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Simplifying Assumptions

IT is a common practice to assume that certain elements of a circuit have a negligible effect on its performance and to ignore them in analysing the circuit and explaining its action. It is often essential to do this if the analysis or explanation is not to become prohibitively complex.

We are, however, a little worried about the light-hearted way in which some authors make simplifying assumptions. All too often they take it for granted that their readers will know by instinct what simplifying assumptions have been made and the conditions under which the resulting error is negligibly small. We fear that sometimes this occurs because the author himself does not know—he has perhaps made use of some well-known equivalent circuit to simplify his problem without stopping to ask himself if the circuit does, in fact, adequately represent matters in his particular case.

More often, however, it arises because the author assumes too much knowledge on the part of his readers. There is a common tendency for authors to forget that readers know less about their subject than they do themselves.

It is not suggested, of course, that it is necessary for an author to mention in detail every simplifying assumption which he makes. Some are so common and so elementary that, except when writing for the beginner, an author can assume his readers to be familiar with them. When they are less common, however, or are widely used only in a particular branch of radio, he must certainly draw attention to them, and he must always deal with them very fully when his simplifying assumptions introduce more than a negligible error.

We have been led to these remarks by a case which recently came to our notice in which a simplifying assumption affects results to quite a small extent in one respect but to a very large extent in another. As far as the wanted output is concerned the real and the ideal circuits behave nearly alike but the waveforms in a branch of the circuit are quite different.

In setting up and adjusting the circuit it is actually very convenient to observe the waveform in this branch and the discrepancy can, therefore, be very misleading. It was through doing this that we came to observe it.

The effect occurs in a circuit which is in common use in the line-scanning circuits of present-day television receivers and it forms a good example of the need for authors to explain their assumptions. It is also interesting in its own right and as we have never seen it previously described we propose to treat it in some detail.

In an electromagnetic-deflection circuit the inductance, stray capacitance and resistance form a damped oscillatory circuit. During fly-back the valves associated with it are cut off and the circuit is allowed to execute very slightly more than one-half cycle of damped oscillation, so that the current changes from an initial value of, say, i_0 in the deflector coil to a new value, say, $-i_1$ at the end of fly-back. The magnitude of i_1 is always less than that of i_0, but in low-loss circuits it can approach it.

The withdrawal of overtime working by a section of the printing industry made it impossible to produce the August issue of *Wireless Engineer*. In order to maintain continuity, this issue is dated August-September. A slight reduction in the number of pages and some delay in publication are still unavoidable. All journals printed in London are similarly affected, to a greater or lesser extent, but journals printed in the provinces are unaffected.

The voltage across the circuit rises to a maximum when the current is changing most rapidly and takes a value v_1, say, when the current is $-i_1$. The voltage is then of opposite polarity to that prevailing through the major part of the fly-back period.

If such a circuit is left to itself it will go on oscillating until all the energy initially stored in it has been dissipated. To prevent this a biased diode is connected across it as in Fig. 1. At the end of fly-back when the voltage v rises to v_1 and equals E_D, it is of the same polarity as E_D and the diode becomes conductive. Ideally it then acts as a switch to connect E_D across the LCR circuit. Since v is then maintained constant by E_D the subsequent decay of current in L is linear and can form the initial part of the next scanning cycle. This reduces the peak current which the driving valve (not shown in Fig. 1) need provide. In addition, E_D can be replaced by an auto-bias circuit and the energy passed to it by the linear decay of current in L can be employed usefully in the second part of the scanning cycle.

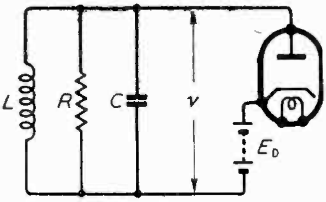

Fig. 1.

It is clear that the diode current should have the waveform sketched in Fig. 2 (a). It is not, of course, exactly like this because the series resistance of L and the internal resistance of the diode have been neglected. These round off the initial rise of current and make the decay exponential but the effects can be mitigated very largely by making E_D a time-variable, instead of a fixed, voltage. The current waveform through the diode should thus be quite close to that of Fig. 1 (a). Instead, it is observed in practice to have the form sketched in (b). This is so very different that it is disconcerting to find it.

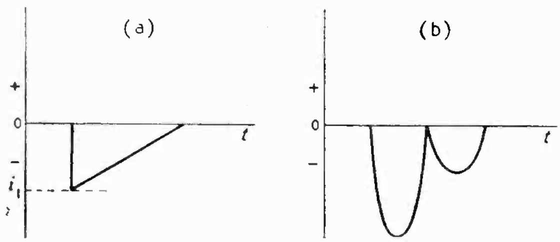

Fig. 2.

In practice, of course, a transformer is nearly always used to couple the driving valve to the deflector coil. It is invariably assumed in published explanations of the circuit that this does not affect matters. This is not true, however, for there is inevitably a leakage field in a transformer and the current produced by the release of the energy stored in this leakage field cannot be controlled by a diode connected to one winding. This is obvious when one thinks about it because the leakage field is merely that part of the field produced by one winding which does not link with the other.

The reason for the waveform of Fig. 2(b) is easily seen by considering the 'equivalent' circuit of Fig. 3. Here L_s represents the secondary inductance of the transformer and L_l the leakage inductance referred to the secondary while L_L is the inductance of the deflector coil and C_p' and C_s are the primary and secondary circuit capacitances, C_p' being the effective value for a unity-ratio transformer. This will be recognized as a well-known equivalent circuit of a transformer.

When the diode conducts the currents have

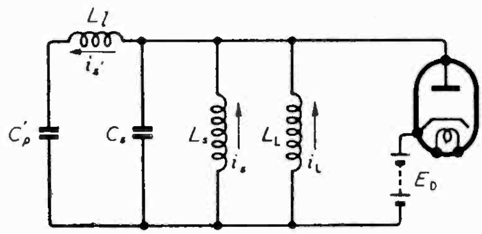

Fig. 3.

the directions shown and initially $i_s' = i_s + i_L$ so that the initial diode current is zero. If the diode holds the voltage across L_L constant at E_D, the current $i_s + i_L$ decays linearly and produces the triangular waveform of diode current of Fig. 1(a). The current i_s', however, flows initially in opposition through the diode and it flows in the oscillatory circuit L_lC_p' which is completed through the diode. The current decays in oscillatory fashion and the total diode current is its sum with the triangular current. The peaks and troughs of the total current are produced by the periodic reversal of the oscillatory current. If this is lightly damped it may exceed the triangular current after a few cycles and, as the diode cannot pass a reverse current there will then be gaps between successive 'half sine waves' of total current. If the diode is connected to the transformer primary a similar effect occurs involving L_l and C_s.

If the diode has zero resistance when conductive the decay of current in L_L is unaffected because C_lL_p' is a separate circuit. Thus the simple explanation is adequate so far as conditions in the deflector coil are concerned but very misleading about the diode current. In practice, the diode has resistance which acts to couple the two circuits together. The oscillatory current in $L_l C_p'$ does then modify the decay of current in L_L. It produces an oscillatory component in it which is sometimes large enough to produce a visibly distorted scan.

The effect is, therefore, not always negligible even for the useful scan so that it is hardly justifiable completely to ignore it even when only this is considered. It is definitely misleading to ignore it in a general treatment because one of the first and most obvious things to do when checking such a circuit is to check the diode-current waveform.

It may be noted in passing that while the diode circuit can be used to recover energy from L_L and L_S it cannot do so from L_l. The energy stored in the leakage field of the transformer cannot be recovered in this way and it can amount to as much as 30% of the total energy in the circuit.

All this affords a good example of the need for care in making simplifying assumptions. It is very tempting to ignore the leakage inductance of a transformer but it is unwise to do so.

W. T. C.

TUNED ABSORPTION CIRCUITS

Analysis and Characteristics

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SUMMARY.—This paper is concerned with the arrangement of coupled circuits in which input and output terminals are connected to the same tuned circuit and a coupled circuit absorbs power over a comparatively narrow band of frequencies.

It shows, with approximate formulæ, the various response curves obtained when the secondary is tuned to give a minimum response to a signal at a frequency f_{min}, and the primary to give a maximum response to a signal at f_{max}. It describes how the shape is affected by the difference between these two frequencies, and shows that if this quantity is less than a certain value (5 per cent of the mean frequency in a practical case) the greater peak will not occur at a frequency anywhere near f_{max}.

The depth and shape of the trough are also discussed.

Introduction

MUCH has already been written on the subject of the conventional band-pass circuit, in which a signal is fed to a resonant circuit, and an output is taken from a second resonant circuit which is coupled fairly weakly to the first. The subject has been analysed quite fully and, in particular, Terman in his "Radio Engineer's Handbook" (1st edition, 1943, pp. 154–172) gives a very full treatment, including a consideration of those cases in which the pass-band is an appreciable fraction of the mean frequency and those in which the degrees of damping in the two circuits are unequal.

It is not intended to deal with this subject in this present article but, in order to emphasize the differences between the sucker circuit and the conventional arrangement, it may be useful to re-state a few of the general results as follows:—

(*a*) The conventional circuit is normally used where a fairly flat-topped response curve, in conjunction with rapid attenuation outside the pass-band, is needed. The coupling coefficient, therefore, is usually adjusted to a value between a third and three times the critical value.

(*b*) The response curve is double-humped if the coupling exceeds the critical value.

(*c*) If the pass-band is a fairly small fraction of the mean frequency and there are no stray reactions round the amplifier valves, the response curve is symmetrical provided that *either* the two resonant circuits are tuned to the same frequency *or* the two circuits have the same magnification factor. An asymmetrical curve can only result if the circuits differ in both respects.

(*d*) The circuit is difficult to tune by simple methods if the coupling exceeds about 0.7 of the critical value, because of the interaction of the two resonant circuits.

(*e*) The response curve cannot show a sudden trough of attenuation over a frequency band which is narrow compared with the pass-band.

(*f*) In the region of the skirts the response curve is twice as steep as that of a single circuit, and the phase-change of sidebands relative to a central carrier is 180 degrees. The phase changes rapidly at the limits of the pass-band with the

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result that these circuits can only be used with caution in television applications.

(*g*) All the symmetrical cases can be considered, mathematically, in terms of quadratic expressions and can therefore be accurately examined with complete generality.

curve which is flat in the pass-band, double humps develop if the coupling coefficient exceeds half the critical value. When it is used for absorption, the depth of the attenuating trough varies with the coupling coefficient, but the effect on the shape of the curve depends to a major

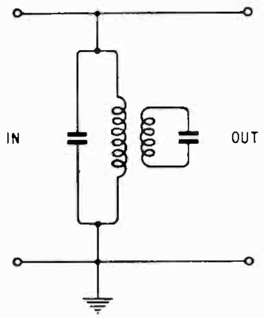


Fig. 1. *Mutual-inductance coupling.*

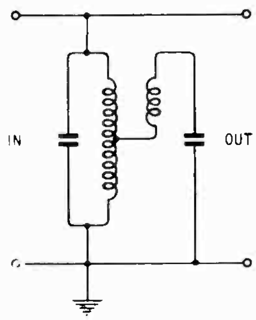


Fig. 2. *Tapped-primary circuit.*

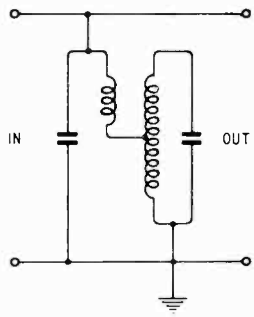


Fig. 3. *Tapped-secondary circuit.*

extent on the relationships between magnification factors, coupling, and detuning, as will be described later.

Absorption Circuit

This name is given to any arrangement in which the input and output connections are taken to the same resonant circuit, and a second circuit is coupled to the first so as to absorb power at some specific frequency. This second circuit may be used to broaden and flatten the response curve of the primary circuit or to make a deep trough at, say, the frequency of an unwanted signal without interfering unduly with the transmission of a wanted signal.

The most common circuit arrangements are shown in Figs 1–3, in which coupling is achieved by means of mutual or common inductance. Capacitance-coupling can, of course, be used, but it is less desirable owing to the fact that an extra component is necessary. In addition, the value of the capacitor, which is required in the case of 'top coupling,' may be inconveniently low. The behaviour of the capacitance-coupled circuit does not differ significantly from that of the circuit with inductance coupling, provided that the pass-band is comparatively narrow.

In order to contrast the chief features of the absorption or sucker circuit with those of the normal type, they may be summarized, in general and qualitative terms, as follows :—

(*a*) The sucker circuit can be used to give a fairly flat-topped response curve without a particularly steep slope outside the pass-band. In this case the coupling coefficient is usually rather less than the critical value. Alternatively, the circuit may be used to attenuate a particular narrow band of frequencies, and for this application a considerably higher value of the coupling coefficient is required.

