SUPPLEMENT
TO THE
POST OFFICE ELECTRICAL ENGINEERS' JOURNAL

Vol. 56 No. 4 January 1964

CONTENTS

CITY AND GUILDS OF LONDON INSTITUTE EXAMINATIONS, 1963

QUESTIONS AND ANSWERS

Answers are occasionally omitted or reference is made to earlier Supplements in which questions of substantially the same form, together with the answers, have been published. Some answers contain more detail than would be expected from candidates under examination conditions.

ENGINEERING SCIENCE, 1963

Students were expected to attempt four questions from Q. 1–6 and two from Q. 7–10.

Q. **1.** *A load of 500 lb is supported by two wires attached to eye bolts, one in the wall and one in the roof, as shown in Fig.* 1 *below.*

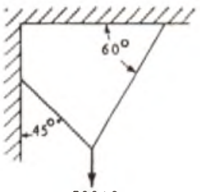
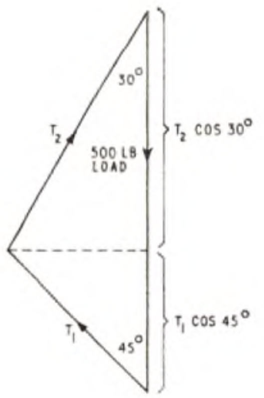


Fig. 1

Find the tension in each wire.

The ultimate strength of the wires is 40 tons/in². Allowing a safety factor of 5, what should be the diameter of the wire from the roof?

A. **1.** The point of connexion of the 500 lb load is in equilibrium under the action of the load and the tension in the wires. The triangle of forces for this point is shown in the sketch, where T_1



represents the tension in the wire connected to the wall, and T_2 the tension in the wire connected to the roof.

By measurement from the scale drawing,

$$T_1 = \underline{260\text{ lb}},\text{ and } T_2 = \underline{365\text{ lb}}.$$

Alternatively,

$$T_1 \cos 45° + T_2 \cos 30° = 500,$$

$$\text{i.e. } \frac{T_1}{\sqrt{2}} + \frac{\sqrt{3}}{2} T_2 = 500.$$

$$\therefore \sqrt{2}\, T_1 + \sqrt{3}\, T_2 = 1{,}000 \quad \dots\dots (1)$$

Also, $T_1 \sin 45° = T_2 \sin 30°$.

$$\frac{T_1}{\sqrt{2}} = \frac{T_2}{2}.$$

$$\therefore \sqrt{2}\, T_1 = T_2.$$

Substituting in equation (1),

$$T_2 + \sqrt{3}\, T_2 = 1{,}000.$$

$$\therefore T_2 = \frac{1{,}000}{\sqrt{3}+1} = \underline{366\text{ lb.}}$$

$$\therefore T_1 = \frac{1{,}000}{\sqrt{2}(\sqrt{3}+1)} = \underline{259\text{ lb.}}$$

Allowing a factor of safety of 5, the stress in the wire from the roof should not exceed 8 tons/in².

Thus, T_2 must not exceed $8 \times 2{,}240 \times A$ lb, where A is the cross-sectional area.

$$\therefore \text{Minimum value of } A = \frac{366}{8 \times 2{,}240} = 0{\cdot}02043 \text{ in}^2.$$

If the diameter of the wire is d in., $A = \dfrac{\pi d^2}{4}$.

$$\therefore d = \sqrt{\frac{4A}{\pi}} = \sqrt{\frac{4 \times 0{\cdot}02043}{3{\cdot}142}} = 0{\cdot}162 \text{ in.}$$

Therefore, the diameter of the wire should not be less than $\underline{0{\cdot}162\text{ in.}}$

Q. **2.** *Explain what is meant by the moment of a force about a point, and state the principle of moments.*

Fig. 2 *below shows a uniform bar 16 ft long weighing 20 lb, which is*

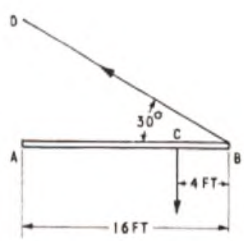


Fig. 2

pivoted at A. A load of 40 lb is attached to the bar at the point C, 4 ft from B. Find the tension in the cord BD required to keep the bar in a horizontal position.

What is the magnitude and direction of the vertical force acting on the bar at the end A?

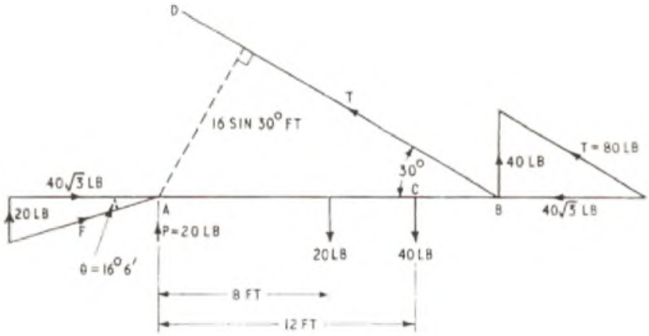
A. 2. The moment of a force about a given point is the product of the force and the perpendicular distance of the point from the line of action of the force. If a rigid body which is free to turn about a fixed point is acted upon by a force whose line of action does not pass through the fixed point, it will experience a turning force or moment.

The principle of moments states that if any number of co-planar forces acting on a rigid body has a resultant, the algebraic sum of their moments about any point in their plane is equal to the moment of the resultant about that point. It follows that if a system of co-planar forces is in equilibrium, the algebraic sum of their moments about any point in their plane is zero, i.e. the sum of the anticlockwise moments is equal to the sum of the clockwise moments.

The bar in Fig. 2 is in equilibrium under the action of

(*a*) the tension in the cord BD,
(*b*) the weight of the bar,
(*c*) the 40 lb load, and
(*d*) the reaction at the pivot A.

The sketch shows the magnitude and direction of these forces.



Taking moments about point A, and letting T be the tension in the cord,

Sum of clockwise moments = Sum of anti-clockwise moments.

Thus, $8 \times 20 + 12 \times 40 = T \times 16 \sin 30° = T \times 8.$

$$\therefore\ T = \frac{160 + 480}{8} = 80 \text{ lb.}$$

$\therefore$ The tension in the cord, required to keep the bar in a horizontal position, is 80 lb wt.

The tension in the cord and the force acting at point A can both be resolved into horizontal and vertical components. If the bar is in equilibrium, the algebraic sum of the vertical components must be zero; the algebraic sum of the horizontal components must also be zero. These components are shown in the sketch, and it will be seen that the weight of the bar and the load must be balanced by the sum of the vertical components of the tension in the cord and the vertical force at A. If P is the vertical force at A,

$$20 + 40 = T \sin 30° + P = 40 + P.$$

$$\therefore\ P = 20 \text{ lb.}$$

The horizontal component of T must be balanced by the horizontal component of the force at A since there are no other horizontal forces in the system.

The horizontal component of T, $T \cos 30° = 40\sqrt{3}$ lb.

$\therefore$ Horizontal component of force at A $= 40\sqrt{3}$ lb.

The actual force at A will act at an angle θ to the horizontal, as shown in the sketch. Let the magnitude of this force be F.

$$\text{Now } \tan\theta = \frac{20}{40\sqrt{3}} = \frac{1}{2\sqrt{3}} = \frac{1}{3{\cdot}464} = 0{\cdot}2887.$$

$$\therefore\ \theta = 16°\ 6'.$$

$$\therefore\ F = \frac{P}{\sin\theta} = \frac{20}{0{\cdot}2773} = 72 \text{ lb.}$$

Therefore the reaction F at A has a magnitude of 72 lb wt. and acts in an upward direction at 16° 6′ to the horizontal.

Q. 3. *Define the newton, the joule and the watt.*

Water is pumped upward at a uniform rate through a vertical distance of 50 metres into a storage tank having a capacity of 30 cubic metres. The power input to the pumping systems is 10 kW. If the overall efficiency is 65 per cent, find the time taken to fill the tank.

A. 3. The newton is defined as that force which will give a mass of 1 kilogramme an acceleration of 1 m/s^2.

The joule is defined as the work done by a force of 1 newton acting through a distance of 1 metre, and is, therefore, equal to 1 newton-metre.

The watt is the unit of power and is defined as a rate of working of 1 newton-metre per second or 1 joule per second.

The mass of water required to fill the tank is 30,000 kg (as, by definition, 1 cm^3 of water has a mass of 1 gramme) and the work done in raising this quantity of water 50 metres against the force of gravity is $30{,}000 \times 9{\cdot}81 \times 50$ joules.

Let the time taken to fill the tank be t seconds, then the rate of working, or the power required, is

$$\frac{\text{work done}}{\text{time}} = \frac{30{,}000 \times 9{\cdot}81 \times 50}{t} \text{ watts.}$$

The power available for pumping if the overall efficiency is 65 per cent is

$$0{\cdot}65 \times 10{,}000 = 6{,}500 \text{ watts.}$$

$$\therefore\ \frac{30{,}000 \times 9{\cdot}81 \times 50}{t} = 6{,}500.$$

$$t = \frac{30{,}000 \times 9{\cdot}81 \times 50}{6{,}500} = \frac{29{,}430}{13} \text{ seconds.}$$

$$= 37{\cdot}8 \text{ minutes.}$$

Q. 4. *Calculate the position of the centre of gravity of the thin uniform lamina shown in Fig. 3.*

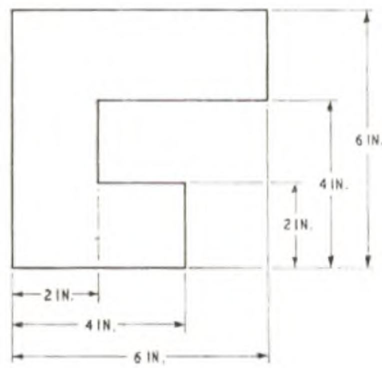
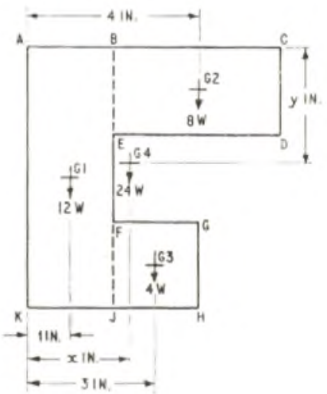


Fig. 3

Explain how you would find, by experiment, the position of the centre of gravity of an irregularly-shaped lamina.

A. 4. Consider the lamina as composed of three rectangles ABJK, BCDE and FGHJ, with their respective centres of gravity at G1, G2 and G3, as shown in sketch (*a*). Let the centre of gravity of



(*a*)

the complete lamina be at G4, x in. to the right of AK and y in. below AC, as shown.

Let the weight per unit area of the lamina be w, then the weight acting vertically down through G1 is

$$w \times \text{AK} \times \text{KJ} = w \times 6 \times 2 = 12w.$$

The weight acting vertically down through G2 is

$$w \times \text{BC} \times \text{CD} = w \times 4 \times 2 = 8w.$$

The weight acting vertically down through G3 is

$$w \times FG \times GH = w \times 2 \times 2 = 4w.$$

Hence, the total weight of the lamina is equal to

$$12w + 8w + 4w = 24w.$$

Now, about any point and for any position of the body, the sum of the moments of the forces acting through G1, G2 and G3 must be equal to the moment about the same point of the total force acting through G4.

For the position shown in sketch (*a*), and taking moments about A,

$$12w \times 1 + 8w \times 4 + 4w \times 3 = 24w \times x$$

Dividing both sides by w,

$$12 + 32 + 12 = 24x$$

$$x = \frac{56}{24} = 2\tfrac{1}{3} \text{ in.}$$

Note: If the lamina was freely suspended at a point in AC $2\frac{1}{3}$ in. from A it would remain in the position shown in sketch (*a*), i.e. with AC horizontal.

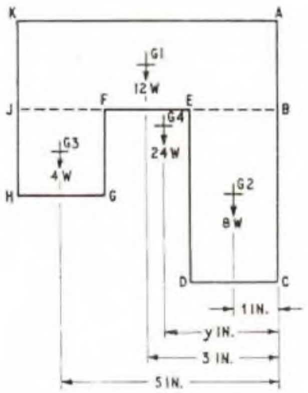
Now consider the body turned through 90°, as shown in sketch (*b*), and again taking moments about A,

$$12w \times 3 + 8w \times 1 + 4w \times 5 = 24w \times y.$$

$$36 + 8 + 20 = 24y.$$

$$y = \frac{64}{24} = 2\tfrac{2}{3} \text{ in.}$$

Note: If the lamina was freely suspended at a point in AK



(*b*)

$2\frac{2}{3}$ in. from A it would remain in the position shown in sketch (*b*), i.e. with AC vertical.

Thus the centre of gravity is $2\frac{1}{3}$ in. to the right of AK and $2\frac{2}{3}$ in. below AC, as shown in sketch (*a*).

An experimental method for finding the position of the centre of gravity of an irregularly-shaped lamina is given in A.4, Engineering Science, 1960, *Supplement*, Vol. 53, No. 4, p.58, Jan. 1961.

Q. 5. *What is meant by the statement that the mechanical equivalent of heat is* 4·18 *joules per calorie?*

The coefficient of friction between a 10 *kg mass and the horizontal table on which it rests is* 0·3. *Find the force necessary to slide the mass over the table with uniform velocity, and the heat generated when the mass moves a distance of* 2 *metres.*

What happens when the force applied is double that required to move the mass?

A. 5. When work is done by a force moving against a resistance, energy is expended generally in the form of heat.

Joule performed experiments, using a calorimeter containing water and a paddle system operated by a falling weight, to establish a relationship between work and heat. The amount of work done on the paddle system by the falling weight was known, and the amount of heat generated could be ascertained from the rise in temperature and the known physical constants of the calorimeter. Thus, he was able to establish that a certain amount of work done on a system results in a certain gain in heat energy of the system. He found that approximately 4·2 joules of mechanical work done resulted in a heat gain of 1 calorie. Later determinations have shown that 4·18 joules of work done is equivalent to 1 calorie of heat, and this relationship is known as the mechanical equivalent of heat.

The force necessary to slide the mass over a horizontal table with uniform velocity will be equal to the limiting friction, which is given by $F = \mu R$, where R is the vertical force between the mass and the table, and μ is the coefficient of friction.

$$F = 0{\cdot}3 \times 10 \times 9{\cdot}81 \text{ newtons}$$
$$= 3 \times 9{\cdot}81 = \underline{29{\cdot}43 \text{ newtons.}}$$

Work done in sliding the mass a distance of 2 metres

$$= \text{force} \times \text{distance}$$
$$= 29{\cdot}43 \times 2 \text{ newton-metres}$$
$$= 58{\cdot}86 \text{ joules.}$$

$$\therefore \text{ Heat generated} = \frac{58{\cdot}86}{4{\cdot}18} \text{ calories}$$
$$= \underline{14{\cdot}06 \text{ calories.}}$$

If the force is doubled, half the force will be used to overcome the frictional force, which remains constant, and the other half will cause the mass to accelerate.

Now F = mass × acceleration.

Therefore, if a is the acceleration of the mass,

$$F = 29{\cdot}43 = 10 \times a.$$

$$\therefore a = \frac{10}{29{\cdot}43} \text{ m/s}^2$$
$$= 0{\cdot}34 \text{ m/s}^2.$$

Therefore, when the force is doubled, the mass will be moved across the table with a constant acceleration of $0{\cdot}34$ m/s².

Q. 6. *Explain what is meant by the terms "tensile stress," "tensile strain," and "Young's modulus."*

During a tensile test on a wire of length 2 *metres and diameter* 0·12 *cm, the following figures were obtained:*

Load in kg	2	4	6	8	10	12
Extension in cm	0·010	0·019	0·028	0·037	0·048	0·063

Comment on the shape of the load/extension graph, and calculate Young's modulus for the material of the wire.

How could the measurements of extension and diameter be made?

A. 6. The tensile stress of a material is the force per unit cross-sectional area induced in the material when it is under tension, i.e. when the external forces applied to it are tending to pull it apart.

Thus, tensile stress = P/A,

where P = the load, and

A = the cross-sectional area normal to the direction of P.

Tensile strain in a material is the ratio of the extension, which takes place when it is put under tension, to the original length of the material.

Thus, tensile strain = x/l,

where x = the extension of the material, and

l = its original length.

As x and l are both measured in the same units, strain is a ratio and has no units.

Within the elastic limit, the relationship between stress and strain is a constant. This constant is known as the modulus of elasticity or Young's modulus.

$$\therefore \text{ Young's modulus} = \frac{\text{stress}}{\text{strain}} = \frac{P/A}{x/l} = \frac{P}{x} \times \frac{l}{A}.$$

Now P/x is the slope of the load/extension graph, and l/A is constant for a particular test provided that l is large compared with the diameter of the wire.

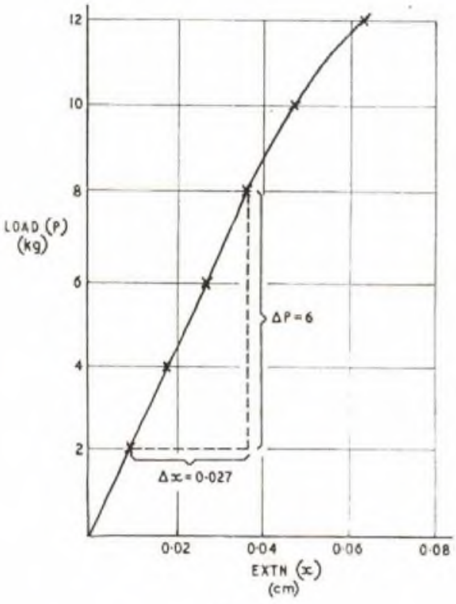
The load/extension graph is shown in sketch (*a*), from which it can be seen that for loads up to 8 kg the graph is a straight line, but above 8 kg the slope gradually decreases. Thus, up to a load of 8 kg, the material is within its elastic limit and if a load within this range were removed the wire would return to its original length. For loads greater than 8 kg, the material is stressed beyond its elastic limit and the wire would not return to its original length if the load were removed, i.e. it would have suffered permanent deformation.

$$\text{From the given data, } \frac{l}{A} = \frac{2}{\pi \times (0{\cdot}12 \times 10^{-2})^2/4}$$
$$= \frac{8 \times 10^8}{144\pi} = \frac{10^8}{18\pi} \text{ m}^{-1}.$$

The slope of the straight portion of the load/extension graph,

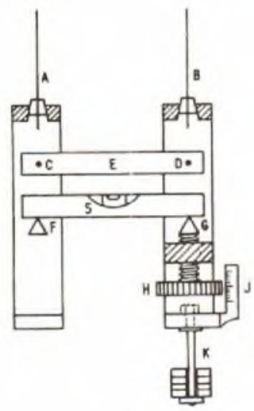
$$\frac{\Delta P}{\Delta x} = \frac{6}{0{\cdot}027 \times 10^{-2}} = \frac{6 \times 10^5}{27} \text{ kg/m.}$$

$$\therefore \text{ Young's modulus} = \frac{10^8 \times 6 \times 10^5}{18\pi \times 27}$$
$$= \underline{3{\cdot}93 \times 10^{10} \text{ kg/m}^2.}$$



(a)

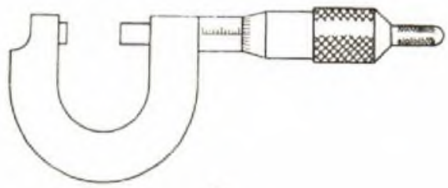
The measurements of extension could be made using the arrangement shown in sketch *(b)*. Two wires, A and B, of the material to be



(b)

tested are clamped to a support at their upper ends, and at their lower ends carry frameworks joined by a spacer, E, which is freely pivoted at C and D so that the two wires can move vertically relative to one another but not horizontally. The framework on wire B carries a hanger, K, on which are placed the weights used to stretch wire B. A spirit level, S, is supported at one end on a fulcrum, F, on the framework attached to wire A, and at the other end on the point of the screw G, which runs in a nut attached to the frame. The screw is operated by a divided head, H, which moves past a scale, J. The screw is first adjusted so that the bubble in the spirit level is at some reference point, and the readings on the scales H and J are noted. A weight is then placed on the hanger K, and the screw adjusted to bring the bubble back to the reference point. The screw has a pitch of 0·05 cm, i.e. for each complete turn it moves 0·05 cm vertically, and the periphery of H is divided into 50 equal divisions. Thus, when the head H is rotated by one division, the screw moves 0·001 cm vertically. The amount by which the right-hand end of the spirit level has to be raised to bring it back to its original position can be ascertained from the number of divisions through which H is turned, and equals the extension of the wire caused by the weight on K. The weight is increased in steps and the screw adjusted at each step to bring the spirit level back to its original position, the reading on H and J being noted at each step.

