SUPPLEMENT

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City and Guilds of London Institute Examinations, 1960

CITY AND GUILDS OF LONDON INSTITUTE EXAMINATIONS, 1960

QUESTIONS AND ANSWERS

Answers are occasionally omitted or reference is made to earlier Supplements in which questions of substantially the same form, together with the answers, have been published. Some answers contain more detail than would be expected from candidates under examination conditions.

ENGINEERING SCIENCE, 1960

Q. 1. *Explain the terms mass, force and weight, giving examples of units in which each may be measured.*

A train weighing 100 *tons has a frictional resistance of* 16 *pounds per ton, and runs on a level track. Find the force required to give it a velocity of* 30 *miles per hour in one minute starting from rest, and the force required to bring it to rest in an equal time.*

A. 1. Mass is the quantity of matter in a body. It is measured by comparison with a standard mass using a beam balance. Typical units are the pound and kilogram. Mass is independent of a body's position in space and for most practical purposes is independent of its velocity.

Force is any action which changes or tends to change the state of rest or uniform motion of a body in a straight line. It is measured in terms of the rate of change of momentum of a body. If a force acting on a body of mass m changes its velocity from v_1 to v_2 in t seconds, the change in momentum is $mv_2 - mv_1$ and the average rate of change of momentum is $(mv_2 - mv_1)/t$. This may be written $m(v_2 - v_1)/t$. Now $(v_2 - v_1)/t$ is the average rate of change of velocity, i.e. the acceleration. Hence, force = mass × acceleration. Typical units of force are the poundal and the newton. The poundal is that force which will accelerate a mass of one pound by 1 ft per second per second and the newton is the force which will accelerate a mass of 1 kilogram by 1 metre per second per second.

The weight of a body is the downward force which a body experiences under the influence of a gravitational field. Strictly speaking, the weight of a body is a force and should be expressed in units of force (e.g. poundals or newtons). In practice, however, units of mass are used to express weights. Thus, a body which has a mass of one pound is said to have a weight of one pound. If a mass of one pound (or any other value) were allowed to fall freely under the action of gravity it would be accelerated downwards at 32·2 ft per second per second (in London); hence the force acting on it would be 1 lb (mass) × 32·2 ft/s² = 32·2 poundals. This unit of force is one pound weight.

It is important to note that the mass of a body is independent of its position but its weight varies from place to place on the earth's surface.

The applied force must:
(*a*) Overcome the frictional resistance.
(*b*) Accelerate the mass of 100 tons.

$$\begin{aligned}
\text{Frictional force} &= 16 \times 100 = 1{,}600 \text{ lb wt.}\\
\text{Average acceleration} &= 30 \text{ m.p.h./minute}\\
&= (44 \text{ ft/s})/60 \text{ s}\\
&= 44/60 \text{ ft/s}^2\\
\text{Accelerating force} &= \text{mass} \times \text{acceleration}\\
&= \frac{100 \times 2{,}240 \times 44}{60} \text{ poundals}\\
&= \frac{100 \times 2{,}240 \times 44}{60 \times 32{\cdot}2} \text{ pounds weight}\\
&= 5{,}110 \text{ lb wt.}\\
\text{Total force} &= 1{,}600 + 5{,}110 = \underline{6{,}710 \text{ lb wt.}}
\end{aligned}$$

The braking force required to stop the train in one minute will also be 5,110 lb, but the friction force will provide part of this force. Net braking force required = 5,110 − 1,600 = <u>3,510 lb wt.</u>

Q. 2. *The pin-jointed frame shown in Fig.* 1 *supports a load of* 2,000 *lb. Find the loads in the members AB and BC, and the values of the reactions R_1 and R_2.*

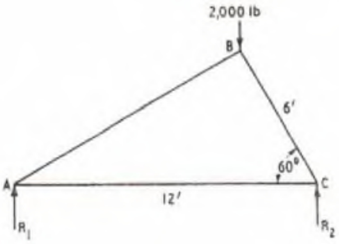
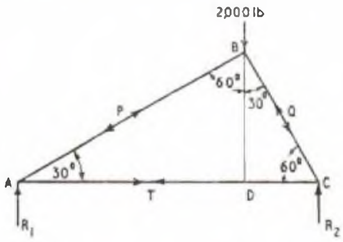


Fig. 1

A. 2. The forces in AB, BC and CA can be resolved into horizontal and vertical components. The forces in the members AB and BC must provide a total vertical component of 2,000 lb to support the load. For equilibrium these two forces must also have equal but opposite horizontal components. Thus, at B the forces must act from A to B and C to B. Hence AB and BC must be thrust members. Let these thrusts be P and Q as shown in the sketch. Since none of the external forces has a horizontal component, the horizontal components of P and Q at points A and C must be counteracted by a tension in AC. Let this tension be T.



From the geometry of the figure, BC/AC = ½ and cos ∠C = ½. Thus the triangle ABC is right-angled. The angles will be as shown in the sketch where BD is a perpendicular dropped on to AC.

For horizontal equilibrium at point B the horizontal components of AB and BC must be equal.

$$\begin{aligned}
\therefore P\sin 60^\circ &= Q\sin 30^\circ\\
P\sqrt{3}/2 &= Q\tfrac{1}{2}\\
Q &= \sqrt{3}\,P \quad \ldots\ldots\ldots\ldots\ldots (1)
\end{aligned}$$

Also, for vertical equilibrium,

$$\begin{aligned}
P\cos 60^\circ + Q\cos 30^\circ &= 2{,}000\\
P\tfrac{1}{2} + Q\sqrt{3}/2 &= 2{,}000\\
P + \sqrt{3}\,Q &= 4{,}000
\end{aligned}$$

Substitute for Q from equation (1).

$$\begin{aligned}
P + \sqrt{3} \times \sqrt{3} \times P &= 4{,}000\\
\therefore P &= \underline{1{,}000 \text{ lb.}}\\
\text{Hence, } Q &= \underline{\sqrt{3} \times 1{,}000 \text{ lb.}}
\end{aligned}$$

At A the vertical component of $P = R_1$.

$$\therefore R_1 = P\sin 30$$
$$= 1{,}000 \times \tfrac{1}{2} = \underline{500 \text{ lb.}}$$
$$\text{Similarly, } R_2 = Q\sin 60$$
$$= \sqrt{3} \times 1{,}000 \times \sqrt{3}/2 = \underline{1{,}500 \text{ lb.}}$$

The horizontal components of P and Q are equal to each other and to the tension in AC.

$$\therefore P\sin 60^\circ = Q\sin 30^\circ = T$$
$$\text{and } T = 1{,}000 \times \sqrt{3}/2 = \sqrt{3} \times 500 \text{ lb.}$$

Q. 3. *Explain what is meant by the moment of a force, and state the principle of moments.*

A man wishing to raise a heavy wheel-shaped casting slightly from the ground uses a 12-ft bar as a lever as shown in Fig. 2, without success. Re-draw the apparatus as it would be used to the best advantage. If the casting can now be lifted by using an effort of 200 lb wt., find the weight of the casting and the load on the fulcrum.

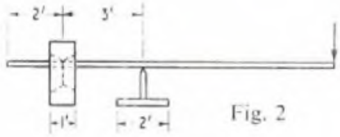


Fig. 2

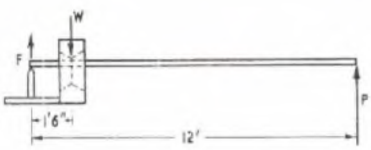
A. 3. If a rigid body has one point fixed, and the body is subjected to a force whose line of action does not pass through that point, then the body will experience a turning force or moment.

The moment of a force about a given point is defined as the product of the force and the perpendicular distance between the point and the line of action of the force.

The principle of moments states that if any number of co-planar forces acting on a rigid body has a resultant, the algebraic sum of their moments about any point in their plane is equal to the moment of the resultant about that point.

It follows from this principle that when any system of co-planar forces is in equilibrium the algebraic sum of their moments about any point in their plane is zero.

To obtain the maximum mechanical advantage the distance from fulcrum to the point of application of the effort must be maximum, and the distance from fulcrum to load must be minimum. Thus, the fulcrum and the point of application of the effort must be placed one at each end of the bar and the load must be as close to the fulcrum as the sizes of the parts allow. This arrangement is shown in the sketch where W is the weight of the casting, P is the effort and F the load on the fulcrum.



Taking moments about the fulcrum and assuming anticlockwise moments are positive,

$$12P - 1{\cdot}5W = 0$$
$$\therefore W = \frac{12 \times 200}{1{\cdot}5} \text{ lb wt.}$$
$$= \underline{1{,}600 \text{ lb wt.}}$$

By taking moments about the point where W acts:

$$(12 - 1{\cdot}5)P - 1{\cdot}5F = 0$$
$$10{\cdot}5P = 1{\cdot}5F$$
$$F = \frac{10{\cdot}5 \times 200}{1{\cdot}5}$$
$$= \underline{1{,}400 \text{ lb wt.}}$$

Q. 4. *Describe a laboratory experiment suitable for finding the centre of gravity of a plane metal sheet of uniform thickness.*

The assembly shown in Fig. 3 consists of three identical thin uniform rods, each of length 8 inches. Find its centre of gravity.

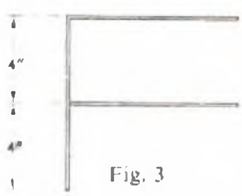
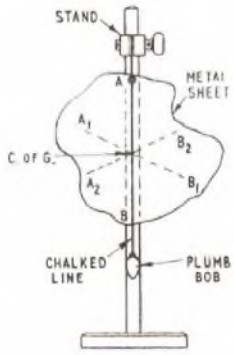


Fig. 3

A. 4. *Determination of centre of gravity of a uniform lamina.*

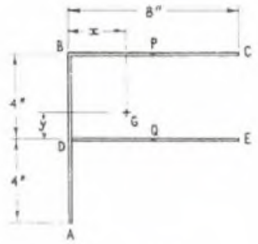
A chalked thread and plumb-bob are suspended from a stand and the metal sheet is then temporarily secured to the thread (point A in sketch (*a*)) close to the edge of the sheet. This could be done by using a small piece of plasticine or adhesive tape. The sheet will take up a position such that its centre of gravity is vertically below the point of suspension. The plumb-line will also pass across the centre of gravity of the sheet and the position of the line can be marked by plucking the chalked thread to produce a mark on the lamina corresponding to AB. By changing the point of suspension successively to A_1 and A_2 further lines A_1B_1 and A_2B_2 can be produced. Each of these lines passes through the centre of gravity and hence they will all intersect in one point. This point is the centre of gravity of the lamina.



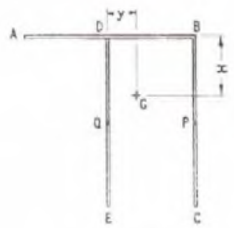
(*a*)

In Fig. 3 each rod may be regarded as having all of its mass concentrated at its centre. Thus, in sketch (*b*), the weight of AB acts through D, the weight of BC through P and the weight of DE through Q.

Let the centre of gravity of the system be at some point, G,



(*b*)



(*c*)

situated y in. above DE and x in. to the right of AB. Thus, if each of the rods weighs w lb the total weight of $3w$ lb may be regarded as acting through G.

By taking moments about B,

$$x \times 3w = 4 \times 2w$$
$$\therefore x = \tfrac{8}{3} \text{ in.}$$

Now consider the system rotated through 90° as shown in sketch (*c*).

By taking moments about D,

$$y \times 3w = 4 \times w$$
$$\therefore y = \tfrac{4}{3} \text{ in.}$$

Hence, the centre of gravity of the arrangement of rods is $2\tfrac{2}{3}$ in. to the right of the left-hand rod, BA, and $1\tfrac{1}{3}$ in. above the lower horizontal rod, DE.

Q. 5. *Explain the meaning of the terms "mechanical advantage," "velocity ratio" and "efficiency" as applied to a machine.*

Sketch a pulley system having a velocity ratio of 4. If the efficiency of this system is 60 per cent, find the effort required and the work done in lifting a load of 200 lb through a vertical distance of 2 ft.

A. 5. A machine is a device used to overcome a resistance at one point by applying a force at another. In general, any machine will have losses and more work must be done by the applied force (effort) than is required to overcome the resistance (load). The sizes of the load and effort forces and the corresponding distances moved by their points of application are usually different.

The relationships between these quantities can be defined in terms of the following characteristics:

Mechanical advantage. This is the ratio of the load to the effort.

$$\text{Mechanical advantage} = \frac{\text{Load}}{\text{Effort}}$$

Velocity Ratio. This is the ratio of the distance moved by the effort to the distance moved by the load.

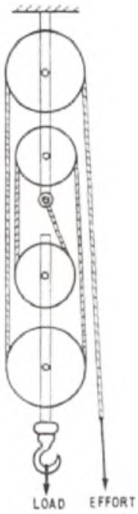
$$\text{Velocity ratio} = \frac{\text{Distance moved by effort}}{\text{Distance moved by load}}$$

Efficiency. This is the ratio of the work output of the machine to the work input to it.

$$\text{Efficiency} = \frac{\text{Work output}}{\text{Work input}}$$

$$= \frac{\text{Load} \times \text{distance moved by load}}{\text{Effort} \times \text{distance moved by effort}}$$

$$= \frac{\text{Mechanical advantage}}{\text{Velocity ratio}}$$



A pulley system having a velocity ratio of 4 is shown in the sketch.

$$\text{Efficiency} = \frac{\text{Mechanical advantage}}{\text{Velocity ratio}}$$

$$\therefore \frac{60}{100} = \frac{200}{\text{Effort}} \times \frac{1}{4},$$

$$\text{and effort} = \frac{200 \times 100}{4 \times 60} = \underline{83{\cdot}4 \text{ lb.}}$$

Distance moved by effort
$= 2 \times 4$ ft

$\therefore$ Work done by effort in lifting load 2 ft
$= 8 \times 83{\cdot}4$ ft lb $= \underline{667 \text{ ft lb.}}$

Q. 6. *State Hooke's law of elasticity.*

A wooden block of mass 10 kilograms rests on a horizontal table as shown in Fig. 4, the coefficient of friction between the block and the table being 0·4.

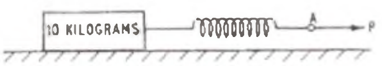


Fig. 4

A force P is applied as shown, and it is found that the end A of the spring moves 6 cm before the block begins to slide.

Draw a diagram to show how the tension in the spring varies as A moves, and hence find the work done in joules by the force P before the block slides. The acceleration due to gravity may be taken as 9·8 metre/sec.^{2}

A. 6. If a specimen of a material of length L and cross-sectional area A is subjected to a load W such that it is extended by a length x then the stress in the material is given by W/A and the strain by x/L. Hooke's law states that for values of strain below the elastic limit for the material, the stress is proportional to the strain,

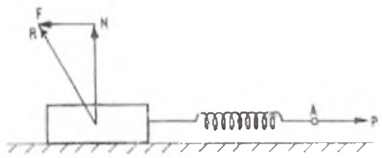
$$\text{i.e. } \frac{W}{A} \propto \frac{x}{L}.$$

It follows from this law that

$$\frac{W}{A} = E\frac{x}{L},$$

where E is a constant for the material known as Young's Modulus of Elasticity.

The block will be subjected to a reaction R at the surface of the table. This can be resolved into two forces as shown in sketch (a); the normal reaction N, which overcomes the weight of the block and the limiting friction force F, which must be overcome by P before the block can slide.



(a)

Since the coefficient of friction is defined as F/N

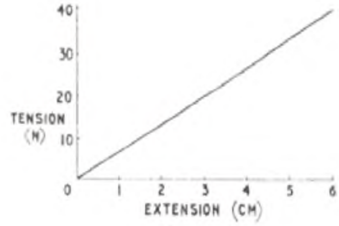
$$0{\cdot}4 = \frac{F}{10}$$

$\therefore$ $F = 4$ kilograms
$= 4 \times 9{\cdot}8$ newtons $= 39{\cdot}2$ newtons.

Hence, P must be 4 kg and the stiffness of the spring is, therefore,

$$S = \frac{39{\cdot}2}{6} \text{ newton/cm.} = 6{\cdot}53 \text{ newton/cm.}$$

The load/extension graph will be of the form $P = Sx$
where P = tension in spring (maximum value = 39·2 newtons)
S = stiffness (6·53 newton/cm.)
x = extension.



(b)

The required graph is shown in sketch (b).

The work done by P is given by the area under the curve.

$\therefore$ Work done $= \frac{1}{2} \times 39{\cdot}2 \times 6 \times 10^{-2}$ joules
$= \underline{1{\cdot}176 \text{ joules.}}$

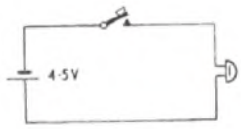
Q. 7. *Compare the properties and uses of the dry (Leclanché) cell with those of the lead-acid accumulator, and draw a circuit diagram to illustrate one practical use of the dry cell.*

Why are cells sometimes connected in series, and sometimes in parallel?

A. 7. *Dry Cells.* The main advantage of dry cells is that they are convenient and completely portable. For very light-load applications they can be made in miniature sizes. They can be hermetically sealed and in normal circumstances there is no risk of electrolyte leakage. Their initial cost is relatively low, but they cannot be recharged and must be discarded and replaced once exhausted. They require no attention during their life and can be stored for relatively long periods in a state ready for immediate use.

Their main disadvantage is that they have a limited capacity and a relatively high internal resistance. If used for long periods the terminal voltage tends to fall due to polarization. Their use is therefore limited to intermittent light-current loads. The e.m.f. is 1·5 volts per cell.

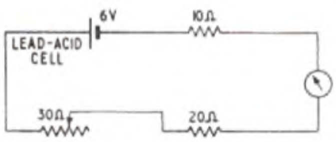
Lead-Acid Accumulators. These have a very low internal resistance and can supply heavy currents for long periods without difficulty. They tend to be somewhat bulky and heavy. Cell ventilation is essential as explosive gases are evolved. Portable enclosed types must have special spill-proof vents, and even with these there is a risk of acid leakage due to seepage. The initial cost is high but cells can be recharged repeatedly and have a long life. Time is required to attend to the cells and charging apparatus must be provided. Accumulators cannot be stored for long periods in a condition ready for immediate use. The e.m.f. is 2 volts per cell.



The sketch shows a circuit using a dry cell to ring a bell in an extension telephone. The push button would be in the main instrument.

Cells are connected in series when a higher voltage than the nominal cell e.m.f. is required. In certain applications, special counter e.m.f. cells are also connected in series with a main battery for voltage-control purposes. Parallel connexion is used when the load current is higher than can be provided by a single cell.

Q. 8. *Calculate the value of the current flowing in the above circuit when the rheostat is set (a) for zero resistance and (b) for maximum resistance.*



On two separate occasions, the ammeter readings taken as in (a) and (b) above were found to differ from the expected values, being respectively 160 mA and 80 mA on the first occasion, and 300 mA and 120 mA on the second. Deduce the probable fault in each case, and state how your deductions could be confirmed.

The resistance of the ammeter may be neglected.

A. 8. When the rheostat is set at zero resistance the total circuit resistance is $10 + 20 = 30$ ohms.

$$\text{The current} = \frac{V}{R} = \frac{6}{30} = \underline{0{\cdot}2 \text{ amp.}}$$

When the rheostat is set at maximum resistance the total circuit resistance is $10 + 20 + 30 = 60$ ohms.

$$\text{The current} = \frac{6}{60} = \underline{0{\cdot}1 \text{ amp.}}$$

Under fault conditions, the above tests were repeated and the currents on the first occasion were 0·16 amp and 0·08 amp. As the current is halved, then the total circuit resistance must have been doubled. Hence the fault is unlikely to be in the resistors as this would require two simultaneous faults. Alternatively the battery terminal voltage could have dropped. The battery voltage would have to be:

(*a*) When the rheostat is set at zero resistance, $V = 0{\cdot}16 \times 30 = 4{\cdot}8$ volt.

(*b*) When the rheostat is set at maximum resistance, $V = 0{\cdot}08 \times 60 = 4{\cdot}8$ volt.

Thus, the voltage of one of the cells must have dropped to 0·8 volts possibly due to a high-resistance internal connexion. Alternatively, the ammeter could be reading low but this is less likely. A short-circuit between two parts of the external circuit is not possible because the load current has decreased. The fault could be confirmed by measuring the battery voltage or replacing it with a battery in good condition.

On the second occasion, the currents were 0·3 amp and 0·120 amp. Assuming the battery voltage remains correct, the total circuit resistance when the rheostat is set at zero is $R = 6/0{\cdot}3 = 20$ ohms.

When the rheostat is set at maximum the total circuit resistance is $R = 6/0{\cdot}12 = 50$ ohms.

This gives a change of resistance of $50 - 20 = 30$ ohms, which indicates that the 30-ohm rheostat is functioning correctly. The most likely fault is that the 10-ohm resistor is short-circuited. This could be confirmed by shorting out the 10-ohm resistor. If no current change occurs then the diagnosis is correct.

