SUPPLEMENT

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Contents

1973–74 CITY AND GUILDS OF LONDON INSTITUTE EXAMINATIONS

QUESTIONS AND ANSWERS

Answers are occasionally omitted or reference is made to earlier Supplements in which questions of substantially the same form, together with the answers, have been published. Some answers contain more detail than would be expected from candidates under examination conditions.

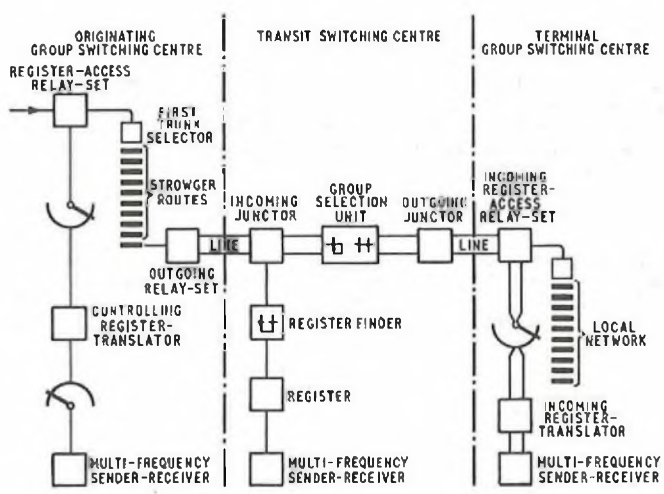
TELEPHONY C, 1973 (*continued*)

Q. 10. (*a*) *Describe the principle of operation of an inter-register signalling system using multi-frequency techniques.*

(*b*) *State the advantages of such a system over a simple 1 v.f. system, and describe a network in which an inter-register signalling system would be used.*

Use sketches to illustrate your answer.

A. 10. (*a*) Sketch (*a*) shows a typical trunking arrangement for an inter-register, multi-frequency (m.f.) signalling system. The connexion illustrated utilizes signalling system m.f. No. 2 (s.s.m.f. 2), and is routed via one transit switching centre (t.s.c.).



(*a*)

In s.s.m.f. 2, each signal consists of two frequencies, applied simultaneously, out of six possible frequencies in the forward direction and five in the backward direction. The forward signals are

(*i*) digits 1–0,
(*ii*) forward prefix, and
(*iii*) class of service 1, 2 or 3.

The backward signals are

(*i*) transit proceed to send,
(*ii*) terminal proceed to send,
(*iii*) congestion,
(*iv*) number received,
(*v*) spare code,
(*vi*) backward prefix,
(*vii*) send *class-of-service* signal,
(*viii*) called-line free (ordinary),
(*ix*) called-line free (coin-collecting box), and
(*x*) called-line busy.

At the originating group switching centre (g.s.c.), the controlling register-translator associates an m.f. sender-receiver (m.f.s.r.) when it has discriminated that a call is to be connected via an m.f. route. It then sends the local Strowger routing digits necessary to gain access to the outgoing m.f. route, and an outgoing relay-set is seized which, in turn, seizes an incoming junctor and an associated register and m.f.s.r. in the t.s.c. The following sequence of interchange of m.f. signals takes place when the controlling register and the transit register, and their associated m.f.s.r.s are connected together.

(*i*) The t.s.c. m.f.s.r. returns a *backward-prefix* signal.
(*ii*) The g.s.c. m.f.s.r. sends a *forward-prefix* signal.
(*iii*) The t.s.c. m.f.s.r. replaces the *backward-prefix* signal with a *transit-proceed-to-send* signal.
(*iv*) On receipt of the *transit-proceed-to-send* signal, the g.s.c. m.f.s.r. responds by sending forward the first three digits of the national number, known as the *A-*, *B-* and *C-digits*. The *forward-prefix* signal is disconnected and replaced by the A-digit signal for a period of 80 ms, after which, the *forward-prefix* signal is reconnected for 80 ms. This is followed by the B-digit signal, the *forward-prefix* signal, and the C-digit signal, each signal having a duration of 80 ms.

For a call routed through more than one t.s.c., this sequence takes place each time the controlling register is connected to a transit register.

The *congestion* signal is returned from the t.s.c. m.f.s.r. if link congestion is encountered.

On receipt of the A-, B- and C-digit information, the t.s.c. switches the call to an outgoing m.f. route to the required terminal g.s.c., and the t.s.c. register and m.f.s.r. then release from the connexion.

The incoming relay-set in the terminal g.s.c. associates an incoming register-translator and m.f.s.r. The following sequence of interchange of m.f. signals takes place when the controlling register and the incoming register, and their associated m.f.s.r.s, are connected together.

(*i*) The terminal g.s.c. m.f.s.r. returns a *backward-prefix* signal.
(*ii*) The controlling g.s.c. m.f.s.r. sends a *forward-prefix* signal.
(*iii*) The terminal g.s.c. m.f.s.r. replaces the *backward-prefix* signal with a *terminal-proceed-to-send* signal of 80 ms duration.
(*iv*) On receipt of the *terminal-proceed-to-send* signal, the controlling g.s.c. m.f.s.r. disconnects the *forward-prefix* signal and sends forward the C-digit signal for a period of 80 ms. (For London calls, the B-digit signal is sent, instead of the C-digit.)
(*v*) The terminal g.s.c. m.f.s.r. then returns the *backward-prefix* signal for 80 ms, followed by a *terminal-proceed-to-send* signal of 80 ms duration.
(*vi*) On receipt of this signal, the controlling g.s.c. m.f.s.r. sends the *forward-prefix* signal for a period of 80 ms, followed by the D-digit signal (C-digit signal for London calls), also for a period of 80 ms.
(*vii*) This process continues until the complete national number has been sent forward.

(*viii*) After receipt of the complete national number, the terminal g.s.c. m.f.s.r. returns the *backward-prefix* signal for a period of 80 ms, followed by a *number-received* signal of 80 ms duration.

The *spare-code* signal is returned from the terminal g.s.c. m.f.s.r. if a non-valid national number is received.

When the incoming register has received the national number, it sends forward the necessary Strowger routing digits, the connexion is switched through at both g.s.c.s, and the controlling and incoming registers, and their associated m.f.s.r.s, release.

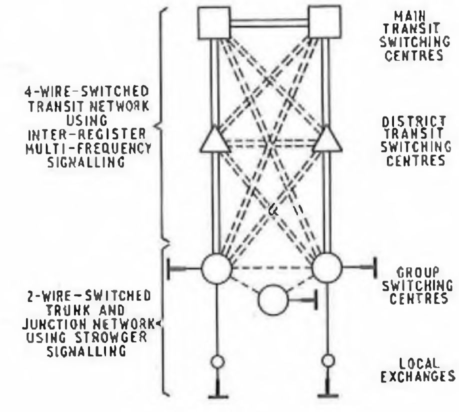
(*b*) The advantages of an inter-register m.f. signalling system over a 1 v.f. system are that

(*i*) a higher signalling speed is achieved,
(*ii*) more links in tandem are possible,
(*iii*) no signal repetition is required at intermediate switching points,
(*iv*) there is security from false signals,
(*v*) a repeat-attempt facility is provided, if the first attempt to set up the call fails, and
(*vi*) class-of-service information can be forwarded, if required.

Sketch (*b*) shows a typical network, the trunk transit network, in which inter-register m.f. signalling is used. Calls are set up from g.s.c. to g.s.c.

(*i*) by means of direct routes,
(*ii*) via an intermediate g.s.c., or
(*iii*) via up to four t.s.c.s.

The solid lines represent 2-wire and 4-wire basic routes, and the broken lines represent possible 2-wire and 4-wire auxiliary routes.



(*b*)

CORRECTION

TELEPHONY C, 1972 (*Supplement*, Vol. 66, Jan. 1974).

A. 4. (*a*) (*ii*) Signalling system d.c. No. 2 is also used on 2-wire amplified audio circuits with negative-impedance amplifiers or 2-wire hybrid-connected amplifiers.

TELECOMMUNICATION PRINCIPLES C, 1973

Students were expected to answer any six questions

Q. 1. (*a*) *Sketch diagrams showing the electric field configuration in*

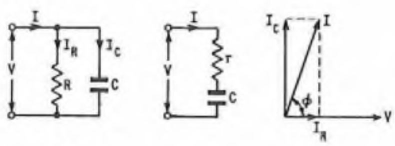
(*i*) *a parallel-plate capacitor, and*
(*ii*) *the plane through, and perpendicular to, a length of air-spaced, 2-wire line.*

Indicate on the sketches the polarity of the applied e.m.f. and the direction of the electric field.

(*b*) *A 159 pF capacitor is connected to a 10-volt supply which has a frequency of 2·5 MHz. The power loss in the capacitor is found to be 50 μW. Calculate, for the capacitor,*

(*i*) *the equivalent shunt loss-resistance,*
(*ii*) *the equivalent series loss-resistance,*
(*iii*) *the power factor, and*
(*iv*) *the Q-factor.*

A. 1. (*a*) (*i*) and (*ii*) See A.1, Telecommunication Principles C, 1969. *Supplement*, Vol. 63, p. 45, July 1970.



(*a*) (*b*) (*c*)

(*b*) (*i*) Power loss in the capacitor configuration (see sketch (*a*))

$= \frac{V^2}{R}$, where V is the supply voltage, and R is the equivalent shunt resistance (ohms).

$$\therefore\ R = \frac{10^2}{50 \times 10^{-6}} = \underline{2\ \text{Mohm.}}$$

(*ii*) From sketch (*a*), the current in the equivalent shunt resistance,

$$I_R, = \frac{V}{R} = \frac{10}{2 \times 10^6} = 5\ \mu\text{A}.$$

Also, the current, I_C, in the capacitor

$= 2\pi fCV$, where f is the supply frequency (hertz), and C is the capacitance (farads).

$$\therefore\ I_C = 2 \times \pi \times 2{\cdot}5 \times 10^6 \times 159 \times 10^{-12} \times 10 = 25\ \text{mA}.$$

Therefore, the magnitude of the total current, I, supplied to the capacitor configuration is given by

$$|I| = \sqrt{(I_R^2 + I_C^2)} = \sqrt{\{(5\times10^{-6})^2 + (25\times10^{-3})^2\}}\ \text{amp},$$
$$\simeq 25\ \text{mA}.$$

Hence, referring to sketch (*b*), the power loss

$= |I|^2 r$, where r is the equivalent series resistance (ohms).

$$\therefore\ r = \frac{50 \times 10^{-6}}{625 \times 10^{-6}} = \underline{0{\cdot}08\ \text{ohm.}}$$

(*iii*) With reference to the phasor diagram in sketch (*c*), power factor

$$= \cos\phi = \frac{I_R}{|I|} = \frac{5 \times 10^{-6}}{25 \times 10^{-3}} = \underline{2 \times 10^{-4}}.$$

(*iv*) $Q\text{-factor} = \frac{1}{\text{power factor}} = \frac{1}{2 \times 10^{-4}} = \underline{5{,}000}.$

Q. 2. (*a*) *A 10 mH coil is connected in series with a capacitor, which may be assumed loss-free, to a variable-frequency source which supplies a constant voltage of 9 volts. The circuit current has a maximum value of 90 mA at a frequency of 80 kHz. Calculate*

(*i*) *the capacitance of the capacitor,*
(*ii*) *the Q-factor of the coil, and*
(*iii*) *the frequencies at which the circuit current will be* $\frac{90}{\sqrt{2}}$ *mA.*

(*b*) *The capacitor in the circuit of (a) is changed for one of poorer quality and, with the rest of the circuit unchanged, a maximum current value of 81 mA occurs at the same frequency of 80 kHz. Determine the power factor of the replacement capacitor.*

A. 2. (*i*) The impedance, Z, of the circuit is given by

$$Z = r + j\omega L + \frac{1}{j\omega C} = r + j\left(\omega L - \frac{1}{\omega C}\right),$$

where r is the resistance of the coil (ohms),
L is the inductance of the coil (henrys),
C is the capacitance of the capacitor (farads), and
$\omega = 2\pi f$, where f is the frequency (hertz).

Now, the circuit current, I, is a maximum when Z is purely resistive; that is, when $\omega L - \frac{1}{\omega C} = 0$, or $C = \frac{1}{\omega^2 L}$, and the maximum value of current occurs at a resonant frequency of 80 kHz.

$$\therefore\ C = \frac{1}{(2\pi \times 80 \times 10^3)^2 \times 10 \times 10^{-3}} = \underline{400\ \text{pF.}}$$

(*ii*) Now, at resonance, $r = \frac{V}{I}$, where V is the supply voltage.

$$\therefore\ r = \frac{9}{90 \times 10^{-3}} = 100\ \text{ohms}.$$

$$Q\text{-factor} = \frac{\omega L}{r} = \frac{2 \times \pi \times 80 \times 10^3 \times 10 \times 10^{-3}}{100},$$
$$= \underline{50.}$$

(*iii*) If the circuit current reduces to $1/\sqrt{2}$ of its value at resonance, the power dissipated reduces to half of its value at resonance. The frequencies at which this occurs, f_1 and f_2, are known as the half-power frequencies, and are related by the expression

$$f_2 - f_1 = \frac{f_0}{Q}, \text{ where } f_0 \text{ is the resonant frequency (hertz),}$$

$$= \frac{80 \times 10^3}{50} = 1{,}600 \text{ Hz.}$$

Now, f_1 and f_2 are spaced equally below and above f_0.

$$\therefore f_1 = f_0 - \frac{1{,}600}{2} = (80 \times 10^3) - 800 = \underline{79 \cdot 2 \text{ kHz,}}$$

$$\text{and } f_2 = f_0 + \frac{1{,}600}{2} = (80 \times 10^3) + 800 = \underline{80 \cdot 8 \text{ kHz.}}$$

(*b*) It is assumed that the capacitor has a series resistance r_c ohms, so that the total series resistance of the circuit is $r + r_c$ ohms.

$$\text{Then, at resonance, } \frac{V}{I} = r + r_c = \frac{9}{81 \times 10^{-3}} \text{ ohms.}$$

$$\therefore r_c = \frac{9}{81 \times 10^{-3}} - 100 = 11 \text{ ohms.}$$

$$\text{Now, power factor} \simeq 2\pi f C r_c = 2 \times \pi \times 80 \times 10^3 \times 4 \times 10^{-10} \times 11,$$

$$= \underline{0 \cdot 0022.}$$

Q. 3. (*a*) *A source, of open-circuit e.m.f.* 6 *volts and frequency* 159 *Hz, has an internal impedance which may be represented by a* 300-*ohm resistor in series with a* 2·5 *μF capacitor. Calculate the values of the components which should be connected across the source to take maximum power from the source, and the value of the maximum power.*

(*b*) *The source described in* (*a*) *is to be used to deliver its maximum power to a* 1,200-*ohm resistor. Explain how this could be achieved, and determine the values of the necessary components.*

A. 3. (*a*) Let the load, Z, for maximum power transfer be $Z = R + jX$, where R is the value of the load resistance (ohms), and X is the value of the load reactance (ohms).

Then, circuit current, I, is given by

$$I = \frac{6}{300 - j\dfrac{1}{2 \times \pi \times 159 \times 2 \cdot 5 \times 10^{-6}} + R + jX}.$$

$$\therefore |I| = \frac{6}{\sqrt{\{(300 + R)^2 + (X - 0 \cdot 4 \times 10^3)^2\}}}.$$

$$\text{Now, power} = |I|^2 R,$$

$$= \frac{36R}{(300 + R)^2 + (X - 0 \cdot 4 \times 10^3)^2}.$$

Thus, power has a maximum value when $X - 0 \cdot 4 \times 10^3 = 0$.

$$\therefore X = 0 \cdot 4 \times 10^3.$$

$$\text{Hence, } 2\pi f L = 0 \cdot 4 \times 10^3,$$

where f is the frequency (hertz), and
L is the load inductance (henrys).

$$\therefore L = \frac{0 \cdot 4 \times 10^3}{2 \times \pi \times 159} = \underline{0 \cdot 4 \text{ H.}}$$

$$\text{For this condition, power} = \frac{36R}{(300 + R)^2}, \qquad \ldots\ldots(1)$$

$$= \frac{36}{\dfrac{300^2}{R} + 600 + R}.$$

This has a maximum value when the denominator is a minimum. Differentiating the denominator and equating to zero gives

$$-\frac{300^2}{R^2} + 1 = 0.$$

$$\therefore R^2 = 300^2.$$

$$\therefore \underline{R = 300 \text{ ohms.}}$$

Hence, for maximum power transfer, the load is an inductor having an inductance of 0·4 H and a resistance of 300 ohms.

The maximum power, W_{max}, is given by equation (1), above.

$$W_{max} = \frac{36 \times 300}{(300 + 300)^2} = \underline{30 \text{ mW.}}$$

(*b*) The magnitude of the source impedance is equal to

$$\sqrt{\left\{300^2 + \left(\frac{1}{2 \times \pi \times 159 \times 2 \cdot 5 \times 10^{-6}}\right)^2\right\}},$$

$$= 500 \text{ ohms.}$$

Maximum power is delivered to a resistive load when its value is also 500 ohms.

This can be achieved by connecting a low-loss transformer between the source and the 1,200-ohm resistor, having a turns ratio of $\sqrt{\left(\frac{1{,}200}{500}\right)}$

$$= \underline{1 \cdot 55 : 1.}$$

Q. 4. (*a*) *State* (*i*) *Thévènin's theorem, and* (*ii*) *Norton's theorem.*

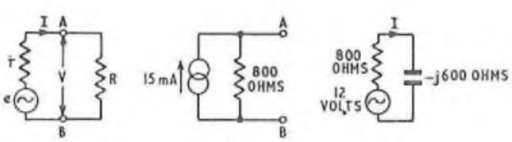
(*b*) *A source is connected to a resistive load. Readings of load current are taken, using a low-resistance ammeter, as the load resistance is varied. The results are given in the table.*

Load resistance (ohms)	200	400	700	1,200
Load current (mA)	12	10	8	6

Assuming the source impedance to be resistive, determine the Norton and Thévènin equivalent circuits for the source.

(*c*) *The resistive load is replaced by a capacitor of reactance* 600 *ohms at the source frequency. Calculate the magnitude of the capacitor current.*

A. 4. (*a*) (*i*) and (*ii*) See A.2, Telecommunication Principles C, 1972. *Supplement*, Vol. 66, p. 44, July 1973.



(*a*) (*b*) (*c*)

(*b*) The source and load can be represented as shown in sketch (*a*) and, from this, it can be seen that

$$V = IR = e - Ir,$$

where V is the external circuit voltage,
e is the source e.m.f.,
I is the load current (amperes),
R is the load resistance (ohms), and
r is the source resistance (ohms).

The relationship, $V = e - Ir$, is a linear law of the form $y = mx + c$, where m is the slope, equivalent to r, and c is a constant, equivalent to e.

Values of V are found from the table.

Load resistance, R (ohms)	200	400	700	1,200
Load current, I (mA)	12	10	8	6
Voltage, $V = IR$ (volts)	2·4	4	5·6	7·2

$$\text{Slope} = r = \frac{7 \cdot 2 - 2 \cdot 4}{(12 - 6) \times 10^{-3}} = \underline{800 \text{ ohms.}}$$

Substituting for $I = 12$ mA in the equation $V = e - Ir$ gives

$$2 \cdot 4 = e - 12 \times 10^{-3} \times 800.$$

$$\therefore \underline{e = 12 \text{ volts.}}$$

The Thévènin equivalent circuit of the given source is a constant-voltage generator, having an e.m.f. of 12 volts, in series with a resistance of 800 ohms, and is the circuit to the left of terminals A and B in sketch (*a*).

Now, the short-circuit current, i.e. the current which would flow if terminals A and B in sketch (*a*) were short circuited, is given by

$$\frac{e}{r} = \frac{12}{800} = 15 \text{ mA}.$$

Therefore, the Norton equivalent circuit is a constant-current generator of infinite impedance, delivering 15 mA, in parallel with a resistance of 800 ohms. The circuit is shown is sketch (*b*).

(*c*) Using the Thévenin equivalent circuit, the circuit can be represented as shown in sketch (*c*).

The capacitor current is given by

$$I = \frac{12}{800 - j600} \text{ amp}.$$

$$\therefore \quad |I| = \frac{12}{\sqrt{(800^2 + 600^2)}} = \underline{12 \text{ mA}}.$$

Q. 5. (*a*) *Describe an experimental method of determining the frequency response of an audio-frequency transformer between a resistive source and a resistive load, and sketch a typical response curve.*

(*b*) *Explain the curve with reference to an equivalent circuit for the transformer.*

(*c*) *How would the response be modified by a significant increase in the source resistance?*

A. 5. (*a*) and (*b*) See A.2, Telecommunication Principles C, 1968. *Supplement*, Vol. 62, p. 22, Apr. 1969.

(*c*) When the source resistance is increased significantly, the current supplied to the primary winding of the transformer and, therefore, the voltage, V, across it, are reduced. The component of the equivalent circuit which is affected is the shunt resistance, R.

$$\text{Power} = \frac{V^2}{R} = \text{hysteresis loss} + \text{eddy current loss},$$

$$= K_1 B^{\gamma} f + K_2 B^2 f^2,$$

where K_1 and K_2 are constants,
B is the magnetic flux density (teslas),
f is the frequency (hertz), and
γ is an empirical coefficient.

