THE RADIO AND ELECTRONIC ENGINEER

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"To promote the advancement of radio, electronics and kindred subjects by the exchange of information in these branches of engineering."

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COMMONWEALTH RELATIONS

In order to fulfil its principal object—the advancement of the science and practice of radio and electronic engineering—the Institution is necessarily international in character. The prefix "British" was not adopted for any national purpose but for reasons given in the Institution's official history.† In fact, members of the Institution are to be found in 64 other countries; in addition subscriptions for *The Radio and Electronic Engineer* account for the *Journal* being read in over 90 countries. These facts are evidence of the Institution's international status.

The qualifications required of applicants do, of course, limit membership of the Institution. Since its foundation thirty-eight years ago, the Institution has steadfastly advocated the need for ensuring the professional competence of radio and electronic engineers by holding examinations and thereby promoting the study of radio and electronic science. This activity is as important as—and indeed essential for—the dissemination and advancement of knowledge, which the Institution provides in its "learned society" functions through *The Radio and Electronic Engineer* and other publications, and by arranging lectures and discussion meetings.

Moreover, in following this British pattern of learned *and professional* society policy, the Institution has played its part in securing recognition of the professional status of the engineering profession as a whole. The success of this policy is shown by the preference given to corporate members of Chartered Institutions for official and other appointments throughout the world.

Admittedly much remains to be done to ensure wider public understanding of the distinction between trained and qualified engineers and those who cannot meet the standards of competence demanded of applicants who wish to acquire the status of Chartered Engineer.

In the Commonwealth particularly there is general agreement on the functions of such Institutions as our own and among Commonwealth engineers there is a shared purpose in developing the aims and objects described. It is a valuable and developing factor in Commonwealth relationship and understanding and, as such, has been especially recognized in the Institution's Charter and Bye-laws which empower the Council to establish Divisions in countries outside Great Britain. Within those Divisions a National Council may be elected which, whilst operating within the framework of the Charter and Bye-laws of the Institution, will manage the affairs of the Division. Where a Division of the Institution is established, the National Council will be able to form zones or sections, publish their own *Proceedings*, and in this and other ways provide full opportunity for members within their Division to participate in the activities of the Institution. Thus, a member may enjoy all the advantages of membership of a professional *and* international Institution, whilst at the same time participating in activities of particular concern in his own country.

By reason of its concentration of membership, Divisions of the Institution are being established in the Commonwealth countries. Common language and common purpose should enable this enterprise to increase the momentum of disseminating as well as advancing knowledge for the benefit of all. It is this object of the Institution which transcends national barriers.

G. D. C.

† "A 20th Century Professional Institution—the Story of the Brit.I.R.E."

B

INSTITUTION NOTICES

Elections to the Council

Corporate members of the Institution are asked particularly to note that the list of Council nominees for election to the 1963–4 Council, notice of the Annual General Meeting and the Annual Report of the Council will be published in forthcoming issues of the *Proceedings of the Brit.I.R.E.* and not in the *Journal—The Radio and Electronic Engineer*—as in previous years.

Following the publication of the Council list of nominations for election, nominations from the membership will be invited. Full details of the procedure to be adopted will accompany this list (see also Bye-law 44).

The Annual General Meeting, the second since Incorporation by Royal Charter, will be held in London at the School of Hygiene and Tropical Medicine on Wednesday, 27th November.

The Institution and the Engineering Institutions Joint Council

As technology has developed the engineering profession has tended to divide into more and more separate divisions. The resulting fragmentation has prevented any single voice from speaking on behalf of the engineering profession. The older Institutions have collaborated on occasions but they could not claim to speak for engineers as a whole. Earlier efforts to form some unifying body have been unsuccessful largely because the older Institutions tended to dominate the affairs by having a greater representation than the smaller Institutions.

The formation of the Engineering Institutions Joint Council, however, is on an equivalent basis. Each Institution has the same number of representatives and contributes the same financial support. In the Engineering Institutions Joint Council there is at last a forum where the general problems of the profession can be discussed.

The main Council has already set up a number of committees and the matters under discussion include a code of conduct for professional engineers; the definition of the technician and the need to establish associations to cater for technicians; the restriction of the use of designatory letters to Corporate Membership; the relationship between Institutions in the United Kingdom and similar bodies in the Commonwealth; the situation created by the proposed Engineers Registration Bill of the Province of Quebec.

These are very early days in the work of the Engineering Institutions Joint Council. Undoubtedly, if it is successful and secures the co-operation of all its members, there will be a rapid increase in its activities and importance. At the present time the Council is being operated with an honorary secretary but there is no doubt that within a very short time it will be necessary to appoint a full time secretary and staff.

This Institution's Council feels very strongly that the E.I.J.C. must have absolute support. To this end the following representatives have been appointed to the committees shown.

The Joint Council: Colonel G. W. Raby, C.B.E., J. L. Thompson and G. D. Clifford (Members).

General Purpose and Finance Committee:
J. L. Thompson (Member).

Education and Training Committee:
H. Arthur (Associate Member).

Fourth International Conference on "Non-destructive Testing"

The British National Committee for Non-destructive Testing in association with The Institution of Mechanical Engineers will hold the Fourth International Conference on "Non-destructive Testing" at Church House, Westminster, London, from 9th–13th September 1963. There will be nine technical sessions at which about 50 papers will be presented and discussed. Sessions will cover "Practical Considerations", "Radiography", "Ultrasonics", "Pressure Parts" and "Other Non-destructive Testing Techniques and Practices".

Details of the Conference and registration forms may be obtained from The Conference Section, The Institution of Mechanical Engineers, 1 Birdcage Walk, London, S.W.1.

E.O.Q.C. International Conference

The theme of the Seventh International Conference of the European Organization for Quality Control is "Cost Reduction through Quality Control". The Conference will be held in Copenhagen on 2nd–4th September 1963.

British participation in the conference is through the National Council for Quality and Reliability on which the Institution is represented. Further particulars may be obtained from the N.C.Q.R., c/o British Productivity Council, Vintry House, Queen Street Place, London, E.C.4.

Completion of Volume 25

This issue completes Volume 25 of the *Journal* which covers the period January–June 1963. An index to the volume will be circulated with the July issue.

Members are reminded that they may have their half-yearly volumes (six issues) bound at 16*s*. 6*d*. per volume (postage extra: 3*s*. Great Britain; 4*s*. Overseas). The issues and the appropriate indexes should be sent to the Publications Department, at 9 Bedford Square, London, W.C.1, with remittance.

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LAMBDA: A Radio Aid to Hydrographic Surveying

Presented at a meeting of the Radar and Navigational Aids Group in London on 14th March 1962.

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AND

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Summary: The word LAMBDA is an abbreviation for "low ambiguity Decca" and refers to a radio position-fixing system used by hydrographic survey ships. A fix is obtained by measuring the distance from the ship to shore stations; the c.w. master transmitter and the receiver are carried on the ship and the two slave stations are sited on land. This paper describes the principles of the LAMBDA system with special reference to the method of lane identification. Circuits and practical details of the equipment are described and notes are included on wave propagation aspects and the performance of the system in the field.

1. Introduction

Since the Second World War several different radio position-fixing systems have been adapted or developed for use in hydrographic surveying. Such systems permit a survey ship to fix her position with respect to a known trigonometrical network ashore. By this means the depth soundings and other observations that the survey comprises can be assigned their correct positions on the earth's surface, so permitting marine charts of the area to be constructed or revised. Accurate radio position-fixing also enables the ship to navigate along the predetermined parallel tracks that must be followed if the area is to be surveyed economically, and to proceed to and from the area of interest with the minimum delay and wasted steaming.

The main contribution that the radio systems have made to hydrographic surveying is the facility of operating without full-time dependence on the visibility of shore landmarks, enabling surveys to be carried out far beyond horizon range from shore with an accuracy greatly superior to that of the astro-fixing on which long-range working previously depended.

In this paper the principles of the Lambda hydrographic surveying system are outlined and certain aspects of technical interest are described.

2. General Principles

2.1. *Two-range Fixing*

In common with the majority of radio position-fixing systems used for hydrographic work (e.g. Decca, Raydist, Lorac, Hi-Fix, Hydrodist, Rana), Lambda is based upon the phase-comparison of continuous-wave signals. It provides position-fixing of the "two-range" form, which for hydrographic purposes has tended to become more widely used than the "hyperbolic" fixing normally associated with phase-comparison systems. In two-range working, the master transmitting station is carried on the ship together with the receiver and the latter furnishes readings which are related to the direct distances from the ship to two slave stations established at known positions ashore. In the case of the Lambda system, the use of the two-station shore "chain" is confined to a single ship at a time; survey ships often operate as single units, however, and this limitation is normally acceptable.

The two-range layout is simpler to deploy ashore than the conventional three or four-station hyperbolic chain, and also has the merit of using simple co-ordinates comprising two families of concentric circular position-lines. Two-range fixes are more readily amenable to direct processing, e.g. by the construction of an over-printed grid, than are the fixes from a pair of distance-difference lines that a hyperbolic system provides. A geometrical characteristic of the two-range layout is the relatively large proportion of the coverage (compared with a hyperbolic system having the same distance between the slave stations) within which a high fix-accuracy can be obtained. The angle of cut between the two circular position-line patterns is large over a wide area, and there is no loss of accuracy through lane expansion such as occurs when similar equipment is operated as a hyperbolic chain.

2.2. *Generation of Circular Phase Patterns*

Lambda employs some of the basic techniques of the Decca Navigator, and the name is derived from the words "Low AMBiguity DeccA". This term refers to the incorporation of a method of lane identification by which the ambiguities in the circular phase patterns are partly resolved and represents an important operational advance over the earlier version of the same basic system known as "Two-Range Decca"[1] which relied solely on mechanical lane-counting by geared-down phasemeter pointers to obtain the whole-lane numbers.

The layout of a Lambda chain is shown in Fig. 1. Since the radiated and phase-comparison frequencies

† The Decca Navigator Company Limited, New Malden, Surrey.

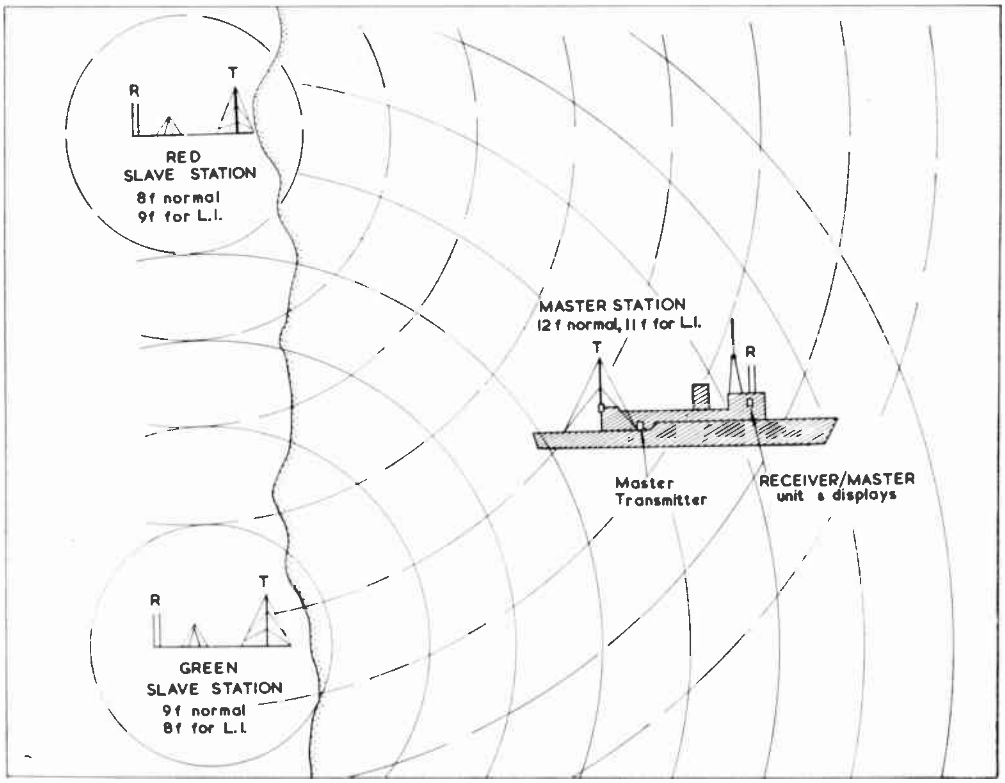


Fig. 1. Chain layout of two-range hydrographic survey system.

are harmonically related, it is convenient to refer to them in a harmonic notation rather than by the actual frequency values. The master transmitter on the ship radiates a c.w. signal of frequency $12f$ where f is approximately 14 kc/s. On shore, the "red" slave station receives the master transmission and radiates a frequency $8f$, in a manner such that the slave and master signals have a constant phase relationship at the common multiple frequency $24f$; a stable pattern of constant phase-difference lines therefore exists about the two stations in the form of a family of confocal hyperbolae.

In two-range working, the hyperbolic pattern as such is not used. Instead, advantage is taken of the fact that, everywhere along the line joining master and slave, the "lanes" corresponding to the whole cycles of phase difference are of constant width equal to half a wavelength at the phase-comparison frequency. If the ship moves so as to alter the master-to-slave distance by half a wavelength at the red frequency ($24f$), for example, the red Decometer will make one revolution and will thus directly indicate the change of one lane in the ship-to-shore distance. At $24f$ the lane-width is roughly 420 metres and the Decometer can be read to less than half of a hundredth of a revolution; the system may therefore be said to be sensitive to a change of a metre or two in the ship's distance from the slave. A similar process takes place in the green co-ordinate ($9f$ slave frequency) at a common comparison frequency $36f$, giving a lane-width of about 280 metres as shown in Table 1.

Used in this manner, the receiver responds to a series of circular position-lines centred on each slave station and spaced at constant intervals of half a wavelength. The lanes are numbered in a manner similar to those of the conventional Decca Navigator and are grouped into "zones" 10 km wide; for convenience, the Decometers are so connected that the readings decrease as the slave stations are approached (in normal Decca the readings increase from master to slave). The display meters are described in greater detail later, together with the method of integrating the successive lanes and zones passed through and the system of lane identification.

The zone, lane and fraction readings for the two patterns can be converted into distance units if the speed of propagation is known, so providing a two-range fix of the ship's position. The readings are normally converted into distances by plotting them on a chart overprinted with the two patterns of circular position-lines, the patterns being coloured and numbered to correspond with the Decometers. The circular position-lines comprising the patterns are drawn at constant radial intervals of one or more

Table 1
Typical values for frequency and lane width.

Function	Carrier frequencies		Phase comparison frequencies		Lane width
	kc/s	harmonic	kc/s	harmonic	(c = 299 650 km/s) metres
Master	177·6	$12f$	—	—	—
Master identification (Lambda system only)	162·8	$11f$	—	—	—
Green	133·2	$9f$	532·8	$36f$	281·2
Red	118·4	$8f$	355·2	$24f$	421·8

Note: All frequencies are harmonically related to a non-transmitted fundamental value f.

lane-widths, and the fractional meter readings are interpolated between the lines as required.

The function of the system as so far described can be usefully summarized in the following way. If a second receiver were placed close to, say, the red slave station, the red Decometer reading of this receiver would show practically no change as the ship moved; this is because the slave station radiates at all times a signal having a constant phase relationship with that received from the master. At the "master" end of the line, however, the shipborne Decometer indicating the phase-difference between the master and slave signals is sensitive to changes in the ship-to-shore distances, since these alter the lengths of the transmission paths from the master to the slave and back to the receiver while the direct path from the master to the receiver remains constant.

2.3. *Practical Aspects of Distance Measurement*

In practice corrections have to be applied for the non-uniform velocity of propagation of radio waves in the groundwave mode and for certain fixed phase shifts. The full expression for measurement of the distance between the ship and a slave station by the Lambda system therefore becomes

$$d = \frac{\lambda_{cf}}{2}(\phi - \alpha - \psi)$$

where

d is the distance from the "electrical centre" of the ship to the mid-point between the receiving and transmitting aerials at the slave station;

$\frac{\lambda_{cf}}{2}$ is the lane-width in metres for the appropriate pattern, assuming free-space velocity;

ϕ is the observed Decometer reading (whole lane number plus fraction);

α is the "locking constant";

ψ is a correction to the free-space value of the velocity of propagation.

The electrical centre of the ship is not necessarily the mid-point between the master transmitting aerial and the receiving aerial. Its exact location varies with different types of vessel and is found by calibration at a known distance and on a number of different headings. The locking constant is the name given to the overall phase shift resulting from two causes, namely the close proximity of the receiver to the master transmitter (placing the former in the "induction field") and, at the slave station, a possible fixed displacement from the nominal zero phase-difference condition that is assumed to exist between the received master signal and the outgoing slave transmission. The value of the locking constant for each pattern is found at the start of a survey by observations at exact known distances from the slaves, and is thereafter subtracted from all observed Decometer readings.

The quantity ψ refers to the dependence of the effective velocity of propagation upon the nature of the medium over which the signals are transmitted—an aspect of the ground-wave mode of propagation which, if it does not normally figure in the communications field, is of fundamental importance when low-frequency radio transmissions form the basis of accurate position or distance determination. A phase-lag results from absorption of energy by an imperfectly conducting earth and has been the subject of theoretical work, notably by Sommerfeld, Bremmer and Norton, described in a paper by Schneider.[2] Extensive work in this field has also been carried out by Dr. B. G. Pressey and his associates of the D.S.I.R. Radio Research Station.[3]

Figure 2 shows a practical set of phase-lag correction curves for the red and green patterns of the Lambda system. The increase in the correction value at short ranges is the result of the complex field existing around the transmitter, and the increase with distance beyond 100 km or thereabouts is the effect of the phase-lag. The effective speed of propagation resulting from the phase-lag varies widely with the electrical characteristics of the medium over which

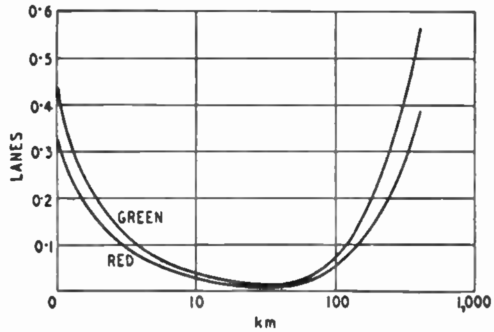


Fig. 2. Phase correction (expressed in lanes) on sea-water transmission path for assumed free-space velocity of 299 776 km/s.

the signals are transmitted; for example, the sum of experience so far points to a mean speed of 299 650 km/s over sea-water transmission paths, while a corresponding figure for land paths of the lowest soil conductivity yet encountered (of the order of $\sigma = 5 \times 10^{-15}$ e.m.u.) amounts to about 298 400 km/s.

In practice it is possible to apply corrections for different path conductivities, and also to some extent for paths of mixed conductivity such as the case where a large island or promontory intervenes between the ship and the shore. The application of the corrections shown in Fig. 2 for transmissions over sea-water leaves, with present knowledge, a residual uncertainty in distance measurement amounting to one or two parts in 10 000. The curves were constructed on a theoretical basis, but have been confirmed by observations at distances up to about 200 km; beyond this distance it is hoped to obtain practical confirmation from trials specially designed for the purpose and from the experience being gained in field operations carried out at long ranges.

2.4. *Lane Integration*

The Lambda system departs from normal Decca practice in employing the "fine" phasemeters operating at $24f$ and $36f$, purely for determining the fractional lane values. These Decometers will be seen at the top of the display unit illustrated in Fig. 4. Lane integration is carried out by the lower pair of meters and these respond to the phase difference between master and slave stations measured at the transmitted slave station frequencies ($8f$ for red, $9f$ for green); thus lane integration is performed with respect to a phase pattern three times coarser than the red fine pattern (and four times for green), so that the number of ambiguous position lines in the patterns is reduced in these ratios compared with the ambiguities there would be if the fine meters were used for lane integration. In practical terms this means that the user only has to know his distance from the stations to within about ± 630 and ± 560 metres for red and green respectively in order to be able to set the lane pointers on the lower meters to the correct numbers when starting off.

Although there are only eight of the basic slave-frequency lanes round the dial on the red meter, its scale is marked in the fine lane units so as to tie up with the fraction meter above, which in turn interpolates to one-hundredth of a fine lane. It is assumed, of course, that the overall accuracy of the phase comparison at the slave frequency is always good enough to indicate the correct fine lane on the lower scale, that is to say always better than $\pm$ 60 deg of phase for red and $\pm$ 45 deg of phase for the green. As well as reducing the basic ambiguity of the system, this arrangement permits operation at a greater range for a given probability of the lane integration breaking down, since no frequency multiplication is involved when comparing phase at the slave frequency, and the discrimination against noise is correspondingly higher.

2.5. *Lane Identification*

The degree of ambiguity indicated above would often be acceptable in hydrographic work since the survey normally starts from a known point, but would be excessive in a system such as Lambda which can be used, under suitable conditions, at distances several hundred miles offshore; the rectification of an error in lane integration could prove extremely costly in time and steaming. Moreover it may not always be possible for a ship moving from one distant survey area to another to know her position on arrival at the new location with the accuracy of better than half a slave-frequency lane in each co-ordinate that is needed for setting the meters correctly, particularly if the chain has been switched off in the interval.

Accordingly the Lambda system incorporates a lane identification facility similar in broad principle to that which forms part of the Decca Navigator, but differing from the latter in requiring less equipment at the shore stations and being accordingly more mobile. Briefly the lane identification process consists in momentarily super-imposing on each fine pattern a coarse phase pattern resulting from phase comparison at frequency $1f$ (using the previous notation). This facility reduces the ambiguity by factors of 8 and 9 compared with the slave-frequency lanes, and requires the user to know his distance from the ship to the shore stations only to about ± 5 km before starting operations. The lane identification transmissions are initiated from the ship by radiating in place of the normal master signal, a transmission of frequency $11f$; the manner in which the required phase-comparison frequency $1f$ is extracted from the master and the slave stations is described later.

3. Equipment

3.1. *The Master Transmitter*

The master transmitting station on the survey ship radiates a stable c.w. transmission at frequency $12f$, replaced intermittently for periods of about one second by a similar transmission at $11f$. The latter serves to trigger the lane identification transmissions and also to resolve frequency-division ambiguities in the slave phase control units. The master equipment comprises a so-called control unit, which contains the early stages of the transmitter and is physically integral with the receiver; a c.w. transmitter unit; an aerial tuning coil unit, and a transmitting aerial. The latter is generally installed near the stern of the ship and comprises a base-insulated tubular mast about 45 ft in height. The mast is normally stayed at three heights, although there are variations in aerial rig from ship to ship depending on the size and configuration of the vessel; survey ships using Lambda, or its immediate predecessor Two-Range Decca, have ranged in size from the M.V. *Calypso* of 130 ft to the R.N. survey ship H.M.S. *Vidal* (315 ft). The stay insulators, which are designed to minimize the effects of salt spray, are inserted at the lower ends of the stays so as to increase the aerial capacitance for which a typical figure is 600 pF.

The aerial tuning coil unit is installed close to the base of the mast and contains provision for manually adjusting the tuning and coupling. The aerial coil circuit includes a relay which effects the necessary change of tuning required by the $11f$ lane identification transmissions. The transmitter unit, which is identical to those used ashore by the slave stations, may be housed in any convenient part of the ship, together with its associated tuned anode circuit which is in a separate box. The tuned anode circuit includes a relay which alters the tuning for the $11f$ transmissions. The transmitter delivers approximately 450 W into the aerial, and a radiated power of 3·0 W is typical. In practice this low level of radiated power, which results from the small size of the aerial in relation to the transmitted wavelength, is offset by the narrow passband of the receiver r.f. circuits (± 15 c/s at -6 dB) which is made possible by the lack of modulation.

3.2. *The Receiver*

Figure 3(a) is a block diagram showing the basic elements of the receiver as it appears in normal operation, i.e. in the absence of lane identification transmissions. Included in this diagram is the oscillator which forms the source of the master transmissions and which is located on the same chassis as the receiver itself. The signals from the red and green slave stations at frequencies $8f$ and $9f$ are received and amplified in separate r.f. channels, containing two-stage crystal filters. Noise-free replicas of the filtered signals are generated by means of locked oscillators, employing temperature-controlled ovens, in the manner shown. In the "red" channel, the slave frequency is multiplied in the receiver by a factor of 3 and is compared, at $24f$, with the 24th harmonic of the $1f$ master oscillator. The Decometer indicating the result of this phase comparison makes one revolution per lane, but is used solely to indicate the lane fractions (hundredths) into which the scale is divided as already described. Similarly the green lane-fraction meter operates at the common multiple $36f$.

The phasemeter movements which drive the lane-counting pointers rotate in response to the phase comparison between the master and the slave signals which is carried out at the slave frequency. Again referring to the red co-ordinate, the 8th harmonic of the output of the master oscillator is compared with the output of the $8f$ oscillator locked to the slave signal of that frequency. The rotor shaft of the red lane-counting Decometer therefore responds to a pattern of lanes whose width is three times greater than that of the "fine" $24f$ lanes. The lane-counting pointer is driven by gearing from the phasemeter rotor in the ratio 8 : 1 and therefore makes one revolution per zone, i.e. one revolution for eight red slave-frequency lanes. The corresponding green meter has 9 : 1 gearing between the rotor and the lane pointer. If the lower red meter is turned manually by means of the reset button, therefore, it will be found that the lane pointer can take up any one of eight equally-spaced positions around the dial; this is the extent of the ambiguities in the two distance measurements which it is the function of the lane identification facility to resolve. Once the lane-counting pointers are set correctly, it may be assumed that their readings correctly identify the fine lanes since phase errors would have to reach the level of ± 60 deg for red and ± 45 deg for green for the lane pointers to indicate the wrong fine lane units on the respective dials.

Turning now to the lane identification system, it has already been noted that the coarse pattern on which this depends is the result of phase comparison at frequency f. Since it is out of the question actually to transmit a frequency of this value (approximately 14 kc/s) from the stations, it has to be extracted from master and slave by other means. This presents no problem with the master channel of the shipborne receiver, since the latter obtains a $1f$ signal directly from the master oscillator on the same chassis. The corresponding $1f$ information from the slave stations is provided by interchanging their frequencies, so that red radiates $9f$ and green $8f$ for lane identification purposes. Given a means of memorizing the phase of the normal slave transmissions while the counter-changed frequencies are being transmitted, a $1f$ beat-note whose phase represents that of the slave $1f$ oscillator can be derived from each slave station. By this means the effect of radiating low-frequency signals from the slave stations is simulated without employing any additional frequencies.

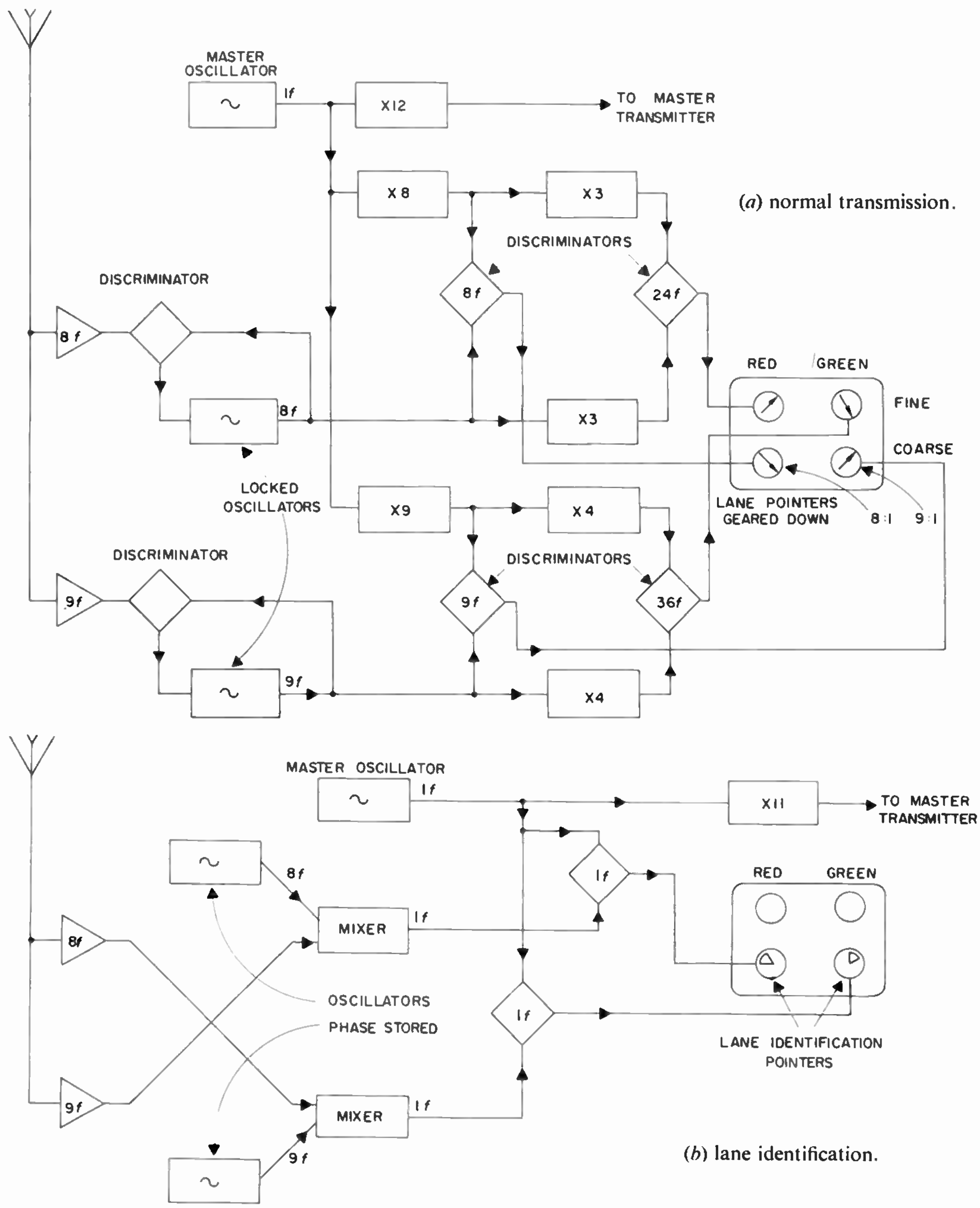


Fig. 3. Block schematic of the main elements of a ship installation.

The lane identification process is initiated by the user at the shipborne receiver pressing a button, the immediate effect of which is to stop the normal $12f$ master transmission from the ship and replace it, for about 1·3 seconds, by a signal of frequency $11f$. The latter forms a trigger signal, in combination with the gap following the cessation of the $12f$ signal, which initiates the counter-change of frequencies at the slave stations already described. The $11f$ transmission also provides the slave stations with a so-called "notch" datum, referred to later. Figure 3(*b*) shows the receiver and the master signal source as they appear under the lane identification condition.

The $8f$ and $9f$ oscillators are sufficiently stable to preserve the phase of the signals normally controlling them, and their outputs mixed with the received signals from the slave stations furnish the red and green slave signals at frequency $1f$ required for lane identification. The phase difference between master and slave at $1f$ is displayed by a phasemeter mounted concentrically with the appropriate lane-counting pointer. The phasemeter drives a sector-shaped pointer which, on receipt of a lane identification transmission, should move so as to enclose the lane-counting pointer. Theoretically the centre of the sector pointer should coincide with the lane pointer when the latter is correctly set, the object of the sector shape being to provide a tolerance for inaccuracies in the $1f$ phase-difference measurement. If the sector pointer does not enclose the lane pointer, the latter is at one of the "ambiguous" positions and is then manually corrected by means of the reset button.

3.3. *Receiver Displays*

The meter display unit forms the basic read-out for the receiver and provides the position-fix information in the form of a pair of lane readings which may be converted into distances, and hence into a fix, by plotting them on a chart bearing the concentric circle patterns centred on the two slave stations. In addition, provision is made for driving, in parallel with the fractional meters, a Marine Automatic Plotter. This instrument is derived from the Decca Flight Log, which it resembles in general principle, and comprises a plotter in which a pen and a roll of chart paper are driven in the horizontal and vertical axes respectively in response to the two fine-pattern phase-difference readings. Servo systems based on magnetic amplifiers are employed and provision is made, by a choice of gear ratio, for a number of alternative chart scales. By recording automatically the track made good, the plotter is a useful aid to various phases of the survey work and can also serve as an additional check on lane integration.

3.4. *Receiver Controls*

The Lambda position-line information is displayed continuously and the taking of a fix does not involve the operation of controls. Such controls as are provided on the front panel of the master/receiver unit are for checking and monitoring purposes and for the detection and correction of differential phase shifts in the various channels. The latter process is known as "referencing" and consists of impressing on the input circuits a 0·5 μs pulse having a recurrence frequency f which contains in-phase components of the required harmonically related frequencies $8f$, $9f$, $11f$ and $12f$; the Decometer stators are then turned manually until their pointers read zero. The controls include a pair of goniometers having an accuracy of 0·3 deg which may be used for calibrating the fractional Decometers and thereby correcting for any errors introduced by drift in the d.c. amplifiers feeding the Decometers and for non-linearity in the operation of the phase discriminators.

Since the equipment is normally operated in remote areas, fault diagnosis and maintenance work have to be carried out on the spot, and switched monitor meters are therefore provided with which to measure voltages and currents at all key points of the circuit. Meters respectively showing the error volts on the $12f$ and $11f$ output discriminator (see Sect. 4.5), and the levels of the outgoing and incoming signals, are mounted in a detachable unit which can be set up facing to the rear so that they can be directly read from the back of the chassis when it is swung out of the bulkhead-mounted housing (see Fig. 4).

Fig. 4. Chart room housing receivers, display units and track plotter. The Decometer unit has upper meters indicating fine pattern readings ($24f$ and $36f$). Whole lanes and lane identification are given by the lower dials.

3.5. *The Slave Station*

In practical details the slave transmitting station is similar to the shipborne master, except for the use of a 100 ft stayed mast for the transmitting aerial, together with an associated earth mat of 100 ft radius. Power for the station is normally supplied by a portable diesel alternator set, usually duplicated. In most cases these sets are rated in excess of the normal demand from the transmitting equipment of about 1 kW to give a margin for tropical operating conditions and also to provide a domestic power supply for the operators.

The transmitter and its associated power supply are identical to those at the master. The signal source at the slave station comprises a pair of phase control units, one of which receives the master signal and maintains the slave transmitter at a constant phase relationship with it. Two 25-ft vertical aerials are used for reception, each comprising an insulated wire held in a self-supporting fibreglass tube. A small radial earth system for the receiving aerials is provided in order to ensure phase stability at this point in the control loop.

3.6. *The Slave Phase Control Unit*

Two phase control units are provided at each station, for stand-by purposes and also to permit a measure of self-monitoring. The latter is possible since the unit essentially comprises a receiver and a means of measuring the phase-difference between the master and slave transmissions. Figure 8 shows the essentials of the phase control unit, in which the principal element is an oscillator of frequency f which is locked at its 12th harmonic to the incoming $12f$ master signal. The diagram represents the operative phase control unit at the red slave station, where the 8th harmonic of the oscillator is amplified and transmitted continuously except during lane identification. The phase control units at the red and green slave stations are identical and are switched by a panel control to the appropriate "colour" on installation.

The locking of the slave to the master, upon which the generation of a stable pattern of position line circles depends, is carried out in two stages. The incoming master signal, having been passed through a two-stage crystal filter, is amplified and its phase is compared with the twelfth harmonic of the slave's oscillator in a discriminator, whose output controls the phase of the oscillator to keep the $12f$ signals phase-locked. A second stage of phase-locking (not shown in the diagram) operates between the radiated output from the aerial and the input to the transmitter; the $8f$ output of the oscillator passes to the transmitter through a reactance stage which is controlled from a phase discriminator comparing the drive and radiated $8f$ signals. This serves to keep the radiated signal locked to the drive signal irrespective of capacitance changes (e.g. through wind) in the transmitting aerial. The two stages of phase locking are repeated at $9f$, which at the red slave station ensures that the slave transmission at that frequency during lane identification shall be similarly phase-stable with respect to the master.

In considering the phase-locking of the slave station, note should be taken of a problem that arises from comparing the phase of the master and the slave signals at the slave carrier frequencies. This is done in the Lambda system for generating the basic position-line patterns, as already described. The comparison of phase at a frequency lower than one or both of the radiated transmissions involves a process of frequency division which, even if it is not performed literally in a dividing circuit, nevertheless takes place in effect. Consider, for example, the phase relationship between the red slave station radiating its normal frequency $8f$, and the $12f$ master transmission to which it is phase-locked. At the slave the actual process of slave-to-master locking is effected at $12f$, but of the 12 cycles or "notches" of the $12f$ signal which occupy 1 cycle of the oscillator frequency, there are only 4 which the oscillator could lock to (at its 12th harmonic) without producing an error in the phase of the $8f$ radiated pattern. In practice all such ambiguities are eliminated simply by ensuring that the master and slave oscillators are in phase at the fundamental frequency f; the correctness or otherwise of this relationship is displayed to the slave station operator, every time the ship initiates a lane identification transmission, by means of a "notch" meter.

The notch meter indicates the phase-difference between the $11f$ signal from the master (i.e. the 11th harmonic of the master oscillator) and the 11th harmonic of the slave oscillator. The slave oscillator is sufficiently stable to be considered as remaining locked to the interrupted $12f$ master signal, so that the 12th harmonics of the two oscillators are already in phase; if the 11th harmonics are seen by the notch meter to be also in phase, the master and slave oscillator outputs will have the correct (i.e. zero) phase relationship at their fundamental frequency. If one of the eleven other possible readings is observed, a 12-position notch control, which operates a phase-shifting network, is turned a corresponding number of clicks in the appropriate direction to bring the 11th harmonic of the oscillator into the right phase. The use of a click device makes it easy for the operator to apply the correction after the 1·3 second lane identification period has finished.

When the meter reads zero, the whole station is correctly "notched", i.e. the slave has the correct phase relationship with the master at the fundamental frequency f and hence also at the transmitted harmonic frequencies. The probability of a notch error developing during a day's work has been shown to be extremely remote. However, to enable the slave operator to check the notch in the event of an interruption in transmission, provision is made for him to request the ship to initiate a lane identification transmission. The request signal takes the form of a momentary phase shift in the slave transmission which is too rapid to introduce a Decometer error, but which serves to trigger a "slave-call" lamp on the receiver.

3.7. *Slave Station Operation*

Once the slave stations have been set up at the start of a day's work, they operate continuously and the operator's task is supervisory. Provision is made for him to carry out periodical checks on voltages and currents at all key points in the circuit. Since lane identification transmissions are normally initiated only on demand from the ship and do not take place on a rhythmic schedule, it is necessary to make some provision for the slave station operator to initiate a lane identification transmission ($11f$ in place of $12f$ from the master) so as to check the "notching" as described above. Pressing a button on the slave phase control unit inserts a shift of approximately 90 deg into the phase of the radiated slave signal for 0·25 second; the result at the ship is a momentary drop in the cosine output voltage of the appropriate slave frequency discriminator to below -10 V and this in turn operates the "slave call" lamp on the master control unit. The duration of the phase shift is short enough to cause only a momentary movement of the fractional Decometers, owing to the relatively long time-constant of the reactor stages controlling the $8f$ and $9f$ locked oscillators.

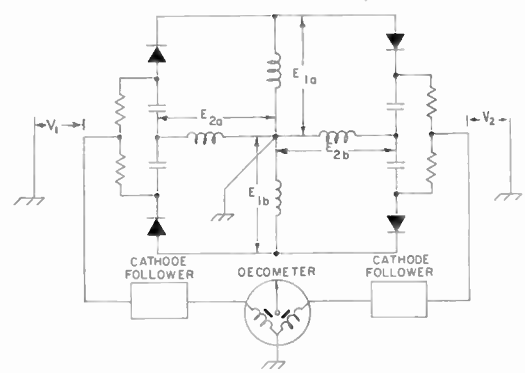
The second phase control unit permits the master/slave phase relationship to be observed, and is a convenient means of checking the value of any manual phase adjustment which may appear necessary, from observations at a distance, when first setting up the system. The r.f. channels of the slave phase control unit are furnished with the referencing facility whereby differential phase shifts may be observed and corrected, as in the shipborne receiver.

When the equipment is to be used at long ranges (say greater than 100 miles) at night, the circuits controlling the interchange of radiated frequency in the slave phase control unit are disconnected by a switch provided for the purpose, in order to prevent false triggering of these circuits by the combined effects of noise and skywave interference. At night the reliability of lane identification may suffer a marked reduction at distances above about 150 miles, but the lane integration facility remains reliable at almost twice this distance and the system can be usefully employed for navigating overnight between one survey area and another. With return of daylight the normal lane identification facility is restored, and any error in lane integration that may have been acquired at long range during the night can thereby be resolved so long as it does not exceed ± 12 fine red lanes or 18 green, i.e. $\pm$ half a cycle of phase-difference at the lane identification frequency f.

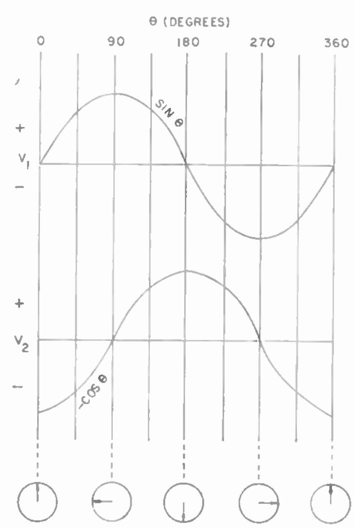
4. Circuit Details

4.1. *The Phase Discriminator*

The shipboard equipment contains ten, and each slave phase control unit four phase discriminator circuits. Figure 5(a) is a simplified circuit diagram of the discriminator, in which two alternating voltages E_1 and E_2 of the same frequency but differing in phase by an angle θ are compared. The two d.c. output voltages V_1 and V_2 are applied to an associated Decometer phasemeter which displays θ directly, or to the input driving one co-ordinate of an automatic plotter. In some instances the output V_1 is used to maintain the phase of a locked oscillator in the correct relationship with a received signal.



(a) Basic circuit.



(b) Relationship between phase-difference angle θ and discriminator output voltages V_1 and V_2.

Fig. 5. Phase discriminator.

The input signal E_1 is split, by the use of a centre-tapped transformer winding, into two voltages E_{1a} and E_{1b} equal in amplitude but opposite in phase. The other signal E_2 is also divided into two parts E_{2a} and E_{2b} of equal amplitude, but in phase quad-between grid and cathode of V1 will be resistive and small compared with that of C. Thus the voltage between grid and cathode, and hence the anode current, must also lead the anode by substantially 90 deg, the magnitude of this current being proportional to the mutual conductance of the valve. V1 therefore represents a capacitive reactance to the circuit and the output voltage will be advanced with respect to the input by an amount depending upon the g_m of the valve which itself is dependent on the amplitude of the control voltage.

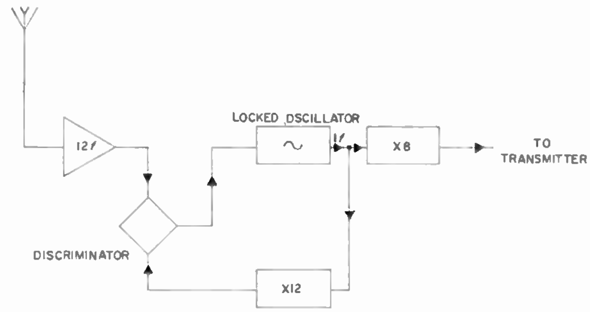
Since the control grid of V1 is coupled to that of V2 through the centre tapped inductance, the voltages on the two grids must be 180 deg out of phase. When V2 is conducting its grid-cathode voltage and also its anode current must therefore lag behind the anode voltage by approximately 90 deg, so that it presents an inductive reactance to the circuit. The typical performance curve in Fig. 7(*d*) shows that the average phase shift is approximately 90 deg per volt.

4.5. *Frequency Interchanging*

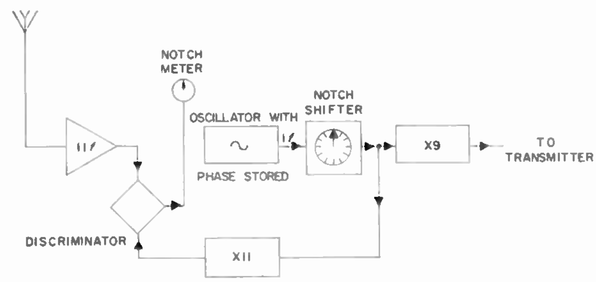
An important feature of the Lambda system is the time-sharing of the transmitted frequencies. The fact that no transmitter is required to handle more than one frequency at a time halves the required transmitter power for a given operating range, simplifies the design of the tuned anode and aerial coils, reduces the voltage across the insulators of the transmitting aerials and reduces the size and weight of the transmitting equipment. The latter fact is important since the shore stations have to be as light and mobile as possible.

The frequency interchanging sequence is initiated by pressing the "check lane" button on the ship's Decometer unit. When this is done, the $12f$ master transmission is interrupted; a transmission at $11f$ is radiated from the master aerial for 1·3 seconds; and the locking of the $8f$ and $9f$ oscillators in the receiver is disconnected so that their outputs, in the free-running state, may be said to "store" the phases of the interrupted slave transmissions. At the slave stations, the cosine output of the $12f$ discriminator is normally −20 V and that of the $11f$ discriminator zero. The frequency interchange at the ship causes a corresponding interchange of these two outputs; as soon as the cosine output from the $12f$ discriminator falls below −10 V and that from the $11f$ discriminator exceeds −10 V, the frequency-change relay RLA in Fig. 8 operates. This double action greatly reduces the chance of accidental triggering of the frequency interchange process by noise since the change will not start until both conditions are fulfilled.

The red and green slave stations employ identical equipment, and the setting of a switch on the front panel of the slave control unit alone determines the



(*a*) During normal transmission.



(*b*) During lane identification.

Fig. 8. Block diagram of slave station control system.

"colour" of the station. The relay switching described below for the red station is reversed for the green: Fig. 9 may be taken to show either the red station in its normal state (radiating $8f$) or the green station radiating $8f$ for the purpose of lane identification. The $8f$ and $9f$ output stages are series-connected and feed the slave transmitter and anode coil, the latter being in turn coupled to the aerial by way of a tuning and coupling coil. In the condition shown, the $9f$ drive is completely cut off due to the high bias voltage (−50 V) applied to the control grid of V1 from the traveller of relay contacts RLA1. The contacts RLA2 provide an a.c. supply to a heavy-duty relay RLB; the latter is located in the anode coil unit and carries two sets of Mycalex-insulated high-voltage contacts (RLB1, RLB2) which are in parallel and serve to connect an additional tuning capacitor (C1) across the main anode tuning capacitor C2. C2 tunes the anode coil at $9f$ and C1 + C2 tunes it at $8f$; thus the value of C1 is $\frac{17}{64}$ of C2. Each of the high-voltage contacts B1 and B2 comprise a pair of tungsten contacts which make and break the current, together with a pair of heavy-duty silver contacts which act as the current carriers by closing after the tungsten contacts make and opening before they break. Guard contacts RLB3 are also provided, referred to below.

The change of tuning of the aerial circuit is more conveniently performed by altering the inductance than the capacitance, owing to the high r.f. voltages obtaining in the aerial circuit for which a peak value of 35 kV is typical. A high-voltage relay RLC, identical to that employed in the anode coil, is used in the aerial circuit, shown in the diagram as de-energized and thus placing the whole winding L1 + L2 across the aerial capacitance which it tunes to resonance at $8f$. In the lane identification condition, contacts RLC1 and RLC2 connect L2 to ground, leaving L1 to tune the aerial to resonance at $9f$. Here, similarly, L2 = $\frac{17}{64}$ of L1. Guard contacts (RLC3) are again provided, and all three sets of guard contacts are paralleled to ensure that the $9f$ output is fully suppressed. When the frequency interchange is started, relay RLA operates, contacts A1 and A2 change over and in turn cause relay RLB to drop out and energize relay RLC. This sequence of events serves to tune the anode coils automatically to $9f$, but the $9f$ drive is not applied to the transmitter until all three relays have completed their movements and the guard contacts have operated. This arrangement eliminates the possibility of the two drive frequencies being applied to the transmitter simultaneously.