Q. 9. *Name three effects of an electric current which might allow its existence to be detected.*

Describe in each case an experiment which could be used to demonstrate the effect.

Q. 10. *Sketch and describe any one piece of apparatus which makes use of a magnetic circuit containing an air-gap.*

How would the operation of the apparatus be affected if the size of the air-gap were increased?

A. 10. For a description of the construction and operation of the Post Office 3,000-type relay see A.2, Telephone Exchange Systems I, 1958, *Supplement*, Vol. 51, No. 3, p. 69, Oct. 1958, and A.1, Elementary Telecommunications Practice, 1958, *Supplement*, Vol. 51, No. 3, p. 63, Oct. 1958.

The size of the air-gap could be increased in two different ways and these would have different effects:

(*a*) The gap could be increased in the non-operated position by increasing the armature angle or by reducing the size of the armature back stop. This would increase the minimum operating current and the operating time for a given current would be increased.

(*b*) The gap could be increased in the operated position by increasing the size of the residual stud or by readjustment of the residual screw, when fitted. This would increase the minimum holding current and the releasing time would be decreased for any given current.

MATHEMATICS A, 1960

Q. 1. (*a*) *Simplify, giving answers with positive indices*

$$(i)\ \frac{3p^{-1/2} \times 4q^{3/4}}{6pq^{1/2}} \qquad (ii)\ \frac{\sqrt[3]{(27a^5b^4)}}{a^{-1/3}b^{-2/3}}$$

(*b*) *If* $\frac{p}{q} = \frac{2}{3}$ *evaluate* $\frac{2p^2 + q^2}{4pq}$ *and explain why* $\frac{2p^2 + q}{4p}$ *cannot be evaluated.*

(*c*) *Simplify to one fraction* $\frac{1}{4a - 5} - \frac{2}{20a^2 - 21a - 5}$

A. 1. (*a*) (*i*)

$$\frac{3p^{-1/2} \times 4q^{3/4}}{6pq^{1/2}} = \frac{3 \times 4}{6} \times \frac{q^{3/4 - 1/2}}{p^{1 + 1/2}}$$
$$= \underline{\frac{2q^{1/4}}{p^{3/2}}}.$$

(*ii*)

$$\frac{\sqrt[3]{27a^5b^4}}{a^{-1/3}b^{-2/3}} = \frac{3a^{5/3}b^{4/3}}{a^{-1/3}b^{-2/3}}$$
$$= 3a^{5/3 + 1/3}b^{4/3 + 2/3}$$
$$= \underline{3a^2b^2}.$$

(*b*)

$$\frac{2p^2 + q^2}{4pq} = \frac{2p^2}{4pq} + \frac{q^2}{4pq}$$
$$= \frac{1}{2} \times \frac{p}{q} + \frac{1}{4} \times \frac{q}{p}$$
$$= \frac{1}{2} \times \frac{2}{3} + \frac{1}{4} \times \frac{3}{2}$$
$$= \frac{1}{3} + \frac{3}{8} = \frac{8 + 9}{24} = \underline{\frac{17}{24}}.$$

Dealing with $\frac{2p^2 + q}{4p}$ in the same way,

$$\frac{2p^2}{4p} + \frac{q}{4p} = \frac{1}{2} \times p + \frac{1}{4} \times \frac{q}{p}.$$

It will be seen that, although the second term of this expression can be evaluated, the first term cannot, since it contains only p, apart from the numerical coefficient, and the value of p is not given. Moreover, it is not possible to rearrange the expression in such a way that it is dependent merely on the ratio of p to q, which is the only information given.

(*c*)

$$\frac{1}{4a - 5} - \frac{2}{20a^2 - 21a - 5} = \frac{1}{4a - 5} - \frac{2}{(4a - 5)(5a + 1)}$$
$$= \frac{5a + 1 - 2}{(4a - 5)(5a + 1)}$$
$$= \underline{\frac{5a - 1}{(4a - 5)(5a + 1)}}$$

Q. 2. (*a*) *The resistance of a short-circuited symmetrical network is given by* $R_{SC} = \frac{2R_1R_2}{R_1 + 2R_2}$. *Make* R_2 *the subject of this formula, expressing the result in a factorized form.*

(*b*) (*i*) *Factorize* $a^{2/3} - b^{2/3}$. (*ii*) *Simplify* $\frac{a^{4/3} - b^{4/3}}{a^{2/3} - b^{2/3}}$

(*c*) *The force F between two magnetic poles varies directly as their strengths m and M and inversely as the square of their distance d apart. If* $F = 6$ *when* $m = 3$ *and* $M = 5$ *with* $d = 2$, *find F when* $m = 4{\cdot}5$, $M = 8$ *and* $d = 3$.

A. 2. (*a*)

$$R_{SC} = \frac{2R_1R_2}{R_1 + 2R_2}$$
$$\therefore\ R_{SC}(R_1 + 2R_2) = 2R_1R_2$$
$$\text{or } R_1R_{SC} + 2R_2R_{SC} = 2R_1R_2$$
$$\therefore\ 2R_2(R_1 - R_{SC}) = R_1R_{SC}$$
$$\text{and } R_2 = \underline{\frac{R_1R_{SC}}{2(R_1 - R_{SC})}}.$$

(*b*) (*i*)

$$a^{2/3} - b^{2/3} = (a^{1/3})^2 - (b^{1/3})^2$$
$$= \underline{(a^{1/3} - b^{1/3})(a^{1/3} + b^{1/3})}.$$

(*ii*)

$$\frac{a^{4/3} - b^{4/3}}{a^{2/3} - b^{2/3}} = \frac{(a^{2/3} - b^{2/3})(a^{2/3} + b^{2/3})}{a^{2/3} - b^{2/3}}$$
$$= \underline{a^{2/3} + b^{2/3}}.$$

(*c*)

$$F \propto \frac{mM}{d^2},$$
$$= k \cdot \frac{mM}{d^2},$$

where k is the constant of proportionality.

Substituting the first set of values:

$$6 = k \times \frac{3 \times 5}{2^2}$$
$$\therefore k = \frac{6 \times 4}{3 \times 5} = \frac{8}{5}$$

For the second condition,

$$F = \frac{8}{5} \times \frac{4{\cdot}5 \times 8}{3^2}$$
$$= \frac{8 \times 36}{5 \times 9}$$
$$= \frac{32}{5} = \underline{6{\cdot}4}.$$

Q. 3. (*a*) *The current gain A in a transistor circuit is shown as*

$$A = \frac{\alpha}{1 - \alpha + \dfrac{R}{r}}$$

Calculate A when $\alpha = 0{\cdot}976$, $R = 1{\cdot}44 \times 10^5$, $r = 1{\cdot}20 \times 10^6$.

(*b*) *Evaluate* (*i*) $(2{\cdot}718)^{3\cdot46}$ (*ii*) $\dfrac{\log 4{\cdot}6}{\log 0{\cdot}13}$

A. 3. (*a*)

$$A = \frac{\alpha}{1 - \alpha + \dfrac{R}{r}}.$$

Substituting the values given:

$$A = \frac{0{\cdot}976}{1 - 0{\cdot}976 + \dfrac{1{\cdot}44 \times 10^5}{1{\cdot}20 \times 10^6}}$$

$$= \frac{0{\cdot}976}{0{\cdot}024 + 1{\cdot}2 \times 10^{-1}} = \frac{0{\cdot}976}{0{\cdot}144}$$

$$= \frac{976}{144} = \frac{244}{36} = \frac{61}{9} = \underline{6\tfrac{7}{9}.}$$

(*b*) (*i*) $(2{\cdot}718)^{3\cdot46}$

Taking logarithms,

$$\log_{10}(2{\cdot}718)^{3\cdot46} = 3{\cdot}46 \times \log_{10} 2{\cdot}718$$

$$= 3{\cdot}46 \times 0{\cdot}4343$$

$$= 1{\cdot}503$$

$$\therefore\ (2{\cdot}718)^{3\cdot46} = \underline{31{\cdot}84.}$$

No.	*Log.*
3·46	0·5391
0·4343	$\bar{1}$·6378 +
	0·1769

(*ii*)

$$\frac{\log 4{\cdot}6}{\log 0{\cdot}13} = \frac{\log_{10} 4{\cdot}6}{\log_{10} 0{\cdot}13}$$

$$= \frac{0{\cdot}6628}{\bar{1}{\cdot}1139}$$

$$= \frac{0{\cdot}6628}{-\,0{\cdot}8861}$$

$$= \underline{-\,0{\cdot}7482.}$$

No.	*Log.*
0·6628	$\bar{1}$·8214
0·8861	$\bar{1}$·9474 −
	$\bar{1}$·8740

Note: No base of logarithms is given in the question. In the case of a ratio of logarithms, the value of the ratio is independent of the base.

Q. 4. (*a*) *Solve the equations*

(*i*) $10x + 13 = \dfrac{3}{x}$ *by factors.*

(*ii*) $5x^2 - 7x + 1 = 0$, *giving results to 3 significant figures.*

(*b*) *Solve for x and y the simultaneous equations*

$$5x + 4y = 39$$
$$3x - 8y = -13$$

A. 4. (*a*) (*i*)

$$10x + 13 = \frac{3}{x}$$

or $10x^2 + 13x = 3$,

or $10x^2 + 13x - 3 = 0$.

$\therefore\ (5x - 1)(2x + 3) = 0$

$5x - 1 = 0$, or $2x + 3 = 0$

$$\therefore\ x = \underline{\frac{1}{5}, \text{ or } -\frac{3}{2}.}$$

(*ii*) $5x^2 - 7x + 1 = 0$

This is most conveniently solved by means of the general formula,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}.$$

Hence,

$$x = \frac{7 \pm \sqrt{49 - 20}}{10}$$

$$= \frac{7 \pm \sqrt{29}}{10}$$

$$= \frac{7 \pm 5{\cdot}385}{10}, \text{ using a table of square roots.}$$

$$= \frac{12{\cdot}385}{10}, \text{ or } \frac{1{\cdot}615}{10}$$

$= \underline{1{\cdot}24, \text{ or } 0{\cdot}162}$, to 3 significant figures.

(*b*)

$5x + 4y = 39$ ....(1)

$3x - 8y = -13$ ....(2)

Multiply equation (1) by 2:

$10x + 8y = 78$ ....(3)

Add equations (2) and (3):

$13x = 65$,

or $x = 5$.

Substitute for x in equation (1):

$25 + 4y = 39$

$4y = 14$,

and $y = 3\frac{1}{2}$.

Thus, $\underline{x = 5 \text{ and } y = 3\tfrac{1}{2}.}$

Note: The student should check the solution in each of the original equations (1) and (2).

Q. 5. *When the potential V volts across the armature of a motor varies, the speed n r.p.m. is assumed to vary according to the law* $n = aV + b$. *Use the values in the table below to confirm the approximate truth of the law, and calculate suitable values for a and b.*

n (*r.p.m.*)	550	720	890	1,080	1,240	1,420
V (*volts*)	60	80	100	120	140	160

A. 5. $\underline{a = 8{\cdot}65,\ b = 25.}$

Q. 6. *On the same axes draw the graphs of*

(*i*) $i_1 = 5 \sin(2\pi t - \pi/4)$ *and*

(*ii*) $i_2 = 10 \sin(4\pi t - \pi/4)$ *by plotting points between* $t = 0$ *to* $t = 1$ *at intervals of* 0·1 *of a second.*

Hence, on the same diagram, sketch as accurately as possible the graph of

(*iii*) $i_3 = 5 \sin(2\pi t - \pi/4) + 10 \sin(4\pi t - \pi/4)$.

State the periodic time and frequency of each of the three curves and estimate the smallest positive value of t for which i_3 *is zero.*

A. 6. The periodic times and frequencies of the three curves are as follows:

	Periodic time (Seconds)	Frequency (Cycles/second)
i_1	1·0	1
i_2	0·5	2
i_3	1·0	1

The smallest positive value of t for which i_3 is zero is $\underline{0{\cdot}075 \text{ seconds}}$.

Q. 7. (*a*) *Evaluate, using tables:*

$$\cos 472^\circ,\ \tan\left(\frac{7\pi}{6}\right),\ \sin(-224^\circ)$$

(*b*) *A man stands at a point A, 800 ft due south from the foot B of a tower. If the elevation of the top of the tower from A is* 13°, *show that the height is* 184·7 *ft. The man now walks in a direction N* 33° *E to reach a point C due east of the tower. Calculate the length of BC and the elevation from C of the top of the tower assuming A, B and C are at the same level.*

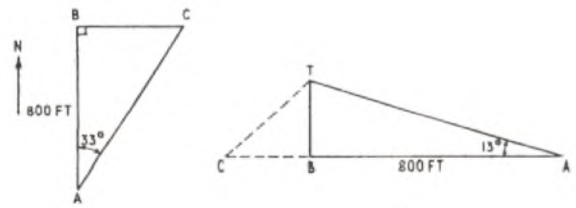
A. 7. (*a*)

$$\cos 472^\circ = \cos(360^\circ + 112^\circ) = \cos 112^\circ$$
$$= -\cos(180^\circ - 112^\circ) = -\cos 68^\circ$$
$$= \underline{-\,0{\cdot}3746.}$$

$$\tan\frac{7\pi}{6} = \tan\left(\pi + \frac{\pi}{6}\right) = \tan\frac{\pi}{6}$$
$$= \tan 30^\circ = \underline{0{\cdot}5774.}$$

$$\sin(-224^\circ) = \sin(360^\circ - 224^\circ) = \sin 136^\circ$$
$$= \sin(180^\circ - 136^\circ) = \sin 44^\circ$$
$$= \underline{0{\cdot}6947.}$$

(*b*) Sketch (*a*) shows a plan view of points A, B and C. Since A and C are due south and east of B, respectively, angle ABC is a right angle. Sketch (*b*) shows, on the right, an elevation in the plane through the tower and point A and, on the left, an elevation in the plane through the tower and point C. Since A, B and C are at the same level angles TBA and TBC are both right angles.



(*a*) (*b*)

From sketch (*b*),

$$\frac{TB}{AB} = \tan \angle BAT$$

$$\therefore\ TB = 800 \times 0{\cdot}2309 = 184{\cdot}72.$$

$\therefore$ Height of tower $\simeq$ 184·7 ft.

From sketch (*a*),

$$\frac{BC}{AB} = \tan 33^\circ$$

$$\therefore\ BC = 800 \times 0{\cdot}6494 = 519{\cdot}52$$
$$\simeq 519{\cdot}5 \text{ ft.}$$

The elevation from C of the top of the tower, T, is given by angle TCB (sketch (*b*)).

$$\tan \angle TCB = \frac{TB}{BC} = \frac{184{\cdot}7}{519{\cdot}5} = 0{\cdot}3555.$$

No.	*Log.*
184·7	2·2665
519·5	2·7156 −
	$\bar{1}$·5509

Hence, elevation of top of tower from C = 19° 34′.

Q. **8.** *Draw on the same axes the graphs of* $y = -5 + 6x - x^2$ *between* $x = 0$ *and* $x = 6$ *and of* $y = \frac{6}{x}$ *between* $x = 1$ *and* $x = 6$.

Read off from your diagram the values of x where the two curves intersect (correct to 2 decimal places). Write down and simplify the equation of which these values of x are two of the roots.

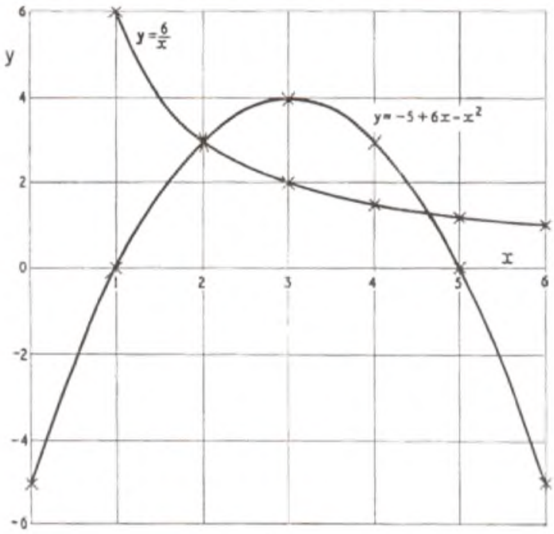
A. 8. The graphs are plotted in sketch (*a*) from the following table of values:

For $y = -5 + 6x - x^2$:

x	0	1	2	3	4	5	6
$-x^2$	0	−1	−4	−9	−16	−25	−36
$6x$	0	6	12	18	24	30	36
−5	−5	−5	−5	−5	−5	−5	−5
y	−5	0	3	4	3	0	−5

For $y = \frac{6}{x}$:

x	1	2	3	4	5	6
$y = \frac{6}{x}$	6	3	2	1·5	1·2	1



(*a*)

The values of x where the two graphs intersect are 2 and approximately 4·65, as read from sketch (*a*). The first value is exact, as seen from the above tables. The second value cannot be determined from sketch (*a*) correct to 2 decimal places and hence it is necessary to draw a second graph, covering a small range of x above and below 4·65, to a larger scale. The range 4·6 to 4·7 is sufficient and the necessary few values are worked out in the following tables:

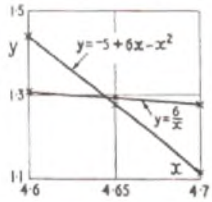
For $y = -5 + 6x - x^2$:

x	4·6	4·65	4·7
$-x^2$	−21·16	−21·6225	−22·09
$6x$	27·6	27·9	28·2
−5	−5	−5	−5
y	1·44	1·2775	1·11

For $y = \frac{6}{x}$:

x	4·6	4·65	4·7
y	1·304	1·290	1·277

These graphs are plotted in sketch (*b*). The point of intersection is read from this sketch as 4·646 and hence the two intersection points occur at x = 2·00 and x = 4·65 (correct to 2 decimal places).



(*b*)

Where the graphs intersect,

$$y = -5 + 6x - x^2 = \frac{6}{x}.$$

$\therefore -5x + 6x^2 - x^3 = 6$, on multiplying through by x.

Therefore, $x^3 - 6x^2 + 5x + 6 = 0$ is the equation of which the above values of x are two of the roots.

Note: The equation is called a cubic and has three roots altogether. The third root is actually $x = -0{\cdot}65$ (to 2 decimal places) and occurs on the graph (not illustrated) in the third quadrant. There is another part of the graph of $y = 6/x$ (for negative values of x) which lies wholly in the third quadrant and which would intersect the graph of $y = -5 + 6x - x^2$ below $x = 0$.

Q. **9.** (*a*) *In Fig. 1 XY is the diameter of a circle centre O. Z is any point on the circumference and XZ produced meets the tangent at Y in T. If angle TYZ = 35°, calculate the values of angles XZO and OZY.*

(*b*) *A circular road tunnel is to be constructed with a cross-sectional radius of 15 ft. If the width of the road is 24 ft, calculate the maximum headroom above the road and the cross-sectional area of the space below the road.*

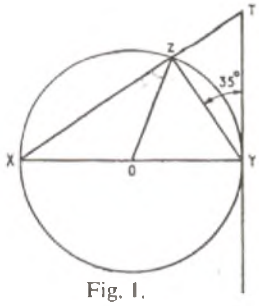


Fig. 1.

A. 9. (*a*) Refer to sketch (*a*).

Since X, Y and Z are all points on the circumference of the circle, OX = OZ = OY = radius of circle.

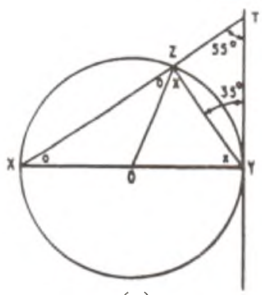
Triangle OXZ is therefore isosceles and hence $\angle OXZ = \angle OZX$. Similarly, triangle OZY is isosceles and $\angle OZY = \angle OYZ$.

$\angle XZY = 90^\circ$, since it is the angle in a semi-circle.

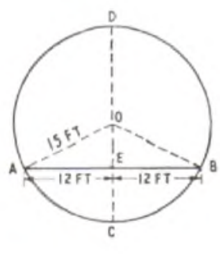
Also, $\angle TYX = 90^\circ$, since it is the angle made by a tangent to the circle with the radius at the point of contact.

$$\therefore\ \angle OZY = \angle OYZ = 90^\circ - 35^\circ = 55^\circ$$
$$\text{and } \angle XZO = 90^\circ - \angle OZY = 90^\circ - 55^\circ = 35^\circ.$$

Thus, $\angle XZO$ = 35° and $\angle OZY$ = 55°.



(*a*)



(*b*)

(*b*) See sketch (*b*).

Let the circle with centre O represent the tunnel, of radius 15 ft, and AB the road, of width 24 ft. Since the headroom is to be a maximum, AB must lie below O as shown and cannot lie above O.

Through O draw the diameter DC at right angles to AB, intersecting AB at E. Then AE = EB = 12 ft and $\angle$AEO = $\angle$BEO = 90°.