(*b*) When the circuit is used to give a response

(*c*) If the two circuits are tuned to the same frequency, the curve will be symmetrical for reasonably narrow pass-bands, whatever the values of the magnification factors may be. Equal magnification factors do not, however, give a symmetrical response curve under any other conditions.

(*d*) In general, the sucker circuit is very much easier to adjust than the normal band-pass circuit because, in the matter of tuning, the resonant circuits are more independent of one another.

(*e*) If the magnification factor of the secondary circuit is considerably higher than that of the primary circuit, the trough will be narrow in comparison with the pass-band.

(*f*) In the region of the skirts, the same value of attenuation is given by the whole circuit as by the primary circuit alone, so that the corresponding phase-change is only 90 degrees. The phase also changes fairly rapidly with change of frequency near the trough, but as there are presumably no wanted frequency components of importance very close to the trough, the circuit is still suitable for television use.

(*g*) Except in the case in which both the resonant circuits are tuned to the same frequency, the equivalent mathematical expressions develop to at least a cubic form and they cannot therefore be interpreted in completely general terms. For this reason the main part of the text which follows is concerned with approximations and graphical illustrations only.

Analysis

The circuit is most conveniently analysed from the starting point of Fig 4, which indicates the reactive components L_1 and C_1 and the loss resistance r_1 of the primary and L_2, C_2, r_2 of the secondary circuits. The following discussion is, however, more conveniently undertaken in terms of X_1 (the reactance of L_1 or C_1 at their present resonant frequency $\omega_1/2\pi$), and similarly for X_2, and the dimensionless expressions

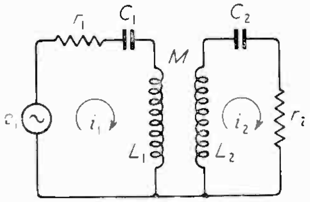


Fig. 4. *Coupled circuits for analysis.*

$\beta_1 = r_1/X_1$ (the reciprocal of the magnification factor)

and $\psi_1 = \dfrac{\omega}{\omega_1} - \dfrac{\omega_1}{\omega}$ (the 'detuning' factor)

so that the total series impedance z_1 of the primary circuit

$$z_1 = r_1 + j\omega L_1 + 1/j\omega C_1$$

can be written

$$X_1\left(\frac{r_1}{X_1} + j\frac{\omega L_1}{\omega_1 L_1} - j\frac{\omega_1 C_1}{\omega C_1}\right)$$

$$= X_1(\beta_1 + j\psi_1) \quad .. \quad .. \quad .. \quad (1)$$

Identical expressions with the suffix 2 in place of 1, will be used to signify the same quantities

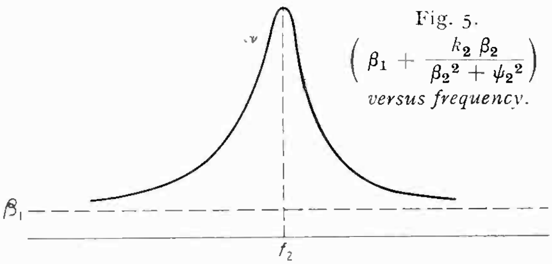


Fig. 5. $\left(\beta_1 + \dfrac{k_2\,\beta_2}{\beta_2^2 + \psi_2^2}\right)$ *versus frequency.*

in the secondary circuit, leading to the corresponding expression

$$z_2 = X_2(\beta_2 + j\psi_2) \quad .. \quad .. \quad .. \quad (2)$$

Internal Impedance

Now, analysing the two circuits when they are coupled together, in the manner of Fig 4, with a mutual reactance $\omega M \approx k\sqrt{X_1 X_2}$ we can write, for the first mesh,

$$e_1 = i_1 X_1(\beta_1 + j\psi_1) + i_2 jk\sqrt{(X_1 X_2)} \quad .. \quad (3)$$

and for the second mesh

$$0 = i_1 jk\sqrt{(X_1 X_2)} + i_2 X_2(\beta_2 + j\psi_2) \quad .. \quad (4)$$

Eliminating i_2, we obtain an expression for the internal impedance, viz.:

$$\frac{i_1}{e_1} = X_1\left\{\beta_1 + j\psi_1 + \frac{k^2}{\beta_2 + j\psi_2}\right\} \quad .. \quad (5)$$

External Conductance

In practice, of course, the circuit is not driven by a low-impedance generator in series with L_1 and C_1, but by a high-impedance (approximately constant-current) source in parallel with C_1. In this case the input current I_1 which is needed to produce the same internal currents is given very closely by

$$I_1 = je_1/X_1 \quad .. \quad .. \quad .. \quad .. \quad (6)$$

and the output voltage E_1, taken across C_1, by

$$E_1 = ji_1 X_1 \quad .. \quad .. \quad .. \quad .. \quad (7)$$

so long as ω is in the region of ω_1, and the magnification factor is high.

Hence $\dfrac{X_1 I_1}{E_1} = \dfrac{e_1}{i_1 X_1} \quad .. \quad .. \quad .. \quad (8)$

and the ratio of the conductance of the whole circuit I_1/E_1 to that of the individual primary reactance $1/X_1$ can be expressed, using equation (5), as

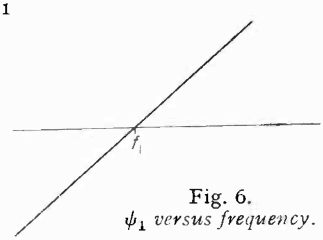


Fig. 6. ψ_1 *versus frequency.*

$$\beta_1 + j\psi_1 + \frac{k^2}{\beta_2 + j\psi_2} \quad .. \quad .. \quad .. \quad (9)$$

This dimensionless expression provides the most suitable means by which the circuit may be examined in general terms. When the circuit is fed as described from a high-impedance (constant-current) source, the response curve is given by the reciprocal of the modulus of expression (9).

To obtain this modulus, the expression (9) must be rewritten in the form

$$\beta_1 + j\psi_1 + \frac{k^2(\beta_2 - j\psi_2)}{\beta_2^2 + \psi_2^2}$$

$$\text{or } \left\{\beta_1 + \frac{k^2\beta_2}{\beta_2^2 + \psi_2^2}\right\} + j\left\{\psi_1 - \frac{k^2\psi_2}{\beta_2^2 + \psi_2^2}\right\} \quad .. \quad (10)$$

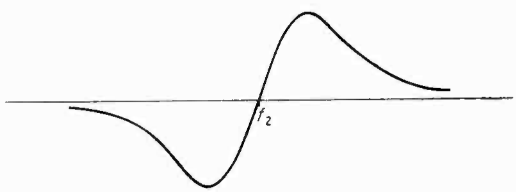
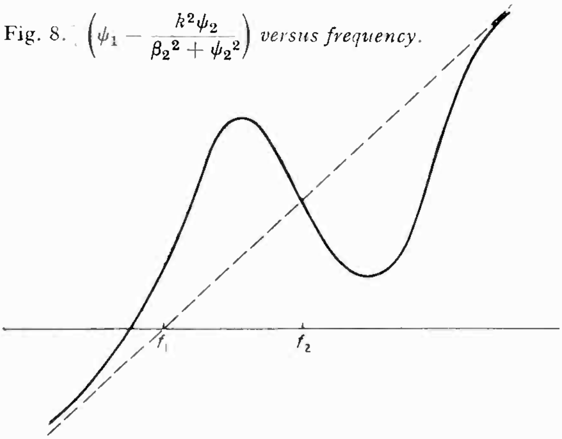


Fig. 7. $\left(\dfrac{k^2\psi_2}{\beta_2^2 + \psi_2^2}\right)$ *versus frequency.*

and the square of the modulus is obtained by squaring and adding the two expressions in the brackets.

Figs 5–8 show the general behaviour of the separate components of expression (10) in the region of f_1 and f_2, the tuned frequencies of the two circuits.

Fig. 8. $\left(\psi_1 - \dfrac{k^2\psi_2}{\beta_2{}^2 + \psi_2{}^2}\right)$ *versus frequency.*



β_1 is constant. $\dfrac{k^2\beta_2}{\beta_2{}^2 + \psi_2{}^2}$ rises to a maximum value of k^2/β_2 when $\psi_2 = 0$ [i.e., at the frequency f_2 $(\omega_2/2\pi)$] and falls to low values at frequencies which are remote from f_2. The behaviour of the sum of the terms in the real bracket of expression (10) is thus represented by Fig 5.

If F represents the mean frequency, then, in the region under consideration, ψ_1 is approximately equal to $2(f - f_1)/F$ and can be represented by a straight line with a slope of $2/F$ crossing the axis at the frequency $f_1 = \omega_1/2\pi$. This is shown in Fig 6.

The value of the term $\dfrac{k^2\psi_2}{\beta_1{}^2 + \psi_2{}^2}$ can be represented by a curve which crosses the axis at the frequency f_2 with a slope of $\dfrac{k^2}{\beta_2{}^2} \cdot \dfrac{2}{F}$. This curve passes through extreme values, $\pm k^2/2\beta_2$, when $\psi_2 = \pm \beta_2$ and returns asymptotically to the axis as shown in Fig 7. The difference between these terms, viz. : $\psi_1 - \dfrac{k^2\psi_2}{\beta_2{}^2 + \psi_2{}^2}$ is represented in Fig 8.

By squaring and adding the ordinates of the curves in Figs 5 and 8, a curve is obtained which represents the square of the attenuation of the circuit under consideration. This curve is shown in Fig 9.

Tuning Effects

At any frequency f, the effect of tuning the primary circuit alone can be examined by varying ψ_1 alone in the mathematical expressions. Similarly, the result of adjusting the secondary can be examined by varying ψ_2 alone. Since $(\psi_1 - \psi_2)$ is approximately equal to

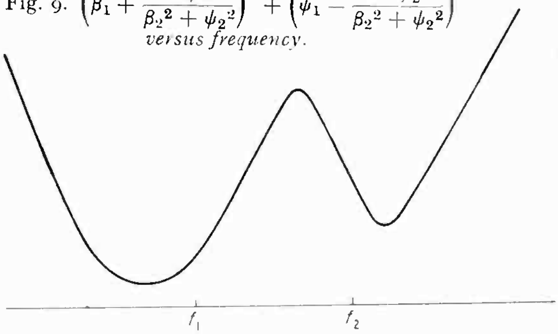
$$2 \cdot \frac{f - f_1}{F} - 2 \cdot \frac{f - f_2}{F} = \frac{2(f_2 - f_1)}{F},$$

the shape of the response curve can be calculated by varying ψ_1 and ψ_2 together in such a way as to keep $(\psi_1 - \psi_2)$ constant.

If we tune the secondary to give minimum primary response at a certain frequency f_{min}, the equivalent mathematical operation involves finding the value of ψ_2 which gives the maximum modulus in expression (10). Now the real component of this expression has a maximum at $\psi_2 = 0$, and the imaginary component a maximum at $\psi_2 \approx -\beta_2$. The greatest modulus will occur at a value of ψ_2 between these limits. If β_2 is small, as is usual when the circuit is used to eliminate unwanted signals, the required tuning point of the secondary circuit is very close to the frequency f_{min}, and it depends only to a small degree on the setting of the primary frequency f_1. The complete expression for ψ_2 at maximum attenuation is a cubic, which is not directly intelligible.

The consideration of the frequency to which the primary circuit must be tuned in order to give maximum response at a certain frequency f_{max} is much easier. ψ_1 does not appear in the real part of expression (10) so that the modulus has a minimum value when the imaginary part vanishes.

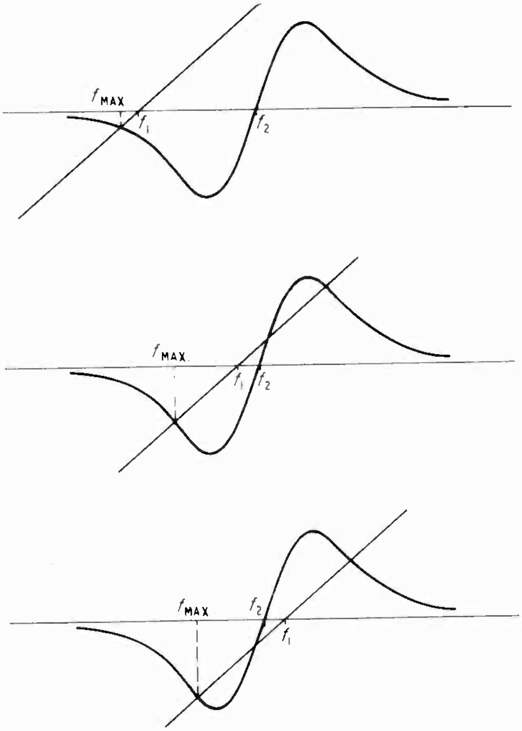
Fig. 9. $\left(\beta_1 + \dfrac{k^2\beta^2}{\beta_2{}^2 + \psi_2{}^2}\right)^2 + \left(\psi_1 - \dfrac{k^2\psi_2}{\beta_2{}^2 + \psi_2{}^2}\right)^2$ *versus frequency.*



The effect in this case is best studied by superimposing the curve of Fig 6 on that of Fig 7, as shown in Figs 10–12. The position, relative to the curve of the line representing ψ_1 (which cuts the axis at a frequency f_1) varies with the frequency to which the primary circuit is tuned. The points at which the line intersects the curve correspond to the values of the frequency for which the imaginary part of expression (10) vanishes.

