hence, radio-frequency power, P_u, for an unmodulated drive is given by

$$P_u = \left(\frac{I_{pk}}{2\sqrt{2}}\right)^2 R \text{ watts,}$$

$$= \left(\frac{50}{10^3 \times 2 \times \sqrt{2}}\right)^2 \times 20 \times 10^3 = 6\cdot25 \text{ W.}$$

The power supplied by the HT

$$= \frac{I_{pk} V}{\pi} \text{ watts,}$$

where V is the HT voltage (V),

$$= \frac{50 \times 1\cdot2 \times 10^3}{10^3 \times \pi} = 19\cdot1 \text{ W.}$$

$$\therefore \eta = \frac{6\cdot25 \times 100}{19\cdot1} = \underline{32\cdot7\%.}$$

(*ii*) When sinusoidal modulation is applied, the anode current varies in proportion to $I_{pk}(1 + m \sin \omega t)$, where m is the modulation factor.

Since radio-frequency power is proportional to $(I_{r.m.s.})^2$, the radio-frequency power developed when modulation is applied

$$= P_u\left\{1 + \left(\frac{m}{\sqrt{2}}\right)^2\right\} \text{ watts,}$$

$$= 6\cdot25\left\{1 + \left(\frac{0\cdot8}{\sqrt{2}}\right)^2\right\} = 8\cdot25 \text{ W.}$$

$$\therefore \eta = \frac{8\cdot25 \times 100}{19\cdot1} = \underline{43\cdot2\%.}$$

Q 5 (*a*) *Briefly discuss the considerations which enter into the design of a short-wave transmitter's output stage.*

(*b*) *Why is each aerial at a high-frequency transmitting station not necessarily permanently associated with a transmitter?*

(*c*) *What do you understand by the term distributed amplifier?*

A 5 (*a*) The power output of a short-wave transmitter should be adequate for the intended application. The components used must be capable of handling the power dissipation and, where necessary, adequate cooling facilities must be provided. In the event of a fault or failure of the drive, causing excessive current in the output stage, it is necessary to provide a protective facility such as an overload relay. To prevent overloading from the reflection of energy, and to obtain the maximum transfer of power to the aerial, careful matching of the impedances between the output stage and feeder, and between the feeder and aerial, is required.

The bandwidth of the coupling circuits must be sufficiently wide for the type of modulation used, yet narrow enough to prevent radiation of any unwanted harmonics. Apart from such harmonics, generated in earlier non-linear stages (for example, the mixer), care must be taken to prevent parasitic oscillations which could cause spurious radiation. Cost and reliability must be considered, together with facilities for maintenance, and the unit should operate as efficiently as possible without introducing distortion.

For the safety of operators, small transmitters are housed in protective cases, while larger units are housed in steel cubicles that prevent the application of the HT supply until all access doors are closed.

(*b*) Amongst the requirements for an aerial used for transmission purposes at a high-frequency transmitting station, it is necessary that the aerial be capable of transmitting the relevant signal frequency and, if it is directional, that it be pointing in the right direction.

Since the transmitter is generally capable of operating over a wide frequency range, while the aerial is often optimized for a limited range, it is sometimes necessary to use more than one aerial. Furthermore, as most directional aerials are fixed installations, multi-directional working demands several aerials.

Because operation to a given station may be required only at infrequent intervals, it is uneconomic to assign permanently a transmitter to one aerial.

A radio station may consist, therefore, of many more aerials than transmitters, interconnexion being made by switching devices. Flexibility for variation in direction of propagation and signal frequency is thus provided.

(*c*) A distributed amplifier, sometimes called a *transmission-line amplifier*, is a wideband amplifier in which the output of each stage is coupled by reactive elements to form a transmission line. The input stages are similarly coupled, the phase delay in the line being such that the contributions from each stage are in phase at the load.

Q 6 (*a*) *With the aid of a phasor diagram, explain why a frequency-modulation receiver will discriminate against the weaker of 2 signals of nearly equal amplitude and frequency.*

(*b*) *An interfering carrier has an amplitude half that of the wanted carrier. If the frequency separation is 5 kHz and the wanted system deviation is 75 kHz, what is*

(*i*) *the peak phase deviation of the wanted carrier by the unwanted carrier,*

(*ii*) *the corresponding peak frequency deviation, and*

(*iii*) *the ratio of the amplitude of the wanted signal to that of the unwanted signal at the output of the discriminator?*

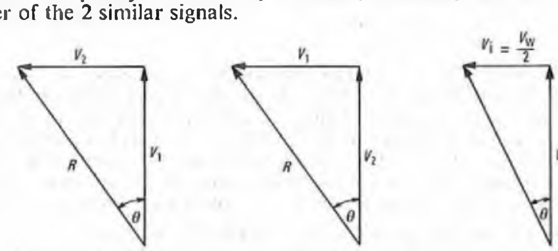
A 6 (*a*) When 2 signals of nearly equal amplitude and frequency are received by a frequency-modulation discriminator, the conditions existing can be represented by the phasor diagrams shown in sketches (*a*) and (*b*).

In sketch (*a*), the phasors V_1 and V_2 represent the 2 similar signals when V_1 is the larger. The resultant, R, is modulated in amplitude and phase at a frequency corresponding to the difference between the frequencies of signals V_1 and V_2, and has an average angular velocity equal to that of V_1. In sketch (*b*), V_2 is the larger of the signals, and the resultant has an average angular velocity equal to that of V_2.

The amplitude modulation of the resultant can be removed by the action of an amplitude limiter and, although the phase modulation cannot be removed in a similar way, it can be shown that this modulation has a negligible effect on the resultant. The phase deviation, θ, is produced by one quarter-cycle of the difference frequency, so that the angular displacement per cycle is 4θ. If the frequency separation is f_s, the average rate of change of phase is $4\theta f_s$, and the average frequency deviation is $4\theta f_s/2\pi$.

When the amplitudes of the 2 signals are similar, then θ approximates to $\pi/4$, so that the frequency deviation approximates to $f_s/2$.

Provided that the signals are similar in frequency, $f_s/2$ is small relative to the system deviation, thus introducing little distortion. The resultant frequency deviation is, therefore, effectively that due to the larger of the 2 similar signals.



(*a*) (*b*) (*c*)

(*b*) (*i*) Sketch (*c*) shows an interfering carrier, V_i, having an amplitude of one half that of the wanted carrier, V_w.

$$\text{Now,} \quad \tan\theta = \frac{V_i}{V_w} = \frac{V_w}{2V_w} = 0\cdot5.$$

$$\therefore \theta = \tan^{-1} 0\cdot5 = 26° \, 34' = \underline{0\cdot4636 \text{ rad.}}$$

(*ii*) When the frequency separation is 5 kHz, the average frequency deviation

$$= \frac{4 \times 0\cdot4636 \times 5 \times 10^3}{2 \times \pi} \text{ Hz} = 1\cdot476 \text{ kHz.}$$

The peak frequency deviation, assuming sinusoidal signals,

$$= 1\cdot476 \times \pi/2 = \underline{2\cdot32 \text{ kHz.}}$$

(*iii*) Since the amplitude of the signal at the output of a discriminator is proportional to the frequency deviation, the ratio of the amplitude of the wanted signal to that of the unwanted signal at the output of the discriminator

$$= \frac{\text{system deviation}}{\text{interference deviation}} = \frac{75 \times 10^3}{2\cdot32 \times 10^3} = \underline{32.}$$

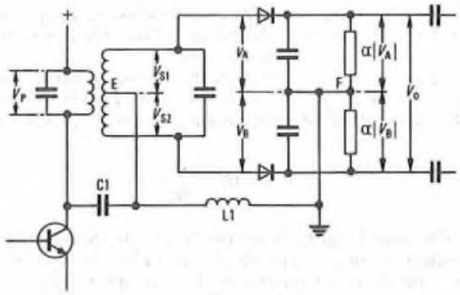
Q 7 (*a*) *Draw the circuit diagram of a Foster-Seeley discriminator with an amplitude limiter preceding it.*

(*b*) *With the aid of phasor diagrams, describe its operation as a frequency-modulation demodulator.*

(*c*) *Which parts of your circuit control the bandwidth over which the discriminator operates?*

A 7 (*a*) Sketch (*a*) shows a transistorized Foster-Seeley discriminator. The preceding amplitude limiter has been omitted for clarity since, in transistorized circuits, a conventional radio-frequency amplifier is normally used, this being overdriven to produce a limiting action.

(*b*) A voltage, having components V_{S1} and V_{S2}, is inductively coupled



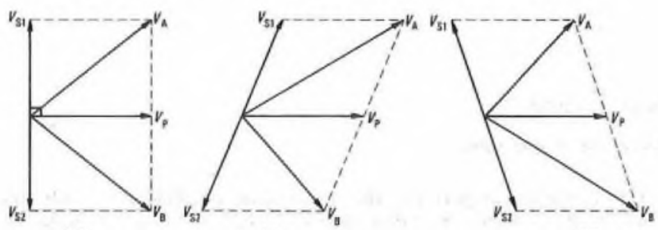
(*a*)

into the secondary winding of the transformer from the primary, while a voltage is simultaneously capacitively coupled via capacitor C1. The reactance of capacitor C1 is low at the signal frequency, so that point E is effectively at the same signal-frequency potential as the collector of the transistor. Point F is at earth potential, so that the voltage across inductor L1 is equal to the primary voltage, V_P. Voltage V_A is the phasor sum of V_P and V_{S1}. Voltage V_B is the phasor sum of V_P and V_{S2}. The rectified load voltages are proportional to the magnitudes of V_A and V_B.

The relationship between the phasors is illustrated in sketch (*b*). At resonance, phasors V_P, V_{S1} and V_{S2} are at right angles, and the magnitudes of phasors V_A and V_B are equal. The output voltage, V_o, is proportional to the difference between the magnitudes of V_A and V_B; that is,

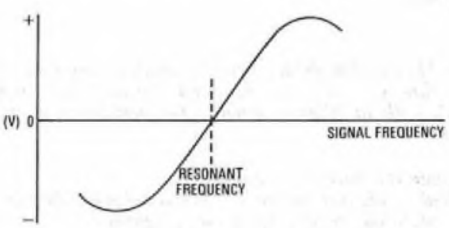
$$V_o = k(|V_A| - |V_B|),$$

so that, at resonance, V_o is zero.



(*b*) (*c*) (*d*)

Above resonance, phasors V_P and V_S are no longer at right angles but become as shown in sketch (*c*). Below resonance, they are as shown in sketch (*d*). As the frequency varies, so do the magnitudes of V_A and V_B, and, hence, that of V_o. The relationship between V_o and the signal frequency is illustrated in sketch (*e*).



(*e*)

(*c*) The bandwidth over which the discriminator operates depends on the coupling between the primary and secondary windings of the transformer and the *Q*-factors of the associated tuned circuits, both of which are resonant at the same frequency.

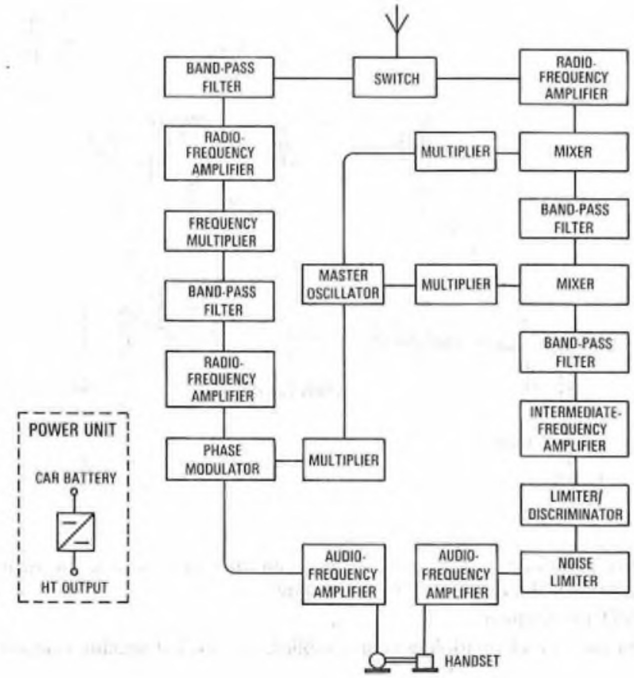
Q 8 (*a*) *Draw the block diagram of a very-high-frequency frequency-modulation transceiver suitable for operation from a car battery. Briefly describe its principle of operation.*

(*b*) *Give typical values, for such a transceiver, of the*

(*i*) *output power,*
(*ii*) *range,*
(*iii*) *audio-frequency bandwidth, and*
(*iv*) *frequency deviation.*

A 8 (*a*) The sketch shows a very-high-frequency frequency-modulation transceiver suitable for operation from a car battery.

For mobile applications, the requirement for rapid channel switching necessitates the use of a stable frequency source for both transmission and reception, and this is provided by means of a master crystal-controlled oscillator. Since it is difficult to frequency modulate a crystal oscillator directly, an indirect technique is adopted. The audio-frequency amplifier contains a correction network whose attenuation increases with frequency, so that the resultant frequency modulation



is independent of audio-frequency variations. The audio-frequency signal is injected into a phase modulator which is driven by a frequency derived from the master oscillator. Since the frequency deviation produced by this method is small, modulation is undertaken at a low frequency, and the modulated signal is frequency multiplied (usually by about 18 times), thus increasing the frequency deviation. Radio-frequency amplification is applied before and after frequency multiplication, while suitable filters prevent the radiation of harmonics.

For reception, a double superheterodyne is generally used, with a high first intermediate frequency of 10·7 MHz, and a low second intermediate frequency of 455 kHz. The received signal is amplified by the radio-frequency amplifier and mixed with a signal derived from the master oscillator. The resultant, after filtering, is mixed with a second crystal-controlled signal which, after further filtering, is detected. The detector may take the form of a ratio detector or, if greater linearity is required, a Foster-Seeley discriminator; the latter must be preceded by an amplitude limiter. Muting is applied to the audio-frequency amplifier to suppress the receiver output when no input signal is being received, while a noise limiter reduces the effects of transients.

A common aerial is used for transmission and reception, but is normally connected only to the receiver, via a switch unit. For transmission, the operator generally operates a PRESS-TO-SPEAK button on the handset which changes over the switch unit and connects the aerial to the transmitting side. HT power is supplied from a d.c.–d.c. convertor driven from the vehicle's battery.

(*b*) Typical values for the parameters specified are given in the table.

Output Power	2–25 W
Range	Up to 50 km
Audio-Frequency Bandwidth	0·3–3·5 kHz
Frequency Deviation	7–15 kHz

Q 9 (*a*) *Draw a dimensioned sketch of a rhombic aerial.*

(*b*) *Why are 3 wires often employed on each leg of a rhombic aerial?*

(*c*) *Upon what factors does the impedance of a rhombic aerial depend?*

(*d*) *Why is it desirable to be able to alter the height of a rhombic aerial?*

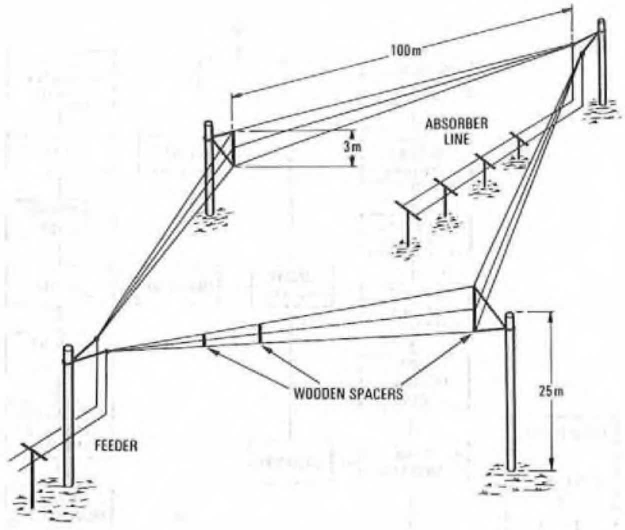
(*e*) *What is the most noticeable difference between a rhombic aerial used for receiving and one used for transmitting?*

A 9 (*a*) A rhombic aerial is shown in the sketch.

(*b*) Three wires are often used in each leg of a rhombic aerial to maintain a constant impedance over the frequency band. The wires are arranged in a vertical formation, diverging from the feed point, and converging to the termination point.

(*c*) The impedance of a rhombic aerial is dependent on

(*i*) the diameter of the wires or, in the case of multiple-wire assemblies, the number of wires and their diameters,



(*ii*) the horizontal separation between the wires which, in turn, depends on the lengths of the wires, and

(*iii*) the frequency.

In the case of multiple-wire assemblies, the vertical spacing is also a factor. The vertical spacing is arranged to diverge at the side extremities to increase the effective diameter of the wire assembly and to offset the diverging width of the rhombic.

(*d*) The angle of elevation, θ (that is, the angle which the direction of maximum radiation makes with the horizontal), is related to the height of the aerial, h, above the ground by the approximation

$$\theta = \sin^{-1}\frac{\lambda}{4h},$$

where λ is the wavelength. Since the angle of elevation is chosen to enable reception at some particular location for a given frequency and ionospheric condition, any change in the ionosphere demands a change in frequency and, hence, in the angle of elevation. The latter may be altered by changing the height of the rhombic aerial.

(*e*) For a rhombic aerial operating as a transmitting aerial, the terminating resistance must be capable of dissipating a high value of power and, so, usually takes the form of an open-wire line (the absorber line) extending back into, but below, the rhombic aerial. For reception purposes, little power is dissipated, so that a carbon resistor may be used for the termination.

Q 10 Describe a method of measuring each of the following, listing the equipment required and indicating each step in the procedure:
(a) the image-channel-rejection ratio of a medium-wave broadcast receiver, and
(b) the effectiveness of the automatic-gain-control system of such a receiver.
Give typical results for each of these measurements.

LINE TRANSMISSION C, 1974

Students were expected to answer any 6 questions

Q 1 (a) Write down an expression for the propagation coefficient of a uniform transmission line in terms of the primary coefficients and frequency.
(b) Hence, derive an approximate expression for the attenuation coefficient at high frequencies in terms of the primary coefficients only.
(c) Explain why, for a high-quality coaxial cable, the attenuation coefficient at high frequencies is substantially proportional to the square root of the frequency.

A 1 (*a*) The propagation coefficient, γ, of a uniform transmission line is given by

$$\underline{\gamma = \sqrt{\{(R + j\omega L)(G + j\omega C)\}}},$$

where R is the loop resistance (Ω/km), L is the loop inductance (H/km), G is the loop leakance (S/km), C is the loop capacitance (F/km), ω is the angular velocity (rad/s) and is equal to $2\pi f$, where f is the frequency (Hz).

(*b*) Now,
$$R + j\omega L = j\omega L\left(\frac{R}{j\omega L} + 1\right),$$

and
$$G + j\omega C = j\omega C\left(\frac{G}{j\omega C} + 1\right).$$

$$\therefore\ (R + j\omega L)(G + j\omega C) = j\omega L j\omega C\left(1 + \frac{R}{j\omega L} + \frac{G}{j\omega C} - \frac{RG}{\omega^2 LC}\right).$$

$$\therefore\ \gamma = j\omega\sqrt{(LC)}\left(1 + \frac{R}{j\omega L} + \frac{G}{j\omega C} - \frac{RG}{\omega^2 LC}\right)^{1/2}$$

At high frequencies, the term containing ω^2 in the denominator may be neglected. Thus, using the binomial expansion and ignoring terms containing ω^2 and higher powers of ω in the denominator,

$$\gamma \simeq j\omega\sqrt{(LC)}\left\{1 + \frac{1}{2}\left(\frac{R}{j\omega L} + \frac{G}{j\omega C}\right)\right\},$$

$$= j\omega\sqrt{(LC)} + \frac{R}{2}\sqrt{\left(\frac{C}{L}\right)} + \frac{G}{2}\sqrt{\left(\frac{L}{C}\right)}.$$

Now, $\gamma = \alpha + j\beta$, where α is the attenuation coefficient (nepers/km), and β is the phase-change coefficient (rad/km). Thus, equating real parts, at high frequencies,

$$\underline{\alpha = \frac{R}{2}\sqrt{\left(\frac{C}{L}\right)} + \frac{G}{2}\sqrt{\left(\frac{L}{C}\right)}\ \text{nepers/kilometre.}}$$

(*c*) As shown in part (*b*), the attenuation coefficient at high frequencies has 2 terms. The first term represents the series loss, and is proportional to R, and the second represents the shunt loss, proportional to G. At high frequencies, the effective resistance becomes proportional to $\sqrt{f}$ due to the skin effect. In modern high-quality cables, the shunt loss is very small, so that the series loss becomes the main factor of the attenuation coefficient.

