SUPPLEMENT

TO THE POST OFFICE ELECTRICAL ENGINEERS' JOURNAL

1971–72 CITY AND GUILDS OF LONDON INSTITUTE EXAMINATIONS

Contents

QUESTIONS AND ANSWERS

Answers are occasionally omitted or reference is made to earlier Supplements in which questions of substantially the same form, together with the answers, have been published. Some answers contain more detail than would be expected from candidates under examination conditions.

TELEPHONY C, 1972 (continued)

Q. **4.** (*a*) *State the circumstances under which each of the following signalling systems would be used.*

(*i*) *Loop-disconnect.*
(*ii*) *Long-distance direct current.*
(*iii*) *1 v.f.*

(*b*) *Describe the principle of operation of a d.c. signalling system suitable for use over long amplified junctions.*

A. **4.** (*a*) (*i*) Loop-disconnect signalling is used where a d.c. signalling path exists and the junction (or line) resistance is within specified limits. These limits are determined by such factors as

(*i*) whether the originating pulses are from a dial or are mechanically generated,
(*ii*) types of equipment used e.g. 3,000-type relays with pre-2,000 type selector mechanisms,
(*iii*) number of tandem junctions,
(*iv*) whether there is mechanical pulse regeneration, and
(*v*) exchange voltage.

(*ii*) Long distance d.c. signalling, e.g. Signalling System Direct Current (S.S.D.C.) No. 2, is used where a d.c. signalling path exists but the junction (or line) resistance is beyond the limits for loop-disconnect signalling. S.S.D.C. No. 2 is only used on 4-wire within-quad amplified audio and 2-wire non-amplified audio circuits.

For a junction outgoing from a 50-volt exchange the pulsing limit is 76,000 microfarad-ohms, the signalling limit is 8,200 ohms and the insulation resistance must be greater than 0·1 Mohm.

(*iii*) A single voice-frequency (1 v.f.) signalling system is used where no d.c. path exists, e.g. a carrier route, or the signalling limits are outside those for long-distance d.c. The 1 v.f. signalling system is a within-speech-band signalling system.

(*b*) The sketch shows the signalling elements of a uni-directional d.c. signalling circuit using double-current working. Relays A and D are of the Carpenter polarized type and are side-stable, i.e. they remain in the make or break position until current flows through the coils in such a direction as to change over the contact. The windings of each relay are connected in such a way that conventional current flow (earth to battery) in the direction of the arrow moves the contact to the position marked *make*. If conventional current flow is in the opposite direction, the contact moves to the *break* position.

With the circuit normal, current flow is from the incoming relay-set (i.c.r.s.) earth via contact GD1, winding 2 of relay A, the positive wire and winding 2 of relay D to battery. Contact A1, therefore, remains at *break* and contact D1 remains at *make* (current in winding 2 of relay D is enough to overcome the effect of the current in bias winding 3). When the outgoing relay set (o.g.r.s.) is seized, relay AA operates and contact AA1 changes over current flow from winding 2 to winding 1 in the relay A and relay D. Contact D1 remains at *make* but contact A1 changes over to *make* to operate relay AA (on the i.c.r.s.). Contact AA1 extends a loop to seize the first selector and relay GD operates to the earth returned on the private-wire (P-wire). Contact GD1 removes the earth connexion from relay A and current now flows round the loop formed by windings 1 and 2 of relays A and D. Contact A1 remains in the *make* position but, since windings 1 and 2 of relay D are in opposition, contact D1 comes under control of bias winding 3 and moves to the *break* position. Contact D1 operates relay DB.
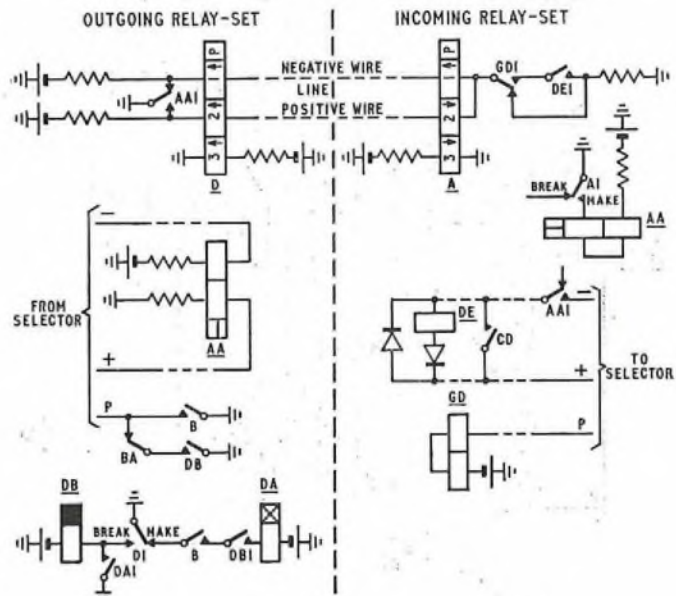
When pulsing takes place, contact AA1 (in the o.g.r.s.) moves to its normal position during each break pulse and this reverses the direction of current in the loop and contact A1, therefore, moves to the *break* position. Thus, contact A1 repeats the pulses to relay AA (in the i.c.r.s.) and contact AA1 (in the i.c.r.s.) repeats the pulses to the selector.

When the called subscriber answers, relay DE operates and contact DE1 reconnects earth to windings 1 and 2 of relay A. Current now flows in the negative wire only. Contact A1 remains in the *make* position but contact D1 now moves to the *make* position. Contact D1 now operates relay DA and contact DA1 holds relay DB. Contacts of relay DA repeat the appropriate supervisory signals.

A backward-busy facility is provided since, if continuity of the line circuit is broken while the circuit is idle, contact D1 moves to *break* and operates relay DB, which engages the circuit.
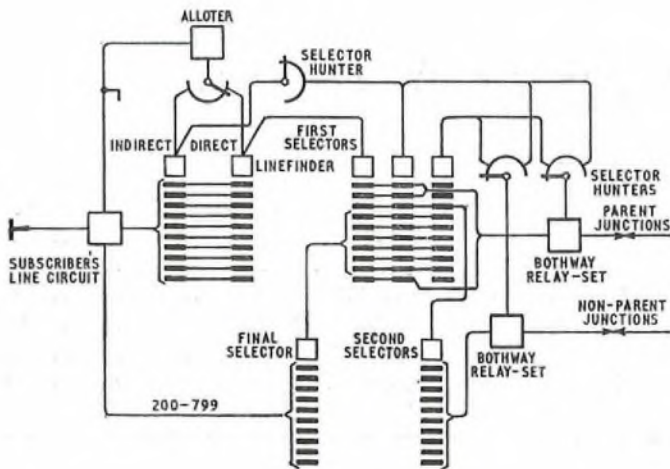
Q. **5.** (*a*) *Sketch the trunking of a unit automatic telephone exchange which has a capacity of 600 connexions.*

(*b*) *With the aid of sketches, explain how the trunk offering facility is given and describe its operation.*

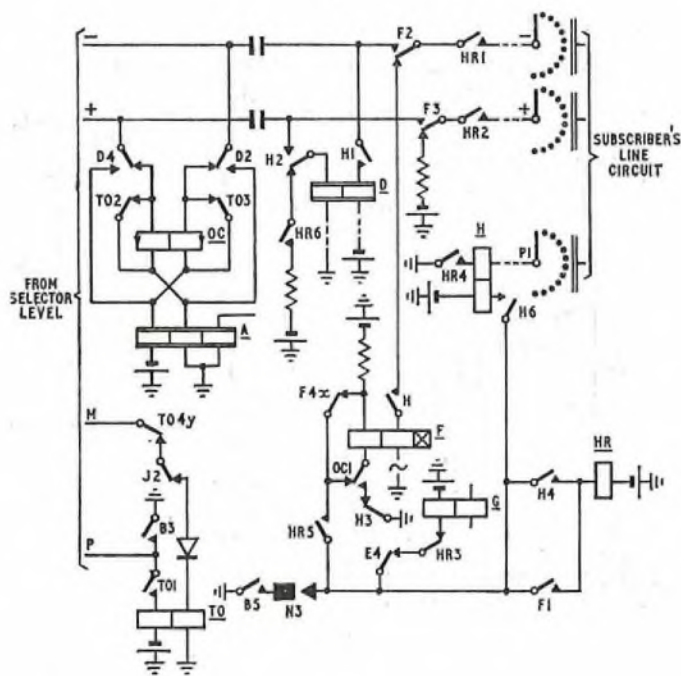(*c*) *What are the advantages of this method of trunk offering?*

A. **5.** (*a*) Sketch (*a*) shows the trunking of a unit automatic telephone exchange with 600 connexions.

(*b*) Sketch (*b*) shows the portions of the circuit concerned with the offering of trunk calls to an engaged subscriber. Relay TO operates to the positive battery discriminating signal from the incoming parent

Note: Basic trunking of a U.A.X. with 600 connexions.
Subscribers' numbering range 200–799.
Junction codes (parent) 1, 9, 0.
Junction codes (non-parent) 80–89.

(*a*)

(*b*)

Q. 6. (*a*) *Draw a trunking diagram of a local call originating from a director exchange and routed via a tandem exchange, as far as the final selector. Indicate where transmission bridges may be encountered.*

(*b*) *Assuming that the called subscriber is busy, explain*

(*i*) *how the busy signal is returned to the calling subscriber,*

(*ii*) *why metering does not commence, and*

(*iii*) *why, if the called subscriber clears while the calling subscriber is still listening to the busy signal, the call is not established.*

A. 6. (*a*) The sketch shows a typical trunking diagram for a local call from one director exchange to another routed via a tandem.

When the subscriber lifts his handset, he is connected to a first code-selector via his uniselector.
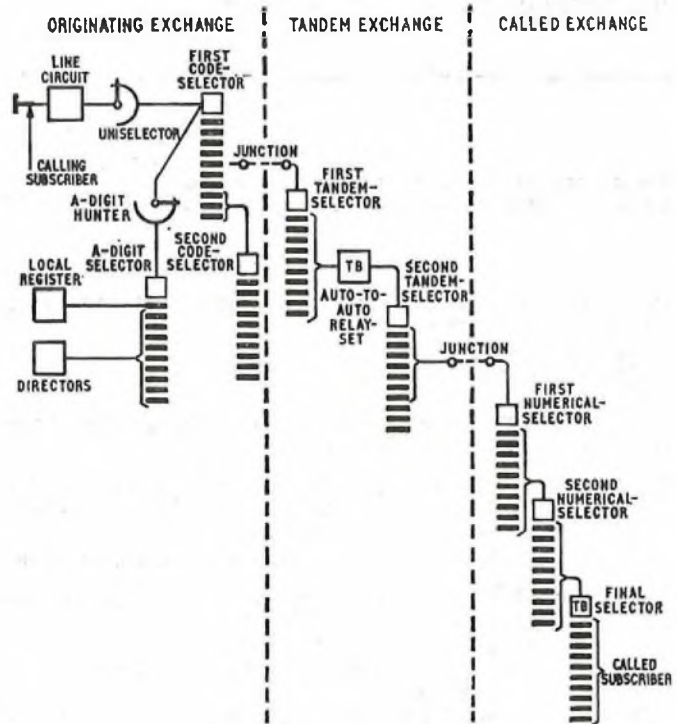
Note: TB indicates Transmission Bridge.

The A-digit hunter associated with the first code-selector hunts for, and seizes, an A-digit selector, which returns dial tone. The first dialled digit steps the A-digit selector to the appropriate level and a director is seized. The next two (and following four) digits are stored in the director which sends out the translation to step the first code-selector and tandem selectors followed by the four numerical digits to step the first and second numerical selectors and final selector, after which the A-digit selector and director clear down.

(*b*) (*i*) When the final selector has been positioned by the final two digits, the testing relay is applied to the called-subscriber's private wire (P-wire). If the line is busy (indicated by an earth on the P-wire), busy tone is applied to the 570-ohm coil of the final selector A-relay and, thence, to line.

The busy tone then uses the speech path from the final selector back to the calling-subscriber's line.

(*ii*) For metering to take place on a local call, it is necessary for the final selector to signal called-subscriber-answer by reversing the polarity applied to the positive and negative wires. This reversal is detected by the first code-selector and is used to commence metering. When the final selector detects that the called line is busy, no reversal of polarity takes place, hence, metering cannot occur.

(*iii*) The check as to whether the called subscriber is busy or free is done immediately after the last digit positions the final selector and, if the state of the line is busy then, busy tone is returned to the calling subscriber. Any subsequent changes of state of the called-subscriber's line are not allowed to further effect the final selector circuitry.

Q. 7. (*a*) *Explain the principle of operation of a crossbar selector, using sketches to illustrate your answer.*

(*b*) *Draw a trunking diagram of a 10,000-line crossbar exchange and explain how a connexion is set up between two telephones on the exchange.*

junction termination, and switches relay OC into circuit. If the required line is engaged, the operator can intercept by operating the RING key. This applies an earth to the junction pair which, by upsetting the balance of current in the two windings of relay OC, allows this relay to operate. Contact OC1 operates relay F and contact F1 operates relay HR. The contacts of relay F and relay HR now extend the operator's circuit to the called-subscriber's circuit irrespective of the condition on the private wire. The operation of relay D at contact HR6 reverses the polarity of the incoming junction to dim the operator's supervisory lamp and releases relay OC. If the called subscriber agrees to accept the trunk call, the local connexion is released and the called subscriber replaces his handset. When the called-subscriber's line circuit is normal, relay H operates to the 1,300-ohm battery on the P1 bank. Contact H2 releases relay D which restores the incoming junction to normal polarity and lights the operator's supervisory lamp. The operator now operates the RING key a second time, and the second operation of relay OC releases relay F. Ringing current is now applied to the called-subscriber's line, and when the handset is lifted, the call proceeds in the normal manner, with the selector functioning as an ordinary final selector.

(*c*) The advantage of this method of trunk offering is that the normal junction can be used for both subscriber- and operator-generated traffic, no special junctions or translation facilities being required. The operator only has to establish one call on which normal subscriber access and trunk-offering access are available, if required.

Q. 8. (*a*) *Describe how a magnetic core with a rectangular hysteresis loop may be used as a storage element.*

(*b*) (*iii*) *Explain how information may be written into the above store and how the store is read.*
(*ii*) *What are the merits of this type of store?*
(*iii*) *With the aid of sketches, describe briefly one other type of electronic store.*

A. 8. (*a*) Sketch (*a*) shows the ideal characteristics of a magnetic core, with two stable states given by points 0 and 1, in which there is a


(*a*)
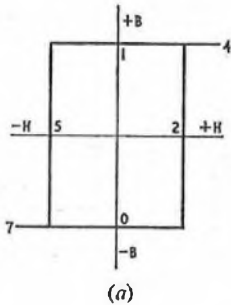
large remnant magnetic field in one direction or the other. Characteristics approximating to this ideal can be obtained with a ferrite core and these are shown in sketch (*b*).


(*b*)
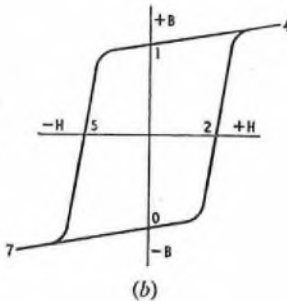
The two stable states of the remnant magnetic field may be used to provide a memory or storage element of the last stimulation applied to the core.

Current is passed through a winding on the core in a particular direction, producing a magnetizing force H which causes a flux to pass through the core as shown in sketch (*c*). If H is made sufficiently

(*c*)
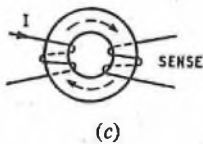
large, saturation occurs and the point 4 of sketch (*b*) is reached. On removal of the current, H falls to zero but the flux density B only falls slightly to the value indicated by the point 1. Although the stimulus has now ceased, the core remembers indefinitely that one has occurred and the direction of the field indicates the sense of the original input. If another current pulse is passed through the winding in the same direction, then B moves from the point 1 to the point 4 and back. However, should a current pulse be passed through the winding in the opposite direction, as in sketch (*d*), the operating point of the flux

(*d*)
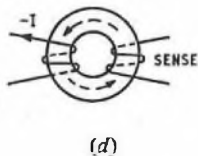
density moves from the point 1 through the point 5 to the point 7, and then back to point 0 when the current pulse ceases. The direction of magnetization has now reversed and a large change in flux has occurred. The core now remembers the sense of the last input (it no longer remembers the sense of any pulses before that, i.e. all previous

information has been destroyed). If another winding is wound on the core, it can be used to detect when a change of flux takes place since any flux change produces an e.m.f. such that

$e = \dfrac{Nd\Phi}{dt}$ where N = number of turns of winding and Φ webers is the instantaneous flux.

The polarity of the e.m.f. depends on the direction of switching, e.g a positive e.m.f. for a transition from the point 0 to the point 4 and a negative e.m.f. for a transition from the point 1 to the point 7.

(*b*) (*i*) If the point 1 is taken as the storage of a logic 1 and the point 0 as the storage of a logic 0, then to ascertain whether a logic 1 or 0 is stored it is merely necessary to write a 0 into the core. If a 0 is already stored, no output is produced but, if a 1 is stored, then an output will be produced. It is, of course, necessary to rewrite the 1 if non-destructive readout is required.

(*ii*) The merits of core storage are

(*a*) very low power consumption,
(*b*) long life expectancy,
(*c*) relatively high immunity from interference as cores are normally operated from low-impedance circuits,
(*d*) the speed of operation is fast e.g. 1 μs, and
(*e*) the information is not lost if the power fails.

(*e*)
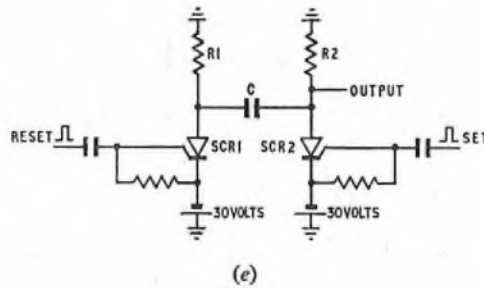
(*iii*) Sketch (*e*) shows a simple toggle using silicon-controlled rectifiers. Assuming SCR1 is conducting and SCR2 is off, then the output at SCR2 anode will be at earth potential (logic 0). If it is desired to store a logical 1, then a positive-going pulse is applied to SCR2 trigger and SCR2 turns on. The potential at the SCR2 anode now falls to $-$ 30 volts (logic 1) and the voltage at the SCR1 anode, already at $-$ 30 volts, falls to $-$ 60 volts due to the effect of capacitor C. Thus SCR1 now becomes reversed biassed and turns off, the potential at its anode rising exponentially to earth.

To reset the toggle, a positive-going pulse is applied to the trigger of SCR1 to turn SCR1 on, which then turns off SCR2 in a manner similar to the way in which SCR2 turned off SCR1.

Q. 9. (*a*) *Give an outline description of a cordless telephone switchboard suitable for use at a large auto-manual centre.*

(*b*) *Describe the method by which incoming calls are allocated queue positions in arrival order. Use sketches to illustrate your answer.*

Q. 10. (*a*) *Describe the principal features of an automatic routine tester for telephone-exchange equipment. Include in your answer a list of the general facilities given.*

(*b*) *With the aid of sketches, describe how connexion is made between the automatic routine tester and the equipment under test.*

(*c*) *What factors are taken into account when determining whether an automatic routine-tester should be provided?*

A. 10. (*a*) Sketch (*a*) shows the principal elements of a routiner. Access to the equipment under test is via an access distributor and two-motion access selector permitting one routiner to have access to up to 4,800 selectors for routining purposes. Each test is applied to the equipment (in this example a selector) in sequence by the test distributor. Each successful test steps the test distributor and a lamp display is given on the control panel to indicate testing progress. Failure to step to another selector in 3—6 minutes is regarded as a fault condition and gives visible and audible alarms (as for a failure during part of the test cycle) on the control panel when the routiner is under manual control.

