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The Rationalized M.K.S. System

THE *Proceedings of the Institute of Radio Engineers* for July 1948, opened with a guest Editorial by S. A. Schelkunoff entitled "The End is in Sight." The end referred to is that of the age of diverse systems of units. In March the Executive Committee of the I.R.E. ratified proposals put forward by the Committees on Wave Propagation and Standards that the rationalized m.k.s. system of units should be adopted as the preferred system. Schelkunoff says that 'this action was prompted by the rapid and unmistakable trend towards universal adoption of this system both in experimental and theoretical investigations and by a desire to shorten the transition period.'

As he says, the rationalized m.k.s. system, after lying dormant for many years, has made astonishing progress during the last few years, not only among engineers but also among physicists. This has been specially noticeable in recent American text books. His concluding remark is that we can now look forward to the universal use of these units. Among the advantages of the rationalized system he states that Maxwell's equations assume a form which is merely a generalization of the one-dimensional transmission-line equations. In recent years waveguides and microwave circuits have closed the gap between circuit and line theories and field theories, and it is desirable to adopt, as far as possible, similar ideas, terminology and units. It is thought that the recommended system fulfils this requirement, and that it is also equally well adapted to electromechanical theories.

The present state of affairs with the electrostatic and electromagnetic c.g.s. systems, with or without Heaviside's suggested rationalization, with the m.k.s. system, also in two or more varieties, superposed upon them, must undoubtedly come to an end, and probably the sooner the better. The transition period will have its difficulties, especially for those who have been brought up under the old régime, who will naturally link the new with the old. We must confess that we always have a rather uneasy feeling on opening a book and finding that it uses the rationalized m.k.s. system, but a new generation may perhaps think of flux density in webers per square metre without any temptation to link it up with such an antiquated conception as lines per square centimetre.

It is probably due to the difficulty of picturing flux density, current density, displacement, etc., in terms of the square metre, that in a German text-book on electromagnetic waves published this year a rationalized practical system has been adopted with the ampere, volt, ohm, etc., as units but with the centimetre as the unit of length and as a consequence with 10 000 kilogrammes as the unit of mass. Since this is equal to ten tons, the system might be referred to as the rationalized c.tt.s. system, although it is often called the centimetre-gram-seven-second system. It was also used in Mie's "Lehrbuch der Elektrizität und des Magnetismus." In this system the unit of $\mathcal{E}$ is the volt/cm, of D the coulomb/cm^2, of H the ampere-turn/cm and of B the weber/cm^2. The permittivity $D/\mathcal{E}$ of space is then $1/(4\pi \times 9 \times 10^{11})$ F/cm, and the permeability B/H is $4\pi/10^9$ H/cm. The unit of force is 10^7 dynes (= 100 newtons) which is referred to as a sthen (sthenos = force). In the book referred to the only mention of mass is in connection with electrons and the mass of an electron is as easily expressed in ten-ton units as in kilogrammes. Although we have drawn attention to the existence of this system we are not advocating it ; it will certainly have to give way to the m.k.s. system.

Apart from rationalization, the change from the c.g.s. to the m.k.s. system involves only changes in the powers of ten and will probably have much less effect on electromagnetic calculations than one might expect at first sight. Speeds will continue to be expressed in kilometres per hour, and revolutions per minute, and capacitances in microfarads and picofarads. There is, therefore, no reason why current densities should not be expressed in amperes per square centimetre or even per square millimetre if found more convenient. Similarly, magnetic flux density can be expressed in microwebers per square centimetre or even in lines per square centimetre where a line is understood to be 10^{-8} weber. The important point is to state quite clearly what units are employed.

The question of rationalization introduces a little more difficulty. In the magnetic case we write $B = 10^{-7} \times 4\pi \times IT/l$ for empty space and in the formula $B = \mu_0 H$ we can either put $\mu_0 = 10^{-7}$ and $H = 4\pi \times$ ampere-turns per metre, or we can put $\mu_0 = 10^{-7} \times 4\pi$ and $H =$ ampere-turns per metre, which is the rationalized form. Similarly, in the electric case, for empty space we have $D = \frac{1}{9 \times 10^9} \cdot \frac{1}{4\pi} \cdot \mathcal{E}$ where $\mathcal{E}$ is in volts per metre, and if we now write $D = \frac{\kappa_0 \mathcal{E}}{4\pi}$ which is the classical form, then $\kappa_0 = \frac{1}{9 \times 10^9}$, whereas if we write $D = \kappa_0 \mathcal{E}$, then $\kappa_0 = \frac{1}{4\pi \times 9 \times 10^9}$. It should be noted that rationalization does not affect B, D or $\mathcal{E}$, but only H, μ and κ.

A consideration of H suggests the advisability of adopting rationalization. For if one gives up the c.g.s. unit, the oersted, one may as well make the complete change and adopt the ampere-turn per metre without the 4π as the unit of H. The rationalized values of κ and μ are in farads per metre and henrys per metre respectively, but for many purposes the values relative to those of empty space will be used ; that is, instead of the permittivity, the dielectric constant, and instead of the permeability, an analogous magnetic constant, which will simply be the present c.g.s. permeability. The greatest inconvenience is likely to be experienced by those engaged in magnetic-circuit design, although if the magnetization curves are already plotted to a base of ampere-turns per cm, as is often the case, it only involves shifting the decimal point.

The basis of the c.g.s. system was the force between unit charges or poles at unit distance in empty space. In the m.k.s. system this leads to impossible figures. Since 1 coulomb $= 3 \times 10^9$ e.s. units the force between two coulombs a metre apart would be $9 \times 10^{18}/10^4$ dynes, which is nearly a million tons. In the rationalized m.k.s. system the force is equal to $q_1 q_2/(4\pi\kappa_0 d^2)$ which, putting $q_1 = q_2 = 1$, $d = 1$ and $\kappa_0 = 1/(4\pi \times 9 \times 10^9)$ gives a force of 9×10^9 newtons which agrees with the above result. The force would be 1 newton in a fictitious medium having 9×10^9 times the permittivity of empty space. Since neither q nor $\mathcal{E}$ is affected by rationalization the force on the charge q in the field $\mathcal{E}$ is equal to $q\mathcal{E}$ newtons in both systems. Although not quite so simple the same is true of the force mH on a pole m in a field H, since the rationalized unit of m has a flux of 1 and not 4π webers, while the rationalized unit of H is 1 and not $1/4\pi$ ampere-turn per metre.

Rationalization does not affect the force on a conductor in a magnetic field which is IB newtons per metre, but the force between two long parallel conductors changes from $2\mu I_1 I_2/d$ to $\mu I_1 I_2/2\pi d$ because of the change in μ. It is, however, always equal to $2\mu' \, I_1 I_2/10^7 d$ where μ' is the relative permeability. The energy in a magnetic field is always equal to $10^7 B^2/8\pi\mu'$ joules per cubic metre, assuming a constant relative permeability μ', and that in an electric field is $0.5\mathcal{E}D$ which is always equal to $2\pi \times 9 \times 10^9 D^2/\kappa'$ where κ' is the dielectric constant.

A question which is exercising the minds of many teachers at present is the best method of approach when developing the fundamental principles of electromagetism to elementary students. There is much to be said for the statement made on the first page of Stratton's "Electromagnetic Theory"; viz., that 'The historical approach is recommended to the beginner, for it is the simplest and will afford him the most immediate satisfaction.' The student must be told of the c.g.s. systems because of their historical importance, and it may be that the best plan is to follow the historical method with its unit charges and unit poles up to a certain point and then change over to the rationalized m.k.s. system. Others may prefer to blot out the past and start straight away with the m.k.s. system. This is likely to cause considerable trouble because of the very small number of textbooks of electrical engineering that employ the rationalized m.k.s. system. We have certainly reached a very interesting and important stage in the development of systems of units.

G. W. O. H.

MUTUAL IMPEDANCE OF PARALLEL AERIALS

By Giorgio Barzilai, M.Sc.

SUMMARY.—Formulae are given for the mutual impedance between two vertical aerials of different lengths, terminated at a perfectly conducting plane, and assuming sinusoidal current distribution.

Using the formulae and a graphical method of integration, some calculations are carried out for aerials of different lengths. Some of the results obtained by the graphical method are compared with the corresponding values obtained by the formulae.

The values so calculated are used to investigate the behaviour of a driven aerial, with a single parasitic element of the same, and of different lengths.

1. Introduction

In this paper, some numerical values are given for the components of the mutual impedance between two vertical aerials of different lengths, assuming sinusoidal current distribution. Brown[1] gives curves for the mutual impedance between two aerials of the same length, and Aharoni[2] reports the analytical expressions for the components of this mutual impedance. However, in technical literature, it appears that no numerical values or formulae are available for the mutual impedance between aerials of different lengths.* Also for aerials of the same length, the numerical values available are usually given in the form of diagrams often so small that they give an insufficient approximation for practical calculations. Tany[3] gives tables for the components of the mutual impedance to three decimal places, but these values refer only to half-wave parallel aerials, and for the particular values of the distances between the two aerials that usually occur in arrays with many elements.

It was thought, therefore, worth while to calculate values of the components of the mutual impedance for different cases, and give these results in the form of a numerical table.

Curves relating to the behaviour of a driven aerial with a single parasitic element, both terminated at right angles by a perfectly conducting plane, and assuming sinusoidal current distribution, were given, for quarter-wave aerials, by Brown[1] and Walkinshaw[4] for a few values of the aerial spacing. No curves appear to exist for the case of a driven aerial with a parasitic element of different length.

In this paper curves are given for four different cases :—namely those of a quarter-wave driven aerial with parasitic elements in turn of one-eighth, one-quarter and one-half wavelength, and a half-wave driven aerial with a quarter-wave parasitic element. In each case curves are given for five values up to a maximum of a quarter wavelength for the element spacing.

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* About four months after this paper was accepted by the Editor, a similar paper, concerning different cases for the numerical calculations but giving the same formulæ for the mutual impedance, was published in *Proc. Inst. Radio Engrs*: C. Russel Cox "Mutual Impedance Between Vertical Antennas of Unequal Heights," Nov. 1947, Vol. 35, No. 11, p. 1367.

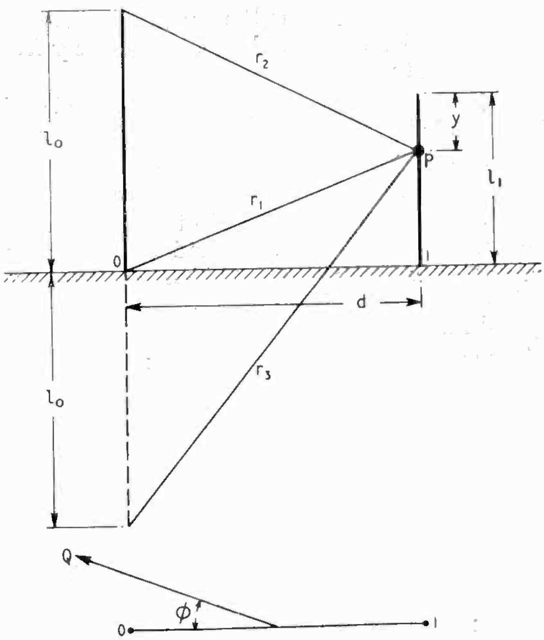

Fig. 1. *Two vertical aerials terminated by a perfectly conducting plane.*

2. Impedance Calculation

The theory of the mutual impedance between two aerials, assuming sinusoidal current distribution, has been developed by many authors,[5, 6] and for the purpose of this paper, it is only necessary to recall the expressions that define the mutual impedance.

Referring to Fig. 1 consider two vertical aerials 0 and 1, of lengths l_0 and l_1, terminated by a perfectly conducting plane. Assuming sinusoidal current distribution, the mutual impedance between the two aerials, $\mathbf{Z}_m = R_m + jX_m$, is defined by the expressions :

$$R_m = -\frac{\int_0^{l_1} \mathbf{F}_y \cdot \mathbf{I}_y \, dy}{I_0 I_1} \quad \ldots \quad \ldots \quad \ldots \quad (1)$$

$$X_m = -\frac{\int_0^{l_1} \mathbf{F}_y \, . \, j\mathbf{I}_y \, dy}{I_0 I_1} \quad \ldots \quad \ldots \quad \ldots \quad (2)$$

where : $\mathbf{I}_y = I_0 \sin my$ is the current at the point P of the aerial 1, situated at the distance y from the open end ;

$\mathbf{F}_y$ is the vertical component of the electric field, produced by the aerial 0 at the point P ;

$m = 2\pi/\lambda$

λ is the wavelength ;

I_0 and I_1 are the moduli of the currents at the antinodes of the aerials 0 and 1 respectively ; i.e., at those points for which $\sin my = 1$; and $\mathbf{F}_y \cdot \mathbf{I}_y$ is the scalar product of vectors $\mathbf{F}_y$ and $\mathbf{I}_y$.

The expression for the field $\mathbf{F}_y$ is[1] :

$$\mathbf{F}_y = -30\, I_0 \left(-\frac{\sin mr_1}{r_1} 2 \cos ml_0 + \frac{\sin mr_2}{r_2} + \frac{\sin mr_3}{r_3}\right) + j\left(-\frac{\cos mr_1}{r_1} 2 \cos ml_0 + \frac{\cos mr_2}{r_2} + \frac{\cos mr_3}{r_3}\right) \quad (3)$$

Where : r_1, r_2, r_3 are the distances as indicated in Fig. 1, and their analytical expressions are :

$$r_1 = \sqrt{d^2 + (l_1 - y)^2} \; ;$$

$$r_2 = \sqrt{d^2 + (l_1 - l_0 - y)^2} \; ;$$

$$r_3 = \sqrt{d^2 + (l_1 + l_0 - y)^2},$$

where d is the distance between the two aerials.

Substituting expression (3) for $\mathbf{F}_y$ in equations (1) and (2) the following expressions for R_m and X_m are obtained :

$$R_m = 30\int_0^{l_1}\left(-\frac{\sin mr_1}{r_1} 2 \cos ml_0 + \frac{\sin mr_2}{r_2} + \frac{\sin mr_3}{r_3}\right) \sin my \, dy \quad \ldots \quad \ldots \quad \ldots \quad (4)$$

$$X_m = 30\int_0^{l_1}\left(-\frac{\cos mr_1}{r_1} 2 \cos ml_0 + \frac{\cos mr_2}{r_2} + \frac{\cos mr_3}{r_3}\right) \sin my \, dy \quad \ldots \quad \ldots \quad \ldots \quad (5)$$

The evaluation of the integrals appearing in equations (4) and (5) can be carried out in terms of the integral sine and cosine functions[2]. After the evaluation, the expressions for R_m and X_m become :

$$\begin{aligned} R_m = 15\{&\cos m(l_1 - l_0)\,[-\mathrm{Ci}\,(K_1) + 2\mathrm{Ci}\,(K_2) - \mathrm{Ci}\,(K_3) + \mathrm{Ci}\,(K_4) - \mathrm{Ci}\,(K_5) + \mathrm{Ci}\,(K_6) - \mathrm{Ci}\,(K_7)] + \\ &\cos m(l_1 + l_0)\,[-\mathrm{Ci}\,(K_1) + 2\mathrm{Ci}\,(K_2) - \mathrm{Ci}\,(K_3) - \mathrm{Ci}\,(K_5) - \mathrm{Ci}\,(K_7) + \mathrm{Ci}\,(K_8) + \mathrm{Ci}\,(K_9)] + \\ &\sin m(l_1 - l_0)\,[+\mathrm{Si}\,(K_1) - \mathrm{Si}\,(K_3) - \mathrm{Si}\,(K_4) - \mathrm{Si}\,(K_5) + \mathrm{Si}\,(K_6) + \mathrm{Si}\,(K_7)] + \\ &\sin m(l_1 + l_0)\,[+\mathrm{Si}\,(K_1) - \mathrm{Si}\,(K_3) + \mathrm{Si}\,(K_5) - \mathrm{Si}\,(K_7) - \mathrm{Si}\,(K_8) + \mathrm{Si}\,(K_9)]\} \quad \ldots \quad (6) \end{aligned}$$

$$\begin{aligned} X_m = 15\{&-\cos m(l_1 - l_0)\,[-\mathrm{Si}\,(K_1) + 2\mathrm{Si}\,(K_2) - \mathrm{Si}\,(K_3) + \mathrm{Si}\,(K_4) - \mathrm{Si}\,(K_5) + \mathrm{Si}\,(K_6) - \mathrm{Si}\,(K_7)] - \\ &\cos m(l_1 + l_0)\,[-\mathrm{Si}\,(K_1) + 2\mathrm{Si}\,(K_2) - \mathrm{Si}\,(K_3) - \mathrm{Si}\,(K_5) - \mathrm{Si}\,(K_7) + \mathrm{Si}\,(K_8) + \mathrm{Si}\,(K_9)] + \\ &\sin m(l_1 - l_0)\,[+\mathrm{Ci}\,(K_1) - \mathrm{Ci}\,(K_3) - \mathrm{Ci}\,(K_4) - \mathrm{Ci}\,(K_5) + \mathrm{Ci}\,(K_6) + \mathrm{Ci}\,(K_7)] + \\ &\sin m(l_1 - l_0)\,[+\mathrm{Ci}\,(K_1) - \mathrm{Ci}\,(K_3) + \mathrm{Ci}\,(K_5) - \mathrm{Ci}\,(K_7) - \mathrm{Ci}\,(K_8) + \mathrm{Ci}\,(K_9)]\} \quad \ldots \quad (7) \end{aligned}$$

where :

$$\mathrm{Si}\,(x) = \int_0^x \frac{\sin u}{u}\,du \qquad \mathrm{Ci}\,(x) = -\int_x^\infty \frac{\cos u}{u}\,du$$

and

$$\begin{matrix} K_1 \\ K_3 \end{matrix} = m\left(\sqrt{d_1^2 + l_1^2} \mp l_1\right) ;$$

$$\begin{matrix} K_4 \\ K_6 \end{matrix} = m\left[\sqrt{d^2 + (l_1 - l_0)^2} \mp (l_1 - l_0)\right] ;$$

$$\begin{matrix} K_5 \\ K_7 \end{matrix} = m\left(\sqrt{d^2 + l_0^2} \mp l_0\right)$$

$$\begin{matrix} K_8 \\ K_9 \end{matrix} = m\left[\sqrt{d^2 + (l_1 + l_0)^2} \mp (l_1 + l_0)\right]$$

$$K_2 = md$$

It is easy to verify, by interchanging l_0 with l_1, that equations (6) and (7) do not change. The equations are therefore consistent with the reciprocal relation existing between the two aerials. By assuming $l_0 = l_1$, formulae (6) and (7) become those given by Aharoni[2] for two aerials of the same length.