But, $\quad B \propto V.$

$$\therefore \quad \frac{1}{R} = K_3 \frac{V^{\gamma}}{V^2} f + K_4 \frac{V^2}{V^2} f^2,$$

where K_3 and K_4 are constants.

$$\therefore \quad \frac{1}{R} = K_3 V^{(\gamma-2)} f + K_4 f^2.$$

This is equivalent to

$$\frac{1}{R} = \frac{1}{R_h} + \frac{1}{R_e},$$

where R_h and R_e are components of the shunt resistance, representing the hysteresis and eddy-current losses respectively.

Thus, $\quad R_e$ is independent of V.

Also, $\quad R_h = \dfrac{V^{(2-\gamma)}}{K_3 f}.$

If $\gamma < 2$, R_h is reduced when V is reduced, so that R is also reduced. The effect of this is to reduce the value of the ratio of the secondary-to-primary voltages, $\dfrac{V_s}{V_p}$, at the frequency $f = \dfrac{1}{2\pi}\sqrt{\left(\dfrac{1}{L_2 C}\right)}$, where L_2 is the effective inductance of the primary winding with the secondary circuit open-circuited, and C is the winding capacitance referred to the primary circuit.

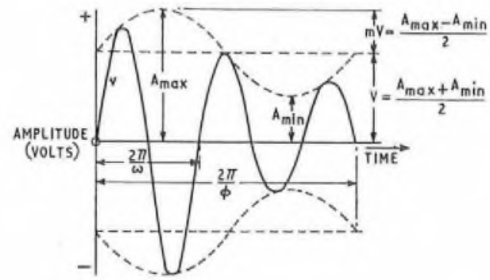
Therefore, the curve of $\dfrac{V_s}{V_p}$ plotted against f becomes flattened about this frequency.

If $\gamma > 2$, the converse is true.

Q. 6. (*a*) *Describe a practical method of measuring the modulation depth of an amplitude-modulated carrier.*

(*b*) *A sinusoidal carrier is amplitude-modulated by a sinusoidal signal. Measurements made on the modulated wave record a total power of* 540 *watts and a modulation depth of* 40 *per cent. Calculate the carrier power and the power in each side-frequency.*

A. 6. (*a*) The modulation depth of an amplitude-modulated carrier can be measured by means of a cathode-ray oscilloscope. The modulated carrier is applied to one pair of deflexion plates, and the modulating frequency to the other. This gives a display of the modulated carrier, as shown in the sketch. A sinusoidal modulating signal has been assumed.



By measuring the maximum, A_{max}, and minimum, A_{min}, values of the trace, the modulation depth, m, is calculated from the equation

$$m = \frac{\dfrac{A_{max} - A_{min}}{2}}{\dfrac{A_{max} + A_{min}}{2}} = \frac{A_{max} - A_{min}}{A_{max} + A_{min}}.$$

(*b*) Let the sinusoidally-modulated carrier be represented by the equation

$$v = V(1 + m \sin \phi t) \sin \omega t,$$

where $\dfrac{\omega}{2\pi}$ is the frequency of the carrier (hertz),

$\dfrac{\phi}{2\pi}$ is the modulating frequency (hertz), and

V is the peak value of the amplitude of the unmodulated carrier (volts).

$$\therefore \quad v = V \sin \omega t + mV \sin \phi t \sin \omega t,$$

$$= V \sin \omega t + \frac{mV}{2} \cos(\omega - \phi)t - \frac{mV}{2} \cos(\omega + \phi)t.$$

Since each component of the modulated carrier is sinusoidal, the total power dissipated in a resistance, R, is the sum of the powers in each component.

$$\therefore 540 = \frac{1}{R}\left(\frac{V}{\sqrt{2}}\right)^2 + \frac{1}{R}\left(\frac{mV}{2\sqrt{2}}\right)^2 + \frac{1}{R}\left(\frac{mV}{2\sqrt{2}}\right)^2,$$

$$= \frac{V^2}{2R}\left(1 + \frac{m^2}{2}\right).$$

But, $\dfrac{V^2}{2R}$ is the carrier power.

$$\therefore \text{ carrier power} = \frac{540}{\left(1 + \dfrac{0{\cdot}4^2}{2}\right)} = \underline{500 \text{ W}}.$$

Hence, the power in each side-frequency is <u>20 W.</u>

Q. 7. *Give a reasoned estimate of the signal bandwidth (before modulation) for the following transmissions:*

(*a*) *commercial speech,*

(*b*) *music, and*

(*c*) *a picture transmission system which scans at* 2 *lines/mm and transmits a* 50 *mm* × 40 *mm picture in* 2 *s with equal definition in both directions.*

A. 7. (*a*) The signal bandwidth for the commercial transmission of speech is 300–3,400 Hz.

The approximate frequency range for speech is 100–10,000 Hz. The transmission bandwidth is reduced to 300–3,400 Hz because of the comparatively poor frequency responses of the transmitters and receivers used, and to increase the availability of channels in the frequency spectrum of the transmission system. The intelligibility of the speech is not seriously affected.

(*b*) The signal bandwidth for music is 50–10,000 Hz.

Music has a wider frequency range than speech and is listened to for pleasure, so that a reduction in bandwidth has a more serious effect.

The microphones and loudspeakers used for music have better response characteristics than are necessary for speech, and a compromise between cost and quality is obtained by using the frequency range 50-10,000 Hz.

However, in both cases the ear is somewhat accommodating, so that aural reception appears better than the bandwidths suggest.

(*c*) It is assumed that the picture width is 50 mm, and that horizontal scanning is used. The number of horizontal lines $= 2 \times 40 = 80$, and for equal definition, the number of elements in a horizontal line $= 2 \times 50 = 100$.

The time taken to scan one horizontal line

$$= \frac{2}{80} = 25 \text{ ms.}$$

If, on one horizontal line, the elements are black and white alternately (this is the worst-case consideration), then the fundamental frequency from the light detector

$$= \frac{100}{2 \times 25 \times 10^{-3}} = 2 \text{ kHz.}$$

Thus, provided it is only necessary to transmit the fundamental frequency, the minimum bandwidth required is 2 kHz.

A picture transmission system must be able to transmit d.c. signals, to allow for periods of constant signal level, due to areas of even optical intensity in the original picture.

Q. 8. *The transistor used in the amplifier circuit shown in Fig. 1 has, at the bias-point, hybrid parameters $h_{ie} = 2$ kohm, $h_{fe} = 40$, and both h_{re} and h_{oe} are negligible. Calculate the voltage gain between input and output, assuming the bias components and coupling capacitor to have negligible effect on the signal, when terminals A and B are*

(*a*) *open-circuited, and*
(*b*) *connected to a 75-ohm resistor.*

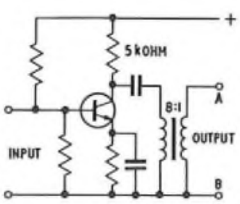
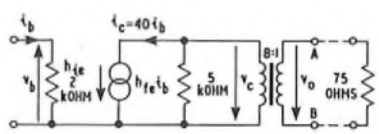


Fig. 1



(*a*)

A. 8. (*a*) Sketch (*a*) shows the *h*-parameter equivalent circuit.

When the output terminals are open-circuited, the effective load in the collector circuit is 5 kohm.

Now, $v_b = h_{ie} i_b = 2{,}000\, i_b$,

and $i_c = h_{fe} i_b = 40\, i_b$.

Also, $v_c = 5{,}000 i_c = 200 \times 10^3 \times i_b$.

$$\therefore \frac{v_c}{v_b} = \frac{200 \times 10^3 \times i_b}{2{,}000 i_b} = 100.$$

$$\therefore \text{voltage gain} = \frac{v_o}{v_b} = \frac{v_o}{v_c} \times \frac{v_c}{v_b} = \frac{100}{8} = 12 \cdot 5.$$

(*b*) With a 75-ohm load across terminals A and B, the effective primary impedance of the transformer is $8^2 \times 75$ ohms.

Thus, the effective collector load is

$$\frac{5{,}000 \times 8^2 \times 75}{5{,}000 + (8^2 \times 75)} = 2 \cdot 45 \text{ kohm.}$$

Then, $$\frac{v_c}{v_b} = \frac{2 \cdot 45 \times 10^3 \times 40 \times i_b}{2{,}000 i_b} = 49.$$

$$\therefore \text{voltage gain} = \frac{v_o}{v_b} = \frac{49}{8} = 6 \cdot 13.$$

Q. 9. *Explain how the electron beam in a cathode-ray tube may be deflected*

(*a*) *electrostatically, and*
(*b*) *magnetically.*

In each case, discuss the factors on which the deflexion sensitivity depends.

A. 9. The electron gun of a cathode-ray tube is designed to produce a beam of electrons, which emerges at a uniform velocity, proportional to the final-anode voltage, E_o, in a straight line along the axis of the tube.

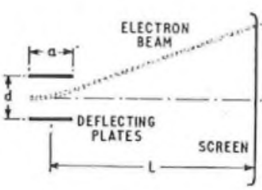
(*a*) To deflect the beam electrostatically, two thin, rectangular, nickel plates are fixed inside the tube parallel to, and equidistant from, the axis, as shown in sketch (*a*). When a voltage, *E*, is applied across the plates, the electric field produced between them deflects the beam as indicated. The path of the electrons between the plates is parabolic, and the electrons continue to the screen in the straight line of the tangent to this parabola at the point of emergence.

The deflexion, *D*, at the screen is proportional to

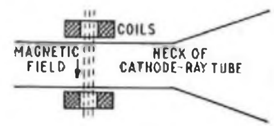
(*i*) the applied voltage, *E*,
(*ii*) the distance, *L*, from the centre of the plates to the screen,
(*iii*) the axial length, *a*, of the plates,
(*iv*) the reciprocal of the distance, *d*, between the plates, and
(*v*) the reciprocal of the axial velocity of the electrons passing between the plates, which, in turn, is proportional to the final-anode voltage, E_o.

Thus, $$D \propto \frac{EaL}{E_o d}.$$

The addition of two more deflecting plates, in the plane at right angles to the first set of plates, gives control of the beam-deflexion in the horizontal direction.



(*a*)



(*b*)

(*b*) To deflect the beam magnetically, two coils, having many turns of fine, insulated wire, are saddled across the neck of the tube between the final anode of the electron gun and the screen, as shown in sketch (*b*). When a current is passed through the coils, which are connected in series-aiding, an approximately uniform magnetic field is produced at right angles to the axis of the tube. The electron beam is deflected in a plane at right angles to both the axis of the tube and the axis of the coils. The path of the beam is similar to that described in part (*a*).

The deflexion is given by the equation

$$D \propto \frac{Bbl}{\sqrt{E_0}}, \text{ where } B \text{ is the flux density (teslas),}$$

b is the breadth of the magnetic field (metres), and
l is the distance from the centre of the magnetic field to the screen (metres).

Q. 10. *Fig. 2 shows the circuit diagram of a transformer-coupled, transistor power amplifier. The 1-ohm load resistor is coupled to the collector circuit via the 10 : 1 transformer. The signal causes the collector current to vary sinusoidally between maximum and minimum instantaneous values of 110 mA and 10 mA respectively. Calculate*

(*a*) *the peak-to-peak collector voltage swing,*
(*b*) *the load power,*
(*c*) *the collector efficiency, and*
(*d*) *the collector dissipation.*

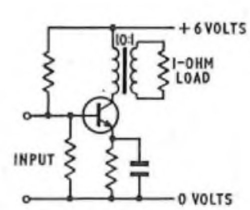


Fig. 2

Students were expected to answer any six questions

Q. 1. *Telegraph signals may be affected by characteristic, fortuitous and bias distortion.*

(*a*) *What are the possible causes of each type of distortion on a multi-channel voice-frequency system?*

(*b*) *With the aid of sketches, describe how each type of distortion would be recognized and measured by inspecting the display on an electronic distortion-measuring set.*

A. 1. (*a*) Fortuitous distortion on a multi-channel voice-frequency (m.c.v.f.) telegraph circuit is mainly caused by interference from neighbouring circuits or channels. Intermodulation, or crosstalk, between adjacent channels, or an irregular transmission path, can also contribute to this type of distortion. When the interference reaches a sufficiently high level to mask or delay a transition, or to give the effect of an additional transition during a character, the circuit is affected by fortuitous distortion.

Bias distortion, which consists of the lengthening of each positive (or negative) element at the expense of elements of the opposite polarity, may be caused by maladjustment of the receive relay, or by inaccuracies in the gain-compensation circuit. If the signal voltages applied to the tongue of the receive relay are not equal, this also causes bias distortion, as may hysteresis effects, or varying earth-currents. Bias distortion may normally be minimized by adjustments to the apparatus.

Distortion due to the inherent electrical characteristics of the circuit is known as characteristic distortion, and may be caused by a low mean-state-of-gain compensation, or by poor filter response. This type of distortion occurs consistently with any given series or combination of signal elements.

(*b*) See A.1, Telegraphy C, 1970. *Supplement*, Vol. 64, p. 43, July 1971.

Q. 2. (*a*) *Under what circumstances would* (*i*) *a telegraph repeating relay, and* (*ii*) *a telegraph regenerative repeater, be used?*

(*b*) *With the aid of a block diagram, describe the operation of a regenerative repeater.*

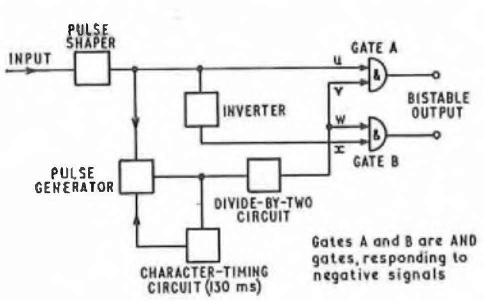
(*c*) *List the essential features of such a repeater, and state why each is important.*

A. 2. (*a*) (*i*) The strength of telegraph signals transmitted over a line is attenuated due to the effects of line inductance, capacitance and resistance. These line constants affect both the value of the line current and the shape of the telegraph signal, so that the response of the receiver may not be uniform. In the limit, the received current may be insufficient to operate the receiver for some elements, and the signal is, therefore, mutilated. This effect limits the transmission speed, or the length of line over which signals can be transmitted satisfactorily. The function of a repeating relay is to receive the attenuated signals at one end of a section of line, and re-transmit them as square-wave signals at the correct voltage levels. The relay cannot, however, correct any time-distortion in the received signal, and the transmitted signal has the same amount of time-distortion as the received signal. A repeating relay is used on a long physical line, where the overall distortion is not excessive.

(*ii*) A regenerative repeater accepts distorted telegraph signals and re-transmits them as undistorted signals. This type of repeater is used for circuits where the amount of distortion could be expected to exceed the margin of the receiver. For example, a repeater would be inserted in a circuit where the number of channels linked in tandem exceeded the limits for a switched network, such as the telex network. By the use of regenerative repeaters, a circuit may include any number of sections, provided that a repeater is introduced before the distortion reaches an unacceptable level.

(*b*) The sketch shows a block diagram of a regenerative repeater.

When the line is idle, the input is at negative potential, so that input u to gate A is negative, and input x to gate B, fed via the inverter, is positive. When a telegraph signal is received, the *start* element, which is of positive polarity, is applied to the pulse generator, and this generates accurate reversals at a frequency of 100 Hz. The divide-by-two circuit



reduces the reversals to accurate narrow pulses of negative potential at 20 ms intervals. The pulses are applied to gates A and B, at inputs v and w, 10 ms after the *start* element has been detected, and are used to examine the incoming signal at the theoretical centre of each character element. If the character element is of negative potential (*mark* signal), gate A conducts, as inputs u and v are at negative potential when the timing pulse is applied, and gate B does not conduct, as input x is at positive potential and input w is at negative potential. The bistable output transmits negative potential to line. After 20 ms, the input is again examined by the timing pulse, and, if the line potential is positive (*space* signal), gate B conducts, as inputs w and x are negative, and gate A does not conduct. The bistable output then transmits a positive signal for the ensuing 20 ms.

Therefore, incoming signals are repeated to the outgoing circuit, and, as the duration of the timing pulses is very short (a few microseconds) and the pulse generator is stable, the incoming signals may bear almost 50 per cent early or late distortion, and still be regenerated as perfect signals.

(*c*) The essential features of a regenerative repeater are

(*i*) an accurate and stable pulse generator, so that the repeater may repeat the signals as perfect telegraph signals at the correct interval,

(*ii*) a short timing pulse, to ensure that the margin of the repeater is as high as possible,

(*iii*) short-*start*-signal rejection, which is required in order to reject spurious *start* signals of less than 10 ms duration,

(*iv*) automatic *stop*-signal insertion, which ensures that the repeater minimizes the effect of badly-distorted received signals, by inserting a *stop* element 130 ms after the receipt of a *start* element, irrespective of the line polarity, and

(*v*) suppression of the automatic *stop*-signal-insertion facility. This is required for circuits on systems, such as telex, where the clearing signal is a long signal of the same polarity as the *start* signal. The reception of five positive code-elements in one character causes the suppression of the automatic *stop*-signal-insertion facility, and the correct positive-potential clearing signal is transmitted.

Q. 3. (*a*) *What factors govern the maximum amount of power which may be transmitted to line for each channel of a multi-channel voice-frequency system?*

(*b*) *What checks and modifications must be made to a* 4-*wire speech circuit before it is suitable for use by such a system?*

A. 3. (*a*) The value of the current transmitted to line must be high enough to minimize crosstalk interference from neighbouring circuits and channels, and give a good signal-to-noise ratio, but must not be so high as to cause interference in adjacent circuits. In addition, the current must not overload the line amplifiers, as this would cause distortion of the waveforms, due to the production of harmonics. The power in each channel is adjusted to a value which ensures that the overall limit of the system is never exceeded.

If the maximum power permitted for a system of N channels is W watts, with a line of characteristic impedance Z ohms, the maximum voltage, E, is obtained from the formula $E = \sqrt{(WZ)}$.

The voltage is at a maximum when all the carrier frequencies reach peak amplitude simultaneously. The maximum permissible voltage/channel, e, is given by

$$e = \frac{E}{N} = \frac{\sqrt{(WZ)}}{N} \text{ volts,}$$

and the maximum power/channel

$$= \frac{e^2}{Z} = \frac{WZ}{N^2 Z} = \frac{W}{N^2} \text{ watts.}$$

The maximum power for a multi-channel voice-frequency (m.c.v.f.) system has been standardized internationally at 5 mW. For an 18-channel system, the maximum power/channel

$$= \frac{5 \times 10^{-3}}{18^2} \simeq 15\ \mu\text{W}.$$

This enables telegraph systems to use telephone circuits with negligible interference to neighbouring circuits. A value of 10 μW/channel is used in practice, as it has been found that the slight decrease in signal-to-noise ratio is more than compensated for by the reduction in inter-channel interference due to cross-modulation.

(*b*) When a 4-wire speech circuit is used for an m.c.v.f. telegraph system, the circuit is split into two 2-wire channels to provide the *go* and *return* circuits, and any bridging equipment not required is removed. The voice-frequency signalling and ringing equipment is disconnected, and, at each end of the circuit, the 2-wire-to-4-wire hybrid termination is detached. Echo-suppressors are removed.

To ensure that the performance of all telegraph channels is similar, the circuits are adjusted so that the attenuation at any frequency does not vary from the attenuation at 800 Hz by more than 1 dB. For frequency-modulated systems, the frequency drift must also be kept low.

Q. 4. (*a*) *Why is synchronization of transmitting and receiving equipment important for*

(*i*) *a fascimile system, and*
(*ii*) *a time-division-multiplex system?*

(*b*) *Explain how this is achieved in one of the above systems.*
(*c*) *How does a start-stop system differ from a synchronous system and what advantages does it offer?*

A. 4. (*a*) (*i*) For a facsimile system, it is necessary for the transmitting and receiving equipment to operate in synchronism so that the received picture matches the document which is being transmitted.

For a description of the effects of incorrect synchronization, see A.9, Telegraphy C, 1971, *Supplement*, Vol. 65, p. 37, July 1972; and A.10 of this examination paper.

(*ii*) A time-division-multiplex system operates on the principle that each of several channels of communication has exclusive use of the transmission line in turn for a short period of time. The time period may be for the duration of a character or for the duration of an element, but as each piece of information is received, it is necessary for the receiver to allocate it to the correct channel. The receiver must, therefore, start by connecting the first character or element to be received to the first output channel, and must thereafter continue at the same speed as the transmitter, so that successive characters or elements are connected to the correct output channels in turn.

