SUPPLEMENT

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Contents

1972–73 CITY AND GUILDS OF LONDON INSTITUTE EXAMINATIONS

QUESTIONS AND ANSWERS

Answers are occasionally omitted or reference is made to earlier Supplements in which questions of substantially the same form, together with the answers, have been published. Some answers contain more detail than would be expected from candidates under examination conditions.

LINE TRANSMISSION C, 1972 (*continued*)

Q. 2. (*a*) *With the aid of sketches, show how the attenuation/frequency characteristic of a cable pair is affected by the addition of loading coils at regular intervals.*

(*b*) (*i*) *Explain how the use of coils restricts the width of the effective transmission band.*

(*ii*) *Derive an expression for the upper limit of frequency in terms of the capacitance of the cable, the inductance of the coils and the spacing between them.*

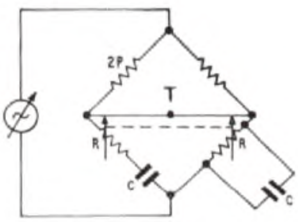
A. 2. See A. 4, Line Transmission C, 1970. *Supplement*, Vol. 64, p. 79, Jan. 1972.

Q. 3. (*a*) *Sketch the circuit of a bridge for the measurement of frequencies in the audio range.*

(*b*) *Derive an expression for the measured frequency in terms of the values of the bridge components at balance.*

(*c*) *Explain what precautions are necessary in order to ensure an accurate result.*

A. 3. (*a*) The sketch shows the basic circuit of a Wien frequency bridge suitable for the measurement of audio frequencies. The fixed



ratio arms are 2P and P. The two variable arms each contain R and C. These are in series in one arm and in parallel in the other. Resistors R are adjustable and ganged together so as to remain in step. The frequency range of the bridge can be altered by changing the value of the fixed capacitors C.

(*b*) At balance,

$$\frac{2P}{\frac{1}{R}+j\omega C} = P\left(R+\frac{1}{j\omega C}\right). \qquad \ldots\ldots(1)$$

$$\therefore \frac{2P}{P} = \left(R+\frac{1}{j\omega C}\right)\left(\frac{1}{R}+j\omega C\right).$$

$$\therefore 2 = 1+\frac{1}{j\omega CR}+j\omega CR+1. \qquad \ldots\ldots(2)$$

Thus, $-\dfrac{1}{j\omega CR} = j\omega CR$,

giving $\omega^2C^2R^2 = 1$.

$$\therefore f = \frac{1}{2\pi CR}.$$

The choice of a ratio of 2 : 1 for the fixed resistive arms can be explained by putting X and Y in place of 2P and P. Then equation (2) becomes,

$$\frac{X}{Y} = 1+\frac{1}{j\omega CR}+j\omega CR+1.$$

Equating imaginary parts gives:

$$-\frac{1}{j\omega CR} = j\omega CR, \text{ as before.}$$

Equating real parts gives:

$$\frac{X}{Y} = 2.$$

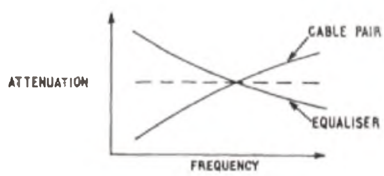
Both real and imaginary parts must be equal for balance.

(*c*) To ensure an accurate result all the components of the bridge must be of high quality and properly calibrated. It is particularly important that the fixed resistance arms should be in the correct 2 : 1 ratio and that the variable resistance arms should be properly in step. Errors due to stray capacitance can be avoided by care in the layout of the components and the insertion of a balanced and screened transformer between the bridge terminals and the source whose frequency is to be measured. Detection of the balance point can be simplified by including a band-pass filter in the source connexion so as to exclude the effect of harmonics.

Q. 4. (*a*) *Explain briefly the need for equalizers in line transmission systems.*

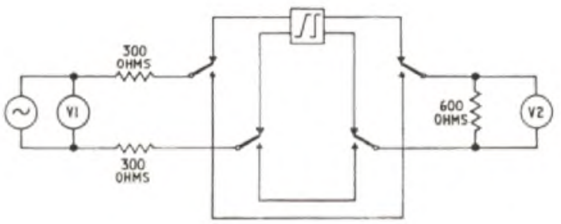
(*b*) *With the aid of diagrams, explain how the insertion-loss/frequency characteristic of a line equalizer can be measured with a reasonable degree of accuracy.*

A. 4. (*a*) Equalizers are necessary in a line transmission system to enable the desired attenuation/frequency characteristic to be achieved. In general, the attenuation of a cable pair rises with increasing frequency, while the gain of an amplifier tends to be independent of frequency. A line equalizer is a network whose attenuation/frequency, characteristic is the inverse of that of the line. Provided that the equalizer and the line are both properly matched in impedance, the combination of the two in series can be arranged to give an overall attenuation which is substantially independent of frequency. Sketch (*a*) illustrates the principle.



(*a*)

(*b*) The insertion loss of a line equalizer is the ratio between the power delivered to the load before the equalizer is connected and the power delivered to the load after it is connected. Sketch (*b*) shows a



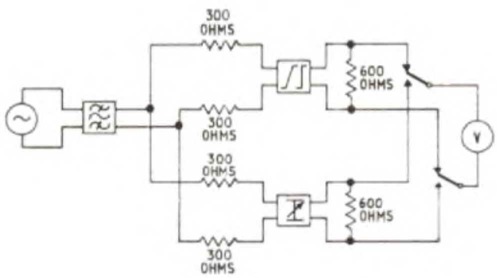
(*b*)

circuit arrangement by which the loss can be measured directly. At each frequency, two measurements are made—one via the equalizer and one via the direct path. Then, assuming the oscillator output to be adjusted to give a constant output V_1,

$$\text{insertion loss} = 20 \log_{10} \frac{V_2\,(\text{direct})}{V_2\,(\text{via equalizer})}\ \text{dB}.$$

The accuracy of this method depends upon the stability of both voltmeters and the calibration of the one reading V_2.

Sketch (*c*) shows a method by which the equalizer is compared directly with an adjustable attenuator. The voltmeter is switched



(*c*)

alternately between the equalizer path and the attenuator path while the attenuator is adjusted until the same reading is given for both paths. The accuracy thus depends only upon the calibration of the attenuator, stability and calibration of the voltmeter being unimportant.

For both methods of measurement, it is important to avoid errors caused by harmonics of the testing frequency and this may be done by using a band-pass filter in the oscillator output circuit or by having a frequency-selective detector in place of the voltmeter. The voltmeter (or detector) should have a high impedance relative to that for which the equalizer is designed. In the sketches, a 600-ohm equalizer is shown.

Q. 5. (*a*) *Explain how a fault on a transmission line can be located by plotting an impedance/frequency characteristic and observing a regular pattern of maximum and minimum values.*

(*b*) *A cable pair having a loop inductance of* 1·5 *mH/km and a loop capacitance of* 0·1 *μF/km shows a spacing of* 3 *kHz between adjacent minimum values. Calculate the distance of the fault from the testing end.*

A. 5. (*a*) See A. 9, Line Transmission C, 1969. *Supplement*, Vol. 63, p. 61, Oct. 1970.

(*b*) Using the figures given,

$$x = \frac{1}{2 \times 3 \times 10^3\sqrt{(1{\cdot}5 \times 10^{-3} \times 0{\cdot}1 \times 10^{-6})}} = 13{\cdot}6\ \text{km}.$$

Distance to fault = 13·6 km.

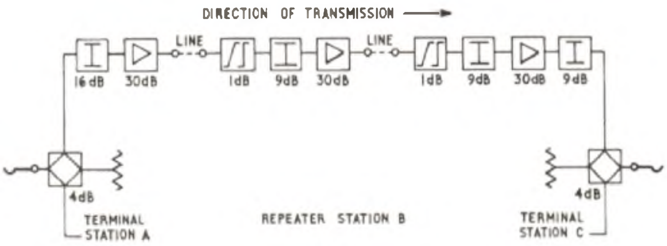
Q. 6. *An audio-frequency* 4-*wire circuit with* 2-*wire terminations is to be set up between terminal repeater stations* 20 *km apart. Each pair of the interconnecting cable has an attenuation of* 2 *dB/km at* 3·4 *kHz and of* 0·5 *dB/km at* 0·3 *kHz. The circuit is to be lined up to a loss of* 3 *dB between the* 2-*wire points.*

(*a*) *Draw* (*i*) *a block diagram for the whole circuit, and* (*ii*) *a level diagram for one direction of transmission.*

(*b*) *Explain why* (*i*) *maximum, and* (*ii*) *minimum, levels are usually specified for a circuit of this kind.*

A. 6. At 3·4 kHz, the cable loss between terminal stations 20 km apart is 40 dB. In a 4-wire circuit of the kind described, it is necessary to place limits on the upper and lower signal levels at any point and these conditions cannot be met without the use of an intermediate repeater.

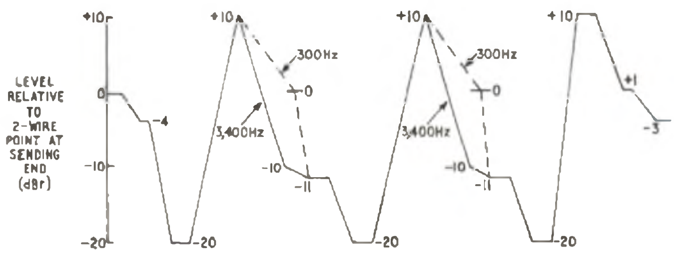
(*a*) Sketch (*a*) shows a block diagram for one direction of transmission between terminal stations A and C with an intermediate station B.



(*a*)

Hybrid transformers and balancing networks are provided at the terminal stations to allow for connexion to the 2-wire ends. Each repeater comprises a 30 dB fixed-gain amplifier and an adjustable attenuator. Line equalizers are fitted where necessary to make the attenuation of the whole circuit substantially independent of frequency.

Sketch (*b*) shows a level diagram for one direction of transmission. All levels are relative to that of the 2-wire point at the sending end



(*b*)

and are thus denoted as dBr. At each 2-wire–4-wire termination, the theoretical loss is 3 dB, but 4 dB has been allowed to cater for the transformer loss. A loss of 1 dB has been assumed for the line equalizers at 3·4 kHz. The whole arrangement is designed to ensure that the planned +10 dBr upper and −20 dBr lower limits of signal level are not exceeded. During the subsequent process of lining up the circuit, minor adjustments to the attenuator settings might have to be made.

(*b*) It is necessary to limit the maximum signal level at any point to minimize the likelihood of interference with other circuits in the same cable. A minimum signal level is necessary to avoid a poor signal-to-noise ratio. In sketch (*b*), values of +10 dBr and −20 dBr have been used.

Note.

A candidate who based his answer on the use of terminal repeaters only and then explained the weakness of the arrangement (i.e. failure to conform to the generally-accepted limitations of signal level) would not have been penalized.

Q. 7. *A main-line coaxial-cable system is to be designed with repeaters spaced* 1,500 *m apart. Power-feeding stations are to be established at* 20 *km intervals along the route and the repeaters each require a constant-current power supply of* 100 *mA d.c.*

(*a*) *With the aid of sketches, show how all the repeaters could be energized, assuming an a.c. mains supply to be available at each of the power-feeding stations.*

(*b*) *Show what stand-by equipment would be needed to guard against failure of the transmission system in the event of a mains breakdown.*

A. 7. (*a*) It is accepted practice to feed power to intermediate repeaters over the centre conductors of the coaxial cables and an essential feature is the use of power-separating filters. The principles are outlined in A.6, Line Transmission C, 1971. *Supplement*, Vol. 65, p. 90, Jan. 1973. For a description of a system working at a constant current of 100 mA d.c. with repeaters at 1,500 m spacing, see *P.O.E.E.J.*, Vol. 66, p. 240, Jan. 1974.

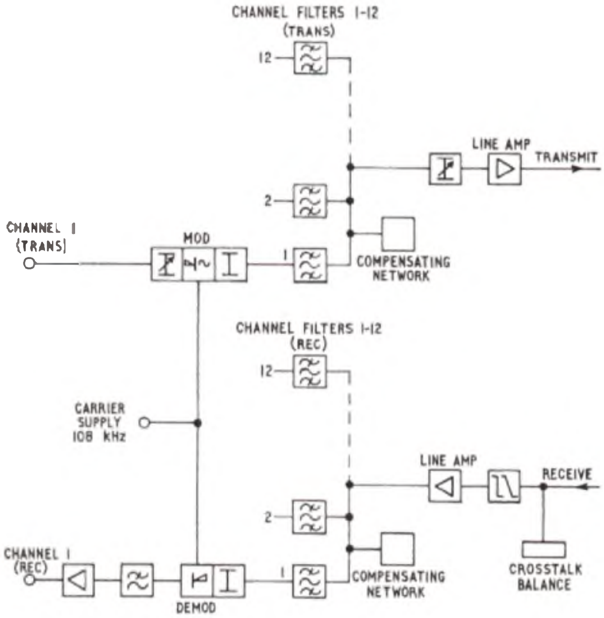
(*b*) See A.10, Line Transmission C, 1967. *Supplement*, Vol. 61, p. 33, July 1968.

Q. 8. *(a) Draw a block schematic diagram of a 12-channel C.C.I.T.T. group equipment for carrier telephony, and outline the function of each part.*

(b) Sketch the attenuation/frequency characteristic of a typical channel filter and show its relationship with those of the adjacent channels.

(c) Explain why filters should have a sharp cut-off and say how this is achieved.

A. 8. (*a*) Sketch (*a*) shows a block diagram of a 12-channel C.C.I.T.T. group equipment for carrier telephony. Channel 1 is



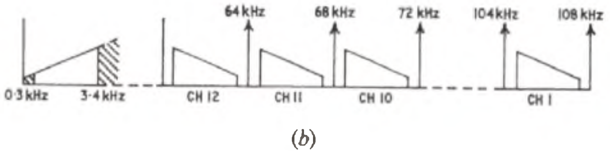
(*a*)

shown in detail and channels 2–12 are similar except for the carrier-supply frequency, which goes up at intervals of 4 kHz from 64 kHz for channel 12 to 108 kHz for channel 1. Individual channels are connected to line through two banks of channel filters (one for transmit and one for receive). Each has a compensating network on the line side to ensure proper operation of the whole bank.

Consider transmission from channel 1. Audio signals, applied to the transmit side, reach the modulator which is fed from the 108 kHz carrier supply. The modulation process gives rise to two sidebands, but the channel filter only allows the lower one to pass, the upper sideband being rejected. The lower sideband, together with the lower sidebands of the other channels, passes to line through the line amplifier.

On the receive side, each lower sideband is selected by the appropriate channel filter. Channel 1 sideband reaches the demodulator which is fed from the 108 kHz supply, and the demodulation process yields the original audio signal plus an unwanted band far above it in frequency. This unwanted band is rejected by the low-pass filter and only the audio signal passes to the receive terminals of channel 1.

Sketch (*b*) shows how the frequency spectrum of each channel is translated into the lower sideband of a carrier frequency. For any

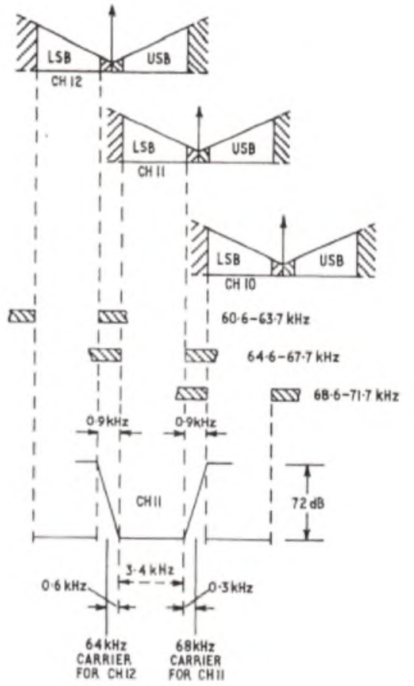


(*b*)

channel, the transmitted frequency range is 0·3 –3·4 kHz and the shaded parts indicate the frequencies above and below this range which are rejected by the channel filters.

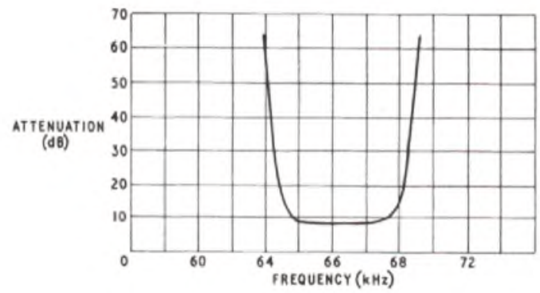
(*b*) Sketch (*c*) shows the filter characteristic required for channel 11 and indicates its relationship with the adjacent channels 10 and 12. The loss of each channel filter is required to be very small and substantially constant over the pass-band and to rise to about 72 dB in the stop-bands. Sketch (*d*) shows the attenuation/frequency characteristic of a typical crystal band-pass filter used in a modern system.

(*c*) The space between adjacent pass-bands is only 0·9 kHz and the rate of rise of the attenuation/frequency characteristic of each channel



(*c*)

filter has to be very large to allow a discrimination of 72 dB to be achieved. For this reason, crystal filters are used. These are expensive items but their provision is justified on economic grounds because they allow the line bandwidth available to be exploited to the best advantage.



(*d*)

Q. 9. *(a) Sketch the circuit elements of a central-battery telephone set and explain its operation (i) when the user is talking, and (ii) when the user is listening.*

(b) Outline the various tests which would be needed before a new design of telephone could be accepted for installation in an existing network.

A. 9. (*a*) See A.5, Line Transmission C, 1971. *Supplement*, Vol. 65, p. 89, Jan. 1973.

(*b*) See A.6, Line Transmission C, 1968. *Supplement*, Vol. 62, p. 70, Oct. 1969.

Q. 10. *Crosstalk between circuits and interference from supplies can often arise within a terminal station containing carrier equipment.*

(a) Give sketches to show how racks of equipment should be cabled in order to avoid such interference.

(b) Show what precautions are taken in the design of power-supply circuits to prevent them from causing interference.

A. 10. (*a*) See A.9, Line Transmission C, 1961. *Supplement*, Vol. 55, p. 30, July 1962.

(*b*) See A.5, Line Transmission C, 1964. *Supplement*, Vol. 58, p. 34, July 1965.