Shape of Response Curve

Three cases which differ qualitatively can occur. In the first case, the line ψ_1 only intersects the other curve at one point, and this represents the frequency f_{max}. In the other two cases there are three intersections.



Figs. 10-12. *Superimposition of curves of Figs.* 6 *and* 7. Fig. 10 (top) *one intersection point only;* Fig. 11 (centre) f_1 *between* f_{max} *and* f_2; *and* Fig. 12 (bottom) f_1 *on side of* f_2 *remote from* f_{max}.

In the first case there will be little difference between f_1 and f_{max}, which means that the peak will be close to the resonant frequency of the primary. Although the reactive component of expression (10) does not fall to zero at any other point, it does dip temporarily where the curves approach one another, but the frequency at which this occurs is so near to f_2 that the real part has quite a high value and the attenuation is still substantial. The presence, therefore, of the secondary circuit produces a trough, which is flanked by a slight peak, on the side of the response curve. This is illustrated in Fig 13.

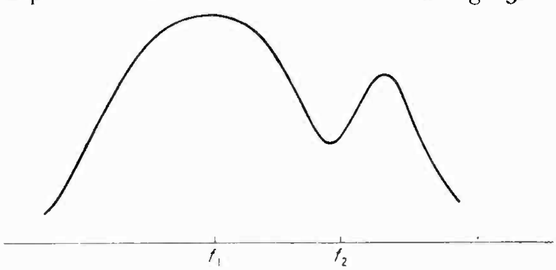


Fig. 13. *Response curve corresponding to Fig.* 10.

The first case merges into the second approximately when the frequency f_1 is such that the line ψ_1 passes through the peak of the other curve. The peak ordinate of the curve has a value $k^2/2\beta_2$ and therefore ψ_1 must be equal to $k^2/2\beta_2$ at this frequency. The transition thus occurs approximately when

$$f_{max} \sim f_{min} = \frac{F}{2} \cdot \frac{k^2}{2\beta_2} \quad \ldots \quad \ldots \quad (11)$$

Taking a particular case, if $k = 0.1, \beta = 0.01 = k^2$ and $F = 60$ Mc/s, the transition will occur when the separation of f_{max} and f_{min} is 15 Mc/s.

In the second case the line ψ_1 cuts the other curve in three points, and it cuts the axis on the same side of f_2 as f_{max}. Here the imaginary part of expression (10) falls to zero at two other points besides f_{max}, but at one (the nearest to f_2) the real component is large and so there is not a third hump on the response curve. At the third point the real component is not as large, but it is greater than at f_{max}, so that the hump at this third point will not be as high as that at f_{max}. This is illustrated in Fig 14.

In the third case, the line ψ_1 again cuts the other curve in three points, but it cuts the axis on the other side of f_2. The response curve is now the mirror image of that which arose in the second case, the small peak occurring at f_{max} and the larger peak on the far side of the trough.

These two cases merge when there is symmetry and $f_1 = f_2$. Here, if ψ denotes the common value of ψ_1 and ψ_2, we find that

$$\psi^2 = k^2 - {\beta_2}^2$$

and that, if k is at all large compared with β_2,

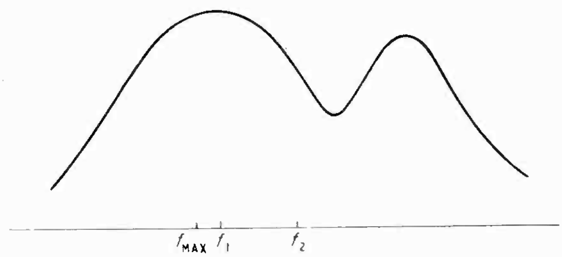


Fig. 14. *Response curve corresponding to Fig.* 11.

ψ is closely equal to k, and

$$f_{max} \sim f_{min} = \frac{kF}{2}$$

In a particular case, if $k = 0.1$, and $F = 60$ Mc/s, the transition will occur when the spacing between f_{max} and f_{min} is 3 Mc/s.

If, therefore, an attempt were made to tune the primary for maximum response at a frequency nearer to f_{min} than that determined by the above expression (within 3 Mc/s in the particular case quoted), it would only result in a response curve with a small peak near f_{max}, the larger peak being spaced well away on the far side of the trough.

Depth and Shape of Trough

The response at the peak is controlled mainly by β_1. The response at the trough is controlled by k^2/β_2 and the relative depth of the trough is therefore approximately $k^2/\beta_1\beta_2$. This, however, is only true up to a point because, as soon as k^2/β_2 becomes comparable with unity, the phase angle of the loaded inductance approaches 45 degrees, and the transference of further damping has little effect. The maximum value that can be obtained for the relative depth of the trough is therefore $1/\beta_1$ (which is equal to Q_1).

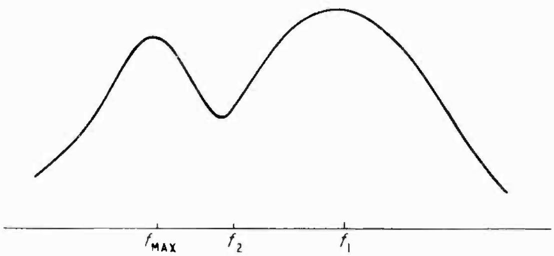


Fig. 15. *Response curve corresponding to Fig. 12.*

The width of the trough at its greatest depth is determined by β_2 only (provided that the limiting effect of the last paragraph is not reached), and hence the width of the band of frequencies attenuated is $F\beta_2$. The total width over which the trough affects the shape of the curve is determined by equality between the terms β_1 and $k^2\beta_2/\psi_2{}^2$, and hence the separation between the shoulders of the trough can be shown to be $Fk\sqrt{(\beta_2/\beta_1)}$ or alternatively

$$F\beta_2\left(\frac{k^2}{\beta_1\beta_2}\right)^{\frac{1}{2}}$$

This latter form of the expression shows that the ratio between the widths of the trough at the bottom and at the shoulders is equal to the square root of the attenuation obtained.

Using the concrete figures assumed in previous examples, the attenuation ratio is 10 to 1 (20 db). The width of the trough is 0.6 Mc/s at the bottom, and nearly 2.0 Mc/s at the shoulders.

The width at the shoulders increases with $\sqrt{k^2/\beta_1\beta_2}$, even when the attenuation is not obtained because of the 'saturation effect' mentioned. As this extra width is usually harmful, it is best in practice to limit k^2/β_2 to about 0.5 in order to get the maximum attenuation that can be obtained without giving rise to wasteful broadening at the top of the trough.

SPIRAL TIME BASE

Magnetic and Electric Deflection

This note describes some work carried out by the Radar Research and Development Establishment in 1939 and described, together with other features of a particular radar set, in British Patent No. 582,419.

THE basic circuit for a discontinuous spiral time base for a cathode-ray tube is shown in idealized form in Fig. 1. The deflector coils, each of inductance L, are used also as the inductances of the tuned circuit which gives electrical quadrature automatically and can easily be adjusted to give mechanical quadrature.[1]

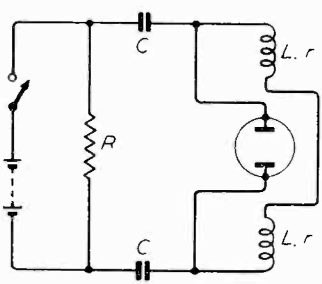


Fig. 1. *Basic circuit of spiral time base.*

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The values of L and C are chosen to resonate at the required frequency of rotation of the time base and the series resistance r of the coils is kept small. The resistance R is made equal to $2L/C$, the value giving critical damping of the circuit.

When the switch is closed there is an extremely short transient period during which stray capacitances are charged, but not the main capacitance of the oscillatory circuit. After this there is a free damped oscillation in the tuned circuit and it has a period which is determined solely by L and C. Neglecting the effect of the series resistance r of the deflector coils, which is small, the current in the coils is 90° out of phase with the voltage across them. As shown in Fig. 1 the magnetic deflection is at 90° to the electric and so it is only necessary to make the amplitudes of the two equal in order to obtain a truly "circular" rotation of decreasing amplitude. With the connections shown the waveform of Fig. 2 (a) is drawn on the screen of the c.r. tube without having to adopt any special biasing arrangements.

It is tuned to $\frac{1}{8}$th of the frequency of LC and controls the number of turns in the spiral trace.

The basic circuit employed for radar purposes is shown in Fig. 4 and various waveforms in the circuit are illustrated in Fig. 5. A multivibrator, comprising V_1 and V_2, feeds a pulse of standard duration to V_3 and is locked by a rectified pulse from the radar transmitter. This locking pulse is shown in Fig. 5(a) and the standard pulse in the output of V_3 in (b). The valves V_6 and V_7 form a cathode-follower so that this waveform can appear at low impedance at point A in Fig. 4. The impedance is only $1/2g_m$ where g_m is the mutual conductance of each of the two valves. The square wave appearing across

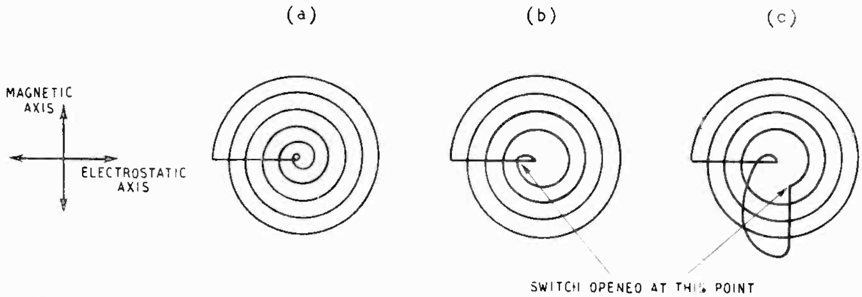
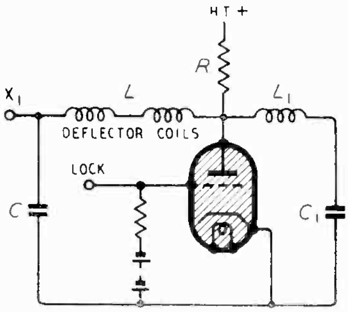


Fig. 2 (left). *Forms of trace given by the circuit of Fig. 1: (b) and (c) illustrate the effect of opening the switch at different points of the cycle.*

Fig. 3 (below). *Simple equivalent of Fig. 1 using a thyratron as a switch.*



Now let the switch be opened after the circuit has made four oscillations. At this instant there is energy stored in the electric fields of the capacitors and the magnetic fields of the coils. The relative amounts of each depends on the part of the cycle at which the switch is opened. However, whatever the prevailing conditions, the critical damping imposed by R ensures that nearly all the energy is dissipated in about one cycle. Once this cycle has been completed there is no energy left anywhere in the system and the process is truly a discontinuous one. The traces obtained when the switch is opened at two different parts of a cycle are shown in (b) and (c) of Fig. 2.

One of the simplest circuits for carrying out the switching action is shown in Fig. 3 but it has the disadvantage for some applications of needing a thyratron valve. The circuit L_1C_1 supplies current to keep the thyratron conductive during the back swing of voltage across LC.