At the end of the lane identification period, the $11f$ master signal is once more replaced by the $12f$, the $11f$ and $12f$ phase discriminator cosine outputs revert respectively to about 0 and -20 V, and the relays at both slave stations return to their normal operating positions. On the ship, the frequency transposition is accomplished in the same manner as at the green slave station (in both cases the higher frequency is normally radiated).

5. Performance

5.1. *Accuracy*

The phase shifts which constitute the locking constant and the phase lag error (see Sect. 2.3) are assumed to be systematic in character and hence to be capable of correction by calibration. This applies also to the variations in reading which on some coastal slave station sites can occur with changes in the bearing of the ship from the slave, due to a complex effect that takes place when the signal from the master passes over the land/sea boundary close to the

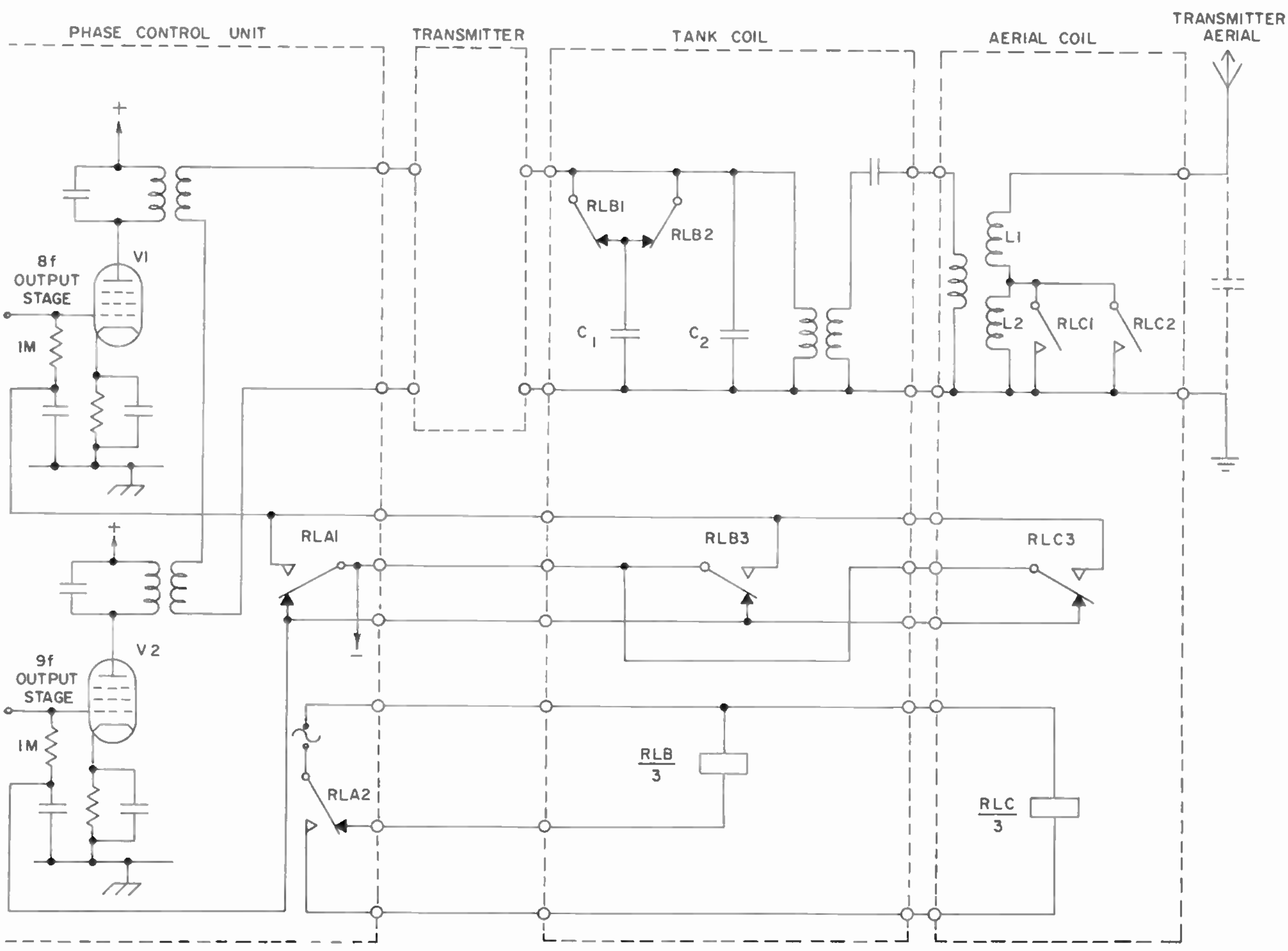


Fig. 9. Frequency interchanging; relay switching circuit. (All relays are shown in operational state.)

station. Trials results suggest that if this effect is neglected an error of about 70 ft r.m.s. may be present in the range measurements. Errors of a random character result from instrumental variations and from instability associated with wave propagation. In summer daytime the equipment contributes most of the random error, which under low-noise conditions is then of the level indicated in Fig. 10.

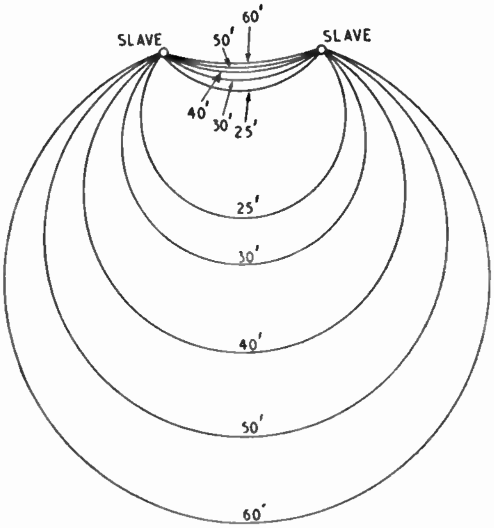


Fig. 10. Fixing accuracy contours in feet for summer day operation for a 0·01 lane deviation.

As a practical guide to the overall performance of the system as a survey aid, and calling upon greater field experience with the previous "Two-range Decca" system, it has been found that errors or variations due to the system itself are seldom detectable when the fixes are plotted at a chart scale of 1 : 70 000 (about 1 in. per nautical mile). This general approximation may be said to hold good for distances up to about 100 miles from shore: at greater distances it will take a correspondingly longer time to build up a clear empirical picture of the system's performance owing to the self-evident difficulty of finding a yardstick against which to compare it.

5.2. *Range*

The Two-range Decca system, of which the first practical example was tested in one of H.M. survey ships in 1953, used the conventional frequency value of $6f$ (about 85 kc/s) for the master transmission, from which the fine patterns at $24f$ and $18f$ for red and green respectively were derived by multiplication. As field experience was gained and further tests were made, it became evident that, whereas the system gave satisfactory coverage up to 75 miles from the slave stations in home waters, this range could not be achieved in the tropics. Accordingly a new version was introduced using the master frequency $12f$, giving fine pattern frequencies $24f$ and $36f$, and this has been retained for Lambda.

Doubling the master frequency had a two-fold advantage: the atmospheric noise level at 170 kc/s is of the order of 5 dB lower than at 85 kc/s, and about 10 dB more power can be radiated from the ship-borne master station. Trials by the then Admiralty Signal and Radar Establishment showed that this arrangement would permit accurate operation out to ranges of at least 75 miles in any part of the world, and also that the margin of radiated power would permit ranges many times greater to be obtained in low-noise areas. For example, in tests at 80 miles from shore, torque remained strong on both Decometers and there was no trace of instability when the master aerial current was reduced from the normal 12 amps to 1 amp.

Field results have not so far been published for the Lambda chains now in operation, but sufficient experience has been gained to show that the performance so far as the basic range measurement is concerned is equal to or better than that of Two-range Decca, and that the introduction of a built-in method of lane identification has enhanced the value of the system as an aid to long-range hydrographic survey.

6. Acknowledgments

An important factor in the evolution of Two-range Decca as a practical survey tool, culminating in the development of the Lambda version of the system described in this paper, has been the close co-operation between the Admiralty Surface Weapons Establishment (formerly the Admiralty Signal and Radar Establishment), the Hydrographic Department of the Royal Navy, and the Decca Navigator Co. Ltd. The authors, on behalf of the Decca Navigator Co. Ltd., gratefully acknowledge this co-operation. Their thanks are due to Mr. H. F. Schwarz, Managing Director of the Decca Navigator Co. Ltd., for permission to publish this paper.

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Absorption of Sound in Sea-Water

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Summary: Laboratory and field data on the absorption of sound in sea-water are reviewed in the light of modern theory, including effects of pressure, temperature and salinity. The effects of temperature and $MgSO_4$ salt concentration appear to be compatible from one investigator to another. There is some dispute, however, about the absolute value of the coefficient. The pressure effect has been measured for pure water, but has not been applied to the ocean as far as the authors know. The importance of this pressure effect on bottom loss measurements is shown to be sizable. An expression is presented for sound absorption in sea-water as a function of frequency, temperature, pressure and salt concentration.

1. Introduction

The object of this paper is to present a practical expression for sound absorption in the ocean, by considering the present state of knowledge and by incorporating a large body of experimental ocean propagation data at the lower frequencies (2 to 25 kc/s). This expression will be obtained as a function of temperature, salinity and pressure.

It should be emphasized that what is being discussed is the absorption component of the attenuation coefficient. Both coefficients are expressed as uniform loss rates in decibels per kiloyard. The attenuation coefficient includes all such losses, whereas the absorption coefficient is that part of the attenuation coefficient which passes directly into heat in the ocean.

First, it is planned to discuss absorption in pure water. Then absorption in solutions of magnesium sulphate is discussed. This salt appears to be the chief contributor to absorption in the ocean. After that the effect of other salts in the ocean on this absorption is considered. Finally, data are presented on low frequency absorption in the ocean (2 to 25 kc/s) and related to existing data. A practical expression is obtained for computing absorption.

2. Absorption in Pure Water[1, 2]

If one were to compute the absorption of sound in pure water based on its shear viscosity, the answer would come out about one-third of the measured absorption. However, the expected dependence on the square of the frequency would hold. Hall[3] and Debye[4] proposed a structural relaxation to explain this excess absorption. According to Hall's theory, there is a time lag in the density effect of an acoustic pressure wave when molecules are rearranged between two states of packing. He computed a time constant of the order of $1 \cdot 6 \times 10^{-12}$ seconds for this relaxation process at 30° C.

Hall used the Bernal and Fowler[5] model of liquid water structure in which the water molecule is considered to be a sphere, but with a negative charge and two positive charges arranged as a triangle of angle 109° inside the sphere. In ice, these molecules form a regular structure with rather large holes. Each molecule is surrounded by a tetrahedron of others so that the hydrogens always point at an oxygen of the neighbour. The nearest approach of two molecules in ice is found to be 2·76 Å from x-ray data.

In liquid water at low temperature, this ice structure is always present. In addition, there is also a more closely-packed structure, leading to the density maximum at 4° C and atmospheric pressure. While the distance between immediate neighbours is 2·76 Å in both, the distance to the next nearest neighbours is 4·2 Å in the close-packed structure and 4·5 Å in the ice structure. Litovitz and Carnevale[6] have shown from studies of the pressure dependence of sound absorption in water that near 0° C there is about 30% ice structure present.

It should be remarked that there are many types of relaxation processes, which give the same elementary form of the dispersion and absorption equation as a function of frequency. Experimental results are sometimes explained by the superposition of several such processes with different relaxation times.

Table 1[7, 8] gives $\alpha/f^2 \times 10^{17}$ (cm^{-1} s^2) for water as a function of temperature at atmospheric pressure. Here α is the absorption coefficient and f is the frequency in cycles per second. The temperature effect is large with α/f^2 being reduced by half in going from 0° C to 15° C.

Table 2[6] gives α/f^2 for water as a function of pressure at 30° C. It may be seen that the pressure effect is also large, with α/f^2 being reduced by about half at

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a pressure of 2000 atmospheres. The pressure in 2000 fathoms of water is about 500 atmospheres.

Table 1

Absorption in water as a function of temperature (atmospheric pressure)

Temperature (°C)	0	5	10	15	20	30	40	50	60
$10^{17}\ \alpha/f^2$ (cm^{-1} s^2)	56·9	44·1	36·1	29·6	25·3	19·1	14·6	11·99	10·2

Table 2

Absorption in water as a function of pressure ($T = 30°$ C)

Pressure (atm.)	0	500	1000	1500	2000
$10^{17}\ \alpha/f^2$ (cm^{-1} s^2)	18·5	15·4	12·7	11·1	9·9

3. Composition of Sea-water

Sea-water has a salinity ranging between 3·3% and 3·8%. Nine kinds of ions constitute 99½% of the salts in solution. These are found in remarkably constant proportion, unless the water is of unusually low salinity. The effect of living organisms is to localize minor constituents and dissolved gases.

Table 3

Composition of major salt ions in sea-water (Based on 34·4‰ salinity)

Cations	%	mmole/kg	Anions	%	mmole/kg
Sodium	30·4	455·0	Chloride	55·2	535·1
Magnesium	3·7	52·5	Sulphate	7·7	27·6
Calcium	1·16	10·2	Bicarbonate	0·35	2·35
Potassium	1·1	9·7	Bromide	0·19	0·81
Strontium	0·04	0·15	Borate	0·07	0·44
	36·40%			63·51%	

Table 4

Salts obtained from sea-water

Salt	Weight g/kg sea-water	Percentage of total salts
NaCl	27·213	77·758
$MgCl_2$	3·807	10·878
$MgSO_4$	1·658	4·737
$CaSO_4$	1·260	3·600
K_2SO_4	0·863	2·465
$CaCO_3$	0·123	0·345
$MgBr_2$	0·076	0·217
	35·000	100·000

The percentage composition of salt ions in sea-water, along with their concentration in millimoles per kilogramme according to Defant,[9] is given in Table 3. Table 4, gives the precipitated salt content in grammes in a kilogramme of sea-water and the corresponding percentage of the total salt content.

It may be seen that sodium chloride constitutes 77·8% of the total salt content, while magnesium sulphate constitutes 4·7% of the total. Nevertheless, Leonard and Wilson[10] found that the acoustic absorption properties of sea-water were due almost entirely to the magnesium sulphate present. Kurtze, Tamm and Kaiser[11, 12] made the most thorough and comprehensive study and measurements on the sound absorption properties of salt solutions. They found that it was not the abundant monovalent salts that showed excess absorption over that of pure water, but the 2·2 valent salts like $MgSO_4$, $CoSO_4$ and $CaCrO_4$.

4. Absorption by Magnesium Sulphate in Solution

In explaining absorption in electrolytes such as magnesium sulphate solutions, Kurtze, Tamm and Kaiser[11, 12] assumed incomplete dissociation and the existence of $MgSO_4H_2O$ complexes which represent the normal undissociated state of the electrolyte (see Fig. 1).

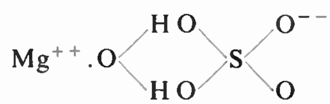

Fig. 1. Magnesium sulphate complex in solution.

The passage of the sound wave creates, through its pressure, two univalent ions,

$$MgSO_4 . H_2O \rightarrow MgOH^+ + HSO_4^- \qquad (\tau < 10^{-9} \text{ seconds})$$

which split up in a subsequent step:

$$MgOH^+ + HSO_4^- \rightarrow Mg^{++} + OH^- + H^+ + SO_4^{--} \qquad (\tau \sim 10^{-6} \text{ seconds}$$

The amount of absorption depends on the number of undissociated molecules available for the process and this in turn depends on the concentration, being directly proportional, up to a point.

The dissociation of the cation ($MgOH^+$) causes an absorption maximum at a lower frequency, 130 kc/s at 20° C, than that of the anion (HSO_4^-) which occurs at 170 Mc/s. Although the dissolved electrolyte alters the structure equilibrium of the water content and thereby changes its absorption, a considerable decrease in the water absorption occurs only for large concentrations of the electrolyte. Thus the resultant absorption in magnesium sulphate solutions will be the sum of the absorption due to the water content and that due to the dissociation processes.

Apparently the relaxation effects at about 170 Mc/s due to the anion may be neglected at sonar frequencies and it is customary to describe the absorption at the lower frequencies quantitatively as the sum of a single relaxation and the pure water contribution. A fit of the excess absorption data of Kurtze, Tamm and Kaiser then gives:

$$\alpha = \frac{50{\cdot}82 n f_T f^2}{f_T^2 + f^2} \quad \text{dB/km} \qquad \ldots\ldots(1)$$

where n is the concentration of the $MgSO_4$ solution in moles/litre ($n \leqslant 0{\cdot}5$), f is the frequency in kc/s and f_T is the relaxation frequency and is given by

$$f_T = 25{\cdot}2 \times 10^{\left(6 - \frac{1548}{T+273}\right)} \quad \text{kc/s} \qquad \ldots\ldots(2)$$

T is temperature in deg C.

The absorption for a 0·1 molar solution at 20° C is plotted in Fig. 2; f_T is 130 kc/s at this temperature. In this figure the smoothed curve represents the data given by Kurtze, Tamm and Kaiser. The effect of the anion absorption at 170 Mc/s is evident, but small. The circled points are computed from eqn. (1) for a single relaxation process plus the residual absorption due to pure water.

5. Pressure Effect for Absorption by Magnesium Sulphate Solutions

Since acoustic waves propagate to great depths in the ocean, where hydrostatic pressures are quite large, it is natural to consider the effect of pressure on absorption. In general, one might expect large pressure effects since it is this sensitivity which causes the acoustic phenomenon.

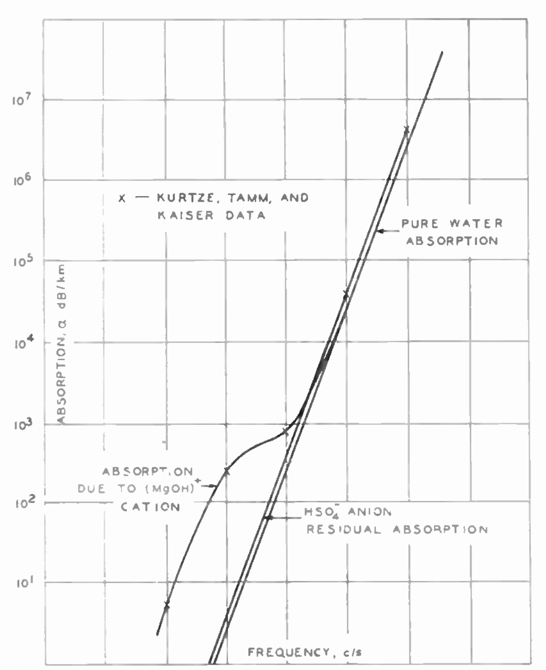

Fig. 2. Absorption in $MgSO_4$ (0·1 molar).

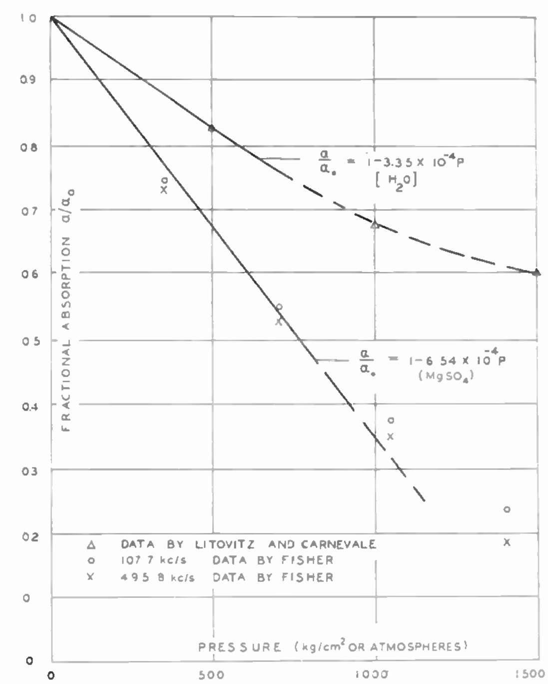

Fig. 3. Pressure effect on absorption in $MgSO_4$ solutions (0·5 molar).

Fisher[13] reported absorption, relaxation frequency and sound velocity studies in a 0·5 molar solution of $MgSO_4$ at frequencies from 100–600 kc/s and pressures up to 20 000 lb/in.^{2} at 26° C. He found that the absorption at 20 000 lb/in.^{2} was less than one quarter that at atmospheric pressure and that the relaxation frequency remained fairly constant within $\pm 10\%$ at about 190 kc/s. Kurtze and Tamm report about 170 kc/s at 26° C and atmospheric pressure. Fisher also found that the sound velocity increased linearly with pressure at the rate 0·1588 m/s/atmosphere.

According to Fisher the theoretical pressure dependence is much greater than the observed pressure effect on absorption. Even then the observed pressure dependence is so large, being twice that of pure water, that the authors advise its use in calculating absorption effects in the ocean. The percentage change of absorption with pressure is plotted in Fig. 3 for Fisher's data. Pressures which correspond to those at the mean ocean bottom at 2000 fathoms are about 500 kg/cm^2 (atmospheres) or 7100 lb/in.^{2}. Down to these pressures a linear fit to the data is given by:

$$\frac{\alpha(P)}{\alpha_0} = 1 - 6{\cdot}54 \times 10^{-4} P \qquad \ldots\ldots(3)$$

where P is pressure in kg/cm^2 or atmospheres

α_0 is the absorption at atmospheric pressure

$\alpha(P)$ is the absorption at pressure P for constant temperature.

6. Influence of other Electrolytes on Absorption by MgSO$_4$ Solutions

Kurtze and Tamm also found that, if sodium chloride is added to a $MgSO_4$ solution, the number of $MgSO_4$ associates is reduced in favour of $Mg-Cl^+$ and $Na-SO_4^-$ associates, the percentage of which is determined by the ion strengths of the two partners. They deduced this from an empirical mixing rule which was observed from the decrease of absorption as sodium chloride was added.

$$\frac{\Delta\alpha}{\alpha} = \frac{\alpha_0-\alpha}{\alpha} = \frac{NaCl}{MgSO_4} \cdot f' \qquad \ldots\ldots(4)$$

f' is an empirical constant which they found to be one-fifth for this combination, but different for combinations of different kinds of salts.

Investigation of synthetic sea-water showed that the absorption was equal to that of a $MgSO_4$ solution of concentration 0·014 moles/litre, which is less than the concentration of Mg^{++} or SO_4^{--} in the synthetic sea-water. Table 5 shows the content of their synthetic sea-water.

Table 5

Synthetic sea-water of Kurtze, Tamm and Kaiser

0·454 moles/litre	Na^{++}	0·530 Cl^-
0·010 ,,	K^+	0·001 Br^-
0·052 ,,	Mg^{++}	0·0275 SO_4^{--}
0·010 ,,	Ca^{++}	0·0025 CO_3^{--}

7. Measurements of Absorption in Sea-water

The ocean is a complex medium. Although the absorption computed from the magnesium sulphate concentration found in the sea can account for a large portion of the observed attenuation, it is still desirable to know how the measured absorption in the ocean relates to the laboratory measurements on synthetic sea-water and magnesium sulphate solutions. Measurements have been made on actual sea-water samples by Wilson and Leonard[10] at frequencies from 50 to 500 kc/s. Liebermann,[14] at 120 to 1000 kc/s and Murphy and Garrison,[15] at 60 to 467 kc/s, have reported measurements over propagation paths in open low salinity water, under very restricted environmental conditions.

This part of the paper is concerned with the analysis of a large number of propagation loss measurements carried out while the authors were at the U.S. Navy Underwater Sound Laboratory. In an effort to enhance the absorption loss determinations, twenty-seven stations with deep isothermal layers (> 350 ft) were occupied at various locations in the North Atlantic, at various times of the year. In this way, the temperature effects could be maximized, while sea surface scattering and other losses could be separated. The deep isothermal layer stations also allowed a predictable inverse square spreading loss correction out to fairly long ranges in the first skip zone, and part of the second, before surface contacts had an appreciable effect.

Thirty thousand individual measurements were made for frequencies between 2 and 25 kc/s, out to ranges of 24 kiloyards, for projector and receiver depths down to 500 ft. Figure 4 illustrates the temperature correlation obtained. It is a plot of propagation loss versus temperature at 25 kc/s and 8000 yds. The propagation loss at 35° F is some 30 dB more than the loss at 70° F. Table 6 shows the data for 16 kc/s as a function of sea state as well. Analysis showed that the sea-state effect was small.

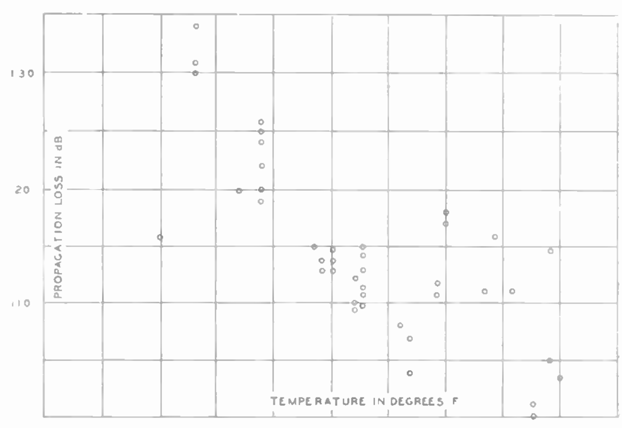

Fig. 4. Propagation loss vs. temperature at 25 kc/s at 8000 yds.

In addition, the depth dependence was found to be small except at 2 kc/s where special depth corrections were applied based on a wave theoretical treatment, first introduced by Bremmer.[16] Figure 5 shows the relative propagation loss as a function of receiver depth for the frequency range 8 to 25 kc/s. Figure 6 shows an example of 2·2 kc/s.

Figure 7 shows how the divergence loss follows a 20 log R behaviour when the aR term is subtracted from the propagation loss, N_w, after the data were fitted to:

$$N_w = 20\log R + aR + 60 \quad \text{dB} \qquad \ldots\ldots(5)$$

where R is in kiloyards.

The attenuation data were then fitted to an absorption formula due to a single relaxation process plus a

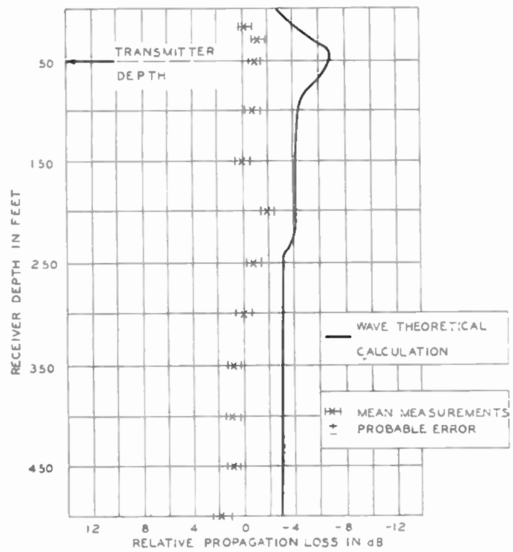

Fig. 5. Relative propagation loss vs. depth at 3000 yd range at 8–25 kc/s.

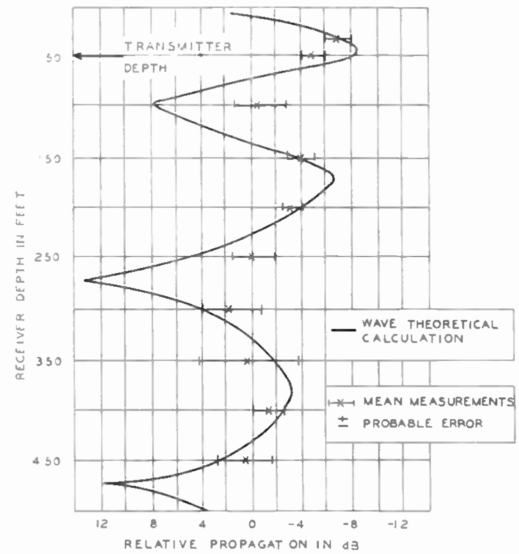

Fig. 6. Relative propagation loss vs. depth at 3000 yd range at 2·2 kc/s.

Table 6

Median attenuation coefficients by stations (dB/kyd) (16 kc/s)

	Sea State						
Temperature (°F)	0	1	2	3	4	5	6
70	1·3						
69				1·5	2·5		
68			1·4				
66			1·9				
64			1·9				
60			2·0				
59			2·8				
57			1·1				
56		1·2					
54			2·0				
53			2·6	1·9			
53			2·0				
52			1·9				
52			1·9				
50					2·1		
50					1·9		
49							2·3
48					2·3		
47				2·3			
44				2·6	2·5		2·8
42						2·7	
38				4·1			
35			3·7				

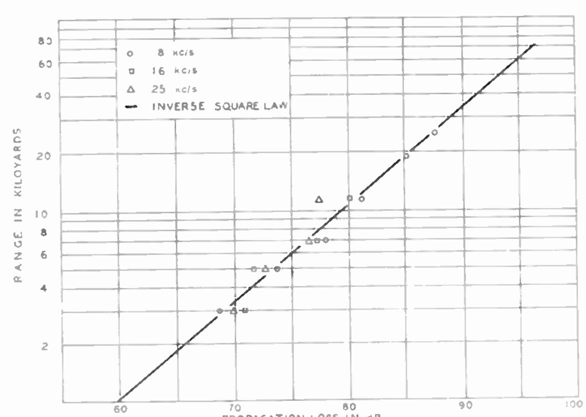

Fig. 7. Propagation loss corrected for average attenuation vs. range.

residual pure water contribution. Pinkerton's data for pure water absorption, given earlier, were used to correct for the residual effect. All the data processing was carried out with a high-speed computing machine, and the formulae finally obtained represent the data in a least-squares fit. The probable error of the average propagation loss turns out to be 3 dB for all the data when eqn. (5) is used with the resulting formula for absorption.

The Wilson and Leonard data for actual sea-water were used for the fit at middle frequencies. Figures 8, 9 and 10 show the two sets of data as attenuation versus frequency plots for three different temperatures. Note that the spread and slow trend in the data at the

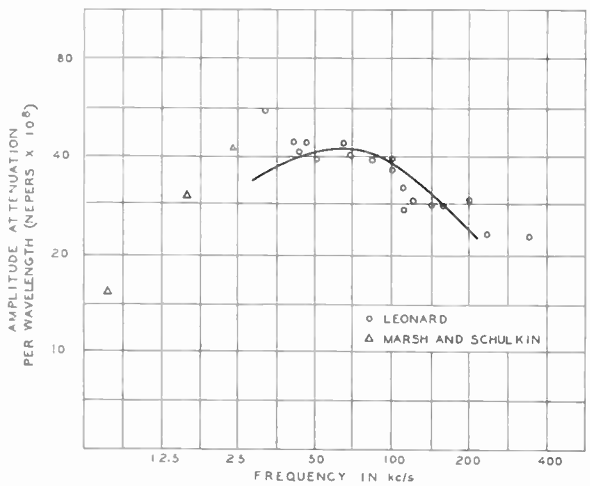

Fig. 8. Attenuation vs. frequency at 41° F (5·1° C).

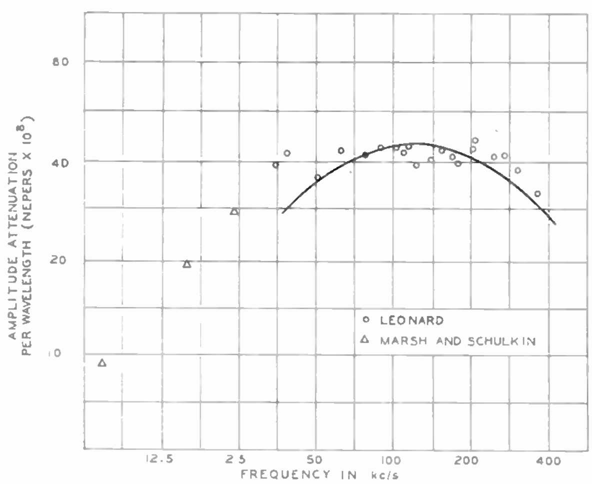

Fig. 9. Attenuation vs. frequency at 59° F (15° C).

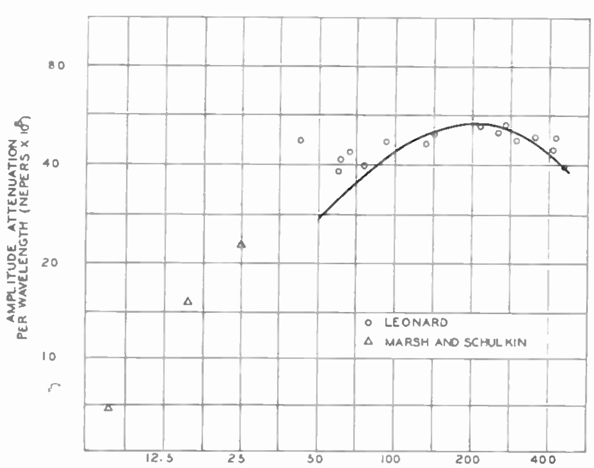

Fig. 10. Attenuation vs. frequency at 71° F (22·5° C).

middle and high frequencies makes the lower frequency data very important in determining the relaxation frequency at each temperature.

The final expression for the absorption α was:

$$\alpha = \frac{81{\cdot}9\times10^{-6}f_T f^2}{f_T^2+f^2}+\frac{3{\cdot}38\times10^{-6}f^2}{f_T}\quad\text{nepers/metre} \qquad \ldots\ldots(6)$$

where f_T, the relaxation frequency, equals

$$21{\cdot}9\times10^{\left(6-\frac{1520}{T+273}\right)}\ \text{kc/s} \qquad \ldots\ldots(7)$$

T is the temperature in deg C.

(To convert† α to dB/kyd multiply by the factor $7{\cdot}943\times10^3$; there are 8·686 dB in one neper.)

Note that the same f_T is used in both terms. While there is no basis for this theoretically, a comparison with Pinkerton's data showed that over the temperature range of interest in the ocean, the approximation is satisfactory to within 10% (see Table 7). In addition the contribution of the second term is usually considerably smaller than that of the first term.

Table 7

Comparison of measured and computed absorption

T 0°	5°	10°	15°	20°	30°	40°	50°	60°
$\alpha/f^2\times10^9$ ($m^{-1}s^2$) (Pinkerton)								
56·9	44·1	36·1	29·6	25·3	19·1	14·6	11·99	10·2
$3{\cdot}38\times10^3/f_T$ ($m^{-1}s^2$)								
57·1	45·3	36·4	29·2	23·8	15·7	11·1	7·9	5·7

f is in kc/s.

As a comparison with other results obtained for sea-water, Fig. 11 shows a plot of the logarithm of the relaxation frequency against the reciprocal of the temperature in deg K. It has Wilson and Leonard's points, Liebermann's data and the Murphy and Garrison point. The equivalent expression obtained from the Kurtze and Tamm data for magnesium sulphate is plotted for reference. Note that the two lines have substantially the same slope but that the relaxation frequencies for magnesium sulphate seem to be a little lower than those for actual sea-water.

The expression for absorption in synthetic sea-water from data by Kurtze and Tamm set up in equivalent form is

$$\alpha = \frac{81{\cdot}9\times10^{-6}f_T f^2}{f_T^2+f^2}+\frac{3{\cdot}38\times10^{-6}f^2}{f_T}\quad\text{nepers/metre} \qquad \ldots\ldots(8)$$

† The profusion of units arises from the attempt in this paper to correlate original laboratory and field data.

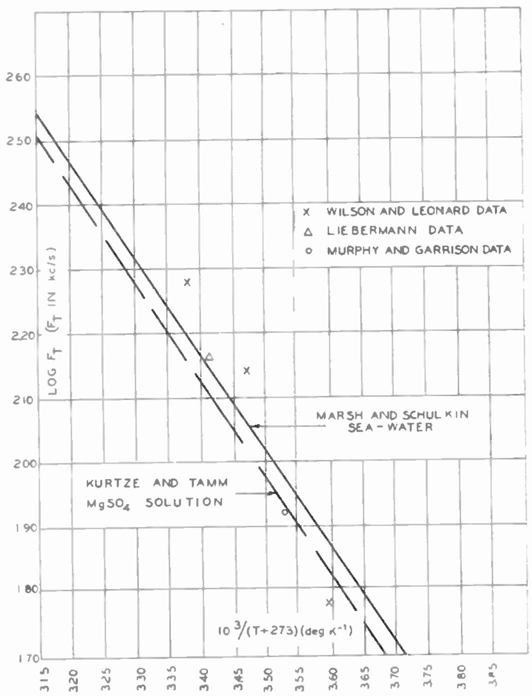

Fig. 11. Dependence of relaxation frequency on temperature for sea-water.

The agreement between eqns. (6) and (8) is considered to be remarkable. Remember, however, that the f_T are different as shown in Fig. 11. (Compare eqns. (2) and (7)). However, determination of the coefficient of the excess absorption term from the low salinity water measurements of other workers shows a value which is lower by 35%.[17]

8. Practical Expression for Absorption Calculations for the Ocean

In addition to temperature, the quantities salinity and pressure have important effects on absorption in the ocean.

In view of the proportionality of absorption with magnesium sulphate concentration, at low concentrations, and in view of the constant proportion of magnesium sulphate and other salts with salinity, the excess absorption is taken as proportional to salinity. The average salinity observed during the measurements reported here was 35°/₀₀. This approximation is not recommended for waters of very low salinity.

The pressure effect observed by Fisher in $MgSO_4$ solutions and shown in Fig. 3 is recommended for use also. This pressure coefficient is twice that observed for pure water. Its use with the residual term for pure water leads to small error because this term has a small contribution at sonar frequencies. This pressure correction leads to 30% less absorption at the bottom in 2000 fathoms of water than near the surface.

The final expression recommended for computation of absorption in sea-water is

$$\alpha = \left(\frac{2{\cdot}34\times10^{-6}Sf_Tf^2}{f_T^2+f^2}+\frac{3{\cdot}38\times10^{-6}f^2}{f_T}\right)\times(1-6{\cdot}54\times10^{-4}P)\ \text{nepers/metre} \quad \ldots\ldots(9)$$

where S is the salinity in parts per thousand,

f_T is $21{\cdot}9\times10^{\left(6-\frac{1520}{T+273}\right)}$ kc/s and T is in deg C,

f is the frequency in kc/s

and P is the pressure in kg/cm^2 or atmospheres.

9. Conclusion

Absorption of sound in sea-water has been reviewed from the viewpoint of arriving at a practical expression in terms of temperature, pressure and salinity.

It has been found that a single relaxation process arising from the hydrolysis of $MgSO_4$ in sea-water approximates well the observed excess absorption over that of pure water. The sodium chloride in sea-water reduces this absorption.

Analysis of propagation data from many measurements at sea over the North Atlantic in deep isothermal layers, when coupled with Wilson and Leonard's data at middle and high frequencies and corrected for Pinkerton's pure water absorption data, leads to an expression for absorption coefficient which agrees well with the Kurtze, Tamm and Kaiser measurements.

Fisher's data on pressure dependence in $MgSO_4$ solutions are used to supply a term that is considerably larger than that in pure water and leads to a 30% reduction in absorption in 2000 fathoms of water.

It is observed that low salinity water leads to considerably less absorption than called for by proportionality to salinity.

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DISCUSSION

Dr. S. S. Srivastava raised the question whether any contribution was made by other solvents in water, particularly sulphates, and also what corrections were being applied to the measurements because of top and bottom reflections.

The authors (*in reply*): Kurtze and Tamm have studied the acoustic absorption effects of solvents in sea-water individually and collectively in the laboratory. They have found that sodium chloride actually reduces the absorption due to magnesium sulphate ions. One of the advantages of working in deep isothermal layers is that top and bottom reflections are not very important for propagation losses at the ranges and frequencies of the experiment. However, the depth dependence shown for 2·2 kc/s data does include the effect of the surface.

Dr. J. W. Horton: The correlation between measurements of absorption in the deep sound channel and in laboratory samples of water is of significance to the operational use of sonar because of the increasing use of systems in which a major portion of the propagation path is at great depth.

It is becoming increasingly desirable that there be general agreement as to the nominal value of attenuation coefficient. This presents difficulties, however, since appropriate account be taken of effects, due to scattering and occluded air, which increase attenuation in surface layers.

The authors thanked Dr. Horton for his contribution.

Mr. M. J. Daintith: One mechanism giving a conversion of acoustic energy to heat is excitation of gas bubbles. This might be expected in surface channels, at least, but there was no evidence in the paper that this effect did, in fact, occur.

The authors (*in reply*): Absorption effects due to air bubbles should have a sharper frequency dependence than those due to ionic relaxations. In the first case, a resonance phenomenon is involved. It is possible that bubble effects can be important under certain circumstances, but it was not necessary to use these to explain the present data.

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Prediction Methods for Sonar Systems

By

R. J. URICK, M.Sc.†

Presented at the Symposium on "Sonar Systems" in Birmingham on 9th–11th July 1963.

Summary: Performance prediction in sonar is centred around the so-called sonar equation relating various parameters defined by the medium, the target, and the equipment. In this paper an active sonar equation valid for both short transient and long-pulse sonars is presented, and a discussion is given of each of the sonar parameters. Of particular interest is a generalized source level for comparing explosive and pulsed sonars, and prediction expressions for reverberation and detection threshold. An illustration of the use of these concepts is a recent determination of the back-scattering coefficient of the deep sea bed using explosive sound sources.

1. Introduction

At a Symposium on "Sonar Systems" it is particularly appropriate to consider the relationships that a design engineer uses—or should use—to predict the performance of the sonar system that he may be designing, or to explain the performance of some system he has already built. The relationships in question involve all the factors that govern how a signal received at a distance is related to what is radiated by the source; they contain implicitly all the effects, processes, and quantitative information that the research scientist is anxious to understand and to measure. These so-called "sonar equations" were first stated (to the writer's knowledge) in an academic fashion during the World War II years, and were gradually made more definitive and useful in the post-war period. At the present time it is the rare report or proposal that does not rely on the sonar equations for performance prediction, or to explain why the system parameters were chosen in the way they were. The equations have their immediate counterparts in radar,[1] though in terms of different units and definitions.

The purpose of this paper is to extend the ordinary formulation of the sonar equations to the case of short transient sounds, and to arrive at a set of relationships that can be used for both the square-topped, long sinusoidal pulses of conventional active sonars and the short transient pulses produced by explosives. Attention will also be given to the quantities occurring in the relationships, so as to attempt to obtain some useful sub-relationships and prediction estimates for use in the equation itself.

2. The Sonar Equation

In the common—or essentially c.w.—formulation, the sonar equations (for the active and the passive case) state simply that the *level* (intensity in decibels) of the signal is equal to the *level* of the background which just masks it for the purpose for which the sonar is intended. The word *level* has been emphasized in order to stress that it is the *intensity* of the signal and background that is being equated; the equality applies to the mean square pressure averaged over some period of time. In units of intensity level, the active sonar equation is‡

$$I_0 - 2(TL) + T = N_1 + (DT - DI)$$

where the left-hand side is the *echo level*, in terms of the source intensity I_0 in the direction of the target, the transmission loss to the target and back again, $2(TL)$ for an adjacent source and receiver (the "monostatic" situation), and the target strength T. The three terms on the right are the *background masking level*, in which N_1 is the spectrum level of the noise or reverberation background, DT is the detection threshold, or the echo-to-background-spectrum-level ratio needed for detection (or any other function), and DI is the directivity index of the receiving array against the particular background that may be occurring. The latter two terms are sometimes combined, and called *processing gain* to include the improvement in signal-noise ratio produced by both the receiving array and the processing-display electronics.

The point of particular note in this well-known equality, and its counterpart for passive sonars,§ is that its units are, and the parameters in it have been measured in terms of, *intensity or time-rate of change of energy density*. This causes trouble whenever the target or the medium distorts the source signature described by the term I_0. For example, this distortion

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‡ The symbols for the sonar parameters have not been standardized; different symbols are in use by different writers, although some are in more common use than others.

§ Where only the one way transmission loss, TL, appears, and the target strength T is absent.

is particularly pronounced for explosive sources against submarine targets, for which the echo is totally dissimilar in wave shape (Fig. 1). Near the source, an explosive produces a quasi-exponentially decaying shock wave, with or without bubble pulses; the echo from a submarine or other extended target is a relatively long noise-like blob that is totally different in appearance from what was generated by the explosive source. In such situations a sonar equation in terms of

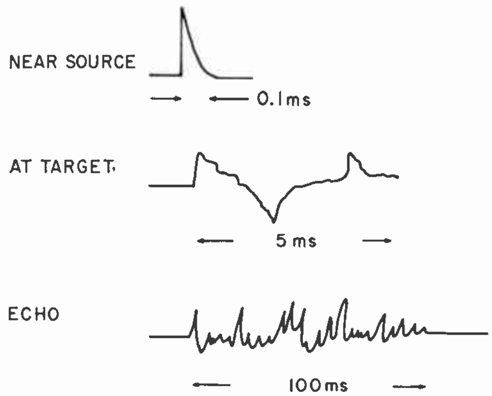

Fig. 1. Effect of target and medium distortion on received signal.

intensity is meaningless, and it becomes more reasonable to base the equality of echo and masking background on *energy density* rather than on its rate of change. Indeed, it is energy density rather than intensity that is involved in conservation of energy considerations, as in inverse-square law spreading under free-field conditions ($TL = 20 \log r$). Energy and its rate of change are equivalent under quasi-steady-state conditions, when the interval over which the energy density is averaged can be very large; this applies for most measurements with relatively long pulses of parameters like target strength and transmission loss, in which all propagation paths to and from the target have an opportunity to contribute.

If we base the equality on energy density rather than intensity we would equate the energy density of the echo to the masking energy density of the background, and obtain

$$E_1 - 2(TL) + T = N_1 + DT - DI + 10 \log t_e$$

where E_1 is the spectrum energy density level of the source, and the term $10 \log t_e$ indicates that the energy density of the background is to be taken over the duration of the echo. This form of a generalized sonar equation has been pointed out in a recent paper.[2] If the term $10 \log t_e$ is brought over to the left-hand side, we again have an equality of intensity, but with the source intensity based on the average energy of the source *over the duration of the echo*,† rather than over the pulse length of the source. We then have

$$(E_1 - 10 \log t_e) - 2(TL) + T = N_1 + DT - DI$$

Another generalization that is desirable to make is to broadband sources, over and above the sinusoidal pulses for which the sonar equations are commonly used. This can be done easily by defining the source energy density to include all the energy emitted by the source within the bandwidth of the receiver, and suitably to define the detection threshold accordingly. We can then write our final form of the active sonar equation as (Fig. 2)

$$E_w - 10 \log t_e - 2(TL) + T = N_1 + DT - DI$$

In practical use, this equality is solved for whatever term occurring in it may be of interest. For a sonar of a given design, the detection range is contained in the transmission loss term, which somehow, by theory or measurement, must be related to range. Other terms are solved for when it is necessary for the system to achieve a desired performance. In the above form, when the parameters are suitably defined as indicated below, the equality may be used for both long-pulse and short transient waveforms.

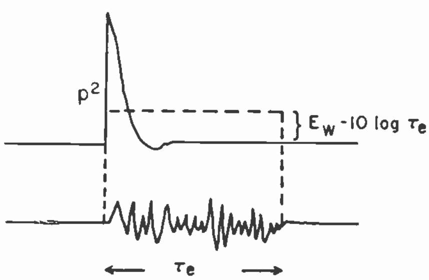

Fig. 2. Detection threshold related to emitted energy.

3. The Sonar Parameters

3.1. *Source Energy Density*, E_w

This is defined as the energy density at some reference distance (usually 1 yard) produced by the source in the direction of the target. In underwater sound, noise backgrounds are usually specified in terms of the intensity of a plane wave having a pressure of one dyne per cm^2; it is therefore convenient to measure E_w in dB relative to the energy density of a plane wave of r.m.s. pressure one dyne per cm^2 for an interval of 1 second. If the source is broad-band, E_w is the energy density in the receiver band w, such that

$$E_w = E_1 + 10 \log w$$

where E_1 is the energy spectrum density at some frequency (depending on spectrum shape) in the band

† Or some portion of it, if, as in some forms of signal processing, the entire echo is not employed in processing.

w. For explosives, E_1 has been established by the work of Weston.[3] For sinusoidal square-topped pulses,

$$E_w = I_0 + 10 \log t_0$$

where I_0 is the source (intensity) level at 1 yard and t_0 is the pulse length.