For $d = 0$ formulae (6) and (7) fail. To find the limits of R_m and X_m for $d = 0$, it is only necessary to remember that, as x approaches zero, the following equations hold :

$$\mathrm{Ci}(x) = E + \log x \; ; \; \sqrt{x^2 + a^2} - a = x^2/2a$$

where $E = 0.57721..$ is the Euler's constant, and a is a positive number.

To calculate R_m and X_m using formulae (6) and (7), is usually very laborious and, for practical purposes, a graphical method of integration gives an adequate approximation. An easy way to evaluate the integrals appearing in equations (4) and (5) is the following : Suppose aerial 1 of Fig. 1 to be divided into q equal segments of length Δl_1. If Δl_1 is sufficiently small, it can be assumed that the functions $\frac{\sin mr}{r}$ and $\frac{\cos mr}{r}$

($r = r_1, r_2, r_3$) are constant along each segment, and their values are equal to the values that these functions assume at the centre of the segment concerned. With such an approximation, the integrals appearing in equations (4) and (5) can be evaluated as follows :

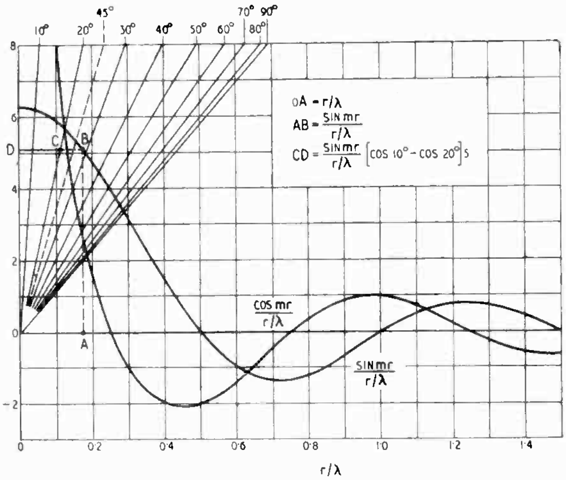

Fig. 2. *Diagram for graphical evaluation of the integrals appearing in equations (4) and (5).*

$$\int_0^{l_1} \frac{\sin mr}{r} \sin my \, dy =$$

$$\sum_{p=1}^{p=q} \left[\frac{\sin mr}{r/\lambda} \right]_{y=(p-\frac{1}{2})\Delta l_1} (\cos m(p-1)\Delta l_1 - \cos mp\Delta l_1) \quad .. \quad .. \quad (8)$$

in which the value of r in $\left[\frac{\sin mr}{r/\lambda}\right]$ must correspond to the midpoint of each segment, that is to $y = (p - \frac{1}{2})\Delta l_1$. Similar expressions can be obtained for the integrals containing the functions $\frac{\cos mr}{r}$

The various operations indicated in equation (8) can be carried out very quickly by graphical methods. Draw the two aerials in their actual relative position as in Fig. 1, assuming for the lengths a certain scale u (u units of length $= \lambda$). The three distances r_1/λ, r_2/λ, r_3/λ, corresponding to the centre of each division, can now be easily recorded on a strip of paper. Plot the two functions $\frac{\sin mr}{r/\lambda}$ and $\frac{\cos mr}{r/\lambda}$ against r/λ, assuming for the abscissae the scale u, and for the ordinate a new scale v (v units of length $= 1$), and draw, starting from the origin, q straight lines forming with the axis of ordinates an angle α_p such that :

$$\tan \alpha_p = \cos m(p-1)\Delta l_1 - \cos mp\Delta l_1 \quad (9)$$

$$p = 1, 2, \ldots\ldots, q.$$

Now using the strip of paper, on which the distances r/λ were recorded, it is easy to evaluate the single terms of the sum indicated in equation (8), as shown in Fig. 2. This diagram is referred to $\Delta l_1 = \frac{\lambda}{36}$ ($m\Delta l_1 = 10°$) and consequently only nine straight lines were drawn ; $\tan \alpha$ was made five times that resulting from equation (9), in order to simplify the graphical evaluation. Obviously the final result must be divided by five. Each straight line is labelled with the number of degrees corresponding to $pm\Delta l_1$ (10°, 20°, ..., 90°). The figure shows the calculation of the term corresponding to the second segment ($p = 2$, $mp\Delta l_1 = 20°$). In the original drawing, the following scales were chosen: $u = 400$ mm, $v = 40$ mm. Therefore the number of millimeters representing the sum of the three integrals appearing in equations (4) and (5) must be multiplied by $30 \frac{1}{v} \cdot \frac{1}{2\pi} \cdot \frac{1}{5} = 0.0239$, in order to obtain the values of R_m and X_m in ohms.

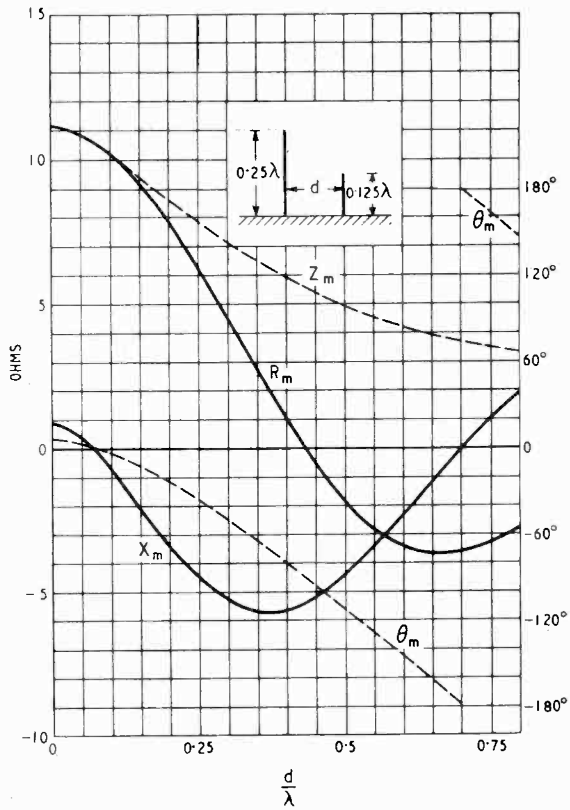

Fig. 3. *Resistive and reactive components, R_m and X_m, modulus Z_m, and phase angle θ_m, of the mutual impedance between two vertical aerials of lengths $l_0/\lambda = 0.25$ and $l_1/\lambda = 0.125$, against the distance d/λ (refer to Fig. 1).*

The scale u is only important for the accuracy of the calculation, and it does not enter in the constant of the diagram.

It will be noted that the diagram of Fig. 2 can be used to calculate the mutual impedance between two parallel aerials of any length provided l_1 is an integral multiple of $\lambda/36$. If l_1 is not an integral multiple of $\lambda/36$ an additional straight line must be added in order to take into account the contribution of the last segment, of length $\Delta'l_1 < \Delta l_1$ ending at the point $y = l_1$. On the diagram of Fig. 2 such a straight line (dotted line labelled 45°) was drawn for $\Delta'l_1 = \lambda/72$ ($m\Delta'l_1 = 45°$) in order to calculate the mutual impedance when $l_1 = \lambda/8$.

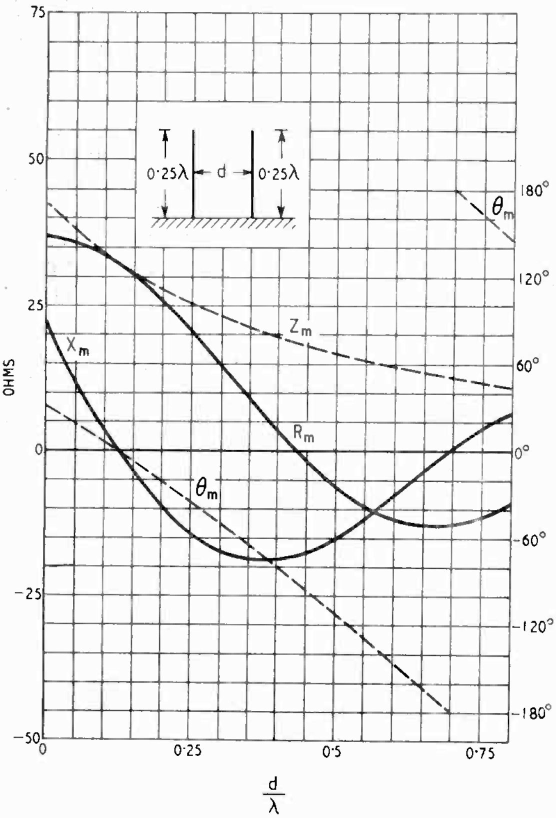

Fig. 4. *The same as Fig. 3, but for $l_0/\lambda = 0.25$ and $l_1/\lambda = 0.25$.*

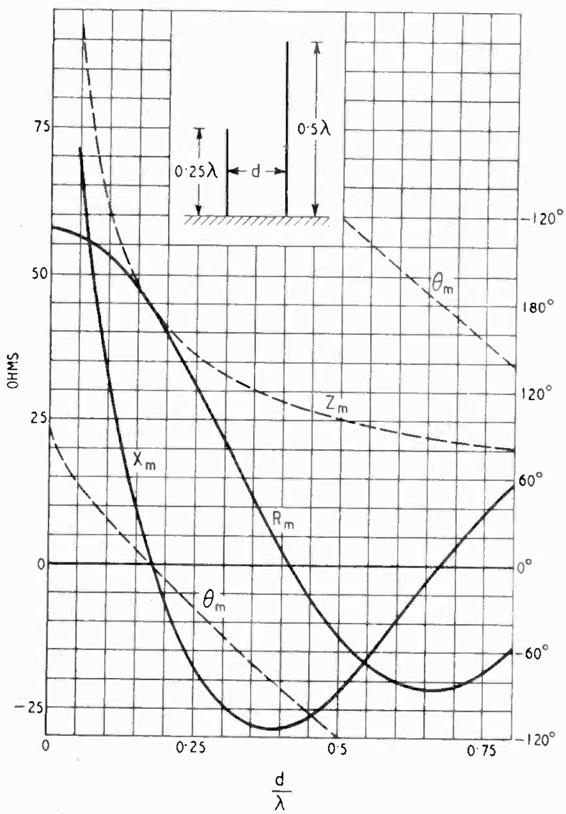

Fig. 5. *The same as Fig. 3, but for l_0/λ 0.25 and $l_1/\lambda = 0.5$.*

If the perfectly conducting plane, in Fig. 1, is removed, and the electrical images of the two aerials are substituted by two real aerials carrying the same currents as the images, the integrals appearing in equations (4) and (5) double their values, and therefore the values of R_m and X_m, calculated by formulae (6) and (7) or by the graphical method, must be multiplied by 2. The diagram of Fig. 2 can be used to calculate the mutual impedance between two parallel aerials ending at different levels, provided the current distributions are symmetrical with respect to the middle points.

Using formulae (6) and (7), and the diagram of Fig. 2, some calculations have been carried out. The results of these calculations are recorded in Table 1. As can be seen from this table, the approximation of the graphical method is good enough for practical purposes. The poorer approximation obtained for X_m when $l_0/\lambda = 0.25$, $l_1/\lambda = 0.50$ and $d/\lambda = 0.05$ or $d/\lambda = 0.10$ is due to the height gradient of the function $\frac{\cos mr}{r/\lambda}$ for small values of r/λ, corresponding to the elements of aerial 1 carrying a heavy current. In other words the approximation is poor when there are terms in the sum (8) which are very predominant. The quantities R_m, X_m, Z_m (modulus of $\mathbf{Z}_m$) and $\theta_m = \tan^{-1} X_m/R_m$, are plotted against d/λ in Figs. 3, 4 and 5, for the values of l_0 and l_1 indicated. It must be remembered that the values of R_m and X_m are referred to the currents at the antinodes ($\sin my = 1$), and therefore when $l_1/\lambda = 0.125$, they are referred to a value of current that does not exist on aerial 1.

From Table I it can be seen that there are cases in which X_m does not have a finite limit for $d = 0$. The mathematical reason for this behaviour can easily be understood. The integrals appearing in equation (5) are not convergent, when in the interval of integration there is a value of y for which the integrating

TABLE I

			Values obtained by formulae (6) and (7)		Values obtained by diagram of Fig. 2	
l_2/λ	l_1/λ	d/λ	R_m	X_m	R_m	X_m
0.125	0.125	0.00	3.36	− ∞	3.2	
,,	,,	0.05	3.29	− 11.31		
,,	,,	0.10	3.10	− 3.79		
,,	,,	0.15	2.79	− 2.02		
,,	,,	0.20	2.38	− 1.66		
,,	,,	0.25	1.90	− 1.69		
0.25	0.25	0.00	36.56*	21.27*		
,,	,,	0.05	35.83	12.13		
,,	,,	0.10	33.67	3.77		
,,	,,	0.15	30.22	− 3.55		
,,	,,	0.20	25.70	− 9.59		
,,	,,	0.25	20.39*	− 14.18*		
0.375	0.375	0.00	92,19	+ ∞		
,,	,,	0.05	90.97	62.96		
,,	,,	0.10	85.26	25.92		
,,	,,	0.15	76.16	0.01		
,,	,,	0.20	64.33	− 19.24		
,,	,,	0.25	50.39	− 33.16		
0.50	0.50	0.00	99.54*	62.71*		
,,	,,	0.05	97.33	35.41		
,,	,,	0.10	90.86	10.66		
,,	,,	0.15	80.55	− 10.72		
,,	,,	0.20	67.11	− 28.06		
,,	,,	0.25	51.42*	− 40.88*	51.6	− 40.5
0.25	0.50	0.00	58.22	+ ∞		
,,	,,	0.05	56.97	73.18	56.5	71.7
,,	,,	0.10	53.30	31.75	53.3	30.6
,,	,,	1.15	47.46	8.03	47.5	8.6
,,	,,	0.20	39.83	− 7.75	40.2	− 7.7
,,	,,	0.25	30.90	− 18.40	30.7	− 18.6
,,	,,	0.35			11.4	− 28.1
,,	,,	0.45			− 6.2	− 26.3
,,	,,	0.50			− 13.1	− 22.2
,,	,,	0.60			− 20.8	− 9.6
,,	,,	0.70			− 20.8	+ 3.3
,,	,,	0.80			− 14.4	+ 13.8
0.25	0.125	0.00	11.08	0.81		
,,	,,	0.05	10.86	0.38	10.8	0.3
,,	,,	0.10	10.20	− 0.72	10.2	− 0.8
,,	,,	0.15	9.16	− 2.12	9.0	− 2.3
,,	,,	c.20	7.82	− 3.48	7.8	− 3.5
,,	,,	0.25	6.22	− 4.62	6.2	− 4.6
,,	,,	0.35			2.7	− 5.7
,,	,,	0.45			− 0.5	− 5.3
,,	,,	0.50			− 1.9	− 4.5
,,	,,	0.60			− 3.5	− 2.6
,,	,,	0.70			− 3.6	− 0.1
,,	,,	0.80			− 2.8	+ 1.8

* These values have been calculated by Tany[3].

Note. The integral sine and cosine tables used are those calculated by Tany[7].

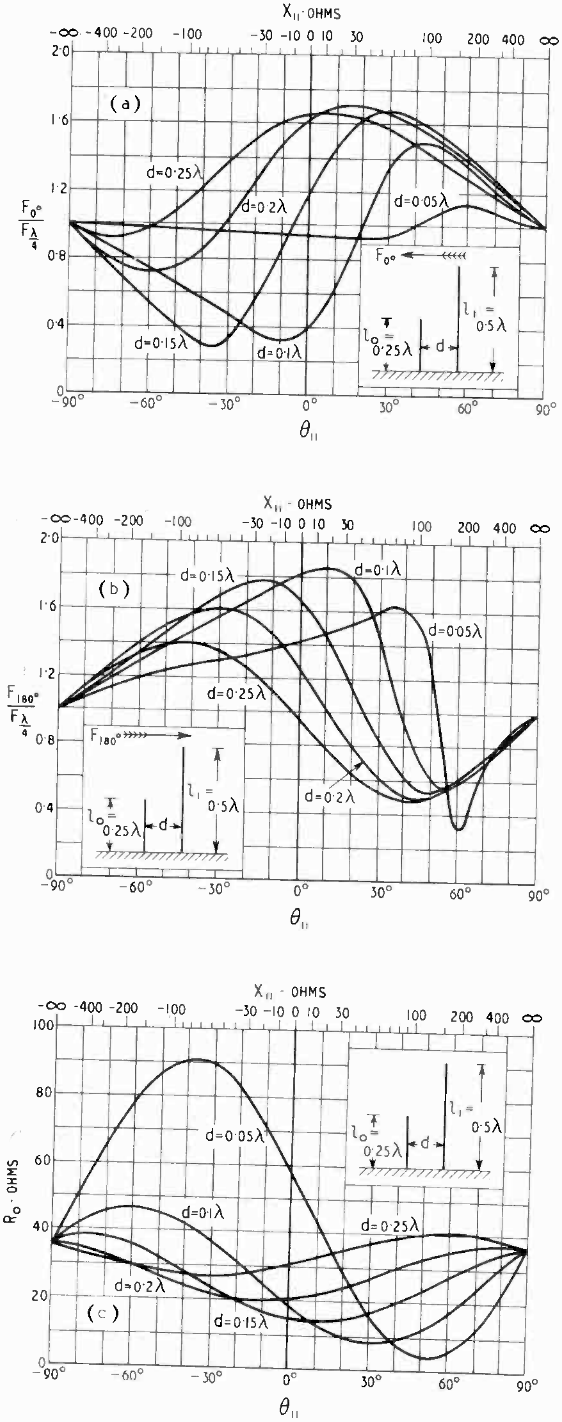

Fig. 9. *The same as Fig. 7, but for $l_0/\lambda = 0.25$ and $l_1/\lambda = 0.5$. When $l_0/\lambda = 0.5$ and $l_1/\lambda = 0.25$ the values of the input resistances of the driven aerial must be multiplied by 2.72. In this case the X_{11} scale is no longer valid.*