The measurement of diameter could be made using a micrometer screw gauge which works on the same principle as the screw described above. This is shown in sketch *(c)*.



(c)

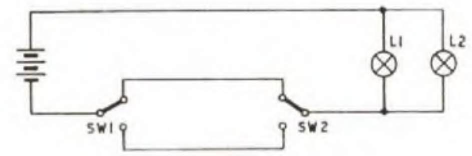
Q. 7. *Write short notes on those properties of a Leclanché cell and of a lead-acid accumulator which must be taken into account when deciding which of the two to use for a particular purpose.*

Two 6-volt 24-watt warning lamps are to be connected so that both can be switched on or off from either of two switches. Draw a circuit for this purpose, and indicate the type of battery to be used.

A. 7. The Leclanché cell has a relatively high internal resistance, which causes the terminal voltage to fall as the load current increases, so limiting the maximum current which it can supply. If current is taken from it for a long period, polarization takes place and this also causes the terminal voltage to fall. The Leclanché cell is, therefore, suitable only for intermittent light loads and where constancy of terminal voltage is not required. It cannot be recharged at the end of its useful life; the dry type must be discarded, while the active materials of the wet type must be renewed. Leclanché cells can be stored for long periods without difficulty and the dry type is available for immediate use unlike the lead-acid accumulator which requires filling with acid and charging before being placed in service.

The lead-acid accumulator has a very low internal resistance and the terminal voltage is substantially independent of the load current. It can have a large capacity and will supply large currents for long periods with constant terminal voltage. The state of its charge can be readily ascertained by the use of a hydrometer and it can be recharged when its capacity has been exhausted. Facilities for recharging must be provided, and the accumulator requires regular maintenance which consists of topping up with distilled water, cleaning, and frequently checking the state of charge. Leclanché cells, on the other hand, require little maintenance.

The sketch shows a circuit by which the two lamps can be connected in parallel and switched on or off from either of the two



switches. The current consumption of each lamp is 24/6 = 4 amp, and, therefore, the total drain on the battery when the lamps are lit will be 8 amp. The most suitable type of battery would be a 6-volt lead-acid accumulator.

Q. 8. *State Ohm's law.*

Find the current supplied by the battery of Fig. 4, and the current flowing in each resistor.

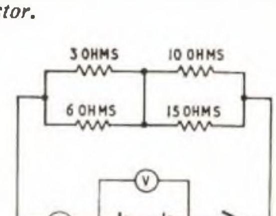


Fig. 4

When the switch was closed the voltmeter reading was 48 volts, but the ammeter reading was found to be 4 amp instead of the calculated value. Suggest one possible reason for this.

A. 8. Ohm's law states that the current flowing in a conductor at constant temperature is directly proportional to the potential difference (p.d.) between its ends, i.e. $V \propto I$, or $V =$ a constant $\times I$. The constant of proportionality is known as the resistance of the conductor.

If $V =$ p.d., in volts, between the ends of the conductor,
$I =$ current, in amp, flowing in the conductor, and
$R =$ resistance of conductor in ohms,
then, $V = IR$.

Assuming that the battery has no internal resistance, that the ammeter has negligible resistance and that the voltmeter has infinite resistance, with the switch in Fig. 4 closed the current, I, supplied by the battery is

$$I = \frac{48}{\dfrac{3 \times 6}{3 + 6} + \dfrac{10 \times 15}{10 + 15}} \text{ amp}$$

$$= \frac{48}{8} = \underline{6 \text{ amp.}}$$

The p.d. across the 3-ohm and 6-ohm resistors in parallel
$= 6 \times 2 = 12$ volts.

$\therefore$ The p.d. across the 10-ohm and 15-ohm resistors is $48 - 12$
$= 36$ volts.

The current in the 3-ohm resistor $= \dfrac{12}{3} = \underline{4 \text{ amp.}}$

The current in the 6-ohm resistor $= \dfrac{12}{6} = \underline{2 \text{ amp.}}$

The current in the 10-ohm resistor $= \dfrac{36}{10} = \underline{3 \cdot 6 \text{ amp.}}$

The current in the 15-ohm resistor $= \dfrac{36}{15} = \underline{2 \cdot 4 \text{ amp.}}$

Some reasons why the observed value of current is 4 amp, instead of 6 amp as calculated, are as follows:

(*a*) 3-ohm resistor disconnected. The parallel 10-ohm and 15-ohm resistors in series with the 6-ohm resistor would give a total circuit resistance of 12 ohms and a current of 4 amp.

(*b*) 15-ohm resistor disconnected. The parallel 3-ohm and 6-ohm resistors in series with the 10-ohm resistor would give a total circuit resistance of 12 ohms and a current of 4 amp.

(*c*) The use of an ammeter having a resistance of 4 ohms instead of one of negligible resistance.

Q. 9. *Describe the construction and action of a moving-coil voltmeter.*

Explain why high resistance is a desirable feature in a voltmeter, and low resistance in an ammeter.

Q. 10. *Write short notes on three of the following topics:*
(*a*) *The earth's magnetic field,* (*c*) *The hot-wire ammeter,*
(*b*) *Electroplating,* (*d*) *The filament lamp.*

MATHEMATICS A, 1963

Students were expected to attempt not more than any six questions.

Q. 1. (*a*) *If* $\dfrac{5a - 4b}{2a - b} = \dfrac{3}{2}$ *find the value of* $\dfrac{a}{b}$.

(*b*) *Simplify giving answers with positive indices:*

(*i*) $\dfrac{3x^{-2}y^3}{12xy^{-2}}$.

(*ii*) $\dfrac{27x^{-1/2}y^{-3/4}z^{1/2}}{18x^{3/2}y^{1/4}z^{-1/2}}$.

(*iii*) $(z^{1/3} - z^{-1/3})(z^{2/3} + 1 + z^{-2/3})$.

(*c*) *Find the value of* θ *between* 0° *and* 360° *which satisfies both equations:*

$\tan \theta = -0 \cdot 4663$, *and* $\cos \theta = -0 \cdot 9063$.

A. 1. (*a*) $$\frac{5a - 4b}{2a - b} = \frac{3}{2}.$$

Dividing the numerator and denominator of the left-hand side by b,

$$\frac{5\dfrac{a}{b} - 4}{2\dfrac{a}{b} - 1} = \frac{3}{2},$$

$\therefore \quad 10\dfrac{a}{b} - 8 = 6\dfrac{a}{b} - 3$, on cross-multiplying,

or $4\dfrac{a}{b} = 5$.

$$\therefore \quad \frac{a}{b} = \underline{\frac{5}{4}}.$$

(*b*) (*i*)
$$\frac{3x^{-2}y^3}{12xy^{-2}} = \frac{y^{3-(-2)}}{4x^{1-(-2)}} = \underline{\frac{y^5}{4x^3}}.$$

(*ii*)
$$\frac{27x^{-1/2}y^{-3/4}z^{1/2}}{18x^{3/2}y^{1/4}z^{-1/2}} = \frac{3z^{1/2-(-1/2)}}{2x^{3/2-(-1/2)}y^{1/4-(-3/4)}} = \underline{\frac{3z}{2x^2y}}.$$

(*iii*) $(z^{1/3} - z^{-1/3})(z^{2/3} + 1 + z^{-2/3})$
$= z^{1/3+2/3} + z^{1/3} + z^{1/3-2/3} - z^{-1/3+2/3} - z^{-1/3} - z^{-1/3-2/3}$
$= \underline{z - \dfrac{1}{z}}.$

(*c*) $\tan \theta = -0 \cdot 4663$,
and $\cos \theta = -0 \cdot 9063$.

From the first equation,

$$\frac{\sin \theta}{\cos \theta} = -0 \cdot 4663.$$

Substituting for $\cos \theta$ from the second equation,
$\sin \theta = -0 \cdot 4663 \times (-0 \cdot 9063)$
$= 0 \cdot 4226$.
$\therefore \theta = 25°$, or $180° - 25°$
$= 25°$, or $155°$.

No.	*Log.*
0·4663	$\bar{1}$·6687
0·9063	$\bar{1}$·9572+
	$\bar{1}$·6259

Since the cosine of θ must be negative, only the value of 155° will satisfy both equations.

Thus, $\theta = \underline{155°}$, for values between 0° and 360°.

Q. 2. (*a*) *If* $x - \dfrac{1}{x} = 5$ *find the value of* $x^2 + \dfrac{1}{x^2}$ *without solving the equation and deduce the value of* $\left(x + \dfrac{1}{x}\right)^2$.

(*b*) *Solve, by completing the square, the quadratic equation*
$2x^2 - 5x - 4 = 0$,
giving roots correct to 2 decimal places.

A. 2. (*a*) $x^2 + \dfrac{1}{x^2} = \underline{27}$; $\left(x + \dfrac{1}{x}\right)^2 = \underline{29}$.

(*b*) $x = \underline{3 \cdot 14, \text{ or } -0 \cdot 64}$.

Q. 3. (*a*) *Evaluate using logarithms* $(0 \cdot 412)^{2 \cdot 36}$.

(*b*) *A triode valve amplification factor* μ *and anode slope resistance* r *is connected as a cathode follower, the value of the cathode resistor is* R. *The voltage gain* G *of the stage is given by*

$$G = \frac{\mu R}{r + (\mu + 1)R}.$$

Make μ *the subject of the formula giving the result in a factorized form.*

A. 3. (*a*) $(0 \cdot 412)^{2 \cdot 36} = \underline{0 \cdot 1234}$.

(*b*) $$\mu = \underline{\frac{G(R + r)}{R(1 - G)}}.$$

Q. 4. *In Fig. 1 below, a triangle ABC has altitudes AX and BY which intersect in H.* $\angle ABH = 23°$, $\angle CBH = 47°$.

Show that $AX^2 + BX^2 = AY^2 + BY^2$.

State what is common to the four points A, B, X, Y, and deduce the values of the angles, $\angle AXY$, $\angle XAY$, $\angle BAX$, *and* $\angle BYX$. *Name the other set of points that all lie on a circle. If* $AX = h$, $HX = x$, $BH = k$ *express HY in terms of h, k, and x.*

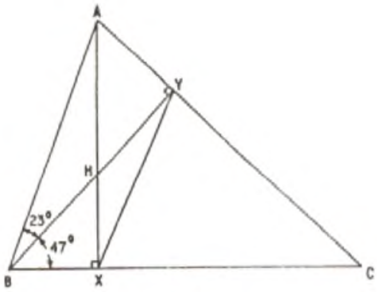


Fig. 1

A. 4. Since AX and BY are altitudes of triangle ABC, the angles at their feet are right angles. Thus, triangles ABX and ABY are both right-angled triangles.

Therefore, in triangle ABX, from the theorem of Pythagoras,

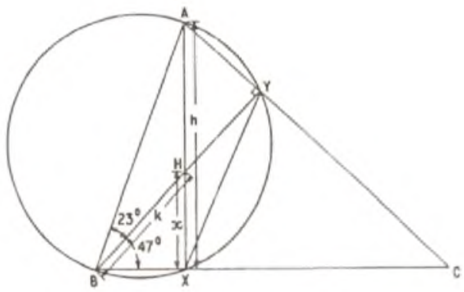
$$AX^2 + BX^2 = AB^2.$$

Similarly, in triangle ABY,

$$AY^2 + BY^2 = AB^2.$$

$$\therefore AX^2 + BX^2 = AY^2 + BY^2. \qquad \text{Q.E.D.}$$

The four points A, B, X and Y are all points which lie on the circumference of the same circle, which has AB as a diameter, as shown in the sketch.



Since AY is a chord of this circle, $\angle$ AXY = $\angle$ ABY = 23°, because angles subtended at the circumference of a circle, standing on the same arc, are equal.

Similarly, as XY is a chord, $\angle$ XAY = $\angle$ XBY = 47°.

In triangle ABX,

$$\angle BAX = 180° - (\angle ABX + \angle AXB)$$
$$= 180° - (23° + 47° + 90°)$$
$$= 180° - 160° = 20°.$$

Again, since BX is a chord of the circle on AB as diameter,

$$\angle BYX = \angle BAX = 20°.$$

Hence, $\angle$ AXY = 23°, $\angle$ BAX = $\underline{20°}$,

$\angle$ XAY = $\underline{47°}$, and $\angle$ BYX = $\underline{20°}$.

Since AX and BY are intersecting chords of the same circle,

$$AH \times HX = BH \times HY,$$

or $(h - x) \times x = k \times HY$.

$$\therefore HY = \underline{\frac{x(h-x)}{k}}.$$

Q. **5.** *(a) Evaluate tan 157°13′, cos 256°27′, sin 325°15′.*

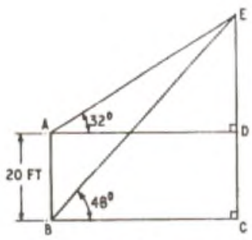


Fig. 2

(b) In Fig. 2 above, from a point A on the roof of a workshop the angle of elevation of the top of a chimney EC is 32° and from B on the ground 20 ft vertically below A it is 48°. Show that $\angle$ AEB = 16°. Apply the sine rule to triangle AEB to find the distance AE and hence calculate the height of the chimney EC.

A. 5. (*a*) tan 157°13′ = $\underline{-0{\cdot}4200}$, cos 256°27′ = $\underline{-0{\cdot}2343}$, and sin 325°15′ = $\underline{-0{\cdot}5700}$.

(*b*) AE = $\underline{48{\cdot}56 \text{ ft.}}$

EC = $\underline{45{\cdot}73 \text{ ft.}}$

Q. **6.** *(a) In Fig. 3 below, the currents i_1 and i_2 are given by the equations*

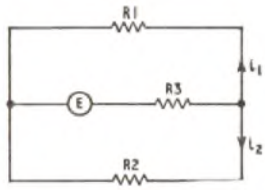


Fig. 3

$$R_3(i_1 + i_2) + R_1i_1 = E, \text{ and } R_1i_1 - R_2i_2 = 0.$$

Find expressions for i_1 and i_2 in terms of R_1, R_2, R_3, and E.

(b) Factorize (i) $3x^2 + 5x - 2$, and (ii) $3x^2 - 10x + 3$.

Hence reduce to one fraction, with the lowest common denominator,

$$\frac{2x-3}{3x^2+5x-2} - \frac{x-2}{3x^2-10x+3}.$$

A. 6. (*a*) $R_3(i_1 + i_2) + R_1i_1 = E$, and $R_1i_2 - R_2i_2 = 0$.

From the second equation, $R_2i_2 = R_1i_1$, or $i_2 = \dfrac{R_1}{R_2}\, i_1$.

Substituting for i_2 in the first equation,

$$R_3i_1 + \frac{R_3R_1}{R_2}\, i_1 + R_1i_1 = E.$$

Multiplying the equation by R_2,

$$R_2R_3i_1 + R_1R_3i_1 + R_1R_2i_1 = ER_2,$$

or $i_1(R_1R_2 + R_2R_3 + R_3R_1) = ER_2$.

$$\therefore\ i_1 = \underline{\frac{ER_2}{R_1R_2 + R_2R_3 + R_3R_1}}.$$

From the second equation,

$$i_2 = \frac{R_1}{R_2}\, i_1 = \frac{R_1}{R_2}\left(\frac{ER_2}{R_1R_2 + R_2R_3 + R_3R_1}\right)$$
$$= \underline{\frac{ER_1}{R_1R_2 + R_2R_3 + R_3R_1}}.$$

(*b*) (*i*) $3x^2 + 5x - 2 = \underline{(3x-1)(x+2)}.$

(*ii*) $3x^2 - 10x + 3 = \underline{(3x-1)(x-3)}.$

$$\frac{2x-3}{3x^2+5x-2} - \frac{x-2}{3x^2-10x+3} = \frac{2x-3}{(3x-1)(x+2)} - \frac{x-2}{(3x-1)(x-3)}$$
$$= \frac{(2x-3)(x-3) - (x-2)(x+2)}{(3x-1)(x+2)(x-3)}$$
$$= \frac{2x^2 - 3x - 6x + 9 - x^2 + 2x - 2x + 4}{(3x-1)(x+2)(x-3)}$$
$$= \underline{\frac{x^2 - 9x + 13}{(3x-1)(x+2)(x-3)}}.$$

Q. **7.** *In Fig. 4 below, C is a point on the circumference of a circle centre O radius 4 in. A circle is drawn with C as centre to cut the first circle in A and B such that AB passes through O. Show that AC = 5·656 in. and calculate the area common to both circles.*

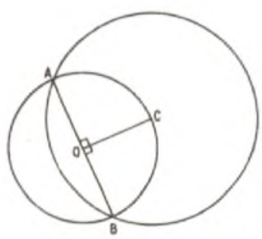
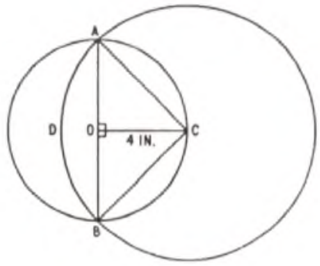


Fig. 4

A. 7 Since AB passes through O, the centre of the circle of radius 4 in., it must be a diameter of this circle and, hence, divides the circle into two equal semicircular areas. Lines AC and CB are drawn, as shown in the sketch.



Since AC = CB, AO = OB and OC is common, the triangles AOC and BOC are congruent. Hence, $\angle$ AOC = $\angle$ BOC = 90°.

Also AO = OC = 4 in.

$\therefore$ By Pythagoras' theorem, $AC^2 = 4^2 + 4^2 = 32$.

$AC = \sqrt{32} = 5{\cdot}657$ in. (from a table of square roots). Q.E.D.

Area common to both circles = semicircle ACB of circle with centre O + segment ADB of circle with centre C.

Area of a semicircle $= \dfrac{\pi r^2}{2}$, where r is the radius of the circle.

$\therefore$ Area of semicircle ACB $= \dfrac{\pi \times 16}{2} = 8\pi$.

Area of a segment of a circle $= \frac{1}{2}r^2(\theta - \sin\theta)$, where θ is the angle subtended at the centre.

Now $\theta = \angle ACB = 90° = \pi/2$ radians, being the angle in a semicircle.

$$\therefore \text{ Area of segment ADB} = \tfrac{1}{2} \times 5{\cdot}657^2\left(\frac{3{\cdot}142}{2} - 1\right)$$
$$= \tfrac{1}{2} \times 32\,(1{\cdot}571 - 1)$$
$$= 0{\cdot}2855 \times 32$$
$$= 9{\cdot}136 \text{ in}^2.$$

Thus, the area common to both circles $= 8\pi + 9{\cdot}136$
$= 25{\cdot}136 + 9{\cdot}136$
$= 34{\cdot}272 \simeq \underline{34{\cdot}27 \text{ in}^2}.$

Q. 8. *Draw the graphs of* $8y = x - 1$ *and* $y = 1/x$ *on the same axes between* $x = -5$ *and* $x = -\frac{1}{2}$ *and between* $x = \frac{1}{2}$ *and* $x = 5$. *Use your graphs to solve the equation* $x^2 - x - 8 = 0$. *Adopt the same procedure to solve the equation* $5x^2 - 2x - 8 = 0$.