Therefore, in a high-quality coaxial cable, the attenuation coefficient at high frequencies is substantially proportional to the square root of the frequency.

Q 2 A cable pair has the following primary coefficients: $R = 30\ \Omega/km$, $L = 1\ mH/km$, $C = 0{\cdot}08\ \mu F/km$, and G is negligible. It is to be loaded with 88 mH coils at 2000 m spacing. The resistance of the loading coils can be neglected.

(a) Explain the purpose of loading.
(b) Calculate the attenuation of the loaded pair when $\omega = 5000$ rad/s.
(c) Calculate the theoretical cut-off frequency.
(d) Estimate the upper usable frequency.

A 2 (*a*) The 2 primary coefficients of a cable pair that cause power loss are the loop resistance, R ohms/kilometre, and the loop leakance, G siemens/kilometre. The losses are equivalent to the series loss, I^2R, and the shunt loss, V^2G, respectively, where I is the current (A) and V is the voltage (V); the series loss predominates.

At any point in a cable pair, V and I are related by the equation $Z_0 = V/I$, where Z_0 is the characteristic impedance of the pair (Ω). If Z_0 can be increased without increasing the resistance, I is reduced, so that the series loss, and hence the attenuation coefficient, decrease. (The voltage will rise and, thus, increase the shunt loss, but this is negligible.)

The characteristic impedance is increased by inserting inductance coils at intervals along the line, this process being known as *loading*. Hence, the attenuation coefficient is reduced.

(*b*) The propagation coefficient, γ, is given by

$$\gamma = \sqrt{\{(R + j\omega L)(G + j\omega C)\}} = \alpha + j\beta, \quad \ldots\ldots (1)$$

where L is the loop inductance (H/km) including the inductance of the loading coils, C is the loop capacitance (F/km), α is the attenuation coefficient (nepers/km), β is the phase-change coefficient (rad/km), and ω is the angular velocity (rad/s).

Now, $R + j\omega L = 30 + j5000(1 \times 10^{-3} + 44 \times 10^{-3})\ \Omega$,

$= 30 + j225\ \Omega$,

$= 227\angle \tan^{-1} 7{\cdot}5 = 227\angle 82^\circ\ 24'\ \Omega$.

Also, $G + j\omega C = 0 + j5000 \times 0{\cdot}08 \times 10^{-6}\ \mathrm{S}$,

$= 0 + j4 \times 10^{-4}\ \mathrm{S}$,

$= 4 \times 10^{-4}\angle \tan^{-1} \infty = 4 \times 10^{-4}\angle 90^\circ\ \mathrm{S}$.

$\therefore\ (R + j\omega L)(G + j\omega C) = (227\angle 82^\circ\ 24')(4 \times 10^{-4}\angle 90^\circ)$,

$= 9{\cdot}08 \times 10^{-2}\angle 172^\circ\ 24'$.

$\therefore\ \sqrt{\{(R+j\omega L)(G+j\omega C)\}} = \sqrt{(9{\cdot}08 \times 10^{-2}\angle 172^\circ\ 24')}$,

$= 3{\cdot}013 \times 10^{-1}\angle 86^\circ\ 12'$.

Hence, from equation (1),

$\alpha = 3{\cdot}013 \times 10^{-1} \cos 86^\circ\ 12'$ nepers/km,

$= \underline{0{\cdot}02 \text{ nepers /km}}$.

(*c*) A coil-loaded line acts as a low-pass filter with series-connected inductances and shunt capacitance. The cut-off frequency of such a filter, f_0 hertz, is given by

$$f_0 = \frac{1}{\pi s \sqrt{(LC)}} \text{ hertz,}$$

where s is the distance between loading coils (km).

$$\therefore\ f_0 = \frac{1}{\pi \times 2 \times \sqrt{\{(1 \times 10^{-3} + 44 \times 10^{-3}) \times 0{\cdot}08 \times 10^{-6}\}}} \text{ Hz},$$

$= \underline{2{\cdot}653 \text{ kHz.}}$

(*d*) The cut-off characteristic is not sharp, and the upper usable frequency is approximately 80% of the cut-off frequency; that is, $2{\cdot}653 \times 0{\cdot}8 \simeq \underline{2{\cdot}1 \text{ kHz.}}$

Q 3 (a) Explain what is meant by near-end crosstalk and far-end crosstalk in a transmission system.

(b) List the main sources of crosstalk in a multiple-pair cable.

(c) Explain how crosstalk could be reduced to an acceptable level in a star-quad cable providing loaded audio circuits between 2 telephone exchanges.

A 3 See A7, Line Transmission C, 1973, *Supplement*, Vol. 67, p. 68, Oct. 1974.

Q 4 A 4-wire audio circuit is to be provided between 2 terminal repeater stations with one intermediate repeater station. The overall loss is to be 3 dB between the 2-wire points. The cable pairs between consecutive stations have a loss of 25 dB at 3400 Hz and 16 dB at 300 Hz.

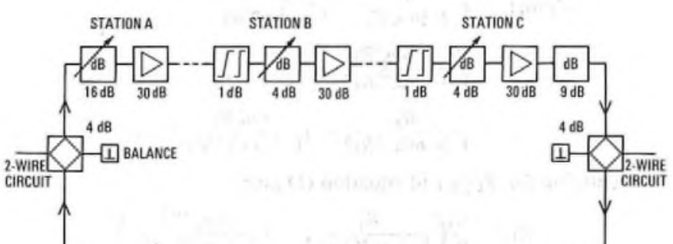
(a) Draw a block diagram to show the arrangement of the various items of equipment which are needed in the circuit.

(b) Explain the function of each item.

(c) Draw a level diagram for one direction of transmission.

(d) Explain the need for imposing upper and lower limits on the signal level, and give typical values.

A 4 (*a*) Sketch (*a*) shows a block diagram of the arrangement of the various items of equipment needed to provide the circuit in one direction. The other direction is similar, but is omitted for simplicity.



(*a*)

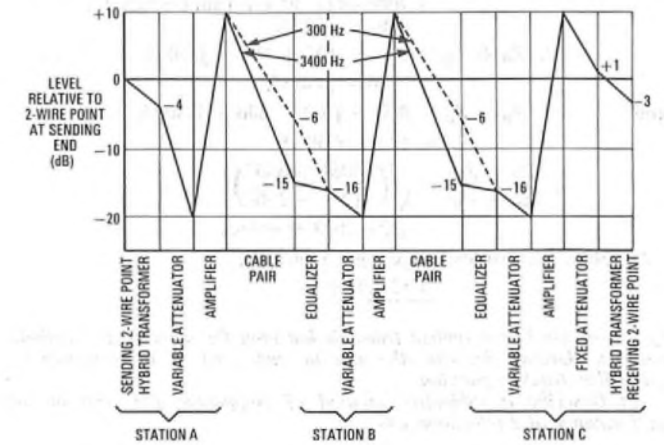
(*b*) Connecting the 2-wire circuits to the 4-wire audio circuit are hybrid transformers, the purpose of which is to combine the uni-directional paths of the 4-wire section with the bi-directional paths of the 2-wire sections. Each hybrid transformer has a loss of about 4 dB.

Each repeater comprises a 30 dB fixed-gain amplifier and an adjustable attenuator, the latter set to the values shown.

At the intermediate and receiving stations (stations B and C), line equalizers are provided to compensate for the attenuation/frequency characteristic of the preceding cable pair, so making the attenuation of the circuit substantially independent of frequency. Each cable pair can then be considered to have its maximum value of loss (that is, 25 dB) over the entire frequency range. Each equalizer introduces a basic loss of approximately 1 dB.

The 9 dB attenuator between the amplifier and hybrid transformer at station C is necessary to reduce the signal level at the 2-wire point to the specified value.

(*c*) Sketch (*b*) shows a level diagram for one direction of transmission.



(*b*)

(*d*) It is necessary to impose limits on the signal level because, if the level is too high, crosstalk can arise between adjacent circuits and, if it is too low, the signal-to-noise ratio becomes so low that the signal is distorted.

A typical range of signal levels is from +10 dB to −20 dB relative to the sending 2-wire point.

Q 5 (a) Draw a block diagram to show the items of equipment needed to assemble a 12-channel CCITT group for carrier telephony.

(b) Explain the function of each item.

(c) Sketch, showing typical values, the attenuation/frequency characteristics of any 2 adjacent channel filters.

(d) Explain why each channel filter needs to have a sharp cut-off characteristic, and say how this is achieved.

A 5 See A8, Line Transmission C, 1972, *Supplement*, Vol. 67, p. 3, Apr. 1974.

Q 6 (a) Define balance return-loss.

(b) Explain how balance return-loss can be measured.

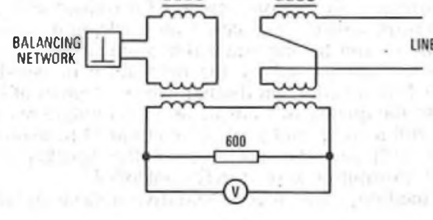
(c) Calculate the balance return-loss between a line having a characteristic impedance of 800 − j100 Ω and a balancing network of impedance 700 ∠ −30° Ω.

A 6 (*a*) Balance return-loss is a measure of the accuracy of the balancing network of a 2-wire–4-wire termination in matching the impedance of the 2-wire line. If Z_0 is the line impedance (Ω), and Z_B is the balancing-network impedance (Ω), then balance return-loss

$$= 20 \log_{10} \frac{Z_0 + Z_B}{Z_0 - Z_B} \text{ dB.}$$

When $Z_B = Z_0$, the balance return-loss is infinite and no reflection occurs.

(*b*) To measure the balance return-loss of a hybrid transformer, the circuit is set up as shown in the sketch.



Two readings are taken from the voltmeter: the first, V_1 volts, with the balancing network disconnected, and the second, V_2 volts, with the balancing network connected. Hence,

$$\text{balance return-loss} = 20 \log_{10} \frac{V_1}{V_2} \text{ dB}.$$

(*c*) Now,

$$Z_0 = 800 - \text{j}100\ \Omega,$$

and

$$\begin{aligned} Z_B &= 700\angle -30^\circ\ \Omega, \\ &= 700\{\cos(-30^\circ) + \text{j} \sin(-30^\circ)\}\ \Omega, \\ &= 606 - \text{j}350\ \Omega. \end{aligned}$$

$$\begin{aligned} \therefore\ Z_0 + Z_B &= 800 - \text{j}100 + 606 - \text{j}350\ \Omega, \\ &= 1406 - \text{j}450\ \Omega, \end{aligned}$$

and

$$\begin{aligned} Z_0 - Z_B &= 800 - \text{j}100 - 606 + \text{j}350\ \Omega, \\ &= 194 + \text{j}250\ \Omega. \end{aligned}$$

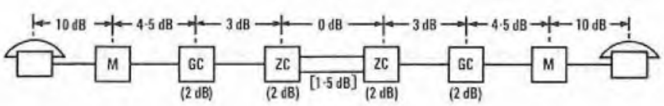
$$\begin{aligned} \therefore\ \left|\frac{Z_0 + Z_B}{Z_0 - Z_B}\right| &= \sqrt{\left(\frac{1406^2 + 450^2}{194^2 + 250^2}\right)}, \\ &= \sqrt{21 \cdot 7638} = 4 \cdot 665. \end{aligned}$$

$$\begin{aligned} \therefore\ \text{balance return-loss} &= 20 \log_{10} 4 \cdot 665 \text{ dB}, \\ &= \underline{13 \cdot 38 \text{ dB}.} \end{aligned}$$

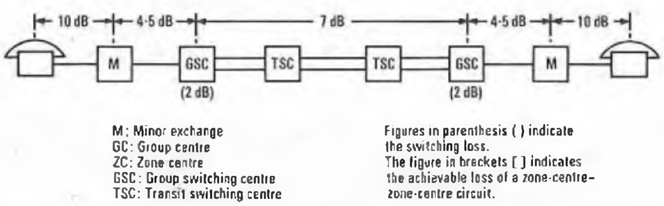
Q 7 (a) Outline a typical transmission plan for a national telephone network showing the loss allowable in each part of a subscriber-to-subscriber trunk connexion.

(b) Describe a subjective method of comparing the transmission performance of 2 telephone sets.

A 7 (*a*) Sketch (*a*) shows the transmission plan introduced by the British Post Office in 1933. The country was divided into 270 group centres, each with a trunk exchange, certain of the larger group centres being designated as zone centres. Each group centre was connected to at least one zone centre, and all zone centres were fully interconnected. The plan shows the overall loss limit for the trunk and junction part of the circuit to be 15 dB, and that for the local network (that is, between the subscriber and the minor exchange) to be 10 dB. The zone-centre–zone-centre circuits were always amplified and usually 4-wire, and the practice was to set them up to have the lowest possible loss with adequate stability, rather than try to achieve relatively unstable zero-loss circuits. In practice, therefore, the actual loss for the trunk and junction part of the circuit was usually about 24–25 dB.



(*a*)



(*b*)

Sketch (*b*) shows a modern plan which is a compromise between cost and quality of transmission. The use of 4-wire switching at transit switching centres avoids the switching losses inherent in the 1933 plan, and the overall practical loss between minor exchanges is about 19–20 dB. Thirty-seven transit switching centres cover the country, and 9 of these are designated main switching centres and are fully interconnected by direct links. Each transit switching centre is directly connected to at least one main switching centre and, also, to other transit switching centres where the traffic justifies such a connexion. Access to a transit switching centre is gained via a group switching centre.

(*b*) A method formerly used to compare subjectively the transmission performance of telephone sets was for trained staff to transmit meaningless words, called *logatones*, over a telephone connexion set up in a laboratory and having adjustable attenuation. The quality of the telephone set was judged by the percentage of words correctly received at the distant end. Even though various degrees of impairment were applied to the quality of transmission to simulate real conditions of use, it was still a somewhat artificial method of measurement since the level of speech and the position of the speaker's mouth in relation to the transmitter were closely controlled.

In a newer method, pairs of representative subjects are asked to use a telephone circuit and give an opinion as to its quality. The subjects may be asked questions over the connexion, or they may engage in normal conversation. This method allows the subjects to hold the telephones in any way they prefer and to converse at any level of speech.

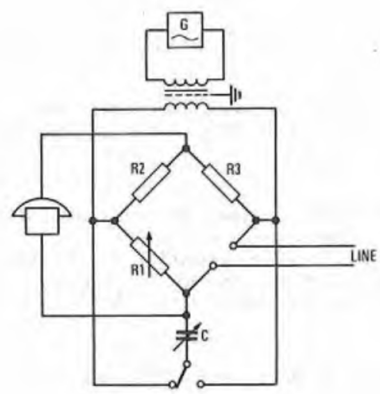
Q 8 The impedance/frequency characteristic of a faulty cable pair terminated in its characteristic impedance shows a series of peaks spaced 1·2 kHz apart. The loop inductance of the pair is 0·8 mH/km and the loop capacitance is 0·08 μF/km.

(a) Draw the circuit diagram of a bridge suitable for making the impedance measurements.

(b) Derive an expression for the line impedance, at a particular frequency, in terms of the bridge components.

(c) Calculate the distance between the cable fault and the testing end.

A 8 (*a*) The sketch shows the circuit diagram of a bridge suitable for making impedance measurements on the faulty cable pair. Variable capacitor C can be connected either across variable resistance R1, when the line impedance has a small negative angle, or across the line itself when its impedance has a small positive angle. A high-impedance telephone receiver acts as the detector, and the signal source is connected to the bridge through a balanced screened transformer.



(*b*) If the bridge is balanced with capacitor C connected across resistance R1, then $R_2 Z_L = R_3 Z_{C//R1}$, where Z_L is the impedance of the line, and $Z_{C//R1}$ is the impedance of the parallel combination of capacitor C and resistance R1.

$$\therefore\ Z_L = \frac{R_3 Z_{C//R1}}{R_2}. \qquad \ldots\ldots (1)$$

Now,

$$Z_{C//R1} = \frac{\dfrac{R_1}{\text{j}\omega C}}{R_1 + \dfrac{1}{\text{j}\omega C}} = \frac{R_1}{1 + \text{j}\omega C R_1}.$$

Rationalizing the expression gives

$$\begin{aligned} Z_{C//R1} &= \frac{R_1}{1 + \text{j}\omega C R_1} \times \frac{1 - \text{j}\omega C R_1}{1 - \text{j}\omega C R_1}, \\ &= \frac{R_1 - \text{j}\omega C R_1^2}{1 + \omega^2 C^2 R_1^2}, \\ &= \frac{R_1}{1 + \omega^2 C^2 R_1^2} - \text{j}\frac{\omega C R_1^2}{1 + \omega^2 C^2 R_1^2}. \end{aligned}$$

Substituting for $Z_{C//R1}$ in equation (1) gives

$$\underline{Z_L = \frac{R_3}{R_2}\left(\frac{R_1}{1 + \omega^2 C^2 R_1^2} - \text{j}\frac{\omega C R_1^2}{1 + \omega^2 C^2 R_1^2}\right).}$$

Similarly, if the bridge is balanced with capacitor C connected across the line, it can be shown that

$$\underline{Z_L = \frac{R_2 R_3 R_1}{R_2^2 + \omega^2 C^2 R_1^2 R_3^2} + \text{j}\frac{\omega C R_3^2 R_1^2}{R_2^2 + \omega^2 C^2 R_1^2 R_3^2}.}$$

(*c*) If β_1 and β_2 radians/kilometre are the phase-change coefficients at frequencies f_1 and f_2 hertz respectively, and ϕ_1 and ϕ_2 radians are the respective phase changes due to reflection at the fault, then the total phase change at f_1 is $2x\beta_1 + \phi_1$ radians and, at f_2, is $2x\beta_2 + \phi_2$ radians, where x is the distance to the fault (km).

If adjacent impedance peaks occur at f_1 and f_2, then the difference between the total phase changes at these frequencies is 2π rad.

$$\therefore\ (2x\beta_1 + \phi_1) - (2x\beta_2 + \phi_2) = 2\pi.$$

$$\therefore\ 2x(\beta_1 - \beta_2) + (\phi_1 - \phi_2) = 2\pi.$$

$$\therefore\ x = \frac{2\pi - (\phi_1 - \phi_2)}{2(\beta_1 - \beta_2)} \text{ kilometres.}$$

But $\phi_1 = \phi_2$.

$$\therefore\ x = \frac{\pi}{\beta_1 - \beta_2} \text{ kilometres.}$$

Now, $\beta_1 - \beta_2 = \omega\sqrt{(LC)}$,

where ω is the difference in angular velocity between f_1 and f_2 (rad/s), L is the loop inductance (H/km), and C is the loop capacitance (F/km).

$$\therefore\ x = \frac{\pi}{2\pi(f_1 - f_2) \times \sqrt{(LC)}} \text{ kilometres,}$$

$$= \frac{1}{2 \times 1{\cdot}2 \times 10^3 \times \sqrt{(0{\cdot}8 \times 10^{-3} \times 0{\cdot}08 \times 10^{-6})}} \text{ km,}$$

$$= \underline{52 \text{ km.}}$$

Q 9 (a) Sketch the cabling layout for a typical rack of mains-operated carrier-transmission equipment.

(b) Show how power can be maintained in the event of mains failure.

(c) Explain how interference between power circuits and transmission circuits is kept to an acceptable level.

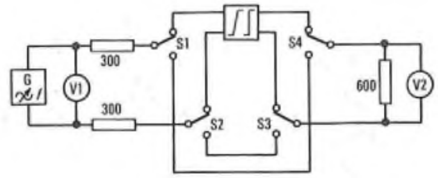
Q 10 (a) Define insertion loss. (Assume source and load impedances to be non-reactive.)

(b) With the aid of sketches, show how the insertion-loss/frequency characteristic of an audio-frequency line equalizer can be measured.

(c) State what precautions are necessary to ensure a reasonable degree of accuracy.

A 10 (*a*) The insertion loss of an item of equipment is the ratio of the power delivered to the load before that equipment is inserted into the line to the power delivered after it has been inserted, and is expressed in decibels.

(*b*) The sketch shows a method of measuring the insertion-loss/frequency characteristic of an audio-frequency line equalizer.



