SUPPLEMENT

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Contents

QUESTIONS AND ANSWERS

Answers are occasionally omitted or reference is made to earlier Supplements in which questions of substantially the same form, together with the answers, have been published. Some answers contain more detail than would be expected from candidates under examination conditions.

TELEGRAPHY B, 1973 (*continued*)

Q. 7. *A telegram is originated by a customer using a public call-office; the telegram is for delivery by hand in a distant city.*

(a) Describe how the telegram is handled at each stage of transmission.

(b) Explain how the customer is charged for the call.

(c) How would the routing of the telegram be changed if the addressee were a telex subscriber?

A. 7. See A. 7, Telegraphy B, 1970. *Supplement*, Vol. 64, p. 27, July 1971.

Q. 8. *(a) Explain how sparking may occur at a contact when an electric circuit is broken.*

(b) Where is this likely to occur in a telegraph circuit and what are the effects?

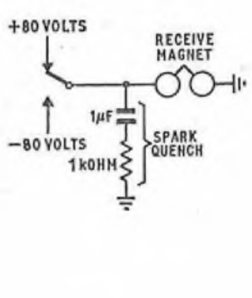
(c) What means are adopted to reduce sparking? Draw a circuit diagram showing typical values and explain how this operates.

A. 8. (*a*) When an inductive circuit is disconnected, the stored electro-magnetic energy causes the generation of an e.m.f. which tends to maintain the direction of the current flow. If the inductance of the circuit is of a high value and the circuit is broken almost instantaneously, an e.m.f. of considerable value may be generated. This may be of sufficient magnitude to break down the insulation of the air gap and cause a spark, or arc, at the point where the circuit is broken.

The stored energy $=\frac{1}{2}LI^2$, where L is the value of the inductance in henrys and I is the value of the current in amperes. The induced e.m.f. $= -L\times$(the rate of change of the current). The negative sign indicates that the e.m.f. is in the reverse direction to the applied e.m.f.

(*b*) Telegraph circuits contain inductors in the form of receive magnets, filters or relays. The telegraph circuit is made or broken by the transmitter contacts at frequent intervals to provide the telegraph signals. Arcing occurs at these contacts, causing the transfer of contact material from one contact to the other, forming a pip on one contact and a crater on the other. These irregularities increase the contact resistance and may reduce the contact gap to such an extent that the contacts weld and become inoperative.

(*c*) Sparking is reduced by connecting a spark-quench circuit across the load. This prevents the formation of an arc by absorbing the energy stored in the inductive circuit. The sketch shows a typical telegraph circuit with a spark-quench circuit consisting of a 1 μF capacitor connected in series with a resistor of 1 kohm; the spark-quench circuit is connected as near to the transmitter contact as possible.



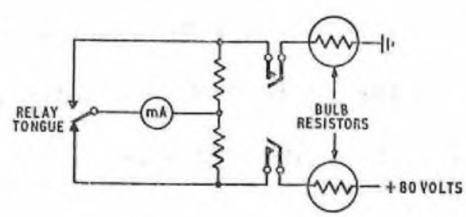
When the inductive circuit is broken, or the polarity is reversed, the induced e.m.f. causes current to flow to charge the capacitor. If the capacitor alone is provided and if it is of sufficient capacitance, the voltage across the capacitor does not rise to a value high enough to cause sparking whilst the contact gap opens. The inductive energy is dissipated by surges of current between the inductor and capacitor. However, a capacitor of high value permits a high charging current to flow when the contacts close, and this current may damage the contacts in a similar manner to the spark. To prevent this, a resistor is connected in series with the capacitor, and this, in turn, limits the efficiency of the quenching circuit, as it reduces the charging rate of the capacitor when the circuit is broken. The values of capacitance and resistance chosen are a compromise depending on the inductance, the value of induced current, the rate of opening of the contact, and the contact material.

Q. 9. *An electromagnetic telegraph relay is to be tested after a long period of use.*

(a) Describe (i) the tests which would be applied to the relay, and (ii) the adjustments which may be necessary.

(b) What other work may be required before the relay can be put back into service?

A. 9. (*a*) (*i*) and (*ii*) The first test to be applied to the relay is the d.c. test. Sketch (*a*) shows the connexions. The relay coil is

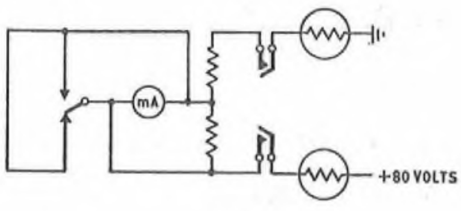


(*a*)

energized and the milliammeter shows whether current is flowing through the contact. Reversal of the coil current causes the milliammeter to give an indication of the opposite polarity to check the other contact. If there is no current flowing, the relay contacts require cleaning and adjusting. If the tongue and contacts are bunched, both bulb resistors glow, indicating that the contacts require adjustment.

With the same circuit connected, but with an alternating current applied to the coil, a test may be made of the bias (or neutrality) of the relay. The meter has a vibrating deflexion, the mean position of which indicates the bias of the relay. The relay contacts should be moved to the left or to the right until the meter vibrates about zero, indicating that the tongue is resting on each contact for the same length of time. More accurate adjustments may be made if the meter shunt is removed; this gives a larger deflexion.

To check the transit time of the relay, the milliammeter is connected as in sketch (*b*) and the coil is again energized by an alternating



(*b*)

current. Using this test, the meter is short-circuited whilst the tongue rests on either contact. Current only passes whilst the tongue is in transit between the contacts. The transit time may be calculated by measuring the deflexion on the meter and applying a formula which depends on the frequency of the vibration, the constants of the ammeter and circuit, and the ratio of the meter deflexion whilst vibrating to the meter deflexion with direct current applied.

The d.c. sensitivity of the relay is tested by connecting the meter as in sketch (*a*) and applying the minimum current to the coil to cause the tongue to move to the opposite contact. Reversal of the coil current should cause the reversal of the current in the meter, showing that the tongue has moved. If the meter does not deflect in either direction, the relay is out of adjustment.

(*b*) Before testing the relay, all dirt and metallic particles should be removed and the relay should be thoroughly cleaned. The contacts should be burnished and surface irregularities removed by the use of a file, a burnisher and a chamois leather. The alignment of the contacts should also be checked. The anti-chatter springs should be checked and the tension adjusted. The relay is then ready for testing.

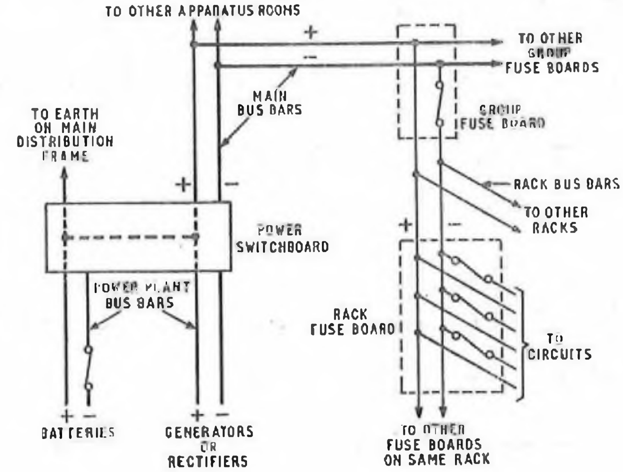
Q. 10. (*a*) *Draw a diagram to illustrate the distribution of power supplies in a telex exchange, showing the location of any protective devices.*

(*b*) *What factors govern*

(*i*) *the sizes of the distribution conductors, and*
(*ii*) *the capacity of the batteries to be provided?*

A. 10. (*a*) A diagram illustrating the distribution of 50-volt power supplies in a telex exchange is shown in the sketch. For ±80-volt signalling supplies, a similar distribution is used.

(*b*) (*i*) The size of a distribution cable depends on the voltage drop that may be allowed and on the current rating of the cable. Typical



values of permissible voltage drop between various stages of the power distribution are as follows.

Mains distribution to motors or rectifiers	0·5 volt.
Rectifier to power switchboard	0·5 volt.
Battery to power switchboard	0·25 volt.
Switchboard to apparatus rooms (80-volt supply)	1 volt.
Switchboard to apparatus room (50-volt supply)	0·15 volt.

Each value is for the maximum load current. The cost of distribution conductors is high and, as the cost is inversely proportional to the voltage drop, the size of the conductor should be calculated for the maximum permissible voltage drop. When the optimum size of the cable has been calculated, a suitable choice is made from the standard sizes of cable. The cable chosen is then checked for current rating to ensure that the cable is capable of carrying fault current up to the maximum value allowed by the fuse without overheating.

(*ii*) Batteries are provided to ensure continuity of operation of the exchange in case the mains supply fails. For small exchanges, the battery normally has a capacity of at least two day-loads; for larger exchanges, which are provided with automatically-starting, engine-driven alternator sets, the batteries are only sufficient to satisfy the busy-hour load. For each type of exchange, the capacity of the battery is calculated on the maximum load which the exchange is designed to carry.

The busy-hour consumption of any group of equipment is determined by calculating the product of the traffic to be carried in erlangs and the average value of the current during a call of average duration.

LINE PLANT PRACTICE B, 1973

Students were expected to answer any six questions

Q. 1. (*a*) *Describe in detail the compacting-factor test for a sample of concrete.*

(*b*) *For what type of concrete mix is this test particularly suited, and why?*

A. 1. See A.1, Line Plant Practice B, 1969. *Supplement*, Vol. 63, p. 34, July 1970.

Q. 2. *Describe the tests that would be carried out on a 0·8 km section of audio-type, paper-insulated cable which does not require balancing.*

A. 2. The tests carried out on an 0·8 km section of audio-type, paper-insulated cable, which does not require balancing, are designed to prove

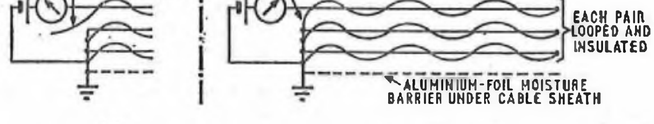
(*a*) the continuity of the conductors,
(*b*) the absence of wire-crosses between pairs,
(*c*) freedom from wire-to-wire and wire-to-earth contacts,
(*d*) the insulation resistance is correct, and
(*e*) the absence of air-leakage under air pressure.

Test for Continuity, Crosses and Contacts

It is possible to carry out tests for all of these conditions in one sequence.

At the remote end of the cable under test, the two wires of each pair are twisted together and insulated with a paper sleeve. All the conductors at the testing end are connected together with tinned copper wire, one end of which is earthed by soldering it to the aluminium-foil moisture barrier under the polyethylene sheath of the cable.

One pair at a time is taken from the earthed bunch; one wire is connected as shown in sketch (*a*), whilst the other wire is insulated.



(*a*) (*b*)

No deflexion on the detector indicates that the pair is clear of wire or earth contacts, and crosses. The insulated wire is returned to the earthed bunch, as shown in sketch (*b*), and a deflexion on the detector indicates continuity of the pair. The tested pair is then placed on one side, and the remaining pairs tested in a similar manner. If, on any pair, a deflexion larger than normal is noted, a contact between the wires of the pair under test is indicated. Any faulty pairs should be reconnected to the earthed bunch before further pairs are tested.

Failure to detect a split pair results in crosstalk, and this cannot be corrected by introducing a further cross; such action results in capacitive unbalance and causes severe crosstalk couplings.

Insulation-Resistance Test

This test is carried out to ascertain the resistance of the insulation between conductors, and confirm that it falls within specified limits for the particular cable under test.

The conductors at the distant end are individually insulated from each other and from earth. At the testing end, the A-wires are bunched together, and so are the B-wires; in the case of quad cables the C- and D-wires are also individually bunched.

The insulation resistance between each group of wires and the remaining groups, joined to the cable earth, is measured with a 500-volt ohmmeter. The insulation resistance of the cable per kilometre in Mohm/km is obtained by multiplying together the reading of the ohmmeter (in Mohms), the number of wires per group, and the length of the cable (km).

The insulation-resistance test proves the absence of contacts between any of the groups, and between any of the wires of the cable and earth. It is possible to prove the absence of contacts between any of the individual wires of each of the groups in the following manner. All the wires of the cable at the testing end are bunched together and earthed. A complete pair (or quad) is then withdrawn, and its insulation resistance is measured with respect to the whole of the remaining wires. This is repeated until each pair has been tested.

Pressure Test

A pressure test is carried out on all joints to ensure the effectiveness of the joint between the sleeve and the cable sheath. Air, which has been dried by being passed through a cylinder containing silica gel, is forced into an air valve, fitted to the joint sleeve, using an air desiccator, the open end of the cable being sealed. When a pressure of approximately 65 kPa has been obtained at both ends of the section, the epoxy-resin wipes, sealing the joint, are smeared with a bubble-forming solution and carefully examined for bubbles.

Following the removal of the desiccator, there should be no observable drop in pressure within 24 h.

Jelly-filled cables cannot be pressure-tested, and a visual inspection must be relied upon, extra care being taken to ensure that the sheath seal is effective.

Q. 3. *With the aid of sketches, describe how a submarine telephone cable with repeaters is laid from a cable ship in deep water.*

Q. 4. (*a*) *With the aid of sketches, describe the equipment used for tracing the route and depth of a buried armoured cable.*

(*b*) *Describe how this equipment would be used.*

A. 4. (*a*) It is important to be able to locate underground cables and cable tracks without the need to excavate, as this is both costly and time consuming. The principle of operation of the present equipment is that a 1 kHz signal is connected to the cable conductors, armouring or sheath, or to a metallic service, the signal setting up electric and magnetic fields along the route of the cable or service. At any point along the line of the route, the cable or service may be located and identified, or its depth determined, by detecting the alternating magnetic field with a search coil, the detected signal being amplified and fed to a headgear receiver.

The 1 kHz signal is produced from a transistor oscillator and the tone can be continuous or interrupted.

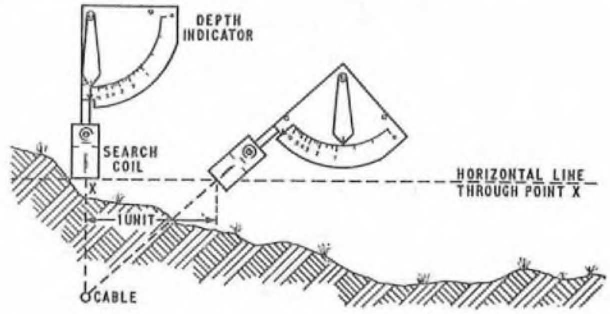
The search coil is a 180 mm long, rectangular coil, wound on a polystyrene former over a flat mu-metal core. A capacitor is connected in parallel with the coil to tune it to have maximum sensitivity at approximately 1 kHz.

A transistor amplifier, with a variable gain of up to 70 dB, is used. The amplifier is connected to the search coil by a coaxial cable, and to a rocking-armature-type headgear receiver.

An aluminium plate, engraved with a quadrant depth-scale, with a freely moving pointer, serves as a depth-indicator.

(*b*) At a suitable point in the line of the route of the cable to be located, whether at the main distribution frame, cabinet, pillar, distribution point, or any other intermediate point, the cable pairs are bunched and connected to the CABLE terminal of the oscillator. If the buried cable is a working cable, the armouring may be used as a metallic path, provided that the armouring is bonded across any joints, and that the d.c. resistance is less than 100 ohms. The REMOTE EARTH terminal is connected to a low-resistance earth, either via an earth-spike driven into the ground to a depth of 0·75–1 m, approximately 8–9 m from the cable, or via existing earths. These existing earths may be used only if they have a low-enough resistance, preferably less than 100 ohms. The cable pairs, or armouring, at the distant end are also connected to a low-resistance earth, so that a strong magnetic field is produced by the current flowing in the cable.

The operator commences the search for the buried cable by moving the search coil from side to side across the line of the route of the cable. Initially, the search coil is operated on its side, with the marking, MAXIMUM, uppermost. In this position, the search coil produces a maximum-amplitude signal in the headgear receiver when it is directly above the cable. To obtain a more accurate location, the



search coil is used upright, so that the marking, MINIMUM, is uppermost. In this position, the operator will not hear any signal when the search coil is directly above the cable; this is known as the *null point*. Where it is necessary to record the line of the route of the cable, marking pegs are placed in the ground directly above the cable, and measurements taken as necessary.

To ascertain the depth of the cable, the depth-indicator is fitted to the search coil, and the cable is accurately located using the null-point method, and its position marked on the ground. The depth-indicator is then held a chosen distance to one side of the cable and tilted, so that the flat base points in the general direction of the cable. The relative positions of the search coil should be in the same horizontal plane. The angle of tilt is adjusted until a further null point is found, and the pointer then indicates the depth of the cable, in the same units as the distance by which the coil was originally displaced. To increase the accuracy, measurement is made on the other side of the cable in a similar manner, and the average of the two readings taken. The sketch illustrates the procedure.

An alternative method is to tilt the coil at one side of the cable until the pointer reads unity, indicating an angle of 45°. The search coil is then moved horizontally until the null point is determined, the distance moved being, then, the same as the depth of the cable.

This apparatus, and method of operation, under good conditions, can detect cable for considerable distances, and at depths of up to 4·5 m.

Q. 5. *A pole route carries an aerial cable 7 m high at pole positions. The terminal span is 50 m, and the cable has a maximum dip of 550 mm. The terminal pole is supported by a strut, in line with the cable. The strut is fixed to the pole 1 m below the cable, and enters the ground 2 m from the base of the pole. If the cable weighs 5 N/m, and the average diameter of the strut is 200 mm,*

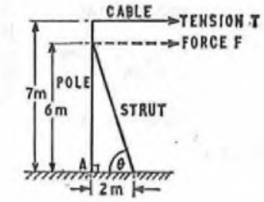
(*a*) *calculate the force in the pole,*
(*b*) *calculate the stress in the strut, and*
(*c*) *state whether the pole and the strut are in tension or compression.*

A. 5. The tension, T, in the cable is given by

$$T = \frac{L^2W}{8d},$$

where L is the length of the span (m),
W is the cable weight (N/m), and
d is the dip (m).

$$\therefore\ T = \frac{50^2 \times 5}{8 \times 550 \times 10^{-3}},\ = 2{,}841\ \text{N}.$$



From the sketch, the force, F, resisted by a strut 1 m below the cable, is given by taking moments about A.

Hence, $$F \times 6 = T \times 7.$$

$$\therefore\ F = \frac{2{,}841 \times 7}{6},\ = 3{,}315\ \text{N}.$$

Now, $F = C_1 \cos\theta$, where C_1 is the force in the strut (N), and θ is the angle of the strut with respect to the ground.

$$\therefore\ C_1 = \frac{3{,}315}{\dfrac{2}{\sqrt{(6^2+2^2)}}} = 10{,}483\ \text{N}.$$

(*a*) The force in the pole, C_2, is given by

$$C_2 = C_1 \sin\theta.$$

$$\therefore\ C_2 = 10{,}483 \times \frac{6}{\sqrt{(6^2+2^2)}} = \underline{9{,}945\ \text{N}.}$$

(*b*) Now, the area of the strut section

$$= \pi \times 100^2 \times 10^{-6}\ \text{m}^2.$$

$$\therefore\ \text{the stress in the strut} = \frac{10{,}483}{\pi \times 100^2 \times 10^{-6}}\ \text{N/m}^2,$$

$$= \underline{3{\cdot}34 \times 10^5\ \text{N/m}^2.}$$

(*c*) By inspection, the pole is in tension, and the strut is in compression.