Students were expected to answer any six questions

Q. **1.** *With the aid of a diagram describe in detail the "fall of potential" method of measuring the resistance of earth electrodes.*

A. 1. See A. 6, Line Plant Practice C, 1969. *Supplement*, Vol. 63, p. 64, Oct. 1970.

Q. **2.** (*a*) *Explain the theory used when assessing the lateral earth pressure on a manhole constructed in a non-cohesive soil.*

(*b*) *A manhole with an overall height of 2·5 m is built in a non-cohesive soil with an angle of friction of 30°. The roof of the manhole is 0·8 m below the surface. If the soil has a bulk density of 1,850 kg/m³, calculate the horizontal pressure on the centre of one wall.*

(*c*) *What would be the horizontal pressure at the same point if a load of 4,500 kg/m² were added at the surface?*

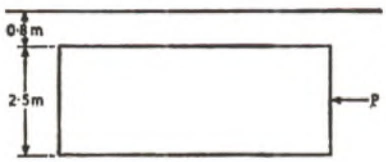
A. 2. (*a*) Rankine's theory of earth pressure is used to assess the lateral earth pressure on a structure constructed in a non-cohesive soil and gives the following relationship:

$$p = wh\frac{1 - \sin\phi}{1 + \sin\phi},$$

where w = density of soil in N/m³,
h = depth of soil in metres,
ϕ = internal angle of friction of soil in degrees, and
p = lateral earth pressure in pascals (Pa). [1 Pa = 1 N/m².]

If a load of x Pa is applied to the surface, the new combined lateral stress due to the effects of earth pressure and surface loading is given by

$$p = (x + wh)\frac{(1 - \sin\phi)}{(1 + \sin\phi)}.$$



(*b*) The sketch shows the position of the manhole built in the ground.

Now $h = (\frac{1}{2} \times 2\cdot5 + 0\cdot8)$ m.

$\phi = 30°$.

$w = 1{,}850$ kg/m³.

Using the Rankine formula,

$$p = 1{,}850 \times 2\cdot05\frac{(1 - 0\cdot5)}{(1 + 0\cdot5)}\text{ kg/m}^2,$$

$= 1{,}264$ kg/m²,

$= \underline{1{,}2401 \text{ Pa.}}$

(*c*) Added load is 4,500 kg/m².

Using second formula,

$$p = \{4{,}500 + (1{,}850 \times 2\cdot05)\}\frac{(1 - 0\cdot5)}{(1 + 0\cdot5)},$$

$= 2{,}764$ kg/m²,

$= \underline{27{,}116 \text{ Pa.}}$

Q. **3.** (*a*) *Discuss the advantages and disadvantages of polyethylene when used in the construction of*

(*i*) *telephone trunk and distribution cables, and*
(*ii*) *submarine cable.*

(*b*) *With the aid of a sketch describe in detail the make-up of an armoured local distribution cable of 50 pairs which makes full use of polyethylene as an insulant and sheath material.*

A. 3. (*a*) (*i*) The advantages of cables having polyethylene sheaths is that they are clean and easy to handle and are extremely flexible. These factors, coupled with the reduced weight, makes it possible to install longer lengths of cable than would be practicable with conventional paper-insulated and lead-sheathed construction. This, in itself, makes the use of polyethylene an economic proposition. Also, because of the waxy texture of polyethylene, the drawing-in of cables is less difficult and it is often possible to fill the duct space to a greater degree. Finally, the thickness of the polyethylene insulant allowable on the smaller-gauge conductors permits a reduction in the overall diameter of the cable compared with paper-insulated cables, again allowing more cables per ductway.

The disadvantages of polyethylene when used as an insulant or sheathing material are as follows.

(*1*) It is by no means a perfect barrier to molecules of water vapour, although it is fairly efficient in the prevention of the passage of water. It is generally recognized that, of the materials at present available, only a complete metallic sheath, such as lead or aluminium, can effectively prevent moisture entering the cable core. Therefore, an aluminium-foil moisture barrier is bonded to the polyethylene sheath.

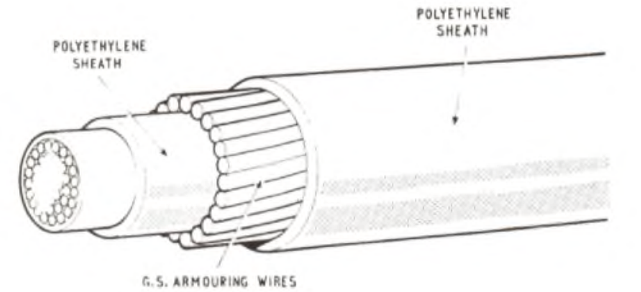
(*2*) It is flammable and cannot be used where arcing may occur.

(*3*) When buried directly in the ground, it is subject to attack by rodents.

(*4*) In its natural state it has a milky-white colour and is affected by ultra-violet light, i.e. sunlight. This is reduced by the addition of carbon black.

One use of polyethylene is as a corrosive protection over lead-sheathed cables. This also permits a reduction in the thickness of the lead sheath due to the added mechanical strength of the polyethylene. The average thickness of the polyethylene protection varies between 1·25 mm and 2·28 mm.

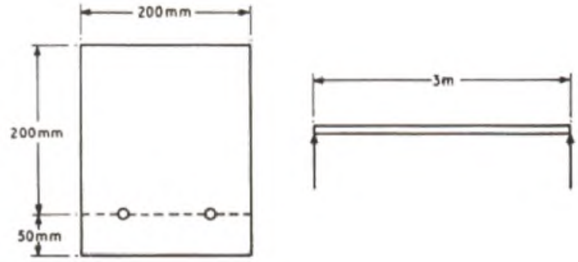
(*ii*) As an insulator for use in submarine cables, polyethylene possesses almost ideal characteristics; it is robust, easily-processed, non-hygroscopic and its electrical characteristics show a very marked improvement over any material previously used. Its dielectric constant is only 2·3 against 3·4 for gutta percha and its leakage losses are so low that for most practical purposes they can be neglected. Above all, it is cheap.



(*b*) The sketch shows the make-up of an armoured local-distribution cable of 50 pairs. This cable has aluminium or copper conductors insulated by an extruded cellular polyethylene covering, coloured for identification purposes. The conductors are uniformly twisted to form pairs and are stranded in layers around a 4-pair centre, successive layers being stranded in opposite directions. An open helical wrapping of tape is applied over each layer and two layers of paper are lapped over the completed core which is sheathed with polyethylene. The sheath has a layer of galvanized mild-steel wire armouring, stranded as shown in the sketch. A further polyethylene sheath is then extruded over the armouring.

Q. **4.** *A reinforced concrete beam of rectangular cross-section 250 mm deep and 200 mm wide is reinforced with two 12 mm diameter steel bars placed at 200 mm from the upper face. The beam is freely supported at its ends over a span of 3 m. If the safe working stresses of steel and concrete are 140 N/mm² and 7 N/mm² respectively and the modular ratio is 15, calculate the total safe uniformly-distributed load which the beam can support.*

A. 4. Referring to the sketches,



area of reinforcing steel $A_s = 2\pi\left(\frac{12}{2}\right)^2$ mm²,

$= 226$ mm².

Area of effective concrete $A_c = 200 \times 200$ mm²

$= 40{,}000$ mm².

Ratio (r) steel to concrete $= \frac{226}{4 \times 10^4} = 0{\cdot}00565.$

Let h be depth of neutral axis.

Then, $$\frac{h}{d} = \sqrt{(r^2m^2 + 2rm)} - rm.$$

If modular ratio (m) = 15,

$$\text{then } \frac{h}{d} = \sqrt{(r^2\,(15^2) + 2 \times r \times 15} - 15r),$$

$$= 0{\cdot}327.$$

$$\therefore\ h = 0{\cdot}327 \times 200 \text{ mm},$$

$$= 65{\cdot}4 \text{ mm}.$$

Steel stress (f_s) = 140 N/mm² or 140×10^6 Pa.
Thus, internal resisting moment M

$$= f_s\, A_s\left(d - \frac{h}{3}\right) = 140 \times 226\ (200\text{–}21{\cdot}8) \text{ Nmm}.$$

$$\therefore\ M = 5{,}638 \text{ Nm}.$$

If load $= W$ N/m, $M = \frac{L^2W}{8}$, where $L = 3$ m is span of beam.

$$\therefore\ 5{,}638 = \frac{9W}{8}.$$

$$\therefore\ W = 5{,}011 \text{ N/m}.$$

Q. 5. *(a) Explain what is meant by the term "non-electrolytic corrosion".*

(b) Briefly describe five of the various forms of non-electrolytic corrosion that can occur on lead-sheathed cables.

A. 5. (*a*) Non-electrolytic corrosion is defined as a form of electro-chemical corrosion in which the anodic and cathodic areas are so close together that the flow of current between them is not measurable.

(*b*) Examples of non-electrolytic corrosion that may occur on lead cable sheaths are as follows:

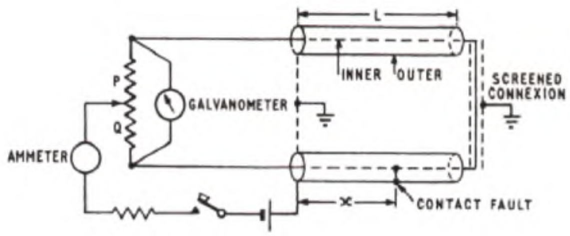
(*i*) Local-cell effects, which are due to non-uniformity of the lead surface or to impurities embedded in the lead.
(*ii*) Differential aeration effects, in which corrosion occurs on those areas of lead sheath to which access of oxygen is restricted.
(*iii*) Drop corrosion, which is a variation of the differential aeration principle.
(*iv*) Crevice corrosion, which is also a variation of the differential aeration principle.
(*v*) Scratch corrosion, where the oxide film is damaged and the exposed lead becomes anodic to the remainder of the surface.
(*vi*) Phenol corrosion, which sometimes occurs on hessian-protected cables and is attributed to denudation of bitumen from the lead sheath by pressure at points where the threads of the hessian tape cross each other. Micro-biological action forms acids in the hessian tape which attack the lead at the bitumen-denuded spots. The result is a large number of pits forming a distinct spiral pattern on the lead sheath. The name phenol corrosion arose because this type of damage was originally attributed to the phenol content of coal tar, which was used as an impregnant instead of bitumen in some European countries.
(*vii*) Acid corrosion, the electro-chemical action of many acids which attack lead and form acid salts. Lead with acetic acid in presence of carbon dioxide produces basic lead carbonate.

Q. 6. *A contact fault has occurred between inner and outer conductors in a coaxial pair in a cable 10 km long. Assuming that a pulse-echo tester is not available:*

(a) state two d.c. methods of locating the fault,
(b) say, with reasons, which of the two is the more accurate, and
(c) with the aid of a diagram, describe the more accurate of the two.

A. 6. (*a*) Two d.c. methods of locating the fault are the Varley test and the Murray test.

(*b*) For a testing length of up to 10 km, the resistance of the inner conductor is very low and the Murray test gives a more accurate result, particularly if a second inner conductor is used for the return path.



(*c*) The circuit for the Murray test is as shown in the sketch. Starting with a small value of current in the circuit, the slide-wire is adjusted for no deflexion on the galvanometer. The testing current is then increased to enable a satisfactory adjustment of the slide-wire to be made. If a good inner conductor is used for the return path, its resistance can be assumed to be equal to that of the faulty inner conductor. Then, at balance:

$$\frac{P}{Q} = \frac{L + L - x}{x}.$$

$$\therefore\ Px = 2QL - Qx.$$

$$\therefore\ x(P + Q) = 2LQ.$$

$$\therefore\ x = \frac{Q}{P + Q} \times 2L.$$

Q. 7. *A 50-pair, 0·63 mm, self-supporting combined aerial cable is erected between two poles 50 m apart at a tension of 4,000 N and a temperature of 15°C. A fall in temperature increases the tension to 4,300 N.*
Young's modulus for steel $E_s = 207 \times 10^3$ *N/mm².*
Coefficient of expansion of steel $= 12 \times 10^{-6}$/°C.
Cross-section of suspension wire = 14 mm².
Mass of the cable = 0·5 kg/m.

(a) Calculate the dip in the cable when tension is 4,000 N.
(b) Calculate the dip in the cable when tension is 4,300 N.
(c) Calculate the temperature when the cable tension is 4,300 N.

A. 7. (*a*) 383 mm.
(*b*) 356 mm.
(*c*) 4·6°C.

Q. 8. *Describe five factors which must be considered when choosing a support structure for an aerial system.*

A. 8. Five factors which should be considered are as follows:

(*a*) The structure must have adequate strength, rigidity and accommodation for the aerial system (or systems) it is to support. For example, a group of twenty 4 m diameter dishes may need to be accommodated at a height of 60 m, at a location where the wind velocity could gust to, say, 58 m/s. Under such an extreme loading condition, it may be desired to limit the rotation of any of the dishes to one degree in any direction.

(*b*) Before a structure can be erected, it is necessary to obtain planning approval. The design must, therefore, take into account the environment in which the radio station is situated. In certain areas of natural beauty, a lattice steel tower may not be tolerable, and a concrete tower may be the only type acceptable.

(*c*) The cost of the structure is all-important and, clearly, the designer should aim at producing the required structure at minimum cost.

(*d*) Limitations of site, should they exist, may well affect the design, e.g. poor soil-bearing properties may require a tower with a large spread at its base, or a restricted site may limit the size of the base. If the choice is between a tower or a guyed mast, then the latter must have an adequate site to accommodate the guy anchors.

(*e*) Maintenance is certainly an important consideration if the structure is to have a useful life-span. Some locations can be very corrosive for steel, particularly in industrial and coastal areas. Concrete towers need very little maintenance compared with steel, but then they usually cost more, and it is often more economical to use steel that is well protected by zinc and paint treatments.

Q. 9. *A beam is simply supported at A and B and carries concentrated and uniformly-distributed loads as shown in Fig. 1 below.*

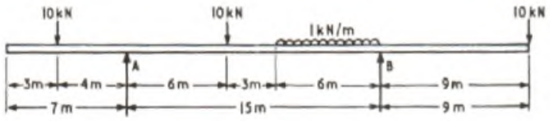


Fig. 1

(*a*) *Construct a shear-force diagram and a bending-moment diagram.*
(*b*) *State the maximum bending moment.*

A. 9. (*a*) It is first necessary to calculate the values of the reactions of the supports at A and B.

Total load A + B = 30 + 6 kN,
= 36 kN.

Take moments about B,

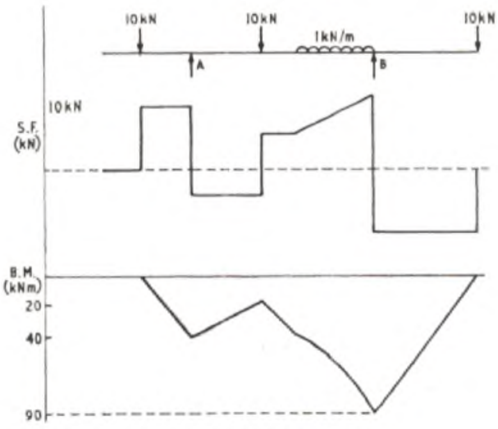
$(10 \times 19) - 15A + (10 \times 9) + (6 \times 3) = 10 \times 9.$

$\therefore A = 13 \cdot 8$ kN,

and $B = 36 - 13 \cdot 8$ kN,

$= 22 \cdot 2$ kN.

(*b*) Sketch (*a*) shows the shear-force diagram and sketch (*b*) the bending-moment diagram. From the latter, the maximum bending moment is 90 kNm.

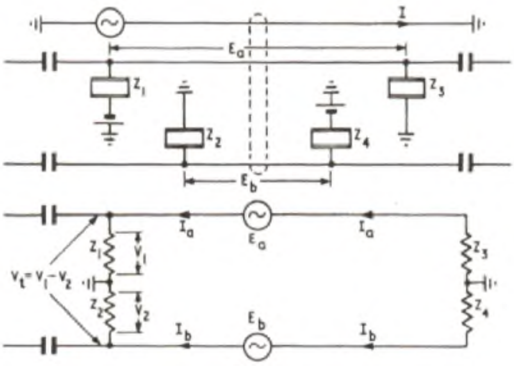


(*b*)

Q. 10. (*a*) *With the aid of a diagram, describe the effect of noise due to a longitudinally-induced voltage in a telephone circuit with an unbalance in the terminating equipment.*

(*b*) *Describe briefly the telephone line conditions which would bring about transverse noise voltages.*

A. 10. (*a*) The sketch shows a power-line carrying a harmonic current *I* producing an external magnetic field which induces longitudinal e.m.f.s E_a and E_b, which may be considered to be equal, in the wires of a nearby telephone line. These e.m.f.s cause currents I_a and I_b to flow in their respective wires from earth at one termination to earth at the other termination. Considering one termination, the products of



I_a and I_b and the individual terminating impedances Z_1 and Z_2 produce voltages v_1, and v_2. If Z_1 and Z_2 are equal, then the instantaneous values of v_1 and v_2 are equal and in phase (neglecting line characteristics) and there is no transverse voltage. If, however, Z_1 and Z_2 are not equal then the difference between v_1 and v_2 appears across the line as the transverse voltage v_t. This voltage is applied to the circuit in the same manner as speech and produces noise in a telephone receiver. In most cases, terminal impedances are not equal and, if their difference is great enough, the transverse voltages become so large as to cause very serious noise interference.

(*b*) Line unbalances also cause transverse noise voltages. Such unbalances are usually caused by fault conditions such as capacitance unbalance, high-resistance joints, disconnexions, and, in the case of open-wire lines, unbalanced insulation resistance.

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TELECOMMUNICATION PRINCIPLES C, 1972 (*Supplements* to Vol. 66, July and Oct., 1973).

A. 2. The polarity of the 2·5-volt generator in sketches (*c*) and (*d*) is shown reversed. Therefore, the generator shown in sketch (*e*) should deliver $2 \cdot 8 - 2 \cdot 5 = 0 \cdot 3$ volts, and, hence, the value of current flowing in the 5·8 kohm resistor in sketch (*f*) should be $\frac{0 \cdot 3}{5 \cdot 8 \times 10^3}$ amp, $= 51 \cdot 7\ \mu$A.