3.2. *Echo Duration, t_e*

This parameter is extremely important for short pulse transient sonars, and is one that must be determined by measurements of particular targets for different ranges and propagation conditions. The echo length of an extended target may be thought of as being the sum of three contributions: (1) the duration of the emitted pulse, (2) a duration produced by multiple transmission paths in the sea, (3) a duration produced by the target itself. Thus, the medium and the target extend the time duration of the initial pulse, and give rise to an echo that sometimes is much longer than the pulse-length generated by the source.

For long pulse sonars, this stretching of the initial pulse-length is inappreciable, and the echo is practically a replica of the source as far as its time duration is concerned. But for short pulse sonars, the last two contributions are overwhelming, and the echo now becomes a long blob relative to the short pulse emitted by the source. For example, consider the underwater explosion of a few pounds of TNT. This generates a shock wave having an exponential tail with a time constant of about 0·1 millisecond. The echo of this short transient, after travelling for a few miles in the sea and after having been scattered by a submarine back toward the source, may have a duration of 100 milliseconds, or a thousand times as long. For short pulse sonars, the echo length is, in short, determined by the target and by the medium, and must be ascertained by field observations on particular targets under particular conditions.

Observations with short pulse sonars show[4] that submarines consist of scatterers of various kinds distributed all along their length. Some of these scatterers return echoes stronger than others, and form "highlights" within the echo. Others lie near the bow and stern. If the length of the target is L and the aspect angle (angle between the direction of the incident sound wave and the heading of the submarine) is θ, various short-pulse sonar observations tend to show that the target contribution to the total echo duration, in the direction back toward the source, is

$$\frac{2L}{c} \cos \theta,$$

where c is the velocity of sound and $L \cos \theta$ is the extension in range of the target. To this, as just indicated, must be added the contribution to the echo duration produced by multi-path effects in the sea (which is a small contribution in deep water, but which is relatively large in shallow water) and the initial pulse length of the source.

3.3. *Transmission Loss, TL*

This parameter summarizes all the effects produced by propagation in the sea on the energy density radiated by the source in any particular direction. It has long been a subject of field measurements, and of attempts at prediction through such aids as the ray diagram. Little can be given in a short space as to its predictability, since it varies so greatly with many factors in the sea and the presence of its boundaries. Very often, in the absence of anything better, spherical spreading (20 log r) plus absorption[5] is a useful rule-of-thumb for a rough prediction under unspecified conditions.

In the energy form of the sonar equations, what is needed is the transmission loss for energy density, or the ratio of the energy density at the field point, to the energy density at the distance at which E_w is specified (usually 1 yard). It is the same as the ordinary transmission loss for intensity under steady state conditions, such that all paths to the distance field point can contribute. It is the transmission loss measured for long pulses or with c.w.; these are the conditions under which most measured data are obtained. Like target strength, the applicable value for this parameter in the energy form of the sonar equation is its steady-state value.

One complication must be mentioned in connection with explosives used as sound sources. If we neglect bubble pulses, which have a negligible contribution to the high frequency energy spectrum, an explosive creates a shock wave whose attenuation is different from that of simple acoustic waves. Measurements in sea water show[6] that the peak pressure p_0 of an explosive pulse varies with range as the $-1{\cdot}13$ power, rather than as the -1 power that applies for acoustic waves in an ideal medium; its time constant t_0 varies with range as the $+0{\cdot}22$ power. The energy spectrum level (for an exponential pulse) can be shown[3] to be proportional to p_0^2 at high frequencies; it will thus fall off with range as the $-2{\cdot}26$ power, or as $22{\cdot}6 \log r$. On the other hand, the low-frequency energy spectrum (neglecting bubble content) of an exponential pulse is proportional to $p_0^2 t_0^2$, and falls off as the $(-2{\cdot}26+0{\cdot}44)$ or $-1{\cdot}82$ power of the range. Thus, high frequency energy (above $f=1/2\pi t_0$) appears to be converted into low frequency energy—a process that is foreign to low amplitude acoustic waves. In short, the extent to which the transmission loss as measured for acoustic waves in the sea can be used for explosive waves is a moot question at the present time; what to use for the echo of an explosive wave from an extended target is even more uncertain.

However, it is possible that the differences involved are small for rough results in many cases.

3.4. *Target Strength, T*

This parameter is defined as the ratio of intensity of the scattered wave from a target, reduced to a distance of 1 yard, to the intensity of the incident wave. It will be shown that this definition applies to energy density as well, provided that (like transmission loss) steady-state conditions (or long pulses) exist, such that all portions of the target contribute to the echo at some one instant of time.

Let I_e be the echo intensity reduced to 1 yard from the target and let I_i be the incident intensity. Then by definition the target strength is

$$T = 10 \log I_e/I_i$$

We wish to find the relationship between this quantity and a similar ratio for energy density:

$$T' = 10 \log E_e/E_i$$

where E_e and E_i are the scattered and incident energy densities, respectively. Since energy density is the product of an average intensity and a time interval, we have

$$T = 10 \log \frac{I_e}{I_i} = 10 \log \frac{E_e/t_e}{E_i/t_0} = 10 \log \frac{E_e}{E_i} + 10 \log \frac{t_0}{t_e}$$

$$= T' + 10 \log \frac{t_0}{t_e}$$

where t_0 and t_e are the durations of the incident pulse and the echo, respectively. For long pulses, $t_e = t_0$ and the two target strengths are equal. For short pulses, for which $t_e > t_0$, the target strength for intensity, T, increases with pulse-length. Physically this is due to the failure of short pulses to insonify the target completely at any one instant of time. This increase of target strength with pulse-length is in agreement with observations of submarine echo with short pulse sonars; the target strength is found to increase with pulse-length, until the target becomes completely insonified at some one instant of time; for back-scattering this occurs when

$$t_0 = \frac{2L}{c} \cos \theta$$

where $L \cos \theta$ is the projected length of the target in the direction of the source and c is the velocity of sound. Above this value of pulse-length, the target strength remains constant. This long-pulse or c.w. target strength is, as shown above, the appropriate value for energy density (Fig. 3).

3.5. *Background Spectrum Level N_1*

This may be either noise or reverberation, measured in a one-cycle band at the equivalent mid-frequency of a broadband receiver and source. For ambient noise in deep water, these levels were established long ago by World War II measurements,[7] and are still considered valid.

For reverberation it is convenient to base the prediction on two factors: (1) a reverberation coefficient per unit area or volume, together with (2) a geometrical computation of the area or volume returning reverberation at some instant of time. Thus, for pulsed sonars, the reverberation level N_0 at a time equivalent to a range r can be written as

$$N_0 = I_0 - 2(TL)_R + S_{S,V} + 10 \log (A, V)$$

$$A = \phi r \,.\, \frac{ct_0}{2}$$

$$V = \psi r^2 \frac{ct_0}{2}$$

where $2(TL)_R$ is the transmission loss for reverberation, often, but not always, equal to simple spreading plus absorption, $S_{S,V}$ is a back-scattering coefficient called scattering strength (analogous to target strength) for surface (S_S) or volume (S_V) reverberation, and A and V are the reverberating area or volume, defined by (1) the plane or solid angle beam widths (ϕ or ψ, respectively), (2) the pulse length t_0, and (3) the range r. Substituting for A and V we obtain the pair of equations

$$N_0 = I_0 - 2(TL)_R + S_S + 10 \log \frac{\phi r c t_0}{2}$$

$$N_0 = I_0 - 2(TL)_R + S_V + 10 \log \frac{\psi r^2 c t_0}{2}$$

for surface and volume reverberation, respectively.

For short transients such as explosive waves, both the average (intensity) level I_0 and pulse-length t_0 are ill-defined (except in a mathematical sense). One notes, however, that since energy density is the product of the average intensity and pulse-length, we have

$$E_0 = I_0 + 10 \log t_0$$

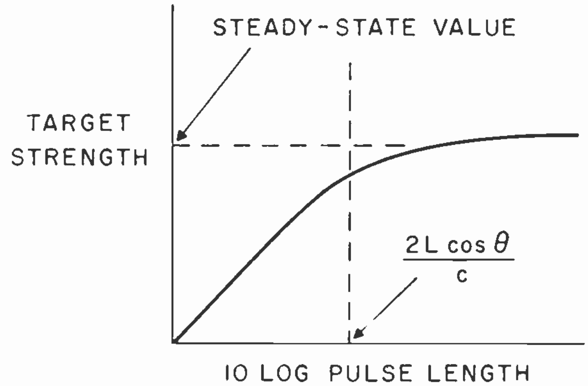

Fig. 3. Target strength vs pulse length.

so that we can write, in terms of energy density, a similar set of equations for short transients defined in terms of their spectrum energy density:

$$N_1 = E_1 - 2(TL)_R + S_S + 10 \log \phi r \frac{c}{2}$$

$$N_1 = E_1 - 2(TL)_R + S_V + 10 \log \psi r^2 \frac{c}{2}$$

These two equations (one for surface and one for volume reverberation) can be derived from basic considerations, as has been done in an earlier paper.[2] The usefulness of the equation for surface reverberation can be seen from some recent work on the back-scattering coefficient of the deep sea bed—summarized in the Appendix—where explosives were used as sound sources, and where coefficients equivalent to those found with pulsed sinusoidal sonars were obtained.

3.6. *Detection Threshold, DT*

This term is meant to replace the well-known term *recognition differential*, which was long ago taken over into underwater sound from auditory acoustics, and which has by now largely ceased to have its original meaning for most sonar systems. The word *detection* is employed to indicate the type of function which many sonar systems are called upon to perform; in what follows, detection describes the decision which the system (including the observer, if there is one) is called upon to make.

A theoretical basis for a prediction formula for detection threshold is the work of Peterson, Birdsall and Fox[8] done at the University of Michigan about 10 years ago. We will here concern ourselves only with two simple detection situations, which may be thought of as being the extremes in regard to the available knowledge of the signal and background. We will also restrict ourselves to (1) sinusoidal or Gaussian signals in Gaussian noise backgrounds, (2) small input signal-to-noise ratios and (3) a large number of statistical sample points ($2wt_e \gg 1$). The essence of the work of Peterson, Birdsall and Fox is to define a *detection index d* based upon probability considerations of signal and noise, and to relate it to the receiver signal-to-noise ratio. The detection index was shown to be determined, for a given signal in a given type of noise, by both the *probability of detection* and the *probability of false alarms*. The latter quantity is the probability of a false decision that a signal is present when it actually is absent at the receiver input terminals. With these two quantities as co-ordinates, curves of equal detection index were drawn to form the so-called *receiver operating characteristic* (ROC) *curves*, which for particular statistics of input signal and noise, relate the detection probability and the false alarm probability for a receiver employing certain optimum criteria for a binary decision of "signal-present" or "signal-absent". Some values of the detection index for various pairs of detection and false alarm probabilities are given in Table 1 (Fig. 4).

Peterson, Birdsall and Fox considered a number of theoretical and practical cases, of which we will here consider only two; these may be regarded as opposite

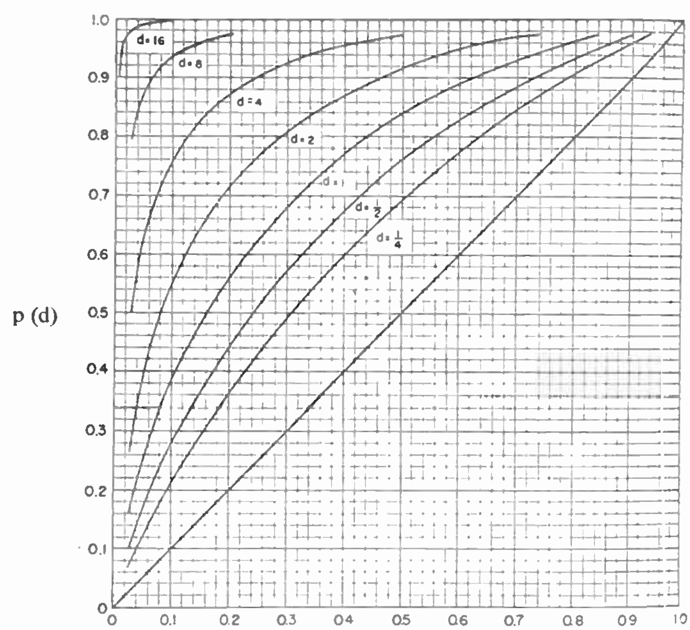

Fig. 4. Detection index for various combinations of detection and false alarm probabilities.

Table 1

Values of d for Various Combinations of Detection and False Alarm Probabilities

		Detection Probability		
		0·5	0·7	0·9
	0·01	6	9	13
False alarm probability	0·05	3	6	9
	0·10	2	4	7

ends of the range of knowledge concerning the signal. At one extreme, the signal is known exactly as to waveform and time of occurrence; at the other, the signal is completely unknown except as to the knowledge that it is a sample of a sine wave or of Gaussian noise. For these two cases, it was shown that the detection index is related to the input signal-to-noise ratio by

$$d = 2wt_e\left(\frac{s_w}{n_w}\right), \text{ signal exactly known}$$

$$d = wt_e\left(\frac{s_w}{n_w}\right)^2, \text{ signal unknown}$$

where s_w/n_w is the ratio of signal power to noise power in the receiver band w, measured at the receiver input.

We will now define detection threshold, for the manner the sonar equation has been written, as the difference between the echo level $S_w = 10 \log s_w$ in the receiver band w and the spectrum level $N_1 = 10 \log n_1$ of the background. Hence we have

$$DT = S_w - N_1 = 10 \log \frac{s_w}{n_1} = 10 \log \frac{d}{2t_e},$$

signal exactly known

$$DT = S_w - N_1 = 10 \log \frac{s_w}{n_1} = 5 \log \frac{d_w}{t_e},$$

signal unknown

These are our prediction equations for detection threshold. They apply to many cases of practical interest. For sonar echo ranging, the first case is that of a point target at a known range in a distortionless medium; the optimum detector is a matched filter, or its equivalent—a correlator, in which the known, noise-free signal is correlated with signal plus noise. The second case is that of an extended target at an unknown range and with an unknown range-rate (Doppler shift); the optimum detector is a square-law detector followed by a matched averager (i.e., of averaging time equal to t_e). Simple corrections to these expressions can be made for multiple pulses, a non-square-law detector, and a mis-matched averager. They have been found to fit the observed detection threshold of the ear and of the p.p.i. displays of radar.[9]

A few words may be said in passing about some of the implications of these relationships. For an exactly known signal in Gaussian noise and with the other conditions as stated at the outset, the first expression gives the best (minimum) possible value of detection threshold. It is not possible to build a better detector, and the only way to do better for a given value of d is to somehow increase the observation time t_e (as by multiple echoes). Also, there is often, in practical cases, only a few decibels of difference between the first case and the second—between an often elaborate correlator and a simple square-law detector—representing extremes of knowledge concerning the signal. In general the only way to improve matters is increase one's knowledge of signal and noise, and to build the optimum detector for the statistics at hand. For Gaussian-noise backgrounds the above relationships serve as simple predictors for detection threshold; for a background of reverberation, no prediction methods appear to be available as yet.

3.7. *Receiving Directivity Index, DI*

This part of the total processing gain has long been separated from "detection threshold" to indicate that portion of the total gain that can be had from the receiving transducer array alone; the dividing line between the two is commonly the transducer terminals. This separation is still convenient to make, although with modern arrays, such as correlative arrays, it is not at all clear where the transducer terminals are located, that is, where the transducer ends and the electronics begins. For additive arrays, prediction expressions for linear and plane configurations were derived in World War II or earlier, and are still useful for simple sonars. They need not be repeated here. They assume, usually tacitly, that the noise background is uncorrelated from element to element of the array, and that the signal has perfect correlation ($r = 1$) along some direction, called the axis of the array. For signals and backgrounds that do not satisfy these conditions, prediction methods apparently have not yet been worked out in a form useful to the sonar engineer. Volume arrays and multiplicative (correlative) arrays have received attention in the acoustic and radio literature, but without summarization in a synoptic form useful to the general design engineer.

4. Conclusions

The purpose of this paper has been to state a generalized form of the sonar equations that can be used by sonar engineers for prediction of range once a system design is established, or to establish the system design in order to achieve a desired range. Only the active sonar equation has been considered, since the passive case requires only simple modification of the echo ranging equation. The emphasis has been placed on a form of the basic relationship that can be used for explosive echo-ranging as well as for echo ranging with pulsed sinusoidal sources. A short discussion has been given of the parameters appearing in the equations.

The engineer will find his greatest difficulty in making a prediction in selecting the proper numerical values of the parameters that apply to a particular case. This is especially troublesome for parameters like transmission loss or scattering strength, which are characteristics of the underwater medium and its boundaries, and therefore can only be determined by measurement at sea, and for which values needed for prediction are often lacking.

In a broad sense the sonar equation with its parameters epitomize the efforts of the research scientist in underwater sound. Its usefulness to the engineer is no better than the existing knowledge of the parameters. He will find, for new sonar systems that are radical departures from old ones, that no more than an order-of-magnitude type of prediction—if that—is possible. The improvement of the prediction ability of the engineer should be one of the aims in life of the applied research scientist.

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6. Appendix

Back-Scattering of Explosive Sound by the Deep Sea Bed

Work has recently been reported[10] on the determination of the back-scattering strength S_S of the deep ocean bottom through the use of shallow explosions of 2·5 lb of tetrytol as sound sources and a nearby shallow hydrophone as receiver. From field recordings, analyses were made of the back-scattered return from the bottom at a depth of about 2500 fathoms between Cape Hatteras and Bermuda.

If the bottom can be considered to be flat, the start of the return arrives from a point underneath the source-receiver at a grazing angle of 90 deg with the bottom; subsequent portions of the return represent back-scattering at decreasing grazing angles —until an angle of 30 deg is reached, when the second bottom bounce arrives.

The scattering levels as measured from tape records were converted into the coefficient scattering strength, by the relationship given previously:

$$N_1 = E_1 - 2(TL)_R + S_S + 10 \log \left(\frac{\phi c}{2}\right)$$

where N_1 is the scattering level in a 1 c/s band, E_1 is the source energy density in a 1 c/s band, TL is the transmission loss, taken to be 20 log r with absorption and shock wave effects neglected, S_S the desired back-scattering strength, ϕ is 2π radians for non-directional transducers, and c is the sound velocity in yards per second.

When plotted against angle, the coefficient, S_S, was found to rise slowly with angle in the angular range 30 deg to 65 deg, as if diffuse or Lambert's "Law" back-scattering was the dominant process of the return. In the range 65 deg to 90 deg the back-scattering strength rose rapidly with angle as normal incidence (90 deg) was approached, suggesting that specular reflection from inclined flat facets of bottom is the principal process by which sound is returned to the vicinity of the source. Alternatively, the increase may represent reflection from sub-bottom layers.

From analyses made in the four bands 0·5–1·0, 1·29–2·23, 2–4, and 4–8 kc/s, little change in scattering strength with frequency could be found that could not be attributed to the effects of absorption and shock-wave loss in the sea water medium. The average back-scattering strength for the soft mud sea bed for all frequencies at an angle of 30 deg was −33 dB.

These results are in agreement with data previously reported by Mackenzie[11] using sinusoidal pulses of length 0·1 to 2·0 seconds at frequencies of 530 and 1030 c/s. Together with earlier observations at higher frequencies, the data of Mackenzie tend to show a Lambert's Law type of variation with angle:

$$S_S = -28 + 10 \log \sin^2 \theta,$$

with negligible change with frequency over the range 530 c/s to 60 kc/s.

The short duration of an explosive pulse permits the back-scattering behaviour of the sea bed to be studied near normal incidence, and the use of explosives with a simple non-directional hydrophone permits information to be obtained over a range of angle and frequency with a minimum of field instrumentation. The results agree with the scattering strengths obtained with pulsed sinusoidal sound sources.

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POINTS FROM THE DISCUSSION

Mr. J. O. Ackroyd†: This paper covers an important field in sonar work. Detection ranges vary greatly due to weather, oceanographic and other conditions and it is a difficult task to evaluate at sea the relative merits of sonars; the paper is valuable in giving an alternative approach.

Sea noise is treated as though it has the same properties as Gaussian noise; while this may be a reasonable assumption when the noise is observed deep down in the water, conditions are very different in a surface ship under way. There noise levels as high as 20 dB above the average are common. The signal fluctuates also.

Clearly, the author appreciates that reverberation may be the limiting factor in practice but remarks that no prediction methods are available for it. There is, however, evidence in the literature that reverberation has a Rayleigh distribution and it seems preferable to include its effect on predicted ranges, always remembering that the treatment of both noise and reverberation are approximate only.

The term "recognition differential" is rather old-fashioned, but many will prefer it to "detection threshold", which does not convey the idea of a difference or margin.

The Author (*in reply*): Mr. Ackroyd's comments appear to pertain to the subject of detection threshold or recognition differential. They make it clear that we lack prediction methods for this parameter whenever the background is non-Gaussian in nature. As he indicates, this situation exists for both self noise, which is spiky, and reverberation, which is Rayleigh-distributed. Perhaps the theoreticians can somehow be persuaded to provide the engineer with some prediction guidelines for these cases.

Personally, I prefer to restrict term recognition differential to auditory detection, and to let detection threshold refer to *all* detection methods, including the ear.

Dr. A. Freedman†: Mr. Urick's curve of target strength versus transmission length (Fig. 3) appears to be incorrect at the short pulse length end. Whereas at longer transmission lengths I would expect the curve to have the rising and then flattening off characteristics shown, the curve should not go down to zero at the short pulse end. When the transmission is short enough to resolve individual echo components, then, apart from the effect of bandwidth limitations, the target strength will not decrease below that corresponding to the largest echo component.

The Author (*in reply*): Dr. Freedman is right in implying that a complex target like a submarine, say, will always produce an echo, even for extremely short pulse-lengths where individual scatterers would produce spikes in the target signature. The target strength T, however, is defined as an average over the duration of the echo. Hence, the target strength could conceivably approach zero with an extended complex target because the echo spikes become averaged over a much longer time interval. The curve referred to showed T going to zero at $t_0 = 0$.

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U.D.C. 621.318.132: 681.32

The Design of a 4096-word One-microsecond Magnetic Film Store

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AND

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Summary: The store is made up of a large number of aluminium plates coated with thin magnetic film. It is word-organized and bi-directional digit currents are used for writing. The word and digit lines consist of flat conductors pressed against the plates and these define the storage positions on the continuous areas of film.

Advantage is taken of the low impedance of the film to provide the word selection at low cost. A store-size ferrite core serves as the selection device for each word and a matrix of diodes directs the drive currents into the chosen lines of cores.

A single set of digit conductors are used for both sensing and write back and these serve all the 4096 words of the store. The digit current disturbance is minimized by a special balancing arrangement.

1. Introduction

Magnetic films have for some time held the promise of providing faster and less expensive storage systems than are possible with ferrite cores. Difficulties with film reproducibility and with the handling of small output voltages in the presence of large operating currents appear to have delayed the development of such storage systems. In the system to be described, films deposited on an aluminium substrate have been used and this has allowed much tighter coupling of the conductors to the film and thereby minimized these noise difficulties. The successful production of these films has been previously reported by Bradley.[1, 2]

The storage system is word-organized with square-loop ferrite cores used as the selection devices. This arrangement is attractive in that the low impedance of the film is compatible with the use of small-sized cores which can be driven from a diode matrix, thus minimizing the number of transistors required.

A single set of digit conductors are used for both reading and writing and one set of sense amplifiers and digit drivers serve the whole store. The low impedance of the digit lines between film plates is maintained by metal bridging pieces and difficulties with the disturbance caused at the sense amplifiers by the digit drivers are avoided by a balanced arrangement of the digit lines.

2. Mode of Operation

Thin magnetic film may be operated in a variety of different ways. Pohm and Mitchell[3] have described some of the possibilities, including coincident-current operation, with films of well-controlled characteristics; Williams,[4] on the other hand, has analysed the behaviour to be expected when the film axes are not correctly aligned and shown novel ways of using poorer quality films. Others have described word-organized systems.[5, 6] In practice it becomes a question of choosing the simplest and cheapest system which is compatible with the quality of film that can be produced. This has led, in the storage system to be described, to the choice of word-selection with single areas for each stored bit and to the use of a bi-directional digit current for writing.

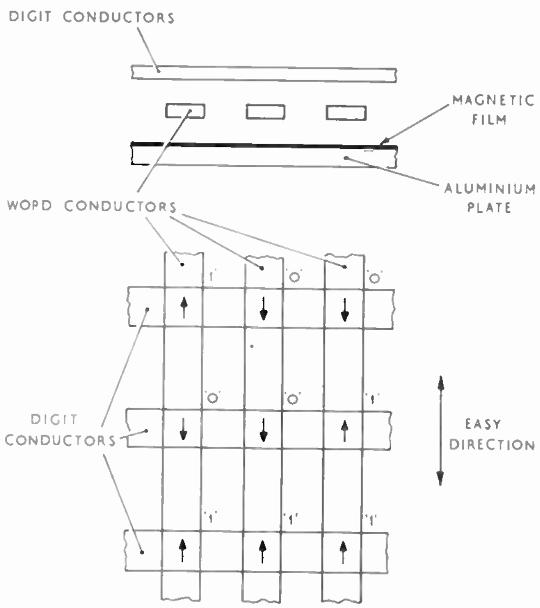
The film is deposited as a continuous area on aluminium plates 4·5 in × 3·25 in and it has a composition of 80 Ni : 17 Fe : 3 Co. Storage spots are defined solely by the conductor pattern, the field spread from flat conductors placed close to the plate being very small. It is considered that the continuous film has better electrical characteristics than when it is divided up into spots; there is also advantage in avoiding registration difficulties.

Figure 1(*a*) shows how the conductors are arranged in relation to the film. The areas of film at each junction of word and digit conductors are the storage locations and, after writing, the magnetization of these areas is in either of the two directions along the easy axis of the film. In the easy direction the film has a rectangular hysteresis loop and the remanent points on this loop correspond to the stored "1" and "0".

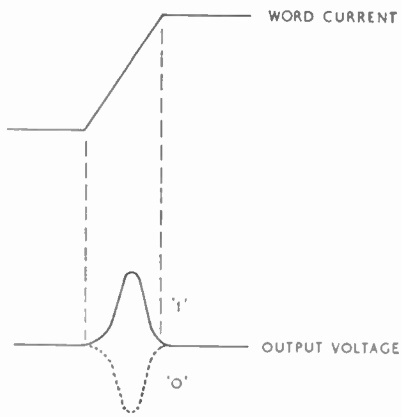
To read the stored information a current is passed through a selected word conductor. This has the effect of applying a hard direction field to the film and thereby causing the magnetization of each of the storage locations on that word to rotate 90 deg into the hard direction. The rotation is thus clockwise or anti-clockwise according to the previously stored state, and correspondingly either a positive or a negative voltage pulse appears on each of the digit lines.

† International Computers and Tabulators (Engineering) Ltd., Stevenage, Hertfordshire.

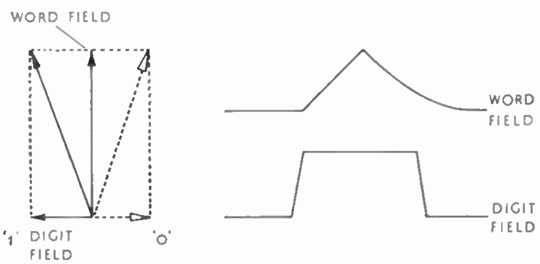
This rotation takes place within the rise-time of the word field and the amplitude of the read-out signal is proportional to the rate of rise of current. With a conductor width of 0·04 in the word current required is 1·5 A and with a rise-time of 80 ns the output voltage is greater than 1 mV.



(*a*) Construction.



(*b*) Reading.



(*c*) Writing.

Fig. 1. Mode of operation.

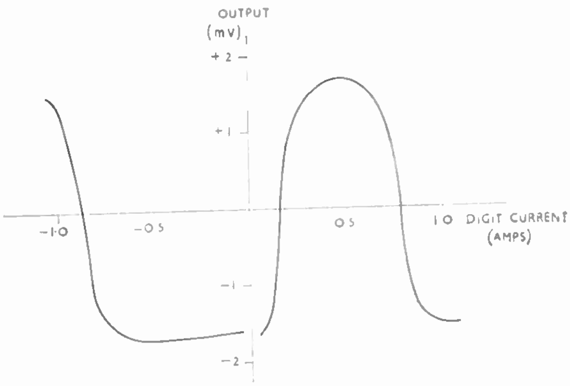
The writing process is illustrated in Fig. 1(*c*). Either a positive or a negative current is established in each of the digit conductors, in accordance with the information to be written, and coincident with these a current is passed through the chosen word conductor. This results in the film magnetization aligning itself with the vector sum of the fields at each of the storage locations on the chosen word. By removing first the word current and then the digit current the storage locations are left set in the desired directions. The speed of writing is limited only by the rise and fall times of the pulses concerned together with the requirement that the digit field be removed last.

The value of digit current required is a function of the accuracy with which the film axes are aligned in relation to the conductor pattern; a tilt of one with respect to the other results in an increase in the value of digit current required to write in one of the two directions. The coercivity of the film in the easy direction sets a limit to the value of digit current that can be used and, if this is exceeded memory positions on other words will be disturbed. The method of testing the film has been, to set it, say, to the "0" state with large currents, then to the "1" state with selected values of current, disturb it with a number of digit current pulses of opposite polarity and finally check the output voltage on reading. Figure 2(*a*) shows a plot of output voltage to digit current for a storage spot where the tilt is such as to make positive writing more difficult.

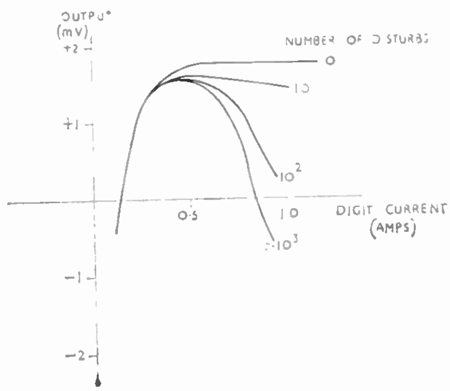
It is of interest to know the number of disturbing digit current pulses required to cause the maximum reduction of output for a particular value of disturbing current. Figure 2(*b*) shows a typical result obtained with different numbers of 0·5 μs wide pulses; from these tests it was concluded that the number required to give the maximum effect was the order of 10^3. This appeared to correspond to a total switching time, under these conditions, of 500 μs.

The possibility of minimizing the effect of digit disturbance by including an additional opposite polarity digit current pulse in the succeeding store cycle was considered. However, it did appear that such a scheme was only fully effective if the two pulses were closely matched in amplitude and duration. The advantage to be gained would be that lower quality film could be used, but this is offset by an addition to the cycle time of the store and the scheme was therefore dropped.

The optimum value of digit current has been found to be in the region of 300 mA and the standard test for the films, carried out spot by spot on an automatic plate tester, has been to write with 300 mA, disturb 10^4 times with 360 mA, and check that the read-out signal is of the correct polarity and above 1 mV.



(*a*) Output voltage against digit current.



(*b*) Effect of total number of 0·5 microsecond disturb pulses.

Fig. 2. Relationship between output voltage and digit write current.

The packing density of the storage elements on a plate is limited by interaction effects. If the elements are placed too close together there is a reduction in the operating range of digit currents, and in the size of the output voltage, when a given element is surrounded by opposing information. From these considerations the packing density has been limited to 32 words and 18 digit lines per plate.

3. Word Selection

Tolerancing and noise difficulties appear to preclude the use of coincident current selection with present magnetic films and a separate system of word selection has to be provided. This can be done by either a diode or a switch core matrix. The relatively large word currents required (1·5 A), together with the speed requirements, make the choice of diode difficult. Ferrite cores are appreciably cheaper and they can result in a more economical selection system as a whole; they have, therefore, been chosen as the word selection device. Slow-speed core stores do not require the last stage of word selection and therefore, to compete directly in price, the film storage medium plus word selection has to be less expensive than wired core planes. This is considered to be the case with the method chosen.

Figure 3 shows the constructional details of the store. Each word conductor links 18 bits on each of two magnetic film plates placed back to back. It is driven by a square-loop switch core similar in size to those normally used as storage cores. The cores are arranged as a matrix with X and Y drive lines,

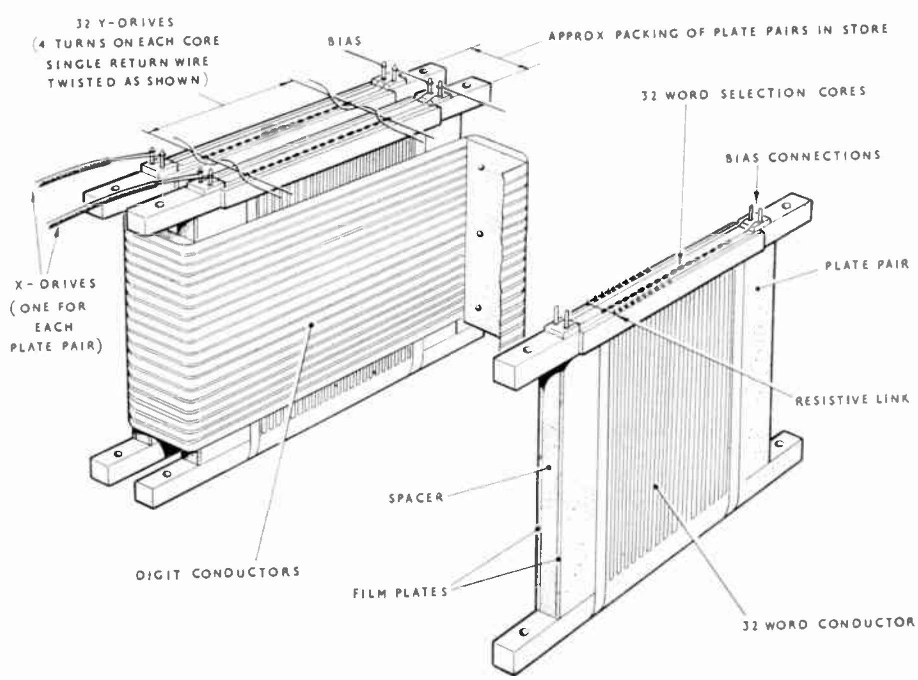


Fig. 3.
Construction details of store.

together with a d.c. bias current line, threading each core. Each of the drives, when energized, provides 3 ampere-turns to the core and this is opposed by the bias, whereas the coincidence of two drives causes the core to switch and make available to the word loop the excess drive current.

During the core switching process the word current rises to a given value and thereafter decays with a time-constant set by the inductance and resistance of the loop. When the drive currents are removed, the bias current switches the core back to its original state and an opposite polarity of word current is produced. The first word current pulse is used for reading and the second for writing.

The integral of the voltage developed by the switch core can be equated to that developed across the inductive and resistive components of the word circuit and, for a single turn and a linear rise of current

$$BA = \frac{I_p}{10}\left(L + \frac{Rt}{2}\right)$$

where B is change of flux density of the core in lines/cm^2; A is the area of core in cm^2; L the inductance of the word loop in nH; R the resistance of the word loop in ohms; t the switching time in ns; I_p is the peak current in amperes.

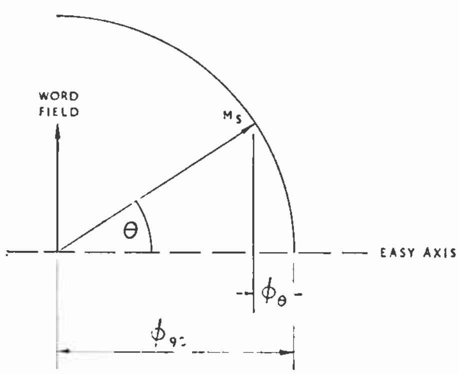


Fig. 4. Relationship between word field and flux change in easy direction.

If it is assumed that resistance, additional to that of the word conductor, is included in the circuit to reduce the time-constant to 100 ns, the relationship becomes:

$$BA = \frac{I_p L}{10}\left(1 + \frac{t}{200}\right)$$

It follows, therefore, that it is the product of the peak word current and the inductance which determines the size of switch core required. The metal substrate of the film assists considerably in minimizing this inductance and, with the construction shown, a value of 45 nH is obtained. With a switching time of 80 ns a core of 0·05 in diameter and 0·045 in thick provides the value of volt-nanoseconds required to establish 1·5 A in the loop.

The drive currents have a rise time of 150 ns and the core switching takes place in the latter portion of this period when the bias has been exceeded. The difference of 1·5 AT between the excess drive ampere turns and the output ampere turns represents the magnetizing current required to switch the core at this speed.

Heating problems which arise, should the same word be selected continuously on each store cycle, are avoided by the fact that the cores are mounted on a metal plate.

The inductance of the cores when the saturation region of the hysteresis loop is being traversed, under the action of a single drive current, sets a limit to the number of cores that can be driven, and causes disturbing word currents to be induced into unselected word loops. Under these drive conditions, the squareness ratio of the cores is approximately 20 and the disturbing word currents are therefore 75 mA.

To evaluate the effect these currents have on the film it is necessary to consider the film switching process. As illustrated in Fig. 4 the magnetization of a storage location can be regarded as a vector of constant amplitude M_S rotated away from the easy axis under the action of a word field in the hard direction H_T. The field H_T exerts a torque $H_T M_S \cos\theta$ on the vector and this is counterbalanced by a restoring torque of internal origin equal to $2K \sin\theta \cos\theta$.

Hence $$H_T M_S \cos\theta = 2K \sin\theta \cos\theta$$

or $$\frac{H_T M_S}{2K} = \sin\theta$$

where M_S and K are constants.[1, 4]

If, therefore, it is assumed that the full word current of 1·5 A produces a field to cause a 90 deg rotation, a current of 1/20th of this value will cause a rotation equal to $\sin^{-1}$ 0·05 or 2·8 deg.

The output voltage is proportional to the rate of change of flux in the easy direction. From the diagram

$$\frac{\phi_\theta}{\phi_{90}} = 1 - \cos\theta$$

where ϕ_θ is the flux change in the easy direction for a rotation θ

ϕ_{90} is the flux change in the easy direction for a rotation of 90 deg.

Therefore with θ = 2·8 deg

$$\frac{\phi_\theta}{\phi_{90}} = 0{\cdot}0012$$

If the rise-time of the disturbing current is taken to be the same as that for the full current, then this ratio

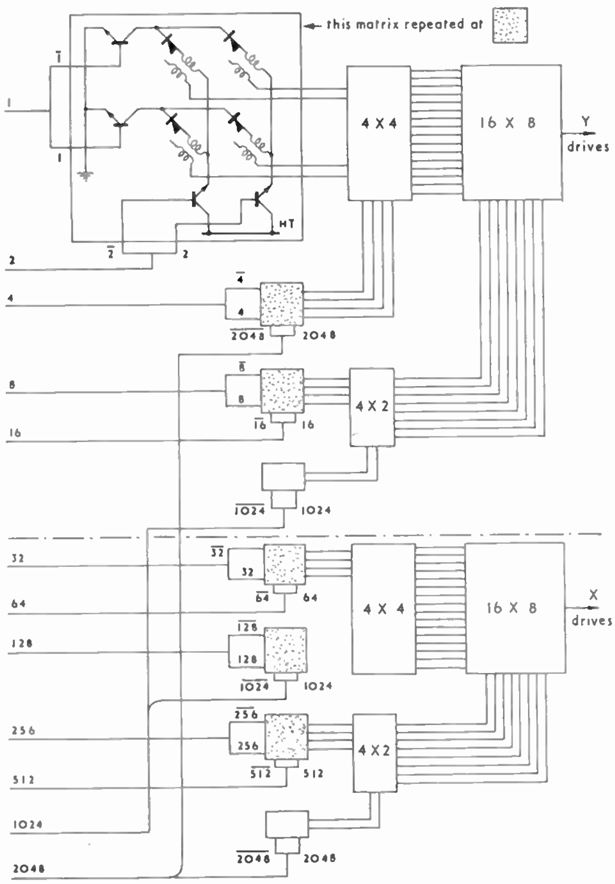


Fig. 5. Decoding scheme for 4096-word store.

represents the ratio of a disturbing output to a full output. A further consideration which has to be taken into account is that the minimum output from a storage position under the worst conditions may be smaller than the full outputs from the locations being disturbed.

From these considerations, together with drive limitations set by the inductance of the unselected cores, the core matrix has been limited to a size of 32×32 and four such matrices are used to make a 4096 word selection system. The core inductance for a single turn is 2·4 nH. Each core is wound with four turns and the inductance of 31 unselected cores is therefore 1·2 μH. By a suitable wiring technique the air inductance of the wire is kept the same in both directions and the total inductance of each line is 2·5 μH.

The decoding circuits which precede the core selection are shown in Fig. 5. A form of diode decoding is used throughout and the method of coupling the output of one decoding stage to the controlling transistors of the next stage is as in Fig. 6. By this means the process of decoding and current amplification is carried out simultaneously.

The last stages of this decoding are two diode matrices, each 8×16, and these provide the drive currents for the four core matrices, each 32×32. The transformers which couple the last diode matrices to the core lines are specially designed to isolate the relatively large voltage swings in the selection system from the store. Without these transformers the voltage changes would cause a disturbance on the digit line

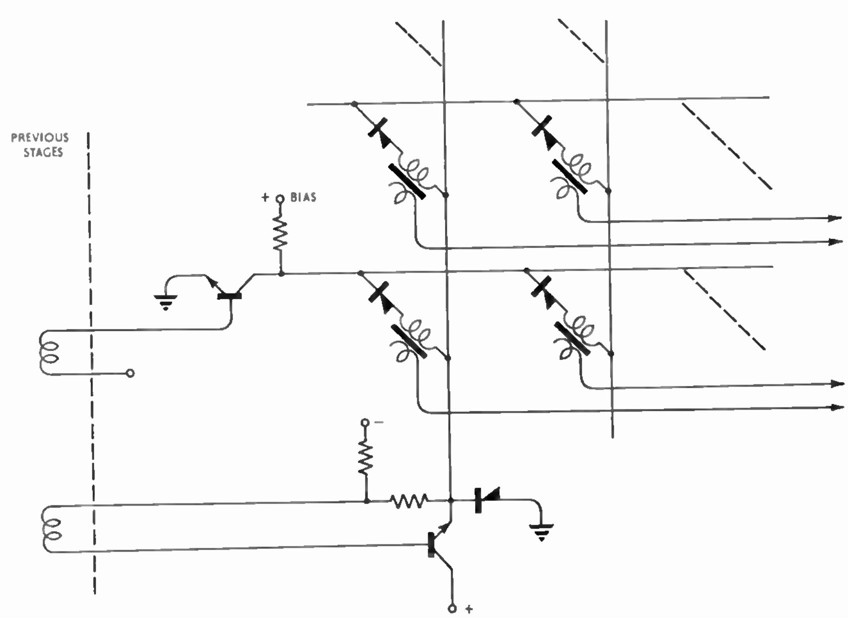


Fig. 6. Method of coupling decoding stages.

due to capacitive coupling between drive and word lines and between word and digit lines.

In the last diode matrices the transistors are arranged as blocking oscillators and these serve to hold on the drive currents for the period between reading and writing. A current-defining transistor is also included to fix the value of current through the cores.

These last matrices are arranged on printed boards with an earthed return sheet to minimize the series inductance. Even so, series losses, mainly due to the other components in the circuit, are significant and of the 50 V available from the supply only 16 V appears across the selected lines of cores. Up to the point at which the selected core starts to switch, the current rise-time is 60 ns. With the core switching there is a further loss and the total rise-time becomes 150 ns. Using better transistors and diodes it may be possible to improve this by a factor of 2.

4. Digit Lines

These can be treated as transmission lines and, if $\omega L > R$ and $\omega C > G$, then the characteristic impedance Z_0 is given by,

$$Z_0 = \sqrt{\frac{L}{C}}$$

and the attenuation in decibels as

$$A = 4{\cdot}34\left(\frac{R}{Z_0} + GZ_0\right)$$

where R, G, L and C are the resistance, conductance, inductance and capacitance of the line.

At the frequencies being considered, and with Melinex or Mylar as the insulating material, the value of G is negligible and can be ignored. The attenuation is therefore determined by the ratio R/Z_0.

In dealing with the small output signals from a large magnetic film store there is inevitably a limit to the total line attenuation that can be tolerated. The resistance of the line cannot be reduced beyond a certain point due to the skin-depth effect. It therefore follows that the only way of minimizing the attenuation of the lines is to keep their characteristic impedance reasonably high. In the case of a small store[6] with a short length of digit line, of therefore lower resistance, closer spacing, and therefore a lower value of Z_0 is permissible and, as will be shown, this can have advantage in reducing the digit current disturbance.

The above considerations have led to a store design in which the word conductors are placed nearest to the film, and the digit conductors further away separated by additional insulation. This is illustrated in Fig. 3. With this construction the plate pairs can be made and tested separately; they are then assembled

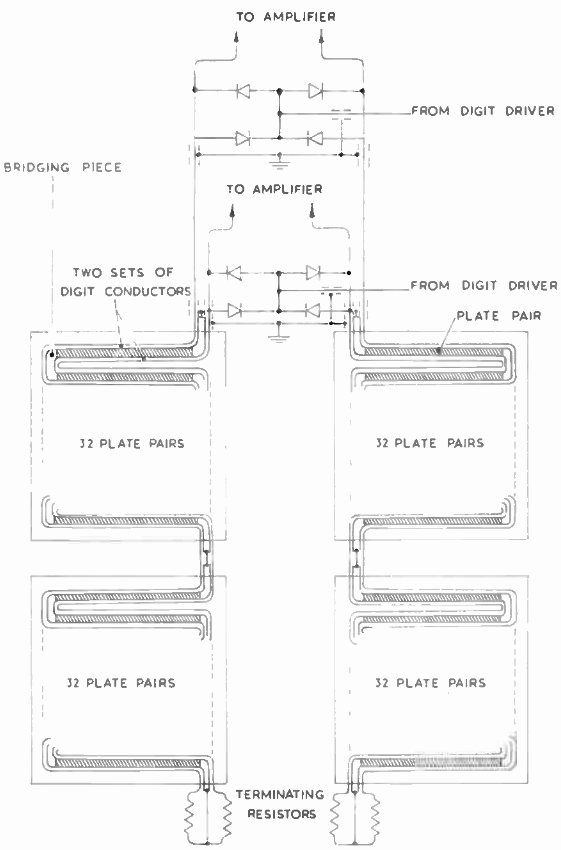


Fig. 7. Arrangement of digit lines.

into a block with a long length of digit conductor sandwiched between the plate pairs and held in position with rubber pads.

The plate pairs are arranged as blocks of 32, to correspond to the size of the selection matrix, but the digit lines of the four blocks are connected together as shown in Fig. 6. This arrangement permits the use of a single set of digit conductors for both writing and reading. The plates form the return circuit for the digit lines and end pieces serve to connect them together in series.

With a digit conductor size of 0·04 in × 0·001 in and a separation to the word conductors of 0·0005 in the line impedance is 15 ohms. The length for 2048 words is 30 ft and this introduces a delay of 75 ns between the furthest address and the digit circuits. With the circuit as shown the maximum attenuation of the read-out signal is less than 2 dB.

A further consideration in dealing with the digit lines arises from the need to minimize the disturbance created when the digit current pulse used for writing is removed. This disturbance sets a minimum limit to the time at which the next reading operation can take place. The method adopted, as shown in Fig. 7, has been to arrange the store as two halves of a balanced circuit. Two pairs of diodes are provided and the digit current of a given polarity divides between the two of them. The diodes have low impedance when conducting and the disturbance is thus kept to a minimum. During reading, the impedance of the diodes to the small read-out voltage is high and there is therefore no loss of signal due to their presence.

Even with this arrangement, it is still necessary to allow 400 ns from the time at which the digit drive current commences to fall to the time at which the next reading operation takes place. Part of this time is due to hole storage in the diodes concerned, but this is coincident with the lines being in a disturbed condition and this is the basic difficulty. The disturbed state arises because the voltage developed across the impedance of the lines by the digit current is the order of 10^4 times the size of the smallest signal. At this voltage level the small reactive component of the line impedance becomes important and a termination of it which avoids reflections is extremely difficult.