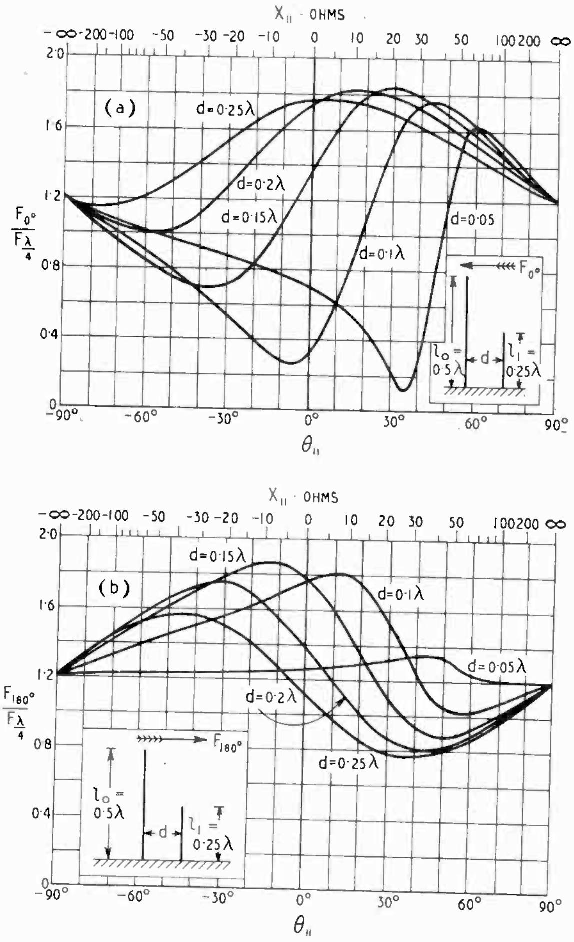

Fig. 10. *The same as Fig. 7, but for $l_0/\lambda = 0.5$ and $l_1/\lambda = 0.25$. For the values of the input resistances see Fig. 9 (c).*

The quantities R_0, $F\phi/F_{\lambda/4}$, for $\phi = 0°$ and $\phi = 180°$, were calculated for the following cases: $l_0/\lambda = 0{\cdot}25$, $l_1/\lambda = 0{\cdot}125$; $l_0/\lambda = 0{\cdot}25$, $l_1/\lambda = 0{\cdot}25$; $l_0/\lambda = 0{\cdot}25$, $l_1/\lambda = 0{\cdot}50$; $l_0'/\lambda = 0{\cdot}50$, $l_1/\lambda = 0{\cdot}25$; for five values of d/λ, and assuming θ_{11} as variable. The results of these calculations are shown in Figs. 7, 8, 9 and 10, respectively. On these diagrams, the upper scale gives the values of X_{11} in ohms, corresponding to the values of θ_{11}. The values of R_0 relating to the case of $l_0/\lambda = 0{\cdot}50$, $l_1/\lambda = 0{\cdot}25$, are easily obtained from Fig. 9(c) by multiplying the ordinate scale by $99{\cdot}54/36{\cdot}56 = 2{\cdot}72$, as obtained from equation (13) The values of η, for the cases of Figs. 7, 8, and 9, are respectively: $\eta = \int_0^{45°} \sin \alpha \, d\alpha = 0{\cdot}293$, $\eta = 1$, $\eta = 2$. For the case $l_1/\lambda = 0{\cdot}5$ and $l_1/\lambda =$

0·25, η = 0·5 and the resulting gains must be multiplied by two.

The reciprocal case of Fig. 7 (l_0/λ = 0·125, l_1/λ = 0·25) was not developed, as it leads to very low values of the input resistance of the driven aerial, and is not of practical interest. The input resistances, in such a case, are obtained from Fig. 7(c), by multiplying the ordinate scale by 3·36/36·56 = 0·092.

From the symmetry of the system of Fig. 1, with respect to the plane of the two aerials, it follows that the directions $\phi = 180°$ and $\phi = 0°$ always correspond to a maximum or minimum field. It is clear that in order to have a maximum for $0° < \phi < 180°$, it is necessary that the phase shift of $\mathbf{I}_1$ with respect to $\mathbf{I}_0$, be less than md. In this case there will exist an angle ϕ such that the resulting field is proportional to the sum of the moduli of the two currents, and therefore it is the maximum obtainable field for those values of currents. From equation (14) it can be seen that the said angle is given by the equation :

$$md \cos \phi = \beta \qquad \ldots \quad \ldots (16)$$

Usually condition (16) cannot be satisfied if the two aerials are too close (closer then 0·20λ — 0·25λ), but when satisfied the corresponding values of θ_{11} are very large ; i.e., the current I_1 is very small, as is shown by equation (12).

In the cases to which Figs. 7, 8, 9 and 10 relate, condition (16) can be satisfied in the following instances :

l_0/λ = 0·25; l_1/λ = 0·125; d/λ=0·25; $\theta_{11} > 53°20'$
,, ,, d/λ=0·20; $\theta_{11} > 84°0'$
,, l_1/λ = 0·25; d/λ=0·25; $\theta_{11} > 55°10'$
,, ,, d/λ=0·20; $\theta_{11} > 87°30'$
l_0/λ or l_1/λ = 0·25; l_0/λ or l_1/λ = 0.50; d/λ=0·25 $\theta_{11} > 59°10'$

In all other cases condition (16) cannot be satisfied for any one value of θ_{11}. The gains corresponding to the cases for which condition (16) is satisfied are very small, and furthermore, they are smaller than the maximum gains relating to the same distance, for $\phi = 0°$ and $\phi = 180°$ as could be seen by calculating the currents by equation (12). Therefore it can be concluded that in the case to which Figs. 7, 8, 9 and 10 relate, the maximum gains obtainable, for each value of d, correspond to $\phi = 0°$ and $\phi = 180°$. It is interesting to note that, referring to Fig. 1, the theoretical behaviour in the case of l_0/λ = 0·50, l_1/λ = 0·50 can be considered the same as that in the case of l_0/λ = 0·25, l_1/λ = 0·25, except that the resistances R_0 must be multiplied by 99·54/36·56 = 2·72, and the gains $F\phi/F_{\lambda/4}$ must be multiplied by $2/\sqrt{2\cdot72}$ = 1·21. This can be understood by referring to the curves of Fig. 11. From these curves, it can be seen that the angles, θ_m, and the ratios Z_m/R_{11}, corresponding to the same value of d/λ, differ very little in the two cases. Therefore what is stated above follows from equations (13) and (14).

From the above curves it can be seen that usually the maximum theoretical gains decrease with the effective height of the aerials. However, when l_0/λ = 0·25 and l_1/λ = 0·125, the maximum theoretical gains are very near to those obtained for l_0/λ = 0·25 and l_1/λ = 0·25, in spite of the very low effective height of the parasitic element. The very low radiation resistance of the parasitic element makes, it however, difficult to obtain practical gains near to the theoretical.

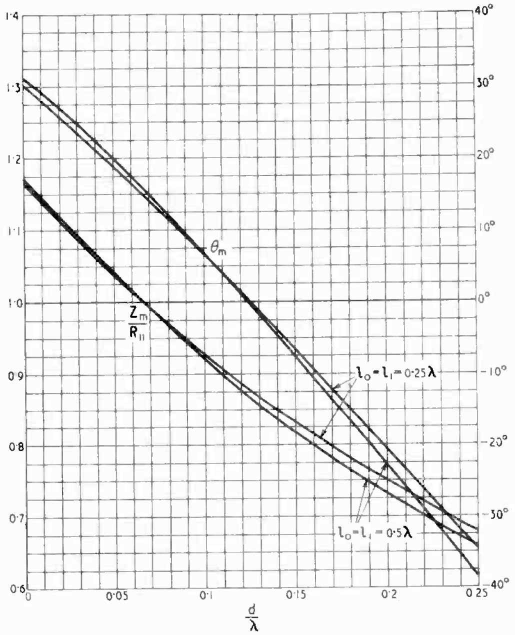

Fig. 11. *Diagram to demonstrate that the behaviour of a driven aerial, with a parasitic element, both half wavelength, can be considered the same as for the case in which both aerials are of a quarter wavelength, except that gains and input resistances must be multiplied by constants.*

It is interesting to note the peculiar behaviour of the systems to which Figs. 9 and 10 relate, when d/λ = 0·05. In these two cases for $\phi = 0°$ and $\phi = 180°$, respectively, the gain is almost constant, and equal to 1, irrespective of the tuning condition of the parasite, while the gains in the opposite direction vary considerably with θ_{11}.

A practical application of the curves of Figs. 7 and 9 could be the following : Suppose a radio station to be working, but not simultaneously, on two wavelengths, one being double the other. Also suppose that it is required, on both wavelengths, to concentrate the power in the same pulse centre, and it is assumed that this quantity is adjustable at will. This assumption corresponds to an adjustment of the transit angle between the control and output fields in practical valves.

Throughout the analysis it is assumed that the effect of space charge in modifying space potentials is negligible.

3. Preliminary Results

(a) *The equations to be solved.*

The equation of motion of an electron between the electrodes is :

$$\frac{d^2x}{dt^2} = \frac{eE_0 \sin \omega t}{md} \quad \ldots \quad (1)$$

which on integration and rearrangement yields the two following fundamental formulae.

$$\frac{v_2}{v_1}\frac{2\phi}{M} = \cos \tau - \cos \tau_2 + \frac{2\phi}{M} \quad \ldots \quad (2)$$

$$\frac{2\phi^2}{M} = \left(\frac{2\phi}{M} + \cos \tau_1\right)(\tau_2 - \tau_1) - \sin \tau_2 + \sin \tau_1 \quad \ldots \quad (3)$$

The quantity ϕ which appears here is the d.c. transit angle ($= \omega d/v$) and is the value that the difference $(\tau_2 - \tau_1)$ would have for zero depth of modulation M. It is remarkable that variations in any of the three quantities, input velocity of electrons, frequency and distance between electrodes can be summed up by the use of this one parameter.

(b) *Relation between i_1 and i_2.*

Consider the charge $i_1\delta t_1$ passing unit area of electrode 1 during the interval of time δt_1. This charge will travel across the space and arrive at electrode 2 and exit during a time interval δt_2 ; i.e., $i_1\delta t_1 = i_2\delta t_2$. If now the time intervals δt_1, δt_2 tend to zero, we have,

$$i_2 = i_1\frac{dt_1}{dt_2} = i_1\frac{d\tau_1}{d\tau_2} \quad \ldots \quad (4)$$

and from equation (3) :

$$i_2 = \frac{2\phi/M + \cos \tau_1 - \cos \tau_2}{2\phi/M + (\tau_2 - \tau_1)\sin \tau_1} i_1 \quad \ldots \quad (5)$$

(c) *Proximity effect of Electrons.*

There is no reason to suppose that any of the predicted theories of electron behaviour are valid in cases where the electron paths cross each other. Proximity effects are then produced, and the present theory, like all those based on the classic assumptions, must be treated with reserve. It will be shown that according to the assumptions of Section 3 it is possible for the electron trajectories to coincide, and for electrons to meet and for two electrons to occupy the same point in space at the same time. This is a palpable absurdity, but it so happens that the conditions of operation which are of immediate engineering interest do not involve this condition ; the remainder of this analysis, therefore, is confined to those cases where individual electrons are not brought into close proximity one to the other. It is therefore necessary to specify the conditions under which any electron in the stream cannot overtake any other electron.

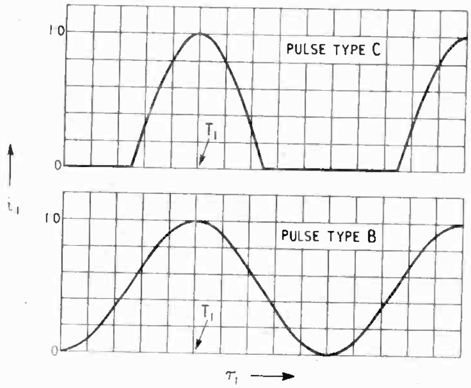

Fig. 1, *Graph of i_1 against τ_1 for two types of pulse.*

The present analysis commences with the conditions expressed in equation (3), namely :—

$$\frac{2\phi^2}{M} = \left(\frac{2\phi}{M} + \cos \tau_1\right)(\tau_2 - \tau_1) - \sin \tau_2 + \sin \tau_1 \quad \ldots \quad (3)$$

We require to determine the value of M which just causes the convection current density to be infinite at the plane of the second plate, and which does not permit of this quantity becoming infinite between the plates. Hence $i_2 = \infty$, and from equations (4) and (5),

$$d\tau_2/d\tau_1 = 0$$

$$F = \frac{2\phi}{M} + (\tau_2 - \tau_1)\sin \tau_1 = 0 \quad \ldots \quad (6)$$

and

$$\frac{2\phi}{M} + \cos \tau_1 - \cos \tau_2 \neq 0 \quad \ldots \quad (7)$$

It is further necessary that equation (6) should have a double root to impose the condition that the convection current density is finite between the plates.

$$\frac{dF}{d\tau_1} = \sin \tau_1\left(\frac{d\tau_2}{d\tau_1} - 1\right) + \cos \tau_1 (\tau_2 - \tau_1) = 0 \quad (8)$$

i.e.,

$$\tau_2 - \tau_1 = \tan \tau_1 \quad \ldots \quad (9)$$

From equations (6) and (9) we have :

$$2\phi/M = -\sin \tau_1 \tan \tau_1 \quad \ldots \quad (10)$$

and from equations (3) and (9) we have :

$$2\phi^2/M = 2 \sin \tau_1 - \sin \tau_1 \tan^2\tau_1 - \sin \tau_2 \quad \ldots \quad (11)$$

ϕ and M, from equations (10) and (11) may be plotted in terms of τ_1, and hence M in terms of ϕ (Fig. 2). It may be observed that for large values of $\tan \tau_1$

$$2\phi^2/M \approx -\sin \tau_1 \tan^2 \tau_1 \approx \tan^3 \tau_1 \quad \ldots \quad (12)$$

and $M \to 2$ as $\phi \to \infty$.

It is therefore unnecessary for the purposes of the present analysis to consider the values of M exceeding those plotted in Fig. 2.

4. Energy Transfer

In general, the electrodes 1 and 2 are connected to a tuned circuit. The system, consisting of the space between the electrodes, the electrodes themselves, and this external circuit, will be denoted by S. If now a pulse of electrons of one of the types already defined enters the space between the electrodes at electrode 1, it will involve the transfer of an amount of energy W_1 per cycle into the system S. By virtue of the alternating field between the electrodes there is interaction between S and the pulse, and the pulse will reach electrode 2 and leave the space with an energy W_2 per cycle differing in general from W_1. It is assumed at this stage that W_2 is converted into heat or d.c. electrical energy.

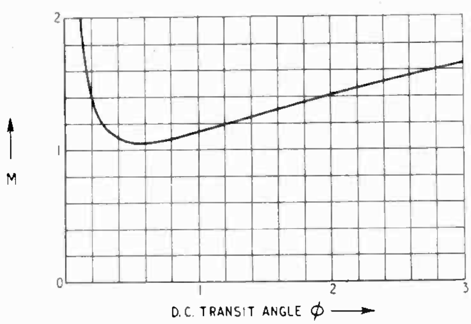

Fig. 2. *Graph of M against φ for no crossing of electrons but infinite current density at electrode 2.*

Clearly it is an object of practical design to make this latter energy W_2 less than the input energy W_1. The efficiency of transfer of energy from the pulse to the system S is then

$$\eta = (W_1 - W_2)/W_1 \quad \ldots \quad (13)$$

The amount of energy $(W_1 - W_2)$ per cycle which is transferred from the pulse to the system S is entirely alternating, but we do not discriminate between energy in the space between the plates and that in the external circuit.

In some previous papers dealing with this subject, the power delivered by the pulse has been calculated by obtaining a value for the current induced in the external circuit from the formula

$$i = ev/d$$

giving the current induced by one electron moving with velocity v perpendicular to the plane of the plates. This gives the energy transfer per cycle. It seems to the writer, however, that the determination of the energy transfer as the difference between the input and output

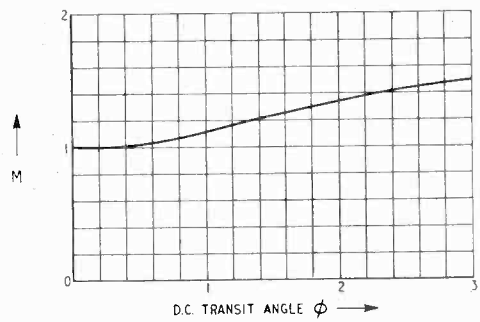

Fig. 3. *Graph of M against φ for maximum efficiency.*

energies is the more straightforward procedure, giving a clearer picture of the phenomenon. This is the method adopted here.

W_1, W_2, and hence η, are functions of ϕ, M, T_1, and f, and the remainder of this section is concerned with evaluating η as a function of ϕ, M and T_1, for two types of pulse C and B. In particular, it is an object to find the maximum value of η, η_{max}, as a function of ϕ, and the corresponding values of M and T_1, also as functions of ϕ. The value of M will, of course, be limited by the conditions of the last section.

Consider, now, the electrons passing electrode 1 during the time interval δt_1. The energy transferred into the space between the electrodes is

$$\delta W_1 = \tfrac{1}{2} N_1 m v_1^2 \, \delta t_1$$

Where N_1 = number of electrons passing electrode 1 per second ;

i.e.,

$$N_1 = i_1/e$$

Thus

$$\delta W_1 = \frac{m}{2e} i_1 v_1^2 \, \delta t_1 = \frac{m}{2e\omega} i_1 v_1^2 \, \delta\tau_1$$

Therefore

$$W_1 = \frac{m}{2e\omega} \int_0^{2\pi} i_1 v_1^2 \, d\tau_1 \quad \ldots \quad (14)$$

Similarly,

$$W_2 = \frac{m}{2e\omega} \int_0^{2\pi} i_2 v_2^2 \, d\tau_2 = \frac{m}{2e\omega} \int_0^{2\pi} i_1 v_2^2 \, d\tau_1 \quad (15)$$

by equation (4).