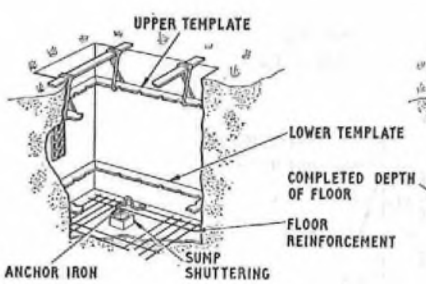
Q. 6. (*a*) *Describe in detail, with sketches, the work of constructing a reinforced-concrete manhole, using straight bars for the reinforcement.*

(*b*) *What are the advantages of straight-bar reinforcement as compared with the earlier types of bent-bar reinforcing?*

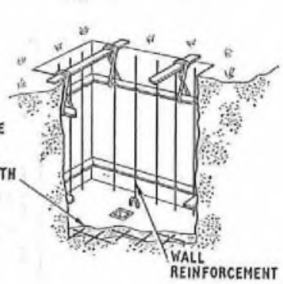
A. 6. (*a*) The manhole is excavated to the required dimensions, allowing a minimum of 300 mm for wall thickness (150 mm per wall) above the internal dimensions of the manhole, using a template placed on the surface to act as a guide. Where necessary, the sides of the excavation are supported with poling boards, held in place by horizontal walings. The bottom of the excavation is levelled, and any soft spots are dug out. The manhole base is then filled with hardcore and consolidated to form a solid foundation of approximately 300 mm depth. A sump hole, having dimensions of 480 mm square and 300 mm deep, is excavated in the foundation material. A minimum depth of 125 mm of concrete is placed in the base, and shuttering is erected to a point above the finished floor level.

For each wall to be constructed, two light, wooden templates, slotted to hold mild-steel reinforcing bars, are placed in position. The lower template is supported by short lengths of reinforcing bars, driven into the walls of the excavation about 300 mm above the foundations; the upper template is positioned 300 mm below roof level. The templates are necessary to ensure that the reinforcing bars remain in position, about 25 mm beneath the surface of the wall, while the concrete is being placed.

The floor concrete, of quality A (one part cement, two parts sand, four parts 19 mm aggregate), is placed on the foundation base to a depth of about 40 mm less than the finished depth. The floor anchor iron is correctly positioned, with its associated reinforcing bars, followed by the floor reinforcement. Sketch (*a*) illustrates the construction arrangements at this stage. The vertical wall-reinforcing bars, which are of such a length that they partially project into the floor and roof, are then placed in position and are held in place by the templates. The floor is brought up to its full thickness as soon as possible, sloping it slightly towards the sump. Sketch (*b*) illustrates the construction at this stage.



(*a*)



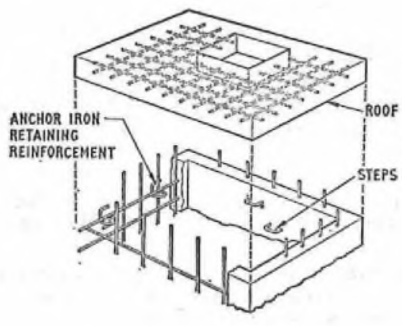
(*b*)

After a minimum period of 12 h, construction of the walls commences. The main framework of the internal wall shuttering consists of corner posts, made from stout wooden timbers, cut to length and temporarily strutted. The first set of shuttering boards are cut to length and placed horizontally on the floor, between the corner posts. A construction joint, which consists of the removal of all dirt, stones and laitance (a scum left on top of new concrete when set), and a rendering of neat cement to a depth of 6 mm, is made between the internal shuttering boards and the walls of the excavation.

The wall concrete, of quality A, is then immediately placed behind the shuttering boards. The next set of shuttering boards is fixed in position, and further concrete is placed. As the construction of the manhole walls proceeds in this way, horizontal mild-steel reinforcing bars, anchor irons and their reinforcing bars, steps, where called for, and foundation bolts for cable bearers are all placed in position. Compaction of the concrete is carried out with a vibrating poker.

Ideally, the erection of the walls should be completed in one operation; if this is not possible, and there is a break of more than two hours, a construction joint is made before the recommencement of concreting, such construction joints being sited so as to give a minimum clearance of 300 mm from any anchor-iron position. Care is taken when compacting concrete newly placed on concrete which has reached its initial set, as any reinforcement hit by a vibratory poker may break its bond and become ineffective.

When the walls have reached the height of the roof, the roof shuttering is erected. A minimum of 12 h should elapse before concreting takes place. Prior to placing the roof concrete, a construction joint is made on the top of the walls. A minimum of 25 mm of concrete of quality A is placed on the roof shuttering. The main reinforcement is then laid on this concrete, and the concrete work is completed as quickly as possible, additional reinforcing bars being placed around the entrance shaft. The ladder hooks and steps are fixed in the shaft shuttering prior to concreting. Sketch (*c*) shows the roof construction with part of the walls; some of the reinforcing bars have been omitted for clarity.



(*c*)

The shuttering within the manhole is removed after a period depending on the type of cement used. The internal walls are finished with a cement wash to fill all voids. The floor, is rendered with cement mortar, 20 mm thick, and given a slight slope to the sump, and the sump-hole grating fitted. The shaft on the manhole is constructed, either in brick or reinforced concrete, such that, when the frame and cover are fitted, it is level with the surrounding surface. Traffic is allowed to pass over the manhole after a further period of time has elapsed.

(*b*) The advantages of straight-bar reinforcement over earlier types of bent-bar reinforcing are that

(*i*) it requires less space and, hence, is easier to store,
(*ii*) it can be cut to size on-site,
(*iii*) alterations to manhole design, duct entry or shaft positions can be carried out more easily with straight bars,
(*iv*) it is cheaper to provide straight bars than have pre-formed beams and bends made, and
(*v*) although more steel reinforcement is used in straight-bar manhole design, the overall cost of reinforcement is lowered.

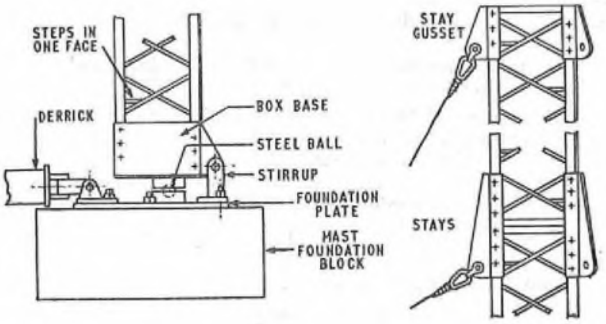
Q. 7. (*a*) *Describe, with sketches, the construction and staying details of a steel lattice aerial mast, 48 m high, composed of twelve 4 m sections.*

(*b*) *Describe the method of erecting such a mast.*

A. 7. (*a*) Lattice steel mast units are made up of 3·8 m long, prefabricated sections. Each section is made up of three angle members with 12 mm zig-zag rod bracing welded either into the root of the angle or to the edge of the angle. The sections are bolted together with 6 mm angle plates to form the whole mast. At stayed joints, jointing plates are used which incorporate stay gussets. Climbing steps are welded into one face of the mast.

A mast of this type is stayed in three directions at mutual angles of 120°. A 48 m high mast would have four vertical stay points, with two stays of 7/2·0 mm and two of 7/2·5 mm stay-wire fitted. The stay-wire is galvanized, standard high-tensile steel wire. Each stay is secured to the stay gusset by means of an 18 mm shackle, and is terminated at the stay anchor by an electrically-welded chain and 22 mm rigging screw, which permits fine and coarse adjustment of the stay. All stays in one vertical plane are secured to a single anchor block, the stay base being approximately two thirds of the mast height.

Sketch (*a*) shows the constructional details of a lattice steel mast.



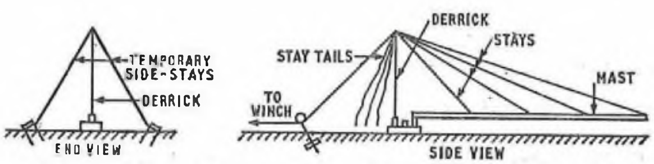
(*a*)

(*b*) The falling-derrick method is used to erect a mast of this size.

The derrick is conveniently formed from sections of steel tube, screwed together. Because the length of the derrick is less than the distance from the base of the mast to one set of stay anchorages, it is not possible to use the full lengths of the stays when fixing them to the top of the derrick. Therefore, short tails are fitted to the stays at appropriate positions, using stay clamps. As an alternative, a set of temporary stays is sometimes provided for the attachment of the mast to the derrick.

A convenient means of carrying out the work, and getting the stay lengths correct, is to lay the derrick on the ground at right angles to the mast and with its foot in the stirrup. To assist this process, the derrick is usually mounted on a universal joint. The geometry of the stays is then similar to that which obtains when the derrick is vertical. Next, the winch line is attached to the derrick, together with two other lines, which act as side stays to keep the derrick in a vertical plane during the raising of the mast.

When all the attachments have been made, the derrick is hoisted into the vertical position by using a short wooden pole as a subsidiary derrick. When vertical, the side lines are fixed to temporary anchorages, and the winch line is tightened to take up the slack in the system. The arrangement is shown in sketch (*b*).



(*b*)

By operating the winch, the derrick is pulled down to ground level, thereby raising the mast. During this process, a close watch is kept on all temporary stays, and if necessary, lifting is interrupted so that adjustments may be made.

Each permanent stay is now released in turn from the derrick and transferred to the anchorage, the top stay being transferred first. When all stays are terminated on their permanent anchorages, they are adjusted to set the mast vertical and to give the designed tensions. The derrick gear and temporary stays are dismantled and the operation is, then, complete.

Q. 8. *A 50 Hz power line runs close to an overhead telephone line as shown in Fig. 1, which is to scale. Fig. 2 shows the relationship between mutual inductance and separating distance. To what value must the fault current in the power line be limited so that the longitudinal induced voltage in the telephone line shall not exceed* 430 *volts?*

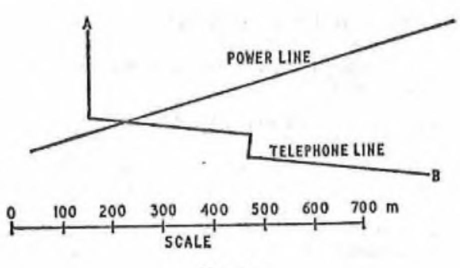


Fig. 1

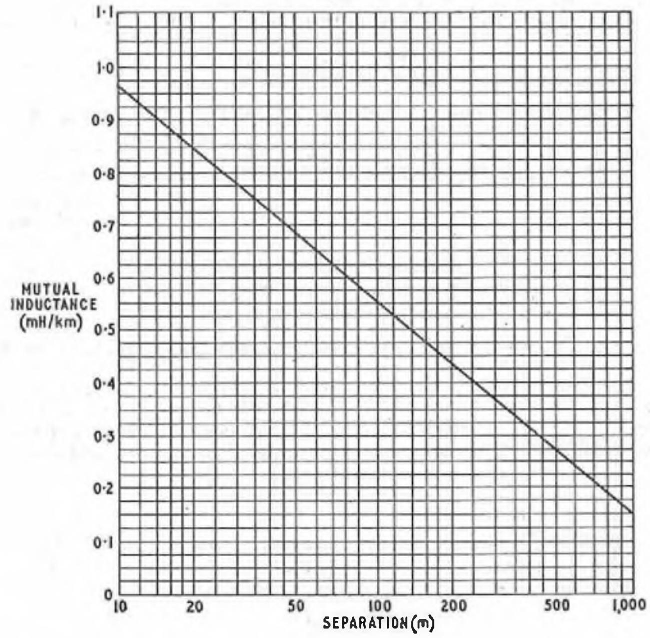


Fig. 2

A. 8. The principle of the solution to this question is exactly as described in A.10, Line Plant Practice B, 1969. *Supplement*, Vol. 63, p. 37, July 1970.

The derived formula, $I = \dfrac{1{,}370}{ML}$ amps, still applies, where I is the current (amperes), M is the mutual inductance (mH/km), and L is the length (km).

The value of $M \times L$ obtained from the derived table of measured values and calculated results is 0·316 mH.

$$\therefore\ I = \underline{4{,}335 \text{ amps.}}$$

Q. 9. *A lead-sheathed cable has failed because of corrosion.*

(*a*) *Explain how it can be ascertained whether the failure was due to electrolytic or non-electrolytic corrosion.*

(*b*) *How can a cable be protected from non-electrolytic corrosion?*

A. 9. (*a*) The cause of corrosion can generally be distinguished by the nature of the surrounding medium, the nature of the attack, and the chemical composition of the corrosion products.

If the corrosion is non-electrolytic, small, local cells are set up by

(*a*) slight variations in the typical concentration of acid or alkaline compounds which may be present in normal soil moisture,

(*b*) variations in the amount of oxygen in the electrolyte,

(*c*) variations in the condition of the lead sheath, or

(*d*) the presence of other metals.

The local cells consist of anodic and cathodic areas which are very close together. Because of this, the anodic products combine with the alkaline substances formed at the adjacent cathodic areas. This inter-diffusion causes the formation of lead carbonates and basic lead carbonates, which are both white in colour, and, in some cases, lead monoxide, which is orange-red. The action forms deep, steep-sided pits in the cable sheath, often running in a line along the sheath.

Severe corrosion, which occurs when a lead cable sheath is in contact with an aqueous solution, such as water containing a strong concentration of acids, alkalies or chlorides, produces a corrosion product which is, essentially, lead carbonate or lead monoxide. Nitrates are sometimes detected, but only small traces are present, and these can be identified by chemical analysis. The corrosion area is subjected to a uniform and superficial attack, in which the cavities are not undercut, but arise from steady erosion. Such cavities are shallow and saucer-shaped.

If the corrosion is electrolytic, the action is more rapid in very wet situations, particularly if the conductivity of the water is increased by dissolved salts. The corrosion product appears as white lead chlorides. In severe cases, where the current density exceeds about 70 amps/m^2, lead peroxide, which is reddish-brown, may be found. The lead peroxide is transformed into lead carbonate if it remains in contact with the surrounding soil for any length of time. The corrosion area is indicated by inter-crystalline attack, the presence of steep-sided cavities and long, corroded grooves in the metal. The cavities may appear at random points, or may be in straight lines along the length of the cable.

(*b*) A cable is protected from non-electrolytic corrosion by the following precautions.

(*i*) Self-generated sheath currents are reduced if the cable track is adequately drained.

(*ii*) Trunk and junction cables with lead sheaths, installed before the advent of polyethylene-sheathed cables, were provided with a covering of bituminous hessian tapes. Such a covering gives protection, and is most effective in combating the effects of strong acid or alkaline aqueous solutions, and the effects of local cells formed by differences in soil or soil-water composition. Protection in the latter case is due to the provision of a more uniform environment for the cable sheath.

Q. **10.** *(a) What factors influence the transmission efficiency of a local line circuit?*

(b) Fig. 3 shows a subscriber's line connected to an exchange using a variety of conductors. From the values given in the table below, find the maximum length of 0·63 *mm conductor which could be used to keep the line within both transmission* (10 *dB*) *and signalling* (1,000 *ohms*) *limits.*

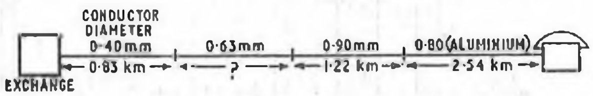


Fig. 3

Conductor diameter (mm)	*Loop resistance to d.c. (ohms/km)*	*Planning attenuation (dB/km)*
0·40	275	2·20
0·63	109	1·38
0·90	55	1·04
0·80 (aluminium)	112	1·70

A. **10.** (*a*) The transmission efficiency of a local line circuit is influenced by the loop resistance and mutual capacitance of a pair. For a given loop resistance, a cable pair with heavy-gauge conductors will extend over a greater distance than a cable pair with lighter conductors, but because of its greater capacitance, gives an inferior transmission performance.

(*b*) For the 0·40 mm diameter conductor section,

the loop resistance $= 275 \times 0{\cdot}83 = 228$ ohms, and
the transmission loss $= 2{\cdot}2 \times 0{\cdot}83 = 1{\cdot}83$ dB.

Similarly, for the 0·90 mm diameter conductor section, the loop resistance is 67 ohms, and the transmission loss is 1·27 dB.

Also, for the 0·80 mm diameter, aluminium conductor section, the loop resistance is 284 ohms, and the transmission loss is 4·32 dB.

The sum of the transmission losses for the three sections is

$$1{\cdot}83 + 1{\cdot}27 + 4{\cdot}32 = 7{\cdot}42 \text{ dB},$$

and the sum of the loop resistances is

$$228 + 67 + 284 = 579 \text{ ohms}.$$

Thus, as the total transmission loss allowable is 10 dB, the maximum permissible loss of the 0·63 mm diameter conductor section is

$$10 - 7{\cdot}42 = 2{\cdot}58 \text{ dB}.$$

Hence, the maximum length of the section is restricted to

$$\frac{2{\cdot}58}{1{\cdot}38} = 1{\cdot}87 \text{ km}.$$

Also, as the total loop resistance allowable is 1,000 ohms, the maximum permissible loop resistance for the 0·63 mm diameter conductor section is $1{,}000 - 579 = 421$ ohms.

Hence, the maximum length of the section is restricted to

$$\frac{421}{109} = 3{\cdot}86 \text{ km}.$$

Therefore, the limiting factor is the transmission loss, and the maximum length of 0·63 mm diameter conductor which can be used is 1·87 km.

COMPUTERS B, 1973

Students were expected to answer any six questions

Q. **1.** *Consider the denary number* 175·671875.
Convert this number into

(a) binary,
(b) octal, and
(c) binary-coded decimal, weighted 8421.
Show all working.

A. **1.** (*a*) To convert the denary (decimal) number, 175·671875, into binary, it is advisable to consider the integer and fractional parts separately.

175_{10} may be converted to binary by repeated division by 2, and noting the remainder.

Quotient	Remainder
2 175	
87	1
43	1
21	1
10	1
5	0
2	1
1	0
0	1

The binary number is obtained by writing down the remainder in the reverse order.

$$\therefore\ 175_{10} = 10\ 101\ 111.$$

0·671875 may be converted to binary by repeated multiplication by 2, and noting the integer part.

Integer Part	Result
	0·671875 × 2
1	·343750
0	·68750
1	·3750
0	·750
1	·50
1	·00

The binary number is obtained by writing down the integer parts in the correct order.

$$\therefore\ 0{\cdot}671875_{10} = 0{\cdot}101\ 011.$$

$$\therefore\ 175{\cdot}671875_{10} = 10\ 101\ 111{\cdot}101\ 011_2.$$

(*b*) Since $8 = 2^3$, each three-bit binary group may be converted, by inspection, to an octal digit.

$$\therefore\ 175{\cdot}671875_{10} = 10\ 101\ 111{\cdot}101\ 011_2 = 257{\cdot}53_8.$$

(*c*) In the binary-coded-decimal (b.c.d.) system, four bits are required to form each decimal digit. The bits are weighted such that the least significant bit has the decimal value 1, the second significant bit the value 2, the third the value 4, and the most significant bit the value 8. Therefore, the decimal numbers 0 to 9 are represented as shown in the table.

Decimal Digit	B.C.D.
0	0000
1	0001
2	0010
3	0011
4	0100
5	0101
6	0110
7	0111
8	1000
9	1001

$\therefore$ $175{\cdot}671875_{10}$ = 0001 0111 0101·0110 0111 0001 1000 0111 0101.

Q. 2. (*a*) *What is meant by the term "two's complement"?*
(*b*) *Why is two's-complement arithmetic used in most modern digital computers?*
(*c*) *Using two's-complement arithmetic, perform the following calculations in binary, showing all working:*
(*i*) $33_{(10)} - 17_{(10)}$,
(*ii*) $58_{(10)} - 73_{(10)}$.

A. 2. (*a*) The two's complement of a binary number is that number which, when added to the original number, will result in an all-zeros answer and a carry from the left-most bit. The two's complement is obtained by finding the one's complement and adding 1. The one's complement is obtained by inverting each bit of the original number.