A. 8. $8y = x - 1$, or $y = \dfrac{x-1}{8}$.

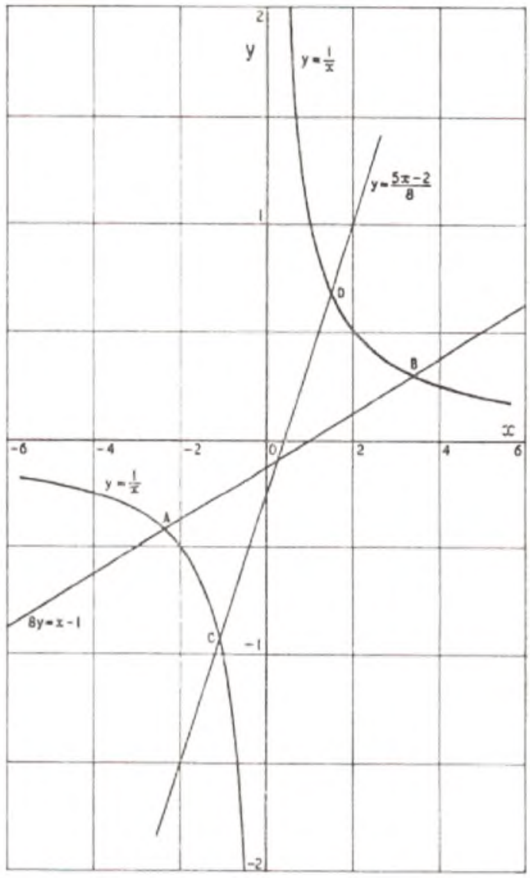
This is of the form $y = mx + c$ and hence its graph will be a straight line. It is, therefore, only necessary to plot two points to draw the graph. In practice, three points would be drawn—the additional one acting as a check. Suitable values are given in the table below to cover the required range:

x	-5	0	5
$x - 1$	-6	-1	4
$y = \dfrac{x-1}{8}$	$-\frac{3}{4}$	$-\frac{1}{8}$	$\frac{1}{2}$

$$y = \frac{1}{x}.$$

Values of y for this function, over the range of x values from $x = -5$ to $x = -\frac{1}{2}$ and from $x = \frac{1}{2}$ to $x = 5$, are given in the following table:

x	-5	-4	-3	-2	-1	$-\frac{1}{2}$	$\frac{1}{2}$	1	2	3	4	5
$y = \dfrac{1}{x}$	$-\frac{1}{5}$	$-\frac{1}{4}$	$-\frac{1}{3}$	$-\frac{1}{2}$	-1	-2	2	1	$\frac{1}{2}$	$\frac{1}{3}$	$\frac{1}{4}$	$\frac{1}{5}$



The graphs of both functions are shown in the sketch.

Where the graphs intersect, i.e. at points A and B, the values of the functions are the same. Hence, at these points

$$y = \frac{x-1}{8} = \frac{1}{x}.$$
$$\therefore\quad x(x-1) = 8,$$
$$\text{or}\quad x^2 - x - 8 = 0.$$

Thus, the abscissae of points A and B must give the solution of the equation $x^2 - x - 8 = 0$.

Reading values from the sketch,

$$x = \underline{-2{\cdot}38, \text{ or } 3{\cdot}38}.$$

To solve $5x^2 - 2x - 8 = 0$, rearrange the equation as follows:

$$5x^2 - 2x = 8,$$
$$x(5x - 2) = 8,$$
$$\text{or } \frac{5x-2}{8} = \frac{1}{x}.$$

Hence, the solution may be obtained by plotting the graph of $y = (5x - 2)/8$, and finding where it intersects the graph of $y = 1/x$, which is already drawn in the sketch.

The graph of $y = (5x - 2)/8$ is conveniently drawn in the same sketch from the following set of values covering a restricted range of x values.

x	-2	0	2
$5x - 2$	-12	-2	8
$y = \dfrac{5x-2}{8}$	$-1\frac{1}{2}$	$-\frac{1}{4}$	1

The graphs of $y = \dfrac{1}{x}$ and $y = \dfrac{5x-2}{8}$ intersect at points C and D shown in the sketch and, reading from the graph, the solution of $5x^2 - 2x - 8 = 0$ is, therefore, obtained as $x = \underline{-1{\cdot}08, \text{ or } 1{\cdot}5}$.

Note: Calculated values of the roots of the two equations, to four significant figures, are as follows:

$x^2 - x - 8 = 0$; $x = -2{\cdot}373$, or $3{\cdot}373$.
$5x^2 - 2x - 8 = 0$; $x = -1{\cdot}081$, or $1{\cdot}481$.

Q. 9. (a) Use the identity $\cos^2\theta + \sin^2\theta = 1$ to establish the following results:

(i) $1 + \tan^2\theta = \sec^2\theta$, and (ii) $\cot^2\theta + 1 = \operatorname{cosec}^2\theta$.

Prove the identities:

(iii) $(\sec\theta + \tan\theta)(\sec\theta - \tan\theta) = 1$, and

(iv) $\dfrac{\cos^2\theta - \cos^4\theta}{1 - \sin^2\theta} = \sin^2\theta$.

(b) Find the values of θ between 0° and 360° which satisfy the equation $3\sin\theta = 2\cos^2\theta$.

A. 9. (*a*)(*i*) $\cos^2\theta + \sin^2\theta = 1$.

Dividing by $\cos^2\theta$,

$$1 + \frac{\sin^2\theta}{\cos^2\theta} = \frac{1}{\cos^2\theta},$$

or $1 + \tan^2\theta = \sec^2\theta$. Q.E.D.

(*ii*) Dividing the identity by $\sin^2\theta$,

$$\frac{\cos^2\theta}{\sin^2\theta} + 1 = \frac{1}{\sin^2\theta},$$

or $\cot^2\theta + 1 = \operatorname{cosec}^2\theta$. Q.E.D.

(*iii*) $(\sec\theta + \tan\theta)(\sec\theta - \tan\theta) = 1$.

Left-hand side $= \sec^2\theta - \tan^2\theta$

$= 1 + \tan^2\theta - \tan^2\theta$, from part (*i*),

$= 1$. Q.E.D.

(*iv*) $$\frac{\cos^2\theta - \cos^4\theta}{1 - \sin^2\theta} = \sin^2\theta.$$

Left-hand side $= \dfrac{\cos^2\theta\,(1 - \cos^2\theta)}{1 - \sin^2\theta}$.

From $\cos^2\theta + \sin^2\theta = 1$,

$1 - \cos^2\theta = \sin^2\theta$, and $1 - \sin^2\theta = \cos^2\theta$.

$\therefore$ Left-hand side $= \dfrac{\cos^2\theta \times \sin^2\theta}{\cos^2\theta} = \sin^2\theta$. Q.E.D.

(*b*) $3\sin\theta = 2\cos^2\theta$.

But $\cos^2\theta = 1 - \sin^2\theta$.

$\therefore 3\sin\theta = 2(1 - \sin^2\theta)$,

or $2\sin^2\theta + 3\sin\theta - 2 = 0$.

$\therefore (2\sin\theta - 1)(\sin\theta + 2) = 0$,

or $\sin\theta = \frac{1}{2}$, or -2.

The second value is inadmissible and hence

$\sin\theta = \frac{1}{2}$.

$\therefore \theta = 30°$, or $180° - 30°$

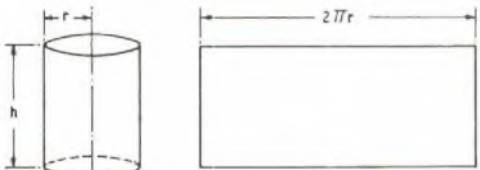
$=$ 30°, or 150°, between 0° and 360°.

Q. 10. A cylinder of radius r, closed at both ends, has a volume 200π. Show that the total surface area S is given by

$$S = \frac{400\pi}{r} + 2\pi r^2.$$

What would be the total surface area of a solid cone of the same volume and base radius?

A. 10. A cylinder of radius r is shown in sketch (*a*). Let the height of the cylinder be h.



(*a*) (*b*)

Then, volume of cylinder $= \pi r^2 h = 200\pi$.

$$\therefore\ r^2 h = 200, \text{ or } h = \frac{200}{r^2}.$$

The total surface area $= 2 \times$ area of one end face $+$ curved surface area.

The curved surface area is shown in the development of sketch (*b*). If the cylinder is imagined to be cut along a vertical line and unrolled into one plane, then the curved area becomes a rectangle of height h and length $2\pi r$.

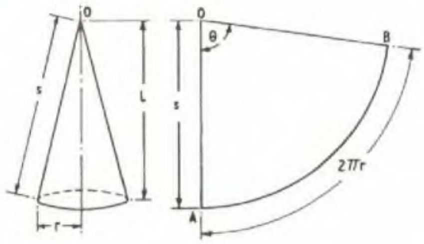
Thus, the total surface area,

$$S = 2\pi r^2 + 2\pi rh$$

$$= 2\pi r^2 + 2\pi r \times \frac{200}{r^2}, \text{ since } h = \frac{200}{r^2},$$

$$= \frac{400\pi}{r} + 2\pi r^2. \quad \text{Q.E.D.}$$

A solid cone of base radius r is shown in sketch (*c*). Let the vertical height of the cone be l.



(*c*) (*d*)

Then, volume of cone $= \frac{1}{3}\pi r^2 l = 200\pi$.

$$\therefore\ r^2 l = 600, \text{ or } l = \frac{600}{r^2}.$$

Total surface area of cone $=$ area of circular base $+$ curved surface area.

The curved surface area of the cone is shown in the development of sketch (*d*). In this instance, if the cone is cut along the slant height and unfolded into one plane, a sector of a circle of radius s is obtained, where s is the slant height. The length of arc of this sector will be the length of the circumference of the circular base of the cone, i.e. $2\pi r$.

Hence, total surface area of cone

$= \pi r^2 +$ area of sector OAB.

From sketch (*c*),

$$s^2 = r^2 + l^2.$$

$$\therefore\ s = \sqrt{r^2 + \frac{600^2}{r^4}} = \frac{1}{r^2}\sqrt{r^6 + 600^2}.$$

Now, area of sector OAB $= \pi s^2 \times \dfrac{\theta}{2\pi}$,

where θ radians is the angle AOB.

But, length of arc $=$ radius $\times$ angle subtended at the centre,

or $2\pi r = s \times \theta$.

$$\therefore\ \theta = \frac{2\pi r}{s}.$$

$$\therefore\ \text{Area of sector OAB} = \pi s^2 \times \frac{2\pi r}{2\pi s} = \pi rs.$$

$\therefore$ Total surface area of cone

$$= \pi r^2 + \pi rs$$

$$= \pi r(r + s)$$

$$= \pi r\left(r + \frac{1}{r^2}\sqrt{r^6 + 600^2}\right)$$

$$= \frac{\pi}{r}\left\{r^3 + \sqrt{r^6 + 600^2}\right\}.$$

TELECOMMUNICATION PRINCIPLES A, 1963

Students were expected to attempt three questions from Q. 1–4 and three questions from Q. 5–10.

Q. 1. State Lenz's Law.

A piece of thick copper wire is made into a single, square continuous loop of side 10 cm which can rotate about an axis in its plane through the centre point of two opposite sides. This single-turn coil can be rotated at 600 rev/min in a uniform magnetic field of 0·1 Wb/m² with its axis of rotation either parallel to the direction of the magnetic field or at right angles to it. What electrical effects will occur in the two cases?

Will the torque required to rotate the coil be the same in each case? Give reasons for your answer.

Calculate the average e.m.f. that will be generated per half revolution.

A. 1. The average e.m.f. per half revolution $=$ 40 mV.

Q. 2. *A parallel-plate tuning capacitor has 11 fixed blades and 10 moving ones. If in the position of maximum capacitance the effective area of one side of each plate is 50 cm² and the air-gap between adjacent fixed and moving plates is 0·5 mm, calculate the maximum capacitance. When the movable blades are rotated to the position of minimum capacitance the effective area of one side of each plate is 10 cm². The capacitance is adjusted from minimum to maximum while a p.d. of 80 volts is maintained across the plates. Calculate:*

(a) the minimum charge on the capacitor,

(b) the maximum charge on the capacitor, and

(c) the average current flowing into the capacitor if the adjustment from minimum to maximum takes 0·1 seconds.

The permittivity of free space is 8·854 × 10⁻¹² farads/metre.

A. 2. The capacitance between two parallel conducting plates, each with an effective area of A m², separated by a distance of d metres is,

$$C = \epsilon_r \frac{\epsilon_0 A}{d} \text{ farads,}$$

where ϵ_r = the relative permittivity of the dielectric between the plates, assumed to be uniform, and

ϵ_0 = the permittivity of free space.

The tuning capacitor has a "sandwich" of alternate moving and fixed plates. Thus, since each side of a moving plate forms one plate of a capacitor, there will be 20 capacitors all connected in parallel. The dielectric is air, so that $\epsilon_r = 1$.

Therefore, maximum capacitance,

$$C_{max} = \frac{20 \times 1 \times 8{\cdot}854 \times 10^{-12} \times 50 \times 10^{-4}}{0{\cdot}5 \times 10^{-3}} \text{ farads}$$
$$= 10^{-10} \times 8{\cdot}854 \times 2$$
$$= \underline{1{,}770{\cdot}8\ \mu\mu\text{F.}}$$

Minimum capacitance,

$$C_{min} = \frac{1}{5} \times 1{,}770{\cdot}8 \simeq 354\ \mu\mu\text{F.}$$

The charge, Q coulombs, is related to the capacitance, C farads, and the voltage produced, V volts, by

$$C = Q/V, \text{ or } Q = CV.$$

(*a*) The minimum charge on the capacitor,

$$C_{min} \times V = 354 \times 10^{-12} \times 80$$
$$= \underline{0{\cdot}02832 \text{ microcoulombs.}}$$

(*b*) The maximum charge on the capacitor,

$$C_{max} \times V = \underline{0{\cdot}1416 \text{ microcoulombs.}}$$

(*c*) The average current flowing

$$= \text{average rate of change of charge}$$
$$= \frac{0{\cdot}11328 \times 10^{-6}}{0{\cdot}1} \text{ amp}$$
$$\simeq \underline{1{\cdot}133\ \mu\text{A.}}$$

Q. 3. *Explain, with reference to a sinusoidal current, the meaning of the terms:*

(a) frequency, (b) periodic time, and (c) amplitude.

Write down the mathematical expression representing a sinusoidal current having a r.m.s. value 100 mA and a frequency of 1 kc/s.

The r.m.s. voltage across an inductor of negligible resistance is 10 volts when this current flows in it. Deduce an expression for this voltage.

Using the same axes, sketch these current and voltage waveforms, showing clearly the phase relationship between them. Mark approximate scale values on the axes.

A. 3. See A.5, Telecommunication Principles A, 1961, *Supplement*, Vol. 54, p. 46, Oct. 1961.

The sinusoidal current is given by,

$$i = \underline{141{\cdot}4 \sin 2{,}000\pi t \text{ mA.}}$$

$$\text{The voltage is } v = \underline{14{\cdot}14 \sin\left(2{,}000\pi t + \frac{\pi}{2}\right) \text{ volts.}}$$

Q. 4. *Describe briefly the principle of operation of a transistor as an amplifying device.*

With the aid of a circuit diagram show how you would determine the basic static characteristics of a transistor.

Sketch the current/voltage characteristics that would result from such measurements, giving typical values on your axes.

A. 4. Information for answering this question can be found in the article Outline of Transistor Characteristics and Applications, Part 1, *P.O.E.E.J.*, Vol. 56, p. 122, July 1963.

Q. 5. *A battery of e.m.f. 6 volts and internal resistance 2·5 ohms, an adjustable resistor and an ammeter are connected in series. When the resistor is adjusted to 45 ohms, the meter reads 100 mA. Find the resistance of the ammeter.*

What value of shunt must be added across this ammeter in order that it still reads 100 mA when 0·5 amp flows in the external circuit?

With this shunt in position, the adjustable resistor in the circuit is altered until the meter reads 50 mA. Calculate this new value of the adjustable resistor and the p.d. across the battery terminals.

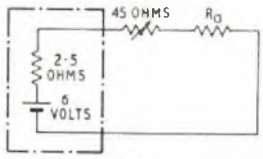
With the shunt removed, how could the ammeter be converted into a voltmeter to measure 100 volts for an indication of 100 mA on the scale?

A. 5. This problem is an application of Ohm's law, which states that the current, in amp, flowing in a resistor multiplied by the value of the resistance, in ohms, is equal to the potential difference, in volts, across it.

The total resistance in the series circuit

$$= 45 + 2{\cdot}5 + R_a = 47{\cdot}5 + R_a.$$

Note that the battery resistance is included in the total loop resistance as shown in sketch (*a*).



(*a*)

Then, as the total voltage is 6 volts and the current is 0·1 amp,

$$0{\cdot}1 \times (47{\cdot}5 + R_a) = 6.$$
$$\therefore R_a = \underline{12{\cdot}5 \text{ ohms.}}$$

The voltage across the ammeter for full scale deflexion

$$= 0{\cdot}1 \times 12{\cdot}5 = 1{\cdot}25 \text{ volts.}$$

This must also be the voltage across the meter shunt. If the shunt has to carry 0·4 amp when the meter carries 0·1 amp, the resistance of the shunt, R_s ohms, is given by

$$R_s \times 0{\cdot}4 = 1{\cdot}25.$$
$$\therefore R_s = \underline{3{\cdot}125 \text{ ohms.}}$$

The combined meter and shunt resistance

$$= \frac{\text{voltage across the combination}}{\text{total current}}$$
$$= \frac{1{\cdot}25}{0{\cdot}5} = 2{\cdot}5 \text{ ohms.}$$

Let the value of the adjustable resistor be X ohms, when 50 mA flows in the meter itself.

The total series resistance $= 2{\cdot}5 + 2{\cdot}5 + X = 5 + X$.

The total current in the meter and shunt is equal to 0·25 amp.

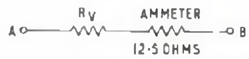
$\therefore$ By Ohm's law, $0{\cdot}25 \times (5 + X) = 6$ volts.

$$24 = 5 + X.$$
$$\therefore X = \underline{19 \text{ ohms.}}$$

The p.d. across the battery terminals

$$= \text{e.m.f.} - \text{battery resistance drop}$$
$$= 6 - (2{\cdot}5 \times 0{\cdot}25)$$
$$= \underline{5{\cdot}375 \text{ volts.}}$$

This ammeter can be converted into a voltmeter by the addition of a suitable series resistor to restrict the current to 100 mA on the full test voltage. The conditions require that the current in the meter must be 0·1 amp when 100 volts is applied across the points AB in sketch (*b*).



(*b*)

$$\text{Thus, } 0{\cdot}1 = \frac{100}{R_v + 12{\cdot}5}.$$
$$\therefore R_v = 1{,}000 - 12{\cdot}5$$
$$= \underline{987{\cdot}5 \text{ ohms.}}$$

Q. 6. *State Faraday's Laws of Electrolysis.*

Define "electrochemical equivalent."

A thin rectangular metal plate 20 cm by 25 cm is to be silver plated *to a depth of* 0·02 *mm on both sides. Describe briefly how this can be done electrolytically.*

Calculate how long the plating will take, assuming a current of 6 *amp.*

The electrochemical equivalent of silver is 0·001118 *gramme/coulomb. The density of silver is* 10·3 *gramme/cm*^{3}.