(*b*) Synchronism for the facsimile system is obtained at each station by using synchronous motors connected to a frequency-controlled mains supply, such as the national grid. If the same frequency-controlled supply is not available at each station, a phonic motor is used, which is controlled by a tuning-fork-controlled oscillator. The motor consists of a stator which is fed at a frequency of 1,020 Hz from the oscillator, together with a polarizing current, to set up a rotating magnetic field. A toothed, soft-iron rotor moves inside the stator, and, if the rotor is run up to speed such that it moves forward one tooth for each cycle of frequency, it will run synchronously.

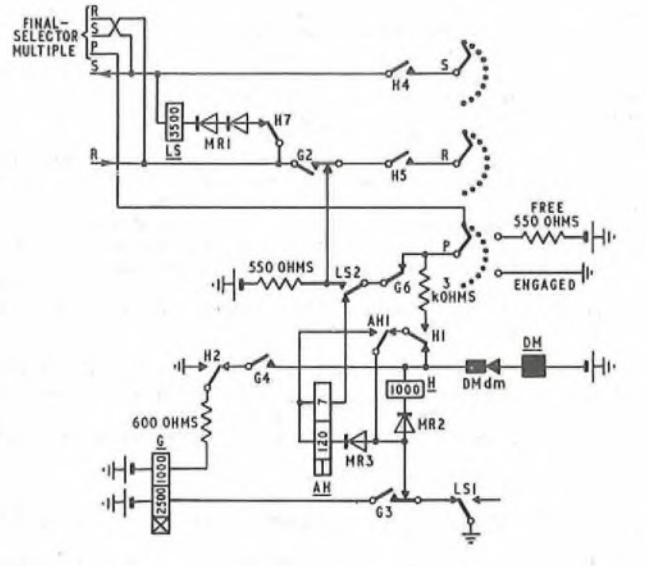
To ensure that the transmitter and receiver are in phase, the drum at the transmitter is started from a pre-determined position by means of a clutch. At the same time, a starting signal is transmitted to line to operate the starting clutch of the distant machine, which then starts from the same relative position.

(*c*) Start-stop working differs from synchronous working in that the transmitter and receiver do not have to be in synchronism. Both mechanisms remain at rest until a *start* signal is sent from the transmitter at the start of each character. The *start* signal starts the mechanism at the receiver, which runs for the length of one character and then returns to the rest position. In this way, the speeds of the transmitter and receiver only require to be accurate to within 0·5 per cent of the standard speed. If start-stop working were not adopted, the slightest speed difference would become cumulative, and the transmitter and receiver would get out of step.

The advantage of start-stop working is that it is possible for two instruments to work together with the minimum of attention to the speed adjustment. In addition, complex switching networks can be designed to connect one station to another such that, when the connexion has been established, the two machines may work to one another in the same manner as two directly-connected teleprinters. There is no requirement for synchronizing the whole system.

Q. 5. (*a*) *With the aid of a circuit diagram, explain the operation of the station line-circuit in an automatic telex exchange when a calling signal is received from the subscriber's station equipment.*
(*b*) *Describe the operation of the switching relays in preventing a double connexion when two station-line-circuit switches try to seize the same group selector simultaneously.*

A. 5. (*a*) The circuit diagram of a station line-circuit is shown in the sketch. When the circuit is disengaged, positive potential is connected to the R-wire from the subscriber's station, and earth potential is connected to the S-wire. Hence, relay LS is operated. When the subscriber wishes to initiate a call, the CALL key at the station is operated, which applies negative potential to the R-wire and, thus, releases relay LS. Contact LS2 disconnects the 550-ohm negative potential from the final-selector-multiple P-wire, thus preventing seizure of the circuit by an incoming call. Contact LS1 connects earth potential, through contacts AH1 and H1, to the drive magnet of the uniselector. The interrupter springs cause the uniselector to drive until a free outlet is found on the P-bank; a disengaged first group-selector is indicated by a 550-ohm negative potential on the P-bank outlet. Relay AH operates from the the earth potential at LS1, through rectifier MR3, the 120-ohm and 7-ohm coils of relay AH in series contacts LS2 and G6, and the negative potential on the free outlet. Contact AH1 disconnects the uniselector drive circuit, so that the uniselector remains on the disengaged contact, and removes the short-circuit from relay H, which operates. Contacts H4 and H5 connect the S- and R-wires to the first group-selector, contact H7 disconnects relay LS, and contact H2 operates relay G. Relay AH releases when earth potential is connected to the P-wire from the succeeding switch. Contact AH1 holds relay H when contact G3 operates, and contact G3 holds relay G. Contact G2 completes the switching of the R-wire.



The station is now connected to the first group-selector and the time-zone equipment. A *proceed-to-select* signal is transmitted to the subscriber to indicate that dialling may begin.

(*b*) Relay AH operates when a disengaged outlet is found. Contact AH1 disconnects the drive magnet and, at the same time, connects a short-circuit across the 120-ohm coil of relay AH, leaving the relay held by the current through the 7-ohm coil. The earth potential through the 7-ohm coil prevents a second uniselector from switching to this outlet, by limiting the current to a value less than that necessary to operate the second AH relay through its 120-ohm and 7-ohm coils in series. If two uniselectors try to seize a free outlet simultaneously, one AH relay operates faster than the other, and applies the 7-ohm earth potential to the outlet, causing the second relay to release. The second switch then continues to search for a free outlet.

Q. 6. (*a*) *Draw a trunking diagram for a typical telex zone exchange.*
(*b*) (*i*) *Explain why routing translators are provided in such an exchange.*
(*ii*) *List the principal facilities provided by such equipment.*

A. 6. (*a*) See A.7, Telegraphy C, 1969. *Supplement*, Vol. 63, p. 53, Oct. 1970.
(*b*) See A.5, Telegraphy C, 1970. *Supplement*, Vol. 64, p. 45, July 1971.

Q. 7. (*a*) *What are the effects of the following on telegraph transmission over radio circuits?*

(*i*) *Fading.*
(*ii*) *Multi-path propagation.*
(*iii*) *Interference.*

(*b*) *Explain measures which may be taken to overcome these defects.*

A. 7. (*a*) (*i*) Fading is the variation in signal strength due to changes in the condition of the propagation path. Propagation at radio frequencies involves reflection from, and refraction by, the ionized layers of the atmosphere. The parameters of these layers vary, so that the degrees of reflection or refraction vary. Hence, the strength of the received signal changes, sometimes to such an extent that the signal is lost altogether.

(*ii*) Multi-path propagation is caused by the signal taking two or more paths through the atmosphere. At the receiving station, the signal may arrive earlier over one path than over a second, reflected, path, the difference in arrival-time amounting perhaps to several milliseconds. The effect of this is to lengthen or shorten the signal element which, in severe cases, could be sufficient to lose the significance of the element and cause mutilation of the message.

(*iii*) Interference is normally caused by transmissions on neighbouring channels, or systems, affecting the channel in use. The power transmitted by a radio channel must be as high as possible, so as to operate the transmitter at maximum efficiency and to ensure that a strong signal is received at the distant terminal. In addition, because of the increasing demand for circuits, the maximum use must be made of the available bandwidth. Interference results in increased distortion of telegraph signals, and may cause mutilation of a message.

(*b*) To overcome these defects, several methods may be used. Diversity reception, whereby the better of two signals is selected from spaced receiving aerials, may be used to combat fading. Dual-frequency transmission may also be used to give a similar effect.

Synchronous transmission, with the *start* and *stop* elements of each character suppressed, may be used to reduce the effect of multi-path propagation. The five code-elements only are transmitted, with the *start* and *stop* elements automatically re-introduced at the receiver. The receiver can provide a better margin for distorted elements, and can also avoid the possible loss of character synchronization due to the loss of a *start* or *stop* element.

Synchronous working can be extended to offer error-detecting facilities, by including one or more additional elements in each character. The number of elements in each character, or the ratio of positive to negative elements, can be fixed, so that, if an element is not received, or additional elements are received, the incorrect ratio alarms the receiver. The detection of errors can also be used to initiate the re-transmission of the mutilated characters until a sequence of signals of correct ratio is received. This system is known as automatic error-correction.

Q. 8. (*a*) *With the aid of a block diagram, explain the functioning of the position circuit and a connecting circuit when an international call is connected at a telex cordless switchboard.*

(*b*) *What indications are given to the operator of the progress of a call over a radio circuit?*

(*c*) *How is the caller charged for this type of call?*

A. 8. (*a*) See A.6, Telegraphy C, 1969. *Supplement*, Vol. 63, p. 52, Oct. 1970.

(*b*) The operator can monitor the progress of a call over a radio circuit by observing an illuminated panel associated with each connecting circuit. A steady lamp-glow indicates that the radio circuit is usable, and a flashing light indicates that the radio circuit is undergoing error-correction by the automatic-error-correcting (ARQ) equipment. A monitoring teleprinter is provided for each connecting circuit, and the operator can judge the quality of the transmission from the printed copy.

(*c*) The charge for a radio telex call depends on the number of characters that could be sent; that is, the charge is proportional to the time for which the circuit is usable. A uniselector mechanism, which responds to pulses generated by the ARQ equipment when the circuit is capable of passing traffic, operates an electrically-reset counting meter in 6 s increments. One meter is associated with each connecting circuit, and the operator reads the meter and enters on the call-charging ticket the total time for which the circuit was available during the call.

Q. 9. (*a*) *What are the advantages and disadvantages of using a private telegraph message-relay system, as compared with those of using the telex network?*

(*b*) *With the aid of a diagram, explain the operation of a push-button, torn-tape, message-relay centre.*

(*c*) *What arrangements are made in such a centre for dealing with a lost or mutilated message?*

A. 9. (*a*) A private message-relay system is preferred to a circuit-switching system, such as telex, when the amount of traffic to be carried is heavy, or when the facilities required are different from those offered by the public network. A private system normally consists of out-station circuits, connected to message-relay centres, with the centres inter-connected by long-distance, high-cost circuits. The long-distance circuits are worked in the duplex mode; that is, messages are transmitted in both directions at the same time, with automatic transmission from each centre. Provision is made for messages to queue at each centre, so that the centre-to-centre circuits are fully utilized to give the greatest economy. As each message is stored at each relay centre, it is possible to accord precedence to certain messages depending on the degree of priority; messages for broadcast to a number of stations can also be accepted. The assurance that a message will retain a place in the queue for eventual transmission is often an advantage when considering delays which may occur on other networks; in many cases, the time of transmission may be much shorter.

The disadvantages of a private network are the restricted number of destinations, the high cost of private circuits, the need to employ labour or to provide expensive equipment in the relay centre, and the requirement for a fixed message-format. Owing to the use of duplex circuits, there is no facility for conversing with the distant station, and, unless arrangements are made for an acknowledgement message to be sent, there is no assurance that a message has reached its destination.

(*b*) See A.7, Telegraphy C, 1970. *Supplement*, Vol. 64, p. 45, July 1971.

(*c*) To deal with lost or mutilated messages, it is customary to number each message between centres automatically. The numbers form part of the preamble to each message and are in sequence. The distant equipment, or operator, checks that the sequence is maintained, and lost or mutilated messages are re-transmitted on request. Monitor teleprinters are connected to provide copies of incoming and outgoing messages for re-transmission. On a manual system, the perforated tapes are also retained for several days, in case they are required. Further safeguards are provided by paper-fail, motor-fail, and incoming-message alarms.

Q. 10. (*a*) *With the aid of sketches, explain how the tonal qualities of a photograph may be converted into electrical signals for transmission over a line.*

(*b*) *Describe the effect on the received photograph if the correct speed, phase and index of co-operation are not maintained between transmitter and receiver.*

A. 10. See A.9, Telegraphy C, 1971. *Supplement*, Vol. 65, p. 37, July 1972.

Index of Co-operation

Apart from speed and phase, the third relationship which must be established to ensure complete synchronization is the index of co-operation. This is the relationship between the length and breadth of the photograph. This ratio must be the same at both the transmitter and receiver; otherwise, the received picture is elongated vertically or horizontally with respect to the original.

LINE PLANT PRACTICE C, 1973

Students were expected to answer any six questions

Q. 1. *A 90 m span of dropwire is tensioned at 40 N. As a result of ice-formation, the tension increases to 240 N. The dropwire consists of two steel conductors, each of cross-sectional area 0·4 mm², with a polyvinyl-chloride sheath. The weight of the dropwire is 0·15 N/m and Young's modulus, E, = 205 kN/mm². Assume that the ice coating is uniform over the length of the dropwire and neglect the strength of the ice. Find the weight of the ice coating per metre run.*

A. 1. The original length of the wire, l_1, is given by

$$l_1 = L + \frac{8d^2}{3L},$$ where L is the span length (metres), and d is the dip (metres).

Now, $$d = \frac{wL^2}{8T_1},$$ where w is the weight of the wire (newtons per metre), and T_1 is the original tension (newtons).

$$\therefore\ l_1 = L + \frac{w^2L^3}{24T_1^2},$$

$$= 90 + \frac{0{\cdot}15^2 \times 90^3}{24 \times 40^2},$$

$$= 90{\cdot}43 \text{ m.}$$

From Hooke's law, $$\frac{l_2 - l_1}{l_1} = \frac{T_2 - T_1}{A} \times \frac{1}{E},$$

where l_2 is the length of the wire when ice-coated (metres), T_2 is the tension when ice-coated (newtons), and A is cross-sectional area (metres²).

$$\therefore\ l_2 = \frac{(240 - 40)\ 90{\cdot}43}{205{,}000 \times 10^6 \times 0{\cdot}8 \times 10^{-6}} + 90{\cdot}43,$$

$$= 90{\cdot}54 \text{ m.}$$

If w_i is the weight of the wire per metre with the ice coating,

then, $$l_2 = L + \frac{w_i^2L^3}{24T_2^2}.$$

$$\therefore\ w_i = \sqrt{\left\{\frac{24(l_2 - L)T_2^2}{L^3}\right\}},$$

$$= \sqrt{\left\{\frac{24 \times (90{\cdot}54 - 90) \times 240^2}{90^3}\right\}},$$

$$= 1{\cdot}012 \text{ N/m.}$$

Hence, the ice coating weighs $1{\cdot}012 - 0{\cdot}15 = \underline{0{\cdot}862 \text{ N/m}}$.

Q. **2.** (*a*) *Discuss the effects of various lubricants on the coefficients of static and dynamic friction between polyethylene-insulated and sheathed cable and* (*i*) *glazed earthenware, and* (*ii*) *polyvinyl-chloride duct.*

(*b*) *What effect does the reduction of friction between cable and duct have on the economics of cable installation?*

(*c*) *Why do the maximum working tensions allowed for drawing-in cable with aluminium conductors differ from those of the equivalent cable with copper conductors?*

A. **2.** (*a*) The table shows the approximate coefficients of friction between polyethylene-sheathed cable and polyvinyl-chloride (p.v.c.) and earthenware ducts, using different lubricants.

Lubricant	Coefficient of Friction			
	P.V.C. Duct (90 mm bore)		Earthenware Duct (92 mm bore)	
	Static	Dynamic	Static	Dynamic
No lubricant	0·38	0·28	0·34	0·33
Water	0·29	0·27	0·26	0·19
Dry French chalk (talc)	0·28	0·22	0·32	0·29
Talc-water slurry	0·20	0·18	0·21	0·17
Liquid paraffin	0·22	0·13	0·15	0·12
Petroleum jelly	0·20	0·12	0·29	0·19
Glycerine	0·195	0·10	0·21	0·17
Bentonite	0·32	0·18	—	—
Graphite	0·225	0·19	—	—

Graphite is rather unpleasant to use and could cause complaints from users. Bentonite sets to a solid when mixed with water and dried, which could cause blocked ducts. Glycerine, being a chemical break-down of sugar, could cause a growth of mould in manholes. Hence, French chalk and liquid paraffin appear to be the most suitable lubricants at the present time.

(*b*) Reducing friction between cable and duct allows longer cable-lengths to be drawn-in, using the same winching equipment. If the length of cable that can be drawn-in is increased, then the number of lengths required to cable between two points is reduced. Therefore, the number of joints required is reduced.

In a straight, unlubricated track, a 20 kN winch draws in 800 m of 254/1·27 paper-core-quad-trunk cable. When the track is lubricated, the same winch draws in 1,200 m of cable. Thus, in a length of 2,400 m, a reduction of one joint is made. The basic cost of jointing this cable is approximately three times the cost of lubrication using French chalk. However, the cost of lubrication using liquid paraffin is about $1\frac{1}{2}$ times the cost of making a joint.

For a 1200/0·6 polyethylene unit-twin cable, the cost savings are more significant. The cost of a joint is $2\frac{1}{2}$ times the cost of lubrication using liquid paraffin, and $11\frac{1}{2}$ times the cost of using French chalk.

(*c*) When electrically-equivalent cables having copper and aluminium conductors are stressed under steadily increasing loads, they have roughly comparable breaking stresses. However, when subjected to repeated impulsive loadings, the aluminium cables only withstand 50 per cent of the number of applications of the load as the equivalent copper cable. The major reason for this difference is that the maximum elongation of copper before breaking is about 15 per cent, but, for aluminium, it is only about 2 per cent. Also, aluminium work-hardens when stressed or bent. Therefore, conditions arise where the stresses in the aluminium conductors do not even out between all of the conductors, as they do in copper cables, and necking and breakage of the individually-stressed conductors occurs.

The force required to pull in a length of aluminium cable is about half that required for the same length of electrically-equivalent copper cable.

Q. **3.** (*a*) *What is Poisson's ratio, and when is it used?*

(*b*) *A steel rod is subjected to a tension of* 12 *kN. If the rod has a diameter of* 24 *mm before tension was applied, find*

(*i*) *the decrease in diameter of the rod, and*

(*ii*) *the reduction in area, expressed as a percentage of the original cross section.*

Assume Young's modulus, E, = 205 *kN/mm*², *and Poisson's ratio,* $m = \frac{10}{3}$.

A. **3.** (*a*) When a longitudinal stress is applied to a bar of elastic material, the resulting longitudinal strain is accompanied by a lateral strain; that is, there is also a change in diameter. The ratio of longitudinal strain to lateral strain is constant within the elastic range of the material, and is known as Poisson's ratio. Its value is of importance in relation to reinforced concrete, because too great a change in reinforcing-bar diameter could lead to a loss of bond between the steel and the concrete.

(*b*) (*i*) Longitudinal strain $= \dfrac{T}{EA}$,

where T is the tension (newtons), and A is the cross-sectional area (metre²),

$$= \frac{12 \times 10^3}{205 \times 10^9 \times \pi \times 12^2 \times 10^{-6}},$$
$$= 0{\cdot}000129.$$

$$\text{Now, lateral strain} = \frac{\text{longitudinal strain}}{m},$$
$$= \frac{0{\cdot}000129 \times 3}{10},$$
$$= 0{\cdot}0000387.$$

$$\text{Hence, decrease in diameter} = 24 \times 0{\cdot}0000387,$$
$$= \underline{0{\cdot}00093 \text{ mm}}.$$

$$\text{New cross-sectional area} = \left(\frac{24 - 0{\cdot}00093}{2}\right)^2 \times \pi,$$
$$= 452{\cdot}3542 \text{ mm}^2.$$
$$\text{Original cross-sectional area} = 12^2 \times \pi,$$
$$= 452{\cdot}3893 \text{ mm}^2.$$
$$\therefore \text{ percentage change} = \frac{452{\cdot}3893 - 452{\cdot}3542}{452{\cdot}3893} \times 100,$$
$$= \underline{0{\cdot}0078 \text{ per cent.}}$$

Q. **4.** (*a*) *Describe the manner in which an a.c. electric railway can interfere with circuits in a nearby telephone cable.*

(*b*) *Explain how the interference is reduced by*

(*i*) *the rails, and*

(*ii*) *the introduction of booster transformers.*

A. **4.** See A.7, Line Plant Practice C, 1970. *Supplement*, Vol. 64, p. 77, Jan. 1972.

Q. **5.** (*a*) *Outline the basic theory of pre-stressed concrete, both post-tensioned and pre-tensioned.*

(*b*) *Describe creep in pre-stressed concrete, and say how its effect is reduced in practice.*

(*c*) *Describe, briefly, one type of pre-stressing anchor.*

A. **5.** (*a*) Pre-stressing is a technique of construction whereby initial compressive stresses are set up in a member to resist, or annul, the tensile stresses produced by the load. Since concrete is a material with a high compressive strength and a relatively-low tensile strength, the advantages of pre-stressing are considerable. In pre-stressed concrete, up to the limit of the working load, the steel pre-stressing wires are not used for reinforcement, but only as a means of producing a compressive strength in the concrete. A member made of pre-stressed concrete is permanently under compression, the stress varying with the load between chosen maxima and minima. As a consequence, there is complete avoidance of cracks under normal loads, and, under an overload, provided it is not greater than the elastic limit, any cracks that occur will close up again without deterioration in the structure. One advantage of pre-stressing is that, under dead load, a section may be designed to the minimum concrete stress at the top fibres and the maximum concrete stress at the bottom fibres. When the live load is applied, the stresses are reversed, giving the maximum concrete stress at the top fibres, and the minimum concrete stress at the bottom fibres.