When the routiner is operating automatically, under clock-control, a fault condition results in the association of the fault recorder with the consequent printing of a fault docket. After printing, the access is stepped to the next selector.

The general facilities provided are:

(*i*) General routine. Under these conditions the routiner automatically proceeds to test, in sequence, all the equipments to which it

75

has access and gives a ROUTINE-FINISHED alarm when all the equipments have been tested.

(*ii*) Continuous routine. Under this condition, the routiner continues to repeat the sequence of tests on any chosen item of equipment.

(*iii*) Group routine. This facility is found on director and junction routiners to enable a particular group to be singled out and tested.

(*iv*) Alarm indications.

(*v*) Lamp indications on the control panel to indicate the particular test being carried out at any instant and also the position of the access-distributor and selector.

(*b*) Connexion to the apparatus to be tested is normally established via access-selectors (or sometimes relays) mounted on, or adjacent to, the main equipment racks. In this way, access is obtained to sufficient

points in each item of equipment to enable control to be effected by the routiner during testing.

Connexion to the access-selectors is obtained via a primary-distributor which is a uniselector having 48 working outlets. The access selectors are either uniselectors or two-motion selectors (see sketches (*b*) and (*c*)).

(*b*)

(*c*)
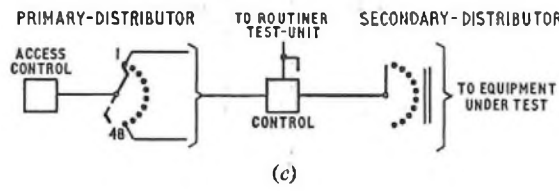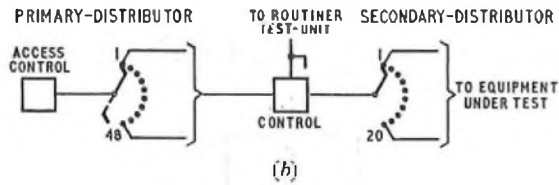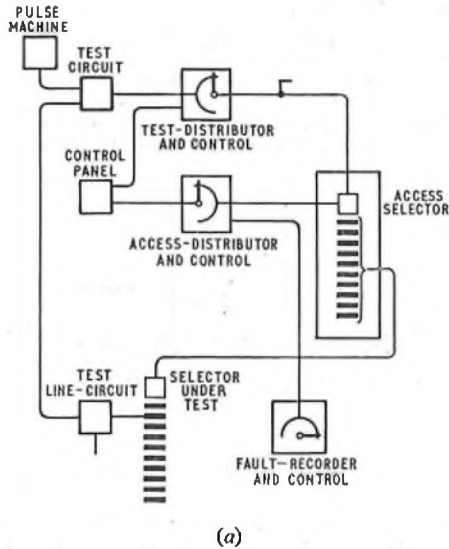
On group-selector and final-selector routiners, for example, it is necessary to have access to both the incoming and outgoing sides of the selector in order to fully test the selector. Access to the outgoing side is obtained via the selector bank and wipers by arranging for one or more outlets of the multiple to be switched from their normal next stage to the routiner when required.

(*c*) Routiners are installed where the complexity or quantity of the equipment to be tested would render testing by manual methods uneconomical. The labour costs of testing using automatic routiners are much less than those when manual testers are used. The critical number of items of equipment of a particular type to justify an automatic routiner is that at which the total annual charges for testing by manual testers equal the charges for testing by routiner, including the cost of the routiner and access equipment.

(*a*)

MATHEMATICS C, 1972

Students were expected to answer any six questions

Q. 1. (*a*) *Solve the simultaneous equations*

$$u + v + 1 = 0, \quad \frac{1}{u} + \frac{1}{v} = \frac{1}{12}$$

given that $u > v$.

(*b*) *A certain circuit involving L henrys inductance, C farads capacitance and R ohms resistance will oscillate if the quadratic $L\lambda^2 + R\lambda + \frac{1}{C} = 0$ has complex roots.*

(*i*) *Express this condition as a formula involving L, R and C.*

(*ii*) *If R^2 is very small compared with $\frac{4L}{C}$ show that $\lambda = \frac{-R}{2L} \pm j\omega$ is a close approximation, where $\omega^2 = \frac{1}{LC}$.*

A. 1. (*a*) $\qquad u + v + 1 = 0.$ (1)

$$\frac{1}{u} + \frac{1}{v} = \frac{1}{12}.$$ (2)

From equation (1),

$$v = -1 - u.$$ (3)

Substituting for v in equation (2) gives

$$\frac{1}{u} + \frac{1}{-1-u} = \frac{1}{12},$$

or, $\quad \dfrac{1}{u} - \dfrac{1}{1+u} = \dfrac{1}{12}.$

$$\therefore \ 1 + u - u = \frac{1}{12}u(1+u),$$

or, $\quad 12 = u + u^2.$

$$\therefore \ u^2 + u - 12 = 0.$$

$$\therefore \ u = \frac{-1 \pm \sqrt{(1+48)}}{2},$$

$$= \frac{-1 \pm 7}{2},$$

$$= 3 \text{ or } -4.$$

Hence, from equation (3) $v = -4$ or 3.

But, $u > v$ and, hence, the solution
$u = -4, v = 3$ is inadmissible.

$$\therefore \ \underline{u = 3, v = -4.}$$

(*b*) $\qquad L\lambda^2 + R\lambda + \dfrac{1}{C} = 0.$

From the general solution to a quadratic equation,

$$\lambda = \frac{-R \pm \sqrt{\left(R^2 - 4\frac{L}{C}\right)}}{2L}.$$

For the roots to be complex, the discriminant must be negative.

$$\therefore \ 4\frac{L}{C} > R^2,$$

or, $\quad \underline{4L > CR^2.}$

If $4\dfrac{L}{C} \gg R^2$,

$$\lambda \simeq \frac{-R \pm \sqrt{\left(-\frac{4L}{C}\right)}}{2L},$$

$$= \frac{-R \pm 2j\sqrt{\left(\frac{L}{C}\right)}}{2L},$$

$$= -\frac{R}{2L} \pm j\sqrt{\left(\frac{1}{LC}\right)}.$$

But, $\omega^2 = \dfrac{1}{LC}$,

or, $\omega = \sqrt{\left(\dfrac{1}{LC}\right)}$.

$\therefore \lambda \simeq -\dfrac{R}{2L} \pm j\omega.$ Q.E.D.

Q. **2.** (*a*) *Write down and simplify the first four terms of the binomial expansion of* $(1 + x)^{-1/3}$ *in powers of* x, *and give an expression for the term in* x^r.

(*b*) *Express the cube root of* $7 \cdot 2$ *as twice the cube of* $\left(1 - \dfrac{1}{10}\right)$, *and so calculate to four significant figures, without using tables, the value of* $\dfrac{1}{3\sqrt{(7 \cdot 2)}}$.

A. **2.** (*a*) $(1 + x)^{-1/3} = 1 - \dfrac{1}{3}x + \dfrac{-\dfrac{1}{3}\left(-\dfrac{1}{3} - 1\right)}{2!}x^2$

$\qquad + \dfrac{-\dfrac{1}{3}\left(-\dfrac{1}{3} - 1\right)\left(-\dfrac{1}{3} - 2\right)}{3!}x^3$

$\qquad + \ldots,$

$= 1 - \dfrac{x}{3} + \dfrac{\dfrac{1 \times 4}{3 \times 3}}{2}x^2 - \dfrac{\dfrac{1 \times 4 \times 7}{3 \times 3 \times 3}}{3 \times 2}x^3 + \ldots,$

$= 1 - \dfrac{x}{3} + \dfrac{2x^2}{9} - \dfrac{14x^3}{81} + \ldots.$

The term in x^r can be stated as

$(-1)^r \dfrac{\dfrac{1 \times 4 \times 7 \times \ldots \times \{1 + 3(r - 1)\}}{3^r}}{r!} x^r,$

$= (-1)^r \dfrac{1 \times 4 \times 7 \times \ldots \times (3r - 2)}{3^r \times r!} x^r.$

(*b*) $\sqrt[3]{(7 \cdot 2)} = \sqrt[3]{(8 \times 0 \cdot 9)},$

$\qquad = 8^{1/3} \times (1 - 0 \cdot 1)^{1/3},$

$\qquad = 2 \times \left(1 - \dfrac{1}{10}\right)^{1/3}.$ Q.E.D.

$\dfrac{1}{3\sqrt{(7 \cdot 2)}} = \dfrac{1}{2 \times (1 - 0 \cdot 1)^{1/3}},$

$\qquad = \dfrac{(1 - 0 \cdot 1)^{-1/3}}{2}.$

But, $(1 - 0 \cdot 1)^{-1/3} = 1 + \left(-\dfrac{1}{3}\right)(-0 \cdot 1)$

$\qquad + \dfrac{-\dfrac{1}{3}\left(-\dfrac{1}{3} - 1\right)}{2!} \times (-0 \cdot 1)^2$

$\qquad + \dfrac{-\dfrac{1}{3}\left(-\dfrac{1}{3} - 1\right)\left(-\dfrac{1}{3} - 2\right)}{3!} \times (-0 \cdot 1)^3$

$\qquad + \dfrac{-\dfrac{1}{3}\left(-\dfrac{1}{3} - 1\right)\left(-\dfrac{1}{3} - 2\right)\left(-\dfrac{1}{3} - 3\right)}{4!} \times (-0 \cdot 1)^4$

$\qquad + \ldots,$

$= 1 + \dfrac{0 \cdot 1}{3} + \dfrac{\dfrac{1}{3} \times \dfrac{4}{3}}{2} \times 0 \cdot 01 + \dfrac{\dfrac{1}{3} \times \dfrac{4}{3} \times \dfrac{7}{3}}{3 \times 2} \times 0 \cdot 001$

$\qquad + \dfrac{\dfrac{1}{3} \times \dfrac{4}{3} \times \dfrac{7}{3} \times \dfrac{10}{3}}{4 \times 3 \times 2} \times 0 \cdot 0001 + \ldots,$

$= 1 + 0 \cdot 033333\dot{3} + \dfrac{0 \cdot 02}{9} + \dfrac{0 \cdot 014}{9 \times 9} + \dfrac{0 \cdot 0035}{9 \times 9 \times 3}$

$\qquad + \ldots,$

$= 1 \cdot 033333 + 0 \cdot 002222\dot{2} + \dfrac{0 \cdot 001555\dot{5}}{9}$

$\qquad + \dfrac{0 \cdot 0001\dot{2}9\dot{6}}{9} + \ldots,$

$= 1 \cdot 035555 + 0 \cdot 0001728 + 0 \cdot 0000144 + \ldots,$

$= 1 \cdot 035743$ to six decimal places.

$\therefore \dfrac{1}{3\sqrt{(7 \cdot 2)}} = \dfrac{1}{2} \times 1 \cdot 035743,$

$\qquad = 0 \cdot 517872.$

Thus, $\dfrac{1}{3\sqrt{(7 \cdot 2)}} = \underline{0 \cdot 5179}$ correct to four significant figures.

Q. **3.** *The power at a point 5 km from the sending end of a long transmission line is 85 per cent of the transmitted power.*

(*a*) *Assuming that the formula* $P = P_0 e^{-kx}$ *gives the power* P *milliwatts received* x *kilometres from the sending end, calculate* k.

(*b*) *Expand* P *in terms of an exponential series in* x *and use this series to calculate to two significant figures the power loss in decibels per kilometre.*

A. **3.** (*a*) $P = P_0 e^{-kx}.$

When $x = 5$ km,

$\qquad P = 85$ per cent of the transmitted power which is clearly P_0.

$\therefore 0 \cdot 85\, P_0 = P_0 e^{-5k},$

or, $e^{-5k} = 0 \cdot 85.$

$\therefore -5k = \log_e 0 \cdot 85.$

$\qquad = \bar{3} \cdot 6974 + 2 \cdot 1401$ from a table of Natural logarithms,

$\qquad = \bar{1} \cdot 8375,$

$\qquad = -0 \cdot 1625.$

$\therefore k = \dfrac{0 \cdot 1625}{5},$

$\qquad = \underline{0 \cdot 0325}.$

(*b*) $P = P_0 e^{-0 \cdot 0325x},$

$\qquad = P_0\left\{1 - 0 \cdot 0325x + \dfrac{(0 \cdot 0325x)^2}{2!} - \dfrac{(0 \cdot 0325x)^3}{3!} + \ldots\right\}.$

If $\qquad x = 1$ kilometre,

$\dfrac{P}{P_0} \simeq 1 - 0 \cdot 0325 + \dfrac{0 \cdot 001056}{2} - \dfrac{0 \cdot 000034}{6} + \ldots.$

$\therefore \dfrac{P}{P_0} \simeq 0 \cdot 9675 + 0 \cdot 000528 - 0 \cdot 0000057,$

$\qquad = 0 \cdot 968033$ to six decimal places.

Thus, the ratio of power received at the end of a 1 km length of line to that transmitted is $0 \cdot 968033$.

$\therefore$ Power loss in decibels per kilometre

$\qquad = 10 \log_{10} \dfrac{P}{P_o},$

$\qquad = 10 \log_{10} 0 \cdot 968033,$

$\qquad = 10 \times \bar{1} \cdot 9859,$

$\qquad = -0 \cdot 141.$

$\therefore$ Power loss $= \underline{0 \cdot 14\ \text{dB/km}}$ to two significant figures.

Note—The negative sign above indicates a power loss. The numerical answer will be the same no matter whether the ratio $\dfrac{P}{P_o}$ or its reciprocal is considered, because $10 \log_{10} \dfrac{P}{P_o} = -10 \log_{10} \dfrac{P_0}{P}.$

Q. 4. (*a*) *Show that* $\dfrac{\cot A + \tan B}{\cot B + \tan A} = \cot A \tan B$ *and derive a similar result for* $\dfrac{\tan A + \tan B}{\cot A + \cot B}$.

(*b*) *By devising a quadratic equation in* $\sin\theta$, *or otherwise, solve the equation* $\sin\theta = 3\cos^2\theta$, *giving solutions between* $-180°$ *and* $+180°$.

A. 4. (*a*) $\dfrac{\cot A + \tan B}{\cot B + \tan A} = \cot A \tan B$.

The left-hand side of the identity

$$= \frac{\cot A + \tan B}{\cot B + \tan A},$$

$$= \frac{\dfrac{\cos A}{\sin A} + \dfrac{\sin B}{\cos B}}{\dfrac{\cos B}{\sin B} + \dfrac{\sin A}{\cos A}},$$

$$= \frac{\dfrac{\cos A \cos B + \sin A \sin B}{\sin A \cos B}}{\dfrac{\cos A \cos B + \sin A \sin B}{\sin B \cos A}},$$

$$= \frac{\cos A \sin B}{\sin A \cos B},$$

$$= \cot A \tan B,$$

$$= \text{right-hand side of identity.} \qquad \text{Q.E.D.}$$

Similarly, $\dfrac{\tan A + \tan B}{\cot A + \cot B}$

$$= \frac{\dfrac{\sin A}{\cos A} + \dfrac{\sin B}{\cos B}}{\dfrac{\cos A}{\sin A} + \dfrac{\cos B}{\sin B}},$$

$$= \frac{\dfrac{\sin A \cos B + \cos A \sin B}{\cos A \cos B}}{\dfrac{\cos A \sin B + \sin A \cos B}{\sin A \sin B}},$$

$$= \frac{\sin A \sin B}{\cos A \cos B},$$

$$= \tan A \tan B.$$

Thus, $\dfrac{\tan A + \tan B}{\cot A + \cot B} = \tan A \tan B$.

(*b*) $\sin\theta = 3\cos^2\theta$.

But, $\sin^2\theta + \cos^2\theta = 1$,

or, $\cos^2\theta = 1 - \sin^2\theta$.

Hence, substituting for $\cos^2\theta$ in the equation gives

$$\sin\theta = 3(1 - \sin^2\theta),$$

or, $3\sin^2\theta + \sin\theta - 3 = 0$.

$$\therefore \sin\theta = \frac{-1 \pm \sqrt{(1 + 36)}}{6},$$

$$= \frac{-1 \pm 6\cdot083}{6},$$

$$= \frac{5\cdot083}{6}, \text{ or, } -\frac{7\cdot083}{6},$$

$$= 0\cdot8472, \text{ the negative solution being inadmissible.}$$

$$\therefore \theta = 57° 54' \text{ or } (180° - 57° 54').$$

Thus, $\theta = 57° 54'$ or $122° 6'$ in the range between $-180°$ and $+180°$.

Q. 5. (*a*) *Find the maximum and minimum value of* $y = x^3 - 3x^2 - 24x - 4$ *and hence, or otherwise, show that the cubic equation* $x^3 - 3x^2 = 24x + 4$ *has three real roots, two of them negative.*

(*b*) *Obtain by a suitable graphical enlargement the positive root of the above cubic equation to two significant figures.*

A. 5. (*a*) $\qquad y = x^3 - 3x^2 - 24x - 4$,

$$\frac{dy}{dx} = 3x^2 - 6x - 24,$$

and, $\dfrac{d^2y}{dx^2} = 6x - 6$.

For maximum or minimum values, $\dfrac{dy}{dx} = 0$.

$$\therefore 3x^2 - 6x - 24 = 0,$$

or, $x^2 - 2x - 8 = 0$,

or, $(x - 4)(x + 2) = 0$.

$$\therefore x = 4, \text{ or, } -2.$$

When $\qquad x = 4$,

$$\frac{d^2y}{dx^2} = 24 - 6 = 18 \text{ which is positive and, hence, a minimum occurs at } x = 4.$$

When $\qquad x = -2$,

$$\frac{d^2y}{dx^2} = -12 - 6 = -18 \text{ which is negative and, hence, a maximum occurs at } x = -2.$$

Hence, $\quad y_{max} = (-2)^3 - 3(-2)^2 - 24(-2) - 4$,

$$= -8 - 12 + 48 - 4,$$

$$= 24.$$

Also, $\quad y_{min} = (4)^3 - 3(4)^2 - 24(4) - 4$,

$$= 64 - 48 - 96 - 4,$$

$$= -84.$$

Thus, the maximum and minimum values are 24 and -84 respectively.

$$x^3 - 3x^2 = 24x + 4,$$

or, $\qquad x^3 - 3x^2 - 24x - 4 = 0$.

This is the condition applying when $y = 0$ in the equation $y = x^3 - 3x^2 - 24x - 4$ and, thus, the roots of the cubic are those of $y = 0$.

Since y has a maximum value of 24 at $x = -2$ and a minimum value of -84 at $x = 4$, the graph of y must cross the x-axis at a value of x intermediate between $x = -2$ and $x = 4$. When $x = 0$, $y = -4$ and, hence, the graph must cross the x-axis at a negative value of x. Thus, one root of the cubic equation is negative.

As x decreases below -2, the only term of y which remains positive is $-24x$ and, hence, the value of y rapidly becomes negative. At $x = -3$, $y = 14$, whilst at $x = -4$, $y = -20$. Hence, a second negative root occurs between $x = -3$ and $x = -4$.