Applying the theorem of Pythagoras to triangle AEO,

$$OE^2 = 15^2 - 12^2 = 3 \times 27.$$

$$\therefore\ OE = \sqrt{3 \times 27} = 9 \text{ ft}.$$

Hence, maximum headroom = DE = DO + OE

$$= 15 + 9 = \underline{24 \text{ ft.}}$$

Cross-sectional area of space below road = segment ABC

$$= \tfrac{1}{2}r^2(\theta - \sin\theta),$$

where r = radius and θ = angle subtended at the centre by the arc of the segment.

$$\text{Now, } \theta = \angle AOB = 2 \times \angle AOE$$

$$\sin \angle AOE = 12/15 = 4/5.$$

$$\therefore\ \angle AOE = 53^\circ\ 8' = 0{\cdot}9274 \text{ radians}$$

$$\text{and } \sin\theta = \sin 106^\circ\ 16' = \sin 73^\circ\ 44' = 0{\cdot}96.$$

$$\therefore\ \text{Area of segment ABC} = \frac{1}{2} \times 15^2\,(1{\cdot}8548 - 0{\cdot}96)$$

$$\simeq \frac{225}{2} \times 0{\cdot}895$$

$$= 100{\cdot}7$$

No.	*Log.*
112·5	2·0512
0·895	$\bar{1}$·9518+
	2·0030

Hence, cross-sectional area of space below road

$$= \underline{100{\cdot}7 \text{ ft.}^2}$$

Q. **10.** (*a*) *Draw the triangle PQR right-angled at Q. If PQ = 2ab and QR = $a^2 - b^2$ express sin R and cos R in terms of a and b. Use your results to show that*

$$\frac{\sin R}{\cos R} = \tan R.$$

(*b*) *Prove that* $$\frac{1}{\cos^2\theta} - \tan^2\theta = 1$$

(*c*) *In Fig. 2 ABCD is a parallelogram, with BX and DY perpendicular to AC. If DY produced cuts AB in Z such that* $\frac{AZ}{Z} = \frac{2}{1}$ *calculate the ratios* (*i*) $\frac{DY}{YZ}$.

(*ii*) $\frac{\text{Area } \triangle AYD}{\text{Area } \triangle AYZ}$

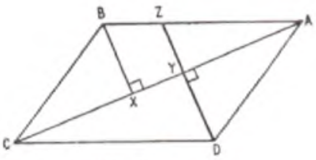


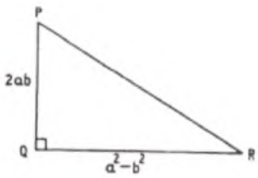
Fig. 2

Show that the ratio of the area of $\triangle$ ABX to that of $\triangle$ ADY is 3 : 2.

A. 10. (*a*) See sketch.

Since PQR is a right-angled triangle,

$$PR^2 = PQ^2 + QR^2$$
$$= (2ab)^2 + (a^2 - b^2)^2$$
$$= 4a^2b^2 + a^4 - 2a^2b^2 + b^4$$
$$= a^4 + 2a^2b^2 + b^4$$
$$= (a^2 + b^2)^2.$$
$$\therefore\ PR = a^2 + b^2$$



$$\therefore\ \sin R = PQ/PR = \underline{2ab/(a^2 + b^2).}$$

$$\cos R = QR/PR = \underline{\frac{a^2 - b^2}{a^2 + b^2}}.$$

$$\frac{\sin R}{\cos R} = \frac{2ab}{a^2 + b^2} \times \frac{a^2 + b^2}{a^2 - b^2}$$

$$= \frac{2ab}{a^2 - b^2} = \frac{PQ}{QR} = \tan R \qquad \text{Q.E.D.}$$

(*b*)

$$\frac{1}{\cos^2\theta} - \tan^2\theta = \frac{1}{\cos^2\theta} - \frac{\sin^2\theta}{\cos^2\theta}$$

$$= \frac{1 - \sin^2\theta}{\cos^2\theta}$$

$$= \frac{\cos^2\theta}{\cos^2\theta} = 1 \qquad \text{Q.E.D.}$$

(*c*) Refer to Fig. 2 of the question.

In triangles ABX, CYD:

AB = CD		(opposite sides of a parallelogram)
$\angle$BXA = $\angle$CYD		(right angles)
$\angle$BAX = $\angle$YCD		(alternate angles)

Hence, the triangles are congruent and BX = DY.

$$(i)\quad \therefore\ \frac{DY}{YZ} = \frac{BX}{YZ}.$$

Now, since triangles AYZ and AXB are similar,

$$\frac{BX}{ZY} = \frac{AB}{AZ}.$$

$$\text{But } \frac{AZ}{ZB} = \frac{2}{1}, \text{ or } \frac{ZB}{AZ} = \frac{1}{2}.$$

$$\therefore\ \frac{ZB}{AZ} + 1 = \frac{3}{2},$$

$$\text{i.e. } \frac{ZB + AZ}{AZ} = \frac{AB}{AZ} = \frac{3}{2}.$$

$$\therefore\ \frac{DY}{YZ} = \underline{\frac{3}{2}}, \text{ from the above relationships.}$$

$$(ii)\quad \frac{\text{Area } \triangle\ AYD}{\text{Area } \triangle\ AYZ} = \frac{\frac{1}{2} \times AY \times DY}{\frac{1}{2} \times AY \times YZ}$$

$$= \frac{DY}{YZ} = \underline{\frac{3}{2}}.$$

$$\frac{\text{Area } \triangle\ ABX}{\text{Area } \triangle\ ADY} = \frac{\frac{1}{2} \times AX \times BX}{\frac{1}{2} \times AY \times DY}$$

$$= \frac{AX}{AY}, \text{ since BX = DY}$$

$$= \frac{AB}{AZ}, \text{ from the similar triangles ABX and AZY}$$

$$= \underline{\frac{3}{2}}, \text{ from (i).}$$

TELEPHONY AND TELEGRAPHY A, 1960

Q. **1.** *Why is a five unit code used in teleprinter systems? How many arrangements or combinations can be obtained from such a code? Is this number sufficient for the characters to be signalled from the keyboard of a page-type teleprinter? If not, describe methods which could be used to expand the code.*

A. 1. The reasons for using a 5-unit code in teleprinter systems are:

(*a*) The number of characters are sufficient for teleprinter purposes.

(*b*) All characters are composed of the same number of signal elements. This simplifies the mechanism for direct printing.

(*c*) All characters occupy the same transmission time. This simplifies the maintenance of synchronization.

(*d*) It produces a reasonably fast message transmission, and a high speed of transmission is desirable in order to economize on circuit occupancy time.

(*e*) As the signal characters consist of combinations of positive and negative potential, signal detection is simple.

The maximum number of combinations that can be obtained from a 5-unit code is $2^5 = 32$. This is insufficient to meet the requirements of a page-type keyboard where, in addition to the alphabet, numerical digits, punctuation marks and functional signals, such as carriage return, are required. In all, a minimum of 50 characters is necessary.

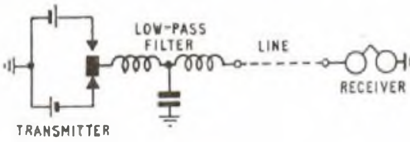
To provide these characters from a 5-unit code it is necessary to employ two characters as functional operational signals, namely letter-figure and figure-letter shifts, to operate one of two sets of printing mechanisms from the same signal characters. These two functional characters are used to operate a shift comb which occupies one of two positions, thereby permitting only one or other set of bell cranks to be operated.

An alternative method of providing additional characters would be to employ a 6-unit code giving 2^6, i.e. 64 combinations. This would be sufficient to provide all the facilities of the 5-unit code without the use of the shift mechanism. The speed of transmission would, however, be slower than that of the 5-unit code.

Q. 2. In a direct-current telegraph circuit what are the relative advantages and disadvantages of using (a) a single-wire circuit with earth return and (b) a loop circuit?

For each case, draw a simple circuit diagram showing a transmitter working to a receiver.

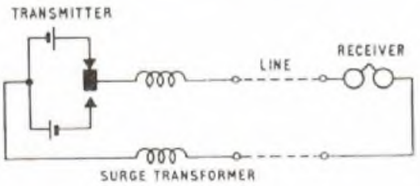
A. 2. The simple single-wire earth-return circuit is shown in sketch (*a*) and has the principal advantage of economy in line plant.



(*a*)

The system can be used for separate transmission and reception by the use of a send/receive switch, but simultaneous transmission and reception are not possible over a single circuit. To provide simultaneous transmission and reception, two physical wires are required and the cost of providing the line is, therefore, doubled. The disadvantages of the single-wire system are that it is electrically unbalanced and is dependent on earth resistance. Although the overall resistance of an earth-return circuit is less than that of a loop circuit, an earth-return circuit can only operate satisfactorily up to a distance of approximately 40 miles. Over 40 miles, the interference from neighbouring circuits causes errors in the received telegraph signals. A further effect arising from the electrical unbalance is that the telegraph circuit interferes with telephone circuits. This interference increases with the length of the telegraph circuit but can be overcome by incorporating low-pass filters at the transmitting end of the telegraph circuit to prevent the higher harmonic frequencies being transmitted. A further disadvantage of the earth-return circuit is that variations in earth potential, e.g. during magnetic storms, may be of sufficient magnitude to make telegraph working impossible.

The simple loop circuit shown in sketch (*b*) has the advantage that



(*b*)

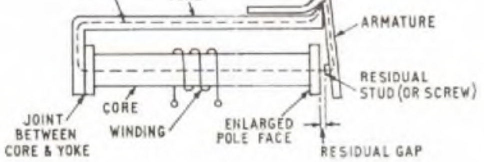
a local-record facility can be provided, but it suffers from high bias-distortion. Double-current working cannot be used and mutilation of signals occurs if both ends attempt to transmit simultaneously. The principal advantages of the loop circuit are that fortuitous distortion is greatly reduced as interference e.m.f.s induced in the loop tend to cancel out, and interference with adjacent telephone channels is greatly reduced. Adjacent circuit interference can be further reduced by fitting surge transformers in the telegraph circuit. To provide simultaneous transmission facilities with a loop circuit it is necessary to employ two pairs of wires, hence increasing the cost. Against this, however, bias distortion is reduced as double-current working can be employed.

Q. 3. Sketch the magnetic circuit of a telephone relay and indicate the component parts.

Explain the importance of (a) a close-fitting joint between the core and yoke and (b) the residual stud or screw.

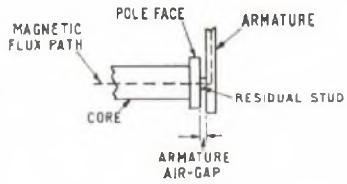
What determines the pull exerted on the armature of such a relay when a current flows through the winding?

A. 3. The magnetic circuit of a telephone relay and its component parts are shown in sketches (*a*) and (*b*). The reluctance of a part of a



(*a*)

magnetic circuit depends on the length and cross-sectional area of the flux path and on the permeability of the medium through which the flux passes. In the magnetic circuit of a relay, the core, yoke and armature are all made of soft iron of low reluctance, and the reluctance of the total magnetic circuit of the relay is,



(*b*)

therefore, largely determined by the designed air-gap between the armature and the pole face, and by any spurious air-gaps between the core and yoke or between the yoke and armature.

The air-gap between the core and the yoke is detrimental to the effective operation of the relay since it adds unwanted reluctance to the magnetic circuit—thereby either necessitating increased ampere-turns to give the correct flux or reducing the flux—and reduces the dependency of the relay's operation on the designed armature air-gap, hence affecting the operation and release of the relay.

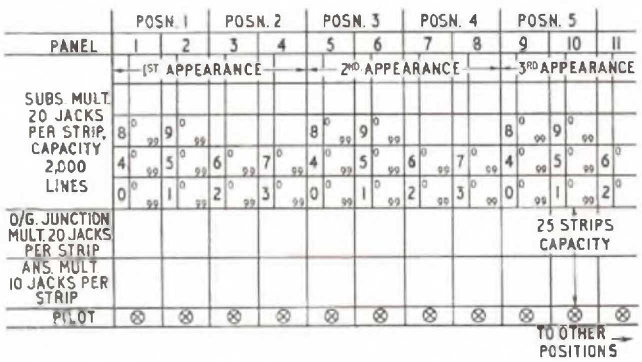
The purpose of the residual stud or screw, which is of non-magnetic material, e.g. phosphor-bronze, is to provide an air-gap in the magnetic circuit between the armature and the pole face of determined length. This prevents a closed metallic path being formed when the relay is operated and reduces the residual magnetism when the current in the relay is ceased. The relay operation is, therefore, more closely controlled and the relay release is more positive and reliable. In relay circuits it is sometimes desirable to use a relay with closely controlled operating and release characteristics, and one way in which this may be achieved is by having an adjustable residual screw to vary the length of the air-gap.

The pull on the armature is proportional to the square of the flux density and the cross-sectional area of the air-gap. It follows that the pull on the armature is determined by the ampere-turns of the coil and the length and cross-sectional area of the air-gap. Since practically all the reluctance of the circuit is concentrated in the air-gap, the pull on the armature can sometimes be increased by increasing the area of the pole face. This is of interest since an increase in pole-face area reduces the flux density and there are obviously limits to which an increase in area can increase the pull on the armature.

Q. 4. Sketch the arrangement of equipment on the face of a manual switchboard suitable for about 1,000 subscribers' lines, and explain how provision is made for future growth of the exchange multiple.

What factors influence the maximum number of subscribers' lines that can be served from such a switchboard?

A. 4. A typical manual switchboard for about 1,000 subscribers' lines would consist of six or eight positions and would employ a three-panel or five-panel repetition depending on traffic loading. The face-equipment layout for such an exchange is shown in the sketch.



Provision for growth is made basically in two ways. Firstly, the multiple cables to the jack strips are provided with tails such that further cables can be added when it is necessary to extend the multiple to additional positions. Secondly, the multiple is laid out on the face equipment so that spare jack strips (dummy or blank strips) are inserted in the space above the multiple initially provided. This facilitates the provision of new multiple cables required for additional subscribers. The same considerations apply to the answering multiple and to the junction multiple.

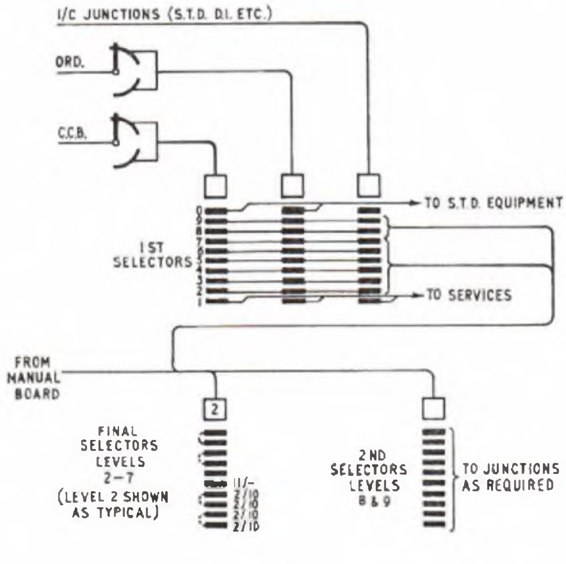
The major factors which limit the number of lines that can be served by such a switchboard are the traffic loading on the positions, the repetition of the multiple and accessibility of the multiple from the operator's position. Additional positions can only be provided to the limit of the available accommodation and the number of panels per repetition of the multiple is limited by the fact that operators can only readily reach jacks in the two panels either side of the position at which they are seated, i.e. a six-panel range. The space available on the face of the switchboard to accommodate the multiple is also limited by the height of multiple that it is possible to install. The height of the multiple, whilst dependent on the height of the switchboard, is also restricted by the fact that the operators must be able to reach the jacks in the multiple without difficulty.

One method of increasing the multiple without increasing the height or spread too greatly is by the use of smaller components, e.g. jacks of smaller diameter. However, mechanical considerations, and the need for clear identification of the circuits on the face of the switchboard, limit the size to which components may be reduced.

Q. 5. *Draw a trunking diagram of a 3-digit step-by-step exchange employing subscribers' uniselectors. What considerations determine the total number of switches of each type to be provided?*

If the group selectors are of the 100-outlet type, explain how you would interconnect them to final-selector units comprising more than ten final selectors.

A. 5. The trunking diagram of a 3-digit step-by-step exchange employing uniselectors is shown in sketch (*a*). The exchange consists

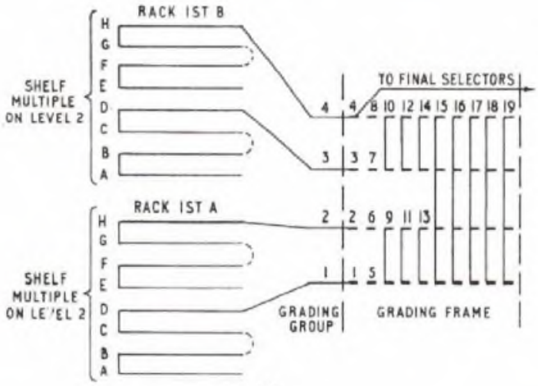


(*a*)

basically of three types of selectors, uniselectors, 1st group selectors and final selectors.

The number of uniselectors provided depends on the number of lines. (Note: coin-box lines and ordinary subscribers' lines are segregated.) The number of 1st group selectors depends upon two factors: firstly, the traffic originated by the exchange subscribers and, secondly, the number of incoming junctions and trunks. The former requirements are calculated from a knowledge of the traffic originated in each of the ordinary and coin-box groups of uniselectors and a standard grade of service, and the latter are calculated on the basis of one group selector per incoming and bothway junction or trunk. The final selectors are provided on the basis of traffic loading on individual final-selector units. These would be 200-line multiple units with 200-outlet-type group selectors or 100-line multiple units with 100-outlet group selectors. The final selectors are of three types, designed to serve either ordinary, 2–10-line P.B.X. or 11-and-over-line P.B.X. multiples, and the actual selector requirements in each type of unit and the number of units of each type would be determined on the basis of the traffic flowing to the units.

Hundred-outlet group selectors have an availability of 10, and with 10 or less than 10 final selectors per unit, full availability conditions would exist and each outlet of the 1st selector level (all banks on the level being commoned) would have individual access to a final selector. If more than 10 final selectors were necessary, limited availability conditions would exist and a grading must be employed. Under these conditions the group selectors would be split into grading groups consisting of approximately equal traffic loading either on a rack or a shelf basis, and these groups would be interconnected to form a grading giving access to final selectors. The principles of such a grading are shown in sketch (*b*), from which it



(*b*)

will be seen that certain outlets have individual access to a final selector and other outlets are commoned to give joint access to a final selector.

Q. 6. *Sketch and describe a circuit element for stepping a two-motion selector vertically under control of pulses from a subscriber's dial.*

Why are the dial "break" pulses longer than the "make" pulses?

Q. 7. *What is the "erlang"? Explain its meaning in terms of circuit occupancy.*

On a group of circuits it is known that 120 *calls are carried in the busy-hour and that the average number of calls simultaneously in progress during this period is six. What is* (*a*) *the traffic carried and* (*b*) *the average duration of the calls?*

A. 7. The erlang is the unit of traffic flow, where traffic flow is defined as the average number of calls in progress simultaneously. Thus, if one call occupies a circuit for one hour the traffic carried is one erlang; alternatively, if 12 calls in the busy hour each occupy the circuit for an average of five minutes, the circuit again carries one erlang of traffic.

For groups of circuits the definition is applied to the average circuit occupancy or holding time for the total number of calls within the hour period. And the traffic is calculated from:

$$A = C \times T,$$

where A is traffic in erlangs, C is the total number of calls during the hour, and T is the average holding time of the calls, expressed in hours.

With groups of circuits, the average number of simultaneous calls in progress over the hour period can be demonstrated to be equal to the traffic in erlangs and this forms an alternative definition for the erlang.

If the total calls carried by the group of circuits in the busy hour is 120 and the average number of simultaneous calls is six, then:

(*a*) The traffic carried equals the number of simultaneous calls, i.e. 6 erlangs.