A. 6. Faraday's laws of electrolysis define the numerical relationships, in electrolytic action, between the extent of the chemical change and the direct current and time required to produce it. There are two laws:

The first law of electrolysis states that the mass of an element, or an ion, liberated or deposited in an electrolytic cell is proportional to the direct current and the time for which it passes.

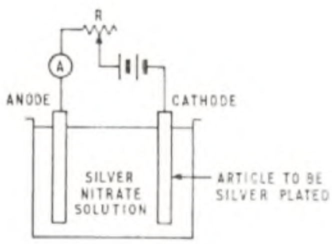
The second law of electrolysis states that the masses of different elements, or ions, liberated or deposited by the same quantity of electricity are directly proportional to the equivalent weights of the substances concerned.

If m is the mass of the substance liberated by a steady current I amp flowing for t seconds, the first law can be expressed as

$$m = ZIt,$$

where Z is a constant, known as the *electrochemical equivalent*, for the substance being liberated. Since It coulombs is the quantity of electricity that has passed, the electrochemical equivalent of a substance is the mass that can be liberated, or deposited, by 1 coulomb of electricity.

The silver plating can be done in an electrolytic tank containing a solution of silver nitrate in water. Two electrodes are suspended in the solution—one, the cathode, being the plate to be coated, and the other, the anode, being a silver plate. A suitable battery, ammeter, and adjustable resistor for controlling the current, are connected in series between the anode and cathode, the positive terminal of the battery being joined to the silver anode plate. A stop-clock is also desirable for timing the current flow.



The circuit is connected up as shown in the sketch with the electrodes in position in the silver-nitrate solution and the resistor, R, adjusted to give the required current of 6 amp. The cathode, which is the electrode to be silver-plated, is now withdrawn, washed, dried and weighed. The mass of silver, in grammes, that must be deposited to give the desired thickness of coating, is calculated from the data given. Then, using Faraday's first law of electrolysis, $m = ZIt$, and knowing m, I, and Z, it is possible to calculate the time, t seconds, during which the current must be allowed to flow to give the correct deposit of silver. After the experiment has been allowed to run for this time, the cathode plate is again withdrawn, washed, dried and weighed. The mass of silver deposited is the difference between the weight of the cathode before and after the experiment, and this can be checked against the calculation of the mass needed.

The area to be plated $= 20 \times 25 \times 2 = 1{,}000\ \text{cm}^2$.

The volume of silver $= 1{,}000 \times 0{\cdot}02 \times 10^{-1} = 2{\cdot}0\ \text{cm}^3$.

The mass of silver deposited $= 2{\cdot}0 \times 10{\cdot}3 = 20{\cdot}6$ grammes.

From Faraday's first law of electrolysis, $m = ZIt$,

$$\text{i.e. } 20{\cdot}6 = 0{\cdot}001118 \times 6 \times t.$$

$$\therefore t = \frac{20{\cdot}6}{0{\cdot}006708}\ \text{seconds}$$

$$= 3{,}070\ \text{seconds}$$

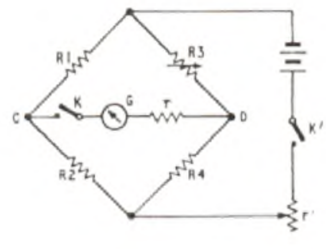
$$= \underline{51\ \text{min}\ 10\ \text{s.}}$$

Q. 7. *Explain the principle of the Wheatstone Bridge for measuring resistance and derive a formula from which an unknown resistance can be determined. What are the advantages of using a "null" method of measurement?*

During the determination of the value of the temperature coefficient of a resistor, the ratio arms are set at 100 *and* 500 *ohms. The adjustable arm, which is connected to the 500-ohm ratio arm at one end of the bridge, is found to be* 665 *ohms to balance the unknown resistor at* 16°C. *If the temperature of the unknown is raised to* 100°C, *the adjustable arm balances the bridge at* 695 *ohms. Find the temperature coefficient of the unknown resistor. Assume that the temperature is constant in all the other components of the bridge.*

If the unknown resistor consists of a 10*-metre length of wire of cross-sectional area* 0·5 *mm*^{2}, *find the resistivity of the material of the wire at* 16°C.

A. 7. The Wheatstone Bridge is an application of Ohm's law and the potentiometer principle. When a current flows through two resistors connected in series, the ratio of the potential differences across them is equal to the ratio of their resistance values, this relationship only being true when the same current flows in each resistor.



The Wheatstone Bridge circuit, shown in the sketch, consists of two pairs of series resistors, R1 and R2, and R3 and R4, connected in parallel across a battery of about 6 volts. The circuit is operated by adjusting one, or more, of the resistor values until the potential difference between points C and D is zero; this is known as a condition of bridge balance. Balance is obtained by noting the reading of a centre-zero galvanometer, M, connecting C and D, and adjusting resistor R3, say, until there is no deflexion of the galvanometer.

In practice, it is usual to connect a switch, K′, in the battery circuit with a limiting resistor, r′, to ensure some current limitation to the bridge. Also, a safety resistor, r, of fairly high value, e.g. 1,000 ohms, together with a key, K, is normally connected in the galvanometer circuit. The switch K′ is first operated to energize the bridge, then K is momentarily depressed and the galvanometer deflexion noted. Resistor R3 is then adjusted in an attempt to move nearer towards the condition of balance as shown by zero deflexion on the galvanometer. Increased sensitivity can be obtained to get a more accurate final balance by short-circuiting the resistor r.

The Wheatstone Bridge is a "null" method of measurement, which means that it does not depend for its accuracy upon a meter calibration. Moreover, in a null method the balance condition is independent of the battery supply voltage, which means, in practice, that small changes in battery voltage do not affect the accuracy of the bridge balance.

The values of resistors R1 and R2 and the adjustable resistor R3 must be accurately known, and must remain constant under operating conditions. This implies that zero temperature-coefficient resistors must be used, and it is essential to keep the bridge-operating currents to a minimum and to make their duration brief to minimize energy dissipation within the bridge. The Post Office box is often used as a Wheatstone Bridge.

The unknown resistance, R_4, can be calculated in terms of R_1, R_2 and R_3 as follows. Since, at balance, the current in the galvanometer is zero, the same current, I_1, flows in R_1 and R_2. Similarly, I_2 flows in R_3 and R_4. But as, at balance, there is no p.d. between C and D, the p.d. across R_1 equals that across R_3.

$$\therefore I_1R_1 = I_2R_3.$$

$$\text{Similarly, } I_1R_2 = I_2R_4.$$

$$\text{By division, } \frac{R_1}{R_2} = \frac{R_3}{R_4}, \text{ or } R_4 = R_3\frac{R_2}{R_1}.$$

The resistors R2 and R1 are known as the "ratio arms" because the range of resistance value that can be measured can be increased by increasing the ratio R_2/R_1.

At 16°C, the value of the unknown resistor, R4, is equal to

$$X_1 = 665 \times \frac{100}{500} = 133\ \text{ohms}.$$

At 100°C, the value of the unknown resistor is equal to

$$X_2 = \frac{695}{5} = 139\ \text{ohms}.$$

The temperature coefficient, K, of the resistor is the change in resistance per °C.

$$\text{Thus, } K = \frac{139 - 133}{100 - 16} = \frac{6}{84} = \underline{0{\cdot}0714.}$$

Note: Although not asked for in the question, the student may wish to compare this answer with the temperature coefficient of resistance, which is the change in resistance per ohm per degree centigrade.

Thus, the temperature coefficient of resistance is equal to

$$\frac{0{\cdot}0714}{133} = 5{\cdot}35 \times 10^{-4}\,.$$

The resistivity of a material (i.e. volume resistivity) is the resistance in ohms between opposite faces of a unit cube of the material. The numerical value depends upon the unit of length employed, e.g. if the unit of length is the metre, the unit of resistivity will be the ohm per metre cube, usually referred to as the ohm-metre.

The resistance of a wire of length l metres, cross-sectional area A m² and resistivity ρ is given by

$$R = \frac{\rho l}{A}\,.$$

Now, $l = 10$ m, and $A = 0{\cdot}5 \times 10^{-6}$ m².

$$\therefore\ 133 = \frac{\rho \times 10}{0{\cdot}5 \times 10^{-6}} \text{ ohms.}$$

$$\text{Thus, } \rho = \frac{0{\cdot}5 \times 133}{10^{7}}$$

$$= \underline{6{\cdot}65 \times 10^{-8} \text{ ohm-metre.}}$$

Q. 8. *Describe briefly the principle of the moving-coil ammeter. What steps are taken to ensure that the angular deflexion is proportional to the current in the coil?*

Two moving-coil meters differ only in that the moving coil in one has 60 *turns and a resistance of* 10 *ohms, while in the other it has* 750 *turns and* 700 *ohms resistance. Find the ratios of their deflexions when:*

(*a*) *each is connected in turn across a battery of* 1·5 *volt e.m.f. and* 50 *ohms internal resistance, and*

(*b*) *the two meters are connected in series across this battery.*

A. 8. For the first part of the answer see A.8, Telecommunication Principles A, 1959, *Supplement*, Vol. 52, p. 58, Jan. 1960.

The two moving-coil meters have identical magnetic field strengths, B, coil movements of equal size and mass, A, and control springs of equal modulus.

∴ Coil deflexion, $\phi \propto$ Number of turns on coil × Current flowing, i.e. $\phi \propto NI$.

Let suffix 1 refer to the 60-turn coil, and suffix 2 refer to the 750-turn coil.

(*a*) By Ohm's law the currents, I_1 and I_2, flowing when each meter is connected across the battery are

$$I_1 = \frac{1{\cdot}5}{10 + 50} = 0{\cdot}025 \text{ amp, and}$$

$$I_2 = \frac{1{\cdot}5}{700 + 50} = 0{\cdot}002 \text{ amp.}$$

$$\therefore\ \frac{\phi_1}{\phi_2} = \frac{60 \times 0{\cdot}025}{750 \times 0{\cdot}002} = \frac{1{\cdot}5}{1{\cdot}5} = \underline{1.}$$

Thus, the deflexions of the two meters are equal.

(*b*) When connected in series, the same current, I, flows in each meter. The ratio of the deflexions will, therefore, be independent of this current and depend only on the ratio of the number of turns.

$$\therefore\ \frac{\phi_1}{\phi_2} = \frac{60I}{750I} = \frac{1}{12{\cdot}5}\,.$$

Therefore, the ratio of the deflexions is $\underline{12{\cdot}5 : 1,}$ the 750-turn coil giving the larger deflexion.

Q. 9. *Describe the principle of operation of a telephone receiver. Explain, either with the aid of graphs or algebraically, why a permanent magnet is necessary.*

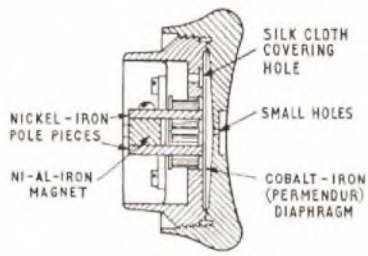
What is the source of power energizing such a receiver?

A. 9. Considerable advances have occurred in telephone receiver design in the last few years. These have included improvements to the well-known stalloy-armature bi-polar receiver and also the development of a basically new design, the rocking-armature receiver.

A modern bi-polar inset receiver is shown in sketch (*a*). The principle of operation of this class of receiver has already been described in A.10, Telecommunication Principles A, 1959, *Supplement*, Vol. 52, No. 4, p.59, Jan. 1960. There it was shown that the permanent magnet is necessary to increase the sensitivity, which is proportional to the square of the strength of the magnetic field in the air-gap, and, by giving the diaphragm a strong magnetic bias, to avoid frequency-doubling.

Improvements in the latest bi-polar receiver include:

(*a*) The use of a permendur diaphragm which has a higher permeability and can be made thinner and ligher than stalloy.



(*a*)

(*b*) Greater precision in the dimensions of the air-gaps between the magnetic diaphragm and the two pole faces.

(*c*) The elimination of objectionable acoustic resonances in cavities within the receiver case.

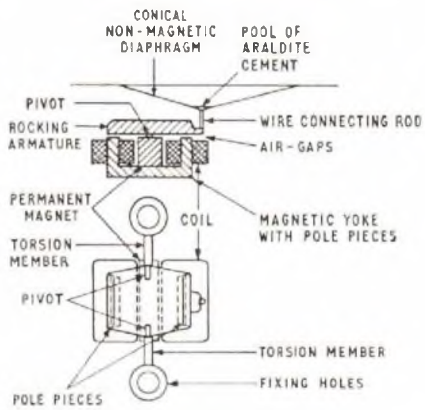
(*d*) A more satisfactory shaping of the ear-cap of the receiver case.

The thinner diaphragm yields a more efficient sound radiator and, together with the other changes, gives an improved overall frequency response.

The only source of power in the receiver is the energy derived from the speech currents passing through its coils.

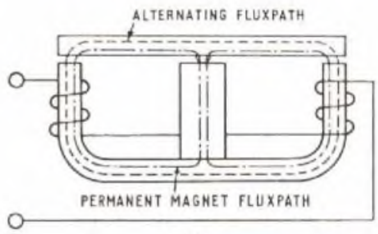
The bi-polar receiver has given years of satisfactory service. It has proved robust and is relatively cheap to produce but is somewhat insensitive. For this reason, the rocking-armature receiver has been developed to supersede it.

The principle of the rocking-armature receiver is illustrated in sketch (*b*), which shows a cross-section and a plan view of the



(*b*)

armature and magnetic unit with the diaphragm removed. The receiver differs in principle from the bi-polar receiver in that the magnetic driving circuit and the acoustic function are separated, so that each part can be designed for the best performance.



(*c*)

The magnetic circuit (see sketch (*c*)) employs a stout armature of high-permeability nickel-iron alloy (Permalloy B) rocking about a central fulcrum located on one pole of a small high-energy anisotropic permanent magnet (Alcomax III). The other pole of this magnet is attached to the centre of a U-shaped yoke, also of Permalloy B, on the two pole pieces of which small coils are fitted, connected in series and wound so that their flux is additive. Thus, the flux due to the permanent magnet follows two paths in parallel, each consisting of one limb of the yoke and its air-gap, and the appropriate half of the armature, whilst the path of the alternating magnetic flux due to the coil currents consists of the yoke, the armature and two air-gaps all in series. The path through the permanent magnet is, unavoidably, of high reluctance.

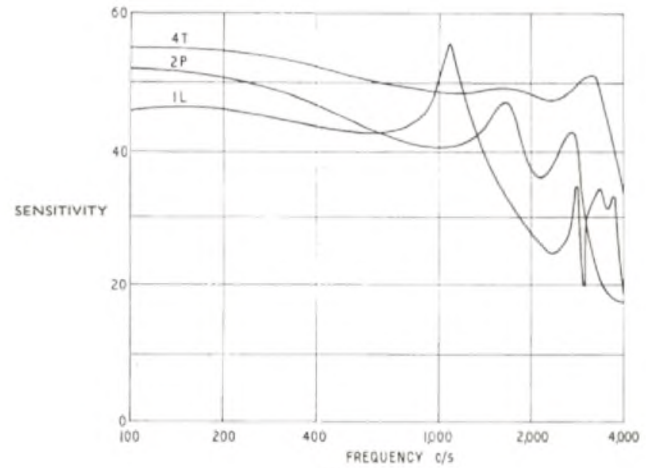
The small size of the magnetic unit and its sturdy design has made possible the use of very small air-gaps; high fluxes are, therefore, produced resulting in increased sensitivity.

The armature has a small ridge pressed into its surface making contact with the permanent magnet on which it can rock. The armature-restoring torque, which must be large because of the considerable forces from the field of the permanent magnet, comes from "torsion rods" extending sideways, as shown in sketch (*b*), and fixed at the outer ends on suitable anchorages in the receiver case. These rods are stiff enough to centre the armature against the unbalanced magnetic pull resulting from maximum deflexion of the armature.

When a current is passed through the coils, the pole pieces acquire "unlike" polarities which are superimposed on the much stronger "like" polarities induced by the permanent magnet, sketch (*c*). The effect of the coil current is, therefore, to weaken the flux in one gap and to strengthen it in the other, so rocking the armature by a push-pull action. A varying current will cause the armature to seesaw in sympathy with the current changes.

The movement generated in the rocking armature is transferred to a light, flaired diaphragm of high effective area by a short wire connecting link. The diaphragm, which is made of a light non-magnetic alloy, is clamped round its outer edge and moves like an elastic piston to impart energy as sound to the air. The diaphragm must be of light construction to act as an efficient radiator, and yet have the necessary stiffness to avoid permanent deformation and be free from spurious resonances. For this reason, it is flared, rather than flat, in shape.

The rocking-armature receiver is small and is of a capsule form of construction, which excludes dirt and moisture and assists in maintaining uniformity in performance. The nominal impedance is 150 ohms at 1 kc/s compared with 350 ohms for the bi-polar type, and the rocking-armature receiver is about 7 db the more sensitive of the two types.



(*d*)

Sketch (*d*) shows a comparison between the frequency response of the receiver type 1L (the pattern in common use), the modern bi-polar receiver 2P and the new rocking-armature receiver.

The sensitivity scale shows the sound output in db relative to 1 dyne/cm² per volt applied to the receiver input terminals.

Note: In order to obtain a true comparison in sensitivities, the curves correspond to a condition in which all three receivers are wound to give a nominal impedance of 350 ohms at 1,000 c/s.

Q. 10. *When an alternating current flows in a resistor, the resistor becomes warm. When it flows in a mica capacitor the capacitor remains cool. Explain the reason for this difference.*

Three heater elements of 300, 400 *and* 500 *ohms, respectively, are available to heat an electric oven from a* 200-*volt supply. How would you connect these three resistors so as to generate the largest possible rate of heating in the oven?*

Calculate the electrical power equivalent to this rate of heating.

Would an a.c. or d.c. supply be preferred? Give reasons for your answer.

A. 10. Whenever a current, I amp, flows in a resistor, R ohms, energy is dissipated as heat according to the law:

$$\text{Energy dissipated} = I^2R \text{ watts}$$
$$= I \times V,$$

where V is the voltage set up across the resistance R.

When the supply is a.c., the current in the resistor and the voltage across it will be in phase, that is to say, the current and voltage rise and fall in step so that work is done at each instant of the cycle. The effective value of the alternating current is the "root mean square" value (the r.m.s. value of the current is the peak value of the current divided by $\sqrt{2}$), which is defined as the value of the direct current that will give the same heating effect in a resistor as the alternating current.

When an alternating current flows in a pure capacitance, i.e. one free from any sort of loss, no energy is dissipated because the voltage across the capacitance is always exactly 90° out of phase (lagging) with the current. When the current is maximum the voltage is zero, and vice versa. Since no energy is dissipated, no heat will be generated in the capacitance. In practice, no capacitor is perfectly free from loss, so a real capacitor can be considered as a pure capacitance that is shunted by a very large resistance whose value is such as to give the loss that actually occurs. Mica is a very low-loss dielectric, so a mica capacitor is almost free from energy dissipation, and hence remains cool when an alternating current flows in it.

Heat will be generated according to the expression,

$$W = I^2R, \text{ or } W = \frac{V^2}{R} \text{ watts.}$$

For maximum heat generation at a constant voltage, V, it follows that R must be a minimum. The lowest value for the equivalent resistance for a combination of three resistors will always be obtained by connecting all three resistors in parallel; therefore, the three heater elements must be connected in parallel to generate the largest possible rate of heating in the oven.

