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CONTENTS

City and Guilds of London Institute Examinations, 1961 and 1962

CITY AND GUILDS OF LONDON INSTITUTE EXAMINATIONS, 1961 and 1962

QUESTIONS AND ANSWERS

Answers are occasionally omitted or reference is made to earlier Supplements in which questions of substantially the same form, together with the answers, have been published. Some answers contain more detail than would be expected from candidates under examination conditions.

TELEGRAPHY C, 1961 (*continued*)

Q. 3. *Explain how a 4-wire telephone circuit is used to provide a multi-circuit voice-frequency telegraph system.*

What line characteristics are necessary for the successful operation of the telegraph system?

What factors determine the amount of power to be transmitted in the line for each v.f. telegraph channel?

A. 3. Before a 4-wire telephone circuit can be used to carry a multi-circuit voice-frequency telegraph system it is necessary to split the 4-wire circuit into two 2-wire channels and to remove any unwanted bridging equipment. At each end of the circuit the 2-wire/4-wire hybrid terminations are disconnected, and the voice-frequency signalling or ringing equipment is also removed. If echo-suppressors are used on the telephone circuit these should also be disconnected. A pair of 2-wire channels, transmitting in opposite directions, is then available for the duplex multi-circuit telegraph system, using the standard repeater gains and transmission levels of the telephone circuit.

The attenuation over the frequency range should be equalized so as not to differ by more than $\pm$ 1 db at any frequency from the attenuation at 800 c/s; this is desirable so that the performance of all telegraph channels will be similar. The transmission levels should also be as stable as possible if amplitude-modulated telegraph equipment is to be used. Similarly for frequency-modulated telegraph equipment, the frequency-drift of any carrier 4-wire circuits should be kept low.

The amount of power transmitted for each voice-frequency channel is determined from conflicting requirements.

It is desirable to have maximum power to minimize the effects of interference from other circuits and obtain a good signal-to-noise ratio. At the same time, too high a power would cause interference with other circuits and would overload line amplifiers, causing distortion and inter-channel interference to the telegraph channels. The aggregate telegraph power is therefore limited to a safe value and this permitted power is divided amongst the telegraph channels.

If the maximum instantaneous power permitted at a given point in the transmission system is W watts then the maximum voltage E across the line impedance Z is obtained from the relation for power, $W = E^2/Z$, whence $E = \sqrt{WZ}$. This occurs when all telegraph carrier supplies reach their peak values simultaneously and the maximum permissible voltage e per channel is $e = E/N = \sqrt{WZ}/N$, where N is the number of channels. The maximum permissible power w per channel is $w = e^2/Z = \frac{WZ}{N^2} \times \frac{1}{Z} = W/N^2$.

Thus, maximum power per channel is governed by the square of the number of channels. For international circuits the C.C.I.T.T. has set the maximum aggregate power at 5 mW measured at a zero reference point. For an 18-channel system the permitted power per channel, measured at a zero reference point, would be $(5 \times 10^{-3})/18^2 = 15\,\mu$W per channel. This power enables telegraph circuits to operate on telephone-type circuits with negligible mutual interference. In practice it is usual to use a smaller power, namely, 10 μW per channel at a zero point, since experience has shown that the slight reduction in signal-to-noise ratio is more than compensated by the resultant reduction in inter-channel interference.

Q. 4. *Describe the principles of operation of a direct-recording (black-and-white) facsimile telegraph system suitable for transmitting diagrams, weathermaps or documents.*

Explain what factors determine the frequency bandwidth requirements.

What line characteristics are necessary for satisfactory transmission?

A. 4. In a typical facsimile apparatus of the direct-recording type, suitable for the transmission of diagrams, documents or weather maps, the design of the transmitting machine closely follows the principles conventionally used for photo-telegraph transmitters. The document to be transmitted is wrapped around a drum which is rotated at uniform speed. A beam of light illuminates a small area of the document which reflects light to a photo-electric cell according to the degree of black or white present in the elemental area being examined. Either the drum or the optical system is given a steady traverse in the direction of the axis of the rotating drum so that the whole area of the document is sequentially scanned. The output signal from the photo-electric cell is modulated and amplified to present the information in a form suitable for transmission by line or by radio.

In direct recording, various methods are available to avoid the need for photographic processing. Special papers are available which will record indelible marks under direct control of the amplified received signals. One paper in extensive use depends upon the principle of electrolysis, in which the signal voltage brings about a chemical reaction which changes the colour of the paper from white to black at the recording point; in another paper the application of a high voltage—the amplified signal voltage—burns away a light-coloured insulating layer to expose a carbon-black interior surface at the recording point. Another class of machine uses ordinary paper which is marked by a small inking wheel under the control of a moving-coil electromagnetic system.

In order that indelible marks shall be recorded in the appropriate positions a scanning process complementary to that at the transmitter must take place in the receiver. For this purpose a sheet of recording paper may be fixed to the drum, which is subsequently rotated and traversed by the recording electrode, stylus or ink-wheel. For repeated transmissions, e.g. weathermaps, or for unattended receiving stations it is preferable to use a roll of recording paper which can be steadily fed out to the recording device: scanning is then conveniently effected between the two electrodes required for the electrolytic recording by making one in the form of a stationary bar and the other as a rotating helix suitably supported.

It is essential that the transmitting drums rotate synchronously and always in phase, in order that a particular scanned area at the transmitter shall be in correspondence with the appropriate area at the receiver. Synchronism is conveniently provided by running the two driving motors from the same frequency-controlled power supply. Where this method is not possible it is usual to depend upon stable oscillators to control the motor speed. Phasing is effected by generating a positional signal at the transmitter and sending this signal to the receiver to control the starting instant of the drum rotation.

The required frequency bandwidth of the transmission system depends on the rate at which information is transmitted. This in turn depends upon the following:

(*i*) The smallest dimension which can be scanned, normally equal to the pitch p of the traversing feed device.

(*ii*) The circumference of the drum (πD), which determines how many of the elemental scanning areas can be transmitted in one revolution.

(*iii*) The speed of revolution of the drum (N) which controls the number of lines scanned per second.

The maximum modulating frequency f would occur if alternate scanned elements were black and white and would be equal to one half of the element scanning rate, or $f=\pi DN/2p$.

When amplitude-modulation is used for transmission it is important that the attenuation of the line remains sensibly stable throughout a transmission, that noise, both uniform and impulsive, is at a low level, that echoes due to impedance mismatch are avoided and that delay distortion is small.

Q. 5. *A direct telegraph circuit is to be provided between two teleprinter stations some* 200 *miles apart. Draw a block schematic diagram of the telegraph system apparatus through which the signal would pass, explaining briefly the purpose of each main block of equipment.*

Why is a local record of signals desirable? Explain with a simple sketch how this can be provided.

Q. 6. *Describe the essential features of any form of tape-relay system. What types of apparatus are used and what are their functions? Mention any special operating procedures which are used in this system.*

A. 6. A tape-relay system is one in which telegraph messages over one channel are routed towards their destination by being retransmitted from a central office over another channel; for this purpose the incoming message is recorded in the form of perforations in a paper tape so that retransmission can take place automatically by machine without the need for manual transmission from a keyboard.

Such systems usually operate in conjunction with teleprinter systems using the standardized five-unit start-stop telegraph alphabet. The essential items of apparatus required are:

(*i*) *Printing Reperforator.* The printing reperforator is a machine basically similar to a teleprinter, but frequently used as a receiving-only machine without a keyboard. In place of the printed page of the teleprinter it provides a perforated paper-tape record in which the five-unit combinations are punched transversely across the tape. The holes are not completely perforated, a small part of the circumference being left uncut to act as a hinge and retain the paper "chad". This "chadless" perforation leaves a suitable surface for the corresponding characters to be also printed on the tape: there is in fact a displacement of several characters between the perforated and printed records of a given combination owing to the practical impossibility of simultaneously perforating and printing at the same point. This printed record is required for the operator to read the address and other forwarding instructions on the tape.

(*ii*) *Automatic Transmitter.* The automatic transmitter is a motor-driven transmitter into which the perforated tape is inserted for machine transmission. The machine "reads" the tape by means of five peckers which explore the tape for the presence or absence of perforations, the peckers having sufficient tension to raise the lids of the chadless perforations. In this way the signals stored in the perforated tape are reconverted into electrical five-unit signals, the start and stop signals being inserted automatically by the transmitter. In order to reduce demands on accommodation and to simplify operating procedure, triple-headed machines are used in large switching centres, the three independent transmitting heads being driven from a single electric motor. Alternatively twin heads may be used in which a second message-tape can be loaded before a first message-tape has completed transmission. Suitable control equipment then ensures that the second transmission will take place over a given channel immediately the first transmission is completed: in this way the channel is kept fully occupied and unnecessary delay to messages is eliminated.

(*iii*) *Message-Numbering Transmitters.* When large numbers of messages are being sent over a channel it is desirable that each message should bear a serial number to guard against possible loss and to enable messages to be traced in the event of queries. For this purpose message numbering is achieved by using a form of automatic transmitter: in one form an automatic transmitter is loaded with a tape perforated with a sequence of serial numbers. By means of a control relay-set the numbering transmitter is arranged to cut into a circuit between message-tape transmissions on a given channel and transmit a serial number, starting from zero at midnight and advancing by one for each transmission.

(*iv*) *Teleprinters.* Teleprinters are used as required for monitoring purposes, for example to check the outgoing message transmission.

In a manual tape-relay system, sometimes referred to as a "torn-tape" system, incoming messages at a tape-relay centre are received on printing reperforators and retransmitted over the next stage by means of automatic transmitters. Each receiving channel terminates in a printing reperforator and each sending channel is operated from an automatic transmitter: the send and receive channels can be operated quite independently under duplex conditions. The tapes may be transferred from an incoming position to an outgoing position by hand or by a tube conveyor system.

A jack-field would be provided with the installation and all apparatus and circuits wired to this. This provides flexibility and patching facilities for redistribution of traffic loads, temporary replacement of faulty apparatus, testing, monitoring and supervisory control.

Other forms of tape-relay systems providing for manual, semi-automatic or fully-automatic operation at the switching centre are in service. For message handling at the central switching centre it is important that proper attention is given to the layout of the message as regards preamble, address and text. For message control it may be necessary to signal one or more of the following conditions: the use of certain five-unit character combinations (shown in brackets) has been adopted for this purpose:

(*i*) start of message (ZCZC),
(*ii*) start of text (GGGG),
(*iii*) end of telegram (ZZZZ),
(*iv*) end of message (NNNN).

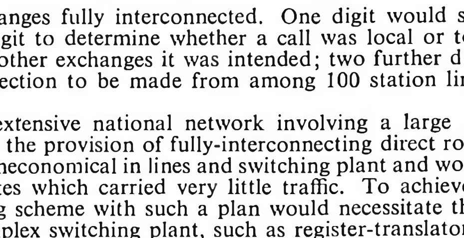
To prevent possible interference from teleprinters it is important that the "who-are-you?" signal should not be present in a perforated tape.

Q. 7. *Describe the basis on which a linked national numbering scheme can be designed for an automatically-switched teleprinter-exchange network. Draw a simplified trunking diagram for a typical exchange.*

What factors, other than the numbering scheme, must be considered in the design of such a network?

A. 7. A linked numbering scheme is one in which a given station may be reached by dialling the same number from any point in the network.

One method of ensuring this would be to design the teleprinter-exchange network in such a way that all exchanges were fully interconnected by direct circuits. Sketch (*a*) shows a simple network of



(*a*)

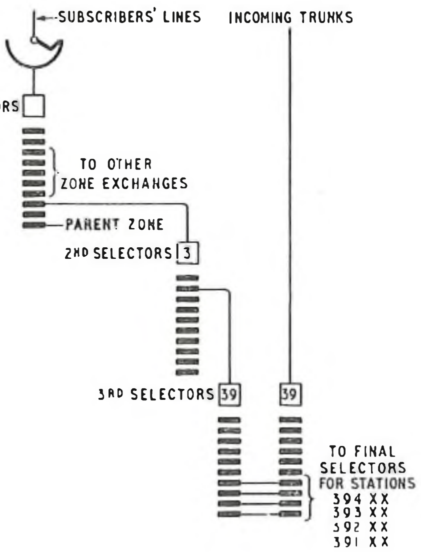
four exchanges fully interconnected. One digit would suffice as a routing digit to determine whether a call was local or to which of the three other exchanges it was intended; two further digits would enable selection to be made from among 100 station lines on any exchange.

In an extensive national network involving a large number of exchanges the provision of fully-interconnecting direct routes would be most uneconomical in lines and switching plant and would include some routes which carried very little traffic. To achieve a limited numbering scheme with such a plan would necessitate the use of a more complex switching plant, such as register-translators in which the dialled routing digits are automatically modified to suit the routing conditions, or discriminating selectors which are able to ignore or suppress certain unwanted digits.

A close approximation to the linked numbering scheme may be attained by basing the network upon a number of major switching centres each of which will act as an intermediate switching point for a large zone of the network; it is an economical advantage if station lines in the immediate area of the zone switching centre are also connected to the appropriate zone exchange but its primary function is to serve as an intermediate switching point. For this purpose all zone exchanges are fully interconnected with one another by direct circuits so that a call can pass from any zone to any other zone for a fixed number of selecting digits. In a network having rather less than 10 zone exchanges a single characteristic digit would suffice to define each zone, and the interconnecting zone circuits would leave first-selector levels at the outgoing exchange to terminate upon second selectors at the incoming zone exchanges.