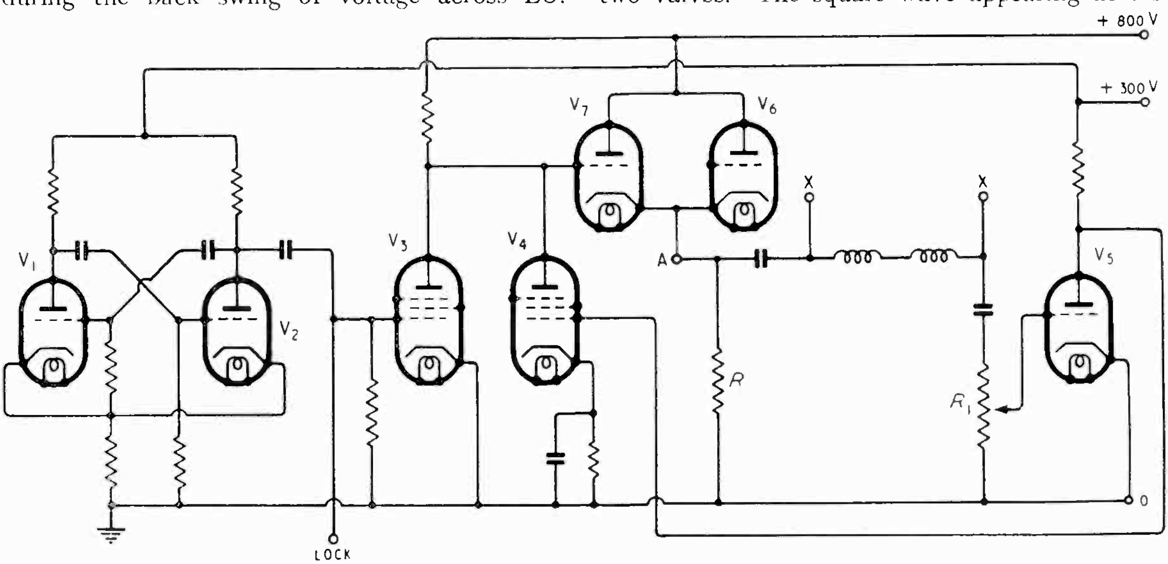


Fig. 4. *Elaborate switch circuit including a feedback circuit with which the coil resistance can be cancelled.*

an impedance $1/2g_m$ acts in this circuit as the equivalent of the switch of Fig. 1.

A feedback circuit is provided with V_4 and V_5. A voltage is derived from the resistance R_1 in series with the tuned circuit, amplified

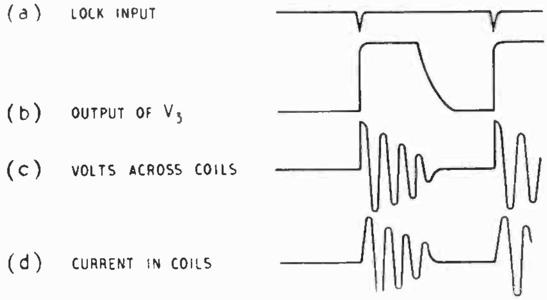


Fig. 5. *Waveforms in the circuit of Fig. 4.*

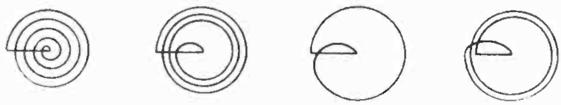
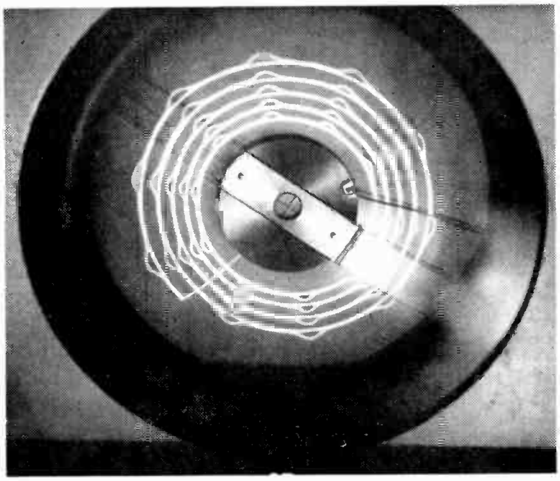


Fig. 6. *Example of the kind of trace obtainable with the circuit of Fig. 4.*

and fed back to the "top-end" of the series-resonant circuit. If there is unity gain in the amplifier V_4, V_5 and there is no phase shift the two voltages exactly cancel.

A small increase in the gain enables the effect of the resistance of the coils to be cancelled, while still greater gain results in a spiral trace increasing outwards. This is a stable condition and the transition from the inward spiral to the outward is quite smooth. The various forms of trace obtainable are shown in Fig. 6. In some equipment a modified feedback circuit was used and also a method of stabilizing the amplitude of the trace. These later developments do not affect the general operation, however.

When deflecting the trace radially it is of no advantage to obtain a greater deflection than that needed to fill the space between the turns of the spiral. Any system which will do this is adequate, therefore. The method adopted in the radar equipment for which this spiral time base was designed was to apply the deflecting signal to a metal-plate fixed to the front of the tube and to rely on the field between this plate and the final anode to produce the deflection. This method may seem somewhat crude but it works and proved quite suitable for the particular application. The plate is visible in the photograph, which also shows the spiral trace and its deflection.



Radial deflection of the trace is obtained by applying the signal to a metal plate fixed to the screen of the c.r. tube.

This information is published with the approval of the Chief Scientist, Ministry of Supply.

REFERENCE

[1] "A New Polar Co-ordinate Cathode-Ray Oscillograph with Extremely Linear Time Scale," by Manfred von Ardenne, *The Wireless Engineer*, January 1937, p. 5.

THE SYNCHRODYNE AS A PRECISION DEMODULATOR

By D. G. Tucker* and R. A. Seymour

(*Post Office Research Station*)

SUMMARY.—Demodulating systems in which the modulation frequency is extracted by filtration *following* the demodulator or detector have advantages over the conventional systems in that the difficult problem of narrow-band h.f. filtration is avoided. When unwanted signals of high level are liable to be present, ordinary detectors cannot be used without preliminary filtration, so that systems of the synchrodyne, homodyne or exalted-carrier types are necessary.

The use of the synchrodyne system is described here for applications where a high degree of precision is required; i.e., where special requirements of low harmonic distortion, constant gain to the modulation-frequency component, etc., exist. It is shown that some measure of control of the phase angle of the local oscillation relative to the incoming carrier is necessary, and that the phase-shifts in various parts of the circuit must be carefully considered and restricted.

Following an analysis of the distortion produced by phase errors, suitable circuits for controlling the phase angle of the local oscillation and for reducing stray phase errors are described.

List of Symbols

θ = phase angle between injected synchronizing signal and the forced oscillation at the same point in the oscillator.

θ' = phase angle as above when modulated due to the a.m. on the injected signal. (N.B. θ' is a time function).

ϕ = stray phase-shifts such that the phase angle between input to the main modulator and the switching signal is $\phi + \theta$.

e_s = injected-signal voltage.

e_i = forced-oscillation voltage.

p (as subscript) = pull-out value.

ω_0 = modulation angular frequency.

ω_s = carrier angular frequency of input signal.

ω_N = natural angular frequency of oscillator.

x = ω_s/ω_N.

m = depth of modulation.

$J_n(a)$ = Bessel function of the first kind, of order n and argument a.

E = amplitude of input signal.

Other symbols used only in one place are defined as they occur.

1. Introduction

THE method of demodulating amplitude-modulated signals known as the 'synchrodyne' is a development of the homodyne method, which is relatively old, having apparently been first published by Colebrook[1] in 1924. In its early form the homodyne consisted merely of a self-oscillating detector, the oscillations being synchronized to the incoming-carrier frequency. Only poor-quality demodulation is obtainable by this method, but a series of patent specifications[3–7] during the thirties detailed improvements which made the system quite satisfactory for many purposes. An article[2] in 1942 outlined the benefits which would be obtained if a really satisfactory homodyne system could be designed. The synchrodyne in its most developed form probably is such a system. Most of its possibilities as a radio receiver have been fully discussed in publications, of which the full list is very long; but those given here[8–15] appear to make some significant contribution to the development of the subject. Another application, of much less popular interest, but much more severe in its performance requirements, is to highly-selective transmission-measuring equipment [16, 17] which is required to measure the voltage level of the sidebands of a test-tone modulated at say 33 c/s and transmitted in the narrow gaps between channels in a multi-channel carrier telephone system.

The synchrodyne process, in its simplest form, is illustrated in Fig. 1. A local oscillator is tuned to a frequency close to that of the a.m. signal required to be demodulated, and is actually synchronized to its carrier by the injection of a suitable amount of the signal into the oscillator circuit. The mechanism of this process, and of the discrimination of the circuit which results in the output of the oscillator being a relatively pure tone, free of the sidebands and interfering signals, is described in earlier publications[18–20]. The output of this oscillator is used as the switching signal of a switching-type modulator to the input of which is applied the a.m. signal. The output contains, among other products which can easily be filtered out, the original modulation-frequency signal. The important point is that no filtration is necessary before demodulation provided that the main transmission path has a linear response, all selectivity and discrimination against other signals being vested in the synchronized oscillator and in the low-pass filter in the output circuit. The frequency-response of the

* Now Royal Naval Scientific Service

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wanted signal need not be impaired by demodulation except in so far as the low-pass filter departs from an ideal response (which need be very little) or has to cut-off below the upper frequency-limit of the required signal owing to overlapping by other signals. This has been adequately discussed in earlier works. The only other distortions to which the demodulated signal is liable are nonlinear ones due to the output of the local oscillator being phase-modulated at the modulation frequency. These affect both applications of the system as will be clear later. These distortions have been largely ignored in earlier works, and the subject of the first part of the present paper is to analyse them in the basic circuit. The second part discusses the circuit arrangements by which the system can be made a demodulator of real precision, of stability and with freedom from distortion.

2. Basic Distortion in a Synchrodyne Demodulator

2.1 *Strict Analysis for Small Depth of Modulation**

In a synchrodyne demodulator distortion arises due to the output of the synchronized oscillator becoming phase-modulated at the modulation frequency of the incoming signal. This occurs because the locking tone is amplitude modulated, and because when the natural frequency of the oscillator is not the same as the forced-oscillation frequency there is a phase angle (θ) between the locking tone and the forced oscillation at the point of injection. This phase angle is given by the equation

$$\sin\theta \approx \frac{1-x}{1-x_p} \qquad \ldots \quad \ldots \quad \ldots \quad (1)$$

* The case of large depth of modulation is discussed in reference 26.

where x is the ratio ω_s/ω_N; ω_N = natural angular frequency; ω_s = locking frequency and p signifies the value at pull-out.

The synchronized range $(1 - x_p)$ is given by

$$1 - x_p = e_s/2e_iQ \qquad \ldots \quad \ldots \quad \ldots \quad (2)$$

where e_s = injected (locking) signal,

e_i = forced-oscillation amplitude at point of injection

Q = effective Q of oscillator tuned circuit.

Thus for fixed natural and synchronized frequencies

$$\sin\theta \propto 1/e_s \qquad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (3)$$

If the injected locking signal is amplitude-modulated (as it is in the synchrodyne demodulator), then e_s is varying at the modulation frequency, so that $\sin\theta$ also varies; i.e., the oscillator is phase-modulated. If we ignore the larger values of θ, which occur towards the edges of the synchronizing range, then $\sin\theta \approx \theta$ so that $\theta \propto 1/e_s$, and if we assume the depth of modulation is small, then we may regard the phase modulation as sinusoidal; i.e., the phase angle in the oscillator is

$$\theta' = \theta - m\theta\cos\omega_0 t \qquad \ldots \quad \ldots \quad \ldots \quad (4)$$

where m = depth of modulation of a.m. signal and ω_0 = angular frequency of modulation.