If R is the equivalent single resistance,

$$\frac{1}{R} = \frac{1}{300} + \frac{1}{400} + \frac{1}{500}$$

$$\text{or } R = \frac{6{,}000}{47} \text{ ohms.}$$

The rate of heat generation in the oven,

$$\frac{V^2}{R} = \frac{(200)^2 \times 47}{6{,}000} = \underline{313 \cdot 3 \text{ watts.}}$$

It is almost immaterial whether a.c. or d.c. is used to supply the oven. The use of a.c. implies that the peak voltage to be withstood by the insulation is $\sqrt{2}$ greater than the equivalent d.c. supply voltage, i.e. 282 volts instead of 200 volts.

The switching of d.c. can be more troublesome than the switching of a.c. because arcs tend to develop across the switch contacts while the switch is opening. Rapid-action wide-break switches are necessary to prevent this happening.

LINE PLANT PRACTICE A, 1963

Students were expected to attempt not more than any six questions.

Q. 1. *Describe briefly how a tree from the forest is converted into a pole suitable for use as a telephone distribution pole. Describe particularly the preservation process used.*

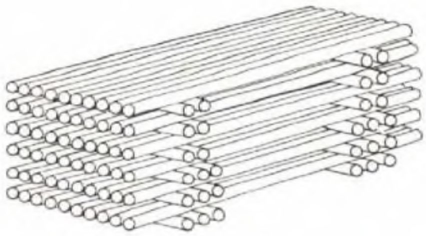
A. 1. The timber selected for use as a pole should be hard grown, with annular rings closely pitched, and cut and felled between November and February when the sap is at its lowest. All bark and branches with any adhering soft wood are removed to leave a clean hard surface. The natural butt of the tree is sawn square, after the timber has been felled, and is left on the pole. The pole is then examined to ensure that it is

(*a*) reasonably straight,

(*b*) free from large shakes and large or dead knots, and

(*c*) free from decay and from infestation by woodworm and other insects.

The pole is next seasoned, by stacking it with others in open formation to allow a free circulation of air, as shown in the sketch. The time required for seasoning may vary from six months to three



years depending on the size of the pole and the state of the weather. During seasoning, the cell spaces will dry out and leave some 25–30 per cent of the original quantity of moisture in the pole.

Identification marks are placed on the butt to indicate the length of the pole, its classification (i.e. light, medium or stout) and the inspecting officer's initials. The cubical content is marked at a point 5 ft from the butt, and at 10 ft from the butt the pole is scored or branded to indicate the owner, the length and classification of the pole, and the last two figures of the year of applying the preservative treatment.

The next step is to apply some form of preservative treatment and usually the Ruping or Empty-Cell process is used by which creosote under pressure is applied to the pole. The apparatus for this treatment consists of a working cylinder in which the poles are placed, a cylinder containing creosote, an air or vacuum pump, and an oil (creosote) pump and storage tank. Poles of similar characteristics and known cubical contents are placed together in the working cylinder, which is then closed and bolted. Air is pumped into the cylinder at a pressure of 60 lb/in^2 for about 20 minutes. Then, without reducing the air pressure, hot creosote is fed into the cylinder until it is full. The pressure is next increased to allow an additional 12 lb of creosote for each cubic foot of timber contained in the cylinder to be pumped in. For this operation it may be necessary to reach a pressure of 150 lb/in^2, taking 1 hour in the process. The amount of creosote fed into the tank is metered, and the amount of creosote forced into the timber is assessed from a knowledge of the volumes of the cylinder and of the poles. When all the sap wood is judged to have been treated, the pressure is released, the surplus creosote drained off, and suction applied to remove excess creosote from the timber.

The poles are then taken out of the cylinder and stacked in open formation to weather for sufficient time to let the surplus creosote drain off and to enable the poles to dry out before they are ready for use.

Q. 2. *What are the two usual types of hole which are dug for the erection of a telephone pole? Describe in detail three different methods of excavating a pole hole.*

Q. 3. *Sketch and describe the ratchet and tongs which are used for tensioning overhead wires. Describe in detail how you would tighten the overhead wires leading into a subscriber's premises which have become slack and are causing faults.*

A. 3. See A.2, Line Plant Practice I, 1958. *Supplement*, Vol. 52, No. 2, p. 34, July 1959.

Q. 4. *A light overhead route of open wires is to be provided along a narrow country road to serve a farm and you are required to carry out the necessary survey. Give an account of how you would proceed. What are the main factors to be considered and to what extent can preparation be made in advance of the actual work on site?*

A. 4. Since it has already been decided that a light overhead route is required along a narrow country road to serve a farm, it is assumed that the preliminary and detailed surveys can be carried out at the same time.

Before commencing the actual physical survey, it is necessary to obtain an ordnance survey map showing plant details of the area concerned, and on it the position of the farm and the nearest distribution pole (D.P.) are identified. A forecast of total requirements along the country road is next obtained; this may indicate the possible use of long-line construction. Also required are the views of the local authority to the proposed erection of the route, and details of any other services, e.g. sewers, water, gas and electricity. It may be possible to carry out joint construction with the local electricity board over part of the route.

Tentative pole sites are plotted on the map, taking particular note of the following factors:

(*a*) Town and country planning schemes.
(*b*) Future road widening operations.
(*c*) Underground power circuits.
(*d*) Overhead power circuits.
(*e*) Nearby airfields.
(*f*) Trunk or special roads, rail and river crossings.
(*g*) Places of special beauty.
(*h*) Sharp bends in the road.

The physical survey can now commence, for which will be required the ordnance survey map, a set of measuring rods, a clinometer and a survey book. First of all the tentative pole sites are checked and the final positions marked on the ground and on the map. The use of measuring rods will help in deciding the final pole positions, and the clinometer can be used to find out the heights of any obstructions.

In selecting the final positions of the poles, further factors should be considered. The span lengths should be kept as nearly normal as possible, but variations may be necessary to avoid entrances to private property; to obtain improved facilities for the siting of a stay or strut; to avoid obstructions, such as trees and buildings; to avoid exposure to traffic, as at narrow sections of the road, bridge approaches, etc., and to avoid having to excavate deep pole holes, such as at sharp bends and ditches. It is also desirable to avoid acute angles in the line by shortening spans. The line should be so planned that road crossings, if not entirely avoided, are reduced to the minimum. The poles should be erected on public property, if possible, for ease of maintenance and wayleave considerations. Care should be taken to avoid siting the poles where there may be a risk of damage to other undertakers' plant during the excavation of the pole holes.

When the most suitable pole sites have been selected, a complete record of all the information required for the preparation of a detailed estimate, the requisitioning of the stores and the guidance of the working party should be compiled in the survey book. This record should include the following:

(*a*) The exact position of each pole, stay and strut.

(*b*) The size and class of each pole, based on the ultimate wire capacity required and the clearance necessary over roads, farm entrances, road overhangs, etc.

(*c*) Advice on the method of excavating pole holes and erecting the poles, and any special tools required.

(*d*) The sizes of stays and struts, the number and type of arms, and the number and type of wires to be erected initially.

(*e*) Particulars of any special stipulation, e.g. game guards.

(*f*) Details of any private wayleave or permission for tree cutting that may be required.

(*g*) Details of any protective measures necessary.

(*h*) Details of where joint construction with the electricity authority is possible.

(*i*) Details of the termination at the farm.

The following preparations can be made in advance of the actual work on site:

(*a*) The necessary wayleave consents, both public and private, should be obtained, and the relevant notices under the Public Utilities Street Works Act should be sent to the appropriate authorities.

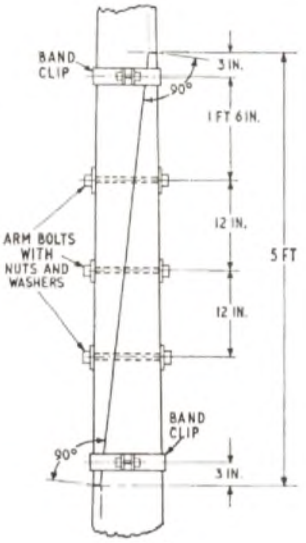
(*b*) Permission for any tree cutting should be obtained.

(*c*) The means of cartage, delivery and layout of materials, and the availability of any special tools likely to be required should be decided upon.

(*d*) It should be checked that a spare cable pair is available from the D.P. to the exchange.

Q. 5. *Describe the assembling and erection of a 75 ft single-spliced pole suitable for supporting a rhombic aerial. How would you ensure that the mast is vertical on completion?*

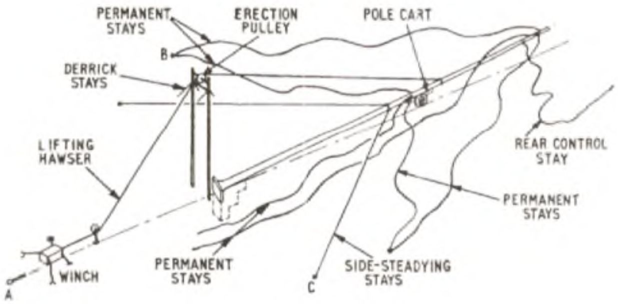
A. 5. To construct a single-spliced pole, two poles are required and these would be selected using suitable sizes for splicing together, the butt of the top pole being sawn off a few feet from the bottom to obtain a truly circular section for marrying to the top of the bottom pole. Pairs of poles are usually obtained from the factory already spliced-cut and drilled. The poles are bolted together on site and fitted with locally-made pole bands, as depicted in sketch (*a*). This fitting together can best be done if the poles are placed on two pole carts, and this also facilitates the placing of the pole for entry into the prepared foundation hole. Lowering of the butt against the sliding board into the hole will be done later with the aid of the lifting-tackle rope. The pole is now ready for fitting with steps and should lie on the carts with the splice-cut at right angles to the line of lift. Steps are fixed 15 in. apart on alternate sides of the pole 15 ft



(*a*)

from the base to 2 ft from the top, except that the third step from the top is a double step to provide a standing work-position. The stepping is started at the splice position to ensure that the steps do not foul the pole bands. The permanent stays are now attached to the pole at two levels, and these will be arranged so that, in the plan view, the individual stays of each set are 120° apart. One top and one second stay should be in line with the direction of lift. Two temporary side-steadying stays are now affixed, and the pole is ready to be positioned with its butt entering the foundation hole and pressing against the sliding planking. Two derrick poles are erected to take the erection pulley, and the powered winch is positioned and anchored. The lifting rope is passed through a ground pulley and over the lifting pulley, and used to lift the butt of the pole off the first pole cart and to lower it for entry into the hole. The lifting rope is now detached from the butt and made off at the lifting point against a step to stop it slipping.

Lifting operations may now commence, the arrangements being as shown in sketch (*b*), where A, B and C are the permanent stay



(*b*)

anchorages. Care should be taken that the pole is raised through a vertical plane, control being exercised by the side-steadying ropes and a head back-steadying rope. When the appropriate height is reached, the lifting hawser will leave the erection pulley.

The pole may be aligned in the vertical position by the use of a theodolite or a plumb bob line, sights being taken in line with one stay and again at right angles to this line. When using a plumb line it will be found helpful if it is suspended with the bob in a bucket of water.

Q. 6. *Describe in detail the construction (make-up) of a 400-pair paper-insulated lead-sheathed cable suitable for use in a local cable network.*

Sketch the cross-section of the cable and explain how the wires are identified.

A. 6. A 400-pair paper-insulated lead-sheathed cable suitable for the local cable network would be of the paper-core unit-twin (P.C.U.T.) type. Its construction is as follows:

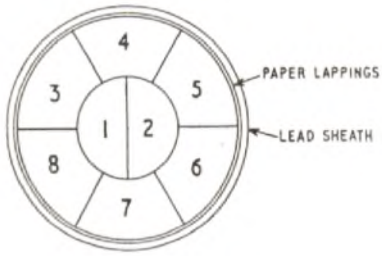
(*a*) *Conductors.* Each conductor in the cable consists of a solid wire of standard annealed-copper, which is smoothly drawn, circular in section and uniform in quality. It should be free from any defects.

(*b*) *Insulation.* Each conductor is insulated with an overlapped lapping of paper, which is printed uniformly with coloured lines in such a manner that rings are formed on the outside of each covered conductor. The insulating paper is of a uniform thickness not less than 0·002 in. and should be free from metallic particles and deleterious substances.

(*c*) *Twinning.* The insulated conductors are uniformly twisted together to form pairs.

(*d*) *Stranding.* The twisted pairs are stranded into eight compact and symmetrical units. Each unit consists of 51 pairs, arranged as a centre of 4 pairs, and then three layers, stranded in the same direction, of 10, 16 and 21 pairs, respectively. The additional pair is provided to allow for the possibility of a manufacturing fault in the unit. The layers are separated by a cotton or rayon whipping, and the outer layer is covered by an open lapping of neutral-coloured insulating paper, bearing the identification number of the unit in black ink, followed by a whipping of cotton or rayon. The units in this cable are numbered 1 to 8.

(*e*) *Make-up.* The eight completed units are laid up into a compact and symmetrical cable, with layers stranded in the same direction as the units. The units are numbered from the centre of the cable, units 1 and 2 forming the core and units 3–8 forming a layer round the core, as shown in sketch (*a*). An overall lapping of insulating



(*a*)

paper to provide at least two thicknesses is applied over the completed core. The colour of the outer paper lapping indicates the composition of the lead sheath as follows:

Lead	.. .. ..	neutral.
Lead Alloy B	.. ..	red.
Lead Alloy E	.. ..	green.

(*f*) *Lead Sheath.* The completed core is dried out, and a longitudinal tape applied showing the manufacturer's name, type of cable (i.e. P.C.U.T.), and year of manufacture. The lead sheath is then applied.

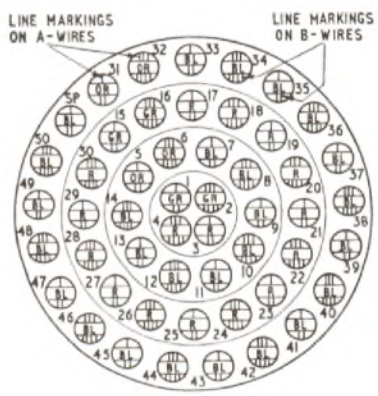
(*g*) *Protection.* If required, the cable may be suitably protected against corrosion by one of the following methods:

(*i*) Lappings of hessian tape suitably impregnated with bitumen.

(*ii*) An over sheath of polythene.

(*h*) *Colour schemes.* The coloured-ink markings on the covered conductors are arranged in groups from 1–4, as shown in sketch (*b*),

(*b*)



(*c*)

the grouping being so arranged that the quantity of ink on each conductor is similar over a length of the cable. Pair 1 consists of a conductor marked with one ring (the A leg) and one with two rings (the B leg). Pair 2 consists of conductors marked with three rings (the A leg) and four rings (the B leg).

The colour scheme used to identify pairs in each unit of this cable is illustrated in sketch (*c*), and is as follows:

(*i*) Centre and even layers—1st pair, green; 2nd pair, green; all remaining pairs, red.

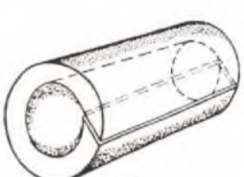
(*ii*) Odd layers—1st pair, orange; 2nd pair, orange; all remaining pairs, blue.

The units are identified by a numbered tape, as described in paragraph (*d*).

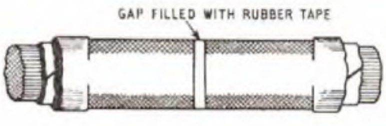
Q. 7. *In what circumstances is an insulating gap used in a telephone cable and what is its purpose? Describe in detail the fitting of an insulating gap in an existing cable and mention any precautions which must be taken. Illustrate your answer with sketches.*

A. 7. An insulating gap is used in an underground lead-sheathed telephone cable when it is found that stray current from another undertaker's plant, e.g. an electric railway, is flowing in the sheath of the cable. It is used to prevent corrosion by electrolytic action at places where the stray current leaves the cable sheath. The insertion of an insulating gap increases the longitudinal resistance of the sheath and so reduces the amount of current flowing in the sheath. Insulating gaps are normally fitted in "pick-up" areas; a gap fitted in a "discharge" area may cause the current leaving the sheath to be concentrated at one point, and hence increase the corrosion damage.

To fit an insulating gap, about 8 in. of the sheath is dressed to be as near as possible of circular cross-section, without appreciably compressing the cable. This length of the cable is then roughened with a rasp. A piece of the sheath ½–¾ in. wide, depending on the diameter of the cable, is removed from the middle of the roughened length. This gap in the sheath is built up to the same thickness as the lead sheath with rubber-wax tape. The prepared length of the sheath is then given a sufficient number of tight layers of tape to ensure that a split rubber sleeve, which is slipped over the tape, has a gap of ¼ in. between the faces of the cut, see sketch (*a*). An aluminium coupling is then placed over the rubber sleeve so that the cut in the sleeve and one of the spaces between the flanges of the coupling are in the relative position as shown in sketch (*b*). Rubber

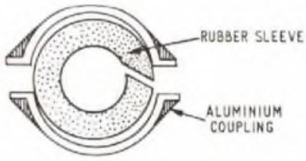


(*a*)



(*c*)

(*b*)



(*d*)

spacers are fitted between the flanges, which are then drawn together with nuts and bolts. The various components are illustrated in sketches (*c*) and (*d*).

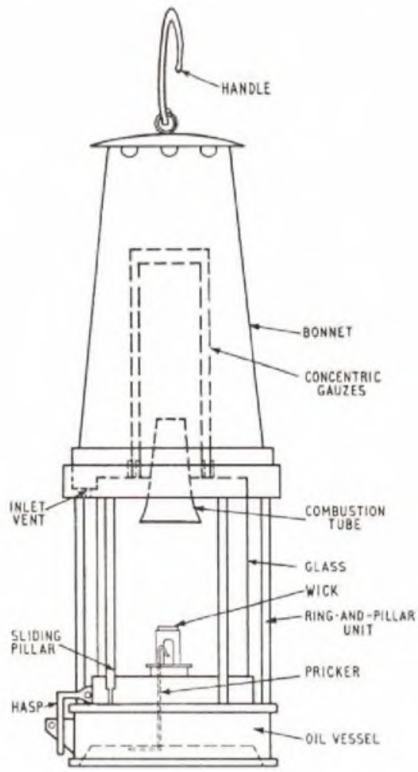
The precautions which must be taken are as follows:

(*a*) To ensure that the sheath remains watertight, the gap in the cable sheath must be filled with tape, the tape over the roughened sheath wound evenly, and the couplings drawn together evenly as the bolts are tightened.

(*b*) As the object of fitting an insulating gap is to break the electrical continuity of the sheath, care has to be taken to see that the gap is not short-circuited by bonding strips, by other cables (which should also be gapped, if necessary), or by chance contact with cable bearers, etc. Cable bearers can be insulated with plastic insulating tape.

Q. 8. *Describe in detail the method and the apparatus used for determining whether or not a manhole is clear of foul air. What other test and precautions must be taken before testing for foul air?*

A. 8. The apparatus used to test for the presence of foul air and asphyxiating gas is an oil-burning safety lamp similar to the type illustrated in the sketch.



The base of the lamp consists of an oil vessel which holds ⅛th pint of paraffin, a flat wick projecting from a burner tube, and a pricker by which the height of the wick projection may be adjusted. The centre portion of the lamp has a glass cylinder through which the wick may be observed, and a ring-and-pillar unit to secure and protect the glass. The top of the lamp consists of a combustion tube to carry exhaust gases to the top of the lamp, two concentric metal gauzes of 20 meshes per inch, and a bonnet with exhaust vents at its top and inlet vents along the ring at its base. A carrying handle is fitted to the bonnet.

The ring-and-pillar has a hasp so that it may be locked to the base of the lamp. One pillar of the ring-and-pillar unit slides into engagement with the bonnet ring and prevents the removal of the bonnet when the base of the lamp is in position.