With pre-stressed concrete, it is possible to obtain lighter members than with reinforced concrete, and considerable savings of concrete and steel are effected.

In the pre-tensioning process, a high-tensile-steel wire is stressed by means of hydraulic jacks, or some other mechanism, and firmly held by wedges. The concrete is then cast around the steel wire and allowed to set. When the concrete has fully hardened, the wedges are removed and the stress is, therefore, transferred from the wires to the concrete, due to adhesion between the steel and the concrete. This method is mostly used for pre-cast units, such as electric-light standards and railway sleepers, where large numbers of a particular unit may be required.

For post-tensioning, the steel wires or rods are coated with bitumen, tar or paper, so that they will not eventually adhere to the concrete. Alternatively, ducts are cast into the concrete, and the reinforcement is then fed into the ducts after the concrete has hardened. When the concrete is quite hard, the wires are stretched at the point where they emerge, and anchored by wedges in their stretched condition. The ducts are then filled under pressure with a cement grout, after which the ends of the beam or member are usually cemented over to provide a smooth surface.

The fundamental advantage of post-tensioning is that the reaction from stressing the wires is taken on the concrete, and there is no loss of stress due to elastic deformation, as with pre-tensioning. Furthermore, as the concrete has hardened, shrinkage in the concrete has already occurred.

(*b*) Creep, in both steel and concrete, is an important factor in pre-stressing. A steel wire, held under stress, shows an increase in elongation, or strain, over a period of time. A loss of stress due to a creep of 15–20 per cent is often allowed for in a pre-stressed structure, but the magnitude of the creep can be reduced by subjecting the wires to a brief overload before use. Creep also occurs in concrete, but this is often a useful property in that it enables a readjustment of stress to take place when local stresses of high intensity might, otherwise, result in failure of the structure.

(*c*) The sketch shows a common type of pre-stressing anchorage. It is operated on the wedge principle, and consists of a split cone fitted into an outer, cylindrical housing. As the cone is forced into the housing, the teeth on the inside of it grip the stressed wires.

Q. 6. (*a*) *Describe, with a sketch, the basic elements of a theodolite.*

(*b*) *Describe how this instrument is used to set out the stay bases for a stayed mast, and to ensure that the mast is vertical.*

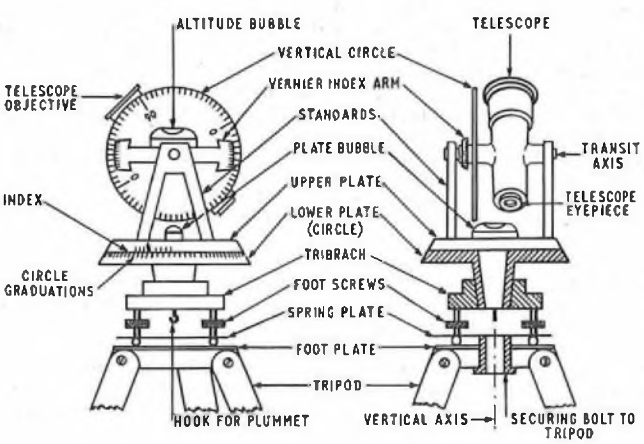
A. 6. (*a*) The essential features of a theodolite are shown in the sketch.

The main parts of a theodolite are the

(*i*) telescope,
(*ii*) levelling head,
(*iii*) lower and upper plates,
(*iv*) vertical measuring circle and standard,
(*v*) bubble tubes,
(*vi*) clamp and target screws, and
(*vii*) supporting tripod.

(*b*) Marking out the stay bases for a mast is done with the aid of a theodolite, a levelling staff, a measuring tape and ranging rods. The theodolite is set up at the mast-centre and truly levelled. With the main-table horizontal-scale clamped at zero, the line of direction for a stay base is sighted and the distance measured with the tape. It is assumed that the site is level, so that there is no need to correct the distance measured for sloping ground. To determine the other stay-base directions, the instrument is traversed horizontally through the angle appropriate to each stay line. Straddling pegs are set in the ground to identify the stay position for excavation purposes.

The mast is observed to be vertical by setting up the theodolite in a radial position along a stay line, at a distance outside the stay-base. The main table and the trunnion axis of the instrument must be truly levelled. With a parallel-sided mast, an edge is viewed in the telescope from the top to the bottom to observe any deflexion from the vertical.

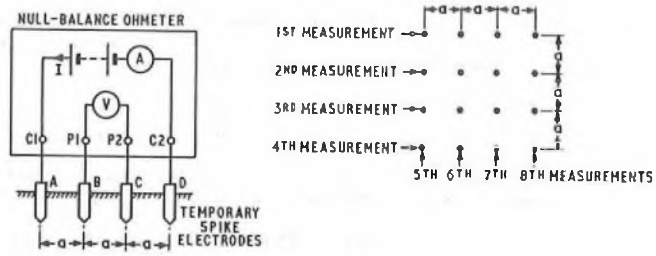


The telescope is then reversed through 180°, both in the horizontal and vertical scales, and the viewing repeated to ensure that the instrument is correctly aligned. A second position for the theodolite is chosen at approximately the same radial distance from the mast, but normal to the first stay line, and the procedure of vertical sighting is repeated.

Q. 7. (*a*) *Explain Wenner's four-electrode method for the measurement of soil resistivity.*

(*b*) *If the average value of a series of measurements is 5 ohms, and the spacing between electrodes is 7 m, calculate the mean soil resistivity.*

A. 7. (*a*) Sketch (*a*) shows the principle of the Wenner method of measuring earth resistivity.



(*a*) (*b*)

Considering a section of earth with a uniform resistivity, ρ ohm m, into which four electrodes A, B, C and D are inserted, a current, I, enters the earth at A and leaves at D. The distances, a, between the electrodes are assumed to be sufficient for the current distribution around the electrodes to be radial.

Thus, $\rho = 2\pi a \frac{V}{I}$ ohm m, where V is the voltage produced between electrodes B and C.

The electrodes are short metal spikes which are placed 6–15 m apart, the spacing depending on the depth at which the resistivity is to be measured. The spikes are not driven into the ground more than a distance $d = \frac{a}{6}$.

Direct current can be used for the measurements, but, due to polarization voltages of the electrodes and stray currents in the earth, the polarity of the current circuit and voltage circuit have to be simultaneously reversed very frequently. Using a null-balance ohmmeter, an integral hand-generator provides a low-frequency a.c. supply, and $\frac{V}{I}$ can be read directly by balancing with the instrument's calibrated rheostat.

Where the resistivity of an area is required, the spikes are arranged in a grid formation, as shown in sketch (*b*), and the average value of eight measurements is used.

(*b*) Substituting the values $a = 7$ m and $\frac{V}{I} = 5$ ohms in the above formula,

$$\rho = 2 \times \pi \times 7 \times 5 = \underline{219 \cdot 91 \text{ ohm m.}}$$

Q. 8. (*a*) *Explain the term pneumatic resistance as applied to a cable-pressurization scheme.*

(*b*) *The unit pneumatic resistance of a cable is 5·7. Calculate the rate of gas flow, under conditions of maximum leakage, at the far end of a 500 m length of this cable. The cable diameter is 46 mm, and dry air is applied at the exchange end of the cable at a pressure of* 0·062 *N/mm²*.

A. 8. (*a*) The pneumatic resistance of a cable is its resistance to gas flow through the spaces between the insulated conductors of that cable. The pneumatic resistance of a cable depends upon the length, size and type of cable concerned.

To determine approximately the volume of gas which has passed through a cable in a given time, the pneumatic analogy to Ohm's law, $F = \frac{P}{R}$, is used, where F is the rate of flow of gas through the cable (m³/h), analogous to electric current flow, P is the total pressure drop along the length of the cable (Pa), analogous to voltage, and R is the total pneumatic resistance, analogous to electrical resistance. The relationship between total pneumatic resistance and unit pneumatic resistance is given by $R = \frac{rl}{d^2}$, where r is the unit pneumatic resistance, d is the diameter of the cable (m), and l is the length of the cable (m).

(*b*) Combining the two equations above,

$$F = \frac{Pd^2}{rl},$$

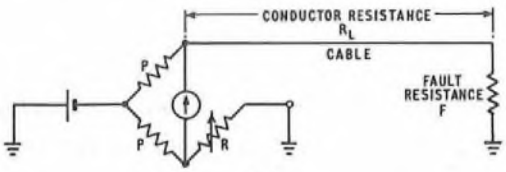
$$= \frac{0 \cdot 062 \times 10^6 \times (46 \times 10^{-3})^2}{5 \cdot 7 \times 500},$$

$$= \underline{0 \cdot 046 \text{ m}^3/\text{h}.}$$

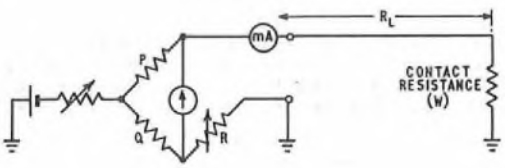
Q. **9.** *An earth fault has occurred in a submarine cable. Describe, with diagrams, the following methods of locating the fault:*
(*a*) *Mance's method, and*
(*b*) *Black's method.*

A. 9. (*a*) *Mance's Method*
A Wheatstone bridge is connected to the cable as shown in sketch (*a*), and the following tests are made.

(*i*) The ratio arms, P, are both set to 100 ohms, and the bridge is balanced, by adjusting the variable-resistance arm, R, to a value R_1 ohms.
(*ii*) The ratio arms are rapidly reset to 1,000 ohms, and the bridge is rebalanced to give a value R_2 ohms.



(*a*)



(*b*)

Tests (*i*) and (*ii*) are repeated in quick succession until consistent results are obtained.

Then, $$x = \frac{1{,}000R_1 - 100R_2}{(1{,}000 + R_2) - (100 + R_1)} \text{ ohms,}$$

where x is the sum of the conductor resistance, R_L, to the fault and the fault resistance, F, (ohms).

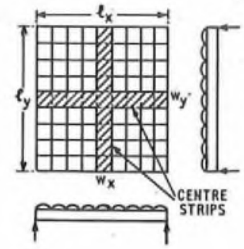
(*b*) *Black's Method*
A Wheatstone bridge, with a milliameter in series with the line, is connected as shown in sketch (*b*).
A negative battery is applied to the line and the deflexion of the galvanometer is observed. The current is reduced by a known ratio and the deflexion again observed. The bridge balancing resistor is then adjusted until similar deflexions are obtained, whichever value of current is applied.

Then, $$R_L = R - \left(\frac{100}{I} + W\right) \text{ ohms,}$$

where R is the balancing resistance (ohms), I is the initial current (amps), and W is the contact resistance (ohms), which can be taken as being equal to the resistance of 1 km of the conductor under test (or 1 nautical mile if this unit is being used).

Q. **10.** *A slab, of effective length* $1 \cdot 4$ *m and effective breadth* $0 \cdot 7$ *m, is simply supported along all four edges. It is subjected to a uniformly-distributed load of* $0 \cdot 20$ *N/mm²*. *Calculate the bending moment per metre run at the centre of the slab, parallel to*
(*a*) *the long side of the slab, and*
(*b*) *the short side of the slab.*

A. 10. Consider the rectangular slab illustrated in the sketch, freely supported along all four edges, having the dimensions l_x and l_y. The two central strips, of unit width, shown shaded, are deflected at mid-span by the same amount under any value of load, since they are coincident at this point. Let the total unit loading be w newtons/metre², the portion of load carried in the directions l_x and l_y be w_x and w_y respectively, I_x and I_y be the respective second moments of area per unit length in those directions, and E be Young's modulus for the material.



Using the general relationship for maximum deflexion, and equating deflexions at mid-span,

$$\frac{5}{384} \times \frac{w_x l_x^4}{EI_x} = \frac{5}{384} \times \frac{w_y l_y^4}{EI_y}.$$

$$\therefore\ w_x l_x^4 = w_y l_y^4, \text{ assuming } I_x = I_y.$$

$$\therefore\ \frac{w_x}{w_y} = \left(\frac{l_y}{l_x}\right)^4 = \left(\frac{1 \cdot 4}{0 \cdot 7}\right)^4 = 16.$$

Since $w = w_x + w_y$, then w is divided between the two strips in the ratio of 16 : 1, i.e. 17 parts.

(*a*) The bending moment per metre run of the short-span strip is, therefore, given by

$$M_x = \frac{16}{17} \times \frac{wl_x^2}{8},$$
$$= \frac{16}{17} \times \frac{0 \cdot 20 \times 10^6 \times (0 \cdot 7)^2}{8},$$
$$= \underline{11 \cdot 53 \text{ kN m.}}$$

(*b*) The bending moment per metre run of the long-span strip is given by

$$M_y = \frac{1}{17} \times \frac{wl_y^2}{8},$$
$$= \frac{1}{17} \times \frac{0 \cdot 20 \times 10^6 \times (1 \cdot 4)^2}{8},$$
$$= \underline{2 \cdot 88 \text{ kN m.}}$$

Note: In the calculations, w has the dimension newton/metre, since the strips under consideration are of unit width.

BASIC MICROWAVE COMMUNICATION C, 1973

Students were expected to answer any six questions

Q. **1.** (*a*) *What factors are of importance in deciding* (*i*) *the lower frequency limit, and* (*ii*) *the higher frequency limit, for microwave communication systems?*
(*b*) *The distance between a microwave transmitter and receiver is R. At mid-path, the line-of-sight just clears an obstacle by a distance, d, measured vertically. Explain briefly how the strength of the received signal may be expected to depend on the frequency selected for the system.*

A. 1. (*a*) (*i*) The following factors are of importance in deciding the lower frequency limit of a microwave communication system.

(1) The bandwidth available is proportional to the carrier frequency, and the channel capacity, therefore, decreases with decreasing frequency.
(2) The wavelength increases as the frequency is reduced and, consequently, the size of waveguide and microwave components becomes inconveniently large.
(3) The beamwidth of a parabolic aerial increases, and the aerial gain is reduced, as the wavelength increases. To compensate for this, aerials with larger diameters are required, with the attendant mounting and windage problems.
(4) To avoid ground reflections, the beam must clear obstacles by a distance dependent upon the wavelength. Thus, greater path clearances and, hence, higher aerials are required at lower frequencies.

(*ii*) The following factors are of importance in deciding the upper frequency limit.

(1) Attenuation due to rain, wet hail and snow becomes increasingly important at high frequencies, and limits the path length which can be used.

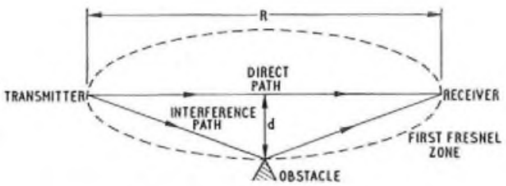
(2) The water-vapour absorption of the radio path increases steadily with frequency, and molecular resonances, such as those of water vapour at about 22 GHz and oxygen at 60 GHz, should be avoided.

(3) The availability of high-frequency components may be limited, and their cost high. Proportionately smaller powers and higher noise factors are produced by oscillators at higher frequencies.

(4) The dimensional accuracy required for components, waveguides and the surfaces of dish aerials increases with frequency.

(5) The tuning of resonant circuits becomes more critical at higher frequencies, and the stability poorer.

(b) The obstacle affects the strength of the received signal if $d < \frac{1}{2}\sqrt{(\lambda R)}$, where λ is the wavelength; that is, if the obstacle is within the first Fresnel zone. The surface of this zone includes all points where the indirect path (or interference path) is half a wavelength longer than the direct path. The sketch illustrates the first Fresnel zone.



The clearance depends upon the wavelength which, in turn, is inversely proportional to the frequency. At high frequencies, the obstacle is outside the first Fresnel zone, and the received-signal strength tends to that of a free-space path. At lower frequencies, the obstacle is embraced by the, now enlarged, Fresnel zone, and the received signal is affected by reflection, diffraction and absorption. The result is frequency-dependent enhancements or degradations of the received-signal strength, depending on the phase and amplitude of the combined interference- and direct-path signals.

Q.2. (*a*) *What is meant by the reflection coefficient due to a resistive load, R, terminating a transmission line of characteristic impedance Z_0?*

(*b*) *A low-loss coaxial cable, of length $\frac{3}{4}\lambda$, where λ is the wavelength, and characteristic impedance 75 ohms, terminates in a resistive load of resistance 150 ohms. The peak voltage across the load is found to be 40 volts. Calculate*

(*i*) *the peak voltage at the input to the line,*
(*ii*) *the peak current at the input to the line,*
(*iii*) *the distance from the load of the nearest current maximum, and*
(*iv*) *the fraction of the incident power reflected by the load.*

A. 2. (*a*) The reflection coefficient, ρ, is the ratio of the voltage reflected back along the line from the termination to the signal voltage incident on the termination. The reflexion coefficient is given by

$$\rho = \frac{R - Z_0}{R + Z_0}.$$

(*b*) (*i*) The voltage at the termination (V)

= the incident voltage (V_i) + the reflected voltage (V_r),

and $V_r = \rho V_i$.

$$\therefore \quad V = V_i(1 + \rho). \qquad \ldots\ldots (1)$$

$$\text{Now, } \rho = \frac{R - Z_0}{R + Z_0} = \frac{150 - 75}{150 + 75} = 0{\cdot}33.$$

From equation (1), $40 = V_i(1 + 0{\cdot}33)$.

$$\therefore \quad V_i = 30 \text{ volts,}$$

$$\text{and } V_r = 0{\cdot}33 \times 30 = 10 \text{ volts.}$$

Since both R and Z_0 are resistive, and ρ is positive, maximum values of the voltage standing wave occur at the load and at multiples of $\lambda/2$ from the load. Minimum values occur at distances of $\lambda/4$, $3\lambda/4$, etc. from the load. The maxima have a common peak value of $|V_i| + |V_r| = 30 + 10 = 40$ volts, and the minima of $|V_i| - |V_r| = 30 - 10 = 20$ volts, since for each distance of $\lambda/4$ along the line, the phase difference between V_i and V_r changes by $90° - (-90°) = 180°$.

As the input to the line is a distance of $3\lambda/4$ from the load, the peak input voltage is 20 volts.

(*ii*)
$$\frac{I_r}{I_i} = -\frac{V_r}{V_i} = -\rho = -0{\cdot}33,$$

where I_r and I_i are the reflected and incident currents, respectively.

$$\text{Now,} \quad I_i = \frac{V_i}{Z_0} = \frac{30}{75} = 0{\cdot}4 \text{ amp.}$$

$$\therefore \quad I_r = -0{\cdot}33 \times 0{\cdot}4 = -0{\cdot}133 \text{ amp.}$$

Maximum values of the current standing wave occur at distances of $\lambda/4$, $3\lambda/4$, etc. from the load, and have a peak value of $0{\cdot}4 + 0{\cdot}133 = 0{\cdot}533$ amp. Minimum values occur at the load and at distances of $\lambda/2$, λ, etc. from the load, and have a peak value of $0{\cdot}4 - 0{\cdot}133 = 0{\cdot}267$ amp.

Therefore, the peak current at the input to the line is 0·533 amp.

(*iii*) From part (*ii*), the distance from the load of the nearest current maximum is $\lambda/4$.

(*iv*) The fraction of the incident power reflected by the load

$$= \left|\frac{V_r I_r \cos\phi_r}{V_i I_i \cos\phi_i}\right|, \text{ where } \phi_r = \phi_i = 0,$$

$$= |\rho \times -\rho|,$$

$$= 0{\cdot}11.$$

Q. 3. (*a*) *Explain clearly, with the aid of a diagram, what is meant by a TEM wave.*

(*b*) *A TM_{01} wave is propagated in a hollow circular waveguide. In separate sketches, clearly indicate*

(*i*) *the E- and H-field patterns in the cross-section of the guide, and*
(*ii*) *the E-field pattern in the length of the guide.*

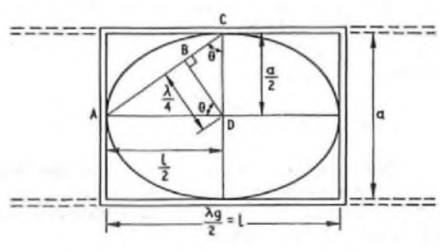
Q. 4. (*a*) *A length, l, of rectangular waveguide, having a broad dimension, a, and a narrow dimension, b, is closed at both ends to form a resonant cavity. Show that the lowest frequency, f_0, at which the cavity can resonate is that which has a free-space wavelength given by*

$$\lambda_0 = \frac{2l}{\sqrt{\left(1 + \frac{l^2}{a^2}\right)}}.$$

(*b*) *State, with reasons, what effect, if any, the narrow dimension has on the performance of the cavity.*

A. 4. (*a*) The lowest resonant frequency supported by the cavity is given by the dominant TE_{10} mode. The closed length of waveguide is half a guide wavelength ($\lambda_g/2$) long, and contains one magnetic loop of the TE_{10} wave-pattern.