By the repeated operation of switches S1–4, readings of the output voltage, V_2 volts, are made with and without the equalizer connected into the circuit. The input voltage, V_1 volts, is maintained constant, and readings are taken for a range of frequencies in the audio-frequency band. Then, for each frequency,

$$\text{insertion loss} = 20 \log_{10} \frac{V_2 \text{ with equalizer not connected}}{V_2 \text{ with equalizer connected}} \text{ dB.}$$

Hence, the insertion-loss/frequency characteristic may be plotted.

(*c*) It is important to avoid errors caused by harmonics of the testing frequency, and this may be done either by using a bandpass filter in the output of the oscillator circuit, or by using a frequency-selective detector in place of the voltmeters. Any voltmeters used must have a high impedance relative to that for which the equalizer is designed.

The accuracy is also dependent on the stability of both voltmeters and the calibration of voltmeter V2.

TELEPHONY C, 1974

Students were expected to answer any 6 questions

Q 1 (a) Pure-chance traffic of 2 erlangs is offered to a number of trunks which must provide a grade of service of not worse than one lost call in 100. What is the least number of trunks which must be provided? The approximation

$$1 + A + \frac{A^2}{2!} \dots + \frac{A^N}{N!} = e^A$$

may be used in your answer.

(b) Give 4 ways by which electromechanical meters can be used to check the adequacy of plant provision in a telephone exchange.

A 1 (*a*) Now,
$$B = \frac{\dfrac{A^N}{N!}}{1 + A + \dfrac{A^2}{2!} + \dfrac{A^3}{3!} + \dots \dfrac{A^N}{N!}},$$

where B is the grade of service, A is the traffic offered (erlangs), and N is the number of trunks.

Therefore, using the approximation given in the question,

$$B = \frac{\dfrac{A^N}{N!}}{e^A} = \frac{A^N}{N!} \times e^{-A}.$$

$$\therefore\ 0{\cdot}01 = \frac{2^N \times e^{-2}}{N!}.$$

$$\therefore\ 1 = \frac{100 \times 2^N \times e^{-2}}{N!}.$$

$$\therefore\ N! = 100 \times 2^N \times e^{-2}.$$

$$\therefore\ \frac{N!}{2^N} = 100 \times e^{-2} = 13{\cdot}53.$$

If $N = 6$, then $N!/2^N = 11{\cdot}25$, which is less than $13{\cdot}53$ and, if $N = 7$, then $N!/2^N = 39{\cdot}38$, which is greater than $13{\cdot}53$.

Therefore, the least number of trunks required is $\underline{7}$.

(*b*) Electromechanical meters can be used in traffic recorders, or as destination-call-count meters, late-choice-call meters and late-choice-unit meters, to check the adequacy of plant provision in a telephone exchange.

Q 2 (a) Draw a trunking diagram of a unit automatic exchange (UAX) and its associated group switching centre (GSC) up to the first trunk-switching stage.

(b) Describe how a single group of junctions can provide for both STD and non-STD traffic for ordinary and call-office subscribers between the UAX and the GSC, and what special arrangements are made if the GSC is handling traffic from UAXs situated in a different charge group. Why are these arrangements necessary?

A 2 See A6, Telephony C, 1973, *Supplement*, Vol. 67, p. 71, Oct. 1974.

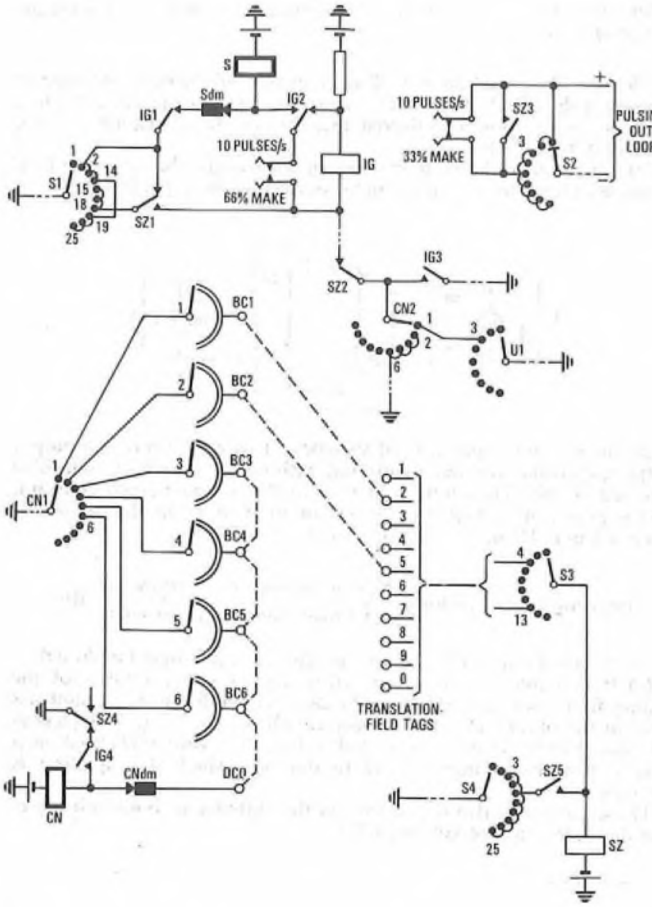
Q 3 (a) Describe, with the aid of a sketch of the circuit elements, how translation and pulsing-out facilities are provided in a director exchange.

(b) List the types of forced release that occur in a director exchange.

A 3 (*a*) The sketch shows the circuit elements of a director concerned with providing the translation and pulsing-out facilities.

Wipers 1–6 are part of the BC-switch, a 2-motion selector that is stepped in response to the second and third digits dialled by the subscriber. There are a total of 600 bank contacts (that is, 6 banks of 100 contacts each), thus permitting one translation to be given for each of the 100 possible BC-switch positions, the 6 wipers allowing up to 6 routing digits per translation.

Assuming that a level-2 director has been chosen, after the subscriber has dialled, say, 237, then the BC-switch will be positioned on outlet 7 of level 3. If a translation of, say, 25 is required, then, for this particular BC-switch position, bank contact BC1 is strapped to translation-field tag 2, and bank contact BC2 is strapped to translation-field tag 5. As no more routing digits are required, bank contacts BC3–BC6 are all strapped to the DCO tag. Translation-field tags 1–0 are connected to bank contacts 4–13 respectively of send-switch arc S3.



Unit-switch arc U1 acts as an incoming-digit distributor and steps once for each incoming digit. After the C-digit has been received, it steps to outlet 3, and an earth from arc U1 is extended via arc CN2 to operate relay IG when the 66% *make*-period impulse springs are next open. These impulse springs are synchronized with a set of 33% *make*-period impulse springs connected in the pulsing-out loop, the 2 spring-sets being arranged so that, when one set is closed, the other is open. The impulse springs operate at 10 pulses/s while the director is seized. The operation of relay IG extends the impulse springs to magnet S via contact IG2. Contact IG3 maintains an earth for the stepping circuit irrespective of the position of wiper CN2, and contact IG4 energizes magnet CN. Uniselector S now steps at 10 pulses/s. When wiper S2 reaches outlet 3, the pulsing-out loop is extended to the impulse springs. When the 66% *make*-period impulse springs close to energize magnet S, the 33% *make*-period impulse springs open to transmit one loop-disconnect pulse to line. When the 33% *make*-period impulse springs close, the others open to release magnet S and step the S-wipers to outlet 4. This process continues as uniselector S steps round the bank, one loop-disconnect pulse being sent to line for each step of uniselector S. In this example, when uniselector S reaches outlet 5 (that is, after 2 loop-disconnect pulses have been sent to line), wiper S3 encounters the earth extended from wiper CN1 via bank-contact BC1 and translation-field tag 2. This marking earth operates relay SZ, which holds via contact SZ5 and arc S4. Contact SZ3 short circuits the 33% *make*-period impulse springs to prevent any more loop-disconnect pulses being sent to line. Contact SZ4 releases magnet CN, and uniselector CN steps to outlet 2 in readiness for the transmission of the next impulse train. Contact SZ2 cuts the operating circuit for magnet S and, also, releases relay IG. The restoration of contact IG1 completes a self-drive circuit for uniselector S, which self-drives to outlet 15. At this point, the self-drive circuit is broken at arc S1 and earth is again applied to operate relay IG via contact SZ1 when the 66% *make*-period impulse springs open. When relay IG re-operates, uniselector S once more steps under control of the 66% *make*-period impulse springs to provide part of the inter-digital pause. When uniselector S reaches outlet 19, the self-drive circuit is re-established at arc S1, and the uniselector self-drives back to outlet 1, the home position, relay IG being released when wiper S1 leaves outlet 18. Relay SZ is released when wiper S4 leaves outlet 25.

The release of contact SZ2 again applies earth to operate relay IG, and the circuit operation is repeated for the transmission of the second digit, in this case digit 5, wiper CN1 extending the marking earth via bank-contact BC2 and translation-field tag 5, which is connected to bank-contact 8 of arc S3.

After the second routing digit has been sent, it is necessary to arrange for uniselector CN to pass over outlets 3–6 since no more routing digits are required. To provide this step-on feature, unused bank-contacts BC3–BC6 are connected to the DCO tag, so that the earth extended by arc CN1, when it reaches outlet 3 after transmission of the second routing digit, energizes magnet CN via interrupter contacts CNdm. Uniselector CN then self-drives to outlet 7 in readiness for transmission of the numerical digits.

Inter-digital-pause timing is provided by uniselector S stepping at 10 pulses/s from outlet 1 to outlet 3 (approximately 200 ms), and from outlet 15 to outlet 19 (approximately 400 ms), plus the self-drive time. The total inter-digital pause is, thus, in the order of 700–800 ms.

(*b*) The types of forced release that occur in a director exchange are

(*i*) forced release without tone, where the subscriber is parked without tone on the first code-selector by the A-digit selector if no dialled digits are received within 20–40 s, and

(*ii*) forced release with tone, where

(1) the subscriber is parked with tone on the first code-selector if a spare code is dialled,

(2) the subscriber is parked with tone on the first code-selector if he pauses for more than 20–40 s between digits, and

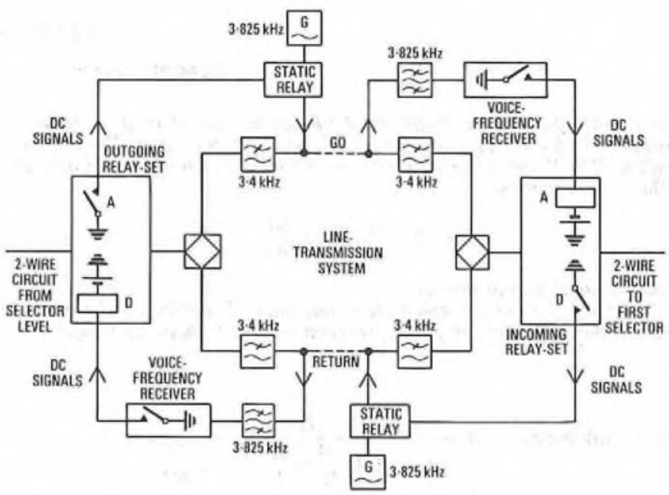
(3) the subscriber is parked with tone on the first code-selector if dialling is commenced before an A-digit selector has been seized and dial tone returned.

The tone referred to in each case above is number-unobtainable tone.

Q 4 (a) Draw a block diagram of a built-in-out-of-speech-band (BIOSB) signalling system, and explain its operation. What are the advantages of a BIOSB signalling system compared with in-band signalling systems?

(b) What are the relative merits of TONE-OFF *or* TONE-ON *idle conditions for BIOSB systems?*

A 4 (*a*) The sketch shows a block diagram of a typical BIOSB signalling system using the TONE-OFF idle-condition signalling method.



Low-pass filters, having a cut-off frequency of 3·4 kHz, are connected in the GO and RETURN paths of the 4-wire section of the circuit to

(*i*) prevent any speech frequencies above 3·4 kHz being transmitted and, thus, interfering with the voice-frequency receivers, and

(*ii*) prevent the 3·825 kHz signalling tone from feeding into the hybrid transformers and, hence, the 2-wire portions of the speech path.

The 3·825 kHz signalling tone is transmitted as soon as the outgoing relay-set is seized, by contact A operating the static relay, and is transmitted over the GO path. Loop-disconnect pulses sent into the outgoing relay-set are, therefore, converted to tone-on/tone-off pulses by the static relay. At the incoming end, the signalling tone passes through the 3·825 kHz band-pass filter and operates a voice-frequency receiver, the output of which is used to operate relay A in the incoming relay-set and, hence, seize the first selector. The voice-frequency receiver and relay A in the incoming relay-set convert the tone-on/tone-off signals back into loop-disconnect signals to step the selectors.

When the called party answers, relay D in the incoming relay-set operates and contact D operates the static relay, thus transmitting 3·825 kHz back along the RETURN path. This transmission lasts for the duration of the conversation. At the outgoing end, the signalling tone passes through the 3·825 kHz filter and operates a voice-frequency receiver, the output of which is used to operate relay D in the outgoing relay-set, which repeats the answering supervisory signal back over the 2-wire circuit.

When the called party clears, the backward signalling tone is disconnected from the RETURN path, thus releasing relay D in the outgoing relay-set. When the caller clears down, contact A releases the static relay, and the resulting disconnexion of signalling tone on the GO path is recognized in the incoming relay-set as a *clear-forward* signal. The incoming relay-set then transmits a *release-guard* signal (tone-on) back to the outgoing relay-set. When the *release-guard* signal ceases, the circuit becomes available for further traffic.

The advantages of a BIOSB signalling system are that

(*i*) the voice-frequency receiver can be simple, since voice immunity is not required,

(*ii*) signalling can take place in either direction while the circuit is being used for speech; for example, periodic metering pulses and coin-box discrimination signals can be sent, and

(*iii*) the outgoing and incoming relay-sets can be relatively simple, as tone-recognition and timing functions are not required.

(*b*) TONE-OFF idle-condition signalling has the following advantages.

(*i*) As loop-disconnect pulses are periods of silence, transient bursts of interference in the transmission path do not give rise to false dial pulses.

(*ii*) Short-duration interruptions of the transmission path do not result in the seizure of idle incoming equipment.

(*iii*) As the TONE-ON condition is maintained during the period that the circuit is seized (except during loop-disconnect pulses), the voice-frequency receiver's automatic-gain control functions more effectively for dial pulses. (This applies only to the GO direction.)

(*iv*) The chance of overloading the common amplifiers is reduced.

TONE-ON idle-condition signalling has the advantage that automatic backward-busying is given during interruption of the transmission path.

In general, an advantage of TONE-OFF (or TONE-ON) idle-condition signalling becomes a disadvantage for TONE-ON (or TONE-OFF) idle-condition signalling respectively. Similarly, in some cases, a particular advantage during an idle condition can become a disadvantage during seizure conditions. However, in the main, TONE-OFF idle-condition signalling tends to offer more advantages than TONE-ON idle-condition signalling.

Q 5 (a) Describe, with the aid of sketches, the basic principles of call timing used on a call routed via an auto-manual switchboard.

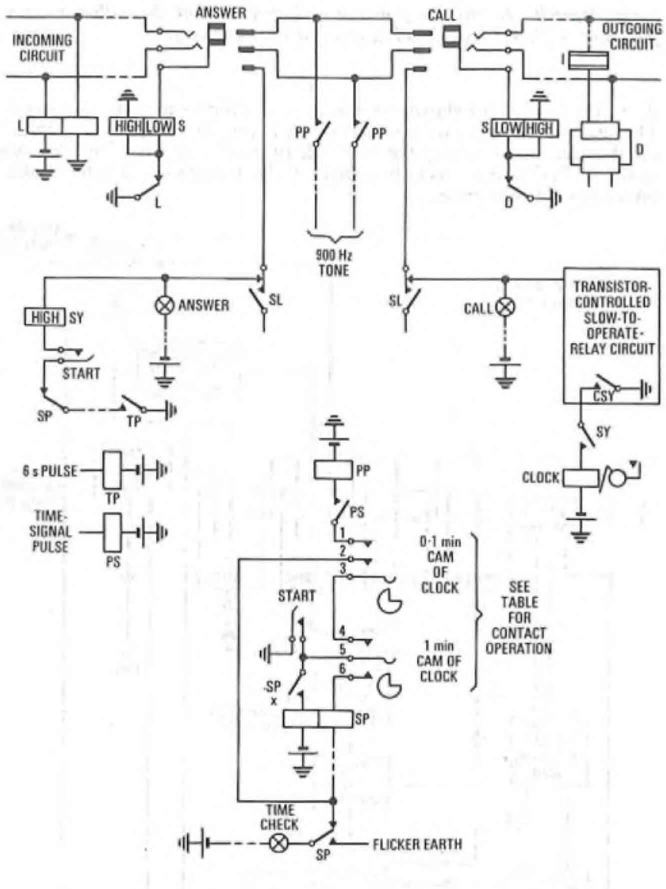
(b) What special problems are associated with timing and charging for calls originating from coin boxes which are routed via an auto-manual switchboard?

A 5 (*a*) The sketch shows the basic circuit elements concerned with call timing at a sleeve-control switchboard. For clarity, the additional circuitry required for coin-box-call timing has been omitted, and that shown is for ordinary calls only.

When the operator has established the connexion and the called subscriber is being rung, the START key is actuated. This allows relay SY to operate once every 6 s via the ANSWER supervisory lamp, the START key, contact SP and contact TP, since relay TP is operating once every 6 s. When the called subscriber answers, relay D in the outgoing circuit operates to dim the CALL supervisory lamp. The change in potential on the sleeve wire is detected by a transistor circuit, and relay CSY is operated after a delay of about 150 ms. This allows the clock to be energized, and thus stepped, once every 6 s. The clock has two 10-step cams, one showing minutes and the other tenths of minutes, and the contacts operated for each step are shown in the table. A total reading of 9·9 min is possible.

Minute Cam		Tenths-of-Minutes Cam	
Interval (min)	Contacts Operated	Interval (min)	Contacts Operated
2–3	4 and 5	0·8–0·0	2 and 3
5–6	4 and 5	0·9–0·0	1, 2 and 3
8–9	4 and 5		
9–0	5 and 6		

At the end of the second minute, clock contacts 4 and 5 make and, after a further 0·8 min, clock contacts 2 and 3 make to extend the earth at the START key to the TIME-CHECK lamp via an SP contact. Thus, the TIME-CHECK lamp glows after 2·8 min have elapsed. After



a further 0·1 min, clock contact 1 makes with contacts 2 and 3, and relay PP operates to extend tone pips to the calling party each time relay PS operates to the synchronizing time-signal pulse. At the end of the third minute, clock contacts 1, 2 and 3, and 4 and 5, break to dim the TIME-CHECK lamp and prevent further operation of relay PP.

If the call continues, the above operation is repeated after 5·8 min and 8·8 min. When the clock steps to the ninth-minute position, clock contacts 5 and 6 make to operate relay SP in series with the TIME-CHECK lamp. Relay SP locks in via an x contact, and other SP contacts connect flicker earth to the TIME-CHECK lamp, to advise the operator that 9 min have elapsed, and disconnect the operate circuit for relay SY, to prevent further stepping of the clock. The operator resets the clock if further timing is required.

When the calling subscriber clears down, relay L releases to light the ANSWER supervisory lamp. The low-resistance earth on the sleeve conductor effectively short-circuits relay SY, and this stops the clock from stepping further. The clock, therefore, indicates the actual time for which the call was effective.