In each zone would be a number of subsidiary exchanges, according to requirements, each serving an appropriate area of the network and each working direct to the zone-centre exchange serving that part of the network. If each zone comprised less than 10 areas each area could be given a two-letter routing code, the first digit being the characteristic digit of the "parent" zone exchange. Circuits from the parent zone to the dependent area exchanges would leave from second-selector levels and zone subscribers would be able to reach all other stations by dialling only the directory number.

Exceptionally, at area exchanges the circuits to the parent zone centre leave the first-selector level (level 1 in sketch (*b*)) and terminate



(*b*)

on first selectors at the zone exchange. This provides equivalent routing for calls originated either by zone-exchange or area-exchange subscribers but necessitates the prefix digit (1) being dialled by area exchange subscribers when making calls via the parent zone exchange. This is catered for by supplying a simple dialling-code list indicating when the prefix digit is to be used.

Direct circuits between area exchanges and zone centres other than the parent zone can be provided when the amount of such traffic to be carried justifies this: these routes leave the first-selector levels in the area exchange and enable subscribers' stations in the distant zone exchange to be reached by dialling only the directory number. For other direct routings the use of register-translators may be necessary.

The arrangements outlined above provide a very economical solution for a teleprinter-exchange network in which the large majority of stations are concentrated at the zone exchange serving the capital city (which is also the outlet to other national networks), while the remaining stations are mainly served by zone exchanges and only the minority of lines are connected to area exchanges.

In designing the network, consideration must be given to two other factors apart from the routing question. The network plan must be such that metering equipment can determine the appropriate call charge depending upon the geographical position of the objective exchange. It must also be ensured that the number of circuits switched in tandem to form a connexion will not exceed the limits specified for satisfactory teleprinter transmission.

Q. 8. *With the aid of a simplified diagram, explain how supervisory signalling is effected over a bothway trunk circuit connecting two teleprinter automatic exchanges.*

Mention briefly the circuit problems associated with bothway working and explain how they may be resolved.

A. 8. The essential supervisory signals required for effecting connexions over a bothway trunk circuit and their electrical characteristics are as follows:

(*i*) Free-line Condition. A +80-volt potential on each transmission path.

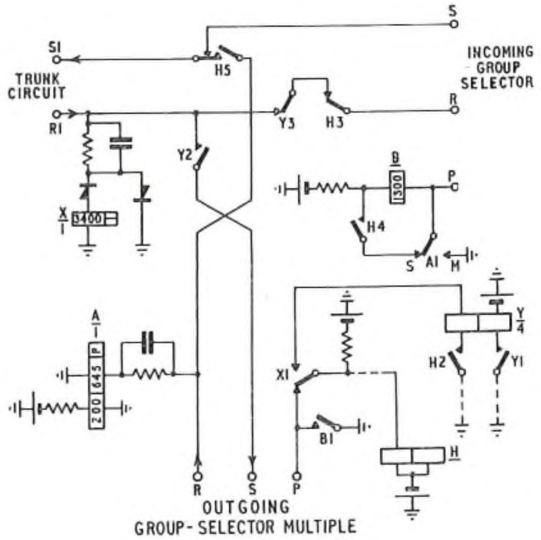
(*ii*) Calling Signal. A −80-volt potential on the forward channel.

(*iii*) Call-confirmation Signal; (*iv*) Proceed-to-select Signal: A combined signal comprising a 20 ms pulse at −80-volt potential over the backward channel.

(*v*) Call-connected Signal. Inversion to −80-volt potential on the backward channel.

(*vi*) Clearing Signal. Inversion to +80-volt potential for at least 325 ms, initiated on either forward or backward channel and followed by reversion to condition (*i*).

A simplified diagram of the circuit elements involved in a bothway trunk relay-set is given in the sketch.



At the outgoing end a battery-testing circuit is provided over the P-wire from the group-selector level multiple. When a free outgoing circuit is seized relay H operates and its contacts H3 and H5 disconnect the path to the incoming selector. When the outgoing group selector extends the R path, relay A is operated to the mark contact, and contact A1 operates relay B. Contact B1 guards the circuit and holds the selectors at the P-wire. With the operation of contact H5 the calling signal is extended over the S1 path of the trunk circuit. At the distant incoming end, H contacts being normal, the call signal is passed to the incoming group selector which generates the call-confirmation pulse for transmission over the backward channel to the outgoing end. The calling signal is also received by the X relay at the incoming end: this X relay, polarized by rectifiers to prevent operation by +80-volt signals, is a high-speed relay whose function at this point is to open the P-wire at contact X1 and so prevent immediately any possibility of the bothway trunk circuit being seized in this direction. At the outgoing end the X relay operates to the call-confirmation pulse and relay Y operates via contacts X1 and H2. Contact Y2 completes the backward path to the selector multiple. The circuit is now through for transmission of dial pulses, teleprinter signals and, later, the clearing signal.

The clearing signal may be originated from either end of the circuit but from the subscriber's termination the clearing signal will appear on both forward and backward channel, causing the restoration of relays A and X and the other relays in sequence. An important feature, omitted from the sketch of the circuit elements, is a delayed guarding feature at clear-down to prevent the trunk circuit from being re-seized over the multiple at either end until the selectors at either end have had ample time to complete their release.

A problem associated with bothway working on trunk circuits is the possibility of a circuit being seized at the same time from both ends, due to the slight delay in the arrival of a calling signal at the incoming end. The possibility of such an occurrence is much reduced by the use of the high-speed X relay and also by reversing the testing sequence at the two ends so that an early-choice circuit at one exchange becomes a late-choice outlet at the distant exchange.

Q. 9. *What forms of signalling code can be used on long d.c. submarine telegraph cables and what are their relative advantages?*

Describe, with a diagram, the construction and operation of either (*a*) *an automatic transmitter* or (*b*) *a highly sensitive receiving relay, for use on such a cable.*

TELEGRAPHY C, 1961 (*continued*)

Q. 10. *Describe, with the aid of a block-schematic diagram, the principles of operation of an automatic error-correcting (ARQ) system suitable for the provision of teleprinter circuits over a radio path. In the system you describe what are the possibilities of the presence of undetectable errors?*

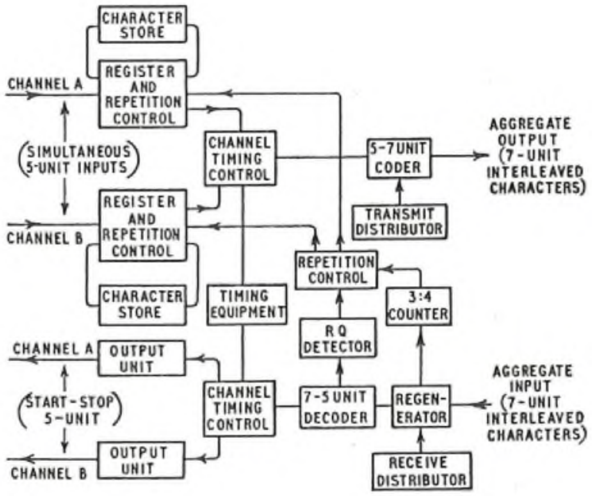
A. 10. In the course of transmission by h.f. radio, telegraph signals are subjected to distorting effects which do not arise on landline circuits. Changes in the nature of the propagation path produce considerable reductions in the level of the received signals due to "fading": these deep fades are often accompanied by electrical noise which at the receiver appears as a marked reduction in the signal-to-noise ratio. Various techniques are applied to transmission and reception radio-telegraphy but for transmission of teleprinter signals by radio an additional form of signal protection is often employed with a view to indicating when mutilation of a signal has occurred. If a duplex radio path is available this indication of signal mutilation is used to initiate an automatic request for the mutilated character to be repeated, also automatically.

In the Van Duuren system of automatic error-correction (ARQ) the five-unit teleprinter signals are converted to seven-unit combinations of special form for transmission by radio. The start and stop signals are dispensed with and the system is worked synchronously. From the seven-unit code, the thirty-five combinations made up of three mark elements plus four space elements are selected so that at the receiver the presence of this 3 : 4 ratio can be detected and used to determine whether the received signal-combination can be regarded as authentic. Defects in transmission would usually convert a mark to a space, or a space to a mark and destroy the 3 : 4 ratio. For the teleprinter alphabet, 32 combinations are required leaving one each to represent the permanent mark and space polarities required for supervisory signals, together with a special "RQ" signal used for initiating the automatic repetition cycle. For economic reasons the synchronous ARQ system is usually operated as a multiplex system, using time-division methods to interleave either two or four 50-baud channels, according to transmission conditions on a given route. The effect of eliminating the start and stop signals, replacing a five-unit code by a seven-unit code and working multiplex is to increase the modulation rate from 50 bauds to 96 bauds (two-channel) or 192 bauds (four-channel).

The sketch shows a block diagram for a 2-channel ARQ system. The input from each 50-baud channel is converted from sequential start-stop signals to simultaneous signals on five wires, using, for example, a tape-reader stepped from the channel timing control. These incoming character-signals are fed to a cyclic store designed to hold four characters usually and to discard them in turn if a repetition is not required. At the same time the signals from the two channels A and B are fed alternately into the coder, under control of the timing distributor, for conversion from 5-unit to 7-unit code. The aggregate interleaved 2-channel signal is fed to the radio transmitter.

On the receive side the aggregate input from the radio receiver is fed into a regenerator, which corrects signal distortion, and into the 3 : 4 ratio counter. Assuming that the ratio is found to be correct the signals are fed into the decoder for reconversion to 5-unit signals. Signals for the two channels A and B are separated by the timing control and fed to the output units where the start and stop signals are added before transmission into the teleprinter receiver.

In the event of a mutilated signal being received this is detected by the 3 : 4 ratio detector. The mutilated signal is not printed: instead



the special 7-unit RQ signal is injected into the transmitting sequence to the distant radio-receiving station. This RQ signal is identified by the RQ detector which initiates a repetition of the last three characters in store, followed by the RQ signal for supervisory purposes. This automatic repetition can take place repeatedly until the correct 3 : 4 ratio is detected, after which transmission continues normally. The four-character RQ cycle is necessary on account of the finite time required for the overall transmission over the double path—send and receive—including radio, landline and equipment.

Infallible error-correction will occur if within any 7-unit combination a space is converted to a mark, or a mark to a space. In the event of a double transposition (mark to space and space to mark) within a 7-unit combination, the 3 : 4 ratio would appear correct and the error would not be detected: the probability of such an occurrence is relatively small.

Correction

In the model answer given to Question 2 of Telegraphy C, 1961 (*Supplement*, Vol. 55, No. 2, p. 40, July 1962), a sketch was given showing a timing chart for a 50-baud 7·5-unit selecting cycle. The words "receiving cam sleeve stops" which are shown above the vertical broken line at 150 ms should have been associated with the vertical broken line at 130 ms, as the receiving cam sleeve stops 130 ms after it has been started.

PRACTICAL MATHEMATICS, 1962

Students were expected to answer any six questions

Q. 1. *Use tables to calculate:*

(*a*) (*i*) $\dfrac{32{\cdot}62 \times 4{\cdot}257 \times 10^{-4}}{\sqrt{23{\cdot}42}}$

(*ii*) $\dfrac{1}{(5{\cdot}6)^2} - \dfrac{1}{35{\cdot}6}$

(*b*) *the percentage error (to three significant figures) when* $(1{\cdot}08)^5$ *is approximated to* 1·40.

A. 1. (*a*) (*i*) $\dfrac{32{\cdot}62 \times 4{\cdot}257 \times 10^{-4}}{\sqrt{23{\cdot}42}}$

$= \underline{0{\cdot}002869}.$

No.	*Log.*
23·42	1·3696
$\sqrt{23{\cdot}42}$	0·6848
32·62	1·5135
4·257	0·6291
10^{-4}	$\bar{4}{\cdot}0000$
	$\bar{2}{\cdot}1426$
	0·6848 –
	$\bar{3}{\cdot}4578$

(*ii*) $\dfrac{1}{(5{\cdot}6)^2} - \dfrac{1}{35{\cdot}6}$

$= \dfrac{1}{31{\cdot}36} - \dfrac{1}{35{\cdot}6}$, from a table of squares.

$= 0{\cdot}03189 - 0{\cdot}02809$, from a table of reciprocals.

$= \underline{0{\cdot}0038}.$

(*b*) $(1{\cdot}08)^5 = 1{\cdot}469$

Percentage error in approximating

$(1{\cdot}08)^5$ to $1{\cdot}40 = \dfrac{(1{\cdot}469 - 1{\cdot}40)}{1{\cdot}469} \times 100$

$= \dfrac{0{\cdot}069}{1{\cdot}469} \times 100$

$= \dfrac{6{\cdot}9}{1{\cdot}469} = 4{\cdot}697.$

No.	*Log.*
1·08	0·0334
$(1{\cdot}08)^5$	0·1670
6·9	0·8388
1·469	0·1670 –
	0·6718

The error is 4·70 per cent (to three significant figures) and would usually be quoted as $\underline{-\ 4{\cdot}70 \text{ per cent}}$ to signify that the approximated figure is less than the true figure.