As a means of further minimizing this disturbing time, consideration was given to the possibility of keeping the digit current on during the reading time, and only changing its polarity, if necessary, before writing, as mentioned by Raffel.[5] However, the presence of the metal substrate has the effect of doubling the digit field under pulse conditions and to work with a possible combination of pulses and d.c. conditions would reduce the yield of satisfactory plates.

5. Timing of Store Cycle

Figure 8 shows the timing details of the store cycle and the type of waveforms involved. The reading operation is as has been described: there is an initial decoding delay followed by the drive current rise-time up to the bias current level; when the bias level is exceeded, the word current rises and the reading operation takes place. If the address selected is at the far end of the digit line, there is a delay of 75 ns before the signal reaches the amplifier.

Further time is then involved in dealing with the store output, passing it through control gates and establishing the digit currents for write-back. These control gates allow new information to be written as required. The core drive currents are removed to allow the selected core to reset, and thereby produce the word write current at a time to coincide with the digit current. There is a complication in writing back in that it is necessary to allow the word current to decay to 10 per cent of its full value before removing digit current. Because the digit current is delayed

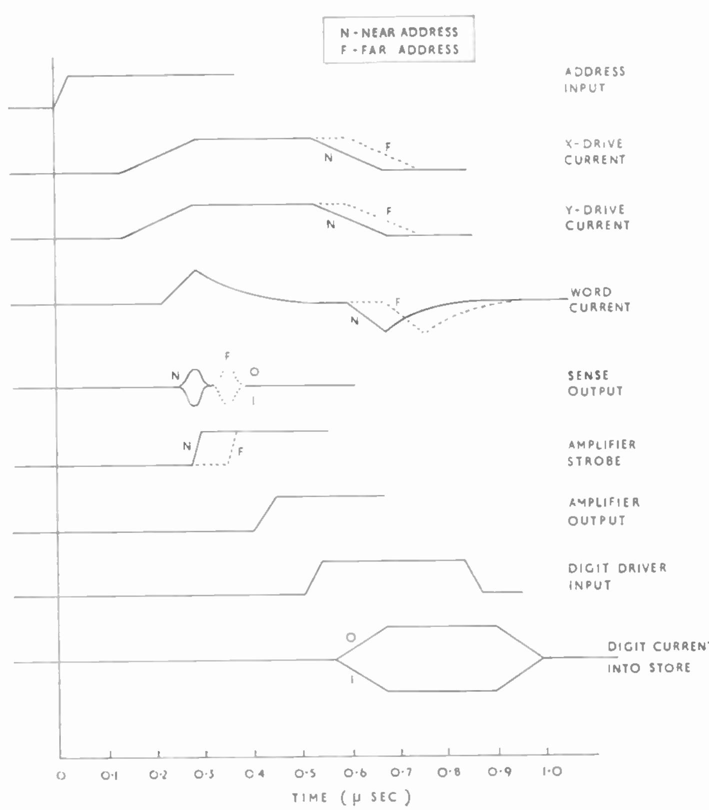


Fig. 8. Timing details of store cycle.

along the length of the line, it is an advantage to vary the writing time according to the address information and thereby secure the best relationship between the word and digit current waveforms.

The next store cycle can commence at 1·0 μs; this allows a slight overlap in that the next address is selected before the digit current has ceased. It does, however, allow the necessary 400 ns disturbing time between the time at which the digit current into the store starts to fall and the next reading time.

6. Conclusions

It is considered that the use of a metal substrate for the film has played a significant part in ensuring the success of this development. Its presence has made possible the low impedance word loop and hence the core drive with its relatively cheap selection system. It has ensured tight coupling of both word and digit lines to the film and hence minimized noise difficulties. In all it has resulted in a storage system with relatively straightforward wiring arrangements.

Possible future developments lie in the direction of further reducing the overall cost of the storage system in terms of cost per bit. In so far as the cost of the electronic circuits tends to be the dominant factor, the aim will be to make the same circuits serve a larger store. It is hoped that this can be done by increasing the packing density of the storage bits and by using a larger selection system.

7. Acknowledgments

The assistance of other members of the staff at International Computers and Tabulators and at the Hirst Research Centre of the General Electric Company Limited, in particular Mr. K. Bingham, is gratefully acknowledged.

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Techniques for the Optimization of Controlled Processes

By

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Summary: Some mathematical representations of processes are considered and a general statement made of the requirements for optimization. By assuming that the process output is a quadratic function of the variables it is shown how various approximation techniques can be adapted to sampled systems. Finally a transformation is given, which enables the past history of a process to be of utility in correcting deviations which occur in a process already optimized. It is indicated that this transformation may also be useful in designing electronic equipment for ease of operation.

1. The Mathematical Representation of a Process

For the purposes of automatic control, processes can be divided into two main classes: finite and infinite. In the former the product, P, is generated over a limited interval of time whilst, in the latter, continuous operation is envisaged. Typical examples of finite processes are the production, say, of a batch of copper nitrate from the reaction between metallic copper and nitric acid, and of a quantity of 2BA hexagon nuts from a hexagonal brass bar. Continuous processes are the manufacture of the various grades of petroleum from crude oils and the mass production of motor cars.

The mathematical representation of these two classes of process can be a general one but, for applications, is best effected in two distinct ways. For finite processes the total product, P, is related to the interactants by the expression:

$$P = \int_{t=0}^{T} \Pi(x_1, x_2, \dots x_n)\,\mathrm{d}t \qquad \dots\dots(1)$$

where x_j ($j = 1, 2, \dots n$) are the interactants and, in general,

$$x_j = \phi_j(x_1, x_2, \dots x_i, \dots x_n, t) \quad \begin{pmatrix} j = 1, 2, \dots n \\ i \neq j \end{pmatrix} \quad \dots(2)$$

T is the time of operation of the process and Π and the ϕ_j are functionals which define the particular interaction involved.

If the integral is taken in the Stieltjes sense, the expression (1) includes those discontinuous cases where the process outputs are discrete ones—such as nuts and motor cars.

The infinite, or continuous, process can be represented more simply, if it is assumed possible to include *all* of the interactants, x_j, then

$$P' = \Pi(x_1, x_2, \dots x_n) \qquad \dots\dots(3)$$

where P' is the production per unit time.

In practice drift occurs because of parameters which are not accounted for in a first order analysis of the situation: for example poisoning or long term corrosion of the equipment, scale deposition in a cooling system etc.

Effects of this type can be accounted for by writing:

$$P' = \Pi(x_1, x_2, \dots x_n, t) \qquad \dots\dots(4)$$

or, if some of the uncontrolled variables, u_i, are known, by:

$$P' = \Pi(x_1, x_2, \dots x_n, u_1, u_2, \dots u_m, t) \quad \dots\dots(5)$$

It should be noted that, just as (2) defines possible interdependencies of the variables in the finite case, so, in (3), (4) and (5), the x_j, u_i and t may not be independent.

2. The Goal of Process Control

The usual objective, claimed in discussions of process control, is "the optimization of the process" and it is worth examining what this may mean. Quite frequently the implication is simply that P or P' is made as large as possible by manipulation of the variables x_j. This definition of optimization, although satisfying to the scientist, may be less attractive to the economist who would prefer that the unit cost of the product be as small as possible.

The latter target, although superficially identical with the former, is often really quite different. Thus, if the "process" is the transport of material from London to Southampton maximum efficiency on one model might involve jet aircraft whereas an equally good result might be produced, at far lower cost, by the use of cargo boats or of barges.

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These complex questions are generally assumed to be outside the scope of an automatic control system although, in the future when general purpose digital computers may be the process monitors, the controlling machine might conceivably modify the actual process to achieve greater efficiency in the above sense.

For the purpose of this paper it will be assumed that the processes under consideration are of the continuous variety and that the optimization goal is max (P'). It will, however, be assumed that constraints on variables are possible.

3. Optimizing the Unconstrained Process

Here the problem may be mathematically formalized as:

given $$P' = \Pi(x_1, x_2, \ldots x_n, t) \qquad \ldots\ldots(6)$$

find P' max from:

$$\frac{\partial P'}{\partial x_1} = \frac{\partial P'}{\partial x_2} = \ldots \frac{\partial P'}{\partial x_n} = 0 \qquad \ldots\ldots(7)$$

Now although (6) and (7) together constitute a complete formal solution to the problem they are, in practice, quite useless. Practical methods for solving the problem posed by equations (7) depend upon the precise nature of the function Π.

For reasonably well behaved processes it may be justifiable to assume that Π is everywhere representable by a quadratic form in the x_i, but, in any case, as the optimum is approached, the expression (6) may be expanded in a convergent Taylor series about the process optimum. This means that to the second order:

$$\Pi \equiv \sum_{i,j=1}^{n} a_{ij}x_i x_j + \sum_{i=1}^{n} b_i x_i + c \qquad (a_{ij} = a_{ji}) \ldots\ldots(8)$$

Equations (7) now became:

$$\sum_{j=1}^{n} a_{ij}x_j + b_i = 0 \qquad (i = 1, 2, \ldots n) \quad \ldots\ldots(9)$$

Thus, if we write the matrix of the quadratic form as:

$$A = \begin{bmatrix} a_{11} & a_{12} & \ldots & a_{1n} \\ a_{21} & a_{22} & \ldots & a_{2n} \\ \cdot & \cdot & \cdot & \cdot \\ a_{n1} & a_{n2} & \ldots & a_{nn} \end{bmatrix}$$

the vector ξ, defining the optimum working point, is the solution of the equation:

$$A\xi = b \qquad \ldots\ldots(10)$$

which is, formally,

$$\xi = A^{-1}b \qquad \ldots\ldots(11)$$

Practical methods of optimization depend upon these relationships. It is clear that, if the coefficients a_{ij}, b_i were known, a digital computer could be used to find immediately the best operating point. In practice, these coefficients are unknown and the optimization process is one of successive trial. Some controllers work in a continuous fashion, so that the x_i can take any values subject to the overall possible precision of the system, and Π can be measured in a similar way. Recently the work of Uttley,[1] Ratz,[2] and others has suggested that a controller in which variables change by discrete amounts and in which the output is sensed in a binary fashion (i.e. "better" or "worse") may have virtue and this leads to a different method of solution.

3.1. *The Continuous Stationary Case*

If it is assumed that experiments may be made upon the process before any attempt is made to optimize it, an examination of equation (8) shows that:

$$a_{ij}\ (= a_{ji}) = \frac{1}{2}\frac{\partial^2 \Pi}{\partial x_i \partial x_j} \qquad \ldots\ldots(12)$$

Now, since Π is a quadratic form, it is a well-known theorem of the calculus of finite differences[3] that:

$$\frac{\partial^2}{\partial x_i \partial x_j} \equiv \frac{1}{\delta x_i \delta x_j}\delta_i . \delta_j$$

where δ_i is the central difference operator, defined by:

$$\delta_i f(x_i) = f(x_i + \tfrac{1}{2}\delta x_i) - f(x_i - \tfrac{1}{2}\delta x_i)$$

and, using the partial shift operator defined by:

$$E_i x_i = x_i + \delta x_i$$

so that $$\delta_i \equiv E_i^{\frac{1}{2}} - E_i^{-\frac{1}{2}}$$

$$\delta_i \delta_j \equiv (E_i^{\frac{1}{2}} - E_i^{-\frac{1}{2}})(E_j^{\frac{1}{2}} - E_j^{-\frac{1}{2}}) = E_i^{\frac{1}{2}}E_j^{\frac{1}{2}} - E_i^{\frac{1}{2}}E_j^{-\frac{1}{2}} - E_i^{-\frac{1}{2}}E_j^{\frac{1}{2}} + E_i^{-\frac{1}{2}}E_j^{-\frac{1}{2}}$$

It is easily seen that:

$$\begin{aligned} a_{ij}(= a_{ji}) &= \frac{1}{2\delta x_i \delta x_j}[\Pi(x_1, x_2, \ldots x_i + \tfrac{1}{2}\delta x_i, \ldots x_j + \tfrac{1}{2}\delta x_j, \ldots x_n) - \\ &\quad - \Pi(x_1, x_2, \ldots x_i + \tfrac{1}{2}\delta x_i, \ldots x_j - \tfrac{1}{2}\delta x_j, \ldots x_n) - \\ &\quad - \Pi(x_1, x_2, \ldots x_i - \tfrac{1}{2}\delta x_i, \ldots x_j + \tfrac{1}{2}\delta x_j, \ldots x_n) + \\ &\quad + \Pi(x_1, x_2, \ldots x_i - \tfrac{1}{2}\delta x_i, \ldots x_j - \tfrac{1}{2}\delta x_j, \ldots x_n)] \qquad \ldots\ldots(13) \end{aligned}$$

It should be noted that, since:

$$E_i^{\frac{1}{2}} . E_i^{-\frac{1}{2}} \equiv I$$

where I is the identity operator,

$$\delta_i^2 \equiv E_i - 2I + E_i^{-1}$$

and

$$\begin{aligned} a_{ii} &= \frac{1}{2(\delta x_i)^2}[\Pi(x_1, x_2, \ldots x_i + \delta x_i, \ldots x_n) - \\ &\quad - 2\Pi(x_1, x_2, \ldots x_i, \ldots x_n) + \\ &\quad + \Pi(x_1, x_2, \ldots x_i - \delta x_i, \ldots x_n)] \qquad \ldots\ldots(14) \end{aligned}$$

The determination of the a_{ij} from (13) and (14) involves performing $(2n^2+1)$ experiments on the process. The final solution of (10) involves, in addition, finding the values of the b_i. These are most easily evaluated from:

$$b_i = \frac{\partial \Pi}{\partial x_i} - 2\sum_{j=1}^{n} a_{ij}x_j \quad \ldots\ldots(15)$$

in which the values of a_{ij} are known from the previous experiments (as are the x_j) whilst:

$$\frac{\partial \Pi}{\partial x_i} \simeq \frac{\delta_i \Pi}{\delta x_i} = \frac{1}{\delta x_i}[\Pi(x_1, x_2, \ldots x_i + \tfrac{1}{2}\delta x_i, \ldots x_n) - \\ - \Pi(x_1, x_2, \ldots x_i - \tfrac{1}{2}\delta x_i, \ldots x_n)] \quad \ldots\ldots(16)$$

This involves performing a further $2n$ experiments so that, in all, $(2n^2+2n+1)$ experiments must be made on the process.

Once these experiments are made, and the a_{ij} and b_i determined, the solution of the linear simultaneous equation set, implied by (10), defines the process optimum uniquely.

In processes where only a few variables are involved this method may be quite acceptable. When, however, many variables are present the time taken for this investigation may be unacceptably long. For this reason the use of methods of successive approximation has been advocated and these have the advantage that, generally speaking, each experiment brings the process nearer to its optimum whereas the previous method does nothing to improve the situation until it is complete.

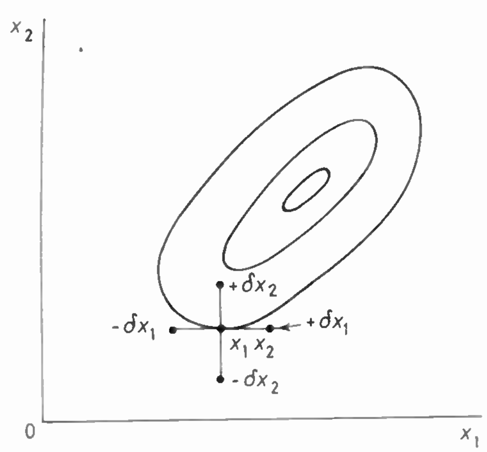

Fig. 1. Two-dimensional response surface.

Two methods of successive approximation are in common use: relaxation and steepest descent. These are most easily visualized by considering the two-dimensional process response surface shown in Fig. 1.

In the relaxation method, as modified by Thom,[4] the process is assumed to be equilibrium at (x_1, x_2), small perturbations $\pm\delta x_1$, are made in x_1, and the values of the responses, $\Pi(x_1 \pm \delta x_1, x_2)$ are found. Now on the assumption that Π is a quadratic form in the variables (x_1, x_2) it is easily shown[5] that the greatest value of Π, for variations in x_1, occurs at:

$$x_1(\max) = x_1 + \\ + \frac{\Pi(x_1-\delta x_1, x_2) - \Pi(x_1+\delta x_1, x_2)}{2[\Pi(x_1-\delta x_1, x_2) - 2\Pi(x_1, x_2) + \Pi(x_1+\delta x_1, x_2)]} \cdot \delta x_1 \quad \ldots\ldots(17)$$

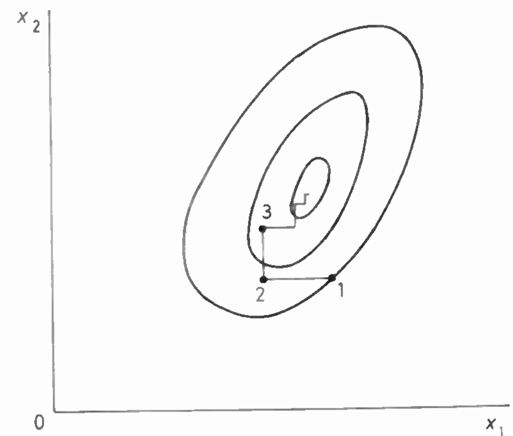

Fig. 2. Slow convergence of relaxation method.

This process is now repeated at the new operating point $(x_1(\max), x_2)$ but the next variation is made along the x_2 axis (i.e. with the variable x_2). In the two-variable case which we are using as an example, the above sequence of operations would be continued until the optimum is reached. This may take a considerable number of trials, as suggested in Fig. 2.

In some process control devices even this simple method is regarded as too complex and an even more primitive technique is adopted. The value of $\Pi(x_1+\delta x_1, x_2)$ is calculated and if

$$\Pi(x_1+\delta x_1, x_2) > \Pi(x_1, x_2)$$

the next trial point is taken to be $(x_1+\delta x_1, x_2)$; if, however,

$$\Pi(x_1+\delta x_1, x_2) < \Pi(x_1, x_2)$$

the new trial point is assumed to be $(x_1-\delta x_1, x_2)$. It is clear that this method, although involving only binary decisions, is likely to be only slowly (if at all) convergent, will not locate the process optimum to a precision which is better than the δx_i, and is likely to oscillate about this point.

The next method in order of sophistication, is that called "steepest descents".[6, 7] Here, starting from an arbitrary point $X(x_1, x_2, \ldots x_n)$, the direction in which Π increases most rapidly is found and the control variables (x_i) are changed so as to move the process

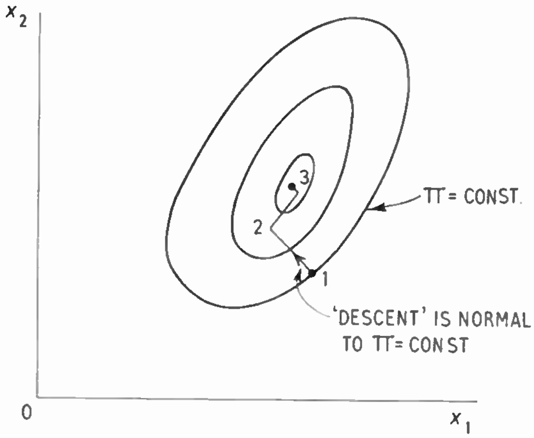

Fig. 3. Steepest "descents".

in this direction until no further increase in efficiency occurs. This is shown, in a two-dimensional case, in Fig. 3.

The direction of steepest descent (or, in this case, ascent) is simply that of the gradient of the scalar field defined by Π = constant. It is well known that the components of the gradient vector at $X(x_1, x_2, \ldots x_n)$ are simply proportional to

$$\frac{\partial\Pi}{\partial x_1}, \frac{\partial\Pi}{\partial x_2}, \cdots \frac{\partial\Pi}{\partial x_n}$$

and, if we call this gradient vector $\boldsymbol{v}$, then, progress along $\boldsymbol{v}$, in the $(x_1, x_2, \ldots x_n)$ space, will lead to a minimum.

Now:

$$\Pi(X+\alpha\boldsymbol{v}) = \Pi(X)+\alpha\sum_{r=1}^{n} v_r\frac{\partial\Pi}{\partial x_r}+$$
$$+\tfrac{1}{2}\alpha^2\sum_{r,s=1}^{n} v_r . v_s\frac{\partial^2\Pi}{\partial x_r\partial x_s}$$
$$+ \text{ terms in } \alpha^3 \text{ etc.} \quad \ldots\ldots(18)$$

where α is a scalar multiplier, and v_r is the component of $\boldsymbol{v}$ in the direction x_r. Since, in our case, Π is assumed to be a quadratic form, higher order terms vanish and the maximum of Π, for variations along $\boldsymbol{v}$, is given by

$$\frac{\partial\Pi}{\partial\alpha}=0$$

or, using (18),

$$\alpha = -\sum_{r=1}^{n}\left(\frac{\partial\Pi}{\partial x_r}\right)^2\Bigg/\sum_{r,s=1}^{n}\frac{\partial\Pi}{\partial x_r}\cdot\frac{\partial\Pi}{\partial x_s}\cdot\frac{\partial^2\Pi}{\partial x_r\partial x_s}$$

so that the *change* in any co-ordinate, x_i say, is:

$$\alpha\frac{\partial\Pi}{\partial x_i} = -\frac{\partial\Pi}{\partial x_i}\cdot\sum_{r=1}^{n}\left(\frac{\partial\Pi}{\partial x_r}\right)^2\Bigg/\sum_{r,s=1}^{n}\frac{\partial\Pi}{\partial x_r}\cdot\frac{\partial\Pi}{\partial x_s}\cdot\frac{\partial^2\Pi}{\partial x_r . \partial x_s} \quad (19)$$

To use (19) in a controller the evaluation of the derivatives would be made via a set of trial perturbations $\pm\delta x_i$ in the co-ordinates and the approximations:

$$\frac{\partial\Pi}{\partial x_i} \simeq \frac{1}{\partial x_i}[\Pi(x_1, x_2, \ldots x_i+\delta x_i, \ldots x_n)-$$
$$-\Pi(x_1, x_2, \ldots x_i, \ldots x_n)]$$

$$\frac{\partial^2\Pi}{\partial x_i^2} \simeq \frac{1}{(\partial x_i)^2}[\Pi(x_1, x_2, \ldots x_i+\delta x_i, \ldots x_n)-$$
$$-2\Pi(x_1, x_2, \ldots x_i, \ldots x_n)+$$
$$+\Pi(x_1, x_2, \ldots x_i-\delta x_i, \ldots x_n)]$$

$$\frac{\partial^2\Pi}{\partial x_i\partial x_j} \simeq \frac{1}{\partial x_i\partial x_j}[\Pi(x_1, x_2, \ldots x_i, \ldots x_j, \ldots x_n)+$$
$$+\Pi(x_1, x_2, \ldots x_i+\delta x_i, \ldots x_j+\delta x_j, \ldots x_n)-$$
$$-\Pi(x_1, x_2, \ldots x_i+\delta x_i, \ldots x_j, \ldots x_n)-$$
$$-\Pi(x_1, x_2, \ldots x_i, \ldots x_j+\delta x_j, \ldots x_n)]$$

It is clear that, for any appreciable number of variables, the work involved is prohibitive.

A simpler variant of the steepest descent process first determines the direction of descent, which involves only the determination of the n values of $\partial\Pi/\partial x_i$ $(i = 1, 2, \ldots n)$ and then values of Π at two points in addition to the starting point $X(x_1, x_2, \ldots x_n)$. The appropriate maximum on the gradient victor is then determined by the same process which was used to obtain (17); the formula is quite complex[8] and will not be reproduced here.

The disadvantage of the method of steepest descents, as with relaxation methods, is that convergence may be very slow. An attempt to remedy this by arranging to find the maximum in a sequence of two-dimensional planes so chosen as to constitute conjugate planes of the quadratic surfaces has been made.[9] This so-called "conjugate gradient" method has the disadvantage

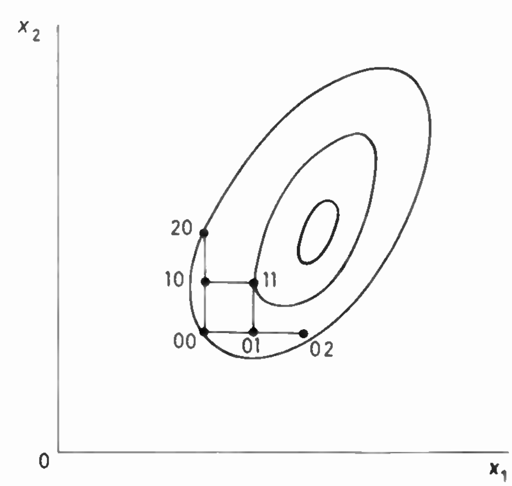

Fig. 4. Sampling points for interpolation method.

that the formulae are even more complex than those of steepest descents but, to counterbalance this, they determine the maximum in precisely n operations.

Various attempts have been made to reduce the complexity of steepest descent and conjugate gradient methods without losing their advantages. In one of these, due to the present author, the method suggested earlier, whereby three points in a line of steepest ascent were used to obtain the maximum by quadratic interpolation, is extended so that the maximum is found in a plane defined by two of the co-ordinates.

Thus, the values of the process response at the points (00) (01) (02) (10) (20) (11) shown in Fig. 4 are used to obtain, via a 2-dimensional finite difference formula, the quadratic in $(x_1 x_2)$ which represents the process. Differentiation then given the maximum without the labour of evaluating derivatives *ab initio*. This evaluation is repeated on all other planes.

A typical example of a quadratic form, so determined, is:

$$\begin{aligned}\Pi(x_1,x_2) = \Pi(0,0)+x_1(\Delta_1-\tfrac{1}{2}\Delta_1^2)\Pi+ \\ +x_2(\Delta_2-\tfrac{1}{2}\Delta_2^2)\Pi+\tfrac{1}{2}x_1^2\Delta_1^2\Pi+ \\ +\tfrac{1}{2}x_2^2\Delta_2^2\Pi+x_1x_2\Delta_1\Delta_2\Pi \quad \ldots(20)\end{aligned}$$

Here, $\Pi(00)$ is the value of Π at the point (0,0) of Fig. 4 and, in terms of the point notation shown in that figure,

$$\begin{aligned}\Delta_1\Pi &= \Pi(01)-\Pi(00)\\ \Delta_2\Pi &= \Pi(10)-\Pi(00)\\ \Delta_1^2\Pi &= \Pi(02)-2\Pi(01)+\Pi(00)\\ \Delta_2^2\Pi &= \Pi(20)-2\Pi(10)+\Pi(00)\\ \Delta_1\Delta_2\Pi &= \Pi(11)-\Pi(10)-\Pi(01)+\Pi(00)\end{aligned}$$

The maximum is located from the simultaneous equations:

$$\frac{\partial\Pi}{\partial x_1} = (\Delta_1-\tfrac{1}{2}\Delta_1^2)\Pi+x_1\Delta_1^2\Pi+x_2\Delta_1\Delta_2\Pi = 0$$

$$\frac{\partial\Pi}{\partial x_2} = (\Delta_2-\tfrac{1}{2}\Delta_2^2)\Pi+x_1\Delta_1\Delta_2\Pi+x_2\Delta_2^2\Pi = 0$$

The method is easily generalized to include as many variables as are desired and presents no difficulties if a digital, or analogue, computer is available to solve the resulting set of simultaneous equations:

$$\frac{\partial\Pi}{\partial x_i} = (\Delta_i-\tfrac{1}{2}\Delta_i^2)\Pi+x_i\Delta_i^2\Pi+\sum_{i\neq j=1}^{n} x_j\Delta_i\Delta_j\Pi = 0$$
$$(i = 1,2,\ldots n) \quad \ldots(21)$$

3.2. *Direct Search in the Continuous, Stationary, Case*

The complexity of even the simpler steepest descent-type methods has led to numerous studies aimed at finding methods of operation in which the derivatives of the optimizing function are not required. A typical example of such a method[10] is as follows (Fig. 5):

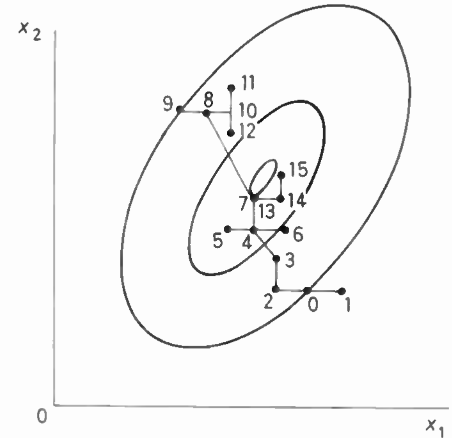

Fig. 5. "Direct search" in two dimensions.

Starting from some initial point

$$X_0 \equiv (x_{01}, x_{02}, \ldots x_{0n})$$

each variable (x_i) is changed by $(\pm\delta x_i)$ both singly and in combination with changes in the other variables. The combination of changes which produces the greatest value of Π is then used to derive a new operating point:

$$X_1 \equiv (x_{11}, x_{12}, \ldots x_{1n})$$

where $\quad x_{1i} = x_{0i} \pm 2\delta x_{0i} \qquad (i = 1,2,\ldots n)$

the $\pm$ sign being that found in the search with interval δx_{0i}.

A new search with permuted intervals $(\pm\delta x_{1i})$ is now made using X_1 as a base and, in this way a point, X_2, is found for which Π is as much greater than at X_1 as possible. From X_2 a move is now made to X_3 where

$$X_3 \equiv (x_{31}, x_{32}, \ldots x_{3n})$$

and $\quad x_{3i} = x_{2i}+(\pm\delta x_{0i}\pm\delta x_{1i}) \quad (i = 1,2,\ldots n)$

The search algorithm is thus:

(1) If the last step was one obtained by permuting changes in variables, the current step is obtained by changing each variable by an amount equal to the sum of the last two changes which it has suffered.

(2) If the last step was obtained as in (1), the new operating point is taken to be that having maximum Π from amongst the points where co-ordinates are $(x_i \pm \delta x_i)$ $(i = 1, 2, \ldots n)$.

(3) If, after a move of type (1), the value of Π is decreased, return to the start of the step and apply procedure (2).

(4) If at any stage no new point can be found in procedure (2) which has a value of Π which is greater than that at the base point, reduce the intervals δx_i to $\frac{1}{2}\delta x_i$ and repeat (2). This halving process is continued until $(1/2^r)\delta x_i$ is less than some pre-assigned lower limit or alternatively until Π is constant within acceptable limits.

The advantage of this method is that the steps can be quite large initially and that they tend, because of procedure (1), to follow directions of steepest descent.

4. The Effects of Constraints

Constraints on the process variables, x_i, may be of two types, the first equalities such as

$$g_j(x_1, x_2, \dots x_n) = 0 \qquad (j = 1, 2, \dots m)$$

and the second inequalities such as

$$g_j(x_1, x_2, \dots x_n) \lesseqgtr 0 \qquad (j = 1, 2, \dots m)$$

In the case of equality-type constraints the methods just described are adequate if

$$P' = \Pi(x_1, x_2, \dots x_n)$$

is replaced by $P'' = \Pi + \sum_{j=1}^{m} \lambda_j g_j$

where the λ_j are Lagrangian multipliers.[11]

When the constraints take the form of inequalities the method of solution depends upon the nature of the process: thus truly quadratic processes lead to simultaneous linear equations and inequalities which can be treated by the methods of linear programming[11]; whereas non-quadratic processes must be handled by introducing "slack" variables z_j defined by:

$$g_j - z_j = 0$$

and then reverting to the Lagrangian method.

5. Monte Carlo Methods

Some methods, which replace systematic searches of the types just described by a random sampling of the process response, have been derived.[12] They are particularly suitable for conditional probability computers[2] but are generally less efficient than methods in which a systematic search is made by dividing up the process control space into a network of equispaced points. It has been shown by Spang[13] that if it is desired to locate the process optimum within a hypercube of side (δx) in a space of (n) dimensions, and if the confidence level on this location is to be 0·9, then the number of sample points (p) which must be chosen at random and tested is

$$p \simeq 2{\cdot}3\left(\frac{d}{\delta x}\right)^n \qquad \dots\dots(22)$$

where d is the side of a hypercube which includes the possible values of the process variables.

It is self-evident that if we test all of the points at intervals (δx) on each control axis we shall certainly locate the process maximum in:

$$p = \left(\frac{d}{\delta x}\right)^n \qquad \dots\dots(23)$$

samples. Thus certainty is attained in less than one half of the trials required for 90% certainty on the Monte Carlo method.

An attempt has been made to improve (22) by using a sequential method[14] but this is still in a state of development in that it does not allow general estimates of error probabilities in the absence of detailed knowledge of the optimization function.

6. Non-stationary Processes

The methods just described provide a formal solution both to the problem of optimizing stationary processes and those whose optimum point changes with time. This is evident when it is recollected that any one of the variables x_i in (4) and (5) could have been re-defined to represent t after which the treatment is as before.

The disadvantage of this approach is, however, that methods such as steepest descents make no use of knowledge gained in previous optimization when re-location of the maximum is made necessary by temporal drift. To overcome this difficulty and, at the same time, to produce a more satisfactory control system for the process, the following result from the theory of quadratic forms suggests itself:

Theorem

Any quadratic form can be expressed as a sum of multiples of squares by means of a linear transformation of the variables.

For, suppose that the coefficient of a_{11} is non-zero. The form can be written

$$\begin{aligned}\Pi(x_1, x_2, \dots x_n) &= \sum_{i,j=1}^{n} a_{ij}x_i x_j + \\ &\qquad + \text{first and zero order terms} \\ &= a_{11}x_1^2 + 2\sum_{j=2}^{n} a_{1j}x_1 x_j + \\ &\qquad + \sum_{i,j=2}^{n} a_{ij}x_i x_j + \text{lower order terms}\end{aligned} \qquad \dots\dots(24)$$

Now put

$$y_1 = x_1 + \sum_{j=2}^{n} \frac{a_{1j}}{a_{11}} x_j \qquad \dots\dots(25)$$

or

$$a_{11}y_1^2 = a_{11}x_1^2 + 2\sum_{j=2}^{n} a_{1j}x_1 x_j + \frac{1}{a_{11}}\left\{\sum_{j=2}^{n} a_{1j}x_j\right\}^2$$

Equation (24) then becomes

$$\Pi(y_1, x_2, \dots x_n) = a_{11} y_1^2 - \frac{1}{a_{11}} \left\{ \sum_{j=2}^{n} a_{1j} x_j \right\}^2 + \\ + \sum_{i,j=2}^{n} a_{ij} x_i x_j + \text{etc.} \quad \dots\dots(26)$$

If now the square term in (26) is expanded, and the terms are re-grouped, we obtain

$$\Pi(y_1, x_2, \dots x_n) = a_{11} y_1^2 + \sum_{i,j=2}^{n} b_{ij} x_i x_j + \\ + \text{lower order terms}$$

The process is now repeated on x_2, x_3 and so on until finally

$$\Pi(y_1, y_2, \dots y_n) = a_{11} y_1^2 + b_{11} y_2^2 + \dots + \\ + \text{lower order terms}$$

A small difficulty may arise if, at the j'th stage say, the term in x_j^2 is absent. The procedure is then as follows:

If there exists an x_i $(i>j)$ for which the coefficient of x_i^2 is non-zero, interchange x_i and x_j and proceed as above. This continues until the stage is reached in which there are terms in $(x_k . x_l)$ but no square terms. We now put

$$x_k' = x_k + x_l, \qquad x_l' = x_k - x_l$$

so that $$\tfrac{1}{4}(x_k'^2 - x_l'^2) = x_k . x_l \quad \dots\dots(27$$

The form now contains square terms and the original method is applicable.

In this treatment the first and zero order terms have been neglected. They are easily dealt with, for suppose that

$$\Pi(y_1, y_2, \dots y_n) = a_{11} y_1^2 + a_1 y + b + \\ + \text{terms in } y_2, y_3, \dots y_n$$

Then if

$$z_1 = y_1 + \frac{1}{2} \frac{a_1}{a_{11}}$$

$$a_{11} z_1^2 = a_{11} y_1^2 + a_1 y_1 + \frac{1}{4} \frac{a_1^2}{a_{11}}$$

whence

$$\Pi(z_1, y_2, \dots y_n) = a_{11} z_1^2 + b - \frac{1}{4} \frac{a_1^2}{a_{11}} + \\ + \text{terms in } y_2, y_3, \dots y_n$$

A repetition of this process will lead, eventually, to

$$\Pi(z_1, z_2, \dots z_n) = c_{11} z_1^2 + c_{22} z_2^2 + c_{33} z_3^2 + \\ + \dots c_{nn} z_n^2 + B$$

where, since the process is assumed to have an optimum, defined in the sense (7), it follows that all c_{ii} must be negative.

The importance of this method lies in its application to processes where optima drift with time. The quadratic form for the control variables is first derived using the generalized form of (20):

$$\Pi(x_1, x_2, \dots x_n, t_0) = \Pi(0, 0, 0, \dots 0, t_0) + \\ + \sum_{i=1}^{n} x_i (\Delta_i - \tfrac{1}{2}\Delta_i^2)\Pi + \tfrac{1}{2} \sum_{i=1}^{n} x_i^2 \Delta_i^2 \Pi + \\ + \sum_{i \neq j = 1}^{n} x_i x_j \Delta_i \Delta_j \Pi \quad \dots(28)$$

Using the transformations (25) and (27) this is reduced to the canonical form

$$\Pi(y_1, y_2, \dots y_n, t_0) = \sum_{i=1}^{n} \alpha_{ii} y_i^2 + \text{terms of lower order}$$

and the optimum, at t_0, found. If now the process drifts with time, a simple variation of each of the transformed co-ordinates y_i so as to minimize Π will result in a return to optimum working.

From the viewpoint of apparatus design, the method just described implies that, by taking a set of readings of the type indicated in Fig. 4, and computing a set of transformations via (25) and (27), linear combinations of the original variables can be deduced which have the property of independence. Since a linear combination of, say, shaft rotations is easily obtained by suitable gearing, it is seen that this method allows the construction of such things as L, R, C bridges in which, instead of a complex balancing operation which involves successive approximation, a simple adjustment of independent controls produces the required balance. At least one example[15] of such a system exists, and it is hoped that the presentation of this general method of design may lead to others.

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Data Processing Centre for Space Experiments at the Radio Research Station

British equipment will be used for the first time to process the "raw" data received from the Anglo-American satellite to be launched by N.A.S.A. next year. The satellite is at present known as S52, and is now at an advanced stage of construction. The equipment, which is being manufactured in this country, will be installed and in working order by mid-July 1963 at a special data processing centre to be set up at the D.S.I.R.'s Radio Research Station, Slough.

The magnetic tapes on which is recorded the information telemetered from the satellite will be sent to Slough from receiving stations throughout the world. They will contain measurements from several experiments in the satellite recorded in a pulsed frequency modulation code together with timing information. The new processing equipment will be required to convert this information into a digital form so that it can be used as input in fast digital computers for further processing and for eventual analysis by those taking part in the experiments. The tapes will first be "edited" to select the satisfactory parts and to assess their overall usefulness. Those tapes that are found to be of adequate quality will then be passed to the main programming and digitizing part of the system where the pulsed frequency signals are separated from the background noise and recorded into a form acceptable to the digital computer.

Facilities are also provided for the experimenter to do some preliminary analysis. The processing equipment will print out data from two selected experiments and will draw graphs of up to four experiments simultaneously, while it is engaged in the main task of preparing the computer's input tape.

The system is being designed in such a way that it can be modified or extended to handle other forms of telemetry if this should be required or to produce input tapes for many of the large scientific computers expected to be in service in the near future.

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The Application of Statistical Techniques to Vibration Analysis and Testing

By

D. E. MULLINGER†

Presented at a meeting of the Electro-Acoustics Group in London on 12th April 1961.

Summary: The realization that most environmental vibration includes random motion has led to new techniques for analysis and simulation. The most important parameters of analysis are spectral density and amplitude distribution; these are defined and analysis methods indicated. Limitations of analysis due to non-stationarity and sampling errors are mentioned, and the methods of finding test levels are described. The simulation of random motion and its attendant difficulties are considered.

1. Introduction

Over the past five years or so, a major change has taken place in the philosophy applied to environmental vibration. Put simply, the modern approach recognizes the essential randomness both of the vibration and of the changes induced in the equipment subjected to this vibration. It is now generally accepted that most environmental vibration has a random character.[1] Periodic motion caused, for example, by motors may be superimposed, but there is no special reason why the aerodynamic excitation of an aircraft, or the forces set up when a car travels a bumpy road should exhibit any such periodic character. It may not always be appropriate to regard the resulting vibration as a classic case of random noise, but it is more closely related to the agitation which produces thermal noise than to simple harmonic motion.

Furthermore, the intensity of vibration on apparently similar journeys will differ slightly in a random fashion. Finally, it is important to realize that apparently identical components will behave differently in this environment, and that these differences too are mainly random. Faced with such a profusion of randomness, the engineer must adopt statistical techniques if he is not to become bogged down in a mass of confusing data.

2. Analysis

Vibration analysis is only useful if it helps in understanding the environment and enables test conditions to be specified. The methods used must be valid for random or periodic time functions, or combinations of the two. It is fairly easy to understand why a statistical method is essential when dealing with random phenomena. For example, the waveform of a random noise time function continually changes in a non-repetitive manner, and so the Fourier spectrum changes likewise. Consequently, n consecutive samples of a random noise will produce n sets of Fourier coefficients at each frequency; and these will form a random distribution which can only be described by their mean value, standard deviation etc,[2] that is, statistically. A random wave-form may be expected to produce occasional large peaks; if these are measured in isolation, the information will not be representative, but statistical methods enable averages and extremes to be properly assessed.

To describe a sinusoidal signal, amplitude and frequency are quoted; similarly a random signal (or a transient) is described by the distribution of amplitudes and the distribution of frequency components. (Phase information is usually required only when comparing signals originating in a common source.) The statistical parameters which can describe either periodic or random signals, and which are most useful for engineering purposes are:

(a) the amplitude distribution or probability density of amplitude,

(b) the spectral density.

If these vary systematically with time, the signal is said to be non-stationary and the variations must be stated for a complete description of the signal.

2.1. *Amplitude Distribution*

Amplitude distribution is the graph of the amplitude probability density, $p(y)$ defined as

$$p(y) = \left.\frac{P(y, y+\Delta y)}{\Delta y}\right|_{\Delta y \to 0} \qquad \ldots\ldots(1)$$

where $P(y, y+\Delta y)$ is the probability that the amplitude will lie within the range bounded by y and $y+\Delta y$. Alternatively, P can be regarded as the proportion of time which the amplitude spends within the given range. The amplitude distribution is really a statistical expression of the waveform of a time function, and

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provides a means of distinguishing various types of periodic or random signals, without requiring human judgement. (See Fig. 1.)

The term random noise is reserved for time functions having close approximations to a Gaussian or normal probability density. It must be understood that since this particular distribution includes all amplitudes from $-\infty$ to $+\infty$, it is not exactly realized in practice, but is merely used as a convenient model. The proportion of time which a random noise waveform spends within a given amplitude range can be found from tables of the area under the normal curve, since

$$P(y_1, y_2) = \int_{y_1}^{y_2} p(y)\,\mathrm{d}y \qquad \ldots\ldots(2)$$

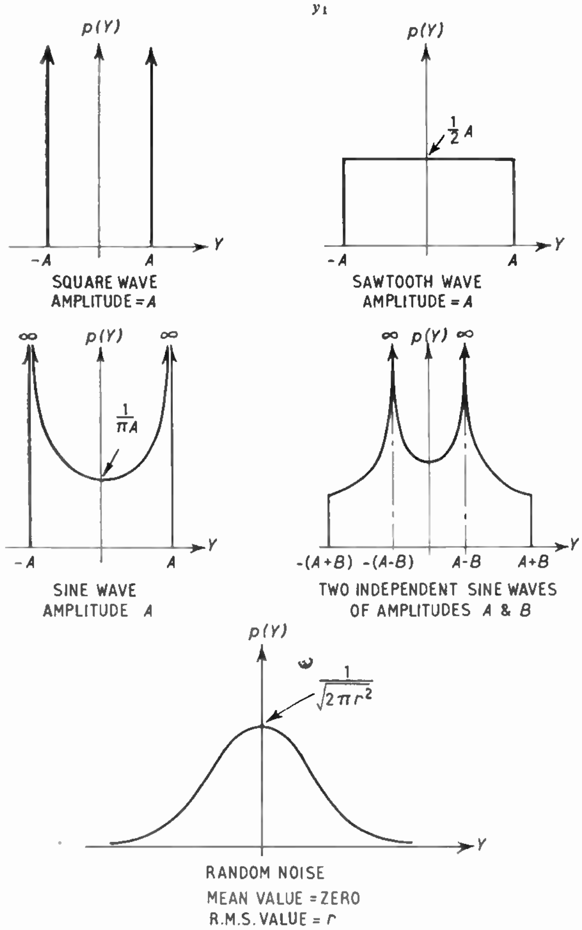

Fig. 1. Amplitude distributions of some elementary time functions.

For example, 68% of the time is spent within the amplitude range $\pm 1r$, r being the r.m.s. value of the waveform, 95% within the range $\pm 2r$, 99·7% within the range $\pm 3r$ and so on. A practical distribution which approximates to the normal up to amplitudes of ± 3 times the r.m.s. value is therefore so close to the model that it is generally regarded as random noise. In addition, at the higher amplitudes it becomes more and more difficult to be sure of deviations from the Gaussian model, since the larger values occur so infrequently.

Whilst many environmental vibration recordings appear to have some of the characteristics of random noise, their amplitude distributions may be no more than approximations to the normal. Non-Gaussian noise distributions can occur if the vibration process is not stationary, or if it is a mixture of random and periodic vibrations. In either case, the departure from a normal distribution indicates a systematic process, and the amplitude distribution can be a powerful aid in understanding the environment. Fortunately it is relatively simple to obtain experimental distributions, and several instruments[3, 4, 5] have been developed for this purpose.

2.2. *Frequency Distribution: Spectral Density*

The most useful method of describing the frequency composition of a random time function is to employ the spectral density function $w(f)$ defined as

$$w(f) = \frac{r^2_{\Delta f}}{\Delta f}_{\Delta f \to 0} \qquad \ldots\ldots(3)$$

where $r_{\Delta f}$ is the r.m.s. value of the time function within the frequency band Δf. Spectral density is sometimes referred to as the power spectrum, but this term can be misleading.

The definition shows that the spectral density of an electrical signal—and nowadays vibration is almost always measured by transducers with electrical outputs—can be obtained by applying the signal to a filter of effective bandwidth Δf, and measuring the mean square value of the output. By scanning the whole frequency range of interest, the spectral density function is determined. A practical filter cannot have an infinitesimally small bandwidth, and experimentally determined densities cannot show spectral detail finer than the resolving bandwidth; that is, practical spectra will show smoothed values of $w(f)$. However, the resolution can be made comparable to the equivalent bandwidths of the dynamic response of equipment subjected to vibration, so the smoothing is no drawback.

The usual wave analyser can be adapted for spectral density measurements, but specialized instruments are available.

A random noise which has a flat spectrum, i.e. w independent of frequency, is called white noise by analogy with the spectrum of white light. Now the total mean square value of any signal is

$$r^2 = \int_0^\infty w(f)\,\mathrm{d}f \qquad \ldots\ldots(4$$

and if the spectrum were truly white, r would be infinite. Therefore any real spectrum which appears to be white must fall off at some high frequency. Nevertheless, a random noise can often be regarded as white for a particular frequency range; that is, white noise is another convenient standard or model. The terms white noise and random noise are sometimes confused, but white noise is a special type of random noise. It is unlikely that a flat spectrum will be generally representative of vibration conditions.

The value of the spectral density function in vibration analysis is that there is no ambiguity of definition, as compared with such a parameter as the "equivalent sine wave" previously in vogue. The function can be used to describe a signal which is random, periodic or a mixture of the two. For example, the spectral density of a sine wave takes the form of an impulse function or Dirac spike at the appropriate frequency; in a practical spectrum, this is smoothed to a peak with a width equal to the analysis bandwidth.