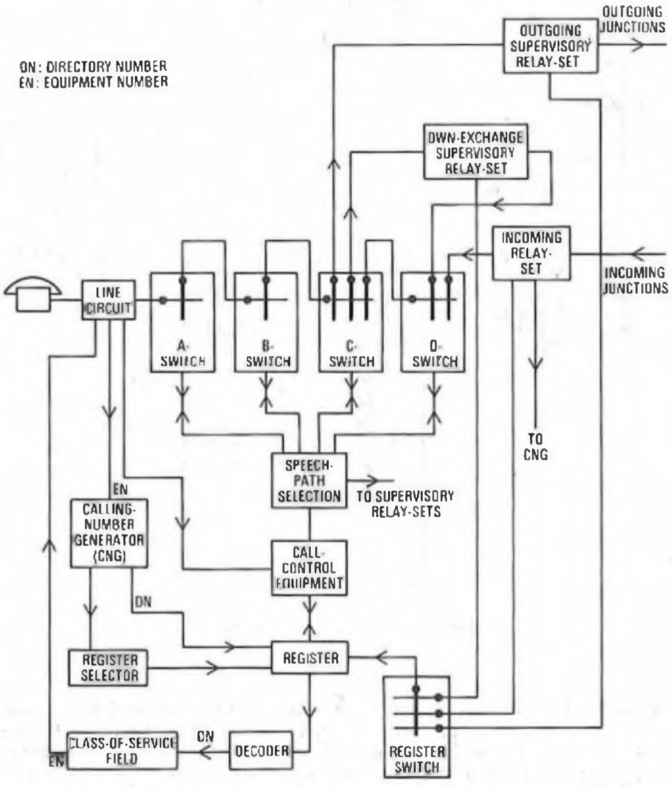
(*b*) Where pay-on-answer-coin-box and ordinary subscribers share the same assistance-level circuits, it is necessary to give some form of indication to the operator when a call has originated from a coin box. This is achieved by arranging that, when the operator answers the call, a positive-battery pulse is connected to the positive wire of the circuit. This is detected in the coin-and-fee-checking (CFC) equipment, which then returns pay tone to the operator. Further positive-battery pulses are then connected to the positive wire by momentary actuation of the position RING key, and these are also detected by the CFC equipment. The first actuation of the RING key causes the CFC equipment to disconnect the discriminating pay tone. A second actuation of the RING key causes the CFC equipment to unlock the coin box's coin slots. Further actuations of the RING key initiate the auditing operation within the CFC equipment; that is, the transmission to the operator of tone pips corresponding to the number of coins inserted into the coin box. If a fraudulent coin-pulse signal is detected by the CFC equipment, number-unobtainable tone is returned to the operator.

On coin-box calls, the START key has a COIN-BOX-START position, and this alters the circuit so that flicker earth is connected to the TIME-CHECK lamp after 2·8 min instead of 9·0 min, thus necessitating resetting of the clock if further timing is required. Thus, the operator is reminded that the caller must insert more money if speech is to continue.

Q 6 (a) Sketch the trunking of a TXE2 exchange and describe briefly how a call is set up between subscribers on the same exchange.

(*b*) *Describe briefly the function and operation of the calling-number generator (CNG). Use sketches to illustrate your answer.*

A 6 (*a*) Sketch (*a*) shows the outline trunking of a TXE2 exchange. The exchange is register-controlled, with the speech paths being set up through a multiple-stage network of reed-relay matrices, known as the A, B, C and D switching stages. Calls are processed sequentially, on a one-at-a-time basis.



(*a*)

When the caller lifts his handset, the operation of the line relay in the line circuit causes the CNG to generate the caller's directory number. This is immediately transferred to, and stored in, a register allocated to the call by the register selector.

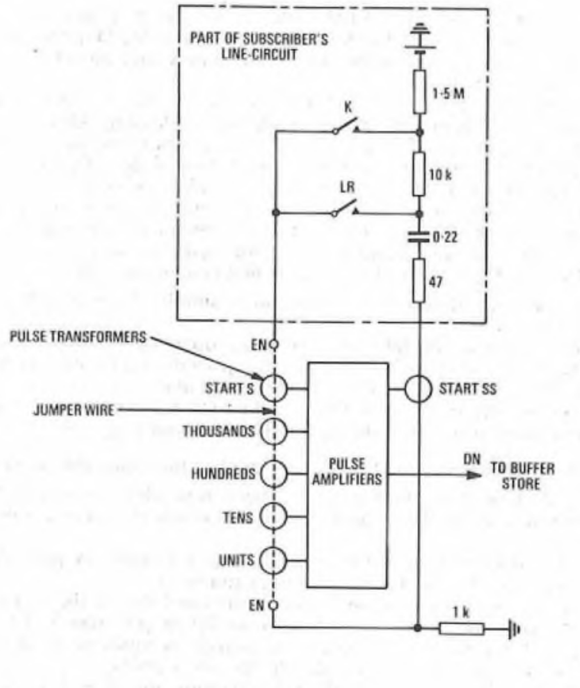
The storage of the caller's identity in the register causes the call-control equipment to set up a path through the switching network from a selected supervisory relay-set to the calling line, via the A-, B- and C-switches, and to the register, via the register switch. The selected supervisory relay-set may be an outgoing or own-exchange relay-set. If the majority of traffic is to the main exchange, then an outgoing supervisory relay-set would be the first choice; it is assumed that this is the case in this example.

Dial tone is returned from the register, and the caller dials the required number, the dialled digits being stored in the register. The register examines the stored digits to determine whether the call is

(*i*) a local call, to be routed to or via the main exchange,
(*ii*) an own-exchange call,
(*iii*) a call to an exchange for which there is a direct route, or
(*iv*) a national-number-dialled call.

As soon as sufficient digits have been received to indicate that the call is to another subscriber on the same exchange, the initial connexion to the outgoing supervisory relay-set is released, and a new connexion is set up to an own-exchange supervisory relay-set during the inter-digital pause. At the end of dialling, when the remaining digits have been stored in the register, the stored digital information is decoded to identify the called line-circuit via the class-of-service field. The call-control equipment tests the called line and, if it is free, a path is set up between the own-exchange supervisory relay-set and the called line-circuit via the D-, C-, B- and A-switches. Thus, a connexion between the calling and called line is established. The register and common equipment are released, leaving the own-exchange supervisory relay-set in control of the call. Ringing current is applied to the called line and ring tone is returned to the caller. When the called subscriber answers, ringing current and ring tone are disconnected and the speech path is established. Metering takes place, and local-call timing commences.

(*b*) Sketch (*b*) shows a simplified circuit diagram of the CNG.



(*b*)

On operation of relay LR in the line circuit, a current pulse passes through a jumper wire threaded through an array of pulse transformers. The pulse transformers are arranged in 4 rows, representing the thousands, hundreds, tens and units part of the directory number. There are 5 transformers in each row, and the jumper wire is threaded through 2 transformers in each row, as well as the start-S transformer. This allows direct coding of the directory number in 2-out-of-5 code. If the exchange has 5-digit or 6-digit directory numbers, only the last 4 digits are generated, as these are sufficient for identification purposes within the exchange.

With the circuit idle, the capacitor is charged to 50 V. The operation of relay LR causes the capacitor to discharge through the jumper wire between the EN tags. The jumper wire forms a single-turn winding on each transformer through which it passes, and the output signals from the secondary windings are amplified and stored. On incoming calls, contact K discharges the capacitor slowly, so that no pulse is generated should relay LR operate due to the calling party clearing first. The $1 \cdot 5\ M\Omega$ resistor ensures that the capacitor cannot charge up between the release of relay K and the operation of relay LR.

Two start signals, S and SS, are generated in response to a pulse from the line circuit. Signal S is fed to an insensitive amplifier, while signal SS is taken to a sensitive amplifier. In the event of signal SS being generated without signal S, a spurious-noise condition is assumed and the CNG is cleared as a precautionary measure.

The function of the CNG is to generate and pass into the register the directory number of the calling line so that, subsequently, the line circuit can be identified or marked within the exchange. It also enables the exchange to determine the class of service particular to that line; for example, PBX, shared service, or outgoing calls barred. As any line circuit can be allocated any directory number, this flexibility gives a more even distribution of traffic throughout the exchange; for example, PBXs can be allocated any directory number and do not need to be within a certain numbering range.

Q 7 (*a*) *Describe the arrangements in a TXK1 exchange for handling incoming traffic to* (*i*) *small PBX groups and* (*ii*) *large PBX groups.*

(*b*) *Describe how subscribers' lines are allocated to TXK1 distributor outlets, and explain any special considerations to be given to the allocation of PBX lines.*

Q 8 (*a*) *Describe the advantages of 4-wire switching in a national trunk network, and give an outline description of the trunk transit network.*

(*b*) *Sketch a trunking diagram showing the routing of a call between 2 local non-director exchanges completed via the transit network. Indicate on the sketch typical signalling systems which might be used on each link of the call.*

(*c*) *Describe the factors to be taken into account when determining whether the normal routing of a trunk call should be via the transit network or not.*

A 8 (*a*) The main advantages of 4-wire switching in a trunk network are that

(*i*) transmission losses at switching stages are reduced, since 4-wire–2-wire and 2-wire–4-wire conversion is not required at intermediate switching points,
(*ii*) echo is reduced, since 2-wire–4-wire hybrid transformers are required only at each end of a trunk circuit,
(*iii*) transmission quality is improved,
(*iv*) up to 5 links in tandem between group switching centres (GSCs) are possible, and
(*v*) fast call-setting-up times can be achieved if a high-speed inter-register signalling system is used.

The trunk transit network comprises a number of transit switching centres (TSCs) distributed throughout the country. Nine TSCs are fully interconnected, and are known as main switching centres (MSCs). The remaining 28 TSCs are known as district switching centres (DSCs) and have access to at least one MSC. Each GSC has access to at least one DSC or MSC. The entire transit network is 4-wire switched, with crossbar equipment being used at the TSCs. Signalling between registers at the GSCs and TSCs is accomplished in multi-frequency form, using Signalling System Multi-Frequency No. 2 (SSMF2). Switching through the transit network is achieved by the originating GSC forwarding the national-numbering-group (NNG) code to its DSC (or MSC), and the DSC (or MSC) then switching the call through to the next TSC, if necessary. Each time the originating GSC is switched through to a further TSC, it forwards the NNG code. This continues until the objective GSC is reached. The routing of a call between GSCs via 5 transit links would be

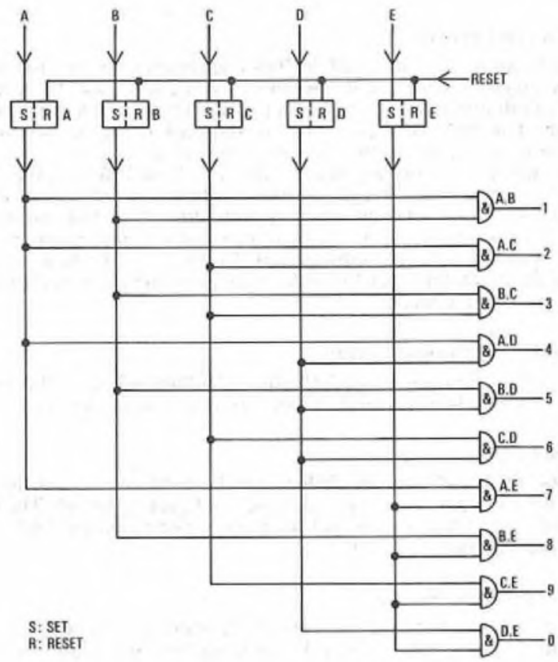
GSC–DSC–MSC–MSC–DSC–GSC.

(*b*) See A10, Telephony C, 1973, *Supplement*, Vol. 67, p. 73, Jan. 1975. Typical signalling systems are: loop disconnect on the local-exchange–GSC links, SSDC3 on the originating GSC–TSC link, and SSAC11 on the TSC–terminal GSC link. The inter-register signalling system would be SSMF2, using multi-frequency sender-receivers.
(*c*) The factors to be taken into account when determining whether a normal routing should be via the transit network or not are
(*i*) the possibility of routing the call via 1 or 2 Strowger links through the normal main network,
(*ii*) the sufficiency of routing digits available from the controlling register for a Strowger routing,
(*iii*) on a 2-link Strowger routing, the sufficiency of switching capacity at the intermediate GSC for the anticipated volume of traffic, and
(*iv*) the volume of traffic justifying the provision of a one-link or 2-link Strowger route.

Q 9 (*a*) *Describe, with the aid of sketches, 2 different types of electronic store, and explain their operation with the aid of graphs.*
(*b*) *Describe a typical application of an electronic store in a telephone exchange.*

A 9 (*a*) See A8, Telephony C, 1972, *Supplement*, Vol. 66, p. 75, Jan. 1974; and A8, Telephony C, 1973, *Supplement*, Vol. 67, p. 71, Oct. 1974.

(*b*) The sketch shows the use of an electronic store in a telephone exchange as part of a 2-out-of-5-code-to-decimal-code converter.



With no signals present on leads A–E, toggles A–E are in the RESET state. The SET outputs from the toggles form the inputs to ten 2-input AND gates. When a signal is simultaneously received on 2 of the leads, the toggles connected to those leads set to give an output signal. The AND gate that then has 2 coincident signals on its inputs opens, thus giving an output on one of the leads marked 1–0. Typically, signals on leads A and D will set toggles A and D, and an output is obtained on lead 4. The logic statement beside each of leads 1–0 gives the toggle combination required to give an output on that lead. The symbol . indicates a logical AND function.
The signals on leads A–E would probably be of short duration, while the output on leads 1–0 lasts until the toggles are reset. Hence, the toggles store the input conditions.

Q 10 *For a 3000-type relay, describe*
(*a*) *the mechanical adjustments required,*
(*b*) *the various factors of safety applied, and how these are used, and*
(*c*) *how, given the normal operating-load equivalent, the operating ampere-turns are determined.*

TELEGRAPHY C, 1974

Students were expected to answer any 6 questions

Q 1 (*a*) *How are Telex service difficulties and faults brought to the attention of the maintenance staff?*
(*b*) *Discuss the action to be taken to deal with each of the cases considered in part* (*a*).

A 1 (*a*) and (*b*) The aim of the maintenance procedure used to deal with service difficulties and faults is to provide for the effective and economical performance of maintenance work, to keep systematic records, to ensure a uniform procedure and to provide statistics for overall control. Faults, complaints and difficulties can originate from a variety of sources, and these are listed below, together with the action taken to deal with them.

Customer Reports

A customer, wishing to report a Telex fault, normally uses the telephone. The operator connects the call to the Telex-faults position in the Telex exchange test-room. The customer's Telex number and the nature of the complaint are recorded on the customer's fault card, and an entry is made in the daily fault register, a registered fault number being issued for each fault. The testing officer tests the customer's circuit by intercepting it at the engineering control board and locating the fault to the exchange, customer's line or station equipment. Details of an exchange fault are passed to the exchange staff by means of a fault docket, and details of a line fault are passed to the voice-frequency terminal or to the associated telephone exchange. If the fault is at the customer's station, the details of the fault are passed to the telegraph repair centre, where a faultsman is detailed to visit the customer. When the fault has been cleared, the fault control is informed, entries are made on the fault card and register, and the card is filed. All Telex customers are entitled to emergency service, with attention to faults being given at any hour of the day.

Other Reports

Fault reports from Telex switchboard operators, or from distant exchanges, are again received at the Telex-faults position, and the fault is dealt with in a similar way to a customer's complaint. A reference, or *slip*, number is normally given to the person reporting the fault so that a check can be made if the fault is not cleared quickly.

Exchange Alarms

Bell and lamp signals are provided in exchanges to indicate blown fuses or equipment faults such as selectors that fail to release. The fault is located by following the floor, gangway and rack lamps leading to the component causing the alarm. Alarms are classified as urgent or non-urgent (prompt and deferred respectively). Urgent-alarm conditions are extended to the nearest manned telephone or Telex exchange during off-duty hours, and justify the calling-out of an emergency fault officer.

Routiners and Testers

A variety of testers are used in Telex exchanges. A manual tester applies tests as its operator throws the keys; if a test fails, the operator writes a fault docket and either attends to the fault himself or circulates the docket to the maintenance officer responsible for the section. A semi-automatic tester automatically performs a cycle of tests on a selector when the START key is operated. If a fault is found, the tester stops and a lamp lights to indicate the test that has failed. As before, the operator writes and circulates a fault docket. A fully-automatic routiner automatically tests selectors in sequence, and prints a fault docket if an irregularity is discovered. In all cases, the fault dockets are numbered and a record card kept to show the faults reported on each piece of equipment.

Routine Inspections and Patrols

Equipment is inspected regularly by patrolling officers who clean, inspect and overhaul selectors. Faults are recorded on dockets.

Hold and Trace

Occasionally, a call that has failed may be held and traced through the switching equipment until the faulty selector is found. The train of selectors is then released and the faulty selector busied and noted for future attention.

Special-Fault Investigation

A close check is kept on the records of each Telex circuit so that recurring faults, faults of long duration and frequent faults can be made the subject of a special investigation.

Q 2 Describe briefly how a Telex subscriber is charged for

(*a*) *a local call through an automatic exchange,*
(*b*) *a manually-connected international-cable call,*
(*c*) *an international call using an error-corrected radio circuit, and*
(*d*) *an international call using automatic ticketing equipment.*

A 2 (*a*) See A4, Telegraphy C, 1971, *Supplement*, Vol. 65, p. 35, July 1972.

(*b*) When a call is received on an international-cable position, the service signal LONDON SWITCHBOARD is transmitted to the calling subscriber to indicate that teleprinter signals can be exchanged between the caller and the operator. The operator records the calling-subscriber's identity on a ticket, together with the called-subscriber's number. The operator selects the called subscriber and, when the *call-connected* signal is received, the operator is able to call for the answer-back code from the calling subscriber and the called subscriber in turn. The 2 sets of answer-back codes are received by the operator and the respective distant subscriber, the teleprinter signals from the caller being printed in black on the operator's teleprinter and the signals from the called subscriber being printed in red.

The operator checks that the correct connexion has been made and operates a DISCONNECT key to put the 2 stations in contact. The operation of this key also starts a counter-type clock associated with the connecting circuit. The clock operates to meter pulses at 6 s intervals up to 999·9 min. The operator's position is now free to attend to further calls.

The connexion can be cleared by either subscriber or by the operator. Whichever party clears, the timing clock stops on receipt of a *clear* signal. When a CLEARED lamp glows, the operator notes the elapsed time from the clock and enters this on the charging ticket. The operation of a CLEAR key clears the connexion and resets the clock to 999·9 min, ready for the next call.

(*c*) International calls over error-corrected radio circuits are controlled by operators using positions in a radio suite. Each position is designed for 2 calls only to be handled simultaneously, and 2 teleprinter receivers are provided so that each call can be monitored continuously under difficult radio conditions. The calls are set up in a similar way to a cable call, except that the clock is controlled by pulses dependent on the availability of the radio channel. Radio circuits normally operate with automatic error-correcting equipment which ensures that messages are unmutilated by retransmitting characters that have been incorrectly received at the distant terminal. Circuit time occupied by retransmission is not charged for, and the timing of each call is based on one nominal 6 s pulse for each 41 characters that are, or could be, sent over the radio circuit. The call ticket is completed by the operator at the conclusion of the call to include the time recorded. Completed tickets are inserted in a chute to a compartment at the rear of the console for collection, pricing and billing.

(*d*) See A4, Telegraphy C, 1972, *Supplement*, Vol. 66, p. 55, Oct. 1973.

Q 3 (a) What is the relationship between the frequency bandwidth and the baud-speed of teleprinter signals?

(b) With the aid of a component diagram and a frequency/attenuation graph, explain the operation of

(i) a low-pass filter suitable for preventing d.c. telegraph signals causing interference in adjacent pairs in a telephone cable, and

(ii) a band-pass filter suitable for use in a multi-channel-voice-frequency transmission system.

Q 4 (a) Using timing diagrams, explain the difference between start-stop and isochronous distortion measurements.

(b) Describe the difference in the principles of design of distortion-measuring sets suitable for measuring each type of distortion.

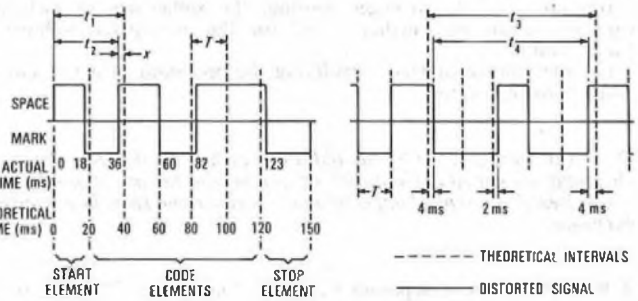
(c) What is the standard test message for voice-frequency telegraph channels, and why has it been chosen?