Q. 2. (*a*) *The following calculation arises in the solution of a traffic problem. Simplify the result, leaving your answer as a fraction in its lowest terms.*

$$\frac{\dfrac{2^3}{3 \times 2}}{1 + 2 + \dfrac{2^2}{2} + \dfrac{2^3}{6}} \times 2$$

(*b*) *The length* (*s*) *measured from its lowest point, of a heavy cable suspended between two points and the vertical height* (*y*) *are connected by the equation*

$(y + c)^2 = s^2 + c^2$, *where c is a constant.*

If $y = 4$ *when* $s = 8$ *show that* $c = 6$. *Find s when* $y = 10$.

A. 2. (*a*)

$$\frac{\dfrac{2^3}{3 \times 2}}{1 + 2 + \dfrac{2^2}{2} + \dfrac{2^3}{6}} \times 2$$

$$= \frac{\dfrac{2^2}{3}}{1 + 2 + 2 + \dfrac{2^2}{3}} \times 2$$

$$= \frac{\dfrac{4}{3}}{5 + \dfrac{4}{3}} \times 2 = \frac{\dfrac{4}{3}}{\dfrac{15 + 4}{3}} \times 2$$

$$= \frac{4}{3} \times \frac{3}{19} \times 2 = \frac{8}{19} = \underline{\frac{2^3}{19}}.$$

(*b*) $(y + c)^2 = s^2 + c^2$.

$\therefore\ y^2 + 2yc + c^2 = s^2 + c^2$

or $y^2 + 2yc = s^2$

$2yc = s^2 - y^2$

$$c = \frac{s^2 - y^2}{2y}.$$

When $y = 4$, and $s = 8$,

$$c = \frac{64 - 16}{8} = 8 - 2 = 6 \quad \text{Q.E.D.}$$

$s^2 = y^2 + 2yc$ from above.

When $y = 10$, and $c = 6$ (constant),

$s^2 = 100 + 2 \times 10 \times 6 = 220.$

$\therefore\ s = \underline{14{\cdot}83}$, from a table of square roots.

Q. 3. (*a*) *Find* i_1 *and* i_2 *when*

$2{\cdot}5i_1 - 3{\cdot}0i_2 = -0{\cdot}75$

$1{\cdot}6i_1 + 0{\cdot}8i_2 = \ \ 0{\cdot}88$

(*b*) *Find the Highest Common Factor and the Lowest Common Multiple of*

$24pq^2r, \qquad 16p^2qr, \qquad 6pqr^3.$

(*c*) *The power, P watts, generated in a circuit is*

$$P = \frac{E^2}{R}, \quad \textit{where E is in volts, and R is in ohms.}$$

Keeping P in watts, find a formula for P in terms of K and Y where K is in megavolts and Y is in megohms.

A. 3. (*a*)

$2{\cdot}5i_1 - 3{\cdot}0i_2 = -0{\cdot}75,$

$1{\cdot}6i_1 + 0{\cdot}8i_2 = \ \ 0{\cdot}88.$

Since the first equation has a common factor 5 and the second equation a common factor 8, it is useful to simplify the equations by dividing them by their respective common factors. Thus

$0{\cdot}5i_1 - 0{\cdot}6i_2 = -0{\cdot}15$ ....(1)

$0{\cdot}2i_1 + 0{\cdot}1i_2 = \ \ 0{\cdot}11$ ....(2)

Multiply the second equation by 6:

$1{\cdot}2i_1 + 0{\cdot}6i_2 = \ \ 0{\cdot}66$ ....(3)

Add equations (1) and (3):

$1{\cdot}7i_1 = 0{\cdot}51$

or $i_1 = 0{\cdot}3$.

Substitute for i_1 in equation (2):

$0{\cdot}06 + 0{\cdot}1i_2 = 0{\cdot}11$

$\therefore\ 0{\cdot}1i_2 = 0{\cdot}11 - 0{\cdot}06$

$$i_2 = \frac{0{\cdot}05}{0{\cdot}1}$$

$= 0{\cdot}5.$

Thus $\underline{i_1 = 0{\cdot}3, \text{ and } i_2 = 0{\cdot}5.}$

Note: These solutions should be checked in each of the original equations.

(*b*) From the beginner's point of view it is convenient to set out the working for the Highest Common Factor (H.C.F.) in tabular form as follows:

2	$24pq^2r$,	$16p^2qr$,	$6pqr^3$.
p	$12pq^2r$,	$8p^2qr$,	$3pqr^3$.
q	$12q^2r$,	$8pqr$,	$3qr^3$.
r	$12qr$,	$8pr$,	$3r^3$.
	$12q$,	$8p$,	$3r^2$.

The H.C.F. is then given as the product of the common, prime, divisors, i.e. $2 \times p \times q \times r = 2pqr$.

Thus, H.C.F. $= \underline{2pqr}$.

Note: For more experienced students it is simpler to arrange the working as follows, each expression being split up into its prime factors:

$24pq^2r = 2 \times 2 \times 2 \times 3pq^2r$

$16p^2qr = 2 \times 2 \times 2 \times 2p^2qr$

$6pqr^3 = 2 \times 3pqr^3.$

The H.C.F. is thus $2pqr$.

The Lowest Common Multiple is $2 \times 2 \times 2 \times 3pq^2r \times 2p \times r^2$

$= \underline{48p^2q^2r^3}.$

(*c*)
$$P = \frac{E^2}{R} \text{ watts,}$$

If K is in megavolts, then

$$K = \frac{E}{10^6}, \text{ or } E = K \times 10^6,$$

Similarly, Y being in megohms,

$$Y = \frac{R}{10^6}, \text{ or } R = Y \times 10^6,$$

$$\therefore\ P = \frac{(K \times 10^6)^2}{Y \times 10^6}$$

$$= \frac{10^6 K^2}{Y} \text{ watts.}$$

Q. 4. (*a*) *Divide* $x^3 + x^2 + 2x - 4$ *by* $x - 1$.

(*b*) *Multiply* $x^2 + 3x$ *by* $x + 2$.

(*c*) *If* $\log 2 = a$, $\log 3 = b$ *find, without using tables,* $\log 6$, $\log 12$, *and* $\log 4{\cdot}5$ *in terms of a and b.*

A. 4. (*a*)

```
             x² + 2x + 4
x - 1 ) x³ + x² + 2x - 4
        x³ - x²
        .   2x² + 2x
            2x² - 2x
            .     4x - 4
                  4x - 4
                  .    .
```

$$\therefore\ \frac{x^3 + x^2 + 2x - 4}{x - 1} = \underline{x^2 + 2x + 4}.$$

(*b*) $(x^2 + 3x)(x + 2) = x^3 + 3x^2 + 2x^2 + 6x$

$= \underline{x^3 + 5x^2 + 6x}.$

(*c*) $\log 6 = \log (2 \times 3)$

$= \log 2 + \log 3 = \underline{a + b}.$

$\log 12 = \log (4 \times 3) = \log (2^2 \times 3)$

$= 2 \log 2 + \log 3 = \underline{2a + b}.$

Alternatively, using the first result,

$\log 12 = \log (2 \times 6) = \log 2 + \log 6 = a + a + b$

$= \underline{2a + b}.$

$$\log 4{\cdot}5 = \log \frac{9}{2} = \log (3^2) - \log 2$$

$= 2 \log 3 - \log 2$

$= \underline{2b - a}.$

Q. 5. (*a*) *Simplify* $15ab + 3bc - 9ba - 2cb$.

(*b*) *Remove the brackets and simplify* $4x - 3[x - (5x + 1)]$.

(*c*) *Factorize* (*i*) $x^2 - x$, (*ii*) $2x^2y - 2y^3$.

(*d*) *Solve for x the equation* $\frac{3}{5}(2x - 3) + \frac{1}{4}(2 - 3x) = \frac{1}{8}(x - 1)$.

A. 5. (*a*) $15ab + 3bc - 9ba - 2cb = \underline{b(6a + c)}$.

(*b*) $4x - 3[x - (5x + 1)] = \underline{16x + 3}$.

(*c*) (*i*) $x^2 - x = \underline{x(x - 1)}$

(*ii*) $2x^2y - 2y^3 = \underline{2y(x - y)(x + y)}$.

(*d*) If $\frac{2}{3}(2x - 3) + \frac{1}{4}(2 - 3x) = \frac{1}{8}(x - 1)$

$$x = \underline{\frac{47}{13}, \text{ or } 3\frac{8}{13}}.$$

Q. 6. (*a*) *The sending-end resistance (R) of a transmission line is given by*

$$R = R_2 + \frac{R_1R_2}{R_1 + R_2}.$$

Calculate R when R_1 = 4,270 ohms. R_2 = 2,320 ohms.

(*b*) *Make R_1 the subject of the formula in (a).*

(*c*) *State the value of R_2 for which R is zero.*

A. 6. (*a*)

$$R = R_2 + \frac{R_1R_2}{R_1 + R_2}$$

$$= 2{,}320 + \frac{4{,}270 \times 2{,}320}{4{,}270 + 2{,}320}$$

$$= 2{,}320 + \frac{4{,}270 \times 2{,}320}{6{,}590}$$

$$= 2{,}320 + 1{,}503$$

$$= \underline{3{,}823 \text{ ohms.}}$$

No.	*Log.*
4,270	3·6304
2,320	3·3655 +
	6·9959
6,590	3·8189 −
	$\bar{3}$·1770

(*b*)

$$R = R_2 + \frac{R_1R_2}{R_1 + R_2}.$$

$$\therefore\ R(R_1 + R_2) = R_2(R_1 + R_2) + R_1R_2$$

$$\text{or } RR_1 + RR_2 = R_1R_2 + R_2^2 + R_1R_2$$

$$RR_1 - 2R_1R_2 = R_2^2 - RR_2$$

$$R_1(R - 2R_2) = R_2(R_2 - R)$$

$$\therefore\ \underline{R_1 = \frac{R_2(R_2 - R)}{R - 2R_2}}.$$

(*c*) When $R = 0$,

$$R_2 + \frac{R_1R_2}{R_1 + R_2} = 0$$

$$\text{or } R_2(R_1 + R_2) + R_1R_2 = 0$$

$$R_1R_2 + R_2^2 + R_1R_2 = 0$$

$$R_2^2 + 2R_1R_2 = 0$$

$$R_2(R_2 + 2R_1) = 0$$

$$\therefore\ R_2 = 0, \text{ or } R_2 + 2R_1 = 0.$$

Thus, the value of R_2 for which $R = 0$ is $\underline{0, \text{ or } -2R_1}$.

Q. 7. (*a*) *The power P in a circuit is proportional to the square of the current i. If P = 36 when i = 3 calculate i when P = 156.*

(*b*) *When a control switch is moved from one position to the next a variable resistance is increased by the same amount. At position 5 the resistance is 5,200 ohms and at position 9 it is 6,400 ohms. Find*

(*i*) *the resistance at position 17,*

(*ii*) *the position of the switch for a resistance of 6,100 ohms.*

A. 7. (*a*) $P \propto i^2$.

$\therefore\ P = ki^2$, where k is a constant.

When $P = 36$ and $i = 3$,

$$k = \frac{P}{i^2} = \frac{36}{9} = 4.$$

$$\therefore\ P = 4i^2$$

When $P = 156$,

$$i^2 = \frac{P}{4} = \frac{156}{4} = 39.$$

$$\therefore\ i = \sqrt{39} = \underline{6{\cdot}245.}$$

(*b*) Let x ohms be the uniform increase in resistance when the control switch is moved one step, i.e. from one position to the next.

In moving from position 5 to position 9 there are 4 steps. Thus the resistance increase will be $4x$ ohms.

$$\therefore\ 4x = 6{,}400 - 5{,}200,$$

$$\text{or } x = \frac{1{,}200}{4} = 300 \text{ ohms.}$$

(*i*) In moving from position 9 to position 17, there are $17 - 9 = 8$ steps. Hence the resistance increases by $8 \times 300 = 2{,}400$ ohms and therefore the resistance at position 17 = 6,400 + 2,400

$$= \underline{8{,}800 \text{ ohms.}}$$

(*ii*) A resistance of 6,100 ohms is an increase of 6,100 − 5,200 = 900 ohms above the resistance at position 5. This is equivalent to 900/300 = 3 steps of the switch and hence the switch position will be $5 + 3 = \underline{8}$.

Q. 8. (*a*) *A triangle ABC is right-angled at B with AB = 2·6 in. and BC = 3·6 in. Find the angles and third side of the triangle by graphical construction and verify your answers by calculation.*

(*b*) *In Fig. 1 the two marked angles are equal. Name the two similar triangles. If AD = 5 cm and DC = 4 cm calculate BC and state the ratio of BD to AB.*

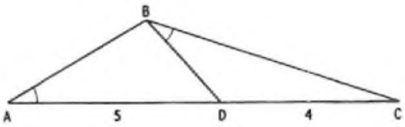
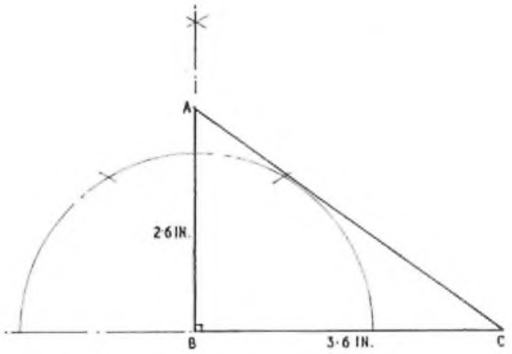


Fig. 1

A. 8. (*a*) The construction is shown in the sketch, which is drawn full-scale.



Construction. Draw the side BC, 3·6 in. long and set out a right angle at B. Draw the line BA, 2·6 in. long. Join AC. Then ABC is the required triangle.