The loop can be considered to be formed by two components propagated at angles of $\pm\theta$ to the axis of the waveguide in the plane of the broad dimension, as shown in the sketch.



The front of one of the waves, AC, moves from B to D, a distance of $\lambda/4$, where λ is the free-space wavelength, in a quarter of a period.

$$\text{Now,} \quad \sin\theta = \frac{BD}{CD} = \frac{\lambda}{2a}, \text{ and}$$

$$\cos\theta = \frac{BD}{AD} = \frac{\lambda}{2l}.$$

$$\therefore \quad \frac{\lambda^2}{4a^2} + \frac{\lambda^2}{4l^2} = 1, \text{ since } \sin^2\theta + \cos^2\theta = 1.$$

$$\therefore \quad \frac{\lambda^2}{4l^2}\left(\frac{l^2}{a^2} + 1\right) = 1,$$

$$\text{or, } \lambda = \frac{2l}{\sqrt{\left(1 + \frac{l^2}{a^2}\right)}}.$$

(*b*) The above formula shows that the resonant frequency is independent of the narrow dimension, b, of the cavity. However, the Q-factor of the resonator is affected by the narrow dimension. The Q-factor depends on the ratio of the energy stored to the wall losses per cycle which, in turn, and in a complex manner, depend upon b. Higher Q-factors are given by larger values of b, but b must always be appreciably less than a, so that secondary resonances, close to the required resonance, are avoided.

Q. 5. (*a*) *Explain briefly why a reflex klystron may function in several different modes. (Constructional details are not required.)*

(*b*) *By means of separate diagrams, show how the klystron output varies (i) in magnitude, and (ii) in frequency, as the reflector voltage is adjusted.*

(*c*) *State, with reasons, what mode is the more desirable for a reflex klystron used as*

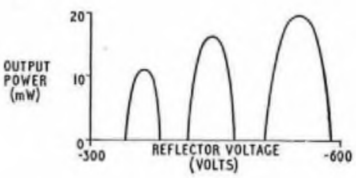
(*i*) *a source for a microwave test-bench, and*

(*ii*) *a local oscillator provided with automatic frequency control.*

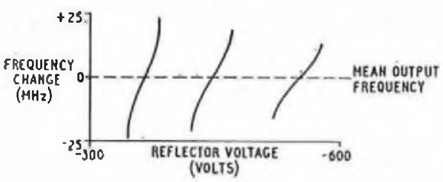
A. 5. (*a*) In a reflex klystron, the number of space-charge wavelengths in the drift distance from the resonator to the reflection point, and back again, must be equal to $(N + \frac{3}{4})$, where N is an integer. Different values of N constitute the different modes of oscillation. Higher values of N correspond to longer drift paths, and are produced by lower reflector voltages.

(*b*) Sketch (*a*) shows how the klystron output varies in magnitude as the reflector voltage is adjusted. As the reflector voltage becomes more negative, so the drift path shortens, and more-pronounced bunching occurs, giving rise to higher output powers. In each mode, the maximum power occurs when the phase of the feedback is exactly 0°. When the reflector voltage is varied either side of the peak, the phase of the feedback changes. The in-phase component is gradually reduced, and the power falls until the feedback can no longer replace the energy losses in the resonator circuit, when the oscillations suddenly cease.

Sketch (*b*) shows how the frequency of the output varies with the reflector voltage. For a given mode, changes in reflector voltage from the optimum point cause a phase shift in the feedback. The quadrature component of the feedback is equivalent to a reactance in the resonator circuit, and the resonant frequency is, therefore, altered.



(*a*)



(*b*)

(*c*) (*i*) For microwave test-bench purposes, and assuming that high output power and good frequency stability are not major considerations, a larger and more-useful electronic tuning range is given by the modes produced by the smaller (i.e. less negative) reflector voltages. For modulation tests, the linearity of the frequency-change/reflector-voltage characteristic is of importance, and the reflector voltage and mode giving the straightest characteristic over the required modulation range is chosen for this purpose.

(*ii*) For use as a local oscillator, the klystron must have adequate power to operate a crystal mixer. The klystron output should also contain as little frequency-modulation noise as possible, as this will appear in the mixer output. Therefore, the highest (i.e. most negative) reflector-voltage mode is preferable, as the power will be adequate, and the output will be less affected by noise on the reflector voltage. The smaller range, and any non-linearity of frequency-correction given by adjustments of the reflector voltage, are, nevertheless, adequate for automatic frequency control.

Q. 6. (*a*) *Give two reasons why little electromagnetic energy is radiated into space from the open end of a hollow waveguide. Explain briefly how its effectiveness as a radiator could be greatly improved.*

(*b*) *Deduce an expression for the power per unit area received at a distance, d, from an isotropic source radiating a total power, P.*

(*c*) *A microwave aerial has a gain of 600 and a beamwidth of 6·5°. State precisely what is meant by these terms.*

A. 6. (*a*) (*i*) The dimensions of the aperture, formed by the open end of a waveguide, are smaller than one wavelength of the guided wave. Therefore, the aperture radiates little energy, and this is distributed over a large angle.

(*ii*) The impedance into space at the open end of a waveguide is higher than that of the guide itself, and energy is, therefore, reflected back along the waveguide.

The effectiveness of a waveguide as a radiator could be improved by adding a flange, or by flaring the waveguide into a horn, so as to increase the aperture size and the directivity of the radiation. Any mismatch incurred can be corrected by a matching device, such as an iris placed behind the horn.

(*b*) An isotropic radiator is one which radiates power uniformly in all directions.

Consider a sphere of radius d, with the source at its centre. The surface area of the sphere is equal to $4\pi d^2$, and the power per unit area is the total power radiated divided by the surface area of the sphere.

$$\text{Thus, power per unit area} = \frac{P}{4\pi d^2}.$$

(*c*) The gain of the aerial is the ratio of the power per unit area at a point in the far-field, and on the axis of the beam, to the power which would be present if the aerial were replaced by an isotropic radiator, radiating the same total power.

The beamwidth is measured diametrically between points, off the axis of the aerial, where the power per unit area is 3 dB below the maximum power per unit area on the axis. In the question, the beamwidth is 6·5°. Assuming that all the radiated energy passes uniformly through a circle on the surface of the sphere considered in part (*b*), then the diameter of the circle subtends an angle of 6·5° at the centre of the sphere.

Q. 7. (*a*) *A receiver has a gain G, a bandwidth B, and a noise factor F. Prove that the noise power, N_r, introduced by the receiver is given by*

$$N_r = (F - 1)kTBG \text{ watts.}$$

(*b*) *When pulse-duration or pulse-position modulation is used, noise introduced into the system can, to some extent, be eliminated at the receiver. In the case of pulse-amplitude modulation, this is not possible. Explain these statements.*

A. 7. (*a*) The noise factor is defined as

$$F = \frac{\text{signal-to-noise power ratio at input}}{\text{signal-to-noise power ratio at output}},$$

$$= \frac{\text{input signal power}}{\text{output signal power}} \times \frac{\text{output noise power}}{\text{input noise power}},$$

$$= \frac{1}{G} \times \frac{GN_i + N_r}{N_i}, \text{ where } N_i \text{ is the input noise power,}$$

$$= 1 + \frac{N_r}{GN_i}.$$

But, $N_i = kTB$, where k is Boltzman's constant,
T is the temperature (K), and
B is the bandwidth (Hz).

$$\therefore\ F = 1 + \frac{N_r}{kTBG}.$$

$$\therefore\ \underline{N_r = (F - 1)\,kTBG \text{ watts.}}$$

(*b*) For pulse-duration modulation, the duration of the pulse is proportional to the amplitude of the modulating wave at the time of sampling. Similarly, for pulse-position modulation, the pulse is displaced in time from its datum position by an amount proportional to the amplitude of the modulating wave at the time of sampling. In both cases, the amplitude of the modulator output is constant, and amplitude variations, but not phase variations, introduced as a result of system noise, can be eliminated by limiting and clipping the pulses at the receiver. However, a wider bandwidth is required in order to transmit the shorter rise-times necessary to give this noise improvement at the receiver.

For pulse-amplitude modulation, the amplitude of the pulse carries information about the amplitude of the modulating wave, and clipping techniques cannot be used. Any amplitude noise, introduced by the transmission system, appears at the output of the receiver, and no noise improvement can be obtained.

Q. 8. (*a*) *Explain why transit time imposes a limit on the use of conventional valves in amplifiers.*

(*b*) *Draw a neat diagram of a travelling-wave tube, indicating the polarity and order of voltages applied to each of its electrodes. How, in this device, is the output isolated from the input?*

A. 8. (*a*) In low-frequency applications, the transit time of electrons in a valve is short compared with a period of the applied signal. At high frequencies, the transit time is an appreciable fraction of this period. This reduces the dynamic impedance between the anode and cathode, and it may become negative at some frequencies. At very high frequencies, the input conductance to the grid increases as the square of the frequency.

Special triodes are made for use at microwave frequencies, in which the transit time is reduced by reducing the gaps between the electrodes, up to a limit imposed by the constructional tolerances and mechanical stability. The voltages which can be applied to the minute gaps are also limited.

(*b*) See A.8, Basic Microwave Communication C, 1969. *Supplement*, Vol. 63, p. 69, Oct. 1970.

Q. **9.** (*a*) *State, with reasons, the order of bandwidth required in the intermediate-frequency (i.f.) amplifier of a receiver to be used for a frequency-modulated speech channel* (300–3,400 *Hz*), *using a deviation ratio of* 8. *In what way would the signal at the output of the receiver be affected if the deviation ratio were reduced to* 6*?*

(*b*) *A broadband characteristic is required in a multi-stage i.f. amplifier. With the aid of a diagram, explain why, for this purpose, it is preferable to use staggered tuning rather than low-Q circuits tuned to the same frequency.*

Q. **10.** (*a*) *Identify the circuit shown in Fig.* 1 *and, with the aid of waveform diagrams, describe its behaviour when triggered by a train of* −5 *volt pulses of short duration.*

(*b*) *What are the effects on circuit performance of*

(*i*) *time constant,* CR_1, *and*

(*ii*) *time constant,* CR_2*?*

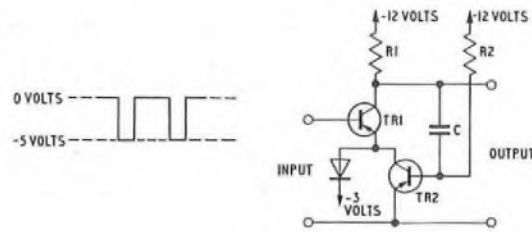


Fig. 1

PRACTICAL MATHEMATICS, 1974

Students were expected to answer any six questions

Q. **1.** (*a*) *An electromagnet exerts a pull, P, given by the formula*

$$P = \frac{B^2A}{8\pi g}.$$

Calculate P when B = 4,280, π = 3·142, *g* = 981 *and A* = 5·96.

(*b*) *By rearranging the formula* $I^2 = \dfrac{E^2}{R^2 + X^2}$, *make X the subject.*

(*c*) *A speedometer reads* 5 *per cent above the true speed. Calculate*

(*i*) *the reading on the speedometer for a true speed of* 75 *km/h, and*

(*ii*) *the true speed for a reading of* 90 *km/h.*

A. 1. (*a*) $P = \dfrac{B^2A}{8\pi g} = \dfrac{4{,}280 \times 4{,}280 \times 5{\cdot}96}{8 \times 3{\cdot}142 \times 981},$

$$= \frac{535 \times 4{,}280 \times 5{\cdot}96}{3{\cdot}142 \times 981}.$$

Using logarithmic tables,

$$\begin{aligned}\log_{10} P &= \log_{10}(535 \times 4{,}280 \times 5{\cdot}96) - \log_{10}(3{\cdot}142 \times 981),\\ &= (2{\cdot}7284 + 3{\cdot}6314 + 0{\cdot}7752) - (0{\cdot}4972 + 2{\cdot}9917),\\ &= 7{\cdot}1350 - 3{\cdot}4889,\\ &= 3{\cdot}6461.\end{aligned}$$

Taking antilogarithms,

$$\underline{P = 4{,}427.}$$

(*b*) $$I^2 = \frac{E^2}{R^2 + X^2},$$

or, $$R^2 + X^2 = \frac{E^2}{I^2}.$$

$$\therefore\ X^2 = \frac{E^2}{I^2} - R^2,$$

$$= \frac{E^2 - R^2I^2}{I^2}.$$

$$\therefore\ \underline{X = \frac{\sqrt{(E^2 - R^2I^2)}}{I}.}$$

(*c*) Let R_T be the true speed and R the corresponding reading on the speedometer.

Then, $R = \dfrac{105}{100} \times R_T = 1{\cdot}05\, R_T.$

(*i*) When $R_T = 75$ km/h,

$R = 1{\cdot}05 \times 75 = \underline{78{\cdot}75 \text{ km/h}}.$

(*ii*) When $R = 90$ km/h,

$$R_T = \frac{R}{1{\cdot}05} = \frac{90}{1{\cdot}05} = \underline{85{\cdot}71 \text{ km/h}}.$$

Q. **2.** (*a*) *Express the following in standard-form notation (e.g.,* $0{\cdot}5\ \mu F = 5 \times 10^{-7} F$*):*

(*i*) 25 *kV in volts,* (*ii*) 35 *mH in henrys,*

(*iii*) 93·5 *MHz in hertz,* (*iv*) 298 *km/s in metres/second,*

(*v*) 300 *mA in ampères.*

(*b*) *Using logarithmic tables, evaluate the expression*

$$\frac{58{\cdot}62 \times 8{\cdot}651}{27{\cdot}68 \times 16{\cdot}16}.$$

A. 2. (*a*) (*i*) 25 kV = 25 × 10^3 volts = $\underline{2{\cdot}5 \times 10^4 \text{ volts.}}$

(*ii*) 35 mH = 35 × 10^{-3} H = $\underline{3{\cdot}5 \times 10^{-2} \text{ H.}}$

(*iii*) 93·5 MHz = 93·5 × 10^6 Hz = $\underline{9{\cdot}35 \times 10^7 \text{ Hz.}}$

(*iv*) 298 km/s = 298 × 10^3 m/s = $\underline{2{\cdot}98 \times 10^5 \text{ m/s.}}$

(*v*) 300 mA = 300 × 10^{-3} amp = $\underline{3{\cdot}0 \times 10^{-1} \text{ amp.}}$

(*b*) $$\frac{58{\cdot}62 \times 8{\cdot}651}{27{\cdot}68 \times 16{\cdot}16}.$$

Taking logarithms, $\log_{10}\left(\dfrac{58{\cdot}62 \times 8{\cdot}651}{27{\cdot}68 \times 16{\cdot}16}\right)$

$$\begin{aligned}&= \log_{10} 58{\cdot}62 + \log_{10} 8{\cdot}651 - (\log_{10} 27{\cdot}68 + \log_{10} 16{\cdot}16),\\ &= 1{\cdot}7680 + 0{\cdot}9371 - (1{\cdot}4422 + 1{\cdot}2084),\\ &= 2{\cdot}7051 - 2{\cdot}6506,\\ &= 0{\cdot}0545.\end{aligned}$$

Taking antilogarithms,

$$\frac{58{\cdot}62 \times 8{\cdot}651}{27{\cdot}68 \times 16{\cdot}16} = \underline{1{\cdot}134}.$$

Q. **3.** *The cost of producing a component is* £3, *and it is sold for* £5. *A batch of* 1,000 *is produced, and the initial cost of setting up the machinery, etc. is* £1,000.

(*a*) *Draw graphs on the same axes, with the same scales, to show the total cost and total sales.*

(*b*) *Estimate from the graphs, marking your results on the diagram,*

(*i*) *the output when total cost and total sales are equal,*

(*ii*) *the total gain on the sale of* 1,000 *components, and*

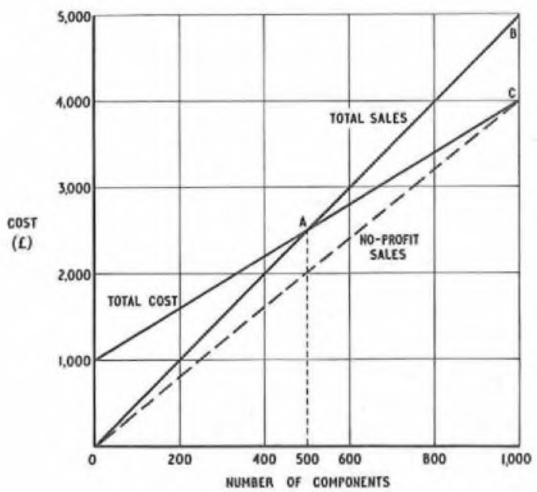
(*iii*) *the sales price which could be charged if no profit were made on the sale of* 1,000 *components.*

A. 3. (*a*) The sketch shows the graphs of the cost of production of the components and the money obtained from sales.

The graph of total cost starts at £1,000 for zero production, as this is the initial setting-up cost, and rises linearly to £4,000 for the production of 1,000 components, since £3 × 1,000 = £3,000, to which is added the setting-up cost.

The graph of total sales rises from zero to £5,000 for the production of 1,000 components at £5 each.

(*b*) (*i*) The output when the total cost and total sales are equal is 500 components (point A on the graph).



(*ii*) The total gain after selling 1,000 components is given by the difference between the total sales and the total cost for 1,000 components (points B and C on the graphs), and is £1,000.

(*iii*) If no profit were made on the sale of 1,000 components, the graph of total sales would be that shown by the dashed line, rising linearly from zero to point C. The selling price of each component would then be

$$\frac{£4{,}000}{1{,}000} = £4.$$

Q. **4.** (*a*) *A storage tank, as shown in Fig. 1, consists of a horizontal cylinder, of diameter d metres and length l metres, with hemispherical ends.*

(*i*) *Write down a formula for its volume, V, in terms of* π, *l and d.*
(*ii*) *If* $l = 3d$ *and* $V = 198\pi$, *calculate the value of d.*

(*b*) *Simplify, giving your answers with positive indices only:*

(*i*) $\dfrac{10p^{-2}q^{-3}}{2r^4}$, (*ii*) $\dfrac{15a^2b}{c^3} \times \dfrac{c^2}{3ab^2}$.

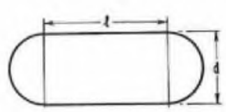


Fig. 1

A. **4.** (*a*) (*i*) The hemispherical ends of the tank, each of diameter *d* metres, together form a sphere of the same diameter, leaving a cylinder of length *l* metres and diameter *d* metres.

Total volume of tank, *V*, = volume of cylinder + volume of sphere.

$$\therefore\ V = \frac{\pi d^2 l}{4} + \frac{4\pi}{3}\left(\frac{d}{2}\right)^3,$$

$$= \frac{\pi d^2}{4}\left(l + \frac{2d}{3}\right)\ \text{m}^3.$$

(*ii*) Substituting the values,

$$198\pi = \frac{\pi d^2}{4}\left(3d + \frac{2d}{3}\right),$$

$$\text{or, } 792 = d^2\left(\frac{11d}{3}\right).$$

$$\therefore\ d^3 = \frac{792 \times 3}{11} = 216.$$

$$\therefore\ d = 6\ \text{m}.$$

(*b*) (*i*) $\dfrac{10p^{-2}q^{-3}}{2r^4} = \dfrac{5}{p^2q^3r^4}$.

(*ii*) $\dfrac{15a^2b}{c^3} \times \dfrac{c^2}{3ab^2} = \dfrac{5a}{bc}$.

Q. **5.** (*a*) *Solve the following simultaneous equations:*

$$5u - 3v = 13,$$
$$12u - 4v = 44.$$

(*b*) *Solve each of the following equations:*

(*i*) $\frac{3}{4}(2t + 1) - \frac{1}{3}(1 - t) = \frac{1}{2}(t + 1)$,

(*ii*) $\dfrac{3}{2a} + \dfrac{1}{a} = 4$.