$y = x^3 - 3x^2 - 24x - 4$
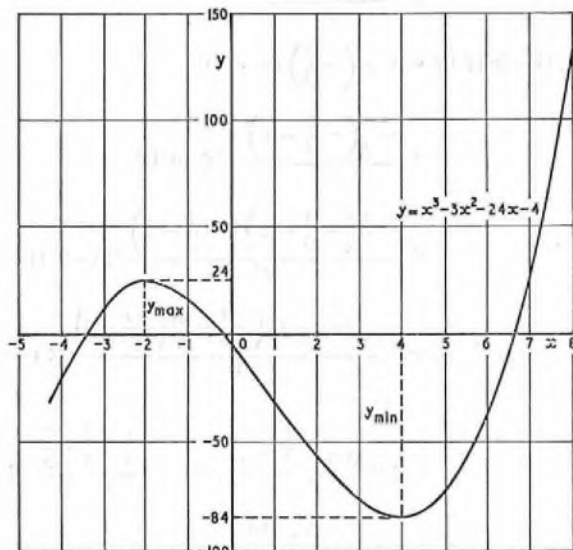
(*a*)

As x increases beyond 4, only the x^3 term of y remains positive, but this term soon becomes greater than the sum of the other three negative terms. Thus, at $x = 6$, $y = -40$ whereas, when $x = 8$, $y = 124$, and the positive value of y increases rapidly beyond this point. Hence, a real-positive root occurs between $x = 6$ and $x = 8$.

Thus, the cubic has three real roots, two of them negative. Q.E.D.

Note:—Although a sketch graph is not strictly necessary to answer part (a), the student would have been well-advised to draw one in this instance, in view of the second part (b) of the question.

A rough sketch of the graph is shown in sketch (a) drawn from the information already derived.

(b) From sketch (a) it is seen that the positive root lies between $x = 6$ and $x = 7$ and is probably closer to the latter. It should, therefore, suffice to draw an accurate graph between, say, $x = 6 \cdot 5$ and $x = 7$ although in view of the fact that only 2-significant figure accuracy is called for, a wider range of $x = 6$ to $x = 7$ with somewhat less cumbersome arithmetic would probably be adequate.

The accurate graph is drawn from the following table of values.

x	$6 \cdot 5$	$6 \cdot 8$	7
x^3	$274 \cdot 625$	$314 \cdot 432$	343
$-3x^2$	$-126 \cdot 75$	$-138 \cdot 72$	-147
$-24x$	-156	$-163 \cdot 2$	-168
-4	-4	-4	-4
y	$-12 \cdot 125$	$8 \cdot 512$	24

The graph is shown in sketch (b), the plotted points having been joined by straight lines. Over such a small range of x values this is not only more practicable but sufficiently accurate, provided enough care has been taken to select the appropriate range.
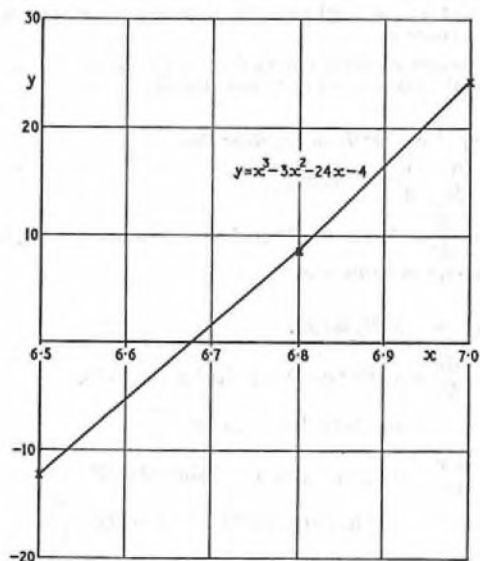
(b)

From the graph, the positive root of $x^3 - 3x^2 = 24x + 4$ is $6 \cdot 7$ correct to two significant figures.

Q. **6.** (a) *If u, v are any two functions of x prove that* $\frac{d}{dx}(uv) = v\frac{du}{dx} + u\frac{dv}{dx}$ *and state without proof a similar formula for differentiating a quotient of two functions.*

(b) *Differentiate the functions*

(i) $y = x \sin x$,

(ii) $y = \frac{x - 1}{(x + 2)^2}$,

(iii) $y = \log_e (\text{cosec } x)$,

simplifying each result where possible.

(c) *Sketch the graph of any one of the three functions in (b).*

A. **6.** (a) Let $y = uv$.

Suppose x increases by a small amount δx, and let δu, δv and δy be the corresponding increases in u, v and y respectively.

Then, $y + \delta y = (u + \delta u)(v + \delta v)$,

and, $\delta y = (u + \delta u)(v + \delta v) - uv$, since $y = uv$,

$= uv + v\delta u + u\delta v + \delta u\delta v - uv$,

$= v\delta u + u\delta v + \delta u\delta v$.

$\therefore \frac{\delta y}{\delta x} = v\frac{\delta u}{\delta x} + u\frac{\delta v}{\delta x} + \delta u\frac{\delta v}{\delta x}$.

In the limit, as $\delta x \to 0$,

$\frac{\delta y}{\delta x} \to \frac{dy}{dx}$, $\frac{\delta u}{\delta x} \to \frac{du}{dx}$, $\frac{\delta v}{\delta x} \to \frac{dv}{dx}$ and $\delta u \to 0$.

$\therefore \frac{dy}{dx} = \lim_{\delta x \to 0} \frac{\delta y}{\delta x}$,

or, $\frac{d}{dx}(uv) = v\frac{du}{dx} + u\frac{dv}{dx}$. Q.E.D.

If $y = \frac{u}{v}$ where, u and v are functions of x,

then, $\frac{dy}{dx} = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$.

[(b) (i) $y = x \sin x$.

$\frac{dy}{dx} = x\frac{d(\sin x)}{dx} + \sin x\frac{d(x)}{dx}$,

$= x \cos x + \sin x$.

(ii) $y = \frac{x - 1}{(x + 2)^2}$.

$\frac{dy}{dx} = \frac{(x + 2)^2\frac{d(x - 1)}{dx} - (x - 1)\frac{d(x + 2)^2}{dx}}{(x + 2)^4}$,

$= \frac{(x + 2)^2 - (x - 1)2(x + 2)}{(x + 2)^4}$,

$= \frac{x + 2 - 2(x - 1)}{(x + 2)^3}$,

$= \frac{-x + 4}{(x + 2)^3}$.

(iii) $y = \log_e (\text{cosec } x)$.

$\frac{dy}{dx} = \frac{1}{\text{cosec } x}\frac{d(\text{cosec } x)}{dx}$.

But, $\text{cosec } x = \frac{1}{\sin x}$.

$\therefore \frac{d(\text{cosec } x)}{dx} = \frac{d\left(\frac{1}{\sin x}\right)}{dx}$,

$= \frac{\sin x\frac{d(1)}{dx} - 1\frac{d(\sin x)}{dx}}{\sin^2 x}$,

$= \frac{\sin x \times 0 - \cos x}{\sin^2 x}$.

$= -\frac{1}{\sin x}\frac{\cos x}{\sin x}$,

$= -\text{cosec } x \cot x$.

Hence, $\frac{dy}{dx} = \frac{1}{\text{cosec } x}(-\text{cosec } x \times \cot x)$,

$= -\cot x$.

(c) (i) The graph of $y = x \sin x$ is shown in sketch (a), together with the normal sine function, shown dotted. It is seen that $y = x \sin x$ has the same periodicity at $y = \sin x$ but the amplitudes increase as x increases. The maxima and minima occur after those of the sine curve but the delay periods become less and less as x increases.

When x is negative,
$$y = (-x) \sin (-x),$$
$$= (-x)(- \sin (x)),$$
$$= + x \sin x,$$
and, hence, the graph of y will be a mirror image in the y axis of the graph of sketch (*a*).

(*a*)
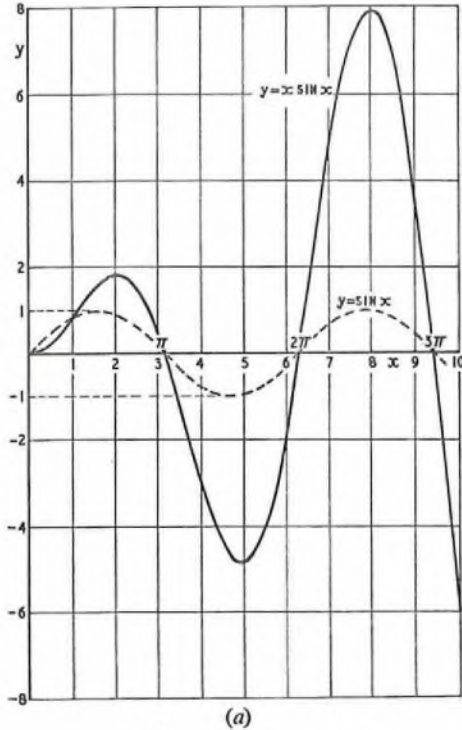
(*ii*)
$$y = \frac{x-1}{(x+2)^2}.$$

When $x = 1$, $y = 0$. As x increases beyond unity, the denominator of y increases more rapidly than the numerator and, hence, the magnitude of y decreases.

For large values of x, $y \simeq \dfrac{x}{x^2} = \dfrac{1}{x}$ and, hence, as $x \to \infty$, $y \to 0$.

Hence, the curve becomes asymptotic to the x-axis. At $x = 4$, a maximum occurs.

When $x = -2$, $y = -\infty$. When $x < 1$, the numerator of y is negative, whilst $(x + 2)^2$ remains positive.

Hence, as x decreases, the curve first descends rapidly to $-\infty$ as shown in sketch (*b*) and then ascends rapidly towards the x-axis.

When x attains large negative values,
$$y \simeq \frac{-x}{x^2} = -\frac{1}{x},$$
and, hence, the curve again becomes asymptotic to the x-axis.

In sketch (*b*) the positive values of y are shown to a much larger scale for clarity.

(*b*)
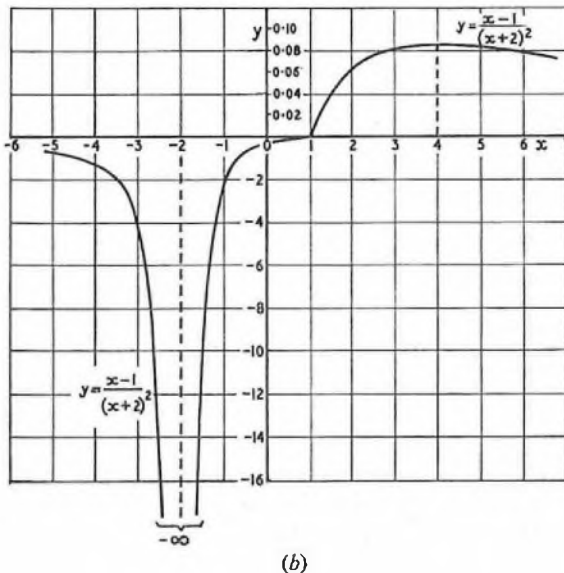
(*iii*)
$$y = \log_e (\operatorname{cosec} x).$$

Since $\operatorname{cosec} x = \dfrac{1}{\sin x}$, the graph of $y = \operatorname{cosec} x$ is the plot of reciprocal values of $\sin x$. The graphs of both functions are shown dotted in sketch (*c*) and it is seen that the graph of $y = \operatorname{cosec} x$

(*c*)
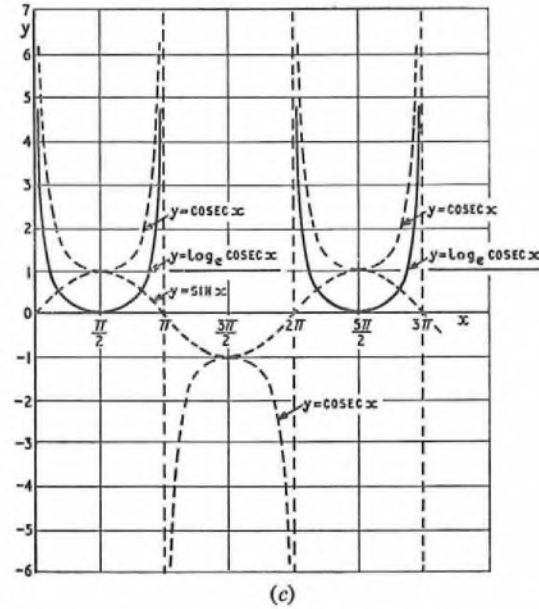
comprises alternate positive and negative half-loops. The values of $y = \operatorname{cosec} x$ approach $\pm \infty$ at 0, $\pm \pi$, $\pm 2\pi$ . . . radians.

The graph of $y = \log_e \operatorname{cosec} x$, shown as a full-line curve is of the same shape except that there are no negative half-loops since the logarithm of a negative number is imaginary. Also since $\log_e 1 = 0$, the minima of the curve all have the value zero instead of unity in the case of $y = \operatorname{cosec} x$.

Note.—Although all three curves have been shown, the student was only required to sketch one, *i.e.* part $c(i)$, (*ii*) or (*iii*).

Q. 7. (*a*) *If $y = x^{-1/2} \sin 2x$, show that*
$$x^2 \frac{d^2y}{dx^2} + x \frac{dy}{dx} = \frac{1}{4}(1 - 16x^2)\, y.$$

(*b*) *Given $\dfrac{dy}{dx} = 2x^3 + 3e^{-2x}$ and $y = 2$ where $x = 0$, derive an expression for y in terms of x.*

A. 7. (*a*) $y = x^{-1/2} \sin 2x.$

$\therefore \quad \dfrac{dy}{dx} = x^{-1/2}\, 2 \cos 2x + \sin 2x(-\tfrac{1}{2} x^{-3/2}),$

$\qquad = \cos 2x\, 2x^{-1/2} - \sin 2x \dfrac{x^{-3/2}}{2}.$

$\therefore \quad \dfrac{d^2y}{dx^2} = \cos 2x\,(-x^{-3/2}) - 2 \sin 2x\, 2x^{-1/2}$

$\qquad\qquad - \sin 2x\,(-\tfrac{3}{4} x^{-5/2}) - 2 \cos 2x \dfrac{x^{-3/2}}{2},$

$\qquad = \cos 2x(-x^{-3/2} - x^{-3/2}) - \sin 2x(4x^{-1/2} - \tfrac{3}{4} x^{-5/2}).$

Hence, $\quad x^2 \dfrac{d^2y}{dx^2} + x \dfrac{dy}{dx}$

$\qquad = x^2 \{-2 \cos 2x\, x^{-3/2} - \sin 2x(4x^{-1/2} - \tfrac{3}{4} x^{-5/2})\}$

$\qquad\quad + x \left\{ \cos 2x\, 2x^{-1/2} - \sin 2x \dfrac{x^{-3/2}}{2} \right\},$

$\qquad = \cos 2x \{-2x^{1/2} + 2x^{1/2}\}$

$\qquad\quad - \sin 2x \{4x^{3/2} - \tfrac{3}{4} x^{-1/2} + \tfrac{1}{2} x^{-1/2}\},$

$\qquad = - \sin 2x (4x^{3/2} - \tfrac{1}{4} x^{-1/2}).$

The right-hand side of the expression to be proved

$\qquad = \tfrac{1}{4}(1 - 16x^2)\, y,$

$\qquad = \tfrac{1}{4}(1 - 16x^2)\, x^{-1/2} \sin 2x,$

$\qquad = - \sin 2x (4x^{3/2} - \tfrac{1}{4} x^{-1/2}),$

$\qquad = $ left-hand side deduced above.

Hence, $x^2\dfrac{d^2y}{dx^2} + x\dfrac{dy}{dx} = \dfrac{1}{4}(1 - 16x^2)\,y.$ Q.E.D.

(b) $\dfrac{dy}{dx} = 2x^3 + 3e^{-2x}.$

$\therefore\ y = \displaystyle\int (2x^3 + 3e^{-2x})\,dx,$

$= \dfrac{2x^4}{4} + \dfrac{3e^{-2x}}{-2} + c,$

where c is a constant.

$\therefore\ y = \dfrac{x^4}{2} - \dfrac{3e^{-2x}}{2} + c.$

But, $y = 2$, when $x = 0$.

$\therefore\ 2 = -\dfrac{3e^0}{2} + c,$

or, $c = 2 + \dfrac{3}{2},$

$= \dfrac{7}{2}.$

$\therefore\ y = \dfrac{x^4}{2} - \dfrac{3e^{-2x}}{2} + \dfrac{7}{2},$

or, $2y = x^4 - 3e^{-2x} + 7.$

Q. 8. (a) *With the same cartesian axes, sketch the curves $y = 5 + 2\cos 3x$ and $y = 10\sin x$ from $x = 0$ to $x = \dfrac{\pi}{2}$.*

(b) *Find the area enclosed between the above curves and the y-axis, within this range of x.*

(c) *Calculate the root-mean-square value of $y = 5 + 2\cos 3x$ from $x = 0$ to $x = \dfrac{\pi}{2}$.*

A. 8. (a) The graphs of the two functions are shown in the sketch.
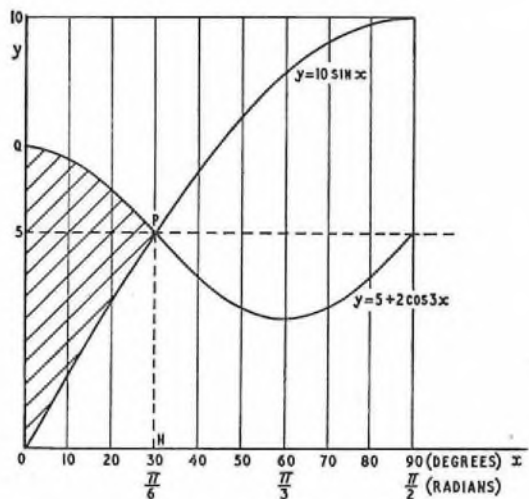
The graph of $y = 5 + 2\cos 3x$ has the same shape as that of a simple cosine curve but is of triple frequency and, hence, there are three quarter-cycles between $x = 0$ and $x = \dfrac{\pi}{2}$, as compared with only one for the function $y = 10\sin x$.

Because of the constant term 5, the graph oscillates about the line $y = 5$ with an amplitude of 2.

(b) The area enclosed between the curves and the y-axis is that shown shaded in the sketch. The graphs intersect at $\text{P}\left(\dfrac{\pi}{6}, 5\right)$.

Area required = Area QPNO under the curve $y = 5 + 2\cos 3x$
  $-$ Area PON under the curve $y = 10\sin x$,

$= \displaystyle\int_0^{\pi/6} (5 + 2\cos 3x)\,dx - \int_0^{\pi/6} 10\sin x\,dx,$

$= \left[5x + \dfrac{2\sin 3x}{3}\right]_0^{\pi/6} - \left[-10\cos x\right]_0^{\pi/6},$

$= \dfrac{5\pi}{6} + \dfrac{2}{3}\sin\dfrac{\pi}{2} + 10\left(\cos\dfrac{\pi}{6} - \cos 0\right),$

$= \dfrac{5\pi}{6} + \dfrac{2}{3} + 10(0\cdot866 - 1),$

$= 3\cdot285 - 1\cdot34,$

$= 1\cdot945.$

Hence, area enclosed between the two curves and the y-axis is $1\cdot945$ square units.

(c) The r.m.s. value of y is the square root of the mean value of y^2.