A. 3. In the sketch, the curve for susceptance should be shown as positive below resonance and negative above, to correspond with the values given in the table.

A. 5. The sketch shows the equivalent circuit for a p-n-p transistor whereas the question referred to an n-p-n device. However, the same results are obtained, using the method described, if the collector supply voltage is considered to be +10 volts instead of −10 volts.

A. 8. The principle of the three-winding transformer bridge is that the secondary windings act as direct ampere-turn ratio arms, and the transfer of resistance or reactance from the primary circuit to the secondary circuit is not dependent on the square of the turns ratio.

Thus, in part (*a*), (*i*), the unknown is a capacitance of $\frac{1{,}200}{2} = 600$ pF in parallel with a resistance of $2 \times 1 \cdot 5$ kohm = 3 kohm.

Similarly, in part (*a*), (*ii*), the unknown is an inductance of 0·083 H in parallel with a resistance of 2×2 kohm = 4 kohm.

Hence, the equivalent series circuit for part (*ii*) becomes an inductance of 0·03 H in series with a resistance of 2·55 kohm.

Also, in part (*b*), the primary-winding current, I_p, becomes 7·5 mA instead of 5·0 mA.

A. 9. The collector power dissipation of a transistor is greatest with no signal applied, and least when a continuous signal is applied (since power is passed from the collector to the load).

In part (*b*), (*i*), the power dissipated in the loaded transistor is equal to the power which would be dissipated with no applied signal, minus the power supplied to the load, i.e. $12 \cdot 5 - 5 = 7 \cdot 5$ W. Subtracting the value of 1 W, for which the transistor requires no heat sink, the sink must dissipate 6·5 W. Hence, sink A would be used.

In part (*ii*), the transistor can be regarded as unloaded and, thus, the sink must dissipate 12·5 W, less the value of 1 W for which no sink is required. Hence, sink B would be used.

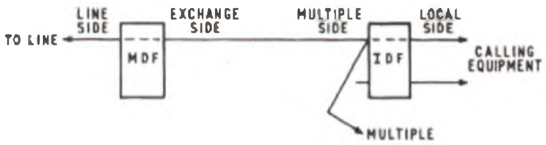
Students were expected to answer any six questions

Q. **1.** (*a*) *With the aid of a block schematic diagram, explain the purpose of the main and intermediate distribution frames used in a telephone exchange.*

(*b*) *Describe the construction of a typical fuse as found on the main frame, and explain its operation.*

(*c*) *Compare the purpose of the fuse with that of the heat coil and protector with which it is associated.*

A. **1.** (*a*) The distribution frames provide flexibility in associating exchange equipment with external lines, as shown in sketch (*a*).



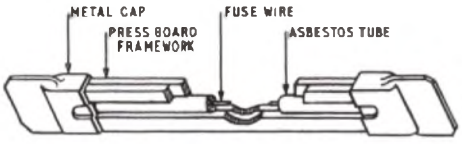
(*a*)

External lines enter the exchange in groups which are basically formed on a geographical basis. For economic reasons, exchange equipment is only allocated to those cable pairs which are actually required to provide service, and cross-connexion on the main distribution frame (m.d.f.) allows any line to be connected to any exchange-multiple number. All numbers appear in sequence on the exchange side.

The multiple side of the intermediate distribution frame (i.d.f.) is cabled directly from the m.d.f., and also presents all multiple numbers in sequence. However, to equalize the loading on the calling-equipment groups, it is desirable to group busy and less-busy callers together. The i.d.f. permits flexibility of this kind by allowing any calling equipment to be associated with any exchange-multiple number.

An important incidental purpose of the m.d.f. is that it accommodates the protective devices which safeguard the exchange equipment from damaging currents or voltages induced into external lines under fault conditions. It also provides a point where a circuit can be interrupted for test purposes.

(*b*) A simple line fuse is shown in sketch (*b*). The fuse is connected in series with the circuit and does not affect its operation. Current flowing through the fuse wire causes its temperature to rise, but the heat generated is safely dissipated. If, however, the current rises to a dangerous value, the wire is heated to its melting point and disconnects the circuit. The fusing current is roughly twice that of the continuous-current rating.



(*b*)

(*c*) In earlier practice (which is now being superseded by the use of delayed-action fuses), the line fuse is augmented by a heat coil and a protector.

The heat coil protects against faults which cause currents to flow in excess of the line-fuse current rating, but less than its fusing current. If currents in this range persist for a long time, they may occasion a fire risk due to the gradual heating of the line equipment. The heat coil has a similar long-term capacity for accumulating heat, and after an extended period, depending on the magnitude of the current, a soldered joint melts and the line is connected safely to earth.

The protector guards the apparatus against the effect of high potentials, such as those arising from a lightning discharge. It consists of two electrodes, one of which is connected to the line and the other to earth. They are separated by a narrow insulating gap. In the presence of a sufficiently-high voltage, the gap breaks down and the line is effectively earthed for the duration of the high voltage, so protecting the apparatus without the fuse blowing. When the high voltage is removed, the insulating properties of the gap are restored and the circuit is restored to a working condition.

Q. **2.** (*a*) *Define the term power rating as applied to a component.*

(*b*) *Two resistors, having the same ohmic value, are used in different circuits in such a way that the p.d. across the second is five times larger than that across the first. Compare the power dissipated in the two resistors.*

(*c*) *If the first resistor is rated at 0·5 W, briefly describe suitable forms of construction for the two resistors.*

(*d*) *List four properties which are desirable in a wire to be used in the construction of a wire-wound resistor.*

A. **2.** (*a*) The power rating of a component is the maximum power which it can dissipate continuously without permanent change in its characteristics.

(*b*) The power P dissipated in a resistor R is given by the product of the potential difference E across it and the current I flowing through it. This can be expressed as $P = EI$.

But, $E = IR$.

$\therefore P = I^2R$, or $\frac{E^2}{R}$, and it is seen that for a given value of resistance the dissipated power varies as the square of the potential across it. In the case given, the powers in the two cases are in the ratio $1^2 : 5^2 = 1 : 25$.

(*c*) A 0·5 W carbon-rod resistor is suitable in the first case, and a wire-wound resistor in the second case.

(*d*) Desirable properties of resistance wire include

(*i*) high specific resistance,
(*ii*) uniformity of cross section,
(*iii*) low temperature coefficient of resistivity,
(*iv*) low temperature coefficient of expansion,
(*v*) malleability, to facilitate winding, and
(*vi*) capability of being terminated on a tag by soldering or other means.

Q. **3.** (*a*) *Describe how lead-sheathed underground cables are damaged by electrolytic corrosion.*

(*b*) *With the aid of sketches, explain how cable networks are safeguarded by the use of*

(*i*) *insulation gaps, and*
(*ii*) *cathodic protection.*

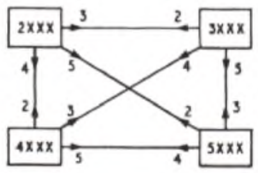
(*c*) *Compare the effects of alternating and direct leakage currents circulating through a cable sheath.*

Q. **4.** (*a*) *With the help of a simple block diagram, show how four automatic telephone exchanges can be linked together to serve a local area.*

(*b*) *Give an account, illustrated by a graph, of typical variations of traffic through a telephone exchange during a twenty-four hour period.*

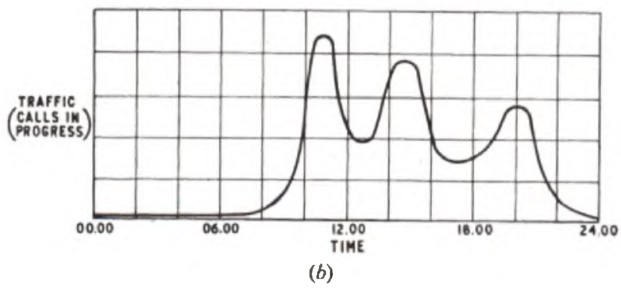
(*c*) *Define the terms busy hour and grade of service.*

A. **4.** (*a*) Sketch (*a*) shows the simplest form of a linked numbering scheme. The initial digit of a customer's number identifies the particular exchange to which he is connected. It is arranged that, on receipt of the first digit, the calling customer is connected via a junction to the appropriate target exchange, or to the home exchange if the local prefix digit is dialled.



(*a*)

(*b*) In a typical exchange area, traffic usually reaches a peak in the morning and again in the afternoon because of business calls. There is likely to be a peak of lesser magnitude during the evening arising from social calls. Through the night the incidence of traffic falls to a very low level. The variation of traffic density with time of day is shown in sketch (*b*).



(*b*)

(*c*) The busy hour is that hour of the day when traffic, measured as the product of the number of calls made and their average duration, is a maximum. It is the period in which the greatest occupancy of equipment occurs.

The grade of service is a measure of the adequacy of the switching plant to meet the demands made on it, and is expressed as the ratio of the number of calls failing due to shortage of plant to the total number of calls attempted.

Q. 5. (*a*) *With the aid of a target diagram, describe the make-up of a small paper-cored star quad cable.*

(*b*) *Explain how individual wires of the cable are identified.*

(*c*) *List four advantages of polythene as compared with lead as a sheath material.*

Q. 6. (*a*) *Describe the operation of a floated-battery scheme for providing direct current to a load.*

(*b*) *What is the purpose of the battery?*

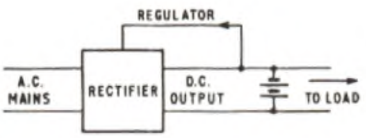
(*c*) *Briefly describe two simple tests, either of which will identify faulty cells of a lead-acid battery.*

A. 6. (*a*) A floated-battery scheme can be used where telecommunication equipment needs a constant-voltage d.c. supply. Normally, the equipment is fed from the output of a mains-driven rectifier, with a secondary battery connected in parallel with the load. Arrangements are made to regulate the output voltage by an automatic voltage-control link from the output to the rectifier (see sketch).

(*b*) The battery serves the following purposes:

(*i*) it maintains service for a period in the event of mains failure,
(*ii*) it provides a reserve of energy to meet temporary overloads, being recharged during a later period of light load,
(*iii*) it stabilizes the output voltage against short-term variations in supply voltage, and
(*iv*) it prevents alternating components at the rectifier output from passing to the load.

(*c*) The condition of the cells of a secondary battery can best be observed by measuring the specific gravity of the dilute sulphuric acid in the cells, using an hydrometer. A faulty cell is identified by a lower specific gravity than that of the other cells.



A check can also be made of the potential difference across each cell when the battery is on load. A faulty cell shows a lower value than good cells, particularly towards the end of discharge.

Q. 7. (*a*) *Describe two telegraph codes, one suitable for hand sending and the other for machine working.*

(*b*) *Draw the circuit diagram of a single-current circuit suitable for transmitting information in one of these codes.*

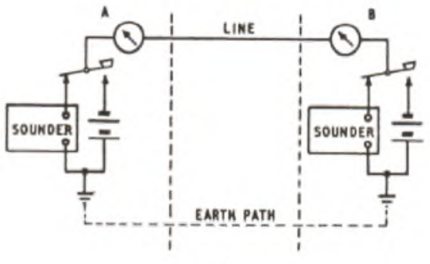
(*c*) *Sketch a graph of the sending-end voltage against time during the transmission of one character over the circuit in* (*b*).

A. 7. (*a*) (*i*) The Morse code is suitable for hand sending. It consists of combinations of two signals of similar character but of different lengths in the ratio of 1 : 3. They are known as the dot and the dash. Signal elements are separated by an interval equal in length to the dot, and letters by an interval equal to 5 dots.

Each alphabetical and numerical character is represented by a unique combination of dots and dashes, between one and five elements in total, so that there is considerable variation in character length. The code is satisfactory for hand sending, but is unsuitable for a machine system which is inherently suitable for operation on a constant-time cycle.

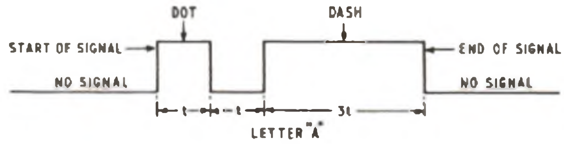
(*ii*) The five-unit (Murray) code is suitable for machine telegraphy. Each character is of similar length, consisting of a combination of five signal elements. All elements are of the same length but may take one of two forms, e.g. positive/negative, current/no current, and so on. Each character is preceded by a common one-unit signal which triggers the receiving machine, and is followed by a 1½-unit signal of opposite kind which stops the receiving machine. In this way, synchronism between the sending and receiving machines is maintained.

(*b*) A single-current circuit suitable for transmitting information in the code is shown in sketch (*a*).



(*a*)

(*c*) A diagram of the sending-end voltage against time for the circuit described in (*b*) is shown in sketch (*b*).



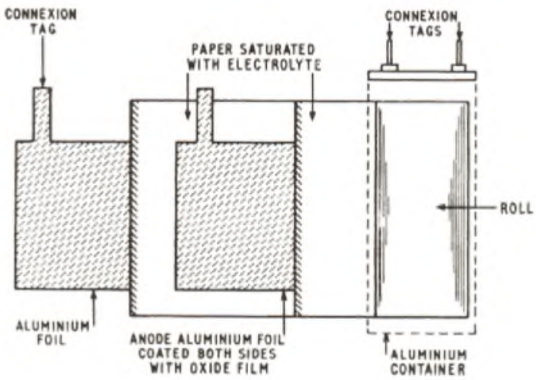
(*b*)

Q. 8. (*a*) *With the aid of a sketch, describe the construction of an electrolytic capacitor, and explain why this type has a high capacitance for its volume compared with most other types.*

(*b*) *Give two typical applications of electrolytic capacitors.*

(*c*) *State the principal precautions which must be taken in their use.*

A. 8. (*a*) The capacitance of a capacitor is dependent, among other things, on the separation between its plates. In an electrolytic capacitor, the plates are separated by an insulating coating of oxide on the plates. The thickness of the coating is controlled during manufacture and



can be limited to the minimum required to withstand the specified working voltage of the capacitor, without breakdown. In this way a much higher value of capacitance can be achieved in a given volume than is possible by the inclusion of a separate insulator.

(*b*) Electrolytic capacitors are widely used when low-impedance paths are required in the presence of d.c. potentials. Typical examples are in the smoothing stages of d.c. power supplies, and for de-coupling in electronic circuitry.

(*c*) Electrolytic capacitors should not be subjected to

(*i*) voltages in excess of their rated voltage,
(*ii*) reverse voltages, and
(*iii*) excessive ambient temperatures.

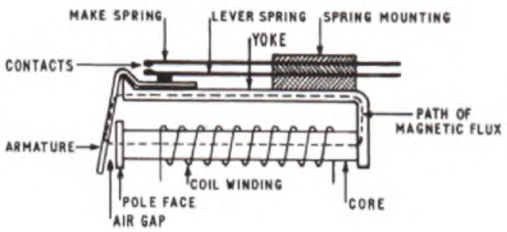
Q. 9. (*a*) *Sketch a simple telephone-type relay, indicating the magnetic circuit, and explain why the relay releases when the current in the coil is interrupted.*

(*b*) *Give reasons for the use, in some relay springs, of*

(*i*) *twin contacts, and*
(*ii*) *contacts with a domed profile.*

(*c*) *Name a suitable material for the contacts if they are required to carry low-level speech signals.*

A. 9. (*a*) The sketch shows a typical relay. When the coil is energized by the passage of a current, a magnetic field is set up which causes the armature to be attracted to the pole face and so operates the spring



set. The lever spring is pressed against the make spring. When the current in the coil is interrupted, the magnetic field collapses and there is no longer a force of attraction between the armature and the pole face. The deflected springs of the spring set, therefore, return to their normal positions and, in doing so, they return the armature to the unoperated position.

(*b*) (*i*) Twin contacts reduce the likelihood of a failure due to dust preventing contact being made. The chance of both pairs of contacts being faulty is much smaller than the chance of one being faulty.

(*ii*) Domed contacts allow for a measure of misalignment. They meet with a slight rolling action which has a self-cleaning effect.

(*c*) Silver, which is fairly cheap and has reasonable resistance to corrosion, is commonly used for speech-path applications.

Q. 10. (*a*) *Describe how speech is transmitted from the lips of a speaker to the ear of a listener over a line equipped with simple telephones which use carbon microphones and polarized receivers.*

(*b*) *What other features are needed in a telephone used in a public service?*

A. 10. (*a*) In face-to-face speech, pressure waves set up by the speaker are radiated through the surrounding air as travelling regions of relative compression and rarefaction. These changing pressures are detected by the ear of a listener, and are interpreted as sounds by the auditory mechanisms of the ear and brain.

In telephony, arrangements are made to simulate the acoustic pressure changes by electrical changes. These signals are transmitted to the receiving point, where they are used to produce a reasonably faithful facsimile of the original, which can then be detected by the listener's ear.

At the speaker's end, the energy in the acoustic wave produces sympathetic mechanical movement of the diaphragm. This, in turn, varies the pressure on a capsule containing two electrodes in contact with granules of carbon. The electrical resistance between the electrodes depends on the pressure exerted by the diaphragm, and, if the capsule is included in a d.c. circuit, the current flowing varies in sympathy with the changes of pressure on the diaphragm, and, therefore, with the speech signal. The changes in current can then be transmitted along a line to the listener's end as electrical signals.

At the listener's end, the signal is applied to a polarized receiver. It is passed through the coils of the receiver and generates magnetic forces which inter-act with the permanent magnetic force to produce a varying force of attraction on a diaphragm.

The diaphragm responds with mechanical movement, and, by its movement, generates pressure signals in the surrounding air which are similar to those existing at the speaker's end.

(*b*) A telephone to be used in a switched public telephone service must include means for

(*i*) calling the exchange and passing routing information,

(*ii*) receiving a calling signal, and

(*iii*) indicating to the exchange that a call has ended.

PRACTICAL MATHEMATICS, 1973

Students were expected to answer any six questions.

Q. 1. (*a*) (*i*) *Add together* $(3a + 4b + c)$ *and* $(5a - 2b - 3c)$.

(*ii*) *Subtract* $(6x - 4y + 2z)$ *from* $(5x - y - 4z)$.

(*iii*) *Express as one fraction* $\frac{pq}{r} - \frac{qr}{p} + \frac{rp}{q}$.

(*b*) (*i*) *Show that* $(x + y)^2 - (x - y)^2 = 4xy$.