The output of the oscillator is used as the switching signal on a modulator, the input of which is the main a.m. signal,

$$E\cos\omega_s t.\,(1 + m\cos\omega_0 t) \qquad \ldots \quad \ldots \quad (5)$$

Let the circuit phase-shift additional to that due to the locking of the oscillator be ϕ so that the phase angle between switching signal and input signal at the modulator is $\phi + \theta - m\theta\cos\omega_0 t$. Then the output of the modulator is, apart from a constant numerical factor,

$$E\cos\omega_s t.\,(1 + m\cos\omega_0 t).\cos(\omega_s t + \phi + \theta - m\theta\cos\omega_0 t) \qquad \ldots \quad (6)$$

$$= E\cos\omega_s t.\,(1 + m\cos\omega_0 t).\,[J_0(m\theta)\cos(\omega_s t + \phi + \theta) + 2J_1(m\theta)\sin(\omega_s t + \phi + \theta).\cos\omega_0 t - 2J_2(m\theta)\cos(\omega_s t + \phi + \theta).\cos 2\omega_0 t + \ldots] \qquad \ldots \quad (6a)$$

$$= E\Big[J_0(m\theta)\Big\{\tfrac{1}{2}\cos(2\omega_s t + \phi + \theta) + \tfrac{1}{2}\cos(\phi + \theta) + \frac{m}{2}\cos(2\omega_s t + \phi + \theta).\cos\omega_0 t + \frac{m}{2}\cos(\phi + \theta).\cos\omega_0 t\Big\} + J_1(m\theta).\Big\{\sin(2\omega_s t + \phi + \theta).\cos\omega_0 t + \sin(\phi + \theta).\cos\omega_0 t + m\sin(2\omega_s t + \phi + \theta)\cos^2\omega_0 t + \frac{m}{2}\sin(\phi + \theta) + \frac{m}{2}\sin(\phi + \theta).\cos 2\omega_0 t\Big\}$$

$$- J_2(m\theta)\Big\{\cos(2\omega_s t + \phi + \theta).\cos 2\omega_0 t + \cos(\phi + \theta).\cos 2\omega_0 t + m\cos(2\omega_s t + \phi + \theta).\cos\omega_0 t.\cos 2\omega_0 t + \frac{m}{2}\cos(\phi + \theta).\cos\omega_0 t + \frac{m}{2}\cos(\phi + \theta).\cos 3\omega_0 t\Big\} + \text{etc.}\ldots\Big]$$

Significant terms are underlined in the foregoing expansion. After filtering-off all h.f. components, we are left with the underlined terms, which may be rearranged thus :—

$$\tfrac{1}{2} E \Big[\cos(\phi+\theta) \,.\, J_0(m\theta) + m \sin(\phi+\theta) \,.\, J_1(m\theta)\Big]$$
$$+ E\Big[\frac{m}{2}\cos(\phi+\theta) \,.\, \{J_0(m\theta) - J_2(m\theta)\} + \sin(\phi+\theta) \,.\, J_1(m\theta)\Big]\cos\omega_0 t$$
$$+ E\Big[\frac{m}{2}\sin(\phi+\theta) \,.\, J_1(m\theta) - \cos(\phi+\theta) \,.\, J_2(m\theta)\Big]\cos 2\,\omega_0 t$$
$$- E\Big[\frac{m}{2}\cos(\phi+\theta) \,.\, J_2(m\theta)\Big]\cos 3\,\omega_0 t \qquad (7)$$

assuming that $J_3(m\theta)$ and higher orders are negligible, which is true for values of θ in the range we are concerned with.

If there were no phase errors or phase modulation the output would be merely

$$\frac{Em}{2}\cos\omega_0 t \qquad \ldots \qquad \ldots \qquad \ldots \qquad (8)$$

with a d.c. term of $E/2$.

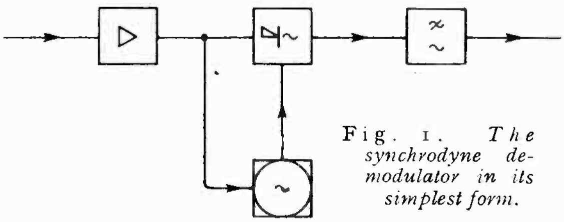

Fig. 1. *The synchrodyne demodulator in its simplest form.*

The remainder of the terms represent some form of distortion, according to the application of the circuit. For instance, in a synchrodyne radio receiver, the variation of the level of the fundamental frequency is relatively unimportant, but the introduction of harmonics of the modulation frequency is a serious form of distortion which will become detectable to the ear if of sufficient magnitude. On the other hand, in the synchrodyne demodulator of a highly-selective transmission-measuring equipment, the harmonics are unimportant since they are filtered off, but variations in the level of the fundamental are direct errors in measurement.

The distortion described in this section has been called 'basic distortion' because it is fundamental to the synchrodyne circuit and is not due to practical imperfections. The only way of keeping it within desired limits is to control the phase angles as much as necessary. Thus, in the transmission-measuring equipment, an automatic phase-control circuit has been added.

We have here assumed that the depth of modulation is small. The modifications necessary when m is not small are comparatively minor.

2.2 *Simplified Analysis*

The analysis of the previous section does not greatly assist the physical understanding of the distortion effect, and in view of the restrictive assumptions made that $m \ll 1$ and $\sin\theta \approx \theta$, there is some justification for a cruder method of analysis that does illustrate how the distortion arises.

In the absence of the phase-modulation effect the output of the modulator includes (before it is filtered) the d.c. and modulation-frequency components ; thus

$$\frac{E}{2}\Big[\cos(\phi+\theta) + m\cos(\phi+\theta) \,.\, \cos\omega_0 t\Big]$$

With the phase-modulation of depth $m\theta$, these terms fluctuate between

$$\frac{E}{2}\Big[\cos(\phi+\theta-m\theta) + m\cos(\phi+\theta-m\theta)\cos\omega_0 t\Big]$$

and $$\frac{E}{2}\Big[\cos(\phi+\theta+m\theta) + m\cos(\phi+\theta+m\theta)\cos\omega_0 t\Big]$$

Thus we have a fluctuation of the d.c. component of peak-to-peak amplitude

$$\frac{E}{2}\Big[\cos(\phi+\theta-m\theta) - \cos(\phi+\theta+m\theta)\Big]$$

i.e., $E\sin(\phi+\theta) \,.\, \sin m\theta$

at frequency ω_0, and also a fluctuation of the amplitude of the component of frequency ω_0 of m times this amount at frequency ω_0. The first introduces mainly an additional output of modulation frequency, and the second introduces second harmonic of the modulation frequency. There is also an additional second-harmonic component due to the lack of symmetry of the first fluctuation term. Fig. 2 illustrates the process.

We may assume, as a first approximation, that the peak-to-peak amplitude of the fluctuations represents the fundamental components, and that half the difference between the positive fluctuation on one half-cycle and the negative fluctuation on the other represents the second-harmonic components. We thus arrive at the following expression for the output, putting $\sin m\theta \approx m\theta$, and regarding the signs of the various terms,

$$\frac{Em}{2}\left[\cos(\phi+\theta)\,.\cos m\theta + \theta\sin(\phi+\theta)\right]\cos\omega_0 t$$
$$+\frac{E}{2}\left[\frac{m^2}{2}\theta\sin(\phi+\theta) - \cos(\phi+\theta)\,.\right.$$
$$\left.(1-\cos m\theta)\right]\cos 2\,\omega_0 t$$
$$-\frac{Em}{2}\left[\cos(\phi+\theta)\,.\,(1-\cos m\theta)\right]\cos 3\,\omega_0 t \qquad \ldots \quad \ldots \quad \ldots \quad (9)$$

This expression is actually a very good approximation to that developed more formally using Bessel functions; it is identical if we put

$$J_0(m\theta) \approx 1$$
$$J_1(m\theta) \approx \tfrac{1}{2}\,m\theta$$
$$J_2(m\theta) \approx (1-\cos m\theta)$$

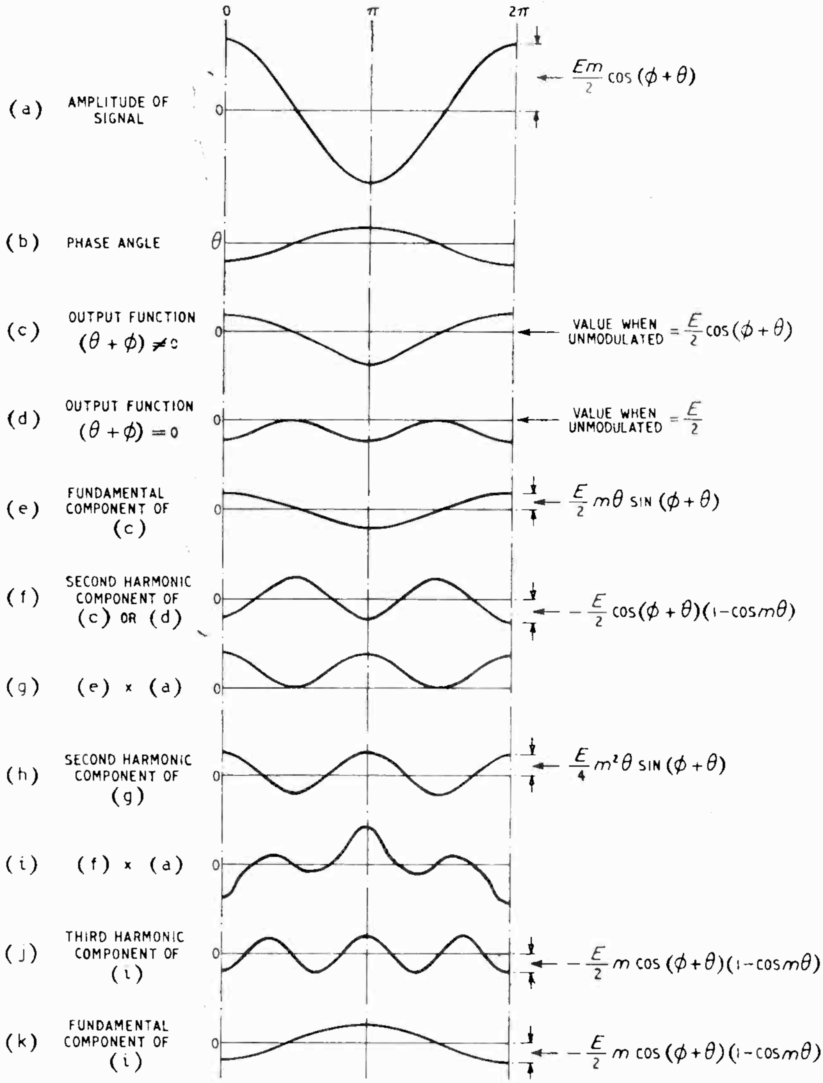

Fig. 2. *The analysis of harmonic production is illustrated by this breakdown of the output into its component waves.*

The first two are quite acceptable approximations, since the usual approximations for small arguments are

$$\left.\begin{aligned} J_0(m\theta) &\approx 1 - \frac{(m\theta)^2}{4} \\ J_1(m\theta) &\approx \tfrac{1}{2}\,m\theta \\ J_2(m\theta) &\approx \tfrac{1}{8}\,(m\theta)^2 \end{aligned}\right\} \quad \ldots \quad \ldots \quad \ldots \quad (10)$$

Unfortunately $1 - \cos m\theta \approx \frac{1}{2}\,(m\theta)^2$ so that there is an error in all the frequency components given by the approximate formula, but the amplitude of this term in question is very small, and is again modified if m is not small compared with unity, due to the unsymmetry of the phase modulation itself.

2.3 *Numerical Results*

The variation of amplitude of the d.c., fundamental, and 2nd and 3rd harmonic components, as given in equation (7), is conveniently plotted against θ, for each of a small number of values of ϕ. Figs. 3–5 give these relationships. Fig. 3(a) shows the resultant output of fundamental modulation frequency, and it can be seen that errors in θ increase the output, while errors in ϕ decrease it. Noting the very enlarged output scale, it can be seen that the dependence of output on θ around $\theta = 0$ is quite small, and even for 0.1-db range of output level, about $\pm$ 8.5° can be tolerated. The value of ϕ is obviously not very critical, and it will, in general, be constant. Fig. 3(b) shows separately the two terms making up the fundamental component. The term $\sin(\phi+\theta)\,.\,J_1(m\theta)$, shown by curves B, represents the output that would be obtained if the modulation were removed from the input to the modulator but not from the injected synchronizing signal. It can be seen that measurement of this curve, in practice, is a simple way of measuring ϕ, since the interval between the two zeros is equal to ϕ.