(*b*) The average holding time in minutes can be obtained by substituting in the formula, $A = \dfrac{C \times T}{60}$.

$$\text{Hence, } T = \frac{60 \times A}{C} = \frac{60 \times 6}{120} = \underline{3 \text{ minutes.}}$$

Q. 8. *Design the best* 4-*group grading for connecting one level of* 100-*outlet selectors to a group of* 18 *junctions.*

What rearrangements would you propose if it became necessary to increase the size of the group to 20 *junctions?*

A. 8. As a 4-group grading is to be designed, it must consist of a combination of individual choices, paired choices and full commons. Let the number of individual choices be A, the number of pairs be B and the number of full commons be C.

$$\text{Then } A + B + C = 10 \quad \ldots\ldots\ldots\ldots\ldots(1)$$

$$\text{and } 4A + 2B + C = 18 \quad \ldots\ldots\ldots\ldots\ldots(2)$$

Subtracting equation (1) from equation (2),

$$3A + B = 8, \text{ or } A = \frac{8 - B}{3} \quad \ldots\ldots\ldots(3)$$

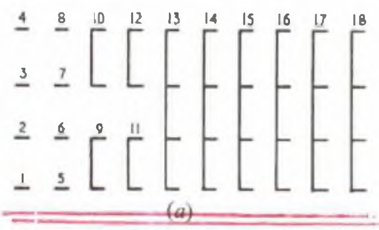
$A = \frac{8 - B}{3}$

By substituting values for B in equation (3), corresponding values of A are obtained. As the number of choices must be either zero or a positive whole number in all cases, solutions to the equation which

	A	B	C	Sum of successive differences
Grading 1	2	2	6	4
Grading 2	1	5	4	5
Grading 3	0	8	2	14

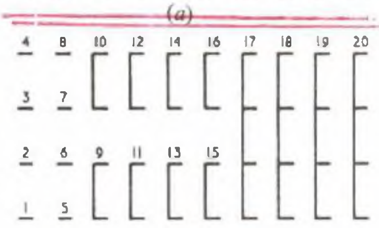
result in fractions or negative numbers are inadmissible. The permissible values of A, B and C obtained by substitution are shown in the table.

The smoothest or best grading is the one with the least sum of differences, in this case grading 1, which is shown in sketch (*a*).



(*a*)

To increase the grading to 20 trunks, either (*a*) two pairs (trunks 9 and 10) can be cut to make four individuals, giving a grading of 3:1:6, with a sum of differences of 7, or (*b*) two full commons (trunks 13 and 14) can be cut to make two pairs, giving a grading of 2:4:4 with a sum of differences of 2. The latter gives the better grading and is shown in sketch (*b*).



(*b*)

Q. 9. *Draw a circuit to show the connexion of protective devices to a subscriber's line at the main distribution frame.*
Sketch each of the devices and give suitable ratings.

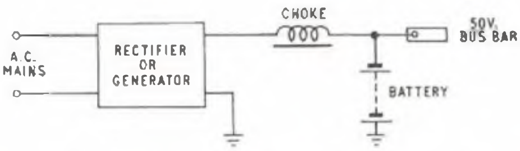
Q. 10. *What advantages does floated-battery working offer when compared with the charge-discharge system of providing a power supply?*
Explain the importance of smoothing in a floated-battery system. How is smoothing achieved?

A. 10. The advantages of a floated-battery system over a charge-discharge system are:

(*i*) The batteries are kept in a fully charged state and are, therefore, immediately available to take the exchange load in an emergency, whereas in the charge-discharge system one battery is being discharged whilst the other is being charged.
(*ii*) Because the batteries are kept fully charged the size of battery required for standby is smaller and, consequently, of lower cost.
(*iii*) The efficiency of power conversion is higher as the losses of energy inherent in the charge-discharge cycle are practically eliminated.
(*iv*) The life of the battery is increased as regular and frequent charge-discharge cycles are avoided.
(*v*) The maintenance costs are lower as less topping-up and fewer specific-gravity checks are necessary.
(*vi*) Neglecting the effect of a.c. mains ripple, which is considered below, a constant voltage supply is available for the equipment.

A mains rectifier or generator does not produce a pure d.c. voltage, but a d.c. voltage upon which an a.c. ripple is superimposed. It is undesirable to have a.c. ripple voltages on the d.c. power supply at the exchange busbars as this produces noise in the transmission circuits. In the charge-discharge system this ripple condition is avoided because there is no direct connexion between the main power supply and the equipment. In the floated-battery system, however, the battery is connected in parallel with the generator or mains rectifier across the exchange busbars, and some method of smoothing-out the ripple voltage is necessary.

To smooth the exchange supply a choke coil, i.e. an iron-cored inductor, is connected in series with the generator or rectifier, as shown in the sketch. The choke coil offers a high impedance to the



a.c. ripple voltage but a low resistance to the d.c. voltage which is required to feed the exchange.

MATHEMATICS B, 1960

Q. 1. (*a*) *By the method of completing the square obtain the roots of the equation*

$$2x - 7 = \frac{17}{x} \text{ to 3 significant figures.}$$

(*b*) *Solve the simultaneous equations*

$$4x + 5y + z = 9, \quad 3x - 2y = 18, \quad x + y + z = 0.$$

A. 1. (*a*)

$$2x - 7 = \frac{17}{x}.$$

$$\therefore 2x^2 - 7x = 17,$$

$$x^2 - \frac{7}{2}x = \frac{17}{2},$$

$$x^2 - \frac{7}{2}x + \left(\frac{7}{4}\right)^2 = \frac{17}{2} + \left(\frac{7}{4}\right)^2.$$

$$\therefore \left(x - \frac{7}{4}\right)^2 = \frac{17}{2} + \frac{49}{16},$$

$$\text{and } x - \frac{7}{4} = \pm\sqrt{8 \cdot 5 + 3 \cdot 0625}.$$

$$\therefore x = \pm\sqrt{11 \cdot 5625} + 1 \cdot 75$$
$$= \pm 3 \cdot 400 + 1 \cdot 75, \text{ from a table of square roots}$$
$$= 5 \cdot 150 \text{ or } -1 \cdot 650$$
$$= \underline{5 \cdot 15 \text{ or } -1 \cdot 65,} \text{ to 3 significant figures.}$$

(*b*)

$$4x + 5y + z = 9 \quad \ldots\ldots (1)$$
$$3x - 2y = 18 \quad \ldots\ldots (2)$$
$$x + y + z = 0 \quad \ldots\ldots (3)$$

Subtracting equation (3) from equation (1):

$$3x + 4y = 9 \quad \ldots\ldots (4)$$

Subtracting equation (4) from equation (2):

$$-6y = 9$$
$$\text{and } y = -1\tfrac{1}{2}.$$

Substituting for y in equation (2):

$$3x + 3 = 18$$
$$\therefore 3x = 15$$
$$\text{and } x = 5.$$

Substituting for x and y in equation (3):

$$5 - 1\tfrac{1}{2} + z = 0$$

$$\text{and } z = 1\tfrac{1}{2} - 5 = -3\tfrac{1}{2}$$

Thus $\underline{x = 5}$, $\underline{y = -1\frac{1}{2}}$, and $\underline{z = -3\frac{1}{2}}$.

Note: These solutions should be checked in each of the original equations (1), (2) and (3).

Q. 2. (*a*) *Show that* 2^{10} *is nearly* 10^3; *hence derive a simple approximation to* $\log_{10}2$, *without using tables.*
(*b*) *If* $q = q_1(1 - e^{-kt})$, *derive a formula giving* t *in terms of* q, q_1, *and* k. *Calculate* t *when* $q = \frac{1}{2}q_1$, $k = 0{\cdot}027$. ($e = 2{\cdot}718$.)

A. 2. (*a*).

$$2^{10} = (2^2)^5 = 4^5 = 16 \times 16 \times 4 = 1{,}024$$
$$10^3 = 1{,}000$$

Thus $2^{10} \simeq 10^3$, being only 2·4 per cent greater.

Taking logarithms to base 10 of each number:

$$\log_{10}(2^{10}) = 10\log_{10}2$$
$$\text{and } \log_{10}(10^3) = 3.$$
$$\text{Thence, as } 2^{10} \simeq 10^3,$$
$$10\log_{10}2 \simeq 3,$$
$$\text{or } \underline{\log_{10}2 \simeq 0 \cdot 3.}$$

(*b*)

$$q = q_1(1 - e^{-kt})$$

$$\text{or } \frac{q}{q_1} = 1 - e^{-kt}$$

$$\therefore e^{-kt} = 1 - \frac{q}{q_1}$$

Taking logarithms to base e,

$$-kt = \log_e (1 - q/q_1),$$

$$\text{or } t = -\frac{\log_e (1 - q/q_1)}{k}.$$

Substituting the values given:

$$t = -\frac{\log_e (1 - \frac{1}{3})}{0{\cdot}027}$$

$$= -\frac{\log_e 2/3}{0{\cdot}027}$$

$$= -\frac{(\log_e 2 - \log_e 3)}{0{\cdot}027}$$

$$= \frac{1{\cdot}0986 - 0{\cdot}6931}{0{\cdot}027}, \text{ from a table of natural logarithms}$$

$$= \frac{0{\cdot}4055}{0{\cdot}027} = \frac{40{\cdot}55}{2{\cdot}7}$$

$$= 15{\cdot}02 \simeq \underline{15.}$$

No.	*Log.*
40·55	1·6080
2·7	0·4314
	1·1766

Q. 3. *The gravitational force on a sphere due to a second sphere is directly proportional to the mass of each sphere, and inversely proportional to the square of the distance between their centres.*

A metal sphere weighs 2 lb at the surface of the Earth. Find to two significant figures the gravitational force on this sphere if it were on the surface of the moon, given the following data:

	Earth	*Moon*
Diameter in miles (approx.)	7,920	2,160
Mass in tons (2 sig. figs.)	$6{\cdot}0 \times 10^{21}$	$7{\cdot}8 \times 10^{19}$

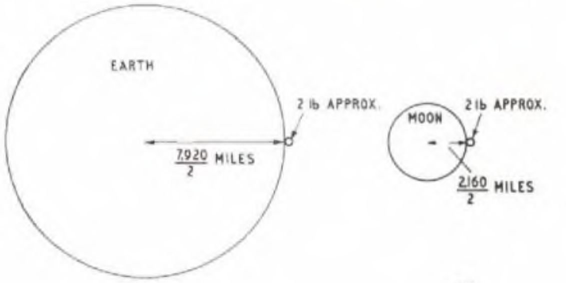
A. 3. Let F be the gravitational force, m_1 and m_2 the masses, and d the distance between the centres of the spheres.

Then, $$F \propto \frac{m_1 m_2}{d^2}$$

$$\text{or } F = k \cdot \frac{m_1 m_2}{d^2},$$

where k is a constant.

A metal sphere weighing 2 lb will clearly have a diameter which is negligibly small compared with the earth's diameter and hence the distance of its centre from the centre of the earth is effectively the earth's radius. A similar assumption is true in the case of the moon.



(a) (b)

See sketches (*a*) and (*b*) in which the size of the 2 lb sphere has been greatly exaggerated.

Let m = mass of 2 lb sphere.

Note: The mass of a body is a measure of the quantity of material in it and is the same, not only at various places on the earth's surface, but also, for example, on the moon; the weight, or gravitational force of attraction towards the centre of the earth (or moon) varies with the distance in accordance with the law stated above.

Substituting the data given for the earth,

$$F_1 = k \cdot \frac{6{\cdot}0 \times 10^{21} \times m}{3{,}960^2},$$

since $3{,}960 = 7{,}920/2$ = distance apart of the centres.

But $F_1 = 2$ lb wt.

$$\therefore k = \frac{2 \times 3{,}960^2}{6 \times 10^{21} \times m}.$$

For the sphere located on the moon's surface,

$$F_2 = k \cdot \frac{7{\cdot}8 \times 10^{19} \times m}{1{,}080^2},$$

since $1{,}080 = 2{,}160/2$ = distance apart of the centres.

$$\therefore F_2 = \frac{2 \times 3{,}960^2}{6 \times 10^{21} \times m} \times \frac{7{\cdot}8 \times 10^{19} \times m}{1{,}080^2}$$

$$= 2{\cdot}6 \times \left(\frac{396}{108}\right)^2 \times 10^{-2} \text{ lb wt.}$$

$$= 2{\cdot}6 \times \left(\frac{11}{3}\right)^2 \times 10^{-2} \text{ lb wt.}$$

$$= 0{\cdot}3495 \text{ lb wt.}$$

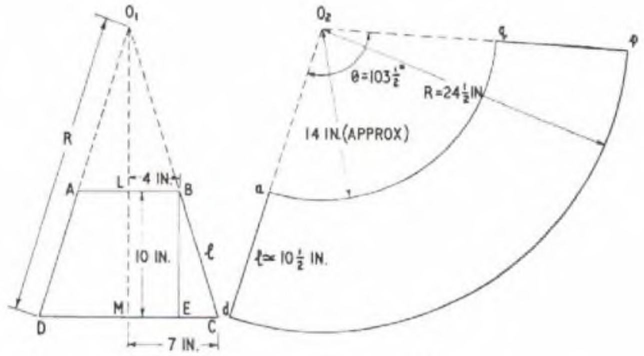
Hence, gravitational force on the metal sphere at the moon's surface $= \underline{0{\cdot}350 \text{ lb wt.}}$, correct to three significant figures.

Q. 4. *A lampshade in the form of the frustrum of a cone is to be 10 in. high and have greatest and least diameters 14 in. and 8 in. respectively.*

If it is to be cut from a circle of material, calculate the radius of this circle. What angle of sector must be removed to leave the correct development?

Mark on a clear sketch (not to scale) how the flat material should be cut, indicating all dimensions.

A. 4. A vertical cross-section of the lampshade, ABCD, is shown in sketch (*a*). Since the lampshade forms a frustum of a cone, if the



(a) (b)

slant sides AD and BC are produced backwards, they will intersect in O_1 the vertex of the cone. O_1LM is the axis of the cone and, hence, LB = 4 in. (radius of the least diameter) and MC = 7 in. (radius of the greatest diameter); also, LM = BE = 10 in. = the height of the lampshade.

Let l in be the slant height. Then, from the right-angle triangle BEC,

$$l^2 = 10^2 + (7-4)^2 = 100 + 9 = 109$$

$$\text{and } l = \sqrt{109} = 10{\cdot}44 \text{ in.}$$

Let $O_1D = R$ in. Imagine the lampshade cut along AD and then opened out flat. Since it is a frustum of a cone, all points on the periphery of the circle of diameter DC will be at the same distance, R, from O_1 and hence these points form the arc of a circle of radius R, as shown in sketch (*b*). Similarly all points on the periphery of the circle of diameter AB form an arc of a circle of radius $(R - l)$. Sketch (*b*) is known as the development of sketch (*a*); $O_2a = O_1A$ and $O_2d = O_1D = R$.

Since the lampshade is to be cut from a circle of material, a part of which is shown in sketch (*b*), the angle, θ, subtended at its centre, O_2, will be such that the arc length aq = circumference of the circle of diameter AB (sketch (*a*)) and the arc length dp = circumference of the circle of diameter DC (sketch (*a*)).

But, length of arc = radius of circle × angle subtended at the centre by the arc.

$$\therefore 2\pi \times 4 = (R - l)\theta,$$

$$\text{and } 2\pi \times 7 = R\theta.$$

Dividing the second equation by the first.

$$\frac{7}{4} = \frac{R}{R - 10{\cdot}44}$$

$$\therefore 7R - 73{\cdot}08 = 4R.$$

$$3R = 73{\cdot}08$$

$$\text{and } R = \underline{24{\cdot}36 \text{ in.}}$$

$$\theta = \frac{14\pi}{R}$$

$$= \frac{14\pi}{24{\cdot}36} \text{ radians.}$$

$$= \frac{14\pi}{24{\cdot}36} \times \frac{180}{\pi} \text{degrees}$$

$$= \frac{14 \times 60}{8{\cdot}12} \text{ degrees}$$

$$= 103{\cdot}5 \text{ degrees}$$

Thus, angle of sector $= \underline{103\frac{1}{2}^\circ}$

No.	*Log.*
14	1·1461
60	1·7782
	2·9243
8·12	0·9096
	2·0147

The flat material should be cut as indicated in sketch *(b)*, i.e. from a circle of radius $24\frac{1}{2}$ in. (approx.) cutting out a sector of angle $103\frac{1}{2}°$ and then cutting along the arc aq at a radius of $24{\cdot}36 - 10{\cdot}44 \simeq 14$ in. In practice, it would be necessary to increase θ slightly in order to leave a slight overlap along the edges ad and qp to allow for jointing these edges by stapling or any other suitable method.
Note: There is a possible ambiguity in the question. If taken literally, the angle of sector that must be removed (from the circle of material) to *leave* the correct development is $360° - 103\frac{1}{2}° = 256\frac{1}{2}°$. It is considered that the answer given, $103\frac{1}{2}°$, is the one intended.

Q. 5. *The voltage/current characteristic of a certain circuit element is indicated by the following measurements:*

v	48	100	169	225	324
i	0·85	2·55	5·6	8·5	14·7

Assuming the formula $i = kv^n$ relates these two quantities, plot suitable variables to obtain a straight-line graph.
From the graph estimate k and n.

A. 5. $n = 1{\cdot}5$ and $k = 0{\cdot}0026$ are the values estimated from the graph.

Q. 6. (*a*) *Write down the rth term and the sum to* n *terms of the series*

(i) $24 + 19 + 14 + \ldots$
(ii) $144 - 48 + 16 - \ldots$

Have either of these series a "sum to infinity"? If so, evaluate it.
(*b*) *How long will it take a sum of money to treble itself at 6 per cent p.a. compound interest?*

A. 6. (*a*) (*i*) $24 + 19 + 14 + \ldots.$
This is an arithmetic progression whose common difference is -5.

r^{th} term $= a + (r - 1)d$

where a and d are the first term and difference, respectively.

$\therefore r^{th}$ term $= 24 - 5(r - 1) = 29 - 5r$

Also $S_n = \frac{n}{2}\{2a + (n - 1)d\}$,

where n = number of terms.

$$\therefore S_n = \frac{n}{2}\{48 - 5(n - 1)\} = \frac{n}{2}(53 - 5n).$$

(*ii*) $144 - 48 + 16 - \ldots.$
This is a geometric progression whose ratio is $-\frac{1}{3}$.

r^{th} term $= ax^{r-1}$,

where a and x are the first term and ratio, respectively.

$\therefore r^{th}$ term $= 144(-\frac{1}{3})^{r-1}$.

$$\text{Also } S_n = \frac{a(x^n - 1)}{x - 1} = \frac{a(1 - x^n)}{1 - x}.$$

The second form is more convenient when $|x| < 1$.

$$\therefore S_n = \frac{144\{1 - (-\frac{1}{3})^n\}}{1 - (-\frac{1}{3})} = 108\{1 - (-\tfrac{1}{3})^n\}.$$

The second series has a sum to infinity because the r^{th} term tends to zero as r becomes very great. In the expression for S_n of this series, $x^n \to 0$ and hence,

$$S_n = \frac{a}{1 - x}, \text{ as } n \to \infty.$$
$$= \frac{144}{1 - (-\frac{1}{3})} = 144 \times \tfrac{3}{4} = 108.$$

(*b*) Let £P be the original sum of money, and let r per cent be the rate of compound interest. Then, after one year, the sum of money increases to an amount

$$P + \frac{r}{100}P = P\left(1 + \frac{r}{100}\right)$$

At the end of the second year, the amount becomes

$$\left\{P\left(1 + \frac{r}{100}\right)\right\} + \frac{r}{100}\left\{P\left(1 + \frac{r}{100}\right)\right\}$$
$$= P\left(1 + \frac{r}{100}\right)\left(1 + \frac{r}{100}\right) = P\left(1 + \frac{r}{100}\right)^2$$

After n years it may similarly be deduced that the amount is $P\left(1 + \frac{r}{100}\right)^n$. If this is to be treble the original sum, then

$$P\left(1 + \frac{6}{100}\right)^n = 3P,$$
$$\text{or } 1{\cdot}06^n = 3.$$
$$\therefore n \log_{10} 1{\cdot}06 = \log_{10} 3$$
$$\text{and } n = \frac{0{\cdot}4771}{0{\cdot}0253} = 18{\cdot}86$$

No.	Log.
0·4771	$\bar{1}{\cdot}6786$
0·0253	$\bar{2}{\cdot}4031$
	1·2755

Thus, it will effectively take 19 years for the sum to treble itself.

Q. 7. (*a*) *Find from first principles an expression for dy/dx, where $y = 5x - 2x^3$.*
(*b*) *Plot the graph of $y = \cos x$ (x in radians) from $x = -\pi$ to $x = +\pi$.*

From the graph, measure the slope of the curve when (i) $x = -\frac{\pi}{4}$, (ii) $x = 2\pi/3$.

State an expression for d/dx ($\cos x$) and verify this for the above x values.

A. 7. (*a*) $\frac{dy}{dx} = 5 - 6x^2$

(*b*) (*i*) At $x = -\pi/4$, gradient $\simeq 0{\cdot}708$
(*ii*) At $x = 2\pi/3$, gradient $= -0{\cdot}864$.

$$\frac{d}{dx}(\cos x) = -\sin x.$$

Q. 8. *State Simpson's Rule for finding an approximation to the area under a curve between two ordinates. On what assumption is the rule based?*
Sketch (but do not plot) the graph of $y = 24x^2 - 8x^3$, and use Simpson's Rule to calculate the area enclosed between this curve and the x-axis.

A. 8. See A.9 Mathematics B, 1959, *Supplement*, Vol. 53, No. 1, p.20, April 1960.
Area under the graph from $x = 0$ to $x = 3$ is 54.