(*b*) Two's-complement arithmetic is used in computers because it simplifies the process of subtraction. The two's complement of a number is a convention used for representing negative numbers. During calculations using two's-complement working, it is not necessary to keep a check of the sign of partial results. If the answer is negative, its two's complement is taken as the magnitude.

In two's-complement, each number is assigned a sign bit: 0 for positive and 1 for negative. To obtain a negative number, the positive number, including the sign bit, is two's compelemented. In two's-complement arithmetic, subtraction can be performed by an addition, e.g. $(33 - 17)_{10}$ is the same as $(33 + (-17))_{10}$.

(*c*) (*i*) $33_{10} - 17_{10}$.

	sign bit	
17_{10}	0	010 001
Two's complement of 17_{10}	1	101 111
Add 33_{10}	0	100 001
	0	010 000

The answer is $+$ 010 000_2 (i.e. 16_{10}).

(*ii*) $58_{10} - 73_{10}$.

	sign bit	
73_{10}	0	1 001 001
Two's complement of 73_{10}	1	0 110 111
Add 58_{10}	0	0 111 010
	1	1 110 001

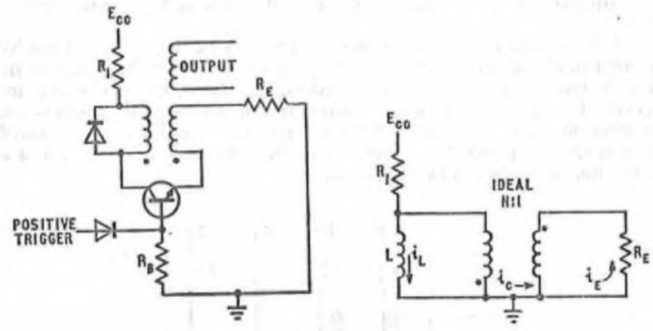
The 1 in the sign bit indicates a negative number and, hence, the result has to be two's complemented again.

Two's-complement answer = 0 0 001 111.

The answer is $-$ 1 111_2 (i.e. -15_{10}).

Q. 3. (*a*) *With the aid of a circuit diagram, explain the operation of a blocking oscillator.*
(*b*) *Give one application of blocking oscillators in digital computers.*
(*c*) *Why is the blocking oscillator no longer used in modern digital computers and what has tended to replace it?*

A. 3. (*a*) The circuit diagram of a typical blocking oscillator is shown in sketch (*a*).



(*a*) (*b*)

The blocking oscillator shown is a monostable device which is stable with the transistor cut-off. The output of the transistor is coupled back to the input via a transformer. The primary inductance of the transformer is used to obtain the timing of the temporarily-stable operating state. The transformer has N times as many turns in the emitter circuit as in the collector circuit and is connected in such a way as to provide polarity inversion. Thus, the collector current flowing in the transformer induces an emitter current which raises the potential on the base. This, in turn, further reduces the collector voltage and, by this regenerative process, the transistor is quickly saturated.

At this point, the equivalent circuit shown in sketch (*b*) becomes valid. Since the time taken to reach saturation is negligible, the pulse amplitude and width can be considered to be determined by this equivalent circuit, in which the transformer is replaced by an ideal transformer in parallel with the magnetizing inductance, L, of the collector winding.

Initially, the current, i_L, through the primary inductance, L, is small. But, since L is finite, i_L grows whilst current i_E begins to fall. Eventually, i_E becomes too small to hold the transistor saturated; i_L ceases to rise, cutting off i_E, and the transistor turns off degeneratively.

The transistor is saturated whilst $\beta I_B > I_C$, and cuts off when $\alpha I_E \simeq I_C$.

(*b*) A monostable blocking oscillator is sometimes used to generate a single pulse of fixed amplitude and width on receipt of a pulse at the trigger input. The special property of blocking oscillators is that of producing large-current or high-voltage pulses. Another of their advantages is the ability to have multiple outputs.

(*c*) Blocking oscillators are no longer used in modern digital computers because of the difficulties associated with transformers, e.g. size, weight, cost and heat dissipation. Blocking oscillators were mainly used as monostable and astable multivibrators and for frequency division, but, in recent years, with the increasing use of integrated circuits and thin-film technology, it is more common to implement these functions with semiconductors, using capacitor-resistor circuits to control the timing.

Q. 4. (*a*) *Draw a circuit diagram of a two-input, diode-transistor, positive-logic* NAND *gate with an emitter-follower output stage, and explain its operation.*
(*b*) *Sketch on the same voltage-against-time axes a typical input- and output-voltage waveform for the emitter follower used in (a).*

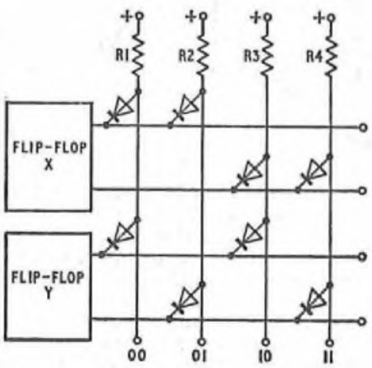
Q. 5. (*a*) *What is meant by the term "microprogram"?*
(*b*) *With the aid of sketches, describe two ways in which microprograms are constructed.*
(*c*) *How is the address of the next microprogram step produced?*

A. 5. (*a*) The concept of microprogramming has been developed to provide a means whereby the programmer may, within limits, design the machine-code instruction-set to suit his requirements. Microprograms translate the machine-code instructions into a greater number of elemental operations which achieve the required effect.

Microprograms are implemented in hardware and in software, but this is transparent to the machine-code programmer. In a microprogrammed machine, the functions used by the programmer are interpreted and performed by an underlying program, the *microprogram* which uses a very primitive order-code implemented in electronic logic.

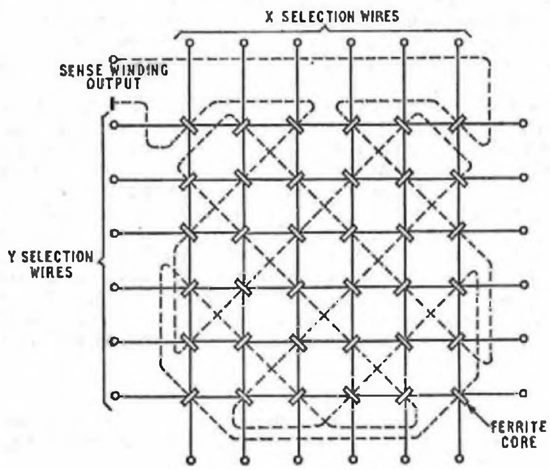
The feature, common to all forms of microprogramming, is that some method is provided for specifying the individual control signals to be actuated in the computer. A *micro-operation*, then, is an operation controlled by one of these individual control signals. In the most elementary form of microprogramming, the instruction format, particularly the operation part, is so designed that the individual bits in the instruction pertain to individual control signals. In the more general form of microprogramming, the bit combinations in the operation part of each instruction are used to represent a set of instructions.

(*b*) Two ways in which microprograms may be implemented are by using a diode matrix, and by software, using a read/write store. In the first instance, the program is wired-in, and the control circuits are arranged so that any given bit combination in the operation part of an instruction initiates a desired set of micro-instructions. The control signals are generated through arrays of diode matrices. A diagram of a diode matrix is shown in sketch (*a*).



(*a*)

In the second instance, microprograms are stored in a storage unit which is typically a core store (see sketch (*b*)). With stored microprograms, the execution of each instruction is somewhat analogous to the functioning of a sub-routine in conventional programming.



(*b*)

(*c*) The sequencing of micro-instructions involves jumps and conditional jumps from one address to another in the microprogram storage unit for obtaining successive micro-instructions. Either a one-address or a two-address format could be used for indicating the jump addresses. In one version of the jumping procedure, a no-address format is used, and certain of the micro-operations, by design of the system, cause the next successive micro-instruction to be taken from a predetermined address. Any of these micro-operations can be conditional, as with conventional jump instructions.

With stored microprograms, the execution of each of the instructions would be accomplished by causing reference to a particular address in the storage unit. The functioning of the system would depend on the microprograms stored in that unit. One of the micro-operations that must be provided is an operation that terminates the microprogram and causes the next micro-instruction to be initiated.

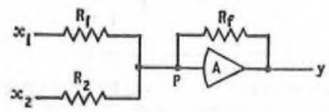
Q. 6. (*a*) *Sketch a graph of the possible outputs of a sense read wire in a typical ferrite-core store, giving, on the same axis, the output of a one and a nought, with typical e.m.f. values.*

(*b*) *With the aid of a block diagram, explain the operation of the major sections of a circuit that will accept the sense-wire output, amplify it, discriminate between a one and a nought, and produce a strobed output.*

Q. 7. (*a*) *Draw a diagram of a circuit using an operational amplifier to sum two analog values, showing clearly the position of input and feedback components.*

(*b*) *With reference to* (*a*), *derive, from basic principles, a formula which gives the output voltage in terms of the input voltage and the input and feedback resistances.* (*Assume an operational amplifier with an infinite gain.*)

A. 7. (*a*) A circuit using an operational amplifier to sum two analog values is shown in the sketch.



(*b*) The operation of the circuit shown in part (*a*) may be analysed by applying Kirchhoff's current law at node P.

The assumption is made that no current is taken by the amplifier itself.

The voltage at node P is V_a.

$$\frac{x_1 - V_a}{R_1} + \frac{x_2 - V_a}{R_2} + \frac{y - V_a}{R_f} = 0. \qquad \ldots\ldots(1)$$

The equation of the amplifier of gain μ is

$$y = -\mu V_a. \qquad \ldots\ldots(2)$$

Substituting equation (2) into equation (1) to eliminate V_a, and rearranging the terms, gives

$$y = -\frac{R_f\left(\frac{x_1}{R_1} + \frac{x_2}{R_2}\right)}{1 + \frac{1}{\mu}\left\{1 + R_f\left(\frac{1}{R_1} + \frac{1}{R_2}\right)\right\}}.$$

If μ is very large, then the following assumption may be made:

$$\frac{1}{\mu}\left\{1 + R_f\left(\frac{1}{R_1} + \frac{1}{R_2}\right)\right\} \ll 1,$$

$$\therefore\ y \simeq -R_f\left(\frac{x_1}{R_1} + \frac{x_2}{R_2}\right),$$

$$\text{or, } y \simeq -R_f \sum_{i=1}^{2} \frac{x_i}{R_i},$$

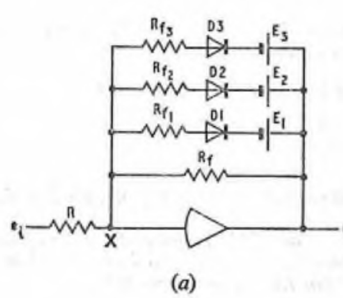
$$\text{or, } y \simeq -\sum_{i=1}^{2} A_i x_i, \text{ where } A_i = \frac{R_f}{R_i}.$$

Q. 8. (*a*) *Why are diode function generators required in analog computers?*

(*b*) *Draw a diagram of a diode function generator that will produce the function* $y = x^2$ *for both positive and negative values of x. With the aid of a graph, illustrate the expected output of the function generator and, by labelling the graph, relate it to your block diagram.*

A. 8. (*a*) Diode function generators are required to produce a linear approximation to the required characteristic in realizing a non-linear simulation. This means that, instead of having a slope that varies continuously as the input varies, the characteristic consists of constant-slope segments with step discontinuities of slope occurring at particular input values. These input values, at which the characteristic changes from one segment to another, are known as break points. The individual segments are obtained by linear methods, and the choice of segment for any particular input is made by a diode switching circuit.

(*b*) A circuit of a diode function generator, which will approximate to the function $y = x^2$ for negative values of x, is shown in sketch (*a*).



(*a*)

It can be seen from sketch (*a*) that the feedback branches containing diodes will be effectively open circuited as long as the diodes are reverse biased, i.e. $e_0 > E_I$. The values of the three batteries are chosen such that $E_3 > E_2 > E_1 > 0$. If $(e_0 - E_1) < 0$, then all the feedback loops are in circuit and the effective feedback resistance is R_{e1}, where $R_{e1} = R_{f3}$ in parallel with R_{f2} in parallel with R_{f1} in parallel with R_f.
Assuming the potential at point X is zero, then,

$$\frac{e_I}{R} + \frac{e_0}{R_{e1}} = 0$$

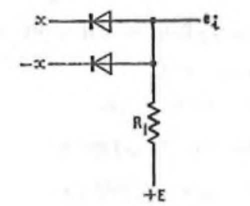
by Kirchhoff's first law, which states that there is no net current at a junction.

$$\therefore \quad e_0 = -\frac{R_{e1} \times e_I}{R}.$$

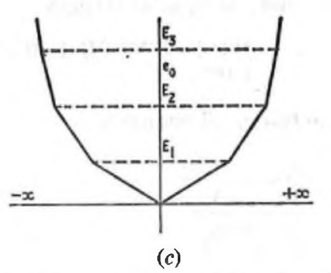
The slope of the function over the range $0 < e_0 < E_1$ is given by

$$\frac{de_0}{de_i} = \frac{R_{e1}}{R}.$$

The first breakpoint occurs when $e_0 = E_1$. Diode D_1 ceases to conduct, switching out the feedback resistance R_{f1}. The effective feedback resistance during the range $E_1 < e_0 < E_2$ is now R_{e2}, where $R_{e2} = R_{f3}$ in parallel with R_{f2} in parallel with R_f.
Similarly, it can be shown that the second breakpoint occurs when $e_0 = E_2$, and the new feedback resistance during the range $E_2 < e_0 < E_3$ is now R_{e3}, where $R_{e3} = R_{f3}$ in parallel with R_f. A further breakpoint will occur when $e_0 = E_3$.
The slope of the function at a particular point is, therefore, dependent on the total effective resistance divided by the input resistance, R. By choosing the appropriate values for the feedback resistances, R_f, R_{f1}, R_{f2} and R_{f3}, the generator can be adjusted to represent the function $y = x^2$. As each breakpoint occurs, the feedback resistance and the slope of the transfer function are increased. The accuracy of the approximation can be improved by increasing the number of breakpoints.
The output voltage, y, cannot be negative since it represents the square of the input. Therefore, owing to the polarity inversion of a function generator the input voltage must always be negative. The circuit to implement this is shown in sketch (*b*). The output of this



(*b*)



(*c*)

circuit, e_I, is always the negative value of the modulus of x, and this is input to the function generator. The expected output of the function generator in series with the negative modulus circuit is shown in sketch (*c*).

Q. **9.** *(a) Explain what is meant by variation of resistance with temperature, and illustrate, with the aid of a sketch, what practical application this effect might have.*
(b) What is meant by the term "fundamental interval"?
(c) Give three main advantages of resistance thermometers over other forms of temperature-measuring elements.

Q. **10.** *A 24-bit digital computer inputs 8-bit bytes into a 24-bit accumulator. A software routine is required to input three bytes, pack them into the accumulator and, then, to store the accumulator into successive store locations in a fixed buffer area, starting at location 100 and finishing at location 600.*

The flow chart of this software routine is shown in Fig. 1.

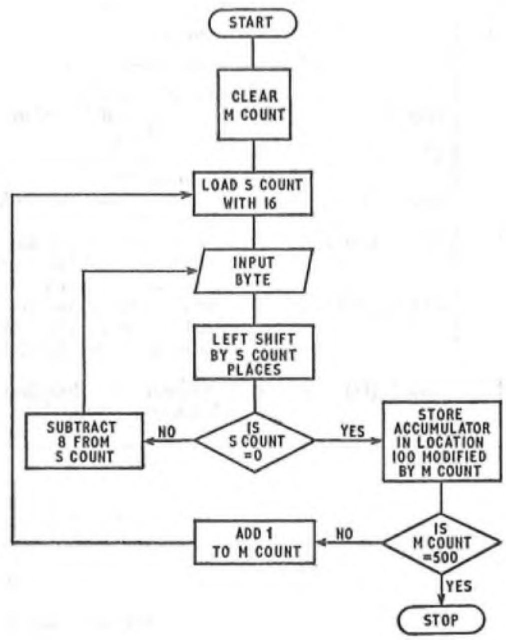


Fig. 1

Devise a code of your own choice and use it to write a program to perform the flow-chart operation.
Give a key to the code used.

A. **10.** The assembly language, devised for this problem, is of the form that would be implemented on a simple computer with one accumulator and the ability to handle only single-address instructions. Because of the restriction of one accumulator, it is necessary to use a temporary store location, TEMP, to hold intermediate results during packing.
The software routine which performs the operation of packing and storing bytes of data is given in Table 1, and the Keycode in Table 2.

Table 1 Software Routine

Label	Instruction		Comments
	CLR		Clear accumulator
	ST	MCOUNT	Set MCOUNT = 0
	ST	TEMP	Set TEMP = 0
L1:	LC	16	Load 16 into accumulator
	ST	SCOUNT	Set SCOUNT = 16
L2:	JSR	READ	Read one byte of data into accumulator
	SHL	SCOUNT	Shift accumulator left SCOUNT places
	A	TEMP	Add TEMP to accumulator
	ST	TEMP	Store accumulater in location TEMP
	L	SCOUNT	Load SCOUNT into accumulator
	BZ	L3	If accumulator equals zero then branch to L3
	SC	8	Subtract 8 from accumulator
	ST	SCOUNT	Store accumulator in location SCOUNT
	B	L2	Branch to L2
L3:	L	TEMP	Load TEMP into accumulator
	STA 100,	MCOUNT	Store accumulator in location (MCOUNT) + 100
	L	MCOUNT	Load MCOUNT into accumulator
	SC	500	Subtract 500 from accumulator
	BZ	L4	If accumulator equals zero then branch to L4
	AC	501	Add 501 to accumulator
	ST	MCOUNT	Store accumulator in location MCOUNT
	CLR		Clear accumulator
	ST	TEMP	Set TEMP = 0.
	B	L1	Branch to L1
L4:	STOP		

Table 2 Keycode

Instruction		Meaning	
CLR		$O \rightarrow A$	Clear accumulator
L	N	$(N) \rightarrow A$	Load contents of location N into accumulator
LC	C	$C \rightarrow A$	Load integer constant, C, into accumulator
ST	N	$(A) \rightarrow N$	Store contents of accumulator in location N
STA C, N		$(A) \rightarrow (N) + C$	Store contents of accumulator in location C modified by the contents of location N
SHL	N	$(A) \times 2^{(N)} \rightarrow A$	Contents of the accumulator are shifted left a number of places given by the contents of location N
A	N	$(A) + (N) \rightarrow A$	The content of location N is added to the accumulator

Instruction		Meaning	
AC	C	$(A) + C \rightarrow A$	The constant C is added to the accumulator
SC	C	$(A) - C \rightarrow A$	The constant C is subtracted from the accumulator
B	N	$N \rightarrow PC$	Unconditional branch
BZ	N	If $(A) = O$ then $N \rightarrow PC$	Branch if accumulator equals zero.
JSR SUB		$PC \rightarrow$ Stack $SUB \rightarrow PC$	Branch to the subroutine SUB

PC = Program Counter
A = Accumulator
(A) = Contents of accumulator
N = Location N
(N) = Contents of location N

MATHEMATICS C, 1973

Students were expected to answer any six questions

Q. **1.** (*a*) *Find the number k, such that the equation*

$$\frac{kt}{t-2} = \frac{t+2}{t-1} \textit{ has two equal roots in } t.$$

(*b*) *solve for x the equations*

$$20x + 10y - z = 0,$$
$$20x + y + 31z = 6,$$
$$55x - 11y + 4z = 0.$$

Express the answer correct to two significant figures.