(*ii*) *If* $x - y = 5$ *and* $xy = 36$, *find the values of* $(x + y)$ *and* $(x^2 - y^2)$.

A. 1. (*a*) (*i*)

$$\begin{aligned}&(3a + 4b + c) + (5a - 2b - 3c)\\ &= 3a + 4b + c + 5a - 2b - 3c,\\ &= 8a + 2b - 2c,\\ &= \underline{2(4a + b - c)}.\end{aligned}$$

(*ii*)

$$\begin{aligned}&(5x - y - 4z) - (6x - 4y + 2z)\\ &= 5x - y - 4z - 6x + 4y - 2z,\\ &= \underline{-x + 3y - 6z}.\end{aligned}$$

(*iii*)

$$\begin{aligned}&\frac{pq}{r} - \frac{qr}{p} + \frac{rp}{q}\\ &= \underline{\frac{p^2q^2 - q^2r^2 + p^2r^2}{pqr}}.\end{aligned}$$

(*b*) (*i*)

$$\begin{aligned}&(x + y)^2 - (x - y)^2\\ &= \{(x + y) + (x - y)\}\{(x + y) - (x - y)\},\end{aligned}$$

from the formula $a^2 - b^2 = (a + b)(a - b)$ for the difference of two squares,

$$\begin{aligned}&= 2x(x + y - x + y),\\ &= 2x \times 2y,\\ &= \underline{4xy}.\end{aligned}$$

Q.E.D.

(*ii*) If $x - y = 5$, and $xy = 36$,

$$(x - y)^2 = 25.$$

Also, from part *b*(*i*),

$$(x + y)^2 - (x - y)^2 = 4xy.$$

$\therefore\ (x + y)^2 - 25 = 4 \times 36$, on substituting the given values.

$$\begin{aligned}\therefore\ (x + y)^2 &= 144 + 25,\\ &= 169.\\ \therefore\ x + y &= \sqrt{(169)} = 13.\end{aligned}$$

Also,

$$\begin{aligned}x^2 - y^2 &= (x + y)(x - y),\\ &= 13 \times 5,\\ &= 65.\end{aligned}$$

Hence, $x + y = \underline{13}$, and $x^2 - y^2 = \underline{65}$.

Q. 2. (*a*) (*i*) *Without using mathematical tables find the approximate value to two significant figures of* $\frac{36\cdot 24 \times 8\cdot 56}{0\cdot 0583 \times 7{,}625}$.

(*ii*) *Evaluate the expression in* (*i*) *using logarithmic tables.*

(*b*) *Evaluate* λ *using the formula* $\lambda = \sqrt{\left(\frac{\phi t - \theta s}{st}\right)}$,

when $\phi = 69\cdot 4$, $t = 6\cdot 2$, $\theta = 0\cdot 35$, $s = 765$.

A. 2. (*a*) (*i*) $\frac{36\cdot 24 \times 8\cdot 56}{0\cdot 0583 \times 7{,}625}$.

There are evidently no simple common factors but, if $36\cdot 24$ is approximated to $36\cdot 25$ it is clear that numerator and denominator will both be divisible by at least a factor of 25.

$$\text{Thus, } \frac{36\cdot 24 \times 8\cdot 56}{0\cdot 0583 \times 7{,}625} \simeq \frac{\overset{1\cdot 45}{\cancel{36\cdot 25}} \times 8\cdot 56}{5\cdot 83 \times \underset{3\cdot 05}{\cancel{76\cdot 25}}},$$

$$= \frac{0\cdot 29 \times 8\cdot 56}{5\cdot 83 \times 0\cdot 61}, \text{ on again dividing by 5,}$$

$$\simeq \frac{8\cdot 56}{20 \times 0\cdot 61}, \text{ assuming that 29 is half of } 58\cdot 3,$$

$$= \frac{0\cdot 428}{0\cdot 61}.$$

If the last fraction is assumed to be $\frac{0\cdot 42}{0\cdot 6}$ (note that the reduction in both numerator and denominator is proportionately much the same amount, thus minimizing error), then

$$\frac{36\cdot 24 \times 8\cdot 56}{0\cdot 0583 \times 7{,}625} \simeq \frac{0\cdot 42}{0\cdot 6},$$

$$= \underline{0\cdot 70}, \text{ to two significant figures.}$$

Note: It is wrong to assume that this answer is correct to two significant figures without further investigation, but it is likely to be in view of the close approximations made.

(*ii*) Using logarithmic tables,

$$\log_{10}\left(\frac{36\cdot 24 \times 8\cdot 56}{0\cdot 0583 \times 7{,}625}\right) = \log_{10}(36\cdot 24 \times 8\cdot 56) - \log_{10}(0\cdot 0583 \times 7{,}625),$$

$$= 1\cdot 5592 + 0\cdot 9325 - (\bar{2}\cdot 7657 + 3\cdot 8823),$$

$$= 2\cdot 4917 - 2\cdot 6480,$$

$$= \bar{1}\cdot 8437.$$

$$\therefore \frac{36\cdot 24 \times 8\cdot 56}{0\cdot 0583 \times 7{,}625} = \underline{0\cdot 6977.}$$

The approximation 0·70 is thus seen to be correct and accurate to about 3 parts in 1,000.

(*b*)

$$\lambda = \sqrt{\left(\frac{\phi t - \theta s}{st}\right)},$$

$$= \sqrt{\left(\frac{\phi}{s} - \frac{\theta}{t}\right)}.$$

Substituting the values given,

$$\lambda = \sqrt{\left(\frac{69\cdot 4}{765} - \frac{0\cdot 35}{6\cdot 2}\right)},$$

$$= \sqrt{(0\cdot 09072 - 0\cdot 05645)},$$

$$= \sqrt{(0\cdot 03427)},$$

$$= \underline{0\cdot 1851}, \text{ from a table of square roots.}$$

Q. 3. *Using tables of squares, square roots and reciprocals only, evaluate the following.*

(*a*) $\sqrt{(0\cdot 274)} - \sqrt{(0\cdot 0953)}$.

(*b*) $\frac{1}{42\cdot 6} - \frac{1}{58\cdot 7}$.

(*c*) $(5\cdot 87)^2 + (0\cdot 682)^2$.

(*d*) $\sqrt{\left(\frac{1}{56\cdot 2} + \frac{1}{63\cdot 8}\right)}$.

A. 3. (*a*) $\sqrt{(0\cdot 274)} - \sqrt{(0\cdot 0953)} = 0\cdot 5235 - 0\cdot 3087$, from a table of square-roots,

$$= \underline{0\cdot 2148}.$$

(*b*) $\frac{1}{42\cdot 6} - \frac{1}{58\cdot 7} = 0\cdot 02347 - 0\cdot 01704$, from a table of reciprocals,

$$= \underline{0\cdot 00643}.$$

(*c*) $(5\cdot 87)^2 + (0\cdot 682)^2 = 34\cdot 46 + 0\cdot 4651$, from a table of squares,

$$= \underline{34\cdot 9251}.$$

(*d*) $\sqrt{\left(\frac{1}{56\cdot 2} + \frac{1}{63\cdot 8}\right)} = \sqrt{(0\cdot 01779 + 0\cdot 01567)}$, from a table of reciprocals,

$$= \sqrt{(0\cdot 03346)},$$

$$= \underline{0\cdot 183}, \text{ from a table of square roots.}$$

Q. 4. (*a*) *A ball race contains 36 metal balls each of diameter 8·5 mm as shown in Fig. 1. Calculate the approximate radius of the circle on which their centres lie.*

(*b*) *A machine contains three gear wheels. The first, with 36 teeth, is meshed to a second with 96 teeth, which is separately meshed to a third of 60 teeth. If the first wheel is rotating at 360 rev/min, calculate the speed of the third.*

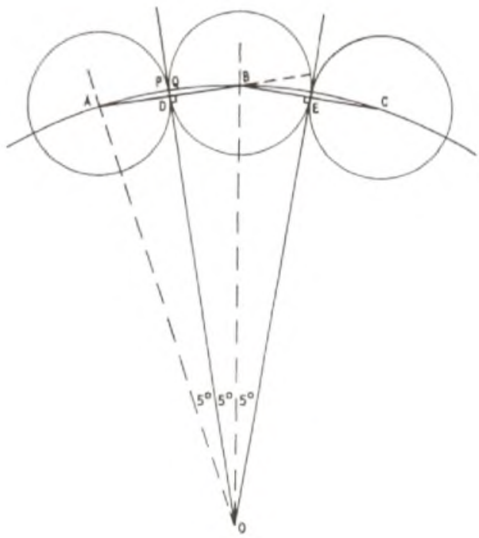
(*c*) *Given the formula $V = IR$,*

(*i*) *find the percentage change in V when I is reduced by 33⅓ per cent and R is increased by 33⅓ per cent.*

(*ii*) *If I is increased by 25 per cent, find the percentage change in R which will keep V constant.*

Fig. 1

A. 4. (*a*) Sketch (*a*) shows an enlargement of three adjacent metal balls of the race (not to scale), with centres A, B and C. Assuming all



(*a*)

balls just to touch one another, the common tangents to the circles, DO, EO etc. will all meet at O, the centre of the circle on which the centres of the balls lie.

Since there are 36 balls, the angle subtended at O by each ball will be $\frac{360}{36} = 10°$ and, hence, $\angle DOB = \angle BOE = 5°$. The difference in length between AD, the radius of one ball, and AP, the semi-arc of the circle with centre O contained within the first ball will be extremely small. Hence, for an approximate answer,

circumference of circle with centre O

$$\simeq 8\cdot 5 \times 36 \text{ mm},$$

$$= 2\pi \times AO.$$

$\therefore$ Radius of circle, AO

$$\simeq \frac{8\cdot 5 \times 36}{2\pi} \text{ mm},$$

$$= \frac{153}{\pi} \text{ mm},$$

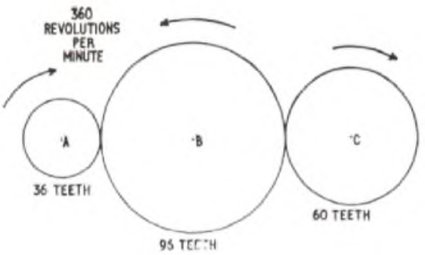
$$= 48\cdot 7 \text{ mm}.$$

Thus, approximate radius of the circle on which the centres of the balls lie is 48·7 mm.

Notes: (*i*) In sketch (*a*), the points P and Q where the circumference of circle with centre O intersects the circumferences of the two small circles appear to be one point due to the smallness of the diagram.

(*ii*) From triangle ADO or BDO, the radius may easily be calculated more accurately as $\frac{4 \cdot 25}{\sin 5^\circ}$, where 4·25 = AD the radius of the ball, and is found to be 48·74 mm.

(*b*) In sketch (*b*), A represents the first gearwheel, with 36 teeth, rotating at 360 rev/min. During one complete revolution of A,



(*b*)

gearwheel B will only rotate through 36 of its teeth and, hence, its speed will be reduced in the ratio of 36 to 96.

$$\therefore \text{ speed of gearwheel B} = 360 \times \frac{36}{96} \text{ rev/min.}$$

The speed of gearwheel C, however, will increase as compared with that of B since it has less teeth than B.

$$\text{Speed of gearwheel C} = 360 \times \frac{36}{96} \times \frac{96}{60} \text{ rev/min,}$$

$$= 6 \times 36 \text{ rev/min,}$$

$$= 216 \text{ rev/min.}$$

Hence, speed of third wheel = 216 rev/min.

(*c*) (*i*) $V = IR$.

If I is reduced by $33\frac{1}{3}$ per cent, its new value will be $66\frac{2}{3}$ per cent of the original, i.e. $\frac{2}{3}I$. Similarly the new value of R will be $\frac{4}{3}R$.

$$\therefore \; V' = \frac{2}{3}I \times \frac{4}{3}R, \text{ where } V' = \text{new value of } V,$$

$$= \frac{8}{9}IR,$$

$$= \frac{8}{9}V.$$

Thus, V has been reduced by one ninth or, $11\frac{1}{9}$ per cent.

(*ii*) If I is increased by 25 per cent, the new value of I will be 125 per cent of I or, $1 \cdot 25\, I$. Let R' be the new value of R for V to remain constant.

$$\therefore \; V = 1 \cdot 25\, I \times R',$$

$$\text{or, } R' = \frac{V}{1 \cdot 25\, I} = \frac{IR}{1 \cdot 25\, I} = \frac{R}{1 \cdot 25}.$$

$$\therefore \; R' = \frac{4R}{5} = 0 \cdot 8R.$$

Hence, R must be reduced by one fifth or 20 per cent.

Q. 5. (*a*) *Expand* $\left(x - \frac{1}{x}\right)^2$ *and, hence, show that if* $\left(x - \frac{1}{x}\right) = 3$, *then* $x^2 + \frac{1}{x^2} = 11$.

(*b*) *Factorize* $a^2 - 2ab - 15b^2$ *and use the result to express as one fraction*

$$\frac{1}{a^2 - 2ab - 15b^2} + \frac{1}{a - 5b}.$$

(*c*) *Find the value of x which satisfies each of the following equations:*

(*i*) $\log x + \log 6 = \log 18$,
(*ii*) $x \log 3 = \log 81$.

A. 5. (*a*)

$$\left(x - \frac{1}{x}\right)^2 = x^2 - 2 + \frac{1}{x^2}.$$

$$\text{But, } x - \frac{1}{x} = 3.$$

$$\therefore \; 9 = x^2 + \frac{1}{x^2} - 2,$$

$$\text{or, } x^2 + \frac{1}{x^2} = 11. \qquad \text{Q.E.D.}$$

(*b*)

$$a^2 - 2ab - 15b^2 = (a - 5b)(a + 3b).$$

Hence,

$$\frac{1}{a^2 - 2ab - 15b^2} + \frac{1}{a - 5b} = \frac{1}{(a - 5b)(a + 3b)} + \frac{1}{a - 5b},$$

$$= \frac{1 + a + 3b}{(a - 5b)(a + 3b)}.$$

(*c*) (*i*)

$$\log x + \log 6 = \log 18.$$

$$\therefore \; \log 6x = \log 18,$$

$$\text{or, } 6x = 18.$$

$$\therefore \; x = 3.$$

(*ii*)

$$x \log 3 = \log 81,$$

$$= \log 3^4.$$

$$\therefore \; x = \frac{4 \log 3}{\log 3},$$

$$= 4.$$

Q. 6. (*a*) *Draw on the same axes the graphs of* $5y = 4x + 8$, $y + 2x = 10$.

(*b*) *From the graphs determine*

(*i*) *the values of x and y at the point where they intersect,*
(*ii*) *the slope of each line, and*
(*iii*) *the area, in square units, of the quadrilateral contained by the two lines and both the axes.*

A. 6. (*a*)

$$5y = 4x + 8,$$

$$\text{or, } y = \frac{4x + 8}{5}.$$

Since this law is of linear form, $y = ax + b$, it is only necessary to plot two points to obtain the graph. The graph may be drawn from the following values (the third point serving as a check):

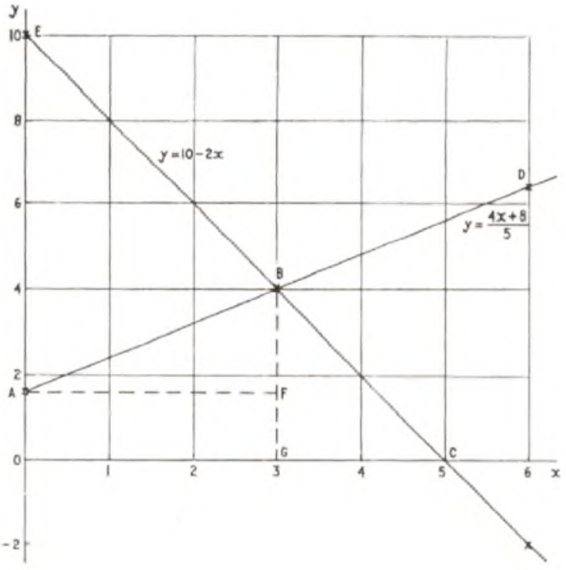
x	0	3	6
y	$1\frac{3}{5}$	4	$6\frac{2}{5}$

$$y + 2x = 10,$$

$$\text{or, } y = 10 - 2x.$$

This is also of linear form and its graph may be plotted from the following values:

x	0	3	6
y	10	4	−2



The two graphs are shown, drawn on the same axes, in the sketch.

(*b*) (*i*) The graphs intersect at point B where $x = 3$, and $y = 4$.

(*ii*) The slope of $y = \frac{4x + 8}{5}$ may be determined from the co-ordinates of A and D as

$$\frac{6\frac{2}{5} - 1\frac{3}{5}}{6 - 0} = \frac{4\frac{4}{5}}{6} = \frac{4}{5}.$$

Similarly, the slope of $y = 10 - 2x$, using the coordinates of points E and C, is given as

$$\frac{0 - 10}{5 - 0} = -2.$$

Hence, the slopes of $5y = 4x + 8$ and $y + 2x = 10$ are $\frac{4}{5}$ and -2 respectively.

Note: These slopes are seen, from part (*a*), to be the coefficients of x when each equation is put into the form $y = ax + b$.

(*iii*) The quadrilateral contained by the two lines and both axes is that of ABCO in the sketch.

The area of the quadrilateral may conveniently be obtained by drawing the ordinate BG through B and then drawing AF parallel to the x-axis.

Then, area of quadrilateral

$= $ area of rectangle AFGO + area of triangle ABF + area of triangle BGC,

$= \text{AO} \times \text{OG} + \frac{1}{2} \times \text{AF} \times \text{BF} + \frac{1}{2} \times \text{GC} \times \text{BG},$

$= 1\frac{3}{5} \times 3 + \frac{1}{2} \times 3 \times \left(4 - 1\frac{3}{5}\right) + \frac{1}{2} \times 2 \times 4,$

$= 4\frac{4}{5} + \frac{3}{2} \times \frac{12}{5} + 4,$

$= 4\frac{4}{5} + 3\frac{3}{5} + 4,$

$= 12\frac{2}{5}$ square units.