There are alternative definitions of spectral density, which indicate other methods of analysis. The relationship with the Fourier spectrum is given by:[6]

$$w(f) = \frac{2|F(f)|^2}{T} \quad {}_{T \to \infty} \qquad \ldots\ldots(5)$$

where $F(f)$ is the Fourier integral spectrum of the time function $y(t)$ over the time T, i.e.

$$F(f) = \int_0^T y(t)\,e^{-j\omega t}\,dt \qquad \ldots\ldots(6)$$

In addition, for stationary signals, spectral density and the auto-correlation function $\psi(\tau)$ are related by Fourier transforms,[6] namely

$$w(f) = 4\int_0^\infty \psi(\tau)\cos 2\pi f\tau\,d\tau \qquad \ldots\ldots(7)$$

$$\psi(\tau) = \int_0^\infty w(f)\cos 2\pi f\tau\,df \qquad \ldots\ldots(8)$$

where

$$\psi(\tau) = \lim_{T\to\infty}\frac{1}{T}\int_0^T y(t)\,y(t+\tau)\,dt \qquad \ldots\ldots(9)$$

These relationships enable spectral density to be obtained by digital computer methods. However, when the vibration record is in analogue form, analogue computation (by filtering) often produces quicker results.

The auto-correlation function adequately describes a time-varying signal, but the interpretation of complex signals in the time domain is difficult for the engineer accustomed to work with frequency responses. It is usual therefore to transform auto-correlation results to the frequency domain, but unless the original data have been suitably modified, errors will be made in the process.[9]

2.3. *Sampling Errors*

When recording vibration it is common practice to sample the output of several transducers. Sample measurements at any one point are then used to estimate the conditions existing throughout the total recording time. Clearly, very short samples will not be representative, but it is not usually realized that the accuracy with which the spectral conditions are estimated depends also upon the resolving bandwidth used in analysis.

The error of an estimate is largely dependent upon the dimensionless quantity[7]

$$2\pi\Delta f T$$

where Δf is the resolving bandwidth, and T is the duration of the sample. This quantity is of course a measure of the information contained in the analysis sample.

In the case of stationary random noise analysed by a single degree of freedom filter (such as a resonant circuit) the standard error of the estimated spectral density is[8]

$$S_e \simeq \frac{1}{\sqrt{2T\Delta f}} \qquad \ldots\ldots(10)$$

and the same accuracy can be obtained by a long sample and narrow band or short sample and wide band. Since the resolving bandwidth should be related to the dynamic bandwidths of the equipment subjected to vibration, such an expression enables a minimum sampling time to be determined. Sometimes the sampling time must be decided by considerations other than those of analysis, and then the analysing bandwidth may not be narrowed indefinitely.

A similar effect occurs when estimating the r.m.s. level of a complex signal from a sample; the resolving bandwidth is now that of the whole spectrum, and the representativeness of the sample depends upon the type of signal. It is interesting to note that for the case of white noise filtered by an integrating RC network (quite a good model for many smooth spectra), eqn. (10) gives the standard error of the mean square value of the sample.

2.4. *Non-stationary Signals*

The mathematical techniques underlying the analysis of random functions usually postulate uniform, infinitely continuing and reproducible processes. Vibration recordings hardly fit these conditions; never infinitely long, not necessarily statistically reproducible, they are often non-uniform. But finite data can be used to estimate the properties of an infinite model, and groups of recordings may be divided into classes which are related. It is the requirement of a stationary process which disturbs most engineers. How can a vibration be satisfactorily analysed when it is not uniform?

By a stationary process is meant one in which the statistical properties do not vary with time. A non-stationary recording, then, is one in which there is a systematic change, linked with some physical change, and it is permissible to divide the data into phases within which the vibration is approximately stationary, and to analyse these separately.

The situation is similar to that existing in the electronic world, where most theory and practice depends on the notion of sinusoidal waves persisting for an infinite time. Oscillators certainly do not generate such signals, yet engineers continue to use them with perfectly satisfactory results, because the output of an oscillator so closely resembles the fictitious time function, during the period of its use.

It follows that having recognized the main sections of a vibration recording, the apparently uniform portions should be examined to see how closely they approximate a stationary process. This is done by sub-dividing and analysing each sub-sample. The results will differ if the vibration is complex, but if the variations are no greater than would be expected from sampling a uniform process, an assumption of regularity is justified.

There may be cases where an obviously non-stationary vibration cannot be ignored, and a full study would raise difficult mathematical problems. Possibly the best approach is to analyse as for a stationary process, but to treat the results only as a rough estimate of conditions. This will be of some use when determining test levels.

3. Analysis Techniques

The time needed for any analysis is many times greater than the duration of the vibration sample. It is usual to record the vibration on magnetic tape, which can be formed into a loop for continuous replay and analysis. The next step is to examine the changes in signal level, and so identify the main sections of the recording. An r.m.s. meter with short time-constant can be used; the variations of r.m.s. level will indicate whether these sections are stationary, although until experience is gained, it is safer to compare the spectra of sub-samples.

3.1. *Spectral Density Analysis*

3.1.1. Filter methods

Equation (3) can be re-written as

$$w(f) = \frac{\frac{1}{T}\int_0^T y_{\Delta f}^2 \, \mathrm{d}t}{\Delta f}$$

where $y_{\Delta f}$ is that part of the time function $y(t)$ in the band Δf. This shows that the operations required to determine a smoothed spectral density by filter methods are, for a given bandwidth, filtering, squaring and averaging. The last two may be carried out by a thermal instrument but since this will have its own time constant, it is preferable to average separately by integrating and then dividing by time.

The final output can be made to drive an XY pen recorder, which if linked with the frequency scanning mechanism, will plot a vibration spectrum.

Earlier it was said that the resolving bandwidth should be comparable with the bandwidth of dynamic responses. In most mechanical equipment the dynamic behaviour is dominated by resonances, and these determine the effective bandwidth of response. These resonances can generally be approximated by single degree of freedom responses, having Q values, over the frequency range 20 c/s to 3 kc/s, of

$$Q = kf^{\frac{1}{3}} \qquad \ldots\ldots(11)$$

where k is normally distributed with mean 2·8 and standard deviation 0·92. Now the effective bandwidth of any filter is the equivalent rectangular bandwidth, defined as the bandwidth of a hypothetical filter of rectangular bandpass characteristic having the same response at the centre frequency as the practical filter, and having the same area under the (response)^2 curve. For single degree of freedom systems the effective bandwidth is $\pi/2$ times the -3 dB bandwidth, i.e.

$$\Delta f = \frac{\pi}{2}\frac{f_c}{Q} \qquad \ldots\ldots(12)$$

where f_c is the centre frequency. It follows that the most probable effective bandwidth is

$$\Delta f \simeq 0{\cdot}6 f^{2/3} \qquad \ldots\ldots(13)$$

and this is a good guide for the choice of resolving bandwidth.

For ease of computation a bandpass filter with rectangular response characteristic would seem to be ideal but since the mechanical resonances are approximately "single degree of freedom", an electrical filter with a resonant characteristic is permissible, and this is easier to obtain than a rectangular one. Particularly stable filters can be built using an amplifier with feedback via a parallel T network; these have the advantage that the Q value is easily adjusted.

3.1.2. Digital computation

Digital computation of spectral density is based on the relationship with the autocorrelation function of equations (7) and (8). The vibration record is digitized by sampling at regular intervals Δt, given by

$$\Delta t \leqslant \frac{1}{2f_{\max}}$$

where f_{max} is the maximum frequency of analysis. The autocorrelation function is computed for a range of time lag τ, using a suitable weighting function, and is then transformed to a spectrum.

Some preliminaries are necessary. If frequencies higher than f_{max} are present in the recording they are not discarded in the analysis, but their content is added to sub-harmonics in the analysis range. This is known as fold-back or aliasing,[9] and to minimize errors, the original data must be sharply cut off at the upper limit of analysis by a filter before digitizing. Also, if a periodicity is present, there will be difficulties in transforming the autocorrelation function (although the type of function will enable the periodicity to be recognized) and filtering may again be needed.

Digital methods are not necessarily more accurate than analogue computation; sampling errors still exist, and the effective resolution is not $1/\Delta t$ as might be supposed, but depends upon the weighting factor. Unless computers are easily available, the filtering method is generally cheaper and more direct.

3.2. *Amplitude Distribution Analysis*

An amplitude distribution can be obtained graphically by dividing an oscilloscope recording into a number of amplitude ranges, and measuring the proportion of time which the waveform spends within each band. However, this is a tedious process, and the resulting histogram is rather crude (Fig. 2).

Electronic techniques permit better accuracy and resolution. One of the simplest methods utilizes the display of the waveform on a cathode-ray tube but without time-base or X shift. The cathode ray tube is viewed through a narrow slit (representing the amplitude band Δy of eqn.(1)) and the amount of time spent within this band is computed. There are several ways of doing this, e.g. a high frequency oscillator can be triggered on whenever the light spot is within the slit, and vice versa; the total number of oscillations is a measure of the time. Alternatively the pulses produced by a viewing photocell as the light spot enters and leaves the slit can be shaped to uniform height, then integrated, the integrator output being proportional to the time. By adding a Y shift

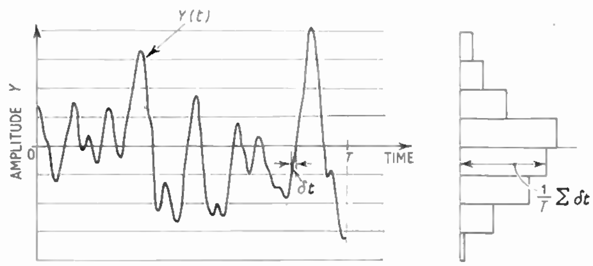

Fig. 2. Graphical determination of amplitude distribution.

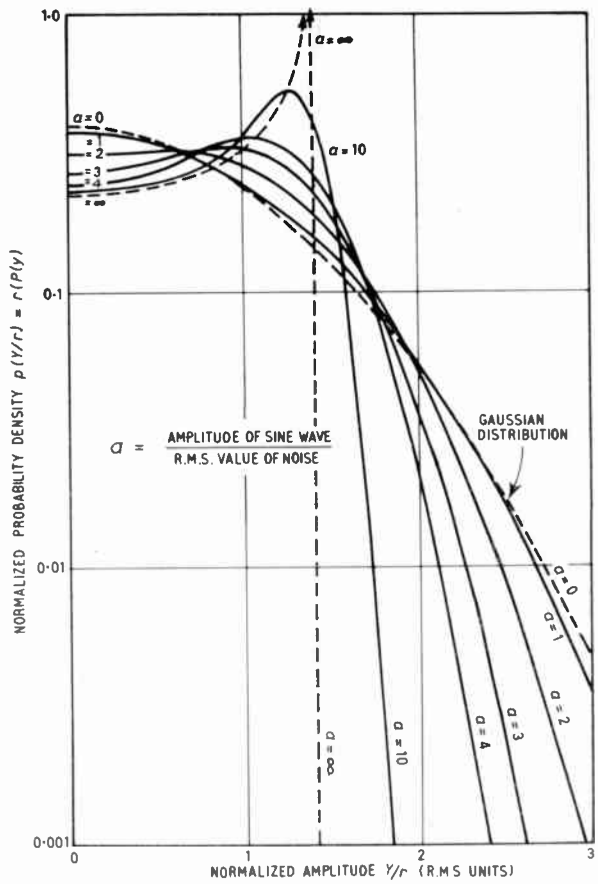

Fig. 3. Probability density of sine wave plus random noise. Based on Rice.[10]

voltage, the waveform can be moved past the slit, and the whole amplitude range can be scanned. An XY recorder can be linked with the scanning voltage, so as to plot the relative time spent in each band, and a complete amplitude distribution can be built up.

In any experimentally obtained distribution, the scanning band will not be infinitesimally small as required by eqn. (1) and so the distribution is blurred or smoothed just as is a practical frequency spectrum. For most purposes the resolution given by a scanning range of 1% of the maximum peak to peak excursion is satisfactory.

When a distribution has been obtained it may not be easily identified. Fortunately this type of analysis is self-diagnostic, and obvious faults in the data processing, due to signal limiting or asymmetrical transmission, are easily discovered. If non-stationary effects have been minimized it is probable that an unfamiliar distribution results from the combination of different types of waveform.

A mixture of random noise and a sinusoid, for instance, gives rise to distributions such as shown in Fig. 3. Knowledge of this characteristic[10] permits one to discriminate between peaky spectra caused by resonant magnification of noise, and those resulting from mains frequency pick-up during recording. When there is only a small periodic content, the amplitude distribution of the whole waveform is not a sensitive indicator of the periodicity, and to improve the indication, the proportion of noise can be reduced by filtering, based on a preliminary spectral analysis.

4. Determination of Test Levels

Test conditions are derived from trials results, and it is essential that several measurements at each monitoring point should be available so that the variations between trials can be properly assessed. Ideally, the distribution of the variations should be found so that an upper limit of the spectra can be obtained which will be exceeded only on rare occasions (perhaps 5% of the total). Where measurements are few, however, it is customary to use the envelope of all available spectra, increased by a suitable safety factor, as the upper limit for the particular monitoring point.

If a dozen or so measurements are possible the spectral limit will be relatively smooth, individual peculiarities being averaged out. It is more usual to find that insufficient effort has been allotted to the vibration trial, and the enveloping spectrum is too peaky for easy reproduction. Further smoothing is then applied according to judgement. This is a risky process which is positively dangerous if it consists of drawing a set of straight lines. If graphical smoothing must be carried out, it should not remove significant detail such as large peaks or troughs merely to present an attractive spectrum. Over-smoothing will lead to difficulties during test, such as an attempt to force equipment to vibrate strongly at an anti-resonance.

With a random motion environment, a spectral limit properly obtained can be used directly for random vibration tests. However if sweep frequency testing is required, an "equivalent" sine wave amplitude must be found. The equivalence will be limited, but if the criterion employed is that of the maximum inertia force or stress level, the effects of resonances will be dominant. Using the Q value of eqn. (11) and taking the lower 95% confidence limit of Q in order to minimize undertesting, the equivalent sine wave amplitude becomes

$$a \simeq 4[w(f)f^{2/3}]^{\frac{1}{2}} \qquad \ldots\ldots(14)$$

where if w is an acceleration density, a is the amplitude of acceleration.

This amplitude can be thought to correspond to the vibration level at the appropriate frequency which is exceeded only about $\frac{1}{4}$% of the total time by the random motion.

5. Test Methods

5.1. *General Techniques*

The means of producing sinusoidal vibration are so well known as to need no description. It is sufficient to say that for equipment testing, linear vibration in the range 30 c/s to 5 kc/s is usual, for which purpose the electro-magnetic vibrator has no serious rival. The techniques of providing random motion, however, are still somewhat novel and are considered more fully below.

Random vibration is needed when the full effects of a random motion environment are to be taken into account, as when checking the functioning of complex equipment. On the other hand, diagnosis is best performed under the more restricted conditions of sinusoidal excitation. There are some cases where there is no choice; limitations of equipment may preclude random motion testing, while a device having a life shorter than the duration of a complete frequency sweep cannot easily be tested under sinusoidal conditions.

5.2. *Simulation of Random Motion*

The method of producing linear random vibration is essentially the same as that for sinusoidal motion. The output of a signal generator is applied to a power amplifier which drives the vibrator. In this case the signal generator is normally a white noise generator followed by spectrum shaping networks.

5.2.1. Equalization

There is no inherent difficulty in this arrangement for if a vibration system has a frequency response which is flat within the working range, an input signal spectrum will be transformed into a corresponding vibration spectrum. This desirable state of affairs does not often occur, due to resonances and other limitations of the vibrator, and additional electrical networks are required to compensate for deviations

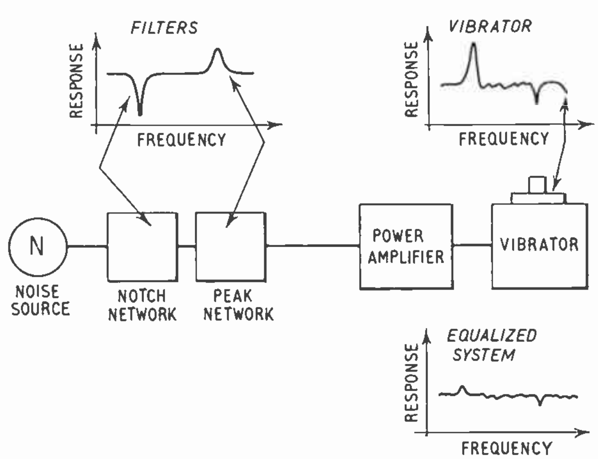

Fig. 4. Equalization by peak and notch filters.

of the frequency response from the ideal. There are two main methods of equalization. The first, variously referred to as the analogue or peak-notch method, compensates by networks which produce approximately the inverse characteristic of the deviations. Each major peak, notch or other large variation requires a separate network, which may be passive or active. The advantage of the method is that the networks can be extremely simple. (Fig. 4.)

The other system divides the working frequency range into a number of bands. By applying the input noise signal to a large number of variable gain bandpass filters, the outputs of which are subsequently combined, the level in each band can be adjusted until the required response is obtained. This method appears more flexible but quite narrow bandwidth filters are required if high Q resonances are to be dealt with. In addition the filter responses must be very steep sided to minimize interaction. Not only does interaction complicate setting-up, but also coherence effects will cause de-randomization in the interacting zones.

Both methods result in a tedious trial and error procedure when setting up. The multiple bandpass system however has the great advantage of permitting automatic operation, when a feedback loop is provided for each band (Fig. 5).

The tolerance on the flatness of the equalized response need not be simply $\pm x$ dB; it is more logical to require the mean square response within any bandwidth as given by eqn. (13) to be within so many per cent of the overall mean square response.

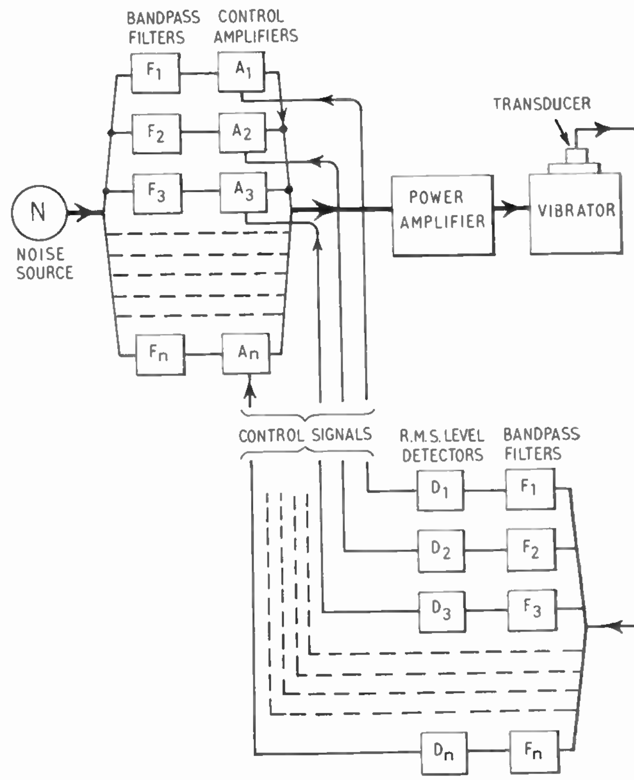

Fig. 5. Automatic equalization by multiple bandpass filters.

5.2.2. Spectrum shaping

Test spectra obtained by the method of Section 4 will, because of averaging effects, be smoother than the spectra of individual measurements. This simplifies spectrum shaping. Since a white noise generator produces signals with flat spectra, shaping networks should agree in their mean square response with the spectral density curve required on test. When this falls or rises smoothly with frequency, simple RC networks provide a means of shaping; for curves with gradients of less than 6 dB per octave, RC step networks[11] in cascade are the solution.

Using various collections of these networks, suitably buffered, it is possible to build up the broad outline of the test spectra, local peaks or troughs being added by resonant circuits.

The restriction of the frequency contents of the vibration is also a part of spectrum shaping; quite conventional high-pass and low-pass filters suffice. Since the functions of spectrum shaping and equalizing are quite independent, the networks performing these duties should also be separate. If they are combined, the dynamic range for equalization will be reduced. Normally only one or two spectra are required, whereas the vibrator has to be equalized after each change of load.

5.2.3 Power amplifiers.

Much has been written of the large amplifiers needed to drive random noise vibrators. It is usual to require the vibrator to be able to produce an amplitude distribution of acceleration which is Gaussian up to ± 3 times the r.m.s. value. This means that instantaneous peaks of current of 3 times r.m.s. must be provided by the drive amplifier and the product of peak voltage and peak current will be 9 times the continuous VA value. For sinusoidal motion, the corresponding ratio is only 2, and it appears that to drive the same vibrator, a random motion system requires an amplifier rated at $4\frac{1}{2}$ times the power of the corresponding sinusoidal drive amplifier.

This is an oversimplification. The continuous power input to a vibrator is governed by the thermal rating, and must be the same in both cases. What is required for random motion is an amplifier with the ability to provide occasional peaks of current and voltage which are large in comparison with the maximum continuous condition, and the rating must be assessed statistically.[12]

6. Conclusions

New parameters and techniques are coming into use for analysis and simulation of the random motion which is an inevitable part of most environmental vibration. Very little statistical knowledge is needed to understand the terms and methods in use, but it must be remembered that vibration problems are, in many cases, statistical problems.

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DISCUSSION

Under the Chairmanship of Dr. J. W. R. Griffiths

Mr. R. Wroe referred to Mr. Mullinger's assertion that the response (for equalization purposes) of a vibration system was largely determined by vibrator response. He asked if Mr. Mullinger considered this to be true when the mass of the test object was large compared to that of the vibrator moving parts.

He then commented on the phenomenon described by Mr. Mullinger when the spectral density plot exhibited a deep "notch" indicating the presence of a node at or near the reference accelerometer. Did Mr. Mullinger consider that a second reference accelerometer could be employed to avoid the over-testing which would occur by straightforward equalization at the "notch"; or whether moving-coil current should be the reference.

Mr. D. E. Mullinger (*in reply*): If the test specification is representative of the environment, and the vibration is measured at the same position as in the trials, the only remaining source of discrepancy is the exciting equipment, which may introduce its own resonances, or cause mismatching due to the source impedance differing from that in the real environment. Equalization is an attempt to minimize these effects, and although source impedance becomes more important as the test mass is increased, this is a question of degree.

There is little to be gained in routine tests by observing the vibration at more points than were measured during trials. A peak or notch may only be equalized if analysis shows the environmental spectrum to be smooth in that frequency region. If a test specimen is difficult to excite because of resonance it is likely to be just as obdurate in its real environment. Over-testing usually comes about by trying to vibrate a system to an arbitrary test specification, and the importance of field measurements cannot be over emphasized.

Mr. P. J. Hurst: Mr. Mullinger pointed out that the bandwidth of random motion analysis equipment should be of the same order as that of the resonant mechanical structure itself. Is it not true that equalization filter bandwidths are similarly determined, and that a high degree of precision in the elimination of peaks and notches is therefore not demanded?

The coherence problem manifests itself in sinusoidal operation as a dip in the frequency response. Is not this avoided by suitable phasing of the components of the filter?

A vibration testing method has been proposed in which the centre frequency of a narrow band of noise is swept through the relevant frequency spectrum. Has Mr. Mullinger any views on this method?

The author (*in reply*): I agree with the first statement. As regards the coherence problem, the statement is correct in principle, but the difficulty is to obtain suitable phasing since the derandomization only disappears when the interacting signals are in phase or anti-phase, and the phase relationship may be a consequence of a desired frequency response. (See also Taylor below.)

The swept narrow-band noise method seems to have all the disadvantages of sweep frequency testing, with none of the realism of wide band excitation. Its appeal lies in the simplicity of control, a single servo loop, but if the specimen can be satisfactorily excited by a narrow noise band, similar results would probably be obtained with a single frequency sweep.

Mr. W. T. Kirkby: Many environmental test engineers have facilities for carrying out vibration tests using sinusoidal excitation but, because of the costs involved, there has been reluctance in some quarters to change to random noise testing. I believe that Mr. Mullinger has fairly recently carried out comparative endurance tests on a sample batch of small components, using first of all random vibration, and subsequently subjecting a second nominally identical batch to endurance tests based on the so-called "equivalent sine wave". Would Mr. Mullinger please tell us the outcome of these tests and also give us his general views on the necessity for random vibration testing, as opposed to sinusoidal testing?

The author (*in reply*): Comparison of sweep frequency and random noise tests on small components has shown that component behaviour varies as much as the nature of the items. When the mode of failure is simple—perhaps a single resonance—failures occur more frequently with the random excitation because the failure frequency is always present. Usually, however, failures are more complex; for example a relay may have several resonant modes of the contact blades, and under random motion these may exist simultaneously, with the result that separation of contacts will occur at a much lower vibration level than in sinusoidal testing. On the other hand, gyroscopes behave in a totally un-typical manner when given sinusoidal vibration, and this method of test is not normally used to determine correct operation. Their behaviour under random excitation is more closely related to operational experience. These differences are justification for the simultaneous existence of two methods of testing—sinusoidal mainly for diagnosis, random motion for checking correct operation.

Mr. D. R. B. Webb: In the latter part of your paper, it is stated that where there was difficulty in providing for a facility which would generate vibration having a wide frequency spectrum, the difficulty could be overcome by dividing the spectrum into narrower bands and testing successively until the whole spectrum was covered. In my experience, in the structural field, this process is likely to lead to complications when computing the cumulative damage caused by the additive effect of successively applying a series of narrow bandwidths. We know for instance that when Miner's cumulative damage law is applied to random fatigue testing, very large scatter can be observed when testing results are correlated with sine wave testing. Could you give an indication of any difficulties you may have experienced in this direction?

The author (*in reply*): For random vibration to be effective, the excitation bandwidth must be much greater than that of individual dynamic responses. In testing small or medium size assemblies it would seem reasonable to work with a bandwidth of 1000 c/s or thereabouts; some overlap would be needed if the spectrum is divided. Difficulties in correlating sinusoidal and random test results will arise if there are several modes of possible failure as mentioned when replying to Mr. Kirkby.

Mr. D. M. Gore: Why are you worried by certain equalization networks causing "coherence", which you say leads to de-randomization of your signal; when surely the environment which we are trying to simulate is in de-randomized form by the time that it reaches the mounting points of the parts in question due to the mechanical filtering of the structure?

The author (*in reply*): I agree that coherence exists in mechanical systems, but it is necessary to have a standard excitation of known properties and of reproducible character. One wishes to avoid an indeterminate or uncontrollable amount of coherence being introduced, which may vary from test to test according to the equalizing conditions.

Mr. W. A. Johnson: I would like to ask three questions: Firstly, we are often faced with the necessity to analyse flight records of short duration. Mr. Mullinger has mentioned that the accuracy of the analysis of such records decreases with reduction of filter bandwidth. Could he give a specimen figure for the accuracy obtainable with a given bandwidth and record length?

Secondly, could Mr. Mullinger say what evidence exists for accepting three times the r.m.s. value as being the upper limit of interest? If one considers simple fatigue it can be predicted that a substantial amount of damage will be caused by peaks above this level.

Thirdly, does Mr. Mullinger subscribe to the view that random motion testing is more expensive than sinusoidal testing? My experience suggests that vibrator power requirements are not necessarily greater. For the cost of a noise generator and very little circuit development work, one is in the random motion field and very substantial savings in expensive testing time can be gained.

The author (*in reply*): If we use equation (10), we find that for a 1-second sample of a stationary noise, analysed with a resonant circuit having an effective bandwidth of 10 c/s, the standard error of the spectral density would be about 22%. This is not an analysis inaccuracy, but indicates how well one could estimate, from the sample, the steady conditions of the continuing signal.

The probability of a noise waveform exceeding three times the r.m.s. value is very low. High amplitudes will always be more damaging, but their severity must depend on the likelihood of occurrence. By setting an amplitude limit, one takes a risk when testing, but the risk is known. There are many other greater uncertainties, and the existence of higher levels may not be shown by experimental evidence.

I heartily agree with the view that random motion testing can be commenced with very simple equipment, and have long advocated such an approach.

Mr. D. A. Drew pointed out that Mr. Mullinger had talked exclusively about environmental testing of components at pre-determined accelerations over pre-determined frequency ranges. He asked whether Mr. Mullinger had any experience of accessories for a vehicle being tested while they were subject to accelerations obtained from a tape recording of an engine or vehicle under actual conditions.

Such a test condition might be set up by measuring the accelerations of the mounting face of the accessory during engine operation and then using the same or a similar accelerometer on the mounting face of the accessory bolted to a vibration table. The table could be fed from a wide-band amplifier, the acceleration measured at the mounting flange being compared with the signal from a tape recording of the acceleration obtained on the previous engine test. The difference signal would be used to vary the loop gain and see that the acceleration spectrum applied to the accessory was the same on the table as was applied on the running engine. Mr. Drew pointed out that such a system would clearly necessitate a very small phase lag in the closed loop containing the amplifier.

The author (*in reply*): Mr. Drew's system is quite feasible, but an isolated tape recording could not be considered as typical of the environment. The main purpose of vibration analysis is to determine the typical conditions. It should be noted too that the methods of equalization are in fact attempts to produce a system with a small phase shift.

Mr. P. H. Taylor: The author's statement that a network which drives a random signal and subsequently recombines the components in a common output derandomizes the signal would appear to bar the use of bridged T configurations and indeed any filter with parallel elements in its series arm. Moreover, a feedback amplifier would seem to be unacceptable in that the input to the first grid is the vector sum of the input signal and the feedband voltage, the latter having suffered phase shift in traversing the loop. Can the author qualify his statement and free us from these very severe restrictions?

S. Goldman, in his book "Frequency Analysis, Modulation and Noise" says, on page 326, "random noise will still be random noise even after going through a linear transmission system having any selectivity characteristic whatsoever". Later on (page 334) he tells us "any two signals, or any two parts of the same signal, which have a specified relationship between their detailed values are coherent" and he goes on to say that coherence makes the process no longer random. Could Mr. Mullinger resolve this apparent contradiction?

As well as these difficulties we have a fundamental philosophical issue; a random signal contains absolutely no information, so it is difficult to understand how a linear passive network can conjure up information (derandomize) not present in the original signal. It is contrary to the laws of entropy.

The author (*in reply*): It was possible to make only a brief reference to the derandomizing caused by combining coherent, or correlated, signals. To produce this effect a network must have two independent (or virtually so) paths of differing phase or time delay characteristics, the signals from which interact when they are added. Such a network will introduce a systematic effect into a noise signal, and this is a de-randomizing influence. An example is that of noise fed to two circuits, one of which transmits the signal directly, the other delays it by a fixed interval. The combined output will include a large number of twin events, separated by the time delay, and this signal will not be random.

If a network can be reduced to an equivalent T or π section, there is only one effective path, and the question of coherence does not arise. Fortunately most networks are of this type, including parallel-T sections and feedback amplifiers, and it is only a small minority, required for special frequency characteristics, which give rise to coherent effects. It is true that some feedback amplifiers may not be reducible to a ladder network, but such an arrangement will usually be unobjectionable in a wide band amplifier, since phase differences between the parallel paths will only be significant at the extremes of the passband where the signal content will be low.

This subject leads to some fascinating philosophical questions, which cannot be dealt with here. However one might suggest that any patterning introduced by a coherent process is characteristic of that process and such information as then exists tells us only of its operation.

Mr. F. J. Fahy: Can Mr. Mullinger justify the use of the term "frequency" when applied to the conception of "completely" random noise and if so what is his definition of frequency? If random noise can be well approximated by superimposition of five sine waves, cannot "shake" testing be done with five oscillators and a single vibrator?

How can a phase change through resonance of the mechanical system be compensated in the peak-notch system?

How does the amount of damping extent in the system being tested affect the case of a random input shake test?

The author (*in reply*): The term "frequency" refers to the frequency of the continuous sine waves of which we visualize the random waveform to be composed. This is justified by Fourier analysis.

Five or more equal amplitude independent oscillators have a combined amplitude distribution which approaches the Gaussian, but only for the whole frequency range; within the bandwidth of a mechanical resonance, the input would be sensibly sinusoidal. It is unlikely that a small number of oscillators would excite all possible modes of vibration of a system and a random noise generator is cheaper and easier to operate.

Since the peak-notch system equalizes with an approximately inverse characteristic, both in amplitude and phase, the phase variation at resonance will be approximately neutralized.

One of the advantages of random motion testing is that the excitation is independent of the damping in the test specimen, although the severity of the vibration depends very much upon the frequency response of the test object.

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On the Improvement of Detection and Precision Capabilities of Sonar Systems

By

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Summary: The paper surveys recent advances in signal processing techniques in sonar, indicating trends in the use of diversified types of pulses, correlation and multiplication techniques and instrumental evaluation of signals.

1. Introduction

Great advances have been made in recent years in signal processing techniques and these improvements will certainly make their impact felt also in the design of sonar equipment. It seems not out of place in a Symposium dedicated to active sonar to review briefly the present generation of sonar equipments and the possible directions of improvements. This review cannot by any means be complete, as it is based only on details of developments which have appeared in the current technical literature, and on advances in allied techniques such as radar and computers.

2. Present Equipment

The currently employed types of sonar which will be discussed are mainly medium-distance type using direct ray propagation and frequencies from 8 to 20 kc/s.[1, 2] Although their range is obviously limited, their reduced size and power consumption make them useful for installation on small craft and it is likely that their use will be continued. Such equipments employ two separate treatments of the signals obtained from the transducers, one for searchlight and one for scanning use. In the searchlight treatment signals from the transducers are phased by rotation of the transducer or by means of a phasing network which is set by the operator at any desired bearing and signals are amplified, compensated at will for own Doppler effect and then brought to the detecting device of which there are three main types.

The best and the most widely used is a loudspeaker which permits the signal to be listened to by an operator. The second is a range recorder which permits the measurement of range and carries out visual integration of successive returns. The third is a Doppler meter which displays the signal on a c.r.t. In the last the vertical shift of the spot is proportional to range, the horizontal deviation is independent of the amplitude of the signal, as this is limited, but is proportional to Doppler frequency.

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An automatic device which repeats the speed of the vessel relative to the sonar bearing permits display of absolute Doppler shift. The main lobe of the transducer is of the order of 5 to 10 deg horizontally, and the duration of the pulse can be changed within the limits of 10 to 100 ms. The receiving bandwidth is the critical bandwidth of the ear. Detection takes place aurally, discrimination against reverberation being obtained with a reduction of the pulse duration for stationary targets and with an aural assessment of Doppler and longer pulses for moving targets. Decisions are of course entrusted entirely to the operator.

As an auxiliary to the searchlight treatment of the signals, continuous scanning is carried out by means of a mechanically switched system of phasing which performs one or more complete scannings during one pulse duration. The signals are then presented as a p.p.i. presentation in range and true or relative bearing.

However the use of a scanning phasing system before detection makes it necessary that all filtering be carried out before scanning and as the filter accommodates both the frequency band of the pulse and the Doppler frequency, the signal-to-noise ratio is necessarily worse than in the case of reception by ear. The scanning processing does not discriminate between signals affected and not affected by Doppler and therefore this means of resolution is absent. In more recent equipment individual transducers are phased by means of a plurality of phase delay networks, one for each discrete direction, and the networks are switched by means of an electronic commutator. This has a greater scanning speed and the number of scans for any one bearing during pulse duration is greater than in the earlier equipment thus giving improved resolution.[2] P.p.i. presentation is not so good for early detection but gives a panoramic presentation of the scene and is particularly useful when more than one target is present.

3. Sonar Performances

The main features expected from a sonar are:

(*a*) Early detection for distant targets.

(*b*) Resolution in range, bearing and Doppler as discrimination against reverberation.

(*c*) Precision in measurement of range, bearing and Doppler for delivering exact data for operational purposes.

(*d*) High resolution in range and bearing to permit the determination of the shape of the target and its identification.

(*e*) High rate of information (scanning for all bearings at the same time).

In order to carry out the indicated operations the sonar obtains information from the target, thus establishing a two-way communication channel. Power is delivered by the sonar and the target modulates the received energy in different ways. Firstly, the target can be present or absent: when it is present the returned pulse can be considered as a carrier amplitude modulated with the pulse spectrum frequencies. Secondly, the returned pulse is delayed in time due to range and it can be said that the envelope is pulse position modulated. Thirdly, the carrier is frequency modulated by the relative speed of the target. Fourthly, the minute structure of the target produces additional amplitude, phase and frequency modulation within the pulse.

The target returns are masked by noise and by returns from other targets (crosstalk). In order to eliminate crosstalk the operator can try to establish a channel in which only the wanted target can receive and return energy by focusing energy into it alone at a given range and bearing. This is done in bearing, but is generally considered impracticable in range, due to the great dimensions necessary for range focusing.[3]

4. Early Detection

If reverberation is absent or sufficient discrimination against it is afforded by target Doppler, target detection is limited by signal/random noise ratio. This is proportional to the ratio between pulse energy and noise spectral density.[4]

The only possible way of increasing detection range is an increase in pulse energy. As duty cycles are small, high pulse energy is favoured by long pulses and convenient storage.

For a long time, capacitors have been used, but a capacitor capable of storing 1000 joules has a weight of about 100 kg. More recently, in power supply equipment for radio links, flywheels have been used for the same purpose with a great reduction in weight, as a flywheel of 20 kg weight can store easily 10 000 joules. Obviously the associated generators must be proportioned for the maximum power and have a low efficiency and an increased weight. The introduction of amplifiers using transistors now makes batteries a possible proposition. A silver-zinc battery can deliver 15 000 joules for 300 ms and a duty cycle of 1 to 100, and has a weight of about 20 kg. Even more attractive appears the possibility of storing energy in a compressed gas, as 20 kg of air at 150 atmospheres stores about 400 000 joules. This of course depends on finding means for transforming it into sound energy in air and then transmitting it into water, with a reasonable efficiency.

In order to make the most effective use of returned energy for detection the receiving bandwidth should be reduced to its optimum value. The critical bandwidth of the ear as used at present is from 80 to 100 c/s for the most favourable pitch, whilst the optimum bandwidth for one second pulse would be 1 c/s, which is very difficult to obtain with classical filters.

Filtering in the time domain by means of cross-correlation techniques is coming into use. The signal is multiplied by a local replica of it or two signals from a split array are multiplied and the output is integrated for a suitable time.[5]

5. Range Resolution

If reverberation is present or the target is stationary, signal/random noise ratio is not the only factor. Reverberation can easily mask the echo from a stationary target return and become the limiting factor. To reduce its effect range resolution is needed. It must be possible to identify signals coming back from points differing little in range. When c.w. pulses are used, targets in the same main lobe can be well separated only if their distance is greater than $r = ct/2$ where c is the velocity of sound and t the pulse duration. For good resolution t must be small. As peak power energy is limited, the pulse energy becomes small and detection range is reduced.

In order to increase the duration of the pulses and increase signal-to-noise ratio with a good signal-to-reverberation ratio many different techniques have been suggested.

The first is the use of long pulses linearly modulated in frequency as in radar. Two targets will be resolved if the difference in frequency of the produced signals can be resolved. If DF is the frequency deviation in a second and the minimum observable frequency difference is df, the minimum noticeable range separation is $(c/2).(df/DF)$ which is smaller than for c.w. pulses of the same duration, whilst the signal-to-noise ratio is not appreciably changed.[6, 7]

A second suggested type of signal is the "chirp" or compressed pulse recently introduced in radar systems.[8, 9, 10] In this new approach a long pulse is transmitted, but this is frequency-modulated linearly with time. The pulse is received by means of a dispersion delay network. This delays the high frequency components at the start of the input pulse more than the low frequency; components in between are delayed proportionally. The net result is a time compression of the pulse. As a passive delay network is assumed, the peak power of the compressed pulse is greater than the peak power of the input pulse by the ratio of the durations of the input and output. The output pulse is filtered in the usual way by a matched filter. The system bandwidth must everywhere accommodate the same signal bandwidth, regardless of the input pulse envelope time duration. Therefore the noise content of the system does not vary and any system parameter that depends on the ratio of signal energy to noise energy is the same before and after signal compression. Nothing is therefore obtained as far as signal energy levels are concerned, but what has been obtained is increased signal resolution at the price of system complexity.

These are not the only possible forms of signal. In order to obtain good range resolution it is possible to keep the target under constant interrogation, thus increasing signal-to-noise ratio as long as any one parameter of the transmitted wave is modulated with time. When the return signal is received it is cross-correlated with a replica of the transmitted signal shift by a varying time T and the cross-correlation function may vary rapidly with T giving a good range definition. To this purpose pseudo-random noise has been suggested, which is a succession of pulses of random frequency at random intervals with a wide frequency band which presents advantages in cross-correlation.

A noise-modulated range measuring system has been described in which the noise modulates the frequency of the transmitter carrier.[11] The signal received from the target is mixed with the transmitted signal, limited and detected by a linear discriminator. If the target range is zero the frequency difference between outgoing and incoming signal is obviously zero. When the target range increases the frequency of the incoming wave will have a tendency to deviate more and more from the outgoing frequency because of the random character of the modulation. By a convenient choice of parameters the averaged output of the demodulator is made nearly proportional to range.

Instruments for measuring range have been developed on the same principle as pulse duration counters and they permit very high precision if good signal-to-noise ratio is available as range error is directly proportional to the square root of the noise-to-signal ratio and to the rise-time of the signal.

6. Doppler Resolution

In c.w. pulses Doppler resolution is better for long pulses than for short pulses. On the other hand range resolution is better for short pulses than for long pulses. In frequency modulation Doppler is equivalent to a change in range. The problem of the choice of the signal to employ for best combined resolution has been examined in terms of auto-correlation functions and it has been shown that the total ambiguity, that is the uncertainty of position of the target signal in terms of range and Doppler, is the same for all types of signals. However signals may exist in which ambiguity is high only in a small domain of range and Doppler. In the surrounding region, although this is wider than in the case of c.w. or f.m. signals, ambiguity is very much lower, thus giving a particularly favourable type of signal.[12] Generally it is considered impossible to have a waveform giving best results for all situations of the target relative to Doppler and reverberation, and it is suggested that a different optimized waveform should be transmitted for each different target situation.

7. Angular Resolution

Angular resolution depends primarily on the size of array. In the sonars indicated above it is impossible to resolve two targets differing by less than 5 to 10 deg and reverberation becomes therefore an important factor in masking target returns. An increase in resolution apart from obvious means, such as increasing the size of the array, is difficult to obtain. Superdirective arrays in which the phase distribution of the transducer velocity on the array is specially tailored to obtain higher directivity do not represent a good solution.[13] Multiplicative arrays, that is arrays in which the transducer is split in half and the two voltages multiplied, give advantages in resolution relative to additive arrays.[4]

A new approach has been suggested in which a wide frequency band is used. It has been shown that if n carrier components in harmonic relation to one another are received from two receiving elements at a given distance, and the voltages multiplied and averaged, the directional pattern obtained is equal to that of a narrow band additive array of $4n$ point receivers and bearing resolution is appreciably increased.[14, 15, 29, 30] The improvement is the sum of two improvements, one given by the wider bandwidth used and the other by the multiplicative technique.

It has been customary for a long time to scan only in the horizontal plane, with a vertical lobe sufficiently wide to take into account the rolling and pitching of the ship. The need to receive signals coming from a long distance with an angle differing appreciably from the horizontal has brought back the practice of tilting the transducer or using spherical arrays.[16] This permits the use of a narrower vertical lobe which affords more protection against reverberation. A possible means of obtaining greater resolution might well be the use of volume arrays.[36]

Another important parameter in sonar efficiency is the precision of measurement of bearing. For a linear transducer split in two halves the bearing error is proportional to

$$\frac{\lambda}{d}\sqrt{\frac{N}{S}}$$

N/S is the output noise to signal power ratio, λ is the wavelength of the transmitted frequency in water and d is the semi-width of the linear transducer.[17, 18] The width of the main lobe is therefore not the only factor in obtaining good bearing accuracy as this can be improved very much with suitable instrumentation and good signal-to-noise ratio. As a means of improving accuracy, besides the well-known systems of sum and difference or of bearing deviation indicators, correlation techniques or straight phase-measuring techniques have been suggested.

8. Integration

Integration on successive returns in a searchlight sonar is carried out by means of a range recorder which is extremely effective in increasing signal-to-noise ratio.[19, 20] Integration by successive returns on a p.p.i. is not so effective as in radar because the time between returns is much greater.

Storage techniques employed in radar which sum voltages due to successive returns, are more difficult to employ in sonar because of the long delays.[21] Storage tubes do have very long "memories" but have inherent limitations due to limited range, difficulty in signal summation, etc.[22]

The problem is also more difficult in sonar because the time during which integration within a pulse or within a sector is effective is proportional to the ratio between the velocity of the energy in the medium and the radial or angular velocity of the target. This is smaller than in radar. In sonar it is necessary to have correlation between returns from adjacent pulses or sectors.

9. Identification

If the energy returned from all parts of a target is summed and treated as whole, the target can be more easily detected against noise but no distinctive features will be discovered.

For identification, range and bearing definition must be so good that the different parts of the target can be resolved. This means a resolution of less than say 10 yd in range and of less than 0·1 deg in bearing. Any improvement effective against reverberation will obviously be a step towards target resolution. Useful data about the target can also be obtained from the minute structure of the Doppler. This can be measured very accurately by means of the period meter which indicates the time elapsed between successive zeros with great precision.[23]

10. Simultaneous Scanning

In a searchlight sonar energy is sent out and received from one sector of space. A target is detected in a time which is equal to $2r/c$ and one bit of information is acquired. If all the space must be searched, transmission and reception must be carried out successively for all sectors but the information per second is still $c/2r$ bits.

This is much less than in radar and in fact far too small. In an ideal scanning sonar reception and transmission are carried out simultaneously for all sectors. In the scanning sonar described above power is transmitted in all directions but reception in each direction is not continuous and takes place once or twice for each pulse, the result being presented on a p.p.i. The bandwidth in scanning operation is greater than in searchlight operation and detection range is therefore smaller. Integration on the p.p.i. is also very much less effective than on a range recorder.

Simultaneous reception from all sectors of space is possible and produces a great amount of information. It favours the use of digital and computer techniques, which permit much simpler handling and processing of a vast number of signals. An application of those techniques to a scanning sonar is indicated in a recent paper.[24]

The signals from the transducers are limited and introduced in digital delay lines, one for each transducer, which advance each polarity or bit through the succession of stages at the clocking pulse rate. Directional beams are obtained by summing signals at different and appropriate stages in the delay lines. In a radical departure from the methods used until now, clipped voltages only are used and information in the amplitude of the signal is suppressed, only phase information being utilized. It has been shown however that the loss in signal-to-noise ratio produced by this treatment is extremely small and more than offset by operational advantages.[25] Instead of summing signals from transducers alternative techniques are used: (1) the array can be split in half and the outputs clipped, multiplied and averaged; (2) every possible pair of receiver outputs can be multiplied and the resultant products are summed and time averaged; (3) every pair of receiver outputs are multiplied together, time averaged and the outputs of the averaging circuits then multiplied together. In certain types of noise conditions the alternative treatments can be advantageous.[26]

11. Storage

Similar techniques are used for storage of information as in radar.[27] The underlying idea is to quantize the signals received, to store them in digital form, to take decisions on the strength of the quantized signals and present the final decision only, thus limiting the amount of information displayed for use by the operator.

The signal plus noise received from a given bearing as a return for a transmitted pulse is compared for a given signal level. If the received level is greater than the reference level, it is assumed that a target is present. If not, it is assumed that the target is absent. The reference signal level is not fixed, but is determined according to a given false alarm probability. A false alarm occurs when an amplitude produced by noise is assumed to be produced by a target.

In order to set the reference level correctly, the receiver input is examined during a period of time when targets are certainly absent and the level which is not overstepped more than once in a time equal, say, to 1000 pulse durations is assumed as a reference level. This gives a false alarm probability for one pulse of 10^{-3}. With this procedure, at every elementary time interval a signal is obtained, which will be $+1$ if a target has been assumed present and -1 if the target is assumed absent. These signals are stored in a magnetic drum. A second pulse is transmitted in space and the succession of returns, treated in the same way, gives rise to a second series of signals which are stored on a second track of the drum.