A. **5.** (*a*)

$$5u - 3v = 13, \qquad \ldots\ldots (1)$$
$$12u - 4v = 44. \qquad \ldots\ldots (2)$$

Multiplying equation (1) by 4 gives

$$20u - 12v = 52. \qquad \ldots\ldots (3)$$

Multiplying equation (2) by 3 gives

$$36u - 12v = 132. \qquad \ldots\ldots (4)$$

Subtracting equation (3) from equation (4) gives

$$16u = 80.$$
$$\therefore\ u = 5.$$

Substituting for *u* in equation (1) gives

$$25 - 3v = 13,$$
$$\text{or, } 3v = 12.$$
$$\therefore\ v = 4.$$

(*b*) (*i*) $\frac{3}{4}(2t + 1) - \frac{1}{3}(1 - t) = \frac{1}{2}(t + 1)$.

Multiplying throughout by 12, which is the lowest common multiple of 4, 3 and 2, gives

$$9(2t + 1) - 4(1 - t) = 6(t + 1),$$
$$\text{or, } 18t + 9 - 4 + 4t = 6t + 6.$$
$$\therefore\ 16t = 1.$$
$$\therefore\ t = 1/16.$$

(*ii*)

$$\frac{3}{2a} + \frac{1}{a} = 4.$$

Multiplying throughout by 2*a* gives

$$3 + 2 = 8a.$$
$$\therefore\ a = 5/8.$$

Q. **6.** (*a*) *Evaluate the following as accurately as tables permit:*

(*i*) *sin* 53° 15′, (*ii*) *cos* 22° 19′, (*iii*) *tan* 72° 17′.

(*b*) *In Fig. 2, calculate the values of angles x and z.*

(*c*) (*i*) *State the sine of* 60° *in terms of* $\sqrt{3}$.

(*ii*) *Calculate the length of the side of an equilateral triangle with an area of* $16\sqrt{3}$ *cm*^{2}.

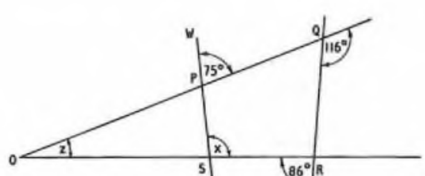
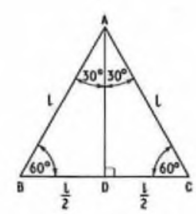


Fig. 2



(*a*)

A. **6.** (*a*) (*i*) sin 53° 15′ = 0·8013.

(*ii*) cos 22° 19′ = 0·9251.

(*iii*) tan 72° 17′ = 3·1303.

(*b*) From Fig. 2, $\angle QRO = 180° - 86° = 94°$,
and $\angle OQR = 180° - 116° = 64°$.

The sum of the angles of triangle OQR is 180°.

$$\therefore \angle z = 180° - (64° + 94°) = \underline{22°}.$$

Also, $\angle OPS = 75°$, since it is opposite $\angle WPQ$. Hence, in triangle OPS,

$$\angle OSP = 180° - (22° + 75°) = 83°.$$
$$\therefore \angle x = 180° - 83° = \underline{97°}.$$

(*c*) (*i*) $$\underline{\sin 60° = \frac{\sqrt{3}}{2}}.$$

(*ii*) An equilateral triangle, ABC, of side length l, is shown in sketch (*a*). AD is the perpendicular bisector of the base, BC, from the vertex, A. An equilateral triangle is equiangular and, hence, each of the angles of triangle ABC is 60°.

$$\text{Area of triangle ABC} = \tfrac{1}{2} \times \text{base} \times \text{perpendicular height},$$
$$= BD \times AD,$$
$$= BD \times AB \sin \angle ABC,$$
$$= \frac{l}{2} \times l \sin 60°.$$
$$\therefore 16\sqrt{3} = \frac{l^2}{2} \times \frac{\sqrt{3}}{2}.$$
$$\therefore \underline{l = 8 \text{ cm}}.$$

Q. 7. (*a*) *Fig. 3 shows a rectangular room whose dimensions are 5 m × 4 m × 3 m. A length of cable is to be attached to the walls from A to X and, then, from X to E. Calculate the shortest length of cable required, and the distance of X above the floor.*

(*b*) *Given that the tangent of an acute angle, A, is $\frac{5}{12}$, calculate, without the use of trigonometric tables, the values of sin A and cos A. Hence, find the values of:*

(*i*) $(\sin A)^2 + \cos A - 1$,
(*ii*) $(\cos A)(1 - \cos A)$.

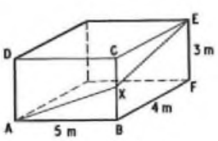
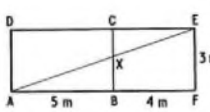
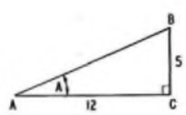


Fig. 3



(*a*)



(*b*)

A. 7. (*a*) The two faces, DCBA and CEFB, are shown developed in sketch (*a*), so that they appear in the same plane. The total cable length from A to E is then AX + XE, and it is clear that the shortest length of cable must occur when AX and XE are in the same straight line.

In triangle AEF, $(AE)^2 = (AF)^2 + (FE)^2$,
$$= 9^2 + 3^2 = 90.$$
$$\therefore AE = 9{\cdot}487 \text{ m}.$$

Thus, the shortest length of cable is $\underline{9{\cdot}487 \text{ m}}$.

Now, $\tan \angle XAB = \tan \angle EAF$,
$$= \frac{3}{9} = \frac{1}{3}.$$
$$\therefore \frac{XB}{AB} = \frac{1}{3}.$$
$$\therefore XB = \frac{5}{3} = 1{\cdot}667 \text{ m}.$$

Thus, the distance of X above the floor is $\underline{1{\cdot}667 \text{ m}}$.

(*b*) Sketch (*b*) shows triangle ABC, with BC = 5 and AC = 12, such that $\tan A = \frac{5}{12}$.

By Pythagoras, $AB^2 = 12^2 + 5^2 = 169$.
$$\therefore AB = 13.$$

Hence, $$\underline{\sin A = \frac{5}{13}}, \text{ and } \underline{\cos A = \frac{12}{13}}.$$

(*i*) $$(\sin A)^2 + \cos A - 1 = \left(\frac{5}{13}\right)^2 + \frac{12}{13} - 1,$$
$$= \frac{25 + (12 \times 13) - 13^2}{13^2},$$
$$= \frac{25 + 13(12 - 13)}{13^2},$$
$$= \underline{\frac{12}{169}}.$$

(*ii*) $$(\cos A)(1 - \cos A) = \frac{12}{13}\left(1 - \frac{12}{13}\right),$$
$$= \underline{\frac{12}{169}}.$$

Q. 8. (*a*) *Solve each of the following equations:*

(*i*) $2 \tan x = 1$,
(*ii*) $\log (x + 3) + \log 2 = \log 8$,
(*iii*) $10^{1{\cdot}31} = x$.

(*b*) (*i*) *Remove the brackets and simplify*
$-4\{3a - 2[b - (2a - b)]\}$.
(*ii*) *Evaluate* $p^3q - p^2(p - 2q) + 3$, *when* $p = -1$, *and* $q = 2$.
(*iii*) *Multiply out* $(a - b)(a^2 + ab + b^2)$.

A. 8. (*a*) (*i*) $$2 \tan x = 1,$$
$$\text{or, } \tan x = 1/2.$$
$$\therefore \underline{x = 26° \, 34'}.$$

(*ii*) $$\log (x + 3) + \log 2 = \log 8.$$
$$\therefore \log (x + 3) = \log 8 - \log 2,$$
$$= \log 8/2.$$
$$\therefore x + 3 = 4,$$
$$\text{or, } \underline{x = 1}.$$

(*iii*) $$10^{1{\cdot}31} = x.$$
$$\therefore \log_{10} x = 1{\cdot}31.$$
$$\therefore \underline{x = 20{\cdot}42}, \text{ from a table of antilogarithms.}$$

(*b*) (*i*) $$-4\{3a - 2[b - (2a - b)]\}$$
$$= -4\{3a - 2[b - 2a + b]\},$$
$$= -4\{3a - 4[b - a]\},$$
$$= -4\{3a - 4b + 4a\},$$
$$= -4\{7a - 4b\},$$
$$= 16b - 28a,$$
$$= \underline{4(4b - 7a)}.$$

(*ii*) $$p^3q - p^2(p - 2q) + 3 = (-1)^3 \times 2 - (-1)^2 \times (-1 - 4) + 3,$$
$$= -2 - (-5) + 3,$$
$$= \underline{6}.$$

(*iii*) $$(a - b)(a^2 + ab + b^2)$$
$$= a^3 + a^2b + ab^2 - a^2b - ab^2 - b^3,$$
$$= \underline{a^3 - b^3}.$$

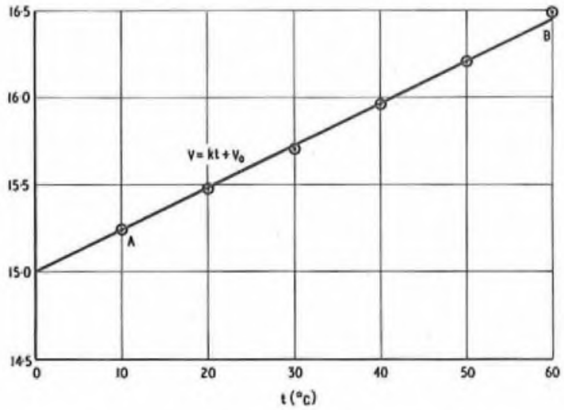
Q. 9. *An experiment was conducted to show the variation in the volume, V, of a given mass of gas as the temperature changes. The results were as follows.*

$V(cm^3)$	15·24	15·48	15·70	15·96	16·20	16·49
$t(°C)$	10	20	30	40	50	60

It is thought that the expression connecting V and t is of the form $V = kt + V_0$. Verify that this expression is approximately true, and estimate values for k and V_0.

A. 9. The variables in the expression, $V = kt + V_0$, are V and t, k and V_0 being constants. Hence, the expression is of the straight-line form, $y = mx + c$.

The sketch shows the graph of V/t. The plotted points lie reasonably close to a straight line. Hence, the law $V = kt + V_0$ is approximately true for the experimental data.



When $t = 0$, $V = V_0$ and, hence, the value of V_0 is given by the intersection of the graph and the vertical axis. From the graph, when $t = 0$, $V = 15$.

The gradient of the line, k, is obtained from the co-ordinates of two widely-separated points which actually lie on the straight line. Reading the co-ordinates of points A and B from the graph,

$$k = \frac{16 \cdot 45 - 15 \cdot 24}{60 - 10} = 0 \cdot 024.$$

Hence, the estimated values of k and V_0 are 0·024 and 15, respectively.

Note: These values will vary with the accuracy with which the points are plotted, and the positioning of the straight line.

Q. 10. (*a*) *An isosceles triangle has an area of 65 cm² and a vertex angle of* 44°. *Calculate*

(*i*) *the length of the base of the isosceles triangle, and*

(*ii*) *the angles of a right-angled triangle with the same area and base as the isosceles triangle.*

(*b*) *Sketch the graphs of* $y_1 = \cos\theta$ *and* $y_2 = 2\sin\theta$ *between* $\theta = 0°$ *and* $\theta = 180°$, *and estimate the value of* θ *where* $y_1 = y_2$.

A. 10. (*a*) Sketch (*a*) shows an isosceles triangle, ABC, in which AB = AC = l, and $\angle$BAC = 44°. AD is the perpendicular bisector of the base and, hence, $\angle$BAD = $\angle$DAC = 22°.

(*i*) The area of a triangle is equal to $\frac{1}{2} ab \sin C$, where a and b are the lengths of any two sides, and C is the angle included by them.

Hence, in triangle ABC, $65 = \frac{l^2}{2} \sin 44°$,

$$\text{or } l^2 = \frac{130}{0 \cdot 6947}.$$

$$\therefore l = 13 \cdot 68 \text{ cm.}$$

Now,

$$\begin{aligned} BD &= l \sin 22°, \\ &= 13 \cdot 68 \times 0 \cdot 3746, \\ &= 5 \cdot 125 \text{ cm.} \end{aligned}$$

$$\therefore \text{ base length, BC, } = 10 \cdot 25 \text{ cm.}$$

(*ii*) A right-angled triangle on base BC, and having the same area as triangle ABC, must have the same height as triangle ABC. Thus, in sketch (*a*), triangle EBC, right-angled at B, is the required triangle.

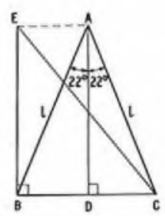
Now,

$$\begin{aligned} \tan \angle ECB &= \frac{EB}{BC}, \\ &= \frac{AD}{BC}, \\ &= \frac{AB \cos 22°}{BC}, \\ &= \frac{13 \cdot 68 \times 0 \cdot 9272}{10 \cdot 25}, \\ &= 1 \cdot 238. \end{aligned}$$

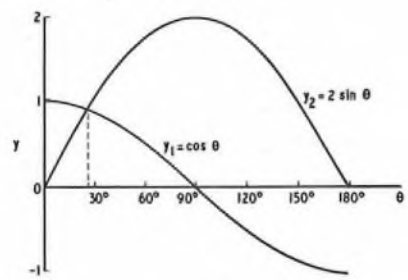
$$\therefore \angle ECB = 51° \, 4'.$$

$$\therefore \angle BEC = 90° - 51° \, 4' = 38° \, 56'.$$

Thus, the angles of the right-angled triangle are 90°, 51° 4′, and 38° 56′.



(*a*)



(*b*)

(*b*) The two graphs are shown in sketch (*b*). They intersect once within the range θ = 0–180°, and from the sketch, the estimated value of θ where $y_1 = y_2$ is 26°.

Note: The true value is 26° 34′.

ENGINEERING SCIENCE, 1974

Students were expected to answer two questions from Q. 1–4, and four questions from Q. 5–10

Q. 1. (*a*) *Explain the meanings and, in each case, give two examples of*

(*i*) *a scalar quantity, and*

(*ii*) *a vector quantity.*

(*b*) *A mass of 50 kg is hung from the middle of a wire which is stretched between two walls, 10 m apart, as shown in Fig. 1. The wire sags 1·5 m in the middle. Find the tension in the section of wire AB.*

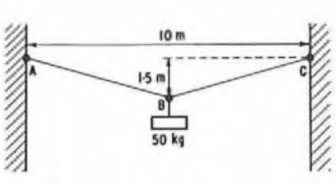
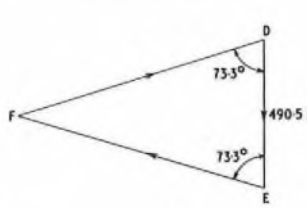


Fig. 1



(*a*)

A. 1. (*a*) (*i*) A scalar quantity is one which is completely defined by its magnitude; for example, power or mass.

(*ii*) A vector quantity is one which is completely defined only when both magnitude and direction are stated; for example, velocity or force.

(*b*) The tension in the section of wire AB can be determined graphically by drawing to scale a triangle of forces. Three forces are acting at point B and are in equilibrium: the tensions in sections of wire AB and BC, and the force due to the 50 kg mass.

Since the mass is suspended from the middle of the wire, the angle which AB and BC make with the vertical is

$$90° - \tan^{-1} \frac{1 \cdot 5}{5} = 73 \cdot 3°.$$

The force due to the mass = 50 × 9·81 = 490·5 N.

Sketch (*a*) shows the triangle of forces drawn to scale (25 N : 1 mm). DE is drawn vertically and represents the force due to the 50 kg mass. FD represents the tension in the section of wire BC, and is drawn at an angle of 73·3° to DE. FE represents the tension in the section of wire AB, and is drawn at an angle of 73·3° to DE.

By measurement of FE, the tension in the section of wire AB = 854 N.

Q. 2. (a) Find the kinetic energy of a mass of 15 kg two seconds after being accelerated from rest at 2 m/s².

(b) Calculate the power used when a mass of 10 kg is made to slide, with uniform speed, a distance of 1·5 m over a horizontal surface in 4 s. The coefficient of friction between the mass and the surface is 0·2.

(c) Find the power supplied by a torque of 4 N m to rotate a disc at 20 rev/s.

A. 2. (*a*) The kinetic energy of a moving body is given by $\frac{1}{2}mv^2$, where m is the mass (kg), and v is the velocity (m/s).

If a body is steadily accelerated, then its velocity after a time, t seconds, is given by $v = u + at$, where u is the initial velocity, and a is the acceleration (m/s²).

Thus, the velocity 2 s after being accelerated at 2 m/s² from rest is given by

$$v = 0 + (2 \times 2) = 4 \text{ m/s}.$$

Therefore, the kinetic energy of the body

$$= \tfrac{1}{2} \times 15 \times 4^2 = \underline{120 \text{ J}}.$$

(*b*) Power is the rate of doing work; that is, the work done divided by the time taken.

The work done in causing a mass of 10 kg to slide, with uniform speed, a distance of 1·5 m over a horizontal surface is determined by the force required to overcome friction and the distance moved.

Now, force = coefficient of friction × weight,

$$= 0{\cdot}2 \times 10 \times 9{\cdot}81 = 19{\cdot}62 \text{ N}.$$

$$\therefore \text{ the work done} = 19{\cdot}62 \times 1{\cdot}5 = 29{\cdot}43 \text{ J}.$$

$$\therefore \text{ the power used} = \frac{29{\cdot}43}{4} = \underline{7{\cdot}36 \text{ W}}.$$

(*c*) Torque is the product of a force and its perpendicular distance from the point of rotation of the force. The work done by a torque is the product of that torque and the distance moved by its point of application. If the disc in the question rotates through 20 rev, then the point of application of the torque moves through 20 × 2π rad.

Thus, the work done by a torque of 4 Nm

$$= 40\pi \times 4 = 503 \text{ J}.$$

As the speed of revolution is 20 rev/s, the work is done in 1 s.

$$\therefore \text{ power supplied} = \underline{503 \text{ W}}.$$

Q. 3. A motorist, travelling at 72 km/h, applies the brakes on his car in order to avoid road-repair works. The speed of the car is reduced uniformly to 18 km/h in a distance of 300 m. He travels at a constant speed of 18 km/h for a distance of 80 m and, then, accelerates uniformly for a period of 10 s to a final speed of 72 km/h.

(a) Find the deceleration when the brakes are applied.

(b) Calculate the time for which the car is travelling at 18 km/h.

(c) Sketch the velocity/time graph of the motion of the car from the time when the brakes are applied.

A. 3. (*a*) Deceleration is the change in speed divided by the time taken. If the speed is reduced uniformly from 72 km/h to 18 km/h, the average speed over the period $= \frac{72 + 18}{2} = 45$ km/h. The car has, therefore, travelled 300 m at an average speed of 45 km/h.

$$\text{Time taken} = \frac{\text{distance travelled}}{\text{average speed}},$$

$$= \frac{300 \times 3{,}600}{45 \times 10^3} = 24 \text{ s},$$

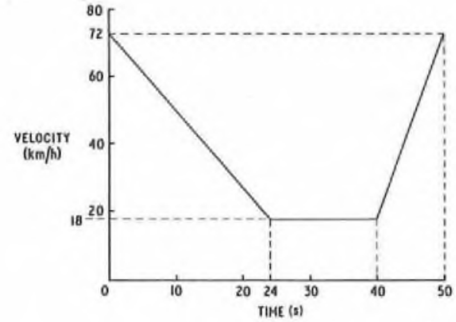
$$\text{and change in speed} = \frac{(72 - 18) \times 10^3}{3{,}600} = 15 \text{ m/s}.$$

$$\therefore \text{ deceleration} = \frac{15}{24} = \underline{0{\cdot}6 \text{ m/s}^2}.$$

(*b*) The time for which the car is travelling at 18 km/h

$$= \frac{80 \times 3{,}600}{18 \times 10^3} = \underline{16 \text{ s}}.$$

(*c*) The velocity/time graph of the motion of the car is shown in the sketch.



Q. 4. Write short notes on two of the following.

(a) The purpose of a flywheel, and the factors which decide its inertia.

(b) The transfer of heat from a hot body to its surroundings.

(c) The function of a screw-jack, and the factors which decide its velocity ratio and mechanical advantage.

A. 4. (*a*) The purpose of a flywheel is to store energy from a source when the supply exceeds the demand, and to give up this energy to a load when the demand exceeds the supply. In this way, the supply is not subjected to sudden fluctuations in demand; nor is the load subjected to sudden fluctuations in supply. An example of this is found in the internal-combustion engine of a car, which does not produce driving energy continuously, but in sudden bursts, as each cylinder fires in turn. Without the flywheel, the energy provided by each piston would be transmitted to the wheels of the car in sudden bursts, and the car would not run smoothly.

The factors which decide the inertia, I, of a flywheel are its mass, m, and its radius of gyration, k. The radius of gyration is the distance from the axis of rotation to a point at which the mass of the flywheel is considered to be concentrated. The inertia is given by

$$\underline{I = mk^2}.$$

(*c*) The function of a screw-jack is to raise a load, W, by the application of an effort, P, which turns the screw of the jack. For each rotation of the screw, the load will be raised by a distance equal to the distance between two successive threads of the screw, known as the pitch, p, of the thread. The effort is usually applied by means of an arm and, if the distance from the axis of the screw to the point of application of the effort on the arm is l, then the effort follows a circular path around the screw, of length $2\pi l$, for each rotation of the screw.