A. 1. (*a*) $$\frac{kt}{t-2} = \frac{t+2}{t-1}.$$

$$\therefore\ kt^2 - kt = t^2 - 4.$$

$$\therefore\ (k-1)t^2 - kt + 4 = 0.$$

This is a quadratic equation in t, and the condition for it to have two equal roots is that $b^2 = 4ac$ in the general equation $ax^2+bx+c = 0$.

Hence, $$(-k)^2 = 4 \times (k-1) \times 4.$$

$$\therefore\ k^2 = 16k - 16,$$

or, $$k^2 - 16k + 16 = 0.$$

$$\therefore\ k = \frac{16 \pm \sqrt{(256 - 64)}}{2},$$

from the general solution to a quadratic equation,

$$= \frac{16 \pm 13{\cdot}86}{2}.$$

Hence, $$k = \underline{14{\cdot}93} \text{ or } \underline{1{\cdot}07}$$

for the equation to have equal roots in t.

(*b*)
$$20x + 10y - z = 0, \qquad \ldots\ldots (1)$$
$$20x + y + 31z = 6, \qquad \ldots\ldots (2)$$
$$55x - 11y + 4z = 0. \qquad \ldots\ldots (3)$$

Multiplying equation (1) by 31 gives

$$620x + 310y - 31z = 0. \qquad \ldots\ldots (4)$$

Adding equations (2) and (4) gives

$$640x + 311y = 6. \qquad \ldots\ldots (5)$$

Multiplying equation (1) by 4 gives

$$80x + 40y - 4z = 0. \qquad \ldots\ldots (6)$$

Adding equations (3) and (6) gives

$$135x + 29y = 0. \qquad \ldots\ldots (7)$$

Multiplying equation (5) by 29 gives

$$18{,}560x + 311 \times 29y = 174. \qquad \ldots\ldots (8)$$

Multiplying equation (7) by 311 gives

$$41{,}985x + 29 \times 311y = 0. \qquad \ldots\ldots (9)$$

Subtracting equation (8) from equation (9) gives

$$23{,}425x = -174,$$
$$\therefore\ x = -0{\cdot}007427.$$

Substituting for x in equation (7) gives

$$29y = 135 \times 0{\cdot}007427.$$
$$\therefore\ y = 0{\cdot}03457.$$

Substituting for x and y in equation (1) gives

$$z = 20 \times (-0{\cdot}007427) + 10 \times 0{\cdot}03457,$$
$$= 0{\cdot}1972.$$

Hence, correct to two significant figures,

$$\underline{x = -0{\cdot}0074,}$$
$$\underline{y = 0{\cdot}035}, \text{ and}$$
$$\underline{z = 0{\cdot}20.}$$

Note: The solution should be checked in each of the three original equations. Because of the small differences involved, the check should be made, by slide-rule, using the values obtained for x, y and z before correction to two significant figures.

Q. **2.** (*a*) *Show graphically, or otherwise, that the cubic equation*

$$x^3 - 6x^2 + 14x - 42 = 0$$

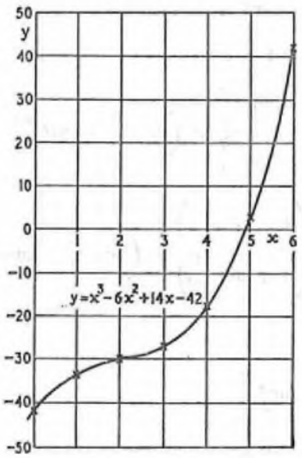
has only one real root, and obtain this to two significant figures.
(*b*) *Using this approximate root, calculate, to two decimal places, the two conjugate complex roots of the cubic equation.*

A. 2. (*a*) The real roots of the equation may be obtained by plotting the function $y = x^3 - 6x^2 + 14x - 42$, over an appropriate range of values of x, to determine the point where the graph crosses the x-axis; that is, where $y = 0$.

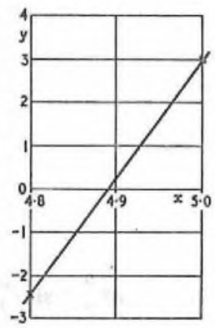
The graph is plotted from the following table of values.

x	x^3	$-6x^2$	$14x$	-42	y
0	0	0	0	-42	-42
1	1	-6	14	-42	-33
2	8	-24	28	-42	-30
3	27	-54	42	-42	-27
4	64	-96	56	-42	-18
5	125	-150	70	-42	3
6	216	-216	84	-42	42

The curve is shown in sketch (*a*), and is seen to cross the x-axis at $x = 4{\cdot}9$.



(*a*)



(*b*)

For values of x greater than 6, the curve rises sharply, and y remains positive, since

$$\begin{aligned} y &= x^3 + 14x - 6x^2 - 42, \\ &= x(x^2 + 7 + 7) - 6(x^2 + 7), \end{aligned}$$

and it is clear that, when $x \geqslant 6$, the positive term must always be greater than the negative term.

Similarly, when $x < 0$, the curve descends steeply, and y remains negative, since, in this case, all the terms of y are negative when x is negative. Thus, since the graph of y only crosses the x-axis at one point, the equation $x^3 - 6x^2 + 14x - 42 = 0$ has only one real root.

The graph shown in sketch (*a*) is almost certainly of sufficient accuracy to determine the root correct to two significant figures, i.e. $x = 4{\cdot}9$. However, as a check, it will suffice to plot one additional point at $x = 4{\cdot}8$, and to draw a larger-scale graph between $x = 4{\cdot}8$ and $x = 5$.

At $x = 4{\cdot}8$,

$$\begin{aligned} y &= 4{\cdot}8^3 - 6 \times 4{\cdot}8^2 + 14 \times 4{\cdot}8 - 42, \\ &= -2{\cdot}448. \end{aligned}$$

The plotted points are shown in sketch (*b*), from which it is verified that the real root is $\underline{x = 4{\cdot}9}$, correct to two significant figures.

(*b*) Since $x = 4{\cdot}9$ is one root of the equation, $(x - 4{\cdot}9)$ must be a factor of $x^3 - 6x^2 + 14x - 42$.

$$\therefore\ x^3 - 6x^2 + 14x - 42 = (x - 4{\cdot}9)(x^2 + ax + b),$$

or,
$$x^2 + ax + b = \frac{x^3 - 6x^2 + 14x - 42}{x - 4{\cdot}9}.$$

The long division is shown below:

$$\begin{array}{r|l} & x^2 - 1{\cdot}1x + 8{\cdot}61 \\ \hline x - 4{\cdot}9 & x^3 - 6x^2 + 14x - 42 \\ & x^3 - 4{\cdot}9x^2 \\ \hline & -1{\cdot}1x^2 + 14x \\ & -1{\cdot}1x^2 + 5{\cdot}39x \\ \hline & 8{\cdot}61x - 42 \\ & 8{\cdot}61x - 42{\cdot}189 \\ \hline & 0 + 0{\cdot}189 \end{array}$$

The slight discrepancy in the last subtraction is due to the fact that $x = 4{\cdot}9$ is only accurate to two significant figures.

Hence, $x^3 - 6x^2 + 14x - 42 = (x - 4{\cdot}9)(x^2 - 1{\cdot}1x + 8{\cdot}61) = 0$, and the solution of $x^2 - 1{\cdot}1x + 8{\cdot}61 = 0$ gives the complex roots of the cubic equation. From the general formula for the solution of a quadratic equation,

$$\begin{aligned} x &= \frac{1{\cdot}1 \pm \sqrt{(1{\cdot}1^2 - 4 \times 8{\cdot}61)}}{2}, \\ &= \frac{1{\cdot}1 \pm j\sqrt{(33{\cdot}23)}}{2}, \\ &= 0{\cdot}55 \pm j2{\cdot}88, \text{ to two decimal places.} \end{aligned}$$

Thus, the two conjugate complex roots of the cubic equation are $\underline{0{\cdot}55 + j2{\cdot}88}$ and $\underline{0{\cdot}55 - j2{\cdot}88}$.

Q. 3. (*a*) *Write down as far as the term in x^4 the binomial series for*

(*i*) $(1 + x)^{-2}$, *and*
(*ii*) $(1 - x)^{1/2}$,

assuming x is numerically less than unity.

(*b*) *The maximum velocity, u metres per second, of the electron stream in an oscilloscope is given by the formula*

$$u = 3 \times 10^8 \sqrt{\left\{1 - \left(1 + \frac{2V}{10^6}\right)^{-2}\right\}},$$

where V volts is the controlling voltage. If V is of the order of 1,000 or less, show that u can be considered to be proportional to the square root of V.

A. 3. (*a*) (*i*)
$$\begin{aligned} (1 + x)^{-2} &= 1 + (-2)x + \frac{(-2)(-3)}{1 \times 2}x^2 \\ &\quad + \frac{(-2)(-3)(-4)}{1 \times 2 \times 3}x^3 \\ &\quad + \frac{(-2)(-3)(-4)(-5)}{1 \times 2 \times 3 \times 4}x^4 + \ldots, \\ &= \underline{1 - 2x + 3x^2 - 4x^3 + 5x^4 - \ldots.} \end{aligned}$$

(*ii*)
$$\begin{aligned} (1 - x)^{1/2} &= 1 + \tfrac{1}{2}(-x) + \frac{\frac{1}{2}(-\frac{1}{2})}{1 \times 2}(-x)^2 \\ &\quad + \frac{\frac{1}{2}(-\frac{1}{2})(-\frac{3}{2})}{1 \times 2 \times 3}(-x)^3 \\ &\quad + \frac{\frac{1}{2}(-\frac{1}{2})(-\frac{3}{2})(-\frac{5}{2})}{1 \times 2 \times 3 \times 4}(-x)^4 + \ldots, \\ &= \underline{1 - \frac{x}{2} - \frac{x^2}{8} - \frac{x^3}{16} - \frac{5}{128}x^4 - \ldots.} \end{aligned}$$

(*b*) From part (*a*) (*i*), $\left(1 + \frac{2V}{10^6}\right)^{-2}$ may be expanded as follows:

$$\left(1 + \frac{2V}{10^6}\right)^{-2} = 1 - \frac{4V}{10^6} + 3 \times \frac{4V^2}{10^{12}} \ldots.$$

If V is of the order of 1,000 or less, the value of the third term of the expansion is 12×10^{-6} or less, and may be neglected in comparison with the second term. Similarly, all terms after the third, being of rapidly diminishing magnitude, may also be neglected.

Hence,
$$\begin{aligned} 1 - \left(1 + \frac{2V}{10^6}\right)^{-2} &\simeq 1 - \left(1 - \frac{4V}{10^6}\right), \\ &= \frac{4V}{10^6}. \end{aligned}$$

$$\therefore\ u \simeq 3 \times 10^8 \sqrt{\left(\frac{4V}{10^6}\right)}.$$

Thus, $\underline{u \propto V^{1/2}}$. Q.E.D.

Q. 4. (*a*) *Assuming the compound-angle formulae, derive a formula for $\cos 3A$ in terms of $\cos A$.*

(*b*) *A current, $6 \sin \omega t - 2 \sin 3\omega t$ milliamperes (at an instant t seconds), passes through an inductance of 2 mH.*

(*i*) *Sketch the current/time graph from $t = 0$ to $t = 2\pi/\omega$.*
(*ii*) *If $\omega = 10^4$, calculate the time in milliseconds at which the current is first zero, after $t = 0$.*

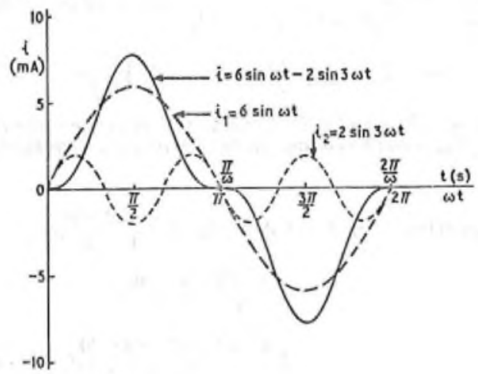
(*iii*) *Derive an expression for the voltage, v volts, across the inductance at the instant t, assuming $v = L\dfrac{di}{dt}$, where L is the inductance in henrys, and i is the current in amperes. Show that it may be expressed in the form $v = a \sin 2\omega t \sin \omega t$, where a is a constant.*

A. 4. (*a*) Now, $\cos 3A = \cos(A + 2A)$,

$$\begin{aligned}
&= \cos A \cos 2A - \sin A \sin 2A,\\
&= \cos A(\cos^2 A - \sin^2 A) - \sin A \times 2 \sin A \cos A,\\
&= \cos A\{\cos^2 A - (1 - \cos^2 A)\}\\
&\quad -2\cos A \sin^2 A,\\
&= 2\cos^3 A - \cos A - 2\cos A\,(1 - \cos^2 A),\\
&= 2\cos^3 A - \cos A - 2\cos A + 2\cos^3 A,\\
&= \underline{4\cos^3 A - 3\cos A.}
\end{aligned}$$

(*b*) Let $i = 6 \sin \omega t - 2 \sin 3\,\omega t$, where i is the current in milliamps.

(*i*) The current/time graph of $i = 6 \sin \omega t - 2 \sin 3\omega t$ is shown in the sketch, together with the graphs, shown dashed, of the component currents, $i_1 = 6 \sin \omega t$ and $i_2 = 2 \sin 3\omega t$.



(*ii*) When the current is zero,

$$i = 6 \sin \omega t - 2 \sin 3\omega t = 0.$$

$$\therefore\ 3 \sin \omega t = \sin 3\omega t.$$

But, $\quad \sin 3\omega t = 3 \sin \omega t - 4 \sin^3 \omega t.$

$$\therefore\ 3 \sin \omega t = 3 \sin \omega t - 4 \sin^3 \omega t,$$

or, $\sin \omega t = 0.$

Hence, $\quad \omega t = 0, \pi, 2\pi \ldots.$

After $t = 0$, the current is first zero at $\omega t = \pi$.

$$\therefore\ t = \frac{\pi}{10^4} \times 10^3 \text{ ms},$$

$$= \underline{0{\cdot}3142 \text{ ms.}}$$

(*iii*) If $\quad i = 6 \sin \omega t - 2 \sin 3\omega t,$

then, $\quad \dfrac{di}{dt} = 6\,\omega \cos \omega t - 6\omega \cos 3\omega t.$

Now, $\quad v = L\dfrac{di}{dt}.$

$$\therefore\ v = L \times 10^{-3} \times 6\omega(\cos \omega t - \cos 3\omega t),$$

$$= \underline{0{\cdot}006\omega L(\cos \omega t - \cos 3\omega t) \text{ volts.}}$$

Let $\omega t = \theta$, so that $3\omega t = 3\theta$. From the multiple-angle formulae,

$\cos(3\theta - \theta) = \cos 3\theta \cos \theta + \sin 3\theta \sin \theta, \quad \ldots\ldots (1)$

and, $\cos(3\theta + \theta) = \cos 3\theta \cos \theta - \sin 3\theta \sin \theta. \quad \ldots\ldots (2)$

Subtracting equation (2) from equation (1),

$$\cos(3\theta - \theta) - \cos(3\theta + \theta) = 2 \sin 3\theta \sin \theta.$$

Now, let $3\theta - \theta = T$, and $3\theta + \theta = S$.

Then, adding and subtracting the formulae for T and S,

$$3\theta = \frac{S + T}{2} \text{ and } \theta = \frac{S - T}{2}.$$

Hence, $\quad \cos T - \cos S = 2 \sin \dfrac{S + T}{2} \sin \dfrac{S - T}{2}.$

Applying this formula, for transforming the difference of two cosines into a product of sines, to the expression for v,

$$\cos \omega t - \cos 3\omega t = 2 \sin 2\omega t \sin \omega t.$$

$$\therefore\ v = 0{\cdot}006\omega L \times 2 \sin 2\omega t \sin \omega t,$$

$$= \underline{0{\cdot}012\omega L \sin 2\omega t \sin \omega t.} \qquad \text{Q.E.D.}$$

Q. 5. (a) Using the substitution, $\tan \theta/2 = t$, show that $\cos \theta = \dfrac{1-t^2}{1+t^2}$, and $\sin \theta = \dfrac{2t}{1+t^2}$. Hence, or otherwise, solve the equation

$$5 \cos \theta - 2 \sin \theta = 3$$

for $0 \leqslant \theta \leqslant 360°$.

(b) Sketch the graph of the curve given in polar co-ordinates by the equation $r = 3(1 + \sin \theta)$ for $0 \leqslant \theta \leqslant 2\pi$.

A. 5. (*a*) Now, $\cos \theta = \cos\left(\dfrac{\theta}{2} + \dfrac{\theta}{2}\right)$,

$$= \cos^2 \frac{\theta}{2} - \sin^2 \frac{\theta}{2}$$

from the addition formula,

$$= \frac{\cos^2 \frac{\theta}{2} - \sin^2 \frac{\theta}{2}}{\cos^2 \frac{\theta}{2} + \sin^2 \frac{\theta}{2}} \times \left(\cos^2 \frac{\theta}{2} + \sin^2 \frac{\theta}{2}\right),$$

$$= \frac{\cos^2 \frac{\theta}{2}\left(1 - \tan^2 \frac{\theta}{2}\right)}{\cos^2 \frac{\theta}{2}\left(1 + \tan^2 \frac{\theta}{2}\right)} \times \left(\cos^2 \frac{\theta}{2} + \sin^2 \frac{\theta}{2}\right).$$

But $\quad \cos^2 \dfrac{\theta}{2} + \sin^2 \dfrac{\theta}{2} = 1.$

$$\therefore\ \cos \theta = \frac{1 - \tan^2 \frac{\theta}{2}}{1 + \tan^2 \frac{\theta}{2}},$$

$$= \underline{\frac{1 - t^2}{1 + t^2}}. \qquad \text{Q.E.D.}$$

Also, $\quad \sin \theta = 2 \sin \dfrac{\theta}{2} \cos \dfrac{\theta}{2},$

$$= 2 \tan \frac{\theta}{2} \cos^2 \frac{\theta}{2},$$

$$= \frac{2 \tan \frac{\theta}{2}}{\sec^2 \frac{\theta}{2}}.$$

But, $\quad \sec^2 \dfrac{\theta}{2} = 1 + \tan^2 \dfrac{\theta}{2}$

from the identity $\sin^2 \dfrac{\theta}{2} + \cos^2 \dfrac{\theta}{2} = 1.$

$$\therefore\ \sin \theta = \frac{2 \tan \frac{\theta}{2}}{1 + \tan^2 \frac{\theta}{2}},$$

$$= \underline{\frac{2t}{1 + t^2}}. \qquad \text{Q.E.D.}$$

Substituting these expressions in $5 \cos \theta - 2 \sin \theta = 3$,

$$5\left(\frac{1 - t^2}{1 + t^2}\right) - 2 \times \frac{2t}{1 + t^2} = 3.$$

$$\therefore\ 5 - 5t^2 - 4t = 3 + 3t^2.$$

$$\therefore\ 8t^2 + 4t - 2 = 0,$$

$$\text{or, } 4t^2 + 2t - 1 = 0.$$

$$\therefore\ t = \frac{-2 \pm \sqrt{(4 + 16)}}{8},$$

$$= \frac{-2 \pm 4{\cdot}472}{8},$$

$$= 0{\cdot}309 \text{ or } -0{\cdot}809.$$

$$\therefore \frac{\theta}{2} = 17° \, 10' \text{ or } (180° + 17° \, 10')$$

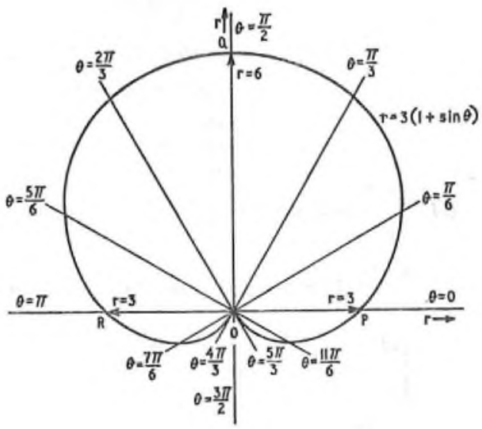
$$\text{or } (180° - 38° \, 58') \text{ or } (360° - 38° \, 58').$$

$$\therefore \theta = \underline{34° \, 20' \text{ or } 282° \, 4'} \text{ for } 0 \leqslant \theta \leqslant 360°.$$

(*b*) Now, $r = 3(1 + \sin\theta),$

$$= 3 + 3\sin\theta.$$

As θ varies from 0, through $\theta = \pi/2$ rad, to $\theta = \pi$ rad, the radius, r, increases from a minimum value of 3 units to a maximum value of 6 units at $\theta = \pi/2$ rad, and decreases again to a value of 3 units at $\theta = \pi$ rad. When $\theta > \pi$, $\sin\theta$ is negative, and, therefore, r decreases to zero at $\theta = 3\pi/2$ rad, and returns to its value of 3 units at $\theta = 2\pi$ rad. The graph of the function is shown in the sketch.