By measurement,

$$AC = \underline{4{\cdot}42 \text{ in.}}$$

$$\angle\ BAC = \underline{54\tfrac{1}{4}^\circ.}$$

$$\angle\ ACB = \underline{36^\circ.}$$

By calculation,

$$AC^2 = 2{\cdot}6^2 + 3{\cdot}6^2 = 6{\cdot}67 + 12{\cdot}96 = 19{\cdot}72.$$

$$\therefore\ AC = \sqrt{19{\cdot}72} = \underline{4{\cdot}44 \text{ in.}}$$

$$\tan \angle\ BAC = \frac{3{\cdot}6}{2{\cdot}6} = 1{\cdot}385.$$

$$\therefore\ \angle\ BAC = \underline{54^\circ\ 10'.}$$

$$\angle\ ACB = 180^\circ - (90^\circ + 54^\circ\ 10')$$

$$= 180^\circ - 144^\circ\ 10' = \underline{35^\circ\ 50'.}$$

No.	*Log.*
3·6	0·5563
2·6	0·4150 −
	0·1413

Note: As a check, it is advisable to determine ∠ ACB also in a similar manner to that used for ∠ BAC.

Thus, $\tan \angle\ ACB = \frac{2{\cdot}6}{3{\cdot}6} = 0{\cdot}7219$ (reciprocal of 1·385),

$$\therefore\ \angle\ ACB = 35^\circ\ 50'.$$

(*b*) Refer to Fig. 1.

The two similar triangles are ABC and BCD. Comparing the angles in these triangles, it is seen that angle BCD is common, angle CBD is equal to angle BAD (given) and, hence, angle BDC equals angle ABC.

From the properties of similar triangles,

$$\frac{BC}{DC} = \frac{AC}{BC}.$$

$$\therefore \quad (BC)^2 = 4 \times 9 = 36$$

$$BC = \underline{6 \text{ cm.}}$$

$$\text{Also,} \quad \frac{BD}{AB} = \frac{DC}{BC} = \frac{4}{6} = \underline{\frac{2}{3}}.$$

Q. **9.** *Copy and complete the following table.*

$x°$	0	20	30	60	80	90
cos x	1·0			0·5		
sin x	0			0·866		
tan ½x	0			0·577		

On the same axes draw the graphs of $y = \sin x$, $y = \cos x$, $y = \tan \frac{1}{2}x$ *Read off from your graphs the values of x at which (i)* $\cos x = \sin x$, *(ii)* $\sin x = \tan \frac{1}{2}x$, *(iii)* $\cos x = \tan \frac{1}{2}x$.

A. 9.

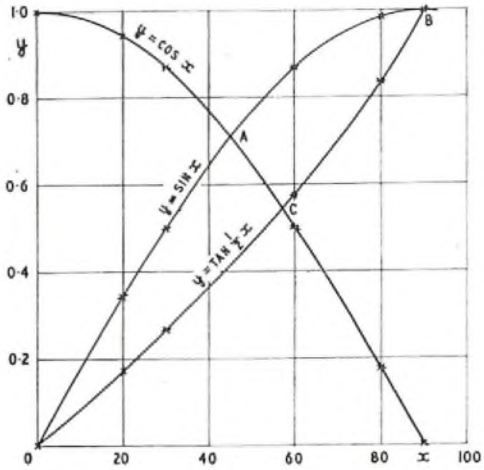
$x°$	0	20	30	60	80	90
cos x	1·0	0·940	0·866	0·5	0·174	0
sin x	0	0·342	0·5	0·866	0·985	1·0
tan ½x	0	0·176	0·268	0·577	0·839	1·0

The graphs of sin x, cos x and tan $\frac{1}{2}x$ are shown in the sketch. From the graph, the following values are obtained:

(*i*) Value of x at which $\cos x = \sin x$ is 45° (point A).

(*ii*) Values of x at which $\sin x = \tan \frac{1}{2}x$ are 0° and 90° (origin and point B).

(*iii*) Value of x at which $\cos x = \tan \frac{1}{2}x$ is 57° (point C).



Q. **10.** *The table shows the resistance and current consumed by electric light bulbs of sizes* 25, 40, 60, 100 *and* 150 *watts.*

Current (amperes)	0·125	0·2	0·3	0·5	0·75
Resistance (ohms)	1,600	1,000	667	400	267

With scales 1 *in.* = 200 *ohms and* 1 *in.* = 0·1 *amperes draw a smooth graph of resistance against current. Find the resistance of the bulb which consumes* 0·4 *amperes and work out its size using the result*

$$Watts = (Amperes)^2 \times Ohms.$$

What size of a bulb would have a resistance of 900 *ohms?*

A. **10.** The resistance of the bulb = 500 ohms.

Wattage of the bulb = 80 watts.

Wattage of the bulb which has 900 ohms resistance = 44 watts (to nearest watt).

ENGINEERING SCIENCE, 1962

Students were expected to attempt four questions from Q.1–6 and two from Q.7–10.

Q. **1.** *A uniform plank,* 20 *ft long, is supported as shown. It is found that the left end of the plank rises if the weight W suspended from the other end exceeds* 60 *lb. Find the weight of the plank.*

If now the suspended weight is reduced to 30 *lb, find the reaction at each support.*

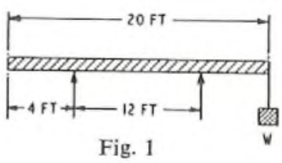
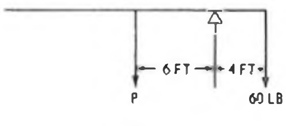


Fig. 1

A. **1.** The condition obtaining when the weight W is 60 lb is shown in sketch (a). The system is balanced on the right-hand support and the left-hand support has no effect. Let the weight of the plank be P lb. Since the plank is uniform its weight acts through its centre of gravity, which is at its mid-point as shown.



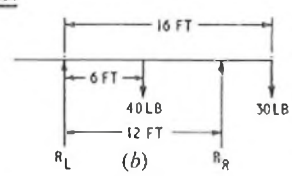
(a)

Equating clockwise and anti-clockwise moments about the right hand support,

$$60 \times 4 = P \times 6$$

$$\therefore \quad P = \underline{40 \text{ lb.}}$$

When $W = 30$ lb there will now be a reaction at each support. Let these reactions be R_L and R_R as shown in sketch (b). The total weight supported is $P + W$, i.e. 40 + 30 = 70 lb. Therefore $R_R + R_L$ must also equal 70 lb.



(b)

Taking moments about the left-hand support, the moment of R_L will be zero and ($R_R \times 12$) anti-clockwise = (30 × 16) + (40 × 6) clockwise.

$$\therefore \quad R_R = \frac{480 + 240}{12} = \underline{60 \text{ lb.}}$$

But $R_L + R_R = 70$ lb. $\therefore$ $R_L = 70 - R_R = \underline{10 \text{ lb.}}$

Q. **2.** *Explain what is meant by the "centre of gravity" of a body. Find the position of the centre of gravity of the brass block shown in Fig. 2, which consists of two rectangles of unequal thickness.*

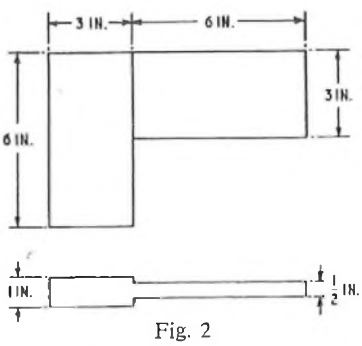
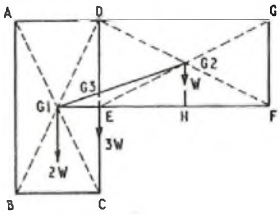


Fig. 2

A. **2.** The centre of gravity of a body is a fixed point through which the weight of the body can be assumed to act whatever its orientation. If the body is freely suspended at a point on its surface a vertical line from the point of suspension will pass through the centre of gravity. Three vertical lines from different points of free suspension will cross at one particular point which is the position of the centre of gravity. The centre of gravity may also be regarded as the point about which the algebraic sum of the moments of its separate portions is zero.

It can be seen from the plan view that the block is symmetrical about a vertical plane through the middle of the cross-section and the centre of gravity will therefore lie on this plane. The block may then be considered as two rigidly connected lamina ABCD and DEFG with their respective centres of gravity at G_1 and G_2 as shown in the sketch.



Let the weight of lamina DEFG be W. Then, since the volume of the thicker rectangle is twice the volume of the thinner and both are of the same material, the weight of lamina ABCD must be $2W$. The position of G_1 can be found by drawing the diagonals AC and BD, and G_2 by drawing the diagonals DF and EG. The centre of gravity G_3 of the whole lamina will lie on the line joining G_1 and G_2. The total weight of the lamina will be $3W$. The sum of the moments about any point due to the forces acting through G_1 and G_2 must be equal to the moment of the force acting through G_3 about the same point. Let the perpendicular distance between G_3 and GF be x in.

Taking moments about F

$$(2W \times 7\tfrac{1}{2}) + (W \times 3) = 3W \times x$$

$$\therefore \quad x = \frac{18}{3} = 6 \text{ in.}$$

$\therefore$ G_3 also lies in the line ED.

Now triangles G_1G_2H and G_1G_3E are similar so that

$$\frac{G_1H}{G_1E} = \frac{4\frac{1}{2}}{1\frac{1}{2}} = 3.$$

Also, $\dfrac{G_2H}{G_3E} = \dfrac{G_1H}{G_1E} \quad \therefore G_3E = \dfrac{G_3H}{3} = \dfrac{1\frac{1}{2}}{3} = \frac{1}{2}$ in.

Thus the centre of gravity of the block lies $\frac{1}{2}$ in. above E, and at the mid-point of the cross-section of the block through DE.

Q. 3. *A load of 500 lb is supported by a pin-jointed frame as shown in Fig. 3.*

Find graphically, or otherwise, the loads in the members AB and AC.

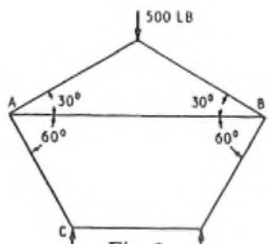
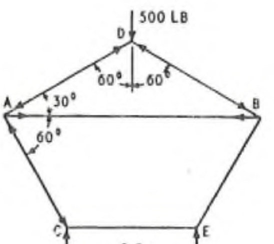
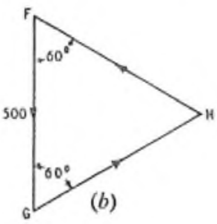


Fig. 3

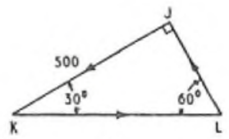


(*a*)

A. 3. The joint at D, as shown in sketch (*a*), must be in equilibrium under the action of the force in the members AD and BD and the 500 lb load. The triangle of forces for joint D is shown in sketch (*b*) and is obtained by drawing FG vertically to represent 500 lb to a suitable scale and then drawing GH and FH at 60° to the vertical, i.e. parallel to members AD and BD. For equilibrium these forces



(*b*)



(*c*)

must act in the direction G to H and H to F and, therefore, members AD and BD are in compression. Triangle FGH is equilateral so forces HF and GH are each equal to 500 lb. Now consider joint A for which the triangle of forces, JKL, is shown at sketch (*c*). Member AD has already been shown to be in compression so the force at joint A must act in the direction D to A, and JK is drawn to scale at 60° to the vertical to represent a force of 500 lb. The forces in members AB and AC are represented by KL and LJ drawn parallel to AB and AC, respectively. For equilibrium these forces must act in the direction K to L and L to J. Therefore, member AB is in tension and member AC is in compression. The triangle JKL is right-angled at J. Hence $\dfrac{JK}{KL} = \cos 30°$.

$$\therefore \quad KL = \frac{JK}{\cos 30°} = \frac{500 \times 2}{\sqrt{3}} = 577{\cdot}5 \text{ lb},$$

$$\text{and } JL = JK \tan 30° = \frac{500}{\sqrt{3}} = 288{\cdot}75 \text{ lb}.$$

Thus, the load in member AC = 288·75 lb compression, and the load in member AB = 577·5 lb tension.

Q. 4. *What quantities are measured in newtons, joules and watts? Give a definition of each of these three units.*

A car weighing 1,000 kg is driven at a steady speed of 45 km/hour up a gradient of one in twenty (i.e. rising 1 m in a slope distance of 20 m). Find the power used in overcoming the force of gravity and the work done against gravity in driving the car a distance of 1 km. The acceleration due to gravity may be taken as 9·81 metres per second per second.

A. 4. Newtons, joules and watts are all derived units used in the m.k.s. system, and are used for the measurement of force, work, and rate of working, respectively.

The force required to give a mass of 1 kg an acceleration of 1 m/s^2 is 1 newton. When a force of 1 newton acts through a distance of 1 m the work done is equal to 1 joule. When a force of 1 newton acts through a distance of 1 m in 1 second the rate of working is 1 joule/s and is equal to 1 watt.