$y^2 = (5 + 2\cos 3x)^2,$

$= 25 + 20\cos 3x + 4\cos^2 3x.$

The mean value of y^2 between $x = 0$ and $x = \dfrac{\pi}{2}$ is given by

$$\dfrac{\displaystyle\int_0^{\pi/2}(25 + 20\cos 3x + 4\cos^2 3x)\,dx}{\dfrac{\pi}{2}}.$$

From the identity $\cos 2\theta = \cos^2\theta - \sin^2\theta = 2\cos^2\theta - 1$, $\cos^2 3x$ may be expressed as $\dfrac{1 + \cos 6x}{2}.$

Hence, mean value of y^2

$= \dfrac{2}{\pi}\displaystyle\int_0^{\pi/2}\{25 + 20\cos 3x + 2(1 + \cos 6x)\}\,dx,$

$= \dfrac{2}{\pi}\displaystyle\int_0^{\pi/2}(27 + 20\cos 3x + 2\cos 6x)\,dx,$

$= \dfrac{2}{\pi}\left[27x + \dfrac{20\sin 3x}{3} + \dfrac{2\sin 6x}{6}\right]_0^{\pi/2},$

$= \dfrac{2}{\pi}\left[\dfrac{27\pi}{2} - \dfrac{20}{3}\right],$

$= 27 - \dfrac{40}{3\pi},$

$= 22\cdot756.$

Hence, $y_{\text{r.m.s.}} = \sqrt{(22\cdot756)},$

$= 4\cdot77.$

**Q. 9.** (a) *Evaluate $\displaystyle\int_3^4 \dfrac{dx}{2x - 5}.$*

(b) *The current taken from a battery charger was recorded at half-hour intervals, with the following results.*

Time in hours . . .	0	0·5	1	1·5	2	2·5	3
Current in amperes . .	5·2	4·3	3·7	3·2	2·8	2·5	2·3

Use Simpson's Rule to evaluate the total ampere-hours taken from the battery charger in this 3-hour period.

A. 9. (a) $\displaystyle\int_3^4 \dfrac{dx}{2x - 5} = \left[\dfrac{\log_e(2x - 5)}{2}\right]_3^4,$

$= \tfrac{1}{2}(\log_e 3 - \log_e 1),$

$= \tfrac{1}{2} \times 1\cdot0986,$

$= 0\cdot5493.$

(b) The graph of current against time is shown in the sketch.
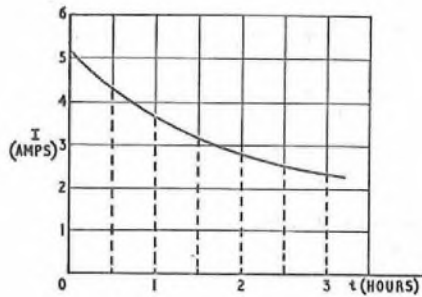
The total ampere-hours taken from the battery charger will be $\int I dt$, where I is the current and t the time. This is given by the area under the curve over the 3-hour period and may be evaluated approximately by Simpson's Rule. This states that, if the area under a curve is divided into an even number of strips of equal width, h, the area will be given by

$\dfrac{h}{3}$ {sum of first and last ordinates

$\quad + 4 \times$ (sum of even ordinates)

$\quad + 2 \times$ (sum of remaining odd ordinates)}.

The intervals given divide the area into six strips of equal width of $\frac{1}{2}$ hour, and, hence, the ordinates may be read directly from the given data.

Hence, the total ampere-hours

$$= \frac{1}{2} \times \frac{1}{3}\{5 \cdot 2 + 2 \cdot 3 + 4(4 \cdot 3 + 3 \cdot 2 + 2 \cdot 5)$$
$$\qquad\qquad + 2(3 \cdot 7 + 2 \cdot 8)\},$$
$$= \frac{1}{6}(7 \cdot 5 + 4 \times 10 \cdot 0 + 2 \times 6 \cdot 5),$$
$$= \frac{1}{6} \times 60 \cdot 5,$$
$$= 10 \cdot 083.$$

$\therefore$ Total ampere-hours $\simeq \underline{10 \cdot 1}$.

Q. **10.** *For a long transmission line the propagation coefficient per kilometre is given by the formula $\gamma = \sqrt{\{(R + j\omega L)(G + j\omega C)\}}$.*

(a) Express first γ^2, and then γ, in the polar form $r \angle \theta$.
(b) Evaluate γ in the form $\alpha + j\beta$, when $R = 40$, $L = 1 \cdot 2 \times 10^{-3}$, $C = 0 \cdot 05 \times 10^{-6}$, $\omega = 10^4$, and G is so small it may be neglected.

A. **10** (a) $\gamma = \sqrt{(R + j\omega L)(G + j\omega C)}$.

(a) $\qquad \gamma^2 = (R + j\omega L)(G + j\omega C)$,

$$= \sqrt{(R^2 + \omega^2 L^2)} \Big/ \tan^{-1} \frac{\omega L}{R}$$

$$\times \sqrt{(G^2 + \omega^2 C^2)} \Big/ \tan^{-1} \frac{\omega C}{G}.$$

$$\therefore \gamma^2 = \sqrt{\{(R^2 + \omega^2 L^2)(G^2 + \omega^2 C^2)\}} \Big/ \left(\tan^{-1} \frac{\omega L}{R} + \tan^{-1} \frac{\omega C}{G}\right).$$

$$\gamma = \sqrt[4]{\{(R^2 + \omega^2 L^2)(G^2 + \omega^2 C^2)\}} \Big/ \frac{1}{2}\left(\tan^{-1} \frac{\omega L}{R} + \tan^{-1} \frac{\omega C}{G}\right).$$

(b) When G is small enough to be neglected,

$$\gamma = \sqrt{\{(R + j\omega L)(j\omega C)\}},$$
$$= \sqrt{(j\omega CR - \omega^2 LC)}.$$

If $\qquad R = 40$, $L = 1 \cdot 2 \times 10^{-3}$, $C = 0 \cdot 05 \times 10^{-6}$ and $\omega = 10^4$,

$$\gamma = (-10^8 \times 1 \cdot 2 \times 10^{-3} \times 0 \cdot 05 \times 10^{-6}$$
$$\qquad + j10^4 \times 0 \cdot 05 \times 10^{-6} \times 40)^{1/2},$$
$$= (-0 \cdot 006 + j0 \cdot 02)^{1/2},$$
$$= \sqrt[4]{\{(-0 \cdot 006)^2 + (0 \cdot 02)^2\}}$$
$$\left[\cos \frac{(\theta + n360°)}{2} + j \sin \frac{(\theta + n360°)}{2}\right],$$

where $\qquad \theta = \tan^{-1}\dfrac{0 \cdot 02}{-0 \cdot 006} = 180° - 73°18' = 106°42'$,

and $n = 0$ or 1.

$$\therefore \gamma = \sqrt[4]{(0 \cdot 000036 + 0 \cdot 0004)}$$
$$\left[\cos \frac{(106°42' + n360°)}{2} + j \sin \frac{(106°42' + n360°)}{2}\right]$$
$$= 0 \cdot 1445 \{\cos (53°21' + n180°) + j \sin (53°21' + n180°)\}.$$

When $\qquad n = 0$,

$$\gamma = 0 \cdot 1445(0 \cdot 5969 + j0 \cdot 8023),$$
$$= 0 \cdot 08626 + j0 \cdot 116.$$

When $\qquad n = 1$,

$$\gamma = 0 \cdot 1445(-0 \cdot 5969 - j0 \cdot 8023),$$
$$= -0 \cdot 08626 - j0 \cdot 116.$$

Hence, $\qquad \gamma = \pm (0.0863 + j0 \cdot 116)$.


LINE PLANT PRACTICE B, 1972

Students were expected to answer any six questions

Q. **1.** *(a) Describe in detail the method of carrying out the following tests on a concrete mix:*

(i) slump test, and
(ii) cube compression test.
(b) Of what use are these tests to the site engineer?

A. **1.** (a) (i) The slump test is a method of determining the consistency of concrete. The equipment required to carry out the test is a standard slump can (see sketch (a)), a tamping rod 600 mm long, 16 mm in diameter with one end rounded, and a metal base-plate 500 mm by 900 mm.

All equipment should be clean and dry. The can should be held firmly whilst the concrete is placed in it. The sample of concrete should be taken at random from different points in the mix and be truly representative of the mix; the sample should not be less than $0 \cdot 03$ m^3 (1 ft^3). The random sample of concrete should be placed in the can in four, approximately equal, layers. Each layer should be tamped with 25 strokes of the rounded end of the tamping rod. The strokes should be distributed in a uniform manner over the surface area of the concrete and, for the second and subsequent layers, the tamping rod should

penetrate into the previous layers. After the top layer has been tamped, the concrete should be struck off level with a trowel and the slump can inverted on to the base-plate. The slump can should be removed immediately by raising it slowly and carefully in a vertical direction, and placed on the base-plate beside the mound of concrete. A straight
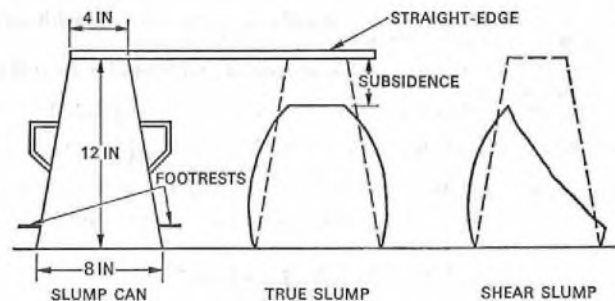
edge is then placed on the slump can, above the mound of concrete, and the extent of the slump measured.

There are two distinct types of slump which may occur, other than a complete collapse. The first is the true slump when uniform subsidence occurs; the second is shear slump when the top half of the mound of concrete shears off and slips down an inclined plane. Where shear slump occurs, an accurate measurement cannot be made and it is necessary to repeat the test to produce a shape more representative of a true slump.

Typical slump measurements for British Post Office work are: not greater than 100 mm for concrete quality A when used for the construction of jointing chambers, 90 mm for concrete quality F when used for the base of any p.v.c. concrete duct blocks and 180 mm for the same quality concrete when used for the infill or surround of any p.v.c. concrete duct blocks. A slump test should normally be taken whenever a new batch of aggregate or sand is started, at all times when the mix seems to be varying from previous mixes, when the mix appears to be overwatered and when the site engineer considers the mix would not meet the slump measurements required. Details of all slump tests and their results should always be recorded.

(*ii*) The cube compression test is a means of checking the compressive strength of concrete used during the progress of construction. Normally, a 150 mm (6 in) cube is prepared for test but, if the aggregate does not exceed 19 mm ($\frac{3}{4}$ in), 100 mm (4 in) cubes may be prepared.

The mould is of steel or cast iron and strong enough to prevent distortion when being filled with concrete. The sides and base are firmly bolted together, or secured with spring-loaded clips, so that no leakage occurs and the concrete cube may be removed without damage. The internal faces and the flanges of the mould should be free from dirt and hardened mortar and should be coated with heavy oil to prevent any leakage during filling and to prevent the concrete cube sticking on removal.

The mould should be filled in three, approximately equal, layers, each layer being uniformly tamped over the whole surface with a tamping bar. The tamping bar should have a mass of 2 kg (4 lb), be approximately 380 mm long and have a 25 mm square end for ramming. The number of strokes per layer is dependent upon the size of the aggregate used in the concrete, but should not be less than 35 strokes per layer for 150 mm cubes and 25 strokes per layer for 100 mm cubes. The concrete within the mould may be compacted by vibration, in which case each layer is vibrated by electrical or pneumatic hammer, or by a suitable vibrating table, until the specified condition is attained. When the top layer has been tamped, any surplus concrete should be struck off with a trowel and the surface finished level.

The test cubes should be stored on site at a place free from vibration, under damp sacking or similar material, and left undisturbed in a temperature of 5–21°C for 24 hours. After 24 hours, the cubes should be marked for later identification, removed from the moulds and, unless required for testing within 24 hours, stored in clean water at a temperature of 10–21°C until transported to the testing laboratory. Tests should be carried out at 24 hours, 7 days or 28 days as specified.

When the cubes are transported to the testing laboratory, they should be wrapped in two or three layers of sacking, saturated with water and enclosed in a polyethylene bag to prevent any loss of moisture during transit. The cube is tested in a saturated condition and positioned between the two platens of the testing machine so that the load is applied to opposite sides of the cube as cast, not to the top and bottom. The load is applied without shock and increased continuously at a rate of 14 MN/m²/min (2,000 lb/in²/min) until the resistance of the cube to increasing load breaks down and no greater load can be sustained. Details of the maximum load applied and the nature of the failure should be recorded. The equivalent compressive strength of the concrete can be calculated by dividing the maximum load applied to the cube during the test by its cross-sectional area and should be expressed in MN/m². To obtain an accurate assessment of the compressive strength of any concrete used in structures, a minimum of three cubes is required. British Post Office requirements are that concrete quality A used for the construction of jointing chambers shall meet the following compressive strengths:

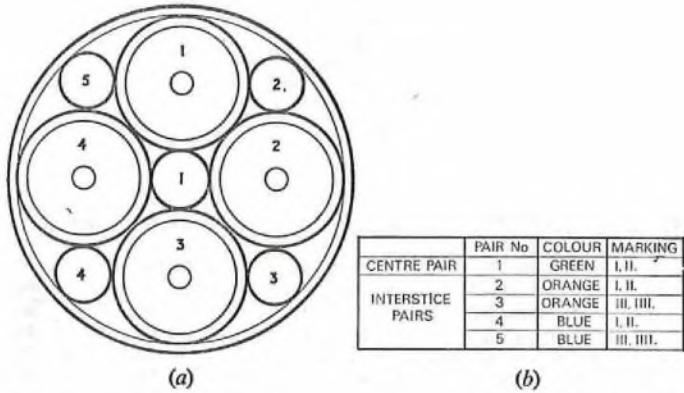| Age of Concrete	Type of Cement Used		
	Portland and Sulphate Resisting	Rapid Hardening	High Alumina
7 days	Over 14 MN/m²	Over 18 MN/m²	Over 35 MN/m²
28 days	Over 21 MN/m²	Over 28 MN/m²	—

(*b*) The slump test assists the site engineer by enabling him to ensure that concrete to be used on site is constant from one mix to another and that the water content is correct. The cube compression test tells the site engineer whether any concrete used for structural work has met the minimum compressive strength, for that mix, as specified. Finally, the fact that such tests are being constantly carried out tends to result in the contractor producing above-average quality concrete rather than face loss of time and money arising out of condemned work.

Q. 2. (*a*) *With the aid of a sketch describe the make-up of a cable containing four coaxial* 1·2/4·4 A *pairs and five* 0·63 mm *paper-insulated pairs.*

(*b*) *Describe briefly the method of jointing one* 1·2/4·4 A *coaxial pair.*

A. 2. (*a*) Sketch (*a*) shows a cross-section of a cable containing four coaxial 1·2/4·4 A pairs and five 0·63 mm paper-insulated pairs. Each individual 0·63 mm pair is marked as shown in sketch (*b*).

	PAIR No	COLOUR	MARKING
CENTRE PAIR	1	GREEN	I, II.
INTERSTICE PAIRS	2	ORANGE	I, II.
	3	ORANGE	III, IIII.
	4	BLUE	I, II.
	5	BLUE	III, IIII.

(*a*)   (*b*)

(*b*) The coaxial pair to be jointed is first isolated from the remainder of the pairs in the cable using rubber jointing formers. The steel tapes on the left-hand pair are removed after 12 mm adhesive brown-paper tape has been bound round close to the joint former. A wire tie is made round the outer steel tape at a point 25 mm from the adhesive paper binding, and both steel tapes are unwound and cut off at this point. The paper and steel tapes of the other (right-hand) pair are unwound back to the former, and positioned clear of the jointing gap.

The coaxial pairs are next cut so that they overlap by 25 mm. The conical outer-conductor opening tool is inserted between the outer conductor and the insulation of each pair, in turn, to open up the outer conductors to within, approximately, 12 mm of the steel tapes. Then, using a pair of flat-nosed adjusting pliers, the outer conductors are flattened for a length of 50 mm from their ends. The two flattened tapes are brought together, overlapping at the centre of the joint, and securely clamped in position. The flattened-tape outer conductors are cut with a pair of scissors so that the two tapes just butt. A strip of silver solder 125 μm thick, 1·5 mm wide, 15 mm long is positioned on the underside of the butting tapes, with the ends bent over the edges of the tape to hold it in position. (See sketch (*c*)). With the brazing tongs

CLAMP
STRIP OF SOLDER
(*c*)
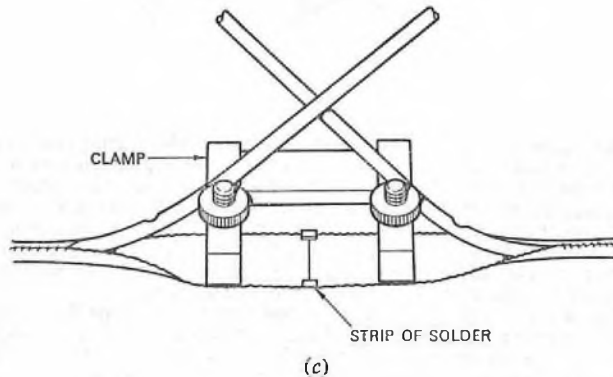
connected and the control unit switched to OUTER-CONDUCTOR TAPES, the carbon electrodes are placed centrally over the joint as shown in sketch (*d*). The brazing current is switched on until the tape glows deep red, then the current is disconnected. The electrodes are allowed to cool before being removed from the tape.

Next, the two centre conductors are laid side by side, and the polyethylene insulation is carefully removed from each conductor for a distance 20 mm either side of the centre jointing point. The insulation can be cut using a heated wire, which ensures that the centre conductor

(*d*)
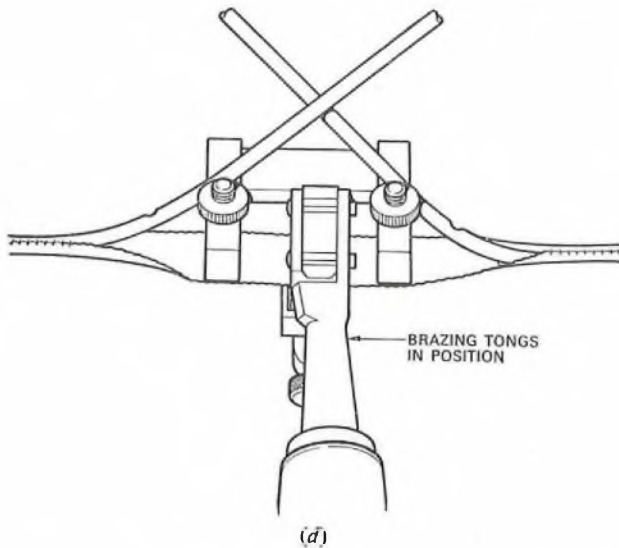
is not damaged. The centre conductors are cut through so that their ends butt together at the jointing point.