A 4 (*a*) Start-stop signal distortion is measured as the maximum difference between the actual and theoretical intervals separating any characteristic instant from the beginning of the *start* element immediately preceding it, expressed as a percentage of the duration of a unit element. Sketch (*a*) shows an example of a distorted start-stop character. It can be seen that the maximum difference, x milliseconds, occurs at the beginning of the second code-element, and that $x = t_1 - t_2$, where t_1 is the theoretical interval separating the beginning of the second code-element from the beginning of the *start* element (ms), and t_2 is the actual interval (ms). Hence, the start-stop distortion

$$= \frac{x}{T} \times 100\% = \frac{t_1 - t_2}{T} \times 100\%,$$

where T is the duration of a unit element (ms). In this example, the start-stop distortion

$$= \frac{40 - 36}{20} \times 100\% = \underline{20\%}.$$



(*a*) (*b*)

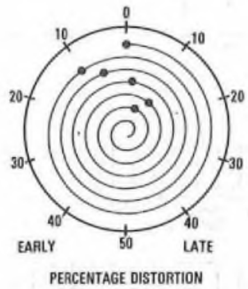
Isochronous distortion is measured as the sum of the differences between each of the earliest and latest transitions and their respective ideal transitions, expressed as a percentage of the duration of a unit element. Sketch (*b*) shows an example of a distorted isochronous signal. The vertical dashed lines represent the ideal transitions, spaced 20 ms apart. It can be seen that t_4 is the difference between the earliest transition and the latest transition (ms), and t_3 is the difference between the respective associated ideal transitions (ms). Hence, the sum of the differences between each of the earliest and latest transitions and their respective ideal transitions is given by $t_3 - t_4$ milliseconds. Therefore, in this example, isochronous distortion

$$= \frac{t_3 - t_4}{T} \times 100\%,$$

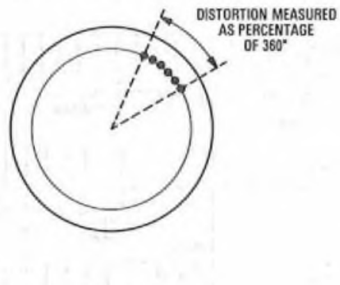
$$= \frac{100 - 92}{20} \times 100\% = \underline{40\%}.$$

(*b*) A telegraph-distortion measuring set is used to determine the amount of distortion present in a telegraph signal. The set consists of a cathode-ray tube with control circuits, such that the transitions, or instants of modulation, are displayed as bright dots on the face of the tube.

For start-stop working, an oscillator is held inoperative until the leading edge of a *start* element is received. The oscillator controls a time base which causes the beam to scan the face of the tube in a diminishing spiral, with each revolution taking 20 ms, starting at the top centre. Transitions occurring during the character cause a bright spot to be displayed at the instant the transition is received. If the signals are without distortion, the dots are displayed vertically beneath the *start* dot. Sketch (*c*) illustrates a display of alternate *mark* and *space* signals having start-stop distortion.

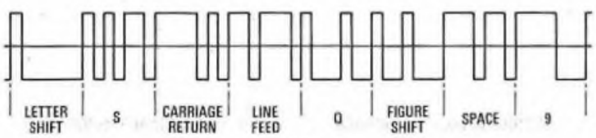


(*c*)



(*d*)

For the measurement of isochronous distortion, each transition causes a bright spot to appear on a circular trace. The oscillator is free-running, and is manually adjusted so that one revolution of the trace is equal to one signal element (20 ms for 50 baud circuits). If the incoming signals are undistorted, they arrive at the same time in each revolution, or multiple of a revolution, and the flashes always occur at the same position on the screen. If the signals are distorted, the distance between the earliest and latest transition is a measure of the isochronous distortion, as illustrated in sketch (*d*). In practice, the time-base rate is adjusted to match the speed of the incoming signal as closely as possible, so that the display remains stationary, although the dots vary in position.



(*e*)

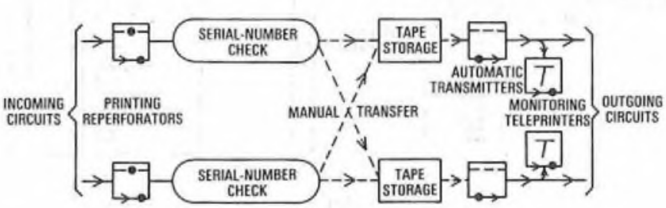
(*c*) The standard test message for voice-frequency telegraph channels is the Q9S signal, which consists of the characters *letter shift*, S, *carriage return*, *line feed*, Q, *figure shift*, *space*, 9 transmitted in that order. Each character consists of 7 elements, the *stop* element being the length of one character element. The message is sent at cadence speed, and successive messages follow each other without a break. This message has been chosen because it contains a wide variety of combinations of long and short positive and negative pulses, as illustrated in sketch (*e*), which ensure that the channel is adequately tested.

Q 5 (*a*) *With the aid of block diagrams, describe the routing of a message through*

(*i*) *a manual message-relay centre, and*
(*ii*) *an automatic message-relay centre.*

(*b*) *How would a multi-address message be processed in each type of centre?*

A 5 (*a*) (*i*) The sketch illustrates the routing of a message through a manual message-relay centre. The incoming message is received on a printing reperforator, which perforates a tape according to the incoming signals and prints the message on the tape. The message normally bears a heading which includes the destination address and a serial number particular to the circuit. The operator checks the serial number, usually by means of a tick sheet, to verify that no message has been lost. The message is then torn from the tape and carried across the office to the automatic transmitter serving the outgoing circuit. If the required channel is in use, the tape is left in a tape-storage rack for transmission when the circuit is free. Monitoring teleprinters give a record of all messages sent, so that lost or mutilated messages can be retransmitted on request.



(*ii*) For the routing of a message through, and the processing of a multi-address message in, an automatic message-relay centre, see A7, Telegraphy C, 1971, *Supplement*, Vol. 65, p. 36, July 1972.

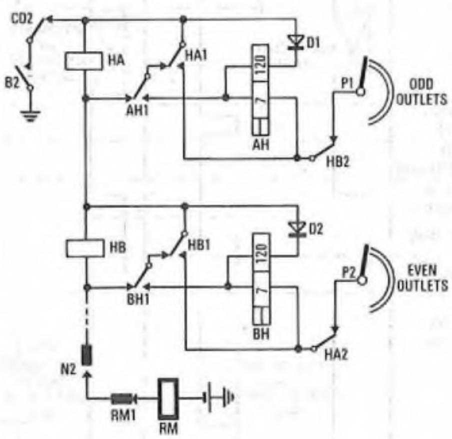
(*b*) For multi-address or broadcast messages, a *tape factory* is used. This consists of an automatic transmitter feeding up to about 6 printing reperforators through a key-selection panel. The message to be duplicated is placed in the transmitter and the appropriate number of keys operated to connect the printing reperforators. When the transmitter is started, the correct number of tapes is produced, and each of these is carried to the channel transmitter corresponding to one of the multi-address circuits, and is transmitted as a normal single-address message.

Q 6 (*a*) *Draw a circuit diagram, and describe the operation of, a 200-outlet group selector when hunting rotarily if, on the first rotary step,*

(*i*) *both outlets are engaged, and*
(*ii*) *both outlets are free.*

(*b*) *Explain the facilities offered by the selector if all the outlets on a level are engaged.*

A 6 (*a*) (*i*) The sketch shows the circuit elements of a group selector used in rotary stepping. When vertical stepping has finished, relay CD releases, connecting earth potential to energize rotary magnet RM through contacts B2 (operated), CD2, HA1, AH1, HB1, BH1, N2 (operated), and RM1. When magnet RM operates, the wipers are moved to the first bank contact; contact RM1 also operates and disconnects magnet RM. If the first 2 outlets are engaged, earth potential is encountered on the first contacts of arcs P1 and P2, and relays AH and BH do not operate. Interaction between magnet RM and contact RM1 causes the selector to step from the first contact to the second, and hunting continues until a free outlet is found.



(*ii*) If both contacts are free, a 550 Ω resistive 50 V potential is present on the first contacts of arcs P1 and P2, and relays AH and BH are energized. When relay AH operates, contact AH1 disconnects the circuit of magnet RM and prevents further rotary movement. In addition, this contact effectively disconnects relay BH by virtue of the resistance of relay HA, removes the short-circuit from relay HA, and short-circuits the 120 Ω coil of relay AH. The connexion of earth potential through the 7 Ω coil of relay AH guards the P-wire from seizure by another selector until relay HA has operated, as the reduced potential on the P-wire is then insufficient to allow a second testing relay to operate. After a short delay, relay HA operates to connect the SEND and RECEIVE wires to the next stage; contact HA1 connects a full earth potential to guard the P-wire and release relay AH. To prevent relay HA releasing when contact B2 opens, contact HA1 holds relay HA to the earth potential returned on the P-wire from the subsequent selector, when this has been seized.

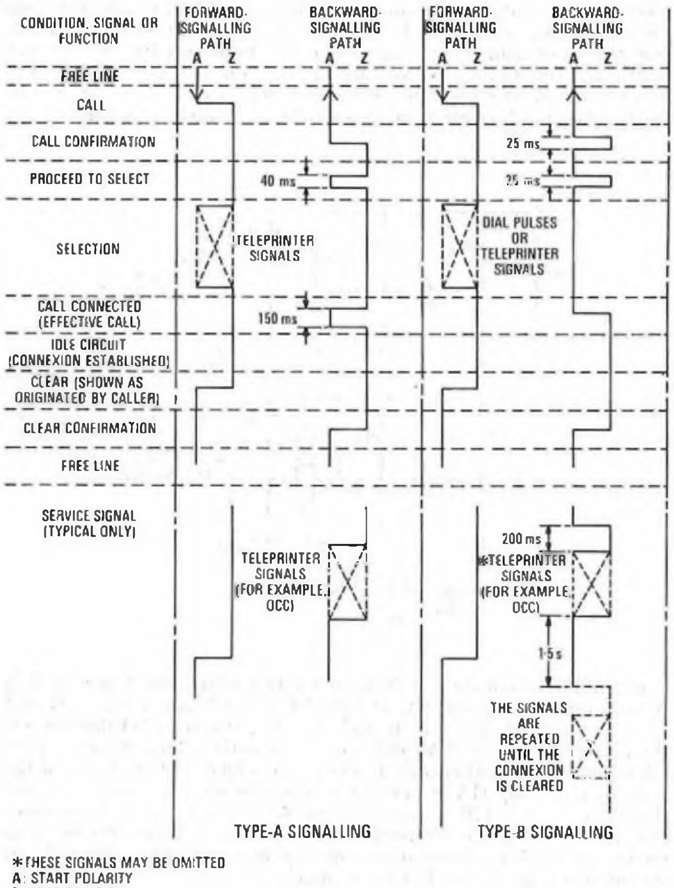
Diode D1 acts as an aid to the prevention of double-switching, as its forward resistance when switched to a free outlet is 80 Ω. If the outlet has already been seized and a 7 Ω earth potential applied, the reduced current flowing through diode D1 increases its forward resistance to 1 kΩ, thus preventing the operation of relay AH.

(*b*) If all the outlets on one level are engaged, the wipers step to the eleventh outlet, and the S-springs in the selector operate. A 550 Ω resistive 50 V potential is connected to the eleventh contact on arc P1, so that relay AH operates and prevents magnet RM from operating further. The S-springs operate an overflow meter, start the service-signal generator if necessary, connect a *call-connected* signal to the calling station to prepare for the reception of the service signal, and connect the service signal NC (no circuits) to the SEND wire for transmission to the calling station. The service signal is followed by positive potential, which acts as a clearing signal to release the call. If the level were unallocated, the service signal NP (no path) would be used.

Q 7 (a) Describe the methods of international Telex signalling known as Type-A, Type-B and Type-C.

(b) Draw the trunking diagram of an international exchange, showing how an inland subscriber may gain access to international routes.

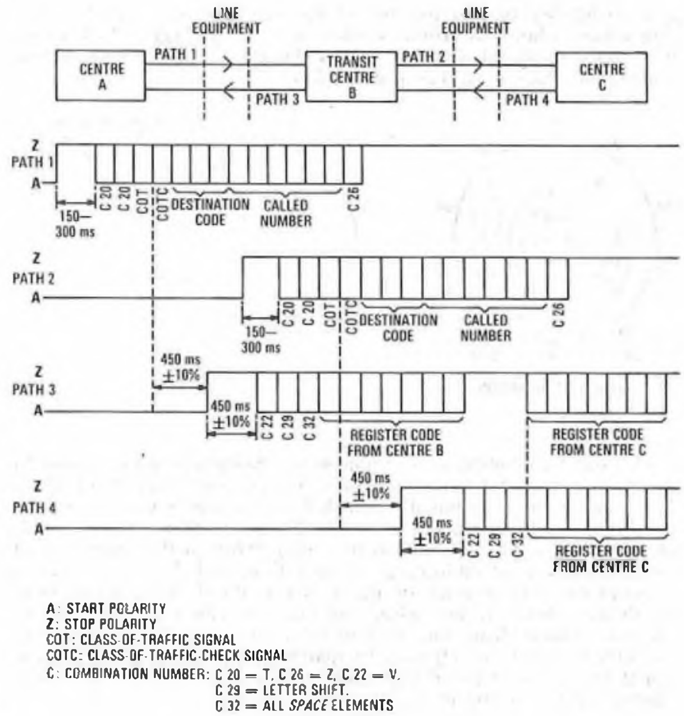
A 7 (*a*) Sketch (*a*) shows the conventions recommended by the CCITT for Type-A and Type-B signalling. Type-A signalling requires the transmission of *stop* polarity on the backward path whilst the call is being set up. This permits teleprinter signals to be transmitted to the caller to indicate the progress of the call; for example, the register code or the called-subscriber's answer-back code. The selection signals in the forward direction consist of teleprinter characters. These are suitable for register systems, since teleprinter characters are stored and translated as readily as dial pulses. The use of teleprinter signals for both directions of transmission whilst establishing a call facilitates the use of all types of synchronous multiplex systems.



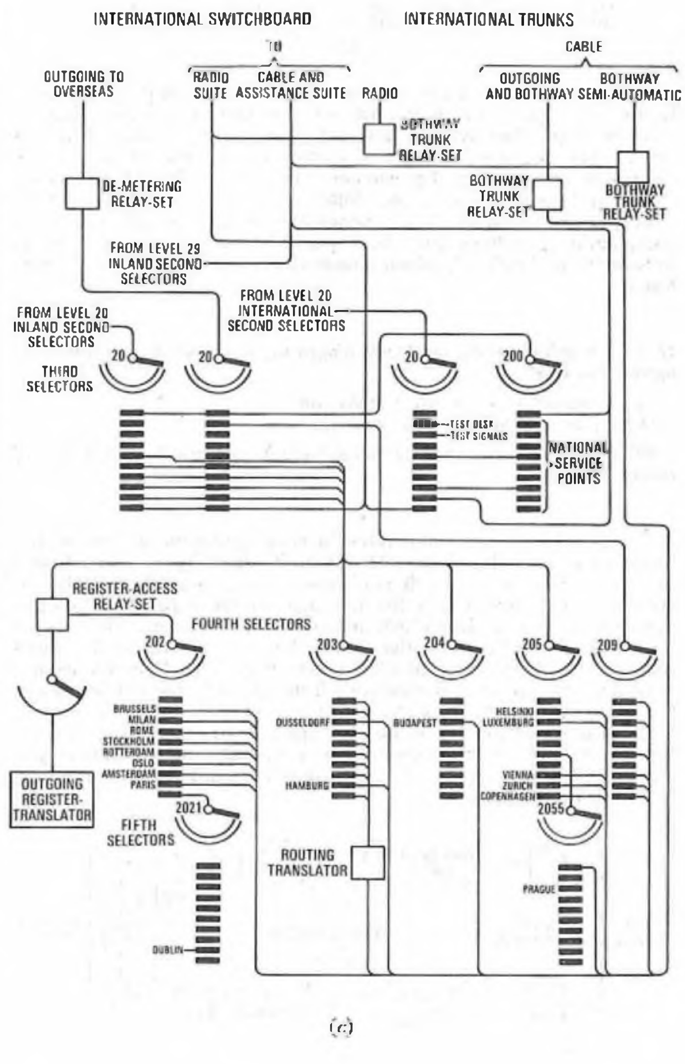
(*a*)

Selection signals for Type-B systems may consist of dial pulses or teleprinter signals. The use of dial pulses allows direct selection, whereby the dial pulses directly operate the selectors at the called exchange. The pulsed call-confirmation signal reduces the possibility of double-seizure or lock-up on bothway circuits.

To eliminate disadvantages associated with Type-A and Type-B signalling, a standardized signalling system, known as Type-C has been adopted. Sketch (*b*) shows a typical exchange of signals via a transit centre during the establishment of a call using Type-C signalling. This system is a fast signalling system using teleprinter signals for all selection and supervisory purposes. Calling and selection signals



(*b*)



(*c*)

consist of a group of characters, and are passed to the next exchange as soon as possible in the stage of the call, until the last tandem exchange is reached. The free-line condition is *start* (*space*) polarity in both directions of transmission. The calling signal consists of an inversion to *stop* (*mark*) polarity for 150–300 ms followed by 2 successive character combinations No. 20 (letter T) and the selection signals. The *class-of-traffic* signal indicates the speed of transmission, the type of call and whether the call has been subject to alternative routing. The *class-of-traffic-check* signal is merely the complement of the *class-of-traffic* signal in having each of the 5 code elements reversed in polarity. The destination code and subscriber's number are limited to a total of 12 characters to ensure that fast forward switching begins as soon as enough digits have been received to determine the route. The *reception-confirmation* signal consists of inversion of the backward-path polarity for 450 ms followed by character combination No. 22 (letter V). Combination Nos. 29 (*letter shift*) and 32 (all *space* elements) together make up the *transmission-confirmation* signal which ensures that the transmission path is satisfactory, and this is followed by the register-code signals.

Alternative backward-path signals can be transmitted to indicate transmission failure, register congestion or dual seizure.

(*b*) Sketch (*c*) shows the trunking diagram of an international exchange.

Q 8 (*a*) *Describe the formation of an error-detecting code suitable for use with Telex radio circuits.*

(*b*) *With the aid of a block diagram, describe the operation of equipment using such a code as part of an error-correcting system.*

Q 9 (*a*) *List the service signals which are provided in a Telex exchange, and explain the meaning of each.*

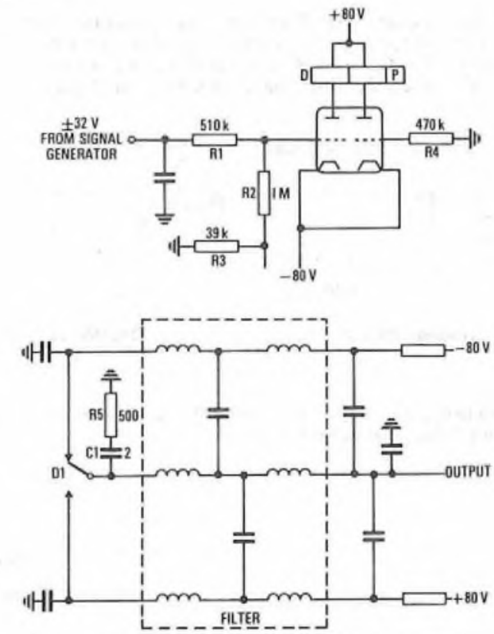
(*b*) *From which items of switching equipment is each service signal connected to the line?*

(*c*) *With the aid of a circuit diagram, describe how the service signals are distributed, explaining the precautions taken to prevent overloading of the signal generator.*

A 9 (*a*) and (*b*) The service signals provided in a Telex exchange, and the equipment that connects the signals to line, are shown in the table.

Signal	Meaning	Connected By
OCC	Subscriber engaged	Group selector, final selector or time-zone equipment
NC	Trunk engaged	Group selector
NP	Spare line or level	Group selector or time-zone equipment
DER	Subscriber's line faulty	Time-zone equipment
ABS	Station closed	Final selector or time-zone equipment
MOM	Wait	Switchboard or service levels
WRU	Who are you? (request for answer-back code)	Time-zone equipment

(*c*) The sketch shows the service-signal distribution circuit. The service signals are fed from the generator to the grid circuit of the double-triode valve; the input impedance of this circuit is high to prevent an excessive current drain on the generator. The ± 32 V input signals, reduced to ± 21 V by the potential divider formed by resistors R1, R2 and R3, are applied to the left-hand grid of the valve. The right-hand grid is maintained at earth potential via resistor R4. When the input signal is -21 V, the right-hand triode conducts, energizes the right-hand coil of relay D, and contact D1 connects -80 V to the output. When the input signal is $+21$ V, the left-hand triode conducts, relay D changes over, and contact D1 connects $+80$ V to the output.



In the output circuit, the spark-quench circuit, comprising resistor R5 and capacitor C1, is provided to prevent excessive sparking which would damage contact D1. This contact feeds up to 40 lines. The filter acts as a radio-frequency suppressor.