Q. 9. (*a*) *State without proof the expansions for $\cos(A + B)$ and $\cos(A - B)$. Hence show that $42 \sin\theta + 24 \cos\theta$ may be expressed in the form $r\cos(\theta - \alpha)$, where α is an acute angle. Calculate r and α.*
(*b*) *If the x-axis is taken as the initial line of a polar co-ordinate system, and the y-axis as the line $\theta = \pi/2$, write down expressions for x, y in terms of r, θ.*
Translate the equation of the parabola $y^2 = 4ax$ into polar co-ordinates, giving r in terms of θ.

A. 9. (*a*)
$\cos(A + B) = \cos A \cos B - \sin A \sin B$
$\cos(A - B) = \cos A \cos B + \sin A \sin B$
$r\cos(\theta - \alpha) = r\{\cos\theta\cos\alpha + \sin\theta\sin\alpha\}$

Comparing this expansion with $42\sin\theta + 24\cos\theta$ and re-writing the latter as $24\cos\theta + 42\sin\theta$, the two expressions will be equal if

$r\cos\alpha = 24$ .... (1)
and $r\sin\alpha = 42$ .... (2)

Squaring and adding equations (1) and (2),

$$r^2(\sin^2\alpha + \cos^2\alpha) = r^2 = 24^2 + 42^2$$
$$\therefore r = \sqrt{576 + 1{,}764} = \sqrt{2{,}340}$$
$$= 48{\cdot}37.$$
$$\text{Also, } \frac{r\sin\alpha}{r\cos\alpha} = \tan\alpha = \frac{42}{24} = \frac{7}{4} = 1\tfrac{3}{4}$$
$$\therefore \alpha = 60°\ 15', \text{ since } \alpha \text{ is acute.}$$

Hence, $r = 48{\cdot}37$ and $\alpha = 60°\ 15'$.

Note: Students may be more familiar with the above equivalence expressed in the following way:

$$24\cos\theta + 42\sin\theta$$
$$= \sqrt{24^2 + 42^2}\left\{\cos\theta\frac{24}{\sqrt{24^2 + 42^2}} + \sin\theta\frac{42}{\sqrt{24^2 + 42^2}}\right\}$$
$$= 48{\cdot}37\left\{\cos\theta\frac{24}{48{\cdot}37} + \sin\theta\frac{42}{48{\cdot}37}\right\}$$
$$= 48{\cdot}37\{\cos\theta\cos\alpha + \sin\theta\sin\alpha\}$$
$$= 48{\cdot}37\cos(\theta - \alpha),$$
$$\text{and } \tan\alpha = \frac{\sin\alpha}{\cos\alpha} = \frac{42}{24}, \text{ as before.}$$

(*b*)
$x = r\cos\theta$
$y = r\sin\theta$
$y^2 = 4ax$

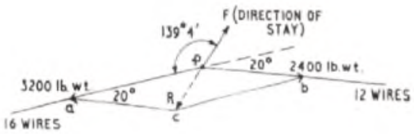
This equation may be converted into polar co-ordinates by making the above substitutions.

$$\therefore\ r^2 \sin^2\theta = 4ar \cos\theta$$
$$\text{or } r = 4a\frac{\cos\theta}{\sin^2\theta}$$
$$= \underline{4a \operatorname{cosec}\theta \cot\theta.}$$

Q. 10. *At a certain telegraph pole an overhead line route changes direction by 20°. Four wires terminate at this pole, leaving twelve continuing along the route.*

Calculate, and indicate on a sketched plan, the correct direction for a stay-wire at this pole, and find the tension in this stay if each wire is tensioned to 200 pounds, and the stay is inclined at 60° to the horizontal.

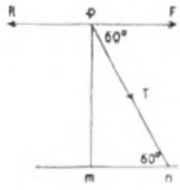
A. 10. See sketch (*a*). Since four wires terminate at the pole and



(*a*)

12 continue, there will be 16 wires in one direction, shown as ap, and 12 in the other at 20° and shown as pb, p representing the top of the pole. Assuming that all wires are attached to the pole at a common point, the tension on one side will be 200 × 16 = 3,200 lb wt, and on the other will be 200 × 12 = 2,400 lb wt, as shown in sketch (*a*).

The resultant of these two forces is shown in sketch (*a*) as the diagonal of the parallelogram of forces apbc. To completely neutralize this force, acting in the horizontal plane, it is necessary to apply an equal and opposite force *F*, by means of the stay wire. This force is shown, in sketch (*b*), in elevation and will result from some force *T* in the stay wire itself.



(*b*)

To find *R*, it is necessary to solve the triangle apc. Using the cosine rule.

$$pc^2 = ap^2 + ac^2 - 2 \times ap \times ac \times \cos \angle pac$$

$$\text{or } R^2 = 3{,}200^2 + 2{,}400^2 - 2 \times 3{,}200 \times 2{,}400 \cos 20^\circ,$$

where *R* represents the resultant force.

$$\therefore\ R^2 = 10{\cdot}24 \times 10^6 + 5{\cdot}76 \times 10^6 - 15{\cdot}36 \times 0{\cdot}9397 \times 10^6$$
$$= 16{\cdot}0 \times 10^6 - 14{\cdot}43 \times 10^6$$
$$= 1{\cdot}57 \times 10^6$$
$$\therefore\ R = 1{,}253 \text{ lb wt.}$$

No.	*Log.*
15·36	1·1864
0·9397	$\bar{1}$·9730 +
	1·1594

Referring to sketch (*b*),

$$F = R = T \cos 60^\circ$$
$$\therefore\ T = \frac{1{,}253}{0{\cdot}5} = 2{,}506 \text{ lb wt.}$$

Also, from triangle apc

$$\frac{\sin \angle apc}{ac} = \frac{\sin \angle pac}{pc}$$

$$\text{or } \sin \angle apc = \frac{2{,}400}{1{,}253} \times \sin 20^\circ$$

$$\therefore\ \angle apc = 40^\circ\ 56'$$

No.	*Log.*
2,400	3·3802
sin 20°	$\bar{1}$·5341 +
	2·9143
1,253	3·0979 −
	$\bar{1}$·8164

Thus, the stay-wire should be in the direction of the force *F* shown in sketch (*a*), at an angle of <u>139° 4′</u> to the 16-wire direction.

The tension in the stay will be <u>2,506 lb wt.</u>

RADIO AND LINE TRANSMISSION A, 1960

Q. 1. *State the approximate values of carrier frequencies you would expect to find used for the following applications:*

(*a*) *a high-quality sound broadcast service to serve a relatively small area*

(*b*) *a group of* 12 *telephone channels to be transmitted over a pair-type cable*

(*c*) *a point-to-point television radio-relay system.*

A radio wave is propagated through a block of polythene, in which its velocity of propagation is 2×10^8 *metres/sec. If its wavelength is found to be 4 cm, what is the frequency of the wave?*

A. 1. (*a*) To provide a high-quality sound-broadcast service the medium-wave band is no longer satisfactory in this country, owing to the very large number of stations broadcasting simultaneously in Europe. As the question specifies that the service area is relatively small, the v.h.f. band will be very suitable, the carrier frequencies lying between 80 and 100 Mc/s.

(*b*) Groups of telephony channels are commonly assembled at 4 kc/s intervals within the frequency band 60–108 kc/s. This band is either transmitted directly, or translated down to the band 12–60 kc/s.

(*c*) The microwave bands are used for point-to-point television radio-relay systems, suitable frequencies lying between 1,000 and 10,000 Mc/s. Within these frequency limits certain bands are specifically allocated for this type of service by international agreement.

The velocity of propagation of a wave *V* is related to its frequency *f* and wavelength λ by the relation:

$$V = f\lambda,$$

where *V* is in metres/second, λ is in metres, and *f* is in cycles/second.

Hence, $f = V/\lambda$.

Substituting the values given,

$$f = \frac{2 \times 10^8}{4 \times 10^{-2}} = \underline{5{,}000 \text{ Mc/s.}}$$

Q. 2. *For each of the two circuit diagrams given, draw an equivalent circuit diagram.*

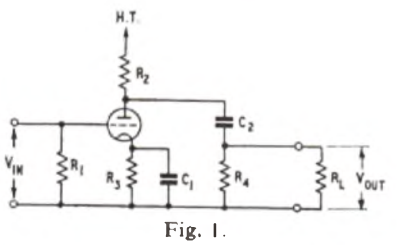


Fig. 1.

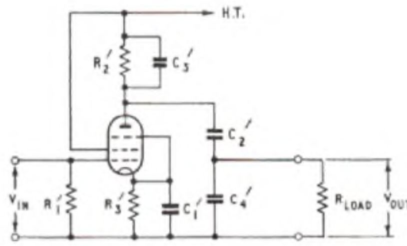


Fig. 2.

In Fig. 1, *which single component could be modified in value to reduce the gain at low frequencies relative to the gain at higher frequencies?*

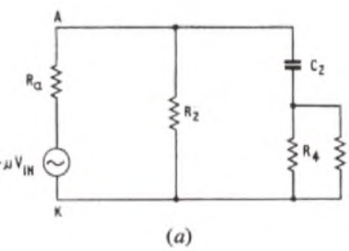
A. 2. In order to draw the equivalent circuit diagram of a triode-valve circuit, the following procedure will be found helpful.

(*i*) Mark on the actual circuit diagram the grid (G), anode (A) and cathode (K).

(*ii*) Start the equivalent circuit diagram between A and K with a resistor R_a and a generator of potential $-\mu e_{gk}$, where R_a is the anode slope resistance of the valve, μ its amplification factor, and e_{gk} the input voltage to the circuit measured between grid and cathode.

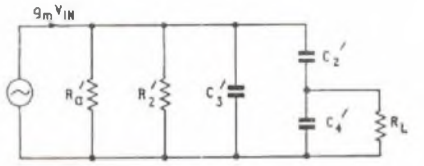
(*iii*) Complete the equivalent circuit diagram by transferring all circuit elements from the actual to the equivalent circuits but omitting the valve and all d.c. sources.

For the circuit diagram of Fig. 1, for frequencies at which capacitor C_1 has negligible reactance compared with that of resistor R_3, e_{gk} is equal to V_{in}. The equivalent circuit therefore becomes as shown in sketch (*a*).



(*a*)

For the pentode valve (Fig. 2), $R_a \gg R_L$ in normal circuits, and the current-source equivalent circuit is usually more convenient. This is drawn in a similar manner to that already described except that the valve is replaced by a current generator which supplies a current $g_m e_{gk}$ in the direction from anode to cathode within the valve, and with the anode slope resistance placed across this generator's terminals. The equivalent diagram for Fig. 2 drawn in this way is shown in sketch (*b*). As before, it is assumed that $1/\omega C'_1 \ll R_3'$.



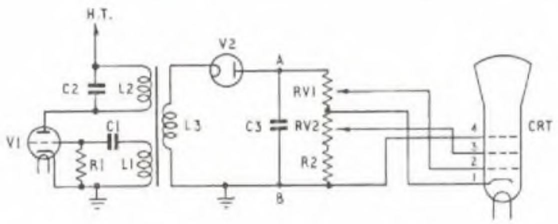
(*b*)

In Fig. 1 of the question, if capacitor C_2 is made smaller in value, its rising reactance at low frequencies would cause the voltage developed across resistor R_4 to fall. Alternatively, if capacitor C_1 is reduced in value, the gain of the stage will be reduced as the frequency is reduced, because of negative-feedback action.

Q. 3. *Draw a circuit diagram, and explain the operation, of a power-supply unit suitable for the cathode-ray tube in an oscilloscope.*

What is the main advantage of an R. F. oscillator e.h.t. generator for such an application?

A. 3. The circuit diagram of a power-supply unit suitable for a cathode-ray tube in an oscilloscope is given in the sketch. The e.h.t.



is derived from an oscillator operating at about 30 kc/s. The triode valve, V1, works as a tuned-anode feedback oscillator; self bias is obtained by means of C1 and R1. A third winding, L3, having a large number of turns, is inductively coupled to L2 so that a high voltage is developed across it when the circuit is oscillating. The voltage is rectified by the half-wave rectifier valve, V2, for which C3 forms the reservoir capacitor. A resistive potential-divider chain is connected across these points, and from this the potentials required by the various electrodes of the cathode-ray tube are tapped off.

These electrodes, numbered on the diagram, are (1) the cathode, (2) the grid, or brilliance electrode, held at about -20 volts relative to the cathode, (3) the focus electrode held at about $+200$ volts relative to the cathode, and (4) the accelerator or final anode at full e.h.t. of about $+2{,}500$ volts relative to the cathode. The potentials on electrodes (2) and (3) are adjustable by means of the controls RV1 and RV2 to permit control of the brilliance and focusing, respectively.

Notice that as the final anode of the cathode-ray tube is at earth potential, the cathode is at $-2{,}500$ volts relative to earth, so that the heater of the cathode-ray tube has to be supplied from a suitably insulated transformer secondary.

The advantages of using an r.f. oscillator-type e.h.t. supply instead of a mains-frequency supply arise from the much higher frequency of the oscillator. Because of the higher frequency, smoothing is easier and a smaller, cheaper, reservoir capacitor can be used. Furthermore, the stored charge, which is proportional to the capacitance of the reservoir capacitor, is smaller and the potential danger to operating personnel is reduced.

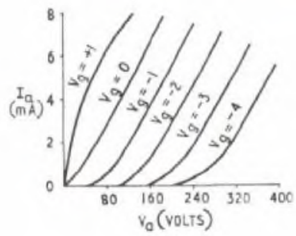
Q. 4. *Sketch sets of curves showing the following valve characteristics:*

(*a*) *I_a against V_a for various values of V_g for a triode valve*
(*b*) *I_a against V_g for various values of V_a for a triode valve*
(*c*) *I_a against V_a for various values of V_g for a pentode valve*
(*d*) *I_a against V_g for various values of V_s for a pentode valve*

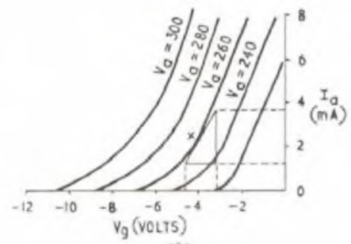
V_a = Anode voltage $\quad I_a$ = Anode current
V_s = Screen voltage $\quad V_g$ = Grid voltage

Explain, with reference to curve (b) what is meant by the term "mutual conductance."

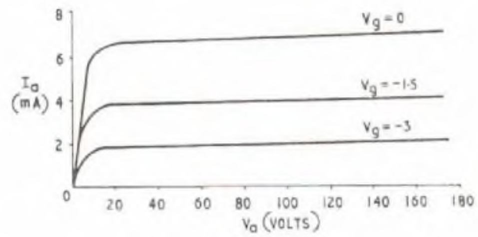
A. 4. The curves showing the required valve characteristics are given in sketches (*a*), (*b*), (*c*) and (*d*).



(*a*)

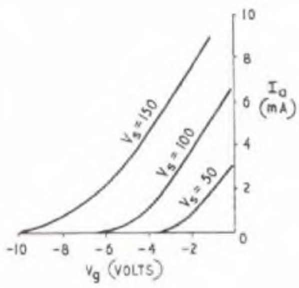


(*b*)



(*c*)

The term "mutual conductance" defines the effectiveness of the control exerted by the grid over the anode current in a triode valve. It is expressed as the ratio of the small change of anode current produced by a small change of grid voltage for a constant anode voltage, so its unit is, therefore, mA/volt. In terms of the grid characteristic of the valve given in sketch (*b*), it will be seen that this is equal to the slope of the curve at the point considered. At the point marked X, on the curve $V_a = 260$ volts, the mutual conductance is equal to $2{\cdot}2/1{\cdot}1 = 2$ mA/volt.



(*d*)

Q. 5. *With the aid of sketches describe the construction, stating the materials used, of two of the following types of fixed capacitors:*

(*i*) *a silvered-mica capacitor having a capitance of about 500 pF*
(*ii*) *a paper capacitor having a capacitance of about $0{\cdot}5\ \mu F$.*
(*iii*) *an electrolytic capacitor having a capacitance of about $16\ \mu F$.*

State one application of each, in a receiver.

A. 5. See A.10, Elementary Telecommunications Practice, 1960, *Supplement*, Vol. 53, No. 3, p.49, Oct. 1960 for details of the construction of a multi-plate silvered-mica capacitor.

See A.7, Elementary Telecommunications Practice, 1959, *Supplement*, Vol. 52, No. 4, p. 64, Jan. 1960 for details of the construction of a foil-type electrolytic capacitor.

Q. 6. *Briefly explain why it is necessary to include a detector stage in a receiver for amplitude-modulated signals.*

Discuss the characteristics of thermionic and semiconductor diodes which make them suitable for the detection of a.m. signals.

A. 6. After amplification by the radio-frequency stages of a radio receiver, an amplitude-modulated radio wave consists of a high-frequency oscillatory current with the audio-frequency modulation impressed upon it. If this current were applied directly to the headphones or loudspeaker, no audible sound would result because:

(*i*) The relatively high inductance of the loudspeaker voice coil and its high shunt stray capacitance would prevent the flow of sufficient current in the voice coil for effective operation.

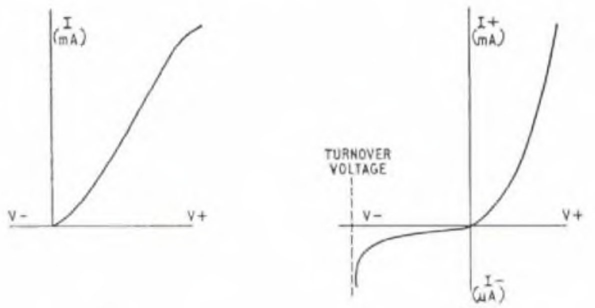
(*ii*) Even if sufficient high-frequency current could be made to flow through the voice coil the inertia of the coil and the cone would prevent the loudspeaker from responding.

(*iii*) Even if radio-frequency sounds could be emitted by the loudspeaker they would be above the range of frequencies that can be heard.

It is necessary, therefore, to provide a device that responds not to the individual cycles of the radio-frequency oscillatory current but only to changes in its amplitude. Such a device is called a detector and in practice, detection of amplitude-modulated waves is usually achieved by rectification of the modulated carrier wave. Such detectors are described as envelope detectors, and should be distinguished from synchronous detectors, which are used in certain types of line transmission equipment.

Circuits employing thermionic and semiconductor diodes are classed as linear detectors although with small signals this is not strictly the case. Detection in both cases is effected by the unidirectional properties of the devices.

The voltage/current characteristics of the thermionic diode and the semiconductor diode are illustrated in sketches (*a*) and (*b*), respectively.



(*a*) (*b*)

The relative merits of detector circuits may be assessed under four headings:

(*i*) The degree of distortion introduced by the detector.
(*ii*) The effect of the detector on the preceding circuit.
(*iii*) The ability of the detector to handle small signals.
(*iv*) The ability of the detector to handle large signals.

The thermionic diode can handle large signals with little distortion but small signals tend to be distorted owing to the bottom-bend curvature of the diode characteristic. Damping of the preceding circuit may usually be avoided by careful choice of the load resistor.

Semiconductor diodes tend to introduce a greater degree of damping of the preceding circuit, but their extremely small self-capitance and their ability to work into low-resistance loads makes them superior to thermionic diodes for certain applications. They also have advantages where size, weight and absence of heater connexions are important considerations.

Q. 7. *Describe, with the aid of a sketch, the constructional features and the principle of operation of the carbon-granule microphone.*

Explain clearly, using a diagram, how the output alternating energy is derived.

A. 7. See A. 10, Telecommunication Principles A, 1959, *Supplement*, Vol. 52, No. 4, p. 59, Jan. 1960.

Q. 8. *Discuss the shortcomings of triode valves and explain why pentode valves are commonly preferred to triodes in the high-frequency sections of medium-wave radio receivers.*

A. 8. The principal difficulty of radio-frequency amplification by means of triodes arises from the capacitance that exists between the control grid and the anode, and which becomes of increasing importance at the higher frequencies. As a result of this inter-electrode capacitance, which may amount to several picofarads, the valve input impedance becomes to some extent dependent on anode-load impedances. Thus, at frequencies at which the anode-load impedance becomes inductive, positive feedback of energy from the anode to the grid circuit may take place. When this positive feedback becomes sufficient it will cause the stage to oscillate. Alternatively, the load impedance may become capacitive, resulting in negative feedback of energy from anode to grid with a tendency to suppress the input signal.

There are three principal reasons why pentode valves are preferred to triodes in the high-frequency sections of medium-wave radio receivers, namely, better stability, increased amplification and improved selectivity.

The cause and effect of instability has already been referred to above. Pentode and tetrode valves each have a screening grid, located between the control grid and anode, which is effectively at earth potential to the signal frequencies. This additional grid reduces the control grid-anode capacitance, so greatly reducing the unwanted coupling and improving the stability of an amplifier using such valves.

Pentode valves commonly have much higher amplification factors than triode valves and this factor, coupled with the improved stability possible with amplifiers employing pentode valves, enables an increased amplification to be achieved.