The velocity ratio is the distance moved by the effort divided by the distance moved by the load.

$$\therefore \underline{\text{velocity ratio} = \frac{2\pi l}{p}}.$$

The mechanical advantage is the ratio of the load to the effort.

$$\therefore \underline{\text{mechanical advantage} = \frac{W}{P}}.$$

As the work done on the load equals the work done by the effort, the further the point of application of the effort from the axis of the screw, the smaller is the effort required for a given load, and, hence, the greater is the mechanical advantage.

Friction also has an effect on the mechanical advantage, since the effort must overcome the frictional forces acting on the screw. Hence, work done by effort equals work done on load plus work done in overcoming friction. Thus, friction is an additional load to be overcome by the effort, and reduces the mechanical advantage.

Q. 5. (a) State, in words, Ohm's law.

(b) Compare conductors and semiconductors in relation to the following electrical properties:

(i) conductivity, and

(ii) temperature coefficient of resistance.

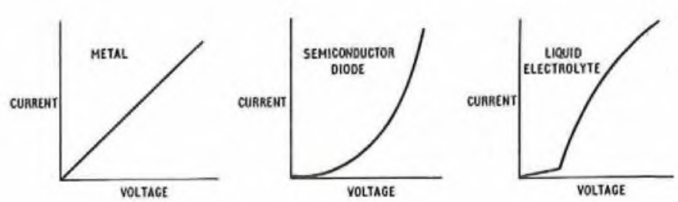


Fig. 2

(c) The graphs in Fig. 2 show the variation of current as a function of voltage for three materials.

(i) Describe the change of resistance with current in each case.
(ii) Is Ohm's law satisfied in each case?

A. 5. (*a*) Ohm's law states that the current in a conductor, at a uniform temperature, is directly proportional to the potential difference between its ends.

(*b*) (*i*) A conductor consists of a material whose atoms have one or two loosely-attached ("free") electrons in their outer shells. As the atoms in a conductor are tightly packed, the free electrons can move easily from one atom to another. Such materials are said to have a high conductivity. A semiconductor does not have a plentiful supply of free electrons and, as a result, movement of charges within the material is difficult. A semiconductor, therefore, has a lower conductivity than a conductor, but not as low as an insulator. Copper, a good conductor, has a conductivity of about $64 \cdot 1 \times 10^6$ S/m, and germanium, a semiconductor, has a conductivity of about 0·01 S/m.

(*ii*) When the temperature of most metallic conductors is raised, there is an increase in the number of collisions between the free electrons and the atoms of the material, which results in an increase in the resistance of the material. The conductor is then said to have a positive temperature coefficient of resistance. With a semiconductor, a rise in temperature results in an increase in the number of charge carriers available, thus reducing its resistance. A semiconductor is, therefore, said to have a negative temperature coefficient of resistance.

(*c*) (*i*) and (*ii*) For the metal, the curve is a straight line and, hence, the current is proportional to the voltage. Therefore, the resistance is constant and Ohm's law is satisfied.

For the semiconductor diode, as the voltage increases, there is an increase in the rate at which the current rises. If the voltage is doubled, the current is more than doubled, and the resistance is, therefore, decreasing. Hence, Ohm's law is not satisfied.

For the liquid electrolyte, the curve is initially linear, and the resistance over this section is, therefore, constant. There is, then, a sudden increase in current, indicating that the resistance of the electrolyte has fallen, but thereafter, as the rate of rise of current decreases, the resistance gradually increases again. Hence, Ohm's law is not satisfied.

Q. 6. *(a) What is the relationship*

(i) between resistance and resistivity, and
(ii) between conductivity and resistivity?

(b) A rectangular bar of silicon has end faces 2 mm by 4 mm, and a length of 15 mm. The resistivity of the silicon is 5 ohm m. Ohmic contacts are bonded on to the end faces.

(i) Calculate the resistance of the silicon bar between the end faces.
(ii) If a current of 2 mA flows through the silicon bar, find the power dissipated.

A. 6. (*a*) (*i*) If a potential difference is applied across the ends of a conductor, a current flows in the conductor. The ratio of the applied potential difference to that current is called the resistance of the conductor.

Resistivity is a property of the material of which the conductor is made, and is expressed as the resistance/unit-volume of the material.

The two are related by the formula

$R = \rho \frac{l}{a}$, where R is the resistance (ohms),
ρ is the resistivity (ohm m),
l is the length of the conductor (m), and
a is the cross-sectional area of the conductor (m²).

(*ii*) Conductivity is the inverse of resistivity.

(*b*) (*i*) From the above formula, the resistance of the silicon bar

$$= 5 \times \frac{15 \times 10^{-3}}{2 \times 4 \times 10^{-6}} = \underline{9{,}375 \text{ ohms.}}$$

(*ii*) Power is the product of the square of the current and the resistance.

$$\therefore \text{ power dissipated} = (2 \times 10^{-3})^2 \times 9{,}375 \text{ W},$$
$$= \underline{37 \cdot 5 \text{ mW.}}$$

Q. 7. *(a) State Faraday's two laws of electrolysis.*

(b) When a current of 2 amps is passed for 35 min through an electroplating bath, $1 \cdot 26 \times 10^{-3}$ kg of copper is deposited on the cathode.

(i) Calculate the electrochemical equivalent of copper.
(ii) If the resistance between the electrodes of the plating bath is 3 ohms, which type of battery would you use to supply the current? Give reasons for your choice.

Q. 8. *(a) Name the SI unit of torque.*

(b) Show, with the aid of diagrams, how the torque on a coil carrying a current in a magnetic field depends on the position of the coil relative to the field.

(c) Sketch the construction of a moving-coil meter and describe its action, using suitable diagrams from part (b).

Q. 9. *(a) Show how a meter, having a full-scale deflexion of 500 µA and a resistance of 50 ohms, may be adapted for use as a meter with a full-scale deflexion of 3 mA.*

(b) The 500 µA meter is used in the circuit shown in Fig. 3 for the measurement of resistance. The resistor, R, is chosen so that full-scale deflexion is obtained when the terminals X and Y are short-circuited.

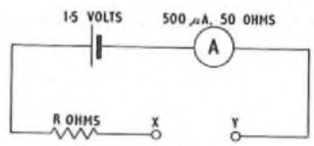
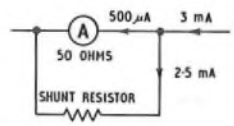


Fig. 3

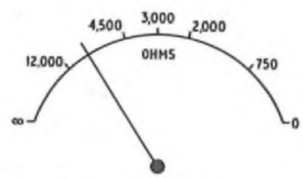
(i) Find the value of R.
(ii) Determine the resistance which, on connexion between X and Y, gives a meter reading of 250 µA.
(iii) Sketch the meter scale, calibrated in resistance values.

A. 9. (*a*) Since the meter has a full-scale deflexion of 500 µA, but is required to register a full-scale deflexion when a current of 3 mA flows in the circuit, the additional current of 2·5 mA must be diverted through a shunt resistor, as shown in sketch (*a*).

The ratio of the meter resistance to the shunt resistance is inversely proportional to the ratio of the currents flowing in the resistances.



(*a*)



(*b*)

$$\text{Hence, value of shunt resistance} = 50 \times \frac{0 \cdot 5}{2 \cdot 5} = \underline{10 \text{ ohms.}}$$

(*b*) For a full-scale deflexion, a current of 500 µA must flow with terminals X and Y short-circuited.

$$\text{Total resistance in circuit} = \frac{1 \cdot 5}{500 \times 10^{-6}} = 3{,}000 \text{ ohms.}$$

$$\therefore \; R = 3{,}000 - 50 = \underline{2{,}950 \text{ ohms.}}$$

(*ii*) Assuming the meter to be a moving-coil type, a meter reading of 250 µA represents half of the full-scale-deflexion current and, as the current is inversely proportional to the resistance, the total resistance must be doubled. Hence, an additional resistance of <u>3,000 ohms</u> must be connected between terminals X and Y.

(*iii*) The meter scale, calibrated in resistance values, is shown in sketch (*b*). The shape of the scale is derived as follows.

For zero deflexion, the resistance in the circuit is <u>infinite.</u>

For full-scale deflexion, the resistance between terminals X and Y is <u>zero.</u>

For half-scale deflexion, the resistance between terminals X and Y is <u>3,000 ohms</u> from part (*b*)(*ii*).

For a deflexion of one-fifth of the full-scale deflexion, the current in the circuit, I, is given by

$$I = 500 \times 10^{-6} \times 1/5 = 100 \; \mu\text{A}.$$

Therefore, the resistance between terminals X and Y, R_{xy}, is given by

$$R_{xy} = \{1 \cdot 5/(100 \times 10^{-6})\} - 3{,}000 = \underline{12{,}000 \text{ ohms.}}$$

Similarly, for a deflexion of two-fifths of the full-scale deflexion,

$$I = 500 \times 10^{-6} \times 2/5 = 200 \; \mu\text{A},$$

$$\text{and } R_{xy} = \{1 \cdot 5/(200 \times 10^{-6})\} - 3{,}000 = \underline{4{,}500 \text{ ohms.}}$$

Similarly, for deflexions of three-fifths and four-fifths of the full-scale deflexion, R_{xy} = <u>2,000 ohms</u> and <u>750 ohms</u>, respectively.

Q. 10. (*a*) *Explain what is meant by the term, self-inductance.*

(*b*) *A circuit, containing a coil in which there is a current of 10 amps, is changed so that the current falls uniformly to zero in 0·2 s. If the self-inductance of the coil is 5 H, what is the induced e.m.f.?*

(*c*) *A coil with an iron core is connected in series with a resistor and a battery. Describe the instantaneous changes that occur when a switch is opened in the circuit.*

A. 10. (*a*) Consider a current flowing in a conductor. Self-inductance is that property of the conductor which opposes any change in that current. The current produces a magnetic field which links with the conductor, and when the current changes, the magnetic field changes. Hence, there is a change in flux linkages, which induces an e.m.f. in the conductor, the direction of which tends to oppose the change in the current.

(*b*) The induced e.m.f., in volts, is given by the inductance of the coil, in henrys, multiplied by the rate of change of the current, in amperes/second.

$$\therefore \text{ induced e.m.f.} = 5 \times \frac{10}{0{\cdot}2} = \underline{250 \text{ volts.}}$$

(*c*) Because of its iron core, the coil is highly inductive. Prior to the switch being opened, steady-state conditions exist, with the current being determined by the voltage of the battery and the total resistance of the circuit. At the instant the switch is opened, the resistance of the circuit rises from a finite value to an infinite value, and the current tends to fall to zero. This causes a sudden collapse of the coil's magnetic field, which induces a high voltage in the coil. This voltage is additive to the battery voltage and tends to maintain the current flowing in the circuit. Thus, current continues to flow for a fraction of a second as the switch contacts open, and may be seen as a spark across the widening contact gap.

MATHEMATICS A, 1974

Students were expected to answer any six questions

Q. 1. (*a*) *Make μ the subject of the formula*

$$G = \frac{\mu R}{r + (\mu + 1)R}.$$

(*b*) *Show that the currents i_1 and i_2 in Fig. 1 are connected by the equations*

$$7i_1 + 3i_2 = 11i_1 - 4i_2 = 6{\cdot}1.$$

Calculate the values of i_1 and i_2.

(*c*) *A quadratic equation has roots α and β. If $\alpha + \beta = -5$ and $\alpha\beta = 6$, find the equation in the form $x^2 + ax + b = 0$.*

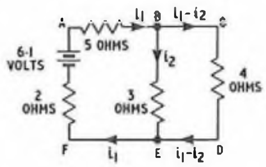


Fig. 1

A. 1. (*a*) $$G = \frac{\mu R}{r + (\mu + 1)R}.$$

$$\therefore G\{r + (\mu + 1)R\} = \mu R,$$

$$\text{or, } Gr + \mu GR + GR = \mu R.$$

$$\therefore G(r + R) = \mu R - \mu GR = \mu R(1 - G).$$

$$\therefore \underline{\mu = \frac{G(r + R)}{R(1 - G)}}.$$

(*b*) With reference to Fig. 1, applying Kirchhoff's law to that part of the network, ABEF, gives

$$6{\cdot}1 = 5i_1 + 3i_2 + 2i_1,$$
$$= 7i_1 + 3i_2. \qquad \ldots\ldots(1)$$

Similarly, applying Kirchhoff's law to that part of the network, ACDF, gives

$$6{\cdot}1 = 5i_1 + 4(i_1 - i_2) + 2i_1,$$
$$= 11i_1 - 4i_2. \qquad \ldots\ldots(2)$$

Therefore, from equations (1) and (2),

$$\underline{7i_1 + 3i_2 = 11i_1 - 4i_2 = 6{\cdot}1.} \quad \text{Q.E.D.} \quad \ldots\ldots(3)$$

Rearranging equation (3) gives

$$7i_2 = 4i_1,$$

$$\text{or, } i_2 = \frac{4}{7}i_1.$$

Substituting for i_2 in equation (1) gives

$$6{\cdot}1 = 7i_1 + \frac{12}{7}i_1.$$

$$\therefore 42{\cdot}7 = 49i_1 + 12i_1.$$

$$\therefore i_1 = \frac{42{\cdot}7}{61} = \underline{0{\cdot}7.}$$

But,
$$i_2 = \frac{4}{7}i_1,$$
$$= \frac{4}{7} \times 0{\cdot}7 = \underline{0{\cdot}4.}$$

(*c*) If α and β are the roots of a quadratic equation, then

$$(x - \alpha)(x - \beta) = 0,$$
$$\text{or, } x^2 - x(\alpha + \beta) + \alpha\beta = 0.$$

Thus, in the equation $x^2 + ax + b = 0$, a, the coefficient of x, is equal to minus the sum of the roots, and b is equal to the product of the roots.

$$\therefore a = -(\alpha + \beta) = 5,$$
$$\text{and } b = \alpha\beta = 6.$$

$\therefore$ $\underline{x^2 + 5x + 6 = 0}$ is the required equation.

Q. 2. *Three alternating voltages are given by:*

$$v_1 = 5 \cos \theta \text{ volts},$$

$$v_2 = 10 \cos\left(\theta - \frac{\pi}{4}\right) \text{ volts},$$

$$v_3 = 12 \cos\left(\theta + \frac{\pi}{2}\right) \text{ volts}.$$

(*a*) *Represent these voltages on a phasor diagram.*

(*b*) *By the method of resolution, using the direction of v_1 as the axis of reference, evaluate their resultant and express your answer in the form $V \cos(\theta + \alpha)$, where α is in radians, correct to two decimal places.*

(*c*) *Verify your result by a graphical method.*

A. 2. (*a*) The three voltages are represented by the phasor diagram in sketch (*a*), with axes X′OX and OY, and using the conventional anticlockwise rotation of phasors. Phasor v_1, of magnitude 5, is set at an angle of θ rad to OX. Phasor v_2, of magnitude 10, lags v_1 by $\pi/4$ rad, and phasor v_3, of magnitude 12, leads v_1 by $\pi/2$ rad.

(*b*) In sketch (*b*), v_1 is drawn on the reference axis, OX. Phasor v_2 lags v_1 by $\pi/4$ rad (45°), and phasor v_3 leads v_1 by $\pi/2$ rad (90°).

Phasor v_2 is resolved into horizontal component OA $= 10 \cos \pi/4$ along OX, and vertical component OB $= 10 \sin \pi/4$ along OY′.

Thus, there are two voltages, v_1 and OA, in the direction OX. Therefore, the total voltage in direction OX, represented by OC, is given by

$$\text{OC} = 5 + 10 \cos 45° = 12{\cdot}07.$$

Similarly, the total voltage in direction OY, represented by OD, is given by

$$\text{OD} = 12 - 10 \sin 45° = 4{\cdot}929.$$

The resultant of the components OC and OD is represented by v.

$$\therefore v^2 = (\text{OC})^2 + (\text{OD})^2,$$
$$= 12{\cdot}07^2 + 4{\cdot}929^2,$$
$$= 170.$$

$\therefore\ v = 13{\cdot}04.$

Also, $\tan\alpha = \dfrac{OD}{OC}$

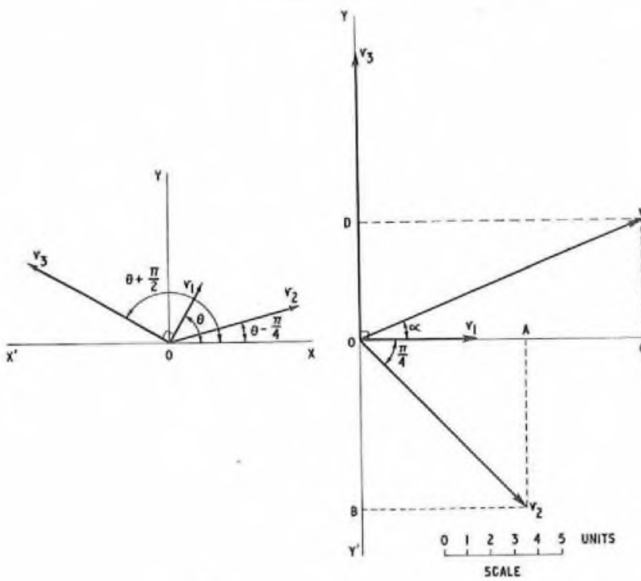
$= \dfrac{4{\cdot}929}{12{\cdot}07} = 0{\cdot}4084.$

$\therefore\ \alpha = 22{\cdot}21°,$

$= 22{\cdot}21 \times \dfrac{\pi}{180}$ rad,

$= 0{\cdot}3877$ rad.

Hence, $\underline{v = 13{\cdot}04 \cos(\theta + 0{\cdot}39) \text{ volts.}}$



(*a*) (*b*)

(*c*) The graphical method is incorporated in sketch (*b*), where v_1, v_2 and v_3 are drawn to scale.

By measurement,

$v = 13{\cdot}05$, and $\alpha = 22{\cdot}5° = 0{\cdot}39$ rad.

Hence, $\underline{v = 13{\cdot}05 \cos(\theta + 0{\cdot}39) \text{ volts.}}$

Q. 3. (*a*) *Using the same axes and scales, plot, on the same diagram, the graphs of $y_1 = 2 \sin\theta$ and $y_2 = \cos 2\theta$ between $\theta = 0°$ and $\theta = 180°$, at intervals of* 30°.

(*b*) *Use your graphs to solve the equation*

$$2 \sin\theta - \cos 2\theta = 0.$$

(*c*) *Plot, on the same diagram, the graph of $y_3 = 2\sin\theta + \cos 2\theta$,* and solve the equation $2\sin\theta + \cos 2\theta = 1{\cdot}3$.

A. 3. (*a*) The values of y_1 and y_2 between $\theta = 0°$ and $\theta = 180°$ are given in the table.

$\theta°$	0	30	60	90	120	150	180
$\sin\theta$	0	0·5	0·866	1·0	0·866	0·5	0
$2\theta°$	0	60	120	180	240	300	360
$y_1 = 2\sin\theta$	0	1·0	1·732	2·0	1·732	1·0	0
$y_2 = \cos 2\theta$	1·0	0·5	−0·5	−1·0	−0·5	0·5	1·0

The sketch shows the graphs of y_1 and y_2.

(*b*) Since $2\sin\theta - \cos 2\theta = 0,$

then, $2\sin\theta = \cos 2\theta,$

or, $y_1 = y_2.$

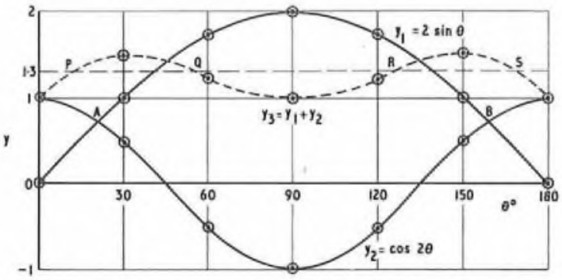
The solution of the equation is, therefore, given by the co-ordinates of the points of intersection, A and B, of the two graphs.

From the sketch, $\underline{\theta = 21°}$ or $\underline{159°.}$

Note: The accurate solution is $\theta = 21° 28'$ or $158° 32'$, obtained by calculation.

(*c*) $y_3 = 2\sin\theta + \cos 2\theta,$

$= y_1 + y_2.$



Thus, the values of y_3 at 30° intervals are the sums of the values of y_1 and y_2 at corresponding values of θ, and are given in the table.

$\theta°$	0	30	60	90	120	150	180
y_3	1·0	1·5	1·232	1·0	1·232	1·5	1·0

The graph of y_3 is shown by the dashed curve in the sketch.