Hence,

$$\eta = \frac{\int_0^{2\pi} i_1 \left\{ 1 - \left(\frac{v_2}{v_1} \right)^2 \right\} d\tau}{\int_0^{2\pi} i_1 \, d\tau_1} \quad \ldots \quad \ldots \quad \ldots \quad (16)$$

The following is the procedure for evaluating this quantity :—

Choosing particular values of ϕ and M, τ_2 is obtained graphically from equation (3) by the method given by Kompfner[3]. τ_2 is tabulated against τ_1. v_2/v_1 can now be calculated from equation (2), and hence $\{1 - (v_2/v_1)^2\}$ can be tabulated against τ_1. The integrand of the numerator is tabulated and plotted against τ_1 for various values of T_1. The integral is then evaluated graphically using a planimeter. In fact, an approximate value of T_1 for maximum η is readily seen on inspection of the curve for $\{1 - (v_2/v_1)^2\}$, and it is not necessary to perform the integration for more than, say, three values of T_1.

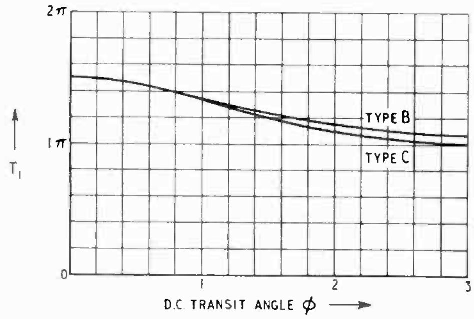

Fig. 4. *Graph of T_1 against ϕ for two types of pulse.*

The process outlined above is repeated for various values of M for each value of ϕ chosen (in this case $\phi = \frac{1}{2}, 1, 1\frac{1}{2}, 2, 2\frac{1}{2}, 3$), and the values of M and T_1 to yield maximum values of η determined for each value of ϕ. These results are summarized in Figs. 3, 4 and 5.

As an example, the complete computation for $\phi = 2\frac{1}{2}$ is given. Five values of M were chosen :

(1) 1.00; (2) 1.11; (3) 1.25; (4) 1.43; (5) 1.67.

(The fifth value exceeds that obtained from Fig. 2. It is given for the purpose of illustration.)

The graph for the solution of equation (3) giving τ_2 in terms of τ_1 is given for the one case $M = 1.67$, and the 'crossing' of the electron 'trajectories' is clearly shown in Fig. 6. The graphs for the other four values of M are similar, but there is no 'crossing.' The Table shows the method of computation of $\{1 - (v_2/v_1)^2\}$ again for the case $M = 1.67$. Others are similar.

Fig. 7 shows curves for $\{1 - (v_2/v_1)^2\}$ against τ_1 for the five values of M under consideration. It is evident from inspection of these that in order to obtain maximum *positive* value for the integral

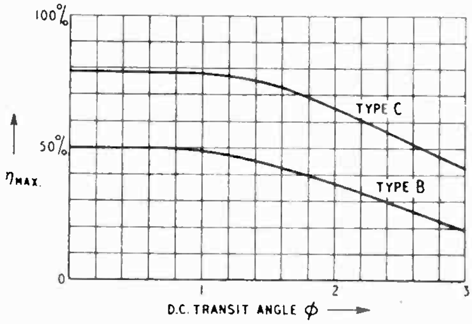

Fig. 5. *Graph of η_{max} against ϕ for two types of pulse.*

$\int_0^{2\pi} i_1\{1 - (v_2/v_1)^2\} d\tau_1$ the peak of the i_1 curve must be near to the peak of the curve for $\{1 - (v_2/v_1)^2\}$; i.e., $T_1 \approx \pi$. It is further evident that the maximum value of the integral will not be attained for the two values of M, 1.0 and 1.11 and the case 1.67 is ruled out as leading to crossing of the electrons. Figs. 8 and 9 show curves for the final stages of the computation.

It should be mentioned that the restriction to the case where there is no crossing of electrons is not in fact serious, as in the particular case of $\phi = 2.5$, η is not increased by increasing M from 1.43 to 1.67, but remains almost unchanged.

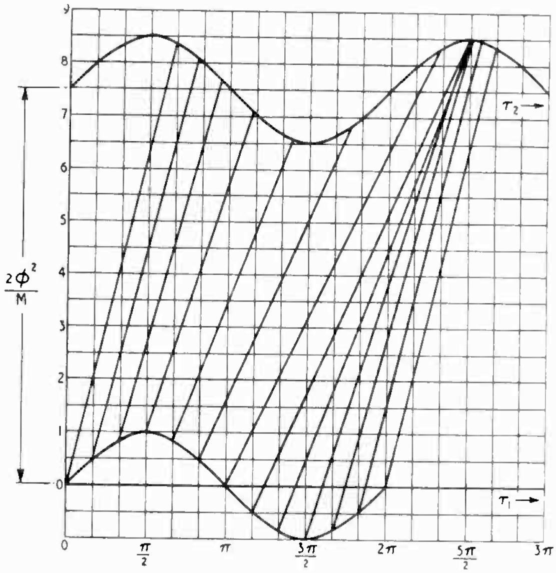

Fig. 6. *Solution of Equation* (3) *for* $\phi = 2.5$; $M = 1.67$.

This general state of affairs was found to hold for values of ϕ in the range $0.5 < \phi < 3.0$.

5. Secondary-Electron Effects

A value for efficiency has been obtained in the last section without taking into consideration any secondary electrons which may be radiated from either of the electrodes. The phenomenon of secondary radiation is extremely complex[4]. It is nevertheless believed that some at least qualitative idea of its effect may be obtained mathematically on making the following drastic simplifying assumptions :—

(1) Secondary electrons are emitted from electrode 2 only.

(2) The velocity of emission is so small that the energy with which electrons are emitted may be neglected.

TABLE

τ_1	τ_2	$\cos \tau_1$	$\cos \tau_2$	$\frac{v_2}{v_1}$	$\left\{1-\left(\frac{v_2}{v_1}\right)^2\right\}$
0.00π	0.66π	1.00	− 0.48	1.49	− 1.23
0.17π	0.79π	0.87	− 0.79	1.55	− 1.40
0.33π	0.94π	0.50	− 0.98	1.49	− 1.23
0.50π	1.14π	0.00	− 0.90	1.30	− 0.69
0.67π	1.39π	− 0.50	− 0.34	0.95	0.10
0.83π	1.76π	− 0.87	0.73	0.47	0.78
1.00π	2.31π	− 1.00	0.56	0.48	0.77
1.17π	2.495π	− 0.37	0.02	0.50	0.51
1.33π	2.515π	− 0.50	− 0.05	0.55	0.28
1.50π	2.495π	0.00	0.02	0.99	0.02
1.67π	2.515π	0.50	− 0.05	1.18	− 0.39
1.83π	2.561π	0.87	− 0.20	1.36	− 0.85
2.00π	2.66π	1.00	− 1.49	1.49	− 1.23

(3) Electrons are emitted instantaneously ; i.e., immediately on impact of the primary electrons.

(4) The number of electrons emitted per second is proportional to the number of primary electrons striking the electrode at all values of the field.

(5) The secondary electrons are emitted parallel to the direction of the electric field, and therefore, also parallel to the direction of the primary electrons.

Suppose now that secondary electrons are emitted at electrode 2 uniformly throughout the cycle. Three things can happen to these electrons under the influence of the field between the electrodes :—

(a) They travel right across the space between the electrodes and arrive at electrode 1 with a negative (or zero) velocity.

(b) They travel out into the space but are then pulled back to electrode 2, and arrive there with positive (or zero) velocity.

(c) They are pulled back into electrode 2 immediately they are emitted and leave only by a negligible amount.

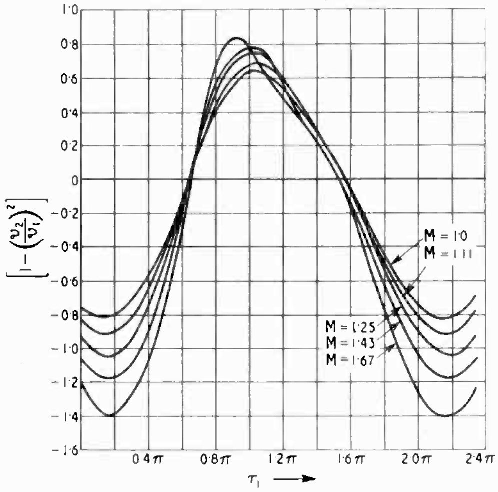

Fig. 7. *Curves of* $[1-(v_2/v_1)^2]$ *against* τ_1.

In cases (a) and (b) energy will be lost by the field. In case (c) no energy will be lost. If v_s denotes the velocity of arrival of secondary electrons at either electrode, it is possible on the basis of the above assumptions, and using the method already given to plot $(v_s/v)^2$ against τ_2 for varying values of ϕ using the values of M determined in the last section, as for example in Fig. 10. In this way the overall efficiency could be determined. However, as this procedure would involve a knowledge of the secondary-radiation coefficient, and as it depends on the validity of the above assumptions, it is thought that a more satisfactory picture of the phenomenon will be obtained in the following manner.

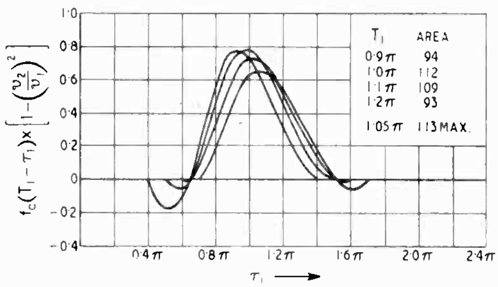

Fig. 8. *Determination of maximum area as* T_1 *is varied for Type C pulse with* $M = 1.43$.

It is clear from Fig. 10 that, in the case of $\phi = 0.5$, when $2n\pi < \tau_2 < (2n+1)\pi$, $(n = 0, 1, 2 \ldots)$ secondary electrons, if they behaved in accordance with the above assumptions, would withdraw no energy from the field. This is a general proposition depending on the fact that any such electron emitted during this time interval immediately experiences a field which prevents it from leaving electrode 2. In fact we have established that case (c) above occurs, for $2n\pi < \tau_2 < (2n+1)\pi$ for all values of ϕ.

We are thus led to take as a measure of the effect of secondary electrons a quantity ν, being the ratio of the number of primary electrons per cycle for which $2n\pi < \tau_2 < (2n+1)\pi, (n = 0, 1, 2, 3 \ldots)$ to the total number of primary electrons per cycle ; i.e.,

$$\nu = \int_0^{\pi} i_2 \, d\tau_2 \Big/ \int_0^{2\pi} i_1 \, d\tau_1 \quad \ldots \quad \ldots \quad \ldots \quad (17)$$

This quantity which we call the coefficient of secondary-electron suppression is such that, when large, it corresponds to the most efficient transfer of energy from the primary electron pulse to the field.

ν is a function of ϕ, M, T_1 and f. It is plotted in Fig. 11 against ϕ for two types of pulse and for those values of M and T_1 which have already been determined to yield η_{max}.

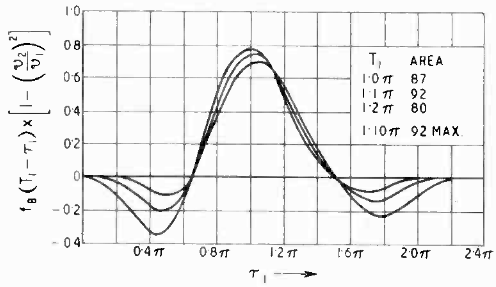

Fig. 9. *Determination of maximum area as T_1 is varied for Type B pulse with $M = 1.43$.*

Of the two curves, that of pulse Type C is the more remarkable in view of the sudden increase in ν for ϕ between 1.0 and 1.4. Fig. 12 illustrates how this occurs. In this figure i_1 is plotted against τ_1, and i_2 against τ_2, for various values of ϕ. This shows the manner in which, as ϕ increases, the current at electrode 2 develops a 'peak' which moves across the line $\tau_2 = 2\pi$ as ϕ increases from 1.0 to 1.2.

6. Suppression of Secondary Electrons[5]

Two measures of efficiency have been established in the previous two sections. It has been shown that for both types of pulse C and B, the efficiency η_{max} in the absence of secondary electrons does not begin to fall until $\phi \approx 1.0$ and has fallen only by about 13% when $\phi \approx 2.0$.

Furthermore when secondary electrons are emitted, there exists a critical value of transit angle $\phi \approx 1.2$, above which there is a considerable increase in secondary-radiation suppression. For ϕ between 1.5 and 2.0, from 65% to 70% of the emitted secondary electrons are suppressed.

These two facts taken in conjunction lead one to suppose that the most efficient transfer of energy from an electron beam, modulated by either of the wave shapes shown, to an output

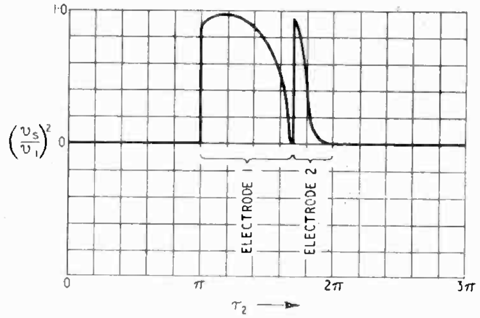

Fig. 10. *Secondary electron energy against τ_2 ; $\phi = 0.5$.*

field will occur when the d.c. transit angle ϕ is about 1.5 if secondary radiation is present.

The value of the overall efficiency actually attained will depend on the value of the secondary-radiation coefficient, and it is to be expected that in any practical application of the above method of suppression it would be advisable to ensure the lowest possible value of the secondary-radiation coefficient for the surface of electrode 2.

The foregoing analysis makes no claim to great accuracy, but it is hoped that it will serve to give a clearer picture of the way in which energy is transferred to the electric field in a valve. The

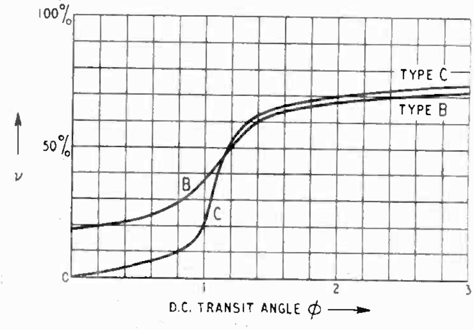

Fig. 11. *Graph of ν against ϕ for two types of pulse.*

procedure adopted for evaluating the various quantities is lengthy and tedious. For this reason no attempt has been made to investigate fully the behaviour of pulse shapes other than the two mentioned, but it is believed that it is possible to utilize pulses having shapes which give almost complete secondary-electron suppression with high primary efficiency.

7. Short-Duration Pulses

A graphical analysis is given above of two types of pulse C and B. A different method of approach may be used in the case of a pulse assumed to be of infinitely short duration.

Assume a pulse of n electrons per cycle passing electrode 1 simultaneously at an entrance angle τ_1. Since all the electrons will travel across the space together and reach electrode 2 simultaneously, we have a case in which 100% efficiency is attainable on making $v_2 = 0$. We will endeavour to find in this case what is the least value of M for a given value of ϕ, and the corresponding values of τ_1 and τ_2 necessary to bring the electron pulse to rest on reaching electrode 2. In this case equations (2) and (3) become:

$$2\phi/M = \cos\tau_2 - \cos\tau_1 \quad \ldots \quad \ldots \quad (18)$$

$$2\phi^2/M = (\tau_2 - \tau_1)\cos\tau_2 - \sin\tau_2 + \sin\tau_1 \quad (19)$$

It is important to note before proceeding that physical considerations determine $\pi < \tau_2 < 2\pi$. This is because it has been assumed that the electrons are brought to rest at the plane of the second electrode on their way from the plane of the first electrode and, if they pass this electrode, cannot return to the space between the electrodes. If, therefore, they are brought to rest at this plane, they must have experienced a deceleration in some interval of time, however small, immediately preceding the time of arrival; that is, the field between the electrodes must have been negative during this time, and whatever the transit angle the field must always be negative just before they arrive.

A condition for minimum M at constant ϕ is now obtained by setting $dM/d\tau_1 = 0$ with $\pi < \tau_2 \leqslant 2\pi$. We have from equations (18) and (19) the following:

$$\frac{1}{M}\frac{dM}{d\tau_1} = \frac{\cos\tau_2 - \cos\tau_1 + (\tau_2 - \tau_1)\sin\tau_1}{\sin\tau_1 - \sin\tau_2 + (\tau_2 - \tau_1)\cos\tau_1} \quad (20)$$

i.e., $\cos\tau_2 - \cos\tau_1 + (\tau_2 - \tau_1)\sin\tau_1 = 0 \quad (21)$

τ_1 and τ_2 cannot in fact be eliminated between equations (18), (19) and (21), but we can obtain useful results in terms of the parameter $\tau = \tau_2 - \tau_1$. τ is the transit angle through which the pulse of electrons travels between the electrodes. Equation (21) now yields,

$$\tan\tau_1 = \frac{1 - \cos\tau}{\tau - \sin\tau} \quad \ldots \quad \ldots \quad \ldots \quad (22)$$

which shows $\tan\tau_1 \geqslant 0$. Also, from equation (18):

$\cos\tau_2 - \cos\tau_1 \geqslant 0$ for $\phi \geqslant 0$, $M \geqslant 0$

and, $\tau_2 - \tau_1 \geqslant 0$

$\therefore$ by equation (21) $\sin\tau_1 \leqslant 0$

$$\therefore \pi \leqslant \tau_1 \leqslant \frac{3\pi}{2}$$

We can also obtain τ_2 in terms of τ, namely:

$$\tan\tau_2 = \frac{1 - \cos\tau - \tau\sin\tau}{\sin\tau - \tau\cos\tau} \quad \ldots \quad \ldots \quad (23)$$

M_{min} and ϕ can now be obtained as functions of τ by substituting from (22) and (23) in (18) and (19). It is less trouble, however, for practical purposes to plot M_{min} and ϕ as functions of τ in two stages. This is done by first plotting τ_1 and τ_2 from equations (22) and (23) and then inserting the values so obtained in equations (18) and (19) to obtain ϕ and M_{min}.