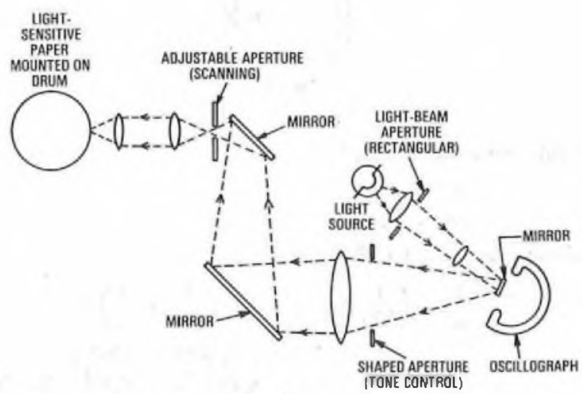
Bulb resistors (not shown) are provided in each output lead to limit the current under fault conditions.

Q 10 (*a*) *With the aid of sketches, describe the operation of a photo-telegraph receiver.*

(*b*) *What changes must be made to the receiver when receiving a picture in the form of a print as compared with the receipt of a negative?*

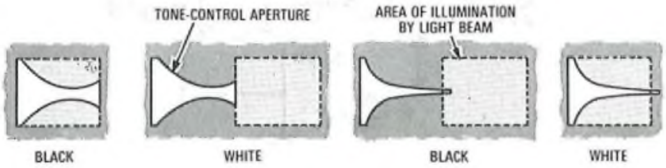
(*c*) *How is the received picture affected if the transmitter and receiver are not in synchronism?*

A 10 (*a*) Sketch (*a*) shows the optical system of a photo-telegraph receiver. A light beam is focused on light-sensitive paper mounted on a drum, as the drum is rotated and moved laterally by a lead screw. The light beam traces a spiral path over the surface of the drum in synchronism with the transmitter, which applies signals to line depending on the tonal quality of the picture to be transmitted. At the receiver, the varying current is applied to an oscillograph which consists of a tensioned loop of fine wire suspended in a magnetic field. The current causes the oscillograph to move depending on the value of the current, and a mirror attached to the movement reflects the light from the lamp. The reflected beam is directed through a specially-shaped tone-control aperture which limits the amount of light allowed to fall on the drum.



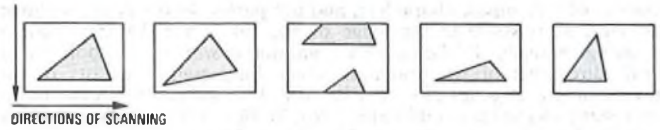
(*a*)

(*b*) Two different shapes of tone-control aperture are used, one for positive or print reception, and the other for negative reception. The 2 shapes are illustrated in sketch (*b*), and arise because the reaction of the emulsion used for the light-sensitive paper bears a logarithmic relationship to the light exposure. If the minimum received current represents *black*, and a print is required, then the minimum current signal must represent maximum light passing the aperture. With this current applied to the oscillograph, the mirror is moved so that the light beam fully illuminates the aperture. For maximum *white* current, the mirror is deflected so that the light beam completely misses the aperture. For negative reception, these conditions are reversed. Sketch (*b*) shows the positions of the light beam for each case.



(*b*)

(*c*) Sketches (*c*)–(*g*) illustrate the effects of the transmitter and receiver not being in synchronism.



(*c*) (*d*) (*e*) (*f*) (*g*)

Sketch (*c*) shows the original picture. The effect of a constant speed-error between the transmitter and receiver is shown in sketch (*d*). Incorrect phasing (that is, the transmitting and receiving scanners not being in the same relative position with regard to the picture and the recording blank) leads to a split reproduction, as shown in sketch (*e*). The effect of an incorrect index of co-operation is shown in sketches (*f*) and (*g*). The index of co-operation is the ratio of the length to the breadth of the picture, and should be the same at the transmitter and receiver. Sketch (*f*) shows the effect when the index of co-operation of the receiver is less than that of the transmitter, and sketch (*g*) shows the effect when it is greater.

MATHEMATICS C, 1974

Students were expected to answer any 6 questions

Q 1 Solve for z, to slide-rule accuracy, the simultaneous equations:

$$3{\cdot}2x - 7{\cdot}8y + 1{\cdot}2z = 5{\cdot}2,$$
$$1{\cdot}8x - 3{\cdot}7z = 7{\cdot}1,$$
$$5{\cdot}2x + 1{\cdot}8y - 1{\cdot}2z = 3{\cdot}9.$$

A 1 Answer: $\underline{z = -1{\cdot}624}$.

Q 2 For the network shown in Fig. 1, the open-circuit impedance, Z_{oc}, and the short-circuit impedance, Z_{sc}, are given by the equations

$$\frac{1}{Z_{oc}} = \frac{1}{2Z_2} + \frac{1}{Z_1 + 2Z_2} \quad \text{and} \quad \frac{1}{Z_{sc}} = \frac{1}{2Z_2} + \frac{1}{Z_1}.$$

(a) If $Z_0{}^2 = Z_{oc}Z_{sc}$, show that

$$\frac{1}{Z_0{}^2} = \frac{1}{4Z_2{}^2} + \frac{1}{Z_1Z_2}.$$

(b) When $Z_1 = 1/j\omega C$ and $Z_2 = j\omega L$, show that
(i) Z_0 is real if $2\omega > 1/\sqrt{(LC)}$, and
(ii) as ω approaches infinity, Z_0 approaches the value $\sqrt{(L/C)}$.

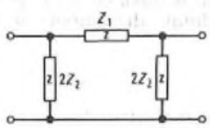


Fig. 1

A 2 (*a*) Now, $Z_0{}^2 = Z_{oc}Z_{sc}$.

$$\therefore \frac{1}{Z_0{}^2} = \frac{1}{Z_{oc}Z_{sc}}.$$

Substituting for $1/Z_{oc}$ and $1/Z_{sc}$ gives

$$\frac{1}{Z_0{}^2} = \left(\frac{1}{2Z_2} + \frac{1}{Z_1 + 2Z_2}\right)\left(\frac{1}{2Z_2} + \frac{1}{Z_1}\right),$$
$$= \left(\frac{Z_1 + 2Z_2 + 2Z_2}{2Z_2(Z_1 + 2Z_2)}\right)\left(\frac{Z_1 + 2Z_2}{2Z_2Z_1}\right),$$
$$= \frac{Z_1 + 4Z_2}{4Z_1Z_2{}^2},$$
$$= \underline{\frac{1}{4Z_2{}^2} + \frac{1}{Z_1Z_2}}. \qquad \text{QED.}$$

(*b*) (*i*) From part (*a*),

$$Z_0{}^2 = \frac{4Z_1Z_2{}^2}{Z_1 + 4Z_2}.$$

When $Z_1 = 1/j\omega C$ and $Z_2 = j\omega L$,

$$Z_0{}^2 = \frac{4\left(\frac{1}{j\omega C}\right)(j\omega L)^2}{\frac{1}{j\omega C} + 4j\omega L},$$
$$= \frac{-4\omega^2L^2}{1 - 4\omega^2LC},$$
$$= \frac{4\omega^2L^2}{4\omega^2LC - 1}.$$
$$\therefore Z_0 = \frac{2\omega L}{\sqrt{(4\omega^2LC - 1)}}.$$

For Z_0 to be real, the denominator must be real. Hence,

$$4\omega^2LC > 1.$$
$$\therefore 4\omega^2 > \frac{1}{LC}.$$
$$\therefore \underline{2\omega > \frac{1}{\sqrt{(LC)}}}. \qquad \text{QED.}$$

(*ii*) From part (*b*) (*i*),

$$Z_0 = \frac{2\omega L}{\sqrt{(4\omega^2LC - 1)}}.$$

As ω approaches infinity, the term $4\omega^2LC$ also approaches infinity. Hence, $4\omega^2LC \gg 1$, and the expression becomes

$$Z_0 \simeq \frac{2\omega L}{\sqrt{(4\omega^2LC)}},$$
$$= \underline{\sqrt{(L/C)}}. \qquad \text{QED.}$$

Q 3 (a) Expand $(1 + x)^{-1/2}$ by the binomial series as far as the term in x^4, and give the $(r + 1)$th term in this expansion.
(b) The period of oscillation, T, of a mass, m, suspended by a helical spring of stiffness s. is given by the formula $T = k\sqrt{(m/s)}$, where k is a constant. Using the expansion obtained in part (a), or otherwise, find, to 2 significant figures, the percentage error in calculating T if there is a $+8{\cdot}0\%$ error in estimating s.

A 3 (*a*) From the binomial series,

$$(1 + x)^{-1/2} = 1 - \tfrac{1}{2}x + \frac{(-\frac{1}{2})(-\frac{3}{2})x^2}{1 \times 2} + \frac{(-\frac{1}{2})(-\frac{3}{2})(-\frac{5}{2})x^3}{1 \times 2 \times 3}$$
$$+ \frac{(-\frac{1}{2})(-\frac{3}{2})(-\frac{5}{2})(-\frac{7}{2})x^4}{1 \times 2 \times 3 \times 4} + \ldots,$$
$$= 1 - \frac{x}{2} + \frac{3x^2}{2^2 \times 2!} - \frac{(3 \times 5)x^3}{2^3 \times 3!}$$
$$+ \frac{(3 \times 5 \times 7)x^4}{2^4 \times 4!} - \ldots,$$

$$= 1 - \frac{x}{2} + \frac{3x^2}{8} - \frac{5x^3}{16} + \frac{35x^4}{128} - \ldots$$

The $(r+1)$th term of the expansion is

$$(-1)^r \times \frac{3 \times 5 \times 7 \ldots \times (2r-1)x^r}{2^r \times r!}.$$

(*b*) $$T = k\sqrt{\left(\frac{m}{s}\right)}.$$

If T' is the calculated value of T when there is a $+8{\cdot}0\%$ error in the value of s, then

$$T' = k\sqrt{\left(\frac{m}{s'}\right)},$$

where s' is the estimated value of s, and is given by

$$s' = s + \frac{8}{100}s.$$

$$\therefore\ T' = k\sqrt{\left\{\frac{m}{s(1+0{\cdot}08)}\right\}} = T(1+0{\cdot}08)^{-1/2}.$$

From part (*a*),

$$(1+0{\cdot}08)^{-1/2} = 1 - \frac{0{\cdot}08}{2} + \frac{3 \times 0{\cdot}08^2}{8} - \ldots,$$

$$= 1 - 0{\cdot}04 + 0{\cdot}0024 - \ldots,$$

$$\simeq 0{\cdot}9624.$$

$$\therefore\ T' = 0{\cdot}9624T.$$

Thus, percentage error in calculating T

$$= \frac{T'-T}{T} \times 100\%,$$

$$= \left(\frac{T'}{T} - 1\right) \times 100\%,$$

$$= (0{\cdot}9624 - 1) \times 100\%,$$

$$= -3{\cdot}8\%$$ to 2 significant figures.

Q 4 (a) By sketching a graph or graphs, show that there is only one root of the equation

$$8e^{-2x} - 3x = 2.$$

(b) By enlargement, or otherwise, find this root approximately, giving its value to 2 significant figures.

A 4 (*a*) Now, $8e^{-2x} - 3x = 2$,

or $8e^{-2x} = 3x + 2$.

Any real roots of the equation $8e^{-2x} - 3x = 2$ will be given by the intersection of the graphs of $y = 8e^{-2x}$ and $y = 3x + 2$.

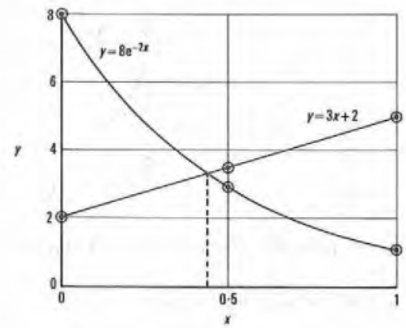
When $x = 0$, the value of $8e^{-2x}$ is 8, and that of $3x + 2$ is 2. When $x = 1$, the value of $8e^{-2x}$ falls to approximately unity, whereas that of $3x + 2$ becomes 5. Thus, it is apparent that the graphs will intersect between $x = 0$ and $x = 1$.

Therefore, the graphs are plotted from the following table and are shown in sketch (*a*).

x	0	0·5	1
e^{-2x}	1	0·3679	0·1353
$y = 8e^{-2x}$	8	2·94	1·08
$y = 3x + 2$	2	3·5	5

From sketch (*a*), it can be seen that the graphs intersect at about $x = 0{\cdot}45$. Thus, the equation $8e^{-2x} - 3x = 2$ has a real root at $x \simeq 0{\cdot}45$.

As x decreases below zero, $8e^{-2x}$ becomes increasingly large, whereas $3x + 2$ decreases linearly towards $-\infty$. Hence, the graphs cannot intersect below $x \simeq 0{\cdot}45$. As x increases above $0{\cdot}45$, $e^{-2x} \to 0$ and, hence, the graph of $y = 8e^{-2x}$ becomes asymptotic to the x axis.



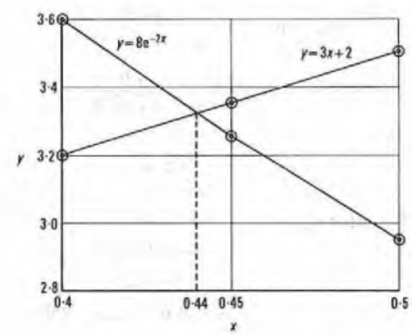
(*a*)

Since $y = 3x + 2$ increases linearly, the graphs do not intersect above $x \simeq 0{\cdot}45$.

Hence, there is only one real root of the equation $8e^{-2x} - 3x = 2$.

(*b*) The root of the equation is more accurately found by drawing an enlargement of the graphs over the range, say, $x = 0{\cdot}4$ to $x = 0{\cdot}5$. The graphs are plotted from the following table and are shown in sketch (*b*).

x	0·4	0·45	0·5
e^{-2x}	0·4493	0·4066	0·3679
$y = 8e^{-2x}$	3·5946	3·2526	2·9430
$y = 3x + 2$	3·2	3·35	3·5



(*b*)

From sketch (*b*), it can be seen that the graphs intersect at $x \simeq 0{\cdot}44$. Thus, the root of the equation is $x = 0{\cdot}44$, to 2 significant figures.

Note: In sketch (*b*), the plotted points of the curve $y = 8e^{-2x}$ are joined by straight lines because the actual departure of the curve from linearity over such a restricted range is very small.

Q 5 (a) Convert the Cartesian equation of the curve

$$x^2 + y^2 = 4x$$

into an equation in polar co-ordinates, taking the origin, O, of the Cartesian system to be the pole and the x-axis to be the initial line $\theta = 0$.

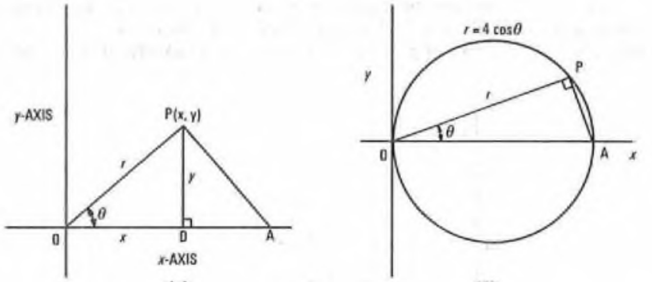
(b) If A is the point on the curve where $\theta = 0$, and P is any other poin on the curve, show that $\angle OPA = 90°$.

(c) Sketch the curve and name it.

(d) Show that, at P,

$$\frac{d^2r}{d\theta^2} + r = 0.$$

A 5 (*a*) If any point, P, on a curve has Cartesian co-ordinates x,y and polar co-ordinates r,θ, then $x = r\cos\theta$ and $y = r\sin\theta$, as illustrated in sketch (*a*).



(a) (b)

Hence, substituting polar for Cartesian co-ordinates in the equation $x^2 + y^2 = 4x$ gives

$$r^2 \cos^2 \theta + r^2 \sin^2 \theta = 4r \cos \theta.$$

$$\therefore\ r^2(\cos^2 \theta + \sin^2 \theta) = 4r \cos \theta.$$

$$\therefore\ r^2 = 4r \cos \theta.$$

$$\therefore\ \underline{r = 4 \cos \theta.}$$

(*b*) With reference to sketch (*a*), if the line PD is drawn perpendicular to the x-axis, then

$$OD = x = r \cos \theta \text{ and } PD = y = r \sin \theta.$$

In triangle OPD, $\angle$OPD and θ are complementary and, hence, $\tan \angle OPD = \cot \theta$.

Also, in triangle PAD,

$$\tan \angle PAD = \frac{PD}{DA},$$

$$= \frac{y}{4 - x},$$

since, when $\theta = 0$, OA $= 4 \cos \theta = 4$.

Therefore, substituting for x and y gives

$$\tan \angle PAD = \frac{r \sin \theta}{4 - r \cos \theta},$$

$$= \frac{r \sin \theta}{4 - 4 \cos^2 \theta},$$

since $r = 4 \cos \theta$,

$$= \frac{r \sin \theta}{4 \sin^2 \theta},$$

since $1 - \cos^2 \theta = \sin^2 \theta$,

$$= \frac{4 \cos \theta \sin \theta}{4 \sin^2 \theta},$$

$$= \cot \theta = \tan \angle OPD.$$

Therefore, $\angle$PAD $= \angle$OPD, and triangles OPD and POA are similar.

$$\therefore\ \underline{\angle OPA = \angle PDO = 90°.} \qquad \text{QED.}$$

(*c*) The curve is shown in sketch (*b*), and is a circle.

(*d*) Since P is any point on the curve, at P,

$$r = 4 \cos \theta.$$

$$\therefore\ \frac{dr}{d\theta} = -4 \sin \theta.$$

$$\therefore\ \frac{d^2r}{d\theta^2} = -4 \cos \theta.$$

$$\therefore\ \frac{d^2r}{d\theta^2} + r = -4 \cos \theta + 4 \cos \theta = 0. \qquad \text{QED.}$$

Q 6 (a) State, without proof, the expansions of sin (A + B) and cos (A + B) in terms of the trigonometrical ratios of angles A and B.

(b) Prove that sin 3θ = 3 sin θ − 4 sin³ θ.

(c) Sketch the waveform v = 5 cos ωt − 2 cos 3 ωt from t = −π/ω to t = + π/ω. Calculate the peak value of this waveform.

A 6 (*a*) $\underline{\sin (A + B) = \sin A \cos B + \cos A \sin B,}$

and $\underline{\cos (A + B) = \cos A \cos B - \sin A \sin B.}$

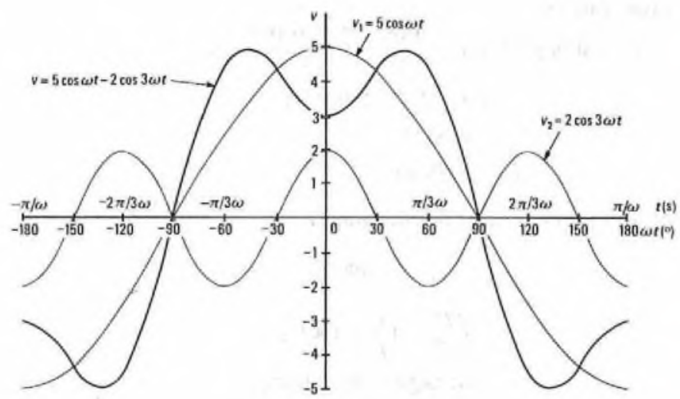
(*b*)
$$\sin 3\theta = \sin (\theta + 2\theta),$$
$$= \sin \theta \cos 2\theta + \cos \theta \sin 2\theta.$$

But
$$\cos 2\theta = \cos^2 \theta - \sin^2 \theta,$$
$$= 1 - 2 \sin^2 \theta,$$

and
$$\sin 2\theta = 2 \sin \theta \cos \theta.$$

$$\therefore\ \sin 3\theta = \sin \theta (1 - 2 \sin^2 \theta) + \cos \theta \times 2 \sin \theta \cos \theta,$$
$$= \sin \theta - 2 \sin^3 \theta + 2 \sin \theta \cos^2 \theta,$$
$$= \sin \theta - 2 \sin^3 \theta + 2 \sin \theta (1 - \sin^2 \theta),$$
$$= \sin \theta - 2 \sin^3 \theta + 2 \sin \theta - 2 \sin^3 \theta,$$
$$= \underline{3 \sin \theta - 4 \sin^3 \theta.} \qquad \text{QED.}$$

(*c*) The waveform $v = 5 \cos \omega t - 2 \cos 3\omega t$ comprises 2 components, one of frequency $\omega/2\pi$ and the other of $3\omega/2\pi$. The waveform can most easily be drawn by separately sketching the 2 cosine curves, $v_1 = 5 \cos \omega t$ and $v_2 = 2 \cos 3\omega t$, and subtracting values of v_2 from the corresponding values of v_1 for the same values of time to obtain the resultant curve, v.