Thus, since force = mass × acceleration,

$$\therefore 1 \text{ newton} = 1 \text{ kg} \times 1 \text{ m/s}^2.$$

$$\text{Work done} = \text{force} \times \text{distance},$$

$$\therefore 1 \text{ joule} = 1 \text{ newton} \times 1 \text{ m}.$$

$$\text{Power} = \text{work done/time},$$

$$\therefore 1 \text{ watt} = 1 \text{ joule/1 s} = 1 \text{ joule/s} = 1 \text{ kg} \times \frac{1 \text{ m}}{\text{s}^2} \times \frac{1 \text{ m}}{\text{s}}.$$

Thus, all three units can be defined in terms of the fundamental units of mass, length and time, which in the m.k.s. system are 1 kg, 1 metre and 1 second.

For every metre the car travels up the incline it rises 1/20 m vertically. In 1 second the car travels 45,000/3,600 metres up the incline. Therefore, in 1 second it rises vertically

$$\frac{45{,}000}{3{,}600 \times 20} = \frac{45}{72} \text{ metres}.$$

The mass of the car is 1,000 kg and therefore the force of gravity which must be overcome in order to raise the car is 1,000 × 9·81 newtons.

The work done per second = force × distance raised per second.

$$\therefore \text{work done per second} = \text{power} = \frac{45}{72} \times 9{,}810 = 6{,}131{\cdot}25 \text{ watts}.$$

When the car is driven a distance of 1 km it will rise a distance of

$$\frac{1{,}000}{20} \text{ m} = 50 \text{ m}.$$

$$\text{Work done} = \text{force} \times \text{distance} = 9{,}810 \times 50 = 490{,}500 \text{ joules}.$$

Q. 5. *State what is meant by the coefficient of friction between two plane surfaces, and describe an experiment by which it may be measured.*

The coefficient of friction between a solid cube of weight 4 lb and the table on which it stands is 0·3. What is the value of the force necessary to make the block slide when the force is applied as in Fig. 4?

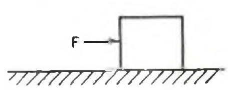


Fig. 4

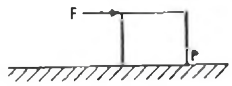


Fig. 5

If a slowly increasing force is applied as in Fig. 5, will the cube first slide or turn over about P?

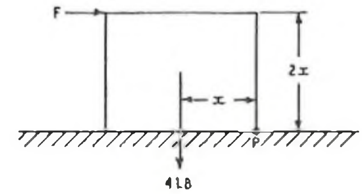
A. 5. The coefficient of friction (μ) between two plane surfaces is the ratio of the force required to just move the surfaces relative to each other (limiting friction) and the normal force between them.

For a description of an experiment by which it may be measured see A.5, Engineering Science, 1959, *Supplement*, Vol. 52, No. 4, p. 54, Jan. 1960.

The force required to make the block slide when applied as in Fig. 4 is equal to the coefficient of friction × weight of the block.

$$\therefore F = \mu R = 0{\cdot}3 \times 4$$
$$= \underline{1{\cdot}2 \text{ lb wt.}}$$

When the force is applied as in Fig. 5, in order to turn the cube over about point P the moment of force F about P must exceed the moment of the weight of the cube about P. Let each side of the



cube be $2x$ in. The weight of the cube acts at its centre point; therefore, taking moments about P, as shown in the sketch,

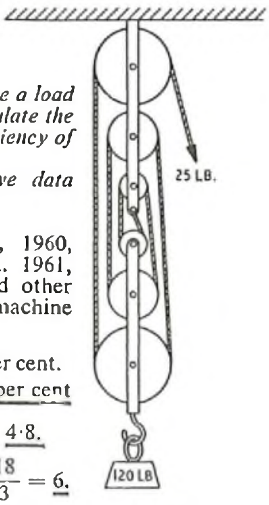
$$F \times 2x = 4 \times x$$
$$F = 2 \text{ lb wt.}$$

Since the cube will slide when F exceeds 1·2 lb wt it will slide before it will turn over.

Q. 6. *Explain the meaning of the term "efficiency" as applied to a machine.*

When using a lifting machine, it is found that an effort of 25 lb must be used through a distance of 18 ft in order to raise a load of 120 lb through a distance of 3 ft. Calculate the speed ratio, mechanical advantage and efficiency of the machine.

Sketch a machine to which the above data might refer.



A. 6. See A.5, Engineering Science, 1960, *Supplement*, Vol. 53, No. 4, p. 58, Jan. 1961, for an explanation of the efficiency and other characteristics of a machine. For the machine specified in this question:

$$\text{Efficiency} = \frac{120 \times 3}{25 \times 18} \times 100 \text{ per cent.}$$
$$= \underline{80 \text{ per cent}}$$

$$\text{Mechanical advantage} = \frac{120}{25} = \underline{4{\cdot}8.}$$

$$\text{Speed ratio} = \text{Velocity ratio} = \frac{18}{3} = \underline{6.}$$

The sketch shows a machine to which the above data might refer.

Q. 7. *A moving-coil meter has a resistance of 50 ohms and gives a full-scale deflexion when a current of 1 mA passes through it. Show how the meter may be used to measure (a) currents up to 100 mA, (b) voltages up to 100 volts.*

Two moving-coil meters are to be built into a piece of equipment to measure the current through a resistance and the voltage across it.

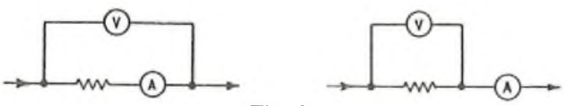
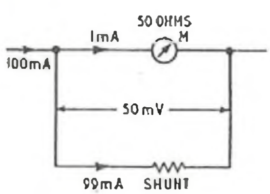


Fig. 6

Explain briefly why, whichever of the two arrangements shown in Fig. 6 is used, one meter will give an incorrect reading.

A. 7. (*a*) In order to measure currents up to 100 mA the meter must be shunted so that at full-scale deflexion 1 mA flows in the meter and 99 mA in the shunt as shown in sketch (*a*). Since the resistance of the meter is 50 ohms, when 1 mA is flowing the potential difference across it will be



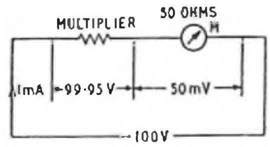
(*a*)

$$\frac{1}{1{,}000} \times 50 = 50 \text{ mV.}$$

This potential difference also appears across the shunt and, since 99 mA is flowing through it, its resistance must be

$$\frac{50}{1{,}000} \div \frac{99}{1{,}000} = \underline{0{\cdot}505 \text{ ohms.}}$$

(*b*) In order to measure voltages up to 100 volts a multiplier must be inserted in series with the meter, as shown in sketch (*b*). It has already been shown that the potential difference required across the meter for full-scale deflexion is 50 mV so that the potential difference across the multiplier must be 99·95 volts. Therefore the resistance of multiplier



(*b*)

$$= 99{\cdot}95 \times 1{,}000 = \underline{99{,}950 \text{ ohms.}}$$

When the voltmeter is connected across both the resistor and ammeter the voltage measured is the potential across the combined resistance of the resistor and ammeter. This will be greater than the potential across the resistor alone. The ammeter, however, will indicate the true current in the resistor.

With the second method of connexion the voltmeter is connected directly across the resistor and thus gives a true reading of the voltage across the resistor. The current flowing in the ammeter, however, is the sum of the current in the resistor and the current through the voltmeter and the reading will be greater than the current in the resistor only.

Q. 8. *State what is meant by the terms "electromotive force, potential difference, one coulomb of electricity".*

What is the reading of the high-resistance voltmeter in the circuit shown in Fig. 7 when switches S_1 and S_2 are both open?

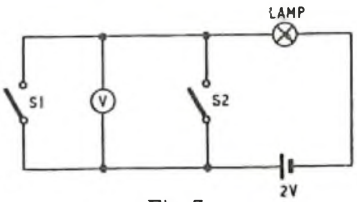


Fig. 7

It is found that on closing switch S_1 the lamp lights brightly and the voltmeter reading is zero; but on opening S_1 and closing S_2, the lamp lights only dimly and the voltmeter reads 0·5 volts. Suggest a reason for this, and a possible remedy.

Calculate the time taken for 10 coulombs to flow in the circuit when S_1 is closed.

A. 8. The electronic theory of the structure of matter postulates that atoms are made up of a central nucleus, consisting of protons and neutrons, around which electrons, equal in number to the number of protons in the nucleus, are in orbit. The proton possesses a positive charge while the electron has an equal negative charge so that the atom is generally electrically neutral. In conductors the electrons in the outer orbits are only loosely bound to the atom and move at random in the material. Charges may be isolated by chemical or electromagnetic means and then aquire potential energy, so producing an electric field. In an electric field a potential difference (p.d.) exists between any two points and a current of electrons will move along a conductor placed in the field until the potential difference at the ends of the conductor is zero.

A current of electricity can then be considered as a movement of electric charges usually in the form of electrons. In electrolytes and semiconductors the current may consist of a movement of positive charges as well as electrons. The magnitude of the current depends on the rate at which the charges are flowing, i.e. on the number of charges passing any point in unit time. The charge carried by an electron is very small and the practical unit of charge is the coulomb. One coulomb of electricity is defined as that quantity of charge which passes a given point in 1 second when a current of 1 ampere is flowing. In order to move the charges from one point to another, work must be done to overcome the resistance of the circuit and when the work done by 1 coulomb in moving between two points is 1 joule the p.d. between the points is 1 volt.

When the terminals of a simple voltaic cell are left disconnected charges accumulate on the plates due to the chemical action and expenditure of energy in the cell, and a p.d. is built up between the plates. The p.d. rises until a steady value is reached which opposes further movement of charges within the cell. The force which causes the charges to move within the cell is termed the electromotive force (e.m.f.). When the cell is not maintaining a current, a state of equilibrium exists and the e.m.f. is exactly equal to the p.d. at the terminals. When the terminals are connected externally, charges flow in the circuit and are replenished by the e.m.f. In practice a simple cell has internal resistance, and when the cell is delivering current the p.d. across its terminals will be less than the e.m.f. due to the voltage drop in its internal resistance. E.m.f. is therefore a measure of the electrical driving force in a circuit and is due to separation of charges either by chemical means, or electromagnetic means as in a transformer or generator.

In Fig. 7, when switches S_1 and S_2 are both open the resistance in the circuit is very high. The current will be very small and for practical purposes can be neglected. There will be no p.d. across the lamp and the voltmeter will therefore indicate the p.d. across the terminals of the cell, which will be equal to its e.m.f. since it is not delivering current. The voltmeter will therefore read 2 volts.

When switch S_1 is closed there is no p.d. across it and the lamp is effectively connected directly across the cell, because the resistance of switch S_1 is zero. When switch S_2 only is closed the p.d. across the contact is 0·5 volts; switch S_2 is, therefore, not a short-circuit but has some resistance. This may be caused by the contact being dirty, the spring pressure being low, or by a badly soldered joint; the contact could, therefore, be cleaned, the spring pressure adjusted or the joint resoldered, to remedy the defect.

Let the resistance of the lamp be R ohms, then when switch S_1 is closed the current flowing will be $2/R$ amp.

$$\text{Now current} = \frac{\text{coulombs}}{\text{time}}, \text{ or } I = \frac{Q}{t}$$

$$\therefore t = \frac{Q}{I}$$

$$\therefore \text{ Time taken for 10 coulombs to flow} = \frac{10}{2/R}$$

$$= \underline{5R \text{ seconds}}$$

Q. 9. *Describe the construction and properties of a 6-volt lead-acid accumulator, and the measures necessary for its maintenance.*

Q. 10. *With the aid of sketches and/or diagrams, show how the heating, chemical and magnetic effects of an electric current may be demonstrated.*

Explain briefly how one of these effects may be used for the measurement of a current.

ENGINEERING DRAWING, 1962

Students were expected to attempt Question 1 and three others.

Q. 1. *The front elevation and plan of a Switch Arm for an electric motor starter are given in Fig. 1. Draw, full size, two views as follows:*

(*i*) *the front elevation,*

(*ii*) *the end elevation as seen when looking in the direction of the arrow A.*

Show eight principal dimensions. Print, neatly, the title and scale. Also state whether your views are in 1st or 3rd angle projection.

This drawing is in first angle projection.

All dimensions in inches.

The sizes of arcs of small radius, not dimensioned, are left to your discretion.