At points P and R, the radius has the value 3 units, and, at Q, it has its maximum value of 6 units.

Q. 6. *The current in an L-R-C circuit at a time, t, is given by*

$$i = \frac{V}{Z}\sin(\omega t - \phi),$$

where $Z^2 = R^2 + (\omega L - 1/\omega C)^2$, *and* $R \tan\phi = \omega L - 1/\omega C$.

(*a*) *Express i in the form:* $a \sin \omega t + b \cos \omega t$.

(*b*) *Find the value of* ω *for which the peak value of the current is a maximum.*

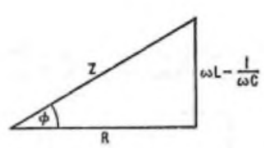
(*c*) *What is this maximum value, and which value of* ϕ *corresponds to it?*

A. 6. (*a*) Now, $i = \dfrac{V}{Z}\sin(\omega t - \phi),$

$$= \frac{V}{Z}(\sin \omega t \cos\phi - \cos\omega t \sin\phi).$$

But $\tan\phi = \dfrac{\omega L - \dfrac{1}{\omega C}}{R},$

and, hence, the impedance vectors may be represented by the impedance triangle shown in the sketch.



From the impedance triangle,

$$\cos\phi = \frac{R}{Z},$$

and $\sin\phi = \dfrac{\omega L - \dfrac{1}{\omega C}}{Z}.$

$$\therefore \quad i = \frac{V}{Z}\left\{\frac{R}{Z}\sin\omega t - \frac{\left(\omega L - \dfrac{1}{\omega C}\right)}{Z}\cos\omega t\right\},$$

$$= \underline{\frac{VR}{Z^2}\sin\omega t - \frac{V\left(\omega L - \dfrac{1}{\omega C}\right)}{Z^2}\cos\omega t.}$$

(*b*) Since $i = \dfrac{V}{Z}\sin(\omega t - \phi)$, the peak value must be $\dfrac{V}{Z}$, that is, when $\sin(\omega t - \phi)$ is unity.

$$\therefore \quad i_{peak} = \frac{V}{Z} = \frac{V}{\left\{R^2 + \left(\omega L - \dfrac{1}{\omega C}\right)^2\right\}^{1/2}}.$$

The peak value will, therefore, attain a maximum when $\left\{R^2 + \left(\omega L - \dfrac{1}{\omega C}\right)^2\right\}^{1/2}$ is a minimum, that is, when

$$\omega L - \frac{1}{\omega C} = 0.$$

$$\therefore \omega L = \frac{1}{\omega C}.$$

$$\therefore \omega^2 = \frac{1}{LC},$$

$$\text{or, } \omega = \frac{1}{\sqrt{(LC)}}.$$

Thus, $\underline{\omega = \dfrac{1}{\sqrt{(LC)}}}$ is the value for which the peak current is a maximum.

(*c*) When $\omega = \dfrac{1}{\sqrt{(LC)}},$

$\underline{i_{peak} = \dfrac{V}{R}}$ is the maximum value.

Also, $\sin\phi = \dfrac{\omega L - \dfrac{1}{\omega C}}{Z} = 0$ when $\omega L - \dfrac{1}{\omega C} = 0.$

Hence, $\underline{\phi = 0}$ corresponds to the maximum value, $\dfrac{V}{R}$, of the peak current.

Q. 7. (*a*) *If u and v are two functions of x, prove that*

$$\frac{d}{dx}\left(\frac{u}{v}\right) = \frac{1}{v^2}\left(v\frac{du}{dx} - u\frac{dv}{dx}\right).$$

(*b*) *State the corresponding formula for differentiating the product of two such functions.*

(*c*) *Differentiate the following, simplifying each result where possible:*

(*i*) $y = x^2 \cos x,$

(*ii*) $y = \dfrac{2x - 7}{(x + 2)^2},$

(*iii*) $y = \log_e\{x + \sqrt{(1 + x^2)}\}.$

A. 7. (*a*) Let $y = \dfrac{u}{v}.$

Suppose that x increases by a small amount, δx, and let δu, δv and δy be the corresponding changes in u, v and y respectively.

Then, $y + \delta y = \dfrac{u + \delta u}{v + \delta v},$

$$\text{or, } \delta y = \frac{u + \delta u}{v + \delta v} - y,$$

$$= \frac{u + \delta u}{v + \delta v} - \frac{u}{v},$$

$$= \frac{v(u+\delta u) - u(v+\delta v)}{v(v+\delta v)},$$

$$= \frac{v\delta u - u\delta v}{v(v+\delta v)}.$$

$$\therefore \frac{\delta y}{\delta x} = \frac{v\dfrac{\delta u}{\delta x} - u\dfrac{\delta v}{\delta x}}{v(v+\delta v)}.$$

As $\delta x \to 0$, $\dfrac{\delta u}{\delta x} \to \dfrac{du}{dx}$, $\dfrac{\delta v}{\delta x} \to \dfrac{dv}{dx}$, $\dfrac{\delta y}{\delta x} \to \dfrac{dy}{dx}$, and $\delta v \to 0$.

Hence, in the limit, when δx is zero,

$$\frac{dy}{dx} = \frac{v\dfrac{du}{dx} - u\dfrac{dv}{dx}}{v^2},$$

or, $$\underline{\frac{d}{dx}\left(\frac{u}{v}\right) = \frac{1}{v^2}\left(v\frac{du}{dx} - u\frac{dv}{dx}\right).}$$ Q.E.D.

(*b*) The corresponding formula for differentiating the product of two such functions is

$$\underline{\frac{d}{dx}(uv) = u\frac{dv}{dx} + v\frac{du}{dx}.}$$

(*c*) (*i*) $y = x^2 \cos x.$

$$\therefore \frac{dy}{dx} = x^2(-\sin x) + 2x\cos x,$$

$$= \underline{x(2\cos x - x\sin x).}$$

(*ii*) $y = \dfrac{2x-7}{(x+2)^2}.$

$$\therefore \frac{dy}{dx} = \frac{(x+2)^2 \times 2 - (2x-7)\times 2\times(x+2)}{(x+2)^4},$$

$$= \frac{2(x+2) - 2(2x-7)}{(x+2)^3},$$

$$= \underline{\frac{-2x+18}{(x+2)^3}.}$$

(*iii*) $y = \log_e\{x + \sqrt{(1+x^2)}\}.$

$$\therefore \frac{dy}{dx} = \frac{1}{x+(1+x^2)^{1/2}} \times \frac{d}{dx}\{x+(1+x^2)^{1/2}\},$$

$$= \frac{1+\frac{1}{2}(1+x^2)^{-1/2}\times 2x}{x+(1+x^2)^{1/2}},$$

$$= \frac{1+\dfrac{x}{(1+x^2)^{1/2}}}{x+(1+x^2)^{1/2}},$$

$$= \frac{x+(1+x^2)^{1/2}}{(1+x^2)^{1/2}\{x+(1+x^2)^{1/2}\}},$$

$$= \underline{\frac{1}{\sqrt{(1+x^2)}}.}$$

Q. 8. (*a*) *Evaluate*

(*i*) $\displaystyle\int_0^2 \frac{dx}{(x+3)^2},$

(*ii*) $\displaystyle\int_0^{\pi/6} \tan 2x dx,$

(*iii*) $\displaystyle\int_0^{\pi/3} \sin 2x \cos x dx.$

(*b*) *Calculate the mean value of the function*

$$y = \sqrt{(x)} - \frac{1}{\sqrt{(x)}} \text{ from } x = 1 \text{ to } x = 4.$$

A. 8. (*a*) (*i*) $$\int_0^2 \frac{dx}{(x+3)^2} = \int_0^2 (x+3)^{-2}dx,$$

$$= \left[\frac{(x+3)^{-1}}{-1}\right]_0^2,$$

$$= -5^{-1} + 3^{-1},$$

$$= \underline{\frac{2}{15}}.$$

(*ii*) $$\int_0^{\pi/6} \tan 2x dx = \int_0^{\pi/6} \frac{\sin 2x}{\cos 2x}dx,$$

$$= \int_0^{\pi/6} \frac{-\frac{1}{2}d(\cos 2x)}{\cos 2x},$$

$$= -\tfrac{1}{2}\Big[\log_e(\cos 2x)\Big]_0^{\pi/6},$$

$$= -\tfrac{1}{2}\left(\log_e \cos\frac{\pi}{3} - \log_e \cos 0\right),$$

$$= -\tfrac{1}{2}(\log_e 0\cdot 5 - \log_e 1),$$

$$= \tfrac{1}{2}\{0 - (-0\cdot 6932)\},$$

$$= \underline{0\cdot 3466.}$$

(*iii*) $$\int_0^{\pi/3} \sin 2x \cos x dx,$$

$$= \int_0^{\pi/3} 2\sin x\cos^2 x\,dx, \text{ since } \sin 2x = 2\sin x\cos x,$$

$$= \int_0^{\pi/3} -2\cos^2 x\, d(\cos x),$$

$$= -2\int_0^{\pi/3} \cos^2 x\, d(\cos x),$$

$$= -2\left[\frac{\cos^3 x}{3}\right]_0^{\pi/3},$$

$$= -\frac{2}{3}\left\{\left(\frac{1}{2}\right)^3 - 1\right\},$$

$$= \underline{\frac{7}{12}}.$$

(*b*) $$y = \sqrt{(x)} - \frac{1}{\sqrt{(x)}},$$

$$= x^{1/2} - x^{-1/2}.$$

Now, $$y_{mean} = \frac{\displaystyle\int_1^4 (x^{1/2} - x^{-1/2})dx}{4-1},$$

$$= \frac{1}{3}\left[\frac{2}{3}x^{3/2} - 2x^{1/2}\right]_1^4,$$

$$= \frac{1}{3}\left\{\left(\frac{16}{3} - 4\right) - \left(\frac{2}{3} - 2\right)\right\},$$

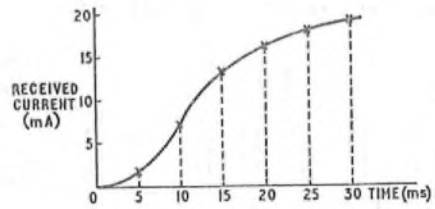
$$= \underline{\frac{8}{9}}.$$

Q. 9. *When a battery is applied to the sending end of a long telegraph line, the received current, at 5 ms intervals, is given by*

Time (ms)	0	5	10	15	20	25	30
Current (mA)	0	1·5	7	13	16	18	19

Use Simpson's rule to evaluate the mean current flowing in these 30 *ms.*

A. 9. The graph of received current against time is shown in the sketch.



Simpson's rule for the approximate evaluation of an area under a curve states that, if the area is divided into an even number, $2n$, of strips, of equal width, h, the area, A, is given by

$$A = \frac{h}{3}\{y_1 + y_{2n+1} + 2(y_3 + y_5 + \ldots) + 4(y_2 + y_4 + \ldots)\},$$

where $y_1, y_2 \ldots y_{2n+1}$ are the ordinates erected at the points of division of the strips.

Thus, referring to the sketch, h is 5 ms, y_1 is the ordinate (equal to zero) at time = zero, and y_{2n+1} is the ordinate (equal to 19 mA) at time = 30 ms; y_3 and y_5 are the odd ordinates at time = 10 ms and time = 20 ms respectively, and y_2, y_4, etc. are the even ordinates at time = 5 ms, time = 15 ms, etc.

Hence, from the given data, the approximate area under the curve from 0–30 ms is given by

$$\frac{5}{3}\{0 + 19 + 2(7 + 16) + 4(1\cdot5 + 13 + 18)\} = 325.$$

$$\text{The mean current flowing} = \frac{\text{area under curve}}{30},$$

$$= \frac{325}{30},$$

$$= \underline{10\cdot83 \text{ mA.}}$$

Q. 10. *(a) If $z = 3 - 4j$, express $2z - z^2$ in the polar form, $r \angle \theta$.*

(b) If $Z_0 = \sqrt{\left(\frac{R + j\omega L}{G + j\omega C}\right)}$, calculate the modulus (magnitude) and the argument (angle) of Z_0 when $R = 88$, $L = 1 \times 10^{-3}$, $C = 0\cdot054 \times 10^{-6}$, and G is negligible.

A. 10. (*a*) $2z - z^2 = z(2 - z),$

$$= (3 - 4j)(-1 + 4j),$$

$$= -3 + 4j + 12j + 16,$$

$$= 13 + 16j,$$

$$= \sqrt{(13^2 + 16^2)} \angle \tan^{-1} \tfrac{16}{13},$$

$$= 20\cdot62 \angle \tan^{-1} 1\cdot231,$$

$$= \underline{20\cdot62 \angle 50° \, 55'.}$$

(*b*)
$$Z_0 = \sqrt{\left(\frac{R + j\omega L}{G + j\omega C}\right)}.$$

When G is negligible, $Z_0 = \sqrt{\left(\frac{R + j\omega L}{j\omega C}\right)}$,

$$= \sqrt{\left(\frac{L}{C} + \frac{R}{j\omega C}\right)},$$

$$= \left(\frac{L}{C} - j\frac{R}{\omega C}\right)^{1/2}.$$

Note: The question omits to give a value to ω, and, hence, it is not possible to determine either the modulus or the argument of Z_0, except in terms of ω. However, in view of the wording of the question, it is presumed that the omission of a numerical value for ω was an oversight. Therefore, assuming ω to be, say, 10^6, the calculation is as follows:

$$Z_0 = \left(\frac{1 \times 10^{-3}}{0\cdot054 \times 10^{-6}} - j\frac{88}{10^6 \times 0\cdot054 \times 10^{-6}}\right)^{1/2},$$

$$= (1\cdot852 \times 10^4 - j1{,}629)^{1/2},$$

$$= \sqrt[4]{\{(1\cdot852 \times 10^4)^2 + (1{,}629)^2\}} \angle \tfrac{1}{2} \tan^{-1}\left(-\frac{1{,}629}{1\cdot852 \times 10^4}\right),$$

$$= \sqrt[4]{(3\cdot565 \times 10^8)} \angle \tfrac{1}{2} \tan^{-1}(-0\cdot08796),$$

$$= \sqrt{(1\cdot859 \times 10^4)} \angle \tfrac{1}{2}(-5° \, 2'),$$

$$= 136\cdot3 \angle -2° \, 31'.$$

Thus, the modulus of Z_0 for $\omega = 10^6$ is 136·3, and the argument is −2° 31′.

COMMUNICATION RADIO C, 1973

Students were expected to answer any six questions

Q. 1. *(a) Briefly explain the terms*

(i) critical frequency,
(ii) maximum usable frequency (m.u.f.), and
(iii) optimum traffic frequency.

(b) What is the relationship between (i) and (ii) in part (a)?

(c) Fig. 1 is a simplified diagram of an ionospheric region in the atmosphere.

(i) Explain how the radio wave incident on the ionized region represented in Fig. 1 is returned to earth when it is below the m.u.f.
(ii) Why does a wave at a frequency above the m.u.f. penetrate the region?
(iii) Where is the maximum electron density in the ionospheric region shown in Fig. 1?

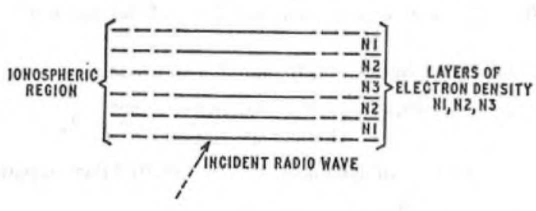


Fig. 1

A. 1. (*a*) (*i*) The critical frequency is the maximum frequency which, for given ionospheric conditions, is reflected when vertically incident.

(*ii*) The maximum usable frequency (m.u.f.) is that frequency which, for given ionospheric conditions, allows communication between two displaced points.

(*iii*) The optimum traffic frequency is approximately 85 per cent of the m.u.f., and allows for fluctuation of ionospheric conditions.

(*b*) The relationship between the critical frequency and the m.u.f. is given by

$$f_m = f_c \sec i,$$

where f_m is the m.u.f. (hertz),
f_c is the critical frequency (hertz), and
i is the angle of incidence (degrees of arc).

(*c*) (*i*) In Fig. 1, the densities of the layers are such that $N_3 > N_2 > N_1$.

Thus, as the incident wave penetrates the ionospheric region, it passes through a progressively more-dense medium. The refractive index of a medium (equal to $\frac{\sin i}{\sin r}$, where r is the angle of refraction), is inversely proportional to the density, so that, as the density increases, sin r, and hence r, increases. The wave is, therefore, progressively bent away from the normal. If the N_3 layer is sufficiently dense, then the wave is eventually totally reflected within the layer and, on passing through the progressively less-dense layers below N_3, bent towards the normal and returned to earth.

(*ii*) The refractive index is also directly proportional to the frequency of the incident wave, so that, as the frequency increases, the angle of refraction decreases. The incident wave is not, then, sufficiently bent for total reflexion, and thus, passes through the N_3 layer. The wave, now passing through the progressively less-dense regions above N_3, is bent towards the normal to take up a direction similar to that of the incident wave.

(*iii*) The maximum electron density is N_3.

Q. 2. (*a*) *Explain clearly why noise factor is an important measure of the performance of a radio receiver.*

(*b*) *If the noise power in a given bandwidth in an aerial is 10 pW, and the noise factor of an amplifier matched to the aerial is 9, what is*

(*i*) *the total noise power in the given bandwidth at the output of the amplifier if it has a power gain of 50, and*

(*ii*) *the contribution of the amplifier to the output noise power in this bandwidth?*

(*c*) *What is the effect on the noise power output from this amplifier if*

(*i*) *the bandwidth is halved, and*

(*ii*) *the aerial and amplifier are mismatched?*

A. 2. (*a*) The source of noise which appears at the output of a receiver may be external, such as electrical ignition systems and other transient disturbances, or internal. Internal noise may be generated by the random movement of electrons in resistors or active devices, such as transistors and valves.

The noise factor, F, of a receiver is a measure of the amount of internal noise introduced by the receiver per unit of gain, and is defined as

$$F = \frac{\text{total noise power at the output}}{\text{output noise power due to input noise power}}.$$

Thus, if P_{no} is the noise power at the output (watts),
P_{ni} is the noise power at the input (watts),
P_{so} is the signal power at the output (watts), and
P_{si} is the signal power at the input (watts),

then,
$$F = \frac{P_{no}}{P_{ni} \times \text{gain}}, \qquad \ldots\ldots(1)$$

$$= \frac{P_{no}}{P_{ni} \times P_{so}/P_{si}} = \frac{P_{si}/P_{ni}}{P_{so}/P_{no}},$$

$$= \frac{\text{input signal-to-noise power ratio}}{\text{output signal-to-noise power ratio}}.$$

(*b*) (*i*) From equation (1),

$$P_{no} = F \times P_{ni} \times \text{gain},$$
$$= 9 \times 10 \times 50 \text{ pW}.$$

$\therefore$ noise power at output $= 4{,}500$ pW.