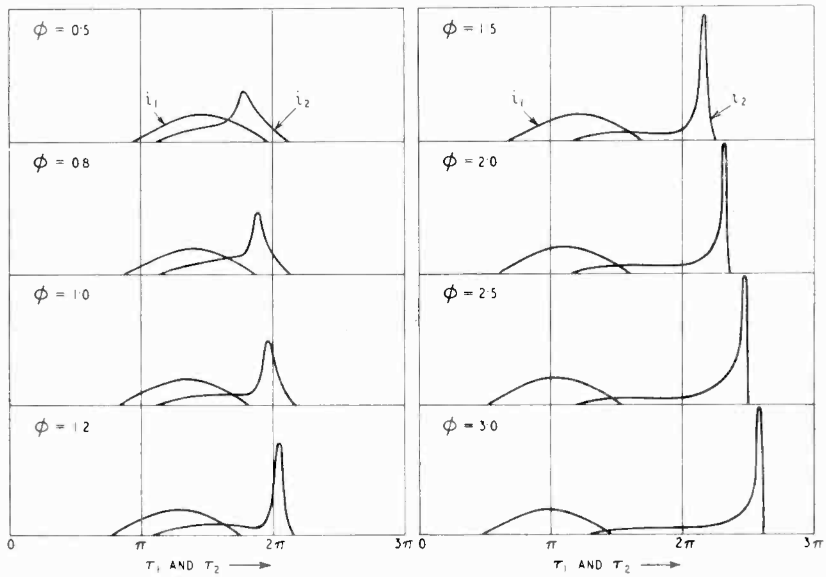

Fig. 12. *Graphs of i_1 and i_2 against τ_1 and τ_2 respectively; pulse Type C.*

It will be seen that as τ increases, τ_2 also increases and reaches the value 2π. When this occurs from equation (23),

$$1 - \cos\tau - \tau\sin\tau = 0 \quad \ldots \quad \ldots \quad \ldots \quad (25)$$

the solution of which is a critical value of transit angle $\tau = 2.33$. For $\tau > 2.33$, the solution obtained from the present set of equations leads to a violation of the condition $\pi < \tau_2 \leqslant 2\pi$. In this case τ_2 remains equal to 2π and equations (18) and (19) become,

$$2\phi/M_{\min} = 1 - \cos \tau_1 \quad \ldots \quad (26)$$

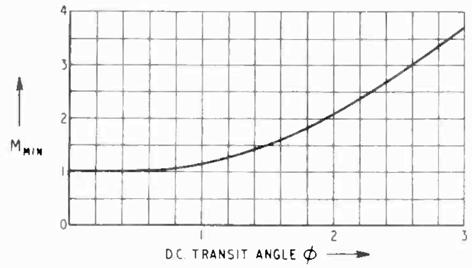

Fig. 13. *Graph of M_{min} against ϕ; short-duration pulse.*

and

$$2\phi^2/M_{\min} = (2\pi - \tau_1) + \sin \tau_1 \quad \ldots \quad (27)$$

or in terms of $\tau(= 2\pi - \tau_1)$, we have the relatively simple expressions:

$$\phi = \frac{\tau - \sin \tau}{1 - \cos \tau} \quad \ldots \quad (28)$$

$$M_{\min} = 2\frac{\tau - \sin \tau}{(1 - \cos \tau)^2} \quad \ldots \quad (29)$$

For $\tau = 2.33$, the critical value referred to above, the corresponding value of ϕ is 0.95. Fig. 13 shows the complete curve for $M_{\min}$ against ϕ, and Fig. 14 those for τ_1 and τ_2 against ϕ with the critical value $\phi = 0.95$ shown.

We have, in this idealized example, a case in which the primary efficiency remains at 100% for all values of ϕ, and in which complete secondary electron suppression occurs for $\phi > 0.95$.

There is no reason in this case why ϕ should not be increased indefinitely. The limit is a merely practical one, for example, circuit losses which are proportional to M^2.

The results of this example indicate the desirability in any practical design of valve for use at very high frequency of using a pulse of as short a duration as possible.

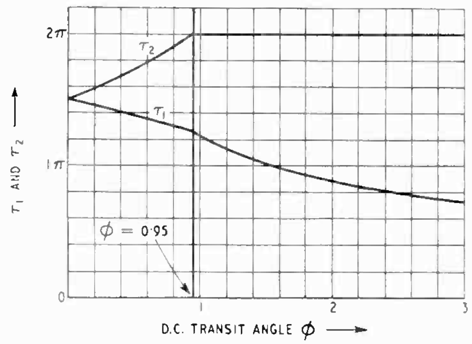

Fig. 14. *Graph of τ_1 and π_2 against ϕ; short-duration pulse.*

8. Acknowledgment

The author wishes to thank the Directors of Rediffusion Ltd., and Central Rediffusion Services Ltd. for permission to publish this paper.

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RHOMBIC AERIAL DESIGN CHART

By R. H. Barker, B.Sc., A.M.I.E.E.

(Signals Research and Development Establishment)

SUMMARY.—A graphical method has been evolved for solving the equation for the angle of elevation at which the gain of a horizontal rhombic aerial is a maximum, consideration being restricted to the vertical plane containing the major axis. The resulting chart has been used to prepare families of curves summarizing all the properties of the aerial as regards the major lobe. An extension has been made to include the effects of a ground plane of finite conductivity and dielectric constant.

1. Introduction

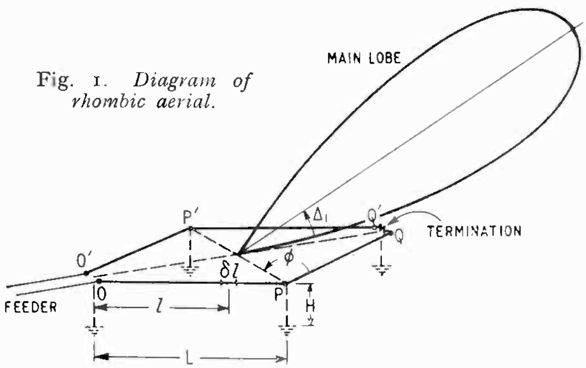
THE rhombic aerial consists of four long wires arranged in the form of a rhombus as illustrated in Fig. 1. The mode of operation of one of the wires constituting the rhombic is fundamentally the same as that of any other type of single-wire aerial whether resonant or not. In the resonant case the standing waves present on the wire may be considered as the resultant of two progressive current waves travelling in opposite directions. The polar diagrams associated with these progressive waves are illustrated in Fig. 2 for the forward wave (a) and for the reflected wave (b). That of the resonant aerial (c) is the vector sum of these two and is, therefore, symmetrical about the centre point. The number of lobes and the directions of minima for the resonant aerial are identical with those for the two progressive current waves.

The rhombic aerial provides a convenient method of terminating the single wires of which it is comprised (Fig. 1). Power is fed into or abstracted from the aerial through a balanced feeder connected to the ends of the wires OP and O′P′. The other two wires PQ and P′Q′ are terminated at QQ′ by a resistive load designed to prevent reflections which would give rise to radiation in undesired directions, principally backwards. The polar diagram usually consists of one large lobe with angle of elevation Δ, together with a number of smaller lobes not indicated in the diagram.

The directivity of a long wire remote from earth and carrying a progressive current wave depends upon two factors which tend to conflict. Each element of the wire behaves as a short Hertzian dipole from which the radiation in any direction is proportional to the cosine of the angle that this direction makes with the normal to the wire. Furthermore the field strength at a remote point is the resultant of the contributions from all the elements. These contributions will all be in phase with each other when the direction of the remote point makes only a very small angle with the direction of the wire. This is because the current travels along the wire with very nearly the same velocity as does the radiation outside it, and the total time for the energy to go from source to receiver is the same for radiation from all elements of the wire.

The effect of this phase addition is that the energy tends to be radiated in the direction of the wire, whereas the cosine term determines that the radiation in this direction is actually zero. Combining these effects results in a polar diagram which is symmetrical about the wire as axis and which has one large lobe at an angle of only a few degrees to the wire. A number of smaller lobes are also present. As the wire is lengthened the phasing becomes the more important factor and the main lobe is depressed to a smaller angle. This leads to the apparent anomaly that an infinitely long wire does not radiate at all but guides all the energy along itself.

Fig. 1. *Diagram of rhombic aerial.*



The polar diagram of a terminated rhombic aerial in free space is the resultant of the four long-wire patterns. The proportions are usually arranged so that the radiation forwards tends to add and the radiation sideways tends to cancel. The polar diagram then has its main lobe along the axis of the rhombic. As a further complication, reflection from the ground must be taken into account. This deflects the major lobe upwards to an angle which is chiefly dependent upon the height of the aerial.

The complete polar diagram includes secondary

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lobes above the main lobe and to the sides. These are not usually of great importance, except perhaps in certain cases where the aerial is used for reception, and the direction may correspond to an interfering signal.

In January 1935, Bruce, Beck and Lowry[1] first published the directivity equation of the horizontal rhombic aerial and discussed methods of compromise design based upon it. Other writers, notably A. E. Harper[2] in his book "Rhombic Antenna Design" have derived expressions of a more convenient form and have provided tables of data and graphs to facilitate computation. In general the formulae are not such that approximations may easily be made, the calculations are very tedious and the results much more detailed than is generally required.

W. R. Piggott[3] has recently evolved a graphical method for the rapid estimation of the relative gain in any direction expressed in terms of the angles of elevation Δ and azimuth β. Contours of constant gain may be quickly plotted on a Δ–β chart. This method of presentation shows up the presence of lobes in the directivity pattern at angles of azimuth other than along the major axis of the rhombic and is of particular value in considering the performance of aerials used for reception. It is unfortunate that by this method the position of the main lobe is the least easy to determine precisely and it is usually this lobe that is of greatest importance in the transmitting case.

The present paper describes a method whereby the precise performance of the aerial as regards its main lobe may be quickly determined, and presents in convenient graphical form a summary of results adequate for many practical purposes. It was prepared originally in order to assist in the correct use of a large number of rhombic aerials of various dimensions over wide frequency ranges. With this in view the basic parameters were chosen so that only one is frequency dependent. These, illustrated in Fig. 1, are as follows :—

Δ_1 = least angle of elevation at which the gain is a maximum on the major axis.

ϕ = semi-angle of the rhombic.

H/λ = height above ground in terms of wavelengths.

L/H = ratio of length of side to height.

Fig. 7 shows the method of presenting results. The relative gain associated with the main lobe is plotted against H/λ for certain useful values of L/H and ϕ. The small figures along the curves indicate the angle of elevation Δ_1 of the main lobe corresponding to these values of H/λ, L/H and ϕ. The chart described in the first part of this paper was evolved so that Fig. 7 could be prepared without a prohibitive expenditure of effort. When any three of the above parameters are known the fourth may be read from it. The chart should be used for cases where Fig. 7 is inadequate and when the effect of finite ground constants has to be included. It also enables the angle of elevation Δ_2 of the second lobe on the major axis to be determined.

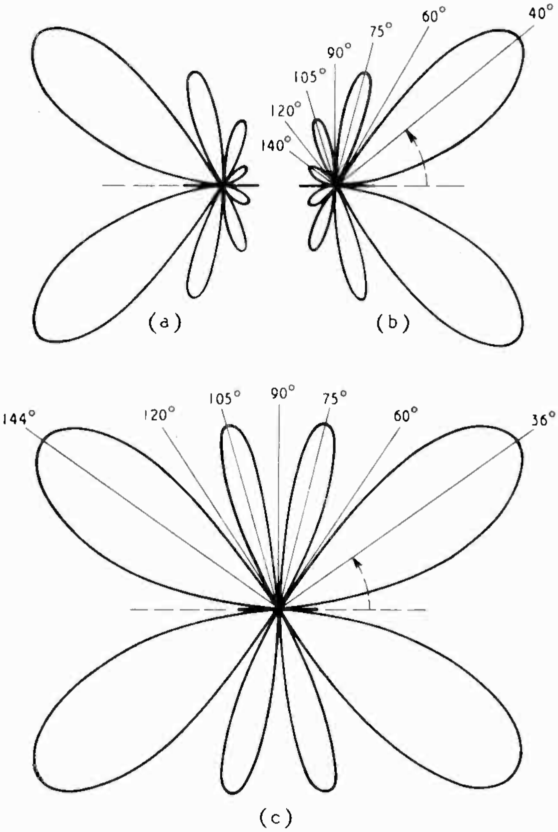


Fig. 2. *Polar diagrams for a progressive wave travelling to the left (a) and to the right (b). Their vector sum gives the diagram (c) for standing waves (resonant operation). Length of aerial = 2λ in each case.*

2. Directivity Equation

A. E. Harper[2] gives the complete directivity equation for rhombic aerials taking into account both vertically and horizontally polarized components. When consideration is restricted to directivity in the vertical plane containing the major axis, the equation is considerably simplified since the symmetry of the system renders it insensitive to vertically polarized waves. The directivity in this plane depends mainly upon the product of two terms, one due to the reflection from the ground, and the other the result of integration with correct phases of the radiations from all the elements comprising the aerial.

(a) *The Ground-reflection Term*

The rhombic is illustrated in Fig. 1. Imagine a short element of length δl situated in the arm OP a distance l from O. The field strength at a remote point, distance S, due to a current I in this element is the resultant of two components, one propagated directly and the other reflected from the ground. The reflected wave suffers a phase delay of $2\left[\frac{2\pi H}{\lambda}\sin\Delta\right]$ due to the additional distance travelled and a phase delay of $\pi - \psi_H$ due to imperfect reflection at the ground. A perfect conductor would produce a phase change of π but since the ground presents a capacitive impedance, ψ_H is actually a negative angle. In addition the reflected wave is attenuated to K_H of the amplitude of the direct wave. The resultant R of these two components takes full account of reflection at the ground.

This reduces to

$$R^2 = 4K_H \sin^2\left[\frac{2\pi H}{\lambda}\sin\Delta - \frac{\psi_H}{2}\right] + h^2 \qquad (1)$$

where $h = 1 - K_H$. In general ψ_H and h are nearly zero. For the moment we will neglect the quantity h. The errors this involves are discussed in more detail later. ψ_H is taken into account by substituting

$$H' = H\left[1 - \frac{\psi_H}{4\pi x}\right]$$

where $x = \frac{H}{\lambda}\sin\Delta$

so that $R^2 \approx 4K_H \sin^2\left[\frac{2\pi H'}{\lambda}\sin\Delta\right]$

or $R \approx \sqrt{2K_H}\sin 2\pi x'$

where $x' = \frac{H'}{\lambda}\sin\Delta$

$$\mathcal{E} = \frac{60\pi}{\lambda S} \cdot 2RI\cos\phi \frac{2\sin^2\left[\frac{\pi L}{\lambda}(1-\cos\theta)\right]}{\frac{\pi}{\lambda}(1-\cos\theta)} \cdot \cos\left[\omega t - \frac{2\pi S}{\lambda} - \frac{2\pi L}{\lambda}(1-\cos\theta)\right]$$

(b) *The Integral Term*

At a point in the vertical plane containing the major axis of the rhombic and at a distance S from the rhombic the horizontal component of field strength due to the radiating element δl carrying current $I\cos\omega t$ is given by

$$\frac{60\pi}{\lambda S} I\cos\phi\sin\left(\omega t - \frac{2\pi S}{\lambda}\right)\delta l$$

where ϕ is the angle between the element and the normal in the horizontal plane to the direction of propagation considered. This angle is equal to the semi-angle of the rhombic (see Fig. 1). The phase of the wave radiated from this element is important. It is retarded (with reference to radiation from an element at O) by $\frac{2\pi l}{\lambda}$ due to the time for the current to flow along the wire and is advanced by $\frac{2\pi l}{\lambda}\cos\theta$ due to the position of this element in front of O. Here, θ is the angle between the direction of propagation and the wire OP.

By geometry $\cos\theta = \cos\Delta\sin\phi$.

Reflection at the ground must be taken into account and we may write the field strength at a distance S assumed large compared to the dimensions of the rhombic as

$$\delta\mathcal{E} = \frac{60\pi}{\lambda S} RI\cos\phi\sin\left[\omega t - \frac{2\pi S}{\lambda} - \frac{2\pi l}{\lambda}(1-\cos\theta)\right]\delta l$$

An element in the arm PQ will contribute similarly except that ϕ is replaced by $(\pi - \phi)$ and the phase delay becomes $\frac{2\pi}{\lambda}(l+L)(1-\cos\theta)$. Elements in O'P' and P'Q' will contribute in phase with those in OP and PQ respectively. Hence the resultant field strength will be

$$\mathcal{E} = \frac{60\pi}{\lambda} \cdot 2RI\cos\phi\int_0^L\left\{\sin\left[\omega t - \frac{2\pi S}{\lambda} - \frac{2\pi l}{\lambda}(1-\cos\theta)\right] - \sin\left[\omega t - \frac{2\pi S}{\lambda} - \frac{2\pi}{\lambda}(l+L)(1-\cos\theta)\right]\right\}dl$$

The current I is usually assumed to be constant along the wire if the rhombic is correctly terminated at QQ'.

On integrating

Since we are not interested in the phase at the remote point the last term may be omitted. In order that the wavelength shall be involved in only one of the parameters finally appearing in the directivity equation, we will use the ratio L/H instead of L/λ and make the substitutions

$$k = bL/2H$$

$$b = \frac{1-\cos\theta}{\sin\Delta} = \frac{1-\cos\Delta\sin\phi}{\sin\Delta}$$

i.e., $2\pi kx = (1-\cos\theta)\,\pi L/\lambda$

The field strength can then be expressed as $\mathscr{E} = \frac{60I}{S}\sqrt{K_H} \cdot \frac{8\cos\phi}{1 - \cos\Delta\sin\phi} \cdot \sin^2 2\pi kx \,.\, \sin 2\pi x' \quad (2)$

The directivity D is that part of the expression which depends upon the aerial parameters.

$$D = \frac{8\cos\phi}{1 - \cos\Delta\sin\phi} \,.\, \sin^2 2\pi kx \,.\, \sin 2\pi x' \quad (3)$$

It is interesting to consider the influence of these terms separately upon the directivity.

The first term $\left[\frac{8\cos\phi}{1 - \cos\Delta\sin\phi}\right]$ varies only slowly with Δ and the positions of the lobes of the polar diagram depend little upon it. The gain tends to be larger at low angles of elevation.