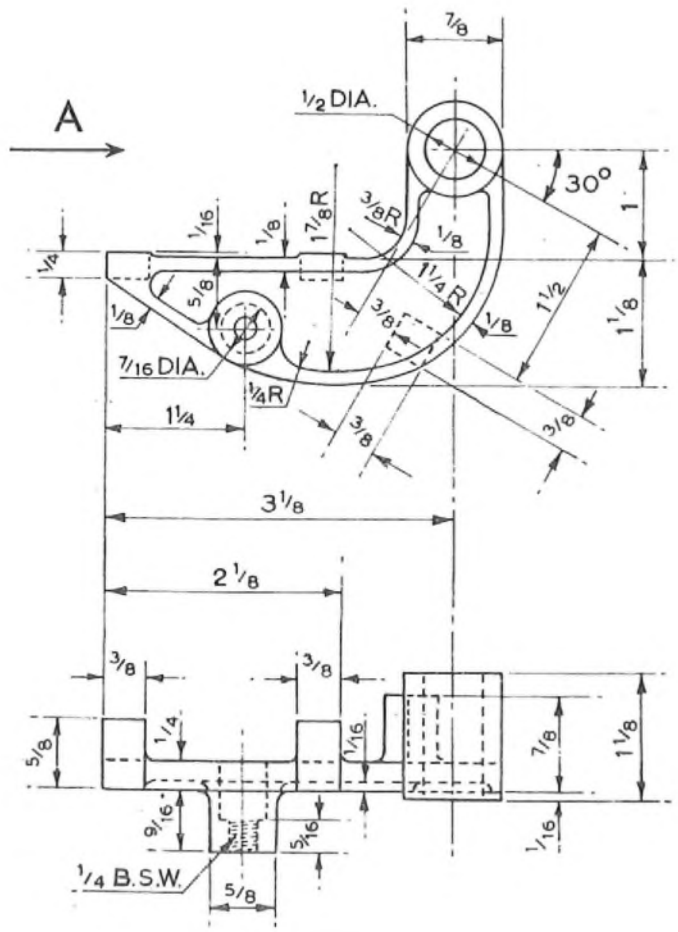
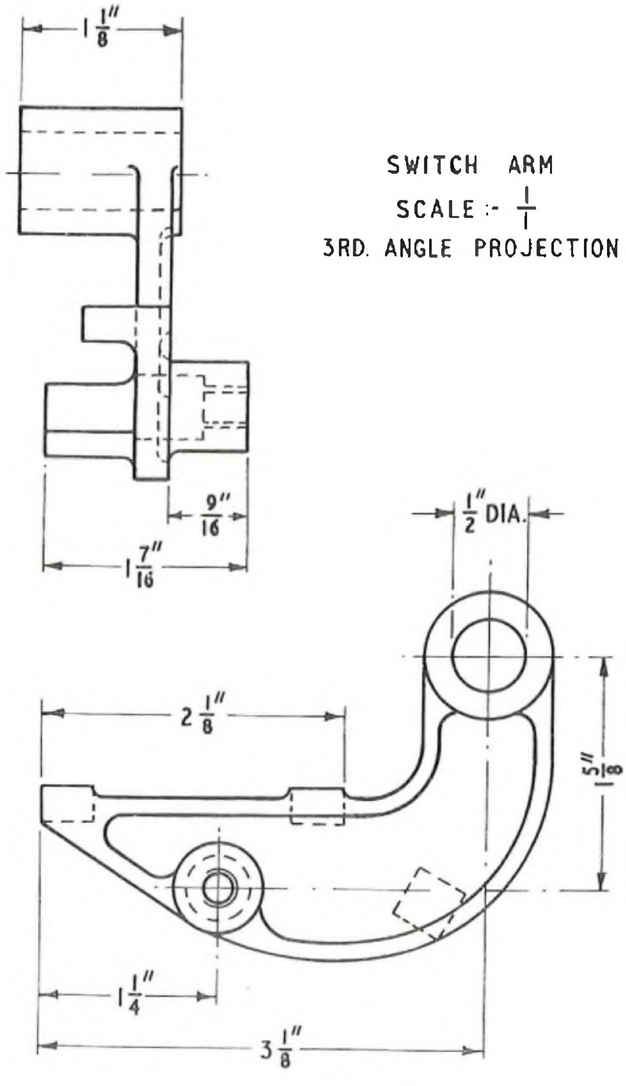


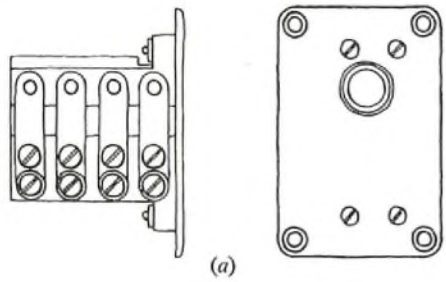
Fig. 1

A. 1. The required views are shown in the sketch. For convenience in printing, the sketch is reproduced at a reduced size from that asked for in the question and the end elevation has been shown above the front elevation instead of at its left-hand side.

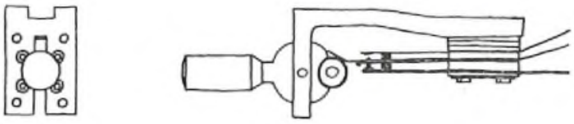


Q. 2. Draw, freehand, the front and side elevations of ONE of the following: a telephone jack, a telephone key, or a single-pole knife switch.

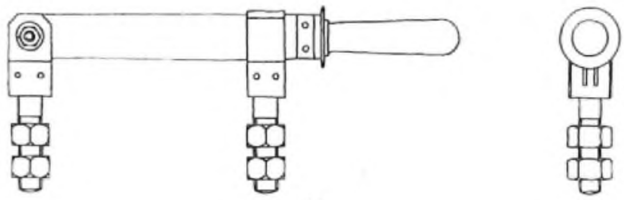
A. 2. The freehand drawings of the telephone jack, telephone key and single-pole knife switch are shown in sketches (*a*), (*b*) and (*c*), respectively.



(*a*)



(*b*)



(*c*)

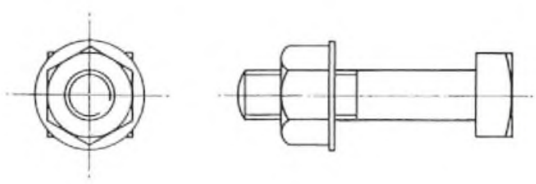
Q. 3. Name the material of which each of the following is made:
The contact springs of a key or jack,
The contact points on these springs,
The insulators between the springs,
The base on which a fuse is mounted,
An insulator for overhead lines.

A. 3. Typical examples of materials commonly used for the purposes stated are given below. The list is not exhaustive.

(*i*) Contact springs of a key or jack: nickel silver, hard.
(*ii*) Contact points on these springs: silver.
(*iii*) Insulators between the springs: synthetic resin-bonded paper (s.r.b.p.) or ebonite.
(*iv*) Base on which a fuse is mounted: any suitable insulating material, e.g. thermosetting-plastic moulding, or ebonite.
(*v*) Insulator for overhead lines: glazed porcelain.

Q. 4. Draw, full size, two views of a ½ in. B.S.W. bolt with a square head and fitted with a hexagonal nut and a washer. Make the length of the bolt 2½ in. and the thread half the length of the bolt. The thread is to be shown, only where it is visible, by a conventional method.

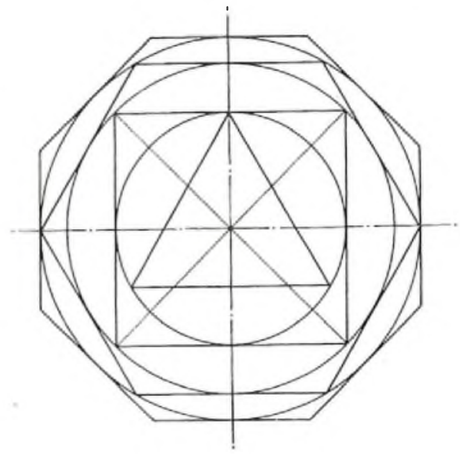
A. 4. The two views are shown in the sketch. For convenience in printing, the sketch is reproduced at a reduced size from that asked for in the question.



Q. 5. Draw a circle 4 in. diameter. Round it draw the circumscribing octagon. Inside the circle draw the inscribed hexagon. Inscribe a circle in this hexagon. In this circle inscribe a square. Draw the inscribed circle in this square and in this last circle inscribe an equilateral triangle.

All construction lines should be shown.

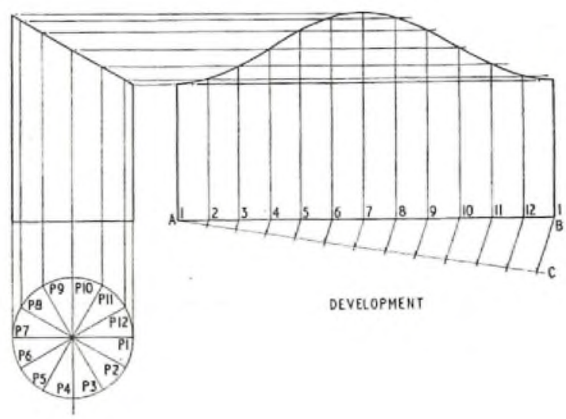
A. 5. The required construction is made as shown in the sketch and described below, although the description was not required in the answer.



Draw the horizontal and vertical centre lines.
Draw a circle of 4 in. diameter.
Draw lines tangential to the circle using a 45° set-square and a tee-square, thus producing the circumscribing octagon.
Using a 60° set-square, draw lines commencing at the intersection of the horizontal centre line with the circle, and continuing from each new intersection.
Draw the inscribed circle such that the sides of the hexagon just drawn are tangential to it.
Draw diagonals through the centre point, using a 45° set-square, to intersect this circle, and draw horizontal and vertical lines from these intersections to complete the square.
Draw the inscribed circle such that the sides of the square just drawn are tangential to it.
Using a 60° set-square, draw lines from the vertical centre line to intersect the circle, and from these intersections draw a horizontal line to complete the equilateral triangle.

Q. 6. A cylinder, 1¾ in. diameter, stands on one end. The top is cut off by a plane at 60 deg. to the axis. The maximum height is 3 in. Draw the development of the curved surface of this solid.

A. 6. The development of the curved surface of the solid is shown in the sketch. The construction is described below although it was not required in the answer.

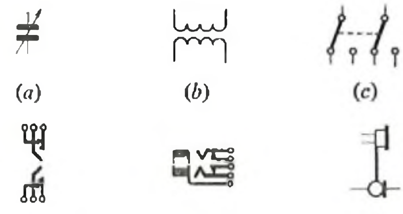


Draw the side elevation and plan of the cylinder.
Divide the plan into 12 equal sectors.
Draw the base line AB equal to πd, where d is the diameter of the cylinder.
Draw the line AC at any suitable angle to AB and mark off 12 equal divisions. Join C to B and draw, from the marked points on AC, lines parallel to CB to intersect the line AB at points 2–12.
Draw vertical lines from P8–P12 on the plan view to intersect with the 60° plane, and draw horizontal lines from these intersections to intersect with verticals drawn from points 2–12 on the base line

ENGINEERING DRAWING, 1962 (*continued*)

AB. The points ot intersection of these horizontal and vertical lines are points on the development curve.

Draw the development curve by joining these points.



(a) (b) (c) (d) (e) (f)

Q. 7. Draw the graphical symbol used in circuit diagrams for each of the following:
Variable capacitor; transformer; double-pole two-way switch; locking key, make-before-break; break jack; handset.

A. 7. The required symbols are shown in sketches (*a*)-(*f*).

TELECOMMUNICATION PRINCIPLES A, 1962

Students were expected to attempt three questions from Q.1-4 and three questions from Q.5-10.

Q. 1. A straight conductor, which is at right angles to a uniform magnetic field of 500 milliwebers/m², carries a direct current of 10 amperes. What is the force on a 10 cm length of the conductor?

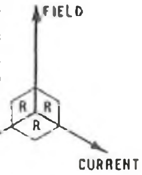
Show on a diagram the relative directions of the magnetic field, the current and the force and state a rule of electro-magnetism from which these directions can be deduced.

In a milliammeter the moving coil is square with 2 cm sides and 100 turns. It rotates so as to cut a uniform radial magnetic field with a strength of 2 webers/m². Calculate the deflecting torque on the coil when it is carrying 10 milliamps.

If, instead of being in a radial magnetic field, this coil were pivoted with its axis perpendicular to a uniform field of flux density 2 webers/m², deduce an expression for the torque on the coil when its plane makes an angle θ with the direction of the field.

A. 1. The force, F, exerted on a conductor carrying a current of i amp in a uniform magnetic field of strength B webers/m² at right angles to the conductor is $i \times B \times L$ newtons, where L is the length of the conductor in metres.

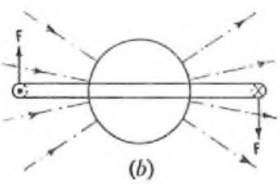
Thus, $F = 10 \times 500 \times 10^{-3} \times 10 \times 10^{-2}$ newtons
$= 0{\cdot}5$ newton.



(*a*)

This force is exerted perpendicular to the conductor and in the direction given by Fleming's left-hand rule (see sketch (*a*)). This states that: "If the thumb, first and second fingers of the left hand are extended so that they are mutually at right angles, and the seCond finger points in the direction of the Current, the First finger in the direction of the magnetic Field then the thuMb points in the direction of Motion when the conductor moves under the action of the field."

Since the magnetic field in which the coil of the milliammeter moves is radial relative to the coil, the lengthwise sides of the coil will always move at right angles to the field, as shown in sketch (*b*).



(*b*)

The ends of the coil will be parallel to the field and so no force will be exerted on them.

The force on each length-wise conductor
$= 2 \times 10^{-2} \times 2 \times 10 \times 10^{-3}$
$= 4 \times 10^{-4}$ newtons.

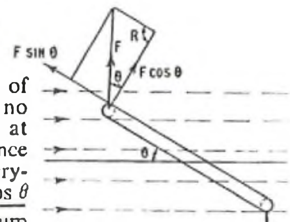
The radial distance to each conductor is
1×10^{-2} metres.