This goes on for a given time duration until the number N of independent signals stored is considered sufficient. The N quantized signals are then summed, taking care that no overlapping of signals takes place. The sum of a given number of N signals on an elementary time interval will be positive if the number of times in which the target has been decided present is greater than the number of times in which it considered absent. According to the level of the sum signal, a decision is taken on the final presence or absence of the target. If a positive decision is taken, the signal operates a time counter which gives the range of the target with the approximation of an elementary time interval.

This system could prove useful for early detection. It is difficult, however, to see how a sonar could dispense with a p.p.i. display for the evaluation of any tactical situation. The final solution might well be, as suggested in radar, a storage system (tube or otherwise) which stores the information and displays it at intervals on another shorter persistence tube.[28]

Auto-correlation and cross-correlation methods of filtering are becoming a very useful alternative to classical filtering with passive networks in many fields as well as in sonar. Special mention however should be made of a method of cross-correlation (the delta correlator) which carries out correlation between signals in a very effective way using digital techniques.[32]

Obtaining a cross-correlation function $C(T)$ means carrying out the operation of multiplication of two functions F_1 and F_2, one of them shifted in time T relative to the other, for all the time t during which the functions exist or are significant and integrating the resulting product, thus obtaining the function $C(T)$. The time necessary to get one point of $C(T)$ is t so unless a multiplicity of devices are employed simultaneously, the time necessary to obtain n points of $C(T)$ is at least $t \times n$, which can be very large. The delta correlator speeds up the process; it samples the functions F_1 and F_2 at given intervals and stores them at much smaller intervals in a quartz delay line. The operation of correlation is performed on the sampled function, which are time-compressed. All n points of $C(T)$ are then obtained in the time t, avoiding the necessity of a multiplicity of correlators or saving the time employed in the repetition of the operation.

12. Conclusions

Improvements in sonar can be obtained by more refined and complex signal processing techniques. Trends, as indicated by scientific literature, include:

(*a*) The use of diversified signals (frequency modulation, pseudo-random noise, etc.) in order to enhance range resolution even with long pulses;

(*b*) The use of auto- and cross-correlation techniques for filtering instead of classical filters;

(*c*) The use of clipped signals and of digital techniques for greater facility of storage and processing;

(*d*) The use of more refined instruments for greater precision in the measurement of target data;

(*e*) The use of instrumental evaluation for reaching decisions and presenting to the operator a limited amount of weighted information.

The price to be paid will be inevitably a greater circuitry complexity but is expected that the use of transistors and printed circuits will permit a reduction rather than an increase in the size and weight of the equipment.

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Manuscript received by the Institution on 18th June 1962. (Paper No. 821/SS26).

POINTS FROM THE DISCUSSION

Mr. M. Schulkin: Have you considered at what point improved signal reliability arising from sophisticated detector techniques is balanced by reduced electronic equipment reliability arising from the complex circuitry and vast numbers of components required? Modern high-speed computers usually have a "down-time" of at least an hour a day for preventive maintenance.

The author (*in reply*): I agree with you that more complex equipment is certainly likely to be less reliable. But in this case, I believe that if a simple equipment will not permit you to detect a submarine and a complex one does, we should use the complex equipment, whilst trying to make it as reliable as possible.

Mr. J. O. Ackroyd: In Section 10 the author makes a strong case for increasing the information rate, which is so slow in present-day sonars that they are overtaken by events. Does this not show a possible danger in his proposal to match the system to the characteristic of a target? While preoccupied with one (possibly imaginary) target there may be a danger of missing a real target of different characteristics.

The author (*in reply*): The danger of missing targets as pointed out by Mr. Ackroyd is quite real. In fact it would be necessary to search at the same time for targets of different characteristics by using a plurality of detectors in parallel and scanning them at high rate.

U.D.C. 621.375.3: 621.315.28

A Magnetic Amplifier for Low-level Telegraph Signals

By

L. A. RIOUAL,

(*Associate Member*)†

Summary: In the design of submerged repeaters for submarine telegraph cables, the need arises for a circuit capable of amplifying very low frequency signals with great reliability. It is highly desirable that the bulk of the apparatus be reduced to a minimum. A repeater circuit is described, in which a magnetic amplifier is used for frequency conversion of the telegraph signals.

1. Introduction

It is often necessary to lay submerged telegraph repeaters in deep sea, in order to avoid the interference which would occur in coastal areas.[1] Valve circuits do not lend themselves readily to such projects because of the high voltages which have to be applied at the cable terminal. For instance, let us assume that a valve repeater needs a d.c. supply of 300 milliamperes at 130 V (a fairly typical figure). If it is placed at the end of a 300-mile section having a resistance of 600 ohms, the voltage drop in the cable section will be 180 V. This means applying 310 V at the cable end, much too high a voltage in the case of a gutta-percha insulated cable. Furthermore, present types of submerged telegraph repeaters tend to be extremely bulky, mainly because of the size of the output transformer and interstage coupling capacitors. It is therefore all the more difficult to design a suitable pressure-resistant casing for that type of repeater.

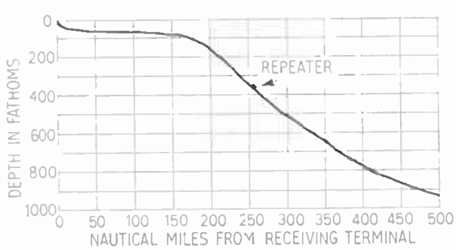

Fig. 1. Typical position of repeater on cable route.

The use of transistor circuits would remove the power supply difficulty. However, the interstage coupling capacitors would still have to be rather large. The output transformer would also remain a problem, although its windings would not have to carry as high a direct current as in the case of a valve amplifier.

A substantial reduction of the size of the output transformer and coupling capacitors would result from the use of a frequency converter. In such a circuit, two tones are transmitted from the repeater to the receiving station, each tone corresponding to a polarity of the signal at the input of the repeater. The choice of the frequencies must be governed by the attenuation over the short cable section, and by the nature of the disturbances against which one must discriminate at the receiving end.

The fact that the frequency at the output of the repeater is much higher than the signal frequency results in a corresponding decrease in the primary inductance of the output transformer, conducive to a great saving of bulk and weight.

2. Application of the Pulse Relaxation Amplifier to D.C./A.C. Conversion

The frequency converter mentioned above must be designed to supply a.c. signals whenever a signal appears at its input. It must respond to signal polarity. It is necessary to minimize the "drift", which is defined as the value of the output when there is no signal at the input. These various requirements can be met by a simple magnetic device, the pulse relaxation amplifier, a description of which has been given in an American paper.[2]

The device consists of a magnetic core made of high-permeability material, such as Mumetal. The core is designed to be easily brought into a state of magnetic saturation. It is standard practice to build such cores of spirally-wound metal strip, so that no appreciable air-gap remains, and reluctance is reduced to a minimum. The strip is given a very thin insulating coating on one side in order to decrease the

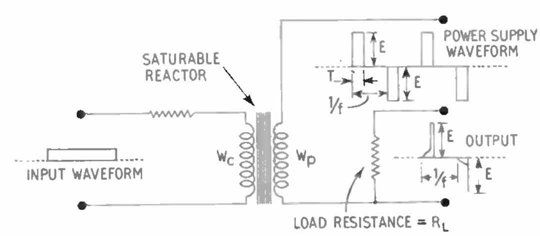

Fig. 2. Basic circuit of pulse relaxation amplifier.

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eddy-current losses, as is done in the case of transformer core laminations.

Let us consider such a toroidal core, carrying two distinct windings. (See Fig. 2.) Winding Wc will be called "control winding" because its purpose is to carry the current produced by the low-frequency telegraph signal appearing at its terminals. Winding Wp is supplied with current pulses of alternate polarities delivered by a generator. Between a positive pulse and a negative pulse, there is an interval called the relaxation period.

The amplitude and duration of the power supply pulses are such that each pulse drives the core into saturation. We assume the hysteresis loop of the core to be "square". (See Fig. 3.) A resistance is placed in series with winding Wp; we will refer to it as the "load resistance". Assuming that positive signal current flows in winding Wc and no pulse is delivered by the generator, the flux level in the core is determined only by the signal current. When a positive pulse appears in Wp, the ampere-turns in Wp will add to those in Wc, and the core will reach saturation sooner than it would have done had there been no current in winding Wc. When the core is in a state of saturation, its inductance becomes negligible, and the current flowing in the load resistance reaches maximum value. Therefore, if the core reaches saturation at an

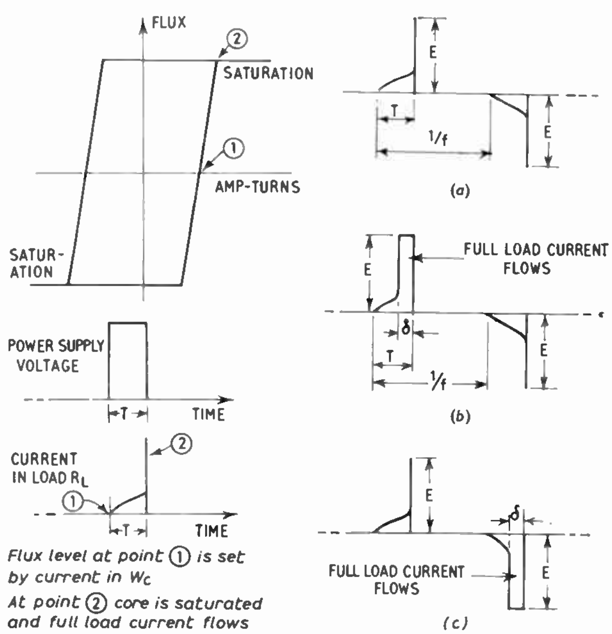


Fig. 3. Pulse relaxation amplifier—basic principle.
(*a*) No current in control winding. Power pulses just saturate the core.
(*b*) Positive signal current in Wc. Core saturates before end of pulse.
(*c*) Negative signal current in Wc. Core saturates before end of pulse.

earlier time, maximum load current will flow for a longer period and more power will be delivered to the load. The device is capable of a power gain.

The sequence of events would be identical in the case of negative signal current and a negative pulse. A positive signal produces a positive pulse of current in the load resistance, and a negative signal produces a negative pulse in that same resistance. During the time interval between two pulses, the flux level in the core is determined by the signal current. The low-frequency (or d.c.) signal is thus "converted" into an a.c. signal. This conversion simplifies the construction of the following stages of amplification.

In order to analyse quantitatively the behaviour of the device, we shall use the following symbols:

- E pulse peak voltage.
- ϕ magnetic flux in core.
- N_p number of turns in winding Wp.
- I_p current in load resistance.
- R_L load resistance.
- L_p inductance of winding Wp.
- B_s saturation flux density of core.
- a cross-sectional area of core.
- T duration of each pulse.
- e basis of the Napierian logarithms.

The voltage across the power supply winding is given by

$$E - I_p R_L = \frac{d\phi}{dt} \cdot \frac{N_p}{10^8} \quad \text{(neglecting winding resistance)} \quad \ldots\ldots(1)$$

The voltage across the load resistance is

$$I_p R_L = E\left(1 - e^{-t\frac{R_L}{L_p}}\right) \quad \ldots\ldots(2)$$

A variation of the magnetic flux in the core can be written

$$\Delta\phi = B \times a \quad \ldots\ldots(3)$$

The power supply pulses just saturate the core; therefore, the limits of integration of the voltage with respect to time are 0 and T. In the absence of a signal we combine equations (1), (2) and (3) and obtain the following expression:

$$\Delta\phi = \frac{10^8 E}{N_p}\int_0^T e^{-t\frac{R_L}{L_p}}\, dt = 2B_s \times a$$

which reduces to

$$2B_s \times a = \frac{10^8 E L_p}{N_p R_L}\left(1 - e^{-T\frac{R_L}{L_p}}\right) \quad \ldots\ldots(4)$$

We assume that the control winding is terminated in such a high impedance that transformer loading of winding Wp does not affect the operation of the device. In fact, the control winding is connected to the output of a transistor amplifier designed for very low drift. The input and output impedances of the transistor amplifier depend on the parameters of the particular transistors used, but representative figures are given on Fig. 4 for transistors of a widely used type.[3] The emitter-coupled differential amplifier has the advantage that the drifts produced by temperature changes in the two transistors tend to cancel. Biasing of the bases from potential divider circuits provides good stability of the operating point.

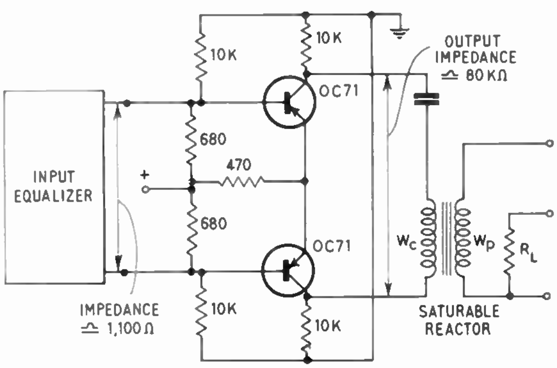


Fig. 4. Emitter-coupled differential stage as pre-amplifier for pulse relaxation amplifier stage. Input impedance is about 1·1 kΩ. Current gain is 30.

Now let us consider what happens when a low-frequency signal appears at the output of the differential pre-amplifier. Current flows in the control winding Wc, and the resulting ampere-turns will be added to the power supply winding ampere-turns. According to the polarity of the signal, either positive or negative supply ampere-turns will be assisted. If we take

I_c signal current.

N_c number of turns on Wc.

μ permeability of the core

l length of the magnetic path.

then during the relaxation period the magnetic flux in the core assumes the value

$$\Delta\phi_c = \mu \times \frac{4\pi N_c I_c}{10l} \times a \qquad \text{......(5)}$$

During the relaxation period, the flux level in the core is set by the current flowing in the control winding.[2] Therefore saturation will be reached when a power supply pulse gives the necessary ampere-turns for a flux variation of

$$B_s \times a - \Delta\phi_c = \Delta\phi_R, \qquad B_s \times a \qquad \text{......(6)}$$

being the flux variation for saturation in the absence of control current, given by equation (4).

The length of time which elapses before saturation occurs being taken as t, we may write, as in equation (4)

$$\Delta\phi_R = \frac{10^8 E L_p}{N_p R_L}\left(1 - e^{\frac{-tR_L}{L_p}}\right) \qquad \text{......(7)}$$

from which t can be obtained.

$$t = 2{\cdot}303\,\frac{L_p}{R_L}\log_{10}\left(\frac{10^8 E L_p}{10^8 E L_p - \Delta\phi_R N_p R_L}\right) \quad \text{...(8)}$$

During the interval of time $T-t$ the core is saturated and full current flows in the load resistance. The output of the pulse relaxation amplifier consists of a train of pulses of duration $T-t$ and amplitude E (see Fig. 5). A train of pulses of duration $T-t = \delta$ and repetition frequency $\omega/2\pi$ can be analysed into a Fourier series. If E is the pulse amplitude, the series is

$$E\left[\frac{\omega\delta}{2\pi} + \frac{2}{\pi}\left(\sin\frac{\omega\delta}{2}\cos\omega t + \frac{1}{2}\sin 2\frac{\omega\delta}{2}\cos 2\omega t + \right.\right.$$
$$\left.\left. + \ldots + \frac{1}{n}\sin n\frac{\omega\delta}{2}\cos n\omega t\right)\right] \quad \text{...(9)}$$

The fundamental component is given by the expression

$$E\left(\frac{\omega\delta}{\pi}\cdot\frac{\sin\omega\delta/2}{\omega\delta/2}\cos\omega t\right) \qquad \text{......(10)}$$

v is the r.m.s. output voltage at the fundamental frequency, the power output at that frequency is v^2/R_1, since the output is taken across the load resistance. From (10)

$$v = 0{\cdot}707E\,\frac{\omega\delta}{\pi}\,\frac{\sin\omega\delta/2}{\omega\delta/2} \simeq 0{\cdot}707E\,\frac{\omega\delta}{\pi} = \sqrt{2}Ef\delta \quad \text{...(11)}$$

Taking $$f = \frac{\omega}{2\pi}$$

and $$\frac{\sin\omega\delta/2}{\omega\delta/2} \simeq 1,$$

for small angles,

$$P_{\text{out}} = \frac{E^2\omega^2\delta^2}{2\pi^2 R_L} = \frac{2E^2 f^2\delta^2}{R_L} \qquad \text{......(12)}$$

The input power is

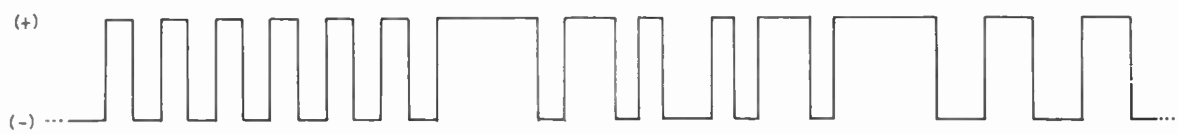
$$P_{\text{in}} = R_c{\cdot}(I_c)^2$$

where R_c is the resistance of the control winding and I_c the control current delivered by the pre-amplifier.

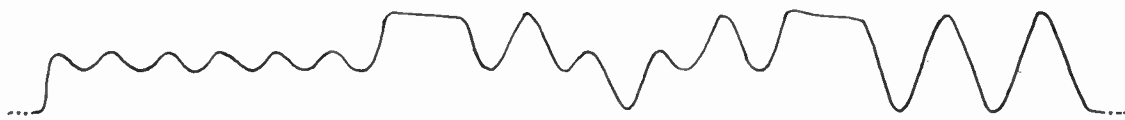
The power gain is given by

$$\frac{P_{\text{out}}}{P_{\text{in}}} = \frac{E^2\omega^2\delta^2}{2\pi^2 I_c^2 R_c R_L} = \frac{2E^2 f^2\delta^2}{R_c R_L I_c^2} \qquad \text{......(13)}$$

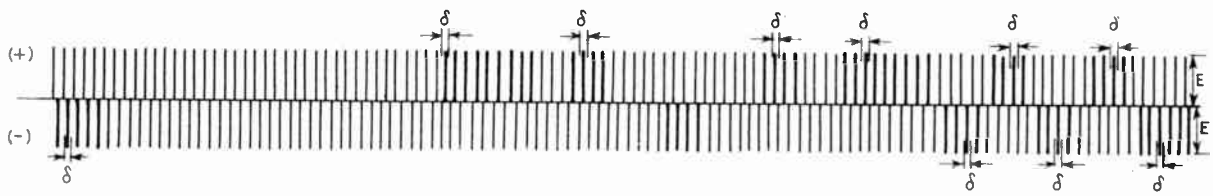
So far we have only considered one polarity of signal and power supply pulse. A positive signal will produce a train of positive pulses, whereas a negative signal will produce a train of negative pulses, provided that both power supply and control windings are wound in the same direction. If we consider the output at a frequency above the fundamental, the



(*a*) Signals transmitted on the cable.



(*b*) Signals received after equalization and pre-amplification (suppressed signals).



(*c*) Pulse relaxation amplifier—pulse output (current in R_L).

(*d*) Outputs of a.f. selective amplifiers (frequencies f_1 and f_2).

Fig. 5.

amplitude will be given by equation (9), so that the r.m.s. amplitude is

$$v_n = 0{\cdot}707E\frac{\omega\delta}{\pi} = \sqrt{2}Ef\delta \qquad \ldots\ldots(14)$$

In order to retain the advantages of frequency-shift, the output of the pulse relaxation amplifier will be taken as one sinusoidal frequency component of the positive train of pulses, and one component, of a different frequency, of the negative train.

The net output of the amplifier being taken as the difference between the positive pulse output and the negative pulse output, the signal received after amplification of the sinusoidal frequency components is the difference between the amplitude of a tone and the amplitude of another tone of a different frequency. The two transistor audio amplifiers have to be adjusted so that they have the same gain at the frequencies considered.

3. Design of the Pulse Relaxation Amplifier

In order to decrease the losses in the magnetic core, and at the same time to minimize the number of ampere-turns required for control, it is necessary to use a core of small cross-section area and short magnetic path. Such toroidal cores are in production for computer and magnetic amplifier applications. The core must be encapsulated in order to avoid mechanical strain, leaving sufficient window area for winding.

A convenient type of core consists of 6 turns of Mumetal strip, $\frac{1}{8}$ in. wide, 0·001 in. thick, and its dimensions are:

inside diameter = 0·75 in
outside diameter = 0·765 in
cross-section area = 0·0048 sq. cm.
mean magnetic path = 6·04 cm.

The saturation flux density of the magnetic material is 6 800 gauss, and the permeability is of the order of 100 000.

N_c = 4000 turns, 46 s.w.g.
R_c = 300 ohms
N_p = 1000 turns, 44 s.w.g.
R_p = 30 ohms
L_p = 100 millihenries
E = 4 V
R_L = 200 Ω
T = 100 microseconds

The duration t of the net signal output pulse can be calculated from equation (8), $\Delta\phi_c$ being obtained from equation (5). The values of v and v_n follow from equations (11) and (14), in which $\delta = T - t$; equation (8) gives the value of t.

Figure 6 shows the graph of alternating output voltage against control current, at various power supply pulse repetition frequencies. There is a "low-gain" region in the neighbourhood of the zero. This is due to the width of the hysteresis curve, as explained in reference 2. The sensitivity of the device can be adjusted, by choosing the number of turns on the control winding, so that the disturbances which are much weaker than the signal do not provide enough control current to produce a change of magnetic flux during the relaxation period, and are thus suppressed. If the signal current delivered by the pre-amplifier is large enough, the output voltage reaches its maximum value (horizontal part of the curves of Fig. 6).

The response of the device to transient signals depends on the time constant of the control winding, which is L_c/R_c. This, being only a few milliseconds, will have little effect on signals transmitted at a repetition frequency of the order of 20 per second. The time-constant of the control winding can be decreased by reducing the number of turns. The reduction in sensitivity, expressed as $d\Delta\phi_c/dI_c$ is proportional to the number of turns removed (equation (5)). The time-constant of the control winding is proportional to the inductance, and therefore to the square of the number of turns on that winding. Writing

$$L_c = kN_c^2$$

$$dN_c = \text{number of turns removed}$$

the new inductance is

$$L_{c1} = k(N_c - dN_c)^2$$

and the time constant is decreased of

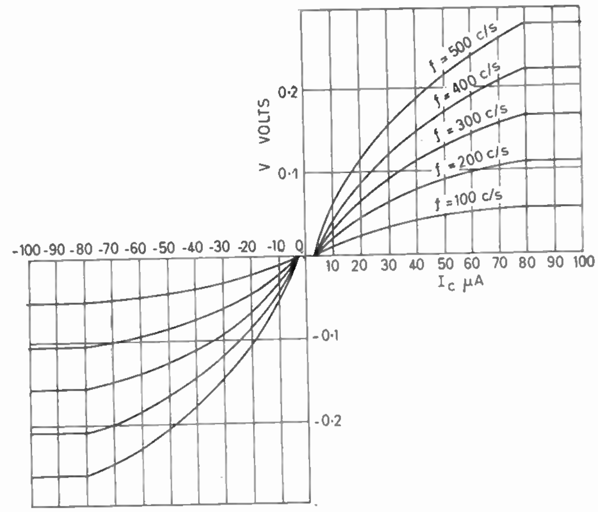


Fig. 6. Calculated characteristics of pulse relaxation amplifiers at different pulse repetition frequencies.

$$\frac{L_c - L_{c1}}{R_c} = \frac{2kN_c dN_c - k\,dN_c^2}{R_c}$$

(assuming that R_c is kept at a fixed value). For instance, assuming that the control winding has 4000 turns, removing 1000 turns will decrease the sensitivity by 25% and the time-constant by 43%, R_c being kept constant.

In each particular application of that type of amplifier, the sensitivity and the control winding time-constant will be determined from the amplitude and repetition frequency of the received signals. The power supply pulse repetition frequency may be limited by the transmission characteristics of the cable connected to the output of the amplifier.

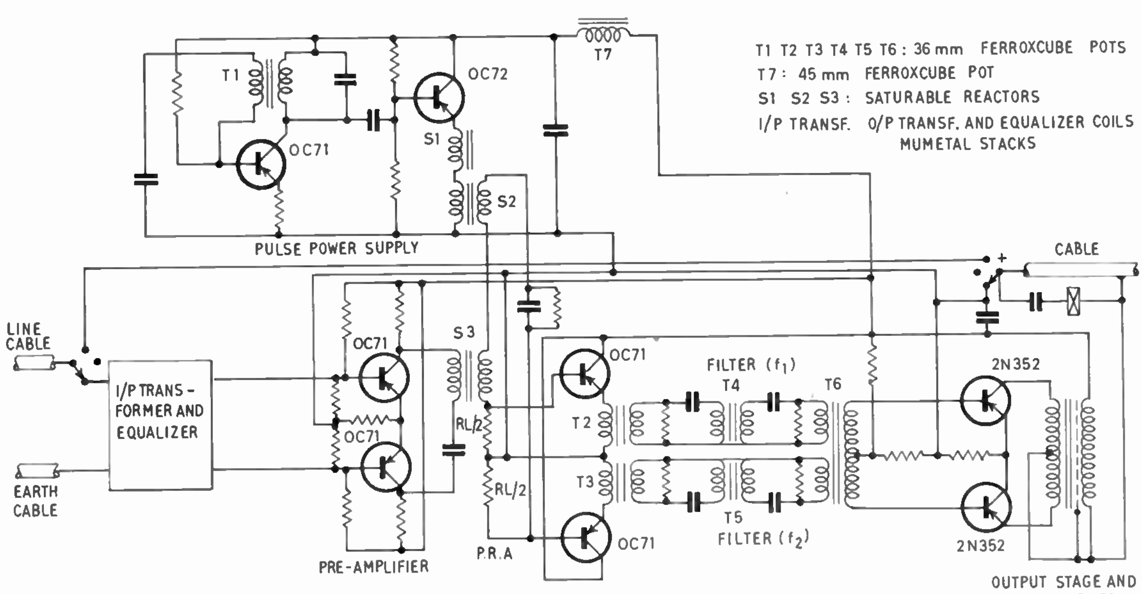


Fig. 7. Suggested circuit of frequency-converting amplifier including the pulse relaxation amplifier and its power supply for a submerged repeater.

4. Pulse Power Supply

The pulse power supply unit consists of a combination of transistors and saturable magnetic cores. A transistor oscillator delivers a sine-wave to the input of a "common collector" stage which has a high input impedance and a low output impedance. The frequency of the sine-wave is equal to the repetition frequency of the power pulses. The load of the common collector stage consists of a saturable reactor and a transformer wound on a saturable magnetic core. (See Fig. 7.) The reactor is designed to saturate after $\frac{1}{4}$ of a cycle, leaving the transformer to carry the whole voltage. The core of the transformer saturates after a time equal to the desired pulse duration T. A pulse of duration T and amplitude E appears across the secondary winding, which is connected to the pulse relaxation amplifier. The saturable cores on which the inductor and transformer are wound are identical to the amplifier core.

A filter prevents the appearance of a part of the sine-wave voltage across the d.c. supply impedance. The shunted capacitor connected to the magnetic amplifier circuit offers a high impedance to the low frequency signal and a very low impedance to the power supply pulses; this prevents the loading of the control winding by transformer coupling to the low-impedance power supply.

5. Conclusion

Although submarine telegraph cables are being superseded by modern repeated coaxial cables, a large number of them will remain in operation for many years. It is desirable to improve on the reliability of their terminal equipment and submerged repeaters.

The pulse relaxation amplifier provides means of converting a d.c. or very-low-frequency signal into an a.c. signal which can be delivered either to an a.c. coupled amplifier or to a rectifier circuit. A high amplification of d.c. signals can thus be obtained without a multi-stage d.c.-coupled amplifier, the drawbacks of which are well known.

An application of the device is the amplification of low-frequency signals in submarine cable telegraphy, in which case the pulse relaxation amplifier, associated with transistor circuits, leads to a reduction in equipment bulk and an improved reliability.

6. References

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7. Appendix

Harmonic Composition of a Train of Rectangular Pulses

Let a train of rectangular pulses (Fig. 8) have an amplitude E volts, width δ seconds, period $\tau = 2\pi/\omega$.

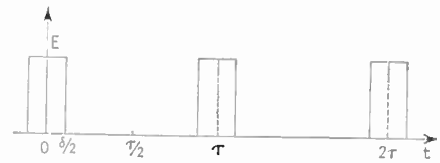


Fig. 8. Train of rectangular pulses for analysis into a Fourier series.

This can be analysed into a Fourier series

$$y = A_0 + A_1 \sin x + A_2 \sin 2x + \ldots + A_n \sin \omega x + $$
$$+ B_1 \cos x + B_2 \cos 2x + \ldots + B_n \cos \omega x$$

with $x = \omega t$ and $n = 1, 2, 3, 4, 5, \ldots$ etc.

We shall now calculate coefficients A_0, A_n and B_n.

$$A_0 = \frac{1}{\omega\tau}\int_0^{\delta/2} E\omega\,dt + \frac{1}{\omega\tau}\int_{\tau-\delta/2}^{\tau} E\omega\,dt = E\frac{\omega\delta}{2\pi}$$

$$A_n = \frac{2}{\omega\tau}\int_0^{\delta/2} E\omega \sin n\omega t\,dt + \frac{2}{\omega\tau}\int_{\tau-\delta/2}^{\tau} E\omega \sin n\omega t\,dt$$

$$= \frac{2E}{\tau}\left[\frac{1}{n\omega}\left(1 - \cos n\omega\frac{\delta}{2}\right) + \right.$$
$$\left. + \frac{1}{n\omega}\left(\cos n\omega\left(\tau - \frac{\delta}{2}\right) - \cos n\omega\tau\right)\right] = 0$$

$$B_n = \frac{2}{\omega\tau}\int_0^{\delta/2} E\omega \cos n\omega t\,dt + \frac{2}{\omega\tau}\int_{\tau-\delta/2}^{\tau} E\omega \cos n\omega t\,dt$$

$$= \frac{2E}{\tau}\left[\frac{1}{n\omega}\left(\sin n\omega\frac{\delta}{2}\right) + \right.$$
$$\left. + \frac{1}{n\omega}\left(\sin n\omega\tau - \sin n\omega\left(\tau - \frac{\delta}{2}\right)\right)\right]$$

$$= \frac{2E}{\pi\omega}\sin n\frac{\omega\delta}{2}$$

and the series is

$$y = E\left[\frac{\omega\delta}{2\pi} + \frac{2}{\pi}\left(\sin\frac{\omega\delta}{2}\cos\omega t + \frac{1}{2}\sin 2\frac{\omega\delta}{2}\cos 2\omega t + \right.\right.$$
$$\left.\left. + \ldots + \frac{1}{n}\sin n\frac{\omega\delta}{2}\cos n\omega\right)\right]$$

Based on a thesis submitted for exemption from the Institution's Graduateship Examinations.

Manuscript first received by the Institution on 2nd January 1961 and in revised form on 13th February 1962. (Paper No. 822.)

U.D.C. 621.372.852.21/2

Properties of Inhomogeneous Cylindrical Waveguides in the Neighbourhood of Cut-off

By

R. A. WALDRON, M.A.

(*Member*)†

Summary: For suitable values of the properties of a dielectric or ferrite rod, concentrically placed in a waveguide of circular cross-section, the phase constant of the H_{11} mode, expressed as a function of frequency, is two-valued over a range of frequencies in the neighbourhood of cut-off. In the two-valued region, one branch of the phase-constant curve has positive slope and the other negative slope, corresponding respectively to forward and backward waves. The field components and power distribution of these waves are discussed, and a method is suggested of separating the two kinds of wave by a mode filter. It is shown that at two values of the frequency where the group velocity is zero there is a high concentration of energy in the guide; this would be infinite in the lossless case, but where losses are present due to imaginary parts of the dielectric constant of the rod, and of the permeability elements in the case of a ferrite rod, they become large when the group velocity is zero, and so prevent the energy concentration from rising beyond a certain level. Over part of the waveguide cross-section the power is negative, and when the group velocity is negative the negative power exceeds the positive power. When the energy in the guide is large compared with the rate of transfer of energy, the system behaves somewhat like a cavity, and a "Q" can be defined.

List of Symbols

r, θ, z	Cylindrical co-ordinates
a	Waveguide radius
b	Radius of ferrite or dielectric rod
ε	Dielectric constant of ferrite or dielectric rod
μ, α	Diagonal and off-diagonal elements of the permeability tensor of the rod, when this is of ferrite. For a dielectric rod, $\mu = 1$ and $\alpha = 0$
ω	2π times working frequency
λ_0	Free-space wavelength
λ_g	Wavelength in the guide
β	Phase constant, $= 2\pi/\lambda_g$
$\bar{\beta}$	Normalized phase constant, $= \lambda_0\beta/2\pi = \lambda_0/\lambda_g$
v_φ	Phase velocity, $= \omega/\beta$
v_g	Group velocity, $= d\omega/d\beta$
c	Velocity of light
P_+	Power in the guide in the forward direction
P_-	Power in the guide in the backward direction
P_a	Power absorbed per half-wavelength ($\lambda_g/2$)
Q	Waveguide "Q", defined by equations (10) and (11)

1. Introduction

The system to be considered in this paper consists of a cylindrical waveguide of radius a, containing a concentric ferrite or dielectric rod of radius b, longitudinally polarized. The mode spectrum of this system has previously been studied by the author,[1] particular attention being paid to the case in which the diagonal element, μ, of the permeability tensor is numerically greater than the off-diagonal element, α. This case includes dielectrics, which, as far as their effects on electromagnetic waves are concerned, may be thought of as degenerate ferrites having $\mu = 1$ and $\alpha = 0$. For waveguides which are not near cut-off, the field components and distribution of power in the cross-section have also been studied.[2, 3] Similar studies to those of references 1, 2, and 3 have also been carried out by Tompkins.[4]

An interesting feature that emerges from these studies is that over a limited range of frequencies in the neighbourhood of cut-off it is possible for the group velocity to be negative.[5] The conditions under which this occurs have been studied by Clarricoats,[6] who reaches similar conclusions to the present writer.[7] It may also be noticed from the results of references 2, 3, and 4 that for a guide not near cut-off it is possible, under certain conditions, for the power over a part of the cross-section to be negative. The author has pointed out[7] that the results of reference 2 demonstrate that as cut-off is approached the negative power increases, and has shown[7] that it increases still further in the neighbourhood of cut-off itself, becom-

† Marconi's Wireless Telegraph Co. Ltd., Research Division, Great Baddow, Essex.

ing greater than the positive power in the region of negative group velocity.

It seemed that the properties of ferrite-loaded and dielectric-loaded waveguides in the neighbourhood of cut-off are likely to exhibit many interesting features, and the studies which form the subject of this paper were undertaken in order to discover those features. Reference 7 describes an earlier stage of the work, in which the relation is examined between the ratio of negative to positive power and the group velocity, in the lossless case. Details of this work which were not discussed, or not fully discussed, in reference 7, are treated below; the work treated in reference 7 is also discussed more briefly, partly for the sake of completeness of the present discussion, partly because the interrelation of the various topics makes it difficult to leave any of them out.

Recently, studies have been made of the loss properties of ferrite-loaded cylindrical waveguides, both away from cut-off and near cut-off; results of these studies have been published.[8] The results given in reference 7 deal only with the lossless case. In the present paper, the modifications of these results produced by losses will be considered, thus effecting an integration of the work of reference 7 with that of reference 8 in the cut-off region.

Until quite recently, negative group velocities in waveguides have only been obtainable by the use of structures periodic along the direction of the guide axis, such as helices and interdigital structures. Waves with negative group velocities are called backward waves; the phase velocity and group velocity are oppositely directed, and if the direction of propagation of the wave-front is regarded as the forward direction, the energy flows backwards. Recently, backward waves in waveguides containing plasma have been reported,[9, 10] and analogous effects have been predicted[11, 12, 13] for waveguides containing ferrites with $|\mu| < |\alpha|$. The backward waves predicted for dielectric- or ferrite-loaded waveguides near cut-off, with $|\mu| > |\alpha|$, are of a different type from those considered in references 9 to 13, depending more on the geometry of the waveguide than on the properties of the materials present.

2. The HSP Waveguide

The term "HSP waveguide" has recently been introduced by the author,[14, 7, 2] and it will be convenient to use it later in this paper. As the term is not yet familiar, a brief explanation is in order here; further discussion may be sought in the references cited, particularly the first. We shall say a guide is simple if only scalar (not necessarily lossless) media are present; perfect if only lossless (not necessarily scalar) media are present; and homogeneous if the electromagnetic fields are confined to a single homogeneous medium. The homogeneous, simple, perfect waveguide, which it is convenient to abbreviate to "HSP waveguide", is one which possesses all these attributes. Consideration shows that it consists of two media, separated by a single boundary surface; one of the media is a perfect conductor, the other any lossless scalar dielectric.

In the present paper we are concerned with guides of circular cross-section. The term "HSP waveguide" in the present context will be restricted to mean a guide consisting of a single circular cylindrical surface $r = a$, with perfect conductor in the region $r > a$ and any lossless scalar dielectric (including vacuum) in the region $r < a$.

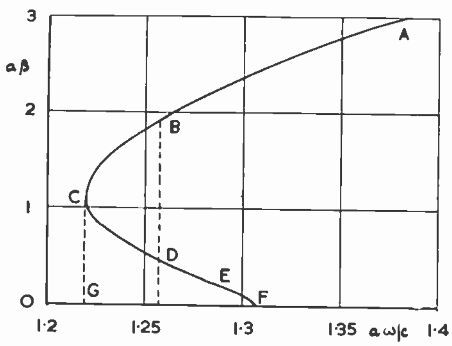


Fig. 1. A typical curve of phase constant against frequency, showing a pronounced bulge into values of frequency below cut-off. Parameter values: $b/a = 0{\cdot}48$, $\epsilon = 15$, $\mu = 1$, $\alpha = 0$.

3. Behaviour of the Phase Constant

In reference 1, a number of tables are given of the normalized phase constant, $\bar{\beta}$, as a function of the ratio b/a of ferrite radius b to guide radius a, for various values of the parameters a/λ_0, ε, μ, and α (defined in the list of symbols). Only the lossless case was considered, i.e. all the parameters were taken to be real. By taking $\bar{\beta}$ as a function of b/a for several values of a/λ_0, with constant ε, μ, and α, it is possible to obtain $\bar{\beta}$ as a function of a/λ_0. It is now convenient to think in terms of $a\beta = 2\pi a/\lambda_0$ times $\bar{\beta}$, instead of $\bar{\beta}$, and to write $a\omega/c$ for $2\pi a/\lambda_0$. A graph can be traced of $a\beta$ against $a\omega/c$; thus the ordinates are proportional to β and the abscissae to ω, a and a/c being constants. This was done for the H_{11} mode in reference 5, and it was found that the curves of $a\beta$ against $a\omega/c$ are similar in form to the curves of $\bar{\beta}$ against b/a from which they are obtained.

A typical curve of $a\beta$ against $a\omega/c$ is illustrated in Fig. 1. It has two distinct parts, the region ABC in which the slope is positive, and the region CDEF in which the slope is negative. When $\mu = 1$ and $\alpha = 0$, curves of this form are obtained[6, 7] for values of a/λ_0 between about 0·25 and 0·1, and values of ε greater than about 10; the limiting values of a/λ_0 depend on ε,

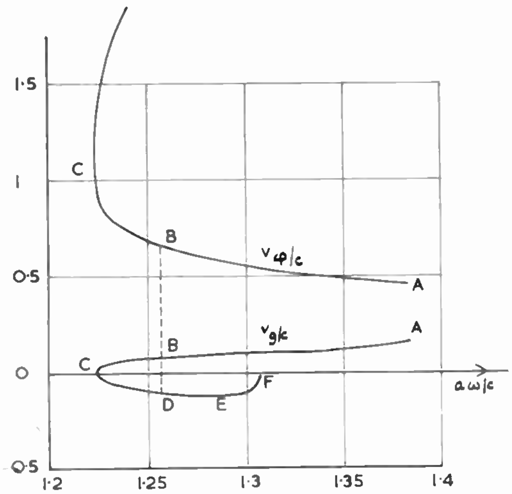


Fig. 2. Phase and group velocities, calculated from the curve of Fig. 1. The lettered points correspond to the lettered points of Fig. 1.

and the minimum necessary value of ε depends on the value of a/λ_0. It is evident from the results of reference 1 that as μ or α increases, the minimum necessary value of ε decreases. It has also been found that as ε increases, the lower limit of a/λ_0 decreases substantially, although no accurate quantitative information is available. However, experience based on the results of reference 1 indicates that the upper limit of a/λ_0 is not greatly dependent on ε, μ, and α; this is confirmed in the case of the dependence on ε by Clarricoats' computations.[6]

Corresponding to a particular value of $a\omega/c$ in the range from G to F there will be two values of phase constant, e.g. those given by the points B and D. There will be a set of electromagnetic field components corresponding to each value of β, and the fields of either set can propagate in the absence of the other. All points on the curve ABCDEF belong to the H_{11} mode, so that the two sets of field components are both H_{11} field patterns. However, in their property of independent existence they behave like separate modes. It is therefore convenient to regard them as distinct sub-modes of the H_{11} mode. Points on the curve lying between A and C, and their associated field patterns, will be said to belong to the normal sub-mode, while points lying between C and F, and their associated field patterns, will be said to belong to the paranormal sub-mode. This terminology was introduced in reference 7. The paranormal sub-mode propagates only in a limited frequency range (G to F) below cut-off, while the normal sub-mode propagates for all frequencies higher than a certain value. In most respects near cut-off, and in all respects above cut-off. the normal sub-mode behaves like a mode of a conventional waveguide; it is normal in the everyday sense. The paranormal sub-mode, however, has upper and lower limiting frequencies, and its phase constant has a negative instead of positive frequency dependence. The term "paranormal" was chosen because since backward waves have been known for some time it can hardly be called abnormal, while on the other hand it is not normal, either in the everyday sense of the word or in the sense in which it is used in the term "normal sub-mode".

The points C, E, and F, in Fig. 1 are special points. E is a point of inflection; this is difficult to fix with any accuracy since the curvature is not great over a wide range of frequencies. At C and F the slope of the curve is infinite. F is the cut-off point, given, as in a conventional waveguide, by $\beta = 0$. Notice that the phase-constant curve for the normal sub-mode does not terminate at $\beta = 0$ but at a finite value of β; it is in this respect that the normal sub-mode differs from modes in conventional waveguides.

4. Phase and Group Velocities

The phase velocity of a wave is given by

$$v_\varphi = \omega/\beta \qquad \text{......(1)}$$

and the group velocity by

$$v_g = \mathrm{d}\omega/\mathrm{d}\beta \qquad \text{......(2)}$$

It is convenient to normalize velocities with respect to that of light, and using the quantities $a\beta$, $a\omega/c$, that we have in Fig. 1, we may write

$$v_\varphi/c = \frac{a\omega/c}{a\beta} \qquad \text{......(3)}$$

$$v_g/c = \frac{\mathrm{d}(a\omega/c)}{\mathrm{d}(a\beta)} \qquad \text{......(4)}$$

The normalized velocities can thus be read off directly from curves of the type of Fig. 1, and typical results are shown in Fig. 2.

The points A, B, C, D, E, F, in Fig. 2 correspond to the points designated by the same letters in Fig. 1. At C and F, corresponding to the infinite slope of the curve of $a\beta$ against $a\omega/c$, the group velocity is zero. At E, the slope of the curve in Fig. 1 is a numerical maximum, giving the minimum of v_g/c in Fig. 2. This is a very flat minimum because the curvature of the phase-constant curve is small in the neighbourhood of E.

The phase velocity takes no special value at C, but goes to infinity at F. There is a point of inflection between C and F on the phase-velocity curve, but this does not coincide with E; this can be seen as follows: Write $v_\varphi = \omega/\beta$ and differentiate twice with respect to β.

$$\frac{\mathrm{d}^2 v_\varphi}{\mathrm{d}\beta^2} = \frac{1}{\beta}\frac{\mathrm{d}^2\omega}{\mathrm{d}\beta^2} - \frac{2}{\beta^2}\frac{\mathrm{d}\omega}{\mathrm{d}\beta} + \frac{2\omega}{\beta^3}$$

At the point of inflection, $d^2\omega/d\beta^2 = 0$ and we have

$$\frac{d^2 v_\varphi}{d\beta^2} = \frac{2}{\beta^2}\left[\frac{\omega}{\beta} - \frac{d\omega}{d\beta}\right]$$

i.e.

$$\frac{d^2 v_\varphi}{d\beta^2} = \frac{2}{\beta^2}[v_\varphi - v_g]$$

For $d^2 v_\varphi/d\beta^2$ to have a point of inflection corresponding to that of the phase-constant curve, therefore, we require $v_\varphi = v_g$, and it is evident from Fig. 2 that this condition is not satisfied.

5. Field Components

The theory of reference 1 can be used to calculate the values of the field components as functions of position in the cross-section. The results of a number of such computations are given in references 2 and 3, and the behaviour of the various fields was studied as the various parameters were changed. No results were obtained close to cut-off, however, and this question will now be considered.

It has already been stated in Section 3 that waves of the normal or paranormal sub-modes can propagate separately. Boundary conditions have been applied to solutions of Maxwell's equations, and from the equations expressing boundary conditions the characteristic equation has been obtained. Solutions of this give values of $a\beta$ for given values of $a\omega/c$. There is nothing in this procedure that requires the simultaneous presence of both sub-modes. All that is necessary is that the boundary conditions should be satisfied, and they are satisfied for any single value of β on the appropriate phase-constant curve.

One naturally asks, if waves corresponding to the points B and D of Fig. 1 are capable of separate existence, how may a waveguide be induced to support one of these and not the other? The answer to this question requires a knowledge of the field components of the two sub-modes; we must hope that there will be some significant difference between the two sets of field patterns which may be exploited to make a mode filter. Such a difference does exist, as will be seen shortly.

Computations of the H_{11}-mode field components have been carried out for a typical pair of points like B and D. The parameter values chosen were

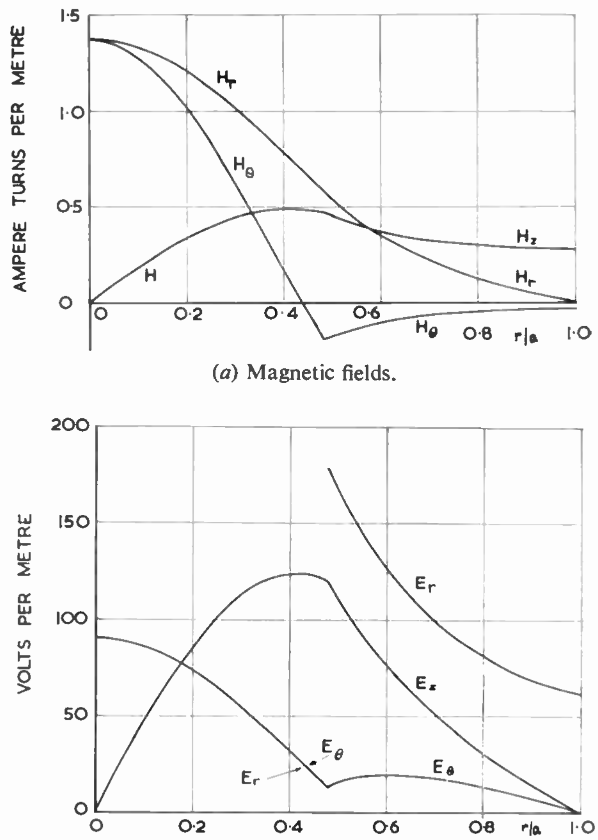
$$a/\lambda_0 = 0{\cdot}2,\ b/a = 0{\cdot}48,\ \varepsilon = 15,\ \mu = 1,\ \alpha = 0.$$

The values of β corresponding to B and D were, respectively, 1·539 and 0·374; these values were obtained by the use of the electronic computer DEUCE as in reference 1. The corresponding values of $a\beta$ are 1·934 and 0·470. The results of these computations are shown in Figs. 3 and 4.

The values of electric and magnetic field are normalized as in reference 2, such that the total power in the guide cross-section is λ_0^2 watts, when λ_0 is measured in metres. If the fields, as given, are divided by the value of λ_0 corresponding to the working frequency, the values of fields so obtained will be those obtaining when the power is 1 watt. By "power" here is meant the net power, as discussed in Section 6.