The integral term ($\sin^2 2\pi kx$) depends upon L/λ, Δ and ϕ. It is zero when $(1 - \cos\Delta\sin\phi)\,L/\lambda$ equals an integer and between each zero rises to unity. If L/λ is large there will be a large number of maxima and minima as Δ varies from 0 to $\pi/2$. Whenever this term is zero the whole directivity expression is zero, but the maxima do not correspond to the lobes of the polar diagram since the ground reflection term is also a rapidly varying function of Δ.

At grazing incidence ($\Delta = 0$) both h and ψ_H become zero so that the ground reflection term R is zero. It is also zero when $(H'/\lambda)\sin\Delta = \frac{1}{2}n$, where n is an integer; it is a maximum when $(H'/\lambda)\sin\Delta = \frac{2n+1}{4}$. When the maxima of the integral and the factor R occur together, the gain will be very near its maximum and the position of the main lobe will be given approximately by putting $n = 0$ in this equation.

In general the position of a lobe depends upon the combined effect of the variations of the three terms. The chart described below enables this to be determined precisely.

3. Description of the Chart

The method is similar to that employed by W. A. Baker[4]. The directivity equation (3) is differentiated with respect to the angle of elevation Δ treating H' as a constant. (This approximation only introduces a second-order error in the final result).

$$\frac{1}{D}\cdot\frac{dD}{d\Delta} = \frac{-\sin\Delta\sin\phi}{1 - \cos\Delta\sin\phi} + \frac{\frac{2\pi H'}{\lambda}\cos 2\pi x'\cos\Delta}{\sin 2\pi x'} + \frac{\frac{2\pi L}{\lambda}\sin\phi\sin\Delta\cos 2\pi kx}{\sin 2\pi kx}$$

Maximum values of D are obtained by equating the right-hand side to zero. H'/λ and L/λ are put in terms of x' and k.

$$\frac{\sin\Delta\sin\phi}{1 - \cos\Delta\sin\phi} = 2\pi x'\cot 2\pi x'\cot\Delta + 4\pi kx\cot 2\pi kx\,\frac{\sin\Delta\sin\phi}{1 - \cos\Delta\sin\phi}$$

$$\therefore \frac{\sin\phi}{b}[1 - 4\pi kx\cot 2\pi kx] = 2\pi x'\cot 2\pi x'\cot\Delta$$

$$\therefore \tan 2\pi x'\left[\frac{1}{2\pi x'} - 2k'\cot 2\pi k'x'\right] = \frac{b\cot\Delta}{\sin\phi} \quad (4)$$

here $k' = kH/H'$; i.e., $k'x' = kx$

The effect of imperfect reflection of the ground is considered in detail later. For the moment we will assume such reflection to be perfect and write equation (4) as

$$f(k, x) = a \quad \ldots \quad (5)$$

where a depends only upon Δ and ϕ. This will have a number of solutions, some of which correspond to maximum gain. The minima have already been discussed and may be obtained most conveniently by inspection of equation (3).

The gain is a maximum for values of Δ equal to Δ_1, Δ_2, etc., which are solutions to equation (5). The chart is designed to facilitate the solution of this equation.

Consider first of all an example in which Δ, ϕ and L/H are assumed known and it is required to find H/λ.

First determine

$$b = \frac{1 - \cos\Delta\sin\phi}{\sin\Delta}$$

$$a = \frac{b\cot\Delta}{\sin\phi}$$

from the assumed values of Δ_1 and ϕ. On the chart (Fig. 3) these are plotted for ϕ equal to 50°, 55°, 60°, 65° and 70°. The vertical scale is logarithmic and a subsidiary scale of $L/2H$ is added so that the relation $k = bL/2H$ can be taken into account by a vertical shift. The horizontal scale is logarithmic in $\sin\Delta$.

Superimposed upon the a and b curves is the function $f(k, x) = \tan 2\pi x\left[\frac{1}{2\pi x'} - 2k\cot 2\pi kx\right]$ plotted against x for values of k between 0 and 2. The same logarithmic vertical scale is used, but to avoid confusion the scale of x (which is also logarithmic) is not carried across the chart. Only the parts of the function $f(k, x)$ corresponding to the first two maxima have been included.

Curves for the first maximum extend between $x = 0$ and $x = 0.25$.

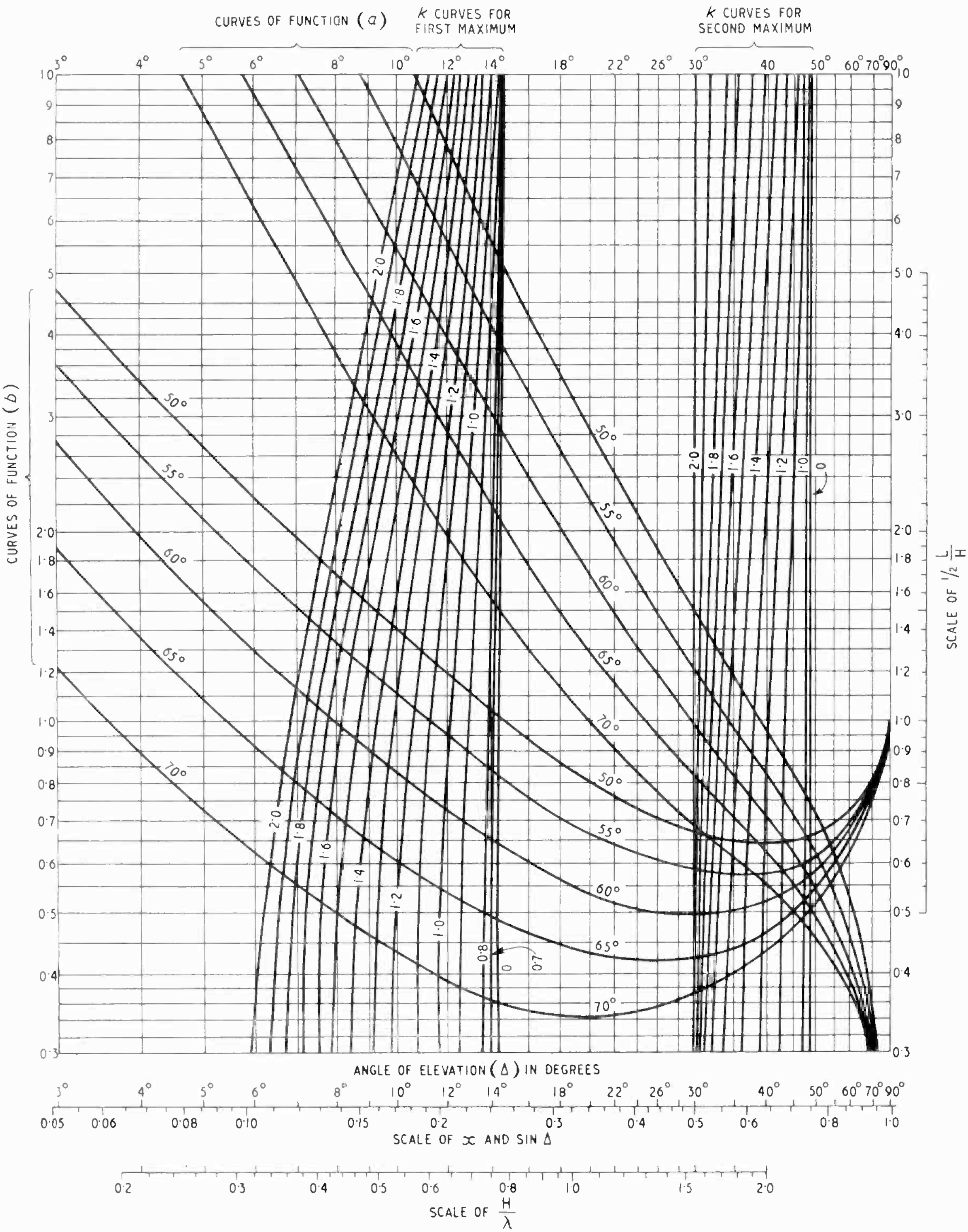


Fig. 3. *Chart for the solution of Equation 5*

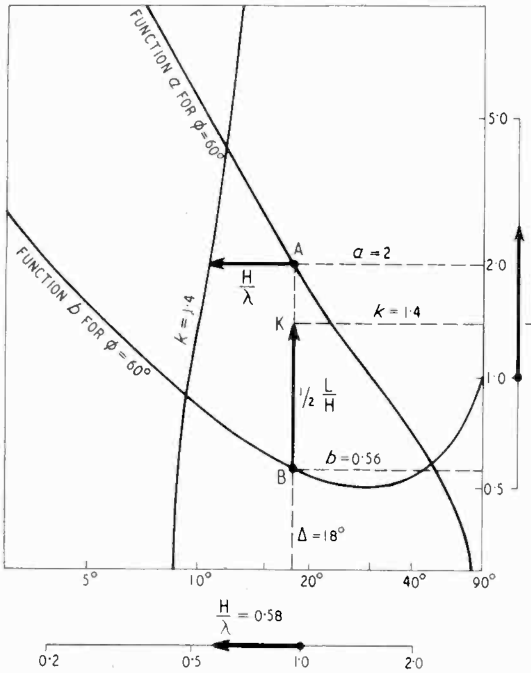


Fig. 4 (*above*). *An illustration of the use of the chart of Fig. 3.*

Fig. 5 (*right*). *Curves of constant F (a, k).*

Curves for the second maximum extend between $x = 0.5$ and $x = 0.75$.

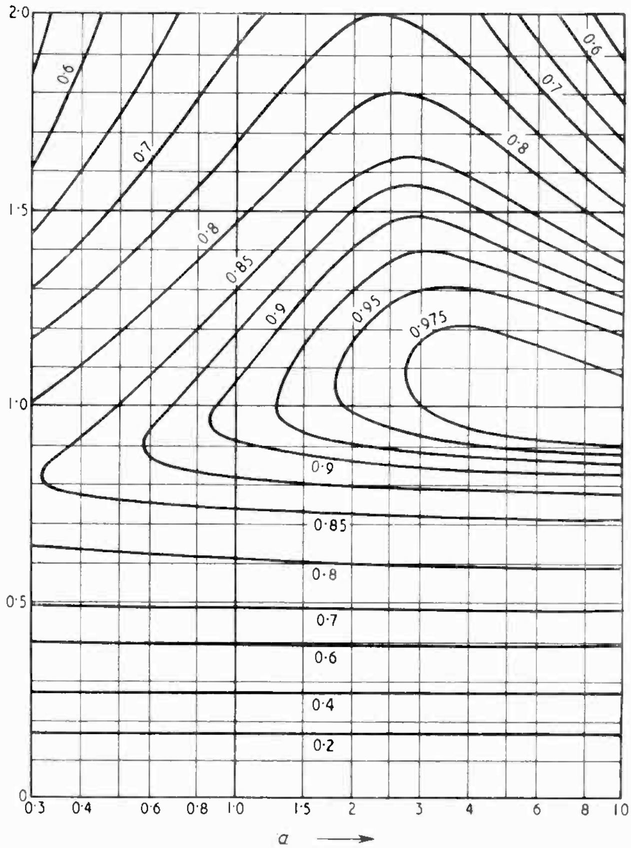
The limiting values correspond to $\tan 2\pi x = 0$ and $\tan 2\pi x = \infty$. Having determined the value of k from the relation $k = bL/2H$, and remembering that $a = f(k, x)$ these curves may be used to determine x directly and hence H/λ. The scale of x has been arranged to coincide with the scale of $\sin \Delta$ so that the relation $x = (H/\lambda) \sin \Delta$ can be taken into account by means of a horizontal shift. In practice it is not necessary to read off the actual value of x and it will be found convenient to use dividers set to the horizontal distance between the point at which the appropriate k curve crosses the a ordinate and the value of Δ concerned and to transfer this distance to the subsidiary scale of H/λ.

Fig. 4 illustrates the use of the chart in the case where $\Delta_1 = 18°$ $\phi = 60°$ and $L/H = 5.0$. Identify points A and B where the Δ co-ordinate intercepts the a and b curves and proceed vertically from B a distance corresponding to multiplication by $L/2H$ so evaluating k, in this case 1.4. The horizontal distance from A to the $k = 1.4$ curve may be transferred to the subsidiary scale to give the value of H/λ ($H/\lambda = 0.58$.)

If L/H is the unknown quantity, the procedure will be very similar except that the value of k will first be deduced using the a and k curves.

When Δ_1 is not known the method becomes less straightforward for neither a nor b can be determined directly. By trial and error a value of Δ_1 must be selected so that the value of k found via a and L/λ coincides with that found via b and $L/2H$. The following example is included for illustration :

Given $\phi = 70°$, $H/\lambda = 1.2$, $L/H = 5.0$ find Δ_1. Make a guess at Δ_1, say 15°. This would correspond to $x = 0.31$ for which there are no $f(k, x)$ values plotted. Clearly Δ_1 must be less than this, try 11°. Then $a = 2.3$ and $k = 0.96$ but also $b = 0.41$ giving $k = 1.03$, so that Δ_1 is still too large. Reducing Δ_1 until the values of k via a and via b coincide we obtain $\Delta_1 = 10°$, $k = 1.08$. This is the value of Δ_1 required. ($a = 2.6$, $b = 0.435$). We may find the angle of the second maximum Δ_2 in a similar manner. It must be such that x lies between 0.5 and 0.75 and actually comes out to be 37°. ($a = 0.59$, $b = 0.415$, $k = 1.04$).



It is convenient to note that H/λ equals H in thousands of feet multiplied by frequency in megacycles per second to an accuracy of nearly 1%.

constant $F(a, k)$ have been plotted in Fig. 5.

In the example given above where $\phi = 70°$ $H/\lambda = 1.2$ and $L/H = 5.0$ it was seen that $a = 2.6$, $b = 0.435$ and $k = 1.08$. From Fig. 4,

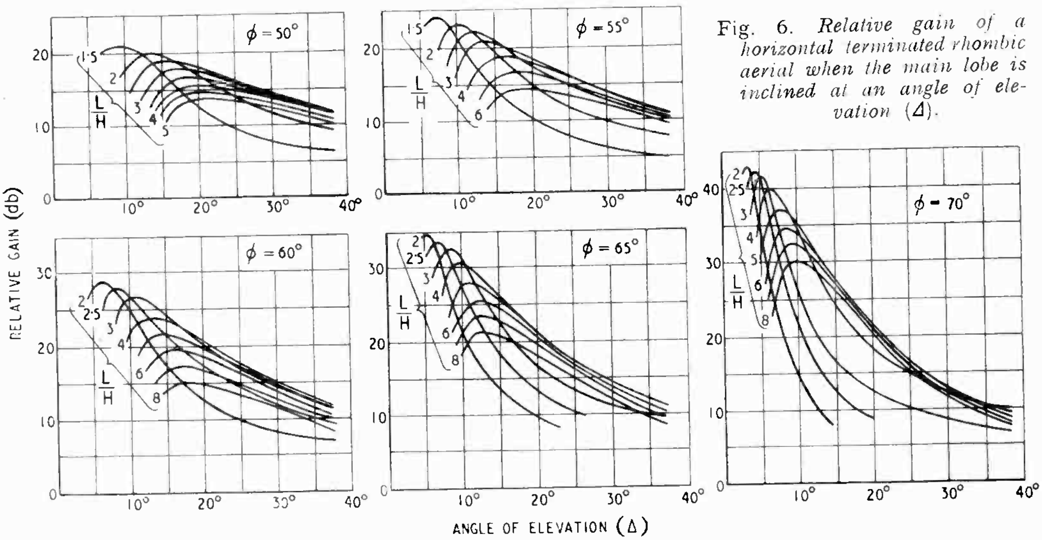


Fig. 6. *Relative gain of a horizontal terminated rhombic aerial when the main lobe is inclined at an angle of elevation* (Δ).

4. Aerial Gain

The method of determining the angle of the main lobe in the polar diagram has been described in the preceding section. For one value of this angle there exists a large number of combinations of the other parameters, ϕ, L/H and H/λ. It is important, therefore, to know how the gain will vary as the aerial dimensions are changed in order to effect the maximum economy in cost of installation consistent with the required performance.

The calculation of the gain may be simplified by eliminating x' between equations (3) and (4).

$$D_{max} = \frac{8 \cos \phi}{1 - \cos \Delta_1 \sin \phi} \cdot F(a, k)$$

$$= \frac{F(a, k) \cos \phi}{b \sin \Delta_1} \quad \ldots \quad \ldots \quad \ldots \quad (6)$$

The function $F(a, k)$ is transcendental and is best expressed in terms of the parameter x and the equations

$$\sin 2\pi x \sin^2 2\pi k x = F(a, k)$$

$$\tan 2\pi x \left[\frac{1}{2\pi x} - 2k \cot 2\pi k x \right] = a$$

Values of x less than 0.25 are relevant to the main lobe and this part of the function has been evaluated for the range of values of a and k with which the chart of Fig. 3 deals. When k is small $F(a, k)$ becomes independent of a and equal to $\frac{(\pi k)^2}{1 + (\pi k)^2}$. The error introduced by this approximation is insignificant when $k < 0.7$. Contours of

$F(a, k) = 0.975$ and hence, using equation (6), $D = 35$.

Since equation (6) does not involve x, the relative gain associated with a lobe at an angle of elevation Δ_1 is independent of the frequency and is determined only by the geometry of the system. The frequency and geometry must of course be related so that the angle of maximum gain is actually equal to the angle Δ_1 selected. Fig. 6 shows the relative gain D_{max} expressed in terms of Δ_1, ϕ and L/H.

A knowledge of the directivity factor D enables the field strength at a distance S to be calculated in terms of the current I in the aerial. From equations (2) and (3).

$$\mathcal{E} = 60D.\ I/S \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (7)$$

It gives no information about the proportion of the total input power that is radiated in the desired direction, though this may be calculated if the input impedance is known.