Fig. 4 (a, b, and c) give the 2nd and 3rd harmonic amplitudes. It is evident that for any given maximum tolerable 2nd harmonic, there is an optimum value of ϕ which gives the greatest tolerance on θ. For example, for a harmonic level of about 0.01, $\phi = 0$ gives a range of θ

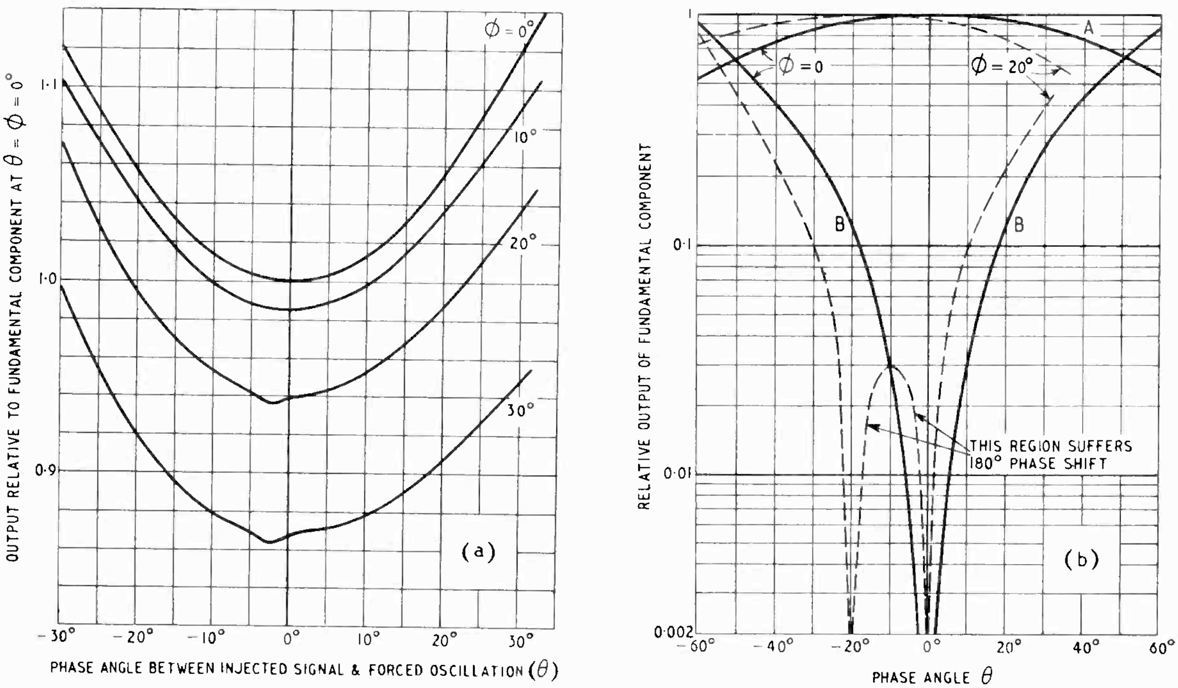

Fig. 3. *Fundamental component of modulation frequency ($m = 0.3$); resultant output (a) and components of the output (b). In the latter, curve A shows the main part of the output and curve B the spurious part.*

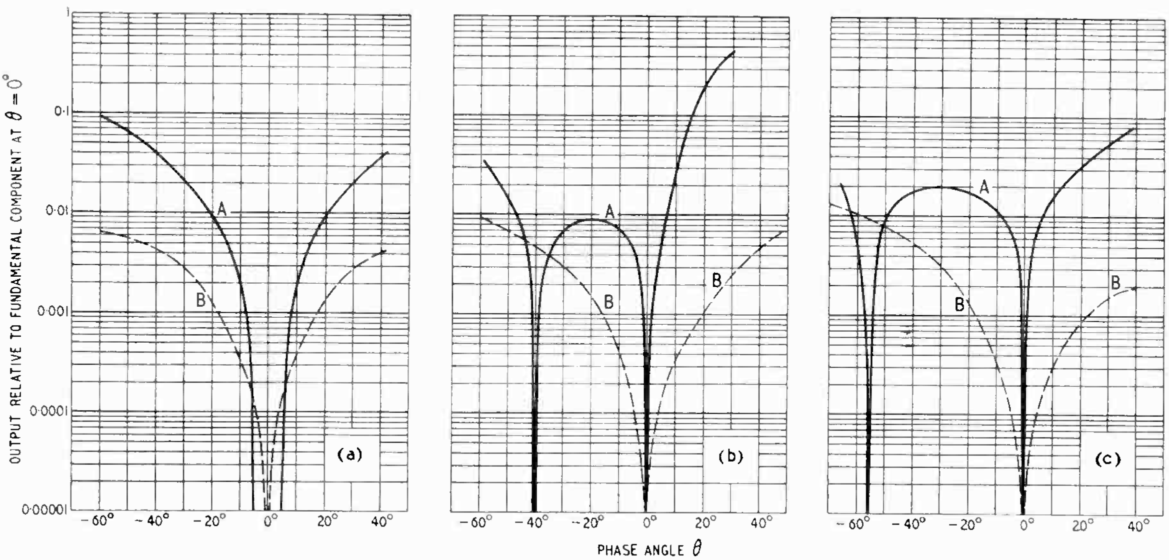

Fig. 4. *Output of harmonics of fundamental modulation frequency for $m = 0.3$; (a) $\phi = 0$, (b) $\phi = 20°$ and (c) $\phi = 30°$. Curves A and B are for second and third harmonics respectively.*

of about 42°, but $\phi = 20°$ gives a range of 54°, and $\phi = 30°$ reduces the range to only 14°.

Fig. 5 shows the d.c. component.

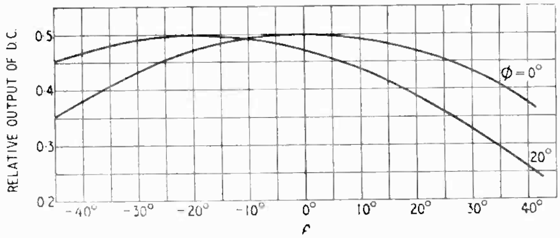

Fig. 5. *Output of d.c. component for m = 0.3.*

3. Practical Methods of Achieving Precision

In designing a synchrodyne demodulator for precision work, the following features are, or may be, desirable :

(a) Stability of the oscillator tuning.

(b) Limitation of the phase angle between the locking signal and the local oscillation to a small value, say 10°.

(c) Limitation of stray phase-shifts in the demodulating circuit to a small value, say 10 or 20°.

(d) Constancy of the overall gain of the demodulator.

Feature (a) is of prime importance in equipment with pre-set or fixed tuning. It is also desirable in any equipment, since the need for frequent retuning is always best avoided.

Features (b) and (c) are necessary, as shown in Section 2, if nonlinear distortion is to be avoided.

Feature (d) is relatively unimportant in a radio receiver, but is supremely important in the measuring-equipment application.

To achieve (b), the use of a phase-control circuit involving a valve reactor has already been proposed[9] and used[17]. The valve reactor forms part of the tuned circuit of the oscillator, and it is unfortunate that it may involve considerable instability of tuning, since its reactance is directly proportional to its mutual conductance, which in turn varies considerably with supply voltage and with age. Instability of tuning due to this cause exceeds all other variations. It is now proposed, therefore, to achieve (a) by the use of differential valve reactors in which (in the quiescent condition) equal and opposite reactances are provided by two valves, only one of which is varied by the control voltage.

To achieve (c), it would be possible in a single equipment to insert suitable phase-shifters to correct the phase angles, though to make them effective at more than one frequency may be difficult. In circuits as hitherto published there have always been transformers, and these have phase-shifts which form the bulk of the stray shift. It is found that in practice the phase-shift of a transformer varies considerably from one batch to another (even from one individual to another) and is, in consequence, somewhat unpredictable. If a design is to be suitable for manufacture in quantity, the need for large individual corrections of phase-shift must be avoided, and it is now proposed that all transformers be eliminated by means described later. The circuit is then left with only capacitance-resistance couplings which can be designed to have negligible phase-shift.

To achieve (d) it is necessary to have an adequate amount of feedback on all amplifier stages in the main transmission path. The most difficult part of the circuit to stabilize for gain (or loss) is the modulator, but this can be given a very good short-term stability (say $< \pm 0.05$ db under normal conditions) and a probably adequate long-term stability (say $< \pm 0.25$ db over a 1-month period) by the use of a valve-modulator with modulated feedback[21].

These various aspects of the circuit design are discussed in the following sections. They are dealt with in much more detail, with experimental results, in a Post Office Research Report[26].

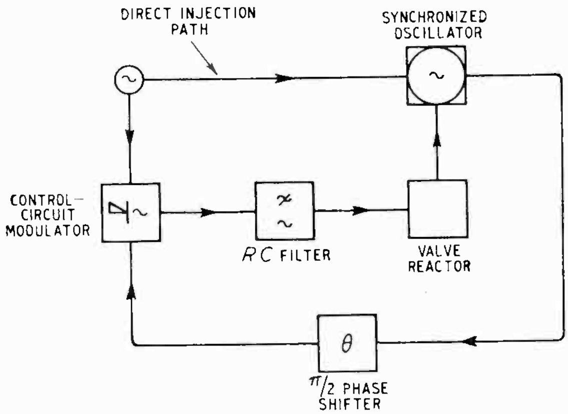

Fig. 6. *Phase-control circuit for synchronized oscillator.*

3.1 *Phase Control by Valve Reactors.*

It is evident from Section 2 that the phase angle between the locking signal and the forced oscillation in the oscillator depends on the amount by which the natural frequency of the oscillator departs from the synchronized frequency. The method of phase control is thus to retune the natural frequency by means of a valve reactor, which is connected as part of the tuning reactances, and whose reactance is controlled by a unidirectional voltage proportional to the sine of the phase angle. This has been described elsewhere[9], and only the block schematic need be repeated here—see Fig. 6.

The valve reactor provides a reactance between anode and earth when the anode is coupled back

Variations in h.t. voltage can be compensated by feeding a suitable current from the h.t. supply through the cathode resistor. This, if correctly adjusted, causes the grid to become more negative relative to cathode, as the h.t. voltage increases,

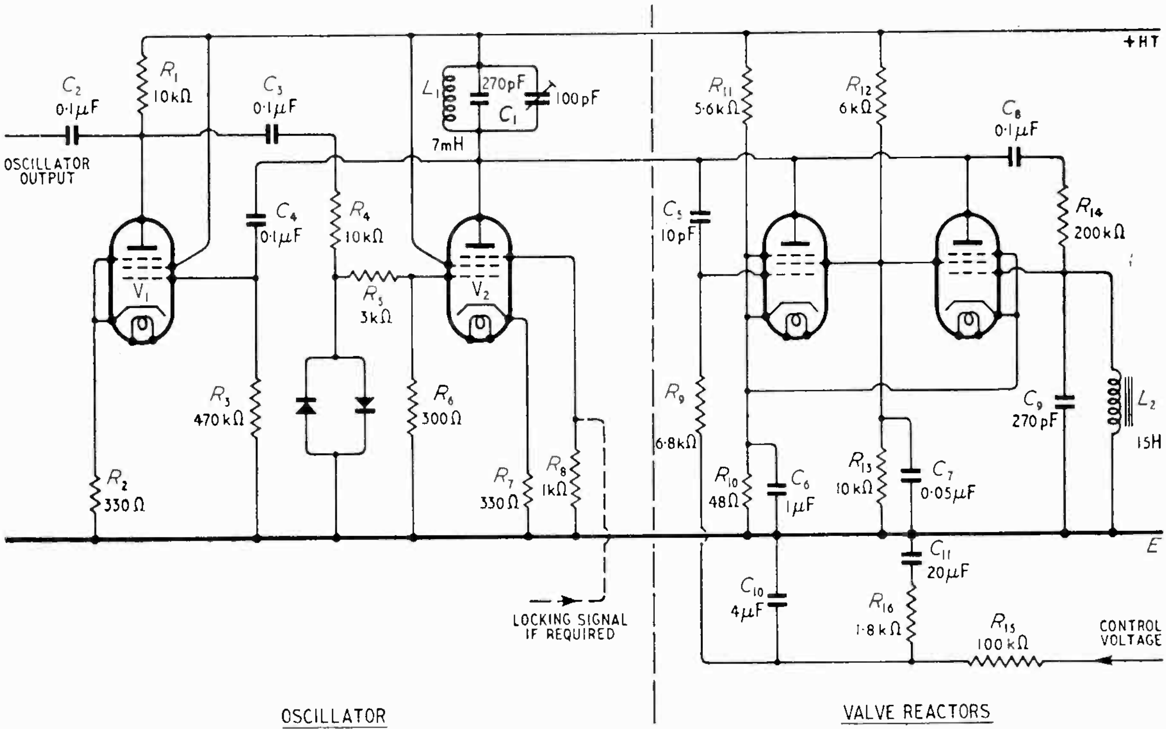

Fig. 7. *Oscillator and differential valve reactor circuits. Component values shown are for an operating frequency of 84 Kc/s.*

to the grid by means of a *RC* circuit giving a suitable phase-shift[22]. When the anode-grid connection is a capacitance, with a resistance from grid to earth, the anode-earth reactance is negative; when the capacitance and resistance are interchanged, the anode-earth reactance is positive. In both cases the reactance magnitude is inversely proportional to the mutual conductance of the valve, and is normally adjusted by varying the bias potential of the control grid. Unfortunately other factors beside this also alter the reactance, namely, h.t. voltage, heater voltage and ageing. Variations of reactance due to such causes may cause the natural frequency in the absence of the control voltage (i.e., before the input signal is connected to the synchrodyne demodulator) to depart greatly from its nominal value, and this may entirely prevent synchronism being attained in a circuit with pre-set tuning although once synchronized by some means it will remain so. The drift of natural frequency may even be sufficient to cause synchronism with unwanted signals. Thus it is generally essential in pre-tuned equipment to reduce these effects.

to an extent such that the mutual conductance is unchanged. It has the additional advantage that a low cathode resistance can be used, thus avoiding a reduction of sensitivity due to negative d.c. feedback.