Q. 7. (*a*) *Solve the two simultaneous equations for u and v:*

$$3u - 4v = -14.$$
$$5u + 3v = 25.$$

(*b*) *Rearrange the following formula to make X the subject*

$$G = \frac{V(R - X)}{X}.$$

(*c*) *In an experiment to determine the internal resistance, r ohms, of a cell, it was observed that, when the cell was connected to an external resistance of* 10 *ohms, the current flowing was three times that when the cell was connected to an external resistance of* 36 *ohms. Calculate the value of r.*

A. 7. (*a*)

$$3u - 4v = -14. \qquad \ldots\ldots(1)$$
$$5u + 3v = 25. \qquad \ldots\ldots(2)$$

Multiplying equation (1) by 3 gives

$$9u - 12v = -42. \qquad \ldots\ldots(3)$$

Multiplying equation (2) by 4 gives

$$20u + 12v = 100. \qquad \ldots\ldots(4)$$

Adding equations (3) and (4) gives

$$29u = 58,$$
$$\text{or, } u = 2.$$

Substituting for u in equation (2) gives

$$10 + 3v = 25,$$
$$\text{or, } 3v = 15.$$
$$\therefore v = 5.$$

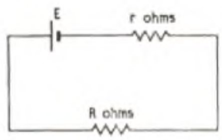
Hence, $u = 2$ and $v = 5$.

Note: This solution should be checked in each of the original equations (1) and (2).

(*b*)

$$G = \frac{V(R - X)}{X}.$$
$$\therefore GX = VR - VX,$$
$$\text{or, } GX + VX = VR.$$
$$\therefore X(G + V) = VR,$$
$$\text{and, } X = \frac{VR}{G + V}.$$

(*c*) The circuit is shown in the sketch, where R ohms is the external resistance, r ohms the internal resistance of the cell and E is the e.m.f. of the cell.



From Ohm's law, the current in the circuit is given as

$$i = \frac{E}{r + R}.$$

Under the first conditions, when $R = 10$ ohms,

$$i_1 = \frac{E}{r + 10}.$$

When $R = 36$ ohms,

$$i_2 = \frac{E}{r + 36}.$$

But, $i_1 = 3 \times i_2$.

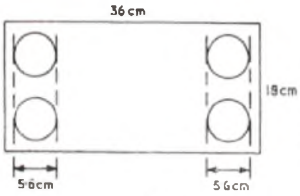
$$\therefore \frac{E}{r + 10} = 3 \times \frac{E}{r + 36},$$
$$\text{or, } r + 36 = 3(r + 10).$$
$$\therefore r + 36 = 3r + 30,$$
$$\text{or, } 6 = 2r.$$
$$\therefore r = 3.$$

Thus, the internal resistance, r, is 3 ohms.

Q. 8. (*a*) *A rectangular gasket* 36 *cm by* 18 *cm has four holes cut in it, each of diameter* 5·6 *cm. Calculate the area of the gasket.*

(*b*) *A vertically-fixed, open, inverted, hollow cone, base radius* 0·3 *m and vertical height* 0·72 *m, is filled with liquid. A solid sphere of radius* 0·20 *m is lowered into the cone and is just submerged when it touches the side of the cone. Calculate the volume of liquid remaining in the cone.*

A. 8. (*a*) Sketch (*a*) illustrates the gasket with four holes in it, the location of the latter being immaterial to the question.



(*a*)

$$\text{Area of gasket} = \text{rectangular area} - 4 \times \text{area of one circular hole},$$

$$= 36 \times 18 - 4 \times \pi \times \frac{d^2}{4},$$

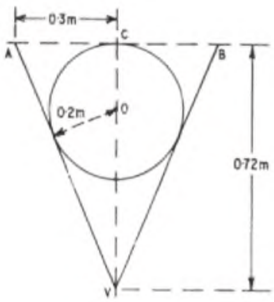
where d = diameter of hole = 5·6 cm.

$$\therefore \text{Area of gasket} = 648 - \pi \times 5{\cdot}6^2 \text{ cm}^2,$$

$$= 648 - 98{\cdot}54 \text{ cm}^2,$$

$$= \underline{549{\cdot}46 \text{ cm}^2}.$$

(*b*) Sketch (*b*) shows a cross-section of the inverted hollow cone with vertex V and base diameter AB. The solid sphere with centre O then just touches the base at C.



(*b*)

Volume of liquid remaining in cone = volume of cone − volume of sphere,

$$= \frac{1}{3}\pi r^2 h - \frac{4}{3}\pi r_s^3,$$

where, r = base radius of cone = 0·3 m.

h = vertical height of cone = 0·72 m,

and, r_s = radius of sphere = 0·2 m.

∴ volume of remaining liquid

$$= \frac{\pi}{3}(0{\cdot}3^2 \times 0{\cdot}72 - 4 \times 0{\cdot}2^3) \text{ m}^3,$$

$$= 1{\cdot}0472\,(0{\cdot}09 \times 0{\cdot}72 - 4 \times 0{\cdot}008) \text{ m}^3,$$

$$= 1{\cdot}0472\,(0{\cdot}0648 - 0{\cdot}032) \text{ m}^3,$$

$$= 1{\cdot}0472 \times 0{\cdot}0328 \text{ m}^3,$$

$$= 0{\cdot}03435 \text{ m}^3.$$

Thus, volume of liquid remaining in the cone is $\underline{0{\cdot}03435 \text{ m}^3}$.

Q. 9. *In Fig. 2, ABC is a right-angled triangle with the right angle at A, with AB = 10 cm, BC = 26 cm. D is the point on BC such that BD = 10 cm and the angle BDO is a right angle.*

(*a*) *Show that AC = 24 cm.*
(*b*) *Name the two similar triangles.*
(*c*) *Calculate the lengths of OD and OC.*
(*d*) *Calculate the area of the figure OABD.*

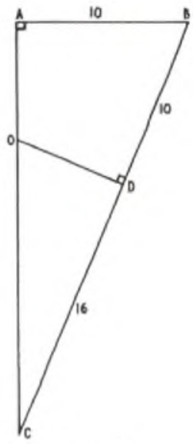
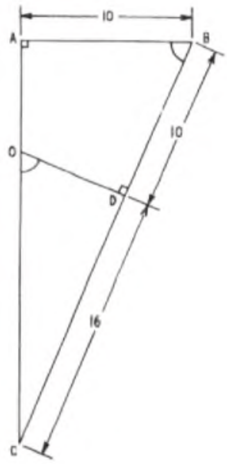


Fig. 2

A. 9. (*a*) Since triangle ABC, shown in the sketch, is right-angled at A then, by the theorem of Pythagoras,



$$BC^2 = AB^2 + AC^2,$$

$$\text{or, } AC^2 = BC^2 - AB^2,$$

$$= 26^2 - 10^2,$$

$$= 676 - 100,$$

$$= 576.$$

$$\therefore AC = \sqrt{(576)} = \underline{24 \text{ cm}}.$$ Q.E.D.

(*b*) Triangles ABC and ODC are similar.

(*c*) Since triangles ODC and ABC are similar,

$$\frac{OD}{AB} = \frac{CD}{AC}.$$

$$\therefore OD = AB \times \frac{CD}{AC},$$

$$= 10 \times \frac{16}{24},$$

$$= 6\frac{2}{3} \text{ cm}.$$

Again, $$\frac{OC}{BC} = \frac{CD}{AC}.$$

$$\therefore OC = BC \times \frac{CD}{AC},$$

$$= 26 \times \frac{16}{24},$$

$$= 17\frac{1}{3} \text{ cm}.$$

Thus, $\underline{OD = 6\frac{2}{3} \text{ cm}}$ and $\underline{OC = 17\frac{1}{3} \text{ cm}}$.

(*d*) Area of quadrilateral OABD

$$= \text{area of triangle ABC} - \text{area of triangle ODC},$$

$$= \frac{1}{2} \times AC \times AB - \frac{1}{2} \times OD \times DC,$$

$$= \frac{1}{2} \times 24 \times 10 - \frac{1}{2} \times 6\frac{2}{3} \times 16,$$

$$= 120 - 53\frac{1}{3},$$

$$= 66\frac{2}{3} \text{ cm}^2.$$

Thus, area of figure OABD $= \underline{66\frac{2}{3} \text{ cm}^2}$.

Q. **10.** (*a*) *Complete the table, giving results in terms of* $\sqrt{(2)}$ *and* $\sqrt{(3)}$ *where appropriate.*

θ	30°	45°	60°
sin θ	$\frac{1}{2}$		
cos θ			$\frac{1}{2}$
tan θ		1	

(*b*) *In Fig.* 3, *angle PQS is a right-angle with PQ* = 1 *unit and* $\angle S = 30°$. *PQR is a right-angled isosceles triangle. RT is perpendicular to PS.*

(*i*) *Show that* $RS = \sqrt{(3)} - 1$.

(*ii*) *Show that* $TR = \frac{1}{2}\{\sqrt{(3)} - 1\}$.

(*iii*) *Write down, in terms of* $\sqrt{(2)}$ *and* $\sqrt{(3)}$, *the sine of* 15° *and evaluate it to three decimal places.*

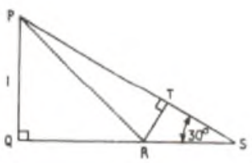


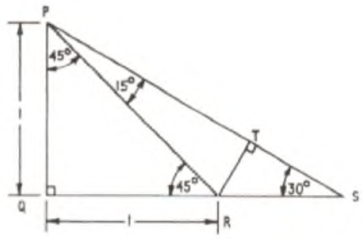
Fig. 3

A. **10.** (*a*)

θ	30°	45°	60°
sin θ	$\frac{1}{2}$	$\frac{1}{\sqrt{(2)}}$	$\frac{\sqrt{(3)}}{2}$
cos θ	$\frac{\sqrt{(3)}}{2}$	$\frac{1}{\sqrt{(2)}}$	$\frac{1}{2}$
tan θ	$\frac{1}{\sqrt{(3)}}$	1	$\sqrt{(3)}$

(*b*) See sketch. Since PQS is a right-angled triangle, and $\angle PSQ = 30°$, then $\angle SPQ = 60°$.

Also, since triangle PQR is a right-angled isosceles triangle, QR = PQ = 1.



Also, $\angle QPR = \angle QRP = 45°$.

Hence, $\angle RPT = \angle SPQ - \angle QPR$,

$$= 60° - 45°,$$

$$= 15°.$$

(*i*) In triangle PQS.

$$\frac{PQ}{QS} = \tan 30° = \frac{1}{\sqrt{(3)}}.$$

$$\therefore\ QS = PQ \times \sqrt{(3)},$$

$$= \sqrt{(3)}.$$

Hence, $RS = QS - QR$,

$$= \underline{\sqrt{(3)} - 1}. \qquad \text{Q.E.D.}$$

(*ii*) In triangle TRS,

$$\frac{TR}{RS} = \sin 30° = \frac{1}{2}.$$

$$\therefore\ TR = \frac{1}{2}RS,$$

$$= \underline{\frac{1}{2}(\sqrt{(3)} - 1)}. \qquad \text{Q.E.D.}$$

(*iii*) In triangle PQR, by Pythagoras,

$$PR^2 = PQ^2 + QR^2,$$

$$= 2.$$

$$\therefore\ PR = \sqrt{(2)}.$$

In triangle PRT, $\angle RPT = 15°$.

$$\therefore\ \sin 15° = \frac{TR}{PR},$$

$$= \frac{\frac{1}{2}\{\sqrt{(3)} - 1\}}{\sqrt{(2)}},$$

$$= \underline{\frac{\sqrt{(3)} - 1}{2\sqrt{(2)}}}.$$

Evaluating,

$$\frac{\sqrt{(3)} - 1}{2\sqrt{(2)}} = \frac{\sqrt{(2)}\{\sqrt{(3)} - 1\}}{2 \times 2},$$

$$= \frac{\sqrt{(6)} - \sqrt{(2)}}{4},$$

$$= \frac{2 \cdot 449 - 1 \cdot 414}{4} \text{ from a table of square roots},$$

$$= \frac{1 \cdot 035}{4},$$

$$= 0 \cdot 25875.$$

Thus, $\sin 15° = \underline{0 \cdot 259}$ to three decimal places.

Students were expected to answer two questions from Q. 1–4 and four questions from Q. 5–10.

Q. **1.** (*a*) *Explain the following terms used with reference to machines:*

(*i*) *velocity ratio,*
(*ii*) *mechanical advantage.*

(*b*) *A machine lifts a mass of* 200 kg *through a vertical distance of* 1·5 *m using a force of* 600 *N and with an efficiency of* 40 *per cent.*

(*i*) *Find the distance moved by the force.*
(*ii*) *Find the work lost as a result of friction.*
(*iii*) *Draw a diagram to show a suitable machine.*

A. 1. (*a*) (*i*) The velocity ratio of a machine is the ratio of the distance moved by the effort, which must be applied to the machine to move the load, to the distance moved by that load.

$$\text{Thus, velocity ratio} = \frac{\text{distance moved by effort}}{\text{distance moved by load}}.$$

(*ii*) The mechanical advantage of a machine is the ratio of the load to be moved by the machine to the effort which must be applied to the machine to move that load.

$$\text{Thus, mechanical advantage} = \frac{\text{load on the machine}}{\text{effort applied to the machine}}.$$

(*b*) For an ideal machine,

$$\text{effort} \times \text{distance moved by effort} = \text{load} \times \text{distance moved by load}.$$

$$\text{Hence,} \quad \frac{\text{distance moved by effort}}{\text{distance moved by load}} = \frac{\text{load}}{\text{effort}}.$$

$$\text{Thus,} \quad \text{velocity ratio } (V.R.) = \text{mechanical advantage } (M.A.),$$

$$\text{or,} \quad \frac{M.A.}{V.R.} = 1.$$

In practice, because of losses in the machine, more work must be done by the effort than is required by the load and the velocity ratio is greater than the mechanical advantage.

$$\text{The efficiency of a machine} = \frac{M.A.}{V.R.} \times 100 \text{ per cent.}$$

(*i*)

$$\text{Load} = 200 \text{ kg} = 200 \times 9{\cdot}81 \text{ N},$$
$$= 1{,}962 \text{ N}.$$

$$\text{Mechanical advantage} = \frac{\text{load}}{\text{effort}},$$
$$= \frac{1{,}962}{600}.$$

$$\text{Efficiency} = \frac{M.A.}{V.R.} \times 100 \text{ per cent.}$$

$$\therefore 40 = \frac{1{,}962}{600 \times V.R.} \times 100.$$

$$\therefore V.R. = \frac{1{,}962}{600} \times \frac{100}{40},$$
$$= 8{\cdot}175.$$

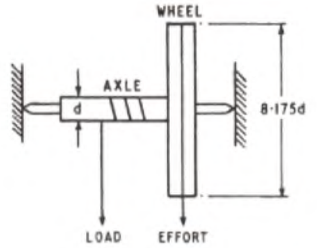
$$\text{Now,} \quad V.R. = \frac{\text{distance moved by effort}}{\text{distance moved by load}}.$$

$$\therefore \text{distance moved by effort} = V.R. \times \text{distance moved by load},$$
$$= 8{\cdot}175 \times 1{\cdot}5 \text{ m},$$
$$= \underline{12{\cdot}26 \text{ m}}.$$

(*ii*)

$$\text{The work lost} = \text{work done by the effort} - \text{work done on the load},$$
$$= 600 \times 12{\cdot}26 - 1{,}962 \times 1{\cdot}5 \text{ J},$$
$$= \underline{4{,}413 \text{ J}}.$$

(*iii*) A suitable machine, consisting of a wheel and axle, is shown in the sketch. As the required velocity ratio is 8·175, the circumference of the wheel must be 8·175 times greater than the circumference of the axle. As the circumference is proportional to the diameter, it follows that the diameter of the wheel must be 8·175 times greater than the diameter of the axle.



Q. **2.** (*a*) *Name and define the SI unit of*

(*i*) *force,*
(*ii*) *energy.*

(*b*) *A body of mass* 50 *kg, originally at rest, is given a constant acceleration of* 4 m/s^2 *for* 10 *s, then an acceleration of* 6 m/s^2 *in the same direction for a further* 20 *s. Find*

(*i*) *the velocity of the body after* 30 *s,*
(*ii*) *the force required during the first* 10 *s,*
(*iii*) *the kinetic energy of the body after* 30 *s.*

A. 2. (*a*) (*i*) The unit of force is called the newton and is that force which, when applied to a body having a mass of one kilogram, gives it an acceleration of one metre per second per second.

(*ii*) The unit of energy is called the joule and is the work done when the point of application of a force of one newton is displaced through a distance of one metre in the direction of the force.

(*b*) (*i*) The final velocity of a body can be obtained from the equation

$v = u + at$, where v = final velocity,
u = initial velocity,
a = acceleration, and
t = time.

As the body starts from rest, $u = 0$ m/s,

and the velocity after 10 s = 4 × 10 = 40 m/s.

The velocity after 10 s may now be taken as the initial velocity so that, after a further 20 s with an acceleration of 6 m/s^2, the velocity will be

$$v = 40 + (6 \times 20).$$

$\therefore$ <u>the velocity of the body after 30 s is 160 m/s.</u>

(*ii*) Force = mass × acceleration.

As the acceleration during the first 10 s is 4 m/s^2, the force required = 50 × 4 N,
= <u>200 N.</u>

(*iii*) Kinetic energy = $\frac{1}{2}$ × mass × $(\text{velocity})^2$.

The velocity of the body after 30 s is 160 m/s.

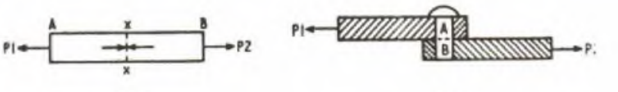
$\therefore$ the kinetic energy after 30 s = $\frac{1}{2} \times 50 \times 160^2$ J,
= <u>640 kJ.</u>

Q. **3.** (*a*) *Distinguish between the terms "tensile stress" and "shear stress".*

(*b*) *A wire* 15 *m long and* 3·0 *mm in diameter is stretched* 60 *mm by a force of* 5·6 × 10^3 *N. Find*

(*i*) *the tensile strain,*
(*ii*) *the tensile stress,*
(*iii*) *Young's modulus of elasticity for the wire.*

A. 3. (*a*) Tensile stress is set up in a body as a reaction to external tensile forces applied to the body which tend to pull it apart. In sketch (*a*), a solid bar, AB, is being pulled in opposite directions by forces P1 and P2. Forces equal and opposite to P1 and P2 exist within the bar at section XX preventing the bar from being pulled apart. The material at section XX is, thus, said to be under tensile stress. Shear stress is set up across a plane section within a body when there are forces tending to make the material on one side of the section slide past the material on the other. In sketch (*b*), a pin is subjected to



(*a*) (*b*)

shear stress across the plane section AB (shown dotted) due to the forces P1 and P2 acting in the directions shown. Thus, the upper part, A, of the pin tends to slide to the left relative to the lower part, B.