The solution of the equation, $2\sin\theta + \cos 2\theta = 1{\cdot}3$, is given by the points of intersection, P, Q, R and S, of the graph of y_3 with the line $y = 1{\cdot}3$.

From the sketch,

$\underline{\theta = 11°, 55°, 126° \text{ and } 169°.}$

Note: Accurate values for θ are $10° 35'$, $54° 43'$, $125° 17'$ and $169° 25'$. It is unlikely that an accuracy better than $\pm 1°$ in the value of θ would be expected for a graphical solution.

Q. 4. (*a*) *Evaluate, as accurately as tables permit:*

(*i*) *sin* $125° 16'$,

(*ii*) *cos* (1·5 *rad*),

(*iii*) *tan* $\frac{5}{9}\pi$,

(*iv*) *cos* $212° 17'$.

(*b*) *Simplify, giving your answers with positive indices only:*

(*i*) $\dfrac{4a^2bc^2 \times 3ab^2c}{6ab^4c^3}$,

(*ii*) $\sqrt{\left(\dfrac{36x^{-2}y^{-3}}{z^{-5}}\right)}$.

(*c*) *If sin A* = 0·8, *calculate, without the use of trigonometric tables, the values of cos A and tan A, assuming angle A is acute.*

A. 4. (*a*) (*i*) $\sin 125° 16' = \sin(180° - 125° 16'),$

$= \sin 54° 44',$

$= \underline{0{\cdot}8165.}$

(*ii*) $\cos(1{\cdot}5 \text{ rads}) = \underline{0{\cdot}0707.}$

Note: This answer is obtained directly from a table of *Radians to Circular Functions.*

(*iii*) $\tan\frac{5}{9}\pi = \tan(\frac{5}{9} \times 180°),$

$= \tan 100°,$

$= -\tan(180° - 100°),$

$= -\tan 80°,$

$= \underline{-5{\cdot}6713.}$

(*iv*) $\cos 212° 17' = \cos(180° + 32° 17'),$

$= -\cos 32° 17',$

$= \underline{-0{\cdot}8454.}$

(*b*) (*i*) $\dfrac{4a^2bc^2 \times 3ab^2c}{6ab^4c^3} = 2a^{(2+1-1)}b^{(1+2-4)}c^{(2+1-3)},$

$= 2a^2b^{-1}c^0,$

$= \underline{\dfrac{2a^2}{b}.}$

(*ii*) $$\sqrt{\left(\frac{36x^{-2}y^{-3}}{z^{-5}}\right)} = 6 \times \sqrt{\left(\frac{z^5}{x^2y^3}\right)},$$
$$= \underline{\frac{6z^{5/2}}{xy^{3/2}}}.$$

(*c*) Since $\sin A = 0{\cdot}8 = \frac{4}{5}$, then angle A is an angle in a right-angled triangle where the opposite side length is 4 units, the hypotenuse length is 5 units and, therefore, the adjacent side length is 3 units.

Hence, $\cos A = \frac{3}{5} = \underline{0{\cdot}6},$

and $\tan A = \frac{4}{3} = \underline{1{\cdot}\dot{3}}.$

Q. 5. (*a*) *A rectangular enclosure uses an existing wall for one side and fencing, of total length 200 m, for the other three sides. If x metres is the length of one of the sides, and the area of the enclosure is 3,648 m², *

(*i*) *show that* $x^2 - 100x + 1{,}824 = 0$, *and*

(*ii*) *calculate the lengths of the sides.*

(*b*) *Solve the following equation, giving your results correct to two decimal places:*

$$2t^2 + 3t - 18 = 0.$$

(*c*) *Show that*

(*i*) $(\cos\theta + \sin\theta)(\cos\theta - \sin\theta) = 2\cos^2\theta - 1$, *and*

(*ii*) $(\cos\theta + \sin\theta)^2 = 1 + 2\cos\theta\sin\theta$.

(*d*) *Use the results of part* (*c*) *to simplify*

$$\frac{2\cos^2\theta - 1}{1 + 2\cos\theta\sin\theta}.$$

A. 5. (*a*) Let x metres be the lengths of the sides adjacent to the wall, and y metres be the lengths of the wall and its opposite side.

(*i*) Now, total length of fencing $= 2x + y = 200$ m, ...(1)

and area of enclosure $= xy = 3{,}648$ m².

$$\therefore\ y = \frac{3{,}648}{x}.$$

Substituting for y in equation (1) gives

$$2x + \frac{3{,}648}{x} = 200.$$
$$\therefore\ 2x^2 + 3{,}648 = 200x.$$
$$\therefore\ \underline{x^2 - 100x + 1{,}824 = 0.}$$ Q.E.D.

(*ii*) From the general solution to a quadratic equation,

$$x = \frac{100 \pm \sqrt{(100^2 - 4 \times 1{,}824)}}{2},$$
$$= \frac{100 \pm 52}{2},$$
$$= 76 \text{ m or } 24 \text{ m}.$$

When $x = 76$ m, $y = \frac{3{,}648}{76} = 48$ m,

and when $x = 24$ m, $y = \frac{3{,}648}{24} = 152$ m.

Hence, the lengths of the sides of the enclosure are either

<u>76 m and 48 m,</u> or <u>24 m and 152 m.</u>

(*b*) From the general solution to a quadratic equation,

$$t = \frac{-3 \pm \sqrt{\{3^2 - 4 \times 2 \times (-18)\}}}{2 \times 2},$$
$$= \frac{-3 \pm 12{\cdot}37}{4},$$
$$= \underline{2{\cdot}34 \text{ or } -3{\cdot}84}, \text{ to two decimal places.}$$

(*c*) (*i*) $(\cos\theta + \sin\theta)(\cos\theta - \sin\theta) = 2\cos^2\theta - 1$.

The left-hand side of the identity

$= \cos^2\theta - \cos\theta\sin\theta + \sin\theta\cos\theta - \sin^2\theta,$
$= \cos^2\theta - \sin^2\theta,$
$= \cos^2\theta - (1 - \cos^2\theta)$, since $\sin^2\theta + \cos^2\theta = 1$,
$= \underline{2\cos^2\theta - 1.}$ Q.E.D.

(*ii*) $(\cos\theta + \sin\theta)^2 = 1 + 2\cos\theta\sin\theta$.

The left-hand side of the identity

$= \cos^2\theta + 2\cos\theta\sin\theta + \sin^2\theta,$
$= \underline{1 + 2\cos\theta\sin\theta}$, since $\sin^2\theta + \cos^2\theta = 1$. Q.E.D.

(*d*) Substituting for the numerator and denominator from part (*c*),

$$\frac{2\cos^2\theta - 1}{1 + 2\cos\theta\sin\theta} = \frac{(\cos\theta + \sin\theta)(\cos\theta - \sin\theta)}{(\cos\theta + \sin\theta)^2},$$
$$= \underline{\frac{\cos\theta - \sin\theta}{\cos\theta + \sin\theta}}.$$

Q. 6. (*a*) *Compile a table to show the values of*

$$y_1 = 9 + 10x - 4x^2$$

at unit intervals between $x = -1$ *and* $x = 4$.

(*b*) *On the same axes, between* $x = -1$ *and* $x = 4$, *plot accurately the graphs of*

(*i*) $y_1 = 9 + 10x - 4x^2$, *and*

(*ii*) $y_2 = 5 - 3x$.

(*c*) *From the graphs drawn in part* (*b*), *determine*

(*i*) *the solution of the equation* $9 + 10x - 4x^2 = 0$, *and*

(*ii*) *the range of values of x for which* $y_1 > y_2$.

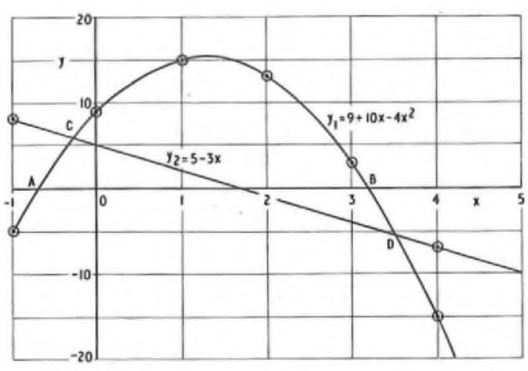
(*d*) *Find the equation whose roots are given by the points of intersection of* $y_1 = 9 + 10x - 4x^2$ *and* $y_2 = 5 - 3x$.

A. 6. (*a*) The table shows the required values of y_1.

x	-1	0	1	2	3	4
9	9	9	9	9	9	9
$10x$	-10	0	10	20	30	40
$-4x^2$	-4	0	-4	-16	-36	-64
y_1	-5	9	15	13	3	-15

(*b*) (*i*) The graph of $y_1 = 9 + 10x - 4x^2$ is plotted from the table of values, as shown in the sketch.

(*ii*) The graph of $y_2 = 5 - 3x$, being a linear law of the form $y = mx + c$, may be plotted from two values only. At $x = -1$, $y_2 = 8$, and at $x = 4$, $y_2 = -7$. The graph is shown in the sketch. As a check, when $x = 0$, $y_2 = 5$.



(*c*) (*i*) The solution of the equation $9 + 10x - 4x^2 = 0$ is given by the abscissae of the points A and B, where the graph of y_1 crosses the x-axis.

From the graph, $\underline{x = -0{\cdot}7 \text{ or } 3{\cdot}2.}$

Note: Accurate values for x are $-0{\cdot}7025$ or $3{\cdot}2025$.

(*ii*) From the sketch, it can be seen that $y_1 > y_2$ over the range between the points of intersection, C and D, of the graphs.

Hence, the range is from $\underline{x = -0{\cdot}27 \text{ to } x = 3{\cdot}53.}$

Note: Accurately, the range is from $x = -0{\cdot}2825$ to $x = 3{\cdot}5325$.

(*d*) If the roots of an equation are given by the points of intersection of the graphs of y_1 and y_2, that is, by the values obtained in part (*c*)(*ii*), then,

$$\{x - (-0{\cdot}27)\}\{x - 3{\cdot}53\} = 0$$

is the required equation.

$$\therefore\ (x + 0{\cdot}27)(x - 3{\cdot}53) = 0.$$
$$\therefore\ x^2 + x(0{\cdot}27 - 3{\cdot}53) - 0{\cdot}27 \times 3{\cdot}53 = 0,$$
$$\therefore\ \underline{x^2 - 3{\cdot}26x - 0{\cdot}95 = 0.}$$

Note: Accurately, the required equation is $x^2 - 3{\cdot}25x - 0{\cdot}9979 = 0$.

Q. 7. (*a*) *Two angles of a triangle are 63° and 42°, and the shortest side is 25 m long. Calculate*

(*i*) *the length of the longest side, and*
(*ii*) *the area of the triangle.*

(*b*) *A cable runs directly from A to B, a distance of 8 km. A factory is built at C, 5 km from A and 6 km from B. If power is to be supplied to C from the existing cable, calculate the length of the shortest link that must be laid.*

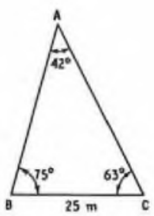
A. 7. (*a*) Since the sum of the angles of a triangle is 180°, the third angle is 180° − (63° + 42°) = 75°. Also, since the shortest side of a triangle is opposite the smallest angle, the triangle is as shown in sketch (*a*).

(*i*) The longest side is opposite the largest angle, and is, therefore, side AC. From the sine rule,

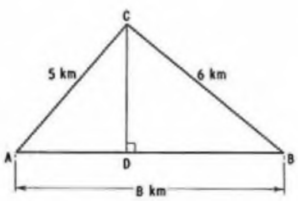
$$\frac{AC}{\sin\angle ABC} = \frac{BC}{\sin\angle BAC}.$$
$$\therefore\ AC = \frac{25}{\sin 42^\circ} \times \sin 75^\circ,$$
$$= \frac{25 \times 0{\cdot}9659}{0{\cdot}6691},$$
$$= \underline{36{\cdot}09\ \text{m.}}$$

(*ii*) The area of a triangle is given by half the product of any two sides and the sine of the included angle.

$$\therefore\ \text{area} = \tfrac{1}{2} \times AC \times BC \times \sin\angle ACB,$$
$$= \tfrac{1}{2} \times 36{\cdot}09 \times 25 \times \sin 63^\circ,$$
$$= \underline{402\ \text{m}^2.}$$



(*a*)



(*b*)

(*b*) The factory at C is located as shown in sketch (*b*), from which it is clear that the shortest distance of C from the cable run, AB, is CD, the line perpendicular to AB through C. From the cosine rule,

$$\cos\angle CAB = \frac{AC^2 + AB^2 - CB^2}{2 \times AC \times AB},$$
$$= \frac{5^2 + 8^2 - 6^2}{2 \times 5 \times 8},$$
$$= 0{\cdot}6625.$$
$$\therefore\ \angle CAB = 48^\circ\ 30'.$$

In triangle CAD, $\dfrac{CD}{AC} = \sin\angle CAD.$

$$\therefore\ CD = 5 \sin 48^\circ\ 30',$$
$$= \underline{3{\cdot}745\ \text{km.}}$$

Q. 8. (*a*) *A voltage, v, has the periodic waveform shown in Fig. 2. Calculate the average value of v.*

(*b*) *Instantaneous values of an alternating current, i amperes, at time t milliseconds, over a half-cycle, are given in the table.*

t	0	1	2	3	4	5	6	7	8	9	10
i	0	7·0	10·8	10·0	5·1	8·0	13·8	17·2	16·6	9·5	0
i^2											

Copy and complete the table to show the values of i^2 for the range given.

(*c*) *Using the values in part (b), divide the given half-cycle into 5 equal intervals, and apply the mid-ordinate rule to determine*

(*i*) *the mean (average) value of i, and*
(*ii*) *the r.m.s. value of i.*

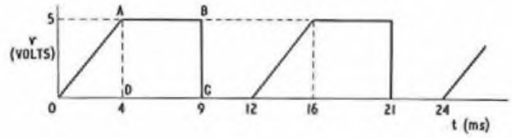


Fig. 2

A. 8. (*a*) As the waveform is periodic, it is sufficient to consider only one period, say, that from $t = 0$ to $t = 12$ ms, to determine the average value of v.

The average value of v between 0–12 ms is given by

$$v_{average} = \frac{\text{area under waveform}}{12},$$
$$= \frac{\text{area of triangle OAD} + \text{area of rectangle ABCD}}{12},$$
$$= \frac{(\tfrac{1}{2} \times 4 \times 5) + (5 \times 5)}{12},$$
$$= \underline{2{\cdot}917\ \text{volts.}}$$

(*b*) The completed table is given below.

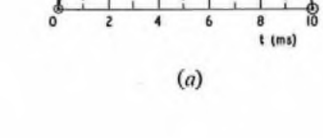
t	0	1	2	3	4	5
i	0	7·0	10·8	10·0	5·1	8·0
i^2	0	49·0	116·6	100·0	26·0	64·0

t	6	7	8	9	10
i	13·8	17·2	16·6	9·5	0
i^2	190·4	295·8	275·6	90·3	0

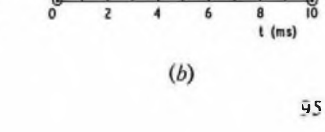
(*c*) The area under a curve may be found approximately by use of the mid-ordinate rule, which states that, if the area under a curve is divided into a number of vertical strips of equal width, and the ordinates at the centre of each strip (the mid-ordinates) are measured, then the area is approximately equal to the width of a strip multiplied by the sum of the mid-ordinates.

(*i*) The graph of i/t is shown in sketch (*a*). The half-cycle is divided into 5 equal intervals by the ordinates at $t = 2$, 4, 6 and 8 ms, and the mid-ordinates of the strips, shown by the dashed lines, occur at $t = 1$, 3, 5, 7 and 9 ms.

$$i_{average} = \frac{\text{area under curve from } t = 0 \text{ to } t = 10}{10},$$
$$= \frac{2(7{\cdot}0 + 10{\cdot}0 + 8{\cdot}0 + 17{\cdot}2 + 9{\cdot}5)}{10},$$
$$= \underline{10{\cdot}34\ \text{amps.}}$$



(*a*)



(*b*)

(*ii*) The r.m.s. value of i is defined as the square root of the mean (or average) value of i^2. The graph of i^2/t is shown in sketch (*b*). Using the mid-ordinate rule,

$$i^2_{average} = \frac{2(49\cdot0 + 100\cdot0 + 64\cdot0 + 295\cdot8 + 90\cdot3)}{10},$$

$$= 119\cdot82 \text{ amps}^2.$$

$$\therefore\ i_{r.m.s.} = \sqrt{119\cdot82} = \underline{10\cdot95 \text{ amps}}.$$

Note: The question does not call for the graphs of i/t and i^2/t to be plotted; neither is this strictly necessary, since the values of i and i^2 required for the application of the mid-ordinate rule are given or derived in the table. Nevertheless, it is good practice to sketch the curves as a safeguard against errors.

Q. 9. (*a*) *The following calculation, carried out on a slide-rule, gives a result having the significant figures, 2658. By suitable approximation and cancelling, insert the decimal point in its correct place in this result.*

$$\frac{47\cdot86 \times 3\cdot16^2 \times \tan 43°}{0\cdot085 \times \sqrt{389\cdot1}}$$

(*b*) *If* $\log_a 40 = 1 + \frac{3}{2} \log_a 4$, *find the value of a.*

(*c*) *In the binary scale, multiply* 10 111 *by* 101, *and convert the result to denary form,.*

(*d*) *The anode current,* I_a *milliamperes, and voltage,* V_a *volts, of a valve are related by the law* $I_a = kV_a^n$, *where k and n are constants. If* $I_a = 4\cdot2$ *when* $V_a = 50$, *and* $I_a = 54\cdot9$ *when* $V_a = 250$, *calculate the values of k and n.*

A. 9. (*a*) Since $3\cdot16^2 \simeq 10$, $\tan 43° \simeq \tan 45° = 1$, and $\sqrt{389\cdot1} \simeq \sqrt{400} = 20$,

$$\frac{47\cdot86 \times 3\cdot16^2 \times \tan 43°}{0\cdot085 \times \sqrt{389\cdot1}} \simeq \frac{48 \times 10 \times 1}{0\cdot08 \times 20},$$

$$\simeq \frac{600 \times 10}{20},$$

$$= 300.$$

$\therefore$ the slide-rule answer is $\underline{265\cdot8}$.

(*b*)
$$\log_a 40 = 1 + \tfrac{3}{2}\log_a 4,$$

or,
$$\log_a 40 - \tfrac{3}{2}\log_a 4 = 1.$$

$$\therefore\ \log_a 40 - \log_a 4^{3/2} = 1.$$

$$\therefore\ \log_a\left(\frac{40}{8}\right) = 1,$$

or,
$$\log_a 5 = 1.$$

$$\therefore\ 5 = a^1.$$

$$\therefore\ \underline{a = 5.}$$

(*c*)

Multiplicand:	10 111
Multiplier:	101
	10 111
	1 011 100
Binary product:	1 110 011

The table shows the conversion of the binary product to denary form.

Binary Digit	Order	Equivalent Denary Value
1	2^0	$1 \times 2^0 = 1 \times 1 = 1$
1	2^1	$1 \times 2^1 = 1 \times 2 = 2$
0	2^2	$0 \times 2^2 = 0 \times 4 = 0$
0	2^3	$0 \times 2^3 = 0 \times 8 = 0$
1	2^4	$1 \times 2^4 = 1 \times 16 = 16$
1	2^5	$1 \times 2^5 = 1 \times 32 = 32$
1	2^6	$1 \times 2^6 = 1 \times 64 = 64$
		Total: 115

Thus, $10\,111_2 \times 101_2 = \underline{115_{10}}.$

(*d*)
$$I_a = kV_a^n.$$

Substituting the given values for I_a and V_a gives

$$4\cdot2 = k50^n, \qquad \ldots\ldots(1)$$

$$\text{and } 54\cdot9 = k250^n. \qquad \ldots\ldots(2)$$

Dividing equation (2) by equation (1) gives

$$\frac{250^n}{50^n} = \frac{54\cdot9}{4\cdot2},$$

$$\text{or, } 5^n = 13\cdot07.$$

$$\therefore\ n\log_{10} 5 = \log_{10} 13\cdot07.$$

$$\therefore\ n = \frac{1\cdot1163}{0\cdot6997} = \underline{1\cdot597.}$$

Substituting for n in equation (1) gives

$$4\cdot2 = k \times 50^{1\cdot597},$$

or,
$$k = \frac{4\cdot2}{50^{1\cdot597}}.$$

$$\therefore\ \log_{10} k = \log_{10} 4\cdot2 - 1\cdot597\log_{10} 50,$$

$$= 0\cdot6232 - 2\cdot713,$$

$$= -2\cdot0898 = \bar{3} + 0\cdot9102.$$

$$\therefore\ \underline{k = 0\cdot00813.}$$

(*to be continued*)

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