The 3 curves are shown in the sketch, from which it is seen that one complete period of v_1 occurs between $t = -\pi/\omega$ and $+\pi/\omega$ whereas the curve of v_2, being 3 times the frequency of v_1, has 3 complete periods. The curve of the resultant, v, has the same frequency as v_1, since the amplitude of v_1 is greater than that of v_2.

Note: Although the curves shown are drawn accurately, students would be expected to produce only a freehand sketch, provided that the salient features were evident.

Now,
$$v = 5 \cos \omega t - 2 \cos 3\omega t.$$

$$\therefore\ \frac{dv}{dt} = -5\omega \sin \omega t + 6\omega \sin 3\omega t.$$

For maximum or minimum values of v, $dv/dt = 0$.

$$\therefore\ 5\omega \sin \omega t = 6\,\omega \sin 3\omega t.$$

But, from part (*b*), $\sin 3\omega t = 3 \sin \omega t - 4 \sin^3 \omega t$.

$$\therefore\ 5 \sin \omega t = 6\,(3 \sin \omega t - 4 \sin^3 \omega t).$$

$$\therefore\ 13 \sin \omega t = 24 \sin^3 \omega t.$$

$$\therefore\ \sin \omega t = 0,$$

or, dividing by $\sin \omega t$,

$$\sin^2 \omega t = \frac{13}{24},$$

whence
$$\sin \omega t = \pm 0{\cdot}7360.$$

$$\therefore\ \omega t = \pm 47° \, 23' \text{ or } \pm 132° \, 36'.$$

Hence, the peak value of the waveform is given by

$$v_{\text{peak}} = 5 \cos 47° \, 23' - 2 \cos (3 \times 47° \, 23'),$$
$$= 5 \times 0{\cdot}6771 - 2 \times (-0{\cdot}7896),$$
$$= \underline{4{\cdot}9647.}$$

Q 7 (*a*) *Differentiate the function*

$$y = \frac{2x+5}{(x-3)^3}$$

and simplify the result.

(*b*) *If* $y = \log_e(\cos x - \sin x)$,

(*i*) *state the range of* x*, within the limits* $-\pi/2 < x < \pi/2$*, for which* y *is real, and*

(*ii*) *show that* $dy/dx = -(\sec 2x + \tan 2x)$.

(*c*) *Find the maximum and minimum values of* $y = x^2e^{-2x}$*, and sketch the graph of this function.*

A 7 (*a*)
$$y = \frac{2x+5}{(x-3)^3}.$$

$$\therefore \frac{dy}{dx} = \frac{(x-3)^3 \times 2 - (2x+5) \times 3 \times (x-3)^2}{(x-3)^6},$$

$$= \frac{2(x-3) - 3(2x+5)}{(x-3)^4},$$

$$= -\frac{4x+21}{(x-3)^4}.$$

(*b*) (*i*) The logarithm of a negative number is a complex quantity and, hence, y is real only when $\cos x > \sin x$.

When $-\pi/2 < x < 0$, $\cos x$ is positive and $\sin x$ is negative. Hence, $\cos x > \sin x$ in this range.

When $0 < x < \pi/2$, $\cos x$ decreases from unity to zero at $x = \pi/2$, whereas $\sin x$ increases from zero to unity at $x = \pi/2$. Hence, in this range, $\cos x > \sin x$ up to the point where $\cos x = \sin x$; that is, where $\tan x = 1$, or $x = \pi/4$.

Therefore, the range of x for which y is real is from $-\pi/2$ to $+\pi/4$.

(*ii*)
$$y = \log_e(\cos x - \sin x).$$

$$\therefore \frac{dy}{dx} = \frac{1}{\cos x - \sin x} \times \frac{d(\cos x - \sin x)}{dx},$$

$$= -\frac{\sin x + \cos x}{\cos x - \sin x}. \qquad \ldots\ldots (1)$$

The right-hand side of the derivative given in the question

$$= -(\sec 2x + \tan 2x),$$

$$= -\left(\frac{1}{\cos 2x} + \frac{\sin 2x}{\cos 2x}\right),$$

$$= -\frac{1 + 2\sin x \cos x}{\cos^2 x - \sin^2 x},$$

$$= -\frac{\sin^2 x + \cos^2 x + 2\sin x \cos x}{\cos^2 x - \sin^2 x},$$

$$= -\frac{(\sin x + \cos x)^2}{(\cos x - \sin x)(\cos x + \sin x)},$$

$$= -\frac{\sin x + \cos x}{\cos x - \sin x},$$

= equation (1) above. QED.

(*c*)
$$y = x^2e^{-2x}.$$

$$\therefore \frac{dy}{dx} = x^2 \times \frac{d(e^{-2x})}{dx} + e^{-2x} \times \frac{d(x^2)}{dx},$$

$$= -2x^2e^{-2x} + 2xe^{-2x},$$

$$= 2e^{-2x}(x - x^2).$$

For maximum or minimum values, $dy/dx = 0$.

$$\therefore 2e^{-2x}(x - x^2) = 0.$$

$$\therefore e^{-2x} = 0 \text{ or } x - x^2 = 0.$$

For e^{-2x} to be zero, $x \to \infty$ and no maxima or minima can occur.

If $x - x^2 = 0$,

then $x = x^2$.

$$\therefore x = 0 \text{ or } 1.$$

Also,
$$\frac{d^2y}{dx^2} = 2e^{-2x} \times \frac{d(x-x^2)}{dx} + (x - x^2) \times \frac{d(2e^{-2x})}{dx},$$

$$= 2e^{-2x}(1 - 2x) + (x - x^2)(-4e^{-2x}),$$

$$= 2e^{-2x}(1 - 2x - 2x + 2x^2),$$

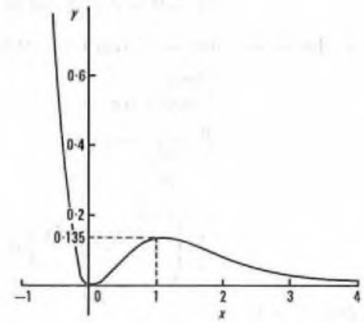
$$= 2e^{-2x}(1 - 4x + 2x^2).$$

When $x = 0$, $d^2y/dx^2 = 2$ and is positive. Hence, a minimum point must occur.

When $x = 1$, $d^2y/dx^2 = -2/e^2$ and is negative. Hence, a maximum point must occur.

Therefore, when $x = 0$, $y_{min} = 0$, and when $x = 1$, $y_{max} = e^{-2} = 0 \cdot 1353$.

The graph of the function is shown in the sketch, and is derived from the following table.



x	$-0 \cdot 5$	$-0 \cdot 1$	0	1	2	3	4
x^2	0·25	0·01	0	1	4	9	16
e^{-2x}	2·7183	1·2214	1	0·1353	0·0183	0·0025	0·0003
$y = x^2e^{-2x}$	0·6796	0·0122	0	0·1353	0·0733	0·0223	0·0054

Q 8 (*a*) *Evaluate*

(*i*) $\displaystyle\int_0^1 3e^{-2x}\,dx$,

(*ii*) $\displaystyle\int_0^{\pi/2\omega} \cos\left(\omega t + \frac{\pi}{6}\right) dt$, *and*

(*iii*) $\displaystyle\int_1^4 \left(x + \frac{1}{\sqrt{x}}\right)^2 dx$.

(*b*) *Find the r.m.s. value of* $y = \cos x$ *from* $x = 0$ *to* $x = 2\pi/3$.

A 8 (*a*) (*i*)
$$\int_0^1 3e^{-2x}\,dx = -\frac{3}{2}\left[e^{-2x}\right]_0^1,$$

$$= -\frac{3}{2}(e^{-2} - e^0),$$

$$= -\frac{3}{2}(0 \cdot 1353 - 1),$$

$$= \frac{3}{2} \times 0 \cdot 8647,$$

$$= 1 \cdot 297.$$

(*ii*)
$$\int_0^{\pi/2\omega} \cos\left(\omega t + \frac{\pi}{6}\right) dt = \left[\frac{\sin(\omega t + \pi/6)}{\omega}\right]_0^{\pi/2\omega},$$

$$= \frac{1}{\omega}\left\{\sin\left(\frac{\pi}{2} + \frac{\pi}{6}\right) - \sin\frac{\pi}{6}\right\},$$

$$= \frac{1}{\omega}\left(\frac{\sqrt{3}}{2} - \frac{1}{2}\right),$$

$$= \frac{0 \cdot 366}{\omega}.$$

(*iii*) $\displaystyle\int_1^4 \left(x + \frac{1}{\sqrt{x}}\right)^2 dx = \int_1^4 \left(x^2 + 2x^{1/2} + \frac{1}{x}\right) dx,$

$$= \left[\frac{x^3}{3} + \frac{2x^{3/2}}{3/2} + \log_e x\right]_1^4,$$

$$= \frac{1}{3}\{64 + 4 \times 8 + 3 \log_e 4 - (1 + 4 + 3 \log_e 1)\},$$

$$= \frac{1}{3}(91 + 3 \log_e 4),$$

$$= 30 \cdot \dot{3} + 1 \cdot 3863,$$

$$= \underline{31 \cdot 720} \text{ to 3 decimal places.}$$

(*b*) If $y = \cos x$, the mean value of y^2 from $x = 0$ to $x = 2\pi/3$

$$= \frac{\int_0^{2\pi/3} \cos^2 x \, dx}{\frac{2\pi}{3}},$$

$$= \frac{3}{2\pi} \int_0^{2\pi/3} \left(\frac{1 + \cos 2x}{2}\right) dx,$$

since $\cos 2x = 2 \cos^2 x - 1$,

$$= \frac{3}{4\pi}\left[x + \frac{\sin 2x}{2}\right]_0^{2\pi/3},$$

$$= \frac{3}{4\pi}\left\{\frac{2\pi}{3} + \frac{\sin (4\pi/3)}{2} - \left(0 + \frac{\sin 0}{2}\right)\right\},$$

$$= \frac{3}{4\pi}\left(\frac{2\pi}{3} - \frac{\sqrt{3}}{4}\right),$$

$$= 0 \cdot 5 - \frac{3 \times \sqrt{3}}{16\pi},$$

$$= 0 \cdot 5 - 0 \cdot 1034,$$

$$= 0 \cdot 3966.$$

$$\therefore\ y_{\text{r.m.s.}} = \sqrt{0 \cdot 3966},$$

$$= \underline{0 \cdot 6298}.$$

Q 9 (a) Three ordinates, y_1, y_2 and y_3, are equally spaced a distance h apart as shown in Fig. 2. Taking the y_2 ordinate as the y-axis, a parabola, $y = ax^2 + bx + c$, is drawn to pass through points A, B and C. Prove that the area bounded by ordinates y_1 and y_3, the x-axis and this parabola is given by the expression

$$\frac{h}{3}(y_1 + 4y_2 + y_3).$$

(b) Use Simpson's rule with 5 ordinates to evaluate the integral

$$\int_0^{\pi/3} \tan x \, dx.$$

Check your result by exact integration.

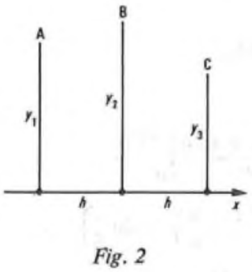
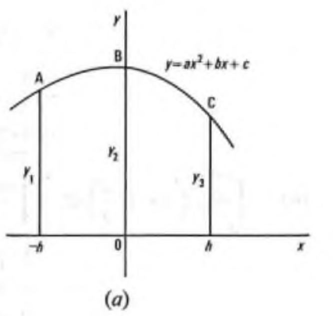


Fig. 2



(*a*)

A 9 (*a*) The parabola $y = ax^2 + bx + c$ is shown in sketch (*a*), with the y_2 ordinate as the y-axis. The abscissa corresponding to ordinate y_1 is then $-h$, and that corresponding to ordinate y_3 is $+h$.

The area under the curve between ordinates y_1 and y_3

$$= \int_{-h}^{h} (ax^2 + bx + c) \, dx,$$

$$= \left[\frac{ax^3}{3} + \frac{bx^2}{2} + cx\right]_{-h}^{h},$$

$$= \frac{ah^3}{3} + \frac{bh^2}{2} + ch - \left(-\frac{ah^3}{3} + \frac{bh^2}{2} - ch\right),$$

$$= \frac{2ah^3}{3} + 2ch. \qquad \ldots\ldots (1)$$

Substituting the co-ordinates of points A, B and C respectively into the equation $y = ax^2 + bx + c$ gives the following 3 equations:

$$y_1 = ah^2 - bh + c, \qquad \ldots\ldots (2)$$

$$y_2 = c, \qquad \ldots\ldots (3)$$

and $\quad y_3 = ah^2 + bh + c. \qquad \ldots\ldots (4)$

Subtracting equation (2) from equation (4) gives

$$y_3 - y_1 = 2bh.$$

$$\therefore\ b = \frac{y_3 - y_1}{2h}. \qquad \ldots\ldots (5)$$

Substituting for b and c, from equations (5) and (3) respectively, in equation (2) gives

$$y_1 = ah^2 - \frac{y_3 - y_1}{2} + y_2.$$

$$\therefore\ ah^2 = \frac{y_1}{2} - y_2 + \frac{y_3}{2}.$$

$$\therefore\ a = \frac{1}{2h^2}(y_1 - 2y_2 + y_3). \qquad \ldots\ldots (6)$$

Substituting for a and c, from equations (6) and (3) respectively, in equation (1) gives the area under the curve

$$= \frac{2h^3}{3} \times \frac{1}{2h^2}(y_1 - 2y_2 + y_3) + 2y_2h,$$

$$= \frac{h}{3}(y_1 - 2y_2 + y_3 + 6y_2),$$

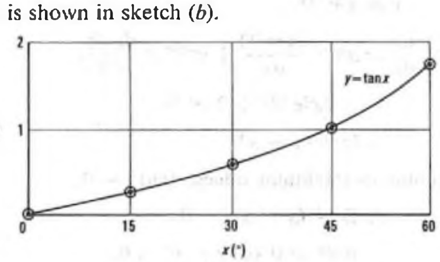
$$= \underline{\frac{h}{3}(y_1 + 4y_2 + y_3)}. \qquad \text{QED.}$$

(*b*) The integral $\int_0^{\pi/3} \tan x \, dx$ gives the area under the curve $y = \tan x$ between the limits $x = 0$ and $x = \pi/3$ rad (60°).

The use of 5 ordinates requires the division of 60° into 4 equal intervals of 15°. The corresponding values of $\tan x$ for each ordinate, obtained from a table of tangents, are given in the following table.

$x°$	0	15	30	45	60
$\tan x$	0	0·2679	0·5774	1·0000	1·7321

The curve is shown in sketch (*b*).



(*b*)

Simpson's rule states that the area under a curve is given by

$\frac{1}{3}$ (width of an interval) × [(sum of the first and last ordinates)
+ 2(sum of the remaining odd ordinates)
+ 4 (sum of the even ordinates)].

Using the tabulated values for the curve in sketch (*b*) gives the area under the curve between $x = 0$ and $x = 60°$ as

$$\frac{1}{3} \times \frac{1}{4} \times \frac{\pi}{3} \times \{0 + 1{\cdot}7321 + 2 \times 0{\cdot}5774 + 4\,(0{\cdot}2679 + 1{\cdot}0000)\},$$

$$= \frac{\pi}{36} \times 7{\cdot}9585 = 0{\cdot}6945.$$

Hence, by Simpson's rule, $\underline{\int_0^{\pi/3} \tan x \, dx = 0{\cdot}6945.}$

Also, by calculation,

$$\int_0^{\pi/3} \tan x \, dx = \int_0^{\pi/3} \frac{\sin x}{\cos x} \, dx,$$

$$= \int_0^{\pi/3} - \frac{d(\cos x)}{\cos x},$$

$$= \Big[- \log_e \cos x \Big]_0^{\pi/3},$$

$$= - \log_e 0{\cdot}5 + \log_e 1,$$

$$= \underline{0{\cdot}6931.}$$

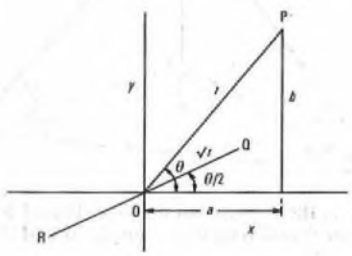
The approximate result obtained by Simpson's rule closely agrees with the calculated result, the error being approximately 0·2%.

Q 10 *(a) Explain, with the aid of a complex plane diagram, why a complex number has 2 distinct square roots.*
(b) Convert to polar form each of the complex expressions $(R + j\omega L)$ and $(G + j\omega C)$. Hence, express $\surd\{(R + j\omega L)(G + j\omega C)\}$ in the polar form $\alpha + j\beta$.
(c) If $\omega = 10^4$, $R = 44$, $L = 1{\cdot}2 \times 10^{-3}$, $C = 0{\cdot}048 \times 10^{-6}$ and G is negligible, calculate α and β, given that α is positive.

A 10 *(a)* In the sketch, if P represents the complex number $z = a + jb$, then, in polar form, the modulus, r, and argument, θ, of z are given by

$$r = \surd(a^2 + b^2) \text{ and } \theta = \tan^{-1}\frac{b}{a}.$$

Hence, $\underline{\surd(r\angle\theta) = \pm\surd(r)\angle\theta/2.}$



The positive root, $\underline{\surd(r)\angle\theta/2}$, is represented in the sketch by point Q. The second root can be expressed as

$$-\surd(r)\angle\theta/2) = (1\angle\pi) \times \{\surd(r)\angle\theta/2\} = \underline{\surd(r)\angle\theta/2 + \pi},$$

since $\quad -1 = 1\angle\pi$.

The second root is, therefore, represented by point R, where OQ = OR, turned through π radians from OQ. This can be verified by squaring; thus,

$$\{\surd(r)\angle\theta/2 + \pi)\}^2 = r\angle\theta + 2\pi = r\angle\theta.$$

Therefore, a complex number has 2 distinct square roots whose moduli are equal, but whose arguments differ by π radians.

(b) $\qquad \underline{(R + j\omega L) = \surd(R^2 + \omega^2 L^2)\angle\tan^{-1}(\omega L/R),}$

and $\qquad \underline{(G + j\omega C) = \surd(G^2 + \omega^2 C^2)\angle\tan^{-1}(\omega C/G).}$

$$\therefore\ \surd\{(R + j\omega L)(G + j\omega C)\} = \{[\surd(R^2 + \omega^2 L^2)\angle\tan^{-1}(\omega L/R)] \times [\surd(G^2 + \omega^2 C^2)\angle\tan^{-1}(\omega C/G)]\}^{1/2},$$

$$= \underline{\{\surd[(R^2 + \omega^2 L^2)(G^2 + \omega^2 C^2)]\angle\tan^{-1}(\omega L/R) + \tan^{-1}(\omega C/G)\}^{1/2}}. \quad \ldots\ldots (1)$$

Let $\qquad Z = \surd\{(R + j\omega L)(G + j\omega C)\} = \alpha + j\beta. \quad \ldots\ldots (2)$

Then, $\qquad |Z| = \surd(\alpha^2 + \beta^2),$

$$= \sqrt[4]{\{(R^2 + \omega^2 L^2)(G^2 + \omega^2 C^2)\}} \text{ from equation (1).}$$

$$\therefore\ \alpha^2 + \beta^2 = \surd\{(R^2 + \omega^2 L^2)(G^2 + \omega^2 C^2)\}. \quad \ldots\ldots (3)$$

But, squaring equation (2) gives

$$\alpha^2 + 2j\alpha\beta - \beta^2 = RG - \omega^2 LC + j(\omega LG + \omega CR). \quad \ldots\ldots (4)$$

Equating the real terms on each side of equation (4) gives

$$\alpha^2 - \beta^2 = RG - \omega^2 LC. \quad \ldots\ldots (5)$$

Adding equations (3) and (5) gives

$$2\alpha^2 = \surd\{(R^2 + \omega^2 L^2)(G^2 + \omega^2 C^2)\} + RG - \omega^2 LC.$$

$$\therefore\ \underline{\alpha = \sqrt{\left[\frac{\surd\{(R^2 + \omega^2 L^2)(G^2 + \omega^2 C^2)\} + RG - \omega^2 LC}{2}\right]}}. \quad \ldots\ldots (6)$$

Subtracting equation (5) from equation (3) gives

$$2\beta^2 = \surd\{(R^2 + \omega^2 L^2)(G^2 + \omega^2 C^2)\} - (RG - \omega^2 LC).$$

$$\therefore\ \underline{\beta = \sqrt{\left[\frac{\surd\{(R^2 + \omega^2 L^2)(G^2 + \omega^2 C^2)\} - (RG - \omega^2 LC)}{2}\right]}}. \quad \ldots\ldots (7)$$

Thus, $Z = \surd\{(R + j\omega L)(G + j\omega C)\}$ is given in the form $\alpha + j\beta$ by using the values for α and β given in equations (6) and (7).