All gases entering or leaving the lamp must pass through the concentric gauzes. The gauzes cool the exhaust gases to a level where ignition of inflammable gases outside the lamp is prevented.

To test for foul air, the base of the lamp is unscrewed and the wick is lit. The base is then replaced and the lamp allowed to warm up for a few minutes until it is burning steadily. The wick is adjusted to the smallest practicable dimensions to obtain maximum sensitivity, and the lamp is lowered to the floor of the manhole for 5 minutes. The lamp is then withdrawn and examined. Foul air and asphyxiating gases are deficient in oxygen and unable to support combustion. Thus, if on removal of the lamp the flame is extinguished or is very feeble, the presence of foul air in the manhole is indicated. If the flame has not altered, the manhole is clear of foul air.

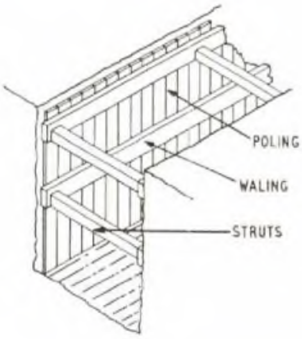
The test with a safety lamp for foul air must not be made until a test for coal gas has been taken and it has been established that no coal gas is present. Before any gas or foul-air tests are commenced, the manhole entrance must be adequately guarded to protect the public. Care should be taken to avoid sparks when removing the manhole cover and to keep away naked lights until the manhole has been proven clear of gas.

Q. 9. *A length of 4-way multiple duct is to be laid across a busy main road to link two existing manholes. Explain how you would carry out this work, particularly as regards the excavation, much of which must necessarily be at extra depth to avoid obstructions. Describe also the subsequent filling in and reinstatement.*

A. 9. It is assumed that the area has already been surveyed and that test pilot holes and other available information from road, water, gas and electricity authorities has revealed that the 4-way duct must be laid at extra depth because of other services. Again, assuming that the busy main road is of normal width, then the excavation of the road and subsequent operations can be done in either two or three sections. This will allow traffic to use the other section of the road during the operations.

Suitable protective barriers and traffic-warning signs should be placed in position, and excavation commenced, using mechanical aids where necessary, at the most suitable manhole. The top layer of material and the second layer of hard core should be laid aside for use during temporary reinstatement. The other excavated materials should be segregated so that they may be replaced in their original positions, and all the excavated material should be protected, as far as possible, from drying out or from gaining water due to being exposed to the weather. If excavated material has to be piled up in a position which interrupts normal surface drainage, provision should be made to redirect the water away from both the excavated material and the excavation. In wet weather, the size of the excavation open at any one time should be limited to the minimum for the satisfactory performance of the work. The width of the trench should be the minimum consistent with the work to be done, and the sides of the excavation should be cut straight.

As the excavation will be at extra depth, sufficient timber and boards should be provided to shore up the sides—one method of



doing this is shown in the sketch. The poling boards are usually 9 in. by 1–1½ in., and are held in position by walings 9 in. by 3 in. The struts are usually 4 in. by 4 in. Wedges are used to tighten the structure. If the soil is firm the poling boards may be spaced out.

Where necessary, other authorities' plant that is exposed by the excavation should be firmly supported by means of ropes attached to stout timber placed across the trench.

After the first length or section of road has been excavated, the manhole wall will be broken into using a hammer and cold chisel or, if convenient, by mechanical means. The bottom of the trench is now well punned and a small hollow made across the trench to take the socket of the first length of 4-way multiple duct. This ensures that the barrel of the duct rests on a firm foundation. The first section of duct is now placed in position with the spigot end through the manhole wall and the socket in the previously-provided hollow.

A hollow is prepared for the second section of duct, and then the spigot of the second section and the socket of the first are wiped clean with a rag, and coated using a brush with a rubber-bitumen emulsion. Both are then forced together using a crowbar and a suitable hard-wood block. The aim is to make a joint which is as watertight as possible.

As laying proceeds, wooden mandrels about 11 in. long are drawn through each completed section of duct. Two sets, of 12 mandrels each, are used for two of the ways and a single mandrel in each of the other two ways.

Filling-in may commence after two or three sections of duct have been laid. Fine sifted-soil, free from stones, is first placed at the sides of the duct and well punned by hand using a special narrow punner or a suitable piece of wood. Filling-in with fine sifted-soil continues until there is a 3 in. cover over the top of the ducts. This is again carefully hand-punned. A layer of subsoil, 9 in. thick when trodden down, is returned to the excavation and well rammed with a 2 cwt power rammer (or its equivalent), giving each square foot at least 12 blows. The rammer should cover the whole of the surface area, subsequent blows overlapping each other. The rest of the subsoil is returned and well rammed in 6–9 in. layers, until only sufficient space is left to take the excavated hard materials and the interim sealing coat. The excavated road surface materials are then returned in 6–9 in. layers, each layer being well rammed. This material should contain sufficient fine material to fill all voids to prevent the passage of water as this might otherwise cause subsidence and cracking. The top of the backfill should be not less than 1½ in. below the surface in carriageways and 1 in. below the surface in footways. The timber used for shoring should be removed as the filling-in proceeds.

Temporary reinstatement is then carried out to form a surface which will seal the backfill against the ingress of moisture, and which will be suitable to carry the anticipated traffic until such times as it is practicable to carry out permanent reinstatement. Before the sealing coat is laid, the edges of the existing surface materials are brushed clean and painted with a bitumen emulsion. Care is taken to cover the whole of the surface that is to be joined to the new material. In carriageways, the sealing coat should be a well-graded ¾ in. tarmacadam, and be well rolled into position. Where necessary, a fluxing oil may be incorporated in the tarmacadam to increase the time during which it remains workable. In footways, the temporary surface should consist of a well-graded ½ in. tarmacadam or fine cold asphalt. A crown should be left on the sealing coat, in each case, to allow for some settlement.

When the first section of road is ready for traffic, the second and third sections should be dealt with in a similar manner, breaking into the second manhole wall as necessary. On completion of the laying of the duct between the manholes, a test mandrel and cylindrical brush is twice drawn through each way to clean it and remove any foreign matter. Finally the manhole walls are made good with cement mortar.

After 4 to 6 months, the temporary reinstatement may be replaced by permanent reinstatement. In general, the surfacing material for permanent reinstatement must provide, when laid, a non-skid surface with a texture, appearance and moisture-sealing properties similar to the surrounding surface. The materials to be used and the thickness of the layers are normally specified by the local authority concerned. If the temporary reinstatement has been properly carried out, only sufficient material need be removed from the trench to accommodate the permanent reinstatement.

Q. 10. *Name the two d.c. tests commonly used for locating earth or contact faults on wires in telephone cables and draw diagrams showing how the apparatus for each test is connected up. What determines the choice of test to be used?*

A. 10. See A.10, Line Plant Practice A, 1959, *Supplement*, Vol. 53, No. 1, p. 16, Apr. 1960, and A.10, Line Plant Practice A, 1961, *Supplement*, Vol. 54, No. 4, p.72, Jan. 1962.

RADIO AND LINE TRANSMISSION A, 1963

Students were expected to attempt not more than any six questions.

Q. 1. *With reference to an amplitude-modulated wave, what are meant by the terms side frequencies, sidebands and depth of modulation?*

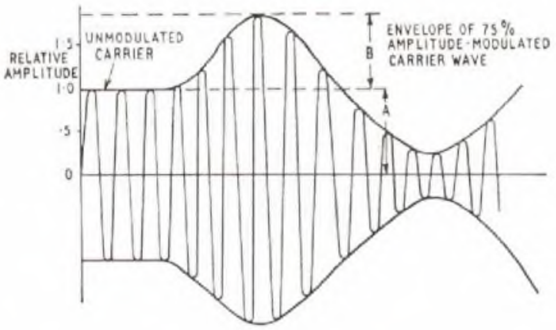
Sketch the waveform of a radio-frequency wave, amplitude-modulated by a sine-wave tone to a depth of 75 per cent.

If a radio-frequency carrier wave is amplitude-modulated by a band of frequencies, 300 c/s to 3,400 c/s, what will be the bandwidth of the transmission and what frequencies will be present in the transmitted wave, if the carrier frequency is 104 kc/s?

A. 1. *Side Frequencies.* When a carrier wave is amplitude-modulated by a low frequency, the resultant waveform has three components. There is one at the original carrier frequency, and two more, one above and one below this frequency. The latter two components, which are called the upper and lower side frequencies, respectively, are displaced from the carrier frequency by an amount equal to the modulating frequency. Thus, for example, if the carrier frequency was f_c and the modulating frequency f_m, the three components of the modulated wave would have the frequencies $f_c - f_m$, f_c and $f_c + f_m$.

Sidebands. If the carrier wave is modulated by a band of frequencies, every frequency present in this band will produce its own pair of side frequencies, and these are collectively called sidebands. There are two sidebands: the upper sideband, which comprises the band of upper side frequencies, and the lower sideband, which consists of the lower side frequencies.

Depth of modulation. If the amplitude of an unmodulated carrier wave is A, and it is amplitude-modulated by a low-frequency wave of amplitude B, then the amplitude of the envelope of the modulated wave will vary, at the frequency of the modulating wave, between the limits $A - B$ and $A + B$. The depth of modulation, m, is equal to B/A.



The sketch shows the waveform of a carrier wave when amplitude-modulated by a sine-wave to a depth of 75 per cent.

If f'_m is the highest modulating frequency, the highest sideband frequency will be $f_c + f'_m$ and the lowest sideband frequency will be $f_c - f'_m$. Therefore, the bandwidth of the transmission is

$$(f_c + f'_m) - (f_c - f'_m) = 2f'_m.$$

Using the figures quoted in the question, the bandwidth occupied by the transmission will be $2 \times 3{,}400$ c/s = 6,800 c/s.

The upper sideband will extend from 104·3 kc/s (= 104 kc/s + 300 c/s) to 107·4 kc/s (= 104 kc/s + 3,400 c/s.)

The lower sideband will extend from 103·7 kc/s (= 104 kc/s − 300 c/s) to 100·6 kc/s (= 104 kc/s − 3,400 c/s).

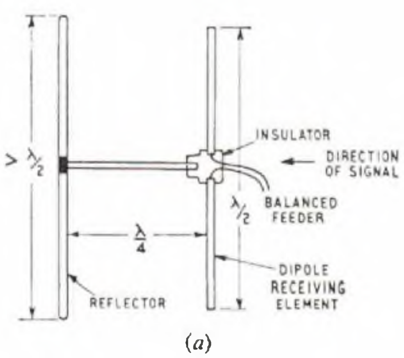
Therefore, the frequencies present in the transmitted wave will be as follows:

(*a*) the lower-sideband components from 100·6 to 103·7 kc/s,
(*b*) the carrier frequency, 104 kc/s, and
(*c*) the upper-sideband components from 104·3 to 107·4 k/cs.

Q. 2. *Sketch and describe a type of aerial commonly used for the reception of television signals. Give approximate dimensions.*

Explain what is meant by the radiation pattern (polar diagram) of an aerial. Sketch the radiation pattern, in the horizontal plane, of the aerial described.

A. 2. An aerial commonly used for the reception of television signals is the half-wave dipole, so called because it consists of two elements, usually made of aluminium tubing, which have a total length equal to one-half the wavelength of the signal being received. This simple dipole is non-directional and responds equally well to signals arriving from any direction in a plane perpendicular to the axis of the dipole. Frequently a tubular parasitic reflector, slightly longer than the dipole element, is fitted about one quarter of a wavelength behind the dipole element. A typical assembly, with approximate dimensions relative to the wavelength, λ, is shown in sketch (*a*).

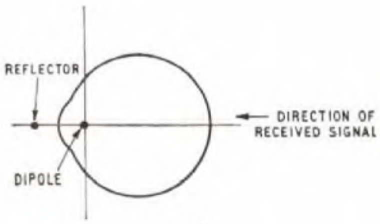


(*a*)

Ideally, the dipole element is connected to the television receiver by means of a balanced feeder, as shown.

The radiation pattern (polar diagram) of an aerial is a graphical representation, using polar co-ordinates, showing the aerial gain for any particular angle of radiation in a given plane. The radiation pattern is drawn on the assumption that the aerial is situated away from any obstacle which would affect its properties.

The polar diagram for a transmitting aerial illustrates the manner in which the strength of the signal varies when received at equal distances around the aerial and in the same plane. For a receiving aerial, the radiation pattern shows the manner in which the strength of the received signal varies as the receiving aerial is rotated about its own axis and in the same plane. It may be shown that the radiation pattern for an aerial array has the same shape whether the array is used for transmission or reception.



(*b*)

The radiation pattern, in the horizontal plane, of the half-wave dipole with reflector is shown in sketch (*b*).

Q. 3. *What is meant by (a) the mutual conductance, (b) the anode a.c. resistance, and (c) the amplification factor of a thermionic valve?*

The following data were obtained for a triode valve:

Grid Voltage, V_g (volts)		0	−0·5	−1·0	−1·5	−2	−2·5
Anode Current I_a (mA)	V_a = 225 *volts*	25	22·5	20	17·5	15	12·5
	V_a = 200 *volts*	20	17·5	15	12·5	10	7·5
	V_a = 175 *volts*	15	12·5	10	7·5	5	2·5

Plot the I_a/V_g characteristic curves and determine the amplification factor, mutual conductance and anode a.c. resistance of the valve.

A. 3. See A.2, Radio and Line Transmission A, 1961, *Supplement*, Vol. 54, No. 4, p. 75, Jan. 1962.

The mutual conductance, g_m = 5 mA/volt.

The anode a.c. resistance, R_a = 5,000 ohms.

The amplification factor, μ = 25.

Q. 4. *Draw circuits suitable for the h.t. power supply of communication-type receivers, using (a) half-wave and (b) full-wave rectifiers.*

Indicate typical values of the smoothing components.

Discuss briefly the advantages and disadvantages of each type of circuit.

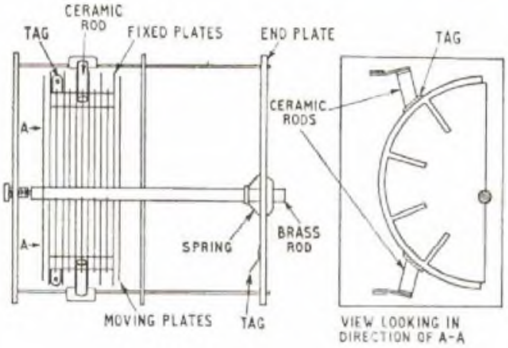
If the half-wave rectifier were being used for a television broadcast receiver, what would be the advantage of using a selenium rectifier?

A. 4. See A.10, Radio and Line Transmission A, 1959, *Supplement*, Vol. 52, No. 4, p. 69, Jan. 1960.

Q. 5. *Sketch and explain the construction of one section of a two-gang variable tuning capacitor for use in a medium-wave broadcast receiver.*

What factor determines the shape of the plates of the capacitor?

If one section of a two-gang variable tuning capacitor is used to tune the aerial circuit to the incoming signal, calculate the maximum and minimum values of the capacitor required to cover the frequency range 500–1,500 kc/s when tuned with an inductance of 150 μH. Hence write down the range of the second section required to tune the frequency-changer oscillator over the range 1,000 kc/s to 2,000 kc/s, if the same value of inductance is used to tune the oscillator circuit.

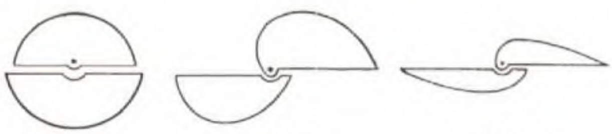


(*a*)

A. 5. The construction of one section of a two-gang variable tuning condenser is shown in sketch (*a*).

The electrodes consist of two sets of interleaved parallel plates, one set of which is fixed and the other movable. The moving plates are clamped between spacing washers on the brass control spindle, which rotates in bearings mounted on the framework of the capacitor. Two end plates rigidly connected by pillars form the framework. Contact between the frame and the moving plates is made through a forked spring working in a groove on the spindle. The fixed plates are spaced and braced by being riveted to two transverse bars, and are attached to the framework by ceramic insulating supports. The fixed and moving plates are made of either aluminium or brass, and are about 0·02 in. thick.

The capacitance is proportional to the area of the overlap of the fixed and moving plates, and, hence, varies with the angle of rotation. The shape of the plates is determined by the need to achieve linear calibration for the scale, or dial, in terms of capacitance, wavelength or frequency. Thus, in sketches (*b*), (*c*) and (*d*) the angle of rotation is directly proportional to the capacitance, wavelength and frequency, respectively.



Straight-line capacitance (*b*) — Straight-line wavelength (*c*) — Straight-line frequency (*d*)

The resonant frequency, f_0, of a tuned circuit having capacitance C and inductance L, is given by

$$f_0 = \frac{1}{2\pi\sqrt{LC}}, \text{ i.e. } C = \frac{1}{4\pi^2 f_0^2 L}.$$

Thus, to tune the signal circuit to resonance at 500 kc/s,

$$C = \frac{1}{4\pi^2 \times (500 \times 10^3)^2 \times 150 \times 10^{-6}} \text{farad}$$
$$= \underline{678 \text{ pF.}}$$

With the inductance, L, constant,

$$f_0 \propto \frac{1}{\sqrt{C}}, \text{ or } C \propto \frac{1}{f_0^2}.$$

This relationship may be used to calculate the capacitance required for resonance at 1,500 kc/s, 1,000 kc/s and 2,000 kc/s in the following manner:

$$\text{Capacitance at 1,500 kc/s} = \text{Capacitance at 500 kc/s} \times \frac{1}{\left(\frac{1{,}500}{500}\right)^2}$$
$$= \text{Capacitance at 500 kc/s} \times 1/9$$
$$= \frac{678}{9} \text{ pF}$$
$$= \underline{75{\cdot}3 \text{ pF.}}$$

$$\text{Capacitance at 1,000 kc/s} = \text{Capacitance at 500 kc/s} \times \frac{1}{\left(\frac{1{,}000}{500}\right)^2}$$
$$= \frac{678}{4} \text{ pF}$$
$$= \underline{169{\cdot}5 \text{ pF.}}$$

$$\text{Capacitance at 2,000 kc/s} = \text{Capacitance at 500 kc/s} \times \frac{1}{\left(\frac{2{,}000}{500}\right)^2}$$
$$= \frac{678}{16} \text{ pF}$$
$$= \underline{42{\cdot}5 \text{ pF.}}$$

Thus, the range of capacitance in the aerial circuit is, say, 70–700 pF, and the range of oscillator capacitance is, say, 40–170 pF.

Q. 6. *Draw the equivalent circuit diagram for the resistance-capacitance-coupled audio-frequency amplifier shown in Fig. 1 at low, middle and high frequencies.*

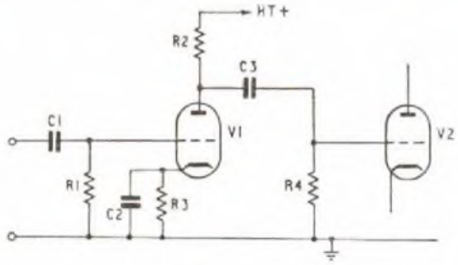
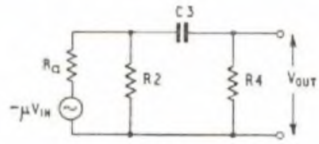


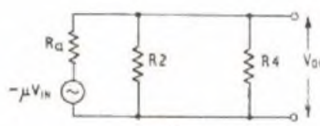
Fig. 1

Use these diagrams to explain the gain/frequency characteristics of such an amplifier. Show that at middle frequencies the gain is dependent upon the parallel combination of R2 and R4.