If two valve reactors are used, one having positive and the other an equal negative reactance, joined in parallel, then any variation of h.t. or heater voltage will affect both valves more or less equally, so that the resultant reactance will not change. The control voltage is applied to one of the valves only, and evidently the overall sensitivity (i.e., change of reactance for a given change of grid voltage) is not affected. There will also be a tendency for both valves to age at the same rate, so that changes of reactance with time are much reduced.

When using a valve reactor with a resistance between anode and grid and capacitance from grid to earth, difficulty usually arises with noise from the h.t. supply, however well it is smoothed, because at very low frequencies the alternating potential of the grid approaches that of the h.t. busbar. It can be appreciated that minute noise voltages can affect the circuit due to the large frequency changes produced by small grid-voltage changes. This trouble can be avoided by the connection of a choke from grid to earth. The circuit arrangement of the complete stabilized valve-reactor circuit is shown, together with its connection to the oscillator circuit, in Fig. 7. The two *RC* phase-shift circuits from anode to grid are C_5, R_9 and R_{14}, C_9; the h.t. compensation circuit is R_{11}, R_{10}; and the choke referred to above is L_2.

As regards the quantitative design of the circuit, the basic formulae for the effective parallel reactance and resistance are

$$X_0 = \frac{1}{g_m} \cdot \frac{Z_1^2 + Z_2^2}{Z_1 Z_2}$$

and $$R_0 = \frac{r_a (Z_1^2 + Z_2^2)}{Z_1^2 + \mu Z_2^2}$$

where Z_1 = impedance between anode and grid,

Z_2 = impedance between grid and earth,

g_m = mutual conductance of valve,

r_a = a.c. resistance of valve,

μ = amplification factor of valve.

For a positive output reactance, Z_1 is a resistance R_1, and Z_2 is the reactance of C_1, so that

$$X_{01} = + \frac{1 + (\omega C_1 R_1)^2}{g_m \omega C_1 R_1}$$

and $$R_{01} = r_a \frac{1 + (\omega C_1 R_1)^2}{\mu + (\omega C_1 R_1)^2}$$

For a negative output reactance, Z_1 is the reactance of C_2 and Z_2 is a resistance R_2, so that

$$X_{02} = - \frac{1 + (\omega C_2 R_2)^2}{g_m \omega C_2 R_2}$$

and $$R_{02} = r_a \frac{1 + (\omega C_2 R_2)^2}{1 + \mu (\omega C_2 R_2)^2}$$

It is evident that one condition for equality of magnitudes of X_{01} and X_{02} is $C_1 = C_2$ and $R_1 = R_2$, but this will make one or both of the parallel resistance components necessarily very low. Thus the *Q*-factor of the oscillator tuned circuit will be made very small. The more useful condition for the differential valve-reactor circuit is $\omega R_1 C_1 = 1/\omega R_2 C_2$, which gives $X_{01} + X_{02} = 0$, and $R_{01} = R_{02}$ = a high value. This is the basis of choice of the component values in the circuit of Fig. 7.

Control of the reactance by means of a unidirectional voltage applied to the grid depends on the relation between mutual conductance and grid voltage v_g. The relation is not linear, but over a large part of the working range it is approximately so, and in a typical small r.f. pentode (CV 138, V888 or SP 41) with 130 volts h.t., and the circuit values of Fig. 7, a typical value for $\frac{dg_m}{dv_g}$ is about 5 mA/V².

It is clear that the amount by which the phase angle θ is reduced by this circuit depends on the magnitude of unidirectional control voltage produced by a given phase error. Therefore, if there is no automatic gain control in the signal path feeding the control modulator, the extent of the control will vary with the signal level.

To determine the magnitude of the phase-modulation effect, described in Section 2, is difficult when the phase-control is added. If the latter has a fairly large time-constant in the *RC* filter in the d.c. (control) lead to the valve reactor, then no phase modulation will be introduced through modulation of the valve reactance by the envelope-frequency component in the control circuit*. But in such a case the control circuit cannot follow the variations of phase due to the amplitude modulation of the injected signal, and it would appear that the results of Section 2 still apply. This is so at relatively high levels of injected signal, but at low levels (e.g., > 40 db below the forced oscillation) the phase modulation is reduced. With very low levels, or with the injected signal completely removed, the variation of fundamental output of the demodulator against phase angle becomes the simple cos $(\phi + \theta)$ relationship, and no harmonics of the modulation frequency are generated.

The control circuit is a feedback loop, and as such is liable to hunt or become unstable. It can be shown that the use of an *LC* filter in the d.c. (control) path will cause instability, but if only a single *RC* section is used, hunting will not necessarily occur. The liability to hunt is obviously a function of the loop gain and is, therefore, greatest if a large degree of phase control is aimed at.

It is important to realize that the injected signal (considered as something added to an existing reactance-valve control circuit) has a large stabilizing influence. The ratio of phase angle with injection alone to that with control and injection is large at small injected voltages, but diminishes to small values at large injected voltages[25]. This corresponds to the effect of a smaller loop gain as the injected voltage is increased; hence the increased stability. This is an advantage of using the combination of control and injection as opposed to either alone.

Although by restricting the d.c. filtering to a single *RC* section the control circuit may be

* It should be noted that the time-constant must be large for another reason also—the phase-control circuit must be free from interference by spurious signals, which could otherwise modulate the oscillator phase and give interference in the output of the demodulator.

quite stable, nevertheless, in the absence of an injected signal, it can become unstable. To prevent this, the loop phase-shift/frequency response can be modified by using the *RC* filter circuit as shown in Fig. 8.

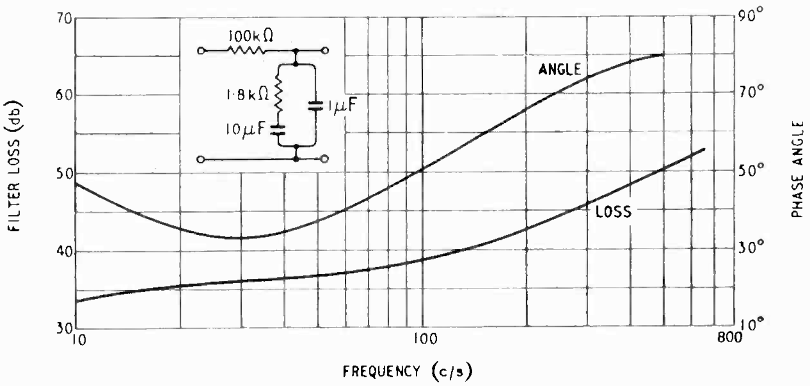

Fig. 8. *Calculated performance of RC filter.*

The use of the combination of control circuit and injected signal has a further advantage over the use of either alone, in that the pull-in frequency range may be increased[25]. As the level of the injected signal is reduced, leaving the control circuit fixed, the pull-in range becomes that of the control circuit alone, which depends on the time-constant, but is generally small. Again, starting from the simple synchrodyne with only the injected signal, the addition of a phase-control circuit causes a large increase in pull-in range, and an even larger increase in pull-out range, (which with plain injection is equal to the pull-in range).

It will have been observed that the addition of a phase-control to the simple synchrodyne which is synchronized by injection, offers certain advantages at the expense of some disadvantages. The further question also arises as to whether the injected signal need be retained when the phase-control is used. The main conclusions are :—

(a) phase-control is necessary for a precision demodulator, where the output of modulation-frequency is required to be constant or the output of harmonics of the modulation frequency is required to be low.

(b) phase-control by reactance valves may cause difficulty with the frequency-stability of the oscillator.

(c) phase-control increases the pull-in frequency range several times, and the pull-out range many times.

(d) the injected signal may be removed provided the phase-control loop has an adequate stability margin.

3.2 *Avoidance of Stray Phase-shifts.*

Stray phase-shifts in the circuit include those which make up the angle ϕ. It is clear from Section 2 that the restrictions which have to be placed on this angle are far less severe than those on the angle θ. Nevertheless, it is essential to make some limit for either the measuring-equipment or radio-receiver application. In the former case, a large value of ϕ reduces the amplitude of the output signal, but is not otherwise important provided it is constant ; and in the latter case, a large value of ϕ causes worse harmonic content over most of the range of θ.

Other stray phase-shifts which may be rather more serious are those in the phase-control loop circuit, since any error in the phase relations at the control modulator will cause the unidirectional control voltage to be zero when the angle θ is different from zero by the amount of the stray shifts.

Factors which cause these stray phase-shifts are :

(a) phase-shifts in coupling circuits,

(b) phase-shifts due to parasitic capacitances,

(c) phase-shift in the oscillator due to harmonic production.

(d) phase-shift due to carrier leak at the control modulator.

The relative magnitudes of (a) and (b) depend on the operating frequency. At frequencies below say 100 kc/s, (a) will predominate, but at frequencies above say 1 Mc/s (b) may predominate. Either or both of these effects will probably be much larger than (c), and all that need be done to deal with (c) is to keep the amplitude of oscillation relatively small so that the harmonic production is small. The method of dealing with (b) is the standard one of keeping anode loads reasonably low, removing avoidable stray capacitances, and using inductance-compensation. It should be possible, provided (a) is dealt with, to operate the circuit with considerable precision and to produce it in quantity, at any rate as a radio receiver, up to 1 or 2 Mc/s at least, and at higher frequencies if only relatively narrow frequency-bands are to be dealt with, so that phase-compensation over a wide frequency-range is unnecessary. The treatment of (d) is to use a well-balanced modulator.

The main cause of phase-shifts under heading (a) is the use of transformers. These occur in many places in the conventional circuit arrangements; the oscillator will probably be a single-valve type and use one in the tuned circuit; the main modulator may, and the control modulator probably will, involve one or two each. All of these, and particularly that in the oscillator, introduce a considerable phase-shift due to their shunt and leakage inductances and self-capacitances; the leakage inductance is probably the dominant factor, and has the largest effect in the oscillator because the transformer there is likely to be on a dust core of low permeability and with poor coupling between the windings. These phase-shifts are difficult to correct except at single frequencies and, moreover, vary considerably from one individual to another, making production in quantity a very expensive and difficult matter. The solution is to use some special transformerless circuits.

A suitable circuit for a transformerless oscillator using two valves with amplitude-limiting on a pair of rectifiers is shown in Fig. 7. It is desirable to avoid limiting on the valves themselves in order to obtain a suitable oscillation level on the reactance valves.

It should be noted that the phase modulation of the oscillator output, due to an injected synchronizing signal, discussed in Section 2, may, in practice, be considerably modified by various performance features (e.g., dependence of phase-shift on amplitude, and detuning due to harmonics[23]) so that the relation of distortion to phase angle may depart considerably[26] from that calculated for a perfect oscillator.

As far as the radio receiver is concerned, there is no difficulty in avoiding transformers in the main signal modulator, since an ordinary valve frequency-changer operates quite satisfactorily provided it has an adequate local-oscillation voltage and provided no d.c. output is required for tuning-whistle-suppression or for a.g.c. This is not adequate for the measuring set, however, since the valve gives a conversion gain which varies considerably with supply voltages and with age. For this purpose a feedback valve modulator can be used with success, as described elsewhere[21].

For the phase-control modulator, however, a d.c. output is required, and this cannot easily be extracted from a valve modulator because it cannot be distinguished from the anode current. So for this purpose, and for the main modulator when a d.c. output component is required, a valve-fed rectifier modulator should be used, as also described elsewhere[24]. The feedback valve modulator is usually preferable, when no d.c. is required, on account of its conversion gain compared with the conversion loss of the rectifier modulator—the difference usually amounts to about 30 db.

The circuits described above are suitable for a precision transmission-measuring equipment up to about 150 kc/s or for a high-quality radio receiver up to about 2 Mc/s. If either application requires operation at higher frequencies, then two main courses are open:—

(a) Use inductance-compensated coupling circuits to correct phase angles, and (b) use the synchrodyne as an i.f. filter and demodulator following a preliminary frequency-changer.

The former course is suitable where operation over only a relatively narrow band is required, and it may still be possible to keep the circuit suitable for production in quantity, since very precise component values may still be avoidable. But where wideband operation is required, the second course is by far the better.

4. Conclusions

The performance of a synchrodyne demodulator is considerably more difficult to predict than was indicated by the earlier work on the subject and, in particular, the question of phase-modulation of the synchronized oscillator is important. An analysis has been made of the performance to be expected from a synchrodyne demodulator incorporating a perfect oscillator, and an indication has been given of how and why this performance is not generally obtained exactly in practice.