A short length of 50 mm insulating paper is first placed round the outer tape to insulate it. A wire-brazing tool is connected to the brazing unit, and the control knob set to 1·2/4·4 A CENTRE CONDUCTOR. The handles of the wire-brazing tool are gently squeezed together until the brazing jaws part sufficiently to insert an ebonite spacer. The two conductors to be jointed are placed centrally in the jaws so that the ends are in alignment, and butt squarely; the thumb-screws are then tightened to secure the conductors. The ends of the handles are squeezed together to separate the conductor ends, and allow a strip of silver solder 1·5 mm wide to be placed between them, at the same time releasing the ebonite spacer (sketch (*c*)). The pressure on the handle is then released to allow

(*e*)
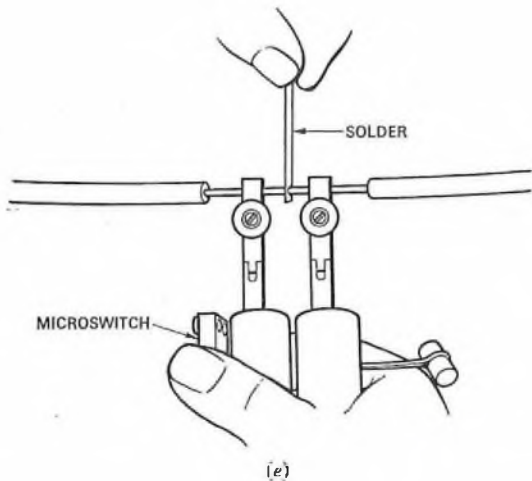
the spring loading of the tool to grip the solder. The brazing current is then connected until a red glow appears in the butted conductors, and it is seen that the conductors have brazed together. The solder is removed, the current disconnected and, when the joint has cooled, any rough edges are removed with fine glass cloth. The outer conductor is then reformed into a tubular shape around the polyethylene insulation. This is partially achieved by using reforming pliers, and completed by using a tube-reforming die appropriate to the size of cable.

The two steel tapes, which were wound up and left at the side of the joint, are now replaced one at a time. The first tape is lapped tightly over the outer conductor to draw its edges close together. The lapping is continued on to the 25 mm of steel tape left on the other side of the joint, the surplus is cut off, and the tape is secured by adhesive paper tape. The second steel tape is lapped over the first and similarly secured, followed by the two paper tapes.

Q. 3. (a) With the aid of a diagram, describe the double-ended Varley method of locating a low-insulation resistance fault on a long length of cable.

(b) In a test on a cable pair 6 km long, the near-end and distant-end readings were 160 ohms and 100 ohms respectively. How far was the fault from the near end?

A. 3. (*a*) The equipment required to carry out a double-ended Varley test consists principally of two Wheatstone bridges, one connected at each end of the cable under test. A complete circuit is shown in the sketch. The test is made in the following manner.
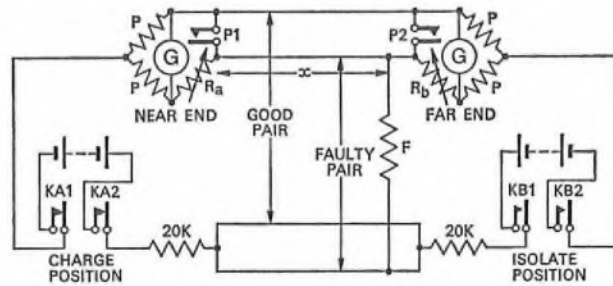
(*i*) The line is charged up from the near-end battery by putting key KA in the CHARGE position. Key KB is left in the ISOLATE position. The line is short-circuited at the far end by inserting plug P2. Plug P1 is left out at the near end.

(*ii*) The bridge resistor at the near end (R_a) is adjusted to give zero deflexion on the galvanometer. When the zero reading is constant—i.e. when the line is charged up to a steady potential—the value of R_a is recorded.

(*iii*) A similar test is then made at the far end by putting key KB in the CHARGE position, and KA at the near end in the ISOLATE position. Plug P2 is taken out and plug P1 inserted. The far-end bridge is balanced and the value of resistor R_b recorded.

These tests are repeated several times to obtain an accurate mean value for R_a and R_b. The distance to the fault is calculated from the following formula:

$$X = \frac{R_b \times L}{R_a + R_b},$$

where X = distance to fault from near end in kilometres,
R_a = near-end Varley reading,
R_b = far-end Varley reading, and
L = length of cable in kilometres.

(*b*) Substituting the given values: $R_a = 160$ ohms, $R_b = 100$ ohms, $L = 6$ km, gives

$$X = \frac{100 \times 6}{160 + 100} \text{ km},$$

$$= 2 \cdot 308 \text{ km.}$$

Q. 4. (a) Describe with sketches a cabinet for housing a maximum of four 100-pair cross-connexion assemblies.

(b) Describe the method of installing such a cabinet and fitting one 100-pair assembly.

A. 4. (*a*) The cross-connexion cabinet, suitable for housing a maximum of four 100-pair assemblies, has dimensions of 1,067 mm (3 ft 6 in) high, 280 mm (11 in) deep and 648 mm (2 ft 1½ in) wide and has one door. The base, door, sides, back, top frame and roof of the cabinet are made of grey iron and are separately cast. The sections are machined and, after dressing with a special compound, are assembled and bolted together. This method of construction is adopted to avoid large castings and the attendant risk of distortion and breakage during handling. In addition, should any part be damaged, such as in a traffic accident, the replacement of individual parts is made easy and is much less costly than replacement of the whole cabinet.

The minimum thickness of all the parts is 8 mm (5/16 in); the door and back castings are dished near their edges to give more rigidity to those parts and improve their appearance. The door is hinged on steel trunnions which turn in non-metallic bushes let into the door; when the door is closed, the edges are seated in a rubber gasket recessed in a groove in the cabinet. The door is secured, when closed, by key-operated triangular-headed bolts. Inside the cabinet are two horizontal bars, one at the top and one at the bottom, secured to lugs on the side castings. The assemblies or cross-connexion strips and jumper-ring bars are bolted to the horizontal bars as required. Provision is also made in the cabinet for desiccators and pressure gauges to be installed. The desiccators contain silica gel and reduce the moisture in the atmosphere within the cabinet. The pressure gauges, when fitted, monitor the air pressure on the exchange-side cable.

(*b*) The site for the cabinet is normally selected at the planning stage and is positioned to cause the minimum of obstruction to pedestrians

and yet allow easy installation and maintenance access. At the works stage, the minimum size of excavation to be taken out should have a length of 763 mm, a width of 380 mm and a depth of 305 mm; the excavation should be timbered as necessary. Care should be taken during excavation to keep the surface materials, foundation materials, and the sub-soil separate.

A wooden template is used to register accurately the positions of the p.v.c. duct and the foundation bolts. The ducts are positioned to give the required depth of cover and to project 30 mm above the proposed base. The foundation bolts are fixed in the template so that they are set vertically in the base and project 40 mm above the proposed base. Where cabinets are installed on sloping ground, it may be necessary for part of the base to be above ground so that the cabinet doors may be fully opened.

Prior to concreting, the ends of the p.v.c. duct should be sealed to prevent the ingress of concrete. Concrete of quality A should be used for the base and should be carefully placed and levelled at a depth of 75 mm below ground level. The cabinet should not be erected on the base until a minimum period of 24 hours has elapsed, if rapid-hardening cement is used, and 48 hours has elapsed if ordinary Portland cement is used. After the correct time-period has elapsed, the duct laid to the jointing chamber should be cleaned with a suitable rag mop and a draw-rope left in the duct. The template used to position the duct and foundation bolts should be removed prior to the erection of the cabinet. The cabinet internal walls should then be wire-brushed or scraped to a height of 40 mm from the base to remove all rust and loose paint. The surface of the concrete base should be thoroughly cleaned and a bed of cement mortar laid to a depth of 13 mm over that part of the base on which the cabinet base flange will rest. The cabinet should then be set on the cement mortar in a truly vertical position and secured by the foundation bolts. The cement mortar should be struck off in line with the internal surface of the cabinet walls, and trowelled on the outside to form a fillet around the flange to assist in the shedding of water. When the mortar has set, the concrete surface inside the cabinet and the internal walls of the cabinet to a height of 40 mm from the base should be thoroughly cleaned and dried. A petroleum/bitumen compound is then run over the concrete base to an approximate depth of 18 mm and served to the internal walls of the cabinet to a minimum height of 40 mm, ensuring good contact between the bitumen of the base and that of the walls.

Backfilling of the excavation should now be undertaken and the surface permanently or temporarily reinstated. The external surface of the cabinet should then be washed with clean water, allowed to dry and then thoroughly brushed with a wire scratch brush to remove all rust and loose paint. The cabinet should then be dusted and painted with two coats of paint of approved type and colour. The internal surfaces should be similarly cleaned and an undercoat and a top coat of approved white enamel paint should be applied. The duct entries within the cabinet are sealed with wooden or cork plugs until cabled. Those parts of the cabinet which may be inaccessible after erection should be given a heavy coat of bitumen-based black paint after cleaning and before erection.

An earlier type of assembly, still in use, consists of five terminal blocks each having 20 exchange (E) side and 20 distribution (D) side pairs. Cross-connexion is achieved either by bridging pins where the D-side and E-side pairs are coincident on the terminal blocks, or by jumper wire.

In the current type of cabinet, the terminal-block type of assembly is replaced by a plastic strip containing a number of holes which locate the wire pairs. The pairs are passed through the holes and jointed by crimping the E- and D-side pairs together, or cross-connected by using a jumper-wire strap.

Assemblies are issued complete with a continuous 9·1 m (30 ft) long tail of cable, the midpoint of which is usually indicated by a wrapping of tape. Where assemblies are to be fitted, cabling operations normally proceed from the cabinet to the jointing chamber.

Before cabling, the area should be guarded to meet the safety requirements, the covers of the jointing chamber removed and the cabinet doors opened. Whether the draw-rope or a cabling rope is used to pull the cable into the duct is dependent upon the size of cable and the likely pulling tensions. When the cable has been pulled into the duct, the assemblies, or the pre-wired strips, together with the horizontal bars and jumper-rings, are bolted into place; duct entries are then sealed to prevent gas entering the cabinet.

Q. **5.** (*a*) *Give a brief account of the causes and effects of cable creepage.*
(*b*) *Describe methods used to minimize the creepage effect.*

A. **5.** (*a*) Cable creepage is the term applied to the movement of an underground telephone cable along its duct, this movement being caused by vibrations set up by road traffic. In some circumstances, the conductors of an underground cable may move independently, or at a different rate, to the cable sheath—this is known as core creepage.

Cable creepage is likely to occur where cables are laid

(*a*) under, or very near, roads carrying heavy traffic,
(*b*) under roads constructed upon unstable subsoils, e.g. clay, shingle,

peat or made-up ground; creepage is likely to be more pronounced in wet subsoils,

(*c*) under roads carried on embankments or with deep ditches alongside,
(*d*) along long straight sections of roads; creepage does, however, occur on winding roads, but is less pronounced, and
(*e*) in self-aligning ducts; creepage does occur in other types of duct but is much less pronounced.

When the ground conditions are conducive to cable creepage, the cables tend to move in the direction of the traffic stream nearest to them. The actual movement of a cable under creepage-inducing conditions is the result of many factors, in addition to those mentioned, including the condition of the particular duct (e.g. wet, dry, silted), the type, size and weight of the cable concerned, and the existence of other cables in the same duct way.

Cable-sheathing faults as a result of creepage usually occur as fractures at kinks or similar points of stress near the plumbed wipes of lead joint-sleeves. Core creepage causes conductor faults such as contacts, due to jointing sleeves being dislodged or insulant being fractured, and disconnexions.

An accepted explanation of creepage is the "surf-riding" theory. This assumes that a vehicle depresses the earth beneath it and, thus, the earth-depression travels along with the vehicle. Should a line of ducts carrying a cable come under the influence of this travelling depression, each duct in turn will be tipped downwards as the depression proceeds (see sketch (*a*)). After the vehicle has passed, the earth recovers slowly and the

(*a*)
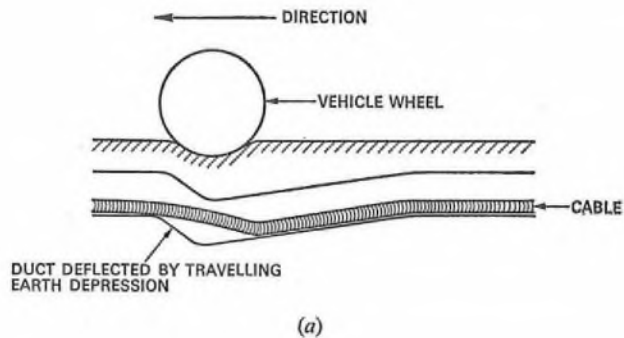
ducts return to their original positions. Thus, a wave-like motion is imparted to the duct-line, and this urges the cable forward in a similar manner to a surf-rider being carried forward by a wave. The same theory can be given to explain the phenomenon of core creepage when the cable sheath is assumed to behave in a similar manner to a duct line under traffic disturbance.

(*b*) To prevent faults caused by cable creepage, anti-creepage measures are taken which fall into two classes; treatment of the ducts in which the cables are laid and anchoring the cable in the appropriate jointing chambers. The various measures are described below.

(i) Treatment of Ducts

Damping is a measure that is usually applied to new ducts when they are laid in situations where cable creepage is expected and the road concerned has only light foundations. The process consists of placing a 75 mm saddle of quality C concrete on the top of the ducts. Normally, damping is only carried out on sections of duct-track not exceeding 500 m.

When new ducts are laid in very unstable soil conditions, e.g. marshy ground, they can be completely surrounded with quality C reinforced concrete. The main object of the surround is to give stability to the duct line; the anti-creepage aspect, although important, is a secondary object.

(ii) Anchoring of Cables

A modern creep-out anchor, shown in sketch (*b*), has the appearance of a shovel and is inserted, shovel-end first, into the duct mouth on the creep-out side of the jointing chamber. A shoulder, acting as a stop, is

(*b*)   (*c*)
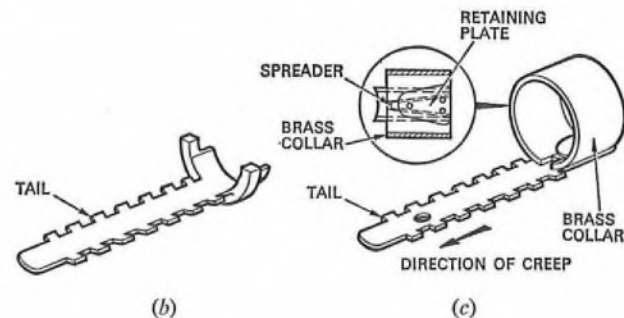
provided to permit the device to pass only a limited distance into the duct. The cable is attached to the device by lashing it to the recessed tail with galvanized-iron bindings. The anchor and wire bindings are given lappings of protecting tape.

A creep-in anchor, see sketch (*c*), consists of a split brass collar, which is fully inserted into the duct mouth on the creep-in side of the jointing chamber. It has a recessed tail similar to that provided with the creep-out anchor except that it has a tapered "spreader" end that engages with the tapered split in the brass collar.

Q. 6. *State, and give the relative importance of, the factors which are taken into consideration in deciding whether to provide telephone service to a new housing estate by means of overhead dropwire served from an aerial cable, underground cable with overhead distribution poles, or wholly underground cable.*

A. 6. In deciding which method of telephone service can best be adopted to serve an estate, it is first necessary to obtain information on the following points:

(*a*) the proposed number and type of dwellings,

(*b*) an up-to-date development forecast of telephone requirements,

(*c*) layout of site, position of dwellings, walkways, roads and amenities of the estate,

(*d*) local geographical features and position of the estate in relation to existing telephone plant, and

(*e*) particulars of any power circuits in the vicinity of the housing estate which may necessitate the provision of some form of protection for the line plant.

The type of plant required to give service should be provided as economically as possible, consistent with sound engineering principles and safety. For estates of houses, low flats and maisonettes, overhead distribution is more economical than underground distribution. It is flexible, and usually involves cabling along one side of the street only. Where subscribers are few and scattered, distribution is provided by means of an aerial cable supported by poles and the service to subscribers is by dropwire connected direct to the cable at pole positions, or from ring-type pole-heads. Where the subscribers are reasonably close together, ring-type distribution poles, spaced about 100 m apart in the footway, are fed by underground cable. Service to the subscriber is provided from the nearest pole to the dwelling by overhead dropwire, when required.

Underground distribution is usually more expensive because it necessitates the laying of distribution cables on both sides of the road and, since it is not known which houses will want telephones over the years, an individual service cable has to be provided from the distribution cable to each dwelling at the outset to avoid disturbance of expensive pavings and cultivated front gardens at a later date.

When plans for providing a telephone service are being prepared, liaison with the estate developer is essential to co-ordinate the provision of telephone plant with building and street works on the estate. It is an advantage to obtain information of work being carried out by the other undertakers as this may influence the final proposals and reduce cost of provision, e.g. by sharing of trenches with other services, or the joint use of poles. It is very often possible to arrange for the, normally, more expensive underground distribution to be used if expense can be saved by the joint use of trenches and the provision of the service cable when the ground is already excavated during building operations.

Q. 7. (*a*) *Describe in detail the method of construction of a brick manhole suitable for a six-way duct route located in the carriageway.*

(*b*) *Give details on one type of frame and cover which could be used on a manhole of this type.*

Q. 8. *Maintenance operations require the opening and closing of a joint on a coaxial cable which feeds power to a repeater station. The joint is located in a joint box in the carriageway. Write an account of the safety measures which have to be taken, and procedures followed, when carrying out this operation.*

A. 8. Where a coaxial joint situated in a carriageway joint-box has to be opened and closed, the following safety precautions should be taken:

(*a*) *Road safety precautions.* The carriageway opening should be adequately guarded. The guard, together with any other obstructions, e.g. maintenance vehicles, joint-box cover, etc., should be marked out with cones, with the addition of red lamps at night, in the approved manner. Roadworks caution signs should also be placed at the appropriate positions to warn oncoming traffic.

The guarding cones, etc., should be removed after the site of operations has been completely cleared, just prior to driving off.

(*b*) *Gas tests.* Before the joint-box is used, tests should be carried out for the presence of gas. The test is made by means of

a combustion-type detector. The joint-box cover is opened a few inches, and samples of the atmosphere to be tested are drawn through the indicator by means of a probe and hose, which are lowered into the joint box, and a rubber aspirator bulb attached to the instrument. The samples of air pass over a platinum filament which is connected in one arm of a Wheatstone bridge and the filament is heated electrically by the bridge current. Before test samples are taken, the bridge is first switched on and balanced by the switch/rheostat with the probe in fresh air. If any combustible gas is present in the sample, it burns, raising the temperature and electrical resistance of the filament, resulting in an out-of-balance current flowing in the indicating meter. The meter is calibrated in terms of the explosive properties of the sample. If the result of this test is negative, i.e. a reading of less than 10 per cent, the cover is removed.

Before work is begun, or a flame used, tests using the gas indicator are made in the mouths of ducts entering the joint-box. Tests should be made at intervals as work progresses and at the resumption from meal breaks.

(*c*) *Power Precautions.* On certain sections of coaxial cable, medium-voltage power is carried to provide power at intermediate repeater stations. It is essential that this power supply should be switched off before the joint is opened. To prevent accidents, the power source is controlled by a system of engraved tokens or keys. The removal of these tokens or keys when the power is switched off prohibits the reconnexion of the power supply whilst work on the cable is in progress. As a further precaution, especially against the isolation of the wrong power supply or work commencing on the wrong cable, the jointer is supplied with a neon tester. This enables the conductors of the pair to be tested for power after the joint is opened. The tester is applied to the conductors immediately they are exposed; no glow on the lamp indicates that dangerous voltages are not present.

(*d*) *Pressurized cables.* Before heat is applied to the joint, a small hole should be cut in the sleeve to release air pressure in the cable.