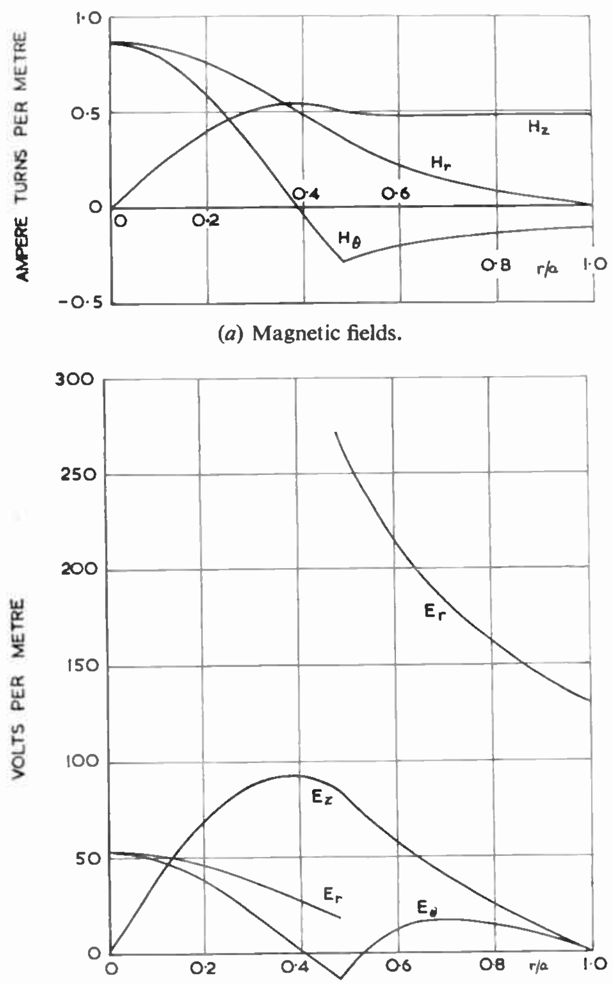
The H_{11}-mode field components for the normal sub-mode (Fig. 3) are similar in form to those previously found[2, 3] for the normal sub-mode well away from cut-off. It will be seen, too, that most of the field components of the paranormal sub-mode (Fig. 4) are of the same general form as those of the normal sub-mode. There is a qualitative difference, however, in the form of E_θ, which can become negative near the surface of the ferrite or dielectric rod in the case of the paranormal sub-mode, with two values of the radius at which it is zero, one just less than the rod radius, the other somewhat greater.

The ranges of values of parameters for which E_θ may become negative has not been studied. It is evident, though, that there must be some range for each parameter. How this range will depend on the values of the other parameters is not easy to say, but we can



(*b*) Electric fields.

Fig. 3. Field components, as functions of radius, corresponding to the points B of Figs. 1 and 2. The values shown are the maximum values.



(*a*) Magnetic fields.

(*b*) Electric fields.

Fig. 4. Field components, as functions of radius, corresponding to the points D of Figs. 1 and 2. The values shown are maximum values.

say that at least in certain cases it will happen that E_θ will be zero just outside, or at any rate somewhere outside, the rod. In these cases, at least, there is hope of achieving a mode filter.

Such a mode filter would probably take the form of a thin ring of moderately resistive material, such that an electric field in it would be rapidly dissipated by the resistive losses of induced currents. A series of rings spaced along the guide might be used if one was not enough by itself. The ring should be of the same radius as the surface on which $E_\theta = 0$ for the paranormal sub-mode. Then on inserting the ring into the waveguide there would be no tendency for current to flow in it due to the paranormal sub-mode, which would therefore continue to propagate undisturbed. The finite azimuthal electric field of the normal sub-mode, however, would induce a current in the ring, and energy would be absorbed.

As well as the H_{11} mode, the guide is also capable of supporting the E_{01} mode; the latter, however, is usually not excited in Faraday-rotation devices. Whether it would be excited by the present mode filter is not likely to be discovered until such a filter is actually made; if it is excited, it would probably be possible to suppress it by another mode filter, or by a modification of the present filter.

A variant of the filter might be to use a ring of metal, with coupling to an external system in which the energy removed from the waveguide could be absorbed or used. The wire of the ring might, for example, continue outside the guide as the core of a coaxial cable, as illustrated in Fig. 5. The radial parts of the wire could be arranged to coincide with the zero of the radial component of electric field in the paranormal H_{11} sub-mode, so that the propagation of this would not be interfered with. Any tendency to excite the E_{01} mode would be suppressed by the radial wires.

The above possible types of mode filter can only act by suppressing the normal sub-mode and allowing the paranormal to propagate undisturbed. The wire ring with external coupling, however, permits active working, with stimulation from an external source. This may provide a method of exciting the normal sub-mode in the absence of the paranormal.

6. Negative and Positive Power

As in reference 2, the power per unit area at a point in the cross-section is given by $\tilde{E}_r\tilde{H}_\theta \cos^2\theta + \tilde{E}_\theta\tilde{H}_r \sin^2\theta$ in the z direction, where $\tilde{E}_r$, $\tilde{H}_\theta$, $\tilde{E}_\theta$ and $\tilde{H}_r$ denote the functions of r and the amplitude factors of the field components, i.e. the field components with the minus signs, j, and functions of θ, z, and t, dropped. In speaking of the dropping of minus signs, it should be understood that this refers to a minus sign which appears in the field component as a factor; where a minus sign appears as the function of r changes sign, it must be retained.

The term $\tilde{E}_r\tilde{H}_\theta \cos^2\theta$ is a maximum on one radius vector of the cross-section, and the term $\tilde{E}_\theta\tilde{H}_r \sin^2\theta$ is zero. On the perpendicular radius vector, $\tilde{E}_\theta\tilde{H}_r \sin^2\theta$ is a maximum and $\tilde{E}_r\tilde{H}_\theta \cos^2\theta$ is zero. Thus plots of

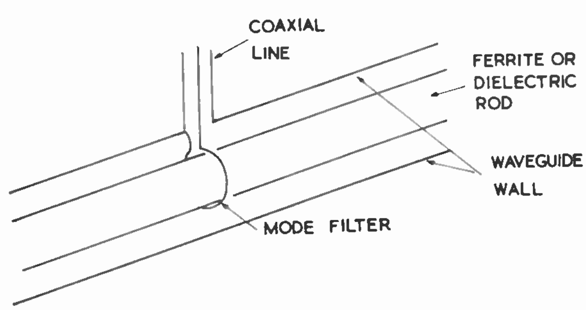


Fig. 5. Mode filter to separate the normal and paranormal sub-modes.

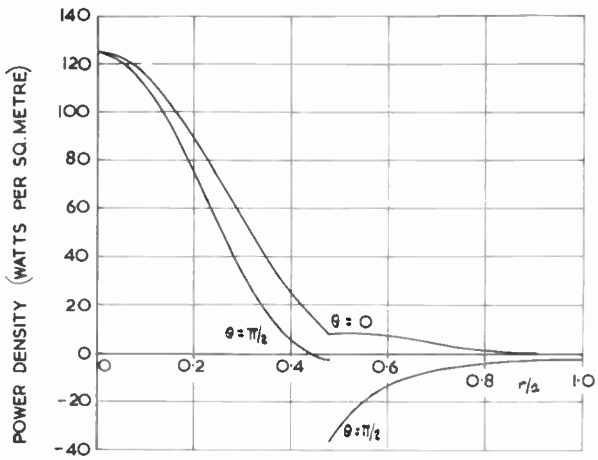


Fig. 6. Power density on the two principal radii of the cross-section, as a function of radius, for the parameter values given by the points B of Figs. 1 and 2.

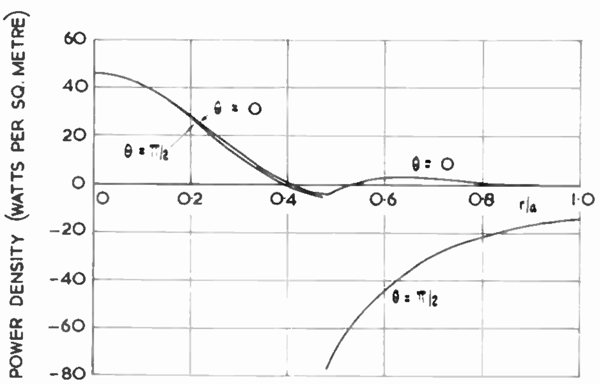


Fig. 7. Power density on the two principal radii of the cross-section, as functions of radius, for the parameter values given by the points D of Figs. 1 and 2.

$\tilde{E}_r\tilde{H}_\theta$ and $\tilde{E}_\theta\tilde{H}_r$ contain, implicitly, all information about the power per unit area at all points in the cross-section. Such plots have been given in reference 2 for a number of parameter values, but no results are given in the neighbourhood of cut-off. Figs. 6 and 7 show such results for a guide near cut-off; the parameter values are the same as for the field components given in Figs. 3 and 4, and the normalization is the same.

It will be noticed in Figs. 3 and 4 that for the normal sub-mode H_θ is negative over part of the guide cross-section, while for the paranormal sub-mode E_θ, as well as H_θ, is negative over part of the cross-section. Where this is so, either $\tilde{E}_r\tilde{H}_\theta \cos^2\theta$ or both $\tilde{E}_\theta\tilde{H}_r \sin^2\theta$ and $\tilde{E}_r\tilde{H}_\theta \cos^2\theta$ is or are negative, corresponding to a negative power. We shall consider the two components of power separately. In general, they are both present at a given point in the cross-section, and may be both positive, both negative, or one positive and one negative. In the last case, we shall refer to a negative power, even though the positive component may be numerically greater so that the net power is positive. For the total power in the guide, we shall define a positive power P_+ and a negative power P_- by

$$P_+ = \tfrac{1}{2}\left\{\int\!\!\int_{R_1} \tilde{E}_r\tilde{H}_\theta \,\mathrm{d}r + \int_{R_2} \tilde{E}_\theta\tilde{H}_r \,\mathrm{d}r\right\} \quad \ldots\ldots(5)$$

$$P_- = \tfrac{1}{2}\left\{\int_{R-R_1} \tilde{E}_r\tilde{H}_\theta \,\mathrm{d}r + \int_{R-R_2} \tilde{E}_\theta\tilde{H}_r \,\mathrm{d}r\right\} \quad \ldots\ldots(6)$$

where R_1, R_2, are the ranges of values of r where $\tilde{E}_r\tilde{H}_\theta$ or $\tilde{E}_\theta\tilde{H}_r$, respectively, is positive and R is the total range of values of the radius, from 0 to a. In general, part of $R-R_2$ will overlap R_1, and in at least part of the overlapping region $|\tilde{E}_r\tilde{H}_\theta|$ will exceed $|\tilde{E}_\theta\tilde{H}_r|$. Thus P_+ and P_- are not the total positive or negative powers in the regions where the net power density is respectively positive or negative; such a definition would appear more logical than the one we have taken, but it would be very difficult to calculate the powers, since the boundaries between the regions of net positive and negative power densities will be very complicated, and would change position with changes of value of the various parameters. The definitions we have adopted for P_+ and P_- enable these quantities to be evaluated much more easily, and the general picture is not much different from that which would have been obtained if the more logical definitions had been used.

Figure 8 shows a rough sketch of the ratio P_-/P_+ as a function of frequency, obtained by substituting the phase constant, as given in such a curve as that of Fig. 1, into the expressions for the field components given in reference 1. The points A, B, C, D, E, and F in Fig. 8 correspond to the points A, B, C, D, E, and F in Fig. 1. At C and F, where the group velocity is zero, the power transfer along the guide should be zero; thus $P_-/P_+ = 1$. The region where P_-/P_+ is greater than 1 corresponds to negative group velocity, i.e. the paranormal sub-mode, while P_-/P_+ is less than 1 for the normal sub-mode, with positive group velocity.

So far, the calculations have been made for the case of an infinitely long waveguide, and we have been

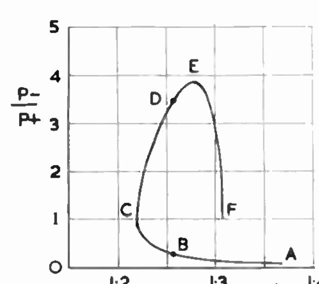


Fig. 8. Ratio of negative to positive power for the same parameter values as in Fig. 1. The points A, B, C, D, E, F, correspond to the points A, B, C, D, E, and F of Fig. 1.

considering a single mode to be present. The field components, if they exist in any transverse plane, will propagate unchanged all along the structure. In this case, the source of the negative power poses no problem—it is at infinity. But now consider a practical situation—a generator, connected to a length of waveguide, with a matched load at the far end. In the waveguide is a length of concentric dielectric or ferrite rod, matched at its ends, and at a point in the middle of the rod the H_{11} mode is flowing. Let us consider the wave to be uncontaminated by other modes, and that the system is lossless. These conditions may, in principle, be approached as closely as we like, so that it is physically meaningful to consider them. The question now arises, where does the power come from that flows towards the generator? All the energy must come from the generator, so that the backward-flowing energy must be derived from the forward-flowing energy, i.e. the energy must flow round and round in loops. It can be shown, however, that the transverse component of power is imaginary, i.e. there is no net transverse energy flow. This holds in the parts of the rod away from the ends, where the normal mode propagates. The transverse power flow, therefore, must take place at the ends of the rod, in the fringing fields. Note that this applies exactly only in the lossless case. When there is loss, this picture must be modified.

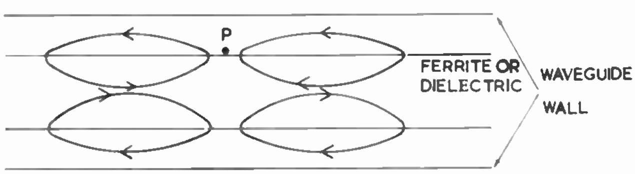


Fig. 9. Flow of energy in loops. At a point such as P, the flow in the transverse plane due to one loop is balanced by that due to the next, so that there is no net transverse flow of energy, if it is assumed that there are no losses.

The flow of energy along the guide may be pictured as in Fig. 9; the energy flows round and round in loops, half a wavelength long. At a point such as P, the transverse power from adjacent loops is equal and opposite in the lossless case, and the net transverse power is therefore zero. At the ends of the rod, the chain of loops is broken, and there are complicated fringing fields. It is reasonable to suppose that it is in these fields that the transverse power flow takes place which enables the normal mode to become established. If loss is present, the amplitude of the waves will steadily decrease along the guide. The loops will then not be equal, and the transverse powers of adjacent loops will not exactly cancel. There will then be a transverse component of power all along the length of the guide. This is borne out by analysis; it is found that when ε, μ, and α are complex, the Poynting vector has a complex transverse component.

When the losses are very slight, the main means of transverse energy flow is still by the fringing fields. When the losses become larger, or in a very long rod, the transverse flow along the length of the guide becomes more important. This will be the case if a lossy dielectric or ferrite rod is used. It will also be the case where the group velocity is small, even if the rod is only very slightly lossy, for the loss becomes very great here, as will be seen in the next section.

It should be noted that not all of the energy flows in the loops. Some does, but some also flows straight along the guide. We shall return to this point in Section 8.

7. Losses

The characteristic equation may be written in the form

$$\Delta(a/\lambda_0,\ \varepsilon,\ \mu,\ \alpha,\ b/a,\ \bar{\beta}) = 0 \qquad \ldots\ldots(7)$$

where a/λ_0, ε, μ, α, and b/a are parameters, and values of $\bar{\beta}$ have to be found such that the function Δ (given in reference 1) vanishes. In principle, this may be written, formally,

$$\bar{\beta} = \mathrm{f}(a/\lambda_0,\ \varepsilon,\ \mu,\ \alpha,\ b/a) \qquad \ldots\ldots(8)$$

in which b/a is to be regarded as the independent variable and $\bar{\beta}$ as the dependent variable, with a/λ_0, ε, μ, and α as parameters. In the preceding sections, losses have been assumed not to occur, so that the parameters are all real. $\bar{\beta}$ is then either real or imaginary, and either propagation takes place or the guide is cut off.

In practice, however, the quantities ε, μ, and α are always complex, and may be written $\varepsilon'-j\varepsilon''$, $\mu'-j\mu''$, $\alpha'-j\alpha''$. We shall only consider cases where ε'', μ'', and α'' are very small. Then, in the propagating region, $\bar{\beta}$ also becomes complex, and may be written $\bar{\beta}'-j\bar{\beta}''$. The imaginary part, $\bar{\beta}''$, is a normalized attenuation constant.

From equation (8) we may write

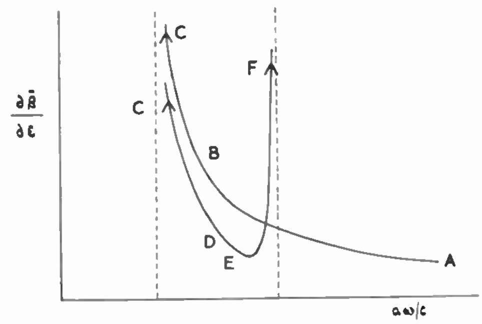
$$\delta\bar{\beta} = \frac{\partial \mathrm{f}}{\partial \varepsilon}\delta\varepsilon + \frac{\partial \mathrm{f}}{\partial \mu}\delta\mu + \frac{\partial \mathrm{f}}{\partial \alpha}\delta\alpha$$

if $\delta\varepsilon$, $\delta\mu$, and $\delta\alpha$ are all small. These may be taken as $-j\varepsilon''$, $-j\mu''$, $-j\alpha''$, and so we obtain

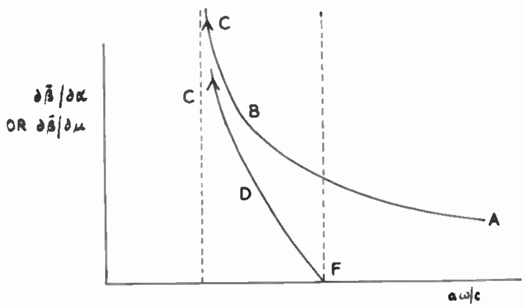
$$\bar{\beta}'' = \frac{\partial\bar{\beta}}{\partial\varepsilon}\varepsilon'' + \frac{\partial\bar{\beta}}{\partial\mu}\mu'' + \frac{\partial\bar{\beta}}{\partial\alpha}\alpha'' \qquad \ldots\ldots(9)$$

ε'', μ'', and α'' will be given as properties of the ferrite rod. The partial derivatives $\partial\bar{\beta}/\partial\varepsilon$, etc., can be obtained from the results of reference 1; details are given in reference 8.

Figure 10 shows typical curves of $\partial\bar{\beta}/\partial\varepsilon$, $\partial\bar{\beta}/\partial\mu$, and $\partial\bar{\beta}/\partial\alpha$ against frequency, obtained in this way. The letters A to F refer to the corresponding letters of Fig. 1. It will be noticed that at the point of zero group velocity for finite $\bar{\beta}$, the partial derivatives all become



(a) Dielectric loss.



(b) Magnetic loss.

Fig. 10. Losses in the neighbourhood of cut-off. The letters A to F refer to the corresponding points of Fig. 1.

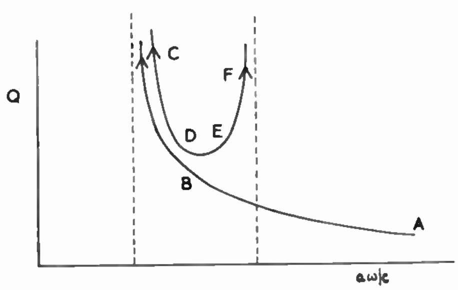


Fig. 11. Waveguide Q. The letters A to F refer to the corresponding points of Fig. 1.

infinite, and that $\partial\bar{\beta}/\partial\varepsilon$ also becomes infinite at cut-off. This is not quite a true picture, because the partial derivatives are calculated from phase-constant values which were obtained for the lossless case, i.e. $\varepsilon'' = \mu'' = \alpha'' = 0$. When these are not zero, the curve of Fig. 1 will be slightly modified in general, and considerably modified near the points of zero group velocity, C and F. We may expect, therefore, that the values of $\partial\bar{\beta}/\partial\varepsilon$, $\partial\bar{\beta}/\partial\mu$, and $\partial\bar{\beta}/\partial\alpha$ will become large at C, but not infinite, while at F $\partial\bar{\beta}/\partial\varepsilon$ will become large but not infinite and $\partial\bar{\beta}/\partial\mu$, $\partial\bar{\beta}/\partial\alpha$, will not vanish but will become very small.

The large values of $\partial\bar{\beta}/\partial\varepsilon$, $\partial\bar{\beta}/\partial\mu$, and $\partial\bar{\beta}/\partial\alpha$ at C, and of $\partial\bar{\beta}/\partial\varepsilon$ at F, lead to comparatively large values of $\bar{\beta}''$, i.e. large losses, even when ε'', μ'', and α'' are small. This may be understood physically by considering the loops of Fig. 9. Normally, the energy flowing in the loops is small compared with the net power flowing in the guide. P_-/P_+ is then small for the normal sub-mode or large for the paranormal sub-mode. But when the group velocity is small, P_- and P_+ are nearly equal and the energy in the loops is large compared with the net energy progressing along the guide in, say, one cycle. Under these conditions, although the probability of absorption of a given quantum in a given time is no higher than normal, the quantum stays in the same locality for a much longer time than normally, and the probability of its being absorbed before leaving the locality is correspondingly much greater.

8. Waveguide "Q"

The energy contained at any instant between two transverse planes may be called stored energy, while the energy transferred may be regarded as energy lost, the loss being made up continuously from the source of supply. This situation is analogous to a cavity, where there is a large stored energy, a small energy lost per cycle, and a continuous supply of energy from a source to make up the loss. Pursuing the analogy, we may define a waveguide "Q" as the ratio of stored energy to energy transferred per cycle, i.e. to net power. We shall define the waveguide "Q" as 2π times the modulus of the energy stored in half a wavelength divided by the energy transferred per cycle. For the moment, losses are neglected. It is shown in reference 7 that this definition leads to the formula

$$Q = \pi\left|\frac{P_+ + P_-}{P_+ - P_-}\right| \qquad \ldots\ldots(10)$$

The behaviour of Q, as a function of frequency, is readily deduced from the curve of P_-/P_+ given in Fig. 8. Thus we obtain the curve of Fig. 11.

In the lossless case, Q becomes infinite at the frequencies of zero group velocity, because here the transferred power becomes zero. When ε'', μ'', and α'' (or only ε'' in the case of a dielectric rod) depart from zero, there is an absorbed power which must be taken into account. Writing P_a for this, we may redefine Q by

$$Q = \frac{\pi(P_+ + P_-)}{|P_+ - P_-| + P_a} \qquad \ldots\ldots(11)$$

P_a can be expressed in terms of P_+, P_-, and the waveguide parameters as follows:

Let the amplitude of a wave be given by

$$A = A_0 \mathrm{e}^{-j\beta' z}\mathrm{e}^{-\beta'' z}$$

where A_0 is the amplitude at $z = 0$, and, like A, is a function of r and θ. Let us take $A = A_1$ at $z = \lambda_g/2$ and expand the factor $\mathrm{e}^{-\beta'' z}$, neglecting second-order and higher-order terms. Hence

$$A_1 = A_0 e^{-j\beta' z}(1-\beta''\lambda_g/2)$$

The decrease in amplitude is $\delta A = A_0 - A_1$. Hence

$$\left|\frac{\delta A}{A_0}\right| = \frac{\beta''\lambda_g}{2} = \bar{\beta}'' \frac{2\pi}{\lambda_0}\frac{\lambda_g}{2} = \frac{\pi\bar{\beta}''}{\bar{\beta}'}$$

The power P is proportional to A^2, so that

$$\frac{\delta P}{P_0} = \frac{2\delta A}{A_0} = \frac{2\pi\bar{\beta}''}{\bar{\beta}'} = \frac{2\pi}{\bar{\beta}'}\frac{\partial\bar{\beta}'}{\partial\varepsilon}\varepsilon''$$

if there is only dielectric loss. If there is also magnetic loss, there will be similar terms involving μ'' and α''. Wall losses and losses in the outer medium ($a > r > b$) are assumed negligible. Now, δP is the absorbed power, and P_0 is the transmitted power, so that

$$\frac{\delta P}{P_0} = \frac{P_a}{|P_+ - P_-|}$$

Hence

$$P_a = \left|P_+ - P_-\right| \frac{2\pi}{\bar{\beta}'}\left\{\frac{\partial\bar{\beta}'}{\partial\varepsilon}\varepsilon'' + \frac{\partial\bar{\beta}'}{\partial\mu}\mu'' + \frac{\partial\bar{\beta}'}{\partial\alpha}\alpha''\right\} \qquad \text{......(12)}$$

Substituting this into equation (11), we obtain

$$Q = \frac{\pi(P_+ + P_-)}{|P_+ - P_-|\left\{1 + \frac{2\pi}{\bar{\beta}''}\left[\frac{\partial\bar{\beta}'}{\partial\varepsilon}\varepsilon'' + \frac{\partial\bar{\beta}'}{\partial\mu}\mu'' + \frac{\partial\bar{\beta}'}{\partial\alpha}\alpha''\right]\right\}} \qquad \text{......(13)}$$

Equation (13) holds good as long as

$$\frac{2\pi}{\bar{\beta}'}\left\{\frac{\partial\bar{\beta}'}{\partial\varepsilon}\varepsilon'' + \frac{\partial\bar{\beta}'}{\partial\mu}\mu'' + \frac{\partial\bar{\beta}'}{\partial\alpha}\alpha''\right\} \ll 1$$

which will usually be the case when ε'', μ'', and α'' are small. Near the points of zero group velocity, however, the partial derivatives are large; also, for non-zero ε'', μ'', α'', $|P_+ - P_-|$ will never be exactly zero. Thus Q will not rise to infinity at C and F, only to some large value.

The significance of the quantity Q is that it is a measure of the concentration of energy in the waveguide, just as the Q of a cavity is a measure of the concentration of energy in the cavity. The system of an empty waveguide, matched into a ferrite-loaded or dielectric-loaded waveguide near cut-off, which is again matched into an empty waveguide at its far end, is very similar to a waveguide matched through a coupling hole into a cavity, which loses energy through another coupling hole into another waveguide. In the cavity, the stored energy is reflected at the ends, travelling backwards and forwards over the same path. In the case of the rod-loaded waveguide, the energy is not exactly reflected at the ends of the length of guide, but moves in loops, so that the backward-flowing energy and the forward-flowing energy do not flow over the same path. But the effect is the same—the energy travels over a closed path many times before escaping, giving a high energy density in the cavity or loaded waveguide.

9. Acknowledgments

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British Electronic Components—1963

Previous exhibitions of the Radio and Electronic Component Manufacturers Federation have shown the progress which is continually being made in reducing the size of electronic components. "Miniaturization" was followed by "microminiaturization" and now "molecular electronics" seems to be the next—and presumably the ultimate—stage. Thus the 18th Components Show, held at Olympia, London, from 21st–24th May, gave a striking insight into the way in which materials technology is becoming ever more important.

Mr. A. F. Bulgin, M.B.E. (Member), chairman of the Federation, said, at the opening of the Exhibition, that the industry has a production of 3000 million individual components a year, worth £137 million. He stressed the importance of reliability and pointed out that one manufacturer had produced a capacitor which has a failure rate of 0·000028% per 1000 hours.

Some of the "microminiature" circuits shown at the Exhibition form part of a British rocket guidance system. A typical example was a module developed by Mullard in which 3438 components were packed into a volume equivalent to a couple of packets of twenty cigarettes. The individual components—more than 1200 transistors and semiconductor diodes, and 2217 resistors and capacitors—are formed by deposition on 164 cigarette-paper thin films which are stacked together. It is claimed that this method of manufacture will eventually become cheaper and much more reliable than conventional wired or printed circuits.

The Ministry of Aviation stand concentrated on the correct use of electronic parts in equipment and on the process control exercised during manufacture. Great care is already taken to provide quality and reliability in electronic parts but further improvements can and will be made. Performance can be impaired or destroyed by injudicious use during equipment design, and overstress—electrical, thermal or mechanical—is a common cause of failure. One section of the exhibition concentrated on some aspects of the misuse of parts which ranged from simple engineering errors to incompatibility of materials.

Semiconductors

A new material that foreshadows an all-electronic ignition system for motor cars was shown for the first time. The material, called "Piezoxide", is based on lead zirconate-titanate, and when compressed develops a high voltage. It can be pressed to a variety of shapes to give the most efficient transfer of energy for particular applications, and the piezo-electric effect can be controlled by the amount of polarization given to the material during manufacture. Mullard gave a demonstration with a slab of Piezoxide firing a conventional sparking plug when subjected to pressure from a lever operated by a rotary cam. The voltage across the plug is proportional to the mechanical pressure and the thickness of the slab: a pressure of 7000 lb/in^2 generates 400V per millimetre of thickness. Piezoxide has obvious applications for transducers in a variety of electronic and electrical equipment.

The Transistor Division of S.T.C. have produced a new range of "Miniflake" transistors—planar epitaxial silicon transistors without cases. Because of their very high packing density potential these new transistors are of special interest to computer manufacturers.

An experimental planar transistor shown by Mullard has a high current gain (between 40 and 120 at a collector current of 10 mA) and operates at frequencies up to 300 Mc/s.

Avalanche rectifiers are designed to protect other circuit components by surge clipping characteristics and simplify the series operation of rectifiers in high voltage applications as no shunting resistors are necessary. The A.E.I. 1200 V all-diffused avalanche rectifier is able to operate in the avalanche region indefinitely, without damage, where it will absorb momentary power surges. The rectifier does not require de-rating in the reverse blocking direction as it is self-protecting against voltage transients. Thus, a controlled avalanche rectifier with peak reverse voltage of 1200 V is equivalent to a conventional rectifier of 2500–3500 peak reverse voltage in many circuits as safety factors are necessary.

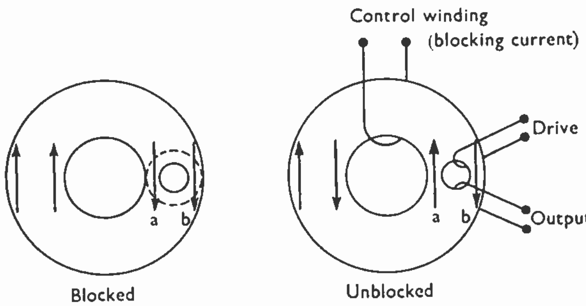
A new approach to the important problem of heat dispersal in semiconductor devices has been developed by Alexander Orba. The "heat exchanger" is a slab of aluminium foil honeycomb, embedded in which, or attached to it in various configurations, is an aluminium plate or block machined to serve as a platform for the transistor or rectifier. Care is taken to ensure intimate thermal contact between device and platform, and platform and honeycomb, thus giving low thermal resistance—the actual value depending on the configurations employed.

The De La Rue "Frigistor" is a solid state thermo-electric cooling module comprising dissimilar semiconductors mounted in couples and connected in series. A direct current passed through the module pumps heat from one couple junction to the other, making one face of the module cold and the other hot. A module capable of removing about 12 watts when the hot junction is at room temperature and the cold junction at 0° C measures only $1\frac{1}{2}$ in. wide by $2\frac{5}{8}$ in. long by $\frac{5}{16}$ in. thick (38·1 × 66·67 × 0·31 mm). The modules are completely silent in operation, have no moving parts and contain no corrosive refrigerants.

Circuit Construction Methods

There have been some remarkable advances in the etching of metals, the techniques of which originated in America; these advances have improved beyond recognition the final finish of an etched piece and have extended enormously the range of metals which can be so treated. Since photographic techniques have also been undergoing steady refinement into the "micro" range, it is now possible to make extraordinarily minute parts, as well as simple pieces, by etching. Microponent Development showed how a drawing is prepared of the part and photographed to the required size. A piece of the appropriate metal is then coated with a photographically-sensitized enamel and an exposure from the negative is made on it. The piece is then etched under suitable controls, the enamel protecting the parts that do not require to be etched.

It seems likely that much of the logical circuitry in existing computers can be made simpler and more reliable by the use of square loop cores, particularly the so-called "multi-aperture devices" or M.A.D.'s which have not yet been fully exploited. The simplest of these devices is the "transfluxor", developed by Plessey.



Operation of the transfluxor.

The area round the small hole can switch as though it were a separate self-contained core. When the large core is saturated, either clockwise or anti-clockwise the transfluxor is "blocked" and the flux in paths "a" and "b" is in the same direction. Provided the signal applied to the drive winding is not sufficient partially to switch the large core, no flux reversal can occur round the small hole and there is no output. Over a range of settings, as the blocking current is progressively reversed, the flux in path "a" reverses before "b" and is then in opposite directions. The drive current can then produce a flux reversal round the small hole and there is an output signal.

Jastac techniques, introduced by J. & S. Engineers, are new methods of introducing one material through another without the necessity for pre-drilling or pre-punching holes. The applications are numerous, covering many fields of industry both electrical and mechanical. One of the methods enables relatively hard materials, for example, a beryllium-copper or brass insert, to be introduced through a relatively softer material such as a plastic laminated board, or an aluminium chassis.

Each insert makes its own hole and secures itself rigidly by means of the design of the insert, the design of the tools employed and the amount of insertion pressure applied. Stud inserts may be introduced through a single sheet of material to secure external parts to the stud, or they can equally well be inserted through two or more overlying sheets, thus combining the functions of a securing rivet and an attachment device. The technique is being sponsored by the National Research Development Corporation.

An improved glass/epoxy copper-clad laminate shown by Formica is claimed to hold several advantages over the earlier grades, including its retention of mechanical properties at elevated temperatures, its high level of bond strength between foil and laminate after thermal shock (a decided advantage where replacement components have to be soldered during servicing) and the fact that it can be successfully plated with the precious metals used in printed circuit practice.

The techniques which have proved so successful for silicon semiconductor device production have now been applied by Ferranti to the production of complete integrated circuits consisting of a number of planar silicon transistors (or diodes) and resistors contained within one or two tiny pieces of silicon and sealed in a reduced height capsule which normally holds only one transistor.

One circuit is a linear amplifier from the "Microlin" range of elements. It consists of six resistors and two transistors enclosed in an eight-lead capsule measuring 0·36 in in diameter and 0·18 in in height.

Integrated circuits such as these are manufactured by producing a number of separate minute regions which function as components (resistors, capacitors, diodes and transistors) within one or more pieces of semiconductor material. The regions are interconnected by evaporated layers in the basic material, and isolated by *p-n* junctions. Connections to other circuits lead off from the case and the complete circuit is hermetically sealed in a metal container which provides a stable environment and ensures a high standard of reliability.

Batteries

Mallory have produced a range of five international 1·5 V size manganese batteries which are particularly suitable where the application involves heavy current drains or continuous use over long periods. Compared with previous types of cell they are stated to have at least three times more life—up to ten times on heavier currents—and can be stored or left in equipment for two years or more without losing energy and without corroding or leaking. Also, their low internal impedance results in a more constant output during discharge.

To add to their range of standard cells Muirhead have produced a new model K-30-A. This is a precision standard cell which has been developed to meet the increasing demands of research and industry for cells

Cut-away view of the manganese battery. The manganese "cathode" is distributed in the depolarizer cylinder mix.

having a much higher order of accuracy and stability than has hitherto been available. After each cell has been aged, it is subjected to a three-month test period, during which its e.m.f. is regularly checked and noted. The stability of a cell satisfying this test is such that the value of e.m.f. obtained from it is always within $\pm 1 \mu V$ of the mean value. The e.m.f. of the cells lies within the range 1·01858 to 1·01864 volts absolute.

Other Items of Interest

Cardiac Pacemaker. For some years considerable attention has been directed to the treatment of heart block by means of a "built-in" electronic pulse generator.† During this time a number of pacemakers have been designed and data have been accumulated regarding the problems associated with such a device. An example was shown by Mallory Batteries.

Maximum circuit reliability is achieved on the basis of the fewer components the greater the reliability, the final arrangement being a transformerless two-transistor bistable circuit with four resistors and three capacitors only. The circuit delivers a 2 millisecond pulse of 5 V peak into a load of 1000 ohms. Electromagnetic coupling was chosen since it is less influenced by the intervening tissue. A simple bias control circuit in the implanted unit is energized by a coupling coil taped to the body.

Vidicon Camera Tube. A new addition to the EMI Electronics range of 1-in. vidicon camera tubes has a considerably higher resolution than a standard tube when operated at the same wall anode voltage and in the same scan and focus coils. This improved performance may be used to give higher definition pictures—given adequate bandwidth—or to give pictures with a greater signal/noise ratio. At 5 Mc/s on a 625-line system the new tube permits over double the depth of modulation of a standard vidicon. Unlike that of a standard vidicon, the resolution of this tube is not reduced by excess beam current, so the beam control can be preset to the level necessary to discharge any likely overload signal.

Miniature 160 W Magnetron. A miniature tunable magnetron developed by Mullard is intended for use in airborne transponder equipments operating at a frequency of approximately 5·65 Gc/s (C-band). Because of its very low temperature coefficient of 50 kc/s per deg C high frequency stability is claimed. With the maximum pulse input of 1·18 kV 0·8 A, the tube gives a pulse power output of 160 W. Maximum pulse duration is 3·0 microseconds at a duty cycle of 0·002.

U.H.F. Tuner. Sidney S. Bird & Sons introduced two new products in the u.h.f. range. The first is a transistor u.h.f. tuner which mechanically is similar to the valved version, now in quantity production. The two transistors used in place of valves result in a 4 dB improvement in the noise factor.

The second new product is a v.h.f. tuner coupled to a u.h.f. tuner. Both are controlled by a common selector, allowing television channels in Bands I, III, IV and V to be switched in with a single tuning control.

Capacitors. EMI Electronics showed a new type of high reliability capacitor which is particularly stable and has exceptional insulation resistance. Originally designed to withstand the extreme conditions experienced in missiles, an analysis of 100 000 of the capacitors, used in computors, yielded a mean failure rate of 0·000028% per 1000 hours. They are made of polyethylene terephthalate film interleaved with either tin or aluminium foil, and a very high capacitance-packing value ensures components of small physical size.

Screened Compartments. The largest single item, at a show generally most notable for the smallness of its exhibits, was a new type of screened compartment (500 cubic feet) of modular construction developed by Belling and Lee. The floor, walls and roof are assembled from panels of a standardized unit size and the compartment embodies the latest techniques in screening and filtering, including waveguide ventilation and a new type of r.f. leakproof door. The modular form of construction offers a wide choice of structures from a range of standard production sub-assemblies.

Conclusions

Compared with the Components Show in 1961, the most striking impression this year was perhaps the extent to which the electronic valve, while by no means obsolete, has been overshadowed by the semiconductor device and its solid state circuit.

It is also interesting to note the decreasing physical size, not only of individual components, but of circuits as a whole. A great deal has been achieved in reducing circuitry to a minimum, but one wonders whether the practical aspect of construction of such elements is beyond the capability of the human hand. Automatic equipment for manufacture will surely become essential.

Research and development departments in industry must be prepared for these advances and the Components Show fulfils a very useful role in acting as a "shop-window" for the industry. The fullest use must be made of these techniques in producing better equipment for both the home and overseas markets.

Much of the impetus to component development comes from defence requirements. It is the manufacturers of capital equipment for non-military purposes, notably computers, who have been foremost in applying the latest techniques, probably because many of these manufacturers have been closely concerned with military work; the domestic radio, television and sound recording industries have so far been lagging in this respect, maybe due to the high costs which are still involved. It was particularly noticeable that virtually no new component techniques were used in audio frequency equipment shown at the recent International Audio Festival and Fair in London.‡ It will be interesting to see if new components and construction techniques are introduced in radio and television sets at the next (1964) National Radio Show.

† See, for instance, J. G. Davies, "Artificial cardiac pacemakers for the long-term treatment of heart block", *J. Brit.I.R.E.*, **24**, No. 6, p. 453, December 1962.

‡ *Proc. Brit.I.R.E.*, **1**, No. 4, p. 100, May–June 1963.

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Data Logging in Power Generating Stations

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Presented at the Convention on "Electronics and Productivity" in Southampton on 19th April 1963.

Summary: The applications of alarm scanning and data logging techniques in the operation of boiler/turbine units in conventional power generating stations are discussed. Consideration is given to the operational requirements with emphasis on the importance of careful selection of the data to be logged. A generalized equipment specification is considered; the problems encountered at the input interface and the methods of dealing with a wide range of different types of input signal are discussed. A description is given of a 200 point alarm scanning and data logging system. An account is given of the practical experience gained as a result of this project.

1. Introduction

The modern oil or pulverized-coal fired power generating station is a complex industrial plant which represents a capital investment of many millions of pounds. A typical station consists of a number of self-contained units, each unit comprising a boiler, turbine and alternator, with their associated auxiliaries. Such a complex requires a complicated control and supervisory system to provide safe and efficient running under all the modes of operation including start-up, loading, normal running and shut-down.

The trend towards very large units poses problems in control and operational safety which can only be solved by the application of advanced control-engineering techniques. Ultimately, the answer undoubtedly lies in the installation of on-line computer systems, programmed to perform the necessary monitoring, control and supervisory functions.[1]

As an interim measure, much can be done to improve safety and productivity if the operator can be relieved of routine supervisory duties by the installation of automatic alarm scanning and data logging equipment. Such a system not only leaves the operator free to concentrate upon the main task of co-ordinating the various control-loops to give efficient operation, but also provides a much more comprehensive check on operating conditions. With suitable instrumentation it is possible to extend the alarm scanning facilities to include the detection of abnormal rates-of-change, thus warning the operator as soon as a dangerous condition begins to develop. Such early-warnings often enable corrective actions to be taken before a catastrophic failure occurs, such as a rupture of a high-pressure steam pipe.

The importance of the selection of the process variables to be monitored and/or logged cannot be overstressed—useless information is worse than no information. Data logging should be confined to those process-variables which yield useful data; in general, these data can be classified under two major headings, namely:

(*a*) Operating information

(*b*) Record information

Operating information will usually be confined to print-outs of alarms, that is, abnormal conditions, and this is best logged quite separately from the record data.

The alarm printer can conveniently be used to print manually requested information such as a summary of all points in alarm and lists of the high and low alarm limit settings; this facility is most useful, particularly when a new operating shift comes on duty.

The selection of logging points will, in general, be determined by the requirements of the station efficiency engineer, and will usually include records of the unit output real and reactive powers, auxiliary power, steam temperatures and pressures and flows, feed-water and cooling-water temperatures and pressures, etc. The long-term variation of certain parameters can give very useful indications of such things as the fouling of boiler tubes or condenser tubes. These trend points can be logged as a secondary automatic logging cycle, or alternatively this may be arranged as an "on-request" function.

2. Equipment Specifications

The precise specification of an alarm-scanning and data-logging equipment will obviously depend upon the nature, size and complexity of the particular power station for which it is required. However, the principal variations will be the number of alarm and logging points, the types of primary measurement instruments and the physical construction and disposition of the equipment. Most other aspects will be common to all systems, so that a generalized specification can be considered.

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2.1. *Environmental Conditions*

The equipment must be suitable for use in conditions which are, at best, of a heavy industrial nature. A fully commissioned, operational station is likely to have ambient temperatures which may vary from 5 to 45° C, with relative humidities in the range of 5% to 90% and the data logging equipment must be suitable for continuous operation under such conditions. Additional hazards are encountered in the way of airborne coal dust and ash, and vibrations transmitted via the building structures from the turbine pedestals and auxiliaries such as fan and pump motors and the pulverizing mills. These are the normal conditions and allowance must be made for accidents such as a burst water pipe or steam pressure-vessel. These conditions, rough as they are, are good compared with those encountered in a station during construction; this is, in fact, the more usual case since most stations have four or more self-contained units in one building, the construction programme extending over a number of years. This means that the first unit is commissioned whilst the second is being erected, and so on; the result is a nightmare medley of cement-mixers, arc-welders and riveters. The installation of the data-logging equipment must be in step with the boiler/turbine unit and it should be operational during the commissioning and acceptance trials of the unit. The equipment is therefore subjected to hazards such as heavy deposits of cement dust, showers of sparks from welding operations and large pieces of metal dropped from above.

2.2. *Reliability*

The equipment must be designed for the highest order of reliability since it will be required for use continuously, 24 hours a day, 7 days a week, year in, year out. This requirement can only be met by careful design, the elimination of electro-mechanical devices wherever possible and the exclusive use of high-quality components, which should be very conservatively rated. Despite these precautions it must be accepted that failures of the equipment will occur. The equipment designer must recognize that speedy fault location is essential if good availability figures are to be obtained. The design must therefore allow for comprehensive self-checking facilities and system-failure alarm annunciators to warn the operating staff that a fault has occurred. The self-checks should include a regular test of the measurement accuracy of the analogue-to-digital converter. The alarm system should cover power supply failures, cessation of input-scanning, jammed printers and, of course, failure to satisfy the self-checks. In the case of a power supply failure it is usually necessary to trip an interlock system which switches off all other supplies in order to avoid damage to the equipment; it is desirable to provide some form of indicator system which enables the offending power supply unit to be quickly identified.

Recognition and localization of faults will obviously depend upon the skill and experience of the maintenance technician since the self-checking facilities cannot cover every eventuality. The significant improvements which have been made in equipment reliability presents a new type of problem; namely, how does the technician gain experience when faults occur at infrequent intervals? Periods of up to six months or even more between faults are by no means exceptional, and no matter how well trained the technician, he may well have forgotten many of the details of the system when he is called upon to clear a fault. This a very real problem which must be solved in the not too distant future. A complete solution to the problem is difficult to visualize, but some obvious requirements include first-class technical handbooks, with quick-reference diagnostic charts and equipment designed around the minimum number of standard plug-in units. Provided that faults can be localized to a particular plug-in module an immediate repair can be effected by the substitution of a spare unit. To minimize equipment down-time it is essential that the spare units are kept with the equipment; not in the central stores half a mile away.

2.3. *Scanning Speeds*

The majority of process variables in a boiler/turbine unit have quite low rates-of-change, at least, by electronic engineering standards they are slow variations. This means that the input sampling rate can be very modest, and alarm-scanning rates of about 5 points per second are more than adequate for most inputs. In cases where more frequent sampling of some inputs is considered necessary, the equipment should be designed to allow interlaced scanning so that the selected input points can be examined more often than the other inputs. The speed of a complete alarm scan will obviously depend upon the number of inputs and also upon the number of new alarms or "returns-to-normal" found during the scan, since these must be printed-out at a speed determined by the type of output device.

On the automatic periodic logging cycles, the speed of logging will be determined solely by the speed of the output printer and the number of characters to be printed for each point. Typically, the average electric typewriter has a typing speed of 10 characters per second; a three digit plus space print-out therefore takes 0·4 seconds.

2.4. *Measurement Accuracy*

When considering the specification of the analogue-to-digital conversion equipment, due regard should be given to the accuracy with which the primary measurements can be made. Typically, measuring devices, transducers, and instrument transmission systems found in modern power stations have accuracies in the range of 0·25% to 2% of full-scale, depending upon the type of primary measurement. There is little point in demanding very high orders of accuracy when specifying the data-logging equipment; the principal requirement is that the measuring system should not appreciably degrade the accuracy of the primary measurements. In point of fact, repeatability is probably a much more important requirement than absolute accuracy. A reasonable specification is an overall accuracy of recording of 1% of full-scale with respect to the input signal. This can be easily met with an analogue-to-digital converter having an accuracy of 0·1% of full-scale which leaves a reasonable margin for error in the signal amplifier and linearizing circuits.

2.5. *Periodic Logging Intervals*

The frequency of automatic logging cycles cannot be decided in an arbitrary manner, and will be determined mainly by the rates-of-change of the process variables. In the case of a base-load station, the routine logs usually need not be more frequent than say once an hour, with the trend points being logged at 24 hour intervals. For stations operating on a two-shift basis, it may be advantageous to have a variable logging interval so that during start-up and loading the logging-interval can be shorter than during normal running conditions, say at 15 minute intervals. Such arrangements can easily be incorporated in the design of the data logging equipment.