(*ii*) The noise power at the output due to the input noise

$$= 10 \times 50 \text{ pW}.$$

$\therefore$ the contribution of the amplifier to the output noise power

$$= 4{,}500 - 500 = 4{,}000 \text{ pW}.$$

(*c*) (*i*) The maximum power, P, that a thermal noise source can supply to a load is when the source and load are matched, and is given by

$$P = \text{k} \times t \times \Delta f \text{ watts}, \qquad \ldots\ldots(2)$$

where k is Boltzmann's constant ($1{\cdot}38 \times 10^{-23}$ J/K),
t is the temperature (K), and
Δf is the bandwidth (Hz).

Thus, if the bandwidth is halved, the noise-power output is halved.

(*ii*) When the aerial and amplifier are not matched, equation (2) is no longer valid, and the noise power transferred to the amplifier is reduced; i.e., P_{ni} is reduced.

From equation (1),

$$P_{no} = F \times \text{gain} \times P_{ni}.$$

Since F and gain remain constant, the net effect is a reduction in total noise output.

Q. 3. (*a*) *From what stage in a superheterodyne receiver is automatic gain control (a.g.c.) derived?*

(*b*) *To what stages in a superheterodyne receiver can a.g.c. be applied?*

(*c*) *Why is delayed a.g.c. preferred to simple a.g.c.?*

(*d*) *List the components in the circuit shown in Fig. 2 which contribute to the a.g.c. action, and indicate the function of each. How does this circuit compensate for a change in input-signal level?*

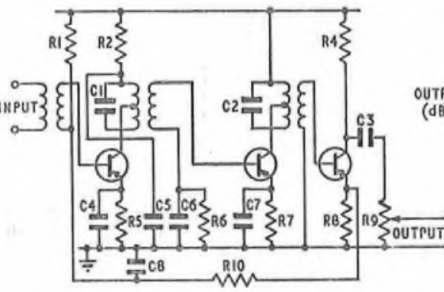
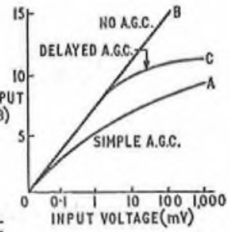


Fig. 2



(*a*)

A. 3. (*a*) Automatic gain control (a.g.c.) reduces any change in amplitude at the receiver output due to variations in carrier amplitude, for a given degree of modulation. A control voltage may, therefore, be obtained by rectification of the radio frequency (r.f.) signal and removal of the audio-frequency (a.f.) signal. The detector or a separate rectifier may be used.

(*b*) A.g.c. may be applied to the r.f. and intermediate-frequency (i.f.) stages such that their gain is controlled. Alternatively, variation of the bias on a diode shunting the first i.f. transformer may be used, or the output of the local oscillator may be varied.

(*c*) Simple a.g.c. controls the gain of the amplifying stages in proportion to the amplitude of the received signal. As illustrated in sketch (*a*), the greater the received signal, the greater the suppression of receiver output (curve A). In comparison with an uncontrolled characteristic (curve B), the effect is merely to reduce the range of variation of output and, unfortunately, to reduce the very weak signals.

Delayed a.g.c. (curve C), however, does not operate until the received signal exceeds some predetermined value. The suppression of weak signals is prevented and, as a bonus, the resultant output characteristic is relatively constant.

(*d*) The components contributing to the a.g.c. action are

(*i*) R8, the resistor across which the a.g.c. control voltage is developed,

(*ii*) R10 and C8, the time constant of which is made long in relation to the a.f. period to prevent the control responding to a.f. variations,

(*iii*) R1, which provides bias voltage to the base of the first transistor,

(*iv*) R2, which determines the potential applied to the collector, and

(*v*) C5, which maintains the R2–C1 junction at earth potential to r.f.

If the receiver output rises, the voltage across R8 becomes more negative. This voltage change is communicated, via R10 and C8, to drive the base of the first transistor more negative. The first-stage collector current rises, and the voltage drop across R2 increases. The collector potential is thus reduced, so that the stage gain, and hence the receiver output, falls.

Q. 4. *A superheterodyne receiver has an intermediate frequency (i.f.) of 10·7 MHz, and the local-oscillator frequency is above the radio frequency. The receiver covers the band of frequency-modulated carriers spaced at 0·5 MHz intervals between 75–97 MHz. Each of these carriers has a modulation index of 5 when the maximum modulating frequency is 15 kHz.*

(*a*) (*i*) *What is the number of frequency-modulated channels in the band?*

(*ii*) *What is the minimum i.f. bandwidth required?*

(*b*) *When the receiver is tuned to the carrier at 75 MHz, what is the band covered by the*

(*i*) *image channel, and*

(*ii*) *adjacent channel?*

(*c*) (*i*) *Which carriers are most susceptible to image-channel interference from within the band?*

(*ii*) *What are the frequencies of the interfering carriers?*

A. 4. (*a*) (*i*) By inspection, the number of carriers

$$= \frac{\text{upper frequency} - \text{lower frequency}}{\text{channel spacing}} + 1.$$

Hence, the number of channels between 75–97 MHz, separated by 0·5 MHz, is $\frac{97 - 75}{0{\cdot}5} + 1 = 45.$

(*ii*) The frequency-modulated wave contains the frequency components f_c, $f_c \pm f_m$, $f_c \pm 2f_m$, $f_c \pm 3f_m$, $f_c \pm 4f_m \ldots f_c \pm nf_m$, where f_c is the carrier frequency and f_m the modulation frequency. Fortunately, not all the components are of significance, and an empirical formula for the bandwidth required for acceptable signal reproduction is $2(f_m+f_d)$ where f_d is the frequency deviation.

Now, modulation index, m, is defined as the ratio of frequency deviation to modulating frequency.

$$\therefore\ f_d = mf_m,$$

so that the bandwidth required is

$$2(f_m + mf_m) = 2(15 + 5 \times 15) = \underline{180\text{ kHz.}}$$

(*b*) (*i*) When the local-oscillator frequency is above the required signal frequency, the image frequency is that which is twice the i.f. above the required signal frequency. Since the i.f. is 10·7 MHz and the required signal frequency is 75 MHz, the image frequency is $75 + 21{\cdot}4 = 96{\cdot}4$ MHz. The deviation about the required signal frequency is ± 90 kHz, so that the image frequency will fall in the band $\underline{96{\cdot}4 \pm 0{\cdot}09\text{ MHz}}$, which is within the band centred on the 96·5 MHz carrier.

(*ii*) The adjacent channel to the 75 MHz channel that falls within the range of the receiver is 75·5 MHz. Since the bandwidth about each carrier is ± 90 kHz, the band covered by the adjacent channel is $\underline{75{\cdot}5 \pm 0{\cdot}09\text{ MHz}}$.

(*c*) (*i*) Consider the image channels of those carriers at the lower end of the receiver range, as shown in the table.

Carrier frequency (MHz)	Image channel (MHz)
75·0 ± 0·09	96·4 ± 0·09
75·5 ± 0·09	96·9 ± 0·09
76·0 ± 0·09	97·4 ± 0·09

The image channel to a 76 MHz carrier (and all above this figure) falls outside the receiver range, so that channels most susceptible to image-channel interference will be centred on 75 MHz and 75·5 MHz.

(*ii*) The interfering frequencies, outlined in part (*c*), (*i*), are between 96·31–96·49 MHz and 96·81–96·99 MHz. These would fall in the the bands centred on the 96·5 MHz and 97 MHz carriers.

Q. 5. (*a*) *What do you understand by the terms*

(*i*) *high-level modulation, and*
(*ii*) *low-level modulation?*

Illustrate your answers with block diagrams of amplitude-modulation transmitters.

(*b*) *What class of operation is usual for the final-stage amplifiers for each of these types of transmitter?*

(*c*) *What do the following abbreviations stand for when used in connexion with amplitude-modulation transmission:*

(*i*) *s.s.b.,*
(*ii*) *d.s.b.,*
(*iii*) *d.s.b.s.c.,*
(*iv*) *i.s.b.?*

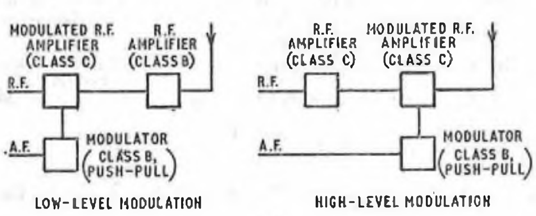
(*d*) *Using sketches of a sinusoidal modulating-signal and a sinusoidal carrier-signal, illustrate the waveform of a modulated signal using modulation types:*

(*i*) *d.s.b.,*
(*ii*) *d.s.b.s.c.,*
(*iii*) *s.s.b.*

A. 5. (*a*) (*i*) High-level modulation is the term used for the system in which modulation of the signal to be transmitted is undertaken in the final amplifying stage, where the signal power is a maximum.

(*ii*) Low-level modulation is when modulation is undertaken prior to the final amplifying stage.

Block diagrams illustrating high- and low-level systems are shown in sketch (*a*).



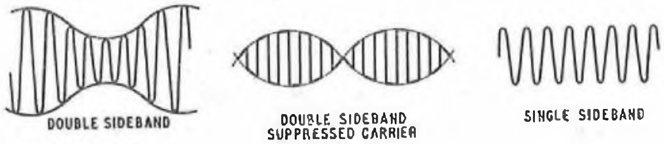
(*a*)

(*b*) The final-stage amplifiers for low- and high-level modulation transmitters are operated in the class-B and class-C modes respectively.

(*c*) The abbreviations respectively stand for

(*i*) single sideband,
(*ii*) double sideband,
(*iii*) double sideband suppressed carrier, and
(*iv*) independent sideband.

(*d*) The waveforms are illustrated in sketch (*b*).



(*b*)

Q. 6. *A class-C amplifier employs a valve, operating at* 1,000 *volts high tension, whose anode current has an angle of flow of* 120°.

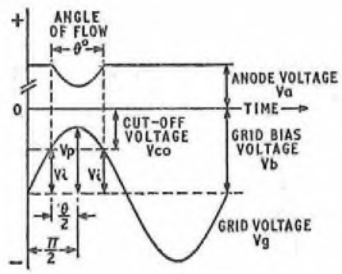
(*a*) *Sketch, on the same axes, the variation of anode and grid voltages with time over one cycle of sinusoidal applied voltage. Indicate the cut-off voltage and the angle of grid-current flow.*

(*b*) *From your sketch, obtain an equation for the angle of flow of anode current in terms of the peak applied signal, cut-off voltage and bias voltage.*

(*c*) *If the cut-off voltage for this valve is* −20 *volts, and the peak applied-signal voltage is* 150 *volts, what bias voltage will give an angle of flow of*

(*i*) 120°, *and*
(*ii*) 150°?

A. 6. (*a*) The variation of anode and grid voltage is shown in the sketch.



(*b*) Referring to the sketch, it may be seen that, for a sinusoid having a peak, V_p volts, the instantaneous voltage, v_i, is defined as

$$v_i = V_p \sin \omega t.$$

Consider the instantaneous voltage when grid current just starts to flow.

$$v_i = V_p \sin\left(\frac{\pi}{2} - \frac{\theta}{2}\right) = V_p \cos\frac{\theta}{2}.$$

Also, $v_i = -V_b - (-V_{co})$, where V_b is the grid bias voltage, and V_{co} is the cut-off voltage.

$$\therefore\ -V_b + V_{co} = V_p \cos\frac{\theta}{2}.$$

$$\therefore\ \underline{\theta = 2\cos^{-1}\frac{V_{co} - V_b}{V_p}.}$$

(*c*) (*i*) If V_{co} is −20 volts and V_p is 150 volts, then, for an angle of flow of 120°,

$$V_b = -V_p \cos\frac{\theta}{2} + V_{co},$$
$$= -150\cos 60 - 20,$$
$$= \underline{-95\text{ volts.}}$$

(*ii*) For an angle of flow of 150°,

$$V_b = -150\cos 75 - 20,$$
$$= \underline{-58{\cdot}7\text{ volts.}}$$

Q. 7. (*a*) *In frequency modulation, the phase of the carrier changes with the modulating frequency.*

(*i*) *What is the relationship between the phase of the carrier and the modulating waveform?*
(*ii*) *If the modulating frequency is halved, how is the change of phase affected?*

(b) Using the phasor representation of a carrier and two side-frequencies, indicate how amplitude modulation and frequency modulation differ.

(c) In frequency modulation, the carrier wave has a constant amplitude. Show, from the phasor representation of part (b), that two side-frequencies and the carrier are not enough to fully represent a frequency-modulated signal.

A. 7. (*a*) (*i*) In frequency modulation, the carrier frequency, f_c, is varied at the rate of the modulating frequency, f_m, by an amount depending on the modulating amplitude.

A sinusoidal carrier may be expressed as $f_c(t) = \cos\theta(t) = \cos\omega_c t$, and the modulating signal may be expressed as $f_m(t) = \cos\omega_m t$.

The instantaneous angular frequency, ω_i, can be expressed as the rate of change of the phase, θ, that is $\frac{d\theta}{dt}$, or as the nominal carrier angular frequency plus the deviation, that is $\omega_c + \Delta\omega\cos\omega_m t$.

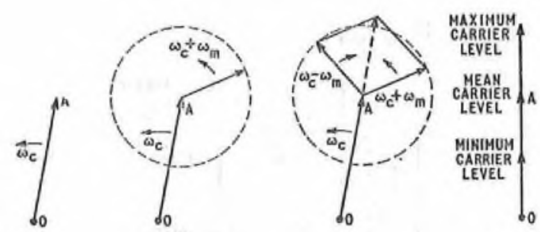
Therefore, the phase variation is given by $\theta(t) = \int\omega_i dt$, and so, $\theta(t) = \omega_c t + \frac{\Delta\omega}{\omega_m}\sin\omega_m t$. The modulated signal may, therefore, be expressed as

$$\cos\theta(t) = \cos\left(\omega_c t + \frac{\Delta\omega}{\omega_m}\sin\omega_m t\right), \qquad \ldots\ldots(1)$$

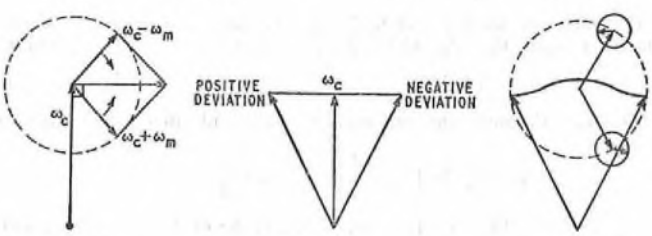
where $\frac{\Delta\omega}{\omega_m}$ is the peak phase deviation, and is known as the modulation index.

(*ii*) From equation (1), provided that the frequency deviation is maintained constant for a given modulating-signal amplitude, it can be seen that, if the modulating frequency is halved, the peak phase-deviation is doubled, and the rate of change of phase is halved.

(*b*) A sinusoidal signal, ω_c, may be represented by a phasor A, having a length proportional to its amplitude, rotating anti-clockwise about some reference, O, as shown in sketch (*a*). For an amplitude-



(*a*) (*b*) (*c*) (*d*)



(*e*) (*f*) (*g*)

modulated wave, the two side-frequencies, $\omega_c + \omega_m$ and $\omega_c - \omega_m$, may be superimposed on the carrier phasor. The upper side-frequency can be represented by a phasor rotating about that of the carrier, as shown in sketch (*b*), and, since it has a higher angular velocity, may be shown rotating anti-clockwise. The lower side-frequency, having an angular velocity below that of the carrier, appears to be moving in the opposite direction and, so, may be represented by a phasor rotating clockwise about the carrier phasor, A, such that the locus of the resultant of the two side frequencies lies along the axis OA. Thus, only the amplitude of phasor A is varied, as illustrated in sketch (*c*), and this is shown more simply in sketch (*d*).

The first-order side frequencies about the carrier in a frequency-modulated wave may also be shown in phasor form, but, in this case, the carrier amplitude must remain constant. This may be approximated by drawing the high-frequency phasor rotating anti-clockwise, and the lower-frequency phasor clockwise, as before, but such that the locus of their resultant lies perpendicular to the unmodulated carrier phasor, as shown in sketch (*e*). This may be simplified as shown in sketch (*f*).

(*c*) The representation in sketch (*f*) is, however, only an approximation, since the carrier amplitude at each extremity is slightly larger than when the carrier is unmodulated. A more accurate representation requires additional side-frequency component phasors. Sketch (*g*) shows the phasor when the second-order side-frequencies are included. It can be seen that the resultant locus is a curve, rather than a perpendicular axis to the unmodulated carrier.

Q. 8. Fig. 3 shows a discriminator.

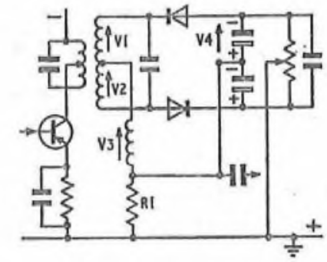


Fig. 3

(a) What type is it?

(b) Draw phasor diagrams relating V_1 with each of V_2, V_3 and V_4 when the signal is

(i) at the carrier frequency, and

(ii) above the carrier frequency.

(c) Draw a typical input/output characteristic for such a discriminator, labelling the axes.

(d) What happens to the output if the incoming carrier drifts from its nominal frequency?

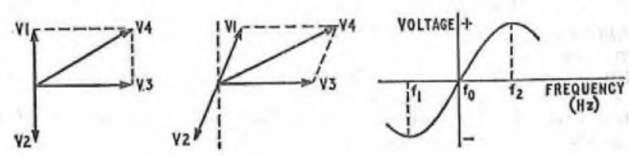
A. 8. (*a*) The circuit shown is a ratio detector.

(*b*) When the frequency of the voltage in the tank circuit is at the carrier frequency, then the voltage, V_3, is in quadrature with voltages V_1 and V_2, as shown in sketch (*a*). The voltage, V_4, is the vector sum of V_1 and V_3.

When the frequency in the tank circuit is above the carrier frequency, then V_3 is no longer in quadrature with V_1 and V_2. The resultant of V_1 and V_3, shown in sketch (*b*), now differs from that in the resonant condition.

(*c*) The characteristic is shown in sketch (*c*), where f_0 is the resonant frequency, and $f_2 > f_0 > f_1$.

(*d*) If the incoming carrier drifts from its nominal frequency, then, from sketch (*c*), the mean voltage output would not be centred on zero. The d.c. component would effectively be removed by the coupling capacitor, and, provided that the drift did not veer too far from zero, the discriminator would still operate over a linear portion of its characteristic, so that no distortion should be introduced. The direct voltage produced by frequency drift may, however, be utilized to control the receiver local-oscillator and, thus, provide automatic frequency control.



(*a*) (*b*) (*c*)

Q. 9. For a rhombic aerial, the approximate relationship between the angle of elevation of the main lobe and the height of the aerial is

$$\sin\delta = \frac{\lambda}{4h}.$$

(a) If the aerial height is 20 m and the leg length is 100 m,

(i) what frequency range is the aerial suitable for,

(ii) how many wavelengths is the leg-length at the extremities of this operating band, and

(iii) what is the angular range of elevation of the main lobe for this aerial?

(b) Indicate a typical gain/frequency characteristic for such an aerial, using typical values for the scales on the axes.

A. 9. (*a*) (*i*) The power radiated by a rhombic aerial is proportional to the leg length, whereas the angle of elevation is inversely proportional to the leg length. An effective compromise is achieved when this length is between 2–8 wavelengths.