(*e*) *Replacement of joint-box cover.* On completion of the work, the cover should be properly replaced and checked to ensure that it is properly seated and presents no danger to the public.

Q. 9. (*a*) *Derive an expression for the relationship between tension and dip in an overhead wire or cable.*

(*b*) *An aerial cable is erected over a span of 60 m to a tension of 3,800 N. What would be the maximum dip if the mass of the cable were 0·5 kg/m?*

A. 9. (*a*) The forces to which a suspended wire is subjected are shown in the sketch. At point B in the span there is a sag of *d* metres.
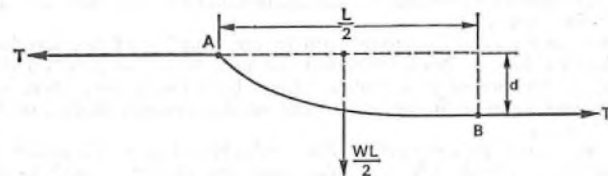
If the sag is small compared with the span of the wire, the weight of the half span can be assumed to act at $\dfrac{L}{4}$ m from point A. Let the mass of the wire be *W* kg/m and the tension be *T* newtons. Then, taking moments about A,

$$Td = \frac{WLg}{2} \times \frac{L}{4}.$$

$$\therefore T = \frac{WL^2g}{8d} \text{ newtons.}$$

(*b*) If $W = 0·5$ kg/m, $L = 60$ m and $T = 3,800$ newtons, substituting in the formula,

$$3,800 = \frac{0·5 \times 60^2 \times 9.81}{8d}.$$

$$\therefore \ d = \frac{0·5 \times 3600 \times 9·81}{8 \times 3800} \text{ m,}$$

$$= 585 \text{ mm.}$$

Q. 10. (*a*) *Describe the apparatus used for measuring current in a cable sheath.*

(*b*) *Explain how this apparatus is used.*

A. 10. (*a*) Apparatus used to measure the current flowing in a cable sheath is required to indicate the magnitude and direction of the current flow. The apparatus must be designed to facilitate easy connexion to the cable and have negligible effect upon the value of the sheath current.

The apparatus is mounted in a portable case and includes a voltmeter circuit and an ammeter circuit, each of which terminates on two terminals to facilitate connexion to the cable sheath. The voltmeter circuit consists of a reflecting galvanometer having a sensitivity of 10 μV/mm and a coil resistance of 10 ohms, range-switching keys, and shunt and series resistors to permit three ranges of sensitivity. A ZERO CHECK key disconnects the galvanometer and shunts it with a 30-ohm resistor, thus enabling it to return quickly to zero. The circuit arrangements of the range-switching keys are such that, irrespective of the sensitivity range in use, the input resistance of the voltmeter circuit is maintained at 10 ohms. Coarse adjustment of the galvanometer zero is made by a control on the torsion head of the galvanometer and fine adjustment by movement of the galvanometer scale which is graduated in millimetre divisions and marked 50–0–50, 100–0–100 and 250–0–250 to correspond with the three sensitivity ranges.

The ammeter circuit consists of a 3-volt primary battery, which also supplies power for the galvanometer lamp, a moving-coil ammeter having scales of 0–0·25 amp and 0–2·5 amp, a 0–10 ohm semi-log variable resistor for current adjustment, a reversal key for reversing the connexions to the ammeter circuit terminals (thus reversing the direction of the calibration current), a key and terminals to enable the calibration current to be drawn from an external power supply if necessary, and a key to disconnect the ammeter circuit from the apparatus terminals. When the internal 3-volt battery is used, the ammeter is switched to the 0–0·25 amp range but, when an external power supply is used, the ammeter is switched to the 0–2·5 amp range.

(*b*) The magnitude of the current flowing in the lead sheath of an underground cable is determined by measuring the potential difference between two points on the cable sheath approximately 1 m apart. The low values of sheath current and sheath resistance normally encountered require the use of a very sensitive measuring instrument. A centre-zero reflecting galvanometer, having a sensitivity of 10 μV/mm, is therefore used. The connexions of the galvanometer and calibrating circuit to the cable sheath are shown in the sketch.
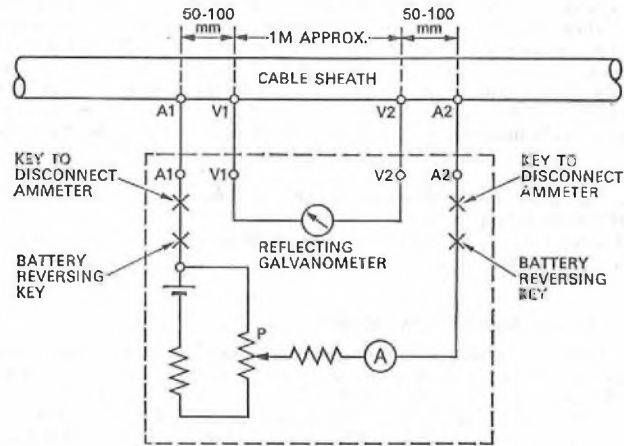
The galvanometer is calibrated by passing a known current through the cable sheath. The calibration current, usually of the order of 0·2 amp, is adjusted by means of the potentiometer, P. The galvanometer deflexion is proportional to the sum or difference of the calibrating current and the sheath current. The galvanometer deflexion is noted, the direction of the calibrating current is reversed and a second galvanometer reading is taken. The number of divisions separating the two galvanometer readings is proportional to twice the calibrating current, since the effect of the normal sheath current cancels when the calibrating current is reversed. The calibration of the galvanometer, is therefore, given by the expression:

$$\text{calibration (mA/division)} = \frac{2 \text{ (calibration current in mA)}}{\text{total swing between two deflexions}}$$

The ammeter circuit is next disconnected by means of the disconnecting key shown in the diagram and the galvanometer indicates the magnitude and direction of the sheath current in mA/division.

RADIO AND LINE TRANSMISSION B, 1972

Students were expected to answer any six questions.

Q. **1.** *Describe, in detail and with the aid of circuit diagrams, the operation of two of the following.*

(*a*) *A diode detector for use in a communications receiver.*

(*b*) *A frequency changer for use in a broadcast receiver.*

(*c*) *A double-balanced ring modulator as used in multi-carrier telephony transmission circuits.*

Q. **2.** (*a*) *Why is the superheterodyne method generally preferred to straight reception in communication receivers?*

(*b*) *Explain the following terms used with reference to superheterodyne reception.*

(*i*) *Image-channel rejection.*

(*ii*) *Adjacent-channel selectivity.*

(*c*) *In which sections of a superheterodyne receiver would image-channel rejection and adjacent-channel selectivity be most effectively accomplished?*

A. 2. (*a*) The superheterodyne receiver is generally preferred for communication purposes because it provides well-defined passband characteristics in the intermediate-frequency (i.f.) amplifier stages. These give constant gain, uniform passband response and adequate rejection of the adjacent channels over a broad tuning range. By contrast, straight receivers cannot provide these characteristics because the tuning variables cause changes in the characteristics of the tuned circuits. At high frequencies, also, it is difficult to obtain adequate channel selectivity.

(*b*) (*i*) *Image-channel rejection*

In a superheterodyne receiver, the local-oscillator frequency is separated from the incoming frequency by a value equal to the i.f. The frequency spectrum illustrating this condition is shown in sketch (*a*). In this case, the local-oscillator frequency is shown to be above that of the carrier. If there is inadequate selectivity of the carrier frequency in the radio-frequency (r.f.) stages, it is possible for the local oscillator to mix with an incoming frequency which is above the local-oscillator frequency.

Suppose, for example, the carrier frequency is 1 MHz and the i.f. is 465 kHz. The local-oscillator frequency would then be 1,465 kHz. However, if another carrier at 1,930 kHz reached the mixer, it would also produce an i.f. signal. In this case, the 1,930 kHz signal is known as the image frequency and discrimination in the r.f. amplifier stages against this frequency is known as image-channel rejection.
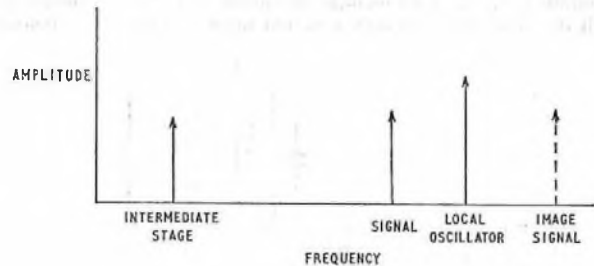
(*a*)

(*ii*) *Adjacent-channel selectivity*

An adjacent channel is situated on one side of the carrier. Internationally agreed frequency allocations provide a fixed spacing of 9 kHz between carriers. Adequate selectivity is, therefore, required in each receiver to provide a uniform passband over the required carrier spectrum and, at the same time, discriminating against the adjacent channels on each side. The process of providing adequate selectivity to avoid adjacent-channel interference is known as adjacent-channel selectivity.

(*c*) Since the mixer in a superheterodyne receiver cannot discriminate between the carrier frequency and the image, it is essential that rejection of the image channel be obtained prior to the frequency changer. If a single r.f. circuit is inadequate, then one or more additional r.f. stages would have to be provided to meet the require-

ments for adequate image rejection. Normally, the r.f. stages in a superheterodyne receiver provide tuning over a fairly wide frequency range. Under these conditions, it is difficult to provide uniformity of tuning response and adequate adjacent-channel selectivity. These problems are overcome in the i.f. stages which include fixed-tuned high-Q circuits providing a uniform passband and the required adjacent-channel selectivity, regardless of the tuning in the r.f. stages.

Q. 3. (*a*) *What is the essential difference between multiplicative and additive frequency mixers?*

(*b*) (*i*) *Give an outline circuit diagram of each of the above, and* (*ii*) *describe how the mixing operations occur.*

A. 3. (*a*) *Multiplicative mixer*

The mixing action in a multiplicative mixer is obtained through the simultaneous modulation of an electron stream by both the local-oscillator and the carrier signals. Suppose, for example, in a multi-grid valve, the mutual conductance through one grid is g_{m1} and through a second grid is g_{m2}. The carrier signal $A_1 \cos \omega_1 t$ is applied to grid 1 and the oscillator signal $A_2 \cos \omega_2 t$ is applied to grid 2. The resultant anode–current variations, due to the two applied signals, then become:

$$g_{m1}A_1 \cos \omega_1 t \times g_{m2}A_2 \cos \omega_2 t,$$

$$= g_{m1}g_{m2}A_1A_2 \cos \omega_1 t \cos \omega_2 t,$$

$$= g_{m1}g_{m2}A_1A_2\tfrac{1}{2}[\cos (\omega_1 + \omega_2)t + \cos (\omega_1 - \omega_2)t].$$

The sum and difference components are thus obtained from the product of the two applied signals.

Additive Mixer

In an additive mixer, the two signals are applied to a non-linear device such as a biased transistor. The non-linear portion of the transistor characteristic can be expressed as a power series, i.e.

$$i_c = a + bv_b + cv_b^2,$$

where i_c is the collector current,

a, b and c are constants determined by the characteristic of the transistor, and

v_b is the applied signal.

If $v_b = A_1 \cos \omega_1 t + A_2 \cos \omega_2 t$,

the carrier and oscillator signals respectively, then

$$i_c = a + b(A_1 \cos \omega_1 t + A_2 \cos \omega_2 t)$$
$$+ c(A_1 \cos \omega_1 t + A_2 \cos \omega_2 t)^2.$$

The second-order term yields:

$$A_1^2 \cos^2 \omega_1 t + 2A_1A_2 \cos \omega_1 t \cos \omega_2 t + A_2^2 \cos^2 \omega_2 t,$$

the middle term of which includes the product of the two signals from which the sum and difference components are obtained as before.
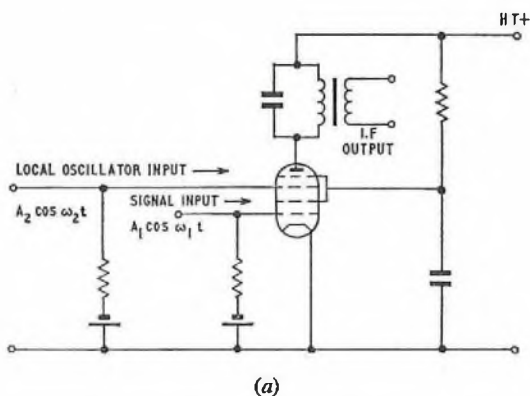
(*a*)

(*b*) The hexode valve shown in sketch (*a*) has a fourth grid which is connected to, and is at the same potential, as the second grid. This effectively isolates the third grid. Modulation of the anode current is then obtained through grid 3 as well as grid 2. Since the mutual conductance of the valve through grid 1 is greater than through grid 3, the received signal is applied to grid 1, whilst the oscillator signal is applied to grid 3. The two signals, thus, simultaneously modulate the anode current, providing the multiplicative

mixing action described in (*a*). The difference product of the two signals is selected by an intermediate-frequency (i.f.) tuned circuit operating as the anode load.

Most practical circuits utilize the triode-hexode valve which incorporates a triode in the same envelope as the hexode. The triode is coupled internally to the fourth grid of the hexode and operates in the local-oscillator circuit.

Additive mixing action can be obtained with a transistor by applying the local oscillator to the emitter whilst the signal is applied to the base of the transistor as shown in sketch (*b*). The required difference
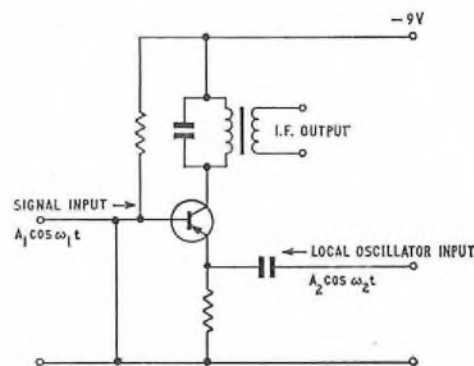
(*b*)

products are obtained through the nonlinearity of the transistor and are selected in the collector circuit, through an i.f. transformer.

In practice, a separate oscillator is not used often. Instead, a portion of the emitter output is coupled back through a resonant circuit to the emitter to provide self-oscillation, at the required local-ocillator frequency, as shown in sketch (*c*).
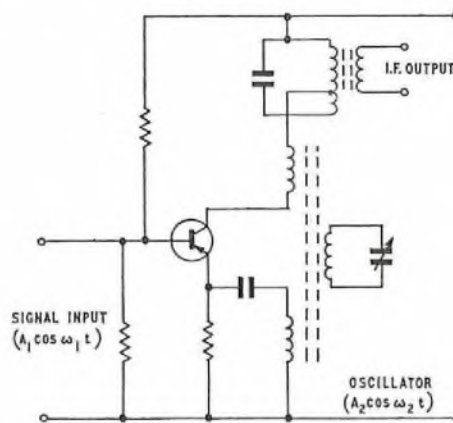
(*c*)

Q. 4. (*a*) *What is the nature of the ionosphere, and how does it affect radio propagation?*

(*b*) *With the aid of simple sketches, explain the following terms used in high-frequency long distance radio transmission.*

(*i*) *Critical frequency at vertical incidence.*
(*ii*) *Maximum usable frequency.*
(*iii*) *Skip distance.*
(*iv*) *Optimum traffic frequency.*

A. 4. (*a*) Ultra-violet radiation from the sun has the effect of separating electrons from their atoms in the outer regions of the earth's atmosphere. A spherical shell of free electrons and ions is thus created in the rarified outer atmosphere. The process is known as ionization and the region in which it occurs is termed the ionosphere. The electron density as a result of this ionization increases with height above the earth's surface and reaches a maximum at between 300 km and 500 km, and is dependent upon the time of day and season. The propagation of radio waves begins to become influenced at about 100 km where there is a fairly distinct layer of electrons, known as the E-layer. Fairly-constant conditions prevail at this height with little diurnal or seasonal variation. Above this layer, the electron density increases but is influenced considerably by the sun. During the day,

there are two main layers, F_1 and F_2, at heights approximately 200 km and 300 km respectively, and at night-time the F_2-layer descends to merge with the F_1-layer providing a maximum electron density at about 250 km. The distribution of electron density with height is shown in sketch (*a*) for day-time conditions and in sketch (*b*) for night-time conditions.


(*a*)   (*b*)
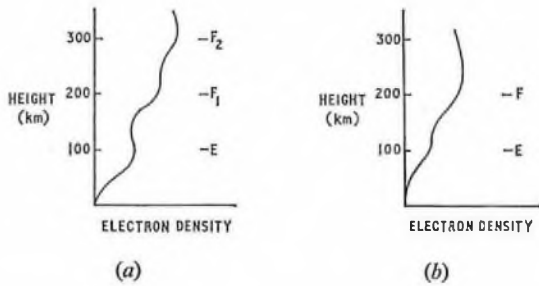
A radio wave passing through the ionosphere transfers energy to the electrons which, in consequence, oscillate in sympathy with, but not in phase with, the radio field. The oscillating electrons re-radiate energy, which in combination with the original field, has the effect of changing the direction of travel. The magnitude of the electron oscillations is inversely proportional to frequency so that the bending effects of the ionosphere and the attenuation through it are greater at lower frequencies. The influence of the ionosphere insofar as it affects radio communications becomes negligible at frequencies above about 30 MHz. Below this frequency, the ionosphere is used as a reflecting medium to provide propagation over distances of several thousands of kilometres.

(b) (i) Critical Frequency at Vertical Incidence

When a signal is transmitted vertically, there is a very limited range of frequencies over which signals are returned from the ionosphere.

The virtual height of each layer can be determined by transmitting short pulses and recording the time elapsed before the return of the pulses to a receiver. A typical curve of virtual height measured against frequency is given in sketch (*c*). The virtual height changes

(*c*)
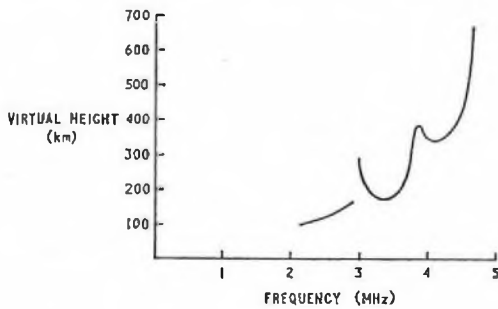
rapidly at particular frequencies where penetration of each layer occurs as the frequency is increased. For example, at 3 MHz the signal penetrates the E-layer and at about 4·5–5 MHz the signal penetrates the ionosphere entirely. The frequency at which penetration of each layer occurs under these circumstances is known as the critical frequency at vertical incidence.

(b) (ii–iv) Maximum Usable Frequency, Skip Distance and Optimum Traffic Frequency

Long-distance radio communications are obtained by beaming transmissions at low angles of elevation and by reflexion from the ionosphere.

For a given frequency, if the angle of elevation of the transmission is exceeded (or the angle of incidence reduced excessively), the radio waves are not refracted sufficiently to cause reflexion. As a corollary, for a given angle of propagation, if the frequency is increased the refraction through the ionosphere reduces and a limiting frequency is reached when communication is not possible. Thus, for communication between the two points T and R shown in sketch (*d*) there is a maximum frequency beyond which communication is not possible. This frequency is termed the maximum usable frequency, and under this condition the receiver is on the edge of the zone of reception. When the frequency is increased, the inside radius of the zone of reception increases and reception is not possible. The receiver is then in the dead zone and the distance between the transmitter and the

point lying on the edge of the zone of reception is known as the skip distance.