The principles of design of a precision synchrodyne demodulator and some desirable new circuit units have been discussed.

5. Acknowledgments

Acknowledgment of assistance is due to several of the authors' colleagues who have taken part at some stage in the work on which this paper is based. Chief among them is J. Garlick, who did the early development of the phase-control circuit, E. Jeynes and C. D. Thompson who have done much of the experimental work.

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CORRESPONDENCE

Letters to the Editor on technical subjects are always welcome. In publishing such communications the Editors do not necessarily endorse any technical or general statements which they may contain.

Wideband Two-phase Networks

SIR,—In his paper in the March issue, Mr. H. J. Orchard gives an analytical method for designing phase-shift networks. This gives a Tchebycheff approximation to a constant phase difference of 90° over a prescribed frequency band. The mathematical treatment is based on the elliptic sn-, cn- and dn- functions, and the frequency-determining poles of the phase-shift networks are given by the equation

$$\left.\begin{aligned} p_\sigma &= \frac{1}{\sqrt{k'}} \cdot \frac{\operatorname{cn}(u_\sigma, k)}{\operatorname{sn}(u_\sigma, k)} \\ u_\sigma &= \frac{4\sigma + 1}{2n} K \,;\ \sigma = 0, 1, 2, \ldots n - 1 \end{aligned}\right\} \qquad (22)$$

For the numerical computation Mr. Orchard uses repeated Landen transformations to get the value of $\operatorname{sn}(u_\sigma, k)$ from the trigonometrical sin-function, and $\operatorname{cn}(u_\sigma, k)$ is then calculated from the relation

$$\operatorname{sn}^2(u_\sigma, k) + \operatorname{cn}^2(u_\sigma, k) = 1 \quad \ldots \quad \ldots \quad \ldots \quad (28)$$

The operations involved are straightforward but somewhat tedious, as the author admits. By means of the Jacobian Theta-functions a formula permitting approximation can be derived, with p_σ expressed as a cotangent.

By using the appropriate Theta-functions equation (22) can be written as

$$p_\sigma = \frac{\cos z_\sigma + q^2 \cos 3z_\sigma + q^6 \cos 5z_\sigma + q^{12} \cos 7z_\sigma + }{\sin z_\sigma - q^2 \sin 3z_\sigma + q^6 \sin 5z_\sigma - q^{12} \sin 7z_\sigma + } \quad (1)$$

where

$$z_\sigma = (-1)^\sigma (2\sigma + 1) \frac{\pi}{4n} \,;\ \sigma = 0, 1, 2 \ldots (n - 1) \quad (2)$$

and q can be calculated from equ. (25) or (26), (27), loc. cit.

By putting

$$p_\sigma = \cot B_\sigma \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (3)$$

and observing that $\cos z + j \sin z = e^{jz}$ we get

$$\frac{\cot B_\sigma + j}{\cot B_\sigma - j} = e^{2jB_\sigma} =$$

$$\frac{e^{jz_\sigma} + q^2 e^{-3jz_\sigma} + q^6 e^{5jz_\sigma} + q^{12} e^{-7jz_\sigma} + \ldots}{e^{-jz_\sigma} + q^2 e^{3jz_\sigma} + q^6 e^{-5jz_\sigma} + q^{12} e^{7jz_\sigma} + \ldots}$$

$$= e^{2jz_\sigma} \frac{1 + q^2 e^{-4jz_\sigma} + q^6 e^{4jz_\sigma} + q^{12} e^{-8jz_\sigma} + \ldots}{1 + q^2 e^{4jz_\sigma} + q^6 e^{-4jz_\sigma} + q^{12} e^{8jz_\sigma} + \ldots}$$

This can be written as

$$B_\sigma = z_\sigma - z'_\sigma \quad \ldots \quad \ldots \quad \ldots \quad (4)$$

where

$$z'_\sigma = \tan^{-1} \frac{(q^2 - q^6) \sin 4z_\sigma + (q^{12} - q^{20}) \sin 8z_\sigma + \ldots}{1 + (q^2 + q^6) \cos 4z_\sigma + (q^{12} + q^{20}) \cos 8z_\sigma + \ldots} \quad (5)$$

In most cases q^6 is a very small quantity, and q^{12} and q^{20} can be discarded altogether.

As Mr. Orchard points out we have $|p_\sigma| = |p_{n-1-\sigma}|^{-1}$ or $|B_\sigma| + |B_{n-1-\sigma}| = \frac{1}{2}\pi$. Thus we need only calculate half the values of p_σ from equations (3), (4), (5), in which case the angle $|4z_\sigma|$ is restricted to the first and second quadrant. We observe that positive values of p_σ correspond to network No. 2, and negative values to network No. 1.

When n is *even* the accuracy of the result can be checked very simply. In this case it is easy to show that the phase-difference between the two sections at the geometrical mid-band frequency $f = \sqrt{f_a f_b}$ can be written as

$$\beta_m = -4 \sum_{\sigma = 0}^{\frac{1}{2}n - 1} B_\sigma \quad \ldots \quad \ldots \quad \ldots \quad (6)$$

This value of β_m should now differ from 90° by the 'tolerance' ϕ_m if the computations are right.

Often the required frequency range $f_a \leqslant f \leqslant f_b$ for 90-degree phase difference is quite large. The value of q can then be more easily calculated if we start with Theta-functions of modulus $(q')^2$. The relation between q and q' is

$$(\log_{10} q) \cdot (\log_{10} q') = \tfrac{1}{2}(\pi \log_{10} e)^2 = 0.930\ 762 \qquad (7)$$

and between f_a, f_b, q' :

$$q' \approx \tfrac{1}{4} \cdot \frac{f_a}{f_b} + \frac{1}{16}\left(\frac{f_a}{f_b}\right)^3 \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (8a)$$

$$\text{or} \ \frac{f_a}{f_b} \approx 4q' - 16(q')^3 \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (8b)$$

In the numerical example given by Mr. Orchard $f_a = 200$ c/s, $f_b = 4000$ c/s, $n = 4$. Equation (8a) gives

$$q' = 0.012\ 5078 \,;\ \log_{10} q' = -1.902\ 819$$

From equation (7) :

$$\log_{10} q = -\frac{0.930\ 762}{1.902\ 819} = 9.510\ 852 - 10 \,;\ q = 0.324\ 229$$

Thus $q^2 = 0.105\ 125$; $q^6 = 0.001\ 162$. (q^{12} is neglected.)

Equations (2) and (5) give respectively

$z_0 = 11_0\ 15',\ z_1 = -33^0\ 45'$ and

$$z'_0 = \tan^{-1} \frac{0.073\ 513}{1 + 0.075\ 156} = 3^\circ\ 54'\ 41''$$

$$z'_1 = \tan^{-1} \frac{-0.075\ 513}{1 - 0.075\ 156} = -4^\circ\ 32'\ 41''$$

This gives

$B_0 = 7^\circ\ 20'\ 19''$; $B_1 = -29^\circ\ 12'\ 19''$;
$B_2 = 60^\circ\ 47'\ 41''$; $B_3 = -82^\circ\ 39'\ 41''$.

The poles in network No. 1 are:

$|\tan B_1| = 0.55900$; $|\tan B_3| = |7.76447$

in network No. 2:

$\tan B_0 = 0.128\ 788$; $\tan B_0 = 1.78890$

These values are in good agreement with those given in the paper, and of more than sufficient accuracy for practical purposes. As a check we may calculate the phase difference at the geometrical mid-band frequency by equation (6): $\beta_m = 4(29^\circ\ 12'\ 19'' - 7^\circ\ 20'\ 19'') = 90^\circ - 2^\circ\ 32'$. Equation (25) (loc. cit.) gives $\phi_m = 2^\circ\ 31'\ 36''$. The angular error in β_m is thus less than 30″. N. O. JOHANNESSON.

Stockholm, Sweden.

An Electrostatic Field Problem

SIR,—I have read with much interest your Editorial in the May issue explaining how the electrostatic force acting upon the surface of a charged sphere concentric within a larger sphere is much greater than the same force pulling inwards upon the inner surface of the encasing sphere.

Might it be permissible to explain the difference in this way? You point out that if the inner charged sphere is in two parts, these will repel one another. To continue this argument: if the inner sphere consisted of a large number of equally-charged particles then each would repel its neighbours, and assuming the sphere to be perfectly regular in shape, it would increase in size without any corresponding inward pull being exerted upon the inner surface of the larger sphere.

This outward force is acting upon the charged sphere whether or not it is in one piece, but does not need to be balanced by a corresponding force from the outer sphere.

I realize that this argument may be fallacious and if so, would appreciate its demolition.

Tunbridge Wells, Kent. F. D. C. BAKER.

For the sake of simplicity we assumed the inner sphere to be split into two similar parts. It could be assumed to consist of a large number of equally-charged mutually-repellent particles, and the sphere would then increase in size. The total inward pull on the outer sphere is independent of the size of the inner sphere, so long as its total charge remains constant. The total outward pull on the surface of the inner sphere, however, would rapidly decrease, since it is inversely proportional to the square of its radius. As Mr. Baker says, there is no question of the two forces balancing. G. W. O. H.

B.B.C. ENGINEERING DIVISION

R. T. B. Wynn, C.B.E., M.A., M.I.E.E., has been appointed Deputy Chief Engineer and will be responsible under the Chief Engineer, H. Bishop, C.B.E., B.Sc.(Eng.), M.I.E.E., M.I.Mech.E., for the general control and direction of all Engineering Departments.

H. L. Kirke, C.B.E., M.I.E.E., who has been head of the Research Department since 1925 becomes Assistant Chief Engineer and will be responsible for the co-ordination and direction of the technical work of the Research, Planning and Installation, Designs and Equipment Departments. He is succeeded in the Research Department by W. Proctor Wilson, C.B.E., B.Sc., M.I.E.E., and E. C. Drewe, M.I.E.E., becomes Assistant Head of the Department.

NEW BOOKS

Acoustic Measurements

By LEO L. BERANEK, S.D., D.Sc.,(Hon.). Pp. 914+vii. John Wiley & Sons, New York and Chapman & Hall Ltd., 37, Essex St., London, W.C.2. Price 56s.

It can be said without reservation that this volume is of outstanding importance to workers in the acoustical field. Accurate and meaningful measurements in the audio-frequency region are difficult as the majority of measuring elements have dimensions which are comparable, over some portion of the spectrum, with the wavelength of the sound being measured. Free-space conditions are difficult to establish in the laboratory and the combined reaction of sound-source environment and measuring device upon the sound field make it necessary to view all acoustic measurements with suspicion.

Over the past years Dr. Beranek has made many contributions to measuring technique, particularly in obtaining correlation between the various parameters that may be used to define the performance of acoustical absorbents, and he is therefore well qualified to produce a text on the subject.

The material is divided into twenty chapters, of which the first is an historical survey which includes a refreshingly reasonable reference to the work of non-U.S. workers, and a review of acoustical terminology. This is largely based upon the proposed ASA/IRE Standards of 1949.

Chapter 2 has about eighteen pages devoted to a discussion and tabulation of the properties of the gasses which form the usual medium for the propagation of sound. In the remainder of the chapter the various wave equations and the non-linear properties of the medium are discussed.

Chapter 3 is a study of the diffraction effects that result from the introduction of an obstacle in a sound field. A thorough understanding of this is a basic necessity.

The primary technique for the measurement of 'the strength' of a plane sound wave and the calibration of microphones are collected together in Chapter 4 and this includes a most valuable discussion of Reciprocity Technique in calibration. The construction of microphones is more fully dealt with in Chapter 5, which also includes a rather brief description of that devastatingly perfect microphone, the human ear.

Six chapters are then devoted to detailed discussion of the measurement of Frequency, Acoustic Impedance, Microphones, Loudspeakers, Rooms and the properties of Acoustical Materials. The relative merits of using acoustical impedance or absorption coefficient to specify the performance of an absorbent are discussed, and it is noted that the earlier feeling in favour of specifying the acoustic impedance as a unique description of a material is not confirmed.

Other chapters, all excellent, are devoted to that deceptively complex problem, Random Noise, to Indicating and Integrating Instruments and Communication System Tests. The final chapter 'The Sound Level Meter' reviews the ASA and British standard meters and indicates some of the limitations inherent in the instruments.

As a minor point it is believed that some data on the construction and performance of 'free field' rooms would be a valuable addition. This is a subject on which the author is an acknowledged authority.

The book can be unreservedly recommended.

J. M.

Television in Your Home

By W. E. MILLER, M.A. (Cantab.). P. 64 with 30 illustrations. Iliffe & Sons Ltd., Dorset House, Stamford Street, London, S.E.1. Price 2s. (Postage 2d.).