2.6. *Presentation of Output Data*

The output devices may be strip-printers, page-printers or electric typewriters for the permanent records and additionally there may be visual displays. The strip-printer is a convenient device for recording the occurrence of alarms; the width of paper is adequate since the number of characters required for each point is small, and the sequential record of alarms as they occur in time is a useful facility. The preferred format of the record is a print-out of the time of the alarm followed by the point indentification number, high or low limit symbol and the measured value. A return-to-normal print-out should also be given in the same format. In a more sophisticated data logger, a page-printer can be used for the alarm records, the additional printing space being used to record the high and low alarm limit settings.

The automatic or manually demanded logging cycles are recorded by means of page-printers or electric typewriters. The page-printer is useful in cases where an off-line digital computer is available for the analysis of the logged data since a reperforator attachment can be fitted, thus allowing the simultaneous production of a punched paper tape for input to the computer. In cases where it is desired to record all the points on a single line of a pre-printed log sheet a long carriage electric typewriter is necessary; if a punched paper tape output is also required a punch output channel must be provided in parallel with the typewriter. The format of the logsheet can be conveniently divided into separate sections for the boiler and turbine points. The record will usually commence with the time of the log followed by a character identifying the type of log and the values of the points, each block of characters being separated by a space. The points which are within limits are usually printed in black and the points which are in alarm are printed in red. This allows rapid identification of abnormal conditions from the logsheet.

Facilities are usually provided which allow the operator to select a particular input for visual display on some form of digital indicator. This type of display will not slow down the alarm scanning rate and permits the semi-continuous examination of a process variable, the reading being up dated on each scan until the display is released. This facility is useful during abnormal conditions or when corrective actions are being taken.

3. The Input Interface

The input interface may be defined as "the plane of interconnection which exists between the plant measurement devices and transducers and the data logging equipment". It is at this interface that difficulties usually arise, not only at the design stage but also during the commissioning of the equipment.

The first requirement is that all the input signals must be translated into digital form; the provision of a converter for each input channel is, certainly at the present time, out of the question because of economic considerations, so that a shared converter must be employed. Consequently, all the input signals must first be translated to the common language of the a.d.c.

The problem is complicated by the wide variety of instrument transmission signals which are in use. At the present there are no generally accepted standards, although a British Standards Technical Committee is actively considering the matter, and an Industry Draft Standard has been circulated for d.c. systems. Without doubt, the most convenient language for the a.d.c. is a direct voltage at a reasonably high level, say 0 to 5 volts. Plant transducers sometimes give outputs which are compatible with the a.d.c. but this is the exception rather than the rule; the signals usually require amplification and often conversion from one form of representation to another. At one time, most process control instruments were pneumatic devices which used an analogue representation of 3 to

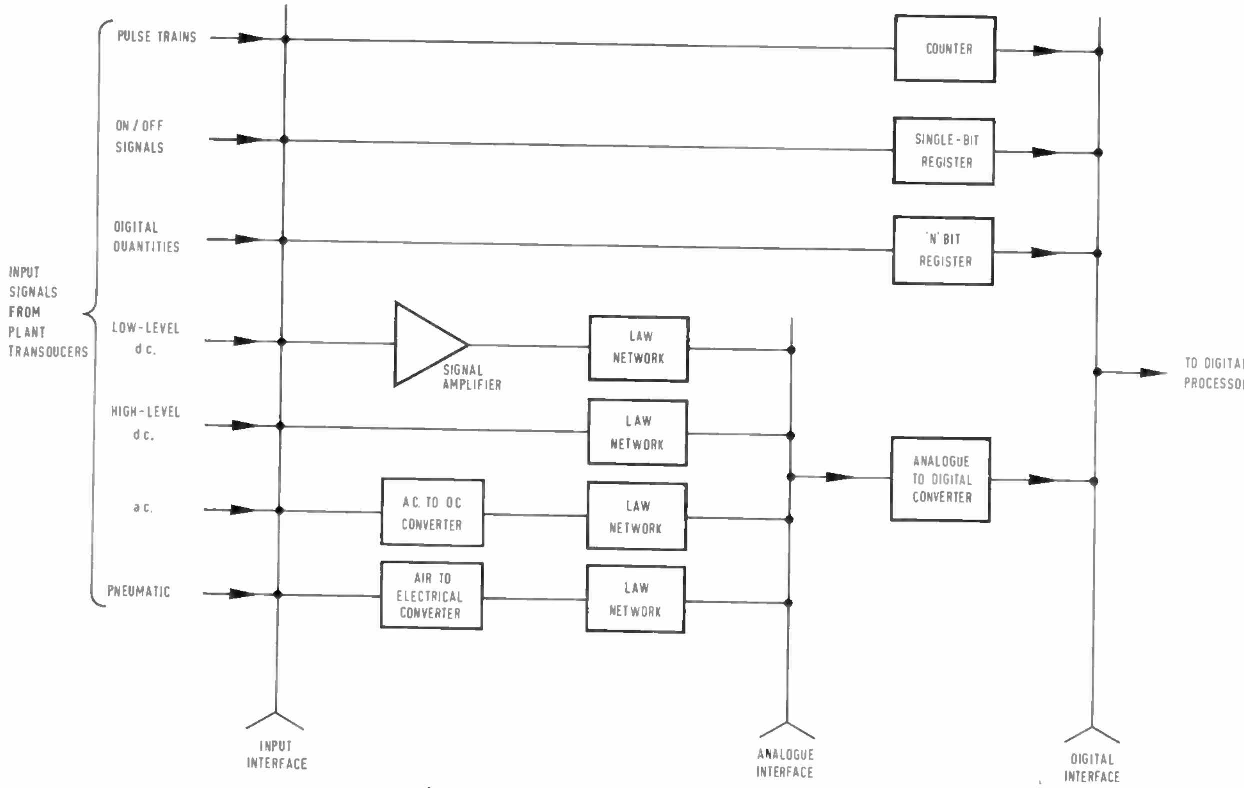

Fig. 1. The structure of the input interface.

15 lb/in^2 and this scaling, with a "live-zero"† and a 5 : 1 ratio between full-scale and zero was adopted by many instrument manufacturers for their electronic process controllers, but there were wide divergences on the choice of the basic analogue and the span. Some manufacturers used a current analogue, typically 3 to 15 mA, 2 to 10 mA or in some cases 4 to 20 mA. Others favoured voltage analogues with spans of 3 to 15 mV and so on. An opposing school-of-thought favoured a "dead-zero" system and used analogues with spans of, for example, 0 to 10 mA. All of the foregoing forms of representation are in use today and must be accepted as inputs by the data logging equipment and converted to the appropriate common language. To complicate the problem even further, there is a growing tendency towards the use of transducers and controllers which use an alternating voltage analogue.[2] Such systems use the r.m.s. value of an alternating voltage to represent the process variables, usually with a nominal span of 0 to 0·5 V. Apart from purely analogue input signals, the data logger must be able to accept other forms of input signals such as on/off signals from pressure switches, limit switches or similar devices, digital inputs from shaft position encoders or manually set decade switches, etc. The structure of the interface is shown in block diagrammatic form in Fig. 1.

† In the live-zero system, a zero value of the measured quantity is represented by a finite value of the analogue signal. For example a gas pressure of 0 lb/in^2 may be represented by a current of 2 mA.

3.1. *Direct Current Inputs*

The output current of a transducer can be simply converted to a voltage within the range of the analogue-to-digital converter by closing the circuit with a precision wire-wound resistor. Although this type of process instrument is not a perfect current source, the output impedance is usually high enough to permit the connection of a load resistance of up to 1 kΩ so that a suitable choice of value allows the potential drop to be matched to the a.d.c. input requirements. Unless the a.d.c. has a very high input-impedance it is necessary to employ a buffer amplifier in order to avoid loading errors. In the case of "live-zero" signals, an "offset" voltage can be added algebraically to the signal at the input of the buffer amplifier to back off the zero reading—in certain cases this is best done as a logical operation subsequent to digitizing; the choice of method largely depends upon the number of inputs with offsets, if there are many different offsets the digital approach is probably the better one.

In practice, the direct-current signal should really be called a uni-directional signal, since it is usually obtained from a rectified mains supply and contains a high percentage of ripple. This ripple must be

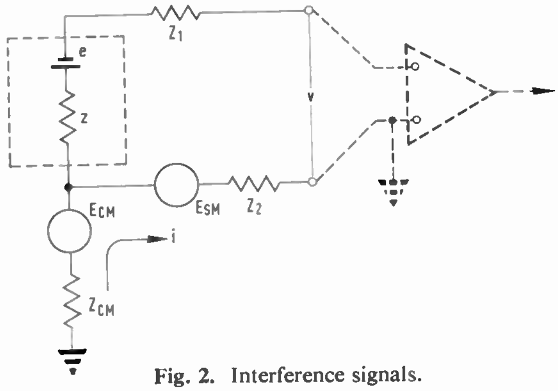

Fig. 2. Interference signals.

removed by suitable filters before the a.d.c., or when a buffer amplifier is used this can be designed as a low pass-band unit.

3.2. *Low-level Direct Voltage Inputs*

Electrical signals are subject to interference, such as noise pick-up which can introduce serious errors in the measurement of the signal. In most cases the primary measurements are made at points which are remote from the data logging equipment. The signals must therefore be transmitted over considerable distances and the connecting cables almost always pick up noise from adjacent power cables or similar interference generators. Such noise can be very troublesome, as it gives rise to spurious input signals and, in extreme cases, causes saturation of amplifiers. Special techniques must therefore be adopted to allow the accurate detection of very small signal voltages which may be masked by noise.

The problem is illustrated in Fig. 2, which shows a d.c. transducer, represented by a battery having an e.m.f. of e volts and an internal impedance of Z ohms. In practice, transducers are seldom completely isolated from "earthed" masses, such as turbine casings or boiler tubes which are, in fact, not at true earth potential. Potentials from these sources, together with noise pick up, cause unwanted voltages to appear at both transducer terminals. These are represented in the figure by a voltage generator, E_{cm} and their source impedance by an impedance, Z_{cm} to true earth. E_{cm} is referred to as the "common-mode" voltage and may be d.c., a.c. or in some cases, d.c. with a superimposed a.c. component. The a.c. component is usually at power frequency or its harmonics and can have an amplitude of many volts, with a source impedance ranging from a few ohms to a few hundred megohms. Pick up also produces a noise signal E_{sm}, which is added algebraically to the true input signal so that the voltage which appears between the lines at the far end of the cable is given by $V = e + E_{sm}$. The noise voltage is referred to as the "series-mode" voltage.

If an amplifier having one earthed input terminal is connected as indicated by the dotted lines in Fig. 2, the common mode voltage E_{cm} produces a current flow through the line impedance Z_2 in series with Z_{cm}. This current in the so-called earth-loop produces a voltage drop across Z_2 which is added to the transducer output signal. This amplifier connection thus causes the common-mode voltage to be converted into a series-mode signal at the amplifier input terminals. For example, if E_{cm} is 100 V, Z_{cm} is 10 MΩ and Z_2 is 100 Ω, then the voltage drop in the line is 1 mV. Typically, a transducer such as a thermocouple has a full-scale signal of 10 mV, so in this case the noise is 10% of the maximum signal—a very serious error.

In order to overcome the earth-loop problem it is necessary to use an amplifier which has a completely floating input stage. Even in these circumstances the common-mode voltage can cause a series-mode voltage to be generated if the distributed line impedances are unbalanced. The lines have resistance, inductance and capacitance to earth distributed throughout the length of the line; these can be approximated by "lumped" constants as shown in Fig. 3. The common-mode voltage generator causes currents to flow and unless all the line impedances are balanced a voltage difference will exist between the amplifier input terminals which will be amplified as if it were a genuine input signal. This condition of unbalance is aggravated in the case of thermocouples because a special compensating cable is often used to connect the couples to a common cold junction at the amplifier input terminals. Such cables may be resistively unbalanced by as much as 3 : 1. This means that ideally the lines should be balanced by inserting padding resistors immediately before the amplifier. Some series-mode noise will still be present however, due to imperfect balancing and pick up, and this noise must be attenuated by means of a filter placed before the amplifier input.

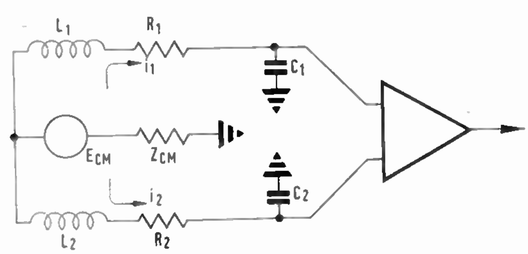

Fig. 3. Common-mode interference.

General precautions should be observed with input signals in order to minimize noise pick-up. These are summarized below:

(*a*) Wherever possible, signals should be produced at, or converted to, a high-level at the point of origin.

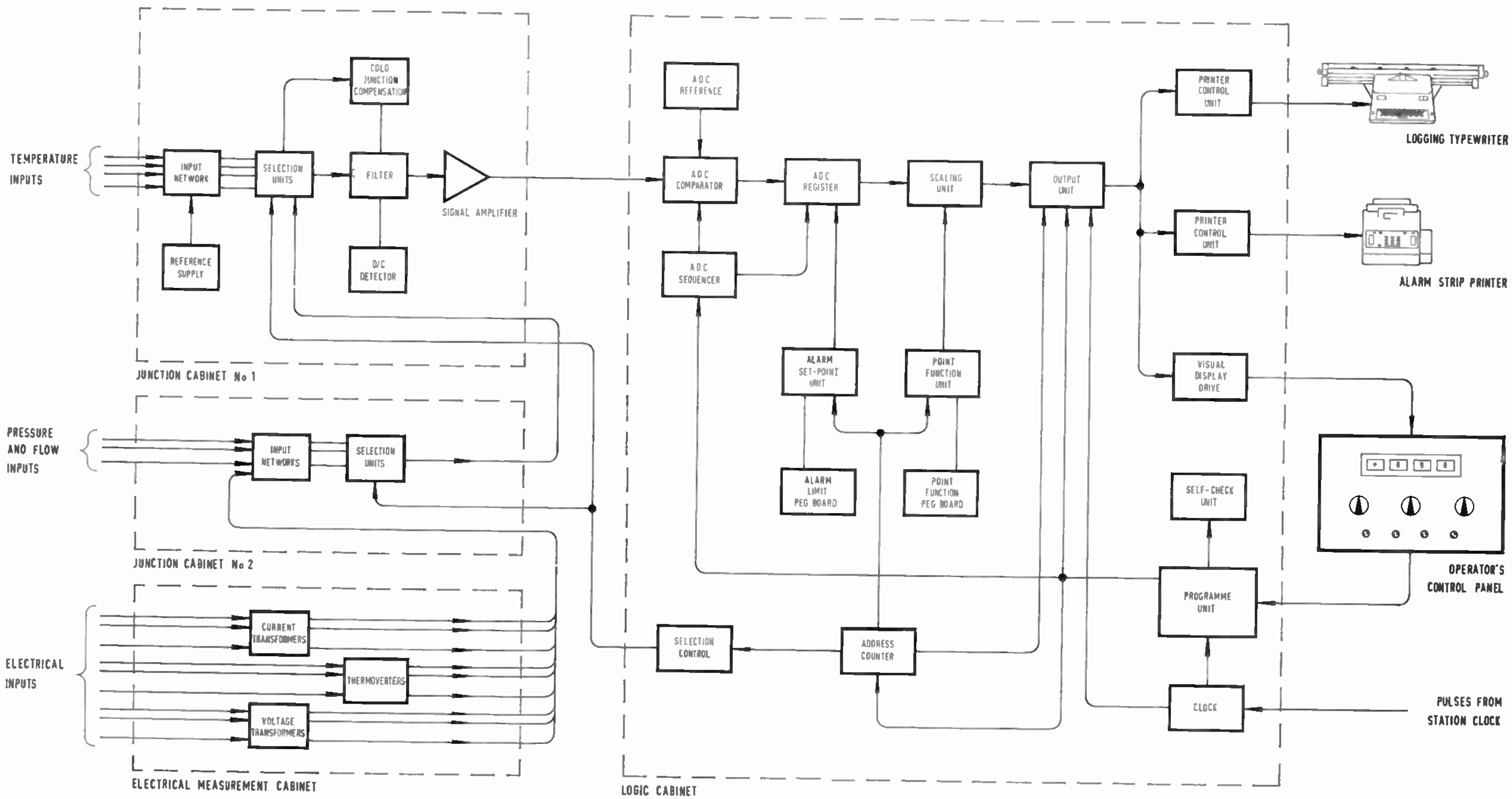

Fig. 4. Block diagram of a 200 point data-logger.

(*b*) Balanced, twisted-pair connecting cables should be used to minimize pick up due to electromagnetic fields.

(*c*) The conductors should be enclosed in a braided copper sheath, or screen, in order to minimize pick up due to electrostatic fields.

(*d*) The connecting cables should be routed to avoid parallel runs close to power cables.

(*e*) Earth loops must be avoided.

(*f*) Care must be exercised to avoid spurious e.m.f.'s due to thermo-electric effects at terminals by eliminating junctions of dissimilar metals or arranging that they cancel each other.

3.2.1. The low-level signal amplifier

A low-level signal amplifier suitable for use in data-logging systems is described elsewhere.[3] It is sufficient here to summarize the essential points of the specification as follows:

(*a*) A high rejection to common-mode interference.

(*b*) Very low voltage drift.

(*c*) Good linearity.

(*d*) A range of programmable, accurate forward-gains.

(*e*) A very high input impedance.

(*f*) A very low output impedance.

3.3. *A.C. Inputs*

Apart from the a.c. input signals from transducers associated with process variables such as pressure and flow, the data logger must be capable of accepting inputs representing the power output of alternators, the auxiliary power, etc. The two types of input are quite distinct, the instrument signals being at relatively low level whilst the power measurements are derived from current and potential transformers, usually at final values of 5 A and 110 V.

The low-level signals may be converted to direct voltage by means of a precision, linear demodulator; such a unit can be realized by the use of a d.c. operational amplifier with a diode bridge limiter in the feedback path. Precision converters based on this principle have been designed using a fully transistorized operational amplifier[3] and linearities of 0·1% have been achieved, with transient responses fast enough to allow the converter to be time-shared between inputs at scanning rates of up to 5 points per second.

For the power measurements, proprietary units are available under the trade name "Thermoverter". The devices are based upon the principle of measurement of the heating effect of the alternating current; by means of a thermo-junction welded to a pure resistive heater element, a low level direct voltage output is obtained which can be amplified by a floating input signal amplifier prior to the a.d.c. For the measurement of three phase power, two Thermoverters may be used in a configuration analogous to the "two-wattmeter" method of measurement.

3.4. *Linearization of Transducer Characteristics*

Many of the input signals will be derived from primary measurement instruments which have non-linear relationships between the output of the transducer and the physical quantity being measured. The non-linearities may be merely a slight departure from strict proportionality over the range or parts of the range; in some cases the measurements may be of an implicit nature involving functions of a variable. Examples of the two types are thermocouples and flowmeters. The slope, or sensitivity, of a thermocouple varies with the temperature difference throughout the range, but the departure from a linear law is seldom greater than a few per cent. On the other hand, many flowmeters have square-law characteristics and the signals require square-root extraction before digitization.

In general, the linearization function requires the signal to be modified depending upon the value of the signal, and this can be best achieved by means of biased diode function generators, which approximate the function by straight line segments.[4] The linearization is best performed at high-level, subsequent to the signal amplifier but before the a.d.c. In practice, several function generators will be required, one for each different class of input, and part of the scan programming will be the selection of the appropriate "law unit".

4. Data Logging Equipment at Northfleet Power Station

The Central Electricity Generating Board's Northfleet power station comprises a total of six boiler/turbine units, each consisting of a pulverized-fuel fired boiler and a 120 MW turbo-alternator set.

The first four units are monitored by alarm-scanning and data-logging equipments which were placed on order early in 1958, and these are relay-logic machines, with electro-mechanical digitizers. The loggers were progressively installed during 1960 and 1961 and are in full-time routine operation. Units 5 and 6 have more modern loggers which are functionally similar to the others but are completely transistorized and incorporate refinements including digital alarm comparison. The description which follows is confined to the equipment installed on units no. 5 and no. 6.

4.1. *General Description*

The basic functional units of the system are illustrated in block diagram form in Fig. 4 and these comprise four major units, namely, two junction cabinets, a power-measurement equipment cabinet and the main logic cabinet. The operator's panel and the output printers are incorporated in the main instrumentation control desk.

4.1.1. The input junction cabinets

The cabinets are similar in construction and appearance, and have been designed to accommodate the necessary terminal units and transducer selection units for a maximum of 128 inputs in each cabinet, in groups of 8 points. The experience gained from the first four installations emphasized the importance of good access to terminals and robust physical construction; the cabinets are therefore fabricated from heavy-gauge sheet steel, with a top-entry gland plate for the incoming cables.

Junction cabinet no. 1 contains the transducer selection units associated with all the temperature measurements, a total of some 90 points, more than half of which are obtained from resistance thermometers. This cabinet also houses a cold-junction compensation unit for the thermocouple inputs, a reference voltage unit for energizing the resistance thermometer bridges and the low-level signal amplifier.

Junction cabinet no. 2 contains the transducer selection units for the pressure and flow measurement inputs, and also receives the output signals from the power measurement cabinet. There are about 70 inputs, leaving space for future expansion.

4.1.2. The power measurement cabinet

The power measurement cabinet is similar in construction to the input junction cabinets and contains interposing current and potential transformers and Thermoverter units. The inputs include turbine auxiliary power, generator real and reactive power, stator current and stator voltage.

4.1.3. The logic cabinet

The logic cabinet accommodates the analogue-to-digital converter, the digital clock, the logic programming unit, the alarm-limit set-point units, the alarm memories, the scan sequencing control unit, the digital scaling unit and the various output decode and drive units. An engineer's monitor panel is provided which allows the system to be operated and checked independently of the operator's control panel.

4.1.4. The control desk

The data-logger section of the control desk houses a 30-in carriage electric typewriter for print-out of the routine logs and a $2\frac{1}{2}$-in strip-printer for the alarm records. The operator's panel carries the manual-command selector switches and a visual display unit with its associated controls. A general view of the control room, showing this section of the desk is given in Fig. 5.

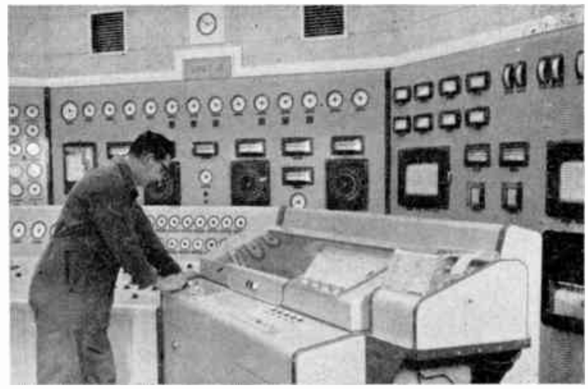

Fig. 5. View of the control desk.

4.2. *Functional Description*

The equipment satisfies the following operational requirements, which are given in the order of priority:

(*a*) The continuous automatic scanning of some 30 inputs and printing records of any points which deviate from pre-set low and high limits. The record gives the point identification number, the time of the alarm and the value. A further record is printed when the point returns to normal. The occurrence of any abnormal condition also operates a visual and aural warning device to attract the attention of the operator.

(*b*) The print-out of a summary of all points in alarm when demanded by the operator.

(*c*) The automatic logging of some 90 inputs at regular hourly intervals synchronized to the power station clock. The log is preceded by the time and an identification symbol. The point values are printed in sequence on a tabulated logsheet, each column headed with the point identity and the scale in engineering units.

(*d*) The printing, when demanded by the operator, of a full logsheet, the log being preceded by the time and an identification symbol.

(*e*) The logging of some 70 "trend" points when demanded by the operator.

4.2.1. Input selection

The techniques employed for the selection of the input data are illustrated in Fig. 6, showing four representative input networks. In practice all the other input networks are connected via the contacts of the selection relays to the common signal busbars, and the size of the relay matrix is correspondingly increased. The input networks are provided as necessary to translate the signals into a form suitable for acceptance by the signal amplifier as described in Section 3. The examples given should illustrate the principles.

Input no. 1 is derived from a transducer which has a current output in the range 0 to 10 mA; this current

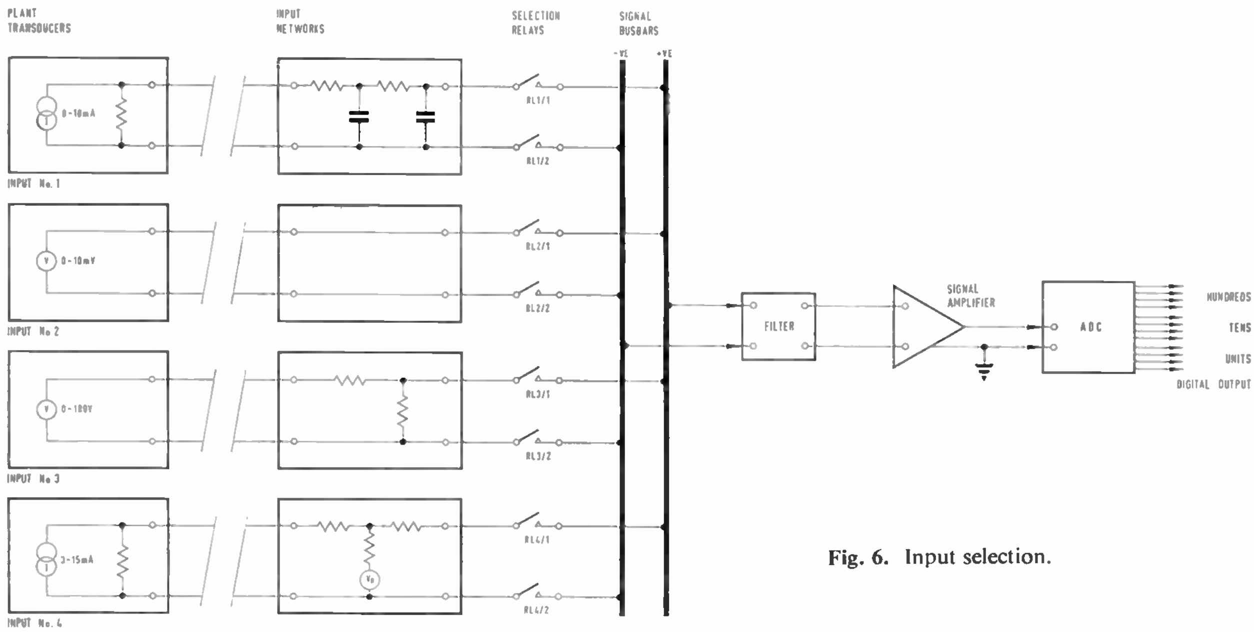

Fig. 6. Input selection.

is converted to a voltage by completing the circuit with a precision resistor. The voltage so obtained is smoothed by a two-stage r-c filter inserted in the input lines before the contacts of the selection relay.

Input no. 2 is a smooth, low-level direct voltage and is therefore suitable for direct connection to the signal amplifier.

Input no. 3 is a smooth, high-level direct voltage and is attenuated in the input network to produce a voltage which matches the amplifier input requirements.

Input no. 4 is obtained from a current source which has a minimum value of 3 mA and thus has a zero-offset; the input network therefore incorporates a floating reference voltage V_r which is summated with the input signal by a resistor network and is scaled so that the offset is reduced to zero. The voltage reference is provided by a single mercury type primary cell which has sufficient capacity to give approximately one year's service before a replacement is necessary.

The input networks are connected to a pair of signal-busbars via the contacts of the selection relays. The coils of the relays are connected in the form of a matrix and the required input is selected by energizing the appropriate X and Y drive lines; semi-conductor diodes, connected in series with the coils ensure that only the selected relay is energized. This matrix method of input selection is very flexible since the inputs can be scanned in any order as determined by the programming unit and do not have to be selected in a fixed sequence. The alarm scanning rate is fixed at 5 points per second and the rate of the input selection during the logging cycles is determined by the speed of the typewriter, the average speed being approximately 2 points per second.

The input selection relays are of the dry-reed type, with gold-plated contacts and have proved to be very reliable low-level switches. The life expectancy is at least 10^8 operations so that even those relays associated with the continuously scanned alarm points should have a minimum life of more than ten years.

4.2.2. The analogue-to-digital converter

The analogue-to-digital converter is an entirely electronic unit. The conversion is performed by circuits whose operation is based upon a potentiometric comparison principle as illustrated in Fig. 7 which shows a simplified four-bit binary comparator.

The analogue input signal is measured by successively comparing its magnitude with accurate potentials in a number of discrete steps, each step corresponding to a digit of fixed significance, for example, half full-scale, quarter full-scale, etc. A precision,

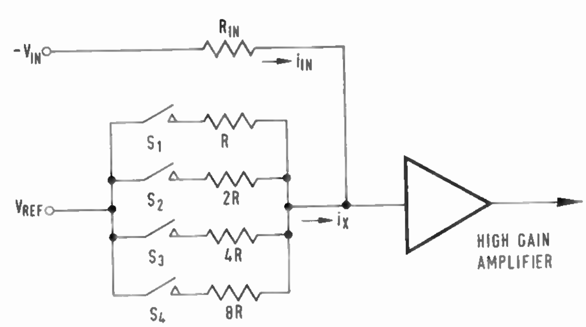

Fig. 7. Simplified four-bit binary comparator.

stable reference voltage is applied to a chain of precision resistors whose values are chosen in geometric progression, so that each resistor, when switched into the circuit, causes a current to flow which is equal to half of the current due to the preceding resistor. The unknown input signal $-V_{IN}$ is applied to a standard resistor R_{IN} and produces a negative current I_{IN} which is compared with the sum of the positive reference currents by means of a very high gain amplifier. The polarity of the amplifier output voltage is determined by the direction of flow of the net input current, thus indicating whether the sum of the reference currents I_X is greater or less than I_{IN}. The sensitivity of the amplifier is such that a current representing one least-significant-digit can switch the amplifier from positive output saturation to negative output saturation. To perform a conversion, the switches are closed one at a time, in sequence, in an attempt to satisfy the null-condition, $I_X + I_{IN} = 0$. The sequence is as follows:

(1) S_1 is closed and left closed if I_X is less than I_{IN}; otherwise it is opened again.

(2) S_2 is closed and left closed if I_X is less than I_{IN}; otherwise it is opened again.

(3) This procedure is repeated for the remaining switches until the null condition is satisfied as closely as possible.

(4) The final state of the switches gives a binary number which is proportional to the unknown analogue input signal; each closed switch represents a 1 and each open switch a 0. Switch S_1 is the most significant digit and S_4 is the least significant digit.

A block diagram of the complete a.d.c. is shown in Fig. 8. The switches in the comparator are transistor-switching circuits controlled by a register which consists of a number of bistables (single-bit memories), one per digit. The bistables are controlled by a sequencer. A trigger pulse initiates the operation which is started by resetting the register to remove the previous contents. The most-significant-digit switch

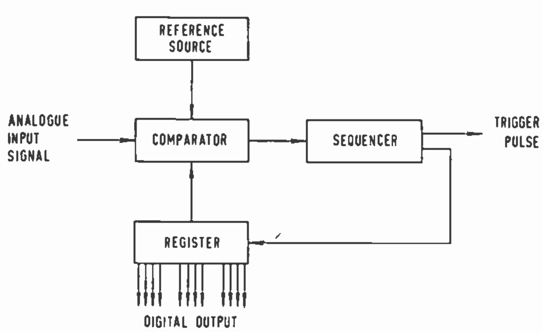

Fig. 8. Block diagram of the complete analogue-to-digital converter.

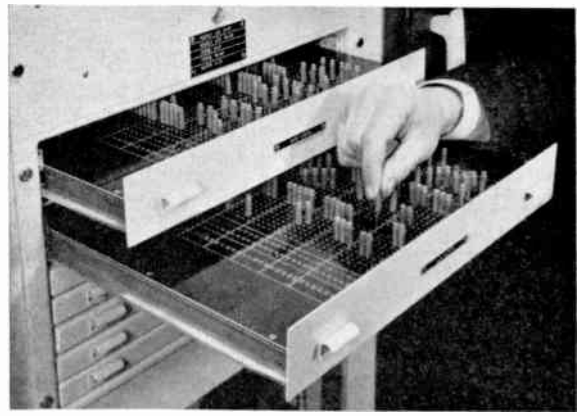

Fig. 9. Alarm limit pegboard.

is then closed and the comparator instructs the sequencer whether this digit is required or not. If it is not required the m.s.d. bistable is reset; otherwise it is left set. This procedure is repeated until the least significant digit has been determined. The register then contains a digital number proportional to the magnitude of the analogue input signal.

The actual converter has 12 binary digits, arranged in three decades of binary-coded decimals and covers the range 0 to 999. The conversion speed is approximately 200 complete readings per second and the accuracy is $\pm 0{\cdot}1\%$ of full-scale. The use of the binary-coded-decimal converter simplifies the decoding to decimal form for the printing of the log sheets.

A digital scaling unit follows the a.d.c. and provides four basic scales which are selected by the point function unit appropriately for each input. The scales are ± 50, 500, 1000 and 2000. The point function unit also determines the type of the input, i.e. alarm only, hourly log only, trend only, or combinations of all three.

4.2.3. Alarm detection

The alarm detection is achieved by comparing the value of each point with preset digital numbers corresponding to the desired minimum and maximum values of the process variable. The limits are set up and stored by means of plug-in diode matrices, one for the low limits and one for the high limits. Each plugboard has a row for each alarm point and columns for three decades of decimals for the limit values; the diode pegs are plugged into the appropriate holes. The plugboards are shown in Fig. 9.

The alarm scanning is carried out in the following manner: the transducer selection matrix is set to the desired input point and a period of some 150 ms is allowed during which the filter and signal amplifier switching transients decay; a signal proportional to the selected input is presented to the analogue input of the a.d.c. comparator unit and the digital value of the limit is set into the a.d.c. register, and the comparator output indicates whether the input is within the limit. This "YES/NO" signal is sent to the program unit which examines the state of the alarm memory for the point concerned; if the alarm state has changed since the previous scan, either an alarm will be initiated or a return-to-normal print-out is arranged as appropriate. The state of each alarm point is stored in a number of single-bit memories (bistables) thus allowing a summary of all the points which are in alarm to be compiled on request.

4.2.4. The program unit

The program unit determines the complete scanning and logging sequences of the whole system. It controls the other units in a fixed logical sequence, ensuring that each point is tested for alarms, logged or visually displayed as required.

4.2.5. The clock unit

The clock unit consists of a logical counter which receives, and counts, the half minute pulses from the station clock. It presents time in units and tens of minutes, and units and tens of hours for printing as required. It also provides the program unit with one hour pulses for triggering the automatic logging cycles.

4.2.6. The output unit

The output unit is used to gate the outputs of the clock (time), address counter (point number) and the a.d.c. (point value) to the logging typewriter, alarm strip-printer or the visual display as instructed by the program unit.

4.2.7. The self-check unit

The self-check unit, or watch-dog as it is commonly called, continuously tries to give an alarm, but is prevented from doing so if the system self-checks are satisfactory. The checks include scanning, a.d.c. conversion accuracy and power supplies. The accuracy of the a.d.c. is checked by digitizing a fixed, accurately known reference voltage once every scan cycle, a "fail" signal being generated if the digital output of the a.d.c. deviates outside very narrow limits.

4.3. *The Practical Aspects of the Installation*

Considerable experience has been gained as a result of this project and some of the more important aspects are worthy of individual discussion.

4.3.1. Power supplies

The instrument power supply in a generating station is far from perfect, and is subject to quite wide variations in voltage. This, in itself, is not of any great significance since the equipment can be easily supplied from constant-voltage transformers, but the supply is usually subject to noise in the form of switching transients and may even have interruptions of a few cycles with which the constant-voltage transformer cannot satisfactorily cope. Unlike the earlier relay loggers, transistor systems are sensitive to fast transients on the power lines so that fast, electronically stabilized power supply units must be employed which can deal with the noise. However, these are unable to deal with interruptions of several cycles which apparently can occur from time to time, and to meet the C.E.G.B.'s specification the equipment had to be capable of working without error on supplies subject to interruptions of up to a quarter of a second. To meet this specification, it was necessary to provide a buffer motor-alternator set, with a flywheel to act as an energy store during the interruptions of the supply.

4.3.2. Transducer input facilities

The installation of the transducer input cables is a major operation, and must commence long before the installation of the data logging equipment. The production schedule must therefore allow for the delivery of the input junction cabinets much earlier than the rest of the equipment. The need for providing adequate space in the cabinets has already been mentioned, but it is of sufficient importance to be restated.

Many of the input signals are derived from instruments which have a current output, and in addition to the data logger it may feed other instruments on the main control panel. These instruments may be installed and commissioned before the data logger is installed, so it is desirable that the precision resistors used to provide voltage signals for the logger are incorporated in the instrument system, and not in the logger junction cabinets.

5. Conclusion

The paper has outlined the development of data-logging systems and has discussed their application to the routine operation of generating units in conventional power stations. The improved reliability which is obtained by the use of solid-state switching elements will undoubtedly lead to more extensive use of such systems, and it is probably true to say that the data logger will soon be a normal feature of power station instrumentation.

For the immediate future, the usefulness of data loggers can be extended by providing systems which include some computing ability as an on-line function. This will probably include provision for alarm detection on rates-of-change, integration of flows and the calculation of turbine heat-rates.

6. References

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3. G. B. Cole and S. L. H. Clarke, "The development of ARCH—a modular computer-control system", Brit.I.R.E. 1963 Convention Paper (to be published in *The Radio and Electronic Engineer*).
4. C. C. Ritchie and R. W. Young, "The design of biased diode function generators", *Electronic Engineering*, **31**, pp. 347–51, June 1959.

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DISCUSSION

Under the chairmanship of Professor E. E. Zepler

Mr. K. A. MacKenzie: I would congratulate Mr. Willison on a very concise and well-thought-out paper which leaves very few points unstated or requiring further clarification. However, a point does arise on one of the photographs of the equipment which he showed. From this it would appear that the cubicles are mounted on a plinth approximately 1 ft 6 in high—presumably as a precaution against flooding—and are completely sealed. If this is so, has he not found any necessity for forced air cooling of the equipment?

Secondly, he mentioned in his paper the transient fluctuations of the instrument power supply in a generating station which led to the use of a buffer motor generator set. Does he feel that this approach is the correct answer to the power supply problem for industrial data processing equipment or should the equipment itself contain circuitry which would make it suitable for direct connection to the customer's supply network, and if so, should one cater for automatic changeover to an auxiliary supply?

The author (*in reply*): A sealed cabinet construction is employed with an air cooling system. The air is sucked through the cabinet via special air filters and an alarm system is incorporated to give a warning of excessive temperature in the cabinet. The alarm lamp is labelled "dirty filters", although of course the trouble could also be caused by a fan failure.

As far as power supplies are concerned the motor generator set with a flywheel is undoubtedly a convenient way of dealing with short interruptions in the supply, but has the disadvantage that maintenance is necessary. Also an automatic changeover system to an auxiliary supply is required to deal with total failure of the supply. My opinion is that a static inverter operating from a floating-charge battery system is a good approach since this avoids the need for rapid changeover to a standby supply with the associated problem of recognizing the failure in time to effect the changeover. Economics—and personal preferences—obviously come into play and I do not feel that it is possible to say which is the best system.

Mr. M. James: I agree that in the past data logging has produced far too much data for sensible interpretation. I would like to suggest that a data logger should be made to do more work for its living and to that end I would suggest two possible techniques.

Firstly, would it not be possible to contain within the logger a form of correction sub-routine which could be called upon to take control action should an important failure occur?

Secondly, has Mr. Willison considered the application of a fault pattern recognition system, in which the logger has been programmed to recognize distinct patterns of failure? It would thereby indicate the prime faults rather than print-out a number of subsidiary dependent failures. This same technique should allow anticipation of failure since if say 90% of the faults making up a particular pattern had occurred then the operator would be told that a certain probability of a major fault occurring in the near future had been established.

The author (*in reply*): In reply to your first point, the short answer is "Yes". This could be a very simple logical operation such as bringing in a standby pump or a more complex function demanding some real computing ability. The ARCH system which is described in another convention paper[3] allows the construction of sophisticated data logging systems such as those suggested by Mr. James.

As far as fault pattern recognition is concerned, this has been considered for nuclear power stations where the problem is very acute, particularly during start-up. Some 2500 alarm points are involved and the operators find it almost impossible to cope with the problem using conventional alarm annunciators. Such a problem requires a stored programme computer with fairly extensive storage capacity. There is obviously scope for smaller scale systems in conventional stations and here again ARCH should make this possible in an economic manner.

Dr. D. A. Bell: The elimination of electromechanical devices Section (2.2.) is usually recommended, but I thought that it was agreed in a discussion yesterday that mechanical contact devices are preferable for intensity of duty up to 100 000 operations per year. Can Mr. Willison tell us whether his applications lie above or below this threshold? Can he also give any figures on first cost and maintenance cost to prove the economic advantage of the solid-state version of the data-logging equipment? Some such information is necessary to convince the potential user, who may argue that a relay failure can often be located by a maintenance man by visual inspection whereas a sophisticated diagnostic routine must be used to locate a fault in a solid-state system.

The author (*in reply*): I believe it was suggested that 100 000 operations a year is a suitable figure for mechanical contact devices but I do not recall any agreement. In this application, we do in fact use reed relays for the input signal selection—we would like to use solid-state switches, but for low-level signals these are not yet an economic proposition. The relays associated with the alarm points have to perform 5 million operations per year, well over the threshold you mentioned, but they still have a life of well over 10 years!

On the cost question, I cannot quote you precise figures, but as a guide the solid-state loggers described in my paper cost about 20% more than the earlier relay logic machines. This should be quickly recovered by reduced maintenance costs—and the priceless improvement in "operator confidence" which a reliable equipment brings. With the exception of the output devices, the solid-state loggers do not require maintenance, whereas the relay machines *must* receive regular attention. The solid-state loggers have so far been running continuously for over 6 months without any maintenance and there has only been one failure—a printer failure due to lack of oil!

Mr. R. J. Dennis: A data logger has been described which is not essential to the operation of the power station—when a fault occurs therefore in the logger the power station will come under conventional observation and control. In the paper the author highlights the problem of servicing reliable equipment when the fault experience is no better than say 1 fault every 6 months. For more direct on-line equipment does this then mean that we should be prepared to spend say as much on supervisory and fault monitoring equipment as on the main functions of the equipment?

The author (*in reply*): This is undoubtedly a very important question and I cannot really give a satisfactory answer. Certainly one must be prepared to spend a certain amount on self-checking and other supervisory devices, but if the fault monitor becomes too complex then equipment will be required to monitor the fault monitor, and so on *ad infinitum!*

Dr. C. B. Newport: What are the measured values of common-mode and series-mode interference? Secondly why are the input networks shown unbalanced in Fig. 6? Finally what form of signal filter is used?

The author (*in reply*): Actual measurements of common-mode and series-mode noise were not made since we had no trouble due to interference. The signal amplifier which we used had a common-mode rejection ratio of 1 000 000 to 1 and an attenuation of 50 dB to series-mode at 50 c/s.

The input networks shown in Fig. 6 are used merely to illustrate the basic principles. In practice I agree that it would appear that balanced networks should be used if a high degree of common-mode rejection is to be achieved. The filter used in the amplifier in this particular system was a prototype L-C section.

Radio Engineering Overseas . . .

The following abstracts are taken from Commonwealth, European and Asian journals received by the Institution's Library. Abstracts of papers published in American journals are not included because they are available in many other publications. Members who wish to consult any of the papers quoted should apply to the Librarian, giving full bibliographical details, i.e. title, author, journal and date, of the paper required. All papers are in the language of the country of origin of the journal unless otherwise stated. Translations cannot be supplied. Information on translating services will be found in the Institution publication "Library Services and Technical Information".

TELEVISION CHANNEL ALLOCATION

The problem of positioning of mutually interfering television stations under certain idealized situations has recently been discussed by an Australian engineer. It is shown that if each channel is to serve the maximum proportion of a large geographical area, optimum spacings exist between the stations; these spacings depend upon transmitting aerial height and the frequency. It is shown, for example, that if all transmitting aerials are 500 ft above average terrain, then the optimum condition exists if the transmitter powers and spacings are adjusted so that each station has a useful rural service range of about 30 miles, at both u.h.f. and v.h.f.; if, on the other hand, the height of the transmitting aerials above average terrain is 2000 ft, the corresponding optimum range is about 60 miles. Although the results of the analysis cannot be used directly in planning a television service, they draw attention to some underlying factors, the significance of which should not be ignored at the detailed planning stage.

"Some considerations of television channel allocation", K. G. Dean. *Proceedings of the Institution of Radio Engineers Australia*, **24**, pp. 304–13, March 1963.

MICROWAVE DIELECTRIC MEASUREMENTS

The dielectric properties of gases at high pressure and of liquids and solids are still only known quantitatively in rough outline from molecular theories and hence measurements of these properties are of considerable importance in connection with electromagnetic energy transmission or storage.

A paper by two engineers from the Birla College of Engineering, Pilani, India, describes measurements of the complex permittivity at X-band wavelengths. Because the samples were of disc shape, the resonant cavity method had to be employed and, as the loss due to the dielectric is often small compared with the loss due to the metal of the cavity, the position is rather unfavourable for measuring a small quantity in the presence of a large parasitic one.

"Design and fabrication of a cavity to measure complex permittivity at X-band", M. Chaudhuri and O. P. N. Calla. *Journal of the Institution of Engineers (India)*, **43**, Part ET 2, No. 5, pp. 47–71, January 1963.

PULSE CODE MODULATION

Two companion papers from staff of the Department of Electrical Engineering, Osaka City University, discuss instantaneous companding for pulse code modulation communication and delta pulse code modulation communication, respectively. (In Δ-p.c.m. the differences between adjacent samples are coded.) In both cases the apparatus has arbitrary instantaneous companding characteristics. Measurements are shown of the signal-to-quantizing-noise ratio. It is concluded that the improvement of signal-to-noise ratio is better in Δ-p.c.m. speech communication than ordinary p.c.m.

"Instantaneous companding for p.c.m. communication" and "Instantaneous companding for Δ-p.c.m. communication", Y. Tanaka, K. Yamashita and S. Hosokawa. *Memoirs of the Faculty of Engineering, Osaka City University*, **4**, pp. 59–69, and 71–78, December 1962.

NOISE IN TELEVISION PICTURES

The disturbing effect of "white noise" on a television picture has been investigated by German engineers using a test pattern consisting of a regular arrangement of small circular dots which display a regular variation in size in the vertical direction and in contrast in the horizontal direction. It is well known that the noise components of higher frequency cause less interference than do those of lower frequency. Below about 0·75 Mc/s the disturbing effect falls off again. A comparison with the picture quality observed subjectively confirms the measurements for permissible interference. When combining several noise components of different frequencies a linear relation is obtained between the subjective disturbing effect and the evaluated root-mean-square value of the noise voltage, provided that the non-linearity of the cathode-ray tube and the resulting occurrence of harmonic and combination frequencies is taken into account. The authors put forward the formula for the subjectively observed interference.

"The subjective disturbing effect of noise in television pictures", F. Below, F. Huert As-Sendra, E. Fritze and E. Semrau. *E.B.U. Review*, 78-A Technical, pp. 49–53, April 1963.

LONG-DISTANCE PROPAGATION EXPERIMENTS

Pulse transmissions at oblique incidence at seven fixed frequencies were carried out between Lindau, Harz and Tsumeb, South West Africa. Simultaneously back-scatter echoes were recorded at the transmitting end. The transit time of the different paths was determined by re-transmission from Tsumeb. The identification of the different paths of propagation was facilitated by these additional measurements. The field strengths of the different paths were measured at the receiving end in Tsumeb. The measured field strength differences correspond to the calculated values in a satisfactory manner.

"Back-scatter and pulse transmission experiments on a radio link from Lindau (Harz)—Tsumeb (South West Africa)", H. Werle, *Archiv der Elektrischen Ubertragung*, **17**, pp. 121–30, March 1963.

SUBJECT INDEX

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Cables et Transmission (France)

Canadian Electronics Engineering

Canadian Journal of Physics

Electrical and Mechanical Engineering Transactions, Institution of Engineers, Australia

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Journal of the Institution of Engineers (India)

L'Onde Electrique (France)

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