Note: The form $\alpha + j\beta$ of a complex quantity is not the polar form as implied in the question. The polar form is $r\angle\theta$ or $r(\cos\theta + j\sin\theta)$, although the latter is often called the trigonometric form. The form $\alpha + j\beta$ is sometimes called the rectangular form.

(c) It is convenient to evaluate the term

$$\surd\{(R^2 + \omega^2 L^2)(G^2 + \omega^2 C^2)\}$$

separately, since this occurs in each of the expressions for α and β.

$$\surd\{(R^2 + \omega^2 L^2)(G^2 + \omega^2 C^2)\} = \omega C\surd(R^2 + \omega^2 L^2),$$

since G is negligible,

$$= 10^4 \times 0{\cdot}048 \times 10^{-6} \times \surd(44^2 + 10^8 \times 1{\cdot}2^2 \times 10^{-6}),$$

$$= 0{\cdot}02189.$$

Hence,

$$\alpha = \sqrt{\left(\frac{0{\cdot}02189 - 10^8 \times 1{\cdot}2 \times 10^{-3} \times 0{\cdot}048 \times 10^{-6}}{2}\right)},$$

from equation (6) and neglecting G,

$$= \underline{0{\cdot}08981} \text{ since } \alpha \text{ is positive.}$$

Similarly,

$$\beta = \sqrt{\left(\frac{0{\cdot}02189 + 10^8 \times 1{\cdot}2 \times 10^{-3} \times 0{\cdot}048 \times 10^{-6}}{2}\right)},$$

from equation (7) and neglecting G,

$$= \underline{0{\cdot}1176.}$$

Students were expected to answer any 6 questions

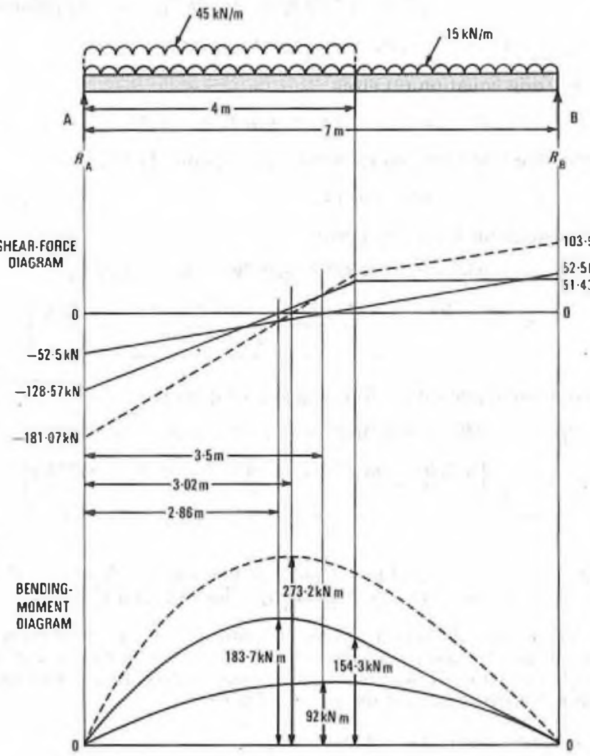
Q 1 A beam, 7 m long, rests on supports at each end and carries a distributed load of 15 kN/m. An additional distributed load of 45 kN/m is carried over a distance of 4 m from the left-hand support.

(a) Draw a shear-force diagram of the loaded beam.

(b) Draw a bending-moment diagram of the loaded beam.

(c) Find the magnitude of the maximum bending moment.

A 1 The sketch shows the loaded beam with reactions R_A and R_B at points A and B respectively.



Now, the reactions at A and B due to the 15 kN/m load are given by

$$R_{A15} = R_{B15} = \frac{15 \times 7}{2} \text{ kN} = 52{\cdot}5 \text{ kN},$$

and, considering the reactions at A and B due to the 45 kN/m load,

$$R_{B45} \times 7 = 45 \times 4 \times 2,$$

where R_{B45} is in kilonewtons.

$$\therefore \ R_{B45} = 51{\cdot}43 \text{ kN}.$$

Thus, $$R_{A45} = (45 \times 4) - 51{\cdot}43 \text{ kN} = 128{\cdot}57 \text{ kN}.$$

Therefore, the total reactions at A and B are given by

$$R_A = 52{\cdot}5 + 128{\cdot}57 \text{ kN} = 181{\cdot}07 \text{ kN},$$

and $$R_B = 52{\cdot}5 + 51{\cdot}43 \text{ kN} = 103{\cdot}93 \text{ kN}.$$

(*a*) The shear force at any point is the alegebraic sum of all the perpendicular components of force acting on the beam either to the left or right of that point. By convention, shear force is considered to be positive when, in a horizontal beam, the right-hand portion tends to move upwards relative to the left-hand portion.

The shear-force diagrams for the 2 loads are shown separately in the sketch, which is not to scale. The resultant is shown dashed.

(*b*) and (*c*) The bending moment at any point is the algebraic sum of the moments about that point of all the forces applied to one side of the point. By convention, bending moments are considered to be positive when they cause the beam to sag.

Now, the maximum bending moment occurs when the shear force is zero, and the shear force is zero at 181·07/(45 + 15) m from A; that is, 3·02 m from A.

Therefore, the maximum bending moment

$$= 181{\cdot}07 \times 3{\cdot}02 - \left(\frac{60 \times 3{\cdot}02}{2}\right) \times 3{\cdot}02 \text{ kN m},$$

$$= \underline{273{\cdot}2 \text{ kN m}}.$$

The bending-moment diagrams for the 2 loads are shown roughly in the sketch, with the resultant shown dashed. The salient points are calculated as indicated below.

The maximum bending moment due to the 15 kN/m load occurs at the centre of the beam and is equal to

$$52{\cdot}5 \times 3{\cdot}5 - 15 \times 3{\cdot}5 \times 1{\cdot}75 = 92 \text{ kN m}.$$

The bending moment at a point 4 m from A due to the 45 kN/m load

$$= 51{\cdot}43 \times 3 = 154{\cdot}3 \text{ kN m},$$

and the diagram is linear to the right of this point.

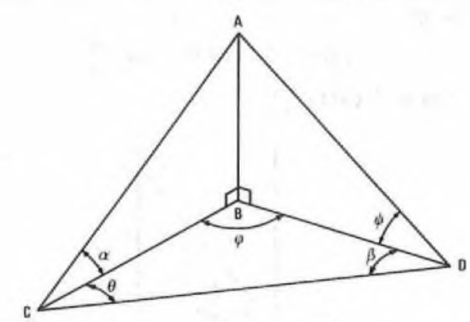
The shear force due to the 45 kN/m load is zero at 128·57/45 = 2·86 m from A. Therefore, the maximum bending moment due to the 45 kN/m load occurs at this point, and is equal to

$$128{\cdot}57 \times 2{\cdot}86 - \left(\frac{45 \times 2{\cdot}86}{2}\right) \times 2{\cdot}86 = 183{\cdot}7 \text{ kN m}.$$

Q 2 An aerial dish is mounted at a point, A, on a supporting mast. Point B is vertically under point A at the base of the mast. Using 2 survey points, C and D, on the ground, show how you would find the height AB using a theodolite which can be set at the same height as the base of the mast. It is only possible to measure the distance between points C and D.

A 2 The sketch illustrates the problem; it is not assumed that the base triangle is isosceles.

Measurement of the ground distance, CD, is carefully done by means of a steel tape. A theodolite is set up at position C, and is carefully levelled with the vertical scale set at zero. With the horizontal scale set at zero, the instrument is sighted on point A and the inclination angle, α, read off the vertical scale, is recorded. A ranging rod is set up vertically at D, and a sight is taken on this, the horizontal angle, θ, being read off the horizontal scale of the instrument.



The theodolite is then moved to position D and levelled as before. By sighting first on A and then C, the angles ψ and β are measured.

Now, $$\phi = 180^\circ - (\theta + \beta),$$

and, by the sine rule,

$$\frac{CD}{\sin \phi} = \frac{BC}{\sin \beta} = \frac{BD}{\sin \theta},$$

$$\therefore \ BC = \frac{CD \sin \beta}{\sin \phi},$$

and $$\underline{AB = BC \tan \alpha}.$$

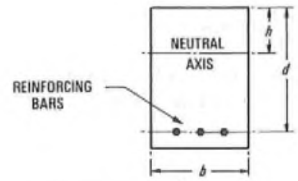
A check, using the calculated side BD and the angle ψ, is made to prove the result.

Since the theodolite can be set at the same height as point B, the height of the theodolite from the ground is irrelevant.

Q 3 (a) What is meant by the term economic percentage of steel in a reinforced-concrete beam?

(b) Calculate the economic percentage of steel in a reinforced-concrete beam using safe working stresses of steel and concrete of 140 MPa and 7 MPa, respectively, with a modular ratio of 15.

A 3 (*a*) In a reinforced-concrete beam where the stresses in both the steel and concrete have reached their maximum allowable values, the steel used is called the *critical amount* or, expressed as a percentage of the cross-sectional area of the beam, the *economic percentage*.



(*b*) Let the beam's cross-section be as shown in the sketch, where h is the depth of the neutral axis (m), d is the depth of the reinforcement (m), and b is the breadth of the beam (m). Then,

$$\frac{h}{d} = \frac{1}{1 + \dfrac{f_s}{f_c m}},$$

where f_s is the stress in the steel (Pa), f_c is the stress in the concrete (Pa), and m is the modular ratio.

$$\therefore \frac{h}{d} = \frac{1}{1 + \dfrac{140 \times 10^6}{7 \times 10^6 \times 15}} = 0{\cdot}43.$$

Now, the compressive force in the concrete above the neutral axis is equal to the total tensile force in the steel reinforcement.

Thus, $\tfrac{1}{2} f_c bh = f_s A_s,$

where A_s is the area of the steel reinforcement (m²).

Now, the ratio, r, of the area of the steel reinforcement to the total area of the beam is given by

$$r = \frac{A_s}{bd},$$

$$\therefore \tfrac{1}{2} f_c bh = f_s rbd.$$

$$\therefore r = \frac{f_c h}{2 f_s d},$$

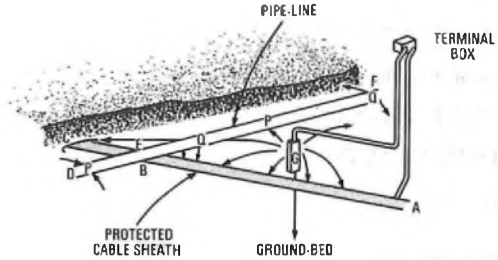
$$= \frac{7 \times 10^6 \times 0{\cdot}43}{2 \times 140 \times 10^6} = 0{\cdot}01075.$$

Therefore, the economic percentage of steel is approximately 1%.

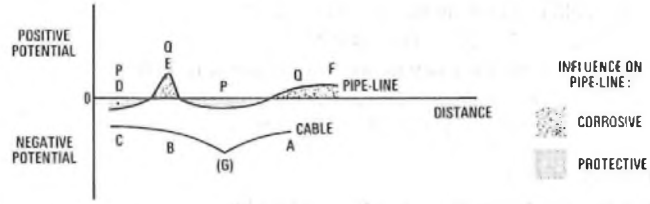
Q 4 (a) Explain what is meant by corrosion interference in connexion with a cathodic-protection scheme.

(b) Describe measures which can be adopted to reduce corrosion interference.

A 4 (*a*) Sketch (*a*) shows a cathodically-protected cable, ABC, in close proximity to, but not touching, a pipe-line, DEF. If cathodic protection is applied satisfactorily, then current passes from the ground-bed to the cable sheath via the main bulk of soil, and a proportion of it enters, on its way, the pipe-line at points such as P. The pipe-line, being of a much lower resistance than the soil, carries the current to the point nearest the cable sheath and, at this point, the current leaves the pipe-line and traverses the soil to the cable sheath. The points where the current leaves the pipe-line are marked Q. Points such as Q can, therefore, be defined as positive, or anodic, zones, whilst points such as P can be defined as cathodic, or negative, zones. Thus, under normal conditions, corrosion takes place at such points as Q. Examination of such cases in practice shows that maximum corrosion of the pipe-line takes place at point E, where the pipe-line crosses the protected cable. Sketch (*b*) shows the potential distribution along both the cable sheath and the pipe-line.



(*a*)



(*b*)

(*b*) Corrosion interference can be reduced by the following measures.

(*i*) A reduction in the current injected into the ground by the cathodic-protection scheme causes a corresponding decrease in the potential existing on the pipe-line.

(*ii*) The effect of injecting a current into the soil via a ground-bed, or sacrificial anode, is to raise the potential of the surrounding earth. The radial potential gradient varies inversely with the distance from the effective centre of the ground-bed. Thus, moving the ground-bed to another site reduces the interference.

(*iii*) The protected cable-sheath and buried pipe-line can be connected by one or more metallic bonds. The bonds should ideally be connected to the structure where the maximum positive potential is measured.

(*iv*) Cathodic protection may be provided for both the buried pipe-line and the cable sheath, instead of for just the cable sheath.

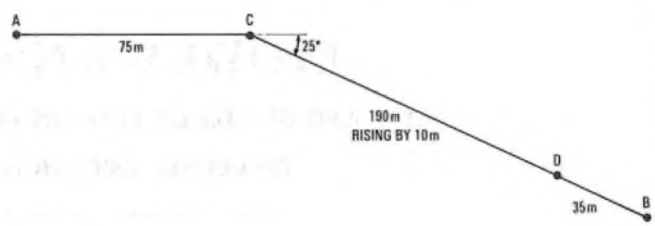
(*v*) Magnesium anodes may be installed at the points on the buried pipe-line where the positive potential is greatest. This is quite satisfactory where the points of positive potential are localized.

(*vi*) Pipes can be wrapped or served cables can be substituted. Alternatively, wrapping may take place *in situ* at crossing points. This is effective only if the measured positive potential at this point is small.

(*vii*) A short section of the cable sheath can be isolated from the cathodic-protection scheme by means of insulating joints and an insulated continuity band.

Q 5 A cable is to be pulled into a duct, AB. The duct runs 75 m horizontally from point A, bends round a short 25° arc, followed by a straight run of 190 m up a slope rising by 10 m. The remaining 35 m are horizontal to point B. If the cable weighs 60 N/m, and the coefficient of friction between the cable and duct is 0·4, compare the pulling tensions when the cable is drawn-in in the direction A–B to that when it is drawn-in from B–A.

A 5 The sketch shows the dimensions and layout of the duct route.



Drawing-In A–B

The pulling tension in section AC is given by

$$T_{AC} = \mu l w \text{ newtons},$$

where μ is the coefficient of friction, l is the length of the section (m), and w is the weight per unit length of the cable (N/m),

$$= 0{\cdot}4 \times 75 \times 60 = 1800 \text{ N}.$$

The pulling tension at bend C is given by

$$T_C = T_{AC} e^{\mu\theta} \text{ newtons},$$

where θ is the angle of the arc (rad),

$$= 1800 \times e^{0{\cdot}4 \times 0{\cdot}4363} = 1800 \times 1{\cdot}1907 = 2143 \text{ N}.$$

The pulling tension up gradient CD is given by

$$T_{CD} = wl(\mu \cos \phi + \sin \phi) \text{ newtons},$$

where ϕ is the angle of the slope,

$$= 60 \times 190 \times \left\{0{\cdot}4 \times \frac{\sqrt{(190^2 - 10^2)}}{190} + \frac{10}{190}\right\} = 5154 \text{ N}.$$

The pulling tension in section DB is given by

$T_{DB} = 0{\cdot}4 \times 35 \times 60 = 840$ N.

Therefore, the total pulling tension in direction A–B

$= T_C + T_{CD} + T_{DB}$ newtons,

$= 2143 + 5154 + 840 = \underline{8137 \text{ N}}.$

Drawing-In B–A

The pulling tension in section BD is given by

$T_{BD} = 0{\cdot}4 \times 35 \times 60 = 840$ N.

The pulling tension down gradient DC is given by

$T_{DC} = wl(\mu \cos\phi - \sin\phi)$ newtons,

$$= 60 \times 190 \left\{0{\cdot}4 \times \frac{\sqrt{(190^2 - 10^2)}}{190} - \frac{10}{190}\right\} = 3954 \text{ N}.$$

Thus, the total tension in section BC is given by

$T_{BC} = T_{BD} + T_{DC}$ newtons,

$= 840 + 3954 = 4794$ N.

The pulling tension at bend C is given by

$T_C = T_{BC}e^{\mu\theta}$ newtons,

$= 4794 \times e^{0{\cdot}4 \times 0{\cdot}4363} = 4794 \times 1{\cdot}1907 = 5708$ N.

The pulling tension in section CA is given by

$T_{CA} = 0{\cdot}4 \times 75 \times 60 = 1800$ N.

Therefore, the total pulling tension in direction B–A

$= T_C + T_{CA}$ newtons,

$= 5708 + 1800 = \underline{7508 \text{ N}}.$

Thus, it is easier to draw-in the cable in the direction B–A.

Q 6 (*a*) *Give an account of the stresses which occur in the sleeve of a cable joint containing air under pressure.*

(*b*) *A lead sleeve of a cable joint contains air at 60 kPa. If the sleeve has an internal diameter of 120 mm, how thick would the sleeve wall have to be if the stress in the lead must not exceed 1·0 MPa?*

A 6 (*a*) See A7, Line Plant Practice C, 1971, *Supplement*, Vol. 65, p. 86, Jan. 1973.

(*b*) The hoop stress, f_h pascals, is given by

$$f_h = \frac{Pd}{2t},$$

where P is the internal air pressure (Pa), d is the internal diameter (m), and t is the thickness of the sleeve wall (m).

$$\therefore\ t = \frac{Pd}{2f_h} \text{ metres},$$

$$= \frac{60 \times 10^3 \times 120 \times 10^{-3}}{2 \times 1{\cdot}0 \times 10^6} \text{ m} = \underline{3{\cdot}6 \text{ mm}}.$$

Q 7 (*a*) *What factors determine whether an existing pole route is strong enough for the erection of an aerial cable?*

(*b*) *Describe methods of strengthening the route, should this be necessary.*

A 7 (*a*) For poles without transverse or angle staying, the maximum stress is caused by transverse winds which give rise to a bending moment on the pole. This moment has a maximum value at ground level, and depends on

(*i*) the maximum wind speed,
(*ii*) the diameters of the wires and cables,
(*iii*) the height of the wires and cables, and
(*iv*) the span lengths.

From these factors, the maximum bending moment is calculated, and this is compared with the maximum safe moment for the pole, which itself increases with the diameter of the pole. For stayed poles, the maximum stress is compressive, and is called a *buckling load*. This is proportional to

(*i*) the maximum tension in the wires and cables, usually taken to be either the breaking weight or half the breaking weight, depending on circumstances,
(*ii*) the pull on the pole, in the case of angle poles, and
(*iii*) the height-to-base-distance ratio of the stay.

The stay converts the horizontal breaking strain into a vertical buckling load. This is compared with the maximum the pole will withstand, which depends on the pole diameter and the stay height.

(*b*) The usual methods of strengthening a route are by

(*i*) transverse staying,
(*ii*) the use of stronger poles, or
(*iii*) shortening the span lengths.

Sometimes, the maximum stress on the route is reduced by combining several wires and cables into one unit, lowering plant, and increasing the height-to-base-distance ratio of the stays.

Published by *The Post Office Electrical Engineers' Journal*, 2–12 Gresham Street, London EC2V 7AG, and printed in Great Britain at the Gresham Press by Unwin Brothers Limited, Old Woking, Surrey.