(*b*) (*i*) The tensile strain of a body is the ratio of the change in length of the body, under stress, to its original length.

$$\text{Thus, the tensile strain} = \frac{\text{increase in length}}{\text{original length}},$$

$$= \frac{60}{15 \times 10^3},$$

$$= \underline{0 \cdot 004}.$$

Note that strain is expressed as a ratio and is, therefore, dimensionless.

(*ii*) Tensile stress is expressed as the average force exerted per square metre of section.

$$\text{Thus, the tensile stress} = \frac{\text{force}}{\text{cross-sectional area}},$$

$$= \frac{5 \cdot 6 \times 10^3}{\pi \times 1 \cdot 5^2 \times 10^{-6}} \text{ N/m}^2,$$

$$= \underline{793 \text{ MN/m}^2, \text{ or } 793 \text{ MPa}}.$$

(*iii*) Young's modulus of elasticity for any given material is a constant and is the ratio of stress within the material to the resulting strain. Young's modulus of elasticity for the wire

$$= \frac{\text{stress}}{\text{strain}},$$

$$= \frac{793}{0 \cdot 004} \text{ MN/m}^2,$$

$$= \underline{198{,}250 \text{ MN/m}^2, \text{ or } 198{,}250 \text{ MPa}}.$$

Q. 4. *A vehicle of mass* 800 *kg is towed, with uniform speed, up an incline of length* 50 *m and height* 3·0 *m, as shown in Fig.* 1. *The coefficient of friction is* 0·2. *Find*

(*a*) *the reaction between the vehicle and the ground,*
(*b*) *the frictional force opposing the motion of the vehicle, and*
(*c*) *the potential energy of the vehicle at the top of the incline.*

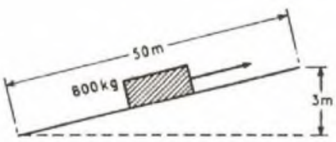
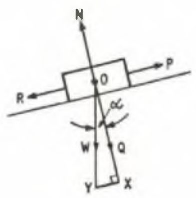


Fig. 1



(*a*)

A. 4. The forces acting on the vehicle are as shown in sketch (*a*). *W* is the gravitational force on the vehicle acting vertically downwards, i.e. the vehicle's weight. *N* is the normal reaction between the vehicle and the surface of the incline, and *Q* is a force equal and opposite to *N* which just keeps the vehicle in contact with the incline. *P* is the towing force acting up the incline and, as the vehicle is not accelerating, *R* is equal and opposite to *P*. *R* is made up of two components, one being the frictional force and the other being the resolved part of *W* acting down the incline.

(*a*) The normal reaction may be found by resolving the components of *W*. To do this, *W* and *Q* form two sides of a right-angled triangle in which $\angle OXY$ is the right angle. The angle α is equal to the angle between the incline and the horizontal. Thus,

$$\sin \alpha = \frac{3}{50},$$

$$= 0 \cdot 06.$$

$$\therefore \alpha = 3° 26'.$$

$$\text{Also, } W = 800 \times 9 \cdot 81 \text{ N},$$

$$= 7{,}848 \text{ N}.$$

$$\text{Now, } \cos \alpha = \frac{Q}{W}.$$

∴ Q, which equals the normal reaction N, $= W \cos \alpha$,

$$= 7{,}848 \times 0 \cdot 9981 \text{ N},$$

$$= 7{,}832 \text{ N}.$$

<u>The reaction between the vehicle and the ground is 7,832 N.</u>

(*b*) The coefficient of friction, μ, for an inclined plane is the ratio of the frictional force opposing the motion of the vehicle to the normal reaction. Thus,

$$\mu = \frac{\text{frictional force}}{N}.$$

$$\therefore \text{frictional force} = \mu N,$$

$$= 0 \cdot 2 \times 7{,}832 \text{ N},$$

$$= \underline{1{,}566 \cdot 4 \text{ N}}.$$

(*c*) The potential energy of the vehicle at the top of the incline is the amount of work which would be done if the vehicle was allowed to fall through a vertical distance of 3 m.

$$\text{Work} = \text{force} \times \text{distance},$$

$$= 7{,}848 \times 3 \text{ J},$$

$$= 23{,}544 \text{ J}.$$

<u>∴ the potential energy of the vehicle at the top of the incline is 23,544 J.</u>

Q. 5. (*a*) *A component carrying a steady current is mounted on a heat sink. What factors determine the temperature of the component?*

(*b*) *Calculate the power dissipated in each resistor in the circuit shown in Fig.* 2.

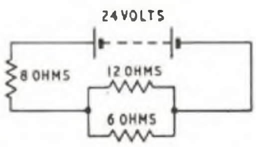


Fig. 2

A. 5. (*a*) The temperature of the component depends on the heat generated by the current passing through the component and the rate at which this heat can be conducted away by the heat sink.

The heat generated is determined by the power dissipated by the component.

$$\text{Power} = I^2R \text{ watts},$$

where I is the current flowing (amps) and R is the resistance of the component (ohms).

The heat generated, therefore, depends on the value of the steady current and the resistance of the component. The rate at which the heat is conducted away from the component by the heat sink increases if

(*i*) there is intimate contact between the heat sink and the component over as large an area as possible to allow the easy passage of heat from the component to the sink,
(*ii*) the material of the heat sink is a good heat conductor,
(*iii*) the surface area of the heat sink is as large as possible to improve heat loss due to convection and radiation,
(*iv*) there is a free flow of air around the heat sink to aid convection currents, and
(*v*) the surface of the heat sink is dark coloured to aid radiation.

The ambient temperature will also effect the rate of heat loss, the loss increasing with decreasing ambient temperature.

(*b*) The equivalent resistance of the 6-ohm and 12-ohm resistors in parallel

$$= \frac{6 \times 12}{6 + 12} \text{ ohms,}$$

$$= 4 \text{ ohms.}$$

$\therefore$ the total resistance of the circuit $= 8 + 4$ ohms,

$$= 12 \text{ ohms.}$$

By Ohm's law, the current, I, in the circuit

$$= \frac{24}{12} \text{ amps,}$$

$$= 2 \text{ amps.}$$

Now, the power dissipated by a resistor $= I^2R$ watts, where R is the value of the resistor (ohms).

$\therefore$ the power dissipated in the 8-ohm resistor $= 2^2 \times 8$ W,

$$= \underline{32 \text{ W.}}$$

Now, the potential difference across the parallel resistors $= 2 \times 4$ volts,

$$= 8 \text{ volts,}$$

and power may also be expressed as $\frac{E^2}{R}$ watts, where E is the potential difference (volts) across the resistor.

$\therefore$ the power dissipated in the 12-ohm resistor $= \frac{8^2}{12}$ W,

$$= \underline{5\tfrac{1}{3} \text{ W.}}$$

Power dissipated in the 6-ohm resistor $= \frac{8^2}{6}$ W,

$$= \underline{10\tfrac{2}{3} \text{ W.}}$$

Q. 6. (*a*) *Explain what is meant by "temperature coefficient of resistance".*

(*b*) *A power cable has a resistance of* 5·0 *ohms at* 0°C. *The metal of the cable has a temperature coefficient of* $4 \cdot 0 \times 10^{-3}/°C$.

(*i*) *If the temperature of the cable falls to* −10°C, *what is the resistance of the wire?*

(*ii*) *If the voltage drop in the cable is* 10 *volts at* 20°C, *what is the resultant power loss?*

A. 6. (*a*) The "temperature coefficient of resistance" may be defined as the change in resistance of a material due to a rise in temperature of 1°C, expressed as a ratio to the resistance of the material at 0°C. For example, if a material has a temperature coefficient of resistance (α) of 0·002 and has a certain resistance (R_0 ohms) at 0°C, then its resistance will increase by $0 \cdot 002 \times R_0$ ohms for every 1°C rise in temperature above 0°C.

(*b*) The resistance (R_T) of the cable at any temperature T°C may be found from the formula

$$R_T = R_0(1 + \alpha T), \text{ where } T = \text{change in temperature from } 0°\text{C.}$$

Therefore, at −10°C

(*i*) $R_T = 5\{1 + 4 \times 10^{-3}(-10)\}$,

$$= 4 \cdot 8 \text{ ohms.}$$

<u>The resistance of the wire at −10°C is 4·8 ohms.</u>

(*ii*) At 20°C, $R_T = 5(1 + 4 \times 10^{-3} \times 20)$,

$$= 5 \cdot 4 \text{ ohms.}$$

The power loss is given by $\frac{V^2}{R_T}$ watts, where V is the voltage drop in the cable.

$\therefore$ power loss $= \frac{10^2}{5 \cdot 4}$,

$$= \underline{18 \cdot 5 \text{ W.}}$$

Q. 7. (*a*) *Explain the meaning of the terms*

(*i*) *primary cell,*
(*ii*) *secondary cell.*

(*b*) *A d.c. source with a constant terminal voltage of* 24 *volts is used to charge eight secondary cells at a constant rate of* 2 *amps. Whilst charging at this rate, the terminal voltage of each cell rises from* 2·1 *volts to* 2·5 *volts.*

(*i*) *Draw a simple charging circuit.*

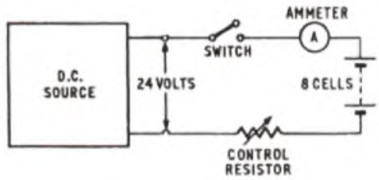
(*ii*) *Find the maximum and minimum resistances of the circuit required to maintain a constant current of* 2 *amps.*

(*c*) *Give the principal ways of assessing the state of charge of a lead-acid cell.*

A. 7. (*a*) (*i*) A primary cell consists basically of two electrodes of dissimilar metals immersed in an electrolyte. The resulting chemical action produces an e.m.f. across the terminals of the two electrodes. Chemical energy may, thus, be converted into electrical energy, but, in the primary cell, this process cannot be reversed. The store of energy in the cell can only be renewed by replacement of the active components.

(*ii*) A secondary cell usually consists of two lead plates covered with lead compounds and immersed in dilute sulphuric acid. Chemical energy is again transformed into electrical energy, but, in this case, the process is reversible and the secondary cell can be recharged from an external source.

(*b*) (*i*) The sketch shows a simple charging circuit for the eight cells. As the voltage of the cells rises, the value of the control resistor must be reduced to maintain the charging current at 2 amps.



(*ii*) The effective voltage in the circuit is the difference between the d.c. source voltage and the total voltage of the eight secondary cells.

To maintain a constant current of 2 amps, the maximum resistance is required when the effective voltage is highest, i.e. when the terminal voltage of the secondary cells is at its lowest. Therefore, when each cell has a terminal voltage of 2.1 volts,

the total voltage $= 8 \times 2 \cdot 1$ volts,

$$= 16 \cdot 8 \text{ volts.}$$

$\therefore$ the effective voltage $= 24 - 16 \cdot 8$ volts,

$$= 7 \cdot 2 \text{ volts.}$$

$\therefore$ if the current is to be maintained constant at 2 amps, the value of

the control resistor $= \frac{7 \cdot 2}{2}$ ohms,

$$= 3.6 \text{ ohms.}$$

<u>The maximum resistance required is 3·6 ohms.</u>

The minimum resistance is required when the difference between the source voltage and the battery is at its lowest. Therefore, when each cell has a terminal voltage of 2·5 volts,

the total voltage $= 8 \times 2 \cdot 5$ volts,

$$= 20 \text{ volts.}$$

$\therefore$ the effective voltage $= 24 - 20$ volts,

$$= 4 \text{ volts.}$$

The value of the control resistor $= \frac{4}{2}$ ohms,

$$= 2 \text{ ohms.}$$

<u>The minimum resistance required is 2 ohms.</u>

(*c*) The principal ways of assessing the state of charge of a lead-acid cell are by

(*i*) reading the specific gravity of the electrolyte on a hydrometer,
(*ii*) inspecting the colour of the positive plate, and
(*iii*) measuring the value of the terminal voltage of the cell.

Q. 8. *(a) An alternating voltage of peak value 5·0 volts and frequency 200 Hz is applied across a 500-ohm resistor. Sketch graphs to show how the current through the resistor and the voltage across it vary with time over a period of 10 ms.*

(b) Describe one method by which an alternating voltage may be generated.

Q. 9. *(a) What properties are desirable in a material to be used as a permanent magnet?*

(b) A vertical wire 0·5 m long moves in air across a uniform horizontal magnetic field with a flux density 0·6 T with a speed of 1·5 m/s.

(i) What is the induced e.m.f. in the wire?

(ii) With the aid of a diagram, show clearly the direction of the induced e.m.f. relative to the motion and the field.

A. 9. (*a*) The properties desirable in a material suitable for use as a permanent magnet are that it should

(*i*) be capable of producing a high magnetic flux density,

(*ii*) retain a high proportion of its magnetic flux after the initial magnetizing force has been removed, and

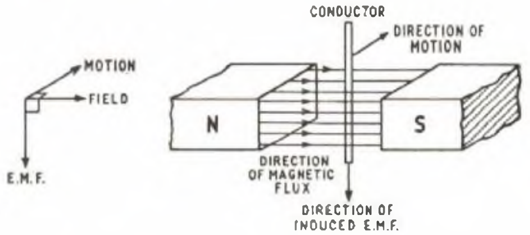
(*iii*) not be easily demagnetized.

(*b*) (*i*) The e.m.f. induced in a wire passing at right angles to, and with constant velocity through, a uniform magnetic field is given by

$e = Blv$, where e = e.m.f. in volts,
B = flux density in tesla,
l = length of conductor in metres, and
v = velocity in metres per second.

$$\therefore\ e = 0\cdot6 \times 0\cdot5 \times 1\cdot5 \text{ volts,}$$
$$= \underline{0\cdot45 \text{ volt.}}$$

(*ii*) The direction of the induced e.m.f. may be determined by applying Fleming's right-hand rule which states that if the thumb and first two fingers of the right hand are positioned so that they are mutually at right angles, then, if the thumb is pointed in the direction of motion and the first finger in the direction of the magnetic flux, the second finger points in the direction of the induced e.m.f. This is illustrated in the sketch.



Q. 10. *Write short notes on two of the following topics:*

(a) mutual inductance,

(b) electrolytic corrosion,

(c) the use of meter shunts and multipliers.

RADIO AND LINE TRANSMISSION A, 1973

Students were expected to answer any six questions

Q. 1. *(a) Sketch the waveform of a carrier wave amplitude modulated by a sinusoidal tone to a depth of 60 per cent.*

(b) This waveform is applied to the input of the detector circuit shown in Fig. 1.

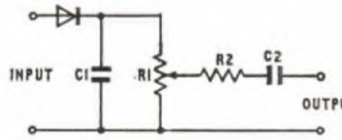
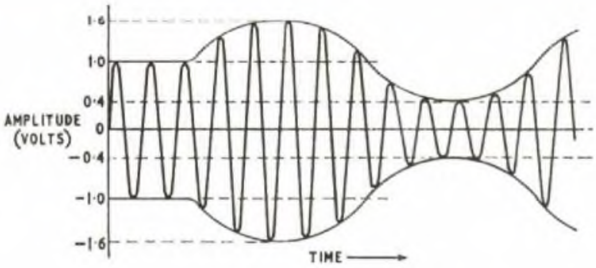


Fig. 1

(i) Explain, with reference to further current and voltage waveforms, the parts played by the various circuit components in the demodulating and filtering processes.

(ii) Sketch the waveform of the signal at the output of the detector.

A. 1. (*a*) The waveform of an amplitude-modulated carrier signal, sinusoidally modulated to a depth of 60 per cent, is shown in the sketch.



(*b*) (*i*) See A. 7, Radio and Line Transmission A, 1972. *Supplement*, Vol. 65, p. 94, Jan. 1973.

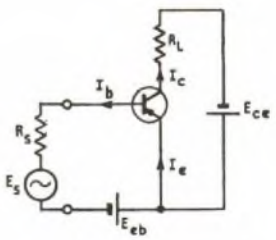
Q. 2. *(a) Explain the principle of operation of a resistance-loaded small-signal transistor amplifier in the common-emitter configuration.*

(b) Draw the circuit diagram of a two-stage common-emitter audio-frequency amplifier.

(c) Briefly explain why thermionic triode amplifiers mainly use R-C interstage coupling.

A. 2. (*a*) Sketch (*a*) shows the basic circuit of a resistance-loaded small-signal transistor amplifier in the common-emitter configuration.

In this circuit, the emitter-base junction is forward biased and the collector-base junction is reverse biased. When an input signal E_s is applied between the base and emitter, it produces a small change in the



(*a*)

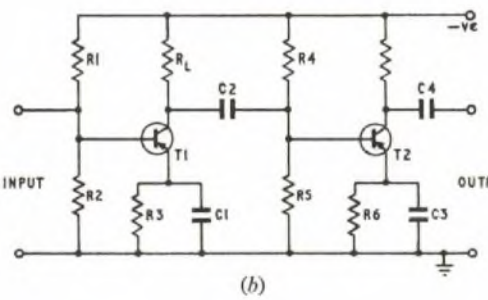
base current I_b. It can be shown that this small change in I_b produces a much larger change in the collector current I_c. Thus, a current gain is produced. Further, the change in collector current produces a corresponding voltage change across the collector load resistor R_L, and it can be shown that

$$\text{voltage gain} = \frac{\text{current gain} \times R_L}{R_{in}},$$

$$\text{and power gain} = \frac{(\text{current gain})^2 \times R_L}{R_{in}}.$$

Since R_L is made larger than the input resistance R_{in}, then current, voltage and power gain can be achieved with this type of circuit.

(*b*) Sketch (*b*) shows the circuit diagram of a two-stage, audio-frequency amplifier using transistors in the common-emitter configuration.