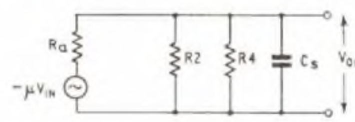
A. 6. The equivalent circuit diagrams for the resistance-capacitance-coupled audio-frequency amplifiers at low, middle and high frequencies are given in sketches (*a*), (*b*) and (*c*), respectively.



(*a*)



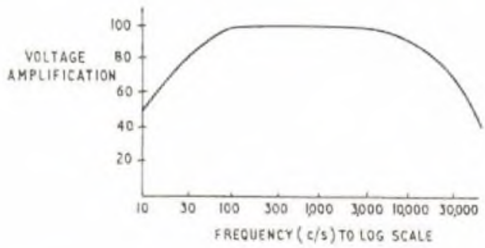
(*b*)



(*c*)

In these circuits it is assumed that valve V1 has an anode slope resistance equal to R_a and an amplification factor equal to μ.

At the low frequencies, sketch (*a*), the shunt capacitance effects of the valves and wiring may be neglected. The effect of the coupling capacitor C3 is, however, not negligible compared with resistor R4, and its impedance increases with decrease of frequency. Thus, the voltage applied to the grid of valve V2 decreases with decrease of frequency, and this accounts for the fall of gain at the low-frequency end of the range in the gain/frequency curve shown in sketch (*d*).



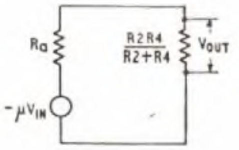
(*d*)

Over the middle frequencies, sketch (*b*), the shunting capacitances have little effect and the coupling capacitor, C3, has negligible reactance in comparison with the resistance of resistor R4. Hence, over this range of frequencies the gain is substantially constant, as shown in sketch (*d*).

At high frequencies, sketch (*c*), the reactance of capacitor C3 is negligible compared with the resistance of resistor R4, but the output capacitance of valve V1 plus the input capacitance of valve V2 and the wiring capacitances between valves V1 and V2 all add up to give an overall effective shunting capacitance C_s. The effect of this capacitance becomes greater as the frequency is increased (i.e. the capacitive reactance decreases with increase of frequency) and, hence, the gain of the amplifier will decrease as the frequency is increased.

From sketch (*b*), it may be seen that the effective anode load, R_L, of valve V1 at the middle frequencies consists of resistors R2 and R4 connected in parallel,

$$\text{i.e. } R_L = \frac{R_2R_4}{R_2 + R_4}.$$



(*e*)

Hence, sketch (*b*) may be redrawn as shown in sketch (*e*).

$$\text{Thus, } V_{out} = \mu V_{in}\,\frac{\dfrac{R_2R_4}{R_2+R_4}}{R_a + \dfrac{R_2R_4}{R_2+R_4}},$$

$$\text{and the gain} = \frac{V_{out}}{V_{in}} = \mu\,\frac{\dfrac{R_2R_4}{R_2+R_4}}{R_a + \dfrac{R_2R_4}{R_2+R_4}}.$$

Q. 7. *State the approximate values of carrier frequencies and bandwidths used for the following applications:*

(*a*) *A sound broadcast service to serve a relatively small area.*
(*b*) *A long-distance overseas point-to-point radio-telephony service.*
(*c*) *The provision of 600 telephone channels over a coaxial cable.*
(*d*) *The provision of 24 voice-frequency telegraph channels over a pair-type cable.*

Briefly explain what determines the bandwidths required in each case.

The velocity of propagation in a cable is 2×10^8 m/s. Calculate the wavelength of a 200 Mc/s television signal through such a cable.

A. 7. (*a*) To provide a high-quality sound broadcast service to serve a relatively small area, transmission in the v.h.f. band will be very suitable, i.e. the carrier frequency would be in the range 80–100 Mc/s. In order to reproduce the overtones and harmonics, which give character to the various instruments of an orchestra, it is necessary to reproduce frequencies within the range from about 30 c/s to 15,000 c/s. Thus, with double-sideband amplitude modulation a radio-frequency bandwidth of 30 kc/s is required. With frequency modulation, the bandwidth will be 150 kc/s.

(*b*) A long-distance overseas point-to-point radio-telephony service would use carrier frequencies in the high-frequency range 3–30 Mc/s (more especially 4–25 Mc/s), and employ single-sideband or independent-sideband transmissions. In either type of transmission, the radio frequency occupied by each telephone channel would be 3 kc/s, such a bandwidth being adequate for commercial speech.

(*c*) Twelve telephone channels are commonly assembled at 4 kc/s intervals to form a "group" within the frequency band 60–108 kc/s. Five such groups may then be assembled to form a "super-group" within the frequency band 312–552 kc/s. Finally, to provide for 600 telephone channels over a coaxial cable, ten such super-groups may be assembled to occupy a bandwidth of 2,400 kc/s (i.e. 600 × 4 kc/s) within the frequency band 60–2,540 kc/s, using carrier frequencies spaced at suitable intervals between 612 and 2,852 kc/s.

(*d*) To provide 24 voice-frequency telegraph channels over a pair-type cable, 24 telegraph channels, each occupying a bandwidth of 100 c/s, are commonly assembled at 120 c/s intervals to occupy 2,880 c/s of bandwidth, within the frequency band 420–3,300 c/s. The 24 carrier frequencies range from the lowest at 420 c/s up to the highest at 3,180 c/s, spaced at 120 c/s.

The velocity of propagation, v, is related to its frequency, f, and wavelength, λ, by

$$v = f \times \lambda,$$

where v is in metres/second, λ is in metres, and f is in cycles/second.

$$\text{Hence, } \lambda = \frac{v}{f}.$$

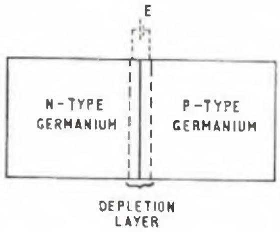
Substituting the values given,

$$\lambda = \frac{2 \times 10^8}{200 \times 10^6} = \underline{1 \text{ metre.}}$$

Q. 8. *Explain the rectifying action of a semiconductor diode.*

Briefly discuss the relative advantages and disadvantages of thermionic and semiconductor diodes for the detection of amplitude-modulated waves.

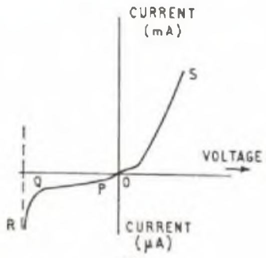
A. 8. If a crystal of n-type germanium and a crystal of p-type germanium are joined, the junction is known as a p–n junction and has properties which enable it to be used as a rectifier. In practice, it is not possible to join two such germanium crystals so that a perfect junction is formed, but it is possible to produce a single crystal having n-type characteristics at one end and p-type characteristics at the other end and having the same characteristics as a perfect junction.



(*a*)

Sketch (*a*), which is for explanatory purposes only and should not be taken as a sketch showing the construction of the device, shows a p–n junction with n-type germanium to the left and p-type germanium to the right. Due to the movement of holes and electrons across the p–n junction, a thin layer, known as a depletion layer, is set up. This consists of n-type germanium positively charged and p-type germanium negatively charged. This condition is equivalent to a potential battery and may be represented by an imaginary battery, E.

If now an actual battery is connected across the p–n junction in such a direction that it assists the barrier potential, the consequent strengthening of the barrier has the effect of increasing the junction resistance. However, due to thermal agitation there is a flow of minority carriers which causes a small current to flow. This small current is known as a reverse current and remains relatively steady until a breakdown voltage is reached. These conditions may be represented by part OPQR of the graph in sketch (*b*). It should be



(*b*)

noted that the reverse-current scale is different from the forward-current scale in the graph.

If the battery is connected across the junction to oppose the potential barrier, the effective resistance of the junction will be lowered so allowing a flow of majority carriers and giving rise to a large current flow. This condition is represented by OS of the graph in sketch (*b*).

It will be seen that when an alternating voltage is applied across such a junction, current will flow during alternate half-cycles, resulting in a rectifying action.

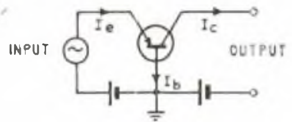
When used for the detection of amplitude-modulated waves, both thermionic and semiconductor diodes can handle large signals with little distortion, but small signals tend to be distorted owing to the curvature of the diode characteristic at low currents and voltages. Damping of the preceding stage may usually be avoided with thermionic diodes by careful choice of the load resistor, but semiconductor diodes tend to introduce a greater degree of damping of the preceding circuit because of their low input impedance. However, the extremely small self-capacitance of the semiconductor diodes and their ability to work into low-resistance loads makes them generally superior to thermionic diodes for certain applications. Further, semiconductor diodes are smaller, lighter and take less power than thermionic diodes.

Q. 9. *Draw circuit diagrams showing how a junction transistor may be used in a single-stage audio-frequency amplifier (a) with common-base connexion, (b) with common-emitter connexion. Mark clearly the input, output and biasing arrangements and the currents flowing. State the input-impedance, output-impedance and current-gain characteristics for each type of amplifier.*

Derive an expression showing how, for small input signals, the current gain of a transistor connected in a common-emitter circuit is related to the gain of the same transistor connected in a common-base circuit.

A transistor connected in a common-emitter circuit shows changes in emitter and collector currents of 1·0 mA and 0·98 mA, respectively. What change in base current produces these changes and what is the current gain of the transistor?

A. 9. Sketch (*a*) shows a p–n–p transistor connected as a common-



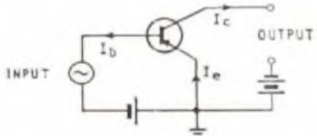
(*a*)

base amplifier, that is, the base connexion is common to both input and output circuits. The emitter is biased positively with respect to the base and this tends to reduce the effect of the emitter–base potential barrier. The collector is biased negatively with respect to the base and this tends to assist the base–collector potential barrier.

The current gain, $\alpha = \frac{I_c}{I_e}$,

where I_e = the change in emitter current, and
I_c = the corresponding change in collector current.

Sketch (*b*) shows a p–n–p transistor connected as a common



(*b*)

emitter amplifier, that is, the emitter is common to both input and output circuits. The bias batteries again assist the base–collector potential barrier and reduce the effect of the emitter–base potential barrier.

The current gain, $\beta = \frac{I_c}{I_b}$,

where I_b = the change in base current, and
I_c = the corresponding change in collector current.

The current-gain, input-impedance and output-impedance characteristics of the common-base and common-emitter arrangements are summarized in the table given below:

Characteristic	Common-Base	Common-Emitter
Current Gain	< 1 (typically 0·98)	High (typically 50)
Input Impedance	Low	Medium
Output Impedance	High	Medium

Assuming that the two circuits are working under corresponding conditions so that the currents in the three electrodes are the same for each type of connexion,

then, since $I_b = I_e - I_c$,

$$I_b = I_e - \alpha I_e = I_e(1 - \alpha)$$

$$= \frac{I_c}{\alpha}(1 - \alpha).$$

$$\therefore \frac{I_c}{I_b} = \frac{\alpha}{1 - \alpha}.$$

But, $\frac{I_c}{I_b} = \beta$.

Therefore, the current gain of a transistor connected in a common-emitter circuit is

$$\beta = \frac{\alpha}{1 - \alpha}.$$

Using the values given, the change in base current is

$$I_b = I_e - I_c$$
$$= 1·0 - 0·98 = 0·02 \text{ mA}.$$

The current gain is

$$\beta = \frac{I_c}{I_b} = \frac{0·98}{0·02} = 49.$$

Q. 10. *By reference to a block schematic diagram, explain the equipment needed to provide a repeatered audio junction circuit on (a) a 2-wire basis, and (b) a 4-wire basis. Quote typical losses for each part of the circuit.*

State the relative advantages of 2-wire and 4-wire operation.

A. 10. See A.8, Radio and Line Transmission A, 1961, *Supplement*, Vol. 54, No. 4, p. 78, Jan. 1962.

TELEPHONY AND TELEGRAPHY A, 1963

Students were expected to attempt not more than any six questions.

Q. 1. *Draw the magnetic circuits of:*
(*a*) *a receiving relay for double-current pulses from a teleprinter, and*
(*b*) *a receiving relay for loop-disconnect pulses from a subscriber's dial.*

Indicate the component parts of the magnetic circuit of each relay.

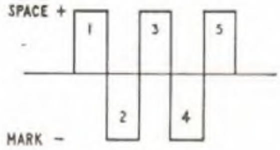
Describe briefly the adjustments which can be made to the magnetic circuit of the loop-disconnect pulsing relay.

A. 1. See A.1, Telephony and Telegraphy A, 1961, *Supplement*, Vol. 54, No. 4, p. 64, Jan. 1962.

Q. 2. *Describe the 5-unit telegraph code used in teleprinter working, making clear the need for:*
(*a*) *start and stop elements, and*
(*b*) *letter and figure shift characters.*

Give details of the time durations of each code element and state the maximum number of characters that may be transmitted in one minute.

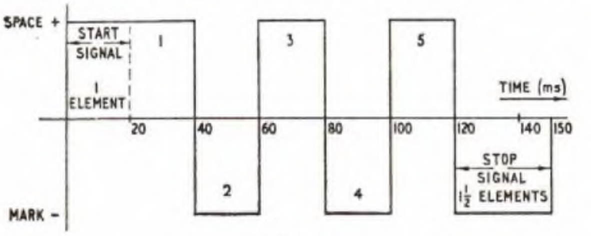
A. 2. Each character of the 5-unit telegraph code used in teleprinter working consists of a unique arrangement of five equal-duration signal elements of positive or negative potential. A positive pulse is termed a "space" element and a negative pulse a "mark" element. A typical signal formation, that for the letter R, is shown in sketch (*a*).



(*a*)

The number of possible discrete combinations of elements in a 5-unit, 2-state-signal-element code is $2^5 = 32$. This would provide an insufficient number of separate characters for practical teleprinter purposes as 26 characters are needed for the alphabet, 10 for the numerical digits, and others are required for punctuation marks, etc. Two of the 32 arrangements are, therefore, used as case-shift signals, i.e. letter-shift and figure-shift signals, and so the 29 available combinations (a character of five space signals is not generally employed) can serve a dual function, one set providing letters and the other numerical digits, punctuation marks, etc. Thus, when a case-shift signal is transmitted, the receiving mechanism operates so that all subsequent characters are interpreted in that particular case.

The transmitting and receiving mechanisms must be in synchronism to correctly interpret the signals received. The difference that can be accepted between the speeds of the transmitting and receiving mechanisms, without resulting in errors, depends upon the length of time the mechanisms are required to work continuously together. A high degree of accuracy would be necessary if errors were cumulative, but the errors can be limited to an acceptable level if they are permitted to accumulate over the period of one character only. Each 5-unit character is, therefore, preceded by a single-element space signal. This is the start signal and it causes the receiving mechanism to be coupled to its continuously-running driving motor to ensure that both mechanisms are in synchronism at the commencement of each character. Each character is also followed by a one-and-a-half-element mark signal. This is the stop signal, which ensures that the receiving mechanism is at rest for a minimum interval before the transmission of the next character. The complete signal element for R would, therefore, be as shown in sketch (*b*) with each element of



(*b*)

the 5-unit code being 20 ms in duration. Thus, every character will have an overall duration of 150 ms, and the maximum transmission rate will be 400 characters per minute.

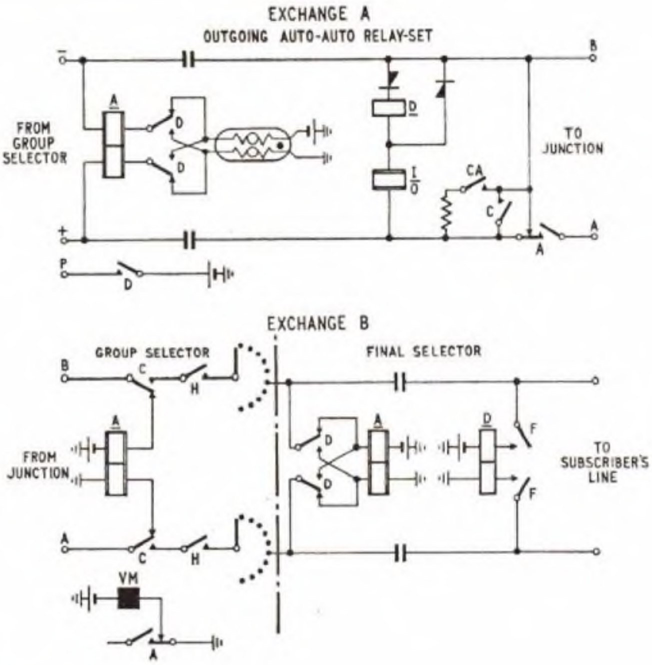
Q. 3. *Describe, with the aid of simple circuit elements, the d.c. signalling arrangements which would be used on a junction between two automatic exchanges which are spaced three miles apart.*

Why would this signalling arrangement not be suitable if the exchanges were 300 *miles apart? What other method of signalling would be used in these circumstances?*

A. 3. With the normal types of junction cable in use, the signalling-path 2-wire loop resistance of a 3-mile junction between two automatic exchanges would be within the prescribed limits for loop-disconnect pulsing over a single link. The normal resistance limit for this signalling system is 1,500 ohms but, with modern type relay-sets and selectors, this may be increased to 2,000 ohms. As loop-disconnect pulsing is a comparatively cheap signalling system, it would normally be used wherever possible unless there were particular reasons for employing a signalling system with additional facilities.

The circuit elements of a single-link loop-disconnect pulsing circuit are shown in the sketch. When the group selector in exchange A has selected a free junction circuit via the outgoing relay-set, the loop is extended forward over the junction to seize the incoming group selector at exchange B. The release and reoperation of relay A at exchange A in response to the dialled digits is repeated forward over the junction as loop disconnexions and connexions, respectively, under the control of the pulsing contact of relay A in the outgoing relay-set. These loop disconnexions and connexions are received by relay A of the incoming group selector at exchange B, and, after the vertical and rotary stepping of the group selector has been completed, the loop is extended forward to the final selector. This, in turn, has its relay A operated and released by the pulse train of subsequently dialled digits. Each pulse in the train is nominally of 100 ms duration with a break-to-make ratio of 2 : 1, i.e. $66\frac{2}{3}$ ms break and $33\frac{1}{3}$ ms make durations.

When the called-subscriber answers, relay D in the final selector operates and reverses the polarity applied to the line via the coils of relay A. This reversal is transmitted back over the junction to exchange A where it operates the relay D in the relay-set. This, in turn, passes the reversal back via relay A, and causes the metering



condition to be applied to the calling-subscriber's meter. As the pulse-repetition relays are connected across the line during speech, high-impedance relays are used. The relays D and I are used to detect supervisory conditions during the setting up of the call and during the speech condition.

Loop-disconnect pulsing would not be satisfactory for a circuit between exchanges 300 miles apart for the following reasons:

(*a*) The ohmic resistance of the signalling path, assuming 4-wire audio-frequency amplified circuits, would be of the order of 14,000 ohms. This, with normal exchange voltages, would preclude satisfactory operation of the pulsing relays.

(*b*) The low d.c. line currents resulting from the high line resistance would result in inadequate operating margins for the relays and make the circuits very susceptible to voltage surges and false pulses.

(*c*) The *CR* value of the circuit would also be very high, and this would result in a poor received-pulse waveform and intolerable distortion of the break/make pulse ratio.

As the signalling-path resistance limits for loop-disconnect pulsing would be exceeded, and as the *CR* value would exceed that permissible for d.c. signalling systems, it would be necessary to employ a.c. signalling techniques for such a long-distance circuit. To obtain a good transmission performance economically, the normal practice would be to provide such a circuit in h.f. plant, e.g. a coaxial or radio carrier system, and this would also make it necessary to employ an a.c. signalling system.