When the leg length is 100 m, the shortest wavelength supported is $\frac{100}{8} = 12{\cdot}5$ m, which corresponds to a frequency of $\frac{3\times10^8}{12{\cdot}5} = 24$ MHz. Similarly, the longest wavelength is $\frac{100}{2} = 50$ m, which corresponds to a frequency of $\frac{3\times10^8}{50} = 6$ MHz. A suitable range is, therefore, 6–24 MHz.

(*ii*) From part (*i*), the wavelengths at the extremities of the band are 12·5 m and 50 m.

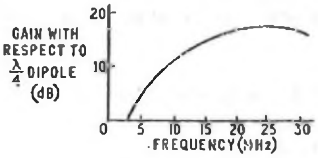
(*iii*) When $\lambda = 12\cdot5$ m, the angle of elevation $= \sin^{-1}\frac{12\cdot5}{80}$,

$$= \sin^{-1}0\cdot156,$$
$$= 9^\circ.$$

When $\lambda = 50$ m, the angle of elevation $= \sin^{-1}\frac{50}{80}$,

$$= \sin^{-1}0\cdot625,$$
$$= 38^\circ41'.$$

(*b*) A typical gain/frequency characteristic is illustrated in the sketch.



Q. 10. *Describe a method of measuring each of the following, numbering each step in the procedure and listing the equipment required:*

(*a*) *the selectivity of a medium-wave broadcast receiver, and*
(*b*) *the image-channel rejection ratio of such a receiver.*
Give typical results for each of these measurements.

CORRECTION

COMMUNICATION RADIO C, 1972 (*Supplement* to Vol. 66, Oct. 1973).

Part (*c*) of the answer to Q.6 should include a control for switched bands. Such a control is essential for the receiver described in the question.

LINE TRANSMISSION C, 1973

Students were expected to answer any six questions

Q. 1. (*a*) *Write down an expression for the characteristic impedance of a uniform transmission line in terms of its primary coefficients.*

(*b*) *Calculate the characteristic impedance at $\omega = 5{,}000$ rad/s, given that $R = 40$ ohms/km, $L = 1\cdot5$ mH/km, $G = 1$ μS/km and $C = 0\cdot1$ μF/km.*

(*c*) *Explain how the characteristic impedance of an actual line can be calculated from the results of two separate impedance measurements.*

A. 1. (*a*) For a uniform transmission line, the characteristic impedance, Z_0, is given by the expression

$$Z_0 = \sqrt{\left(\frac{R + j\omega L}{G + j\omega C}\right)} \text{ ohms},$$

where ω is the angular velocity (rad/s),
R is the loop resistance (ohms/km),
L is the loop inductance (H/km),
G is the loop leakance (S/km), and
C is the loop capacitance (F/km).

R, L, G and C are known as the primary coefficients.

(*b*) For the values given,

$$R + j\omega L = 40 + j7\cdot5 = 40\cdot7 \angle 10\cdot6^\circ,$$

and
$$G + j\omega C = (1 + j500) \times 10^{-6} = 500 \times 10^{-6} \angle 89\cdot9^\circ.$$

$$\therefore\ |Z_0| = \sqrt{\left(\frac{40\cdot7}{500 \times 10^{-6}}\right)} = 285\cdot3 \text{ ohms},$$

and
$$\phi = \frac{10\cdot6^\circ - 89\cdot9^\circ}{2} = -39\cdot7^\circ, \text{ where } \phi \text{ is the argument of } Z_0.$$

Hence,
$$Z_0 = 285\cdot3 \angle -39\cdot7^\circ \text{ ohms}.$$

(*c*) The characteristic impedance of a transmission line can be calculated from bridge measurements of the input impedance when the far end is (*i*) short-circuited (Z_{sc}), and (*ii*) open-circuited (Z_{oc}). The characteristic impedance of the line is the geometric mean of these two measurements.

Thus,
$$Z_0 = \sqrt{(Z_{oc} \times Z_{sc})}.$$

Q. 2. *At a particular frequency, a cable pair has a characteristic impedance of 600 ohms and is half a wavelength long. It has a transmission loss of 3 dB when properly terminated. Calculate its sending-end impedance when the far end is open-circuited.*

A. 2. The sending-end impedance, Z_s, of a transmission line is given by

$$Z_s = Z_0\left(\frac{1 - \eta_c k \angle -2\beta l}{1 + \eta_c k \angle -2\beta l}\right),$$

where Z_0 is the characteristic impedance (ohms),
η_c is the current reflexion coefficient,
k is the attenuation experienced by a signal travelling from the sending end to the termination and back, expressed as a voltage or current ratio,
β is the phase-change coefficient, and
l is the length of the line in terms of the wavelength, λ.

Now, the total phase change is given by $2\beta l$, where $\beta = 2\pi/\lambda$.

$$\therefore \text{ total phase change} = 2 \times \frac{2\pi}{\lambda} \times \frac{\lambda}{2} = 2\pi \text{ rad, or, effectively, } 0\ .$$

The total attenuation for both directions is 6 dB. This represents a voltage or current ratio of 1 : 2. Therefore, $k = 0\cdot5$.

Now, $\eta_c = \frac{Z_0 - Z_T}{Z_0 + Z_T}$, where Z_T is the terminating impedance,

$$= \frac{600 \angle 0^\circ - \infty}{600 \angle 0^\circ + \infty} = \frac{\frac{600 \angle 0^\circ}{\infty} - 1}{\frac{600 \angle 0^\circ}{\infty} + 1} = -1.$$

$$\therefore\ Z_s = 600\left\{\frac{1 - (-1) \times 0\cdot5 \angle 0^\circ}{1 + (-1) \times 0\cdot5 \angle 0^\circ}\right\},$$

$$= 600\left(\frac{1 + 0\cdot5}{1 - 0\cdot5}\right),$$

$$= 1{,}800 \text{ ohms}.$$

Note: This question can also be answered by consideration of the phasor diagrams for current and voltage. The principle of this method is illustrated in A. 2, Line Transmission C, 1969, *Supplement*, Vol. 63, p. 59, Oct. 1970.

Q. 3. *A coaxial cable, designed for use up to 60 MHz, comprises a copper conductor held centrally within an outer copper conductor by means of polyethylene disks attached to the centre conductor at regular intervals.*

(*a*) *Make sketches to show how the primary and secondary coefficients of this cable would vary over the range 0–60 MHz.*

(*b*) *Explain why these variations occur.*

(*c*) *How would the attenuation be affected if the spacing of the disks were reduced?*

A. 3. (*a*) and (*b*) See A.1, Line Transmission C, 1971, *Supplement*, Vol. 65, p. 87, Jan. 1973; and A.3, Line Transmission C, 1970, *Supplement*, Vol. 64, p. 79, Jan. 1972.

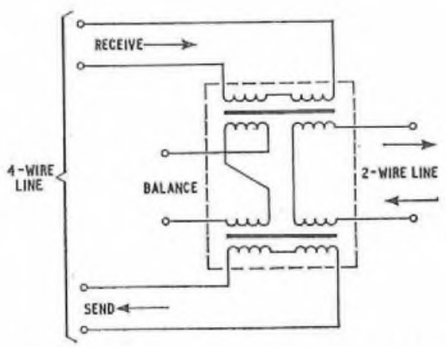
(*c*) The dielectric of the coaxial pair is made up partly of polyethylene, and partly of air. A reduction in the spacing of the disks increases the amount of polyethylene in a given length of cable. As the permittivity of polyethylene is greater than that of air, the effective loop capacitance is increased. This reduces the characteristic impedance of the pair and increases its attenuation. In practice, the polyethylene disks are about 1·2 mm thick and are spaced about 33 mm apart. Hence, the amount of polyethylene is small in relation to the amount of air, and therefore, a reduction in the spacing would, in general, only have a small effect on the loop capacitance. However, the effect on the attenuation of the pair at high frequencies could be significant.

Q. 4. (*a*) *Draw the circuit diagram of a two-wire-to-four-wire terminating unit.*

(*b*) *Explain the function of such a unit in a four-wire, audio-frequency, repeatered circuit.*

(*c*) *Calculate the balance return-loss for a line having an impedance of* (600 − *j*100) *ohms, connected to the two-wire terminals, when a network having an impedance of* 700 $\angle 30°$ *ohms, is connected to the balance terminals.*

A. 4. (*a*) A circuit diagram of a two-wire-to-four-wire terminating unit is shown in the sketch.



(*b*) Repeatered circuits are used in the four-wire configuration because amplifiers are unidirectional devices and, hence, for amplification purposes, the send- and receive-direction signals on a transmission line must be separated into two unidirectional paths. A terminating unit is used at each end of the four-wire section in order to combine the unidirectional paths with the bi-directional path of the two-wire section.

The terminating unit allows signals from the two-wire section to pass into the four-wire section and, also, allows signals, arriving on the four-wire RECEIVE path, to pass into the two-wire section. The attenuation experienced by these signals is, theoretically, 3 dB, but in practice, the balance is rarely a perfect match for the two-wire line, and losses occur in the transformers. Therefore, it is usual to allow for a loss of 4 dB in these paths.

The terminating unit, when properly balanced, prevents instability in the four-wire section by presenting a very high impedance between the four-wire RECEIVE and SEND paths.

(*c*) The balance return-loss is a measure of the quality of the impedance match between a balancing network, having an impedance Z_B, and a two-wire line, having an impedance Z_0. If $Z_B = Z_0$, the balance is perfect, and the balance return-loss is infinite.

Now, balance return-loss

$$= 20 \log_{10} \left|\frac{Z_0 + Z_B}{Z_0 - Z_B}\right| \text{dB},$$

$$= 20 \log_{10} \left|\frac{600 - \text{j}100 + 700 \cos 30° + \text{j}700 \sin 30°}{600 - \text{j}100 - (700 \cos 30° + \text{j}700 \sin 30°)}\right|,$$

$$= 20 \log_{10} \left|\frac{600 - \text{j}100 + 606 + \text{j}350}{600 - \text{j}100 - 606 - \text{j}350}\right|,$$

$$= 20 \log_{10} \left|\frac{1206 + \text{j}250}{-6 - \text{j}450}\right|,$$

$$= 20 \log_{10} \left(\frac{1206^2 + 250^2}{6^2 + 450^2}\right)^{1/2},$$

$$= 20 \log_{10} 2 \cdot 74,$$

$$= \underline{8 \cdot 74 \text{ dB.}}$$

Q. 5. (*a*) *Draw the circuit diagram of an a.c. bridge, suitable for measuring the impedance of an audio-frequency cable pair.*

(*b*) *Derive the modulus and angle of the measured impedance in terms of the bridge components.*

(*c*) *State what precautions are necessary in order to ensure a high degree of accuracy.*

A. 5. See A.4, Line Transmission C, 1971, *Supplement*, Vol. 65, p. 88, Jan. 1973.

Q. 6. (*a*) *Describe the transmission plan for a typical telephone network, showing the losses allowable in each part of a trunk connexion between two subscribers.*

(*b*) *What are the advantages of four-wire switching in a trunk network?*

A. 6. See A.6, Line Transmission C, 1969. *Supplement*, Vol. 63, p. 60, Oct. 1970.

Q. 7. *A carrier telephony system is to be set up over star-quad cables, and it is essential to reduce crosstalk to a minimum.*

(*a*) *Explain how crosstalk could arise.*

(*b*) *What steps should be taken to reduce crosstalk to an acceptable level?*

A. 7. (*a*) Crosstalk can be caused by

(*i*) capacitance unbalance,
(*ii*) resistance unbalance,
(*iii*) inductive coupling,
(*iv*) low insulation resistance, or
(*v*) wire-to-wire contacts.

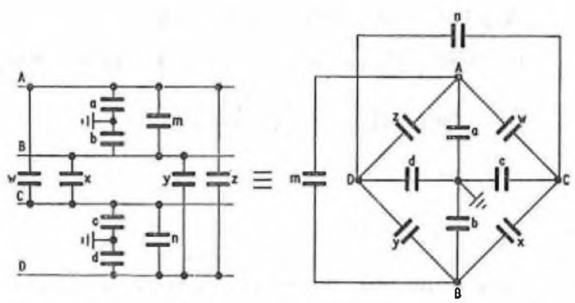
(*b*) Some of these factors can be minimized by sound design, and by ensuring uniformity during manufacture; others by careful installation and maintenance procedures.

The four wires of each quad are taken from the same reel, thus eliminating any possible differences in diameter due to the drawing die, and all are insulated with paper ribbons cut from the same roll. The identification marks, printed on the paper in the form of one, two, three or four rings, are spaced in such a way as to use the same amount of ink in a given length for each wire of the quad, thus obviating any difference in insulation resistance attributable to the ink itself.

All quads have different lengths of lay, some being laid clockwise and some anti-clockwise. The cable is built up in layers of quads, each layer being laid helically over the one below it and in the opposite direction of lay. Thus, all quads in a layer have the same direction of lay, but two adjacent layers have clockwise and anti-clockwise quads respectively. This arrangement of layers and quads minimizes the possibility of inductive coupling between quads. The completed cable core is thoroughly dried in an oven before the sheathing is applied, and it is important that moisture should not be allowed to enter the core and, hence, reduce the insulation resistance, during the subsequent jointing operations.

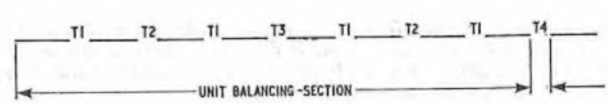
In practice, the main source of crosstalk is capacitance unbalance. It is necessary to make measurements on each cable length and, then, to select quads for jointing in such a way as to even out and minimize the overall unbalance figures. Such joints are called *test-selected joints*.

Sketch (*a*) shows the various capacitances involved in a star quad with wires A, B, C and D. No crosstalk occurs within this quad if $w = x = y = z$, $a = b$, $c = d$, and $m = n$.



(*a*)

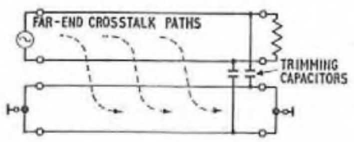
Sketch (*b*) shows a typical unit balancing-section for a carrier cable-system. This comprises eight lengths of cable, each about 160 m long, and every joint is test selected. The cable lengths are first jointed in



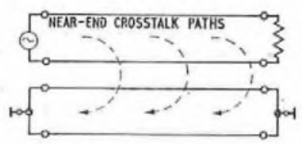
(*b*)

pairs (joints T1) and, then, in groups of four (joints T2). A selected joint, T3, completes the unit balancing-section. A number of such sections are then selectively jointed together (joints T4) to build up the cable to the required length. It is possible to carry out much of the measurement and selection work in the factory before the cable lengths are delivered to the site, and installed. However, it is usually necessary to test and select for joints T3 and T4 *in situ*.

For a carrier system, it is usual to use separate cables for each direction of transmission. On completion of the balancing process, small trimming capacitors are added at the receiving end, as required, in order to compensate for any residual unbalance, as shown in sketch (*c*). For this purpose, a special balancing-frame is provided at the terminal station. Although there are various paths over which far-end crosstalk can occur, they all have the same electrical length. Thus, all the components of the total far-end crosstalk arrive in the same phase, and their effect can be substantially eliminated by the use of simple capacitors. This contrasts with the conditions for near-end crosstalk, where the various paths have different lengths (see sketch (*d*)) and, thus, the components of the total near-end crosstalk cover a range of phase relationships. Near-end crosstalk is avoided by making use of the screening effect of the lead sheaths.



(*c*)



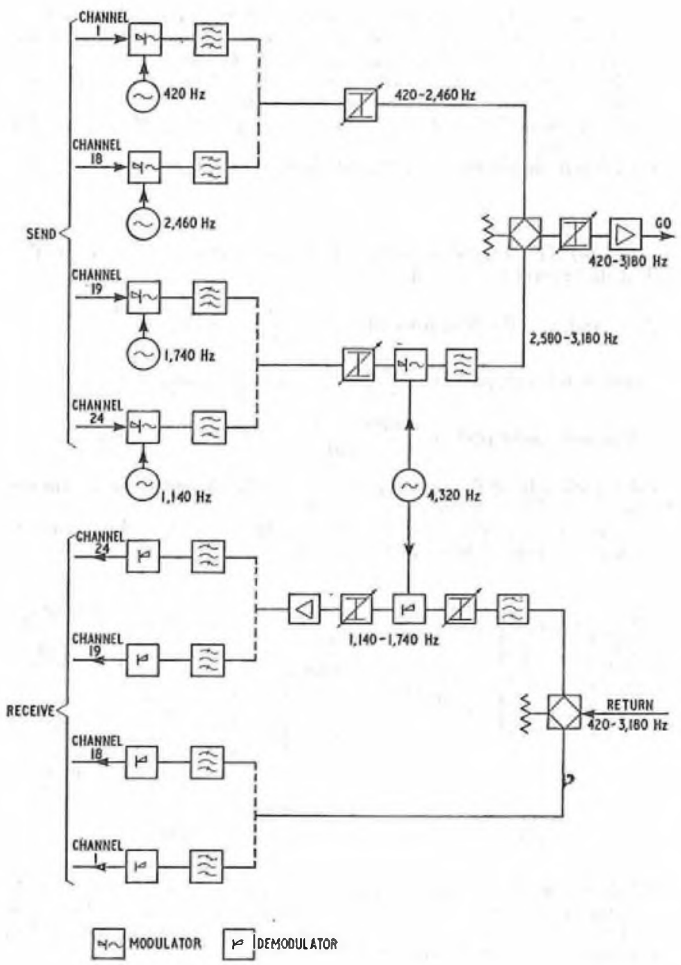
(*d*)

Q. 8. (*a*) *Draw the block diagram for one end of a 24-channel, voice-frequency telegraph system.*

(*b*) *Explain the function of each block in your diagram.*

(*c*) *Why do the requirements for the channel filters differ from those used in a multi-channel carrier-telephony system?*

A. 8. (*a*) The block diagram for one end of a 24-channel, voice-frequency telegraph system is shown in the sketch.



(*b*) A 24-channel, voice-frequency telegraph system comprises an 18-channel system in the frequency range 420–2,460 Hz, plus a six-channel system in the range 1,140–1,740 Hz. A group modulator, operating at a frequency of 4,320 Hz, converts the six-channel system to a higher frequency band for transmission to line. Thus, the whole 24-channel system has line frequencies between 420–3,180 Hz, at a spacing of 120 Hz. Each channel has a nominal width of 120 Hz, which is appropriate for a working speed of 50 bauds. All of the line frequencies are odd harmonics of 60 Hz. This arrangement minimizes the possibility of inter-channel interference, since the most dominant harmonic (the second harmonic) in any channel falls midway between two higher channels in the frequency spectrum.

In each channel, the line signals are in the form of bursts of the carrier frequency. The sending teleprinter's transmitting contacts are arranged to shunt the channel carrier-frequency with a low impedance for a space element, and to remove the shunt for a mark element. A band-pass filter is provided at the output of each channel to provide proper separation between channels.

(*c*) In any carrier system, it is essential to make the best use of the available bandwidth, and the cost of the channel filters is an important consideration. For a telegraph system, the required capacity can be obtained by using fairly inexpensive channel filters with a gradual cut-off characteristic, because the bandwidth needed for each channel is comparatively small. For a carrier-telephony system, however, a much larger bandwidth is required for each channel, and it becomes economic to provide more expensive filters with a sharper cut-off characteristic, in order to make the best use of the frequency spectrum available.