(*b*)

(*c*) Thermionic triode amplifiers mainly use R-C interstage coupling because the alternative types of interstage coupling, namely choke-capacitance and transformer, have poorer gain/frequency characteristics and are more bulky and expensive. Thus, the gain/frequency characteristic of an R-C coupled triode amplifier is fairly independent of frequency over the audio range 100–10,000 Hz, whereas the characteristics for choke and transformer couplings tend to have peaks.

Q. **3.** (*a*) *State whether the transistor is basically a current- or a voltage-operated device.*

(*b*) *Sketch the circuit of a self-biased transistor oscillator incorporating a tuned circuit with feedback by mutual inductance.*

(*c*) *Briefly explain why such an oscillator is self-starting and produces oscillations of constant amplitude.*

(*d*) *If the capacitance of the tuned circuit is halved, determine the percentage change in the frequency of oscillation.*

A. 3. (*a*) The transistor is basically a current-operated device.

(*b*) See A. 3, Radio and Line Transmission A, 1972. *Supplement*, Vol. 65, p. 92, Jan. 1973.

(*c*) The oscillator is basically a common-emitter amplifier with feedback between the collector output and the base input.

When switching on, oscillations are initiated in the base circuit at the frequency of resonance of the circuit consisting of inductor L2 and capacitor C2. These oscillations are amplified by the common-emitter amplifier section and the amplified signals are fed back from the collector output into the base circuit via capacitor C1, inductor L1 and the mutual inductance between inductors L1 and L2. The conventional bias and stabilization circuits (resistor R3, capacitor C3 and resistors R1 and R2) produce the self-biasing required to produce oscillations of constant amplitude.

(*d*) The frequency of resonance is given by $f = \frac{1}{2\pi\sqrt{(LC)}}$ Hz, where L is in henries and C is in farads. From this formula, the frequency of resonance is proportional to $\frac{1}{\sqrt{(C)}}$. Thus, if f_1 is the resonant frequency for tuning capacitance C, and f_2 is the resonant frequency for tuning capacitance $\frac{C}{2}$, then:

$$\frac{f_1}{f_2} = \frac{\frac{1}{\sqrt{(C)}}}{\frac{1}{\sqrt{\left(\frac{C}{2}\right)}}} = \frac{\sqrt{\left(\frac{C}{2}\right)}}{\sqrt{(C)}} = \frac{1}{\sqrt{(2)}}.$$

$$\therefore f_2 = f_1\sqrt{2} = 1{\cdot}414 f_1.$$

Thus, f_2 is 41·4 per cent greater than f_1.

Q. **4.** (*a*) *State the advantages of using logarithmic units in radio and line-transmission work.*

(*b*) *A telecommunication circuit consists of three items of equipment connected in tandem by line and radio links as shown in Fig. 2.*

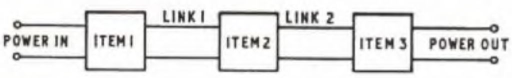


Fig. 2

The gains of items 1 and 3 are 23 dB and 16 dB respectively.
The losses in links 1 and 2 are 30 dB and 42 dB respectively.

If the input power to item 2 is 316 mW and the output from item 2 is 12,600 mW determine, using decibel notation,

(*i*) *the gain of item 2,*
(*ii*) *the output power from item 1,*
(*iii*) *the input power to item 1,*
(*iv*) *the input power to item 3,*
(*v*) *the output power from item 3, and*
(*vi*) *the overall gain or loss of the circuit.*

A. 4. (*a*) See A. 10, Radio and Line Transmission A, 1972. *Supplement*, Vol. 65, p. 95, Jan. 1973.

(*b*) (*i*) The gain of item 2 $= 10 \log_{10} \frac{12{,}600}{316}$,

$\simeq 10 \log_{10} 40 \simeq$ 16 dB.

(*ii*) Input to item 2 = 316 mW = $10 \log_{10} 316$ dB, relative to 1 mW (dBm),

$\simeq$ + 25 dBm.

Output from item 1 = (input to item 2) + (loss in link 1),

= (25 + 30) dBm = + 55 dBm.

(*iii*) Input to item 1 = (output from item 1) − (gain in item 1),

= (55–23) dBm = + 32 dBm.

(*iv*) Input power to item 3 = (output from item 2) − (loss in link 2).

Output from item 2 = 12,600 mW = $10 \log_{10} 12{,}600$ dBm,

$\simeq$ + 41 dBm.

∴ input power to item 3 = (41–42) dBm,

= − 1 dBm.

(*v*) Output power from item 3 = (input to item 3) + (gain in item 3),

= (−1 + 16) dBm = + 15 dBm.

(*vi*) Input to item 1 = + 32 dBm.

Output from item 3 = + 15 dBm.

There is, therefore, an overall loss of (32–15) = 17 dB.

Q. **5.** (*a*) *Sketch a circuit for determining the static characteristics of a triode valve.*

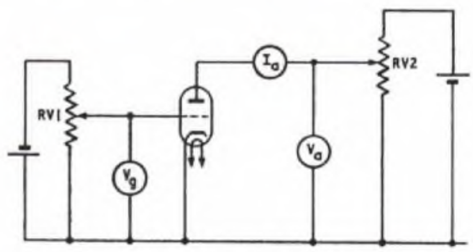
(*b*) *The following data were obtained for a triode valve.*

Anode Voltage	*Anode Current (mA) at grid voltages V_g =* 0 *volts*	−1 *volts*	−2 *volts*	−3 *volts*
50	3·8	1·2		
100	11·4	5·3	0·6	
150	20·6	12·9	5·8	1·0
200		21·6	14·4	6·8
250			23·2	15·4

Plot the anode characteristic (I_a/V_a) for each value of grid voltage.

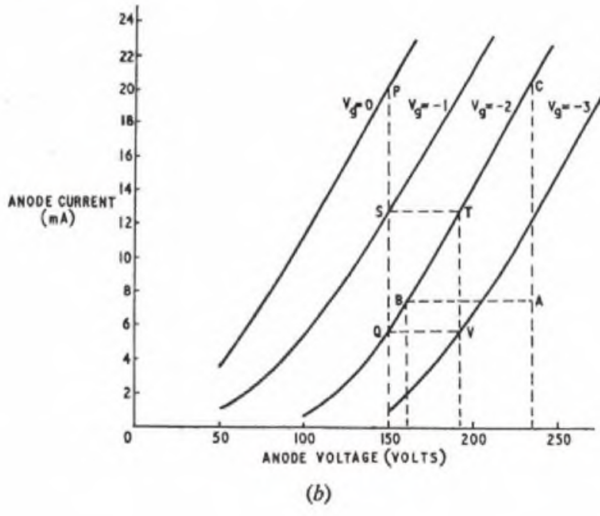
(*c*) *Use the anode characteristic to determine the anode a.c. resistance, mutual conductance and amplification factor.*

A. 5. (*a*) A circuit suitable for determining the static characteristics of a triode valve is given in sketch (*a*).



(*a*)

(*b*) The anode characteristics for each value of grid voltage are given in sketch (*b*).



(*b*)

(*c*) (*i*) Anode a.c. resistance $r_a = \frac{\text{change in } V_a}{\text{change in } I_a}$ with V_g constant,

where V_a is the anode voltage,
I_a is the anode current, and
V_g is the grid voltage.

For $V_g = -2$, and using the straight part of the characteristic,

$$r_a = \frac{AB}{AC} = \frac{(235\text{–}160) \text{ volts}}{(20\cdot5\text{–}7\cdot5) \text{ mA}} = \underline{5\cdot8 \text{ kohms.}}$$

(*ii*) Mutual conductance $g_m = \frac{\text{change in } I_a}{\text{change in } V_g}$ with V_a constant.

Taking $V_a = 150$ volts, and using points P and Q on the curves for $V_g = 0$ and $V_g = -2$,

$$g_m = \frac{(20\cdot5 - 5\cdot7) \text{ mA}}{[0 - (-2)] \text{ volts}} = \underline{7\cdot4 \text{ mA/volt.}}$$

(*iii*) Amplification factor $\mu = \frac{\text{change in } V_a}{\text{change in } V_g}$ to give the same change in I_a.

Using points Q, S, T, V, then a change of 1 volt in V_g produces a change in I_a of $(13 - 5\cdot7)$ mA. Also, a change in V_a of $(192 - 150)$ volts produces the same change in I_a.

$$\therefore \mu = \frac{(192 - 150) \text{ volts}}{0 - (-1) \text{ volts}} = \underline{42}.$$

Rough check: $\mu = g_m r_a$,

$$= \frac{7\cdot4}{10^3} \times 5\cdot8 \times 10^3 = \underline{43}.$$

Q. 6. (*a*) *Draw a labelled block diagram of a simple communication system in which a 2-wire-to-4-wire termination is used.*

(*b*) *State the purpose of the 2-wire-to-4-wire termination.*

(*c*) *Draw the circuit of a 2-wire-to-4-wire termination and briefly outline its operation.*

(*d*) *State the order of loss in decibels that you would expect in the 2-wire-to-4-wire termination.*

A. 6. (*a*) and (*c*). See A. 5, Radio and Line Transmission A, 1972. *Supplement*, Vol. 65, p. 93, Jan. 1973.

(*b*) Long-distance line circuits need amplifiers, and since amplifiers are essentially one-way devices, it is necessary to split the circuit into go and return sections at the points where amplifiers are to be inserted. Unless a special 2-wire-to-4-wire termination is used at these points, then oscillations will be produced.

In the case of an overseas radio-telephone system, a 2-wire-to-4-wire termination is used where the 2-wire line circuit is split into the separate circuits to the radio transmitter and radio receiver.

(*d*) The 2-wire-to-4-wire termination produces a loss of about 3·5 dB.

Q. 7. (*a*) *State the function of a telephone receiver.*

(*b*) *With the aid of a sketch,* (*i*) *describe the essential features of a telephone receiver using a non-magnetic diaphragm, and* (*ii*) *outline its operation.*

(*c*) *State the frequency range over which you would expect such a telephone receiver to have an effective response.*

A. 7. (*a*) The function of the telephone receiver is the opposite to that of the microphone. Thus, at the microphone the sound waves are converted into electrical signals for transmission via the line to the telephone receiver. At the telephone receiver, the electrical signals produced at the microphone are re-converted to produce the original sound waves.

(*b*) See A. 4, Radio and Line Transmission, A, 1971. *Supplement*, Vol. 64, p. 88, Jan. 1972.

(*c*) This type of telephone receiver has a flat sensitivity/frequency response over the range 200–3,400 Hz.

Q. 8. (*a*) *Explain why a parallel tuned-circuit is used in the radio-frequency amplifier section of a radio receiver.*

(*b*) *The voltage/frequency characteristic of a tuned circuit is given in the table below.*

Frequency (kHz)	185	190	195	198	200	202	205	210	215
Voltage	48	98	150	192	200	192	150	98	48

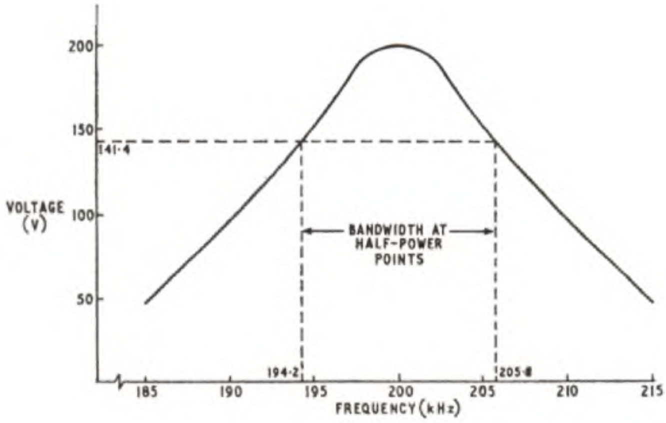
Plot the voltage/frequency characteristic and determine the bandwidth between the half-power points.

(*c*) *Discuss the possible effects if this tuned circuit were the only frequency-selective device in a radio receiver for the reception of*

(*i*) *Morse-code telegraphy transmission,*
(*ii*) *a double-sideband telephony transmission, and*
(*iii*) *a high-quality broadcast transmission.*

A. 8. (*a*) See A. 1, Radio and Line Transmission A, 1971. *Supplement*, Vol. 64, p. 87, Jan. 1972.

(*b*) The voltage/frequency characteristic is plotted in the sketch.



The half-power points are the points at which the response is 3 dB below the maximum. Using voltages:

$$20 \log_{10} \frac{200}{V} = 3,$$

where V is the voltage at the 3 dB points.

$$\therefore 3/20 = \log_{10} \frac{200}{V},$$

$$\text{i.e. } \frac{200}{V} = \text{antilog}_{10}\, 3/20,$$

$$= 1\cdot414.$$

$$\therefore V = \frac{200}{1\cdot414} = 141\cdot4.$$

From the graph, this voltage is obtained at frequencies of about 194·2 kHz and 205·8 kHz.

$\therefore$ bandwidth at half-power points $= 205\cdot8 - 194\cdot2 = \underline{11\cdot6 \text{ kHz.}}$

(*c*) (*i*) A Morse-code telegraphy transmission only requires a bandwidth of about 0·5 kHz, so the above circuit provides a bandwidth which is too wide. Hence, many other unwanted signals are likely to be received, so causing interference.

(*ii*) A double-sideband telephony transmission requires a bandwidth of about 6 kHz. Hence, the circuit in part (*b*) again provides a bandwidth which is too large and other unwanted signals are likely to be received and cause interference.

(*iii*) A high-quality broadcast transmission requires a bandwidth of 10 kHz or more in the medium wave band. Hence, the circuit in part (*b*) is just satisfactory for the reception of a high-quality medium-wave broadcast transmission, but would not be satisfactory at very high frequencies.

Q. 9. (*a*) *Discuss the limitations of a triode valve when used in the radio-frequency stage of a high-frequency (3–30 MHz) radio receiver.*

(*b*) *List the advantages and disadvantages of the composition type of resistor compared with those of the wire-wound type of resistor for use at high frequencies.*

(*c*) *State with reasons what types of capacitor you would expect to find in the high-frequency section of a radio receiver.*

A. 9. (*a*) The principal limitation of a triode valve when used in the radio-frequency (r.f.) stage of a high-frequency radio receiver is its instability. This instability arises from the inter-electrode capacitance between the grid and anode which becomes very effective at r.f. and results in self-oscillations.

The gain and selectivity of r.f. circuits using triode valves are also limited by the instability that can be caused by the grid-anode capacitance.

(*b*) The advantages and disadvantages of the composition type of resistor compared with those of the wire-wound type of resistor for use at high frequencies are given in the table below.

Advantages	*Disadvantages*
Almost non-inductive	Limited power dissipation
Negligible self-capacitance	Poor stability
Large range of values	Poor tolerance
Light weight	Noisy
Cheap	
Easily soldered into circuit	

(*c*) The two types of capacitor generally found in the r.f. section of a radio receiver are:

(*i*) air dielectric, and
(*ii*) mica dielectric.

The reasons for using these types of capacitors are the good stability, close tolerance, low loss and high working voltages possible with air and mica dielectric. These characteristics are essential for components used in the r.f. section of a radio receiver.

Q. **10.** (*a*) *Sketch suitably labelled input/output waveform diagrams to illustrate what is meant by the following conditions of amplification:*

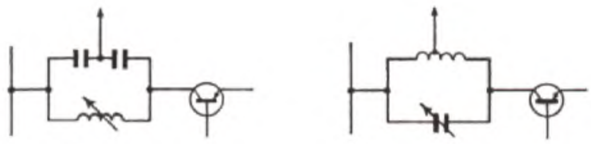
(*i*) *class A,*
(*ii*) *class B, and*
(*iii*) *class C.*

(*b*) *Draw the circuit of a tuned-collector transistor amplifier and briefly explain its operation.*

(*c*) *Draw simple sketches to illustrate two methods of tapping the parallel tuned-circuit of the tuned transistor amplifier.*

A. **10.** (*a*) and (*b*). See A. 4, Radio and Line Transmission A, 1972. *Supplement*, Vol. 65, p. 92, Jan. 1973.

(*c*) Two methods of tapping the parallel tuned-circuit of the tuned-collector transistor amplifier are given in sketches (*a*) and (*b*).



(*a*) (*b*)

TELECOMMUNICATION PRINCIPLES A, 1973

Students were expected to answer six questions, at least one being from Q. 9 and 10

Q. **1.** *Calculate, using Kirchhoff's laws, the current flowing through the 1-ohm resistor in Fig. 1. Check your answer by a separate calculation using the Superposition Theorem.*

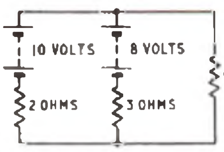
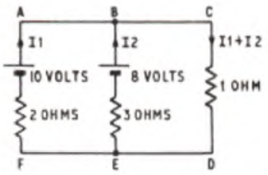


Fig. 1



(*a*)

A. **1.** *Using Kirchhoff's laws*

Let currents I_1 and I_2 be as shown in sketch (*a*). Then by Kirchhoff's first law, the current in the 1-ohm resistor $= I_1 + I_2$.

For loop ABCDEF, using Kirchhoff's second law,

$$(I_1 + I_2) \times 1 + I_1 \times 2 = 10.$$

$$\therefore\ 3I_1 + I_2 = 10. \qquad \ldots\ldots (1)$$

For loop BCDE, $(I_1 + I_2) \times 1 + I_2 \times 3 = 8$.

$$\therefore\ I_1 + 4I_2 = 8. \qquad \ldots\ldots (2)$$

Multiplying equation (2) by 3 and subtracting from equation (1) gives

$$-11I_2 = -14.$$

$$\therefore\ I_2 = 1 \cdot 28 \text{ amps.}$$

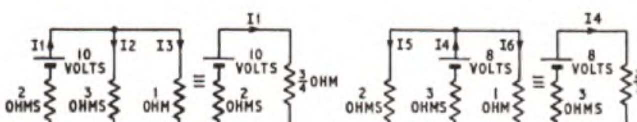
Substituting for I_2 in equation (2) gives

$$I_1 = 8 - 5 \cdot 1 = 2 \cdot 9 \text{ amps.}$$

$\therefore$ current in 1-ohm resistor $= I_1 + I_2 = \underline{4 \cdot 18 \text{ amps.}}$

Using the Superposition Theorem

Each battery in turn is replaced by a short circuit, giving the circuits shown in sketches (*b*) and (*d*), which are equivalent to sketches (*c*) and (*e*) respectively.