The use of the aerial for reception should also be considered. For a wave arriving along the direction of the major axis, only the horizontally-polarized component is effective. If this component is of strength $\mathcal{E}_0$, each element δl of the aerial will act as a generator of voltage equal to $\mathcal{E}_0 \cos \phi \delta l$. If the aerial is terminated by a matched receiver this element will be effective in inducing a voltage δV across the receiver terminals.

$$\delta V = \frac{1}{2} \mathcal{E}_0 \cos \phi \delta l.$$

Compare this with the equation giving the field strength due to a current I in the radiating element δl used as a basis for the original calculation.

$$\delta \mathcal{E} = \frac{60\pi}{\lambda S} I \cos \phi \delta l$$

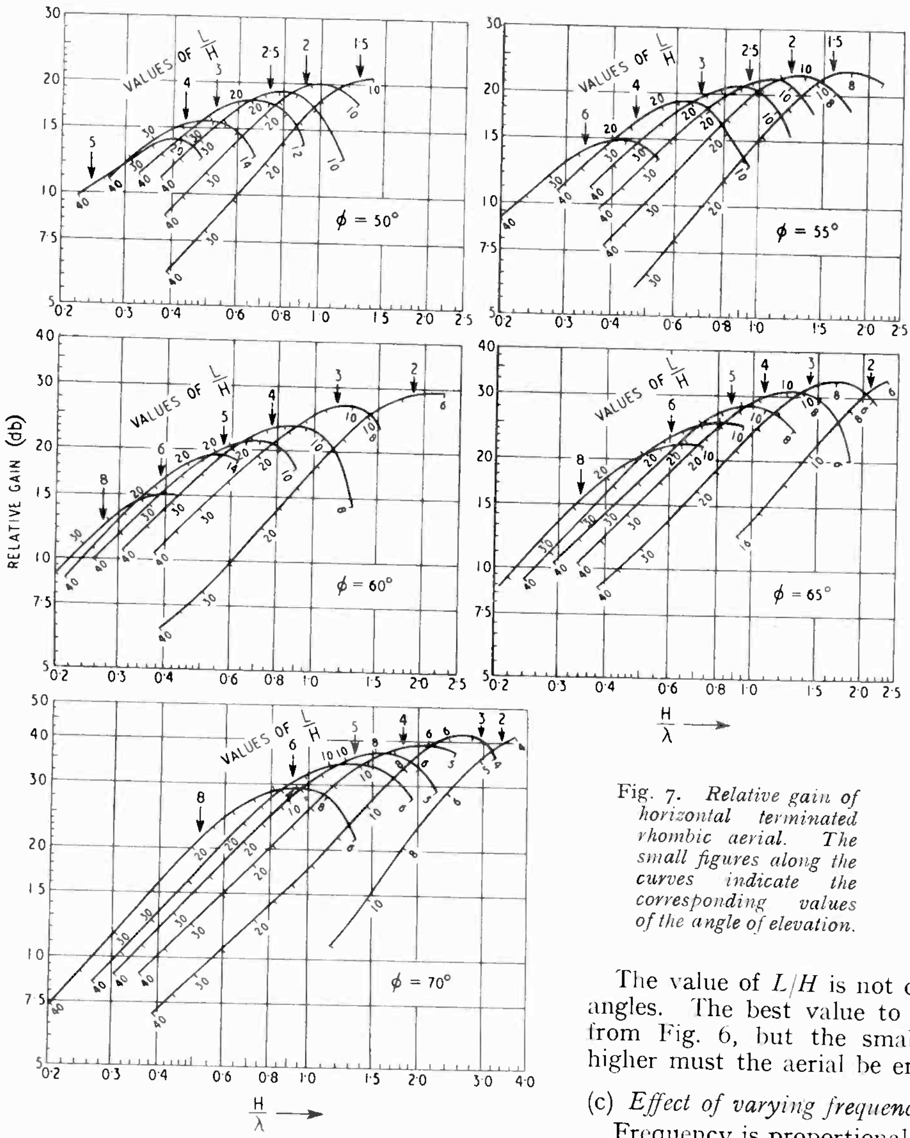


Fig. 7. *Relative gain of horizontal terminated rhombic aerial. The small figures along the curves indicate the corresponding values of the angle of elevation.*

The same directivity factor will be applicable in both cases so that the voltage across the terminals of a matched receiver is

$$V = \frac{\lambda}{2\pi} \cdot D \cdot \mathcal{E}_0 \quad \ldots \quad \ldots \quad \ldots \quad (8)$$

5. General Performance

By making repeated use of the chart, including the information expressed in Fig. 6, it has been found practicable to express all the properties of the rhombic aerial as regards the first lobe in the major axis in the five families of curves shown in Fig. 7. These correspond to ϕ equal to 50°, 55°, 60°, 65° and 70°; and give the gain as a function of H/λ for various values of L/H. In addition the angle of the lobes corresponding to the values of ϕ, L/H and H/λ selected may be found from the small scales marked along the curves.

The following general conclusions may be drawn from studying Figs. 6 and 7.

(a) *Effect of varying ϕ*

The value of ϕ is not at all critical, but long narrow rhombics (large ϕ) provide greater gain at the expense of increased length. The optimum value of H/λ is practically independent of ϕ. Small values of ϕ should be used for high angles of elevation.

(b) *Effect of varying L/H*

The value of L/H is not critical except at low angles. The best value to choose may be seen from Fig. 6, but the smaller it is made, the higher must the aerial be erected.

(c) *Effect of varying frequency*

Frequency is proportional to H/λ (for fixed H) and varying the frequency corresponds to moving along the appropriate curve of Fig. 7. As the frequency is raised the angle of the major lobe decreases, and the gain increases up to a maximum beyond which it falls off quickly.

(d) *Effect of varying height*

Considering fixed-frequency working at a definite angle of elevation, the effect of varying height may be observed by inspection of the locus of points on the Δ_1 scales. The optimum height is fairly critical, particularly for low values of ϕ. In spite of this, however, the actual values of H/λ for maximum gain is practically independent of ϕ and is approximately as depicted in Fig. 8. It has been shown that the ground reflection term is a maximum when $(H/\lambda) \sin \Delta = 1/4$. The values of H/λ from Fig. 8 are slightly less than this, however, on account of the slowly varying factor $\frac{\cos \phi}{1 - \cos \Delta \sin \phi}$ which displaces the maximum to a lower angle. The integral term is assumed to be maximized.

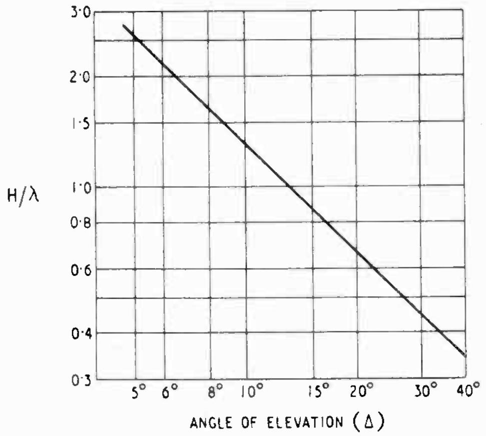


Fig. 8. *Optimum value of* H/λ.

6. Effect of Ground Constants

The data expressed in Figs. 6, 7 and 8 are based upon the assumption of a perfectly-reflecting ground surface. For most practical purposes this is sufficiently accurate. The finite resistance and dielectric constant of the ground do however introduce attenuation and phase change in the reflected wave which slightly alter the directivity pattern.

K_H is the ratio of the reflected to incident ray for horizontally-polarized waves.

$\pi - \psi_H$ is the phase change for horizontally-polarized waves.

These quantities depend upon the frequency, the angle of incidence and upon the conductivity and dielectric constant of the ground. Charts for their determination are given by Harper[2] (page 17) and Terman[5] (page 700) and also in more convenient form by McPetrie[6].

In the calculations based upon the directivity factor D_{max} used in the construction of Figs. 6, 7 and 8, a perfectly reflecting ground was assumed. Inspection of equations (3) and (4) shows that those results may be directly applied providing an effective height H' is used instead of H, where $H' = H\left[1 - \frac{\psi_H}{4\pi x}\right]$. The gain associated with the lobe must also be multiplied by $\sqrt{K_H}$. This correction factor was derived from the ground reflection term [equation (1)] in which h^2 was neglected. The error involved in so doing will now be considered.

Values of K_H less than 0.7 will very seldom be met in practice and then only over very poor ground and at very high frequencies. Furthermore we are only concerned with the magnitude of this term in the vicinity of a maximum and a reasonable lower limit for $\sin 2\pi x$ may be taken as 0.7. Using these figures the error in neglecting h^2 is only 3.2% and its effect upon the position and the gain of the lobe will be quite insignificant.

7. Acknowledgments

Acknowledgments are due to Dr. J. S. McPetrie for his encouragement and advice in the preparation of this paper and to the Chief Scientist, Ministry of Supply, for permission to publish.

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BOOK REVIEWS

Elektrische Wellen

By W. O. Schumann. Pp. 340, with 248 figures. Carl Hanser Verlag, München.

The author has been Professor of Theoretical Electrotechnics at Munich since 1924 and this book is based on courses of lectures that he has given. It is described as an introduction to the propagation and radiation of electromagnetic waves, but it is more than an introduction. The first chapter is devoted to the Maxwell equations and vector algebra. The statement on the first page that $A[BC] = C[BA]$ is incorrect. Successive chapters deal with energy transmission, Poynting vector, plane waves in dielectrics and on lines, reflection, transmission in ionized media without and with a magnetic field, waveguides and the various types of waves, complex permeability and permittivity, waves on single wires and dielectric rods, concentric cables, etc., spherical waves and the reciprocity law. The treatment is very detailed and well illustrated. The system of units employed is an unusual one first employed by G. Mie; it is a rationalized practical system in which, however, the centimetre is maintained as the unit of length. This necessitates the unit of mass being increased to 10^4 kg (i.e., ten metric tons), in order that $0.5\,mv^2$ may still give the energy in joules. On p. 17 it is stated that in this system the permeability of empty space is $\frac{1}{4\pi.10^9}$ H/cm; this is incorrect, it should be $\frac{4\pi}{10^9}$ H/cm. This is easily seen, since in $B = \mu H$ the removal of the 4π from the H necessitates its introduction into the μ. A possible explanation of these blunders is indicated in the preface where the author thanks someone for correcting the proofs in his absence; they certainly detract from the value of the book.

G. W. O. H.

Fundamentals of Electrical Engineering

By V. P. Hessler and J. J. Carey. Pp. 241 + x, with 195 figures. McGraw-Hill Book Co. Inc. Aldwych House, London, W.C.2. Price 21s (in U.K.).

This is one of an electrical and electronic engineering series of which Professor Terman acts as consulting editor. It is stated to be for use in beginning courses, but a background of electricity and magnetism from a physics course is assumed. It deals solely with fundamentals and there is no mention of electrical machines or thermionic valves. Successive chapters deal with fundamental concepts, units, electric circuits, resistance, batteries, network theorems, magnetic fields and magnetic circuits, induced e.m.f., electric fields, capacitance, transients, energy in fields, magnetostatics, and flux mapping. The treatment differs very much from the classical method, a magnetic pole making its first appearance on page 204.

As stated in the preface, 'since it has always been a source of endless confusion, the question of unit systems is given extensive consideration. The text is written in the mks unrationalized system. . . . The unrationalized form was chosen because it permitted parallel solutions in the mks and cgs systems.' The first three chapters are devoted largely to systems of units, and it is stated that the prime purpose of the book is to 'present the science of electricity and magnetism in such a manner as to clarify the consideration of units.' It is important to bear this in mind as one might otherwise form an unfavourable opinion because of the emphasis given to the subject of systems of units. This book certainly brings home to one the difficulties which students will have to face during the next few years due to the change over from the c.g.s. to either the unrationalized or the rationalized m.k.s. system. The authors introduce yet another variety which they call the subrationalized system, but on examination one finds that this is what is usually referred to as the rationalized m.k.s. system with the permeability of space taken as $4\pi/10^7$ instead of $1/10^7$, so that H is equal to the ampere-turns per metre without any 4π.

They maintain that complete rationalization would leave the permeability at $1/10^7$ but alter the values of the current and other electrical units by factors involving 4π—an unthinkable procedure. When the Institute of Radio Engineers of America recently decided to adopt the rationalized m.k.s. system, they undoubtedly meant what the authors call the subrationalized system. Although the authors refer to it and devote a section to it, they do not adopt it, but use throughout the book the unrationalized m.k.s. system, in which the permittivity of space $\kappa_0 = 1.113 \times 10^{-10}$ $(= 10^7/c^2)$ and the permeability $\mu_0 = 10^{-7}$.

On page 23 the statement that 'Potential difference is the agent in an electric circuit that causes a current to flow' is hardly correct; this is the function of the electromotive force, and a current can flow in a circuit in which there is no potential difference. The statement that the resistivity of copper is 10.37 ohms *per mil-foot* and its conductivity 0.0964 mho *per mil-foot* indicates that the mil-foot fulfils the same dual function in America as the inch-cube does here, the mil-feet being put in series for the resistance and in parallel for the conductance.

The chapters dealing with complex circuits and network theorems are very complete and include the solution of numerous problems by the application of Kirchhoff's laws, superposition, Thévenin's theorems and star-delta transformation; in these things one is not troubled with systems of units. By complex the authors mean merely complicated circuits; there is no suggestion of reactance or impedance; in fact it should be emphasized that alternating currents are not dealt with at all in the book, which might preferably have been called 'the fundamentals of direct-current electrical engineering.' The growth of current in circuits containing inductance and capacitance is considered for a steady applied voltage in the chapter on transients.

When we come to the chapters on magnetism we have to adjust ourselves to flux density B in webers per square metre and H in a monstrosity called pragilberts per metre; this latter is defined as 'that value of the field intensity which will set up a flux density of 10^{-7} weber per sq m in air.' For the uninitiated we may add that pragilberts is another way of saying 4π times the ampere-turns. The following statement on page 103 is almost incredible: 'It is often convenient in studying magnetic phenomena to consider that flux lines exert forces on each other directly. For this purpose, *parallel flux lines directed opposite are considered to attract.* Also, all flux lines are considered to act as stretched rubber bands.'

In the chapter on electric fields the authors seem to use the term displacement in a different sense from that employed by Clerk Maxwell, who introduced the term and the conception. Maxwell says 'The amount of displacement is measured by the quantity of electricity which crosses unit area while the displacement increases from zero to its actual amount.' There is no ambiguity about this, and in the m.k.s. system the unit of this displacement is a coulomb per square metre. The authors talk about 'electrostatic flux density or, as it is often called, displacement density' and then say that the total flux from a unit charge is 4π. This is surely very misleading. If flux density is the same as displacement density (Maxwell's displacement), the total flux from a unit charge can only be 1 coulomb and the formula $D = Q/r^2$ should be $Q/4\pi r^2$. The same misuse of displacement occurs in dealing with a capacitor, when it is stated that the total displacement between the plates is $4\pi Q$. This has apparently been done in order to rationalize Maxwell's $4\pi D = \epsilon_0 \mathcal{E}$ and write $D = \epsilon_0 \mathcal{E}$ in parallel with the magnetic $B = \mu_0 H$.

A very strange term is 'pracoulomb'; the coulomb or ampere-second is the practical unit in the m.k.s. system and it is very undesirable to call some other unit a pracoulomb. As we have already mentioned, the concept of magnet poles is first mentioned on p. 204 where it is developed from a solenoid. It is unusual but interesting to find in a book of this type a chapter on the mapping of fields by the curvilinear-square method; a number of useful examples of the process are given.

It was Professor Kennelly who suggested calling the e.m. units abhenrys, abfarads, abohms, etc., and the e.s. units stathenrys, statfarads, statohms, etc. The practical unit of H would have been the praoersted. The suggestion was not officially adopted and we were under the impression that the usage was dying out. This is evidently not the case, for the authors make frequent use of it.

At the end of nearly every chapter in the book there are a number of questions and problems; we note that after calculating the energy stored in an air capacitor, the student is asked what change takes place in it if the capacitor is now 'dunked in a sea of acetone.' Appendix X gives dielectric constants but the table does not contain acetone; we have learnt, however, that a dunker is an American baptist who believes in triple immersion ! G. W. O. H.

Microwave Magnetrons

Edited by George B. Collins. Pp. 806 + xviii, with 526 illustrations. (Vol. 6, M.I.T. Radiation Laboratory Series). McGraw-Hill Publishing Co., Ltd., Aldwych House, London, W.C.2. Price 54s. (in U.K.).

The editor of this book and his ten co-authors set themselves a formidable task—'to present in usable form this large amount of theoretical and practical knowledge' which was acquired during the war not only in their own laboratories but also by many workers elsewhere. The acknowledgments of the British work which initiated magnetron development for radar are generous; we even read of 'the 10 cm. magnetron perfected by the British in 1940': But of the large amount of experimental and development work done in this country after that date there is scarcely a hint, apart from Sayers' introduction of strapping in 1941.

Many readers may turn to this book to learn something about the magnetron for the first time. For them the introductory account (Chapter 1), will be of special importance. It is the more regrettable that this should be one of the less satisfactory parts of the book. The review of early types of magnetron from 1921 to 1940 cannot be said to have achieved its object of 'pointing out the significant steps that have led to the present highly efficient sources of microwaves.' There is, for example, no mention of pre-war work in England or in France. Errors in matters of detail may be exemplified from the sub-section on what are called (not very happily) 'cyclotron-type magnetrons.' On p. 4 we are told that the object of tilting the magnetic field is to cause the retarded electrons 'to spiral out of the end of the anode.' The highest power reported in pre-war literature from such magnetrons at 10 cm was not 'about 1 W' but about 20 W.

The description of the functions of the resonant system of a typical 'microwave' magnetron is concise and illuminating, though there are some slips which may confuse the beginner. The section on 'The Space Charge' is marred by the attempt to explain the 'sorting' of the retarded from the accelerated electrons in terms of the special case of Fig. 1.22. It is difficult to understand why the author did not point out that what is significant is not the velocity of an electron but the direction of the resultant electric field in which it moves. The remainder of Chapter 1 is much better, though there are several misprints which are not self-evident and a curious confusion between B and H which change their meanings in the course of a paragraph in Sec. 1.10.

The remaining 18 chapters of the book are divided into five Parts dealing with Resonant Systems (Chap. 2-5), Analysis of Operation (Chap. 6-9), Design (Chap. 10-13), Tuning and Stabilization (Chap. 14-16) and Practice (Chap. 17-19).

The treatment of resonant systems, unstrapped, strapped, and 'rising-sun', as well as of output circuits, is thorough and comprehensive.

Chapter 6 (Interaction of the Electrons and the Electromagnetic Field) integrates a large amount of theoretical work done during the war in England and the United States into an account which admirably combines completeness with compactness. Perhaps the weakest point in this account is the attempt (pp. 211–212) to justify the hypothesis of negligible initial velocity.