Finally, the anode impedance of a pentode valve used in the high-frequency section of an amplifier is much higher than that of a triode valve. Since this anode impedance is effectively in parallel with the tuned-anode circuit of the amplifier, then it follows that the pentode valve will have far less effect in reducing the selectivity of the tuned-anode circuit than would a triode valve.

Q. 9. *Describe, with the aid of sketches, the construction and features of a low-power alloy junction transistor.*

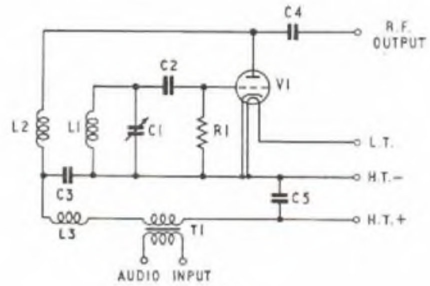
Discuss the materials used.

A. 9. See A. 5, Radio and Line Transmission A, 1959, *Supplement*, Vol. 52, No. 4, p. 67, Jan. 1960.

Q. 10. *With the aid of a circuit diagram, describe how an oscillator may be anode-modulated by an external audio-frequency source.*

Briefly explain what you understand by the sidebands of an amplitude-modulated wave, illustrating your answer with reference to a carrier modulated by a single frequency tone.

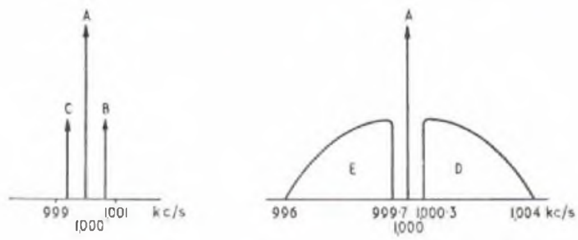
A. 10. Sketch (*a*) is a circuit diagram of an anode-modulated radio-frequency oscillator. The tuned circuit L1, C1 is connected to the grid of the triode valve V1, and oscillations are maintained by inductive feedback from coil L2, coupled to coil L1. The grid



(*a*)

capacitor C2 and leak resistor R1 provide self-bias. The output is taken through an h.t. blocking capacitor, C4. The output is amplitude-modulated by applying an audio-frequency signal to the primary of the audio transformer T1, the secondary being connected in series with the h.t. supply to the anode of V1. The radio-frequency choke L3 and coupling capacitor C3 prevent radio-frequency currents entering the audio circuits; C5 is an audio-frequency decoupling capacitor which keeps audio-frequency currents out of the h.t. supply unit.

The sidebands of an amplitude-modulated wave are the component waves that appear in addition to the carrier wave when the carrier wave is modulated by a band of frequencies. Sketch (*b*)



(*b*) (*c*)

shows the spectrum of a 1,000 kc/s carrier wave amplitude-modulated by a 1 kc/s sine-wave tone. The spectrum now consists of three component waves, the carrier A of frequency 1,000 kc/s and two additional waves B and C, of lower amplitude than A and having frequencies 1,001 kc/s and 999 kc/s, i.e. 1,000 $\pm$ 1 kc/s. These waves are known as the upper side-frequency (B) and the lower side-frequency (C). Sketch (*c*) shows the spectrum of a 1,000 kc/s carrier wave amplitude-modulated by commercial quality speech having components in the band 0·3 to 4 kc/s. Each single-frequency component in the complex speech wave generates an upper and lower side-frequency and the band of side-frequencies corresponding to the whole speech wave are called the upper sideband (D) and the lower sideband (E). The upper sideband extends from (1,000 + 0·3) kc/s to (1,000 + 4) kc/s and the lower sideband from (1,000 − 0·3) kc/s to (1,000 − 4) kc/s.

Q. 1. *Describe in detail with the aid of a sketch the "Empty Cell" process of preserving poles.*

State the advantages of this process over the "Full Cell" process.

A. 1. For a description of the "Empty Cell" or Ruping process see A.1, Line Plant Practice A, 1959, *Supplement*, Vol. 53, No. 1, p. 12, Apr. 1960.

Q. 2. *Describe in detail, with sketches, two methods which may be used in strengthening an overhead pole route in an exposed situation.*

A. 2. The forces to which a pole line is most vulnerable in an exposed situation are wind forces acting horizontally and at right-angles to the direction of the line. The forces are due to:

(*i*) The direct wind force on the poles and fittings.

(*ii*) The wind force acting on the wires or aerial cable attached to the poles.

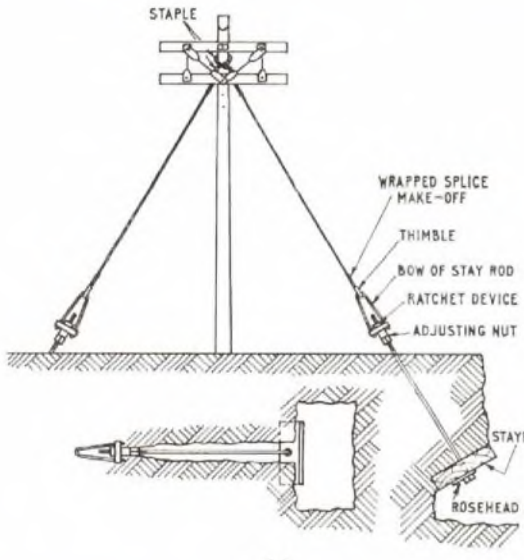
The amount of strengthening any given pole line will need to resist these forces must be estimated by calculation of the total force induced on each pole from a local knowledge of the maximum wind velocity that is likely to occur and the number of wires and cables to be erected.

Two methods by which support may be given to any pole to provide additional lateral strength are:

(*i*) Transverse stays ("rocking" or "wind" stays).

(*ii*) Struts.

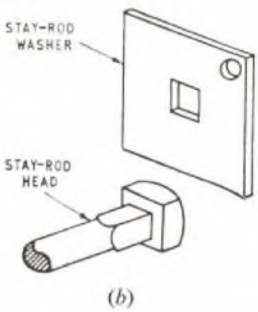
Transverse Stays. Transverse stays are fitted in pairs, one on each side of the pole and at right angles to the direction of the line. The spread of the stay should not be less than half the height of attachment to the pole and preferably equal to the height. A pole fitted with transverse stays is shown in sketch (*a*).



(*a*)

Stays are usually composed of galvanized stranded-steel wire attached to the pole at the resultant-load point by means of a single turn round the pole and made off by means of a wrapped splice. The lower end of the stay wire is made off in a similar manner round a thimble linked in the bow of the stay rod, the whole of which is anchored to a stayblock buried in the ground.

The stay rod consists of a long bolt with a headed adjusting nut which fits into the bow crosshead and secures the two parts together. The other end of the bolt is formed with a square neck and head. A large square washer with a central square hole, which keys on to the neck of the bolt, is provided thus spreading the load on the stayblock. The washer also has a small circular hole punched at one corner so that it may be secured to the stayblock by means of a rosehead nail, see sketch (*b*). By this means, the stay rod is prevented from turning when the adjusting nut is operated. The adjusting nut and the crosshead of the bow are provided with a ratchet device to prevent the malicious release of the stay.

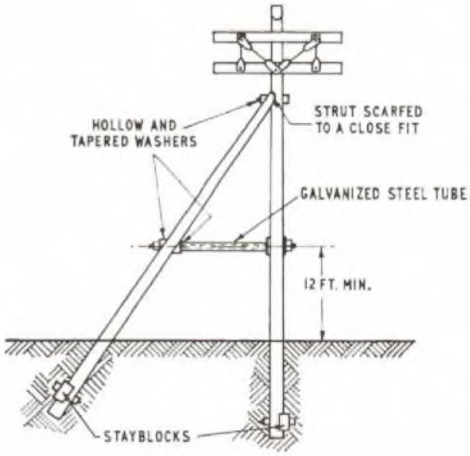


(*b*)

The hole for the stayblock should be excavated and undercut (as shown in sketch (*a*)) to provide a firm bed of undisturbed soil at right-angles to the stay for the stayblock to rest upon. A narrow channel should be cut in the soil as shown, to allow the stayrod to align with the stay wire when the rod is assembled on the stayblock and placed in position.

The other stay should be fitted with the stay wire crossing over the wire of the first at the head of the pole as shown in sketch (*a*). Each stay wire should be stapled to the pole with a single staple fitted on the opposite side of the pole to the make-off.

Struts. A strut consists of another pole of one class lighter, or a lighter pole of the same class, erected at an angle to the line pole in the form of a prop (see sketch (*c*)). As with a stay, the spread of the strut should be made equal to the height of attachment as far as conditions will permit. The structure should be built so that it will resist wind forces from either side of the line. This condition is achieved by fitting a stayblock as near as possible to the butt end of the strut and by fitting a tie-bolt between pole and strut.



(*c*)

The strut pole should be scarfed at the smaller end and brought to a feather edge to closely fit the line pole just below the lowest arm or aerial-cable bracket. The cut surface should be well treated with creosote and tar mixture to make a watertight joint and thus prevent decay of the timber. The head of the strut should be fixed to the pole by a bolt passing through the bottom of the scarf. A tie-bolt should be fitted about midway between the head of the strut and the ground. The tie-bolt should not, however, be lower than 12 ft above ground because it may then form an obstruction or hazard.

The tie-bolt should be assembled through a length of galvanized-steel tubing fitted between the pole and strut which, bearing against the bolt washers, acts as a spacer to give the structure rigidity. Hollow and tapered washers should be used on the bolts against the strut so as to provide nut seatings approximately at right-angles to the axis of the bolts.

Slots should be cut near the bottom of both pole and strut and suitable stayblocks fitted. The blocks should be securely attached by means of bolts and washers.

Q. 3. *What factors influence the siting of a distribution pole?*

State the various methods of providing overhead distribution to subscribers and describe one method in detail.

A. 3. The following factors influence the siting of a distribution pole:

(*i*) The forecast number of telephone subscribers.

(*ii*) The position should be such that all subscribers planned to be served can be served from the pole without difficulties due to trees, buildings, power wires, etc.

(*iii*) The pole should not obstruct views from windows, should be in a safe position so far as pedestrian and vehicular traffic is concerned, and should preferably be on public property.

(*iv*) The position should be discussed with and agreed by the local council surveyor or his representative. The surveyor will be particularly interested in the exact position of the pole in the footway, i.e. close to the fence or close to the kerb line.

(*v*) A site near to an existing underground track is an advantage but not a controlling factor in determining the final position of a distribution pole.

Methods of providing overhead distribution are:

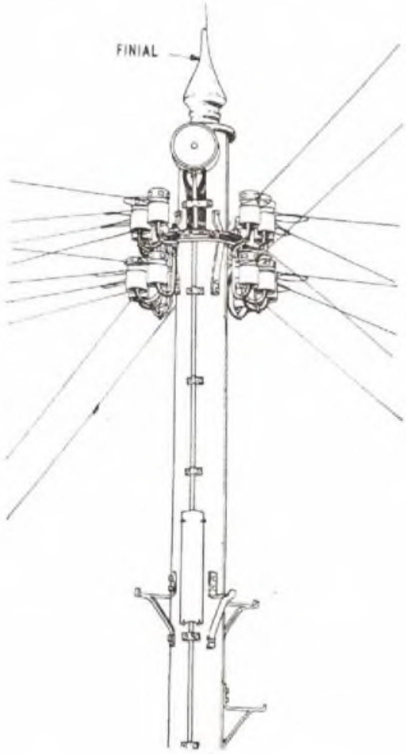
(*a*) Ring-type distribution pole using open wires or insulated wires (drop-wire).

(*b*) Armed pole serving on overhead route of poles with feeders to subscribers' premises.

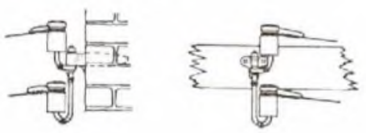
(*c*) Cross-armed pole serving a combination of single span feeders to subscribers' premises and a route, or routes, of feeder poles.

The following is a description of the ring-head method of overhead distribution using open wires. The distribution pole is served by an underground cable terminated on a terminal block fitted to the pole. The individual circuits are connected to the open-wire spans by means of pole leads.

The pole head consists of a channelled ring of steel or aluminium alloy, fixed to the pole by means of brackets. A slot is provided in the upper flange of the steel ring to enable the pole leads to pass into the channel without being chafed; on the alloy pole head, the pole leads are fed to the ring by means of channels in two of the brackets. Holes are provided in the ring to take a maximum of 15 double-J spindles. The alloy ring is slightly larger than the steel ring and a quarter segment of the ring is without spindle holes; this provides a gap which gives easy access to the leads and terminal block. If more than 15 circuits are required to be served from one pole an additional ring may be fitted below the first to provide a maximum of 20 circuits. The method of terminating the wires on the pole is shown in sketch (*a*).



(*a*)



(*b*) (*c*)

Typical terminations at the subscribers' premises are shown in sketch (*b*) and (*c*).

Q. 4. *Describe the principles of a cable-lashing machine and give a brief account of its use in the erection of an aerial cable (assume the suspension wire is already in position).*

A. 4. See A.3, Line Plant Practice I, 1958, *Supplement*, Vol. 52, No. 2, July 1959.

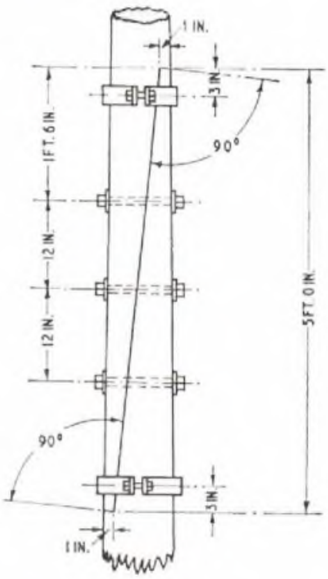
Q. 5. *Describe a typical stayed, spliced, pole mast* 100 *ft in height suitable for supporting a light aerial.*

Sketch the construction of the splice.

How deep would the butt of the mast be buried?

A. 5. The pole would consist of a 34 ft light pole, a 36 ft medium pole, and a 40 ft heavy pole, selected so that their dressed diameters at their top and butt ends allow them to be spliced to form a smoothly tapering mast. The total length of the poles is 110 ft, allowing 5 ft for each splice.

The splice joints are usually shaped at contractors works, and are of the form shown in the sketch. Three arm bolts, carrying spiked



timber-connecting washers at the scarf faces, run through each splice. The "toe" of the scarf is inclined so as to grip the mating pole, and the joint is completed by bolting mild-steel bands (2 in. × ¼ in.) at each end of the splice. Before assembly, the cut surfaces are treated with a tar, tallow and pitch composition.

It is usual to step the pole for its full height. Stays are attached at the top and at the centre of each splice, running radially outwards in three directions with 120° between two adjacent stays. If a head load is carried the direction of one of the sets of stays is arranged to directly oppose it. A stay of 7-strand 14 S.W.G. wire is employed, but heavier wire may be used for the top, or any other loaded stay point. It is usual to attach each set of stays to one stay rod, via stay swivels.

A spliced pole depends upon its stays for stability, and it is usual to bury the butt to a nominal depth of 4 ft 6 in.

Q. 6. *What precautions must be taken in the mixing of concrete? Describe the process of excavating for, and the construction of, a medium size reinforced-concrete surface joint-box.*

A. 6. *Mixing concrete*

When mixing concrete the following precautions must be taken:

(*i*) The quality of the ingredients should be carefully inspected. It is important that all the ingredients—cement, aggregate, sand, water—should be of high quality and special attention should be paid to cleanliness at all stages of the mixing and placing. Cement may be considered good if it feels warm, silky and free from lumps. Aggregate should be clean, screened river ballast, gravel, stone or other approved material of the nature of cubes (not flakes), and well graded in size. It should be free from dirt, floury stone dust, loam and earthy or like materials. Sand should be obtained from river or pits and should be clean, sharp, gritty and free from loam, organic matter and adherent coatings. The water used must be clean and fit for human consumption.

(*ii*) The ingredients should be measured in gauging boxes or by other means, and mixing may be done by machine or manually. The mixing must be thorough. When manual mixing is employed, the platform on which the operation is performed should be large enough to permit the material to be moved completely from one place to another in the course of being turned over.

Before adding the water, the dry material should be repeatedly turned over until the mixture is an even colour throughout. Mixing should then be continued while gradually adding the water, until the correct proportion of water is included. The whole mass should then be turned until thoroughly mixed.

The water should be applied through a rose or sprinkler only to the extent required to obtain a wet mixture with a plasticity that will permit convenient placing and compacting throughout the structure.

(*iii*) Normal Portland-cement concreting should not be undertaken when the temperature is below 40°F. High-alumina cement concrete work can be carried out successfully at temperatures down to 12°F. While mixing, the aggregate and sand is kept at a temperature above the freezing point of water and the mixing water used may with advantage be lukewarm. The concrete, when placed, is protected against frost for at least four hours; the protection may then be dispensed with, since whilst setting, the concrete will generate sufficient heat to prevent the frost damaging the cement.

(*iv*) In warm or windy weather, concrete should be prevented from drying too rapidly by shielding it from the sun and wind. Portland-cement concrete should be kept moist for 7 days (3 days in the case of rapid-hardening Portland cement and 24 hours for high-alumina cement) by frequent sprinkling with a hose or watering can, or by covering the concrete with a saturated sackcloth or wet sand.

(*v*) Concrete should be placed (not thrown) in position as quickly as possible after being mixed and should be lightly tamped and worked to fill all cavities. Concrete when mixed should be used within 30 minutes.

Building a reinforced-concrete surface joint-box

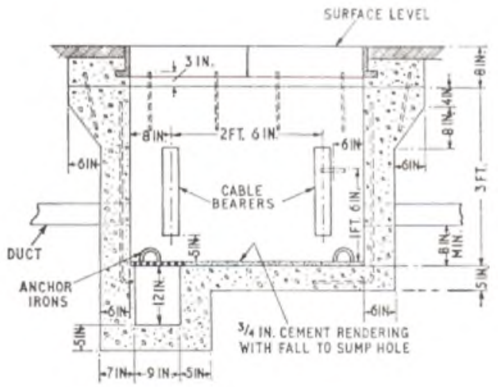
The position for the box having been determined, a pilot hole should be excavated to the limits of the finished joint-box in each of the four directions. Precautions to protect the workmen and public from danger should be taken. Having fixed the final position by examining the pilot hole, excavation of the ground to the full extent required may proceed. The full extent is 12 in. wider and longer than the finished inside dimensions of the joint box and about 6 in. deeper than the finished floor level. Additional excavation will be required for the sump-hole. Excavation work may be done manually or with mechanical aids according to the situation and condition of the soil. In loose, very wet or unstable soil, wooden shuttering may be required to prevent the sides of the excavation from falling in; but in solid ground shuttering will not be necessary.

When the hole has been excavated to the required dimensions the floor should be rammed hard; hardcore may be necessary for this purpose. Laying of the concrete floor (including the sump-hole) may now commence, the concrete being placed, not thrown, into position and well tamped. Reinforcing bars for the floor and the floor-to-wall connexions, and for anchor irons, are laid in their appropriate positions. The floor is allowed to set.

The first part of the wall shuttering is now placed in position and neat cement mortar is laid on the base where the walls are to be built. Concreting of the walls and placing of the reinforcing bars, anchor rings, steps, bearer bolts, duct entries, etc., now proceed stage by stage up to the top level of the joint-box. The concrete is well tamped and consolidated as the work proceeds. The frame for the cover is bedded into the top of the walls at the appropriate level of the roadway or footway.

The concrete is now allowed to set, taking the normal precaution (see first part of answer), and when set the shuttering is removed; the whole of the inside of the joint-box is rendered with a mixture of 3 parts of sand and 1 of cement.

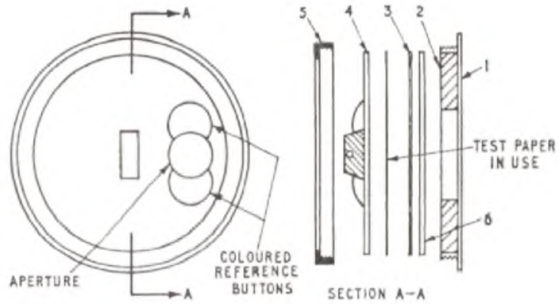
The sketch shows a typical joint-box.



Q. 7. *Describe with the aid of sketches a gas-leak indicator, and explain fully how and when it should be used.*

A. 7. The gas-leak indicator described is designed for detecting very small quantities of coal gas or other gas containing carbon monoxide. It is based on the visible chemical reaction of a palladium-chloride solution with carbon monoxide.

The indicator consists of six parts (numbered in the sketch):



(*i*) A moulded bakelite base (1).

(*ii*) A sorbo-rubber disk (2) forming a cushion for the pressure plate.

(*iii*) A pressure plate of synthetic moulding material (3).

(*iv*) A face plate having two reference buttons, one on either side of an aperture in the plate (4). The face plate and two reference buttons are coloured, the three colours constituting the reference scale used in the test. The face plate also has a handle which enables it to be rotated with respect to the test paper.