(*b*) (*c*) (*d*) (*e*)

By Ohm's law, $I_1 = \dfrac{10}{2 \cdot 75} = 3 \cdot 64$ amps.

$$\therefore\ I_3 = \frac{3}{4} \times 3 \cdot 64 = 2 \cdot 73 \text{ amps.}$$

Also, $I_4 = \dfrac{8}{3 \cdot 67} = 2 \cdot 18$ amps.

$$\therefore\ I_6 = \frac{2}{3} \times 2 \cdot 18 = 1 \cdot 45 \text{ amps.}$$

The total current in the 1-ohm resistor $= I_3 + I_6$,

$$= \underline{4 \cdot 18 \text{ amps.}}$$

Q. **2.** (*a*) *Explain the meaning of the term efficiency as applied to*

(*i*) *an electric motor,*
(*ii*) *a hydraulic pump.*

(*b*) *A pump driven by an electric motor raises water at the rate of 20 kg/s through a vertical height of 10 m. The efficiency of the pump and pipe system is 60 per cent and the efficiency of the motor is 78 per cent. Calculate*

(*i*) *the power input to the motor,*
(*ii*) *the current taken from a 240-volt d.c. supply,*
(*iii*) *the electrical energy used over a period of 12 hours of continuous operation.*

A. **2.** (*a*) Efficiency is defined as the ratio of the useful output power obtained from a machine to the input power required to drive it. It can be expressed as a fraction or as a percentage.

(*i*) For an electric motor,

$$\text{efficiency} = \frac{\text{electrical equivalent of mechanical output power}}{\text{electrical input power to drive motor}}.$$

(*ii*) For a hydraulic pump,

$$\text{efficiency} = \frac{\text{mechanical equivalent of hydraulic output power}}{\text{mechanical input power to drive pump}}.$$

(*b*) The overall efficiency of the motor-pump system

$$= (\text{efficiency of motor}) \times (\text{efficiency of pump}),$$

$$= 0 \cdot 78 \times 0 \cdot 6 = 0 \cdot 468.$$

$$\therefore\ \frac{\text{output power from pump}}{\text{input power to motor}} = 0 \cdot 468.$$

(*i*) The output power in the raised water

$= (20 \times 9 \cdot 81) \times 10,$

$= 1{,}962$ W.

$\therefore$ input power $= \frac{1{,}962}{0 \cdot 468}$ W,

$= 4 \cdot 2$ kW.

(*ii*) Current input $= \frac{\text{power input}}{\text{supply voltage}},$

$= \frac{4{,}200}{240},$

$= 17 \cdot 5$ amps.

(*iii*) Energy for 12 hours operation

$= 4 \cdot 2 \times 12$ kWh,

$= 50 \cdot 4$ kWh,

$= 50 \cdot 4 \times 10^3 \times 3{,}600$ J,

$= 181 \cdot 5$ MJ.

Q. 3. (*a*) *Explain what is meant by the terms*

(*i*) *self-inductance,*
(*ii*) *mutual inductance.*

(*b*) *State one practical application where mutual inductive effect is an advantage and one where it is a disadvantage.*

(*c*) *Two windings, A and B, are arranged so that mutual inductance exists. The self-inductance of A is 0·2 H and the mutual inductance is 0·05 H. The current in A increases uniformly from 0 to 16 amps in 0·04 s. Calculate the average value of induced e.m.f.*

(*i*) *in A, and* (*ii*) *in B.*

(*d*) *Would the mutual inductance be affected if the windings were moved further apart? Give reasons for your answer.*

A. 3. (*a*) (*i*) A circuit is self-inductive if a change of current, accompanied by a change of flux, produces an induced e.m.f. in the circuit.

(*ii*) If a change of current in a circuit, accompanied by a change in the flux linked with a second circuit, causes an e.m.f. to be induced in the second circuit, the two circuits are mutually inductive.

(*b*) Mutual inductance is used to advantage in a transformer where it couples the primary and secondary windings. It is a disadvantage between two transmission circuits as it leads to energy transfer, resulting in crosstalk.

(*c*) (*i*) The e.m.f. induced in winding A due to its self-inductance

$= -L\frac{di}{dt}$, where L is the self-inductance (henrys) and $\frac{di}{dt}$ is the rate of change of current (amperes/second),

$= -0 \cdot 2 \times \frac{16}{0 \cdot 04}$ volts,

$= -80$ volts.

(*ii*) The e.m.f. induced in B due to the mutual inductance

$= -M\frac{di}{dt}$, where M is the mutual inductance (henrys),

$= -0 \cdot 05 \times \frac{16}{0 \cdot 04}$ volts.

$= -20$ volts.

(*d*) The mutual inductance would decrease if the windings were moved apart since fewer lines of flux would link the two windings.

Q. 4. (*a*) *Explain the meaning of the terms*

(*i*) *electric field strength,*
(*ii*) *relative permittivity.*

(*b*) *A capacitor consists of two metal plates each of effective area 0·1 m² and spaced 2 mm apart. Calculate the capacitance if the space between the plates is completely filled with*

(*i*) *air,*
(*ii*) *mica having a relative permittivity of 6.*

(*c*) *For the capacitor in* (*b*) (*ii*), *calculate the charge and the energy stored when 500 volts d.c. is applied between the plates.*

(*d*) *State one typical application for a capacitor having air as its dielectric.*

A. 4. (*a*) (*i*) An electric field exists between two charged surfaces at different electric potentials, such as between the adjacent surfaces of the plates of a charged capacitor. The electric field strength at a point in the field is the potential gradient at that point, measured in volts per metre. If the dielectric between the charges is uniform, as in a parallel-plate capacitor, the electric field strength is uniform between the plates and the voltage gradient is the potential difference between the plates divided by their distance apart in metres.

(*ii*) If the dielectric is, say, mica then the capacitance between the plates will be greater than that which would exist if the dielectric were free space because a greater electric flux occurs in the mica dielectric than in free space.

The ratio of the capacitance with the dielectric to that without it is the relative permittivity ϵ_r of the dielectric.

(*b*) The capacitance, C, is given by the formula

$C = \frac{\epsilon_0 \epsilon_r A}{d}$, where C is the capacitance (farads),
ϵ_0 is the permittivity of free space (farads per metre),
ϵ_r is the relative permittivity,
A is the effective area of the plates (metre²),
and d is the distance between the plates (metres).

ϵ_0 is a constant and is equal to $8 \cdot 85 \times 10^{-12}$ F/m.

(*i*) For air, $\epsilon_r = 1$.

$\therefore$ capacitance $= \frac{8 \cdot 85 \times 10^{-12} \times 1 \times 0 \cdot 1}{2 \times 10^{-3}} = 442 \cdot 5$ pF.

(*ii*) For mica, $\epsilon_r = 6$.

$\therefore$ capacitance $= \frac{8 \cdot 85 \times 10^{-12} \times 6 \times 0 \cdot 1}{2 \times 10^{-3}} = 2{,}655$ pF.

(*c*) For any capacitance, the charge is given by

charge (coulombs) = capacitance (farads) × potential difference (volts).

For (*b*) (*ii*),

charge $= 2{,}655 \times 10^{-12} \times 500$ coulombs,

$= 1 \cdot 3275\ \mu$C.

Energy stored $= \frac{1}{2} \times$ capacitance $\times$ (potential difference)²,

$= \frac{2{,}655 \times 10^{-12} \times 25 \times 10^4}{2}$ joules,

$= 332\ \mu$J.

(*d*) A typical application of an air-dielectric capacitor is the tuning capacitor in a radio receiver.

Q. 5. *In the circuit shown in Fig. 2, the instantaneous value of the current, i, in amperes is given by i = 28·28 sin 2,000 t, where t is in seconds.*

(*a*) *Determine for this current*

(*i*) *the r.m.s. value,*
(*ii*) *the frequency,*
(*iii*) *the periodic time.*

(*b*) *For the circuit shown, give the expression for the instantaneous value of the voltage across*

(*i*) *the capacitor,*
(*ii*) *the inductor.*

(c) Sketch one cycle of the current and of each of the voltage waveforms in (b) on the same axes to show the phase relationship.

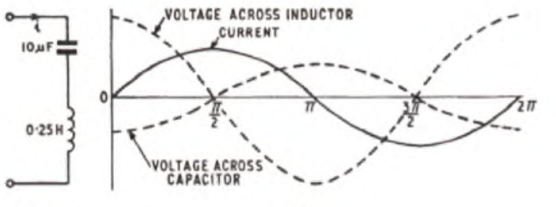


Fig. 2 (*a*)

A. 5. The general expression for the instantaneous value of a sinusoidal current is

$$i = I \sin 2\pi ft,$$

where I is the peak value of the current (amperes),
f is the frequency (hertz), and
t is the time in seconds from a point where $i = 0$.

(*a*) (*i*) If $i = 28{\cdot}28 \sin 2{,}000\, t$,

then peak current $= 28{\cdot}28$ amps.

$$\therefore \text{ r.m.s. value} = \frac{28{\cdot}28}{\sqrt{(2)}} = \underline{20 \text{ amps.}}$$

$$(ii)\ \text{Frequency} = \frac{2{,}000}{2\pi} = \underline{319 \text{ Hz.}}$$

$$(iii)\ \text{Periodic time} = \frac{1}{\text{frequency}} = \underline{3{\cdot}14 \text{ ms.}}$$

(*b*) The voltage across a reactance is given by the product of reactance and current flowing. This relationship applies to instantaneous values, as well as r.m.s. values, of current.

(*i*) The reactance of a capacitor is given by

$$X_C = \frac{1}{2\pi fC} \text{ ohms, where } C \text{ is the capacitance (farads).}$$

$$\therefore X_C = \frac{1}{2{,}000 \times 10 \times 10^{-6}} = 50 \text{ ohms.}$$

Now the instantaneous voltage, v_C, across a capacitor lags the current, i, by $\frac{\pi}{2}$ radians and

$$v_C = iX_C.$$

$$\therefore v_C = \left\{28{\cdot}28 \sin\left(2{,}000\, t + \frac{\pi}{2}\right)\right\} \times 50,$$

$$= \underline{1{,}424 \sin\left(2{,}000\, t + \frac{\pi}{2}\right) \text{ volts.}}$$

(*ii*) The instantaneous voltage, v_L, across an inductor leads the current by $\frac{\pi}{2}$ radians and,

$v_L = iX_L$, where X_L is the reactance of the inductor (ohms).

Now $X_L = 2\pi fL$ ohms, where L is the inductance (henrys).

$$\therefore X_L = 2{,}000 \times 0{\cdot}25,$$

$$= 500 \text{ ohms.}$$

$$\therefore v_L = \left\{28{\cdot}28 \sin\left(2{,}000\, t - \frac{\pi}{2}\right)\right\} \times 500$$

$$= \underline{14{,}240 \sin\left(2{,}000\, t - \frac{\pi}{2}\right) \text{ volts.}}$$

(*c*) Sketch (*a*) shows the phase relationship of the current and voltage waveforms. The amplitudes of the voltage waveforms are not drawn to scale.

Q. 6. (a) Describe with the aid of sketches the construction and principle of operation of a moving-coil instrument.

(b) Explain briefly how the same instrument may be modified for use as

(i) an ammeter,
(ii) a voltmeter.

(c) Draw a simplified circuit diagram of a multi-range instrument with switching arrangements to give two current ranges and two voltage ranges only.

A. 6. See A.6, Telecommunication Principles A, 1970. *Supplement*, Vol. 64, Apr. 1971, and A.5, Telecommunication Principles A, 1971. *Supplement*, Vol. 65, Oct. 1972.

Q. 7. (a) State briefly what is meant by the following types of semi-conducting material:

(i) p-type,
(ii) n-type.

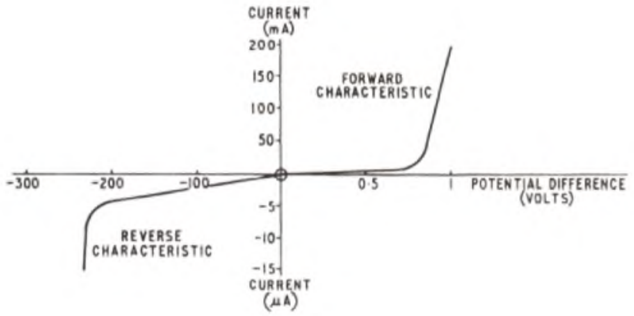
(b) Sketch a typical forward and reverse characteristic for a p–n junction, indicating approximate values on the axes of the graph.

(c) Explain briefly how a p–n junction rectifies alternating current.

A. 7. (*a*) (*i*) A p-type semi-conductor is a crystal, such as germanium or silicon, which is tetravalent (i.e. has four valence or bonding electrons in the outermost orbit of each atom), to which has been added a trace of a trivalent impurity element such as indium or aluminium. The impurity atoms form part of the crystal structure, but each can offer only three valence electrons to the four presented by the adjacent germanium (or silicon) atoms. Consequently, an incomplete covalent bond exists which has a vacancy for an electron, this vacancy being known as a *hole*. The incomplete bond can attract an electron from a nearby complete bond, thus filling the original hole, but creating another. This process continues, the holes moving about at random within the crystal structure. The number of holes is far greater than the number of free electrons and, thus, the holes are known as majority carriers and the free electrons as minority carriers.

(*ii*) In an n-type semi-conductor, the impurity element is pentavalent, such as arsenic or antimony. The impurity atoms again form part of the crystal structure, but each now offers five valence electrons to the four presented by the germanium (or silicon). Since only four covalent bonds can form, a free electron is provided which moves at random within the crystal. The electrons, in this case, are the majority carriers.

(*b*) The sketch shows a typical characteristic for a silicon junction diode.



(*c*) When a p–n junction is formed, holes, due to their random movement, diffuse across the p–n boundary into the n-type semi-conductor and electrons similarly diffuse into the p-type semi-conductor. Consequently the region in the p-type material adjacent to the boundary becomes negatively charged and the corresponding region in the the n-type material becomes positively charged. Thus, a potential barrier is set up across the boundary which inhibits the further diffusion of majority carriers.

When the junction is forward biased (i.e. a potential difference is applied across the junction in such a direction that the p-type material is made positive with respect to the n-type material), the electric field thus set up causes electrons in the p-type material to drift towards the applied positive potential, and holes in the n-type material to drift towards the applied negative potential. The potential barrier is, therefore, reduced and hence, allows a large number of majority carriers to cross the junction. Thus, the p–n junction conducts.

When the junction is reverse biased (i.e. the n-type material is made positive with respect to the p-type material), the potential barrier is increased so that majority carriers cannot cross the junction and hence, the p–n junction becomes an insulator.

Therefore, if an alternating current is applied to the p–n junction, conduction takes place, during alternate half-cycles only, in the direction in which the applied alternating current forward biases the junction. Hence, rectification occurs.

Q. 8. (a) Describe one of the following experiments. Set out your answer in the form of a laboratory report, which should include a circuit diagram, details of equipment and procedure, typical results, and conclusions.

(i) Measurement of resistance by any form of Wheatstone bridge.
(ii) Illustration of the relationship between induced e.m.f., number of turns and flux in inductive circuits.

(iii) Determination of the static characteristics of a vacuum diode.

(b) Comment upon any likely sources of error in your experiment.

A. 8. See A.1, Telecommunication Principles A, 1969. *Supplement*, Vol. 62, Jan. 1970, and A.2, Telecommunication Principles A, 1971. *Supplement*, Vol. 65, Oct. 1972.

Q. 9. *(a) (i) Draw a simplified circuit diagram of a common-emitter transistor amplifier and describe its action.*

(ii) Sketch and describe typical input and output characteristics for the amplifier.

(b) Give the construction for a load-line for the amplifier in (a) with a resistance load.

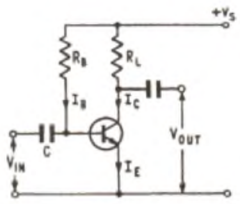
A. 9. (*a*) (*i*) A simplified circuit is shown in the sketch (*a*) for a common-emitter single-stage amplifier. Resistor R_L in the collector circuit is the load. The input, V_{in}, is connected between the base and emitter. Capacitor C passes an a.c. input, but isolates the input from the d.c. biasing circuit, which consists of resistor R_B delivering a bias current, I_B, given by the formula $I_B = \frac{V_S}{R_B}$, where V_S is the supply voltage.

The value of capacitor C is chosen such that its reactance at the operating frequency range is low. The collector current I_C is approximately equal to the emitter current I_E. The collector current, $I_C = \alpha I_E$, where α is the current gain of the transistor, typically of the order of 0·98. Thus I_B is small, being approximately $\frac{1}{50} I_E$, since $I_E = I_C + I_B$.

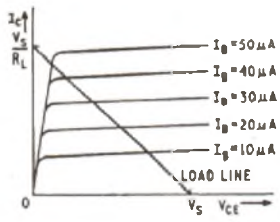
The input signal V_{in} creates an alternating base current i_B. This causes an alternating component in I_C much larger than i_B, which, in turn, gives an alternating voltage, V_{out}, across the load resistor, R_L, greater than the input, V_{in}.

(*ii*) The input resistance is approximately 1,000 ohms for a common emitter circuit, and is nearly constant over the operating range.

An output characteristic is the relationship between the collector–emitter voltage, V_{CE}, and the collector current, I_C, for a particular



(*a*)



(*b*)

value of quiescent base current, I_B. The curve is almost linear and horizontal over the working range, but falls sharply towards zero for low values of collector–emitter voltage. A characteristic curve can be drawn for various values of quiescent base current, giving a family of output characteristics as shown in sketch (*b*).

(*b*) A load-line can be drawn across the output characteristics using the equation

$$V_{out} = V_{CE} = V_S - I_C R_L.$$

Now, when $V_{CE} = 0$, then $\underline{I_C = \frac{V_S}{R_L}}$,

and when $I_C = 0$, then $\underline{V_{CE} = V_S}$.

The load-line is a straight line drawn through these two points, and its slope equals $\frac{1}{\text{load resistance}}$.

Q. 10. *(a) Describe the principle of the thermocouple ammeter.*

(b) What precautions must be taken to obtain a reliable result when using the thermocouple instrument?

(c) State two applications for which the thermocouple instrument is especially suitable.

A. 10. See A.9, Telecommunication Principles A, 1972. *Supplement*, Vol. 66, Apr. 1973.

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