In Chapter 7 (Space Charge as a Circuit Element) the radio engineer will find himself on more familiar ground. In the part of the discussion which refers to Fig. 7 (c) there is some confusion over signs and arrow directions; and in Fig. 7.21 the output figures appear to be too large by a factor of 2.

The discussion of the complex and important subject of mode selection in Chapter 8 is of great interest and is well illustrated by numerous oscillograms. The brief review of noise in magnetrons (Chapter 9) will serve to emphasise the magnitude of this unsolved problem.

In Part III, and especially in Chapter 10, a comprehensive statement is made of the philosophy and practice of what may be called the M.I.T. school of magnetron design. There is much of value here, but the approach adopted is not one which appeals to the personal judgment of the reviewer. It has sometimes seemed that the technique of 'scaling,' which achieved outstanding practical results in Americans hands, was in some danger of being regarded as an end rather than a means. That a less restricted viewpoint is in fact possible is well illustrated by the interesting results presented at the end of Chapter 10.

While the treatment of design of the resonant system (Chapter 11) is excellent, that of the cathode (Chapter 12) is less happily balanced. The constructional details illustrated at the end of Chapter 12 will be welcomed. The absence of any adequate treatment of magnet design is a surprising feature of the book.

The three chapters covering tuning (mechanical and electronic) and frequency stabilization—a part of the magnetron art to which the American contribution was particularly large and successful—will be of special interest to British readers. While a praiseworthy attempt has evidently been made to eliminate 'lab. slang' from this book, the survival of 'cookie- cutter' in Chapter 14 may puzzle the non-American reader.

There is much interesting data in Chapter 17 on 'Construction,' especially on brazing technique. Whether it would suffice to enable 'an experimenter new to the field of microwave magnetrons . . . to build tubes' would depend greatly on his talent and experience. In the reference (p. 650) to the use of 'selenium copper' for magnetron blocks in England, 'tellurium copper' is evidently intended. A surprising omission is the lack of any reference to the gold-wire seal, though the facts which make its use sometimes desirable are clearly stated on pp. 672-673. It may also be noted that the pumping schedule given on p. 693 is more elaborate than has been found necessary in British practice.

The discussion on 'Measurements' in Chapter 18 is useful, particularly on 'cold' measurements, though fairly elementary. A concise and well laid-out selection of data on 'Typical Magnetrons,' including some for c.w. operation as well as some recent experimental designs of the rising-sun type, conclude the book. But this chapter would have been more accurately headed 'Typical American Magnetrons.'

In general the book is admirably produced and illustrated. Misprints, incorrect cross-references, and other minor errors, are commoner than they should be in a book of this kind; but in the prevailing circumstances, the choice was evidently between having the book like this and not having it at all. In spite of the blemishes in a part of its content, it would have been a great misfortune if we had not had it at all.

E. C. S. M.

CORRESPONDENCE

Letters to the Editor on technical subjects are always welcome. In publishing such communications the Editors do not necessarily endorse any technical or general statements which they may contain.

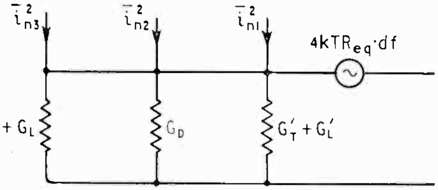
Valve Noise and Transit Time

SIR,—I was interested to read, in the above article by Campbell, Francis and James, in your May, 1948 issue, some criticism of the conclusion reached by Bakker* that his theoretical result for induced grid noise has been substantiated experimentally.

If Bakker's description of the experimental procedure is correct, then there would be appreciable error in his results at the higher frequencies. In order that his experiment should give a satisfactory result it is necessary to replace the 'noisy' input admittance of the test valve by an equal passive admittance at a known temperature. According to Bakker's description, the valve under test was biased to cut off and no other circuit modification is mentioned. It is reasonable to assume that the circuit capacitance was corrected to allow for the change in valve capacitance although there is no specific mention of this. On the other hand, if a resistor of the same value as the value input conductance had been substituted the fact would surely have been mentioned; and the difference in saturated diode current readings would then have been a measure of the 'excess noise.'

If I am correct in thinking there was this omission in the experimental procedure, then this alone would account for an error of the order of 2 $(4kTG_T)$ at 50 Mc/s whereas the total theoretical effect is 4.8 $(4kTG_T)$.

This can be shown as follows:



In the diagram

G_T = Test Valve input conductance due to transit time.

G_L = Test Valve input conductance due to lead inductance.

G_D = Circuit Losses.

$G'_T + G'_L$ = Input conductance of first stage of receiver (EF8).

R_{eq} = Equivalent noise resistance of receiver.

$\overline{i^2_{n1}}$ = Induced grid noise of receiver input circuit.

$\overline{i^2_{n2}}$ = Thermal noise current generator $(4kTG_D.df)$.

$\overline{i^2_{n3}}$ = Induced grid noise of test valve.

$G = G_T + G_L + G_D + G'_T + G'_L$

Then the first measurement gives:

$$\overline{i^{2\prime}_D} = \overline{i^2_{n1}} + \overline{i^2_{n2}} + \overline{i^2_{n3}} + 4kTR_{eq} \cdot G^2$$

the second measurement gives:

$$\overline{i^2_D}'' = \overline{i^2_{n1}} + \overline{i^2_{n2}} + 4kTR_{eq}\,(G_D + G'_T + G'_L)^2$$

Hence

$$\overline{i^2_D}' - \overline{i^2_D}'' = \overline{i^2_{n3}} + 4kTR_{eq}\{G^2 - (G_D + G'_T + G'_L)^2\}$$

$\therefore$ Error in deduced value of $\overline{i^2_{n3}}$

$$= 4kTR_{eq}\,(G_T + G_L)\,(G_T + G_L + 2G_D + 2G'_T + 2G'_L)$$

If we take, as Bakker has done, $G_L = 2G_T$

Error =

$$4kTG_T \times 3R_{eq}\,(G_T + G_L + 2G_D + 2G'_T + 2G'_L).$$

Although accurate data for the EF8 is not available a conservative estimate for R_{eq} is 1,500 ohms and for $(G'_T + G'_L)$ is 100 μmhos at 50 Mc/s. From Bakker's measurements $(G_T + G_L)$ = 200 μmhos and an estimate of 50 μmhos for G_D can be taken.

$$\therefore \text{ At 50 Mc/s, Error} \approx 4kTG_T\,(3 \times 1{,}500 \times 500 \times 10^{-6}) = 4kTG_T \times 2.25.$$

It is assumed throughout that the receiver bandwidth is narrow compared with the bandwidth of the test circuit.

According to the above analysis the equivalent saturated diode current I_D, in Bakker's paper, is given by

$$2eI_D = \overline{i^2_{n3}} + B\omega^4 + CG_D\omega^2.$$

If we neglect the third term (which may not be justifiable) and correct Bakker's results at a current of 9.6 mA for the second term $B\omega^4$, assuming an error of 2 in 5 at $\omega = 4 \times 10^8$, then the points are found to lie just as closely to a straight line as originally—without the low-frequency point being seriously out, as shown by Bakker.

It appears, therefore, that the experimental agreement obtained by Bakker is of little significance.

Great Malvern. N. HOULDING.
Worcs.

* "Fluctuations and Electron Inertia," C. J. Bakker. *Physica*, January 1941, p. 23.

WIRELESS PATENTS

A Summary of Recently Accepted Specifications

The following abstracts are prepared, with the permission of the Controller of H.M. Stationery Office from Specifications obtainable at the Patent Office, 25, Southampton Buildings, London, W.C.2, price 2/- each.

DIRECTIONAL AND NAVIGATIONAL SYSTEMS

594 525.—Automatic gain control system for a radiolocation set, with four directive aerials which are switched to give quadrantal scanning

Marconi's W.T. Co. Ltd. (assignees of A. V. Bedford). Convention date (U.S.A.) 23rd January, 1943.

594 530.—Radio-navigational system in which a periodic phase-sweep is applied by aerial-switching, to a wave radiated at constant frequency, in order to indicate its direction of origin

Standard Telephones and Cables Ltd., C. W. Earp and C. E. Strong. Application date 4th August 1944.

594 567.—Super-regenerative circuit, particularly for receiving the pulsed signals used in radiolocation, for presentation to a cathode-ray indicator.

Hazeltine Corpn. (assignees of H. A. Wheeler). Convention date (U.S.A.) 14th April, 1944.

594 704.—Direction-finding system in which a constantly-rotating aerial is used to phase-modulate the incoming wave by means of the doppler effect.

Marconi's W.T. Co. Ltd. (assignees of D. G. C. Luck). Convention date (U.S.A.) 31st May, 1943.

594 710.—Direction-finding system utilizing pulsed signals radiated from spaced beacon stations at different repetition-frequencies, so as to distinguish direct-path signals from those arriving via the Heaviside layer.

Marconi's W.T. Co. Ltd. (assignees of B. Schmurak and G. D. Hulst Junr.). Convention date (U.S.A.) 30th December, 1943.

594 712.—Driving the deflection coils of a cathode-ray indicator in synchrony with the scanning aerial of a radiolocation set.

Marconi's W.T. Co. Ltd. (assignees of W. A. Tolson and W. J. Poch). Convention date (U.S.A.) 31st December, 1941.

594 765.—Radio-altimeter equipment using a frequency-modulated exploring-wave, the rate-of-change of frequency being also periodically varied.

Marconi's W.T. Co. Ltd. (assignees of I. Wolff) Convention date (U.S.A.) 31st July, 1942.

594 788.—Radiolocation set in which the sensitivity of the exploring wave is constantly varied over a given vertical angle, so as to give a direct indication of elevation.

K. Hopkinson. Application date 26th February, 1946.

594 880.—Time-base circuits for radiolocation sets, in which the echo-traces are expanded in order to ascertain the nature of the target from the deformation it produces.

Standard Telephones and Cables Ltd. (assignees of D. D. Grieg). Convention date (U.S.A.), 18th September, 1942.

594 888.—Generating and calibrating the time-base sweep for the cathode-ray indicator of a radiolocation set.

Standard Telephones and Cables Ltd. (assignees of H. G. Busignies). Convention date (U.S.A.), 14th September, 1942.

594 893.—Aerodrome equipment, controlled by a motor-driven aerial, for reproducing the position of an incoming plane relatively to a scale model of the landing ground, and televising this information back to the pilot.

H. D. Gracias. Application date 14th February, 1945.

595 002.—Navigational system in which a mobile craft determines its position in terms of the phase-difference between the synchronized signals it receives from a number of spaced beacon stations.

Marconi's W.T. Co. Ltd. (assignees of H. S. Huff). Convention date (U.S.A.) 6th January, 1943.

595 006.—Impulse-generator, suitable for radiolocation, in which a discharge tube of the rhumbatron type is utilized as a blocking-oscillator.

Standard Telephones and Cables Ltd. (assignees of R. R. Buss). Convention date (U.S.A.) 17th July, 1941.

595 022.—Navigational system in which the distance, azimuth, and course of an approaching plane is transmitted to it from the airport by successive characteristic pulsed signals on a common carrier.

Standard Telephones and Cables Ltd. (assignees of E. M. Deloraine, H. G. Busignies and P. R. Adams). Convention date (U.S.A.) 26th April, 1944.

595 062.—Construction of light-transparent reflector for the directive aerial of a radiolocation set used for aircraft spotting in association with a searchlight.

Marconi's W.T. Co. Ltd. (assignees of C. W. Hansell). Convention date (U.S.A.) 27th January, 1943.

595 074.—Direct-ray direction finder in which a c.r. indicator responds to the leading edge only of pulsed signals.

Marconi's W.T. Co. Ltd. (assignees of L. E. Norton). Convention date (U.S.A.) 28th August, 1942.

595 094.—Differentiating desired signals from 'clutter' in radiolocation sets using p.p.i. presentation.

C. W. Oatley, W. S. Elliott and H. Pursey. Application date 2nd May, 1945.

595 168.—Sensitive receiver for direction-finding systems using phase or frequency-modulated signals.

Standard Telephones and Cables Ltd. and C. W. Earp. Application date 11th June, 1945.

595 216.—Directional system in which a beacon radiates a rotating beam which is frequency or phase modulated in step with the phase of its rotation.

Standard Telephones and Cables Ltd. and E. O. Willoughby. Application date 16th May, 1945.

595 224.—Frequency-changing system, particularly for converting pulses of high repetition-frequency into pulses of lower frequency in radiolocation.

Hazeltine Corporation (assignees of C. J. Hirsch). Convention date (U.S.A.) 16th August, 1944.

595 352.—Non-contact-making coupling, which allows the relative rotation of two sections of a waveguide, feeding the aerial say of a radiolocation set.

E. C. Cork and M. Bowman-Manifold. Application date 1st September, 1942.

595 653.—Flanged waveguide or horn for feeding centi-

metre waves to the reflecting-mirror of a directional aerial.

A. Bolton. Application date 16th June, 1945.

595 724.—Method of feeding centimetre waves to the scanning aerial of a radiolocation set through a waveguide which is offset from the scanning-plane and the focus of the parabolic reflector.

Western Electric Co. Inc. Convention date (U.S.A.) 6th November, 1943.

RECEIVING CIRCUITS AND APPARATUS

(See also under Television)

595 008.—Gain-control system, particularly for removing undesired low-frequency modulations from the rotating radio-beam signals received on a grounded-grid amplifier.

Marconi's W.T. Co. Ltd. (assignees of L. E. Norton). Convention date (U.S.A.) 28th January, 1942.

595 497.—Receiver for frequency-modulated signals in which a control-bias, derived from the signal, is utilized to give an unambiguous indication of any mis-tuning of the circuits.

The British Thomson-Houston Co. Ltd. Convention date (U.S.A.) 23rd July, 1943.

595 574.—Device for receiving a facsimile signal and for simultaneously recording several copies of it.

Faximile Inc. Convention date (U.S.A.) 30th March, 1942.

595 601.—Method of forming and processing the fine point of the contact wire of a crystal rectifier particularly for centimetre waves.

Western Electric Co. Inc. Convention date (U.S.A.) 10th March, 1943.

595 602.—Receiver for phase-modulated signals comprising a piezo-electric crystal and a tuned network coupled to a pair of opposed diodes.

Marconi's W.T. Co. Ltd. (assignees of M. G. Crosby). Convention date (U.S.A.) 3rd April, 1943.

TELEVISION CIRCUITS AND APPARATUS

For Transmission and Reception

595 260.—Construction and operation of a picture-dissecting tube including a photo-electric cathode and an electron-multiplier, for use in television.

Farnsworth Television and Radio Corporation. Convention date (U.S.A.) 26th April, 1943.

595 730.—Television system in which discrete portions of frequency-modulated sound-signals are interpolated, separately from the synchronizing pulses, during the fly-back periods of scanning.

Marconi's W.T. Co. Ltd. (assignees of G. L. Freidendall and A. C. Schroeder). Convention date (U.S.A.) 24th March, 1944.

TRANSMITTING CIRCUITS AND APPARATUS

(See also under Television)

595 138.—Automatic gain-control system for the short-wave link channels between a distributing network of transmitters for broadcasting television over a wide area.

The General Electric Co. Ltd. D. C. Espley and D. O. Walter. Application date 6th November, 1944.

595 308.—Magnetron with say three resonant anodes, each coupled to a separate aerial, for radiating a polyphase high-frequency rotating field.

R. K. Sas. Application date 23rd September, 1944.

595 376.—Construction and mounting of the tuning screw or plunger used with waveguides and cavity-resonators.

Marconi's W.T. Co. Ltd. (assignees of V. D. Landon). Convention date (U.S.A.) 29th October, 1943.

595 575.—Facsimile signalling system in which the copy to be transmitted is mounted on a rotating drum which is scanned by the electron-beam of an iconoscope tube.

Faximile Inc. Convention date (U.S.A.) 22nd November, 1941.

595 698.—Frequency-modulating transmitter in which the inter-electrode capacitance of a valve is utilized to increase the angular-deviation produced by a given signal-voltage.

Marconi's W.T. Co. Ltd. (assignees of G. L. Usselman). Convention date (U.S.A.) 16th May, 1944.

SIGNALLING SYSTEMS OF DISTINCTIVE TYPE

594 235.—Signalling system in which the phase of the signal source is periodically reversed in order to minimize the effect of interference or jamming.

Standard Telephones and Cables Ltd. (assignees of N. H. Young, Jr.). Convention date (U.S.A.), 16th April, 1943.

594 523.—Resistance-capacitance coupled amplifier for the scanning-frequencies developed in facsimile-signalling systems.

Marconi's W.T. Co. Ltd. (assigness of M. Artzt). Convention date (U.S.A.), 18th November, 1942.

594 566.—Modulation system in which the spacing of successive pulses is determined by the values derived at equal intervals of time from the signal voltage.

"Patelhold" Patentverwertungs &c. A.G. Convention date (Switzerland), 16th March, 1944.

594 797.—Generating trains of pulses from a cavity-magnetron oscillator, and modulating them in space or time, particularly by speech or television signals.

Marconi's W.T. Co., Ltd. (assignees of J. Evans). Convention date (U.S.A.), 12th August, 1943.

594 798.—Filtering and selecting the different signals in a multi-channel system using a single carrier-wave modulated at different repetition frequencies.

D. E. Bridges. Application date 9th April, 1945.

594 803.—Multi-stage wide-band amplifier, with graded couplings to produce a given 'step-function' response, particularly for short signalling pulses.

Standard Telephones and Cables Ltd., and W. A. Montgomery. Application date 14th May, 1945.

594 892.—Generating pulses of constant repetition, modulating them in space or time, and then converting them into a sine wave before detecting the imposed signal.

Marconi's W.T. Co., Ltd. (assignees of D. S. Bond). Convention date (U.S.A.), 26th September, 1942.

CONSTRUCTION OF ELECTRONIC-DISCHARGE DEVICES

594 460.—Construction of gas-filled discharge tube including a resonator which operates as a safety switch or device in a high-frequency transmission line.

The M-O Valve Co., Ltd., B. N. Clack and N. L. Harris. Application date 30th June, 1944.

594 549.—Construction and operation of a gas-filled discharge tube as a frequency-modulator by voltage or capacitance control.

"Patelhold" Patentverwertungs &c., A.G. Convention date (Switzerland), 13th December, 1943.