Q. 4. *Describe the essential facilities provided for use by operators at manual exchanges catering for:*

(*a*) 200 *subscribers, and* (*b*) 1,000 *subscribers.*

What determines the maximum number of subscribers lines which can be presented to one operator?

A. 4. The essential facilities provided for use by operators are common to all manual exchanges but the method of providing them may vary with the type of manual system used. A manual exchange catering for 200 subscribers would generally be of the central battery signalling (CBS) type, but an exchange catering for 1,000 lines would almost invariably be of the central battery (CB) type.

Each subscriber's line circuit and each incoming junction circuit is terminated in the answering field on a jack, which is associated with a calling indicator. On small exchanges, the indicators may be electrically-operated mechanical devices, but lamps are generally used. The calling lamps are mounted in strips of ten directly above the strips of jacks (also in tens) with which they are associated. Similarly, each subscriber's line circuit and each outgoing junction circuit terminates on a jack in the subscribers' multiple or in the outgoing junction field, respectively.

To answer an incoming calling signal, the operator is provided with an answering plug-ended cord, and to obtain access to an outgoing junction circuit or the subscribers' multiple the operator is provided with a calling plug-ended cord. These two cords are interconnected by the cord-circuit equipment, which includes a three-position key and a dial key. Each operating position is provided with a number of such cord circuits, consisting of pairs of cords and associated keys and equipment, up to a maximum number of 16.

Having inserted an answering plug into an answering jack, the operator throws forward the three-position key which enables her to speak to the subscriber and obtain the objective subscriber's number. She then seizes the required subscriber's line, or an outgoing junction circuit, by inserting the calling cord into the outgoing jack. The operator either rings the objective subscriber's bell, or signals over the outgoing junction to a distant manual exchange, by throwing the three-position key into the backward, non-locking, or ringing, position. If dialling access to an automatic exchange is required over the outgoing junction, the dial key is thrown which connects the dial, mounted on the position keyshelf, into the cord circuit and disconnects the ringing circuit.

Supervisory signals are provided to indicate to the operator the state of the calling and called lines during conversation and the setting up and clearing down of the connexion. These supervisory signals are displayed on lamps associated with the answering and calling cords.

Engaged-testing facilities are also provided to enable the operator to determine whether the objective subscriber's line or the outgoing junction circuit is free. This facility is generally provided at manual exchanges by electrical conditions on the bush of the outgoing jack. If the line is engaged, tapping the tip of the calling cord on the jack bush causes current to flow in the cord circuit and an audible click is produced in the operator's headset.

Call-charging facilities are generally provided on each position. For metering local calls, a non-locking plunger key is provided with each cord circuit, and depression of the key causes an electrical pulse to operate the calling-subscriber's meter. For charging timed calls, a number of the cord circuits are fitted with timing clocks. The number of cord circuits so equipped on a position depends on the traffic to be carried and varies from four to ten.

A 200-line exchange would generally be a single, or at most a two-position, exchange so that an operator would be able to reach every jack terminating the incoming and outgoing lines. These would, therefore, only require a single appearance on the exchange positions. At a 1,000-line exchange a number of positions would be required, and the answering fields, the subscribers' multiple, and the outgoing junction field would have to be spread over the positions according to the traffic load. As each operator must be able to interconnect every calling line appearing on her position with every outgoing line in the subscribers' multiple or outgoing junction field, the multiple and the outgoing junction field have to have repeated appearances along the exchange positions.

The maximum number of subscribers' lines presented to an operator is determined by the following factors:

(*a*) The size of multiple which can be accommodated on the position panels. This depends on the physical dimensions of the jacks and indicators as the number of circuits that can be terminated on a position depends upon the number of strips which can be accommodated on the switchboard face. The size of multiple is also restricted by the physical length of the position cords and the number of position panels within reach of any one operator.

(*b*) The traffic loading presented to the operator by the lines. This is dependent on the subscribers' originating calling rate and the complexity of the calls to be handled.

Q. 5. *Sketch and describe the construction of the mechanical arrangements provided on a two-motion selector for operating contacts when the wipers are:*

(*a*) *off-normal,*
(*b*) *raised to a particular level, and*
(*c*) *stepped into any level.*

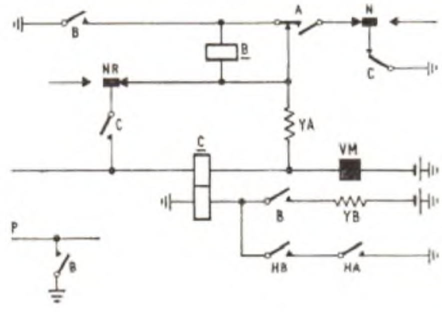
What other mechanically-operated spring-sets may be fitted and under what circumstances do they operate?

Q. 6. *The vertical-stepping circuit of a two-motion group selector includes two slow-to-release relays.*

Explain why these relays need to be slow to release, illustrating your answer with a sketch of the circuit element concerned.

A. 6. The two relays which have the slow-to-release feature in the vertical-stepping circuit of a two-motion group selector are relays B and C. The element of the circuit, which illustrates the need for the slow-to-release feature, is shown in the sketch.

The basic circuit operation is that when relay A operates to the loop seizure condition, the contact of relay A operates relay B, a contact of which in turn operates relay C. When pulsing com-



mences, relay A releases and places a short circuit across relay B making it slow to release. At the same time, it completes the operate circuit for the vertical magnet. The wipers, on taking the first vertical step, operate the N springs which operate relays HA and HB (not shown). Contacts of relays HA and HB place a short circuit across relay C making it slow to release, thereby ensuring that it remains operated during the pulse train. The operate and release sequence of relay A continues during the pulse train and causes the wipers to step vertically to the desired level. At the end of the pulse train, the short circuit across relay C allows the relay to release slowly, breaking the vertical-stepping circuit and making the rotary-stepping circuit (not shown). Whilst the switching operation is in progress, relay B remains operated, safeguarded by its own contact, during the periods when it is short-circuited by the contact of relay A.

The purpose of the slow-to-release feature of the relay B is to ensure that the guarding earth placed on the P-wire by the seizure of the selector is maintained until switching is completed and that a holding earth is passed back from the following switching stage.

Q. 7. *In what circumstances would a grading be used between automatic selectors?*

Give a sketch of a 6-group, 27-trunk grading from a 10-availability selector level.

A. 7. A grading is employed to interconnect two stages of automatic switching when limited-availability conditions exist, i.e. when the number of outlets from selectors in the first stage is less than the required number of trunks to, and hence the number of selectors in, the second stage.

For a 6-group grading, the outlets of the selectors may be graded as individual choices, pairs, trebles or full commons to the trunks to the subsequent stage.

Let a = the number of outlets connected as individual choices,
b = the number of pairs,
c = the number of trebles, and
d = the number of full commons.

Then, for a grading of 27 trunks from 10-availability selectors,

$$6a + 3b + 2c + d = 27 \quad (1)$$
$$a + b + c + d = 10 \quad (2)$$

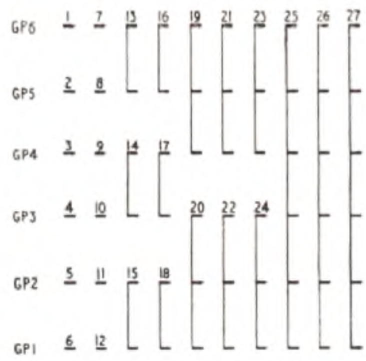
Subtracting equation (2) from equation (1),

$$5a + 2b + c = 17 \quad (3)$$

Values of a, b and c which satisfy equation (3), and which are admissible under equations (1) and (2), are given in the following table:

a	b	c	d	Sum of successive differences
0	7	3	0	14
0	8	1	1	15
1	3	6	0	11
1	4	4	1	6
1	5	2	2	7
1	6	0	3	14
2	0	7	1	15
2	1	5	2	8
2	2	3	3	1*
2	3	1	4	6
3	0	2	5	8
3	1	0	6	9

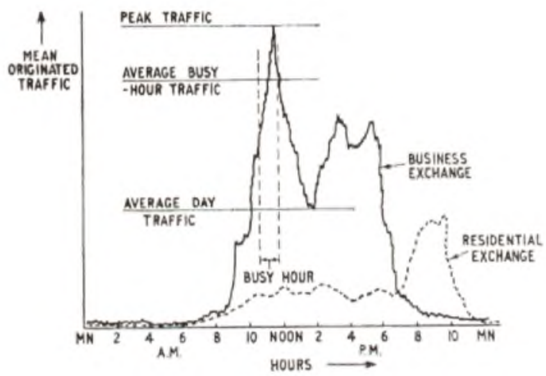
The preferred grading is the one with the least sum of successive differences, which is obtained by adding the differences between *a* and *b*, *b* and *c*, and *c* and *d*. Thus, the preferred grading is the one indicated by*, and is shown in the sketch.



Q. 8. *Explain the factors which can influence the incidence of telephone traffic during a period of:*
(*a*) *24 hours, and* (*b*) *one year.*
Illustrate your answer with simple graphs of traffic against time.
Describe briefly the factors which determine the quantity of switching equipment provided to carry this traffic.

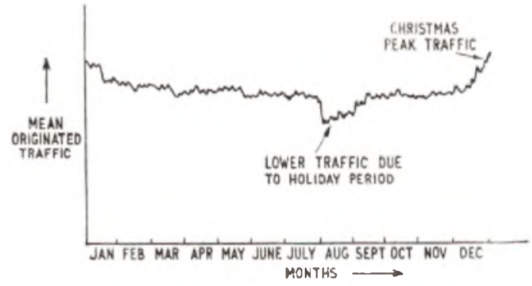
A. 8. Variations in traffic over a 24-hour period and over a year for typical exchanges are shown in sketches (*a*) and (*b*), respectively. In practice, the traffic patterns vary, often widely, from exchange to exchange and at the same exchange from day to day and year to year. The incidence of traffic depends on two closely-related functions, namely, the level and the distribution of the traffic. Both of these depend on the traffic characteristics of the subscribers, the relative numbers of business and residential subscribers, the industrial, commercial and social activities of the area, and the geographical location of the exchange. It is often difficult, therefore, to distinguish between the effect of these factors particularly with respect to daily and yearly variations of traffic.

So far as the traffic over a 24-hour period is concerned, the principal factors are the calling rate and the average call duration of the several categories of subscribers, the ratio of business and residential subscribers, and the industrial, commercial and social activities of the area. The characteristics of business and residential exchanges can be seen in sketch (*a*). As business subscribers invariably have a higher calling rate than residential subscribers, the level of traffic and the length of peak-traffic periods vary, as well as the time at which the high-traffic periods occur. This effect is sometimes offset by the longer duration of "off-peak calls" from residential subscribers. The activities in the area may have the effect of giving traffic peaks at particular times during the day. The peak period may be fixed, for example, because of Stock Exchange or market activity (although in the latter case the traffic will peak on market days only), or it may move from day to day, for example, with the times of high tide in fishing ports and for exchanges serving harbours.



(*a*)

On the yearly scale, however, the traffic is affected more by seasonal variations, holidays, etc., and by the geographical location of the exchange, than by the traffic characteristics of the subscribers and the ratio of business to residential subscribers. The last two features tend to fix the traffic level but not the distribution. Exchanges in city and urban areas tend to have yearly traffic distributions of the type shown in sketch (*b*). Holiday resorts, etc., would tend to have a pronounced high-traffic period during the holiday season. Adverse weather and special events also affect the traffic distribution but tend to have only transitory effects.



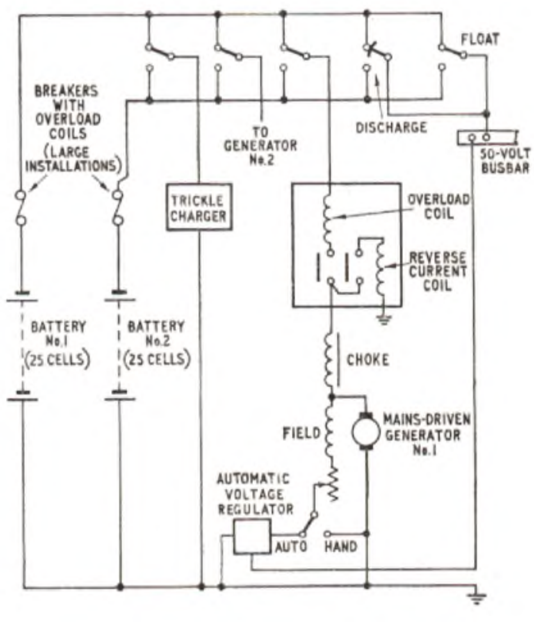
(*b*)

For economic reasons, switching equipment is not provided to carry the absolute peak traffic; conversely, to provide only sufficient equipment to carry the average level of traffic during the day would give the subscribers a most inferior service during the busiest parts of the day. An hourly average of traffic, over two successive busiest half-hours out of the three in the busiest period of the day during which traffic measurements are made, is taken as the basis for equipment provision purposes—this period is termed the busy-hour. Switching equipment is provided to carry the busy-hour traffic at a standard grade of service. The grade of service is, by definition, the ratio of the number of calls that are deliberately allowed to fail at the first attempt, due to the limitation of plant, to the total number of attempted calls, e.g. a grade of service of 0·5 per cent (sometimes shown as 0·005) is equivalent to 1 lost call in 200.

Q. 9. *Draw a schematic diagram of the trunking of a 4-digit automatic exchange in which subscribers' uniselectors are used. Indicate the points at which connexions pass through the main and intermediate distribution frames.*
Why are these connexions passed via the frames?

Q. 10. *Draw a block schematic diagram to show the equipment arrangements of a floated-battery telephone exchange power plant in which power is supplied by a mains-driven motor-generator set with automatic voltage control.*
Explain briefly how the power plant is safeguarded against the following occurrences:
(*a*) *mains failure,*
(*b*) *connexion of the motor-generator to the battery when its output voltage greatly exceeds that of the battery, and*
(*c*) *an earth fault on the main exchange busbar.*

A. 10. The block schematic diagram of a floated-battery power plant driven by a mains motor-generator with automatic voltage control is shown in the sketch.



The power plant is safeguarded against the eventualities quoted by the following means:

(*a*) To prevent the generator being driven by the battery as a motor in the event of mains failure or similar fault condition, a circuit breaker with a reverse-current coil is placed between the generator and the battery feed line to the exchange busbar. When current flows in the direction reverse to normal the breaker is tripped.

(*b*) The breaker also has an overload coil that trips the breaker when the current from the generator exceeds a pre-determined value, which would occur if the generator output voltage greatly exceeded the voltage of the battery. In the event of this or a fault condition such as described in (*a*) occurring, the battery would remain connected to the exchange busbar.

(*c*) If an earth fault occurred on the exchange busbar, the high current flowing would have two effects. Firstly, the current in the overload coil would trip the breaker and disconnect the generator, and, secondly, the battery fuse (or breaker) would blow thereby disconnecting the battery from the busbar.

MATHEMATICS B, 1963

Students were expected to attempt not more than any six questions.

Q. 1. (*a*) *The currents x, y and z flowing in the branches of a network are given by the equations:*

$$r_1x + Rz = r_4y.$$
$$r_2(x - z) - Rz = r_3(z + y).$$
$$r_1x + r_2(x - z) = E.$$

Derive an expression for z in terms of r_1, r_2, r_3, E and R, and hence write down the conditions for z to be zero.

(*b*) *In discussing the deflexion of a beam, the equation*

$$\frac{14a^4}{45} + \frac{y^4}{24} = \frac{a^2y^2}{3} \text{ occurs.}$$

Find y in terms of a.

A. 1. (*a*)

$$r_1x + Rz = r_4y \quad \ldots\ldots(1)$$
$$r_2(x - z) - Rz = r_3(z + y) \quad \ldots\ldots(2)$$
$$r_1x + r_2(x - z) = E \quad \ldots\ldots(3)$$

From equation (1), $y = \dfrac{r_1x + Rz}{r_4}$.

Substituting for y in equation (2),

$$r_2(x - z) - Rz = r_3\left(z + \frac{r_1x + Rz}{r_4}\right),$$

$$\text{or } r_2x - r_2z - Rz = r_3z + \frac{r_1r_3}{r_4}x + \frac{r_3R}{r_4}z.$$

$$\therefore x\left(r_2 - \frac{r_1r_3}{r_4}\right) - z\left(r_2 + r_3 + R + \frac{r_3R}{r_4}\right) = 0 \quad \ldots\ldots(4)$$

From equation (3), $x(r_1 + r_2) - r_2z = E$,

$$\text{or } x = \frac{E + r_2z}{r_1 + r_2}.$$

Substituting for x in equation (4),

$$\left(\frac{E + r_2z}{r_1 + r_2}\right)\left(r_2 - \frac{r_1r_3}{r_4}\right) - z\left(r_2 + r_3 + R + \frac{r_3R}{r_4}\right) = 0.$$

Multiplying the above equation by $r_4(r_1 + r_2)$,

$$(E + r_2z)(r_2r_4 - r_1r_3) - z(r_1 + r_2)\{r_2r_4 + r_3r_4 + R(r_3 + r_4)\} = 0.$$

$$\therefore z[r_2(r_2r_4 - r_1r_3) - (r_1 + r_2)\{r_2r_4 + r_3r_4 + R(r_3 + r_4)\}] = E(r_1r_3 - r_2r_4),$$

$$\text{or } z\{r_2^2r_4 - r_1r_2r_3 - r_1r_2r_4 - r_1r_3r_4 - r_2^2r_4 - r_2r_3r_4 - R(r_1 + r_2)(r_3 + r_4)\} = E(r_1r_3 - r_2r_4).$$

$$z - r_1r_2(r_3 + r_4) - r_3r_4(r_1 + r_2) - R(r_1 + r_2)(r_3 + r_4) = E(r_1r_3 - r_2r_4).$$

$$\therefore z = \underline{\frac{E(r_2r_4 - r_1r_3)}{r_1r_2(r_3 + r_4) + r_3r_4(r_1 + r_2) + R(r_1 + r_2)(r_3 + r_4)}}.$$

For z to be zero, the numerator must be zero.

$$\therefore r_2r_4 - r_1r_3 = 0, \text{ or } r_1r_3 = r_2r_4.$$

$$\therefore \underline{\frac{r_1}{r_2} = \frac{r_4}{r_3}}.$$

(*b*)

$$\frac{14a^4}{45} + \frac{y^4}{24} = \frac{a^2y^2}{3},$$

$$\text{or } \frac{y^4}{8} - a^2y^2 + \frac{14a^4}{15} = 0.$$

This is of quadratic form with y^2 as the unknown, and hence may be solved from the general formula for a quadratic equation.

$$\therefore y^2 = \frac{a^2 \pm \sqrt{a^4 - 4 \times \frac{1}{8} \times \frac{14a^4}{15}}}{\frac{1}{4}}$$

$$= \frac{a^2 \pm a^2\sqrt{1 - \frac{7}{15}}}{\frac{1}{4}}$$

$$= 4a^2\left(1 \pm \sqrt{\tfrac{8}{15}}\right) = 4a^2(1 \pm 0{\cdot}7303)$$

$$= 6{\cdot}921a^2, \text{ or } 1{\cdot}079a^2.$$

Whence, $y = \underline{\pm 2{\cdot}631a, \text{ or } \pm 1{\cdot}038a}$.

(*To be continued*)

Published by *The Post Office Electrical Engineers' Journal*, G.P.O., 2–12 Gresham Street, London, E.C.2,
and printed in Great Britain at The Baynard Press by Sanders Phillips and Co., Ltd., Chryssell Road, London S.W.9