(*v*) A metal clamping or bezelring (5).

(*vi*) The test papers which are a grade of chemical filter paper 7 cm in diameter (6).

A phial of palladium-chloride solution, bearing a label giving the date of expiration of useful life, is also provided.

Before the indicator is used the date on the phial of palladium-chloride solution should be checked to ensure that it has not expired. A fresh test paper is placed on top of the pressure plate, the indicator assembled, and a small quantity of palladium-chloride solution applied to the portion of test paper visible through the aperture of the face plate. Immediately after the paper has been moistened, the indicator is lowered into the atmosphere to be tested and remains there for five minutes. If the manhole or joint-box is more than 3 ft 6 in. deep it is lowered to a point midway between roof and floor; if less than 3 ft 6 in. deep the test is taken at all duct mouths connected to duct lines. After exposure the indicator is withdrawn and the colour of the exposed portion of the test paper is compared with the colour of the face plate and reference buttons.

If the colour of the exposed portion of the test paper is:

(*a*) Not darker than the lighter reference button, work can proceed.

(*b*) Intermediate between the two reference buttons, caution is indicated. Half-hourly repeat tests must be made while work proceeds. Work should not be undertaken for periods longer than two hours.

(*c*) As dark or darker than the darker reference button, the atmosphere is dangerous and work must not be undertaken. The test paper should be endorsed by the person taking the test, with his name, location, date and time of test and result of the test.

Tests with the indicator must be made in each of the following circumstances:

(*a*) Before any underground structure is entered.

(*b*) After pumping or baling out an underground structure.

(*c*) Immediately before a flame of any description is brought near an underground structure.

(*d*) Whenever there is any sign of stuffiness and smell of gas whilst work is in progress.

(*e*) Whenever regulations require repeat tests.

Q. 8. *Describe in detail the construction (make-up) of a 50-pair plastic-insulated and sheathed local distribution cable and the process of jointing such a cable to another of the same size and type.*

A. 8. The conductors of a 50-pair plastic-insulated and plastic-sheathed local-distribution cable are of annealed-copper wire of weights $6\frac{1}{2}$ lb, 10 lb and 20 lb/mile. Each wire is insulated with grade two polythene which was originally 0·015 in. thick but is now graded according to conductor size, e.g. $6\frac{1}{2}$ lb is covered with 0·008 in., 10 lb is covered with 0·010 in. and 20 lb with 0·012 in. Each conductor is identified by the colour of its insulation and two conductors are twinned to form a pair. The core is formed of layers of pairs with alternate clockwise and anti-clockwise lays, the centre and each layer except the last being given an open helical lapping of melinex plastic tape. The complete core is lapped with two paper tapes.

A 50-pair cable is made up with 3 centre pairs, 9 first layer pairs, 16 second layer pairs, and 22 third layer pairs. The colour scheme used is as follows:

Centre: First pair, orange/white; second pair, red/grey; third pair, green/black.

Layers: First pair, orange/white; second pair, red/grey; third pair, blue/brown; fourth pair, red/grey; fifth pair, blue/brown; and so on until the last pair of the layer which is green/black. The A-wire is always the colour quoted first for each pair.

The completed core is sheathed with an extrusion of grade two polythene-compound, containing butyl rubber and carbon black, to a thickness of 0·060 in.

The normal method of jointing a 50-pair plastic-insulated and sheathed cable is by means of the "U" or "back" joint, as this method requires only one expanding plug and access to the joint is easier. A lead sleeve of suitable length and diameter is cut from one of standard size, the ends of which should be trued and cleared of any ragged edges of lead using a shavehook and file. A reinforcing collar should be placed on one end and the sleeve bossed internally to make it a tight fit. The portion of the lead sleeve within the reinforcing collar should be burnished with glass paper. A disc of lead should be soldered to the opposite end to that prepared as described above.

The ends of the two cables should be cleaned with a dry rag. They are then passed through the expanding plug for a length of 18 in. The sheath is stripped off for 13 in. and the outer paper lappings removed. Three inches of the sheath at the butt should be roughened with glass paper until all longitudinal scores in the sheath have been removed. When the cores are exposed their free ends are tied to preserve the formation until jointing starts. A water-barrier sleeve is then passed over each cable and is positioned so that the sheath protrudes inside the rubber sleeve approximately ¼ in. from the small end of the tapered gland. The cables are then secured in an upright position with the open end of the sleeves uppermost ready for filling with sealing compound. The conductors should be separated as much as possible to permit the compound to flow round the conductors. A quantity of water-barrier compound, sufficient for filling the sleeves and subsequent topping up, should be heated in a ladle until a temperature of 270°F is reached. It is then poured into the sleeves up to the level of the open end and allowed to cool. During cooling the rubber sleeves should be squeezed slightly to help the wax penetrate into the spaces around the conductors. During cooling, shrinkage takes place and this should be made good by topping up.

The cable cores can now be tied about ½ in. from the water barriers. To facilitate the formation of the "U" when jointing, a temporary support of wire or tape should be used. The overall length of the "U" formed by the pairs should be about 5 in. measured from the water barriers. The wire-jointing sequence will normally follow the colour code. The insulation should be stripped from the wires as close to the base of the twist as possible so that no insulation is trapped in the twist. The normal crank-handle method of twisting the stripped wires is used. The twists are folded down and insulated with polythene sleeves placed over the wires before jointing. The sleeved joints are built up in banks during jointing and on completion, each bank is tied with thread.

The expanding plug is now moved up the cables to the water barriers, and the prepared lead sleeve is slipped over the joint and the expanding plug so that the outer pressure plate is located inside about $\frac{1}{16}$ in. from the sleeve end. The bolts should now be tightened a little, one at a time in rotation, until no slip can be detected as the cables are pulled. A local pressure test of 5–10 lb should be applied to the joint on completion to check the sealing of the expanding plug. The efficiency of the water barriers can be checked by nicking the sheath of each cable during the pressure test and applying soap suds. The nick should be taped over on completion of the test.

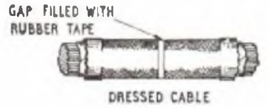
Q. 9. *Explain what is meant by:*

(*a*) *electrolytic corrosion* (*c*) *bonding*
(*b*) *insulating gap* (*d*) *burn-out*

A. 9. (*a*) *Electrolytic corrosion.* Liquids which are capable of passing an electric current are called electrolytes. If two solid conductors are connected to a source of direct e.m.f. and immersed in an electrolyte, a current will flow through the electrolyte from the positive conductor (the anode) to the negative conductor (the cathode). With the passage of current, chemical changes occur in the electrolyte and on the surface of the conductors and this results in the metal of both the anode and the cathode being eaten away. This eating away of the metal is called electrolytic corrosion. The majority of the corrosion occurs at the anode (anodic corrosion). Corrosion at the cathode (cathodic corrosion) is usually due to secondary effects and only takes place in the presence of certain alkalis.

The telephone engineer is concerned with electrolytic corrosion since the sheaths of underground telephone cables very often carry stray currents arising from various causes, e.g. electric tramway and electric railways, and where these stray currents leave the cable sheath and pass into the earth, anodic corrosion occurs. The eating away of the sheath eventually allows water to enter the cable core and so causes a cable fault.

(*b*) *Insulating Gap.* An insulating gap is a device used to prevent or minimize the flow of current in an underground-cable sheath. It consists essentially of a break in the continuity of the sheath. To fit an insulating gap, about 8 in. of the sheath is dressed to be as near as possible of circular cross-section, without appreciably compressing the cable. This length of the cable is then roughened with a rasp. A piece of the sheath either ½ in. or ¾ in. wide, depending on the diameter of the cable, is removed from the middle of the roughened length. This gap in the sheath is built up to the same thickness as the lead sheath with rubber-wax tape. The prepared length of the sheath is then given a sufficient number of tight layers of tape to ensure that a split rubber sleeve which is slipped over the tape has a gap of ¼ in. between the faces of the cut, see sketch (*a*). An aluminium coupling is then placed over the rubber sleeve so that the cut in the sleeve and one of the spaces between the flanges of the coupling are in the relative position as shown in sketch (*b*). Rubber spacers are fitted between the flanges which are then drawn together with nuts and bolts. The various components are illustrated in sketches (*c*) and (*d*).



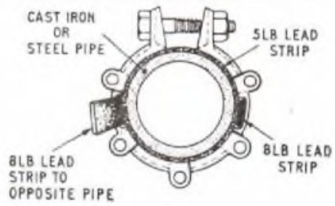
(*a*)

(*b*)

(*c*)

(*d*)

(*c*) *Bonding.* To prevent stray currents, which may be flowing in iron pipes or the armouring of armoured cables, from being forced to travel by the lead sheath of cables, should the continuity of the pipes or armouring be interrupted, it is arranged that a metallic connexion is effected which bridges the break in continuity. Such breaks may occur in iron conduits at a manhole or in armoured cable when this is jointed. The bonding of pipes is effected by the use of bonding clips and lead strip as shown in sketch (*e*). The



(*e*)

ends of the pipes are cleaned and wrapped with 5 lb lead strip 1 in. wide. A bonding clip is loosely fitted over the strip at the end of each pipe, and the clips are connected with a strip of 8 lb lead 1 in. wide fitted in the slots provided. The unused slots are packed with pieces of 8 lb lead strip. All surfaces are liberally coated with petroleum jelly at each stage of the work, and the nuts on the clips are tightened to force the strips of lead together and into contact with the cleaned surface of each pipe.

The continuity of armoured cable should be maintained, at jointing points, by soldering a strip of 8 lb lead, 1 in wide, to the termination of the armouring at each side of the joint.

(*d*) *Burn-out.* The term burn-out refers to the following conditions. Sometimes an underground cable (such as a supply-main or feeder-cable for a d.c. traction system) develops a fault which produces intense heat, sufficient to melt the lead sheath of an adjacent telephone cable. In some cases an electric arc may strike between the faulty power cable and the telephone cable or duct. It often happens that the conductors in the cable are burned and broken, in addition to the melting of the sheath.

Q. 10. *Give an account of the periodic tests which should be made on (a) local lines, (b) junction cables, and (c) trunk cables.*

What results would you expect from these tests when the plant is in good condition?

A. 10. See A.10, Line Plant Practice I, 1956, *Supplement*, Vol. 50, No. 1, p. 4, Apr. 1957.

Q. 1. A parallel plate air-spaced capacitor is charged from a 200-volt battery, which is then removed and the capacitor is lowered into a bath of oil without any leakage of electric charge in the process.

What change occurs in:

(a) the potential difference across the plates
(b) the electric flux density between the plates
(c) the energy stored in the capacitor?

Take the effective area of each of the two plates as 200 cm² and their distance apart as 0·1 cm. The relative permittivity of the oil is 5·0.

The experiment is repeated, but this time the 200-volt battery remains connected. Explain what happens in the battery circuit as the capacitor

(d) enters the oil
(e) is withdrawn from it.

The permittivity of free space in m.k.s. units is $8{\cdot}854 \times 10^{-12}$ F/m.

A. 1. (*a*) The charge, Q coulombs, the voltage, V volts, and the capacitance, C farads, of a capacitor are related by the expression $C = Q/V$. This is a fundamental relationship and applies for any condition in the capacitor.

The capacitance between two parallel plates, each of effective area A m² and d metres apart, when immersed in a dielectric of relative permittivity, ϵ_r, is

$$C = \epsilon_r \times \epsilon_0 \times A/d,$$

where ϵ_0 is the permittivity of free space in m.k.s. units. From this it is clear that the capacitance is proportional to the permittivity of the dielectric.

When the charged plate is lowered into the oil, no loss of charge occurs. There is no change in the dimensions of the plate and so the capacitance will increase in the ratio of the increase of the relative permittivity of oil compared with air.

From the equation $C = Q/V$, since Q is constant,

$$\frac{V_{oil}}{V_{air}} = \frac{C_{air}}{C_{oil}} = \frac{1 \times \epsilon_0 \times A/d}{5 \times \epsilon_0 \times A/d} = \frac{1}{5}.$$

Therefore, $V_{oil} = \dfrac{200}{5} = \underline{40 \text{ volts.}}$

(*b*) The electric flux density is given by the charge per unit area (coulombs per square metre). The flux density is independent of the permittivity of the dielectric. In the given problem, the area of the cross section of the electric field is constant throughout and the charge remains constant; therefore the charge per unit area remains constant. Hence, the flux density is unaltered as the capacitor is lowered into the oil, its value being Q/A coulombs per square metre.

(*c*) The energy stored in a capacitor is given by $\frac{1}{2}CV^2$ joules, where C is in farads and V in volts.

The energy when the dielectric is air divided by the energy when it is oil gives the ratio 5 because,

$$\frac{\text{Energy}_{air}}{\text{Energy}_{oil}} = \frac{\frac{1}{2}C_{air}(200)^2}{\frac{1}{2}C_{oil}(40)^2} = \frac{1 \times (200)^2}{5 \times (40)^2} = 5.$$

Lowering the capacitor into oil therefore reduces the energy stored in it to one fifth. A force will have to be exerted to withdraw the capacitor from the oil and, as the voltage then returns to its original value, 200 volts, the energy will also return to its original value.

(*d*) While the battery remains connected, the voltage across the plates will remain constant. The relationship $C = Q/V$ must still apply as the capacitor is lowered into the oil, but this time V is constant. The capacitance, C, will increase by five times, since the oil has a relative permittivity of 5, and therefore the charge held by the capacitor must increase by five times. This increase in charge means that a current must flow from the battery into the capacitor while the plates are being lowered into the oil. When the plates are fully immersed the capacitance stops increasing, and the current stops flowing. The electrical energy held by the capacitor has increased by five times as a result of the immersion in oil.

(*e*) When the capacitor is withdrawn, current flows back into the battery, because the charge must be reduced to keep the voltage constant as dictated by the constant voltage of the battery. If a secondary cell is used, the current from the capacitor charges the battery slightly.

Q. 2. Explain why a high voltage can be generated in a circuit containing a low voltage battery and an iron-cored inductance. What determines the magnitude of this high voltage?

Is the iron core essential to the generation of a high voltage?

What happens to the energy supplied to the circuit by the battery?

A. 2. Whenever a changing current is flowing in the windings of an inductor an e.m.f., known as the e.m.f. of self-induction, is induced in the turns of the coil. This is an illustration of Faraday's Law of Electromagnetic Induction which states that, when the magnetic flux linked with an inductor is changing, an e.m.f. is induced in that conductor and its magnitude is proportional to the rate of change of flux linkage. The direction of the e.m.f. is, from Lenz's Law, such as to oppose the cause producing it.

The magnitude of the self-induced e.m.f. is proportional to the rate of change of flux linkage. The flux linkage is the product of the number of turns on the coil and the effective flux cutting them. The effective flux is proportional to the number of turns on the coil producing the flux and, also, to the current flowing in the windings. It follows that the flux linkage is proportional to the (number of turns)² times the current flowing.

Hence, the self-induced e.m.f. is proportional to the rate of change of current times the square of the number of turns on the coil.

The flux generated by the coil for a given current can be increased by reducing the magnetic reluctance of the path taken by the flux. If a core of high-permeability magnetic material is inserted inside the coil to provide a low-reluctance path for the flux, the flux will increase for a given value of current so increasing the self-induced e.m.f. in the coil. This simply means that the inductance of the coil is increased by the insertion of the iron core.

The rate of change of flux is also determined by the rate of change of current in the coil. A rapid break would lead to a larger e.m.f. of self-induction than would a slow break.

The factors that will lead to the generation of a high e.m.f. of self-induction in a coil are therefore:

(*a*) There must be a large number of turns on the coil.
(*b*) The reluctance of the magnetic path must be as low as possible.
(*c*) A high rate of change of the current flowing through the inductor is needed.

Factors (*a*) and (*b*) together mean that the coil must have a high self-inductance if a large e.m.f. is to be generated by a rapidly changing current. However, it is the product of the inductance and the rate of change of current that determines the magnitude of the self-induced e.m.f. An iron core is not essential to a high inductance: a very large number of turns on the coil could give the same effect, but such a coil would have a large d.c. resistance.

The energy supplied to create the flux comes from the supply battery. When the circuit is broken, the energy stored in the magnetic field of the inductor is transferred, by the e.m.f. of self-induction, to electrical energy again; as the switch opens, the induced e.m.f. may be high enough to ionize the air between the opening contacts. This conducting path can then momentarily carry a current which heats the air, so setting up a spark which is visible. The electrical energy is dissipated as heat in the resistance of the spark gap and of the coil winding. The energy dissipated may be sufficient to destroy the switch-contact material. A spark-quench circuit is often included across a switch contact so that this energy is dissipated slowly and switch-contact damage is avoided.

Q. 3. Explain briefly the meaning of the terms "reactance," "impedance" and "resonance."

Find the impedance of the series circuit given in Fig. 1 in terms of L, C and R at a frequency of f c/s.

Sketch the reactance/frequency curves for

(a) the inductance
(b) the capacitance

Hence, explain why the reactive component of the circuit may be either positive or negative.

If L = 0·01 henrys, R = 50 ohms and C = 25 pF at what frequency is the reactance zero? What is then the impedance of the circuit?

Fig. 1.

A. 3. The frequency of resonance is $\underline{10^6/\pi \text{ c/s}}$.

At resonance the impedance is purely resistive and is $\underline{50 \text{ ohms}}$.

Q. 4. What is the meaning of the term "root-mean-square" value of an alternating wave?

Write down the equations representing sinusoidal voltages of:

(a) 50 volts r.m.s. and frequency 100 c/s.
(b) the same frequency and amplitude as (a) but leading it in phase by 90°.

Plot these wave forms on the same pair of axes and hence draw the waveform of the current that would flow in a 100-ohm resistor, if the two voltages were applied in series across it.

A. 4. The "root-mean-square" (r.m.s.) value of an alternating wave is the numerical value of a direct current of constant value which gives the same heating effect, in a purely resistive load, as an alternating current having the waveform and frequency of the alternating wave. The r.m.s. value is derived by adding the squares of the ordinates of instantaneous values of the wave and finding their mean value. The squaring process turns negative ordinates into positive because $[(+1) \times (+1)] = 1$ and $[(-1) \times (-1)] = +1$, so that the r.m.s. equivalent is not zero, as is the true average value of a sinusoidal wave. It should be noted that the heating effect in a resistor is independent of the direction of the current flowing in it.

The typical equation for a sinusoidal wave having peak value V and instantaneous value v is

$$v = V \sin \theta$$

If $\theta = 2\pi ft$, where f is the frequency of the alternating wave and t is the time from an arbitrary zero, this becomes the equation to an alternating voltage wave.

As $\sin \theta = 0$ when $\theta = 0$, which will occur when time $t = 0$, this sine wave passes through the origin of co-ordinates. The time for one cycle to pass is $1/f$ seconds.

The equation for a wave leading this by $\alpha°$ is $v = V \sin (2\pi ft + \alpha)$.

When $t = 0$ the value of v is given by $v = V \sin \alpha$, which is not zero. Therefore the wave does not this time go through the origin when $t = 0$.

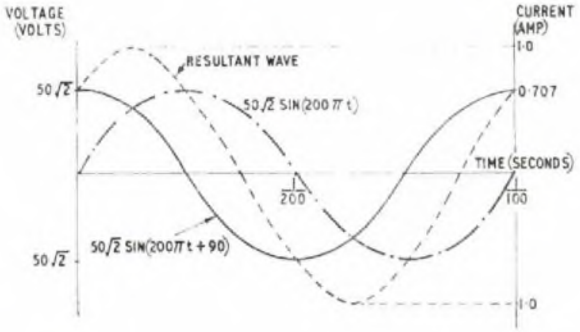
It can readily be shown that the r.m.s. value of a sine wave is $1/\sqrt{2}$ times the maximum, or peak, value.

(*a*) The r.m.s. voltage is 50, so that the peak voltage is $50\sqrt{2}$.

As the frequency is 100 c/s, the equation is

$$v = \underline{50\sqrt{2} \sin 200\, \pi t.}$$

This is plotted in the sketch.



(*b*) To make the above wave lead by 90°, a phase angle of $\alpha = 90°$ must be added to the angle $2\pi ft$.

$$\therefore\ v = \underline{50\sqrt{2} \sin (200\, \pi t + 90°).}$$

This is also plotted in the sketch.

If the two voltages are applied in series their effect will be additive. The resultant can be obtained by simply adding the ordinates of the two waves, as in the sketch. If the load is a pure resistance, the alternating current must be in phase with the voltage across the resistance. Hence, the wave shape of the current must be identical to that of the voltage, albeit drawn to a different scale, which is found by dividing the voltage by the resistance of the load.