Hence, the torque due to one conductor
= force × radius
$= 4 \times 10^{-6}$ newton metres.

For a coil of 100 turns there will be 200 lengthwise conductors.

∴ Total deflecting torque
$= 200 \times 4 \times 10^{-6}$
$= 8 \times 10^{-4}$ newton metres.

If the coil is in a parallel magnetic field, instead of a radial field, the force exerted on the conductors will always be perpendicular to the field. As shown in sketch (*c*) this force can be resolved into two forces at right angles. The component $F \sin \theta$ will be radial, i.e. through the axis of rotation of the coil and so will exert no torque. The component $F \cos \theta$ is at right angles to the radius and hence exerts a torque. When the coil is carrying 10 mA, this torque is $(8 \times 10^{-4}) \cos \theta$ newton metre. This torque is maximum when $\cos \theta = 1$, i.e. when $\theta = 0$, and the plane of the coil is parallel to the magnetic field; it is zero when the plane of the coil is perpendicular to the field, since in that position cos 90 = 0.



(*c*)

Q. 2. Calculate the capacitance between two air-spaced parallel plates each of effective area 150 cm², spaced 1 mm apart. A potential difference of 100 volts is maintained between the plates. What is
(a) the electric field strength between the plates,
(b) the charge held by the capacitance?

This capacitor is connected in a circuit which gives it a steady charging current of 1·0 microamperes. Draw a graph showing the relation between the voltage across the capacitor and the time during which the charging current has been flowing. Show values on your axes.

The permittivity of free space in m.k.s. units is 8·854 × 10⁻¹² farads/m.

A. 2. The capacitance, C farads, between two parallel plates each of effective area A m² separated by a distance d metres, with all the space between them filled by a dielectric material of relative permittivity ϵ_r, is given by:

$$C = \frac{\epsilon_r\, \epsilon_0\, A}{d} \text{ farads,}$$

where $\epsilon_r = 1$ for air.
ϵ_0 = absolute permittivity of free space
$= 8{\cdot}854 \times 10^{-12}$ farads/m,
$A = 150 \times 10^{-4}$ m²,
$d = 10^{-3}$ m.

$$\therefore\ C = \frac{1 \times 8{\cdot}854 \times 10^{-12} \times 150 \times 10^{-4}}{10^{-3}} = 133 \times 10^{-12} \text{ farads}$$
$$= 133\ \mu\mu\text{F.}$$

The electric field strength (or electric force) in the dielectric is the electric potential gradient (or potential drop per unit length) across it. If a potential difference of V volts is maintained between the parallel plates of the capacitor d metres apart, the potential gradient $= V/d$ volts/m.

When $V = 100$ volts and $d = 10^{-3}$ m,
electric field strength $= 100/10^{-3} = 10^5$ volts/m.

The charge Q coulombs held by a capacitor of C farads charged to a potential of V volts is:

$Q = CV$ coulombs
$= (133 \times 10^{-12} \times 100)$
$= 0{\cdot}0133$ microcoulombs

A current of i amp consists of a flow of electricity at the rate of i coulombs/second. Hence, the charge Q coulombs that collects on the plates of the capacitor C farads in t seconds is:

$Q = it$ coulombs.

Therefore the p.d. (V volts) between the plates of the capacitor after t seconds of charging at i amp is given by:

$$V = \frac{Q}{C} = \frac{it}{C} = \left(\frac{i}{C}\right)t.$$

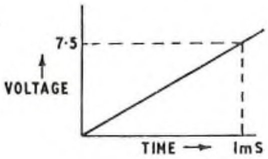
Now $\left(\frac{i}{C}\right)$ is constant.

$$\therefore \quad \frac{i}{C} = \frac{10^{-6}}{133 \times 10^{-12}} = \frac{10^{6}}{133} = 7{,}500.$$

Thus the law relating V and t in this problem is

$$V = \underline{7{,}500\ t.}$$

This can be represented by a straight line with a gradient such that the voltage increases steadily at a rate of 7,500 volts/second, as shown in the sketch.



Q. 3. *Explain the following terms with reference to sketches of typical I_a/V_a and I_a/V_g curves for a thermionic triode valve:*

(a) the anode slope resistance (anode a.c. resistance),

(b) the mutual conductance,

(c) the amplification factor.

Draw the circuit of a single valve amplifier with a resistance as anode load. Derive an expression for the voltage amplification of this stage, and hence show why this is less than the amplification factor of the valve.

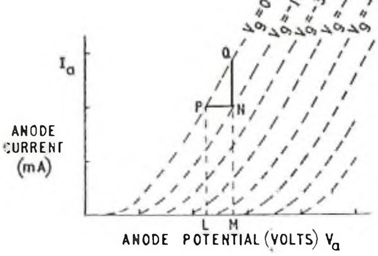
A. 3. If in a thermionic triode valve the grid–cathode potential difference is denoted by V_g, the anode–cathode potential difference by V_a and the anode–cathode current by I_a, the following definitions can be written down in terms of small changes, denoted by δ, in V_g, V_a, I_a.

(*a*) Anode slope resistance, r_a, $= \dfrac{\delta V_a}{\delta I_a}$, for a fixed grid voltage V_g.

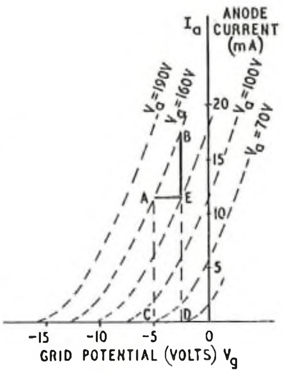
(*b*) Mutual conductance, g_m, $= \dfrac{\delta I_a}{\delta V_g}$, for a fixed anode voltage V_a.

(*c*) Amplification factor, μ, $= \dfrac{\delta V_a}{\delta V_g}$, where V_g produces a given small change in I_a with V_a constant and V_g produces the same change in I_a with V_a constant.

Typical families of I_a/V_a and I_a/V_g curves for a triode are shown in sketches (*a*) and (*b*).



(*a*)



(*b*)

The I_a/V_a curves are almost straight over much of their length, so that, given a fixed value of grid voltage, the anode-current change is proportional to the anode-voltage change producing it. In sketch (*a*) this means that QN/PN is constant. The inverse of this, PN/QN, has the dimensions of resistance, i.e. voltage ÷ current, and is known as the anode slope resistance, r_a.

The I_a/V_g curves are also straight over most of their length, V_g being negative. The ratio BE/AE $= I_a/V_g$ is the gradient of the curve for a given anode voltage. This is the mutual conductance, g_m.

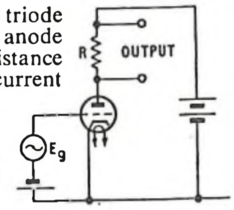
The amplification factor (μ), defined as in (*c*), can be written as follows:

$$\frac{\delta V_a}{\delta V_g} = \frac{\delta V_a}{\delta I_a} \times \frac{\delta I_a}{\delta V_g}$$

$$= \text{anode slope resistance} \times \text{mutual conductance}$$

or $\qquad \mu = r_a \times g_m.$

The simplest basic circuit of a single-valve triode amplifier is given in sketch (*c*). Let R be the anode load resistance and r_a the anode slope resistance of the valve. Then the change of the anode current I_a due to an alteration in the grid voltage E_g will be



(*c*)

$$I_a = \frac{\text{change of anode voltage produced by } E_g}{\text{the total resistance of the anode circuit}}$$

$$= \frac{\mu E_g}{R + r_a}.$$

The change in voltage across the load R ohms, i.e. output voltage V_{out}, due to a change in voltage E_g between the grid and cathode is I_aR.

$$V_{out} = I_aR = \frac{\mu E_g R}{R + r_a}.$$

The voltage amplification

$$= \frac{\text{Output voltage across load } R}{\text{Input voltage grid–cathode}} = \frac{V_{out}}{E_g} = \underline{\frac{\mu R}{R + r_a}}.$$

Now, since $(R + r_a)$ must always exceed R, the voltage amplification $\dfrac{\mu R}{R + r_a}$ must always be less than μ, the amplification factor of the valve.

Q. 4. *State Ohm's Law.*

A battery is connected across terminals AB of the circuit in Fig. 1 with the result that a potential difference of 10 volts is maintained across the 10-ohm resistor.

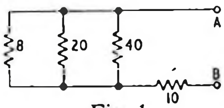


Fig. 1

Calculate from first principles the voltage across AB.

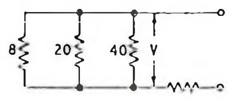
The voltage measured between the terminals of the battery when it is disconnected from A and B is found to be 18·2 volts. Calculate the internal resistance of the battery.

The positions of the battery and the 20-ohm resistor are now interchanged in the circuit. Calculate the resulting voltage across AB.

A. 4. Ohm's law states that the flow of an electric current in a circuit is proportional to the potential difference across the circuit.

As a potential difference of 10 volts exists across the 10-ohm resistor, by Ohm's law the current in this resistor = 10/10 = 1 amp. This is also the battery current.

Let V be the voltage across each of the parallel resistors shown in sketch (*a*). The current in each resistor will be V divided by its resistance. The total current will be the sum of the currents of the three parallel resistors. This will be 1 amp.



(*a*)

$$\therefore \quad \frac{V}{8} + \frac{V}{20} + \frac{V}{40} = 1 \text{ or } V(0{\cdot}125 + 0{\cdot}05 + 0{\cdot}025) = 1$$

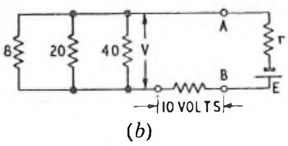
$$V(0{\cdot}2) = 1$$

$$V = 5 \text{ volts.}$$

The voltage across AB $= V + 10$

$= \underline{15 \text{ volts.}}$

The battery is equivalent to a series resistance r (the battery resistance) in series with an e.m.f. E volts, as shown in sketch (*b*).



(*b*)

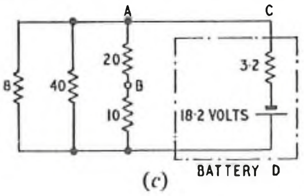
The battery current is 1 amp; hence the voltage drop in the internal resistance r of the battery is $r \times 1$ volts.

The battery e.m.f. must be equal to the total voltage drop round the whole circuit, which is given by the voltage across the battery terminals plus that across the internal resistance of the battery.

$$\therefore \quad E = 15 + r \times 1$$

$$18{\cdot}2 = 15 + r, \text{ and } r = \underline{3{\cdot}2 \text{ ohms.}}$$

The rearranged circuit is shown in sketch (*c*). The three parallel



(*c*)

resistors are 8, 40, 30 ohms. The single equivalent resistor R for this parallel combination is given by:

$$\frac{1}{R} = \frac{1}{8} + \frac{1}{40} + \frac{1}{30} = \frac{15 + 3 + 4}{120} = \frac{11}{60}$$

$$\text{or } R = \frac{60}{11} = 5{\cdot}45 \text{ ohms.}$$

The total equivalent resistance in the circuit is 5·45 ohms + battery resistance = 8·65 ohms.

$$\therefore \text{ Battery current} = \frac{18{\cdot}2}{8{\cdot}65} \text{ amp} = 2{\cdot}1 \text{ amp.}$$

Now the voltage across the battery terminals CD

$$= \text{e.m.f.} - \text{internal voltage drop}$$
$$= 18{\cdot}2 - (2{\cdot}1 \times 3{\cdot}2)$$
$$= 18{\cdot}2 - 6{\cdot}72 = 11{\cdot}48 \text{ volts.}$$

Voltage across AB is $\left(\frac{20}{20 + 10}\right)$ of voltage across battery terminals

$$= \frac{2}{3} \times 11{\cdot}48 = \underline{7{\cdot}65 \text{ volts.}}$$

Q. 5. *Explain what is meant by the phase difference between the alternating voltage across a circuit and the current flowing in it.*

Illustrate your answer by sketching on the same axes two cycles of the waveforms of:

(a) a sinusoidal voltage of frequency 50 c/s,

(b) a current of the same frequency lagging the voltage by 90°.

What is the periodic time of the voltage?

When an alternating voltage is applied in turn across

(a) a resistance,

(b) a capacitor,

(c) an inductor,

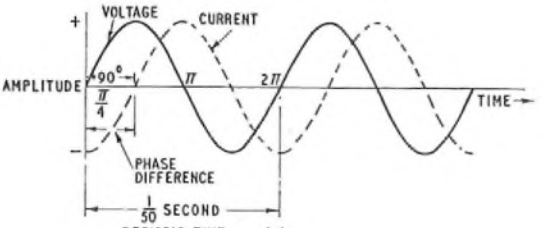
what is the phase relationship between the current and the voltage in each case?

Explain the reason for the difference.

A. 5. The phase difference between the alternating voltage across a circuit and the current flowing in it is the fraction of a cycle between successive zeros in voltage and current. One whole cycle is taken as 360° so that the phase difference is measured in degrees (or radians) and must be between 0 and 360° (or 0 and 2π radians).

The phase difference can be leading or lagging. The voltage leads the current if a voltage zero occurs earlier in time than the current zero in the same cycle. The voltage lags the current when the reverse condition applies.

The required sketch is shown in sketch (*a*).