Q. 9. (*a*) *Draw block diagrams to show how power may be provided for the transmission equipment in a repeater station.*

(*b*) *Explain what precautions are taken to ensure the continuity of the supply in the event of a mains failure.*

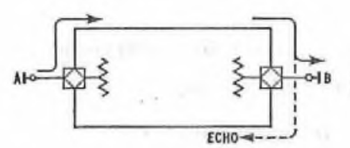
(*c*) *Show how electrical noise from the power plant is prevented from interfering with the transmission circuits.*

A. 9. See A.10, Line Transmission C, 1967. *Supplement*, Vol. 61, p. 33, July 1968.

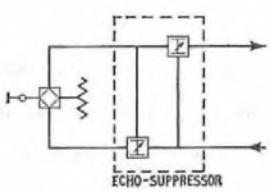
Q. 10. (*a*) *Why are echo-suppressors necessary on some types of telephone circuit?*

(*b*) *Explain the basic principles of an echo-suppressor for use on such circuits.*

A. 10. (*a*) Sketch (*a*) shows a four-wire connexion between two subscribers, A and B. Ideally, the loss between the four-wire RECEIVE and SEND pairs across a terminating unit should be infinite, but in



(*a*)



(*b*)

general, this is not the case, because of the practical difficulty of making the balancing network a perfect match for the two-wire line.

Assume subscriber A to be speaking, and signals to be passing to subscriber B over the path shown by the full arrows. Any imperfection in the matching of the balancing network at B causes part of the received signal to be passed back to subscriber A over the path shown by the broken arrow. For a connexion which is electrically short, that is, the delay time is small, subscriber A merely hears this signal as sidetone. However, where the delay time is long, for example, of the order of 200 ms, the signal appears as a distinct echo of his own voice. Similar considerations apply when subscriber B is talking, and such echoes can cause a serious impediment to meaningful conversation.

(*b*) For long circuits, echo-suppressors are provided in order to attenuate the return path when the forward path is activated by a speech signal. The arrangement is shown in sketch (*b*).

Echo-suppressors were originally used on long four-wire audio circuits, but these are now largely superseded by high-velocity carrier circuits, where the delay time is small. The need for echo-suppressors remains, however, on transoceanic-cable and satellite circuits.

Students were expected to answer any six questions

Q. 1. *When* 4 *erlangs of traffic are offered to a full-availability group of* 10 *trunks, the grade of service given is* 0·005, *and when* 10 *erlangs of traffic are offered to a full-availability group of* 20 *trunks, the grade of service given is* 0·002. *For each case, calculate the traffic carried by the first, second and last trunks, and use these values to sketch graphs showing the traffic carried by each trunk.*

A. 1. *First case*

Traffic offered to first outlet, $A = 4$ erlangs.

Traffic offered to second outlet = traffic lost from first outlet,

$$= A\left(\frac{A}{1+A}\right) = 4\left(\frac{4}{1+4}\right),$$

$$= 3\cdot2 \text{ erlangs.}$$

Traffic offered to third outlet = traffic lost from second outlet,

$$= A\left(\frac{\frac{A^2}{2}}{1+A+\frac{A^2}{2}}\right) = 4\left(\frac{\frac{16}{2}}{1+4+\frac{16}{2}}\right),$$

$$= 2\cdot46 \text{ erlangs.}$$

Thus, traffic carried by first outlet $= 4 - 3\cdot2 =$ 0·8 erlang,

and traffic carried by second outlet $= 3\cdot2 - 2\cdot46 =$ 0·74 erlang.

Traffic carried by the last outlet $= B\left(\frac{N}{1-B} - A\right)$,

where B is the grade of service, and
N is the total number of trunks.

If B is small, then $\frac{N}{1-B} \simeq N$.

Thus, last-outlet traffic $= B(N - A) = 0\cdot005(10 - 4)$
$=$ 0·03 erlang.

Second case

Similarly, traffic offered to first outlet, $A = 10$ erlangs.

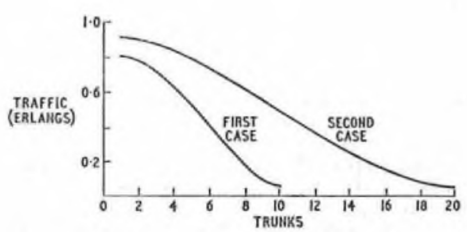
Traffic offered to second outlet $= 10\left(\frac{10}{1+10}\right) = 9\cdot09$ erlangs.

Traffic offered to third outlet $= 10\left(\frac{\frac{100}{2}}{1+10+\frac{100}{2}}\right) = 8\cdot2$ erlangs.

Thus, traffic carried by first outlet $= 10 - 9\cdot09 =$ 0·91 erlang,

and traffic carried by second outlet $= 9\cdot09 - 8\cdot2 =$ 0·89 erlang.

Also, last-outlet traffic $= 0\cdot002\,(20 - 10) =$ 0·02 erlang.



The sketch shows graphs of the traffic carried by each trunk in each case.

Q. 2. (*a*) (*i*) *List the facilities provided by a final-selector routiner.*
(*ii*) *Draw a block diagram of a final-selector routiner and, in addition, indicate how access is gained to the selectors to be tested.*
(*iii*) *Describe how the routiner carries out a typical test on the final selector.*
(*b*) *What are the advantages and disadvantages of an automatic routiner compared with an artificial-traffic equipment?*

A. 2. (*a*) See A.10, Telephony C, 1972. *Supplement*, Vol. 66, p. 75, Jan. 1974.

(*b*) The advantages of artificial-traffic equipment are that it

(*i*) tests interconnecting wiring between switching-equipment stages,
(*ii*) can detect faults on any level or outlet, and
(*iii*) can be used to flood-test particular areas of the switching equipment.

The advantages of an automatic routiner are that

(*i*) latent faults can be detected before they affect service, since a more stringent functional test is applied,
(*ii*) it can be used with a fault-docket printer, and left unattended,
(*iii*) it does not occupy calling equipment, and
(*iv*) it ensures that every item of equipment is tested.

The disadvantages of an automatic routiner are that it

(*i*) cannot be used for quality-of-service measurements,
(*ii*) is not mobile, and
(*iii*) does not test inter-switching-stage wiring.

Q. 3. (*a*) *Explain why it is necessary to provide voice immunity in an in-band signalling system, and say how this is achieved in a single-frequency system. Use sketches to illustrate your answer.*
(*b*) *Draw a block diagram of a single-voice-frequency, in-band signalling system and give a brief description of its operation.*

A. 3. See A.7, Telephony C, 1971. *Supplement*, Vol. 65, p. 33, July 1972.

Q. 4. (*a*) *Describe the principle of operation of a crossbar switch, and explain how a crosspoint, once operated, is maintained in that condition.*
(*b*) *Describe how a switch with six horizontal operating bars may be arranged to form a 26-outlet switch. Use sketches to illustrate your answers.*

Q. 5. (*a*) *A telephone dial may operate between 7–12 pulses/s with a break percentage of 63–72 per cent. What is the duration of* (*i*) *the minimum break pulse, and* (*ii*) *the minimum make pulse?*
(*b*) (*i*) *Describe the effect of line resistance, leakance and capacitance on pulsing, using sketches to illustrate your answer.*
(*ii*) *What other factors have an effect on the pulsing performance of a telephone link?*
(*c*) *Sketch the circuit and describe the operation of one form of pulse corrector.*

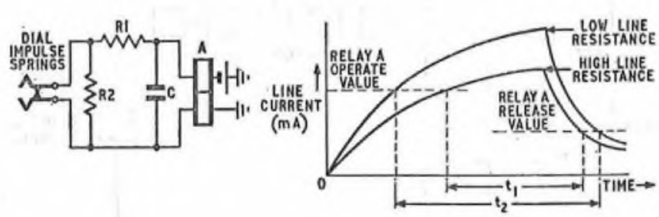
A. 5. (*a*) The minimum make and break periods occur at the fastest dial speed, i.e. 12 pulses/s.

At 12 pulses/s, the time for one pulse $= \frac{1}{12}\text{s} = 83$ ms.

Minimum break period $= \frac{63}{100} \times 83 \text{ ms} =$ 52·3 ms.

Minimum make period $= \frac{(100-72)}{100} \times 83 \text{ ms} =$ 23·2 ms.

(*b*) (*i*) Sketch (*a*) shows a typical pulsing circuit with line resistance $R1$, leakance $R2$, and line capacitance C.
Sketch (*b*) shows the effect on the operate and release time of relay A of low and high values of line resistance.



(*a*) (*b*)

With a low line-resistance, the current reaches the A-relay-operate value more quickly than with a high line-resistance. Hence, relay A operates sooner, but releases slightly later. For a given make pulse, the relay is operated for time t_2.

With a high line-resistance, relay A operates later, due to the slower build up of current, and releases slightly earlier. For the same make pulse, the relay is operated for time t_1. Hence, an increase in line resistance tends to increase the break portion of a loop-disconnect pulse.

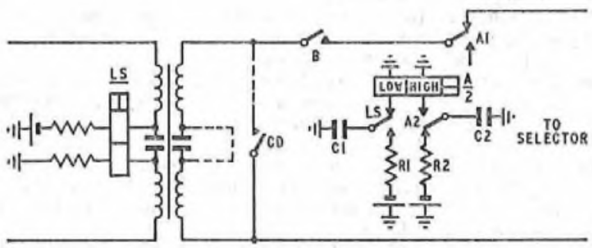
Line leakance allows a small pre-operate current to flow in relay A, and prevents the current from falling to zero during break periods. Thus, relay A is operated for longer during each make pulse than if the line had no leakance. Hence, the effect is to reduce the break portion of a loop-disconnect pulse.

Line capacitance generally only affects the release of relay A, since the line discharges quickly when the impulse springs make. However, when the impulse springs break, the line capacitance recharges via relay A; this has the effect of keeping relay A operated for a slightly longer period. The overall effect is to reduce the break period of the pulse from relay A.

(*ii*) Other factors which have an effect on pulsing performance are

(1) the dial speed and ratio,
(2) relay A operate and release times,
(3) transmission bridges with pulse repetition, and
(4) the exchange battery voltage.

(*c*) Sketch (*c*) shows a simple pulse-corrector element in a transmission bridge.



(*c*)

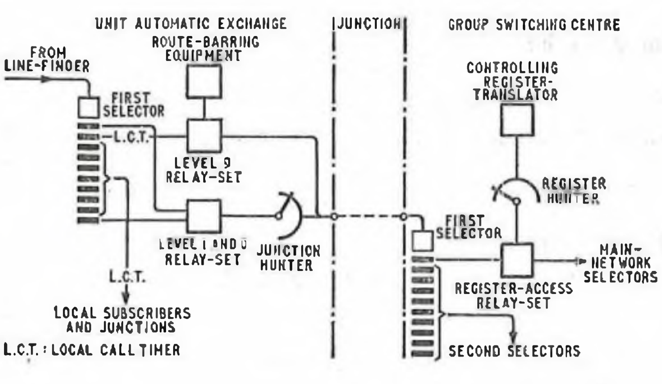
On seizure, and during each make pulse, capacitor C1 charges to −50 volts via resistor R1 when relay LS is operated. At each break pulse, when relay LS releases, capacitor C1 discharges via the low-resistance coil of relay A, which then operates. Contact A1 breaks the forward loop, while contact A2 connects capacitor C2 (already charged to −50 volts) to the high-resistance coil of relay A. Capacitor C2 now discharges via the A-relay coil and holds this relay operated for, typically, 66 ms (this time depends on the time constant of the high-resistance coil of relay A and capacitor C2). Thus, one 66 ms break pulse is transmitted forward. On the release of relay A, capacitor C2 recharges via resistor R2, but, before the sequence can be repeated, relay LS must re-operate to a make pulse.

Thus, the circuit functions as a fixed-break pulse-corrector, that is, it gives a fixed 66 ms break period, but does not change the pulse speed. Fixed-break pulse-correctors are normally only used where the pulsing originates from a controlled source, such as a register.

Q. 6. (*a*) *With the aid of sketches, describe the arrangements at a unit automatic exchange (u.a.x.) and its group switching centre (g.s.c.) to provide subscriber trunk dialling facilities to telephones served by the u.a.x. Include in your answer details of the method of provision of coin-box discrimination and charging-group identification at the g.s.c.*

(*b*) *State the major advantages in not providing a separate group of level-0 junctions at the u.a.x.*

A. 6. (*a*) The sketch shows a simplified trunking diagram of the arrangements at a unit automatic exchange (u.a.x.) and its group switching centre (g.s.c.) for providing s.t.d. facilities to a subscriber served by the u.a.x.



A subscriber, originating a call, is connected to a first selector in the u.a.x., which returns dial tone. If the subscriber dials a trunk call (initial digit 0), the first selector steps to level 0, and a level-1-and-0 relay-set is seized. The level-1-and-0 relay-set seizes a first selector at the g.s.c. via the junction-hunter and junction. The level-1-and-0 relay-set then repeats the digit 0 over the junction to step the g.s.c. first selector to level 0, thus seizing a register-access relay-set, which has access to controlling registers and the main trunk network. At the u.a.x., the level-1-and-0 relay-set receives a class-of-service signal from the line-finder which indicates whether the call originates from an ordinary or a coin-box subscriber. The level-1-and-0 relay-set, after sending the digit 0 forward, sends a class-of-service discrimination digit, which is accepted by the register-access relay-set in the g.s.c. This digit is in the range 1–6, and indicates to the register-access relay-set the charging group where the call has originated and whether it is an ordinary, or a coin-box, call.

After the level-1-and-0 relay-set has sent forward the class-of-service digit, it then repeats the rest of the subscriber's dialled digits following the initial digit 0, to enable the controlling register to set up the call through the network.

(*b*) The major advantages of providing a common group of junctions from the u.a.x. to the g.s.c., rather than separate groups for levels 1, 9 and 0, are that

(*i*) a common group of junctions has a better traffic-carrying capacity for a fixed number of circuits than have segregated groups of circuits, and

(*ii*) the u.a.x. register-access relay-sets at the g.s.c. can be common to all u.a.x.s served by the g.s.c., irrespective of the originating charging-group.

Q. 7. (*a*) *Describe the arrangement made to ensure that a subscriber is given a full initial metering period when originating an s.t.d. call.*

(*b*) *List the features desirable in a signalling system used to operate meters located at a subscriber's premises, and describe a signalling system which would meet these requirements.*

(*c*) *Describe a subscriber's private meter for use with such a system. Use sketches to illustrate your answers.*

A. 7. (*a*) See A.6, Telephony C, 1971. *Supplement*, Vol. 65, p. 33, July 1972.

(*b*) See A.6, Telephony C, 1969. *Supplement*, Vol. 63, p. 58, Oct. 1970.

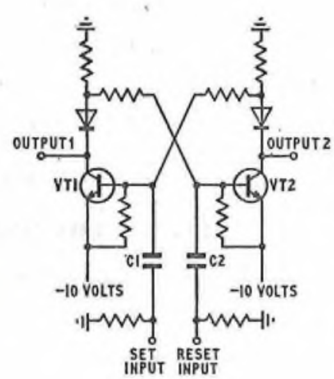
Q. 8. (*a*) *With the aid of sketches of voltage waveforms, describe a circuit in which transistors may be used as an electronic store.*

(*b*) *Describe those factors taken into account when selecting a transistor to be used for relay operation. Explain why it is sometimes necessary to provide protection for the transistor, and describe how this is done.*

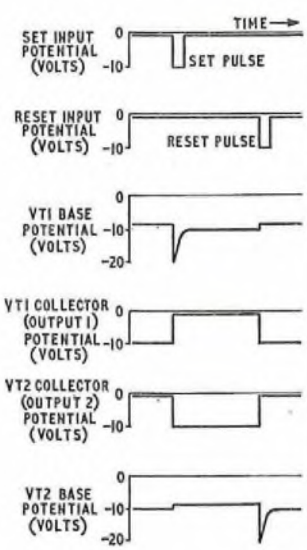
Use sketches to illustrate your answers.

A. 8. (*a*) Sketch (*a*) shows a typical toggle circuit, using transistors, which can function as an electronic store. The voltage waveforms at various points in the circuit are shown in sketch (*b*).

Assuming transistor VT1 to be in its conducting state, then transistor VT2 is non-conducting, since the collector of VT1, and, hence, the base of VT2, is at −10 volts. Therefore, as the collector of VT2 is



(*a*)

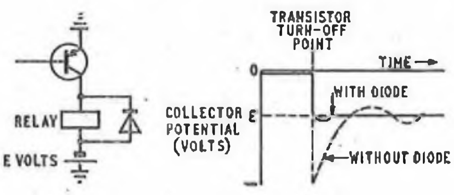


(*b*)

at earth potential, VT1 is maintained in a stable conducting condition, since its base–emitter junction is forward biased.

Referring to sketch (*b*), if a 10-volt negative-going pulse is applied to the SET input, capacitor C1 transfers this pulse to the base of VT1, the potential of which then falls from just above -10 volts to approximately -20 volts, before rising exponentially back to -10 volts. Thus, VT1 switches off, and, as its collector potential rises towards earth, the base–emitter junction of VT2 becomes forward biased, and, hence, VT2 conducts. Therefore, the collector potential of VT2 falls from earth to -10 volts, which maintains VT1 in the non-conducting condition. Thus, the SET pulse is stored, in that the collector of VT2 is now at a potential of -10 volts.

When it is desired to erase the memory, a 10-volt negative-going pulse is applied to the RESET input. Capacitor C2 transfers this pulse to the base of VT2, the potential of which then falls from just above -10 volts to approximately -20 volts, before rising exponentially back to -10 volts. Thus, VT2 switches off, and the base–emitter junction of VT1 now becomes forward biased, since the collector potential of VT2 rises towards earth and VT1 now conducts, its -10-volt collector potential maintaining VT2 in the non-conducting state. Thus, the reset pulse erases the store.



(*c*) (*d*)

(*b*) Sketch (*c*) shows a relay operated by a transistor. A transistor used for such a purpose must possess

(*i*) sufficiently-high current gain, such that, with minimum base current flowing, the collector current is enough to ensure that the transistor is saturated,
(*ii*) a collector that can withstand at least the supply voltage, *E* volts,
(*iii*) a low collector leakage current, such that, when the transistor is turned off, this current is below the minimum hold-current for the relay,
(*iv*) sufficient power dissipation at the maximum value of collector current, and
(*v*) rapid turn-on and turn-off times.

The diode across the relay is necessary to protect the transistor when it turns off, since a high voltage is induced across the relay when the current through it collapses. If this voltage is greater than the maximum collector voltage allowed for the transistor, then the transistor could be destroyed. Because the induced voltage across the relay tends to make the collector-side of the relay negative, when the voltage exceeds *E* volts, the diode becomes forward biased and, so, clamps the collector voltage at a value which cannot exceed *E* volts (see sketch (*d*)).

However, the diode provides a low-resistance path for eddy currents in the winding and, so, increases the release lag of the relay.

Q. 9. (*a*) *Describe the facilities given by an s.t.d. controlling register-translator.*

(*b*) *Explain, with the aid of sketches, the operation of any type of translator used to give s.t.d. translations.*

A. 9. (*a*) An s.t.d. controlling register-translator has facilities for

(*i*) storing up to nine digits,
(*ii*) examining up to the first five digits for translation purposes,
(*iii*) sending the fee digit and up to six translation digits, followed by the appropriate repeated digits,
(*iv*) force-releasing the forward equipment, and providing a signal to the register-access relay-set to cause it to return number-unobtainable tone if a spare or barred code is received, or there is delayed dialling-in during the first seven digits,
(*v*) applying a 4s time-out period after the seventh digit (and eighth, if received), such that, if no further digits are received in that time, dialling-in is assumed to have finished, and sending is completed,
(*vi*) delaying the sending of the penultimate digit until the final digit has been stored,
(*vii*) associating a signalling system multi-frequency No. 2 sender/receiver, if required,
(*viii*) alternative routing, and
(*ix*) disconnecting itself from the call on completion of sending.

(*b*) See A.2, Telephony C, 1970. *Supplement*, Vol. 64, p. 39, July 1971.

(*to be continued*)

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