For communication purposes, it is undesirable to operate under the above conditions as the ionospheric layer is never constant. The frequency is, therefore, reduced so that the zone of reception lies nearer to the transmitter and envelopes the receiver. Further reduction of the frequency causes increased attenuation through the ionosphere. For given conditions of time of day, year etc., there is an optimum frequency of operation. This frequency is known as the optimum traffic frequency.

(*d*)
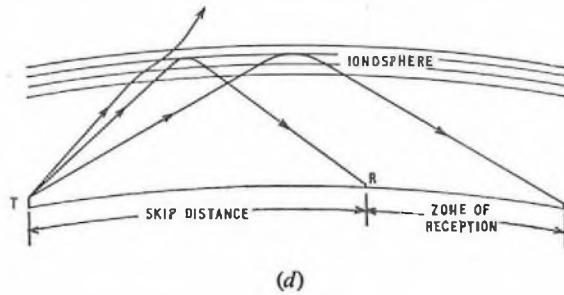
Q. 5. The radiation pattern of a half-wave dipole in any plane containing the dipole is a figure-of-eight.

(a) Explain the term radiation pattern.

(b) Starting from an elementary doublet, describe in simple terms how the radiation pattern of a half-wave dipole is produced when measured at a distant point.

A. 5. (a) Radiation Pattern

The radiation pattern of an aerial is the measurement of the electric-field intensity over the surface of a sphere having the aerial at its centre.

In practice, it is not always possible or necessary to plot complete three-dimensional radiation patterns. Two-dimensional patterns in two planes at right angles to each other generally suffice. For example, the radiation pattern of a horizontal dipole could be represented by a pattern in the horizontal and vertical planes which cross at the centre of the dipole. In this particular case the horizontal radiation pattern is a figure-of-eight and the vertical radiation pattern is a circle.

Radiation patterns are normally, although not necessarily, plotted in polar co-ordinates and measurements are usually obtained by recording field strength whilst the aerial is rotated. The radiation pattern normally of use to the systems engineer is known as the far-field radiation pattern since this is one parameter determining the field strength at the far end of the radio system, and is the effective pattern determining the performance of the aerial in terms of directivity and discrimination against interfering signals.

(b) The Elementary Doublet

The elementary doublet is a dipole whose length is short compared with a half-wavelength so that the current along its length can be regarded as uniform. The elementary doublet is of use only to determine the radiation patterns of practical aerials. Its radiation pattern in the plane containing the doublet is given by:

$$E(r_1\theta) = \frac{60\pi Il \sin \theta}{\lambda r} \text{ volts/m,}$$

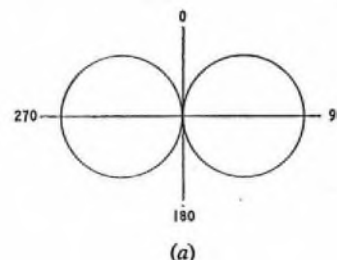where E = field strength,

l = length of dipole (m),

λ = wavelength (m),

I = current through dipole (amperes), and

$r_1\theta$ = polar coordinates of the point of measurement (r in m).

Thus, the radiation pattern is a figure-of-eight as shown in sketch (*a*).

(*a*)

The radiation pattern of a half-wave dipole can be obtained by considering the dipole as a series of elementary doublets, each with a current magnitude corresponding to that of the dipole at the doublet's particular point on the dipole.

The current distribution patterns are shown in sketches (*b*) and (*c*). The patterns are of similar shape, but of different magnitude, and

CURRENT DISTRIBUTION
OF A HALF-WAVE
DIPOLE

APPROXIMATION TO (*a*)
BY A SERIES OF
ELEMENTARY DOUBLETS

(*b*) (*c*)
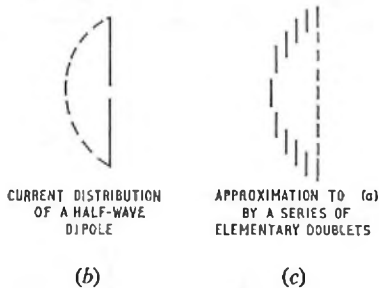
radiate from different points along the dipole. The summation of the patterns approximates to the radiation pattern of the dipole.

The radiation pattern of a dipole can be more exactly evaluated by reducing the elementary doublets in size so that they become infinitesimal. The summation of the radiations is then evaluated by calculus, which is beyond the scope of this subject. The resultant radiation pattern of the dipole is very similar to that of the elementary doublet. It provides slightly greater gain, 1·64 as compared with 1·5, and has a smaller width of lobes at the half-power points, 78° as compared with 90°.

Q. **6.** (*a*) *Derive a formula giving the voltage gain of an amplifier which has a small proportion β of the output voltage fed back in antiphase with the input voltage.*

(*b*) *State advantages and disadvantages of using negative feedback in an audio-frequency amplifier.*

(*c*) *If the gain of an amplifier without feedback is 64 dB, calculate the resultant gain in decibels when negative feedback is applied from the output to the input and β = 0·1. (It can be assumed that the amplifier has equal input and output impedances, which are unaltered by the application of feedback.)*

A. **6.** (*a*) The normal voltage gain of the amplifier shown in the sketch is A. If a fraction $β$ of the output voltage V_2 is fed back in
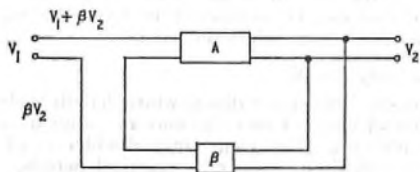
series with the input voltage V_1, the voltage applied to the amplifier input terminals becomes:

$$V_1 + βV_2.$$

Thus, the output voltage $V_2 = A(V_1 + βV_2)$,

$$= AV_1 + AβV_2.$$

$$\therefore V_2(1 - Aβ) = AV_1,$$

$$\text{and } \frac{V_2}{V_1} = \frac{A}{1 - Aβ}.$$

This is the general expression for the overall voltage gain of the amplifier with feedback.

When the feedback is negative, $β$ becomes $-β$.

Therefore,
$$\frac{V_2}{V_1} = \frac{A}{1 + Aβ}.$$

(*b*) The principal advantage of using negative feedback in an audio-frequency amplifier is the reduction of distortion for a given level of output. Let D, expressed as a fraction of the fundamental output, be the level of distortion. When negative feedback is applied, the resultant level of distortion D_0 becomes:

$$D_0 = \frac{D}{1 + Aβ}.$$

Thus, the original distortion is reduced in the ratio of $\frac{1}{1 + Aβ}$.

A further advantage of the use of negative feedback is the improvement in the gain-frequency response.

Assume, for example, that the gain varies from about 1,000 to 10,000 times over the usable frequency band. When 1 per cent of the output voltage is fed back in antiphase, the gain is reduced to 9·1 and 9·9, respectively. The gain variation over the band is, thus, reduced from 10 times to 1·09 times.

(*c*) Gain of the amplifier = 64 dB.

If A is the voltage gain,

then $20 \log_{10} A = 64,$

$$\log_{10} A = 3·2.$$

$$\therefore A = 1,585.$$

When $β = 0·1,$

$$\frac{V_2}{V_1} = \frac{1,585}{1 + 158·5},$$

$$= \frac{1,585}{159·5}.$$

$$\therefore 20 \log_{10} \frac{V_2}{V_1} = 19·95 \text{ dB}.$$

Resultant gain of amplifier with feedback = 19·95 dB.

Q. **7.** (*a*) *With the aid of a circuit diagram, describe in detail the operation of an oscillator of your choice.*

(*b*) *What factors affect adversely the frequency stability of the oscillator chosen, and how can frequency variations be minimized?*

Q. **8.** (*a*) *With the aid of a circuit diagram, describe a resistance-capacitance coupled pentode amplifier stage for use at audio frequencies.*

(*b*) *In such an amplifier stage, the following component values were used*

Anode load resistor	100 *kohm*
Interstage coupling capacitor	0·1 *μF*
Grid resistor of second stage	500 *kohm*.

If the total circuit shunt capacitance is 100 pF,

(*i*) *draw the equivalent circuits of this amplifier at low, middle and high frequencies*

(*ii*) *calculate the frequencies at which the gain falls 3 dB relative to that in the middle of the usable frequency band.*

A. **8.** (*a*) The circuit of a resistance-capacitance coupled amplifier stage is given in sketch (*a*). The pentode valve V1 has an anode load

(*a*)
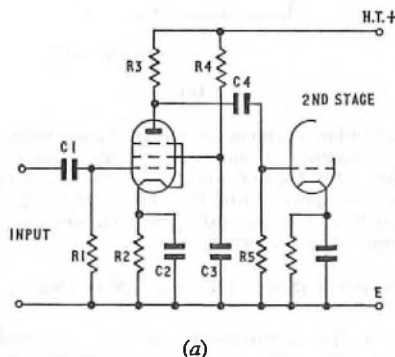
resistor R3 and a screen voltage-dropping resistor R4. R2 is a cathode-bias resistor decoupled at audio frequencies by the capacitor C2. The screen feed is decoupled by C3.

The input signal is coupled through C1 to V1 grid which, under d.c. conditions, is maintained at earth potential by R1. The d.c. current through the valve develops a voltage across R2 which provides the correct grid-cathode bias. The amplified signal is developed across R3 and is coupled to the second stage via the coupling capacitor C4. R5 is the grid resistor of the second valve. Both C2 and C3 are required to be of sufficiently low reactance at the lowest frequency of operation to effectively short circuit the resistors R2 and R4, respectively, otherwise the signals developed across the resistors reduce the effective gain of the amplifier.
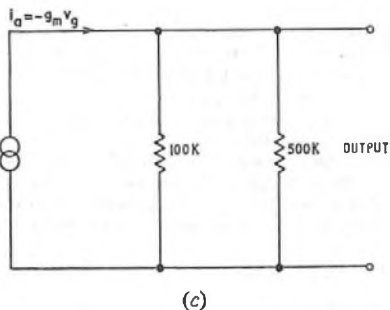
(*b*) Since the anode a.c. resistance r_a is not given, it is assumed that $r_a \geqslant 100$ k ohms.

The equivalent circuit for low frequencies is shown in sketch (*b*).

(b)
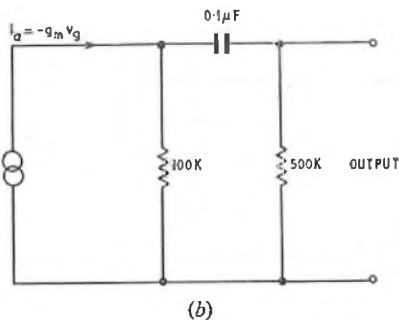
At the middle of the frequency band, when the effects of the reactances are assumed to be negligible, the effective load is the anode-load resistor in parallel with the grid resistor of the second stage, as shown in sketch (*c*).


(c)

$$R_L = \frac{100 \times 500}{100 + 500},$$

$$= 83 \cdot 3 \text{ kohms.}$$

The gain falls by 3 dB at low frequencies when the reactance of C4 becomes equal to R3 in series with R5.

Thus,

$$\frac{1}{\omega C_4} = R_3 + R_5.$$

$$\therefore \frac{1}{2\pi f \times 0 \cdot 1 \times 10^{-6}} = 100,000 + 500,000.$$

$$\therefore f = \frac{10^6}{2\pi \times 0 \cdot 1 \times 600,000} = 2 \cdot 65 \text{ Hz.}$$

The gain falls by 3 dB at high frequencies when the reactance of the total shunt capacitance (100 pF) becomes equal in magnitude to the total load resistance, i.e. R3 in parallel with R5 [see sketch (*d*)].

(*d*)
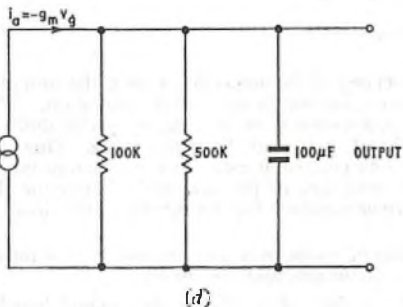
Thus, $\dfrac{1}{\omega \times 100 \times 10^{-12}} = 83,300.$

$$\therefore f = \frac{10^{12}}{2\pi \times 100 \times 83,300},$$

$$= 19 \cdot 1 \text{ kHz.}$$

Q. 9. (*a*) *Why are multi-channel carrier-cable systems in common use in national telephony networks?*

(*b*) *With the aid of a block schematic diagram, describe how a 12-channel transmission system is assembled.*

(*c*) *Describe briefly with the aid of a circuit diagram the type of modulator used in the system described in* (*b*).

A. 9. (*a*) The traffic density in the national telephone network increases at a rate of about 12 per cent per annum. In order to accommodate this traffic growth, the national network has to be increased accordingly. However, it becomes more economical to increase the traffic-handling capacity of the cables rather than to increase the number of cables. The capacity of a cable is increased by extending its bandwidth and translating telephone channels to higher frequency bands, allocating a 4 kHz band to each channel. By this means, an assembly of channels can be amplified simultaneously by one repeater and transmitted over a single cable. As many as 2,700 channels are accommodated in this manner in a frequency band of about 12 MHz. The telephone channels are assembled at each end of the cable by means of multiplexing equipment which assembles carriers at the required 4 kHz intervals.

(*b*) *Channel translating equipment*

The assembly of 12 channels for line transmission is illustrated in sketch (*a*). The audio input signal level is adjusted to the correct level into the channel modulator which translates the signal to a channel in the frequency band 60–108 kHz. This frequency band accommodates twelve channels spaced at intervals of 4 kHz. The lower sideband generated by the modulator is selected by a band-pass filter and combined with the other channels to provide a basic group of twelve channels. Sketch (*a*) shows, as an example, the assembly of

(*a*)
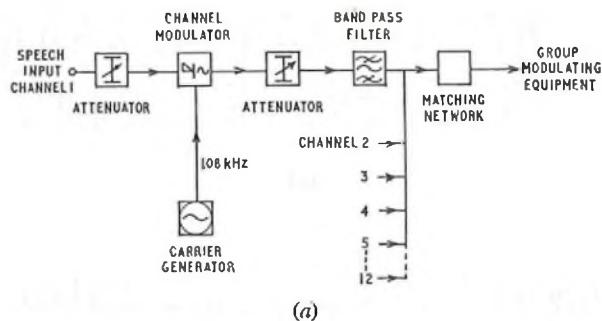
channel 1 which modulates a 108 kHz carrier to produce two sidebands occupying the band 104–112 kHz. The lower sideband is selected by the band-pass filter to give the channel allocation of 104–108 kHz. Channels 2–12 are selected in a similar manner, and require carriers at 4 kHz intervals down to 64 kHz, and in each case the lower sideband of each modulated signal is selected to provide bands of 104–100 kHz, 100–96 kHz down to 64–60 kHz. The assembled group of channels is passed through a matching network and thence into a group modulator.

Group modulating equipment

For transmission over a pair of lines, this channel group is translated down to the frequency range 12–60 kHz. This is obtained by modulating a 120 kHz carrier and selecting the lower sideband through a 60 kHz low-pass filter. This arrangement is shown in sketch (*b*).

(*b*)
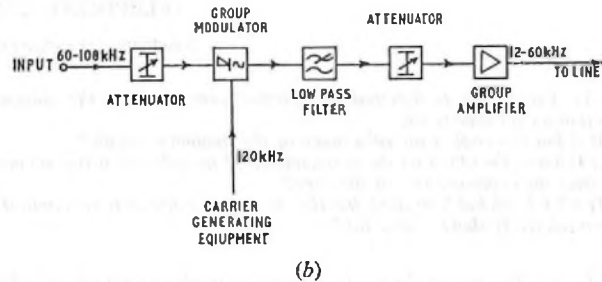
The output attenuator is for final level adjustment into the group amplifier which provides the output level required for transmission along the line.

(*c*) A suitable type of modulator for the above system is the Cowan modulator, the circuit of which is shown in sketch (*c*).

(*c*)   (*d*)   (*e*)

A. 2. (*a*) The circuit elements concerned with vertical stepping under the control of dialled pulses are shown in sketch (*a*).

(*a*)
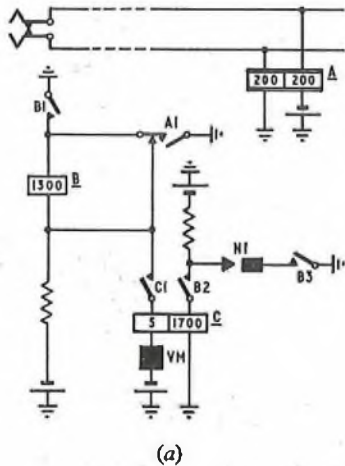
The A relay is pre-operated by the subscriber's loop before dialling commences; contact A1 operated operates relay B and B2 operated operates relay C. On dialling, the dial springs break the loop and the A relay releases. A1 in releasing short-circuits relay B and energizes the vertical magnet via B1 operated, A1 released, C1 operated, the 5-ohm coil of relay C, vertical magnet coil to battery. The vertical magnet steps and, in stepping, operates the N1 springs, thus short-circuiting C relay via B2 and B3 operated. Both B and C relays are, thus, made slow to release.

On the next make period of the dial springs, A relay re-operates and A1 removes the short circuit from B relay, thus re-energizing it. A1 also de-energizes the vertical magnet, which releases, and C relay which holds because of its slow-release feature.

The next break period of dial springs again releases relay A; A1 released again short-circuits relay B and re-energizes relay C and the vertical magnet. The vertical magnet steps again and this process is repeated for the whole of the vertical pulse train.

At the end of the vertical pulse train, A relay re-operates, releasing the vertical magnet and leaving C relay un-energized. After its slow-release period, relay C releases, initiating the change-over to rotary stepping.

(*b*) The required current/time charts are shown in sketch (*b*).

(*b*)
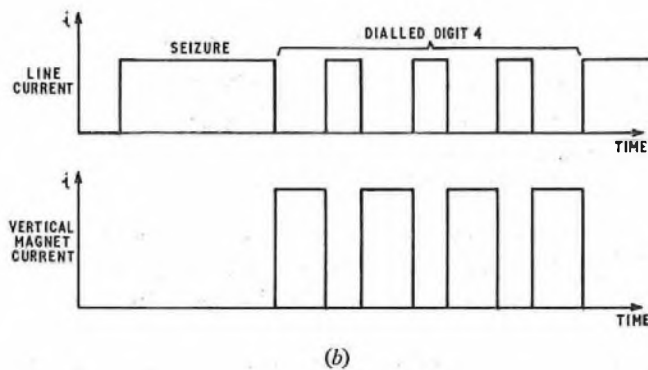
(*c*) If no relay were interposed between the vertical magnet and the line wires, then the current available to energize the magnet would be limited by the resistance of the line. This would introduce a severe limitation on the permissible line resistance or would necessitate a higher battery voltage to drive sufficient current through such a resistance. Another reason is that the passing of current of such a magnitude through the line resistance would result in a considerable increase in the wasteful dissipation of power in the line and would increase the power consumption of the exchange.

Q. **3.** (*a*) *What purposes are served by the steady line current that flows in a telephone circuit while the handset is in use?*
   (*b*) (*i*) *Why is it necessary to prevent this current from flowing in the telephone receiver?*
     (*ii*) *With the aid of a sketch of the telephone circuit, explain how this is achieved.*

A. 3. (*a*) The steady line current which flows whilst the handset is in use serves two purposes. It acts as a transmitter-feeding current to energize the transmitter and it holds the A relay in the exchange until the disconnexion of the circuit signals a forward clear.

(*b*) (*i*) It is necessary to prevent this steady current from flowing in the receiver since d.c. would desensitize the receiver.