(*a*)

The periodic time = time for one cycle of alternation

$$= \frac{1}{\text{frequency}} = \frac{1}{50} \text{ second.}$$

(*a*) When an alternating voltage is applied to a resistance the current is in phase with the alternating voltage.

(*b*) When an alternating voltage is applied to a capacitor the current leads the voltage across the capacitor by 90°.

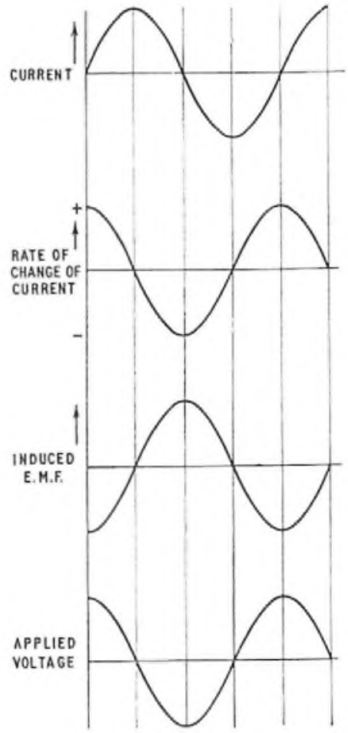
(*c*) When an alternating voltage is applied to an inductor the current flowing in the inductor lags by 90° on the voltage across the inductor.

The fact that the current leads the voltage by 90° in a capacitance can be explained physically as follows:

The effect of an alternating voltage across the capacitor is to alter the charge on the plates at the frequency of the applied voltage, the movement of electrons round the outer circuit being the alternating flow of current. The instantaneous magnitude of this current will be proportional to the instantaneous voltage, which will be the resultant of the voltage of the source at that instant and the potential difference due to the charge already stored in the capacitance. To charge a plate negatively, additional electrons must be supplied to it from a source of negative potential. The first few electrons flow in without difficulty, but once on the plate they tend to repel additional electrons trying to enter. The repelling force increases as more electrons build up on the plate. When the negative repelling force equals the charging force the instantaneous current is zero; so that, as the current falls from maximum negatively to zero, the voltage across the plates has risen from zero to maximum negatively. This corresponds to the current having a 90° lead over the voltage.

The net energy taken from the source by a loss-free capacitor is zero because the flow of energy alternates back-and-forth between the source and the plates.

For an inductor having zero resistance the fact that the current lags by 90° on the voltage across its terminals is due to the self-induced e.m.f. generated in the turns of the coil when the current is changing. This e.m.f. is proportional to the rate of change of the magnetic field, i.e. to the rate of change of the current flowing. A sine curve has its maximum gradient (largest rate of change) at the point of zero amplitude; therefore the maximum self-induced e.m.f. will occur at instants of zero current. The direction of the self-induced e.m.f. will always be such as to oppose the change of current. Now the applied e.m.f. from the alternating source must at all times be equal and opposite to the self-induced e.m.f., otherwise the flow of current could not be maintained. The applied e.m.f. must therefore lead the current by 90°. The waveforms referred to are shown relative to each other in sketch (*b*).



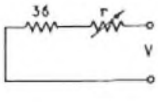
(*b*)

Q. 6. *A circuit containing a heater having a constant resistance of 36 ohms is to be supplied from a d.c. source which varies between 20 and 30 volts. It is necessary to maintain the current in the heater within the limits 0·25 and 0·3 amp despite this variation in supply voltage. This is done by adding a series resistance to the circuit with a high temperature coefficient of resistance of 0·002 per °C relative to its resistance at room temperature.*

Calculate the maximum and minimum values of the series resistor that are necessary to meet the specified conditions.

What rise in temperature in the resistor will give the required range of control?

A. 6. The circuit consists of two resistors in series as shown in the sketch. The variable resistor r has a resistance which depends upon its temperature.



When the voltage V is at its minimum of 20 volts the current will have the minimum tolerable value of 0·25 amp. Then by Ohm's law if r_1 is the value of the variable resistor,

$$20 = 0{\cdot}25(36 + r_1).$$
$$\therefore r_1 = 80 - 36 = \underline{44 \text{ ohms.}}$$

When the voltage is at maximum, 30 volts, the current will be maximum, 0·3 amp. If r_2 is then the maximum value of the variable resistor

$$30 = 0{\cdot}3(36 + r_2),$$

and $r_2 = 100 - 36 = \underline{64 \text{ ohms}}.$

The resistance r is related to its temperature t by the expression,

$$r = r_o(1 + at),$$

where a = the temperature coefficient of resistance,
t = the temperature rise in °C,
r_o = the initial resistance,
and r = the resistance at t°C.

Now, r_o = 44 ohms, r = 64 ohms and $a = 2 \times 10^{-3}$.

Therefore the rise in temperature t°C is given by:

$$64 = 44\{1 + (2 \times 10^{-3})t\}$$

$$\text{or } 20 = 88 \times 10^{-3} \times t.$$

$$\therefore t = \frac{10^4}{44} = \underline{227{\cdot}2°\text{C}}.$$

Q. 7. *Which of the terms (a) and (b) in the following sentence is correct?*

"When the magnetic field linking a coil of wire is changing (a) a current, (b) an electromotive force is induced in it."

Give reasons for your answer.

On what factors does the magnitude of this effect depend?

Describe the principle of the transformer and give an example of its use.

A. 7. The term (*b*) is correct. An electromotive force (e.m.f.) is generated in a conductor which is cutting a magnetic field. If the ends of the conductor are joined so that a closed circuit is formed, then the generated e.m.f. will drive a current round the circuit; the current is, however, the result of the presence of the e.m.f. The generation of an e.m.f. is not dependent upon the existence of a current in the conductor.

Faraday's law of electromagnetic induction states that, when the magnetic field linking a coil of wire is changing, an e.m.f. is induced in the coil proportional to the rate of change of the magnetic field linking the coil and to the number of turns on the coil.

The rate of change of the field linkage will be proportional to the speed of the coil relative to the magnetic field and to the magnitude of the field.

Faraday's law applies equally where a coil of wire rotates in a steady magnetic field (as in a d.c. generator) and where there is no physical movement of the coil but the change of flux linkage is obtained by varying the magnetic field itself. Such a situation exists in a transformer where the magnetic field is created by a varying current in a second coil of wire wound in close proximity to the first coil, e.g. on top of it. The coil creating the field is known as the primary and is energized by connecting an alternating voltage across its terminals with the result that an alternating current passes through it. This sets up an alternating magnetic field in the core which induces an e.m.f. in the other coil, the secondary. If there are N_1 and N_2 turns on the primary and secondary, respectively, and if they are wound closely together, e.g. one on top of the other, then the voltage V_2 induced in the secondary as a result of applying V_1 across the primary coil is given by $V_2/V_1 = N_2/N_1$, i.e. the ratio of the output voltage to the input voltage is proportional to the turns ratio of the two coils. Such an arrangement of two tightly-coupled coils is known as a transformer. It provides a means whereby the alternating-voltage in a circuit can be stepped up or down by a required factor.

A practical transformer, e.g. one for use on a 50 c/s power supply, has a highly-efficient iron core in the shape of a closed loop on which the primary and secondary coils are wound. The magnetic-field linkage is thereby enormously increased over that obtainable with an air core because of the high permeability of the iron. The core is usually built up from a large number of thin laminations of stalloy, a material which gives low magnetic losses when rolled into thin sheets.

Q. 8. *Draw, and describe the principle of, the potentiometer circuit in which a resistance wire fitted with a sliding contact can be used in conjunction with a standard cell to measure d.c. voltages.*

Over what voltage range can such a device operate?

How can this range be increased without in any way altering the construction of the potentiometer.

In a measurement using a 1 m bridge as a potentiometer, the standard cell was found to give a balance when the sliding contact was 30 cm from one end of the metre wire. When the potentiometer was used to measure the voltage of another source of d.c. it was at first found impossible to obtain a balance point on the metre bridge. After a simple alteration in the two connexions to the unknown voltage a balance was obtained 85 cm from one end of the metre wire. What do you suppose was the alteration to the connexions?

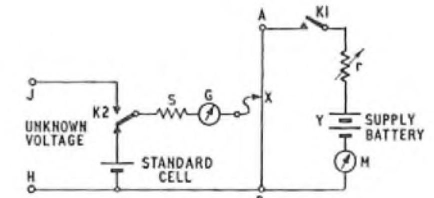
Calculate the two possible values of the voltage required.

The voltage of the standard cell is 1·018 volts.

A. 8. The potentiometer is widely used in d.c. electrical circuits as a means of obtaining a voltage which is a known fraction of a given voltage.

The potentiometer principle can be applied to measure a d.c. voltage as follows:

The uniform resistance wire AB, as shown in sketch (*a*), carries a current supplied by battery Y which must have a voltage greater



(*a*)

than that to be measured. It is convenient to provide a switch, K1, an ammeter, M, and an adjustable resistor, r, so that the current in AB can be controlled throughout the experiment.

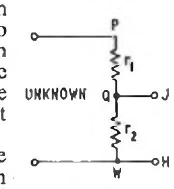
As AB is a uniform wire, the potential difference between any two points on it x cm apart will be proportional to x. A standard cell of known voltage can be used to calibrate the wire, i.e. determine the voltage drop per centimetre for a given current in the ammeter, by connecting the standard cell with a centre-zero galvanometer, G, in series between two points on the wire, e.g. X and B. A safety resistor, S, should also be in the circuit to limit the current taken from the standard cell. The position of X is moved along AB until there is no deflexion of the galvanometer needle when the switch K2 is in the unoperated position.

The potential difference between X and B must then be that of the standard cell, which under these conditions is carrying no current. Thus, if E is the known e.m.f. of the standard cell, and XB = C cm at balance, the voltage drop per cm on the wire AB = E/C volts/cm.

The current in the wire (i.e. in ammeter M) must be kept constant throughout the experiment, otherwise the voltage calibration of AB alters.

Any source of steady voltage less than the p.d. across AB can now be measured by operating a change-over switch, K2, to interchange it with the standard cell. With the unknown voltage across JH, the slider X is re-adjusted until a balance of zero is obtained on G. If XB then = x cm the unknown voltage = xE/C.

The simple potentiometer voltmeter is limited to measuring steady voltage less than the voltage drop across AB. The range can be multiplied if a known fraction of the unknown voltage is applied across HJ, using a potential divider PQW as shown in sketch (*b*). This increases the range by the ratio $(r_1 + r_2)/r_2$. The range of measurement can also be increased by a small amount without the additional potentiometer by increasing the voltage drop across AB, i.e. by increasing the current supplied by battery Y.



(*b*)

The most likely reason for being unable to locate a balance is that the connexions to the unknown voltage were reversed. No balance can be obtained under such conditions because the potential gradient along AB will be opposite in sign to the voltage being measured. In the problem, two values can be deduced for the unknown voltage because the end of the wire from which the 85 cm is measured is not specified.

The calibration using the standard cell gives $\frac{1{\cdot}018}{30}$ volts/cm.

$$= 0{\cdot}0339 \text{ volts/cm}.$$

$\therefore$ The unknown voltage can be either $85 \times 0{\cdot}0339 = \underline{2{\cdot}882 \text{ volts}},$

or $15 \times 0{\cdot}0339 = \underline{0{\cdot}51 \text{ volts}}.$

This ambiguity is removed when the end of the 1 m wire connected to the unknown voltage is specified.

Q. 9. *Describe the principle of operation of a simple primary cell. Explain what determines the value of the e.m.f. of a primary cell and show why this e.m.f. is independent of the size of the cell.*

Why is a simple primary cell of little practical use? How are its defects overcome in the Leclanché cell?

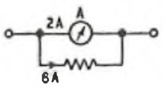
Q. 10. *Describe briefly the principle of the moving-iron ammeter. Explain why it can measure a.c. or d.c.*

A moving-iron ammeter has a resistance of 0·2 *ohms and gives full-scale deflexion for a current of* 2·0 *amperes.*

How can this meter be used to give full-scale deflexion for 10 *amperes? Calculate the value of any additional component needed to achieve this.*

A. 10. For the first part of this answer either the attraction or repulsion type of meter can be described. See Model Answer Book, *Telecommunications (Principles) A*, A69 and A70, p.102.

The meter will require a shunt to carry 8·0 amp when it is showing full-scale deflexion with 2·0 amp flowing, as shown in the sketch.



Now the p.d. across the meter which has a resistance of 0·2 ohms and is carrying 2·0 amp = 0·4 volts.

∴ By Ohm's law the resistance needed in the shunt

$$= \frac{0{\cdot}4}{8} = \underline{0{\cdot}05 \text{ ohms.}}$$

Correction.

In the model answer given to Question 9 of Telecommunication Principles A, 1961 (*Supplement*, Vol. 54, No. 3, p. 48, Oct. 1961) sketch (*b*) shows, incorrectly, the relation between the anode current and the anode potential of a thermionic diode.

The sketch (*b*) should be ignored, therefore, and the third paragraph of the answer be replaced by the following:

The anode current is the result of electrons being drawn from the space charge by the positive potential of the anode. The current only flows in the external circuit when the anode has a positive potential. When the anode potential is zero the anode current is zero. As a small positive potential is applied some of the negative space charge is drawn to the anode. Further increase in the anode potential attracts more electrons and the anode current is proportional to (anode potential)$^{3/2}$. This law holds until saturation point is reached.

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