The current wave becomes,

$$i = \frac{50\sqrt{2}}{100}\left[\sin 200\, \pi t + \sin (200\, \pi t + 90°)\right] \quad \ldots\ldots\ldots (1)$$

The resultant voltage wave is shown in the sketch. With the ordinate scale divided by 100 it represents the current in the 100-ohm resistor. The peak value of this current is 1 amp, its complete equation being

$$i = 1 \times \sin (200\, \pi t + 45°).$$

This equation can easily be deduced from equation (1) above by using the identity $\sin A + \sin B = 2 \sin \left(\frac{A + B}{2}\right) \cos \left(\frac{A - B}{2}\right)$.

where $A = 200\, \pi t$ and $B = (200\, \pi t + 90)$

Then $\frac{A + B}{2} = (200\, \pi t + 45°)$ and $\frac{A - B}{2} = 45°$ and, because $\cos 45 = 1/\sqrt{2}$, the peak value of the current wave becomes

$$\frac{50\sqrt{2}}{100} \times 2 \times \frac{1}{\sqrt{2}} = 1.$$

Q. 5. *An a.c. supply of adjustable frequency is connected across the circuit AB in Figure 2. At a certain frequency the voltage across the inductance L2 is 10 volts and the current in the inductance L1 is 20 milliamps.*

Draw the vector diagram relating currents and voltages in the circuit for this condition and, hence, determine the supply frequency.

Deduce from the vector diagram the current taken from the supply at this frequency.

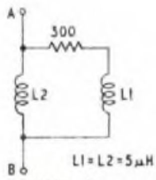
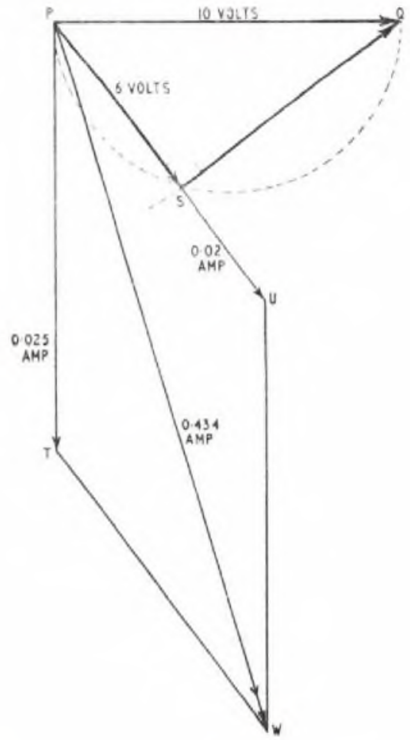


Fig. 2.

A. 5. The circuit of Fig. 2 consists of two inductive arms connected in parallel. At the frequency which has to be determined, the a.c. supply gives 10 volts across both arms. Since the current in L1 is given, 0·02 amp, this is also the current in the 300-ohm resistor. Therefore, the voltage across this resistor is $300 \times 0{\cdot}02 = 6$ volts.

The starting point for the drawing of the vector diagram is the current in the arm containing the resistor (see the sketch).



Let PU represent this current of 0·02 amp to a convenient scale.

Then let PS, in phase with the current PU, represent the voltage drop across 300-ohm resistor. PS = 6 volts drawn to a convenient scale.

The voltage SQ across the inductor L1 must lead the current in L1 by 90°. Draw the direction of SQ at right-angles to PS. The length of SQ, representing the voltage across L1, is given by the reactance of L1 at the unknown frequency times the current in L1. If f is the frequency, voltage across L1

$$= 2\pi f \times 0{\cdot}005 \times 0{\cdot}02 \ldots\ldots\ldots\ldots\ldots\ldots\ldots (1)$$

The resultant voltage across L1 and the 300-ohm resistor is then PQ, the vector sum of PS and SQ. This voltage is given as 10 volts. Therefore, PQ = 10 volts.

As PSQ is a right angle, PQ is the diameter of a semi-circle.

To construct the voltage circle diagram, draw a semi-circle to a suitable scale with diameter to represent 10 volts. Let PQ be the diameter. With centre P, strike an arc of radius 6 volts, to cut the circle at S and join SQ. This construction is shown in the sketch.

By Pythagoras' theorem, since $\angle$PSQ is a right angle.

$$PQ^2 = PS^2 + SQ^2$$

$$\text{or}\quad 10^2 = 6^2 + SQ^2,$$

$$\text{whence } SQ = 8 \text{ volts.}$$

But SQ, representing the voltage drop across the inductor, is given by equation 1.

$$\therefore\ 8 = 2\pi f \times 0{\cdot}0001.$$

$$\text{Therefore, } f = \underline{\frac{40{,}000}{\pi} \text{ c/s.}}$$

The second inductor L2 takes current in parallel with the circuit of L1. Since there is no resistance in the arm L2, the current i_2 in L2 must lag by exactly 90° on the voltage across L2.

This voltage, 10 volts, is PQ in the vector diagram.

Hence PT, at right angles to PQ at P, must represent the current in L2. The current i_2 is given by

$$i_2 = \frac{10}{2\pi f L_2}.$$

But f is now known to be $40{,}000/\pi$ c/s, and $L_2 = 5$ mH.

$$\text{Hence,}\quad i_2 = \frac{10}{2\pi \times \frac{40{,}000}{\pi} \times 5 \times 10^{-3}}$$

$$= 25 \text{ mA.}$$

PT is then drawn to a scale length of 25 mA.

The parallelogram PUWT is completed and the diagonal PW is the vector sum of the two currents represented by PU and PT.

PW, therefore, represents the total current from the supply at the frequency of $40{,}000/\pi$ c/s. From the measurement of the length of PW and with reference to the scale used, the total current = 43·4 mA.

The student should note that the "circle diagram" offers the neatest way of solving many vector problems involving two perpendicular vectors representing series-connected components supplied from a constant-voltage source. In this problem an inductor and resistor are in series, with a 10-volt supply. The circle-diagram type of solution would also apply to a capacitor in series with a resistor with a supply of constant voltage across the circuit.

Q. 6. *Explain the term "permeability" of a magnetic material.*

Plot the B/H curve for a specimen of iron having the characteristics given in the table below.

H amp-turns per metre	200	300	400	600	800	1600	2,400	4,000
B webers per sq. metre	0·4	0·8	1·2	1·6	1·75	2·0	2·1	2·1

A circular toroidal core of mean diameter 5 cm is made of iron rod of 1 cm diameter section. A radial air-gap of 0·1 cm circumferential length is left in the core. Calculate the current needed in a coil of 1,000 turns uniformly spaced around this core to produce a flux density of 1·5 webers/sq. metre in the air-gap.

What is the maximum flux that can be produced in this air-gap when the iron is just saturated? The permeability of free space in m.k.s. units is $4\pi \times 10^{-7}$ H/m.

A. 6. The current required is 1·53 amp.
The maximum flux is 165 microwebers.

Q. 7. *Explain the meaning of the term "sideband" in connection with an amplitude modulated wave.*

Describe a simple way of demodulating such a modulated wave. Illustrate your answer with waveform diagrams.

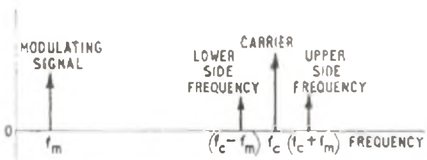
A. 7. When a carrier wave, frequency f_c, is modulated by an audio frequency f_m, the modulated output wave contains three components:

(*a*) The original carrier frequency, f_c.

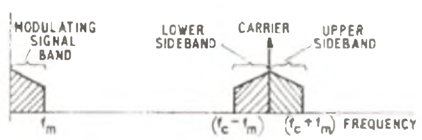
(*b*) The summation frequency $(f_c + f_m)$.

(*c*) The difference frequency $(f_c - f_m)$.

This is illustrated in sketch (*a*) where the frequency scale is the horizontal axis, the vertical axis being purely diagrammatic to represent the presence of a wave. The sum and difference frequencies are known as the side frequencies of the modulated wave. They are actually single frequencies, and there would be no frequency components at all in the gaps above and below the carrier. If the carrier were modulated with a band of speech signals, however, the sum and difference signals would themselves be bands of frequencies of the same width above and below the carrier as the original modulating band of speech. These are known as the sidebands, as shown in sketch (*b*).



(*a*)

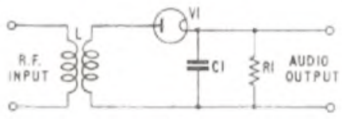


(*b*)

It should be noted that, in the case of a modulating band of frequencies, the lower sideband is "inverted" with respect to the original audio frequency band, i.e. the lowest frequency in the speech band is nearest to the carrier f_c in both the upper and lower sidebands.

To demodulate a modulated wave it is necessary to use a circuit that will respond only to the frequency of the original modulating signal, which is usually at an audio frequency, while eliminating the high-frequency carrier wave.

For this purpose, a rectifier is used, the simplest form being a single diode rectifier circuit as shown in sketch (*c*). The diode valve, V1,



(*c*)

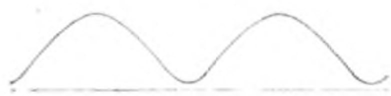
connects one side of the input circuit to a load resistor R1 shunted by a capacitor, C1.

The diode conducts only in one direction, so that it passes current at every positive peak of the carrier f_c. These peaks of unidirectional current charge the capacitor C1, and the charge then drains away continuously through the load R1. The variation of the carrier amplitude due to the modulation f_m is relatively slow compared with the periodic time of f_c, with the result that if the time-constant of the R1,C1 combination is suitably chosen, the modulating frequency appears faithfully reproduced across R1. The voltage across R1, varying now only at the modulating frequency, provides an output that can be carried on to further amplifying stages as required.

The waveform of the modulated input wave is shown in sketch (*d*). Sketch (*e*) shows the peaks of unidirectional current passed by the diode; current can only pass when the instantaneous r.f. voltage across the diode exceeds that across the R1,C1 load, so that this current appears as a series of short bursts which charge C1. Sketch (*f*) shows the final modulating frequency as it appears across R1, where all the carrier component is removed.

(*d*)

(*e*)



(*f*)

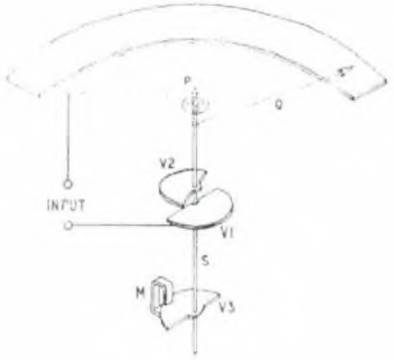
Q. 8. *Describe the principle of operation of an electrostatic voltmeter.*

What factors determine the sensitivity of this type of instrument? Give two applications of this class of instrument.

A. 8. The electrostatic voltmeter uses the principle that the force of attraction between two oppositely charged conducting plates is determined by their potential difference.

The sketch shows the essential features of the instrument, the construction of which resembles that of an air-spaced variable capacitor.

V1 is a stationary metal vane; V2 is a vane made of light metal fixed to the spindle S and free to rotate so as to overlap V1. Rotation of the spindle is controlled by a spiral spring, P, and oscillations of the movement are damped by eddy currents induced in an auxiliary vane V3 operating between the pole pieces of a permanent magnet M. A pointer, Q, attached to the spindle indicates the deflection on a calibrated scale. The fixed and moving vanes are insulated from each other.



A voltage applied between the vanes V1 and V2 sets up charges on them which are equal in magnitude and opposite in sign. The resulting force of attraction between the vanes produces a torque tending to cause the spindle to rotate, which it does until the control torque from the spiral spring P is equated to the deflecting torque; the movement then settles at this angular deflection.

Energy stored in the capacitor formed by the two plates charged to voltage V and having capacitance C is then given by

$$W = \tfrac{1}{2}CV^2$$

The increase in energy from say, W_1 to W_2, produces an increase x in the deflection. If T is the mean torque during this increase, the energy stored in the spring will be Tx so that

$$Tx = W_1 - W_2.$$

Since capacitance C is dependent upon the angular deflection, the change in energy stored $= \frac{1}{2}V^2 \times$ change in capacitance.

The angular deflection is therefore proportional to V^2 so that the instrument has a non-linear scale. This can be overcome to some extent by shaping the vanes to regulate the change of capacitance with angular rotation. If, for example, the change of capacitance per unit deflection could be made proportional to $1/V$ then the scale would be perfectly linear with voltage. In practice, this cannot be achieved at the low end of the scale where V is small because $1/V$ is then very large and, consequently, the scale is always compressed near zero.

The sensitivity of this type of instrument is determined by the change of capacitance per unit deflection. This can be made larger by increasing the effective area of the vanes, since larger vanes carry a greater charge. Increasing the number of vanes is also a convenient way of achieving this effect. Sensitivity is also increased by bringing the vanes closer together and by decreasing the controlling torque exerted by the spring. The vanes will always move so as to store maximum energy.

The instrument is suitable for both a.c. and d.c. voltage measurement because the energy stored is determined by V^2. An instrument calibrated on d.c. will read r.m.s. values of alternating current. The electrostatic voltmeter is chiefly useful for the measurement of large voltages in low-power circuits where the current required to operate an electromagnetic instrument is not obtainable without upsetting the conditions of the circuit being measured.

Typical applications are:

(a) The measurement of e.h.t. voltage in a cathode-ray tube circuit.

(b) The measurement of the alternating grid voltage in a high-power audio amplifier.

Q. 9. *With the aid of sketches explain the operation of single-phase power rectifiers supplying:*

(a) *Half-wave rectification.*

(b) *Full-wave rectification using*

(i) *only two rectifiers*

(ii) *bridge-connected rectifiers.*

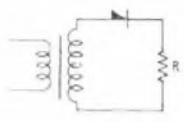
In each case sketch the approximate waveforms of the voltages across a non-reactive load when no smoothing circuit is connected between the rectifier and the load.

Compare the current and peak inverse voltage ratings required for the individual rectifier arms in each of these circuits, for the same power delivered to a given load.

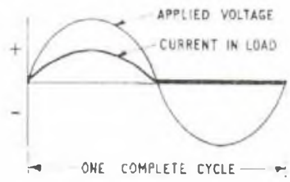
A. 9. An ideal rectifier is a device which will pass current in one direction only. For a practical rectifier in the conducting direction of polarity its resistance is low, while in the non-conducting direction it is very high. Ideally, a single rectifier can be likened to a switch which is "on" in the conducting direction and "off" in the reverse direction. The current rating of a rectifier is usually determined by the temperature rise within the rectifying element, because the forward resistance is not zero and so heat must be generated in the conducting half-cycle. This is particularly so in a metal rectifier. In a thermionic-valve rectifier the limit is the current itself, which cannot exceed the emission that the cathode can provide.

The voltage rating is limited by the physics of the rectifying device, and if the voltage to be rectified exceeds the permissible reverse voltage for one rectifier, then two, or more must be connected in series. The peak reverse voltage across a rectifier is usually the same value as the peak voltage generated across the load during the forward cycle.

(a) Sketch (a) shows a half-wave rectifier circuit, which is the simplest of all rectifier arrangements. The single rectifier and the load are connected in series across the a.c. supply: in one direction the rectifier interposes a negligible resistance, and the current shown in sketch (b) flows for one half-cycle. During the reverse half-cycle no current flows in the load. The result is a pulsating intermittent current, which only flows for half the time. The transformer supplying this circuit will be unable to drive any appreciable current round the rectifier path during the reverse half-cycle, so that all the power is delivered during the positive half-cycle only.



(a)

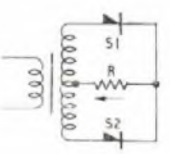


(b)

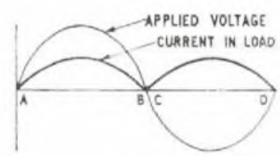
(b) (i) A more efficient arrangement than the half-wave circuit is the full-wave rectifier in which both half-cycles of the a.c. supplied appear as d.c. in the load. Full-wave rectification can be obtained with either two or four rectifiers, as each is capable of being joined in a symmetrical circuit. The reverse voltage to be withstood by a rectifier is, however, less in the 4-rectifier circuit than in the two.

In the circuit of sketch (c), two half-wave rectifiers, S1 and S2 are connected symmetrically, with the load resistor, R, in series with the conducting direction of each of them. This circuit can be considered as two half-wave rectifiers side-by-side. The supply transformer must be able to give double the voltage of that of one half-wave rectifier: a centre-tapped transformer winding will produce the same result as two separate windings.

Current flows in each side of the circuit in turn in this full-wave 2-rectifier system, the load being the only common section of the two paths. Sketch (d) shows the waveforms under ideal conditions. The half-wave AB flows in S1 and the half-wave CD in S2. While one rectifier is conducting, the other experiences reverse voltage.

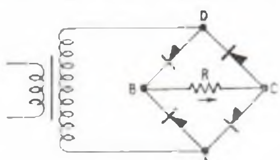


(c)



(d)

(ii) Sketch (e) shows the bridge-connected rectifier circuit, in which four identical rectifiers are so connected that one half-wave flows in the path AB–load–CD while the other half flows DB–load–CA. Both half-waves flow through this resistor R in the same direction. The waveform diagrams have the same shape as sketch (d). One advantage of the bridge circuit is that the peak reverse voltage across any rectifier is lower than in the other types described here, and the supply voltage needed is lower than for the circuit of sketch (c).



(e)

The rating of a rectifier is its current-carrying capacity, which is limited only by the temperature rise of the rectifier, i.e. by the r.m.s. current passing through it. The voltage rating is the reverse voltage which the rectifier can withstand without damage.

The question requires that expressions for the r.m.s. current through a rectifier in the forward direction and the reverse voltage across it be determined for each of the three circuits described, under the condition when each circuit is delivering the same power into a resistance as load. It must be noted that, in a full-wave rectifier circuit, power flows into the load at every half-cycle of the supply alternating current. In a half-wave rectifier circuit, power is only delivered every half-cycle, alternate half-cycles being quiescent. The active half-cycle must therefore supply twice the power in a half-wave rectifier circuit that it does in a full-wave rectifier to obtain equal rates of power delivery into a resistive load. The current in the active half-wave will therefore be greater in a half-wave rectifier than in a full-wave and the peak voltage will be greater.

The relative values can be determined as follows:

Let W be the power delivered to the load resistance R ohms for each complete cycle of alternating supply voltage, assumed to be sinusoidal.

If i is the r.m.s. current in the load R after rectification,

$$W = i^2R, \text{ and so } i = \sqrt{W/R}.$$

This relation applies whatever the type of rectifier circuit, and i is the d.c. current which, flowing continuously, would give W watts in the load R ohms.

(*a*) *Half-wave Rectifier.* Since in the circuit shown in sketch (*a*) all the power is delivered during one half-cycle only, the rate of delivery of power during the conducting half-cycle must be $2W$.

If I_1 is the peak of the sinusoidal current in this half-cycle, the root-mean-square value for half-cycle is $I_1/\sqrt{2}$.

The power delivered into a resistance R by this current for this half-cycle is $(I_1/\sqrt{2})^2R$ and is equal to $2W$ from the above argument.

Therefore, I_1 = peak current $= \sqrt{4W/R} = 2\sqrt{W/R}$.

The peak voltage due to this current is then $R(2\sqrt{W/R})$

$$= 2\sqrt{WR}.$$

Although this has been calculated as the peak voltage in the conducting direction, the alternating supply must reach this peak in both half-cycles, in opposite polarity. The reverse voltage, which is obstructed by the rectifier, therefore has the same peak value.

It follows from the above reasoning that, because the r.m.s. current into the load taken over a whole cycle is $\sqrt{W/R}$ and the peak current in a half-wave rectifier circuit is $2\sqrt{W/R}$, the r.m.s. equivalent of a half-wave rectifier output taken over a whole cycle is half the peak value.

(*b*) In *full-wave rectifier circuits*, both halves of the a.c. input contribute equally to the d.c. output, the rate of power flow per half-cycle is uniform at $W/2$ per half-cycle. In calculations, careful distinction must be drawn between the r.m.s. current in the load R and the r.m.s. current in one rectifier, the latter being the factor that determines the rectifier rating.

(*i*) In the 2-rectifier full-wave circuit, all the peak reverse voltage must appear across one rectifier. Each rectifier in turn must carry the full value of load current, but only for half of each cycle; each rectifier therefore delivers half the total power into the load, i.e. $W/2$.

Therefore, (r.m.s. current in one rectifier)$^2 \times R = W/2$.

Thus, the r.m.s. current per rectifier $= \sqrt{W/2R} = \dfrac{1}{\sqrt{2}}\sqrt{\dfrac{W}{R}}$

This half-wave of current is sinusoidal in shape; therefore its peak value is $\sqrt{2} \times$ r.m.s. value.

$$\therefore \text{ Peak current } = \sqrt{W/R}.$$

This will give a peak voltage of $R\sqrt{W/R}$ across R ohms $= \sqrt{WR}$, which is also the peak reverse voltage across the rectifier.

(*ii*) In the 4-rectifier full-wave circuit, there are two rectifiers in series in each alternate conducting path: the peak voltage is therefore $\sqrt{WR}/2$ for each individual rectifier.

The whole current for each half-cycle will flow through both rectifiers in each series circuit, so that the current rating is the same as in the circuit of *b*(*i*), i.e. $\sqrt{W/2R}$.

(*To be continued*)

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