(*ii*) The sketch shows the outline of a telephone circuit. For the duration of a call, a steady line-current flows through the telephone via

(*a*)
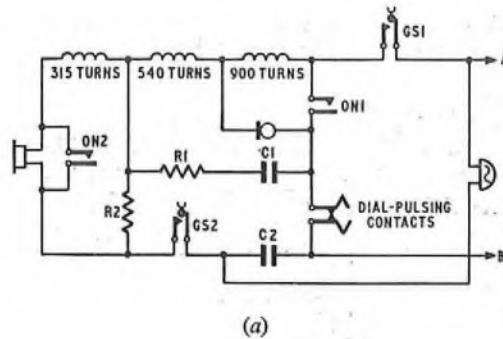
GS1 operated, 900-turn coil, transmitter and dial-pulsing contacts. This steady current (d.c.) is prevented by capacitor C2 from flowing in the receiver circuit. Alternating speech currents flow via GS1, 900-turn coil, 540-turn coil, 315-turn coil, receiver, GS2 and C2. Capacitor C2 may be regarded as a short-circuit to a.c. and a disconnexion to d.c.

Q. **4.** (*a*) *Why, in the busy hour, is the traffic carried less than the traffic offered?*
   (*b*) *What essential differences would there be in the method of connecting 18 trunks from*

     (*i*) *the banks of subscribers' uniselectors,*
     (*ii*) *a level of* 100-*outlet group selectors?*

   (*c*) *Why would arrangement* (*i*) *give the better grade of service if the traffic offered were the same in each case?*

A. 4. (*a*) Telephone circuits are provided in sufficient numbers to provide a pre-determined grade of service to traffic offered in the busiest period of the day. Thus, it is accepted that a small percentage of calls will fail and that a call originating in the busy hour has a statistical chance of failure equal to the grade of service.

The reason that it is not possible to provide a zero grade of service (no traffic lost) is that telephone calls originate at pseudo-random intervals and although, over a long period, the average number of calls in progress may remain approximately constant, at any particular instant there may be many more, or less, calls in progress than the average. To cater for these peaks caused by the random nature of traffic would require a very large number of circuits, most of which would be idle except for these very short-duration peaks. This is economically unacceptable and circuits are provided to meet the average traffic flow over the busiest hour of the day at the specified grade of service.

(*b*) (*i*) The banks of a subscriber's uniselector have an availability of 25. Thus, with 18 trunks, full-availability conditions would apply. The 18 trunks would occupy the first 18 outlets, the remaining outlets would be spare and every uniselector would have access to all 18 trunks.

(*ii*) The banks of a 100-outlet group selector have an availability of 10 and so, with 18 trunks, limited-availability conditions would apply and a grading would be necessary. A grading similar to that shown in the sketch would be used and every selector would have access to only 10 of the 18 trunks.

(*c*) The full-availability scheme would provide the better grade of service because the average traffic carried per trunk would be higher. This is so because, during peak traffic periods, all traffic offered has access to all trunks. With the limited-availability scheme, it is sometimes the case that offered traffic is lost due to congestion whilst trunks to which that traffic does not have access are still free.

Q. 5. (*a*) *Draw a block schematic diagram of a floated-battery power-plant for a telephone exchange.*

(*b*) *With the aid of sketches of the circuit elements concerned, explain how the following functions are performed.*

   (*i*) *Reduction of mains voltage.*
   (*ii*) *Conversion of alternating current to direct current.*
   (*iii*) *Smoothing.*

(*c*) *Why is a longer life predicted for a battery which is floated compared with one subjected to charge-discharge cycles?*

A. 5. (*a*) A block diagram of a floated-battery power plant for a telephone exchange is shown in sketch (*a*).

(*a*)
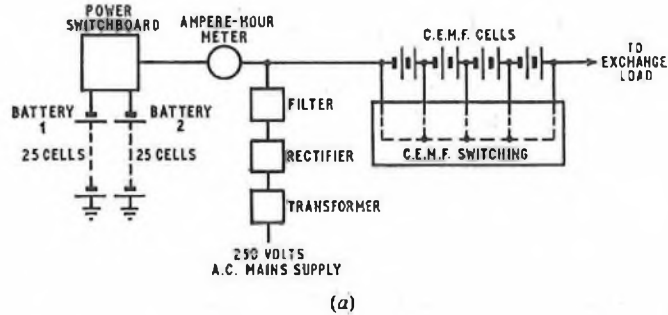
(*b*) (*i*) Mains voltage at 250 volts a.c., is reduced to exchange-battery voltage, —50 volts, by a 5:1 step-down transformer, as shown in sketch (*b*).

(*ii*) Alternating current derived from the mains is converted to d.c. by a full-wave rectifier, as shown in sketch (*c*). During one half-cycle

(*b*)   (*c*)   (*d*)
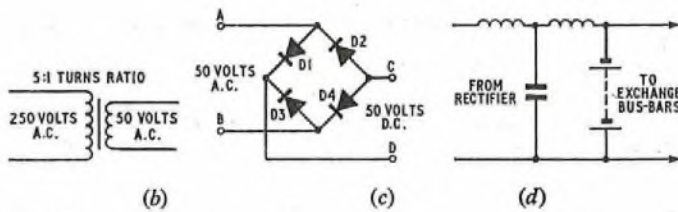
of the incoming a.c. waveform, current flows from terminal A via D1 to point D, through the load to terminal C, D4 and to terminal B. Diodes D2 and D3 are reverse-biassed and are non-conducting. During the next half-cycle, the incoming current flow is reversed and the path is from terminal B via D3, terminal D, load, terminal C, D2 to A; D1 and D4 do not conduct. Thus, current in the load flows in one direction only, irrespective of the direction of the input current.

(*iii*) The output of a full-wave rectifier consists of direct current but included with this is a ripple alternating component. This ripple may be smoothed by the circuit shown in sketch (*d*). Considering the steady d.c. component only, such a circuit is equivalent to a very low series resistance—the resistance of the chokes—and the capacitor is, effectively, a disconnexion. However, for the a.c. component, the circuit acts as a large series reactance (the inductors) together with a low shunt reactance (the capacitor) and the battery. Thus, a.c. ripple is both attenuated and shunted, and is considerably reduced.

(*c*) The main cause of cell deterioration under charge/discharge is the gassing which occurs as cells are recharged. Such gassing loosens the active material on the plates, thus both reducing the capacity and building up a sludge in the bottom of the cell. Under float systems, a cell is fully charged at all times and no gassing occurs. Also, the danger of sulphation occurring is reduced since this affects discharged cells only.

Q. 6. (*a*) *Sketch the stepping mechanism of a 25-point uniselector and describe its mechanical operation under self-drive conditions.*

   (*b*) (*i*) *At what point in the travel of the armature should the magnet interrupter contacts be set to open?*
        (*ii*) *What is the effect of advancing the time of opening?*

A. 6. (*a*) The stepping mechanism of a uniselector is shown in the sketch. When the magnet coil is energized, the armature is attracted to the coil against the pressure of the restoring spring. As the armature pivots about the knife-edge, the armature-extension pushes the pawl forward. As the pawl moves forward, it rides over the long face of the next tooth of the ratchet wheel and drops into engagement with the short

face. The ratchet-wheel and wiper assembly are prevented from moving backwards by the detent which is tensioned against the ratchet.
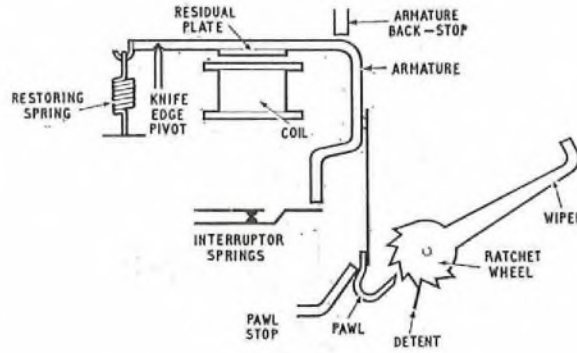
Partway through the armature stroke, the armature breaks the interrupter springs and disconnects the magnet circuit. The armature is restored by the tension of the restoring spring and, in restoring, the pawl pulls the ratchet-wheel and wiper assembly round one step. During the step, the detent rides up the long face of the ratchet tooth and drops into engagement with the short face of the next tooth. As it restores, the armature allows the interrupter springs to remake and the magnet circuit is reconnected. The process is then repeated.

With the armature normal (resting against the armature back-stop) the pawl is lightly wedged between the pawl stop and the ratchet. By this means, the ratchet-wheel is positively restrained at the end of the armature stroke and the momentum of the wiper assembly is prevented from carrying the assembly beyond the bank contact position.

(*b*) (*i*) The interrupter contacts are set to open part-way through the armature stroke such that the armature will just complete its stroke under its own momentum.

(*ii*) If the time of opening were advanced further, the armature would not travel far enough to engage the pawl on the next tooth. The armature would partially operate and release without stepping the wiper assembly and would continue until either the call attempt was discontinued, the coil burned out, or some outside agency intervened.

Q. 7. (*a*) *Explain the essential differences between*

   (*i*) *morse code and 5-unit telegraph code*
   (*ii*) *single-current working and double-current working.*

(*b*) *Sketch current/time graphs to show how a letter of the alphabet may be signalled telegraphically by the possible combinations of the above. (Ignore the effects of distortion.)*

Q. 8. (*a*) *What purposes are served by the double-ended cord circuits of a manual switchboard?*

(*b*) *Explain why the number of cord circuits provided on a manual switchboard determines the traffic capacity in erlangs.*

(*c*) *How does the cord circuit provide calling signals to*

   (*i*) *a local telephone*
   (*ii*) *a distant switchboard over a manual junction?*

Q. 9. (*a*) *What are the characteristics desirable in a relay used for the reception of telegraph signals?*

(*b*) *What constructional features do these characteristics demand?*

(*c*) *How can contact bounce be minimized in a polarized relay?*

A. 9. (*a*) The major desirable features in a telegraph receiving relay are as follows.

   (*i*) It requires a high sensitivity.
   (*ii*) It requires a rapid response time.
   (*iii*) Its operate and release characteristics must be symmetrical.
   (*iv*) It is required to be polarized, i.e. directional.

(*b*) The main features of a commonly-used telegraph relay are shown in the sketch.

The sensitivity of the relay is achieved by the use of two permanent magnets. The arrangement is such that the relay is just side-stable in the unenergized condition in either side position. The flux necessary to initiate change-over is, thus, only that necessary to tip the magnetic balance whilst the major portion of flux is due to the permanent magnets.

The rapid response time is dependant upon a number of factors:

   (*i*) an efficient magnetic circuit,
   (*ii*) short armature travel,
   (*iii*) bounce-free contacts, and
   (*iv*) small mass of moving parts.

The magnetic circuits shown are efficient, closed, iron paths except for small air-gaps. The armature travel, shown exaggerated in the sketch, is short and the armature assembly is very light. Bounce-free contacts are provided.

The polarized feature and the symmetrical operate and release characteristics are achieved by the use of a symmetrical mechanical design and the use of permanent magnets in opposition.

(*c*) Contact bounce is minimized by the use of contacts mounted on resilient anti-chatter springs. These permit the fixed contact to move slightly and follow the moving contact whilst the free end of the spring dissipates energy as friction against the buffer plate.
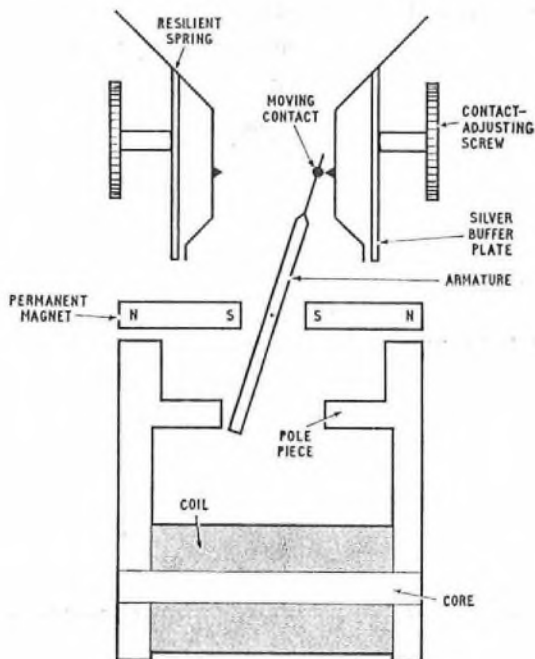
Q. **10.** (*a*) *With the aid of sketches of the circuit elements concerned, describe how a uniselector hunts for and seizes a disengaged outlet.*

(*b*) *What would happen if the P-wire of the seized outlet were disconnected as a result of a wiring fault?*

(*c*) *At what point or points is such a fault most likely to arise?*

A. **10.** (*a*) The circuit concerned is shown in the sketch. When a call is originated, a loop is extended forward to operate L relay via K1 and K2 normal. L in operating prepares the uniselector stepping circuit at L1 whilst L2 completes the initial step circuit and places a fast guard on the final-selector multiple P-wire. The drive magnet is energized from earth, L2 operated, home contact of P-arc, P-wiper L1 operated, K5 normal, DMdm, DM to battery. The drive magnet is energized, the DMdm interrupter springs break and DM releases, stepping the wipers to the first contact.

On stepping, the L2 earth is removed from the P-wiper and thus from the DM. Re-energization of DM is thus dependent on the condition

on the P-wire outlet. Also the K relay, previously short-circuited by virtue of the L2 earth being applied to both sides of the coil, is no longer in this condition. The L2 earth is still applied to one side of the coil but the other side is connected via K5 and L1 operated to the P-wiper.

Thus an earth on the P-bank contact (the busy condition) prevents the operation of K relay and re-energizes DM via K5, L1 operated, P-wiper to earth. In due course, DMdm springs break and the wipers are stepped to the next contact. This process is repeated so long as the wipers are stepping over engaged contacts with earthed P-wires.

When the wipers step on to a contact without an earth on the P-wire there is no earth available for the re-energization of DM and so no further drive is possible. K relay now operates from L2 operated to the DM battery but DM will not operate in series with the K relay coil. In operating, K1 and K2 disconnect the L relay and extend the line wires to the wipers, K4 prepares the meter circuit, K3 applies a fast guard earth to busy the P-wire until the backward guard condition is returned whilst K5 prevents the guarding earth from re-energizing the DM. After the slow release of L, the L2 earth is removed and K holds to the backward guard earth on the P-wire.

(*b*) Such a wiring fault, occurring after a call had been set up, would leave the K relay disconnected, leading to its release. The release of K would complete the self-drive homing circuit of DM from the earth on the homing arc via L1, K5 released, DMdm to DM. Drive would cease on the home contact.

However, K1 and K2 would reconnect L to the line after the L operate lag and a new call would be originated. If the same contact were seized the switching sequence would be completed again but the call would fail after the slow release of L because of the lack of backward guard.

If the operate lag of L were less than the drive-to-home time of DM, the home-drive circuit would be cut at L1 and testing for a free outlet would recommence over the latter part of the bank.

(*c*) The likely places for a P-wire disconnexion to occur are:

(*i*) flexibility points in the cabling to the first selector racks and

(*ii*) the B relay contact in the first selector which provides the guarding earth.
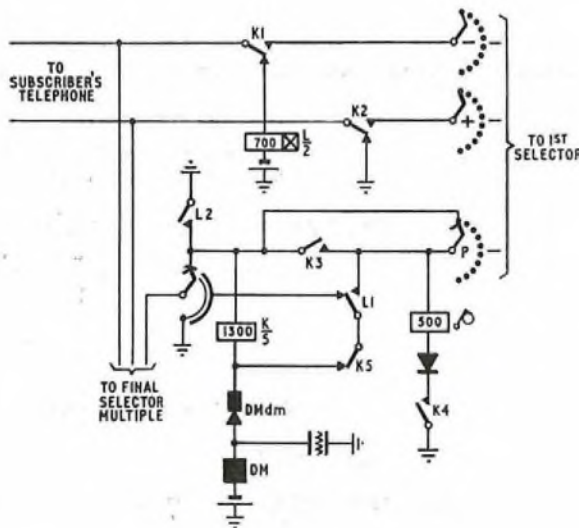
LINE TRANSMISSION C, 1972

Students were expected to answer any six questions

Q. **1.** (*a*) *Show how the characteristic impedance and the attenuation coefficient of a uniform transmission line can be calculated from its primary coefficients.*

(*b*) *A coaxial cable has a loss of 6 dB/km at 1 MHz and of this loss 1 dB/km is due to the dielectric. Estimate the loss of the cable at 4 MHz, stating clearly the assumptions made.*

A. **1.** (*a*) For a uniform transmission line, the characteristic impedance Z_0 can be expressed as:

$$Z_0 = \sqrt{\left(\frac{R + j\omega L}{G + j\omega C}\right)} \text{ ohms} \qquad \ldots\ldots(1)$$

and the propagation coefficient as:

$$\gamma = \sqrt{\{(R + j\omega L)(G + j\omega C)\}} \qquad \ldots\ldots(2)$$

where ω is the angular velocity (rad/s),

R is the loop resistance (ohms/km),
L is the loop inductance (H/km),
G is the loop leakance (S/km) and
C is the loop capacitance (F/km).

R, L, G and C are known as the primary coefficients.

The propagation coefficient γ is a complex quantity, and

$$\gamma = \alpha + j\beta, \qquad \ldots\ldots(3)$$

where α is the attenuation coefficient (nepers/km) and β is the the phase-change coefficient (rad/km).

The characteristic impedance can be calculated by direct substitution in equation (1). It is generally preferable to derive values for $R + j\omega L$

and $G + j\omega C$ as the first step rather than to attempt rationalization of equation (1) itself.

The attenuation coefficient can be calculated by direct substitution in equation (2) followed by derivation of the real part α using equation (3). Again, it is convenient to derive values for $R + j\omega L$ and $G + j\omega C$ as the first step.

(*b*) For a coaxial cable at 1 MHz, it can be assumed that

$$\alpha \simeq \frac{R}{2}\sqrt{\left(\frac{C}{L}\right)} + \frac{G}{2}\sqrt{\left(\frac{L}{C}\right)} \text{ nepers/km} \quad \ldots\ldots(4)$$

(See A.3, Line Transmission C, 1970, *Supplement*, Vol. 64, p. 79, Jan. 1972).

In equation (4), the first term (containing R) represents the series loss and the second term (containing G) represents the shunt or dielectric loss. At 1 MHz, it is assumed that the skin effect is developed to the point where the series loss is proportional to $\sqrt{(f)}$. It is also assumed that the dielectric loss is proportional to f.

Thus, total loss = series loss + shunt (or dielectric) loss,

$$= k_1\sqrt{(f)} + k_2 f,$$

where k_1 and k_2 are constants.

For the coaxial cable at 1 MHz,

$$\text{total loss} = 6 \text{ dB/km},$$
$$\text{dielectric loss} = 1 \text{ dB/km} = k_2 \times 10^6, \text{ and}$$
$$\text{series loss} = 5 \text{ dB/km} = k_1 \times 10^3.$$

Hence, $\qquad k_1 = 5 \times 10^{-3} \text{ and } k_2 = 1 \times 10^{-6}.$

Thus, for the coaxial cable at 4 MHz,

$$\text{total loss} = 5 \times 10^{-3}\sqrt{(4\times10^6)} + 1\times10^{-6}\times4\times10^6,$$
$$= 10 + 4,$$
$$= \underline{14 \text{ dB/km.}}$$

(*to be continued*)